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Lean Six Sigma and Statistical Tools for Engineers and Engineering Managers

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Abstract

This book is intended to be a textbook for advanced undergraduate students, graduate students in engineering, and mid-career engineering professionals. It can also be a reference book, or be used to prepare for the Six Sigma Green Belt and Black Belt certifications by organizations such as American Society for Quality.

The book focuses on the introduction of the basic concepts, processes, and tools used in Lean Six Sigma. A unique feature of this book is the detailed discussion on Design for Six Sigma aided by computer modeling and simulation. The authors present several sample projects in which Lean Six Sigma and Design for Six Sigma were used to solve engineering problems or improve processes based on their own research and development experiences in engineering design and analysis.

Keywords

Lean, Six Sigma, Design for Six Sigma, statistics, probability, DMAIC, engineering, management, optimization, process improvement, quality control, design of experiment, response surface method, Monte Carlo analysis, hypothesis test, robust design, computer-aided design, data analysis.

Contents

Preface

As practitioners and trainers of Lean Six Sigma, we have long held the view that it should be taught more widely in colleges as part of the basic curriculum of engineering and management disciplines. Lean Six Sigma has proven to be a valuable tool for improving efficiency and quality of products and services. College graduates who can apply its principles are needed across a broad range of industries, in private sectors, government agencies, and nonprofit organizations. The education in Lean Six Sigma will provide students with the necessary skills to respond to this demand.

While most of traditional science and engineering courses are taught as deterministic knowledge, Six Sigma helps students form a statistical view of the real world in which a product or a process is built and operated. The students will learn to anticipate and deal with the undesirable variations that cause the performance of a product or process to deviate from its design intent. This unique statistical mindset will benefit them in solving practical problems.

While a college-level textbook needs to be self-contained, covering the concepts and process of Lean Six Sigma, statistical background, major tools including computer simulation, and practical examples, it does not have to be overly extensive. An entry-level Lean Six Sigma textbook should be easy to follow, even suitable for self-learning. These were the considerations we had in mind while writing this book.

Over the past 15 years, many Lean Six Sigma books have been published, each providing a unique perspective. Some of the well-written books in Lean Six Sigma are as follows:

- W. Brussee, (2012). *Statistics for Six Sigma Made Easy! Revised and Expanded*, 2nd ed., New York, NY: McGraw-Hill Education.
- H. S. Gitlow, R. J. Melnvck, and D. M. Levine, (2015). A Guide to Six Sigma and Process Improvement for Practitioners and Students: Foundations, DMAIC, Tools, Cases, and Certification, 2nd ed., Upper Saddle River, NJ: Pearson FT Press.

- T. Pyzdek and P. A. Keller, (2014). *The Six Sigma Handbook*, 4th ed., New York, NY: McGraw-Hill.
- E. A. Cudney, S. Furterer, and D. Dietrich, eds., (2013). *Lean Systems: Applications and Case Studies in Manufacturing, Service, and Healthcare*, Boca Raton, FL: CRC Press.
- M. J. Franchetti, (2015). Lean Six Sigma for Engineers and Managers: With Applied Case Studies, Boca Raton, FL: CRC Press.

Brussee's book covered many topics of statistics in the context of Six Sigma, while both Pyzdek and Keller's book and Gitlow *et al.*'s book aimed to teach Six Sigma to people with backgrounds from different sectors. Cudney *et al.* focused on the process and applications of Lean in the service sector, and Franchetti concentrated mainly on case studies (a total of 11 case studies).

However, we did not find a Lean Six Sigma textbook specifically written for engineering students or practitioners that has a comprehensive treatment of Lean Six Sigma using advanced statistical and computer-aided tools with plenty of examples and real-world case studies from engineering research and development. This book was the result of our attempt. Drawing on our extensive experiences in the application of Lean Six Sigma in the automotive industry, we incorporated the following unique features into this book:

1. It provides many examples so that readers can learn by themselves.
2. The DMAIC process of Lean Six Sigma is illustrated in detail with an example that does not require much technical background to understand, namely, the example of Krisys robot kits. This makes it easy to understand when, what, why, and how to carry out tasks necessary for the completion of a Lean Six Sigma project.
3. The book thoroughly explores Design for Six Sigma (DFSS). Chapter 7 is devoted completely to DFSS, and two pertinent case studies are included in Chapter 8.
4. Statistical tools are employed, as presented in Chapter 4 and the case studies. The related statistical knowledge is included for readers who wish to understand those tools.

5. Advanced analysis methods such as modeling, simulation, Monte Carlo analysis, design of experiments, and response surface method are employed in case studies. Since we have first-hand knowledge as engineers in the automotive industry, the case studies demonstrate practical applications of Lean Six Sigma in product development.
6. Computer software is used to demonstrate how Lean Six Sigma projects can be executed. Chapter 5 contains a number of examples for the usage of Excel, MATLAB, Minitab, and R. Readers can easily follow along with screen captures to fully understand the concepts.

The purpose of this book is two-fold. First, it is designed for undergraduate, graduate students, and practitioners to learn Lean Six Sigma. In addition, it serves as preparation material for the Lean Six Sigma Green Belt and/or Black Belt certification.

Both authors have many years of experience with Lean Six Sigma. Wei Zhan completed several Six Sigma projects as a system engineer in the automotive industry. He is currently an associate professor at Texas A&M University and is an ASQ-certified Six Sigma Black Belt. He has published many papers in the area of Lean Six Sigma. Xuru Ding is a Master Black Belt of DFSS at General Motors, leading and coaching DFSS and robust optimization projects in various engineering disciplines. She has also taught cooperate-level training classes in robust optimization and DFSS at General Motors since 1991. These experiences allowed the authors to share with the reader valuable insights on the implementation of Lean Six Sigma.

The materials in this book have been used in ESET 329 of Texas A&M University for six semesters. They were also used in a continuing education workshop, Lean Six Sigma Green Belt Training offered at Texas A&M University. Survey data and comments from the students of ESET 329 and the workshop were studied thoroughly, in order to improve the manuscript.

It is only fitting that this book is the result of a Lean Six Sigma project. We reached out to the target users, students, to collect Voice of Customer (VOC). In the spring semester of 2015, seven student teams at Texas A&M University used the improvement of the draft manuscript as their semester-long Lean Six Sigma project for the Six Sigma and Applied Statistics course (ESET 329). The students' recommendations and questions were considered as VOC and revisions to the content were made accordingly. Many typographical and other errors were found and corrected with the help of the students. As a result, the manuscript has improved significantly since the first draft.

Acknowledgments

First and foremost, I thank God for making everything possible. In addition, I'd like to thank my wife Jessica and my daughters Lilian and Jennifer for their patience and support throughout the process of writing this book.

—Wei Zhan

I would like to thank my mentor Dr. Hilario L. Oh, who first introduced me to the concept of robust design many years ago, thus began my career as a DFSS practitioner.

—Xuru Ding

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CHAPTER 1

Introduction

Since the mid-1980s, there has been increasing interest in the application of the Six Sigma methodology in many industries [1–6]. **Six Sigma** is a well-accepted tool for process improvement and quality control. It is a structured, disciplined, data-driven methodology and process for improving business performance, with an emphasis on **voice of the customer** (**VOC**) and using statistical analysis tools. In the early 2000s, Six Sigma was combined with the Lean principle to become **Lean Six Sigma** to achieve business and operational excellence [7, 8]. In this chapter, the history, rationale, and benefits of Lean Six Sigma will be discussed.

1.1 History of Lean Six Sigma

The use of statistics in manufacturing started in the 1920s at Bell Laboratories by Dr. Walter A. Shewhart, Dr. Harold Dodge, and Dr. Harry Romig. The military adopted the use of statistics in the 1940s during World War II. By the 1950s, Drs. Edwards Deming, Joseph M. Juran, and Armand V. Feigenbaun had made significant contributions to the quality engineering field by developing the **Total Quality Management** (**TQM**) system. Dr. Deming first taught quality control to the Japanese, and they were the first ones to embrace TQM. In the 1950s, Dr. Genechi Taguchi popularized the concept of "design of experiments" in order to improve product quality. The Japanese manufacturing industry made significant improvements in quality due to the wide use of statistical methods such as TQM. The Japanese were so successful in quality control that American customers preferred to buy products made in Japan rather than those made in the United States. As a result, Japanese manufacturers enjoyed large market shares in the United States, especially in the markets for automobiles and consumer electronics. To meet its customers' needs, Toyota created the **Toyota Production System** (**TPS**) [9] to deliver products of the right quality, in the right quantity, and at the right price.

This marked the beginning of **Lean production** [10–13]. The key concept of Lean is to identify and eliminate the non–value-added steps in a process. These steps waste resources and add the chance of defects; eliminating them can accelerate the process and reduce the cost.

In 1980, NBC aired a documentary, *If Japan Can…Why Can't We?*, which raised the awareness of the quality issues among the American industries. During the 1980s, U.S. corporations made tremendous strides to catch up with their Japanese counterparts. Six Sigma was developed partly due to this endeavor [4, 5, 14].

In 1986, Six Sigma was first introduced by Motorola, where William Smith and Mikel Harry created the quality improvement process. In 1989, Motorola established the Six Sigma Research Institute, followed soon by AlliedSignal and General Electric (GE). Black & Decker, DuPont, Dow Chemical, Federal Express, Boeing, Johnson & Johnson, Ford Motor Company, General Motors, and many other companies initiated Six Sigma afterwards. Most of the early proponents of Six Sigma were from the manufacturing and technology industries. As the methodology progressed, however, it spread to pharmaceuticals, financial institutes, toy makers, clothing retailers, military, and many other sectors. By the late 1990s, about two-thirds of the Fortune 500 companies had embraced Six Sigma initiatives aimed for reducing cost and improving quality. Today, many companies require their employees to go through the Six Sigma training. It is fair to say that Six Sigma has become an internationally accepted management system, a process improvement methodology, a technical problem-solving tool, and a global common language across different sectors.

In general, Six Sigma focuses on reducing the variation and defects in the performance of a product or process, as illustrated in Figure 1.1.

The performance of mass-produced products varies slightly from unit to unit; when this variation exceeds the tolerance, the product is a defect. Defects have to be tested, repaired, replaced, recycled, or trashed, adding waste and cost to the manufacturer. After the product is deployed in the field, a defect would cause customer dissatisfaction, repair charge, and potential loss of market share. To both the manufacturer and the consumer, a consistent product performance adds value, which is why Six Sigma is so important. Six Sigma also allows more flexibility in choosing the nominal values for design parameters.

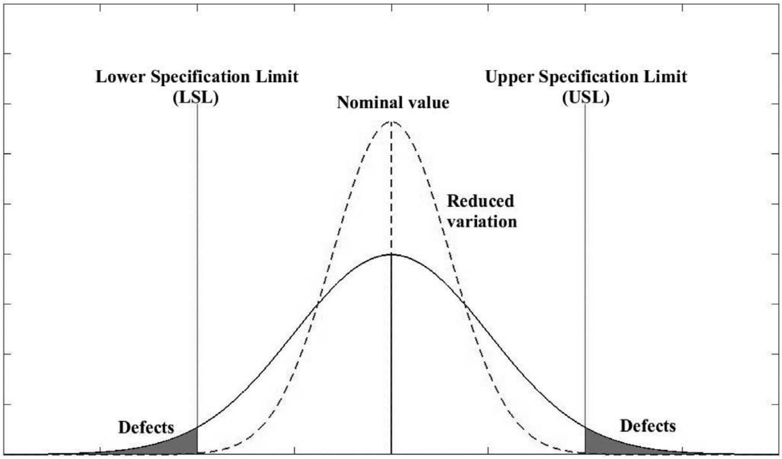

Figure 1.1 Improve the quality of a product by reducing the variation

In the narrow statistical sense, a Six Sigma process is one that produces 3.4 **defective parts per million opportunities** (**DPMO**). However, the objective of a Six Sigma project can also be a five sigma process, a three sigma process, or an eight sigma process, depending on the situation.

Many practitioners realized that Six Sigma alone might not be enough for some process improvement. Sometimes, we need to shift the mean in addition to reducing the variation. For example, for a process in manufacturing or service, it may be desirable to reduce the average and the variation of the cycle time. M. George was credited as the first person to propose the combination of Lean and Six Sigma in his book *Lean Six Sigma: Combining Six Sigma Quality with Lean Production Speed* [7]. This new methodology, **Lean Six Sigma**, takes advantage of both: waste elimination and process acceleration in Lean, and variation reduction in Six Sigma. Lean Six Sigma is more effective than Lean or Six Sigma alone. As a result, it has become more and more popular as a tool for improving business and operational excellence over the last decade.

Even though statistics is a major part of the methodology [15, 16], Lean Six Sigma is more than just a statistical tool for reducing product variation. It is also a process that can make a business more successful. Companies and organizations employ Lean Six Sigma to raise the quality of their products, to eliminate the waste in their processes, to maintain the competitiveness of their products, and to improve the financial bottom line. To survive in today's global market, companies must continuously improve; no company is too big to fail!

1.2 Optimal Quality Cost

J.M. Juran introduced the concept of **optimal quality control** in his book *Quality Planning and Analysis* [17]. The total cost related to product quality was broken down into two categories: the **cost of failure** and the **cost of appraisal/prevention**. These costs can be plotted as functions of product quality, which ranks from 0 percent (all bad) to 100 percent (all good), as illustrated in Figure 1.2.

The cost of failure is a monotonically decreasing function, and the cost of appraisal and prevention is a monotonically increasing function. The cost of failure curve tells us that we will pay higher costs as the product quality lowers. The cost of appraisal and prevention will go up as more effort is needed to further improve the end product customers receive. While it is tempting to go after 100 percent good in quality, sometimes the high cost associated with it may make the product unprofitable, this is particularly true for organizations that are just starting their quality-improvement effort. As organizations become more mature in quality control, the cost of appraisal and prevention may not go up as steep as Figure 1.2 shows. Juran argued that we should consider

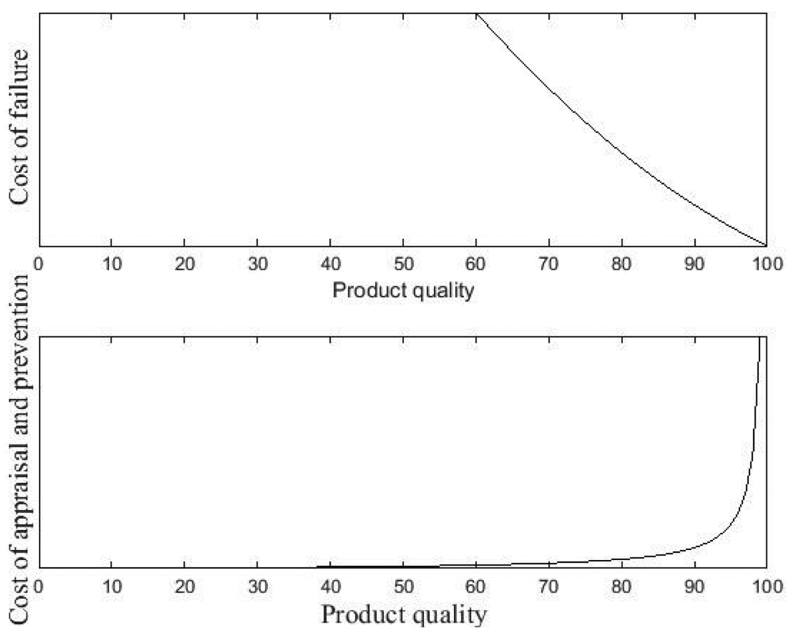

Figure 1.2 Cost in two categories

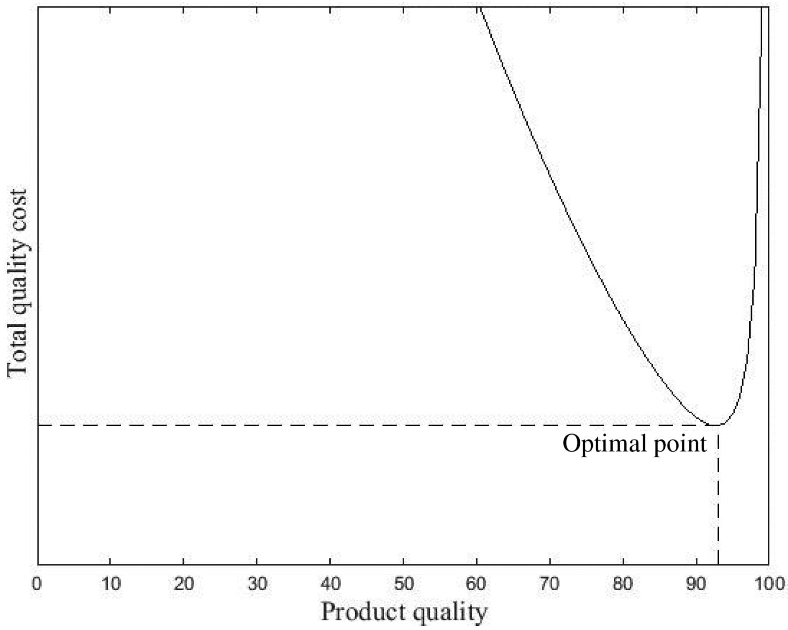

Figure 1.3 Optimal Quality Cost [17]

Source: Reprinted with permission by McGraw Hill.

the sum of the two costs for decision making. The sum is defined as the **total quality costs**, which is a function of the product quality.

A typical total quality cost function is plotted in Figure 1.3. The objective is to minimize the total quality costs by operating around the optimal point.

It is commonly accepted in industry that continuous efforts to achieve stable and predictable process results are vitally important for businesses to remain successful. In order to maximize the benefits of Lean Six Sigma, businesses must invest in training, change organizational infrastructure, and shift the corporate culture. Every employee must think about how he or she can impact the customers and to improve communication within the business using a common language. All these require a resource commitment by the management as well as the rest of the organization. In the end, companies and organizations must take everything into consideration when weighing options to minimize the total quality cost of their products or services. We can see in Figure 1.3 that the optimal point is usually close to 100 percent good quality, but

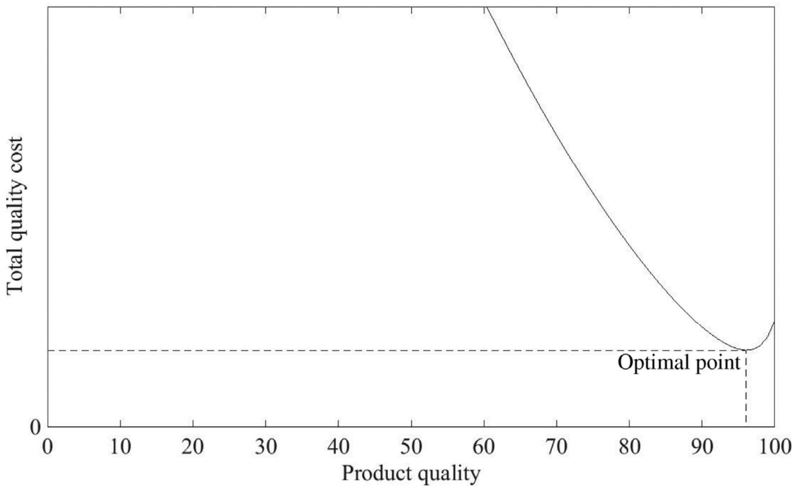

Figure 1.4 Optimal point shifting toward 100% good quality

not at 100 percent. However, it is moving closer and closer to 100 percent good, as technologies evolve, global competition increases, and better infrastructures for quality control are established in organizations. In addition, the cost for 100 percent good quality is also becoming lower. Therefore, the total quality curve in Figure 1.3 may have become what is shown in Figure 1.4.

1.3 Benefits of Lean Six Sigma

Many corporations reported significant improvement in their financial bottom line after deploying Lean Six Sigma [14]. GE announced savings of $350 million in 1998 due to the implementation of Six Sigma. Over 5 years, GE saved $12 billion, adding $1 to its earnings per share. In 2005, Motorola attributed over $17 billion in savings to Six Sigma over the years. Honeywell (AlliedSignal) recorded more than $800 million in savings in 1 year. Pulakanam reported in an article [18] in the ASQ *Quality Management Journal*, "An average return of more than $2 in direct savings for every dollar invested on Six Sigma." Lean manufacturing made Toyota one of the most competitive and profitable companies in the world.

Many companies and organizations also reported positive influences of Lean Six Sigma in sales growth, operating income, profits, employee turnover, employee satisfaction, and customer satisfaction.

Although Lean Six Sigma can be used in many different ways to improve processes, it has limitations and boundaries and is by no means a universal tool that can solve all the problems in the world. While Lean Six Sigma does have its drawbacks, it is important to understand that some of the myths circulated by opponents of Lean Six Sigma are not true [19]. Below is a list of such myths.

Myth 1: Lean Six Sigma mostly finds application in large organizations.

As a matter of fact, Lean Six Sigma contains a large number of tools and techniques that also work well in small to medium-sized organizations.

Myth 2: Lean Six Sigma is only suitable for the manufacturing sector.

Actually, Lean Six Sigma has been successfully implemented in many other sectors such as pharmaceuticals, financial institutes, services, and military. A process is anything that has a beginning and an end. If there is a process involved, it can be streamlined with Lean Six Sigma!

Myth 3: Introduction of Lean Six Sigma has the effect of stifling creativity which is needed in R&D.

In reality, research, development, and other creative work can be rigorous and follow systematic approaches. Creativity and Lean Six Sigma do not have to be contradictory.

Myth 4: Software development does not need Lean Six Sigma, because there will be no variation in the results of any software.

On the contrary, the actual inputs to software can deviate from the design intent, causing variation in the result.

Myth 5: Lean Six Sigma is for incremental improvement, not for new products.

In fact, Design for Six Sigma (DFSS) is specifically for new product development.

Lean Six Sigma changes the way people think and work; it changes the culture of corporations. It has become both a business model and a business culture. Nowadays, many companies in manufacturing and pharmaceutical industries mandate the use of Lean Six Sigma tools in their everyday business operation. Engineers and engineering managers need to have a more thorough knowledge of Lean Six Sigma as more businesses, government agencies, and organizations adopt the methodology to improve their products, processes, and services. Lean Six Sigma provides a systematic approach for improving a process, identifying the root cause of a problem, finding a robust design, performing statistical data analysis, and assessing a measurement system, all of which are relevant to the tasks of engineers.

What do you need to know about Lean Six Sigma as a manager/leader?

- Product development/improvement process, technology innovation and planning, R&D: if you know Lean Six Sigma, you will be able to plan more efficiently and make better decisions.

What do you need to know as a Lean Six Sigma practitioner?

- The Lean Six Sigma process (DMAIC), systematic way of problem solving, process mapping, process improvement, new product development, robust design, among others.

What are the characteristics of Lean Six Sigma?

1. The strong emphasis on defining the project, especially on understanding what customers need
2. The thoroughness in validating the measurement system
3. The use of data and statistical tools to identify root causes
4. The creativity in solution development
5. The emphasis on establishing controls to maintain improvements
6. The never-ending effort to improve processes
7. A strong support from the management

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CHAPTER 2

Probability and Statistics

In engineering designs, we often face two problems: how to predict the results based on the knowledge of probability and how to draw a conclusion about the entire population from a set of sampled data. We apply the probability theory to solve the first problem and statistics to solve the second. Statistics is widely used in Lean Six Sigma projects. The fundamental concepts of statistics required for learning Lean Six Sigma are covered in this chapter. More advanced tools, such as hypothesis testing, are discussed in Chapter 4.

2.1 Why is Statistics Relevant to Engineers?

Designing products and conducting tests are important parts of an engineer's job, whose decision making usually involves analysis of measured data. Since error exists in all measurement systems that collect data, multiple data points are usually recorded during a test when possible. This provides us with more reliable information for understanding the underlying phenomenon observed in the test. The collected data can then be analyzed with statistical techniques, shedding light on a problem that might otherwise be very difficult to understand.

Many parameters that affect the performance of a product or process will not be perfect as designed. For instance, physical and chemical properties of a material, manufactured dimensions of a part, temperature and humidity of the operating environment of the system, usage and preference of a customer, all have variation to some degree. The question is what is the impact of these variations on the product performance? An engineer cannot automatically assume that the variations are negligible, that somehow the design should magically work in presence of the variation. In order to design and build high-quality products, it is critical that we understand how parameter variation affects an engineering design.

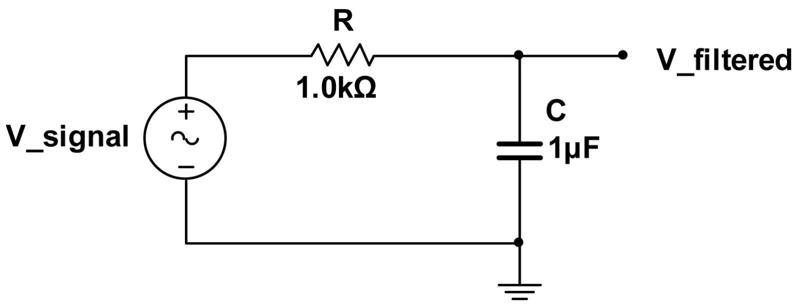


Figure 2.1 Design of a low-pass filter

Example 2.1 A simple low-pass filter circuit is illustrated in Figure 2.1. It is designed to block noises above 159.15 Hz and allow the signal below it to pass.

The cutoff frequency f_c is calculated as

$$f_c = \frac{1}{2\pi RC} = \frac{1}{2\pi \times 10^3 \times 10^{-6}} = 159.15\,Hz$$

To implement this low-pass filter design, it isn't enough to select the nominal values of resistance and capacitance. If we select the resistor made by Vishay with the part number CRCW08051K00JNEA, it has a 5 percent tolerance. Similarly, the Vishay capacitor VJ0805Y105MXQTW1BC has a tolerance of 20 percent. Using these tolerances, we can calculate the worst case cutoff frequencies (i.e., the furthest away from the nominal of 159.15 Hz) for this circuit:

$$f_{c1} = \frac{1}{2\pi RC} = \frac{1}{2\pi \times (1+0.05) \times 10^3 \times (1+0.20) \times 10^{-6}} = 126.31\,Hz$$

$$f_{c2} = \frac{1}{2\pi RC} = \frac{1}{2\pi \times (1-0.05) \times 10^3 \times (1-0.20) \times 10^{-6}} = 209.41\,Hz$$

The designer must consider if these cutoff frequencies are acceptable for the product. ■

Of course, other factors such as cost need to be considered as well. An interesting question arises when we design a product: Do we really need to design for the worst case scenario caused by parameter variations? To answer this question, we need to review some basic concepts in probability theory.

2.2 Probability, Sampling, and Random Variables

Probability is the relative possibility that an event will occur. It can be calculated as

$$P(A) = \frac{Actual\ occurences\ of\ Event\ A}{Total\ possible\ occurences} \tag{2.1}$$

Population is a well-defined collection of objects that are of interest in a specific case. If all the information for all objects in the population is available, we have a **census**. A census is usually time-consuming and costly to obtain. Oftentimes a **sample**, which is a subset of the population, is randomly selected from a given population instead. When the sample is randomly selected and the sample size is large enough, one can hypothesize that the sample carries information that is representative of the population. In other words, if an experiment is repeated sufficiently many times, say N times, and an event A occurs n times, then approximately

$$P(A) \approx \frac{n}{N} \tag{2.2}$$

An **experiment** is any activity which has uncertain outcomes. The **sample space** (Ω) of an experiment is the set of all possible outcomes of that experiment.

An **event** is a subset of the sample space Ω. A **simple event** is an element in Ω that consists of a single outcome. If an event contains more than one outcome, it is called a **compound event**.

A more rigorous interpretation of probability is as follows:

Probability is a function P(•) that gives a precise measure P(A) of the chance that an event A will occur, with the sample space Ω as the **domain** and the set of real numbers [0, 1] as the **range**.

Example 2.2 The experiment is to inspect 20 samples out of the 100 products. Each sample is either defective or not defective.

In this case, the sample space Ω consists of all the possible outcomes, that is, {0 defects, 1 defect, 2 defects, 3 defects, …, 20 defects}, while {0 defects}, {1 defect}, …, {20 defects} are the simple events.

There are many compound events in this sample space. For example, {less than 5 defects}, which consists of the first 5 simple events in Ω; {more than 10 defects}, which consists of simple events 12 to 20; {between 10 to 15 defects, inclusive}, which consists of {10 defects}, {11 defects}, {12 defects}, {13 defects}, {14 defects}, and {15 defects}. ■

In practice, we often use Equation (2.2) to attain an approximation for probability of an event. In order to get an approximation with sufficient accuracy, we need to ensure that the selection of samples is representative of the population, which is also a requirement for calculating other statistical quantities. Two commonly used sampling methods are *simple random sampling* and *stratified sampling*.

Simple random sampling assigns any particular subset of a given size the same chance of being selected. **Stratified sampling** first divides the entire sample space, Ω, into nonoverlapping subsets, followed by simple sampling within each subset.

Example 2.3 When polling for a local mayoral race, a simple sampling method can be used. Every eligible voter is assigned a specific number, and a certain percentage of the voters are randomly selected by a computer to participate in the poll. For example, assume that a total of 10,000 voters participate in the poll. Each voter is assigned a unique number between 1 and 10,000. A computer program such as Excel or MATLAB can be used to randomly generate 5 percent of the total voters, that is, 500 numbers. The voters who were assigned these 500 numbers earlier will be surveyed for their opinion. ■

Example 2.4 The U.S. presidential election is not determined by nationwide popular votes. Instead, the election is determined by the votes cast by the Electoral College. Therefore, it makes more sense to use the stratified sampling method than the simple sampling method in polling for the presidential race. The total population that is eligible to vote is about 200 million. If we decide to poll 0.01 percent of the voters, that would be 20,000. We can divide these 20,000 people among the 50 states and Washington DC according to the electors allocated to each state and DC. For example, out of the total 538 electors, there are 55 (or 55/538 = 10.22%) for California, 38 (or 38/538 = 7.06%) for Texas, 29 (or 29/538 = 5.39%) for

New York, …, 3 (or 3/538 = 0.56%) for Wyoming, and 3 (or 3/538 = 0.56%) for DC. Therefore, out of the 20,000 voters to be polled, 2045 should be from California, 1413 from Texas, 1078 from New York, …, 111 from Wyoming, and 111 from DC. Within each state or district, simple sampling can be applied. In this case, the stratified sampling polling method would be more accurate than the simple sampling method for predicting the U.S. presidential election. ■

A **random variable** is a function with the sample space Ω as its domain, and the real numbers R, or a subset of R, as its range. A random variable is said to be **discrete** if it takes finitely many or infinitely but countably many values. A random variable is said to be **continuous** if it takes uncountable many values and the probability for the random variable to be equal to any given value is 0.

Example 2.5 The number of people voting for a particular presidential candidate is a discrete random variable, since there are only a finite number of possible results. A digital signal is a discrete random variable, since it can only take 0 or 1 as its value. An analog voltage signal in an electronic circuit is a continuous random variable, since it can take uncountably many different values. However, if a digital multimeter with certain resolution is used to measure the analog voltage, the measurement would be a discrete random variable, since there are only a finite number of possible readings from the digital multimeter. ■

2.3 Set Theory

For any two events A and B, various operations can be defined in a similar way as in the set theory:

1. **Intersection**: $A \cap B$ is the event that consists of the sample points that are in both events A and B.
2. **Union**: $A \cup B$ is the event that consists of the sample points that are in A or B or both.
3. **Complement**: A^c (or A') is the event that consists of the sample points that are not in A.

The complement of the sample space Ω is the empty set, denoted by Φ.

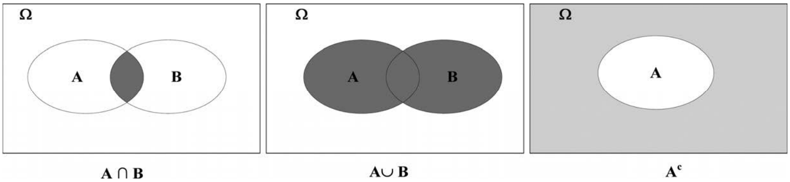


Figure 2.2 Venn diagram for set operations

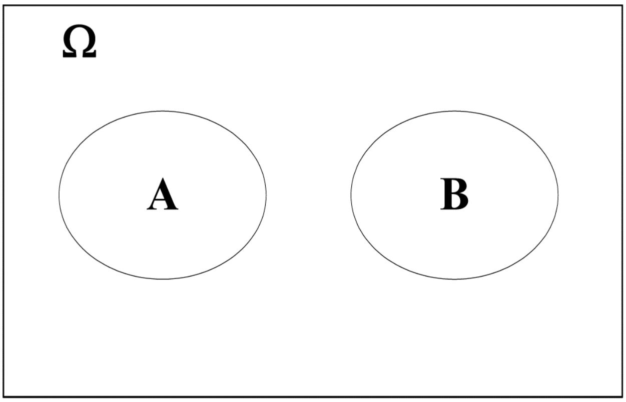


Figure 2.3 Mutually exclusive events

These basic set operations are illustrated in Figure 2.2 using Venn diagrams.

Events A and B are said to be **mutually exclusive** or **disjoint** if $A \cap B = \Phi$, in other words, A and B do not have any common elements. This can be illustrated by the Venn diagram in Figure 2.3.

For any three events *A, B, C,* we have

Commutative Law*:*

$$A \cup B = B \cup A, \quad A \cap B = B \cap A \tag{2.3}$$

Associative Law*:*

$$A \cup (B \cup C) = (A \cup B) \cup C = A \cup B \cup C, \quad A \cap (B \cap C) = (A \cap B) \cap C = A \cap B \cap C \tag{2.4}$$

Distributive Law:

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C), \quad A \cup (B \cap C) = (A \cup B) \cap (A \cup C) \tag{2.5}$$

De Morgan's Laws*:*

$$(A \cap B)' = A' \cup B', \quad (A \cup B)' = A' \cap B' \tag{2.6}$$

These can all be proved using Venn diagrams.

Example 2.6 Consider the experiment of rolling a die twice. The sample space is expressed as

Ω = {(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)}

We can define events A, B, C as the following:

A = {sum of the two numbers is less than 6}

B = {both numbers are even}

C = {at least one of the numbers is 4}

A, B, C can also be written as

A = {(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (3,1), (3,2), (4,1)}

B = {(2,2), (2,4), (2,6), (4,2), (4,4), (4,6), (6,2), (6,4), (6,6)}

C = {(1,4), (2,4), (3,4), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (5,4), (6,4)}

With these definitions, it is easy to perform the set operations, for example,

$A \cap B$ = {(2,2)}

$A \cap C$ = {(1,4), (4,1)}

$B \cap C$ = {(2,4), (4,2), (4,4), (4,6), (6,4)}

$A \cap B \cap C = \Phi$ ■

2.4 Calculus of Probabilities

The probability of the sample space is always 1, that is, $P(\Omega) = 1$.

For any event A, $1 \geq P(A) \geq 0$.

If $A \subset B$, then $P(A) \leq P(B)$

If $A_1, A_2, A_3, \ldots, A_n$ is a collection of mutually exclusive events, then

$$P(A_1 \cup A_2 \cup A_3 \cup \ldots \cup A_n) = \sum_{i=1}^{n} P(A_i) \qquad (2.7)$$

This is true even if n is infinity.

$$P(A^c) = 1 - P(A) \tag{2.8}$$

$$P(\Phi) = 1 - P(\Omega) = 0 \tag{2.9}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \tag{2.10}$$

Events $A_1, \ldots, A_n$ are **mutually independent** if for every k (k = 2, 3, . . . , n) and every subset of indexes $i_1, i_2, \ldots, i_k$:

$$P(A_{i_1} \cap A_{i2} \cap \ldots \cap A_{i_k}) = P(A_{i_1})(A_{i_2}) \ldots P(A_{i_k}) \tag{2.11}$$

When n = 2, then two events A and B are **independent** if and only if,

$$P(A \cap B) = P(A)P(B) \tag{2.12}$$

If A and B are independent, then

$$P(A \cup B) = P(A) + P(B) - P(A)P(B) \tag{2.13}$$

Example 2.7 Using Venn diagram to prove that

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C) \tag{2.14}$$

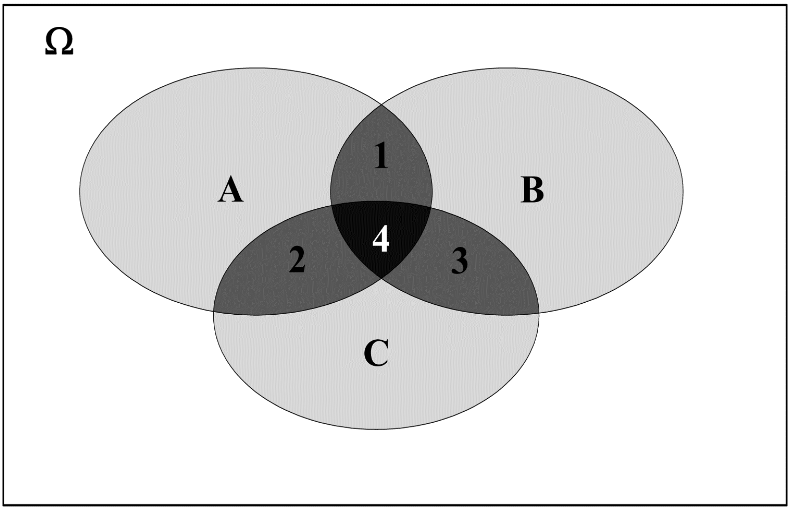


Figure 2.4 Venn diagram illustration of **P(A ∪ B ∪ C)**

Proof: We can use the areas to represent the probability of different parts of the Venn diagram. As such, $P(A \cup B \cup C)$ is the total area of the shaded parts; $P(A)$, $P(B)$, and $P(C)$ are the area of events A, B, and C, respectively. Clearly, $P(A) + P(B) + P(C)$ is more than $P(A \cup B \cup C)$, because parts 1, 2, and 3 in Figure 2.4 are counted twice and part 4 is counted three times. In other words, we have

$$P(A) + P(B) + P(C) = P(A \cup B \cup C) + area1 + area2 + area3 + 2 \times area4$$

Note that

$$P(A \cap B) = area1 + area4$$
$$P(A \cap C) = area2 + area4$$
$$P(B \cap C) = area3 + area4$$
$$P(A \cap B \cap C) = area4$$

Therefore,

$$P(A \cap B) + P(A \cap C) + P(B \cap C) - P(A \cap B \cap C)$$
$$= area1 + area2 + area3 + 2 \times area4.$$

It follows that

$$P(A) + P(B) + P(C) = P(A \cup B \cup C) + P(A \cap B) + P(A \cap C) + P(B \cap C) - P(A \cap B \cap C)$$

That is,

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C).$$ ■

The **conditional probability** of event A given that event B, which has a nonzero probability, has occurred is defined as

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \tag{2.15}$$

Events A and B are independent if and only if, $P(A|B) = P(A)$. This can be interpreted as "whether B has occurred or not has no impact on the probability of A." In other words, the outcome of A has nothing to do with that of B.

Mixing up the concepts of two events being independent and two events being disjoint is a common mistake. The difference in the two concepts is illustrated in the following example

Example 2.8 Assume that $P(A) \neq 0$ and $P(B) \neq 0$. If A and B are independent events, prove that A and B are not disjoint.

Proof: Since $P(A) \neq 0$ and $P(B) \neq 0$, we have $P(A)\ P(B) \neq 0$. If A and B are independent events, then $P(A \cap B) = P(A)\ P(B) \neq 0$, which means that A and B are not disjoint. ■

2.5 System Probability as a Function of Subsystem Probabilities

Sometimes we would like to calculate the probability of a system functioning normally within a specified period of time, given the probabilities of the subsystems functioning normally within the same time period. Here we make the assumption that the subsystems are mutually independent.

Let us first consider two basic configurations: the series and the parallel.

The series configuration is depicted in Figure 2.5.

For this system to work, each subsystem must work. If we denote the events that the system, the subsystem 1, and subsystem 2 work as S, S_1, and S_2 respectively, then

$$P(S) = P(S_1 \cap S_2) = P(S_1)\ P(S_2) \tag{2.16}$$

Similarly, if we have n subsystems connected in series, then the system probability is the product of the subsystem probabilities,

$$P(S) = P(S_1)\ P(S_2) \cdots P(S_n) \tag{2.17}$$

Figure 2.5 Series configuration

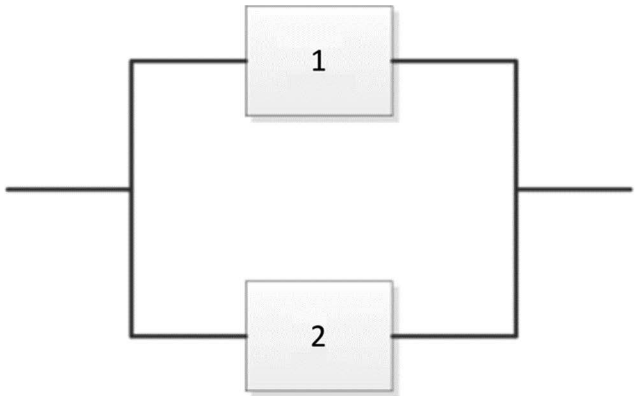


Figure 2.6 Parallel configuration

The parallel configuration is plotted in Figure 2.6. For the system to work, at least one of the subsystems must work. Therefore,

$$\begin{aligned} P(S) &= P(S_1 \cup S_2) = P(S_1) + P(S_2) - P(S_1 \cap S_2) \\ &= P(S_1) + P(S_2) - P(S_1)P(S_2) \end{aligned} \qquad (2.18)$$

Alternatively, we can first calculate the probability of the failure of the system. The system fails only if both subsystems fail, which implies that

$$P(S^c) = P(S_1^c \cap S_2^c) = P(S_1^c)P(S_2^c) = [1 - P(S_1)]\,[1 - P(S_2)]$$

Therefore, $P(S) = 1 - P(S^c) = 1 - [1 - P(S_1)]\,[1 - P(S_2)]$.

This analysis can be easily extended to the case of n parallel-connected subsystems with the following formula:

$$P(S) = 1 - [1 - P(S_1)]\,[1 - P(S_2)] \cdots [1 - P(S_n)] \qquad (2.19)$$

Now that we know how to calculate the system probability as a function of subsystem probabilities for the two basic configurations, we are ready to consider more complicated cases. The following procedure can be applied:

1. First, break down the system into subsystems which are either in series or parallel.
2. Write the probability of the system (being a success) as a function of probabilities of the subsystems.
3. Continue this process until you reach a point where you can calculate the probabilities of all subsystems.

Example 2.9 Derive the system probability as a function of the subsystem probabilities for the system described in Figure 2.7:

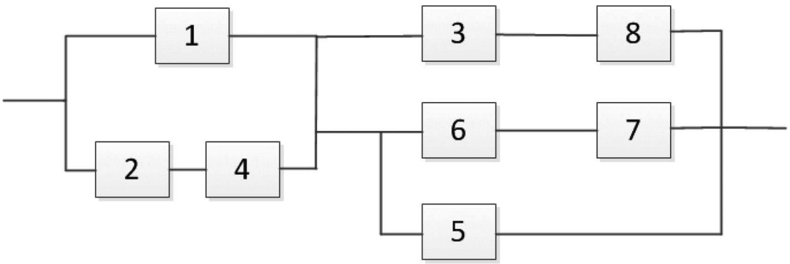


Figure 2.7 System configuration for Example 2.9

Let us first divide the system into two subsystems consisting of (1, 2, 4) and (3, 5, 6, 7, 8), denoted by S_{124} and S_{35678}, respectively. Clearly, these two subsystems are connected in series. Therefore, $P(S) = P(S_{124} \cap S_{35678}) = P(S_{124})P(S_{35678})$.

Now the original problem becomes the calculation of two subsystem probabilities. For each subsystem, we apply the similar technique to divide them into two even simpler subsystems. S_{124} is divided into S_1 and S_{24} with parallel connection. As a result, we have

$$P(S_{124}) = P(S_1 \cup S_{24}) = P(S_1) + P(S_{24}) - P(S_1)P(S_{24})$$

Subsystem S_{24} is comprised of subsystems 2 and 4 in series, so $P(S_{24}) = P(S_2)P(S_4)$.

Subsystem S_{35678} consists of three subsystems, S_{38}, S_{67}, *and* S_5, connected in parallel; therefore, according to Equation (2.19) we have

$$P(S_{35678}) = 1 - [1 - P(S_{38})][1 - P(S_{67})][1 - P(S_5)]$$

$$= 1 - [1 - P(S_3)P(S_8)][1 - P(S_6)P(S_7)][1 - P(S_5)]$$

Alternatively, we can divide S_{35678} into S_{38} and S_{567}, and further divide S_{567} into S_{67} and S_5. The same result will be obtained. Substituting the subsystem probabilities back into the system probability to get

$$P(S) = [P(S_1) + P(S_2)P(S_4) - P(S_1)P(S_2)P(S_4)]\{1 - [1 - P(S_3)P(S_8)][1 - P(S_6)P(S_7)][1 - P(S_5)]\}. \quad \blacksquare$$

2.6 Law of Total Probability and Bayes' Theorem

The law of total probability and Bayes' theorem can provide a quick solution to some problems involving conditional probabilities.

Events $A_1, \ldots, A_k$ are **mutually exclusive** if no two events have any common outcomes. The events are **exhaustive** if $A_1 \cup \ldots \cup A_k = \Omega$, where Ω is the entire sample space.

2.6.1 *Law of Total Probability*

Theorem 2.1 (Law of Total Probability) Let $A_1, \ldots, A_k$ be mutually exclusive and exhaustive events. Then for any event B,

$$P(B) = P(B|A_1)P(A_1) + \ldots + P(B|A_k)P(A_k) = \sum_{i=1}^{k} P(B|A_i)P(A_i) \qquad (2.20)$$

The Law of Total Probability can be easily understood using a Venn diagram and the fact that $P(B|A_i)P(A_i) = P(A_i \cap B)$.

Example 2.10 For any event B, B and B^c are mutually exclusive and exhaustive; therefore,

$$P(A) = P(A|B)P(B) + P(A|B^c)P(B^c) = P(A \cap B) + P(A \cap B^c)$$

This result can also be easily illustrated with Venn diagrams. ■

2.6.2 *Bayes' Theorem*

Theorem 2.2 (Bayes' Theorem) Let $A_1, A_2, \ldots, A_n$ be a collection of n mutually exclusive and exhaustive events. For any event B with $P(B) > 0$, the conditional probability of A_i given that B has occurred is

$$P(A_i|B) = \frac{P(A_i \cap B)}{P(B)} = \frac{P(B|A_i)P(A_i)}{P(B)} = \frac{P(B|A_i)P(A_i)}{\sum_{j=1}^{n} P(B|A_j)P(A_j)} \qquad (2.21)$$

for $i = 1, 2, 3, \ldots, n$.

Example 2.11 A company has three different plants making the same product. Sixty-five percent of the products are made in plant 1, 20 percent in plant 2, and the remaining 15 percent in plant 3. The defect rates in plants 1, 2, and 3 are 1, 2, and 3 percent respectively. What is the probability that a product purchased by a customer is defective? If a consumer purchases a defective product, what is the probability that it was made in plant 2?

Let A_i = {product purchased by a customer is from plant # i} for i = 1, 2, 3
B = {product purchased by a customer is defective}
then the given information implies the following:

$P(A_1) = 0.65$, $P(A_2) = 0.20$, $P(A_3) = 0.15$, $P(B|A_1) = 0.01$, $P(B|A_2) = 0.02$, $P(B|A_3) = 0.03$.

Substituting these values into the equation for the Law of Total Probability, we get

$$P(B) = P(B|A_1)P(A_1) + P(B|A_2)P(A_2) + P(B|A_3)P(A_3)$$

$$= (0.01)(0.65) + (0.02)(0.20) + (0.03)(0.15) = 0.015$$

That is, 1.5 percent of the products will be defective. According to Bayes' theorem,

$$P(A_2 \mid B) = \frac{P(B \mid A_2)P(A_2)}{P(B)} = \frac{(0.02)(0.2)}{0.015} = 0.267$$

which means that the probability of the defective product was made in plant 2 is 26.7 percent. ■

2.7 Probability Distributions and Cumulative Distribution Functions

The **probability distribution** or **probability mass function** of a discrete random variable is defined as

$$\boxed{p(x) \triangleq P(X = x) = P\,(\text{union of all events } s \text{ in } \Omega \text{ that have } X(s) = x) \quad (2.22)}$$

The **cumulative distribution function**, $F(x)$, of a discrete random variable X with probability mass function $p(x)$ is defined as

$$F(x) \triangleq P(X \le x) = \sum_{y:y\le x} p(y) \tag{2.23}$$

Since $\{X > x\}$ is the complement of $\{X \le x\}$, we can calculate $P(X > x)$ in terms of $F(x)$ as follows:

$$P(X > x) = 1 - P(X \le x) = 1 - F(x) \tag{2.24}$$

Similarly, for any two numbers a and b with $a \le b$, we have

$$P\,(a \le X \le b) = F\,(b) - F\,(a-) \tag{2.25}$$

where "$a-$" represents the largest possible value that is strictly less than a.

The **probability distribution** or **probability density function** of a continuous random variable X is a non-negative function $f\,(x)$ such that for any two numbers a and b with $a \le b$,

$$P\ (a \le X \le b) = \int_a^b f(x)\,dx \tag{2.26}$$

This means that the probability of X being inside an interval is equal to the area under the density curve over the interval.

The **cumulative distribution function** $F(x)$ of a continuous random variable X is defined as

$$F(x) = P(X \le x) = \int_{-\infty}^{x} f(y)\,dy \tag{2.27}$$

For each x, $F(x)$ is the area under the probability density curve, $f(x)$, to the left of x.

Since $\{X > x\}$ is the complement of $\{X \le x\}$, we can calculate $P(X > x)$ in terms of $F(x)$ as follows:

$$P(X > x) = 1 - P(X \le x) = 1 - F(x) \tag{2.28}$$

For any two numbers a and b with $a < b$,

$$P(a \leq X \leq b) = F(b) - F(a) \tag{2.29}$$

as shown in Figure 2.8.

Let p be a number between 0 and 1. The **(100p)th percentile** of the distribution of a continuous random variable X, denoted by *percentile*(p), is defined by $p = F(percentile(p))$

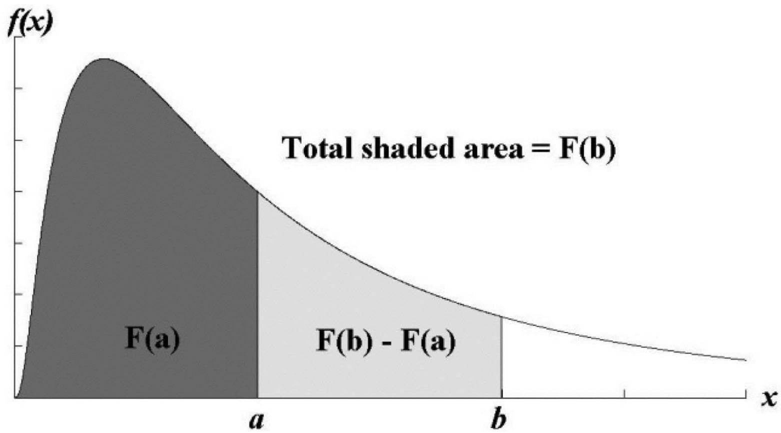


Figure 2.8 Probability of a continuous random variable being between two numbers

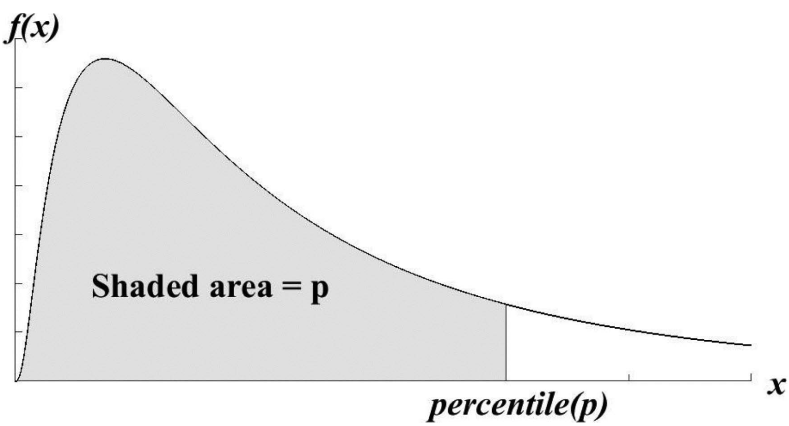


Figure 2.9 (100p)th percentile

Percentile **(p)** is a value on the measurement axis such that ($100p$) percent of the area under the graph of $f(x)$ lies to its left and $100(1 - p)$ percent lies to the right (Figure 2.9).

The **median** of a continuous distribution, denoted by $\tilde{\mu}$, lies at the fiftieth percentile.

2.8 Expected Value and Variance

The expected value or the mean value of a discrete random variable X is defined as

$$E(X) = \mu = \sum_{all\ possible\ x} xp(x) \tag{2.30}$$

where $p(x)$ is the probability mass function of X.

The **expected value** of a continuous random variable X is defined as

$$E(X) = \mu = \int_{-\infty}^{\infty} x f(x) dx \tag{2.31}$$

Similarly, the expected value of any function $h(X)$ of a random variable X denoted by $E[h(X)]$, is defined for discrete random variables as

$$E(h(X)) = \sum_{all\ possible\ x} h(x)p(x) \tag{2.32}$$

and for continuous random variables as

$$E(h(X)) = \int_{-\infty}^{\infty} h(x) f(x) dx \tag{2.33}$$

Using the definitions for the expected values, it is easy to prove that for any constant a and b,

$$E(aX + b) = a E(X) + b \tag{2.34}$$

where X is a discrete or continuous random variable.

The **variance** of a discrete or continuous random variable X is defined as

$$\sigma^2 = V(X) = E[(X-\mu)^2] \tag{2.35}$$

The **standard deviation** of X is $\sigma = \sqrt{V(X)}$. It can be shown that

$$V(aX+b) = a^2\ V(X) \tag{2.36}$$

$$\sigma_{aX+b} = |a|\,\sigma_X \tag{2.37}$$

2.9 Probability Distributions

Many probability distributions can be used in Lean Six Sigma. We only list a few commonly encountered ones. The reader is encouraged to find more information in reference books listed at the end of this chapter.

2.9.1 Normal Distribution

A continuous random variable is said to have a **normal distribution** if its distribution density function is

$$f(x;\mu,\sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/(2\sigma^2)}, \qquad -\infty < x < \infty \tag{2.38}$$

where μ *is* a real number and σ is a positive number.

It can be shown that $E(X) = \mu$ and $V(X) = \sigma^2$, so the parameters μ and σ are the mean and the standard deviation of X. The notation $X \sim N(\mu, \sigma^2)$ is used to denote that X has a normal distribution with mean of μ and standard deviation of σ.

Two probability density functions of normal distributions with different parameter values are plotted in Figure 2.10. Note that the probability density functions are centered at their mean values and the standard deviations indicate how wide spread the probability density functions are.

The cumulative distribution function is given by

$$F(x) = P(X \le x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}\sigma} e^{-(y-\mu)^2/(2\sigma^2)}\, dy \tag{2.39}$$

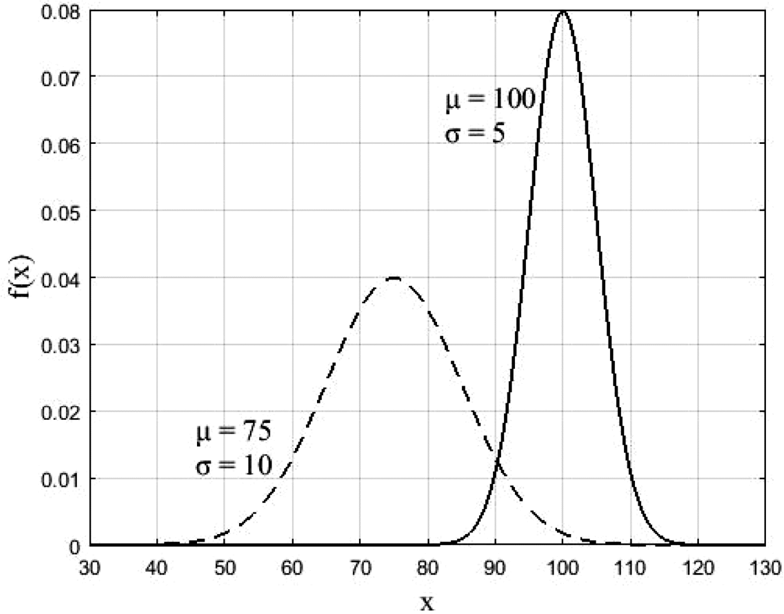


Figure 2.10 Probability density functions of normal random variables

The probability of a normal random variable X with parameters μ and σ being between a and b is determined by

$$P(a \le X \le b) = \int_a^b \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/(2\sigma^2)}\, dx = F(b) - F(a) \tag{2.40}$$

However, the closed form expression of *F(x)* cannot be obtained, except for a few limited special cases such as $x = 0$ and $x = \infty$. Instead of a closed form expression for the integration, we can use numerical methods to obtain an approximation of $P(a \le X \le b)$ for any given values a and b. The accuracy of such approximation is more than enough for most practical cases.

