


Practical Applications of Engineering Economics

Kal Renganathan Sharma

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Contents

Dedication

This book is dedicated
to my eldest son
R Hari Subrahmanyan Sharma
(alias Ramkishan)
who turned nine on Aug-13-2010.

About the Author

Kal Renganathan Sharma serves as Adjunct Professor of Chemical Engineering at the Roy G. Perry College of Engineering at Prairie View A & M University, Prairie View, TX. He instructs CHEG 2003 Economic Analysis and Technical Applications. He received his B.Tech. from Indian Institute of Technology, Chennai, India, in 1985, MS and Ph.D degrees in 1987 and 1990 from West Virginia University; Morgantown, WV, all three degrees in chemical engineering. He received the once in a school time prize from nursery to higher secondary school for the Best All Round Student of the Penultimate year from the Rev. Brothers of St. Gabriel at RSK Higher Secondary School, Trichy; India. He has held a number of high level academic positions in India and USA.

Dr. Sharma, KR, is the author of 10 books, 4 Book-Chapters, 19 journal articles, 503 conference papers and 108 other presentations. He has instructed over 2500 students in 83 semesterwise courses in India and USA over the past 15 years.

Honors received include Honorary Fellowship, Australian Institute of High Energetic Materials, Melbourne, Australia, 2010, Fellow, Indian Chemical Society; Kolkata, India, 2007, Who's Who in America, 2009–2010, Marquis Who's Who, Upper Saddle River, NJ, International Biographical Center, Cambridge, UK, 2007, member, New York Academy of Sciences, New York, NY, 1996, Phi Kappa Phi, 1990, Sigma Xi, 1995, cash award, SASTRA University, Thanjavur, India, 2005, cash award, Monsanto, Indian Orchad, MA, 1990, Who's Who in Plastics and Polymers, CRC Press, Baco Raton, FL, 2000. He is a naturalized citizen of USA.

Preface

The economy has changed rapidly. Both the nation's economy and the world economy has undergone changes since the World War II. The end of cold war has given impetus to rise of globalization. China and India are now invited to attend G20 meetings. Engineering education imparts a variety of skills to the student. Skills from economics can be synergistically applied. The engineering economy is a field of endeavor that explains different methods to evaluate alternates available to the business owner. Engineering Economy is the study of the feasibility and evaluation of the cost of possible solutions to engineering problems. When benefits outweigh costs the alternate becomes a acceptable one. The lowest cost among alternates can be selected by using different methods discussed in detail in the textbook. This is calculated at a certain interest rate over a certain prescribed period of time.

The inverse problem of determining the interest rate, i, given the F, A and N is explained in detail in this work. The binomial series expansion of $(1+i)^N$ is used to obtain mathematical expressions. N can be estimated given the future amount, F, uniform series of payments, A and interest rate i by obtaining the logarithms where necessary.

The fundamental principles, concepts and methods of engineering economy are provided. Current trends and issues are reflected in the worked examples and end of chapter exercises. Alnernates are drawn from case studies. These include the Keystone pipeline project, CW of World Trade Center, six different methods to prepare bioethanol, biodiesel plant in Taiwan, micro power plant, combined cycle power plant, meglumine antimonoate drug from supercritical extraction, on-shore oil well, solar hats, stirling dish solar power plant, photovoltaic panel solar power plant, liquefaction plants, copper chlorine thermochemical cycle for hydrogen, energy efficient windows, continuous mass polymerization process to manufacture ABS, renovation of Macy's department store, heap leaching and agitation leaching of gold, rechargeable batteries for electric car, PEVs

and HEV, hybrid electric vehicles, four different cooling tower designs, oil refinery, economics of life insurance, social security, printing press vs. e-book, anti-allergic cream, sequestration by dimethyl carbonate formation, microfiltration, SWRO, sea water reverse osmosis plant, catalytic process to manufacture CNTs, carbon nanotubes, hexane extraction of rice bran oil, municipal garbage collection truck, activated carbon from bamboo, polypropylene gas manufacture, custom foam fabricator, car rental, coffee house, frozen yoghurt parlor, short path distillation, olive oil business, bonds from private sector, Chevy volt, public internet, gas sales and convenience store, condominiums at Galveston beach, outpatient drug treatment center, overseas travel ticket, snack factory. Break-even points in flower shop, soda beverage, cars, quandratic price-demand relations for tomatoes, peaches, inelastic supply of common salt, dualistic relations of price and demand, Le-Chatellier-Samuelson principle are discussed in detail. G20 nations and GDP estimates by PPP, purchasing power parity and the changing world order in economies is touched upon. Life-cycle costs of aviation lighting, LED, photocopier, HVAC systems are discussed. Depreciation, taxes, replacement analysis are discussed.

Unique Features

Optimization concepts are discussed in detail. Brainstorming can be used to develop alternates for a given problem.

- The five methods of analysis, PW, FW; AW, IRR and ERR can be used to evaluate the alternates. Evaluation of alternates can be used to invest in the more profitable process, selection of the lower cost alternate, quantitate the savings accrued from implementation of a process improvement and assessment of environmental impact of chemical processes. Alternates can be mutually exclusive, independent projects and screening by other criteria.
- When the PWs of both alternates are within 20% of each other the projects can be adjudged as too close to call within the sensitivity of the estimates of the capital and operating expenses and expected revenues.

- Rudiments of household finance are discussed.
- The capitalization worth factors and sinking fund factors are calculated and presented as annuity tables as well as in graphical form.
- In addition to P/A, F/A, A/F, A/P, F/P, P/F closed form analytical expressions to calculate the interest rate, i and pay period, N has been developed.
- Benefit/Cost Ratio of public sector projects are discussed.

CHAPTER 1

Replacement Analysis and Depreciation

Topics Covered

- Straight-Line Method
- Declining Balance Method
- Units of Production Method
- MACRS
- Property Class, Class Life
- Replacement Analysis
- EUAC

1.1 Overview

***D**epreciation* can be used for accounting purposes. It can be used to establish an equivalent series of annual deductions to the value of an asset acquired as a single payment or transaction. Depreciation can also serve as a critical item in the determination of the ATCF (*After-Tax Cash Flow*). Estimation of the ATCF (after-tax cash flows) and *tax consequences* on decision analysis are described in detail in Chapter 2.0. Clever accountants can associate their depreciation calculations of assets to real-time deterioration of physical properties of the asset. For example, as soon as a new car rolls out of the dealership, it depreciates in value. Every year the book value of the car depreciates by a certain amount. The end of the useful life of the car occurs at, say, 6.67 years for an automobile with a 4-cylinder engine, or perhaps 10 years for an automobile with a 6-cylinder engine, or maybe 13.33 years for a car with an 8-cylinder engine.

Depreciation is a noncash cost. It is a deduction from before-tax income. The purpose of the use of depreciation is to establish the equivalence of the portion of the asset's value that is used in the production of income in a given year as annual deductions. A property or piece of equipment is said to be depreciable if the following criteria are met:

(i) it is used in business or used to bring in income;
(ii) it has a definite useful life greater than six months;
(iii) it is subject to wear and tear upon repeated use or deteriorates under natural causes;
(iv) it is not inventory, investment assets, in trade stock, etc.;
(v) it is physically tangible, such as a computer or intangible, such as IPR (intellectual property rights);
(vi) a depreciation schedule can be drawn up according to the equivalence establishment between the asset's value and annual deductions and/or the physical condition of the asset.

Cost Basis (Adjusted) is the amount that is used as a basis for further calculations. This is after the adjustments for allowable increases or decreases. For example, an addition of a second floor to a single-story house with a useful life greater than six months results in an increase in the initial cost basis. A loss by theft or hazard or casualty decreases the original value.

The amount paid to purchase an asset is called the *initial cost basis*. The sales tax and transportation costs of acquisition of the asset can be included in this basis. Any cost incurred that is needed to commission the asset can be included here. This is also referred to as *unadjusted cost basis*.

The *book value* of an asset is the current worth of the object as shown by the accountant of the enterprise. It is obtained from the initial cost basis after subtracting the depreciation deductions. The book value can also be looked at as the capital that remains in the asset that can be recovered during the useful life of the equipment, building, or object. The book value and market value of an item may be different from each other. The *market value* of an item is the amount the buyer is willing to pay for that item from the seller. The *salvage value* is the estimated market value of the item at the end of the study period. This can appear as a cash inflow

in cash flow diagrams. The *useful life* of an equipment is the period over which the equipment is serviceable or functional.

The number of years over which an equipment or building is recovered is referred to as the *recovery period*. Usually, the useful life of the equipment is equal to the recovery period. Under a MACRS (modified accelerated cost recovery system), this period is the "property class" for the GDS (general depreciation system). It is the "class life" in the ADS (alternative depreciation system).

1.2 MACRS (Modified Accelerated Cost Recovery System)

The IRS (Internal Revenue Service) requires the use of MACRS (modified accelerated cost recovery system) for calculation of depreciation of assets [1]. All assets are categorized into classes. This ordains the period over which an asset's cost shall be recovered. MACRS was created with the TRA86, Tax Reform Act of 1986 (more discussion follows on TRA86 in Chapter 2.0). MACRS is applicable to tangible, depreciable property that was placed in service after December 31, 1986.

With the arrival of TRA86, ACRS was replaced with MACRS as the method for calculating depreciation in the United States. It is similar to ACRS. Before ACRS came about, most equipment purchases were depreciated by the method of straight line. This method allowed for the depreciation of an asset over its useful life. ACRS was in use between 1981–1986. It came about with the ERTA (Economic Recovery Tax Act) of 1981. Property classes were established in ERTA. Calculations were based on zero for salvage value. Shorter recovery periods were used to calculate annual depreciation. The result was an accelerated "write-off" of capital costs. The depreciation under ACRS was twice than that using the method of straight line.

MACRS differs from ACRS in two ways: (i) the number of property classes was expanded; and (ii) the semester convention was introduced to simplify the first and final years of the useful life of the item. MACRS allows for faster depreciation of capital assets. This can stimulate purchase of capital equipment driven by the lower after-tax present worth of the equipment. The tax deductible depreciation expense can be taken sooner.

This results in an increase in the present worth of the capital equipment. The enterprise is allowed to retain more of its cash inflows or receipts earlier in the depreciation cycle. It is called a "front-loaded" tax benefit.

There are two systems under MACRS for calculating depreciation deductions. The main system is called the GDS (*General Depreciation System*) and the alternative system is called an *Alternative Depreciation System*. Generally, GDS is used unless the law specifically requires the use of ADS. Longer recovery periods can be realized using the ADS method. The method of straight line is used in ADS for depreciation. Tax-exempt property and capital equipment used overseas are examples of items whose depreciation has to be according to ADS. Items qualified under GDS can be depreciated using ADS by suitable election. Each system differs in the class life of the item. For example, office equipment (Table 1.1) is depreciated with a class life of seven years, whereas automobiles are depreciated with a class life of five years and industrial steam is depreciated over a class life of 15 years. The MACRS classes have a predetermined schedule, which is used to determine the fraction of the asset's cost that is depreciated each year (Tables 1.1–1.4).

- The 3-, 5-, 7-, and 10-year classes use 200% and the 15- and 20-year classes use 150% declining balance depreciation.
- All classes convert to straight-line depreciation in the optimal year, shown with an asterisk (*).
- A half-year depreciation is allowed in the first and last recovery years.
- If more than 40% of the year's MACRS property is placed in service in the last three months, then a mid-quarter convention must be used with depreciation tables that are not shown here.

The useful life is 39 years for nonresidential real property. Depreciation is straight line using the mid-month convention. Thus, a property placed in service in January would be allowed 11½ months' depreciation for Recovery Year 1.

Example 1.1 Depreciation Charge of Offshore Drilling Assets

Calculate the depreciated charge in Tax Year 3 of offshore drilling equipment with an *unadjusted cost basis* of $100 million.

Table 1.1 MACRS "Property Class" and "Class Life"

IRS Asset Classes	Asset Description	ADS Class Life	GDS Class Life
00.11	Office furniture, fixtures, and equipment	10	7
00.12	Information systems: computers/ peripherals	6	5
00.22	Automobiles, taxis	3	5
00.241	Light general-purpose trucks	4	5
00.25	Railroad cars and locomotives	15	7
00.40	Industrial steam and electric distribution	22	15
01.11	Cotton gin assets	10	7
01.21	Cattle, breeding or dairy	7	5
13.00	Offshore drilling assets	7.5	5
13.30	Petroleum refining assets	16	10
15.00	Construction assets	6	5
20.10	Manufacture of grain and grain mill products	17	10
20.20	Manufacture of yarn, thread, and woven fabric	11	7
24.10	Cutting of timber	6	5
32.20	Manufacture of cement	20	15
20.1	Manufacture of motor vehicles	12	7
48.10	Telephone distribution plant	24	15
48.2	Radio and television broadcasting equipment	6	5
49.12	Electric utility nuclear production plant	20	15
49.13	Electric utility steam production plant	28	20
49.23	Natural gas production plant	14	7
50.00	Municipal wastewater treatment plant	24	15
80.00	Theme and amusement park assets	12.5	7

The GDS class life of offshore drilling assets from Table 1.1 can be seen as five years.

From Table 1.2 for Tax Year 3, the applicable percentage for five-year life equipment is 19.2%.

Depreciation Charge = 0.192*$100 = $19.2 million.

Table 1.2 Property Class of Personal Properties

Property Class	Personal Property (all property except real estate)
3-year property	Special handling devices for food and beverage manufacture Special tools for the manufacture of finished plastic products, fabricated metal products, and motor vehicles Property with ADR class life of 4 years or less
5-year property	Information systems; computers/peripherals Aircraft (of non-air transport companies) Computers Petroleum drilling equipment Property with ADR class life of more than 4 years and less than 10 years
7-year property	All other property not assigned to another class Office furniture, fixtures, and equipment Property with ADR class life of more than 10 years and less than 16 years
10-year property	Assets used in petroleum refining and certain food products Vessels and water transportation equipment Property with ADR class life of 16 years or more and less than 20 years
15-year property	Telephone distribution plants Municipal sewage treatment plants Property with ADR class life of 20 years or more and less than 25 years
20-year property	Municipal sewers Property with ADR class life of 25 years or more
Property Class	**Real Property (real estate)**
27.5-year property	Residential rental property (does not include hotels and motels)
39-year property	Nonresidential real property

The information needed to calculate the depreciation deduction under MACRS are: (i) unadjusted cost basis; (ii) when the property was placed in service; (iii) property class and recovery period; (iv) method used for depreciation, GDS or ADS under MACRS; and (v) quarter, semester, or annual.

1.3 Methods of Depreciation

1.3.1 Method of Straight Line

The method of *straight line* for calculation of depreciation is simple in construct. A uniform amount is deducted every year over the useful life of the capital equipment.

Table 1.3 MACRS Applicable Percentage for Property Class

Recovery Year	3-Year Property	5-Year Property	7-Year Property	10-Year Property	15-Year Property	20-Year Property
1	33.33	20.00	14.29	10.00	5.00	3.750
2	44.45	32.00	24.49	18.00	9.50	7.219
3	14.81 *	19.20	17.49	14.40	8.55	6.677
4	7.41	11.52 *	12.49	11.52	7.70	6.177
5		11.52	8.93 *	9.22	6.93	5.713
6		5.76	8.92	7.37	6.23	5.285
7			8.93	6.55 *	5.90 *	4.888
8			4.46	6.55	5.90	4.522
9				6.56	5.91	4.462 *
10				6.55	5.90	4.461
11				3.28	5.91	4.462
12					5.90	4.461
13					5.91	4.462
14					5.90	4.461
15					5.91	4.462
16					2.95	4.461
17						4.462
18						4.461
19						4.462
20						4.461
21						2.231

$$d_j = \frac{B - S}{N} \tag{1.1}$$

Where dj is the depreciation deduction in the jth year, B is the unadjusted cost basis, N is the useful life of the capital equipment, and S is the salvage value. Further, the book value BVj at the end of year j can be seen to be

$$BV_j = B - d_j^* = B - \sum_{k=1}^{j} d_k \tag{1.2}$$

Table 1.4 MACRS Percentage for Real Property (Real Estate) Table

Recovery Year	Month 1	Month 2	Month 3	Month 4	Month 5	Month 6	Month 7	Month 8	Month 9	Month 10	Month 11	Month 12
1	2.461	2.247	2.033	1.819	1.605	1.391	1.177	0.963	0.749	0.535	0.321	0.107
2–39	2.564	2.564	2.564	2.564	2.564	2.564	2.564	2.564	2.564	2.564	2.564	2.564
40	0.107	0.321	0.535	0.749	0.963	1.177	1.391	1.605	1.819	2.033	2.247	2.461

where dj* is the cumulative deduction through year j. The cumulative deduction through year j can be seen to be

$$d_j^* = j^*\left(\frac{B-S}{N}\right) \tag{1.3}$$

Example 1.2 Method of Straight Line for Depreciation of HRA (Helical Ribbon Agitator)

The unadjusted cost basis of an HRA (helical ribbon agitator) used in a continuous polymerization plant to manufacture ABS engineering thermoplastic is $10 million. Determine the per-year depreciation deduction using the method of straight line. Tabulate the depreciation amounts and show the book value at the end of each year. The useful life of the HRA is 15 years. The salvage value of the HRA is $1.5 million.

1.3.2 Method of Declining Balance

The method of declining balance is also called a *Matheson formula* or method of constant percentage. The annual depreciation deduction is calculated as a percentage of the book value of the capital equipment at the beginning of the year. The depreciation is a constant fraction of the book value remaining after the prior deductions. This fraction f lies between the values of 0 and 1, i.e., $0 \leq f \leq 1.0$.

$$d_j = B(1-f)^{j-1}(f) \tag{1.4}$$

The book value of the capital equipment at the end of year j can be seen to be as shown in Table 1.6.

The cumulative deduction through year j, d_j^* can be seen to be

$$d_j^* = Bf + Bf(1-f) + Bf(1-f)^2 + ... + Bf(1-f)^{j-1} \tag{1.5}$$

Eq. (1.5) is a sum of a geometric series and can be seen to be

$$d_j^* = B(1-(1-f)^j) \tag{1.6}$$

When f = 1/N, the depreciations by the method of declining balance is equal to that of the depreciation by the method of straight line.

Table 1.5 Depreciation Deductions and Book Value at EOY

EOY, j	d_j	BV_j (millions)
0	0	$10.0000
1	$566,667	$9.4333
2	$566,667	$8.8667
3	$566,667	$8.3000
4	$566,667	$7.7333
5	$566,667	$7.1667
6	$566,667	$6.6000
7	$566,667	$6.0333
8	$566,667	$5.4667
9	$566,667	$4.9000
10	$566,667	$4.3333
11	$566,667	$3.7667
12	$566,667	$3.2000
13	$566,667	$2.6333
14	$566,667	$2.0667
15	$566,667	$1.5000

Table 1.6 Depreciation and Book Value by the Method of Declining Balance

Year, j	Depreciation	Book Value
0	0	B
1	B(f)	$B(1-f)$
2	$Bf(1-f)$	$B(1-f)^2$
3	$Bf(1-f)^2$	$B(1-f)^3$
….		
j	$Bf(1-f)^{j-1}$	$B(1-f)^j$

Example 1.3 Method of Declining Balance Line for Depreciation of HRA (Helical Ribbon Agitator)

The unadjusted cost basis of an HRA (helical ribbon agitator) used in a continuous polymerization plant to manufacture ABS engineering thermoplastic is $10 million. Determine the per-year depreciation deduction

Table 1.7 Method of Declining Balance (i) f = 2.5/15

EOY, j	d_j	BV_j (millions)
0		$10.0000
1	$1,666,667	$8.3333
2	$1,388,889	$6.9444
3	$1,157,407	$5.7870
4	$964,506	$4.8225
5	$803,755	$4.0188
6	$669,796	$3.3490
7	$558,163	$2.7908
8	$465,136	$2.3257
9	$387,613	$1.9381
10	$323,011	$1.6151
11	$269,176	$1.3459
12	$224,313	$1.1216
13	$186,928	$0.9346
14	$155,773	$0.7789
15	$129,811	$0.6491

using the method of declining balance. Tabulate the depreciation amounts and show the book value at the end of each year. The useful life of the HRA is 15 years. The salvage value of the HRA is 0. The fraction f used in the calculations are

(i) 2.5/N
(ii) 2.0/N

1.3.3 Method of Units of Production

The methods of straight line and declining balance are based on time periods. This works well for capital equipment with a useful life of N years. An underlying assumption is that the capital equipment is used at a *uniform rate*. For example, Jack and Jill bought the same two new cars in the same year. Jack parked the car in his garage. Jill drove her car 25,000 miles per year. The depreciation amounts predicted by the methods of straight line and declining balance would yield the *same* deductions

Table 1.8 Method of Declining Balance (ii) f = 2.0/15

EOY, j	d_j	BV_j (millions)
0		$10.0000
1	$1,333,333	$8.6667
2	$1,155,556	$7.5111
3	$1,001,481	$6.5096
4	$867,951	$5.6417
5	$752,224	$4.8895
6	$651,927	$4.2375
7	$565,004	$3.6725
8	$489,670	$3.1829
9	$424,381	$2.7585
10	$367,796	$2.3907
11	$318,757	$2.0719
12	$276,256	$1.7957
13	$239,422	$1.5562
14	$207,499	$1.3487
15	$179,832	$1.1689

for Jack and Jill. A better method was needed to account for more usage by Jill. The method of units of production is developed with the basis of functional use rather than merely time. Thus, the depreciation per unit of production can be given as

$$UnitDepreciation = \frac{(B - S)}{\#units} \tag{1.7}$$

In the example of Jack and Jill, the depreciation will be based on *per mile* of car driven.

Example 1.4 Hexane Extraction of Rice-Bran Oil
(Please see Example 4.11)

Rice-bran lipids can be extracted using hexane solvent from rice bran at a yield of about 26%. Rice is milled to produce the white long-grain rice sold in supermarkets. The outer layers of the rice kernel are removed.

Table 1.9 Cost Data for Hexane Extraction Plant for Rice-Bran Oil

Description	Hexane Extraction Unit
Capital Investment	$220,000
Manufacturing Cost	$5.18 /year/kg of rice bran
Materials Cost	$0.22/year/kg of rice bran
Labor Cost	$9.62/hr
Revenue from Rice-Bran Oil	$8.08/kg
Revenue from Rice Protein	$4.928/kg
Production Rate	30.96 gal rice-bran oil/day
Useful Life of Equipment	10 years
Operational Hours Per Year	8332 hrs
% Protein in Rice Bran	14.6%
Interest Rate, i	3.0%

These layers include the hull, the germ, and the bran. The bran includes the testa, peripcarp, nucellus, and aleurone layer. Rice bran becomes rancid and must be discarded in landfills. Parboiled rice bran has higher lipid levels than unstabilized rice bran. Rice-bran oil consists largely of saponifiable compounds. Hexane costs about $1.15 per gallon. It is an excellent solvent for nonpolar lipids. Calculate the PW of a hexane extraction unit. The following information was obtained from the *Journal of the American Oil Chemists' Association* [2] and modified suitably for the PW analysis in Example 4.11. Use the method of declining balance to calculate the depreciable amount at a factor of 0.25.

Annual Labor Cost = 8332h*$9.62 = $80,154

Rice Bran Processed = 30.96*3.7854*365/0.26 = 164,525 kg/year

Annual Revenue = 30.96*3.7854*365*$8.08 = $345,634 (from rice-bran oil)

$$+ \$4.928*164525*0.146 = \$118{,}374 \tag{1.8}$$

$$\text{Annual Materials \& Labor Cost} = 164{,}525*(5.18+0.22) = \$888{,}436 \tag{1.9}$$

(P/A,3%,10) = 8.530203 (From Table A-7)

Table 1.10 Depreciable Amounts of Capital Equipment and Contributions to PW

Hexane Oil Extraction						
			f	0.25		
Capital Equipment Cost			$220,000			
		k	d_k	BV_k	PW_k	(P/F,3%,k)
		1	$55,000	$165,000	$53,398.07	0.970874
		2	$41,250	$123,750	$38,882.09	0.942596
		3	$30,938	$92,813	$28,312.21	0.915142
		4	$23,203	$69,609	$20,615.67	0.888487
		5	$17,402	$52,207	$15,011.42	0.862609
		6	$13,052	$39,155	$10,930.64	0.837484
		7	$9,789	$29,366	$7,740.89	0.790789
		8	$7,342	$22,025	$5,795.54	0.789409
		9	$5,506	$16,519	$4,221.90	0.766752
		10	$4,130	$12,389	$3,072.85	0.744094
				PW	($187,981.27)	

The depreciable amounts and book values at a depreciation factor of 0.25 are tabulated below in Table 1.10. The PW contribution from each of the depreciable amounts is also shown as a separate column at an interest rate of 3%.

$$PW = -\$187{,}981.27 + (-888{,}436 + 345{,}634 + 118{,}374 - 80{,}154)\ 8.530203 + 12{,}389 = -4.4798 \text{ millions.}$$

The hexane extraction of rice-bran oil is not a profitable venture. However, the PW has increased by $44,401 when the depreciation charges are taken into account by the method of declining balance every year, as opposed to a one-time charge.

1.4 Replacement Analysis

Often the question arises as to whether to keep an asset that is owned at present *in service* or consider *immediate replacement* of it. There are good reasons for considering a replacement. These reasons can range from

obsolescence as a result of the arrival of new technology, or physical deterioration, vandalism and theft, a change in needs of the organization, etc. The alternatives available are as follows:

(i) Keep the equipment in service;
(ii) Scrap the equipment without replacement;
(iii) Replace the equipment but store it as backup;
(iv) Retrofit the equipment and overhaul it;
(v) Scrap the equipment and replace it.

In order to better evaluate the economic merits of replacement items, the following jargon is introduced:

Useful Life: The time period of an asset kept in service is referred to as the useful life of the equipment.

Technoeconomic Life: The period of time in years that it takes for the minimum equivalent uniform annual cost (EUAC) of owning and working the equipment.

Ownership Life: This is the time that elapsed between the acquisition and final disposition of the equipment by the owner.

Physical Life: This is the time that elapsed between the original acquisition and final disposition over the entire life of the equipment.

Most studies on replacement analysis emphasize the *future* and not the past. Exceptions to this rule of thumb is when the tax consequences require deliberations of matters in the past. Only present and future cash flows are used during replacement analysis. Past decisions are relevant to a limited extent—only to the extent they affect the current situation. Sunk costs are an increasingly important factor. It is obtained as a difference between the book value of the equipment and the market value of the equipment.

In order to obtain the fair market value of a piece of equipment, an impartial *third party* is approached. The opportunity cost approach can also be used. The opportunity cost is the avenue foregone by choosing to hold on to an asset for more time. When a price hike is needed to meet the competition's bids, the realizable market value is increased to the level of the challenger. The defender thus has a price that is competitive with the challenger.

The EUAC (equivalent uniform annual cost) is minimized by the economic life of the challenger. The economic life of the defender is usually taken as one year. This basis allows for complete analysis between alternatives with different lives. As long as the marginal cost of the defender is less than the minimum EUAC of the challenger over its economic life, the defender may be retained for times greater than its economic life. Tax consequences ought to be considered during replacement analysis. Replacement of equipment often results in profit or loss from the transfer of depreciable property.

Example 1.5 Photocopier Before Tax

A leading law partner's office photocopier has a current market value of $5000. It can be in service for five more years. The salvage value after five years of the copier is zero. The operating and maintenance expenses are $1500 per year. The law partners can purchase an e-STUDIO with scanning capability for $15,000. After five years, the market value of the e-STUDIO is $7500. The operating and maintenance expenses are $800 per year. Should the law partners replace the old photocopier with the e-STUDIO for a study period of five years? Use a before-tax MARR of 10%.

