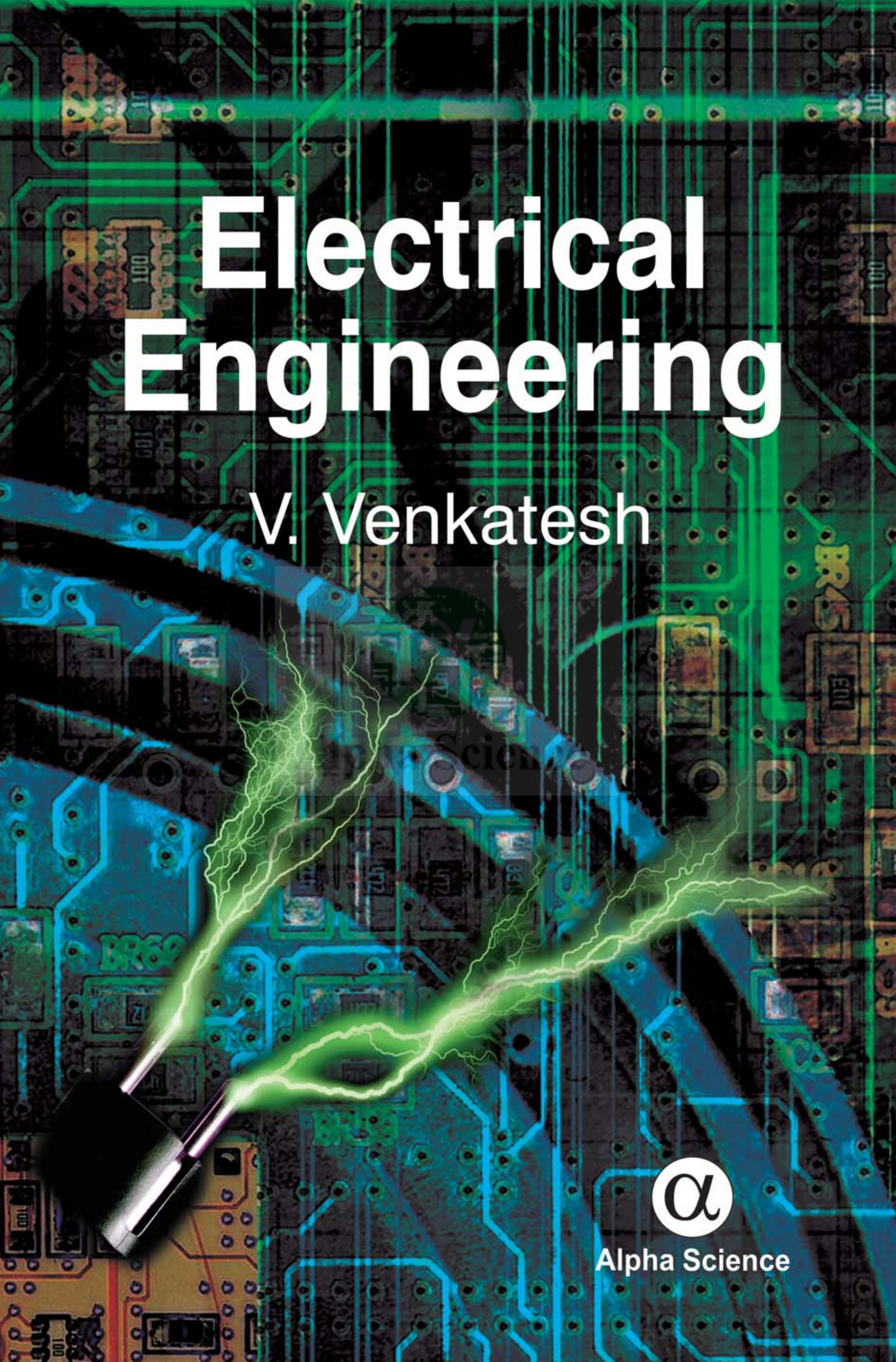

Electrical Engineering

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Alpha Science

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Alpha Science International Ltd.
Oxford, U.K.

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324 pgs. | 234 figs. | 10 tbls.

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www.alphasci.com

ISBN 978-1-84265-858-1
E-ISBN 978-1-78332-044-8

Printed in India

Preface

Electrical Engineering is a semi-professional and professional engineering science that deals with the study and/or application of electricity, electromagnetism and the equipments for measurement, power generation and distribution and also safety mechanisms.

The goal of this book is to provide the reader with a comprehensive introduction to the fundamental aspects of Electrical Engineering, which helps any student to understand the concept.

The most important prerequisite to the material of this book is a solid background in teaching this subject since twenty five years. Familiarity with the subject helped to simplify the complicated issues.

In this book, the approach is made simple, clear and precise. Concepts are clearly explained, derivations are made clear and numerous examples are solved in such a way that the students can understand the proper approach and helps for self learning.

This book includes important points, many solved examples, multiple choice questions, review questions and exercises with answers.

Also, this book is useful to teach and to set the question paper for tests/quizzes/university examination question paper.

I am thankful to Narosa Publishing House Pvt. Ltd., New Delhi for the encouragement in bringing out this book.

Also, I am thankful to Dr. Suresh Kumar D.S., Director, Channabasaveshwara Institute of Technology, Gubbi, Tumkur for his support.

I would like to thank my wife Smt. Kamala V., my son Naganand V. and daughter Deepthi V. for their cooperation.

V. Venkatesh


Alpha Science

Contents

6. Domestic Wiring 6.1

7. DC Machines 7.1

8. Transformers 8.1

9. Synchronous Generators 9.1

10. Three-phase Induction Motors 10.1

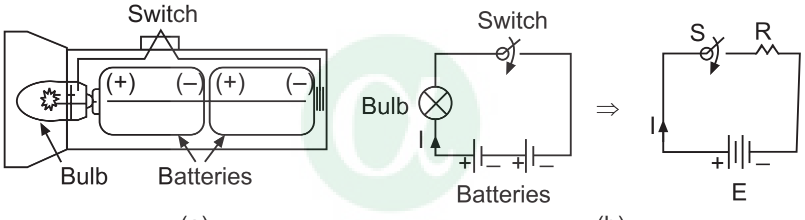
Chapter

1

DC Circuits

1.1 IMPORTANT TERMS

1. **Electric Circuit:** Electric circuit is a closed conducting path. To understand the concept of a circuit, observe the inner parts of an electric torch shown in Fig. 1.1(a).



(a) (b)

Fig. 1.1 (a) Electric torch (b) Circuit diagrams

It consists of two batteries or cells, a bulb and a switch (SPST, single-pole and single-throw). The circuit diagrams are shown in Fig. 1.1(b). If the switch (S) is closed, the circuit will be completed, current (I) flows from the batteries to the lamp and thus the lamp glows.

The circuit element is either **active** element called the **source** (emf), which supplies energy to a circuit or **passive** element called the **load**, which consumes energy. Examples for active elements are batteries and passive elements are resistance, inductance and capacitance.

An electric circuit or network is an interconnection of various electrical elements which forms a closed path for continuous flow of current.

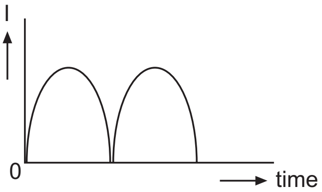


Fig. 1.2 Direct current

A battery provides a steady voltage, so that current flows in one direction and thus called direct current (dc). A circuit, which deals with dc is called dc circuit.

2. **Current:** According to atomic theory, an atom consists of protons (positively charged) in the central nucleus and equal number of revolving electrons (negatively charged) in various orbits. By the application of an external force, electrons of the outermost orbits get removed from the parent atom. The continuous drift or movement of electrons in a particular direction due to an external force causes the flow of current.

   'The continuous flow of electrons in any conductor in a particular direction constitutes electric current (I)'.

   Also, removal of electrons makes the atom positively charged and addition of an electron makes the atom negatively charged. The unit of charge Q is **coulomb (C).**

   'The rate of flow of electric charges through a cross-sectional area of a conductor is called the current'.

   i.e., $$I = \frac{dQ}{dt}$$

   The unit of current is **ampere (A)**. Ampere is that current, which if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section and placed 1 metre apart in vacuum produces between these conductors a force of 2×10^{-7} newton per metre length.

   Also, 1 ampere is that amount of current which flows through a circuit, when 1 coulomb of charge per second is transferred through a point in a circuit.

Important Points

- Current flows from higher potential to lower potential or from positive to negative (flow of current is analogous to flow of water through a pipe line).
- The term current or electric current represents the conventional current, which flows in opposite direction to that of electron current.

3. **Electromotive Force or Voltage:** Electromotive force or EMF (E) is that which tends to produce an electric current in a circuit. EMF is not a force, but it represents the energy. It is a driving influence that causes the current to flow.

   The term voltage (V) means potential difference (pd) between two points in an electrical circuit required to drive the current between them.

   The unit of emf and voltage is **volt (V),** which is defined as the potential difference across a unit resistance causing a unit current.

4. **Resistance:** Resistance (R) is defined as that property of a substance, which opposes the flow of electrons or electric current through it. Its unit is **ohm (Ω)**, which is defined as the resistance between two points of a conductor when a potential difference of 1 volt, applied produces a current of 1 ampere.

   Also, ohm is the resistance of a circuit in which a current of 1 ampere generates heat at the rate of 1 watt.

The value of resistance depends upon the following factors:

(i) The resistance of a conductor varies directly as its length (l).

(ii) The resistance of a conductor varies inversely as its cross sectional area (A).

Therefore, $$R = \rho \frac{l}{A}$$

where ρ is a constant of proportionality called specific resistance or resistivity of the material and its unit is **ohm-metre (Ω - m)**. For popper, $P = 1.72 \times 10^{-8}$.

Effect of Temperature on Resistance

(i) As temperature increases the resistance also increases for pure metals and said to have positive temperature co-efficient. Examples are Silver, Copper and Aluminum. They are called the good conductors as they have low resistance.

(ii) As temperature increases the resistance decreases for insulators and said to have negative temperature co-efficient. Examples are Mica, Porcelain, Carbon, etc. They are called the insulators as they have high resistance.

Important Points

- As current flows through a resistance, it causes a voltage drop across it and its unit is volt (V).
- The reciprocal (or inverse) of resistance is called the conductance (G). The SI unit of conductance is **siemen (S).**
- Resistance is connected in series to limit the current. The resistance connected across the supply is called the load or the load resistance. Example: Filament lamp.
- The resistance (or resistor) may be either a fixed resistance or variable resistance (called the rheostat).
- The practical voltage source may have a small resistance called the internal resistance (r), represented schematically by connecting r in series with the ideal voltage source.

Fixed resistance

Variable resistance

EMF/voltage source and battery

Practical voltage source

Fig. 1.3 Circuit symbols

1.2 OHM'S LAW

Ohm's Law states that the potential difference between the two ends of a conductor is directly proportional to the current flowing through it provided its temperature and other physical parameters remain unchanged.

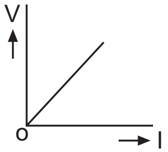


Fig. 1.4 V-I Characteristics

Alternatively, Ohm's law states that the ratio of voltage to current is constant, provided its temperature and other physical parameters remain unchanged.

If V is the potential difference and I is the current,

then, $V \propto I$

or $\frac{V}{I} = R$

or $V = IR$

where R is the constant of proportionality called the resistance of the conductor. The unit of resistance is ohm (Ω).

The relationship between voltage and current, that is, V-I Characteristics (as shown in Fig. 1.4) is linear and also bilateral (i.e. irrespective of direction of flow of current). (Note: A resistance is a linear and bilateral circuit element)

Limitations of Ohm's Law

- It cannot be applied for nonlinear and semi-conducting devices such as diodes, voltage regulators, etc.
- It cannot be applied for non-metallic conductors such as silicon carbide, semi-conductors.
- It cannot be applied for electrolytes.
- It cannot be applied for discharge lamps.

1.3 RESISTANCES CONNECTED IN SERIES

Let R_1, R_2 and R_3 be the resistances connected in series (that is, resistances are connected end to end) and V is the supply voltage.

In series connection, the current flowing through each resistance is same and the voltage drop across each resistance is different.

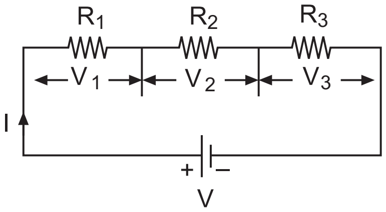


Fig. 1.5 Resistances in series

Applying Ohm's Law,

Voltage drop across resistance R_1, $V_1 = IR_1$

Voltage drop across resistance R_2, $V_2 = IR_2$

Voltage drop across resistance R_3, $V_3 = IR_3$

The sum of the voltage drops is equal to the supply voltage

i.e.,
$$\begin{aligned} V &= V_1 + V_2 + V_3 \\ &= IR_1 + IR_2 + IR_3 \\ &= I(R_1 + R_2 + R_3) \end{aligned}$$

or
$$\frac{V}{I} = R_1 + R_2 + R_3$$

or
$$R = R_1 + R_2 + R_3$$

Thus, the total or effective resistance (R) of a series circuit is equal to the sum of all the individual resistances, which are connected in series.

Voltage Divider

Let R_1 and R_2 be the two resistances connected in series and the applied voltage is V.

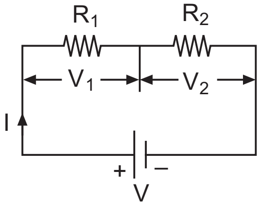


Fig. 1.6 Resistances in series

∴ Current,
$$I = \frac{V}{R_T} = \frac{V}{R_1 + R_2}$$

As current I flows through R_1 and R_2, it causes voltage drop V_1 and V_2 respectively, given by

$$V_1 = IR_1 = \frac{V}{R_1 + R_2} R_1 = V \frac{R_1}{R_1 + R_2}$$

Similarly, $$V_2 = IR_2 = V\frac{R_2}{R_1 + R_2}$$

Thus, with resistances in series the applied voltage divides among the resistances in the direct ratio of their resistances.

1.4 RESISTANCES CONNECTED IN PARALLEL

Let R_1, R_2 and R_3 be the resistances connected in parallel (that is similar ends are connected together) and V is the supply voltage.

In parallel connection, the current flowing through each resistance is different and the voltage drop across each resistance is same and equal to supply voltage.

I_1 = Current through resistance R_1

I_2 = Current through resistance R_2

I_3 = Current through resistance R_3

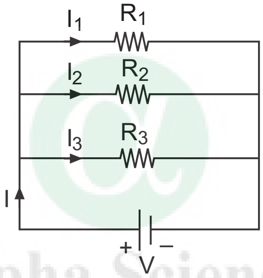


Fig. 1.7 Resistances in parallel

The sum of the currents is equal to the total current I

i.e., $$I = I_1 + I_2 + I_3$$

$$= \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$= V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

or $$\frac{I}{V} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

or $$\frac{1}{R} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

Thus, the reciprocal of effective resistance of a parallel circuit is equal to the sum of the reciprocal of individual resistances which are connected in parallel.

Important Point

For *n* number of identical resistances, each of resistance R, which are connected in parallel, the equivalent resistance of the parallel combination is R/*n*.

Current Divider

Let R_1 and R_2 be the two resistances connected in parallel across supply voltage V. Current I_1 flows through R_1 and current I_2 flows through R_2. Total current is I.

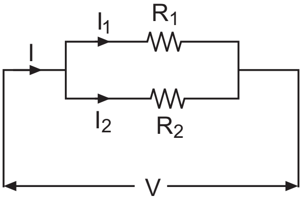


Fig. 1.8 Two resistances in parallel

As R_1 and R_2 are connected in parallel, voltage drop across each resistance is same and equals to supply voltage,

i.e., $$V = I_1 R_1 = I_2 R_2$$

Considering, $$V = I_2 R_2$$

or $$I_2 = \frac{V}{R_2} = \frac{I_1 R_1}{R_2}$$

Total current, $$I = I_1 + I_2$$

$$= I_1 + \frac{I_1 R_1}{R_2}$$

$$= \frac{I_1 R_2 + I_1 R_1}{R_2}$$

$$= I_1 \left(\frac{R_2 + R_1}{R_2} \right)$$

or $$I_1 = I \frac{R_2}{R_1 + R_2}$$

Similarly, $$I_2 = I \frac{R_1}{R_1 + R_2}$$

Thus, in parallel circuit current divides in the reverse ratio of resistance.

$\therefore$ Effective resistance, $R = \dfrac{R_1 \; R_2}{R_1 + R_2}$

1.4.1 Concept of Electrical Load

Let two lamps L_1 and L_2 of resistances R_1 and R_2 respectively are connected in parallel across a supply voltage V. Lamps L_1 and L_2 are controlled by switches S_1 and S_2 respectively.

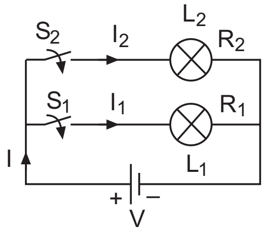


Fig. 1.9 Circuit diagram

If all the switches are opened, the resistance is infinity and thus current will not flow, that is, $I = 0$

If switch S_1 is closed, resistance R_1 is connected and thus a current $I_1(= I)$ flows.

If switch S_2 is closed, resistance R_2 is connected and thus a current I_2 flows through it.

As R_1 is in parallel with R_2, the equivalent resistance decreases and the total current increases, that is, $I = I_1 + I_2$.

Thus, the connection of resistances in parallel across the supply is called the electrical load.

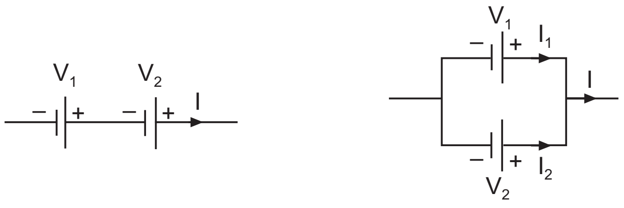
Important Point

If a lamp is connected to a supply and light output is insufficient, then a second lamp is to be added, it must be connected in parallel to the first lamp. This is the correct way of connection of lamps or loads which are to be added. Domestic wiring (all the loads such as lamp, fan, etc. are connected in parallel) is an example.

1.4.2 Concept of Series and Parallel Connection of Voltage Sources

If voltage sources V_1 and V_2 are connected in series as shown in Fig. 1.10(a), then, $V = V_1 + V_2$ and current remains the same. Series connection is used to increase the potential differential or applied voltage.

If voltage sources V_1 and V_2 are connected in parallel as shown in Fig. 1.10(b), then, $I = I_1 + I_2$. Parallel connection is used to increase the total current.



(a) (b)

Fig. 1.10 (a) Batteries in series **(b)** Batteries in parallel

Comparison between Series and Parallel Circuits

- A parallel circuit can run several devices using the full voltage of the supply, varying the current to the need of each device.
- If a break in a parallel circuit occurs, it will continue to work, whereas in a series circuit everything will go off. Also, it is impossible to control the lamp individually in series circuit and thus series circuits are not practical for use in home lighting.
- Resistors connected in parallel yield an inversely additive total resistance, which is usually less than the smallest resistor. Thus effective resistance decreases with inclusion of resistances in parallel and effective resistance increases in case of series connection.
- Batteries connected in parallel do not add their voltages together, but their capacities are added up. Batteries connected in series within a circuit, increases the total voltage applied across the circuit.

1.4.3 Measurement of Current and Voltage

Current (I) is measured by ammeter A, which is connected in series and voltage or voltage drop (V) is measured by voltmeter V, which is connected across the resistance.

$$\therefore \quad \text{Resistance,} \qquad R = \frac{V}{I} \text{ ohm}$$

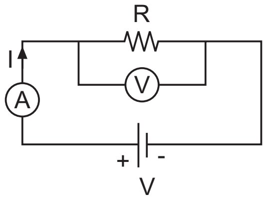


Fig. 1.10 (c) Circuit diagram

1.5 ELECTRICAL POWER

Power is the work done per second. Its unit is **joule/second** or **newton-metre/second** (N-m/s).

i.e., Power, $$P = \frac{\text{work}}{\text{time}}$$

The power consumed in an electrical circuit is 1 watt if potential difference applied across the circuit is 1 volt which causes 1 ampere of current to flow through the circuit.

i.e., $$P = VI$$

According to Ohm's Law, $V = IR$

$$\therefore \quad P = I^2 R$$

or $$P = \frac{V^2}{R}$$

The practical unit of electrical power is **watt (W)** or **kilo watt (kW)**.

Important point: Both for a series and a parallel circuit, powers are additive.

1.6 ELECTRICAL ENERGY

Electrical energy is the total amount of work done. Its unit is **joule (J)**. Electrical energy is the consumption of electrical power of 1 watt for 1 second. Thus,

Electrical Energy = Power × time = $VI\,t = I^2 R \times t$ watt-second or joule.

Practical unit of electrical energy is **kilo watt-hour (kWh)** also, called **BOARD OF TRADE UNIT (BOTU)** or **'unit'**.

1 kWh = 1kW × 1hour = 1000W × (60 minutes × 60 seconds) = 3.6×10^6 watt sec or J.

Solved Examples

Example 1.1 *Find the resistance of an electrical appliance, which takes 5A when connected to a 110V supply. Also, find the power consumed by the resistor.*

Solution: Given: $I = 5A$ and $V = 110$ volt

Resistance, $$R = \frac{V}{I} = \frac{110}{5} = 22 \text{ A}$$

Power consumed, P = VI = 110 × 5 = 550W = **0.55 kW**

Example 1.2 *If an electric heater is connected to a 240V supply, it draws a current of 8A for 12 hours. What is the energy consumed in kWh?*

Solution: Given: $I = 8A, \quad V = 240 \text{ volt}, \quad t = 12 \text{ hours}$

Power, $P = VI = 240 \times 8 = 1920W = 1.92 \text{ kW}$

$\therefore$ Energy, $Pt = 1.92 \times 12 = \mathbf{23\ kWh}$

Example 1.3 *What is the power output of a 240V lamp carrying a rated current of 0.25A? Suppose a similar lamp is connected in parallel with this lamp, what is the supply current required to give the same power output in each lamp?*

Solution:

Given: $V = 240$ volt, Current with single lamp, $I = 0.25$ A

Power output of lamp, $P = VI = 240 \times 0.25 = 60W$

If a second lamp of 60W is connected in parallel to the first one,

Total power output, $P_T = 60W + 60W = 120W$

$$\therefore \text{ Supply current, } \quad I = \frac{P_T}{V} = \frac{120}{240} = \mathbf{0.5\ A}$$

Example 1.4 *An electric device having a resistance of 8 ohm takes a current of 5A. Find the rate of heat dissipated and the heat dissipated in 30 seconds.*

Solution:

Given: $I = 5A$ and $R = 8\Omega$

Rate of heat dissipation, $P = I^2 R = 5^2 \times 8 = \mathbf{200W}$

Heat dissipated in 30 sec., $P \times t = 200 \times 30 = \mathbf{6000J}$

Example 1.5 *A 100 W lamp is used for six hours a day. Find (i) Energy consumed per month and (ii) Cost of energy if each unit costs ₹ 5.*

Solution:

Given: P = 100W, consumption: 6 hours, No. of days–30 in that month, cost/unit = ₹ 1.20

$$\text{Energy consumption/day} = \frac{P}{1000} \times \text{No.of hours/day} = \frac{100}{1000} \times 6 = 0.6 \text{ kWh}$$

$\therefore$ Energy consumption/month, $0.6 \times 30 = \mathbf{18\ kWh}$

Cost of energy/month, ₹ 5 × 18 kWh = **₹ 90/-**

Example 1.6 *A circuit takes a current of 5A at 100V supply. What value of resistance is to be connected in series to limit the current to 10A?*

Solution: Given: Initial current $I_1 = 5$ A, $V = 100$ volt, Final current $I_2 = 2$ A

$$\text{With initial current } I_1, \text{ resistance } R_1 = \frac{V}{I_1} = \frac{100}{5} = 20\Omega$$

With final current I_2, resistance $R_2 = \dfrac{V}{I_2} = \dfrac{100}{2} = 50\Omega$

∴ Value of resistance to be connected in series,

$$R = R_2 - R_1 = 50 - 20 = \mathbf{30\Omega}$$

Example 1.7 *A device consists of 10 batteries each with an emf of 2V and an internal resistance of 0.02Ω. Calculate the rate at which heat is developed in an external circuit of resistance 0.25Ω when the batteries are connected (i) all in series and (ii) all in parallel.*

Solution: Given: Number of batteries: 10, EMF of each battery = 2V, $r = 0.02\Omega$, $R_L = 0.25\Omega$

(i) When all are connected in series:

Equivalent emf, $V = 2 \times 10 = 20$ V

Internal resistance, $r = 0.02 \times 10 = 0.2\Omega$

Total resistance $R_t = r + R_L = 0.2\Omega + 0.25\Omega = 0.45\Omega$

The circuit current, $I = \dfrac{V}{R_t} = \dfrac{20}{0.45} = 44.4A$

∴ Rate of heat developed, $I^2R_L = 44.4^2 \times 0.25 = \mathbf{493A}$

(ii) When all are connected in parallel:

Equivalent emf, $V = 2$

Internal resistance, $r = \dfrac{0.02}{10} = 0.002\Omega$

Total resistance, $R_t = r + R_L = 0.002 + 0.25 = 0.252\Omega$

The circuit current, $I = \dfrac{V}{R_t} = \dfrac{20}{0.252} = 7.94$ A

∴ Rate of heat developed, $I^2R_L = 7.94^2 \times 0.25 = \mathbf{15.76A}$

Example 1.8 *Two resistances, $R_1 = 2500\Omega$ and $R_2 = 4000\Omega$ are connected in series and a voltage of 100V is applied. The voltage drop across R_1 and R_2 are measured successively by a voltmeter having a resistance of 50,000Ω. Find the sum of two readings.*

Solution:

Given: $R_1 = 2500\Omega$, $R_2 = 4000\Omega$, V = 100 volt and resistance voltmeter $R_v = 50{,}000\Omega$ Circuit diagram is drawn.

Reading of voltmeter V_1:

$$R_P = R_1 \parallel R_V = \frac{R_1\, R_V}{R_1 + R_V} = \frac{2500 \times 50000}{2500 + 50000} = 2380.95\Omega$$

$$V_1 = V\frac{R_P}{R_P + R_2} = 100 \times \frac{2380.95}{2380.95 + 4000} = \mathbf{37.31V}$$

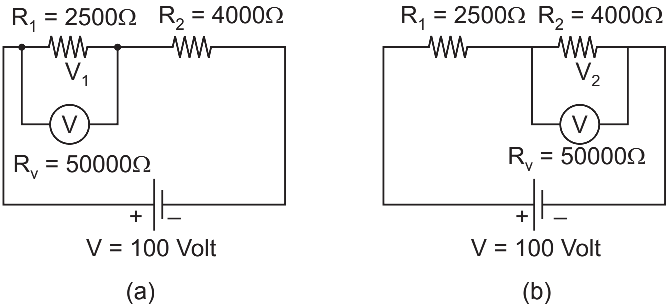


Fig. 1.11 Circuit diagrams

Reading of voltmeter V_2:

$$R_P = R_2 \| R_V = \frac{R_2\ R_V}{R_2 + R_V} = \frac{4000 \times 50000}{4000 + 50000} = 3703.7\Omega$$

$$V_2 = V\frac{R_P}{R_P + R_2} = 100 \times \frac{3703.7}{3703.7 + 2500} = \mathbf{59.7V}$$

Sum of two readings = $V_1 + V_2 = 37.31 + 59.7 =$ **97V**

Example 1.9 *The insulation resistance of a single core cable is connected in series with a voltmeter across a 480V supply, the voltmeter reads 5V. When connected in series with 50000Ω resistance across 240V supply, voltmeter reads 90V. Determine value of resistance.*

Solution:

Given: External resistance $R_e = 50000\Omega$

Let, resistance of voltmeter = R_v and Insulation resistance = R_i

Case (i): Supply voltage, $V_1 = 480V$, Reading of voltmeter $V_{r1} = 5V$

Case (ii): Supply voltage, $V_2 = 240V$, Reading of voltmeter $V_{r2} = 90V$

With resistances in series the applied voltage divides among the resistances in the direct ratio of their resistances.

From Case(i), $$\frac{R_i}{R_v} = \frac{V_1 - V_{r1}}{V_{r1}} = \frac{480 - 5}{5} = 95$$

or $$R_i = R_v \times 95$$

To find R_v:

From Case (ii), $$\frac{R_v}{R_e} = \frac{V_{r2}}{V_2 - V_{r2}} = \frac{90}{240 - 90} = 0.6$$

or $$R_v = R_e \times 0.6 = 50000 \times 0.6 = 30000\Omega = 0.3\ M\Omega$$

∴ $$R_i = 0.3\ M\Omega \times 95 = \mathbf{2.85\ M\Omega}$$

Example 1.10 *Two resistances are connected in parallel and a voltage of 200V is applied to the terminals. The total current taken is 25A and power dissipated in one of the resistors is 1500W. What is the resistance of each resistance?*

Solution:

Given: $V = 200$ volt, $I = 25A$

Let power dissipate in resistance R_1 be $P_1 = 1500W$

Current through resistance R_1, $I_1 = \frac{P_1}{V} = \frac{1500}{200} = 7.5A$

$\therefore$ Resistance, $R_1 = \frac{200}{7.5} = \mathbf{26.67\Omega}$

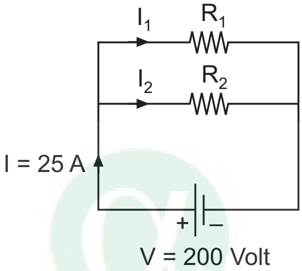


Fig. 1.12 Circuit diagram

Current through resistance R_2, $I_2 = I - I_1 = 25 - 7.5 = 17.5A$

$\therefore$ Resistance, $R_2 = \frac{200}{17.5} = \mathbf{11.43\Omega}$

Example 1.11 *Two resistances R_1 and R_2 give combined resistance of 25Ω when connected in series and 1Ω when connected in parallel. Find R_1 and R_2.*

Solution:

Given: $R_1 + R_2 = 25\Omega$

or $R_1 = (25 - R_2)$

$$R_1 \parallel R_2 = \frac{R_1 R_2}{R_1 + R_2} = 6\Omega$$

Substituting for R_1 and $(R_1 + R_2)$ in above equation

$$6 = \frac{(25 - R_2)R_2}{25}$$

Cross multiplying, $150 = 25R_2 - R_2^2$

or $$150 - 25R_2 + R_2^2 = 0$$

or $$R_2^2 - 25R_2 + 150 = 0$$

$\therefore$ $$R_2 = \frac{-(-25) \pm \sqrt{(-25)^2 - 4 \times 1 \times 150}}{2 \times 1}$$

$$= \frac{25 \pm 5}{2} = 15 \text{ or } 12$$

$\therefore$ $$\mathbf{R_2 = 15\Omega \text{ and } R_1 = 12\Omega}$$

Note: The solution for the equation $ax^2 + bx + c = 0$ is $x = \dfrac{-b \pm \sqrt{(b)^2 - 4ac}}{2a}$

Example 1.12 *A voltage of 200V is applied to a tapped resistor of 500Ω. Determine the resistance between two tapping points connected to a circuit to get 0.1A at 25V. Also, find the total power consumed.*

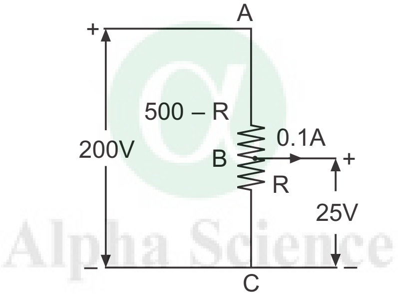


Fig. 1.13 Circuit diagram

Solution:

Based on the data, circuit diagram is drawn.

$$\text{Current in AB} = 0.1 + \frac{25}{R} = \frac{200 - 25}{500 - R}$$

or $$0.1 + \frac{25}{R} = \frac{175}{500 - R}$$

or $(500 - R)(0.1R + 25) = 175R$

or $0.1R^2 + 150R - 12500 = 0$

$\therefore$ $$R = \frac{-150 \pm \sqrt{(150)^2 - [4 \times 0.1 \times (-12500)]}}{2 \times 0.1}$$

Taking +ve value, $\mathbf{R = 79.15\Omega}$

$$\text{Current in AB} = 0.1 + \frac{25}{R} = 0.1 + \frac{25}{79.15} = 0.42\text{A}$$

$\therefore$ Total power consumed, $P = VI = 200 \times 0.42 =$ **84W**

Example 1.13 *A resistance of 5Ω is connected in series with a parallel combination of 2Ω and 10Ω. The total combination is connected across 200V supply. Find (i) the effective resistance and (ii) total current in the circuit (iii) Total power dissipated in the circuit.*

Solution:

Given: $R_1 = 5\Omega$, $R_2 = 2\Omega$, $R_3 = 10\Omega$, $V = 200$ volt. Circuit diagram is drawn.

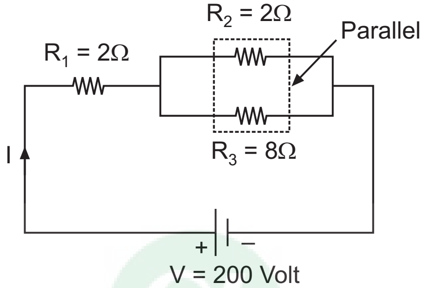


Fig. 1.14 Circuit diagram

(i) Effective resistance, $R = R_1 + \dfrac{R_2 R_3}{R_2 + R_3}$

$$= 5 + \frac{2 \times 10}{2 + 10} = \mathbf{6.67\Omega}$$

(ii) Total current in the circuit, $I = \dfrac{V}{R} = \dfrac{200}{6.67} =$ **30A**

(iii) Total power dissipated in the circuit, $P = I^2R_T = 30^2 \times 6.67 =$ **6003W**

Example 1.14 *A resistance R is connected in series with a parallel circuit comprising two resistances of 12Ω and 8Ω respectively. The total power dissipated in the circuit is 70 W when the applied voltage is 20V. Calculate R.*

Solution:

Given: P = 70 W and V = 20 volt

As per the data given, the circuit diagram is drawn [Fig. 1.15 (a)]

$$12\Omega \parallel 8\Omega = \frac{12 \times 8}{12 + 8} = 4.8\Omega$$

Total resistance, $R_T = 4.8 + R$

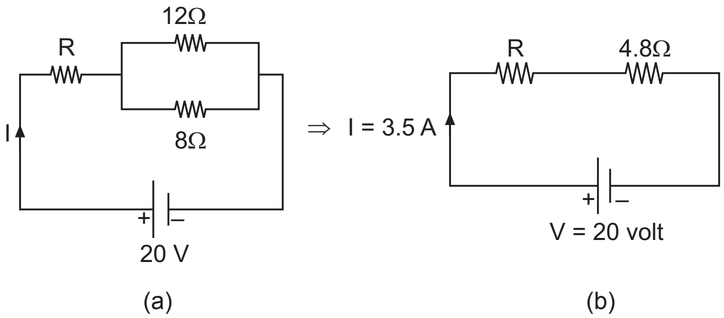


Fig. 1.15 Circuit diagrams

Applying Ohm's Law, $R_T = \frac{V}{I}$

where $I = \frac{P}{V} = \frac{70}{20} = 3.5\text{ A}$

Then $R_T = \frac{20}{3.5} = 5.7\ \Omega$

$\therefore$ $5.7 = 4.8 + R$

or $R = \mathbf{0.9\Omega}$

Example 1.15 *In the circuit shown Fig. 1.16(a) find the value of resistance R when the power dissipated by 12Ω resistance is 36W.*

Solution:

$$12\Omega \parallel 18\Omega \parallel 36\Omega = \frac{12 \times 18 \times 36}{(12 \times 18) + (18 \times 36) + (12 \times 36)} = 6\ \Omega$$

Total resistance, $R_T = 6 + R$

Applying Ohm's Law, $R_T = \frac{V}{I}$

To find I:

For resistors in parallel,

Let the voltage drop across 12Ω resistor = V_{AB}

Since 12Ω, 18Ω and 36Ω resistor are in parallel,

Voltage drop across each of them = $V_{AB.}$

Given that power dissipated in 12Ω resistor = 36W

i.e., $36 = \frac{V_{AB}^2}{12} = \left(\text{because } P = \frac{V^2}{R}\right)$

or $V_{AB}^2 = 36 \times 12 = 432$

or $V_{AB} = \sqrt{432} = 20.78\text{ volt}$

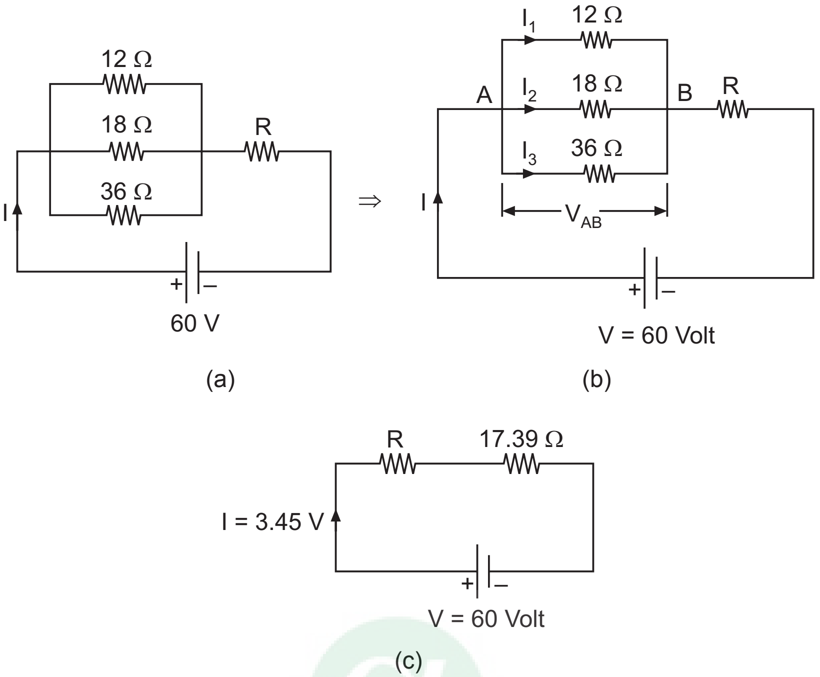


Fig. 1.16 Circuit diagrams

Applying Ohm's Law,

Current flowing through 12Ω resistor, $I_1 = \dfrac{V_{AB}}{12} = \dfrac{20.78}{12} = 1.73\ \text{A}$

Current flowing through 18Ω resistor, $I_2 = \dfrac{V_{AB}}{18} = \dfrac{20.78}{18} = 1.15\ \text{A}$

Current flowing through 36Ω resistor, $I_3 = \dfrac{V_{AB}}{36} = \dfrac{20.78}{36} = 0.57\text{A}$

Total current,

$$I = I_1 + I_2 + I_3$$
$$= 1.73 + 1.15 + 0.57 = 3.45\ \text{A}$$

Applying Ohm's Law,

$$R_T = \frac{V}{I} = \frac{60}{3.45} = 17.39\ \Omega$$

$\therefore$ $R_T = 6 + R$

or $17.39 = 6 + R$

or $\mathbf{R = 11.39\Omega}$

Example 1.16 *A resistance of 10Ω is connected in series with two resistances each of 15Ω arranged in parallel. What resistance should be shunted across this parallel combination so that the total current taken shall be 1.5A with 20V applied voltage?*

Solution:

As two 15Ω resistances are connected in parallel, $15\Omega \parallel 15\Omega = \frac{15}{2} = 7.5\Omega$

Let R be the resistance to be shunted across parallel combination.

Then, total resistance, $R_T = \frac{V}{I}$

i.e., $$10 + \frac{7.5\,R}{7.5 + R} = \frac{20}{1.5}$$

or $$\frac{7.5\,R}{7.5 + R} = \frac{20}{1.5} - 10$$

or $$\frac{7.5\,R}{7.5 + R} = \frac{5}{1.5}$$

or $$(7.5 + R) \times 5 = 7.5\,R \times 1.5$$

or $$\mathbf{R = 6\Omega}$$

Example 1.17 *A battery supplies 48.4 mA to a 25Ω load and 25.9 mA to a 50Ω load. Determine the emf and the internal resistance, assuming that both will remain constant. Also, find terminal voltage of the battery when it is connected to a load of 40Ω.*

Solution:

Let E = EMF of the battery in mV and r = internal resistance of battery in ohm

Then, $$\frac{E}{25 + r} = 48.3 \text{ mA and } \frac{E}{50 + r} = 25.9 \text{ mA}$$

$$\therefore \quad \frac{50 + r}{25 + r} = \frac{48.3}{25.9} = 1.865$$

or $$(50 + r) = 1.865\,(25 + r)$$

or $$(50 + r) = 46.625 + 1.865\,r$$

or $$0.865\,r = 3.38$$

or $$\boldsymbol{r = 3.9\Omega}$$

$$\therefore \quad E = 48.3 \text{ mA } (25 + 3.9) = \mathbf{1390mV = 1.39V}$$

With load of $R_L = 40\Omega$,

Terminal voltage, $$V = E \times \frac{R_L}{R_L + r}$$

$$= 1.39 \times \frac{40}{40 + 3.9} = \mathbf{1.27V}$$

Example 1.18 *Find the value of R and the current flowing through it in the network shown below when the current is zero in the branch OA.*

Solution: Given that current in branch AO is zero. Thus A and O are at the same potential.

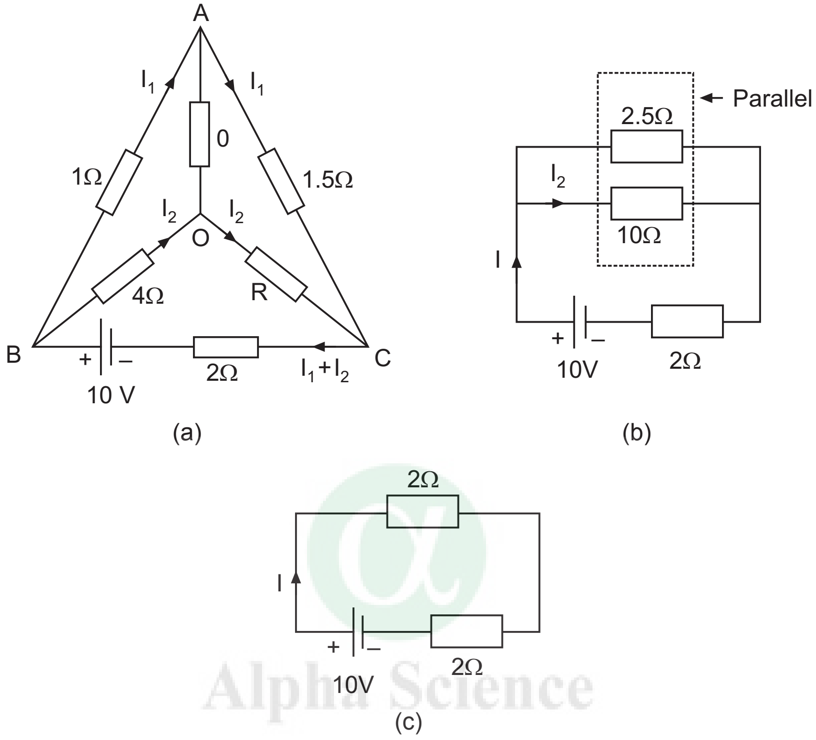


Fig. 1.17 Circuit diagrams

To find R:

Let I_1 be the current in the branches BA and AC and I_2 be the current in the branches BO and OC (that is resistance R).

∴ $$1 \times I_1 = 4 \times I_1$$

and $$1.5 \times I_2 = R \times I_2$$

∴ $$\frac{R}{4} = 1.5$$

or $$\mathbf{R = 6\Omega}$$

To find current I_2 through R:

1Ω and 1.5Ω are in series ⇒ 1Ω + 1.5Ω = 2.5Ω

4Ω and R are in series ⇒ 4Ω + 6Ω = 10Ω

Now 2.5Ω and 10Ω resistances are in parallel as shown in Fig. 1.17(b),

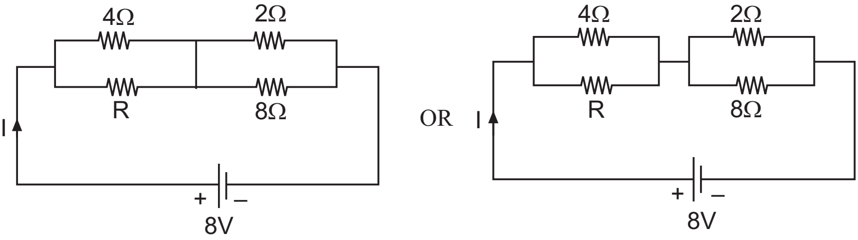
i.e., $$R_p = \frac{2.5 \times 10}{2.5 + 10} = 2\Omega$$

Total current, $$I = \frac{10}{2+2} = 2.5\text{A} \qquad \text{[as shown in Fig. 1.17(c)]}$$

Therefore, referring to Fig. 1.17(b), current through R is

$$I_2 = 2.5 \times \frac{2.5}{10+2.5} = \mathbf{0.5A}$$

Example 1.19 *If the total power dissipated in the network shown below is 16W, find the value of R and the total current.*



(a) (b)

Fig. 1.18 (a) and (b) Circuit diagrams

Solution: $$P = 16\text{W}$$

$$4\Omega \parallel R = \frac{4\,R}{4+R}$$

$$2\Omega \parallel 8\Omega = \frac{2\times 8}{2+8} = 1.6\ \Omega$$

Total resistance, $$R_T = \frac{4\,R}{4+R} + 1.6\Omega$$

To find R_T :

Given that total power dissipated in the circuit P = 16W

i.e., $$16 = \frac{V^2}{R_T} = \frac{8^2}{R_T}$$

or $$R_T = 4\Omega$$

∴ $$4 = \frac{4\,R}{4+R} + 1.6\Omega$$

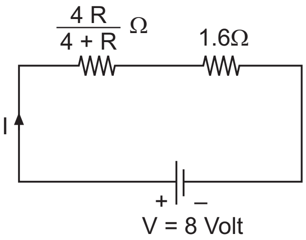


Fig. 1.18 (c)

or $\mathbf{R = 6\Omega}$

Applying Ohm's Law,

Total current, $I = \dfrac{V}{R_T} = \dfrac{8}{4} = \mathbf{2A}$

Example 1.20 *Calculate the equivalent resistance across terminals of the supply and find the total current for the circuit shown.*

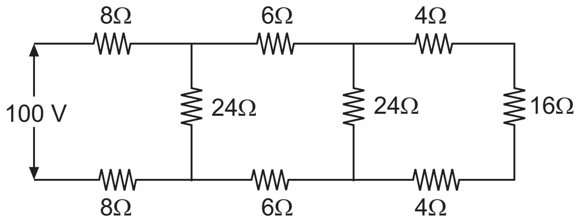


Fig. 1.19 (a) Circuit diagram

Solution: To find equivalent resistance: Start from RHS.

First Reduction: 4Ω, 16Ω and 4Ω are in series

i.e., $4\Omega + 16\Omega + 4\Omega = 24\Omega$

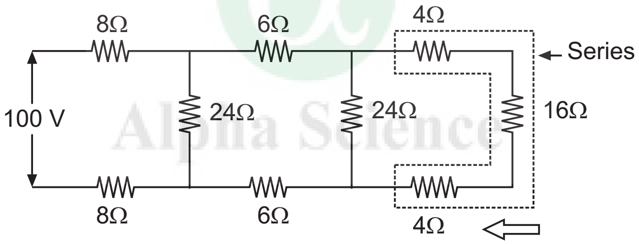


Fig. 1.19 (b) First reduction

Second Reduction: 24Ω || 24Ω;

$$\frac{24 \times 24}{24 + 24} = 12\Omega$$

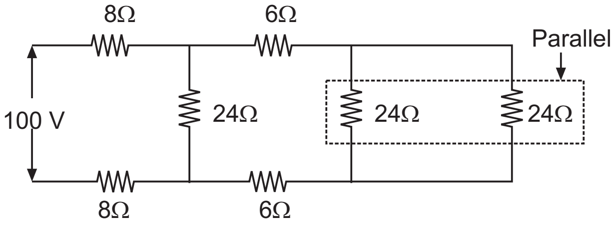


Fig. 1.19 (c) Second reduction

Third Reduction: 6Ω, 12Ω and 6Ω are in series,

i.e., $6\Omega + 12\Omega + 6\Omega = 24\Omega$

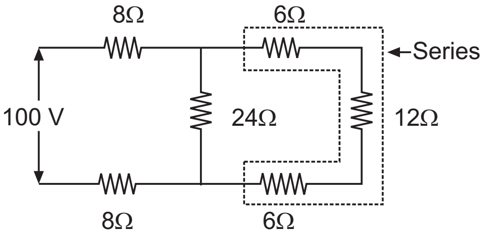


Fig. 1.19 (d) Third reduction

Fourth Reduction: $24\Omega \parallel 24\Omega$

$$= \frac{24 \times 24}{24+24} = 12\Omega$$

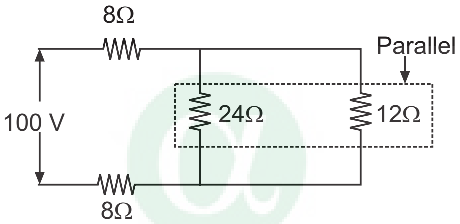


Fig. 1.19 (e) Forth reduction

Fifth Reduction: Equivalent resistance across terminals of supply,

$$R_{eq} = 8\Omega + 12\Omega + 8\Omega = \mathbf{28\Omega}$$

Applying Ohm's Law, total current $I = \frac{V}{R_{eq}} = \frac{100}{28} = \mathbf{3.57A}$

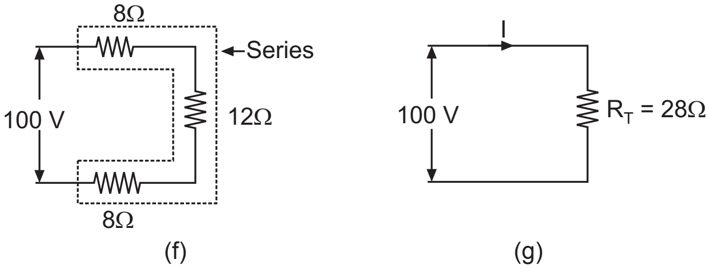


Fig. 1.19 (f) Fifth reduction (g) Circuit diagram

Example 1.21 *Two voltmeters A and B having resistance of 5.2kΩ and 15kΩ respectively are connected in series across 240V. What will be the reading of each voltmeter?*

Solution: As per the data given, circuit diagram is drawn.

Total resistance, $R_T = R_A + R_B = 5.2\text{k}\Omega + 15\text{k}\Omega = 20.2\text{k}\Omega$

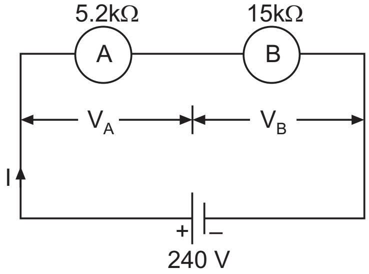


Fig. 1.20 Circuit diagram

Current, $$I = \frac{V}{} = \frac{240}{20.2\times10} = 11.88\times10 \quad A$$

∴ Reading of voltmeter A = $I \times R_A = 11.88 \times 10^{-3} \times 5.2 \times 10^3 = \mathbf{61.77\ V}$

and Reading of voltmeter B = $I \times R_B = 11.88 \times 10^{-3} \times 15 \times 10^3 = \mathbf{178.23\ V}$

(Verification: $V = V_A + V_B = 61.77 + 178.23 = 240V$)

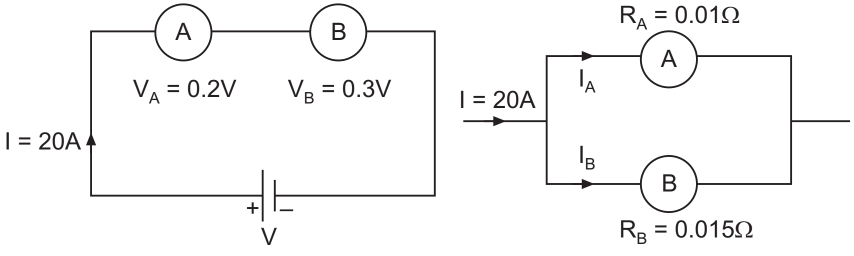
Example 1.22 *A current of 20A flows through two ammeters A and B in series. The potential difference across A is 0.2V and across B is 0.3V. Find how the same current will divide between A & B when they are in parallel.*

Solution: Circuit diagram for series connection is drawn.

Let the resistance of ammeter A and B be R_A and R_B respectively.

Potential difference across ammeter A = $V_A = IR_A$

Potential difference across ammeter B, $V_B = IR_B$



(a) (b)

Fig. 1.21 (a) and (b) circuit diagrams

Considering $V_A = IR_A$

or $0.2 = 20\ R_A$

or $$R_A = \frac{V_A}{I} = \frac{0.2}{20} = 0.01\ \Omega$$

Considering $V_B = I R_B$,

or $0.3 = 20 R_B$

or $R_B = \dfrac{V_B}{I} = \dfrac{0.3}{20} = 0.015\ \Omega$

Now connecting the ammeters in parallel as shown in Fig. 1.21(b),

Applying current division rule,

Current through ammeter A, $I_A = I\dfrac{R_B}{R_A + R_B} = 20\dfrac{0.015}{0.01 + 0.015} = \mathbf{12\ A}$

Current through ammeter B, $I_B = I\dfrac{R_A}{R_A + R_B} = 20\dfrac{0.01}{0.01 + 0.015} = \mathbf{8\ A}$

(Verification: $I = I_1 + I_2 = 12 + 8 = 20$ A)

1.7 KIRCHHOFF'S LAWS

1.7.1 Important Terms Related to Circuits

(i) Junction: It is that point in a circuit, where two or more circuit elements are joined.

(ii) Branch: It is that part of a circuit which lies in between two junctions.

(iii) Loop: It is the closed path of a circuit.

(iv) Mesh: It is an elementary form of a loop and cannot be divided further into another loop.

1.7.2 Kirchhoff's Current Law (KCL)

Statement: It states that the algebraic sum of the currents meeting at a point or junction is zero.

i.e. $\Sigma I = 0$

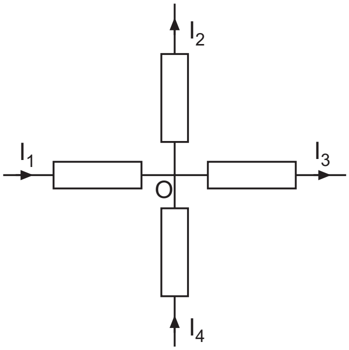


Fig. 1.22 Junction

Explanation: Let currents I_1, I_2 and I_4 are the incoming currents flowing towards the junction O and I_3 is outgoing current, flowing away from the junction O.

Incoming currents are taken as positive and outgoing currents as negative.

Applying KCL at junction O,

$$I_1 + I_2 + (-I_3) + I_4 = 0 \quad \text{...(1.1)}$$

Equation (1.1) can also be written as

$$I_1 + I_2 + I_4 = I_3$$

Therefore, sum of the incoming currents is equal to the sum of the outgoing currents.

Important Point

It is another version of principle of conservation of charge.

1.7.3 Kirchhoff's Voltage Law (KVL)

Statement: It states that the algebraic sum of the emfs and the voltage drops around a closed circuit (or loop) is zero.

Alternatively, the algebraic sum of voltages around a closed circuit is zero.

i.e., $\Sigma V = 0$

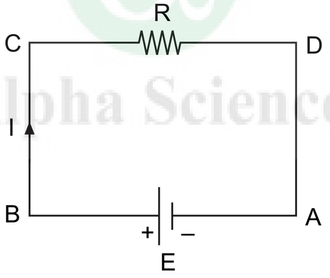


Fig. 1.23 Closed circuit

Explanation: Consider a closed circuit ABCDA, consisting of a resistance R connected across an emf source E.

If a current I flows through the resistance, voltage drop IR occurs across it, which is taken as –ve.

Applying KVL, $E + (-I\,R) = 0$

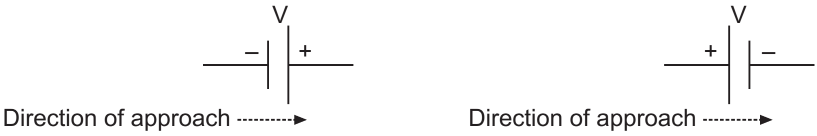
Important Points

- It is based on the principle of conservation of energy.
- KVL is concerned with both linear and non-linear circuits.

1.7.4 Sign Convention

1. **Considering the voltage source**

    (i) If we approach from –ve terminal of the voltage source towards the +ve terminal as shown in Fig. 1.24 (a) then there is a rise in potential, which is taken as positive.



(a) (b)

Fig. 1.24

    (ii) If we approach from +ve terminal of voltage source towards the –ve terminal as shown in Fig. 1.24(b) then there is a fall in potential, which is taken as negative.

2. **Considering the voltage drop**

    If a current flows through a resistance, voltage drop occurs.

    (i) If the current flows along with the direction of our approach as shown in Fig. 1.25(a) then there is a fall in potential, which is taken as negative.

(a) (b)

Fig. 1.25

    (ii) If the current flows in opposite direction of our approach as shown in Fig. 1.25(b) then there is a rise in potential, which is taken as positive.

1.8 APPLICATIONS OF KIRCHHOFF'S LAWS

1. Branch Current Method
2. Loop or Mesh Current Method

1. **Branch Current Method**

Procedure to solve problems using branch current method

Step 1: Name the closed circuit, say, ABCDA starting from the –ve terminal of the voltage source.

Step 2: As current flows from higher potential to lower potential, suitably mark all the branch currents. If the assumed direction of current is wrong, its mathematical value will become negative.

Step 3: Apply KCL at a point or junction if different currents meet at a junction.

Step 4: Assume a suitable direction of approach, either clockwise direction or anticlockwise direction as per your convenience and apply KVL to the closed circuits as per the requirement and write the corresponding equations (the number of equations should be minimum).

Step 5: Solve the simultaneous equations to find the unknowns.

Illustrative Example: Apply KVL to the circuit shown in Fig. 1.26 (a) and write the equations.

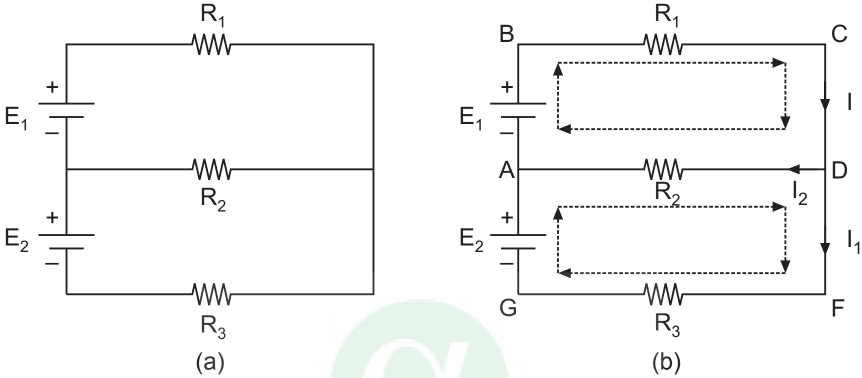


Fig. 1.26 Circuit diagrams

Step 1: The first circuit is named as ABCDA taking A at the –ve terminal of EMF E_1 and the second circuit is named as GADFG taking G at the –ve terminal of EMF E_2 as shown in Fig. 1.26 (b).

Step 2: The branch currents are marked such that current I flows through resistance R_1, I_2 flows through R_2 and I_1 flows through R_3.

Step 3: Applying KCL at junction D,

$$I = I_1 + I_2 \qquad ...(1.2)$$

Step 4: Approaching in clockwise direction, apply KVL to the closed circuit ABCDA.

Branch AB contains EMF E_1 and as approached from A → B, that is, from –ve to +ve, E_1 is taken as +ve.

Branch BC contains resistance R_1. As the current I flows through R_1 along with our direction of approach, from B → C, there is a fall in potential and taken as –ve.

Branch DA contains resistance R_2. As the current I_2 flows through R_2 along with our direction of approach, from D → A, there is a fall in potential and taken as –ve.

Therefore, $E_1 - I\,R_1 - I_2\,R_2 = 0$

Substituting for I from equation (1.2),

$$E_1 - (I_1 + I_2)\,R_1 - I_2\,R_2 = 0$$

or $$E_1 - I_1R_1 - I_2R_1 - I_2\,R_2 = 0$$

or $$E_1 - I_1R_1 - I_2(R_1 + R_2) = 0$$

or $$-I_1R_1 - I_2(R_1 + R_2) = -E_1 \qquad ...(1.3)$$

Similarly applying KVL to the closed circuit GADFG taking E_2 as +ve.

$$E_2 + I_2R_2 - I_1R_3 = 0$$

or $$I_2R_2 - I_1R_3 = -E_2 \qquad ...(1.4)$$

Applying KVL to the closed circuit GABCDFG,

$$E_2 + E_1 - I\,R_1 - I_1\,R_3 = 0$$

Substituting for I from equation (1.2),

$$E_2 + E_1 - (I_1 + I_2)\,R_1 - I_1\,R_3 = 0$$

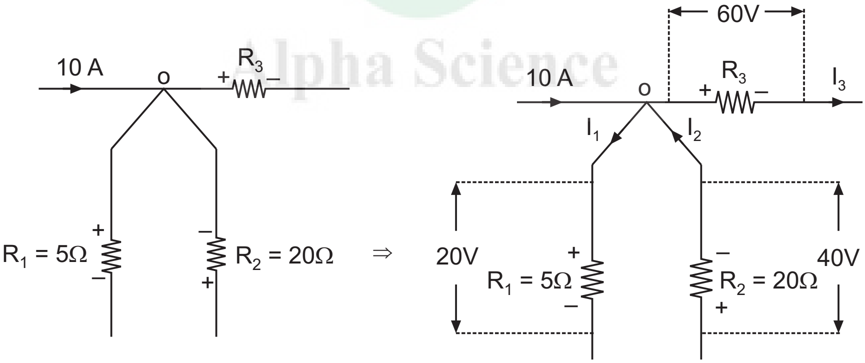
or $$E_2 + E_1 - I_1\,R_1 - I_2R_1 - I_1\,R_3 = 0$$

or $$E_1 + E_2 - I_1(R_1 + R_3) - I_2R_1 = 0$$

or $$-I_1(R_1 + R_3) - I_2R_1 = -E_1 - E_2 \qquad ...(1.5)$$

Solved Examples

Example 1.23 *A portion of the network has the configuration shown in Fig. 1.27(a). The voltages across three resistors are known to be respectively 20V, 40V and 60V having the polarities indicated. If $R_1 = 5\Omega$, $R_2 = 20\Omega$, find R_3.*



(a) (b)

Fig. 1.27 Circuit diagrams

Solution: Let the currents flowing through resistances R_1, R_2 and R_3 be I_1, I_2 and I_3 respectively.

Applying Ohm's Law,

Current through R_1, $$I_1 = \frac{20}{5} = 4\text{ A}$$

Current through R_2, $$I_2 = \frac{40}{20} = 2\text{ A}$$

Applying KCL at point O, $10 - I_1 + I_2 - I_3 = 0$

i.e., $10 - 4 + 2 - I_3 = 0$

or $I_3 = 8\text{ A}$

$$\therefore \qquad R_3 = \frac{60}{I_3} = \frac{60}{8} = \mathbf{7.5\ \Omega}$$

Example 1.24 *Find the current in all branches of the network shown below.*

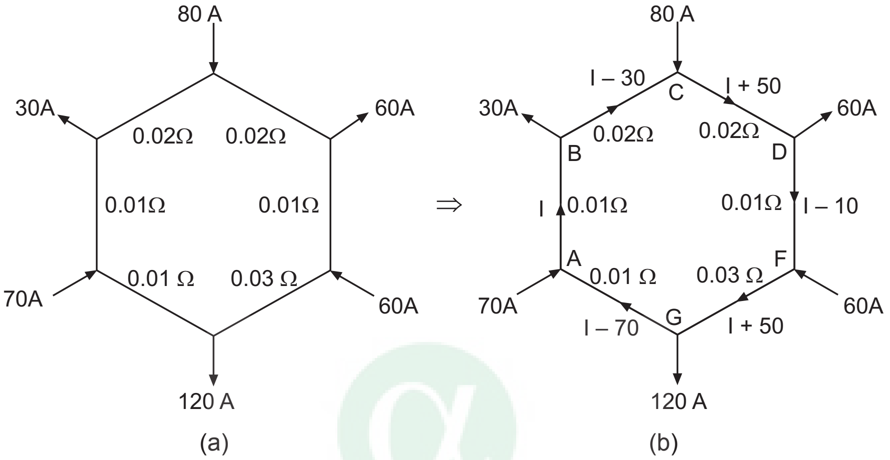


Fig. 1.28 Circuit diagrams

Solution: Assume a current I in branch AB. Applying KCL at all the junctions A, B, C, D, F and G, the current in all the branches are expressed in terms of I as shown in circuit diagram.

Applying KVL to the closed circuit ABCDFGA,

$$0.01(I) + 0.02\,(I - 30) + 0.02\,(I + 50) + 0.01(I - 10) + 0.03(I + 50) + 0.01(I - 70) = 0$$

or $I = -11\text{ A}$

Therefore, current through various are tabulated in Table 1.1.

Table 1.1

Branch	Current	Direction
AB	I = – 11 A	B → A
BC	I – 30 = – 11 – 30 = – 41 A	C → B
CD	I + 50 = – 11 + 50 = 39 A	C → D
DF	I – 10 = – 11 – 10 = – 21 A	F → D
FG	I + 50 = – 11 + 50 = 39 A	F → G
GA	I – 70 = – 11 – 70 = – 81 A	A → G

Example 1.25 *Two batteries with internal resistance 0.2Ω and 0.25Ω respectively are joined in parallel and a resistance of 1Ω is connected across the terminals. Find the current supplied by each battery.*

Solution: Let I_1 and I_2 be the currents supplied by the batteries

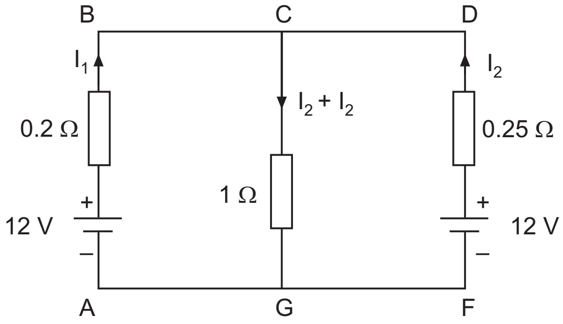


Fig. 1.29 Circuit diagram

Applying KCL at point C,

Current through 1Ω resistor = $I_1 + I_2$

Applying KVL to the closed circuit ABCGA,

$12 - 0.2\,I_1 - 1\,(I_1 + I_2) = 0$

or $12 - 0.\,2I_1 - 1\,I_1 - 1\,I_2 = 0$

or $12 - 1.2\,I_1 - 1\,I_2 = 0$

or $-1.2\,I_1 - 1\,I_2 = -12$ (multiplying with –ve sign)

or $1.2\,I_1 + 1\,I_2 = 12$ ...(1.6)

Applying KVL to the closed circuit GCDFG,

$1(I_1 + I_2) + 0.25\,I_2 - 12 = 0$

or $I_1 + I_2 + 0.25\,I_2 = 12$

or $I_1 + 1.25\,I_2 = 12$ ...(1.7)

Solving equations (1.6) and (1.7),

$1.2\,I_1 + 1\,I_2 = 12$ (multiplying by 1)

$I_1 + 1.25\,I_2 = 12$ (multiplying by 1.2)

$\Rightarrow$ $1.2\,I_1 + 1\,I_2 = 12$

$1.2\,I_1 + 1.5\,I_2 = 14.4$

Subtracting, $-0.5\,I_2 = -2.4$

Therefore, $I_2 = \dfrac{-2.4}{-0.5} = \mathbf{4.8\ A}$

Substituting the value of I_2 in equation (1.6),

$1.2\,I_1 + (1 \times 4.8) = 12$

or $1.2\,I_1 = 12 - 4.8 = 7.2$

or $I_1 = \dfrac{7.2}{1.2} = \mathbf{6\ A}$

Example 1.26 *Two storage batteries A and B are connected to supply a load of 0.3Ω. The open circuit emf of A is 11.7 V and that B is 12.3 V. The internal resistances are 0.06Ω and 0.05Ω respectively. Find the current supplied to the load.*

Solution: As per the data given, the circuit diagram is drawn as shown in Fig. 1.30, connecting internal resistance of 0.06Ω in series with battery A and internal resistance of 0.05Ω in series with battery B. The load resistance R_L is connected in parallel with the batteries so that total current I flows through it.

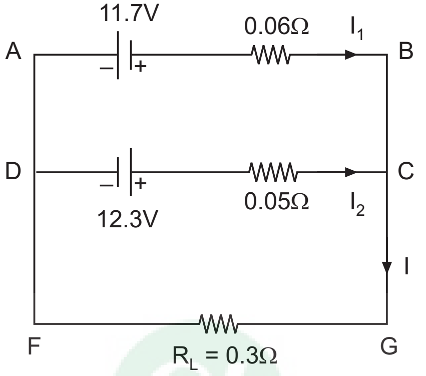


Fig. 1.30 Circuit diagram

Applying KCL at point C, $I_1 + I_2 = I$ ...(1.8)

Applying KVL to the closed circuit ABCDA,

$$11.7 - 0.06\,I_1 + 0.05\,I_2 - 12.3 = 0$$

or $$-0.6 - 0.06\,I_1 + 0.05\,I_2 = 0$$

or $$-0.06\,I_1 + 0.05\,I_2 = 0.6 \text{ (multiplying with –ve sign)}$$

or $$0.06\,I_1 - 0.05\,I_2 = -0.6 \quad \text{...(1.9)}$$

Applying KVL to the closed circuit FABGF,

$$11.7 - 0.06\,I_1 - 0.3\,I = 0$$

Substituting for I from equation (1.8),

$$11.7 - 0.06\,I_1 - 0.3\,(I_1 + I_2) = 0$$

or $$11.7 - 0.06\,I_1 - 0.3\,I_1 - 0.3\,I_2 = 0$$

or $$11.7 - 0.36\,I_1 - 0.3\,I_2 = 0$$

or $$-0.36\,I_1 - 0.3\,I_2 = -11.7$$

or $$0.36\,I_1 + 0.3\,I_2 = 11.7 \quad \text{...(1.10)}$$

Solving equations (1.9) and (1.10),

$$I_2 = 25.5 \text{ A}$$

Substituting the value of I_2 in equation (1.9), $0.06\,I_1 - (0.05 \times 25.5) = -0.6$

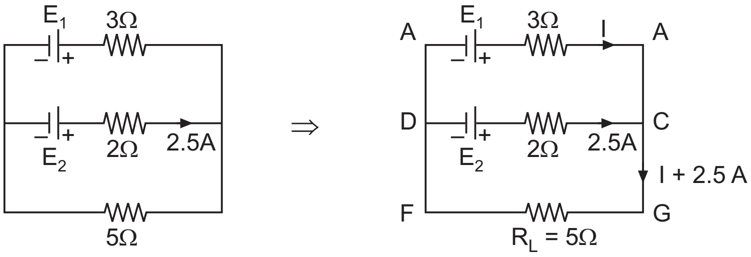
or $$0.06\ I_1 = 0.675$$

or $$I_1 = \frac{0.675}{0.06} = 11.25\ \text{A}$$

According to equation (1.8), current supplied to the load is

$$I = I_1 + I_2 = 11.25 + 25.5 = \mathbf{36.75\ A}$$

Example 1.27 *In the circuit below, find I, E_1, and E_2 when the power dissipated in 5Ω resistor is 125W.*



(a) (b)

Fig. 1.31 Circuit diagrams

Solution: Let current I flows through 3Ω resistance.