Before computer programs such as Excel were widely used in statistical analysis, we relied largely on the lookup tables of the nominal distribution. Since there are infinitely many values that μ and σ can take, it is impractical to generate lookup tables for all normal distributions. However, one can standardize the random variable to have a normal distribution with $\mu = 0$ and $\sigma = 1$, which is named the **standard normal random variable** and denoted by Z. From the lookup table of $\Phi(z)$, defined as

$\Phi(z) = P(Z < z)$, one can calculate probabilities involving x as explained in the following.

Using the normal distribution density function and the cumulative function, it can be shown that

$$Z = (X - \mu)/\sigma \tag{2.41}$$

is a normal random variable with $\mu = 0$ and $\sigma = 1$. In other words, Z so defined is a standard normal distribution. This property allows us to first convert a normal distribution to the standard normal distribution, then use the lookup table for the standard normal distribution, which is called **standard normal distribution table** or the **Z table**, to find the probabilities of interest. Therefore, only one lookup table is needed for all the normal distributions. A part of the standard normal distribution is shown in Table 2.1.

Table 2.1 Standard normal distribution table

z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545

z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890

Let us discuss how to use the Z table. The graph of the standard normal distribution is called the **standard normal curve** or the ***z* curve**. The area to the left of the point z, under the z curve, is the cumulative distribution function $F(z) = P(Z < z)$, which is denoted by $\Phi(z)$, as illustrated in Figure 2.11. The sum of the numbers in the first column and the numbers in the first row in Table 2.1 are the z values. The value in the corresponding cell is the value $\Phi(z)$.

Example 2.12 Let X be a standard normal random variable. Find $P(X < 0.84)$ and $P(X \geq 0.84)$.

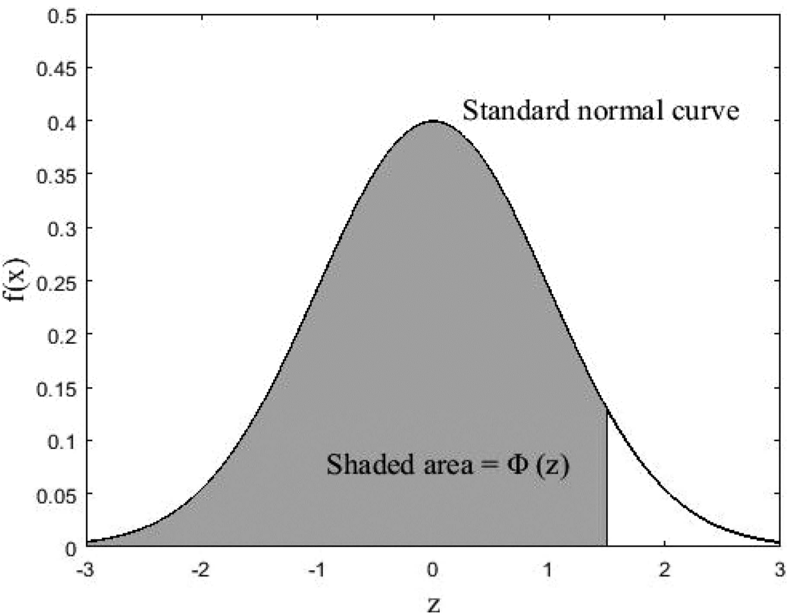


***Figure 2.11** z curve and Φ (z)*

In the first column of Table 2.1, find the ones and tenth digits of 0.84, which is 0.8, then go to the first row and find the hundredth digit 4. The cell in the intersection of the corresponding column and row has a value of 0.7995, which is the probability $P\,(X < 0.84)$.

$$P\,(X \geq 0.84) = 1 - P\,(X < 0.84) = 1 - 0.7995 = 0.2005. \quad \blacksquare$$

Note that the z curve is symmetric about 0. Therefore, it can be seen from Figure 2.12 that

$$\Phi\,(-z) = 1 - \Phi\,(z) \tag{2.42}$$

For this reason, some of the standard normal tables do not include the negative values of z.

Example 2.13 Let X be a standard normal random variable. Find $P\,(X < -0.84)$.

$$P\,(X < -0.84) = \Phi\,(-0.84) = 1 - \Phi\,(0.84) = 1 - 0.7995 = 0.2005. \quad \blacksquare$$

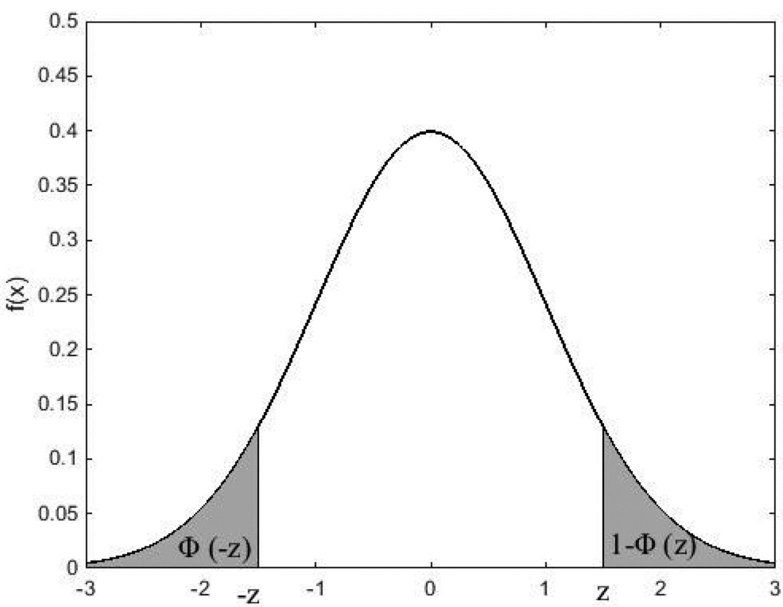


Figure 2.12 ***$\Phi\,(-z) = 1 - \Phi\,(z)$***

Example 2.14 Let X be a standard normal random variable. Find $P(0.15 < X < 0.63)$.

According to the relationship depicted in Figure 2.8,

$$P(0.15 < X < 0.63) = P(X < 0.63) - P(X < 0.15)$$

Using Table 2.1, we can find $P(X < 0.63) = 0.7357$ and $P(X < 0.15) = 0.5596$. Therefore,

$$P(0.15 < X < 0.63) = 0.7357 - 0.5596 = 0.1761. \quad \blacksquare$$

Example 2.15 Suppose $X \sim N(2, 4^2)$, find the probability of $X < -8.0$.

The notation $X \sim N(2, 4^2)$ means that X has a normal distribution with $\mu = 2$ and $\sigma = 4$. Since X does not have a standard normal distribution, we cannot use Table 2.1 directly. We must first use Equation (2.41) to transform X into the standard normal random variable $Z = (X - 2)/4$.

The event $X < -8.0$ is equivalent to $X - 2 < -8.0 - 2$, which in turn is the same as

$$(X - 2)/4 < (-8.0 - 2)/4 \text{ or } Z < (-8.0 - 2)/4 = -2.5$$

Therefore, $P(X < -8.0) = P(Z < -2.5) = \Phi(-2.5) = 1 - \Phi(2.5) = 1 - 0.9938 = 0.0062.$ ■

Another usage of Table 2.1 is to find the z value corresponding to a given probability or the area under the z curve to the left of z. We use the following example to demonstrate.

Example 2.16 Let X be a standard normal random variable. Given $P(X < a) = 0.7088$, find a, that is, the 70.88th percentile.

In Table 2.1, first find the value 0.7088 in the cells other than the first column and first row. Once we identify the cell with 0.7088, we go to the left to find the first two digits of z which are 0.5, and then go to the top to find the third digit 0.05. Adding these two numbers, we have $a = 0.55$. ■

It is worth noting that the numbers increase from left to right and from top to bottom. If you cannot find the exact probability number, use the number that is closest to the probability. If necessary, you can do a linear interpolation between two adjacent numbers in the table.

Nowadays, software packages such as Minitab, MATLAB, and Excel can be used to conduct statistical analysis instead of the lookup tables. These software packages completely change the way probability and statistics are taught. For example, the transformation in Equation (2.41) used to be a must-learn skill for analyzing normal random variables. With the capabilities of the software packages, it has become optional. The accuracy of the result with the software can be higher than that provided by the lookup tables; no linear interpolation is necessary either. As a practitioner, one should definitely learn how to use Excel to perform basic statistical analysis due to its wide availability.

Example 2.17 Reproduce the results in Table 2.1 and Examples 2.12–2.16.

To create Table 2.1, fill Column A with the first two digits of the *z* values, starting from 0. Then fill Row 1 with the third digits of the *z* values, from 0.00 to 0.09. The cumulative probability values are calculated using the Excel function NORM.S.DIST. Figure 2.13 captures the screen when creating the cumulative probability for cell F10. Once the formula is typed in one cell, you can copy and paste it to other cell to complete Table 2.1. To learn more about "$" and its purpose, use the

F10 *fx* =NORM.S.DIST($A10+F$1,TRUE)

	A	B	C	D	E	F	G	H	I
1	**z**	**0**	**0.01**	**0.02**	**0.03**	**0.04**	**0.05**	**0.06**	**0.0**
2	**0.0**	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.52
3	**0.1**	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.56
4	**0.2**	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.60
5	**0.3**	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.64
6	**0.4**	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.68
7	**0.5**	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.71
8	**0.6**	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.74
9	**0.7**	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.77
10	**0.8**	0.7881	0.7910	0.7939	0.7967	**0.7995**	0.8023	0.8051	0.80

Figure 2.13 Screen capture for Example 2.17

Help function in Excel. One can easily increase the resolution of the z values and display more digits of the cumulative probability than what are shown in Table 2.1. Note that earlier versions of Excel used NORMSDIST for this function, and in future versions of Excel, the function name may change. To make sure that you are inserting the correct function, type "= N" in a cell, and a list of functions that have names start with N will be displayed. You can click on a function to find out what it is for.

The results in Example 2.12 can be obtained using Excel as illustrated by the screen captures in Figure 2.14. The results are displayed for six digits after the decimal point to show that more accurate results than Table 2.1 are possible. Once the formulas are set up in Excel, you can change the z value to update the probabilities. For example, you can type in 0.8432 as the z value to calculate the probability without having to do any linear interpolation as you would if you used Table 2.1.

In the Excel program shown in Figure 2.14, if you type "–0.84" in B1, you will see 0.200454 in B2 as the probability of $X < -0.84$.

With slight modification, one can calculate the probability in Example 2.14 as illustrated in Figure 2.15.

B2 fx =NORM.S.DIST(B1,TRUE)

Book1

	A	B	C	D	E	F
1	z	0.84				
2	P(X<z)	0.799546				
3	P(X>z)	0.200454				

B3 fx =1-B2

Book1

	A	B	C	D	E
1	z	0.84			
2	P(X<z)	0.799546			
3	P(X>z)	0.200454			

Figure 2.14 Screen captures for Example 2.17

B5 fx =B4-B3

Book1

	A	B	C	D
1	a	0.15		
2	b	0.63		
3	P(X<a)	0.559618		
4	P(X<b)	0.735653		
5	P(a<X<b)	0.176035		

Figure 2.15 Screen capture for Example 2.17

D2 | =NORM.DIST(A2,B2,C2, TRUE)

Excel examples

	A	B	C	D	E	F
1	a	mean	stdev	P(X<a)		
2	-8.0	2	4	0.00621		

Figure 2.16 Screen capture for Example 2.17

B2 | =NORM.S.INV(A2)

Excel examples

	A	B	C	D	E	F
1	P(X<a)	a				
2	0.7088	0.549882				

Figure 2.17 Screen capture for Example 2.17

Without the transformation from X to Z, the probabilty in Example 2.15 can be obtained with Excel using the NORM.DIST function, as captured in Figure 2.16.

The calculation in Example 2.16 requires the use of NORM.S.INV function with Excel, as shown in Figure 2.17.

The result is a = 0.549882. ■

2.9.2 Student's t-Distribution

A continuous random variable is said to have a ***t*-distribution** if its probability density function is in the form

$$f(x) = \frac{\Gamma\left(\frac{v+1}{2}\right)}{\sqrt{\pi v}\,\Gamma\left(\frac{v}{2}\right)}\left(1+\frac{x^2}{v}\right)^{-(v+1)/2}, \quad -\infty < x < \infty \tag{2.43}$$

where v is the **degree of freedom**, Γ is the gamma function defined as

$$\Gamma(a) = \int_0^\infty t^{a-1} e^{-t}\, dt \tag{2.44}$$

Probability density functions for Student's t-distribution with various v values are plotted in Figure 2.18.

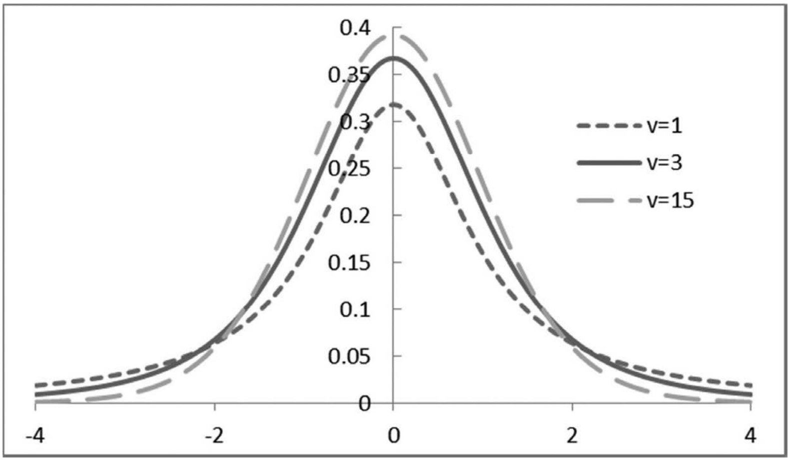


Figure 2.18 Probability density function of Student's t-distribution

Student's t-distribution is useful for analyzing whether the population mean is above or below certain values.

Assuming X is a random variable and n samples of its values are taken, the probability that

$$t = \frac{(\bar{X} - \mu)\sqrt{n}}{s} \tag{2.45}$$

is between a and b is equal to the area under the Student's t-density function between a and b.

Example 2.18 Ten measurements of a random variable are as follows:

$$x_1 = 10.0,\ x_2 = 11.1,\ x_3 = 9.9,\ x_4 = 10.1,\ x_5 = 11.2,\ x_6 = 9.7,\ x_7 = 11.5,$$
$$x_8 = 9.8,\ x_9 = 10.1,\ x_{10} = 11.2.$$

Find the probability that the random variable has a population mean greater than 10.0.

The average of the samples is 10.46, and the sample standard deviation is 0.6979.

$$P(\mu > 10) = P(-\mu < -10) = P\left(\bar{X} - \mu < \bar{X} - 10\right) = P\left(\frac{\bar{X} - \mu}{s}\sqrt{n} < \frac{\bar{X} - 10}{s}\sqrt{n}\right)$$
$$= P\left(t < \frac{10.46 - 10}{0.6979}\sqrt{10}\right) = P(t < 2.084)$$

Using Excel function `T.DIST (2.084, 9, TRUE)`, we find that the probability of the event {X has a population mean greater than 10.0} is 96.66 percent. ■

Example 2.18 illustrates how one can use data to draw a conclusion about the population; this is a typical use of statistics. We can also use the data in this example in a slightly different way. For a given probability, can we conclude that the population mean is greater than 10.0? This is the subject of hypothesis test, which will be discussed in Section 4.3.2.

2.9.3 F-Distribution

A continuous random variable is said to have an ***F*-distribution** if its probability density function is

$$f(x)=\begin{cases}\dfrac{\Gamma(\frac{v_1+v_2}{2})\left(\dfrac{v_1}{v_2}\right)^{v_1/2} x^{(\frac{v_1}{2}-1)}}{\Gamma(v_1/2)\Gamma(v_2/2)\left(1+\dfrac{v_1 x}{v_2}\right)^{(v_1+v_2)/2}}, & \text{if } x>0 \\ 0, & \text{if } x\le 0\end{cases} \tag{2.46}$$

where v_1 and v_2 are the two degrees of freedom. Probability density functions of the F-distribution with various degrees of freedom values are plotted in Figure 2.19.

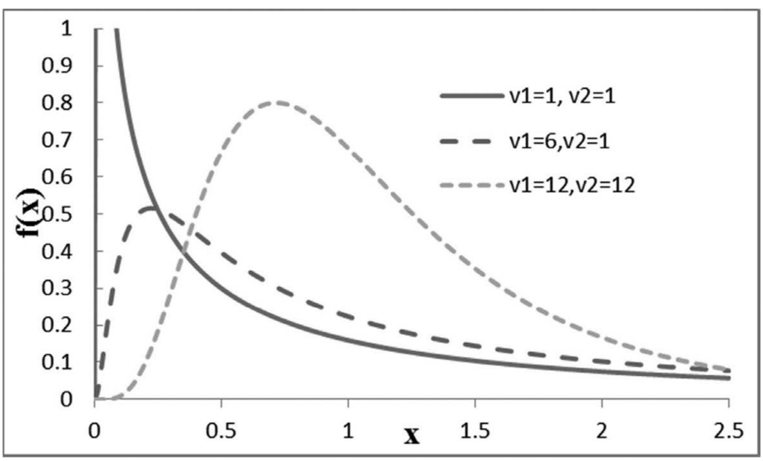


Figure 2.19 Probability density function for F-distribution

F-distributions can be used to compare the population standard deviation of two random variables based on sampled values. If X and Y are two independent normal random variables, with m samples for X and n samples for Y, then the following random variable

$$f = \frac{S_1^2}{S_2^2}$$

has an *F*-distribution with $v_1 = m - 1$ and $v_2 = n - 1$. S_1, S_2 are the sample standard deviations of X and Y, respectively.

2.9.4 Chi-square Distribution

A continuous random variable is said to have a **Chi-square (or χ^2) distribution** if its probability density function is

$$f(x) = \begin{cases} \dfrac{x^{(\frac{v}{2}-1)} e^{-x/2}}{2^{v/2}\Gamma(v/2)}, & \text{if } x > 0 \\ 0, & \text{if } x \le 0 \end{cases} \tag{2.47}$$

where v is the degree of freedom and is a positive number, and $\Gamma(\bullet)$ is the gamma function as defined in Equation (2.44). Probability density functions for χ^2 distribution with various degree of freedom values are plotted in Figure 2.20.

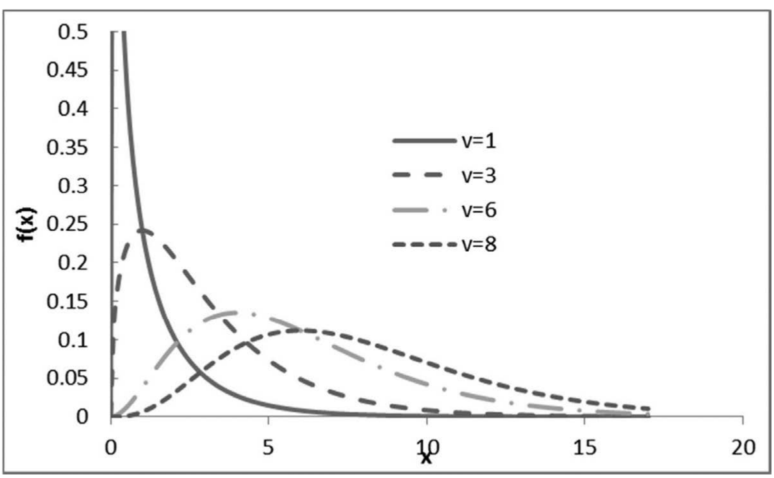
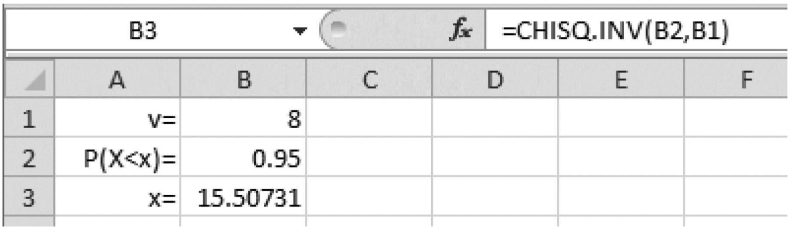


Figure 2.20 Probability density function for χ^2 distribution



B3		fx	=CHISQ.INV(B2,B1)			
	A	B	C	D	E	F
1	v=	8				
2	P(X<x)=	0.95				
3	x=	15.50731				

Figure 2.21 Screen capture for Example 2.19

If $Z_1, Z_2, \ldots, Z_n$ are standard normal random variables, then the sum of squares of these variables is a random variable with n degrees of freedom.

Example 2.19 Let X be a χ^2 random variable with 8 degrees of freedom. Use Excel to find the x value such that $P(X < x) = 95\%$.

The solution to this problem is shown in Figure 2.21. ■

2.9.5 Binomial Distribution

A discrete random variable has a **binomial distribution** if its probability mass function is of the form

$$p(x) = \begin{cases} \dfrac{n!}{x!(n-x)!} q^x (1-q)^{n-x} & x = 0,1,2,\ldots,n \\ 0 & \textit{otherwise} \end{cases} \tag{2.48}$$

where q is a number between 0 and 1. The binomial distribution can be used to model the probability of exactly x successes in n trials. Each trial has two possible outcomes, success or failure, with q as the probability of one trial being a success.

Example 2.20 The defect rate of certain product is 2 percent. If 20 random samples of the product are tested, what is the probability of having less than or equal to 3 defective products?

Let X be the random variable of the number of non–defective products among the 20 samples. Then X is a binomial random variable and $n = 20$. With a defect rate of 2 percent, $q = 0.98$. We need to calculate $P(X \geq 17)$.

$$P(X \geq 17) = p(17) + p(18) + p(19) + p(20)$$

This can be calculated either by directly evaluating the four terms using the binomial probability mass function or by typing "= 1 − BINOM.DIST(16, 20, 0.98, 1)" in an Excel cell. This gives us the probability of 0.9994. ■

2.9.6 Lognormal Distribution

A continuous random variable X is said to have a **lognormal distribution** if *ln (X)* is a normal random variable. The probability density function of a lognormal random variable is of the form

$$f(x;\mu,\sigma)=\begin{cases}\dfrac{1}{\sqrt{2\pi}\sigma x}e^{-[\ln(x)-\mu]^2/(2\sigma^2)} & x>0\\ 0 & x\leq 0\end{cases} \tag{2.49}$$

It can be verified that μ is the mean of *ln(X)*, not of X. Similarly, σ is the standard deviation of *ln(X)*. Three probability density functions of lognormal distribution are plotted in Figure 2.22, with different values of μ and σ.

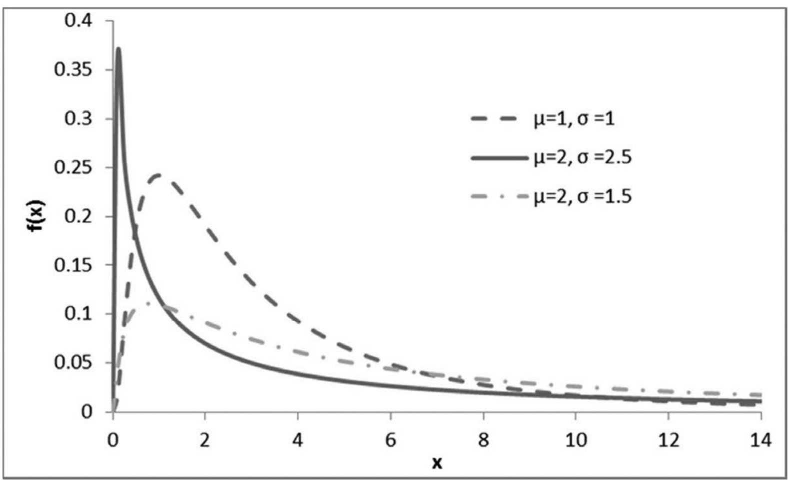


Figure 2.22 Probability density functions for lognormal distribution

2.10 The Central Limit Theorem

There are several reasons why the normal distribution is one of the most important distributions in statistics. First, many random variables are normal, including the weights and heights of a human population, measurements of dimensions, etc. Second, many useful hypothesis tests require that the underlying random variables have normal distributions. Last but not least, the average of the sample values can be approximated with normal distributions even for random variables that are not normal. This last reason is the essence of the **Central Limit Theorem (CLT)**.

> **Theorem 2.3 (Central Limit Theorem)** If a random variable X has a mean μ and a finite variance σ^2, then the average of the n samples $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} x_i$ has a distribution that can be approximated by a normal distribution with mean μ and variance $\frac{\sigma^2}{n}$ if n is sufficiently large.

The size of n depends on the underlying distribution. For most cases, the approximation is sufficiently accurate for $n > 30$.

Example 2.21 A sensor output has a mean value of 40 mV and a standard deviation of 5 mV. If 35 values are measured and the average is calculated for these measurements, what is the probability that the average value is between 39 and 41 mV?

According to the Central Limit Theorem, the average value is approximately a normal distribution with a mean of 40 mV and a standard deviation of $5/\sqrt{35}$ = 0.8452 mV.

$$P\left(39 < \bar{X} < 41\right) = 0.7633$$

which can be calculated using the transformation in Equation (2.41) and Table 2.1, or alternatively, by typing "= NORM.DIST(41, 40, 0.8452,1) − NORM.DIST(39, 40, 0.8452,1)" in an Excel cell. ■

2.11 Statistical Data Analysis

When dealing with large amounts of data, it is usually helpful to use some numerical measures as indicators of certain properties of the entire data set. The two main properties of a data set are its center position and variability.

2.11.1 The measure of center position of the data

The mean, the mode, and the median are common indicators of the center position of a data set, as defined in the following.

Mean or average: Given a set of data $x_1, x_2, \dots x_n$, the sample mean or average is defined as

$$\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \tag{2.50}$$

Note that the sample mean is sensitive to the presence of outliers in the data set.

Mode: The mode of a data set is the value(s) that appears most frequently.

Median: The median of a data set is determined by first sorting the data in ascending order followed by repeatedly eliminating the smallest and largest numbers until there are one or two data points remaining. The median is either the remaining data point or the average of the two remaining data points.

Note that the median is insensitive to the presence of outliers in the data set—sometimes too insensitive.

2.11.2 The measure of variability

The range and the standard deviation are common measures of the variability of the values in a data set.

Range: The range of a data set is defined as the difference between the largest value and the smallest value of the data set.

Standard deviation: The sample standard deviation of a data set is defined as

$$s = \sqrt{\frac{\sum_{i=1}^{n}\left(x_i - \overline{x}\right)^2}{n-1}} \tag{2.51}$$

where $\bar{x}$ is the sample mean, n is the sample size, and s^2 is the sample variance.

2.11.3 Other measurements of data

Quartiles: The median divides the data set into two subsets: the smaller half and the larger half. The median for each subset can be found and are known as the **lower and upper fourth**. The lower fourth (Q_1), the median (Q_2), and the upper fourth (Q_3), also known as the **first, second and third quartiles**, divide the data into four quarters.

Trimmed mean: To calculate the trimmed mean, certain percentage of the sorted data from each end is eliminated first and then the average is calculated for the remaining data. For example, a 10 percent trimmed mean is the average of the data after eliminating the smallest 10 percent data and the largest 10 percent data. The trimmed mean can be viewed as a compromise between the mean and the median.

2.12 Graphical Methods

Sometimes it is easier to draw a conclusion from a data set using graphical methods instead of numerical measurements, as we shall explain in the following.

2.12.1 Stem-and-Leaf Plots

Stem-and-leaf is a graphical method of displaying quantitative data. A steam-and-leaf plot consists of the **stem**, the **leaf**, and a vertical line that separates the stem and the leaf. Other information such as the total number of data points, the **stem or leaf unit**, and the title of the plot may be included as well.

A stem-and-leaf plot can be created as follows:

1. Determine the digits for the stem. The stem may have one or more digits. The trailing digits become the leaves.
2. Draw a vertical line. List the stem values on the left side of the line in an incremental, ascending manner.
3. Record the data points one at a time by finding the row of the corresponding stem value and recording the leaf value to the right side of the vertical line.
4. Indicate the stem or leaf unit, the total number of data points, the title, etc.

The leaf values may or may not be sorted. The decimal point should be left out, since the stem or leaf unit is specified. If the leaf has one digit only, no space between leaf values is needed. If the leaf has more than one digit, a space is left between two leaf values. The stem value can be in increments of 1, 10, or other values.

Example 2.22 Construct the stem-and-leaf plot for the following set of data.

15.2	9.2	12.1	8.8	9.5	18.9	19.2	30.9	45.8	25.6
24.5	15.2	10.0	49.9	41.7	23.4	24.8	31.2	26.8	27.3

We choose the tens digit as the stem unit and the ones and one-tenth digits as leaf digits. In this example, the stem values are chosen to be 0 to 4 with 1 to 4 repeated twice. For instance, 1L is the stem for those numbers whose tens digit is 1 and leaf values are less than 50. 1H is the stem for those numbers whose tens digit is also 1 but leaf values are greater than or equal to 50.

0H	92	88	95		
1L	21	00			
1H	52	89	92	52	
2L	45	34	48		
2H	56	68	73		
3L	09	12			stem digit: tens digit
3H					total number of points: 20
4L	17				
4H	58	99			

Alternatively, we can construct the following stem-and-leaf plot for the same set of data, the only difference being the designation of the stem values.

0	92	88	95				
1	52	21	89	92	52	00	stem digit: tens digit
2	56	45	34	48	68	73	total number of points: 20
3	09	12					
4	58	99	17				

■

2.12.2 Histogram

A **histogram** is used to display the distribution of a data set. To construct a histogram, the horizontal axis is first divided into equal intervals, called **bins**. For example, the horizontal axis can consist of 5 bins of a width of 4, from 0 to 20 (excluding 20), where the bins are defined as [0, 4), [4, 8), [8, 12), [12, 16), and [16, 20). (Here, the brackets mean inclusive and parentheses mean exclusive.) The frequency for each bin is calculated by counting how many numbers in the data set fall inside the bin. Then a rectangle is drawn for each bin, with the bin value as its width and the frequency as its height. Alternatively, the frequencies can be replaced by relative frequencies, that is, the frequencies divided by the total number of data points in the set. As a rule of thumb, the number of bins should be close to the square root of the total number of data points in the data set. If a software package is used, one can easily try different numbers of bins until a satisfactory result is achieved. Usually, the number of bins should be between 5 and 20.

Figure 2.23 shows the histogram of the data set in Example 2.22. The graph was created with Excel. Other software packages such as MATLAB and Minitab can also be used to create histograms.

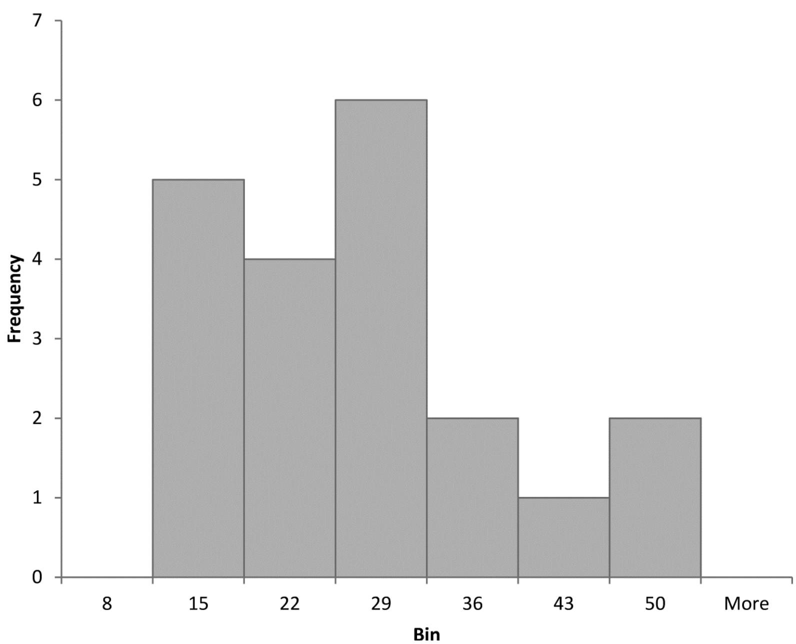


Figure 2.23 A histogram of data in Example 2.22

2.12.3 Box-and-Whisker Plot

The **box-and-whisker plot**, also known as **boxplot**, is determined by five values: the smallest value (minimum), lower fourth (first quartile Q_1), median (second quartile Q_2), upper fourth (third quartile Q_3), and the largest value (maximum). Q_1, Q_2, and Q_3 were defined earlier in Section 2.11.

We use the following example to illustrate the construction of a box-and-whisker plot.

Example 2.23 Display the following data set with a box-and-whisker plot:

10	20	23	34	45	46	46	46	48	51
52	52	56	68	70	90	100	110	120	130

First, we find the five values needed for the plot:

$$x_{min} = 10,\ Q_1 = 45.5,\ Q_2 = 51.5,\ Q_3 = 80,\ x_{max} = 130.$$

We draw a horizontal number line extending from x_{min} to x_{max}. A rectangle with Q_1 and Q_3 as the two sides is plotted, its height being arbitrary. Then add a vertical line segment at Q_2 inside the rectangle. Draw a horizontal line (whisker) from x_{min} to the left or Q_1 side of the rectangle and another horizontal line (whisker) from x_{max} to the right or Q_3 side of the rectangle. Figure 2.24 shows the completed box-and-whisker plot.

There are variations of the box-and-whisker plot. Some depict the outliers and extreme outliers on the plot. To determine the outliers and extreme outliers, the fourth spread is first defined.

$$f_s = Q_3 - Q_1 \tag{2.52}$$

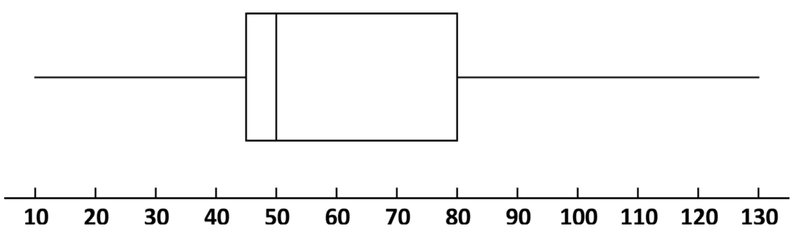


Figure 2.24 A box-and-whisker plot example

■

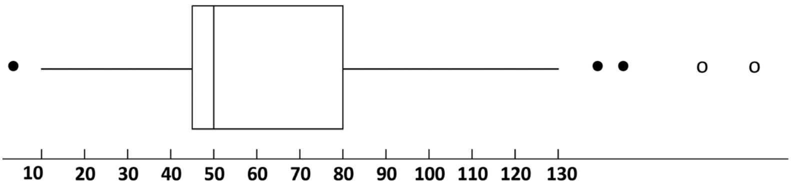


Figure 2.25 A box-and-whisker plot with outliers

Any number x_i that satisfies one of the following conditions is said to be an **outlier**.

$$x_i > Q_3 + 1.5 f_s \quad or \quad x_i < Q_1 - 1.5 f_s \tag{2.53}$$

Furthermore, x_i is an **extreme outlier** if it satisfies one of the following conditions.

$$xi > Q3 + 3 fs \quad or \quad xi < Q1 - 3 fs \tag{2.54}$$

Those outliers that are not extreme outliers are called **mild outliers**.

In Figure 2.25, the whiskers are shown to go from the box to the smallest and largest points that are not outliers. Mild outliers are indicated by solid dots and extreme outliers by small circles.

Further readings

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CHAPTER 3

DMAIC: The Process of Lean Six Sigma

Every product or process has an intended function with an expected outcome—called a **nominal value**—and every outcome has some variation. If you sign your name twice, the signatures will not look exactly the same. Similarly, two vehicles of the same model and year are not exactly the same. These are known as **variations**. Such variation in a set of data can be measured by levels of "**sigma**," as we shall explain. Individual customers don't experience the average performance of a product; instead, they each experience a variation of the product, some better than the average and some worse. By reducing the variation of a product or process, more consumers can receive products or services of higher quality. This chapter discusses the DMAIC process of Lean Six Sigma that helps reduce the variation and improve a product or service. A real-world example with the Krisys robot is used to illustrate the generic execution steps of a Lean Six Sigma project. Details of the tools used in this chapter can be found in Chapter 4.

3.1 Introduction

Lean Six Sigma is a structured, disciplined, data-driven methodology and process, which focuses on improving business performance based on **Voice of Customer** (VOC) and statistical analysis. Lean Six Sigma can be applied to any situation where there is a process, such as in the fields of engineering, service, health care, military, and many others. The essence of Lean Six Sigma is the improvement through reduction of the variation and waste in a process [1, 2]. However, it is not exclusively limited to the improvement of existing processes. For example, **Design for Six Sigma (DFSS)** is best practiced when designing a new product or a new process [3, 4].

Lean Six Sigma uses a number of unique tools as well as some common ones such as project management, quality function deployment, cause-and-effect diagrams, gage R&R, statistical process control (SPC), and lean manufacturing.

3.2 Six Sigma as a Statistical Measure

The Greek letter σ (sigma) represents the standard deviation of random variables. A Six Sigma process implies that the interval between the **Upper Specification Limit (USL)** and the **Lower Specification Limit (LSL)** contains the mean value ±6 × standard deviation of the random variable; therefore, the probability of having any defects is extremely small (Figure 3.1).

Suppose that the mean of a normal distribution is at the center of the specification limits. The probability for success when the specification limits are mean $\pm n\sigma$ is simply the area under the normal curve between $-n\sigma$ and $n\sigma$, for $n = 1, 2, \ldots, 6$. Consequently the probability for defects is the area outside of the interval $(-n\sigma, n\sigma)$. The probability for defects is sometimes multiplied by 10^6 to calculate the **Defects per Million Opportunities (DPMO)**.

Assuming that the mean value of the distribution is centered at the midpoint between the USL and LSL, Table 3.1 summarizes the DPMO corresponding to n sigma levels, where $n = 1, 2, \ldots, 6$.

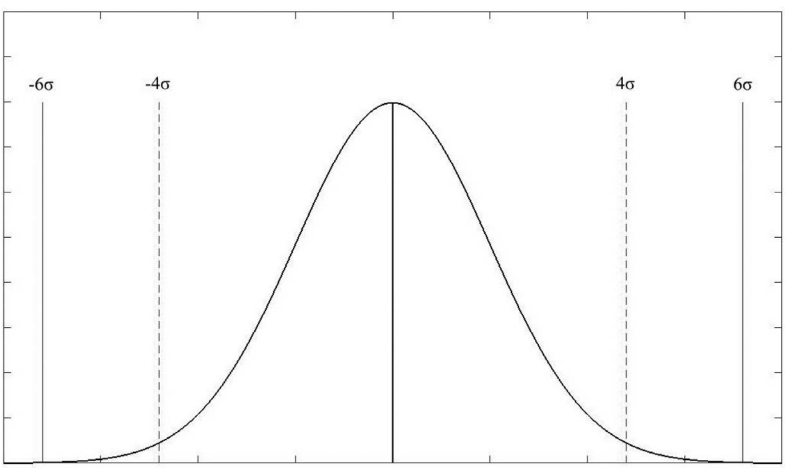

Figure 3.1 Six Sigma and USL/LSL

Table 3.1 DPMO and probabilities of success

Sigma	Defects per Million Opportunities	Probability of Success (%)
1	317,311	68.27
2	45,500	95.45
3	2,700	99.73
4	63.3	99.9937
5	0.57	99.99994
6	0.00197	99.9999998

It was noted in the early days of Six Sigma that over time the center might shift to the left or right by as much as 1.5 times of sigma, as illustrated in Figure 3.2. When that happens, the calculation of the probability of success needs to be modified accordingly, as illustrated in Figure 3.3.

The positions of the specification limits remain the same in the plot, but the normal distribution curve is shifted to the right by 1.5σ. The case of the mean shifted to the left can be treated similarly. The DPMO numbers are identical whether it is shifted to the left or right, since the curve is

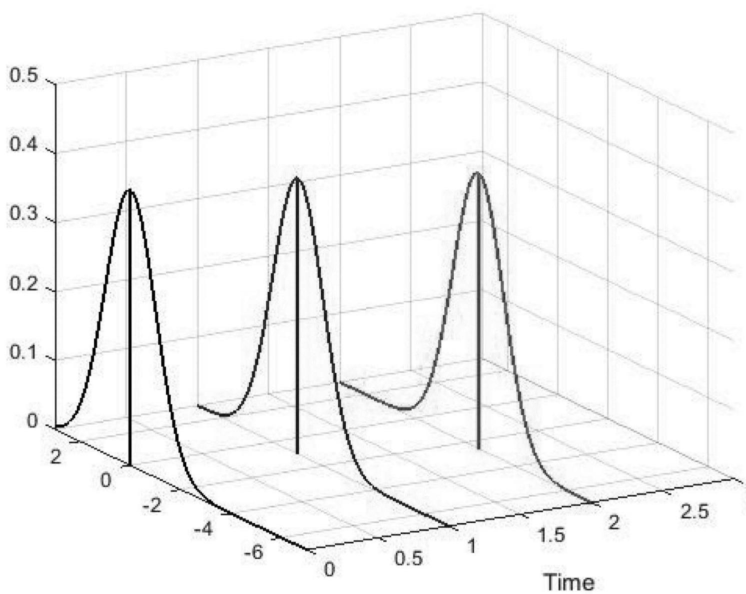

Figure 3.2 Shift of the process mean over time

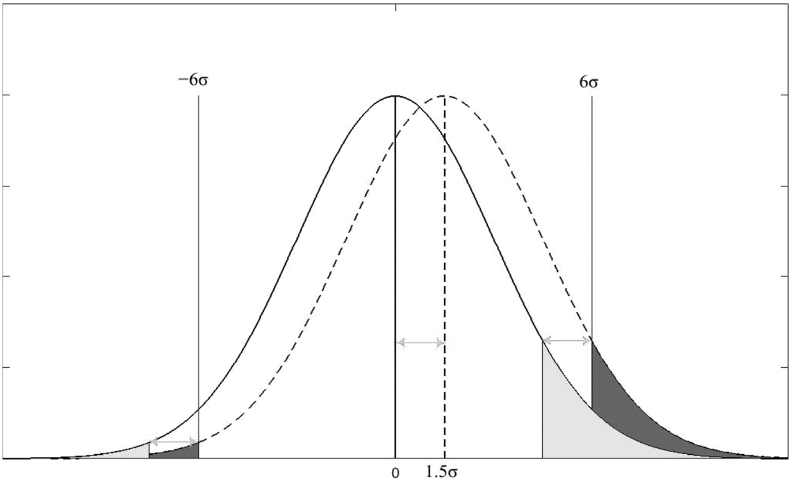

Figure 3.3 Six Sigma and USL/LSL with 1.5σ shift to the right

symmetric. The area to the left of the -6σ line is given by Φ $(-6 - 1.5)$, where Φ (z) is the cumulative distribution function of the standard normal random variable. The area to the right of the 6σ line is $1 - \Phi\ (6 - 1.5)$. Therefore, the probability of success, that is, the area under the shifted density function between the specification lines, is given by

$$1 - [\Phi\,(-6 - 1.5) + 1 - \Phi\,(6 - 1.5)] = \Phi\,(4.5) - \Phi\,(-7.5)$$

The DPMOs and probabilities of success for processes with 1.5σ shift in mean are summarized in Table 3.2. Note that the DPMOs are greater than those without the shift in mean.

Table 3.2 DPMO and probabilities of success with 1.5σ shift in mean

Sigma	Defects per Million Opportunities	Probability of Success (%)
1	697,672	30.233
2	308,770	69.123
3	66,811	93.319
4	6,210	99.379
5	233	99.977
6	3.4	99.99966

Example 3.1 Find the sigma level for a normal process that has a 97 percent success rate with a 1.5σ shift in mean.

This calculation can be easily done in Excel. Type a positive value in cell A1 and "=100*(NORM.S.DIST(A1−1.5, 1)−NORM.S.DIST(−A1−1.5,1))" in cell B1. Change the value in A1 until you have 97 percent in B1.The value in A1 is the sigma level that we are looking for. The answer is 3.3808. ■

One may ask, Why do we have to achieve Six Sigma? or Why isn't a 97 percent success rate good enough? It all depends on the situation. If it is a matter of life or death as are medical operations, airplane landings, or space explorations, a 97 percent success rate is not acceptable. Even Six Sigma may not be good enough. On the other hand, there are many situations where 97 percent success rate is sufficient; examples are the tolerance bands for resistors or capacitors in a noncritical product. The essence of Six Sigma is not about the specific value of DPMO. Instead, it is about how to achieve the required success rate through continuous improvement to reduce the variation in the process.

3.3 The DMAIC Process

Lean Six Sigma can be viewed as a tool, but more importantly, it is a process [5–7]. It consists of five phases: **Define**, **Measure**, **Analyze**, **Improve**, and **Control**, commonly called **DMAIC**. Figure 3.4 illustrates the DMAIC process, with the main tasks of each phase specified.

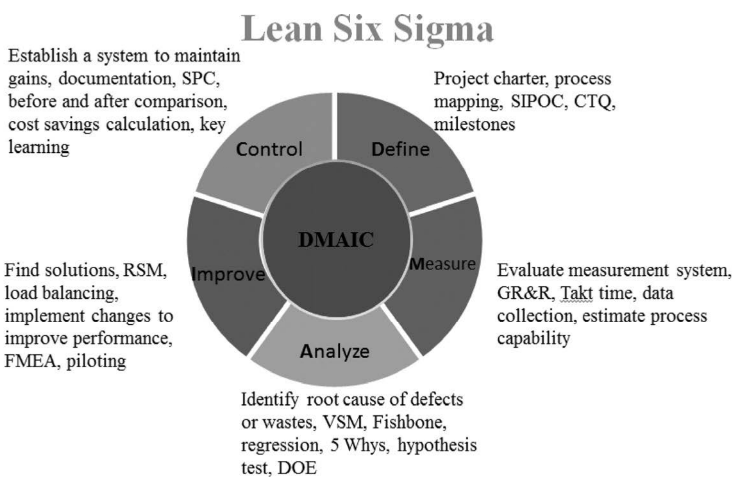

Figure 3.4 The Six Sigma DMAIC process

Each of the five phases will be discussed in a separate subsection. For easier understanding, we will use a Lean Six Sigma project conducted by a group of students at Texas A&M University as an example [8]. Additional case studies will be discussed in Chapter 8.

3.3.1 The Krisys Robot

The Krisys robot, shown in Figure 3.5, is a platform used in one of the digital electronics courses in the Electronic System Engineering Technology (ESET) program at Texas A&M University [8]. It has a microcontroller on a printed circuit board to control two motors in order to move the platform along a trajectory embedded with signals that the robot can sense. A group of students made an effort to commercialize the product with the objective to educate students in product development and innovation. Their intention was to make the Krisys robot kits, and sell them to high schools and colleges interested in using them in course projects for their science and engineering classes. The original design had not considered making a profit in mass production, leaving room for improvement. This new student team was able to make the robot more profitable by following the DMAIC process.

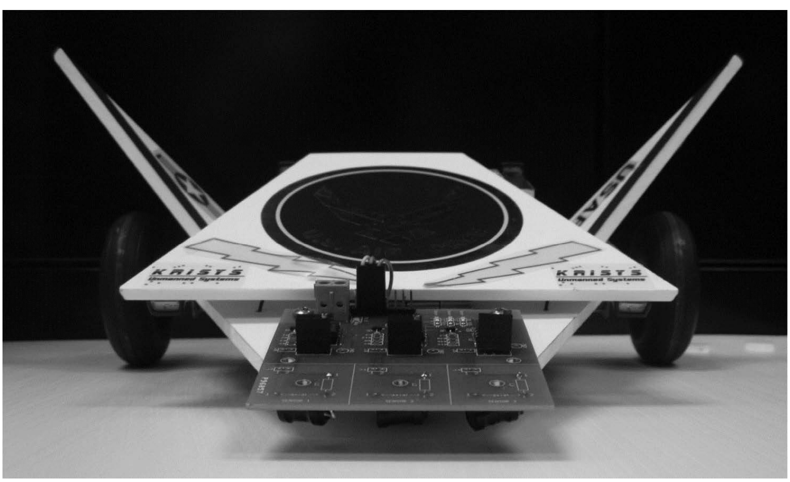

Figure 3.5 A Krisys robot

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3.3.2 *Define*

The main objective of the Define phase is to establish the **project scope** and collect background information about the current process. A **project charter**, typically a one-page summary of the project, is created in this phase. The project charter will be approved by the sponsor or manager of the project before it is started. The **Suppliers-Inputs-Process-Outputs-Customers (SIPOC)** analysis is conducted once the project is approved, to make sure that the project team fully understands the current process, who the **stakeholders** are, and what the customers want from the process. The VOC is collected and analyzed, and the **Critical-to-Quality (CTQ)** analysis is conducted to achieve a high-level understanding of what is critical to the success of the process.

The project charter should contain the following:

- A problem statement.
- A clearly stated business case for the intended Six Sigma project.
- A well-defined project scope.
- Identification of project sponsor.
- A specific **performance metric** or metrics that can be used to evaluate the performance improvement before and after the project.
- Identification of the stakeholders, process owner, project team, and project champion.
- Resources required.
- **Potential risks** of the project not being able to achieve the intended result within the specified project duration.

Let us see how the Define phase was executed in the Krisys project.

Project Charter for the Krisys Robot Kit

Problem statement: The current Krisys robot kit operation is not competitive enough in the marketplace, mainly due to its high cost. As a part of the commercialization effort, the cost must be significantly reduced.

Project scope: The project will focus on improving the design of the current process of purchasing, making, shipping, and supporting the kit. Circuit optimization will not be considered in this project. Due to the time limitation, the improvement identified in this project will be implemented in a small scale only. Large-scale implementation will be carried out in a future project.

Business case: Reducing the cost will improve the profit margin.

Project sponsor: ESET program.

Project duration: The project will be completed in 7 weeks.

Resource: Ten students and a faculty advisor.

Project champion: The faculty advisor for the student team.

Metric: The total cost of the Krisys robot kits will be used as the performance measure.

The project goal: To reduce the cost of the kit by 25 percent.

Deliverables: Cost-reduction recommendations.

Project risks: The team is not familiar with the detailed design of the Krisys robot or the pricing strategies of the vendors.

The SIPOC diagram and process map are illustrated in Figures 3.6 and 3.7.

SIPOC

Suppliers	Inputs	Process	Outputs	Customers
Electronic part suppliers	Parts cost, data sheet	Krisys robot kit	Krisys kits	High schools
Motor supplier	Motor cost, data sheet		Revenue	Students
PCB supplier	Shipping cost		Student worker cost	Teachers
Mail carrier	Website hosting		Kit cost	ETID
TAMU IT	Customer feedback		Support docs	
ETID faculty	PCB cost		Revised designs	
	Customer special request			

Figure 3.6 SIPOC for Krisys robot kit

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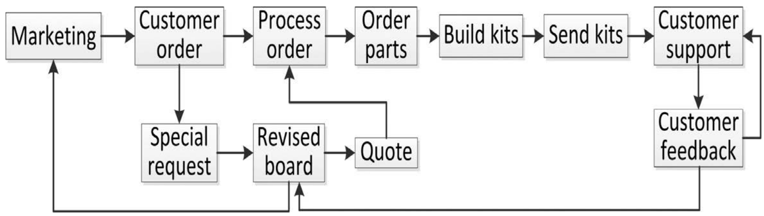

Figure 3.7 Krisys robot kit process

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The stakeholders of the Krisys robot process: ESET faculty members, ESET students, high schools, and, other colleges. The stakeholders are consulted in the project for information and feedback.

The process owner: The ESET faculty member in charge of the Krisys operation.

A high-level understanding of critical components in the success of the process was also investigated in this phase. After conducting the CTQ analysis, the CTQ tree was created as in Figure 3.8.

The CTQ analysis concluded that there were six aspects of the operation that are critical to quality: optimized design, low-cost parts, simplified kit creation, effective marketing, customer improvement, and low-cost carrier. Among these, effective marketing was not included in the scope of the project and would not be analyzed further.

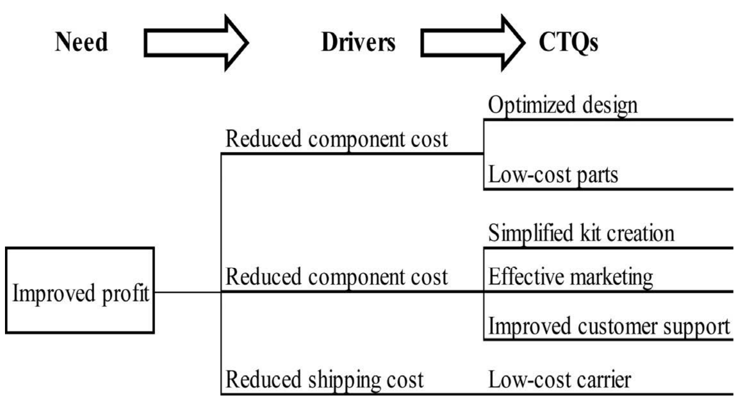

Figure 3.8 CTQ for Krisys robot kit

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3.3.3 *Measure*

In the Measure phase, data need to be collected so that the performance of the current process such as **process capability index** and **process performance index** can be established. The data will be further analyzed in the next phase (Analyze).

To this end, a data-collection plan should be created and the sampling method should be determined. Before measurements are taken, **measurement system analysis (MSA)** may be necessary.

The problem statement in the Define phase may need revision based on the analysis of the data. Any change made to the project charter, however, needs to be approved by the sponsor and the project champion.

Let us see how the Measure phase was carried out in the Krisys project.

To establish the performance of the current process, the cost of the current Krisys robot kit operation had to be calculated. The total cost included the kit cost, the labor cost, and the shipping cost. The profit per kit was the difference between the selling price and the total cost. The electronic part list for the kit was used, and the prices for the parts at Mouser, which was where the parts were purchased, were identified based on the price information posted on the Mouser website. Prices for other components such as motors and wheels were also collected.

The Six Sigma team also recorded the time required for putting together a kit: 30 minutes on average with standard deviation of seven minutes for making a kit, and 8 minutes for another person to double check the kit. The time for double checking had a standard deviation of 20 seconds. There was a large variation in customer support time.

Excluding the customer support cost, the current total cost for the Krisys robot kit operation was $120.

3.3.4 *Analyze*

In the Analyze phase, root causes of the problem are identified. It is critical that the analysis is supported by data collected in the Measure phase. If necessary, one can go back to the Measure phase to collect more data. Specific tasks such as process analysis, data analysis, cause-and-effect analysis, hypothesis testing, regression, and design of experiments can be carried out when it is appropriate. At the completion of this phase, a hypothesis will have been tested and validated.

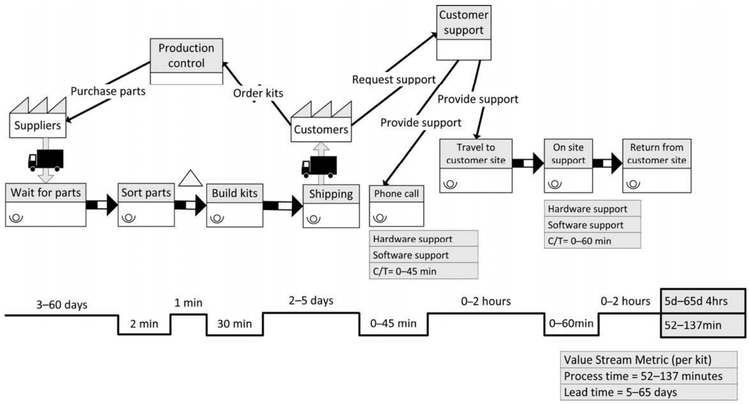

Figure 3.9 Current value stream mapping for Krisys robot kit

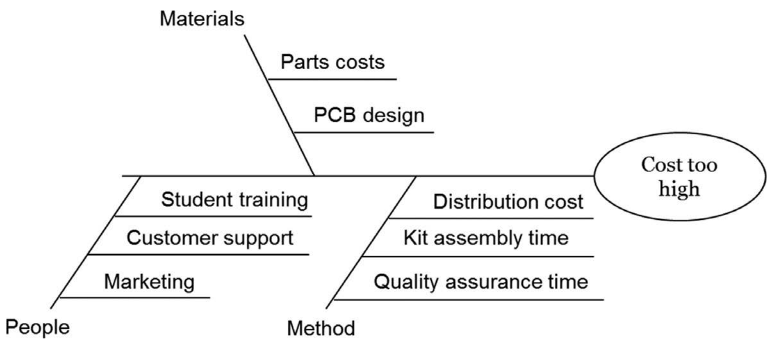

Figure 3.10 Cause-and-effect diagram of the Krisys robot kit

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Let us see how the Analyze phase was conducted in the Krisys project. The team created the value stream map for the Krisys kits operation, as shown in Figure 3.9.

