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# CK-12 Texas Instruments Trigonometry Teacher's Edition



# Texas Instruments Trigonometry Teacher's Edition

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CHAPTER **1**

# TE Introduction to Trigonometry - TI

## CHAPTER OUTLINE

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### 1.1 TRIGONOMETRY TI RESOURCES FLEXBOOK

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# 1.1 Trigonometry TI Resources Flexbook

## *Teacher Edition*

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### Introduction

This flexbook contains Texas Instruments (TI) Resources for the TI-83, TI-83 Plus, TI-84, and TI-84 SE. All the activities in this flexbook supplement the lessons in the student edition. Teachers may need to download programs from [www.timath.com](http://www.timath.com) that will implement or assist in the activities. Each activity included is designed to help the teacher with each activity in the TI Resources Flexbook, Student Edition. All activities are listed in the same order as the Student Edition.

There are also corresponding links in the 1st and 2nd editions of Trigonometry.

- Trigonometry, first edition: <http://www.ck12.org/flexr/flexbook/809>
- Trigonometry, second edition: <http://www.ck12.org/flexr/flexbook/2410>

Any activity that requires a calculator file or application, go to <http://www.education.ti.com/calculators/downloads> and type the name of the activity or program in the search box.

## CHAPTER

**2****TE Trigonometry and Right  
Angles - TI****CHAPTER OUTLINE**

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- 2.1 TRIGONOMETRIC RATIOS**
  - 2.2 ROUND AND ROUND SHE GOES**
  - 2.3 TRIGONOMETRIC PATTERNS**
-

## 2.1 Trigonometric Ratios

*This activity is intended to supplement Trigonometry, Chapter 1, Lesson 3.*

**ID: 9534**

**Time required: 35 minutes**

### Activity Overview

*Students will use a graphing calculator to discover the relationship between the trigonometric functions: sine, cosine and tangent and the side length ratios of a right triangle.*

#### Topic: Trigonometric Functions

- *Solve any right triangle given an angle and the length of an opposite or adjacent side.*
- *Use technology to obtain the sine, cosine, or tangent of any angle.*

#### Teacher Preparation and Notes

*Students will use the graphing calculator to discover the relationship between the trigonometric functions: sine, cosine and tangent and the side length ratios of a right triangle. Prior to beginning the activity, students should download the CabriJr file **TRIG** to their graphing calculators.*

*This activity is designed as an introduction to the world of trigonometry. Students will explore the trigonometric ratios (sine, cosine, tangent) of a right triangle.*

- *This activity requires students to drag a point in CabriJr. If students are not familiar with this function of the CabriJr application, extra time should be taken to explain this.*
- *This activity is intended to be **teacher-led**. You may use the following pages to present the material to the class and encourage discussion. Students will follow along using their handhelds, although the majority of the ideas and concepts are only presented in **this** document; be sure to cover all the material necessary for students' total comprehension.*
- *The student worksheet is intended to guide students through the main ideas of the activity. It also serves as a place for students to record their answers. Alternatively, you may wish to have the class record their answers on separate sheets of paper, or just use the questions posed to engage a class discussion.*
- *To download Cabri Jr, go to <http://www.education.ti.com/calculators/downloads/US/Software/Detail?id=258#>.*
- *To download the calculator file, go to <http://www.education.ti.com/calculators/downloads/US/Activities/Detail?id=9534> and select **TRIG**.*

#### Associated Materials

- Student Worksheet: Trig Ratios <http://www.ck12.org/flexr/chapter/9699>
- Cabri Jr. Application
- TRIG.8xv

*In this activity, students will explore:*

- *Sine, cosine and tangents of angles*

#### 2.1. TRIGONOMETRIC RATIOS



- Side length ratios of a right triangle
- Finding the missing side of a triangle when an angle and side are given.

**Note:** Students will need to exit the application in order to find the decimal values for the ratios and the trigonometric values for angle A. One option is for students to go through the **TRIG** application first, fill in all the ratios on the worksheet, and copy down the measures of angle A for each triangle. Then they would go back and complete the rest of the worksheet. Alternatively, students could work in pairs with one using the application and the other performing the calculations.

## Problem 1 – Trigonometric Ratios

Students will open the **TRIG** file using the CabriJr application. Given a triangle, students will find the relationship between the ratios of the side lengths and the trigonometric functions.

Record the following ratios and trigonometric values to two decimal places.

$$\frac{BC}{AC} = \frac{4.0}{5.9}, \frac{AC}{AB} = \frac{5.9}{7.1}, \frac{BC}{AB} = \frac{4.0}{7.1}$$

$$\sin A = \underline{0.56}, \cos A = \underline{0.83}, \tan A = \underline{0.67}$$

**Note:** Make sure all student calculators are in **DEGREE** mode.

**Make sure students find the decimal values below and compare the answers with the trig values.**

$$\frac{BC}{AC} = 0.67$$

$$\frac{AC}{AB} = 0.83$$

$$\frac{BC}{AB} = 0.56$$

Students are asked to repeat this process for two more different triangles by moving point **B** to a different location (to grab a point, press the **ALPHA** button and use the arrow keys to move it.)

**Answers will vary. Students should check their answers with the trig values. Answers should match the trig identities.**

Based upon your answers hypothesize which ratio goes with each trigonometric function.

$$\sin A = \frac{BC}{AB}$$

$$\cos A = \frac{AC}{AB}$$

$$\tan A = \frac{BC}{AC}$$

A good acronym to use to help remember these relationships is SOHCAHTOA

$$\sin A = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

$$\cos A = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

$$\tan A = \frac{\text{Opposite}}{\text{Adjacent}}$$

## Problem 2 – Trigonometry, What Is It Good For?

One of the uses of trigonometry is finding missing side lengths of a triangle.

To find the length of side  $BC$  in the triangle to the right, write the sine relationship.

$$\sin 27 = \frac{BC}{24}$$

Now solve for  $BC$  and calculate using your handheld.

$$24 \sin 27 = BC$$

$$24 \cdot 0.45 = BC$$

$$10.90 = BC$$

To find the length of side  $AC$  in the triangle to the right, write the cosine relationship.

$$\cos 48 = \frac{AC}{19}$$

Now solve for  $AC$  and calculate using your handheld.

$$19 \cos 48 = AC$$

$$19 \cdot 0.67 = AC$$

$$12.71 = AC$$

To find the length of side  $AC$  in the triangle to the right, write the tangent relationship.

$$\tan 50 = \frac{10}{b}$$

Now solve for  $AC$  and calculate using your handheld.

$$b \tan 50 = 10$$

$$b = \frac{10}{\tan 50} = \frac{10}{1.19} = 8.4$$

## Solutions

1.  $AC = 14.62$
2.  $BC = 19.51$
3.  $AC = 49.52$
4.  $AC = 95.09$

## 2.2 Round and Round She Goes

*This activity is intended to supplement Trigonometry, Chapter 1, Lesson 7.*

**ID:** 12385

**Time Required:** 20 minutes

### Activity Overview

*In this activity, students will explore relationships on the unit circle. Students will identify coordinates of points given an angle measure in degrees.*

**Topic:** Unit Circle

- Right triangle trigonometry and the unit circle
- Special right triangles
- Cosine and sine on the unit circle

### Teacher Preparation and Notes

- *The first problem and second problem engage students in the exploration of the connection between angle measure and the coordinates of points in the first quadrant.*

### Associated Materials

- Student Worksheet: Round and Round She Goes <http://www.ck12.org/flexr/chapter/9699>, scroll down to the second activity.

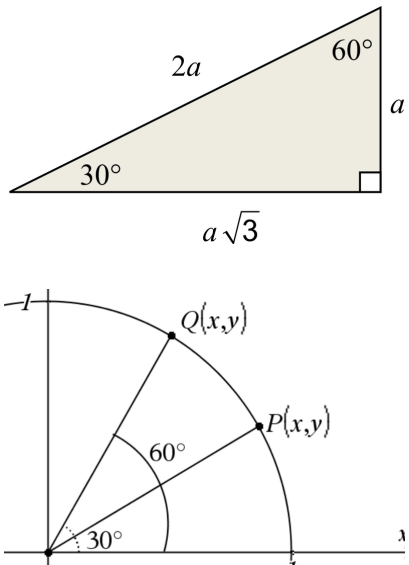
### Problem 1 – Introduction to the Unit Circle

Students are introduced to the concept of the unit circle. Right triangle relationships are explored to develop an understanding of the patterns involved.

Special right triangles are addressed to help students understand the exact values they will likely be expected to know.

Students often have difficulty with remembering some of these special values. Ask students if they can see a pattern that might help them remember that  $\frac{\sqrt{2}}{2}$  goes with the 45 – 45 – 90 triangle. Students may point out that there is a pattern of twos—two angles and two sides are the same.

