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Hydraulic Engineering

Fundamental Concepts

Gautham P. Das

MOMENTUM PRESS ENGINEERING

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Dedication

To my son whose strength and wisdom is infinite.

Abstract

Hydraulic Engineering: Fundamental Concepts includes hydraulic processes with corresponding systems and devices. The hydraulic processes include the fundamentals of fluid mechanics and pressurized pipe flow systems. This book illustrates the use of appropriate pipeline networks, along with various devices like pumps, valves, and turbines. The knowledge of these processes and devices is extended to design, analysis, and implementation.

Keywords

Continuity Equation, Bernoulli's Equation, General Energy Equation, Series and Parallel Pipeline Systems, Pumps.

Contents

Preface

The objective of this book is to present the various applications of fluid mechanics in the form of hydraulic engineering systems. Primary emphasis is on fluid properties, flow systems, pipe networks, and pump selection. Its main purpose is to augment lecture courses and standard textbooks on fluid mechanics and hydraulic engineering by illustrating a wide array of exercise problems with solutions. This book is directed to anyone in an engineering field with the ability to apply the principles of hydraulic engineering and fluid mechanics.

The units used in this textbook are both in the metric and in English units. This will enable the student get a better understanding of practical applications in the field. The reader of this textbook should have a basic understanding of algebra and calculus.

CHAPTER 1

Fundamental Concepts

All matter encountered on an everyday basis exists in one of three forms: solid, liquid, or gas. Generally, these forms are distinguished by the bonds between adjacent molecules (or atoms) that compose them. Thus, the molecules that make up a solid are relatively close together and are held in place by the electrostatic bonds between the molecules. Therefore, solids tend to keep their shape, even when acted on by an external force.

By contrast, gas molecules are so far apart that the bonds are too weak to keep them in place. A gas is very compressible and always takes the shape of its container. If the container of a gas is removed, the molecules would expand indefinitely.

Between the extremes of solid and gas lies the liquid form of matter. In a liquid, molecules are bonded with enough strength to prevent indefinite expansion but without enough strength to be held in place. Liquids conform to the shape of their container except for the top, which forms a horizontal surface free of confining pressure except for atmospheric pressure. Liquids tend to be incompressible, and water, despite minute compressibility, is assumed to be incompressible for most hydraulic problems.

In addition to water, various oils and even molten metals are examples of liquids and share the basic characteristics of liquids.

1.1 Force and Mass

An understanding of fluid properties requires a careful distinction between mass and weight. The following definitions apply:

Mass is the property of a body of fluid that is a measure of its inertia or resistance to a change in motion. It is also a measure of the quantity of fluid.

The symbol m (for mass) is used in this book.

Weight is the amount that a body of fluid weighs, that is, the force with which the fluid is attracted toward Earth by gravitation.

The symbol w (for weight) is used in this book.

An equivalent unit for force is kg·m/s^2, as indicated above. This is derived from the relationship between force and mass

$$F = m * a \tag{1.1}$$

where a is the acceleration expressed in units of m/s^2. Therefore, the derived unit for force is

$$F = ma = \text{kg·m/s}^2 = \text{N} \tag{1.2}$$

Thus, a force of 1.0 N would give a mass of 1.0 kg an acceleration of 1.0 m/s^2. This means that either N or kg·m/s^2 may be used as the unit for force. In fact, some calculations in this book require the ability to use both or to convert from one to the other. Similarly, besides using the kg as the standard unit of mass, the equivalent unit N·s^2/m may be used. This can be derived again from $F = ma$.

$$m = \frac{F}{a} = \frac{\text{N}}{\text{m}/\text{s}^2} = \frac{\text{N} \cdot \text{s}^2}{\text{m}} \tag{1.3}$$

1.2 Surface Tension and Capillarity

All liquids have **surface tension,** which is manifested differently in different liquids. Surface tension results from a different molecular bonding condition at the free surface compared to bonds within the liquid. In water, surface tension results in properties called **cohesion** and **adhesion.** Cohesion enables water to resist a slight tensile stress; adhesion enables it to adhere to another body.

Capillarity is a property of liquids that results from surface tension in which the liquid rises up or is depressed down a thin tube. If adhesion predominates over cohesion in a liquid, as in water, the liquid will wet the surface of a tube and rise up. If cohesion predominates over adhesion in a liquid, as in mercury, the liquid does not wet the tube and is depressed down (Figure 1.1).

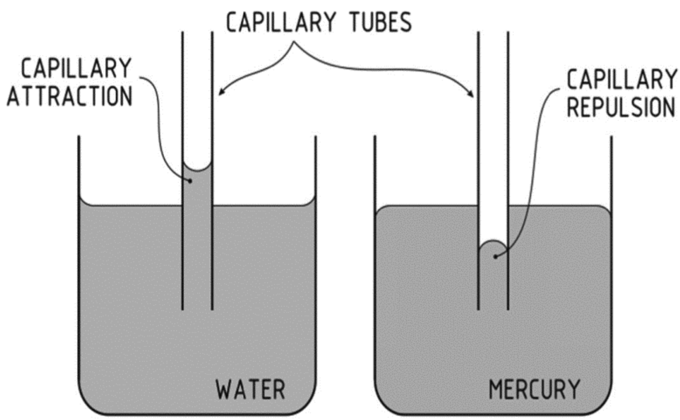


Figure 1.1 Examples of adhesion and cohesion of water and mercury in a glass test tube

1.3 Density and Specific Weight

Density is defined as mass per unit volume and is denoted by the expression

$$\rho = \frac{m}{V} \tag{1.4}$$

where ρ (rho) = density (kg/m^3, slugs/ft^3)

m = mass (slugs, kg)

V = volume (ft^3, m^3).

Density varies with water as shown in Table 1.1.

The slug is a unit of mass associated with Imperial units and U.S. customary units. It is a mass that accelerates by 1 ft/s^2 when a force of one pound-force (lb$_F$) is exerted on it.

$$1 \text{ slug} = 1\frac{\text{lb}_F \cdot \text{s}^2}{\text{ft}} \leftrightarrow 1 \text{ lb}_F = 1\frac{\text{slug} \cdot \text{ft}}{\text{s}^2} \tag{1.5}$$

In general, density can be changed by changing either the pressure or the temperature. Increasing the pressure always increases the density of a material. Increasing the temperature generally decreases the density, but there are notable exceptions to this generalization. For example, the density of water increases between its melting point (0 °C) and 4 °C; similar behavior is observed in silicon at low temperatures.

Table 1.1 Density of water at various temperatures

Temperature	Density		Specific Weight	
°F/°C	slugs/ft³	g/cm³	lb/ft³	kg/l
32/0	1.94	0.99987	62.416	0.999808
39.2/4.0	1.94	1	62.424	1
40/4.4	1.94	0.99999	62.423	0.999921
50/10	1.94	0.99975	62.408	0.999681
60/15.6	1.94	0.99907	62.366	0.999007
70/21	1.94	0.99802	62.3	0.99795
80/26.7	1.93	0.99669	62.217	0.996621
90/32.2	1.92	0.9951	62.118	0.995035
100/37.8	1.92	0.99318	61.998	0.993112
120/48.9	1.94	0.9887	61.719	0.988644
140/60	1.91	0.98338	61.386	0.983309
160/71.1	1.91	0.97729	61.006	0.977223
180/82.2	1.88	0.97056	60.586	0.970495
200/93.3	1.88	0.96333	60.135	0.96327
212/100	1.88	0.95865	59.843	0.958593

The **specific weight** (also known as the unit weight) is the weight per unit volume of a material. The symbol of specific weight is γ.

$$\gamma = \frac{w}{V} \tag{1.6}$$

where

γ = specific weight of the material (weight per unit volume, typically lb/ft^3, N/m^3 units)

w = weight (lb, kg)

V = volume (ft^3, m^3)

Alternatively, specific weight can be defined by the expression

$$\gamma = \rho g \tag{1.7}$$

where

ρ = density of the material (mass per unit volume, typically kg/m^3)

g = acceleration due to gravity (rate of change of velocity, given in m/s^2, and on Earth usually given as 32.2 ft/s^2 or 9.81 m/s^2)

Since specific weight is function of density and density is dependent on temperature, the specific weight will vary with temperature as shown in Tables 1.2 and 1.3.

Table 1.2 Specific weight of water at various temperatures in °C

Temperature (°C)	Specific Weight (kN/m^3)
0	9.805
5	9.807
10	9.804
15	9.798
20	9.789
25	9.777
30	9.765
40	9.731
50	9.69
60	9.642
70	9.589
80	9.53
90	9.467
100	9.399

Table 1.3 Specific Weight of Water at Varying Temperature in °F

Temperature (°F)	Specific Weight (lb/ft^3)
32	62.42
40	62.43
50	62.41
60	62.37
70	62.3
80	62.22
90	62.11
100	62
110	61.86
120	61.71
130	61.55
140	61.38
150	61.2
160	61
170	60.8
180	60.58
190	60.36
200	60.12
212	59.83

Quite often the specific weight of a substance must be found when its density is known and vice versa. The conversion from one to the other can be made using the following equation:

$$\gamma = \rho * g \tag{1.8}$$

where g is the acceleration due to gravity.

This equation can be justified by referring to the definitions of density and specific gravity and by using the equation relating mass to weight,

$$w = mg \tag{1.9}$$

The definition of specific weight is

$$\gamma = \frac{w}{V} \tag{1.10}$$

Multiplying both the numerator and the denominator of this equation by g, yields

$$\gamma = \frac{wg}{Vg} \tag{1.11}$$

$$\text{But } m = \frac{w}{g} \tag{1.12}$$

Therefore

$$\gamma = \frac{mg}{V} \tag{1.13}$$

Because

$$\rho = \frac{m}{V} \tag{1.14}$$

$$\gamma = \rho g \tag{1.15}$$

1.4 Specific Gravity

When the term **specific gravity** is used in this book, the reference fluid is pure water at 4 °C. At this temperature, water has its greatest density. Then, specific gravity can be defined in either of two ways:

Specific gravity is the ratio of the density of a substance to the density of water at 4 °C.

Specific gravity is the ratio of the specific weight of a substance to the specific weight of water at 4 °C.

These definitions for specific gravity (s.g.) can be shown mathematically as

$$\text{s.g.} = \frac{\gamma_s}{\gamma_{w@4°C}} = \frac{\rho_s}{\rho_{w@4°C}} \tag{1.16}$$

where the subscript s refers to the substance whose specific gravity is being determined and the subscript w refers to water. The properties of water at 4 °C are constant, having the following values:

$$\rho_{w@4°C} = 1{,}000\,\text{kg/m}^3 \qquad \rho_{w@4°C} = 1.94\ \text{slugs/ft}^3$$

$$\gamma_{w@4°C} = 9.81\,\text{kN/m}^3 \qquad \gamma_{w@4°C} = 62.4\,\text{lb/ft}^3$$

1.5 Pressure

Pressure (denoted as p or P) is the ratio of force to the area over which that force is distributed.

Pressure is force per unit area applied in a direction perpendicular to the surface of an object. Gage pressure is the pressure relative to the local atmospheric or ambient pressure. Pressure is measured in any unit of force divided by any unit of area. The SI unit of pressure is the newton per square meter (N/m^2), which is called the Pascal (Pa). Pounds per square inch (psi) is the traditional unit of pressure in U.S. customary units.

Mathematically:

$$P = \frac{F}{A}\ or\ p = \frac{dF}{dA} \tag{1.17}$$

where

P is the pressure

F is the normal force

A is the area of the surface on contact

$$1\,\text{Pa} = 1\frac{\text{N}}{\text{m}^2} = 1\frac{\text{kg}}{\text{m}^2} \tag{1.18}$$

$$1\,\text{psi} = 6.8947\,\text{kPa}$$

1.6 Viscosity

Viscosity is a property arising from friction between neighboring particles in a fluid that are moving at different velocities. When the fluid is forced through a tube, the particles which comprise the fluid generally move faster near the axis of the tube and more slowly near its walls: therefore some stress (such as a pressure difference between the two ends of the tube), is needed to overcome the friction between particle layers and keep the fluid moving. For the same velocity pattern, the stress required is proportional to the viscosity.

Viscosity is sometimes confused with density, but it is very different. While density refers to the amount of mass per unit volume, viscosity refers to the ability of fluid molecules to flow past each other. Thus, a very dense fluid could have a low viscosity or vice versa.

The properties of viscosity and density are well illustrated by the example of oil and water. Most oils are less dense than water and therefore float on the water surface. Yet, despite its lack of density, oil is more viscous than water. This property of viscosity is called absolute viscosity. It is designated by the Greek letter mu (μ) and has the units lb·s/ft^2 (kg·s/m^2). Because it has been found that in many hydraulic problems, density is a factor, another form of viscosity, called kinematic viscosity, has been defined as absolute viscosity divided by density. It is usually denoted by the Greek letter nu (ν). It has the units ft^2/s or m^2/s.

$$\nu = \frac{\mu}{\rho} \tag{1.19}$$

1.7 Flow Rate

Flow rate or rate of flow is the quantity of fluid passing through any section of pipeline or open channel per unit time.

It can be expressed in terms of

Volume flow rate
Mass flow rate
Weight flow rate

1.7.1 *Volume Flow Rate*

The **volumetric flow rate** (also known as volume flow rate, rate of fluid flow, or volume velocity) is the volume of fluid which passes per unit of time; usually represented by the symbol Q. The SI unit is m^3/s (cubic meters per second). In U.S. customary units and British Imperial units, volumetric flow rate is often expressed as ft^3/s (cubic feet per second) or gallons per minute.

Volume flow rate is defined by the limit

$$Q = \lim_{\Delta t \to 0} \frac{\Delta V}{\Delta t} = \frac{dV}{dt} \tag{1.20}$$

i.e., the flow of volume of fluid V through a surface per unit time t

Since this is the time derivative of volume, a scalar quantity, the volumetric flow rate is also a scalar quantity. The change in volume is the amount that flows after crossing the boundary for some time duration, not simply the initial amount of volume at the boundary minus the final amount at the boundary; otherwise the change in volume flowing through the area would be zero for steady flow.

Volumetric flow rate can also be defined by

$$Q = v * A \tag{1.21}$$

where
v = flow velocity of the substance elements
A = cross-sectional vector area or surface

1.7.2 *Mass Flow Rate*

Mass flow rate is the mass of a substance which crosses a fixed plane per unit of time.

The unit of mass flow rate is kilogram per second in SI units, and slug per second or pound per second in U.S. customary units. The common symbol is $\dot{m}$ (pronounced "m-dot")

Mass flow rate is defined by the limit

$$\dot{m} = \lim_{\Delta t \to 0} \frac{\Delta m}{\Delta t} = \frac{dm}{dt} \tag{1.22}$$

i.e., the flow of mass $\dot{m}$ through a plane per unit time t.

Mass flow rate can be calculated as

$$\dot{m} = \rho * Q \tag{1.23}$$

ρ = mass density of the fluid

Q = volume flow rate

1.7.3 *Weight Flow Rate*

Weight flow rate is defined as the weight of any fluid passing through any section per unit time. It is denoted by the symbol W.

Weight flow rate can calculated as

$$W = \gamma * Q \tag{1.24}$$

1.8 Principle of Continuity

The method of calculating the velocity of the flow of a fluid in a closed pipe system depends on the principle of continuity. If a fluid flow from section 1 to section 2 occurs at a constant rate, that is, the quantity of fluid flowing past any section in a given amount of time is constant, it is referred to as steady flow. If there is no fluid added, stored, or removed between section 1 and section 2, then the mass of the fluid flowing in section 2 will be the same as the mass of the fluid flowing in section 1. This can be expressed in terms of mass flow rate

$$M_1 = M_2 \tag{1.25}$$

However, $M = \rho A v$

So Eq. (1.25) will become

$$(\rho A v)_1 = (\rho A v)_2 \tag{1.26}$$

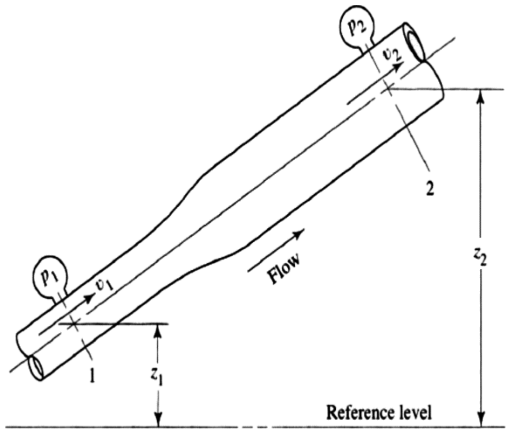


Figure 1.2 Element of fluid moves from section 1 to section 2

Source: King, H.W. and Brater, E.F. (1963). *Handbook of Hydraulics*, 5th edition, New York, McGraw Hill.

This is the mathematical statement of the principle of continuity and is called the continuity equation. It is used to relate the fluid density, flow area, and velocity of flow at two sections of the system in which there is steady flow. If the fluid flowing through the pipe is considered incompressible, then ρ_1 and ρ_2 are equal (Figure 1.2). The equation then becomes

$$(Av)_1 = (Av)_2 \tag{1.27}$$

or, because $Q = Av$

$$Q_1 = Q_2 \tag{1.28}$$

1.9 Conservation of Energy—Bernoulli's Equation

Law of Conservation of Energy: "Energy can be neither created nor destroyed. It can be transformed from one form to another."

Potential energy
Kinetic energy
Pressure energy

In the analysis of a pipeline problem, account for these three principles for all the energy within the system.

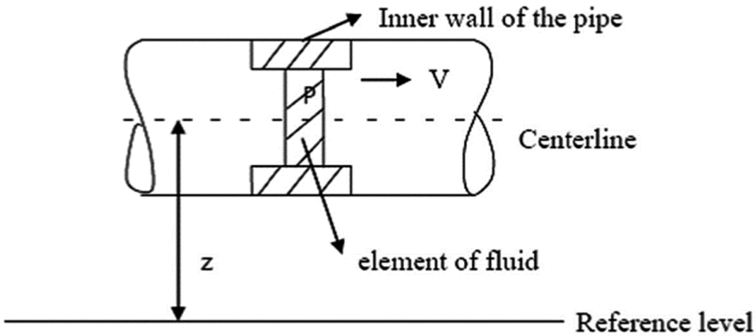


Figure 1.3 Fluid element inside a pipe

An element of fluid, at any point inside a pipe in a flow system (Figure 1.3):

Is located at a certain elevation (z)
Has a certain velocity (v)
Has a certain pressure (P)

The element of fluid would possess the following forms of energy:

Potential energy: Due to its elevation, the potential energy of the element relative to some reference level is

$$PE = w_e z \tag{1.29}$$

w_e = weight of element
z = elevation of fluid

Kinetic energy: Due to its velocity, the kinetic energy of the element is

$$KE = \frac{w_e v^2}{2g} \tag{1.30}$$

Flow energy (pressure energy or flow work): Amount of work necessary to move an element of a fluid across a certain section against the pressure (P).

$$PE = \frac{w_e P}{\gamma} \tag{1.31}$$

Total amount of energy of these three forms possessed by the element of fluid:

$$E = \text{PE} + \text{KE} + \text{FE} \tag{1.32}$$

$$E = w_e z + \frac{w_e v^2}{2g} + \frac{w_e P}{\gamma} \tag{1.33}$$

Considering the fluid flow through the sections as shown in Figure 1.2

$$\text{Total energy in section 1}: E_1 = w_e z_1 + \frac{w_e {v_1}^2}{2g} + \frac{w_e P_1}{\gamma} \tag{1.34}$$

$$\text{Total energy in section 2}: E_2 = w_e z_2 + \frac{w_e {v_2}^2}{2g} + \frac{w_e P_2}{\gamma} \tag{1.35}$$

If no energy is added to the fluid or lost between the sections 1 and 2, then the principle of energy requires that

$$w_e z_1 + \frac{w_e {v_1}^2}{2g} + \frac{w_e P_1}{\gamma} = w_e z_2 + \frac{w_e {v_2}^2}{2g} + \frac{w_e P_2}{\gamma} \tag{1.36}$$

The weight of the element is common to all terms and can be divided out, resulting in the following equation known as Bernoulli's equation.

$$z_1 + \frac{{v_1}^2}{2g} + \frac{P_1}{\gamma} = z_2 + \frac{{v_2}^2}{2g} + \frac{P_2}{\gamma} \tag{1.37}$$

Each term in Bernoulli's equation is one form of the energy possessed by the fluid per unit weight of fluid flowing in the system. The units in each term are "energy per unit weight." In the SI system, the units are N·m/N and in the U.S. customary system, the units are lb·ft/lb.

The force unit appears in both the numerator and denominator and it can be cancelled. The resulting unit thus becomes the meter (m) or foot (ft) and can be interpreted to be a height. In fluid flow analysis, the terms are typically expressed as "head" referring to a height above a reference level (Figure 1.4).

P/γ = pressure head
z = elevation head
$v^2/2g$ = velocity head

} Summation of the terms is called total head.

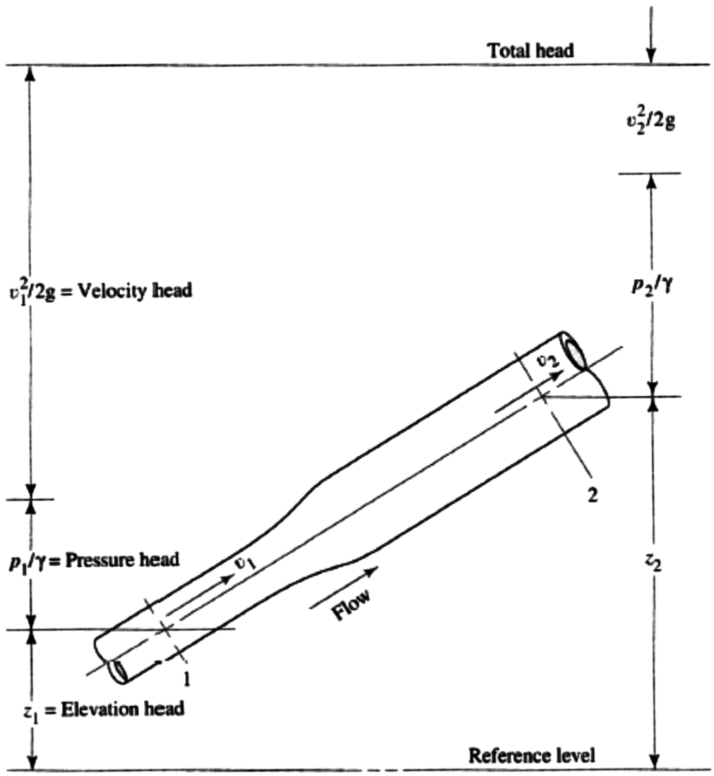


Figure 1.4 Element of fluid moves from section 1 to section 2
Source: King, H.W. and Brater, E.F. (1963). *Handbook of Hydraulics*, 5th edition, New York, McGraw Hill.

1.9.1 Restrictions on Bernoulli's Equation

It is stated only for incompressible fluids, since the specific weight of the fluid is assumed to be the same at two sections of interest.

There can be no mechanical devices between sections 1 and 2 that would add energy to or remove energy from the system, since the equation states that the total energy of the fluid is constant.

There can be no heat transferred into or out of the fluid.

There can be no energy loss due to friction.

Example Problem 1

Water at 10 °C is flowing from section 1 to section 2. At section 1, which is 25 mm in diameter, the gage pressure is 345 kPa and the velocity of flow is 3.0 m/s. Section 2, which is 50 mm in diameter, is 2.0 m above section 1. Assuming there are no energy losses in the system, calculate the pressure P_2.

Given:

P_1 = 345 kPa	P_2 = ?
v_1 = 3 m/s	v_2 = ?
$Ø_1$ = 25 mm	$Ø_2$ = 50 mm
T_{water} = 10 °C	$z_2 - z_1$ = 2.0 m

$$Q = A_1 \times V_1$$

$$\left(\frac{25\,\text{mm}}{1{,}000\,\text{m}}\right)^2 \frac{\pi}{4} \times 3 \text{ m/s} = 1.47 * 10^{-3} \text{ m}^3/\text{s}$$

From continuity equation $Q = A_1 \times V_1 = A_2 \times V_2$

$$\left(\frac{25\,\text{mm}}{1{,}000\,\text{m}}\right)^2 \frac{\pi}{4} \times 3 \frac{\text{m}}{\text{s}} = \left(\frac{50\,\text{mm}}{1{,}000\,\text{m}}\right)^2 \frac{\pi}{4} \times v_2$$

$$P_1 = 345 \text{ kPa} = 345 \text{ kN/m}^2$$

$$z_1 + \frac{v_1^{\,2}}{2g} + \frac{P_1}{\gamma} = z_2 + \frac{v_2^{\,2}}{2g} + \frac{P_2}{\gamma}$$

$$z_1 + \frac{3.0^2}{2 * 9.81} + \frac{345\,\text{kN/m}^3}{9.81\,\text{kN/m}^3} = z_2 + \frac{0.75^2}{2 * 9.81} + \frac{P_2}{\gamma}$$

$$z_1 - z_2 - \frac{0.75^2}{2 * 9.81} + \frac{3.0^2}{2 * 9.81} + \frac{345\,\text{kN/m}^3}{9.81\,\text{kN/m}^3} = \frac{P_2}{\gamma}$$

$$\frac{P_2}{\gamma} = 35.1\,\text{m} + 0.458\,\text{m} + 0.0286\,\text{m} - 2\,\text{m}$$

$$\frac{P_2}{\gamma} = 33.5\,\text{m Pressure Head}$$

$$P_2 = \left(9.81\,\text{kN/m}^3\right) * \left(33.5\,\text{m}\right) = 328.9\,\text{kPa}$$

Example Problem 2

A siphon draws fluid from a tank and delivers it through a nozzle at the end of the pipe (Figure 1.5).

Calculate the volume flow rate through the siphon and the pressure at points B, C, D, and E. Diameter of siphon = 40 mm

Diameter of nozzle = 25 mm

Assume that there are no energy losses in the system.

Reference points A and F (most convenient points)

$$z_A + \frac{v_A^2}{2g} + \frac{P_A}{\gamma} = z_F + \frac{v_F^2}{2g} + \frac{P_F}{\gamma}$$

Points A and F are open to atmosphere; hence, $P_A = 0$ and $P_F = 0$.

Velocity at point A is zero.

Elevation difference between A and F = 1.8 + 1.2 = 3.0 m

$$z_A - z_F = 3.0\,\text{m}$$

$$z_F - z_A = -3.0\,\text{m}$$

$$z_A = z_F + \frac{v_F^2}{2g}$$

$$v_F = \sqrt{(z_A - z_F) * (2 * 9.81)}$$

$$v_F = 7.67\,\text{m/s}$$

$$Q = A \times V$$

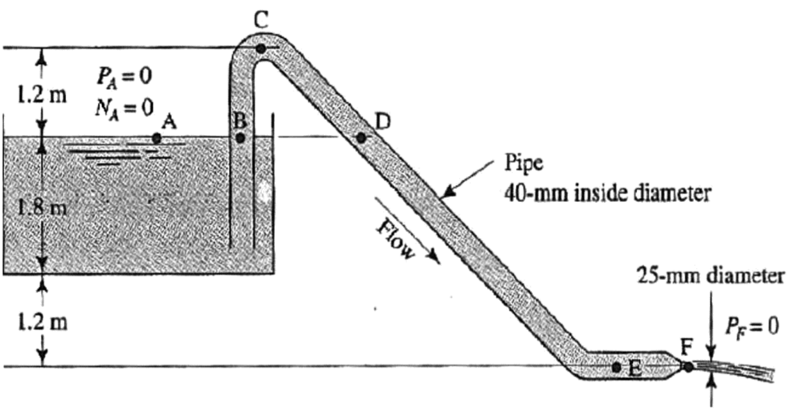


Figure 1.5 Example Problem 2

Source: Mott, R.L. (1972). *Applied Fluid Mechanics*, 6th Edition, Prentice Hall, New Jersey.

$$Q = \left(\frac{25\,\text{mm}}{1{,}000\,\text{m}}\right)^2 \frac{\pi}{4} * 7.67\frac{\text{m}}{s} = 3.77 \times 10^{-3}\ \text{m}^3/\text{s}$$

Determine the pressures between B, C, D and E.

Write the Bernoulli's equation between A and B.

$$z_A + \frac{v_A{}^2}{2g} + \frac{P_A}{\gamma} = z_B + \frac{v_B{}^2}{2g} + \frac{P_B}{\gamma}$$

$$Q = A_B \times v_B$$

$$3.77 \times 10^{-3}\ \text{m}^3/\text{s} = \left(\frac{40\ \text{mm}}{1{,}000\ \text{m}}\right)^2 \frac{\pi}{4} \times v_B$$

$$v_B = 3.0\,\text{m/s}$$

$$0 = \frac{(3.0)^2}{2 * 9.81} + \frac{P_B}{\gamma}$$

$$P_B = -4.50\,\text{kPa}$$

(Negative sign indicates that the pressure is below atmospheric pressure.)

Write the Bernoulli equation between A and C.

$$z_A + \frac{v_A{}^2}{2g} + \frac{P_A}{\gamma} = z_C + \frac{v_C{}^2}{2g} + \frac{P_C}{\gamma}$$

$v_C = v_B$: Since pipe diameter is the same

$$3.0\,\text{m} = 4.2\,\text{m} + \frac{(3.0)^2}{2 * 9.81} + \frac{P_C}{9.81}$$

$$P_C = -16.27\,\text{kPa}$$

Similarly, write the Bernoulli equation between A and D.

$$z_A + \frac{v_A{}^2}{2g} + \frac{P_A}{\gamma} = z_D + \frac{v_D{}^2}{2g} + \frac{P_D}{\gamma}$$

$$P_D = -4.50\,\text{kPa}$$

Similarly, write the Bernoulli equation between A and E.

$$z_A + \frac{v_A{}^2}{2g} + \frac{P_A}{\gamma} = z_E + \frac{v_E{}^2}{2g} + \frac{P_E}{\gamma}$$

$$P_E = 24.93\,\text{kPa}$$

1.9.2 Summary of the Results of Example

1. Velocity from nozzle and therefore the volume flow rate delivered by siphon depends on the elevation difference between the free surface of fluid and the outlet of the nozzle.
2. The pressure at point B is below atmospheric pressure even though it is on the same level as point A, which is exposed to the atmosphere. Bernoulli's equation shows that the pressure head at B is decreased by the amount of the velocity head. That is, some of the energy is converted.
3. The velocity of flow is the same at all points where the pipe size is the same, when steady flow exists.
4. The pressure at point C is the lowest in the system because point C is at the highest elevation.
5. The pressure at point D is the same as that at point B, because both are on the same elevation and the velocity head at both points is the same.
6. The pressure at point E is the highest in the system because point E is at the lowest elevation.