The cause-and-effect diagram was created by the team, shown in Figure 3.10. The team decided to focus on the parts cost, student training, customer support, kit assembly time, and quality assurance time.

Parts cost:

The current process used Mouser Electronics™ as the vendor for all the electronic parts. The main reasons for using one vendor was to save on shipping fees and simplify the purchasing process, which made sense when orders were small and profitability was not important. As a commercialized product, the sales volume was expected to increase, and profitability became critical to success, and purchasing all parts from a single vendor was no longer the best choice. To compare the prices of electronic parts, two additional vendors were selected: DigiKey™ and Allied Electronics™. For illustration purposes, prices for five parts are listed in the following table:

Part number	Qty/kit	Mouser cost/kit ($)	Digikey cost/kit ($)	Allied cost/kit ($)
80-C322C104K1R	7	0.70	2.87	1.428
80-C330C334K5R	2	1.66	1.66	0.394
299-100-RC	1	0.10	0.10	0.015
598-SK100M050ST	1	0.14	0.21	0.133
538-90120-0132	3	5.31	4.05	2.055

The comparison showed that some parts were cheaper at one vendor and other parts were cheaper at another vendor. Creating such a chart made it easier to find significant price differences among various vendors. Purchasing parts from vendors who offered the cheapest prices could reduce the part cost significantly. The shipping cost increase was negligible once the volume reached a high level. For example, an additional $10 shipping cost for 1,000 parts meant $0.01 increase in cost per kit. This cost increase could easily be negated with saving from a single part: If we purchased the second part on the list, 80-C330C334K5R, from Allied Electronics instead of Mouser Electronics, a saving of $1.66 – $0.394 = $1.266 would be achieved.

The team also discovered that some parts were overdesigned. For example, some capacitors in the original design had a maximum voltage of 100 V, but the highest voltage on the Krisys robot platform was about 12 V. The original design had not focused on cost because making profit had not been an objective. The small volume before the commercialization of the kit made it unnecessary to optimize the cost.

In addition, the team found that the two motors used by the Krisys robot had encoders that were not used. After negotiations, the vendor of the motors agreed to provide replacement motors without the encoder, at a lower price.

Cost associated with customer support:

The current process required students and professors to travel to high schools to provide customer support. This was not an efficient method for customer support.

Labor cost:

Because the parts were not well-organized or clearly labeled, it took the student workers a long time to assemble a part kit. Student workers also spent more time labeling the electronic components.

3.3.5 Improve

In the Improve phase, **potential solutions** are developed based on the analysis conducted in the Analyze phase to reduce or eliminate the effects of the root causes. These solutions will be evaluated and then implemented. The root causes identified in the Analyze phase will be validated with data after the implementation of the improvement.

Failure Mode Effects and Analysis (FMEA) can be done to evaluate the risks for all potential failures. High-risk items will be addressed with recommended solutions and reevaluated. Before the large-scale implementation, a small-scale pilot test can be performed to gauge the effectiveness of the solution for improvement.

Let us see how the Improve phase was executed in the Krisys project.

Solutions for reducing the part cost:

- Buy parts from three different companies that offer the lowest prices. When ordering hundreds and thousands of kits, the shipping cost is negligible.

- Optimize the design by selecting the lowest cost parts that meet the requirements.
- Negotiate with the motor vendor for a price reduction for a motor without encoder.

Solutions for reducing the cost of customer support:

- Provide instructions in pdf, PowerPoint, and YouTube to reduce the cost of customer support. There is a one-time cost, but the investment is sound, since it eliminated the face-to-face visits and the time spent on explaining the processes over the phone.
- Host instructional material such as video tutorials and printable instructions at a website. The website also allows users to provide feedback for further improvement of the service and the design of the Krisys robot.
- Implement an online ordering system through the website.
- Provide buyers with an option of paying $2 for small electronic spare parts. These parts may be misplaced or lost, and replacing them can cause delays and significant shipping cost.

Solutions for reducing kit assembly time:

- Organize parts into bins.
- Pregroup parts by type.
- Provide training to new student workers.
- Build inventory for parts that have long lead time.
- Build kits using parts in inventory while the student workers have spare time, before an order comes in.
- Do not label every part in a kit, so that the high school or college students can learn about color coding of resistors. This reduces the time for making the kit and provides learning opportunities for the user.

The new VSM is illustrated in Figure 3.11.

Compared to the current VSM, there are several changes. Inventory is increased, in particular for the parts that have longer lead time. Student workers build Krisys kits when they have spare time, before an order comes in. Since a typical order is less than 50 kits, most of the time there are enough kits in the inventory ready for immediate shipping. Instructions

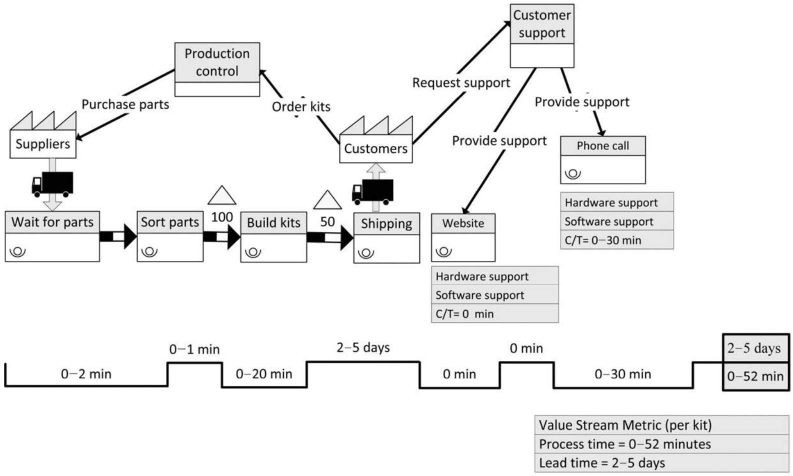

Figure 3.11 The new value stream mapping for Krisys kits

and YouTube videos are posted on a website to provide support for customers, significantly reducing the time and cost needed since on-site support is completely eliminated. These modifications to the process reduce the average lead time and its variation.

VSM will be discussed in more detail in Chapter 4.

3.3.6 Control

In the Control phase, one should make **before and after comparisons** to evaluate the improvement of the new process over the current process, the metrics detailed in the Define phase should be used, and a plan for maintaining the improvement should be developed and evaluated. In addition, the new process should be documented and standardized. It is necessary to make sure that the changes to the process are made according to the change control and compliance requirement of the organization. The data from the new process should be monitored by the process owner using tools such as SPC. Since Six Sigma is a continuous improvement effort, suggestions for a next-phase project can be made in this phase. It is also common to recommend replication of the process in other parts of the organization to multiply the effects of the improvement. The money belt, typically a person with expertise in finance will provide estimations and methods to track the financial impact of the project. The project should be documented including the major findings, recommendations, and lessons learned; typically, these materials are presented to appropriate people in the organization before the project is closed.

For the Krisys robot project, the following activities were completed in the Control phase:

- The cost under the new process was compared with that of the current process. The total operation cost was reduced by 30 percent.
- A website was developed that collected customer feedback on services, suggestions, and the design of the website.
- The future state VSM was compared to the current state VSM to determine how much improvement was made in lead-time reduction.
- A plan was created to collect data on kit completion time and error.
- A plan was created to collect data every other semester to monitor the price fluctuation of the electronic parts. Purchase strategy was modified based on the latest price information.
- Manuals and YouTube videos were created to help customers.
- A project report was written, and a project presentation was made.

Recommendations for a next-phase project were made as follows:

- Continue the investigation of new parts distributors.
- Study how to increase sales.
- Study how to increase customer satisfaction.
- Make this project an example for teaching Six Sigma.

The DMAIC process can go through several iterations if necessary. However, any changes made to the project charter will need to be approved by the management.

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CHAPTER 4

Lean Six Sigma Tools

While Chapter 3 introduced a number of tools which can be used in the DMAIC process of Lean Six Sigma, this chapter will discuss the usage of more generic tools not specifically for Lean Six Sigma projects. They are grouped according to where they can be applied in the DMAIC process; however, some tools can be used in more than one phase, thus making it impossible to follow this rule strictly. The reader may find it helpful to refer to Chapter 3 for their applications in Lean Six Sigma projects.

4.1 SWOT, Affinity Diagram, SIPOC, VOC, CTQ

4.1.1 SWOT Analysis

One can conduct a **Strengths, Weaknesses, Opportunities, and Threats (SWOT) analysis** to evaluate the challenges and opportunities faced in a project or by an organization [1]. The SWOT matrix is basically a matrix with two rows and columns, as illustrated in Figure 4.1.

The rows represent internal and external factors, and the columns represent the factors in favor of or harmful to the organization or products.

	Favorable effects	**Unfavorable effects**
internal factors	**Strengths**	**Weaknesses**
External factors	**Opportunities**	**Threats**

***Figure 4.1** SWOT analysis*

Strengths are the internal factors that are in favor of the organization, hence advantageous. **Weaknesses** are the internal factors that are harmful to the organization. **Opportunities** are the external factors in favor of the organization, and **Threats** are the external factors against the organization. For example, a company may have strength in making a low-cost product, a weakness in innovation, an opportunity in selling its product in a cost-sensitive market, and a threat from other companies outsourcing their product to low-production-cost countries.

The SWOT analysis can be applied in Lean Six Sigma projects to evaluate challenges and opportunities for the products or processes so that areas for improvement can be identified. The impact of the Lean Six Sigma project can be increased by the SWOT analysis. SWOT is useful for developing a high-level strategy and selecting the appropriate Lean Six Sigma projects.

4.1.2 Affinity Diagram

The **affinity diagram** is a tool for organizing a large number of ideas according to their natural relationships [1]. It was invented in the 1960s by the Japanese anthropologist Jiro Kawakita. This tool is typically used in a brainstorming session where a team's consensus is required or for sorting a large amount of information into logical categories. The affinity diagram taps a team's creativity and intuition. The procedure for creating an affinity diagram is as follows:

1. Write ideas on sticky notes, each on a separate note. To prevent the process from being dominated by a few team members, it is recommended that team members do not discuss with each other in this step.
2. After all of the ideas are captured on the sticky notes, organize the sticky notes into logical groups according to their contents. Give each group a name that is appropriate for all the sticky notes in the group.
3. Have a discussion. Make necessary changes or additions. Regroup if necessary.
4. Document the result for further analysis.

Figure 4.2 Affinity diagram

A simple example of affinity diagram for improving the writing of this book is illustrated in Figure 4.2.

4.1.3 SIPOC

SIPOC is the acronym for **Suppliers, Inputs, Process, Outputs, and Customers** [2, 3]. SIPOC analysis is conducted in the Define phase, with the main purpose of capturing the information related to the process to be improved. The process is usually illustrated with a simple flowchart of the main steps. Inputs are things that are provided to the process, outputs are the outcomes of the process, while suppliers and customers provide or receive the inputs or outputs. The SIPOC analysis forces the team to have a good understanding of the current process before they are set out to improve it. Through this analysis, the team will also learn where to gather useful information about the process, who the stakeholders are, and who will be impacted if any change is made to the current process.

The complexity of a **current process mapping** needs to be appropriate; it should not be too simple, containing just 1 or 2 steps, nor should it be too detailed, containing 50 steps. Five to 15 steps are proper

SIPOC

Suppliers	Inputs	Process	Outputs	Customers
Electronic part suppliers	Parts cost, data sheet	Krisys robot kit	Krisys kits	High schools
Motor supplier	Motor cost, data sheet		Revenue	Students
PCB supplier	Shipping cost		Student worker cost	Teachers
Mail carrier	Website hosting		Kit cost	ETID
TAMU IT	Customer feedback		Support docs	
ETID faculty	PCB cost		Revised designs	
	Customer special request			

Figure 4.3 A SIPOC example [4]

Source: Reprinted with permission by Association for Industry, Engineering, and Management Systems

for most processes. The SIPOC diagram also serves the purpose of documentation. Managers and other teams can review the SIPOC to understand the current process at a high level. Figure 4.3 is the example of SIPOC discussed in Section 3.3.2.

4.1.4 VOC

VOC stands for **Voice of Customer**; it is the "**root wants**" of the customers [1]. For a Lean Six Sigma project to be successful, it is critical that information from the customer is collected and thoroughly analyzed before attempting to improve the process.

VOC includes the direct input or feedback from customers on what they want; however, the raw data from customers must be analyzed to become the true **customer wants**, that is, the **root wants**. Not all the root wants are from customers; for example, government regulations are a part of the root wants, which customers may not mention.

One of the common mistakes is to replace the collection and analysis of VOC with a brainstorming session by the project team. This may allow the team to come up with most of the VOC information quickly, but they may miss some important customer needs or overemphasize the internal needs of their organization such as cost reduction. In other words, the team may have a bias which can only be corrected by the true voice of the customers. What the team overlook may be important to customers, and the project may miss the opportunity of creating an excellent product or process if it doesn't address customers' concern.

Another common mistake is to take the raw data from the customers for process improvement without thoroughly analyzing it. Two examples below illustrate how to process the information from customers to derive the VOC.

Example 4.1 In the automotive industry, there is a well-known specification for vehicle acceleration, the time it takes to go from 0-to-60 mph. While this information may be from internal or external customers, it is not a customer root want, but a requirement. The root want is "fast acceleration," and a quick 0-to-60 mph acceleration is only one way of meeting the customer need. If we just use this requirement, we may be missing the customer root want related to the vehicle acceleration in other situations. The right way of capturing this customer input is to keep "fast acceleration" as a customer root want and leave the specific requirements such as "time from 0-to-60 mph" open for further study at the requirement derivation stage.

Figure 4.4 illustrates that not capturing the root want may lead to an inferior design.

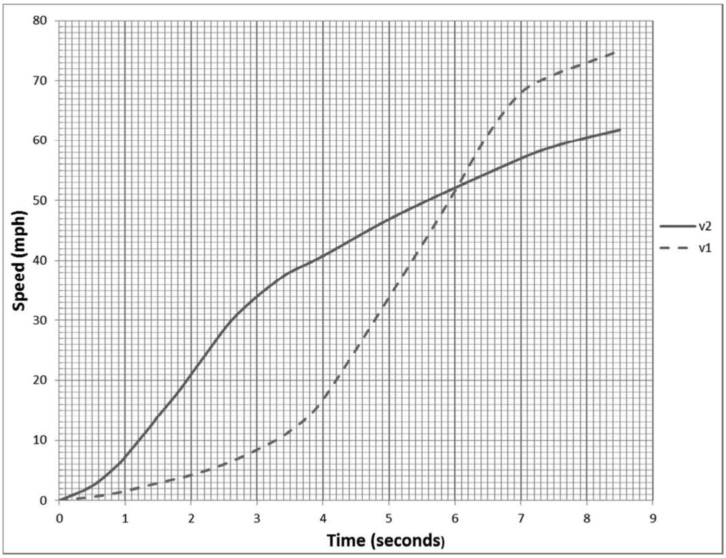

Figure 4.4 Vehicle speed trajectories

Let us say there are two design options, whose 0-to-60 mph traces, *v1* and *v2*, are shown in Figure 4.4. The 0-to-60 mph times for *v1* and *v2* are 6.45 seconds and 7.80 seconds, respectively. Since vehicle 1 has a shorter 0-to-60 mph time than vehicle 2, it seems that vehicle 1 has a faster acceleration. Is this really what the customers want?

Let us look at the distances the two vehicles traveled starting from standstill. Distance is speed integrated over time; therefore, the distances can be plotted based on the speed information, as illustrated in Figure 4.5.

Vehicle 2 travels a much longer distance than vehicle 1 in the same amount of time for the first 8 seconds, which means that vehicle 2 will pull ahead of vehicle 1 if they took off from the same position. Even though customers tell us they want a shorter 0-to-60 mph time, they may like vehicle 2 better than vehicle 1.

This example shows that VOC is not just a direct recording of the raw data from customers; oftentimes these raw data need to be interpreted and analyzed to obtain useful information. In a sense, we need to

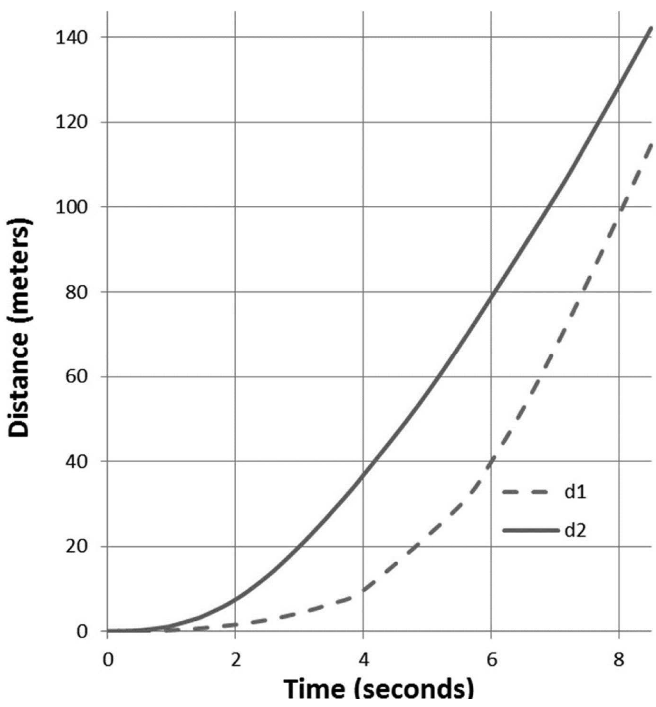

Figure 4.5 Distances traveled over time by two vehicles

read the mind of the customer. In this example we should capture "fast acceleration" as a customer want. The time from 0-to-60 mph is one of the design requirements to meet, and other requirements may include "the distance traveled in 4, 5, 6 seconds starting from the standstill position shall be above x, y, z meters," etc. Together, they address customers' need for fast acceleration. ■

Example 4.2 In a high school Science, Technology, Engineering, and Math (STEM) outreach project sponsored by NASA, the students were asked to use only wood to build a space station model for display in museums. This became the design requirement discussed during the VOC analysis. The team found additional information on why the material was limited to wood: in a similar project carried out by another high school team previously, the space station model had been made of steel, and it was difficult to move the model to museums because of its heavy weight. Using wood was actually a solution to a specific problem, rather than a customer root want. After further discussion with the sponsor, the true customer root wants were identified as "lightweight, durable, and low cost." As a result, a combination of different materials could be used, as long as the customer wants on weight, durability, and cost were satisfied. ■

Raw data for VOC can be obtained in a number of ways. Focus groups, interviews, surveys, customer complaints, customer requirements, and other customer feedback such as warranty data are all valid methods for collecting VOC information.

4.1.5 CTQ Tree

CTQ stands for **Critical to Quality** [1, 3]. Before a CTQ tree can be built, we first need to identify the customer. Then we summarize the high-level **customer need** based on the VOC analysis. Typically, this need is somewhat general, subjective, and not easy to quantify. Starting from this general need, a tree is created with two to three levels. The "**drivers**," which are more specific than the need and easier to quantify, are then derived from the customer need. Next, CTQs are developed from the drivers; they are more specific and easier to quantify than the drivers.

Example 4.3 The CTQ tree for an automobile repair shop can start with "auto repair" as the need. The drivers identify attributes that customers care about, which may include price, problem fixing, after-service support, and convenience in this case. The CTQ related to "price" may be "competitive labor cost" and "parts with better quality." The CTQs related to the driver "problem fixing" may be "identifying the root cause" and "finding a solution." The CTQ related to the driver "after-service support" may be "warranty for parts" and "warranty for labor." The CTQ related to "convenience" may be "options for customers to drop off, pick up, or waiting," "short service time," and "option for online appointment." The CTQ Tree is constructed as shown in Figure 4.6.

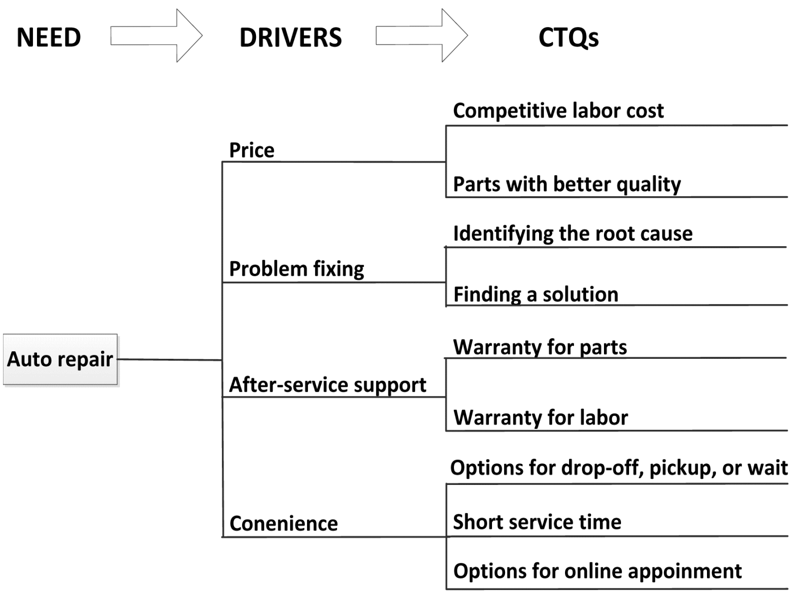

Figure 4.6 A CTQ tree example

■

4.2 C_p, C_{pk}, GR&R, Pareto Diagram, Prioritization Matrix, Normality Check, Monte Carlo Analysis

4.2.1 C_p, C_{pk}

Two widely used measurements for the performance of a process are **process capability index**, C_p, and **process performance index**, C_{pk} [1]. They are defined as follows:

$$C_p = \frac{USL - LSL}{6} \tag{4.1}$$

$$C_{pk} = min\left\{\frac{USL - X}{3}, \quad \frac{X - LSL}{3}\right\} \tag{4.2}$$

where X is the mean value of the process output.

The process capability index is a ratio of the width of the specification limits and 6σ. If the width is greater than 6σ, then the process capability index is greater than one. The larger the process capability index value is, the lower the process defect rate will be. If the output is centered inside the specification limits, then $C_p = C_{pk}$. In general, $C_p \geq C_{pk}$.

Example 4.4 For a six sigma normal process without a shift in its mean, find its C_p and C_{pk}. If a normal process without a shift in its mean has a 1.5 process capability index, calculate the sigma level for the process. If a six sigma process has a 1.5σ shift in its mean, what are its C_p and C_{pk}?

A normal process without a shift in its mean has its mean centered within the specification limits where $LSL = \mu - 6\sigma$ and $USL = \mu + 6\sigma$. It follows from the definition of the process capability index that $C_p = 2$. Similarly, $C_{pk} = 2$.

If the process capability index is 1.5, then

$$\frac{USL - LSL}{6\sigma} = 1.5$$

and USL – LSL = 9σ. Since its mean is centered between the specification limits, we have $LSL = \mu - 4.5\sigma$ and $USL = \mu + 4.5\sigma$. Therefore, it is a 4.5 sigma process.

For a six sigma process with a 1.5σ shift in its mean, we assume the shift is to the right. It follows that $LSL = \mu - 6\sigma - 1.5\sigma = \mu - 7.5\sigma$ and $USL = \mu + 6\sigma - 1.5\sigma = \mu + 4.5\sigma$. Therefore, $C_p = 2$ and $C_{pk} = min\{4.5/3, 7.5/3\} = 1.5$. Similarly, if the mean shifts to the left, it can be calculated that $C_p = 2$ and $C_{pk} = 1.5$. ■

In general, a process with C_{pk} greater than 1.33 is called **capable**; it is **capable with tight control** if C_{pk} is between 1.00 and 1.33, and **incapable** if C_{pk} is less than 1.

4.2.2 GR&R

GR&R is the acronym for **Gage Repeatability and Reproducibility** [1, 5, 6]. A **gage** is any device used to measure certain characteristics of a component, and GR&R is a part of the **Measurement System Analysis (MSA)**, which is typically conducted before collecting data using a measurement system. There are four aspects of a gage that are important to MSA: resolution, accuracy, repeatability, and reproducibility.

1. **Resolution** is the smallest amount of change that a gage can detect, also known as sensitivity or discrimination.
2. **Accuracy** is an indication of how close the measurements are to the true value.
3. **Repeatability** is the variation in measurements on the same part by the same operator at or near the same time, also known as the **equipment variation.** Repeatability can be used to indicate precision.
4. **Reproducibility** is the amount of variation in measurements by different operators at different times, also known as the appraiser variation.

The **ANalysis Of VAriance** (**ANOVA**) is the most common method used for GR&R analysis. The ANOVA for GR&R is conducted with N number of operators and M number of random parts. The parts are first labeled for identification purpose and the operators will take measurements of the parts with L number of replications. We can use Excel, but the most convenient way is to use statistical analysis software such as Minitab. For a given confidence level, Minitab can calculate the contribution from the operators (reproducibility), from the part-to-part variation, from the measurement equipment (repeatability), and from the interaction between operators and parts. The R&R is the sum of repeatability and reproducibility. An example for GR&R analysis using Minitab can be found in Chapter 5 (Example 5.13).

4.2.3 Pareto Diagram

Pareto diagrams, or **Pareto charts**, are graphical representations of data that identify the vital factors separated from many trivial factors [1, 2, 7]. In Figure 4.7, the vertical axis represent the factors, and the horizontal axis is the response we are interested in which can be the occurrence, importance, or impact of the factors on the outcome. Sometimes the graph is rotated by 90 degrees.

The Pareto diagram can also be modified slightly to reflect the importance of each factor. This is done by multiplying the response of each factor by a weighing factor, producing the weighted Pareto diagram.

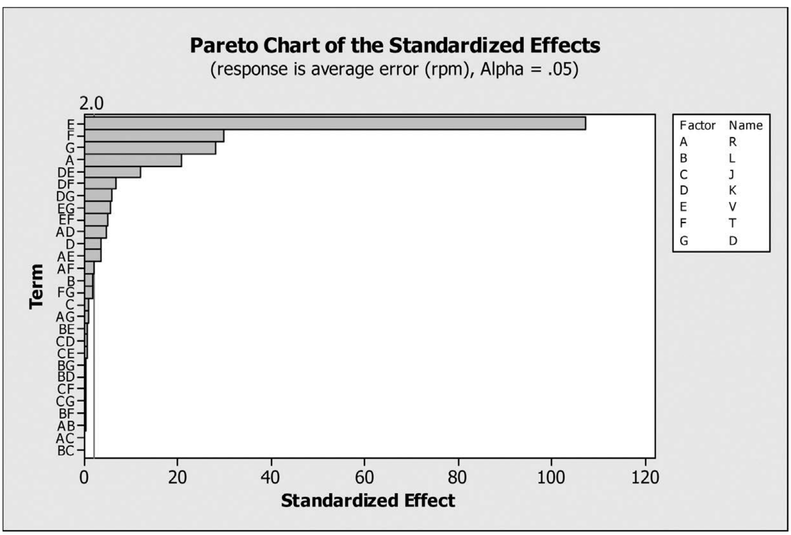

Figure 4.7 A Pareto chart example

4.2.4 Prioritization Matrix

The principle of the **prioritization matrix** is similar to that of the Pugh matrix (Appendix A). The prioritization matrix is typically used to determine how much impact the inputs have on the outputs [1]. The procedure for constructing it is as follows:

1. Identify the inputs and outputs.
2. Create a matrix with the inputs in the first column and the outputs in the first row.
3. Assign a weight to each output.
4. For each output, assign a rank order for each input. Number "1" means that the input has the least influence on the output, and the largest number in ranking indicates the most significant influence.
5. Calculate the weighted averages for each input and assign it as the composite ranking.

Example 4.5 Let x_1, x_2, and x_3 be the inputs and y_1, y_2, y_3, and y_4 be the outputs. The relative importance or the weights for the outputs are 0.2, 0.4, 0.1, and 0.3 respectively. Note that the weights add up to 1 in this case, which is not necessary; you may choose integers to simplify the calculations. Suppose that the influence on y_1 has the rank order of x_1, x_3, x_2 from the most significant to the least significant, the influence on y_2 has the rank order of x_1, x_2, x_3, the influence on y_3 has the rank order of x_3, x_1, x_2, and the influence on y_4 has the rank order of x_2, x_3, x_1. The prioritization matrix can be constructed as shown in Figure 4.8.

Based on the composite rankings, x_1 is the most important input and x_3 is the least important input.

	y_1	y_2	y_3	y_4	Composite ranking
Weights	0.2	0.4	0.1	0.3	
x_1	3	3	2	1	2.3
x_2	1	2	1	3	2
x_3	2	1	3	2	1.7

Figure 4.8 A prioritization matrix example

■

Substituting the outputs with the criteria and the inputs with the options with the weight equal to 1, then the priority matrix becomes a Pugh matrix, only with the rows and columns switched in their roles.

4.2.5 Normality Check

Many probabilistic theories are based on the assumption that the random variable has a normal distribution, also referred to as **Gaussian distribution**. Although many random variables have normal distributions, it is still necessary to check whether the random variables under consideration have normal distribution or not. The **normality check** is a graphical method that allows us to examine, at a given confidence level, if the data is from a normal distribution [1, 8–10]. This method can be extended to more general cases where the distribution may or may not be normal distribution. In this more general setting, the method is then called a **probability plot**. The rationale for the probability plot method is that if the sample was selected from the standard normal distribution, the sample percentiles should be reasonably close to the corresponding population percentiles.

The procedure for constructing a plot for normality check is as follows:

1. Sort the data in ascending order. The ith number is considered to be the $[100(i - 0.5)/n]$th sample percentile, where n is the total number of data points and $i = 1, 2, \ldots, n$. These are the vertical coordinates (y) of the points in the plot.
2. Calculate the $[100(i - 0.5)/n]$th percentile for the standard normal distribution. These are the horizontal coordinates (z) of the points in the plot.
3. Plot y vs z.

If the z–y coordinates are very close to a straight diagonal line passing through the origin with a slope of 1, then the data can be modeled by a random variable with standard normal distribution.

In a more general case, if X is a normal random variable with μ as its mean and σ as its standard deviation, then $Y = (X - \mu)/\sigma$ is a standard normal random variable as shown in Section 2.9.1. If (z, y) coordinates are close to the straight diagonal line passing through the origin with a slope of 1, then it follows that the (z, x) coordinates are very close to a straight line with μ as its x-intercept and σ as its slope, and vice versa. Therefore, if the (z, x) coordinates are close to a straight line, then the data can be modeled by an underlying random variable with a normal distribution. The x-intercept is the estimated mean value and the slope of the straight line is the estimated standard deviation. This graphical method can be implemented in Excel. The goodness of fit can be indicated in Excel by the R^2 value, also called the **coefficient of determination**. The closer R^2 is to 1, the better the fitting is.

Example 4.6 Given the following data set, use normality check to determine if the data are from a normal distribution. If they are, find the estimated mean and estimated standard deviation for the population.

0.6399	0.8059	0.091	0.5194	−0.803	0.1725	0.3382	−1.547	−0.446	−1.13	0.3619	0.4945
−0.551	−0.676	0.2032	2.0713	−1.547	−1.304	−0.501	0.7936	−0.511	0.0105	−0.499	−0.912
0.5627	−1.033	0.8431	−0.625	−0.772	−1.945	1.5952	0.4143	−1.037	−1.238	−0.14	0.9913

There are 36 numbers in the data set, so n = 36. We first sort the data in ascending order to be the vertical (y) coordinates of the points:

−1.945	−1.547	−1.547	−1.304	−1.238	−1.13	−1.037	−1.033	−0.912	−0.803	−0.772	−0.676
−0.625	−0.551	−0.511	−0.501	−0.499	−0.446	−0.14	0.0105	0.091	0.1725	0.2032	0.3382
0.3619	0.4143	0.4945	0.5194	0.5627	0.6399	0.7936	0.8059	0.8431	0.9913	1.5952	2.0713

Next we need to calculate the [100(i – 0.5)/36]th percentile of the standard normal distribution, for i =1, 2,. . . , 36. The results will be the horizontal (x) coordinates of the points, and can be calculated in Excel as shown in Figure 4.9:

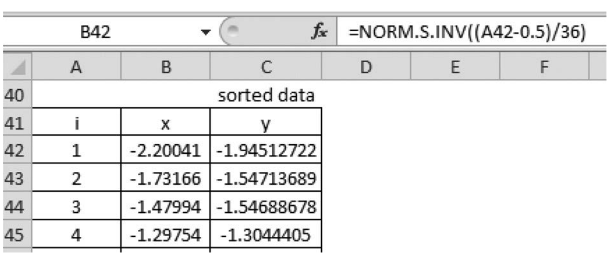

Figure 4.9 Excel screen capture for Example 4.7

The resulting *x* coordinates are listed in the following table:

−2.2	−1.732	−1.48	−1.298	−1.15	−1.025	−0.913	−0.812	−0.719	−0.631	−0.549	−0.469
−0.393	−0.319	−0.246	−0.175	−0.105	−0.035	0.0348	0.1046	0.175	0.2462	0.3186	0.3928
0.4693	0.5485	0.6314	0.7189	0.8122	0.9132	1.0246	1.1503	1.2975	1.4799	1.7317	2.2004

The points with the (*x, y*) coordinates so calculated are plotted in Figure 4.10 with Excel. The equation for the trend line and the R^2 value are shown in the graph. The R^2 value of 0.9812 indicates that the straight line is a good fit for the data, leading to the conclusion that the data can be modeled by a normal random variable. Based on the equation, the *y*-intercept is −0.1752, which is the estimated mean for the normal random variable. The estimated standard deviation is the slope of the trend line, which is 0.9182.

Other methods are readily available for use if statistical software such as Minitab is used, in which case the user can select specific methods such as the Anderson–Darling method.

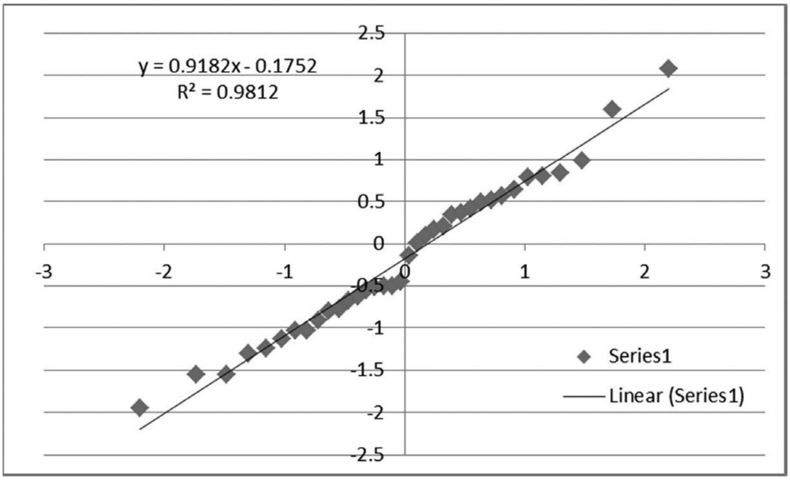

Figure 4.10 Probability plot for Example 4.7

■

4.2.6 Monte Carlo Analysis

Monte Carlo analysis is a numerical analysis usually performed by computers [11, 12]; it uses randomly generated inputs to calculate the output and draw conclusions based on the statistical properties of the results. This analysis can quickly generate a large amount of computer experimental data without physically conducting the experiments. The procedure for Monte Carlo analysis is as follows:

1. Define inputs to the system being analyzed.
2. Randomly generate the values of each input according to its probability distributions (assuming that you know the probability distribution of each input).
3. Simulate the system with the input values thus generated and obtain the outcome data (Some kind of simulation model is usually required to produce the outcome for each set of input).
4. Draw conclusions based on the simulation results.

Example 4.7 A well-known example for Monte Carlo analysis is to estimate the value of π. Consider the unit circle in the first quadrant and the square with sides equal to 1 as shown in the Figure 4.11.

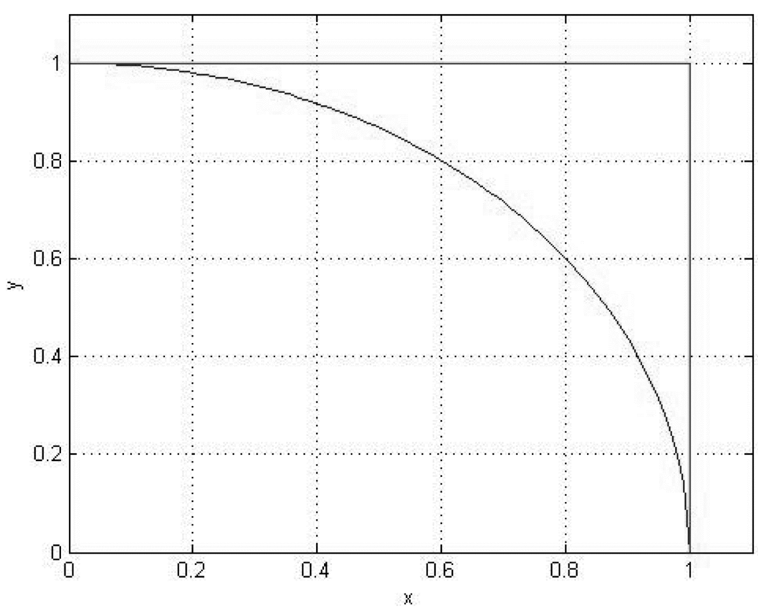

Figure 4.11 Monte Carlo analysis example (Example 4.7)

The area of the unit square is 1, and the area inside the quarter unit circle is $\pi/4$. If the coordinates of a point (x, y) are randomly generated from a uniform distribution independently, with the x, y values between 0 and 1, then the probability for the point to fall inside the quarter unit circle is $\pi/4$. If N such points are randomly generated using some software and N is very large, then the occurrences of points falling inside the quarter circle divided by N should approach the value of $\pi/4$. From this we can estimate the value of π.

In this example, the inputs are the x and y coordinates of the point, whose values can be generated in Excel or other random number generator. In Excel, click on **Data**, **Data Analysis**, choose **Random Number Generation**, and then click **OK**. Assign 2 to Number of Variables; assign a large number, say 1,000, to Number of Random Numbers; select Uniform from Distribution menu and assign parameters to be between 0 and 1. Now we have randomly generated 1,000 pairs of numbers between 0 and 1. Next we calculate the value of $x^2 + y^2$; if it's less than 1, then it is inside the quarter circle. Use COUNTIF in Excel to count the total number of points that are inside the quarter circle. Assume this sum is 791 (this number can be different every time because the numbers are randomly generated), then we have

$$\frac{791}{1000} \approx \frac{\pi}{4}$$

This provides us with an approximation of the value for π to be $791 \times 4/1000 = 3.16$. In general, the larger the number N is, the more accurate the approximation will be. ■

Other examples of Monte Carlo analysis from practical Lean Six Sigma projects can be found in Chapter 8.

4.3 Confidence Intervals, Hypothesis Testing, Cause-and-Effect Analysis, FMEA, DOE, VSM

4.3.1 Confidence Intervals

In Lean Six Sigma projects, we often need to estimate parameters such as the mean of a random variable. There are two types of estimations: point estimation and interval estimation. Point estimation involves a single value, for example, the sample average can be used as a point estimation for the population mean. Point estimations are usually simple to understand; however, they usually do not provide information about how good the estimations are. The interval estimation provides an interval together with a **confidence level** (CL) [1, 9, 10, 13], instead of a single value.

Let us start with the interval estimation of the mean value for a normal distribution with a known population standard deviation σ. Let $x_1, x_2, \ldots, x_n$ be n samples from the same normal distribution. Using the definitions of normal distribution, mean, and standard deviation, one can show $\bar{X} = \sum_{i=1}^{n} x_i$ is a normal distribution with μ as its mean and $\sigma / \sqrt{n}$ as its standard deviation. Based on the discussion in Section 2.9.1, the following random variable

$$Z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}}$$

has a standard normal distribution. Therefore, we can find the probability of Z being between any two numbers. Typically, we use this property in a reverse manner, that is, given a probability, we can find a symmetric interval $(-w, w)$ that Z lies in with the probability. This probability is defined as the **confidence level (CL)** and the interval is defined as the **confidence interval (CI)**.

Typical values for confidence levels are 90, 95, and 99 percent. Here, we use 95 percent as an example to show how the CI is determined. Since the standard normal distribution is symmetric, (1 − 95%)/2 = 2.5% of the area under the standard normal density curve should be to the right of w and 97.5 percent of the area should be to the left of w. In other words, w should be the 97.5 percentile. The value of w can be obtained by typing "=NORM.S.INV(0.975)" in a cell in Excel, it is approximately 1.96. Similarly, for confidence levels of 90 percent and 99 percent, we can use NORM.S.INV(0.95) and NORM.S.INV(0.995) to get w = 1.645 and w = 2.576 respectively. Now let us return to the 95 percent confidence level. We can summarize what we have discussed with the following formula:

$$P(-1.96 < Z < 1.96) = 0.95$$

which means that Z is a number between -1.96 and 1.96 with a probability of 95 percent. We can rewrite the inequalities as

$$-1.96 < \frac{\bar{X} - \mu}{\sigma / \sqrt{n}} < 1.96$$

which is equivalent to

$$\bar{X} - 1.96\frac{\sigma}{\sqrt{n}} < \mu < \bar{X} + 1.96\frac{\sigma}{\sqrt{n}}$$

The result is summarized in the following theorem.

Theorem 4.1 The confidence interval for the population mean of a normal distribution with a known standard deviation is

$$\left(\bar{X} - 1.96\frac{\sigma}{\sqrt{n}},\ \bar{X} + 1.96\frac{\sigma}{\sqrt{n}} \right) \tag{4.3}$$

at 95 percent confidence level.

It is worth noting that the CI is not a fixed interval, instead the values of the two ends are random variables. For one set of samples, we have one interval; for another set of samples, we may have a different interval, even though the two sample sets are from the same population. All we can say is that for all data sets, 95 percent of the time the population mean will be contained in the CI calculated this way.

For other confidence levels, the derivation of CI calculation remains the same; simply change the number 1.96 to the appropriate value determined by the confidence level.

If the samples are not from a normal distribution, but n is large enough, we can use the Central Limit Theorem to establish the CI. The procedure is exactly the same except that it is an approximation in this case. For 95 percent confidence level, the CI is given by Equation (4.3).

In the above derivation, we assumed that the population standard deviation σ is known.If this is not the case, that is, we have a normal random variable with unknown mean and standard deviation. In this case, we can replace the population standard deviation σ with the sample standard deviation s. With the above substitution, the following random variable can be shown to have a t-distribution.

$$T = \frac{\overline{X} - \mu}{s / \sqrt{n}}$$

Following the similar steps as in the derivation of CI for normal variables with known standard deviations, we have the following theorem:

Theorem 4.2 The confidence interval for the population mean of a normal distribution with an unknown standard deviation is

$$\left(\overline{X} - t_{\alpha/2, n-1} \frac{s}{\sqrt{n}},\ \overline{X} + t_{\alpha/2, n-1} \frac{s}{\sqrt{n}} \right) \tag{4.4}$$

where $t_{\alpha/2, n-1}$ is the value which has an area of $\alpha/2$ under the t-distribution probability density function with $n - 1$ degree of freedom to its right side, also known as the **t-critical value**. α is the **significance level**, which is equal to (1 − confidence level), and s is the sample standard deviation.

The CI for standard deviation estimation of a normal distribution is given in the following theorem without a proof.

Theorem 4.3 The CI for the standard deviation of a normal distribution is given by

$$\left(\frac{s\sqrt{n-1}}{\sqrt{\chi^2_{\frac{\alpha}{2}, n-1}}},\ \frac{s\sqrt{n-1}}{\sqrt{\chi^2_{1-\frac{\alpha}{2}, n-1}}} \right)$$

where n is the sample size, s is the sample standard deviation, α is the significance level, which is equal to (1 − confidence level), and $\chi^2_{\gamma,\nu}$ is the χ^2 value such that area γ is under the χ^2 probability density function with ν degree of freedom, to the right of the χ^2 value.

There are many other CI estimation methods for other parameters such as proportion. For more CI estimation methods the reader can refer to the reference books listed at the end of this chapter.

4.3.2 Hypothesis Testing

In **hypothesis testing**, we use statistical tools to analyze a given set of data in order to determine whether a hypothesis should be rejected or not [1, 3, 10, 13]. We have used hypothesis testing in Section 4.2 to check the normality of a data set, or more generally, to decide whether a data set can be modeled by a specific probability distribution. In this section, we will discuss another type of hypothesis testing: to determine whether a parameter is greater, equal to, or less than a certain value. The parameter can be the mean or variation of a population, for example. Such hypothesis testing typically involves a null hypothesis denoted by H_0 and an alternative hypothesis H_1 that is contradictory to H_0.

It is important to understand that there is a risk involved in hypothesis testing. We can make mistakes when we draw a conclusion on a hypothesis. There are two types of errors in hypothesis testing. **Type I error** incorrectly rejects the null hypothesis when it is actually true. The probability of making a type I error, denoted by α, is defined to be the **significance level** of the test; it is also known as the **producer's risk**. Typical values for α are 0.1, 0.05, and 0.01. **Type II error** incorrectly accepts the null hypothesis when it is actually false. The probability of making a type II error, denoted by β, is also known as the **consumer's risk**. In general, for a given set of data, reducing α will cause β to increase, and vice versa. However, increasing the sample size will reduce both α and β.

The fundamental concept of hypothesis testing is based on the fact that if we assume the parameter equal to certain value, then we can calculate the probability of certain test statistic being in a region. If the data falls into a low probability region, then we have enough confidence to reject the hypothesis. The actual application of this concept requires the following steps:

1. Select a null hypothesis.
2. Set a significance level α, which is the probability of making type I error.

3. Determine the **region of rejection**.
4. Calculate the test statistic using the given data. If the test statistic fails in the region of rejection, then the null hypothesis is rejected. Otherwise it is not rejected.

We use the case of hypothesis testing for population mean to illustrate this process. Let's say the **null hypothesis** is that the population mean is less than or equal to certain value μ_0. Next we select a significance level α. Assuming the population mean is equal to μ_0, calculate the $(1 - \alpha)100$th percentile denoted by x_0. Then the region of rejection is $x > x_0$, as illustrated in Figure 4.12.

If the sample mean is in the region of rejection, we will reject the null hypothesis, which means we'll conclude that the population mean is greater than μ_0. The rationale behind the conclusion is that if the null hypothesis were true, then the sample mean would only have a small probability (α) of falling into the region of rejection. If we reject the null hypothesis, the probability of our conclusion being wrong is α. The probability of the null hypothesis being false is $1 - \alpha$, which is the confidence

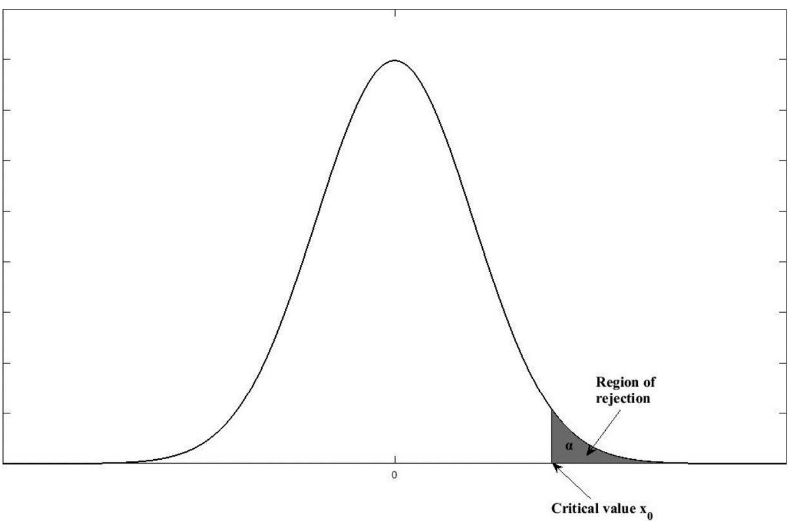

Figure 4.12 Significance level, critical value, and region of rejection

level. If $\alpha = 0.05$, then the probability of our conclusion being wrong is 5 percent and our confidence level for rejecting the hypothesis is 95 percent.

What happens when the population mean is less than μ_0? The probability density curve in Figure 4.12 will be shifted to the left in this case, which will make the probability of $x > x_0$ even smaller. Therefore, if we reject the null hypothesis, the probability of our conclusion being wrong will be smaller than α.

It is worth noting that when the test statistic is not in the region of reject, we are not accepting the null hypothesis; we just cannot reject it. For this reason, if we want to prove a claim, we usually choose the opposite of the claim to be the null hypothesis. The claim we want to prove would then become the alternate hypothesis. If the data support the rejection of the null hypothesis, then we can conclude that the claim we want to prove is correct with a high probability.

An alternative way of testing a hypothesis is to calculate the so-called *p*-value. Instead of selecting a significance level α, a test statistic calculated from the data is used to determine the probability of the test statistic being right on the border line of the region of rejection. If the significance level α is chosen to be smaller than the *p*-value, then we would fail to reject the hypothesis. If the significance level α is chosen to be greater than the *p*-value, then we would be able to reject the hypothesis. The *p*-value is shown in Figure 4.13 as the shaded area to the right of the test statistic.

In the above case we discussed, the **null hypothesis** and **alternate hypothesis** are

$$H_0 : \mu \leq \mu_0 \qquad\qquad H_1 : \mu > \mu_0$$

This is called the **right one-tailed test**, since the region of rejection is on the right side, with an area of α. It is also referred to as the **upper-tailed test**. Similarly, we have the **left one-tailed test**, or the **lower-tailed test**, with the following null hypothesis and alternate hypothesis:

$$H_0 : \mu \geq \mu_0 \qquad\qquad H_1 : \mu < \mu_0$$

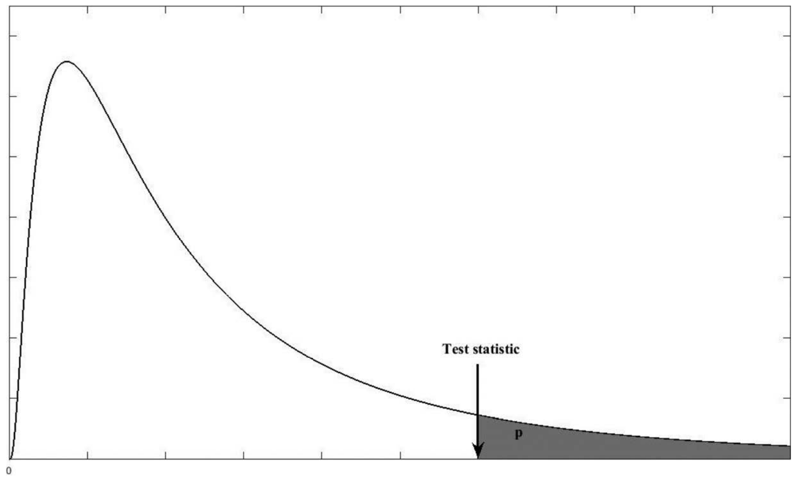

Figure 4.13 p-value

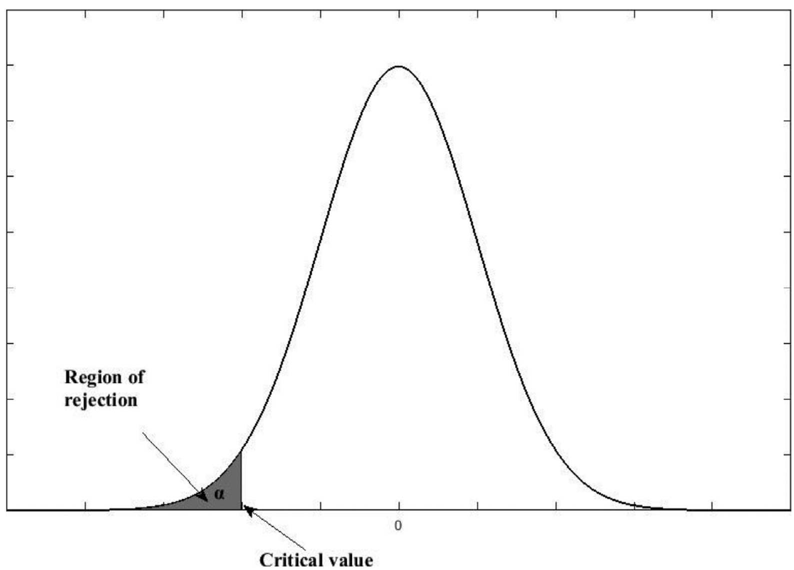

Figure 4.14 Left one-tailed test

In this case, the region of rejection is on the left side, with an area of α as shown in Figure 4.14.

The third case is

$$H_0 : \mu = \mu_0 \qquad\qquad H_1 : \mu < \mu_0 \ \ or \ \ \mu > \mu_0 \, .$$

This is called a **two-tailed test** since the region of rejection has two sides, one on the left and one on the right. Each side has an area of $\alpha/2$, as shown in Figure 4.15.

The most common hypothesis testing involves the population mean and population standard deviation for normal distributions. The proportion in binomial distribution is also widely used in hypothesis testing. We will examine some of these hypothesis tests.

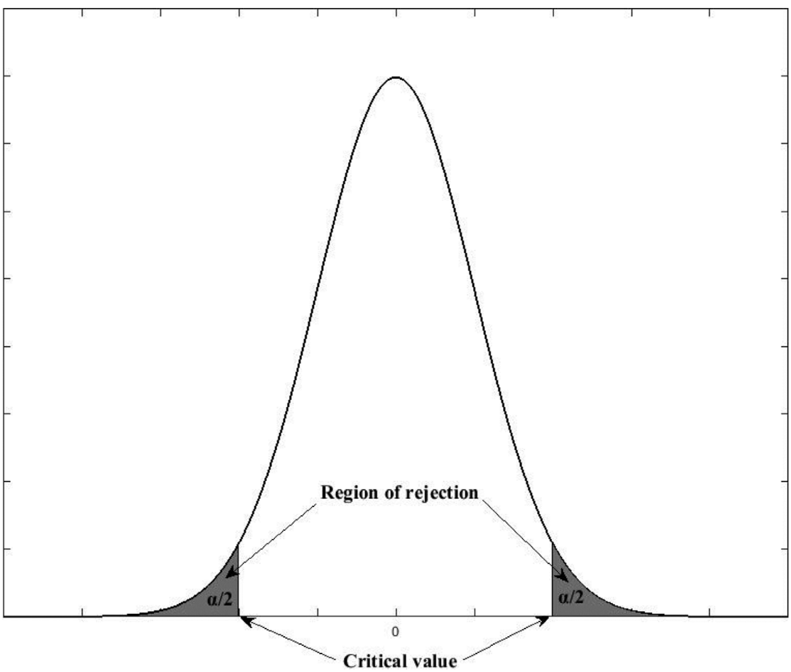

Figure 4.15 Two-tailed test

Z-Test:

If the population is known to have a normal distribution with a known standard deviation σ, and we are interested in comparing the population mean to a fixed value μ_0, then we can use the following test statistic:

$$Z = \frac{\bar{X} - \mu_0}{\sigma / \sqrt{n}} \tag{4.5}$$

where $\bar{X}$ is the sample average, σ is the population standard deviation, μ_0 is the value given in the null hypothesis, and n is the number of sample points.

Theorem 4.4 Given a normal distribution with a known standard deviation, three types of hypothesis tests can be performed with significance level α.

1. For right one-tailed test, if $Z > Z_\alpha$, then the null hypothesis $H_0 : \mu \le \mu_0$ is rejected. Otherwise, there is insufficient evidence to reject the null hypothesis.
2. For left one-tailed test, if $Z < -Z_\alpha$, then the null hypothesis $H_0 : \mu \ge \mu_0$ is rejected. Otherwise, there is insufficient evidence to reject the null hypothesis.
3. For two-tailed test, if $Z < -Z_{\alpha/2}$ or $Z > Z_{\alpha/2,}$ then the null hypothesis $H_0 : \mu = \mu_0$ is rejected. Otherwise, there is insufficient evidence to reject the null hypothesis.

When the sample size is large, according the Central Limit Theorem in Section 2.10, the *Z*-test can be applied without assuming the normality of the distribution and without knowing the population standard deviation. In this case, the population standard deviation σ is replaced with the sample standard deviation *s*.

t-Test:

If the samples are drawn from a normal random distribution, but the population standard deviation is unknown and the sample size is small, then the *Z*-test cannot be used. The test statistic defined by

$$T = \frac{\bar{X} - \mu_0}{s / \sqrt{n}} \tag{4.6}$$

has a *t*-distribution, where *s* is the sample standard deviation.