Defender

$$PW(10\%) = -\$5{,}000 - \$1{,}500(P/A, 10\%, 5) \tag{1.10}$$

$(P/A,10\%, 5) = 3.7908$ (From Appendix A)

$$PW\ (10\%\ MARR) = -\$10{,}686.2 \tag{1.11}$$

Challenger

$$PW(10\%) = -\$15{,}000 + \$7{,}500 - \$800(P/A, 10\%, 5)$$

$$= -10{,}532.6 \tag{1.12}$$

PW of the challenger is greater than the PW of the defender. But the numbers are too close to call. They are within 1.4% of each other. At least 20% difference is needed to prefer one over the other.

The complete replacement analysis requires knowledge of the technoeconomic lives of the alternatives (min. EUAC). The EUAC of a new piece of equipment can be estimated should the fixed costs, maintenance

and operational expenses, and year-by-year market values be known. In order to find the EUAC of the challenger, the total marginal cost of the challenger on an annual basis is needed. The minima of such values identifies the economic life. The PW can be estimated as

$$PW_j = C_F - BV_j(P/F,i\%,j) + \sum_{ki=1}^{j} E_k(P/F,i\%,k) \qquad (1.13)$$

The total marginal cost is the equivalent worth at the end of year j. This is the increase in PW of the total cost from year j-1 to year j.

$$C^j_T = (PW_j - PW_{j-1})(F/P,i\%,j) \qquad (1.14)$$

Combining Eq. (4.35) and Eq. (4.34)

$$C^j_T = SV_{j-1} - SV_j + iSV_{j-1} + E_j \qquad (1.15)$$

The technoeconomic life of the e-STUDIO in Example 1.5 is estimated and shown in Table 1.11. The technoeconomic life of the defender in Example 1.5 is shown in Table 1.12. The following points may be noted during the estimate of EUAC of the defender:

(i) The time until the next major overhaul is the likely life resulting in minima in EUAC estimates.
(ii) The technoeconomic life can be taken as one year when the salvage value is zero and the recurring expenses are expected to increase.

Table 1.11 Technoeconomic Life of the E-STUDIO

	EOY 1	EOY 2	EOY 3	EOY 4	EOY 5
Market Value	$13,500	$12,000	$10,500	$9,000	$7,500
Operational & Maintenance Expenses	$800	$800	$800	$800	$800
Marginal Costs					
Recurring Expenses	$800	$800	$800	$800	$800
Depreciation	$1,500	$1,500	$1,500	$1,500	$1,500
Interest on Capital	$1,500	$1,350	$1,200	$1,050	$900
Total Cost	$3,800	$3,650	$3,500	$3,350	$3,200

Table 1.12 Technoeconomic Life of the Defender in Example 1.5

	EOY 1	EOY 2	EOY 3	EOY 4	EOY 5
Market Value	$4,000	$3,000	$2000	$1000	0
Operational & Maintenance Expenses	$1,500	$1,500	$1,500	$1,500	$1,500
Marginal Costs					
Recurring Expenses	$1,500	$1,500	$1,500	$1,500	$1,500
Depreciation	$1,000	$1,000	$1,000	$1,000	$1,000
Interest on Capital	$500	$400	$300	$200	$100
Total Cost	$3,000	$2,900	$2,800	$2,700	$2,600

(iii) The defender may be retained as long as its marginal cost is less than the minima of the best challenger.

When the defender is retained for times greater than when the total cost exceeds the minima in EUAC estimates for the challenger, the replacement becomes more imminent. Procrastination of replacement decisions can come about when the technology changes rapidly and solutions with better performance are offered. When the defender and challenger have different useful lives, the analysis centers around whether now is a good time for seeking replacement. As discussed in the previous chapter, comparison of apples and oranges is not appropriate. The repeatability and co-termination assumptions may be used in order to arrive at an appropriate study period.

Abandonment is retirement of the equipment or asset from service without replacement. Consider projects that have positive cash flows and a finite period of service. The question arises as to whether the project is profitable to pursue. Should it be profitable, what is the best year to abandon the project? What is the technoeconomic life of the equipment or asset? Both problem types, i.e., cost-dominated project or benefit-dominated project can be treated in this manner. The year the PW undergoes a maxima can be chosen as the year for abandonment.

Example 1.6 Best Year for Abandonment

Ms. Lisa Stone purchased a TEM, transmission electron microscope, for $80,000. The depreciation and book value for a five-year period is shown

Table 1.13 Microscopy Lab Services

	Depreciation	Book Value	
EOY	Depreciation	$80,000	Receipts
1	$32,000	$48,000	$43,000
2	$19,200	$28,800	$40,000
3	$11,520	$17,280	$35,000
4	$6,912	$10,368	$25,000
5	$4,147	$6,221	$15,000

in Table 1.13. Each year the receipts from microscopy studies of premium samples are also shown in Table 1.13. At a MARR of 10%, when should Lisa abandon the TEM?

Keep for 1 year

$$\begin{aligned} PW &= -80{,}000 + (\$48{,}000 + \$43{,}000)(P/F,10\%,1) \\ &= -80{,}000 + (\$48{,}000 + \$43{,}000)(0.90901) \\ &= \$2{,}719.91 \end{aligned} \tag{1.16}$$

Keep for 2 years

$$\begin{aligned} PW &= -48{,}000 + (\$40{,}000 + \$28{,}800)(P/F,10\%,2) \\ &= -48{,}000 + (\$40{,}000 + \$28{,}800)(0.826446) \\ &= \$8{,}859.48 \end{aligned} \tag{1.17}$$

Keep for 3 years

$$\begin{aligned} PW &= -\$28{,}800 + (\$35{,}000 + \$17{,}280)(P/F,10\%,3) \\ &= -80{,}000 + (\$48{,}000 + \$43{,}000)(0.751315) \\ &= \$10{,}478.75 \end{aligned} \tag{1.18}$$

Keep for 4 years

$$\begin{aligned} PW &= -17{,}280 + (\$25{,}000 + \$10{,}368)(P/F,10\%,4) \\ &= -80{,}000 + (\$48{,}000 + \$43{,}000)(0.683013) \\ &= \$6{,}876.80 \end{aligned} \tag{1.19}$$

Keep for 5 years

$$\begin{aligned} PW &= -10{,}368 + (\$15{,}000 + \$6{,}221)(P/F,10\%,5) \\ &= -10{,}368 + (\$15{,}000 + \$6{,}221)(0.620921) \\ &= \$2{,}808.44 \end{aligned} \tag{1.20}$$

The PW is a maxima EOY 3. So the recommendation is to abandon the TEM at the end of year three.

Tax consequences can affect the decision for seeking replacement. Tax considerations during replacement analysis are paramount. Taxes ought to be considered not only for each year of operation of the equipment, but also in relation to the salvage value of the equipment. MS Excel spreadsheets can be used to study problems where the receipts and depreciation amounts change each and every year.

Eq. (4.13) upon taking into account the tax consequences can be written as follows:

$$\begin{aligned} PW_j = C_F - ((1-t)SV_j + tBV_j)(P/F,i\%,j) \\ + \sum_{ki=1}^{j} ((1-t)E_k - td_k)(P/F,i\%,k) \end{aligned} \tag{1.21}$$

The annual taxes and tax consequences on sale of equipment are included in Eq. (4.42). The total marginal cost for each year is then written as follows:

$$C^j_T = (1-t)(SV_{j-1} - SV_j + iSV_{j-1} + E_j) + i(t)(BV_{j-1}) \tag{1.22}$$

The salvage value of the equipment must be compared with the book value of the equipment. This is necessary to assess the tax ramifications. This can be included in the opportunity cost of retaining the defender. The net ATCF, when the defender is retained after taxes, is written as follows:

$$\mathrm{ATCF_j} = -\mathrm{SVj} + \mathrm{t(SVj - BVj)} \tag{1.23}$$

1.5 Summary

Estimation of ATCF (after-tax cash flows) and *tax consequences* on decision analysis are described in detail in Chapter 2.0. *Depreciation* can be used to establish an equivalent series of annual deductions to the value of an asset acquired as a single payment or transaction. *Cost basis (adjusted)* is the amount that is used as the basis for further calculations. The *book value* of an asset is the current worth of the object as shown by the accountant of the enterprise. The *useful life* of an equipment is the period over which the equipment is serviceable or functional.

The IRS (Internal Revenue Service) requires the use of MACRS (modified accelerated cost recovery system) for calculation of depreciation of assets [1]. There are two systems under MACRS for calculating depreciation deductions: The main system is called the GDS (*general depreciation system*) and the alternative system is called an *alternative depreciation system.* The MACRS classes have a predetermined schedule, which is used to determine the fraction of the asset's cost that is depreciated each year. (Tables 1.1–1.4). A worked example is shown in calculating the depreciation of offshore drilling equipment, a helical ribbon agitator, equipment for hexane extraction of rice-bran oil, etc.

Three different methods of calculating depreciation are discussed. These are: (i) method of straight line; (ii) method of declining balance; and (iii) method of units of production. A constant amount is deducted every year in the method of straight line. A constant percentage of the remaining balance is deducted every year in the method of declining balance. Deduction is based on units produced in the method of units of production rather than a time horizon or balance-based procedure.

Jargon needed for performing *replacement analysis* includes: useful life, technoeconomic life, ownership life, and physical life. EUAC is the equivalent uniform annual cost of equipment. The EUAC equivalent uniform annual cost is minimized by the economic life of the challenger. Tax consequences must be considered. A worked example on the photocopier is used to calculate the PW of the challenger and PW of the defender. The complete replacement analysis requires knowledge of the technoeconomic lives of the alternatives (min. EUAC). The EUAC of a new piece of equipment can be estimated, should the fixed costs,

maintenance and operational expenses, and year-by-year market values be known. *Abandonment* is the retirement of the equipment or asset from service without replacement. The year the PW undergoes a maxima can be chosen as the year for abandonment.

The worked example on the TEM microscope is used to show how to calculate the year of abandonment. Exercises on a Versailles-type home, hydrogen from biomass, hybrid electric vehicle, olive oil business, short path distillation for paraffin wax fractionation, coffeehouse, propylene bag manufacture, steam boiler, sugar mill, HVAC, abandonment of a pretzel maker, automatic bagger, Keystone XL project, bioethanol from sugarcane bagasse, sequestration by dimethyl carbonate formation, and a baggage conveyor system at an airport are also developed.

1.6 References

[1] U.S. Department of the Treasury, Internal Revenue Service Publication 946, *How to Depreciate Property*. Washington, DC: U.S. Government Printing Office.

CHAPTER 2

Taxes, Tariffs, and Duties

Topics Covered

- Income Tax
- State and City Taxes
- Sales Tax
- Custom Duties
- Tariffs
- ATCF, BTCF
- Tax Consequences on PW, AW, FW, IRR, and ERR Methods
- Tax Reform under Presidents Reagan, Clinton, Bush, and Obama

2.1 Overview

From time immemorial, taxes have been collected by monarchs and kings from their subjects. According to Benjamin Franklin, "Nothing in the world is certain but death and taxes!" Ratified on February 13, 1913, the 16th Amendment to the Constitution of the United States under the presidency of William Taft, states:

> *The Congress shall have the power to lay and collect taxes on incomes, from whatever source derived, without apportionment among the several states, and without regard to any census or enumeration.*

The 16th Amendment allowed for direct taxation by the federal government based on incomes of individuals, business entities, and other organizations. The Civil War was funded by revenue raised by authority from the *Revenue Act of 1861*. The Internal Revenue Service (IRS) is the implementing government agency for tax collection. The *Internal Revenue*

Code of 1986 is the main text of the statutory law governing income tax, payroll taxes, gift taxes, estate taxes, and excise duties.

Income taxes can have a significant effect on the estimated cash flows in a project. Income taxes can be a cash outflow. This can be used during evaluation of profitability of projects. The cash flows discussed in Chapter 4.0 and Chapter 5.0 are *BTCF* (before-tax cash flow). The Tax Reform Act of 1986 (TRA 86) provided a general procedure for calculation of *ATCF* (after-tax cash flow). Salient changes haven been made to TRA 86 in the *Omnibus Budget Reconciliation Act of 1993* (OBRA 93) and *Taxpayer Relief Act of 1997*.

2.2 Tax Reform

Under the presidency of Ronald Reagan, Congress passed the *Tax Reform Act of 1986* (TRA 86) [1]. The income tax code was simplified, the tax base was expanded, and many a tax shelter was eliminated. Every taxpayer saw a decrease in tax bills. The number of tax deductions and number of tax brackets was reduced. The top income tax rate was decreased from 50% to 28%. The lowest income tax rate was increased from 11% to 15%. The upper income level of the lowest tax rate was increased from $5,720 per year to $29,750 per year. Additionally, 15 levels of tax brackets were reduced to four levels of tax brackets. Capital gains were taxed at the same rate as ordinary income. Interest on credit card debt was no longer deductible.

Incentives were provided for owner-occupied housing compared to rental housing by increasing the home mortgage interest deduction. The IRA (individual retirement account) deduction was restricted. Depreciation deductions were curtailed. The act required people claiming children as dependents on their tax returns to obtain and list a social security number for each child claimed as proof of the child's existence.

The *Omnibus Budget Reconciliation Act of 1993* (OBRA 93) [2] was passed by Congress and signed by President Bill Clinton. The top rate was increased again. A 36%, 39.6% rate for individuals in the top 1.2% of wage earners was created. Corporations were taxed at 35%. Medicare taxes were not capped. Fuel tax for vehicles was increased by 4.3 cents per gallon. The taxable portion of social security benefits was expanded.

Limits on itemized deductions were indefinitely extended. Earned income tax credit was introduced.

The *Taxpayer Relief Act of 1997* [3] came about under the presidency of Bill Clinton. Several federal taxes were cut back. Top capital gains rates fell from 28% to 20%. The 15% tax bracket was lowered to 10%. Starting in 1998, a $400 tax credit for each child under the age of 17 was introduced. This was increased to $500 in 1999. This incentive was phased out for high-income families. Profits from the sale of a personal residence of up to $500,000 for married couples filing jointly and $250,000 for singles were declared *tax exempt*. Estate tax exemption was increased from $600,000 to $1 million gradually by the year 2006. Small businesses and farms owned by families can claim tax exemptions of $1.3 million. The $10,000 annual gift tax exclusion was adjusted for inflation starting from the year 1999. Tax relief was provided for retirement accounts and college savings plans. Credits were given for lifelong learning.

President George W. Bush administered federal tax cuts between 2001 and 2003 for all taxpayers. The lowest income tax rate was brought down from 15% to 10%. The 27% was changed to 25%, the 30% was changed to 28%, and the 35% rate was brought down to 33%. The top rate was changed from 39.6% to 35%. Tax credit for having children went from $500 to $1000. The marriage penalty was reduced. Tax cuts will expire in 2011 unless Congress enacts legislation. The Bush administration tax policy has led to income inequality. The gap is formed between the top 1% and the bottom 99%. Education and skill sets were attributed as causative factors. However, noted economists like P. Krugman pointed out the inequality between the top and the bottom cannot be explained by education alone. Wages of highly educated earners have fallen far behind those of the wealthiest top echelons. The U.S. national debt rose from 2001 to 2008. This was attributable to lower tax receipts. From a budget *surplus* the debt went to $400 billion in *deficit*.

President Barack Obama has introduced stimulus bills. These are intended to bring the nation out of the "great recession." The CBO (congressional budget office) reported that in 2009, the budget deficit increased by $1.9 trillion during fiscal year (FY) 2009, vs. a $1 trillion increase in FY 2008. This was because of a decline in tax receipts due to the recession, tax cuts, bailout efforts, additional spending, increase in spending of Medicare, Medicaid, unemployment insurance, social security, defense, and war

expenses in Afghanistan and Iraq. The federal debt is about $12.7 trillion as of March 2010. This is about 80% of the GDP. The federal debt is expected to double between 2008 and 2015. It may become 100% of the GDP. Fifteen cents on every tax dollar in 2008 go toward payment of interest on debt. The exchange value of the dollar is expected to fall. The *Economic Stimulus Act of 2008* [4] provides tax rebates to stimulate the economy. Tax credits, extended unemployment benefits, and monies for infrastructure programs were introduced. The 2010 budget proposal of President Obama has health care, clean energy, education, and infrastructure as priorities. The national debt is proposed to be increased every year between 2010–2019 by $900 billion each year. Tax cuts are removed for the wealthiest top echelon and marginal rates returned to President Clinton levels.

2.3 Types of Taxes

The following different types of taxes have been selected by leaders and politicians:

(i) **Income Tax**: This is collected by the IRS as a function of gross income less allowable deductions. Income-based taxes can also be collected by state governments and local governments of cities.

(ii) **Property Tax:** This is assessed based on the value of the property owned, such as land, buildings, jewelry, etc. These taxes are levied by local governments, such as municipalities or counties.

(iii) **Sales Tax:** Sales tax can be levied by state or local governments. The amount of sales tax is a certain percentage of the dollar value of transactions of goods and services. This can be added to the cost of items acquired.

(iv) **Excise Taxes:** Nonessentials, when transacted, incur a duty called excise tax. This can be collected by the federal government. Part of it is borne by the manufacturer and some of it is passed along to the customer making the purchase.

(v) **Luxury Tax:** A good example of a luxury tax is the one levied in a five-star hotel such as a Hilton in a big city. This is in addition to the hotel charges incurred. The amount of luxury tax is a certain percentage of the bill.

(vi) **Tariffs:** The federal line subscription charge due to a telephone company (such as AT&T) for providing landline telephone services is a good example of a tariff.

2.4 Tax Consequencs of Evaluation of a Single Project

The MARR (minimum acceptable rate of return) discussed in earlier chapters is based on before-tax calculations. The MARR after tax can be estimated as

$$MARR(after-tax) = MARR(before-tax)^{*}(1 - TaxRate) \quad (2.1)$$

The value of money calculated by the interest rate, i (%), needs to be modified for tax. The cost of capital is a weighted average

$$C_{WACC} = i'(1 - t_r)\gamma + (1 - \gamma)i \quad (2.2)$$

Where C_{WACC} is the weighted average cost of capital, i' (%) is the interest rate before taxation, g is the fraction of the capital that is borrowed, and i (%) is the interest rate after taxation. Interest on borrowed capital is tax deductible.

Business enterprises usually distinguish between *gross income* and *net income*. Gross income is the revenues less the expenses before tax. Net income is the profit made after payment of taxes.

The federal corporate income tax rate structure in 2010 is shown in Table 2.1.

A flat rate of 35% is levied for incomes of more than $18.33 million. The state tax rates are usually lower than that of the federal income tax and range about 6–12%. There are two kinds of tax consequences for the sale of an asset:

(i) **Depreciation Recapture:** When the sale value is greater than the book value of the asset;
(ii) **Capital Loss:** When the sale value is lower than the book value of the asset.

Table 2.1 Corporate Federal Income Tax Rates (2010)

Taxable Income Range	Tax Rate
$0–$50,000	15%
$50,000–$75,000	25%(R > $50,000) + $7,500
$75,000–$100,000	34%(R > $75,000) + $13,750
$100,000–$335,000	39%(R > $100,000) + $22,250
$335,000–$10 million	34%(R > $335,000) + $113,900
$10 million–$15 million	35%(R > $10 million) + $3.4 million
$15 million–$18.33 million	38%(R > $15 million) + $5.15 million
$18.33 million and up	35% (flat rate)

The cash flow diagram can be modified to reflect the tax consequences. The term ATCF is introduced. This is called after-tax cash flow. This is different from the BTCF, which is the before-tax cash flow. As can be recalled from Chapters 4.0 and 5.0, the income each year of the enterprise is revenues, R_k less the expenses, E_k incurred each year, and the depreciation of capital equipment for that year d_k. Thus,

$$BTCF_k = R_k - E_k - d_k \tag{2.3}$$

Let the tax consequence in the k^{th} year be denoted by T_k. Then the $ATCF_k$ for the k^{th} year can be written as

$$ATCF_k = BTCF_k \pm T_k \tag{2.4}$$

It can be seen that T_k can be calculated from Table 2.1 as

$$T_k = -t_r (R_k - E_k - d_k) \tag{2.5}$$

Where t_r is the tax rate. Combining Eqs. (2.3–2.5)

$$\begin{aligned} ATCF_k &= R_k - E_k - d_k - t_r (R_k - E_k - d_k) \\ &= (1 - t_r)(R_k - E_k) + t_r d_k \end{aligned} \tag{2.5}$$

The $ATCF_k$ as given by Eq. (2.5) has two components: (i) a component that represents the *net income* accrued in the k^{th} year. This is

gross income ($R_k - E_k$) less the tax paid to the government, $t_r(R_k - E_k)$. The second component reflects a tax savings attributable to depreciation, $+t_r d_k$.

Example 2.1 Present Worth of a Sugar Mill After Tax Consequences

A business plan (Example 4.5) was presented that would get a sugar mill installed in rural Texas. The costs of raw materials such as sugar cane and utility costs rise every year due to inflation. The revenue from the sale of sugar rises and falls in synchrony with the boom-and-bust cycles of the economy. The EOY (end of year) revenues and costs are shown in Table 2.2.

The capital cost of the sugar mill, including land and all the necessary equipment, is estimated at $8 million. What is the PW of the sugar business at an interest rate of 3%, after taking into account the tax consequences? The tax rate given in Table 2.1 can be considered.

Another table was created in an MS Excel spreadsheet. The results are shown in Table 2.3. The $BTCF_k$ (before-tax cash flow) was calculated by subtracting the costs from the revenue column. It can be seen from the column under Profit (Before Tax) in Table 2.3 that the profits for the

Table 2.2 EOY Receipts and Costs

Year	Receipts Millions $	Cost Millions $
1	2.3	1.00
2	2.5	1.10
3	2	1.21
4	1.5	1.33
5	2	1.46
6	2.5	1.61
7	3	1.77
8	3.5	1.95
9	4	2.14
10	3.5	2.36
11	3	2.59

Table 2.3 PW Analysis of Sugar Mill after Tax Consequences (All Quantities in Millions $)

			Before Tax		After Tax	
Year	Receipts	Cost	Profit	T_k	Profit	PW_k
1	$2.30	$1.00	$1.30	$0.44	$0.86	0.83301
2	$2.50	$1.10	$1.40	$0.48	$0.92	0.870959
3	$2.00	$1.21	$0.79	$0.27	$0.52	0.477155
4	$1.50	$1.33	$0.17	$0.06	$0.11	0.099102
5	$2.00	$1.46	$0.54	$0.18	$0.35	0.3051
6	$2.50	$1.61	$0.89	$0.30	$0.59	0.491656
7	$3.00	$1.77	$1.23	$0.42	$0.81	0.65923
8	$3.50	$1.95	$1.55	$0.53	$1.02	0.808234
9	$4.00	$2.14	$1.86	$0.63	$1.23	0.939038
10	$3.50	$2.36	$1.14	$0.39	$0.75	0.560864
11	$3.00	$2.59	$0.41	$0.14	$0.27	0.193703
	PW					−1.76195

11 years are greater than $335,000 and less than $10 million. So the tax rate can be seen from Table 2.1 as

$$T_k = \$113{,}900 + 0.34(Pr_k - \$335{,}000) \tag{2.6}$$

This was calculated and shown as a separate column T_k in Table 2.3. The profit after tax is calculated by subtracting the tax consequence column from the profit (before tax) column. The after-tax profit is shown as a separate column in Table 2.3. Since the profits vary from year to year, the (P/F, 3%,k) multiplier is used for each year in order to calculate the time value of money; k is varied from 1 to 11. The contribution to the PW for each k^{th} year is calculated at PW_k = Profit/$(1+i)^k$ and shown as a separate column PW_k in Table 2.3. This column is summed up and the PW of the project is calculated taking into account the initial $8 million of capital investment.

It can be seen that PW has gone from positive (+$1.451 million) to negative territory (−$1.76 million). It is not profitable to start the sugar mill under the current tax rate.

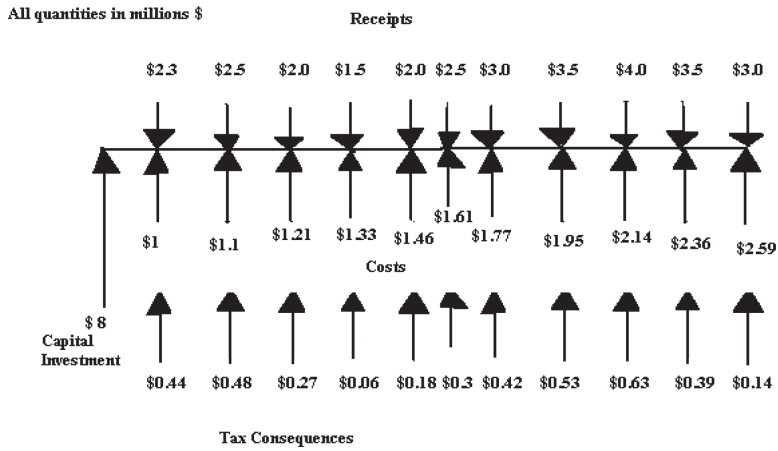

Figure 2.1 Cash Flow Diagram for Sugar Mill in a Boom-and-Bust Economy with Tax Consequences

The cash flow diagram after taking into account the tax consequences is shown in Figure 2.1.

Example 2.2 PW of ABS Plant Taking into Account Inflation and Tax Consequences

The inflation rates for the past 10 years, 1999–2009, are given below in Table 2.4. This table was prepared using the information provided by the Bureau of Labor Statistics (Example 4.8).

In 1999 dollars, calculate the PW (present worth) of an ABS (acrylonitrile butadiene styrene) engineering thermoplastic continuous polymerization plant. The capital cost of a new plant in 1999 dollars was 80 million. The selling price of ABS is $4 per pound. The raw materials cost rises every year because of inflation (Table 2.5). The price in year 1998 was $2.8 per pound of ABS sold. The process cost also goes up with inflation due to the increase in utility costs and wages to workers. The process cost can be taken as 30 cents per pound of ABS produced in 1998. The production rate from the two CSTR and falling strand devolatilizer plant is 10,000 lb/hr. Around 85% of the time the plant can be expected to run. The selling price of ABS remains flat during the 10-year period. An interest rate of 11% may be assumed. Consider the tax consequences. The corporate tax rate given in Table 2.1 may be used.

Table 2.4 Inflation Rates for 1999–2009

Year	(%) Inflation
1999	2.19
2000	3.38
2001	2.83
2002	1.59
2003	2.27
2004	2.68
2005	3.39
2006	3.24
2007	2.85
2008	3.85
2009	−0.34

The inflation rates given in Table 2.4 were used to calculate the materials cost and process cost. This is shown in the column in Table 2.5 under Cost–Materials and Cost–Process. Each row was $(1+f_i)$ times the previous row, where f_i is the inflation rate. Inflation rate was calculated by dividing the values provided under the column (%) inflation by 100. The revenue was calculated at a flat price of $4 per lb. of ABS multiplied with the quantity of ABS produced, i.e., 74.46 million lbs. This was arrived at from the production rate of 10,000 lb/hr multiplied with the operating time of the plant 24*365*0.85, where 0.85 is the utilization factor. The inflation effects on the materials and process costs resulted in loss in years 2008 and 2009. In other years there was a profit. The profit was obtained by subtracting the total expenses of materials and process from the revenue from sales of ABS. After-tax profit was calculated using the tax rates given in Table 2.1.

