Supply and Operations Management Collection

M. Johnny Rungtusanatham and Joy M. Field, *Editors*

Managing and Improving Quality

Integrating Quality, Statistical Methods and Process Control

Amar Sahay

BUSINESS EXPERT PRESS

Managing and Improving Quality

Managing and Improving Quality

Integrating Quality, Statistical Methods and Process Control

Amar Sahay

BEP BUSINESS EXPERT PRESS

Managing and Improving Quality: Integrating Quality, Statistical Methods and Process Control

First published in 2016 by
Business Expert Press, LLC
222 East 46th Street, New York, NY 10017
www.businessexpertpress.com

ISBN-13: 978-1-63157-341-5 (paperback)
ISBN-13: 978-1-63157-342-2 (e-book)

Business Expert Press Supply and Operations Management Collection

Collection ISSN: 2156-8189 (print)
Collection ISSN: 2156-8200 (electronic)

Cover and interior design by Exeter Premedia Services Private Ltd., Chennai, India

First edition: 2016

10 9 8 7 6 5 4 3 2 1

Printed in the United States of America.

Abstract

Quality is a discipline that focuses on product and service excellence. This book is about improving the quality of products and services. The improved quality and reliability lead to higher perceived value and increased market share for a company, thereby increasing revenue and profitability.

The book discusses the concepts and dimensions of quality, costs of poor quality, the importance of quality in this highly competitive global economy, and quality programs—Six Sigma and Lean Six Sigma that focus on improving quality in industries. The text integrates quality concepts, statistical methods, and one of the major tools of quality—Statistical Process Control (SPC)—a major part of Six Sigma control phase. A significant part of the book is devoted to process control and the tools of SPC—control charts—used for monitoring, controlling, and improving the processes by identifying the causes of process variation. The fundamentals of control charts, along with SPC techniques for variables and attributes, and process capability analysis and their computer applications are discussed in detail.

This book fills a gap in this area by showing the readers comprehensive and step-wise solutions to model and solve quality problems using computers.

Keywords

capability analysis, common cause of variation, control charts, control charts for attributes, control charts for variables, control limits, cost of quality, lean six sigma, pattern analysis, p-chart, process spread, process variation, rational subgroup, R-Chart, run charts, s-chart, Six Sigma, special causes of variation, specification limits, statistical process control, three-sigma limits, total quality management (TQM), $\bar{x}$ chart

Contents

Preface

This book provides an overview of the field of quality, the importance of quality in today's competitive global economy, and one of the major tools used to manage and improve quality of products and services—statistical process control (SPC). In this book, we explore quality programs used in industry today with a focus on SPC. We discuss one of the major quality and process improvement tools known as control charts. Computerized application, and implementation of various control charts. is one of the major focuses of this text. SPC is a part of overall quality program—Lean Six Sigma.

Quality is a discipline that focuses on product and service excellence. Both manufacturing and service companies have quality programs. The quality is closely related to the variation in both products and processes. Variation is an inherent part of products and processes that create these products and services. For example, no two parts produced by a production process are the same, and no machine can dispense exactly the same amount of beverage in two cans. This is because of the variation. You may recall that statistics is the tool that allows us to study variation. Most of the quality programs are data driven and almost all data show variation that can be studied using statistics. One of the major objectives of the quality programs is to reduce the variation in products and processes to the extent that the likelihood of producing a defect is virtually nonexistent. This means improving quality and meeting or exceeding customers' expectations.

There is a close relationship between quality, profitability, and market share. Quality is achieved through customers' perception; therefore, organizations must understand customer needs and expectations in order to meet and exceed them. Customer needs and expectations can be achieved through quality improvement. Quality is important to the consumers. In today's highly competitive and global economy, a company cannot survive and stay in business unless they are able to provide high quality products and services. Improving quality can help organizations increase their market share and profitability. The improved quality and reliability in products and services lead to higher perceived value and increased market share for a company. This leads to increased revenue and profitability.

This book provides a comprehensive coverage of quality. We first explore what is quality before discussing various statistical tools and methods that are used to monitor and improve the quality of products and services. We explain the term *quality* from the perspective of a manufacturer, a design engineer, a service provider, and the end user of a product or service—the customer.

Quality has been defined from several perspectives. For example, quality may have a different meaning to the engineer who designs the product, or to the manufacturer involved in the production of a product. Although we define quality from many different perspectives, the final judge of the product or service quality is the customer, and therefore, quality is the customer's perception of the degree to which the product or service meets his or her expectations.

The book begins with an introductory chapter where we explain quality, dimensions of quality, and its importance in this highly competitive global economy. This chapter also discusses the quality costs and costs of poor quality. Quality is important, because quality—both good and bad—costs money. There is a cost involved with improving the quality of products and services; because, poor quality can significantly affect an organization's competitiveness and market share. In his book *Quality is Free*, Phillip Crosby has described quality costs or the costs of quality as having two components: (1) costs of good quality (or the cost of conformance), and (2) costs of poor quality (or the cost of nonconformance). We will explore the costs of poor quality and its impact on the organizations in the first chapter.

To improve quality, it is important to have an understanding of systems and processes. A system converts inputs into useful products or services through a conversion process. The products or services are the output of the system. Quality is concerned with the variation in the output of a system or the products and the processes of the system. The quality of the products and services are improved by reducing the defects and variation. Quality is inversely proportional to the variation in the products and processes. This means that as the variation is reduced and controlled, significant improvement in quality can be achieved. The quality programs including Total Quality Management (TQM), Six Sigma, and Lean Six Sigma all focus on quality improvement by reducing variation and removing waste from the system. Almost all types of organizations and systems have two things in common: waste and variation. The quality programs focus on removing the waste and reducing variation and defects to improve quality.

While a complete coverage of the quality programs is beyond the scope of this book, we devote a chapter describing these programs. We also provide sufficient statistical background for the reader to understand the principles of quality in a separate chapter.

In the remaining chapters of the book, we turn our attention to SPC and control charts. These are graphical tools for monitoring the process, identifying the causes of process variation, and taking necessary actions to control and improve the process. Before going into the details of control charts and their applications, we explain the run chart, a tool used to describe the variation of the process output in the form of a time series plot. The run charts are an excellent way of understanding the variation and pattern of variation in the process.

The chapters on the run chart is followed by chapters on fundamentals of control charts, why and how control charts work, control charts for variables, computerized applications of control charts, additional SPC techniques for variables, control charts for attributes, and process capability analysis.

Who Can Benefit from This Book?

SPC is an integral part of quality improvement program in industries. This book provides an overview of one of the major tools used to manage and improve quality of products and services. The topics are dealt with in a concise and simple to understand format. Throughout the book, we emphasize the computer applications and implementation of quality programs in the real world. The book is unique in the sense that it shows the importance of SPC, and its place, in overall quality improvement programs. The reader is also provided with necessary statistical background to understand the subject matter and is introduced to quality programs used in industry today. Computerized application and implementation of various SPC tools is one of the major focuses of this text.

This book fills a gap in this area by showing the readers comprehensive and step-wise solutions to model process quality and solve quality problems. Where applicable, we provide data files, computer instructions, computer output, and interpretation of results.

This book is written with a wide audience in mind both managers and future professionals. Also, undergraduate and graduate data analysis and statistics students and MBAs, as well as audience in engineering taking a course in quality and process control will find the book to be useful.

Six Sigma professionals and those implementing Six Sigma in their companies will find the book to be very useful. The quality professionals and particularly those implementing Six Sigma quality in their companies will find the book to be a valuable resource.

Acknowledgments

I would like to thank the reviewers who took the time to provide excellent insights which helped shape this book.

I would especially like to thank Mr. Karun Mehta, a friend and engineer. His expertise and tireless efforts in helping to prepare this text is greatly appreciated.

I would like to thank Dr. Roger Lee, a senior professor and colleague for administering invaluable advice and suggestions.

Thanks to all of my students for their input in making this book possible. They have helped me pursue a dream filled with lifelong learning.

I am indebted to senior acquisitions editor, Scott Isenberg; director of production, Sheri Dean; marketing manager, Charlene Kronstedt; and all the reviewers and collection editors for their counsel and support during the preparation of this book. I acknowledge the help and support of Exeter Premedia Services–Chennai, India–team for reviewing and editing the manuscript.

I would like to thank my parents who always emphasized the importance of what education brings to the world. Lastly, I would like to express a special appreciation to my wife Nilima, to my daughter Neha and her husband David, my daughter Smita, and my son Rajeev for their love and support.

CHAPTER 1

Introduction to Quality

Quality as a Field

This chapter provides an overview of the field of quality, the importance of quality in today's competitive global economy, and statistical process control. The chapter explores these topics and shows how various statistical tools can be used in improving the quality of the products and services.

Quality is a discipline that focuses on product and service excellence. Both manufacturing and service companies have quality programs. Quality is closely related to the variation in both products and processes, and statistics is the tool that allows us to study variation. Most of the quality programs are data driven and almost all data show variation that can be studied using statistics. One of the major objectives of the quality programs is to reduce the variation in the product and process to the extent that the likelihood of producing a defect is virtually nonexistent. This means improving quality and meeting or exceeding customer's expectations. The improved quality and reliability in products and services lead to higher perceived value and increased market share, thereby, increasing revenue and profitability.

Before discussing various statistical tools and methods that are used to monitor and improve the quality of products and services, we explain the term *quality* and outline many different ways quality has been defined. Some of the definitions of quality are presented here.

Quality Defined

Quality means different things to different people. Therefore, quality can be defined from several different perspectives. From the perspective of the customer or the end user of the product or service, the quality of a product or service is the customer's perception of the degree to which the product or service meets his or her expectations. This also means that the quality of a product or service can be determined by the extent to which the product or service satisfies the needs and requirements of the customers. This definition is a customer-driven quality approach that aims at meeting or exceeding customer expectations.

Quality has also been defined from several other perspectives. For example, quality may have a different meaning to the engineer who designs the product, or to the manufacturer involved in the production of a product. Thus quality can be defined from the perspective of the manufacturer or the designer. Quality has a transcendental definition and can also be product based, user based, manufacturing based, and value based (Garvin). Following are some of the other ways quality has been defined:

- *Transcendent*: Quality is something that is intuitively understood but nearly impossible to communicate, such as beauty or love.
- *Product-based*: Quality is found in the components and attributes of a product.
- *User-based*: If the product or service meets or exceeds customer's expectations, it has good quality.
- *Manufacturing-based*: If the product conforms to design specifications, it has good quality.
- *Value-based*: If the product is perceived as providing good value for the price, it has good quality.

Quality has also been defined as:

- Meeting or exceeding customer expectation
- Fitness for intended use
- Conformance to specifications
- Inversely proportional to variation
- Total customer service and satisfaction
- The degree or standard of excellence of something

These definitions of quality show that although we can define quality from many different perspectives, the final judge of the product or service quality is the customer, and therefore, quality is the customer's perception of the degree to which the product or service meets his or her expectations.

Dimensions of Quality

The dimensions of quality specify the characteristics the product or service should possess in order to be high quality. Garvin has identified eight dimensions of quality described here. These dimensions describe the product quality that is critical to developing high quality products or services. The recognition of these dimensions by the management and the selection of these dimensions along which the business will compete is critical to business success.

1. *Performance*: Will the product do the job?
2. *Features or added features*: Does it have features beyond the basic performance characteristics?
3. *Reliability*: Is it reliable? Will it last a long time?
4. *Conformance*: Does the product conform to the specifications? Is the product made exactly as the design specified?
5. *Serviceability*: Can it be fixed easily and cost effectively?
6. *Durability*: Can the product tolerate stress without failure?
7. *Aesthetics*: Does it have sensory characteristics such as taste, feel, sound, look, and smell?
8. *Perceived quality*: What is the customer's opinion about the product or service? How customers perceive the quality of the product or service?

Importance of Quality

There is a close relationship between quality, profitability, and market share. Quality is achieved through customer's perception, therefore, organizations must understand customer needs and expectations to meet

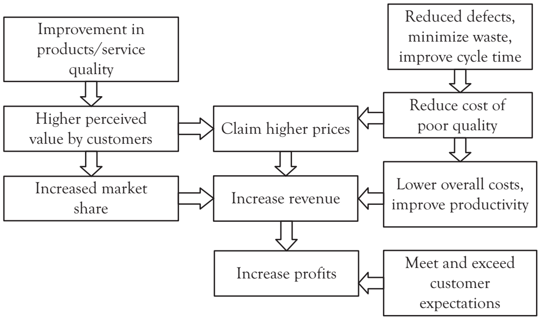


Figure 1.1 Quality, profitability, and market share

and exceed them. Customer needs and expectations can be achieved through quality improvement. Quality is important to the consumers. In today's highly competitive and global economy, a company cannot survive and stay in business unless they are able to provide high quality products and services. Figure 1.1 shows how improving quality can help organizations increase their market share and increase profitability.

Costs of Quality and Costs of Poor Quality

Quality is also important because the quality—both good and bad—costs money. There is a cost involved with improving the quality of products and services, because poor quality can significantly affect an organization's competitiveness and market share. In his book *Quality is Free*, Phillip Crosby has described quality costs or the costs of quality (COQ) as having two components: (1) costs of good quality (or the cost of conformance) and (2) costs of poor quality (or the cost of nonconformance). These are shown in Figure 1.2.

The focus of many quality programs is to reduce the cost of poor quality. Since the cost of poor quality is significant, reducing this cost will lead to increased revenue and improved productivity. A quality program should be focused on preventing poor quality. A *prevention system* is focused on preventing the poor quality and is far superior to a *detection system* that detects the defects and nonconformities in the products after they are produced.

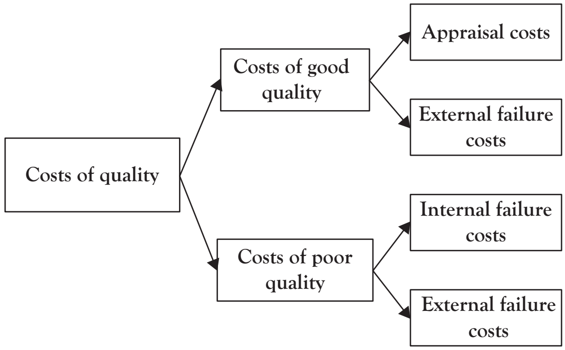


Figure 1.2 Costs of quality

The major components of costs of good quality—prevention costs and appraisal costs, and the costs of poor quality—internal failure and external failure costs, are explained in Table 1.1.

Detection Versus Prevention Quality Systems

Figure 1.3 shows the quality costs under detection and prevention systems (Griffith 2000). The costs under the detection system are similar to the costs that are measured for the first time in a company that has no formal quality prevention system in place. In the detection system, the costs of internal failure (e.g., scrap, rework, repair, and retest) are almost equal to the appraisal costs (e.g., inspection, testing, and auditing). The internal failure and appraisal costs tend to increase simultaneously. Since no or little prevention efforts are in place, more inspection is performed that finds more defects. On the other hand, as more defects are produced, more inspection is required. In a detection system, the external failure costs are small because of high inspection. The prevention costs are also small in a detection system.

A prevention quality system focuses on preventing failures and defects. Several companies have reported significant reduction in cost of poor quality through Six Sigma quality, which is a prevention quality program.

Table 1.1 Quality costs

Prevention cost Attempts to prevent poor quality from being produced. These costs include: • Quality planning and engineering • Product and process design • Process control • New product review • Manufacturing engineering tasks • Quality Training • Vendor relations • Variability analyses • Design reviews and manufacturing planning • Designing equipment and processes to measure and control quality	**Appraisal costs** Related to functions that appraise or evaluate. These are the cost of: • Inspection and testing of incoming material • Inspection and testing of products • Staffing inspectors and supervisors • Maintaining the accuracy of test equipment • Maintaining test or inspection records • Performing audits and field tests
Internal failure cost Related to failure or nonconformance that occurs in-house. These costs include the cost of: • Scrap • Rework • Retest • Loss in profit due to substandard product • Repairs • Failure analysis • Downtime	**External failure cost** Related to failures or nonconformance in the customer's facility. These costs include: • Returned products or material that must be inspected, reworked, or scrapped • Customer complains • Cost of testing, legal services, settlements • Other costs related to product liability • Customer dissatisfaction (not directly measurable)

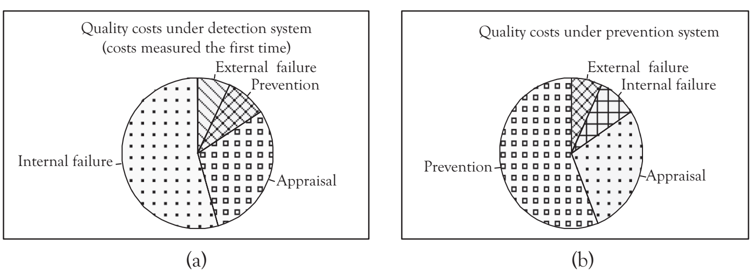


Figure 1.3 Quality costs: Detection system versus prevention system (a) Quality costs in a detection system, (b) Quality costs in a prevention system

Systems and Processes

The quality methods and tools are applied to the systems and the processes that make the systems. The system and the processes within the system are responsible for creating the products or services. Therefore, it is important to understand the systems and the processes.

Systems

A system usually consists of a group of interacting, interrelated, or interdependent processes forming a complex whole. Thus a system is a collection of processes with a specific mission or purpose. Figure 1.4 shows the model of a basic system. A process can be viewed as a part of a *system*.

Some examples of systems are electronic manufacturing or food processing companies which produce electronic or food products. Such systems are usually a collection of interacting or interrelated processes; for example, both the electronic manufacturing and food processing plants may consist of a number of departments including manufacturing engineering, marketing, design engineering, sales, transportation, warehousing, finance and accounting, and distribution systems.

All these departments can be viewed as processes. In manufacturing or food processing companies, the raw materials are converted into useful products, which are outputs. Such systems as shown in Figure 1.4 have a feedback through which the companies receive information about their products from the customers and market. This information is helpful in changing or modifying their processes and products to adapt to the needs and requirements of their customers.

The other types of systems are the service systems. These systems exist to provide various types of services to their customers. Examples of such systems are education institutions, government organizations, technical call centers, health care organizations, hospitals, and insurance companies. These systems also consist of a number of processes and provide services

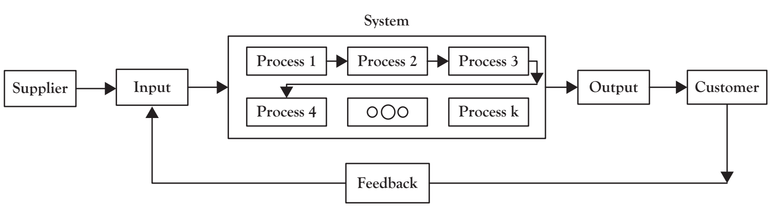


Figure 1.4 A basic system

through the collection of processes. The outputs of the service systems are usually intangible.

Processes

In many cases, the focus of statistical analysis has been to draw conclusions or make decisions about the population using the sample data. The other aspect of statistical analysis is to study and reduce the variation in the products or processes studied using data. Statistics and statistical methods enable us to study variation in the processes. Almost all data show variation and controlling or minimizing variation in products and processes lead to improved product quality. The variance in a process is an important measure of the quality of the products and processes. A large variation in any product, process, or both is not desirable and is an indication that the process be improved by finding ways to reduce the process variance. As variation in the products and processes is reduced, the product or the process becomes more consistent. Therefore, one of the major objectives of quality programs like Six Sigma is to reduce variation in product, process, or service.

In this text, we will study how the variations in the processes affect the product quality. To study this, we will explore the relationship between the variation and product quality, and the statistical tools that are used to study, monitor, and control the variation. This area comes under statistical process control.

Since the quality of products and services is related to the variation in products and the processes that create the products, we will first define and study the processes. A process can be a chemical process, or a manufacturing process. The processes in general use the inputs that go through a transformation to produce outputs or useful products.

Any organization, or any of its parts, can be viewed as a process. *A process is a transformation of inputs into outputs*. Some examples of processes include electronic and appliance manufacturing processes, computer and car assembly lines, and chemical processing plants. A process in its simplest form is shown in Figure 1.5.

A process usually consists of a sequence or network of activities that depicts the flow of the complete procedure required to transform the

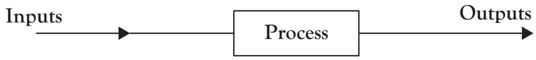


Figure 1.5 A process in its simplest form

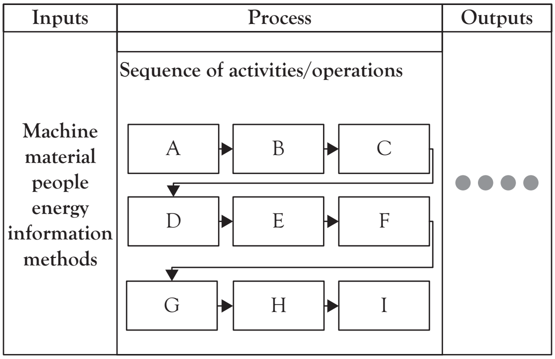


Figure 1.6 An input-output process

inputs into outputs (useful product or service). The transformation is achieved by flows through network of activities that are performed by various resources. Figure 1.6 shows an input output process.

Outputs of Processes and Variation

The processes, as discussed earlier, take inputs and convert them into outputs using some type of transformation process. Systems, on the other hand, may consist of a number of processes. It is important to note the following characteristics of the system outputs and the outputs produced by the processes of the system:

1. The outputs of the process always vary.
2. The products produced by the same processes are different. This means that no two products are identical and the measured quality characteristic of products vary. For example, the volumes of two beverage cans labeled 16 oz. are not exactly the same; the two tires that are 13.0 inches in radius are not both exactly the same radius. Figure 1.7 shows the measurements of the diameters of a 13.0-inch radius tires that are manufactured by the same process. The radius in this case is a critical quality characteristic of the product or the

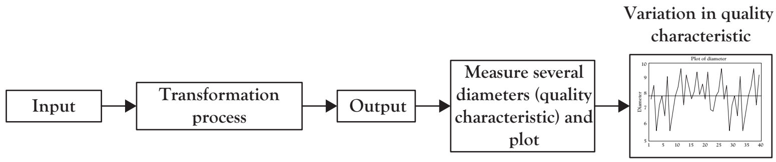


Figure 1.7 Variation in quality characteristic (diameters of tires)

output. Notice how the measured radius varies from product to product (the last block in Figure 1.7 that shows the measured values of several products). Similarly, the computers and calculators made by the same processes are not exactly the same. Although the products look alike, they always vary in critical quality characteristics. The variation in many cases is not noticeable. The variations in product characteristic do not affect the functionality as long as the variation is within a certain limit.

3. Variation is an inherent characteristic in products and processes.

As long as the variation in products and processes that produce these products is within certain limit, the product is acceptable. When the variation increases beyond the desired or set limits, the product quality, functionality, and reliability are affected. This is the reason why the variation in the products and processes must be monitored and controlled. The variation in the products and processes can be studied using statistical tools.

Sources of Variation in Products and Processes

The major sources of variation in the products and processes are attributed to the following factors:

1. Materials
2. Men (Operator)
3. Machines
4. Methods
5. Measurement
6. Environment

These sources of variation are shown in Figure 1.8(a) and (b) where (a) shows the general categories that are common sources of variation

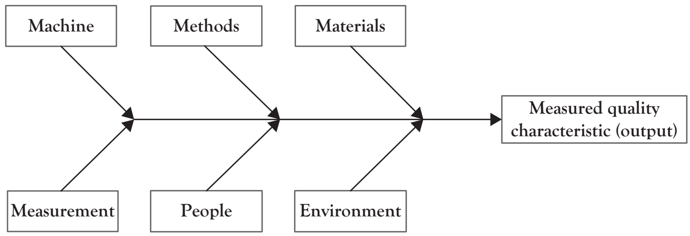


Figure 1.8(a) Sources of variation in products and processes

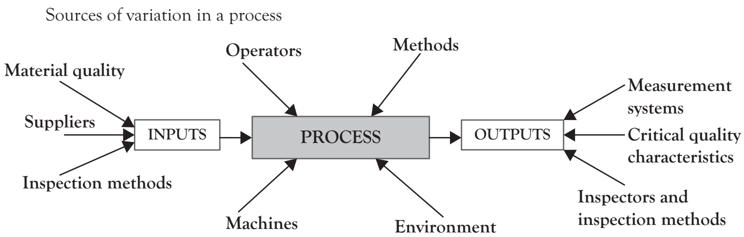


Figure 1.8(b) Sources of variation in products and processes

and (b) shows the details. Note that all categories may not apply to all products or services.

In the rest of the chapter, we will see how the variation in the products and processes are studied, measured, and controlled.

Measuring Variation

The variation in the data is measured using the variance and standard deviation. The Greek letter σ^2 (read as sigma-squared) represents the variance of a population data and σ represents the standard deviation. The corresponding symbols for the variance and standard deviation of a sample data are s^2 and s. The standard deviation σ is a measure of spread or deviation around the mean as shown in Figure 1.8(c). We may have two or more sets of data all having the same average, but their spread or variability may be different. This is shown in Figure 1.8(d). It can be seen from this figure that the data sets A and B have the same mean but different variations—curve B has less spread or variability than curve A. The more variation the data has, the more spread out the curve will be. We may also have a case where two sets of data have the same variation but different mean.