Following a similar reasoning as for the *Z*-test, we have the following theorem:

> **Theorem 4.5** Given a normal distribution with an unknown standard deviation and a small sample size, three types of hypothesis tests can be performed with significance level α:
> 1. For right one-tailed test, if $T > t_{\alpha,n-1}$, then the null hypothesis $H_0 : \mu \leq \mu_0$ is rejected. Otherwise, there is insufficient evidence to reject the null hypothesis.
> 2. For left one-tailed test, if $T < -t_{\alpha,n-1}$, then the null hypothesis $H_0 : \mu \geq \mu_0$ is rejected. Otherwise, there is insufficient evidence to reject the null hypothesis.
> 3. For two-tailed test, if $T < -t_{\frac{\alpha}{2},n-1}$ or $T > t_{\frac{\alpha}{2},n-1}$ then the null hypothesis $H_0 : \mu = \mu_0$ is rejected. Otherwise, there is insufficient evidence to reject the null hypothesis.

The definitions of $t_{\alpha,n-1}$ and $t_{\frac{\alpha}{2},n-1}$, known as the t – critical values, are similar to those of Z_α and $Z_{\alpha/2}$ with the standard normal probability density function replaced by the *t*-distribution.

Example 4.8 The average daily failure rate for a product was 1 percent. A Lean Six Sigma team worked on the process improvement. The new process has been tested for 16 days with the average daily failure rate of 0.85 percent and a sample standard deviation of 0.3 percent. Can we make the conclusion that the process mean has been improved with 95 percent confidence level?

We choose the null hypothesis and alternate hypothesis as

$$H_0 : \mu \geq \mu_0 \qquad\qquad H_1 : \mu < \mu_0$$

This matches case 2 of Theorem 4.5. The test statistic is calculated as

$$T = \frac{\bar{X} - \mu_0}{s / \sqrt{n}} = \frac{0.0085 - 0.01}{0.003 / \sqrt{16}} = -2.0$$

We can use Excel to calculate the t-critical value: $t_{0.05,15} = 1.7531$. Since $T < -t_{0.05,\,15}$, we can reject the null hypothesis and accept the alternate hypothesis according to Theorem 4.5. In other words, based on the data, we can conclude with 95 percent confidence that the process has been improved by the team. ■

Let us revisit Example 2.18 from the perspective of hypothesis testing.

Example 4.9 Ten measurements of a random variable are as follows:

$x_1 = 10.0,\ x_2 = 11.1,\ x_3 = 9.9,\ x_4 = 10.1,\ x_5 = 11.2,\ x_6 = 9.7,\ x_7 = 11.5,$
$x_8 = 9.8,\ x_9 = 10.1,\ x_{10} = 11.2$

Can we make the conclusion with 95 percent confidence level that the random variable has a population mean greater than 10.0?

We choose the null hypothesis and alternate hypothesis as

$$H_0 : \mu \leq 10.0 \qquad\qquad H_1 : \mu > 10.0$$

The average of the samples is 10.46, and the sample standard deviation is 0.6979.

$$T = \frac{\bar{X} - \mu_0}{s / \sqrt{n}} = \frac{10.46 - 10}{0.6979 / \sqrt{10}} = 2.084$$

Using Excel function T.INV (0.95, 9), we find that $t_{\alpha,n-1} = 1.833$. Since $T > t_{\alpha,n-1}$, the null hypothesis is rejected and the alternative hypothesis is accepted, that is, at 95 percent confidence level, we can conclude that the population mean is greater than 10.0. ■

F-Test:

Let $x_1, x_2, \ldots, x_m$ be samples from a normal random variable X and $y_1, y_2, \ldots, y_n$ be samples from a normal random variable Y. Suppose X and

Y are independent. To compare the population standard deviations of X and Y, an F-test statistic is defined as

$$F = \frac{s_1^2}{s_2^2}$$

where s_1 and s_2 are the sample standard deviations for X and Y, respectively.

Theorem 4.6 Let $x_1, x_2, \ldots, x_m$ be samples from a normal random variable X and $y_1, y_2, \ldots, y_n$ be samples from a normal random variable Y. Suppose X and Y are independent. Three types of hypothesis tests can be performed on the population statandard deviations of X and Y with significance level α:

1. If $F \geq F_{\alpha,m-1,n-1}$, then the null hypothesis $\sigma_1 \leq \sigma_2$ is rejected. Otherwise, there is insufficient evidence to reject the null hypothesis.
2. If $F \leq F_{1-\alpha,m-1,n-1}$, then the null hypothesis $\sigma_1 \geq \sigma_2$ is rejected. Otherwise, there is insufficient evidence to reject the null hypothesis.
3. If $F \geq F_{\alpha/2,m-1,n-1}$, or $F \leq F_{1-\alpha/2,m-1,n-1}$ then the null hypothesis $\sigma_1 = \sigma_2$ is rejected. Otherwise, there is insufficient evidence to reject the null hypothesis.

Example 4.10 Eight and 10 samples were taken for normal random variable X in January and February, respectively.

January			9.9	11	9.8	10.5	11.1	9.9	11.4	9.8
February	9.1	11.7	9.9	10.1	11.2	9.7	11.5	9	10.1	12.5

The sample standard deviation for January is calculated to be 0.6628. The sample standard deviation for February is calculated to be 1.1764. Do the samples indicate a significant increase in variation from January to February at 95 percent confidence level? What if we change the confidence level to 90 percent?

The null hypothesis is

$$H_0 : \sigma_f \leq \sigma_j, \qquad v_1 = 9, v_2 = 7$$

The alternative hypothesis is

$$H_1 : \sigma_f > \sigma_j$$

The F-test statistic is

$$f = \frac{1.1764^2}{0.6628^2} = 3.1506$$

At a 95 percent confidence level, the F statistic needs to be greater than the critical value of 3.6767 to reject the null hypothesis. This can be found from either an F distribution table or by using the Excel function `F.INV(0.95,9,7)`. Since f is not greater than the critical value of 3.6767, we fail to reject the null hypothesis at 95 percent confidence level.

If we change the confidence level to 90 percent, the critical value is reduced to 2.7246 ("`= F.INV(0.9, 9,7)`"). Since $f = 3.1506 > 2.7247$, we will reject the null hypothesis and accept the alternative hypothesis.

In summary, at a 90 percent confidence level we can draw the conclusion that the variation in February is greater than that of January; such a conclusion cannot be drawn at a 95 percent confidence level, however. ■

Chi-Square Test:

For a normal distribution, the Chi-square distribution can be used for hypothesis testing of the population standard deviation, whether it is greater than, less than, or equal to a number σ_0. The test statistic is calculated as

$$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2} \tag{4.7}$$

where n is the number of sample points and s is the sample standard deviation.

Similar to the Z-test, t-test, and F-test discussed earlier, there are three scenarios with the null hypothesis and alternate hypothesis:

$$\text{Right one-tailed test:} \quad H_0 : \sigma \le \sigma_0, \ H_1 : \sigma > \sigma_0$$

$$\text{Left one-tailed test:} \quad H_0 : \sigma \ge \sigma_0, \ H_1 : \sigma < \sigma_0$$

$$\text{Two-tailed test:} \quad H_0 : \sigma = \sigma_0, \ H_1 : \sigma > \sigma_0 \ \ or \ \ \sigma < \sigma_0$$

Theorem 4.7 Given a normal distribution, three types of hypothesis tests can be performed on the population standard deviation with significance level α.

1. For right one-tailed test, if $\chi^2 > \chi^2_{\alpha,n-1}$, then the null hypothesis $H_0: \sigma \leq \sigma_0$ is rejected. Otherwise, there is insufficient evidence to reject the null hypothesis.
2. For left one-tailed test, if $\chi^2 < \chi^2_{1-\alpha,n-1}$, then the null hypothesis $H_0: \sigma \geq \sigma_0$ is rejected. Otherwise, there is insufficient evidence to reject the null hypothesis.
3. For two-tailed test, if $\chi^2 < \chi^2_{1-\frac{\alpha}{2},n-1}$ or $\chi^2 > \chi^2_{\frac{\alpha}{2},n-1}$, then the null hypothesis $H_0: \sigma = \sigma_0$ is rejected. Otherwise, there is insufficient evidence to reject the null hypothesis.

Example 4.11 One hundred fifty measuremes were taken for the dimension of a product. The sample standard deviation is calculated to be 0.0101. The vendor of this product claims that the standard deviation is less than or equal to 0.01, which is chosen as the null hypothesis. At 5 percent significance level, can we reject the null hypothesis based on the sample data?

Since we are testing whether the standard deviation is less than or equal to a value, it is a right one-tailed test.

$$H_0 : \sigma^2 \leq 0.01^2, \quad H_1 : \sigma^2 > 0.01^2$$

The test statistic is given by

$$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2} = \frac{149 \times 0.0101^2}{0.01^2} = 151.99$$

According to Theorem 4.7, this test statistic needs to be compared with $\chi^2_{\alpha,n-1}$, where α = 0.05 and n = *150*. Using Excel, $\chi^2_{0.05,149} = 121.787$. Since the test statistic is greater than $\chi^2_{0.05,149}$, the null hypothesis is rejected and the standard deviation of the measured dimension is deemed greater than 0.01. ■

Paired t-Test:

Let $x_1, x_2, \ldots, x_n$ be samples from a normal random variable X and $y_1, y_2, \ldots, y_n$ be samples from a normal random variable Y. Suppose X and Y are independent. Let $D = X - Y$. Then the following variable has a t-distribution with degree of freemdom of $n - 1$:

$$T = \frac{\bar{D}\sqrt{n}}{s_D} \tag{4.8}$$

where $\bar{D}$ and s_D are the sample average and standard deviation of D, respectively. The following theorem can be used to compare the population means μ_X and μ_Y.

Theorem 4.8 Let $x_1, x_2, \ldots, x_n$ be samples from a normal random variable X and $y_1, y_2, \ldots, y_n$ be samples from a normal random variable Y. Suppose X and Y are independent. Three types of hypothesis tests can be performed on the population means of X and Y with significance level α:

1. If $T \geq t_{\alpha,n-1}$, then the null hypothesis $\mu_X \leq \mu_Y$ is rejected. Otherwise, there is insufficient evidence to reject the null hypothesis.
2. If $T \leq -t_{\alpha,n-1}$, then the null hypothesis $\mu_X \geq \mu_Y$ is rejected. Otherwise, there is insufficient evidence to reject the null hypothesis.
3. If $T \geq t_{\alpha/2,n-1}$ or $T \leq -t_{\alpha/2,n-1}$, then the null hypothesis $\mu_X = \mu_Y$ is rejected. Otherwise, there is insufficient evidence to reject the null hypothesis.

Example 4.12 Two sets of data were collected before and after a Lean Six Sigma project.

Before	5.1	5.2	5.1	5.5	5.9	5.3	5.1	4.7
After	4.9	5.4	4.9	4.8	5.2	5.5	5.0	4.3

Can we conclude, with confidence level of 95 percent, that the population mean has been reduced?

Let us select the hypotheses as

$$H_0: \mu_b \le \mu_a, \; H_1: \mu_b > \mu_b$$

where μ_b and μ_a are the population means of before and after, respectively. It can be calculated that the difference between the "before" and "after" data has a sample average of 0.2375 and sample standard deviation of 0.3503. The degree of freedom is 7. Using Excel function "=T.INV(0.95,7)", we can calculate the *t*-critical value as $t_{0.05,7} = 1.8946$. The test statistic is

$$T = \frac{\bar{D}\sqrt{n}}{s_D} = \frac{0.2375\sqrt{8}}{0.3503} = 1.9179$$

Since $T > t_{0.05,7}$, we can conclude with 95 percent confidence level that the population mean has been reduced. ■

More detailed discussions on hypothesis tests can be found in the reference books [1, 9, 10, 13–15].

In many software packages, the *p*-value is calculated when conducting hypothesis testing. ***p*-value** is defined as the smallest significance level α at which the null hypothesis can be rejected. That is, if *p*-value ≤ α, the null hypothesis is rejected; If *p*-value > α, the hypothesis is not rejected.

4.3.3 *Cause-and-Effect Analysis*

Cause-and-effect diagram:

The **cause-and-effect** diagram, also known as the **fishbone diagram**, is a graphical method that can be used to analyze the root cause of a problem [7]. It is typically created in combination with other tools such as brainstorming, affinity diagram, and prioritization matrix. It starts from a problem statement, followed by sorting the possible causes of the problem into several categories such as **machine**, **material**, **measurement**, **method**, **manpower**, and **environment**. Each category contains more detailed causes. This information is represented in the diagram that resembles a fishbone, hence the name "fishbone diagram." The following example can be found in Section 3.3.4 (Figure 3.10, reprinted here as Figure 4.16).

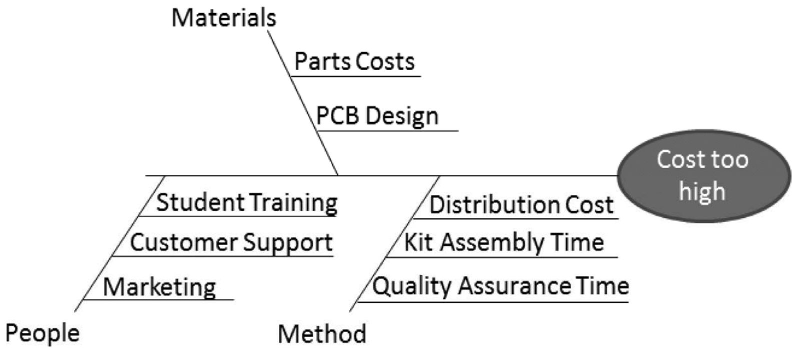

Figure 4.16 A cause-and-effect diagram example

After the fishbone diagram is created, the next step is to prioritize the causes so that the project team can focus on the most important ones.

Additional examples of the cause-and-effect diagram can be found in Ishikawa's book: *Guide to Quality Control* [7].

5 Whys:

Another effective tool for cause-and-effect analysis is the **5 Whys** method. It was first introduced by Toyota as a part of the Toyota Production System [1]. It states that one can ask "why" five times to get to the root cause of a problem. When asking the 5 Whys questions, the focus should be on the failure of the process where the problem resides. The concept of 5 Whys is illustrated by the following example.

Example 4.13 We have a problem with a vehicle: its engine cannot be started. We ask the first "why" and find that the vehicle's battery is dead. If we stopped at this point, the solution would seem to be replacing the battery. We ask the second "why," "Why is the battery dead?" The answer is that it is not properly charged by the alternator. Instead of replacing the alternator, we continue to ask the third "why," and discover that the alternator belt is slipping and almost broken. The fourth "why" unearthed the fact that the belt has not been replaced as recommended by the manufacturer. After the fifth "why," we know that the vehicle is not maintained properly by the owner. This is the root cause. By fixing the root cause, we can not only solve this problem but also prevent other problems from occurring. ■

4.3.4 FMEA

Failure Mode and Effects Analysis (FMEA) is an effective tool for risk assessment [16–18]. It is used to identify possible ways for failures to occur (**failure modes**), the **potential effects** of the failures, the **severity** of the consequence of the failures, the probability of failures occurring (**occurrence**), and the probability of detecting the failures (**detectability**). In addition to identifying the potential failures, FMEA is also used to prioritize actions that need to be taken. It is a common practice to use cause-and-effect analysis method during the process of FMEA analysis. An FMEA example is illustrated in Figure 4.17.

The header of the document contains information on the product, the FMEA, and those responsible for the documentation. The main body has a number of columns. The functions of a process or product are identified in the first column. For every function of the process, its potential failure modes are identified in the second column. For each failure mode, the effect of the failure is captured in the third column. The fourth column indicates the severity or the impact of the failure. This column is numerical, and the assignment of the numerical values can vary from one industry to another or even from one company to another. An example for severity criteria is given as follows:

10: Extremely Hazardous, 9: Hazardous, 8: Very High, 7: High, 6: Moderate, 5: Low, 4:Very Low, 3: Minor, 2: Very Minor, 1: None

Failure Mode and Effects Analysis

Item: Fan Motor   Responsibility: M. Johnson   FMEA #: A23901
Model: FM5609A   Prepared by: M. Johnson   Pages: 1 of 1
Team: M. Johnson (System Design), A. Lopez (Software), T. Katz (Mechinical engineering), L. Morgan (Production)   FMEA Date: 3/2/15
Revison: rev 1.0

Process Function	Potential Faiure mode	Potential Effects of Failure	Sev	Class	Potential Causes /mecahnisms of Failure	Occ	Current Process Control	Det	RPN	Recommended Actions	Responsibility and Target completion date	Action Results				
												Actions Taken	S	O	D	RPN
Drive HVAC fan at desired speeds	Speed too high	Temperature too high/low	3		Volateg too high	4	None	5	60							
	Speed too low	Temperature too low/high	3		Voltage too low	4	None	5	60							
					Motor performance deteriorating	4	None	9	108	Use back emf to measure speed	M. Johnson, A. Lopez	3/23/2015	3	4	3	36
	Inconsistant speed	Inconsistant temperature	2		Voltage fluctuating	7	None	5	70							
					Motor brush wear	3	None	10	60							

Figure 4.17 An example of FMEA document

The fifth column of the FMEA document is optional, indicating if the failure mode is safety- related or not. The sixth column indicates the potential causes for the failure. The seventh column is the occurrence, which is used to indicate the possibility of the failure occurring. An example for occurrence criteria is given as follows:

10: 1 in 2, 9: 1 in 3, 8: 1 in 8, 7: 1 in 20, 6: 1 in 80, 5: 1 in 400, 4: 1 in 2000, 3: 1 in 15,000, 2: 1 in 150,000; 1: 1 in 1,500,000

The eighth column captures what is currently being done to prevent or reduce the occurrence of the failure. The ninth column is detectability, indicating the possibility of the failure being detected. An example for detectability criteria is given as follows:

10: Almost Impossible, 9: Very Remote, 8: Remote, 7: Very Low, 6: Low, 5: Moderate, 4: Moderately High, 3: High, 2: Very High, 1: Almost Certain

The tenth column is the **Risk Priority Number** (**RPN**), which is calculated as product of S (severity), O (occurrence), and D (detectability). Based on the FMEA result, recommended actions to address the high RPN issues are recorded in the eleventh column. The twelfth column has the target completion dates for the recommended actions and the people responsible for the actions. The next five columns are for the next round of RPN evaluation, after the recommended actions are taken. The RPN values are the most important indicators in the FMEA form, items with high RPN values usually become high priority tasks to look into. Although different industries or companies have different general criteria for acceptable RPN values, the typical acceptable threshold is between 75 and 100.

It is a common mistake to oversimplify FMEA and think that the RPN numbers are the only output of the analysis. In fact, FMEA documents and guides the design effort during the product development and manufacturing processes. In some cases, failure modes with relatively low RPN also need to be investigated. For example, if the severity of a failure mode is very high even though its RPN is not, actions may be required to reduce its occurrence or detection level. On the other hand, for a failure mode whose consequence is not very severe but its occurrence is very frequent, that failure can be very annoying to the customer. A low RPN value alone is not enough a reason for ignoring the failure mode.

FMEA can be performed at the design stage for the design FMEA (**DFMEA**) and the manufacturing process planning stage for the process FMEA (**PFMEA**).

4.3.5 DOE

Design of experiments (DOE) is a method widely used in science and engineering for planning efficient experiments [19–23]. The purposes of the experiments include understanding the relationship between a set of inputs and outputs of a system, fitting regression models, and design optimization.

In the context of Lean Six Sigma and Design for Six Sigma, DOE helps us determine the impact of inputs or **factors** of a process on the output or **response** of the process. If there is only one factor, we can vary it and measure the response to understand the effect of the factor on the response, using techniques such as regression. More often the response is affected by multiple factors, and we would like to know the effect of each factor and possibly their **interactions** on the response. An interaction of two factors is important when their influence on the response is not additive. For instance, if the product AB of factors A and B causes the response to change significantly, rather than A and B separately, then the response is not an additive function of A and B. In that case, we can still use the regression techniques to uncover the impact of the factors on the response, but more experimental data will be needed. Under tight constraints on time and budget cost, it may not be feasible to conduct many experiments. DOE is a technique that can considerably reduce the number of experiments needed to study which factors and interactions have more significant impact on the response.

In a DOE, each factor takes several levels of values. The most popular choice is the **two-level DOE**, where a minimum and a maximum value are specified for each factor. The two levels are usually denoted by 1, −1, +, −, or 1, 2. There are two types of DOEs, full factorial and fractional factorial.

Full-factorial DOE

The **full-factorial DOE** contains all combinations of each factor taking the two levels.

Example 4.14 The full-factorial DOE for the four-factor, two-level experiments is given as follows:

StdOrde	RundOr	CenterP	Blocks	A	B	C	D
16	1	1	1	1	1	1	1
7	2	1	1	−1	1	1	−1
13	3	1	1	−1	−1	1	1
9	4	1	1	−1	−1	−1	1
5	5	1	1	−1	−1	1	−1
11	6	1	1	−1	1	−1	1
2	7	1	1	1	−1	−1	−1
4	8	1	1	1	1	−1	−1
3	9	1	1	−1	1	−1	−1
6	10	1	1	1	−1	1	−1
10	11	1	1	1	−1	−1	1
15	12	1	1	−1	1	1	1
1	13	1	1	−1	−1	−1	−1
12	14	1	1	1	1	−1	1
14	15	1	1	1	−1	1	1
8	16	1	1	1	1	1	−1

Note that there are 16 runs and each one can be replicated, causing the total number of runs to be large. To reduce the number of runs, we can use the ½-factorial DOE matrix, which will be discussed later. In general, for N factors each with two levels, the full-factorial DOE has 2^N combinations without replicates. ■

Unless the experiments are carried out using computer simulation, it is recommended to have two or more **replicates** for each experiment. If there are many factors and some factors are known to have insignificant effects or their effects are completely known, then we can use the **blocking** technique in DOE to fix those factors. To reduce the impact of unknown factors, the order of experiments should be **randomized**. After the experiments are completed, the outcome can be used to create a Pareto chart, the **main effect chart**, and the **interaction chart** to identify the main effect and the interaction. Nowadays, most DOE matrices are generated with computer software, so are the charts. However, in order to illustrate the basic concept, let us go through the following simple example of calculating the main effects.

Example 4.15 Consider a full-factorial DOE of two factors A and B, each with two levels. The first two columns in the matrix define the DOE tests. There are four runs in total. The third column is the interaction factor AB. The tests were run, and the response y was recorded in the last column. Assume that we would like to maximize the value of the response y.

A	B	AB	y
1	1	1	3
1	−1	−1	14
−1	1	−1	6
−1	−1	1	4

To calculate the main effect of factor A, we take the average of the y values when $A = 1$, that is, the average of 3 and 14, which is 8.5. We then take the average value of y when $A = -1$, which is 5. The main effect of A is $8.5 - 5 = 3.5$. Similarly, the main effect of B is -4.5, and the main effect of AB is -6.5.

Since the absolute value of AB is the largest, the DOE analysis tells us that the interaction between A and B has the most significant effect on the response y. Factor B is the second most significant factor, while A is the least significant. To maximize the response, we would choose A and B with different signs so that $AB = -1$; factor B should be set at -1 and factor A at 1. ■

Fractional-factorial DOE

Most of the time, responses are only affected by a small number of main effects and lower order interactions; hence, not all the experimental data in a full-factorial design are useful information. When the amount of experiments in a full-factorial DOE is too many, one may choose to use the **fractional-factorial DOE** instead. A fractional-factorial DOE eliminates certain experiments from the full-factorial DOE, hence has considerably fewer experiments. The drawback of using fractional-factorial DOE is that we may not be able to determine the impact of certain interactions.

Example 4.16 Four-factor, two-level, ½-factorial DOE

StdOrder	RundOrder	CenterPt	Blocks	A	B	C	D
6	1	1	1	1	−1	1	−1
3	2	1	1	−1	1	−1	1
4	3	1	1	1	1	−1	−1
7	4	1	1	−1	1	1	−1
1	5	1	1	−1	−1	−1	−1
2	6	1	1	1	−1	−1	1
8	7	1	1	1	1	1	1
5	8	1	1	−1	−1	1	1

From the full-factorial DOE test matrix in Example 4.13, if we choose the rows with $ABCD = 1$, where $ABCD$ is the interaction among A, B, C, and D, then we have only one-half of the experiments. We call it the "½ factorial" because its number of experiments is one-half of that of the full-factorial DOE. Alternatively we can choose the rows with $ABCD = -1$. In either case, by reducing the total number of runs, we no longer have the information on interaction $ABCD$. The data analysis with this fractional-factorial DOE will only reveal the effects of the single factors, the interactions of any two factors, and the interactions of any three factors. One can further reduce the runs by using the ¼-factorial DOE, in which case, we will lose the information of the interactions of some three factors and of all four factors. There is a trade-off between the reduced runs and the information coming out of the DOE analysis. However, if we know that the three-factor and four-factor interactions are not significant based on physics or other experience, then such a trade-off is justified. ■

Taguchi design:

Among the fractional-factorial DOEs, one type deserves special attention: **orthogonal arrays** (**OA**). These highly fractional DOEs were promoted by Dr. Genichi Taguchi, hence also referred to as **Taguchi design** [23–25]. In the orthogonal arrays, the number of tests is greatly reduced. For each level of a particular factor, all levels of each of the other factors are tested at least once. OA are also **balanced**, meaning that in the test matrix each factor takes different levels for the same number of times. Figure 4.18 shows the Taguchi OA of eight experiments for seven factors, each with two levels (L8 array). For more OA matrices, refer to Appendix B of this book.

Taguchi method is best used when there are less than 50 factors, few interactions between variables, and only a small number of factors have significant impact on the response. The Taguchi design also contains some unique aspects such as the treatment of **uncontrollable input** (the **inner array factors** or **noise**) and **controllable input** (the **outer array factors** or **signal**) and the **signal-to-noise ratio**. In Chapter 7, we shall discuss this subject in more detail.

Run	Columns						
	1	2	3	4	5	6	7
1	1	1	1	1	1	1	1
2	1	1	1	2	2	2	2
3	1	2	2	1	1	2	2
4	1	2	2	2	2	1	1
5	2	1	2	1	2	1	2
6	2	1	2	2	1	2	1
7	2	2	1	1	2	2	1
8	2	2	1	2	1	1	2

Figure 4.18 Taguchi method: L8 array

More examples of DOE analysis can be found in Chapters 7 and 8. It is also a common practice to use the DOE data to create a response surface, which will be discussed in Section 4.4.1.

4.3.6 Value Stream Mapping

Value Stream Mapping (VSM) is a Lean method for analyzing a process and eliminating wastes in the process [2, 26–28]. Lean focuses on **value**, defined as what is important to the customer, through the elimination of waste and acceleration in the velocity of the processes. **Value-added** work is something that customers are willing to pay for, including tasks that add desired functionalities or features to the product. It can also be tasks that result in lower price, short lead time, and good quality. **Non–value-added** work, or **waste** (translated from a Japanese word ***muda***), includes tasks that have no value to the customer. Whether a particular task is value-added or non–value-added, we should judge from the customers' perspective. By analyzing the process and identifying what the value-added and non–value-added steps are, VSM allows one to focus on the elimination of waste and the increase of the process speed.

There are seven commonly known categories of waste: over production, inventory, defects, motion, excessive processing, waiting, and transport. Over production is producing more than needed. Inventory includes raw materials, work in process (WIP), and finished products, since extra inventory costs money. Motion refers to the physical activities of the operator. Unnecessary motions for the operator should be avoided. Transport muda is about inefficient move of the work piece or product. Some also add unused talents as the eighth muda.

To identify the waste in the process, we first need to understand how the process works. A value stream map can be divided into three sections: the information flow, the process flow, and the timeline and summary statistics, as illustrated in Figure 4.19. This VSM is from the Krisys Robot Kit project presented in Chapter 3, where more background information on the project can be found. The VSM in Figure 4.19 was created using Microsoft Visio, which has a group of standard symbols in the stencil accessible by clicking **Shapes> More Shapes> Business> Business process> Value Stream Map Shapes**.

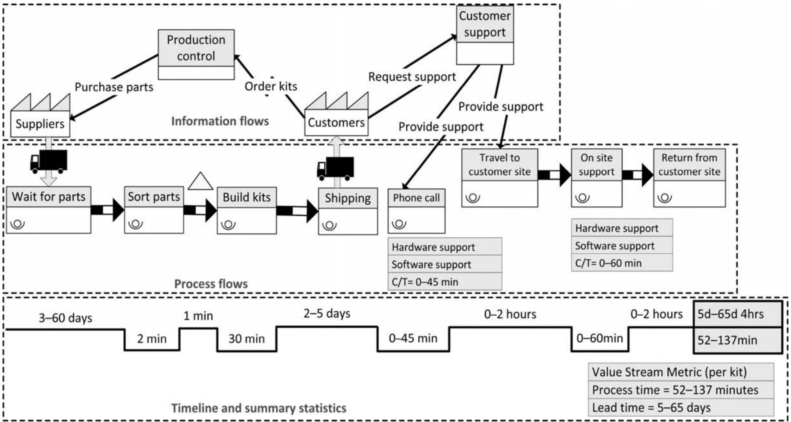

Figure 4.19 A VSM example

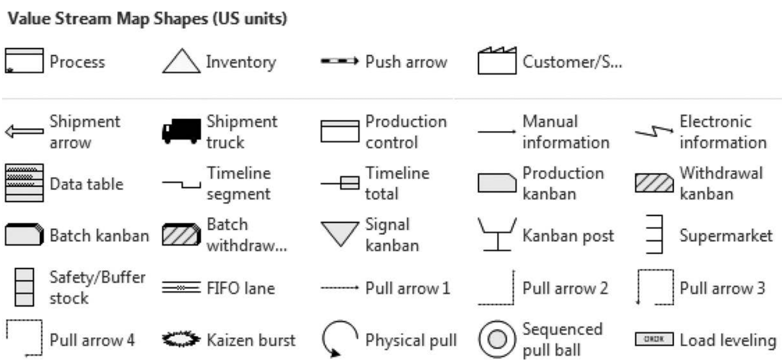

Figure 4.20 VSM symbols

The legend for the VSM shapes used in Visio can be found in Figure 4.20.

Most of the symbols are self-explanatory. A **production kanban** is a visual signal representing a trigger for producing a specific number of parts. A kaizen burst highlights the need for improvement that is critical to the overall process. More information can be found in [28].

The procedure for VSM analysis is as follows:

1. Understand how the process works.
2. Make observation of the information flow and the process flow.
3. Measure the average time for each step in the process.
4. Analyze which steps of the process are value-adding and which are not value-adding.

5. Draw the VSM for the current process, which is known as the **current state map**.
6. Identify improvement opportunities by eliminating the waste (muda or non–valued-added steps) in the process.
7. Modify the current process to make it more efficient. The modified VSM is also known as the **future state map**.

After the VSM was created for the Krisys Robot Kit project, it was easy to conclude what areas needed improvement to make the process more efficient. The value-added steps only took 52 to 137 minutes while the non–value-added steps took 5 to 65 days. The majority of the lead time was spent on ordering parts from suppliers and shipping kits to customers. This example confirms the well-known **Pareto principle** or the **80/20 rule**: 80 percent of the problem arises from 20 percent of the process issues. The Krisys team decided that certain parts with lead time of 60 days should be ordered long before the customer orders came in. The team was able to significantly reduce the lead time by optimizing the inventory with the forecast of the orders.

One of the tools that help eliminating waste is **5S: Sort**, **Set in order**, **Shine**, **Standardize**, and **Sustain**. 5S can improve the productivity by maintaining an orderly workplace and using visual cues to achieve more consistent operational results.

Additional important concepts used in VSM are listed below:

Work in Process (WIP): Average number of units in the process at any point in time.

Lead Time (LT): The average time it takes from start to finish in making a product or completing a service (unit: time period such as seconds, minutes, hours, days, etc.)

Cycle Time (CT): The average time between two successive finished products or completed service (unit: time period such as seconds or minutes per piece). Cycle time reduction is one of the most common objectives of Lean Six Sigma projects.

Takt Time: The available time in a specified time period divided by customer demand (how many products should be produced or customers should be served) in the same time period (unit: time period such as seconds or minutes per piece).

Throughput: The production output rate, which is the reciprocal of cycle time (unit: pieces per unit time period such as second, minute, etc.)

The LT, CT, and WIP are related by the following formula:

$$LT = CT \times WIP \tag{4.9}$$

Many practitioners are confused by the definitions of LT, CT, and Takt. The following simple example illustrates how each one is calculated.

Example 4.17 A customer receives a service that requires her to go through three consecutive steps, each taking 10, 15, and 12 minutes to complete. It is desirable that as many as eight customers can be served in each day. Each day, the three service people all work 8 hours, with 1 hour lunch break and 1 hour for other breaks. Find the LT, CT, and Takt. Will they be able to serve as many as eight customers a day?

If there is someone right before the customer, then she will spend 10 minutes in counter 1 and wait for 5 minutes before she goes to counter 2. She will then spend 15 minutes at counter 2 and 12 minutes at counter 3, assuming the time to move from one counter to the next is negligible. So, the total time the customer needs is 10 + 5 + 15 + 12 = 42 minutes, which is the LT. To calculate the CT, we start timing when the first customer has completed the service. At that point, the second customer has spent 12 minutes at counter 2. She will need 3 more minutes at counter 2 and 12 minutes at counter 3 to complete her service. Therefore, the CT is equal to 3 + 12 = 15 minutes.

If a customer walks in with nobody right in front of him waiting for service, then the LT is 10 + 15 + 12 = 37. So, the LT is between 37 and 42 minutes.

The total available time in a day is 8 − 1 − 1 = 6 hours; here for simplicity we assume that the three workers all take breaks at the same time. Takt = 6 × 60 (minutes)/8 = 45 minutes per customer. Since the worst case LT is 42 minutes, the answer is yes, they will be able to serve eight customers per day. ■

4.4 RSM, Regression

4.4.1 Response Surface Method

The purpose of the **Response Surface Method (RSM)** is to fit a set of input–output data with a mathematic model, so that the output can be estimated from the values of the inputs [29, 30]. The resultant model can then be used to solve optimization problems, or to understand the effects of the inputs (factors) on the output (response). In Lean Six Sigma projects, RSM can help find the optimal values or ranges for several factors such that the response is optimal, which typically means that the output achieves the largest or the smallest value. For example, the response could be the total cost or the number of defects to be minimized. If the response represents the efficiency, profit, or customer satisfaction level, then we would like to maximize it. RSM can be effectively used when either the input–output relationship is too difficult to characterize with a first-principle mathematical model or the mathematical model is too complex for finding a closed-form expression of the optimal solution. The RSM is typically used in combination with DOE. After the DOE analysis, we select the factors that have significant impact on the response to be included in the response surface, and set the other factors at their nominal values. If possible, we would narrow the major factors to less than three. This is because RSM relies on graphics to show the relationship between the inputs and outputs, and the highest dimension for easy-to-understand graphics is three.

Figure 4.21 shows the cost function of an electric vehicle plotted as a "surface," its "height" changing with two factors, the specific energy, and the battery weight [31]. The optimal- or low-cost region of the design is depicted by the oval shape in the horizontal plane.

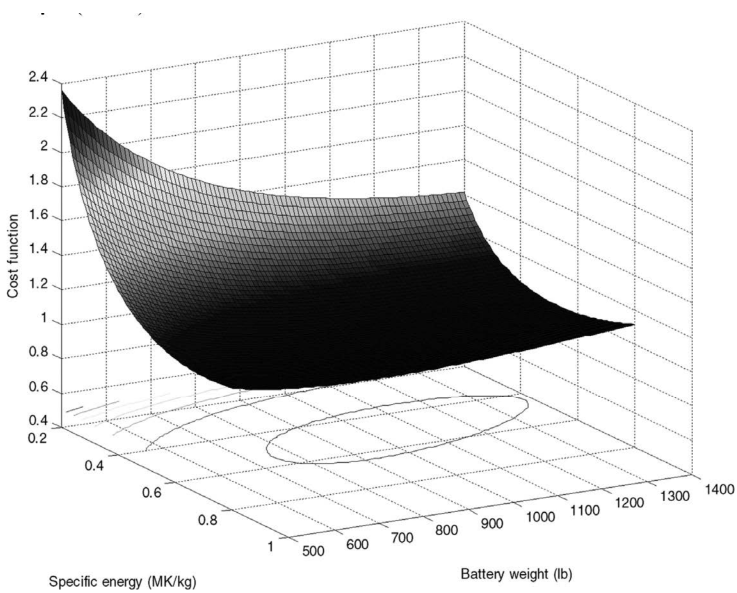

Figure 4.21 A RSM example [31]

Source: Reprinted with permission by SCIYO.

More examples of RSM analysis can be found in Chapter 8 and the references listed at the end of this chapter.

4.4.2 Regression

Regression is used to find the underlying relationship between the factors and response from a data set [9, 10, 13, 15]. We use a simple linear model to illustrate the concept of regression. Given a data set of n pairs (x_1, y_1), (x_2, y_2), . . . , (x_n, y_n), first, we plot the n points in a two-dimensional Cartesian coordinate system, shown in Figure 4.22.

The question to be answered is, Is there a linear relationship between x and y? Note that there are a lot of variations in y, which can be divided into two categories: one caused by the change in x and the other caused by some random noise in the data. We propose the following model.

$$y = mx + b + \varepsilon \tag{4.10}$$

where ε is a random variable. This model can be interpreted as "The variation in y is caused by the change of x, as described by $mx + b$, and a random noise ε." To find the two parameters m and b, we use the **principle of least squares.**

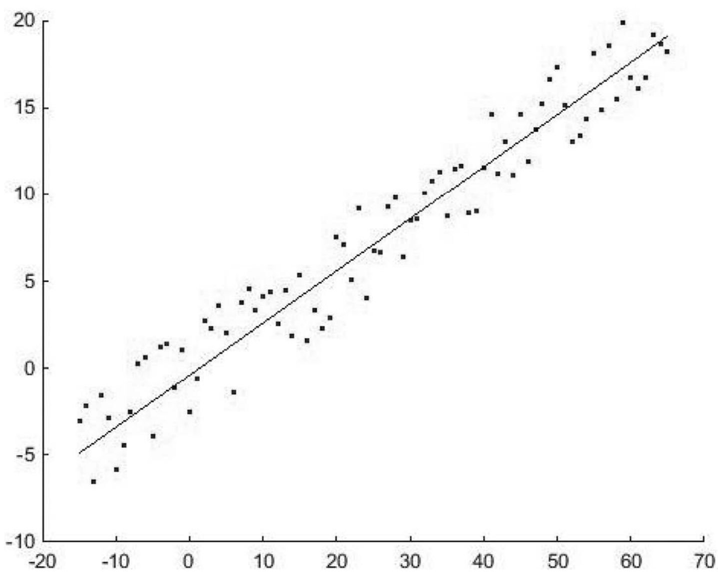

Figure 4.22 Linear regression

First, we define the error for each point (x_i, y_i) as the difference between y_i and the value of the straight line $y = mx + b$ at x_i. Our objective is to minimize the sum of squared errors by choosing the appropriate values for m and b. Mathematically, this is an optimization problem defined as follows:

$$\min_{m,b} \sum_{i=1}^{n} [y_i - (mx_i + b)]^2$$

Define the sum of the error square terms as

$$f(m, b) = \sum_{i=1}^{n} [y_i - (mx_i + b)]^2$$

The optimal values of m and b can be found by taking the partial derivatives of f with respect to m and b and then set the partial derivatives equal to 0. Solving for m and b, we get

$$m = \frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{\sum_i (x_i - \bar{x})^2} \tag{4.11}$$

$$b = \frac{\sum_i y_i - m\sum_i x_i}{n} \tag{4.12}$$

where $\bar{x}$ and $\bar{y}$ are the sample average for x_i and y_i, respectively.

Let us compare the total variation in y and the portion caused by ε, the noise in the data. The **total sum of squares** (*TSS*), which is an indication for the total variation in y, is given by

$$TSS = \sum_i (y_i - \bar{y})^2 = \sum_i y_i^2 - \frac{1}{n}\left(\sum_i y_i\right)^2 \tag{4.13}$$

The **residual sum of squares** (*RSS*), which represents the variation in y caused by the noise, is given by

$$RSS = \sum_i (y_i - \hat{y})^2 = \sum_i \left[y_i - (b + m x_i)\right]^2 \tag{4.14}$$

We then use the ratio of *RSS* over *TSS* as a measure for how well the linear model fits the data. If the fit is perfect, then $RSS = 0$. If there is no relationship between x and y, then the total sum of square should not be reduced by the line fitting effort, that is, *RSS* should be equal to *TSS*. Based on this insight, the **coefficient of determination** R^2 is used to indicate how good the fit is.

$$R^2 = 1 - \frac{RSS}{TSS} \tag{4.15}$$

If R^2 is close to 1, then we have a good regression model; if it is close to 0, then we should look for a different model.

Similarly, the principle of least square can be applied to **multivariable model** and **nonlinear model**. For example, the principle of least square can be directly applied to the following model.

$$y = a_0 + a_1x_1 + a_2x_2 + a_3x_3 + \varepsilon \tag{4.16}$$

$$y = a_0 + a_1x + a_2x^2 + a_3x^3 + \varepsilon \tag{4.17}$$

Equation (4.17) is nonlinear in the x but linear in the parameters a_0, a_1, . . . , a_3, and ε. The formulas will be different from those for the linear single variable case, but the derivation steps are the same.

Example 4.18 A temperature sensor (thermistor) is characterized by measuring its resistance at various temperatures. The test data is given in the following table:

T (degree C)	0	10	14	18	21	22	24	25	26	27
R (Ohms)	6,980	4,560	3,759	3,040	2,700	2,495	2,260	2,150	2,050	1,950

We can use linear regression to find the relationship between the resistance and the temperature. In Excel, insert a scattered chart of the data and add a trend line, with the equation and R^2 value displayed as shown in Figure 4.23.

The coefficient of determination has a value close to 1, indicating that the trend line is a good fit of the data. The equation of the trend line, $y = -181.74\,x + 6593$, provides us an easy way to predict the resistance at a given temperature, or predict the temperature at a given resistance. This method is commonly used in the design of electronic instrumentation. ■

Since both authors are runners, we use the following example to show how regression can be used in running.

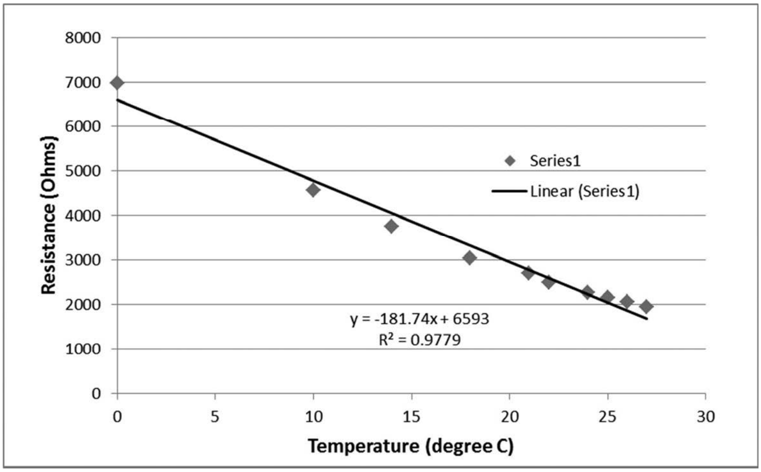

Figure 4.23 Linear regression model for a temperature sensor

Example 4.19 For distance runners, it is important to control the pace of running. Many prediction methods are available for runners to use. For example, Runner's World gives the following formula:

$$t_2 = 1.06 t_1 \times \frac{d_2}{d_1}$$

where t_1, t_2, are times and d_1, d_2 are distances, d_2 being longer between the two. This formula allows a runner to predict his or her marathon time using his or her half marathon time for instance. How was this formula generated? Regression! Runner's World collected a lot of data from runners and used regression to come up with this formula. This formula is simple to use; however, it only reflects the average of all runners who contributed their data. For those runners who are trying to shed a few minutes off from their marathon time, this formula is not going to help much. In contrast, if we can collect more relevant data from a certain age group of male or female runners, the regression model would better predict the performance of runners in the same age and gender group.

If a runner keeps his own record of times for various shorter distances, then a regression model can be derived to predic his marathon time and the pace he should target. Table 4.1 is an example of such record.

We can use regression to find a formula based on the data in Table 4.1. This time we use the "Power" option in Excel trend line selection, and the result is shown in Figure 4.24 as $y = 6.5509x^{0.0959}$, where x is the distance

Table 4.1 Marathon time prediction

Miles	Time	Pace	Pace in minutes
3	21:58	7:19	7.317
6	47:22	7:44	7.733
7	54:58	7:52	7.867
8	1:03:38	7:57	7.950
9	1:13:16	8:08	8.133
13	1:49:30	8:25	8.417

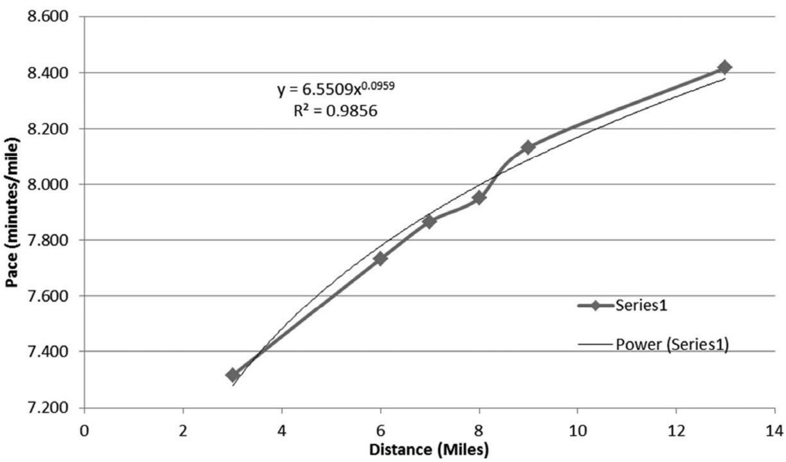

Figure 4.24 Race pace prediction using a regression model

■

and y is the pace (minutes per mile). Let $x = 26.2$, then $y = 8.96$ minutes per mile. His predicted marathon time is $26.2 \times 8.96 = 234.75$ minutes or 3:54:45.

4.5 Statistical Process Control

All processes have variation. It is critical to know if the variation is due to random variation or some changes of the process. The latter would require corrective actions. **Statistical process control** (SPC) is a statistic analysis method for measuring, monitoring, and controlling processes [1, 32, 33]. SPC uses different types of control charts to identify instability and unusual circumstances and to diagnose problems. A **control chart** plots the **process average**, **upper control limit** (*UCL*), **lower control limit** (*LCL*), and certain process metrics. There are many control charts that can be plotted in SPC; two most commonly used ones are $\bar{X}$ **chart** and *R* **chart**, where the averages and ranges are the process metrics respectively.

The procedure for plotting the $\bar{X}$ chart and *R* chart is as follows:

1. Select **subgroups** such that specific information can be attached to the subgroups. For instance, we can select subgroups according to the timestamps or the machine/operator that is involved in processing the parts. This allows us to pinpoint where or when a problem might have happened. The size of the subgroup *n* is typically chosen to be greater than three. The sampling frequency (e.g., out of every 100) is selected. Sufficient data are required before we can plot the control charts. A typical value is more than 25 subgroups.
2. Calculate the average $\bar{X}$ for each subgroup.
3. Calculate the range *R* for each subgroup.
4. Calculate the average of $\bar{X}$, which is the center line for the $\bar{X}$. chart. Calculate the average of *R*, which is the center line for the *R* chart.
5. Calculate the *UCL*s and *LCL*s:
   For the X-bar chart: $LCL = \bar{\bar{X}} - A_2\bar{R} \quad UCL = \bar{\bar{X}} + A_2\bar{R}$ (4.18)
   For the R chart: $LCL = D_3\bar{R} \quad UCL = D_4\bar{R}$ (4.19)

where A_2, D_3, and D_4 are given by Table 4.2.

It can be shown that for normal distributions, the standard deviation can be estimated by $A_2\bar{R}/3$. So, the *UCL* and *LCL* for $\bar{R}$ are $+/-3\sigma$ from the mean. It can also be shown that the standard deviation for *R* can be estimated by a function of *n*, $\hat{\sigma}_R(n)$. Defining

$$D_3 = 1 - 3\hat{\sigma}_R(n), \quad D_4 = 1 + 3\hat{\sigma}_R(n), \tag{4.20}$$

it is clear that the *LCL* and *UCL* for the *R* chart are the $+/-3\sigma$ from the mean. The details of the proof and calculation of A_2 and $\hat{\sigma}_R(n)$ is out of the scope of this book.

It can be calculated that the probability of the sample average being between *LCL* and *UCL* is 99.73 percent. Based on this observation, we can claim that the probability of the sample average $\bar{X}$ falling outside of the

Table 4.2 Coefficients for X-bar and R charts

n	A_2	D_3	D_4
2	1.88	0	3.267
3	1.023	0	2.574
4	0.729	0	2.282
5	0.577	0	2.114
6	0.483	0	2.004
7	0.419	0.076	1.924
8	0.373	0.136	1.864
9	0.337	0.184	1.816
10	0.308	0.223	1.777
11	0.285	0.256	1.744
12	0.266	0.283	1.717
13	0.249	0.307	1.693
14	0.235	0.283	1.717

range between *LCL* and *UCL* is very small. If this does happen, then we have a high level of confidence that something other than the random variation in the process has occurred. This is the basis for many of the rules for determining if something caused the changes in the process mean and/or standard deviation; these causes are called **special causes of variations**. If the process mean and standard deviation do not change, the variations in the process are called **common cause of variation**. Therefore, SPC charts provide us with a way to differentiate the common causes and the special causes of variations. Based on the above discussion, we have the following basic rule:

1. The basic rule violation: A point is outside of the +/−3σ control limits.

In addition to this basic rule, there are many other rules, some of which are related to the +/−1σ or +/− 2σ lines. Those lines can be plotted on the SPC chart by dividing the region from the center line to the +/− 3σ lines into three equally sized regions. Some of the popular rules are listed as follows:

2. Run violation: Seven consecutive points on one side of the center line
3. Trend violation: Upwards or downwards movement of seven points
4. Two of three consecutive points above 2σ

5. Two of three consecutive points below -2σ
6. Fourteen points in a row, alternating direction
7. Fifteen points in a row within +/-1σ

It is worth noting that typically only a few rules from No. 2 to 7 are used. Using too many rules can cause false detection of special causes of variation, which can be annoying to the user. A process is said to be **out of control** or **unstable** if any of the rules we selected to use for SPC charts is violated. Otherwise, the process is said to be **in control** or **stable**.

Similar to the discussion we had in "Hypothesis Testing" (Section 4.3.2), we can define the following errors:

Type I error: The chart indicates the process is out of control, but the process is actually in control.

Type II error: The chart indicates the process is in control, but the process is actually out of control.

We have calculated the probability of a point being inside of the +/-3σ limits as 99.73 percent. Therefore, the probability of type I error (α) related to rule 1 is 1 − 99.73% = 0.27%.

Example 4.20 A battery manufacturer measures the battery voltage at the end of the production line. They randomly select a subgroup of 5 out of every 100 batteries for voltage measurement. The measurement data are given in the following table:

Subgroup #	Battery voltages					x bar	R
1	1.500	1.514	1.518	1.495	1.503	1.506	0.023
2	1.505	1.490	1.497	1.508	1.508	1.502	0.018
3	1.489	1.511	1.500	1.504	1.484	1.498	0.026
4	1.497	1.504	1.511	1.497	1.495	1.501	0.016
5	1.508	1.489	1.506	1.490	1.502	1.499	0.019
6	1.505	1.491	1.507	1.505	1.508	1.503	0.017
7	1.498	1.490	1.495	1.503	1.511	1.500	0.021
8	1.495	1.508	1.511	1.515	1.486	1.503	0.030
9	1.491	1.508	1.493	1.492	1.483	1.494	0.025
10	1.507	1.496	1.485	1.528	1.503	1.504	0.043

Subgroup #	Battery voltages					x bar	R
11	1.509	1.497	1.505	1.482	1.504	1.499	0.026
12	1.497	1.503	1.480	1.500	1.519	1.500	0.038
13	1.498	1.495	1.508	1.510	1.485	1.499	0.025
14	1.601	1.491	1.515	1.500	1.494	1.520	0.110
15	1.500	1.507	1.496	1.486	1.506	1.499	0.021
16	1.517	1.491	1.518	1.504	1.503	1.507	0.026
17	1.509	1.503	1.503	1.495	1.491	1.500	0.019
18	1.488	1.492	1.490	1.495	1.487	1.490	0.008
19	1.498	1.514	1.493	1.501	1.511	1.503	0.021
20	1.500	1.505	1.499	1.501	1.501	1.501	0.005
21	1.510	1.510	1.497	1.493	1.490	1.500	0.020
22	1.493	1.517	1.485	1.489	1.512	1.499	0.031
23	1.498	1.487	1.493	1.506	1.494	1.496	0.019
24	1.514	1.509	1.514	1.519	1.488	1.509	0.031
25	1.494	1.498	1.504	1.504	1.495	1.499	0.011
26	1.495	1.503	1.497	1.499	1.501	1.499	0.007

Since n = 5, we have

$$A_2 = 0.577,\ D_3 = 0, \text{ and } D_4 = 2.114.$$

The averages of $\overline{X}$ and R are

$$\overline{\overline{X}} = 1.501, \overline{R} = 0.025.$$

The control limits are calculated as follows:

$$\overline{X}: \ LCL = \overline{\overline{X}} - A_2\overline{R} = 1.501 - 0.577 \times 0.025 = 1.487$$

$$UCL = \overline{\overline{X}} + A_2\overline{R} = 1.501 + 0.577 \times 0.025 = 1.516$$

$$R: \ LCL = D_3\overline{R} = 0 \times 0.025 = 0; \ UCL = D_4\overline{R} = 2.114 \times 0.025 = 0.054$$

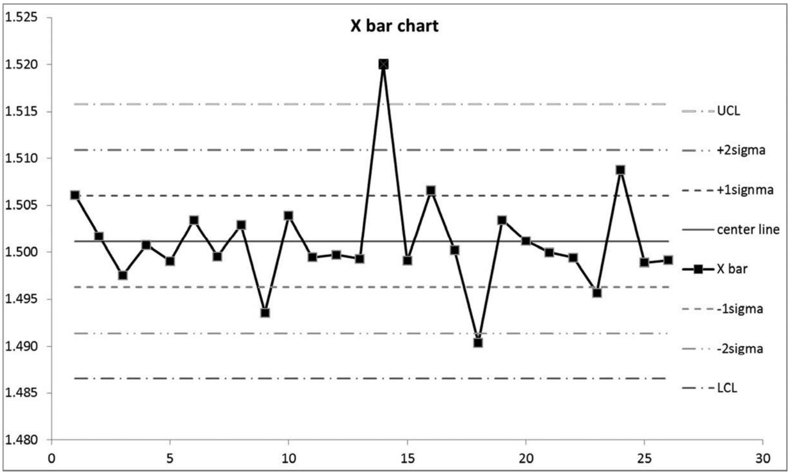

Figure 4.25 X bar chart for Example 4.20

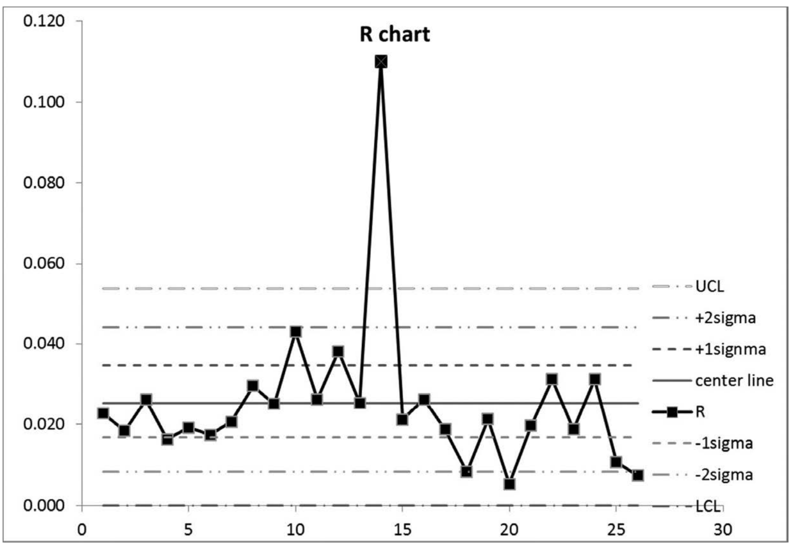

Figure 4.26 R chart for Example 4.20

Inspection of the $\overline{X}$ chart and R chart (Figures 4.25 and 4.26) reveals that the process is out of control, since rule 1 was violated in both charts. We leave the other rules for the reader to check. ■

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Further readings

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CHAPTER 5

Excel, MATLAB, Minitab, and other simulation tools

As we have learned in Chapter 4, Lean Six Sigma projects use statistical analysis tools extensively. Most projects have data that are more than what we want to handle with paper and pencil. Fortunately, there are quite a few software packages that are inexpensive and easy to learn and can help us analyze data. In this chapter, we will focus on three software packages: Excel, MATLAB [1, 2], and Minitab [3–7]. R will be briefly discussed due its free availability [8]. Excel is probably the most accessible among all the statistical analysis tools. While not obvious to most, Excel is extremely capable of performing statistical analysis. MATLAB is not necessarily low in price, but it deserves our attention because many engineers use it for engineering design and analysis. Minitab specializes in statistical analysis and is inexpensive as well. We will demonstrate the usage of these three software packages in the statistical analysis and graphing discussed in Chapters 2 to 4.