Applying KCL at point C, current flowing through load resistor R_L is (I + 2.5)

Given that power dissipated in 5Ω resistor is 125W

i.e., $$125 = (I + 2.5)^2 \times 5$$

or $$\mathbf{I = 2.5\ A}$$

Applying KVL to the closed circuit ABCDA,

$$E_1 - 3\ I + (2 \times 2.5) - E_2 = 0$$

or $$E_1 - 3\ I + 5 - E_2 = 0$$

or $$E_1 - E_2 = 3\ I - 5 \qquad \text{...(1.11)}$$

Applying KVL to the closed circuit DCGFD,

$$E_2 - (2 \times 2.5) - 5\ (I + 2.5) = 0$$

or $$E_2 - 5 - 5\ I - 12.5 = 0$$

or $$E_2 = 5\ I - 17.5 \qquad \text{...(1.12)}$$

Substituting the value of I in equation (1.12),

$$E_2 = (5 \times 2.5) - 17.5$$

$$\mathbf{= 30\ V}$$

Substituting the value of E_2 and I in equation (1.11),

$$E_1 - 30 = (3 \times 2.5) - 5$$

or $$\mathbf{E_1 = 32.5\ V}$$

Example 1.28 *Find the value of R and the current flowing through it in the network shown below when the current is zero in the branch OA using Kirchhoff's Laws.*

(Note: The same example was solved using Kirchhoff's Law)

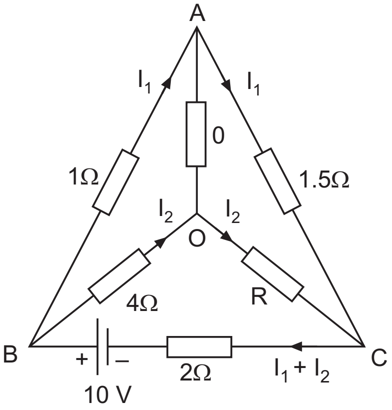


Fig. 1.32 Circuit diagram

Solution: Given that current in branch AO is zero.

Let I_1 and I_2 be the currents flowing through the branch BA and BO respectively. Applying KCL at the following junctions:

'A', current in branch BA = current in branch AC = I_1

'O', current in branch BO = current in branch OC

= current in resistance R = I_2

'C', current in branch CB = $I_1 + I_2$

Applying KVL to the following closed circuits:

'BAOB', $-I_1 + 0 + 4\,I_2 = 0$

or $I_1 = 4I_2$

'ACOA', $-\,1.5\,I_1 + R\,I_2 = 0$

Substituting, $I_1 = 4I_2$,

$-\,1.5 \times 4I_2 + R\,I_2 = 0$

or $\mathbf{R = 6\Omega}$

'BOCB', $10 - 4I_2 - R\,I_2 - 2\,(I_1 + I_2) = 0$

or $2\,(I_1 + I_2) + 4I_2 + R\,I_2 = 10$

Substituting for $I_1 = 4I_2$

and $R = 6\Omega$,

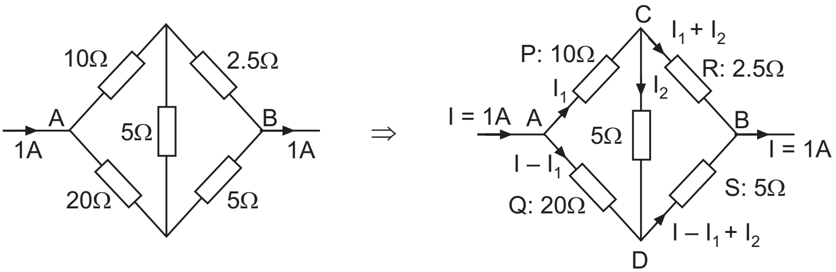
$2(4I_2 + I_2) + 4I_2 + 6\,I_2 = 10$

or $(2 \times 5\, I_2) + 10\, I_2 = 10$

or $20\, I_2 = 10$

or $I_2 = \mathbf{0.5\ A}$

Example 1.29 *Find the currents in all the resistors of the circuit shown below. Also, find the voltage across AB.*



(a) (b)

Fig. 1.33 Circuit diagrams

Solution: *The above circuit is called Wheatstone bridge. This example can be solved by two methods.*

Let I_1 and I_2 be the currents through P:10Ω and 5Ω resistances respectively.

Method I : Applying KVL to the closed circuit ACDA,

$$-10I_1 - 5I_2 + 20\,(I - I_1) = 0$$

or $-10I_1 - 5I_2 + 20\,(1 - I_1) = 0$

or $-10I_1 - 5I_2 + 20 - 20\,I_1 = 0$

or $-30I_1 - 5I_2 = -20$

or $30I_1 + 5I_2 = 20$ ...(1.13)

Applying KVL to the closed circuit CBDC,

$$-2.5(I_1 - I_2) + 5\,(I - I_1 + I_2) + 5I_2 = 0$$

or $-2.5(I_1 - I_2) + 5(1 - I_1 + I_2) + 5I_2 = 0$

or $-2.5I_1 + 2.5I_2 + 5 - 5I_1 + 5I_2 + 5I_2 = 0$

or $7.5\, I_1 - 12.5\, I_2 = 5$ ...(1.14)

Solving equations (1.13) and (1.14),

$$\mathbf{I_2 = 0}$$

Substituting $I_2 = 0$ in equation (1.13),

$$30\, I_1 = 20$$

or $\mathbf{I_1 = 0.667\ A}$

Current through R: 2.5Ω resistance, $I_1 - I_2 = 0.667 - 0 = \mathbf{0.667\ A}$

Current through S: 5Ω resistance, $I - I_1 + I_2 = 1 - 0.667 + 0 = \mathbf{0.333\ A}$

$\therefore$ Voltage drop across AB, $V_{AB} = I \times R_T$

To find total resistance R_T: As $I_2 = 0$, 10Ω and 2.5Ω resistances are in series = 12.5Ω. Also, 20Ω and 5Ω resistances are in series = 25Ω

$$R_T = 12.5\Omega \parallel 25\Omega = \frac{12.5 \times 25}{12.5 + 25} = 8.33\Omega$$

$$\therefore \quad V_{AB} = 1 \times 8.33 = \mathbf{8.33\ V}$$

Method II: If the bridge is balanced, $\frac{P}{Q} = \frac{R}{S}$

i.e., $$\frac{10}{20} = \frac{2.5}{5}$$

or $$0.5 = 0.5$$

Then, $$\mathbf{I_2 = 0}$$

Therefore, 10Ω and 2.5Ω resistances are in series ⇒ 10Ω + 2.5Ω = 12.5Ω,

20Ω and 5Ω resistances are in series ⇒ 20Ω + 5Ω = 25Ω.

Now 12.5Ω and 25Ω resistances are in parallel.

Let *x* and *y* be the currents, which flow through 12.5Ω and 25Ω respectively as shown in Fig. 1.33(c).

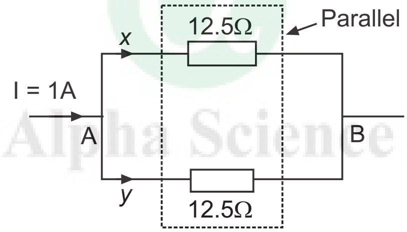


Fig. 1.33 (c) Circuit diagram

$$x = 1 \times \frac{25}{12.5 + 25} = \mathbf{0.667\ A}$$

$$y = I - 0.667 = 1 - 0.667 = \mathbf{0.333\ A}$$

Voltage drop across AB, $V_{AB} = x \times 12.5 = 0.667 \times 12.5 = \mathbf{8.33\ V}$

or $V_{AB} = y \times 25 = 0.333 \times 25 = \mathbf{8.33\ V}$

Example 1.30 *For the circuit shown in Fig. 1.34, calculate the potential difference between X and Y.*

Solution: Let currents I_1 and I_2 flows through loop 1 and loop 2 respectively.

Applying KVL to the loop 1,

$$2 - 2I_1 - 3I_1 = 0$$

or $$-5I_1 = -2$$

or $$I_1 = 0.4\ A$$

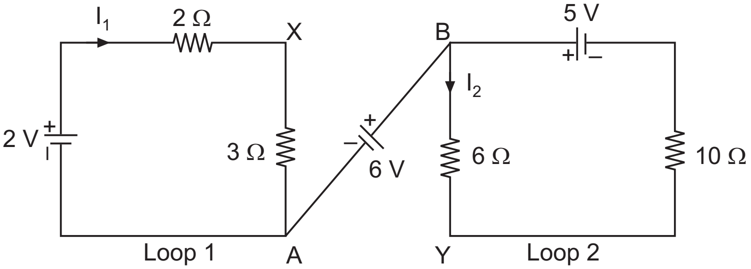


Fig. 1.34 Circuit diagram

Potential difference between X and A, $V_{XA} = -(0.4 \times 3) = -1.2$ V

Applying KVL to the loop 2,

$$5 - 6 I_2 - 10 I_2 = 0$$

or $$-16 I_2 = -5$$

or $$I_2 = 0.315 \text{ A}$$

Potential difference between B and Y, $V_{BY} = -(0.315 \times 6) = -1.875$ V

∴ Potential difference between X and Y,

$$V_{XY} = V_{XA} + V_{AB} + V_{BY}$$
$$= -1.2 + 6 - 1.875 = \mathbf{2.925\ V}$$

Example 1.31 *Using Kirchhoff's Laws, find the current flowing through the ammeter 'A' having a resistance of 10Ω in the given network shown below.*

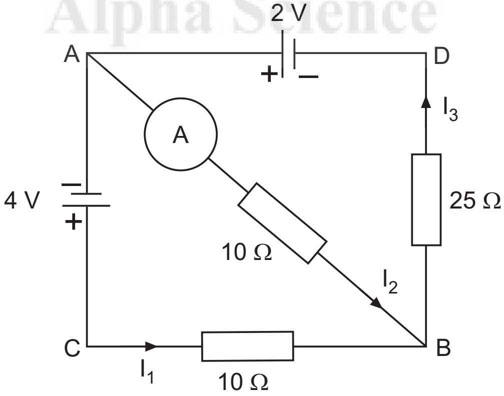


Fig. 1.35 Circuit diagram

Solution: Applying KCL at junction B, $I_1 + I_2 = I_3$ ...(1.15)

Applying KVL to the closed circuit ACBA,

$$4 - 10 I_1 + 10 I_2 = 0$$

or $$-10 I_1 + 10 I_2 = -4$$

or $$10 I_1 - 10 I_2 = 4 \quad \text{...(1.16)}$$

Applying KVL to the closed circuit DABD,

$$2 - 10\,I_2 - 25\,I_3 = 0$$

Substituting for I_3 from equation (1.15),

$$2 - 10\,I_2 - 25\,(I_1 + I_2) = 0$$

or $$-10\,I_2 - 25\,I_1 - 25\,I_2 = -2$$

or $$-25\,I_1 - 35\,I_2 = -2$$

or $$25\,I_1 + 35\,I_2 = 2 \qquad \text{...(1.17)}$$

Solving equations (1.16) and (1.17),

$$I_1 = 0.2667\text{ A} \quad\text{and}\quad I_2 = -0.1333\text{ A}$$

Therefore, current through ammeter is I_2 = **0.1333 A** and its direction is from B to A.

2. **Loop or Mesh current method:** It is also called Maxwell's loop current method. In this method continuous currents are assumed in various loops called loop currents. It uses KVL. This method is applicable for the networks having large number of junctions and simplifies the calculations.

(Note: Equations can be written in matrix form and solved using Cramer's rule)

Procedure to solve problems using loop current method

Step 1: Identify the loops and name them.

Step 2: Loop currents are assumed in each loop.

Step 3: Same sign conventions can be followed.

Step 4: Apply KVL to each loop and write equations.

Step 5: Solve the simultaneous equations to find the unknowns.

Illustrative example: Apply KVL to the circuit shown in Fig. 1.36 and write the equations.

Two loops are identified; loop1 (ABCDA) taking A at the –ve terminal of EMF E_1 and loop2 (CFGDC) taking G at the –ve terminal of EMF E_2 as shown in Fig. 1.36.

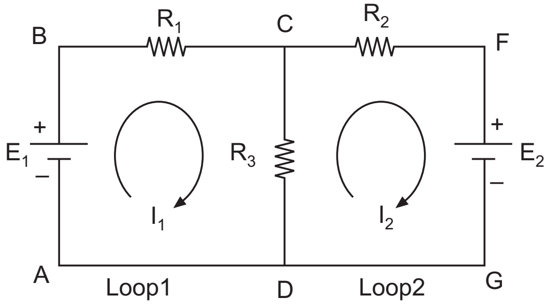


Fig. 1.36 Circuit diagram

Two loop currents, I_1 and I_2 are assumed in loop1 and loop2 respectively. Important point to be observed is that the branch CD containing resistance R_3 carries two loop currents I_1 and I_2.

Sign convention is same as discussed in branch current method.

Approaching in clockwise direction, apply KVL to the loop1. As current I_2 flows through R_3, there is a rise in voltage drop and taken as +ve.

$$E_1 - I_1 R_1 - I_1 R_3 + I_2 R_3 = 0$$

or $$-I_1(R_1 + R_2) + I_2 R_3 = -E_1$$

Similarly applying KVL to the loop 2, (as current I_1 flows through R_3, there is a rise in voltage drop and taken as +ve).

$$-I_2 R_2 - E_2 - I_2 R_3 + I_1 R_3 = 0$$

or $$I_1 R_3 - I_2 (R_2 + R_3) = E_2$$

Solved Examples

Example 1.32 *Find the current through 8Ω resistance for the network shown below using mesh/loop current method.*

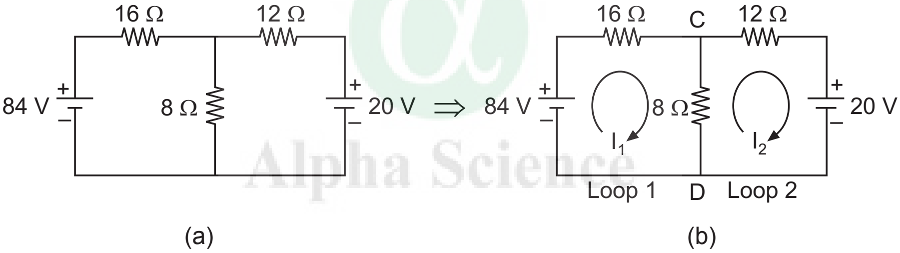


Fig. 1.37 Circuit diagrams

Solution: Consider loop currents I_1 and I_2 in loops 1 and 2 respectively.

Applying KVL to loop 1,

$$84 - 16 I_1 - 8 I_1 + 8 I_2 = 0$$

or $$-24 I_1 + 8 I_2 = -84 \quad \text{...(1.18)}$$

Applying KVL to loop 2,

$$-8 I_2 + 8 I_1 - 12 I_2 - 20 = 0$$

or $$8 I_1 - 20 I_2 = 20 \quad \text{...(1.19)}$$

Solving equations (1.18) and (1.19),

$$I_2 = 0.5 \text{ A}$$

Substituting the value of I_2 in equation (1.19),

$$I_1 = 3.65 \text{ A}$$

Therefore, current through 8Ω resistance = $I_1 - I_2$ = 3.65 – 0.5 = **3.15 A**

Example 1.33 *For the circuit shown below determine the voltage V_1 and V_3.*

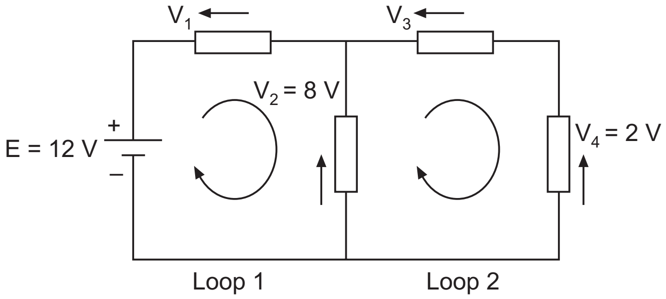


Fig. 1.38 Circuit diagram

Solution: Applying KVL to loop 1,

$$E = V_1 + V_2$$

or

$$V_1 = E - V_2$$

$$= 12 - 8 = \mathbf{4\ V}$$

Applying KVL to loop 2,

$$-V_2 + V_3 + V_4 = 0$$

or

$$V_3 = V_2 - V_4$$

$$= 8 - 4 = \mathbf{6\ V}$$

Multiple Choice Questions

Choose the Correct Answers

1. As the temperature increases, the resistance of a conductor made of a metal
   (a) increases (b) decreases
   (c) remains constant (d) none of these
2. Conductance of an insulator is
   (a) very high (b) very low (c) medium (d) none of these
3. On increasing its temperature, the resistance of an insulator
   (a) increases (b) decreases
   (c) remains constant (d) none of these
4. The following material has positive temperature coefficient
   (a) wood (b) Carbon (c) Mica (d) none of these
5. A wire of resistance R is stretched to double its length. The new resistance of the wire is
   (a) R/4 (b) 4R (c) R/2 (d) none of these

6. If resistance of a 700 m long cable is 100Ω, then the resistance of 8 km similar cable is

   (a) 100Ω (b) 2186Ω (c) 8000Ω (d) 1143Ω

7. Electrical load means connecting a number of resistances in

   (a) parallel (b) series

   (c) series-parallel (d) none of these

8. If a 220 V, 100W filament lamp is connected across 110V, it consumes

   (a) 12.5W (b) 25W (c) 50W (d) 100W

9. Resistance is

   (a) active element (b) passive element

   (c) unilateral element (d) none of these

10. The current in a circuit having constant resistance is doubled. The power consumed by resistance increases

    (a) two times (b) decreases to half

    (c) four times (d) none of these

11. A 5Ω resistor draws a current of 2 A. If this resistor is replaced by 10Ω resistor, the current drawn will be

    (a) < 2A (b) > 2 A (c) $= 2$ A (d) none of these

12. Three resistances of 4Ω, 6Ω and 9Ω are connected in parallel. Which resistance will consume maximum power?

    (a) 4Ω (b) 6Ω (c) 9Ω (d) both (a) and (b)

13. Which of the following statements is true both for a series and parallel circuit?

    (a) resistances are additive (b) currents are additive

    (c) voltages are additive (d) powers are additive

14. Two resistances R_1 and R_2 give combined resistance of 4.5Ω when in series and 1Ω when in parallel, the resistances are

    (a) 2Ω and 2.5Ω (b) 1Ω and 3.5Ω

    (c) 1.5Ω and 3Ω (d) 4Ω and 0.5Ω

15. One kWh of electrical energy is equal to

    (a) 36×10^2 J (b) 36×10^3 J (c) 36×10^4 J (d) none of these

16. Kirchhoff's Voltage Law applies to a circuit with

    (a) linear elements (b) non-linear elements

    (c) both (a) and (b) (d) none of these

17. A practical voltage source is represented by

    (a) a resistance in parallel with an ideal voltage source

    (b) a resistance in series with an ideal current source

(c) a resistance in series with an ideal voltage source

(d) none of these

18. A voltage drop of 10 V develops across 1 kΩ resistor, the power consumed in the resistance is

(a) 0.1W (b) 0.1 kW (c) 1W (d) 1 kW

19. As per Ohm's Law

(a) V = I (b) V α 1/R (c) I α (d) V = IR

20. When several resistances are connected in parallel, the algebraic sum of branch currents is

(a) zero (b) infinity (c) one (d) none of these

Answers:—

1. (a)	2. (b)	3. (b)	4. (d)	5. (d)	6. (c)	7. (a)	8. (b)	9. (b)	10. (c)
11. (a)	12. (c)	13. (d)	14. (c)	15. (c)	16. (c)	17. (c)	18. (a)	19. (d)	20. (a)

Review Questions

1. Define the following terms: (i) EMF, (ii) Current, (iii) Resistance, (iv) Electrical Power and (v) Electrical Energy. Mention their unit.
2. What are the factors affecting the resistance?
3. Obtain the equivalent resistance when various resistances are connected in (i) Series and (ii) Parallel.
4. A lamp or bulb is connected to a source through a switch. It is found that the light output is insufficient and needs to connect a second lamp to give more light. Give the appropriate lamp connection. Justify your answer.
5. State Ohm's Law. What are its limitations?
6. State and explain Kirchhoff's Laws as applied to dc circuits.

Exercises

1. A 10Ω resistor takes a current of 3 A. Find (i) the power dissipated by the resistor and (ii) the energy consumed in 5 minutes. (**Ans:** 90W and 2700 J)
2. A current of 8A from a supply of 240 V flows through a heater. Determine the energy consumed in kWh. (**Ans:** 23 kWh)
3. Two resistances of 22Ω and 44Ω are connected in parallel across 110V. Find current drawn from each branch and thus total current.

(**Ans:** $I_1 = 5A$, $I_2 = 2.5A$ and $I = 7.5A$)

4. Two coils are connected in parallel and a voltage source of 200 V is applied between the terminals. The total current taken is 25 A and the power dissipated in one of the coils is 1500W. Calculate the resistance of each coil.
(**Ans:** 26.7Ω and 11.4Ω)

5. Three resistances of 2Ω, 3Ω and 8Ω are connected in series. If the current is 1.5 A, calculate voltage across each resistance and hence the supply voltage.
(**Ans:** 3 V, 4.5 V, 12 V and 19.5 V)

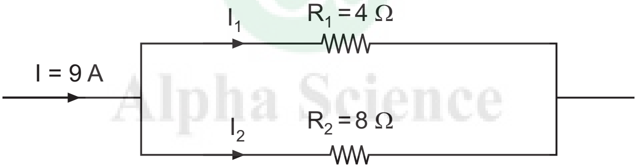
6. Three resistances of 40Ω, 50Ω and 70Ω are connected in series. If the supply voltage is 100V, calculate the circuit current. (**Ans:** I = 0.625 A)

7. Three resistances of 2.2Ω, 4.7Ω and 6.8Ω are connected in parallel across supply voltage of 12 V, calculate the effective resistance and circuit current.
(**Ans:** R = 1.23 Ω and I = 9.76 A)

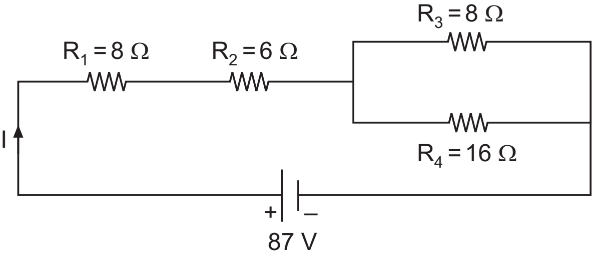
8. A resistance R is connected in series with a parallel circuit comprising of 8Ω, 12Ω and 24 V. The total power dissipated in the circuit is 80W when the applied voltage is 20 V. Calculate R. (**Ans:** 1Ω)

9. A 8Ω resistance is in series with a parallel combination of two resistors 12Ω and 6Ω. If the current in the 6Ω resistor is 5 A, determine total power dissipated in the circuit. (**Ans:** 675W)

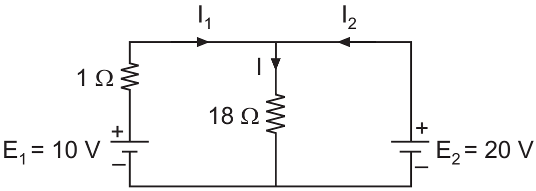
10. For the circuit shown below. Calculate power dissipated by each resistance.
(**Ans:** 72W and 140W)



11. For the circuit shown, calculate the supply current. (**Ans:** I = 4.5 A)

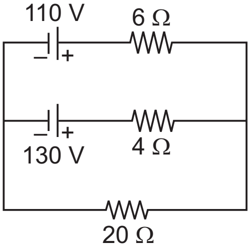


12. For the circuit shown determine below determine current through resistance and I_1 and I_2. (**Ans:** I = 1.11 A, I_1 = – 10 A and I_2 = 11.1 A)

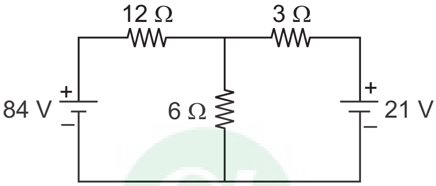


13. For the network shown below determine the value and direction of current in each battery and voltage across the load resistance.

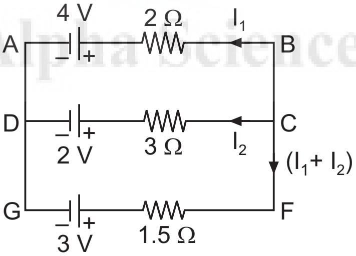
(**Ans:** $I_1 = 0.18$ A, $I_2 = 5.27$ A and V = 108.9 V)



14. For the circuit shown below, find the value of current through 6Ω resistor using KVL. (**Ans:** 4 A)



15. Determine the magnitude and direction of the current in the 2V battery for the circuit shown below. (**Ans:** $I_2 = 0.52$ A)



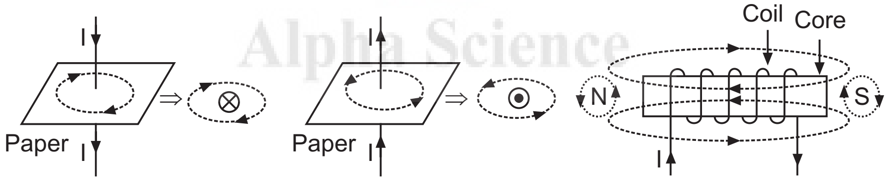
Chapter 2

Electromagnetism

2.1 CONCEPT OF ELECTROMAGNETISM

Electromagnetism deals with the interrelationship between electricity and magnetism. If a current I flows through a wire, a magnetic field sets up as shown in Fig. 2.1(a) and 2.1(b). The direction of magnetic lines will be given by Right-hand thumb rule or corkscrew rule.

If a current I flows through a coil wound on a core as shown in Fig. 2.1(c), it becomes magnetized and it is called the electromagnet. This phenomena is called the electromagnetism. Electromagnet finds application in most of the electrical machines where permanent magnet cannot be used due to the limitations such as size, shapes, etc.



(a) Current flow towards paper (b) Current flow away from paper (c) Electromagnet

Fig. 2.1 Concept of electromagnetism

On the other hand, whenever the magnetic flux linked with a conductor or coil changes, an emf is induced in it and sets up a current I if the conductor forms a closed circuit as shown in Fig. 2.1(d). This phenomena is called the electromagnetic induction.

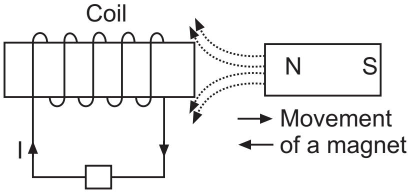


Fig. 2.1 (d) Electromagnetic induction

2.2 IMPORTANT TERMS

1. **Magnetic Field:** The magnetic field is defined as the space or region around a magnet in which the magnetic effects can be detected. The magnets may be permanent magnets or electromagnets.
2. **Magnetic Flux:** The entire magnetic lines of force in a magnetic field are called magnetic flux (ϕ).

   In SI system of units, a unit North pole is supposed to radiate out a flux of 1 **weber (Wb)**.The weber is that magnetic flux which, linking a circuit of 1 turn, induces in it an emf of 1 volt when the flux is reduced to zero at a uniform rate in 1 second.
3. **Permeability:** Permeability is a certain property of a medium, which is defined as the ability of a material to conduct magnetic flux through it. It is denoted by μ.

   Every medium possess

   (i) Absolute permeability (μ_0) and

   (ii) Relative permeability (μ_r)

In order to measure relative permeability, vacuum or free space is considered as a reference medium, which has an Absolute Permeability given by

$$\mu_0 = 4\,\pi \times 10^{-7} \text{ henry/m}$$

Relative Permeability: The ratio of flux density produced in a material (magnetic core) to the flux density produced in vacuum (or non-magnetic core) by the same magnetic field strength is termed as the Relative Permeability (μ_r). It is given by

$$\mu_r = \frac{\text{permeability of material}}{\text{permeability of free space}} = \frac{\mu}{\mu_0}$$

or $$\mu = \mu_0\,\mu_r$$

(Note: For vacuum or free space, $\mu_r = 1$)

4. **Magnetic Flux Density**: Magnetic flux density is defined as the total flux passing through an area at right angles to the lines of force. It is given by

$$B = \frac{\phi}{A} \text{ weber/m}^2 \text{ or tesla (T)}$$

where ϕ = Flux in weber and A = Area in m^2

5. **Magnetizing Force (H):** Magnetizing force or magnetic field strength or magnetic field intensity at any point is defined as the force experienced by a unit North pole placed at that point.

   Mathematically, $$H = \frac{NI}{l} \text{ ampere-turns/metre}$$

   Relationship between B, μ and H:

$$B = \mu\,H$$

6. **Equation for Flux (Ohm's Law of Magnetic Circuit):** Consider a toroid having relative permeability μ_r. Let l be the length of the magnetic path in metre and A be the area of cross-section in m^2.

   If a coil of N turns carries current of I ampere, it sets up a flux ϕ, given by

$$\phi = \frac{NI}{l / \mu_0 \mu_r A} \text{ weber}$$

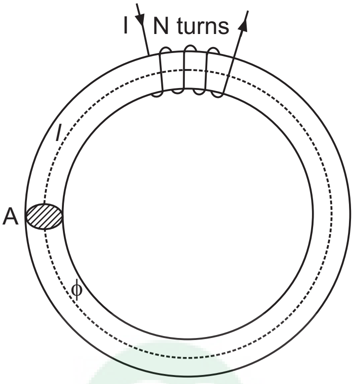


Fig. 2.2 Toroid

In the above equation:

(i) The numerator NI, which produces magnetization in the magnetic circuit is called the magneto motive force (mmf). Its unit is **ampere-turns (AT)**.

(ii) The denominator $\dfrac{l}{\mu_0 \mu_r A}$, which offers opposition for setting up of flux (or resistance offered to a magnetic circuit) is called the Reluctance (S). Its unit is **ampere-turns per weber (AT/Wb)**.

$$\therefore \qquad \text{Flux} = \frac{\text{mmf}}{\text{Reluctance}} \text{ weber}$$

7. **Electromagnetic Induction:** The phenomenon by which an emf and hence current is induced in a conductor that is cut by a magnetic flux is called the electromagnetic induction.

Electrical machines like generators/alternators, transformers and induction motors work on Faraday's Laws of Electromagnetic Induction (EMI).

Production of induced emf and current: Consider a steel rod wound with a coil of N turns and the terminals are connected to a Galvanometer (G) as shown in Fig. 2.3 (a).

If the North pole of a bar magnet is moved towards the coil as shown in Fig. 2.3(a), the Galvanometer (G) deflects indicating that an emf is induced and current sets up due to the movement of the magnet, which causes a change in the flux linking with the coil.

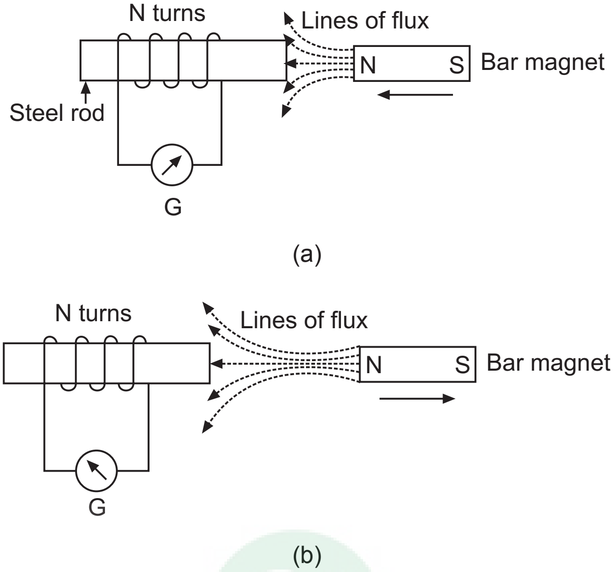


Fig. 2.3

Now, if the North pole of the bar magnet is moved away from the coil as shown in Fig. 2.3(b), the Galvanometer (G) deflects in reverse direction indicating that the current sets up in opposite direction.

An important point to be noted is that if the movement of the bar magnet is stopped, there is no change in the flux, even though the flux is linking with the coil.

2.3 FARADAY'S LAWS OF ELECTROMAGNETIC INDUCTION

Statement of First Law: When the magnetic flux linked with a circuit changes an emf is induced in it.

Alternatively, whenever a conductor cuts across magnetic lines of flux, an emf is induced in that conductor.

Statement of Second Law: The magnitude of the induced emf is equal to the rate of change of magnetic flux linkages.

Explanation: Consider a coil of N turns and a flux through it changes from an initial value ϕ_1 Wb to a final value ϕ_2 Wb in time *t* seconds.

$\therefore$ Flux linkages = number of turns × flux linking with the coil

Initial flux linkages = $N\phi_1$ and Final flux linkages = $N\phi_2$

$$\therefore \quad \text{Induced emf,} \qquad e = \frac{N\phi_2 - N\phi_1}{t} \text{ Wb/second}$$

$$= \frac{N(\phi_2 - \phi_1)}{t} \text{ volt}$$

In differential form,

$$e = -N\frac{d\phi}{dt} \text{ volt}$$

The direction of induced emf and current is given by Fleming's Right-Hand Rule.

(Note: The negative sign given to the right hand side of the expression is to signify the fact that the induced emf sets up a current in such a way that the magnetic effect produced by it opposes the very cause of producing it.)

2.4 FLEMING'S RIGHT-HAND RULE

Stretch the right-hand fingers such that the index finger (or forefinger), the middle-finger and the thumb at right angles to each other.

If the index finger points in the direction of magnetic lines of flux (or field) and the thumb points in the direction of motion of conductor relative to the magnetic field, then the middle-finger points in the direction of the induced emf.

Application: DC Generator.

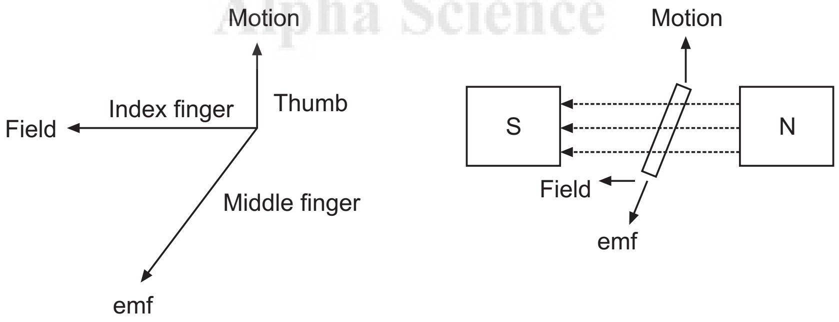


Fig. 2.4 Fleming's right-hand rule

2.5 LENZ'S LAW

The direction of induced emf is such that it tends to set up a current opposing the motion or the change of flux responsible for inducing that emf.

'In effect, that electro magnetically induced current always flows in such a way that the action of the magnetic field set up by it tends to oppose the very cause, which produces it'.

2.6 FORCE ON A CURRENT CARRYING CONDUCTOR PLACED IN A MAGNETIC FIELD

Whenever a current carrying conductor is placed in a magnetic field, it experiences a mechanical force, which acts in a direction perpendicular to both the direction of current and the magnetic field. This principle is used in measuring instruments, dc motor, etc.

If l is the length of a conductor in metre placed in a magnetic field of uniform flux density B Wb/m^2 and carries a current I ampere, the force exerted is given by

$$F = B\, I\, l \text{ newton}$$

The direction of force can be determined by Fleming's Left-Hand Rule.

Important Point

It is based on Newton's Third Law of Motion.

2.7 FLEMING'S LEFT-HAND RULE

Stretch the left-hand fingers such that the index finger (or forefinger), the middle finger and the thumb at right angles to each other.

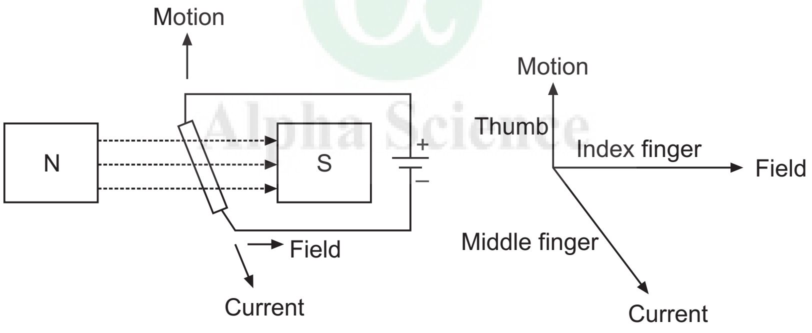


Fig. 2.5 Fleming's left-hand rule

If index finger points in the direction of magnetic lines of flux and the middle finger points in the direction of current, then the thumb points in the direction of the mechanical force exerted by the conductor.

Application: DC Motor and Measuring Instruments.

Solved Examples

Example 2.1 *A coil of resistance 100 Ω is placed in a magnetic field of 1 mWb. The coil has 100 turns, a Galvanometer of 400 Ω is connected in series with it. Find the average emf and current if the coil is moved in 1/10th second from the given field to a field of 0.2 mWb.*

Solution:

Given: Resistance of the coil, $R_c = 100\ \Omega$, Initial flux, $\phi_1 = 1$ mWb, N = 100, Resistance of the Galvanometer, $R_G = 400\ \Omega$, $dt = 1/10^{th}$ of second = 0.1 sec, Final flux, $\phi_2 = 0.2$ mWb.

Average emf, $$e = -N\frac{d\phi}{dt}$$

where $$d\phi = \phi_2 - \phi_1 = 0.2 - 1 = -8 \text{ Wb}$$

$\therefore$ $$e = -0.8 \text{ mWb} = -0.8 \times 10^{-3} \text{ Wb}$$

$$= -100\frac{\left(-0.8 \times 10^{-3}\right)}{0.1} = \mathbf{0.8\ V}$$

Induced current $$= \frac{e}{R_t}$$

where $$R_t = R_c + R_G = 100 + 400 = 500\ \Omega$$

$\therefore$ Induced current $$= \frac{0.8}{500} = 1.6 \times 10^{-3} \text{ A} = \mathbf{1.6\ mA}$$

Example 2.2 *A magnetic flux of 400μWb passing through a coil of 1200 turns is reversed in 0.1 s. Calculate the average value of the emf induced in the coil.*

Solution: Given: N = 1200

As flux is reversed, $\phi = (400 \times 10^{-6}) - (-400 \times 10^{-6}) = 800 \times 10^{-6}$ Wb, $t = 0.1$ sec.

Average emf induced, $$e = \frac{N\phi}{t}$$

$$= \frac{200 \times (800 \times 10^{-6})}{0.1} = \mathbf{1.6\ V}$$

Example 2.3 *A short coil of 200 turns surrounds the middle of a bar magnet. If the magnet sets up a flux of 80 μWb, calculate the average value of the emf induced in the coil when the latter is removed completely from the influence of the magnet in 0.05 s.*

Solution: Given: N = 200, $\phi = 80\ \mu\text{Wb} = 80 \times 10^{-6}$ Wb and $t = 0.05$ second

Average emf induced, $$e = \frac{N\phi}{t}$$

$$= \frac{200 \times \left(80 \times 10^{-6}\right)}{0.05} = \mathbf{0.32\ V}$$

2.8 DYNAMICALLY INDUCED EMF

Consider a conductor and a magnetic field. EMF can be induced in two ways; either the magnetic field remains stationary and the conductor is in motion or the conductor remains stationary and the magnetic field is in motion. Thus, the emf induced due to relative motion between the conductor and the magnetic field is called the dynamically (or motionally) induced emf.

Example: DC Generator and Alternator.

Expression for magnitude of dynamically induced emf from first principles:

Case (i):

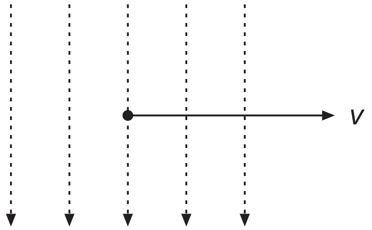


Fig. 2.6

Let l be the length of the conductor in metre moves at right angles to a uniform magnetic field of flux density B Wb/m^2 with a velocity v m/s.

If the conductor moves through a small distance dx in a small time dt, then

$$\text{Area swept by the conductor} = l \times dx$$

$$\therefore \quad \text{Flux cut} = \text{Flux density} \times \text{area swept by the conductor}$$

$$= \text{B} \times (l \times dx)$$

$$= \text{B}\, l\, dx$$

According to Faraday's Laws of EMI, emf induced in the conductor is given by

$$e = \frac{\text{flux cut}}{\text{time}}$$

$$= \text{B}\, l \frac{dx}{dt}$$

Now, velocity, $$v = \frac{dx}{dt} \text{ m/s}$$

$$\therefore \quad e = \text{B}\, l\, v \text{ volt} \qquad \text{...(2.1)}$$

Case (ii):

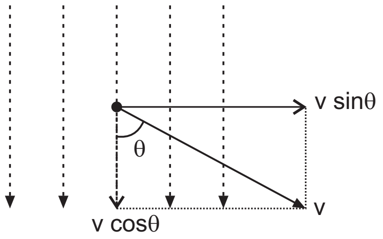


Fig. 2.7

Let the same conductor moves at an angle θ (with respect to the direction of magnetic field) in the same uniform magnetic field with same velocity.

The velocity v can be resolved into components;

(i) **Component $v\cos\theta$:** It is parallel to the magnetic field and hence does not induce any emf.

(ii) **Component $v\sin\theta$:** It is perpendicular to the magnetic field and hence induces an emf.

Therefore, the emf equation (2.1) can be rewritten as

$$e = B\,l\,v\sin\theta \text{ volt} \qquad \ldots(2.2)$$

Solved Examples

Example 2.4 *A conductor of active length 30 cm carries a current of 100 A and lies at right-angles to a magnetic field of density 0.4 T. Calculate the force in newton exerted on it. If the force causes the conductor to move at a velocity of 10 m/s. Calculate (a) the emf induced in it and (b) the power developed by it in watt.*

Solution: Given: $l = 30$ cm $= 30 \times 10^{-2}$ m, $I = 100$ A, $B = 0.4$ T and $v = 10$ m/s.

Force acting on the conductor,

$$F = B\,I\,l \qquad \text{(Note: This is motor action)}$$

$$= 0.4 \times 100 \times (30 \times 10^{-2}) = \mathbf{12\ N}$$

(a) The emf induced, $e = B\,l\,v$ (Note: This is generator action)

$$= 0.4 \times (30 \times 10^{-2}) \times 10 = \mathbf{1.2\ V}$$

(b) Power developed, $P = e \times I$

$$= 1.2 \times 100 = \mathbf{120\ W}$$

Example 2.5 *A straight conductor has an active length of 20 cm and moves in uniform magnetic field of 0.5 T at the rate of 5 m. Find the generated emf, if the motion of the conductor is (a) parallel, (b) perpendicular and (c) at an angle of 30° to the direction of the field.*

Solution: Given: (Active) length, $l = 20$ cm $= 20 \times 10^{-2}$ m,

$B = 0.5$ T (i.e. Wb/m^2), rate of movement of conductor, $v = 5$ m

According to equation.(2.2), dynamically induced emf,

$$e = B\,l\,v\sin\theta$$

(a) If the motion of the conductor is parallel, emf will not be induced.

Substituting the data given and $\theta = 0$ in the above equation,

$$e = 0.5 \times 0.2 \times 5\sin(0) = \mathbf{0}$$

(b) If the motion of the conductor is perpendicular, maximum emf will be induced

Substituting the data given and $\theta = 90^0$ in the above equation,

$$e = 0.5 \times 0.2 \times 5\sin(90°) = \mathbf{0.5\ V}$$

(c) If the motion of the conductor is at angle of 30°, an emf will be induced.

Substituting the data given and $\theta = 30°$ in the above equation,

$$e = 0.5 \times 0.2 \times 5 \sin(30°) = \mathbf{0.25\ V}$$

2.9 STATICALLY INDUCED EMF

The conductor remains stationary. If the flux linking with it is changed by variation of the current, an emf is induced called the statically induced emf.

Alternatively, an emf will be induced by variation of magnetic flux due to alternating current is called the statically induced emf.

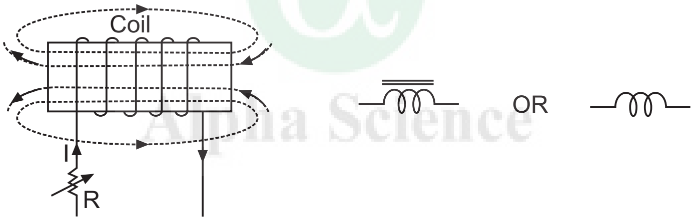
Examples: Choke or Transformer.

(Note: This device will not have any rotating part)

2.10 SELF- INDUCTANCE

Definition: The property of a coil due to which it opposes any variation of current.

Alternatively, the ability of a coil due to which it induces an emf in itself by variation of current through it is called the self-inductance.



(a) (b)

Fig. 2.8 (a) Self inductance (Solenoid) (b) Schematic representation

If a current of I ampere flowing through the coil is changed by means of a variable resistance, then the flux linked with its own turns will also change, which induces an emf called the self-induced emf or the back emf as it is induced due to the opposition to any variation of current.

Alternatively, an emf induced in the coil due to the change of its own flux linked with it is called the self-induced emf (e_L) or the back emf (e_b).

Example: Choke or Ballast.

Coefficient of Self-Inductance (L): Coefficient of self-inductance of a coil is defined as the weber-turns per ampere in the coil. Consider a coil of N turns carrying a current of I ampere and the flux produced is ϕ weber.

By definition, coefficient of self-inductance of a coil is given by weber-turns per ampere,

i.e.,
$$L = \frac{N\phi}{I}$$

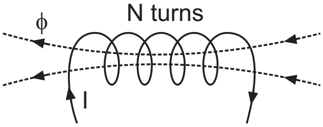


Fig. 2.9

The unit of self-inductance is **henry (H)**. A coil is said to have a self-inductance of 1 henry, if a current of 1ampere flowing through it produces a flux linkage of 1 weber-turn in it.

$$\therefore \qquad L = \frac{N\phi}{I} \text{ henry} \qquad ...(2.3)$$

In terms of Reluctance (S): Consider a toroid having relative permeability $\mu_{r.}$ Let l be the length of the magnetic path in metre and A be the area of cross-section in m^2. If the coil of N turns carries a current of I ampere, it sets up a flux, given by

$$\phi = \frac{\text{mmf}}{\text{Reluctance}} \text{ weber}$$

$$= \frac{NI}{\dfrac{l}{\mu_0 \mu_r A}} \text{weber}$$

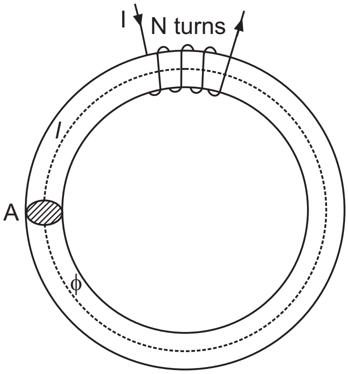


Fig. 2.10 Toroid

Substituting for ϕ in equation (2.3),

$$L = \frac{N}{I} \times \frac{NI}{\dfrac{l}{\mu_0 \mu_r A}}$$

$$= \frac{N^2}{\dfrac{l}{\mu_0 \mu_r A}}$$

$$= \frac{N^2}{S} \text{ henry}$$

where S is the reluctance expressed in AT/Wb.

Also, $$L = \frac{N^2 \mu_0 \mu_r A}{l}$$

Expression for self-induced emf (in differential form):

By definition, $$L = \frac{N\phi}{I} \text{ henry}$$

or $$N\phi = L\,I$$

Multiplying both the sides by –ve sign,

$$-N\phi = -L\,I$$

Differentiating the above equation both sides with respect to *t*

$$-\frac{d}{dt}(N\phi) = -L\frac{dI}{dt} \quad \text{(Assuming that L is constant)}$$

or $$-N\frac{d}{dt} = -L\frac{d}{dt}$$

Now, $$\text{induced emf} = -N\frac{d\phi}{dt} \text{ volt}$$

Therefore, induced emf or self-induced emf or back emf is given by

$$e \text{ or } e_L \text{ or } e_b = -L\frac{dI}{dt} \text{ volt}$$

A coil has a self-inductance of 1 henry, if 1 volt is induced in it when current through it changes at the rate of 1 ampere per second.

2.11 MUTUAL INDUCTANCE

Definition: It is defined as the ability of a coil or circuit to produce an emf in a nearby coil by induction, when the current in the first coil changes.

Consider two magnetically coupled coils 1 and 2 which, are close to each other, whenever the current I in coil 1 changes, the flux linking with coil 2 changes and an emf called mutually induced emf is produced in coil 2.

This action being reciprocal; the coil 2 can also induce an emf in coil 1 when the current in coil 2 changes. This ability of reciprocal induction is measured in terms of the Coefficient of Mutual Inductance.

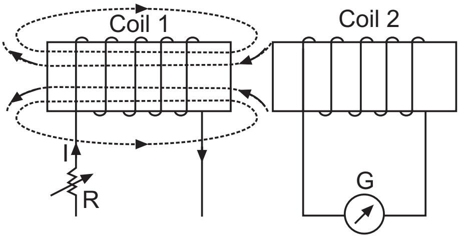


Fig. 2.11 Magnetically coupled coils

Example: Transformer.

Coefficient of Mutual Inductance (M): Consider two magnetically coupled coils 1 and 2 having N_1 and N_2 turns respectively. Coefficient of mutual inductance between the two coils is defined as the weber-turns in a coil due to 1 ampere current in the other.

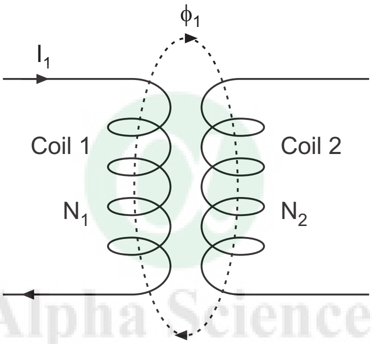


Fig. 2.12

Let a current of I_1 ampere flows in coil 1 having N_1 turns will produce a flux ϕ_1 weber in it and assumed that this entire flux links with coil 2 having N_2 turns.

The coefficient of mutual inductance between two coils is defined as weber-turns in coil 2 for unit current in coil 1.

i.e.,
$$M = \frac{N_2\phi_1}{I_1}$$

The unit of mutual inductance is **henry (H)**. Hence, two coils are said to have a mutual inductance of 1 henry, if 1 ampere current flowing in one coil produces flux linkage of 1weber-turn in the other.

$$\therefore \qquad M = N_2\frac{\phi_1}{I_1} \text{ henry} \qquad ...(2.4)$$

In terms of Reluctance(S): Consider two magnetically coupled coils 1 and 2 wound on an iron ring of relative permeability μ_r.

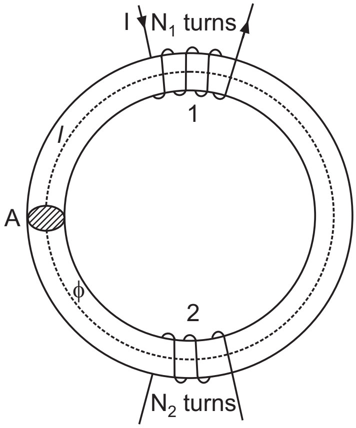


Fig. 2.13 Magnetically coupled coils

Let l be the length of magnetic path in metre and A be the cross-sectional area in m^2.

By definition, flux, $$\phi = \frac{\text{mmf}}{\text{Reluctance}} \text{ weber}$$

Flux in coil 1, $$\phi_1 = \frac{N_1 I_1}{\dfrac{l}{\mu_0 \mu_r A}} \text{ weber}$$

or flux/ampere, $$\frac{\phi_1}{I_1} = \frac{N_1}{\dfrac{l}{\mu_0 \mu_r A}}$$

Substituting the above equation in equation (2.4), assuming that entire flux ϕ_1 links with coil 2 having N_2 turns, then weber-turns in it due to flux/ampere in coil 1 is given by

$$M = N_2 \times \frac{N_1}{\dfrac{l}{\mu_0 \mu_r A}}$$

$$= \frac{N_1 N_2}{\dfrac{l}{\mu_0 \mu_r A}}$$

$$= \frac{N_1 N_2}{S} \text{ henry} \qquad \text{(where, S = Reluctance)}$$

Also, $$L = \frac{N_1 N_2 \ \mu_0 \ \mu_r \ A}{l}$$

Expression for mutually induced emf (in differential form):

By definition, $$M = N_2 \frac{\phi_1}{I_1} \text{ henry}$$

or $$N_2\,\phi_1 = M\,I_1$$

Multiplying both the sides by –ve sign,

$$-N_2\,\phi_1 = -M\,I_1$$

Differentiating the above equation both sides with respect to t,

$$-\frac{d}{dt}(N_2\phi_1) = -M\frac{dI_1}{dt} \qquad \text{(Assuming that M is constant)}$$

or $$-N_2\frac{d\phi_1}{dt} = -M\frac{dI_1}{dt}$$

Now, mutually induced emf, $$e_M = -N_2\frac{d\phi_1}{td}$$

$$\therefore \qquad e_M = -M\frac{dI_1}{dt}$$

2.12 INDUCTANCES CONNECTED IN SERIES

In series connection, two coils 1 and 2 having self-inductances of L_1 and L_2 respectively are connected in series. Let M be the mutual inductance between them.

Case (1) Series aiding or cumulative coupled connection: In this case the current flow through two coils is in same direction and thus the flux due to two inductive coils is in the same direction.

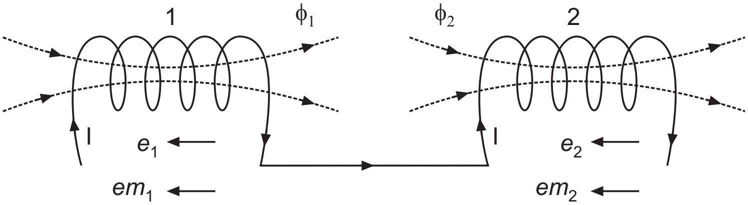


Fig. 2.14

Self-induced emf in coil 1, $$e_1 = -L_1\frac{dI}{dt} \text{ volt}$$

Mutually induced emf in coil 1 due to change of current in 2,

$$e_{m1} = -M\frac{dI}{dt} \text{ volt}$$

Self-induced emf in coil 1, $$e_2 = -L_2\frac{dI}{dt} \text{ volt}$$

Mutually induced emf in coil 2 due to change of current in 1,

$$e_{m2} = -M\frac{dI}{dt} \text{ volt}$$

Induced emf, $$e = -\frac{dI}{dt}\left(L_1 + L_2 + 2M\right) \text{ volt} \quad ...(2.5)$$

If L is the equivalent inductance, then total emf induced,

$$e = L\frac{dI}{dt} \text{ volt} \quad ...(2.6)$$

Equating equations (2.5) and (2.6), the equivalent inductance is

$$L = L_1 + L_2 + 2M$$

Case (2) Series opposition or differentially coupled connection: In this case the current flow through the coils is in opposite direction and thus the flux due to one inductive coils opposes the flux due to the other coil.

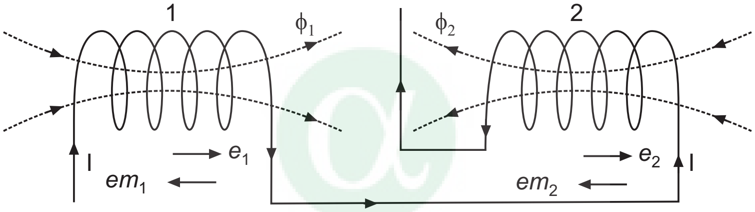


Fig. 2.15

As explained in the previous case, $e_1 = -L_1\frac{dI}{dt}$ volt and $e_{m1} = M\frac{dI}{dt}$ volt

$$e_2 = -L_1\frac{dI}{dt} \text{ volt and } e_{m2} = M\frac{dI}{dt} \text{ volt}$$

Induced emf, $$e = -\frac{dI}{dt}\left(L_1 + L_2 - 2M\right) \text{ volt}$$

Equating equations (2.5) and (2.6), the equivalent inductance is

$$L = L_1 + L_2 - 2M$$

2.13 COEFFICIENT OF MAGNETIC COUPLING (K)

Two coils are said to be magnetically coupled if full or a part of the flux produced by one coil links with the other.

If L_1 and L_2 are the self inductances of the two coils and M is the mutual inductance between them, then the coefficient of magnetic coupling is given by

$$K = \frac{M}{\sqrt{L_1 L_2}}$$

Thus, coefficient of magnetic coupling is defined as the ratio of actual mutual inductance between the two coils to the maximum possible value.

Expression for K: Let two coils 1 and 2 having N_1 and N_2 number of turns and carrying currents I_1 and I_2 respectively. The flux produced by the coil 1 is ϕ_1 and coil 2 is ϕ_2.

The mutual inductance between them is given by

$$M = \frac{N_2 k_1 \phi_1}{I_1}$$

and $$M = \frac{N_1 k_2 \phi_2}{I_2}$$

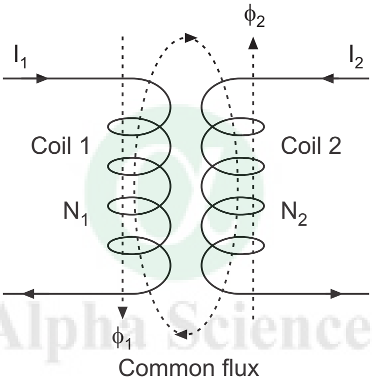


Fig. 2.16

Multiplying them,

$$M \times M = \frac{N_2 k_1 \phi_1}{I_1} \times \frac{N_1 k_2 \phi_2}{I_2}$$

or $$M^2 = k_1 k_2 \left(\frac{N_1 \phi_1}{I_1}\right) \times \left(\frac{N_2 \phi_2}{I_2}\right)$$

where k_1 is the fraction of ϕ_1 linking with coil 2 and k_2 is the fraction of flux ϕ_2 linking with coil 1.

In above equation,

$$\left(\frac{N_1 \phi_1}{I_1}\right) = L_1 \Rightarrow \text{self inductance of coil 1}$$

$$\left(\frac{N_2 \phi_2}{I_2}\right) = L_2 \Rightarrow \text{self inductance of coil 2}$$

$$\therefore \qquad M^2 = k_1 k_2 L_1 L_2$$

$$\text{or} \qquad M = \sqrt{k_1 k_2}\sqrt{L_1 L_2}$$

If the entire flux produced by one coil links with other, $k_1 = k_2 = K$. Then, maximum mutual inductance existing between them is given by

$$M = K\sqrt{L_1 L_2}$$

$$\text{or} \qquad K = \frac{M}{\sqrt{L_1 L_2}}$$

Important Points

- If the entire flux produced by one coil links with other, then $k_1 = k_2 = K = 1$(unity) and maximum mutual inductance between the coils is $M = \sqrt{L_1 L_2}$.
- If K = 1(maximum), the coils are said to be tightly coupled and if K is a fraction, the coils are said to be loosely coupled.
- If there is no common flux between the two coils they are said to be magnetically isolated, i.e., $M = 0$ and $K = 0$.

2.14 ENERGY STORED IN A MAGNETIC FIELD

Let i be the instantaneous value of current in t seconds and L be the coefficient of self-inductance. Then induced emf at that instant is given by

$$e = L\frac{di}{dt} \text{ volt} \qquad \text{...(2.7)}$$

$\therefore$ Instantaneous power, $p = e\,i$

Substituting for e from equation (2.7),

$$p = L\frac{di}{dt}i$$

$$= Li\frac{di}{dt} \qquad \text{...(2.8)}$$

Then work done in time dt to overcome this opposition is given by

$$dw = p.dt$$

Substituting for p from equation (2.8),

$$dw = Li\frac{di}{dt}dt$$

$$= Li\,di$$

Therefore, the total work done W in establishing the maximum steady current I is given by

$$\int_0^w dw = \int_0^I L\,i\,di$$

or $$W = \frac{1}{2} L I^2$$

If L is in henry and I in ampere, then energy stored in magnetic field is given by

$$W = \frac{1}{2} L I^2 \text{ joule}$$

Solved Examples

Example 2.6 *Calculate the inductance and energy stored in the magnetic field of a air-cored solenoid 100 cm long 5 cm in diameter and wound with 1000 turns, if the current rises from 0 to 10 A.*

Solution: Given: $l = 100$ cm $= 100 \times 10^{-2}$ m, diameter $d = 5$ cm $= 5 \times 10^{-2}$ m,

N = 1000, I = 10 A and $\mu_r = 1$ for air-cored solenoid.

Self-inductance, $$L = \frac{N^2 \mu_0 \mu_r A}{l} \qquad \text{...(2.9)}$$

where μ_0 = Absolute permeability = $4\pi \times 10^{-7}$ H/m

Cross-sectional area, $$A = \frac{\pi d^2}{4} = \frac{\pi \times (5 \times 10^{-2})^2}{4} = 1.963 \times 10^{-3} \text{ m}^2$$

Substituting the values for μ_0, A and the given data in equation (2.9),

$$L = \frac{(4\pi \times 10^{-7}) \times 1 \times (1.963 \times 10^{-3}) \times (1000)^2}{100 \times 10^{-2}}$$

$$= \mathbf{2.467 \times 10^{-3}\ H = 2.467\ mH}$$

Energy stored in magnetic field, $$W = \frac{1}{2} L I^2$$

$$= \frac{1}{2} \times (2.467 \times 10^{-3}) \times (10)^2 = \mathbf{0.123\ J}$$

Example 2.7 *A coil of 1500 turns, gives rise to a magnetic flux of 2.5 mWb, when carrying a certain current. If the current is reversed in 0.2 seconds, what is the average value of the emf induced in the coil?*

Solution: Given: N = 1500, $\phi = 2.5$ m Wb $= 2.5 \times 10^{-3}$ Wb.

As current is reversed in 0.2 second, $dt = 0.2$.

Average emf induced, $$e = N\frac{d\phi}{dt} \text{ (ignoring negative sign)}$$

Reversal of current causes the reversal of flux, i.e., change in flux, $d\phi = (2.5 \times 10^{-3}) - (-2.5 \times 10^{-3}) = 5 \times 10^{-3}$ Wb

$$\therefore \qquad e = 1500\left(\frac{5\times10^{-3}}{0.2}\right) = \mathbf{37.5\ V}$$

Example 2.8 *A coil of 300 turns wound on a core of non-magnetic material has an inductance of 10 mH. Calculate (a) The flux produced by a current of 5 A and (b) The average value of the emf induced when a current of 5 A is reversed in 8 m second.*

Solution: Given: N = 300, L = 10 mH = 10×10^{-3} H, I = 5 A

and $t = 8$ ms $= 8 \times 10^{-3}$ second

Flux produced, $\phi = \dfrac{L \times I}{N}$ $\left(\text{because, } L = \dfrac{N\phi}{I}\right)$

$$= \frac{(10\times10^{-3})\times5}{300} = \mathbf{0.167 \times 10^{-3}\ Wb = 0.167\ mWb}$$

Average emf induced, $e = L\dfrac{dI}{dt}$

As the current changes from 5 A to – 5 A, $dI = 5 - (-5) = 10$

Then, average rate of change of current $= \dfrac{dI}{dt} = \dfrac{10}{8\times10^{-3}} = 1250$ A/s

$$\therefore \qquad e = 0.01 \times 1250 = \mathbf{12.5\ V}$$

Example 2.9 *A flux of 0.5 mWb is produced in a coil of 900 turns wound on a wooden ring due to a current of 3 A. Calculate (a) the inductance of the coil, (b) the average emf induced in the coil when a current of 5 A is switched off, assuming that the current to fall to zero in 1 ms and (c) the mutual inductance between the coils, if a second coil of 600 turns was uniformly wound over the first coil.*

Solution: Given: $\phi = 0.5$ mWb $= 0.5 \times 10^{-3}$ Wb, $N_1 = 900$ and $N_2 = 600$, I = 3 A,

Initial current, $I_1 = 5$ A and Final current, $I_2 = 0$, $t = 1$ ms $= 1 \times 10^{-3}$ s

Also, when second coil is wound over the first coil, then flux remains the same

(a) Inductance of the coil, $L = \dfrac{N_1\phi}{I}$

$$= \frac{900\times\left(0.5\times10^{-3}\right)}{3} = \mathbf{0.15\ H}$$

(b) Average induced emf, $e = L \times$ average rate of change of current

The average rate of change of current $= \left(\dfrac{I_2 - I_1}{t}\right) = \dfrac{0-5}{1\times10^{-3}} = -\,5000$ A/s

$\therefore$ Average induced emf, $e = (0.15 \times -5000) = \mathbf{-750\ V}$

(c) Mutual inductance between the coils, $M = N_2 \dfrac{\phi}{I}$

$$= 600 \times \frac{\left(0.5 \times 10^{-3}\right)}{3} = \mathbf{0.1\ H}$$

Example 2.10 *If an emf of 5 volt is induced in a coil when the current in an adjacent coil varies at a rate of 80 A/s, what is the value of the mutual inductance of the two coils?*

Solution: Given: $e = 5$ volt and $\dfrac{di}{dt} = 80$ A/m

Average emf induced, $e = M\dfrac{di}{dt}$

or mutual inductance, $M = \dfrac{e}{\frac{di}{dt}} = \dfrac{5}{80} = \mathbf{0.0625\ H = 62.5\ mH}$

Example 2.11 *A non-magnetic ring having a mean diameter of 30 cm and cross-sectional area of 4 cm^2 is uniformly wound with two coils A and B one over the other. A has 90 turns and B has 240 turns. Calculate the mutual inductance between the coils. Also, calculate the emf induced in B when a current of 6 A is reversed in 0.02 seconds.*

Solution: Given: Mean diameter $D = 30$ cm $= 30 \times 10^{-2}$ m, $A = 4$ cm$^2 = 4 \times 10^{-4}$ m^2

$N_A = 90 \times N_B = 240$, Length of Iron path, $l = \pi D = \pi \times (30 \times 10^{-2}) = 0.942$ m

For non-magnetic material, $\mu_r = 1$, $dI = 6$ A, $dt = 0.02$

Also, if windings are wound one over the other, the flux remains constant.

Mutual-inductance, $M = \dfrac{N_A N_B\ \mu_0 \mu_r A}{l}$

$$= \frac{90 \times 240 \times \left(4\pi \times 10^{-7}\right) \times 1 \times \left(4 \times 10^{-4}\right)}{0.942}$$

$$= \mathbf{1.15 \times 10^{-5}\ H}$$

Mutually induced emf, $e_M = M\dfrac{dI}{dt}$

$$= (1.15 \times 10^{-5}) \frac{\left[6 - (-6)\right]}{0.02} = \mathbf{6.9 \times 10^{-3}\ V}$$

Example 2.12 *Two coils having 50 and 500 turns are wound side by side on a closed Iron circuit of cross-section 50 cm^2 and mean length 120 cm. Estimate the self-inductance of each coil, mutual inductance between them if the relative permeability of Iron is 1000. If the current in one coil grows steadily from zero to 5 A in 0.01 s, find the induced emf in the other.*

Solution: Given:

$$N_1 = 50 \times N_2 = 500$$

$$\left.\begin{aligned} A &= 50\ \text{cm}^2 = 50 \times 10^{-4}\ \text{m}^2 \\ l &= 120\ \text{cm} = 120 \times 10^{-2}\ \text{m} \\ \mu_r &= 1000 \end{aligned}\right\}\text{Same for both the coils}$$

$$dI = 5\ \text{A},\ dt = 0.01$$

Self-inductance, $L = \dfrac{N^2}{S}$

Now reluctance, $S = \dfrac{l}{\mu_0 \mu_r A}$

$$= \frac{120 \times 10^{-2}}{\left(4\pi \times 10^{-7}\right) \times 1000 \times \left(50 \times 10^{-4}\right)} = 190985.93\ \text{AT/Wb}$$

$\therefore$ Self-inductance of first coil, $L_1 = \dfrac{N_1^2}{S}$

$$= \frac{(50)^2}{190985.93} = \mathbf{0.013\ H}$$

Self-inductance of second coil, $L_2 = \dfrac{N_2^2}{S}$

$$= \frac{(500)^2}{190985.93} = \mathbf{1.309\ H}$$

Mutual Inductance, $M = \dfrac{N_1 N_2}{S}$

$$= \frac{50 \times 500}{190985.93} = \mathbf{0.13\ H}$$

$\therefore$ Mutually induced emf, $e_M = M\dfrac{dI}{dt}$

$$= 0.13\frac{(5-0)}{0.01} = \mathbf{65\ V}$$

Example 2.13 *Two identical coils A and B each having 1000 turns lie in parallel plane such that 60 % of the lines of force produced by one coil links with the other. A current of 5 A in coil A produces a flux of 0.05 mWb. If the current in the coil A reverses from 6 A in 0.01 second, what will be the magnitude of emf induced in B? Also, calculate the self-inductance of each coil.*

Solution: Given:

$$N_A = N_B = 1000$$

$$\phi_1 = 0.05\ \text{mWb} = 0.05 \times 10^{-3}\ \text{Wb},\ I_1 = 5\text{A}$$

$$\phi_2 = 0.6 \times (0.05 \times 10^{-3}) = 3 \times 10^{-5} \text{ Wb}$$

$$\frac{dI_1}{dt} = \frac{[6-(-6)]}{0.01} = 1200 \text{ A/s}$$

Mutually induced emf in coil B, $e_{MB} = M\dfrac{dI_1}{dt}$

Now mutual inductance, $M = N_2 \dfrac{\phi_2}{I_2} = 1000\dfrac{(3 \times 10^{-5})}{5} = 0.006 \text{ H}$

$\therefore$ $e_{MB} = 0.006 \times 1200 =$ **7.2 V**

Self-inductance of both the coils, $L = N_1 \dfrac{\phi_1}{I_1}$

$$= 1000\frac{(0.05 \times 10^{-3})}{5} = \mathbf{0.01\ H}$$

Example 2.14 *The two windings of a transformer have inductance of 6 H and 0.06 H respectively with a coefficient of coupling K = 0.9. Find the emf induced in both windings when the primary current increases at the rate of 100 ampere/sec.*

Solution: Given: $L_p = 6$ H, $L_q = 0.06$ H, K = 0.9, $\dfrac{dI_1}{dt} = 100$ A/sec

Self induced emf in primary, $e_L = L\dfrac{dI_1}{dt}$

$= 6 \times 100 =$ **600 V**

Mutually induced emf, $e_M = M\dfrac{dI_1}{dt}$

where mutual inductance, $M = K\sqrt{L_p L_q} = 0.9\sqrt{6 \times 0.06} = 0.54 \text{ H}$

$\therefore$ Mutually induced emf, $e_M = 0.54 \times 100 =$ **54 V**

Example 2.15 *Two coils A of 11450 turns and B of 14500 turns lie in parallel planes so that 65% of flux produced in A links coil B. It is found that a current of 6A in coil A produces a flux of 0.7 mWb while the same current in coil B produces 0.9 mWb. Determine (i) Mutual inductance and (ii) Coefficient of coupling.*

Solution: Given: $N_1 = 11450$, $N_2 = 14500$, $\phi_2 = 0.65\phi_1$,

$I = 6$A, $\phi_1 = 0.7$ mWb. $= 0.7 \times 10^{-3}$ Wb and $\phi_2 = 0.9$ mWb $= 0.9 \times 10^{-3}$ Wb.

Mutual inductance, $M = \dfrac{N_2 k\phi_1}{I} = \dfrac{14500 \times (0.65 \times 0.7 \times 10^{-3})}{6} = \mathbf{1.1\ H}$

Coefficient of coupling, $K = \dfrac{M}{\sqrt{L_1 L_2}}$

Now $$L_1 = \frac{N_1 \phi_1}{I} = \frac{11450 \times 0.7 \times 10^{-3}}{6} = 1.336 \text{ H}$$

$$L_1 = \frac{N_2 \phi_2}{I} = \frac{14500 \times 0.9 \times 10^{-3}}{6} = 2.175 \text{ H}$$

$\therefore$ $$K = \frac{1.1}{\sqrt{1.336 \times 2.175}} = \mathbf{0.645}$$

2.15 COMPARISON BETWEEN MAGNETIC AND ELECTRIC CIRCUITS

Table 2.1

S.No.	Magnetic Circuit	Electric Circuit
1.	$\text{Flux} = \dfrac{\text{MMF}}{\text{Reluctance}}$ weber	$\text{Current} = \dfrac{\text{EMF}}{\text{Resistance}}$ ampere
2.	MMF, AT	EMF, volt
3.	Reluctance, AT/weber	Resistance, ohm(Ω)
4..	Flux Density	Current Density
5..	Permeability	Conductivity
6.	Flux does not actually flow	Electric current flows
7.	Requires electrical energy for establishing the flux	Flow of current consumes electrical energy due to opposition offered by resistance
8.	For a particular temperature, permeability depends upon flux density or total flux established through the material	If temperature is maintained constant, then resistance remains constant and independent of current strength
9.	When mmf is zero, flux will not be zero due to residual magnetism	When emf is zero, current will not exist

Additional Information

Relative permeability μ_r for pure Iron is 5000, mild steel is 2000 and silicon steel is 7000.