CHAPTER 2

General Energy Equation

It was identified in the last chapter that the Bernoulli's equation has a few limitations that cannot account for the flow of water through pipes. In fluid systems, there are different types of devices and components. These devices and components occur in most fluid-flow systems and they add energy to the fluid, remove energy from the fluid, or cause undesirable losses of energy from the fluid.

In this chapter, pumps, fluid motors, friction losses as fluid flows in pipes and tubes, energy losses from changes in the size of the flow path, and energy losses from valves and fittings are discussed.

In later chapters, details about how to compute the amount of energy losses in pipes and specific types of valves and fittings are discussed. The method of using performance curves for pumps and to apply them properly is discussed in detail.

2.1 Pumps

A pump is a common example of a mechanical device that adds energy to a fluid. A pump is a device that moves fluids by mechanical action. Pumps can be classified into three major groups according to the method they use to move the fluid: direct lift, displacement, and gravity pumps.

An electric motor or some other prime power device drives a rotating shaft in the pump. The pump then takes this kinetic energy and delivers it to the fluid, resulting in fluid flow and increased fluid pressure. Pumps operate via many energy sources, including manual operation, electricity, engines, or wind power, and come in many sizes, from microscopic for use in medical applications to large industrial pumps.

2.2 Fluid Motors

Fluid motors, turbines, rotary actuators, and linear actuators are examples of devices that take energy from a fluid and deliver it in the form of work, causing the rotation of a shaft or the linear movement of a piston. The major difference between a pump and a fluid motor is that when acting as a motor, the fluid drives the rotating elements of the device. The reverse is true for pumps.

The hydraulic motor shown in Figure 2.1 is often used as a drive for the wheels of construction equipment and trucks and for rotating components of material transfer systems, conveyors, agricultural equipment, special machines, and automation equipment. The design incorporates a stationary internal gear with a special shape. The rotating component is like an external gear, sometimes called a *gerotor*, which has one fewer teeth than the internal gear. The external gear rotates in a circular orbit around the center of the internal gear. High-pressure fluid entering the cavity between the two gears acts on the rotor and develops a torque that rotates the output shaft. The magnitude of the output torque depends on the pressure difference between the input and output sides of the rotating gear. The speed of rotation is a function of the displacement of the motor (volume per revolution) and the volume flow rate of fluid through the motor.

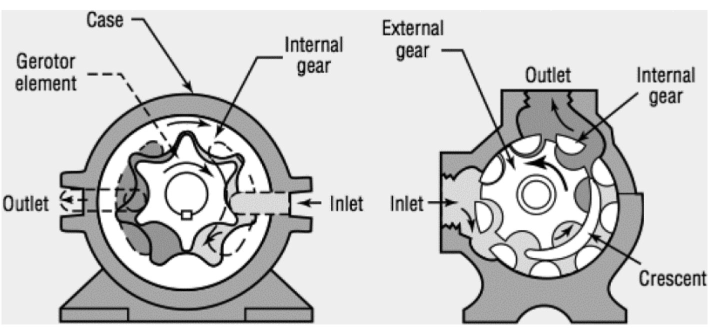


Figure 2.1 Rotor and external gear

2.3 Fluid Friction

A fluid in motion is subject to frictional resistance to flow. Part of the energy generated by that resistance is converted into thermal energy (heat), which is dissipated through the walls of the pipe in which the fluid is flowing. The magnitude of the energy loss is dependent on the properties of the fluid, the flow velocity, the pipe size, the smoothness of the pipe wall, and the length of the pipe.

2.4 Valves and Fittings

Elements that control the direction or flow rate of a fluid in a system typically set up local turbulence in the fluid, causing energy to be dissipated as heat. Whenever there is a restriction, a change in flow velocity, or a change in the direction of flow, these energy losses occur. In a large system, the magnitude of losses due to valves and fittings is usually small compared with frictional losses in the pipes. Therefore, such losses are referred to as minor losses.

Energy losses and additions in a system are accounted for in this book in terms of energy per unit weight of fluid flowing in the system. This is also known as "head," as de- scribed in Chapter 6. As an abbreviation for head, symbol *h* is used for energy losses and additions. Specifically, the following terms are used throughout the next several chapters:

h_A = *Energy added to* the fluid with a mechanical device such as a pump; this is often referred to as the *total head on* the pump
h_R = *Energy removed* from the fluid by a mechanical device such as a fluid motor
h_L = *Energy losses from* the system due to friction in pipes or minor losses due to valves and fittings

The magnitude of energy losses produced by fluid friction, valves, and fittings is directly proportional to the velocity head of the fluid. This can be expressed mathematically as

$$h_L = K\left(\frac{v^2}{2g}\right) \tag{2.1}$$

The term *K* is the *resistance coefficient.* The value of *K* for fluid friction can be determined using the Darcy equation. In the following chapters, the various methods of calculating the value of K for many kinds of valves and fittings, and changes in flow cross section and direction are provided. Most of these are found from experimental data.

2.5 General Energy Equation

The general energy equation as used in this text is an expansion of Bernoulli's equation, which makes it possible to solve problems in which energy losses and additions occur. The terms E'_1 and E'_2 denote the energy possessed by the fluid per unit weight at sections 1 and 2, respectively. The respective energy additions (h_A), removals (h_R), and losses (h_L) are shown. For such a system, the expression of the principle of conservation of energy is

$$E'_1 + h_A - h_L - h_R = E'_2 \tag{2.2}$$

Figure 2.2 shows how the terms of this equation are related to a typical section of a pipe system.

The energy possessed per unit weight is given by the following equation:

$$E' = z + \frac{v^2}{2g} + \frac{P}{\gamma} \tag{2.3}$$

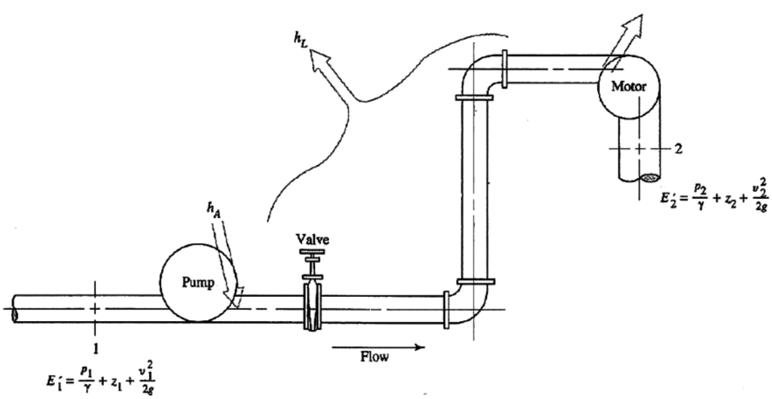


Figure 2.2 Fluid flow system demonstrating the general energy equation

Source: Mott, R.L. (1972). *Applied Fluid Mechanics,* 6th Edition, Prentice Hall, New Jersey.

Substituting the values from Eq. (2.2) into Eq. (2.3) yields the following equation:

$$z_1 + \frac{v_1^2}{2g} + \frac{P_1}{\gamma} - h_L - h_R + h_A = z_2 + \frac{v_2^2}{2g} + \frac{P_2}{\gamma} \tag{2.4}$$

This is the form of the energy equation that is used in this book. As with Bernoulli's equation, each term in Eq. (2.4) represents a quantity of energy per unit weight of fluid flowing in the system. Typical SI units are N·m/N, or meters. U.S. customary system units are lb·ft/lb, or feet.

It is essential that the general energy equation be written in the direction of flow, that is, from the reference point on the left side of the equation to that on the right side. Algebraic signs are critical because the left side of Eq. (2.4) states that an element of fluid having a certain amount of energy per unit weight at section 1 may have energy added ($+h_A$), energy removed ($-h_R$), or energy lost ($-h_L$) from it before it reaches section 2. There it contains a different amount of energy per unit weight, as indicated by the terms on the right side of the equation.

For example, in Figure 2.2, reference points are shown to be points 1 and 2 with the pressure head, elevation head, and velocity head indicated at each point. After the fluid leaves point 1 it enters the pump, where energy is added. A prime mover such as an electric motor drives the pump, and the impeller of the pump transfers the energy to the fluid ($+h_A$). Then the fluid flows through a piping system composed of a valve, elbows, and the lengths of pipe, in which energy is dissipated from the fluid and is lost ($-h_L$). Before reaching point 2, the fluid flows through a fluid motor, which removes some of the energy to drive an external device ($-h_R$). The general energy equation accounts for all of these energies.

Example Problem 2.1

Water flows from a large reservoir at the rate of 1.20 ft^3/s through a pipe system as shown in Figure 2.3. Calculate the total amount of energy lost from the system because of the valve, the elbows, the pipe entrance, and fluid friction.

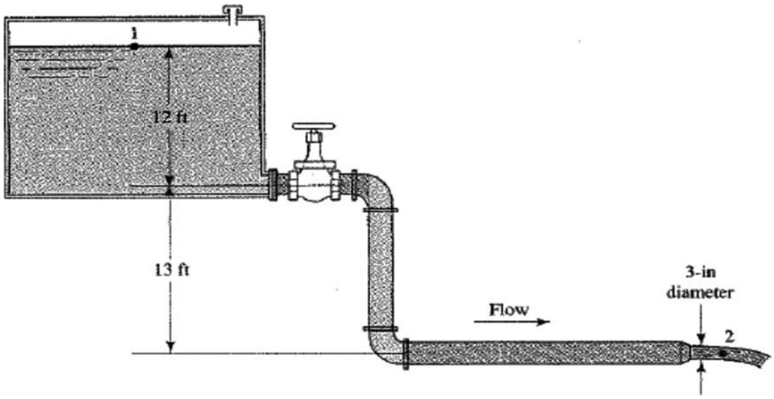


Figure 2.3 Example Problem 2.1

Source: Mott, R.L. (1972). *Applied Fluid Mechanics*, 6th Edition, Prentice Hall, New Jersey

Step 1. Identify two points in the system with the most number of known values

Step 2. Points 1 and 2 are identified in the above figure as

$P_1 = 0$:Surface of reservoir exposed to the atmosphere

$P_2 = 0$: Free stream of fluid exposed to the atmosphere

$v_1 = 0$ (approximately): Surface area of reservoir is large, and hence it can be assumed to be negligible

$h_A = h_R = 0$: No mechanical device in the system

Step 3. Apply the energy equation between points 1 and 2.

$$z_1 + \frac{v_1^2}{2g} + \frac{P_1}{\gamma} - h_L - h_R + h_A = z_2 + \frac{v_2^2}{2g} + \frac{P_2}{\gamma}$$

Step 4. The general energy equation becomes

$$z_1 + 0 + 0 - h_L - 0 + 0 = z_2 + \frac{v_2^2}{2g} + 0$$

Step 5. Since the objective is to determine energy loss in the system, solve for h_L.

$$h_L = (z_1 - z_2) - \frac{v_2^2}{2g}$$

$$z_1 - z_2 = 25 \text{ ft}$$

Given Q = 1.20 ft^3/s and 3-in. diameter pipe is 0.0491 ft^2

$$v = \frac{Q}{A} = \frac{1.20\ \text{ft}^3\ /\ \text{s}}{0.0491\ \text{ft}^2} = 24.0\,\text{ft/s}$$

$$h_L = (25\,\text{ft}) - \frac{(24.0\ \text{ft/s})^2}{2(32.2\ \text{ft/s}^2)}$$

$$h_L = 25\,\text{ft} - 9.25\,\text{ft}$$

$$h_L = 15.75\,\text{ft} = 15.75\frac{(\text{lb}\cdot\text{ft})}{\text{lb}}$$

Example Problem 2.2

The volume flow rate through the pump, as shown in Figure 2.4, is 0.014 m^3/s. The fluid being pumped is oil with a specific gravity of 0.86. Calculate the energy delivered by the pump to the oil per unit weight of oil flowing in the system. Energy losses in the system are caused by the check valve and friction losses as the fluid flows through the piping. The magnitude of such losses has been determined to be 1.86 N·m/N.

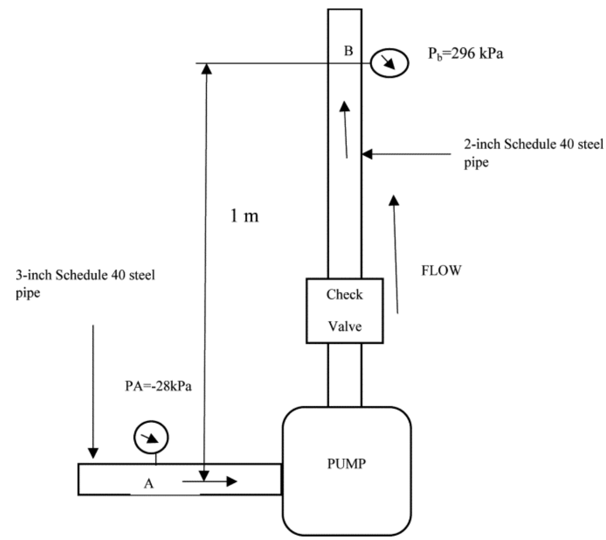


Figure 2.4 Example Problem 2.2

Source: Mott, R.L. (1972). *Applied Fluid Mechanics*, 6th Edition, Prentice Hall, New Jersey.

Step 1. Objective is to determine the energy delivered by the pump, h_A

Step 2. Apply energy equation

$$z_A + \frac{v_A^2}{2g} + \frac{P_A}{\gamma} - h_L - h_R + h_A = z_B + \frac{v_B^2}{2g} + \frac{P_B}{\gamma}$$

Step 3. Rewrite the equation to solve the unknown h_A

$$h_A = (z_B - z_A) + \frac{(v_B^2 - v_A^2)}{2g} + \frac{(P_B - P_A)}{\gamma} + h_L + h_R$$

Step 4. Determine specific weight

$$\gamma = \frac{sg}{\gamma_w} = \frac{0.86}{9.81\,\text{kN/m}^3} = 8.44\,\text{kN/m}^3$$

Step 5. Determine v_A and v_B

$$v_A = \frac{Q}{A_A} = \frac{(0.014)\text{m}^3/\text{s}}{(4.768\times10^{-3})\text{m}^2} = 2.94\,\text{m/s}$$

$$v_B = \frac{Q}{A_B} = \frac{(0.014)\text{m}^3/\text{s}}{(2.168\times10^{-3})\text{m}^2} = 6.46\,\text{m/s}$$

Step 6. Calculate h_A

$$h_A = (1.0\ \text{m}) + \frac{(6.46^2 - 2.94^2)\ \text{m}^2}{2 * 9.81\,\text{kN/m}^3} + \frac{(296-(-28))\ \text{kN/m}^2}{8.44\,\text{kN/m}^3} + 1.86\,\text{N·m/N} + 0$$

$$h_A = 1.0\ \text{m} + 1.69\,\text{m} + 38.4\,\text{m} + 1.86\,\text{N·m/m} + 0$$

$$h_A = 42.9\ \text{m or } 42.9\ \text{N·m/N}$$

That is, the pump delivers 42.9 N·m/N of energy to each newton of oil flowing through it.

2.6 Power Generated by Pumps

Power is defined as the rate of doing work. In fluid mechanics this statement can be modified to consider that power is the rate at which energy is being transferred. The unit for power in the SI system is watt (W), which is equivalent to 1.0 N·m/s or 1.0 joule (J)/s.

In Example Problem 2.2 it was determined that the pump was delivering 42.9 N·m/N of energy to each newton of oil as it flowed through the pump. To calculate the power delivered to the oil, it is necessary to determine how many newtons of oil are flowing through the pump in a given amount of time. This is called the weight flow rate, *W*, which was defined in the previous chapter, and is expressed in units of N/s. Power is calculated by multiplying the energy transferred per newton of fluid by the weight flow rate. This is

$$P_A = h_A W \tag{2.5}$$

However, from the previous chapter it was identified that

$$W = \gamma Q$$

Therefore

$$P_A = h_A \gamma Q \tag{2.6}$$

Using the information from Example Problem 2.2

$$h_A = 42.9\,\text{N·m/N}$$

$$\gamma = 8.44\ \text{kN/m}^3 = 8.44 \times 10^3\ \text{N/m}^3$$

$$Q = 0.014\ \text{m}^3/\text{s}$$

$$P_A = 42.9\frac{\text{N}\cdot\text{m}}{\text{N}} * 8.44 \times 10^3\ \frac{\text{N}}{\text{m}^3} * 0.014\frac{\text{m}^3}{\text{s}}$$

$$P_A = 5069.0\ \text{N·m/s}$$

Because 1.0 W = 1.0 N·m/s, this can be expressed in watts:

$$P_A = 5069\ \text{W} = 5.07\ \text{kW}$$

The unit for power in the U.S. customary system is lb·ft/s. Because it is common practice to refer to power in horsepower (hp), the conversion factor required is

$$1\ \text{hp} = 550\ \text{lb·ft/s}$$

In Eq. (2.6) the energy added h_A is expressed in feet of the fluid flowing in the system. Then, expressing the specific weight of the fluid in lb/ft^3 and the volume flow rate in ft^3/s would yield the weight flow rate γQ in lb/s. Finally, in the power equation, $P_A = h_A \gamma Q$, power would be expressed in lb·ft/s.

To convert these units to the SI system, the following factors are used:

$$1.0 \text{ lb-ft/s} = 1.356 \text{ W}$$

$$1 \text{ hp} = 745.7 \text{ W}$$

2.7 Mechanical Efficiency of Pumps

The term efficiency is used to denote the ratio of the power delivered by the pump to the fluid to the power supplied to the pump. Because of energy losses due to mechanical friction in pump components, fluid friction in the pump, and excessive fluid turbulence in the pump, not all of the input power is delivered to the fluid. Then, using the symbol e_M for mechanical efficiency

$$e_M = \frac{\text{Power delivered to fluid}}{\text{Power put into pump}} = \frac{P_A}{P_I} \quad (2.7)$$

The value of e_M should always be less than 1.0.

Continuing with the data of Example Problem 2.2, calculate the power input to the pump if e_M is known. For commercially available pumps, the value of e_M is published as part of the performance data. It is assumed that for the pump in this problem the efficiency is 82 percent, then

$$P_I = P_A/e_M = 5.07/0.82 = 6.18 \text{ kW}$$

The value of the mechanical efficiency of pumps depends not only on the design of the pump, but also on the conditions under which it is operating, particularly the total head and the flow rate.

2.8 Power Delivered to Fluid Motors

The energy delivered by the fluid to a mechanical device such as a fluid motor or a turbine is denoted in the general energy equation by the term h_R. This is a measure of the energy delivered by each unit weight of fluid as it passes through the device. The power delivered is found by multiplying h_R by the weight flow rate, W.

$$P_R = h_R \gamma Q \quad (2.8)$$

where P_R is power delivered by the fluid to the fluid motor.

2.9 Mechanical Efficiency of Fluid Motors

As was described for pumps, energy losses in a fluid motor are produced by mechanical and fluid friction. Therefore, not all the power delivered to the motor is ultimately converted to power output from the device. Mechanical efficiency is then defined as

$$e_M = \frac{\text{Power output from motor}}{\text{Power delivered by fluid}} = \frac{P_o}{P_R} \qquad (2.9)$$

Here again the value of e_M should be less than 1.0.

Example Problem 2.3

Water at 10 °C is flowing at a rate of 115 L/min through the fluid motor, as shown in Figure 2.5.

The pressure at A is 700 kPa and the pressure at B is 125 kPa. It is estimated that due to friction in the piping there is an energy loss of 4 N·m/N of water flowing. (1) Calculate the power delivered to the fluid motor by the water. (2) If the mechanical efficiency of the fluid motor is 85 percent, calculate the power output.

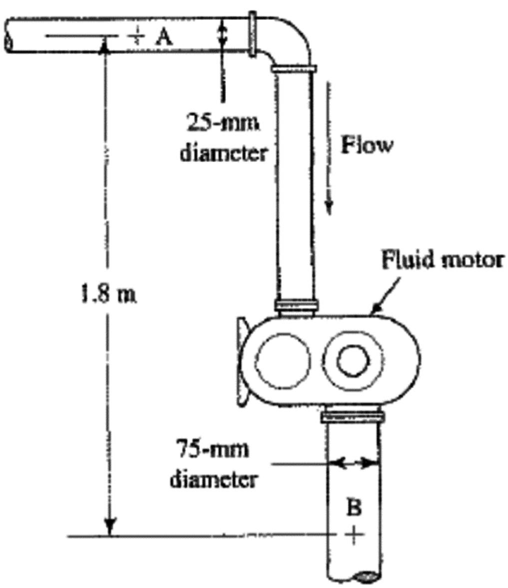


Figure 2.5 Example Problem 2.3

Source: Mott, R.L. (1972). *Applied Fluid Mechanics,* 6th Edition, Prentice Hall, New Jersey.

Step 1. Objective is to determine the energy delivered by the pump, h_A

Step 2. Apply energy equation

$$z_A + \frac{v_A^2}{2g} + \frac{P_A}{\gamma} - h_L - h_R + h_A = z_B + \frac{v_B^2}{2g} + \frac{P_B}{\gamma}$$

Step 3. Rewrite the equation to solve the unknown, h_A

$$h_R = (z_A - z_B) + \frac{(v_A^2 - v_B^2)}{2g} + \frac{(P_A - P_B)}{\gamma} - h_L$$

Step 4. Determine pressure head

$$\frac{(P_A - P_B)}{\gamma} = \frac{(700-125)(10^3)\text{N} / \text{m}^2}{9.81{*}10^3\ \text{kN} / \text{m}^3} = 58.6\,\text{m}$$

Step 5. Determine v_A and v_B

$$Q = (115.0\,\text{L} / \text{min}) * \frac{(1.0\text{m}^3 / \text{s})}{(60{,}000\,\text{L/min})} = 1.92 * 10^{-3}\ \text{m}^3/\text{s}$$

$$v_A = \frac{Q}{A_A} = \frac{1.92 * 10^{-3}\ \text{m}^3/\text{s}}{(4.909 {*} 10^{-4})\ \text{m}^2} = 3.91\ \text{m/s}$$

$$v_B = \frac{Q}{A_B} = \frac{1.92 * 10^{-3}\ \text{m}^3/\text{s}}{(4.418 {*} 10^{-3})\text{m}^2} = 0.43\,\text{m/s}$$

Step 6. Given h_L = 4.0 N·m/N

Step 7. Calculate h_R

$$h_R = (1.8\ \text{m}) + \frac{(3.91^2 - 0.43^2)\ \text{m}^2}{2 * 9.81\text{kN/m}^3} + 58.6\,\text{m} - 4.0\,\text{N·m/N}$$

$$h_R = 1.8\ \text{m} + 0.77\,\text{m} + 58.6\,\text{m} - 4.0\,\text{N·m/m}$$

$$h_R = 57.2\ \text{m}$$

Step 8. Calculate P_R

$$P_R = h_R \gamma Q = (57.2\,\text{m}) * \left(9.81 * 10^{-3}\right)\frac{\text{N}}{\text{m}^3} * 1.92 * 10^{-3}\ \frac{\text{m}^3}{\text{s}} = 1080\,\text{N·m/s}$$

$$P_R = 1.08\,\text{kW}$$

Step 9. Calculate the power output

Efficiency (e_M) is given as 85% = 0.85.

P_R is determined as 1.08 kW.

P_o is calculated as follows:

$$e_M = \frac{P_o}{P_R}$$

$$P_o = P_R * e_M$$

$$P_o = 0.85 * 1.08 = 0.92 \text{ kW}$$

CHAPTER 3

Types of Flow and Loss Due to Friction

Fluid flow in circular and noncircular pipes is commonly encountered in practice. Water in a city is distributed by extensive piping networks. Oil and natural gas are transported hundreds of miles by large pipelines. Blood is carried throughout our bodies by arteries and veins. The cooling water in an engine is transported by hoses to the pipes in the radiator where it is cooled as it flows. Thermal energy in a hydronic space heating system is transferred to the circulating water in the boiler, and then it is transported to the desired locations through pipes.

Fluid flow is classified as external and internal, depending on whether the fluid is forced to flow over a surface or in a conduit. Internal and external flows exhibit very different characteristics. In this chapter, internal flow where the conduit is completely filled with the fluid and the flow is driven primarily by a pressure difference is explained. This should not be confused with open-channel flow where the conduit is partially filled by the fluid and thus the flow is partially bounded by solid surfaces, as in an irrigation ditch, and flow is driven by gravity alone.

3.1 Reynolds Number

Reynolds number (N_R) is a dimensionless quantity that is used to help distinguish the different flow patterns in the pipe flow. The main types of flow in pipes are

a. Laminar flow
b. Turbulent flow
c. Critical flow

It has been demonstrated that the character of flow in a round pipe depends on four variables: fluid density ρ, fluid viscosity μ, pipe diameter D, and average velocity of flow v. Osborne Reynolds was the first to demonstrate that laminar or turbulent flow can be predicted if the magnitude of a dimensionless number, now called the Reynolds number (N_R), is known. It is defined by the equation

$$N_R = \frac{vD\rho}{\mu} = \frac{vD}{\vartheta} \tag{3.1}$$

As kinematic viscosity $\vartheta = \dfrac{\mu}{\rho}$, the variation in the formula occurs.

The Reynolds number is the ratio of the inertia force on an element of fluid to the viscous force. The inertia force is developed from Newton's second law of motion, $F = ma$.

Flows having large Reynolds numbers, typically because of high velocity and/or low viscosity, tend to be turbulent. Those fluids having high viscosity and/or moving at low velocities will have low Reynolds numbers and will tend to be laminar. The following section gives some quantitative data with which to predict whether a given flow system will be laminar or turbulent.

For practical applications in pipe flow, if the Reynolds number for the flow is less than 2,000, the flow will be laminar. If the Reynolds number is greater than 4,000, the flow can be assumed to be turbulent. In the range of Reynolds numbers between 2,000 and 4,000, it is impossible to predict which type of flow exists; therefore, this range is called the *critical region.* Typical applications involve flows that are well within the laminar flow range or well within the turbulent flow range, so the existence of this region of uncertainty does not cause great difficulty. If the flow in a system is found to be in the critical region, the usual practice is to change the flow rate or pipe diameter to cause the flow to be definitely laminar or definitely turbulent. More precise analysis is then possible.

When N_R is greater than about 4,000, a minor disturbance of the flow stream will cause the flow to suddenly change from laminar to turbulent. Therefore, for practical applications in this book, the following assumption is made:

If $N_R < 2{,}000$, the flow is laminar.
If $N_R > 4{,}000$, the flow is turbulent.

3.2 Laminar Flow

As the water flows from a faucet at a very low velocity, the flow appears to be smooth and steady. The stream has a fairly uniform diameter and there is little or no evidence of mixing of the various parts of the stream. This is called laminar flow, a term derived from the word layer, because the fluid appears to be flowing in continuous layers, with little or no mixing from one layer to the adjacent layers. Laminar flow is a flow regime wherein mixing is characterized by high-momentum diffusion and low-momentum convection.

When laminar flow exists, the fluid seems to flow as several layers, one on another. Because of the viscosity of the fluid, a shear stress is created between the layers of fluid. Energy is lost from the fluid by the action of overcoming the frictional forces produced by the shear stress. Because laminar flow is so regular and orderly, a relationship between the energy loss and the measurable parameters of the flow system can be derived. This relationship is known as the *Hagen–Poiseuille equation*:

$$h_L = \frac{32\mu L v}{\gamma D^2} \tag{3.2}$$

h_L = energy loss
μ = dynamic viscosity
L = length of pipe
D = diameter for pipe
γ = specific weight of fluid

It should be noted that the Hagen–Poiseuille equation is valid only for laminar flow ($N_R < 2{,}000$).

3.3 Turbulent Flow

Turbulent flow is a flow regime characterized by chaotic property changes. This includes low-momentum diffusion, high-momentum convection, and rapid variation of pressure and flow velocity in space and time.

Although there is no theorem relating the Reynolds number (N_R) to turbulence, flows at Reynolds numbers larger than 4,000 are typically turbulent. In this case, the energy losses in the energy equation are given by the Darcy equation:

$$h_L = f * \frac{L}{D} * \frac{v^2}{2g} \qquad (3.3)$$

h_L = energy loss due to friction

f = friction factor

D = diameter of pipe

v = velocity of fluid

g = acceleration due to gravity

For turbulent flow of fluids in circular pipes it is most convenient to use Darcy's equation to calculate the energy loss due to friction. Turbulent flow is rather chaotic and is constantly varying. For these reasons experimental data are used to determine the value of f. Tests have shown that the dimensionless number f is dependent on two other dimensionless numbers, the Reynolds number and the relative roughness of the pipe. The relative roughness is the ratio of the pipe diameter D to the average pipe wall roughness ε (Greek letter epsilon). Figure 3.1 illustrates pipe wall roughness (exaggerated) as the height of the peaks of the surface irregularities. The condition of the pipe surface is very much dependent on the pipe material and the method of manufacture. Because the roughness is somewhat irregular, averaging techniques are used to measure the overall roughness value.