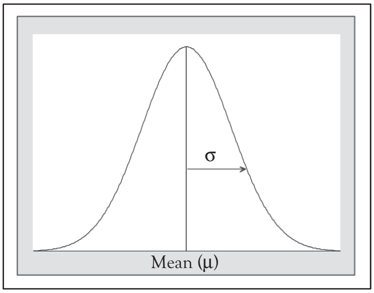


Figure 1.8(c) The measure of variation—standard deviation σ

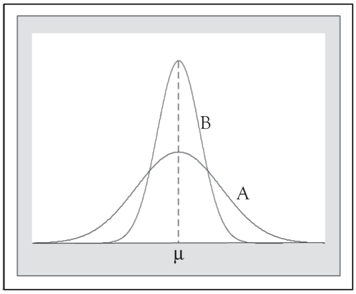


Figure 1.8(d) Data sets A and B with same mean but different variation

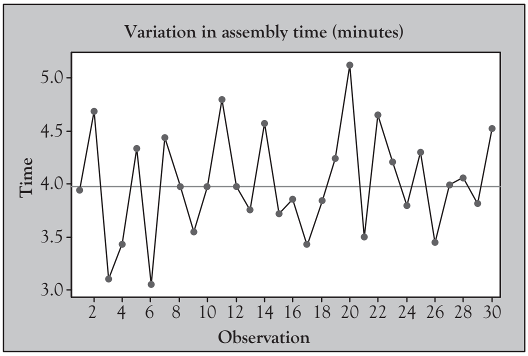


Figure 1.8(e) Plot showing the variation in assembly time

The variation in the data can also be plotted using a graph. Suppose that the average time to assemble a product is 4.0 minutes. The average assembly time for all the products is not going to be exactly 4.0 minutes. This means that the assembly time will vary from product to product.

The variation in the assembly time can be studied using a graph that is shown in Figure 1.8(e). This graph shows the assembly time of a sample of 30 products. Note how the assembly time varies around the average of 4.0 minutes.

Summary

This chapter provided an overview of the field of quality, various ways quality has been defined, and the importance of quality in today's competitive global economy. Quality is a discipline that focuses on product and service excellence. Both manufacturing and service companies have quality programs. Quality is closely related to the variation in both products and processes, and statistics is the tool that allows us to study variation. Most of the quality programs are data driven and almost all data show variation. One of the major objectives of the quality programs is to reduce the variation in the product and process to the extent that the likelihood of producing a defect is virtually nonexistent. This means improving quality and meeting or exceeding customer's expectations. The chapter described the dimensions of product quality. These are the characteristics that the product and service should possess in order to be of high quality. The COQ were also discussed. Cost of poor quality is a significant percent of the sales dollars in companies. Reducing these costs leads to improved quality, higher perceived value by the customer, and increased market share. The chapter emphasized on the importance of prevention quality programs like Six Sigma and Lean Six Sigma quality programs. These programs have been applied with tremendous success in a large number of companies. The tools of quality are applied to systems and processes within the systems. These systems and processes are responsible for creating goods and services. The chapter provided an overview of the systems and processes. Finally, the sources of variation in products and processes were discussed and it was shown that there is always a variation when we measure the critical quality dimensions. No two products are exactly the same. There is always some degree of variation in them. Quality is all about studying, reducing, and controlling the variations to improve product or service quality.

CHAPTER 2

Quality Programs in Use Today: Lean Six Sigma and Total Quality Management

Introduction—A Brief History of Quality

The quest for quality traces its roots back to medieval Europe in the 13th century when the craftsmen started organizing into unions called guilds. These guilds developed rules for producing products of high quality. The concept of inspection was introduced and inspection committees were formed to produce flawless products. The craftsmen model of guild existed until the early 19th century across medieval Europe. The craftsmanship model was replaced by the factory system that originated in Great Britain. The factory system was the product of the Industrial Revolution. Since then quality has evolved in different forms. Table 2.1 provides a brief historical perspective (http://asq.org/learn-about-quality/history-of-quality/overview/overview.html). It outlines the major developments in the field of quality and its current state.

Total Quality Management

Total quality management (TQM) approach to quality is a management approach to long-term success through customer satisfaction. In a TQM effort, all members of an organization participate in improving processes, products, services, and the culture in which they work. The methods for implementing this approach come from the teachings of quality leaders such as Philip B. Crosby, W. Edwards Deming, Armand V. Feigenbaum, Kaoru Ishikawa, and Joseph M. Juran (http://asq.org/learn-about-quality/total-quality-management/overview/overview.html). TQM has been a popular quality program and was widely adapted before the emergence

Table 2.1 A brief history of quality

Middle ages	Skilled craftsmanship
Industrial revolution	Introduction of inspection methods and separate quality departments
Early 20th century	Statistical methods at Bell System
1900 to 1920s	Taylor and scientific management Ford assembly line Shewhart and statistical process control
World War II	Quality control Efficient use of limited resources Quality control methods
1950s	Quality emphasis and education—Quality gurus Deming, Juran, and Feigenbaum
Postwar Japan	Evolution of total quality management
1960s to 1980s	Japanese quality revolution The Toyota production systems
1980s to 1990s	Quality awareness in the United States. Manufacturing industry during 1980s: from *Little Q* to *Big Q*—total quality management Just-in-time and lean manufacturing Total quality management (TQM) *The U.S. response to Japanese quality revolution, applications of statistical methods and managerial approaches for organization wide quality continued....* Business process reengineering (BPR) Emergence of quality management in service industries, government, health care, and education Birth of Six Sigma Malcolm baldrige national quality award.
Current	Six Sigma becomes the part of the corporate business plan that is key to achieving business objectives and quality excellence with top leadership support and involvement Six Sigma must address the *voice of the customer* (VOC) and critical to quality (CTQ) characteristics Lean Six Sigma—removal of waste and defects from processes Design for Six Sigma (DFSS)—incorporate quality early in the design phase Six Sigma and Big data

of Six Sigma and Lean Sigma quality programs. In Table 2.2, we provide a comparison between Six Sigma and TQM. As can be seen, the TQM lacks some of the critical components and has some drawbacks compared to Six Sigma.

Six Sigma provides a very structured way of solving problem. Almost all problems in manufacturing, service, and other areas can be defined in terms of DMAIC (Define, Measure, Analyze, Improve, and Control). Because of the systematic nature of problem solving approach, well-defined statistical tools, and verifiable return on investment, Six Sigma approach to quality has become very popular.

Table 2.2 Six Sigma and TQM

Six Sigma	TQM
Six Sigma is a team based approach owned by business leaders, champions, and other stake holders	Total employee involvement through worker empowerment and team work
Six Sigma efforts are focused on building strong competitive advantage driven by customer wants and needs	Process-centered: Focus on process thinking (a process is a series of activities that converts inputs into outputs). In TQM, process steps are defined, performance measures are monitored to detect variations
In Six Sigma, VOC and CTQ characteristics must be addressed	Customer focus but the tools for meeting customer requirements are not rigorous or well defined
Six Sigma uses simple to advanced statistical tools that are outlined in each phase of Six Sigma project	Simple improvement tools are applied
Six Sigma is based on a strategic improvement methodology known as DMAIC, which stands for Define, Measure, Analyze, Improve, and Control	Tools of improvements not standardized
The benefits or cost savings from Six Sigma can be quantified rather quickly	Little or no financial accountability
Cost savings is one of the major requirements that must be addressed in the project charter. Six Sigma projects require verifiable return on investment	No method for quantifying cost savings

Six Sigma, Lean Sigma, and Design for Six Sigma

Six Sigma, Lean Sigma, and DFSS are the most sought after quality programs in industry today. More and more companies are realizing that it is possible to achieve dramatic improvements in cost, quality, and time by using the Lean Six Sigma and DFSS. Several companies including Toyota, General Electric, Motorola, and others have accomplished impressive results using one or more of the techniques mentioned. Six Sigma and DFSS are a customer-driven quality approach that aims at meeting or exceeding customer expectations.

Six Sigma employs a well-structured continuous methodology to reduce process variation and decrease defects within the business processes using simple to advanced statistical tools and techniques. It improves quality through defect removal and process optimization. The improved quality leads to higher perceived value for the products or services the company offers that help companies achieve increased market share. Companies have reported significant savings by reducing the costs of poor quality.

The term *sigma* (denoted by the Greek letter, σ) is a metric based on the statistical measure called *standard deviation* and is a measure of variability. In Six Sigma, the *metric* σ is a measurement of certain quality characteristics; for example, percent defect. The term *sigma* also refers to the population that falls within plus or minus six standard deviations of the mean. Statistically, Six Sigma equates to 3.4 defects per million. Thus a Six Sigma process is capable of producing 3.4 defects per million opportunities (DPMO). In practice, this refers to the maximum acceptable range of noncompliance.

The other approach to achieving excellence in products and services is based on the removal of waste from service and manufacturing processes is '*Lean*' approach commonly known as *Lean Sigma*. Many companies have reported significant improvement through the removal of waste or nonvalue added activities. Companies have also reported that bringing the two concepts—Lean and Six Sigma—together deliver faster results. While the objective of Lean is to create flow and eliminate waste, Six Sigma improves process capability and reduces defects and variation that leads to improved quality and cost savings for the companies. If a company just applies Six Sigma, it cannot maximize the potential of the organization. A combined approach—*Lean Six Sigma* is currently very popular and is a widely used quality program.

Any process has the following things in common: Variation, Waste, and Delay. The removal of these will make the process much more efficient. Controlling variation makes the process consistent and defect free thereby improving quality. Removing waste and delay from the process will improve flow and reduce the cycle time. Six Sigma is used to reduce defects and variation from the products and processes. Lean Sigma is an approach to reducing or eliminating the waste, improving the flow, and reducing the cycle time. A combined approach—*Lean Six Sigma*—is needed to reduce variation, waste, and delay in the processes.

The term 'lean' has its root in 'Just-in-time Manufacturing' or 'Lean Manufacturing'; a philosophy of production that emphasizes on the minimization of the amount of all the resources (including time) used in the various activities of the enterprise. It involves:

- identifying and eliminating nonvalue-adding activities,
- employing teams of multiskilled workers, and
- using highly flexible, automated machines.

The other quality program gaining popularity now is DFSS. The goal of DFSS is to address and incorporate quality issues early in the design or redesign process using robust design methodologies. It integrates engineering design and statistical methods to predict and improve quality before production. DFSS is a way of understanding the key product characteristics to design and build successful products. The success of companies depend on designing, developing and launching new products of superior quality, getting to the market quickly (reduced cycle time), bringing innovation in products, and understanding the customer's needs and requirements. Research shows that approximately 5 percent of all new-product ideas survive to production, and only about 10 percent of these are successful. Therefore, actively building quality in every phase of the product development process, and predicting and optimizing critical quality characteristics are keys to ensuring product success. DFSS is a systematic method to build quality and key customer requirements in all stages of product development. These key or critical quality characteristics (CTQs) and customer requirements can be measured, verified, and optimized. DFSS is an approach to meet or exceed customer needs, requirements, and expectations using the VOC. This research focuses on one of the major tools—Quality Function Deployment and House of Quality (HOQ)—used in DFSS and also in Six Sigma to design products and services to meet and exceed customer requirements by identifying and addressing CTQs and VOC early in the design phase.

To achieve Six Sigma quality level, the companies must determine where the Lean, Six Sigma, and DFSS activities occur in the life cycle of the product. In other words, the companies must determine when to apply the Lean Six Sigma or DFSS approach.

There is a need for an integrated approach to achieve the overall objectives. It is also important for the companies to identify and initiate appropriate projects based on Six Sigma, Lean, or DFSS depending on the objectives and priorities. Sometimes a combination of these methodologies (Lean, Six Sigma, and DFSS) is needed as an integrated approach to achieve the overall objectives of improving quality, reducing defect and becoming a Six Sigma company, reducing cost, eliminating waste, providing speed and reliability of delivery, incorporating flexibility and innovation in products and services, and meeting or exceeding customers' expectations.

Six Sigma

Six Sigma is a business strategy that employs well-structured continuous improvement methodology and statistical tools to reduce defects and process variability. It is a quality discipline that focuses on product and service excellence. The objective of a Six Sigma program is to reduce the variation in the product and process to the extent that the likelihood of producing a defect is virtually nonexistent. This means improving quality, and meeting or exceeding customers' expectations.

Six Sigma seeks to find and eliminate causes of defects and errors in manufacturing and service processes by focusing on outputs that are critical to customers and a clear financial return for the organization. Six Sigma can be viewed as:

- A customer-focused approach to create near perfect processes, products, and services all aligned to delivering what the customer wants.
- A project based approach where majority of projects are selected for measurable bottom line or customer impact.
- A methodology that uses well-defined set of statistical tools and process improvement techniques by well-trained people in an organization.
- A business strategy that has evolved from a focus on process improvement using statistical tools to a comprehensive framework for managing a business.

Six Sigma is about improving quality by reducing variation. One of the major objectives of Six Sigma is to reduce variation in products and processes. According to Robert W. Galvin, Chairman Emeritus of Motorola:

> We quickly learned that if we could control variation, we could get all the parts and process to work and get to an end result of 3.4 defects per million opportunities, or a Six Sigma level. Our people coined the term and it stuck. It was shorthand for people to understand that if you can control the variation, you can achieve remarkable results.

A commonly accepted customer-driven definition of quality is that *the quality of a product or service is a customer's perception of the degree to which the product or service meets his or her expectations*. Six Sigma is a customer-driven quality approach that aims at meeting or exceeding customer expectations. The underlying principles of Six Sigma are *customer focus, a project and team-based approach, and a process-focused approach based on continuous improvement.* Successful implementation of Six Sigma requires creating a culture that demands excellence and perfection in everything companies do. For more information, see the list of bibliography.

Business Success of Six Sigma

A number of organizations—including banks and hospitals—have successfully implemented Six Sigma program within their corporate structure. Among the companies who have reported significant success with Six Sigma are GE, Texas Instruments, Honeywell, Boeing, IBM, Caterpillar, 3M, Xerox, Raytheon, Citibank, Home Depot, and the U.S. Air Force. The list goes on. Six Sigma has been successfully applied to many service industries including health care and financial services. The savings resulting from Six Sigma initiative range from $150 million to $800 million for some of the big companies.

Based on the industry research, and current trends, Six Sigma and related methodologies are considered as one of the most sought after emerging technologies and programs by industries today.

A survey conducted in 2010 by the consulting firm Compdata indicates that the deployment of Lean and Six Sigma methods remains strong in the manufacturing sector. The section of the survey on safety and cost-cutting procedures showed that nearly 70 percent of the 1,100 companies surveyed have implemented Lean practices in their operations. The survey also found that 58.6 percent of the companies are using Six Sigma, which is almost unchanged from the 58.5 percent who reported the same in 2009.

A survey by *Quality Digest* on Six Sigma shows some interesting facts. Approximately 2,577 quality professionals took part in the survey. The survey also compiled data on Six Sigma's effect on companies, support for Six Sigma by top management, cost of implementing this technique, and the functional areas where Six Sigma is being implemented. A majority of the respondents acknowledge that Six Sigma has improved their companies' profitability. Nearly 80 percent of the respondents agreed that the management fully supported the Six Sigma program. The results of the survey also indicate that Six Sigma is making its way into small companies. The survey also shows that an increasing number of companies are implementing—or plan to implement—Six Sigma techniques. With the overwhelming success of Six Sigma, many companies are now moving toward Lean Six Sigma and DFSS.

To achieve the overall objectives of improving quality, reducing variation and defect, reducing cost, eliminating waste, providing speed and reliability of delivery, and incorporating flexibility and innovation in products and services, the goal of many companies is to become a Six Sigma company using the proven techniques of Lean Six Sigma and DFSS.

Six Sigma: Current Trends

The following are some impressive data reported in iSixSigma LLC website (http://www.isixsigma.com). The research report states:

- Over the past 20 years, use of Six Sigma, the popular business improvement methodology, has saved Fortune 500 companies an estimated $427 billion, according to research published in the January–February 2007 issue of iSixSigma Magazine.
- Corporate-wide Six Sigma deployments save an average 2 percent of total revenue per year.
- Six Sigma adoption has increased phenomenally in recent years.
- Six Sigma started out slowly in the late 1980s but then took off in the mid-1990s once people started seeing successes at companies like GE and AlliedSignal.
- About 53 percent of Fortune 500 companies are currently using Six Sigma and that figure rises to 82 percent when you look at just the Fortune 100.
- The market for Six Sigma training and consulting is very much open where 47 percent of the Fortune 500 have not yet embraced the methodology.
- Six Sigma has a 20-year record of accomplishment of impressive results and is still expanding.

Statistical Basis of Six Sigma

Traditionally, manufacturing processes have been controlled using a three-sigma process. In such a process, 3σ upper and lower limits must be inside the customer's specification limits for a capable process. A ±3σ process results in 99.73 percent of the items being within the 3σ limits, and 0.27 percent of the items being out of specification. This means that even if all the special causes of variations are removed and the process has only random fluctuations (natural causes of variation for which we have no control), the process will produce 0.27 percent (2700 defects per million) defective or nonconforming items. This level of nonconformity is not acceptable in today's highly competitive global market.

As outlined previously, one of the major objectives of a Six Sigma program is to reduce variation in products and processes. In a three-sigma process where three standard deviations just fit within the specification limits, the process will begin producing nonconforming products if there is a shift in the process mean. This process is very sensitive to the process shift. Figure 2.1 compares a three-sigma process to a Six Sigma process. In a three-sigma process, Six Sigmas (or three Sigmas on either side of the mean) fit within the specification limits; whereas, in a Six Sigma process, 12 Sigmas (or Six Sigmas on either side of the mean) fit within the specification limits.

In a Six Sigma process, even if the process mean shifts by as much as 1.5 sigma on either side of the mean, the majority of the products will remain within specification limits. In fact, in a Six Sigma process, a shift

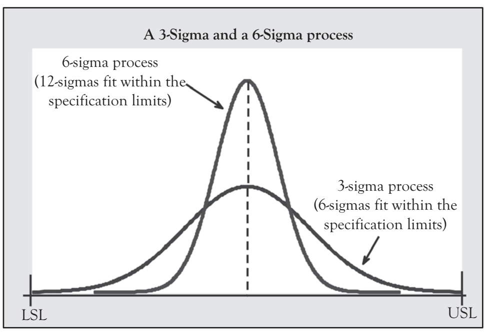


Figure 2.1 Comparing a three-sigma process to a Six Sigma process

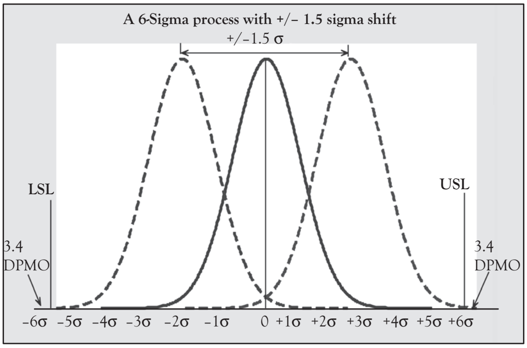


Figure 2.2 A Six Sigma process

in the process mean of 1.5 sigma on either side of the mean results in 3.4 nonconforming products per million. Figure 2.2 shows a Six Sigma process where 12 standard deviations (Six Sigmas on either side of the mean) fit within the specification limits. It also shows the shift in the process mean by as much as ±1.5 σ.

In a Six Sigma process, variation is reduced to an extent that a spread of 12 Sigmas (Six Sigmas on either side of the mean) fits within the process specifications. The specification limits (also called tolerance) are established during the design phase of the product or process, or the customer provides them. A Six Sigma quality level represents 3.4 DPMO. This means that if a process is operating at a Six Sigma level, it will produce no more than 3.4 defects per million. This is only possible when the variation in the process is reduced significantly. *To be exact, to achieve a Six Sigma process, the variation must be reduced to one-half, or 50 percent that of a three-sigma level quality*. With the variation reduced to this level, even a drift in the process on either side of the mean or target value will not allow the process to go out of control. It is important to note here that no process can be controlled exactly at the target or the mean value. It is natural for a process to drift from its mean or target value in due course. This drift or shift can be as much as 1.5 standard deviations on either side of the mean or the target. In a Six Sigma process, the process variation is equal to half of the design specification or tolerance so that a shift in the process mean of as much as 1.5 standard deviations on either side of the target will keep the process well within the tolerance, and the likelihood of producing nonconforming products is virtually nonexistent.

Metrics and Measurements in Six Sigma

A metric is a measurement of certain quality characteristic. In Six Sigma, metrics provide ideas regarding the overall quality level or information on overall performance. Metrics are used to evaluate and communicate quality performance. For example, a company that has achieved Six Sigma quality level produces no more than 3.4 defects per million. Here, defects per million opportunities or DPMO is a metric and is a measure of quality level. Refer to Table 2.3 that shows the sigma values translated to number of defects. A metric tells the customers and other stakeholders about the level of quality (e.g., a four-sigma level of quality) for the company and conveys expectations for employees and suppliers. Thus, a metric (or a measurement) can be seen as a performance indicator that provides information on overall performance that is helpful in evaluating and identifying opportunities for improvement.

Industries use different metrics or performance measures. While many of these metrics seem to focus on manufacturing, these can be applied universally. Some of the commonly used metrics in use are:

- DPMO
- PPM (parts per million defective)
- % Defects
- % Good quality
- Rolled throughput yield (%)
- First pass yield (%)

The metrics most commonly used by industries are DPMO and PPM. According to a survey conducted by Aberdeen Group approximately half or 49 percent of best in class companies use DPMO as their performance measure. About the same percent (49 percent) of the companies in the same category used PPM defective as a performance measure. The survey also indicated that those companies that measure DPMO and PPM achieved better performance than those who measured *% good* or *% defective* as metrics. Since DPMO is one of the most commonly used metric, it is explained next.

Defects per Million Opportunities

A Defect per Unit (DPU) is the measure of output that focuses on the final product and ignores the process—or the processes—that makes the product. A product can be produced using different processes, or a service can be provided using different steps or processes. In such cases, different processes may have different numbers of opportunities for error. This makes it difficult to compare the DPUs for different processes. Therefore, Six Sigma defines the quality performance as defects per million opportunities (DPMO), which is defined as:

$$\text{DPMO} = (\text{Number of defects/Opportunities for error}) \times 1{,}000{,}000$$

Quality Level and Percent Nonconforming for a Noncentered Process (A Process Mean Shift of ±1.5 σ)

Table 2.3 provides the numerical values in parts per million nonconforming for different quality level with a process mean shift of ±1.5 σ.

Figure 2.3 shows the nonconforming parts per million for different sigma quality levels. The amount of improvement for different sigma quality levels can be seen from the graph.

As can be seen from the Figure 2.3, an improvement from a three-sigma to Six Sigma level in quality leads to a 70 times improvement.

Table 2.3 Percent nonconforming for a noncentered process (A process mean shift of ± 1.5σ)

Specification limits (Quality level)(σ)	Percent within specification limits (%)	Percent outside specification limits (Nonconforming) (%)	Nonconforming PPM
±2σ	69.1229	30.8771	308771
±3σ	93.3190	6.681	66811
±4σ	99.3790	0.621	6210
±5σ	99.97674	0.02326	233
±6σ	99.999660	0.000340	3.4

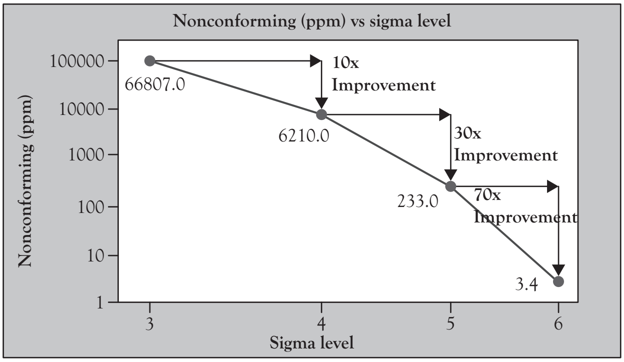


Figure 2.3 Defect rates in parts per million for different sigma quality level

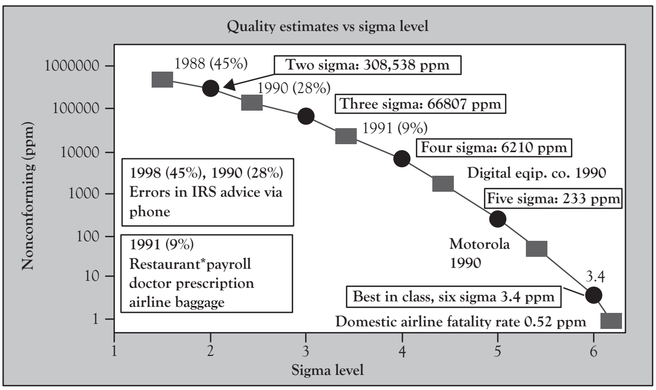


Figure 2.4 Improvement in quality for different levels of sigma

Source: Six-Sigma Programs, Selected Topics in Assurance Related Technologies, Volume 6, Number 5.

Service Successes of Six Sigma

Most of the techniques of Six Sigma were developed in manufacturing but the applications are not limited to manufacturing operations only. Six Sigma has been applied successfully in many service industries including banking, health care, airlines, and several financial companies. Six Sigma techniques can be applied to virtually any process. Figure 2.4 shows the application areas of Six Sigma with achieved improvement in quality level for some of the service industries.

Six Sigma Methodology

The Six Sigma approach is a collection of managerial and statistical concepts and techniques that focus on reducing process variation and preventing product deficiencies. As mentioned earlier, the variability in the process is described by sigma (σ). This sigma is the standard deviation of measurements around the process mean. The process that has achieved the six-sigma capability will have much smaller variation. In many processes, we establish a relationship between the response (output) y and input variables x_1, x_2, x_3,…, x_n; that is,

$$y = f(x_1, x_2, \ldots\ldots, x_n)$$

The Six Sigma approach identifies the process variables that cause variation in the products. Some of these input variables are controllable and are critical to maintaining quality. These variables must be controlled within a specified range.

Six Sigma is based on a strategic improvement methodology known as DMAIC which stands for **D**efine, **M**easure, **A**nalyze, **I**mprove, and **C**ontrol. The steps in DMAIC methodology contain several simple to advanced statistical tools—and other process-improvement tools—used to carry out or execute Six Sigma projects. The DMAIC process is briefly explained next followed by a detailed explanation of each. Figure 2.5 shows the DMAIC model. Each of the five phases in DMAIC is described in the next section.