Multiple Choice Questions

Choose the Correct Answer

1. Magnetic flux can be obtained by
(a) current carrying conductor (b) permanent magnet
(c) both (a) and (b) (d) none of these

2. AT/m is the unit of
(a) mmf (b) reluctance
(c) magnetizing force (d) magnetic flux density

3. The unit of magnetic flux is
(a) weber (b) weber/second (c) weber/m (d) weber/m^2

4. Magnetic flux sets up more easily in
(a) air (b) wood (c) iron (d) vacuum

5. Magnetic flux is the ratio of mmf to
(a) current (b) inductance (c) resistance (d) reluctance

6. The unit of flux density is
(a) Newton/metre(b) weber (c) weber/m^2 (d) Tesla/m2

7. The unit of inductance is
(a) Weber (b) Farad (c) Weber/m^2 (d) Henry

8. The unit of MMF is
(a) weber-turns (b) weber/m (c) ampere/m (d) ampere-turns

9. If a current carrying conductor is placed in a magnetic field it experiences- force
(a) mechanical (b) electrical (c) chemical (d) both (a) and (b)

10. The unit of reluctance is
(a) AT/Wb (b) Wb/AT (c) weber-turns (d) AT

11. The emf induced in a coil of N turns is
(a) $N(d\phi/dt)$ (b) $N(d\phi/di)$ (c) $-N(d\phi/dt)$ (d) $L(d\phi/dt)$

12. Direction of induced emf and current is given by
(a) Fleming's Right-Hand Rule (b) Fleming's Left-Hand Rule
(c) both (a) and (b) (d) none of the above

13. Fleming's Right-Hand Rule is applicable for
(a) transformer (b) motor (c) generator (d) none of these

14. Fleming's rules cannot be applied for .... induced emf
    (a) chemically (b) dynamically (c) statically (d) motionally
15. According to Lenz's Law, the current in a coil always .... the cause of producing it.
    (a) helps for (b) opposes
    (c) both (a) and (c) (d) none of these
16. A generator works on the production of .... induced emf
    (a) chemically (b) dynamically (c) statically (d) both (a) and (c)
17. The principle of statically induced emf is utilized in
    (a) motor (b) generator (c) battery (d) transformer
18. If the number of turns of a coil is doubled, its inductance
    (a) increases two times (b) decreases to half
    (c) increases four times (d) becomes zero
19. If the Iron core of an Iron-cored coil is removed, then the inductance of air core will
    (a) increase (b) decrease (c) remains the same (d) be doubled
20. The magnitude of statically induced emf depends upon
    (a) the rate of change of flux (b) the magnitude of flux
    (c) the coil resistance (d) none of these
21. An inductor of 5 mH has its core length of 10 cm. If the core length is doubled, what will be the value of inductance if all other quantities remaining the same?
    (a) 2.5 mH (b) 5 mH (c) 1.25 mH (d) 10 mH
22. An inductor of 3.5 mH has a coil of 500 turns. If the number of turns is doubled, what will be the value of inductance if all other quantities remaining the same?
    (a) 7 mH, (b) 2.5 mH (c) 14 mH (d) 3.5 mH
23. An emf of 7.2V is induced in a coil of 6mH. The rate of change of current is
    (a) 12 A/s (b) 120 A/s (c) 1200 A/s (d) 12000 A/s
24. The mutual inductance between two magnetically coupled coils is 100 mH. If the number of turns of first coil is reduced to half and that of the second is doubled, what will be the value of mutual inductance?
    (a) 200 mH (b) 100 mH (c) 50 mH (d) 400 mH
25. Coefficient of mutual inductance is defined as the ability of a coil or circuit to produce an emf in a nearby coil by induction, when the current in the
    (a) first coil changes (b) other coil changes.
    (c) first coil remains constant (d) second coil remains constant

26. If coefficient of magnetic coupling < 1, then the coils are said to be
    (a) tightly coupled (b) loosely coupled
    (c) no common flux (d) not magnetically coupled

27. The ideal coefficient of coupling is
    (a) zero (b) one (c) 0.75 (d) 0.5

28. When all the flux due to current in one coil links with other coil, the mutual inductance between coils is given by
    (a) $M = \sqrt{L_1 L_2}$ (b) $M = K\sqrt{L_1 L_2}$
    (c) $M = L_1 L_2$ (d) $M = L_1 L_2/2$

29. The flux linkage between the coils is maximum, when m =
    (a) $1/\sqrt{L_1 L_2}$ (b) $\sqrt{L_1 L_2}$ (c) $L_1 = L_2$ (d) L_1/L_2

30. The maximum value of co-efficient of coupling is
    (a) 100% (b) more than 100%
    (c) 90% (d) none of these

Answers:—

1. (c) 2. (c) 3. (a) 4. (c) 5. (d) 6. (c) 7. (d) 8. (d) 9. (a) 10. (a)
11. (c) 12. (a) 13. (c) 14. (c) 15. (b) 16. (b) 17. (d) 18. (c) 19. (b) 20. (a)
21. (a) 22. (c) 23. (c) 24. (b) 25. (a) 26. (b) 27. (b) 28. (a) 29. (b) 30. (a)

Review Questions

1. Define the following terms: (i) Permeability and (ii) Magnetic flux density.
2. Explain the electromagnetic induction?
3. State and explain Faraday's Laws of Electromagnetic Induction.
4. Distinguish between statically induced emf and dynamically induced emf. Give examples.
5. Derive the expression for the magnitude of dynamically induced emf.
6. State (i)Fleming's Right-Hand Rule (ii)Fleming's Left-Hand Rule and (iii) Lenz's Law. Mention the applications for (i) and (ii).
7. Explain Fleming's Right-Hand Rule and Fleming's Left-Hand Rule as applied to electrical machines.
8. Define (i) Self-inductance and (ii) Mutual inductance. Mention their unit and write the formulae to calculate each of them.
9. Derive the expressions for calculating the self-inductance and mutual inductance.

10. Derive the expressions for self-induced emf and mutually induced emf
11. Derive an expression for the energy stored in an inductor of self inductance L henry carrying a current of I ampere.
12. What do you mean by coefficient of magnetic coupling?
13. Define coefficient of coupling and find its relation with L_1, L_2 and M.
14. With usual notations derive an expression for coefficient of magnetic coupling.
15. Bring out the comparison between magnetic circuit and electric circuit.

Exercises

1. The current in a coil decreases from 10 A to 4 A in 0.1 second. If the inductance of the coil is 4H, what is the emf induced in the coil? (**Ans:** 240 V)
2. A flux of 0.4 mWb is created by a current of 10 A flowing through a coil of 100 turns. If the current reverses in 0.01 second, what is the inductance of the coil? (**Ans:** 4 mH)
3. An air cored coil of length 2.5 cm has an average cross-sectional area of 2 cm^2. How many number of turns are required to give an inductance of 400μH? (**Ans:** 200)
4. A current-carrying conductor is situated at right angles to a uniform magnetic field having a density of 0.3 T. Calculate the force, (in newton /m length)on the conductor when the current is 200 A. (**Ans:** 60 N/m)
5. A conductor of 150 mm long is carrying a current of 60 A at right angles to a magnetic field. The force on the conductor is 3 N. Calculate the density of the field. (**Ans:** 50 A)
6. Calculate the inductance of a circuit in which 30 V are induced when the current varies at the rate of 200 A/s. (**Ans:** 0.15 H)
7. A certain coil is wound with 50 turns and a current of 8 A produces a flux of 200 μWb. Calculate (a) the inductance of the coil corresponding to a reversal of the current, (b) the average emf induced when the current is reversed in 0.2 second. (**Ans:** 1.25 μWb, 0.1 V)
8. A large electromagnet is wound with 1000 turns. A current of 2 A in this winding produces a flux through the coil of 0.03 Wb. Calculate the inductance of the electromagnet. If the current in the coil is reduced from 2 A to zero in 0.1 second, what average emf will be induced in the coil ? Assume that there is no residual flux. (**Ans:** 15 H, 300 V)
9. A coil consists of 750 turns and a current of 10 A in the coil gives rise to a magnetic flux of 1200 μWb. Calculate the inductance of the coil, and determine the average emf induced in the coil when this current is reversed in 0.01 second. (**Ans:** 0.09 H, 180 V)

10. If two coils have a mutual inductance of 400 μH, calculate the emf induced in one coil when the current in the other coil varies at a rate of 30000 A/m.
(**Ans:** 12 V)

11. A coil with 250 turns carries a current of 2A and produces flux of 0.3 mWb. When this current is reduced to zero in 2 millisecond, the voltage induced in a nearby coil is 60 V. calculate (i) Self-inductance of each coil, (ii) Mutual inductance between the coils. Assume coefficient of coupling as 0.7.
(**Ans:** L_1 = 0.037 H, M = 0.06 H, L_2 = 0)

12. If the current through a coil having an inductance of 0.5 H is reduced from 5A to 2A in 0.05 second, calculate the mean value of the emf induced in the coil. (**Ans:** –30 V)

13. A coil of 1500 turns carrying a current of 5 A produces a flux of 2.5 mWb. If the current is reversed in 0.2 second (or in 1/20th of a second), find the average value of emf induced in the coil. Also, find the self-inductance of the coil.
(**Ans:** L = 0.75 H and *e* = 37.5 V)

14. Two magnetically coupled coils have a co-efficient of coupling 0.8. the current in coil P is 3A and total flux 0.4 mWb. The voltage induced in coil Q is 85V when current in coil is reduced to zero in 3 msec. The number of turns of coil P is 300. Determine L_1, M, L_2 and number of turns of coil Q.
(**Ans:** L_1 = 40 mH, M = 0.085 H, L_2 = 282 mH and N_B = 80)

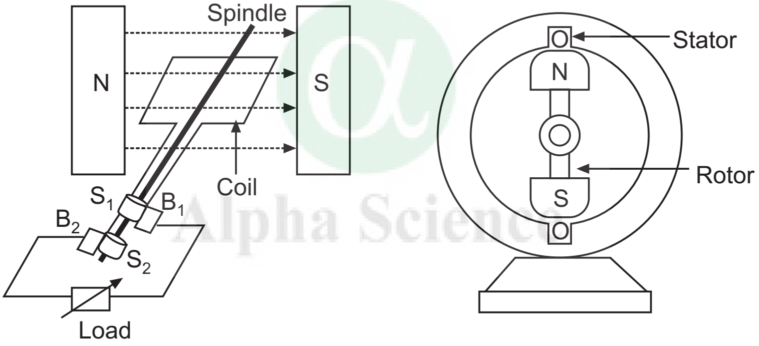
15. The winding of an electromagnet is wound with 96 turns and has a resistance of 50 Ω. The exciting voltage is 250V and flux linking the coil is 5 mWb. Find the energy stored in the magnetic field. Then, if the current is reversed in 0.1 second, what emf is induced in the coil? (**Ans:** W = 1.2 J and *e* = –9.6 V)

Chapter 3

Single-phase AC Circuits

3.1 GENERATION OF SINUSOIDAL VOLTAGE

Sinusoidal voltage (or emf) is generated either by rotating a conductor or a coil by means of a spindle in a uniform magnetic field as shown in Fig. 3.1(a) or by rotating a magnetic field, called the rotor within a stationary coil called the stator as shown in Fig. 3.1(b).



(a) (b)

Fig. 3.1 (a) and (b)

Referring Fig. 3.1(a), the terminals of the coil are brought out and connected to two insulated slip rings S_1 and S_2, which are attached to the spindle. On these slip rings, Carbon brushes B_1 and B_2 are fixed, which are connected to external load circuit.

Referring Fig. 3.1(b), the terminals of the coil can be directly brought out without slip rings and brushes.

The value of the emf generated depends upon the following factors:

(i) The number of turns of the coil,

(ii) The strength of the magnetic field and

(iii) The speed at which the coil (or rotor) rotates.

3.2 EQUATION OF ALTERNATING EMF AND CURRENT

Assume a rectangular coil of N turns rotating in a uniform magnetic field with an angular velocity ω radian/second in anticlockwise direction.

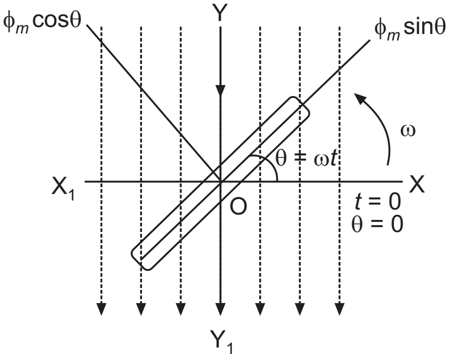


Fig. 3.2

When the coil coincides with the X-axis, maximum flux ϕ_m links with it. In time t seconds, the coil rotates through an angle $\theta = \omega t$. In this deflected position, the maximum flux linking vertically downwards can be resolved into two components;

(i) Component $\phi_m \sin \omega t$: This is parallel to the plane of the coil and will not induce emf.

(ii) Component $\phi_m \cos \omega t$: This is perpendicular to the plane of the coil and will induce an emf.

∴ Flux linkages of the coil in this deflected position = $N\phi_m \cos \omega t$

According to Faraday's Laws of EMI,

emf induced in the coil = rate of change of flux linkages of the coil.

Therefore, instantaneous emf induced in the coil is given by

$$e = -\frac{d}{dt}(\mathrm{N}\,\phi_m \cos\omega t)$$

$$= -N\phi_m \frac{d}{dt}(\cos\omega t)$$

$$= -\mathrm{N}\,\phi_m(-\sin\omega t)\,\omega$$

$$= \mathrm{N}\,\phi_m\,\omega \sin\omega t$$

$$= \mathrm{N}\,\phi_m\,\omega \sin\theta \qquad \text{...(3.1)}$$

The value of θ is maximum when the coil rotates through an angle 90°,

i.e., $\theta = 90°$

$$\therefore \qquad \sin\theta = 1$$

Hence, maximum value of emf E_m is given by

$$E_m = N\phi_m \omega \text{ volt} \qquad ...(3.2)$$

Substituting for angular velocity, $\omega = 2\pi n$ in equation (3.2), (n = speed of rotation in revolutions/second and $\phi_m = B_m a$, where B_m = maximum flux density and a = area of each turn)

$$E_m = N B_m a\, 2\pi n \text{ volt}$$
$$= 2\pi B_m a n N \text{ volt}$$

Substituting equation (3.2) in equation (3.1), the equation for alternating emf is given by

$$e = E_m \sin\theta \text{ volt}$$

or $$e = E_m \sin\omega t \text{ volt} \qquad ...(3.3)$$

Equation of alternating current

If the coil circuit is completed by connecting a load resistance R ohm then due to instantaneous emf, an instantaneous current i will set-up, given by

$$i = I_m \sin\theta \text{ ampere}$$

or $$i = I_m \sin\omega t \text{ ampere} \qquad ...(3.4)$$

3.3 IMPORTANT DEFINITIONS

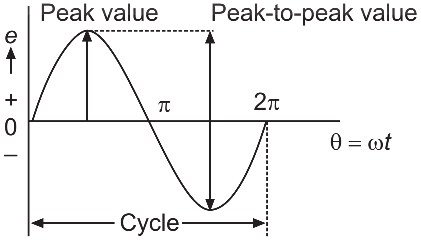


Fig. 3.3 Waveform for alternating quantity

1. **Waveform:** The graph between an alternating quantity (voltage or current) and time is called the waveform.
2. **Sinusoidal EMF:** The emf induced varies as the sine function of the time angle (ωt) and if emf induced is plotted against time, a curve of sine wave shape is obtained as shown in Fig. 3.4. This is known as the sinusoidal emf.

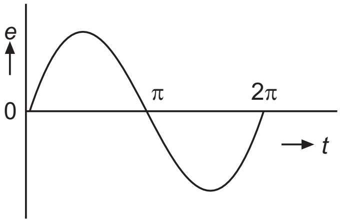


Fig. 3.4 Sinusoidal emf

3. **Alternating Quantity:** An alternating quantity is one, which acts in alternate positive and negative directions, whose magnitude undergoes a definite series of changes in definite intervals of time.
4. **Instantaneous Value:** The magnitude of a waveform at any instant in time is termed as instantaneous value. EMF or Voltage and Current are represented by e or v and i respectively.
5. **Cycle:** Each repetition of a variable quantity repeating at equal intervals is termed as cycle. The values of a sine wave repeat after every 2π radians.

   Alternatively, one complete set of positive and negative values of the function is called a cycle.
6. **Frequency:** The number of cycles that occur in one second is termed as frequency (f). Its unit is **hertz ($\mathbf{H_Z}$)**, that is, number of revolutions per second.

   Angular velocity, $\omega = 2\pi f$ radians/second

   $\therefore \quad e = E_m \sin 2\pi f t$ volt
7. **Period or Time Period or Periodic Time:** Time taken to complete one cycle is termed as period (T) or time period or periodic time. Its unit is **second**.

   Relationship between frequency (f) and period (T):

   $$f = \frac{1}{T}$$

   $$\therefore \qquad e = E_m\, 2\pi \frac{1}{T} t \text{ volt}$$
8. **Peak Value or Amplitude or Maximum Value:** The maximum instantaneous value of an alternating quantity measured from its zero value is termed as Peak value. Examples: E_m, V_m and I_m.

Solved Examples

Example 3.1 *A coil of 100 turns is rotated at 1500 r/min in a magnetic field having a uniform density of 0.05 T, the axis of rotation being at right angles to the direction of the flux. The mean area per turn is 40 cm^2. Calculate (a) the frequency, (b) the period, (c) the maximum value of the generated emf and (d) the value of the generated emf. When the coil has rotated through 30° from the position of zero emf.*

Solution: Given: $N = 100$, $n = 1500$ r/min or rpm (i.e. revolutions per minute),

$B = 0.05$ T, Mean Area $A = 40 \text{ cm}^2 = 40 \times 10^{-4} \text{ m}^2$, $\theta = 30°$

(a) When the coil rotates through one revolution, the emf generated in the coil undergoes one cycle of variation.

By definition, frequency = number of cycles per second

or f = number of revolutions per second

$$= \frac{1500}{60} = \mathbf{25\ Hz}$$

(b) By definition, period is the time taken to complete one cycle

i.e., $$T = \frac{1}{f} \quad \left(\text{because } f = \frac{1}{T}\right)$$

$$= \frac{1}{25} = \mathbf{0.04\ second}$$

(c) Maximum emf generated,

$$E_m = 2\pi B A n N$$

$$= 2\pi \times 0.05 \times (40 \times 10^{-4}) \times \left(\frac{1500}{60}\right) \times 100$$

$$= \mathbf{3.14\ V}$$

(d) The value of the generated emf when the coil has rotated through an angle 30° from the position of zero emf is given by

$$e = E_m \sin\theta$$

Given that $\theta = 30°$

Then, sin $\theta = \sin 30° = 0.5$

$$e = 3.14 \times 0.5 = \mathbf{1.57\ V}$$

3.4 CONCEPT OF AVERAGE VALUE AND RMS VALUE

Figure 3.5 (a) shows a current waveform. Assuming n number of mid-ordinates $i_1, i_2, i_3 \ldots i_n$ in first half-cycle and applying graphical or mid-ordinate method,

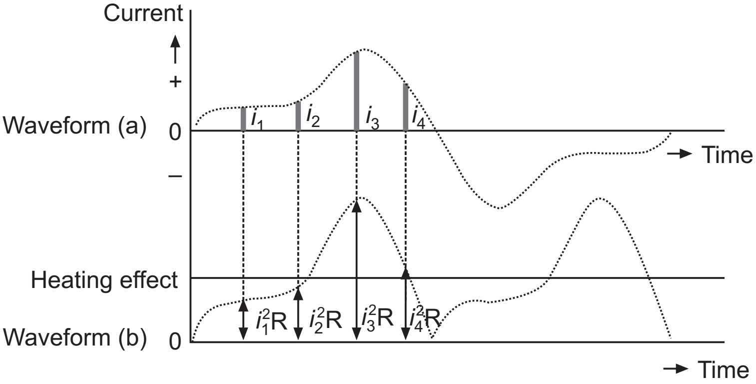


Fig. 3.5 Current waveform

The average value of current over first half-cycle,

$$I_{av} = \frac{i_1 + i_2 + \ldots\ldots + i_n}{n}.$$

Now, if the same current as represented in waveform (a) is passed through a resistance R ohm, (Example: filament lamp or heater coil), it causes a heating effect as follows; i_1 causes heating effect of i_1^2R, i_2 causes i_2^2R, etc. as shown in waveform (b).

Then, average heating effect $= \dfrac{i_1^2\mathrm{R} + i_2^2\mathrm{R} + \ldots\ldots + i_n^2\mathrm{R}}{n}$.

If I is the direct current passed through the same resistance R ohm, to produce the same heating effect as produced by alternating current.

Then, $$I^2 R = \frac{i_1^2 R + i_2^2 R + \ldots.. + i_n^2 R}{n}$$

or $$= \frac{R\,(i_1^2 + i_2^2 + \ldots.. + i_n^2)}{n}$$

or $$I^2 = \frac{i_1^2 + i_2^2 + \ldots.. + i_n^2}{n}$$

or $$I = \sqrt{\frac{i_1^2 + i_2^2 + \ldots.. + i_n^2}{n}}$$

= root-mean square or rms value of the current.

Hence, rms value or effective value of an alternating current is measured in terms of the direct current that produces the same heating effect in the same resistance.

Definition

The average value of an alternating current is expressed by that steady current which transfers across any circuit the same charge as is transferred by that alternating current.

In case of symmetrical alternating current (i.e., half cycles are exactly similar), the average value over a complete cycle is zero. Hence, average value is obtained by adding or integrating instantaneous values of current over first half cycle only.

Expression

Consider an alternating current as shown in waveform Fig. 3.6 varying sinusoidally.

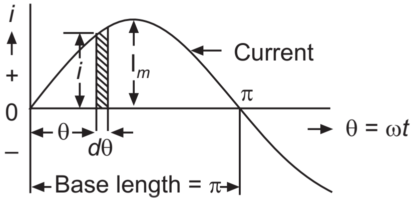


Fig. 3.6 Waveform

The instantaneous value of current is given by

$$i = I_m \sin\theta$$

where I_m = maximum value of current and

θ = angle in radians from instant of zero current.

Assume a small interval $d\theta$ radian represented by a shaded strip in first half-cycle.

Area of the shaded strip = $i.d\theta$

Therefore, area enclosed by the current wave over first half-cycle is given by

$$\int_0^\pi i\,.\,d\theta$$

$$= \int_0^\pi I_m \sin\theta\,.\,d\theta$$

$$= I_m \int_0^\pi \sin\theta\,.\,d\theta$$

$$= I_m\,[-\cos\theta]_\theta^\pi$$

$$= I_m\,[-\cos\pi + \cos 0]$$

$$= 2\,I_m$$

Average value of current over a first half-cycle is given by

$$I_{av} = \frac{\text{Area enclosed over first half-cycle}}{\text{Length of base over first half-cycle}}$$

$$= \frac{2I_m}{\pi}$$

$$= \mathbf{0.637\ I_m}$$

∴ Average value of sinusoidal voltage or current = 0.637 × Maximum Value.

(Note: For half rectified wave, Average value = 0.318 I_m).

3.4.1 Root-Mean Square Value or Effective Value or Virtual Value

Definition

The rms value of an alternating current is given by that steady current which when flowing through a given circuit for a given time produces the same heat as produced by the alternating current when flowing through the same circuit for the same time.

Expression

Figure 3.7 shows a waveform of an alternating current varying sinusoidally. The instantaneous value of current is given by

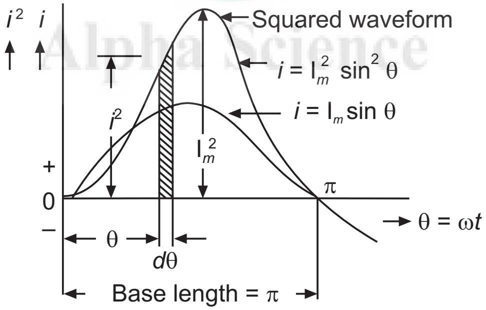


Fig. 3.7 Waveforms

$$i = I_m \sin\theta$$

where I_m = maximum value of current

and θ = angle in radian from instant of zero current.

The squared waveform is drawn as shown in Fig. 3.7.

The square of the current is given by

$$i^2 = I_m^2 \sin^2\theta$$

Assume a small interval $d\theta$ radian represented by a shaded strip in first half-cycle of the squared waveform.

The area of the shaded strip = $i^2 \cdot d\theta$

Therefore, area enclosed by the square of the current wave over first half-cycle is given by

$$\int_0^{\pi} i^2 \,.\, d\theta$$

$$= \int_0^{\pi} I_m^2 \sin^2 \theta \,.\, d\theta$$

$$= \mathrm{I}_m^2 \int_0^{\pi} \sin^2 \theta \,.\, d\theta$$

$$= \mathrm{I}_m^2 \int_0^{\pi} \left[\frac{1 - \cos 2\theta}{2}\right] d\theta$$

$$= \frac{\mathrm{I}_m^2}{2} \int_0^{\pi} [1 - \cos 2\theta]\, d\theta$$

$$= \frac{I_m^2}{2}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi} \qquad \left(\text{Note:} \left[\frac{\sin 2\theta}{2}\right]_0^{\pi} = 0\right)$$

$$= \frac{\mathrm{I}_m^2 \pi}{2}$$

Average value of square of the current over first half-cycle is given by

$$\frac{\text{Area enclosed over first half-cycle}}{\text{Length of base over first half-cycle}}$$

$$= \frac{\mathrm{I}_m^2 \pi}{2} \times \frac{1}{\pi}$$

$$= \frac{\mathrm{I}_m^2}{2}$$

$$\therefore \qquad \mathrm{I}_{\text{rms}} = \mathrm{I} = \sqrt{\text{Mean or average of square of current}}$$

$$= \sqrt{\frac{\mathrm{I}_m^2}{2}}$$

$$= \frac{\mathrm{I}_m}{\sqrt{2}}$$

$$= \mathbf{0.707\ I}_{\boldsymbol{m}}$$

∴ RMS value of sinusoidal voltage or current = 0.707 × Maximum value

(Note: For half rectified wave, RMS value = 0.5 I_m)

(The above method of approach is Analytical Method or Integral Method).

Importance of rms value

1. The measuring instruments like ammeter and voltmeter reads the rms value of alternating current and alternating voltage.
2. In electrical engineering works, unless indicated, the given values of voltage and current are taken as the rms values.

3.5 FORM FACTOR OF A SINE WAVE

$$K_f = \frac{\text{RMS value}}{\text{Average value}}$$

$$= \frac{0.707 \times \text{Maximum value}}{0.637 \times \text{Maximum value}} = \mathbf{1.11}$$

3.6 PEAK OR CREST OR AMPLITUDE FACTOR

$$K_p \text{ or } K_c \text{ or } K_a = \frac{\text{Maximum volue}}{\text{RMS value}}.$$

$$= \frac{\text{Maximum volue}}{0.707 \times \text{Maximum value}} = \mathbf{1.414}$$

Solved Examples

Example 3.2 *An alternating current of sinusoidal waveform has rms value of 10 A. What are the peak values of this current over one cycle?*

Solution: Given: Current, $I = 10$ A

By definition, $I = 0.707\, I_m$

or $$I_m = \frac{I}{0.707} = \frac{10}{0.707} = \mathbf{14.14\ A}$$

∴ The peak values are **14.14 A** and **–14.14 A**

Example 3.3 *An alternating voltage has the equation v = 141.4 sin (377t) volt; what are the values of (a) rms voltage, (b) frequency and (c) the instantaneous voltage when t = 3 ms?*

Solution: Given: $v = 141.4 \sin (377\, t)$ volt

This equation is of the form $v = V_m \sin \omega t$

where ω = angular velocity

$= 2\pi f$ radians/second

(a) By definition, rms value of voltage is given by

$$V = V_m\ 0.707$$

As per given data, $V_m = 141.4$

$\therefore$ $V = 141.4 \times 0.707$

$= 99.969 \approx \mathbf{100\ V}$

(b) Angular velocity, $\omega = 2\pi f$

or frequency, $f = \dfrac{\omega}{2\pi} = \dfrac{377}{2\,\pi} = \mathbf{60\ Hz}$

(c) Instantaneous voltage, $v = V_m \sin \omega t$

Given that $t = 3$ ms $= 3 \times 10^{-3}$ second

$\therefore$ $v = 141.4 \sin (377 \times 3 \times 10^{-3}) = \mathbf{2.791\ V}$

Example 3.4 *If the waveform of a voltage has a form factor of 1.15 and peak factor of 1.5, if the peak value is 4.5kV, calculate the average and rms values of the voltage.*

Solution: Given: $K_f = 1.15$, $K_p = 1.5$, $V_m = 4.5$ kV = 4500 V

Peak factor, $K_p = \dfrac{\text{Maximum value}}{\text{RMS value}} = \dfrac{V_m}{V}$

or RMS value of voltage, $V = \dfrac{V_m}{K_p} = \dfrac{4500}{1.5} = \mathbf{3000\ V}$

Form factor, $K_f = \dfrac{\text{RMS volue}}{\text{Average value}} = \dfrac{V}{V_{av}}$

or average value of voltage, $V_{av} = \dfrac{V}{K_f} = \dfrac{3000}{1.15} = \mathbf{2608.69\ V}$

Example 3.5 *The equation for an alternating voltage is given by v = 0.04 sin(2000 t + 60°) volt. Calculate the frequency and instantaneous voltage when t = 160 μ second. What is the time represented by a 60° phase angle?*

Solution: Given: $v = 0.04 \sin (2000\text{ t} + 60°)$ volt

This equation is of the form $v = V_m \sin (\omega t + \phi)$ volt

$\therefore V_m = 0.04$, $\omega = 2000$ rad/sec, $t = 160$ μsecond $= 160 \times 10^{-6}$ second and $\phi = 60°$

To find frequency:

Angular velocity, $\omega = 2\pi f$

or frequency, $f = \dfrac{\omega}{2\pi} = \dfrac{2000}{2\pi} = \mathbf{318.3\ Hz}$

$$\text{Instantaneous voltage, } v = 0.04 \sin [(2000 \times 160 \times 10^{-6}) + 60°]$$

$$= 0.04 \sin (0.32 \text{ rad} + 60°)$$

$$= 0.04 \sin \left(0.32 \times \frac{180°}{\pi} + 60°\right) \text{ (because } \pi = 180°\text{)}$$

$$= 0.04 \sin (18.3° + 60°)$$

$$= \mathbf{0.0384\ V = 38.4\ mV}$$

To find time representing 60° phase angle: By the definition of time, one cycle of 360° represents time period T.

$$\text{Now,} \quad T = \frac{1}{f} = \frac{1}{318.3} = 3.14 \text{ ms}$$

∴ 360° represents time period of 3.14 ms.

Then, 60° phase angle represents time t, given by

$$t = \frac{60°}{360°} \times 3.14 \text{ ms} = \mathbf{0.52\ ms}$$

3.7 PHASOR REPRESENTATION OF AN ALTERNATING QUANTITY

For the sake of convenience, alternating quantities are represented in the form of vectors or phasors rotating in anticlockwise direction. A vector is a physical quantity having both magnitude and direction.

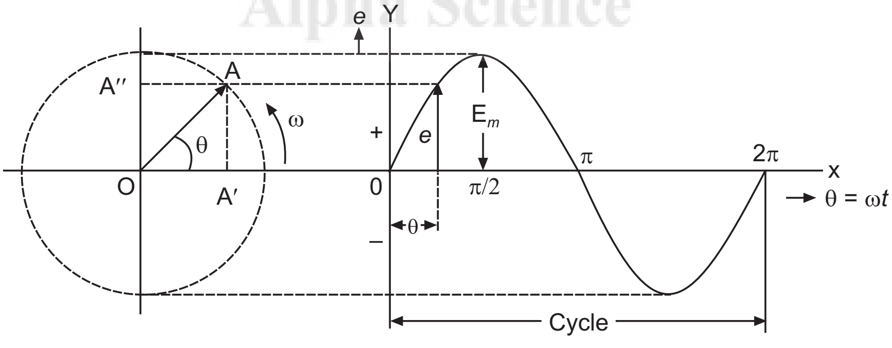


Fig. 3.8 Phasor representation

Let OA represents a vector of alternating emf e. Its angle with X-axis gives its phase and the projection on Y-axis gives the instantaneous value.

Starting from zero if it rotates through an angle θ in anticlockwise direction with an angular velocity ω, the instantaneous value of emf in this deflected position is given by

$$OA'' = AA' = OA \sin \theta$$

or $$e = E_m \sin \theta$$

where E_m = maximum value of emf (at $\theta = 90°$) and thus represents its magnitude.

Therefore, as the vector OA rotates through one revolution or 2π radians, one cycle of emf waveform will be obtained.

3.7.1 Phase

An alternating quantity changes in magnitude and direction at every instant. Thus, the condition of an alternating quantity at any particular instant is called the phase.

Phase of an alternating quantity at any particular instant is defined as the fractional part of a period or cycle through which the quantity has advanced from a selected origin.

3.7.2 Phase Difference

If two alternating quantities, say, e and i are considered simultaneously with same frequency, they may not pass through a particular point at the same time and thus these two quantities have phase difference.

The phase difference is measured by the angular difference between points, wherever the two curves cross the base or reference line in the same direction.

To explain phase the difference, consider two alternating quantities emf e and current i represented by vectors OA and OB respectively as shown in Fig. 3.9.

Case (i): The emf e is ahead in phase and thus it leads the current i by an angle ϕ or current i lags behind the emf e by an angle ϕ. Thus, the vector OA leads the vector OB by an angle ϕ or the vector OB lags behind the vector OA by an angle ϕ.

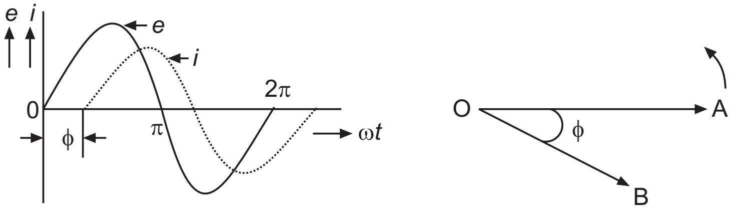


Fig. 3.9 Waveforms and phasor diagrams

[Note: If emf, $$e = E_m \sin \omega t$$

and current, $$i = I_m \sin (\omega t - \phi)$$

Thus, emf e leads current i by an angle ϕ or current i lags behind emf e by an angle ϕ].

Hence, a leading alternating quantity reaches its maximum value earlier than the other, and a lagging alternating quantity reaches its maximum value later than the other.

Case (ii): Two alternating quantities *e* and *i* will be in phase with each other as they pass through zero value at the same instant and rise in the same direction. Thus the vector OA and the vector OB are in phase.

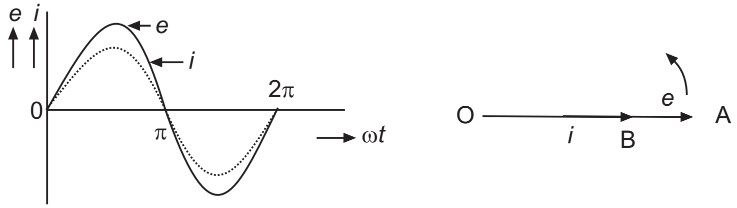


Fig. 3.10 Waveforms and phasor diagrams

Case (iii): Two alternating quantities *e* and *i* pass through zero value at the same time instant but rise in opposite direction and thus they are in phase opposition or in quadrature**.** Thus, the vector OA and the vector OB are in phase opposition.

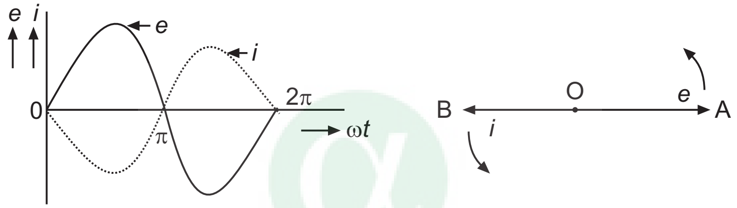


Fig. 3.11 Waveforms and phasor diagrams

3.8 ANALYSIS OF SINGLE-PHASE AC CIRCUITS

3.8.1 Circuit with Resistance (R) only

Figure 3.12 (a) shows an ac circuit with a pure resistance R only (that is, Non-Inductive Resistance).

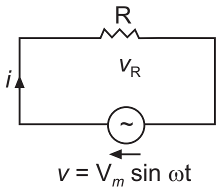


Fig. 3.12 (a) Pure resistive circuit

The applied voltage, $v = V_m \sin \omega t$ volt ...(3.5)

When an alternating current *i* flows through the resistance R, a voltage drop occurs across it, given by

$$v_R = v = i\,R \text{ volt}$$

Substituting for v from equation (3.5),

$$V_m \sin \omega t = i\,R$$

or $$i = \frac{V_m}{R} \sin \omega t \qquad ...(3.6)$$

Current i is maximum when $\sin \omega t = 1$

i.e., maximum current, $I_m = \dfrac{V_m}{R}$

Substituting for $\dfrac{V_m}{R}$ in equation (3.6), the equation for current is

$$i = I_m \sin \omega t \text{ ampere} \qquad ...(3.7)$$

By comparing equations (3.5) and (3.7), current is in phase with the applied voltage.

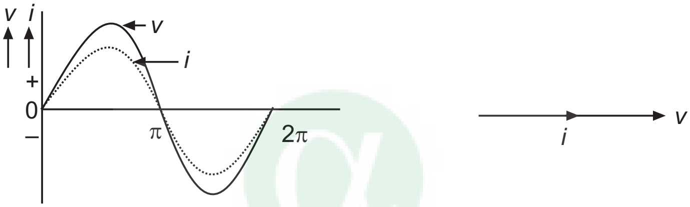


Fig. 3.12 (b) Waveforms and phasor diagrams

Expression for Power:

Instantaneous voltage, $v = V_m \sin \omega t$

Instantaneous current, $i = I_m \sin \omega t$

Instantaneous power, $p = v \times i$

$$= V_m \sin \omega t \times I_m \sin \omega t$$

$$= V_m I_m \sin^2 \omega t$$

$$= V_m I_m \frac{(1 - \cos 2\omega t)}{2}$$

$$= \frac{V_m I_m}{2} - \frac{V_m I_m}{2} \cos 2\,\omega t$$

Thus, instantaneous power consists of

(i) Constant part $\Rightarrow \dfrac{V_m I_m}{2}$

(ii) Fluctuating part $\Rightarrow \dfrac{V_m I_m}{2} \cos 2\,\omega t$, its average value over a complete cycle is zero.

Therefore, average (or total) power over a complete cycle is given by

$$
\begin{aligned}
P &= \frac{V_m I_m}{2} \\
&= \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \\
&= V\,I \text{ watt} \\
&= I^2\,R \text{ watt}
\end{aligned}
$$

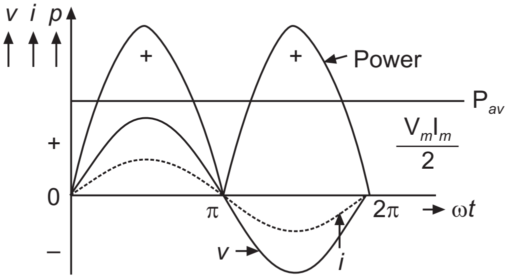


Fig. 3.13 Power waveform

3.8.2 Circuit with Inductance (L) only

Figure 3.14 shows an ac circuit with a coil of pure inductance L only.

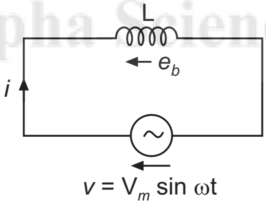


Fig. 3.14 Pure inductive circuit

The applied voltage, $v = V_m \sin \omega t$ volt ...(3.8)

When an alternating voltage is applied to a pure inductive coil, a back emf or self-induced emf is produced, given by

$$e_b = -L\frac{di}{dt} \text{ volt} \quad ...(3.9)$$

As the pure inductance will not have any resistance drop, the applied voltage has to overcome this back emf only. Hence, at every instant,

$$v = -e_b \quad ...(3.10)$$

Substituting equation (3.8) and equation (3.9) in equation (3.10),

$$V_m \sin \omega t = -\left(-L\frac{di}{dt}\right)$$

$$= L\frac{di}{dt}$$

or $$di = \frac{V_m}{L} \sin \omega\, t \,.\, dt$$

$\therefore$ $$i = \frac{V_m}{L} \int \sin \omega t \,.\, dt$$

$$= \frac{V_m}{L}\left[\frac{-\cos \omega t}{\omega}\right]$$

$$= \frac{V_m}{\omega L}(-\cos \omega t)$$

$$= \frac{V_m}{\omega L} \sin\left(\omega t - \frac{\pi}{2}\right) \qquad ...(3.11)$$

Current i is maximum when $\sin\left(\omega t - \frac{\pi}{2}\right) = 1$

i.e., maximum current, $I_m = \frac{V_m}{\omega L} = \frac{V_m}{X_L}$

where X_L is called the inductive reactance and its unit is **ohm (Ω)**

Substituting for $\frac{V_m}{X_L}$ in equation (3.11), the equation for current is

$$i = I_m \sin\left(\omega t - \frac{\pi}{2}\right) \text{ampere} \qquad ...(3.12)$$

By comparing equations (3.8) and (3.12), current **lags behind** the applied voltage by $\left(\frac{\pi}{2}\right)$ radians or an angle 90°.

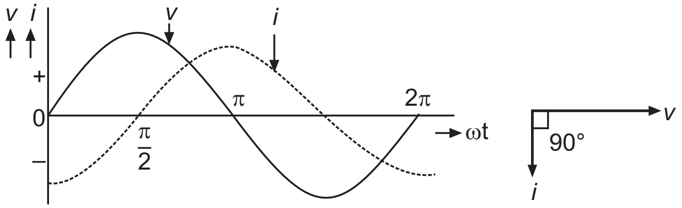


Fig. 3.15 Waveforms and phasor diagrams

Expression for Power:

Instantaneous voltage, $v = V_m \sin \omega t$

Instantaneous current, $i = I_m \sin\left(\omega t - \frac{\pi}{2}\right) = I_m(-\cos \omega t)$

Instantaneous power, $p = v \times i$

$$= V_m \sin \omega t \times I_m(-\cos \omega t)$$

$$= -V_m I_m \sin \omega t \cos \omega t$$

$$= -\frac{V_m I_m}{2} \sin 2\omega t$$

Therefore, average (or total) power over a complete cycle is given by

$$P = -\frac{V_m I_m}{2} \int_0^{2\pi} \sin 2\omega t \, dt = \mathbf{0}$$

i.e., average power consumed by a pure inductance over a complete cycle is **zero**.

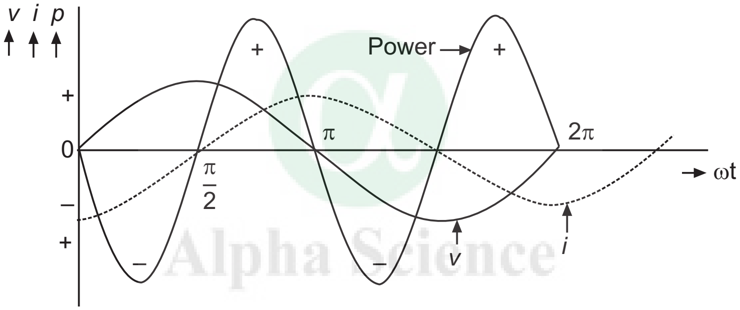


Fig. 3.16 Power waveform

(Note: Average of a sinusoidal quantity of double frequency over a complete a complete cycle is zero.)

3.8.3 Circuit with Capacitance (C) only

Figure 3.17 shows an ac circuit with a pure capacitance C only. The applied voltage or potential difference between plates is

$$v = V_m \sin \omega t \text{ volt} \qquad ...(3.13)$$

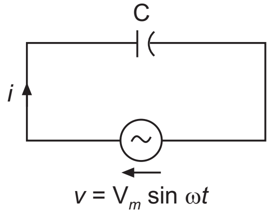


Fig. 3.17 Pure capacitive circuit

When an alternating voltage is applied to the plates of a capacitor, it is charged first in one direction and then in the opposite direction as the voltage reverses.

Therefore, instantaneous charge is given by

$$q = c\,v$$

where c = capacitance

Substituting for v from equation (3.13),

$$q = c\,V_m \sin \omega t \qquad \text{...(3.14)}$$

The capacitor current is given by

$$i = \frac{dq}{dt}$$

Substituting for q from equation (3.14),

$$i = \frac{d}{dt}(cV_m \sin \omega t)$$

$$= cV_m \frac{d}{dt}(\sin \omega t)$$

$$= cV_m \cos \omega t \,.\, \omega$$

$$= \omega\, c\, V_m \cos \omega t$$

$$= \frac{V_m}{1/\omega c} \sin\left(\omega t + \frac{\pi}{2}\right) \qquad \text{...(3.15)}$$

Current i is maximum when $\sin\left(\omega t + \frac{\pi}{2}\right) = 1$

i.e., maximum current, $I_m = \dfrac{V_m}{1/\omega c} = \dfrac{V_m}{X_c}$

where X_c is called the capacitive reactance and its unit is **ohm (Ω)**.

Substituting for $\dfrac{V_m}{X_c}$ in equation (3.15), the equation for current is

$$i = I_m \sin\left(\omega\, t + \frac{\pi}{2}\right) \text{ ampere} \qquad \text{...(3.16)}$$

By comparing equations (3.13) and (3.16), current **leads** the applied voltage by $\left(\frac{\pi}{2}\right)$ radians or an angle 90°.

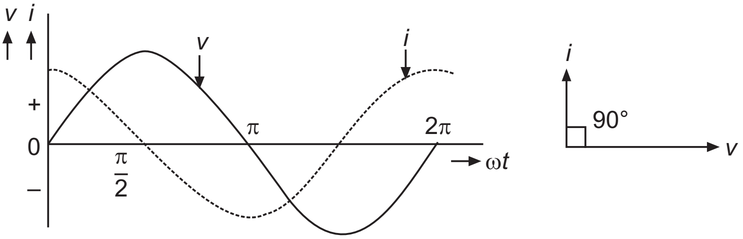


Fig. 3.18 Waveforms and phasor diagrams

Expression for Power:

Instantaneous voltage, $v = V_m \sin \omega t$

Instantaneous current, $i = I_m \sin\left(\omega t + \frac{\pi}{2}\right) = I_m \cos \omega t$

Instantaneous power, $p = v \times i$

$$= V_m \sin \omega t \times I_m \cos \omega t$$

$$= V_m I_m \sin \omega t \,.\, \cos \omega t$$

$$= \frac{V_m I_m}{2} \sin 2\omega t$$

Therefore, average (or total) power over a complete cycle is given by

$$P = \frac{V_m I_m}{2} \int_0^{2\pi} \sin 2\omega t \, dt = \mathbf{0}$$

i.e., average power consumed by a pure capacitance over a complete cycle is **zero**.

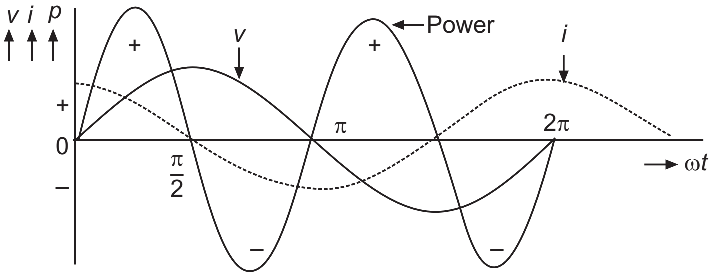


Fig. 3.19 Power waveform

Table 3.1 Summary

S.No.	Circuit Element	Phase angle between voltage and current	Average Power Consumption
1	R	Current is in phase with the voltage, phase angle is zero	$P = VI$ watt or $P = I^2R$ watt
2	L	Current lags behind the voltage by an angle 90°	Zero
3	C	Current leads the voltage by an angle 90°	Zero

Solved Examples

Example 3.6 *A 30 μF capacitor is connected across a 400 V, 50 Hz supply. Calculate (a) the reactance of the capacitor and (b) the current.*

Solution: Given: $C = 30\ \mu F = 30 \times 10^{-6}$ F, $V = 400$ volt, $f = 50$ Hz.

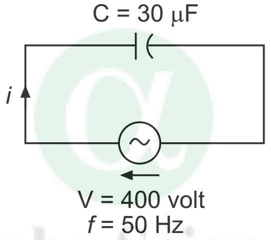


Fig. 3.20

(a) Capacitive reactance, $X_C = \dfrac{1}{2\pi f C}$

$$= \frac{1}{2\pi \times 50 \times (30 \times 10^{-6})} = \mathbf{106.11\ \Omega}$$

(b) Current, $I = \dfrac{V}{X_C} = \dfrac{400}{106.11} = \mathbf{3.77\ A}$

Example 3.7 *A coil having an inductance of 0.2 H and negligible resistance is connected across a 100 V ac supply. Calculate the current when the frequency is (a) 30 Hz and (b) 500 Hz.*

Solution: Given: $L = 0.2$ H, $V = 100$ volt

(a) When frequency is 30 Hz;

$$X_L = 2\pi f L = 2\pi \times 30 \times 0.2 = 37.699\ \Omega$$

$$\therefore \quad I = \frac{V}{X_L} = \frac{100}{37.699} = \mathbf{2.6\ A}$$

(b) When frequency is 500 Hz;

$$X_L = 2\pi f L = 2\pi \times 500 \times 0.2 = 628.318\ \Omega$$

$$\therefore \qquad I = \frac{V}{X_L} = \frac{100}{628.318} = \mathbf{0.159\ A}$$

3.8.4 R-L Series Circuit

Figure 3.21 shows an ac circuit with a resistance R and an inductance L connected in series. The applied voltage V and the current drawn I are expressed in RMS values.

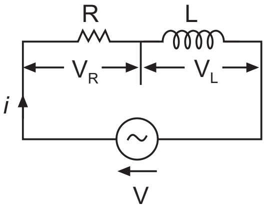


Fig. 3.21 R-L Series circuit

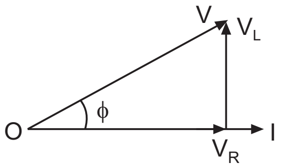


Fig. 3.22 Vector diagram

Vector diagram is drawn applying KVL to the closed circuit. As R and L are connected in series and the current flowing through them is same, the current I is taken as reference.

As the current I flows through R and L, voltage drop occurs across them as given below:

(i) Voltage drop across resistance, $V_R = I\,R$ volt. V_R is in phase with the current I.

(ii) Voltage drop across inductance, $V_L = I\,X_L$ volt. V_L leads the current I by an angle 90°.

(because in inductive circuit current lags behind the voltage, as current is taken as reference, voltage V_L leads current).

Therefore, the applied voltage V is the **vector sum** of V_R and V_L,

i.e.,
$$V = \sqrt{V_R^2 + V_L^2}$$
$$= \sqrt{(IR)^2 + (IX_L)^2}$$
$$= I\sqrt{R^2 + X_L^2}$$

or
$$I = \frac{V}{\sqrt{R^2 + X_L^2}}$$
$$= \frac{V}{Z}\ \text{ampere}$$

where Z is called the impedance of the circuit and its unit is **ohm (Ω).**

(Note: The opposition offered by a circuit to the flow of alternating current through it is known as the impedance Z of the circuit and its unit is ohm.)

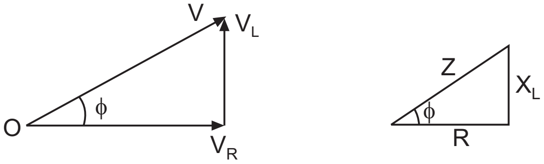


Fig. 3.23 (a) Voltage triangle and impedance triangle

From voltage triangle, $\tan\phi = \frac{V_L}{V_R} = \frac{I\,X_L}{I\,R} = \frac{X_L}{R}$

or $$\phi = \tan^{-1}\frac{X_L}{R}$$

From impedance triangle, $\phi = \cos^{-1}\frac{V_R}{V} = \cos^{-1}\frac{R}{Z}$

If the applied voltage is given by

$$v = V_m \sin \omega t$$

Then the equation for current is

$$i = I_m \sin(\omega t - \phi)$$

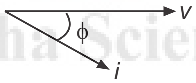


Fig. 3.23 (b) Phasor diagram

That is, if a circuit contains resistance and inductance in series, called the inductive circuit, the current lags behind the voltage by an angle ϕ as shown in Fig. 3.23 (b).

Expression for Power:

Instantaneous voltage, $v = V_m \sin \omega t$

Instantaneous current, $i = I_m \sin(\omega t - \phi)$

Instantaneous power,
$$\begin{aligned} p &= v \times i \\ &= V_m \sin \omega t \times I_m \sin(\omega t - \phi) \\ &= V_m\, I_m [\sin \omega t \,.\, \sin(\omega t - \phi)] \\ &= \frac{1}{2}\, V_m\, I_m [\cos\phi - \cos(2\omega t - \phi)] \\ &= \frac{1}{2}\, V_m\, I_m\, \text{co}\phi - \frac{1}{2}\, V_m\, I_m \cos(2\omega t - \phi) \end{aligned}$$

In the above equation, instantaneous value of power consists of

(i) **Constant part:** $V_m I_m \cos\phi \Rightarrow$ it contains no reference to ωt, it remains constant and hence contributes to Real Power.

(ii) **Pulsating component:** $\frac{1}{2} V_m I_m \cos(2\omega t - \phi) \Rightarrow$ the average value of a cosine curve over a complete cycle is zero

Therefore, average (or total) power over a complete cycle is given by

$$\begin{aligned} P &= \frac{1}{2} V_m I_m \cos\phi \\ &= \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\phi \\ &= V_{rms} I_{rms} \cos\phi \text{ watt} \\ &= V I \cos\phi \text{ watt} \end{aligned}$$

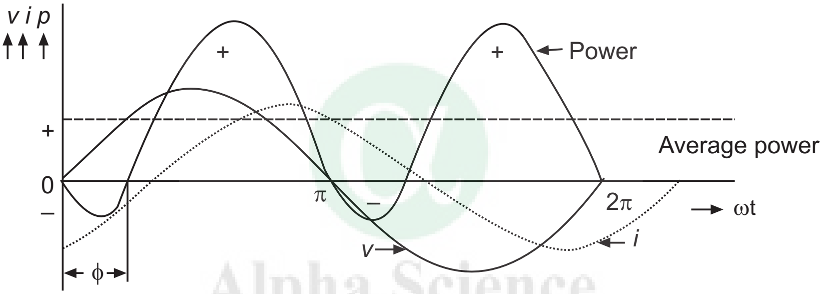


Fig. 3.24 Power waveform

3.8.5 Apparent, Real and Reactive Power and Power Factor

Apparent Power: The product of the voltage and current is called the Apparent Power.

i.e., $S = V I$ volt-ampere (VA) or kilo volt-ampere (kVA)

Real Power: The average or total power over a complete cycle is called the Real Power or the Active Power or the True Power. It is given by

$P = V I \cos\phi$ watt (W) or kilo watt (kW)

Also, Active power = Apparent power × *pf* watt

i.e., $P = S \times \cos\phi$ watt

Reactive Power: It is the product of the applied voltage and reactive component of current.

i.e., $Q = V I \sin\phi$ volt-ampere reactive (VAR) or (kVAR)

Power Triangle: Power triangle is drawn assuming that $X_L > X_C$

$$P \text{ (or W)} = VA \cos\phi$$

$$Q = VAR = VA \sin\phi$$

$$S = VA = \sqrt{W^2 + VAR^2}$$

- I cos ϕ is known as active component of current and it is in phase the with applied voltage. It also called wattful component.
- I sin ϕ is known as reactive component of current and it is in quadrature with the applied voltage. It is also called wattles component.

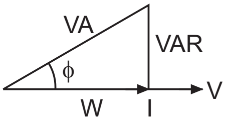


Fig. 3.25 Power triangle

Power Factor: Power Factor (*pf*) is defined as

(i) Cosine of the angle between voltage and current (or cosine of the angle lag or lead).

(ii) Power Factor = $\dfrac{\text{Active power in Watt}}{\text{Apparent power in VA}} = \dfrac{P}{S} = \dfrac{VI\cos\phi}{VI} = \cos\phi$

(iii) Power Factor = $\cos\phi = \dfrac{R}{Z}$ (referring the impedance triangle)

Practical importance of power factor (or disadvantages of low power factor):

(i) The phase difference between the voltage and current depends upon the nature of the load and not upon the alternator.

(ii) For electrical machines like alternators and transformers, the rating is expressed as

$$kVA = \frac{kW}{pf}$$

Thus, for the required value of power at low power factor, kVA rating becomes more, the size of the machine increases and becomes more expensive.

(iii) For transmission and distribution of a fixed value of power at a fixed voltage, the conductors should carry large current at low *pf* and naturally it will be of large size.

(iv) At low power factor, I^2R loss increases and thus the efficiency of the machines reduces.

Causes for low power factor:

(i) Electrical machines like single-phase induction motors, 3-phase induction motors are inductive loads, which work at low *pf* for small loads.

(ii) Industrial heating furnaces, arc lamps operate at low *pf*.

Remedy: Power factor will be improved by installing capacitors bank or using special machines.

3.8.6 R-C Series Circuit

Figure 3.26 shows an ac circuit with a resistance R and a capacitance C connected in series. The applied voltage V and the current drawn I are expressed in RMS values.

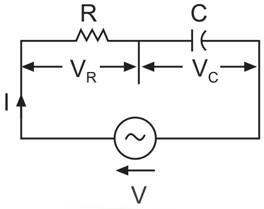


Fig. 3.26 R-C series circuit

Vector diagram is drawn applying KVL to the closed circuit. As R and C are connected in series and the current flowing through them is same, the current I is taken as reference.

As the current I flows through R and C, voltage drop occurs across them as given below:

(i) Voltage drop across resistance, $V_R = I R$ volt. V_R is in phase with the current I.

(ii) Voltage drop across capacitance, $V_C = I X_C$ volt. V_C lags behind the current I by an angle 90° *(because in capacitive circuit, current leads the voltage, as current is taken as reference, voltage V_C lags behind current).*

Therefore, applied voltage V is the vector sum of V_R and V_C,

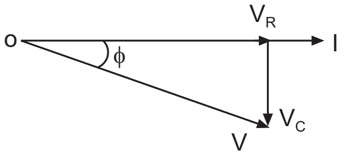


Fig. 3.27 Vector diagram

i.e., $$V = \sqrt{V_R^2 + V_C^2}$$

$$= \sqrt{(IR)^2 + (IX_C)^2}$$

$$= I\left(\sqrt{R^2 + X_C^2}\right)$$

or, $$I = \frac{V}{\sqrt{R^2 + X_C^2}}$$

$$= \frac{V}{Z} \text{ ampere}$$

where Z is called the impedance of the circuit and its unit is **ohm (Ω)**.

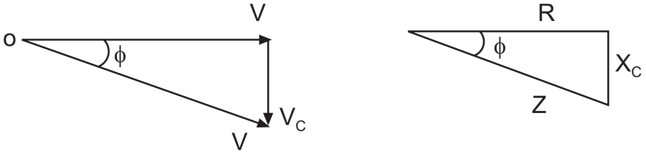


Fig. 3.28 (a) Voltage triangle and impedance triangle

From voltage triangle, $\tan\phi = \frac{V_C}{V_R} = \frac{I\,X_C}{I\,R} = \frac{X_C}{R}$

or $$\phi = \tan^{-1}\frac{X_C}{R}$$

From impedance triangle, $$\phi = \cos^{-1}\frac{V_R}{V} = \cos^{-1}\frac{R}{Z}$$

If the applied voltage is given by

$$v = V_m \sin \omega t$$

Then, equation for current is

$$i = I_m \sin(\omega t + \phi)$$

That is, if a circuit contains resistance and capacitance in series, called the capacitive circuit, the current leads the voltage by an angle ϕ as shown in Fig. 3.28 (b).

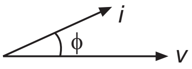


Fig. 3.28 (b) Phasor diagram

Solved Examples

Example 3.8 *A resistance of 7 Ω is connected in series with a pure inductance of 31.4 mH and the circuit is connected to a 100 V, 50 Hz sinusoidal supply. Calculate (a) the circuit current and (b) the phase angle.*

Solution: Given: $R = 7\ \Omega$, $L = 31.4\text{ mH} = 31.4 \times 10^{-3}$ H,

$V = 100$ volt, $f = 50$ Hz

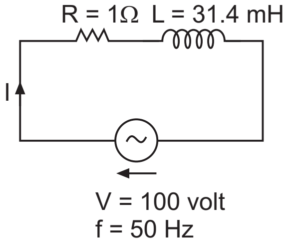


Fig. 3.29 Circuit diagram

(a) Circuit current, $I = \dfrac{V}{Z}$

Now impedance, $Z = \sqrt{R^2 + X_L^2}$

where inductive reactance, $X_L = 2\pi f L = 2\pi \times 50 \times (31.2 \times 10^{-3}) = 9.8\ \Omega$

then, $Z = \sqrt{7^2 + 9.8^2} = 12\ \Omega$

∴ $I = \dfrac{100}{12} = \mathbf{8.33\ A}$

(b) From impedance triangle,

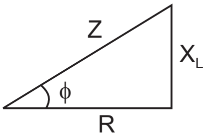


Fig. 3.30 Impedance triangle

Phase angle, $\phi = \tan^{-1}\dfrac{X_L}{R} = \tan^{-1}\dfrac{10}{7} = \mathbf{55°(lag)}$

Example 3.9 *A pure inductance of 318 mH is connected in series with a pure resistance of 75 Ω. The circuit is supplied from a 50 Hz sinusoidal source and the voltage across the 75 Ω resistor is found to be 150 V. Calculate the supply voltage using KVL and Ohm's law.*

Solution: Given: $L = 318\text{ mH} = 318 \times 10^{-3}\text{ H}$, $R = 75\ \Omega$, $f = 50\text{ Hz}$, $V_R = 150$ volt

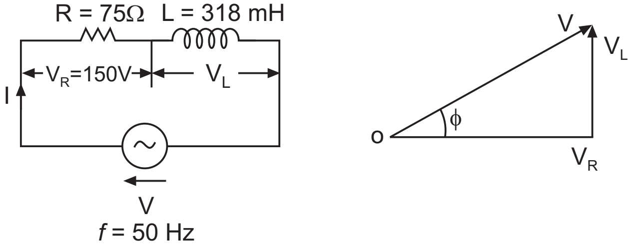


Fig. 3.31 Circuit diagram and voltage triangle

From the voltage triangle, applied voltage,

$$V = \sqrt{V_R^2 + V_L^2}$$

Now $\quad V_L = I\,X_L$

To find I and X_L:

Applying Ohm's Law, voltage drop across resistance,

$$V_R = I\,R$$

or current, $\quad I = \frac{V_R}{R} = \frac{150}{75} = 2\text{ A}$

Inductive reactance, $\quad X_L = 2\pi f L$

$$= 2\pi \times 50 \times (318 \times 10^{-3}) = 99.9\ \Omega$$

Thus, $\quad V_L = 2 \times 99.9 = 199.8\text{ V}$

$\therefore \quad V = \sqrt{150^2 + 199.8^2} = \mathbf{249.84\ V}$

Example 3.10 *A voltage of 100 V at 50 Hz is applied to a R-L series circuit. The current in the circuit is 5 A lagging behind the voltage by 35°. Write the expression for the voltage and current.*

Solution: Given: $V = 100$ volt, $f = 50$ Hz, $I = 5$ A, $\phi = 35°$

(Note: V and I are given in RMS values)

Expression for voltage will be in the form,

$$v = V_m \sin \omega t \text{ volt}$$

Maximum value of applied voltage,

$$V_m = \sqrt{2}\,V_{rms} = \sqrt{2} \times 100 = 141.42\text{ V}$$

Angular velocity, $\quad \omega = 2\pi f = 2\pi \times 50 = 314.15$ rad/sec

$\therefore$ Expression for voltage, $\quad \boldsymbol{v = 141.42 \sin 14.15\, t}$ **volt**

For R-L series circuit, expression for current will be in the form,

$$i = I_m \sin(\omega t - \phi)$$

Maximum value of current, $I_m = \sqrt{2}\, I_{rms} = \sqrt{2} \times 5 = 7.07$ A

Expressing ϕ in radian, $\phi = \dfrac{\pi}{180°} \times 35° = 0.61$ (Note: $180° = \pi$)

$\therefore$ Expression for current, $\boldsymbol{i = 7.07 \sin (314\, t - 0.61)}$ **ampere**

Example 3.11 *A current of* $i = 2.8 \sin\left(94.3\, t + \dfrac{\pi}{6}\right)$ *is flowing through a resistance of 10 Ω connected in series with an inductance of 1H. Obtain an expression for voltage across the R and L combination in the form* $v = V_m \sin(\omega t + \theta)$.

Solution: Given: $i = 2.8 \sin\left(94.3\, t + \dfrac{\pi}{6}\right)$ ampere, R = 10 Ω and L = 1 H

Given equation for current is in the form,

$$i = I_m \sin\left(\omega t + \frac{\pi}{6}\right),$$

where $I_m = 2.8$ ampere and $\omega = 94.3$ rad/sec.

Required to obtain equation for voltage in the form,

$$v = V_m \sin(\omega\, t + \theta)$$

To find V_m and θ:

Maximum value of voltage, $V_m = I_m Z$

Now $Z = \sqrt{R^2 + X_L^2}$

where $X_L = \omega L = 94.3 \times 1 = 94.3\ \Omega$

Then, $Z = \sqrt{R^2 + X_L^2} = \sqrt{10^2 + 94.3^2} = 94.82\ \Omega$

$\therefore$ $V_m = 2.8 \times 94.82 = 265.49$ V

Phase angle between voltage and current,

$$\phi = \tan^{-1} \frac{X_L}{R} = \tan^{-1} \frac{94.3}{10} = 83.94°$$

As per the given equation for current, current leads a reference by $\dfrac{\pi}{6} = 30°$, and as per the voltage equation, voltage should lead the reference by an angle θ.

$\therefore$ $\theta = 83.94° + 30° = 113.94°$

or $= 113.94° \times \dfrac{\pi}{180°} = 0.633\, \pi$ rad

$\therefore$ Expression for voltage, $\boldsymbol{v = 265.49 \sin (94.3\, t + 0.633\, \pi)}$ **volt**

Example 3.12 *An inductor coil is connected to a supply of 250 V at 50 Hz and takes a current of 5 A. The coil dissipates 750 W. Calculate (a) the resistance and inductance of the coil and (b) the power factor of the coil.*

Solution: Given: Supply voltage = 250 V, f = 50 Hz

Current, I = 5 A, Power dissipated P = 750 W

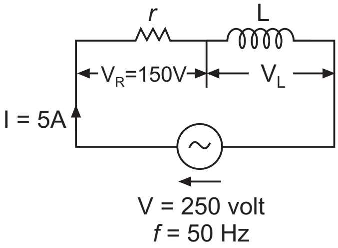


Fig. 3.32 Circuit diagram

(a) To find resistance and inductance of the coil:

$$\text{Resistance of the coil,}\quad r = \frac{P}{I^2} = \frac{750}{5^2} = 30\ \Omega$$

$$\text{Inductance of coil,}\quad L = \frac{X_L}{2\pi f} \quad (\text{because } X_L = 2\pi f L)$$

$$\text{Now}\quad X_L = \sqrt{Z^2 - r^2}$$

$$\text{where impedance of the coil, } Z = \frac{V}{I} = \frac{250}{5} = 50\ \Omega$$

$$\text{Then,}\quad X_L = \sqrt{Z^2 - r^2} = \sqrt{50^2 - 30^2} = 40\ \Omega$$

$$\therefore\quad L = \frac{X_L}{2\pi f} = \frac{40}{2\pi \times 50} = \mathbf{0.127\ H}$$

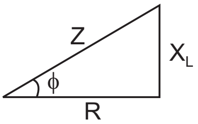


Fig. 3.33 Impedance triangle

(b) From impedance triangle, $pf = \frac{R}{Z} = \frac{30}{50}$

$= \mathbf{0.6\ (lag)}$

Example 3.13 *The potential difference measured across a coil is 20 V when a direct current of 2 A is passed through it. With an alternating current of 2 A at 40 Hz, the pd across the coil is 140 V. If the coil is connected to a 230 V, 50 Hz supply, calculate (a) the current, (b) the pf* and *(c) the active power.*

Solution: Given: With dc supply; *pd* = 20 V and I = 2 A

With ac supply; Case (i) I = 2 A, $f_1 = 40$ Hz and *pd* = 140 V = V_1 (say)

Case (ii) V = 230 volt, f = 50 Hz

With dc supply; resistance of the coil is to be determined;

i.e.,
$$R = \frac{\text{pd with dc supply}}{I} = \frac{20}{2} = \mathbf{10\ \Omega}$$

With ac supply;

Case (i): Impedance,
$$Z_1 = \frac{V_1}{I} = \frac{140}{2} = \mathbf{70\ \Omega}$$

Reactance,
$$X_{L1} = \sqrt{Z_1^2 - R^2} = \sqrt{70^2 - 10^2} = 69.28\ \Omega$$

Also,
$$X_{L1} = 2\pi f_1 L$$

or
$$L = \frac{X_{L1}}{2\pi f_1} = \frac{69.28}{2\pi \times 40} = 0.276\ \text{H}$$

Case (ii):

(a) Current,
$$I = \frac{V}{Z}$$

Now impedance,
$$Z = \sqrt{R^2 + X_L^2}$$

where
$$X_L = 2\pi f L = 2\pi \times 50 \times 0.276 = 86.7\ \Omega$$

Then,
$$Z = \sqrt{R^2 + X_L^2} = \sqrt{10^2 + 86.7^2} = 87.275\ \Omega$$

∴
$$I = \frac{V}{Z} = \frac{230}{87.275} = \mathbf{2.64\ A}$$

(b) Power factor,
$$\cos\phi = \frac{R}{Z} = \frac{10}{87.275} = \mathbf{0.114}$$

(c) Active power,
$$P = VI\cos\phi = 230 \times 2.64 \times 0.114 = \mathbf{69.22\ W}$$

Example 3.14 *A coil having resistance of R ohm and inductance of L henry are connected is connected across a variable frequency alternating current supply of 240V. An ammeter in the circuit showed 60A when the frequency was 50 Hz and 40A when the frequency was 100Hz. Find the values of R and L.*

Solution: Given: Supply voltage, V = 240 volt

Case (i): At frequency $f_1 = 50\text{Hz}$, current $I_1 = 60\text{A}$

Case (ii): At frequency $f_2 = 100$ Hz, current $I_2 = 40\text{A}$

To find the values of R and L, impedance Z has to be calculated.

***Important point*:** As frequency changes, reactance changes

Case (i): $$Z_1 = \frac{V}{I_1} = \frac{240}{60} = 4\ \Omega$$

Also, $$Z_1 = \sqrt{R^2 + X_1^2}$$

or $$Z_1^2 = R^2 + X_1^2$$

$$= R^2 + (2\pi f_1 L)^2$$

or $$4^2 = (2\pi \times 50L)^2$$

or $$16 = (100\ \pi)^2\, L^2$$

Case (ii) $$Z_2 = \frac{V}{I_2} = \frac{240}{40} = 6\ \Omega$$

Also, $$Z_2^2 = R^2 + X_2^2 = R^2 + (2\pi f_2\, L)^2$$

or $$6^2 = (2\pi \times 100\ L)^2$$

or $$36 = (200\ \pi)^2\, L^2$$

∴ $$(200\ \pi)^2\, L^2 - (100\ \pi)^2\, L^2 = 36 - 16$$

∴ $$= 20$$

or inductance, $$L = \sqrt{\frac{20}{(200\ \pi)^2 - (100\ \pi)^2}} = \mathbf{8.22\ mH}$$

or resistance, $$R = \sqrt{Z_1^2 - X_1^2}$$

where $$X_1 = 2\pi f_1 L = 2\pi \times 50 \times (8.22 \times 10^{-3}) = 2.58\ \Omega$$

Thus, $$R = \sqrt{4^2 - 2.58^2} = \mathbf{3.05\ \Omega}$$

Example 3.15 *An emf given by 100 sin* $\left(314\, t - \frac{\pi}{4}\right)$ *V is applied to a circuit and current is 20 sin (314 t – 1.57) A. Find (a) the frequency and (b) the circuit elements.*

Solution: Given: $v = 100 \sin\left(314\, t - \frac{\pi}{4}\right)$ V and $i = 20 \sin(314\, t - 1.57)$ A

Given equations are in the form,

$$v = V_m \sin(\omega t - \phi_1)$$

and $$i = I_m \sin(\omega t - \phi_2)$$

(a) To find the frequency:

$$\omega = 2\pi f$$

or $$f = \frac{\omega}{2\pi} = \frac{314}{2\pi} = \mathbf{50\ Hz}$$

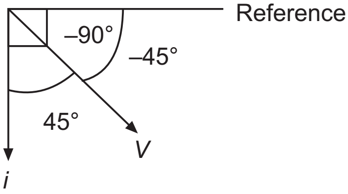


Fig. 3.34 Phasor diagram

(b) To find the circuit elements:

Voltage lags a reference by an angle $\phi_1 = -\frac{\pi}{4}$

$$= -\frac{180}{4} = -45°$$

Current lags the reference by an angle $\phi_2 = -1.57 = -1.57 \times \frac{180}{\pi}$

$$= -89.95° \approx -90°$$

Therefore, angle between voltage and current is given by

$$\phi = (\phi_1 - \phi_2) = [(-45°) - (-90°)] = 45° \text{ lag}$$

Thus, the circuit is inductive and the circuit elements are resistance and inductance.