The PW contribution of each cell in the profit column was calculated using the single payment compound amount factor of $\frac{\mathrm{Pr}_k}{(1+i)^{Year-1999}}$ where k is varied from 0 to 10 and Pr_k was the profit earned in each year. All the PW contributions from 1999 to 2009 were added, using the SUM command in an MS Excel spreadsheet on a desktop computer. The capital cost of $80 million was also subtracted as a one-time charge. No salvage value was attributed in 2009. The PW of the ABS plant in 1999

Table 2.5 Effect of Inflation on Monomer Prices in an ABS Engineering Thermoplastic Plant

			Year	(%) Inflation	Cost Materials	Cost Process	Revenue Millions $
			1999	2.19	2.861	0.3066	297.84
Product Rate	10000	lb/hr	2000	3.38	2.958	0.3169	297.84
ABS Price	4	$/lb	2001	2.83	3.042	0.3259	297.84
Materials	2.8	$/lb	2002	1.59	3.090	0.3311	297.84
Process Cost	0.3	$/lb	2003	2.27	3.160	0.3386	297.84
			2004	2.68	3.245	0.3477	297.84
Production	7E+07	lbs	2005	3.39	3.355	0.3595	297.84
	74.46	million lb	2006	3.24	3.464	0.3711	297.84
i	11	%	2007	2.85	3.562	0.3817	297.84
			2008	3.85	3.700	0.3964	297.84
			2009	−0.34	3.687	0.3950	297.84

Table 2.6 PW of ABS Engineering Thermoplastic Plant after Accounting for Inflation and Tax Consequences

								Before Tax	After Tax	
			Year	(%) Inflation	Cost Materials	Cost Process	Revenue Millions $	Profit Millions $	Profit	PW Million $
			1999	2.19	2.861	0.3066	297.84	61.95891	40.27329	$40.27
Product Rate	10000	lb/hr	2000	3.38	2.958	0.3169	297.84	53.98613	35.09098	$31.61
ABS Price	4	$/lb	2001	2.83	3.042	0.3259	297.84	47.08507	30.60529	$27.57
Materials	2.8	$/lb	2002	1.59	3.090	0.3311	297.84	43.09806	28.01374	$25.24
Process Cost	0.3	$/lb	2003	2.27	3.160	0.3386	297.84	37.31542	24.25502	$21.85
			2004	2.68	3.245	0.3477	297.84	30.33336	19.71668	$17.76
Production	7E+07	lbs	2005	3.39	3.355	0.3595	297.84	21.26489	13.82218	$12.45
	74.46	million lb	2006	3.24	3.464	0.3711	297.84	12.30385	4.20635	$3.79
i	11	%	2007	2.85	3.562	0.3817	297.84	4.16607	1.41646	$1.28
			2008	3.85	3.700	0.3964	297.84	−7.14037	−7.14037	($6.43)
			2009	−0.34	3.687	0.3950	297.84	−6.10344	−6.10344	($5.50)
Capital Cost of Plant										($80)
								Total		$89.90

dollars was seen to be $89.90 million after taking into account the tax consequences. This has decreased from a PW of $157.07 million for the same plant before tax consequences. The results are shown in Table 2.5.

Example 2.3 Profitability of Liquefaction Plants

The conversion of natural gas to liquid (GTL) fuel using the Fischer-Tropsch process was first developed in 1923. Syngas ($CO + H_2$) was converted into synthetic fuels. At a Society of Petroleum Engineers symposium, a novel method of evaluating the economics of GTL plants was presented (Example 4.15). The capital expenses (CAPEX) are pegged to the production of one barrel of hydrocarbon liquid per day (BLPD). Annual operational expenses (OPEX) are expressed as a certain percentage of the CAPEX. It is assumed that the overall thermal efficiency of GTL plants is 60% and the plant operates 330 days per annum. Consider one instance of CAPEX of $40,000/ BLPD and OPEX of 7% of CAPEX. The plant is considered to be in existence for eight years. The price of crude oil is $37.65/bbl. Calculate the IRR for the project after taking into account the tax consequences. The tax rate given in Table 2.1 can be used.

Basis of Production: 1 Bbl a Day of liquid fuel/oil

Annual Revenue = $37.65*330* 1 = $12,424.5

Capital Expenses = $40,000

Annual Expenses = 0.07*$40,000 = $2800

PW = −$40,000 + $12,424.5*(P/A,i%,8) – 2800*(P/A,i%,8)

The PW for different interest rates was calculated using an MS Excel Spreadsheet on a desktop computer. The P/A values for the interest rates used in the simulation were read from the annuity tables given in Appendix A. The tax rate can be seen from Table 2.1 to be 15%. The values are given in Table 2.6 below. The PW as a function of interest rate is plotted

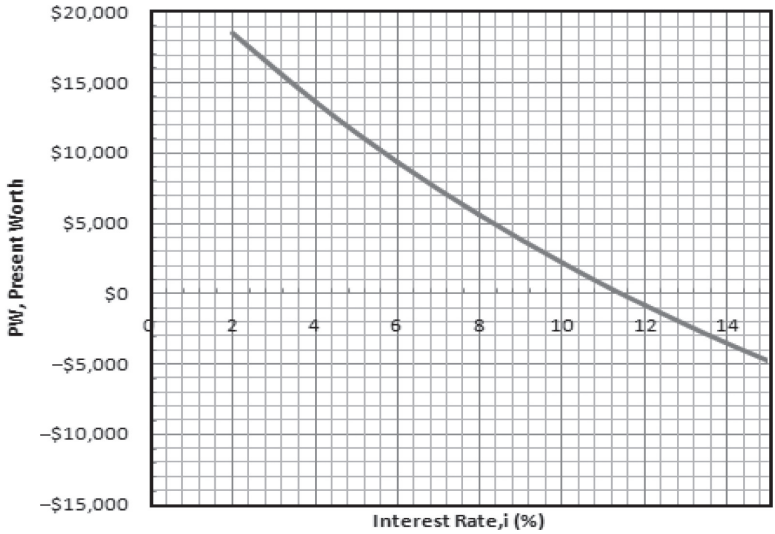

Figure 2.2 IRR Calculations for GTL Plant after Tax Consequences

Table 2.7 IRR Analysis for GTL Plant at 1 Bbl per Day with Tax Consequences

N	8		CAPEX	−$40,000
P/A	i	PW	Revenue	$12,424.50
7.325481	2	$18,484.80	OPEX	−$2,800
6.732745	4	$13,635.73	Tax	$1,443.68
6.2098	6	$9,357.61		
5.7466	8	$5,568.25		
5.3349	10	$2,200.21		
4.9676	12	−$804.61		
4.4873	15	−$4,733.86		

in Figure 2.2 The IRR can be read from Figure 2.2 as 11.3%. Should the MARR be less than the IRR, the project is considered *profitable*.

2.5 Summary

Enacted in 1913, the 16th Amendment allows for direct taxation by the federal government based on the incomes of individuals, business entities, and other organizations. Income taxes can have a significant effect on the

estimated cash flows in a project. Income taxes can be a cash outflow. BTCF is before-tax cash flow and ATCF is after-tax cash flow. The Tax Reform Act of 1986 (TRA 86) provided a general procedure for calculation of ATCF (after-tax cash flow). Salient changes haven been made to TRA 86 in the Omnibus Budget Reconciliation Act of 1993 (OBRA 93) and Taxpayer Relief Act of 1997.

The different types of taxes are income tax, property tax, sales tax, excise tax, luxury tax, and tariffs. The tax consequences can be reflected in the MARR calculation by calculation of an after-tax MARR. The cost of capital can be expressed as an weighted average of amounts before taxation and after taxation. The $ATCF_k$ includes the tax rate, t_r as shown in Eq. (2.5). This includes the depreciation in addition to expenses and revenue. Worked examples on a sugar mill, a manufacturing plant for ABS engineering thermoplastic, effects of inflation, and tax consequences on the profitability of liquefaction plants are shown. Exercises are developed on tax consequences on an outpatient drug treatment center, gas station and convenience store, public Internet, video chat, and printing services, olive oil business, short path distillation for paraffin wax fractionation, frozen yogurt parlor, coffeehouse, car rental, turnaround business, onshore oil well, custom foam fabricator, and propylene bag manufacturer.

2.6 References

[1] Tax Reform Act of 1986 (TRA) (Pub.L. 99–514, 100 Stat. 2085, enacted October 22, 1986).
[2] Pub.L. 103–66, 107 Stat. 312, enacted August 10, 1993.
[3] Pub.L. 105–34, Taxpayer Relief Act of 1997.

CHAPTER 3

Benefit-to-cost Ratio and Public Sector Initiatives

Topics Covered

- Examples of Public Sector Projects
- Costs, Benefits, and Disbenefits
- B/C Ratio by PW
- B/C Ratio by AW
- B/C Ratio by FW
- B/C Ration by CW

3.1 Introduction

The evaluation methods of a single project and methods for selection of alternates discussed in earlier chapters are largely for the private sector and/or for-profit organizations. *Public sector* alternatives are different from private sector alternatives. Public sector projects are usually owned by the government, which provides for the good of its citizens. Public sector projects involve a lot of scrutiny and care of officials of government or international agencies such as the World Bank. The methods discussed for private sector enterprises may not be applicable for public sector projects. Public sector projects are built for the common good of millions of citizens. This is usually undertaken by the monarch or the federal government, and by state and local governments as well. *Au contraire* private sector projects, such as the *Chevy Volt* of General Motors Corp., are for the profit of the shareholders of the corporation. The characteristics of public sector projects are the large size of investment, lofty ideals carried in the vision, and undertaken

for the common good. The project durations are longer for public sector projects. The public sector does not have a profit objective. There are cash flow diagrams with costs, benefits, disbenefits, etc. Private sector projects have the ulterior motive of profit. Cash flow diagrams with costs, revenues, and taxes are available. Examples of public sector initiatives include the nation's economic infrastructure such as airports, canals, dams, dikes, pipelines, railroads, interstate highways, tunnels, and artificial harbors.

Some specific examples of public sector projects in the history of mankind are as follows:

3.1.1 Marshall Plan

The Marshall Plan was implemented in 1947 and dispensed $13 billion in assistance. This economic plan was used to reconstruct western Europe after World War II The project duration was four years.

3.1.2 Taj Mahal

One of the wonders of the world, the mogul emperor Shah Jahan, Akbar's grandson, built this monument as a mausoleum for his wife, the begum Mumtaj Mahal. She died in childbirth. The construction cost of the tomb was 32 million rupees. The period of construction was 20 years. The building is entirely constructed of imported white marble. To this day, the Taj Mahal is a renowned tourist attraction.

3.1.3 Great Wall of China

Considered by some as the mightiest work of mankind, this wall stretches 5,500 miles and was maintained from the 5th century B.C. through the 16th century B.C. The wall was built during the Ming Dynasty. Stones and rammed earth were used for its construction.

3.1.4 Great Pyramid of Giza

The construction of the Great Pyramid of Giza took 20 years to complete. About 200,000 workers participated. The accuracy achieved on the pyramid was 0.05%. For about 3,800 years, this was the tallest man-made

construction in the world. The Egyptian pharaoh Khufu built this as a tomb for himself and his family. The pyramid is composed of 2.5 million limestone blocks, each weighing about 36 tons.

3.1.5 Eiffel Tower

The Eiffel Tower has been visited by 200 million people since its completion in 1889 to commemorate the centennial of the French Revolution. It is the most-visited paid monument in the world. Before the construction of the Chrysler Building in New York City in 1930, this was the world's tallest construction. It is a feat of engineering.

3.1.6 Suez Canal

The Suez Canal provides the *critical* connection between the Mediterranean Sea and the Red Sea. The canal allows for water transportation between Europe and Asia. If this canal were shut down for some reason, the ships would have to take a lengthy detour around Africa. The Suez Canal is a little over 100 miles in length. The Cold War between the United States and the erstwhile USSR led to the Suez Crisis in 1956. An event such as this one clearly sets apart projects such as these from those intended to make a profit for privately owned enterprises. So many nations are involved, so many treaties are signed, and so many "peacekeeping" forces are in place that such a project cannot be listed as a private sector project. As it is for the good of the word traders at large and offers a shorter path between East and West, this can be considered a public sector project. L. B Pearson was awarded the Nobel Peace Prize for averting World War III arising from the Suez Crisis in 1956. About 7.5% of the world sea trade is carried out via the Suez canal.

3.1.7 Hoover Dam

President Franklin D. Roosevelt dedicated the Hoover Dam to the nation in 1935. The dam, located in Nevada, produces hydroelectric power. Commissioned 75 years ago, the public sector project to this day serves as an example that renewable energy projects stem more from government than from private entrepreneurs. After the dark days of the Great Depression, this public sector project lifted the nation's morale.

3.1.8 *Apollo Space Program*

The first humans landed on the moon by this project. To gain supremacy in the "space race" with the Soviet Union, President John F. Kennedy shared his vision in an address to Congress in 1961 "… I believe that this nation should commit itself to achieving the goal, before this decade is out, of landing a man on the Moon and returning him back safely to the earth. No single space project in this period will be more impressive to mankind, or more important for the long-range exploration of space; and none will be so difficult or expensive to accomplish." He further went on to say "We choose to go the Moon in this decade and do the other things, not because they are easy, but because they are hard." This project improved the world's pride. The greedy for-profit approach of some private sector companies would not have come up with such a project. This can be considered an excellent public sector project.

3.1.9 *NASA Mars Science Laboratory Mission*

A large mobile laboratory, the NASA rover *Curiosity* will land on the planet Mars. The duration of the study is planned at 23 months. Dozens of samples drilled from rocks or scooped ground will be analyzed. The *Curiosity* will carry the most advanced payload scientific gear ever used on a Martian surface. The focus of the investigations is on whether favorable conditions for microbial life existed or does exist!. The launch is slated for December 18, 2011, and landing is expected around August 2012. The rover's electrical power will be supplied by U.S. Department of Energy radioisotope power generator. Electricity is generated from heat by the radioactive decay of plutonium 238. A suite of instruments named Sample Analysis at Mars will be used to analyze samples of materials collected and delivered by the rover's arm. The suite of instruments includes a gas chromatograph, mass spectrometer, and tunable laser spectrometer capable of measuring isotope ratios. Isotope ratios can be used for better understanding the history of the Martian atmosphere and water. An X-ray diffraction and fluorescence instrument called CheMin will be used to examine samples gathered by the rover's robotic arm. Quantitation of minerals in rocks and soil and measurement of bulk composition will be the objects of the study. The Mars Hand Lens Imager will be used to obtain close-up photographs of rocks, soil, and ice (if present), with spatial resolution smaller than the width of a human hair.

3.1.10 Interstate Highways

This is the largest public works project in history. These are the interstate highways that allow for driving long distances on the road for vacation or business. The system has a total length of 46,876 miles. About 70% of the construction and maintenance costs of highways are collected through user fees and tolls. The rest come from fuel taxes and other taxes levied by the government.

After landslide electoral victories of Republican presidents, the image of the public sector project is poor in the United States. Lists of public sector projects include hospitals, clinics, parks and recreational sites, water, electricity, gas, sewer, sanitation, utilities, schools, convention centers, sports arenas, transportation highways, bridges, waterways, police and fire protection, food stamps and public assistance welfare programs, codes and standards, etc.

The government usually assigns the tasks in a public sector project to contractors. The contractors who work on these projects take on the work under a lump-sum fixed price or cost reimbursement basis. Sometimes the public and private form a public-private partnership. The *BOT* (build-operate-transfer) arrangement may require the contractor to be completely or partially responsible for the construction, operation, and maintenance activities for the time of the project duration. After the project duration, the government becomes the owner. The title of ownership is thus transferred to the government.

3.2 B-C-D Method

The ratio of benefit to cost can be used in selecting public sector projects. The *disbenefits* are not considered in private sector projects, but can be considered during the evaluation of public sector projects. Public sector projects have

(i) Costs: These are expenses incurred by the government during the procurement, construction, and execution phases of the public sector project.
(ii) Benefits: The advantages availed by the citizens include income, savings, comfort of commute, etc.
(iii) Disbenefits: Expected unwanted consequences to the owners.

Disbenefits can cause public controversy and agitation or even picketing. Benefits to one group of taxpayers are disbenefits to another group of taxpayers. For example, a project to expand a four-lane highway into a six-lane highway may decrease congestion and decrease the commute time. This project offers advantages to the citizens who commute using that highway. Another section of citizens receives the disbenefits of higher prices at the gasoline pump due to the fuel tax levied to garner revenue for the project. A public sector project to light a section of highway with laser lighting in order to decrease the life-cycle costs may draw praise from the citizens driving through the highway. The same project incurs criticism from other taxpayers who do not drive on that highway but who still have to pay higher taxes for the highway lighting.

The finances to commission and operate a public sector project are drawn from

(i) Taxes;
(ii) User Fees;
(iii) Tolls;
(iv) Bonds;
(v) Loans;
(vi) Gifts from Charitable Trusts.

Usually, the interest rates for public sector projects are lower than for the cash flows in private sector projects. This is because a number of these public sector projects are tax exempt. Municipalities are allowed to float tax-exempt bonds for private investors as well. The loans are *subsidized* by the government. Tax credits are provided that result in a lower interest rate.

The selection criteria for projects in the private sector are solely based on the MARR (minimum acceptable rate of return). The public sector projects are screened and selected based on several criteria, such as economic and noneconomic parameters, special interests, political philosophy, etc. Public sector projects are discussed in town hall meetings.

The *Flood Control Act of 1936* was signed into law by President Franklin D. Roosevelt. Civil engineering projects such as dams, dikes, levees, and

other flood control measures can be undertaken by the U.S. Army Corps of Engineers with authority. The act was established to protect people and property on about 100 million acres. The *economic benefits* had to exceed the *costs*. Local interests had to meet the ABC requirements for local projects. Under the authority of this act, 375 major reservoirs have been built. These reservoirs have saved billions of dollars in property damage and protected millions of people from tension, anxiety, injury, and disabilities. Rivaled only by the highway system, these are significant landmarks of technical skills and the human spirit.

The B/C (benefit-to-cost ratio) was defined for evaluation of public sector projects based on the Flood Control Act of 1936.

$$\frac{B}{C} = \frac{PW(benefits)}{PW(cost)} \tag{3.1}$$

Or Eq. (3.1) may be defined in terms of AW (annualized worth), FW (future worth), or CW (capitalized worth):

$$\frac{B}{C} = \frac{FW(benefits)}{FW(cost)} \tag{3.2}$$

or

$$\frac{B}{C} = \frac{AW(benefits)}{AW(cost)} \tag{3.3}$$

or

$$\frac{B}{C} = \frac{CW(benefits)}{CW(cost)} \tag{3.4}$$

The costs are indicated by a *plus sign* and not negative, as discussed in the evaluation of single projects in earlier chapters. *Disbenefits* can be subtracted from the *benefits* in the numerator. The B/C ratio is always positive. When the B/C ratio is ≥ 1, the project can be considered acceptable. When the B/C ratio is < 1, the project is considered unacceptable.

The disbenefits may be subtracted from the benefits as follows:

$$\frac{B}{C} = \frac{PW(benefits) - PW(disbenefits)}{PW(cost)} \tag{3.5}$$

Or Eq. (3.5) may be defined in terms of AW (annualized worth), FW (future worth), or CW (capitalized worth):

$$\frac{B}{C} = \frac{FW(benefits) - FW(disbenefits)}{FW(\text{cost})} \tag{3.6}$$

Or

$$\frac{B}{C} = \frac{AW(benefits) - AW(disbenefits)}{AW(\text{cost})} \tag{3.7}$$

Or

$$\frac{B}{C} = \frac{CW(benefits) - CW(disbenefits)}{CW(\text{cost})} \tag{3.8}$$

There is some discussion in the literature on whether to add the disbenefits to the costs in the denominator! A modified B/C ratio may be defined as

$$\frac{B}{C} = \frac{PW(benefits) - PW(disbenefits) - PW(O\&M\ expenses)}{investment} \tag{3.9}$$

Or Eq. (3.9) may be defined in terms of AW (annualized worth), FW (future worth), or CW (capitalized worth):

$$\frac{B}{C} = \frac{FW(benefits) - FW(disbenefits) - FW(O\&M\ expenses)}{investment} \tag{3.10}$$

$$\frac{B}{C} = \frac{AW(benefits) - AW(disbenefits) - AW(O\&M\ expenses)}{investment} \tag{3.11}$$

$$\frac{B}{C} = \frac{CW(benefits) - CW(disbenefits) - CW(O\&M\ expenses)}{investment} \tag{3.12}$$

Example 3.1 Firefighting in Yellowstone National Park in 1988

Fire broke out in Yellowstone National Park in the states of Wyoming, Montana, and Idaho in 1988. There was discussion in the news as to

whether to fight the fires. The costs for this firefighting project were about $120 million. At a prime interest rate of 3.0%, calculate the B/C ratio based on AW (annualized worth). The revenues from wolf watchers alone is about $8.5 million per year. Allow for the payback period up to the year 2010.

$$\text{AW (Benefits)} = \$8.5 \text{ million} \tag{3.13}$$

$$\text{AW (Costs)} = (A/P, i\%, N) = (A/120, 3.25\%, 22)*120 \tag{3.14}$$

From annuity tables (Appendix A), $(A/P, 3\%, 22) = 0.062747$

$= 7.53$ million

$$\text{From Eq. (3.3) B/C} = 8.5/7.53 = 1.12 \tag{3.15}$$

As the B/C ratio is greater than 1, the firefighting efforts can be selected.

Example 3.2 Replacement for the World Trade Center (WTC)

One proposal to replace the World Trade Center (WTC) suggested that the mayor of New York City build another high-rise to replace the WTC, which was destroyed by terrorists in September 2001. In a typical week, 50,000 people are expected to work in these towers. Additionally, 200,000 visitors are expected to go the top floor to obtain a glimpse of York City from the 110th floor. The cost of constructing the building is estimated at $10 billion. Assume a net profit of collecting rent from the occupants is $250 per person per week and a visitor fee of $10. The funds are obtained by the mayor selling New York City tax-exempt bonds. The interest rate is 2%. The project horizon is 20 years. Is this a favorable public sector project?

The PW-based B/C ratio can be used. Eq. (3.1) can be used to evaluate the B/C ratio. No disbenefits are considered in this analysis.

$$\text{Rental Fee collected per year} = 250*50{,}000*52/10^6 = \$\ 650 \text{ million} \tag{3.16}$$

$$\text{Visitor Fee collected per year} = 200{,}000*10*52/10^6 = \$104 \text{ million} \tag{3.17}$$

$$(P/A, 2\%, 20) = 16.35143$$
$$\text{(from Appendix A, 2\% annuity Tables)} \tag{3.18}$$

$$PW\ (\text{Benefits}) = 16.35143^{*}(650+104)$$
$$= 12.329\ \text{billions.} \tag{3.19}$$

$$\text{From Eq. (3.1), } B/C = 12.329/10 = 1.2329 \tag{3.20}$$

It can be seen from Eq. (3.20) that the B/C ratio is greater than 1. This is an *acceptable* public sector project.

3.3 Summary

Public sector alternates are different from private sector alternates. Public sector projects are usually owned by the government and are operated for the good of its citizens. The evaluation methods of a single project and methods for selection of alternatives discussed in earlier chapters are largely for the private sector and/or for-profit organizations. In this chapter, the B/C ratio method is introduced to evaluate public sector projects. Benefit-to-cost ratios may be calculated based on the PW, FW, AW, and CW of the benefits, and the PW, FW, AW, and CW of the costs incurred. In addition to benefits and costs, another item—disbenefits—is introduced. Ten examples of successful public sector projects are discussed. Worked examples and exercises on firefighting in Yellowstone National Park, a replacement for the WTC, solar thermal power plants from photovoltaic and SD (Stirling dish) technology, windmills, Keystone pipeline projects, and expansion of federal highways using high-performance nanostructured concrete are discussed.

3.4 References

[1] http://en.wikipedia.org

CHAPTER 4

Spider-Plots and Break-Even Analysis

Topics Covered

- Perform Break-Even Analysis
- Calculate Break-Even Interest Rate
- Calculate Payback Period
- Construct Spider Plots
- Interpret Sensitivity Analysis
- What Is a "Decision Reversal?"

4.1 Overview

In Chapter 5 of An Introduction to Engineering Economics, decisions were made based upon comparison of alternatives. The alternatives were compared using one of the five methods described in Chapter 4.0: (i) PW (present worth); (ii) FW (future worth); (iii) AW (annual worth); (iv) IRR (internal rate of return); and (v) ERR (external rate of return). It can be seen from the worked examples and exercises that the dollar-and-cents number arrived at as the worth can differ with variation in the data provided for calculation of costs and revenue. An EW (equivalent worth) can refer to PW, FW, or AW. When two alternatives, A and B, are compared with each other, the EW_A, equivalent worth of Project A is a function of several factors. Consider one factor x.

$$EW_A = f(x) \tag{4.1}$$

The EW_B, equivalent worth of project B is also a function of the same factor x. At the break-even point

$$EW_A = EW_B \tag{4.2}$$

The factors can be several. These can be

(i) Capital Expenses;
(ii) Annual Revenue;
(iii) Annual Cost;
(iv) Interest Rate;
(v) Rate of Return;
(vi) Salvage Value;
(vii) Useful Life, N;
(viii) Capacity Utilization.

Each of these eight factors can be varied one at a time by 10% increments and 10% decrements. The EW (equivalent worth) can be calculated and plotted as a function of the increments and decrements for each of the eight factors. Such a plot is called a *spider plot*. The purpose of a spider plot is to bring out the sensitivity of the factors to the decision or estimate made of the project's worthiness.

Example 4.1 Break-Even Analysis for the PEV Chevy Volt

The CEO of General Motors Corp. recently unveiled the Chevrolet Volt, a plug-in electric vehicle (PEV). It has a better range. For up to the first 40 miles, the Volt is powered by electric energy stored in its onboard lithium-ion batteries, which are charged by connection to an electrical power outlet. The 16 kWh battery pack can be fully charged by connecting to a residential electrical outlet operated at 12–240 Volt AC. The cost of the vehicle is about $41,000. A $7,500 federal tax credit can be expected for purchase of the PEV. The fuel efficiency is 230 mpg of gasoline plus 25 kWh per 100 miles. A 2010 Toyota Corolla costs about $18,880 and is a standard gasoline-powered vehicle. It provides a fuel efficiency of 32 mpg. What is the break-even point in miles that must be driven by

the owner before the Volt becomes comparable with the Corolla in a total life-cycle cost? The useful lives of both vehicles may be taken as seven years. The interest rate may be taken as 7%. The household electric utility charges 12.5c per kWh. The cost of gasoline at the pump is $4.75 per gallon.

Let the number of miles driven per year by passenger
= X miles/year (4.3)

Annual Fuel Expenses for Volt

$$= \frac{0.125*25X}{100} + \frac{4.75X}{230} = 0.051902X \tag{4.4}$$

$$(P/A,7\%,7) = 0.5(4.7665+5.9713) = 5.3689 \tag{4.5}$$

$$PW_{volt} = -\$41,000+\$7,500 - 5.3689*0.051902X$$
$$= -33,500 - 0.27866X \tag{4.6}$$

$$PW_{corolla} = -18,880 - 5.3689*4.75X/32$$
$$= -18,880 - 0.796946X \tag{4.7}$$

At the break-even point

$$PW_{volt} = PW_{corolla} \tag{4.8}$$

Combining Eq. (9.6) and Eq. (9.7), it can be seen that X is

$$X = \frac{33,500 - 18,880}{(0.796946 - 0.27866)} = 28,208.2 miles \tag{4.9}$$

Example 4.2 Spider Plot for Life-Cycle Cost of Chevy Volt

Construct the spider plot for the PEV Chevy Volt as discussed in Example 4.1. Assume that the passenger drives 30,000 miles every year.

$$PW_{volt} = -33,500 - 0.27866X = -\$41,859.7 \tag{4.10}$$

The information needed to construct a spider plot was calculated using an MS Excel spreadsheet on a desktop computer and shown in Table 4.1.