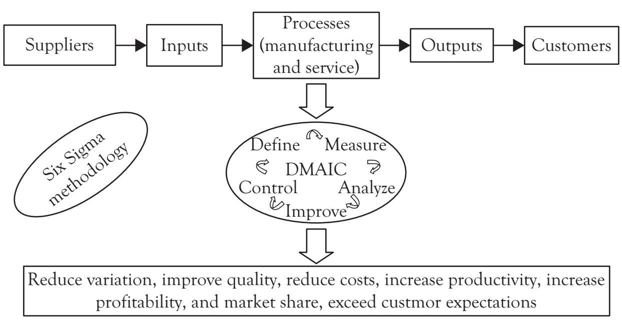


Figure 2.5 The DMAIC model

Five Phases of Six Sigma

The five phases of Six Sigma and their major objectives are outlined here.

- *Define*: identify or define the problem, identify critical customer requirements, define the project, form project teams, and create a project charter.
- *Measure*: outline the process, determine the characteristics CTQ, identify the metrics, assess the measurement system, collect data, and measure the current process capability.
- *Analyze*: analyze data to determine the root cause(s) of the problem, and the key input and output process variables.
- *Improve*: conduct formal experiments (using Design of Experiments, and other statistical tools), isolate key input process variables from a large number of controllable and uncontrollable factors, and determine the process settings to optimize product or process.
- *Control*: measure the new process capability, document the improved process, and impose control techniques on the critical factors to maintain the gain.

The preceding is a brief overview of Six Sigma. A complete understanding of each of the previous five phases is critical to successful implementation of Six Sigma quality program.

Lean Six Sigma

Traditional Six Sigma is well understood and consistently deployed across a number of industries. Six Sigma employs a well-structured continuous methodology to reduce process variation and decrease defects within the business processes using simple to advanced statistical tools and techniques. It improves quality through variation reduction, defect removal, and process optimization. The improved quality leads to higher perceived value for the products or services the company offers. This helps companies achieve increased market share. As noted earlier, companies have reported significant savings by reducing the costs of poor quality.

The other approach to achieving excellence in products and services is based on the removal of waste from service and manufacturing processes. This is *Lean* approach. Many companies have reported significant improvement through the removal of waste or nonvalue added activities. Companies have also reported that bringing the two concepts—Lean and Six Sigma—together deliver faster results. While the objective of Lean is to create flow and eliminate waste, Six Sigma improves process capability and reduces defects and variation that leads to improved quality and cost savings for the companies. If a company just applies Six Sigma, it cannot maximize the potential of the organization. Lean is really an enabler for Six Sigma (Chowdhury 2001).

The term *lean* has its root in *Just-in-time Manufacturing* or *Lean Manufacturing*—a philosophy of production that emphasizes on the minimization of the amount of all the resources (including time) used in the various activities of the enterprise. It involves:

- identifying and eliminating nonvalue-adding activities,
- employing teams of multiskilled workers, and
- using highly flexible, automated machines.

The major focus of lean is eliminating or reducing waste from the process. Eliminating waste in a *manufacturing process* involves the following steps:

- Make only what is needed.
- Reduce waiting by coordinating flows and balancing loads.
- Reduce or eliminate material handling.
- Eliminate all unneeded production steps.
- Reduce setup times and increase production rates.
- Eliminate unnecessary human motions.
- Eliminate defects.

This philosophy can be applied to any process, manufacturing, or service because both have waste and variation. Companies have realized significant improvement in quality, cost, productivity, profitability, and cycle time (speed) through the removal of waste or nonvalue added activities and variation reduction from their process. Thus, lean is for reducing waste, improving flow, and reducing cycle time. Six Sigma is for reducing defects and variation. The combined approach is what is known as *Lean Six Sigma* that is described in quality disciple as a philosophy, a culture, and a journey to excellence.

Difference Between Lean and Six Sigma

- Lean is an approach that seeks to improve flow in the value stream and eliminate waste or nonvalue adding steps. *It is about doing things quickly*.
- Six Sigma uses a powerful framework (DMAIC) and simple to advanced statistical tools to uncover root causes of the problem to understand and reduce variation. *It is about doing things right (defect-free)*. Table 2.4 outlines the differences between Lean and Six Sigma approaches.

Table 2.4 Difference between Lean and Six Sigma

	Lean	Six Sigma
Theory	Reduce waste	Reduce variation
Application guidelines	Identify value Identify value stream Flow Pull Perfection	Define Measure Analyze Improve Control
Focus	Flow	Problem
Assumptions	Waste removal will improve performance Many small improvements are better than systems analysis	A problem exists Figures and numbers are valued System output improves if variation in all processes is reduced
Primary effect	Reduced flow time	Uniform process output
Secondary effects	Less waste Fast throughput Less inventory Improved quality	Less variation Uniform output Less inventory Improved efficiency Improved productivity Improved quality
Criticism	Statistical analysis not used and valued as much as in Six Sigma	System interaction not considered Process improved independently

Source: Lean Six Sigma: some basic concepts (NHS Institute for Innovation and Improvement).

- Lean means "*using less to do more by determining the value of any given process by distinguishing value-added steps from non-value-added and eliminating waste so that ultimately every step adds to the process*" (Miller 2005).

Lean and Six Sigma Approach

The Six Sigma and Lean differ in their approaches and objectives. Table 2.5 outlines the difference between the Lean and Six Sigma objectives.

Lean Six Sigma Project Selection: Problems and Opportunities for Lean Six Sigma Projects

The following are some of the reasons for the companies to initiate Lean Six Sigma projects:

- High COPQ
- High costs (operation, material)
- Excessive defects
- Customer complaints
- Low customer satisfaction
- Declining revenue or profitability

Table 2.5 Lean and Six Sigma objectives

Lean	Six Sigma
Specify value What is important in the eyes of the customer?	Define What is important?
Identify the value stream What is the entire value stream?	Measure How are we doing?
Flow How will the material and information flow through our process?	Analyze What is wrong?
Pull How can we let the customer pull products, rather than pushing product?	Improve What needs to be done?
Perfect How can we optimize our processes?	Control How do we sustain the improvements?

- Declining market share
- Declining sales
- Low throughput yield
- Increase in warranty costs
- Increase in merchandize return for refunds.

Design for Six Sigma (DFSS)

The other quality program in use and gaining popularity is the DFSS. The goal of DFSS is to address and incorporate quality issues early in the design or redesign process using robust design methodologies. It integrates engineering design and statistical methods to predict and improve quality before production. DFSS is a way of understanding the key product characteristics to design and build successful products. The success of companies depend on designing, developing and launching new products of superior quality, getting to the market quickly (reduced cycle time), bringing innovation in products, and understanding the customer's needs and requirements. Research shows that approximately 5 percent of all new-product ideas survive to production and only about 10 percent of these are successful. Therefore, actively building quality in every phase of the product development process, predicting, and optimizing critical quality characteristic (CTQs) are keys to ensuring product success. DFSS is a systematic method to build quality and key customer requirements in all stages of product development. These key quality characteristics (CTQs) and customer requirements can be measured, verified, and optimized. DFSS is an approach to meet or exceed customer needs, requirements, and expectations using voice of customer (VOC).

Companies who have successfully employed a Six Sigma program have found that once they achieve five-sigma quality level (233 defects per million opportunity) they must design or redesign their products, processes and services by means of DFSS to surpass this quality level (Chowdhury 2001; Miller 2005). Also, the cost to correct potential design problems to reduce the defect level for achieving a higher quality level (above four-sigma) is usually greater than the projected cost savings of the further improvement effort. It is therefore important that the quality be built-in during the design phase; and the quality issues must be addressed early in the design process. To achieve Six Sigma quality level, the companies must determine where the Six Sigma and DFSS activities occur in the life cycle of the product. In other words, the companies must determine when to apply the DFSS approach. Unlike Six Sigma, the DFSS is not standardized and does not yet have a structured problem solving approach as Six Sigma. DFSS is not as well deployed as Six Sigma in industry. The underlying principles of DFSS methodology are outlined here.

- DFSS is a systematic methodology to design new products or processes so that quality is built into every phase of product design. It is also used for improving existing products through redesign.
- The roots of DFSS are in systems engineering. It combines systems engineering methodology with statistical methods to achieve *built-in quality* objectives.
- DFSS optimizes the CTQ characteristics to achieve the best system performance. CTQs are the selected few measurable quality characteristics that are key to a specific product, process, or service that must be controlled to meet or exceed customer expectation.
- DFSS uses Robust Design, Design of Experiment (DOE), Design for Manufacturability, Simulation and several other tools to optimize product design.
- DFSS balances the cost and quality.
- DFSS reduces the development cycle time in the long run.
- In DFSS, both engineering methods and statistics are used to optimize the design requirements.
- Like Six Sigma, DFSS also uses a collection of tools. These tools must be understood in context to the engineering design for achieving DFSS objectives.

The DFSS methodology has been identified by a five-step process: DMADV; which stands for **D**efine, **M**easure, **A**nalyze, **D**esign, and **V**erify. These are explained briefly:

1. *Define*: determine the project need, identify the project goals and objectives, determine customers' needs and requirements, and include the voice of customers (VOCs).
2. *Measure*: determine the characteristics CTQ, prioritize customer needs and requirements, and assess customers' needs and CTQ metrics.
3. *Analyze*: evaluate the process options to meet customers' needs and CTQs.
4. *Design*: design the product and process to meet the customer requirements, and include customer requirements in the development process.
5. *Verify*: check the design to ensure that the customers' requirements are met.

The DFSS is also identified by the *IDOV* process, which stands for **I**dentify, **D**esign, **O**ptimize, and **V**alidate. Unlike the Six Sigma process, the DFSS is relatively new and not standardized; therefore, there are inconsistencies in the methodology, tools, and models companies employ. Some authors also argue that DFSS is a complex systems engineering analysis methodology, enhanced with statistical methods, and cannot be fully executed using a simple four-step IDOV process. The IDOV or the DMADV process may be applied to a single requirement or CTQ, but not to an overall product development process. The IDOV process is explained as follows:

1. *Identify*: Identify customer requirements, and address the VOC. Prioritize customer requirements, use HOQ to identify and define CTQs.
2. *Design*: Identify product design parameters and characteristics; build a database about the product and related process, and *design in* key customer requirements.
3. *Optimize*: Optimize the design to achieve a balance of quality, cost, and time to market. Create *robust* design that will minimize the impact of variation in the production process.
4. *Validate*: Using data, demonstrate that the product and process is capable, the process capability meets appropriate sigma level, satisfies the CTQs, and meets the customer's requirements and expectations.

The process of DFSS can be divided into four categories described in the following (Creveling, Slutsky, and Antis 2003). These are very similar to the IDOV process described earlier. The DFSS process can be briefly described using the following four categories:

1. Concept development and concept engineering
2. Design development
3. Design optimization
4. Design verification

Six Sigma or Design for Six Sigma?

Figure 2.6 shows the stages in the life cycle of a product where DFSS and conventional Six Sigma are used. Once the product is released over to manufacturing for production, Six Sigma methodology is used to achieve constant incremental improvements by reducing or minimizing the causes of process variation. When Six Sigma improvement projects are applied based on the assumption that the design of a current product, process, or service is correct, and the design satisfies the functional requirements of the customers, it may be difficult to achieve improvement beyond a four- or five-sigma level. Significant design or redesign effort may be required to achieve further improvement. In addition, the design changes are much more expensive at this level. This is why DFSS should be considered early

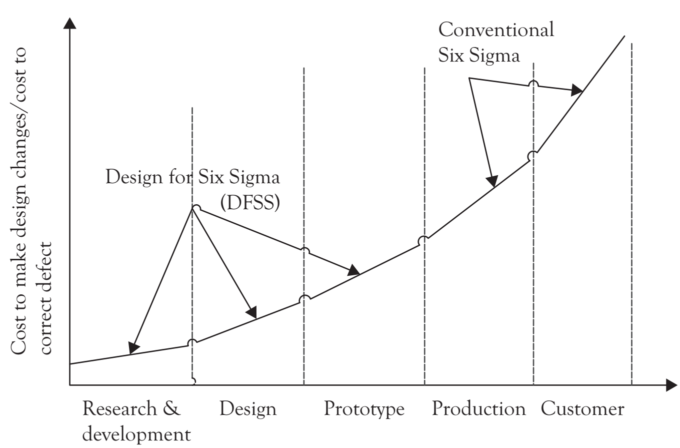


Figure 2.6 Six Sigma and design for Six Sigma in a product's life cycle

in the design stage. Figure 2.6 shows the cost to correct defects and make design changes at different stages of a product life cycle.

Summary

More and more companies are realizing that it is possible to achieve dramatic improvements in cost, quality, and time by incorporating Lean Six Sigma and Design for Six Sigma (DFSS) quality programs. A number of companies including Toyota, General Electric, Motorola have accomplished impressive results using one or more of the quality programs mentioned in this chapter. However, using only one method—Lean, Six Sigma, or DFSS—has limitations. Six Sigma eliminates defects during the production phase, but does not address the importance of quality effort in the research and design phase of a product. Also, Six Sigma does not address the question of how to optimize the process flow, and the Lean principles do not address the use of advanced statistical tools required to reduce variation, defects and achieve the process capabilities needed to be truly *lean*.

Unlike Six Sigma, the DFSS is not standardized and is not deployed well in industry. The goal of DFSS is to address and incorporate quality issues early in the design or redesign process using robust design methodologies. Companies who have successfully employed the Six Sigma program have found that once they achieve five-sigma quality levels (233 defects per million opportunities), they must design or redesign their products, processes, and services by means of DFSS to surpass this quality level (Chowdhury 2001). Also, the cost to correct the potential design problems to reduce the defect level to achieve higher quality level (above four-sigma) is usually greater than the projected cost savings of the further improvement effort (Peplinski 2004). It is therefore important that the quality must be built in the design phase, and the quality issues must be addressed early in the design process. To achieve Six Sigma quality level, the companies must determine where the Lean, Six Sigma, and DFSS activities occur in the life cycle of the product. There is a need for an integrated approach to achieve the overall objectives.

CHAPTER 3

Statistical Methods Used in Quality

Introduction

This chapter provides an overview of statistical methods used in quality improvement. We will discuss a number of simple to complex statistical techniques that are used in modeling, studying, and solving quality problems. Since variation reduction is one of the major objectives of quality improvement programs, the emphasis in this chapter is to introduce the graphical and numerical tools of descriptive statistics that are used to study the variation in process data. The other objective of this chapter is to get insight into several of the discrete and continuous probability distributions and their properties. A good knowledge and understanding of probability distributions is critical in being able to apply these distributions in solving quality problems.

Statistics is studied under two broad categories: (1) *descriptive statistics*, and (2) *inferential statistics*. Descriptive statistics involves the methods of collection, presentation, and characterization of a set of data in order to properly describe the various features of that set of data. There are two ways we can describe the collected data: (1) through charts and graphs and (2) using numerical methods. Charts and graphs fall under the category of graphical methods. Graphical techniques include charts and graphs, such as bar charts, pie charts, histograms, polygons, scatter diagrams, and so on. In addition, a number of simple yet very effective graphical methods including the histogram, stem-and-leaf plot, and box plots are very useful in summarizing and presenting the data. These tools are also used in studying the variation in the process. Simple numerical methods, for example, the measures of central tendency, and measures of variation often used to model and describe the quality characteristics of a process are described in this chapter.

Inferential statistics is the process of using *sample* statistics to draw conclusions about the *population* parameters. *Interference* problems are those that involve inductive generalizations.

Population denotes the entire measurements that are theoretically possible. It is also known as the universe and is the totality of items or things under consideration. For example, total number of light bulbs manufactured by a company in a given period of time, or number of people who can vote in a country, and so on.

Sample is the portion of the population that is selected for analysis (a subset of population).

A population is described by its *parameters,* whereas a sample is described by its *statistics*. A *parameter* is a summary measure that is computed to describe the characteristic of a population. A *statistic* is a summary measure that is computed to describe the characteristic of a sample.

Population parameters are the population mean, population variance, population standard deviation, and population proportion. These are expressed using Greek symbols. The symbols used to describe the population parameters are: *the population mean* μ (read as *mu*), *population variance* (σ^2), *population standard deviation* (σ, read as *sigma*), and *population proportion*, p. Note that each one is denoted by a specific symbol.

A sample is described by its statistics. These statistics are *sample mean* ($\bar{x}$, read as *x-bar*), *sample variance* (s^2), *sample standard deviation* (s), *sample median*, and sample proportion, $\bar{p}$ (read as p-bar). It is important to know the distinction between the population parameters and the sample statistics and the way they are described.

The purpose of this chapter is to introduce the statistical methods commonly used in quality. We also introduce the concept of probability distribution and discuss a number of both discrete and continuous probability distributions that form the basis for the control charts we will be presenting and applying later in this text.

Data and Graphical Methods

In this section, we discuss the methods of describing data using graphical techniques. Graphical techniques involve describing data using charts and graphs. This area comes under descriptive statistics. There are two methods of describing data: graphical and numerical. Before studying the graphical techniques in greater depth, a review of (1) data and types of data; (2) collection, presentation, organization of data; and (3) concept of frequency distribution are presented.

Classification of Data

Data are collections of any number of related observations. All data are some form of measurement. Data can be classified as *qualitative* or *quantitative* data. Quantitative data are numerical data that can be expressed in numbers. For example: data collected on temperature, sales, demand, length, height, and volume are all examples of quantitative data.

Qualitative data are data for which the measurement scale is categorical. Qualitative data are also known as *categorical data*. Examples of qualitative data include the color of your car, response to a yes or no question, or the product rating using a Likert scale of 1 ot 5, where the numbers correspond to a category (excellent, good, etc.). Data can also be classified as:

- discrete or
- continuous

Discrete data are the result of a counting process. These are integers. For example, cars sold by Toyota in the last quarter, the number of houses sold last quarter in a city, or the number of defective parts produced by a manufacturing process. All these are expressed in whole numbers and are examples of discrete data.

Continuous data are the data that can take any value within a given range. These are measured on a continuum or scale that can be divided infinitely. Continuous data are numerical measurements of quantities such as length, volume, temperature, stock price, or time. More powerful statistical tools are available to deal with continuous data as compared to discrete data. Therefore, continuous data are preferred wherever possible.

Collection and Presentation of Data

There are two methods used for describing data. These are:

- tables and
- graphs

The purpose of collecting data is to draw conclusions or to make decisions. To draw meaningful conclusion, the data are organized, grouped, plotted, and analyzed. Organizing data into groups is known as the *frequency distribution*.

Data are collected through actual measurements or observations or can be obtained from company records. This information can be organized in a way that can be used to make decisions or draw conclusions. When data are arranged in a compact, usable form, decision makers can obtain reliable information and use it to make decisions.

Organizing Data

Data can be arranged in different ways. For example, the collected data may be arranged from the lowest to highest value. This arrangement is often called *ordered array*.

Another way to organize or classify the data is by using a frequency distribution. In a frequency distribution, the data are divided into similar categories or classes, and the number of observations in each category or class is then counted. This is also known as *grouping*.

The reasons behind organizing the data in the form of a frequency distribution are: (1) to obtain a compact representation of the data; (2) to be able to see some characteristics of the data, for example, the spread or variation in the data; (3) to observe the pattern in the data; and (4) to find out what values occur most frequently.

Organizing Data: An Example

A manufacturer of televisions is interested in the survival time of one of its components. A sample of 200 television components was tested. The results are shown in Table 3.1. The table shows the life (in hours) of these components rounded to the nearest hours. Note that the lifetime is a variable. In statistics the quality characteristics on which we collect data are known as *variables*. Thus, when we collect data on length, diameter, volume, stock values—they are all variables. Statistics allows us to study variation, and all data show variation.

The data of Table 3.1 are called *raw data* (data which are not arranged and analyzed). The life times of the components are listed in the order in which they occurred. This is *ungrouped data*. Ungrouped data enable us

Table 3.1 Lifetime of 200 television components (in hours)

314	330	371	365	267	307	371	297	291	398
276	253	286	344	385	349	269	304	319	283
430	253	378	306	376	308	339	368	289	344
340	298	330	311	318	358	354	406	369	254
322	242	331	236	344	418	328	393	267	305
325	282	315	328	319	353	336	384	298	398
343	203	373	297	276	333	257	367	296	349
322	325	252	345	373	317	307	289	363	340
309	246	302	260	292	231	338	372	226	365
271	302	331	374	355	336	312	354	329	345
276	329	379	288	356	302	263	364	337	361
416	360	337	273	298	390	215	382	329	306
306	279	414	262	372	303	346	331	362	366
387	304	302	280	287	368	281	329	309	310
375	346	413	309	283	299	335	330	376	260
277	366	345	409	312	266	383	289	294	370
359	363	243	339	323	297	333	299	302	384
370	357	314	348	257	291	358	409	337	347
215	277	313	300	322	304	282	410	390	332
373	280	339	349	363	297	274	334	359	330

to study the sequence of values; for example, *low* or *high* values. The data may also be helpful in determining some causes of variation. However, for a large data set, the ungrouped data do not provide much information.

When the data are arranged in an increasing order of magnitude; that is, in *rank order* we refer to such data as *data array* or *ordered array*. A data array arranges the values in increasing or decreasing order.

Summarizing Quantitative Data: Frequency Distribution

A frequency distribution provides a compact representation of data. This is also known as grouping. Compact representation is obtained by arranging the data into groups or *class intervals* usually of *equal width* and then recording or counting the number of observations in each interval. Counting the number of observations in each group is called the *class frequency*. For example, examine the data in Table 3.2, We can divide this data into nine class intervals with a width of 30 and tabulate the results as follows:

Class interval	Frequency
200–230	4
230–260	11
260–290	30
….. and so on.	…..

The earlier class frequency is an example of a frequency distribution. The class interval of 200–230 means that this interval contains all the values from 200 to 230 (not including 230). If you count the number of observations between 200 and 230 in Table 3.2; you will find there are four observations in this group. The count of 4 is known as the frequency. The class interval can also be written in a formal way as:

$$200 \leq x < 230$$

This means that the values in this class interval include the value 200 but not 230. The value 200 is known as the *lower class boundary* or *lower class limit,* and the value 230 is known as the *upper class boundary* or *upper class limit.*

Table 3.2 Data array: Data of Table 3.1 arranged in increasing order (read row-wise)

203	215	215	226	231	236	242	243	246	252
253	253	254	257	257	260	260	262	263	266
267	267	269	271	273	274	276	276	276	277
277	279	280	280	281	282	282	283	283	286
287	288	289	289	289	291	291	292	294	296
297	297	297	297	298	298	298	299	299	300
302	302	302	302	302	303	304	304	304	305
306	306	306	307	307	308	309	309	309	310
311	312	312	313	314	314	315	317	318	319
319	322	322	322	323	325	325	328	328	329
329	329	329	330	330	330	330	331	331	331
332	333	333	334	335	336	336	337	337	337
338	339	339	339	340	340	343	344	344	344
345	345	345	346	346	347	348	349	349	349
353	354	354	355	356	357	358	358	359	359
360	361	362	363	363	363	364	365	365	366
366	367	368	368	369	370	370	371	371	372
372	373	373	373	374	375	376	376	378	379
382	383	384	384	385	387	390	390	393	398
398	406	409	409	410	413	414	416	418	430

There are several other possibilities of grouping or constructing frequency distributions using the information in Table 3.2. The following information is helpful while grouping or forming a frequency distribution: when dividing the data into class intervals, 5 to 15 class intervals are recommended. If there are *too many class intervals*, the class frequency (count) is low and the savings in computational effort is small. If there are too few class intervals, the true characteristic of the distribution may be obscured and some information may be lost.

The *number of class intervals* should be governed by the *amount* and *scatter* of data present. A small sample or a uniform distribution (a distribution that has a constant frequency) would suggest that fewer class intervals are needed.

Rules for Constructing a Frequency Distribution

The following is a summary of creating a frequency distribution:

- Decide on the number of classes (or class intervals) and the width of each class.
- Decide how many classes to use and the width of each class. Note that the width of each class interval should be equal.
- Use the previously shown formula ($K = 1 + 3.33 \log n$) to determine the approximate number of classes. *In general, the number of classes should be between* 5 *and* 15. (The number of classes depends upon the number of observations and the range of data collected.)
- The class width is of equal size, the number of classes determines the width of each class. Use the following formula to determine the class width:

  From the data in Table 3.2, approximate the number of classes using,

$$\textit{Width of class interval} = \frac{(\textit{Largest value in the data} - \textit{Smallest value})}{\textit{Number of class intervals}}$$

$$K = 1 + 3.33 \log_{10} n = 1 + 3.33 \log_{10} 200$$
$$K = 1 + 3.33\ (2.3010)$$
$$= 8.66 \text{ or approximately 8 or 9 classes}$$

- Note that the number of observations, $n = 200$ and $K =$ number of classes. Using these values, the class width is: $(430 - 203) / 8 = 28.375$.

This width is approximate. You may choose to have a width of 30 rather than 28.375. From the data in Table 3.2, suppose we decide to divide the data into eight class intervals with a class width of 30. Using a class width of 30, the frequency distribution is shown in Table 3.3. The first column shows the class intervals. The lowest value in the data

Table 3.3 Frequency distribution of data from Table 3.2

Class interval	Frequency (f)
200–230	4
230–260	11
260–290	30
290–320	46
320–350	49
350–380	40
380–410	14
410–440	6
	$\sum f = 200$

is 203 so we decided the first class interval to be 200–230. This decision is arbitrary. Note that the interval of 200–230 contains the lowest value of 203 in the data set. The class interval should be chosen in such a way that no results fall on the class boundary. Also, the class intervals should include all the data values.

The second column in Table 3.3 contains the frequency or the number of observations in each class. This is obtained by sorting the data from the lowest to the highest number (as seen in Table 3.2) and counting the number of observations in each class. The frequency distribution is shown in Table 3.3. Note that the sum of the frequencies must be equal to the number of observations (200 in this case).