5.1 Excel

There are two places in Excel where you can find statistical analysis tools. One resides in the functions available in Excel such as AVERAGE, MAX, MIN, MEDIAN, MODE.MULT, which you can find by typing the equal sign (=) in a cell. Excel will then display a list of functions for you to choose from. If the function you need is not displayed on the list, click on "**More Functions…**" and you will see the window shown in Figure 5.1. We will use some examples to illustrate how to use Excel to conduct statistical analysis.

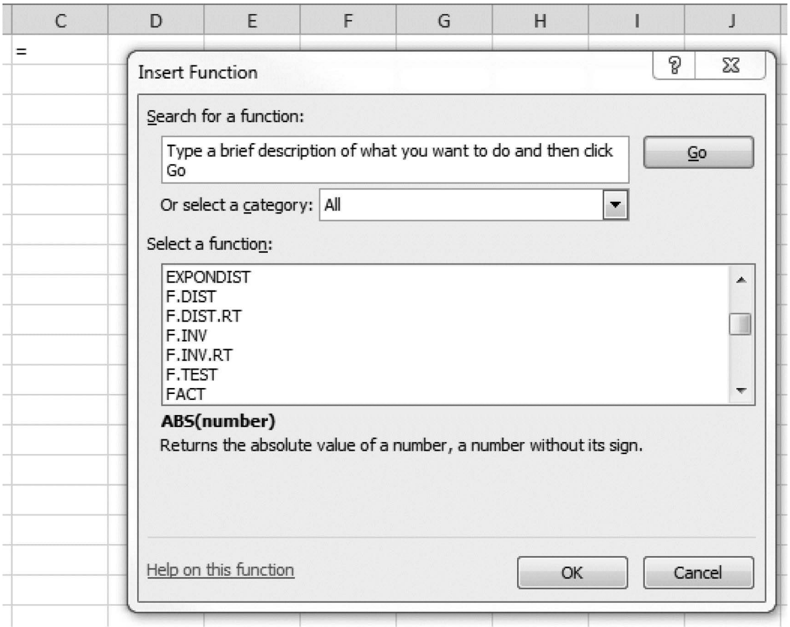


Figure 5.1 Inserting a function in a cell in Excel

Example 5.1 Most of the Excel functions are straightforward to use, but there are some confusing ones, one of which can be used to calculate the modes of a data set.

To find the mode of data in a column (cell A1:A8), first highlight a few cells, say B1:B5. Then type "= MODE.MULT(", highlight A1:A8, then press CTRL+SHIFT+ENTER. If done correctly, you will see that 4, 2, and 8 are the three modes of the data, as shown in Figure 5.2. By pressing CTRL+SHIFT+ENTER, we entered the data as an array. Note that the curly brackets were added in the cell formula as a result. If we do not enter the data as an array, the result will only have the first mode, which is 4 in this case. It may take a few trials, because we didn't know the number of modes, which is how many cells in column B we should highlight. If we don't highlight enough cells, the result will be incomplete. The last two cells with #N/A tell us that there are no more modes.

If you need to know how to use a function, you can double click on the link that appears after you selected the function name. An Excel Help window will pop up with detailed instructions for how to use that function. The instructions can be confusing sometimes, as is the case in this example.

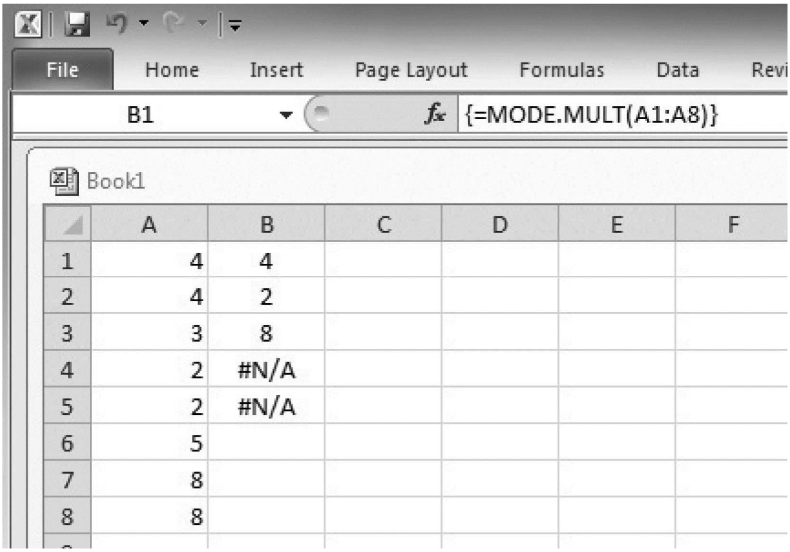


Figure 5.2 Calculating mode in Excel

Warning: Don't try to manually edit one of the cells in B1:B5. You may have to go to the task manager to stop Excel! ■

Some Excel functions that are useful in Six Sigma projects are listed as follows:

BETA.DIST, BETA.INV, CHISQ.DIST, CHISQ.DIST.RT, CHISQ.INV, CHISQ.INV.RT, CHITEST, CONFIDENCE.NORM, CONFIDENCE.T, F.DIST, F.DIST.RT, F.INV, F.INV.RT, F.TEST, LOGNORM.DIST, LOGNORM.INV, NEGBINOM.DIST, NORM.DIST, NORM.INV, NOR.S.DIST, NORM.S.INV, PERCENTILE.EXC, PERCENTILE.INC, QUARTILE.EXC, QUARTILE.INC, RAND, RANDBETWEEN, STDEV, T.DIST, T.DIST.2T, T.DIST.RT, T.INV, T.INV.2T, T.TEST, TREND, TRIMMEAN, WEIBULL.DIST, Z.TEST.

Most of the above functions are self-explanatory, and detailed information can be found in Excel Help if needed.

Example 5.2 Generate 200 random values for a normal distribution with a mean of 2 and a standard deviation of 3. Use the data to find the sample mean, sample standard deviation, range, fifteenth percentile, lower-fourth Q_1, upper-fourth Q_3, median, 10 percent trimmed mean, and plot the histogram.

Before we use the **Data Analysis** tools in Excel, we need to make sure the **ToolPak** is selected, it is under **File>Options>Add-Ins**. If not, you need to select **Analysis ToolPak** and then click **OK**, as shown in Figure 5.3.

Now if you click on the **Data** tab, **Data Analysis** should appear in the top far right corner. Click on it, and the window shown in Figure 5.4 will appear.

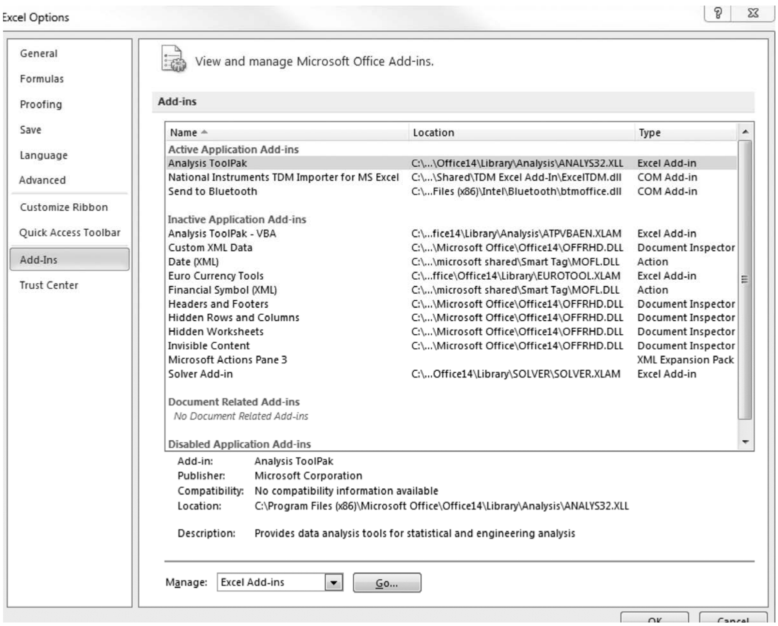


Figure 5.3 Setting up Analysis ToolPak in Excel

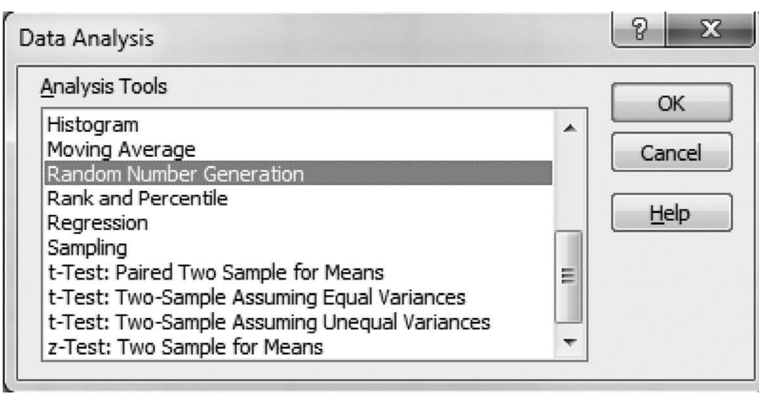


Figure 5.4 Data Analysis window

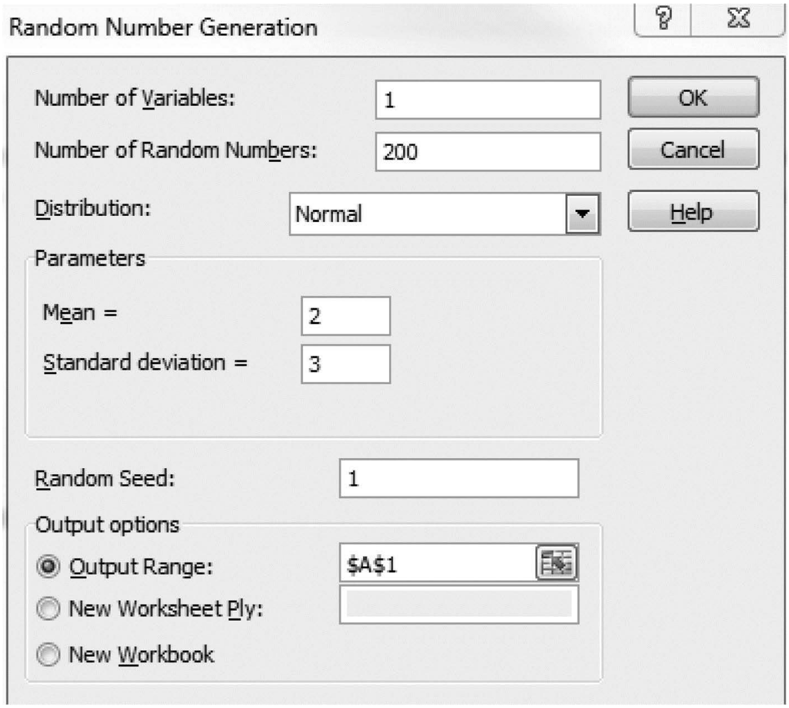
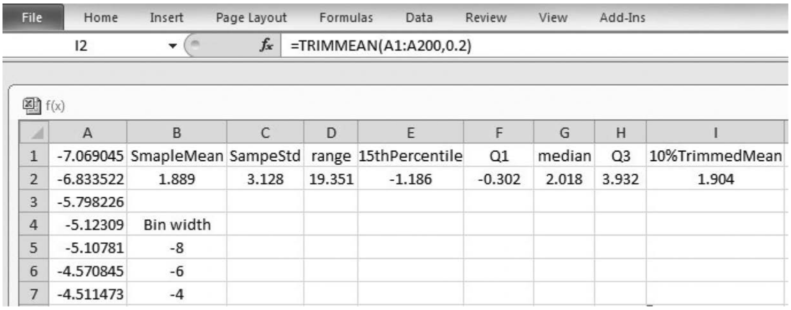


Figure 5.5 Random number generation using Excel

Click on **Help**, you can read the information about how each function works. Select **Random Number Generation** and click **OK**, then fill in the form that pops up, shown in Figure 5.5. The numbers should be self-explanatory except for the **Output Range**, which simply asks where you want to start storing the random numbers generated. If you click the A1 cell, $A$1 will appear in the **Output Range** input area. After you complete the form, click **OK**. A1:A200 will be filled with random numbers that comes from a normal distribution. It is worth noting that if **Random Seed** is not selected, then you will get a different result each time you repeat the steps in this example. In order for the readers to compare results with the answers in this book, we assigned **Random Seed** with value 1.

After the 200 numbers are generated, the following formulas should be typed in cells B2:I2:

B2: =AVERAGE(A1:A200)	C2: =STDEV.P(A1:A200)
D2: =MAX(A1:A200)-MIN(A1:A200)	E2: =PERCENTILE.INC(A1:A200, 0.15)
F2: =QUARTILE.INC(A1:A200,1)	G2: =QUARTILE.INC(A1:A200,2)
H2: =QUARTILE.INC(A1:A200,3)	I2: =TRIMMEAN(A1:A200,0.2)



	A	B	C	D	E	F	G	H	I
1	-7.069045	SmapleMean	SampeStd	range	15thPercentile	Q1	median	Q3	10%TrimmedMean
2	-6.833522	1.889	3.128	19.351	-1.186	-0.302	2.018	3.932	1.904
3	-5.798226								
4	-5.12309	Bin width							
5	-5.10781	-8							
6	-4.570845	-6							
7	-4.511473	-4							

Figure 5.6 Basic statistical calculation in Excel

The data in column A has been sorted so that we can easily check the calculated results; but the sorting is only optional. The results are shown in the captured screen of Figure 5.6.

Manual calculations show that some of the results are slightly different. This is due to slightly different definitions. For example, we define 10 percent trimmed mean as the average value of the data after 10 percent of the smallest and 10 percent of the largest values are trimmed. Excel apparently defines this as a 20 percent trimmed mean. In this case we can still use Excel to do the calculation according to the definition given in this book by calculating the 20 percent trimmed mean in Excel as our 10 percent trimmed mean. There are other small differences that are not easy to get around. For example, we define the first quartile as the median of the lower half. If the total number of data points in the lower half is an even number, then we calculate the average of the two middle points. Excel does an interpolation of the two middle points, by the weighted average. The point further away from the median carries a 0.25 weight, and the point closer to the median carries a 0.75 weight. The difference between the two methods for calculating the quartiles is insignificant, but it does exist.

Note that the mean and the standard deviation of the sample (1.889, 3.128) are slightly different from the mean and the standard deviation of the population (2, 3). It is expected that when the sample size is larger, the sample mean and standard deviation will be closer to the population mean and standard deviation.

To plot the histogram, we need to create a bin width column. Since the data is between −8 and 13, we type −8, −6, −4, . . ., 10, 12, 14 in B5:B16. Now click on **Data** and then **Data Analysis**. Select **Histogram** then click **OK**. Fill out the Histogram form as in Figure 5.7 and click **OK**.

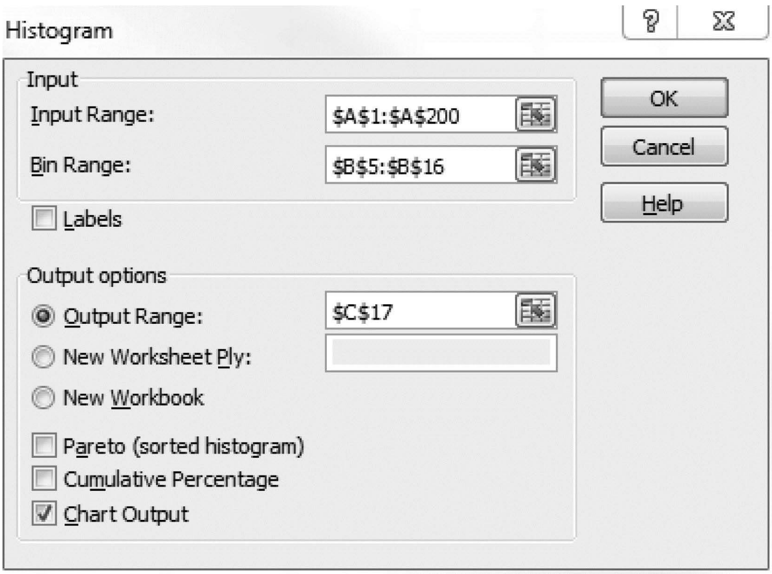


Figure 5.7 Creating histogram in Excel

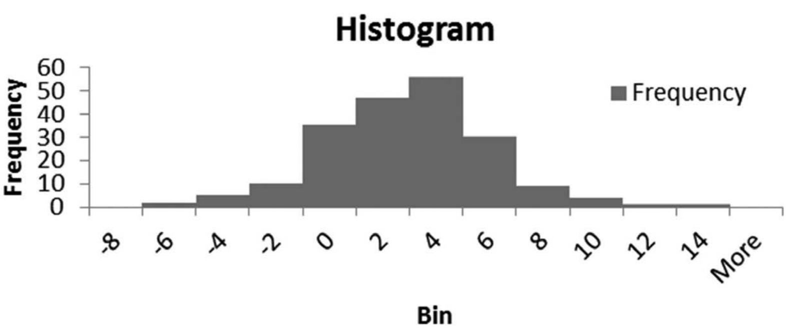
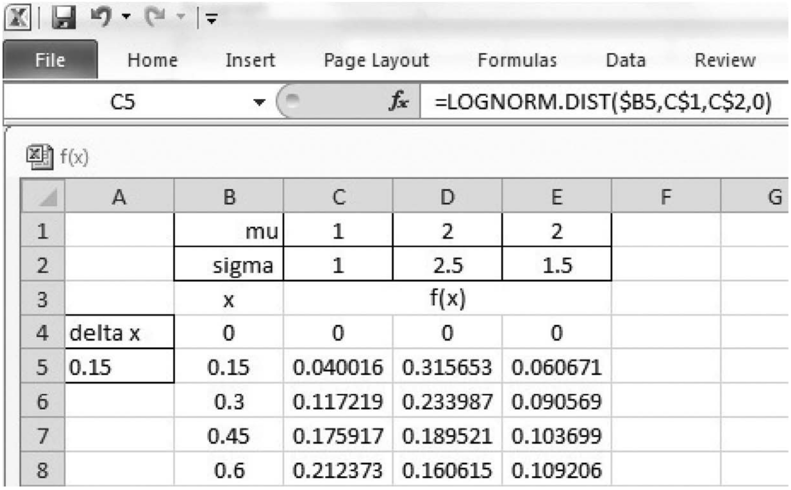


Figure 5.8 An example of histogram created in Excel

After generating the histogram, right click on the bars, select **Format Data Series**, and change **Gap Width** to 0. The final result is shown in Figure 5.8.

The function QUARTILE.INC can be used to create the box-and-whisker plot in Example 2.23. ■

Example 5.3 Plot the probability density functions for three lognormal distributions with means equal to 1, 2, and 2 and standard deviations equal to 1, 2.5, and 1.5, respectively.



	A	B	C	D	E	F	G
1		mu	1	2	2		
2		sigma	1	2.5	1.5		
3		x	f(x)				
4	delta x	0	0	0	0		
5	0.15	0.15	0.040016	0.315653	0.060671		
6		0.3	0.117219	0.233987	0.090569		
7		0.45	0.175917	0.189521	0.103699		
8		0.6	0.212373	0.160615	0.109206		

Figure 5.9 Plotting lognormal distribution in Excel

We can use the function LOGNORMA.DIST(x, mu, sigma, 0) as illustrated in Figure 5.9, where *x* is the variable's value, mu is the mean, sigma is the standard deviation, and the last parameter 0 implies that it is not the cumulative function, but instead it is the pdf.

The means and standard deviations are specified in the first two rows. The increment in the *x* values is specified in A5. The *x* values are started from 0 and incremented by delta *x* each step by typing "=B4+A$5" in B5 and copying and pasting to the cells below B5. f(0)=0, so we type "0" in C4, D4, and E4. In C5, we type "=LOGNORM.DIST($B5, C$1, C$2,0)". The $ is used so that we can copy and paste to the rest of the C column and columns D and E. You can compare the results when you do and don't use the $. After the values *x* and *f(x)* are calculated, we can insert a graph using the data. The graph is shown in Figure 2.22. In fact, all the probability density functions in Section 2.9 were created in Excel using similar method. ■

Example 5.4 The SPC $\bar{X}$ chart and *R* chart in Example 4.20 can be created with Excel.

First, generate five columns of random numbers, 26 values for each column, from a normal distribution with a certain mean and standard deviation. Put the 26 by 5 matrix in B4:F29. A4:A29 contain the subgroup number 1 to 26, as illustrated in Figure 5.10.

G4 =AVERAGE(B4:F4)

	A	B	C	D	E	F	G	H
1								
2								
3	subgroup #	battery voltages					x bar	R
4	1	1.500	1.514	1.518	1.495	1.503	1.506	0.023
5	2	1.505	1.490	1.497	1.508	1.508	1.502	0.018
6	3	1.489	1.511	1.500	1.504	1.484	1.498	0.026
7	4	1.497	1.504	1.511	1.497	1.495	1.501	0.016

Figure 5.10 Setting up data for control chart in Excel

In Example 4.20, the number in the first column and fourteenth row was manually changed to a large number to illustrate the unstable condition. The same thing can be done here. X bar and R in columns G and H are the average and the range of each subgroup. For example, G4 is "=AVERAGE(B4:F4)", and H4 is "=MAX(B4:F4)-MIN(B4:F4)".

The fourth row from J to X is coded as follows:

J4 ($\overline{\overline{X}}$): =AVERAGE(G4:G29)

K4 (LCL for $\overline{\overline{X}}$): =J4-K1*AVERAGE(H4:H29)

L4 (UCL for $\overline{\overline{X}}$): =J4+K1*AVERAGE(H4:H29)

M4 ($\overline{\overline{X}}$ -1σ): =J4-(J4-K4)/3

N4 ($\overline{\overline{X}}$ -2σ): =J4-2*(J4-K4)/3

O4 ($\overline{\overline{X}}$ +1σ): =J4+(L4-J4)/3

P4 ($\overline{\overline{X}}$ +2σ): =J4+2*(L4-J4)/3

R4 ($\overline{R}$): =AVERAGE(H4:H29)

S4 (LCL for R): =Q1*R4

T4 (UCL for R): =S1*R4

U4($\overline{R}$ -1σ): =R4-(R4-S4)/3

V4($\overline{R}$ -2σ): =R4-2*(R4-S4)/3

W4($\overline{R}$ +1σ): =R4+(T4-R4)/3

X4($\overline{R}$ +2σ): =R4+2*(T4-R4)/3

The fifth to twenty-ninth rows for J:X, excluding Q, are just copies of the fourth row. Figure 5.11 shows the Excel calculations with the cell formula in K4 displayed.

K4 =J4-K1*AVERAGE(H4:H29)

	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
1		A2=	0.577					D3=	0	D4=	2.114					
2		X bar								R						
3		center line	LCL	UCL	LCL_1	LCL_2	UCL1	UCL2		center line	LCL	UCL	LCL_1	LCL_2	UCL1	UCL2
4		1.501	1.487	1.516	1.496	1.491	1.506	1.511		0.025	0.0	0.054	0.017	0.008	0.035	0.044
5		1.501	1.487	1.516	1.496	1.491	1.506	1.511		0.025	0.0	0.054	0.017	0.008	0.035	0.044
6		1.501	1.487	1.516	1.496	1.491	1.506	1.511		0.025	0.0	0.054	0.017	0.008	0.035	0.044
7		1.501	1.487	1.516	1.496	1.491	1.506	1.511		0.025	0.0	0.054	0.017	0.008	0.035	0.044
8		1.501	1.487	1.516	1.496	1.491	1.506	1.511		0.025	0.0	0.054	0.017	0.008	0.035	0.044

Figure 5.11 Calculations for control chart in Excel

The $\overline{X}$ chart can be plotted using A4:A29, J4:29, K4:29, L4:29, M4:29, N4:29, O4:29, P4:29. The R chart can be created in a similar manner. The results are shown in Figures 4.15 and 4.16. ■

Example 5.5 In Section 4.3.2, we presented several theorems where critical values of the normal distribution, the *t*-distribution, and the χ^2 distribution were used for hypothesis testing. Here, we will explain how these values can be calculated with Excel. Given the significance level α, calculate the following critical values for hypothesis testing: Z_α, $Z_{\alpha/2}$, $t_{\alpha,n-1}$, $t_{\alpha/2,n-1}$, $\chi^2_{\alpha,n-1}$, $\chi^2_{1-\alpha,n-1}$, $\chi^2_{1-\alpha/2,n-1}$, $\chi^2_{\alpha/2,n-1}$.

In the Excel file, as shown in Figure 5.12, the significance level α is specified in E2 and the sample size is specified in I2. When the user types values into E2 and I2, all the critical values will be automatically updated. The formulas in C5:J5 are implemented as follows:

```
C5: =NORM.S.INV(1-E2)          D5: =NORM.S.INV(1-E2/2)
E5: =T.INV(1-E2,I2-1)          F5: =T.INV(1-E2/2,I2-1)
G5: =CHISQ.INV(1-E2,I2-1)      H5: =CHISQ.INV(E2,I2-1)
I5: =CHISQ.INV(E2/2,I2-1)      J5: =CHISQ.INV(1-E2/2,I2-1)
```

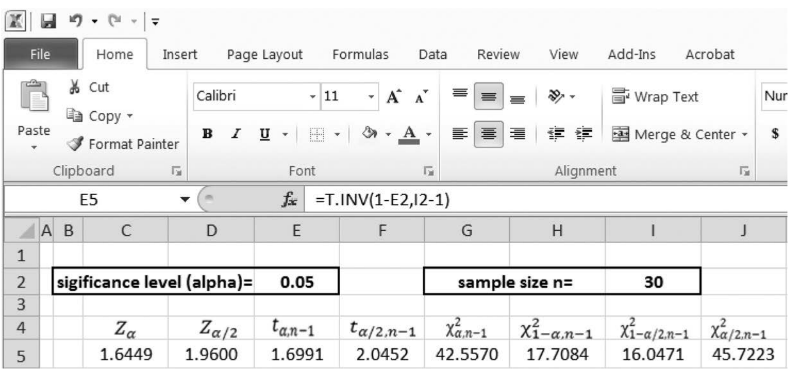


Figure 5.12 Critical value calculations in Excel

■

Example 5.6 The probabilities of the type II errors (β) are usually more complicated than probabilities of type I errors, since it is more than one number. We use the two-tailed test in Theorem 4.4 to illustrate how to calculate β.

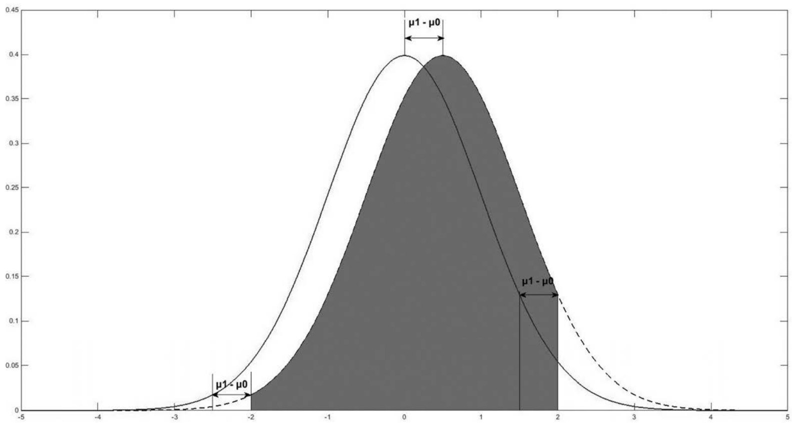
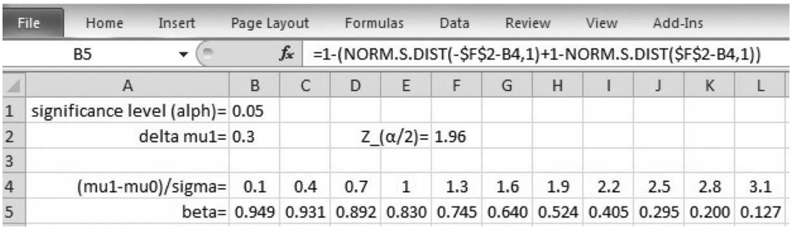


Figure 5.13 Concept of type II error calculation

The type II error is the probability for the mean to be different from μ_0 in the hypothesis, but the test statistic falls outside of the region of rejection. The β value depends on the actual mean μ_1, as indicated by the shaded area in Figure 5.13.

We can use Excel to create a curve that indicates the β value as a function of the actual mean μ_1. To use the standard normal distribution, we will plot β as a function of $\frac{\mu_1 - \mu_0}{\sigma}$.

The significance level α is defined in Excel cell B1. The results including the graph will be automatically updated when a different value for α is typed in B1. The formula in cell F2 is "=NORM.S.INV (1-B1/2)". This calculates the two-tailed z-critical value. The region of rejection is defined by Z > $Z_{\alpha/2}$ or Z < $-Z_{\alpha/2}$. B2 defines the step size for the horizontal axis. C4 is defined as "=B4+$B$2", and D4 is defined as "=C4+$B$2". The formula in cell B5 is shown in Figure 5.14. It calculates the area under the normal distribution centered at *mu1* and outside of the region of rejection. Since this region is outside of the region of rejection, the null hypothesis $\mu = \mu_0$ will not be rejected; however, if the actual distribution is centered at a different location, then the hypothesis should be rejected. So, the probability of this happening is exactly the probability of the type II error.


File Home Insert Page Layout Formulas Data Review View Add-Ins

B5 f_x =1-(NORM.S.DIST(-$F$2-B4,1)+1-NORM.S.DIST($F$2-B4,1))

	A	B	C	D	E	F	G	H	I	J	K	L
1	significance level (alph)=	0.05										
2	delta mu1=	0.3			Z_(α/2)=	1.96						
3												
4	(mu1-mu0)/sigma=	0.1	0.4	0.7	1	1.3	1.6	1.9	2.2	2.5	2.8	3.1
5	beta=	0.949	0.931	0.892	0.830	0.745	0.640	0.524	0.405	0.295	0.200	0.127

Figure 5.14 Calculations of type II error probability using Excel

Adjusting *delta mu1* to an appropriate value, you will have a beta curve shown in Figure 5.15:

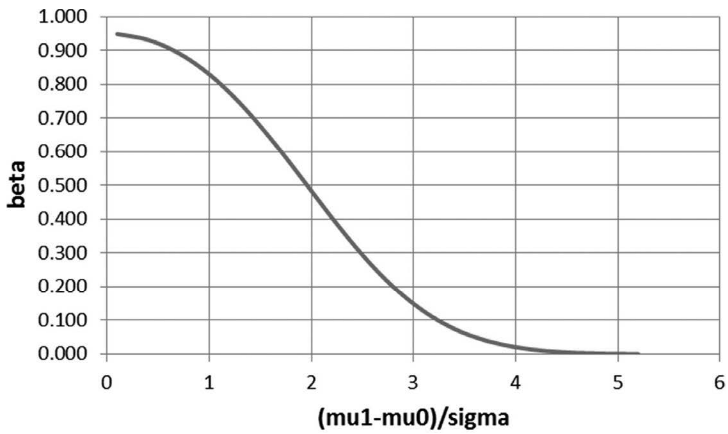


Figure 5.15 Type II error probability curve

For each value of *mu1*, we can use this curve to find the probability of the type II error. ■

More examples using Excel to calculate probabilities for various distributions can be found in Chapters 2 and 3. There are several examples in Chapter 4 where Excel is used for normality check, FMEA, DOE, and regression. Additional statistical analysis tools in Excel are available in Data Analysis to conduct ANOVA, *t*-test, *F*-test, and *Z*-test.

5.2 MATLAB

Example 5.7 Generate two set of random numbers from normal distributions with means at 4 and 6 and standard deviations at 1 and 3, respectively. Add outliers manually to the data sets. Create the boxplots for the two data sets.

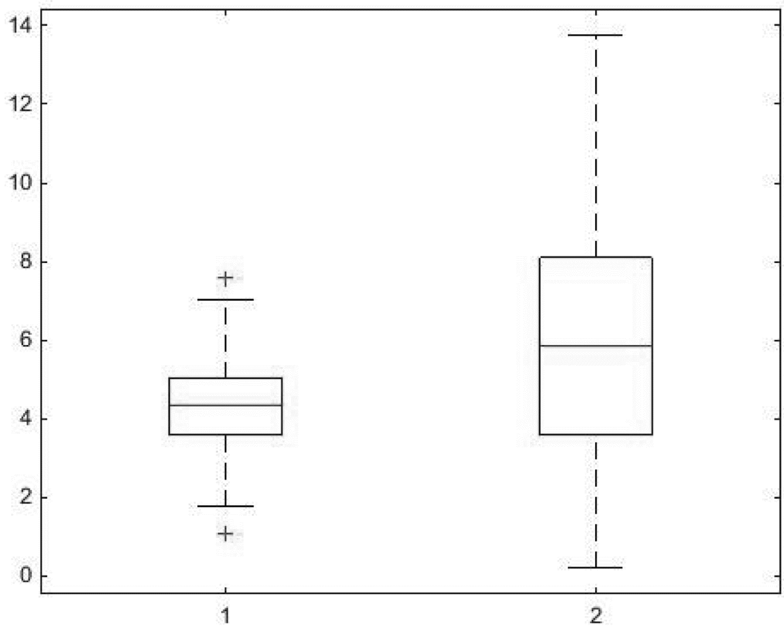


Figure 5.16 A box-and-whisker plot created using MATLAB

These tasks can be completed by running the following codes in MATLAB:

```
mu1 = 4; mu2 = 6; s1 = 1; s2 = 3;
rng('default')
x = mu1+s1*randn(50,1);
y = mu2+s2*randn(50,1);
boxplot([x,y])
```

The resultant Boxplot is shown in Figure 5.16.

To understand the functions in the codes, such as `randn`, `rng`, and `boxplot`, you can type `help <function name>` (without the < >, for example, `help randn`) in the MATLAB command window to read the documentation. ■

Example 5.8 Calculate the mean, the standard deviation, the range, the fifteenth, twenty-fifth, fortieth, fiftieth, seventy-fifth, and ninetieth percentiles, the median, the first and third quartiles, the process capability index C_p, and the process performance index C_{pk} (with USL = 7, LSL = 1) for the first data set (x) in Example 5.7. Plot the histogram of x.

Add the following codes to the codes in Example 5.7.

```
R = range (x) % or R=max(x)-min(x)
%range function may not be available for
  student version of MATLAB.
M = mean(x)
S = std(x)
Percentiles = prctile(x,[15 25 40 50 75 90])
MD = median(x)
Q = quantile(x,[.25 .50 .75])
CpCpk = capability(x,[1, 7])
figure
hist(x, [1:8])
```

You should get the following result in the command window:

```
R = 6.5227
M = 4.2840
S = 1.2625
Percentiles = 2.9311  3.5664  3.9534  4.3222
  5.0347  5.5600
MD = 4.3222
Q =  3.5664   4.3222  5.0347
Cp = 0.7921
Cpk = 0.7171
```

A histogram will be generated by MATLAB as illustrated in Figure 5.17.

Note that the median can be calculated in three different ways, with the same result. ■

Example 5.9 In Example 5.6, we used Excel to calculate the probability for type II errors. The same task can be done in MATLAB with the following codes.

```
alpha=0.05; delta_mu1=0.3;
Z_alpha_half = icdf('normal',1-alpha/2,0,1)
delta=[0.1:delta_mu1:5.2]
beta= 1-(cdf('normal',-Z_alpha_half, del-
  ta,1)+...
      1-cdf('normal', Z_alpha_half-delta,
  0,1))
plot(delta, beta), grid on
```

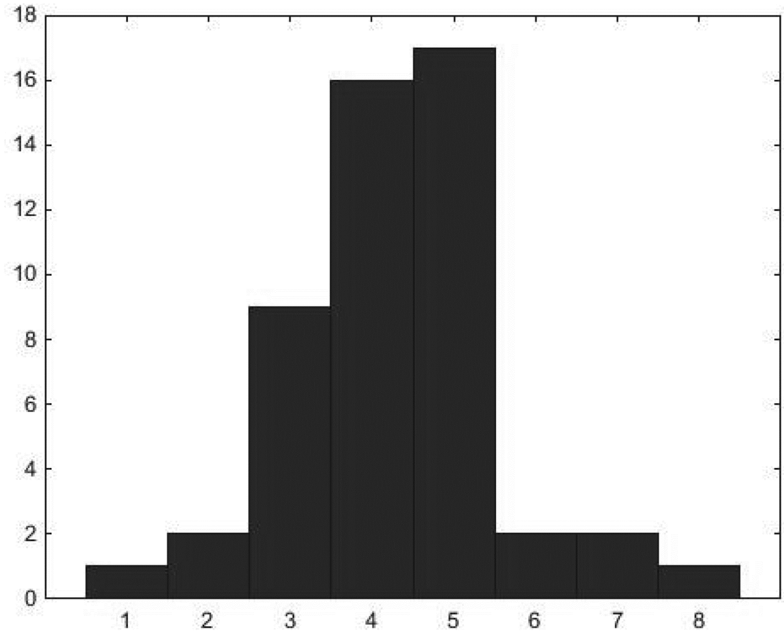


Figure 5.17 A histogram created using MATLAB

The codes can also be easily modified to plot a class of probabilities for type II errors for different significance levels by using a "for loop" in MATLAB. ■

Example 5.10 Generate 26 sets of subgroups of 5 random numbers from a normal distribution with a mean of 1.5 and a standard deviation of 0.01. Change the first number in the fourteenth subgroup to 1.62, second number in the fifteenth subgroup to 1.61, and the third number in the sixteenth subgroup to 1.62. Plot the Xbar–R control charts and check if the process is stable using the following two rules: "1 point above or below the UCL and LCL," and "6 consecutive points increasing or decreasing."

The following code does what is required:

```
rng('default');
x=1.5+0.01*randn(26,5)
x(14,1)=1.62; x(15,2)=1.61; x(16,3)=1.62;
controlchart(x,'chart', {'xbar' 'r'},
  'rules', {'we1' 'we2' 'we5' 'n3'})
```

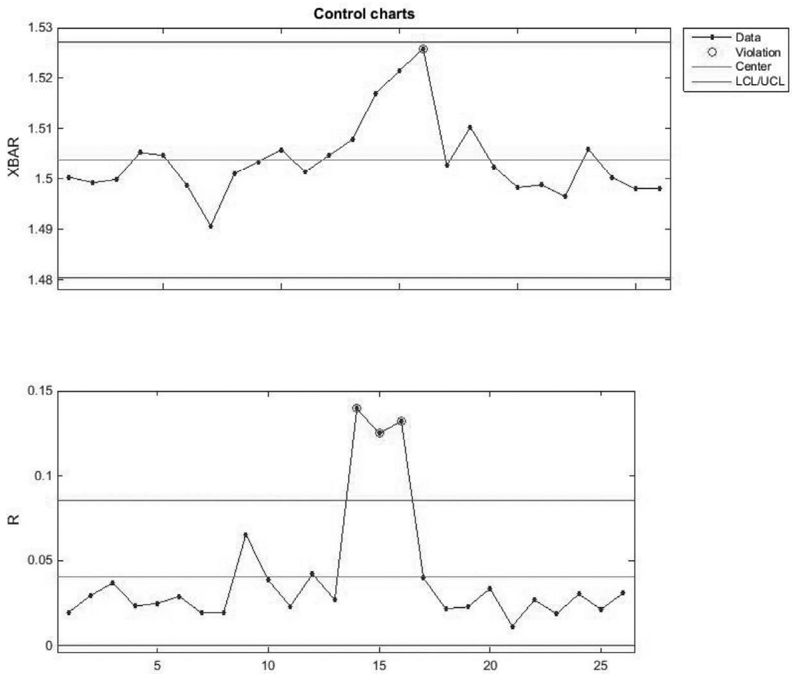


Figure 5.18 An Xbar–R chart created using MATLAB

The Xbar–R charts with points of violations of the rules marked are shown in Figure 5.18.

We will leave it to the reader to figure out which points are in violation of which rules. ■

We can reproduce the results in Example 4.18 using the MATLAB function regress or operator “\” to find the coefficients in the linear regression model. All probability density functions introduced in Section 2.9 can be reproduced in MATLAB using functions `pdf` or `makedist`. The *Z*-critical, *t*-critical, *F*-critical, and χ^2-critical values can be calculated using MATLAB functions `zscore` or `icdf`.

You can find many more useful MATLAB functions by clicking on Help > Statistics Toolbox. A list of functions and other tools that can be used for statistical analysis will pop up. More advanced MATLAB users can also design graphical user interfaces so that some commonly used analysis can be easily done by a user who has no knowledge of MATLAB.

5.3 Minitab

Example 5.11 Generate 500 random numbers from a lognormal distribution. Use the data points to plot the histogram, the boxplot, the stem-and-leaf graph, and the graphical summary. Use the probability plot to verify that it is from a lognormal distribution.

Click on **Calc >Random Data > Lognormal**, a pop-up window appears as shown in Figure 5.19.

Type `500` in "**Number of rows of data to generate:**" and `C1` in the area "**Store in column(s):**" then click **OK**. The 500 data points will be created and stored in column C1.

To plot the histogram, click on **Graph>Histogram**. In the pop-up window, select **Simple**. In the area of "**Graph variables**," type `C1` or double click C1 on the left, then click **OK**. The histogram will appear as in Figure 5.20.

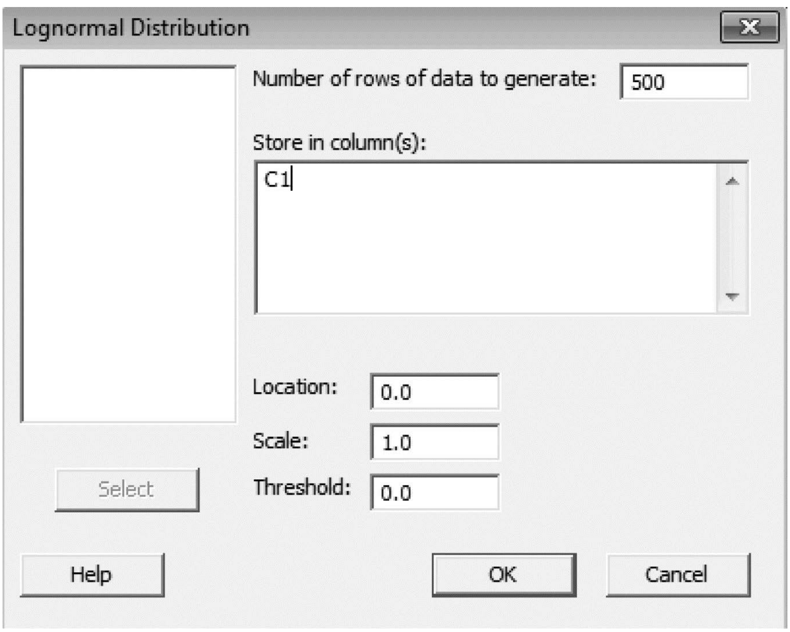


Figure 5.19 A window for generating random numbers in Minitab

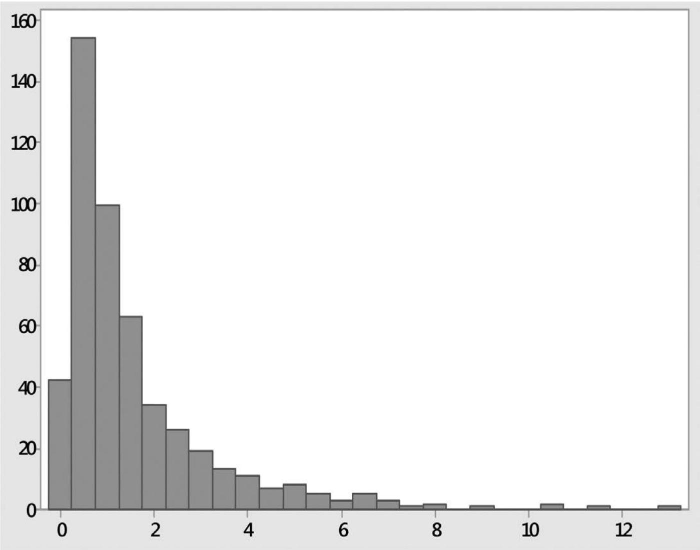


Figure 5.20 A histogram created using Minitab

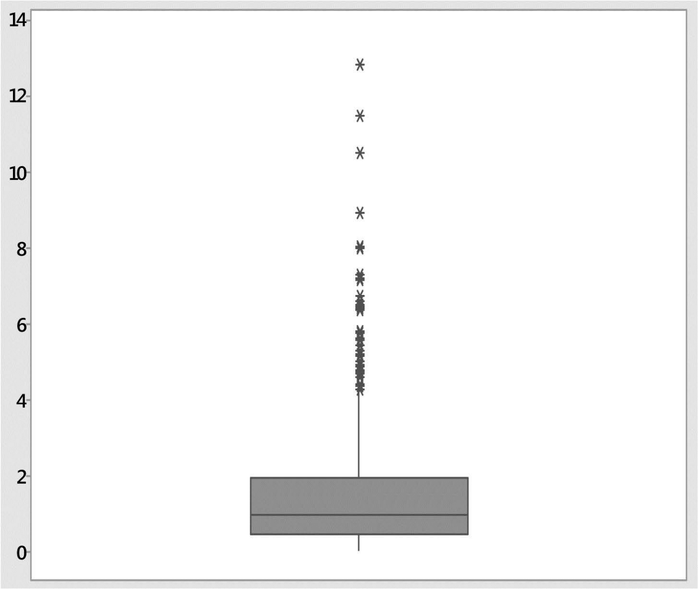


Figure 5.21 A box plot created using Minitab

To create the boxplot, select **Graph>Boxplot**. In the pop-up window, select **Simple**. With the cursor in the area "**Graph variable:**", double click on C1 or highlight C1 then **Select**. Then click **OK**. The boxplot will appear as in Figure 5.21.

To create the stem-and-leaf graph, select **Graph>Stem-and-Leaf:** then double click on C1. Click **OK**. The following stem-and-leaf graph will appear in the Session window.

```
(254) 0
   0001111111111111111111112222222222222222222222222222222233333333333333333+
246   1
   000000000000000000000111111111112222222222333333333344444444444444445555 5+
125   2   000000011111122222233333333334455566666666789999
77    3   000000011112222233344556677778899
44    4   0001222346677899
28    5   001234556778
16    6   344567
10    7   122
7     8   009
4     9
4     10  45
2     11  4
1     12  8
```

To plot the graphical summary for the data, select **Stat>Basic Statistics>Graphical Summary**. The graph shown in Figure 5.22 will appear. In this graphical summary, you can find the *p*-value, mean, standard deviation,

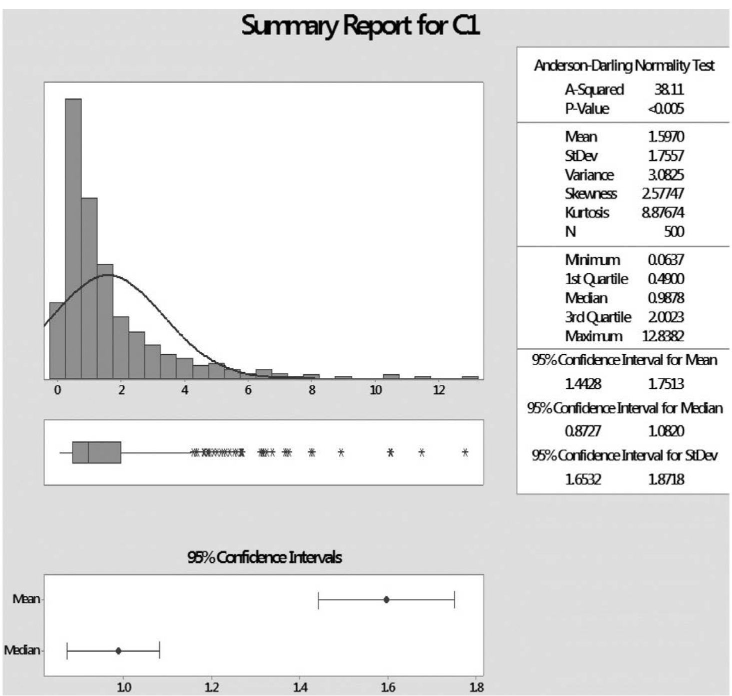


Figure 5.22 A graphical summary created using Minitab

min, max, first quartile, median, third quartile, confidence intervals for mean, median, and standard deviation, histogram, and boxplot.

To use the probability plot for checking if the data is from a lognormal distribution, select **Graph>Probability Plot>Simple** and double click on C1. Click on **Distribution**, and select **Lognormal** in the pop-up window. Click **OK**, and **OK**. The probability plot as in Figure 5.23 will be generated.

The *p*-value, which is equal to (1 − the confidence level), is shown on the plot. A *p*-value greater than the significance level means that the null hypothesis that the data is from a lognormal distribution cannot be rejected. You can try to check if the data are from a normal distribution. The *p*-values will be less than 0.005, which tells us that we should reject the null hypothesis for the given significance level.

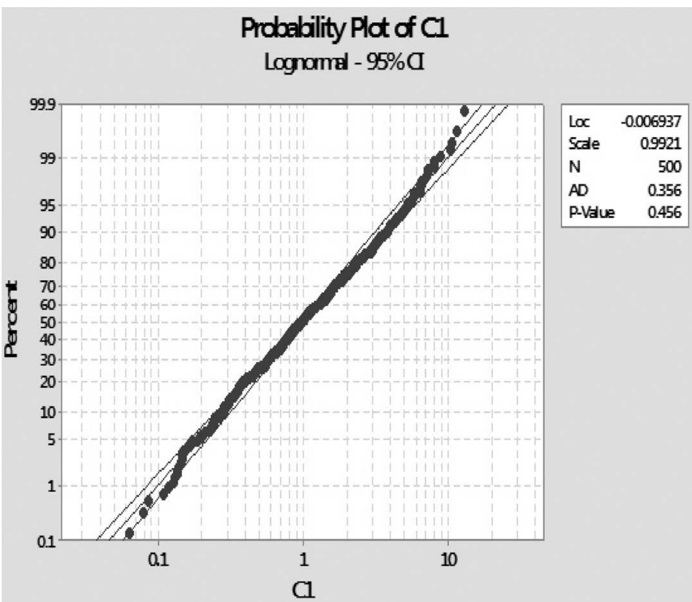


Figure 5.23 A probability plot created using Minitab

■

Example 5.12 This example explains how a cause-and-effect diagram can be created using Minitab.

First, type in all the text in C1 to C6 as shown in Figure 5.24. Select **Stat>Quality Tools>Cause-and-Effect**. Fill out the pop-up window as in Figure 5.24, then click **OK**.

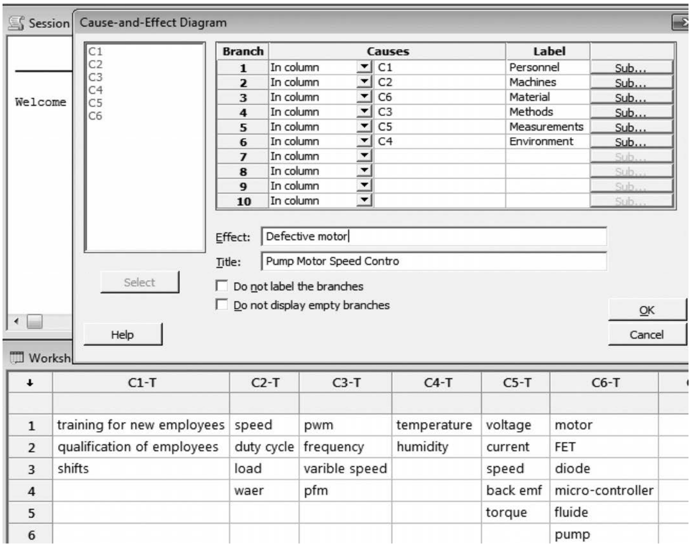


Figure 5.24 Setting up a cause-and-effect diagram in Minitab

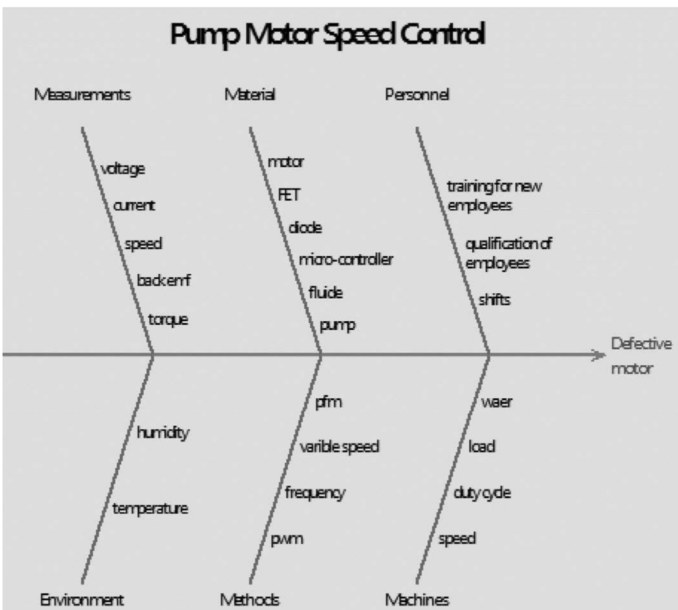


Figure 5.25 A cause-and-effect diagram created using Minitab

The cause-and-effect diagram in Figure 5.25 will appear.

■

Example 5.13 This example illustrates the use of gage R&R analysis.

Select **Stat>Quality Tools>Gage Study>Create Gage R&R Study Worksheet**. Use the default values in the pop-up window, click **OK**. Three columns will be created with C1, C2, and C3 specifying the **RunOrder**, **Parts**, and **Operators**, respectively. Type in your test result for each run in the fourth column (C4) named as *y*. To conduct the gage R&R analysis, select **Stat>Quality Tools>Gage Study>Gage R&R Study (Crossed)**. Fill out the pop-up window as in Figure 5.26.

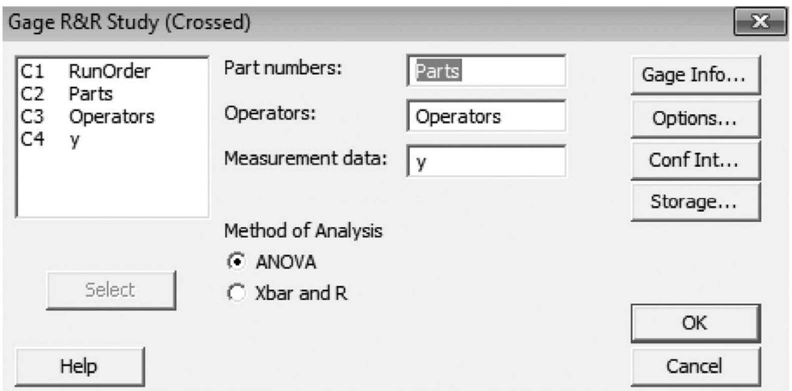


Figure 5.26 Setting a GR&R analysis in Minitab

Click **OK**. You will get the numerical results in the Session window.

Gage R&R

Source	VarComp	%Contribution (of VarComp)
Total Gage R&R	0.0568160	76.51
Repeatability	0.0535694	72.14
Reproducibility	0.0032465	4.37
Operators	0.0032465	4.37
Part-To-Part	0.0174452	23.49
Total Variation	0.0742612	100.00

A graph shown in Figure 5.27 will also be generated.