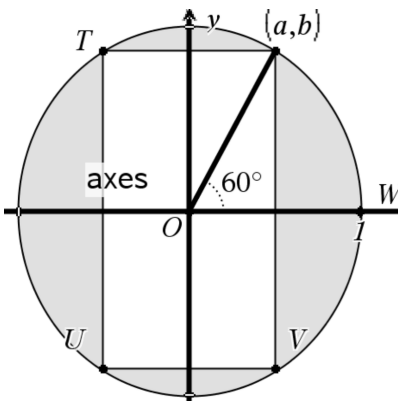
Similarly, ask them how they might remember that  $\frac{\sqrt{3}}{2}$  goes with the 30 – 60 – 90 triangle. Students may state that there is a pattern of threes—all angles involved are multiples of 3. Next, once students realize that  $\frac{\sqrt{3}}{2}$  and  $\frac{1}{2}$  go together in an ordered pair, it is easy to point out using a unit circle sketch that a 60° angle involves an  $x$ -value that is  $\frac{1}{2}$  and the 30° angle involves a  $y$ -value that is  $\frac{\sqrt{3}}{2}$ .



## Problem 2 – Extending the Pattern

Students use a visual model to extend what they established in Quadrant I to Quadrants II, III, and IV.

It is very helpful for students to think about symmetry as they move on to these other quadrants. Construction of rectangles in the unit circle is helpful for many students to make this extension.



## Solutions

1.  $x = \cos \theta$
2.  $x = \sin \theta$
3.  $\frac{1}{2}$
4.  $\frac{\sqrt{2}}{2}$
5.  $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
6.  $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
7.  $\frac{\sqrt{3}}{2}$

8.  $\frac{1}{2}$

9.  $\frac{1}{2}$

10.  $\frac{\sqrt{3}}{2}$

11.  $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

12.  $\frac{\sqrt{2}}{2}$

13.  $\frac{\sqrt{2}}{2}$

14.  $(-a, b)$

15.  $(-a, -b)$

16.  $(a, -b)$

17.  $120^\circ$

18.  $240^\circ$

19.  $300^\circ$

## 2.3 Trigonometric Patterns

*This activity is intended to supplement Trigonometry, Chapter 1, Lesson 8.*

**ID: 12434**

**Time required: 15 minutes**

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### Activity Overview

*Students will use the unit circle to examine patterns in the six trigonometric functions.*

**Topic: Trigonometry**

- Unit Circle
- Right Triangle Trigonometry

### Teacher Preparation and Notes

- *The angle measures will vary from the exact values in the answer tables due to Cabri Jr. only being able to measure coordinates to one decimal place. Make students aware that they may need to use the closest angle measure possible for some values.*
- *The questions in the document are starting points for discussions about the patterns in the values of trig functions. Teachers should be prepared to discuss patterns beyond the ones in the document.*
- *To download Cabri Jr, go to <http://www.education.ti.com/calculators/downloads/US/Software/Detail?id=258#>.*
- *To download the calculator file, go to <http://www.education.ti.com/calculators/downloads/US/Activities/Detail?id=12434> and select UNITC.*

### Associated Materials

- Student Worksheet: Trigonometric Patterns <http://www.ck12.org/flexr/chapter/9699>, scroll down to the third activity.
- Cabri Jr. Application
- UNITC.8xv

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### Trigonometric Patterns

Students will move the triangle on the unit circle to find the angle measures listed in the table on the student worksheet. Students will record the values and then answer questions about patterns in the results. Because the *Cabri Jr.* file only measures the angle less than  $90^\circ$ , there is opportunity for some further student learning. This means that when the angle being displayed is  $30^\circ$  and the point is in the second quadrant, the angle being observed is really  $150^\circ$  ( $180^\circ - 30^\circ$ ).

Students can write the ratios on their worksheet and then use the Home screen to do their calculations.

Note that due to rounding in *Cabri Jr.*, the answers are approximate, not exact.

#### Discussion Questions

#### 2.3. TRIGONOMETRIC PATTERNS

- What happens at  $90^\circ$ ,  $180^\circ$ ,  $270^\circ$ , and  $360^\circ$  ?
- Why is the tangent function undefined for some angle measures?

## Extension – Patterns in reciprocal functions

In this problem, students will repeat the activity for the co-trigonometric functions. Students should notice that these functions are the reciprocals of the functions from the first part of the activity by looking at the given formulas. This will help them in calculating these functions because they can use simply find the reciprocals on the Home screen instead of finding new ratios.

### Discussion Questions

- Are any of the functions undefined? For what values?

Do any of the patterns match the patterns from the functions in the activity?

## Solutions

TABLE 2.1:

$\theta$	$\sin \theta$	$\cos \theta$	$\tan \theta$
$30^\circ$	0.5	0.866	0.577
$45^\circ$	0.707	0.707	1
$60^\circ$	0.866	0.5	1.732
$90^\circ$	1	0	Undefined
$120^\circ$	0.866	-0.5	-1.732
$135^\circ$	0.707	-0.707	-1
$150^\circ$	0.5	-0.866	-0.577
$180^\circ$	0	-1	0
$210^\circ$	-0.5	-0.866	0.577
$225^\circ$	-0.707	-0.707	1
$240^\circ$	-0.866	-0.5	1.732
$270^\circ$	-1	0	Undefined
$300^\circ$	-0.866	0.5	-1.732
$315^\circ$	-0.707	0.707	-1
$330^\circ$	-0.5	0.866	-0.577
$360^\circ$	0	1	0

- $0^\circ < \theta < 180^\circ$
- $180^\circ < \theta < 270^\circ$
- Positive  $0^\circ < \theta < 90^\circ$ ,  $180^\circ < \theta < 270^\circ$  because sine and cosine have same sign Negative  $90^\circ < \theta < 180^\circ$ ,  $270^\circ < \theta < 360^\circ$  because sine and cosine have different signs
- $\cos(330^\circ)$
- Possible response:  $\cos(45^\circ) = \cos(315^\circ)$ ;  $\cos(60^\circ) = \cos(300^\circ)$
- $\tan(225^\circ)$

7. Possible response:  $\tan(60^\circ) = \tan(240^\circ)$ ,  $\tan(30^\circ) = \tan(210^\circ)$

8. Answers will vary. Sample response: The values in the first quadrant are repeated in the other quadrants, but have different signs.

9. Answers will vary. Sample response: The values of sine and cosine switch within a quadrant, such as  $\sin(30^\circ) = \cos(60^\circ)$ , but the signs may be opposite in some quadrants.

### Extension

**TABLE 2.2:**

$\theta$	$\sin \theta$	$\cos \theta$	$\tan \theta$
$30^\circ$	1.155	2	1.732
$45^\circ$	1.414	1.414	1
$60^\circ$	2	1.155	0.577
$90^\circ$	Undefined	1	0
$120^\circ$	-2	1.155	-1.732
$135^\circ$	-1.414	1.414	-1
$150^\circ$	-1.155	2	-1.732
$180^\circ$	-1	Undefined	Undefined
$210^\circ$	-1.155	-2	1.732
$225^\circ$	-1.414	-1.414	1
$240^\circ$	-2	-1.155	0.577
$270^\circ$	Undefined	-1	0
$300^\circ$	2	-1.155	-0.577
$315^\circ$	1.414	-1.414	-1
$330^\circ$	1.144	-2	-1.732
$360^\circ$	1	Undefined	Undefined



## CHAPTER

**3****TE Circular Functions - TI****CHAPTER OUTLINE**

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**3.1 FIND THAT SINE****3.2 VERTICAL AND PHASE SHIFTS****3.3 GETTING TRIGGY WITH IT**

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## 3.1 Find that Sine

This activity is intended to supplement *Trigonometry, Chapter 2, Lesson 3*.

ID: 9733

Time required: 35 minutes

### Activity Overview

Students will use sinusoidal regression to determine equations to model various data sets and use the equations to make inferences.

**Topic: Trigonometric Functions**

- Calculate the trigonometric line of best fit to model bivariate data and use it to predict a value of one.

**Teacher Preparation and Notes**

- Prior to beginning the activity, students should download the **KANSTEMP** program to their handhelds.
- This investigation has students using sinusoidal regression with data sets and making inferences with the created equations.
- Students should already be familiar with the properties of sine graphs.
- This activity is intended to be teacher-led.
- To download the calculator file, go to <http://www.education.ti.com/calculators/downloads/US/Activities/Detail?id=9733> and select **KANSTEMP**.

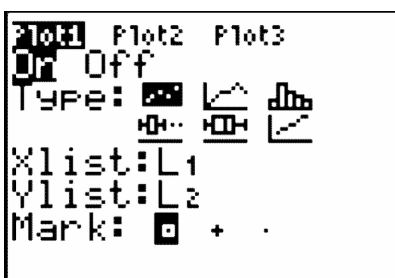
**Associated Materials**

- Student Worksheet: Find that Sine <http://www.ck12.org/flexr/chapter/9700>
- KANSTEMP.8xp

### Problem 1 – Temperature graphs

Students start this activity by running the **KANSTEMP** activity to store the necessary data for this activity in lists. For Problem 1, students will only use the data in lists  $L_1$  and  $L_2$ . The other lists will be used later in this activity.

Students should plot the data using the settings shown to the right. To obtain an acceptable window, press **ZOOM** and select **9:ZoomStat**.