Next, we want to illustrate the frequency distribution in a graphical form as shown in Figure 3.1. This is commonly known as a *histogram*. A histogram is a graph of a frequency distribution. This graph is useful because it shows the pattern that is not so obvious when the data are in a table form. The histogram is also useful for the study of probability distributions (to be discussed later).

In a histogram, the class intervals are plotted on the horizontal axis and the frequencies are plotted on the vertical axis. The histogram is a series of rectangles (see Figure 3.1), each proportional in width to the range of values within each class and is also proportional in height to the number of observations falling within each class.

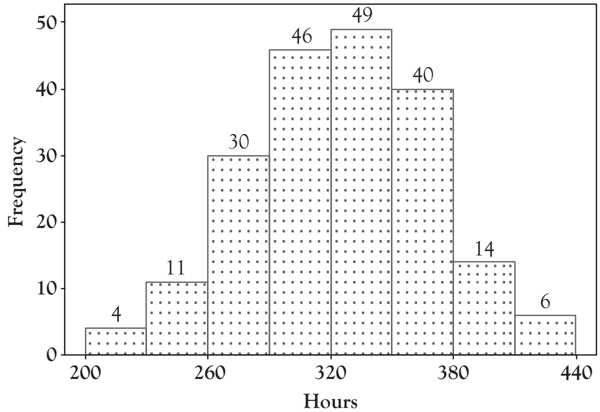


Figure 3.1 Histogram of lifetime data

Applications of Histogram in Quality

In quality, histograms are useful in

- Determining the shape of the distribution, that is, whether the shape is symmetrical or skewed
- Determining the concentration of data points, that is, which intervals or classes have more values in them
- Detecting process problems including a shift in the process
- Evaluating process capability (ability of the process to be within its specification limits)
- Determining how well centered the process is, or how close the data values are to the target value, and determining the process variation

Using Histograms to Detect the Shift and the Variation in the Process

The following histograms demonstrate how the data from a process can be examined to detect a shift and the variation in the process, and how the process capability of the process can be evaluated. The data are collected from a manufacturing process that produces a piston ring. The finished inside diameter of the ring is of interest. Historical data indicate that the diameter of the ring is normally distributed (has a bell-shaped pattern) with a mean of 5.01 cm and standard deviation of 0.03 cm. The specification limits on the diameter are 5 ± 0.05 cm. The data were collected for different shifts, operators, and machines and were examined using a histogram.

The histograms in Figure 3.2 are useful in examining the possible problems and the causes behind them. Once the problems are identified, proper actions should be taken to stabilize the process. Note that the process capability can only be determined once the process is stable and under control. The possible reasons and problems are explained for each plot.

Evaluating Process Capability Using Histograms

Histograms can also be used to assess the process capability (the ability of a process to be within its specifications). The process capability can be determined once the process is stable (e.g., see Figure 3.2(h) and (i)). These plots show that the process is stable and within control. The difference between Figure 3.2(h) and (i) is the variation. Figure 3.2(h) has more variation compared to 3.2(i) but, in both cases the process is centered. A *capable process* is one that shows small variation compared to the specification limits. Figure 3.2(i) is the most desirable where the process is stable, centered, and close to the target with very small variation. An example of a process that is not capable would be Figure 3.2(g).

Histogram with Fit and Groups

This option can be used to compare the mean and variability of two sets of data. Suppose we want to compare the variability in diameter of the shafts produced by two manufacturers. A sample of 124 shafts from manufacturer 1 and a sample of 200 shafts from manufacturer 2 were measured. The created histograms with fit and group shown in Figure 3.3 are useful in measuring the variation of two processes.

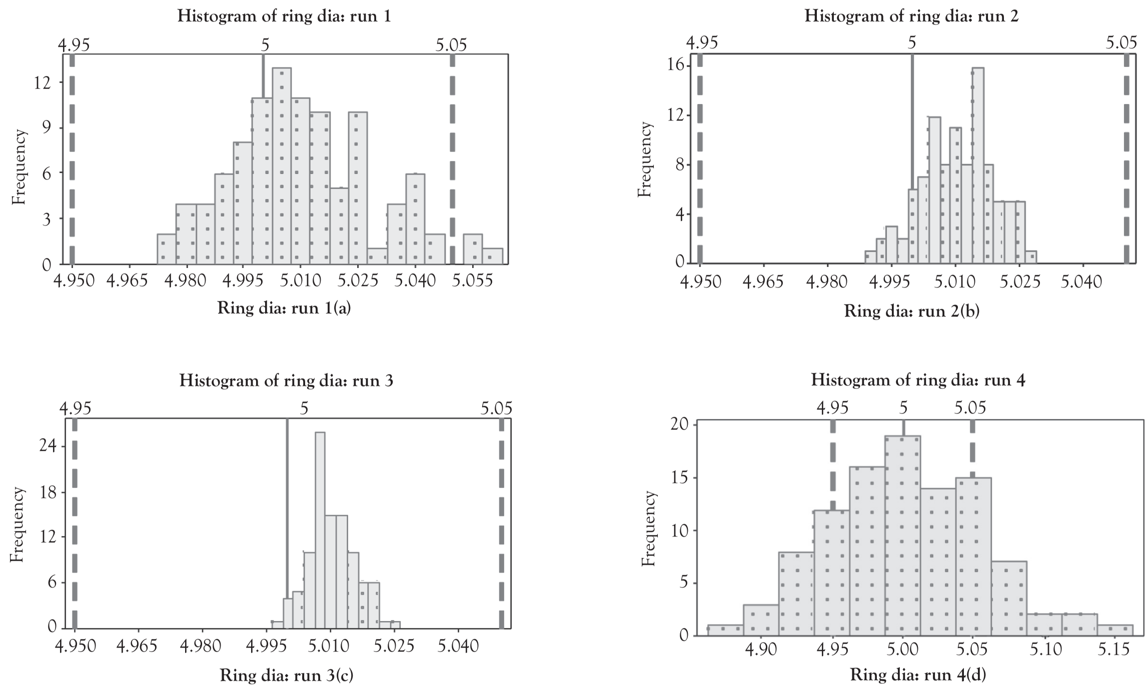
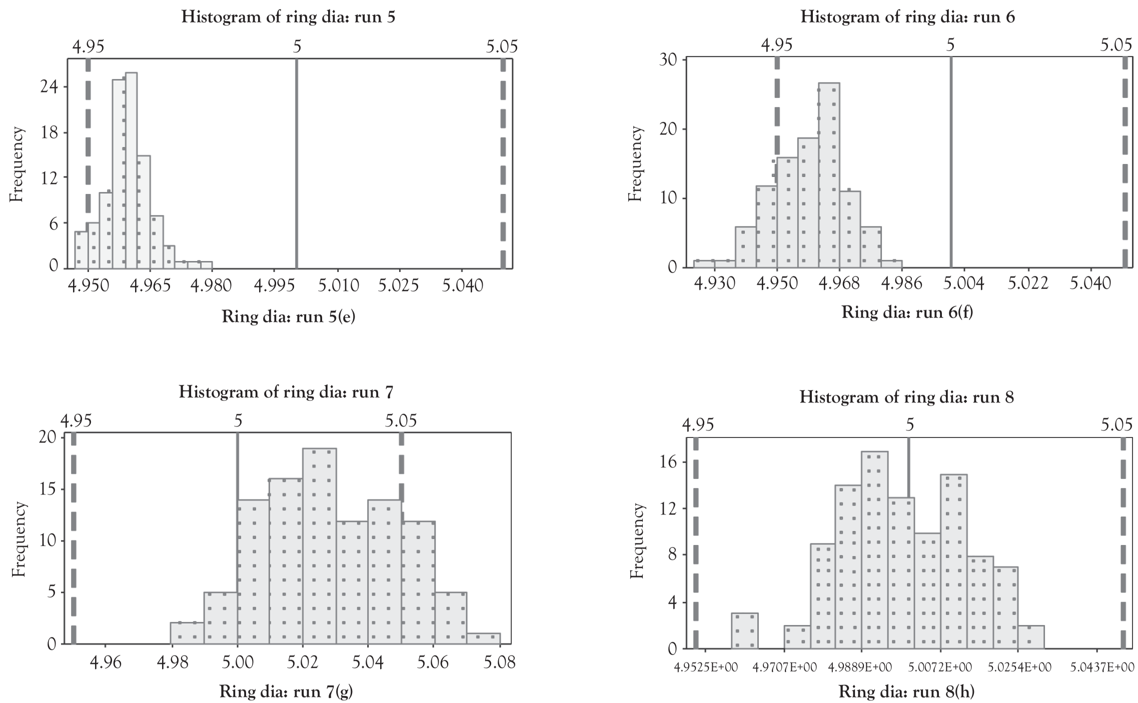
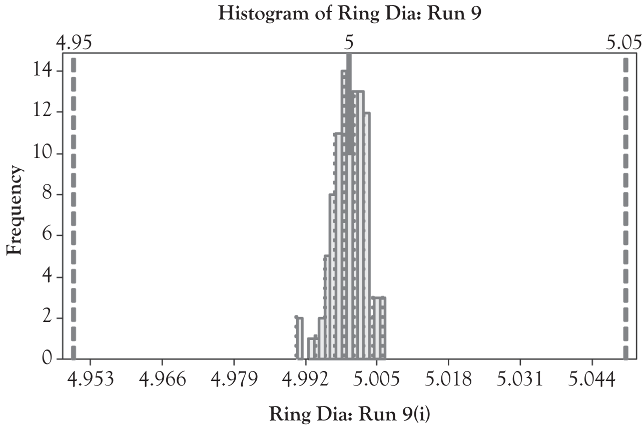


Figure 3.2 (a) Most values within specification, some assignable causes may be present. (b) Process within control but has slightly shifted to the right. (c) Process variation has reduced compared to (a) and (b), but there is a shift to the right. (d) Process out of control and has large variation



(Continued) (e) The process has shifted to the left; products out of specification. (f) Process shift to the left; more variation compared to (e). (g) Process out of control and has large variation. (h) Process within control but has large variation



(Continued) Figure 3.2 (i) Process stable and close to the target (desirable)

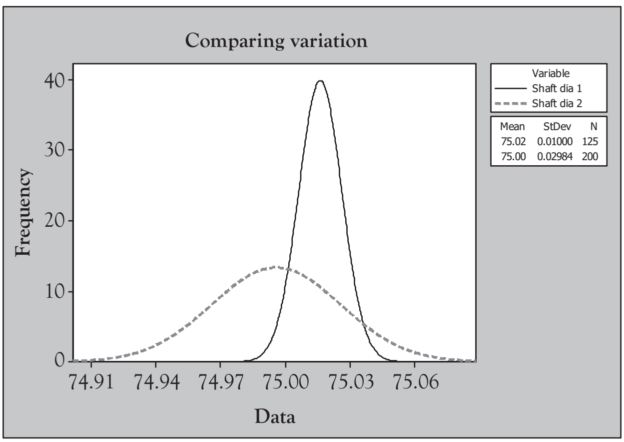


Figure 3.3 Histogram with fit and groups

Stem-and-Leaf Plot

Stem-and-leaf plots are very efficient way of displaying data, checking the variation and shape of the distribution. Stem-and-leaf plots are obtained by dividing each data value into two parts: a stem and a leaf. For example, if the data are two-digit numbers, that is, 34, 56, 67, and so on, then the first number (the tens digit) is considered the stem value, and the second number (the ones digit) is considered the leaf value. Thus, in data value 56, 5 is the stem and 6 is the leaf. In a three-digit data value, the first two digits are considered as the stem and the last digit as the leaf.

Figure 3.4 shows a stem-and-leaf plot of number of defects created using a statistical package.

In a stem-and-leaf plot, the actual data values indicate the frequency of each row. The first column (in Figure 3.4) shows the cumulative count, the second (middle) column shows the stem values, and the values next to the stem are leaf values. The first row has the following values:

5 5 23333

This means that there are five observations in this row, the stem value is 5 and the leaves are 23333. The five observations are 52, 53, 53, 53, and 53. They all have a common stem 5. Similarly, the second row shows:

6 5 4

which means there are six observations up to the second row (five in the first row and one in the second row); the stem is 5 and the leaf is 4 making the value in the second row 54. Note that the first column shows the cumulative count. Refer to Figure 3.4, column one again. The values are 5, 6, 12, 26, 48, and 76. This means that there are 76 observations up to row six. The next number is 30 which is enclosed in a parenthesis: (30). This indicates that there are 30 observations in this row (the row with the parenthesis) and it contains the *median value* of the data. Once the

```
Stem-and-Leaf Display: No. of Defects (out of 1000)
Stem-and-leaf of No. of Defects (out of 1000)  N = 200
Leaf Unit = 1.0
  5    5  23333
  6    5  4
 12    5  666777
 26    5  88888889999999
 48    6  0000000000011111111111
 76    6  2222222333333333333333333333
(30)   6  444444444444444555555555555555
 94    6  6666666666677777777777777777
 66    6  888888888888888999999
 45    7  00000000000001111111
 25    7  2222222222333
 12    7  44444555
  4    7  667
  1    7  8
```

Figure 3.4 Stem-and-leaf plot of number of defects

median is determined, the count begins from the bottom row. Look into the bottom row that shows

1 7 8

This indicates there is one observation in this row which is 78. The next to the last row shows

4 7 667

which means there are four observations up to this row (from bottom) and the values are 76, 76, and 77. The cumulative count continues upward just before the median row. The plot provides useful information. You can see from Figure 3.4 that the shape of the data is very close to symmetrical, the minimum value is 52 and the maximum value is 78. To find the total number of observations, add the observations in the median row which is (30) and the observations above and below the median row, that is, 76 + 30 + 94 = 200.

As can be seen, the stem-and-leaf display is an excellent way of showing the variation and the distribution (shape) of the data. It takes into account all the observations. It also can be used to extract a lot of information from the data including the minimum, maximum, the median, and the values lying above or below certain values. The one drawback of this plot is that it does not take into account the time order of observation.

Using Numerical Measures to Summarize Data Sets

This section deals with the numerical methods of describing and analyzing data, that is, describing data using one or more measures like the mean, median, variance, and so on.

The numerical methods of describing data can be divided into following categories: (1) measures of central tendency or measures of location, (2) measures of position, (3) measures of variation or dispersion, and (4) the measures of shape.

The important measures of central tendency are the mean and median. The mean or the average is a statistical constant which enable us to comprehend the significance of the whole. It provides an idea about the concentration of the data in the central part of the distribution. The requirements for the ideal measures of central tendency are that they should be (1) uniquely defined, (2) based on all observations, (3) affected as little as possible by fluctuations in sampling, and (4) suitable for further mathematical treatment.

The median is the value of the variable that divides the data into two equal parts, such that half of the value lies above the median and the other half below it.

Calculating the Mean

The mean of a data set is sum of the values divided by the number of observations. The mean of n observations $x_1, x_2, \ldots, x_n$ is given by

$$Mean = \frac{x_1 + x_2 + x_3 + \ldots\ldots + x_n}{n}$$

or,

$$Mean = \frac{\sum all\ \ values}{n} = \frac{\sum x}{n}$$

It is important to distinguish whether the summary statistic, such as the mean is being calculated from a sample data or a population data. The formulas for the sample and population mean are given next.

Sample Mean	*Population Mean*
$\bar{x} = \frac{\sum x_i}{n}$	$\mu = \frac{\sum x_i}{N}$

Example 3.1

The number of accidents for the past 6 months on a particular highway is given as follows.

5 8 10 7 10 14

The sample mean $\bar{x}$ is calculated as

$$\bar{x} = \frac{\sum x}{n} = \frac{5+8+10+7+10+14}{6} = 9$$

The previous calculation shows that the average number of accidents was nine. The mean can be interpreted in the following ways:

- It provides a single number presenting the whole data set
- It gives us the significance of the whole
- It is unique because every data set has only one mean
- It is useful for comparing different data sets in terms of the average

Calculating the Median

The *median* is another measure of central tendency. The median is the middle value of a data set when the data are arranged in increasing (or decreasing) order. The median divides the data into two equal parts, such that half of the values lie above the median and the other half below it. The median is the value that measures the central item in the data. For the ungrouped data (data not grouped into a frequency distribution), the median is calculated based on whether the number of observations is odd or even.

Calculating Median When the Number of Observations Is Odd

If the number of observations is odd, the median can be calculated by

- arranging the data in increasing order and
- locating the middle value after the values have been arranged in ascending order of magnitude.

Note that there is a distinct median when the number of observations is odd. Unlike the mean, the median is not affected by extreme values.

Example 3.2

Suppose we have the following observations arranged in increasing order:

1	2	3	4	5	6	7
8.2	8.3	8.9	9.6	9.8	10.2	12.0

The number of observations is 7 ($n = 7$), which is odd; therefore, the middle value or the median is the fourth value which is 9.6.

Suppose the following data are the annual incomes of eight employees (in thousands of dollars) of a manufacturing company for the past year. Find the median.

1	2	3	4	5	6	7	8
70	62	60	45	40	56	38	35

The number of observations is: $n = 8$ *(even)*

- Arrange the data in increasing order

1	2	3	4	5	6	7	8
35	38	40	45	56	60	62	70

- The location of the median for even observations is given by

$$\frac{n+1}{2} = \frac{8+1}{2} = 4.5$$

Therefore, the median is the average of 4th and 5th values

$$Median = \frac{45+56}{2} = 50.5$$

Measures of Variation

The measures of central tendency provide an idea about the concentration of the observations about the central part of distribution. The measures of central tendency (mean and median) are not sufficient to give us a complete description of the data. They must be supported by other measures. These measures are the *measures of variation* or *measures of dispersion*. They tell us about the variation or dispersion of the data values around the average. We may have two or more sets of data all having the same average, but their spread or variability may be different. This is shown in Figures 3.5 and 3.6. Figure 3.5 shows that the data set A and B have the same mean but different variations. In this figure, curve B has less spread or variability than curve A. The more variation the data has, the more spread out the curve will be. We may also have a case where two

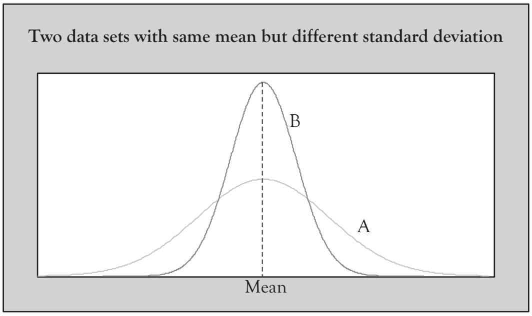


Figure 3.5 Data sets A and B with same mean but different variation

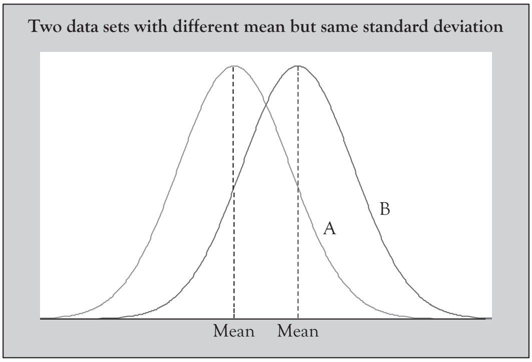


Figure 3.6 Data sets A and B with same variation but different mean

sets of data have the same variation but different means. You can see this in Figure 3.6.

If we measure only the mean of different data, we miss other important characteristics. Mean, median, and mode tell us only part of what we need to know about the characteristic of the data. In order to better analyze and understand the data, we must also measure its dispersion; that is, its spread or variability.

Dispersion or the variation is often used to compare two or more sets of data. In statistical quality control, one of the major objectives is to measure and reduce variability. This is done by extracting data from the process at different stages or time intervals, and analyzing the data by different means in order to measure and reduce variation. As the variation in the product and process are reduced, they become more consistent. Quality is inversely proportional to the variation. One way of improving quality is reducing variation.

The variability in the data is measured using the following measures. These are known as the *measures of variation*.

1. Range
2. Variance
3. Standard Deviation
4. Coefficient of Variation
5. Interquartile Range (IQR)

Out of these, the variance and the standard deviation are the most widely used.

Sample Variance (Denoted by s^2)

Sample variance is the sum of the squared differences between each of the observations and the mean. It is the average of squared distances from the mean. Suppose we have n number of observations $x_1, x_2, x_3, \ldots, x_n$ then the variance, s^2 is

$$s^2 = \frac{(x_1 - \overline{x})^2 + (x_2 - \overline{x})^2 + \ldots\ldots + (x_n - \overline{x})^2}{n-1}$$

or,

$$s^2 = \frac{\sum (x_i - \overline{x})^2}{n-1} \tag{3.1}$$

where, $\overline{x}$ = sample mean

n = sample size or number of observations

x_i = ith value of the random variable x (note that x_1 is the first value of the data point, x_2 is the second value of the data point and so on).

$\sum (x_i - \overline{x})^2$ is the sum of all squared differences between each of x_i values and the mean.

Another formula to calculate the sample variance

$$s^2 = \frac{\sum x^2 - \frac{\left(\sum x\right)^2}{n}}{n-1} \tag{3.2}$$

In Equation (3.2),

$\sum x^2$ is the sum of the squared values of the variable, x and
$\sum x$ is the sum of the values of the variable, x

Mathematically, both Equations (3.1) and (3.2) are identical and provide the same result. Equation (3.2) is computationally easier than Equation (3.1) for manual calculation because the first equation requires calculating the mean from the data, then subtracting the mean from each observation, squaring the values, and finally adding them. This process is tedious for large data set. The second equation simplifies the calculation.

Calculating the Sample Variance, s^2

The following data represents the length of a sample of parts in millimeters (mm).

5 8 10 7 10 14

Calculate the variance using Equation (3.1). Note that n = 6 (the number of observations)

Solution: First, calculate the sample mean using the following formula.

$$\overline{x} = \frac{\sum x}{n} = \frac{5+8+10+7+10+14}{6} = 9$$

Next, subtract each observation from the mean, square, and add the squared values. The calculations can be performed using Table 3.4.

The sample variance can now be calculated, using Equation (3.1) as

$$s^2 = \frac{\sum (x_i - \overline{x})^2}{n-1} = \frac{48}{5} = 9.6(mm)^2$$

Table 3.4 Calculation for variance

x_i	$(x_i - \bar{x})^2$	x_i^2
5	$(5-9)^2 = 16$	25
8	$(8-9)^2 = 1$	64
10	$(10-9)^2 = 1$	100
7	$(7-9)^2 = 4$	49
10	$(10-9)^2 = 1$	100
14	$(14-9)^2 = 25$	196
$\sum x = 54$	$\sum (x_i - \bar{x})^2 = 48$	$\sum x^2 = 534$

Note that the unit in which the data is measured (mm in this case) is also squared because we are taking each value in the data, subtracting the mean from it, and then squaring it. This results in squared unit which is a difficult configuration to interpret. This is the reason we take the square root of the variance. The value that is obtained by taking the square root of the variance is known as the *standard deviation*. Usually, we use the standard deviation to measure and compare the variability of two or more sets of data, not the variance.

Calculating the Variance Using Equation (3.2)

The following example shows the calculation of variance using Equation (3.2). Using this equation,

$$s^2 = \frac{\sum x^2 - \frac{\left(\sum x\right)^2}{n}}{n-1} = \frac{534 - \frac{(54)^2}{6}}{5} = \frac{48}{5} = 9.6$$

The values used in this equation are calculated in Table 3.4.

The variance obtained by this method is same as using Equation (3.1) in the previous example. Note the following features of variance:

- Variance can never be negative.
- If all the values in the data set are the same, the variance and standard deviation are zero, indicating no variability.

- Usually, no random phenomena will ever have the same measured values; therefore, it is important to know the variation in the data.

Calculating the Sample Standard Deviation, s

The sample standard deviation (denoted by s) is calculated by taking the square root of the variance. The standard deviation can be calculated using the following formulas.

$$s = \sqrt{s^2} = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}} \quad \text{or,}$$

$$s = \sqrt{s^2} = \sqrt{\frac{\sum x^2 - \frac{\left(\sum x\right)^2}{n}}{n-1}} \tag{3.3}$$

Example 3.3

Calculate the standard deviation of the data in the previous example.

***Solution*:** To calculate the standard deviation, we first calculate the variance. Using the variance in the previous example, standard deviation can be calculated as:

$$s = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}} = \sqrt{\frac{48}{5}} = 3.1$$

The sample standard deviation, $s = 3.1$ mm

The standard deviation is used as a measure of variation because it is expressed in the original units of measurements (not in the squared units as in the variance).

Relationship Between the Mean and Standard Deviation

There are two important rules in statistics that relate the mean and standard deviation: (1) the Chebyshev's Rule and (2) Empirical Rule. These rules combine the mean and the standard deviation in a data set and provide specific conclusions. The Empirical Rule provides more definite results and applies to symmetrical or bell-shaped data. The histogram, stem-and-leaf, and box plot discussed earlier provide information about the shape (symmetry or lack in symmetry) in the data.

The empirical rule applies to *symmetrical* or *bell-shaped distribution*. This is also known as the *normal distribution*. Unlike Chebyshev's theorem that applies to any shape (skewed or symmetrical), the empirical rule applies to symmetrical shape only. This rule states that if the data are symmetrical or bell shaped:

- Approximately 68 percent of the observations will lie between the mean and ± 1 standard deviation.
- Approximately 95 percent of the observations will lie between the mean and ± 2 standard deviations.
- Approximately 99.7 percent of the observations will lie between the mean and ± 3 standard deviations.

For a population data, the mean and standard deviation are denoted by μ (read as mu) and σ (sigma). Using these symbols, the earlier empirical rule can be stated as

$\mu \pm 1\sigma$ will contain approximately 68 percent of the observations
$\mu \pm 2\sigma$ will contain approximately 95 percent of the observations
$\mu \pm 3\sigma$ will contain approximately 99.7 percent of the observations
Note: for a sample data, the mean and standard deviation are denoted by $\bar{x}$ and s.