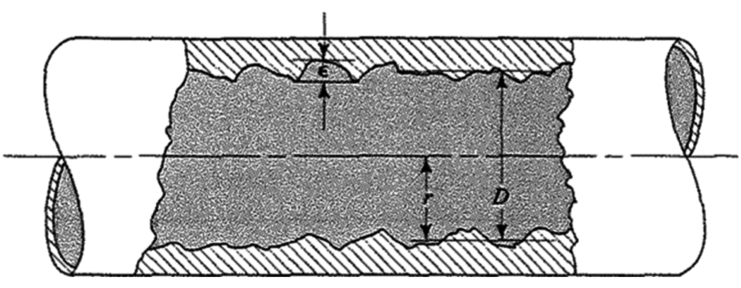


Figure 3.1 Pipe wall roughness

Source: Mott, R.L. (1972). *Applied Fluid Mechanics,* 6th Edition, Prentice Hall, New Jersey.)

For commercially available pipe and tubing, the design value of the average wall roughness ε has been determined as shown in Table 3.1. These are only average values for new, clean pipe. Some variation should be expected. After a pipe has been in service for a time, the roughness could change due to the formation of deposits on the wall or due to corrosion.

Glass tubing has an inside surface that is virtually hydraulically smooth, indicating a very small value of roughness. Therefore, the relative roughness, D/ε, approaches infinity. Plastic pipe and tubing are nearly as smooth as glass, and the values of roughness are listed in the table; however, variations should be expected. Copper, brass, and some steel tubing are drawn to its final shape and size over an internal mandrel, leaving a fairly smooth surface. For standard steel pipe (such as Schedule 40 and Schedule 80) and welded steel tubing, the roughness value is listed for commercial steel or welded steel. Galvanized iron has a metallurgically bonded zinc coating for corrosion resistance. Ductile iron pipe is typically coated on the inside with a cement mortar for corrosion protection and to improve surface roughness.

Table 3.1 Pipe roughness—design values

Material	Roughness ε (m)	Roughness ε (ft)
Glass	Smooth	Smooth
Plastic	3×10^{-7}	1×10^{-6}
Drawn tubing, copper, brass, steel	1.5×10^{-6}	5×10^{-6}
Steel, commercial or welded	4.6×10^{-5}	1.5×10^{-4}
Galvanized iron	1.5×10^{-4}	5×10^{-4}
Ductile iron, coated	1.2×10^{-7}	4×10^{-4}
Ductile iron, uncoated	2.4×10^{-4}	8×10^{-4}
Concrete, well made	1.2×10^{-4}	4×10^{-4}
Riveted steel	1.8×10^{-3}	6×10^{-3}

3.4 Moody Diagram

One of the most widely used methods for evaluating the friction factor employs the Moody diagram shown in Figure 3.2. The diagram shows the friction factor f plotted versus the Reynolds number N_R, with a series of parametric curves related to the relative roughness D/ε. Both f and N_R are plotted on logarithmic scales because of the broad range of values encountered. At the left end of the chart, for $N_R < 2{,}000$, the straight line shows the relationship $f = 64/N_R$ for laminar flow. For

2,000 < N_R < 4,000, no curves are drawn because this is the critical zone between laminar and turbulent flow, and it is not possible to predict the type of flow. The change from laminar to turbulent flow results in values for friction factors within the shaded band. Beyond N_R = 4,000, the family of curves for different values of D/ε is plotted. Several important observations can be made from these curves:

i. For a given Reynolds number of flow, as the relative roughness D/ε is increased, the friction factor f decreases.
ii. For a given relative roughness D/ε, the friction factor f decreases with increasing Reynolds number until the zone of complete turbulence is reached.
iii. Within the zone of complete turbulence, the Reynolds number has no effect on the friction factor.
iv. As the relative roughness D/ε increases, the value of the Reynolds number at which the zone of complete turbulence begins also increases.

Figure 3.2 is a simplified sketch of Moody's diagram on which the various zones are identified. The laminar zone at the left has already been discussed. At the right of the dashed line downward across the diagram is the zone of complete turbulence. The lowest possible friction factor for a given Reynolds number in turbulent flow is indicated by the smooth pipes line.

Between the smooth pipes line and the line marking the start of the complete turbulence zone is the transition zone. Here, the various D/ε lines are curved, and care must be exercised to evaluate the friction factor properly. As shown in the example, the value of the friction factor for a relative roughness of 500 decreases from 0.0420 at N_R = 4,000 to 0.0240 at $N_R = 6.0 \times 10^5$, where the zone of complete turbulence starts.

Table 3.2 Example values to determine friction factor

N_R	D/ε	f
6.7×10^3	150	0.0430
1.6×10^4	2,000	0.0284
1.6×10^6	2,000	0.0171
2.5×10^5	733	0.0233

Source: Mott, R.L. (1972). *Applied Fluid Mechanics,* 6th Edition, Prentice Hall, New Jersey

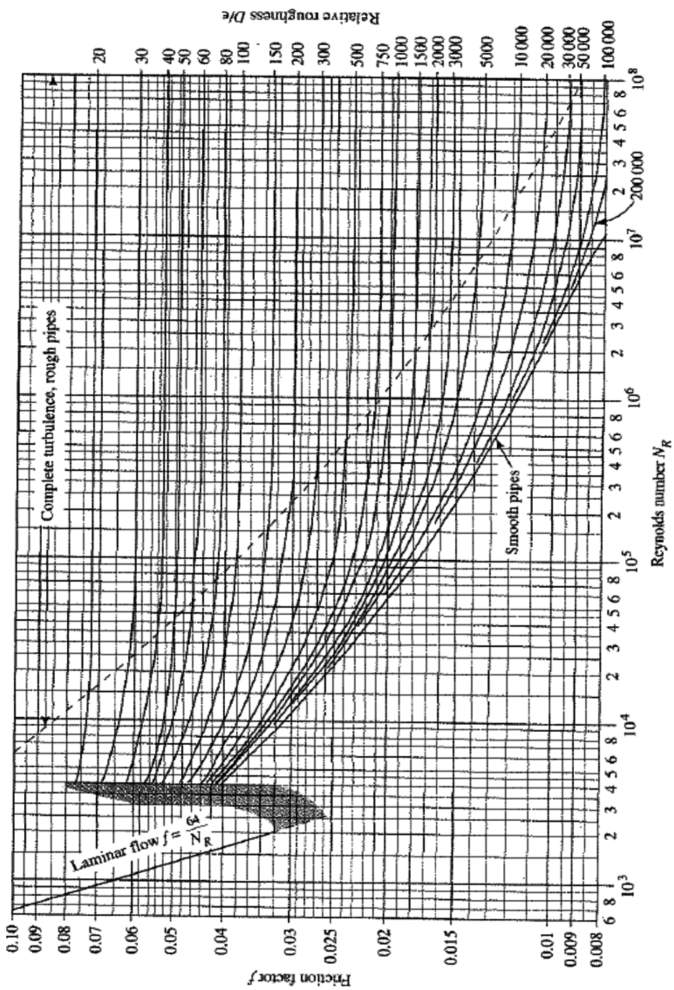


Figure 3.2 Moody diagram

3.5 Friction Factor Equations

The Moody diagram in Figure 3.2 is a convenient and sufficiently accurate means of determining the value of the friction factor when solving problems by manual calculations. However, additional equations are often used to determine the friction factor.

In the *laminar flow zone,* for values below 200, the friction factor can be found from the equation

$$f = \frac{64}{N_R} \tag{3.4}$$

This relationship, developed in Figure 3.2, plots in the Moody diagram as a straight line on the left side of the chart. Of course, for Reynolds numbers from 2,000 to 4,000, the flow is in the critical range and it is impossible to predict the value of *f.*

The value of the friction factor for turbulent flow can be determined by

$$f = \frac{0.25}{\left\{ \log\left(\frac{1}{3.7\left(\frac{D}{\varepsilon}\right)} \right) + \frac{5.74}{N_R^{\ 0.9}} \right\}^2} \tag{3.5}$$

The value determined from Eq. (3.5) will be similar to the value from the Moody diagram shown in Figure 3.2.

Example Problem 3.1

In a chemical processing plant, benzene at 50 °C (s.g. = 0.86) must be delivered to point B with a pressure of 550 kPa. A pump is located at point A, 21 m below point B, and the two points are connected by 240 m of plastic pipe having an inside diameter of 50 mm. If the volumetric flow rate is 110 L/min, calculate the required pressure at the outlet of the pump (Figure 3.3).

Step 1. Apply the energy equation between A and B

$$z_A + \frac{v_A^{\ 2}}{2g} + \frac{P_A}{\gamma} - h_L - h_R + h_A = z_B + \frac{v_B^{\ 2}}{2g} + \frac{P_B}{\gamma}$$

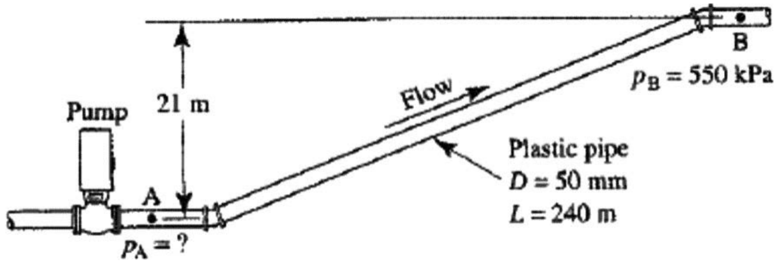


Figure 3.3 Example Problem 3.1

Step 2.

i. Velocity will be the same between the two points as the diameter of the pipe does not change.
ii. No energy added between the two points; hence h_A=0
iii. No energy removed between the two points; hence h_R=0

Rewrite energy equation to solve for P_A

$$P_A = \left\{ (z_B - z_A) + \frac{v_B^{\ 2} - v_A^{\ 2}}{2g} + \frac{P_B}{\gamma} + h_L \right\} * \gamma$$

Step 3.

$$z_B - z_A = +21 \text{ m}$$

$$Q = 110 \frac{\text{L}}{\text{min}} * \frac{1 \text{ m}^3/\text{s}}{(60{,}000 \text{ L/min})} = 1.83 * 10^{-3} \text{ m}^3/\text{s}$$

For a 50-mm diameter pipe, D = 0.050 m and A = 1.963 × 10^{-3} m^2

$$v = \frac{Q}{A} = \frac{1.83 * 10^{-3} \text{ m}^3/\text{s}}{1.963 * 10^{-3} \text{ m}^2} = 0.932 \text{ m/s}$$

For benzene at 50 °C with a specific gravity = 0.86

$$\rho = (0.86)\left(1{,}000 \frac{\text{kg}}{\text{m}^3}\right) = 860 \frac{\text{kg}}{\text{m}^3}$$

$$\mu = 4.2 * 10^{-4} \text{ Pa} \cdot \text{s}$$

$$N_R = \frac{(0.932)(0.050)(860)}{4.2 * 10^{-4}}$$

$$N_R = 9.54 * 10^4$$

From Table 3.1, a plastic pipe's roughness is $\varepsilon = 3.0 \times 10^{-7}$

$$\frac{D}{\varepsilon} = \frac{0.050\,\text{m}}{3.0 * 10^{-7}} = 1.667 * 10^{5}$$

From Moody diagram: $f = 0.018$

Step 4. Determine h_L from Darcy's equation

$$h_L = f * \frac{L}{D} * \frac{v^2}{2g} = 0.018 * \frac{240}{0.015} * \frac{(0.932)^2}{2 * 9.81}$$

$$h_L = 3.83 \text{ m}$$

Step 5. Solve for P_A

$$P_A = \left\{(21 \text{ m}) + \frac{550}{9.81} + 3.83\right\} * 9.81$$

$$P_A = 759 \text{ kPa}$$

3.6 Hazen–Williams Formula

The Darcy equation presented in this chapter for calculating energy loss due to friction is applicable for any Newtonian fluid. An alternate approach is convenient for the special case of the flow of water in pipeline systems.

The Hazen–Williams formula is one of the most popular formulas for the design and analysis of water systems. Its use is limited to the flow of water in pipes larger than 2 in. and smaller than 6 ft in diameter. The velocity of flow should not exceed 10 ft/s. Also, it has been developed for water at 60 °F. Use at temperatures much lower or higher would result in some error.

The Hazen–Williams formula is unit-specific. In the U.S. customary unit system it takes the form

$$v = 1.32\, C_h R^{0.63} s^{0.54} \tag{3.6}$$

where

v = average velocity (ft/s)

C_h = Hazen–Williams coefficient (dimensionless)

R = hydraulic radius of flow conduit (ft) = cross-sectional area/wetted perimeter

s = ratio of h_L/L: energy loss/length of conduit

The use of the hydraulic radius in the formula allows its application to noncircular sections as well as circular pipes. Use $R = D/4$ for circular pipes. The coefficient C_h is dependent only on the condition of the surface of the pipe or conduit. Table 3.3 gives typical values. Note that some values are described as for pipe in new, clean condition, whereas the design value accounts for the accumulation of deposits that develop on the inside surfaces of pipe after time, even when clean water flows through them. Smoother pipes have higher values of C_h than rougher pipes.

Table 3.3 Hazen–Williams Coefficient, C $_h$

Type of Pipe	C_h Average for New, Clean Pipe	C_h Design Value
Steel, ductile iron, or cast iron with centrifugally applied cement or bituminous lining	150	140
Plastic, copper, brass, glass	140	130
Steel, cast iron, uncoated	130	100
Concrete	120	100
Corrugated steel	60	60

The Hazen–Williams formula for SI units is

$$v = 0.85 C_h R^{0.63} s^{0.54} \tag{3.7}$$

Example Problem 3.2

For what velocity of flow of water in a new, clean, 6-in. Schedule 40 steel pipe would an energy loss of 20 ft of head occur over a length of 1,000 ft? Compute the volumetric flow rate at that velocity. Then refigure the velocity using the design value of C_h for steel pipe.

Solution:

Part a:

Step 1. $v = 1.32 C_h R^{0.63} s^{0.54}$

Step 2. $s = h_L/L = (20)/(1{,}000 \text{ ft}) = 0.02$

Step 3. $R = D/4 = (0.5054)/4 = 0.126$ ft

Step 4. $C_h = 130$

Step 5. $v = 1.32(130)(0.126)^{0.63}(0.02)^{0.54} = 5.64 \text{ ft/s}$

Step 6. $Q = Av = (0.2006 \text{ ft}^2)(5.64 \text{ ft/s}) = 1.13 \text{ ft}^3\text{/s}$

Part b: Reconfigure for design value

$$C_h = 100 \text{ (design value)}$$

The allowable volume flow rate to limit the energy loss to the same value of 20 ft per 1,000 ft of pipe length would be

$$v = 5.64 \text{ ft/s } (100/130) = 4.34 \text{ ft/s}$$

$$Q = Av = (0.2006 \text{ ft}^2)(4.34 \text{ ft/s}) = 0.869 \text{ ft}^3\text{/s}$$

3.7 Nomograph for Solving Hazen–Williams Equation

The nomograph shown in Figure 3.4 allows the solution of the Hazen–Williams formula to be done by aligning known quantities with a straight edge and reading the desired unknowns at the intersection of the straight edge with the appropriate vertical axis.

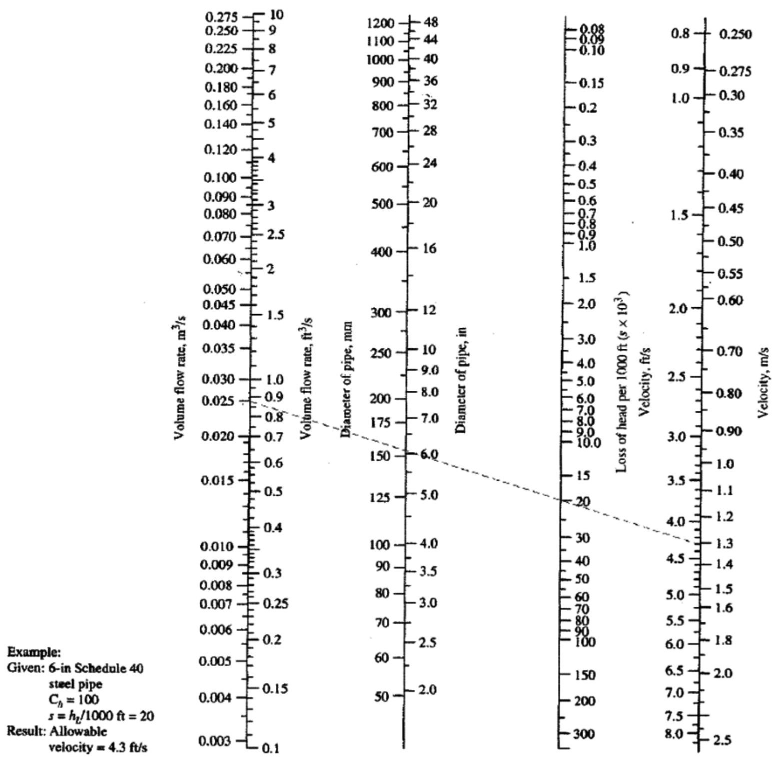


Figure 3.4 Nomograph for Hazen–Williams equation

Example Problem 3.3

Specify the required size of Schedule 40 steel pipe to carry 1.20 ft^3/s of water with no more than 4.0 ft of head loss over a 1,000-ft length of pipe. Use the design value for C_h.

Solution:

Table 3.3 suggests C_h = 100. Using Figure 3.4, place a straight edge from Q = 1.20 ft^3/s on the volume flow rate line to the value of s = 4.0 ft/1,000 ft on the energy loss line. The straight edge then intersects the pipe size line at approximately 9.7 in. The next larger standard pipe size listed in Appendix C is the nominal 10-in. pipe with an inside diameter of 10.02 in.

Returning to Figure 3.4 and slightly realigning Q = 1.20 ft^3/s with D = 10.02 in., an average velocity of v = 2.25 ft/s. This is relatively low for a water distribution system, and the pipe is quite large. If the pipeline is long, the cost for piping would be excessively large.

Hence, allow the velocity of flow to increase to approximately 6.0 ft/s for the same volume flow rate; use Figure 3.4 to show that a 6-in. pipe could be used with a head loss of approximately 37 ft per 1,000 ft of pipe. The lower cost of the pipe (in comparison with the 10-in. pipe) would have to be compared with the higher energy cost required to overcome the additional head loss.

CHAPTER 4

Minor Losses

Energy losses are proportional to the velocity head of the fluid as it flows around an elbow, through an enlargement or contraction of the flow section, or through a valve. Experimental values for energy losses are usually reported in terms of a resistance coefficient K as follows:

$$h_L = K\left(\frac{v^2}{2g}\right) \tag{4.1}$$

In Eq. (4.1), h_L is the minor loss, K is the resistance coefficient, and v is the average velocity of flow in the pipe in the vicinity where the minor loss occurs. In some cases, there may be more than one velocity of flow, as with enlargements or contractions. It is most important to know which velocity is to be used with each resistance coefficient.

The resistance coefficient is dimensionless because it represents a constant of proportionality between the energy loss and the velocity head. The magnitude of the resistance coefficient depends on the geometry of the device that causes the loss and sometimes on the velocity of flow. In the following sections, the process for determining the value of K and for calculating the energy loss for many types of minor loss conditions will be described.

As in the energy equation, the velocity head $v^2/2g$ in Eq. (4.1) is typically in the SI units of meters (or N·m/N of fluid flowing) or in the U.S. customary units of feet (or ft·lb/lb of fluid flowing). Because K is dimensionless, the energy loss has the same units.

The different types of minor losses are the following:

1. Sudden enlargement
2. Gradual enlargement
3. Sudden contraction
4. Gradual contraction

5. Entrance loss
6. Exit loss
7. Resistance coefficient for valves and fittings
8. Pipe bends

4.1 Sudden Enlargement

As a fluid flows from a smaller pipe into a larger pipe through a sudden enlargement, its velocity abruptly decreases, causing turbulence, which generates an energy loss (Figure 4.1(a) and (b)). The amount of turbulence, and therefore the amount of energy loss, is dependent on the ratio of the sizes of the two pipes.

The minor loss is calculated from the equation

$$h_L = K\left(\frac{v_1^2}{2g}\right) \tag{4.2}$$

Here v_1 is the average velocity of flow in the smaller pipe ahead of the enlargement. Tests have shown that the value of the loss coefficient K is dependent on both the ratio of the sizes of the two pipes and the magnitude of the flow velocity. This is illustrated graphically in Figure 4.2 and in tabular form in Table 4.1.

By making some simplifying assumptions about the character of the flow stream as it expands through the sudden enlargement, it is possible to analytically predict the value of K from the following equation:

$$K = [1 - (A_1/A_2)^2 = [1-(D_1/D_2)^2]^2)] \tag{4.3}$$

The subscripts 1 and 2 refer to the smaller and larger sections, respectively, as shown in Figure 4.1. Values for K from this equation agree well with experimental data when the velocity v_1 is approximately 1.2 m/s

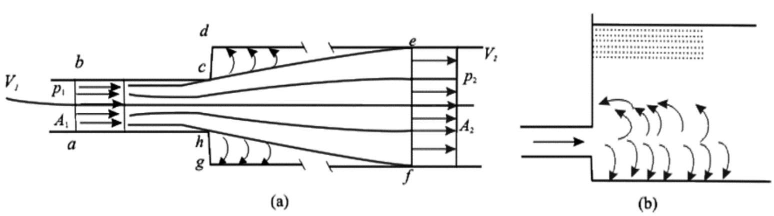


Figure 4.1 Sudden enlargement

(4 ft/s). At higher velocities, the actual values of K are lower than the theoretical values. We recommend that experimental values be used if the velocity of flow is known.

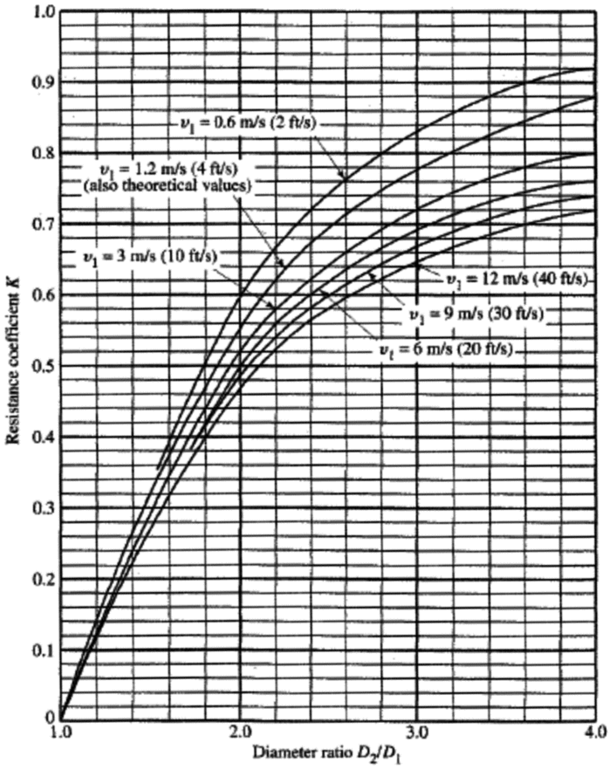


Figure 4.2 Resistance coefficient—sudden enlargement

Source: King, H.W. and Brater, E.F. (1963). *Handbook of Hydraulics*, 5th edition, New York, McGraw Hill.

Table 4.1 Resistance coefficient—sudden enlargement

	Velocity v_1						
D_2/D_1	***0.6 m/s 2 ft/s***	***1.2 m/s 4 ft/s***	***3 m/s 10 ft/s***	***4.5 m/s 15 ft/s***	***6 m/s 20 ft/s***	***9 m/s 30 ft/s***	***12 m/s 40 ft/s***
1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1.2	0.11	0.10	0.09	0.09	0.09	0.09	0.09
1.4	0.26	0.25	0.23	0.22	0.22	0.21	0.20
1.6	0.40	0.38	0.38	0.34	0.33	0.32	0.32
1.8	0.51	0.48	0.45	0.43	0.42	0.41	0.40
2.0	0.60	0.56	0.52	0.51	0.50	0.48	0.47
2.5	0.74	0.70	0.65	0.63	0.62	0.60	0.58

	Velocity v_1						
D_2/D_1	0.6 m/s 2 ft/s	1.2 m/s 4 ft/s	3 m/s 10 ft/s	4.5 m/s 15 ft/s	6 m/s 20 ft/s	9 m/s 30 ft/s	12 m/s 40 ft/s
3.0	0.83	0.78	0.73	0.70	0.69	0.67	0.65
4.0	0.92	0.87	0.80	0.78	0.76	0.74	0.72
5.0	0.96	0.91	0.84	0.82	0.80	0.77	0.75
10.0	1.00	0.96	0.89	0.86	0.84	0.82	0.80
∞	1.00	0.98	0.91	0.88	0.86	0.83	0.81

Source: King, H.W. and Brater, E.F. (1963). *Handbook of Hydraulics*, 5th edition, New York, McGraw Hll; Table 6-7.

4.2 Gradual Enlargement

If the transition from a smaller to a larger pipe can be made less abrupt than the square-edged sudden enlargement, the energy loss is reduced. This is normally done by placing a conical section between the two pipes as shown in Figure 4.3. The sloping walls of the cone tend to guide the fluid during the deceleration and expansion of the flow stream. Therefore, the size of the zone of separation and the amount of turbulence are reduced as the cone angle is reduced.

The energy loss for a gradual enlargement is calculated from

$$h_L = K\left(\frac{v_1^2}{2g}\right) \tag{4.4}$$

where v_1 is the velocity in the smaller pipe ahead of the enlargement. The magnitude of K is dependent on both the diameter ratio D_2/D_1 and the cone angle θ. Data for various values of θ and D_2/D_1 are given in Figure 4.4 and Table 4.2.

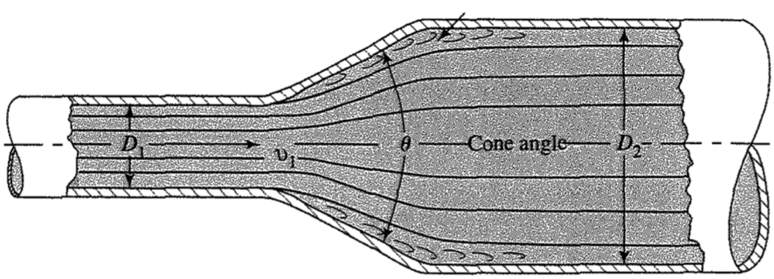


***Figure 4.3** Gradual enlargement*

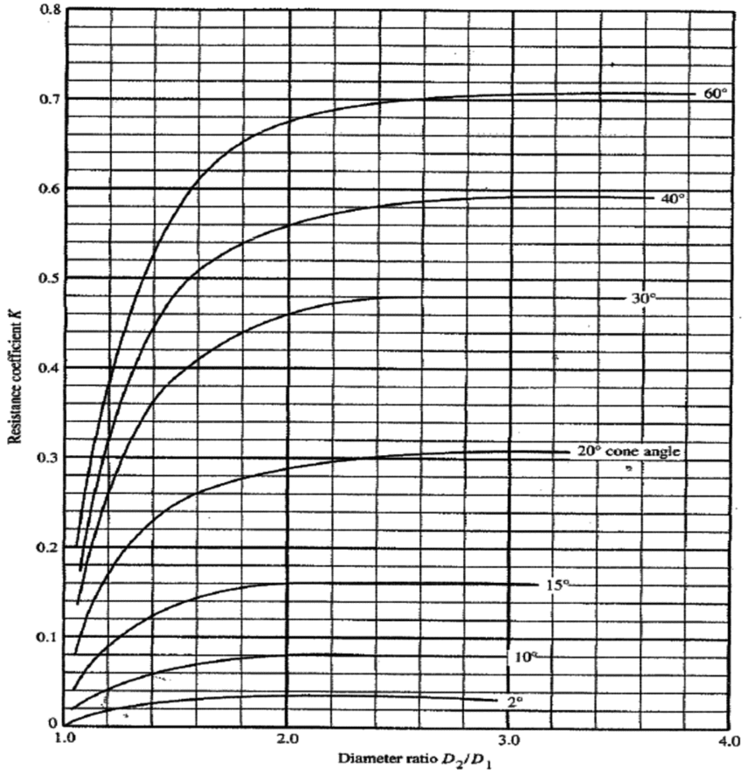


Figure 4.4 Resistance coefficient—gradual enlargement

Source: King, H.W. and Brater, E.F. (1963). *Handbook of Hydraulics*, 5th edition, New York, McGraw Hill.

Table 4.2 Resistance coefficient—gradual enlargement

	Angle of Cone U											
D_2/D_1	2°	6°	10°	15°	20°	25°	30°	35°	40°	45°	50°	60°
1.1	0.01	0.01	0.03	0.05	0.10	0.13	0.16	0.18	0.19	0.20	0.21	0.23
1.2	0.02	0.02	0.04	0.09	0.16	0.21	0.25	0.29	0.31	0.33	0.35	0.37
1.4	0.02	0.03	0.06	0.12	0.23	0.30	0.36	0.41	0.44	0.47	0.50	0.53
1.6	0.03	0.04	0.07	0.14	0.26	0.35	0.42	0.47	0.51	0.54	0.57	0.61
1.8	0.03	0.04	0.07	0.15	0.28	0.37	0.44	0.50	0.54	0.58	0.61	0.65
2.0	0.03	0.04	0.07	0.16	0.29	0.38	0.46	0.52	0.56	0.60	0.63	0.68
2.5	0.03	0.04	0.08	0.16	0.30	0.39	0.48	0.54	0.58	0.62	0.65	0.70
3.0	0.03	0.04	0.08	0.16	0.31	0.40	0.48	0.55	0.59	0.63	0.66	0.71
∞	0.03	0.05	0.08	0.16	0.31	0.40	0.49	0.56	0.60	0.64	0.67	0.72

Source: King, H.W. and Brater, E.F. (1963). *Handbook of Hydraulics*, 5th edition, New York, McGraw Hill, Table 6-8.