Students will use a sinusoidal regression to find a sine function that models the data. They should store the equation in  $Y_1$ .

```
SinReg
y=a*sin(bx+c)+d
a=26.7502224
b=.4764967356
c=-1.79869635
d=51.09629231
```

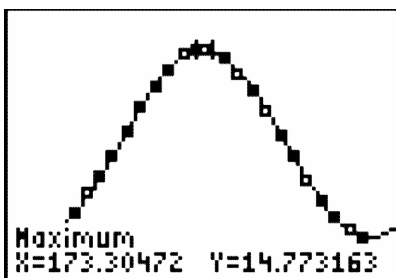
After students have determined and graphed the sine regression equation,  $f(x) = 26.75 \sin(0.47x - 1.80) + 51.1$ , emphasize the need to check for its reasonableness of fit when compared to the scatter plot of the data.

## Problem 2 – Hours of Sunlight

The amount of light a location on the Earth receives from the Sun changes each day depending upon the time of year and latitude of that location. The amount of daily sunshine Kansas City experiences has been recorded in the lists where the calendar day is in  $L_3$ , and the hours of sunlight is  $L_4$ . Remind students to change the settings for **Plot 1** so that the **XList** is  $L_3$  and the **YList** is  $L_4$ .

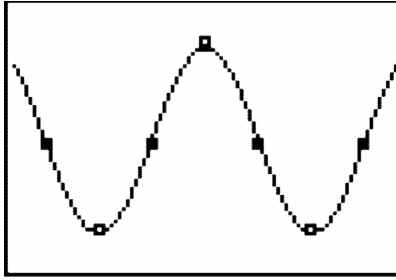
Students will again use the sinusoidal regression to find an equation to model the data.

Students will use their equation to calculate the winter and summer solstices and spring and fall equinoxes. To calculate the equinox dates, students can use **Trace** to find the  $x$ - values when the  $y$ - value is 12. To find the solstice dates, students can use the **minimum** and **maximum** functions from the Calc menu.



## Problem 3 – Tides

The Bay of Fundy has the highest tides in the world. If a tape measure were attached at the water line of a pier, and the water level height were recorded over a period of eighteen hours, data like that in  $L_5$  and  $L_6$  would be generated. Remind students to change the settings for **Plot 1** so that the **XList** is  $L_5$  and the **YList** is  $L_6$ .



After finding an equation to model the data, students will use the model to predict future events. Students can use the **Trace** feature, the **value** feature (students must adjust the window to include  $x = 49$  on the screen), or enter  $Y_1(49)$  on the home screen.

## Solutions

### Problem 1

- $y = 26.75 \sin(0.47x - 1.80) + 51.10$

### Problem 2

- $y = 2.8 \sin(0.02x - 1.38) + 11.97$
- Vernal Equinox - 81.94 calendar days (March 22)
- Autumnal Equinox - 264.97 calendar days (September 22)
- Summer Solstice ( 1<sup>st</sup> maximum of the function) -173.3 calendar days (June 22)
- Winter Solstice ( 1<sup>st</sup> minimum of the function) - 357.76 calendar days (December 22)

### Problem 3

- $y = 4.44 \sin(0.52x - 3.11) + 6.22$
- $y = 4.52$  feet

### Additional Practice

$$y = 150 \sin(0.52x - 2.09) + 650$$

## 3.2 Vertical and Phase Shifts

This activity is intended to supplement Trigonometry, Chapter 2, Lesson 4.

ID: 9608

Time required: 45 minutes

### Activity Overview

Students explore vertical and phase shifts of sine and cosine functions, after a brief review of period and amplitude. Students will use values in lists to change the values of parameters in trigonometric functions; they will determine the effect that each change has upon the shape of the graph. They will then use this knowledge to write a sine function as a cosine function.

**Topic: Trigonometric Functions**

- Analyze and predict the effects of changes in  $A$ ,  $B$  and  $C$  on the graphs of  $A \sin(Bx + C)$ ,  $A \cos(Bx + C)$ , and  $A \tan(Bx + C)$  and interpret  $A$ ,  $B$ , and  $C$  in terms of amplitude, period, and phase shifts.
- Approximate the amplitude, frequency, and phase shift of the primary trigonometric functions by graphing.

### Teacher Preparation and Notes

- Students should already be familiar with the graphs of the sine and cosine functions, and they should also have some experience in determining the period (in radians) and identifying the amplitude of a trigonometric function.
- The graphs of trigonometric functions that students will view on their calculators have been constructed such that the horizontal scale is in multiples of  $\frac{\pi}{2}$ .
- This activity is intended to be **introduced in a whole-class setting** and completed by students individually.
- The programs utilize the **Transformation Graphing Application**.
- To download the **Transformation Graphing Application** and calculator files, go to <http://www.education.ti.com/calculators> and select **PHSESHFT**, **TRIGCOMB**, and the **Transformation Graphing** from the Applications header.

### Associated Materials

- Student Worksheet: Vertical and Phase Shifts <http://www.ck12.org/flexr/chapter/9700>, scroll down to the second activity.
- Transformation Graphing Application
- PHSESHFT.8xp and TRIGCOMB.8xp

Before beginning the activity, students should clear out any functions from the o screen, turn off all Stat Plots, and make sure that the calculator is in Radian mode. They will also utilize the **Transformation Graphing App**. To start this app, press the **APPS** and select **Transfrm** from the list and press **ENTER** twice to activate the application.

### Problem 1 – Amplitude

This problem allows students to review the amplitude of a function of the form  $f(x) = a \sin(x)$ . By changing the value of  $A$ , students should conclude that the sine curve is vertically stretched by a factor of  $|a|$ . Thus, amplitude =  $|a|$ . Be sure to ask what effect the sign of  $a$  has on the graph (if  $a$  is negative, then the curve is reflected over the  $x$ -axis).

### Problem 2 – Period

This problem allows students to review the period of a function of the form  $f(x) = \sin(bx)$ . By changing the value of  $B$ , students will find that the value of  $b$  affects the horizontal stretch of this function and thus changes the period of the function.

After some examination, students should be able to identify the relationship: period =  $\frac{2\pi}{b}$ .

### Problem 3 – A simple phase shift

The graph of  $f(x) = \sin(x + c)$  is explored next. Students might predict that a change in  $c$  will result in a horizontal shift a certain number of units (here called a phase shift), but how that number of units relates to  $c$  will not be immediately clear. This is explored in Problem 5.

### Problem 4 – Vertical shift

The parameter  $d$  in the graph of  $f(x) = \sin(x) + d$  is now considered. Before students change the value of  $D$ , ask them to predict what will happen by first considering a different function,  $y = x^2$ . Ask them how to obtain the graph of  $y = x^2 + 3$  from the graph of  $y = x^2$  (translate the graph up three units). Students should confirm that the vertical shift is equal to this parameter; that is, vertical shift =  $d$ .

### Problem 5 – Combining transformations

While students may be wondering why they cannot declare “phase shift =  $-c$ ,” have them again consider the graph of  $y = x^2$ . Ask them how to obtain from it the graph of  $y = (x + 3)^2$  (translate three units to the left). Then display the equations  $y = [4(x + 3)]^2$  and  $y = (4x + 3)^2$ , and elicit from students that only the former represents a shift three units to the left. At this point students should realize that the phase shift of  $f(x) = a \sin(bx + c) + d$  depends on two parameters:  $b$  and  $c$ .

To establish exactly how  $b$  and  $c$  determine the phase shift, students may change the values of the variables. Encourage students to conjecture the relationship on their own, but if they need help, have them consider the phase shift when  $b = 1$  and  $c = 2$ , when  $b = 2$  and  $c = 1$ , and when  $b = 2.5$  and  $c = -5$ . (It is  $-2$ ,  $-0.5$ , and  $2$ , respectively.) Examining these values, students should conclude that phase shift =  $-\frac{c}{b}$ .

**Note:** It is easiest to identify this relationship if parameters  $a$  and  $d$  are left as initially set ( $a = 1$  and  $d = 0$ ). After the relationship is determined, changing the other values can verify that neither  $a$  nor  $d$  affects the phase shift.

#### 3.2. VERTICAL AND PHASE SHIFTS

## Problem 6 – Bringing it all together

Students are asked to summarize their findings from Problems 1–5. Make sure students have correctly completed these answers before proceeding further.

Next, students are asked to return to Problems 1–5 and replace each “ sin ” in the function definition to “ cos ” to verify that the same characteristics hold true.

Finally, students will apply what they have learned about vertical and phase shifts. They are given the equations and graphs of two sine functions and asked to find equations of cosine functions that coincide. Students should observe that the values of  $a$ ,  $b$ , and  $d$  remain the same for each sine/cosine pair; the only difference occurs in the value of  $c$ . Because these functions are periodic, there are infinitely many equations that satisfy each condition. Be sure to check students' equations.

## Solutions

### Problem 1

- students should conclude that the sine curve is vertically stretched by a factor of  $|a|$
- if  $a$  is negative, then the curve is reflected over the  $x$ - axis
- amplitude =  $|a|$

### Problem 2

Students will find that the value of  $b$  affects the horizontal stretch of this function and thus changes the period of the function. After some examination, students should be able to identify the relationship: period =  $\frac{2\pi}{b}$ .