Table 4-1 Simulation Results for Spider Plot of Life-Cycle Cost of PEV Chevy Volt

	Spider Plot							
					$4.75	Interest Rate		
% Change	MSRP	PW	Miles/Year	Tax Credit	Gas Price	Interest Rate	P/A	PW
−40%	24,600	−$25,460	−$38,516	−$44,860	−$40,529	4.20%	5.957943	−$42,777
−30%	28,700	−$29,560	−$39,352	−$44,110	−$40,862	4.90%	5.807402	−$42,542
−20%	32,800	−$33,660	−$40,188	−$43,360	−$41,194	5.60%	5.662627	−$42,317
−10%	36,900	−$37,760	−$41,024	−$42,610	−$41,527	6.30%	5.523344	−$42,100
0%	41,000	−$41,860	−$41,860	−$41,860	−$41,860	7.00%	5.3689	−$41,860
10%	45,100	−$45,960	−$42,696	−$41,110	−$42,192	7.70%	5.260219	−$41,690
20%	49,200	−$50,060	−$43,532	−$40,360	−$42,525	8.40%	5.135899	−$41,497
30%	53,300	−$54,160	−$44,368	−$39,610	−$42,858	9.10%	5.016109	−$41,310
40%	57,400	−$58,260	−$45,204	−$38,860	−$43,190	9.80%	4.900643	−$41,131

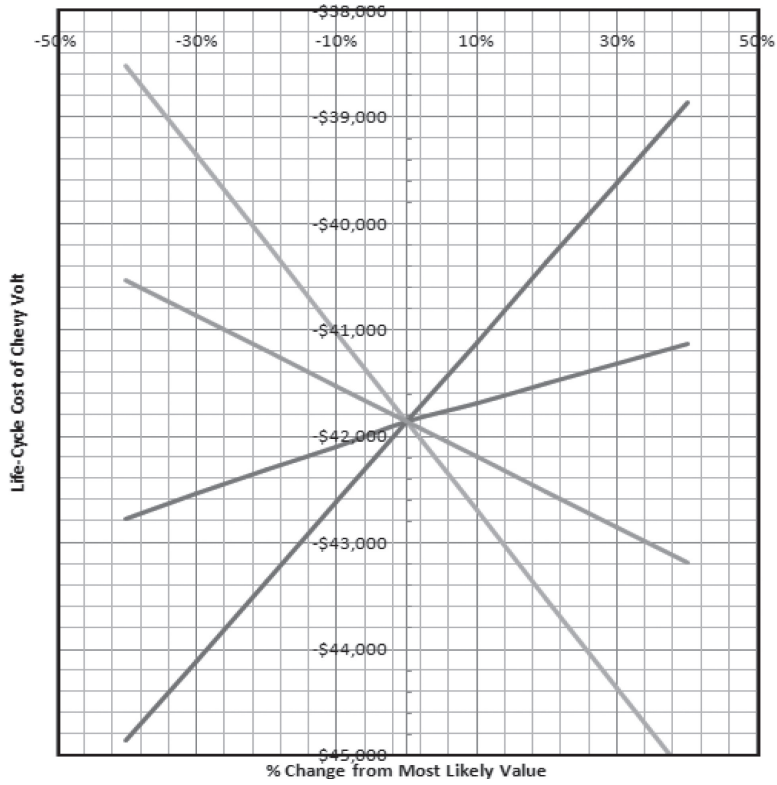

Figure 4.1 Spider Plot of Life-Cycle Cost of 2011 Chevy Volt

The effect of percentage change in likely values of the miles driven per year by the commuter, tax credit given by the federal government for the purchase of the PEV, gas price, and interest rate are shown in Figure 4.1.

APPENDIX A

Discrete Compounding—Interest and Annuity Tables, Table A-1 to Table A-23

APPENDIX A Discrete Compounding—Interest and Annuity Tables

Discrete Compounding	i	1/8%				Table	A-1
	Single Payment	Uniform Series				Gradient	Series
	Compound Amount Factor	Present Amount Factor	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Gradient Present Worth Factor	Gradient Uniform Series Factor
N	F/P	P/F	F/A	P/A	A/F	A/P	P/G	A/G
1	1.001250	0.998752	1.000000	0.998752	1.000000	1.001250	0.000000	0.000000
2	1.002502	0.997505	2.001250	1.996256	0.499688	0.500938	0.997505	0.499688
3	1.003755	0.996259	3.003752	2.992516	0.332917	0.334167	2.990023	0.999167
4	1.005009	0.995016	4.007506	3.987531	0.249532	0.250782	5.975070	1.498433
5	1.006266	0.993773	5.012516	4.981305	0.199501	0.200751	9.950164	1.997502
6	1.007523	0.992533	6.018781	5.973837	0.166147	0.167397	14.912827	2.496356
8	1.010044	0.990056	8.035088	7.955187	0.124454	0.125704	27.790981	3.493442
10	1.012571	0.987586	10.056438	9.931592	0.099439	0.100689	44.539810	4.439694
12	1.015104	0.985121	12.082845	11.903066	0.082762	0.084012	65.289669	5.485114
14	1.017643	0.982663	14.114321	13.869620	0.070850	0.072100	89.870983	6.479700
16	1.020189	0.980211	16.150879	15.831267	0.061916	0.063166	118.314253	7.473454
18	1.022741	0.977765	18.192531	17.788020	0.054968	0.056218	150.600053	8.466375
20	1.025299	0.975325	20.239291	19.739889	0.049409	0.050659	186.709028	9.458464
22	1.027864	0.972891	22.291171	21.686888	0.044861	0.046111	226.621897	10.449720
24	1.030435	0.970464	24.348183	23.629028	0.041071	0.042321	270.319452	11.440142
26	1.033013	0.968042	26.410342	25.566323	0.037864	0.039114	317.782556	12.429733
28	1.035597	0.965627	28.477659	27.498783	0.035115	0.036365	368.992144	13.413490
30	1.038188	0.963217	30.550148	29.426421	0.032733	0.033983	423.929225	14.406415
36	1.045998	0.956024	36.798772	35.180522	0.027175	0.028425	610.916556	17.365193
40	1.051238	0.951259	40.380818	38.992699	0.024396	0.025646	753.867237	19.333548
44	1.056504	0.946518	45.203462	42.785B75	0.022122	0.023372	911.278094	21.298574
48	1.061797	0.941800	49.437409	46.560143	0.020228	0.021478	1083.001425	23.260268
52	1.067116	0.937106	53.692566	50.315599	0.018625	0.019875	1268.890640	25.218633
56	1.072461	0.932435	57.969039	54.052337	0.017251	0.018501	1468.800256	27.173668
60	1.077834	0.927787	62.266935	57.770449	0.016060	0.017310	1632.585338	29.125373
66	1.085943	0.920859	68.754180	63.312897	0.014545	0.015795	2028.968705	32.046689
72	1.094113	0.913983	75.290231	68.813958	0.013282	0.014532	2405.771346	34.960514
84	1.110638	0.900384	88.510226	79.693153	0.011298	0.012548	3248.746985	40.765698
96	1.127412	0.886987	101.929890	90.410478	0.009811	0.011061	4207.788030	46.540933
108	1.144440	0.873790	115.552239	100.968341	0.008654	0.009904	5279.254029	52.286231
∞				$00.003000		0.001250		

Discrete Compounding			i	1/4%			Table	A-2
	Single	Payment	Uniform	Series			Gradient	Series
	Compound Amount Factor	Present Amount Factor	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Gradient Present Worth Factor	Gradient Uniform Series Factor
N	F/P	P/F	F/A	P/A	A/F	A/P	P/G	A/G
1	1.002500	0.997506	1.000000	0.997506	1.000000	1.002500	0.000000	0000000
2	1.005006	0.995019	2.002500	1.992525	0.499376	0.501876	0.995019	0.499376
3	1.007519	0.992537	3.007506	2.985062	0.332501	0.335001	2.980093	0.998335
4	1.010038	0.990062	4.015025	3.975124	0.249064	0.251564	5.950280	1.496879
5	1.012563	0.987593	5.025063	4.962718	0.199002	0.201502	9.900653	1.995006
6	1.015094	0.985130	6.037625	5.947848	0.165628	0.168128	14.826305	2.492717
8	1.020176	0.980223	8.070351	7.910745	0.123910	0.126410	27.583909	3.486891
10	1.025283	0.975340	10.113253	9.863864	0.098880	0.101380	44.184201	4.479401
12	1.030416	0.970482	12.166383	11.807254	0.082194	0.084694	64.588583	5.470246
14	1.035574	0.965648	14.229791	13.740963	0.070275	0.072775	88.758742	6.459427
16	1.040759	0.960837	16.303529	15.665040	0.061336	0.063836	116.656656	7.446943
18	1.045969	0.956051	18.387648	17.579533	0.054384	0.056884	148.244586	8.432795
20	1.051206	0.951289	20.482201	19.484488	0.048823	0.051323	183.485079	9.416982
22	1.056468	0.946550	22.587240	21.379955	0.044273	0.046773	222.340961	10.399506
24	1.061757	0.941835	24.702818	23.265980	0.040481	0.042981	264.775340	11.380365
26	1.067072	0.937143	26.828986	25.142609	0.037273	0.039773	310.751599	12.359560
28	1.072414	0.932475	28.965799	27.009891	0.034523	0.037023	360.233399	13.337092
30	1.077783	0.927830	31.113309	28.867871	0.032141	0.034641	413.184675	14.312960
36	1.094051	0.914034	37.620560	34.386465	0.026581	0.029081	592.498783	17.230581
40	1.105033	0.904950	42.013204	38.019863	0.023802	0.026302	728.739878	19.167346
44	1.116125	0.895957	46.449939	41.617154	0.021529	0.024029	878.016195	21.097459
48	1.127328	0.887053	50.931208	45.178695	0.019634	0.022134	1040.055196	23.020922
52	1.138644	0.878238	55.457459	48.704842	0.018032	0.020532	1214.588473	24.937736
56	1.150073	0.869510	60.029141	52.195947	0.016659	0.019159	1401.351692	26.847903
60	1.161617	0.860869	64.646713	55.652358	0.015469	0.017969	1600.084536	28.751424
66	1.179150	0.848068	71.660119	60.772676	0.013955	0.016455	1920.067161	31.594251
72	1.196948	0.835458	78.779387	65.816858	0.012694	0.015194	2265.556855	34.422136
84	1.233355	0.810797	93.341920	75.681321	0.010713	0.013213	3029.759228	40.033118
96	1.270868	0.786863	108.347387	85.254603	0.009230	0.011730	3886.283161	45.584438
108	1.309523	0.763637	123.809259	94.545300	0.008077	0.010577	4829.012470	51.076177
∞				400.000000		0.002500		

Discrete Compounding		i	1/2%			Table	A-3
	Single	Payment	Uniform	Series			Gradient	Series
	Compound Amount Factor	Present Amount Factor	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Gradient Present Worth Factor	Gradient Uniform Series Factor
N	F/P	P/F	F/A	P/A	A/F	A/P	P/G	A/G
1	1.005000	0.995025	1.000000	0.995025	1.000000	1.005000	0.000000	0.000000
2	1.010025	0.990075	2.005000	1.985099	0.498753	0.503753	0.990075	0.498753
3	1.015075	0.985149	3.015025	2.970248	0.331672	0.336672	2.960372	0.996675
4	1.020151	0.980248	4.030100	3.950496	0.248133	0.253133	5.901115	1.493766
5	1.025251	0.975371	5.050251	4.925866	0.198010	0.203010	9.802597	1.990025
5	1.030378	0.970518	6.075502	5.896384	0.164595	0.169595	14.655188	2.485453
8	1.040707	0.960885	8.141409	7.822959	0.122829	0.127829	27.175522	3.473816
10	1.0S1140	0.951348	10.228026	9.730412	0.097771	0.102771	43.386491	4.458855
12	1.061678	0.941905	12.335562	11.618932	0.081066	0.086066	63.213598	5.440569
14	1.072321	0.932556	14.464226	13.488708	0.069136	0.074136	86.583463	6.418959
16	1.083071	0.923300	16.614230	15.339925	0.060189	0.065189	113.423805	7.394026
13	1.093929	0.914136	18.785788	17.172768	0.053232	0.058232	143.663429	8.365770
20	1.1048S6	0.905063	20.979115	18.987419	0.047666	0.052666	177.232212	9.334192
22	1.115972	0.896080	23.194431	20.784059	0.043114	0.048114	214.061088	10.299292
24	1.127160	0.887186	25.431955	22.562866	0.039321	0.044321	254.082034	11.261071
26	1.138450	0.878380	27.691911	24.324018	0.036112	0.041112	297.228055	12.219530
23	1.149873	0.869662	29.974522	26.067689	0.033362	0.038362	343.433173	13.174669
30	1.161400	0.861030	32.280017	27.794054	0.030979	0.035979	392.632412	14.126490
36	1.196681	0.835645	39.336105	32.871016	0.025422	0.030422	557.559832	16.962050
40	1.220794	0.819139	44.158847	36.172228	0.022646	0.027646	681.334687	18.835851
44	1.245394	0.802959	49.078770	39.408232	0.020375	0.025375	815.608700	20.696404
48	1.270489	0.787098	54.097832	42.580318	0.018485	0.023485	959.918810	22.543721
52	1.296090	0.771551	59.218031	45.689747	0.016887	0.021887	1113.816153	24.377814
56	1.322207	0.756311	64.441404	48.737757	0.015518	0.020518	1276.865682	26.198696
60	1.348850	0.741372	69.770031	51.725561	0.014333	0.019333	1448.645795	28.006382
66	1.389825	0.719515	77.964972	56.096976	0.012826	0.017826	1721.795671	30.693199
72	1.432044	0.698302	86.408856	60.339514	0.011573	0.016573	2012.347790	33.350414
84	1.520370	0.657735	104.073927	68.453042	0.009609	0.014609	2640.664052	38.576285
96	1.614143	0.619524	122.828542	76.095218	0.008141	0.013141	3324.184603	43.684540
108	1.713699	0.583533	142.739900	83.293424	0.007006	0.012006	4054.374734	48.675808
∞				200.000000		0.005000		

Discrete Compounding			i	3/4%			Table	A-4
	Single	Payment	Uniform	Series			Gradient	Series
	Compound Amount Factor	Present Amount Factor	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Gradient Present Worth Factor	Gradient Uniform Series Factor
N	F/P	P/F	F/A	P/A	A/F	A/P	P/G	A/G
1	1.007500	0.992556	1.000000	0.992556	1.000000	1.007500	0.000000	0.000000
2	1.015056	0.985167	2.007500	1.977723	0.498132	0.505632	0.985167	0.498132
3	1.022663	0.977833	3.022556	2.955556	0.330846	0.338346	2.940834	0.995019
4	1.030339	0.970554	4.045225	3.926110	0.247205	0.254705	5.852496	1.490660
5	1.038067	0.963329	5.075565	4.889440	0.197022	0.204522	9.705813	1.985056
6	1.045852	0.956158	6.113631	5.845598	0.163569	0.171069	14.486603	2.478207
8	1.061539	0.941975	8.213180	7.736613	0.121756	0.129256	26.774672	3.460774
10	1.077583	0.928003	10.344339	9.599580	0.096671	0.104171	42.606406	4.438362
12	1.093807	0.914238	12.507586	11.434913	0.079951	0.087451	61.873975	5.410971
14	1.110276	0.900677	14.703404	13.243022	0.068011	0.075511	84.471969	6.378602
16	1.126992	0.887318	16.932282	15.024313	0.059059	0.066559	110.297350	7.341258
18	1.143960	0.874156	19.194718	16.779181	0.052098	0.059598	139.249401	8.298939
20	1.161184	0.861190	21.491219	18.508020	0.046531	0.054031	171.229686	9.251648
22	1.178667	0.848416	23.822296	20.211215	0.041977	0.049477	206.142000	10.199387
24	1.196414	0.835831	26.188471	21.889146	0.038185	0.045685	243.892326	11.142158
26	1.214427	0.823434	28.590271	23.542189	0.034977	0.042477	284.388789	12.079964
28	1.232712	0.811220	31.028233	25.170713	0.032229	0.039729	327.541619	13.012807
30	1.251272	0.799187	33.502902	26.775080	0.029848	0.037348	373.263101	13.940690
36	1.308645	0.764149	41.152716	31.446805	0.024300	0.031800	524.992356	16.694617
40	1.348349	0.741648	46.446482	34.446938	0.021530	0.029030	637.469330	18.505834
44	1389256	0.719810	51.900856	37.358730	0.019268	0.026768	758.281493	20.297304
48	1.431405	0.693614	57.520711	40.184782	0.017385	0.024885	886.840449	22.069062
52	1.474833	0.678043	63.311068	42.927618	0.015795	0.023295	1022.585224	23.821150
56	1.519578	0.658077	69.277100	45.589689	0.014435	0.021935	1164.981167	25.553611
60	1.565681	0.638700	75.424137	48.173374	0.013258	0.020758	1313.518881	27.266492
66	1.637471	0.610698	84.996134	51.906955	0.011765	0.019265	1546.786356	29.799212
72	1.712553	0.583924	95.007028	55.476849	0.010526	0.018026	1791.246287	32.288177
84	1.873202	0.533845	116.426928	62.153965	0.008589	0.016089	2308.128298	37.135657
96	2.048921	0.488062	139.856164	68.258439	0.007150	0.014650	2853.935242	41.810731
108	2.241124	0.446205	165.483223	73.839382	0.006043	0.013543	3419.904093	46.315449
∞				133.333333		0.007500		

Discrete Compounding			i	1%			Table	A-5
	Single	Payment	Uniform	Series			Gradient	Series
	Compound Amount Factor	Present Amount Factor	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Gradient Present Worth Factor	Gradient Uniform Series Factor
N	F/P	P/F	F/A	P/A	A/F	A/P	P/G	A/G
1	1.010000	0.990099	1.000000	0.990099	1.000000	1.010000	0.000000	0.000000
2	1.020100	0.980296	2.010000	1.570395	0.497512	0.507512	0.980296	0.497512
3	1.030301	0.970590	3.030100	2.940985	0.330022	0.340022	2.921476	0.993367
4	1.040604	0.960980	4.060401	3.901966	0.246281	0.256281	5.804417	1.487562
5	1.051010	0.951466	5.101005	4.853431	0.196040	0.206040	9.610280	1.980100
6	1.061520	0.942045	6.152015	5.795476	0.162548	0.172548	14.320506	2.470980
8	1.082857	0.923483	8.285671	7.651678	0.120690	0.130690	26.381197	3.447766
10	1.104622	0.905287	10.462213	9.471305	0.095582	0.105582	41.843498	4.417923
12	1.126825	0.887449	12.682503	11.255077	0.078849	0.088849	60.568677	5.381454
14	1.149474	0.869963	14.947421	13.003703	0.066901	0.076901	82.422147	6.338360
16	1.172579	0.852821	17.257864	14.717874	0.057945	0.067945	107.273358	7.288645
13	1.196147	0.836017	18.614748	16.398269	0.050982	0.060982	134.995693	8.232314
20	1.220190	0.819544	22.019004	18.045553	0.045415	0.055415	165.466356	9.169370
22	1.244716	0.803396	24.471586	19.660379	0.040864	0.050864	198.566280	10.099819
24	1.269735	0.787566	26.973465	21.243387	0.037073	0.047073	234.180020	11.023667
26	1.295256	0.772048	29.525631	22.795204	0.033869	0.043869	272.195662	11.940918
28	1.321291	0.756836	32.129097	24.316443	0.031124	0.041124	312.504724	12.851580
30	1.347849	0.741923	34.784892	25.807708	0.028748	0.038748	355.002069	13.755660
36	1.430769	0.696925	43.076878	30.107505	0.023214	0.033214	494.620685	16.428485
40	1.488864	0.671653	48.886373	32.834686	0.020456	0.030456	596.856056	18.177608
44	1.549318	0.645445	54.931757	35.455454	0.018204	0.028204	705.585308	19.900614
48	1.612226	0.620260	61.222608	37.973959	0.016334	0.026334	820.146005	21.597590
52	1.677689	0.596058	67.768892	40.394194	0.014756	0.024756	939.917523	23.268629
56	1.745810	0.572800	74.580982	42.719992	0.013408	0.023408	1064.318789	24.913834
60	1.816697	0.550450	81.669670	44.955038	0.012244	0.022244	1192.806145	26.533314
66	1.928460	0.518548	92.846015	48.145156	0.010771	0.020771	1392.095930	28.914558
72	2.047099	0.488496	104.709931	51.150391	0.009550	0.019550	1597.867334	31.238614
84	2.306723	0.433515	130.672274	56.648453	0.007653	0.017653	2023.315308	35.717045
96	2.599273	0.384723	159.927293	61.527703	0.006253	0.016253	2459.429786	39.972722
103	2.928926	0.341422	192.892579	65.857790	0.005184	0.015184	2898.420284	44.010288
∞				100.000000		0.010000		

Discrete Compounding			i	2%			Table	A-6
	Single	Payment	Uniform	Series			Gradient	Series
	Compound Amount Factor	Present Amount Factor	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Gradient Present Worth Factor	Gradient Uniform Series Factor
N	F/P	P/F	F/A	P/A	A/F	A/P	P/G	A/G
1	1.020000	0.980392	1.000000	0.980392	1.000000	1.020000	0.000000	0.000000
2	1.040400	0.961169	2.020000	1.941561	0.495050	0.515050	0.961169	0.495050
3	1.061208	0.942322	3.060400	2.883883	0.326755	0.346755	2.845813	0.986799
4	1.082432	0.923845	4.121608	3.807729	0.242624	0.262624	5.617350	1.475249
5	1.104081	0.905731	5.204040	4.713460	0.192158	0.212158	9.240273	1.960401
6	1.126162	0.887971	6.308121	5.601431	0.158526	0.178526	13.680130	2.442256
8	1.171659	0.853490	8.582969	7.325481	0.116510	0.136510	24.877924	3.396080
10	1.218994	0.820348	10.949721	8.982585	0.091327	0.111327	38.955100	4.336736
12	1.268242	0.788493	13.412090	10.575341	0.074560	0.094560	55.671156	5.264242
14	1.319479	0.757875	15.973938	12.106249	0.062602	0.082602	74.799921	6.178621
16	1.372786	0.728446	18.639285	13.577709	0.053650	0.073650	96.128815	7.079899
IB	1.428246	0.700159	21.412312	14.992031	0.046702	0.066702	119.458125	7.968108
20	1.485947	0.672971	24.297370	16.351433	0.041157	0.061157	144.600334	8.843282
22	1.545980	0.646839	27.298984	17.658048	0.036631	0.056631	171.379470	9.705459
24	1.608437	0.621721	30.421862	18.913926	0.032871	0.052871	199.630495	10.554683
26	1.673418	0.597579	33.670906	20.121036	0.029699	0.049699	229.198718	11.391000
28	1.741024	0.574375	37.051210	21.281272	0.026990	0.046990	259.939244	12.214460
30	1.811362	0.552071	40.568079	22.396456	0.024650	0.044650	291.716444	13.025117
36	2.039887	0.490223	51.994367	25.488842	0.019233	0.039233	392.040453	15.380865
40	2.208040	0.452890	60.401983	27.355479	0.016556	0.036556	461.993132	16.888504
44	2.390053	0.418401	69.502657	29.079963	0.014388	0.034388	533.516529	18.346534
48	2.587070	0.386538	79.353519	30.673120	0.012602	0.032602	605.965718	19.755595
52	2.800328	0.357101	90.016409	32.144950	0.011109	0.031109	678.784892	21.116377
56	3.031165	0.329906	101.558264	33.504694	0.009847	0.029847	751.497527	22.429619
60	3.281031	0.304782	114.051539	34.760887	0.008768	0.028768	823.697534	23.696103
66	3.694974	0.270638	134.748679	36.468103	0.007421	0.027421	930.300004	25.509964
72	4.161140	0.240319	158.057019	37.984063	0.006327	0.026327	1034.055703	27.223409
84	5.277332	0.189490	213.866607	40.525516	0.004676	0.024676	1230.419116	30.361591
96	6.692933	0.149411	284.646659	42.529434	0.003513	0.023513	1409.297343	33.136988
108	8.488258	0.117810	374.412879	44.109510	0.002671	0.022671	1569.302514	35.577419
∞				50.000000		0.020000		

Discrete Compounding			i	3%			Table	A-7
	Single	Payment	Uniform	Series			Gradient	Series
	Compound Amount Factor	Present Amount Factor	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Gradient Present Worth Factor	Gradient Uniform Series Factor
N	F/P	P/F	F/A	P/A	A/F	A/P	P/G	A/G
1	1.030000	0.970874	1.000000	0.970874	1.000000	1.030000	0.000000	0.000000
2	1.060900	0.942596	2.030000	1.913470	0.492611	0.522611	0.942596	0.492611
3	1.092727	0.915142	3.090900	2.828611	0.323530	0.353530	2.772879	0.980297
4	1.125509	0.888487	4.183627	3.717098	0.239027	0.269027	5.438340	1.463061
5	1.159274	0.862609	5.309136	4.579707	0.188355	0.218355	8.888776	1.940905
6	1.194052	0.837484	6.468410	5.417191	0.154598	0.184598	13.076197	2.413833
8	1.266770	0.789409	8.892336	7.019692	0.112456	0.142456	23.480611	3.344963
10	1.343916	0.744094	11.463879	8.530203	0.087231	0.117231	36.308790	4.256498
12	1.425761	0.701380	14.192030	9.954004	0.070462	0.100462	51.248181	5.148499
14	1.512590	0.661118	17.086324	11.296073	0.058526	0.088526	68.014129	6.021042
16	1.604706	0.623167	20.156881	12.561102	0.049611	0.079611	86.347700	6.874214
18	1.702433	0.587395	23.414435	13.753513	0.042709	0.072709	106.013671	7.708116
20	1.806111	0.553676	26.870374	14.877475	0.037216	0.067216	126.798659	8.522862
22	1.916103	0.521893	30.536780	15.936917	0.032747	0.062747	148.509387	9.318577
24	2.032734	0.491934	34.426470	16.935542	0.029047	0.059047	170.971082	10.095401
26	2.156591	0.463695	38.553042	17.876842	0.025938	0.055938	194.025984	10.853482
28	2.287928	0.437077	42.930923	18.764108	0.023293	0.053293	217.531971	11.592982
30	2.427262	0.411987	47.575416	19.600441	0.021019	0.051019	241.361285	12.314074
36	2.898278	0.345032	63.275944	21.832252	0.015804	0.045804	313.702840	14.368780
40	3.262038	0.306557	75.401260	23.114772	0.013262	0.043262	361.749945	15.650163
44	3.671452	0.272372	89.048409	24.254274	0.011230	0.041230	408.997183	16.862891
43	4.132252	0.241999	104.408396	25.266707	0.009578	0.039578	455.025473	18.008395
52	4.650886	0.215013	121.696197	26.166240	0.008217	0.038217	499.519146	19.090215
56	5.234613	0.191036	141.153768	26.965464	0.007084	0.037084	542.248093	20.108985
60	5.891603	0.169733	163.053437	27.675564	0.006133	0.036133	583.052609	21.067416
66	7.034882	0.142149	201.162741	28.595040	0.004971	0.034971	640.440671	22.396914
72	8.400017	0.119047	246.667242	29.365088	0.004054	0.034054	693.122552	23.603626
84	11.976416	0.083497	365.880536	30.550086	0.002733	0.032733	784.543373	25.680562
96	17.075506	0.058563	535.850186	31.381219	0.001866	0.031866	858.637702	27.361515
208	24.345588	0.041075	778.186267	31.964160	0.001285	0.031285	917.601260	28.707192
∞				33.333333		0.030000		