The empirical rule is graphically shown in Figure 3.7.

In quality control, the processes were traditionally controlled at 3-sigma level. This means that for a process that is controlled at 3-sigma level, 99.73 percent of the products would be within the specification.

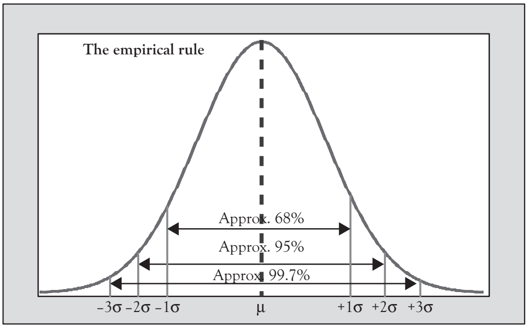


Figure 3.7 Areas under the normal curve

Measures of Position

Other important measures in describing the data are percentile and quartiles. These are known as *measures of position* and are described here.

Quantitative data are sometimes summarized in terms of percentiles. A percentile is a point below which a stated percentage or proportion of observations lie. Quartiles are special percentiles which divide the observations into groups of successive size, each containing 25 percent of the data points. The quartiles are denoted by Q_1: the first quartile or 25th percentile; Q_2: the second quartile or 50th percentile (which is also the median); and Q_3: the third quartile or 75th percentile. Another measure calculated using the quartiles is *IQR* which is the difference between the third quartile and the first quartile and encompasses the middle 50 percent of the values. One very useful data summary called *the five measure summary* provides a visual display of the data in the form of a plot known as the *box plot*. This is a plot of the minimum and maximum values and three quartiles, Q_1, Q_2, and Q_3. The box plot shows the data extremes, the range, the median, the quartiles, and the IQR.

The pth percentile of a data set is a value, such that at least p percent of the values are less than or equal to this value, and at least (100 – p) percent of the values are greater than or equal to this value.

A percentile tells us how the data values are spread out over the interval from the smallest value to the largest value.

If a score on a SAT test states that a student is at 85th percentile, it means that 85 percent who took the test had scores at or below this value and 15 percent of those who took the test scored higher than this value. The percentile value provides us a comparison in relation to other values.

The quartiles divide the data into four parts. For a large data set, it is often desirable to divide the data into four parts. This can be done by calculating the quartile. Note that

Q_1 = first quartile or 25th percentile

Q_2 = second quartile or 50th percentile or the median

Q_3 = third quartile or 75th percentile

The first quartile or Q_1 is the value such that 25 percent of the observations are below Q_1 and 75 percent of the values are above Q_1. The other quartiles can be interpreted in a similar way. Using the formula here, we can determine the percentile and quartile for any data set.

Calculating Percentiles and Quartiles

To find a percentile or quartile:

- arrange the data in increasing order
- find the location of the percentile using the following formula

$$L_p = (n+1)\frac{P}{100} \tag{3.4}$$

where L_p = location of the percentile

n = total number of observations

P = desired percentile

Example 3.4

The data in Table 3.5 show the monthly income of 20 part-time employees of a company. Find the median, the first quartile, and the third quartile values of the income data. Table 3.6 shows the sorted values from Table 3.5.

Table 3.5 Monthly income

Monthly income							
2038	1758	1721	1637	2097	2047	2205	1787
2287	1940	2311	2054	2406	1471	1460	1500
2250	1650	2100	1850				

Table 3.6 Sorted income data

Sorted data (read row-wise)							
1460	1471	1500	1637	1650	1721	1758	1787
1850	1940	2038	2047	2054	2097	2100	2205
2250	2287	2311	2406				

Table 3.7 Descriptive statistics of income data using MINITAB

Calculations using MINITAB						
Variable	N	Mean	Median	TrMean	StDev	SE Mean
Data	20	1928.5	1989.0	1927.9	295.2	66.0
Variable	Minimum	Maximum	Q_1	Q_3		
Data	1460.0	2406.0	1667.8	2178.8		

Note that the number of observations, $n = 20$.

Table 3.7 shows the descriptive statistics for the data in Table 3.5. This table shows the median, first quartile, and the third quartile values. We have explained earlier how to obtain these values using the percentile formula.

The median or Q_2 (50th percentile) is located at

$$L_p = (n+1)\frac{P}{100} = 21\left(\frac{50}{100}\right) = 10.5$$

This means that the median is located half way between the 10th and 11th value, or the average of the 10th and 11th value in the sorted data. This value is (1940 + 2038)/2 = 1989. Therefore,

Median or Q_2 = 1989

The first quartile (Q_1) or 25th percentile is located at

$$L_p = (n+1)\frac{P}{100} = 21\left(\frac{25}{100}\right) = 5.25$$

Therefore, Q_1 is the 5th value in the sorted data, plus 0.25 times the difference between the 5th and the 6th value, which is

$$1650 + (0.25)\ (1721 - 1650) = 1667.75 \text{ or,}$$
$$Q_1 = 1667.75 \text{ or } 1667.8$$

The third quartile (Q_3) or 75th percentile is located at

$$L_p = (n+1)\frac{P}{100} = 21\left(\frac{75}{100}\right) = 15.75$$

Thus Q_3 is the 15th value in the sorted data, plus 0.75 times the difference between the 15th and the 16th value, which is

$$2100 + (0.75)\ (2205 - 2100) = 2178.75$$

Therefore, $$Q_3 = 2178.75 \text{ or } 2178.8$$

The calculated values of the median, Q_1, and Q_3 are the same as in Table 3.7.

The Box Plot

A box plot uses a five-number summary as a graphical representation of data. These five numbers are as follows:

- Smallest or the minimum data value
- Q_1: the first quartile, or 25th percentile
- Q_2: the second quartile, or the median or 50th percentile
- Q_3: the third quartile, or 75th percentile
- Largest or the maximum data value

In our earlier discussions, we already explained percentiles and quartiles. Calculating these five measures from the data and constructing a box plot are explained subsequently.

Example 3.5

The utility bill for a sample of 50 customers ($n = 50$) rounded to the nearest dollar was collected. The data were sorted using computer software. Table 3.8 shows the sorted data. Construct a box plot of the utility bill data.

***Solution*:** The descriptive statistics of the data in Table 3.8 was calculated using the MINITAB software. The results are shown in Table 3.9. You should verify the results provided using the formulas. Figure 3.8 shows the box plot of the data.

The box plot simultaneously shows several features of the data including the central tendency, variability, the shape of the distribution (symmetrical or skewed), and the outliers in the data. In a box plot, Q_1, Q_2, and Q_3 are enclosed in a box. Q_2 is the median. If Q_2 or the median divides the box in approximately two halves, and if the distance from the X_{min} to Q_1 and Q_3 to X_{max} are equal or approximately equal, then the data are symmetrical. In case of right skewed data, the Q_2 line will not divide the box into two halves. Instead, it will be closer to Q_1 and the distance from Q_3 to X_{max} will be greater than the distance from X_{min} to Q_1.

Table 3.8 Sorted data

Sorted data

82	90	95	96	102	108	109	111	114	116
119	123	127	128	129	130	130	135	137	139
141	143	144	147	148	149	149	150	151	153
154	157	158	163	165	166	167	168	171	172
175	178	183	185	187	191	197	202	206	213

Table 3.9 Descriptive statistics of utility bill data

Descriptive Statistics: C1

Variable	N	Mean	Median	TrMean	StDev	SE Mean
Utility Bill	50	147.06	148.50	146.93	31.69	4.48
Variable	Minimum	Maximum	Q_1	Q_3		
Utility Bill	82.00	213.00	126.00	168.75		

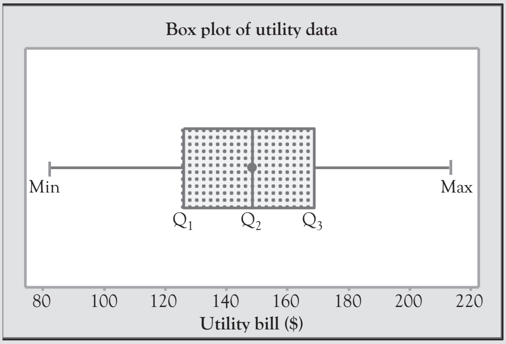


Figure 3.8 Box plot of the utility bill data

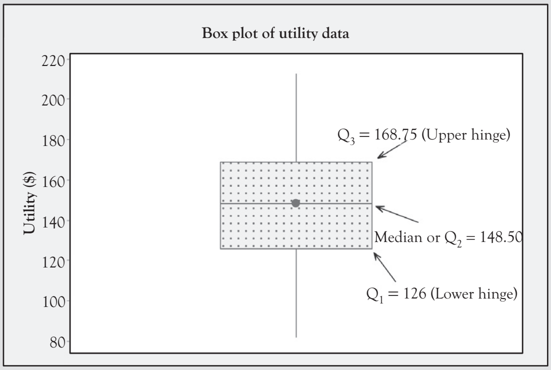


Figure 3.9 Box plot of utility data

In a box plot, the Q_1 or 25th percentile is also known as the lower quartile, Q_2 or 50th percentile is known as middle quartile, and Q_3 or 75th percentile is known as the upper quartile.

In a box plot, the three quartiles Q_1, Q_2, and Q_3 make a box. The top and the bottom sides of the rectangle are called the *upper* and *lower hinges* which are drawn at the quartiles Q_3 and Q_1 (see Figure 3.9). The middle 50 percent of the observations—those between Q_1 and Q_3—fall inside the box and the difference between the upper and the lower hinge or Q_3 and Q_1 is known as the *IQR*.

From Figure 3.9,

$$IQR = Q_3 - Q_1 = 168.75 - 126 = 42.75$$

The median or Q_2 is 148.50 (also shown in Figure 3.9). To construct the *tails* of the box, two sets of limits called *inner fences* and *outer fences* are used neither of which actually appear on the box plot. These fences are located at a distance of 1.5 (IQR) from the hinges. The vertical lines extending from the box are known as *whiskers*. The two whiskers extend to the most extreme observation to the fences. The lower whisker extends to the most extreme observation inside the lower inner fence, whereas the upper whisker extends to the most extreme observation inside the upper inner fence. The lower inner and upper inner fences can be calculated as shown here.

$$\text{Lower inner fence} = \text{Lower hinge} - 1.5\ (\text{IQR})$$
$$\text{Upper inner fence} = \text{Upper hinge} + 1.5\ (\text{IQR}) \qquad (3.5)$$

The lower and upper inner fences are used to determine the outliers in the data.

Applications of Box Plot

Consider a shaft manufacturing process. Suppose you took five samples each of size 36. Four machines were used in the production of these shafts. You want to check the consistency of the diameters with respect to the machines. Figure 3.10 shows the box plot.

Suppose you took five samples each of size 36. Four machines were used in the production of these shafts. You want to check the consistency

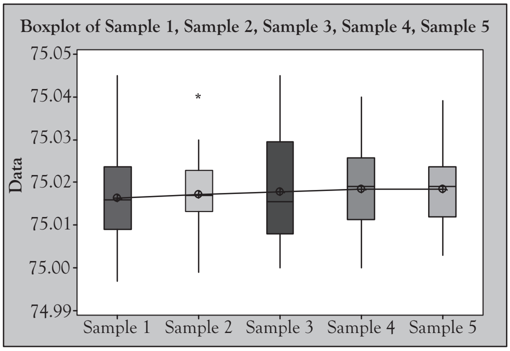


Figure 3.10 Box plot of samples 1 through 5

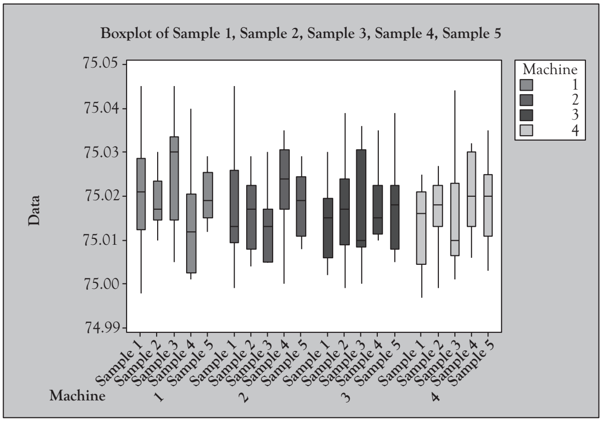


Figure 3.11 Box plot of samples versus machines

of the diameters with respect to the machines. A box plot in Figure 3.11 can be constructed to check the consistency and distribution of the diameters with respect to the machines.

Probability Distributions

The graphical and numerical techniques discussed in the previous sections are used to describe the sample data. These methods help us draw conclusions about the process from which data are collected.

The major objective of this section is to gain insight into several of the discrete and continuous probability distributions and their properties. A good knowledge and understanding of probability distributions is critical in being able to apply these distributions in data analysis, decision making, modeling, quality, and computer simulation. The probability distributions are used to calculate the probability of occurrence of certain phenomenon such as, the probability that the next produced item will be defective, or the probability or the percent of items not meeting the required specifications. In quality, a number of distributions are applied in solving quality problems. The probabilities are calculated using a number of probability distributions. This section provides an overview of probability distribution. Knowledge of these distributions is critical in modeling and solving quality problems.

Probability Distribution and Random Variable

The probability distribution is a model that relates the value of a random variable with the probability of occurrence of that value.

A random variable is a numerical value that is unknown and may result from a random experiment. The numerical value is a variable and the value achieved is subject to chance and is, therefore, determined randomly. Thus *a random variable is a numerical quantity whose value is determined by chance. Note that a random variable must be a numerical quantity.*

Types of random variables: Two basic types of random variables are *discrete* and *continuous* variables, which can be described by discrete and continuous probability distributions.

Discrete Random Variable

A random variable that can assume only integer value or whole number is known as discrete. An example would be the number of customers arriving at a bank. Another example of a discrete random variable would be rolling two dice and observing the sum of the numbers on the top faces. In this case, the results are 2 through 12. Also, note that each outcome is a whole number or a discrete quantity. The random variable can be described by a discrete probability distribution.

Table 3.10 shows the discrete probability distribution (in a table form) of rolling two dice and observing the sum of the numbers. In rolling two dice and observing the sum of the numbers on the top faces, the outcome is denoted by x which is the *random variable* that denotes the sum of the numbers.

In Table 3.10, the outcome X (which is the sum of the numbers on the top faces) takes on different values each time the pair of dice is rolled. On each trial, the sum of the numbers is going to be a number between 2 and 12 but we cannot predict the sum with certainty in advance.

Table 3.10 Outcome (X) and Corresponding Probabilities P(X)

X	2	3	4	5	6	7	8	9	10	11	12
$P(X)$	1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36	3/36	2/36	1/36

In other words, the outcomes or the occurrence of these numbers is a chance factor. The probability distribution is the outcomes X_i, and the probabilities for these outcomes $P(X_i)$. The probability of each outcome of this experiment can be found by listing the sample space or all 36 outcomes. These can be shown both in a tabular or a graphical form. Figure 3.12 shows the probability distribution graphically.

In summary, the relationship between the values of a random variable and their probabilities is summarized by a probability distribution. A probability distribution of a random variable is described by the set of possible random variable's values and their probabilities. *The probability distribution provides the probability for each possible value or outcome of a random variable.* A probability distribution may also be viewed as the shape of the distribution. The basic foundation of probability distributions is the laws of probability. Note that most of the phenomenon in real-world situation are random in nature. In a production situation, finding the number of defective product might be seen as a random variable because it takes on different values according to some random mechanism.

Continuous Random Variable

The random variable that might assume any value over a continuous range of possibilities is known as continuous random variables. Some examples of continuous variable are physical measurements of length, volume, temperature, or time. These variables can be described using continuous distributions.

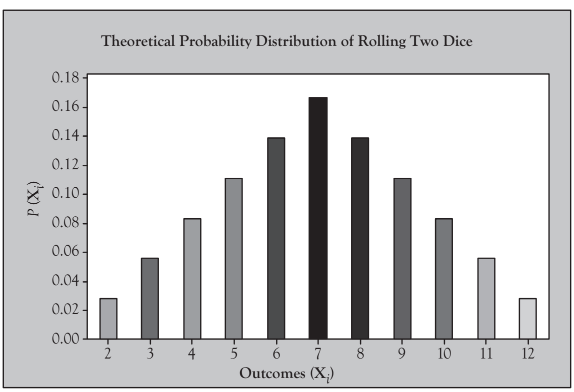


Figure 3.12 Probability distribution of rolling two dice

The *continuous probability distribution* is usually described using a probability density function. *The probability density function, f*(*x*), describes the behavior of a random variable. It may be viewed as the shape of the data. Figure 3.13 shows the histogram of the diameter of a machined part with a fitted curve. It is clear that the diameter can be approximated by certain pattern that can be described by a probability distribution.

The shape of the curve in the Figure 3.13 can be described by a mathematical function, $f(x)$, or a probability density function. The area below the probability density function to the left of a given value, x, is equal to the probability of the random variable (the diameter in this case) shown on the x-axis. The probability density function represents the entire sample space; therefore, the area under the probability density function must equal one.

The probability density function, $f(x)$, must be positive for all values of x (as negative probabilities are impossible). Stating these two requirements mathematically,

$$\int_{-\infty}^{\infty} f(x) = 1$$

and, $f(x) > 0$ for continuous distributions. For discrete distributions, the two conditions can be written as

$$\sum_{i=1}^{n} f(x) = 1.0 \text{ and, } f(x) > 0.$$

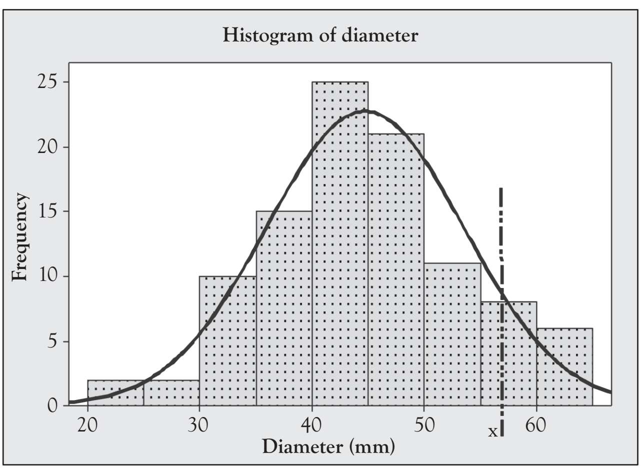


Figure 3.13 Diameter of machined parts

To demonstrate how the probability density function is used to compute probabilities, consider Figure 3.14. The shape in the figure can be well approximated by a *normal distribution*. Assuming a normal distribution, we would like to find the probability of a diameter below 40 mm. The area of the shaded region represents the probability of a diameter, drawn randomly from the population having a diameter less than 40 mm. This probability is 0.307 or 30.7 percent using a normal probability density. Figure 3.15 shows the probability of the diameter of one randomly selected machined part having a diameter greater than or equal to 50 mm. but less than or equal to 55 mm.

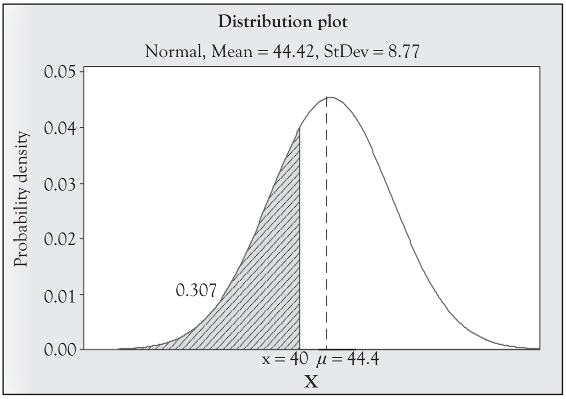


Figure 3.14 An example of calculating probability using probability density

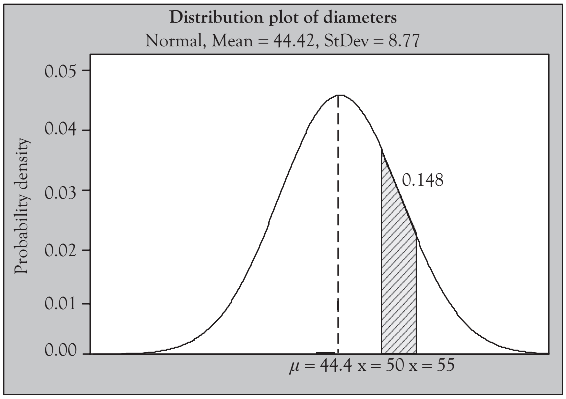


Figure 3.15 Another example of calculating probability using probability density

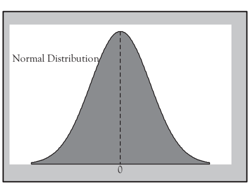
There are a number of distributions used in modeling and solving quality problems. Several of the control charts we are going to study later are based on the normal, binomial and Poisson distributions. Here we discuss some of the important continuous and discrete distributions.

Some Important Continuous Distributions

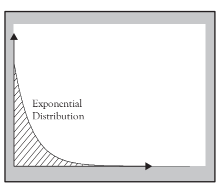
In this section, we will discuss the *continuous probability distributions*. When the values of random variables are not countable but involve continuous measurement, the variables are known as continuous random variables. Continuous random variables can assume any value over a specified range. Some examples of continuous random variables are as follows:

- Length of time to assemble an electronic appliance
- Life span of a satellite power source
- Fuel consumption in miles-per-gallon of new model of a car
- Inside diameter of a manufactured cylinder
- Amount of beverage in a 16-ounce can
- Waiting time of patients at an outpatient clinic

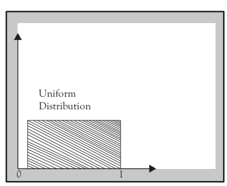
In all the previous cases, each phenomenon can be described by a random variable. The variables could be any value within a certain range and are not discrete whole numbers. The graph of a continuous random variable x is a smooth curve. This curve is a function of x, denoted by $f(x)$ and is commonly known as a *probability density function*. The probability density function is a mathematical expression that defines the distribution of the values of the continuous random variable. The following figures show the three continuous distributions.



Normal Distribution



Exponential Distribution



Uniform Distribution

One of the most widely used and important distribution in quality is the *normal distribution*. The other distribution of importance is the exponential distribution. We discuss both of these here. We will leave the other important distributions such as, the uniform, gamma, and Weibull distribution for the interested readers to explore further.

The Normal Distribution

Background: A continuous random variable X is said to follow a normal distribution with parameters μ and σ, and the probability density function of X is given by:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$$

where $f(x)$ is the probability density function, μ = mean, σ = standard deviation, and e = 2.71828, which denotes the base of the natural logarithm. The distribution has the following properties:

The normal curve is a bell-shaped curve. It is symmetrical about the line $x = \mu$. The mean, median, and mode of the distribution have the same value.

The parameters of normal distribution are the mean $\mu$and standard deviation σ. The interpretation of how the mean and the standard deviation are related in a normal curve is shown in Figure 3.16.

Figure 3.16 states the area property of the normal curve. For a normal curve, approximately 68 percent of the observations lie between the mean and $\pm 1\sigma$ (one standard deviation), approximately 95 percent of all observations lie between the mean and $\pm 2\sigma$ (two standard deviations), and approximately 99.73 percent of all observations falls between the mean and $\pm 3\sigma$ (three standard deviations). This is also known as the *empirical rule*.

The shape of the curve depends upon the mean (μ) and standard deviation (σ). The mean μ determines the location of the distribution, whereas the standard deviation σ determines the spread of the distribution. Note that larger the standard deviation (σ), more spread out is the curve (see Figure 3.17).

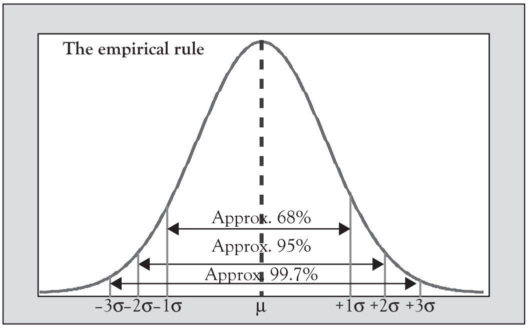


Figure 3.16 Areas under the normal curve

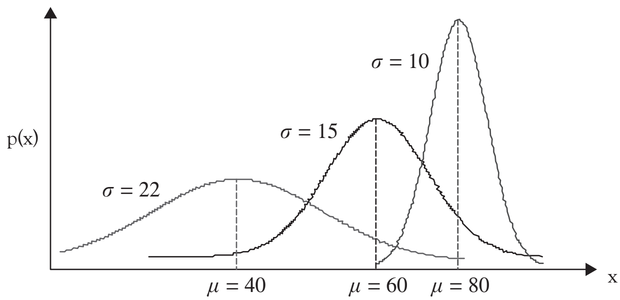


Figure 3.17 Normal curve with different values of mean and standard deviation

The Standard Normal Distribution

To calculate the normal probability, $P(x_1 \le X \le x_2)$ where X is a normal variate with parameters μ and σ, we need to evaluate:

$$\int_{x_1}^{x_2} \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}\,dx$$

To evaluate the earlier expression, none of the standard integration techniques can be used. However, the expression can be numerically evaluated for $\mu = 0$ and $\sigma = 1$. When the values of the mean μ and standard deviation σ are 0 and 1, respectively, the normal distribution is known as the *standard normal distribution*.

The normal distribution with $\mu = 0$ and $\sigma = 1$ is called a standard normal distribution. Also, a random variable with standard normal distribution is called a *standard normal random variable* and is usually denoted by Z.

The probability density function of Z is given by

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-Z^2/2\sigma^2} \qquad -\infty < z < \infty$$

The cumulative distribution function of Z is given by:

$$P(Z \le z) = \int_{-\infty}^{z} f(y)dy$$

which is usually denoted by Φ (z).