### Problem 3

Students might predict that a change in  $c$  will result in a horizontal shift a certain number of units (here called a phase shift), but how that number of units relates to  $c$  will not be immediately clear.

### Problem 4

Students should confirm that the vertical shift is equal to this parameter. (vertical shift =  $d$ ).

### Problem 5

While students may be wondering why they cannot declare “phase shift =  $-c$ ,” have them again consider the graph of  $y = x^2$ . Ask them how to obtain from it the graph of  $y = (x + 3)^2$  (translate 3 units to the left). Then display the equations  $y = [4(x + 3)]^2$  and  $y = (4x + 3)^2$ , and elicit from students that only the former represents a shift 3 units to the left. Students should realize that the phase shift of  $f(x) = a \sin(bx + c) + d$ , depends on two parameters:  $b$  and  $c$ .

To establish exactly how  $b$  and  $c$  determine the phase shift, students may be to change the values. Encourage students to conjecture the relationship on their own, but if they need help, have them consider the phase shift when  $b = 1$  and  $c = 2$ , when  $b = 2$  and  $c = 1$ , and when  $b = 2.5$ , and  $c = -5$ . (It is -2, -0.5, and 2, respectively.) Examining these values, students should conclude that phase shift =  $-\frac{c}{b}$ .

**Note:** It is easiest to identify this relationship if parameters  $a$  and  $d$  are left as initially set ( $a = 1$  and  $d = 0$ ). After the relationship is determined, dragging the sliders can verify that neither  $a$  nor  $d$  affects the phase shift.

### Problem 6

- amplitude =  $|a|$ , period =  $\frac{2\pi}{b}$ , phase shift =  $-\frac{c}{b}$ , vertical shift =  $d$

- $f(x) = 1.5 \cos\left(x + \frac{3\pi}{4}\right) + 4$
- $g(x) = 3 \cos\left(2x - \frac{\pi}{2}\right) - 5$

Students should observe that the values of  $a$ ,  $b$ , and  $d$  remain the same for each sine/cosine pair; the only difference occurs in the value of  $c$ . Since these functions are periodic, there are infinitely many that satisfy each condition. Be sure to check students' equations.



## 3.3 Getting Triggy With It

This activity is intended to supplement *Trigonometry, Chapter 2, Lesson 6*.

ID: 9774

Time required: 45 minutes

### Activity Overview

Students systematically explore the effect of the coefficients on the graph of sine or cosine functions. Terminology describing the graph—amplitude, period, frequency, phase shift, baseline, and vertical offset—is introduced, then reinforced as the student calculates these values directly from the graph using the graphing calculator.

**Topic: Trigonometric Functions #38; Equations**

- Approximate the zeros, minima and period of the primary trigonometric functions by graphing.
- Approximate the amplitude, frequency, and phase shift of the primary trigonometric functions by graphing.
- Given the equation of a primary trig function, state its range, amplitude, frequency, period and phase shift.
- Describe how the graph of a trigonometric function  $y = f(x)$  changes under transformations.

### Teacher Preparation and Notes

- Students should already have been introduced to the basic sine and cosine graphs.

### Associated Materials

- Student Worksheet: Getting Triggy with It <http://www.ck12.org/flexr/chapter/9700>, scroll down to the third activity.

### Problem 1 – A general trigonometric function

Students should start by opening the *Transformation Graphing* app by pressing **APPS** and choosing **Transfrm** from the list. (Hint: Press **ALPHA 4** to jump to the Ts.)

In  $Y_1$ , they should enter the general sine function  $A * \sin(B * X + C) + D$ . Next, adjust the window settings so that  $x$  goes from -0.5 to 20 and  $y$  goes from -10 to 10.

```

Plot1 Plot2 Plot3
M1Y1 A*sin(B*X+C)
+D
M1Y2 =
M1Y3 =
M1Y4 =
M1Y5 =
M1Y6 =

```

The *Transformation Graphing* app will allow students to change the values of  $A$ ,  $B$ ,  $C$ , and  $D$  by pressing  $\leftarrow$  or  $\rightarrow$ . They can also move up or down to a different variable by pressing  $\uparrow$  or  $\downarrow$ .

Students can use the **zero**, **minimum**, and **maximum** commands ( 2<sup>nd</sup> [CALC]) to find these features of the graph. The graph has many zeros.

---

## Problem 2 – The effect of the coefficients

By changing the coefficients systematically, students can figure out which coefficients affect which graph features. Students will examine each of the coefficients individually to see the effects of each on the graph of the function.

The value of  $A$  is the **amplitude** of the graph. The amplitude of the graph of a sine or cosine function is the “height” of the wave. It is equal to half of the difference between its maximum and minimum values.

The value of  $B$  is the **frequency** of the graph. The frequency affects the **period**. The period of the graph of a sine or cosine function is the “width” of one cycle of the wave. It is the distance from one peak of the curve to the next. When  $B = 1$ , the period of the graph is  $2\pi$ .

The value of  $C$  shifts the graph horizontally. Because the graph extends infinitely in both directions, shifting it horizontally does not actually move it. The value of  $C$  is called the **phase shift**, which shifts the point where a new cycle of the graph starts horizontally.

The phase shift of a sine function is the horizontal distance from the  $y$ - axis to the first point where the graph intersects the baseline. The phase shift of a cosine function is the horizontal distance from the  $y$ - axis to the top of the first peak.

The value of  $D$  shifts the graph vertically and affects the **baseline**. The baseline is the midpoint of the vertical oscillation. In a basic sine graph, the baseline is the  $x$ - axis,  $y = 0$ . The value of  $D$  is called the **vertical offset** and is equal to distance from the  $x$ - axis to the baseline.

---

## Problem 3 – A closer look at amplitude, period, and frequency

Next, students will enter the general cosine function shown in  $Y_1$ . They will use this function to study the amplitude, period, and frequency of trigonometric graphs in more detail.

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# CHAPTER **4** TE Trigonometric Identities - TI

## CHAPTER OUTLINE

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**4.1 TRIGONOMETRIC IDENTITIES**

**4.2 TRIG PROOFS**

**4.3 WHAT'S THE DIFFERENCE?**

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## 4.1 Trigonometric Identities

*This activity is intended to supplement Trigonometry, Chapter 3, Lesson 1.*

**ID: 9848**

**Time required: 45 minutes**

### Activity Overview

*Students will verify, prove, and explore trigonometric identities symbolically, numerically, and graphically.*

**Topic: Trigonometric Identities**

- Use the Pythagorean Theorem to prove the trigonometric identities  $\sin^2 \theta + \cos^2 \theta = 1$  and  $\sec^2 \theta = 1 + \tan^2 \theta$ .
- Verify trigonometric identities by graphing.

### Teacher Preparation and Notes

- Prior to beginning the activity, students should download the **VERITRIG** program to their handhelds. This investigation has students exploring, proving, and verifying trigonometric identities.
- Students should already be familiar with right triangle trigonometric properties, similar triangles, and the Pythagorean Theorem since much of the activity is based on these fundamental theorems.
- Students should begin by clearing out any functions from the  $Y =$  screen and turning off all Stat Plots.
- To download the calculator file, go to <http://www.education.ti.com/calculators/downloads/US/Activities/Detail?id=9848> and select **VERITRIG**.

### Associated Materials

- Student Worksheet: Trigonometric Identities <http://www.ck12.org/flexr/chapter/9701>
- VERITRIG.8xp

### Problem 1 – Prove

In this problem, students will prove two of the basic trigonometric identities using the Pythagorean Theorem.

Press **PRGM** to access the **Program** menu.

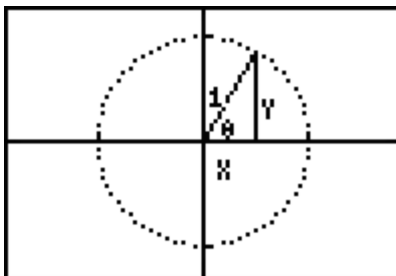
Then choose the **VERITRIG** program and press **ENTER**. Select **1: PROVE ID 1**.

Students will see a circle with a radius of 1 with a right triangle in the first quadrant.

Students will label the right triangle using the **Text** tool. Press  $2^{nd}$  [**PRGM**] and arrow down to select **0:Text**.

Now students can position the cursor near the middle of the hypotenuse and enter in the radius. Then press  $2^{nd}$  [**APLHA**] to lock the alpha key on and enter in  $x$ ,  $y$ , and  $\theta$  in their respective places in a right triangle.

If students make a mistake, they can press  $2^{nd}$  [**PRGM**] and select **ClrDraw**. Then they need to run the program again.



On the worksheet, students will prove:  $\cos^2 \theta + \sin^2 \theta = 1$ .

1.  $x^2 + y^2 = 1$
2.  $\cos \theta = \frac{x}{1}$  and  $\sin \theta = \frac{y}{1}$
3. Substituting  $x = \cos \theta$  and  $y = \sin \theta$  into  $x^2 + y^2 = 1$  gives  $\cos^2 \theta + \sin^2 \theta = 1$ .