Impedance, $$Z = \frac{V_m/\sqrt{2}}{I_m/\sqrt{2}} = \frac{100/\sqrt{2}}{20/\sqrt{2}} = 5\,\Omega$$

(Note: Impedance is calculated in terms of RMS values.)

From impedance triangle,

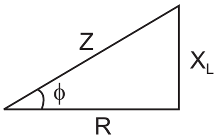


Fig. 3.35 Impedance triangle

Resistance, $R = Z\cos\phi = 5 \times \cos(45°) = \mathbf{3.53\ \Omega}$

Inductance, $L = \dfrac{X_L}{2\pi f}$ (because $X_L = 2\pi f L$)

where $X_L = \sqrt{Z^2 - R^2} = \sqrt{5^2 - 3.53^2} = 3.53\ \Omega$

$\therefore$ $L = \dfrac{X_L}{2\pi f} = \dfrac{3.53}{2\pi \times 50} = \mathbf{0.011\ H}$

Example 3.16 *A coil having a resistance 6 Ω and an inductance of 0.03 H is connected across a 50 V, 60 Hz supply. Calculate (a) the current, (b) the phase angle between the current and the applied voltage, (c) the apparent power and (d) the active power.*

Solution: Given: $R = 6\ \Omega$, $L = 0.03$ H, $V = 50$ volt and $f = 60$ Hz

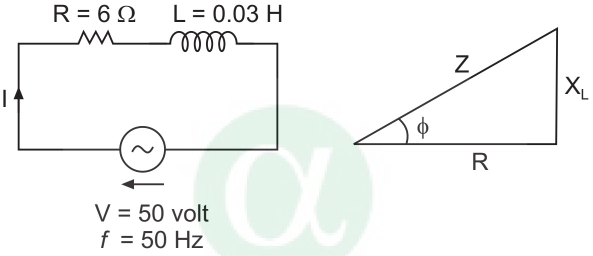


Fig. 3.36 Circuit diagram and impedance triangle

(a) Current, $I = \dfrac{V}{Z}$

Now, $Z = \sqrt{R^2 + X_L^2}$

where $X_L = 2\pi f L = 2\pi \times 60 \times 0.03 = 11.31\ \Omega$

$\therefore$ $Z = \sqrt{R^2 + X_L^2} = \sqrt{6^2 + 11.31^2} = 12.8\ \Omega$

$\therefore$ $I = \dfrac{V}{Z} = \dfrac{50}{12.8} = \mathbf{3.91\ A}$

(b) Phase angle, $\phi = \cos^{-1}\dfrac{R}{Z} = \cos^{-1}\dfrac{6}{12.8} \approx \mathbf{62°}$

(c) Apparent power, $S = VI = 50 \times 3.91 = \mathbf{195.5\ VA}$

(d) Active power, $P = \text{Apparent power} \times \cos\phi.$

$= 195.5 \times \cos 62° = \mathbf{91.78\ W}$

Example 3.17 *A capacitor of 8 μF takes a current of 1 A when the alternating voltage applied across it is 250 V, calculate (a) the frequency of the applied voltage, (b) the resistance to be connected in series with the capacitor to reduce the current in the circuit to 0.5 A at the same frequency and (c) the phase angle of the resulting circuit.*

Solution: Given: $C = 8\ \mu F = 8 \times 10^{-6}$ F, $I = 1$ A,

$V = 250$ volt, New current $I_1 = 0.5$ A

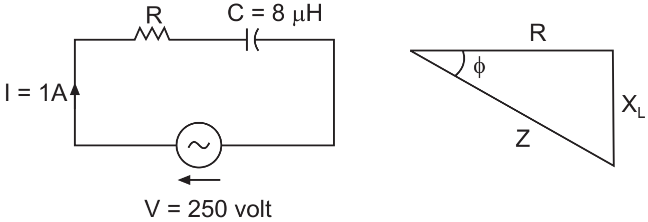


Fig. 3.37 Circuit diagram and impedance triangle

(a) Frequency, $$f = \frac{1}{2\pi CX_C}$$

where $$X_C = \frac{V}{I} = \frac{250}{1} = 250\ \Omega$$

∴ $$f = \frac{1}{2\pi CX_C} = \frac{1}{2\pi(8\times10^{-6})\times 250} = \mathbf{79.57\ Hz}$$

(b) Let R be the resistance connected to get a new current $I_1 = 0.5$ A

Then the impedance, $$Z = \frac{V}{I_1} = \frac{250}{0.5} = 500\ \Omega$$

Also, $$Z = \sqrt{R^2 + X_C^2}$$

i.e., $$500 = \sqrt{R^2 + 250^2}$$

or $$R = \sqrt{500^2 - 250^2} = \mathbf{433\ \Omega}$$

(c) From impedance triangle, $\phi = \cos^{-1}\dfrac{R}{Z} = \cos^{-1}\dfrac{433}{500} = \mathbf{30°}$

Example 3.18 *A resistor of 100 Ω is connected in series with a condenser of 50 μF to supply at 200 V, 50 Hz. Find (i) Impedance, (ii) Current, (iii) pf (iv) Phase angle and (v) Voltage across R and C.*

Solution: Given: $R = 100\ \Omega$, $C = 50\ \mu F = 50 \times 10^{-6}$ F, $V = 200$ volt, $f = 50$ Hz

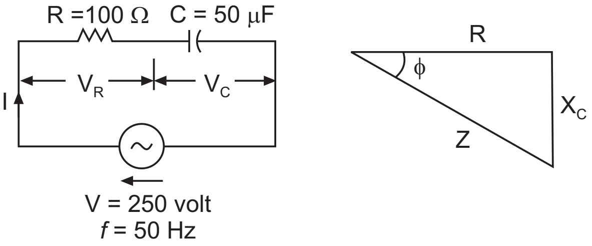


Fig. 3.38 Circuit diagram and impedance triangle

(i) Impedance, $Z = \sqrt{R^2 + X_C^2}$

where $X_C = \dfrac{1}{2\pi f C} = \dfrac{1}{2\pi \times 50 \times \left(50 \times 10^{-6}\right)} = 63.66\ \Omega$

$\therefore$ $Z = \sqrt{100^2 + 63.66^2} = \mathbf{118.54\ \Omega}$

(ii) Current, $I = \dfrac{V}{Z} = \dfrac{200}{118.54} = \mathbf{1.687\ A}$

(iii) Power factor, $\cos\phi = \dfrac{R}{Z} = \dfrac{100}{118.54} = \mathbf{0.84}$

(iv) Phase angle, $\phi = \cos^{-1}(0.84) = \mathbf{32.85°}$

(v) Voltage across Resistance, $V_R = I R = 1.687 \times 100 = \mathbf{168.7\ V}$

Voltage across Capacitance, $V_C = I X_C = 1.687 \times 63.66 = \mathbf{107.39\ V}$

3.8.7 R-L-C Series Circuit

Figure 3.39 shows an ac circuit with a Resistance R, an Inductance L and a Capacitance C connected in series. The applied voltage V and the current drawn I are expressed in RMS values.

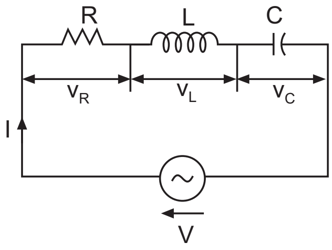


Fig. 3.39 R-L-C Series circuit

Vector diagram is drawn applying KVL to the closed circuit. As R, L and C are connected in series and the current flowing through them is same, current I is taken as reference.

As current flows through R, L and C, voltage drop occurs across them as given below:

(i) Voltage drop across resistance, $V_R = I\,R$ volt. V_R is in phase with current I.

(ii) Voltage drop across inductance, $V_L = I\,X_L$ volt. V_L leads the current I by an angle 90°.

(iii) Voltage drop across capacitance, $V_C = I\,X_C$ volt. V_C lags the current I by an angle 90°.

V_R is represented by the vector OA, V_L by the vector AB and V_C by the vector AC. It is observed that the vectors AB and AC are in direct phase opposition.

Assuming that $X_L > X_C$

$\Rightarrow$ $V_L > V_C$

$\Rightarrow$ $AB > AC$

$\therefore$ $AD = AB - AC$

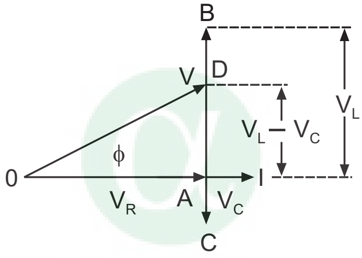


Fig. 3.40 Vector diagram

Then,
$$OD = \sqrt{OA^2 + AD^2}$$
$$= \sqrt{OA^2 + (AB - AC)^2}$$

i.e.,
$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$
$$= \sqrt{(I\,R)^2 + (I\,X_L - I\,X_C)^2}$$
$$= \sqrt{R^2 + (X_L - X_C)^2}$$

or
$$I = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}}$$

or
$$I = \frac{V}{Z} \text{ ampere}$$

where Z is called the impedance of the circuit and its unit is **ohm (Ω)**

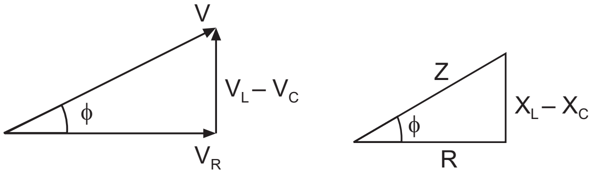


Fig. 3.41 Voltage triangle and impedance triangle

From Voltage triangle,

$$\tan\phi = \frac{V_L - V_C}{V_R} = \frac{I(X_L - X_C)}{IR} = \frac{(X_L - X_C)}{R}$$

or $$\phi = \tan^{-1}\frac{(X_L - X_C)}{R}$$

From Impedance triangle, power factor is given by

$$\cos\phi = \frac{R}{Z}$$

or $$\phi = \cos^{-1}\frac{V_R}{V} = \cos^{-1}\frac{R}{Z}$$

Power, $$P = V\,I\cos\phi \text{ watt}$$

Thus, in a series circuit containing resistance, inductance and capacitance, if the inductive reactance is greater than capacitive reactance, the current lags behind the voltage by an angle ϕ and the circuit is called the inductive circuit.

If $V_L = V_C \Rightarrow X_L = X_C \Rightarrow Z = R$, then the R-L-C series circuit is in electrical resonance.

Thus, $$P = I^2 R \text{ watt}$$

Solved Examples

Example 3.19 *A circuit having resistance of 12 Ω, an inductance of 0.15 H and a capacitance of 100 μF in series, is connected across a 100 V, 50 Hz supply. Calculate: (a) the impedance, (b) the current, (c) the voltage across R, L and C and (d) the phase difference between the current and the supply voltage.*

Solution: Given: $R = 12\ \Omega$, $L = 0.15$ H, $C = 100\ \mu F = 100 \times 10^{-6}$ F

$V = 100$ volt, $f = 50$ Hz

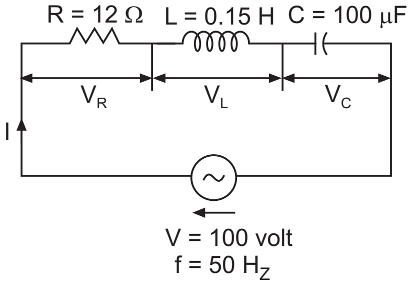


Fig. 3.42 (a) Circuit diagram

(a) Impedance, $$Z = \sqrt{R^2 + \left(X_L - X_C\right)^2}$$

where $$X_L = \text{inductive reactance} = 2\pi f L = 2\pi \times 50 \times 0.15$$
$$= 47.12\ \Omega$$

$$X_C = \text{capacitive reactance} = \frac{1}{2\pi\ f\ C}$$

$$= \frac{1}{2\pi \times 50 \times (100 \times 10^{-6})} = 31.83\ \Omega$$

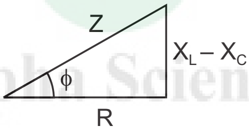


Fig. 3.42 (b) Impedance triangle

∴ $$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$= \sqrt{12^2 + (47.1 - 31.83)^2}$$

$$= 19.42\ \Omega$$

(b) Current, $$I = \frac{V}{Z} = \frac{100}{19.42} = \mathbf{5.15\ A}$$

(c) Voltage drop across Resistance, $V_R = I\,R = 5.15 \times 12 = \mathbf{61.8\ V}$

Voltage drop across Inductance, $V_L = I\,X_L = 5.15 \times 47.12 = \mathbf{242.66\ V}$

Voltage drop across Capacitance, $V_C = I\,X_C = 5.15 \times 31.83 \approx \mathbf{164\ V}$

(d) Phase difference, $\phi = \cos^{-1}\dfrac{R}{Z} = \cos^{-1}\dfrac{12}{19.42} = \mathbf{51.83°}$

Example 3.20 *A 230 V, 50 Hz ac supply is applied to a coil of 0.06 H inductance and 2.5 Ω resistance connected in series with a 6.8 μF capacitor. Calculate: (i) impedance, (ii) current, (iii) phase angle between current and voltage, (iv) power factor and (v) power consumed.*

Solution: Given: $V = 230$ volt, $f = 50$ Hz,

$$R = 2.5\ \Omega,\ L = 0.06\ \text{H}$$

$$C = 6.8\ \mu\text{F} = 6.8 \times 10^{-6}\ \text{F}$$

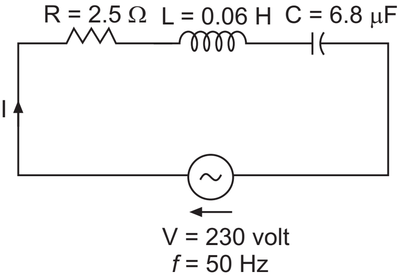


Fig. 3.43 (a) Circuit diagram

(i) Impedance, $Z = \sqrt{R^2 + (X_L - X_C)^2}$

where inductive reactance, $X_L = 2\pi f L$

$$= 2\pi \times 50 \times 0.06 = 18.85\ \Omega$$

capacitive reactance, $X_C = \dfrac{1}{2\pi f\, C}$

$$= \frac{1}{2\pi \times 50 \times (6.8 \times 10^{-6})} = 468.1\ \Omega$$

$\therefore$ $$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$= \sqrt{2.5^2 + (18.85 - 468.1)^2}$$

$$= \mathbf{449.26\ \Omega}$$

(ii) Current, $I = \dfrac{V}{Z}$

$$= \frac{230}{449.26} = \mathbf{0.51\ A}$$

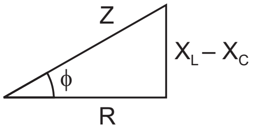


Fig. 3.43 (b) Impedance triangle

(iii) Phase angle between current and voltage,

$$\phi = \cos^{-1}\frac{R}{Z} = \cos^{-1}\frac{2.5}{449.26} = \mathbf{89.68^\circ}$$

(iv) Power factor, $\cos\phi = \cos 89.68^\circ = \mathbf{0.0055}$

(v) Power consumed, $P = V I \cos\phi = 230 \times 0.51 \times 0.0055 = \mathbf{0.655\ W}$

3.8.8 R-L Parallel Circuit

Figure 3.44 shows an ac circuit containing a resistance R and an inductance L connected in parallel. The applied voltage V and the current drawn I are expressed in RMS values.

Vector diagram is drawn applying KCL at point A. As R and L are connected in parallel, voltage drop across them will be the same and equal to the applied voltage and thus voltage V is taken as reference.

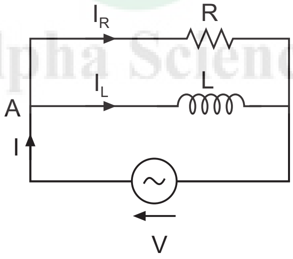


Fig. 3.44 Circuit diagram

Current in resistance, $I_R = \dfrac{V}{R}$ ampere. I_R is in phase with voltage V.

Current in inductance, $I_L = \dfrac{V}{X_L}$ ampere. I_L lags the voltage V by 90°.

$$\therefore \quad \text{Total Current,} \quad I = \sqrt{I_R^2 + I_L^2}$$

$$= \sqrt{\left(\frac{V}{R}\right)^2 + \left(\frac{V}{X_L}\right)^2}$$

$$= V\sqrt{\left(\frac{1}{R^2}\right)+\left(\frac{1}{X_L^2}\right)}$$

or $$\frac{I}{V} = \sqrt{\left(\frac{1}{R^2}\right)+\left(\frac{1}{X_L^2}\right)}$$

∴ $$\frac{V}{I} = Z = \frac{1}{\sqrt{\frac{1}{R^2}+\frac{1}{X_L^2}}} \text{ ohm}$$

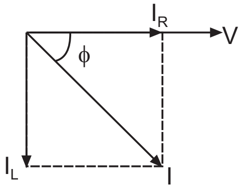


Fig. 3.45 Vector diagram

Referring vector diagram, $\tan\phi = \frac{I_L}{I_R}$

or $$\phi = \tan^{-1}\frac{I_L}{I_R} = \tan^{-1}\frac{R}{X_L}$$

Also, $$\cos\phi = \frac{I_R}{I}$$

or $$\phi = \cos^{-1}\frac{I_R}{I} = \cos^{-1}\frac{Z}{R}$$

3.8.9 R-C Parallel Circuit

Figure 3.46 shows an ac circuit containing a resistance R and a capacitance C connected in parallel. The applied voltage V and the current I are expressed in RMS values.

Vector diagram is drawn applying KCL at point A. As R and C are connected in parallel, voltage drop across them will be the same and equal to applied voltage and thus voltage V is taken as reference.

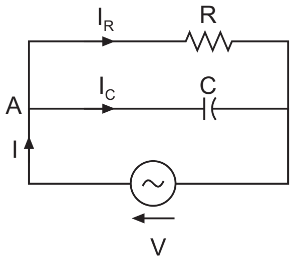


Fig. 3.46 Circuit diagram

Current in resistance, $I_R = \frac{V}{R}$ ampere. I_R is in phase with voltage V

Current in Capacitance, $I_C = \frac{V}{X_L}$ ampere, I_C leads the voltage V by 90°

$$\therefore \text{ Total Current,} \qquad I = \sqrt{I_R^2 + I_C^2}$$

$$= \sqrt{\left(\frac{V}{R}\right)^2 + \left(\frac{V}{X_C}\right)^2}$$

$$= V\sqrt{\left(\frac{1}{R^2}\right) + \left(\frac{1}{X_C^2}\right)}$$

or
$$\frac{I}{V} = \sqrt{\left(\frac{1}{R^2}\right) + \left(\frac{1}{X_C^2}\right)}$$

or
$$\frac{V}{I} = Z = \frac{1}{\sqrt{\left(\frac{1}{R^2}\right) + \left(\frac{1}{X_C^2}\right)}} \text{ ohm}$$

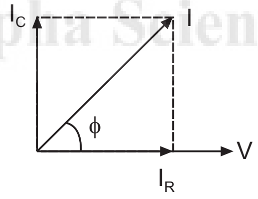


Fig. 3.47 Vector diagram

Referring vector diagram, $\tan\phi = \frac{I_C}{I_R}$

or
$$\phi = \tan^{-1}\frac{I_C}{I_R} = \tan^{-1}\frac{R}{X_C}$$

Also,
$$\cos\phi = \frac{I_R}{I}$$

or
$$\phi = \cos^{-1}\frac{I_R}{I} = \cos^{-1}\frac{Z}{R}$$

Solved Examples

Example 3.21 *A circuit consists of a 120 Ω resistor in parallel with a 40 μF capacitor and is connected to a 240 V, 50 Hz supply. Calculate: (a) the branch current and supply current, (b) the circuit phase angle, (c) the circuit impedance, (d) the active power and (e) the power factor.*

Solution: Given: R = 120 Ω, C = 40 μF = 40×10^{-6} F, V = 240 volt, f = 50 Hz.

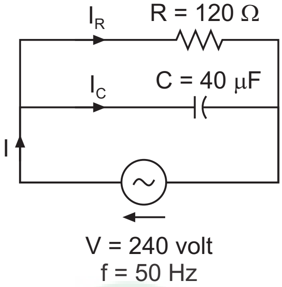


Fig. 3.48 Circuit diagram

(a) To find the branch current and supply current:

Current in resistance, $I_R = \frac{V}{R} = \frac{240}{120} = \mathbf{2\ A}$

Current in Capacitance, $I_C = \frac{V}{X_C}$

where $X_C = \frac{1}{2\pi f\, C} = \frac{1}{2\pi \times 50 \times (40 \times 10^{-6})} = 79.58\ \Omega$

Then, $I_C = \frac{V}{X_C} = \frac{240}{79.58} = \mathbf{3\ A}$

∴ Total Current, $I = \sqrt{I_R^2 + I_C^2} = \sqrt{2^2 + 3^2} = \mathbf{3.6\ A}$

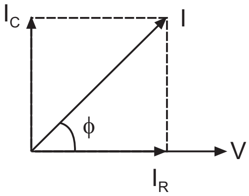


Fig. 3.49 Vector diagram

(b) Circuit phase angle, $\phi = \tan^{-1}\frac{I_C}{I_R} = \tan^{-1}\frac{3}{2} = \mathbf{56.309°}$

(c) Circuit impedance, $Z = \frac{V}{I} = \frac{240}{3.6} = \mathbf{66.66\ \Omega}$

(d) Active power, $W = V\,I\cos\phi = 240 \times 3.6 \times \cos(56.309°) = \mathbf{479.27\ W}$

(e) Power factor, $pf = \cos(56.309°) = \mathbf{0.555}$

Example 3.22 *An inductive load takes a current of 40 A at pf 0.7 lagging from 440 V, 50 Hz supply. What value must a shunting capacitor have to raise the pf to 0.9 lagging?*

Solution: Given: Current taken by inductive load, I = 40 A

Initial power factor: $\cos\phi_1 = 0.7$ (lagging)

$$V = 440 \text{ volt},\ f = 50 \text{ Hz}$$

Required power factor: $\cos\phi_2 = 0.9$ (lagging)

To improve the *pf* a capacitor is to be connected in parallel and its value is to be determined.

Voltage drop across capacitor, $V = I_C X_C$

or $$X_C = \frac{V}{I_C}$$

Also, $$X_C = \frac{1}{2\pi f C}, \text{ where } C = \text{value of shunt capacitor}$$

$$\therefore \quad \frac{1}{2\pi f C} = \frac{V}{I_C}$$

or $$C = \frac{I_C}{2\pi f V}$$

To find I_C:

Before *pf* correction (that is, before connecting the shunt capacitor):

Lagging reactive component of motor,

$$I\sin\phi_1 = I\sqrt{1-\cos\phi_1^2}$$
$$= 40\sqrt{1-0.7^2}$$
$$= 28.56 \text{ A}$$

After *pf* correction (that is, after connecting the shunt capacitor);

Lagging reactive component of motor,

$$\begin{aligned} I \sin \phi_2 &= I \sqrt{1 - \cos \phi_2^2} \\ &= 40\sqrt{1 - 0.9^2} \\ &= 17.44 \text{ A} \end{aligned}$$

Therefore, leading reactive current drawn by capacitor is

$$\begin{aligned} I_C &= I \sin \phi_1 - I \sin \phi_2 \\ &= 28.56 - 17.44 \\ &= 11.12 \text{ A} \end{aligned}$$

$\therefore$ Value of shunt capacitor, $C = \dfrac{11.12}{2\pi \times 50 \times 440} = \mathbf{80.4 \times 10^{-6}\ F = 804\ \mu F}$

3.9 COMPLEX NOTATION

3.9.1 Complex Form

A vector will be specified in terms of its X-component and Y-component. For example, voltage V_1 is represented by vector OV_1 as shown in Fig. 3.50. Its X-axis (horizontal component) is a_1 called real component and Y-axis (vertical component) is b_1, called imaginary component.

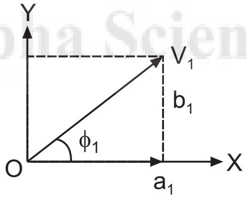


Fig. 3.50

Expressing in complex form,

$$V_1 = a_1 + j\, b_1$$

The above representation is called rectangular form. Symbol ***j*** is called **operator**, which indicates that component b_1 is perpendicular to component a_1. The magnitude (or modulus) of vector is

$$V_1 = \sqrt{a_1^2 + b_1^2}$$

Important Points

Voltage is a scalar quantity. Sinusoidal voltage is represented by a complex scalar or phasor with amplitude and phase angle.

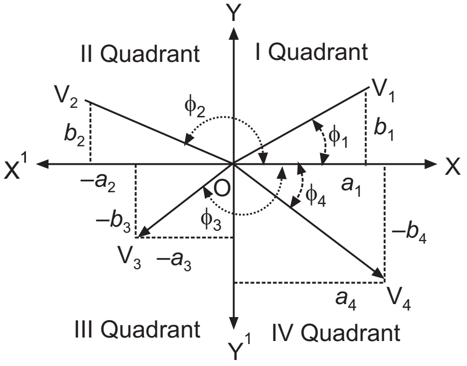


Fig. 3.51

Its phase angle with X-axis, $\phi = \tan^{-1}\left(\frac{b_1}{a_1}\right)$

Thus, expressing in polar form, $V_1 = v_1 \angle \phi_1$

Representation of V_1, V_2, V_3 and V_4 in rectangular and polar form is given in table 3.2.

Table 3.2

Rectangular form	Polar form
$V_1 = a_1 + j\,b_1$	$v_1 \angle \phi_1$
$V_2 = -\,a_2 + j\,b_2$	$v_2 \angle \phi_2$
$V_3 = -\,a_3 - j\,b_3$	$v_3 \angle -\phi_3$
$V_4 = a_4 - j\,b_4$	$v_4 \angle -\phi_4$

Trigonometrically a vector can be represented as follows:

X-component of V is $V\cos\phi$ and Y-component of V is $V\sin\phi$.

$$\therefore \quad V = V\cos\phi + j\,V\sin\phi$$

$$= V(\cos\phi + j\sin\phi)$$

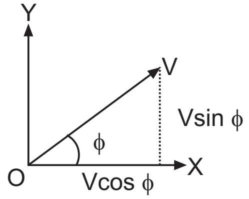


Fig. 3.52

3.9.2 Significance of operator *j*

Symbol *j* indicates the anticlockwise rotation of a vector through 90°

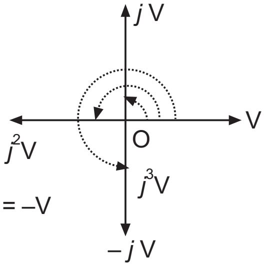


Fig. 3.53

It is assigned a value of $\sqrt{-1}$

$j = 90°$ anticlockwise rotation $= \sqrt{-1}$

$j^2 = 180°$ anticlockwise rotation $= (\sqrt{-1})^2 = -1$

$j^3 = 270°$ anticlockwise rotation $= (\sqrt{-1})^3 = (-\sqrt{-1}) = -j$

$j^4 = 360°$ anticlockwise rotation $= (\sqrt{-1})^4 = +1$

3.9.3 Representation of Impedance (Z) as a complex number

Considering a series R-L circuit, its impedance is expressed in the following forms

Rectangular (R) form: $Z = R + jX$

where R is the real component and X is the imaginary component;

Polar (P) form: $Z = |Z| \times \theta$ or $z \angle \theta$

where $|Z|$ or z is the magnitude and θ is the phase angle wrt the reference axis.

3.9.4 Conversion

Rectangular to Polar form:

$$Z = R + jX \qquad \text{(This is in Rectangular form)}$$

$$Z = \sqrt{R^2 + X^2} = z \text{ and } \theta = \tan^{-1}\frac{X}{R}$$

i.e., $$Z = z\angle\theta \qquad \text{(This is in Polar form)}$$

Polar to Rectangular form:

$$Z = z\angle\theta \qquad \text{(This is in Polar form)}$$

$$Z = z(\cos\theta + j\sin\theta)$$

$$= z\cos\theta + jz\sin\theta$$

i.e., $Z = R + jX$ (This is in Rectangular form)

For Addition and Subtraction of complex numbers, Rectangular form should be used.

Let, $Z_1 = R_1 + jX_1$ and $Z_2 = R_2 + jX_2$

Addition: $Z_1 + Z_2 = (R_1 + R_2) + j(X_1 + X_2)$

Subtraction: $Z_1 - Z_2 = (R_1 - R_2) + j(X_1 - X_2)$

For Multiplication and Division of complex numbers, Polar form should be used.

Let $Z_1 = z_1 \angle\theta_1$ and $Z_2 = z_2 \angle\theta_2$

Multiplication: $Z_1 \times Z_2 = z_1 \times z_2 \angle\theta_1 + \theta_2$

Division: $\dfrac{Z_1}{Z_2} = \dfrac{z_1}{z_2} \angle\theta_1 - \theta_2.$

Solved Examples

Example 3.23 *Two impedances $Z_1 = (20 + j\,31.41)\ \Omega$ and $Z_2 = (50 - j\,79.577)\ \Omega$ are connected in series across 230 V supply. Find the current drawn from the supply.*

Solution: Given: $Z_1 = 20 + j\,31.41\ \Omega$, $Z_2 = (50 - j\,79.577)\ \Omega$ and $V = 230$ volt

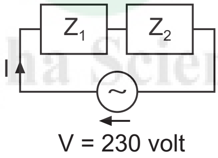


Fig. 3.54 Circuit diagram

Current drawn, $I = \dfrac{V}{Z_T}$

Now total impedance, $Z_T = Z_1 + Z_2$

$= (20 + j\,31.41) + (50 - j\,79.577)$

$= (20 + 50) + j\,(31.41 - 79.577)$

$= (70 - j\,48.167)\ \Omega$

Expressing Z_T in polar form,

$Z_T = (70 - j\,48.167)\ \Omega$

$$= \sqrt{70^2 + (-48.167)^2},\ \tan^{-1}\frac{-48.167}{70}$$

$= 84.97 \angle -34.53°\ \Omega$

$$\therefore \qquad I = \frac{V}{Z_T} = \frac{230}{84.97} \angle 0 - (-34.53°) = \mathbf{2.706 \angle 34.53° \ \Omega}$$

Example 3.24 *A box contains a two element series circuit. A voltage (40 – j 30) volt drives a current of (40 – j 3) A in the circuit. What are the values of the two elements? Supply frequency is 50 Hz.*

Solution: Given: $V = (40 - j\,30)$ volt, $I = (40 - j3)$A, $f = 50$ Hz

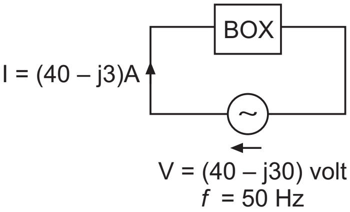


Fig. 3.55 Circuit diagram

To find the circuit elements the impedance is to be determined;

$$Z = \frac{V}{I}$$

Expressing voltage and current in polar form,

Voltage, $\quad V = (40 - j\,30)$

$$= \sqrt{(40)^2 + (-30)^2},\ \tan^{-1}\left(\frac{-30}{40}\right) = -36.86°$$

$$= 50 \angle{-36.86°}\ \text{V}$$

Current, $\quad I = (40 - j\,3)$

$$= \sqrt{(40)^2 + (-3)^2},\ \tan^{-1}\left(\frac{-3}{40}\right) = -4.28°$$

$$= 40.11 \angle{-4.28°}\ \text{A}$$

$$Z = \frac{V}{I} = \frac{50}{40.11} \angle -36.86° - (-4.28°)$$

$$= 1.246 \angle{-32.58°}$$

$$= 1.246\,(\cos 32.58° - j \sin 32.58°)$$

$$= 1.246 \cos 32.58° - j\,1.246 \sin 32.58°$$

$$= (1.05 - j\,0.67)\ \Omega$$

Therefore, the circuit elements are Resistance and Capacitance

Resistance, $\quad \mathbf{R = 1.05\ \Omega}$

Capacitive reactance, $X_C = 0.67\ \Omega$

$$\therefore \text{Capacitance, } C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi \times 50 \times 0.67} = \mathbf{0.00475\ F = 4750\ \mu F}$$

Example 3.25 *An alternating voltage (80 + j 60) V is applied to a circuit in which the current is (–4 + j 10) A. Find (i) the impedance of the circuit, (ii) the power consumed and (iii) the phase angle of circuit. Draw the phasor diagram.*

Solution: Given: V = (80 + *j* 60) volt and I = (– 4 + *j* 10) ampere

(i) To find the impedance of the circuit:

Impedance, $Z = \dfrac{V}{I}$

Expressing voltage and current in polar form;

$$V = (80 + j\,60)$$

$$= \sqrt{(80)^2 + (60)^2},\ \tan^{-1}\left(\frac{60}{80}\right)$$

$$= 100\ \angle 36.86^\circ\ \text{V}$$

Current, $I = (-4 + j\,10)$

$$= \sqrt{(-4)^2 + (10)^2},\ \tan^{-1}\left(\frac{10}{-4}\right)$$

Now $\tan^{-1}\left(\dfrac{10}{-4}\right) = \tan^{-1}(-2.5^\circ) - 68.2^\circ$; this lies in the second quadrant.

Thus, angle made by current = 180° – 68.2° = 111.8°

Then, $I = 10.77\ \angle 111.8^\circ$ A

$$\therefore \quad Z = \frac{V}{I} = \frac{100}{10.77}\ \angle 36.9^\circ - 111.8^\circ)$$

$$= \mathbf{9.28\ \angle{-74.9^\circ}}$$

$$= \mathbf{(2.42 - j\,8.96)}\ \Omega$$

(ii) Power consumed, $P = I^2 R = (10.77)^2 \times 2.42$

$$= \mathbf{280.7\ W}$$

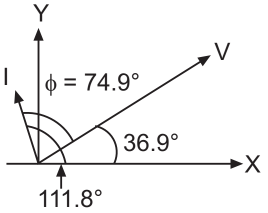


Fig. 3.56 Vector diagram

(iii) To find the phase angle of circuit:

Referring Fig. 3.56, the phase angle between voltage and current is

$$\phi = \mathbf{74.9° \text{ (leading)}}$$

Phasor/Vector diagram:

- Voltage makes an angle 36.9° with a reference.
- Current makes an angle 111.8° with a reference.
- Angle between v and $i \Rightarrow \phi = 74.9°$ (leading)

Example 3.26 *Two impedances $Z_1 = (150 + j\ 157)\ \Omega$ and $Z_2 = (100 + j\ 110)\ \Omega$ are connected in parallel across a 220 V, 50 Hz supply. Find total current and its pf.*

Solution: Given: $Z_1 = (150 + j\ 157)\ \Omega$, $Z_2 = (100 + j\ 110)\ \Omega$

$V = 220$ volt, $f = 50$ Hz

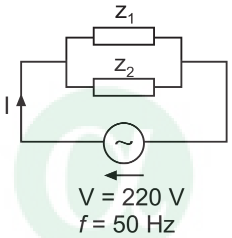


Fig. 3.57(a) Circuit diagram

Total current, $$I = \frac{V}{Z_T}$$

Total impedance, $$Z_T = \frac{Z_1 \times Z_2}{Z_1 + Z_2}$$

Expressing Z_1 and Z_2 in polar form,

$$Z_1 = (150 + j\ 157)$$

$$= \sqrt{(150)^2 + (157)^2},\ \tan^{-1}\left(\frac{157}{150}\right) = 46.3°$$

$$= 217.138\ \angle 46.3°\ \Omega$$

$$Z_2 = (100 + j\ 110)\ \Omega$$

$$= \sqrt{(100)^2 + (110)^2},\ \tan^{-1}\left(\frac{110}{100}\right)$$

$$= 148.66\ \angle 47.72°\ \Omega$$

Adding, $$Z_1 + Z_2 = (150 + j\ 157) + (100 + j\ 110)$$

$$= 250 + j\ 267$$

$$= \sqrt{(250)^2 + (267)^2},\ \tan^{-1}\left(\frac{267}{250}\right) = 46.88°$$

$$= 365.77\ \angle 46.88°\ \Omega$$

Multiplying, $Z_1 \times Z_2 = (217.138 \times 148.66)\ \angle\ 46.3° + 47.72°$

$$= 32279.735\ \angle 94.02°\ \Omega$$

$$Z_T = \frac{Z_1 \times Z_2}{Z_1 + Z_2} = \frac{32279.735}{365.77}\ \angle 94.02° - 46.88°$$

$$= 88.25\ \angle 47.14°\ \Omega$$

$$\therefore \qquad I = \frac{V}{Z_T} = \frac{220}{88.25}\ \angle 0 - 47.14°$$

$$= \mathbf{2.49\ \angle{-47.14°}}\ \Omega$$

Power factor, $\cos(47.14) =$ **0.68**

Note: This problem can also be solved by determining current flowing through each branch/impedance, say, I_1 and I_2 as given below

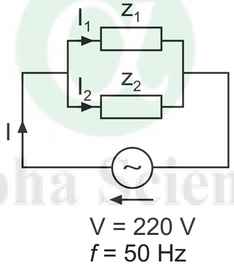


Fig. 3.57 (b) Circuit diagram

$$I_1 = \frac{Z_2}{Z_1 + Z_2}$$

$$I_2 = \frac{Z_1}{Z_1 + Z_2}$$

Converting I_1 and I_2 into rectangular form and then adding them,

$$I = I_1 + I_2$$

Then I will be converted to polar form.

Multiple Choice Questions

Choose the Correct Answer

1. If an alternating voltage given by $v = 20 \sin (314.16\,t + \pi/3)$V, its frequency is ......

   (a) 25 Hz (b) 45 Hz (c) 50 Hz (d) 60 Hz

2. The ac voltage is $v = 20 \sin 157\,t$. The frequency is......

   (a) 50 Hz (b) 75 Hz (c) 25 Hz (d) 100 Hz

3. An ac voltage is $v = 100 \sin 314\,t$. Average value of its half wave is......

   (a) 70.7 V (b) 50 V (c) 63.7 V (d) 100 V

4. A series circuit consists of 4.7 kΩ, 5.6 kΩ, 9 kΩ and 10 kΩ resistors. Which resistor has the highest voltage drop?

   (a) 4.7 kΩ (b) 5.6 kΩ (c) 9 kΩ (d) 10 kΩ

5. When a 10Ω resistor is connected across an alternating supply, given by $v = 100 \sin (314\,t + 37°)$V, the power dissipated by the resistance is

   (a) 500 W (b) 1000 W (c) 10,000 W (d) 10 W

6. A sinusoidal voltage is represented as 141.42 sin 314 t. RMS values of voltage and frequency are respectively

   (a) 141.42 V, 314 Hz (b) 100 V, 50 Hz
   (c) 200 V, 100 Hz (d) 100 V, 100 Hz

7. The power dissipated in each of three parallel branches is 1 watt. The total power dissipation of the circuit is

   (a) 1 W (b) 4 W (c) 3 W (d) 9 W

8. If $e_1 = A \sin \omega t$ and $e_2 = B \sin (\omega t - \phi)$, then

   (a) e_1 lags e_2 (b) e_2 lags e_1
   (c) e_1 is in phase with e_2 (d) none of these

9. A sinusoidal voltage v_1 leads another sinusoidal voltage v_2 by 180°. Then ......

   (a) v_2 leads v_1
   (b) both have their zero values at the same time
   (c) both have their peak values at the same time
   (d) all of them

10. RMS value of sinusoidal current is given by

    (a) $I_m/2$ (b) $I_m/\sqrt{2}$ (c) $I_m \times \sqrt{2}$ (d) $I_m \times 2$

11. Average value of sinusoidal voltage or current ...... times Maximum Value.

    (a) 0.737 (b) 0.637 (c) 0.557 (d) 1

12. The peak factor of a sinusoidally varying voltage is

(a) 1.414 (b) 1.11 (c) 0.866 (d) 0.707

13. The RMS value of a sinusoidal current is 10 A. Its peak value is ......

(a) 7.07 A (b) 14.14 A (c) 10 A (d) 1.11 A

14. The form factor of a sine wave is.......

(a) 1.414 (b) 1.11 (c) 0.866 (d) 0.707

15. Average power consumed by a pure capacitance over a complete cycle is

(a) one (b) 100% (c) zero (d) none of these

16. Average power consumed by a pure inductance over a complete cycle is

(a) zero (b) one (c) 50% (d) infinity

17. The reactance of an inductor rises with ......

(a) voltage (b) frequency

(c) both (a) and (b) (d) none of these

18. In an ac circuit electrical energy is consumed in

(a) L (b) C (c) L and C (d) R

19. In R-L series circuit R = 10 Ω and X = 10 Ω. The phase angle between V and I is

(a) 45° (b) 60° (c) 30° (d) 36.9°

20. The unit of impedance is

(a) ohm-metre (b) ohm/metre (c) ohm m^2 (d) ohm

21. In a certain R-L circuit, $V_R = 2V$ and $V_L = 3V$, the magnitude of total voltage is

(a) 2V (b) 3V (c) 5V (d) 3.6V

22. In R-L series circuit, the phase difference between V and I increases if

(a) X_L decreases (b) X_L increases

(c) R increases (d) supply frequency decreases

23. The meaning of low power factor of an ac circuit is

(a) it draws more reactive power (b) it draws more active power

(c) it draws less current from supply (d) it causes less voltage drop

24. In an ac circuit the ratio of kW/kVA represents ....

(a) power (b) power factor (c) form factor (d) peak factor

25. The power factor of a pure resistive circuit is

(a) zero (b) unity (c) lagging (d) leading

26. At what power factor the active and apparent power of an ac circuit are equal in magnitude?

    (a) 0.8 (b) 0.5 (c) 0.25 (d) 1

27. The power factor is lagging when

    (a) voltage lags the current (b) current lags the voltage

    (c) voltage lags power (d) current lags power

28. The reactive power in a single phase ac circuit is give by

    (a) EI cosϕ (b) EI (c) EI sinϕ (d) none of these

29. The maximum value of power factor of an ac circuit lies in between

    (a) 1 and 0 (b) +1 and –1

    (c) +0.25 and –0.25 (d) +10 and –10

30. The power factor of a load can be improved by

    (a) inductor (b) capacitor

    (c) both capacitor and inductor (d) none of these

31. At 50 Hz of frequency, the impedance of R-L series circuit is 25Ω. If the frequency is increased to 60 Hz, the impedance becomes

    (a) > 25 Ω (b) 25 Ω (c) < 25Ω (d) none of these

32. If a sinusoidal alternating voltage is applied to a pure resistance, the frequency of power is .... the applied voltage.

    (a) half of (b) twice of (c) thrice of (d) same as

33. If the impedance of the circuit is $15.5\angle -30°$ Ω, then the nature of the circuit is

    (a) resistive (b) inductive (c) capacitive (d) none of these

34. In R-L series circuit, the phase difference between the applied voltage and current increases if

    (a) inductive reactance is decreased (b) inductive reactance is increased

    (c) resistance is increased (d) resistance is decreased

35. If the resistance and the reactance of a series R-C circuit are 7.5 Ω each, then

    (a) the voltage leads the current by 84.3°
    (b) the current leads the voltage by 5.7°
    (c) the voltage leads the current by 5.7°
    (d) the current leads the voltage by 84.3°

36. When the frequency of the applied voltage in series R-C circuit is increased the capacitance reactance

    (a) increase (b) decease
    (c) becomes zero (d) remains same

37. By adding more resistance to an RC circuit
    (a) the real power increases (b) the real power decreases
    (c) power factor decreases (d) phase difference increases

38. In an R-L-C series circuit, $v(t) = 20 \sin(314t + 5\pi/6)$ and $i(t) = 10 \sin(314\ t + 2\pi/3$. The power factor of the circuit is
    (a) 1 (b) 0.25 (c) 0.5 (d) 0.45

39. An R-L-C series circuit is said to be in electrical resonance when
    (a) $X_L > X_C$ (b) $X_L < X_C$ (c) $X_L = X_C$ (d) All of them

40. An alternating voltage of $(100 + j60)$ V is applied to a circuit, the current is $(4 + j\ 10)$ A, the power consumed by the circuit is
    (a) 100 W (b) 60 W (c) 10 W (d) 200 W

Answers:—

1. (c) 2. (c) 3. (c) 4. (d) 5. (a) 6. (b) 7. (c) 8. (b) 9. (b) 10. (b)
11. (b) 12. (a) 13. (b) 14. (b) 15. (c) 16. (a) 17. (b) 18. (d) 19. (a) 20. (d)
21. (d) 22. (b) 23. (a) 24. (b) 25. (b) 26. (d) 27. (b) 28. (c) 29. (b) 30. (b)
31. (a) 32. (b) 33. (c) 34. (b) 35. (d) 36. (b) 37. (a) 38. (c) 39. (c) 40. (d)

Review Questions

1. With a neat sketch briefly explain how an alternating voltage is produced when a coil is rotated in a magnetic field.
2. Derive an expression for alternating emf and current.
3. Define the following terms: (a) Instantaneous value; (b) Cycle; (c) Frequency; (d) Time period.
4. In AC circuit, define: (i) Amplitude, (ii) frequency, (iii) form factor and (iv) power factor.
5. State and explain the following terms: (a) RMS value; (b) Average value; (c) Form factor; (d) Peak factor.
6. Prove from fundamentals the expression for average power is equal to VI cos ϕ in ac circuits.
7. Define average and rms value of an alternating current and find their relation with maximum value, if the alternating quantity is sinusoidal.
8. Show that the current through purely resistive circuit is in phase with supply voltage.

9. Prove from the fundamental principles, that the power absorbed by a pure inductor coil is zero, when connected to ac supply.
10. Show that a pure capacitance does not consume any power. Draw the waveforms of voltage, current and power, when alternating voltage is applied to the pure capacitance.
11. With respect to an ac circuit, differentiate between (i) phase and phase difference; (ii) Reactance and Impedance; (iii) Lagging and Leading power factor.
12. Draw the phasor diagram for RL series circuit and derive the expression for real power.
13. Derive an expression for average power in a single-phase series R-L circuit and there from explain the term power factor.
14. Define power factor. What are the disadvantages of low power factor?
15. Explain the causes for low power factor and mention the remedial measures.

Exercises

1. A coil when connected to 200 V, 50 Hz supply takes a current of 10 A and dissipates 1200 W. Find the resistance and Inductance of the coil.

   (**Ans:** $R = 12\ \Omega$, $L = 0.051$ H)

2. A choke coil takes a current of 2 A, lagging 60° behind the applied voltage of 200 V at 50 Hz. Calculate the inductance and impedance of the coil. Also, determine the power consumed when it is connected across 100 V, 25 Hz.

   (**Ans:** $L = 0.275$ H, $Z = 66.1\ \Omega$, $P = 112.5$ W)

3. A series R-L circuit takes 384 W at a power factor of 0.8 from a 120 V, 60 Hz supply. What are the values of R and L? (**Ans:** $R = 24\ \Omega$, $L = 0.047$ H)

4. A current $i = \sin(31\ t - 10)$ A produces a potential difference $v = 220 \sin(31\ t + 20)$ V in a circuit. Find values of circuit parameters assuming series combination. (**Ans:** $R = 190.5\ \Omega$, $L = 3.57$ H)

5. Find an expression for the current and calculate power when a voltage $v = 283 \sin 100\ \pi t$ is applied to a coil having $R = 50\ \Omega$ and $L = 0.159$ H.

   (**Ans:** $I = 4 \sin(100\ \pi\ t - \pi/4)$, $P = 400$ W)

6. A voltage of 200 V is applied to a series circuit consisting of a resistor, inductor and a capacitor. The respective voltage across these components are 170 V, 150 V, 100 V. The current is 4 A. Find the power factor of the circuit.

   (**Ans:** $pf = 0.96$)

7. A resistance of 5 Ω, an inductance of 10 mH and a capacitor of 200 μF are connected in series and the combination is connected across a 230 V, 50 Hz supply. Calculate the current flowing through the circuit and the power factor. (**Ans:** I = 16.8 A, *pf* = 0.365)

8. A coil of resistance 10 Ω and inductance of 0.1 H are connected in series with a condenser of capacitance of 150 μF across 200 V, 50 Hz. Calculate (i) X_L (ii) X_C (iii) Z and (iv) current.

   (**Ans:** X_L = 81.14 Ω; X_C = 21.22 Ω; Z = 14.276 Ω; I = 14 A)

9. Two impedances $Z_1 = (10 + j\,15)\,\Omega$ and $Z_2 = (6 - j\,8)\Omega$ are connected in parallel. The total current supplied is 15 A. What is the power taken by each branch? (**Ans:** P_1 = 736.16 W, P_2 = 1432.2 W)

10. An alternating voltage of $(160 + j\,120)$ V is applied to a circuit and current in circuit is $(6 + j8)$ A. Find (i) the values of circuit elements of circuit (ii) *pf* of the circuit and (iii) power consumed.

    (**Ans:** R = 19.2 Ω, C = 570 μF, *pf* = 0.96, P = 1920 W)

11. Two circuits A and B are connected in parallel across 200 V, 50 Hz supply. Circuit A consists of 10 Ω resistance and 0.12 H inductance in series. Circuit B consists of 20 Ω resistance in series with 40 μF capacitor. Calculate (i) current in each branch, (ii) total current and (iii) power factor of the circuit. Draw phasor diagram.

    (**Ans:** $I_1 = 5.128\angle{-75.14}°$A, $I_2 = 2.437\ \angle 75.89°$A, $I = 3.22\ \angle{-53.64}°$A, *pf* = 0.592)

12. A current of average value 18.019 A is flowing in a circuit to which a voltage of peak value 141.42 V is applied. Determine (i) impedance in polar form and (ii) power. Assume voltage lags current by 30°.

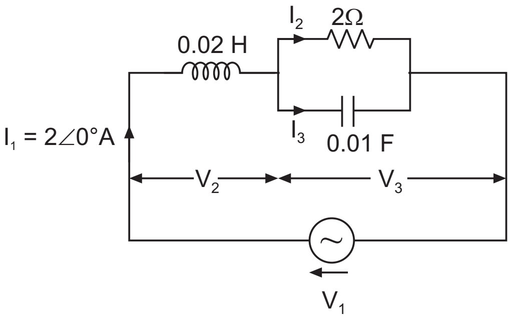
    (**Ans:** $Z = 5\angle{-30}°\ \Omega$ and P = 1.732 kW)

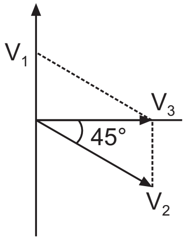
13. A parallel circuit comprises a resistor of 20 ohm in series with an inductive reactance of 15 ohm an one branch and a resistor of 30 ohm in series with a capacitive reactance of 20 ohm in other branch. Determine the current and power dissipated in each branch, if the total current drawn by parallel circuit is $10\angle{-30}$ amp.

    (**Ans:** I_1 = 7.176 A, I_2 = 4.97 A, P_1 = 1029.9 W and P_2 = 741.62 W)

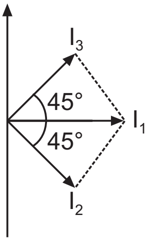
14. The circuit diagram shown below is operating at ω = 50 rad/sec. Construct two phasor diagrams one for 3 voltages and other for 3 currents.

    (**Ans:** $V_2 = 2\angle 90°$V, $V_3 = 2.82\angle{-45}°$V, V_1 = 2V $I_2 = 1.414\ \angle{-45}°$A and $I_3 = 1.414\ \angle 45°$A)





Vector diagram of voltages



Vector diagram of currents

Chapter

4

Three-phase AC Circuits

4.1 ADVANTAGES (OR NECESSITY) OF THREE-PHASE SYSTEM

1. For bulk power generation, transmission and distribution, three-phase (or poly phase or multi-phase) system is more suitable.
2. The efficiency of three-phase machine is higher than single phase machine.
3. For the given capacity, three-phase machine is more economical than single phase machine.
4. For the given capacity, three-phase ac machine is smaller in size than single-phase ac machine.
5. The performance of three-phase ac motors are superior than single-phase ac motors with respect to self-starting, torque and power factor.
6. Parallel operation of three-phase alternators will give better performance.

4.2 GENERATION OF THREE-PHASE VOLTAGE

Three identical coils R-R^1, Y-Y^1 and B-B^1 are fixed to one another at 120°. When these coils are simultaneously rotated in anticlockwise direction in a magnetic field then, emf will be induced in each coil is of same magnitude but displaced from each other by 120° apart.

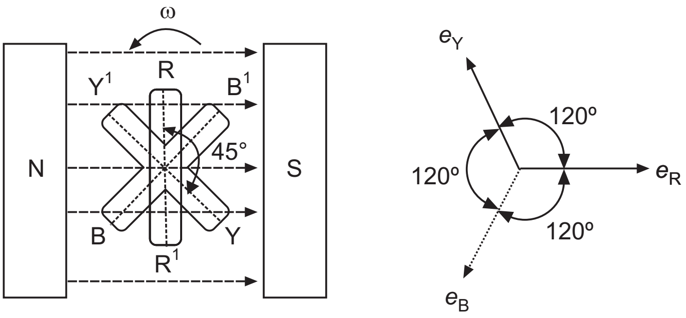


Fig. 4.1 Generation three-phase voltage

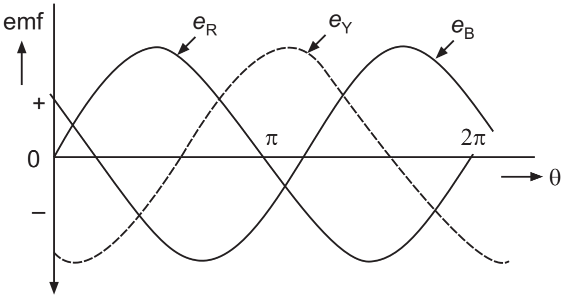


Fig. 4.2 Waveforms

The vector or phasor sum of three emfs is zero.

Expressing in rms values,

$$E_R + E_Y + E_B = 0$$

To Prove that sum of the instantaneous values of three emfs is zero:

Taking e_R as reference and E_m as the maximum value of emf,

The instantaneous emf induced in coil R-R^1:

$$e_R = E_m \sin \theta \qquad ...(4.1)$$

The instantaneous emf induced in coil Y-Y^1:

$$e_Y = E_m \sin (\theta - 120°) \qquad ...(4.2)$$

The instantaneous emf induced in coil B-B^1:

$$e_B = E_m \sin (\theta - 240°)$$

or $$e_B = E_m \sin (\theta + 240°) \qquad ...(4.3)$$

Substituting $t = 0$ in equations (4.1), (4.2) and (4.3),

$$e_R = 0$$

$$e_Y = E_m \sin(-120°) = -0.866\ E_m$$

i.e., e_Y lags e_R by 120° or one-third of a cycle

$$e_B = E_m \sin(120°) = 0.866\ E_m$$

i.e., e_B lags e_Y by 120°

$$\therefore \quad e_R + e_Y + e_B = 0 + (-0.866) + 0.866 = 0$$

Therefore, sum of the instantaneous values of three emfs is zero.

4.3 DEFINITION OF PHASE-SEQUENCE, BALANCED SUPPLY AND LOAD

4.3.1 Phase-sequence

The order or sequence in which the voltages in three-phases reach their maximum values is called the phase-sequence. It is important in determining direction of rotation of ac motors, parallel operation of alternators, etc.

Generally the phase sequence is of the order RYB (identified as Red, Yellow and Blue colors for cables or wires) or abc. The other possible phase sequence is RBY.

4.3.2 Balanced Supply

Balanced supply means all the three voltages (that is, the voltage in each phase) are of same magnitude but displaced from each other by 120° apart. Otherwise, it is called unbalanced supply.

4.3.3 Balanced Load

Balanced load means all the three impedances are exactly equal including both resistive and reactive components. The phase currents have the same peak value and displaced by 120°. Otherwise it is called unbalanced load.

4.4 STAR (Y) CONNECTION

In three-phase star connection, similar ends, that is, either starting or finishing ends of three identical coils are joined at a point N known as the Neutral (it carries no current). Thus, it may have three-phase three wire or three-phase four wire. Applicable for both industrial and domestic loads.

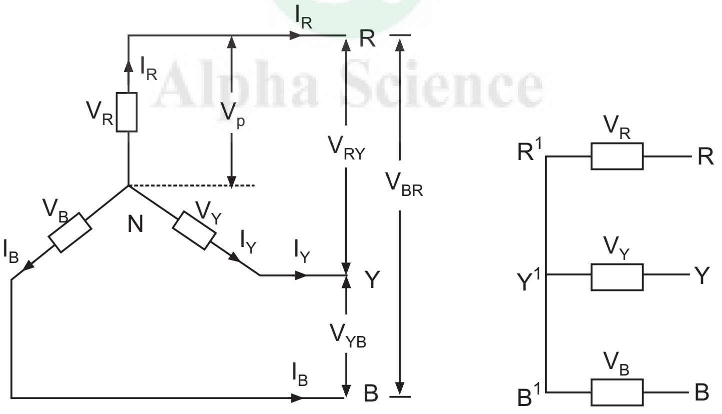


Fig. 4.3 (a) Star connection, three-phase, three wire

Assuming three-phase, 3 wire balanced star connected supply, the voltage induced in each coil or phase is called the phase voltage and current in each coil is called the phase current.

i.e., Phase voltage, $V_{ph} = V_R = V_Y = V_B$

and Phase current, $I_{ph} = I_R = I_Y = I_B$

The voltage available between any pair of terminals is called the line voltage and current in each line is called the line current.

i.e., Line voltage, $V_L = V_{RY} = V_{YB} = V_{BR}$

(Note: This is called double subscript notation)

Line current, $I_L = I_1 = I_2 = I_3$

or $I_L = I_R = I_Y = I_B$

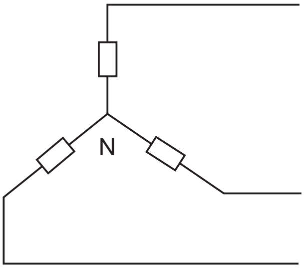


Fig. 4.3 (b) three-phase 4 wire Y-connection

Relationship between line and phase values:

$$V_L = \sqrt{3}\ V_{ph}$$

and $I_L = I_{ph}$ (by inspection)

To prove that $V_L = \sqrt{3}\ V_{ph}$ in three-phase star-connected system:

Let V_R, V_Y and V_B be the phase voltages (V_{ph}), which are 120° apart.

Considering the line voltage, $V_{RY} = V_R - V_Y$ (vector difference), it is determined by compounding phase voltages V_R and V_Y reversed (or $-V_Y$). After completing the parallelogram, the resultant represents V_{RY} (*voltage of terminal R w.r.t. terminal Y*).

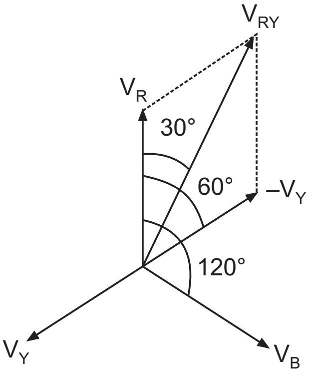


Fig. 4.4 Phasor diagram

Angle between V_R and V_Y reversed is 60°, and the line voltages are 30° ahead of their respective phase voltages.

According to the Law of parallelogram,

$$V_L = V_{RY} = 2 \times V_{ph} \times \cos\left(\frac{60°}{2}\right)$$

$$= 2 \times V_{ph} \times \cos 30°$$

$$= 2 \times V_{ph} \times \frac{\sqrt{3}}{2}$$

$$\therefore \qquad V_L = V_{RY} = \sqrt{3}\ V_{ph}$$

Hence the proof.

Important Point

If I_m is the maximum value of current, the instantaneous current in each phase for balanced star connected system is given by

$$i_R = I_m \sin \omega t$$

$$i_Y = I_m \sin (\omega t - 120°)$$

$$i_B = I_m \sin (\omega t - 240°)$$

Therefore, instantaneous value of current in neutral is given by

$$i_R + i_Y + i_B = \mathbf{0}$$

4.5 DELTA (Δ) OR MESH CONNECTION

In three-phase delta connection, dissimilar ends, that is, starting end of one phase is connected to the finishing end of the other phase and so on to form a closed circuit. Neutral is not available. Thus, it is three-phase three wire system.

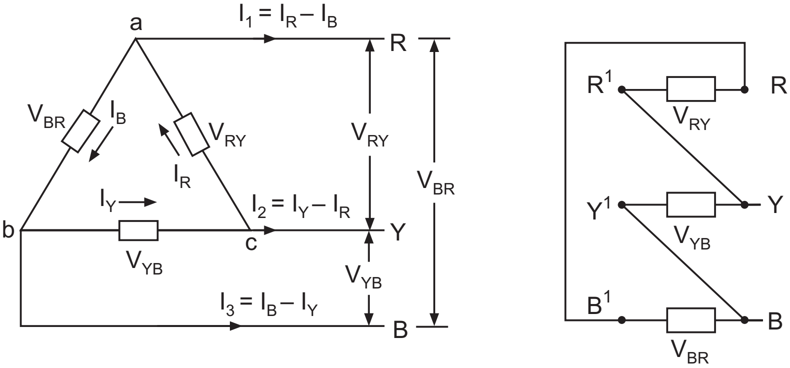


Fig. 4.5 Delta connected system

Assuming three-phase balanced delta connected supply, the voltage induced in each coil or phase is called the phase voltage and current in each coil is called the phase current.

i.e., Phase voltage, $V_{ph} = V_{RY} = V_{YB} = V_{BR}$

and Phase current, $I_{ph} = I_R = I_Y = I_B$

The voltage available between any pair of terminals is called the line voltage and current in each line is called the line current.

i.e., line voltage, $V_L = V_{RY} = V_{BR} = V_{YB}$

and line current, $I_L = I_1 = I_2 = I_3$

Applying KCL at point *a*, $I_1 = I_R - I_B$

Applying KCL at point *b*, $I_2 = I_Y - I_R$

Applying KCL at point *c*, $I_3 = I_B - I_Y$

Relationship between line and phase values:

$$V_L = V_{ph} \text{ (by inspection)}$$

and $$I_L = \sqrt{3}\ I_{ph}$$

To prove that $I_L = \sqrt{3}\ I_{ph}$ in three-phase delta-connected system:

Let I_R, I_Y and I_B be the phase currents (I_{ph}), which are 120° apart.

Assuming inductive load, I_{ph} lags its respective voltage (V_{ph} or V_L) by an angle ϕ in every phase.

Considering line current, $I_1 = I_R - I_B$ (vector difference), it is determined by compounding phase currents I_R and I_B reversed (or $-I_B$).

After completing the parallelogram, the resultant represents I_L. Angle between I_R and I_B reversed is 60°.

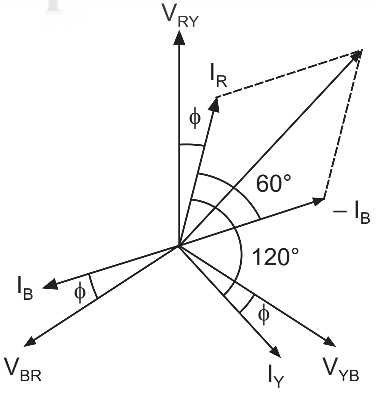


Fig. 4.6 Phasor diagram

According to the Law of parallelogram,

$$I_1 = I_R - I_B = 2 \times I_{ph} \times \cos\left(\frac{60°}{2}\right)$$

$$= 2 \times I_{ph} \times \cos 30°$$

$$= 2 \times I_{ph} \times \frac{\sqrt{3}}{2}$$

$$\therefore \quad I_L = I_R - I_B = \sqrt{3}\ I_{ph}$$

Hence the proof.

4.6 POWER IN BALANCED THREE-PHASE CIRCUITS

Let V_{ph}, I_{ph}, V_L and I_L are the rms values of phase voltage, phase current, line voltage and line current respectively. The phase angle between phase voltage and phase current is ϕ.

Power/phase, $P_{ph} = V_{ph} \times I_{ph} \times \cos\phi$

$\therefore$ Total power, $P = 3 \times V_{ph} \times I_{ph} \times \cos\phi$ watt ...(4.4)

Case (i) For **Star** connection, $V_L = \sqrt{3}\ V_{ph}$ or $V_{ph} = \frac{V_L}{\sqrt{3}}$ and $I_{ph} = I_L$

Substituting for V_{ph} and I_{ph} in equation (4.4),

$$P = 3 \times \frac{V_L}{\sqrt{3}} \times I_L \times \cos\phi$$

$$= \sqrt{3}\ V_L\ I_L \cos\phi \text{ watt}$$

Case (ii) For **Delta** connection, $I_L = \sqrt{3}\ I_{ph}$ or $I_{ph} = \frac{I_L}{\sqrt{3}}$ and $V_{ph} = V_L$

Substituting for V_{ph} and I_{ph} in equation (4.4),

$$\therefore \quad P = 3 \times V_L \times \frac{I_L}{\sqrt{3}} \times \cos\phi$$

$$= \sqrt{3}\ V_L\ I_L \cos\phi \text{ watt}$$

(Note: Reactive power, $Q = \sqrt{3}\, V_L\, I_L \sin\phi$ VAR

Apparent power, $S = \sqrt{3}\ V_L\, I_L$ VA)

Solved Examples

Example 4.1 *Three identical coils, each of R = 20 Ω and L = 0.5 H are connected in star to a three-phase 400 V, 50 Hz supply. Determined the line current and power consumed.*

Solution: Given: $R = 20\ \Omega$, $L = 0.5$ H, $V_L = 400$ V, $f = 50$ Hz

For Y-connection, $V_L = \sqrt{3}\, V_{ph}$

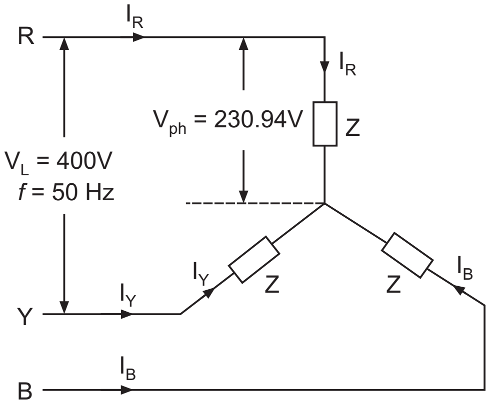


Fig. 4.7 Circuits diagram

or $$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.94 \text{ V}$$

and $$I_L = I_{ph}$$

To find Line current I_L; $$I_L = I_{ph} = \frac{V_{ph}}{Z}$$

Now $$Z = \sqrt{R^2 + X_L^2}$$

where $$X_L = 2\pi f L = 2\pi \times 50 \times 0.5 = 157.07\Omega$$

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{20^2 + 157.07^2} = 158.34\Omega$$

∴ $$I_L = I_{ph} = \frac{V_{ph}}{Z} = \frac{230.94}{158.34} = \mathbf{1.46\ A}$$

Power consumed, $$P = \sqrt{3}\ V_L I_L \cos\phi$$

where $$\cos\phi = \frac{R}{Z} = \frac{20}{158.34} = 0.126$$

∴ $$P = \sqrt{3} \times 400 \times 1.46 \times 0.126 = \mathbf{127.45\ W}$$

Example 4.2 *A balanced three-phase star-connected load of 150 kW takes a leading current of 100 A with a line voltage of 1100 volt. Find the circuit constants of the load per phase.*

Solution: Given: $P = 150 \text{ kW} = 150 \times 1000$ watt

$I_L = 100 \text{ A} = I_{ph}$ for Y-connection

$V_L = 1100 \text{ V}, f = 50$ Hz

Given that current **is leading**, therefore, circuit constants are **Resistance** and **Capacitance**.