The energy loss calculated from Eq. (4.4) does not include the loss due to friction at the walls of the transition. For relatively steep cone angles, the length of the transition is short, and therefore, the wall friction loss is negligible. However, as the cone angle decreases, the length of the transition increases and wall friction becomes significant. Taking both wall friction loss and the loss due to the enlargement into account, the minimum energy loss with a cone angle of about 7° is obtained.

4.2.1 *Diffuser*

Another term for an enlargement is a *diffuser.* The function of a diffuser is to convert kinetic energy (represented by the velocity head, $v^2/2g$) to pressure energy (or otherwise called pressure head, p/γ) by decelerating the fluid as it flows from the smaller to the larger pipe. The diffuser can be either sudden or gradual, but the term is most often used to describe a gradual enlargement.

An ideal diffuser is one in which no energy is lost as the flow decelerates. Of course, no diffuser performs in the ideal fashion. If it did, the theoretical maximum pressure after the expansion could be computed from Bernoulli's equation,

$$z_A + \frac{v_A^2}{2g} + \frac{P_A}{\gamma} = z_B + \frac{v_B^2}{2g} + \frac{P_B}{\gamma}$$

If the diffuser is horizontal, the elevation terms get cancelled out. Then pressure recovery for an ideal diffuser is calculated from the equation,

$$\Delta p = P_B - P_A = \gamma\left(\frac{v_A^2 - v_B^2}{2g}\right) \tag{4.5}$$

In a *real diffuser,* energy losses do occur and the general energy equation must be used:

$$z_A + \frac{v_A^2}{2g} + \frac{P_A}{\gamma} - h_L = z_B + \frac{v_B^2}{2g} + \frac{P_B}{\gamma}$$

The pressure increases and becomes

$$\Delta p = P_B - P_A = \gamma\{\left(\frac{v_A^2 - v_B^2}{2g}\right) - h_L\} \tag{4.6}$$

4.3 Sudden Contraction

The energy loss due to a sudden contraction, such as that sketched in Figure 4.5, is calculated from

$$h_L = K\left(\frac{v_2^2}{2g}\right) \tag{4.7}$$

Here v_2 is the velocity in the small pipe downstream from the contraction. The resistance coefficient K is dependent on the ratio of the sizes of the two pipes and on the velocity of flow, as Figure 4.6 and Table 4.3 show.

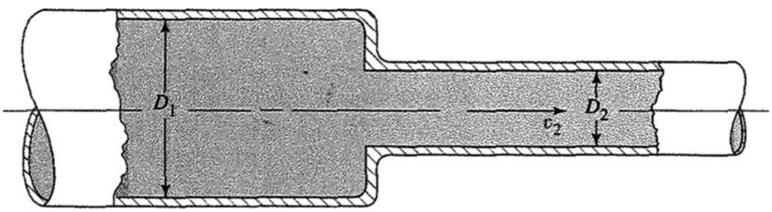


Figure 4.5 Sudden contraction

Source: King, H.W. and Brater, E.F. (1963). *Handbook of Hydraulics*, 5th edition, New York, McGraw Hill.

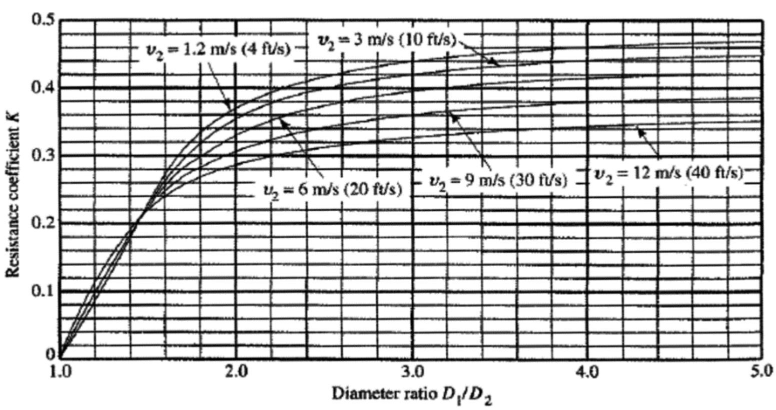


Figure 4.6 Resistance coefficient—sudden contraction

Source: King, H.W. and Brater, E.F. (1963). *Handbook of Hydraulics*, 5th edition, New York, McGraw Hill.

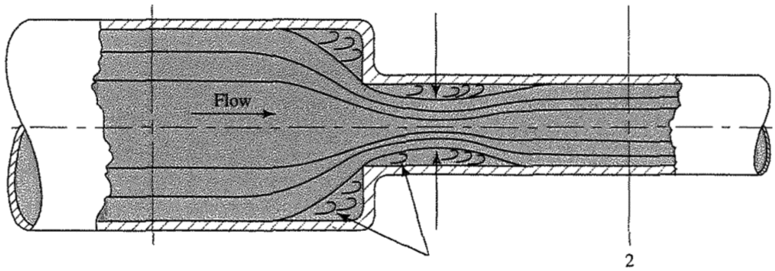
***Table 4.3* Resistance coefficient—sudden contraction**

	Velocity V_2								
D_1/D_2	0.6m/s 2 ft/s	1.2 m/s 4 ft/s	1.8 m/s 6 ft/s	24 m/s 8 ft/s	3 m/s 10 ft/s	4.5 m/s 15 ft/s	6 m/s 20 ft/s	9 m/s 30 ft/s	12 m/s 40 ft/s
1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1.1	0.03	0.04	0.04	0.04	0.04	0.04	0.05	0.05	0.06
1.2	0.07	0.07	0.07	0.07	0.08	0.08	0.09	0.10	0.11
1.4	0.17	0.17	0.17	0.17	0.18	0.18	0.18	0.19	0.20
1.6	0.26	0.26	0.26	0.26	0.26	0.25	0.25	0.25	0.24
1.8	0.34	0.34	0.34	0.33	0.33	0.32	0.31	0.29	0.27
2.0	0.38	0.37	0.37	0.36	0.36	0.34	0.33	0.31	0.29
2.2	0.40	0.40	0.39	0.39	0.38	0.37	0.35	0.33	0.30
2.5	0.42	0.42	0.41	0.40	0.40	0.38	0.37	0.34	0.31
3.0	0.44	0.44	0.43	0.42	0.42	0.40	0.39	0.36	0.33
4.0	0.47	0.46	0.45	0.45	0.44	0.42	0.41	0.37	0.34
5.0	0.48	0.47	0.47	0.46	0.45	0.44	0.42	0.38	0.35
10.0	0.49	0.48	0.48	0.47	0.46	0.45	0.43	0.40	0.36
∞	0.49	0.48	0.48	0.47	0.47	0.45	0.44	0.41	0.38

Source: King, H.W. and Brater, E.F. (1963). *Handbook of Hydraulics*, 5th edition, New York, McGraw Hill.

$$h_L = k\left(\frac{v^2}{2g}\right)$$

Vena contracta



Turbulence zones

Figure 4.7 Vena contracta formed in a sudden contraction
Source: King, H.W. and Brater, E.F. (1963). *Handbook of Hydraulics*, 5th edition, New York, McGraw Hill.

The mechanism by which energy is lost due to a sudden contraction is quite complex. Figure 4.7 illustrates what happens as the flow stream converges. The lines in the figure represent the paths of various parts of the flow stream called streamlines. As the streamlines approach the contraction, they assume a curved path and the total stream continues to neck down for some distance beyond the contraction. Thus, the effective minimum cross section of the flow is smaller than that of the smaller pipe. The section where this minimum flow area occurs is called the vena contracta. Beyond the vena contracta, the flow stream must decelerate and expand again to fill the pipe. The turbulence caused by the contraction and the subsequent expansion generates the energy loss.

4.4 Gradual Contraction

The energy loss in a contraction can be decreased substantially by making the contraction more gradual. Figure 4.8 shows such a gradual contraction formed by a conical section between the two diameters with sharp breaks at the junctions. The angle θ is called the cone angle.

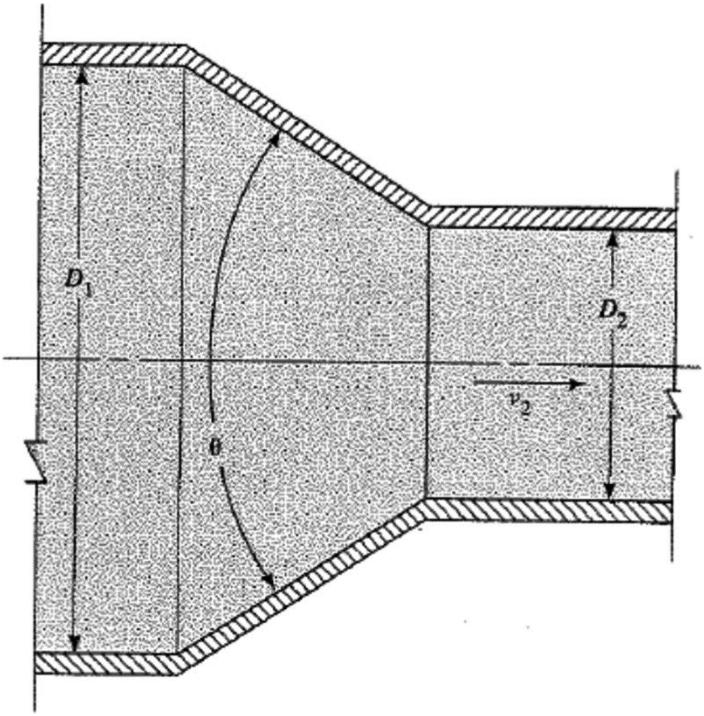


Figure 4.8 Gradual contraction

Source: King, H.W. and Brater, E.F. (1963). *Handbook of Hydraulics*, 5th edition, New York, McGraw Hill.

Figure 4.9 shows the data for the resistance coefficient versus the diameter ratio for several values of the cone angle. The energy loss is computed from Eq. (4.7), where the resistance coefficient is based on the velocity head in the smaller pipe after the contraction. These data are for Reynolds numbers greater than 1×10^5. Note that for angles over the wide range of 15° to 40°, $K = 0.05$ or less, a very low value. For angles as high as 60°, K is less than 0.08.

As the cone angle of the contraction decreases below 15°, the resistance coefficient actually increases, as shown in Figure 4.10. The reason is that the data include the effects of both the local turbulence caused by flow separation and pipe friction. For the smaller cone angles, the transition between the two diameters is very long, which increases the friction losses.

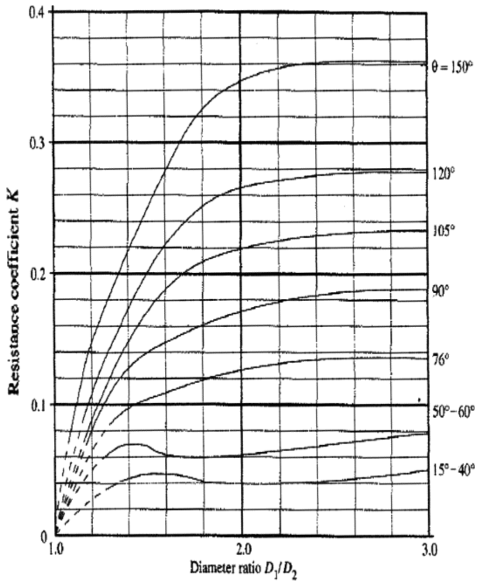


Figure 4.9 Resistance coefficient—gradual contraction with $\theta \geq 15°$

Source: King, H.W. and Brater, E.F. (1963). *Handbook of Hydraulics*, 5th edition, New York, McGraw Hill.

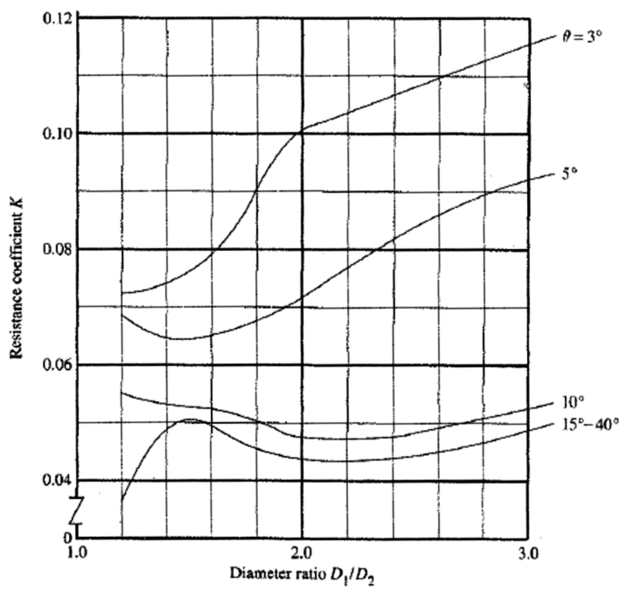


Figure 4.10 Resistance coefficient—gradual contraction with $\theta \leq 15°$

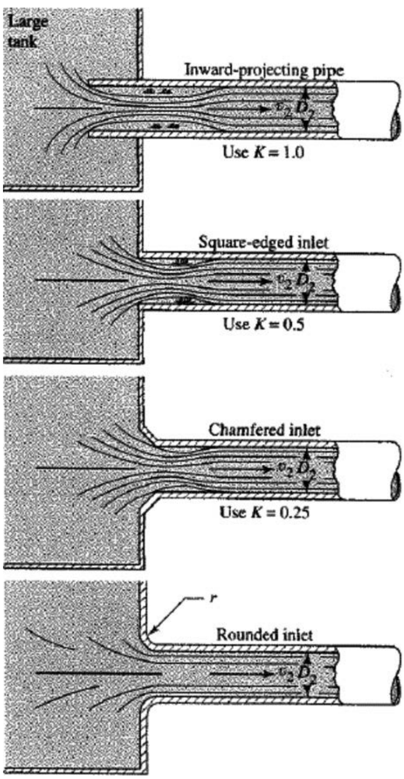
Source: King, H.W. and Brater, E.F. (1963). *Handbook of Hydraulics*, 5th edition, New York, McGraw Hill.

4.5 Entrance Loss

A special case of a contraction occurs when a fluid flows from a relatively large reservoir or tank into a pipe. The fluid must accelerate from a negligible velocity to the flow velocity in the pipe. The ease with which the acceleration is accomplished determines the amount of energy loss, and therefore, the value of the entrance resistance coefficient is dependent on the geometry of the entrance.

Figure 4.11 shows four different configurations and the suggested value of K for each. The streamlines illustrate the flow of fluid into the pipe and show that the turbulence associated with the formation of a vena contracta in the tube is a major cause of the energy loss.

Here v_2 is the velocity of flow in the pipe.



r/D_2	K
0	0.50
0.02	0.28
0.04	0.24
0.06	0.15
0.10	0.09
>0.15	0.04 (Well-rounded)

Figure 4.11 Entrance resistance coefficient

Source: King, H.W. and Brater, E.F. (1963). *Handbook of Hydraulics*, 5th edition, New York, McGraw Hill.

In summary, after selecting a value for the resistance coefficient from Figure 4.11, the energy loss at an entrance can be calculated from

$$h_L = K\left(\frac{v_2^2}{2g}\right) \tag{4.8}$$

4.6 Exit Loss

As a fluid flows from a pipe into a large reservoir or tank, as shown in Figure 4.12, its velocity is decreased to very nearly zero. In the process, the kinetic energy that the fluid possessed in the pipe, indicated by the velocity head $v_1^2/2g$, is dissipated. Therefore, the energy loss for this condition is

$$h_L = K\left(\frac{v_1^2}{2g}\right) \tag{4.9}$$

This is called the exit loss. The value of $K = 1$ is used regardless of the form of the exit where the pipe connects to the tank wall.

4.7 Resistance Coefficients for Valves and Fittings

Many different kinds of valves and fittings are available from several manufacturers for specification and installation into fluid flow systems. Valves are used to control the amount of flow; and they may be globe valves, angle valves, gate valves, butterfly valves, any of several types of check valves, and many more. Fittings direct the path of flow or cause a change in the size of the flow path, and these can include elbows of several designs, tees, reducers, nozzles, and orifices. See Figures 4.13 and 4.14.

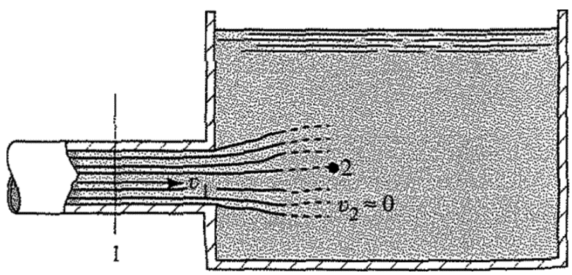


Figure 4.12 Exit loss as the fluid flows into a static reservoir

Source: King, H.W. and Brater, E.F. (1963). *Handbook of Hydraulics*, 5th edition, New York, McGraw Hill.

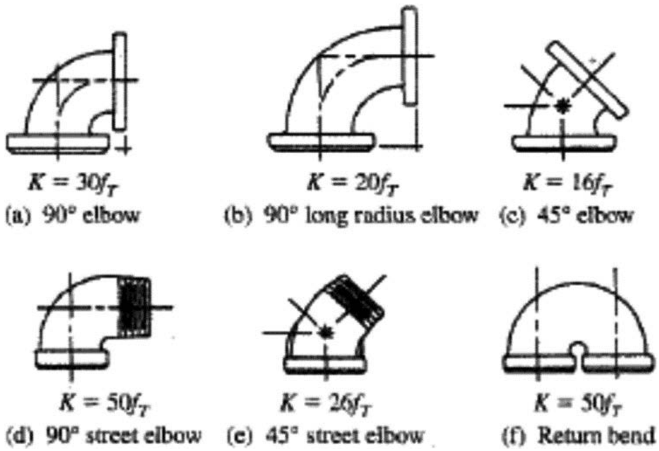


Figure 4.13 Pipe elbows

Source: Crane Valves, Signal Hill, CA.

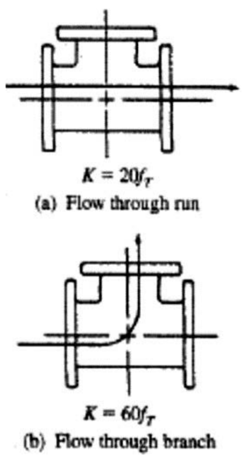


Figure 4.14 Standard tees

Source: Crane Valves, Signal Hill, CA.

It is important to determine the resistance data for the particular type and size chosen, because the resistance is dependent on the geometry of the valve or fitting. Also, different manufacturers may report data in different forms.

Energy loss incurred as fluid flows through a valve or fitting is computed from Eq. (4.10), as used for the minor losses already discussed. However, the method of determining the resistance coefficient K is different. The value of K is reported in the form

$$K = f_T \left(\frac{L_e}{D} \right) \tag{4.10}$$

The value of L_e/D, called the equivalent length ratio, is reported in Table 4.4, and it is considered to be constant for a given type of valve or fitting. The value of L_e is called the equivalent length and is the length of straight pipe of the same nominal diameter as the valve that would have the same resistance as the valve. The term D is the actual inside diameter of the pipe.

The term f_T is the friction factor in the pipe to which the valve or fitting is connected, taken to be in the zone of complete turbulence. Note in Figure 3.2, the Moody diagram, that the zone of complete turbulence lies in the far right area where the friction factor is independent of Reynolds number. The dashed line running generally diagonally across the diagram divides the zone of complete turbulence from the transition zone to the left.

Values for f_T vary with the size of the pipe and the valve, causing the value of the resistance coefficient K to also vary. Table 4.5 lists the values of f_T for standard sizes of new, clean, commercial steel pipes.

Some system designers prefer to compute the equivalent length of pipe for a valve and combine that value with the actual length of pipe. Equation (4.10) can be solved for L_e:

$$L_e = \left(\frac{KD}{f_T} \right) \tag{4.11}$$

Table 4.4 Resistance in valves and fittings expressed as equivalent length in pipe diameters, L_e/D

Type	Equivalent Length in Pipe Diameters L_e/D
Globe valve—fully open	340
Angle valve—fully open	150
Gate valve—fully open	8
—¾ open	35
—½ open	160
—¼ open	900
Check valve—swing type	100
Check valve—ball type	150
Butterfly valve—fully open, 2–8 in	45
—10–14 in	35

Type	Equivalent Length in Pipe Diameters L_e/D
—16–24 in	25
Foot valve—poppet disc type	420
Foot valve—hinged disc type	75
90° standard elbow	30
90° long radius elbow	20
90° street elbow	50
45° standard elbow	16
45° street elbow	26
Close return bend	50
Standard tee—with flow through run	20
—with flow through branch	60

Source: Crane Valves, Signal Hill, CA.

Table 4.5 Friction factor in zone of complete turbulence for new, clean, commercial steel pipes

Nominal Pipe Size (in)	Friction Factor f_T	Nominal Pipe Size (in)	Friction Factor f_T
½	0.027	3½, 4	0.017
¾	0.025	5	0.016
1	0.023	6	0.015
1¼	0.022	8–10	0.014
1½	0.021	12–16	0.013
2	0.019	18–24	0.012
2½, 3	0.018		

Source: Crane Valves, Signal Hill, CA.

It is given that $L_e = (L_e/D)$. Note, however, that this would be valid only if the flow in the pipe is in the zone of complete turbulence.

If the pipe is made from a material different from new, clean, commercial steel pipe, it is necessary to compute the relative roughness D/ε, and then use the Moody diagram to determine the friction factor in the zone of complete turbulence.

4.8 Resistance Coefficient for Pipe Bends

It is frequently more convenient to bend a pipe or tube than to install a commercially made elbow. The resistance to flow of a bend is dependent on the ratio of the bend radius r to the pipe inside diameter D. Figure 4.15 shows that the minimum resistance for a 90° bend occurs when the ratio

r/D is approximately 3. The resistance is given in terms of the equivalent length ratio L_e/D, and therefore, Eq. (4.10) must be used to calculate the resistance coefficient. The resistance shown in Figure 4.15 includes both the bend resistance and the resistance due to the length of the pipe in the bend.

When the r/D ratio is computed, r is defined as the radius to the centerline of the pipe or tube, called the mean radius (see Figure 4.16). That is, if R_o is the radius to the outside of the bend, R_i is the radius to the inside of the bend, and D_o is the outside diameter of the pipe or tube, then

$$r = R_i + D_o/2$$

$$r = R_o - D_o/2$$

$$r = (R_o + R_i)/2$$

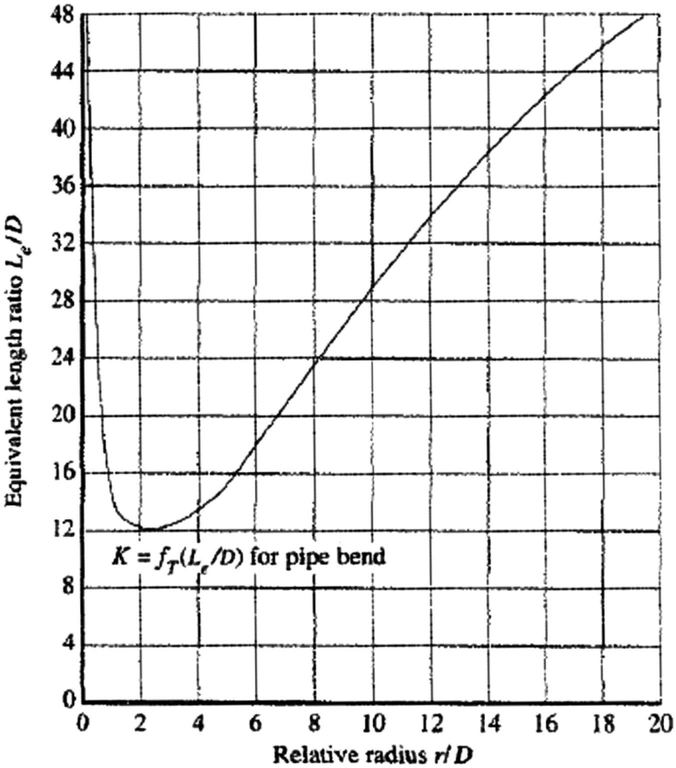


Figure 4.15 Resistance due to 90° pipe bends

Source: Beij, K.H. (1938). "Pressure Losses for Fluid Flow in 90° Pipe Bends." *Journal of Research of the National Bureau of Standards 21* (July): 1–18.

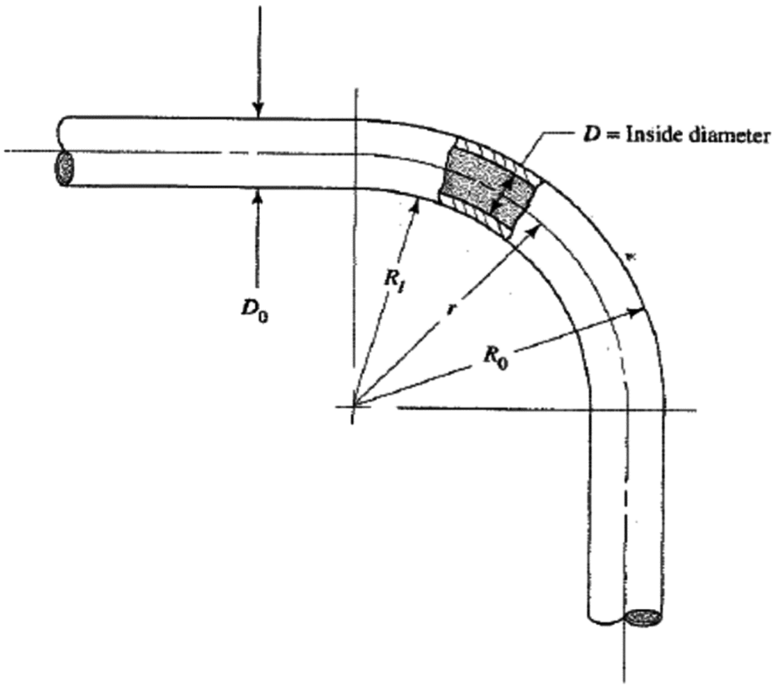


Figure 4.16 Resistance due to 90° pipe bends

Source: King, H.W. and Brater, E.F. (1963). *Handbook of Hydraulics*, 5th edition, New York, McGraw Hill.

Example Problem 1

Calculate the pressure in tank A, as shown in Figure 4.17. Account for all minor losses.

Solution:

Step 1. Write the energy equation

$$z_A + \frac{v_A^{\,2}}{2g} + \frac{P_A}{\gamma} - h_L = z_B + \frac{v_B^{\,2}}{2g} + \frac{P_B}{\gamma}$$

Step 2. $v_A = v_B$ (pipe diameter does not change)

$P_B = 0$ (pressure in tank B = 0, as it is open to atmosphere)

$$P_A = (\gamma(z_B - z_A) + h_L)$$

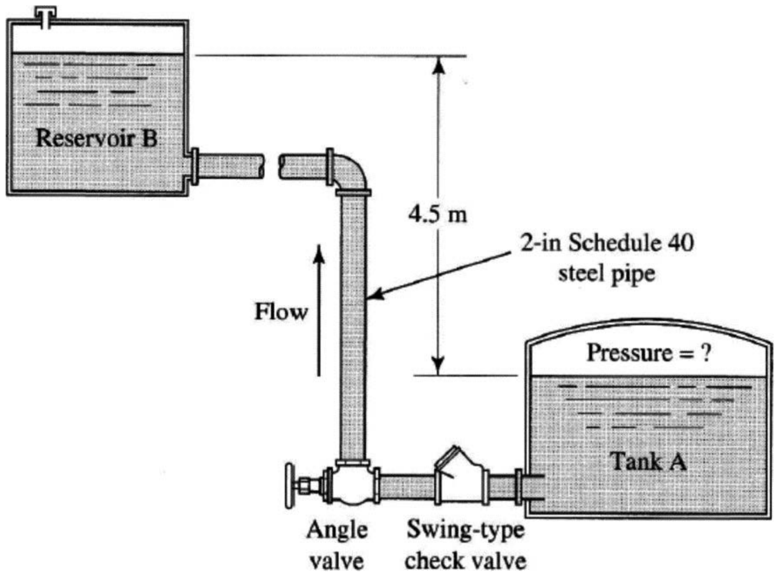


Figure 4.17 Example 1

Step 3. Determine the h_L

Minor and major losses from tank A to tank B would be = *entrance loss* + *check valve* + *swing valve* + *friction* + *elbow* + *exit*

Step 4.

h_L = *entrance loss* + *check valve* + *swing valve* + *friction* + *elbow* + *exit*

$$h_L = 1h_v + 100 f_T h_v + 150 f_T h_v + f \frac{38}{0.0525} h_v + 30 f_T h_v + 1 f_T h_v$$

$$h_L = h_v (7.32 + 724 f)$$

From Table 4.5

$f_T = 0.019$ (2-in. Schedule 40 steel pipe)

$$h_v = \frac{v^2}{2g} (\text{in pipe})$$

$$v = \frac{Q}{A} = \frac{435\,\text{L/min}}{2.168 \times 10^{-3}\,\text{m}^2} \times \frac{1\,\text{m}^3/\text{s}}{60{,}000\,\text{L/min}} = 3.344\,\text{m/s}$$

$$h_v = \frac{3.44^2}{2 * 9.81} = 0.570\,\text{m}\pi$$

$$N_R = \frac{vD\rho}{\mu} = \frac{(3.344)(0.0525)(820)}{1.70 \times 10^{-3}} = 8.47 \times 10^4;$$

$$\frac{D}{\varepsilon} = \frac{0.0525}{4.60 \times 10^{-5}} = 1{,}141$$

From Moody diagram

$$f = 0.022$$

Step 5.

$P_A = (\gamma(z_B - z_A) + h_L)$ = (0.82)(9.81 kN/m^3)[4.5 m + 0.570 (7.32 + 724(0.022))]

P_A = 143.5 kPa

CHAPTER 5

Series and Parallel Pipeline Systems

As discussed in the previous chapter, fluid flow systems have numerous minor losses. In addition to these losses, pipeline systems for analysis purposes are classified into series pipeline systems and parallel pipeline systems.