When finished, students can press 2<sup>nd</sup> [QUIT] to exit the drawing.

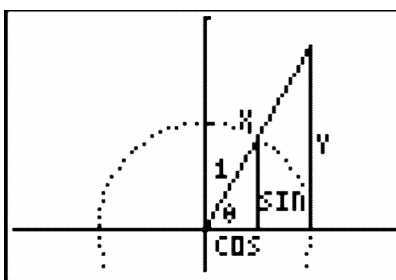
## Problem 2 – Prove

Students need to run the **VERITRIG** program and select **2: PROVE ID 2**.

Students will see a new version of the unit circle with the original right triangle and then a similar triangle drawn to incorporate the original triangle.

Again, they need to use the **Text** tool and label the original triangle.

Students are told on the worksheet that the length of the base of the large triangle is 1, the height is  $Y$  and the hypotenuse is  $X$ . Students need to use the **Text** tool to label the large similar triangle accordingly.



Using the ratio given on the worksheet, students will prove  $\sec^2 \theta = 1 + \tan^2 \theta$ . You may need to explain to students that (small  $\Delta$ ) and (large  $\Delta$ ) reference where that part of the ratio came from and is not actually part of the ratio.

$$\begin{aligned} \frac{1}{\cos \theta} &= \frac{X}{1} & \frac{Y}{\sin \theta} &= \frac{1}{\cos \theta} \\ 1 &= (\cos \theta)(X) & (Y)(\cos \theta) &= (\sin \theta)(1) \\ X &= \frac{1}{\cos \theta} = \sec \theta & Y &= \frac{\sin \theta}{\cos \theta} = \tan \theta \end{aligned}$$

Substituting  $Y = \tan \theta$  and  $X = \sec \theta$  into the Pythagorean Theorem based on the larger triangle:  $1^2 + Y^2 = X^2 \rightarrow 1 + (\tan \theta)^2 = (\sec \theta)^2$        $1 + \tan^2 \theta = \sec^2 \theta$

---

### Problem 3 – Numerical verification

Now that students have proved the two identities, they will numerically verify the two identities.

Students need to run the **VERITRIG** program and select **3: NUM EXPLORE**.

Students will see the same circle, but with three triangles in the first quadrant. When they press **TRACE**, they will see the  $x$ - values for cosine and the  $y$ - values for sine for each angle measurement.

Students should observe that the values for cosine and sine repeat but change signs depending on what quadrant the angle is in.

These values are stored in the lists of the calculator with the angle measurements in  $L_1$  , the cosine values in  $L_2$  , and the sine values in  $L_3$  .

To numerically verify  $\cos^2 \theta + \sin^2 \theta = 1$  ; students are to enter  $L_2^2 + L_3^2$  at the top of  $L_4$  .

Students will discover that they need to enter “  $\frac{1}{\cos \theta}$  ” to get  $\sec \theta$  . The value of  $\sec \theta$  is undefined at  $0$  and  $\frac{3\pi}{2}$  , and the calculator will not evaluate these values within a list. The same thing will happen to  $\tan \theta$  , which is equal to  $\frac{\sin \theta}{\cos \theta}$  .

However, students should arrive at the fact that they can graph the identities to verify the identity as the screen shots illustrate. They should set the calculator to the trig window by pressing **ZOOM** and select **7:ZTrig**. (*Note:* Students will need to use the **ClrDraw** command to erase the unit circle and turn off **Plot1**.)

---

### Problem 4 – Verifying trig identities using graphing.

The previous problem leads into utilizing graphing techniques to verify various trig identities.

Students are to determine whether the identity

$\sin^2 x = 1 - \cos^2 x$  is true by entering  $\sin^2 x$  into  $Y_1$  and  $1 - \cos^2 x$  into  $Y_2$  . To set the bubble animation for the second graph, students need to press **ENTER** to the left of the equals sign.

Students need to select **ZTrig** for a trig window.

If the bubble runs over the same graph as the first, the equation is a trig identity.

Explain to students that they can check any equation using this method.

## 4.2 Trig Proofs

*This activity is intended to supplement Trigonometry, Chapter 3, Lesson 2.*

**ID: 9776**

**Time required: 30 minutes**

### Activity Overview

*Students will perform trigonometric proofs and use the graphing capabilities of the calculator for verification.*

**Topic: Trigonometric Identities**

- *Use fundamental trigonometric identities to prove more complex trigonometric identities.*
- *Verify trigonometric identities by graphing.*

### Teacher Preparation and Notes

- *Students should already be familiar with the Pythagorean trigonometric identities as well the fact that  $\text{tangent} = \frac{\text{sine}}{\text{cosine}}$ ,  $\text{cosecant} = \frac{1}{\text{sine}}$ ,  $\text{secant} = \frac{1}{\text{cosine}}$ , and  $\text{cotangent} = \frac{1}{\text{tangent}}$ .*
- *This activity is intended to be **teacher-led**. You may use the following pages to present the material to the class and encourage discussion. Students will follow along using their calculators, although the majority of the ideas and concepts are only presented in **this** document; be sure to cover all the material necessary for students' total comprehension.*
- *The worksheet is intended to guide students through the main ideas of the activity, while providing more detailed instruction on how they are to perform specific actions using the tools of the calculator. It also serves as a place for students to record their answers. Alternatively, you may wish to have the class record their answers on separate sheets of paper, or just use the questions posed to engage a class discussion.*

### Associated Materials

- Student Worksheet: Trig Proofs <http://www.ck12.org/flexr/chapter/9701>, scroll down to the second activity.

### Problem 1 – Using the Calculator for verification

Prove:  $(1 + \cos x)(1 - \cos x) = \sin^2 x$ .

$$\begin{aligned} (1 + \cos(x))(1 - \cos(x)) &= 1 - \cos^2(x) \\ &= [\sin^2(x) + \cos^2(x)] - \cos^2(x) \\ &= \sin^2 x \end{aligned}$$

To verify this proof graphically, you will determine if the graph of the expression on the left side of the equation coincides with the graph of the expression on the right side of the equation.

Enter the left side of the equation  $(1 + \cos x)(1 - \cos x)$  in  $Y_1$ .

Enter the right side of the equation  $(\sin x)^2$  in  $Y_2$ .

Arrow over to the icon to the left of  $Y_2$  and press **ENTER** to change it to a thick line. Now  $Y_1$  will be graphed as a thin line and  $Y_2$  will be graphed as a thick line. This will help you distinguish the two graphs (and make sure that they are equal).

```

Plot1 Plot2 Plot3
\Y1=(1+cos(X))(1
-cos(X))
\Y2=(sin(X))²
\Y3=
\Y4=
\Y5=
\Y6=

```

Press **ZOOM** and select **ZTrig** to set the window size.

The two graphs coincide, so the two sides of the equation are equal, as we proved. Note that the calculator is only **verifying** what we have proven. The graph in no way constitutes a proof.

## Problems 2-5

For problems 2 through 5, prove the equation given and then verify it graphically. For  $\cot x$ , type  $(\frac{1}{\tan x})$ . For  $\sec x$ , type  $(\frac{1}{\cos x})$ .

2.  $\sin x \cdot \cot x \cdot \sec x = 1$

3.  $\frac{\sec^2 x - 1}{\sec^2 x} = \sin^2 x$

4.  $\tan x + \cot x = \sec x(\csc x)$

5.  $\frac{\sin^2 x - 49}{\sin^2 x + 14 \sin x + 49} = \frac{\sin x - 7}{\sin x + 7}$

## Solutions

### Problem 2

$$\begin{aligned} \sin(x) \cdot \cot(x) \cdot \sec(x) &= \sin(x) \cdot \frac{\cos(x)}{\sin(x)} \cdot \frac{1}{\cos(x)} \\ &= 1 \end{aligned}$$

### Problem 3

#### 4.2. TRIG PROOFS



$$\begin{aligned}
 \frac{\sec^2(x) - 1}{\sec^2(x)} &= \frac{[1 + \tan^2 x] - 1}{\sec^2(x)} \\
 &= \frac{\tan^2(x)}{\sec^2(x)} \\
 &= \frac{\left(\frac{\sin(x)}{\cos(x)}\right)^2}{\frac{1}{\cos^2(x)}} \\
 &= \sin^2(x)
 \end{aligned}$$

**Problem 4**

$$\begin{aligned}
 \tan(x) + \cot(x) &= \frac{\sin(x)}{\cos(x)} + \frac{\cos(x)}{\sin(x)} \\
 &= \frac{\sin^2(x)}{\cos(x) \cdot \sin(x)} + \frac{\cos^2(x)}{\cos(x) \cdot \sin(x)} \\
 &= \frac{\sin^2(x) + \cos^2(x)}{\cos(x) \cdot \sin(x)} \\
 &= \frac{1}{\cos(x) \cdot \sin(x)} \\
 &= \sec(x) \cdot \csc(x)
 \end{aligned}$$