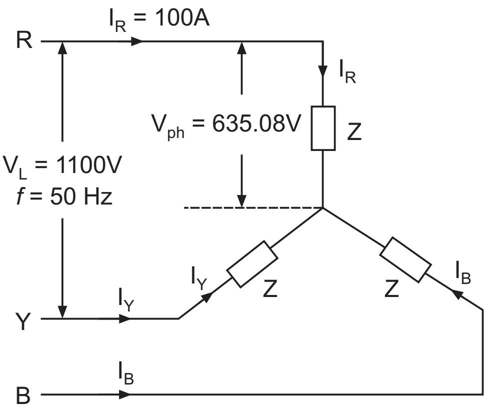


Fig. 4.8 Circuit diagram

To find the impedance of the circuit;

For Y-connection, $V_L = \sqrt{3}\, V_{ph}$ and $I_L = I_{ph}$

Impedance, $Z = \sqrt{R^2 + X_C^{\,2}}$

Also, $Z = \dfrac{V_{ph}}{I_{ph}}$

where $V_{ph} = \dfrac{V_L}{\sqrt{3}} = \dfrac{1100}{\sqrt{3}} = 635.08 \text{ V}$

$\therefore$ $Z = \dfrac{635.08}{100} = 6.35\ \Omega$

pf, $\cos\phi = \dfrac{P}{\sqrt{3} \times V_L\ I_L} = \dfrac{150 \times 1000}{\sqrt{3} \times 1100 \times 100} = 0.78$

$\therefore$ $R = Z\cos\phi = 6.35 \times 0.78 = \mathbf{4.953\ \Omega}$

$X_C = Z\sin\phi = 6.35 \times 0.625 = 3.968\ \Omega$

Also, $X_C = \dfrac{1}{2\pi f C}$

or $C = \dfrac{1}{2\pi f X_C} = \dfrac{1}{2\pi \times 50 \times 3.968} = \mathbf{8 \times 10^{-4}\ F}$

Example 4.3 *Three coils each of impedance 25 ∠–60° Ω are connected in delta across a 400 V 3- phase 50 Hz supply. Find (i) the current in each phase, (ii) the line current and (iii) the power consumed.*

Solution: Given: $Z_{ph} = 25\ \angle{-60^\circ}\ \Omega$

$$V_L = 400 \text{ volt} = V_{ph} \text{ for delta connection,}$$
$$f = 50 \text{ Hz}$$

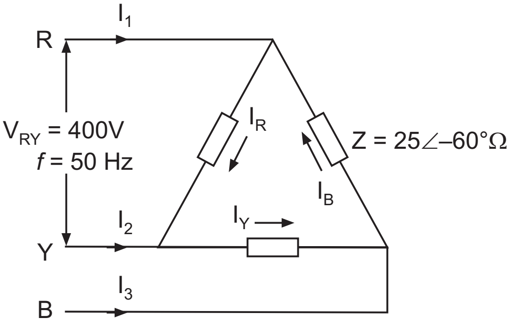


Fig. 4.9 Circuit diagram

(i) Current in each phase, $I_{ph} = \dfrac{V_{ph}}{Z_{ph}} = \dfrac{400}{25}\angle 0 - (-60°)$

$= 16 \angle 60° = \mathbf{16\ A}$

(ii) For delta-connection, Line current, $I_L = \sqrt{3}\ I_{ph} = \sqrt{3} \times 16 = \mathbf{27.71\ A}$

(iii) Power consumed, $P = \sqrt{3}\ V_L I_L \cos\phi$

$= \sqrt{3} \times 400 \times 27.71 \times \cos 60° = \mathbf{9599\ W}$

Example 4.4 *Three Impedances each of (15 – j 20)Ω are connected in mesh across a three-phase 400 V ac supply. Determine the phase current, line current, active power and reactive power drawn from supply.*

Solution: Given: $Z_{ph} = (15 - j\,20)\ \Omega$

$V_L = 400\ V = V_{ph}$ for delta-connection

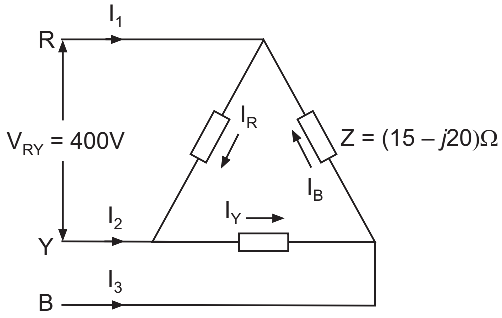


Fig. 4.10 Circuit diagram

For Y-connection, $V_L = V_{ph}$ and $I_L = \sqrt{3}\, I_{ph}$

Phase current, $$I_{ph} = \frac{V_{ph}}{Z_{ph}}$$

Expressing impedance in polar form,

$$Z_{ph} = (15 - j\,20)\Omega$$
$$= \sqrt{15^2 + 20^2} \text{ and } \tan^{-1}\frac{-20}{15}$$
$$= 25\,\angle{-53.13^\circ}\ \Omega$$

$$\therefore \quad I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{400}{25}\angle 0 - (-53.13^\circ) = \mathbf{16\ \angle 53.13^\circ\ A}$$

Line current, $$I_L = \sqrt{3}\ I_{ph} = \sqrt{3} \times 16 = \mathbf{27.7\ A}$$

Active power, $$P = \sqrt{3}\ V_L I_L \cos\phi$$
$$= \sqrt{3} \times 400 \times 27.7 \cos(53.13^\circ) = \mathbf{11514.7\ W}$$

Reactive power, $$Q = \sqrt{3}\ V_L I_L \sin\phi$$
$$= \sqrt{3} \times 400 \times 27.7 \sin(53.13^\circ) = \mathbf{15352.88\ VAR}$$

Example 4.5 *A 400 volt, three-phase is connected to a balanced load of three impedances each consisting of 20 ohm resistance and 15 ohm inductive reactance. Calculate (i) the line current, (ii) power factor and total power in kW when the load is connected in star and delta.*

Solution: Given: $V_L = 400$ volt, $R = 20\ \Omega$, $X_L = 15\Omega$

$$\therefore \quad Z_{ph} = \sqrt{R^2 + X_L^2} = \sqrt{20^2 + 15^2} = 25\Omega$$

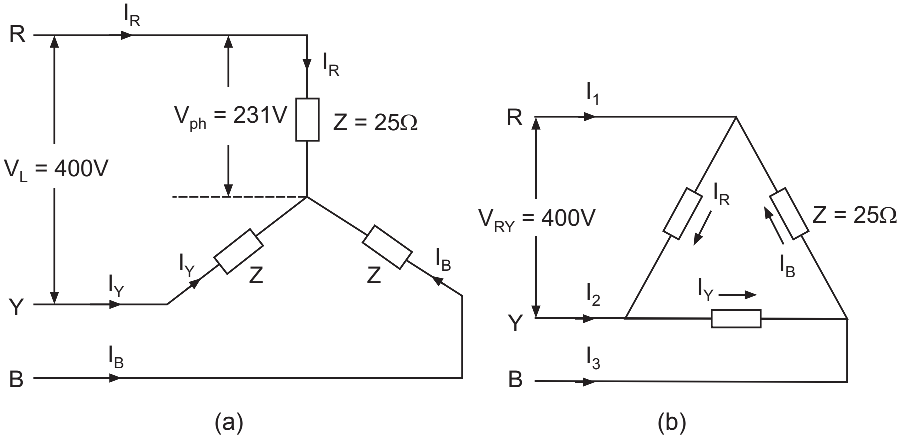


Fig. 4.11 (a) Star connection (b) Delta connection

Case (i): When the load is **star** connected:

$$V_L = \sqrt{3}\,V_{ph}$$

or $$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 231\text{V}$$

Line current, $$I_L = I_{ph}$$

$$= \frac{V_{ph}}{Z_{ph}} = \frac{231}{25} = \mathbf{9.24\ A}$$

Power factor of load, $$\frac{R}{Z_{ph}} = \frac{20}{25} = \mathbf{0.8\ (lag)}$$

Power, $$P = \sqrt{3}\,V_L I_L \cos\phi$$

$$= \sqrt{3} \times 400 \times 9.24 \times 0.8$$

$$= 5121.33\text{ W} = \mathbf{5.12\ kW}$$

Case (ii): When the load is **delta** connected:

$$V_L = V_{ph} = 400\text{V}$$

Line current, $$I_L = \sqrt{3}\,I_{ph}$$

where $$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{400}{25} = 16\text{ A}$$

Then, $$I_L = \sqrt{3}\,I_{ph} = \sqrt{3} \times 16 = \mathbf{27.71\ A}$$

Power, $$P = \sqrt{3}\,V_L I_L \cos\phi$$

$$= \sqrt{3} \times 400 \times 27.71 \times 0.8$$

$$= \mathbf{15358.44\ W = 15.36\ kW}$$

Example 4.6 *A three-phase 400 volt supply is connected to a three-phase balanced delta load. The line current is 20 A and the total power absorbed by the load is 10 kW. Find (i) the impedance per phase, (ii) the power factor and (iii) total power consumed if the same impedances are star connected.*

Solution: Given: Delta connection, $V_L = 400$ volt, $I_L = 20$ A and $P = 10$ kW

For **delta** connection, $$V_L = V_{ph} = 400\text{ V}$$

and $$I_L = \sqrt{3}\,I_{ph}$$

or $$I_{ph} = \frac{I_L}{\sqrt{3}} = \frac{20}{\sqrt{3}} = 11.55\text{ A}$$

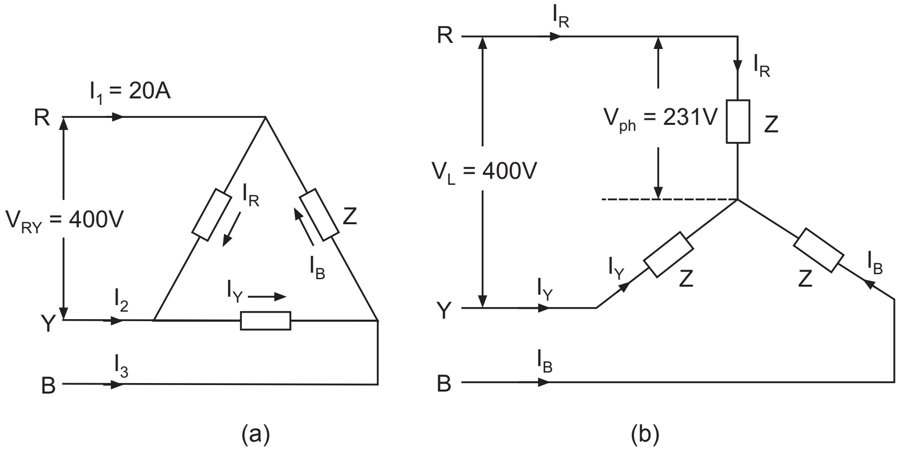


Fig. 4.12 (a) Delta connection (b) Star connection

(i) Impedance per phase, $$Z_{ph} = \frac{V_{ph}}{I_{ph}} = \frac{400}{11.55} = \mathbf{34.64\ \Omega}$$

(ii) Power factor, $$\cos\phi = \frac{P}{\sqrt{3}\, V_L\, I_L} = \frac{10000}{\sqrt{3}\times 400\times 20} = \mathbf{0.72}$$

(iii) When **star** connected, $$V_L = \sqrt{3}\, V_{ph}$$

or $$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 231\ \text{V}$$

and $$I_L = I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{231}{34.64} = 6.67\ \text{A}$$

Power consumed, $$P = \sqrt{3}\, V_L\, I_L \cos\phi$$

$$= \sqrt{3} \times 400 \times 6.67 \times 0.72 = \mathbf{3.33\ kW}$$

4.7 MEASUREMENT OF THREE-PHASE POWER USING TWO WATTMETERS

[***Concept of Wattmeter:*** *It is used to measure the power. It consists of a current coil (CC) and a voltage coil or potential coil (pc). CC is connected in series and carries the load current, pc is connected in parallel with the load and pd appears across it as shown in Fig. 4.13*].

Figure 4.14 shows a balanced three-phase star-connected supply is connected to a three-phase balanced inductive load and thus, the phase currents lag behind its respective phase voltages by an angle ϕ.

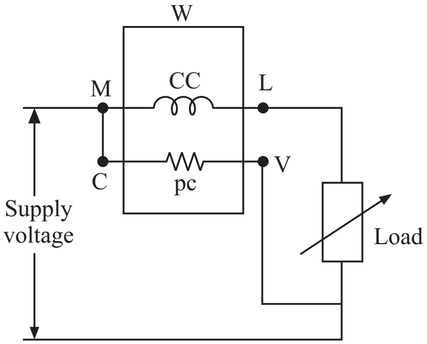


Fig. 4.13 Wattmeter

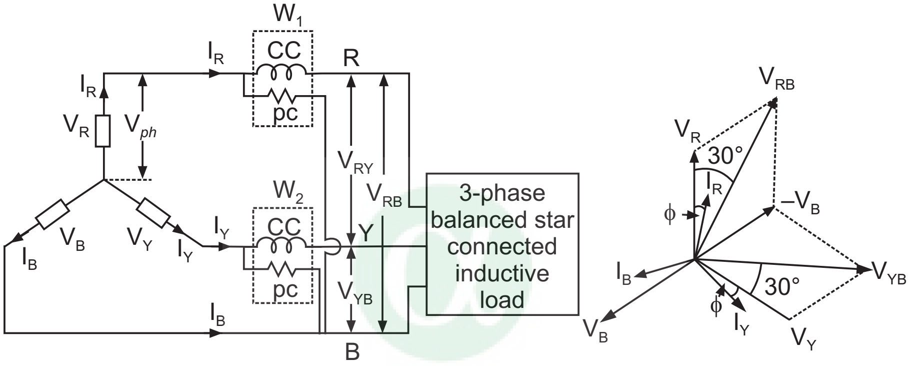


Fig. 4.14 Circuit diagram and phasor diagram

Reading of Wattmeter W_1: Its current coil (CC) is connected to R line and potential coil (pc) is connected across R and B lines.

Current through its current coil is I_R and potential difference across its potential coil is $V_{RB} = V_R - V_B$

This is found by compounding V_R and V_B reversed.

The phase difference between V_{RB} and $I_R = (30° - \phi)$

∴ Reading of wattmeter $W_1 = V_{RB}\, I_R \cos(30° - \phi)$ watt

or Reading of wattmeter $W_1 = V_L\, I_L \cos(30° - \phi)$ watt

(Because $V_{RB} = V_L$ and $I_R = I_L$)

Reading of Wattmeter W_2: Its current coil (CC) is connected to Y line and potential coil (pc) is connected across Y and B lines.

Current through its current coil is I_Y and potential difference across its potential coil is $V_{YB} = V_Y - V_B$

This is found by compounding V_Y and V_B reversed.

The phase difference between V_{RB} and $I_R = (30° - \phi)$

$\therefore$ Reading of wattmeter $W_2 = V_{YB}\, I_Y \cos(30° + \phi)$ watt

or Reading of wattmeter $W_2 = V_L\, I_L \cos(30° + \phi)$ watt

(Because $V_{YB} = V_L$ and $I_Y = I_L$)

Adding the readings of two wattmeters,

$$\begin{aligned} W_1 + W_2 &= V_L\, I_Y \cos(30° - \phi) + V_L\, I_Y \cos(30° + \phi) \\ &= V_L\, I_L[\cos(30° - \phi) + \cos(30° + \phi)] \\ &= V_L\, I_L[\cos 30° \cos\phi + \sin 30° \sin\phi \\ &\qquad + \cos 30° \cos\phi - \sin 30° \sin\phi] \\ &= V_L\, I_L\, [2 \cos 30° \cos\phi] \\ &= V_L\, I_L \left[2\frac{\sqrt{3}}{2}\cos\phi\right] \\ &= \sqrt{3}\; V_L\, I_L \cos\phi \text{ watt} \\ &= \text{Total Power P in watt} \end{aligned}$$

Thus, two wattmeters are sufficient to measure total power in a three-phase system.

4.8 DETERMINATION OF POWER FACTOR USING TWO WATTMETERS

Assuming Lagging power factor;

Reading of wattmeter $W_1 = V_L\, I_L \cos(30° - \phi)$ watt

Reading of wattmeter $W_2 = V_L\, I_L \cos(30° + \phi)$ watt

Adding the readings of two wattmeters,

$$W_1 + W_2 = \sqrt{3}\; V_L\, I_L \cos\phi \qquad \text{...(4.5)}$$

Subtracting the readings of two wattmeters,

$$\begin{aligned} W_1 - W_2 &= V_L\, I_Y \cos(30° - \phi) - V_L\, I_Y \cos(30° + \phi) \\ &= V_L\, I_L\, [\cos(30° - \phi) - \cos(30° + \phi)] \\ &= V_L\, I_L\, [\cos 30° \cos\phi + \sin 30° \sin\phi \\ &\qquad - \cos 30° \cos\phi + \sin 30° \sin\phi] \\ &= V_L\, I_L\, [2 \sin 30° \sin\phi] \\ &= V_L\, I_L \left[2\frac{1}{2}\sin\phi\right] \\ &= V_L\, I_L \sin\phi \qquad \text{...(4.6)} \end{aligned}$$

Dividing equation (4.6) by (4.5),

$$\frac{(W_1 - W_2)}{(W_1 + W_2)} = \frac{V_L I_L \sin\phi}{\sqrt{3}\, V_L I_L \cos\phi} = \frac{1}{\sqrt{3}} \tan\phi$$

or
$$\tan\phi = \sqrt{3}\,\frac{(W_1 - W_2)}{(W_1 + W_2)}$$

or
$$\phi = \tan^{-1}\left[\sqrt{3}\,\frac{(W_1 - W_2)}{(W_1 + W_2)}\right]$$

∴
$$pf = \cos\phi = \cos\left\{\tan^{-1}\left[\sqrt{3}\,\frac{(W_1 - W_2)}{(W_1 + W_2)}\right]\right\}$$

Effect of variation of power factor on wattmeter readings:

(i) When $\phi = 0°$, power factor = 1: The readings of two wattmeters are positive and equal (the nature of load is pure resistive).

(ii) When $\phi = 60°$, power factor = 0.5: The reading of any one of the wattmeters is zero.

(iii) When $\phi > 60°$, power factor < 0.5: The pointer of any one of the wattmeters kicks back. The reading is obtained after reversing CC or pc connection. The reading of this wattmeter is taken as negative.

(iv) When $\phi < 60°$, power factor > 0.5: The readings of both the wattmeters are positive and the sum of these readings gives the total power.

Measurement of Reactive Power using two Wattmeters:

Active power, $P = W_1 + W_2 = \sqrt{3}\, V_L I_L \cos\phi$ watt

and
$$\tan\phi = \sqrt{3}\,\frac{(W_1 - W_2)}{(W_1 + W_2)}$$

As the tangent of the angle of lag between phase current and phase voltage of a circuit is equal to the ratio of the reactive power to the true power in watt;

Reactive power, $Q = \sqrt{3}\,(W_1 - W_2)$ VAR

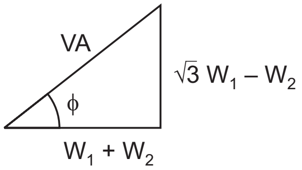


Fig. 4.15

Solved Examples

Example 4.7 *Two wattmeters are used to measure the power in a three-phase balanced system. Calculate the pf when (a) both the meters read equal, (b) one of the meters reads zero and (c) one reads twice the other.*

Solution: Let W_1 and W_2 are the two wattmeters used to measure the three-phase power.

Power factor using two wattmeters W_1 and W_2 is given by

$$pf = \cos\phi = \cos\left\{\tan^{-1}\left[\sqrt{3}\,\frac{(W_1 - W_2)}{(W_1 + W_2)}\right]\right\} \qquad \text{...(4.7)}$$

(a) When both the meters read equal: (Note: The load is Resistive)

Given that $W_1 = W_2$;

Substituting for W_1 in equation (4.7),

Power factor, $$\cos\phi = \cos\left\{\tan^{-1}\left[\sqrt{3}\,\frac{(W_2 - W_2)}{(W_2 + W_2)}\right]\right\}$$

$$= \cos\left\{\tan^{-1}\left[\sqrt{3}(0)\right]\right\}$$

$$= \cos(0)$$

$= \mathbf{1}$ or **Unity**

(b) When one of the meters reads zero:

Let, $W_2 = 0$;

Substituting W_2 in equation (4.7),

Power factor, $$\cos\phi = \cos\left\{\tan^{-1}\left[\sqrt{3}\,\frac{(W_1 - W_2)}{(W_1 + W_2)}\right]\right\}$$

$$= \cos\left\{\tan^{-1}\left[\sqrt{3}\,\frac{(W_1 - 0)}{(W_1 + 0)}\right]\right\}$$

$$= \cos\left[\tan^{-1}\left(\sqrt{3}\right)\right]$$

$$= \cos(60^\circ)$$

$$= \mathbf{0.5}$$

(c) When one reads twice the other:

Let $W_1 = 2\,W_2$;

Substituting for W_1 in equation (4.7),

Power factor, $$\cos\phi = \cos\left\{\tan^{-1}\left[\sqrt{3}\,\frac{(2W_1 - W_2)}{(2W_1 + W_2)}\right]\right\}$$

$$= \cos\left\{\tan^{-1}\left[\sqrt{3}\,\frac{(W_2)}{(3W_2)}\right]\right\}$$

$$= \cos\left[\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)\right]$$

$$= \cos(30°) = \mathbf{0.866}$$

Example 4.8 *Two wattmeters are used to measure power input to a three-phase balanced circuit. What would be the reading of each wattmeter if (i) $\phi = 60°$, (ii) = 30°, (iii) $\phi = 0°$ and (iv) $\phi = 90°$.*

Solution: In general, $W_1 = V_L I_L \cos(30° - \phi)$ watt

and $W_2 = V_L I_L \cos(30° + \phi)$ watt

(i) Substituting $\phi = 60°$,

$$W_1 = V_L I_L \cos(30° - 60°) = V_L I_L \cos(-30°) = \mathbf{0.866\ V_L\ I_L}$$

$$W_2 = V_L I_L \cos(30° + 60°) = V_L I_L \cos(90°) = \mathbf{0}$$

(ii) Substituting $\phi = 30°$,

$$W_1 = V_L I_L \cos(30° - 30°) = V_L I_L \cos(0°) = \mathbf{V_L\ I_L}$$

$$W_2 = V_L I_L \cos(30° + 30°) = V_L I_L \cos(60°) = \mathbf{0.5\ V_L\ I_L}$$

(iii) Substituting $\phi = 0°$,

$$W_1 = V_L I_L \cos(30° - 0°) = V_L I_L \cos(30°) = \mathbf{0.866\ V_L\ I_L}$$

$$W_2 = V_L I_L \cos(30° + 0°) = V_L I_L \cos(30°) = \mathbf{0.866\ V_L\ I_L}$$

(iv) Substituting $\phi = 90°$,

$$W_1 = V_L I_L \cos(30° - 90°) = V_L I_L \cos(-60°) = \mathbf{0.5\ V_L\ I_L}$$

$$W_2 = V_L I_L \cos(30° + 90°) = V_L I_L \cos(120°) = -\,\mathbf{0.5\ V_L\ I_L}$$

Example 4.9 *Two wattmeters are connected to measure power input to a three-phase balanced circuit indicate 8 kW and 0.8 kW; the latter reading being obtained after reversing the current coil connection. Find (i) pf of load, (ii) active power, (iii) reactive power and (iv) apparent power.*

Solution: Given: $W_1 = 8$ kW and $W_2 = -0.8$ kW (because given that reading is obtained after reversing CC connection)

(i) Power factor is given by

$$\cos\phi = \cos\left\{\tan^{-1}\left[\sqrt{3}\,\frac{(W_1 - W_2)}{(W_1 + W_2)}\right]\right\}$$

$$= \cos\left\{\tan^{-1}\left[\sqrt{3}\,\frac{(8-(-8))}{(8+(-8))}\right]\right\}$$

$$= \cos\left\{\tan^{-1}\left[\sqrt{3}\,\frac{(8.8)}{(7.2)}\right]\right\}$$

$$= \cos(64.71) = \mathbf{0.427}$$

(ii) Active power, $P = W_1 + W_2$

$$= 8 + (-0.8) = \mathbf{7.2\ kW}$$

(iii) Reactive power, $Q = \sqrt{3}\,(W_1 - W_2)$

$$= \sqrt{3} \times [8 - (-0.8)] = \mathbf{15.24\ kVAR}$$

(iv) Apparent power, $S = \dfrac{kW}{\cos\phi} = \dfrac{7.2}{0.427} = \mathbf{16.86\ kVA}$

Example 4.10 *Each of two wattmeters connected to measure the input to a three-phase reads 10 kW on a balanced load when the pf is unity. What does each instrument reads when the pf falls to (i) 0.866 lag and (ii) 0.5 lag. The total three-phase power remains unchanged.*

Solution: Given: $W_1 = W_2 = 10$ kW

When *pf* is unity, $\cos\phi = 1;\ \phi = 0$; hence, $\tan\phi = 0$

Therefore, total power $= W_1 + W_2$

$$= 10\ \text{kW} + 10\text{kW} = 20\ \text{kW} \qquad \text{...(4.8)}$$

(i) When *pf*, $\cos\phi = 0.866$

$$= \cos^{-1}(0.866) = 30°$$

Therefore, $\tan\phi = \tan(30°) = 0.577$

Also, $\tan\phi = \sqrt{3}\,\dfrac{(W_1 - W_2)}{(W_1 + W_2)}$

or $0.577 = \sqrt{3}\,\dfrac{(W_1 - W_2)}{20}$

or $W_1 - W_2 = 6.66 \qquad \text{...(4.9)}$

Solving equations (4.8) and (4.9),

$$\begin{aligned} W_1 + W_2 &= 20 \\ W_1 - W_2 &= 6.66 \\ \hline 2W_1 &= 26.66 \\ \hline \end{aligned}$$

or $\mathbf{W_1 = 13.33\ kW}$

Substituting for W_1 in equation (4.4);

$$13.33 + W_2 = 20$$

or $\mathbf{W_2 = 6.67\ kW}$

(ii) When *pf*, $\cos\phi = 0.5$

or $\phi = 60°$

Therefore, $\tan \phi = \tan (60°) = 1.732$

Also, $\tan \phi = \sqrt{3}\,\dfrac{(W_1 - W_2)}{(W_1 + W_2)}$

or $1.732 = \sqrt{3}\,\dfrac{(W_1 - W_2)}{20}$

or $W_1 - W_2 = 19.99$ ...(4.10)

Solving equations (4.8) and (4.10),

$$\begin{array}{r}W_1 + W_2 = 20\\ W_1 - W_2 = 19.99\\ \hline 2W_1 = 39.99\\ \hline\end{array}$$

or $\mathbf{W_1 = 19.99\ kW}$

Substituting for W_1 in equation (4.4),

$19.99 + W_2 = 20$

or $\mathbf{W_2 = 0.01\ kW}$

Example 4.11 *Three coils each of impedance of 20 ∠60° ohm are connected in star to 400 V, three-phase, 50 Hz supply. Find the readings on each of the two wattmeters connected to measure the power input.*

Solution: Given: $Z = 20\angle 60°\ \Omega,\ V_L = 400\ \text{V},\ f = 50\ \text{Hz}$

Total power, $P = W_1 + W_2$

$= \sqrt{3}\ V_L I_L \cos \phi$

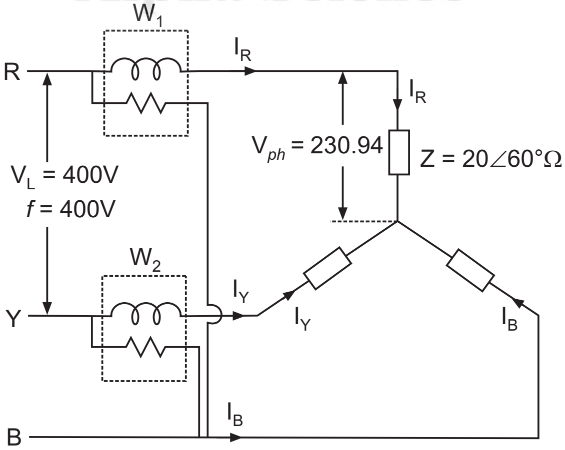


Fig. 4.16 Circuit diagram

For Y-connection, $V_L = \sqrt{\ }\ V_{ph}$ and $I_L = I_{ph}$

Now $I_L = I_{ph} = \dfrac{V_{ph}}{Z}$

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.94 \text{ V}$$

$$\therefore \quad I_L = I_{ph} = \frac{V_{ph}}{Z} = \frac{230.94}{20} \angle 0 - 60° = 11.547 \angle -60° \text{ A}$$

$$P = W_1 + W_2$$

$$= \sqrt{3} \times 400 \times 11.547 \times \cos(60°)$$

$$\text{or} \quad W_1 + W_2 = 4000 \text{ watt} \qquad ...(4.11)$$

$$\text{Now} \quad \tan\phi = \sqrt{3}\,\frac{(W_1 - W_2)}{(W_1 + W_2)}$$

$$\text{where} \quad \tan(60°) = 1.732$$

$$\therefore \quad 1.732 = \sqrt{3}\,\frac{(W_1 - W_2)}{4000}$$

$$\text{or} \quad W_1 - W_2 = 4000 \text{ watt} \qquad ...(4.12)$$

Solving equations (4.11) and (4.12),

$$\mathbf{W_1 = 4000 \text{ watt}} \text{ and } \mathbf{W_2 = 0}$$

Example 4.12 *A three-phase Y connected supply with a phase voltage of 230V is supplying a balanced delta load. The load draws 15 kW at 0.8 pf lagging. Find the line current and the current in each phase of the load. What is load impedance per phase?*

Solution: Given: Y connected supply, $V_{ph} = 230$ V, P = 15 kW, *pf* = 0.8 lagging

For star connected supply, $V_L = \sqrt{3}\, V_{ph} = \sqrt{3} \times 230 = 400\text{V}$

When the load is connected in delta,

$$P = \sqrt{3}\, V_L I_L \cos\phi$$

$$\text{or} \quad I_L = \frac{P}{\sqrt{3}\, V_L \cos\phi} = \frac{15 \times 10^3}{\sqrt{3} \times 400 \times 0.8} = \mathbf{27.06\,A}$$

$$I_{ph} = \frac{I_L}{\sqrt{3}} = \frac{27.06}{\sqrt{3}} = \mathbf{15.625\ A}$$

$$\therefore \text{ Load impedance per phase, } Z_{ph} = \frac{V_{ph}}{I_{ph}} = \frac{400}{15.625} = \mathbf{25.6\ \Omega}$$

Example 4.13 *The power input to a 2000 V, 50 Hz, three-phase motor running on full load is measured by two wattmeters which indicate 300 kW and 100 kW respectively. Calculate (i) the input, (ii) the power factor and (iii) the line current.*

Solution: Given: Input Voltage = $V_L = 2000$ V, $f = 50$ Hz

$W_1 = 300$ kW and $W_2 = 100$ kW

(i) Input,

$$P = W_1 + W_2$$
$$= 300 + 100 = \mathbf{400\ kW}$$

(ii) Power factor,

$$pf = \cos\phi$$

where

$$\phi = \tan^{-1}\frac{\sqrt{3}\,(W_1 - W_2)}{W_1 + W_2}$$

$$= \tan^{-1}\frac{\sqrt{3}\,(300 - 100)}{300 + 100} = \tan^{-1}(0.866) = 40.89°$$

∴

$$pf = \cos(40.89°) = \mathbf{0.755}$$

(iii) Line current;

$$I_L = \frac{P}{\sqrt{3}\,V_L\cos\phi} \quad (\text{because } P = \sqrt{3}\,V_L I_L \cos\phi)$$

$$= \frac{400\times 1000}{\sqrt{3}\times 2000\times 0.755} = \mathbf{152.94\ A}$$

Multiple Choice Questions

Choose the Correct Answer

1. A three-phase equipment has a size .... of a single-phase equipment for same capacity

(a) bigger than (b) same as
(c) smaller than (d) none of these

2. The voltage $V_{AB} = 50\angle 30°$ volt. Then V_{BA} is .... volt.

(a) $50\angle{-180°}$ (b) $50\angle{-150°}$ (c) $50\angle{-30°}$ (d) $50\angle{-210°}$

3. Electrical displacement between different phases in a six-phase system is

(a) 60° (b) 120° (c) 240° (d) none of these

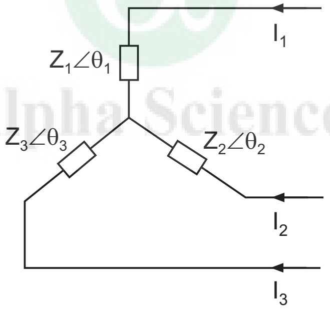
4. The phase sequence RBY denotes that

(a) emf of phase-B lags that of phase-R by 120°
(b) emf of phase-B leads that of phase-R by 120°
(c) both (a) and (b) are correct
(d) none of these

5. The resistance measured between any two terminals of a balanced three-phase star connected load is 2 R. The resistance of each phase will be ....

(a) R (b) 2 R (c) 4 R (d) 6 R

6. The rated voltage of a three-phase system is given as
   (a) rms phase voltage (b) peak phase voltage
   (c) rms line-to-line voltage (d) peak line-to-line voltage
7. The algebraic sum of instantaneous phase voltages in three-phase balanced system is
   (a) zero (b) line voltage
   (c) phase voltage (d) none of these
8. The frequencies of a three-phase voltage in a three-phase balanced system is
   (a) different (b) same (c) zero (d) none of these
9. The angle between line voltage and phase voltage for a balanced star connected circuit is
   (a) 30° (b) $30° + \phi$ (c) 60° (d) 120°
10. In three-phase balanced star connected load, the neutral current is equal to
   (a) zero (b) I_{ph} (c) I_L (d) unpredictable
11. A three-phase three wire star connected system has three unequal impedances. Then the sum of the line currents is equal to
   (a) $3\ I_{ph}$ (b) $\sqrt{3}\ I_{ph}$ (c) I_{ph} (d) zero
12. The figure represents



   (a) unbalanced star connected supply
   (b) balanced star connected load
   (c) balanced star connected supply
   (d) unbalanced star connected load
13. In a balanced three-phase load, the power factor of three-phases are
   (a) different (b) same (c) zero (d) none of these
14. If three unequal impedances are connected in three-phase delta, then
   (a) line currents are equal
   (b) phase currents are unbalanced and line currents are balanced
   (c) sum of the line currents is zero
   (d) phase voltages are unequal

15. In star connection, the relation between line and phase current is

    (a) $I_L < I_{ph}$ (b) $I_L = \sqrt{3}\, I_{ph}$ (c) $I_L > I_{ph}$ (d) $I_L = I_{ph}$

16. In a balanced delta connected load, the resistance between any two terminals is 20 ohm. Then, the resistance of an each phase will be .... ohm.

    (a) 10 (b) 20 (c) 30 (d) 60

17. In a 3ϕ system, if the instantaneous voltage phase R and Y are +60 V and –40 V respectively, then instantaneous voltage of phase B is

    (a) –20 V (b) 40 V (c) 120 V (d) none of the above

18. In a three-phase balanced delta connected system, the relation between line voltage V_L and phase voltage V_{ph} is ....

    (a) $V_L = V_{ph}/\sqrt{3}$ (b) $V_L = \sqrt{3}\, V_{ph}$ (c) $V_L = V_{ph}$ (d) none of these

19. The power consumed by a three-phase star connected load is given by

    (a) $V_{ph}\, I_{ph} \cos\phi$ (b) $\sqrt{3}\, V_{ph}\, I_{ph} \cos\phi$

    (c) $3\, V_L\, I_L \cos\phi$ (d) $\sqrt{3}\, V_L\, I_L \cos\phi$

20. The sum of the two wattmeters readings in a three-phase balanced system is

    (a) $V_{ph}\, I_{ph} \cos\phi$ (b) $3\, V_L\, I_L \cos\phi$

    (c) $\sqrt{3}\, V_L\, I_L \cos\phi$ (d) none of these

21. The total power consumed by a three-phase balanced load is given by

    (a) $W_1 - W_2$ (b) $(W_1 + W_2)/2$

    (c) $\sqrt{3}\,(W_1 - W_2)$ (d) none of these

    where W_1 and W_2 are wattmeter readings.

22. W_1 and W_2 are the readings of two wattmeters used to measure power of a three-phase balanced load. The active power drawn by the load is

    (a) $W_1 + W_2$ (b) $W_1 - W_2$ (c) $\sqrt{3}(W_1 + W_2)$ (d) $\sqrt{3}(W_1 - W_2)$

23. In two wattmeter method of power measurement the load is resistive, if the wattmeters readings are W_1 and W_2 then

    (a) $W_1 > W_2$ (b) $W_1 < W_2$

    (c) $W_1 = W_2$ (d) $W_1 = 0$ and $W_2 = 0$

24. When two wattmeters used to measure a three-phase power, give equal readings, then the power factor of the circuit is ....

    (a) 0.5 (b) 0 (c) 0.866 (d) 1

25. In a three-phase balanced supply system, the sum of the instantaneous values of three voltages at any instant is ....

    (a) maximum (b) zero (c) minimum (d) none of these

26. In measurement of three-phase power using two wattmeters, if one of the wattmeters reads zero, then the power factor of load is ....

    (a) zero (b) 0.5 (c) 0.8 (d) 1

27. In measurement of three-phase power using two wattmeters, if one of the wattmeters reads negative value, then the power factor of the circuit is

(a) 0.5 (b) >0.5 (c) <0.5 (d) none of these

28. In measurement of three-phase power using two wattmeters, the wattmeters indicate equal and opposite readings when the load power factor angle is .... degrees lagging.

(a) 60° (b) 0° (c) 30° (d) 90°

29. A three-phase star connected load consumes P watt of power from a 400 V supply. If the balanced load is connected in delta across that same supply, then power consumption is

(a) 3 P (b) $\sqrt{3}$ P (c) P/3 (d) P

30. In, three-phase system, power equation $\sqrt{3}$ VI cos ϕ, ϕ is the angle between

(a) V_L and I_L (b) V_L and I_{ph} (c) V_{ph} and I_L (d) V_{ph} and I_{ph}

Answers:—

1. (c) 2. (b) 3. (a) 4. (a) 5. (a) 6. (c) 7. (a) 8. (b) 9. (d) 10. (a)
11. (d) 12. (d) 13. (b) 14. (d) 15. (d) 16. (c) 17. (a) 18. (c) 19. (d) 20. (c)
21. (d) 22. (a) 23. (c) 24. (b) 25. (b) 26. (b) 27. (c) 28. (d) 29. (a) 30. (d)

Review Questions

1. Define the three-phase system. Draw the waveforms and phasor diagram. Mention four advantages of three-phase system.
2. What are the advantages of 3ϕ ac system over 1ϕ ac system.
3. Obtain the relationship between the phase and line values of voltages and currents in a balanced star connected system.
4. For a three-phase connection, find the relation between line and phase values of currents and voltages. Also derive the equation for the three-phase power.
5. Derive the relationship between line and phase values of balanced star and delta connected load with balanced supply.
6. Obtain the relationship between line and phase values of voltage and current in a delta connected three-phase balanced system.
7. Show that two wattmeters are sufficient to measure 3ϕ power for balanced 3ϕ power system with the neat circuit diagram and phasor diagram.
8. Starting from expressions of two wattmeters of three-phase power measurement, deduce expression for load power factor.

9. Explain the effect of power factor on the two wattmeter readings connected to measure the three-phase power.
10. Two wattmeters W_1 and W_2 are used to measure power in a three-phase balanced circuit. Mention the conditions under which

    (i) $W_1 = W_2$; (ii) $W_2 = 0$ (iii) $W_1 = 2W_2$

Exercises

1. Three coils, each having resistance of 10 Ω and an inductance of 0.02 H are connected in star across 440 V, 50 Hz, three-phase supply. Calculate the line current and total power consumed. (**Ans:** $I_L = 21.5$ A and P = 13.93 W)
2. Three identical coils of each of R = 20 ohms and L = 0.05 H are connected in (i) star, (ii) delta, to a three-phase, 400 V, 50 Hz supply. Determine in each case (i) the line current, (ii) total power consumed.
   (**Ans:** For Y-connection, 9.24 A and 5120 W;
   for delta-connection,27.7 A and 15350 W)
3. A three-phase, 400 V motor takes an input of 40 kW at 0.45 *pf* lag. Find the reading of the two wattmeters connected to measure the input.
   (**Ans:** $W_1 = 42.915$ kW and $W_2 = -2.915$ kW)
4. The power input to a three-phase induction motor running on 400 V, 50 Hz supply was measured by two wattmeter method and the readings were 3000 W and –1000 W. Calculate (i) Total input power (ii) *pf* (iii) line current.
   (**Ans:** P = 2000 W, *pf* = 0.2773 and $I_L = 10.4$ A)
5. Each of two wattmeters connected to measure the input to a three-phase circuit reads 20 kW. What does each instrument reads, when the power factor is 0.866 lagging with the total three-phase power remaining unchanged in the altered condition. (**Ans:** $W_1 = 26.667$ kW and $W_2 = 13.33$ kW)
6. A three-phase load of three equal impedances connected in delta across a balanced 400V supply takes a line current of 10A at a power factor of 0.7 lagging. Calculate from the first principles : (i) The phase current (ii) The total power (iii) The total reactive VAR.
   (**Ans:** $I_{ph} = 69.27$ A, P = 4849.74 W, Q = 4947.47 VAR)
7. The power flowing in a three-phase, 400 V, three-wire balanced star connected load system is measured by two wattmeter methods. The reading in wattmeter A is 750 W and wattmeter B is 1500 W. What is the power factor of the system and load current per phase? (**Ans:** *pf* = 0.866, $I_L = 3.75$ A)
8. Three similar coils each having resistance of 10 ohm and reactance of 8 ohm are connected in star across a 400 V, three-phase supply. Determine the (i) line current, (ii) total power and (iii) reading of two wattmeters connected to measure.
   (**Ans:** $I_L = 18.04$ A, P = 9.76 W, $W_1 = 7137$ kW and $W_2 = 2.626$ W)

9. A balanced three-phase star connected load draws power from a 440V, three-phase supply. Two wattmeters connected to indicate $W_1 = 4.2$ kW and $W_2 = -0.8$ kW. Calculate the power and power factor.

(**Ans:** P = 3.4 kW and *pf* = 0.364)

10. A balanced three-phase star-connected, 210 kW load takes a leading current when connected across a balanced three-phase 1.1 kV, 50 Hz supply. Determine the circuit parameters per phase. (**Ans:** R = 2.73 Ω and C = 1106 μF)

Chapter

5

Measuring Instruments

5.1 CLASSIFICATION

1. **Absolute Instruments:** This gives the magnitude of quantity to be measured in terms of constants of instrument and deflection only. Previous calibration is not required. Example: Tangent Galvanometer.
2. **Secondary Instruments:** The magnitude of electrical quantity is obtained by the deflection of the instrument only when they are pre-calibrated by comparison with an Absolute instrument.

   Further, secondary instruments are classified as given below:

   (i) **Indicating Instrument:** It indicates instantaneous value of an electrical quantity to be measured at that time by a pointer.

   Examples: Ammeter (it measures the current) Voltmeter (it measures the voltage), Wattmeter (it measures the power).

   (ii) **Recording Instrument:** It continuously records on a graph over a selected period of time.

   (iii) **Integrating Instrument:** It measures and registers over a set of dials and gearing mechanism.

   Example: Energy meter.

5.2 ESSENTIALS OF INSTRUMENTS

1. **Deflecting or Operating Torque (T_d):** It causes the moving system to move from its zero position.
2. **Controlling or Restoring Torque (T_c):** It opposes the deflecting torque and increases with deflection of the moving system. It is also called the retarding torque.

   At the final deflection position of pointer,

   Controlling torque (T_C) = Deflecting torque (T_d)

Examples: Spring control and Gravity control.

3. **Damping Torque**: It acts on moving system only when it is moving and always opposes its motion. It is necessary to bring the pointer to rest quickly.

Example: Air friction, Eddy currents, etc.

5.3 DYNAMOMETER TYPE WATTMETER

Dynamometer or Electrodynamic type wattmeter is a moving-coil instrument, which indicates the power both in ac and dc circuit.

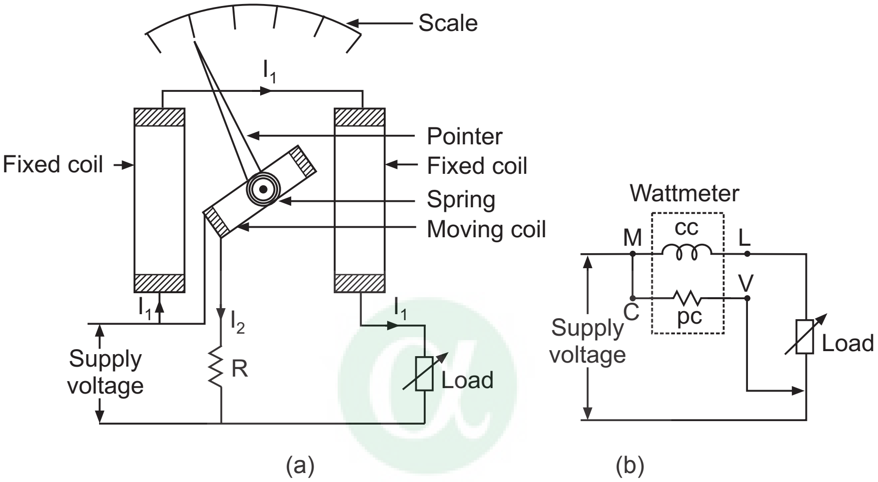


Fig. 5.1 (a) Dynamometer type wattmeter (b) Connection diagram

Principle: Whenever a current carrying conductor is placed in a magnetic field, it is experienced by a mechanical force.

An electrodynamic instrument is a moving-coil instrument in which the operating torque is produced by a fixed coil (or electromagnet) but not by a permanent magnet.

Construction: It consists of two coils, which are

1. Fixed coil and
2. Moving coil.

The fixed coil is air-cored (to avoid hysteresis loss) arranged in two equal sections, placed close to each other. The moving coil is mounted on a spindle and placed in between the two equal sections of fixed coil. A pointer made of Aluminum is attached to the spindle, which moves on a graduated scale. The two equal sections of fixed coil are connected in series with the load and carries the load current (I_1) and thus called Current Coil (CC).

The moving coil is connected across the load, carries a current (I_2) proportional to the voltage (V) and thus called potential coil (pc). Current I_2 is passed through the moving coil by means of two Phosphor – Bronze hair springs, which also provide the necessary controlling torque. To limit current (I_2) through it a high resistance R is connected in series with moving coil.

Working: The magnetic fields are set-up due to the current flowing in fixed and moving coil and hence two fluxes will set-up. These two fluxes will intersect and produces the deflecting torque. As a result, the moving coil rotates and the pointer moves on the graduated scale. The pointer reaches a steady deflection position when the deflecting torque (T_d) is proportional to the controlling torque (T_c).

If the direction of current is reversed, the magnetic field will also get reversed so that direction of deflecting torque (T_d) remains the same. Hence, it can be used to measure power consumed either by an ac circuit or a dc circuit.

Measurement of Power

Case (i) DC Circuit: Let V = Voltage across potential coil

Current through fixed coil or CC = Load current = I_1

Current through potential coil or *pc* = $I_2 \propto V$

Deflecting torque, $T_d \propto BI_2$ (where B = flux density)

As the fixed coil is air-cored, flux density $B \propto I_1$

$$\therefore \quad T_d \propto I_1 I_2$$
$$\propto I_1 V = VI_1$$
$$\propto \text{Power}$$

Case (ii) AC Circuit: Let the instantaneous voltage across the load,

$$v = V_m \sin\theta \text{ volt}$$

Assuming lagging *pf*, instantaneous current flowing through CC or the load is

$$i_1 = I_m \sin(\theta - \phi) \text{ ampere}$$
$$\propto I_1$$

Current through moving coil = $i_2 \propto v$

Average deflecting torque,

$$T_d \propto \text{Average of } i_1 \, i_2$$
$$\propto \text{Average of } i_1 . v$$
$$\propto V_m \sin\omega t \times I_m \sin(\theta - \phi)$$
$$\propto VI_1 \cos\theta$$
$$\propto \text{Power}$$

Controlling torque, $T_c \propto \theta$

At steady deflection position or equilibrium,

$$T_d \propto T_c$$

or

$$\theta \propto \text{Power}$$

Therefore, the **scale is uniform.**

Advantages

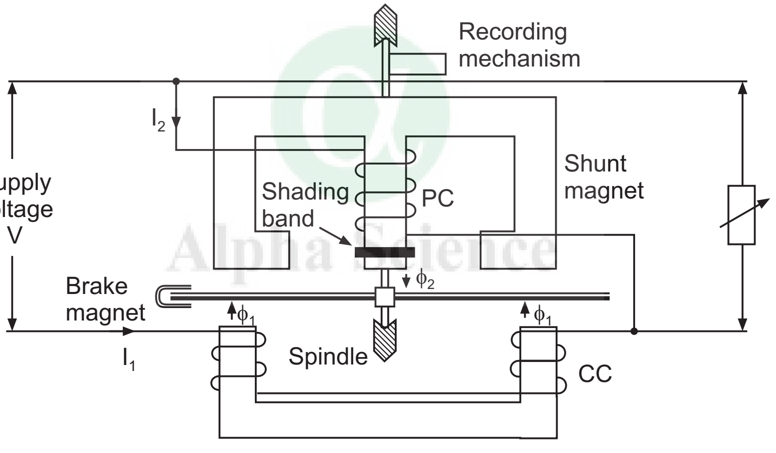
1. As air core is used, it is free from hysteresis and eddy current losses.
2. Precision grade accuracy.
3. It can be used to measure power consumed both on ac and dc circuit.

Disadvantages

1. More friction loss due to heavy moving coil.
2. Gives error at low power factor.
3. More expensive.

5.4 SINGLE-PHASE INDUCTION TYPE ENERGYMETER

Single-phase energy meter or watt-hour meter is used to measure the quantity of electrical energy supplied to a single-phase ac circuit in a given time (measured in kWh). Here electromechanical induction meter is discussed.



Series magnet

Construction: It consists of the following systems:

1. Moving System,
2. Operating System and
3. Recording System.

1. **Moving System**: It consists of a light weight Aluminum disc mounted on a vertical spindle and free to rotate.
2. **Operating System:** It consists of the following:
   (i) **Series magnet:** It is a U shaped Iron or Silicon Steel laminated assembly placed below the Aluminum disc and it is wound with current coil (CC).

(ii) **Shunt magnet:** It is an M shaped Iron or Silicon Steel laminated assembly placed above the Aluminum disc and it is wound with pressure coil or potential coil (pc) or voltage coil.

(iii) **Brake magnet:** It is a permanent magnet placed at the edge of Aluminum disc, which controls the speed of the disc.

3. **Recording System:** It consists of a set of dials and gearing mechanism. It records the number of revolutions of the Aluminum disc.

Working: When the load current I_1 flows through current coil of series magnet, it produces an alternating flux ϕ_1 proportional to I_1 and exactly in-phase with it. When current I_2 proportional to the voltage flows through potential coil of shunt magnet, it produces an alternating flux ϕ_2 proportional to voltage and lags behind it by 90°. These two fluxes ϕ_1 and ϕ_2 induce emfs in the Aluminum disc and hence produces circulating eddy currents.

The reaction between circulating eddy currents and fluxes produce a deflecting (or driving) torque, which causes the disc to rotate and its speed is controlled by brake magnet.

The function of Copper shading band is to ensure flux produced by pc is exactly in quadrature with supply voltage.

The meter constant of energy meter is given by revolutions/kWh.

Computation of Electrical Energy:

Let, V = Supply Voltage in volt,

I = Load or Line current in ampere

N = Speed of the Aluminum disc.

$\cos\theta$ = power factor of load

$\therefore$ Power consumed, $P = VI\cos\theta$ watt

Average deflecting torque, $T_d \propto P$

or $$T_d = K_1 VI\cos\theta,$$

where K_1 is a constant

Braking torque, $T_b \propto N$

or $$T_b = K_2 N,$$

where K_2 is some other constant

The Aluminum disc rotates at approximately steady speed N when

$$T_d = T_b$$

i.e., $$K_2 N = K_1 VI\cos\theta$$

Multiplying by time t both the sides,

$$K_2 N \times t = K_1 VI\cos\theta \times t$$

or $$N \times t = \frac{K_1}{K_2} VI\cos\theta \times t$$

or $$N \times t = K\,P \times t$$

where $K = \dfrac{K_1}{K_2}$ is another constant

Hence, the total number of revolutions in a given period of time *t* gives the electric energy consumed.

(Note: Single-phase induction type energy meter is used only for the electric circuits energized by ac Supply.)

Advantages

1. Simple and strong in construction.
2. It has high torque to weight ratio and thus frictional errors are less and gives accurate readings.
3. Less maintenance.

Creep: In an induction type meter, creep is a phenomenon that can adversely affect accuracy, that occurs when the meter disc rotates continuously with potential applied and the load terminals open circuited.

Multiple Choice Questions

Choose the Correct Answer

1. The moving coil in a dynamometer wattmeter is connected
   (a) in series with fixed coil (b) across the supply
   (c) in series with the load (d) none of these
2. An electro dynamic instrument is a
   (a) moving iron instrument (b) moving coil instrument
   (c) induction (d) electrostatic
3. Dynamometer wattmeter is basically
   (a) an integrating instrument (b) an indicating instrument
   (c) a digital instrument (d) not an instrument
4. The pointer in dynamometer type wattmeter is made of ....
   (a) four (b) three (c) two (d) none of these
5. The fixed coil in dynamometer type wattmeter is arranged in .... equal sections
   (a) copper (b) aluminum
   (c) phosphor bronze (d) platinum
6. Dynamometer type wattmeter is used to measure ....
   (a) only dc power (b) only ac power
   (c) both ac and dc power (d) both active and reactive power

7. In dynamometer type wattmeter, the fixed coil is air-cored to avoid
   (a) copper loss (b) friction loss
   (c) hysteresis loss (d) none of these
8. In dynamometer type wattmeter, a resistance R is connected in series with *pc* to ....
   (a) increase the range (b) get uniform scale
   (c) limit the current through it (d) none of these
9. In dynamometer type wattmeter, Phosphor – Bronze hair springs provide
   (a) controlling torque (b) deflecting torque
   (c) both (a) and (b) (d) none of these
10. In dynamometer type wattmeter, at steady deflection position ....
    (a) $T_d < T_c$ (b) $T_d = T_c$ (c) $T_d > T_c$ (d) $T_d \propto (1/T_c)$
    where T_d = deflecting torque and T_C = controlling torque
11. When the pointer of an indicating instrument comes to rest in final deflection position, then
    (a) only controlling torque acts (b) only deflecting torque acts
    (c) both torques act (d) none of these
12. In the energy meter, constant speed of rotation of the disc is provided by
    (a) a shunt magnet (b) series magnet
    (c) braking magnet (d) none of these
13. The meter constant of energy meter is given by
    (a) rev./kW (b) rev./watt (c) rev./kWh (d) rev./kVA
14. One unit of electrical energy is equivalent to
    (a) 1 kWh (b) 3600 W-sec (c) 100 WH (d) 10 kWh
15. Induction type single phase energy meter can be used on
    (a) ac only (b) dc only
    (c) both ac and dc (d) none of these
16. In electricity bill, the units consumed represents
    (a) kW consumed (b) Wh consumed
    (c) kWh consumed (d) watts consumed
17. Exact value of true quantity being measured can be obtained from measuring instruments by
    (a) cleaning the instrument frequently
    (b) making proper connections
    (c) proper maintenance
    (d) proper calibration

18. Under no load condition, the revolution of the disc due to kinetic energy of an energy meter can be blocked by
    (a) brake magnet (b) electromagnet
    (c) creeping hole with brake magnet (d) copper shading band

19. Single phase energy meter is
    (a) electro dynamic type of instrument
    (b) induction type of instrument
    (c) electrolytic type of instrument
    (d) motor type of instrument

20. Slow but continuous rotation of the disc of an energy meter is called
    (a) sweeping (b) crawling (c) creeping (d) friction error

Answers:—

1. (b)	2. (b)	3. (b)	4. (c)	5. (b)	6. (c)	7. (c)	8. (c)	9. (a)	10. (c)
11. (c)	12. (c)	13. (c)	14. (a)	15. (a)	16. (c)	17. (d)	18. (a)	19. (b)	20. (c)

Review Questions

1. With a neat diagram, explain the working of dynamometer type wattmeter.
2. Explain the principle of operation of dynamometer type of wattmeter.
3. With the help of a neat figure, explain the single phase Induction type energy meter.

Chapter

6

Domestic Wiring

6.1 SERVICE MAINS, METER BOARD AND DISTRIBUTION BOARD

The line bringing electrical power from the supplier to the energy meter installed at the consumer premises is called the service connection. It may be either overhead conductors or underground cables.

The consumer has to pay for the electrical energy consumed. Thus, the supplier's service lines will be connected to the input terminals of the energy meter provided by the supply authority.

After the energy meter, the service line is connected to a cut-out. The cut-out serves two purposes, for protection and enabling the authority to disconnect the supply if the consumer fails to pay the bill in time. The energy meter and the cut-out are the supply authority's property and are sealed.

The consumer's distribution starts after the energy meter and cut-out. The leads from the output terminals are connected to the consumer's main switch from which various loads are connected, called the internal wiring.

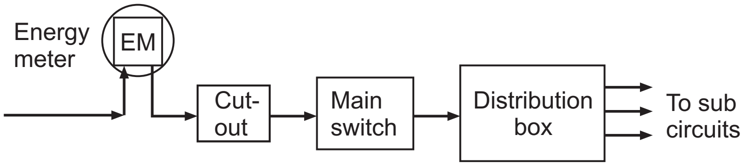


Fig. 6.1 Block diagram

The distribution board, refers to an equipment which consists of bus bars and possible switches, fuse links and automatic protective equipment, bypass equipment, for connecting, controlling and protecting a number of branch circuits fed from one main circuit of a wiring installation in a building or premises for easy and safe handling of incoming power supply.

(Note: Standard voltage for domestic consumer is single-phase ac 240 V, 50 Hz two wire.)

6.2 DIFFERENT TYPES OF WIRINGS

Factors to be considered for selection of wiring:

1. Insulation or Electrical safety: Wiring should not permit the flow of leakage current, which leads to electrical shock.
2. Risk of fire: Wiring should not catch fire in case of short circuit.
3. Moisture absorption: Wiring should not absorb moisture.
4. Mechanical safety: Wiring should withstand for a long time and protect from physical damage.
5. Installation, Inspection and Replacement.
6. Appearance.
7. Cost :This includes initial cost and maintenance cost.

Types of Wiring:

1. Cleat wiring
2. Wood casing and Capping wiring
3. CTS or TRS wiring
4. Lead covered or Metal sheathed wiring
5. Conduit wiring
    (i) Surface (or Open) conduit and
    (ii) Concealed or buried conduit.

1. Cleat Wiring

The Porcelain cleats are in the form of two halves. The first half is provided with grooves to accommodate the wire or conductors and the other half is put over it and both of them are firmly held together by means of screws.

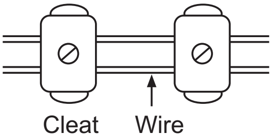


Fig. 6.2 Cleat wiring

The first half is fixed on wooden plugs which are previously cemented on walls or ceilings. VIR(Vulcanized Indian Rubber) Conductors (or wires) are accommodated in cleats.

Cleats support the wiring at intervals of 0.6 m approximately.

Advantages:

1. It provides good insulation.
2. Wires are kept apart in separate grooves.
3. Reduced risk of fire.
4. Easy to install, inspect and to replace.
5. Low cost.

Disadvantages:

1. Needs maintenance as the wiring is exposed to atmosphere.
2. May not provide good mechanical support.
3. This type of wiring gives bad appearance.
4. Wiring should not be installed near water and gas pipes.

Application:

It is applicable for temporary installation.

2. Wood Casing and Capping Wiring

It consists of two separate wooden strips, one is called as the Casing, which is grooved to accommodate wire or conductor and the other is called, the Capping which is put over casing and both of them are firmly held together by means of screws spaced at regular intervals.

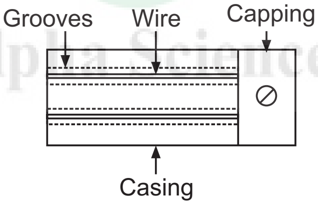


Fig. 6.3 Casing and Capping

The casing is fixed on wooden plugs which are previously cemented on walls or ceilings. VIR Conductors (Wires) are accommodated in grooves provided on Casing. Entire fixture is then painted or varnished.

Advantages:

1. Wood provides good insulation.
2. Wires are kept apart in separate grooves.
3. Easy to install, inspect and to replace.
4. Better appearance.
5. Comparatively cheap.

Disadvantages:

1. Risk of fire.
2. Absorbs moisture.
3. If seasoned wood is not used, there may be trouble from white ants.
4. Requires skilled workmanship.
5. Wiring should not be installed near water and gas pipes.

Application:

Applicable for permanent wiring system under dry conditions.

3. CTS or TRS Wiring

Teak wood battens are fixed on wooden plugs, which are previously cemented on walls or ceilings.

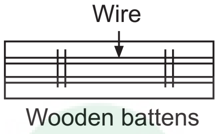


Fig. 6.4 Wiring

Cab – Tyre – Sheathed (CTS) or Tough Rubber Sheathed (TRS) Conductors are insulated conductors, which are completely covered by means of a thick coating of tough rubber. The wiring is accommodated on wooden battens and held firmly by lead or steel strips. The entire fixture is then varnished or painted.

Advantages:

1. Provides good insulation.
2. Reduced risk of fire.
3. Easy to install, inspect and to replace.
4. Better appearance.
5. Economical.

Disadvantages:

1. Wiring should not be exposed to direct Sun light, rain or damp places.
2. Sharp bends should be avoided.
3. Must be adequately protected against mechanical injury.
4. Wiring should not be installed near water and gas pipes.

Applications:

It was most commonly used.

4. Lead Covered or Metal Sheathed Wiring

Rubber insulated conductors are further provided with an outer sheath of a thick coating of molten Lead Alloy, which gives protection from moisture and mechanical injury. Lead sheathed wires are accommodated on wooden battens.

Advantages:

1. It is moisture-proof due to outer sheath of molten Lead Alloy.
2. Reduced risk of fire.
3. It is mechanically robust.

Disadvantages:

1. Expensive.
2. As the outer sheath is made of Lead, which is good conductor; there must be provision of efficient Earthing.
3. Wiring should not be installed near water and gas pipes.

Applications:

It was applicable for damp places where CTS wiring could not be used.

5. Conduit Wiring:

VIR Conductors are accommodated in tubes called the conduits. Wires are drawn inside conduits by means of GI wire.

The Conduits may be metallic such as Steel or Galvanized Iron (GI) tubes or PVC pipes; if PVC pipes are employed, it is called the Flexible conduit system.

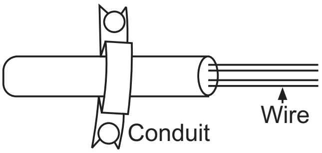


Fig. 6.5 Conduit wiring

Types:

(i) Surface or Open Conduit System: The conduits are fixed on the surface of the wall and the ceiling by saddles or pipe hooks.

(ii) Concealed or Buried Conduit System: The conduits are buried inside the plastering of the walls and ceiling and hence hidden from view.

This system requires junction boxes.

Advantages:

1. It provides very good insulation.
2. Risk of fire is completely eliminated.
3. It is moisture-proof.
4. It provides good mechanical support.
5. Easy to replace.

Disadvantages:

1. Initial cost is more.
2. Skilled workers are required.
3. Wiring should not be installed near water and gas pipes.

Applications:

It is considered as the best type of wiring and most popular one.

Specification of Wires:

The current flowing through a wire causes heat, which is proportional to square of current. Therefore it requires proper insulation to withstand this temperature safely.

Resistance is given by

$$R = \rho \frac{1}{A}$$

where ρ = specific resistance of material,

l = length of conductor

A = cross sectional area

Hence, if the current is more, resistance of conductor should be less and should have larger area of cross section.

As per Metric System, wire is specified as Standard Wire Gauge (SWG). For solid (or single) conductor, the size of wires are denoted by their area of cross-section in square millimeters.

Example: 1.5 sq. mm, 2.5 sq. mm., etc.

For the sake of convenience, the size of wire is referred to the number rather than area of cross section. Higher the number of gauge, smaller is the diameter.

Example: Smallest wire gauge is of number 40 having diameter of 0.0048 inch. The largest wire gauge is of number 0,000,000 (seven zero) having diameter of 0.5 inch.

For flexible wires, which consists of a number of wires stranded together, the wires of standard size of diameter 0.1930 mm (equivalent of 36 SWG wire of diameter 0.0076 inch) are used.

Example: Let the specification of a flexible wire is given by 40/0.007, that is, the flexible wire has 40 strands of 36 SWG, stranded, each strand having a diameter of 0.1930 mm (0.0076 inch), together giving a net cross sectional area of 1.171 sq. mm. This has current carrying capacity of 7 A.

6.3 TWO-WAY CONTROL OF LAMP

Two-way (or two point) control is used to control a single lamp from two different places. It finds application in stair case lighting. This can be accomplished by means of two 2-way switches (S_1 and S_2).

Circuit diagram:

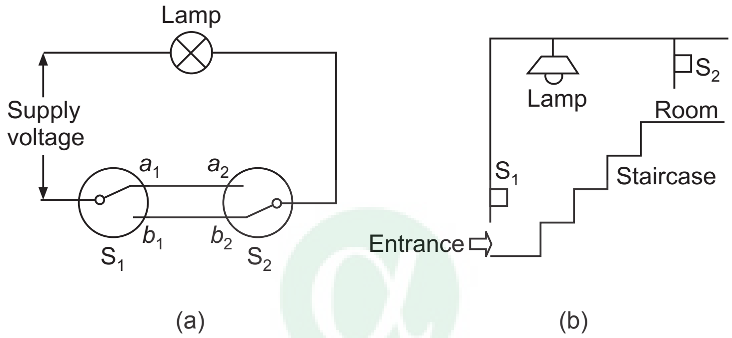


Fig. 6.6 (a) Two-way control (b) Staircase lighting

Table 6.1 Switching Table

S. No.	Position of switch S_1	Position of switch S_2	State of Lamp ON or OFF
1	a_1	b_2	OFF
2	b_1	b_2	ON
3	b_1	a_2	OFF
4	a_1	a_2	ON

Explanation:

Referring SI. No.1: Switches S_1 and S_2 are in dissimilar positions, the circuit is incomplete and thus Lamp is switched OFF.

Referring SI. No.2: Switches S_1 and S_2 are in similar positions, the circuit is completed and thus Lamp is switched ON.

Referring SI. No.3: Switches S_1 and S_2 are in dissimilar positions, the circuit is incomplete and thus Lamp is switched OFF.

Referring SI. No.4: Switches S_1 and S_2 are in similar positions, the circuit is completed and thus Lamp is switched ON.

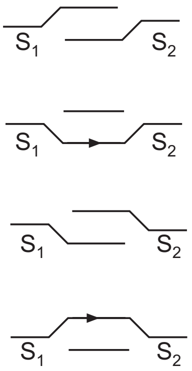


Fig. 6.7 Various positions

Therefore, if two 2-way switches are in similar positions, circuit is completed and lamp is switched ON and when they are in dissimilar positions, circuit is incomplete and lamp is switched OFF.

Stair Case Lighting: Initially the two 2-way switches S_1 (located at the entrance of the stair case) and S_2 (located in the up stair room) are in dissimilar position. A person intending to climb the up stairs, operates the switch S_1 so that 2-way switches will be connected in similar position and thus the lamp glows. After reaching the up stair room, the person operates the switch S_2 and again the 2-way switches will be connected in dissimilar position and the lamp will be switched off.

Two Way Control of Fluorescent Lamp:

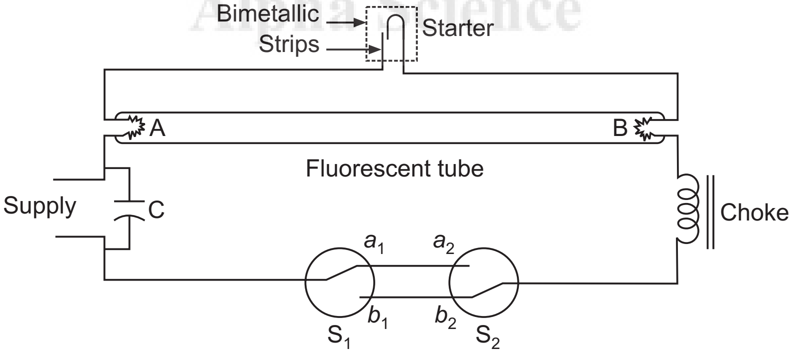


Fig. 6.8 Connection diagram

6.4 THREE-WAY CONTROL OF LAMP

Three-way control is used to control a single lamp from three different places, then three-way (or three point) control of lamp is opted. Examples: Big corridors, Workshops, etc. This can be accomplished by two 2-way switches (S_1 and S_2) and one intermediate switch (S_i).

Circuit diagram:

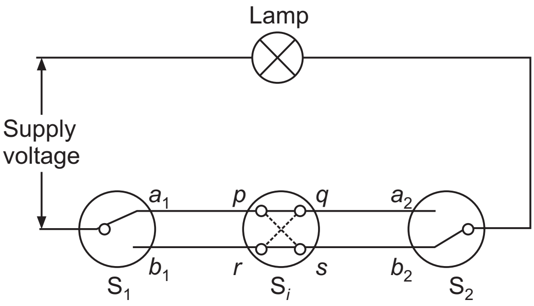


Fig. 6.9 Three-way control

Table 6.2 Switching Table

Sl. NO	Position of switch S_1	Position of switch S_i	Position of switch S_2	State of Lamp ON or OFF
1	a_1	pq-rs (straight)	a_2	ON
2	b_1	pq-rs (straight)	b_2	ON
3	a_1	pq-rs (straight)	b_2	OFF
4	b_1	pq-rs (straight)	a_2	OFF
5	a_1	ps-qr (cross)	a_2	OFF
6	b_1	ps-qr (cross)	b_2	OFF
7	a_1	ps-qr (cross)	b_2	ON
8	b_1	ps-qr (cross)	a_2	ON

Explanation:

When two 2-way switches S_1 and S_2 are in similar positions and intermediate switch S_i is in straight position then the circuit is completed and lamp is switched ON.

When two 2-way switches S_1 and S_2 are in dissimilar positions and intermediate switch S_i is in straight position then the circuit is incomplete and lamp is switched OFF.

When two 2-way switches are in similar positions, and the intermediate switch is in the position of cross connection, then the circuit is incomplete and lamp is switched OFF.

When two 2-way switches are in dissimilar positions, and the intermediate switch is in the position of cross connection, then the circuit is completed and lamp is switched ON.

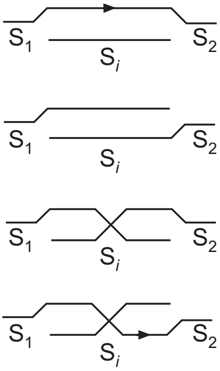


Fig. 6.10 Various positions

Wiring Materials and Accessories

1. Switches: It is necessary to make or brake an electric circuit.
2. Lamp Holder: It is necessary to hold the lamp required for lighting purpose.
3. Ceiling Rose: It is necessary to provide a tapping to the lamp holder through flexible wire.
4. Plugs: They are necessary for tapping power from socket outlets.
5. Main Switch: This is used at the consumer's premises so that consumer may have self-control of the entire distribution circuit.

6.5 PROTECTIVE DEVICES

6.5.1 Fuse

Fuse is essentially a short piece of metal inserted in a circuit, which melts when an excessive current flows in the circuit and thus isolates the device from supply mains. Thus, fuse is a protective device.

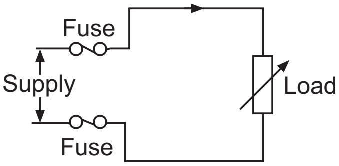


Fig. 6.11 Fuse

Important terms

1. **Minimum fusing current:** It is that minimum value of current at which the fuse element melts. It depends upon the following factors:

   - (i) Material,
   - (ii) Length and
   - (iii) Diameter.

2. **Current rating of fusing element:** It is that value of current at which the fusing element can normally carry without melting.

   Hence current rating of fusing element < minimum fusing current.

3. **Fusing factor:**

$$\text{Fusing factor} = \frac{\text{Minimum fusing current}}{\text{Current rating of fusing element}} > 1.$$

Fuse element material: Most suitable material for the fuse element is a low melting point material such as Tin-Lead Alloy.

Types of fuse:

1. Low Voltage fuse:
   - (i) Semi-enclosed rewirable fuse and
   - (ii) HRC Cartridge fuse
2. High Voltage fuse

Advantages of fuse:

1. It is the cheapest form of protection,
2. It requires low maintenance and
3. It interrupts enormous short circuit currents without noise.

Disadvantage:

Time is lost in replacing fuse after operation.

6.5.2 MCB

A circuit breaker is a switch that automatically turns off itself, if the current more than the specified value flows through it.

Fig. 6.12 Image of MCB

Circuit breakers used in residential and light commercial installations are referred to as miniature circuit breakers (MCB's).They are commonly used in the electrical consumer units as protective devices and installed in consumer units.

MCB consists of a movable handle in the form of a lever. Under normal working conditions, the lever will be in ON position. If a fault occurs either due to over loading of the circuit or short circuit, MCB trips and switches OFF automatically, isolating the electrical equipment from the supply. Then it can be manually reset.

6.6 ELECTRIC SHOCK, PRECAUTIONS AGAINST SHOCK-EARTHING

Electric Shock

If a live wire carrying electric current is in contact with the metal parts of an electric equipment and if a person happens to touch the such metal parts, then the current (leakage current) flows through that person and the person experiences electric shock. It may cause injury, burns, unconsciousness or even death if the shock is severe.

As the leakage is given by V/R, severity depends upon the voltage and the human body resistance (it is approximately 100 kΩ under dry condition and 1 kΩ under wet condition). Also, current at low frequency may cause severe shock than that of high frequency.

A human being can sustain about 30 mA for a short duration of 25 millisecond. If the current is of the range 100 mA, the severity of shock will be more.

Precautions against shock:

1. Electric appliances and switches should not be touched with wet hands.
2. Proper grounding should be provided.
3. Three pin sockets are to be used and ground point should be provided to all the sockets.
4. The earth resistance should be periodically checked and it should not exceed 5Ω.
5. 'Ground Fault Interrupt' may be provided for extra safety.
6. Worn wiring and cords should be replaced.

Earthing or Grounding

Meaning: Earthing or Grounding means, connection of the metal frame of an electrical equipment or machinery to the general mass of earth so that it will ensure at all times an immediate and safe discharge of electrical energy due to leakage or faults.

Earthing is done by connecting the metal frame or casing of electrical apparatus or machinery to the earth by means an earth wire having low resistance, called 'Earth wire' or 'Earth electrode'. Its function is to prevent the risk of shock due to leakage or faults.

Necessity of Earthing

1. If human body happens to come in contact with metal frame of an electrical equipment or machinery, the leakage current due to faults flows through human body to earth, which causes electric shock. The electrical shock may cause disability or even death. Hence to protect human being from shock, earthing is done.
2. For the purpose of protection, all electrical equipments and machines shall be earthed.

Working (or functioning)

With effective earthing, if a person happens to touch the metal frame of an electrical equipment or machinery, when the live wire is in touch with it, it forms a parallel path, thus a circuit will be completed through human body and earth wire.

As earth wire has very low resistance compared to human body, the leakage current flows through earth wire and hence human body will be protected. Thus earth wire works as a protective device against any electrical shock in case of faults in the circuit.

6.7 TYPES OF EARTHING

1. Pipe earthing and
2. Plate earthing

6.7.1 Pipe Earthing

The GI Pipe is used as an earth electrode. It is buried at a depth not less than 3 metre in a moist place as for as possible, near by water tap or drainage and should be away from building and foundation by 0.6 m. Also, for effectiveness, salt water is poured periodically through a funnel provided.

The pit area around the pipe must be completely filled by a layer of charcoal (80 mm) and a layer of common salt (30 mm) all around to increase earth's conductivity.

Size of pipe depends upon the current to be carried and type of soil.

(i) For ordinary soil; 38 mm dia. and 2 m length.

(ii) For dry and rocky soil; 38 mm dia. and 2.75 m length.