Discrete Compounding			i	4%			Table	A-8
	Single	Payment	Uniform	Series			Gradient	Series
	Compound Amount Factor	Present Amount Factor	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Gradient Present Worth Factor	Gradient Uniform Series Factor
N	F/P	P/F	F/A	P/A	A/F	A/P	P/G	A/G
1	1.040000	0.961538	1.000000	0.961538	1.000000	1.040000	0.000000	0.000000
2	1.081600	0.924556	2.040000	1.886095	0.490196	0.530196	0.924556	0.490196
3	1.124864	0.888996	3.121600	2.775091	0.320349	0.360349	2.702549	0.973860
4	1.169859	0.854804	4.246464	3.629895	0.235490	0.275490	5.266962	1.450995
5	1.216653	0.821927	5.416323	4.451822	0.184627	0.224627	8.554670	1.921611
6	1.265319	0.790315	6.632975	5.242137	0.150762	0.190762	12.506243	2.385715
8	1.368569	0.730690	9.214226	6.732745	0.108528	0.148528	22.180581	3.294434
10	1.480244	0.675564	12.006107	8.110896	0.083291	0.123291	33.881352	4.177264
12	1.601032	0.624597	15.025805	9.385074	0.066552	0.106552	47.247729	5.034348
14	1.731676	0.577475	18.291911	10.563123	0.054669	0.094669	61.961794	5.865859
16	1.872981	0.533908	21.824531	11.652296	0.045820	0.085820	77.744120	6.672000
18	2.025817	0.493628	25.645413	12.659297	0.038993	0.078993	94.349770	7.453002
20	2.191123	0.456387	29.778079	13.590326	0.033582	0.073582	111.564686	8.209125
22	2.369919	0.421955	34.247970	14.451115	0.029199	0.069199	129.202421	8.940654
24	2.563304	0.390121	39.082604	15.246963	0.025587	0.065587	147.101194	9.647901
26	2.772470	0.360689	44.311745	15.982769	0.022567	0.062567	165.121228	10.331203
28	2.998703	0.333477	49.967583	16.663063	0.020013	0.060013	183.142351	10.990917
30	3.243398	0.308319	56.084938	17.292033	0.017830	0.057830	201.061832	11.627426
36	4.103933	0.243669	77.598314	18.908282	0.012887	0.052887	253.405199	13.401810
40	4.801021	0.208289	95.025516	19.792774	0.010523	0.050523	286.530302	14.476511
44	5.616515	0.178046	115.412877	20.548841	0.008665	0.048665	317.870049	15.469001
48	6.570528	0.152195	139.263206	21.195131	0.007181	0.047181	347.244554	16.383223
52	7.686589	0.130097	167.164718	21.747582	0.005982	0.045982	374.563809	17.223239
56	8.992222	0.111207	199.805540	22.219819	0.005005	0.045005	399.805372	17.993187
60	10.519627	0.095060	237.990685	22.623490	0.004202	0.044202	422.996648	18.697232
66	13.310685	0.075128	307.767116	23.121810	0.003249	0.043249	454.084674	19.638803
72	16.842262	0.059374	396.056560	23.515639	0.002525	0.042525	481.016968	20.455195
84	26.965005	0.037085	649.125119	24.072872	0.001541	0.041541	523.943092	21.764876
96	43.171841	0.023163	1054.30	24.420919	0.000949	0.040949	554.931180	22.723600
108	69.119509	0.014468	1702.99	24.638308	0.000587	0.040587	576.894913	23.414551
∞				25.000000		0.040000		

Discrete Compounding			i	5%			Table	A-9
	Single	Payment	Uniform	Series			Gradient	Series
	Compound Amount Factor	Present Amount Factor	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Gradient Present Worth Factor	Gradient Uniform Series Factor
N	F/P	P/F	F/A	P/A	A/F	A/P	P/G	A/G
1	1.050000	0.952381	1.0000	0.9524	1.000000	1.050000	0.000000	0.000000
2	1.102500	0.907029	2.0500	1.8594	0.487805	0.537805	0.907029	0.487805
3	1.157625	0.863838	3.1525	2.7232	0.317209	0.367209	2.6347	0.957486
4	1.215506	0.822702	4.3101	3.5460	0.232012	0.282012	5.1028	1.439053
5	1.276282	0.783526	5.5256	4.3295	0.180975	0.230975	8.2369	1.902520
6	1.340096	0.746215	6.8019	5.0757	0.147017	0.197017	11.9680	2.357904
8	1.477455	0.676839	9.5491	6.4632	0.104722	0.154722	20.9700	3.244510
10	1.628895	0.613913	12.5779	7.7217	0.079505	0.129505	31.6520	4.099085
12	1.795856	0.556837	15.9171	8.8633	0.062825	0.112825	43.6241	4.921902
14	1.979932	0.505068	19.5986	9.8986	0.051024	0.101024	56.5538	5.713289
16	2.182875	0.458112	23.6575	10.8378	0.042270	0.092270	70.1597	6.473629
18	2.406619	0.415521	28.1324	11.6896	0.035546	0.085546	84.2043	7.203360
20	2.653298	0.376889	33.0660	12.4622	0.030243	0.080243	98.4884	7.902965
22	2.925261	0.341850	38.5052	13.1630	0.025971	0.075971	112.8461	8.572976
24	3.225100	0.310068	44.5020	13.7986	0.022471	0.072471	127.1402	9.213968
26	3.555673	0.281241	51.1135	14.3752	0.019564	0.069564	141.2585	9.826553
28	3.920129	0.255094	58.4026	14.8981	0.017123	0.067123	155.1101	10.411383
30	4.321942	0.231377	66.4388	15.3725	0.015051	0.065051	168.6226	10.969139
36	5.791816	0.172657	95.8363	16.5469	0.010434	0.060434	206.6237	12.487191
40	7.039989	0.142046	120.80	17.1591	0.008278	0.058278	229.5452	13.377471
44	8.557150	0.116861	151.14	17.6628	0.006616	0.056616	250.4175	14.177699
43	10.401270	0.096142	188.03	18.0772	0.005318	0.055318	269.2467	14.894307
52	12.642808	0.079096	232.86	18.4181	0.004294	0.054294	286.1013	15.533724
56	15.367412	0.065073	287.35	18.6985	0.003480	0.053480	301.0894	16.102291
60	18.679186	0.053536	353.58	18.9293	0.002828	0.052828	314.3432	16.606179
66	25.031896	0.039949	480.64	13.2010	0.002081	0.052081	331.2877	17.253650
72	33.545134	0.029811	650.90	19.4038	0.001536	0.051536	345.1485	17.787688
84	60.242241	0.016600	1184.84	19.6680	0.000844	0.050844	365.4727	18.582093
96	108.186410	0.009243	2143.73	19.8151	0.000466	0.050466	378.5555	19.104364
108	194.287249	0.005147	3865.74	19.8971	0.000259	0.050259	386.8236	19.441246
∞				20.0000		0.050000		

Discrete Compounding			i	7%			Table	A-10
	Single	Payment	Uniform	Series			Gradient	Series
	Compound Amount Factor	Present Amount Factor	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Gradient Present Worth Factor	Gradient Uniform Series Factor
N	F/P	P/F	F/A	P/A	A/F	A/P	P/G	A/G
1	1.060000	0.943396	1.0000	0.9434	1.000000	1.060000	0.000000	0.000000
2	1.123600	0.889996	2.0600	1.8334	0.485437	0.545437	0.889996	0.485437
3	1.191016	0.839619	3.1836	2.6730	0.314110	0.374110	2.5692	0.961176
4	1.262477	0.792094	4.3746	3.4651	0.228591	0.288591	4.9455	1.427234
5	1.338226	0.747258	5.6371	4.2124	0.177396	0.237396	7.9345	1.883633
6	1.418519	0.704361	6.9753	4.9173	0.143363	0.203363	11.4594	2.330404
3	1.593848	0.627412	9.8975	6.2098	0.101036	0.161036	19.8416	3.195208
10	1.790848	0.558395	13.1808	7.3601	0.075868	0.135868	29.6023	4.022007
12	2.012196	0.4-96969	16.8699	8.3833	0.059277	0.119277	40.3369	4.811261
14	2.260904	0.442301	21.0151	9.2950	0.047585	0.107585	51.7128	5.563521
16	2.540352	0.393646	25.6725	10.1059	0.038952	0.098952	63.4592	6.279428
18	2.854339	0.350344	30.9057	10.8276	0.032357	0.092357	75.3569	6.959705
20	3.207135	0.311805	36.7856	11.4699	0.027185	0.087185	87.2304	7.605148
22	3.603537	0.277505	43.3923	12.0416	0.023046	0.083046	98.9412	8.216625
24	4.048935	0.246979	50.8156	12.5504	0.019679	0.079679	110.3812	8.795065
26	4.549333	0.219810	59.1564	13.0032	0.016904	0.076904	121.4684	9.341450
28	5.111687	0.195630	68.5281	13.4062	0.014593	0.074593	132.1420	9.856809
30	5.743491	0.174110	79.0582	13.7648	0.012649	0.072649	142.3588	10.342211
36	8.147252	0.122741	119.1209	14.6210	0.008395	0.068395	170.0387	11.629766
40	10.235718	0.097222	154.76	15.0463	0.006462	0.066462	185.9568	12.358976
44	12.985482	0.077009	199.76	15.3832	0.005006	0.065006	199.9130	12.995559
48	16.393872	0.060998	256.56	15.6500	0.003898	0.063893	212.0351	13.548543
52	20.696885	0.048316	328.28	15.8614	0.003046	0.063046	222.4823	14.026655
56	26.129341	0.038271	418.82	16.0288	0.002388	0.062388	231.4272	14.438196
60	32.987691	0.030314	533.13	16.1614	0.001876	0.061876	239.0428	14.790945
66	46.793670	0.021370	763.23	16.3105	0.001310	0.061310	248.3341	15.225419
72	66.377715	0.015065	1039.63	16.4156	0.000918	0.060918	255.5146	15.565374
84	133.565004	0.007487	2209.42	16.5419	0.000453	0.060453	265.2163	16.033015
96	268.759030	0.003721	4462.65	16.6047	0.000224	0.060224	270.7909	16.308135
108	540.795972	0.001849	8996.60	16.6358	0.000111	0.060111	273.9357	16.466591
∞				16.6667		0.060000		

Discrete Compounding			i	7%			Table	A-11
	Single	Payment	Uniform	Series			Gradient	Series
	Compound Amount Factor	Present Amount Factor	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Gradient Present Worth Factor	Gradient Uniform Series Factor
N	F/P	P/F	F/A	P/A	A/F	A/P	P/G	A/G
1	1.07	0.934579	1.0000	0.9346	1.000000	1.070000	0.000000	0.000000
2	1.14	0.873439	2.0700	1.8080	0.483092	0.553092	0.873439	0.483092
3	1.23	0.816298	3.2149	2.6243	0.311052	0.381052	2.5060	0.954929
4	1.31	0.762895	4.4399	3.3872	0.225228	0.295228	4.7947	1.415536
5	1.40	0.712986	5.7507	4.1002	0.173891	0.243891	7.6467	1.864950
6	1.50	0.666342	7.1533	4.7665	0.139796	0.209796	10.9784	2.303217
8	1.72	0.582009	10.2598	5.9713	0.097468	0.167468	18.7889	3.146541
10	1.97	0.508349	13.8164	7.0236	0.072378	0.142378	27.7156	3.946071
12	2.25	0.444012	17.8885	7.9427	0.055902	0.125902	37.3506	4.702516
14	2.58	0.387817	22.5505	8.7455	0.044345	0.114345	47.3718	5.416727
16	2.95	0.338735	27.8881	9.4466	0.035858	0.105858	57.5271	6.089681
18	3.38	0.295864	33.9990	10.0591	0.029413	0.099413	67.6219	6.722474
20	3.87	0.258419	40.9955	10.5940	0.024393	0.094393	77.5091	7.316307
22	4.43	0.225713	49.0057	11.0612	0.020406	0.090406	87.0793	7.872471
24	5.07	0.197147	58.1767	11.4693	0.017183	0.087189	96.2545	8.392336
26	5.81	0.172195	68.6765	11.8258	0.014561	0.084561	104.9814	8.877332
28	6.65	0.150402	80.6977	12.1371	0.012392	0.082392	113.2264	9.328943
30	7.61	0.131367	94.4608	12.4090	0.010586	0.080586	120.9718	9.748684
36	11.42	0.087535	148.9135	13.0352	0.006715	0.076715	141.1990	10.832126
40	14.97	0.066780	199.64	13.3317	0.005009	0.075009	152.2928	11.423349
44	19.63	0.050946	266.12	13.5579	0.003758	0.073758	161.6609	11.923737
43	25.73	0.038867	353.27	13.7305	0.002831	0.072831	169.4981	12.344666
52	33.73	0.029651	467.50	13.8621	0.002139	0.072139	176.0037	12.696732
56	44.21	0.022621	617.24	13.9626	0.001620	0.071620	181.3685	12.989630
60	57.95	0.017257	813.52	14.0392	0.001229	0.071229	185.7677	13.232092
66	86.96	0.011499	1228.03	14.1214	0.000314	0.070814	190.8927	13.517933
72	130.51	0.007662	1850.09	14.1763	0.000541	0.070541	194.6365	13.729757
84	293.93	0.003402	4184.65	14.2371	0.000239	0.070239	199.3046	13.998952
96	661.93	0.001511	9442.52	14.2641	0.000106	0.070106	201.7016	14.140475
108	1490.50	small	21284.26	14.2761	small	0.070047	202.9099	14.213226
∞				14.2857		0.070000		

Discrete Compounding			i	8%			Table	A-12
	Single	Payment	Uniform	Series			Gradient	Series
	Compound Amount Factor	Present Amount Factor	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Gradient Present Worth Factor	Gradient Uniform Series Factor
N	F/P	P/F	F/A	P/A	A/F	A/P	P/G	A/G
1	1.08	0.925926	1.0000	0.9259	1.000000	1.080000	0.000000	0.000000
2	1.17	0.857339	2.0800	1.7833	0.480769	0.560769	0.857339	0.480769
3	1.26	0.793832	3.2464	2.5771	0.308034	0.388034	2.4450	0.948743
4	1.36	0.735030	4.5061	3.3121	0.221921	0.301921	4.6501	1.403960
5	1.47	0.680583	5.8666	3.9927	0.170456	0.250456	7.3724	1.846472
6	1.59	0.630170	7.3359	4.6229	0.136315	0.216315	10.5233	2.276346
8	1.85	0.540269	10.6366	5.7466	0.094015	0.174015	17.8061	3.098524
10	2.16	0.463193	14.4866	6.7101	0.069029	0.149029	25.9768	3.871314
12	2.52	0.397114	18.9771	7.5361	0.052695	0.132695	34.6339	4.595747
14	2.94	0.340461	24.2149	8.2442	0.041297	0.121297	43.4723	5.273051
16	3.43	0.291890	30.3243	8.8514	0.032977	0.112977	52.2640	5.904626
18	4.00	0.250249	37.4502	9.3719	0.026702	0.106702	60.8426	6.492028
20	4.66	0.214548	45.7620	9.8181	0.021852	0.101852	69.0898	7.036948
22	5.44	0.183941	55.4568	10.2007	0.018032	0.098032	76.9257	7.541181
24	634	0.157699	66.7648	10.5288	0.014978	0.094978	84.2997	8.006612
26	7.40	0.135202	79.9544	10.8100	0.012507	0.092507	91.1842	8.435184
28	8.63	0.115914	95.3388	11.0511	0.010489	0.090489	97.5687	8.828883
30	10.06	0.099377	113.2832	11.2578	0.008827	0.088827	103.4558	9.189712
36	15.97	0.062625	187.1021	11.7172	0.005345	0.085345	118.2839	10.094897
40	21.72	0.046031	259.06	11.9246	0.003860	0.083860	126.0422	10.569919
44	29.56	0.033834	356.95	12.0771	0.002802	0.082802	132.3547	10.959166
48	40.21	0.024869	490.13	12.1891	0.002040	0.082049	137.4428	11.275840
52	54.71	0.018280	671.33	12.2715	0.001490	0.081490	141.5121	11.531766
56	74.43	0.013436	917.84	12.3321	0.001090	0.081090	144.7454	11.737337
60	101.26	0.009876	1253.21	12.3766	0.000798	0.080798	147.3000	11.901538
66	160.68	0.006223	1996.03	12.4222	0.000501	0.080501	150.1432	12.086679
72	254.98	0.003922	3174.78	12.4510	0.000315	0.080315	152.1076	12.216516
84	642.09	0.001557	8013.62	12.4805	0.000125	0.080125	154.3714	12.368973
96	1616.89	0.000618	20198.63	12.4923	small	0.080050	155.4112	12.440590
108	4071.60	0.000246	50882.56	12.4969	small	0.080020	155.8801	12.473468
∞				12.5000		0.080000		

Discrete Compounding			i	8%			Table	A-13
	Single	Payment	Uniform	Series			Gradient	Series
	Compound Amount Factor	Present Amount Factor	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Gradient Present Worth Factor	Gradient Uniform Series Factor
N	F/P	P/F	F/A	P/A	A/F	A/P	P/G	A/G
1	1.08	0.925926	1.0000	0.9259	1.000000	1.080000	0.000000	0.000000
2	1.17	0.857339	2.0800	1.7833	0.480769	0.560769	0.857339	0.480769
3	1.26	0.793832	3.2464	2.5771	0.308034	0.388034	2.4450	0.948743
4	1.36	0.735030	4.5061	3.3121	0.221921	0.301921	4.6501	1.403960
5	1.47	0.680583	5.8666	3.9927	0.170456	0.250456	7.3724	1.846472
6	1.59	0.630170	7.3359	4.6229	0.136315	0.216315	10.5233	2.276346
8	1.85	0.540269	106366	5.7466	0.094015	0.174015	17.8061	3.098524
10	2.16	0.463193	14.4866	6.7101	0.069029	0.149029	25.9768	3.871314
12	2.52	0.397114	18.9771	7.5361	0.052695	0.132695	34.6339	4.595747
14	2.94	0.340461	24.2149	8.2442	0.041297	0.121297	43.4723	5.273051
16	3.43	0.291890	30.3243	8.8514	0.032977	0.112977	52.2640	5.904626
18	4.00	0.250249	37.4502	9.3719	0.026702	0.106702	60.8426	6.492028
20	4.66	0.214548	45.7620	9.8181	0.021852	0.101852	69.0898	7.036948
22	5.44	0.183941	55.4568	10.2007	0.018032	0.098032	76.9257	7.541181
24	6.34	0.157699	66.7648	10.5288	0.014978	0.094978	84.2997	8.006612
26	7.40	0.135202	79.9544	10.8100	0.012507	0.092507	91.1842	8.435184
28	8.63	0.115914	95.3388	11.0511	0.010489	0.090489	97.5687	8.828883
30	10.06	0.099377	113.2832	11.2578	0.008827	0.088827	103.4558	9.189712
36	15.97	0.062625	187.1021	11.7172	0.005345	0.085345	118.2839	10.094897
40	21.72	0.046031	259.06	11.9246	0.003860	0.083860	126.0422	10.569919
44	29.58	0.033834	356.95	12.0771	0.002802	0.082802	132.3547	10.959166
48	40.21	0.024869	490.13	12.1891	0.002040	0.082040	137.4428	11.275840
52	54.71	0.018280	671.33	12.2715	0.001490	0.081490	141.5121	11.531766
56	74.43	0.013436	917.84	12.3321	0.001090	0.081090	144.7454	11.737337
60	101.26	0.009876	1253.21	12.3766	0.000798	0.080798	147.3000	11.901538
66	160.68	0.006223	1996.03	12.4222	0.000501	0.080501	150.1432	12.086679
72	254.98	0.003922	3174.78	12.4510	0.000315	0.080315	152.1076	12.216516
84	642.09	0.001557	8013.62	12.4805	0.000125	0.080125	154.3714	12.368973
96	1616.89	0.000618	20198.63	12.4923	small	0.080050	155.4112	12.440590
108	4071.60	0.000246	50882.56	12.4969	small	0.080020	155.8801	12.473468
∞				12.5000		0.080000		

Discrete Compounding			i	10%			Table	A-14
	Single	Payment	Uniform	Series			Gradient	Series
	Compound Amount Factor	Present Amount Factor	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Gradient Present Worth Factor	Gradient Uniform Series Factor
N	F/P	P/F	F/A	P/A	A/F	A/P	P/G	A/G
1	1.10	0.909091	1.0000	0.9091	1.000000	1.100000	0.000000	0.000000
2	1.21	0.826446	2.1000	1.7355	0.476190	0.576190	0.826446	0.476190
3	1.33	0.751315	3.3100	2.4869	0.302115	0.402115	2.3291	0.936556
4	1.46	0.683013	4.6410	3.1699	0.215471	0.315471	4.3781	1.381168
5	1.61	0.620921	6.1051	3.7908	0.163797	0.263797	6.8618	1.810126
6	1.77	0.564474	7.7156	4.3553	0.129607	0.229607	9.6842	2.223557
8	2.14	0.466507	11.4359	5.3349	0.087444	0.187444	16.0287	3.004479
10	2.59	0.385543	15.9374	6.1446	0.062745	0.162745	22.8913	3.725461
12	3.14	0.318631	21.3843	6.8137	0.046763	0.146763	29.9012	4.388402
14	3.80	0.263331	27.9750	7.3667	0.035746	0.135746	36.8005	4.995529
16	4.59	0.217629	35.9497	7.8237	0.027817	0.127817	43.4164	5.549341
18	5.56	0.179859	45.5992	8.2014	0.021930	0.121930	49.6395	6.052560
20	6.73	0.148644	57.2750	8.5136	0.017460	0.117460	55.4069	6.508075
22	8.14	0.122846	71.4027	8.7715	0.014005	0.114005	60.6893	6.918885
24	9.35	0.101526	88.4973	8.9847	0.011300	0.111300	65.4813	7.288054
26	11.92	0.083905	109.1818	9.1609	0.009153	0.109159	69.7940	7.618650
28	14.42	0.069343	134.2099	9.3066	0.007451	0.107451	73.6495	7.913716
30	17.45	0.057309	164.4940	9.4269	0.006079	0.106079	77.0766	8.176226
36	30.91	0.032349	299.1268	9.6765	0.003343	0.103343	85.1194	8.796497
40	45.26	0.022095	442.59	9.7791	0.002259	0.102259	889525	9.096234
44	66.26	0.015091	652.64	9.8491	0.001532	0.101532	91.8508	9.325816
43	97.02	0.010307	960.17	9.8969	0.001041	0.101041	94.0217	9.500090
52	142.04	0.007040	1410.43	9.9296	0.000709	0.100709	95.6351	9.631318
56	207.97	0.004809	2069.65	9.9519	0.000483	0.100483	96.3264	9.729423
60	304.48	0.003284	3034.82	9.9672	0.000330	0.100330	97.7010	9.802294
66	539.41	0.001854	5384.08	9.9815	0.000186	0.100186	98.5910	9.877416
72	955.59	0.001046	9545.94	9.9895	0.000105	0.100105	99.1419	9.924575
84	299S.06	0.000333	29980.63	9.9967	small	0.100033	99.6866	9.971982
96	9412.34	0.000106	94113.44	9.9989	small	0.100011	99.8874	9.989800
108	29539.97	small	295389.66	9.9997	small	0.100003	99.9601	9.996344
∞				10.0000		0.100000		

Discrete Compounding			i	11%			Table	A-15
	Single	Payment	Uniform	Series			Gradient	Series
	Compound Amount Factor	Present Amount Factor	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Gradient Present Worth Factor	Gradient Uniform Series Factor
N	F/P	P/F	F/A	P/A	A/F	A/P	P/G	A/G
1	1.11	0.900901	1.0000	0.9009	1.000000	1.110000	0.000000	0.000000
2	1.23	0.811522	2.1100	1.7125	0.473934	0.583934	0.811622	0.473934
3	1.37	0.731191	3.3421	2.4437	0.299213	0.409213	2.2740	0.930553
4	1.52	0.658731	4.7097	3.1024	0.212326	0.322326	4.2502	1.369951
5	1.69	0.593451	6.2278	3.6959	0.160570	0.270570	6.6240	1.792259
6	1.87	0.534541	7.9129	4.2305	0.126377	0.236377	9.2972	2.197642
8	2.30	0.433926	11.8594	5.1461	0.084321	0.194321	15.2246	2.958469
10	2.84	0.352184	16.7220	5.8892	0.059801	0.169801	21.5217	3.654416
12	3.50	0.285841	22.7132	6.4924	0.044027	0.154027	27.8388	4.287932
14	4.31	0.231995	30.0949	6.9819	0.033228	0.143228	33.9449	4.861865
16	5.31	0.188292	39.1899	7.3792	0.025517	0.135517	39.6953	5.379382
16	6.54	0.152822	50.3959	7.7016	0.019843	0.129843	45.0074	5.343894
20	8.06	0.124034	64.2028	7.9633	0.015576	0.125576	49.8423	6.258975
22	9.93	0.100869	81.2143	8.1757	0.012313	0.122313	54.1912	6.628289
24	12.24	0.081705	102.1742	8.3481	0.009787	0.119787	58.0656	6.955518
26	15.08	0.066314	127.9988	8.4881	0.007813	0.117813	61.4900	7.244300
28	18.53	0.053822	159.8173	8.6016	0.006257	0.116257	64.4965	7.498181
30	22.89	0.043683	199.0209	8.6938	0.005025	0.115025	67.1210	7.720564
36	42.82	0.023355	380.1544	8.8786	0.002630	0.112630	73.0712	8.230038
40	65.00	0.015384	581.83	8.9511	0.001719	0.111719	75.7789	8.465918
44	98.68	0.010134	887.96	8.9988	0.001126	0.111126	77.7534	8.640440
43	149.80	0.006676	1352.70	9.0302	0.000739	0.110739	79.1799	8.768322
52	227.40	0.004397	2058.20	9.0509	0.000486	0.110486	80.2024	8.861229
56	345.21	0.002897	3129.21	9.0646	0.000320	0.110320	80.9305	8.928219
60	524.06	0.001908	4755.07	9.0736	0.000210	0.110210	81.4461	8.976199
66	980.20	0.001020	8901.86	9.0816	0.000112	0.110112	81.9482	9.023507
72	1333.39	0.000545	16658.08	9.0860	small	0.110060	82.2425	9.051616
84	6414.02	0.000156	58300.17	9.0895	small	0.110017	82.5127	9.077811
96	22439.13	small	203982.98	9.0905	small	0.110005	82.5021	9.086631
108	78502.18	small	713647.08	9.0908	small	0.110001	82.6311	9.089533
∞				9.0909		0.110000		