**Problem 5**

$$\begin{aligned}
 \frac{\sin^2(x) - 49}{\sin^2(x) + 14\sin(x) + 49} &= \frac{(\sin(x) - 7) \cdot (\sin(x) + 7)}{(\sin(x) + 7) \cdot (\sin(x) + 7)} \\
 &= \frac{\sin(x) - 7}{\sin(x) + 7}
 \end{aligned}$$

## 4.3 What's the Difference?

*This activity is intended to supplement Trigonometry, Chapter 3, Lesson 5.*

**ID:** 12556

**Time Required:** 15–20 minutes

### Activity Overview

*Students explore the angle difference formula for cosine. Students will apply the formula and compare their results to interactive unit circle diagrams that assist the student in visualizing the problems involved. The derivations of the angle difference and sum formulas for cosine are optional extensions included with this activity.*

**Topic:** Cosine Difference Identity

- Angle Sum and Difference Identity Derivation (optional extensions)
- Unit Circle
- Sine and Cosine values
- Verification of Equivalence by Graphing

**Teacher Preparation and Notes**

- *If the extensions are used during class, the activity will take approximately 30–45 minutes to complete.*
- *It will be necessary to load the UNITCIRC Cabri jr. files to the graphing calculators before beginning this activity.*
- *The first and second problems engage students in an exploration of the difference formula for cosine. Problem 1 is devoted to unit circle review and developing an understanding of the angle difference diagram included in the activity.*
- *Problem 2 engages students in the application of the angle difference formula for cosine. Students find the cosine for angles such as  $15^\circ$  from well-known angles on the unit circle, such as  $45^\circ$  and  $60^\circ$ .*
- *The extensions of this activity have students derive the angle sum and difference formulas for cosine.*
- *To download Cabri Jr, go to <http://www.education.ti.com/calculators/downloads/US/Software/Detail?id=258#>.*
- *To download the calculator file, go to <http://www.education.ti.com/calculators/downloads/US/Activities/Detail?id=12556> and select UNITCIRC.*

**Associated Materials**

- Student Worksheet: What's the Difference? <http://www.ck12.org/flexr/chapter/9701>, scroll down to the third activity.
- Cabri Jr. Application
- UNITCIRC.8xv

### Problem 1 – Exploring the Angle Difference Formula for Cosine

One of the great things about using the unit circle is that the  $y$ - coordinate is always the sine of the angle and the  $x$ - coordinate is always the cosine of the angle.

4.3. WHAT'S THE DIFFERENCE?

The *Cabri Jr.* file titled **UNITCIRC** is useful in exploring this topic and then for exploring the angle difference formula for cosine.

Students will press **ALPHA** to grab points *A* and *B* and move them around the circle. Remind them that their results are limited to the resolution of the sketch. In other words, just because the *x*- and *y*- coordinates are only displayed to the tenth, they really go on for a long while.

In this part of the activity, the students answer a variety of questions related to the angle difference diagram.

## Problem 2 – Applying the Angle Difference Formula

Students find cosine values for angle measures such as  $15^\circ$  and  $105^\circ$ , which take advantage of angles with well known values (for many students) of sine and cosine.

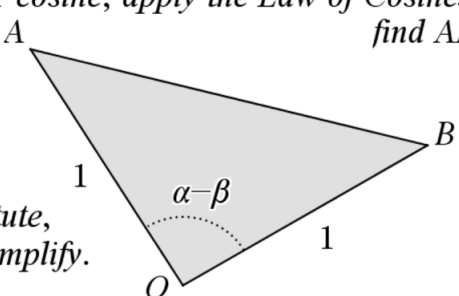
Again, remind the students about the *Cabri Jr.* application only measuring the nearest tenth to account for any discrepancies between their calculated results and their graphical results. Also, be sure to have the students set their graphing calculators to **Degree** mode.

## Extension #1– Deriving the Angle Difference Formula for Cosine

Students use the Law of Cosines to derive the angle difference formula for cosine. A unit circle representation is provided to help students visualize the problem and to provide the necessary background to set up the derivation.

Guidance regarding how to begin the derivation will be helpful to students. Show students how to set up their work for the first derivation and students should be able to follow that example for the remaining derivations in this activity.

First, to find the angle difference formula for cosine, apply the Law of Cosines to find *AB*.

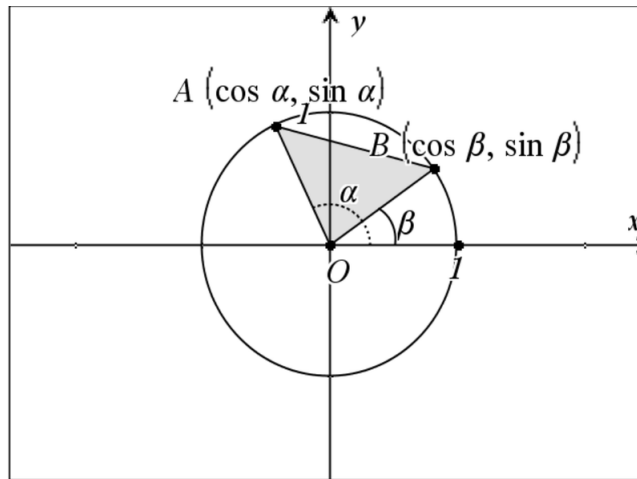


Substitute, then simplify.

$$(AB)^2 = (AO)^2 + (BO)^2 - 2(AO)(BO)\cos(\alpha - \beta)$$

## Extension #2 – Angle Sum Formula for Cosine

It is important to take some time here to discuss with students why  $\cos(-y) = \cos(y)$  and  $\sin(-y) = -\sin(y)$  and explain these two situations involved in this formula derivation.



## Solutions

1. cosine
2. sine
3. 0.98
4. -0.17
5. 0.34
6. 0.94
7. 0.98
8. 0.17
9. answers may vary—relationship is not easy to quickly obtain from the interactive graph page
10. 0.97
11. 0.26
12. -0.26
- 13.

$$\begin{aligned}
 (AB)^2 &= AO^2 + BO^2 - 2 \cdot AO \cdot BO \cdot \cos(AOB) \\
 &= 1 + 1 - 2 \cos(\alpha - \beta) \\
 &= 2 - 2 \cos(\alpha - \beta)
 \end{aligned}$$

14.

$$\begin{aligned}
 (AB)^2 &= (\cos(\alpha) - \cos(\beta))^2 + (\sin(\alpha) - \sin(\beta))^2 \\
 &= \cos^2(\alpha) - 2 \cos(\alpha) \cos(\beta) + \cos^2(\beta) + \sin^2(\alpha) - 2 \sin(\alpha) \sin(\beta) + \sin^2(\beta) \\
 &= 1 - 2 \cos(\alpha) \cos(\beta) + 1 - 2 \sin(\alpha) \sin(\beta) \\
 &= 2 - 2 \cos(\alpha) \cos(\beta) - 2 \sin(\alpha) \sin(\beta)
 \end{aligned}$$

15.  $\cos(\alpha - \beta) = \cos(\alpha) \cos(\beta) + \sin(\alpha) \sin(\beta)$
16.  $\cos(\alpha + \beta) = \cos(\alpha) \cos(\beta) - \sin(\alpha) \sin(\beta)$

## CHAPTER

**5****TE Inverse Functions and  
Trigonometric Equations - TI****CHAPTER OUTLINE**

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**5.1 WHAT'S YOUR INVERSE?**

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## 5.1 What's your Inverse?

*This activity is intended to supplement Trigonometry, Chapter 4, Lesson 3.*

**Time Required: 30 minutes**

### Activity Overview

This activity is intended to be teacher-led, where students will use their graphing calculators to graph the six inverse trig functions. The inverse reciprocal properties will be derived as a class.

#### Topics Covered

- Graphing inverses
- Finding the inverse of a function

#### Teacher Preparation and Notes

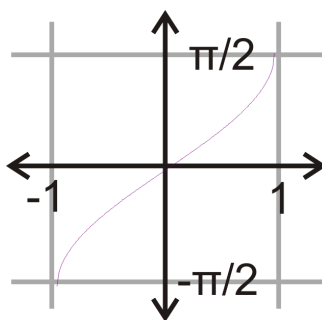
- Make sure students have cleared  $Y =$  menu before starting.
- Students will need to know how to find the inverse of a function algebraically. It might be helpful to have a warm-up covering this topic as a quick review.
- Go over the definition of a restricted domain and make sure students understand why they are necessary to find the inverse of trigonometric functions.
- Make sure students' calculators are in **Radians**.