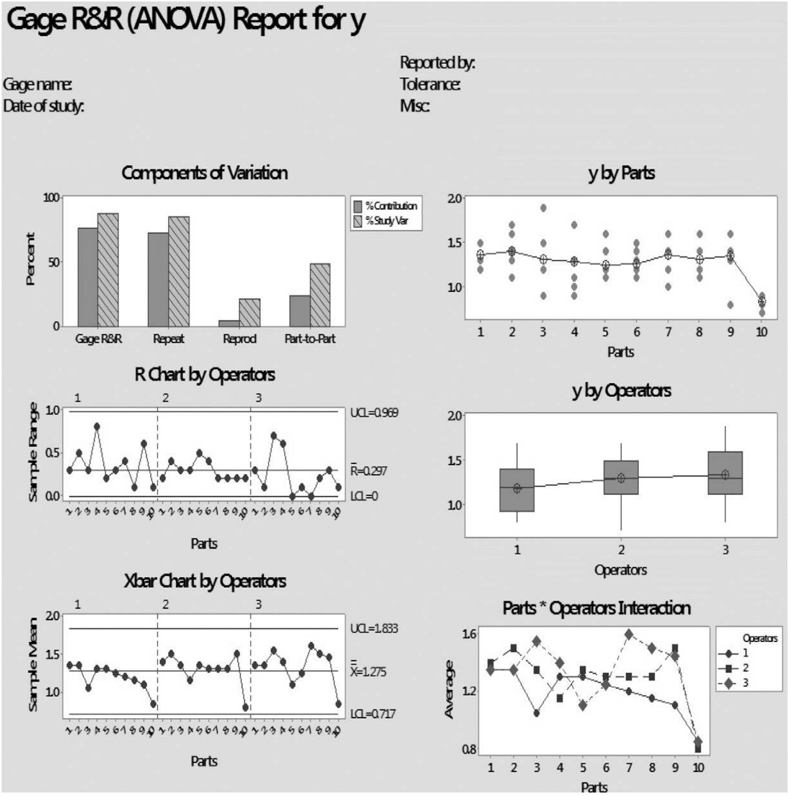


Figure 5.27 GR&R analysis result from a Minitab session

The results indicate that 76.51 percent of the variation is from the measurement system (gage R&R) and 23 percent is from part-to-part variation. Out of the 76.51 percent, the main contributor is repeatability, with 72.14 percent contribution. This tells us that the main problem is with the measurement equipment. According to AIAG, if the gage R&R contribution is more than 9 percent, the measurement system is not acceptable. ■

Example 5.14 In this example, we will illustrate how to create a Taguchi DOE matrix and analyze the results.

Select **Stat>DOE>Taguchi>Create Taguchi Design**. In the pop-up window, select **2-Level Design**, change **Number of factors** to 5. Click on the **Designs** button and select **L8**. Click **OK** and **OK**. The 8 × 5 matrix

Worksheet 2 ***

↓	C1	C2	C3	C4	C5	C6	C7
	A	B	C	D	E		
1	1	1	1	1	1	1.5	1.4
2	1	1	1	2	2	3.0	2.9
3	1	2	2	1	1	2.6	2.7
4	1	2	2	2	2	4.0	4.1
5	2	1	2	1	2	5.6	5.5
6	2	1	2	2	1	2.5	2.4
7	2	2	1	1	2	4.5	4.4
8	2	2	1	2	1	6.4	6.2

Figure 5.28 Creating a Taguchi design matrix in Minitab

will be created in the first five columns and eight rows as shown in the Figure 5.28. This is the design of experiment test matrix. After the experiments are completed, fill in the results in columns C6 and C7.

To analyze the test results, select **Stat>DOE>Taguchi>Analyze Taguchi Design**. With the cursor in the area "**Response data are in:**", double click on C6 then C7. Then click on the **Terms** button. Click on the **>>** sign then **OK** and **OK**. Four graphs will appear, as shown in Figure 5.29. The main effects plots for means and SN ratios indicate the impact on outcomes from each of the five factors A to E. The interaction plots indicate the impact on the outcomes for two factors changing simultaneously. Parallel lines in an interaction plot indicate no interaction. The more the lines depart from parallel, the higher the degree of interaction. The four graphs indicate that factor C has only minor impact on the mean response; factor D has minor impact on the SN ratio, and the interaction between factors B and D is insignificant.

There is more information displayed in the Session window. For example, it states, "The following terms cannot be estimated and were removed: A*B A*C A*D A*E B*C C*D C*E D*E." ■

Example 5.15 This example illustrates how to plot Xbar–R control charts.

Select **Calc>random Data>Normal….** Type `26` in "**rows of data to generate:**". Type `C1 C2 C3 C4 C5` in "**Store in column(s):** ". Type in

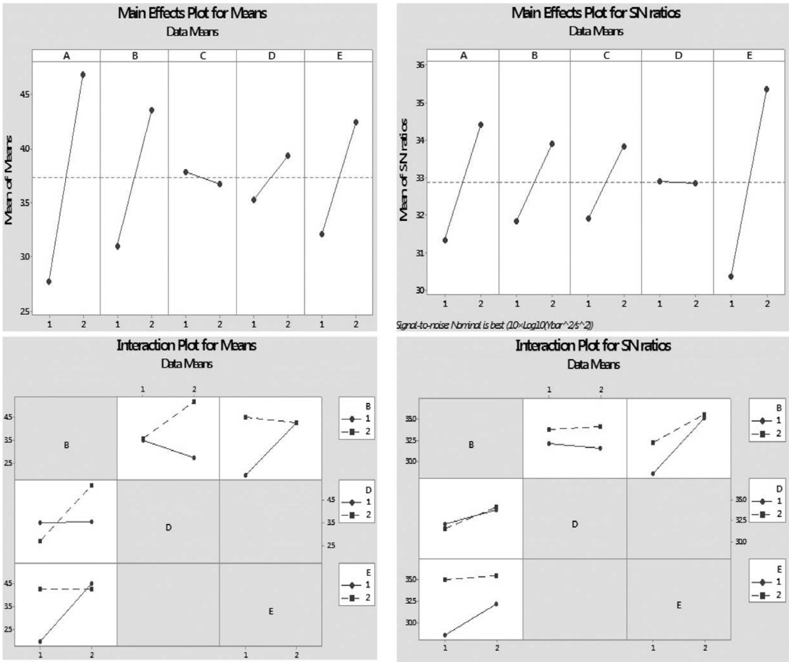


Figure 5.29 Taguchi design analysis using Minitab

1.5 as the mean and 0.01 as the standard deviation. Click **OK**. To create an unstable condition, we change the number in the fourteenth row and first column to a number larger than the original one, say, 1.6. Click **Stat>Control Charts>Variable Charts for Subgroups>Xbar-R....** In the pop-up window, select "**Observation for a subgroup are in one row of columns:**" from the pull-down menu. Click on the blank area underneath the pull-down menu, then double click on C1, C2, . . . , C5. Click on "**Xbar-R Options...**" then click the **Test** tab; select rules that you like to apply. Click **OK** and **OK**. The Xbar–R chart will be created as shown in Figure 5.30.

Rule 1 was violated for the points marked red. You may try different rules and data sets; sometimes violations of rules may happen even for a data set created from a normal distribution. This is a low-probability event (type I error), but it does occur. ■

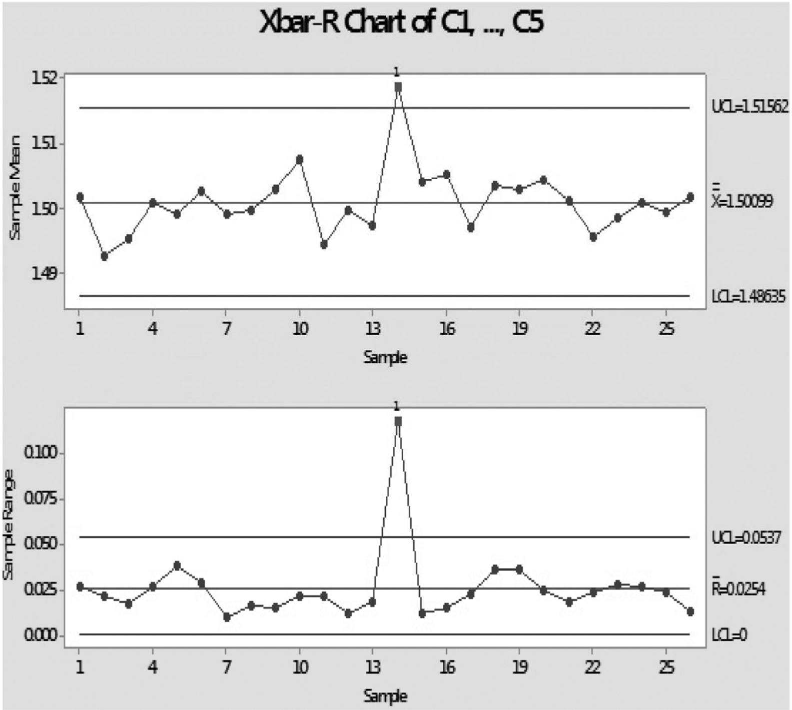


Figure 5.30 ***Control charts created using Minitab***

5.4 R and Other Software Tools

There are many other software packages available, among which are R, JMP, SPSS, Maple, Mathematica, and Mathcad. The American Statistical Association's website has a long list of statistical analysis software packages: http://www.amstat.org/careers/statisticalsoftware.cfm

R is a free software package that is worth considering if your organization does not provide commercial software for statistical analysis. R is similar to MATLAB in its statistical analysis capability and many other areas. In the following example we will use R to plot a pdf and a histogram.

Example 5.16 Plot the probability density function for a normal distribution. Generate 500 random numbers from a standard normal distribution and plot the histogram for the data set.

We use the following codes to plot the desired graphs. If not familiar with R, the reader is encouraged to use the **Help** function in R to comprehend the commands in the code.

```
#plotting pdf for a normal distribution
plot(function(x) dnorm(x, mean =0, sd=4, log =
  FALSE), -15, 15, main = "Probability density
  function for a normal distribution")
#generate 500 random numbers from standard
  normal distribution
N<- rnorm(500, mean = 0, sd = 1)
#plot the histogram
hist(N, freq = FALSE)
```

The results are displayed in Figure 5.31.

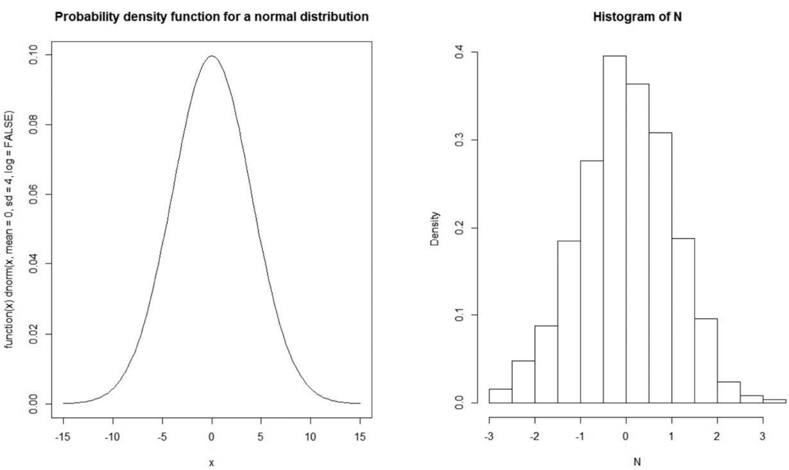


Figure 5.31 Probability density function and histogram created using R

■

Once you have the basic knowledge in this book, it will be relatively easy to master a new statistical software.

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Further readings

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CHAPTER 6

Lean Six Sigma Projects: Strategic Planning and Project Execution

In Chapters 1 to 5, we covered the technical knowledge necessary for completing a Lean Six Sigma project. There are additional skills essential to the success of Lean Six Sigma projects and, at a higher level, the organizationwide deployment of Lean Six Sigma as a strategy [1–4]. For example, how do you select a Lean Six Sigma project? How do you decide whether a potential project should be a go or no-go? How do you form a team for a Lean Six Sigma project? How do you manage a project? What should be the expected results? As a manager, Lean Six Sigma Black Belt, or Master Black Belt, you need to be able to answer these questions with accuracy and confidence. As an engineer, you may also find these skills valuable. When proposing a potential project, you need to understand the criteria for project approval used by the decision makers, so that your proposal can gain their support. This chapter will focus on strategic planning and deployment, project selection, team formation, determining a project scope, project management, milestones and deliverables, and project summary.

6.1 Strategic Planning and Deployment

Every organization should have a **vision statement** and a **mission statement**. A **vision statement** describes what the organization wants to achieve over time. It provides guidance and inspiration to its members with what the organization is focused on achieving in the future. A **mission statement** defines the present state or purpose of the organization. It describes why the organization exists, what it does, who the customers are, and how the organization is currently achieving its goals. Based on the vision and mission statements, a strategic plan can be developed using tools discussed in Section 4.1 such as SWOT. If Lean Six Sigma is a process that your organization wants to follow, then a strategic plan for Lean Six Sigma deployment should be developed [5–8].

There are various paths to Lean Six Sigma deployment. A particular organization may choose to start the Lean Six Sigma deployment with small-scale trials and then extend to the entire organization. Alternatively, it may decide to deploy Lean Six Sigma only to a certain part of the organization where Lean Six Sigma is needed. To be able to complete the projects, the organization needs internal experts with the relevant knowledge and skills, which can be achieved by hiring people with required experience or training the current employees.

For organizations with little or no existing expertise in Lean Six Sigma, the management may choose a few people who are both interested in quality control and have some level of experience in managing teams to go through the Lean Six Sigma Green Belt training [9]. The training can be done either through workshops offered by various organizations or by hiring a consultant to conduct in-house training. These Green Belts are required to complete three to four Lean Six Sigma projects before they can move on to the Black Belt training. Black Belts with good management skills may be designated as Master Black Belts. Before starting a large-scale deployment of Lean Six Sigma in the entire organization, the management team needs to know the basic concept of Lean Six Sigma and why it can help their organization. This can be accomplished through an in-house Yellow Belt training, offered by a consultant, for the management team. The different belts and respective qualifications are described as follows:

Yellow Belt:

What: Have a high-level understanding of Lean Six Sigma, but does not necessarily possess the technical skills required to complete a Lean Six Sigma project. This is the awareness training.

Who: The management team and possibly every employee in the role of making, improving, and managing the products or service

Why: It is critical that the management team understand the importance and benefits of Lean Six Sigma. Without their commitment and support, it is impossible to have a successful deployment. Managers also need to be able to make decisions on which project is appropriate for Lean Six Sigma. Employees who are not working on Lean Six Sigma projects directly should also have some level of understanding of Lean Six Sigma, since they may be asked to assist Lean Six Sigma projects. An organizationwide awareness is also important for building a Lean Six Sigma culture. High quality means no sloppy job, no negligence, and no low standards are allowed at any link of a chain; hence, it is rarely the responsibility of one part of an organization. If the vision of the organization calls for a customer-focused continuous improvement process in its mission, and Lean Six Sigma is selected as the methodology, then all employees should know when Lean Six Sigma should be used and support the projects from their own positions.

Green Belt:

What: Have the technical skills necessary to apply the concepts and methods to Lean Six Sigma projects. However, Green Belt projects are limited in scale, and they are less complex in terms of statistical tools required.

Who: Six Sigma project team members and team leaders

Why: This is usually the largest group doing majority of the work in Lean Six Sigma projects. Proper training must be provided to employees who are motivated to become Green Belts.

Black Belt:

What: Have more advanced technical skills in Lean Six Sigma and have the ability to use advanced statistical tools in large-scale projects

Who: Black Belts are typically full-time positions that lead Black Belt projects, training, and coaching others.

Why: You need technical leaders with advanced knowledge and experience of Lean Six Sigma to train and coach the Green Belts, as well as work in large-scale Black Belt projects.

Master Black Belt:

What: Have the same technical skills required for Black Belts; in addition, they possess more managerial skills and are able to develop organizationwide plans for training, deployment, and overall management of the Lean Six Sigma effort. Typically, Master Black Belts are also full-time positions.

Who: Master Black Belts are leaders in charge of planning and managing the overall Lean Six Sigma deployment within the organization. They set organization policies and procedures related to Lean Six Sigma. They are able to provide training and coaching to both Green Belts and Black Belts.

Why: Master Black Belts are needed for large-scale deployment of Lean Six Sigma. They are leaders who can transform the organization into a Lean Six Sigma organization.

6.2 Project Selection

Before a Lean Six Sigma project is started, it needs to be approved by a decision maker. This decision maker may be a manager, a Lean Six Sigma Black Belt, or a Lean Six Sigma Master Black Belt. He or she may also be the project champion or sponsor.

The decision maker may be given a proposal for a potential Lean Six Sigma project. He or she needs to know how to decide whether it is a "go" or "no-go" project. In addition to some level of technical skills in Lean Six Sigma as presented in the first five chapters, this also requires the knowledge of making a business decision. A Lean Six Sigma practitioner should know the major factors considered by the decision maker while evaluating a potential Lean Six Sigma project.

As a team member or team leader, it is helpful to know what the decision makers will be considering before they make their decisions. This will increase the chance of your project being approved or funded.

To evaluate a potential Lean Six Sigma project, the following steps of **project portfolio analysis** can be used:

- Identify project benefits, risks, costs, and available resources.
- Translate benefits and costs into monetary amounts.
- Conduct benefit–cost analysis.

- Evaluate the team leader and team members based on their skills and track records.
- Make a decision based on data analysis.

Benefits, costs, risks, and available resources are the main factors to be considered. It is desirable to attach monetary amounts to both the benefits and the costs, which may not always be trivial. For example, a project's financial benefits may not be clear now, but the project may have strategic importance. The decision maker must make a call based on his intuition. Costs include materials, labor, and other resources; they all need to be translated to monetary amounts. The benefit–cost analysis can be conducted using any of the following methods: **return on asset (ROA), return on investment (ROI), net present value (NPV), or payback period,** which are defined as follows:

$$\text{ROA} = \frac{\text{Net income}}{\text{Total asset}} \tag{6.1}$$

$$\text{ROI} = \frac{\text{Net income}}{\text{Total investment}} \tag{6.2}$$

$$\text{NPV} = \sum\nolimits_{t=0}^{n} \frac{CF_t}{(1+r)^t} \tag{6.3}$$

where n is the number of years in the period considered, r is the annual interest rate of capital, and CF_t is the cash flow in t-th year.

$$\text{Payback period} = \frac{\text{Initial investment}}{\text{Annual cash flow}} \tag{6.4}$$

Each organization may have some criteria for these financial metrics, such as a ROI greater than 10 percent, a payback period less than 5 years, and a NPV greater than \$20K. If these minimum requirements are not met, the project is deemed as a no-go.

The risk of a project can be estimated by the risk-assessment matrix as illustrated in Table 6.1. Each risk is assigned an estimated probability and severity, with higher severity numbers implying more serious consequences. The fourth column is the product of the second and the third columns. The total risk is the sum of the risk assessment for each risk.

Table 6.1 Risk-assessment matrix

Project risk	Estimated probability (%)	Severity	Risk assessment
Risk 1	1	1	1
Risk 2	5	2	10
Risk 3	1	6	6
Risk 4	12	1	12
		Total risk	29

Projects that can be completed with the available resources, have the highest benefit, lowest cost, shortest payback period, and the lowest risk are good candidates for Lean Six Sigma project. However, there may be conflicts among these factors. A Pugh matrix with benefit, cost, risk, available resources, and team leader and team members' skills and track records as criteria can be used to select the high-priority project. Based on this analysis, the project may be approved, postponed, or rejected.

6.3 Lean Six Sigma Team

The success of a project depends heavily on its execution by the team, and team formation is the first task of the Lean Six Sigma project. The team may be formed with either a top-down or a bottom-up approach. The top-down approach identifies a project and assigns the project to a team by a decision maker, such as a Black Belt, a Master Black Belt, or a manager. The bottom-up approach identifies a project by the team leader or a group of team members. Then the team leader or the team presents the case to the decision maker for approval.

The team's mission must be defined and approved by managers and agreed upon by team members. Buy-ins from stakeholders must then be secured before the project is approved. It is also important to make sure that the resources needed to complete the project are readily available. For Green Belt projects, the size of the team is typically between 4 and 10 people. Black Belt project teams may be larger. Sometimes, a Black Belt project may involve several Green Belt projects.

The selection of the team leader is critical to the success of the project. The team leader should have enough knowledge in Lean Six Sigma, have the necessary managerial skills to pull the team together, be able to resolve conflicts, and make decisions at critical times. Selection of team members should consider different aspects such as specialty, experience, availability, motivation, and personality. Typically, a cross functional team is necessary. This ensures that the team has expertise for all major tasks in the project. It is desirable to mix team members with more experience in Lean Six Sigma, such as those who have completed Lean Six Sigma training, with ones with less experience, such as those currently receiving Lean Six Sigma training. The team's strengths and weakness need to be evaluated by the decision maker. Members of a good project team should have the following characteristics:

- Ability to learn from each other
- Trust and tolerate each other
- Highly motivated by the project
- Put the team's interest above their own
- Have the perseverance to work through difficult time
- Be responsible and willing to take initiatives
- Be able to deliver assigned tasks in a timely manner

Team building is necessary if the members have never worked together before. Team members need to know each other's personalities, strengths, and weaknesses. Rules for resolving potential conflict and running team meetings should be established. Typically, meetings are held weekly by the team and updates to management are made monthly.

A team leader, a timekeeper for meetings, and a person responsible for making meeting minutes should be determined. A meeting should always include an agenda with time allocated for major items. It should also record the time, location, and attendees of the meeting as well as the author of the meeting minutes. Each action item should be assigned to specific team members with due dates. In the subsequent meeting, the team should first discuss the meeting minutes from the previous meeting followed by approval or revision of the minutes. The team members will then report back on the status of the assigned action items. It is important to seek consensus among team members and keep good project documentation.

Last but not least, the organization should establish a mechanism to acknowledge the achievements of good teams and reward them accordingly.

6.4 Project Scope and Project Management

Project scope is established in the Define phase of the DMAIC process [9–12]. Since DMAIC is iterative, revisions are allowed for each phase. However, any change made to the project scope, including the objective, duration, and resources needed, has to be reasonably justified and approved by the project champion. The project scope should be treated as a contract between the team and the project champion.

It is often tempting for the project champion to change the requirements or for teams to add features during the project execution. This incremental expansion of the project scope is called **scope creep** and can have detrimental consequences. The team must learn to manage scope creep; otherwise a Green Belt project may become a Black Belt project, or in the worst case, it may become the whole career of those involved. However, because the DMAIC process is a continuous improvement process, it is desirable to make incremental improvements in a Lean Six Sigma project. In this way, the organization can benefit immediately. If further improvement can be achieved, a new Lean Six Sigma project can be started as a second-phase project instead of changing the scope of the original project.

A Lean Six Sigma project should always be managed to follow the DMAIC process. Project management tools can be used throughout the project, including work breakdown structure (WBS), critical path method (CPM), Gantt chart, responsibility-assignment matrix (also known as responsible, accountable, consulted, informed, or RACI, matrix), cost performance index (CPI), and the schedule performance index (SPI). Regular meetings, reports, and reviews should be held. The risks of the project should be evaluated constantly with tools such as the failure mode and effects analysis (FMEA).

6.5 Milestones, Deliverables, and Project Summary

Project milestones are points that mark the major events of a project. For Lean Six Sigma projects, the natural choice for milestones is the end of each phase of the DMAIC process, while other milestones can be added if necessary. **Project deliverables** are the objects produced in the project, which can be tangible or intangible. They can be requirements, designs, design documentation, test reports, SIPOC, CTQ, and FMEA. Both milestones and deliverables should be clearly specified and reviewed periodically.

Many companies use the **phase-and-gate system**, which divides a project into multiple **phases** with **gates** as the points where the achievements made in the project are assessed. For Lean Six Sigma, one can use DMAIC as the five phases. **Gate reviews** are conducted to assess the milestones and deliverables, analyze the critical path, predict project success, discuss how to avoid mistakes in the subsequent phase manage risks. Decisions are then made on further actions. If the project does not pass a gate review, then that phase may be delayed or the project may be canceled.

The potential financial benefits of the project need to be evaluated by the so-called **Money Belt** who is a financial expert of the organization. In the Control phase, documentation, presentation, records of the lessons learned, and contribution to the knowledge base are made before wrapping up the project. Typically, documentation includes a one-page executive summary with graphs.

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CHAPTER 7

Design for Six Sigma

Design for Six Sigma (DFSS) is a natural extension of Six Sigma to the upstream of product development process, when designs can be optimized for quality assurance. In this chapter, we provide a compact introduction to DFSS, including its importance, concept, basic techniques, and process.

In Section 7.1, we introduce the reader to the concept of DFSS with an example, showing how DFSS can provide competitive advantages to businesses. We then briefly review the history of DFSS and contrast it with Six Sigma. Section 7.2 explains the five phases of DFSS in more detail, and presents the basic methodology illustrated with examples. Section 7.3 is a discussion on how to systematically implement DFSS in an enterprise setting. Two case studies from the automobile industry are offered in Sections 8.2 and 8.3, demonstrating the impact of DFSS in real-world applications. The appendixes of this book include the description of the Pugh decision-making technique and a collection of orthogonal arrays commonly utilized in design of experiment (DOE), which are impotant to DFSS.

7.1 Overview: Quality at Low Costs

7.1.1 Why Do We Need DFSS?

While the practice of Lean Six Sigma has proven to be a powerful tool for reducing variation in performance and quality of products or services, it also has limitations. First, the design of a product or process can be inherently sensitive to certain sources of variation, making it more challenging or costly to deliver consistent performance and quality. Without changing the design, the improvement through Lean Six Sigma may be limited. This will be shown by the examples in this chapter and the DFSS case studies in Chapter 8. Second, sources of variation such as those in usage and operating environment can be very difficult, if not impossible, to control. For example, while airplane pilots are well trained to fly airplanes with consistent procedures, drivers of automobiles have much more variation in how they operate their vehicles. Also, controlling user variation would usually limit the market potential of the product or process. It is therefore desirable to design a robust product or process that is less sensitive to unwanted variation sources, which is the subject of DFSS. Let's start with the following example.

Example 7.1 Design of a Wheel Cover [1]

A wheel cover is one of the thousands of parts that make up a car. There are at least two expectations a customer has for a wheel cover: ease of removal if he or she has to change the tire and good retention on the wheel so that the cover does not fall off when the vehicle hits a bump or turns a sharp corner.

Figure 7.1 shows the backside of a wheel cover. The cover has three clips, each with two prongs, spaced around an imaginary circle. The diameter of this circle, which we call the clip diameter, is larger than the diameter of the rim of the wheel. When the cover is pressed onto the wheel, the clips act as a spring and deform, pushing the cover to the rim and holding it there.

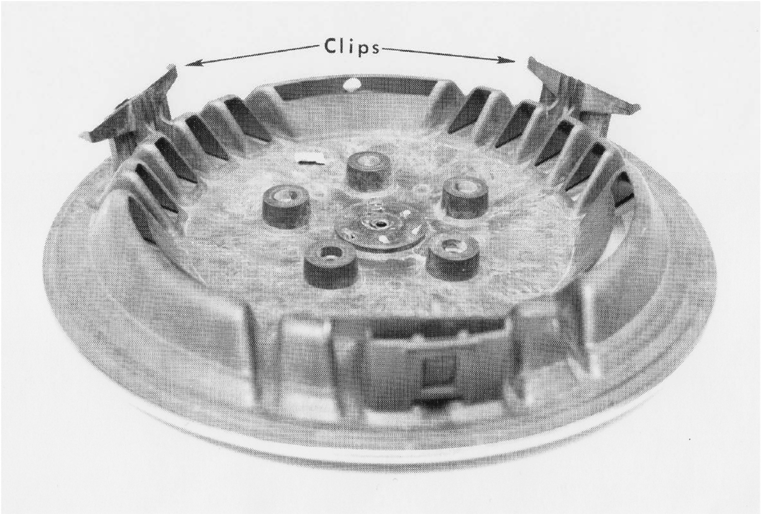

Figure 7.1 Backside of a wheel cover

Source: Reprinted with permission by ESD—The Engineering Society of Detroit

Two variables determine the retention force F: the stiffness k of the clips and the deflection u of the clips. The clips deflect when they are inserted into the rim, and the amount of deflection is the difference between the radius of the circle spanned by the clips and the radius of the rim. By mechanics,

$$F = ku \tag{7.1}$$

Measurement data indicate that the deflection u has a significant amount of variation, since both the rim and clip diameters have their manufacturing tolerances. It is not uncommon for this variation to be up to several millimeters. We classify this deflection variation as a **noise factor**, which implies that the variation is not intended by design, and it affects the performance of the product. However, the nominal values of the clip stiffness and the clip deflection are subject to design, making them **control factors** (We shall define noise factors and control factors more precisely in Section 7.2.4).

Customer survey data show that for the wheel cover to be removable, the retention force should be no more than 60 N. This is shown in Figure 7.2. On the other hand, the data show that covers will fall off at a significant rate when the retention force is less than 30 N, as shown in Figure 7.3. In general, vehicles driven on bumpy roads and with sharp turns need a higher retention force than those cruising on a freeway. Combining

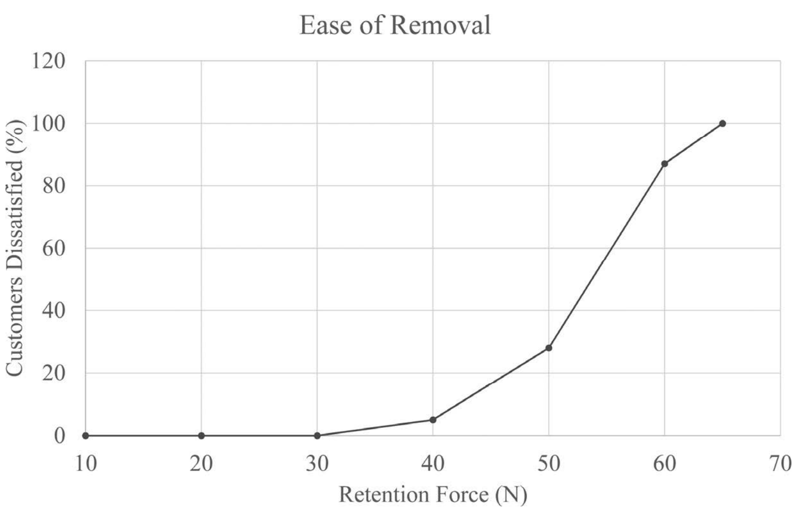

Figure 7.2 Customer survey data on easy wheel cover remover

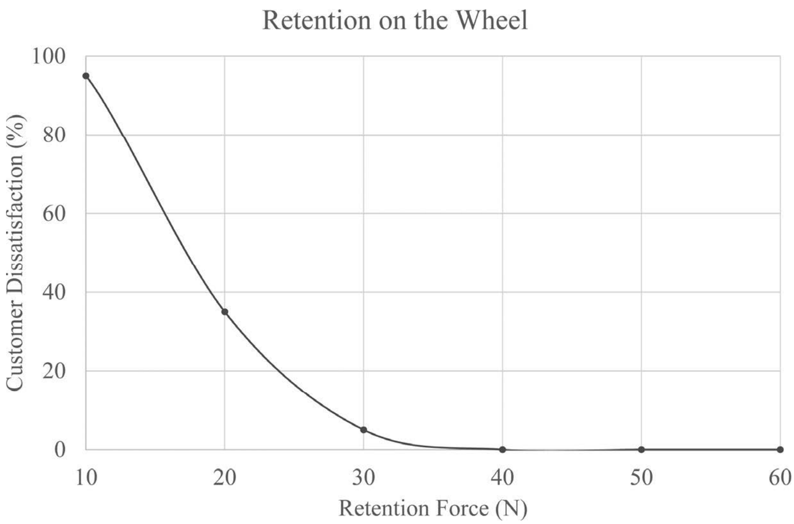

Figure 7.3 Customer survey data on wheel cover retention

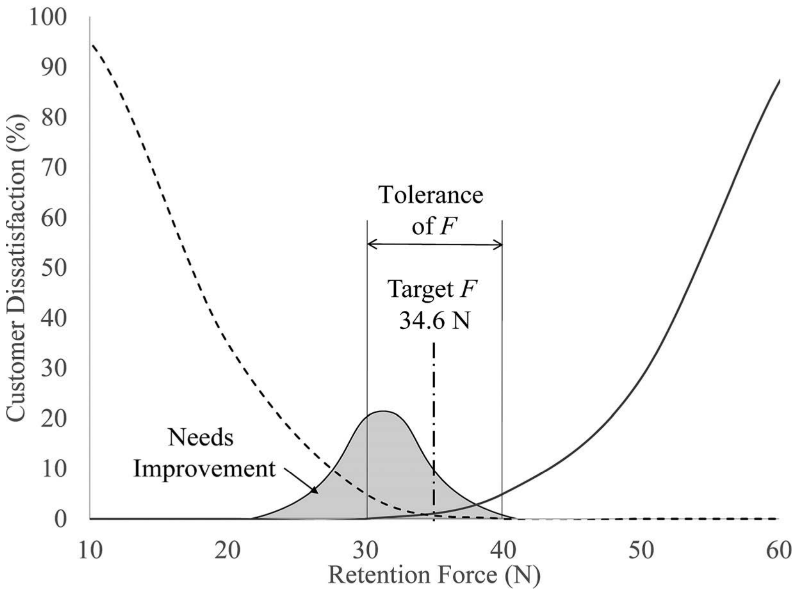

Figure 7.4 Setting the target for retention force

these two competing requirements, a retention force of 34.6 *N* is deemed optimal for most of the customers, as indicated in Figure 7.4. Deviation from this target will lead to some degree of customer dissatisfaction and higher warranty costs to the car maker. The range for the required retention force is therefore set at 30 to 40 *N*.

In mass production, it is impossible to be exactly on target all the time. Typically, the retention force of the wheel cover varies, as shown by the shaded distribution in Figure 7.4, having a mean deviated from the target and a spread around the mean. For the product to satisfy most if not all customers, we must aim for the mean of the retention force to match the target value, while also reducing the standard deviation. Shifting the mean of the retention force toward the target value is less challenging than reducing its variation; therefore, we will focus on how the variation can be reduced.

For a deflection of 6.65 *mm* and a clip stiffness of 5.2 *N/mm*, the nominal retention force is 34.6 *N*, which is on target. However, in mass production, we get a distribution of deflection as shown in Figure 7.5 by the shaded distribution along the horizontal axis. This variation in the deflection *u* projects to a variation in the retention force wider than the required 30 to 40 *N*, resulting in less-satisfied customers.

The usual practice is to tighten the tolerance, say, by sorting the larger covers to match with the larger rims and smaller covers with smaller rims. However, the factory would need extra manpower and storage

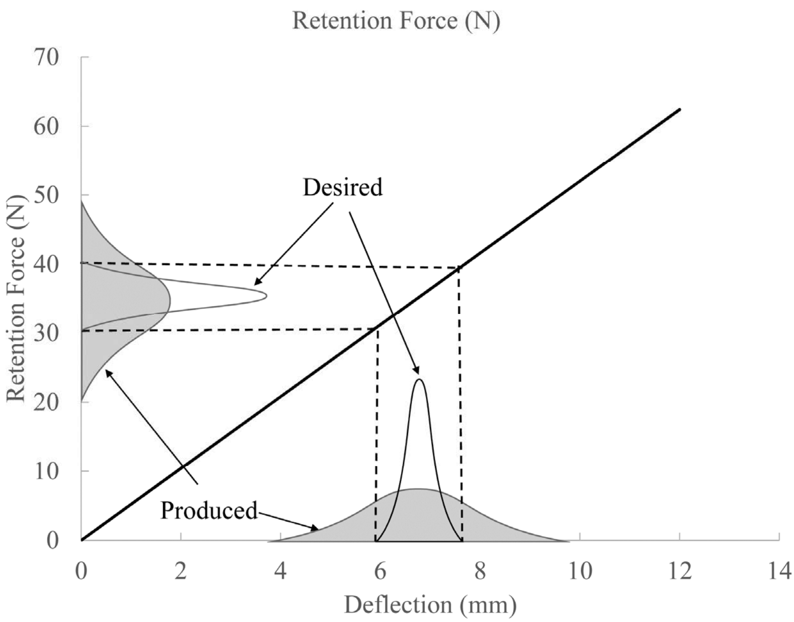

Figure 7.5 Variation in deflection transmitted to retention force

room for sorting. Sorting may give us the quality we aim for, but at an extra cost. Moreover, when the customer has the tires rotated for maintenance, the matched covers may become unmatched again, causing problems.

The Six Sigma approach would aim for the reduction of the variation in the clip deflection. However, the wheels may be produced by one supplier and the covers by another. Data must be collected and analyzed to determine which part contributed most to the variation. Then the supplier at fault needs to improve its manufacturing process to meet a tighter tolerance specification. This may be time-consuming, and the price that the supplier charges for better parts may increase.

Let us examine how DFSS will achieve the required quality at no extra cost. At the center of this problem is the sensitivity of the retention force to the variation in the clip deflection. If this sensitivity can be lowered by design, then we can reduce the variation of the retention force without tightening the manufacturing tolerance of the deflection.

Consider using clips which are less stiff, for example, $k = 3.10\ N/mm$, and increasing the clip diameter so that the deflection u is 11.15 mm. With this design, the nominal retention force is still 34.6 N, but the variation range of the retention force is much smaller than when the stiffer clips are used. In Figure 7.6, the same variation in the deflection projects to retention forces between 30 and 40 N, with which most customers will be satisfied. With this soft-clip design, we achieve the same quality as sorting but without the extra cost. The soft-clip design is more robust to the variation in the diameters of the rim and the clips.

In fact, by choosing still softer clips of 2.0 N/mm and increasing the clip diameter until the nominal deflection is 17.3 mm, we can even relax the tolerance on the deflection and still achieve the same quality, as shown in Figure 7.6. This design is even more robust, allowing more variation in the amount of clip deflection. It can be produced at a lower cost, since we can purchase parts from less-expensive suppliers, require a less-precise assembly process, and lower the skill level of the labor needed. This can lead to extra profit, without sacrificing quality of the product. ■

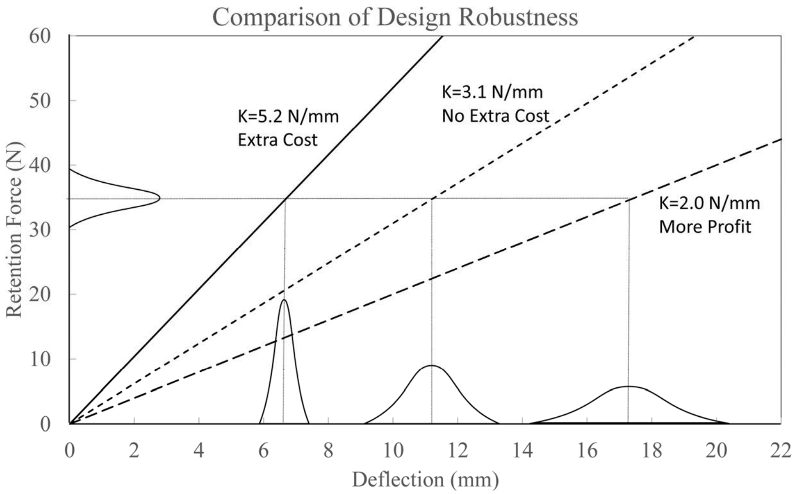

Figure 7.6 Using DFSS to achieve quality at low cost

There are numerous examples similar to this one in the day-to-day work of engineers, where simple math models based on physics are used to design products or processes. The opportunity for DFSS is tremendous.

Compared with the traditional quality control methods, DFSS has the following advantages:

- It is carried out early in the product development cycle, when major capital expenditures such as tooling and testing have not been spent. Design mistakes may be corrected at this stage with much less-severe consequences. Moreover, decisions made at this stage have significant impact on the product quality and reliability, as illustrated by Example 7.1.
- There is more design freedom to address the quality concerns at the design stage. Once in production, design changes will be very costly.
- DFSS can be carried out without using many production parts, especially if the mathematical model of the design is available to simulate the product performance. In Example 7.1, the mathematical model is $F = ku$. This makes DFSS especially suited for the development of new technologies and new products.

- DFSS can exploit opportunities of lowering costs, by relaxing tolerances of the variables the design is not sensitive to. We saw this strategy used in Example 7.1, with the soft clips of 2.0 *N/mm*.

7.1.2 A Brief History of DFSS

Design for Six Sigma is the integration of Six Sigma and **robust design**. It moves Six Sigma upstream into the product or process design phase, combined with the concept and methods of robust design.

The concept of robust design was pioneered by Dr. Genichi Taguchi [2]. After World War II, Japan was faced with the challenge of producing high-quality products when there was a shortage of high-quality material, capable manufacturing equipment, and skilled workers. Dr. Taguchi, then a manager at the Electrical Communication Lab of Nippon Telephone and Telegraph, was assigned to solving this problem. While Dr. Edward Deming taught statistical process control in Japan, Dr. Taguchi developed the foundations of robust design through practical applications during the 1950s and early 1960s. **DOE** was the main approach of robust design at that time, and Dr. Taguchi made heavy use of the orthogonal arrays for DOE (Appendix B). He developed "recipes" for DOE and the subsequent data analysis, which could be followed by engineers without broad statistical knowledge. With robust design, Dr. Taguchi emphasized reducing variation of product performance as used by the customers at a low cost. This was, and still is, very relevant to industries for mass production and consumption.

By the 1980s, Japanese products were gaining a reputation for their quality and reliability, especially in the electronic and automotive industries. American industries followed suit. Some universities started offering courses on robust design, and companies such as AT&T, Ford Motor Company, and General Motors Corporation started to learn and apply robust design methods in their research and development activities. The methodology evolved too, extending beyond DOE with physical parts to using computer simulation models for robust design. Traditional optimization algorithms were combined with the "probabilistic method" in the search for robust solutions [3]. As Six Sigma process moved upstream to the development of products/processes, robust design became a key enabler of institutionalized quality engineering.

7.1.3 Phases of DFSS

The process of DFSS has a number of acronyms, such as

- DMAIC: Define–Measure–Analyze–Improve–Control; this is the same as the Six Sigma process
- DMADV: Define–Measure–Analyze–Design–Verify
- IDOV: Identify–Design–Optimize–Validate
- DMADOV: Define–Measure–Analyze–Design–Optimize–Validate
- DCDOV: Define–Concept–Design–Optimize–Validate
- DMADOV: Define–Measure–Analyze–Design–Optimize–Validate
- IDDOV: Identify–Define–Develop–Optimize–Verify
- ICOV: Identify–Characterize–Optimize–Verify

Although the expressions vary, the underlying philosophy is the same. In this book, we will use IDDOV [4]. More specifically, the five phases of DFSS are as follows:

Phase 1. Identify opportunity of improving quality by design. The opportunity may come from a new technology making its way from research to a commercial product or process, or from redesigning a product or process that has poor quality. The business case should be considered at this stage.

Phase 2. Define requirements. This includes understanding customer needs and translating those needs to measurable requirements on the product or process.

Phase 3. Develop concepts. One or more new concepts may be proposed and evaluated, and the concept that has the most potential of achieving the goal is selected.

Phase 4. Optimize design. This phase will fine-tune the design of the product or process, such that it is not only "on-target," but also "capable," with its nominal performance fulfilling the design intent, and its variation within the range that customers are satisfied with. The optimized tolerances are also determined here as part of the cost-effective design.

Phase 5. Verify design. This phase ensures that the optimized design indeed delivers as expected.

In Example 7.1, the DFSS opportunity was recognized when variation of the wheel cover retention force caused customer complaints, and the traditional quality control methods proved to be too costly. The requirements on the retention force were defined based on customer survey data, as shown in Figure 7.4. The new concept was to change the design of the clips so that the sensitivity of the retention force to variation in the clip deflection was reduced; this approach is in contrast to reducing the variation of the deflection itself. When optimizing the design, different values of the clip stiffness were evaluated for their robustness, in addition to other considerations such as durability (not shown in Example 7.1). Finally, the new design of the clips needed to be verified, not only with the nominal clip deflection, but also with the variation in the deflection, therefore proving its robustness.

7.1.4 Contrast between Six Sigma and Design for Six Sigma

Design for Six Sigma and Six Sigma are similar in their basic philosophy: Quality is crucial to customer satisfaction and business competitiveness; performance consistency is a major part of quality; quality characteristics should be measured quantitatively. They also share some tools such as quality function development (QFD), failure mode and effect analysis, cause-and-effect diagram, fault tree analysis, DOE, and statistical data analysis.

On the other hand, DFSS differs from Six Sigma in one important aspect: timing of the application. While Six Sigma activities occur during the production phase and support sales and services, DFSS is part of the research and development of a product or process. This is illustrated in Figure 7.7.

As the result of this difference in timing, DFSS has more design latitude. Many aspects of the design can be optimized at this stage. For instance, while Six Sigma accepts the performance tolerance specification

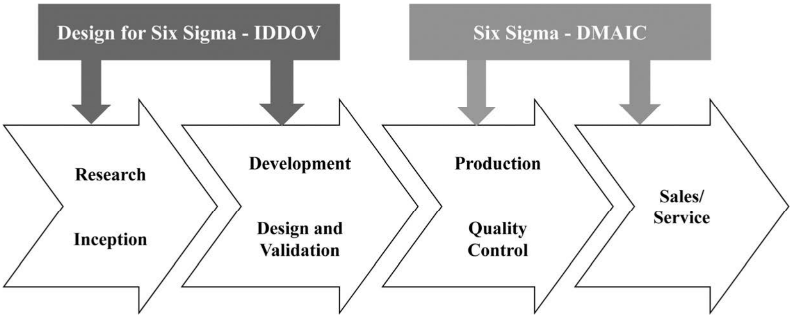

Figure 7.7 Product development process and timing of DFSS and SS

and strive to meet the specs, DFSS can redefine the performance specifications based on customer needs. DFSS can also change the values of the design parameters to reduce its sensitivity to major sources of variation, as demonstrated by Example 7.1. In that example, the stiffness of the wheel cover clips was changed to make it less sensitive to the manufacturing variation in the clip deflection. DFSS is fire prevention, while Six Sigma is fire fighting. DFSS focuses on the first-time quality, while Six Sigma focuses on continuous improvement and maintenance.

One of the major challenges to implementing DFSS is the shift in mentality. Rather than reacting to problems and fixing them, DFSS requires us to anticipate potential downstream problems, far before the product or process is built. For instance, when designing a consumer product with DFSS, we need to anticipate variations from raw materials, manufacturing processes, operating environment such as temperatures, aging, and the ways customers use the product. Field data may or may not be available for a new technology or product, making it difficult to evaluate the robustness of the design.

DFSS therefore relies more on the prediction methods and tools. Mathematical modeling, computer simulation, DOE, statistical simulation, and robust optimization are being used widely for DFSS, as we will discuss in Section 7.3.

DFSS and Six Sigma are coherent. When a DFSS project is complete, a set of specifications should be established. Six Sigma can then be called upon to execute the production, ensuring that the specs are achieved effectively.

7.2 Process of DFSS

In this section, we shall explain the details of the DFSS phases and the commonly used methods, illustrated by examples.

7.2.1 Phase 1: Identify Opportunity—Starting from Customer Needs

This phase is similar to the "Define" phase of the Six Sigma process. However, there are some differences between the two as explained in the following.

DFSS projects should focus on what is important to the customer. Since DFSS is conducted at the design stage, it is critical to understand who the customers are, what the intended function of the product or process is, and what the customers are looking for in the product or process.

"Customers" can be external such as consumers and government agencies, or internal such as plant workers. For example, when designing a car, drivers and passengers are the car manufacturer's external customers. Besides, the government regulates the safety of the car, imposing requirements as the advocate for car buyers; such government agency is also an external customer. On the other hand, the safety and health of the workers in automobile plants require that the design of parts and their manufacturing processes be ergonomical. In this sense, the auto workers become car manufacturer's internal customers.

Each customer desires certain things from a product or process. Collectively, those needs are called "the Voice of Customer," or VOC. During the first phase of a DFSS project, customer surveys may be conducted to understand the VOC. Also, data on the existing design can be analyzed so that its weakness can be corrected by the new design. In the U.S. automotive industry, data from J.D. Power Initial Quality Survey, J.D. Power Vehicle Dependability Survey, Consumer Reports, and auto manufacturers' vehicle warranty repair data can all be used to identify opportunities for DFSS.

In Example 7.1, the main VOCs are "easy removal" and "reliable retention" of the wheel cover. There may be additional VOC for the cover, such as being lightweight and corrosion-resistant, etc.

As with Six Sigma, at the end of the first phase, a DFSS project team should have identified the objectives, scope, duration, resources, stakeholders, and risks of the project.

7.2.2 Phase 2: Develop Requirements—Translating VOC to Specifications

The VOC collected in Phase 1 needs to be translated into quantifiable specifications. Any vague customer preference should be made concrete, expressed in terms of a measurable quantity. In Example 7.1, "easy removal" was translated into the engineering requirement of "retention force must be less than 40 *N*," while "reliable retention" was expressed as "retention force must be greater than 30 *N*."

There are several challenges to this step. First, it is nontrivial to translate all VOC into quantifiable specifications accurately. Often, a VOC is vague and subjective, stating a preference in a layperson's language. Different customers may mean different things even when the same expressions are used. "The room should be comfortable" may mean a temperature of 25°C to an elderly, but 20°C to a teenager; and it may also mean that the humidity needs to be in certain range. The word "noisy" not only relates to high volume of the sound, but also its frequencies. Since the entire project starts and ends with the requirements, it is important to find the correct objective interpretation of the VOC, and derive specifications true to it.

Second, the specification must be on a variable that can be "measured" *at the design stage*. If prototypes can be made for testing, one can measure the performance. However, if no prototype samples are available, one has to replace the real measurements with predictions. A physics-based mathematical model or an empirical model may be used for this purpose. In Example 7.1, Equation (7.1) is a mathematical model predicting the retention force, based on theory of mechanics. Conversely, the vehicle DFSS case studies in Sections 8.2 and 8.3 used both physics-based and empirical models to predict the performance of the products.

In some cases, a failure mechanism cannot be easily modeled mathematically due to its complexity. For instance, when two plastic parts in a car rub against each other, they may produce a squeaky noise, which annoys customers. While this noise can be measured, it is difficult to predict with a mathematical model. In such cases, surrogate variables may be used: instead of predicting the noise, one may predict the lack of clearance and the relative motion between the parts. When using surrogate variables, one must make sure that the failure mechanism is well represented.

As in the second phase of Six Sigma process, QFD is an effective tool for translating the VOC to objective performance requirements. At the end of this phase, the DFSS team should have identified performance variables that are critical to quality and can be predicted reliably, and established their specifications based on the VOC.

Example 7.2 VOC and Functional Requirements for an Electric Bike Battery

Compared to its man-powered counterparts, an electric bike has additional VOCs for its battery system. For instance, customers want the battery to be lightweight, fast-charging, powerful, durable, and affordable. They also want the bike to travel reasonably long distance with each charge, to operate around the year, and so on. Each of these desires needs to be interpreted accurately and precisely in engineering or financial teams. Lightweight means that the mass of the battery must be less than certain kilograms; fast-charging means that the charging time should be less than certain minutes; greater power corresponds to the fast battery discharge; etc. Table 7.1 lists the metrics for each of the VOCs. Note that they are all measurable.

Table 7.1 Translating electric bike battery VOC to requirements

VOC	Engineering Metric on Battery System	Specification for Model I	Specification for Model II
Lightweight	Power and energy density		
Fast-charging	Charging time to 90% battery capacity		
Powerful	Maximum discharge rate and duration		
Durable	Cycle life and calendar life		
Inexpensive	Battery cost		
Adequate travel range per charge	Charge capacity		
Can be safely used in all seasons	Operating temperature and humidity range		
Convenient charging	Charger can use residential power supply of 110 V		
Easy to maintain	Maintenance frequency		

■

The developer of the battery system then needs to determine the specifications of the metrics. In this example, different types of bike riders may have different needs; hence, different battery models can be designed to meet the needs. For instance, one battery model may be designed for people who use the bike in cities no more than 20 km a day, while another can be for dispatchers and delivery workers who ride up to 120 km a day. Each model of the battery system would have its own set of specifications. The blanks in Table 7.1 should be filled based on customer preference and market research data.

7.2.3 Phase 3: Develop Concepts—Searching for Robust DNA

This phase is unique to Design for Six Sigma. At the early development stage, there are usually more than one proposed design concepts. The objective of this phase is to evaluate them with evidence, to find the best concept satisfying the requirements identified in Phase 2 and other business objectives. However, it is not the optimization of a particular design concept.

Professor Stuart Pugh [5, 6] created a method to help concept selection. He emphasized the importance of this step: if the wrong design concept was chosen, no amount of subsequent efforts on details could save the situation. His **Pugh matrix** has been used for concept selection. It "scores" each design concept against a set of criteria, in comparison with a "datum" or a baseline design. The scores for all the candidate designs are then used to make trade-off decisions, and the best overall design concepts will be chosen. For more details, see "Appendix A. Pugh Concept Selection Technique" at the end of this book.

For a DFSS project, robustness should be part of the criteria. Some design concepts are less sensitive to variations in their production and usage, therefore easier to achieve high levels of quality. Including this criterion in the evaluation will help achieve quality at lower costs.

During the concept selection, new ideas and concepts may emerge, as a result of better understanding of the advantage and disadvantage of the existing concepts. It is important to be open-minded and keep as many design options open as possible, to provide the best candidate designs for evaluation.

Note that the "scoring" of the concepts may require a good understanding of the failure modes of each concept. FMEA, fault tree analysis, and fishbone diagram may be utilized to help with the evaluation.

Example 7.3 Concept Selection of Electric Bike Battery

Let's continue with the electric bike battery design discussed in Example 7.2. Suppose that we now need to choose the chemistry of the battery for Model I, which can travel 20 km with each charge at a city biking speed.

Four battery technologies are being compared: lead acid such as in the 12 V battery of a car, nickel cadmium such as in common AA batteries, nickel metal hydride and lithium–manganese used in batteries of today's consumer electronics and electric vehicles.

Table 7.2 shows a hypothetical exercise of choosing the battery technology with the Pugh matrix. Each technology is scored against the lead-acid type on each metric. The result shows that the lithium–manganese battery provides the best performance, but costs more. In fact, lithium–manganese batteries are the most commonly used in electric bikes. However, in this example, Model I only needs to travel 20 km per charge; therefore, nickel metal hydride may also be a good choice. In some markets, even nickel cadmium batteries can be used as a low-cost solution. A decision should be made based on the market data and the sales strategy.

Table 7.2 Pugh Matrix for choosing battery for electric bike

	Lead Acid Battery	Nickel Cadmium Battery	Nickel Metal Hydride	Lithium–Manganese Battery
Power density	0	+2	+3	+6
Energy density	0	+2	+3	+6
Maximum discharge rate and duration	0	+2	+3	+6
Cycle life and calendar life	0	+2	+3	+3
Battery cost	0	−1	−2	−4
Operating temperature and humidity range	0	−1	−2	−2
Charger can use residential 110 V power supply	0	0	0	0
Maintenance frequency	0	+2	+2	+2

	Lead Acid Battery	Nickel Cadmium Battery	Nickel Metal Hydride	Lithium–Manganese Battery
Charging time to 90% capacity	0	+1	+2	+2
Total positive	0	+11	+16	+25
Total negative	0	−2	−4	−6
Total neutral	0	1	1	1

■

7.2.4 Phase 4: Optimize Design—Achieving Quality at Low Cost

Now that the best concept is chosen, we need to optimize the detailed design to achieve the project objectives.

At the core of DFSS is the concept of robust design. A robust design is less sensitive to non-negligible variation sources in the raw materials, manufacturing process, operating environment, and usage. While quality control methods work on reducing or eliminating the variations, robust design searches for the nominal values of the design variables, such that the sensitivity of the performance with respect to the major variation sources can be reduced. In essence, robust design is to make a design more immune to critical variation sources, negating the effects of those unwanted variation on the quality of the product.

Let us reexamine Example 7.1. The sensitivity of the retention force F with respect to the clip deflection u is given by

$$\frac{\mathrm{d}F}{du} = k \tag{7.2}$$

Larger values of the clip stiffness k, or stiffer clips, will make the retention force more sensitive to the variation in u. In order to reduce this sensitivity, smaller values of k, or softer slips, should be chosen.

Similar to the traditional optimization problem, we can formulate robust optimization as "searching for the values of a set of independent variables to maximize (or minimize) the objective function, subject to constraints." However, for robust optimization, we also need to include the measure of robustness in the objective function, which requires statistical estimates.

7.2.4.1 How to Measure or Predict Performance?

By convention, the performance variable we will optimize is called the **response.** In general, the response is a function of a number of independent variables, or factors. In Example 7.1, the retention force is the response we are interested in. There are two options to evaluate the response: experimental measurement and mathematical prediction. If a mathematical model is available to predict the response with sufficient accuracy, then experiments are not necessary at this point of the project, which saves costs. However, if no such predictive model exists, experiments must be conducted to measure the response. In some complex cases, experiments and predictive models can be combined to estimate the system response(s).

In Example 7.1, the response F was predicted by Equation (7.1). It can also be measured with a force gauge if needed.