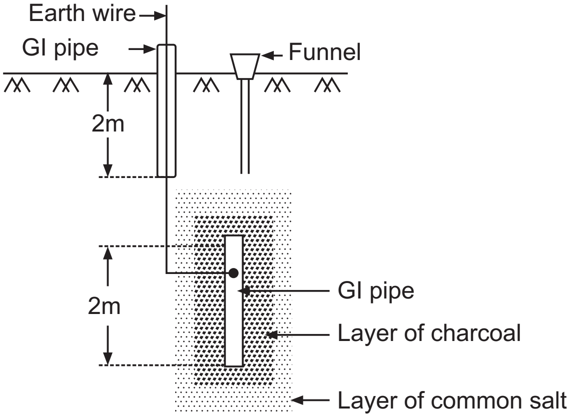


Fig. 6.13 Pipe earthing

Advantage:

Due to circular cross-section of GI pipe, it will have more surface contact with the soil. Thus, it can carry large leakage current for the same dimension.

Disadvantage:

If the soil Specific Resistivity is more, the length of GI pipe should be increased. This will increase the cost.

6.7.2 Plate Earthing

The GI or Copper Plate is used as an earth electrode. Burried at a depth not less than 3 metre in a moist place as for as possible, near by water tap or drainage and should be away from building and foundation by 0.6 m.

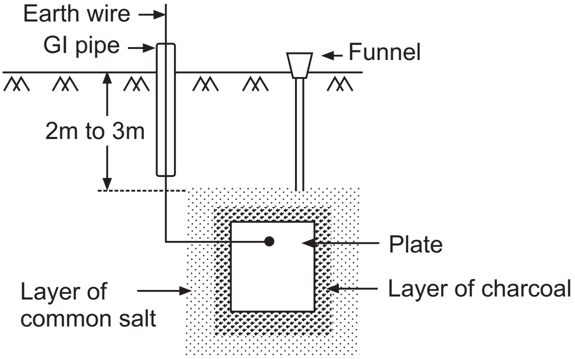


Fig. 6.14 Plate earthing

The pit area around the plate shall be completely filled by a layer of charcoal (80 mm) and a layer of common salt (30 mm) all around to increase earth's conductivity. Also, for effectiveness, salt water is poured periodically through a funnel provided.

Plate dimension:

(i) GI Plate: 0.3 m × 0.3 m × 3.2 mm thickness.

(ii) Copper Plate: 0.3 m × 0.3 m × 6.35 mm thickness.

Advantages:

1. Conductivity can be increased by increasing the dimension of the plate and depth of embedding.
2. Cheaper if GI pipe is opted.

Disadvantages:

1. Sometimes the earth wire may get disconnected with earth plate, which could not be noticed.
2. Becomes expensive if Copper plate is opted.

Multiple Choice Questions

Choose the Correct Answer

1. For stair case lighting, usually .... control is used.

   (a) one way (b) two way (c) three way (d) none of these

2. An intermediate switch is used in .... of lamps.

   (a) single way control (b) two way control

   (c) three way control (d) both (a) and (b)

3. The resistivity of a fuse material is of the order of .... μΩ-m

   (a) 10^2 (b) 10^{-2} (c) 10^4 (d) 10^{-4}

4. The minimum fusing current of a fuse wire is 2.1 A and fusing factor is 1.1. Then, the rated carrying current of the fuse element is

   (a) 2.2 A (b) 2.31 A (c) 1.909 A (d) 0.5238 A

5. A fuse is a

   (a) current limiting device (b) protective device

   (c) voltage limiting device (d) none of these

6 The fusing factor of a fuse is

   (a) zero (b) $= 1$ (c) > 1 (d) < 1

7. The fuse material is made of ....

   (a) an alloy of tin and lead (b) aluminum

   (c) copper (d) zinc

8. The important properties of a fuse wire are ....
   (a) low resistivity and high melting point
   (b) high resistivity and low melting point
   (c) low resistivity and low melting point
   (d) high resistivity and high melting point
9. The primary function of fuse is to ....
   (a) protect the appliance (b) open the circuit
   (c) prevent excessive current (d) protect the line
10. Charcoal and salt used in earthing ....
   (a) not to alter the earthing resistance
   (b) to increase the earthing resistance
   (c) to decrease earthing resistance
   (d) none of these
11. Coke can be used as a sandwich between salt of an earthing system, to.......
   (a) by pass the current
   (b) avoid melting of salt
   (c) improve conductivity
   (d) hold moisture content
12. Earthing brings the potential of the body of the equipment to ......
   (a) 0 (b) 110 V (c) 220 V (d) 400 V
13. The earthing resistance for effective earthing is of the order of .... Ω.
   (a) 0.5 (b) 5 (c) 50 (d) 500
14. A good earthing should provide .... resistance in earthing plate.
   (A) low (b) high (c) medium (d) none of these
15. Severity of electric shock depends upon
   (a) current and earthing resistance
   (b) voltage and earthing resistance
   (c) body resistance and body resistance
   (d) voltage and body resistance

Answers:—

1. (b) 2. (c) 3. (b) 4. (c) 5. (b) 6. (c) 7. (a) 8. (b) 9. (b) 10. (c)
11. (c) 12. (a) 13. (b) 14. (a) 15. (d)

Review Questions

1. Discuss the supply mains, meter board and distribution board.
2. Describe briefly:
    - (i) Conduit system of wiring
    - (ii) CTS System of wiring
3. With neat diagrams explain any three types of wiring.
4. With relevant circuit diagrams and switching tables, explain two-way control of lamp.
5. With diagrams, explain the three way control of lamp.
6. What is the necessity of earthing? With a neat diagram explain the pipe earthing.
7. With a neat sketch explain the plate earthing method.
8. Define the following terms with respect to a fuse:
    - (i) Rated current
    - (ii) Fusing current
    - (iii) Fusing factor
9. Write a note on MCB.
10. What are the precautions to be taken to prevent electric shock?

Chapter
7
DC Machines

7.1 CONCEPT OF ELECTRICAL MACHINES

Electrical machines of rotating type such as generators or motors transfer energy either from mechanical to electrical or electrical to mechanical form. This process is called the electro-mechanical energy conversion (EMEC), which is reversible. In these machines, the magnetic field couples the electrical and mechanical systems.

Electrical machines which are related to an electrical energy of direct type are called dc machines.

7.2 DC GENERATOR

7.2.1 Working Principle as a Generator

An electrical generator is a machine, which converts mechanical energy into electrical energy.

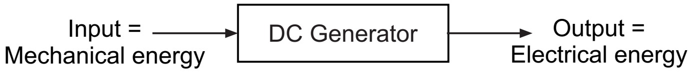


Fig. 7.1 DC Generator

This energy conversion is based on the production of dynamically or motional induced emf according to Faraday’s Laws of Electromagnetic Induction. The change in the flux associated with the conductor exists by the relative motion between the conductor and the flux and this can be achieved by rotating the conductor with respect to the flux. Thus, voltage gets generated in the conductor, as long as the relative motion between the conductor and the flux exist.

7.2.2 Construction

The important parts are Field System and Armature.

Field System: It is a stationary part and its function is to produce the necessary flux. It consists of the following parts:

1. Yoke
2. Pole Core and Pole Shoe
3. Field Coils

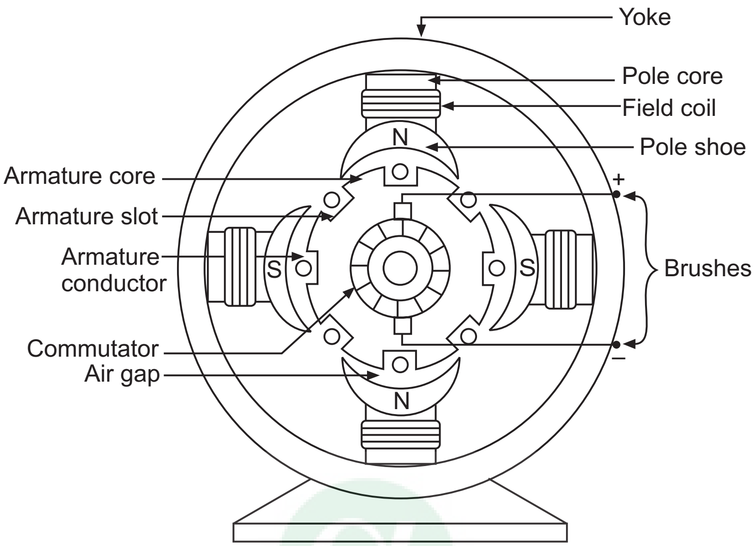


Fig. 7.2 Parts of DC generator

1. **Yoke**: It is a heavy protective frame made of Cast Iron or Steel, which houses all the parts of the generator.

   The functions of yoke:

   (i) To provide mechanical support

   (ii) To carry the magnetic flux

2. **Pole Core and Pole Shoe**: Pole core is made of Cast steel (or Wrought iron or Steel alloy) laminations. It is fixed to the yoke. It is provided with pole shoe for the following reasons:

   (i) To distribute the flux uniformly in the air gap

   (ii) To reduce the reluctance path by a large cross sectional area

   (iii) To support the field coils

3. **Field Coils or Exciting Coils:** Field coils are insulated copper wire, wound round the poles and carry dc

Function of Poles and Field Coils: As the field coils carry direct current, the poles become magnetized and produce the necessary flux. This process is called the excitation (or energization).

Armature: It is a rotating part and its shaft is mechanically coupled with a prime mover such as water turbine or steam turbine or diesel engine.

It consists of the following parts:

1. Armature Core
2. Armature Conductors or Winding
3. Commutator
4. Brushes
5. Bearings

1. **Armature Core:** It is a cylindrical drum shaped body, made of high permeability Silicon Steel laminations (to reduce eddy current loss). The outer surface is provided with slots.
2. **Armature Conductors or Windings**: These are insulated copper conductors placed in armature slots.

   Function of armature core and armature conductors: As armature rotates (by means of a prime mover) in the magnetic field, the armature conductors cut the magnetic flux and thus an emf is induced in it according to the Faraday's Laws of EMI.
3. **Commutator:** It is of cylindrical structure and is made of wedge shaped segments of hard-drawn copper, insulated from each other by a thin layers of mica. The number of commutator segments is equal to number of coils.
4. **Brushes:** Brushes are made of carbon and are in the shape of rectangular block. They are housed in brush-holders. The brushes are pressed on the commutator segments by means of springs.

   Carbon brushes are used as they lubricate and polish the commutator and also they reduce sparking.

   Functions of commutator and brushes: The nature of current induced in the armature conductor is alternating. The commutator and brushes collect the alternating current from the armature conductors, convert into unidirectional current (that is, direct current) and supplies to external load circuit.
5. **Bearings:** For smooth rotation of armature, bearings are used. Usually ball bearings are used. For heavy duty machines, roller bearings are used. Bearings are packed in lubricating oil for smoother operation and to reduce bearing wear.

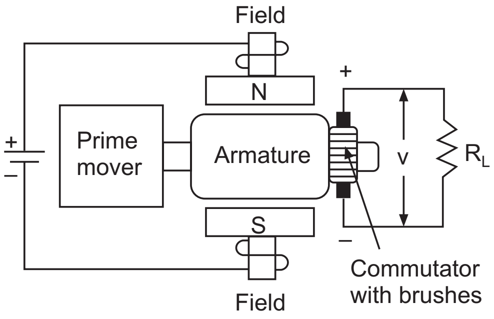


Fig. 7.3 DC Generator

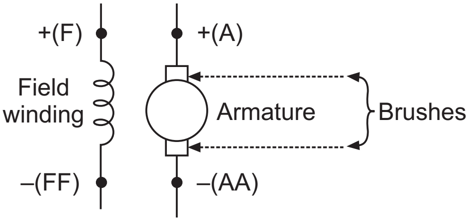


Fig. 7.4 Symbolic representation of dc machine

Working of a dc machine as a dc generator

Armature is mechanically coupled to a prime mover. As armature rotates at rated speed in uniform magnetic field, armature conductors cut the flux and thus an emf is induced (dynamically induced emf) in it according to Faraday's Laws of EMI.

The nature of induced emf and current will be alternating in nature. The commutator and brushes collect the alternating current from the armature conductors and convert into unidirectional current (that is, direct current) and supply to the external load R_L as shown in Figure 7.5(a) and 7.5(b).

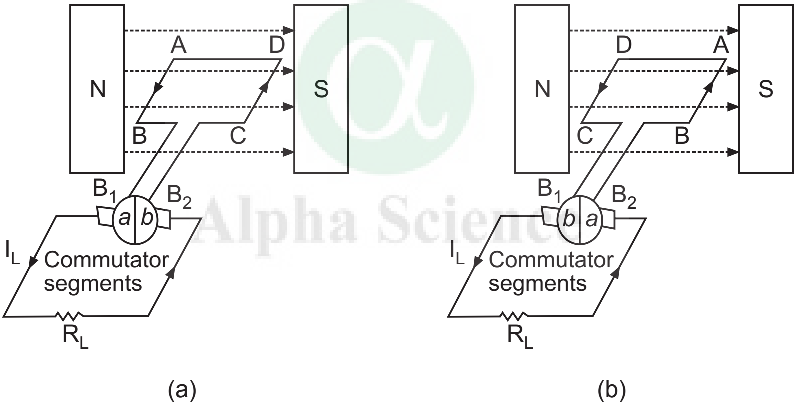


Fig. 7.5 (a) First half-cycle (b) Second half-cycle

To explain the process of conversion of ac to dc known as commutation, assume a single turn coil ABCD rotating in uniform magnetic field. The terminals of the coil are connected to the commutator segments.

Figure 7.5(a) represents the first half-cycle. The commutator segments '*a*' and '*b*' are in contact with brushes B_1 and B_2 respectively. Thus, at this position, segment '*a*' and brush B_1 become positive with respect to segment '*b*' and brush B_2.

Figure 7.5(b) represents the second half-cycle as the coil rotates. The commutator segments '*b*' and '*a*' are in contact with brushes B_1 and B_2 respectively. Thus, at this position, segment '*b*' and brush B_1 become positive with respect to segment '*a*' and brush B_2.

Thus, the direction of the load current I_L through the load is unidirectional.

Important Points

- Slip rings and brushes provide alternating or ac output.
- Commutator and brushes provide unidirectional or dc output.

7.2.3 Types of Armature Windings

1. Lap winding and
2. Wave winding.

1. Lap winding: The finishing end of one coil is connected to a commutator segment and to the starting end of the adjacent coil situated under the same pole and so on until all the coils have been connected. Hence it doubles or laps behind with its succeeding coils.

In lap winding, number of parallel paths = number of poles and

Number of brush sets = number of poles

It is suitable for low voltage and high current generator.

2. Wave winding: The winding progresses in one direction around the armature in series of waves.

In wave winding, number of parallel paths = 2

Number of brush sets = 2

It is suitable for high voltage and low current generator.

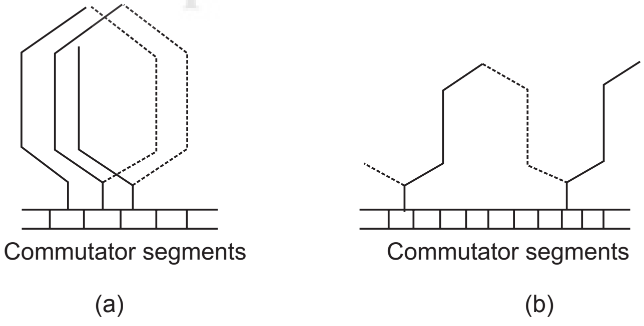


Fig. 7.6 (a) Lap winding (b) Wave winding

7.2.4 EMF Equation

Let, p = Number of poles

ϕ = Flux/pole in weber

Z = Number of armature conductors

= Number of slots × Number of conductors/slot

A = Number of parallel paths

N = Armature rotation in rpm (revolutions per minute)

According to Faraday's Laws of Electromagnetic Induction,

$$\text{Average emf induced/conductor} = \frac{d\phi}{dt} \text{ volt} \qquad \text{...(7.1)}$$

Now $d\phi$ = Flux cut/conductor in one revolution of armature

$= \phi p$

$$\text{Number of revolutions of armature/second} = \frac{N}{60}$$

$$\therefore \quad dt = \text{time taken for one revolution} = \frac{60}{N}$$

Substituting for $d\phi$ and dt in equation (7.1),

$$\text{Average emf induced/conductor} = \frac{\phi p}{60/N} \text{ weber/second}$$

or $$\text{Average emf induced/conductor} = \frac{\phi p N}{60} \text{ volt}$$

If the number of armature conductors/parallel path $= \dfrac{Z}{A}$;

Then average emf generated, $$E_g = \frac{\phi p N}{60} \times \frac{Z}{A} \text{ volt}$$

For lap winding, number of parallel paths, A = p

$\therefore$ Average emf generated, $$E_g = \frac{\phi p N}{60} \times \frac{Z}{p}$$

$$= \frac{\phi NZ}{60} \text{ volt}$$

For wave winding; number of parallel paths, A = 2

$\therefore$ Average emf generated, $$E_g = \frac{\phi p N}{60} \times \frac{Z}{2}$$

$$= \frac{\phi NZp}{120} \text{ volt}$$

Important Points

- The generated emf depends upon flux (ϕ) and speed (N).
- The total power generated by a given machine is same whether the armature is lap wound or wave wound.
- The no-load terminal voltage is nothing but the emf generated.

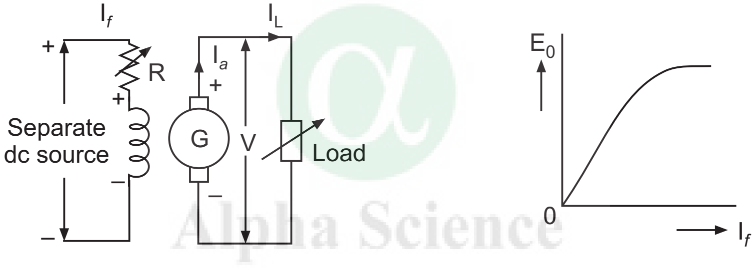
7.2.5 Classification of Generators

According to the type of excitation, generators are classified as follows:

1. Separately excited generator
2. Self-excited generator
   - (i) Shunt generator
   - (ii) Series generator
   - (iii) Compound generator
     - (a) Cumulative compound
     - (b) Differential compound

1. **Separately Excited Generator**: To produce the necessary flux, the field is excited from a separate dc source. The field current (I_f) is passed through the field winding by means of a variable resistance R. The emf or voltage is built up to rated value as represented below.

   As I_f increases → excitation increases → flux increases → emf increases



(a) (b)

Fig. 7.7 (a) Separately excited generator (b) E_0 vs I_f characteristic

Applications: As it requires a separate dc source for the purpose of excitation, its application is restricted to few special cases.

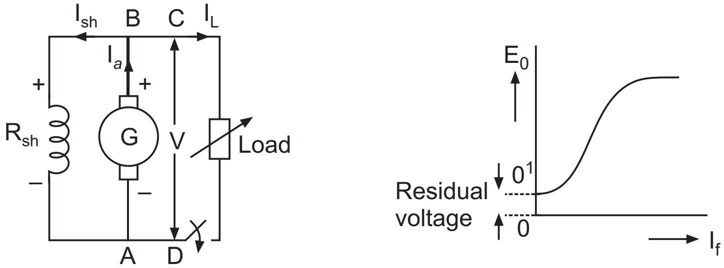
2. **Self-Excited Generator**: To produce the necessary flux, the field is excited by the output of generator itself. Due to residual magnetism, traces of flux are present in the poles. The armature is rotated at its full speed by means of a prime mover. The armature conductors cut this traces of flux, a small amount of emf is induced and hence a small value of field current I_f will set-up, which is partly or fully passed through the field coils, then the excitation is further increased. The emf or voltage is built up to rated value as represented below.

   → I_f increases → excitation increases → flux increases → emf increases →
   ↑ ↓

Meaning of residual magnetism: When the field is excited by a separate dc source, the field becomes magnetized and produces the necessary flux. When the separate dc source is disconnected, the field gets demagnetized, but traces of flux will be present, which is called the residual magnetism.

A small voltage induced when field current is zero due to residual magnetism is called the residual voltage.

(i) Shunt Generator: The field windings are connected across or in parallel with the armature windings. Full voltage of generator appears across field windings. Thus, shunt field winding consists of several number of turns of thin copper wire to have high resistance.



(a) (b)

Fig. 7.8 (a) Shunt generator (b) E_0 vs If characteristic

A part of armature current (I_a) flows through field winding called shunt field current (I_{sh}) for the purpose of excitation. The voltage (V) is built up to the rated value (this can be read by voltmeter connected across the armature terminals and thus called the terminal voltage). After closing the load switch S, a load current (I_L) flows through the load.

Applications: It finds application for battery charging and lighting purposes.

(Note: Rated values such as rating, voltage, current, rpm, etc., are given on the name plate details, supplied by the manufacturer.)

Calculation of E_g in terms of terminal voltage:

Let, R_a = Resistance of armature

R_{sh} = Resistance of shunt field

V = Terminal voltage

I_a = armature current, I_f or I_{sh} = field/shunt field current and I_L = load current

P = Power delivered

V_{cd} = Contact Drop or Brush Contact Drop (BCD)

(Note: It is small and considered if given only)

Applying KVL to the loop ABCDA,

EMF generated = terminal voltage + armature resistance drop + brush contact drop

i.e.,
$$E_g = V + I_a R_a + V_{cd} \text{ volt}$$

Applying KCL at point B,

$$I_a = I_{sh} + I_L$$

where $$I_{sh} = \frac{V}{R_{sh}} \text{ and } I_L = \frac{P}{V}$$

Power developed or Power delivered,

$$P = E_g \times I_a = V \times I_L \text{ watt}$$

(ii) Series Generator: The field windings are connected in series with the armature windings.

Series field carries full-load current for the purpose of excitation.

$\therefore$ Armature current, I_a = series field current

$$I_{se} = \text{full-load current } I_L$$

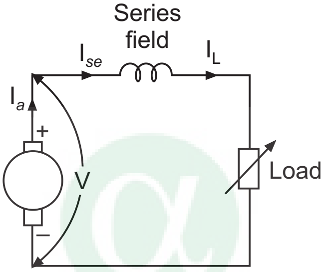


Fig. 7.9 Series generator

Thus, to build up the voltage, load current should be increased. Thus, it cannot be used for generation purpose.

The field winding consists of a few numbers of turns of thick copper wire to have low resistance, so that voltage drop across series field winding is minimum.

Applications: Used as boosters on dc feeders, for arc lamps, etc.

(iii) Compound Generator: It consists of both Series and Shunt field windings.

(a) Cumulative Compound: The series and shunt field windings are connected such that the currents I_{se} and I_{sh} flowing through them are in the same direction.

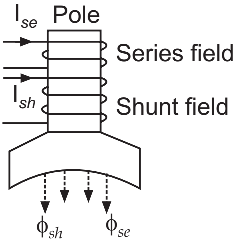


Fig. 7.10 (a) Cumulative compound

Hence, series field flux ϕ_{se} and shunt field flux ϕ_{sh} are additive.

$\therefore$ Total flux, $\phi_T = \phi_{se} + \phi_{sh}$

(b) Differential Compound: The series and shunt field windings are connected in such a way that the currents I_{se} and I_{sh} flowing through them are in opposite direction.

Hence, series field flux ϕ_{se} and shunt field flux ϕ_{sh} are subtractive.

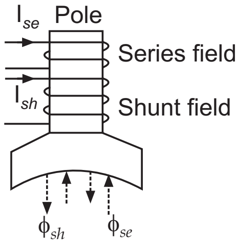


Fig. 7.10 (a) Differential compound

$\therefore$ Resultant flux, $\phi_R = \phi_{se} \sim \phi_{sh}$

(Note: The direction of current through shunt field should not be reversed. Otherwise, it will destroy residual flux.)

Types of connections:

- Long shunt
- Short shunt

Long Shunt Compound Generator: In long shunt connection, series field winding is connected in series with armature winding. To this series combination shunt field winding is connected in parallel.

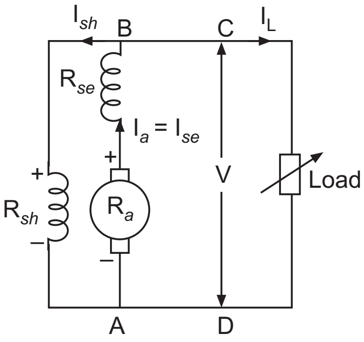


Fig. 7.11 Long shunt connection

Calculation of E_g in terms of Terminal Voltage:

Let R_{se} = Resistance series field, Ω

Applying KVL to the loop ABCDA,

E_g = terminal voltage + armature resistance drop + series field drop + brush contact drop

i.e., $E_g = V + I_a R_a + I_{se} R_{se} + V_{cd}$ volt

In long shunt connection, armature current I_a flows through series field;

i.e., in long shunt, $I_{se} = I_a$

Then series field drop $= I_a R_{se}$

$$\therefore \quad E_g = V + I_a R_a + I_a R_{se} + V_{cd} \text{ volt}$$
$$= V + I_2(R_a + R_{se}) + V_{cd} \text{ volt}$$

Applying KCL at point B,

$$I_{se} = I_a = I_{sh} + I_L$$

where $I_{sh} = \dfrac{V}{R_{sh}}$ and $I_L = \dfrac{P}{V}$

Power developed or Power delivered,

$$P = E_g \times I_a = V \times I_L \text{ watt}$$

Short Shunt Compound Generator: In short shunt connection, shunt field winding is connected in parallel with armature winding. To this parallel combination series field winding is connected in series.

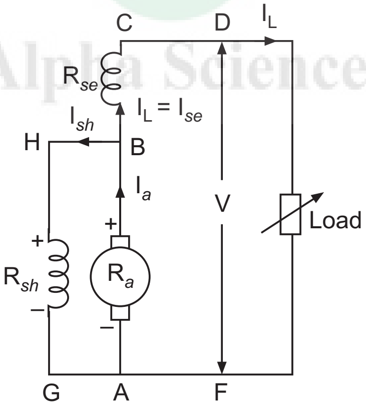


Fig. 7.12 Short shunt connection

Calculation of E_g in terms of Terminal Voltage:

Applying KVL to the loop ABCDFA,

E_g = terminal voltage + armature resistance drop + series field drop + brush contact drop

i.e., $E_g = V + I_a R_a + I_{se} R_{se} + V_{cd}$ volt

In short shunt connection, load current I_L flows through series field;

i.e., in short shunt, $I_{se} = I_L$

Then, series field drop $= I_L R_{se}$

$$\therefore \quad E_g = V + I_a R_a + I_L R_{se} + V_{cd} \text{ volt}$$

Applying KCL at point B,

$$I_a = I_{sh} + I_L$$

Power developed or Power delivered,

$$P = E_g \times I_a = V \times I_L \text{ watt}$$

Determination of I_{sh}

Applying KVL to the outer loop GHBCDFAG,

$$I_{sh} R_{sh} = I_{se} R_{se} + V \qquad \text{(Neglecting brush contact drop)}$$

or
$$I_{sh} = \frac{V + I_L R_{se}}{R_{sh}}$$

and
$$I_L = \frac{P}{V}$$

Applications: Cumulative compound generators are used for domestic lighting and to transmit power over a long distance.

Differential compound generator will not find application due to its characteristics. Rarely used for arc welding purpose.

Solved Examples

Example 7.1 *A dc shunt generator has an induced voltage on open circuit of 127 volt. When the machine is on load the voltage is 120 V. Find the load current if the field circuit resistance is 15 Ω and armature resistance is 0.02 Ω.*

Solution: Given: Induced voltage on open circuit,

$$E_g = 127 \text{ volt}$$

On load, voltage (terminal),

$$V = 120 \text{ volt}$$

$$R_{sh} = 15\ \Omega$$

and
$$R_a = 0.02\ \Omega$$

EMF generated is given by

$$E_g = V + I_a R_a$$

i.e.,
$$127 = 120 + (I_a \times 0.02)$$

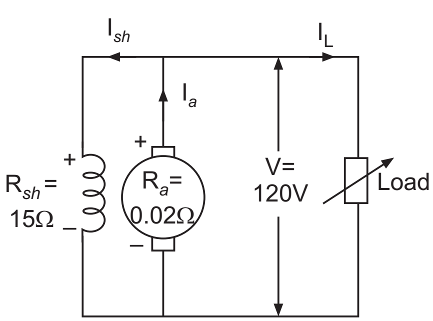


Fig. 7.13 Circuit diagram

or $$(I_a \times 0.02) = 127 - 120 = 7$$

or $$I_a = \frac{7}{0.02} = 350\text{ A}$$

Also, $$I_a = I_{sh} + I_L$$

where $$I_{sh} = \frac{V}{R_{sh}} = \frac{120}{15} = 8\text{ A}$$

∴ $$350 = 8 + I_L$$

or $$\mathbf{I_L = 342\ A}$$

Example 7.2 *A 4 pole wave wound dc generator has 48 slots, each slot having 24 conductors. The flux/pole is 0.018 weber. At what speed the armature be rotated to get an induced emf of 220 V. What will be the induced emf developed if the armature winding is lap connected and runs at the same speed?*

Solution: Given: $p = 4$,

Type of winding: Wave winding,

Number of slots = 48,

Number of conductors = 24

$\phi = 0.018$ Wb.

and $E_g = 220$ volt

With **wave winding**, speed of armature is given by

$$N = \frac{E_g \times 120}{\phi Z p} \quad \left(\text{Because } E_g = \frac{\phi N Z p}{120}\right)$$

Now Z = Number of slots × Number of conductors/slot

$= 48 \times 24 = 1152$

∴ $$N = \frac{E_g \times 120}{\phi Z\ p} = \frac{220 \times 120}{0.018 \times 1152 \times 4} = \mathbf{318\ rpm}$$

When armature is **lap connected,** induced emf is

$$E_g = \frac{\phi NZ}{60} = \frac{0.018 \times 318 \times 1152}{60} = 109.9\text{ V} \approx 110\text{ V}$$

Example 7.3 *A 4 pole generator has 36 slots with 10 conductors/slot. The flux and speed are such that an average emf generated in each conductor is 1.7 volts. The current in each parallel path is 10A. Determine the total power generated when the armature winding is i) lap connected and ii) wave connected.*

Solution: Given: $p = 4$

Number of slots = 36

Number of conductors = 10

emf/conductor = 1.7 volt

Current/parallel I_a/A = 10A

Power generated, $P = E_g \times I$ watt

(i) **For lap winding**, $A = P = 4$

No. of conductors/parallel path $= \frac{Z}{A} = \frac{36 \times 10}{4} = 90$

E_g = emf/conductor × No. of conductors/parallel path

$= 1.7 \times 90 = 153$ V

Current, I = Current/parallel path × No. of parallel paths

$= 10 \times 4 = 40$ A

∴ $P = 153 \times 40 =$ **6120 W**

= **6.12 kW**

(ii) **For wave winding,** $A = 2$

No. of conductors/parallel path $= \frac{Z}{A} = \frac{36 \times 10}{2} = 180$

E_g = emf/conductor × No. of conductors/parallel path

$= 1.7 \times 180 = 306$ V

Current, I = Current/parallel path × No. of parallel paths

$= 10 \times 2 = 20$A

∴ $P = 306 \times 20 =$ **6120 W**

= **6.12 kW**

Example 7.4 *A 4 pole wave wound shunt generator delivers 200 A at a terminal voltage of 250 V. R_a and R_{sh} are 0.05 Ω and 50 Ω respectively. Neglecting brush drop, determine (i) I_a, (ii) current per parallel path, (iii) E_g and (iv) Power developed.*

Solution: Given: Type of winding: Wave winding,

$$A = 2 \qquad p = 4,$$
$$I_L = 200\text{ A},\ V = 250\text{ volt}$$
$$R_a = 0.05\ \Omega$$
and $$R_{sh} = 50\ \Omega$$

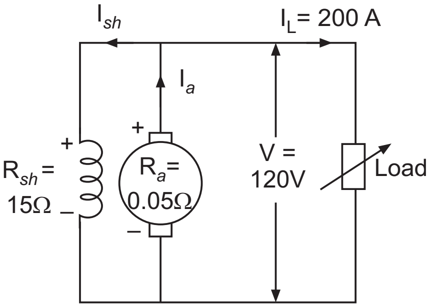


Fig. 7.14 Circuit diagram

(i) Total armature current, $I_a = I_{sh} + I_L$

where $$I_{sh} = \frac{V}{R_{sh}} = \frac{250}{50} = 5\text{A}$$

∴ $$I_a = 5 + 200$$
$$= \mathbf{205\ A}$$

(ii) For wave winding, no. of parallel paths A = 2

Current/parallel path $$= \frac{I_a}{A} = \frac{205}{2}$$
$$= \mathbf{102.5\ A}$$

(iii) Generated emf, $$E_g = V + I_a R_a$$
$$= 250 + (205 \times 0.05)$$
$$= \mathbf{260.25\ V}$$

(iv) Power developed, $$P = E_g \times I_a$$
$$= 260.25 \times 205$$
$$= \mathbf{53351\ W = 53.35\ kW}$$

Example 7.5 *A 4 pole shunt generator with lap connected armature having field and armature resistance of 50 Ω and 0.1 Ω respectively, supplies 60, 110 V 40 W lamps. Calculate the total armature current, current per parallel path and generated emf Allow a contact drop of 1 volt/brush.*

Solution: Given: $p = 4$, Lap winding, $R_{sh} = 50\ \Omega$,

$$R_a = 0.1\ \Omega,$$

Number of lamps = 60,

Wattage of each lamp = 40 W, V = 110 volt

Brush contact drop = 1 volt/brush

Total armature current, $I_a = I_{sh} + I_L$

where $$I_{sh} = \frac{V}{R_{sh}} = \frac{110}{50} = 2.2 \text{ A}$$

and $$I_L = \frac{P}{V}$$

Now P = Number of lamps × Wattage of each lamp

$= 60 \times 40 = 2400$ W

Then $$I_L = \frac{P}{V} = \frac{2400}{110} - 21.8 \text{ A}$$

∴ $I_a = 2.2 + 21.8 =$ **24 A**

For lap winding, no. of parallel paths = Number of poles = 4

Current/parallel path $= \dfrac{I_a}{A} = \dfrac{24}{4}$

= **6 A**

Generated emf, $E_g = V + I_a R_a + V_{cd}$

(Note: A pair of brushes is considered)

$= 110 + (24 \times 0.1) + (1 \times 2) =$ **114.4 V**

Example 7.6 *A 4 pole wave wound 750 rpm dc shunt generator has an armature and field resistance of 0.4 Ω and 200 Ω respectively. The armature has 720 conductors and flux per pole is* 2.895×10^{-2} *weber. If the load resistance is 10 Ω, determine the terminal voltage of generator.*

Solution: Given: $p = 4$,

Type of winding: Wave winding

$N = 750$ rpm $\quad R_a = 0.4\ \Omega$,

$R_{sh} = 200\ \Omega \quad Z = 720$,

$\phi = 2.895 \times 10^{-2}$ Wb, $\quad R_L = 10\ \Omega$

EMF generated, $E_g = V + I_a R_a$

or $V = E_g - I_a R_a$

Now emf generated with wave winding is given by

$$E_g = \frac{\phi NZp}{120} = \frac{\left(2.895 \times 10^{-2}\right) \times 750 \times 720 \times 4}{120} = 521.1 \text{ V}$$

The shunt and load resistances are in parallel (because armature is the source and R_a is the resistance of the source),

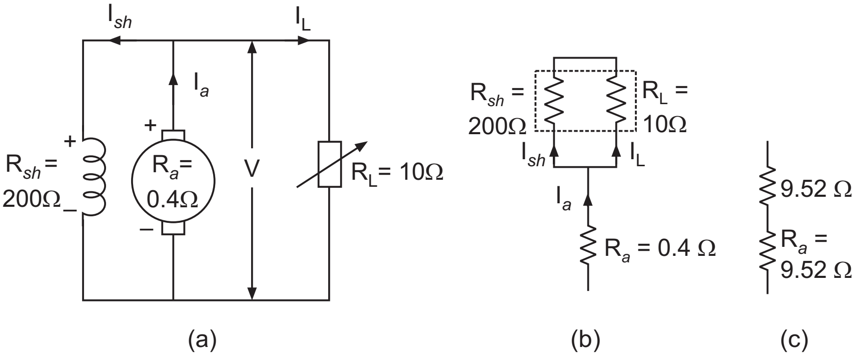


Fig. 7.15 Circuit diagram (a), (b) and (c)

∴ Combined resistance of R_{sh} and R_L $= \frac{R_{sh} \times R_L}{R_{sh} + R_L} = \frac{200 \times 10}{200 + 10}$

$= 9.52\ \Omega$

Now R_a and Combined resistances are in series.

∴ Total resistance, $R_T = R_a$ + combined resistance

$= 0.4 + 9.52$

$= 9.92\ \Omega$

Armature current, $I_a = \frac{E_g}{R_T} = \frac{521.1}{9.92} = 52.53$ A

∴ Terminal Voltage, $V = 521.1 - (52.53 \times 0.4)$

$=$ **500.08 V**

Example 7.7 *A long shunt compound generator delivers a load current of 50 A at 500 V. It has R_a, R_{se} and R_{sh} of 0.05 Ω, 0.03 Ω and 250 Ω respectively. Calculate E_g.*

Solution: Given: Long shunt compound generator

$I_L = 50$ A, $V = 500$ volt

$R_a = 0.05\ \Omega$, $R_{se} = 0.03\ \Omega$

and $R_{sh} = 250\ \Omega$

EMF generated, $E_g = V + I_a R_a + I_{se} R_{se}$

For long shunt, $I_{se} = I_a$

Series field drop, $I_{se} R_{se} = I_a R_{se}$

∴ $E_g = V + I_a R_a + I_L R_{se}$

$I_a = I_{sh} + I_L$

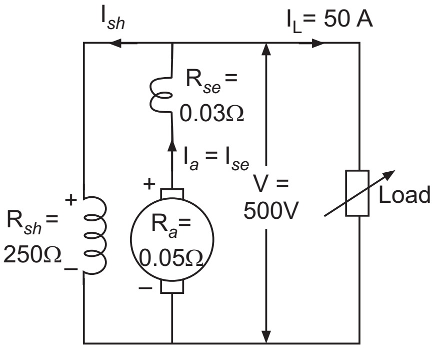


Fig. 7.16 Circuit diagram

where $I_{sh} = \dfrac{V}{R_{sh}} = \dfrac{500}{250} = 2\text{ A}$

Then, $I_a = 2 + 50 = 52\text{ A}$

∴ $E_g = 500 + (52 \times 0.05) + (52 \times 0.03) = \mathbf{504.16\ V}$

Example 7.8 *A short shunt cumulative compound dc generator supplies 7.5 kW at 230 V. The R_{sh}, R_{se} and R_a are 100 Ω, 0.3 Ω and 0.4 Ω respectively. Calculate the induced emf and load resistance.*

Solution: Given: Short shunt compound generator

$P = 7.5\text{ kW} = 7.5 \times 1000\text{ W},$

$V = 230\text{ volt}$

$R_{sh} = 100\ \Omega, \quad R_{se} = 0.3\ \Omega$

and $R_a = 0.4\ \Omega$

EMF generated, $E_g = V + I_a R_a + I_{se} R_{se}$

For long shunt, $I_{se} = I_L$

Series field drop, $I_{se}R_{se} = I_L R_{se}$

∴ $E_g = V + I_a R_a + I_L R_{se}$

Now $I_a = I_{sh} + I_L$

where $I_L = \dfrac{P}{V} = \dfrac{7.5 \times 1000}{230} = 32.6\text{ A}$

Series field drop, $I_L \times R_{se} = 32.6 \times 0.3 = 9.78\text{ V}$

∴ $I_{sh} = \dfrac{V + \text{series field drop}}{R_{sh}} = \dfrac{230 + 9.78}{100} = 2.39\text{ A}$

∴ $I_a = 2.39 + 32.6 = 34.99 = 35\text{ A}$

Thus, $E_g = 230 + (35 \times 0.4) + (32.6 \times 0.3) = \mathbf{253.8\ V}$

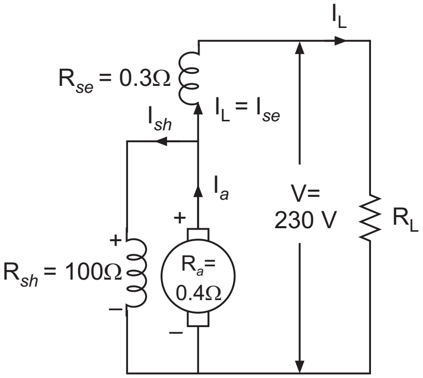


Fig. 7.17 Circuit diagram

Load resistance, $$R_L = \frac{P}{I_L^2} = \frac{7.5 \times 1000}{(32.6)^2} \qquad \text{(Because } P = I_L^2 R_L\text{)}$$

$$= \mathbf{7.05\ \Omega}$$

7.2.6 Concept of Armature Reaction

When the dc generator is supplying a load, its load current flows through the armature conductors and produces a flux which effects the distribution of flux due to main field.

Thus, the effect of armature flux on the distribution flux due to main poles is called armature reaction.

Its effects are

(i) It demagnetizes the main flux or weakens the main flux and

(ii) It cross magnetizes of destroys the main flux.

Thus, armature reaction causes Armature Reaction Drop (ARD). Armature reaction effect exists in dc motors also.

If ARD is given/considered, then emf in terms of terminal voltage will be

$E_g = V + I_aR_a + ARD + V_{cd}$ volt ⇒ for shunt generator

$E_g = V + I_a(R_a + R_{se}) + ARD + V_{cd}$ volt ⇒ for cumulative compound generator

$E_g = V + I_aR_a + I_LR_{se} + ARD + V_{cd}$ volt ⇒ for differential compound generator

7.3 DC MOTOR

7.3.1 Working Principle as a Motor

An electrical motor is a machine, which converts electrical energy into mechanical energy.

Fig. 7.18 DC Motor

It is based on the principle that when a current carrying conductor is placed in a magnetic field, it exerts a mechanical force, whose direction is given by Fleming's Left-hand rule and its magnitude is given by

$$F = B\,I\,l \text{ newton}$$

Figure 7.19(a) represents a conductor placed in the magnetic field. Figure 7.19(b) represents a current carrying conductor placed in the magnetic field. It sets up its own flux. At the left side pole tips or *AB-side,* addition of flux takes place and at the right side pole tips or *CD-side,* cancellation of flux takes place. Thus, gathering of flux takes place at *AB-side* and the current carrying conductor exerts a mechanical force.

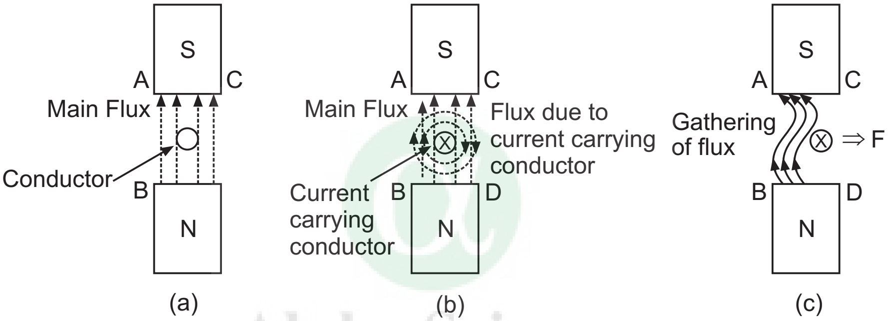


Fig. 7.19 Principle of dc motor

Thus, when dc supply is given to armature, it flows through brushes and commutators, becomes alternating current in armature, thus, develops torque and starts rotating.

7.3.2 Back EMF and its Significance

When the armature carries an armature current (I_a), it rotates in a magnetic field, cuts the flux and hence an emf is induced in it according to Faraday's Law of Electromagnetic Induction, whose direction is given by Fleming's Right hand rule.

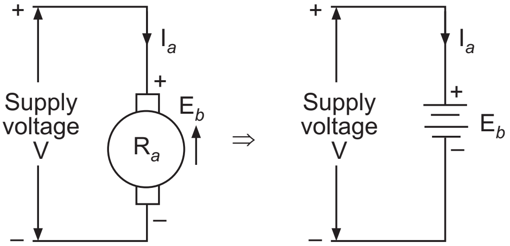


Fig. 7.20 Concept of back emf

It is in opposition to the supply voltage (V) and hence termed as Back emf (E_b). It is equivalent to a battery of emf E_b connected across the supply mains of V volt. Hence voltage has to drive I_a against the opposition offered by back emf E_b.

Significance: Back emf is given by

$$E_b = \frac{\phi NZ}{60} \times \left(\frac{p}{A}\right)$$

$$= K \phi N.$$

or $$E_b \alpha \phi N \qquad ...(7.2)$$

Also, back emf is given by

$$E_b = V - I_a R_a$$

or $$I_a = \frac{V - E_b}{R_a} \qquad ...(7.3)$$

By observing equations (7.2) and (7.3),

(i) If the mechanical load on the motor is reduced, then the armature speed increases, E_b increases and hence I_a decreases.

(ii) If the mechanical load is applied or increases on the motor, then the armature speed decreases, E_b decreases and hence I_a increases.

Hence, back emf E_b acts as 'Governor' and makes a motor self-regulating so that it draws as much armature current I_a as required.

Condition for Maximum Power Developed:

Back emf in terms of supply voltage is given by

$$E_b = V - I_a R_a \text{ volt} \qquad ...(7.4)$$

Multiplying the above equation by I_a both sides,

$$E_b I_a = V I_a - I_a^2 R_a \text{ watt} \qquad ...(7.5)$$

Now, in equation (7.4), $E_b I_a$ = Electrical power converted to Mechanical power or total armature output in watt.

or $$E_b I_a = P_m$$

$$VI_a = \text{Input to the armature, watt}$$

and $$I_a^2 R_a = \text{Armature copper loss, watt}$$

Hence equation (7.5) is rewritten as

$$P_m = VI_a - I_a^2 R_a \text{ watt} \qquad ...(7.6)$$

Differentiating equation (7.6) with respect to I_a,

$$\frac{dP_m}{dI_a} = \frac{d}{d I_a}\left(VI_a - I_a^2 R_a\right)$$

or $$\frac{dP_m}{dI_a} = V - 2 I_a R_a$$

For maximum power developed, $\frac{dP_m}{dI_a} = 0$

i.e., $V - 2 I_a R_a = 0$

or $V = 2 I_a R_a$

or $$I_a R_a = \frac{V}{2} \qquad ...(7.7)$$

Substituting equation (7.7) in equation (7.4),

$$E_b = V - \frac{V}{2}$$

$$= \frac{V}{2}$$

Thus, mechanical power developed by the dc motor is maximum when back emf is equal to half of the supply voltage.

7.3.3 Types of DC Motors

1. DC Series Motor,
2. DC Shunt Motor And
3. DC Compound Motor
    - (i) Cumulative Compound
    - (ii) Differential Compound

Back emf E_b in terms of supply voltage

Let, V = supply voltage,

I_L = input current to load,

I_a = armature current,

I_{sh} = shunt field current

1. **DC Shunt Motor:** Shunt field is connected in parallel with armature.

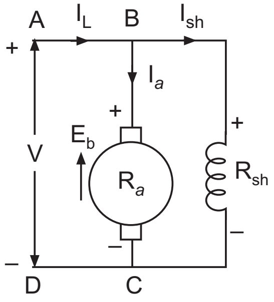


Fig. 7.21 Shunt motor

Applying KCL at point B,

$$I_L = I_a + I_{sh}$$

or $$I_a = I_L - I_{sh}$$

Applying KVL to the loop ABCDA,

$$E_b = V - I_a R_a - V_{cd} \text{ volt}$$

2. **DC Series Motor:** Series field winding and armature are connected in series.

Load current = Series field current = Armature current

i.e., $$I_L = I_{se} = I_a$$

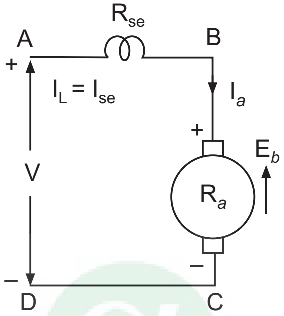


Fig. 7.22 Series motor

Applying KVL to the loop ABCDA,

$$\begin{aligned} E_b &= V - I_a R_a - I_{se} R_{se} - V_{cd} \text{ volt} \\ &= V - I_a (R_a + R_{se}) - V_{cd} \text{ volt} \end{aligned}$$

7.3.4 Speed of DC Motor

In general, back emf, $$E_b \propto \phi N$$

or $$N = \frac{E_b}{\phi}$$

The speed of a dc motor varies as mechanical load. As load increases, speed decreases and vice versa.

Let, $$N_1 = \text{Initial speed so that } N_1 = \frac{E_{b1}}{\phi_1}$$

where $$E_{b1} = V - I_{a1} R_a \text{ and } \phi_1 = \text{initial flux}$$

and $$N_2 = \text{New speed so that } N_2 = \frac{E_{b2}}{\phi_2}$$

where $$E_{b2} = V - I_{a2} R_a$$

and $$\phi_2 = \text{corresponding flux}$$

Taking ratio, $$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\phi_1}{\phi_2}$$

For **Series** motor; flux varies as armature current,

i.e., $\phi_1 \propto I_{a1}$ and $\phi_2 \propto I_{a2}$

$$\therefore \qquad \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{I_{a1}}{I_{a2}}$$

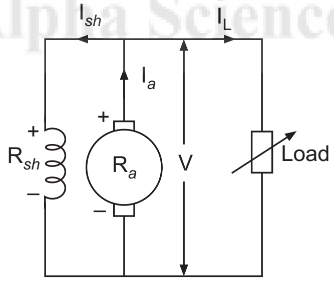
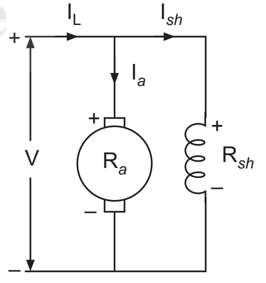
For **Shunt** motor; flux remains approximately constant,

i.e., $\phi_1 \approx \phi_2$

$$\therefore \qquad \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}}$$

7.3.5 Comparison between DC Generator and DC Motor

Table 7.1

Sl. No.	Particulars	DC Generator	DC Motor
1	Energy conversion	M.E→ $\boxed{\text{G}}$ →E.E	E.E→ $\boxed{\text{M}}$ →M.E
2	Fleming's Rules	Fleming's Right-hand Rule	Fleming's Left-hand Rule
3	Power	It generates power	It consumes power
4	Nature of load	Electrical load	Mechanical load
5	Shunt Machine	 $I_a = I_{sh} + I_L$ $E_g = V + I_a R_a$	 $I_a = I_L - I_{sh}$ $E_b = V - I_a R_a$

Solved Examples

Example 7.9 *A 100 kW belt driven shunt generator running at 300 rpm on 220V bus-bar, continues to run as a motor when the belt breaks, taking 10 kW. What will be its speed?*

Given: $R_a = 0.025\Omega$, $R_{sh} = 60\Omega$,

BCD = 1V per brush and ARD = 0

Solution: Given: Shunt machine

When working as generator, $P_g = 100$ kW, $N_g = 300$ rpm, $V = 220$ volt

When working as motor, $P_m = 10$ kW, $R_a = 0.025\ \Omega$ and $R_{sh} = 60\ \Omega$

$$\text{BCD} = V_{cd} = 1 \text{ V/brush}$$

$$\text{ARD} = 0 \text{ (i.e., armature reaction drop = 0)}$$

Let, N_m = speed of m/c when working as motor

E_m = back emf produced when m/c is working as motor

E_g = emf generated when m/c is working as generator

For Shunt m/c (flux remains approximately constant);

$$\frac{N_m}{N_g} = \frac{E_b}{E_g}$$

or $$N_m = \frac{E_b}{E_g} \times N_g$$

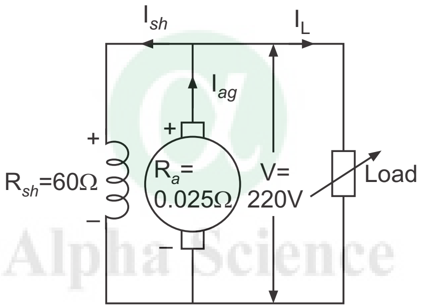


Fig. 7.23 Generator action

When M/C is working as **Generator,**

$$E_g = V + I_{ag} R_a + V_{cd}$$

Armature current, $$I_{ag} = I_{sh} + I_L$$

where $$I_{sh} = \frac{V}{R_{sh}} \times \frac{220}{60} = 3.67 \text{ A}$$

and $$I_L = \frac{P_g}{V} \times \frac{100 \times 1000}{220} = 454.45 \text{ A}$$

Then $$I_{ag} = 3.67 + 454.55 = 458.22 \text{ A}$$

∴ $$E_g = 220 + (458.22 \times 0.025) + (1 \times 2) = 233.46 \text{ V}$$

When M/C is working as **Motor**,

$$E_b = V - I_{am} R_a - V_{cd}$$

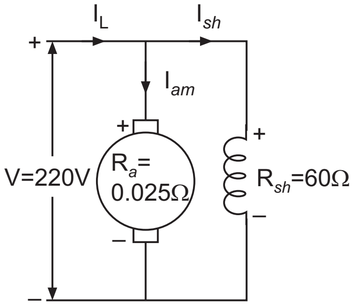


Fig. 7.24 Motor action

and $$I_L = \frac{P_m}{V} = \frac{10 \times 1000}{220} = 45.45 \text{ A}$$

Armature current, $$I_{am} = I_L - I_{sh}$$

$$= 45.45 - 3.67 = 41.78 \text{ A}$$

$$E_b = 220 - (41.78 \times 0.025) - (1 \times 2) = 217.955 \text{ V}$$

∴ $$N_m = \frac{E_m}{E_g} = N_g = \frac{217.955}{233.46} \times 300 = 280 \text{ rpm}$$

Example 7.10 *A 120 V dc shunt motor has $R_a = 0.2\ \Omega$ and $R_f = 60\ \Omega$. It runs at 1800 rpm when it takes full load current of 40 A. Find the speed of the motor when it is operating with half full-load.*

Solution: Given: Shunt motor

$$V = 120 \text{ volt},$$

$$R_a = 0.2\ \Omega$$

and $$R_f = 60\ \Omega$$

Initial speed of motor when running at full-load = N_1 = 1800 rpm

Full load current = I_{L1} = 40 A

Let, N_2 = New speed of motor when running at half full load,

E_{b1} = Back emf developed at full load and

E_{b2} = Back emf developed at half full load

For shunt motor (flux remains approximately constant);

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}}$$

or $$N_2 = \frac{E_{b2}}{E_{b1}} N_1$$

Back emf, $$E_{b1} = V - I_{a1} R_a$$

Now $$I_{a1} = I_{L1} - I_f$$

where $I_f = \dfrac{V}{R_f} = \dfrac{120}{60} = 2\text{ A}$

Then $I_{a1} = 40 - 2 = 38\text{ A}$

∴ $E_{b1} = 120 - (38 \times 0.2) = 112.4\text{ V}$

Back emf $E_{b2} = V - I_{a2}R_a$

Now $I_{a2} = I_{L2} - I_f$

where $I_{L2} = \dfrac{I_{L1}}{2} = \dfrac{40}{2} = 20\text{ A}$

Then $I_{a2} = 20 - 2 = 18\text{ A}$

$E_{b2} = 120 - (18 \times 0.2) = 116.4\text{ V}$

∴ $N_2 = \dfrac{116.4}{112.4} \times 1800 = \mathbf{1864\ rpm}$

Example 7.11 *A 4 pole, 500 V shunt motor has 720 wave connected conductors on its armature. The full load armature current is 60 A and flux per pole is 0.03 weber. The armature resistance is 0.2Ω and contact drop is 1 volt/brush. Calculate full-load speed of motor.*

Solution: Given: Shunt motor

$p = 4$, V = 500 volt, Z = 720, wave connected

$I_a = 60$ A (full-load armature current), $\phi = 0.03$ Wb.

$R_a = 0.2\ \Omega$, BCD = $V_{bcd} = 1$ volt/ brush

Back emf, $E_b = \dfrac{\phi NZ}{60} \times \left(\dfrac{p}{A}\right)$

or $E_b = \dfrac{\phi NZp}{120}$ (Because for wave winding, A = 2)

or $N = \dfrac{E_b \times 120}{\phi Zp}$

Also, back emf, $E_b = V - I_a R_a - BCD$

$= 500 - (60 \times 0.2) - (1 \times 2)$

$= 486\text{ V}$

∴ $N = \dfrac{486 \times 120}{0.03 \times 720 \times 4}$ **= 675 rpm**

Example 7.12 *A series motor runs at 600 rpm when taking 110A from a 250V supply. The resistance of the armature circuit is 0.12Ω and that of series winding is 0.03Ω. The useful flux per pole for 110A is 0.024Wb and that for 50A is 0.0155 Wb. Calculate the speed when the current has fallen to 50A.*

Solution: Given: Series motor

Initial speed $N_1 = 600$ rpm,

Initial current, $I_{a1} = 110$ A $V = 250$ volt,

$R_a = 0.12\ \Omega$ and $R_{se} = 0.03\ \Omega$

Initial flux, $\phi_1 = 0.024$ Wb.

Final current, $I_{a2} = 50$A,

Corresponding flux, $\phi_2 = 0.0155$ Wb

Let N_2 be the final speed corresponding to $I_{a2} = 50$A

$$\therefore \quad \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\phi_1}{\phi_2}$$

$$\text{or} \quad N_2 = \frac{E_{b2}}{E_{b1}} \times \frac{\phi_1}{\phi_2} \times N_1$$

Initial back emf, $E_{b1} = V - I_{a1}(R_a + R_{se})$

$= 250 - 110 \times (0.12 + 0.03)$

$= 233.5$ V

Final back emf, $E_{b2} = V - I_{a2}(R_a + R_{se})$

$= 250 - 50 \times (0.12 + 0.03)$

$= 242.5$ V

$$N_2 = \frac{242.5}{233.5} \times \frac{0.024}{0.0155} \times 600 = \mathbf{965\ rpm}$$

7.3.6 Torque

Definition: It is the turning or twisting moment of a force about an axis and measured by the product of the force F and radius *r* at which the force acts.

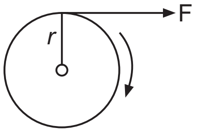


Fig. 7.25

i.e., Torque developed, $T = F \times r$ newton-m

$\therefore$ Work done/second $= T \times \omega$ joule/second

or Power developed $= T \times \omega$ watt

where ω = angular velocity

$= 2\pi N$ radians/second,

N = speed in revolutions per second

Types:

(i) Armature torque or total torque (T_a)

(ii) Shaft torque (T_{sh})

1. **Armature Torque or Total Torque (T_a):** This is the torque developed by the entire armature but not available for doing useful work, because certain percentage is required for supplying Iron and Friction losses.

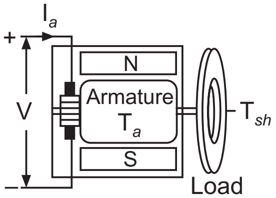


Fig. 7.26 DC Motor

Expression for Armature Torque: Let T_a be the torque developed by the armature of dc motor running at (N/60) rps, with an angular velocity of $\omega = 2\pi\,(N/60)$ radians/second.

Then Power developed by the armature due to armature torque is

$$P_d = T_a \times \omega$$

or

$$P_d = T_a \times \frac{2\pi N}{60} \text{ watt} \qquad ...(7.8)$$

Also, Electrical Power equivalent to Mechanical Power is

$$P_m = E_b I_a \text{ watt} \qquad ...(7.9)$$

where E_b = back emf,

I_a = armature current

Equating equations (7.8) and (7.9),

$$T_a \times \frac{2\pi N}{60} = E_b I_a$$

or

$$T_a = \frac{E_b I_a \times 60}{2\pi N} \text{ N-ms} \qquad ...(7.10)$$

$$= \frac{1}{9.81} \frac{E_b I_a \times 60}{2\pi N} \text{ kg-m}$$

Also, back emf, $E_b = \dfrac{\phi N Z}{60} \times \dfrac{p}{A}$

Substituting for E_b in equation (7.10),

$$T_a = \frac{I_a \times 60}{2\pi N} \times \frac{\phi N Z}{60} \times \left(\frac{p}{A}\right)$$

$$= \frac{1}{2\pi} \times \frac{\phi Z p}{A} \times I_a \text{ N-m} \qquad ...(7.11)$$

or $$T_a = K\,\phi\,I_a \text{ where } K = \frac{1}{2\pi} \times \frac{Zp}{A}$$

or $$T_a \propto \phi\,I_a$$

Hence, total or armature torque developed is directly proportional to the product of flux/pole and armature current.

For **series** motor, $\phi \propto I_a$

$$T_a \propto I_a^2$$

For **shunt** motor, ϕ remains approximately constant and therefore,

$$T_a \propto I_a$$

2. **Shaft Torque (T_{sh}):** This is available at the shaft for doing the useful work and hence also called Useful Torque.

   Expression for Shaft Torque: The power obtained due to shaft torque is called as BHP (Brake Horse Power).

   [Note: 1 HP = 735.49875 W = ≈ 735.5 W (metric) and 747 W (British).]

Now, $$\text{Output of Motor} = T_{sh} \times \frac{2\,\pi\,N}{60} \text{ watt}$$

or $$T_{sh} = \frac{\text{Output of Motor in watt}}{2\pi\,(N/60)} \text{ N-m}$$

or $$T_{sh} = \frac{\text{BHP} \times 735.5}{2\pi\,(N/60)} \text{ N-m} \qquad ...(7.12)$$

If T_f = Torque due to Iron and Friction loss,

Then $$T_f = \frac{\text{Iron and Friction loss in watt}}{2\pi\,(N/60)}$$

Then $$T_{sh} = T_a - T_f \qquad ...(7.13)$$

∴ Lost torque, $$T_a - T_{sh} = \frac{\text{Rotational losses watt}}{2\pi\,(N/60)} \qquad ...(7.14)$$

Solved Examples

Example 7.13 *A 4 pole, 220 V lap connected dc shunt motor has 36 slots, each containing 16 conductors. It draws a current of 40 A from the supply. The field resistance and armature resistance are 110 Ω and 0.1 Ω respectively. The motor develops an output power of 6 kW. The flux per pole is 40 mWb. Calculate (a) the speed (b) torque developed by the armature and (c) the shaft torque.*

Solution: Given: Shunt motor

$p = 4$, V = 220 volt, lap connection, No. of slots = 36, conductors/slot = 16,

$I_L = 40A$, $R_{sh} = 110\ \Omega$, $R_a = 0.1\ \Omega$, $P = 6$ kW and $\phi = 40$ mWb.

(a) To find the speed N:

For lap winding, $$E_b = \frac{\phi N Z}{60}$$

or $$N = \frac{E_b \times 60}{\phi Z} \qquad \text{...(7.15)}$$

Now $$E_b = V - I_a R_a$$

where $$I_a = I_L - I_{sh}$$

$$I_{sh} = \frac{V}{R_{sh}} = \frac{220}{110} = 2\text{ A}$$

Then $$I_a = 40 - 2 = 38\text{ A}$$

∴ $$E_b = 220 - (38 \times 0.1) = 216.2\text{ V}$$

Substituting the value of E_b in equation (7.15),

$$N = \frac{216.2 \times 60}{40 \times 10^{-3} \times (36 \times 16)} = \mathbf{563\ rpm}$$

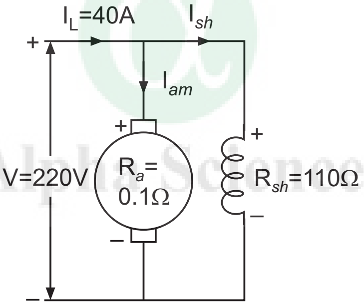


Fig. 7.27 Motor action

(b) Total torque, $$T_a = \frac{E_b I_a \times 60}{2\pi N}\text{ N-m}$$

$$= \frac{216.2 \times 38 \times 60}{2\pi \times 563} = \mathbf{139.42\ N\text{-}m}$$

(c) Shaft torque, $$T_{sh} = \frac{\text{Output of Motor in watt}}{2\pi\left(\frac{N}{60}\right)}$$

$$= \frac{6 \times 1000}{2\pi\left(\frac{563}{60}\right)} = \mathbf{101.77\ N\text{-}m}$$

Example 7.14 *A 240 V, 4 pole shunt motor running at 1000 rpm gives 15 HP with an armature current of 50 A. The armature winding is wave-connected and has 540 conductors. Its resistance is 0.1 Ω and drop at each brush is 1 V. Find (i) useful torque, (ii) total torque, (iii) useful flux per pole and (iv) rotational losses.*

Solution: Given: Shunt motor

$$p = 4, \quad V = 240 \text{ volt}, N = 1000 \text{ rpm},$$

$$P = 15 \text{ HP}, \quad I_a = 50 \text{ A},$$

$$Z = 540, \text{ wave wound},$$

$$R_a = 0.1\ \Omega, \quad V_b = 1 \text{ V/brush}$$

(i) Useful torque,

$$T_{sh} = \frac{\text{Output of Motor in watt}}{2\pi\left(\frac{N}{60}\right)}$$

$$= \frac{15 \times 735.5}{2\pi\left(\frac{1000}{60}\right)} = \mathbf{105.35\ N\text{-}m}$$

Total torque,

$$T_a = \frac{E_b I_a \times 60}{2\pi\ N} \text{ N-m}$$

where

$$E_b = V - I_a R_a - V_b$$

$$= 240 - (50 \times 0.1) - (1 \times 2) = 233 \text{ V}$$

$$\therefore \quad T_a = \frac{233 \times 50 \times 60}{2\pi 1000} = \mathbf{111.249\ N\text{-}m}$$

(iii) Back emf,

$$E_b = \frac{\phi N Zp}{120} \quad \text{(Because for wave winding, A = 2)}$$

or

$$\phi = \frac{120\ E_b}{N\ Zp} = \frac{120 \times 233}{1000 \times 540 \times 4} = \mathbf{12.95\ mWb}$$

(iv)

$$(T_a - T_{sh}) = \frac{\text{Rotational losses in watt}}{2\pi\ \frac{N}{60}}$$

or

$$\text{Rotational losses} = \left(T_a - T_{sh}\right) \times 2\pi\frac{N}{60} = \left(111.249 - 105.35\right) \times 2\pi\frac{1000}{60}$$

$$= \mathbf{617.74\ W}$$

Example 7.15 *A 220V dc Shunt motor runs at 500 rpm taking an armature current of 50 A. Find the speed if the torque is doubled.* R_a *is 0.2 Ω.*

Solution: Given: Shunt motor

$$V = 220 \text{ volt}$$

$$\text{Initial speed} = N_1 = 500 \text{ rpm}$$

Initial armature current = $I_{a1} = 50$ A

$R_a = 0.2\ \Omega$

Let, T_{a1} = Torque corresponding to speed N_1

and E_{b1} = Corresponding back emf

N_2 = New speed when torque is doubled,

T_{a2} = Torque corresponding to speed N_2

E_{b2} = Corresponding back emf

i.e., T_{a2} and back emf E_{b2}

For shunt motor, as flux remains approximately constant,

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}}$$

or
$$N_2 = \frac{E_{b2}}{E_{b1}} N_1 \qquad \text{...(7.15)}$$

Now $E_{b1} = V - I_{a1}$

$R_a = 220 - (50 \times 0.2) = 210$ V

$E_{b2} = V - I_{a2}\, R_a$

To find I_{a2}

As per the data given, $\dfrac{T_{a2}}{T_{a1}} = 2$

i.e.,
$$\frac{T_{a2}}{T_{a1}} = \frac{I_{a2}}{I_{a1}} = 2$$

or $I_{a2} = 2\, I_{a1} = 2 \times 50 = 100$ A

$\therefore$ $E_{b2} = 220 - (100 \times 0.2)$

$= 200$ V

Substituting E_{b1}, E_{b2} and N_1 in eqn. (7.15),

$$N_2 = \frac{200}{210} \times 500 = 476.19$$

$$\approx \mathbf{476\ rpm}$$

Example 7.16 *A 120 V dc shunt motor has an armature resistance of 0.2 Ω and a field resistance of 60 Ω. It runs at 1800 rpm taking a full load current of 40 A. Find the speed on half load condition.*

Solution: Given: Shunt motor

$V = 120$ volt $\quad R_a = 0.2\ \Omega,\ R_{sh} = 60\ \Omega$

Initial speed = $N_1 = 1800$ rpm

Initial full load current = $I_{L1} = 40$ A

Let, T_{a1} = Torque corresponding to speed N_1

and E_{b1} = Corresponding back emf

N_2 = New speed at half load,

T_{a2} = Torque corresponding to speed N_2 and

E_{b2} = Corresponding back emf

For shunt motor (flux remains approximately constant);

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}}$$

or $$N_2 = \frac{E_{b2}}{E_{b1}} N_1 \qquad ...(7.16)$$

Now $E_{b1} = V - I_{a1} R_a$

where $I_{a1} = I_{L1} - I_{sh}$

Then $I_{a1} = 40 - 2 = 38$ A

∴ $E_{b1} = 120 - (38 \times 0.2) = 112.4$ V

$E_{b2} = V - I_{a2} R_a$

As per the data given, $\frac{T_{a2}}{T_{a1}} = \frac{1}{2}$

i.e., $$\frac{T_{a2}}{T_{a1}} = \frac{I_{a2}}{I_{a1}} = \frac{1}{2}$$

or $$I_{a2} = \frac{1}{2} I_{a1} = \frac{1}{2} \times 38 = 19 \text{ A}$$

∴ $E_{b2} = 120 - (19 \times 0.2) = 116.2$ V

Substituting E_{b1}, E_{b2} and N_1 in eqn. (7.16),

$$N_2 = \frac{116.2}{112.4} \times 1800 = 1860.85$$

≈ 1861 rpm

Example 7.17 *A 6 pole dc shunt motor has a lap connected armature with 492 conductors. The resistance of the armature is 0.2 Ω and the flux per pole is 50 mWb. The motor runs at 20 revolutions per second, when it is connected to 500V supply for a particular load. What will be the speed of the motor, when the load is reduced by 50%. Neglect contact drop and magnetic saturation.*

Solution: Given: Shunt motor

p = 6, type of winding: lap winding, Z = 492,

$R_a = 0.2\Omega$, $\phi = 50$ mWb.,

Initial speed, N_1 = 20 rps, V = 500 volt, load is reduced by 50%,

$V_{cd} \Rightarrow$ Ignored

Let, E_{b1} = Back emf corresponding to the initial speed N_1,

N_2 = Speed at 50% of a particular load,

and E_{b2} = Corresponding back emf

For Shunt motor (flux remains approximately constant);

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}}$$

or
$$N_2 = \frac{E_{b2}}{E_{b1}} N_1 \qquad \text{...(7.17)}$$

For lap winding, $E_{b1} = \phi NZ$ (Note: N is given in rps)

$= (50 \times 10^{-3}) \times 20 \times 492 = \mathbf{492\ V}$

Also, $E_{b1} = V - I_{a1} R_a$

i.e., $492 = 500 - (I_{a1} \times 0.2)$

or $I_{a1} = 40$ A

Given that load is reduced by 50%,

i.e.,
$$I_{a2} = \frac{I_{a1}}{2} = \frac{40}{2} = 20\ \text{A}$$

$\therefore$
$$E_{b2} = V - I_{a2} R_a$$
$$= 500 - (20 \times 0.2)$$
$$= 496\ \text{V}$$

Substituting $E_{b1,}$ E_{b2} and N_1 in eqn. (7.17),

$$N_2 = \frac{496}{492} \times 20\ \text{rps} \approx \mathbf{20\ rps}$$

Example 7.18 *A 440V dc shunt motor takes an armature current of 20A and runs at 500 rpm. The armature resistance is 0.6Ω. If the flux density is reduced by 30% and torque is increased by 40%, what are the new values of armature current and speed?*

Solution: Given: Shunt motor

V = 440V, initial armature current, I_{a1} = 20A,

initial armature speed, N_1= 500 rpm, $R_a = 0.6\Omega$

Let, E_{b1} = initial back emf

ϕ_1 = initial flux

T_{a1} = initial armature torque

N_2 = new speed

E_{b2} = corresponding back emf

With variation in flux, $\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\phi_1}{\phi_2}$

or $$N_2 = \frac{E_{b2}}{E_{b1}} \times \frac{\phi_1}{\phi_2} \times N_1 \qquad \text{...(7.18)}$$

$$E_{b1} = V - I_{a1} R_a$$
$$= 440 - (20 \times 0.6) = 428 \text{ V}$$
$$E_{b2} = V - I_{a2} R_a$$

To find I_{a2}:

$$T_{a1} \propto \phi_1 I_{a1}$$
$$T_{a2} \propto \phi_2 I_{a2}$$

$\therefore$ $$\frac{T_{a2}}{T_{a1}} = \frac{\phi_2 I_{a2}}{\phi_1 I_{a1}} \qquad \text{...(7.19)}$$

Given that flux is reduced by 30%,

i.e., $\phi_2 = (100\% - 30\%)\ \phi_1 = 70\%,\ \phi_1 = 0.7\ \phi_1$

When torque is increased by 40%,

$$\frac{T_{a2}}{T_{a1}} = 100\% + 40\% = 140\% = 1.4$$

Substituting these values in equation (7.19),

$$1.4 = \frac{0.7\phi_1 I_{a2}}{\phi_1 \times 20}$$

or $I_{a2} = 40\text{A}$

$\therefore$ $E_{b2} = 440 - (40 \times 0.6) = 416 \text{ V}$

Substituting all the values in equation (7.18),

$$N_2 = \frac{416}{428} \times \frac{\phi_1}{0.7\phi_1} \times 500$$
$$\approx \mathbf{694\ rpm}$$

Example 7.19 *A 220 V, dc series motor is running at a speed of 800 rpm and draws 100 A. Calculate the speed at which the motor will run when developing half of the torque. Total resistance of the armature and field is 0.1. Assume that the magnetic circuit is unsaturated.*

Solution: Given: Series motor

$V = 220 \text{ V},$

Initial speed $N_1 = 800 \text{ rpm}$

Initial current $I_{a1} = 100 \text{ A}$

$R_a + R_{se} = 0.1\ \Omega$

Let, T_{a1} = torque corresponding to speed N_1

E_{b1} and ϕ_1 = corresponding back emf and flux

N_2 = New speed at half of the torque T_{a2}

E_{b2} and ϕ_2 be the corresponding back emf and the flux

For Series motor, $\phi_1 \; \alpha \; I_{a1}$ and $\phi_2 \; \alpha \; I_{a2}$

$$\therefore \qquad \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{I_{a1}}{I_{a2}}$$

$$\text{or} \qquad N_2 = \frac{E_{b2}}{E_{b1}} \times \frac{I_{a1}}{I_{a2}} N_1 \qquad \text{...(7.20)}$$

where $E_{b1} = V - I_{a1}$

$R_T = 220 - (100 \times 0.1) = 210 \text{ V}$

$E_{b2} = V - I_{a2} R_T$

In general for series motor, $T_a \; \alpha \; I_a^{\,2}$

As per the data given, $\dfrac{T_{a2}}{T_{a1}} = \dfrac{1}{2}$

$$\text{i.e.,} \qquad \frac{T_{a2}}{T_{a1}} = \frac{I_{a2}^2}{I_{a1}^2} = \frac{1}{2}$$

$$\text{or} \qquad I_{a2}^2 = \frac{1}{2} I_{a1}^2 = \frac{1}{2} \times (100)^2$$

$$\text{or} \qquad I_{a2} = \frac{100}{\sqrt{2}} = 70.7 \text{ A}$$

$$E_{b2} = (220 - 70.7 \times 0.1) = 212.93 \text{ V}$$

Substituting E_{b1}, E_{b2} I_{a1}, I_{a2} and N_1 in equation (7.20),

$$N_2 = \frac{212.93}{210} \times \frac{100}{70.7} \times 800$$

$$= 1147.3 \approx \mathbf{1147 \text{ rpm}}$$

Example 7.20 *A 460 V series motor runs at 500 rpm taking a current of 40 A. Calculate the speed if the load is reduced so that motor is taking 30 A. The total armature and field circuit resistance is 0.8 Ω. Assume flux and field current to be proportional.*

Solution: Given: Series motor

$V = 460$ V, Initial speed $N_1 = 500$ rpm, initial current

$I_{a1} = 40 \text{ A}$ $R_a + R_{se} = 0.8 \; \Omega$

Let, T_{a1} = torque corresponding to speed N_1

E_b and ϕ_1 = corresponding back emf and flux

N_2 = New speed

and T_{a2} = Corresponding torque

E_{b2}, ϕ_2 = corresponding back emf and flux

For Series motor, $\phi_1 \alpha I_{a1}$ and $\phi_2 \alpha I_{a2}$

$$\therefore \quad \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{I_{a1}}{I_{a2}}$$

$$\text{or} \quad N_2 = \frac{E_{b2}}{E_{b1}} \times \frac{I_{a1}}{I_{a2}} N_1 \qquad \text{...(7.21)}$$

where $E_{b1} = V - I_{a1}$

$R_T = 460 - (40 \times 0.8) = 428\text{V}$

$E_{b2} = V - I_{a2} R_T = 460 - (30 \times 0.8) = \mathbf{436\ V}$

Substituting E_{b1}, E_{b2} I_{a1}, I_{a2} and N_1 in equation (7.21),

$$N_2 = \frac{436}{428} \times \frac{40}{30} \times 500 = \mathbf{679\ rpm}$$

7.3.7 Motor Characteristics and Applications

1. Armature Torque v/s Armature Current (T_a v/s I_a) or Electrical Characteristics
2. Speed v/s Armature Current (N v/s I_a)
3. Speed v/s Armature Torque (N v/s T_a) or Mechanical Characteristics

Series Motor

1. **Torque v/s Armature Current (T_a v/s I_a) or Electrical Characteristics:** For Series motor, $T_a \propto I_a^2$ (because flux $\phi \propto I_a$). Therefore, as I_a increases, T_a increases as square of I_a. Hence, the curve is a parabola, represented by ob.