#### Associated Materials

- Student Worksheet: What's your Inverse <http://www.ck12.org/flexr/chapter/9702>

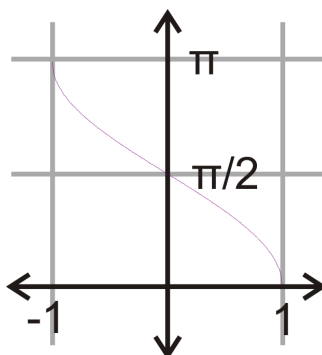
### Problem 1

For the domain and range of  $y = \sin^{-1} x$ , review what the domain and range are of  $y = \sin x$ . The domain is all real numbers and the range is between -1 and 1. For the inverse, they will be switched, so the domain is between -1 and 1 and the range should be all real numbers. However, from looking at the graph, we know this is not true. Hence, there is a restricted domain on  $y = \sin x$  so that it can have an inverse (recall that  $y = \sin x$  is periodic). The domain is restricted to  $[-\frac{\pi}{2}, \frac{\pi}{2}]$ , and the range of  $y = \sin^{-1} x$  would be between  $-\frac{\pi}{2}$  and  $\frac{\pi}{2}$ .



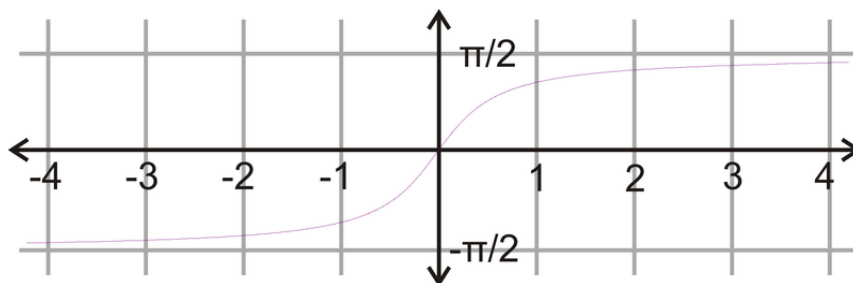
## Problem 2

Again, review the domain and range of  $y = \cos x$ , which are all real numbers for the domain and between -1 and 1 for the range. By looking at the graph, domain of the inverse is also between -1 and 1 and the range is between 0 and  $\pi$ .



## Problem 3

The domain of  $y = \tan x$  is all real numbers, except for every odd multiple of  $\frac{\pi}{2}$  ( $x \neq \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \dots$ ). The range is all real numbers. So, the range of  $y = \tan^{-1} x$  is between two of these asymptotes ( $-\frac{\pi}{2}, \frac{\pi}{2}$ ) and there are horizontal asymptotes. The domain is all real numbers.



**For secant, cosecant and cotangent guide students through how to derive the equation that is needed to plug into the calculator.**

## Problem 4

Prove  $\cos^{-1} x = \sec^{-1} \left(\frac{1}{x}\right)$ . Walk students through these steps.

$$y = \cos^{-1} x$$

$$\cos y = x$$

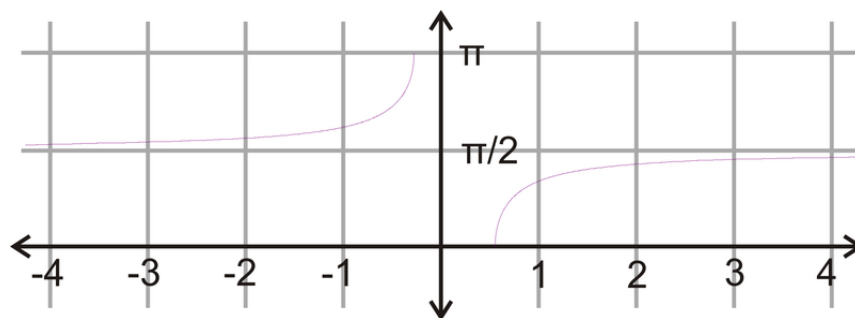
$$\frac{1}{\cos y} = \frac{1}{x}$$

$$\sec y = \frac{1}{x}$$

$$y = \sec^{-1} \left( \frac{1}{x} \right)$$

This means that  $\cos^{-1} x = \sec^{-1} \left( \frac{1}{x} \right)$  and  $\sec^{-1} x = \cos^{-1} \left( \frac{1}{x} \right)$ .

In Y1, students should input  $\cos^{-1} \left( \frac{1}{x} \right)$  in order to graph  $y = \sec^{-1} x$ .



## Problem 5

Prove  $\sin^{-1} x = \csc^{-1} \left( \frac{1}{x} \right)$ . Walk students through these steps.

$$y = \sin^{-1} x$$

$$\sin y = x$$

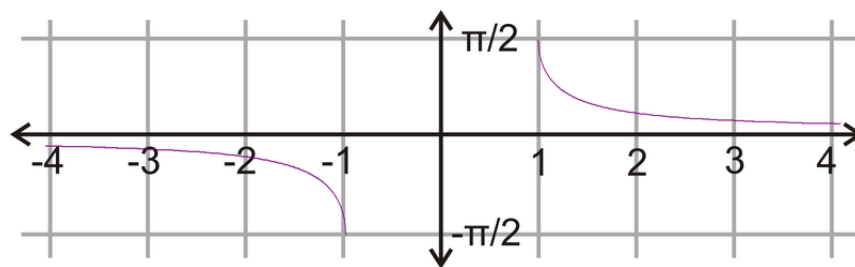
$$\frac{1}{\sin y} = \frac{1}{x}$$

$$\csc y = \frac{1}{x}$$

$$y = \csc^{-1} \left( \frac{1}{x} \right)$$

This means that  $\sin^{-1} x = \csc^{-1} \left( \frac{1}{x} \right)$  and  $\csc^{-1} x = \sin^{-1} \left( \frac{1}{x} \right)$ .

In Y1, students should input  $\sin^{-1} \left( \frac{1}{x} \right)$  in order to graph  $y = \csc^{-1} x$ .





## Problem 6

Tangent and cotangent have a slightly different relationship. Recall that the graph of cotangent differs from tangent by a reflection over the  $y$ -axis and a shift of  $\frac{\pi}{2}$ . As an equation, the relationship would be  $\cot x = -\tan\left(x - \frac{\pi}{2}\right)$ . Students will need to take the inverse of  $y = -\tan\left(x - \frac{\pi}{2}\right)$  to find how to graph  $y = \cot^{-1} x$  in their calculators.

$$y = -\tan\left(x - \frac{\pi}{2}\right)$$

$$x = -\tan\left(y - \frac{\pi}{2}\right)$$

$$-x = \tan\left(y - \frac{\pi}{2}\right)$$

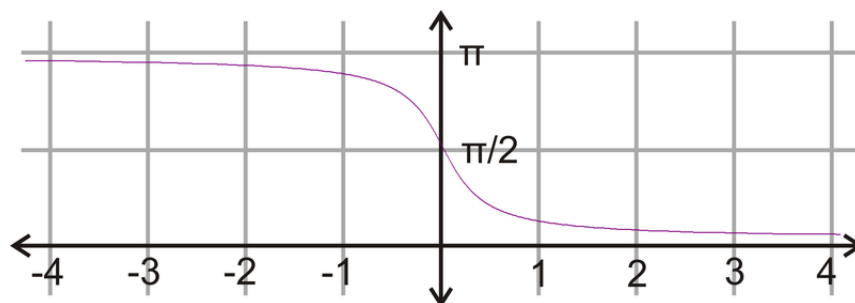
$$\tan^{-1}(-x) = y - \frac{\pi}{2}$$

$$\frac{\pi}{2} + \tan^{-1}(-x) = y$$

$$\frac{\pi}{2} - \tan^{-1} x = y$$

This means that  $\cot^{-1} x = \frac{\pi}{2} - \tan^{-1} x$  and  $\tan^{-1} x = \frac{\pi}{2} - \cot^{-1} x$ .

Because tangent is an odd function, or  $\tan(-x) = -\tan x$ , then its inverse is also odd. In Y1 students should input  $\frac{\pi}{2} - \tan^{-1} x$  in order to graph  $y = \cot^{-1} x$



## CHAPTER

**6****TE Triangles and Vectors - TI****CHAPTER OUTLINE**

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**6.1 ANALYZING HERON'S FORMULA**

**6.2 SINE. IT'S THE LAW**

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## 6.1 Analyzing Heron's Formula

*This activity is intended to supplement Trigonometry, Chapter 5, Lesson 2.*

**Time Required: 20 minutes**

### Activity Overview

In this activity, students will use their graphing calculators to determine the relationship between Heron's Formula and the basic area formula.

#### Topics Covered

- Finding the area of a triangle
- Points of intersection
- Interpreting a graph

#### Teacher Preparation and Notes

- Make sure students have cleared  $Y =$  menu before starting.
- You may need to remind students how to **TRACE** and find points of intersection.

#### Associated Materials

- Student Worksheet: Analyzing Heron's Formula <http://www.ck12.org/flexr/chapter/9703>

### Problem 1: The 3, 4, 5 right triangle

- Students should know this is a right triangle, with hypotenuse 5. Make sure that they have that the legs are 3 and 4.
- The area of this triangle is 6.
- With Heron's Formula,  $A = \sqrt{s(s-a)(s-b)(s-c)}$ , and  $Y1 = \sqrt{x(x-3)(x-4)(x-5)}$  students might get confused with the parenthesis. Make sure all students change their window to the dimensions to the right before graphing.