Note that a factor can be either a continuous variable or a discrete one. A mathematical model may accept both types of factors, while only discrete values (or **levels**) are used in the experimental approach.

7.2.4.2 What Are Control Factor, Noise Factor, and Their Ranges?

The response variable is a function of a number of independent variables of two categories: those that are subject to design (**control factors**), and those that vary but are not subject to design (**noise factors**).

A control factor is an independent variable that affects the response, and its value can be chosen as part of the design. In Example 7.1, the nominal values of the stiffness k and the deflection u of the clips are control factors, since we can choose their values to obtain the retention force of 34.6 N.

A noise factor is a variable that also affects the response, but we cannot control its value or choose not to control for some reason. For instance, the variation of the clip deflection in Example 7.1 is a noise factor. Although controlling this variation is possible through sorting, it is costly. If one can find a robust design that tolerates the variation in deflection, it will be more cost-effective. For this reason, variation in the clip deflection is treated as a noise factor.

Other examples of noise factors are given below:

- When making a car body, sheet metal thickness variation within the specification of a metal gauge.
- When regulating the temperature of a room, the variation in the outside temperature.

For a DFSS project, it is important to estimate the range of each noise factor, in order to evaluate its effect on the variation of the response. Also, one must determine the range of choice for a control factor, which should be as large as feasible to increase the design flexibility, therefore maximizing the chance of finding a robust solution.

In Example 7.1, the variation of the clip deflection is a noise factor, with a range of $\pm 3\ mm$, or $\sigma_u = 1\ mm$ if we assume a $\pm 3\sigma$ variation range. The nominal values of the stiffness k and the deflection u of the clips have reasonably large ranges, as long as other requirements such as durability of the clips are met.

7.2.4.3 How to Evaluate Robustness?

The robustness of the design can be measured by how much the value of the response deviates from its average, when the noise factors vary randomly. A small deviation in the response implies a more robust design. This deviation can be quantified with the variance or standard deviation of the response.

Several methods are commonly used to estimate the variance, as explained below.

a) Taylor expansion method

Suppose that y is a differentiable function of $x = (x_1, x_2, \ldots, x_n)$, where x_i, $i = 1, 2, \ldots, n$ are random variables. In other words, y can be expressed as $y = f(x_1, x_2, \ldots, x_n)$.

The first-order Taylor expansion of y about the mean value $x^o = (x_1^o, x_2^o, \ldots, x_n^o)$ is given by

$$\Delta y = y - y^o \approx \left(\frac{\partial f}{\partial x_1}\Big|_{x=x^o}\right)(x_1 - x_1^o) + \left(\frac{\partial f}{\partial x_2}\Big|_{x=x^o}\right)(x_2 - x_2^o) + \ldots + \left(\frac{\partial f}{\partial x_n}\Big|_{x=x^o}\right)(x_n - x_n^o)$$

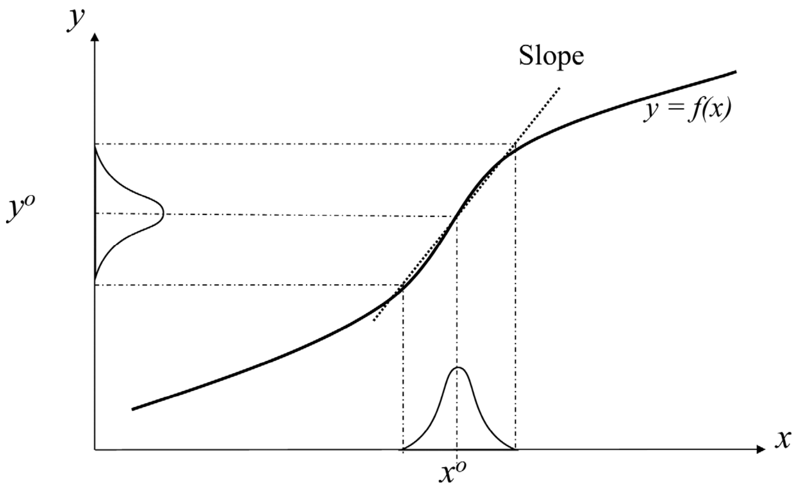

Figure 7.8 Transmission of variation

The variation of x is transmitted to the variation of y, mostly through the slope of the function at the mean value of x, as illustrated in Figure 7.8. This slope corresponds to the first-order derivative of y with respect to x, evaluated at x^o. The magnitude of the variation of y thus transmitted depends on (1) the magnitude of the variation in x and (2) how sensitive y is with respect to x.

Assuming that the x variables are statistically independent. Then the mean and the variance of y can be approximated by the following equations [2]:

$$y^o \approx f(x_1^o, x_2^o, \ldots, x_n^o) \tag{7.3}$$

$$\begin{aligned}\sigma_y^2 &\approx \left(\frac{\partial f}{\partial x_1}\Big|_{x=x^o}\right)^2 \sigma_{x_1}^2 + \left(\frac{\partial f}{\partial x_2}\Big|_{x=x^o}\right)^2 \sigma_{x_2}^2 + \ldots + \left(\frac{\partial f}{\partial x_n}\Big|_{x=x^o}\right)^2 \sigma_{x_n}^2 \\ &= \sum_{i=1}^{n} \left(\frac{\partial f}{\partial x_i}\Big|_{x=x^o}\right)^2 \sigma_{x_i}^2\end{aligned} \tag{7.4}$$

where y^o and x^o are the mean values of y and x respectively and σ_y and σ_{x_i} are the standard deviations of y and x_i respectively.

We can also estimate how much each x_j contributes to the variation of y in percentage terms. The contribution to σ_y^2 by x_j is

$$C_j = \frac{(\frac{\partial y}{\partial x_j}|_o)^2 \sigma_{x_j}^2}{\sum_{i=1}^{n} (\frac{\partial f}{\partial x_i}|_{x=x^o})^2 \sigma_{x_i}^2} \tag{7.5}$$

If the variation of a variable, say x_j, is negligible, then $\sigma_{x_j} \approx 0$ and the corresponding term $(\frac{\partial y}{\partial x_j}|_o)^2 \sigma_{x_j}^2$ in Equation (7.4) and Equation (7.5) will be zero; hence, it won't contribute to the variation of the response. The magnitude of σ_y^2 is a measure of the design robustness.

Example 7.4 Variation Analysis of the Wheel Cover

Consider the wheel cover retention force discussed in Example 7.1. The nominal value of the retention force is $F = 34.6\ N$, since $F = ku = 5.2 \times 6.65 \approx 34.6\ N$.

Assuming $\sigma_u = 1\ mm$ and $\sigma_k = 0.2\ N/mm$, we can estimate the standard deviation of the retention force as follows:

$$\frac{\partial F}{\partial u} = k = 5.2N / mm, \frac{\partial F}{\partial k} = u = 6.65mm,$$

$$\sigma_y = \sqrt{k^2\sigma_u^2 + u^2\sigma_k^2} = \sqrt{5.2^2 \times 1^2 + 6.65^2 \times 0.2^2} = 5.4N$$

The σ_y above is a measure of the robustness of the retention force. At the three-sigma level, the variation will cause the retention force to be outside the customer-preferred range of 30 to 40 *N*, as shown in Figure 7.9.

Using Equation (7.5), the contribution by the variation of *u* is calculated as $(5.2^2 \times 1^2)/5.4^2 = 93\%$, while the contribution by the variation of *k* is $(6.65^2 \times 0.2^2)/5.4^2 = 7\%$. It is clear that the variation of *u* is the main cause of the problem, its effect being amplified by the stiffness *k*. A more robust design would need to reduce the sensitivity of the retention force to *u*. This knowledge indicates that softer clips are more robust. ■

If *y* is not a differentiable function of *x*, or the variation of *x* is too big compared to the linear range of the function, then the Taylor expansion method may not provide an accurate estimate of σ_y^2. In that case, **Monte Carlo Simulation (MCS)** is a good alternative.

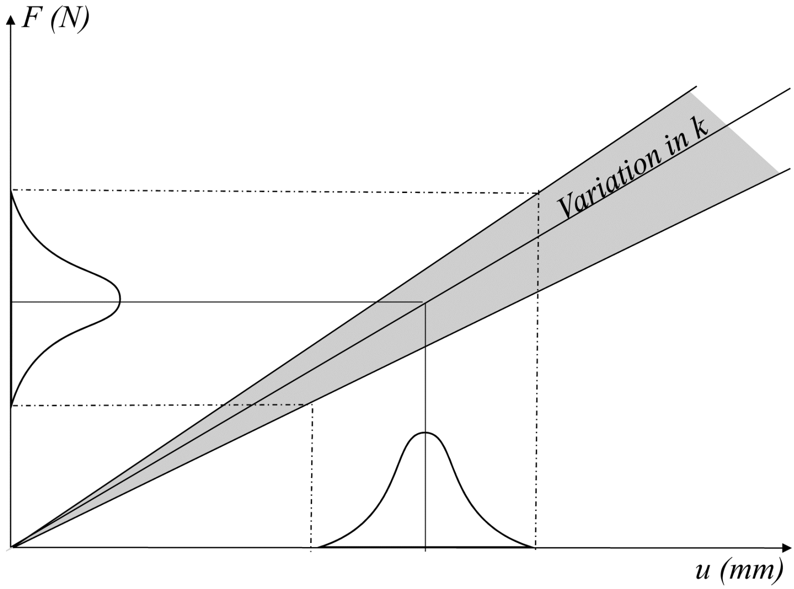

Figure 7.9 Variation caused by both deflection and stiffness of clips

b) Monte Carlo Simulation

MCS imitates the effects of random variation on a system described by a mathematical model. In reality, the parameters in the system and the inputs to the system may have variation, while in the mathematical model they are usually assumed deterministic. If the input to the model is the same, the predicted output will also be the same. For instance, in Example 7.1, if the values of stiffness k and deflection u do not change, then the value of the retention force F does not change either.

In order to evaluate the robustness of the system, one needs to introduce variation of the noise factors into the calculation. Suppose that U is a random variable of normal distribution with a mean of μ_U and a variance of σ_U^2, there is a numerical method that can generate many random values of U, such that their probability distribution matches that of $N(\mu_U, \sigma_U^2)$. For each of these random values of U, Equation (7.1) will produce a value of F. If we have 10,000 such F values, we can estimate the mean and the variance of F, or calculate the probability for F to be between 30 and 40 N with relative accuracy. This approach of simulating the random variation with a mathematical model of the response is called

MCS. In the past decades, as **computer-aided engineering** (**CAE**) advanced, MCS found numerous applications in various fields, from financial planning to automobile design. Algorithms for random number generation have been built into many computer simulation software packages, making MCS a powerful tool for robust design.

MCS does not require the response function to be continuous, and the noise factors may be continuous or discrete. Usually, the number of simulation runs is between hundreds and thousands, depending on the nature of the problem. This may put a burden on the computing system. For instance, a vehicle crash-worthiness simulation that evaluates the structural damage and occupant injury takes hours to complete. It is not practical therefore to conduct thousands of simulations to evaluate the probability of meeting certain requirement. In that case, a response surface model may be created first, and the MCS can be conducted with the response surface model instead. The case studies in Sections 8.2 and 8.3 will illustrate that approach.

c) Experimental method

If no mathematical model of the product or process is available, one needs to conduct experiments to measure the robustness of the design.

Ideally, for a candidate design, we would want to run many experiments, each with different noise-factor levels. The response values from all these experiments would be analyzed statistically; the mean and standard deviation of the response could be estimated as in MCS. However, this approach is usually not feasible, due to the limitation on time and cost of running large number of experiments with prototypes. Instead, DOE methods should be employed to reduce the number of experiments neccesary.

There are various DOE matrices for this purpose. For example, Dr. Taguchi favored orthogonal arrays for robust design [2, 7]. If there are two or more noise factors, we can choose a suitable orthogonal array to run the experiments. For readers' convenience, some frequently used orthogonal arrays are listed in Appendix B.

For a given design (with a combination of control-factor values), we vary the levels of the noise factors according to a DOE matrix, conduct tests with repeats, and obtain several measurements of the response. For

Table 7.3 Full-factorial DOE matrix for two noise factors

Experiment	N_1	N_2	y
1	Low	Low	y_1
2	Low	High	y_2
3	High	Low	y_3
4	High	High	y_4

instance, if we have two noise factors N_1 and N_2, each with two levels (low and high, or Level 1 and Level 2), and if we choose the full-factorial design, a total of four experiments will be conducted, and the response measured, shown in Table 7.3.

We can then calculate the average value μ_y and the variance σ_y^2 of the response as follows:

$$\mu_y = \frac{y_1 + y_2 + y_3 + y_4}{4}$$

$$\sigma_y^2 = \frac{(y_1 - \mu_y)^2 + (y_2 - \mu_y)^2 + (y_3 - \mu_y)^2 + (y_4 - \mu_y)^2}{3}$$

The variance of y thus calculated is a measure of the spread in y. Since we only conducted four experiments and the values of the noise factors were not random, this is not an accurate estimate of the true probabilistic variance of the response. However, for the purpose of robust design, this metric can often distinguish a good design from a poor one.

On the other hand, the average value of the response μ_y is a measure of the "bias." Comparing μ_y with the intended target value of the response, we know how much "mean shift" is needed in order to achieve the target response. The goal for robust design is to choose the optimal control-factor levels, such that the bias and the standard deviation of the response are both minimized.

In practice, the three methods described above are usually adequate for DFSS. There are other sophisticated methods of estimating variation or probability distribution of the response; see [3, 8] for more details. Note that when designing a new product or process, it is usually not feasible to have a large sample size to estimate accurately the statistical characteristics of the responses. For one thing, all mathematical models are approximations of the real system, and the responses predicted by those models have error. This prediction error can be greater than the sampling error due to the limited sample size in MCS, or greater than the neglected higher order terms in the Taylor expansion method. Similarly, in experimental approach, the parts used may be prototypes, and may not be exactly the same as produced during large-scale production. In all these cases, the error in the data of the response can be significant enough that no method can guarantee accurate estimates of the mean, the variance, or the probability distribution of the response. In this context, "Six Sigma" means "a very high level of consistency," rather than "<3.4 defects per million opportunities." This is different from the Six Sigma practice, where true production data is usually available for analysis.

7.2.4.4 How to Formulate the Objective Function?

There are three types of commonly used objective functions for robust optimization: **nominal the best**, **smaller the better**, and **larger the better**. "Nominal the best" means that the nominal value of the response needs to match a target value with minimum bias and variation; the "smaller the better" aims to minimize the response values, and the "larger the better" wants to maximize the response values. Depending on whether the response is predicted with a mathematical model or measured during experiments, the objective function for robust optimization will have various forms.

When a mathematical model is used to predict the response and its variation, the objective function can be expressed as follows:

For "nominal-the-best" problems, search for control-factor values such that $\{(\mu_y - y_t)^2 + \sigma_y^2)\}$ is minimized. The first term in this objective function represents the bias of the mean response μ_y from the target value y_t, and the second term measures the variation of y.

For "smaller-the-better" problems, we want to minimize $(\mu_y + n\sigma_y)$, where n is the sigma level of y variation. For example, if the goal is to achieve a Six Sigma design, then $n = 6$.

Similarly, for “larger-the-better” problems, $(\mu_y - n\sigma_y)$ should be maximized.

There may also be a number of constraints for the optimization problem, which may contain statistical characteristics of response variables such as the mean and the standard deviation.

In the experimental approach, the **signal-to-noise ratio**, or *S/N*, can be used as the objective function. *S/N* for “nominal-the-best” problems is defined by Dr. Taguchi as a logarithm function of the ratio of the mean squared over the variance of the response [2]:

$$\eta_{NTB} = \frac{S}{N} = 10\log_{10}(\frac{\mu_y^2}{\sigma_y^2}) \tag{7.6}$$

From Equation (7.6), smaller variation of the response makes the *S/N* ratio larger; therefore, one should choose the design that maximizes the *S/N* ratio.

For a “smaller-the-better” problem, the objective is to maximize the *S/N* ratio defined by

$$\eta_{STB} = \frac{S}{N} = -10\log_{10}(\frac{1}{n}\sum_{i=1}^{n} y_i^2) \tag{7.7}$$

where y_i, $i=1,2,\ldots,n$ are the measurements of the response when the noise factors take on various levels such as in a DOE for noise factors.

For a “larger-the-better” problem, the objective is to maximize the *S/N* ratio defined by

$$\eta_{LTB} = \frac{S}{N} = -10\log_{10}(\frac{1}{n}\sum_{i=1}^{n}\frac{1}{y_i^2}) \tag{7.8}$$

where y_i, $i=1,2,\ldots,n$ are the measurements of the response when noise factors varies.

We summarize these commonly used objective functions in Table 7.4.

Table 7.4 Summary of robust optimization objective functions

Problem Type	Objective Function Analytical	Objective Function Experimental (dB)
Nominal-the-best	$\min\{(\mu_y - y_t)^2 + \sigma_y^2)\}$	$\max\{10\log_{10}(\frac{\mu_y^2}{\sigma_y^2})\}$
Smaller-the-better	$\min(\mu_y + N\sigma_y)$	$\max\{-10\log_{10}(\frac{1}{n}\sum_{i=1}^{n} y_i^2)\}$
Larger-the-better	$\max(\mu_y - N\sigma_y)$	$\max\{-10\log_{10}(\frac{1}{n}\sum_{i=1}^{n}\frac{1}{y_i^2})\}$

7.2.4.5 Robust Optimization Process

When a mathematical model is used to predict the response, optimization algorithms can be used to search for the optimal solution. However, since the objective function includes statistics such as the mean and the standard deviation of the response, in each iteration of the robust optimization, we need to calculate those statistics of the response. Taylor expansion method, MCS, or other methods can be applied in the estimation.

The optimization process is depicted in Figure 7.10. Note that the noise factors are needed in the step "estimate mean and variance of response." This is where MCS or other probabilistic methods are utilized. In practice, some of the constraints may also contain statistics, and should be evaluated here too.

If experimental data rather than mathematical prediction are available, the robust optimization can be carried out with either the response surface modeling method or the Taguchi methods.

The response surface modeling starts with a DOE of all the factors, control and noise. The DOE matrix can be full factorial, fractional factorial, orthogonal array, etc., depending on the specifics of the project. After the experiments are completed, the measured data can be used to fit a response surface model. With that model, one can choose the analytical objective function from Table 7.4 and the process described in Figure 7.10. This is similar to the situation when we have a mathematical model to predict the response, except that the model is fit with experimental data.

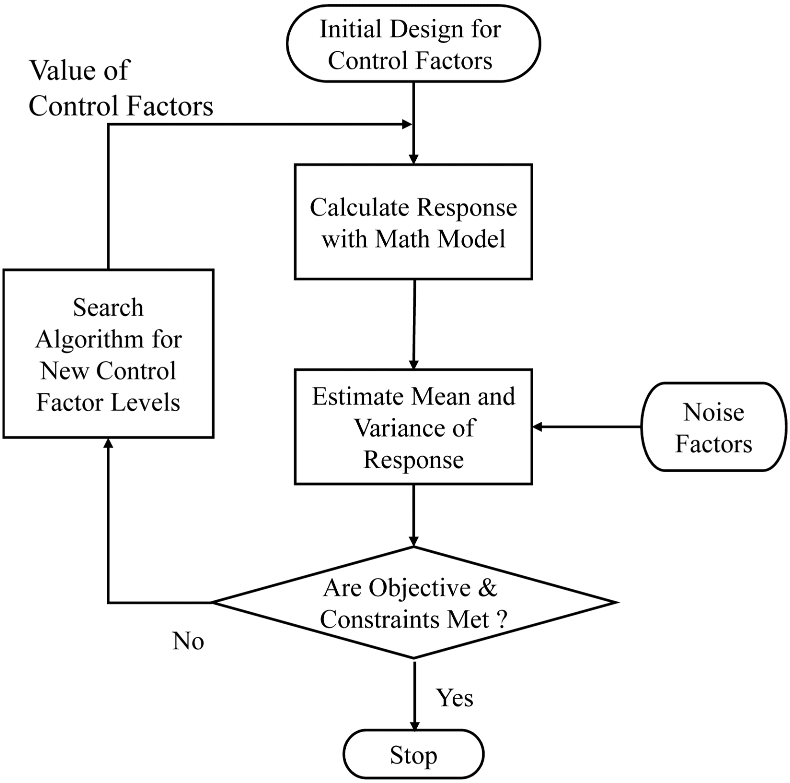

Figure 7.10 Process of robust optimization with a math model

The Taguchi methods, on the other hand, rely more on a graphic method to find the robust design solution. The following describes the basic process of the graphic method:

Step 1. Choose a DOE matrix for the control factors (Dr. Taguchi preferred orthogonal arrays), based on the number of control factors and their levels, and the number of experiments allowed. This DOE matrix for the control factors is also called the "inner array". In Figure 7.11, the inner array is at the left-hand side of the matrix. It has eight combinations of up to seven control factors, A to G, each with two levels. In Appendix B, we list some commonly used orthogonal arrays. For more elaborate treatment of orthogonal arrays, refer to [2] and [7].

Step 2. Choose a DOE matrix for the noise factors. This is called the "outer array." In Figure 7.11, the flipped upper right matrix is the outer array.

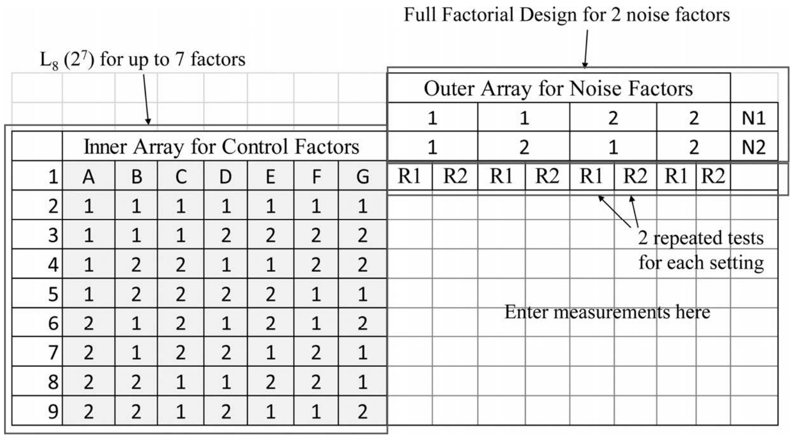

Figure 7.11 Structure of a Taguchi robust design matrix

Step 3. Arrange the inner array and the outer array as shown in Figure 7.11. Each row has a unique combination of the control factors, but with multiple tests, each having a different combination of the noise-factor settings and two repeats. For instance, if the outer array has four combinations of the noise factors, then the first row of the inner array needs to be run eight times.

Example 7.5 Robust Design of the Wheel Cover with DOE

For the wheel cover design examined in Examples 7.1 and 7.4, we can use the inner and outer arrays as shown in Table 7.5. The nominal values of the clip stiffness k and the clip deflection u are control factors. k has two levels at 3.1 and 5.2 *N/mm,* while u has two levels at 6 and 11 mm. The outer array is the full-factorial design with two combinations. The variation of the clip deflection Δu is the noise factor with two levels at −3 *mm* and +3 *mm;* the outer array is very simple in this case.

Each row in the DOE matrix corresponds to one combination of the control factors. Combined with the noise-factor levels, there are two different test settings for each row. In addition, each test setting is repeated twice, so we have a total of four measurements of the retention force, recorded in the shaded area of Table 7.5.

Step 4. Run experiments and record data. The response measurements are entered in the data area.

Table 7.5 DOE matrix for the wheel cover robust design example

Number	Nominal k, N/mm	Nominal u, mm	$\Delta u = -3$ mm		$\Delta u = 3$mm	
			1st F (N)	2nd F (N)	1st F (N)	2nd F (N)
1	3.1	6	9.6	9.6	27.9	27.5
2	3.1	11	25.6	25.2	43.8	42.9
3	5.2	6	16.1	15.3	46.5	46.5
4	5.2	11	42.2	41.2	72.4	72.7

Step 5. Calculate the mean, the standard deviation, and the *S/N* ratio for each data row. For the jth row,

$$\mu_{y,j} = \frac{1}{n}\sum_{i=1}^{n} y_{i,j} \tag{7.9}$$

$$\sigma_{y,j} = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(y_{i,j} - \mu_{y,j})^2} \tag{7.10}$$

where n is the total number of measurements of the response y in each row, and the summations are carried out for each row of data. For instance, in Figure 7.11, each row has eight tests, and the summations are for the eight measurements.

Depending on the objective of the project, one of the *S/N* ratios in Table 7.4 can be chosen. Then use the chosen formula to calculate the *S/N* ratio for each row. Enter the calculated average, the standard deviation, and the *S/N* as in the shaded area of Table 7.6. Then calculate the averages of each column of $\mu_{y,j}$, $\sigma_{y,j}$, and η_j as in the last row of Table 7.6.

Table 7.6 Average, standard deviation, and S/N of a DOE

	Control Variable							Average, STD, and *S/N* (η)		
Experiment	A	B	C	D	E	F	G	$\mu_{y,j}$	$\sigma_{y,j}$	η_j
1	1	1	1	1	1	1	1			
2	1	1	1	2	2	2	2			
3	1	2	2	1	1	2	2			
4	1	2	2	2	2	1	1			
5	2	1	2	1	2	1	2			
6	2	1	2	2	1	2	1			

	Control Variable							Average, STD, and S/N (η)		
Experiment	A	B	C	D	E	F	G	$\mu_{y,j}$	$\sigma_{y,j}$	η_j
7	2	2	1	1	2	2	1			
8	2	2	1	2	1	1	2			
Overall average								μ_y	σ_y	η_y

Table 7.7 Data calculation for the wheel cover DOE

			$\Delta u = -3$ mm		$\Delta u = 3$mm				
Number	Nominal *k*, N/mm	Nominal *u*, mm	1st *F*	2nd *F*	1st *F*	2nd *F*	Average	σ	*S/N*
1	3.1	6	9.6	9.6	27.9	27.5	18.7	10.5	5.03
2	3.1	11	25.6	25.2	43.8	42.9	34.4	10.4	10.38
3	5.2	6	16.1	15.3	46.5	46.5	31.1	17.8	4.86
4	5.2	11	42.2	41.2	72.4	72.7	57.1	17.8	10.11
Overall average							35.3	14.1	7.6

Example 7.5 (cont.) From the data in Table 7.5, we calculate the average, the standard deviation, and the "nominal-the-best" *S/N* for the wheel cover retention force, summarized in Table 7.7.

Step 6. Analyze effect of factors

We first define **additive function**. An additive function is a function of factors *A*, *B*, *C* . . . , such that the response can be expressed as

$$y = m + a(A) + b(B) + c(C) + \ldots \tag{7.11}$$

where *m* is the overall mean of *y*; *a* is a function of factor *A* only, representing the deviation of *y* from its overall mean caused by changes of *A*; *b* is a function of factor *B* only, representing the deviation of *y* from the overall mean caused by changes of *B*, and so on.

The Taguchi methods assume that the overall average of the response μ_y and the *S/N* ratio η_y (when noise factors vary) can be approximated by additive functions of the control factors. Although this assumption is not always valid, additive models are adequate for a large number of applications [2].

To estimate the effect of each control factor on the overall mean and *S/N* is straightforward. For instance, in Table 7.6, Experiments 1 to 4 are all run with factor $A = 1$. If we average $\mu_{y,j}$ for $j = 1,2,3,4$, the result is the average value of y when $A = 1$. Experiments 5 to 8 are run with $A = 2$. If we average $\mu_{y,j}$ *for* $j = 5,6,7,8$, the result is the average value of y when $A = 2$. From these two average values, we obtain the effect of factor A on y when A changes from level 1 to level 2. If the two averages differ significantly, then factor A has a large effect on response y. This comparison also shows how to change A to make y larger or smaller.

Similar calculations can be carried out for factor B, except that the experiments for averaging are different. Experiments 1, 2, 5, and 6 have $B = 1$; if we average $\mu_{y,j}$ for $j = 1,2,5,6$, the result is the average value of y when $B = 1$. Likewise, if we average $\mu_{y,j}$ for $j = 3,4,7,8$, then the result is the average value of y when $B = 2$. These two averages can be compared to show the effect of factor B on the response.

The same analysis can also be carried out for *S/N*. Since *S/N* is a measure of robustness of the design, its factor analysis indicates which factors influence the robustness the most and how they influence it.

Although the factor analysis is simple enough to be carried out with a calculator, we usually use commercial software such as Minitab© to analyze the data and plot the results.

Example 7.5 (Cont.) For the wheel cover example, the factor effects are calculated and summarized in Table 7.8.

The factor plots are also called "***main effects plots.***" Figure 7.12 presents the effects of the control factors on the retention force F. Both the stiffness and the deflection of the clips affect the retention force significantly in the range explored in the DOE. By contrast, their effects on the *S/N* ratio are very different (Figure 7.13): *S/N* changes from 4.9 to 10.3 *dB* as the nominal deflection varies from 6 to 11 *mm*, while remains between 7.7 and 7.5 *dB* as the nominal stiffness k changes from 3.1 to 5.2 *N/mm*. This means that we can maximize the *S/N* ratio by choosing $u = 11$ *mm*, regardless of what k value is chosen.

Table 7.8 Factor effects of the wheel cover DOE

	$k = 3.1$ N/mm	**$k = 5.2$ N/mm**	**$u = 6$ mm**	**$u = 11$ mm**
Average $F(N)$	26.5	44.1	24.9	45.7
S/N (*dB*)	7.7	7.5	4.9	10.3

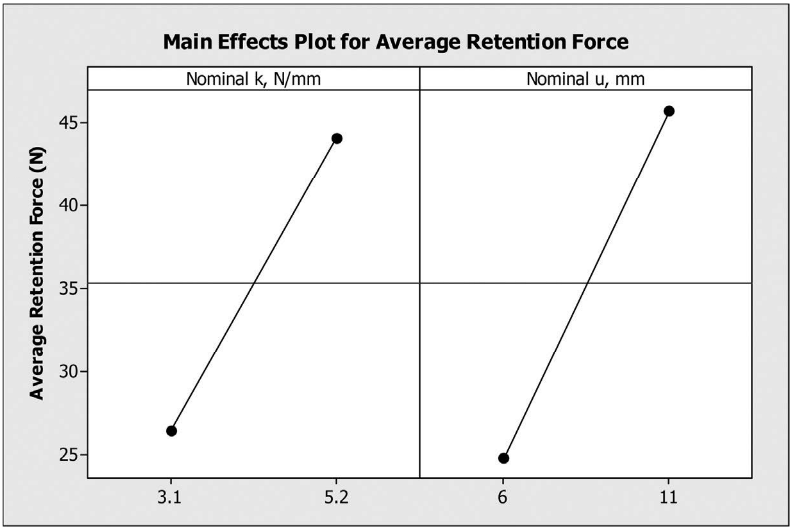

Figure 7.12 Main effects plot of average retention force

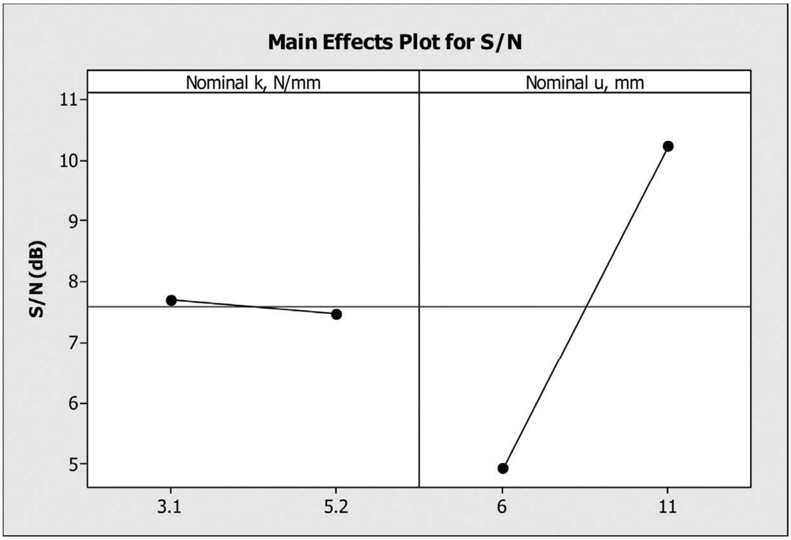

Figure 7.13 Main effects plot of S/N

One advantage of the Taguchi methods is the insights provided by the intuitive factor plots of the mean response and the *S/N*. Those plots can be used to find control factors of four types:

a. Have high impact on the mean response, but low impact on the *S/N* ratio. These control factors can be employed to adjust the mean response, without changing its variation. They are also called "sliding factors," or "adjustment factors."

b. Have low impact on the mean response, but high impact on the *S/N* ratio. The levels of these control factors should be chosen to increase the *S/N* (to reduce the variation of the response), without changing the mean response.

c. Affect neither the mean response nor the *S/N* ratio. These control factors may be chosen based on other considerations, such as cost.

d. Affect both the mean response and the *S/N* ratio. We need to be careful when choosing levels for these control factors, due to the coupled effects.

Note that the "a type" and the "b type" factors have decoupled effects on the mean and the variation of the response. They make the task of robust design more straightforward.

Table 7.9 summarizes the usage of different types of control factors.

Step 7. Choose the optimal level for control factors

Based on the factor effect analysis, we can now select the right values for the control factors to achieve a robust design. If there are adjustment factors that only impact the mean response, we can carry out this step as follows:

1) Maximize the *S/N* ratio by choosing the right level for "b type" and "d type" factors;
2) Shift the mean response toward the target value by choosing the right level for the adjustment factors.

Table 7.9 Usage of the control factors

Control Factor	Has High Impact on μ_y	Has Low Impact on μ_y
Has High Impact on *S/N*	Affects both, choose level carefully	Choose level to increase *S/N*
Has Low Impact on *S/N*	"Adjustment factor"; choose level to adjust μ_y	Low impact, choose cost-saving level

Example 7.5 (Cont.) In the wheel cover example, we first choose u = 11 *mm* to maximize the *S/N* ratio. Then we try k = 3.1 *N/mm*, which has an average retention force of 34.4 *N* (second row of the data, Table 7.7).

More examples of the Taguchi methods can be found in [2] and [7].

7.2.4.6 Predicting Results

Predicting the results of the robust optimization is easy if a mathematical model has been used. MCS can again be conducted to estimate the statistical distribution of the response and the constraints. The results can then be compared with the requirements we have determined at the start of the project.

If the Taguchi DOE approach was used, the predictions of the mean response and the *S/N* ratio for the optimal robust design rely on the assumption of additive models given by Equation (7.11). We illustrate it with the wheel cover example:

Example 7.5 (Cont.) Based on the additive model, the predicted average retention force is

$$\begin{aligned} \text{Average } F_{predicted} &= \text{Overall average of } F + (\Delta F \text{ due to } u = 11) \\ &\quad + (\Delta F \text{ due to } k = 3.1). \end{aligned}$$

From Tables 7.7 and 7.8,

$$\begin{aligned} (\Delta F \text{ due to } u = 11) &= (\text{Average } F \text{ when } u = 11) - (\text{Overall average of } F) \\ &= 45.7 - 35.3 = 10.4\ (N) \end{aligned}$$

$$\begin{aligned} (\Delta F \text{ due to } k = 3.1) &= (\text{Average } F \text{ when } k=3.1) - (\text{Overall average of } F) \\ &= 26.5 - 35.3 = -8.8\ (N) \end{aligned}$$

Therefore, when u = 11 *mm* and k = 3.1 *N/mm*, the average retention force is estimated to be

$$\text{Average } F_{predicted} = 35.3 + 10.4 - 8.8 = 36.9\ (N)$$

Similarly, the *S/N* ratio when u = 11 *mm* and k = 3.1 *N/mm* can be estimated by its additive model:

$$
\begin{aligned}
S/N_{predicted} &= \textit{Overall average of } S/N + \{\Delta(S/N) \textit{ due to } u = 11\} \\
&\quad + \{\Delta(S/N) \textit{ due to } k = 3.1\} \\
&= 7.6 + (10.3 - 7.6) + (7.7 - 7.6) = 10.4 \ (dB)
\end{aligned}
$$

Note the importance of choosing the right levels for the control factors to be included in the DOE. In this example, if u = 11 mm were not part of the DOE, we might not have found the satisfactory design solution. In that case, subsequent DOEs are needed, to further explore the design space. Unlike the analytical approach where a math model is available for optimization, the DOE-optimization process may be iterative: if the data from the first DOE does not yield a robust solution, a second DOE should be conducted, with additional levels or even additional control factors. The expectation is that each iteration will lead to new knowledge and eventually we'll find a solution. ■

7.2.4.7 Tolerance Design

If the optimized design still has more variation in its performance than allowed by the requirements, tolerances of the noise factors should be optimized too. For those noise factor(s) that the response y is most sensitive to, tolerance(s) should be tightened. This is equivalent to reducing σ_{x_i} of those noise factors with large $(\frac{\partial y}{\partial x_i})^2$. For the noise factors that y is no longer sensitive to, their tolerances may be relaxed to lower the cost.

If the mathematical model of the response as a function of the noise factors is available, one can optimize the tolerances with a similar process as described in Figure 7.10. The tolerances of the noise factors are now the independent variables in the iterative search. In each iteration, a new set of tolerances are used to simulate the variation of the noise factors, and the resultant mean and variance of the response can be estimated.

If we do not have a mathematical model relating noise factors to the response, then experiments are needed to understand the sensitivity of the response with respect to the noise factors. Set the control factors at their optimal levels determined by the robust optimization described in

Subsection 7.2.4.5, and vary the noise factors with a DOE matrix. Then use the factor effect plots to identify noise factors that have the most impact on the response, and reduce their tolerance ranges. For those noise factors that do not affect the response very much, their tolerances can be relaxed.

In Example 7.1, we discussed how the tolerance of the clip deflection could be optimized to achieve a high quality at a low cost. It illustrated the concept of tolerance design. For more examples of tolerance design, refer to [2] and [7].

7.2.5 Phase 5: Verify Design—Ensuring Requirements Are Met

In this phase, the predictions of the mean and the variation of the response are confirmed with tests, which should be conducted with production parts and in the real operating environment of the product or process. It is important to include the noise factors in the experiments, so that the robustness of the quality can be verified.

Why is the verification phase necessary? There are several reasons:

- During the optimization phase, mathematical models, or prototype parts, or reworked parts, in controlled operating environment are used to predict the response and its variation; they are only approximations of the production parts and the real operating environment. The predictions thus obtained may have large errors and need verification.
- In the Taguchi methods, additive models are used to predict the mean response and the *S/N* ratio. An additive model assumes that any interaction (or "cross-terms") between two or more factors is negligible. While this assumption simplifies the data analysis and optimization, it may not always be valid. In case that a significant interaction between control factors exists, the predictions of the optimal mean response and *S/N* will be incorrect. We need verification tests to validate that the solution we find is indeed optimal and performs as predicted.

If the product development cycle is long, we may not have the true verification of the robust design until the product or process is in the market. For instance, in the United States, J.D. Power surveys vehicle quality by sending questionnaires to vehicle buyers, and publishes the result. The true verification of a good design is therefore months or even years after a DFSS project is carried out. However, before the new vehicle model is in production, there are validation tests using parts produced with near-production tooling, and there are preproduction vehicles that are built and tested but not sold to the public. We can collect and analyze data from those tests, to assess the design.

7.3 Implementing DFSS

7.3.1 The Drive for DFSS Implementation

While the theory of DFSS is not complicated, its implementation can be challenging. More important than technical environment, such as having the expertise in statistics, computer modeling and simulation, optimization, and so on, is the mindset of the organization. The leadership of the organization must understand the necessity and benefits of DFSS, and commit to its implementation. In addition, DFSS requires support from all who participate in the design and development of a product or process; it can only be successful when all of them share the vision of "quality at low costs" and engage in its realization.

For an organization, a powerful drive for DFSS is the competitive pressure. This pressure is measured by external or internal metrics. External metrics can be ratings by consumer advocates such as J.D. Power and Consumer Reports, who review products and services and collect feedback from consumers; it can also be a government agency such as the U.S. NHTSA, or stock analysts of Wall Street firms. The external metrics show the winners and losers in a marketplace, and can be vital to the success or even the survival of an organization. No one wants to be a loser. Under this pressure, a weak organization would seek to improve its products or services, including quality and reliability. This is an important reason why the quality of the American-made automobiles improved dramatically since the 1980s—when the U.S. automakers' survival was threatened by their Japanese counterparts.

To motivate an organization to embrace DFSS, it is essential to use these driving metrics. Without the competitive pressure and the desire for product excellence, it is not likely that DFSS will take hold in the organization.

As a tactic, in an organization new to DFSS, good case studies should be developed to demonstrate the power of DFSS and to win over the skeptics.

7.3.2 *Quality Products at Low Costs—A Competitive Advantage*

For many enterprises, the internal metric for quality is the warranty cost they pay to repair or maintain their products or services. For instance, the U.S. automobile companies typically spend millions of dollars each year on warranty. This cost reduces profit, and directly impacts the financial results of a company.

Organizations which can provide high-quality products or services at low costs will be very competitive in the global marketplace. However, the traditional point of view believes that "quality is expensive," which often leads to a trade-off between cost and quality. DFSS has shown that high quality can be achieved without high costs; it may even be achieved at lower costs. Depending on the situation, the following strategies can be considered:

- Focus on quality improvement, limiting the cost to be no more than that of the current design.
- Focus on cost reduction, maintaining the same level of quality.
- Balance quality improvement and cost reduction, to achieve both.

The wheel cover example illustrates all these potentials. The key is to carry out DFSS early on, while anticipating the downstream variation that may affect the quality of the product or service.

7.3.3 *How to Choose DFSS Projects?*

DFSS should be applied when a product or process has new or persistent old problems, and is being redesigned. If resources are limited, it is important to prioritize potential DFSS projects. Below is a list of candidates for DFSS:

- Product or process that uses a new technology. A device of new technology may be sensitive to new sources of variation. It may also introduce new interface with the rest of the system. The new technology may have been used in another industry, where the operating condition is very different from the new application. For instance, when consumer electronic devices are integrated into a vehicle, they no longer work in a protected environment such as an office or home. They have to operate in cold and hot temperatures as well as at dry and wet locations, withstand vehicle shakes and vibration, encounter electromagnetic interferences, and even survive a car crash. It deserves a careful review, to decide if DFSS should be used to make the product robust.
- Product or process that has a "hardy perennial problem." This may be a quality problem that appears again and again, in generations of designs, making a comeback each time it was thought to have been solved. A different design concept may be needed to cure the problem, and DFSS is a powerful process to make sure that the new design is robust inherently. To identify those hardy perennials, one can analyze the historic warranty data or customer feedback data.
- Product or process that is at a high risk of failing new regulations. When government agencies change regulatory requirements, if a product or service does not meet them, it may be prohibited for sale, let alone to compete. In addition, the product or service may be inspected or tested by government agencies on a random basis, making it even more important to have a robust design with consistent performance. DFSS is a suitable approach in this case.
- Product or process that is going to a new market. It is critical to understand the customers, and not only from the marketing point of view. The new customers may have different needs and preferences. For instance, in a country

where the passenger usually sit in the back seats of a vehicle while a chauffeur drives the car, the seating comfort requirements for the back seats need to be considered for an upgrade. The government regulation in the new market may also be different from that of the existing markets, which may be more or less stringent. The common warranty policies in the new market may vary as well, presenting opportunities for DFSS.

- Product or process whose cost needs to be slashed. For various reasons, cost reduction is a constant effort in a market economy. How to reduce the cost while maintaining a high quality level of a product or service is a challenge. While cutting cost by sacrificing the quality may produce short-term financial gains, it is not a viable strategy in the long run. With DFSS, however, alternative designs can be developed to ensure that the quality does not suffer as a result of the cost-cutting.
- When a product or process is complex. It is easy to overlook potential failure modes in this situation, especially those failures caused by unexpected sources of variation that the system is sensitive to. The systematic approach of DFSS can guide the design process, and minimize the risks brought by the complexity.

7.3.4 DFSS Support to Manufacturing and Service

As emphasized throughout this chapter, DFSS should be implemented early at the design stage of a product or process. Upon the completion of a DFSS project, two sets of knowledge should be passed on to the manufacturing or the service organization:

1. Functional requirements that are important to customers, and how to measure them. These requirements came from the VOC, and have already been translated to quantitative specifications. The manufacturing and service facilities need to check if the requirements are met, to maintain a high quality level.

2. Tolerances of noise factors that the product is sensitive to. These are the few variables that can negatively impact the performance of the product, as identified during the optimization phase of DFSS. In the process of making and using a product, many factors may deviate from the design intent, but not all of them are equally important. To deliver consistent quality, the knowledge of which factors are more sensitive is valuable.

The above knowledge can be used by a Six Sigma team, to help achieve its objectives.

7.3.5 Making DFSS a Habit

DFFS is most effective if its philosophy becomes a natural habit of an organization, so that DFSS can be implemented whenever the opportunity arises, and for a long period of time. In other words, DFSS is most effective when it is institutionalized. This does not mean that every person in the organization needs to be an expert in DFSS, but that everyone has the mindset of designing robustness into the product or process, understands the concept of DFSS, recognizes opportunities for applying DFSS, and supports DFSS projects from his or her own position.

Of cause, an organization needs DFSS experts, who can lead or coach DFSS projects and provide advice to the management on the strategy of quality improvement. The DFSS experts should also have knowledge about the product or process being designed, so that their advices are more pertinent. To be a good DFSS project leader or coach, one needs to know not only the DFSS methods and techniques, but also how the product or process works. The integration of the DFSS methods and the product knowledge is critical to the success of the projects.

To make DFSS a natural habit, it needs to be built into the standard workflow of the organization, especially for large companies. The knowledge gained from DFSS projects should be applied to future designs of similar products, and the best practice may be integrated into the standard work procedure. These steps will make DFSS a part of the collective memory of the organization, and change its culture toward designing-in high product quality.

A variation database is valuable in facilitating DFSS projects. Data on variation of the material properties, manufacturing process capabilities, ergonomic limits, operating environment, and customer usage can all be part of the database. The source of the data can be public database provided by government agencies, commercial database that can be purchased, or proprietary information such as a company's own production database or the customer feedback records. An easy-to-access database can reduce the time needed for evaluating the robustness of a design.

It should be clear that the above implementation measures all require the commitment of the management. Without firm support of the management, the resource will not be available to DFSS, and the effort to institutionalize DFSS will be a false start.

7.4 Conclusions

DFSS is a rigorous process for the design and development of a product or process. It starts from understanding customer needs, translated into quantitative measurable requirements. It anticipates the downstream sources of variation, and chooses design concepts that have the potential of being robust against the variation. It optimizes the design in the statistical sense to achieve consistent quality, in presence of variation in building and using the product or process. It also optimizes the tolerances so that they are tightened only when necessary, therefore saving costs.

Besides technical expertise, DFSS calls for the change in culture of an organization. The traditional view that "quality is the responsibility of manufacturing" can be persistent, and the paradigm of designed-in high quality needs constant effort to be implanted into the culture. Strong leadership is required to lead this change.

The drive to product excellence is never ending. There are always new customer expectations, new regulatory challenges, new technology to be applied, and DFSS has the competitive advantage of producing high-quality products at lower costs.

As an introduction to DFSS, this chapter is self-contained, focuses on the most commonly used concepts and methods of DFSS. For a more detailed and technically oriented treatment of DFSS, refer to [8].

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CHAPTER 8

Case Studies

This chapter provides three real-world case studies to help readers better understand the Lean Six Sigma and DFSS processes. Compared with the examples in the previous chapters, these case studies are more complex in their technical content, and more advanced tools such as DOE, RSM, and Monte Carlo Analysis are employed. From these case studies, readers can learn the practical usage of some of the tools introduced in Chapters 4 and 7. All three case studies relied heavily on mathematical modeling and computer simulation, which enabled convenient system evaluation and redesign, therefore helped achieve the desired outcome timely and at low costs.

Note that the focus of the case studies in Sections 8.2 and 8.3 is to illustrate the concept and the process of DFSS, not to present the specific details of the designs. Data in those case studies have been transformed, and do not represent any vehicle designs or performance measurements.

8.1 Lean Six Sigma Project—Pulse Width Modulation Control for Motor Speed

Pulse Width Modulation (PWM) is widely used in industries to effectively control analog devices such as motors using digital control signals. For instance, in a typical automobile, a number of subsystems use DC motors as actuators, including power-steering mechanisms, fans for heating, ventilation, and air conditioning (HVAC), sunroofs, antilock brakes, and power windows. The power source for the motors of these subsystems is the lead-acid battery and the alternator, whose voltage is maintained at a relatively constant level. To control the motor speed, a PWM signal is applied to the power supply of the motor, as illustrated in Figure 8.1.

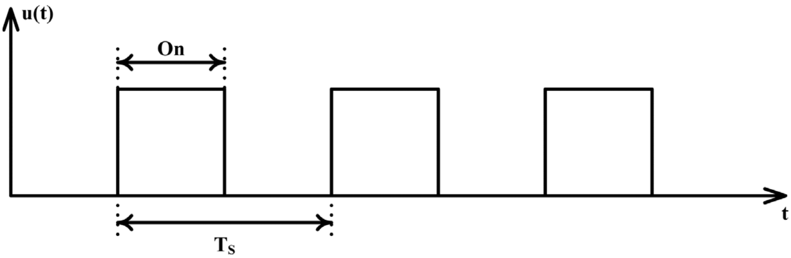

Figure 8.1 Pulse width modulation control

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In these systems, the motor speed is controlled by changing the **duty cycle** of the PWM signal, which is defined as follows:

$$Duty\ cycle = \frac{T_{on}}{T_s} \tag{8.1}$$

Some of these systems use the open-loop control without a speed feedback to reduce the cost. In such cases, the duty cycle is calculated according to the following rule of thumb:

$$Duty\ cycle = \frac{Desired\ voltage}{Nominal\ voltage} \times 100\% \tag{8.2}$$

where the desired voltage is the value of the voltage such that if it is applied to the motor, the desired speed will be achieved, and the nominal voltage is the voltage value that is available. For most vehicles, the nominal voltage is 14 V. The actual voltage could be anywhere between 10 and 16 V. For example, a vehicle has a 14 V nominal voltage from the battery and alternator system. Suppose that to control the motor speed to a desired level, we need a 7 V power source. Since the 7 V power source is not directly available, we can use PWM with a duty cycle of 7/14 × 100% = 50% to achieve the desired speed using the fixed nominal voltage of 14 V. This seems to be a good solution, but field data shows that the motor speed controlled this way is not consistent, which in turn has a negative impact on the system performance. A Lean Six Sigma project team was formed to study this problem [1].

8.1.1 *Define*

Project Charter

Problem statement: The rule of thumb Equation (8.1) used in the PWM control causes inconsistent motor speed. This variation must be reduced to improve the motor speed control performance.

Business case: Reducing the speed variation will improve the performance of subsystems that use motors as actuators. This will help companies to produce more competitive products and better satisfy their customers.

Project scope: The project will focus on the improvement of the PWM control of DC permanent magnetic motors with brushes. The impacts of the motor part-to-part variation, the fluctuation in voltage output of the battery/alternator subsystem, the load torque, and the PWM duty cycle on the average motor speed over time will be studied. Among these factors, the main cause for the speed variation will be identified, and an improved control method will be proposed. Due to the limitation on project duration and available resources, the investigation will rely mainly on computer simulation instead of laboratory testing. This project is the first phase of a series of efforts to improve the motor control performance. The temperature and the PWM frequency are assumed to be constant, to limit the scope of this project.

Project sponsor: Automotive original equipment manufacturers (OEMs).

Process owner: Automotive suppliers for OEMs.

Project duration: The project will be completed in two months.

Resource: Two engineers working full-time.

Project champion: The engineering manager.

Project goal: To reduce the average motor speed error and variation by 60 percent without incurring additional cost.

Metric: The standard deviation of the average motor speed and the mean value of the difference between the average speed and the desired speed will be used as the metrics for performance measurement.

Deliverables: Improved PWM control design, design document, simulation model, and simulation results.

Project risks: The analysis relies mostly on mathematical modeling and computer simulation. Inaccuracy of the model will directly affect the conclusions of the project.

The SIPOC diagram and the process map were developed as shown in Figures 8.2 and 8.3:

SIPOC

Suppliers	**I**nputs	**P**rocess	**O**utputs	**C**ustomers
OEMs	Requirements	Motor PWM control	Motor durability	OEMs
Subsystem suppliers	Regulations		NVH	Subsystem suppliers
Motor suppliers	Noise benchmark		Risk (FMEA)	Government
Electronics group	Motor parameters		Motor speed	Motor suppliers
NVH group	Data sheet for electronic parts		Algorithm	Electronic group
Software group				Software group
Government				NVH group

Figure 8.2 Motor PWM SIPOC

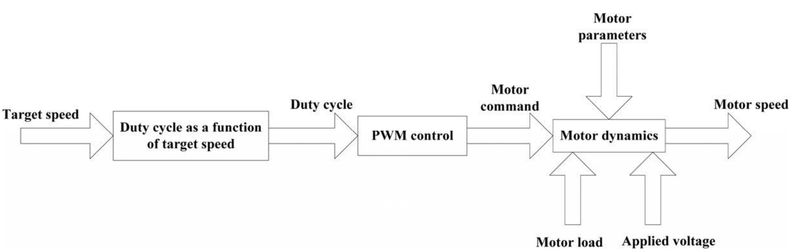

Figure 8.3 Motor PWM process
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The critical-to-quality tree was developed as shown in Figure 8.4.

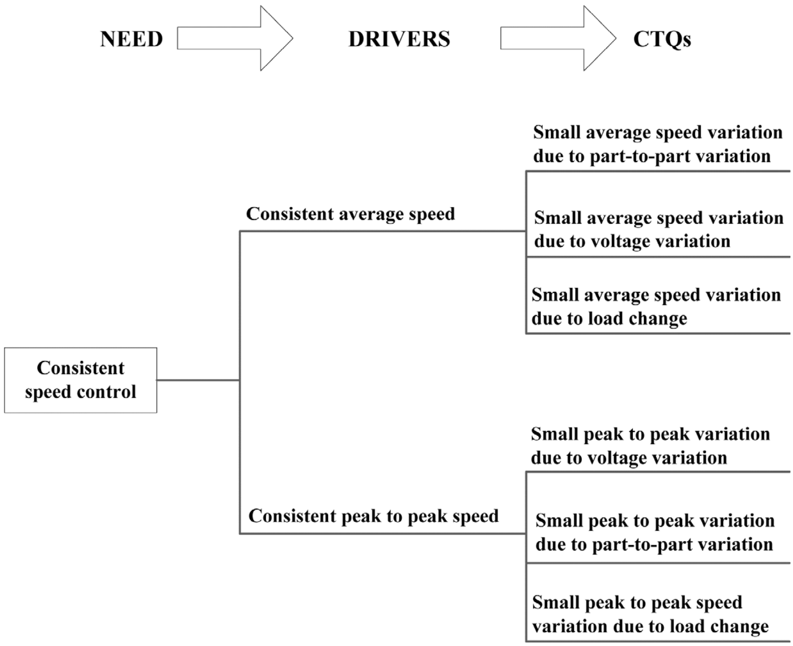

Figure 8.4 CTQ tree for motor PWM control
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8.1.2 Measure

Since modeling and simulation were the main tools to be used in this project, we started by developing the first principle model for DC permanent magnetic motors with brushes:

$$\begin{aligned} \frac{di_a(t)}{dt} &= \frac{1}{L_a} e_a(t) - \frac{R_a}{L_a} i_a(t) - \frac{1}{L_a} e_b(t) \\ T_m(t) &= K_i i_a(t) \\ e_b(t) &= K_b \tfrac{d\theta(t)}{dt} \\ \frac{d^2\theta(t)}{dt^2} &= \frac{1}{J} T_m(t) - \frac{1}{J} T_L(t) \end{aligned} \tag{8.3}$$

where R_a and L_a are the resistance and inductance of the motor coil, respectively, J is the rotor inertia, K_i is the torque gain, K_b is the back emf gain, e_b is the induced back emf voltage, e_a is the applied voltage, T_m is the motor torque, T_L is the load torque, θ is the rotor angle, and i_a is the current through the motor coil.

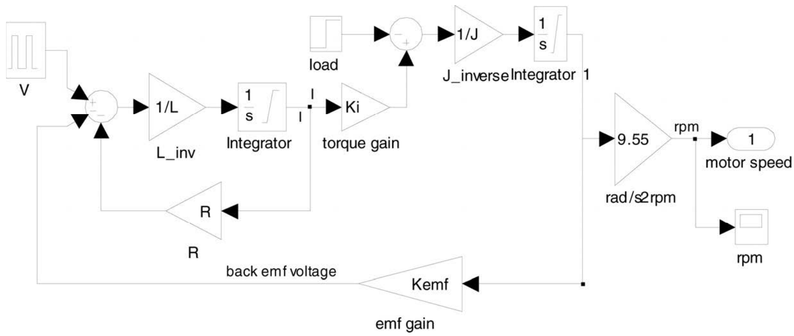

Figure 8.5 The Simulink model for DC PM motor speed control
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In Equation (8.3), the first equation is based on Kirchhoff's voltage law; the second is the formula for the force on a current-carrying wire; the third is based on Faraday's law, and the fourth is based on Newton's second law.