When the random variable X is normally distributed with mean μ and variance σ^2, that is; $x \sim N(\mu, \sigma^2)$, we can calculate the probabilities involving x by standardizing. The standardized value is known as the *standard* or *standardized normal distribution* and is given by:

As indicated previously, if x is normally distributed with mean μ and standard deviation σ, then

$$z = \frac{x - \mu}{\sigma} \qquad (3.6)$$

is a *standard normal random variable* where,

z = distance from the mean to the point of interest (x) in terms of standard deviation units

x = point of interest

μ = the mean of the distribution, and

σ = the standard deviation of the distribution.

Finding Normal Probability by Calculating Z Values and the Standard Normal Table

Example 3.6

The inside diameter of a piston ring is normally distributed with a mean of 5.07 cm and a standard deviation of 0.07 cm. What is the probability of obtaining a diameter exceeding 5.15 cm?

The required probability is the shaded area shown in Figure 3.18. To determine the shaded area, we first find the area between 5.07 and 5.15 using the z score formula and then subtract the area from 0.5. See the calculations here.

$$z = \frac{x - \mu}{\sigma}$$

$$z = \frac{5.15 - 5.07}{0.07} = 1.14 \rightarrow 0.3729$$

Note: 0.3729 is the area corresponding to z = 1.14. This can be read from the table of normal distribution provided in Appendix A. There are other variations of this normal table.

The required probability is

$$p(x \geq 5.15) = 0.5 - 0.3729 = 0.1271$$

or, there is 12.71 percent chance that piston ring diameter will exceed 5.15 cm.

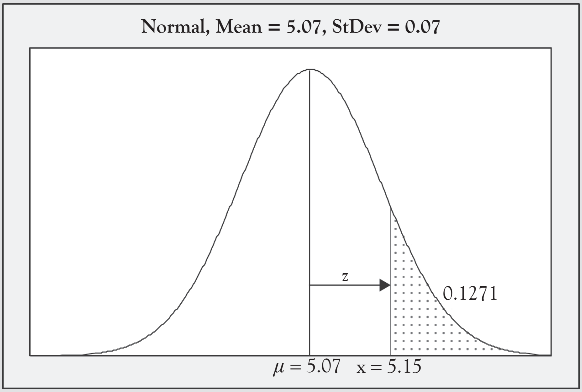


Figure 3.18 Area exceeding 5.15

Example 3.7

The measurements on certain type of PVC pipes are normally distributed with a mean of 5.01 cm and a standard deviation of 0.03 cm. The specification limits on the pipes are 5.0 ± 0.05 cm. What percentage of the pipes is not acceptable?

The percentage of acceptable pipes is the shaded area shown in Figure 3.19.

The required area or the percentage of acceptable pipes is explained here.

$$z_1 = \frac{x-\mu}{\sigma} = \frac{4.95-5.01}{0.03} = -2.0 \Rightarrow 0.4772$$

$$z_2 = \frac{x-\mu}{\sigma} = \frac{5.05-5.01}{0.03} = 1.33 \Rightarrow 0.4082$$

Note: the values 0.4772 and 0.4082 are obtained from the normal table in Appendix A.

The area 0.4772 is the area between the mean 5.01 and 4.95 (see Figure 3.19). The area left of 4.95 is 0.5 – 0.4772 = 0.0228.

The area 0.4082 is the area between the mean 5.01 and 5.05 (Figure 3.19). The area right of 5.05 is 0.5 – 0.4082 = 0.0918.

Therefore, the percentage of pipes not acceptable = 0.0228 + 0.0918 = 0.1146 or 11.46%.

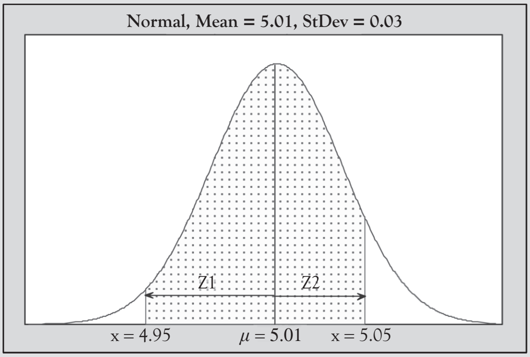


Figure 3.19 The percentage acceptable (shaded area)

The Exponential Distribution

The exponential distribution has wide applications in modeling. In the later section, we will describe a distribution of importance—the *Poisson distribution* which is often used to describe the *number* of arrivals (or occurrences) over a specified time period. The exponential distribution is used to describe such phenomenon as the time between failures of components, the time between arrivals of customers, or telephone calls, or the lifetime of certain types of components in a machine. This distribution is widely used in quality and reliability engineering to describe the time to failure of certain types of components.

If X is the random variable that represents the *number* of arrivals over a specified period T, then X is said to follow a Poisson distribution, and if Y represents the *time between successive arrivals*, then Y follows an exponential distribution. Thus the Poisson and exponential distributions are closely related.

The exponential distribution is an appropriate model to use when the *failure rate is constant*. For example, the time between failures of a computer chip is a continuous random variable and the failure rate is assumed to be a constant. This means that the probability of the chip failing in the next 48 hours would be the same now as it was in the past and this probability should be the same at any time in the future provided that the chip is functioning properly at a future time.

Probability Density of an Exponential Distribution

If the random variable X follows an exponential distribution then the probability density function is given by:

$$f(x) = \frac{1}{\mu} e^{-x/\mu} \qquad \text{where } x > 0 \text{ and } \mu > 0 \qquad (3.7)$$

Cumulative probabilities for exponential distribution is given by:

$$P(x \le x_0) = 1 - e^{-x/\mu} \qquad \text{for } x > 0 \qquad (3.8)$$

The mean and standard deviation of the exponential distribution are equal and are given by:

$$\text{Mean} = \mu$$

$$\text{Standard deviation,} \qquad \sigma = \mu \tag{3.9}$$

The parameter $1/\mu$ in Equation (3.7) is often referred to as the *failure rate (time between failures)* and is related to the Poisson distribution. Consider another example where the *number of arrivals per unit time* in the Poisson distribution and the *time between arrivals* in the exponential distribution can both be used to describe the same thing. For example, if the number of arrivals per unit time follows a Poisson distribution with mean or average of 10 arrivals per hour, then we can say that the time between arrivals is exponentially distributed with mean time between arrivals being 1/10= 0.1 hour or 6 minutes.

Unlike the normal distribution, which is described by its location and shape parameters (μ and σ, respectively), exponential distribution is described by only one parameter, μ. Each value of μ determines a unique exponential distribution. *The distribution has no shape or location parameter; it is described by a scale parameter which is* $(1/\mu)$. In the section that follows, we will investigate the shapes of the exponential distribution.

Investigating the Exponential Distribution

Objective: Investigate the general shape of the exponential distribution and observe how the shape of the distribution changes as we change the characteristic scale parameter, (μ) of the distribution.

Figure 3.20 shows the plot of the density functions of the exponential distribution for different values of (μ = 0.5, 1.0, 1.5, 2.0).

From Figure 3.20, it can be seen that the exponential distribution curve steadily decreases as the value of the random variable *x* increases. The larger the value of *x*; the probability of observing a value of *x* at least this large decreases exponentially. Note also that the distribution is not symmetrical and unlike the normal random variable, the exponential random variable is always greater than zero.

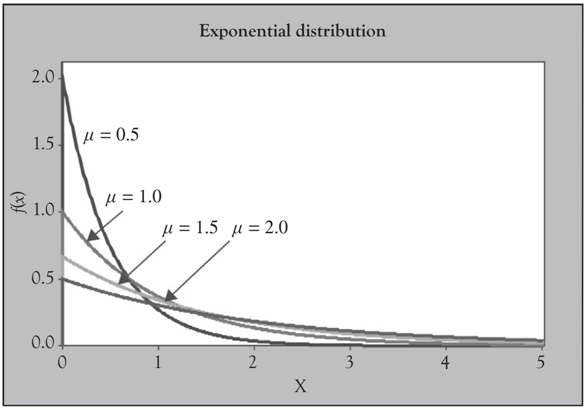


Figure 3.20 Graph of exponential distribution for different values of μ

Finding Exponential Probabilities

The probabilities for exponentially distributed random variables are found by evaluating the areas between the points of interest of the exponential curve described in Figure 3.20. Suppose X is an exponentially distributed random variable with parameter μ, then

$$P(X \geq x) = e^{-x/\mu} \qquad \text{for } x \geq 0 \qquad (3.10)$$

$$P(X \leq x) = 1 - e^{-x/\mu} \qquad \text{for } x > 0 \qquad (3.11)$$

$$P(x_1 \leq X \leq x_2) = e^{-x_1/\mu} - e^{-x_2/\mu} \qquad \text{for } x_1, x_2 > 0 \ (3.12)$$

The previous equations are used to find the probability between the points of interest in exponential distribution. These probabilities are explained in the following graph. Figure 3.21 demonstrates the probability, $P(X \geq x)$ for an exponential random variable. In Figure 3.22, the shaded probability is $P(X \leq x)$ for an exponential random variable.

The exponential distribution is widely used in the area of reliability engineering to describe the time to failure of a system or a component. Note that in the exponential distribution, if μ *is the mean time to failure,* then $1/\mu$ is the failure rate of the system.

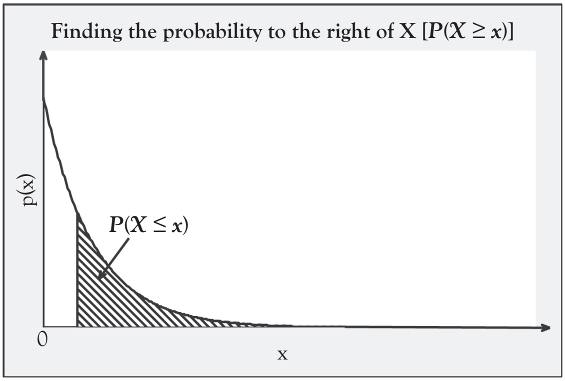


Figure 3.21 Finding the probability $P(X \geq x)$

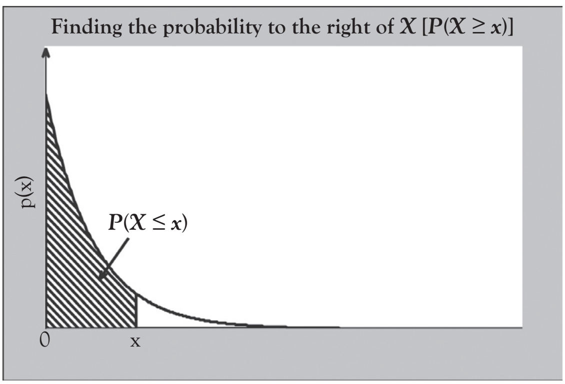


Figure 3.22 Finding the probability $P(X \leq x)$

Example 3.8

The useful life of an electronic component is described by the exponential distribution with a mean life of 1500 days.

(a) Find the probability that the component will fail before its expected life of 1500 days.

(b) The component has 2 years or 730 days of warranty. What percentage of the components will fail during the warranty period?

Solution:

(a) Note that the life of the component (x) follows an exponential distribution with μ = 1500. The probability to be evaluated is shown in the Figure 3.23. From this figure, we can see that

$$P(X \leq x) = 1 - e^{-x/\mu}$$
$$P(x \leq 1500) = 1 - e^{-1500/1500} = 0.632$$

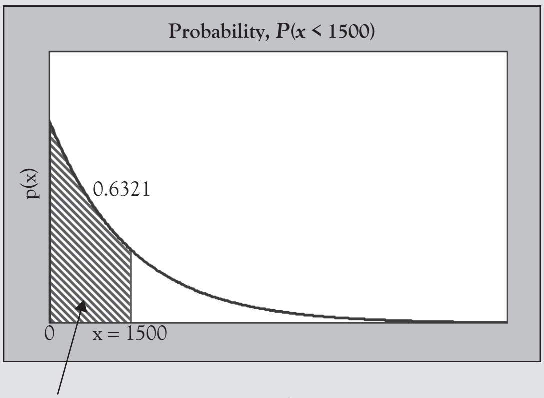
There is a 63.2 percent chance that the component will fail within 1500 hours.

(b) We want to find the probability, $P(x \leq 730)$. This probability can be evaluated as

$$P(x \leq 730) = 1 - e^{-730/1500} = 1 - e^{-0.4867} = 1 - 0.6147 = 0.3853$$

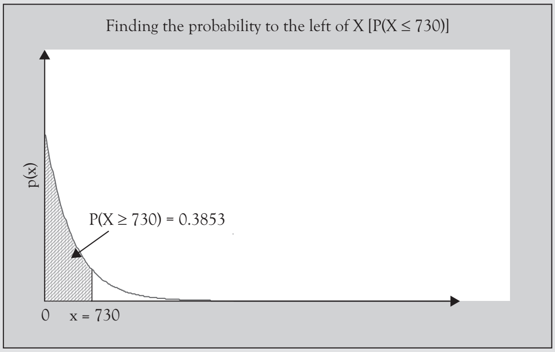
Figure 3.24 shows the graph with the shaded probability.

The preceding probability indicates that the manufacturer will have to replace approximately 38.5 percent of the components during the warranty period. Note that the average life of the components is approximately four years. But this high percentage (38.5 percent) of the components failure is due to the fact that the exponential distribution is positively skewed and there is high concentration of probability on the lower end of the distribution.



$$P(x < 1500) = 1 - e^{-1500/1500} = 1 - e^{-1} = 0.632$$

Figure 3.23 Probability, P(x < 1500)



$$P(x \le 730) = 1 - e^{-730/1500} = 1 - e^{-0.4867} = 1 - 0.6147 = 0.3853$$

Figure 3.24 Probability P(X ≤ 730)

Some Important Discrete Distributions

A number of discrete probability distributions are used to solve quality problems. Here we discuss some of these distributions. The widely used discrete distributions include the binomial, hypergeometric, and Poisson distributions.

The Binomial Distribution

The binomial distribution is a very widely used discrete distribution which describes discrete data resulting from an experiment known as a *Bernoulli* process.

Bernoulli Trials and Bernoulli Distribution

In many situations, the experiment or the process under study consists of *n* number of trials. Each trial has only two possible outcomes: success (S) and failure (F). We can denote this as

$x_j = 1$ if the experiment results in a success (S)
$x_j = 0$ if the experiments results in a failure (F)

The earlier situation is the basis of the Bernoulli distribution. The Bernoulli distribution may be defined as:

A random variable x that takes only two values 1 and 0 with probabilities p and q respectively; or,

$$P(x = 1) = p \text{ and}$$
$$P(x = 0) = q$$

where p and q are Bernoulli variates that follow a Bernoulli distribution. In the previous expression, p can also be referred to as the probability of success and q the probability of failure, such that $p = (1 - q)$.

Example 3.9

1. Outcomes of *n* number of tosses of a fair coin is a Bernoulli process because:
   - Each toss has only two possible outcomes, heads (H) and Tails (T), which may be denoted as a success or failure.
   - Probability of outcome remains constant over time, that is, for a fair coin the probability of success (or probability of getting a head) remains 1/2 for each toss regardless of the number of tosses.
   - The outcomes are independent of each other, that is, the outcome of one toss does not affect the outcome of any other toss.
2. Consider a manufacturing process in which the parts produced are inspected for defects. Each part in a production run may be classified as defective or nondefective. Each part to be inspected can be considered as a single trial that results in a success (if the part is found to be defective) or a failure (if it is nondefective). This is also an example of a Bernoulli trial.

Binomial Probabilities

A random variable that denotes *x* number of successes in *n* Bernoulli trials is said to have a Binomial distribution in which the probability of *x* successes is given by the following expression:

$$p(x) = \binom{n}{x} p^x (1-p)^{n-x} \qquad \text{where, } x = 0, 1, \ldots, n \qquad (3.13)$$

The *mean or expected value* and the variance of the Binomial distribution are given by

$$E(x) = \mu = np$$

$$\sigma^2 = np(1-p)$$

In the preceding expression,

$p(x)$ = probability of x number of successes
n = number of trials
p = probability of success
$(1-p) = q$ is the probability of failure

Example 3.10: Calculating Binomial Probabilities

A product is supposed to contain 5 percent defective items. Suppose a sample of 10 items is selected. What is the probability of finding (a) exactly two, (b) more than two.

***Solution*:** To calculate the Binomial probabilities, we must know n (the number of trials) and p (the probability of success). For this problem,

$$n = 10 \quad p = 0.05$$

(a) The probability of finding two defects; that is, $p\ (x = 2)$ can be calculated using the Binomial formula

$$p(x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

$$p(x=2) = \frac{10!}{2!(10-2)!}(0.05)^2(0.95)^8$$

$$= (45)(0.05)^2(0.95)^8$$

$$= 0.0746 \text{ or } 7.46\%$$

This probability can be easily calculated using a *binomial distribution table or a statistical package*. In order to calculate the binomial probabilities using the binomial table, you must know the number of trials, n, and the probability of success, p. A binomial probability distribution table for $n = 10$ and $p = 0.05$ would look like the one shown in Table 3.11. In this table, $p = 0.05$ and $n = 10$. We can read the probabilities for $x = 0$ through $x = 10$. The probabilities for $n = 10$, $p = 0.05$, and $x = 0, \ldots, 10$ are listed in Table 3.11. We will demonstrate the calculations in this example using the probability values in this table.

Note that if the number of trials n is 10, then the values under the x column will be from 0 through 10. This is because there cannot be more than 10 successes for $n = 10$ trials.

From the table below, the probability of $x = 2$ or $p(x = 2)$ can be read. This probability is 0.0746, which is same as the value we calculated using the formula earlier.

The binomial table below can be used to calculate the probabilities of $p(x > 2)$, $p(x = 3)$, $p(x \geq 3)$, $p(x \leq 3)$, and $p(x < 3)$. All the probability values are read from the binomial table (Table 3.11).

Table 3.11 Binomial distribution table for n = 10 and p = 0.05

n	x	$p = 0.05$
10	0	0.5987
	1	0.3151
	2	0.0746
	3	0.0105
	4	0.0010
	5	0.0001
	6	0.0000
	7	0.0000
	8	0.0000
	9	0.0000
	10	0.0000

(b) Probability of more than two defects

$$\begin{aligned} p(x>2) &= p(x=3)+p(x=4)+p(x=5)+p(x=6) \\ &\quad + p(x=7)+p(x=8)+p(x=9)+p(x=10) \\ &= 0.0105+0.0010+0.0001+0.0000+ \\ &\quad 0.0000+0.0000+0.0000+0.0000 \\ &= 0.0116 \end{aligned}$$

This probability can also be calculated easily, using the following expression

$$\begin{aligned} p(x>2) &= 1-p(x\le 2) \\ &= 1-[p(x=0)+p(x=1)+p(x=2)] \\ &= 1-[0.5987+0.3151+0.0746]=0.0116 \end{aligned}$$

The probability $p(x>2)=1-p(x\le 2)$ because the sum of the probabilities over 10 trials must add up to 1.0 (verify this by adding the second column of Table 3.11). Therefore, the probability, $p(x>2)$, can be calculated by subtracting the probabilities of $p(x=0)$, $p(x=1)$, and $p(x=2)$ from 1.0 or $p(x>2)=1-[p(x=0)+p(x=1)+p(x=2)]$. Calculating the probability this way reduces the computation significantly. The binomial distribution has applications in control charts. The control chart for proportion defective is based on this distribution.

The Poisson Distribution

A random variable *X* is said to follow a Poisson distribution if it assumes only nonnegative values and its probability density function is given by:

$$p(x)=\frac{e^{-\mu}\mu^{x}}{x!} \tag{3.14}$$

where, $x = 0,1,2,......,n$

The *mean or expected value* and the variance of the Poisson distribution are given by

$$Expected\ \ value = \mu$$
$$Variance,\ \sigma^2 = \mu$$

where μ represents the mean and variance of the distribution where $\mu > 0$. The Poisson distribution occurs when there are events which do *not* occur as outcomes for a fixed number of trials of an experiment (unlike that of the binomial distribution), but which occur at random points of time and space. *The Poisson distribution is the correct distribution to apply when n is very large (that is, the area of opportunity is very large) and an event has a constant and very small probability of occurrence.*

One of the major applications of Poisson distribution in quality control is in modeling the number of defects or nonconformities that may occur in one unit of a product. Any random phenomenon that can be described to occur on per unit area, per unit volume, or per unit time can be modeled using the Poisson distribution.

Example 3.11

It has been found that during the morning hours on any given day, a bank teller can expect an average of three customers per minute at the counter. If the teller can handle a maximum of two customers per minute, what is the probability that during any given minute of the morning hour, the teller will be swamped? Assume that the arriving customers follow a Poisson distribution.

The probability that a teller will be swamped is the probability when the number of customers during any given minute exceeds two. We need to find the probability,

$$p(x > 2) \text{ given } \mu = 3$$

which can be calculated as

$$\begin{aligned} p(x > 2) &= 1 - p(x \leq 2) \\ &= 1 - [p(x = 0) + p(x = 1) + p(x = 2)] \\ &= 1 - [0.0498 + 0.1494 + 0.22400] \\ &= 0.5768 \end{aligned}$$

The probabilities of x = 0, x = 1, x = 2 can be obtained from the Poisson probability table or a statistical package. In this table, we first locate $\mu = 3$ and then read the probabilities for $p(x=0)$, $p(x=1)$ and $p(x=2)$.

Therefore, the chances are about 1 in 2 that during any given minute of the morning hour, the teller will not be able to handle the arriving customers adequately.

Example 3.12

Suppose that the number of weld defects per foot in welded joints follows a Poisson distribution with an average of 1 defect per 10 feet.

(a) What is the probability that during an inspection, the inspector will find at least two defects in a 5-foot joint?

The average number of defects is 1 per 10 feet. Therefore, there is an average of 0.5 defects in 5-foot joint. We need to find the probability

$$p(x \geq 2) \text{ when } \mu=0.5$$

$$\begin{aligned} p(x \geq 2) &= 1 - p(x < 2) \\ &= 1 - [p(x=0) + p(x=1)] \\ &= 1 - [0.6065 + 0.3033] \\ &= 0.0902 \end{aligned}$$

[$p(x = 0)$ and p(x = 1) are obtained from a Poisson table for μ = 0.5 or a statistical package.]

(b) What is the probability that the inspector will find between 8 and 12 defects (inclusive) in a 50-foot joint?

The average number of defects per 50 feet will be 5. Therefore, μ = 5.0. The probabilities can be obtained from the Poisson table or a statistical package.

$$\begin{aligned} p(8 \leq x \leq 12) &= p(x=8) + p(x=9) + p(x=10) + p(x=11) + p(x=12) \\ &= 0.0653 + 0.0363 + 0.0181 + 0.0082 + 0.0034 \\ &= 0.1313 \end{aligned}$$

There are other probability distributions both discrete and continuous that are extensively used in modeling and solving quality problems.

Summary

In this chapter, we provided an overview of statistical methods used in quality programs. A number of statistical techniques both graphical and numerical were presented. These descriptive statistics tools are used in modeling, studying, and solving quality problems. The graphical and numerical tools of descriptive statistics presented in this chapter are also used to describe variation in the process data. The graphical tools of descriptive statistics we presented in this chapter include the concept of frequency distribution, histograms, stem-and-leaf plot, and box plot. These are simple but effective tools to study variation in the quality characteristic of interest. A number of numerical measures presented in this chapter are the measures of central tendency that include the mean and the median. Statistics deals with variation and one of the objectives of quality improvement is to study and reduce variation in products and processes. We presented a number of statistical measures including the variance and standard deviation. Standard deviation is a measure of variation. When combined with the mean it provides useful information. We discussed the empirical rule that provides a relationship between the mean and standard deviation in a data set. In the second part of this chapter, we introduced the concept of probability distribution and random variable. A number of probability distributions both discrete and continuous were discussed with their properties and applications. We discussed the normal and exponential probability distributions and their applications in quality. We also studied some discrete distributions including the binomial and Poison distribution, their properties and applications in quality. The probability distributions have wide applications in process control.

CHAPTER 4

Making Inferences About Process Quality

Introduction

In the previous chapter, we discussed probability distributions of discrete and continuous random variables and studied several of the distributions. The understanding and knowledge of these distributions are critical in decisions involving quality-related problems. This chapter extends the concept of probability distribution to that of sample statistics. *Sample statistics* are measures calculated from the sample data to describe a data set. The commonly used sample statistics are the sample size (n), sample mean ($\bar{x}$), sample variance (s^2), sample standard deviation (s), and sample proportion ($\bar{p}$). Note that some quality characteristics are expressed in proportion or percent. For example, percent of defects produced. Proportions are also used in poll results. Proportion is perhaps the most widely used statistics after the mean.

Since the above measures are calculated from the population data, they are called the *population parameters*. These parameters are population size (N), population mean (μ), population variance (σ^2), population standard deviation (σ), and population proportion (p). In most cases, the sample statistics are used to estimate the population parameters. The reason for this estimation is that the parameters of the population are unknown and they must be estimated. In estimating these parameters, we take samples and use the sample statistics to estimate the unknown population parameters. For example, suppose we want to know the average height of women in a country. To do this, we would take a reasonable sample of women, measure their heights, and calculate the average. This average will serve as an estimate. To know the average height of the population (or the population mean), we need to measure the height of every women in the country which is not practical. In most cases, we don't know the true value of a population parameter. We estimate these parameters using the sample statistics.

In this chapter, we will answer questions related to samples and sampling distributions. In sampling theory, we need to consider several factors and answer questions such as, why do we use samples? What is a sampling distribution and what is the purpose behind it?

Samples are used to make inferences about the population and this can be done through sampling distribution. The probability distribution of a sample statistic is called its *sampling distribution*. We will also study the *central limit theorem* and see how the amazing results produced by it are applied in analyzing and solving problems involving control charts.

The concepts of sampling distribution form the basis for the inference procedures we are going to discuss in this chapter. It is important to note that a population parameter is always a constant, whereas a sample statistic is a random variable. Similar to the other random variables, each sample statistic can be described using a probability distribution.