5.1 Series Pipeline Systems

Series pipeline systems are those with a single flow path: all of the fluid must travel the same route. Because of that, conservation of mass and conservation of energy principles are used to develop the generalized form of the energy equation

$$z_1 + \frac{v_1^{\,2}}{2g} + \frac{P_1}{\gamma} - h_{\mathrm{L}} - h_{\mathrm{R}} + h_{\mathrm{A}} = z_2 + \frac{v_2^{\,2}}{2g} + \frac{P_2}{\gamma}$$

The first three terms of the left-hand side of the equation represent the energy possessed by the fluid at point 1 in the form of pressure head, velocity head, and elevation head. The same can be applied for the terms on the right-hand side as the energy possessed by the fluid at point 2. The term h_{A} is the energy added to the fluid by the pump, and this is termed as the *total head on the pump* and is used to select the different types of pumps. This will be discussed in the next chapter.

The term h_{L} denotes the energy lost from the systems anywhere between points 1 and 2. This can include major and minor losses.

$$h_{\mathrm{L}} = \sum_{i=1}^{n} K_i \left(\frac{v^2}{2g}\right) + f \frac{L}{D} \frac{v^2}{2g}$$

System analysis and design problems can be classified into three classes as follows:

Class I: The system is completely defined in terms of the size of pipes, the types of minor losses that are present, and the volume flow rate of fluid in the system. The typical objective is to compute the pressure at some point of interest, to compute the total head on a pump, or to compute the elevation of a source of fluid to produce a desired flow rate or pressure at selected points in the system.

Steps to solve Class I problems: Flow rate (Q) and pipe diameter (D) are known; head loss (h_L) is unknown.

i. Write down energy equation: $P_1/\gamma + z_1 + v_1^2/2g + h_A - h_L - h_R = P_2/\gamma + z_2 + v_2^2/2g$
ii. Identify all the terms that make up h_L, such as pipe losses, $h_L = f(L/D)v^2/2g$, and minor losses, $h_L = Kv^2/2g$.
iii. By using Q, $Q = v \times \pi / 4 \times D^2$, and D, solve for the velocity, v, and determine the Reynolds number, N_R, and the loss coefficient, K.
iv. By using the Reynolds number, N_R, and D/ε, determine the friction factor, f, by using the Moody diagram or the appropriate friction factor equation. For minor losses, find f_T (if required) by using D/ε only.
v. Determine h_L and find the required output (e.g., h_A, etc.).

Example Problem 1: Class I Type Problem

Calculate the power supplied to the pump, as shown in Figure 5.1, if its efficiency is 76 percent. Methyl alcohol at 25 °C is flowing at the rate of 54 m^3/s. The suction line is a standard 4-in. Schedule 40 steel pipe, 15 m long. The total length of a 2-in. Schedule 40 steel pipe in the discharge line is 200 m. Assume that the entrance from reservoir 1 is through a square-edged inlet and that the elbows are standard. The valve is a fully open globe valve. The elevation difference between the two surfaces of the reservoirs was measured as 10 m.

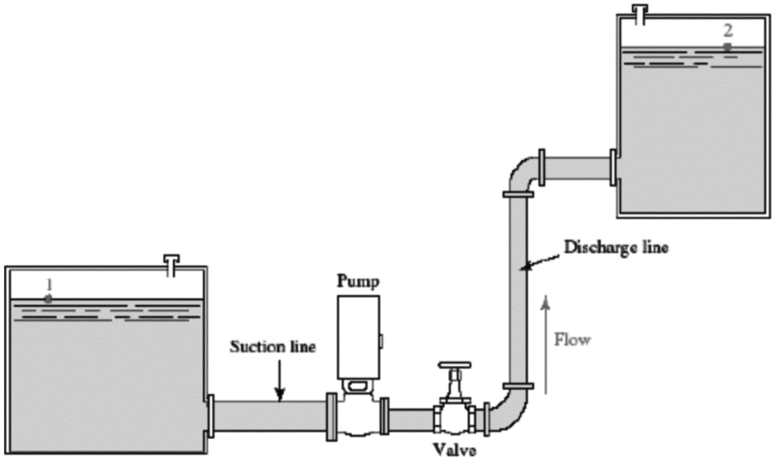


Figure 5.1 Example Problem 1

Source: Mott, R.L. (1972). *Applied Fluid Mechanics,* 6th Edition, Prentice Hall, New Jersey.

Step 1. Write the energy equation

$$z_1 + \frac{v_1^2}{2g} + \frac{P_1}{\gamma} - h_L + h_A = z_2 + \frac{v_2^2}{2g} + \frac{P_2}{\gamma}$$

Step 2. Determine the equation to calculate the total head

$v_1 = v_2 = 0$: velocity at the surface of the reservoir can be negligible

$P_1 = P_2 = 0$ open to atmosphere

$$h_A = z_2 - z_1 + h_L$$

Step 3. Calculate the overall head loss

$$h_L = h_1 + h_2 + h_3 + h_4 + h_5 + h_6$$

h_L = total energy loss per unit weight of fluid flowing

h_1 = entrance loss

h_2 = friction loss in suction line

h_3 = energy loss in valve

h_4 = energy loss in 90° elbows

h_5 = friction loss in the discharge line

h_6 = exit loss

Step 4. Equation for energy loss

$$h_1 = K\left(\frac{v_s^2}{2g}\right) \text{(entrance loss)}$$

$$h_2 = f_s\left(\frac{L}{D}\right)\left(\frac{v_s^2}{2g}\right) \text{(friction loss in suction line)}$$

$$h_3 = f_{dT}\left(\frac{L_e}{D}\right)\left(\frac{v_d^2}{2g}\right) \text{(valve)}$$

$$h_4 = f_{dT}\left(\frac{L_e}{D}\right)\left(\frac{v_d^2}{2g}\right) \text{(two} 90^\circ \text{ elbows)}$$

$$h_5 = f_d\left(\frac{L}{D}\right)\left(\frac{v_d^2}{2g}\right) \text{(friction loss in discharge line)}$$

$$h_6 = 1.0\left(\frac{v_d^2}{2g}\right) \text{(exit loss)}$$

Step 5. Determine velocity in suction and discharge line

$$Q = \frac{54 \text{ m}^3}{\text{hr}} \times \frac{1 \text{ hr}}{3{,}600 \text{ s}} = 0.015 \text{ m}^3/\text{s}$$

$$v_s = \frac{Q}{A_s} = \frac{0.015 \text{ m}^3}{\text{s}} \times \frac{1}{8.213*10^{-3} \text{m}^2} = 1.83 \text{ m/s}$$

$$\frac{v_s^2}{2g} = \frac{(1.83^2)}{2*9.81} = 0.17 \text{ m}$$

$$v_d = \frac{Q}{A_d} = \frac{0.015 \text{ m}^3}{\text{s}} \times \frac{1}{2.168*10^{-3} \text{ m}^2} = 6.92 \text{ m/s}$$

$$\frac{v_d^2}{2g} = \frac{(6.92^2)}{2*9.81} = 2.44 \text{ m}$$

Step 6. Determine the friction factor in suction and discharge line

Methyl alcohol at 25 °C

Suction line

$$N_R = \frac{vD\rho}{\mu} = \frac{(1.83)(0.1023)(789)}{5.60*10^{-4}} = 2.60*10^5$$

For steel pipe: $\varepsilon = 4.6 \times 10^{-5}$

$$\frac{D}{\varepsilon} = 224$$

From Moody diagram: $f_s = 0.018$

Discharge line

$$N_R = \frac{vD\rho}{\mu} = \frac{(6.92)(0.0525)(789)}{5.60 * 10^{-4}} = 5.12 * 10^5$$

For steel pipe: $= 4.6 \times 10^{-5}$

$$\frac{D}{\varepsilon} = 1{,}141$$

From Moody diagram: $f_d = 0.020$

Step 7. Determine total energy loss

$h_1 = 0.5(0.17 \text{ m}) = 0.09 \text{ m}$ (for a square-edged inlet, $K = (0.5)(0.17)$)

$$h_2 = 0.018\left(\frac{15}{0.1023}\right)(0.17) = 0.45 \text{ m}$$

$$h_3 = 0.019(340)(2.44) = 15.76 \text{ m}\left(\frac{L_e}{D} = 340:\right.$$

fully open globe valve)

$$h_4 = 2{*}0.019(30)(2.44) \text{ m} = 2.78 \text{ m(two } 90^\circ \text{ elbows)}$$

$$h_5 = 0.020\left(\frac{200}{0.0525}\right)(2.44) = 185.9 \text{ m}$$

$$h_6 = 1.0(2.44) = 2.44 \text{ m(exit loss)}$$

$$h_L = h_1 + h_2 + h_3 + h_4 + h_5 + h_6 = 207.4 \text{ m}$$

Step 8. Determine total head

$$h_A = z_2 - z_1 + h_L$$

$$h_A = 10 \text{ m} + 207.4 = 217.4 \text{ m}$$

Step 9. The power

$$\text{Power} = \frac{h_A \gamma Q}{e_M} = \frac{(217.4\,\text{m})(7.74 * 10^3\ \text{N}/\text{m}^3)(0.015\,\text{m}^3)}{0.76}$$

$$= 33.2 \times 10^{-2}\ \text{N}.\frac{\text{m}}{\text{s}} = 33.2\,\text{kW}$$

Class II: The system is completely described in terms of its elevations, pipe sizes, valves and fittings, and allowable pressure drop at key points in the system. The objective is to determine the volume flow rate of the fluid that could be delivered by a given system.

Steps to solve Class II problem: Pipe diameter (D) and pressure drop, ΔP, are known; flow rate, Q, is unknown.

i. Write down the energy equation, $P_1/\gamma + z_1 + v_1^2/2g + h_A - h_L - h_R = P_2/\gamma + z_2 + v_2^2/2g$
ii. Separate known variables from unknown variables. Put the known variables on the left-hand side of the equation and the unknowns on the right side.
iii. Identify all the terms that make up h_L, such as pipe losses, $h_L = f(L/D)v^2/2g$, and minor losses, $h_L = Kv^2/2g$.
iv. Since flow rate, Q, is unknown, express h_L as a function v and f, and solve for v.
v. Use D/ε to estimate an initial guess value for f, and find v.
vi. Use the calculated v to determine the Reynolds number, N_R, and determine a new f value.
vii. Find v (velocity) and repeat step v_i until the f value converges to a steady value.
viii. Determine the flow rate, Q.

Example Problem 2: Class II Type (Iterative Method)

Hydraulic oil is flowing in a steel tube with an outside diameter of 2 in. and a wall thickness of 0.083 in (Figure 5.2). A pressure drop of 68 kPa is observed between two points in the tube 30 m apart. The oil has a specific gravity of 0.90 and dynamic viscosity of 3.0×10^{-3} Pa·s. Calculate the velocity of flow of the oil.

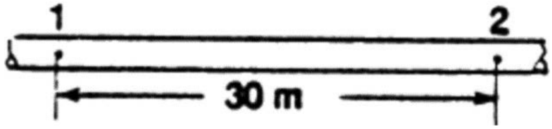


Figure 5.2 Example Problem 2

Step 1. Apply the energy equation between the two points

$$z_1 + \frac{v_1^2}{2g} + \frac{P_1}{\gamma} - h_L = z_2 + \frac{v_2^2}{2g} + \frac{P_2}{\gamma}$$

$z_1 = z_2$: horizontal pipe

$v_1 = v_2$: same size diameter pipe

Step 2. Write the equation for head loss

$$h_L = \frac{(P_2 - P_1)}{\gamma} = \frac{\left(68 \frac{\text{kN}}{\text{m}^2}\right)}{\left(9.81 \frac{\text{kN}}{\text{m}^3}\right) 0.90} = 7.70\,\text{m}$$

$$7.70\,\text{m} = h_L = f \frac{L}{D} \frac{v^2}{2g}$$

$$v = \sqrt{\frac{2gDh_L}{fL}} = \sqrt{\frac{2(9.81)(0.04658)(7.70)}{f(30)}} = \sqrt{\frac{0.235}{f}}$$

$$\frac{D}{\varepsilon} = \frac{0.04658}{1.5 \times 10^{-6}} = 31{,}053$$

Step 3. Make assumptions for f and solve for v
1st Iteration: Try $f = 0.03$; then $v = 2.80$ m/s

$$N_R = \frac{vD\rho}{\mu} = \frac{2.80 * 0.0468 * 900}{3 \times 10^{-3}} = 3.92 \times 10^4$$

Use the Moody diagram and determine the friction factor.
From the Moody diagram: $f = 0.022$ (new f)
2nd Iteration: Try $f = 0.022$; then $v = 3.27$ m/s

$$N_R = \frac{vD\rho}{\mu} = \frac{3.27 * 0.0468 * 900}{3 \times 10^{-3}} = 4.58 \times 10^4$$

Use the Moody diagram and determine the friction factor.

From the Moody diagram: $f = 0.0210$ (new f)

3rd Iteration: Try $f = 0.0210$; then $v = 3.345$ m/s

$$N_R = \frac{vD\rho}{\mu} = \frac{3.345 * 0.0468 * 900}{3\times 10^{-3}} = 4.69\times 10^{4}$$

Use the Moody diagram and determine the friction factor.

From the Moody diagram: $f = 0.0210$ (no change)

Hence, the velocity of flow of oil = 3.34 m/s.

Class III: The general layout of the system is known along with the desired volume flow rate. The size of the pipe required to carry a given volume flow rate of a given fluid is to be determined.

Steps for Class III: Pressure drop, ΔP, and flow rate, Q, are known; D is unknown.

i. Write down the energy equation: $P_1/\gamma + z_1 + v_1^2/2g + h_A - h_L - h_R = P_2/\gamma + z_2 + v_2^2/2g$
ii. Separate known variables from the unknown variables. Put the known variables on the left-hand side of the equation and the unknowns on the right side.
iii. Identify all the terms that make up h_L, such as pipe losses, $h_L = f(L/D)v^2/2g$, and minor losses, $h_L = Kv^2/2g$.
iv. Solve for D by using $Q = v \times \pi/4 \times D^2$, and express it in terms of h_L and f.
v. Make an initial guess value for f (between 0.01 and 0.1; $f = 0.02$ is usually a good initial guess), and find D.
vi. Determine N_R and D/ε, and compute a new f value.
vii. Find D and repeat Step vi until the f value converges to a steady value.
viii. Determine D.
ix. Alternatively the flow diameter can be determined using the equation listed below:

$$D = 0.66\left[\varepsilon^{1.25}\left(\frac{LQ^2}{gh_L}\right)^{4.75} + vQ^{9.4}\left(\frac{L}{gh_L}\right)^{5.2}\right]^{0.04} \qquad (5.1)$$

Example 3: Type III Problem

Compute the required size of a new Schedule 40 pipe that will carry 0.014 m^3/s of water at 15 °C and limit the pressure drop to 13.79 kPa over a length of 30.5 m of horizontal pipe.

Solution:

Step 1. Write the energy equation

$$z_1 + \frac{v_1^2}{2g} + \frac{P_1}{\gamma} - h_L = z_2 + \frac{v_2^2}{2g} + \frac{P_2}{\gamma}$$

$v_1 = v_2$

$z_1 = z_2$

$$h_L = \frac{(P_1 - P_2)}{\gamma} = \frac{13.79\,\text{kPa}}{9.81\,\text{kN/m}^3} = 1.402\,\text{m}$$

Step 2. State the given information

$Q = 0.014\ \text{m}^3/\text{s}$; $L = 30.5$ m; $g = 9.81\ \text{kN/m}^3$

$h_L = 1.402$ m; $\varepsilon = 4.72 \times 10^{-5}$ m; $v = 1.15 \times 10^{-6}\ \text{m}^2/\text{s}$

Step 3. Using Eq. (5.1)

$$D = 0.66\left[{}^{1.25}\left(\frac{LQ^2}{gh_L}\right)^{4.75} + vQ^{9.4}\left(\frac{L}{gh_L}\right)^{5.2}\right]^{0.04}$$

$$D = 0.66\left[(4.72\times10^{-5})^{1.25}\left(\frac{30.5 * 0.014^2}{9.81 * 1.402}\right)^{4.75} + (1.15\times10^{-6})(0.014)^{9.4}\left(\frac{30.5}{9.81 * 1.402}\right)^{5.2}\right]^{0.04}$$

$D = 0.098$ m

From the Appendix it is determined that $D = 0.098$ m would correspond to a 4-in. Schedule 40 pipe.

5.2 Parallel Pipe System

When two or more pipes in parallel connect two reservoirs, as shown in Figure 5.3, for example, then the fluid may flow down any of the available pipes at possible different rates. But the head difference over each pipe will always be the same. The total volume flow rate will be the sum of the flow in each pipe. The analysis can be carried out by simply treating each pipe individually and summing flow rates at the end.

When the principle of steady flow is applied to the above system, the following conclusion may be reached

$$Q_1 = Q_2 = Q_a + Q_b + Q_c \tag{5.2}$$

To analyze the pressure between the two points, the energy equation is applied

$$z_1 + \frac{v_1^2}{2g} + \frac{P_1}{\gamma} - h_L = z_2 + \frac{v_2^2}{2g} + \frac{P_2}{\gamma}$$

Solving for pressure drop

$$P_1 - P_2 = \left[\left(z_2 - z_1 \right) + \frac{\left(v_2^2 - v_1^2 \right)}{2g} + h_L \right]$$

All elements converging in the junction at the right side of the system have the same total energy per unit weight. That is, they all have the same total head. Therefore, each unit weight of fluid must have the same amount of energy.

This can be stated mathematically as

$$h_{L1-2} = h_a = h_b = h_c \tag{5.3}$$

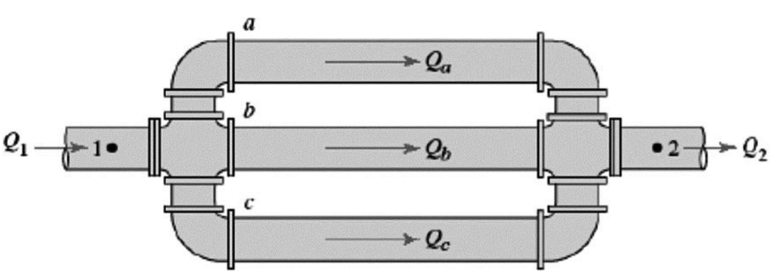


Figure 5.3 System with three branches

5.2.1 System with Two Branches

A common parallel piping system includes two branches arranged as shown in Figure 5.4. The analysis of this type of system is relatively simple and straightforward, although some iteration is typically required. Because velocities are unknown, friction factors are also unknown.

Figure 5.4 is used to illustrate the analysis of flow in two branches. The basic relationships that apply here are similar to Eqs. (5.2) and (5.3) except there are only two branches instead of three.

These relationships are

$$Q_1 = Q_2 = Q_a + Q_b \tag{5.4}$$

$$h_{L1-2} = h_a = h_b \tag{5.5}$$

Below are the solution method for systems with two branches when the total flow rate and the description of the branches are known:

1. Equate the total flow rate to the sum of the flow rates in the two branches, as stated in Eq. (5.4). Then express the branch flows as the product of the flow area and the average velocity; that is,

$$Q_a = A_a v_a \text{ and } Q_b = A_b v_b$$

2. Express the head loss in each branch in terms of the velocity of flow in that branch and the friction factor. Include all significant losses due to friction and minor losses.
3. Compute the relative roughness for each branch, estimate the value of the friction factor for each branch, and complete the calculation of head loss in each branch in terms of the unknown velocities.

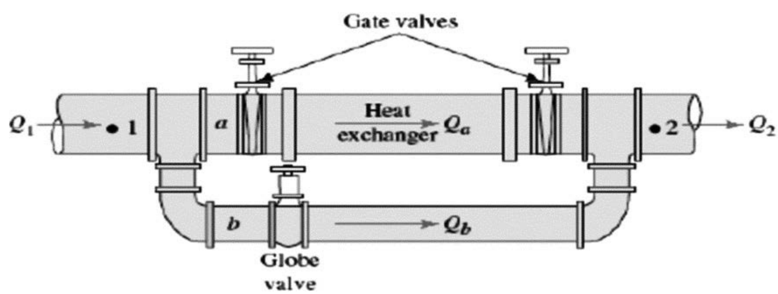


Figure 5.4 System with two branches

4. Equate the expression for the head losses in the two branches to each other as stated in Eq. (5.5).
5. Solve for one velocity in terms of the other from the equation in Step 4.
6. Substitute the result from Step 5 into the flow rate equation developed in Step 1, and solve for one of the unknown velocities.
7. Solve for the second unknown velocity from the relationship developed in Step 5.
8. If there is doubt about the accuracy of the value of the friction factor used in Step 2, compute the Reynolds number for each branch and reevaluate the friction factor from the Moody diagram or compute the values for the friction factor as described in the previous sections.
9. If the values for the friction factor have changed significantly, repeat Steps 3 to 8, using the new values for friction factor.
10. When satisfactory precision has been achieved, use the now-known velocity in each branch to compute the volume flow rate for that branch. Check the sum of the volume flow rates to ensure that it is equal to the total flow in the system.
11. Use the velocity in either branch to compute the head loss across that branch, employing the appropriate relationship from Step 3. This head loss is also equal to the head loss across the entire branched system.

Example Problem 4

In Figure 5.5, 100 gal/min of water at 60 °F is flowing in a 2-in. Schedule 40 steel pipe at section 1. The heat exchanger in branch "a" has a loss coefficient of $K = 7.5$ based on the velocity head in the pipe. All three valves are wide open. Branch "b" is a bypass line composed of 1¼-in. Schedule 40 steel pipe. The elbows are standard. The length of pipe between points 1 and 2 in branch "b" is 6 m. Because of the size of the heat exchanger, the length of pipe in branch "a" is very short and friction losses can be neglected. For this arrangement, determine (a) the volume flow rate of water in each branch and (b) the pressure drop between points 1 and 2.

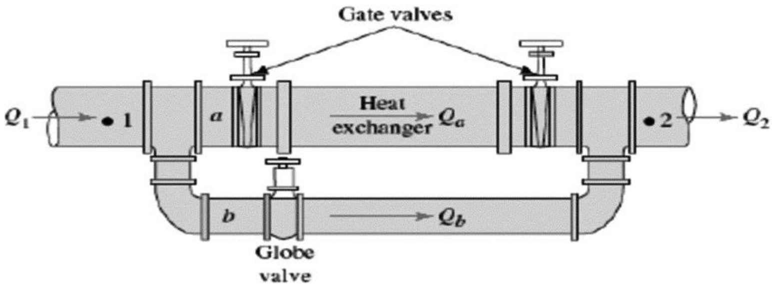


Figure 5.5 Example Problem 4

Step 1. Relate the two flow rates

$$Q_1 = A_a v_a + A_b v_b$$

From the given data, $A_a = 0.02333$ ft^2, $A_b = 0.01039$ ft^2, and $Q_1 = 100$ gal/min

$$Q_1 = 100\,\text{gal / min} \times \frac{1\ \text{ft}^3/\text{s}}{449\ \text{gal/min}} = 0.223\ \text{ft}^3/\text{s}$$

Step 2. Head losses in the two branches are equal. Considering the branch "a"

$$h_a = 2K_1\left(\frac{v_a^2}{2g}\right) + K_2\left(\frac{v_a^2}{2g}\right)$$

where

$K_1 = f_{aT}(L_e/D)$ = resistance coefficient for each gate valve
K_2 = resistance coefficient for the heat exchanger = 7.5 (given)
The following data are known:
f_{aT} = 0.019 for a 2-in. Schedule 40 steel pipe
f_{aT} = 8 for a fully open gate valve

Then

$$K_1 = f_{aT}(L_e/D) = (0.019)(8) = 0.152$$

Then

$$h_a = 2(0.152)\left(\frac{v_a^2}{2g}\right) + 7.5\left(\frac{v_a^2}{2g}\right) = 7.8\left(\frac{v_a^2}{2g}\right)$$

For branch "b"

$$h_b = 2K_3\left(\frac{v_b^2}{2g}\right) + K_4\left(\frac{v_b^2}{2g}\right) + K_5\left(\frac{v_b^2}{2g}\right)$$

where

$K_3 = f_{bT}(L_e/D)$ = resistance coefficient for each elbow
$K_4 = f_{bT}(L_e/D)$ = resistance coefficient for the globe valve
$K_5 = f_b(L/D)$ = friction loss in the pipe for branch "b"
f_{bT} = 0.022 for 1¼-in. Schedule 40 pipe
L_e/D = 30 for each elbow
L_e/D = 340 for fully open globe valve
Then
$K_3 = 0.66$
$K_4 = 7.48$
$K_5 = 173.5\,f_b$

$$h_b = 2(0.66)\left(\frac{v_b^2}{2g}\right) + (7.48)\left(\frac{v_b^2}{2g}\right) + (173.5f_b)\left(\frac{v_b^2}{2g}\right)$$

$$= (8.80 + 173.5f_b)\left(\frac{v_b^2}{2g}\right)$$

$$\frac{D}{\varepsilon} = \left(\frac{0.1150}{1.5\times10^{-4}}\right) = 767$$

From the Moody diagram and using the iterative method described in previous sections, $f_b = 0.023$

$$h_b = (12.80)\left(\frac{v_b^2}{2g}\right)$$

Step 3. Determine the velocity
For the system described in the problem

$$h_a = h_b = 7.80\left(\frac{v_a^2}{2g}\right) = (12.80)\left(\frac{v_b^2}{2g}\right)$$

$$v_a = 1.281v_b$$

$$Q_1 = A_a v_a + A_b v_b = A_a(1.281v_b) + A_b v_b = v_b(1.281A_a + A_b)$$

Solving for v_b

$$v_b = \frac{Q_1}{(1.281A_a + A_b)} = \frac{0.223 \text{ ft}^3/\text{s}}{(1.281(0.02333) + 0.01039)\text{ft}^2}$$

$$v_b = 5.54 \text{ ft/s}$$

$$v_a = 1.281 * 5.54 \frac{\text{ft}}{\text{s}} = 7.09 \frac{\text{ft}}{\text{s}}$$

The calculations were made with an assumed value of f_b; hence, the accuracy of the assumption needs to be checked.

$$N_{Rb} = v_b D_b / \vartheta$$

Kinematic viscosity for water at 15 °C = $\vartheta = 1.21 \times 10^{-5}$ ft^2 / s

$$N_{Rb} = \frac{(5.54)(0.1150)}{1.21 \times 10^{-5}} = 5.26 \times 10^4$$

Relative roughness was determined as 767; from the Moody diagram the new f_b = 0.025. This value is significantly different from the initial assumed value. Hence

$$h_b = [8.80 + 173.9(0.025)].\left(\frac{v_b^2}{2g}\right) = 13.15 \frac{v_b^2}{2g}$$

Equating the head losses in the two branches
$h_a = h_b$

$$7.80\left(\frac{v_a^2}{2g}\right) = 13.15 \left(\frac{v_b^2}{2g}\right)$$

Solving for velocities gives

$$v_a = 1.298 v_b$$

Substituting the equation for v_b used earlier gives

$$v_b = \frac{0.223 \text{ ft}^3/\text{s}}{(1.298(0.02333) + 0.01039)\text{ft}^2} = 5.48 \text{ ft/s}$$

$$v_a = 1.298 v_b = 1.298(5.48) = 7.12 \text{ ft/s}$$

Recomputing the Reynolds number for branch "b"

$$N_{Rb} = \frac{(5.48)(0.1150)}{1.21\times10^{-5}} = 5.21\times10^{4}$$

Using the Moody diagram, f_b = 0.023 (no change).

Step 4. Determine the flow rates

$$Q_a = v_a A_a = 0.02333 * 7.12 = 0.166\frac{\text{ft}^3}{\text{s}} = 74.5 \text{ gal/min}$$

$$Q_b = v_b A_b = 0.01039 * 5.48 = 0.057\frac{\text{ft}^3}{\text{s}} = 25.5 \text{ gal/min}$$

Part b: Determine the pressure drop

Apply the energy equation between points 1 and 2

$$z_1 + \frac{v_1^2}{2g} + \frac{P_1}{\gamma} - h_L = z_2 + \frac{v_2^2}{2g} + \frac{P_2}{\gamma}$$

$$\frac{P_1}{\gamma} - h_L = \frac{P_2}{\gamma}$$

$$\frac{P_1}{\gamma} - \frac{P_2}{\gamma} = h_L$$

$$h_{L_{1-2}} = h_a = h_b$$

$$h_a = 7.80\left(\frac{v_a^2}{2g}\right) = 7.80\left(\frac{7.12^2}{64.4}\right) = 6.14 \text{ ft}$$

$$P_1 - P_2 = 62.4\frac{\text{lb}}{\text{ft}^3}\times 6.14\,\text{ft}\times\frac{1\text{ ft}^2}{144\text{ in.}^2} = 2.66 \text{ psi}$$

5.2.2 *System with Three or More Branches*

Hardy Cross Method

The Hardy Cross method is an iterative method for determining the flow in pipe network systems where the inputs and outputs are known, but the flow inside the network is unknown. The introduction of the Hardy Cross method for analyzing pipe flow networks revolutionized municipal water supply design. Before the method was introduced, solving complex pipe systems for distribution was extremely difficult due to the nonlinear relationship between head loss and flow (Figure 5.6).