Discrete Compounding			i	12%			Table	A-16
	Single	Payment	Uniform	Series			Gradient	Series
	Compound Amount Factor	Present Amount Factor	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Gradient Present Worth Factor	Gradient Uniform Series Factor
N	F/P	P/F	F/A	P/A	A/F	A/P	P/G	A/G
1	1.12	0.892857	1.0000	0.8929	1.600000	1.120000	0.000000	0.000000
2	1.25	0.797194	2.1200	1.6901	0.471698	0.591698	0.797194	0.471698
3	1.40	0.711780	3.3744	2.4018	0.296349	0.416349	2.2208	0.924609
4	1.57	0.635518	4.7793	3.0373	0.209234	0.329234	4.1273	1.358852
5	1.78	0.567427	6.3528	3.6048	0.157410	0.277410	6.3970	1.774595
6	1.97	0.506631	8.1152	4.1114	0.123226	0.243226	8.9302	2.172047
8	2.48	0.403883	12.2997	4.9676	0.081303	0.201303	14.4714	2.913144
10	3.11	0.321973	17.5437	5.6502	0.056984	0.176984	20.2541	3.584653
12	3.90	0.256675	24.1331	6.1944	0.041437	0.161437	25.9523	4.189653
14	4.89	0.204620	32.3926	6.6282	0.030871	0.150871	31.3624	4.731688
16	6.13	0.163122	42.7533	6.9740	0.023390	0.143390	36.3670	5.214664
18	7.69	0.130040	55.7497	7.2497	0.017937	0.137937	40.9080	5.642737
20	9.65	0.103667	72.0524	7.4694	0.013879	0.133879	44.9676	6.020203
22	12.10	0.082643	92.5026	7.6446	0.010811	0.130811	48.5543	6.351407
24	15.18	0.065882	118.1552	7.7843	0.008463	0.128463	51.6929	6.640645
26	19.04	0.052521	150.3339	7.8957	0.006652	0.126652	54.4177	6.892097
28	23.88	0.041369	190.6989	7.9344	0.005244	0.125244	56.7674	7.109764
30	29.96	0.033378	241.3327	8.0552	0.004144	0.124144	58.7821	7.297419
36	59.14	0.016910	484.4631	8.1924	0.002064	0.122064	63.1970	7.714091
40	93.05	0.010747	76709	8.2438	0.001304	0.121304	65.1159	7.898791
44	146.42	0.006830	1211.81	8.2764	0.000825	0.120825	66.4659	8.030756
48	230.39	0.004340	1911.59	8.2972	0.000523	0.120523	67.4068	8.124083
52	362.52	0.002758	3012.70	8.3103	0.000332	0.120332	68.0576	8.189498
56	570.44	0.001753	4745.33	8.3187	0.000211	0.120211	68.5046	8.234991
60	837.60	0.001114	7471.64	8.3240	0.000134	0.120134	68.8100	8.266414
66	1771.70	0.000564	14755.31	8.3286	small	0.120068	69.0948	8.296060
72	3497.02	0.000286	29133.47	8.3310	small	0.120034	69.2530	8.312738
84	13624.29	small	113527.42	8.3327	small	0.120009	69.3880	8.327167
96	53079.91	small	442324.25	8.3332	small	0.120002	69.4281	8.331525
108	206798.05	small	1723308.79	8.3333	small	0.120001	69.4398	8.332811
∞				8.3333		0.120000		

Discrete Compounding			i	13%			Table	A-17
	Single	Payment	Uniform	Series			Gradient	Series
	Compound Amount Factor	Present Amount Factor	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Gradient Present Worth Factor	Gradient Uniform Series Factor
N	F/P	P/F	F/A	P/A	A/F	A/P	P/G	A/G
1	1.13	0.884956	1.0000	0.8850	1.000000	1.130000	0.000000	0.000000
2	1.28	0.783147	2.1300	1.6681	0.469484	0.599484	0.783147	0.469484
3	1.44	0.693050	3.4069	2.3612	0.293522	0.423522	2.1692	0.918724
4	1.63	0.613319	4.8498	2.9745	0.206194	0.336194	4.0092	1.347871
5	1.84	0.542760	6.4803	3.5172	0.154315	0.284315	6.1802	1.757133
6	2.08	0.480319	8.3227	3.9975	0.120153	0.250153	8.5818	2.146774
8	2.66	0.376160	12.7573	4.7988	0.078387	0.208387	13.7653	2.868510
10	3.39	0.294588	18.4197	5.4262	0.054290	0.184290	19.0797	3.516188
12	4.33	0.230706	25.6502	5.9176	0.038986	0.168986	24.2244	4.093592
14	5.53	0.180677	34.8827	6.3025	0.028667	0.158667	29.0232	4.605039
16	7.07	0.141496	46.6717	6.6039	0.021426	0.151426	33.3841	5.055231
18	9.02	0.110812	61.7251	6.8399	0.016201	0.146201	37.2714	5.449112
20	11.52	0.086782	80.9468	7.0248	0.012354	0.142354	40.6854	5.791725
22	14.71	0.067963	105.4910	7.1695	0.009479	0.139479	43.6486	6.088088
24	18.79	0.053225	136.8315	7.2829	0.007308	0.137308	46.1960	6.343090
26	23.99	0.041683	176.8501	7.3717	0.005655	0.135655	48.3685	6.561406
28	30.63	0.032644	227.9499	7.4412	0.004387	0.134387	50.2090	6.747431
30	39.12	0.025565	293.1992	7.4957	0.003411	0.133411	51.7592	6.905235
36	81.44	0.012279	618.7493	7.5979	0.001616	0.131616	55.0446	7.244755
40	132.78	0.007531	1013.70	7.6344	0.000986	0.130986	56.4087	7.388775
44	216.50	0.004619	1657.67	7.6568	0.000603	0.130603	57.3349	7.488128
43	352.99	0.002833	2707.63	7.6705	0.000369	0.130369	57.9580	7.555941
52	575.54	0.001737	4419.57	7.6789	0.000226	0.130226	58.3738	7.601801
56	938.41	0.001066	7210.85	7.6841	0.000139	0.130139	58.6495	7.632569
60	1530.05	0.000654	11761.95	7.6873	small	0.130085	58.8313	7.653068
66	3185.50	0.000314	24496.13	7.6899	small	0.130041	58.9936	7.671582
72	6632.05	0.000151	51008.09	7.6911	small	0.130020	59.0792	7.681450
84	28746.78	small	221121.41	7.6920	small	0.130005	59.1471	7.689386
96	124603.60	small	958481.50	7.6922	small	0.130001	59.1652	7.691537
108	540097.16	small	4154585.87	7.6923	small	0.130000	59.1699	7.692108
∞				7.6923		0.130000		

Discrete Compounding			i	15%			Table	A-18
	Single	Payment	Uniform	Series			Gradient	Series
	Compound Amount Factor	Present Amount Factor	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Gradient Present Worth Factor	Gradient Uniform Series Factor
N	F/P	P/F	F/A	P/A	A/F	A/P	P/G	A/G
1	1.15	0.869565	1.0000	0.8696	1.000000	1.150000	0.000000	0.000000
2	1.32	0.756144	2.1500	1.6257	0.465116	0.615116	0.756144	0.465116
3	1.52	0.657516	3.4725	2.2832	0.287977	0.437977	2.0712	0.907127
4	1.75	0.571753	4.9934	2.8550	0.200265	0.350265	3.7864	1.326257
5	2.01	0.497177	6.7424	3.3522	0.148316	0.298316	5.7751	1.722815
6	2.31	0.432328	8.7537	3.7845	0.114237	0.264237	7.9368	2.097190
8	3.06	0.326902	13.7268	4.4873	0.072850	0.222850	12.4807	2.781329
10	4.05	0.247185	20.3037	5.0188	0.049252	0.199252	16.9795	3.383196
12	5.35	0.186907	29.0017	5.4206	0.034481	0.184481	21.1849	3.908205
14	7.08	0.141329	40.5047	5.7245	0.024688	0.174688	24.9725	4.362408
16	9.36	0.106865	55.7175	5.9542	0.017948	0.167948	28.2960	4.752246
18	12.38	0.080805	75.8364	6.1280	0.013186	0.163186	31.1565	5.084312
20	16.37	0.061100	102.4436	6.2593	0.009761	0.159761	33.5822	5.365137
22	21.64	0.046201	137.6316	6.3587	0.007266	0.157266	35.6150	5.S01020
24	28.63	0.034934	184.1678	6.4338	0.005430	0.155430	37.3023	5.797894
26	37.86	0.026415	245.7120	6.4906	0.004070	0.154070	38.6918	5.961234
28	50.07	0.019974	327.1041	6.5335	0.003057	0.153057	39.8283	6.096002
30	66.21	0.015103	434.7451	6.5660	0.002300	0.152300	40.7526	6.206627
36	153.15	0.006529	1014.3457	6.6231	0.000986	0.150986	42.5872	6.430061
40	267.86	0.003733	1779.09	6.6418	0.000562	0.150562	43.2830	6.516777
44	468.50	0.002134	3116.63	6.6524	0.000321	0.150321	43.7235	6.572548
43	819.40	0.001220	5456.00	6.6585	0.000183	0.150183	43.9997	6.608016
52	1433.14	0.000698	9547.58	6.6620	0.000105	0.150105	44.1715	6.630357
56	2506.57	0.000399	16703.77	6.6640	small	0.150060	44.2778	6.644316
60	4384.00	0.000228	29219.99	6.6651	small	0.150034	44.3431	6.652977
66	10140.46	0.000099	67596.37	6.6660	small	0.150015	44.3967	6.660157
72	23455	small	156363.27	6.6664	small	0.150005	44.4221	6.663597
84	125493	small	836611.58	6.6666	small	0.150001	44.4396	6.665997
96	671418	small	4476110.18	6.6667	small	0.150000	44.4434	6.666524
108	3592252	small	23948338	6.6667	small	0.150000	44.4442	6.666637
∞				6.6667		0.150000		

Discrete Compounding			i	17%			Table	A-19
	Single	Payment	Uniform	Series			Gradient	Series
	Compound Amount Factor	Present Amount Factor	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Gradient Present Worth Factor	Gradient Uniform Series Factor
N	F/P	P/F	F/A	P/A	A/F	A/P	P/G	A/G
1	1.17	0.854701	1.0000	0.8547	1.000000	1.170000	0.000000	0.000000
2	1.37	0.730514	2.1700	1.5852	0.460829	0.630829	0.730514	0.460829
3	1.60	0.624371	3.5389	2.2096	0.282574	0.452574	1.9793	0.895759
4	1.87	0.533650	5.1405	2.7432	0.194533	0.364533	3.5802	1.305103
5	2.19	0.456111	7.0144	3.1993	0.142564	0.312564	5.4046	1.689298
6	2.57	0.389839	9.2068	3.5892	0.108615	0.278615	7.3538	2.048889
8	3.51	0.254782	14.7733	4.2072	0.067690	0.237690	11.3465	2.696946
10	4.81	0.208037	22.3931	4.6586	0.044657	0.214657	15.1661	3.255494
12	6.58	0.151974	32.8239	4.9884	0.030466	0.200466	18.6159	3.731841
14	9.01	0.111019	47.1027	5.2293	0.021230	0.191230	21.6178	4.133982
16	12.33	0.081101	66.6488	5.4053	0.015004	0.185004	24.1628	4.470211
18	16.88	0.059245	93.4056	5.5339	0.010706	0.180706	26.2790	4.748777
20	23.11	0.043280	130.0329	5.6278	0.007690	0.177690	28.0128	4.977605
22	31.63	0.031616	180.1721	5.6964	0.005550	0.175550	29.4166	5.164085
24	43.30	0.023096	248.8076	5.7465	0.004019	0.174019	30.5423	5.314941
26	59.27	0.016872	342.7627	5.7831	0.002917	0.172917	31.4378	5.436152
28	81.13	0.012325	471.3778	5.8099	0.002121	0.172121	32.1456	5.532939
30	111.06	0.009004	647.4391	5.8294	0.001545	0.171545	32.7016	5.609786
36	284.90	0.003510	1669.9945	5.8617	0.000599	0.170599	33.7373	5.755547
40	533.87	0.001873	3134.52	5.8713	0.000319	0.170319	34.0965	5.807288
44	1000.4-1	0.001000	5878.88	5.8765	0.000170	0.170170	34.3088	5.838327
48	1874.66	0.000533	11021.50	5.8792	small	0.170091	34.4330	5.856735
52	3512.89	0.000285	20658.19	5.8807	small	0.170048	34.5052	5.867546
56	6582.76	0.000152	38716	5.8815	small	0.170026	34.5468	5.873845
60	12335.36	small	72555	5.8819	small	0.170014	34.5707	5.877488
66	31642.21	small	186125	5.8822	small	0.170005	34.5887	5.880267
72	81167	small	477450	5.8823	small	0.170002	34.5964	5.881466
84	534087	small	3141685	5.8823	small	0.170000	34.6011	5.882196
96	3514332	small	20672532	5.8824	small	0.170000	34.6019	5.882326
108	23124538	small	136026689	5.8824	small	0.170000	34.6020	5.882348
∞				5.8824		0.170000		

Discrete Compounding			i	19%			Table	A-20
	Single	Payment	Uniform	Series			Gradient	Series
	Compound Amount Factor	Present Amount Factor	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Gradient Present Worth Factor	Gradient Uniform Series Factor
N	F/P	P/F	F/A	P/A	A/F	A/P	P/G	A/G
1	1.19	0.840336	1.0000	0.8403	1.000000	1.190000	0.000000	0.000000
2	1.42	0.706165	2.1900	1.5465	0.456621	0.646621	0.706165	0.456621
3	1.69	0.593416	3.6061	2.1399	0.277308	0.467308	1.8930	0.884612
4	2.01	0.498663	5.2913	2.6386	0.188991	0.378991	3.3890	1.284401
5	2.39	0.419049	7.2966	3.0576	0.137050	0.327050	5.0652	1.656575
6	2.84	0.352142	9.6830	3.4098	0.103274	0.293274	6.8259	2.001864
8	4.02	0.248671	15.9020	3.9544	0.062885	0.252885	10.3421	2.615366
10	5.69	0.175602	24.7089	4.3389	0.040471	0.230471	13.5943	3.133089
12	8.06	0.124004	37.1802	4.6105	0.026896	0.216896	16.4340	3.564462
14	11.42	0.087567	54.8409	4.8023	0.018235	0.208235	18.8228	3.919559
16	16.17	0.061837	79.8502	4.9377	0.012523	0.202523	20.7806	4.208552
18	22.90	0.043667	115.2659	5.0333	0.008676	0.198676	22.3543	4.441260
20	32.43	0.030836	165.4180	5.1009	0.005045	0.196045	23.6007	4.626812
22	45.92	0.021775	236.4385	5.1486	0.004229	0.194229	24.5763	4.773434
24	65.03	0.015377	337.0105	5.1822	0.002967	0.192967	25.3325	4.388345
26	92.09	0.010859	479.4306	5.2060	0.002036	0.192086	25.9141	4.977732
28	130.41	0.007668	681.1116	5.2228	0.001468	0.191468	26.3584	5.046793
30	184.68	0.005415	966.7122	5.2347	0.001034	0.191034	26.6958	5.099826
36	524.43	0.001907	2754.9143	5.2531	0.000363	0.190363	27.2867	5.194381
40	1051.67	0.000951	5529.83	5.2582	0.000181	0.190181	27.4743	5.225087
44	2108.95	0.000474	11094.47	5.2607	small	0.190090	27.5779	5.242285
48	4229.16	0.000238	22253.48	5.2619	small	0.190045	27.6345	5.251805
52	8480.90	0.000118	44631.06	5.2625	small	0.190022	27.6653	5.257026
56	17007.08	small	89506	5.2628	small	0.190011	27.6819	5.259865
60	34104.97	small	179495	5.2630	small	0.190006	27.6908	5.261399
66	96845.96	small	509731	5.2631	small	0.190002	27.6970	5.262476
72	275031	small	1447525	5.2631	small	0.190001	27.6994	5.262896
84	2217914	small	11673228	5.2632	small	0.190000	27.7006	5.263120
96	17885797	small	94135768	5.2632	small	0.190000	27.7008	5.263153
108	144235389	small	759133621	5.2632	small	0.190000	27.7008	5.263157
∞				5.2632		0.190000		

Discrete Compounding			i	21%			Table	A-21
	Single	Payment	Uniform	Series			Gradient	Series
	Compound Amount Factor	Present Amount Factor	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Gradient Present Worth Factor	Gradient Uniform Series Factor
N	F/P	P/F	F/A	P/A	A/F	A/P	P/G	A/G
1	1.21	0.826446	1.0000	0.8264	1.000000	1.210000	0.000000	0.000000
2	1.46	0.683013	2.2100	1.5095	0.452489	0.662489	0.683013	0.452489
3	1.77	0.564474	3.6741	2.0739	0.272175	0.482175	1.8120	0.873683
4	2.14	0.466507	5.4457	2.5404	0.183632	0.393632	3.2115	1.264144
5	2.59	0.385543	7.5892	2.9260	0.131765	0.341765	4.7537	1.624635
6	3.14	0.318631	10.1830	3.2446	0.098203	0.308203	6.3468	1.956106
8	4.59	0.217629	17.1189	3.7256	0.058415	0.268415	9.4502	2.536575
10	6.73	0.148644	27.2738	4.0541	0.036665	0.246665	12.2269	3.015942
12	9.85	0.101526	42.1416	4.2784	0.023730	0.233730	14.5721	3.405932
14	14.42	0.069343	63.9095	4.4317	0.015647	0.225647	16.4804	3.718763
16	21.11	0.047362	95.7799	4.5364	0.010441	0.220441	17.9932	3.966430
18	30.91	0.032349	142.4413	4.6079	0.007020	0.217020	19.1694	4.160153
20	45.26	0.022095	210.7584	4.6567	0.004745	0.214745	20.0704	4.310022
22	66.26	0.015091	310.7813	4.6900	0.003218	0.213218	20.7526	4.424813
24	97.02	0.010307	457.2249	4.7128	0.002187	0.212187	21.2640	4.511950
26	142.04	0.007040	671.6330	4.7284	0.001489	0.211489	21.6445	4.577564
28	207.97	0.004809	385.5479	4.7390	0.001015	0.211015	21.9256	4.626616
30	304.48	0.003284	1445.1507	4.7463	0.000692	0.210692	22.1321	4.663052
36	955.59	0.001046	4545.6848	4.7569	0.000220	0.210220	22.4726	4.724192
40	2048.40	0.000488	9749.52	4.7596	0.000103	0.210103	22.5717	4.742368
44	4390.93	0.000228	20904.42	4.7608	small	0.210048	22.6229	4.751882
48	9412.34	0.000106	44815.92	4.7614	small	0.210022	22.6490	4.756805
52	20176.19	small	96072.35	4.7617	small	0.210010	22.6623	4.759327
56	43249.46	small	205945	4.7618	small	0.210005	22.6690	4.760610
60	92709.07	small	441467	4.7619	small	0.210002	22.6724	4.761258
66	290960.77	small	1385523	4.7619	small	0.210001	22.6746	4.761678
72	913160	small	4348374	4.7619	small	0.210000	22.6753	4.761826
84	8994377	small	42830364	4.7619	small	0.210000	22.6757	4.761895
96	38592213	small	421867676	4.7619	small	0.210000	22.6757	4.761904
108	872609615	small	4155283878	4.7619	small	0.210000	22.6757	4.761905
∞				4.7619		0.210000		

Discrete Compounding			i	25%			Table	A-23
	Single	Payment	Uniform	Series			Gradient	Series
	Compound Amount Factor	Present Amount Factor	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Gradient Present Worth Factor	Gradient Uniform Series Factor
N	F/P	P/F	F/A	P/A	A/F	A/P	P/G	A/G
1	1.25	0.800000	1.0000	0.8000	1.000000	1.250000	0.000000	0.000000
2	1.56	0.640000	2.2500	1.4400	0.444444	0.694444	0.640000	0.444444
3	1.95	0.512000	3.8125	1.9520	0.262295	0.512295	1.6640	0.852459
4	2.44	0.409600	5.7656	2.3616	0.173442	0.423442	2.8928	1.224932
5	3.05	0.327680	8.2070	2.6893	0.121847	0.371847	4.2035	1.563065
6	3.81	0.262144	11.2588	2.9514	0.088819	0.338819	5.5142	1.868332
8	5.%	0.167772	19.8419	3.3289	0.050399	0.300399	7.9469	2.387248
10	9.31	0.107374	33.2529	3.5705	0.030073	0.280073	9.9870	2.797098
12	14.55	0.068719	54.2077	3.7251	0.018448	0.268448	11.6020	3.114516
14	22.74	0.043980	86.9495	3.8241	0.011501	0.261501	12.8334	3.355948
16	35.53	0.028147	138.1085	3.8874	0.007241	0.257241	13.7482	3.536596
18	55.51	0.018014	218.0446	3.9279	0.004586	0.254586	14.4147	3.669792
20	86.74	0.011529	342.9447	3.9539	0.002916	0.252916	14.8932	3.766726
22	135.53	0.007379	538.1011	3.9705	0.001858	0.251858	15.2326	3.836462
24	211.76	0.004722	843.0329	3.9811	0.001186	0.251186	15.4711	3.886125
26	330.87	0.003022	1319.4890	3.9879	0.000758	0.250758	15.6373	3.921182
29	516.99	0.001934	2063.9515	3.9923	0.000485	0.250485	15.7524	3.945735
30	807.79	0.001238	3227.1743	3.9950	0.000310	0.250310	15.8316	3.962816
36	3081.49	0.000325	12321.9516	3.9987	0.000081	0.250081	15.9481	3.988314
40	7523.16	0.000133	30088.66	3.9995	small	0.250033	15.9766	3.994682
44	18367.10	small	73464.40	3.9998	small	0.250014	15.9895	3.997604
48	44841.55	small	179362.20	3.9999	small	0.250006	15.9954	3.998930
52	109476.44	small	437901.77	4.0000	small	0.250002	15.9980	3.999525
56	267276.47	small	1069102	4.0000	small	0.250001	15.9991	3.999790
60	652530.45	small	2610118	4.0000	small	0.250000	15.9996	3.999908
66	2439206.11	small	9956820	4.0000	small	0.250000	15.9999	3.999973
72	9495568	small	37982267	4.0000	small	0.250000	16.0000	3.999992
84	138178697	small	552714784	4.0000	small	0.250000	16.0000	3.999999
96	2010764683	small	8043058730	4.0000	small	0.250000	16.0000	4.000000
∞				4.0000		0.250000		

Discrete Compounding			i	30%			Table	A-23
	Single	Payment	Uniform	Series			Gradient	Series
	Compound Amount Factor	Present Amount Factor	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Gradient Present Worth Factor	Gradient Uniform Series Factor
N	F/P	P/F	F/A	P/A	A/F	A/P	P/G	A/G
1	1.30	0.769231	1.0000	0.7692	1.000000	1.300000	0.000000	0.000000
2	1.69	0.591716	2.3000	1.3609	0.434783	0.734783	0.591716	0.434783
3	2.20	0.455166	3.9300	1.8161	0.250627	0.550627	1.5020	0.827068
4	2.86	0.350128	6.1870	2.1662	0.161629	0.461629	2.5524	1.178277
5	3.71	0.269329	9.0431	2.4356	0.110582	0.410582	3.6297	1.490308
6	4.83	0.207176	12.7560	2.6427	0.078394	0.378394	4.6656	1.765447
8	8.16	0.122589	23.8577	2.9247	0.041915	0.341915	6.4800	2.215595
10	13.79	0.072538	42.6195	3.0915	0.023463	0.323463	7.8872	2.551219
12	23.30	0.042922	74.3270	3.1903	0.013454	0.313454	8.9173	2.795171
14	39.37	0.025398	127.9125	3.2487	0.007818	0.307818	9.6437	2.968501
16	66.54	0.015028	218.4722	3.2832	0.004577	0.304577	10.1426	3.089214
18	112.46	0.008892	371.5180	3.3037	0.002692	0.302692	10.4788	3.171834
20	190.05	0.005262	630.1655	3.3158	0.001587	0.301587	10.7019	3.227541
22	321.18	0.003113	1067.2796	3.3230	0.000937	0.300937	10.8482	3.264623
24	542.80	0.001842	1806.0026	3.3272	0.000554	0.300554	10.9433	3.2B9037
26	917.33	0.001090	3054.4443	3.3297	0.000327	0.300327	11.0045	3.304959
28	1550.29	small	5164.3109	3.3312	0.000194	0.300194	11.0437	3.315261
30	2620.00	small	8729.9855	3.3321	0.000115	0.300115	11.0687	3.321879
36	12646.22	srnall	42150.7285	3.3331	small	0.300024	11.1007	3.330486
40	36118.86	small	120392.88	3.3332	small	0.300008	11.1071	3.332226
44	103159.09	small	343860.30	3.3333	small	0.300003	11.1096	3.332907
60	6864377.17	small	22881254	3.3333	small	0.300000	11.1111	3.333325
∞				3.3333		0.300000		

APPENDIX B

Continuous Compounding—Interest and Annuity Tables, Table B-1 to B-25

APPENDIX B Continuous Compounding—Interest and Annuity Tables

Continuous Compounding			i	1/16%	Table	B-1
	Single Payment	Uniform	Series	Uniform	Series	
	Compound Amount Factor	Present Amount Factor	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor
N	F/P	P/F	F/A	P/A	A/F	A/P
1	1.000625	0.999375	1.000000	0.999375	1.000000	1.000625
2	1.001251	0.998751	2.000625	1.998126	0.499844	0.500469
3	1.001877	0.998127	3.001876	2.996253	0.333125	0.333750
4	1.002503	0.997503	4.003753	3.993756	0.249766	0.250391
5	1.003130	0.996880	5.006256	4.990636	0.199750	0.200375
6	1.003757	0.996257	6.009386	5.986893	0.166406	0.167032
8	1.005013	0.995012	8.017527	7.977540	0.124727	0.125352
10	1.006270	0.993769	10.028181	9.965700	0.099719	0.100344
12	1.007528	0.992528	12.041349	11.951377	0.083047	0.083672
14	1.008788	0.991288	14.057035	13.934573	0.071139	0.071764
16	1.010050	0.990050	16.075243	15.915291	0.062207	0.062833
18	1.011314	0.988813	18.095975	17.893536	0.055261	0.055886
20	1.012578	0.987578	20.119234	19.869309	0.049704	0.050329
22	1.013845	0.986344	22.145024	21.842614	0.045157	0.045782
24	1.015113	0.985112	24.173348	23.813453	0.041368	0.041993
26	1.016383	0.983881	26.204208	25.781831	0.038162	0.038787
28	1.017654	0.982652	28.237609	27.747750	0.035414	0.036039
30	1.018927	0.981425	30.273554	29.711213	0.033032	0.033657
36	1.022755	0.977751	36.396678	35.586897	0.027475	0.028100
40	1.025315	0.975310	40.491537	39.491797	0.024697	0.025322
44	1.027882	0.972875	44.596645	43.386947	0.022423	0.023048
48	1.030455	0.970446	48.712029	47.272371	0.020529	0.021154
52	1.033034	0.968022	52.837714	51.148093	0.018926	0.019551
56	1.035620	0.965605	56.973726	55.014138	0.017552	0.018177
60	1.038212	0.963194	61.120091	58.870531	0.016361	0.016986
66	1.042113	0.959589	67.359108	64.637072	0.014846	0.015471
72	1.046028	0.955997	73.621564	70.382030	0.013583	0.014208
84	1.053903	0.948854	86.217151	81.807516	0.011599	0.012224
96	1.061837	0.941765	98.907559	93.147632	0.010110	0.010736
108	1.069830	0.934728	111.693504	104.403014	0.008953	0.009578
∞				1599.500052		0.000625