   After the saturation of poles, flux ϕ remains approximately constant,

   i.e., $T_a \propto I_a$

   The curve is a straight line, represented by bc. Therefore, dc series motor exerts huge starting torque and thus can be started with heavy mechanical load.

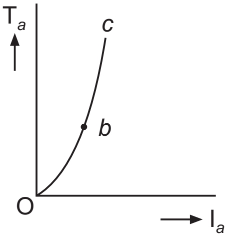


Fig. 7.28 T_a v/s I_a

2. **Speed v/s Armature Current (N v/s I_a):** Speed is given by

$$N \propto \frac{E_b}{\phi}$$

If change in E_b is small and can be ignored,

$$N \propto \frac{1}{\phi}$$

For Series motor, $\phi \propto I_a$

Therefore, as I_a increases flux increases and hence speed decreases or vice versa. Thus, a dc series motor is a variable speed motor.

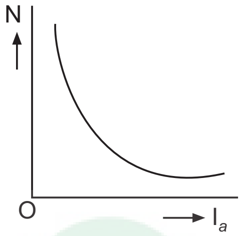


Fig. 7.29 N v/s I_a

It is important that a dc series motor should never be started without mechanical load. Because without mechanical load, I_a is small, then flux is small and thus motor develops excessive speed, even to a dangerous value.

3. **Speed v/s Torque (N vs T_a) or Mechanical Characteristics:** It is noticed from the above characteristics, when speed is high, torque is small and vice versa.

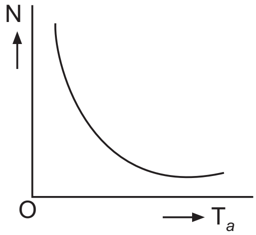


Fig. 7.30 N v/s T_a

Summary of characteristics:

(i) High starting torque.

(ii) Variable speed.

(iii) Adjustable variable speed.

Applications of series motor:

1. Traction work (that is, Electric trains).
2. Cranes and Hoists.
3. Conveyors.
4. Rapid transit system.

Shunt Motor

1. **Torque v/s Armature Current (T_a v/s I_a) or Electrical Characteristics:**

   For Shunt motor, $T_a \propto I_a$ (because flux ϕ = approximately constant).

   Therefore as I_a increases, T_a increases proportionally. Thus the curve is a straight line, that is, shunt motor exerts medium starting torque.

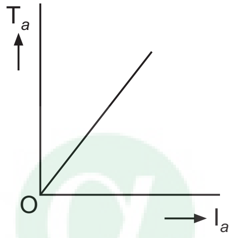


Fig. 7.31 T_a v/s I_a

   It is important that, as starting load requires heavy starting current, dc shunt motor should never be started on heavy loads.

2. **Speed v/s Armature Current (N/I_a):** Speed is given by

$$N \propto \frac{E_b}{\phi}$$

   For shunt motor, flux remains approximately constant (because I_{sh} is approximately constant).

$\therefore$ $$N \propto E_b$$

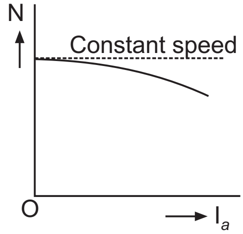


Fig. 7.32 N v/s I_a

If change in E_b is small and can be ignored, then speed should remain approximately constant.

But, in practice $E_b = V - I_a R_a$ volt

Therefore, as I_a increases, armature resistance drop ($I_a R_a$) increases and hence E_b decreases. As E_b decreases the speed decreases and hence the curve slightly drops. Thus, shunt motor is considered as a constant-speed motor.

3. **Speed v/s Torque (N/T_a) or Mechanical Characteristics:**

It is noticed from the above characteristics, as T_a increases, speed slightly decreases.

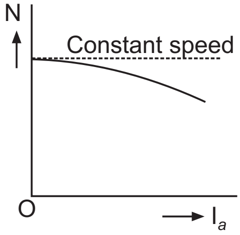


Fig. 7.33 N v/s T_a

Summary of characteristics:

(i) Medium starting torque.

(ii) Approximately constant speed or slightly drooping speed.

(iii) Adjustable speed.

Applications of series motor:

1. Driving constant speed line shafting.
2. Centrifugal and Reciprocating pumps.
3. Lathes.
4. Fans and Blowers.

Compound Motor

(i) Cumulative Compound Motor

1. **Torque v/s Armature Current (T_a v/s I_a) or Electrical Characteristics:**

For Cumulative compound motor, $T_a \propto \phi_T I_a$

where total flux, $\phi_T = \phi_{se} + \phi_{sh}$

Now flux due to series field winding, $\phi_{se} \propto I_a$ and flux due to shunt field winding ϕ_{sh} remains constant.

Therefore, as I_a increases, ϕ_{se} increases, ϕ_T increases and T_a also increases. Due to presence of series field winding, it exerts high starting torque, less than series motor and more than shunt motor.

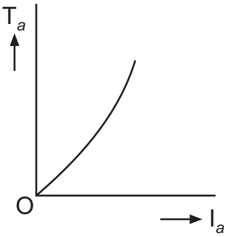


Fig. 7.34 T_a v/s I_a

2. **Speed v/s Armature Current (N/I_a):**

   Speed is given by

$$N \propto \frac{E_b}{\phi}$$

   If change in E_b is small and can be ignored

$$N \propto \frac{1}{\phi_T}$$

   For Cumulative compound motor,

   Total flux, $\phi_T = \phi_{se} + \phi_{sh}$

   Now flux due to series field winding,

$$\phi_{se} \propto I_a$$

   Flux due to shunt field winding ϕ_{sh} remains constant.

   Therefore, as I_a increases, ϕ_{se} increases, ϕ_T increases and hence speed decreases and a cumulative compound motor is a variable speed motor.

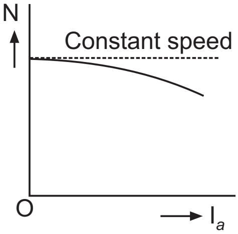


Fig. 7.35 N v/s I_a

3. **Speed v/s Armature Torque (N v/s T_a) or Mechanical Characteristics:**

   As series excitation helps shunt excitation, the characteristic lies in between shunt and series motors.

Summary of characteristics:

(i) High starting torque (due to the presence of series field)

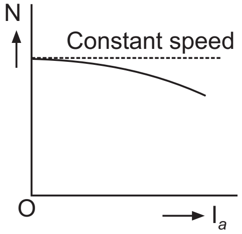


Fig. 7.36 N v/s T_a

(ii) Variable speed.

(iii) Adjustable speed.

Applications of cumulative compound motors:

1. Intermittent high starting torque loads.
2. Shearing and punching.
3. Elevators and lifts.
4. Rolling mills.
5. Heavy machine tools.

(ii) Differential Compound Motor:

1. Torque v/s Armature Current (T_a v/s I_a) or Electrical Characteristics:

For Differential compound motor, $T_a \propto \phi_R I_a$

where resultant flux, $\phi_R = \phi_{se} \sim \phi_{sh}$

Now flux due to series field winding, $\phi_{se} \propto I_a$ and flux due to shunt field winding ϕ_{sh} remains constant.

Therefore, as I_a increases, ϕ_{se} increases, ϕ_R increases and hence T_a increases, but not rapidly.

Thus, a differential compound motor has low starting torque.

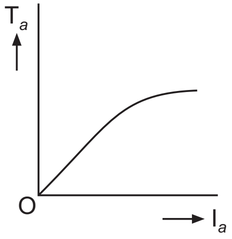


Fig. 7.37 T_a v/s I_a

2. Speed v/s Armature Current (N/I_a): Speed is given by

$$N \propto \frac{E_b}{\phi}$$

If change in E_b is small and can be ignored, thus

$$N \propto \frac{1}{\phi_R}$$

For differential compound motor, resultant flux $\phi_R = \phi_{se} \sim \phi_{sh.}$

Now flux due to series field winding $\phi_{se} \propto I_a$ and flux due to shunt field winding ϕ_{sh} remains constant.

Therefore, as I_a increases, there will be overall decrease in flux and hence speed increases slightly.

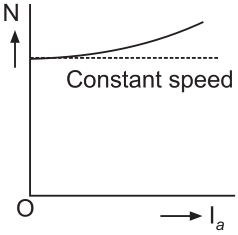


Fig. 7.38 N v/s I_a

3. **Speed vs Armature Torque (N/T_a) or Mechanical Characteristics:**

   As T_a increases, speed increases slightly. However, in practice as speed remains approximately constant, torque remains approximately constant

   Summary of characteristics:

   (i) Low starting torque and

   (ii) Slight increase in speed.

Applications of differential compound motors: Because of its characteristics, differential compound motor is not commonly used. However, it is used for research work, in laboratories and for special purposes.

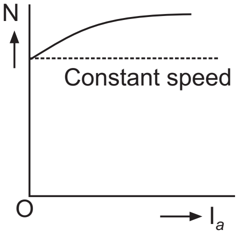


Fig. 7.39 N v/s T_a

7.3.8 Necessity of Starter

Let, V = Terminal voltage, volt

I_a = Armature current, A

$$R_a = \text{Armature resistance, } \Omega$$

Back emf, $$E_b = V - I_a R_a$$

or $$I_a = \frac{V - E_b}{R_a}$$

When the motor is at rest, $E_b = 0$.

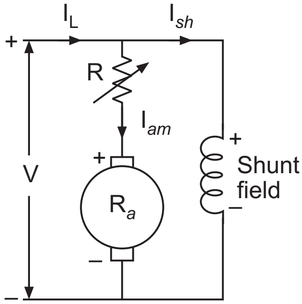


Fig. 7.40 Starter

Hence if full voltage is applied across the armature, it draws large armature current because armature resistance is very small. This excessive armature current will blow out the fuses and dangerous to the armature circuit.

To limit the excessive initial armature current to a safe value at the time of starting, an external or starting variable resistance is connected in series with the armature circuit.

With external or starting resistance R in armature circuit,

$$I_a = \frac{V - E_b}{R_a}$$

This starting resistance is gradually decreased. As the motor gains speed, develops back emf which then regulates armature current. This method is also used for speed control of dc shunt motor.

Three point starter is commonly used.

Three-Point Starter:

Construction:

It consists of

1. A series resistance in the form of studs and a brass arc.
2. A moving starter arm on which a soft Iron piece is attached.
3. A 'Holding coil' or 'No-Voltage release coil' in series with field winding.
4. An 'Over Load Release Coil' in series with the motor.

Working: The supply switch is closed and the starter arm is moved towards right slowly. As soon as the starter arm comes in contact with first stud of starting resistance and Brass arc, supply voltage is applied to shunt field winding and the armature. Thus, the motor starts rotating. At this instant, full external resistance is included in the armature circuit.

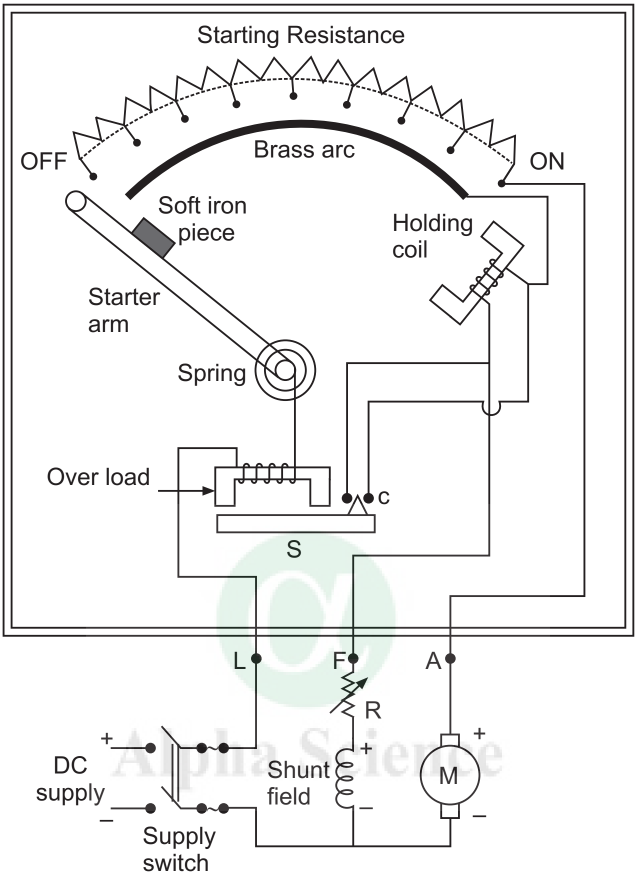


Fig. 7.41 Three-point starter

As the starter arm is further moved gradually on the studs, the resistance decreases gradually and hence the speed of the motor increases.

In ON position, the starting resistance is fully decreased (called cut-out position), the motor reaches almost its rated speed. In this position, the No-voltage release coil or Holding coil becomes magnetized, attracts the soft Iron piece attached on the arm in ON position against the spring action. If the supply voltage fails or disconnection in the field circuit occurs, No-voltage release coil becomes demagnetized, could not attract the soft Iron piece and hence the arm comes back to OFF position due to spring action.

If the motor is overloaded, large current passes through Over-load release coil, it becomes magnetized, attracts soft Iron piece 'S' and shorts contacts 'C' and thus shorts No-voltage release coil. Hence it releases the arm which comes back to OFF position due to spring action.

Multiple Choice Questions

Choose the Correct Answer

1. By applying Fleming's right hand rule to an electric generator, you can find
   (a) direction of magnetic field (b) direction of induced emf
   (c) direction of rotor motion (d) law of induction
2. The yoke of a dc generator is made of ......
   (a) silicon steel (b) soft iron (c) wrought iron (d) cast steel
3. The armature of a dc generator is made of
   (a) soft iron (b) wrought iron (c) silicon steel (d) cast steel
4. The armature of a dc generator is laminated to reduce
   (a) eddy current loss (b) hysteresis loss
   (c) copper loss (d) friction loss
5. Commutator in dc machine is made up of ......
   (a) iron segments
   (b) copper segments
   (c) both iron laminations and copper segments
   (d) none of these
6. The function of a commutator in dc generator is
   (a) to collect current from conductors
   (b) to change dc to ac
   (c) to conduct the current to brushes
   (d) to change ac to dc
7. The brushes in a dc generator are made of carbon because
   (a) they are cheap
   (b) their resistance is high
   (c) they lubricate and polish the commutator
   (d) they are the good conductors
8. Carbon brushes are used in dc machines because
   (a) they have longer life (b) they reduce armature reaction
   (c) they have small resistance (d) they reduce sparking
9. High voltage generators use ...... winding
   (a) lap (b) wave
   (c) either lap or wave (d) none of these

10. For a ‘*p*’ pole lap wound armature of dc machines, the number of parallel paths are equal to ....

    (a) 2 (b) $2p$ (c) p (d) $p/2$

11. In wave winding, the number of parallel paths is equal to

    (a) $p/2$ (b) 2 (c) p (d) $2p$

12. In lap winding, the number of brush sets required is equal to

    (a) number of poles

    (b) number of pair of poles

    (c) number of commutator segments

    (d) 2

13. In wave winding, the number of brush sets required is equal to

    (a) number of poles

    (b) number of pair of poles

    (c) number of commutator segments

    (d) 2

14. The emf generated by a dc generator depends upon

    (a) the flux only (b) the speed only

    (c) both the flux and speed (d) none of these

15. A dc generator generates 220 V when lap connected. The same generator generates, if wave connected, generates ....

    (a) 220 V (b) 440 V (c) 110 V (d) 880 V

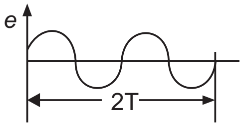
16. In lap connected dc generator having 4 poles, the current flowing in each conductor is 10 A. The total current is ......

    (a) 10 A (b) 20 A (c) 30 A (d) 40 A

17. A 4 pole lap connected dc generator develops a certain voltage at 1200 rpm. To generate the same voltage, the speed with which it has to rotate to generate the same voltage when wave connected is ......

    (a) 1200 rpm (b) 2400 rpm (c) 600 rpm (d) 6000 rpm

18. In one revolution, a generator generates voltage as shown in the figure. The number of poles of the generator is



    (a) 4 (b) 2 (c) 8 (d) 6

19. The current in armature conductors of a dc generator is

    (a) pure dc (b) pulsating dc

    (c) ac (d) pulsating dc plus pure dc

20. The series field of a short shunt dc generator is excited by ...... current

    (a) shunt (b) armature (c) load (d) external

21. In dc shunt generator, the armature and field windings are connected

    (a) in parallel (b) in series

    (c) in series-parallel (d) none of these

22. Residual magnetism is necessary in a dc ....

    (a) shunt generator (b) separately excited generator

    (c) shunt motor (d) series motor

23. Fleming's left hand rule is applicable to

    (a) dc generator (b) transformer

    (c) dc motor (d) both (a) and (b)

24. The back emf in a dc motor is maximum at

    (a) no-load (b) half full-load

    (c) full-load (d) square of full-load

25. Mechanical power developed by the dc motor is maximum when back emf is equal to

    (a) to supply voltage (b) two times of supply voltage

    (c) half of the supply voltage (d) zero

26. The back emf in a dc motor ....

    (a) aids the applied voltage (b) opposes the applied voltage

    (c) aids the armature current (d) opposes the field current

27. In dc motor, the electrical equivalent of the mechanical power developed is....

    (a) $V I_L$ (b) $E_b I_a$ (c) $I_a^2 R_a$ (d) VI_a

28. The relationship between the applied voltage and back emf is

    (a) $V = E_b + I_a R_a$ (b) $V = E_b - I_a R_a$

    (c) $V = E_b$ (d) none of these

29. The speed of a dc motor is ....

    (a) directly proportional to the flux

    (b) inversely proportional to the flux

    (c) inversely proportional to applied voltage

    (d) none of these

30. The torque developed by a dc motor is directly proportional to ....

    (a) $V I_a$ (b) ϕI_a (c) $I_a R_a$ (d) $E_b I_a$

31. The torque developed by a dc series motor is 20 N-m when drawing 20 A. If the current is doubled, the torque developed becomes

    (a) 20 N-m (b) 40 N-m (c) 80 N-m (d) 160 N-m

32. The torque of a shunt motor is proportional to
    (a) armature current (b) applied voltage
    (c) square of armature current (d) none of these
33. The shaft torque of a dc motor is less than its armature torque due to the following losses
    (a) copper and iron (b) copper and mechanical
    (c) iron and friction (d) copper, iron and mechanical
34. When the supply terminals of a dc shunt motor are reversed, then the motor
    (a) will stop
    (b) will run at its normal speed in the same direction as before
    (c) direction of rotation will reverse
    (d) will run at high speed in same direction
35. When the field winding of a dc motor is opened, when it is running at a particular speed, its speed becomes ......
    (a) zero (b) infinity (c) rated speed (d) none of these
36. When load is removed, ... motor will run at the highest speed
    (a) shunt (b) cumulative compound
    (c) differential compound (d) series
37. The speed of a dc ...... motor is almost constant
    (a) series (b) shunt
    (c) compound motors (d) none of these
38. DC .... motor should never be started on no-load
    (a) series (b) shunt
    (c) cumulative compound (d) differential compound
39. A dc motor is still used in industrial applications because it is
    (a) cheap (b) simple in construction
    (c) provides fine speed control (d) none of these
40. A dc motor draws a large current at starting due to
    (a) high value of R_a (b) low back emf
    (c) low flux in shunt field (d) none of these
41. Which dc motor is preferred for constant speed line shafting
    (a) cumulative compound motor (b) differential compound motor
    (c) shunt motor (d) series motor
42. For the movement of trains, dc .... motors are used
    (a) shunt (b) series
    (c) compound motors (d) none of these

43. The speed of a series motor at no-load is
    (a) zero (b) 1500 rpm (c) 3000 rpm (d) infinity
44. At the instant of starting a dc motor, its back emf is
    (a) zero (b) maximum (c) minimum (d) infinity
45. A dc motor draws a large current at the time of starting because ......
    (a) R_a is high (b) $E_b = 0$ (c) R_f is small (d) none of these
46. The function of a starter in a dc motor is to ......
    (a) control its speed
    (b) increase its torque
    (c) limit the starting current to a safe value
    (d) none of these
47. The current drawn by armature of a dc motor is
    (a) V/R_a (b) E_b/R_a
    (c) $(V - E_b)/R_a$ (d) $(E_b - V)/R_a$
48. For 'P' pole lap wound armature dc machine, no. of parallel ports ......
    (a) 2 (b) 2P (c) P (d) P/2

Answers:—

1. (b) 2. (d) 3. (c) 4. (a) 5. (b) 6. (d) 7. (c) 8. (d) 9. (b) 10. (c)
11. (b) 12. (a) 13. (d) 14. (c) 15. (b) 16. (d) 17. (c) 18. (a) 19. (c) 20. (c)
21. (a) 22. (a) 23. (c) 24. (a) 25. (c) 26. (b) 27. (b) 28. (a) 29. (b) 30. (b)
31. (c) 32. (a) 33. (c) 34. (b) 35. (b) 36. (d) 37. (b) 38. (a) 39. (c) 40. (b)
41. (c) 42. (b) 43. (d) 44. (a) 45. (b) 46. (c) 47. (c) 48. (c)

Review Questions

1. Explain the working principle of a dc machine as a generator and motor with suitable diagrams.
2. Explain with a neat sketch the constructional features of a dc machine and mention the function of each part.
3. What are the functions of yoke, armature, poles and brushes in a dc generator?
4. Derive the emf equation of dc generator.
5. Sketch and explain the external characteristics of dc shunt generator and mention its applications.

6. What is back emf ? Explain its significance under no-load and full-load condition in a dc motor.
7. Derive an expression for armature torque developed in a dc motor.
8. Derive an expression for the torque developed in a dc motor and show that the torque is proportional to the product of flux and armature current.
9. Sketch and explain the characteristics of a dc shunt motor.
10. Discuss the characteristics, of T_a/I_a and N/I_a for a series motor.
11. Justify the following statements: (*i*) a resistance must be kept in series with armature at the time of starting a dc motor and (*ii*) a series motor should never be started without a mechanical load.
12. From the characteristics of dc series motor, justify (a) a series motor should not be started on no-load and (b) a series motor has high starting torque.
13. What is the necessity of a starter for dc motor? With a neat diagram explain the working of a three point starter.

Exercises

1. 1.A 20 kW compound generator works on full load with a terminal voltage of 250 V. $R_a = 0.05\ \Omega$, $R_{se} = 0.025\ \Omega$ and $R_{sh} = 100\ \Omega$. Find emf if generator is short shunt. (**Ans**: 256.13 V)
2. A shunt machine connected to 250 V mains has an armature resistance (including brushes) of 0.12 Ω and the resistance of field circuit is 100 Ω. Find the ratio of speed as a generator to the speed of a motor. The line current in each case is 80 A. (**Ans:** 1.07)
3. A 4 pole, lap wound, 220 V dc shunt motor has an armature resistance of 0.4 Ω and shunt field resistance of 220 Ω. It is running at 1500 rpm, when it is taking 40 A from supply. Find the speed of the motor when the motor is taking 10 A from supply. (**Ans:** 1588 rpm)
4. A 4 pole, 250 V series motor has a wave wound armature with 1254 conductors. The flux per pole is 22 mWb, when motor is taking 50 A. Armature resistance is 0.2 Ω and series field resistance is 0.2 Ω. Calculate speed of motor. (**Ans:** 250 rpm)
5. A 230 V dc shunt motor takes a no-load current of 2 A and runs at 1100 rpm. If full load current is 40 A, find speed at full load. Assume flux remains constant, and $R_a = 0.25\ \Omega$. (**Ans:** 977 rpm)
6. The armature of a shunt motor has a lap winding accommodated in 60 slots, each containing 20 conductors. If the useful flux/pole is 28 mWb, calculate the total torque developed in N-m when the armature current is 50 A. (**Ans:** $T_a = 267.38$ N-m)

7. A 4 pole, dc shunt motor takes 30 A from 220 V supply. The armature and shunt field resistances are 0.5 Ω and 110 Ω respectively. The armature is lap wound having a total of 300 conductors. Flux/pole is 20 mWb, calculate the speed and the torque developed. (**Ans:** N = 2060 rpm, T_a = 26.73 N-m)
8. A 4 pole, 250 V, wave wound shunt motor gives 10 kW when running at 1000 rpm and drawing armature of 60 A. It has 560 conductors. Its armature resistance is 0.2 Ω. Assuming a drop of 1 V per brush, determine (a) total torque, (b) useful torque and (c) useful flux per pole.

   (**Ans:** 135.22 N-m, 95.49 N-m, 0.0126 Wb)
9. A 4 pole dc shunt motor takes 22.5 A from 250 V supply. R_a = 0.5 Ω and R_f = 125 Ω. Armature is wave wound with 300 conductors. If flux per pole is 0.02 Wb, calculate (i) Speed, (ii) Torque developed and (iii) Power developed. (**Ans:** 1199 rpm, 39.15 N-m, 4914.87 W)

Chapter

8

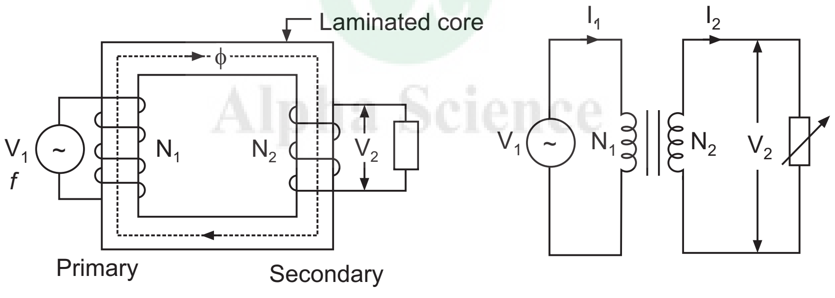
Transformers

8.1 DEFINITION

Transformer is a static or stationary device, which transfers electrical energy from one circuit to another without change in frequency.

8.2 WORKING PRINCIPLE

The physical basis is mutual induction between two circuits linked by a common magnetic flux.



(a) (b)

Fig. 8.1 (a) Transformer (b) Symbolic representation

It consists of two separate inductive coils or windings placed on a common laminated core. These windings are electrically separated and magnetically linked through a path of low reluctance. The two coils possess high mutual inductance.

If the primary is connected to a source of alternating voltage, an alternating flux with the same frequency as that of supply voltage will set up in the laminated core, most of which will be linked with the other coil called secondary and hence mutually induced emf is produced in it according to the Faraday's Laws of Electromagnetic Induction.

If the secondary circuit is closed (with a load), then a current flows through it. Hence electrical power is transferred from primary to secondary without change in frequency.

8.3 CONSTRUCTION

It consists of two parts:

(i) Magnetic core and

(ii) Windings or Coils.

The magnetic core is made of Silicon Steel laminations, which are assembled to provide a continuous magnetic path with minimum air gap. Each lamination is insulated by a core plate varnish or by a layer of an Oxide. The vertical portion is called the limbs. Top and bottom portion is called the yoke.

It consists of two separate coils or windings called Primary and Secondary, which are wound on the opposite limbs. The windings are insulated from each other and from the laminated core.

Large rating transformers are kept in a suitable container, which serves two purposes– provides insulation between transformer and tank and cooling purpose. Bushings are used to bring out the terminals of windings from the tank.

Types of Transformers according to the Construction

1. **Core Type Transformer:** It has single magnetic circuit. The primary and secondary windings are wound on the opposite limbs. The windings occupy a major portion of the core. It is applicable for high voltages as it provides more space for insulation.

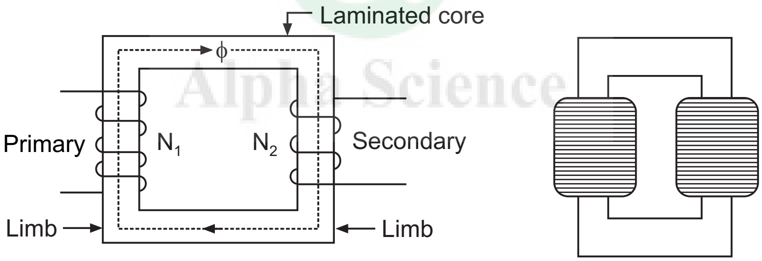


Fig. 8.2 Core type transformer

2. **Shell Type Transformer:** It has double magnetic circuit. The primary and secondary windings are wound on the central limb. The windings occupy a smaller portion of the core. It is applicable for low voltages.

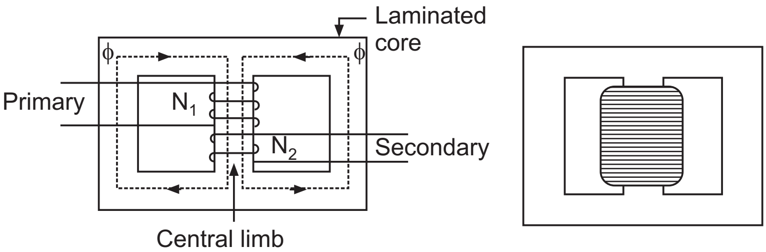


Fig. 8.3 Shell type transformer

3. **Berry of Spiral Type Transformer:** Basically it is a modified shell type transformer.

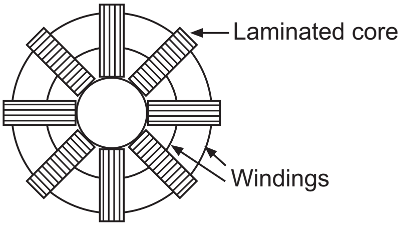


Fig. 8.4 Berry type transformer

It has distributed magnetic circuit. It has more than two independent magnetic circuits comprising of laminations in groups.

8.4 EMF EQUATION

When the primary is energized by a sinusoidally varying voltage V_1 volt and frequency of f Hz, a sinusoidally varying flux will set up in the laminated core, whose frequency is same as supply frequency.

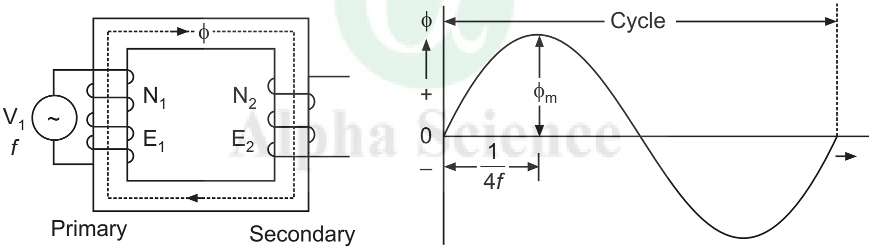


Fig. 8.5 (a) Transformer (b) Waveform

By observation, the flux increases from zero to maximum value ϕ_m in one fourth of a cycle,

i.e., $\frac{1}{4f}$ of second (because $T = \frac{1}{f}$ second).

$$\therefore \quad \text{Average rate of change of flux} = \frac{\phi_m}{\frac{1}{4f}} \text{ weber/second}$$

$$= 4f\phi_m \text{ volt}$$

According to Faraday's Laws of EMI,

Average rate of change of flux = Induced emf

$$\therefore \quad \text{Average emf/turn} = 4f\phi_m \text{ volt} \qquad \text{...(8.1)}$$

Expressing equation (8.1) in rms value;

RMS value of emf/turn = $4 f \phi_m \times 1.11 = 4.44 f \phi_m$ volt

Therefore, rms value of induced emf in the entire primary winding consisting of N_1 number of turns is

$$E_1 = 4.44 f \phi_m N_1 \text{ volt} \quad \text{...(8.2)}$$

Similarly, rms value of induced emf in the entire secondary winding consisting of N_2 number of turns is

$$E_2 = 4.44 f \phi_m N_2 \text{ volt} \quad \text{...(8.3)}$$

Also, $\phi_m = B_m A$

where B_m = maximum flux density and A = cross sectional area of the core

Substituting in equations (8.2) and (8.3),

$$E_1 = 4.44 f B_m A N_1 \text{ volt}$$

$$E_2 = 4.44 f B_m A N_2 \text{ volt.}$$

Important Points

- The voltage/turn is constant for a given transformer.
- The number of turns on primary depends upon input voltage.

8.4.1 Voltage Transformation Ratio (K)

It is defined as

$$K = \frac{E_2}{E_1}$$

where $E_1 = 4.44 f \phi_m N_1$

and $E_2 = 4.44 f \phi_m N_2$

Substituting for E_1 and E_2 in above equation,

$$K = \frac{4.44 f \ \phi_m \ N_2}{4.44 f \ \phi_m \ N_1} = \frac{N_2}{N_1}$$

Types of transformer based on K:

(i) If $K > 1$;

Then $N_2 > N_1$

or $E_2 > E_1$

Thus, the transformer is called the Step-Up Transformer, which is used at the generating station to step-up the voltage.

(ii) If $K < 1$,

Then $N_2 < N_1$

or $E_2 < E_1$

Thus, the transformer is called the Step-Down Transformer, which is used for distribution purpose.

Important Points

- Secondary current is less in step-up transformer and more in step-down transformer.
- If $K = 1 \Rightarrow N_2 = N_1$, it is called 1:1 transformer. It is used for isolation purpose.

8.4.2 Ideal Transformer

It is one, which is assumed to have no losses.

Let, V_1 = Supply voltage to primary, volt,

I_1 = Current drawn by primary, ampere,

V_2 = Secondary terminal voltage, volt,

I_2 = Current flowing in the secondary, ampere.

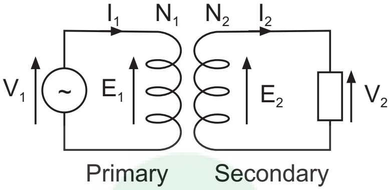


Fig. 8.6 Ideal transformer

Then, $$V_1 I_1 = V_2 I_2$$

or $$\frac{V_1}{V_2} = \frac{I_2}{I_1}$$

or $$\frac{N_1}{N_2} = \frac{I_2}{I_1}$$

i.e., $$\frac{1}{K} = \frac{I_2}{I_1}$$

Important Point: Current Ratio = Inverse of Voltage Ratio.

Solved Examples

Example 8.1 *Find the number of turns required on the HT side of 415 / 240 volt, 50 Hz 1-phase transformer. If the cross-sectional area of the core is 25 cm² and maximum flux density is 1.3 Wb/m².*

Solution: Given: Voltage: 415 / 230 volt f = 50 Hz,

↓ ↓

Primary *Secondary* $A = 25\text{ cm}^2 = 25 \times 10^{-4}\text{ m}^2$

High Tension (HT) *Low Tension (LT)* $B_m = 1.3\text{ Wb/m}^2$

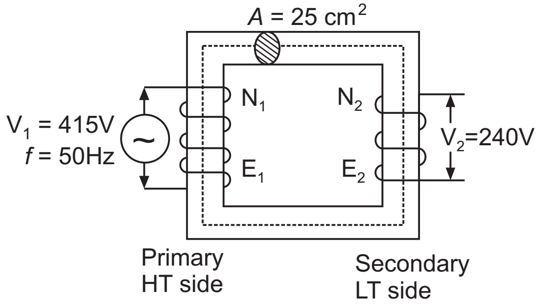


Fig. 8.7

EMF equation for primary or HT side is given by

$$E_1 = 4.44\,f\,B_m A\,N_1$$

$$N_1 = \frac{E_1}{4.44\,f\,B_m A}$$

$$= \frac{415}{4.44 \times 50 \times 1.3 \times (25 \times 10^{-4})} = 575.19$$

$$\approx \mathbf{575}$$

Example 8.2 *A 125 kVA transformer has primary voltage of 2000 V at 60 Hz. Primary turns are 182 and secondary turns are 40. Calculate neglecting losses, (i) No-load secondary emf, (ii) full-load primary and secondary current and (iii) maximum flux in core.*

Solution: Given: Rating = 125 kVA

V_1 = 2000 volt, f = 60 Hz

N_1 = 182, N_2 = 40

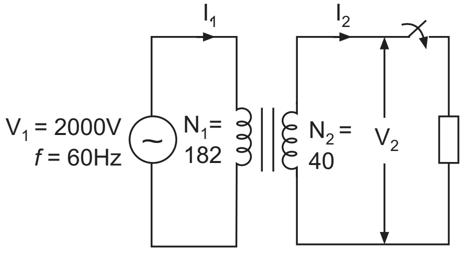


Fig. 8.8

(i) To calculate, E_2:

Voltage transformation ratio, $K = \dfrac{E_2}{E_1}$

or $E_2 = K\,E_1$ $\left(\text{where, } K = \dfrac{E_2}{E_1} = \dfrac{40}{182}\right)$

$$= \frac{40}{182} \times 2000 = \mathbf{439.56\ V}$$

(ii) Full-load primary current, $$I_1 = \frac{\text{kVA} \times 1000}{V_1}$$

$$= \frac{125 \times 1000}{2000} = \mathbf{62.5\ A}$$

To find secondary current:

Given that losses can be neglected, that is, the given transformer is an ideal transformer.

Then, $$\frac{1}{K} = \frac{I_2}{I_1}$$

or $$I_2 = \frac{I_1}{K}$$

$$= \frac{62.5}{\frac{40}{182}} = \mathbf{284.37\ A}$$

(iii) To find flux in the core (ϕ_m):
EMF Equation for Primary is given by

$$E_1 = 4.44 f \phi_m N_1$$

or $$\phi_m = \frac{E_1}{4.44\ f\ N_1}$$

$$= \frac{2000}{4.44 \times 60 \times 182} = \mathbf{0.0412\ Wb}$$

$$= \mathbf{41.2\ mWb}$$

Example 8.3 *A 50 kVA single phase transformer has 600 turns on primary and 40 turns on the secondary. The primary winding is connected to 2.2 kV, 50 Hz supply. Calculate, (i) Secondary voltage on no load (ii) Primary and Secondary current at full-load.*

Solution: Given: Rating = 50 kVA, $N_1 = 600$, $N_2 = 40$

$$V = 2.2\ \text{kV} = 2200\ \text{V},\ f = 50\ \text{Hz}$$

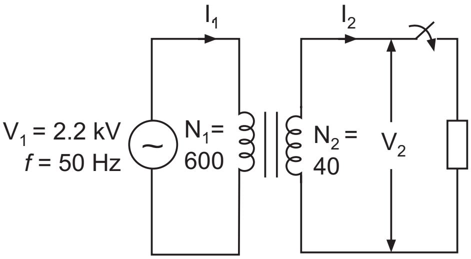


Fig. 8.9

(i) To find secondary no-load voltage:

Voltage transformation ratio, $K = \dfrac{V_2}{V_1}$

or $V_2 = K\,V_1$ $\left(\text{where, } K = \dfrac{N_2}{N_1} = \dfrac{40}{600}\right)$

$$= \frac{40}{600} \times 2200 = \mathbf{146.67\ V}$$

(ii) Full-load primary current, $I_1 = \dfrac{\text{kVA} \times 1000}{V_1}$

$$= \frac{50 \times 1000}{2200} = \mathbf{22.727\ A}$$

Secondary current at full-load, $I_2 = \dfrac{\text{kVA} \times 1000}{V_2}$

$$= \frac{50 \times 1000}{146.67} = \mathbf{340.9\ A}$$

Example 8.4 *A 1-phase transformer with 10:1 turns ratio and rated at 25 kVA, 1200 / 120 volt, 50 Hz is used to step-down the voltage of distribution system. The LT voltage is to be kept constant at 120 V. Find the value of load impedance on LT side so that transformer is fully loaded. Find also the value of maximum flux if the LT side has 25 turns.*

Solution: Given: Rating = 25 kVA

Voltage: 1200 / 120 volt

$$\begin{array}{cc} \downarrow & \downarrow \\ \text{Primary} & \text{Secondary} \\ \text{High Tension (HT)} & \text{Low Tension (LT)} \end{array}$$

$f = 50$ Hz

LT Voltage $= V_2 = 120$ V

$N_2 = 25$

Load impedance, $Z_L = \dfrac{V_2}{I_2}$

where $I_2 = \dfrac{\text{kVA} \times 1000}{V_2} = \dfrac{25 \times 1000}{120} = 208.33$ A

$\therefore$ $Z_L = \dfrac{120}{208.33} = \mathbf{0.576\ \Omega}$

To find ϕ_m:

EMF Equation for Primary is given by

$$E_2 = 4.44 f \phi_m N_2 \text{ volt}$$

or

$$\phi_m = \frac{E_2}{4.44\ f\ N_2}$$

$$= \frac{120}{4.44 \times 50 \times 25} = \mathbf{0.021\ Wb}$$

Example 8.5 *A 1-phase transformer with cross-sectional area of 150 cm^2. With frequency of 50Hz, the maximum flux density is 1.1 T. Calculate the output in kVA if secondary winding consists of 66 turns and connected to a load impedance of 4Ω. Any voltage drop may be ignored.*

Solution: Given: $A = 150\text{ cm}^2 = 0.015\text{ m}$, $f = 50\text{ Hz}$, $B_m = 1.1\text{ T}$,

$N_2 = 66$, $Z_L = 4\Omega$

Output $= V_2 I_2 \times 10^{-3}$ kVA

Given that voltage drop may be ignore,

$$\therefore \quad V_2 = E_2 = 4.44 f B_m A N_2$$

$$= 4.44 \times 50 \times 1.1 \times 0.015 \times 66 = \mathbf{241.76\ V}$$

Secondary current at given load,

$$I_2 = \frac{V_2}{Z_L} = \frac{241.76}{4} = 60.44\text{ A}$$

$$\therefore \quad \text{Output} = 241.76 \times 60.44 \times 10^{-3} = \mathbf{14.61\ kVA}$$

8.5 TRANSFORMER WITH RESISTANCE AND LEAKAGE REACTANCE

In practice, the entire flux, (ϕ) linking with primary may not link with secondary, because a part of it forms a magnetic path through air, which is called as leakage flux (say ϕ_{L1}). It is proportional to primary current I_1 and its effect is similar considering a coil of N_1 turns having inductive reactance X, that is, ϕ_{L1} is replaced by its equivalent inductive reactance X_1.

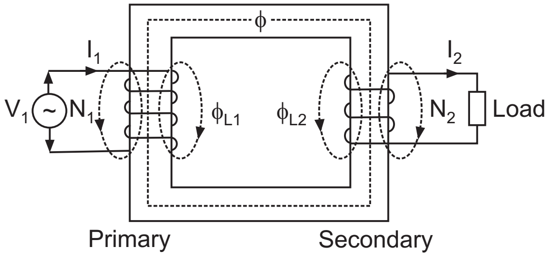


Fig. 8.10

Similarly, for secondary leakage flux ϕ_{L2} is replaced by its equivalent inductive reactance X_2.

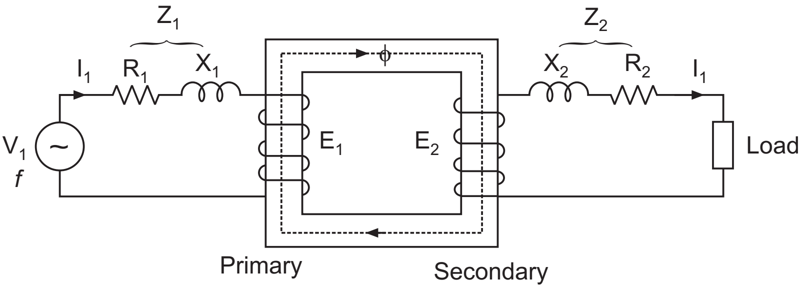


Fig. 8.11 Transformer with resistance and leakage reactance

Hence a transformer has primary resistance (R_1) and leakage reactance (X_1), secondary resistance (R_2) and leakage reactance (X_2).

Transferring all the secondary values to primary;

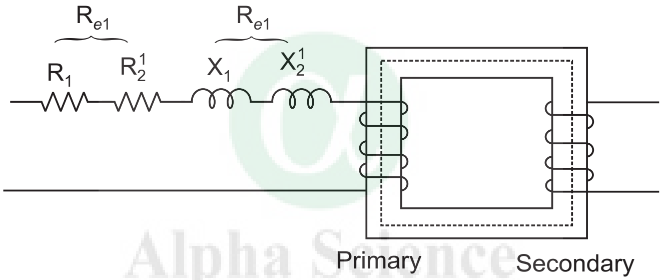


Fig. 8.12 Parameters referred to primary

Equivalent resistance of secondary as referred to primary,

$$R_2^1 = \frac{R_2}{K^2}$$

where K = Voltage transformation ratio

Equivalent reactance of secondary as referred to primary,

$$X_2^1 = \frac{X_2}{K^2}$$

Equivalent or Effective or Total resistance of transformer as referred to primary,

$$R_{e1} = R_1 + R_2^1$$

Equivalent or Effective or Total Reactance of transformer as referred to primary,

$$X_{e1} = X_1 + X_2^1$$

Similarly transforming all the values to the secondary,

Equivalent or Effective or Total resistance of transformer as referred to secondary,

$$R_{e2} = R_1 K^2 + R_2$$

Equivalent or Effective or Total resistance of transformer as referred to secondary,

$$X_{e2} = X_1 K^2 + X_2$$

8.6 LOSSES

As the transformer is a static or stationary device, mechanical (or rotational) losses are absent and it has only two losses;

1. Iron or Core Loss
   (i) Hysteresis Loss and
   (ii) Eddy Current Loss
2. Copper Loss

1. Iron or Core Loss (W_i): This loss occurs in a transformer due to the alternating flux, which sets up in the core.

(i) Hysteresis Loss: As the transformer core is subjected to the alternating flux, power is consumed for continuous reversal of the molecular magnets. This power is dissipated in the form of heat which is called the Hysteresis Loss.

This loss can be minimized by using good quality of material, say, Silicon Steel (which, has high permeability and low hysteresis loss). Hysteresis Loss is given by

$$W_h = P\, B_{\max}{}^{1.6} f \text{ watt where, P is a constant.}$$

(ii) Eddy Current Loss: The transformer core is made of a magnetic material, say, Iron or Steel, which is a good conductor. If the solid core is subjected to an alternating flux, it gets linked with the flux and induces eddy currents, which circulate throughout the length perpendicular to the flux. Eddy currents cause power loss which is dissipated in the form of heat, called the Eddy Current Loss.

This loss can be minimized by laminating the core. Laminated core is made of thin insulated iron sheets so that it offers high resistance (as $R \alpha 1/A$) and path for eddy current gets minimized. Eddy current loss is given by

$$W_e = Q\, B_{\max}^2\, f^2 \text{ watt where Q is a constant}$$

(Note: Eddy current loss varies as square of current)

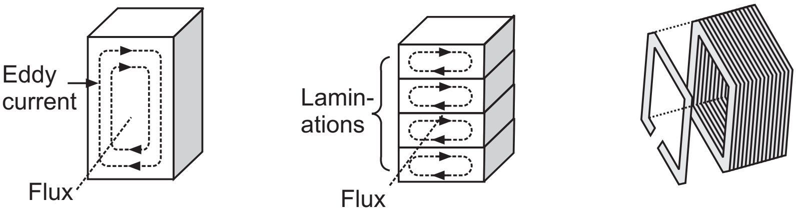
$$\therefore \quad \text{Iron loss } W_i = W_h + W_e \text{ watt}$$

In general, Iron or Core loss depends upon the following factors:

(i) Flux density in the core ($B_{\max}$) which depends upon supply voltage and

(ii) Supply frequency (f).

As these remain practically constant, Iron loss will remain constant from no-load to Full-load, that is, Iron loss is independent of load and hence it is also called the constant loss.



(a) Solid core (b) Laminated core (c) Transformer core

Fig. 8.13 Concept of lamination (for reference only)

2. **Copper Loss (W_C):** This loss is due to the Ohmic resistance of both primary and secondary windings. It varies as square of current and hence called the variable loss.

   Copper Loss (as referred to primary) is given by

$$W_C = I_1^2 R_{01} \text{ watt}$$

Rating of a Transformer: The Iron loss and Copper loss appear in the form of heat which results in temperature rise. The cause for producing heat is the current.

Thus, the output rating is specified as the product of output voltage and output current, that is, VA or kVA, which indicates that when the transformer is operated under this specified rating, its temperature rise should not be excessive.

Also, the power factor of the secondary depends upon the load. Therefore, the rating of transformer is generally expressed in VA or kVA.

8.7 EFFICIENCY

Efficiency of a transformer at a particular load and power factor is defined as

$$\eta = \frac{\text{Output}}{\text{Input}}$$

The output and input are expressed in the same unit, either watt or kilo-watt. As the transformer is a static device, it has no mechanical losses and its efficiency is high.

Another way of expressing its efficiency is

$$\eta = \frac{\text{Output}}{\text{Output} + \text{Losses}}$$

or

$$\eta = \frac{\text{Input} - \text{Losses}}{\text{Input}}$$

$$= 1 - \frac{\text{Losses}}{\text{Input}}$$

Input = Output + Losses

or Output = Input – Losses

Fig. 8.14

where $$\text{Losses} = \text{Iron loss } (W_i) + \text{Copper loss } (W_C)$$

Important Point

To determine efficiency at different loads,

$$\eta = \frac{n\left(\text{Output in watt}\right)}{n\left(\text{Output in watt}\right) + W_i + n^2 W_C} \times 100$$

$$= \frac{n\left(\text{kVA} \times 1000 \times pf\right)}{n\left(\text{kVA} \times 1000 \times pf\right) + W_i + n^2 W_C} \times 100$$

where n is the fraction of load, that is, $n = 1$ for Full-load, $n = 2$ for half Full-load, $n = \frac{3}{4}$ for three fourth of Full-load and $n = \frac{1}{4}$ for one fourth of full-load.

Condition for Maximum Efficiency:

Let the efficiency be given by

$$\eta = 1 - \frac{\text{Losses}}{\text{Input}} \qquad \text{...(8.4)}$$

where $$\text{Losses} = \text{Iron Loss} + \text{Copper Loss}$$

Considering the primary side,

Let, V_1 = Primary voltage (or Supply voltage) volt

I_1 = Input current, ampere

$\cos \phi_1$ = power factor

R_{01} = Equivalent resistance of transformer as referred to primary, Ω

Iron Loss = W_i watt

Copper Loss = $W_C = I_1^2 R_{01}$ watt

Primary Input = $V_1 I_1 \cos \phi_1$ watt

Substituting for losses and input in equation (8.4),

$$\eta = 1 - \left[\frac{\left(I_1^2 R_{01} + W_i\right)}{V_1 I_1 \cos\phi_1}\right]$$

$$= 1 - \frac{I_1^2\ R_{01}}{V_1\ I_1\ \cos\phi_1} - \frac{W_i}{V_1\ I_1\ \cos\phi_1}$$

$$= 1 - \frac{I_1\ R_{01}}{V_1\ \cos\phi_1} - \frac{W_i}{V_1\ I_1\ \cos\phi_1} \qquad \text{...(8.5)}$$

Differentiating equation (8.5) both the sides with respect to $I_{1,}$

$$\frac{d\eta}{dI_1} = \frac{d}{d\ I_1}\left[1 - \frac{I_1\ R_{01}}{V_1\ \cos\phi_1} - \frac{W_i}{V_1\ I_1\ \cos\phi_1}\right]$$

$$= 0 - \frac{R_{01}}{V_1\ \cos\ \phi_1} + \frac{W_i}{V_1\ I_1^2\ \cos\ \phi_1}$$

For maximum efficiency, $\frac{d}{d\eta} = 0$

i.e., $-\frac{R_{01}}{V_1\ \cos\ \phi_1} + \frac{W_i}{V_1\ I_1^2\ \cos\ \phi_1} = 0$

or $$\frac{R_{01}}{V_1\ \cos\ \phi_1} = \frac{W_i}{V_1\ I_1^2\ \cos\ \phi_1}$$

or $I_1^2\ R_{01} = W_i$

or $I_2^2\ R_{02} = W_i$ (as referred to secondary)

Therefore, for the given terminal voltage and power factor, the efficiency is maximum when copper loss (variable loss) is equal to Iron loss (constant loss).

Output current corresponding to maximum efficiency is

$$I_2 = \sqrt{\frac{W_i}{R_{02}}}$$

Load corresponding to maximum efficiency:

Let W_i = Iron loss and W_C = Full-load copper loss

As copper loss varies square of load,

$$W_C \propto (\text{F.L kVA})^2 \qquad \text{...(8.6)}$$

Let x be the load in kVA corresponding to maximum efficiency

Therefore, $W_C \propto x^2$

At maximum efficiency,

$$W_C = W_i,$$

$$W_i \propto x^2 \qquad \text{...(8.7)}$$

Taking the ratio of equations (8.6) and (8.7),

$$\frac{W_C}{W_i} = \left(\frac{\text{Full-load kVA}}{x}\right)^2$$

or $$x = \text{Full-load kVA} \times \sqrt{\frac{W_i}{W_C}}$$

$$= \text{Full-load kVA} \times \sqrt{\frac{\text{Iron loss}}{\text{Full-load Copper loss}}}$$

8.8 TRANSFORMER ON NO-LOAD

The primary is connected to supply voltage V_1 and secondary is left opened. The no-load current called the exciting current I_0 has to supply Iron loss in the core and very small amount of copper loss in the primary. Thus I_0 lags behind V_1 by an angle φ_0, which is less than 90° and has two components, namely

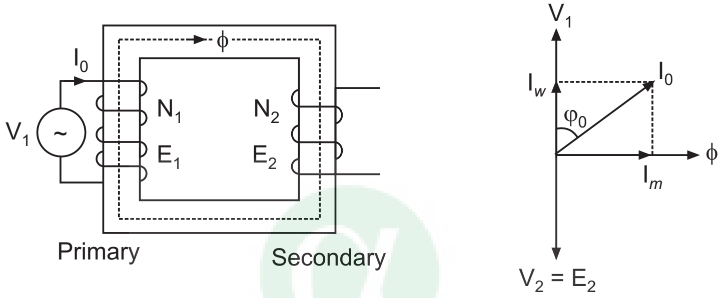


Fig. 8.15 (a) Transformer on no-load (b) Vector diagram

(i) Active or working component I_w and

(ii) Magnetizing or reactive component I_m

Active or working component I_w is in phase with the applied voltage V_1 and supplies a small amount of Copper loss in the primary. It is given by

$$I_w = I_0 \cos \varphi_0$$

Magnetizing or reactive component I_m lags behind applied voltage V_1 by angle 90^0 and its function is to sustain the alternating flux in the core. It will not consume any power and thus also called the wattless component. It is given by

$$I_m = I_0 \sin \varphi_0$$

Therefore, I_0 is the vector sum of I_w and I_m

i.e., $$I_0 = \sqrt{I_w^2 + I_m^2}$$

8.9 TRANSFORMER ON LOAD

Suppose, an inductive load is connected across the secondary, a current I_2 flows through it and sets up its own flux which opposes main flux. Hence primary induced emf E_1 reduces and V_1 momentarily exceeds E_1. This results in greater primary current I_1^1 called load component of primary current. It is in phase opposition to I_2.

Current I_1^1 sets up its own flux which cancels out flux due to I_2. Thus, the main flux in the core remains constant.

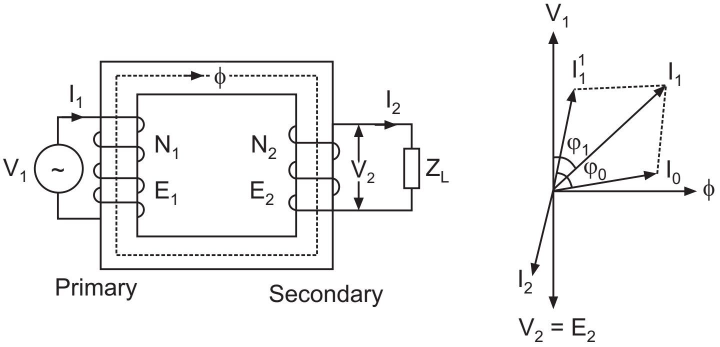


Fig. 8.16 (a) Transformer on load (b) Vector diagram

8.10 VOLTAGE REGULATION

Concept of total approximate voltage drop in a transformer: The primary applied voltage V_1 has to supply the following;

(i) Resistive drop ($I_1 R_1$),

(ii) Leakage reactance drop ($I_1 X_1$) and

(iii) Has to overcome the back emf in the primary winding.

Similarly voltage drop occurs in the secondary.

The total drop in the transformer is the sum of these two. This total drop can be referred either to the primary or the secondary.

Hence, if the primary voltage V_1 is kept constant, the secondary terminal voltage will decrease from no-load to full-load.

Therefore, due to resistive and reactive drops, the secondary terminal voltage decreases as the load on the transformer increases when power factor is lagging and it increases when power factor is leading, assuming that primary voltage remain constant.

Definition

The voltage regulation of a transformer is defined as the change in the secondary terminal voltage from no-load to full-load, keeping the primary voltage constant.

It is usually expressed as a percentage of the secondary no-load voltage as given below

$$\%\text{Regulation} = \frac{\text{Secondary Voltage (NL)} - \text{Secondary Voltage (FL)}}{\text{Secondary Voltage (NL)}} \times 100$$

If V_{02} = No-load secondary terminal voltage

V_2 = Secondary terminal voltage at full-load

$$\therefore \quad \%\text{Regulation} = \frac{V_{02} - V_2}{V_{02}} \times 100$$

Important Points

- A good transformer should have high efficiency and low regulation.
- Regulation of a transformer depends upon the power factor of load.

Solved Examples

Example 8.6 *A 40 kVA single phase transformer has core loss of 450 W and Full-load Copper loss of 850 W. If the power factor of the load is 0.8, calculate (a) Full-load efficiency, (b) load corresponding to maximum efficiency and (c) maximum efficiency at unity power factor.*

Solution: Given: Rating = 40 kVA

Iron loss, W_i = 450 watt,

Copper loss, W_C = 850 watt, $pf = 0.8$

(a) Full-load efficiency,
$$\eta_{FL} = \frac{n(\text{kVA} \times 1000 \times pf)}{n(\text{kVA} \times 1000 \times pf) + W_i + n^2 W_C} \times 100$$

$$= \frac{1(40 \times 1000 \times 0.8)}{1(40 \times 1000 \times 0.8) + 450 + (1 \times 850)} \times 100$$

$$= \mathbf{96.096\%}$$

(b) Load corresponding to maximum efficiency is given by

$$x = \text{Full-load kVA} \times \sqrt{\frac{\text{Iron loss}}{\text{Full-load copper loss}}}$$

$$= 40\sqrt{\frac{450}{850}} = \mathbf{29.104\ kVA}$$

(c) Maximum Efficiency,
$$\eta_{\max} = \frac{\text{Output}}{\text{Output + Losses}} \times 100$$

$$\text{Output at UPF} = (\text{kVA} \times 1000) \times \cos\phi$$
$$= (40 \times 1000) \times 1 = 40000\ \text{W}$$

At maximum efficiency, $W_C = W_i = 450$ W

$\therefore$ Losses $= 2 \times W_i = 2 \times 450 = 900$ W

$$\therefore \quad \eta_{\max} = \frac{40000}{40000 + 900} \times 100 = \mathbf{97.799\%}$$

Example 8.7 *A 25 kVA, 2000/200 V transformer has Iron and copper losses of 350 W and 225 W respectively at* $^3\!/_4$ *th Full-load. Determine the efficiency of the transformer at half load, 0.8 pf. What is the value of Copper loss at maximum efficiency?.*

Solution: Given: Rating = 25 kVA

Voltage: 2000/200 volt

Iron loss, W_i = 350 watt, Copper loss, W_C = 225 watt at $^3\!/_4$ th Full-load

Efficiency of a transformer is given by

$$\eta = \frac{\text{Output}}{\text{Output} + \text{Losses}} \times 100$$

Output at half full-load and 0.8 *pf* = $\frac{1}{2}$ (kVA × 1000) × cos ϕ

$$= \frac{1}{2}(25 \times 1000) \times 0.8$$

$$= 10000 \text{ W}$$

$$\text{Losses} = \text{Iron Loss} + \text{Copper Loss}$$

$$\text{Iron loss} = 350 \text{ W (Irrespective of load)}$$

Given that Copper loss at $^3/_4$th full-load = 225 W

i.e., Copper loss at Full-load,

$$W_C = \frac{225}{\left(\frac{3}{4}\right)^2} = 400 \text{ W}$$

$$\therefore \quad \text{Copper loss at half Full-load} = \left(\frac{1}{2}\right)^2 \times W_C = \left(\frac{1}{2}\right)^2 \times 400 = 100 \text{ W}$$

$$\therefore \quad \text{Losses} = 350 + 100 = 450 \text{ W}$$

$$\therefore \quad \eta = \frac{10000}{10000 + 450} \times 100$$

$$= \mathbf{95.69\ \%}$$

At maximum efficiency, Copper loss = Iron loss

i.e., Copper loss = **350 W**

Example 8.8 *The maximum efficiency at Full-load and* UPF *of a 1ϕ, 25 kVA, 500/1000 V, 50 Hz transformer is 98%. Determine its efficiency at (i) 75% load, 0.9 pf, (ii) 50% load, 0.8 pf, (iii) 25% load, 0.6 pf.*

Solution: Given: Rating = 25 kVA, V = 500/1000 volt, ϕ = 50Hz, η_{max} = 98% = 0.98

From the given data, W_i and W_C are to be determined.

Maximum efficiency, $$\eta_{\max} = \frac{(\text{kVA} \times 1000 \times pf)}{(\text{kVA} \times 1000 \times pf) + 2W_i}$$

(Because at $\eta_{\max}$, $W_i = W_C$)

i.e., $$0.98 = \frac{(25 \times 1000 \times 1)}{(25 \times 1000 \times 1) + 2W_i}$$

or $$2W_i = 510.2$$

∴ $$W_i = W_C = \frac{510.2}{2} = 255.1 \text{ W}$$

In general, the efficiency at any load is given by

$$\eta = \frac{n(\text{kVA} \times 1000 \times pf)}{n(\text{kVA} \times 1000 \times pf) + W_i + n^2 W_C} \times 100$$

(i) At $\eta = 0.75$, $pf = 0.9$,

$$\eta = \frac{0.75(25 \times 1000 \times 0.9)}{0.75(25 \times 1000 \times 0.9) + 255.1 + (0.75^2 \times 255.1)} \times 100$$

$$= \mathbf{97.69\%}$$

(ii) At $\eta = 0.5$, $pf = 0.8$,

$$\eta = \frac{0.5(25 \times 1000 \times 0.8)}{0.5(25 \times 1000 \times 0.8) + 255.1 + (0.5^2 \times 255.1)} \times 100$$

$$= \mathbf{96.9\%}$$

(iii) At $\eta = 0.25$, $pf = 0.6$,

$$\eta = \frac{0.25(25 \times 1000 \times 0.6)}{0.25(25 \times 1000 \times 0.6) + 255.1 + (0.25^2 \times 255.1)} \times 100$$

$$= \mathbf{93.25\%}$$

Example 8.9 *A 600 kVA single-phase transformer has an efficiency of 92% at full-load and* UPF. *The pf is 0.9 at half full-load. Determine its efficiency at 75% of full-load at 0.9 power factor lag.*

Solution: Given: Rating = 600 kVA

Efficiency at full-load = η_{FL} = 92 % at UPF

Efficiency at half full-load = η_h = 92 % at pf = 0.9

(Note: Output and losses are expressed in kVA and kW respectively)

Determination of Iron and Copper losses:

Considering full-load condition;

$$\eta_{FL} = \frac{\text{Output}}{\text{Input}}$$

Output at full-load and UPF = kVA × pf = 600 × 1 = 600 kW

$$\text{Input} = \frac{\text{Output}}{\eta_{FL}} = \frac{600}{0.92} = 652.173 \text{ kW}$$

$$\therefore \quad \text{Losses} = \text{Input} - \text{Output} = 652.173 - 600 = 52.17 \text{ kW}$$

Let X = Iron loss and Y = FL Copper loss;

Then $$X + Y = 52.17 \text{ kW} \qquad \text{...(8.8)}$$

Considering half full-load condition;

$$\eta_h = \frac{\text{Output}}{\text{Input}}$$

Output at half full-load and 0.9 pf, $= \left(\frac{1}{2}\right)$ kVA × pf = 600 × 0.9 = 270 kW

$$\text{Input} = \frac{\text{Output}}{\eta_h} = \frac{270}{0.92} = 293.48 \text{ kW}$$

$$\therefore \quad \text{Losses} = \text{Input} - \text{Output} = 293.48 - 270 = 23.48 \text{ kW}$$

Now Copper Loss at half full-load $= \left(\frac{1}{2}\right)^2 \times$ FL Copper loss $= \left(\frac{1}{2}\right)^2 Y = 0.25\,Y$

$$\therefore \quad X + 0.25\,Y = 23.48 \text{ kW} \qquad \text{...(8.9)}$$

Solving equations (8.8) and (8.9),

$$X + Y = 52.17$$

$$\underline{X + 0.25\,Y = 23.48}$$

$$0.75\,Y = 28.69$$

or $$\underline{Y = 38.25}$$

i.e., **Copper loss = 38.25 kW**

Substituting for Y in equation (8.8),

$$X + 38.25 = 52.173$$

or $$X = 13.923$$

i.e., **Iron loss = 13. 923 kW**

To find efficiency at 75% full-load and $pf = 0.9$;

Output = (0.75) kVA × pf = (0.75) × 600 × 0.9 = 405 kW

Copper Loss = $(0.75)^2 \times 38.25 = 21.516$ kW

$$\text{Iron Loss} = 13.923 \text{ kW}$$

$$\text{Input} = \text{Output} + \text{Losses} = 405 + 13.923 + 21.516$$

$$= 440.439 \text{ kW}$$

$$\therefore \quad \eta = \frac{\text{Output}}{\text{Input}} \times 100$$

$$= \frac{405}{440.439} \times 100 = \mathbf{91.95\%}$$

Example 8.10 *A 50 kVA, 400/200 V single phase transformer has an efficiency of 98% at full-load 0.8 pf and 97 % at 1/2 full-load 0.8 pf. Determine the Iron and full-load Copper losses and voltage regulation, if the terminal voltage on Full-load is 195 V.*

Solution: Given: Rating: 50 kVA, Voltage: 400/200 volt

$$\left.\begin{array}{l} \text{FL efficiency} = \eta_{\text{FL}} = 98\%, \\ \text{Efficiency at } \frac{1}{2} \text{ full-load} = \eta_h = 97\% \end{array}\right\} pf = 0.8$$

Secondary terminal voltage at Full-load = 195 V

Determination of Iron and Full-load Copper loss:

Considering full-load condition;

$$\eta_{\text{FL}} = \frac{\text{Output}}{\text{Input}}$$

Output at full-load and 0.8 pf = kVA × pf = 50 × 0.8 = 40 kW

$$\text{Input} = \frac{\text{Output}}{\eta_{\text{FL}}} = \frac{40}{0.98} = 40.816 \text{ kW}$$

$$\therefore \quad \text{Losses} = \text{Input} - \text{Output} = 40.816 - 40 = 0.816 \text{ kW}$$

Let X = Iron Loss and Y = FL Copper loss

$$\therefore \quad X + Y = 0.816 \text{ kW} \qquad \text{...(8.10)}$$

Considering half full-load condition;

$$\eta_h = \frac{\text{Output}}{\text{Input}}$$

Output at half full-load and 0.8 $pf = \frac{40}{2} = 20$ kW

$$\text{Input} = \frac{\text{Output}}{\text{Input}} = \frac{20}{0.97} = 20.61 \text{ kW}$$

$$\therefore \quad \text{Losses} = \text{Input} - \text{Output} = 20.61 - 20 = 0.61 \text{ kW}$$

Now Copper loss at half full-load $= \left(\frac{1}{2}\right)^2 \times \text{F.L Copper loss} = \left(\frac{1}{2}\right)^2 \text{Y} = \mathbf{0.25\ Y}$

$$\therefore \quad X + 0.25\,Y = \text{Losses}$$

$$\text{i.e.,} \quad X + 0.25\,Y = 0.61 \text{ kW} \qquad \text{...(8.11)}$$

Solving equations (8.10) and (8.11),

$$\begin{aligned} X + Y &= 0.816 \\ \underline{X + 0.25Y} &= \underline{0.61} \\ 0.75\,Y &= 0.206 \end{aligned}$$

$$\text{or} \quad \underline{Y = 0.27466}$$

i.e., **Copper Loss = 0.2746 kW = 274.66 W**

Substituting for Y in equation (8.10),

$$X + 0.27466 = 0.816$$

$$\text{or} \quad X = 0.54133$$

i.e., **Iron Loss = 0. 5413 kW = 541.33 W**

Determination of voltage regulation:

$$\% \text{ Regulation} = \frac{V_{02} - V_2}{V_{02}} \times 100$$

$$= \frac{200 - 195}{200} \times 100 = \mathbf{2.5\%}$$

8.11 CONCEPT OF AUTOTRANSFORMER

It is a single winding transformer. The single winding is wound on a laminated core. The single winding acts as the primary and a part of it acts as the secondary.