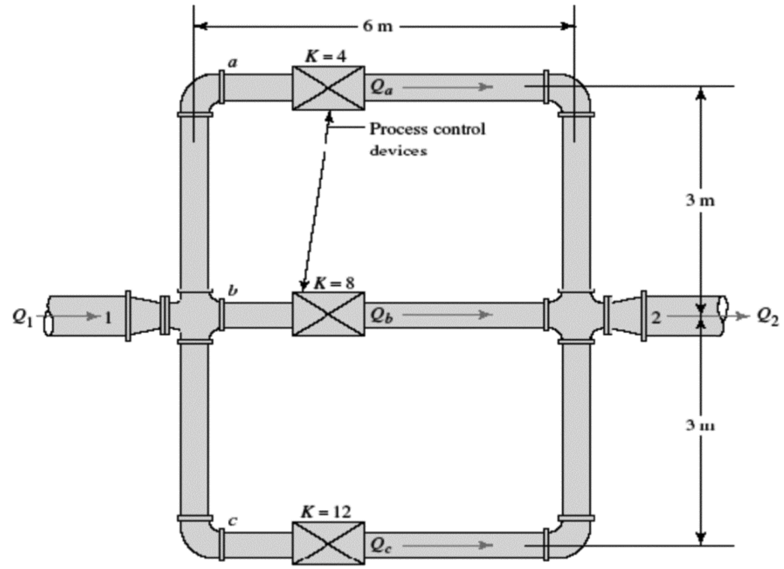


Figure 5.6 System with three branches

The Cross technique requires that the head loss terms for each pipe in the system be expressed in the form

$$h = KQ^n \tag{5.6}$$

From previous chapters it was determined that both friction losses and minor losses are proportional to the velocity head. Then, using the continuity equation, the velocity can be expressed in terms of the volume flow rate. That is

$$v = Q / A$$

$$v^2 = Q^2 / A^2$$

The Cross iteration technique requires that initial estimates for the volume flow rate in each branch of the system be made (Figure 5.7).

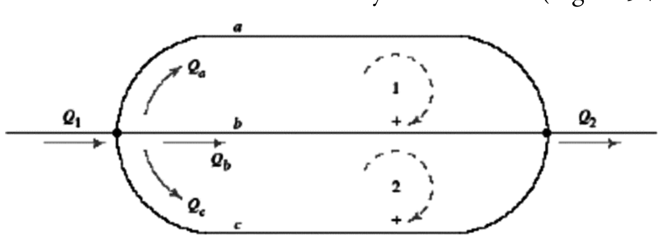


Figure 5.7 Schematic representation of a three-pipe system

Two factors that help in making these estimates are as follows:

- At each junction in the network, the sum of the flow into the junction must equal the flow out.
- The fluid tends to follow the path of least resistance through the network. Therefore, a pipe having a lower value of K will carry a higher flow rate than those having higher values.
- If the flow in a given pipe of a circuit is clockwise, Q and h are positive. If the flow is counterclockwise, Q and h are negative.

The Cross technique for analyzing the flow in pipe networks is presented in a step-by-step form as follows:

1. Express the energy loss in each pipe in the form

$$h = KQ^2$$

2. Assume a value for the flow rate in each pipe such that the flow into each junction equals the flow out of the junction.
3. Divide the network into a series of closed-loop circuits.
4. For each pipe, calculate the head loss using the assumed value of Q.
5. Proceeding around each circuit, algebraically sum all values for h using the following sign convention:
   i. If the flow is clockwise, h and Q are positive.
   ii. If the flow is counterclockwise, h and Q are negative.
   iii. The resulting summation is referred to as Σh.
6. For each pipe, calculate $2KQ$.
7. Sum all values of $2KQ$ for each circuit, assuming all are positive. This summation is referred to as $\Sigma(2KQ)$.

$$\Delta K = \frac{\sum h}{\sum (2kQ)}$$

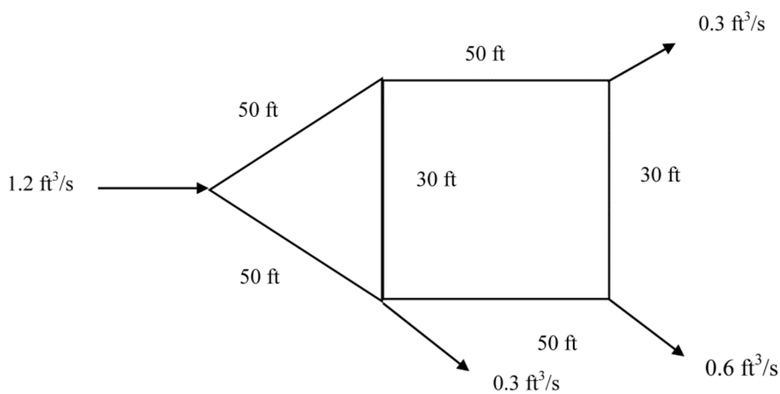
8. For each pipe, calculate a new estimate for Q from

$$Q' = Q - \Delta Q$$

9. Repeat Steps 4 to 8 until ΔQ from Step 8 becomes negligibly small. The Q' value is used for the next cycle of iteration.

Example Problem 5

Find the flow rate of water at 60 °F in each; all pipes are 2½-in. Schedule 40 steel pipes, as shown in the figure below.



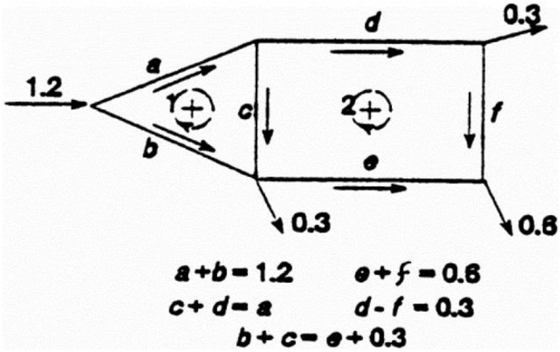
For a 2½-in. Schedule 40 commercial steel pipe: D = 0.2058; A = 0.03326 ft^2

$$h = KQ^2 = f\frac{L}{D}\frac{v^2}{2g} = f\frac{LQ^2}{D2gA^2}$$

$$K = \frac{fL}{D2gA^2}$$

Based on the length of the pipes

$$K_a = K_b = K_d = K_e = \frac{f(50)}{(0.2058)(64.4)(0.03326)^2} = 3{,}410f$$



As described in the previous section, the clockwise flow is shown as positive (+) sign.

$$K_c = K_f = \frac{f(30)}{(0.2058)(64.4)(0.03326)^2} = 2{,}046f$$

Values of f can be computed as

$$\frac{D}{\varepsilon} = 1{,}372$$

$$N_R = \frac{vD}{\vartheta} = \frac{QD}{A\vartheta}$$

For water at 60 °F: $\vartheta = 1.21\times10^{-5}\ \text{ft}^2/\text{s}$

$$N_R = \frac{Q(0.2058)}{(0.03326)1.21\times10^{-5}} = \left(5.114\times10^5\right)Q$$

From Chapter 3, use the equation

$$f = \frac{0.25}{\left\{\log\left(\dfrac{1}{3.7\left(\dfrac{D}{\varepsilon}\right)}\right) + \dfrac{5.74}{N_R^{\,0.9}}\right\}^2}$$

$$f = \frac{0.25}{\left\{\log\left(\dfrac{1}{1.970\times10^{-4}}\right) + \dfrac{5.74}{N_R^{\,0.9}}\right\}^2}$$

For Trial 1: Flow equation at nodes

$Q_a + Q_b = 1.2$
Try $Q_a = 0.5$ and $Q_b = 0.7$

$Q_c + Q_d = 0.5$
Try $Q_c = 0.1$ and $Q_d = 0.4$

$Q_d - Q_f = 0.3$
Try $Q_d = 0.4$ and $Q_f = 0.1$

$Q_e + Q_f = 0.6$
Try $Q_f = 0.1$ and $Q_e = 0.5$

$Q_b + Q_c = Q_e + 0.3$
$0.7 + 0.1 = 0.5 + 0.3$ (check)

Compute the values of f using the equation above

Pipe	f	K	Equation of K
a	0.0197	67.18	$3{,}410\ f_a$
b	0.0194	66.00	$3{,}410\ f_b$
c	0.0233	47.65	$2{,}046\ f_c$
d	0.0200	68.25	$3{,}410\ f_d$
e	0.0197	67.18	$3{,}410\ f_e$
f	0.0233	47.65	$2{,}046\ f_f$

Trial	Circuit	Pipe	Q	N_R	$[N_R]$	*f*	*K*	$H = +/-KQ^2$	[2*KQ*]	Delta Q
1	1	a	0.5	255,700	2.56E+05	0.019666	67.05953	16.76488	67.05953	−8.89E−02
		b	−0.7	−357,980	3.58E+05	0.019298	65.80544	−32.2447	92.12761	
		c	0.1	51,140	5.11E+04	0.023251	47.5708	0.475708	9.514161	
								−15.0041	**168.7013**	
		c	−0.1	−51,140	5.11E+04	0.023251	47.5708	−0.47571	9.514161	−4.18E−02
	2	d	0.4	204,560	2.05E+05	0.019965	68.0809	10.89294	54.46472	
		e	−0.5	−255,700	2.56E+05	0.019666	67.05953	−16.7649	67.05953	
		f	0.1	51,140	5.11E+04	0.023251	47.5708	0.475708	9.514161	
								−5.87194	**140.5526**	
2	1	a	0.588939	301,183.3	3.01E+05	0.019475	66.40968	23.03412	78.22246	−4.66E−03
		b	−0.61106	−312,497	3.12E+05	0.019435	66.27397	−24.7464	80.99492	
		c	1.42E−01	72,505.02	7.25E+04	0.022143	45.3048	0.910666	12.8464	
								−0.80165	**172.0638**	
		c	−1.42E−01	−72,505	7.25E+04	0.022143	45.3048	−0.91067	12.8464	−6.56E−03
	2	d	0.441778	225,925	2.26E+05	0.019826	67.60576	13.19444	59.73341	
		e	−0.45822	−234,335	2.34E+05	0.019777	67.43934	−14.1601	61.80445	
		f	0.141778	72,505.02	7.25E+04	0.022143	45.3048	0.910666	12.8464	
								−0.96565	**147.2307**	
3	1	a	5.94E−01	303,565.9	3.04E+05	0.019466	66.38036	23.38967	78.80646	2.02E−07
		b	−6.06E−01	−310,114	3.10E+05	0.019443	66.30183	−24.3808	80.41116	
		c	1.48E−01	75,859.19	7.59E+04	0.022015	45.04355	0.991123	13.36319	
								3.49E−05	**172.5808**	

Trial	Circuit	Pipe	Q	N_R	$[N_R]$	*f*	*K*	*H* = +/−*KQ*^{2}	[2*KQ*]	Delta Q
		c	−1.48E−01	−75,859.2	7.59E+04	0.022015	45.04355	−0.99112	13.36319	−1.32E−03
		d	4.48E−01	229,279.2	2.29E+05	0.019806	67.53813	13.57553	60.55959	
	2	e	−4.52E−01	−230,981	2.31E+05	0.019796	67.50446	−13.7709	60.97863	
		f	1.48E−01	75,859.19	7.59E+04	0.022015	45.04355	0.991123	13.36319	
								−0.19539	**148.2646**	

As the ΔQ is negligibly small after three trials, the following Q can be accepted.

Summary of results from table:

$Q_a = 0.594$
$Q_b = 0.606$
$Q_c = 0.148$
$Q_d = 0.448$
$Q_f = 0.148$
$Q_e = 0.452$

CHAPTER 6

Pumps and Turbines

Hydraulic pumps are used in hydraulic drive systems and can be hydrostatic or hydrodynamic. A hydraulic pump is a mechanical source of power that converts mechanical power into hydraulic energy (hydrostatic energy, i.e., flow, pressure). It generates flow with enough power to overcome pressure induced by the load at the pump outlet. When a hydraulic pump operates, it creates a vacuum at the pump inlet, which draws liquid from the reservoir into the inlet line to the pump and by mechanical action delivers this liquid to the pump outlet and forces it into the hydraulic system. In the previous chapter, the general energy equation was introduced to determine the energy added by a pump to the fluid as follows:

$$h_A = z_B - z_A + \frac{v_B^2 - v_A^2}{2g} + \frac{P_B - P_A}{\gamma} + h_L$$

h_A is the total head on the pump.

Some pump manufacturers refer to this as the total dynamic head (TDH).

TDH is the total equivalent height that a fluid is to be pumped, taking into account friction losses in the pipe.

As discussed in previous chapters, the power delivered to the fluid by the pump can be calculated by the equation

$$P_A = h_A \gamma Q$$

The efficiency of the pump was determined by the equation

$$e_M = \frac{P_A}{P_I}$$

6.1 Types of Pumps

Pumps are categorized into two types on the response to changes in discharge pressure as shown in Figure 6.1. The basic difference between the two types is their response to changes in discharge pressure.

6.1.1 *Dynamic Pumps*

Dynamic pumps are pumps in which the energy is added to the water continuously and the water is not contained in a set volume. They are used in conditions where high volumes are required and a change in flow is not a problem. As the discharge pressure on a dynamic pump is increased, the quantity of water pumped is reduced. One type of dynamic pump is the centrifugal pump. These are the most common pump used in water systems. Dynamic pumps can be operated for short periods of time with the discharge valve closed.

6.1.2 *Displacement Pumps*

Displacement pumps are pumps in which the energy is added to the water periodically and the water is contained in a set volume. Displacement pumps are used in conditions where relatively small, but precise, volumes are required. Displacement pumps will not change their volume with a change in discharge pressure. Displacement pumps are also called positive displacement pumps. The most common positive displacement pump is the diaphragm pump used to pump chlorine and fluoride solutions. Operating a displacement pump with the discharge valve closed will damage the pump.

6.2 Dynamic Pumps/Centrifugal Pumps

Centrifugal pumps are used to transport fluids by the conversion of rotational kinetic energy to the hydrodynamic energy of the fluid flow. The rotational energy typically comes from an engine or electric motor. The fluid enters the pump impeller along or near to the rotating axis and is accelerated by the impeller, flowing radially outward into a diffuser or volute chamber (casing), from where it exits. Volute is a spiral-shaped casing surrounding a pump impeller that collects the liquid discharged by the impeller. For example, a snail shell is volute shaped as shown in Figure 6.2. The shape of the case helps to determine the direction of rotation of the pump.

The direction of rotation can be determined when looking into the suction side of the volute case. For example, in Figure 6.3, the direction of rotation is counterclockwise.

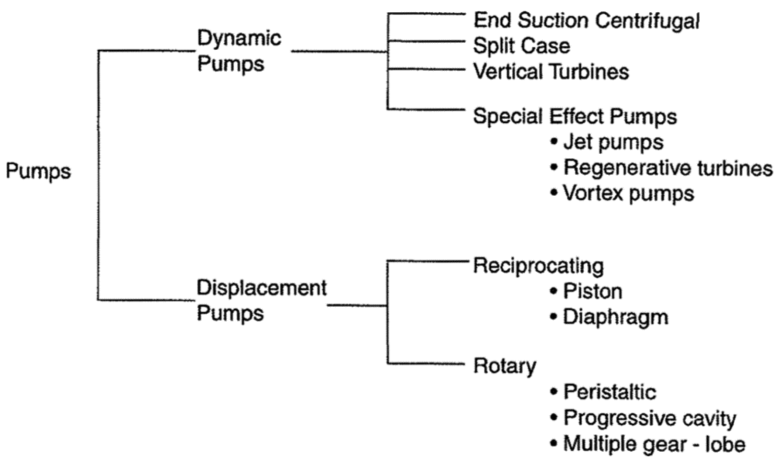


Figure 6.1 Different types of pumps

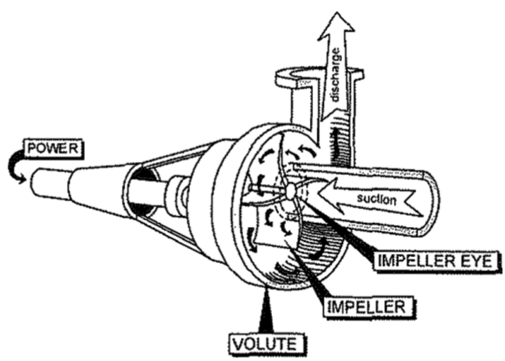


Figure 6.2 Pump case or volute

Source: Grundfos Pumps, Downers Grove, IL.

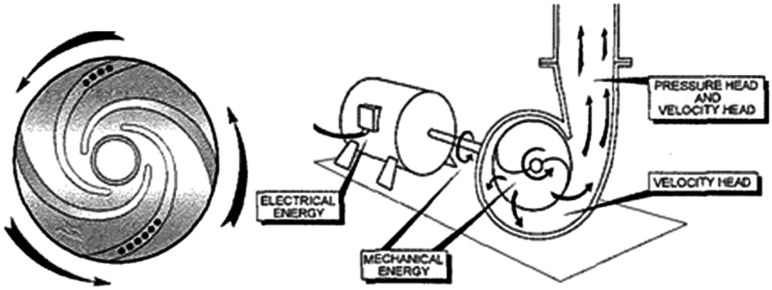


Figure 6.3 Direction of flow
Source: Grundfos Pumps, Downers Grove, IL.

In summary, there are two theories that explain how a centrifugal pump works:

1. Energy transfer—the transfer of energy from the shaft to the impeller and from the impeller to the water
2. Centrifugal force—the force used to throw the water from the impeller

6.2.1 *Types of Centrifugal Pumps*

Centrifugal pumps can be divided into one of three classifications based on the configurations.

a. End-suction centrifugal pumps—The most common style of centrifugal pump. The center of the suction line is centered on the impeller eye. The impeller is attached directly onto the end of the motor shaft. End-suction centrifugal pumps are further classified as either frame-mounted or close-coupled (Figure 6.4).

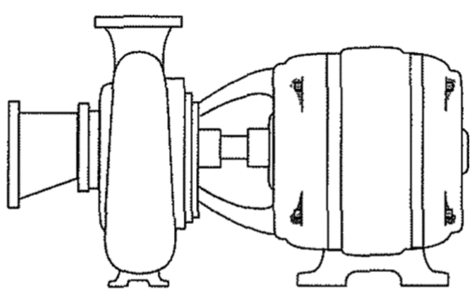


Figure 6.4 End-suction centrifugal pump
Source: Grundfos Pumps, Downers Grove, IL.

i Frame-mounted pumps—End-suction centrifugal pumps are designed so that the pump bearings and pump shaft are independent of the motor. This type of pump requires a coupling between the pump and the motor to transfer energy from the motor to the pump (Figure 6.5).

ii Close-coupled pumps—A close-coupled pump has only one shaft and one set of bearings: the motor shaft and bearings. The pump impeller is placed directly onto the motor shaft. Close-coupled pumps require less space and are less expensive than frame-mounted pumps (Figure 6.6).

b. Split case pumps—A centrifugal pump is designed so that the volute case is split horizontally. The case divides on a plane that cuts through the eye of the impeller. Split case pumps are unique. The case has a row of bolts that allow half of the case to be removed, providing access to the entire rotating assembly for inspection or removal. These pumps are normally found as fire service pumps and circulation pumps in medium-to-large communities (Figure 6.7).

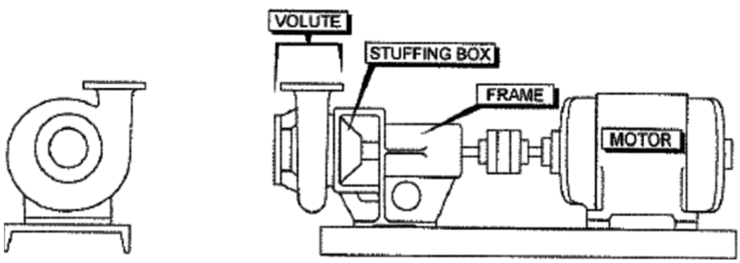


Figure 6.5 Frame-mounted pump
Source: Grundfos Pumps, Downers Grove, IL.

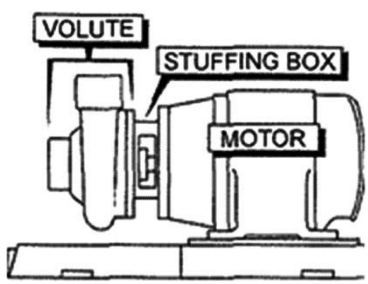


Figure 6.6 Close-coupled pump
Source: Grundfos Pumps, Downers Grove, IL.

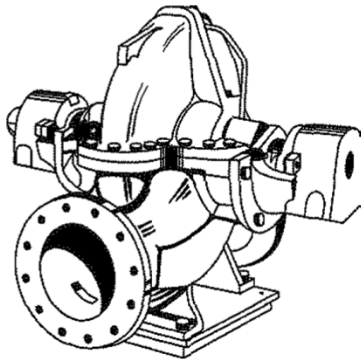


Figure 6.7 Split case suction centrifugal pumps

Source: Grundfos Pumps, Downers Grove, IL.

c) Vertical turbine pumps—A classification of centrifugal pumps that are primarily mounted with a vertical shaft; the motor is commonly mounted above the pump. Vertical turbine pumps are either mixed or axial flow devices (Figure 6.8). There are four styles of vertical turbines: line shaft, axial flow, can turbine, and submersible turbine. The primary difference between the vertical turbine and the submersible turbine is the position of the motor. The pumping assembly is the same. Submersible turbine pumps can range from 5 gpm to 100 gpm or more.

  i) Line shaft turbine pumps—A type of vertical turbine. In this type of vertical turbine, the motor is mounted above the ground, and the pump units mounted below the water surface. A column extends from the pump to a discharge head found just below the motor. A shaft extends on a straight line from the center of the motor to the pump. The pump may be mounted a few feet to several hundred feet away from the motor.

  ii) Axial flow pumps—A type of vertical turbine that uses a propeller instead of an impeller. In axial flow pumps, the energy is transferred into the water so that the direction of the flow is directly up the shaft.

  iii) Can turbine pumps—A type of line shaft turbine. The pump assembly is mounted inside of a sealed can. The inlet is mounted opposite the outlet on the discharge head. The can must always be under pressure.

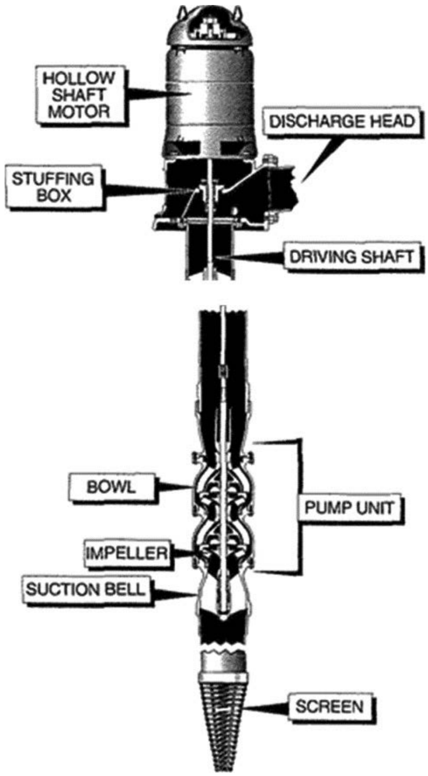


Figure 6.8 Vertical turbine pumps
Source: Grundfos Pumps, Downers Grove, IL.

iv) Submersible turbine pumps—A style of vertical turbine pump in which the entire pump assembly and motor are submersed in the water. The motor is commonly mounted below the pump.

6.3 Positive Displacement Pumps

Although there are several different types of positive displacement pumps available, this section is limited to those commonly used in water systems.

6.3.1 Diaphragm Pumps

A diaphragm pump is operated by either electric or mechanical means (Figure 6.9). Two-valve assemblies: a suction-valve assembly and a discharge-valve assembly. When the diaphragm is pulled back, a vacuum is created in the chamber in front of the diaphragm. This vacuum causes the discharge valve to be forced closed against its seat. The vacuum allows atmospheric pressure to push fluid up against the outside of the suction valve, opening the valve and filling the chamber. When pressure is returned to the diaphragm, forcing it toward the front of the chamber, the increased pressure causes the suction valve to be forced closed and the discharge valve to be forced open. The fluid is pushed out of the chamber, and the pumping cycle starts over.

6.3.2 *Peristaltic Pumps*

A peristaltic pump is a type of positive displacement pump used for pumping a variety of fluids, such as chemicals and sludges. The fluid is contained within a flexible tube fit inside a circular pump casing. A rotor with a number of rollers (or shoes) is attached to a rotating arm that compresses the flexible tube. As the rotor turns, the part of the tube under compression closes, thus forcing the fluid to be pumped through the tube. Since they have no moving parts in contact with the fluid, peristaltic pumps are inexpensive to manufacture. Their lack of valves, seals, and glands makes them comparatively inexpensive to maintain, and the

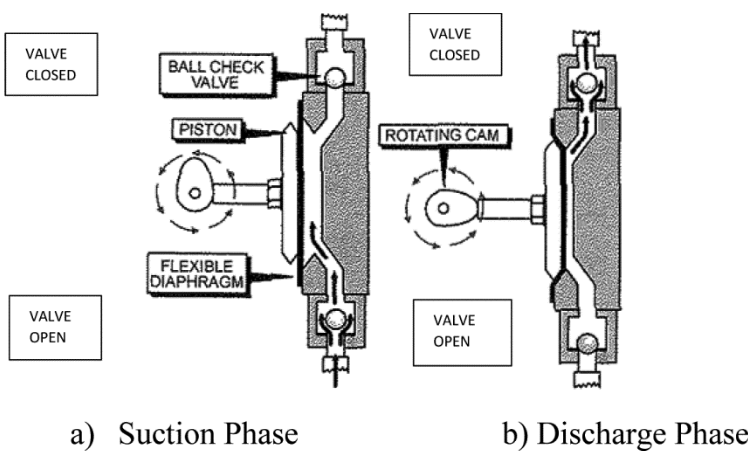


Figure 6.9 Diaphragm pumps

Source: Grundfos Pumps, Downers Grove, IL.

use of a hose or tube makes for a relatively low-cost maintenance item compared with other pump types. It is important to select tubing with appropriate chemical resistance toward the liquid being pumped. Types of tubing commonly used in peristaltic pumps include polyvinyl chloride (PVC), silicone rubber, and fluoropolymer. These are generally used for small, relatively precise flow, such as in a laboratory setting, as opposed to a larger volume in a commercial or industrial setting.

6.3.3 *Progressive Cavity Pumps*

A progressive cavity pump moves fluid by means of a rotary screw or a rotor turning within a stationary stator. The flow rate is proportional to the rotation rate of the pump. Progressive cavity pumps are designed to transfer fluid or fluids with suspended solids. They are frequently used to pump sludge, but can be used to meter large volumes of chemicals in a precise manner.

6.4 Cavitation

Cavitation is the condition where vapor bubbles are formed in a flowing liquid when the pressure of the liquid falls below its vapor pressure. Once the bubbles reach an area where the pressure increases above vapor pressure, the bubbles collapse thereby creating small areas of high temperature and emitting shock waves.

Cavitation in a centrifugal pump occurs when the inlet pressure falls below the design inlet pressure or when the pump is operating at a flow rate higher than the design flow rate. When the inlet pressure in the flowing liquid falls below its vapor pressure, bubbles begin to form in the eye of the impeller. Once the bubbles move to an area where the pressure of the liquid increases above its vapor pressure, the bubbles collapse thereby emitting a "shock wave." These shock waves can pit the surface of the impeller and shorten its service life. The collapse of the bubbles also emits a pinging or crackling noise that can alert the operator that cavitation is occurring. Cavitation is undesirable because it can damage the impeller, cause noise and vibration, and decrease pump efficiency.

6.5 Performance Data for Centrifugal Pumps

Centrifugal pumps have a strong dependency on the variability between the capacity and the pressure that must be developed by the pump. This makes their performance ratings somewhat complex. The pump performance is normally described by a set of curves. This section explains how these curves are interpreted and the basis for the curves.

6.5.1 *Standard Curves*

Performance curves are used by the customer to select a pump matching the requirements for a given application. The data sheet contains information about the head (*H*) at different flows (*Q*) (see Figure 6.10). The requirements for head and flow determine the overall dimensions of the pump.

In addition to head, the power consumption (*P*) is also to be found in the data sheet. The power consumption is used for dimensioning of the installations which must supply the pump with energy. The power consumption is like the head shown as a function of the flow. Information about the pump efficiency (η) and net positive suction head (NPSH) can also be found in the data sheet. The NPSH curve shows the minimum inlet required for cavitation. The efficiency curve is used for choosing the most efficient pump in the specified operating range. Figure 6.10 shows an example of performance curves in a data sheet.

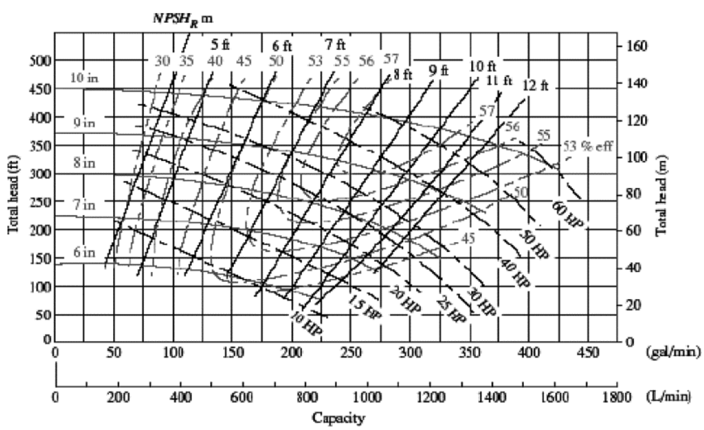


Figure 6.10 Typical performance curves for a centrifugal pump. Head (H), capacity (Q), efficiency (η), and NPSH are shown as function of the flow

Source: Goulds Pumps, Inc., Seneca Falls, NY.

6.5.2 *Net Positive Suction Head*

NPSH is a term describing conditions related to cavitation, which is undesired and harmful. Cavitation is the creation of vapor bubbles in areas where the pressure locally drops to the fluid vapor pressure. The extent of cavitation depends on how low the pressure is in the pump. Cavitation generally lowers the head and causes noise and vibration. Cavitation first occurs at the point in the pump where the pressure is lowest, which is most often at the blade edge at the impeller inlet, see Figure 6.11.