Continuous Compounding			i	1/12%	Table	B-2
	Single Payment	Uniform	Series	Uniform	Series	
	Compound Amount Factor	Present Amount Factor	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor
N	F/P	P/F	F/A	P/A	A/F	A/P
1	1.000834	0.999167	1.000000	0.999167	1.000000	1.000834
2	1.001668	0.998335	2.000834	1.997502	0.499792	0.500625
3	1.002503	0.997503	3.002502	2.995005	0.333056	0.333889
4	1.003339	0.996672	4.005005	3.991677	0.249688	0.250521
5	1.004175	0.995842	5.008344	4.987519	0.199667	0.200500
6	1.005013	0.995012	6.012519	5.982532	0.166320	0.167153
8	1.006689	0.993356	8.023382	7.970071	0.124636	0.125469
10	1.008368	0.991701	10.037599	9.954300	0.099625	0.100459
12	1.010050	0.990050	12.055176	11.935225	0.082952	0.083786
14	1.011735	0.988401	14.076119	13.912851	0.071042	0.071876
16	1.013423	0.986755	16.100432	15.887184	0.062110	0.062944
18	1.015113	0.985112	18.128122	17.858229	0.055163	0.055997
20	1.016806	0.983471	20.159194	19.825992	0.049605	0.050439
22	1.018502	0.981834	22.193655	21.790478	0.045058	0.045892
24	1.020201	0.980199	24.231509	23.751693	0.041269	0.042102
26	1.021903	0.978566	26.272762	25.709641	0.038062	0.038896
28	1.023608	0.976937	28.317420	27.664329	0.035314	0.036148
30	1.025315	0.975310	30.365489	29.615762	0.032932	0.033766
36	1.030455	0.970446	36.530216	35.450585	0.027375	0.028208
40	1.033895	0.967216	40.657191	39.324290	0.024596	0.025430
44	1.037347	0.963997	44.797946	43.185104	0.022322	0.023156
48	1.040811	0.960789	48.952526	47.033070	0.020428	0.021262
52	1.044286	0.957592	53.120979	50.868232	0.018825	0.019659
56	1.047773	0.954405	57.303349	54.690630	0.017451	0.018285
60	1.051271	0.951229	61.499684	58.500309	0.016260	0.017094
66	1.056541	0.946485	67.820471	64.191069	0.014745	0.015578
72	1.061837	0.941765	74.172942	69.853446	0.013482	0.014316
84	1.072508	0.932394	86.973568	81.093618	0.011498	0.012331
96	1.083287	0.923116	99.902843	92.221948	0.010010	0.010843
108	1.094174	0.913931	112.962060	103.239549	0.008853	0.009686
∞				1199.500069		0.000834

Continuous Compounding			i	1/8%	Table	B-3
	Single Payment	Uniform	Series	Uniform	Series	
	Compound Amount Factor	Present Amount Factor	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor
N	F/P	P/F	F/A	P/A	A/F	A/P
1	1.001251	0.998751	1.000000	0.998751	1.000000	1.001251
2	1.002503	0.997503	2.001251	1.996254	0.499688	0.500938
3	1.003757	0.996257	3.003754	2.992511	0.332917	0.334168
4	1.005013	0.995012	4.007511	3.987523	0.249531	0.250782
5	1.006270	0.993769	5.012523	4.981293	0.199500	0.200751
6	1.007528	0.992528	6.018793	5.973821	0.166146	0.167397
8	1.010050	0.990050	8.035110	7.955159	0.124454	0.125705
10	1.012578	0.987578	10.056473	9.931550	0.099438	0.100689
12	1.015113	0.985112	12.082897	11.903006	0.082762	0.084012
14	1.017654	0.982652	14.114393	13.869539	0.070850	0.072100
16	1.020201	0.980199	16.150973	15.831163	0.061916	0.063167
18	1.022755	0.977751	18.192652	17.787888	0.054967	0.056218
20	1.025315	0.975310	20.239441	19.739728	0.049408	0.050659
22	1.027882	0.972875	22.291354	21.686694	0.044860	0.046111
24	1.030455	0.970446	24.348403	23.628799	0.041070	0.042321
26	1.033034	0.968022	26.410601	25.566055	0.037864	0.039114
28	1.035620	0.965605	28.477961	27.498473	0.035115	0.036366
30	1.038212	0.963194	30.550496	29.426067	0.032733	0.033983
36	1.046028	0.955997	36.799279	35.180018	0.027174	0.028425
40	1.051271	0.951229	40.991247	38.992080	0.024395	0.025646
44	1.056541	0.946485	45.204227	42.785130	0.022122	0.023373
48	1.061837	0.941765	49.438325	46.559261	0.020227	0.021478
52	1.067159	0.937067	53.693647	50.314570	0.018624	0.019875
56	1.072508	0.932394	57.970298	54.051148	0.017250	0.018501
60	1.077884	0.927743	62.268387	57.769090	0.016060	0.017310
66	1.085999	0.920811	68.755948	63.311264	0.014544	0.015795
72	1.094174	0.913931	75.292350	68.812026	0.013282	0.014532
84	1.110711	0.900325	88.513145	79.690555	0.011298	0.012549
96	1.127497	0.886920	101.933746	90.407123	0.009810	0.011061
108	1.144537	0.873716	115.557174	100.964142	0.008654	0.009905
∞				799.500104		0.001251

Continuous Compounding			i	1/4%	Table	B-4
	Single Payment	Uniform	Series	Uniform	Series	
	Compound Amount Factor	Present Amount Factor	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor
N	F/P	P/F	F/A	P/A	A/F	A/P
1	1.002503	0.997503	1.000000	0.997503	1.000000	1.002503
2	1.005013	0.995012	2.002503	1.992516	0.499375	0.501878
3	1.007528	0.992528	3.007516	2.985044	0.332500	0.335003
4	1.010050	0.990050	4.015044	3.975093	0.249063	0.251566
5	1.012578	0.987578	5.025094	4.962671	0.199001	0.201504
6	1.015113	0.985112	6.037672	5.947783	0.165627	0.168130
8	1.020201	0.980199	8.070440	7.910634	0.123909	0.126412
10	1.025315	0.975310	10.113396	9.863695	0.098879	0.101382
12	1.030455	0.970446	12.166593	11.807016	0.082192	0.084695
14	1.035620	0.965605	14.230081	13.740643	0.070274	0.072777
16	1.040811	0.960789	16.303913	15.664627	0.061335	0.063838
18	1.046028	0.955997	18.388140	17.579015	0.054383	0.056886
20	1.051271	0.951229	20.482814	19.483855	0.048821	0.051325
22	1.056541	0.946485	22.587987	21.379195	0.044271	0.046774
24	1.061837	0.941765	24.703713	23.265081	0.040480	0.042983
26	1.067159	0.937067	26.830044	25.141561	0.037272	0.039775
28	1.072508	0.932394	28.967034	27.008683	0.034522	0.037025
30	1.077884	0.927743	31.114735	28.866492	0.032139	0.034642
36	1.094174	0.913931	37.622646	34.384509	0.026580	0.029083
40	1.105171	0.904837	42.015804	38.017471	0.023801	0.026304
44	1.116278	0.895834	46.453113	41.614285	0.021527	0.024030
48	1.127497	0.886920	50.935019	45.175309	0.019633	0.022136
52	1.138828	0.878095	55.461968	48.700901	0.018030	0.020534
56	1.150274	0.869358	60.034414	52.191412	0.016657	0.019160
60	1.161834	0.860708	64.652814	55.647192	0.015467	0.017970
66	1.179393	0.847894	71.667588	60.766497	0.013953	0.016456
72	1.197217	0.835270	78.788378	65.809585	0.012692	0.015195
84	1.233678	0.810584	93.354434	75.671633	0.010712	0.013215
96	1.271249	0.786628	108.364092	85.242214	0.009228	0.011731
108	1.309964	0.763379	123.830863	94.529941	0.008076	0.010579
∞				399.500208		0.002503

Continuous Compounding			i	1/4%	Table	B-5
	Single Payment	Uniform	Series	Uniform	Series	
	Compound Amount Factor	Present Amount Factor	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor
N	F/P	P/F	F/A	P/A	A/F	A/P
1	1.005013	0.995012	1.000000	0.995012	1.000000	1.005013
2	1.010050	0.990050	2.005013	1.985062	0.498750	0.503763
3	1.015113	0.985112	3.015063	2.970174	0.331668	0.336681
4	1.020201	0.980199	4.030176	3.950373	0.248128	0.253141
5	1.025315	0.975310	5.050377	4.925683	0.198005	0.203018
6	1.030455	0.970446	6.075692	5.896128	0.164590	0.169603
8	1.040811	0.960789	8.141766	7.822523	0.122823	0.127836
10	1.051271	0.951229	10.228605	9.729750	0.097765	0.102778
12	1.061837	0.941765	12.336417	11.618000	0.081061	0.086073
14	1.072508	0.932394	14.465412	13.487461	0.069130	0.074143
16	1.083287	0.923116	16.615805	15.338321	0.060184	0.065196
18	1.094174	0.913931	18.787809	17.170764	0.053226	0.058239
20	1.105171	0.904837	20.981642	18.984975	0.047661	0.052673
22	1.116278	0.895834	23.197524	20.781133	0.043108	0.048121
24	1.127497	0.886920	25.435675	22.559420	0.039315	0.044327
26	1.138828	0.878095	27.696320	24.320012	0.036106	0.041118
28	1.150274	0.869358	29.979685	26.063086	0.033356	0.038368
30	1.161834	0.860708	32.285999	27.788817	0.030973	0.035986
36	1.197217	0.835270	39.344946	32.863661	0.025416	0.030429
40	1.221403	0.818731	44.169943	36.163290	0.022640	0.027652
44	1.246077	0.802519	49.092410	39.397582	0.020370	0.025382
48	1.271249	0.786628	54.114319	42.567831	0.018479	0.023492
52	1.296930	0.771052	59.237676	45.675304	0.016881	0.021894
56	1.323130	0.755784	64.464532	48.721245	0.015512	0.020525
60	1.349859	0.740818	69.796978	51.706873	0.014327	0.019340
66	1.390968	0.718924	77.998305	56.074832	0.012821	0.017833
72	1.433329	0.697676	86.449399	60.313699	0.011567	0.016580
84	1.521962	0.657047	104.131548	68.419302	0.009603	0.014616
96	1.616074	0.618783	122.907100	76.052872	0.008136	0.013149
108	1.716007	0.582748	142.843667	83.241898	0.007001	0.012013
∞				199.500417		0.005013

Continuous Compounding			i	3/4%	Table	B-6
	Single Payment	Uniform	Series	Uniform	Series	
	Compound Amount Factor	Present Amount Factor	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor
N	F/P	P/F	F/A	P/A	A/F	A/P
1	1.007528	0.992528	1.000000	0.992528	1.000000	1.007528
2	1.015113	0.985112	2.007528	1.977640	0.498125	0.505653
3	1.022755	0.977751	3.022641	2.955391	0.330836	0.338365
4	1.030455	0.970446	4.045396	3.925837	0.247195	0.254723
5	1.038212	0.963194	5.075851	4.889031	0.197011	0.204540
6	1.046028	0.955997	6.114063	5.845029	0.163557	0.171086
8	1.061837	0.941765	8.213993	7.735648	0.121743	0.129272
10	1.077884	0.927743	10.345660	9.598119	0.096659	0.104187
12	1.094174	0.913931	12.509543	11.432861	0.079939	0.087467
14	1.110711	0.900325	14.706129	13.240288	0.067999	0.075527
16	1.127497	0.886920	16.935911	15.020806	0.059046	0.066574
18	1.144537	0.873716	19.199393	16.774815	0.052085	0.059613
20	1.161834	0.860708	21.497083	18.502711	0.046518	0.054046
22	1.179393	0.847894	23.829498	20.204881	0.041965	0.049493
24	1.197217	0.835270	26.197163	21.881710	0.038172	0.045700
26	1.215311	0.822835	28.600611	23.533574	0.034964	0.042492
28	1.233678	0.810584	31.040382	25.160844	0.032216	0.039744
30	1.252323	0.798516	33.517025	26.763888	0.029836	0.037364
36	1.309964	0.763379	41.173805	31.431238	0.024287	0.031815
40	1.349859	0.740818	46.473130	34.428142	0.021518	0.029046
44	1.390968	0.718924	51.933844	37.336473	0.019255	0.026783
48	1.433329	0.697676	57.560861	40.158850	0.017373	0.024901
52	1.476981	0.677057	63.359247	42.897814	0.015783	0.023311
56	1.521962	0.657047	69.334220	45.555828	0.014423	0.021951
60	1.568312	0.637628	75.491157	48.135287	0.013247	0.020775
66	1.640498	0.609571	85.079916	51.862242	0.011754	0.019282
72	1.716007	0.582748	95.110026	55.425201	0.010514	0.018042
84	1.877611	0.532592	116.576487	62.087681	0.008578	0.016106
96	2.054433	0.486752	140.064537	68.176729	0.007140	0.014668
108	2.247908	0.444858	165.764558	73.741701	0.006033	0.013561
∞				132.833958		0.007528

Continuous Compounding			i	1%	Table	B-7
	Single Payment	Uniform	Series	Uniform	Series	
	Compound Amount Factor	Present Amount Factor	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor
N	F/P	P/F	F/A	P/A	A/F	A/P
1	1.010050	0.990050	1.000000	0.990050	1.000000	1.010050
2	1.020201	0.980199	2.010050	1.970249	0.497500	0.507550
3	1.030455	0.970446	3.030252	2.940694	0.330006	0.340056
4	1.040811	0.960789	4.060706	3.901483	0.246263	0.256313
5	1.051271	0.951229	5.101517	4.852713	0.196020	0.206070
6	1.061837	0.941765	6.152788	5.794477	0.162528	0.172578
8	1.083287	0.923116	8.287133	7.649988	0.120669	0.130719
10	1.105171	0.904837	10.464594	9.468756	0.095560	0.105610
12	1.127497	0.886920	12.686043	11.251511	0.078827	0.088877
14	1.150274	0.869358	14.952368	12.998964	0.066879	0.076929
16	1.173511	0.852144	17.264476	14.711816	0.057922	0.067973
18	1.197217	0.835270	19.623292	16.390751	0.050960	0.061010
20	1.221403	0.818731	22.029759	18.036441	0.045393	0.055443
22	1.246077	0.802519	24.484840	19.649544	0.040842	0.050892
24	1.271249	0.786628	26.989516	21.230706	0.037051	0.047102
26	1.296930	0.771052	29.544791	22.780558	0.033847	0.043897
28	1.323130	0.755784	32.151686	24.299721	0.031103	0.041153
30	1.349859	0.740818	34.811243	25.788803	0.028726	0.038777
36	1.433329	0.697676	43.116638	30.081457	0.023193	0.033243
40	1.491825	0.670320	48.936967	32.803430	0.020434	0.030485
44	1.552707	0.644036	54.994829	35.418673	0.018184	0.028234
48	1.616074	0.618783	61.299916	37.931370	0.016313	0.026363
52	1.682028	0.594521	67.862320	40.345543	0.014736	0.024786
56	1.750673	0.571209	74.692539	42.665055	0.013388	0.023438
60	1.822119	0.548812	81.801506	44.893618	0.012225	0.022275
66	1.934792	0.516851	93.012616	48.073695	0.010751	0.020801
72	2.054433	0.486752	104.916983	51.068578	0.009531	0.019582
84	2.316367	0.431711	130.979611	56.545276	0.007635	0.017685
96	2.611696	0.382893	160.365142	61.402672	0.006236	0.016286
108	2.944680	0.339596	193.497236	65.710796	0.005168	0.015218
∞				99.500833		0.010050

Continuous Compounding			i	2%	Table	B-8
	Single Payment	Uniform	Series	Uniform	Series	
	Compound Amount Factor	Present Amount Factor	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor
N	F/P	P/F	F/A	P/A	A/F	A/P
1	1.020201	0.980199	1.000000	0.980199	1.000000	1.020201
2	1.040811	0.960789	2.020201	1.940988	0.495000	0.515202
3	1.061837	0.941765	3.061012	2.882753	0.326689	0.346891
4	1.083287	0.923116	4.122849	3.805869	0.242551	0.262752
5	1.105171	0.904837	5.206136	4.710706	0.192081	0.212282
6	1.127497	0.886920	6.311307	5.597627	0.158446	0.178647
8	1.173511	0.852144	8.589077	7.319129	0.116427	0.136628
10	1.221403	0.818731	10.959806	8.973130	0.091242	0.111444
12	1.271249	0.786628	13.427285	10.562276	0.074475	0.094677
14	1.323130	0.755784	15.995464	12.089112	0.062518	0.082719
16	1.377128	0.726149	18.668453	13.556079	0.053566	0.073768
18	1.433329	0.697676	21.450528	14.965526	0.046619	0.066820
20	1.491825	0.670320	24.346142	16.319707	0.041074	0.061276
22	1.552707	0.644036	27.359928	17.620790	0.036550	0.056751
24	1.616074	0.618783	30.496710	18.870857	0.032790	0.052992
26	1.682028	0.594521	33.761505	20.071909	0.029620	0.049821
28	1.750673	0.571209	37.159540	21.225866	0.026911	0.047112
30	1.822119	0.548812	40.696251	22.334576	0.024572	0.044774
36	2.054433	0.486752	52.196201	25.406619	0.019158	0.039360
40	2.225541	0.449329	60.666319	27.259134	0.016484	0.036685
44	2.410900	0.414783	69.841887	28.969221	0.014318	0.034519
48	2.611696	0.382893	79.781662	30.547831	0.012534	0.032736
52	2.829217	0.353455	90.549291	32.005071	0.011044	0.031245
56	3.064854	0.326280	102.213724	33.350273	0.009783	0.029985
60	3.320117	0.301194	114.849655	34.592051	0.008707	0.028908
66	3.743421	0.267135	135.803931	36.278024	0.007364	0.027565
72	4.220696	0.236928	159.429811	37.773348	0.006272	0.026474
84	5.365556	0.186374	216.102296	40.275844	0.004627	0.024829
96	6.820958	0.146607	288.147146	42.244378	0.003470	0.023672
108	8.671138	0.115325	379.734099	43.792881	0.002633	0.022835
∞				49.501667		0.020201

Continuous Compounding			i	3%	Table	B-9
	Single Payment	Uniform	Series	Uniform	Series	
	Compound Amount Factor	Present Amount Factor	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor
N	F/P	P/F	F/A	P/A	A/F	A/P
1	1.030455	0.970446	1.000000	0.970446	1.000000	1.030455
2	1.061837	0.941765	2.030455	1.912210	0.492501	0.522955
3	1.094174	0.913931	3.092291	2.826141	0.323385	0.353839
4	1.127497	0.886920	4.186465	3.713062	0.238865	0.269320
5	1.161834	0.860708	5.313962	4.573770	0.188183	0.218638
6	1.197217	0.835270	6.475796	5.409040	0.154421	0.184876
8	1.271249	0.786628	8.906692	7.006252	0.112275	0.142730
10	1.349859	0.740818	11.487905	8.510450	0.087048	0.117503
12	1.433329	0.697676	14.228732	9.927050	0.070280	0.100735
14	1.521962	0.657047	17.139043	11.261153	0.058346	0.088801
16	1.616074	0.618783	20.229316	12.517565	0.049433	0.079888
18	1.716007	0.582748	23.510682	13.700809	0.042534	0.072988
20	1.822119	0.548812	26.994956	14.815146	0.037044	0.067498
22	1.934792	0.516851	30.694685	15.864589	0.032579	0.063033
24	2.054433	0.486752	34.623193	16.852917	0.028882	0.059337
26	2.181472	0.458406	38.794626	17.783690	0.025777	0.056231
28	2.316367	0.431711	43.224007	18.660259	0.023135	0.053590
30	2.459603	0.406570	47.927284	19.485780	0.020865	0.051319
36	2.944680	0.339596	53.855174	21.684931	0.015660	0.046115
40	3.320117	0.301194	76.182973	22.945870	0.013126	0.043581
44	3.743421	0.267135	90.082527	24.064223	0.011101	0.041555
48	4.220696	0.236928	105.754231	25.056113	0.009456	0.039910
52	4.758821	0.210136	123.424028	25.935840	0.008102	0.038557
56	5.365556	0.186374	143.346668	26.716088	0.006976	0.037431
60	6.049647	0.165299	165.809382	27.408107	0.006031	0.036486
66	7.242743	0.138069	204.985668	28.302215	0.004878	0.035333
72	8.671138	0.115325	251.888197	29.049037	0.003970	0.034425
84	12.428597	0.080460	375.267495	30.193875	0.002665	0.033119
96	17.814273	0.056135	552.110671	30.992602	0.001811	0.032266
108	25.533722	0.039164	805.585197	31.549854	0.001241	0.031696
∞				32.835833		0.030455

Continuous Compounding			i	4%	Table	B-10
	Single Payment	Uniform	Series	Uniform	Series	
	Compound Amount Factor	Present Amount Factor	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor
N	F/P	P/F	F/A	P/A	A/F	A/P
1	1.040811	0.960789	1.000000	0.960789	1.000000	1.040811
2	1.083287	0.923116	2.040811	1.883906	0.490001	0.530812
3	1.127497	0.886920	3.124098	2.770826	0.320092	0.360903
4	1.173511	0.852144	4.251595	3.622970	0.235206	0.276017
5	1.221403	0.818731	5.425106	4.441701	0.184328	0.225139
6	1.271249	0.786628	6.646508	5.228329	0.150455	0.191266
8	1.377128	0.726149	9.240887	6.710261	0.108215	0.149025
10	1.491825	0.670320	12.051344	8.078258	0.082978	0.123789
12	1.616074	0.618783	15.095876	9.341078	0.066243	0.107054
14	1.750673	0.571209	18.393978	10.506807	0.054366	0.095176
16	1.896481	0.527292	21.966770	11.582911	0.045523	0.086334
18	2.054433	0.486752	25.837128	12.576281	0.038704	0.079515
20	2.225541	0.449329	30.029838	13.493276	0.033300	0.074111
22	2.410900	0.414783	34.571746	14.339769	0.028925	0.069736
24	2.611696	0.382893	39.491936	15.121181	0.025322	0.066132
26	2.829217	0.353455	44.821914	15.842515	0.022311	0.063121
28	3.064854	0.326280	50.595811	16.508391	0.019764	0.060575
30	3.320117	0.301194	56.850598	17.123071	0.017590	0.058401
36	4.220696	0.236928	78.917783	18.697813	0.012671	0.053482
40	4.953032	0.201897	96.862471	19.556196	0.010324	0.051135
44	5.812437	0.172045	117.920757	20.287661	0.008480	0.049291
48	6.820958	0.146607	142.632885	20.910974	0.007011	0.047822
52	8.004469	0.124930	171.632836	21.442127	0.005826	0.046637
56	9.393331	0.106459	205.664594	21.894745	0.004862	0.045673
60	11.023176	0.090718	245.601231	22.280441	0.004072	0.044882
∞				24.503333		0.040811

Continuous Compounding			i	5%	Table	B-11
	Single Payment	Uniform	Series	Uniform	Series	
	Compound Amount Factor	Present Amount Factor	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor
N	F/P	P/F	F/A	P/A	A/F	A/P
1	1.051271	0.951229	1.000000	0.951229	1.000000	1.051271
2	1.105171	0.904837	2.051271	1.856067	0.487503	0.538774
3	1.161834	0.860708	3.156442	2.716775	0.316812	0.368084
4	1.221403	0.818731	4.318276	3.535506	0.231574	0.282845
5	1.284025	0.778801	5.539679	4.314306	0.180516	0.231787
6	1.349859	0.740818	6.823704	5.055125	0.146548	0.197819
8	1.491825	0.670320	9.592631	6.430133	0.104247	0.155518
10	1.648721	0.606531	12.652768	7.674292	0.079034	0.130305
12	1.822119	0.548812	16.034742	8.800053	0.062365	0.113636
14	2.013753	0.496585	19.772402	9.818684	0.050576	0.101847
16	2.225541	0.449329	23.903154	10.740380	0.041835	0.093107
18	2.459603	0.406570	28.468342	11.574364	0.035127	0.086398
20	2.718282	0.367879	33.513655	12.328985	0.029839	0.081110
22	3.004166	0.332871	39.089588	13.011793	0.025582	0.076853
24	3.320117	0.301194	45.251947	13.629624	0.022098	0.073370
26	3.669297	0.272532	52.062407	14.188661	0.019208	0.070479
28	4.055200	0.246597	59.589129	14.694498	0.016782	0.068053
30	4.481689	0.223130	67.907443	15.152199	0.014726	0.065997
36	6.049647	0.165299	98.489165	16.280149	0.010153	0.061424
40	7.389056	0.135335	124.613214	16.864565	0.008025	0.059296
44	9.025013	0.110803	156.521199	17.343043	0.006389	0.057660
48	11.023176	0.090718	195.493701	17.734788	0.005115	0.056386
52	13.463738	0.074274	243.094822	18.055522	0.004114	0.055385
56	16.444647	0.060810	301.234962	18.318117	0.003320	0.054591
60	20.085537	0.049787	372.247490	18.533111	0.002686	0.053957
∞				19.504166		0.051271

Continuous Compounding			i	6%	Table	B-12
	Single Payment	Uniform	Series	Uniform	Series	
	Compound Amount Factor	Present Amount Factor	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor
N	F/P	P/F	F/A	P/A	A/F	A/P
1	1.061837	0.941765	1.000000	0.941765	1.000000	1.061837
2	1.127497	0.886920	2.061837	1.828685	0.485004	0.546841
3	1.197217	0.835270	3.189333	2.663955	0.313545	0.375382
4	1.271249	0.786628	4.386551	3.450583	0.227970	0.289806
5	1.349859	0.740818	5.657800	4.191401	0.176747	0.238584
6	1.433329	0.697676	7.007659	4.889078	0.142701	0.204538
8	1.616074	0.618783	9.962950	6.164908	0.100372	0.162208
10	1.822119	0.548812	13.295031	7.296468	0.075216	0.137053
12	2.054433	0.486752	17.051942	8.300071	0.058644	0.120481
14	2.316367	0.431711	21.287848	9.190188	0.046975	0.108812
16	2.611696	0.382893	26.063818	9.979650	0.038367	0.100204
18	2.944680	0.339596	31.448709	10.679841	0.031798	0.093634
20	3.320117	0.301194	37.520157	11.300854	0.026652	0.088489
22	3.743421	0.267135	44.365695	11.851643	0.022540	0.084376
24	4.220696	0.236928	52.084018	12.340150	0.019200	0.081036
26	4.758821	0.210136	60.786403	12.773416	0.016451	0.078288
28	5.365556	0.186374	70.598315	13.157689	0.014165	0.076001
30	6.049647	0.165299	81.661214	13.498508	0.012246	0.074082
36	8.671138	0.115325	124.055079	14.306667	0.008061	0.069897
40	11.023176	0.090718	162.091464	14.704606	0.006169	0.068006
44	14.013204	0.071361	210.445187	15.017636	0.004752	0.066588
48	17.814273	0.056135	271.914816	15.263874	0.003678	0.065514
52	22.646380	0.044157	350.058030	15.457571	0.002857	0.064693
56	28.789191	0.034735	449.397523	15.609939	0.002225	0.064062
60	36.598234	0.027324	575.682771	15.729796	0.001737	0.063574
∞				16.171666		0.061837