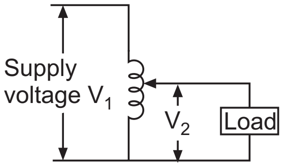


Fig. 8.17 Autotransformer

The winding is tapped at a suitable point to obtain the desired output voltage across the secondary.

Advantages

1. Requires less copper due to single winding.
2. Smaller in size as compared with two winding transformers.
3. It is more economical.
4. Higher efficiency.

Disadvantages

1. As the primary and secondary are not insulated from each other, in case of a fault it causes damage to the device connected.
2. Low impedance offers higher short circuit currents in case of short circuit at the secondary.

Applications

1. For starting the rotating machines such as induction motors.
2. For interconnecting systems which approximately operates at same voltage.

Multiple Choice Questions

Choose the Correct Answer

1. Induced emf on secondary of a transformer is
   (a) dynamically induced emf (b) self induced emf
   (c) mutually induced emf (d) none of these
2. Primary and Secondary windings of a transformer are ......
   (a) electrically connected and magnetically linked
   (b) electrically separated and magnetically separated
   (c) electrically connected and magnetically separated
   (d) electrically separated and magnetically linked
3. The two windings in a transformer are magnetically linked through a path of
   (a) low reluctance (b) low resistance
   (c) high reluctance (d) high resistance
4. The voltage/turn of primary is .... to the voltage/turn of secondary
   (a) greater than (b) less than
   (c) equal to (d) none of these
5. The emf induced in the primary winding consisting of N_1 turns is
   (a) $4.44 f \phi N_1$ (b) $4.44 f \phi_m N_1$ (c) $2.22 f \phi N_1$ (d) $2.22 f \phi_m N_1$
6. The transformation ratio in a transformer is ....
   (a) V_1/V_2 (b) N_2/N_1 (c) N_1/N_2 (d) I_2/I_1
7. Low voltage winding of a step-down transformer is
   (a) primary winding (b) secondary winding
   (c) neither primary nor secondary (d) both primary nor secondary

8. Compared to the secondary of a loaded step-up transformer, the primary has ...... voltage and .... current.

   (a) lower and higher (b) higher and lower

   (c) lower and lower (d) higher and higher

9. An ideal transformer does not change ......

   (a) voltage (b) current (c) power (d) none of these

10. In a step-up transformer ...... remains constant.

    (a) voltage (b) current (c) power (d) none of these

11. Increase or decrease of voltage by the transformer depends on

    (a) size of transformer (b) type of transformer

    (c) transformation ratio (d) all of these

12. A 10 kVA, 2000/200 V draws a full-load secondary current of ....

    (a) 100 A (b) 50 A (c) 200 A (d) 500 A

13. When load on a transformer is reduced, ... decreases.

    (a) eddy current loss (b) hysteresis loss

    (c) copper loss (d) friction loss

14. Losses which do not occur in a transformer are ......

    (a) copper losses (b) magnetic losses

    (c) friction losses (d) none of these

15. The efficiency of a transformer is very high, because ......

    (a) the core is made of silicon steel

    (b) there are no moving parts

    (c) the coupling is magnetic

    (d) the core is laminated

16. The core of a transformer is made of ....

    (a) silicon steel (b) annealed copper

    (c) seasoned wood (d) aluminum

17. The copper loss of a certain transformer at half full-load is 200 W. Then the copper loss at full-load will be

    (a) 100 W (b) 200 W (c) 400 W (d) 800 W

18. A transformer has 200 W Iron loss at full-load. The Iron loss at half full-load is

    (a) 100 W (b) 200 W (c) 400 W (d) 300 W

19. In a transformer, iron loss .... from no-load to full-load coil is,

    (a) varies (b) will remain constant

    (c) varies as square of current (d) none of these

20. Transformer core is laminated in order to
    (a) simplify its construction (b) minimize eddy current loss
    (c) reduce cost (d) reduce hysteresis loss
21. When the load on the transformer is increased, the flux in the core ......
    (a) decreases (b) increases
    (c) remains constant (d) none of these
22. At maximum efficiency of a transformer
    (a) hysteresis loss = eddy current loss
    (b) iron loss = full-load copper loss
    (c) power factor of load is leading
    (d) power factor of load is lagging
23. A transformer is working at its maximum efficiency with iron-loss of 500 W, then its copper-loss will be
    (a) 500 W (b) 250 W (c) 300 W (d) 400 W
24. Regulation and efficiency of a transformer should be respectively
    (a) high, high (b) high, low (c) low, high (d) low, low

Answers:—

1. (c) 2. (d) 3. (a) 4. (c) 5. (b) 6. (b) 7. (b) 8. (a) 9. (c) 10. (c)
11. (d) 12. (b) 13. (c) 14. (c) 15. (b) 16. (a) 17. (d) 18. (b) 19. (b) 20. (b)
21. (c) 22. (b) 23. (a) 24. (c)

Review Questions

1. With a neat diagram describe the construction of a single-phase core type transformer.
2. With a neat diagram explain the construction and working principle of a single-phase transformer.
3. With neat sketches explain the constructional details of core type and shell type transformers.
4. Explain the principle of operation of a single-phase transformer and derive its emf equation.
5. What are the different losses that occur in a transformer? How do they vary with load? How are these losses minimized?
6. Explain various losses in a transformer and derive the condition for maximum efficiency.

7. Develop an expression for the efficiency of a single-phase and obtain the condition for maximum efficiency.
8. Give the reasons for the following:
   (i) Iron loss remains the same irrespective of load on transformer with constant supply.
   (ii) Efficiency of transformer is high compared to other electrical machines.
   (iii) Terminal voltage of transformer decreases with increase in load.
   (iv) The core of a transformer is made up of laminations.
9. Define voltage regulation of a transformer.
10. Explain the construction of an autotransformer. Mention its advantages, disadvantages and applications.

Exercises

1. A single-phase transformer has 350 primary turns and 1050 secondary turns. The net cross-sectional area of core is 55 cm^2. If primary is connected to 400 V, 50 Hz. Find B_{max},voltage induced in the secondary.

   (**Ans:** $B_{max} = 0.936$ Wb/m^2 , $E_2 = 1200$ V)

2. A 25 kVA, single-phase transformer has 500 turns on the primary and 40 turns on the secondary winding. The primary is connected to 300 volt,50 Hz supply. Calculate (i) Primary and Secondary turns on full-load; (ii) The secondary emf; (iii) the maximum flux in the core.

   (**Ans:** $I_1 = 8.33$ A, $I_2 = 104.125$ A, $E_2 = 240$ V, $\phi_m = 0.27$ Wb).

3. A single phase transformer has 1000 turns on its primary and 400 turns on the secondary side. An ac voltage of 1250 volt, 50 Hz is applied to its primary side, with the the secondary left open circuited, calculate (i) the secondary emf, (ii) maximum value of flux density, given that the effective cross section area of core as 60 cm^2. (**Ans:** $E_2 = 500$ V, $B_m = 0.938$ Tesla)

4. A 50 kVA transformer has an efficiency of 98% at full-load, 0.8 *pf* and an efficiency of 96.9% at ¼ full-load, UPF. Determine the Iron loss and the full-load Copper loss. (**Ans:** $W_C = 426$ W and $W_I = 374$ W)

5. A single-phase transformer working at UPF has an efficiency of 92% at both half-load and full-load of 500 kW. Determine the efficiency at 80% of full-load. (**Ans:** $\eta = 97\%$)

6. Determine the efficiency of a 150 kVA transformer at 50% full-load of 0.8 *pf* lag, if the Copper loss at full-load is 1600 W and the Iron loss 1400 W.

   (**Ans:** $\eta = 97\%$)

7. In a 25 kVA, 2000/200 V transformer, the Iron and full-load Copper losses are 350 W and 400 W respectively. Calculate the efficiency at UPF at full-load and half full-load. Find also Copper loss for maximum efficiency.

(**Ans:** 96.52% and 350 W)

8. A 600 kVA single-phase transformer has an efficiency of 92% both at full-load and half full-load at UPF. Determine its efficiency at 75% of full-load at 0.9 power factor lag. (**Ans:** 91.63%)

Chapter 9

Synchronous Generators

9.1 PRINCIPLE OF OPERATION

Synchronous generator or alternator or ac generator is a machine, which converts mechanical energy into electrical energy.

In commercial alternator, armature is stationary, and as field is rotating. As the field system rotates, the stationary armature conductors are cut by the flux and alternating emf is induced in it according to Faraday's Laws of EMI. For bulk power generation (e.g., 11 kVA, 500 MW) three phase alternators are used.

Advantages of Stationary Armature:

(i) Easy to insulate the stationary armature conductors as alternators are designed to generate high voltage.

(ii) The output terminals can be directly connected to the load without any slip rings and brushes.

(iii) Gives good mechanical support.

(iv) The rotor can be rotated at high speed as it is robust in construction.

Comparison between DC Generator and Alternator:

Similarity: DC Generator and Alternator operate on Faraday's Laws of EMI.

Differences: In dc generator, field is stationary and armature is rotating. The output is supplied to load circuit by means of commutator and brushes. In alternator, the field is rotating called the rotor and the armature is stationary called the stator. The output is directly connected to load.

9.2 CONSTRUCTION

Commercial alternator consists of mainly two parts

1. Stationary armature called Stator and
2. Rotating field called Rotor.

1. **Stator or Stationary Armature:**

   It consists of the following parts-

   (i) Stator frame

   (ii) Stator core

   (iii) Stator or Armature winding

   (i) Stator Frame: It is a Cast-Iron protective frame. It houses all the parts of alternator. It is in cylindrical shape.

   (ii) Stator Core: It is made of special Steel Alloy (or Silicon Steel) laminations. Slots are provided on its inner periphery to accommodate three-phase armature windings.

   (iii) Stator or Armature Winding: These are insulated Copper conductors accommodated in stator slots. The stator of an alternator is wound for the same number of poles as on rotor.

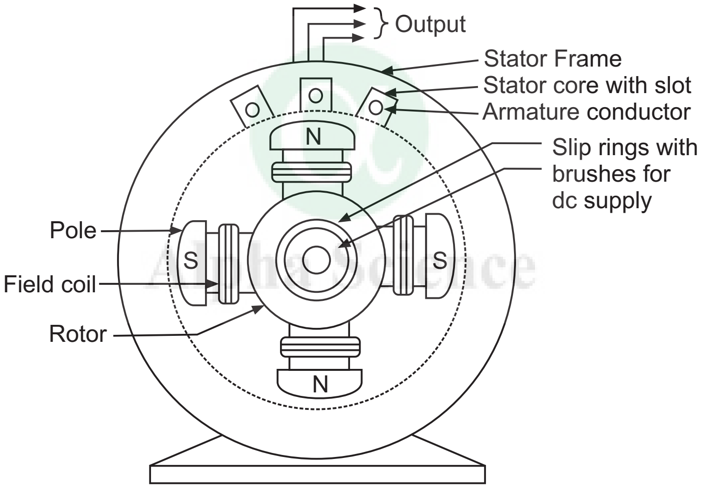


Fig. 9.1 Alternator parts

2. **Rotor-Rotating Field:** It is a rotating part, driven by a prime mover such as water turbine or steam turbine or diesel engine. It is excited by a separate dc source. As the field is rotating, direct current is supplied by means of two slip ring and brushes.

9.3 TYPES OF ROTORS

(i) Salient Pole Rotor

(ii) Non-Salient Pole or Smooth Cylindrical Rotor

(i) **Salient Pole or Projecting Pole Rotor**: The rotor is like a heavy magnetic flywheel made of Cast-Iron or Steel. The salient poles or projecting poles are made of thick Steel laminations. The poles are provided with pole shoe. The poles carry field coils or windings, which are connected to a dc source by means of slip rings and brushes for the purpose of excitation.

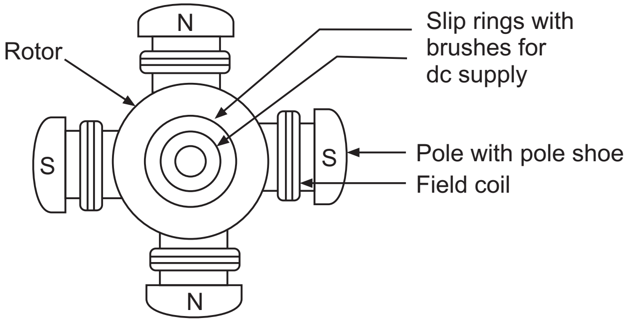


Fig. 9.2 Projecting pole rotor

Projecting pole rotor is characterized by **large diameter** and **short axial length**.

Advantages

1. Less expensive and
2. It provides sufficient space for field ampere-turns.

Disadvantages

1. If the projecting poles are driven at high speed, it will cause excessive air friction loss.
2. By construction, the rotor is not robust and if driven at high speed, it cannot withstand the mechanical stress.

Application

Used for low or medium speed alternator.

Example: Hydroelectric power plants, diesel power plants and gas driven turbines.

(ii) **Non-Salient Pole or Smooth Cylindrical Rotor:** The rotor consists of a smooth solid forged Steel cylinder. Along the outer periphery, slots are milled out at regular intervals to accommodate field coils. Field coils are connected to a dc source by means of slip rings and brushes for excitation purpose.

Thus the poles are non-salient, that is, they do not project out from surface of rotor and hence called smooth cylindrical rotor.

Projected pole rotor is characterized by **small diameter** and **long axial length**.

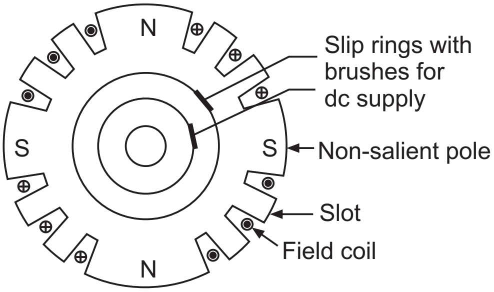


Fig. 9.3 Non-salient pole rotor

Advantages

1. Gives better balance,
2. Quieter operation (that is, makes less noise) and
3. Less air friction loss.

Application: Used for high-speed alternator, called the turbo alternator.

Example: Steam turbines.

Table: 9.1 Comparison between salient pole and non-salient pole rotors

Sl. No.	Salient Pole Rotors	Non-Salient Pole Rotors
1.	Poles are projecting	Poles are non-projecting
2.	Air gap is non-uniform	Air gap is uniform
3.	Makes noise when running	Quieter operation
4.	Large diameter and short axial length	Small diameter and very long axial length
5.	Mechanically weak	Mechanically robust and stable
6.	Applicable for low speed alternators	Applicable for high speed alternators

9.4 RELATIONSHIP BETWEEN SPEED AND FREQUENCY

Let, p = Number of poles,

N = Rotor Speed in rpm

f = Frequency of generated emf in Hz

In one revolution of rotor, stator conductor is cut by $\left(\frac{p}{2}\right)$ North poles and $\left(\frac{p}{2}\right)$ South poles.

$$\therefore \quad \text{Number of cycles/revolution} = \frac{p}{2}$$

$$\text{and Number of revolutions/second} = \frac{N}{60}$$

$$\therefore \quad f = \frac{p}{2} \times \frac{N}{60}$$

$$= \frac{pN}{120} \text{ Hz}$$

Hence, N is called the synchronous speed. This is the speed at which the alternator must run to generate emf of required frequency.

Important Point: The frequency of supply in India is 50 Hz.

9.5 CONCEPT OF WINDING FACTORS

1. **Pitch of a winding:** The distance between the beginning of two consecutive turns is known as 'Pitch of winding'.
2. **Pole-pitch:** It is equal to the number of armature conductors per pole.
3. **Coil-span or Coil-pitch:** It is the distance measured in terms of armature slots between the sides of a coil.
4. **Full-pitch:** For Full-pitch winding, pole-pitch = coil-pitch

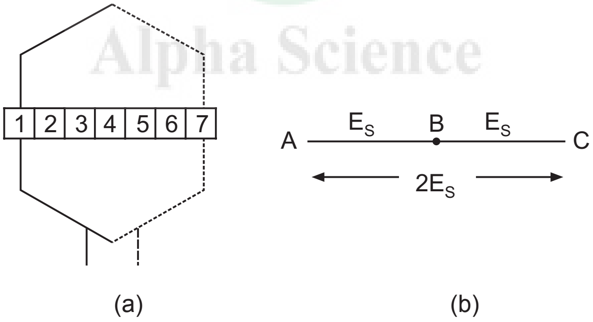


Fig. 9.4 (a) Full-pitch winding (b) Vector diagram

Under full-pitch, one side of winding is under the centre of North pole and other the centre of South pole. Thus, both the sides of the coil are 180° apart and hence emf induced in the two sides of the coil are directly added.

Example: Coil sides occupy slots number 1 to 7 and hence comprise full-pitch winding.

If E_S = Induced emf in each coil-side,

then total emf induced in the coil is given by

$$E_S + E_S = 2E_S$$

5. Short-pitch or Fractional-pitch winding:

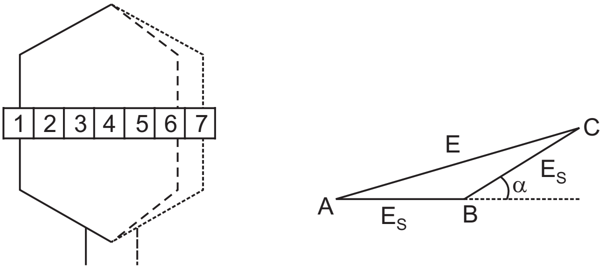


Fig. 9.5 (a) Short-pitch winding (b) Vector diagram

Example: The coil sides occupy slots 1 to 6, then it is called the short-pitch or fractional-pitch winding.

This is employed to:

(i) save Copper at the end connections,

(ii) improve the waveform of emf generated and

(iii) increase overall efficiency

In this winding the total emf induced in the coil is slightly reduced, because the coil is short-pitched by an angle α. Hence resultant emf is the vector sum, given by

$$E = 2\,E_S \cos\frac{\alpha}{2} \text{ where } \alpha = \text{angle of short pitch}$$

Hence a correction factor called the Pitch factor (K_P) or Coil-span factor (K_C) is introduced, which is defined as

$$K_P = \frac{\text{Vector sum of induced emf/coil}}{\text{Arithmatic sum of induced emf/coil}} \leq 1$$

Distribution factor (K_d) or Breadth factor (K_b) or Winding factor (K_w): In practical generator, the coils of each phase of armature are not concentrated or placed together in one slot. They are distributed in other adjacent slots, which lie under each pole.

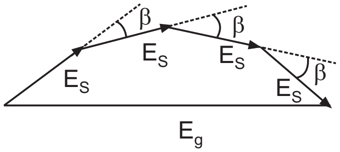


Fig. 9.6 Distributed winding

Hence, the coils are displaced from each other by an angle β. Then the emf induced in the coil sides are not in phase with each other, but differ by the same angle β. Then the total emf induced is slightly reduced.

Hence a correction factor called the Distribution factor is introduced, which is defined as

$$K_d = \frac{\text{emf induced with distributed winding}}{\text{emf induced with concentrated winding}} < 1$$

or $$K_d = \frac{\sin m\frac{\beta}{2}}{m\sin\frac{\beta}{2}}$$

where m = Number of slots/phase/pole

and slot angle $$\beta = \frac{180°}{\text{Number of slots/pole}}$$

where 180^0 = Electrical angle/pole

9.6 EMF EQUATION

Let, Z = Number of stator conductors/phase,

= 2 T, where T = Number of turns/phase

p = Number of poles,

f = Frequency of induced emf, Hz,

ϕ = Flux/pole, Wb,

N = Rotational speed of rotor, rpm

According to Faraday's Laws of Electromagnetic Induction,

$$\text{Average emf induced/conductor} = \frac{d\phi}{dt}\ \text{Wb/second} \qquad \text{...(9.1)}$$

Now $d\phi$ = Flux cut/stator conductor in one revolution of rotor = ϕp Wb.

$$\text{Number of revolutions of rotor/second} = \frac{N}{60}$$

$$\therefore \quad \text{Time taken for one revolution of rotor} = dt = \frac{60}{N}$$

Substituting for $d\phi$ and dt in equation (9.1),

$$\text{Average emf induced/conductor} = \frac{\phi p}{\frac{60}{N}}\ \text{Wb/second}$$

$$\text{or Average emf induced/conductor} = \frac{\phi pN}{60}\ \text{volt} \qquad \text{...(9.2)}$$

Frequency of induced emf is given by

$$f = \frac{pN}{120} \text{ Hz}$$

or $$N = \frac{120\,f}{p}$$

Substituting for N in equation (9.2),

Average emf induced/conductor $= \dfrac{\phi p}{60} \times \dfrac{120\,f}{p} = 2\phi f$ volt

For Z number of stator conductors in series,

$$\begin{aligned} \text{Average emf induced/phase} &= 2\,\phi f Z \\ &= 2\,\phi f \times 2\,T \text{ (because } Z = 2T) \\ &= 4\,\phi\, f\, T \text{ volt} \end{aligned}$$

$$\begin{aligned} \text{RMS value of induced emf/phase} &= \text{Average emf induced/phase} \times \text{form factor} \\ &= 4\,\phi f T \times 1.11 \\ &= 4.44\,\phi f T \text{ volt} \end{aligned}$$

The actual induced emf/phase available is given by introducing Pitch factor (K_p) and Distribution factor (K_d),

$$E_{ph} = 4.44\,\phi f T\, K_P\, K_d \text{ volt}$$

If the alternator is star-connected, then Line emf is given by

$$E_L = \sqrt{3}\, E_{ph} \text{ volt}$$

Solved Examples

Example 9.1 *A 12 pole,500 rpm, Y-connected alternator has 60 slots with 20 conductors per slot. The flux per pole is 0.02 Wb and is distributed sinusoidally. The winding factor is 0.97. Calculate (i) frequency (ii) phase emf and (iii) line emf.*

Solution: Given: $p = 12$, N = 500 rpm,

Number of slots = 60, Number of conductors/slot = 20

$\phi = 0.02$ Wb., $K_w = 0.97$

Frequency, $f = \dfrac{pN}{120} = \dfrac{12 \times 500}{120}$ **= 50 Hz**

Phase emf, $E_{ph} = 4.44\,\phi f T\, K_P\, K_w$

$$\begin{aligned} Z &= \text{number of stator conductors/phase} \\ &= \text{Number of slots/phase} \times \text{Number of conductors/slot} \\ &= \frac{60}{3} \times 20 = 400 \end{aligned}$$

$$\therefore \quad T = \frac{Z}{2} = \frac{400}{2} = 200$$

Assuming pitch factor, $K_p = 1$,

$$E_{ph} = 4.44 \times 0.02 \times 50 \times 200 \times 1 \times 0.97$$

$$= \mathbf{861.36\ V}$$

Line emf, $E_L = \sqrt{3} \times E_{ph}$

$$= \sqrt{3} \times 861.36 = \mathbf{1491.92\ V}$$

Example 9.2 *A 2 pole, three-phase alternator running at 3000 rpm has 42 armature slots with 2 conductors in each slot. Calculate the flux per pole required to generate a line voltage of 2300 V. Distribution factor is 0.952 and pitch factor is 0.956.*

Solution: Given: $p = 2$, N = 3000 rpm,

Number of slots = 42, Number of conductors/slot = 2

$E_L = 2300$ V, $K_d = 0.952$ and $K_P = 0.956$

$$E_{ph} = 4.44\, \phi f T K_P K_d$$

or

$$\phi = \frac{E_{ph}}{4.44\, f\, T K_P K_d}$$

Now

$$E_{ph} = \frac{E_L}{\sqrt{3}} = \frac{2300}{\sqrt{3}} = 1327.9\text{V}$$

Frequency,

$$f = \frac{pN}{120} = \frac{2 \times 3000}{120} = 50 \text{ Hz}$$

To find T: Z = number of stator conductors/phase

= Number of slots/phase × Number of conductors / slot

$$= \frac{42}{3} \times 2 = 28$$

$$\therefore \quad T = \frac{Z}{2} = \frac{28}{2} = 14$$

$$\therefore \quad \phi = \frac{1327.9}{4.44 \times 50 \times 14 \times 0.956 \times 0.952} = \mathbf{0.469\ Wb}$$

Example 9.3 *A three-phase star connected synchronous generator driven at 900 r/min is required to generate a line voltage of 460 V at 60 Hz on open circuit. The stator has two slots per pole per phase and 4 conductors per slot. Calculate (i) the number of poles and (ii) the useful flux per pole.*

Solution: Given: N = 900 rpm, $E_L = 460\text{V}$, $f = 60$ Hz

Number of slots/pole/phase = 2,

Number of conductors/slot = 4

(i) To find number of poles:

$$f = \frac{pN}{120}$$

or
$$p = \frac{120\,f}{N}$$

$$= \frac{120 \times 60}{900} = 8$$

(ii) To find useful flux/pole:

$$E_{ph} = 4.44\,\phi f T K_P K_d$$

or
$$\phi = \frac{E_{ph}}{4.44 f\,T\,K_P K_d}$$

where
$$E_{ph} = \frac{E_L}{\sqrt{3}} = \frac{460}{\sqrt{3}} = 265.58\text{ V}$$

To find T;
$$\begin{aligned} Z &= \text{number of stator conductors/phase} \\ &= \text{Number of slots/pole/phase} \times \text{Number of poles} \\ &\quad \times \text{Number of conductors/slot} \\ &= 2 \times 8 \times 4 = 64 \end{aligned}$$

∴
$$T = \frac{Z}{2} = \frac{64}{2} = 32$$

To find K_w;
$$K_w = \frac{\sin m\frac{\beta}{2}}{m \sin\frac{\beta}{2}}$$

where
$$m = \text{Number of slots/phase/pole} = 2$$

and slot angle,
$$\beta = \frac{180°}{\text{Slots/pole}} = \frac{180°}{\text{Slots/pole/phase} \times 3} = \frac{180°}{2 \times 3} = 30°$$

So,
$$K_w = \frac{\sin\left(2 \times \frac{30°}{2}\right)}{2 \times \sin\left(\frac{30°}{2}\right)} = 0.965$$

Assuming pitch factor, $K_p = 1$,

$$\phi = \frac{265.58}{4.44 \times 60 \times 32 \times 1 \times 0.965}$$

$$= \mathbf{0.03228\ Wb = 32.28\ mWb}$$

9.7 VOLTAGE REGULATION

Voltage regulation of an alternator at a particular power factor is defined as the increase in terminal voltage when full-load is thrown-off, assuming that field current and speed remaining the same.

The percentage regulation is defined as the ratio of change in terminal voltage from full load to no load and rated terminal voltage.

i.e., $$\% \text{ Regulation} = \frac{E_0 - V}{V} \times 100$$

where E_0 = No-load terminal voltage

and V = Full-load rated terminal voltage.

(Note: For leading power factor, $E_0 < V$, hence % regulation is –ve.)

Significance of voltage regulation: Knowledge about voltage regulation is important as it shows how an alternator can maintain its voltage under various conditions of load from no-load to full-load.

It is observed that the change of terminal voltage from full-load to no-load is more in case of lagging and leading power factor loads as compared to unity power factor.

Important Point

Power factor of an alternator depends upon the nature of load.

Determination of E_0: When the alternator is loaded, the terminal voltage reduces or voltage drop occurs due to the following reasons:

(i) Voltage drop due to armature resistance R_a

(ii) Voltage drop due to leakage reactance X_L

(iii) Voltage drop due to armature reaction.

Note: The drop due to armature reaction is theoretically accounted by assuming reactance X_a in the armature winding.

The vector sum of X_L and X_a gives synchronous reactance X_S

$\therefore$ Synchronous Impedance,

$$Z_S = (R_a + jX_S) = \sqrt{R_a^2 + X_s^2}\ \text{ohm}$$

If $V = V_{ph}$ = Terminal voltage/phase on load,

E_0 = Generated emf/phase on no-load

and I_a = Armature current/phase

Then, $E_0 = V + I_a Z_S$ volt

1. **For lagging power factor**

- I_a lags behind the terminal voltage V by an angle ϕ.
- Drop due to $I_a R_a$ is in phase with I_a.
- Drop due to $I_a X_S$ is at right angles to I_a.

$$\therefore \quad E_0 = \sqrt{(V\cos\phi + I_aR_a)^2 + (V\sin\phi + I_aX_S)^2}$$

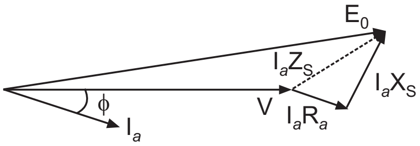


Fig. 9.7 Vector diagram

2. **For leading power factor**

- I_a leads the terminal voltage V by an angle ϕ.
- Drop due to $I_a R_a$ is in phase with I_a.
- Drop due to $I_a X_S$ is at right angles to I_a.

$$\therefore \quad E_0 = \sqrt{(V\cos\phi + I_aR_a)^2 + (V\sin\phi - I_aX_S)^2}$$

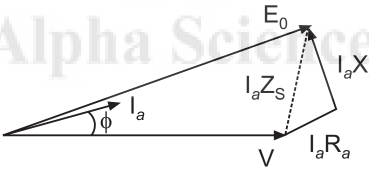


Fig. 9.8 Vector diagram

3. **For UPF**

- I_a is in phase with the terminal voltage V.
- Drop due to $I_a R_a$ is in phase with I_a.
- Drop due to $I_a X_S$ is at right angles to I_a.

$$\therefore \quad E_0 = \sqrt{(V + I_aR_a)^2 + (I_aX_S)^2}$$

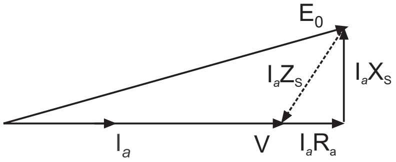


Fig. 9.9 Vector diagram

Multiple Choice Questions

Choose the Correct Answer

1. Alternator generates .... voltage
   (a) ac (b) dc
   (c) Both ac and dc (d) none of these
2. A commercial alternator has
   (a) rotating armature and stationary field
   (b) stationary armature and rotating field
   (c) both armature and field are rotating
   (d) both armature and field are stationary
3. The stator of an alternator is wound for .... as on rotor.
   (a) more number of poles than (b) less number of poles than
   (c) the same number of poles (d) none of these
4. The rotor of synchronous generator has
   (a) 4 slip rings (b) 3 slip rings
   (c) 2 slip rings (d) no slip rings
5. The field winding of an alternator is .... excited
   (a) dc (b) ac
   (c) both ac and dc (d) none of these
6. A salient pole field construction is used for alternator having ....
   (a) low and medium speed (b) large speed
   (c) very large speed (d) none of these
7. Salient pole type rotors are
   (a) smaller in axial length
   (b) larger in axial length
   (c) smaller diameter
   (d) larger diameter and smaller in axial length
8. High speed alternators are driven by
   (a) diesel engine (b) hydraulic turbines
   (c) steam turbines (d) none of these
9. For full-pitch coil, the pitch factor K_P is ....
   (a) 1 (b) >1
   (c) < 1 (d) none of these

10. Usually modern alternators have short pitched windings so as to
    (a) increase the machine rating (b) improve the voltage waveform
    (c) improve generated voltage (d) none of these

11. Distribution factor is always
    (a) equal to one (b) more than one
    (c) zero (d) less than one

12. Distribution factor for a winding having 3 slots/pole/phase and a slot angle of 20 degrees is ....
    (a) 0.96 (b) 1 (c) 0.5 (d) 0.707

13. The disadvantage of a short-pitched coil in an alternator is that
    (a) harmonica are introduced (b) wave form become non sinusoidal
    (c) voltage round the coil is reduced (d) none of these

14. A 6 pole 50 Hz synchronous machine runs at
    (a) 750 rpm (b) 1000 rpm
    (c) 1500 rpm (d) 1440 rpm

15. The frequency of induced emf generated depends on
    (a) speed (b) number of poles
    (c) flux (d) both (a) and (b)

16. The frequency of supply in India is
    (a) 60 Hz (b) 25 Hz
    (c) 50 Hz (d) 75 Hz

17. A 50 Hz alternator will run at the greatest possible speed if it is wound for .... poles
    (a) 8 (b) 6 (c) 4 (d) 2

18. The highest speed at which a 50 Hz AC Generator can be operated is
    (a) 3000 rpm (b) 1500 rpm (c) 3600 rpm (d) 1800 rpm

19. The number of cycles generated in a 6 pole alternator in one revolution is
    (a) 3 (b) 6
    (c) 50 (d) 2

20. If the alternator runs at a speed N rpm, then the time taken for 1 revolution is
    (a) N/60 sec (b) 60/N sec
    (c) 60N sec (d) none of these

21. When the alternator is loaded, its terminal voltage ....
    (a) increases (b) decreases
    (c) does not change (d) none of these

22. The power factor of an alternator is determined by its
    (a) speed
    (b) load
    (c) excitation
    (d) prime mover

Answers:—

1. (a) 2. (b) 3. (c) 4. (c) 5. (a) 6. (a) 7. (d) 8. (c) 9. (a) 10. (b)
11. (d) 12. (a) 13. (c) 14. (b) 15. (d) 16. (c) 17. (d) 18. (a) 19. (a) 20. (b)
21. (b) 22. (b)

Review Questions

1. With neat diagram, explain the constructional features of a three-phase alternator.
2. What are the advantages of stationary armature in alternator?
3. What are the advantages of rotating field synchronous generator?
4. List the differences between salient pole and non-salient pole rotors.
5. Bring out the comparison between a dc generator and an alternator?
6. Derive the emf equation of an alternator. What is the necessity of considering pitch factor and distribution factor for emf equation?
7. With usual notations, obtain the relationship between speed and frequency.

Exercises

1. A three-phase star-connected alternator with 12-poles generates 1100 volt on open circuit at a speed of 500 rpm. Assuming 180 turns/phase, a distribution factor of 0.96 and full-pitched coils, find the useful flux per pole. (**Ans:** $\phi = 0.0165$ Wb)
2. Calculate the phase emf induced in a 4-pole, three-phase, 50 Hz, star-connected alternator with 36 slots and 30 conductors per slot. The flux per pole is 0.05 weber. Assume winding factor of 0.95. (**Ans:** $E_{ph} = 1898$ V)
3. Calculate the line emf induced in a 4-pole, three-phase, 50 Hz star-connected alternator with 48 slots and 20 conductors per slot. The flux per pole is 0.05 weber, assume winding factor 0.96. (**Ans:** $E_L = 2953$ V)
4. A 6 pole, three-phase, star-connected alternator has an armature with 90 slots and 8 conductors per slot. It revolves at 1000 rpm. The flux per pole being 0.05 weber. Calculate the emf generated if the winding factor is 0.97 and all the conductors in each phase are in series. (**Ans:** $E_{ph} = 1292$ V and $E_L = 2237.8$ V)

Chapter 10

Three-phase Induction Motors

10.1 CONCEPT OF ROTATING MAGNETIC FIELD

A three-phase induction motor consists of a stator and a rotor (as shown in Fig. 10.1). The stator consists of a three-phase winding and when connected to three-phase supply, a resultant flux of constant magnitude will set up, which rotates around the stator at a speed called the synchronous speed N_S, given by

$$N_S = \frac{120\,f}{p}$$

where f = frequency of supply in Hz and p = number of pole

The resultant flux rotates in space as if actual magnetic poles were being rotated mechanically.

(Note: The resultant flux at any instant = $1.5\,\phi_m$, where ϕ_m = maximum value of flux due to any one of the three-phases.)

10.2 CONSTRUCTION AND TYPES OF ROTORS

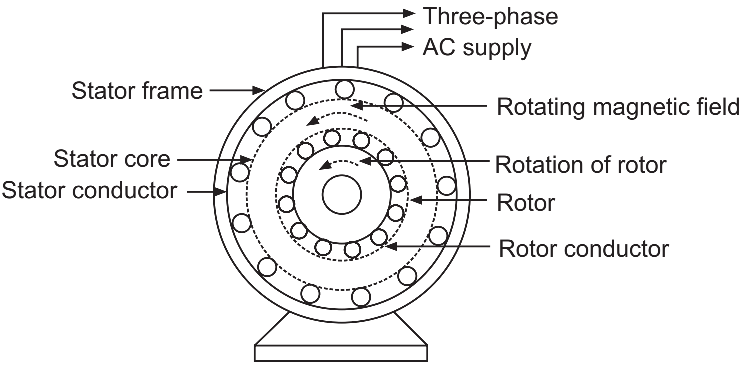


Fig. 10.1 Three-phase induction motor

Important Point

The air gap between stator and rotor is small, in mm (0.4 to 4 mm).

It consists of mainly two parts:

1. Stator and
2. Rotor

1. Stator: It consists of the following parts–

   (i) Stator frame,
   (ii) Stator core and
   (iii) Stator winding.

   (i) Stator Frame: It is made of Cast-Iron or Steel protective frame to give mechanical stability. It houses all the parts of alternator.

   (ii) Stator Core: It is made of special Steel Alloy or Silicon Steel laminations. Slots are provided on its inner periphery to accommodate three-phase armature windings. It is wound for a definite number of poles.

   When the stator windings are connected to three-phase supply, it produces a magnetic flux of constant magnitude and rotates at synchronous speed, given by

$$N_S = \frac{120\, f}{p}$$

   where f = frequency of supply, Hz and p = number of poles

   (iii) Stator Winding: These are insulated Copper conductors accommodated in stator slots. Usually it is distributed winding.

2. Rotor: It is a rotating part. The special feature of this motor is that the rotor is not connected to the supply mains. It works on the principle of Electromagnetic induction.

   Types of rotors:

   (i) Squirrel cage rotor (or Cage rotor) and
   (ii) Slip ring rotor or Phase wound rotor.

   (i) Squirrel Cage Rotor (or Cage Rotor): It is a cylindrical laminated core with parallel slots on its outer periphery for carrying rotor conductors.

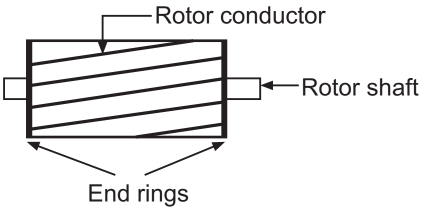


Fig. 10.2 Cage rotor

The rotor conductors are in the form of heavy Copper bars, which are placed in each slot. These rotor bars are permanently short circuited at both the ends by means of two heavy Copper end-rings and hence not possible to connect external resistance at the time of starting.

The rotor bars are not usually parallel to the shaft, but given a slight skew for the following reasons:

(i) To run quietly (with minimum noise) by reducing magnetic hum.

(ii) To reduce the locking tendency between the stator teeth and rotor teeth due to direct magnetic attraction between them.

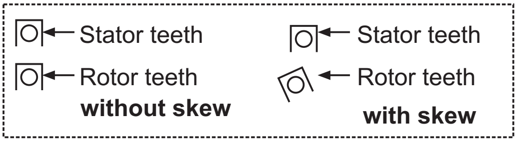


Fig. 10.3 Illustration of skewing of slot

Applications of Squirrel Cage Induction Motor: In this motor,

- The rotor resistance/phase is small and fixed.
- The reactance/phase is very large at standstill as the frequency of rotor current equals the supply frequency.
- Not possible to connect external resistance to rotor circuit, starting current/phase is very large.

Thus, if started against heavy load, it is dangerous to the rotor circuit and the other interconnected apparatus. Also, starting current lags behind rotor emf by a large angle.

Therefore, squirrel cage motors are not useful if required to start against heavy mechanical load. It can be started with medium load.

Important applications:

1. Pump sets
2. Floor mills
3. In electrical devices where smoother operation is required
4. Where approximately constant speed is required

(ii) Slip Ring Rotor (or Phase Wound Rotor): It is a cylindrical laminated core with parallel slots on its outer periphery for carrying rotor conductors.

Three phase star-connected windings are placed in these slots. The number of rotor poles are equal to the number of stator poles. The open terminals of star-connected winding are brought out and connected to three insulated slip rings mounted on the shaft itself with Carbon brushes pressed on them. Hence, provision is made for connecting three-phase star-connected external (or starting) resistance at the time of starting to reduce starting current. This resistance is gradually decreased as the rotor picks up speed.

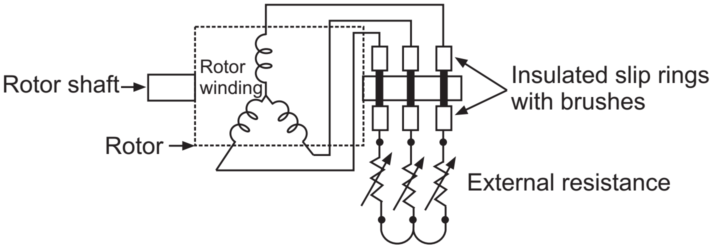


Fig. 10.4 Slip ring rotor

Applications of Phase Wound (or Slip ring) Induction Motor: In this motor, provision is made for connecting a three-phase star-connected external (or starting) resistance to the rotor circuit at the time of starting. Thus, rotor impedance increases. Hence starting current/phase is reduced. Also, power factor of rotor circuit improves.

Therefore, phase wound (or slip ring) induction motors are used if required to start against heavy mechanical load and where speed control is required from rotor side.

Important applications:

1. In industries
2. For cranes and hoists

Table 10.1 Comparison between squirrel cage rotor and phase wound rotor

Sl. No	Squirrel Cage Rotor	Phase Wound Rotor
1	Absence of slip rings and brushes (**advantage**)	Consists of slip rings and brushes (**disadvantage**)
2	Less mechanical losses and more efficient(**advantage**)	More mechanical losses and less efficient (**disadvantage**)
3	Rotor is robust and needs less maintenance (**advantage**)	Rotor is not robust and needs more maintenance (**disadvantage**)
4	Less expensive(**advantage**)	More expensive(**disadvantage**)
5	Not possible to connect external resistance at the time of starting and has medium starting torque (**disadvantage**)	Possible to connect external resistance at the time of starting and thus exhibits high starting torque (**advantage**)

10.3 PRINCIPLE OF OPERATION

When three-phase stator winding is connected to three-phase supply, a magnetic flux of constant magnitude and rotating at synchronous speed will set-up, which is given by

$$N_S = \frac{120\,f}{p}$$

where f = frequency of supply, Hz and p = Number of poles.

As the flux passes through the air gap, it sweeps over the rotor conductors which are yet stationary and hence flux is cut by the stationary rotor conductor. Due to the relative speed between the rotating flux and stationary conductors, an emf is induced in the rotor circuit according to Faraday's Laws of EMI.

(i) The frequency of rotor emf is same as supply frequency

(ii) Its magnitude is proportional to the relative velocity between the flux and the conductors

(iii) The direction is given by Fleming's Right-hand rule.

As the rotor forms a closed circuit, rotor current sets up whose direction is given by Lenz's Law, is such as to oppose the very cause of producing it. The cause for the rotor current is the relative speed between the rotating flux and stationary rotor conductors. Hence to reduce it, the rotor starts rotating in the same direction as that of the flux and tries to catch hold of the rotating flux, but it never succeeds. In this way the rotor rotates.

Important Points

- If two terminals of supply lines are reversed, then the direction of rotation also gets reversed. Thus, the direction of rotation depends on the phase sequence of supply.
- If the rotor terminals are not short-circuited and supply is given to the stator, the motor will not start because the rotor current will not set-up.

10.4 SLIP

The rotor runs at a speed, which is always less than the synchronous speed of the stator flux.

This difference between the synchronous speed of the stator flux and actual speed of the rotor is called the slip. It is usually expressed in percentage.

If N_S = Synchronous speed of the stator field and N = Actual speed of the rotor;

Then Slip, $$S = \frac{N_S - N}{N_S} \times 100 \qquad ...(10.1)$$

and Slip speed $= N_S - N$

From equation (10.1), the rotor speed or actual speed of rotor is given by

$$N = N_S(1 - S)$$

Important Point: $N < N_S$

Significance of slip: When the motor is at standstill (that is, at rest),

$$N = 0$$

and $$S = 1$$

Thus, the slip will never be zero as the actual speed of rotor will never be equal to the synchronous speed of stator flux because, the rotor starts rotating to reduce the relative speed between the rotating stator flux and the stationary rotor conductors. Thus, induction motors are also called as the asynchronous motors.

10.5 FREQUENCY OF ROTOR EMF OR ROTOR CURRENT UNDER RUNNING CONDITION

When the stator is connected to three-phase supply having frequency of f in Hz and the rotor is yet stationary, the frequency of rotor emf and rotor current is same as supply frequency, that is, f Hz.

But when the rotor starts rotating, frequency of rotor emf and rotor current, f_r in Hz depends upon the relative speed or slip speed.

Let, N = Actual speed of rotor

p = Number of poles

Synchronous speed of stator field is given by

$$N_S = \frac{120\,f}{p} \qquad \text{...(10.2)}$$

Then $$N_S - N = \frac{120\,f_r}{p} \qquad \text{...(10.3)}$$

By definition of Slip, $$S = \frac{N_S - N}{N_S}$$

Substituting equations (10.2) and (10.3) in the above equation,

$$S = \frac{120\,f_r}{p} \times \frac{p}{120\,f}$$

$$= \frac{f_r}{f}$$

or $$f_r = S \times f$$

Therefore, the frequency of rotor emf or rotor current under running condition is equal to the slip times the supply frequency.

Solved Examples

Example 10.1 *The frequency of the emf in the stator of 4 pole induction motor is 50 Hz, and that in rotor 1.5 Hz. What is the slip and what speed is the motor is running?*

Solution: Given: $p = 4$, Frequency of emf in stator, $f = 50$ Hz and Frequency of rotor, $f_r = 1.5$ Hz

Actual speed, $N = N_S (1 - S)$

Synchronous speed, $N_S = \dfrac{120\,f}{p} = \dfrac{120 \times 50}{4} = 1500$ rpm

Slip, $S = \dfrac{f_r}{f} = \dfrac{1.5}{50} = 0.03$

∴ $N = 1500\,(1 - 0.03) = \mathbf{1445\ rpm}$

Example 10.2 *A 4 pole, three-phase induction motor operates from a supply whose frequency is 50 Hz. Calculate: (i) The speed at which the magnetic field of the stator is rotating; (ii) The speed of the rotor when the slip is 0.04; (iii) The frequency of the rotor currents when slip is 0.03; (iv) The frequency of the rotor currents at standstill, with the reason behind it.*

Solution: Given: $p = 4$ and $f = 50$ Hz,

(i) The speed of the stator magnetic field is given by

$$N_S = \frac{120\,f}{p} = \frac{120 \times 50}{4} = \mathbf{1500\ rpm}$$

(ii) Speed of rotor when slip = 0.04;

$$N = N_S (1 - S) = 1500\,(1 - 0.04) = \mathbf{1440\ rpm}$$

(iii) Frequency of rotor current when slip = 0.03;

$$f_r = S \times f = 0.03 \times 50 = \mathbf{1.5\ Hz}$$

(iv) Frequency of rotor current at standstill, f = **50 Hz**

[**Reason:** When the stator connected to three-phase supply having frequency of f in Hz and rotor is yet stationary, the frequency of rotor emf and rotor current is same as supply frequency, i.e. f Hz.]

Example 10.3 *A 4 pole, 50 Hz three-phase induction motor has a slip of 1% at no load. When operated at full-load, the slip is 2.5%. Find the change in speed from no load to full-load.*

Solution: Given: $p = 4$, $f = 50$ Hz

No-load slip $= S_0 = 1\% = 0.01$

Full-load slip $= S_f = 2.5\% = 0.025$

Change in speed = (No-load speed) – (Full-load speed)

$= N_0 - N$

Now no-load slip, $S_0 = \dfrac{N_s - N_0}{N_s}$

i.e., $0.01 = \dfrac{N_s - N_0}{N_s}$

where Synchronous speed, $N_S = \dfrac{120\,f}{p} = \dfrac{120 \times 50}{4} = 1500$ rpm

$\therefore$ $0.025 = \dfrac{1500 - N}{1500}$

or $N_0 = 1485$ rpm

To find actual speed of rotor;

Full load slip, $S_f = \dfrac{N_s - N}{N_s}$

i.e., $0.025 = \dfrac{1500 - N}{1500}$

or $N = 1463$ rpm

$\therefore$ Change in speed = 1485 – 1463 = **22 rpm**

Example 10.4 *An 8 pole alternator runs at 750 rpm, and supplies power to a 6 pole, three-phase induction motor which runs at 970 rpm What is the slip of induction motor?*

Solution: Given: Number of poles of Alternator, $p_A = 8$

Speed of Alternator, $N_A = 750$ rpm

Number of poles of induction motor, $p_m = 6$

Speed of induction motor, $N_m = 970$ rpm

Slip, $S = \dfrac{N_S - N_m}{N_S}$

Given that alternator supplies power to induction motor,

$\therefore$ frequency of supply to induction motor = frequency of alternator output = f

Considering the Alternator, $$f = \frac{p_A N_A}{120} = \frac{8 \times 750}{120} = 50 \text{ Hz}$$

Synchronous speed, $$N_S = \frac{120 \times 50}{6} = 1000 \text{ rpm}$$

∴ $$S = \frac{1000 - 970}{1000} = \mathbf{0.03}$$

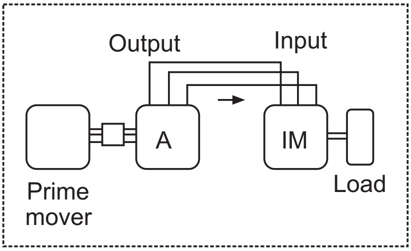


Fig. 10.5

Example 10.5 *A 6 pole alternator running at 1200 rpm supplies a 10 pole three-phase induction motor. If the rotor emf make 3 alterations per second, find the rotor speed.*

Solution: Given: Number of poles of Alternator = $p_A = 6$.

Speed at which Alternator is driven = $N_A = 1200$ rpm

Number of poles of Induction motor = $p_m = 10$

Frequency of rotor emf, $f_r = 3$ alterations/second

Speed of Induction motor is given by

$$N = N_S(1 - S)$$

where $$N_S = \frac{120 f}{p_m}$$

Given that alternator supplies power to induction motor,

∴ frequency of supply to induction motor = f = frequency of alternator output.

Considering the Alternator, $$f = \frac{p_A N_A}{120} = \frac{6 \times 1200}{120} = 60 \text{ Hz}$$

∴ $$N_S = \frac{120 \times 60}{10} = 720 \text{ rpm}$$

Slip, $$S = \frac{f_r}{f} = \frac{3}{60} = 0.05$$

∴ $$N = 720\,(1 - 0.05)$$
$$= \mathbf{684 \text{ rpm}}$$

Example 10.6 *The power input to a 2000 V, 50 Hz, three-phase motor running on full-load at an efficiency of 90% is measured by two Wattmeters which indicate 300 kW and 100 kW respectively. Calculate (i) the input, (ii) the pf, (iii) the line current and (iv) HP Output.*

Solution: Given: Input voltage = $V_L = 2000$ volt, $f = 50$ Hz

Efficiency, $\eta = 90\% = 0.9$

$W_1 = 300$ kW and $W_2 = 100$ kW

(i) Input, $P = W_1 + W_2$

$= 300 + 100 =$ **400 kW**

(ii) Power factor, $pf = \cos\phi$

where $$\phi = \tan^{-1}\frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2}$$

$$= \tan^{-1}\frac{\sqrt{3}(300-10)}{300+100} = \tan^{-1}(0.866) = 40.89^0$$

$\therefore$ $pf = \cos(40.89°) =$ **0.755**

(iii) Line current,

$$I_L = \frac{p}{\sqrt{3}\,V_L\cos\phi} \qquad \text{(Because } P = \sqrt{3}\,V_L I_L \cos\phi\text{)}$$

$$= \frac{400 \times 1000}{\sqrt{3} \times 2000 \times 0.755} = \mathbf{152.9\ A}$$

(iv) Output in kW, $P =$ Input in kW $\times \eta = 400 \times 0.9 = 360$ kW

$\therefore$ $$\text{Output in HP} = \frac{360 \times 1000}{735.5}$$

$=$ **489.46 HP**

Advantages of induction motors:

1. Simple in construction,
2. Reliable,
3. Efficient,
4. Less maintenance and
5. Low cost.

Disadvantages of induction motors:

1. Difficult to achieve speed variation (in dc motor, speed variation is very easy)
2. Speed reduces with increase in load to some extent,

3. Less starting torque (as compared to dc motor) and
4. As it is an induction machine, it operates at lagging power factor.

Comparison between three-phase induction motor and transformer

Similarities:

1. The stator and rotor of three-phase induction motor is similar to primary and secondary of transformer respectively, because electrical power is transformed from stator to rotor by induction. Hence it is called as induction motor.
2. When the stator is energized by three-phase supply and the rotor is yet stationary, induction motor exactly resembles a transformer with its secondary short circuited.

Differences:

As the three-phase induction motor is a rotating machine, the nature of load applied is mechanical whereas the transformer is a static device, its secondary is electrically loaded.

10.6 NECESSITY OF STARTER

At the time of starting, if an induction motor is directly connected to the supply, it draws excessive initial current (about 5 to 7 times) the full load current and produces only 1.5 to 2.5 times the full-load torque. This large starting current is objectionable as it causes large line drop and affects the operation of other interconnected apparatus. Hence starter is must for an induction motor.

Types:

- DOL starter
- Star-delta Starter
- Autotransformer Starter

10.7 STAR-DELTA STARTER

Construction

It consists of a changeover switch, which is in the form of double-throw type (with interlocks to prevent motor starting with the switch in the RUN position)

It connects the stator winding in Star at the instant of Starting and Delta for normal running. It is also provided with an Over load relay, protection against low voltage and Stop button switch for manual operation.

Working

At the instant of starting, the changeover switch is thrown to Star. For Star connection, applied voltage is given by

$$V_L = \sqrt{3}\, V_{ph}$$

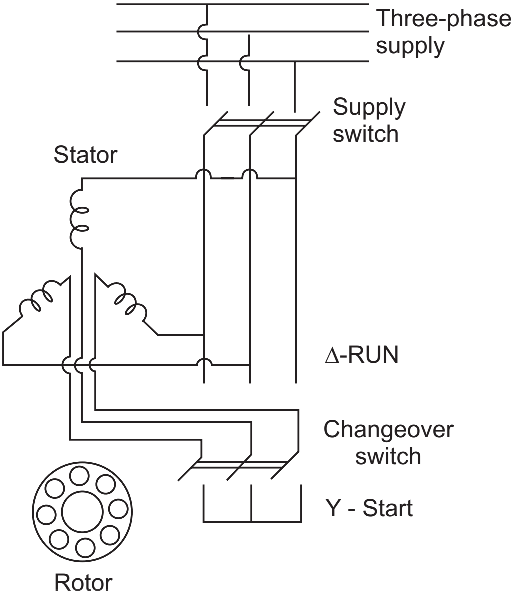


Fig. 10.6 Star-delta starter

Then the applied voltage across each phase of stator winding is reduced, which is given by

$$V_{ph} = \frac{V_L}{\sqrt{3}}$$

Starting line current drawn due to Star-Delta Starter = $\frac{I_{St}}{\sqrt{3}}$

Starting line current that would have drawn if connected directly across supply $= \sqrt{3}\ I_{St}$

Taking the ratio; $\frac{I_{St}/\sqrt{3}}{\sqrt{3}\ I_{St}} = \frac{1}{3}$

$\therefore$ Starting line current drawn by Star-Delta Starter is given by

$\frac{1}{3}$ × Starting line current that would have been drawn if connected directly across supply

As the torque, $T \propto V^2$

The starting torque is reduced to

$$\left(\frac{1}{\sqrt{3}}\right)^2 = \frac{1}{3}$$

$$= 33.33\% \text{ of the normal torque}$$

As the motor attains speed, then the changeover switch is thrown to Delta for normal RUN operation.

Advantages:

1. Absolutely no power loss.
2. User friendly and
3. Economical.

Applications: This starter is used for squirrel Cage Induction motors, which are built to run with delta connected stator winding.

10.8 CONCEPT OF SINGLE-PHASE INDUCTION MOTOR

It consists of a stator and a rotor. The stator is provided with single-phase winding and the rotor may be of cage type.

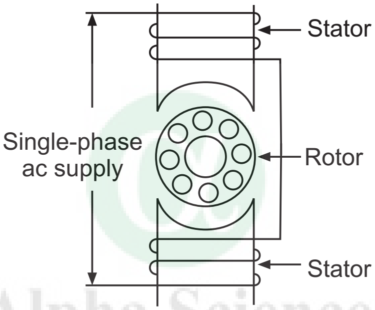


Fig. 10.7 Single-phase induction motor

When the stator is fed with single-phase ac supply, its stator winding produces an alternating flux. The alternating or pulsating flux acting on a cage rotor cannot produce rotation. Thus, single-phase induction motor is not self starting. If an initial start is given, then a torque will be developed and motor starts rotating. This behavior is explained by ‘Double Field Revolving Field’.

To make the single-phase induction motor self starting, it is temporarily converted into a two-phase motor at the time of starting. It is provided with an extra winding known as starting or auxiliary winding in addition to the main winding. These windings are spaced 90° electrically apart. As the phase difference between the current in the two windings is 90° the motor behaves like a two-phase motor. These two currents produce a revolving flux, the motor becomes self starting.

Types of single-phase induction motors:

1. Split phase motor
2. Capacitor start induction run motors
3. Capacitor start and capacitor run motors
4. Shaded pole motor

Types of single-phase ac motors:

1. Repulsion motor
2. AC Series motor
3. Unexcited synchronous motor
   (a) Reluctance motor and
   (b) Hysterisis motor

Universal Motor: By special design, fractional *hp* series motor, which operates satisfactorily either on 50 Hz ac supply or dc supply and hence called the universal motor. It operates at approximately same speed and output on either ac or dc supply. It consists of two salient poles. The entire magnetic path is laminated as it operates on ac. The armature is wound type having laminated core, commutator and brushes.

Multiple Choice Questions

Choose the Correct Answer

1. An induction motor works with ....

(a) dc only (b) ac only

(c) Both ac and dc only (d) none of these

2. When a three-phase supply is given to the stator of a three-phase induction motor, a ......magnetic field is produced.

(a) stationary (b) alternating

(c) rotating (d) none of these

3. The air-gap between the stator and rotor of a three-phase induction motor ranges from

(a) 0.4 mm to 4 mm (b) 1 cm to 2 cm

(c) 2 cm to 4 cm (d) 4 cm to 6 cm

4. If ϕ_m is the maximum value of flux due to any one of the three-phases in an induction motor, the resultant flux ϕ_r at any instant is

(a) $\frac{5}{2}\phi_m$ (b) $\frac{3}{2}\phi_m$

(c) $\frac{2}{3}\phi_m$ (d) $\frac{1}{2}\phi_m$

5. The rotor of an induction motor revolves in .... direction of stator flux.

(a) same (b) opposite

(c) not determinable (d) none of these

6. If two of the supply lines of a three-phase induction motor are reversed, then ....
   (a) the speed increases
   (b) the speed decreases
   (c) the direction of rotation is reversed
   (d) motor stops
7. The rotor of a three-phase induction motor rotates in the same direction as that of stator rotating field. This can be explained by
   (a) Faraday's Laws of EMI (b) Lenz's Law
   (c) Newton's Law of motion (d) Fleming's Right-hand rule
8. The frame of induction motor is usually made of ......
   (a) Silicon steel (b) Cast iron
   (c) Aluminium (d) Bronze
9. If the rotor terminals of a three-phase slip ring induction motor are not short-circuited and supply is given to the stator, the motor will
   (a) not start (b) start running
   (c) run at high speed (d) run at low speed
10. Compared to a slip ring induction motor, the starting torque of a squirrel cage induction motor is
   (a) same (b) high
   (c) very high (d) medium
11. Phase wound induction motors are less extensively used than squirrel cage induction motor because,
   (a) slip rings are required on the rotor circuit
   (b) rotor windings are generally star connected
   (c) they are costly and require greater maintenance
   (d) none of these
12. External resistance is connected to the rotor of a three-phase wound induction motor in order to
   (a) reduce starting current
   (b) collector current
   (c) to connect as a star connected load
   (d) none of these
13. The number of poles in a three-phase induction motor is determined by ....
   (a) supply frequency (b) motor speed
   (c) supply voltage (d) both (a) and (b)

14. In a three-phase induction motor, the slip speed is given by .....
    (a) N_S
    (b) N
    (c) $N_S - N$
    (d) $N - N_S$
15. Slip of an induction motor at standstill is .....
    (a) zero
    (b) one
    (c) infinity
    (d) none of these
16. Slip of an induction motor at standstill is .....
    (a) zero
    (b) unity
    (c) greater than unity
    (d) negative
17. When the rotor of a three-phase induction motor is blocked, the slip is .....
    (a) zero
    (b) 0.5
    (c) 0.1
    (d) 1
18. The rotor of three-phase induction motor always runs at
    (a) synchronous speed
    (b) less than synchronous speed
    (c) more than synchronous speed
    (d) none of these
19. Synchronous speed of three-phase induction motor is given by
    (a) $N_S = 120 f P$
    (b) $N_S = 120 f / P$
    (c) $N_S = 120 P/f$
    (d) $N_S = f P/120$
20. The frequency of rotor current or emf is given by
    (a) $f_2 = sf_1$
    (b) $f_2 = 000f_1/s$
    (c) $f_2 = (1 - S) f_1$
    (d) $f_2 = s/f_1$

21. The difference between synchronous speed and actual speed is 100 rpm and synchronous speed is 1500 rpm, then the value of slip is
    (a) 2%
    (b) 10%
    (c) 6.66%
    (d) 15%
22. A supply of 50 Hz is given to a three-phase IM having 4 poles. If the IM runs at 1440 rpm, the slip is .....
    (a) 3%
    (b) 4%
    (c) 5%
    (d) 3.33%
23. An induction motor under full-load has a slip of about ....
    (a) 0.03
    (b) 0.1
    (c) 0.3
    (d) zero
24. Normal speed of a three-phase, 400 V, 50 Hz induction motor can be ....
    (a) 1455 rpm
    (b) 1550 rpm
    (c) 1500 rpm
    (d) 1050 rpm

25. A 4 pole, 50 Hz induction motor runs at a speed of 1440 rpm. The frequency of the rotor induced emf is ....

    (a) 3 Hz (b) 2.5 Hz
    (c) 2 Hz (d) 4 Hz

26. If a 4 pole induction motor has a synchronous speed of 1500 rpm, the frequency of supply to the stator is

    (a) 25 Hz (b) 50 Hz
    (c) 100 Hz (d) 12.5 Hz

27. Induction motor works at

    (a) lagging *pf* (b) leading *pf*
    (c) *upf* (d) zero *pf*

28. In large induction motors starter is necessary because

    (a) stating torque is very high
    (b) motor takes 5-7 times its full-load current
    (c) it runs at dangerously high speed
    (d) it may run in reverse direction

Answers:—

1. (b) 2. (c) 3. (a) 4. (b) 5. (a) 6. (c) 7. (b) 8. (b) 9. (a) 10. (d)
11. (c) 12. (a) 13. (d) 14. (c) 15. (b) 16. (b) 17. (d) 18. (b) 19. (b) 20. (a)
21. (c) 22. (b) 23. (a) 24. (a) 25. (c) 26. (b) 27. (a) 28. (b)

Review Questions

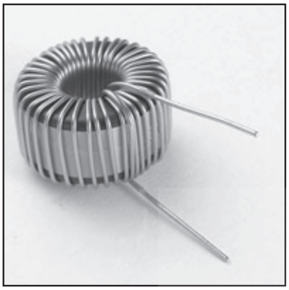
1. What do you mean by rotating magnetic field? Explain the production of torque in a three-phase induction motor.
2. Explain the constructional features of induction motor.
3. Explain the principle of operation of a three-phase induction motor and give reasons for ‘an induction motor can not run at synchronous speed’.
4. Define slip. Derive an expression for frequency of rotor emf.
5. What is slip in an induction motor? Why slip is never zero in an induction motor?
6. Why three-phase induction motors are called asynchronous motors? Explain the principle of operation of three-phase induction motor.

7. With the help of neat figures of stator and rotors, explain construction of a squirrel cage and slip ring induction motor.
8. What are the advantages and disadvantages of three-phase induction motor?
9. Discuss the application of Squirrel cage and Slip ring induction motor.
10. Bring out the comparison between three-phase induction motor and transformer.
11. Why are starters necessary for an induction motor? With a neat diagram explain a star delta starter for a three-phase induction motor.

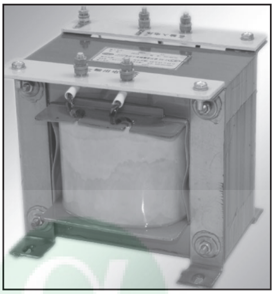
Exercises

1. A three-phase, 4-pole, 440 V and 50 Hz induction motor runs with a slip of 4%. Find the rotor speed and frequency of induced emf.
   (**Ans:** N =1440 rpm and $f_r = 2$ Hz)
2. A three-phase, 6 pole, 60 Hz induction motor has a slip of 3% at full-load. Find the synchronous speed and frequency of rotor current at full-load.
   (**Ans:** $N_S = 1200$ rpm, $f_r = 1.8$ Hz)
3. The frequency of the emf in the stator of a 4-pole induction motor is 50 Hz and in the rotor is $1\frac{1}{2}$ Hz. What is the slip and at what speed is the motor running?
   (**Ans:** S = 3%, N = 1455 rpm)
4. A 6 pole alternator is driven at 1200 rpm; (i) What is the frequency of the generated emf? (ii) If this alternator supplies power to a 10 pole induction motor, find its speed when slip is 3%.
   (**Ans:** $f = 60$ Hz, N = 698 rpm)
5. A three-phase induction motor has 6 poles and runs at 960 rpm on full-load. It is supplied from an alternator having 4 poles and running at 1500 rpm. Calculate the full-load slip of the motor.
   (**Ans:** S = 4%)
6. If the electromagnetic force in the stator of an 8-pole induction motor has a frequency of 50 Hz and that in rotor 1.5 Hz, at what speed is the motor running and what is the slip?
   (**Ans:** N = 728 rpm and S = 3%)
7. A 10 pole induction motor is supplied by a 6-pole alternator, which is driven at 1200 rpm If the motor runs with a slip of 3%, what is its speed?
   (**Ans:** N = 698 rpm)

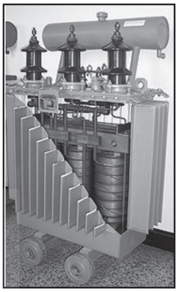
Appendix



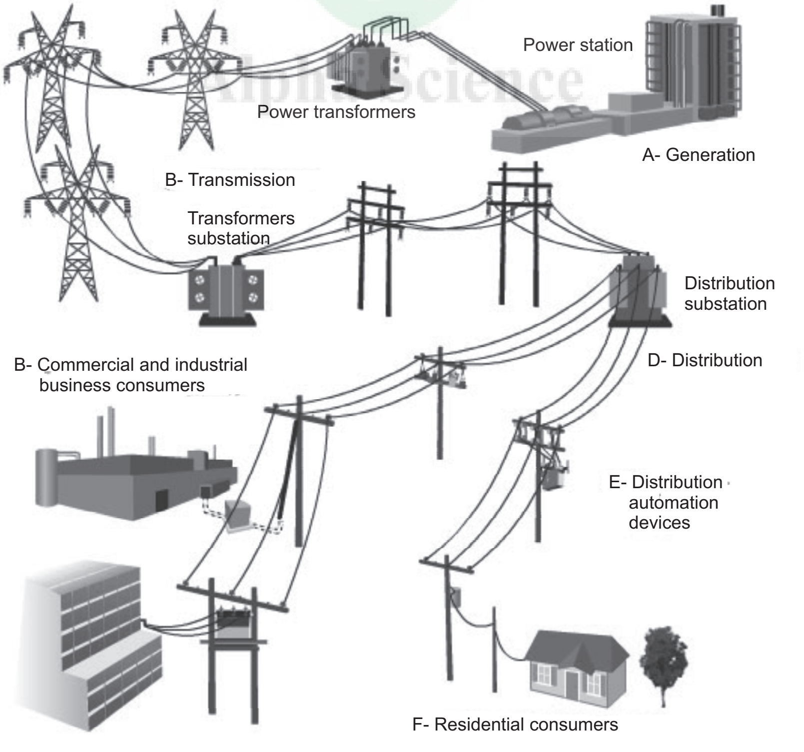
Choke



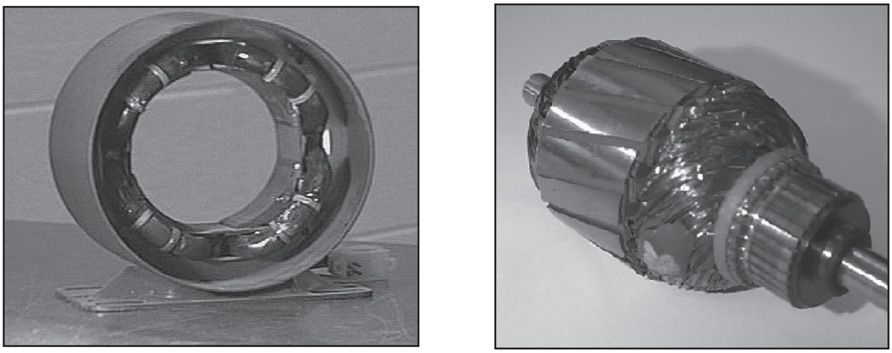
Single-phase Transformer



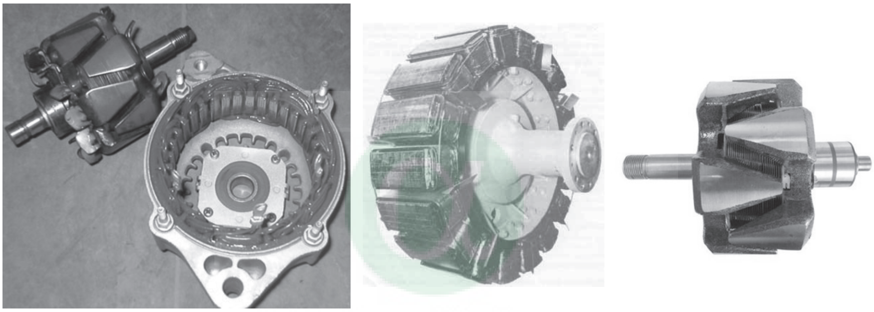
Three-phase Transformer



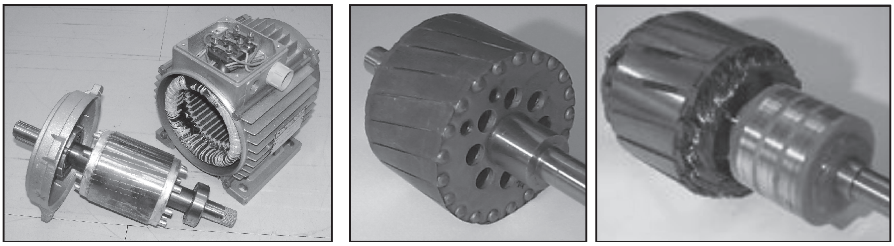
Application of Transformer in Transmission and Distribution



DC Machines: Field and Armature



Alternator: Stator, Salient Pole Rotor and Smooth Cylindrical Rotors



Three-phase Induction Motor: Stator, Squirrel Cage Rotor and Slip Ring Rotors

Index