Besides sampling and sampling distribution, other key topics in this chapter include point and confidence interval estimates of means and proportions. We also discuss the concepts of hypothesis testing which is directly related to the control charts in statistical process control.

Statistical Inference and Sampling Techniques

Statistical Inference

The objective of statistical inference is to draw conclusions or make decisions about a population based on the samples selected from the population. To be able to draw conclusion from the sample, the distribution of the samples must be known. Knowledge of sampling distribution is very important for drawing conclusion from the sample regarding the population of interest.

Sampling Distribution

Sampling distribution is the probability distribution of a sample statistic (sample statistic may be a sample mean $\bar{x}$, a sample variance s^2, a sample standard deviation s, or sample proportion, $\bar{p}$).

As indicated earlier, in most cases the true value of the population parameters are not known. We must draw a sample or samples and calculate the sample statistic to estimate the population parameter. The sampling error of the sample mean is given by

$$\text{Sampling Error} = \bar{x} - \mu$$

Suppose we want to draw a conclusion about the mean of certain population. We would collect samples from this population, calculate the mean of the samples, and determine the probability distribution (shape) of the sample means. This probability distribution may follow a normal or a t-distribution, or other distribution. The distribution will then be used to draw conclusion about the population mean.

- *Sampling distribution of the sample mean* ($\bar{x}$) is the probability distribution of all possible values of the sample mean, $\bar{x}$.
- *Sampling distribution of sample proportion*, $\bar{p}$ is the probability distribution of all possible values of the sample proportion, $\bar{p}$.

The process of sampling distribution is illustrated in Figure 4.1.

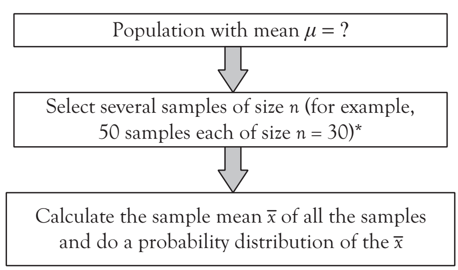


Figure 4.1 Process of sampling distribution

* 50 samples each of size $n = 30$ means that 50 different samples are drawn, where each sample will have 30 items in it. Also, a probability distribution is similar to a frequency distribution. Using the probability distribution, the shape of the sample means is determined.

Example 4.1 Examining the Distribution of the Sample Mean, $\bar{x}$

The assembly time of a particular electrical appliance is assumed to have a mean, $\mu = 25$ minutes, and a standard deviation, $\sigma = 5$ minutes.

1. Draw 50 samples each of size 5 ($n = 5$) from this population using MINITAB statistical software or any other statistical package.
2. Determine the average or the mean of each of the samples drawn.
3. Draw a histogram of the sample means and interpret your findings.
4. Determine the average and standard deviation of the 50 sample means. Interpret the meaning of these.
5. What conclusions can you draw from your answers to (3) and (4)?

Solution to (1): Table 4.1 shows 50 samples each of size 5 using MINITAB.

Table 4.1 Fifty samples of size 5 (n = 5)

Sample						Mean, $\bar{x}$
1	24.13	26.53	33.99	17.09	23.39	25.03
2	26.55	30.49	30.57	26.83	28.46	28.58
3	15.39	26.33	23.04	21.12	26.82	22.54
4	15.99	27.09	27.73	24.95	21.90	23.53
5	26.16	26.16	21.37	36.40	27.25	27.47
6	26.68	26.55	26.72	26.24	26.31	26.50
7	30.37	15.32	25.11	24.10	31.68	25.32
8	31.23	24.27	33.72	30.80	25.31	29.06
9	24.87	16.56	31.46	31.51	16.64	24.21
10	26.14	31.61	25.19	24.10	17.42	24.89
:						
:						
48	22.38	15.57	30.79	19.98	26.68	23.08
49	13.67	26.49	25.37	30.01	23.00	23.71
50	26.01	24.35	21.94	16.89	23.73	22.58

Solution to (2)**:** The last column shows the mean of each sample drawn. Note that each row represents a sample of size 5.

Solution to (3)**:** Figure 4.2 shows the histogram of the sample means shown in the last column of Table 4.1. The histogram shows that the sample means are normally distributed. Figure 4.2 is an example of the sampling distribution of the sample means $\bar{x}$.

In a similar way, we can do the sampling distribution of other statistics such as, the sample variance or the sample standard deviation. As we will see later, the sampling distribution provides the distribution or the shape of the sample statistic of interest. This distribution is useful in drawing conclusions.

Solution to (4)**:** The mean and standard deviation of the sample means shown in the last column of Table 4.1 were calculated using a computer package. These values are shown in Table 4.2.

The mean of the sample means is 24.98, which indicates that $\bar{x}$ values are centered at approximately the population mean of $\mu = 25$.

However, the standard deviation of 50 sample means is 2.285, which is much smaller than the population standard deviation, $\sigma = 5$.

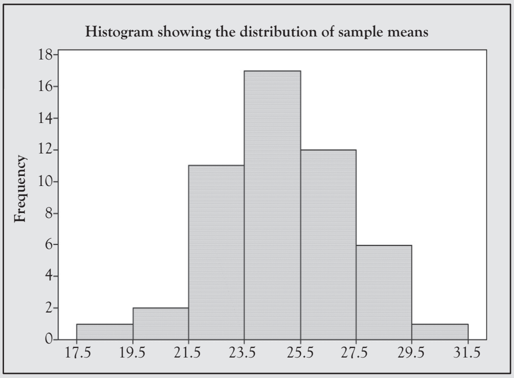


Figure 4.2 Sampling distribution of the sample means

Table 4.2 Mean and standard deviation of sample means

Descriptive statistics: Sample mean	
Mean	StDev
24.978	2.285

Thus we conclude that $\bar{x}$—or the sample mean values—have much less variation than the individual observations.

Solution to (5): Based on parts (3) and (4), we conclude that the sample mean, $\bar{x}$ follows a normal distribution, and this distribution is much narrower than the population of individual observations, which has a standard deviation, $\sigma = 5$. This is apparent from the standard deviation of $\bar{x}$ value, which is 2.285 (see Table 4.2). In general, the mean and standard deviation of the random variable $\bar{x}$ are given by

Mean of the sample mean, $\bar{x}$ is

$$\mu_{\bar{x}} = \mu \quad \text{or} \quad E(\bar{x}) = \mu \tag{4.1}$$

The standard deviation of the sample mean $\bar{x}$ is

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \tag{4.2}$$

For our example, $\mu = 25$, $\sigma = 5$, and $n = 5$. Using these values

$$\mu_{\bar{x}} = \mu = 25$$

and,
$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{5}{\sqrt{5}} = 2.236$$

From Table 4.2, the mean and the standard deviation of 50 sample means were 24.978 and 2.285 respectively. These values will get closer to 25 and 2.236 if we take more and more samples of size 5.

Standard Deviation of the Sample Mean or the Standard Error

Both Equations (4.1) and (4.2) are of considerable importance. Equation (4.2) shows that the standard deviation of the sample mean, $\bar{x}$ (or the sampling distribution of the random variable $\bar{x}$) varies inversely as the square root of the sample size. Since the standard deviation of the mean is a measure of the scatter of the sample means, it provides the precision that we can expect of the mean of one or more samples. The standard deviation of the sample mean $\sigma_{\bar{x}}$ is often called the *standard error*

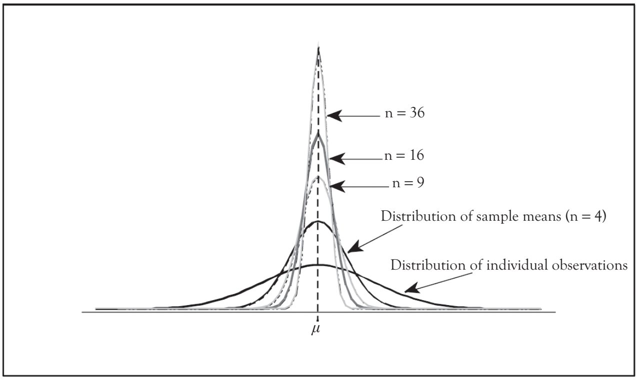


Figure 4.3 Probability distribution of sample means (n = 4, 9, 16, and 36) compared to individual observations

of the mean. Using Equation (4.2), it can be shown that a sample of 16 observations ($n = 16$) is twice as precise as a sample of 4 ($n = 4$). It may be argued that the gain in precision in this case is small, relative to the effort in taking additional 12 observations. However, doubling the sample size in other cases may be desirable.

Figure 4.3 shows a comparison between the probability distribution of individual observations and the probability distributions of means of samples of various sizes drawn from the underlying population.

Note that as the sample size increases, the standard error becomes smaller and hence the distribution becomes more peaked. It is obvious from Figure 4.3 that a sample of one does not tell us anything about the precision of the estimated mean. As more samples are taken, the standard error decreases, thus providing greater precision. The sample size plays an important role in designing the control charts.

Central Limit Theorem

The other important concept in statistics and sampling is the *central limit theorem*. The theorem states that as the sample size (n) increases, the distribution of the sample mean ($\bar{x}$) approaches a normal distribution.

This means that if samples of large size ($n \geq 30$) are selected from a population, then the sampling distribution of the sample means is approximately normal. This approximation improves with larger samples.

The Central Limit Theorem has major applications in sampling and other areas of statistics. It tells us that if we take a large sample ($n \geq 30$), we can use the normal distribution to calculate the probability and draw conclusion about the population parameter.

- Central Limit Theorem has been proclaimed as "the most important theorem in statistics"* and "perhaps the most important result of statistical theory."
- The Central Limit Theorem can be proven to show the *amazing result* that the mean values of the sum of a large number of independent random variables are normally distributed.
- The probability distribution resulting from "a large number of individual effects … would tend to be Gaussian."

The results mentioned earlier are useful in drawing conclusions from the data. For a sample size of $n = 30$ (large sample), we can always use the normal distribution to draw conclusions from the sample data.

- For a large sample, the sampling distribution of the sample mean ($\bar{x}$) follows a normal distribution and the probability that the sample mean ($\bar{x}$) is within a specified value of the population mean (μ) can be calculated using the following formulas:

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} \quad \text{(for an infinite population)} \qquad (4.3)$$

or

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}}} \quad \text{(for a finite population)} \qquad (4.4)$$

---

* Ostle (1979, 76).

In the above equations, n is the sample size and N is the population size. In a finite population, the population size N is known, whereas in an infinite population, the population size is infinitely large. Equation (4.3) is for an infinite population, and Equation (4.4) is for a finite population.

Review of Estimation, Confidence Intervals, and Hypothesis Testing

Estimation and hypothesis testing come under inferential statistics. *Inferential statistics* is the process of using *sample* statistics to draw conclusions about the population parameters. *Interference* problems are those that involve inductive generalizations. For example, we use the *statistics* of the sample to draw conclusions about the *parameters* of the population from which the sample was taken. An example would be to use the average grade achieved by one class to estimate the average grade achieved in all 10 sections of the same course. The process of estimating this average grade would be a problem of inferential statistics. In this case, any conclusion made about the 10 sections would be a generalization which may not be completely valid so it must be stated how likely it is to be true.

Statistical inference involves generalization and a statement about the probability of its validity. For example, an engineer or a scientist can make inferences about a population by analyzing the samples. Decisions can then be made based on the sample results. Making decisions or drawing conclusions using sample data raises question about the likelihood of the decisions being correct. This helps us understand why probability theory is used in statistical analysis.

Tools of Inferential Statistics

Inferential tools allow a decision maker to draw conclusions about the population using the information from the sample data. There are two major tools of inferential statistics; *estimation* and *hypothesis testing*. Figure 4.4 shows the tools of inferential statistic.

- Estimation is the simplest form of inferential statistics in which a sample statistic is used to draw conclusion about an unknown population parameter.

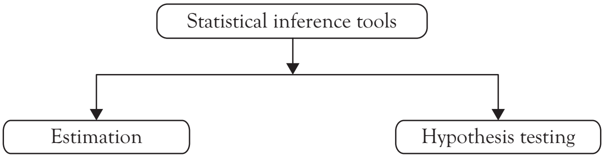


Figure 4.4 Tools of inferential statistics

- An estimate is a numerical value assigned to the unknown population parameter. In statistical analysis, the calculated value of a *sample statistic* serves as the estimate. This statistic is known as the estimator of the unknown parameter.
- Estimation or parameter estimation comes under the broad topic of *statistical inference*.
- The objective of parameter estimation is to estimate the unknown population parameter using the sample statistic. Two types of estimates are used in parameter estimation: *point estimate* and *interval estimate*.

The parameters of a process are generally unknown; they change over time and must be estimated. The parameters are estimated using the techniques of estimation theory. Hypothesis testing involves making a decision about a population parameter using the information in the sample data. These techniques are the basis for most statistical methods.

Estimation

There are two types of estimates: (a) the point estimates, which are single-value estimates of the population parameter, and (b) the interval estimates or the *confidence intervals* which are a range of numbers that contain the parameter with specified degree of confidence known as the *confidence level*. Confidence level is a probability attached to a confidence interval that provides the reliability of the estimate. In the discussion of estimation, we will also consider the *standard error* of the estimates, the *margin of error*, and the *sample size* requirement.

Point Estimate

As indicated, the purpose of a point estimate is to estimate the value of a population parameter using a sample statistic. The population parameters are μ, σ, p, and so on.

a. The point estimate of the population mean (μ) is the sample mean ($\bar{x}$), $\bar{x} = \frac{\sum x}{n}$.

b. The point estimate of the population standard deviation (σ) is the sample standard deviation (s).

$$s = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}} \qquad \text{or,} \qquad s = \sqrt{\frac{\sum x_i^2 - \frac{(\sum x_i)^2}{n}}{n-1}}$$

c. The point estimate of a population proportion (p) is the sample proportion ($\bar{p}$): $\bar{p} = \frac{x}{n}$ where x = no. of successes and n = sample size.

Interval Estimate

An *interval estimate* provides an interval or range of values that is used to estimate a population parameter. To construct an interval estimate, we find an *interval* about the point estimate so that we can be highly confident that it contains the parameter to be estimated. An interval with high confidence means that it has a high probability of containing the unknown population parameter that is estimated.

An interval estimate acknowledges that the sampling procedure is subject to error, and therefore, any computed statistic may fall above or below its population parameter target.

The interval estimate is represented by an interval or range of possible values so it implies the presence of uncertainty. An interval estimate is represented in one of the following ways:

$$16.8 \le \mu \le 18.6$$

or

$$(16.8 \text{ to } 18.6)$$

A formal way of writing an interval estimate is $L \leq \mu \leq U$, where L is the lower limit and U is the upper limit of the interval. The symbol μ indicates the population mean μ is estimated. The interval estimate involves certain probability known as the confidence level.

Confidence Interval Estimate

In many situations, a point estimate does not provide enough information about the parameter of interest. For example, if we are estimating the mean or the average salary for the students graduating with a bachelor's degree in business, a single estimate which would be a point estimate may not provide the information we need. The point estimate would be the sample average and will just provide a single estimate that may not be meaningful. In such cases, an interval estimate of the following form is more useful:

$$L \leq \mu \leq U$$

It also acknowledges sampling error. The end points of this interval will be random variables since they are a function of sample data.

To construct an interval estimate of unknown parameter β, we must find two statistics L and U such that

$$P\{L \leq \beta \leq U\} = 1 - \alpha \tag{4.5}$$

The resulting interval $L \leq \beta \leq U$ is called a 100 $(1 - \alpha)$ percent confidence interval for the unknown parameter β. L and U are known as the lower and upper confidence limits, respectively, and $(1 - \alpha)$ is known as the confidence level. A confidence level is the probability attached to a confidence interval. A 95 percent confidence interval means that the interval is estimated with a 95 percent confidence level or probability. This means that there is a 95 percent chance that the estimated interval would include the unknown population parameter being estimated.

Interpretation of Confidence Interval

The confidence interval means that if many random samples are collected and a 100 $(1 - \alpha)$ percent confidence interval computed from each sample for β, then 100 $(1 - \alpha)$ percent of these intervals will contain the true value β.

In practice, we usually take one sample and calculate the confidence interval. This interval may or may not contain the true value, and it is not reasonable to attach a probability level to this specific event. The appropriate statement would be that β lies in the observed interval $[L, U]$ with confidence 100 $(1 - \alpha)$. That is, we don't know if the statement is true for this specific sample, but the method used to obtain the interval $[L, U]$ yields correct statement 100 $(1 - \alpha)$ percent of the time. The interval $L \leq \beta \leq U$ is known as a two-sided or two-tailed interval. We can also build one-sided interval. The length of the observed confidence interval is an important measure of the quality of information obtained from the sample. The half interval $(\beta - L)$ or $(U - \beta)$ is called the accuracy of the estimator. A two-sided interval can be interpreted in the following way:

> *The wider the confidence interval, the more confident we are that the interval actually contains the unknown population parameter being estimated. On the other hand, the wider the interval, the less information we have about the true value of β. In an ideal situation, we would like to obtain a relatively short interval with high confidence.*

Confidence Interval for the Mean, Known Variance σ^2 (or σ Known)

The confidence interval estimate for the population mean is centered around the computed sample mean $(\bar{x})$. The confidence interval for the mean is constructed based on the following factors:

a. The size of the sample (n),
b. The population variance (known or unknown), and
c. The level of confidence.

Let X be a random variable with an unknown mean μ and known variance σ^2. A random sample of size n $(x_1, x_2, \ldots, x_n)$ is taken from the population. A 100 $(1 - \alpha)$ percent confidence interval on μ can be obtained by considering the sampling distribution of the sampling mean $\bar{x}$. We know that the sample mean $\bar{x}$ follows a normal distribution as the sample size n increases. For a large sample n the sampling distribution of the sample mean is almost always normal. The sampling distribution is given by:

The distribution of the earlier is normal and is shown in Figure 4.5.

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} \tag{4.6}$$

To develop the confidence interval for the population mean μ, refer to Figure 4.5.

From this figure we see that:

or

$$P\{-z_{\alpha/2} \le z \le z_{\alpha/2}\} = 1 - \alpha$$

This can be rearranged to give:

$$P\left\{-z_{\alpha/2} \le \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} \le z_{\alpha/2}\right\} = 1 - \alpha$$

This leads to:

$$P\left\{\bar{x} - z_{a/2}\sigma/\sqrt{n} \le \mu \le \bar{x} + z_{a/2}\sigma/\sqrt{n}\right\} = 1 - a$$

$$\left\{\bar{x} - z_{a/2}\sigma/\sqrt{n} \le \mu \le \bar{x} + z_{a/2}\sigma/\sqrt{n}\right\} \tag{4.7}$$

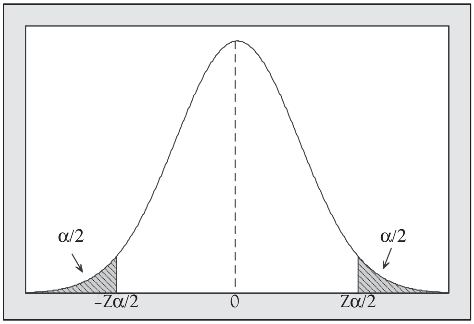


Figure 4.5 Distribution of the sample mean

Equation (4.7) is a 100 $(1 - \alpha)$ percent confidence interval for the population mean μ.

The Confidence Interval Formula to Estimate the Population Mean μ for Known and Unknown Population Variances or Standard Deviations

The confidence interval is constructed using a normal distribution. The following two formulas are used when the sample size is large:

(a) Known population variance (σ^2) or known standard deviation (σ)

$$\bar{x} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \le \mu \le \bar{x} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \tag{4.8}$$

Note that the margin of error is given by

$$E = z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \tag{4.9}$$

(b) Unknown population variance (σ^2) or unknown standard deviation (σ)

$$\bar{x} - t_{n-1,\alpha/2}\frac{s}{\sqrt{n}} \le \mu \le \bar{x} + t_{n-1,\alpha/2}\frac{s}{\sqrt{n}} \tag{4.10}$$

(c) If the population variance is unknown and the sample size is large, the confidence interval for the mean can also be calculated using a normal distribution using the following formula:

$$\bar{x} \pm z_{\alpha/2}\frac{s}{\sqrt{n}} \tag{4.11}$$

In the previous confidence interval formula, s is the sample standard deviation.

Confidence Interval for the Mean When the Sample Size is Small and the Population Standard Deviation σ is Unknown

When σ is unknown and the sample size is small, use *t*-distribution for the confidence interval. The *t*-distribution is characterized by a single parameter, the number of *degrees of freedom (df)* and its density function provides a bell-shaped curve similar to a normal distribution.

The confidence interval using t-distribution is given by

$$\bar{x} - t_{n-1,\alpha/2}\frac{s}{\sqrt{n}} \leq \mu \leq \bar{x} + t_{n-1,\alpha/2}\frac{s}{\sqrt{n}} \tag{4.12}$$

where $t_{n-1,\,\alpha/2}$ = *t*-value from the *t*-table for $(n - 1)$ degrees of freedom and $\alpha/2$.

Confidence Interval for Estimating the Population Proportion p

The confidence interval for the proportion is constructed based on the following:

a. The sampling distribution of the sample proportion ($\bar{p}$) follows a normal distribution when the sample size is large.
b. The value of sample proportion.
c. The level of confidence, denoted by z.

The confidence interval formula is given by (Assumption: sample size n is large so that normal approximation can be used)

$$\bar{p} - z_{\alpha/2}\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} \leq p \leq \bar{p} + z_{\alpha/2}\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} \tag{4.13}$$

In the previous formula, note that p = population proportion and $\bar{p}$ is the sample proportion given by $\bar{p} = x/n$.

Sample Size Determination

Sample Size (n) to Estimate μ

Determining the sample size is an important issue in statistical analysis. To determine the appropriate sample size (n), the following factors are taken into account:

a. The margin of error, E (or, tolerable error level, or the accuracy requirement) For example, suppose we want to estimate the population means salary within \$500 or within \$200. In the first case, the error, $E = 500$; in the second case, $E = 200$. A smaller value of the error E means more precision is required which in turn, will require a larger sample. In general, smaller the error, E, larger the sample size.
b. The desired reliability or the confidence level.
c. A good guess for σ.

Both the margin of error E and reliability are arbitrary choices that have an impact on the cost of sampling and the risks involved. The following formula is used to determine the sample size:

$$n = \frac{(z_{\alpha/2})^2 \sigma^2}{E^2} \tag{4.14}$$

E = margin of error or accuracy (or, maximum allowable error), n = sample size

Sample Size (n) to Estimate p

The sample size formula to estimate the population proportion p is determined similar to the sample size for the mean. The sample size is given by

$$n = \frac{(z_{\alpha/2})^2 p(1-p)}{E^2} \tag{4.15}$$

p = population proportion (if p is not known or given, use $p = 0.5$).

Example 4.2

A quality control engineer is concerned about the bursting strength of a glass bottle used for soft drinks. A sample of size 25 (n = 25) is randomly obtained, and the bursting strength (X10 in pounds per square inch, psi) is recorded. The strength is considered to be normally distributed. Find a 95 percent confidence interval for the mean strength using both t-distribution and normal distribution. Compare and comment on your results.

26, 27, 18, 23, 24, 20, 21, 24, 19, 27, 25, 20, 24, 21, 26, 19, 21, 20, 25, 20, 23, 25, 21, 20, 21

***Solution*:**

First, calculate the mean and standard deviation of 25 values given. You should use your calculator or a computer to do this. The values are

$$\bar{x} = 22.40$$
$$s = 2.723$$

The confidence interval using a t-distribution can be calculated using the following formula:

$$\bar{x} \pm t_{n-1,\frac{\alpha}{2}} \left(\frac{s}{\sqrt{n}} \right)$$

A 95 percent confidence interval using the previous formula

$$22.40 \pm (2.064) \left(\frac{2.723}{\sqrt{25}} \right)$$
$$21.28 \leq \mu \leq 23.52$$

The value 2.064 is the t-value from the t-table (Appendix B) for $n - 1 = 24$ degrees of freedom and $\alpha/2 = 0.025$

The confidence interval using a normal distribution can be calculated using the formula here.

$$\bar{x} \pm Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

$$22.40 \pm 1.96\left(\frac{2.723}{\sqrt{25}}\right)$$

For a 95 percent confidence interval $Z_{\alpha/2} = Z_{0.025} = 1.96$ from the Normal Table in Appendix A

This interval is

$$21.33 \leq \mu \leq 23.47$$

The confidence interval using the t-distribution is usually wider. This happens because with smaller sample size, there is more uncertainty involved.

Example 4.3

The average life of a sample of 36 tires of particular brand is 38,000 miles. If it is known that the average lifetime of the tires is approximately normally distributed with a standard deviation of 3,600 miles, construct 80 percent, 90 percent, 95 percent, and 99 percent confidence intervals for the average tire life. Compare and comment on the confidence interval estimates.

***Solution*:** Note the following data:

$$n = 36 \qquad \bar{x} = 38{,}000 \qquad \sigma = 3{,}600$$

Since the sample size is large ($n \geq 30$), and the population standard deviation σ is known, the appropriate confidence interval formula is

$$\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

The confidence intervals using the previous formula are shown subsequently.

(a) 80 percent confidence interval

$$38{,}000 \pm 1.28\left(\frac{3{,}600}{\sqrt{36}}\right)$$
$$37{,}232 \leq \mu \leq 38{,}768$$

(b) 90 percent confidence interval

$$38{,}000 \pm 1.645\left(\frac{3{,}600}{\sqrt{36}}\right)$$
$$37{,}013 \leq \mu \leq 38{,}987$$

(c) 95 percent confidence interval

$$38{,}000 \pm 1.96\left(\frac{3{,}600}{\sqrt{36}}\right)$$
$$36{,}824 \leq \mu \leq 39{,}176$$

(d) 99 percent confidence interval

$$38{,}000 \pm 2.58\left(\frac{3{,}600}{\sqrt{36}}\right)$$
$$36{,}452 \leq \mu \leq 39{,}548$$

Note that the z values in the earlier confidence interval calculations are obtained from the normal table. Refer to the normal table for the values of z. Figure 4.6 shows the confidence intervals graphically.