Continuous Compounding			i	7%	Table	B-13
	Single Payment	Uniform	Series	Uniform	Series	
	Compound Amount Factor	Present Amount Factor	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor
N	F/P	P/F	F/A	P/A	A/F	A/P
1	1.072508	0.932394	1.000000	0.932394	1.000000	1.072508
2	1.150274	0.869358	2.072508	1.801752	0.482507	0.555015
3	1.233678	0.810584	3.222782	2.612336	0.310291	0.382799
4	1.323130	0.755784	4.456460	3.368120	0.224393	0.296902
5	1.419068	0.704688	5.779590	4.072808	0.173023	0.245531
6	1.521962	0.657047	7.198657	4.729855	0.138915	0.211423
8	1.750673	0.571209	10.352935	5.913690	0.096591	0.169099
10	2.013753	0.496585	13.981218	6.942868	0.071525	0.144033
12	2.316367	0.431711	18.154737	7.837591	0.055082	0.127590
14	2.664456	0.375311	22.955427	8.615426	0.043563	0.116071
16	3.064854	0.326280	28.477534	9.291644	0.035115	0.107624
18	3.525421	0.283654	34.829469	9.879519	0.028711	0.101219
20	4.055200	0.246597	42.135934	10.390593	0.023733	0.096241
22	4.664590	0.214381	50.540369	10.834900	0.019786	0.092294
24	5.365556	0.186374	60.207771	11.221162	0.016609	0.089117
26	6.171858	0.162026	71.327930	11.556961	0.014020	0.086528
28	7.099327	0.140858	84.119157	11.848892	0.011888	0.084396
30	8.166170	0.122456	98.832570	12.102684	0.010118	0.082626
36	12.428597	0.080460	157.618030	12.681885	0.006344	0.078853
40	16.444647	0.060810	213.005574	12.952882	0.004695	0.077203
44	21.758402	0.045959	286.290485	13.157698	0.003493	0.076001
48	28.789191	0.034735	383.255936	13.312494	0.002609	0.075117
52	38.091837	0.026252	511.553815	13.429487	0.001955	0.074463
56	50.400445	0.019841	681.308563	13.517908	0.001468	0.073976
60	66.686331	0.014996	905.916131	13.584735	0.001104	0.073612
∞				13.791547		0.072508

Continuous Compounding			i	8%	Table	B-14
	Single Payment	Uniform	Series	Uniform	Series	
	Compound Amount Factor	Present Amount Factor	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor
N	F/P	P/F	F/A	P/A	A/F	A/P
1	1.083287	0.923116	1.000000	0.923116	1.000000	1.083287
2	1.173511	0.852144	2.083287	1.775260	0.480011	0.563298
3	1.271249	0.786628	3.256798	2.561888	0.307050	0.390337
4	1.377128	0.726149	4.528047	3.288037	0.220846	0.304133
5	1.491825	0.670320	5.905175	3.958357	0.169343	0.252630
6	1.616074	0.618783	7.397000	4.577140	0.135190	0.218477
8	1.896481	0.527292	10.763746	5.675642	0.092904	0.176192
10	2.225541	0.449329	14.714661	6.611723	0.067959	0.151247
12	2.611696	0.382893	19.351101	7.409399	0.051677	0.134964
14	3.064854	0.326280	24.792015	8.089133	0.040336	0.123623
16	3.596640	0.278037	31.176986	8.668365	0.032075	0.115362
18	4.220696	0.236928	38.669819	9.161954	0.025860	0.109147
20	4.953032	0.201897	47.462740	9.582562	0.021069	0.104356
22	5.812437	0.172045	57.781328	9.940981	0.017307	0.100594
24	6.820958	0.146607	69.890304	10.246405	0.014308	0.097595
26	8.004469	0.124930	84.100318	10.506671	0.011891	0.095178
28	9.393331	0.106459	100.775925	10.728454	0.009923	0.093210
30	11.023176	0.090718	120.344931	10.917446	0.008309	0.091597
36	17.814273	0.056135	201.883361	11.332675	0.004953	0.088240
40	24.532530	0.040762	282.547229	11.517248	0.003539	0.086826
44	33.784428	0.029599	393.631681	11.651275	0.002540	0.085828
48	46.525474	0.021494	546.609164	11.748599	0.001829	0.085117
52	64.071523	0.015608	757.278703	11.819271	0.001321	0.084608
56	88.234673	0.011333	1047.397575	11.870589	0.000955	0.084242
60	121.510418	0.008230	1446.928327	11.907854	0.000691	0.083978
∞				12.006666		0.083287

Continuous Compounding			i	9%	Table	B-15
	Single Payment	Uniform	Series	Uniform	Series	
	Compound Amount Factor	Present Amount Factor	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor
N	F/P	P/F	F/A	P/A	A/F	A/P
1	1.094174	0.913931	1.000000	0.913931	1.000000	1.094174
2	1.197217	0.835270	2.094174	1.749201	0.477515	0.571689
3	1.309964	0.763379	3.291392	2.512581	0.303823	0.397997
4	1.433329	0.697676	4.601356	3.210257	0.217327	0.311502
5	1.568312	0.637628	6.034686	3.847885	0.165709	0.259883
6	1.716007	0.582748	7.602998	4.430634	0.131527	0.225701
8	2.054433	0.486752	11.196615	5.449978	0.089313	0.183487
10	2.459603	0.406570	15.498956	6.301405	0.064520	0.158695
12	2.944680	0.339596	20.649794	7.012578	0.048427	0.142601
14	3.525421	0.283654	26.816466	7.606599	0.037291	0.131465
16	4.220696	0.236928	34.199313	8.102767	0.029240	0.123415
18	5.053090	0.197899	43.038186	8.517201	0.023235	0.117409
20	6.049647	0.165299	53.620238	8.863366	0.018650	0.112824
22	7.242743	0.138069	66.289254	9.152507	0.015085	0.109260
24	8.671138	0.115325	81.456820	9.394018	0.012276	0.106451
26	10.381237	0.096328	99.615693	9.595744	0.010039	0.104213
28	12.428597	0.080460	121.355812	9.764241	0.008240	0.102415
30	14.879732	0.067206	147.383459	9.904981	0.006785	0.100959
36	25.533722	0.039164	260.514026	10.202744	0.003839	0.098013
40	36.598234	0.027324	378.003772	10.328470	0.002645	0.096820
44	52.457326	0.019063	546.405281	10.416186	0.001830	0.096004
48	75.188628	0.013300	787.780118	10.477384	0.001269	0.095444
52	107.770073	0.009279	1133.749771	10.520080	0.000882	0.095056
56	154.470015	0.006474	1629.638251	10.549868	0.000614	0.094788
60	221.406416	0.004517	2340.409797	10.570650	0.000427	0.094602
∞				10.618610		0.094174

Continuous Compounding			i	10%	Table	B-16
	Single Payment	Uniform	Series	Uniform	Series	
	Compound Amount Factor	Present Amount Factor	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor
N	F/P	P/F	F/A	P/A	A/F	A/P
1	1.105171	0.904837	1.000000	0.904837	1.000000	1.105171
2	1.221403	0.818731	2.105171	1.723568	0.475021	0.580192
3	1.349859	0.740818	3.326574	2.464386	0.300610	0.405781
4	1.491825	0.670320	4.676432	3.134706	0.213838	0.319009
5	1.648721	0.606531	6.168257	3.741237	0.162120	0.267291
6	1.822119	0.548812	7.816978	4.290049	0.127927	0.233098
8	2.225541	0.449329	11.652850	5.235963	0.085816	0.190987
10	2.718282	0.367879	16.337994	6.010412	0.061207	0.166378
12	3.320117	0.301194	22.050442	6.644477	0.045330	0.150501
14	4.055200	0.246597	29.049855	7.163606	0.034424	0.139594
16	4.953032	0.201897	37.586744	7.588633	0.026605	0.131776
18	6.049647	0.165299	48.013724	7.936615	0.020827	0.125998
20	7.389056	0.135335	60.749266	8.221519	0.016461	0.121632
22	9.025013	0.110803	76.304492	8.454779	0.013105	0.118276
24	11.023176	0.090718	95.303688	8.645756	0.010493	0.115664
26	13.463738	0.074274	118.509359	8.802114	0.008438	0.113609
28	16.444647	0.060810	146.852828	8.930130	0.006810	0.111980
30	20.085537	0.049787	181.471620	9.034940	0.005511	0.110681
36	36.598234	0.027324	338.479830	9.248529	0.002954	0.108125
40	54.598150	0.018316	509.629002	9.334181	0.001962	0.107133
44	81.450869	0.012277	764.953565	9.391595	0.001307	0.106478
48	121.510418	0.008230	1145.853053	9.430081	0.000873	0.106044
52	181.272242	0.005517	1714.088316	9.455879	0.000583	0.105754
56	270.426407	0.003698	2561.795716	9.473171	0.000390	0.105561
60	403.428793	0.002479	3826.426553	9.484763	0.000261	0.105432
∞				9.508332		0.105171

Continuous Compounding			i	11%	Table	B-17
	Single Payment	Uniform	Series	Uniform	Series	
	Compound Amount Factor	Present Amount Factor	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor
N	F/P	P/F	F/A	P/A	A/F	A/P
1	1.116278	0.895834	1.000000	0.895834	1.000000	1.116278
2	1.246077	0.802519	2.116278	1.698353	0.472528	0.588806
3	1.390968	0.718924	3.362355	2.417277	0.297411	0.413689
4	1.552707	0.644036	4.753323	3.061313	0.210379	0.326657
5	1.733253	0.576950	6.306030	3.638263	0.158578	0.274856
6	1.934792	0.516851	8.039283	4.155114	0.124389	0.240667
8	2.410900	0.414783	12.133842	5.032910	0.082414	0.198692
10	3.004166	0.332871	17.235976	5.737358	0.058018	0.174296
12	3.743421	0.267135	23.593627	6.302691	0.042384	0.158662
14	4.664590	0.214381	31.515747	6.756381	0.031730	0.148008
16	5.812437	0.172045	41.387317	7.120475	0.024162	0.140440
18	7.242743	0.138069	53.688051	7.412668	0.018626	0.134904
20	9.025013	0.110803	69.015709	7.647159	0.014489	0.130768
22	11.245859	0.088922	88.115147	7.835341	0.011349	0.127627
24	14.013204	0.071361	111.914513	7.986362	0.008935	0.125213
26	17.461527	0.057269	141.570348	8.107558	0.007064	0.123342
28	21.758402	0.045959	178.523795	8.204821	0.005601	0.121880
30	27.112639	0.036883	224.570625	8.282876	0.004453	0.120731
36	52.457326	0.019063	442.536806	8.436130	0.002260	0.118538
40	81.450869	0.012277	691.883417	8.494488	0.001445	0.117723
44	126.469352	0.007907	1079.045698	8.532073	0.000927	0.117205
48	196.369875	0.005092	1680.195368	8.556279	0.000595	0.116873
52	304.904923	0.003280	2613.604799	8.571868	0.000383	0.116661
56	473.428075	0.002112	4062.916361	8.581908	0.000246	0.116524
60	735.095189	0.001360	6313.272884	8.588375	0.000158	0.116436
∞				8.600074		0.116278

Continuous Compounding			i	12%	Table	B-18
	Single Payment	Uniform	Series	Uniform	Series	
	Compound Amount Factor	Present Amount Factor	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor
N	F/P	P/F	F/A	P/A	A/F	A/P
1	1.127497	0.886920	1.000000	0.886920	1.000000	1.127497
2	1.271249	0.786628	2.127497	1.673548	0.470036	0.597533
3	1.433329	0.697676	3.398746	2.371225	0.294226	0.421723
4	1.616074	0.618783	4.832075	2.990008	0.206950	0.334447
5	1.822119	0.548812	6.448150	3.538820	0.155083	0.282580
6	2.054433	0.486752	8.270269	4.025572	0.120915	0.248412
8	2.611696	0.382893	12.641069	4.840175	0.079107	0.206604
10	3.320117	0.301194	18.197445	5.480965	0.054953	0.182450
12	4.220696	0.236928	25.260983	5.985028	0.039587	0.167084
14	5.365556	0.186374	34.240500	6.381538	0.029205	0.156702
16	6.820958	0.146607	45.655704	6.693444	0.021903	0.149400
18	8.671138	0.115325	60.167271	6.938798	0.016620	0.144117
20	11.023176	0.090718	78.615089	7.131800	0.012720	0.140217
22	14.013204	0.071361	102.066862	7.283621	0.009797	0.137294
24	17.814273	0.056135	131.879909	7.403047	0.007583	0.135080
26	22.646380	0.044157	169.779719	7.496992	0.005890	0.133387
28	28.789191	0.034735	217.959820	7.570891	0.004588	0.132085
30	36.598234	0.027324	279.208733	7.629022	0.003582	0.131078
36	75.188628	0.013300	581.885963	7.739016	0.001719	0.129215
40	121.510418	0.008230	945.203086	7.778782	0.001058	0.128555
44	196.369875	0.005092	1532.350587	7.803389	0.000653	0.128149
48	317.348329	0.003151	2481.224634	7.818616	0.000403	0.127900
52	512.858511	0.001950	4014.675693	7.828038	0.000249	0.127746
56	828.817511	0.001207	6492.846696	7.833868	0.000154	0.127651
60	1339.430764	0.000747	10497.755418	7.837475	0.000095	0.127592
∞				7.843331		0.127497

Continuous Compounding			i	13%	Table	B-19
	Single Payment	Uniform	Series	Uniform	Series	
	Compound Amount Factor	Present Amount Factor	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor
N	F/P	P/F	F/A	P/A	A/F	A/P
1	1.13883	0.878095	1.00000	0.878095	1.000000	1.138828
2	1.29693	0.771052	2.13883	1.64915	0.467546	0.606374
3	1.47698	0.677057	3.43576	2.32620	0.291057	0.429885
4	1.68203	0.594521	4.91274	2.92072	0.203552	0.342381
5	1.91554	0.522046	6.59477	3.44277	0.151635	0.290464
6	2.18147	0.458406	8.51031	3.90118	0.117505	0.256333
8	2.82922	0.353455	13.1761	4.65716	0.075895	0.214723
10	3.66930	0.272532	19.2273	5.24005	0.052009	0.190838
12	4.75882	0.210136	27.0753	5.68950	0.036934	0.175762
14	6.17186	0.162026	37.2536	6.03604	0.026843	0.165671
16	8.00447	0.124930	50.4542	6.30325	0.019820	0.158648
18	10.3812	0.096328	67.5743	6.50928	0.014799	0.153627
20	13.4637	0.074274	89.7780	6.66814	0.011139	0.149967
22	17.4615	0.057269	118.575	6.79062	0.008434	0.147262
24	22.6464	0.044157	155.922	6.88507	0.006413	0.145242
28	29.3708	0.034047	204.359	6.95789	0.004893	0.143722
28	38.0918	0.026252	267.178	7.01404	0.003743	0.142571
30	49.4024	0.020242	348.650	7.05733	0.002868	0.141697
36	107.8	0.009279	769.080	7.13630	0.001300	0.140129
40	181.3	0.005517	1298.526	7.16340	0.000770	0.139598
44	304.9	0.003280	2189.069	7.17951	0.000457	0.139285
48	512.9	0.001950	3686.987	7.18909	0.000271	0.139100
52	862.6	0.001159	6206.528	7.19479	0.000161	0.138990
56	1451.0	0.000689	10444.464	7.19817	small	0.138924
60	2440.6	0.000410	17572.790	7.20019	small	0.138885
∞				7.20314		0.138828

Continuous Compounding			i	15%	Table	B-20
	Single Payment	Uniform	Series	Uniform	Series	
	Compound Amount Factor	Present Amount Factor	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor
N	F/P	P/F	F/A	P/A	A/F	A/P
1	1.16183	0.860708	1.00000	0.860708	1.000000	1.161834
2	1.34986	0.740818	2.16183	1.60153	0.462570	0.624404
3	1.56831	0.637628	3.51169	2.23915	0.284763	0.446597
4	1.82212	0.548812	5.08001	2.78797	0.196850	0.358684
5	2.11700	0.472367	6.90212	3.26033	0.144883	0.306717
6	2.45960	0.406570	9.01912	3.66690	0.110876	0.272710
8	3.32012	0.301194	14.3364	4.31803	0.069753	0.231587
10	4.48169	0.223130	21.5139	4.80040	0.046482	0.208316
12	6.04965	0.165299	31.2026	5.15775	0.032049	0.193883
14	8.16617	0.122456	44.2809	5.42248	0.022583	0.184417
16	11.02318	0.090718	61.9348	5.61860	0.016146	0.177980
18	14.8797	0.067206	85.7651	5.76389	0.011660	0.173494
20	20.0855	0.049787	117.9326	5.87152	0.008479	0.170314
22	27.1126	0.036883	161.354	5.95125	0.006198	0.168032
24	36.5982	0.027324	219.967	6.01032	0.004546	0.166380
26	49.4024	0.020242	299.087	6.05408	0.003344	0.165178
28	66.6863	0.014996	405.886	6.08650	0.002464	0.164298
30	90.0171	0.011109	550.051	6.11052	0.001818	0.163652
36	221.4	0.004517	1361.927	6.15125	0.000734	0.162568
40	403.4	0.002479	2486.673	6.16385	0.000402	0.162236
44	735.1	0.001360	4536.093	6.17076	0.000220	0.162055
48	1339.4	0.000747	8270.380	6.17455	0.000121	0.161955
52	2440.6	0.000410	15074.696	6.17663	small	0.161901
56	4447.1	0.000225	27472.967	6.17777	small	0.161871
60	8103.1	0.000123	50064.089	6.17840	small	0.161854
∞				6.17916		0.161834

Continuous Compounding			i	17%	Table	B-21
	Single Payment	Uniform	Series	Uniform	Series	
	Compound Amount Factor	Present Amount Factor	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor
N	F/P	P/F	F/A	P/A	A/F	A/P
1	1.18530	0.843665	1.00000	0.843665	1.000000	1.185305
2	1.40495	0.711770	2.18530	1.55544	0.457602	0.642907
3	1.66529	0.600496	3.59025	2.15593	0.278532	0.463837
4	1.97388	0.506617	5.25554	2.66255	0.190275	0.375580
5	2.33965	0.427415	7.22942	3.08996	0.138324	0.323629
6	2.77319	0.360595	9.56907	3.45056	0.104503	0.289808
8	3.89619	0.256661	15.6293	4.01144	0.063982	0.249287
10	5.47395	0.182684	24.1437	4.41066	0.041419	0.226723
12	7.69061	0.130029	36.1060	4.69481	0.027696	0.213001
14	10.80490	0.092551	52.9123	4.89706	0.018899	0.204204
16	15.18032	0.065875	76.5243	5.04102	0.013068	0.198373
18	21.3276	0.046888	109.6979	5.14348	0.009116	0.194421
20	29.9641	0.033373	156.3051	5.21641	0.006398	0.191703
22	42.0980	0.023754	221.786	5.26832	0.004509	0.189814
24	59.1455	0.016907	313.783	5.30527	0.003187	0.188492
25	83.0963	0.012034	443.034	5.33157	0.002257	0.187562
28	116.7459	0.008566	624.624	5.35029	0.001601	0.186906
30	164.0219	0.006097	879.750	5.36361	0.001137	0.186442
36	454.9	0.002198	2449.287	5.38465	0.000408	0.185713
40	897.8	0.001114	4839.848	5.39050	0.000207	0.185511
44	1772.2	0.000564	9558.523	5.39347	0.000105	0.185409
48	3498.2	0.000286	18872.612	5.39497	small	0.185358
52	6905.0	0.000145	37257.485	5.39573	small	0.185332
56	13629.6	0.000073	73546.975	5.39612	small	0.185318
60	26903.2	0.000037	145177.991	5.39631	small	0.185312
∞				5.39651		0.185305

Continuous Compounding			i	13%	Table	B-22
	Single Payment	Uniform	Series	Uniform	Series	
	Compound Amount Factor	Present Amount Factor	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor
N	F/P	P/F	F/A	P/A	A/F	A/P
1	1.20925	0.826959	1.00000	0.826959	1.000000	1.209250
2	1.46228	0.683861	2.20925	1.51082	0.452642	0.661892
3	1.76827	0.565525	3.67153	2.07635	0.272366	0.481615
4	2.13828	0.467666	5.43980	2.54401	0.183830	0.393080
5	2.58571	0.386741	7.57808	2.93075	0.131960	0.341209
6	3.12677	0.319819	10.16379	3.25057	0.098389	0.307638
8	4.57223	0.218712	17.0716	3.73376	0.058577	0.267826
10	6.68589	0.149569	27.1728	4.06420	0.036802	0.246051
12	9.77668	0.102284	41.9436	4.29017	0.023842	0.233091
14	14.29629	0.069948	63.5427	4.44470	0.015737	0.224987
16	20.90524	0.047835	95.1268	4.55038	0.010512	0.219762
18	30.5694	0.032712	141.312	4.62265	0.007077	0.216326
20	44.7012	0.022371	208.847	4.67207	0.004788	0.214038
22	65.3659	0.015299	307.603	4.70587	0.003251	0.212501
24	95.5835	0.010462	452.013	4.72898	0.002212	0.211462
26	139.770	0.007155	663.180	4.74479	0.001508	0.210757
28	204.384	0.004893	971.968	4.75560	0.001029	0.210278
30	298.867	0.003346	1423.50	4.76299	0.000702	0.209952
36	934.5	0.001070	4461.13	4.77387	0.000224	0.209474
40	1998.2	0.000500	9544.56	4.77659	0.000105	0.209354
44	4272.7	0.000234	20414.4	4.77786	small	0.209299
48	9136.2	0.000109	43657.0	4.77846	small	0.209273
52	19535.7	small	93356.1	4.77874	small	0.209260
56	41772.8	small	199626.5	4.77887	small	0.209255
60	89321.7	small	426862.1	4.77893	small	0.209252
∞				4.77898		0.209250

Continuous Compounding			i	21%	Table	B-23
	Single Payment	Uniform	Series	Uniform	Series	
	Compound Amount Factor	Present Amount Factor	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor
N	F/P	P/F	F/A	P/A	A/F	A/P
1	1.23368	0.810584	1.00000	0.810584	1.000000	1.233678
2	1.52196	0.657047	2.23368	1.46763	0.447692	0.681370
3	1.87761	0.532592	3.75564	2.00022	0.266266	0.499944
4	231637	0.431711	5.63325	2.43193	0.177517	0.411195
5	2.85765	0.349938	7.94962	2.78187	0.125792	0.359470
6	3.52542	0.283654	10.80727	3.06553	0.092530	0.326208
8	5.36556	0.186374	18.6819	3.48182	0.053528	0.287206
10	8.16617	0.122456	30.6668	3.75535	0.032609	0.266287
12	12.42860	0.080460	48.9074	3.93507	0.020447	0.254125
14	18.91585	0.052866	76.6689	4.05316	0.013043	0.246721
16	28.78919	0.034735	118.9208	4.13075	0.008409	0.242087
18	43.8160	0.022823	183.227	4.18172	0.005458	0.239136
20	66.6863	0.014996	281.098	4.21522	0.003557	0.237236
22	101.4940	0.009853	430.053	4.23723	0.002325	0.236003
24	154.4700	0.006474	656.758	4.25169	0.001523	0.235201
26	235.097	0.004254	1001.795	4.26119	0.000998	0.234676
28	357.809	0.002795	1526.927	4.26743	0.000655	0.234333
30	544.572	0.001836	2326.16	4.27153	0.000430	0.234108
36	1919.8	0.000521	8211.49	4.27716	0.000122	0.233800
40	4447.1	0.000225	19026.46	4.27843	small	0.233731
44	10301.0	0.000097	44077.9	4.27898	small	0.233701
48	23861.0	small	102106.2	4.27921	small	0.233688
52	55270.8	small	236521.1	4.27931	small	0.233682
56	128027.5	small	547875.4	4.27936	small	0.233880
60	296558.6	small	1269086.0	4.27938	small	0.233679
∞				4.27939		0.233678

Continuous Compounding			i	25%	Table	B-24
	Single Payment	Uniform	Series	Uniform	Series	
	Compound Amount Factor	Present Amount Factor	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor
N	F/P	P/F	F/A	P/A	A/F	A/P
1	1.28403	0.778801	1.00000	0.778801	1.000000	1.284025
2	1.64872	0.606531	2.28403	1.38533	0.437823	0.721849
3	2.11700	0.472367	3.93275	1.85770	0.254275	0.538301
4	2.71828	0.367879	6.04975	2.22558	0.165296	0.449322
5	3.49034	0.286505	8.76803	2.51208	0.114051	0.398076
6	4.48169	0.223130	12.25837	2.73521	0.081577	0.365602
8	7.38906	0.135335	22.4947	3.04432	0.044455	0.328480
10	12.18249	0.082085	39.3715	3.23181	0.025399	0.309425
12	20.08554	0.049787	67.1966	3.34552	0.014882	0.298907
14	33.11545	0.030197	113.0725	3.41449	0.008844	0.292869
16	54.59815	0.018316	188.7090	3.45633	0.005299	0.289325
18	90.0171	0.011109	313.413	3.48170	0.003191	0.287216
20	148.4132	0.006738	519.014	3.49709	0.001927	0.285952
22	244.6919	0.004087	857.993	3.50642	0.001166	0.285191
24	403.4288	0.002479	1416.876	3.51208	0.000706	0.284731
26	665.142	0.001503	2338.318	3.51552	0.000428	0.284453
28	1096.633	0.000912	3857.518	3.51760	0.000259	0.284285
30	1808.042	0.000553	6362.26	3.51886	0.000157	0.284183
36	8103.1	0.000123	28525.91	3.52038	small	0.284060
40	22026.5	0.000045	77547.52	3.52065	small	0.284038
44	59874.1	0.000017	210802	3.52075	small	0.284030
48	162755	small	573025	3.52079	small	0.284027
52	442413	small	1557651	3.52080	small	0.284026
56	1202604	small	4234140	3.52081	small	0.284026
60	3269017	small	11509591	3.52081	small	0.284026
∞				3.52081		0.284025

Continuous Compounding			i	30%	Table	B-25
	Single Payment	Uniform	Series	Uniform	Series	
	Compound Amount Factor	Present Amount Factor	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor
N	F/P	P/F	F/A	P/A	A/F	A/P
1	1.34986	0.740818	1.00000	0.740818	1.000000	1.349859
2	1.82212	0.548812	2.34986	1.28963	0.425557	0.775416
3	2.45960	0.406570	4.17198	1.69620	0.239694	0.589553
4	3.32012	0.301194	6.63158	1.99739	0.150794	0.500652
5	4.48169	0.223130	9.95170	2.22052	0.100485	0.450344
6	6.04965	0.165299	14.43339	2.38582	0.069284	0.419143
8	11.02318	0.090718	28.6492	2.59900	0.034905	0.384764
10	20.08554	0.049787	54.5521	2.71599	0.018331	0.368190
12	36.59823	0.027324	101.7503	2.78020	0.009828	0.359687
14	66.68633	0.014996	187.7510	2.81543	0.005326	0.355185
16	121.51042	0.008230	344.4544	2.83477	0.002903	0.352762
18	221.4064	0.004517	629.987	2.84539	0.001587	0.351446
20	403.4288	0.002479	1150.261	2.85121	0.000869	0.350728
22	735.0952	0.001360	2098.261	2.85441	0.000477	0.350335
24	1339.4308	0.000747	3825.631	2.85616	0.000261	0.350120
26	2440.602	0.000410	6973.104	2.85712	0.000143	0.350002
28	4447.067	0.000225	12708.174	2.85765	small	0.349937
30	8103.084	0.000123	23158.15	2.85794	small	0.349902
36	49020.8	small	140113.10	2.85824	small	0.349866
40	162755	small	465198.50	2.85828	small	0.349861
44	540365	small	1544520	2.85829	small	0.349859
48	1794075	smali	5127994	2.85829	small	0.349859
52	5956538	small	17025545	2.85830	small	0.349859
56	19776403	small	56526808	2.85830	small	0.349859
60	65659969	small	187675619	2.85830	small	0.349859
∞				2.85830		0.349859

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Practical Applications of Engineering Economics

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