A *Simulink* model was created based on Equation (8.3), as shown in Figure 8.5. The parameters in the *Simulink* model were specified in an *mfile*. This allowed us to run hundreds of iterations of simulation using the *for loop* in MATLAB. The motor parameters could also be randomly generated in the *mfile* before simulating the motor dynamics.

The *Simulink* model was used to find the voltage that would produce a desired average motor speed, say 2,000 rpm, as illustrated in Figure 8.6. In this case, the desired voltage was found to be 5.69 V. For comparison, if the nominal battery voltage was 14 V, and if the rule of thumb Equation (8.2) was used, the PWM duty cycle would have been $5.69/14 \times 100 = 40.6\%$. However, the simulation result showed that a 40.6 percent duty cycle would have resulted in an average motor speed of 4,100 rpm, far exceeding the desired speed of 2,000 rpm. Other values for the desired speed were also simulated and the conclusion was consistent: the rule of thumb in Equation (8.2) was not accurate enough for our purpose.

Through simulation, we also found that the PWM duty cycle affected average speed while the PWM frequency affected the peak-to-peak variation of the speed. Since the PWM frequency was assumed to be constant as stated in the project scope, the peak-to-peak variation was not

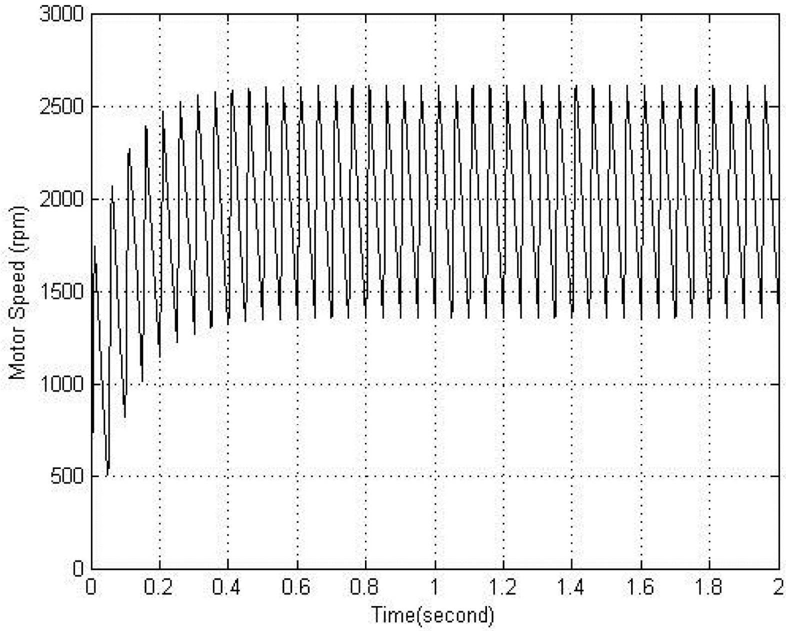

Figure 8.6 A typical PWM control motor speed trace

considered as a performance metric. The duty cycle calculated by Equation (8.2) was replaced by the more accurate one from the simulation of the model.

To measure the performance of the current (baseline) process, 1,000 sets of random values for the motor parameters were generated in MATLAB, assuming that each parameter had a normal distribution.

1. Resistance: normal distribution, mean value 1.0e − 1 Ω, σ = 5.0e − 3 Ω
2. Inductance: normal distribution, mean value 1.0e − 4 H, σ = 5.0e − 5 H
3. Inertia: normal distribution, mean value 9.0e − 5 kgm^2, σ = 4.5e − 6 kg m^2
4. Torque gain: normal distribution, mean value 2.0e − 2 Nm/A, σ = 1.0e − 3 Nm/A
5. Back emf gain: normal distribution, mean value 2.0e − 2 V/(rad/s), σ = 1.0e − 3 V/(rad/s)
6. Applied voltage: normal distribution, mean value 12 v, σ = 1.5V
7. Load: normal distribution, mean value 0.3 Nm, σ = 0.015 Nm

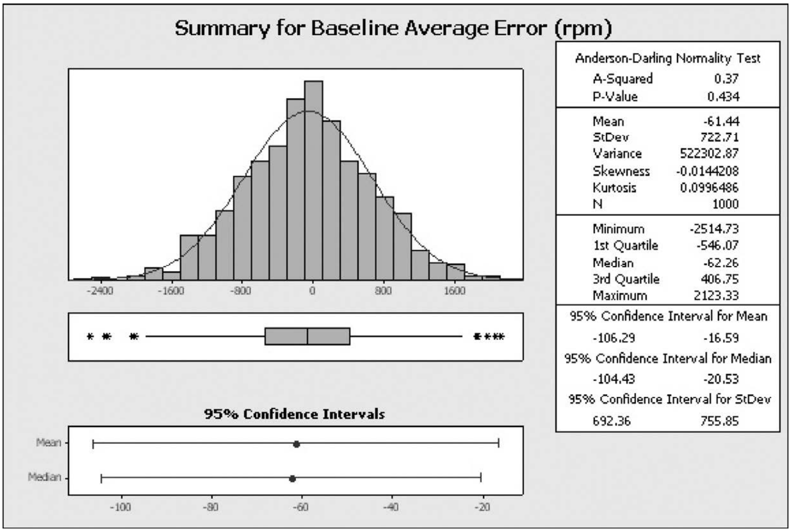

Figure 8.7 Average error statistics for baseline
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Each set of these parameters represented one particular motor. For each motor, the average motor speed was estimated with randomly generated battery voltage. The 1,000 values of such average motor speeds were compared with the desired speed of 2,000 rpm, and the differences were defined as the errors. The statistics of the error are displayed in Figure 8.7.

In summary, the performance metrics of the baseline process were given below:

The mean error was –61.44 rpm.
The standard deviation of the error was 722.71 rpm.

8.1.3 Analyze

There were five motor parameters, whose deviation from their nominal values represented the part-to-part variation. In addition, the battery voltage was a noise factor that could be measured but not controlled, and the PWM duty cycle was a factor that could be controlled. Altogether, there were seven factors, too many to analyze using the techniques such as the response surface method discussed in Section 4.4.1. Instead, the design of experiments technique presented in Section 4.3.5 was applied first to determine which factor(s) had a more significant impact on the outcome.

The minimal and maximal values for each factor were as follows:

1. Resistance: 0.085 ohm, 0.115 ohm
2. Inductance: 8.5e – 5 H, 1.15e – 4 H
3. Inertia: 7.65e – 5 kg m^2, 1.04e – 4 kg m^2
4. Torque gain: 0.017 Nm/A, 0.023 Nm/A
5. Back emf gain: 0.017 V/(rad/s), 0.023 V/(rad/s)
6. Load: 0.255 Nm, 0.345 Nm
7. Applied voltage: 7.5 v, 16.5 v
8. Duty cycle: 18%, 38%

Since simulation was used to conduct the DOE, and each simulation needed only seconds to run, the number of experiments was not a limiting consideration. Also, there was no need for replica of each experiment because the results would be exactly the same for replicas. Therefore, a seven-factor, full-factorial DOE was performed. Since each factor has two levels, the total number of tests was $2^7 = 128$. Figure 8.8 shows the Pareto chart from the DOE analysis. It can be seen that the top three factors having significant impact on the outcome are E, F, and EF, representing the battery voltage, the duty cycle, and the interaction between the battery voltage and the duty cycle, respectively. The motor coil resistance was in the fourth place.

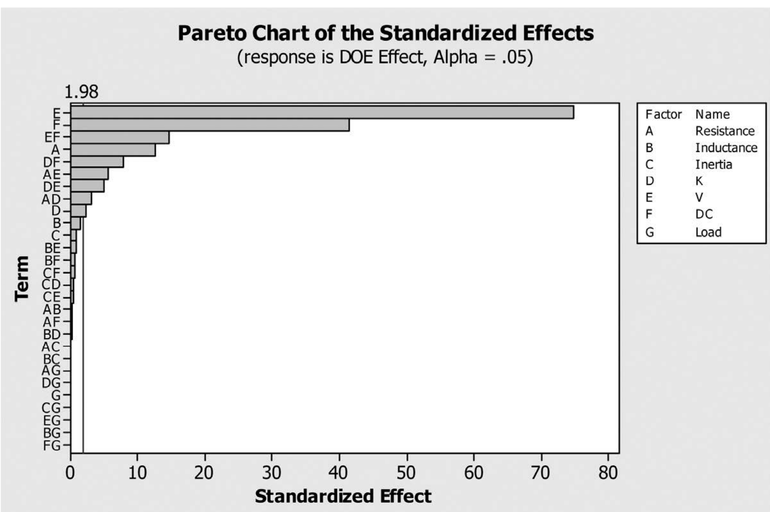

Figure 8.8 Pareto chart of the seven factors

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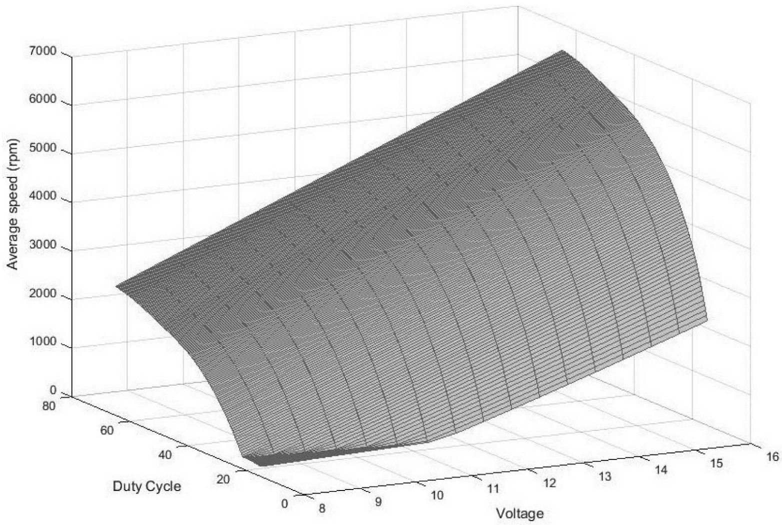

Figure 8.9 surface method for motor speed

Tightening the tolerance for the motor coil resistance would increase the cost of the motor, violating the project constraint of "no additional cost." Therefore, we left this parameter alone and considered the other two, the battery voltage and the PWM duty cycle. The response surface method was used to analyze the relationship between the motor speed and these two factors. The other parameters were assumed to be at their nominal values. While this simplifying assumption was not realistic since all the parameters had variation in them, it was adopted to simplify the analysis. The effect of this assumption will be considered later, when we evaluate the performance of the improved process.

The motor speed is plotted as a function of the battery voltage and the PWM duty cycle, as in Figure 8.9. We can see from the graph that the motor speed changes steeply as the duty cycle and the battery voltage change.

8.1.4 Improve

A horizontal plane representing the desired speed of 2,000 rpm is added in the response surface plot (Figure 8.9). On the one hand, the motor speed must follow the physical laws and stay on the response surface. On the other hand, the desired speed is 2,000 rpm. To achieve the desired

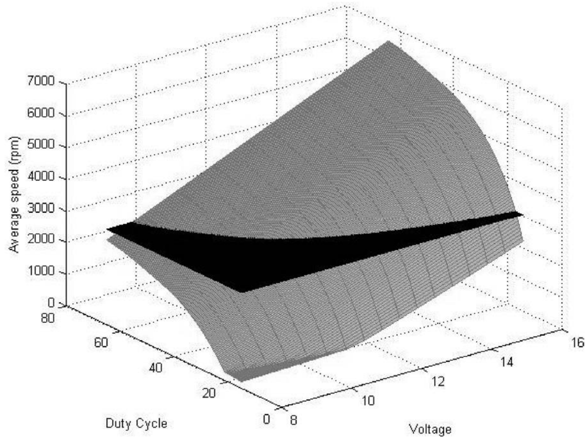

Figure 8.10 Improvement idea: follow the intersection curve

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speed without violating the physical laws, the choice of the PWM duty cycle and the battery voltage must lie on the intersection of the response surface and the horizontal plane (Figure 8.10).

This intersection specifies the ideal relationship between the PWM duty cycle and the battery voltage, as shown in Figure 8.11. In other words, if we choose the PWM duty cycle as a function of the battery voltage as defined in Figure 8.11, then the average motor speed will be maintained at the desired speed.

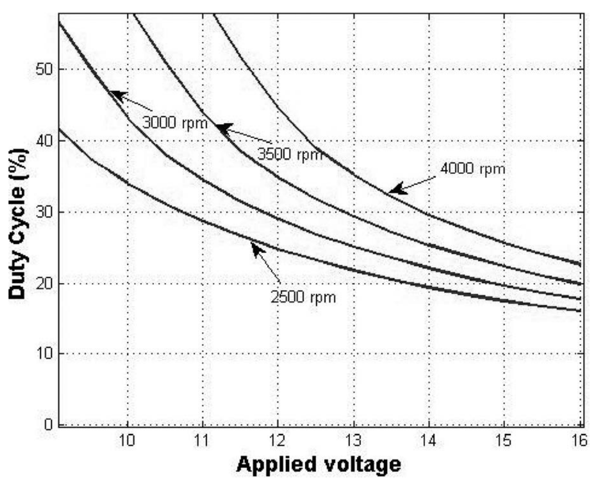

Figure 8.11 New design for determining PWM duty cycle

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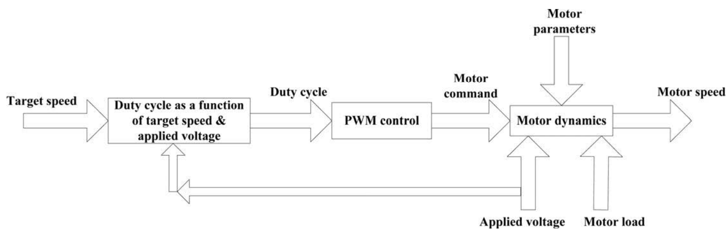

Figure 8.12 The improved process for motor PWM control
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The improved process for the PWM motor speed control is illustrated in Figure 8.12.

8.1.5 Control

The performance of the proposed new process needed to be evaluated and compared with the baseline performance. To this end, the same 1,000 sets of values for the motor parameters generated during the "Measure" phase were used in the simulation, together with the new PWM control algorithm. The error statistics are shown in Figure 8.13.

The mean and the standard deviations of the error for the baseline and the new processes are summarized in Table 8.1 for comparison. It is worth noting that during the evaluation of the new process, all parameters were randomly generated. This validated the simplifying assumption adopted in the RSM used in the Analyze phase.

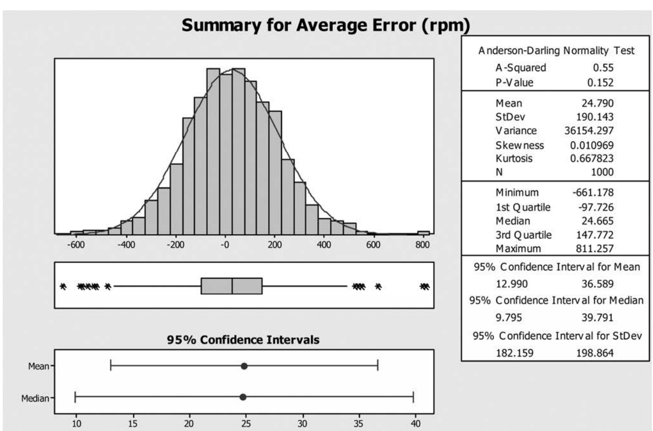

Figure 8.13 Average error statistics of the improve process
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Table 8.1 Before-and-after comparison for motor PWM control

	Current Process (rpm)	Improved Process (rpm)	Improvement (%)
Mean error	−61.44	24.79	60
Standard deviation of errors	722.71	190.143	74

The mean error was reduced by 60 percent and the standard deviation of the error was reduced by 74 percent, both meeting the target of 60 percent reduction specified in the project goal. Since the project was carried out with computer simulations, there was no need for SPC. These results must be validated in the next phase project with real-world tests.

The results from this project was presented to the sponsor and summarized in the final report. Recommendations were made for further improvement of motor PWM control, these included modeling the temperature, adding low-cost measurement for motor speed, and developing algorithm for load detection.

8.2 DFSS Project—Vehicle-Occupant Safety Robust Design for Frontal Impact

To protect occupants from severe injuries caused by crash accidents, automobiles are equipped with occupant-restraint systems. Typically, a driver-restraint system for the frontal impact consists of the air bag located inside the steering wheel, the shoulder and lap seat belts, the steering column which collapses to absorb the impact energy, and the knee bolster to protect the knees of the driver. The performance of such a system is evaluated by a government agency such as the U.S. **National Highway Traffic Safety Administration, or NYTSA.** NYTSA has a **New Car Assessment Program (NCAP)** that tests new vehicle models as they come to the vehicle market, and publish the results as safety ratings. Car buyers can then use the ratings to help make purchase decisions.

These safety performance measures include the accelerations at the critical locations of the test dummy such as its head and chest, the forces or moments on joints such as dummy neck, pelvis, and knees, and deflection of the dummy ribs. NHTSA provides formulae to calculate the "star ratings," derived from real-life automotive accident injury data [2].

8.2.1 Phase 1: Identify Opportunity

It is important to design the restraint system for absorbing and dissipating the crash impact energy. For the frontal impact in which the vehicle is driven to collide with a solid barrier, the power and timing of the airbag inflator deployment, the geometry of the airbag such as its shape and size, the force and timing of the seatbelt tightening, the yield strength of the steering column under the crash load, all affect the dynamic movement of the test dummy. In addition, the results of the frontal-crash tests for the same vehicle model, if conducted with more than one car, can vary significantly. Since the test vehicles are randomly chosen, there are variations in their components within manufacturing specifications, as well as variation in the test setup within ranges allowed by the test procedure. For instance, the seated hip position of the test dummy can vary by more than one inch, the seat back angle by 1 to 2 degrees, the airbag inflator power by 10 percent, etc. If the restraint system is not designed to be robust, the vehicle crash test results and the field performance of the restraint system will not be consistent. It is therefore a challenge for the auto manufactures to produce vehicles with consistent safety performance, and here lies the opportunity for DFSS [3].

The objective of this project was to consistently achieve a five-star rating in the U.S. NCAP frontal-crash test, given variation in the components and in the dummy's posture.

8.2.2 Phase 2: Define Requirements

In this project, the objective was set by the vehicle-development team to achieve a five-star rating, based on a study of the competing vehicles' safety ratings. The star rating was already in a measurable form, its calculation published by NHTSA [4].

Traditionally, when designing a new vehicle, auto manufactures had to run crash tests of prototype vehicles, to assess the occupant-protection performance. The tests were destructive and very expensive, each costing as much as U.S.$500,000. In the last 20 years, however, the U.S. auto industry has been using **computer-aided-engineering (CAE)** models to simulate crash tests. These simulation models include the components of the restraint systems, as well as the test dummies of different sizes

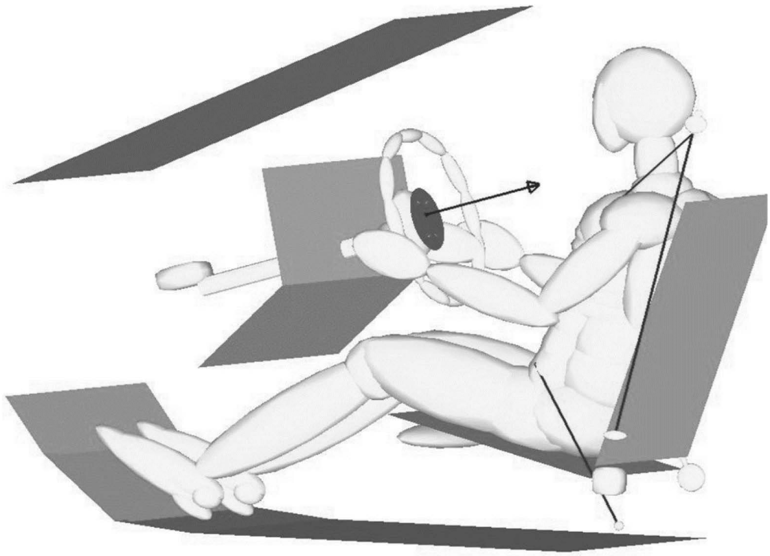

Figure 8.14 Typical CAE simulation of occupant-restraint system for frontal impact

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(Figure 8.14). The computer simulation helps evaluate and improve the occupant-safety performance measured by the "star rating" of NCAP as well as many other technical specifications [4][1]. Using the CAE simulation for the design of occupant-restraint system enables fast design evolution without building prototypes. The same CAE models can also simulate the variation of the random parameters, which is necessary for assessing the robustness of a design.

8.2.3 Phase 3: Develop Concepts

In this project, the most advanced design concept was already chosen, using the up-to-date technologies of the airbag and the seat belt systems. Therefore, the emphasis of the DFSS project was on the optimization of the chosen design concept.

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[1]This case study was carried out in 2001, and the reference to the vehicle safety standards in this section are all based on the U.S. NHSTA documentations of that time.

8.2.4 Phase 4: Optimize Design

There were a number of parameters in the occupant-restraint system, as shown in Table 8.2. Some of them could be changed by design, such as the airbag size and the airbag inflator output; those variables were control factors. Others might vary randomly in a test, such as the dummy's position and posture; those variables were noise factors. A control factor could also have a small random variation around its nominal value. In that case, the nominal value was a control factor and the random variation around the nominal was a noise factor.

The range of the control factors were determined by the constraints of packaging, availability from the suppliers, as well as other performance requirements. The range of the noise factors were determined by available data or the best estimates of their variation. These ranges are summarized in Table 8.3 and Table 8.4.

Table 8.2 Typical factors in frontal occupant-restraint system

Factor	Type
Airbag size and shape (diameter, volume, etc.)	Control
Airbag tether length (ties the outer surface of the air bag when deployed; affects the volume of the airbag)	Control
Airbag vent area (controls how fast the air bag is inflated when deployed)	Control
Airbag inflator output	Control and noise
Steering column stroke (controls how much the column can collapse)	Control and noise
Twist shaft level (affects steering column collapse load)	Control
Seat belt pretension spool (controls how tight the seat belt is when deployed)	Control
Seat belt pretension firing time (controls the timing of seat belt tightening)	Control
Knee bolster stiffness (affects how much impact energy can be absorbed by the knee bolster)	Control and noise
Vehicle crash pulses (time trajectory of the vehicle deceleration after a collision)	Noise
Airbag firing time (controls how fast the air bag is deployed after a collision)	Control
Dummy position and orientation (test variation allowed by the test procedure)	Noise
Frictions between dummy and seat belt and airbag (affect the dynamic motion of the dummy after deployment of the airbag and the seat belt)	Noise
Friction between seat belt and routing rings/buckles (affects the seat belt tightening motion)	Noise

Table 8.3 Design factors and their ranges

Factor	Range (Scaling Factor of the Baseline Design)
Airbag tether length	0.86~1.48
Airbag vent area	1.366~3.534
Twist shaft level	0.62~1.49
Knee bolster stiffness	1.0~1.5
Airbag inflation output	1.0~1.4
Pretension spool	2.0~4.4
Pretension firing time	0.5~1.0
Steering column strokes	0.62~1.88

Table 8.4 Noise factors and their ranges

Factor	Range
Airbag firing time (sec)	+/−0.003
Friction coefficient between airbag and dummy	+/−0.35
Friction coefficient between seat belt and buckle/ring	+/−0.2
Dummy position and posture	Per FMVSS test procedure [4]
Steering column build load (N)	+/−444

In this project, we needed to meet requirements for three major frontal-crash tests (named **load-cases**) described below, which shared the same design and noise factors:

1. The frontal NCAP test. This was conducted with the fiftieth percentile dummies in both the driver and the passenger seats, and with the seat belts on. The vehicle was driven to a rigid barrier at 35 miles per hour, or 56.3 km per hour. Performance metrics included the following:
   a. Head injury index, denoted by **HIC**
   b. Chest acceleration for 3 millisecond moving average, denoted by **Chest G**
   c. Star rating, which was a function of HIC and Chest G, with five-star the best and one-star the worst

2. Frontal impact at 25 miles per hour, or 40 km per hour, with unbelted fiftieth percentile dummies in both front seats
3. Frontal impact at 25 miles per hour, or 40 km per hour, with unbelted fifth percentile dummies in both front seats.

For load-cases 2 and 3 above, there were a number of requirements on head injury, chest acceleration, neck injury, and so on. Vehicles had to meet those requirements in order to be sold in the United States. For a complete list of the requirements, refer to [4].

To search for a robust solution to the problem, numerous computer simulations of the three load-cases would be needed. However, each simulation took hours, making it infeasible to complete the project in a timely manner. Conversely, if high-fidelity response surface models (RSMs) were developed from the original CAE models, then a number of techniques could be used to achieve the objectives in time for the vehicle application.

Therefore, the RSM approach was chosen. RSMs were derived as approximations of the original CAE models of the three load-cases. For each load-case, 150 DOE simulations were conducted, and the data was fit with third-order polynomial regression models for the key response variables such as HIC, Chest G, etc. Those polynomials were deemed accurate enough for design optimization. However, further verification would be needed with more accurate models or tests after the candidate solution was found.

Polynomial RSMs are very efficient to compute, and Monte Carlo simulations (MCS) can be carried out easily with them for a given design. We used the noise factors listed in Table 8.4 for the MCS of the RSMs, to assess the scatter in the response variables. The probability distribution of each noise factor was assumed to be normal, with the mean at the middle of its range and the standard deviation as one-sixth of its variation range. MCS estimated that the initial design had mostly four-star ratings when the variation of the noise factors was simulated.

To formulate a robust optimization problem, one can use $(\mu - 3\sigma)$ of the response as a measure of performance, where μ and σ represent the mean and the standard deviation. If $(\mu - 3\sigma)$ of the star rating was five, then the crash test outcome would be five star with a very high probability. This metric was chosen for its simplicity, and for the reason that the CAE simulation of occupant safety systems was not very accurate in predicting the real test results; therefore, higher sigma levels such as 6 sigma would not be meaningful.

The objective function for robust optimization then became

$$\underset{x}{Max}(\mu_y - 3\sigma_y) \tag{8.4}$$

where x is the set of design factors listed in Table 8.3 within their ranges, μ_y is the mean of the interpolated star rating, and σ_y is the standard deviation of the interpolated star rating.

The solution to Equation (8.4) must also be subject to other constraints derived from the requirements for load-cases 2 and 3. Note that those constraints also contained statistics estimated from MCS of their own RSMs.

Since the RSMs were nonlinear functions, we used global search algorithms for the optimization. In each search iteration, 5,000 MCS were performed to evaluate the mean and the standard deviation of the star rating and the other responds. This sample size of 5,000 for MCS provided sufficient accuracy in estimating the mean and the standard deviation of the responses. At the end of the optimization process, a new design was chosen. Figure 8.15 shows the clusters of the star rating of the initial and the final designs. The improvement was due to the increase in the mean and the decrease in the standard deviation of the star rating.

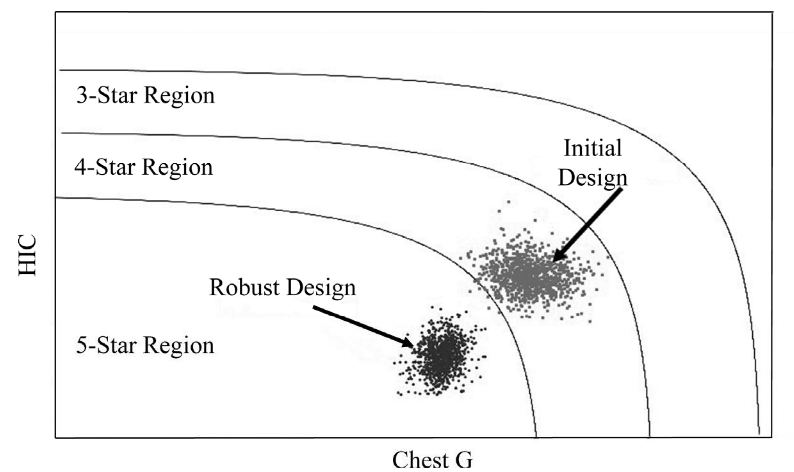

Figure 8.15 Comparison of the star-rating ranges

8.2.5 Phase 5: Verify Design

The new design was verified with the original CAE models for all the load-cases. Besides the nominal design, a number of scenarios of variation were also simulated, with the noise factors set at values that led to extremes in the response. The results of those simulations agreed with the predictions by the RSMs, and the new design was recommended to the design engineers. The final design based on those recommendations was tested when the vehicle was produced, with satisfactory results.

8.3 DFSS Project—Reliability Improvement of Car Window Regulator Operation

This project was an effort by a car manufacturer to reduce the warranty cost in repairing the cable-drive vehicle glass guidance system [5], shown in Figure 8.16. The mechanism was designed to pull the window glass up and down, powered either by an electric motor or by a hand operation.

8.3.1 Phase 1: Identify Opportunity

In defect cases, the power window glass would jam and the electric motor would break. Here, the customers were the vehicle occupants, who needed the windows to move up and down smoothly and fast. They certainly did not want the glass to be stuck at an open position when it was raining outside.

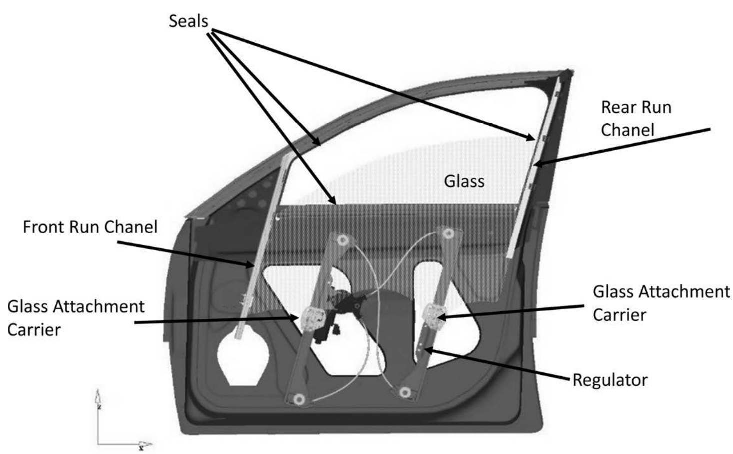

Figure 8.16 Schematic of a cable-drive glass guidance system

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The warranty data indicated that this problem was caused by variation, as only some vehicles had this issue. In other words, the design might be too sensitive to certain random factors in the system, making its operation nonrobust. DFSS was employed to solve the problem.

8.3.2 Phase 2: Define Requirements

The customer requirements relevant to this project are listed in the VOC column of Table 8.5.

The fifth VOC in Table 8.5 was based on the consideration for packaging space, since many components were located inside the doors of a vehicle.

The VOCs of the window regulator were translated to engineering and financial requirements, as shown in Table 8.5. The specific values for the requirements (not shown) were established based on the customers' feedback and the competitive studies of similar vehicles in the market. In this project, the focus was the reliability improvement of the system.

Table 8.5 VOC for a window regulator system

	VOC	Requirement
1	Windows can be opened/closed reliably	Probability of failure be less than a specific value.
2	Power window can be opened/closed fast enough	Glass travel be less than a specific period when being opened/closed fully.
3	Window seals do not leak water in car wash or in rain, as well as isolate air-borne noises	When closed, pressure between the window glass and the seals be greater than a specific value; this includes static condition (when the car is not moving) and the dynamic condition (when the car is moving at highway speed).
4	Energy-efficient	The power rating of the electric motor be less than a specific value. The weight of the regulator be less than a specific value.
5	Compact	The dimensions of the system be such that it can fit into available space in the door.
6	Low-cost	The piece price and installation cost be below a specific amount.

8.3.3 Phase 3: Develop Concepts

Two concepts of the window regulator were considered: the dual-cable design (Figures 8.16 and 8.17) and the cross-arm design (not shown).

In comparison, the dual-cable regulator weighed less, was more compact in dimension, and cost less. However, it was more sensitive to variation in the system. On balance, the dual-cable concept was chosen, but its operation reliability needed to be improved via DFSS.

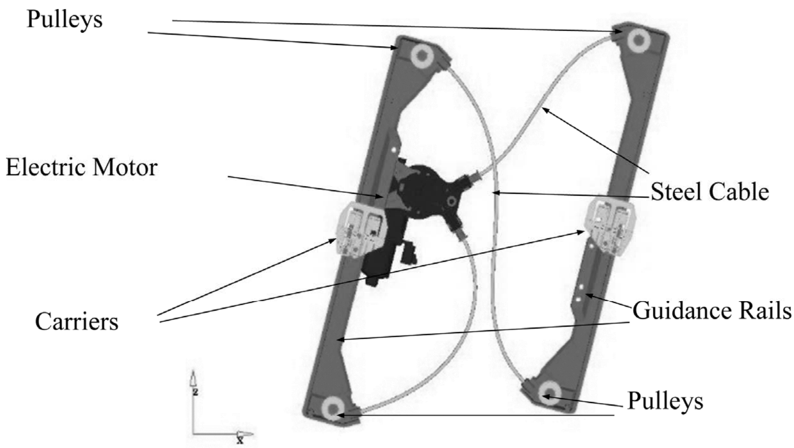

Figure 8.17 Schematic of a dual-cable regulator
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8.3.4 Phase 4: Optimize Design

Let's examine how the system works. As shown in Figures 8.16 and 8.17, the window glass was supported by two carriers. The carriers were driven by the electric motor through cables and pulleys, and moved along the run channels. Around the window opening, the seal strips were attached to the door metal panels, and pressed by the glass when in contact. While the electric motor provided the torque necessary for driving the mechanism, the amount of the torque might deviate from its designed value due to the build variation. On the other hand, a number of factors contributed to the resistance to the glass motion:

- Glass gravity.

- Friction between the glass and the seal strips around the edges of the window opening.
- Friction in the regulator including the pulleys and the cables.
- Friction between the glass carriers and the run channels.

The frictions had variation, depending on the stiffness, the geometry, and the surface coating of the seals, the deflection of the metal panel to which the seal strips were attached, the environment temperature, the compression of the seals by the glass edges, the pulley lubrication and alignment, the cable pretention, etc. Most of these factors had variation from the manufacturing process or the vehicle operating environment.

There was a probability for the required torque to overcome the resistance to exceed the torque output of the electric motor, causing the motor to slow down, stall, or break. In Figure 8.18, the area where the two probability density functions overlap corresponds to the shortage of the motor torque. A highly reliable system should have the two density distributions well separated.

Therefore, the challenge was to design the motor and its position so that it could always overcome the resistant torque in presence of the variation, driving the glass with sufficient speed, while meeting the requirements on weight, power rating, and packaging of the motor.

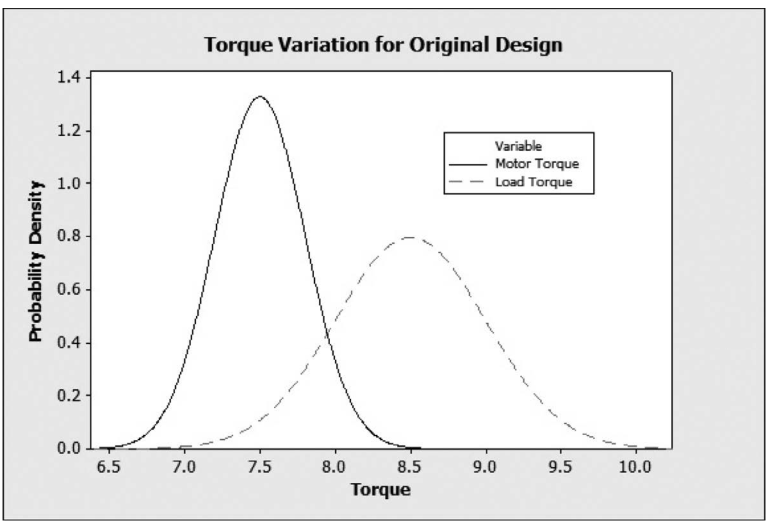

Figure 8.18 Torque distributions of original design

Traditionally, the reliability of the system was evaluated by testing, a practice both costly and time-consuming. With the help of CAE software, however, the system was simulated with finite element models, making it possible to assess and optimize the design so that it was robust against the random variation of the noise factors.

Step 1: Determine the response and the input parameters. The focus in this case was the probability for the required torque load τ_L to be less than the torque provided by the motor τ_m. The required torque load τ_L could be expressed as

$$\tau_L = f(F_1, F_2, p, \mu_r) \tag{8.5}$$

where p was the regulator cable pretention, μ_r was the friction coefficient of the regulator components, and F_1 and F_2 were the forces at the two carriers. F_1 and F_2 were expressed as

$$F_1 = f_1(\delta_1, \delta_2, \mu_s, k) \tag{8.6}$$

$$F_2 = f_2(\delta_1, \delta_2, \mu_s, k) \tag{8.7}$$

where δ_1 was the glass position in the cross-car direction, δ_2 was the regulator position in the forward–afterward direction, μ_s was the friction coefficient of the seal strips, and k was the seal stiffness.

Another important factor was the motor output torque τ_m, whose nominal value was a design choice. The motor torque also had a variation range around its nominal value.

In order for the window regulator to function properly, one must have

$$y = \tau_m - \tau_L > 0 \tag{8.8}$$

The condition expressed in Equation (8.8) should be satisfied even in presence of the variations we discussed earlier. Explicitly, we can rewrite the project objective function as

$$y_{mean} - 3\sigma_y > 0 \tag{8.9}$$

where y_{mean} is the mean of $\tau_m - \tau_L$, and σ_y is the standard variation of $\tau_m - \tau_L$; both were estimated with fast-running mathematical models.

Based on the physics of the mechanism, it was decided that the functions for τ_L, F_1, and F_2 could all be represented by second-order polynomials with sufficient accuracy in the design space of interest. The specific values of the coefficients in Equations (8.5) to (8.7) could be determined by regression, while the data needed for regression could be obtained either by conducting a DOE with hardware tests or computer simulations. The latter was chosen due to its feasibility.

Step 2: Build and validate two CAE models of the system, one for the quasi-static analysis and the other for the dynamic analysis. Those models contained all the parts contributing to the motion of the glass: the regulator, the seals, the glass, and the metal panels for attaching the seals. Together, the two models calculated the required torque to move the glass up and down at the required speed [5]. The models predicted the forces at the carriers F_1, F_2 and the torque load τ_L for a given system design. Those predictions were carefully validated with test measurements of the same system.

Step 3: Conduct DOE to derive RSMs. There were six input variables in Equations (8.5) to (8.8); each could change within a range, representing either the design choices or the random variation. Those ranges are summarized in Table 8.6.

Table 8.6 Input parameters for DOE

Factor	Type
Glass position δ_1	Noise
Regulator position δ_2	Control/noise
Seal friction coefficient μ_s	Control/noise
Seal stiffness k	Control/noise
Cable pretension p	Control/noise
Friction coefficient of regulator μ_r	Noise
Torque provided by motor τ_m	Control/noise

A DOE of 25 simulations was conducted with six factors using the CAE models, and the data was used to fit the regression models for F_1, F_2, and τ_L. As predicted, those second-order polynomial RSMs were good approximations of the original CAE models in the ranges considered.

Step 4: Estimate variation. To estimate the mean and the standard deviation of $y = \tau_m - \tau_L$, one can use a number of methods, including MCS and the first-order approximation as given in Equation (7.3) and Equation (7.4). The analysis of variation (ANOVA) indicated that the regulator friction had the highest impact on $y = \tau_m - \tau_L$, followed by the seal friction, the seal stiffness, the glass position, and the regulator position in descending order.

Step 5: Robust optimization. With the RSM for y, the best design was found so that $y_{mean} - 3\sigma_y > 0$ and other requirements were also met. The new design reduced the mean and the variation in the load torque resisting the glass motion. The resultant probability density function of the load torque is to the left of that of the motor torque, with little overlap. This meant that the probability for the motor to have sufficient power to drive the window was very close to 100 percent, Figure 8.19.

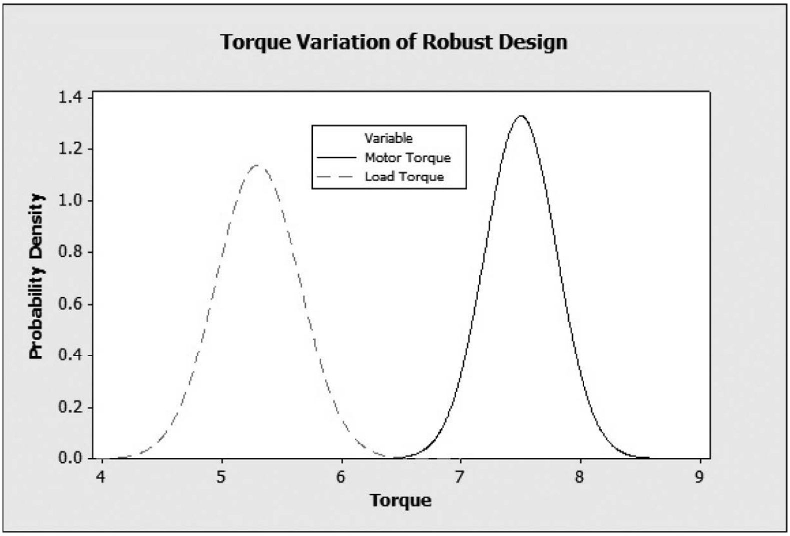

Figure 8.19 Torque distributions of robust design

8.3.5 *Phase 5: Verify Design*

The robust design solution was verified with the original CAE models, followed by hardware tests. Ultimately, it was confirmed by the reduction in the warranty cost after the new design was implemented in vehicles.

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APPENDIX A

Pugh Concept Selection Technique

The Pugh matrix was invented by Stuart Pugh for decision making, when multiple design concepts were being compared [1, 2]. The method is widely used today in engineering and nonengineering fields. Here we'll present the basics of the technique.

In the first column of the Pugh matrix of Table A.1, all the criteria for judging the design concepts are listed. These criteria can be functional requirements or business requirements we would like the product to meet. The functional requirements are usually related to the performance of the product, while the business requirements are related to the cost and productivity. In the top row, alternative design concepts of the product are listed, starting from the "**Datum**" or the "**Baseline Concept.**" The Datum may be the current design, or any other reference concept. Each alternative concept is compared with the Baseline on how it can meet each of the requirements, and scored accordingly as follows:

- If it performs as well as the Datum, the score is 0.
- If it performs better than the baseline, the score is a positive number, such as 1, 2, 3, depending on how much it outperforms the Baseline.
- If it performs worse than the Baseline, a negative score is entered, such as −1, −2, −3, depending on how severely it underperforms.

After scoring, the scores of each alternative concept in its column are summed up, to be the total positive, total negative, and the total neutral scores of that concept. Then we can compare different concepts and make a trade-off decision.

In the following example, five concepts are evaluated, with the first being the Baseline or reference. Alternative Concept 4 is the best, having the most positive score and the least negative score.

Table A.1 An example of Pugh Matrix

	Baseline Concept	Alternative Concept 1	Alternative Concept 2	Alternative Concept 3	Alternative Concept 4
Criteria 1	0	0	1	−1	1
Criteria 2	0	1	1	−2	−1
Criteria 3	0	0	−1	0	2
Criteria 4	0	−2	2	0	0
Criteria 5	0	−1	−1	−3	2
Criteria 6	0	3	2	2	1
Total Positives	0	4	6	2	6
Total Negatives	0	−3	−2	−6	−1
Total Neutrals	0	2	0	2	1

APPENDIX B

Frequently Used Orthogonal Arrays for DOE

Orthogonal arrays are commonly used in DOE by Dr. Taguchi for their simplicity and the ease of data analysis. Listed below are some frequently encountered orthogonal arrays, which can be generated with Minitab©.

1. $L_4(2^3)$ has four combinations of up to three factors, each with two levels (1, 2), as shown in Table B.1.

Table B.1 Orthogonal array L_4 (2^3)

	Factors		
Number	A	B	C
1	1	1	1
2	1	2	2
3	2	1	2
4	2	2	1

2. L_8 (2^7) has eight combinations of up to seven factors, each with two levels (1, 2), as in Table B.2.

Table B.2 Orthogonal array L_8 (2^7)

	Factors						
Number	A	B	C	D	E	F	G
1	1	1	1	1	1	1	1
2	1	1	1	2	2	2	2
3	1	2	2	1	1	2	2
4	1	2	2	2	2	1	1
5	2	1	2	1	2	1	2
6	2	1	2	2	1	2	1
7	2	2	1	1	2	2	1
8	2	2	1	2	1	1	2

3. L_{12} (2^{11}) has 12 combinations of up to 11 factors, each having two levels (1, 2), as shown in Table B.3.

Table B.3 Orthogonal array L_{12} (2^{11})

	Factors (Two-Level)										
Number	A	B	C	D	E	F	G	H	J	K	L
1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	2	2	2	2	2	2
3	1	1	2	2	2	1	1	1	2	2	2
4	1	2	1	2	2	1	2	2	1	1	2
5	1	2	2	1	2	2	1	2	1	2	1
6	1	2	2	2	1	2	2	1	2	1	1
7	2	1	2	2	1	1	2	2	1	2	1
8	2	1	2	1	2	2	2	1	1	1	2
9	2	1	1	2	2	2	1	2	2	1	1
10	2	2	2	1	1	1	1	2	2	1	2
11	2	2	1	2	1	2	1	1	1	2	2
12	2	2	1	1	2	1	2	1	2	2	1

4. L_{18} $(2^1 \times 3^7)$ has 18 combinations of up to 8 factors, one with 2 levels (1, 2) and 7 with 3 levels (1, 2, 3), as in Table B.4.

Table B.4 Orthogonal array L_{18} $(2^1 \times 3^7)$

	Factors							
Number	A	B	C	D	E	F	G	H
1	1	1	1	1	1	1	1	1
2	1	1	2	2	2	2	2	2
3	1	1	3	3	3	3	3	3
4	1	2	1	1	2	2	3	3
5	1	2	2	2	3	3	1	1
6	1	2	3	3	1	1	2	2
7	1	3	1	2	1	3	2	3
8	1	3	2	3	2	1	3	1
9	1	3	3	1	3	2	1	2

	Factors							
Number	A	B	C	D	E	F	G	H
10	2	1	1	3	3	2	2	1
11	2	1	2	1	1	3	3	2
12	2	1	3	2	2	1	1	3
13	2	2	1	2	3	1	3	2
14	2	2	2	3	1	2	1	3
15	2	2	3	1	2	3	2	1
16	2	3	1	3	2	3	1	2
17	2	3	2	1	3	1	2	3
18	2	3	3	2	1	2	3	1

5. L_{18} ($6^1 \times 3^6$) has 18 combinations of up to 7 factors, 1 with 6 levels (1, 2, 3, 4, 5, 6) and 6 with 3 levels (1, 2, 3), as in Table B.5.

Table B.5 Orthogonal array L_{18} ($6^1 \times 3^6$)

	Factors						
Number	A	B	C	D	E	F	G
1	1	1	1	1	1	1	1
2	1	2	2	2	2	2	2
3	1	3	3	3	3	3	3
4	2	1	1	2	2	3	3
5	2	2	2	3	3	1	1
6	2	3	3	1	1	2	2
7	3	1	2	1	3	2	3
8	3	2	3	2	1	3	1
9	3	3	1	3	2	1	2
10	4	1	3	3	2	2	1
11	4	2	1	1	3	3	2
12	4	3	2	2	1	1	3
13	5	1	2	3	1	3	2
14	5	2	3	1	2	1	3
15	5	3	1	2	3	2	1
16	6	1	3	2	3	1	2
17	6	2	1	3	1	2	3
18	6	3	2	1	2	3	1

6. L_{36} ($2^3 \times 3^{13}$) has 36 combinations of up to 16 factors, 3 with 2 levels (1, 2) and 13 with 3 levels (1, 2, 3), shown in Table B.6.

Table B.6 Orthogonal array L_{36} ($2^3 \times 3^{13}$)

	Factors																						
Number	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**J**	**K**	**L**	**M**	**N**	**O**	**P**	**Q**	**R**	**S**	**T**	**U**	**V**	**W**	**X**
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2	2	2	2
3	1	1	1	1	1	1	1	1	1	1	1	3	3	3	3	3	3	3	3	3	3	3	3
4	1	1	1	1	1	2	2	2	2	2	2	1	1	1	1	2	2	2	2	3	3	3	3
5	1	1	1	1	1	2	2	2	2	2	2	2	2	2	2	3	3	3	3	1	1	1	1
6	1	1	1	1	1	2	2	2	2	2	2	3	3	3	3	1	1	1	1	2	2	2	2
7	1	1	2	2	2	1	1	1	2	2	2	1	1	2	3	1	2	3	3	1	2	2	3
8	1	1	2	2	2	1	1	1	2	2	2	2	2	3	1	2	3	1	1	2	3	3	1
9	1	1	2	2	2	1	1	1	2	2	2	3	3	1	2	3	1	2	2	3	1	1	2
10	1	2	1	2	2	1	2	2	1	1	2	1	1	3	2	1	3	2	3	2	1	3	2
11	1	2	1	2	2	1	2	2	1	1	2	2	2	1	3	2	1	3	1	3	2	1	3
12	1	2	1	2	2	1	2	2	1	1	2	3	3	2	1	3	2	1	2	1	3	2	1
13	1	2	2	1	2	2	1	2	1	2	1	1	2	3	1	3	2	1	3	3	2	1	2
14	1	2	2	1	2	2	1	2	1	2	1	2	3	1	2	1	3	2	1	1	3	2	3
15	1	2	2	1	2	2	1	2	1	2	1	3	1	2	3	2	1	3	2	2	1	3	1
16	1	2	2	2	1	2	2	1	2	1	1	1	2	3	2	1	1	3	2	3	3	2	1
17	1	2	2	2	1	2	2	1	2	1	1	2	3	1	3	2	2	1	3	1	1	3	2
18	1	2	2	2	1	2	2	1	2	1	1	3	1	2	1	3	3	2	1	2	2	1	3
19	2	1	2	2	1	1	2	2	1	2	1	1	2	1	3	3	3	1	2	2	1	2	3

	Factors																						
Number	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**J**	**K**	**L**	**M**	**N**	**O**	**P**	**Q**	**R**	**S**	**T**	**U**	**V**	**W**	**X**
20	2	1	2	2	1	1	2	2	1	2	1	2	3	2	1	1	1	2	3	3	2	3	1
21	2	1	2	2	1	1	2	2	1	2	1	3	1	3	2	2	2	3	1	1	3	1	2
22	2	1	2	1	2	2	2	1	1	1	2	1	2	2	3	3	1	2	1	1	3	3	2
23	2	1	2	1	2	2	2	1	1	1	2	2	3	3	1	1	2	3	2	2	1	1	3
24	2	1	2	1	2	2	2	1	1	1	2	3	1	1	2	2	3	1	3	3	2	2	1
25	2	1	1	2	2	2	1	2	2	1	1	1	3	2	1	2	3	3	1	3	1	2	2
26	2	1	1	2	2	2	1	2	2	1	1	2	1	3	2	3	1	1	2	1	2	3	3
27	2	1	1	2	2	2	1	2	2	1	1	3	2	1	3	1	2	2	3	2	3	1	1
28	2	2	2	1	1	1	1	2	2	1	2	1	3	2	2	2	1	1	3	2	3	1	3
29	2	2	2	1	1	1	1	2	2	1	2	2	1	3	3	3	2	2	1	3	1	2	1
30	2	2	2	1	1	1	1	2	2	1	2	3	2	1	1	1	3	3	2	1	2	3	2
31	2	2	1	2	1	2	1	1	1	2	2	1	3	3	3	2	3	2	2	1	2	1	1
32	2	2	1	2	1	2	1	1	1	2	2	2	1	1	1	3	1	3	3	2	3	2	2
33	2	2	1	2	1	2	1	1	1	2	2	3	2	2	2	1	2	1	1	3	1	3	3
34	2	2	1	1	2	1	2	1	2	2	1	1	3	1	2	3	2	3	1	2	2	3	1
35	2	2	1	1	2	1	2	1	2	2	1	2	1	2	3	1	3	1	2	3	3	1	2
36	2	2	1	1	2	1	2	1	2	2	1	3	2	3	1	2	1	2	3	1	1	2	3

7. L_{36} ($2^{11} \times 3^{12}$) has 36 combinations of up to 23 factors, 11 with 2 levels (1, 2) and 12 with 3 levels (1, 2, 3), as in Table B.7.

Table B.7 Orthogonal array L_{36} ($2^{11} \times 3^{12}$)

	Factors															
Number	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**J**	**K**	**L**	**M**	**N**	**O**	**P**	**Q**
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	2	2	2	2	2	2	2	2	2	2	2	2
3	1	1	1	1	3	3	3	3	3	3	3	3	3	3	3	3
4	1	2	2	1	1	1	1	1	2	2	2	2	3	3	3	3
5	1	2	2	1	2	2	2	2	3	3	3	3	1	1	1	1
6	1	2	2	1	3	3	3	3	1	1	1	1	2	2	2	2
7	2	1	2	1	1	1	2	3	1	2	3	3	1	2	2	3
8	2	1	2	1	2	2	3	1	2	3	1	1	2	3	3	1
9	2	1	2	1	3	3	1	2	3	1	2	2	3	1	1	2
10	2	2	1	1	1	1	3	2	1	3	2	3	2	1	3	2
11	2	2	1	1	2	2	1	3	2	1	3	1	3	2	1	3
12	2	2	1	1	3	3	2	1	3	2	1	2	1	3	2	1
13	1	1	1	2	1	2	3	1	3	2	1	3	3	2	1	2
14	1	1	1	2	2	3	1	2	1	3	2	1	1	3	2	3
15	1	1	1	2	3	1	2	3	2	1	3	2	2	1	3	1
16	1	2	2	2	1	2	3	2	1	1	3	2	3	3	2	1
17	1	2	2	2	2	3	1	3	2	2	1	3	1	1	3	2
18	1	2	2	2	3	1	2	1	3	3	2	1	2	2	1	3
19	2	1	2	2	1	2	1	3	3	3	1	2	2	1	2	3
20	2	1	2	2	2	3	2	1	1	1	2	3	3	2	3	1
21	2	1	2	2	3	1	3	2	2	2	3	1	1	3	1	2
22	2	2	1	2	1	2	2	3	3	1	2	1	1	3	3	2
23	2	2	1	2	2	3	3	1	1	2	3	2	2	1	1	3
24	2	2	1	2	3	1	1	2	2	3	1	3	3	2	2	1
25	1	1	1	3	1	3	2	1	2	3	3	1	3	1	2	2
26	1	1	1	3	2	1	3	2	3	1	1	2	1	2	3	3
27	1	1	1	3	3	2	1	3	1	2	2	3	2	3	1	1
28	1	2	2	3	1	3	2	2	2	1	1	3	2	3	1	3
29	1	2	2	3	2	1	3	3	3	2	2	1	3	1	2	1
30	1	2	2	3	3	2	1	1	1	3	3	2	1	2	3	2
31	2	1	2	3	1	3	3	3	2	3	2	2	1	2	1	1
32	2	1	2	3	2	1	1	1	3	1	3	3	2	3	2	2
33	2	1	2	3	3	2	2	2	1	2	1	1	3	1	3	3
34	2	2	1	3	1	3	1	2	3	2	3	1	2	2	3	1
35	2	2	1	3	2	1	2	3	1	3	1	2	3	3	1	2
36	2	2	1	3	3	2	3	1	2	1	2	3	1	1	2	3

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Index

Lean Six Sigma and Statistical Tools for Engineers and Engineering Managers

Wei Zhan • Xuru Ding

The book focuses on the introduction of the basic concepts, processes, and tools used in Lean Six Sigma. A unique feature is the detailed discussion on Design for Six Sigma aided by computer modeling and simulation. The authors present several sample projects in which Lean Six Sigma and Design for Six Sigma were used to solve engineering problems or improve processes based on their own research and development experiences in engineering design and analysis.

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Dr. Wei Zhan is an associate professor and program coordinator for the Electronic Systems Engineering Technology program at Texas A&M University. He received his BS degree in electrical engineering from Beijing University of Aeronautics and Astronautics and his MS and DSc degrees in systems science and mathematics from Washington University in St. Louis. He has been teaching Lean Six Sigma to undergraduate engineering technology students at Texas A&M since 2008. Before joining Texas A&M, he worked in the automotive industry for eleven years as a system design engineer. He is an ASQ certified Six Sigma Black Belt.

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