The NPSH value is absolute and always positive. Hence, it is not necessary to take the density of different fluids into account because NPSH can be stated in meters (m) or in feet (ft). Distinction is made between two different NPSH values: $NPSH_R$ and $NPSH_A$.

$NPSH_A$ stands for NPSH available and is an expression of how close the fluid in the suction pipe is to vaporization. $NPSH_A$ is dependent on the vapor pressure of the fluid being pumped, energy losses in the suction piping, the location of the fluid reservoir, and the pressure applied to the fluid in the reservoir. This can be expressed as

$$NPSH_A = h_{sp} \pm h_s - h_f - h_{vp} \quad (6.1)$$

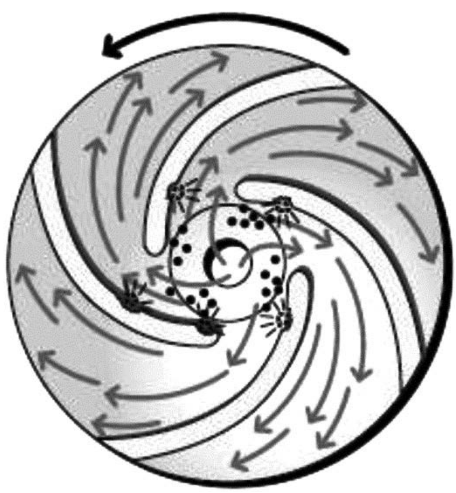


Figure 6.11 Cavitation

Source: Grundfos Pumps, Downers Grove, IL.

h_{sp} = Static pressure head above the fluid in the reservoir, expressed in meters or feet of the liquid; $h_{sp} = p_{sp} / \gamma$

p_{sp} = static pressure above the fluid in the reservoir

h_s = elevation difference from the level in the reservoir to the centerline of the pump suction inlet, expressed in meters or feet

If the pump is below the reservoir, h_s is considered positive and if the pump is above the reservoir, h_s is considered negative.

h_f = head loss in the suction piping due to friction and minor losses, expressed in meters or feet

h_{vp} = vapor pressure head of the liquid at the pumping temperature, expressed in meters or feet; $h_{vp} = p_{vp} / \gamma$

p_{vp} = vapor pressure of the liquid at the pumping temperature

$NPSH_R$ is the minimum pressure required at the suction port of the pump to keep the pump from cavitation. $NPSH_R$ is a function of the pump and must be provided by the pump manufacturer. $NPSH_A$ must be greater than $NPSH_R$ for the pump system to operate without cavitation and is generally given by the equation

$$NPSH_A > 1.10\ NPSH_R \tag{6.2}$$

The data given in pump catalogs for NPSH are for water and apply only to the listed operating speed. If the pump is operated at a different speed, the NPSH required at the new speed can be calculated from

$$(NPSH_R)_2 = \left(\frac{N_2}{N_1}\right)^2 (NPSH_R)_1 \tag{6.3}$$

where the subscript 1 refers to catalog data and the subscript 2 refers to conditions at the new operating speed. The pump speed in rpm is N.

Example Problem 1

Determine the available NPSH for the system shown in Figure 6.12(a). The fluid reservoir is a closed tank with a pressure of –20 kPa above water at 70 °C. The atmospheric pressure is 100.5 kPa. The water level in the tank is 2.5 m above the pump inlet. The pipe is a 1.5-in. Schedule 40 steel pipe with a total length of 12.0 m. The elbow is standard and the valve is a fully open globe valve. The flow rate is 95 L/min.

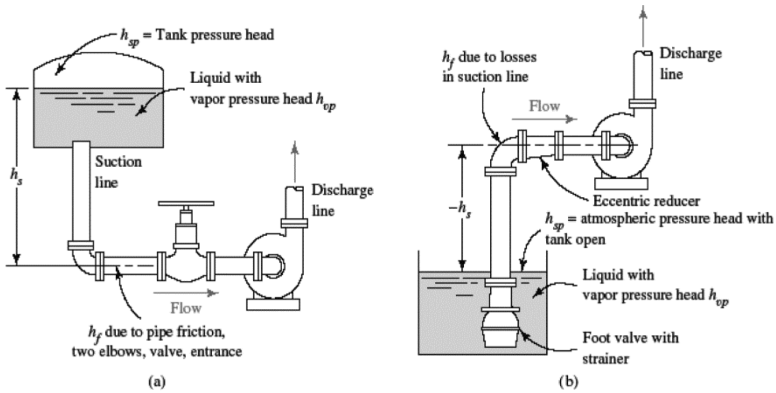


Figure 6.12 The pump suction-line details and definitions of terms for computing NPSH

Solution:

Absolute pressure = Atmospheric pressure + Tank gage pressure

$$p_{abs} = 100.5 \text{ kPa} - 20 \text{ kPa} = 80.5 \text{ kPa}$$

However, it is given that

$$h_{sp} = {p_{abs}}/{\gamma} = \frac{80.5\times10^3 \text{ N/m}^2}{9.59\times10^3 \text{ N/m}^3} = 8.39 \text{ m}$$

Based on the elevation of the tank

$$h_s = +2.5 \text{ m}$$

To find the friction loss, we must find the velocity, Reynolds number, and friction factor:

$$v = \frac{Q}{A} = \frac{95 \text{ L/min}}{1.314\times10^{-3} \text{ m}^2} \times \frac{1.0 \text{ m}^3}{60{,}000 \text{ L/min}} = 1.21 \text{ m/s}$$

$$N_R = \frac{vD}{\vartheta} = \frac{1.21\times0.0409}{4.11\times10^{-7}} = 1.20\times10^5 \text{ (turbulent)}$$

$$\frac{D}{\varepsilon} = \frac{0.0409}{4.6\times10^{-5}} = 889$$

From the Moody diagram, f = 0.0225; from previous chapters, f_T = 0.021. From Figure 6.12(a).

$$h_f = f\left(\frac{L}{D}\right)\left(\frac{v^2}{2g}\right) + 2f_T(30)\left(\frac{v^2}{2g}\right) + f_T(340)\left(\frac{v^2}{2g}\right) + 1.0\frac{v^2}{2g}$$

Pipe Elbow Valve Entrance

The velocity head is

$$\frac{v^2}{2g} = \left(\frac{1.21^2}{2\times 9.81}\right) = 0.0746 \text{ m}$$

Then the friction loss is

$$h_f = 0.0225\left(\frac{12}{0.0409}\right)(0.0746) + 2(0.021)(30)(0.0746) + 0.021(340)(0.0746) + 0.0746$$

$$h_f = 1.19 \text{ m}$$

From Table 6.1: h_{vp} = 3.25m at 70 °C

$NPSH_A = h_{sp} \pm h_s - h_f - h_{vp}$ = 8.9 m + 2.5 m – 1.19m – 3.25 m = 6.45 m

It is known that $NPSH_A > 1.10\ NPSH_R$

$$NPSH_R < NPSH_A/1.10$$

$$NPSH_R < 6.45/1.10 = 5.86 \text{ m}$$

Table 6.1 Vapor pressure and vapor pressure head

Temperature (°C)	Temperature (°F)	Vapor Pressure (kPa)	Vapor Pressure (psi)	Vapor Pressure Head (m)	Vapor Pressure Head (ft)
0	32	0.6113	0.08854	0.06226	0.2043
5	40	0.8726	0.1217	0.08894	0.2807
10	50	1.2281	0.1781	0.1253	0.4109
15	60	1.7056	0.2563	0.1795	0.5917
20	70	2.3388	0.3631	0.2338	0.8393
30	80	4.2455	0.5069	0.4345	1.173
40	90	7.3814	0.6979	0.758	1.618
50	100	12.344	0.9493	1.272	2.205
60	120	19.932	1.692	2.066	3.948
70	140	31.176	2.888	3.25	6.775
80	160	47.373	4.736	4.967	11.18
90	180	70.117	7.507	7.405	17.55
95	202	84.529	11.52	9.9025	27.59
100	212	101.32	14.69	10.78	35.36

6.5.3 *Efficiency Curves*

Pumps efficiency varies throughout its operating range. This information is essential for calculating the motor power. The B.E.P. (best efficiency point) is the point of highest efficiency of the pump. All points to the right or left of the B.E.P have a lower efficiency (Figure 6.13). The impeller is subject to axial and radial forces, which get greater the further away the operating point is from the B.E.P. These forces manifest themselves as vibration depending on the speed and construction of the pump. The point where the forces and vibration levels are minimal is at the B.E.P.

In selecting a pump, one of the concerns is to optimize pumping efficiency. It is good practice to examine several performance charts at different speeds to see if one model satisfies the requirements more efficiently than another. Whenever possible the lowest pump speed should be selected, as this will save wear and tear on the rotating parts. Note: The pump performance curves are based on data generated in a test rig using water as the fluid. These curves are sometimes referred to as water performance curves (Figure 6.14).

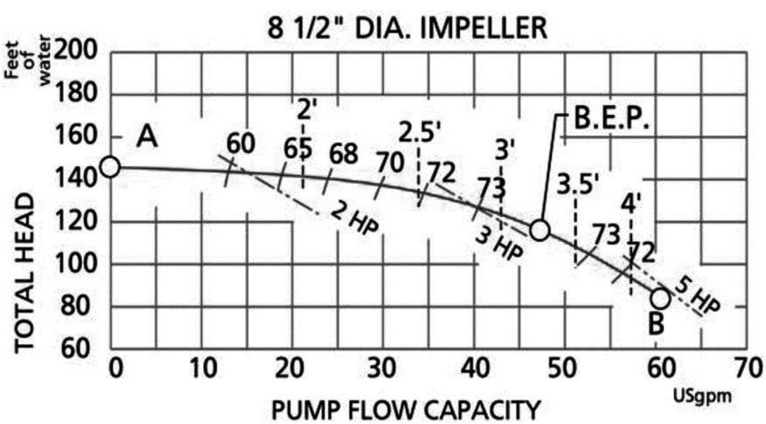


Figure 6.13 Typical performance curve for specific impeller size

Source: Goulds Pumps, Inc., Seneca Falls, NY.

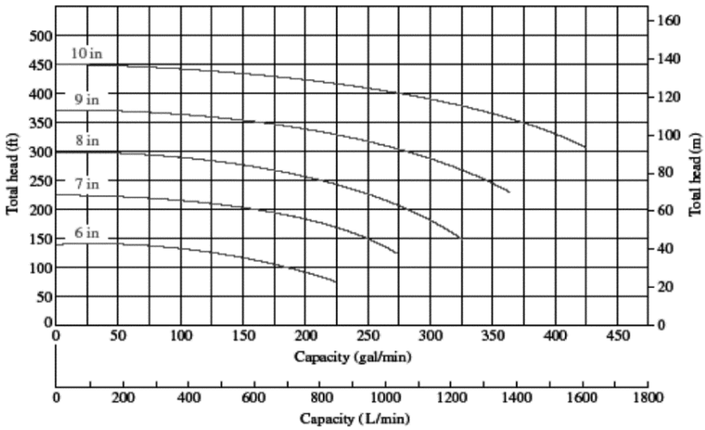


Figure 6.14 Pump performance varies as the size of the impeller varies

Source: Goulds Pumps, Inc., Seneca Falls, NY.

6.5.4 Horsepower Curves

The horsepower curves are shown on the pump performance chart and give the power required to operate the pump within a certain range. For example (Figure 6.15), all points on the 20 hp performance curve will be attainable with a 20 hp motor. The horsepower can be calculated with the total head, flow, and efficiency at the operating point.

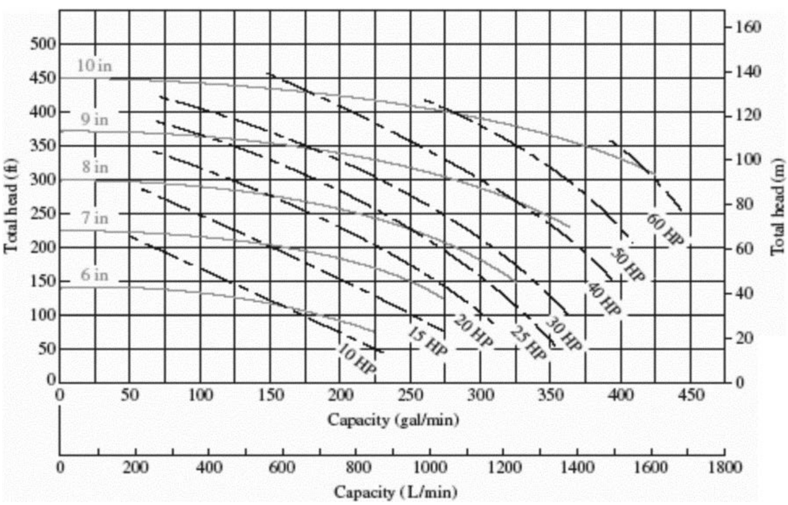


Figure 6.15 Curves showing the power required to drive the pump have been added

Source: Goulds Pumps, Inc., Seneca Falls, NY.

6.5.5 *Affinity Laws for Centrifugal Pumps*

It is important to understand the manner in which capacity, head, and power vary when either speed or impeller diameter is varied. These relationships, called affinity laws, are listed here. The symbol N refers to the rotational speed of the impeller, usually in revolutions per minute (r/min, or rpm). The affinity laws are useful as they allow prediction of the head discharge characteristic of a pump or fan from a known characteristic measured at a different speed or impeller diameter. The only requirement is that the two pumps or fans are dynamically similar; that is, the ratios of the fluid forced are the same.

Law 1. With impeller diameter (D) held constant:

Law 1a. Flow is proportional to shaft speed:

$$\frac{Q_1}{Q_2} = \frac{N_1}{N_2} \tag{6.4}$$

Law 1b. Pressure or head is proportional to the square of shaft speed:

$$\frac{H_1}{H_2} = \left(\frac{N_1}{N_2}\right)^2 \tag{6.5}$$

Law 1c. Power is proportional to the cube of shaft speed:

$$\frac{P_1}{P_2} = \left(\frac{N_1}{N_2}\right)^3 \tag{6.6}$$

Law 2. With shaft speed (N) held constant:

Law 2a. Flow is proportional to the impeller diameter:

$$\frac{Q_1}{Q_2} = \frac{D_1}{D_2} \tag{6.7}$$

Law 2b. Pressure or head is proportional to the square of impeller diameter:

$$\frac{H_1}{H_2} = \left(\frac{D_1}{D_2}\right)^2 \tag{6.8}$$

Law 2c. Power is proportional to the cube of impeller diameter:

$$\frac{P_1}{P_2} = \left(\frac{D_1}{D_2}\right)^3 \tag{6.9}$$

where

- Q is the volumetric flow rate (e.g., GPM or L/s),
- D is the impeller diameter (e.g., in or mm),
- N is the shaft rotational speed (e.g., rpm),
- H is the pressure or head developed by the fan/pump (e.g., psi or Pascal), and
- P is the shaft power (e.g., W).

These laws assume that the pump/fan efficiency remains constant, i.e., $\eta_1 = \eta_2$, which is rarely exactly true, but can be a good approximation when used over appropriate frequency or diameter ranges (i.e., a fan will not move anywhere near 1,000 times as much air when spun at 1,000 times its designed operating speed, but it may increase the flow by 99 percent when the operating speed is doubled). The exact relationship between speed, diameter, and efficiency depends on the particulars of the individual fan or pump design. Product testing or computational fluid dynamics become necessary if the range of acceptability is unknown, or if a high level of accuracy is required in the calculation. Interpolation from accurate data is also more accurate than the affinity laws. When applied to pumps, the laws work well for the constant diameter variable speed case (Law 1) but are less accurate for the constant speed variable impeller diameter case (Law 2).

Example Problem 2

Assume that the pump for which the performance data are plotted in Figure 6.16 was operating at a rotational speed of 1,750 rpm and that the impeller diameter was 330 mm. First determine the head that would result in a capacity of 5,670 L/min (1,500 gal/min) and the power required to drive the pump. Then, compute the performance at a speed of 1,250 rpm.

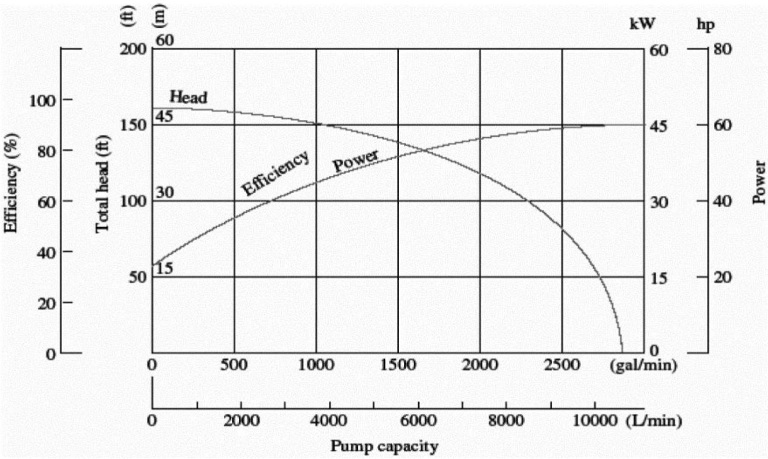


Figure 6.16 Complete performance rating of a pump, superimposing head, efficiency, and power curves and plotting all three versus capacity

From Figure 6.16 projecting upward from Q_1 = 5,670 L/min (1,500 gal/min) gives

$$\text{Total head} = 40 \text{ m} = h_{a1}$$

$$\text{Power required} = 37 \text{ kW} = P_1$$

When the speed is changed to 1,250 rpm, the new performance can be computed by using the affinity laws:

Capacity: $Q_2 = Q_1 (N_2/N_1) = 5{,}670(1{,}250/1{,}750) = 4{,}050$ L/min

Head: $h_{a2} = h_{a1}(N_2/N_1)^2 = 40(1{,}250/1{,}750)^2 = 20.4$ m

Power: $P_2 = P_1(N_2/N_1)^3 = 37(1{,}250/1{,}750)^3 = 13.5$ kW

Note the significant decrease in the power required to run the pump. If the capacity and the available head are adequate, large savings in energy costs can be obtained by varying the speed of operation of a pump.

6.6 Coverage Chart for Centrifugal Pumps

A coverage chart (Figure 6.17) makes it possible to do a preliminary pump selection by looking at a wide range of pump casing sizes for a specific impeller speed. This chart helps to narrow down the choice of pumps that will satisfy the system requirements.

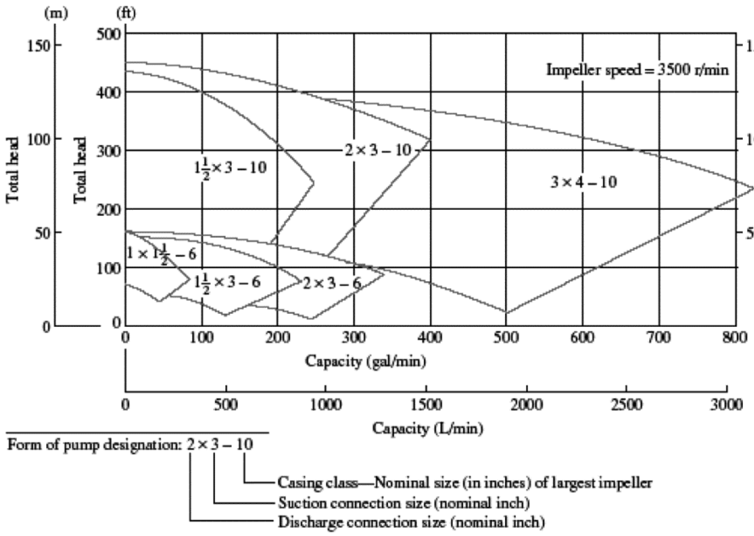


Figure 6.17 A composite rating chart for one line of pumps, which allows the quick determination of the pump size

Source: Goulds Pumps, Inc., Seneca Falls, NY.

6.7 Composite Performance Chart

Figure 6.18 shows a typical pump performance chart for a given model, casing size, and impeller rotational speed. A great deal of information is crammed into one chart and this can be confusing at first. The performance chart covers a range of impeller sizes, which are shown in even increments of 1 in. from 6 in. to 10 in. Impellers are manufactured to the largest size for a given pump casing and machined or “trimmed” to the required diameter when the pump is sold.

Figures 6.19 to 6.24 show the composite performance charts for six other medium-sized centrifugal pumps.

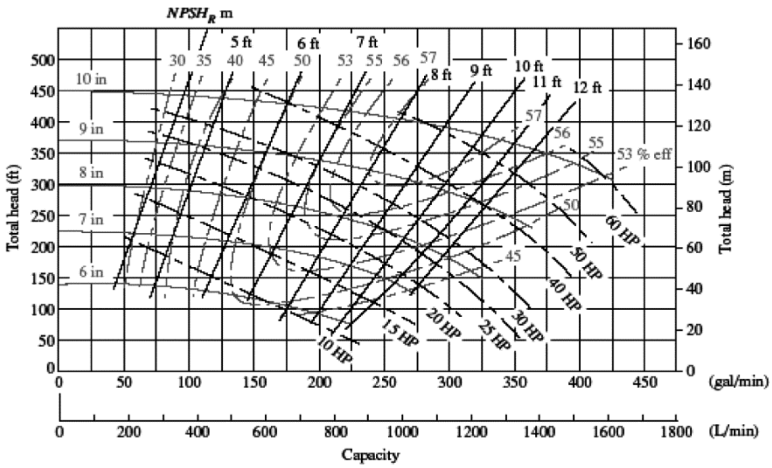


Figure 6.18 All the data are put together on one chart, so the user can see all important parameters at the same time

Source: Goulds Pumps, Inc., Seneca Falls, NY.

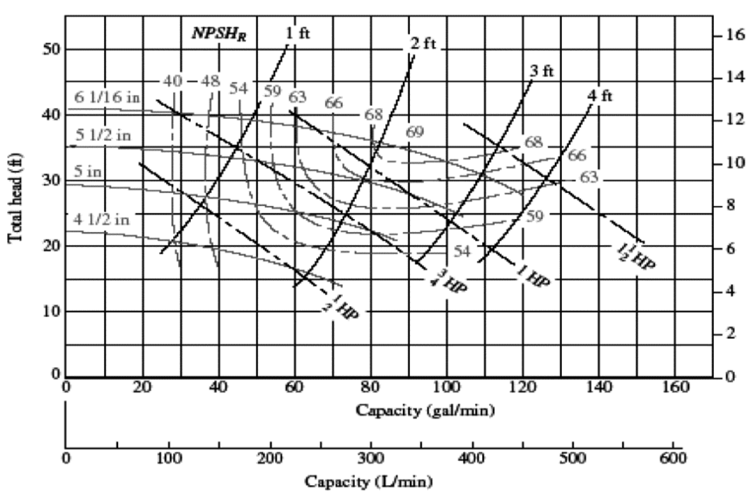


Figure 6.19 The performance of a 1.5 × 3, 6 centrifugal pump at 1,750 rpm.

Source: Goulds Pumps, Inc., Seneca Falls, NY.

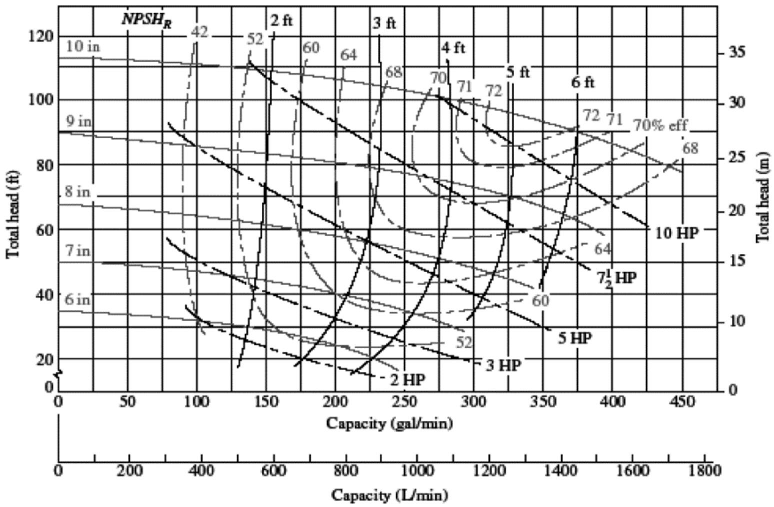


Figure 6.20 The performance of a 3 × 4, 10 centrifugal pump at 1,750 rpm

Source: Goulds Pumps, Inc., Seneca Falls, NY.)

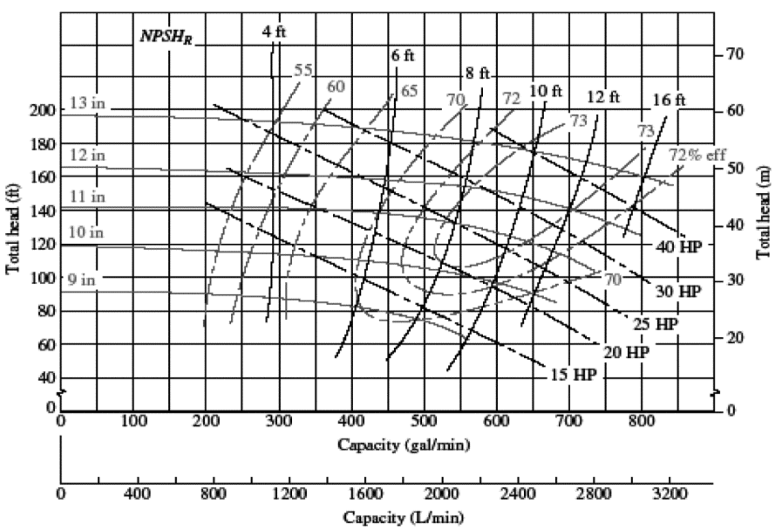


Figure 6.21 The performance of a 3 × 4, 13 centrifugal pump at 1,750 rpm

Source: Goulds Pumps, Inc., Seneca Falls, NY.

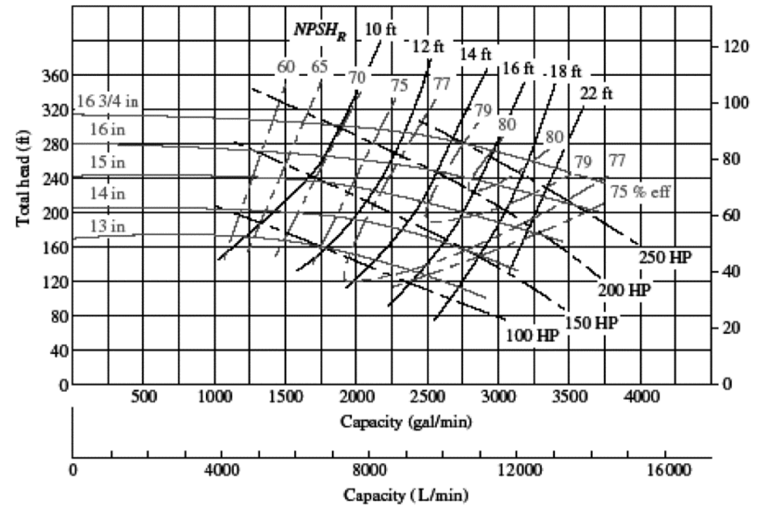


Figure 6.22 Shows the performance of a 6 × 8, 17 centrifugal pump at 1,780 rpm

Source: Goulds Pumps, Inc., Seneca Falls, NY.

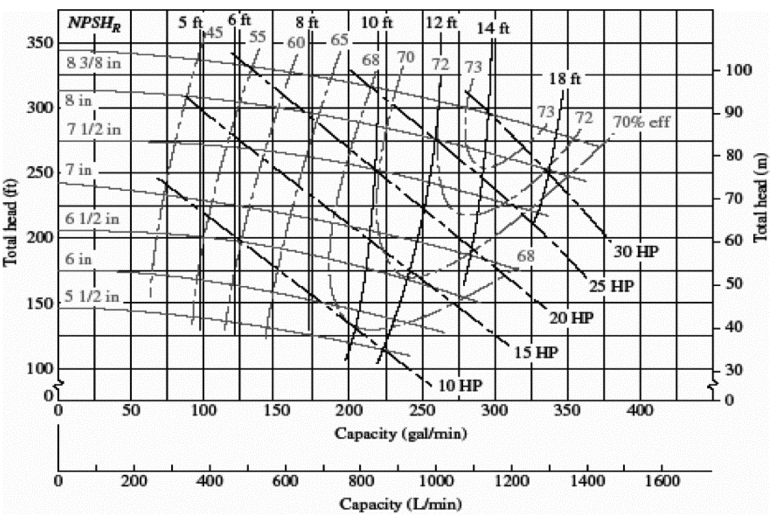


Figure 6.23 The performance of a 2 × 3, 8 centrifugal pump at 3,560 rpm.

Source: Goulds Pumps, Inc., Seneca Falls, NY.

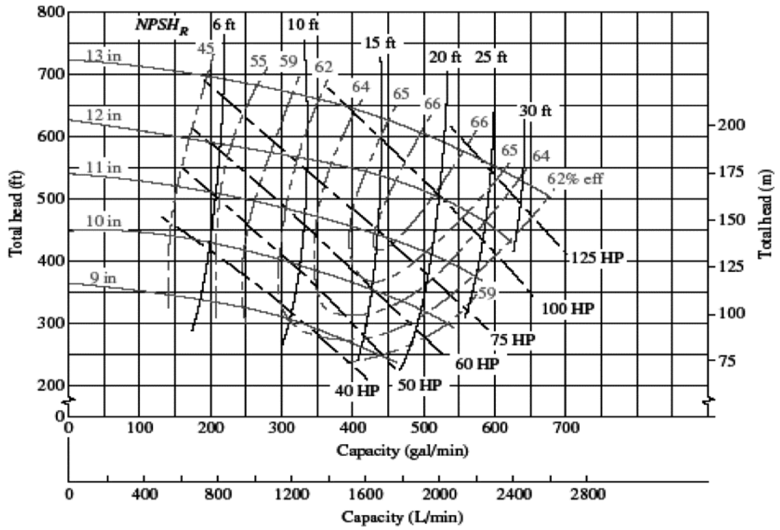


Figure 6.24 The performance of a 1.5 × 3, 13 centrifugal pump at 3,560 rpm

Source: Goulds Pumps, Inc., Seneca Falls, NY.