WINDOW

$$Xmin = -1$$

$$Xmax = 8$$

$$Xscl = 1$$

$$Ymin = -1$$

$$Ymax = 10$$

$$Yscl = 1$$

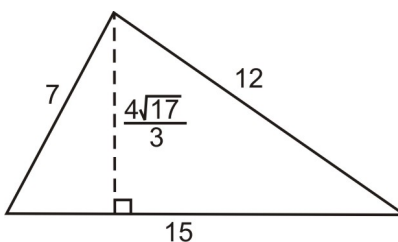
$$Xres = 1$$

- The graph is to the right. Have students analyze the domain and range and why there are blank spaces in the graph. The domain is  $(-\infty, 0)$ ,  $[3, 4]$ , and  $[5, \infty)$  and the range is all real numbers greater than zero. If you have students zoom in further, they will see that there are no  $x$ -intercepts and one  $y$ -intercept at  $(0, 0)$ .



- $Y = 6$  represents the area of this triangle. The horizontal line crosses the graph at  $(-0.435, 6)$  and  $(6, 6)$ . The first point, however, is invalid because  $x$  cannot be negative here. Ask students why. Explain that  $x$  is actually  $s$  and that  $s$  cannot be negative, because by definition it is  $\frac{1}{2}(a + b + c)$ .
- Specifically, the point  $(6, 6)$  represents  $(s, \text{area})$ . So, from this system of equations, we have determined what  $s$  is such that we have the correct area, in this case, also 6. If we can find the area in more than one manner, this will always work as a way to solve for  $s$ .

## Problem 2



For this triangle, the area is,  $A = \frac{1}{2}(15) \left( \frac{4\sqrt{17}}{3} \right) = 5 \cdot 2 \sqrt{17} = 10 \sqrt{17} \approx 41$

Here, students would graph  $y = \sqrt{x(x-7)(x-12)(x-15)}$  and  $y = 10 \sqrt{17}$  to determine what  $x$ , or  $s$ , is. The graph of the two functions looks like:

Again,  $s$  cannot be negative, so we eliminate the negative point of intersection. Therefore the answer is  $(17, 10 \sqrt{17})$ .

### 6.1. ANALYZING HERON'S FORMULA



## 6.2 Sine. It's the Law

*This activity is intended to supplement Trigonometry, Chapter 5, Lesson 3.*

**ID: 11851**

**Time Required: 15 minutes**

### Activity Overview

*In this activity, students will explore the Law of Sines. Students will derive the formula through exploration and solve some application problems. As an extension, students will prove the Law of Sines through guided questions.*

**Topic: Right Triangles #38; Trigonometric Ratios**

- *Law of Sines*

### Teacher Preparation and Notes

- *This activity is geared towards geometry students and only the simplest case of the Law of Sines is explored. The ambiguous case is not explored in this activity.*
- *This activity was written to be explored with the Cabri Jr. app on the TI-84.*
- *To download Cabri Jr, go to <http://www.education.ti.com/calculators/downloads/US/Software/Detail?id=258#>.*
- *To download the calculator file, go to <http://www.education.ti.com/calculators/downloads/US/Activities/Detail?id=11851> and select LAW1.*

### Associated Materials

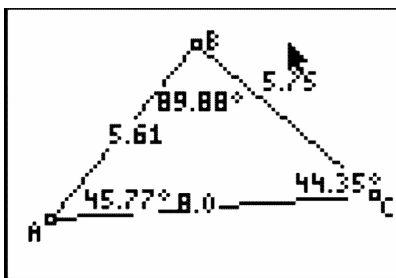
- Student Worksheet: Sine. It's the Law. <http://www.ck12.org/flexr/chapter/9703>, scroll down to the second activity.
- Cabri Jr. Application
- LAW1.8xv

### Problem 1 – Law of Sines

Students will begin this activity by looking at a triangle and investigating the ratio of the sine of an angle to the length of the opposite side. In *LAW1.8xv*, students are given triangle  $ABC$  with the measures of angles  $A, B$ , and  $C$ , and the measure of sides  $a, b$ , and  $c$ .

Students will collect data in the tables on their accompanying worksheet and asked what they notice about the last three columns of the table in Question 2. Discuss round off errors with the students.

Students are asked to compare the three columns to discover the Law of Sines.



## Problem 2 – Application of the Law of Sines

In Problem 2, students are asked to apply what they have learned about the Law of Sines.

## Extension – Proof of the Law of Sines

As an extension, students are asked to prove the Pythagorean theorem through guided questions.

## Solutions

1.

**TABLE 6.1: Sample Answers**

Position	$a$	$b$	$c$	$A$	$B$	$C$
1	5.88	8.00	5.12	47.05	93.35	39.60
2	8.97	8.00	4.01	89.99	63.44	26.57
3	8.16	8.00	3.01	81.90	76.71	21.39
4	4.84	8.00	4.00	27.41	130.24	22.35

2.

**TABLE 6.2: Sample Answers**

Position	$\sin(A)$	$\sin(B)$	$\sin(C)$	$\frac{\sin(A)}{a}$	$\frac{\sin(B)}{b}$	$\frac{\sin(C)}{c}$
1	0.73135	0.99829	0.63742	0.12347	0.12478	0.12449
2	1	0.89446	0.44729	0.11148	0.11180	0.11154
3	0.99002	0.97321	0.36471	0.12132	0.12165	0.12116
4	0.46035	0.76334	0.38026	0.09511	0.09541	0.09506

3. They are approximately equal (not exactly equal due to round off errors).

4.  $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

5.  $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

6. Distance from Tower A : 2.45 mi ; Distance from Tower B : 3.99 mi

7. 36.95 ft tall

8. Distance from lighthouse  $A$  : 6.65  $mi$  ; Distance from lighthouse  $B$  : 9.38  $mi$

9.  $\sin(A) = \frac{BD}{c}$

10.  $\sin(C) = \frac{BD}{a}$

11.  $BD$ ;  $c \cdot \sin(A) = BD$  and  $a \cdot \sin(C) = BD$

12.

$$c \cdot \sin(A) = a \cdot \sin(C)$$

$$\sin(A) = \frac{a \cdot \sin(C)}{c}$$

$$\frac{\sin(A)}{a} = \frac{\sin(C)}{c}$$



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**CHAPTER 7**

# TE Polar Equations and Complex Numbers - TI

## CHAPTER OUTLINE

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### 7.1 POLAR NECESSITIES

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## 7.1 Polar Necessities

*This activity is intended to supplement Trigonometry, Chapter 6, Lesson 4.*

**ID: 12558**

**Time Required: 15 minutes**

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### Activity Overview

*Students will explore what is necessary to understand the calculus of polar equations. Students will graphically and algebraically find the slope of the tangent line at a point on a polar graph. Finding the area of a region of a polar curve will be determined using the area formula.*

**Topic: Polar Equations**

- Find the slope of a polar equation at a particular point.
- Find the area of polar equation.

### Teacher Preparation and Notes

- Make sure each students' calculator is in RADIANS (RAD) and POLAR (POL) in the MODE menu.

### Associated Materials

- Student Worksheet: Trigonometric Patterns <http://www.ck12.org/flexr/chapter/9704>, scroll down to the third activity.

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### Plotting Coordinates #38; Exploring Polar Graphs

Students begin the activity by plotting points on a polar graph. This should be a refresher of polar coordinates for most students. Students practice using the calculator to graph a polar equation.

#### Discussion Questions

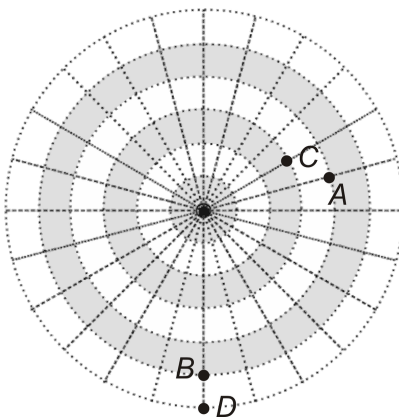
- What do you think it means to have a negative angle, like  $(-\frac{\pi}{3}, 3)$  ?
- What about if  $r$  was negative? For example, move to  $(\frac{\pi}{2}, -6)$  .

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### Solutions

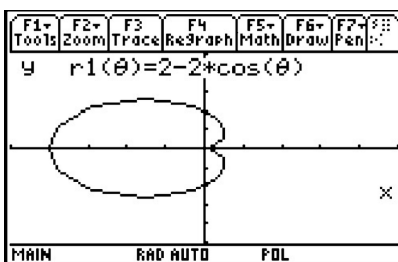
1. See image below.

7.1. POLAR NECESSITIES



2. If  $r(\theta) = \cos(\theta)$ ,  $r\left(\frac{\pi}{3}\right) = 0.5$ .

3. a heart or cardioid



4. A circle is in the form  $r = a$ , where  $a$  is a constant.

A polar rose with even petals is in the form  $r(\theta) = a \cdot \sin(n\theta)$ , where  $n$  is even.

A polar rose with odd petals is in the form  $r(\theta) = a \cdot \sin(n\theta)$ , where  $n$  is odd.

A limaçon with an inner loop comes from  $r(\theta) = b + a \cdot \cos(\theta)$ , where  $b < a$ .