6.8 Operating Point of a Pump and Pump Selection

The operating point of a pump is defined as the volume flow rate it will deliver when installed in a given system. The total head developed by the pump is determined by the system resistance corresponding to the same volume flow rate. Figure 6.25 illustrates the concept.

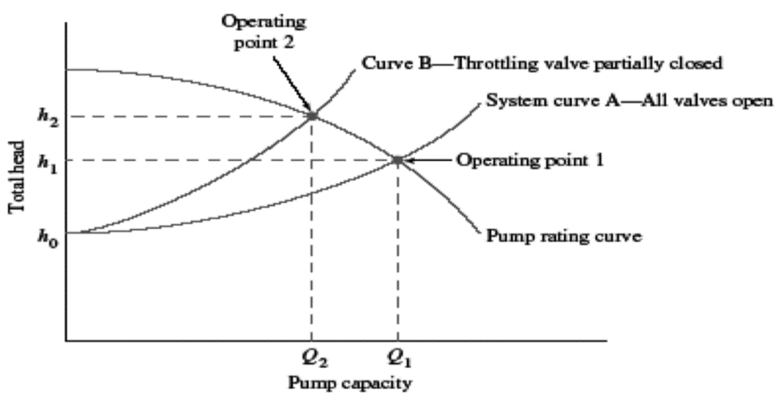


Figure 6.25 Operating point of a pump

The pump is holding the fluid at the elevation of the destination point in the system and maintaining the fluid pressure at that point. This point is called the total static head, where

$$h_o = \frac{p_2 - p_1}{\gamma} + (z_2 - z_1) \qquad (6.10)$$

This equation, derived from the energy equation, illustrates that the pump must develop a head equal to the pressure head difference between the two reference points plus the elevation head difference before any flow is delivered. As the flow increases with its corresponding increase in total head, the system curve eventually intersects the pump rating curve. Where the system curve and the pump rating curve intersect is the true operating point of the pump in this system.

Example Problem 3

A centrifugal pump must deliver at least 945 L/min of water at a total head of 91 m (300 ft) of water. Specify a suitable pump. List its performance characteristics.

Solution:

From Figure 6.18 it can be determined that the pump with a 0.23-m (9-in.) impeller will deliver approximately 1,040 L/min (275 gal/min) at 91 m (300 ft) of head. At this operating point, the efficiency would be 57 percent, near the maximum for this type of pump. Approximately 28 kW (37 hp) would be required. The $NPSH_R$ at the suction inlet to the pump is approximately 2.8 m (9.2 ft) of water.

Example Problem 4

Figure 6.26 shows a system in which a pump is required to deliver at least 850 L/min of water at 15 °C from a lower reservoir to an elevated tank maintained at a pressure of 240 kPa gage. Length of suction line is 2.44 m and discharge line is 110 m. Design the system and specify a suitable pump.

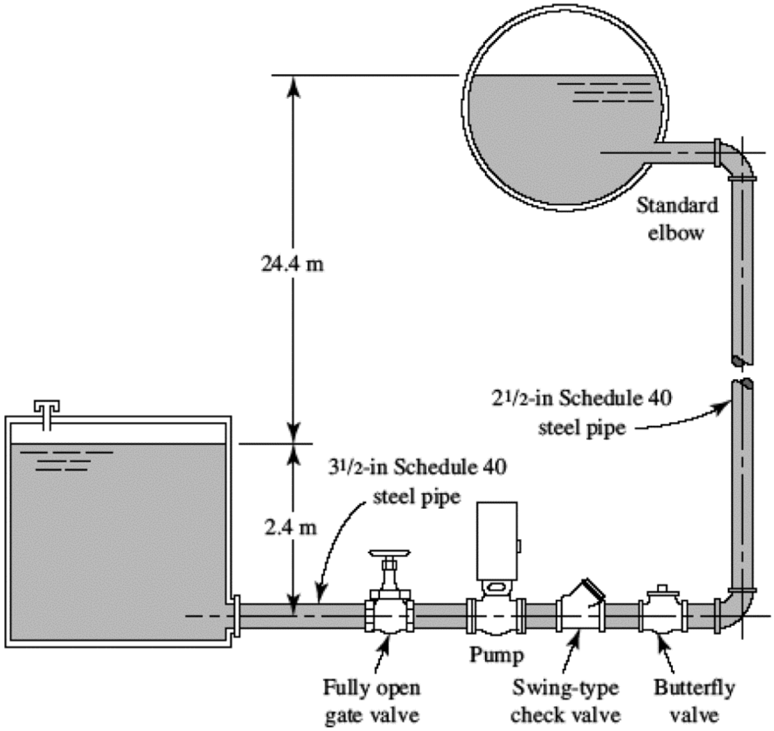


Figure 6.26 Example Problem 4

Solution:

Step 1. fluid—water at 15 °C (Q = 850 L/min minimum)
Source: lower reservoir; p = 0 kPa
Elevation = 2.4 m above pump inlet
Destination: upper reservoir; p = 240 kPa
Elevation = 26.8 m above pump inlet

Step 2. Water at 15 °C:

$$\gamma = 9.80\ \text{kN/m}^3;\ \nu = 1.12 \times 10^{-6}\ \text{m}^2\text{/s};\ h_{vp} = 0.182\ \text{m}.$$

Step 3. Figure 6.26 shows the proposed layout.

Step 4. Design decisions—Suction line is 2.4 m long; discharge line is 110 m.

Step 5. Suction pipe is 3.5-in. Schedule 40 steel pipe; D = 0.09 m;

$$A = 6.381 \times 10^{-3}\ \text{m}^2$$

Discharge line is 2.5-in. Schedule 40 steel pipe; D = 0.063 m; A = 3.09 × 10^{-3} m^2

Step 6. Reference point 1 is the surface of the lower reservoir. Reference point 2 is the surface of the upper reservoir. Computing all the major and minor losses through the system, the head loss h_L was determined to be 41.42 m.

The result for the total dynamic head is given by

$$h_a = (z_2 - z_1) + \frac{p_2}{\gamma} + h_L = 24.4 + 24.5 + 41.42 = 90.32\,\text{m}$$

Step 7. Total static head is

$$h_o = (z_2 - z_1) + \frac{(p_2 - p_1)}{\gamma} = 24.4 + 24.4 = 48.9\,\text{m}$$

Step 8. Pump selection—From Figure 6.17: 2 × 3, 10 centrifugal pump operating at 3,500 rpm. The desired operating point lies between the curves for the 8-in. and 9-in. impellers. We specify the 9-in. impeller diameter so the capacity is greater than the minimum of 850 L/min or 0.014 m^3/s.

6.9 Pumps Operating in Parallel or Series

Many fluid flow systems require largely varying flow rates that are difficult to provide with one pump without calling for the pump to operate far off its best efficiency point.

When two or more pumps are arranged in parallel, their resulting performance curve is obtained by adding their flow rates at the same head as indicated in Figure 6.27. Centrifugal pumps in parallel are used to overcome larger volume flows than one pump can handle alone.

When two (or more) pumps are arranged in series, their resulting pump performance curve is obtained by adding their heads at the same flow rate as indicated in the Figure 6.28. Directing the output of one pump to the inlet of a second pump allows the same capacity to be obtained at a total head equal to the sum of the ratings of the two pumps. This method permits operation against unusually high heads.

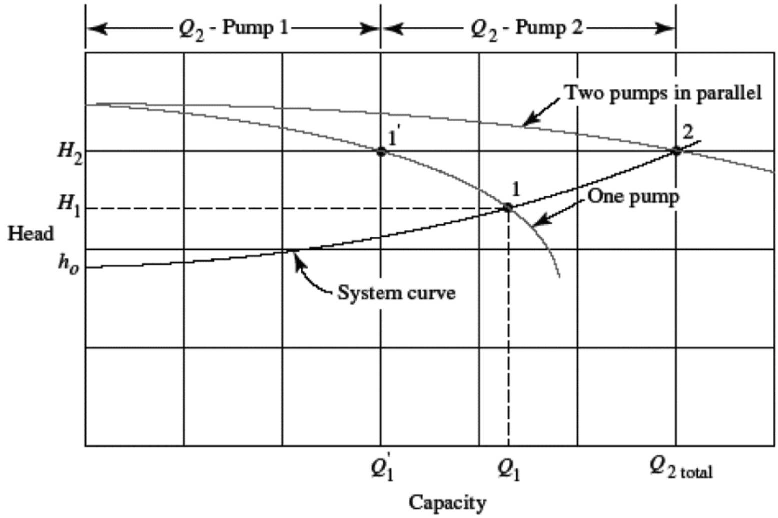


Figure 6.27 Performance of two pumps in parallel

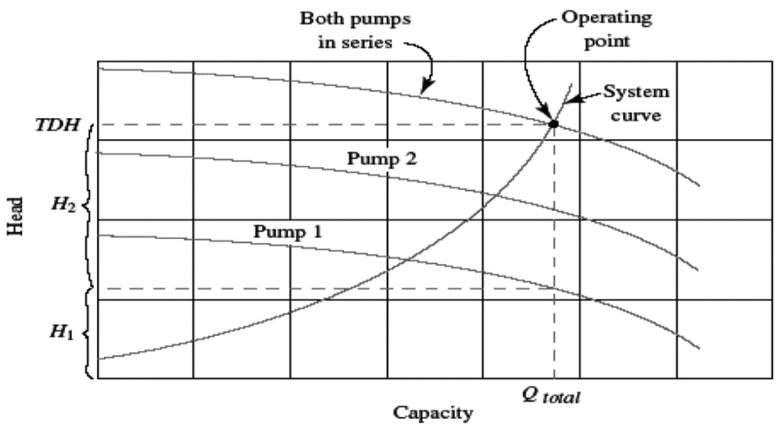


Figure 6.28 Performance of two pumps in series

6.10 Life Cycle Cost of Pumps

The term life cycle cost (LCC) refers here to the consideration of all factors that make up the cost of acquiring, maintaining, and operating a pumped fluid system.

Good design practice seeks to minimize LCC by quantifying and computing the sum of the following factors:

1. Initial cost to purchase the pump, piping, valves and other accessories, and controls.
2. Cost to install the system and put it into service.
3. Energy cost required to drive the pump and auxiliary components of the system over the expected life.
4. Operating costs related to managing the system including labor and supervision.
5. Maintenance and repair costs over the life of the system to keep the pump operating at design conditions.
6. Cost of lost production of a product during pump failures or when the pump is shut down for maintenance.
7. Environmental costs created by spilled fluids from the pump or related equipment.
8. Decommissioning costs at the end of the useful life of the pump, including disposal of the pump and cleanup of the site.

APPENDIX A

Properties of Water

Table A.1 U.S. customary system units (14.7 psia)

Temperature	Dynamic Viscosity (μ)	Kinematic Viscosity (ν)
(°F)	(lb_f·s/ft^2) × 10^{-5}	(ft^2/s) × 10^{-5}
32	3.732	1.924
40	3.228	1.664
50	2.73	1.407
60	2.344	1.21
70	2.034	1.052
80	1.791	0.926
90	1.58	0.823
100	1.423	0.738
120	1.164	0.607
140	0.974	0.511
160	0.832	0.439
180	0.721	0.383
200	0.634	0.339
212	0.589	0.317

Table A.2 Metric units (101 kPa abs)

Temperature	Dynamic Viscosity (μ)	Kinematic Viscosity (ν)
(°C)	(Pa·s, N·s/m^2) × 10^{-3}	(m^2/s) × 10^{-6}
0	1.787	1.787
5	1.519	1.519
10	1.307	1.307
20	1.002	1.004
30	0.798	0.801
40	0.653	0.658
50	0.547	0.553
60	0.467	0.475
70	0.404	0.413
80	0.355	0.365
90	0.315	0.326
100	0.282	0.29

APPENDIX B

Properties of Common Liquids

Table B.1 SI units (101 kPa and 25 °C)

Fluid	Specific Gravity (s.g.)	Specific Weight (γ) (kN/m^3)	Density (kg/m^3)	Dynamic Viscosity (Pa·s)	Kinematic Viscosity (m^2/s)
Acetone	0.787	7.72	787	3.16E−04	4.02E−07
Ethyl alcohol	0.787	7.72	787	1.00E−03	1.27E−06
Methyl alcohol	0.789	7.74	789	5.60E−04	7.10E−07
Propyl alcohol	0.802	7.87	802	1.62E+01	2.02E−02
Aqua ammonia	0.91	8.93	910		
Benzene	0.876	8.59	876	6.03E−04	6.88E−07
Carbon tetrachloride	1.59	15.60	1,590	9.10E−04	5.72E−07
Castor oil	0.96	9.42	960	6.51E−01	6.78E−04
Ethylene glycol	1.1	10.79	1,100	1.62E−02	1.47E−05
Gasoline	0.68	6.67	680	2.87E−04	4.22E−07
Glycerine	1.258	12.34	1,258	9.60E−01	7.63E−04
Kerosene	0.823	8.07	823	1.64E−03	1.99E−06
Linseed oil	0.93	9.12	930	3.31E−02	3.56E−05
Mercury	13.54	132.83	13,540	1.53E−03	1.13E−07
Propane	0.495	4.86	495	1.10E−04	2.22E−07
Seawater	1.03	10.10	1,030	1.03E−03	1.00E−06
Turpentine	0.87	8.53	870	1.37E−03	1.57E−06
Fuel oil, medium	0.852	8.36	852	2.99E−03	3.51E−06
Fuel oil, heavy	0.906	8.89	906	1.07E−01	1.18E−04

Table B.1 SI units (101 kP 14.7 psia and 77 °F)

Fluid	Specific Gravity (s.g.)	Specific Weight (γ)	Density (slugs/ft^3)	Dynamic Viscosity (lb·s/ft^2)	Kinematic Viscosity (ft^2/s)
Acetone	0.787	48.98	1.53	6.60E−06	4.31E−06
Ethyl alcohol	0.787	49.01	1.53	2.10E−05	1.37E−05
Methyl alcohol	0.789	49.10	1.53	1.17E−05	7.65E−06
Propyl alcohol	0.802	49.94	156	4.01E−05	2.57E−07
Aqua ammonia	0.91	56.78	1.77		
Benzene	0.876	54.55	1.7	1.26E−05	7.41E−06
Carbon tetrachloride	1.59	98.91	3.08	1.90E−05	6.17E−06
Castor oil	0.96	59.69	1.86	1.36E−02	7.31E−03
Ethylene glycol	1.1	68.47	2.13	3.38E−04	1.59E−04
Gasoline	0.68	42.40	1.32	6.00E−06	4.55E−06
Glycerine	1.258	78.50	2.44	2.00E−02	8.20E−03
Kerosene	0.823	51.20	1.6	3.43E−05	2.14E−05
Linseed oil	0.93	58.00	1.8	6.91E−04	3.84E−04
Mercury	13.54	844.90	26.26	3.20E−05	1.22E−06
Propane	0.495	30.81	0.96	2.30E−06	2.40E−06
Seawater	1.03	64.00	2	2.05E−05	1.03E−05
Turpentine	0.87	54.20	1.69	2.87E−05	1.70E−05
Fuel oil, medium	0.852	53.16	1.65	6.25E−05	3.79E−05
Fuel oil, heavy	0.906	56.53	1.76	2.24E−03	1.27E−03

APPENDIX C

Dimensions of Steel Pipe

Table C.1 Schedule 40

Nominal Pipe Size (in.)	Diameter			Diameter		Flow Area	
		Internal					
	External (in.)	(in.)	(ft)	External (mm)	Internal (mm)	(ft^2)	(m^2)
1/8	0.41	0.27	0.0224	10.3	6.8	0.000394	3.66E−05
1/4	0.54	0.36	0.0303	13.7	9.2	0.000723	6.72E−05
3/8	0.68	0.49	0.0411	17.1	12.5	0.00133	1.24E−04
1/2	0.84	0.62	0.0518	21.3	15.8	0.00211	1.96E−04
3/4	1.05	0.82	0.0687	26.7	20.9	0.0037	3.44E−04
1	1.32	1.05	0.0874	33.4	26.6	0.006	5.57E−04
1¼	1.66	1.38	0.115	42.2	35.1	0.01039	9.65E−04
1½	1.9	1.61	0.1342	48.3	40.9	0.01414	1.31E−03
2	2.38	2.07	0.1723	60.3	52.5	0.02333	2.17E−03
2½	2.88	2.47	0.2058	73	62.7	0.03326	3.09E−03
3	3.5	3.07	0.2557	88.9	77.9	0.05132	4.77E−03
3½	4	3.55	0.2957	101.6	90.1	0.06868	6.38E−03
4	4.5	4.03	0.3355	114.3	102.3	0.0884	8.21E−03
5	5.56	5.05	0.4206	141.3	128.2	0.139	1.29E−02
6	6.63	6.07	0.5054	168.3	154.1	0.2006	1.86E−02
8	8.63	7.98	0.6651	219.1	202.7	0.3472	3.23E−02
10	10.75	10.02	0.835	273.1	254.5	0.5479	5.09E−02
12	12.75	11.94	0.9948	323.9	303.2	0.7771	7.22E−02
14	14	13.13	1.094	355.6	333.4	0.9396	8.73E−02

Nominal Pipe Size (in.)	Diameter			Diameter		Flow Area	
	External (in.)	Internal		External (mm)	Internal (mm)	(ft^2)	(m^2)
		(in.)	(ft)				
16	16	15	1.25	406.4	381	1.227	1.14E−01
18	18	16.88	1.406	457.2	428.7	1.553	1.44E−01
20	20	18.81	1.568	508	477.9	1.931	1.79E−01
24	24	22.63	1.886	609.6	574.7	2.792	2.59E−01

Table C.2 Schedule 80

Nominal Pipe Size (in.)	Diameter			Diameter		Flow Area	
		Internal					
	External (in.)	(in.)	(ft)	External (mm)	Internal (mm)	(ft^2)	(m^2)
1/8	0.41	0.22	0.0183	10.414	5.588	0.000264	2.45E−05
1/4	0.54	0.3	0.0250	13.716	7.62	0.000491	4.56E−05
3/8	0.68	0.42	0.0350	17.272	10.668	0.000962	8.94E−05
1/2	0.84	0.55	0.0458	21.336	13.97	0.001650	1.53E−04
3/4	1.05	0.74	0.0617	26.67	18.796	0.002987	2.77E−04
1	1.32	0.96	0.0800	33.528	24.384	0.005027	4.67E−04
1¼	1.66	1.28	0.1067	42.164	32.512	0.008936	8.30E−04
1½	1.9	1.5	0.1250	48.26	38.1	0.012272	1.14E−03
2	2.38	1.94	0.1617	60.452	49.276	0.020527	1.91E−03
2½	2.88	2.32	0.1933	73.152	58.928	0.029356	2.73E−03
3	3.5	2.9	0.2417	88.9	73.66	0.045869	4.26E−03
3½	4	3.36	0.2800	101.6	85.344	0.061575	5.72E−03
4	4.5	3.83	0.3192	114.3	97.282	0.080006	7.43E−03
5	5.56	4.81	0.4008	141.224	122.174	0.126188	1.17E−02
6	6.63	5.76	0.4800	168.402	146.304	0.180956	1.68E−02
8	8.63	7.63	0.6358	219.202	193.802	0.317524	2.95E−02
10	10.75	9.56	0.7967	273.05	242.824	0.498474	4.63E−02
12	12.75	11.38	0.9483	323.85	289.052	0.706336	6.56E−02
14	14	12.5	1.0417	355.6	317.5	0.852211	7.92E−02
16	16	14.31	1.1925	406.4	363.474	1.11688	1.04E−01

Nominal Pipe Size (in.)	Diameter			Diameter		Flow Area	
		Internal					
	External (in.)	(in.)	(ft)	External (mm)	Internal (mm)	(ft^2)	(m^2)
18	18	16.13	1.3442	457.2	409.702	1.41904	1.32E−01
20	20	17.94	1.4950	508	455.676	1.75538	1.63E−01
24	24	21.56	1.7967	609.6	547.624	2.53527	2.36E−01

APPENDIX D

Dimensions of Type K Copper Tubing

	External (in.)	Internal		External (mm)	Internal (mm)	Flow Area	
		(in.)	(ft)			(ft2)	(m2)
1/8	0.25	0.18	0.0150	6.35	4.57	0.000177	1.64E−05
1/4	0.375	0.277	0.0231	9.53	7.04	0.000418	3.89E−05
3/8	0.5	402	33.5000	12.70	10,211.63	881.412384	8.19E+01
1/2	0.625	0.527	0.0439	15.88	13.39	0.001515	1.41E−04
5/8	0.75	0.652	0.0543	19.05	16.56	0.002319	2.15E−04
3/4	0.875	0.745	0.0621	22.23	18.92	0.003027	2.81E−04
1	1.125	0.995	0.0829	28.58	25.28	0.005400	5.02E−04
1¼	1.375	1.245	0.1037	34.93	31.63	0.008454	7.85E−04
1½	1.625	1.481	0.1234	41.28	37.62	0.011963	1.11E−03
2	2.125	1.959	0.1632	53.98	49.76	0.020931	1.94E−03
2½	2.625	2.435	0.2029	66.68	61.85	0.032339	3.00E−03
3	3.125	2.907	0.2422	79.38	73.84	0.046091	4.28E−03
3½	3.625	3.385	0.2821	92.08	85.99	0.062495	5.81E−03
4	4.125	3.857	0.3214	104.78	97.98	0.081138	7.54E−03
5	5.125	4.805	0.4004	130.19	122.06	0.125926	1.17E−02
6	6.125	5.741	0.4784	155.59	145.83	0.179764	1.67E−02
8	8.125	7.583	0.6319	206.39	192.62	0.313624	2.91E−02
10	10.125	9.449	0.7874	257.20	240.02	0.486966	4.52E−02
12	12.125	11.315	0.94291629	308.00	287.42	0.698291	6.49E−02

APPENDIX E

Conversion Factors

Table E.1 Conversion factors length, area, and volume

Length Conversion Factors		
Length		
To convert from	**to**	**multiply by**
mile (U.S. statute)	kilometer (km)	1.609347
inch (in.)	millimeter (mm)	25.4
inch (in.)	centimeter (cm)	2.54
inch (in.)	meter (m)	0.0254
foot (ft)	meter (m)	0.3048
yard (yd)	meter (m)	0.9144
Area Conversion Factors		
Area		
To convert from	**to**	**multiply by**
square foot (sq ft)	square meter (sq m)	0.09290304
square inch (sq in.)	square meter (sq m)	0.00064516
square yard (sq yd)	square meter (sq m)	0.83612736
acre (ac)	hectare (ha)	0.4047
Volume Conversion Factors		
Volume		
To convert from	**to**	**multiply by**
cubic inch (cu in.)	cubic meter (cu m)	0.00001639
cubic foot (cu ft)	cubic meter (cu m)	0.02831685
cubic yard (cu yd)	cubic meter (cu m)	0.7645549
U.S. liquid		
gallon (gal)	cubic meter (cu m)	0.00378541
gallon (gal)	liter	3.785
fluid ounce (fl oz)	milliliters (mL)	29.57353
fluid ounce (fl oz)	cubic meter (cu m)	0.00002957

Table E.2 Conversion factors for force, pressure, and mass

Force Conversion Factors		
Force		
To convert from	**to**	**multiply by**
kip (1,000 lb)	kilogram (kg)	453.6
kip (1,000 lb)	newton (N)	4,448.22
pound (lb) avoirdupois	kilogram (kg)	0.4535924
pound (lb)	newton (N)	4.448222
Pressure or Stress Conversion Factors		
Pressure or stress		
To convert from	**to**	**multiply by**
kip per square inch (ksi)	megapascal (MPa)	6.894757
pound per square foot (psf)	kilogram per square meter (kg/sq m)	4.8824
pound per square foot (psf)	pascal (Pa)	47.88
pound per square inch (psi)	pascal (Pa)	6,894.76
pound per square inch (psi)	megapascal (MPa)	0.00689476
Mass Conversion Factors		
Mass (weight)		
To convert from	**to**	**multiply by**
pound (lb) avoirdupois	kilogram (kg)	0.4535924
ton, 2,000 lb	kilogram (kg)	907.1848
grain	kilogram (kg)	0.0000648
Mass (weight) per length		
kip per linear foot (klf)	kilogram per meter (kg/m)	0.001488
pound per linear foot (plf)	kilogram per meter (kg/m)	1.488
Mass per volume (density)		
pound per cubic foot (pcf)	kilogram per cubic meter (kg/cu m)	16.01846
pound per cubic yard (lb/cu yd)	kilogram per cubic meter (kg/cu m)	0.5933

Table E.3 Conversion factors density

	Multiply by										
	Convert to										
Convert from	**kg/m^3**	**g/cm^3**	**oz/in.^{3}**	**oz/gal (imperial)**	**oz/gal (U.S.)**	**lb/ft^3**	**lb/in.^{3}**	**lb/yd^3**	**lb/gal (imperial)**	**lb/gal (U.S.)**	**ton/yd^3**
kg/m^3	1	0.001	0.000578	0.1604	0.1335	0.0624	0.000036	1.6855	0.01	0.00835	0.00075
g/cm^3	1,000	1	0.578	160.4	133.52	62.43	0.036	1,685.6	10.02	8.35	0.752
oz/in.^{3}	1,730	1.73	1	277.4	231	108	0.0625	2,916	17.34	14.44	1.302
oz/gal (imperial)	6.24	0.0062	0.0036	1	0.827	0.389	0.000225	10.51	0.0625	0.052	0.0047
oz/gal (U.S.)	7.49	0.00749	0.0043	1.2	1	0.468	0.00027	12.62	0.075	0.0625	0.0056
lb/ft^3	16.02	0.016	0.0093	2.57	2.14	1	0.000579	27	0.1605	0.134	0.0121
lb/in.^{3}	27,680	27.68	16	4,438.7	3,696	1,728	1	46,656	277	231	20.82
lb/yd^3	0.593	0.00059	0.00034	0.095	0.079	0.037	0.000021	1	0.00595	0.00495	0.00045
lb/gal (imperial)	99.78	0.0998	0.0577	16	13.32	6.23	0.0036	168.2	1	0.833	0.0751
lb/gal (U.S.)	119.8	0.12	0.069	19.2	16	7.48	0.0043	201.97	1.2	1	0.09
ton/yd^3	1,328.9	1.329	0.768	213.1	177.4	82.96	0.048	2,240	13.32	11.1	1

Table E.3 Conversion factors dynamic viscosity (metric units)

	Multiply by				
	Convert to				
Convert from	**Poiseuille (Pa·s)**	**Poise (dyne·s/cm²) (g/cm·s)**	**centiPoise**	**kg/m·h**	**kg_f·s/m²**
Poiseuille (Pa·s)	1	10	103	3.63×10^3	0.102
Poise (dyne·s/cm²) (g/cms)	0.1	1	100	360	0.0102
centiPoise	0.001	0.01	1	3.6	0.00012
kg/m·h	2.78×10^{-4}	0.00278	0.0278	1	2.83×10^{-5}
kg_f·s/m²	9.81	98.1	9.81×10^3	3.53×10^4	1
lb_f·s/in.²	6.89×10^3	6.89×10^4	6.89×10^6	2.48×10^7	703
lb_f·s/ft²	47.9	479	4.79×10^4	1.72×10^5	0.0488
lb_f·h/ft²	1.72×10^5	1.72×10^6	1.72×10^8	6.21×10^8	1.76×10^4
lb/ft·s	1.49	14.9	1.49×10^3	5.36×10^3	0.152
lb/ft·h	4.13×10^{-4}	0.00413	0.413	1.49	4.22×10^{-5}

Table E.4 Conversion factors dynamic viscosity (English units)

Multiply by					
	Convert to				
Convert from	$lb_f \cdot s/in.^2$	$lb_f \cdot s/ft^2$	$lb_f \cdot h/ft^2$	lb/ft·s	lb/ft·h
Poiseuille (Pa·s)	1.45×10^{-4}	0.0209	5.8×10^{-6}	0.672	$2.42\ 10^3$
Poise (dyne s/cm² = g/cms)	1.45×10^{-5}	0.00209	5.8×10^{-7}	0.0672	242
centiPoise	1.45×10^{-7}	2.09×10^{-5}	5.8×10^{-9}	0.000672	2.42
kg/m·h	4.03×10^{-8}	5.8×10^{-6}	1.61×10^{-9}	0.000187	0.672
$kg_f \cdot s/m^2$	0.00142	20.5	5.69×10^{-5}	6.59	$2.37\ 10^4$
$lb_f \cdot s/in.^2$	1	144	0.04	$4.63\ 10^3$	$1.67\ 10^7$
$lb_f \cdot s/ft^2$	0.00694	1	0.000278	32.2	$1.16\ 10^5$
$lb_f \cdot h/ft^2$	25	$3.6\ 10^3$	1	$1.16\ 10^5$	$4.17\ 10^8$
lb/ft·s	0.000216	0.0311	8.63×10^{-6}	1	$3.6\ 10^3$
lb/ft·h	6×10^{-8}	$8.633\ 10^{-6}$	2.4×10^{-9}	0.000278	1

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Fundamental Concepts

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Dr. Gautham P. Das is an associate professor of civil and environmental engineering at Wentworth Institute of Technology in Boston, Massachusetts. Prior to starting his teaching career, Dr. Das worked for environmental consulting firms in North Carolina and Boston. His expertise lies in water resources, hydraulic engineering and environmental remediation. He is active in the Water Environment Federation, the New England Water Environment Association and the American Society of Civil Engineers. He has authored numerous technical papers on various civil and environmental engineering subjects that have been published in peer-reviewed journals and presented at technical conferences. http://gauthampdas.us/

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