Figure 4.6 shows that larger the confidence level, the wider is the length of the interval. This indicates that for a larger confidence interval, we gain confidence. There is higher chance that the true value of the parameter being estimated is contained in the interval but at the same time, we lose accuracy.

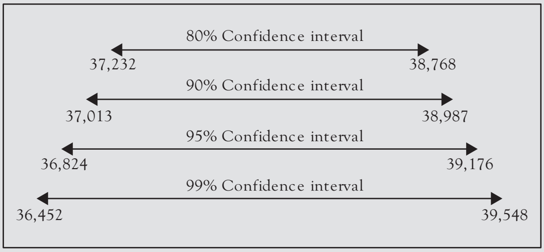


Figure 4.6 Effect of increasing the confidence level on the confidence interval

Example 4.4

During an election year, ABC news network reported that according to its poll, 48 percent voters were in favor of the democratic presidential candidate with a margin of error of ±3 percent. What does this mean? From this information, determine the sample size that was used in this study.

***Solution*:** The polls conducted by the news media use a 95 percent confidence interval unless specified otherwise. Using a 95 percent confidence interval, the confidence interval for the proportion is given by

$$\bar{p} \pm 1.96\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

The sample proportion, $\bar{p} = 0.48$. Thus, the confidence interval can be given by

$$0.48 \pm 1.96\sqrt{\frac{0.48(1-0.48)}{n}}$$

Since, the margin of error is ±3 percent, it follows

$$1.96\sqrt{\frac{0.48(1-0.48)}{n}} = 0.03$$

Squaring both sides and solving for n gives

$$n = 1066$$

Thus 1066 voters were polled.

Example 4.5

A pressure seal used in an assembly must be able to withstand a maximum load of 6,000 pounds per square inch (psi) before bursting. If the average maximum load of a sample of seals taken from a shipment is less than 6,000 psi, then the quality control must reject the entire shipment. How large a sample is required if the quality engineer wishes to be 95 percent confident that the error in estimating this quantity is no more than 15 psi, or the probability that the sample mean differs from the population mean by no more than 15 psi is 0.95. From the past experience, it is known that the standard deviation for bursting pressures of this seal is 150 psi.

***Solution*:** The sample size n can be calculated using

$$n = \left(\frac{z_{\alpha/2}\sigma}{E} \right)^2$$

Since, $Z_{\alpha/2} = Z_{0.025} = 1.96$, $\sigma = 150$, and the error $E = 15$, the required sample size will be as follows:

$$n = \left(\frac{(1.96)150}{15} \right)^2 \approx 385$$

Hypothesis Testing

A *hypothesis* is a statement about a *population parameter*. This statement may come from a claim made by a manufacturer, a mathematical model, a theory, design specifications, and so on. For example, an automobile manufacturer may claim that they have come up with a new fuel injection system design that provides an improved average mileage of 50 miles a gallon. In such a case, we may want to test the claim by taking sample data. The claim can formally be written as a hypothesis testing problem, and can be tested using a hypothesis testing procedure. Several population parameters may be of interest. Sometimes we may be interested in testing the average or the mean. In other cases, we may be interested in testing a variance, a standard deviation, or a population proportion.

Many problems require us to decide whether or not a statement about some parameter is true or false. This statement about the population parameter is called a *hypothesis*.

Hypothesis testing is the decision-making procedure about a statement being true or false. The statement is about a population parameter of interest, such as, a population mean, population variance, or a population proportion. It involves making a decision about a population parameter based on the information contained in the sample data.

Hypothesis testing is one of the most useful aspects of statistical inference because many types of decision problems can be formulated as hypothesis testing problems.

In the chapters that follow, we will see that the control charts used in statistical process control are closely related to the hypothesis testing. The tests are also used in several quality control problems and form the basis of many of the statistical process techniques to be discussed in the coming chapters.

Example 4.6: Testing a Single Population Mean—An Example

An automobile manufacturer claims that their new hybrid model will provide 60 miles per gallon on the average because of an improved design. A consumer group wants to test whether this claim is correct. They would test a hypothesis stated formally as follows:

$$H_0 : \mu = 60\,mpg$$
$$H_1 : \mu \neq 60\,mpg$$

Here, H_0 is known as the null hypothesis and H_1 (also written H_a) is called the alternate hypothesis. The hypothesis written with an ‘equal to’ sign under the null hypothesis and a ‘not equal to’ sign under the alternate hypothesis is known as a two-sided or two-tailed test. A hypothesis can also be one-sided or one-tailed. The alternate hypothesis is opposite of the null hypothesis. To test the validity of previous claim:

- The consumer group would gather the sample data and calculate the sample mean, $\bar{x}$
- Compare the difference between the hypothesized value of (μ) and the value of the sample mean ($\bar{x}$).
- If the difference is small, there is a greater likelihood that the hypothesized value of the population mean is correct. If the difference is large, there is less likelihood that the claim about the population mean is correct.

In most cases, the difference between the hypothesized population parameter and the actual sample statistic is neither so large that we reject our hypothesis nor it is so small that we accept it. Thus, in hypothesis testing, clear cut solutions are not the rule.

Note that in hypothesis testing, *the decision to reject or not to reject the hypothesis is based on a single sample* and therefore, there is always a chance of not rejecting a hypothesis that is false, or rejecting a hypothesis that is true. In fact, we always encounter two types of errors in hypothesis testing. These are:

$$\text{Type I Error} = \alpha = P\,\{\text{Reject } H_0 \mid H_0 \text{ is true}\}$$

$$\text{Type II Error} = \beta = P\,\{\text{Fail to reject } H_0 \mid H_0 \text{ is false}\} \qquad (4.16)$$

We also use another term known as the power of the test defined as

$$\text{Power} = 1 - \beta = P\,\{\text{Reject } H_0 \mid H_0 \text{ is false}\}$$

Thus, the type I error, denoted by a Greek letter α (“alpha”) is the probability of rejecting a true null hypothesis and the type II error, denoted by a Greek letter β (“beta”) is the probability of not rejecting a null hypothesis when it is false. In a hypothesis testing situation there is always a possibility of making one of these errors.

The power of the test is the probability that a false null hypothesis is correctly rejected. The type I error, α is selected by the analyst. Increasing the type I error α will decrease the type II error β and decreasing the type I error will increase the type II error β. Thus, the type I error is controlled by the analyst and the type II error is determined based on the type I error. The type II error is a function of the sample size. The larger the sample size n, the smaller is the β.

The *type I error* α is also known as the level of significance. In hypothesis testing, we specify a value of the type I error or the level of significance and then design a test that will provide a small value of the type II error. Recall that the *type I error is the probability of rejecting a true null hypothesis*. Since we don't want this to happen, α is set to a low value of 1 percent, 5 percent or 10 percent in general cases. If you set the value of $\alpha = 5$ percent, it means that there is a 5 percent chance of making an incorrect decision, and a 95 percent chance of making a right decision. In a hypothesis testing, there is never a 100 percent chance of making a right decision because the test is based on one sample (large or small).

What Does It Mean to Test a Hypothesis at a 5 Percent Level of Significance?

Suppose we want to test a hypothesis about a population mean and the sample size is large ($n \geq 30$) so that the sample mean follows a normal distribution. If the level of significance α is set at 5 percent, it means that we will reject the null hypothesis if the difference between the sample statistic (in this case, $\bar{x}$) and the hypothesized population mean μ is so large that it would occur on the average only five or fewer times in every 100 samples (see Figure 4.7). This figure shows that if the sample statistic falls in the 'do not reject' area, we will not reject the null hypothesis. On the other hand, if the sample value falls in the rejection areas, we will reject the null hypothesis. Rejecting the null hypothesis means that the alternate hypothesis is true.

From Figure 4.7, we can see that that by selecting a significance level α, the areas of rejection and nonrejection are determined. In other words, we set the boundaries that determine when to reject and when not to reject the null hypothesis.

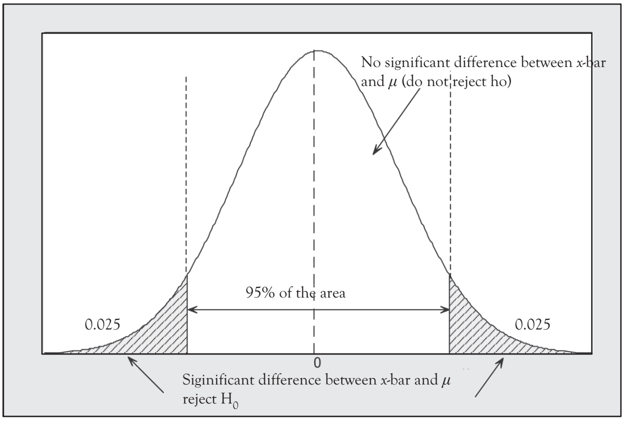


Figure 4.7 Rejection and nonrejection areas

Note that the value of α is selected by the analyst and it must be determined before you conduct the test. If we don't have a significance level α, we don't know when to reject and when not to reject a hypothesis.

In a hypothesis testing situation, there is always a possibility of making one of the two types of errors (that is, there is always a chance of rejecting a true null hypothesis; similarly there is always a chance of accepting a false null hypothesis). In a hypothesis testing, we must consider the cost and the risk involved in making a wrong decision. Also, we would like to minimize the chance of making either a type I or type II error, therefore, *it is desirable to have the probabilities of both type I and type II error to be low*.

In quality control, α is also known as *producer's risk* because it denotes the probability of rejecting a good lot. The type II error β is referred as the *consumer's risk* as it indicates the probability of accepting a bad lot.

Testing a Single Population Mean

Testing a population mean involves testing a one-sided or a two-sided test. The hypothesis is stated as:

$H_0: \mu = \mu_0$ $H_1: \mu \neq \mu_0$ Two-tailed or Two-sided Test	$H_0: \mu \geq \mu_0$ $H_1: \mu < \mu_0$ Left-tailed or Left-sided Test	$H_0: \mu \leq \mu_0$ $H_1: \mu > \mu_0$ Right-tailed or Right-sided Test

Note that μ_0 is the hypothesized value. There are three possible cases for testing the population mean. The test statistics or the formulas used to test the hypothesis are given here.

***Case (1)*:** Testing a single mean with known variance or known population standard deviation σ and large sample: in this case, the sample mean $\bar{x}$ follows a Normal distribution and the *test statistic* is given as follows:

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} \tag{4.17}$$

***Case (2)*:** Testing a single mean with unknown variance or unknown population standard deviation σ and large sample: in this case, the sample mean $\bar{x}$ follows a Normal distribution and the *test statistic* is given by

$$z = \frac{\bar{x} - \mu}{s / \sqrt{n}} \tag{4.18}$$

***Case (3)*:** Testing a single mean with unknown variance or unknown population standard deviation σ and small ($n < 30$) sample. In this case, the sample mean, $\bar{x}$ follows a *t*-distribution and the *test statistic* is given by

$$t_{n-1} = \frac{\bar{x} - \mu}{s / \sqrt{n}} \tag{4.19}$$

Note that s is the sample standard deviation and n is the sample size.

There are different ways of testing a hypothesis. These will be illustrated with examples.

Example 4.7

Examples: Formulating the Correct Hypothesis

A hypothesis test can be formulated as a one-sided or a two-sided test. If we have a one-sided test, it can be a left-sided or a right-sided test.

Example of a Left-Sided Test

Suppose a tire manufacturer claims that the average mileage provided by a certain type of tire is at least 60,000 miles. A product research group has received some complains in the past and wants to check the claim. They are interested in testing whether the average mileage is below 60,000 miles. They would formulate their hypothesis as follows:

$$H_0 : \mu \geq 60,000$$
$$H_1 : \mu < 60,000$$

The hypothesis is formulated as shown earlier because the claim of at least 60,000 miles is $\mu \geq 60,000$ and the opposite of this statement is $\mu < 60,000$ miles. Since the null hypothesis is written with an 'equal to' sign, and this is the claim made by the manufacturer, the statement $\mu \geq 60,000$ is written under the null hypothesis, and the statement $\mu < 60,000$ is written under the alternate hypothesis.

Note that the alternate hypothesis is opposite of the null hypothesis. This is an example of a left sided test. The left-sided test will reject the null hypothesis (H_0) below a specified hypothesized value of μ.

The alternate hypothesis is also known as *the research hypothesis*. If you are trying to establish certain hypothesis, then it should be written as the alternate hypothesis.

The statement about the null hypothesis contains the claim or the theory. Therefore, rejecting a null hypothesis is a strong statement. This is the reason that the conclusion of a hypothesis test is stated as *reject the null hypothesis* or *do not reject the null hypothesis.*

Example of a Right-Sided Test

A car manufacturer has made a significant improvement in the fuel injection system that is expected to provide an improved gas mileage. The average mileage before the modification was 24 miles or less. The research group expects that the modified system will provide significant improvement in the gas mileage. The group would like to test the following hypothesis to show the improvement:

$$H_0 : \mu \leq 24$$
$$H_1 : \mu > 24$$

Example of a Two-Sided Test

A robot welder in an assembly line takes on average 1.5 minutes to finish a welding job. If the average time taken to finish the job is higher or lower than 1.5 minutes it will disrupt other activities along the production line. Since there has been too much variation in the time it takes to perform the welding job by the robot, the line supervisor wants to take a sample to check if the average time taken by the robot is significantly higher or lower than the average of 1.5 minutes. The supervisor would be testing the following hypothesis:

$$H_0 : \mu = 1.5$$
$$H_1 : \mu \neq 1.5$$

Example: A new production method will be implemented if a hypothesis test supports the conclusion that the new method results in reduced production cost per hour. If the current average operating cost per hour is \$600 or more, write the appropriate hypothesis. Also, state and explain the type I and type II error in this situation.

a. Write the appropriate null and alternate hypotheses.
   The appropriate hypotheses to test the claim:

$$H_0 : \mu \geq 600$$
$$H_1 : \mu < 600$$

b. State and explain the type I and type II error in this situation
   Type I error: Reject $H_0 : \mu \geq 600$ and conclude that the average production cost is less than \$600 ($\mu <$ \$600). Type II error would be to conclude that the average operating cost is at least \$600 when it is not.

Example 4.8

In a production line, automated machines are used to fill beverage cans with an average fill volume of 16 ounces. If the mean weight falls above or below this figure, the production line must be stopped and some remedial action be taken. A quality control inspector samples 30 cans every hour; opens and weighs the content, tests the appropriate hypothesis and makes a decision whether to shut down the line for making adjustments. Write the appropriate hypothesis to be tested in this situation and perform the hypothesis test. A significance level of α = 0.05 is selected for the test. The sample results indicate a sample mean of 16.32 oz. and the standard deviation is assumed to be 0.8 oz.

***Solution*:** For this problem, the given data are:

$$n = 30, \alpha = 0.05$$
$$\sigma = 0.8,\ \bar{x} = 16.32$$

1. *State the null and alternate hypothesis*

$$H_0 : \mu = 16$$
$$H_1 : \mu \neq 16$$

   Note that this is a two-sided test.
2. *Determine the sample size or use the given sample size.*
   The given sample size is n = 30 (large sample)
3. *Determine the appropriate level of significance (α) or use the given α.*
   The given level of significance, $\alpha = 0.05$
4. *Select the appropriate distribution and test statistic to perform the test*
   The sample size is large and the population standard deviation is known, therefore, use normal distribution with the following test-statistic

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

5. Based on step 3, find the critical value or values and the area or areas of rejection. Show the critical value(*s*) and the area or areas of rejection and nonrejection using a sketch

   This is a two-sided test. The level of significance, $\alpha = 0.05$ must be split into two halves for a two-tailed test with each tail area 0.025. The critical value (z-value) for an area of 0.475 is 1.96 from the normal or z table. The sketch is shown in Figure 4.8.

6. Write the decision rule

$$\text{Reject } H_0 \text{ if } z > 1.96$$
$$\text{or,} \quad \text{if } z < -1.96$$

7. Use the test data (sample data) and find the value of the test statistic.

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{16.32 - 16}{0.8/\sqrt{30}} = 2.19$$

8. *Find out if the value of the test statistic is in rejection or nonrejection region; make appropriate decision and state your conclusion in terms of the problem.*

$$z = 2.19 > Z_{critical} = 1.96 \text{ therefore, reject } H_0$$

   There is an evidence of over filling or under filling. The line should be shut down.

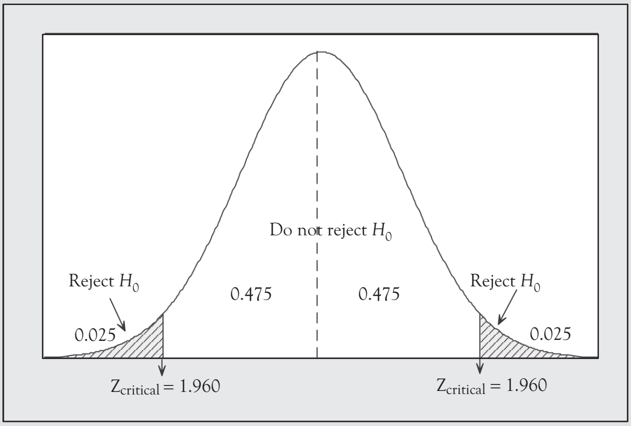


Figure 4.8 Critical values for a two-sided test

Example 4.9: Testing the Hypothesis Using the P-Value Approach

In the z-value approach of testing the hypothesis, we compared the test statistic value to the critical value of z to either reject or not to reject the hypothesis. For example, in the previous example, the critical value z was ±1.96 and the test statistic value 2.19. Since the test statistic value of 2.19 is greater than the critical value of +1.96, we rejected the null hypothesis. In this method of testing the hypothesis, we compared the similar terms (test statistics value of z to the critical z value). This method of stating conclusion in the hypothesis test requires a predefined level of significance which may not tell if the computed test statistic value is barely in the rejection region or it is far into the region. In other words, the information may be inadequate sometimes.

To overcome this problem, another approach of testing the hypothesis is suggested which is widely used in testing hypothesis. This is known as p-value (probability value) approach. This method compares a probability to the given probability.

The p-value is the probability (assuming that the null hypothesis is true) of getting the value of the test statistic at least as extreme as, or more extreme than the value actually observed. The p-value is the smallest level of significance at which the null hypothesis can be rejected. A small p-value for example, $p = 0.05$ or less is a strong indicator that the null hypothesis is not true. The smaller is the value of p, the greater the chance that the null hypothesis is false.

You may recall that to test a hypothesis you must have the level of significance α, which is decided by the analyst before conducting the test. This α is the same as the type I probability, and is often called the given level of significance. In a hypothesis testing situation, the type I probability is given or known. We then calculate the p-value or the probability based on the sample data. This is also the observed level of significance. We then compare the given level of significance α to the p-value (observed level of significance) to test and draw the conclusion about the hypothesis.

If the computed p-value is smaller than the given level of significance α, the null hypothesis H_0 is rejected. If the p-value is greater than a then H_0 is not rejected. For example, a p-value of 0.002 indicates that there is less chance that H_0 is true, while a p-value of 0.2356 indicates a less likelihood of H_0 being false. The p-value also provides us insight into the strength of the decision. This tells us how confident we are in rejecting the null hypothesis.

Example 4.10: Testing the Hypothesis Using P-Value Approach

We will test the hypothesis using p-value for the following two-sided test:

$$H_0 : \mu = 15$$
$$H_1 : \mu \neq 15$$

The data for the problem: $n = 50$, $\bar{x} = 14.2$, $s = 5$, $\alpha = 0.02$

Decision Rule for p-value approach

If $p \geq \alpha$, do not reject H_0
If $p < \alpha$; reject H_0

First; using the appropriate test statistic formula, calculate the test statistic value.

$$Z = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{14.2 - 15}{5/\sqrt{50}} = -1.13$$

This test statistic value of $z = -1.13$ will be converted into a probability that we call p-value. This is shown in Figure 4.9.

Area corresponding to $z = 1.13$ is 0.3708 (from z-table in Appendix A).

Probability of $z > 1.13 = 0.5 - 0.3708 = 0.1292$

Probability of $z < -1.13 = 0.5 - 0.3708 = 0.1292$

For a two-sided test, the p value is the sum of the earlier two values, that is, $0.1292 + 0.1292 = 0.2584$. Since, $p = 0.2584 > \alpha = 0.02$; do not reject H_0.

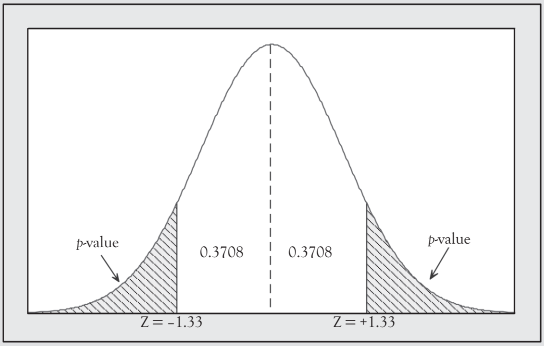


Figure 4.9 P-values for the example problem

Hypothesis Testing Involving Two Population Means

This section extends the concepts of hypothesis testing to two populations. Here we are interested in testing two population means. For example, we may be interested in comparing the average salaries of male and female employees for the same job, or we may be interested in the difference between the average starting salaries of business and engineering majors. In these cases, we would like to test whether the two population means are equal or there is no difference between the two populations (two-sided test). In other cases, we may want to test if one population is larger or smaller than the other population (one-sided test). The hypothesis testing procedures or steps are very similar to those for testing the single mean but the data structure and the test statistics or the formulas to test these hypotheses are different. In testing hypothesis involving two populations, the samples will be drawn from both populations. The hypotheses tested are explained here.

Hypothesis Testing for the Equality of Two Means or the Differences Between Two Population Means

Basic Assumptions

1. The populations are independent
2. The population variances are equal $(\sigma_1^2 = \sigma_2^2)$

The hypothesis for testing the two means can be a two-sided test or a one-sided test. The hypothesis is written in one of the following ways:

a. Test if the two population means are equal or there is no difference between the two means: *a two-sided test*

$$H_0 : \mu_1 = \mu_2 \text{ or } H_0 : \mu_1 - \mu_2 = 0$$
$$H_1 : \mu_1 \neq \mu_2 \text{ or } H_1 : \mu_1 - \mu_2 \neq 0 \tag{4.20}$$

b. Test if one population mean is greater than the other: *a right-sided test*

$$H_0 : \mu_1 \leq \mu_2 \text{ or } H_0 : \mu_1 - \mu_2 \leq 0$$
$$H_1 : \mu_1 > \mu_2 \text{ or } H_1 : \mu_1 - \mu_2 > 0 \tag{4.21}$$

c. Test if one population mean is smaller than the other: *a left-sided test*

$$H_0 : \mu_1 \geq \mu_2 \text{ or } H_0 : \mu_1 - \mu_2 \geq 0$$
$$H_1 : \mu_1 < \mu_2 \text{ or } H_1 : \mu_1 - \mu_2 < 0 \tag{4.22}$$

Since we want to study two population means, the sampling distribution of interest is the sampling distribution of the difference between the sample means $(\bar{x}_1 - \bar{x}_2)$ and the test statistic is based on the information (data) we have. The following three cases and test statistics are used to test the means. To test two means, the test statistics are selected based on the following cases:

***Case 1**: Sample sizes n_1 and n_2 are large (≥ 30) and the population variances σ_1^2 and σ_2^2 are known*

If the sample sizes n_1 and n_2 are large (≥ 30) and the population variances σ_1^2 and σ_2^2 are known, then the sampling distribution of the difference between the sample means follows a normal distribution and the test statistic is given by

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \tag{4.23}$$

where $\bar{x}_1$ $\bar{x}_2$ = sample means from populations 1 and 2 respectively
n_1, n_2 = sample size of sample 1 and 2

σ_1^2, σ_2^2 = variances of first and second populations, respectively (known in this case)

***Case 2**: Sample sizes n_1 and n_2 are large (≥ 30) and the population variances σ_1^2 and σ_2^2 are unknown*

If the sample sizes n_1 and n_2 are large (≥ 30) and the population variances σ_1^2 and σ_2^2 are unknown, then the sampling distribution of the difference between the sample means follows a normal distribution and the test statistic is given by

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \tag{4.24}$$

where $\bar{x}_1$ $\bar{x}_2$ = sample means from populations 1 and 2, respectively
n_1, n_2 = sample size of sample 1 and 2, respectively
s_1^2, s_2^2 = sample variances of 1st and 2nd sample respectively

***Case 3**: Sample sizes n_1 and n_2 are small (≥ 30) and the population variances σ_1^2 and σ_2^2 are unknown*

If the sample sizes n_1 and n_2 are small (< 30) and the population variances σ_1^2 and σ_2^2 are unknown, then the sampling distribution of the difference between the sample means follows a *t*-distribution and the test statistic is given by

$$t_{n_1+n_2-2} = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{S_p^2(\frac{1}{n_1} + \frac{1}{n_2})}} \tag{4.25}$$

where $\bar{x}_1$ $\bar{x}_2$ = sample means from populations 1 and 2, respectively
n_1, n_2 = sample size of samples 1 and 2, respectively
s_1^2, s_2^2 = sample variances of first and second sample, respectively

$n_1 + n_2 - 2$ = degrees of freedom (df)
s_p^2 is the 'pooled' or combined variance, given by

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} \tag{4.26}$$

Important Note

In Equations (4.20), (4.21), and (4.22) the difference $(\mu_1 - \mu_2)$ is zero in most cases. Also, these equations are valid under the following assumptions:

- The populations are independent and normally distributed
- The variances of the two populations are equal; that is $(\sigma_1^2 = \sigma_2^2)$

The assumption that the two population variances are equal may not be correct. In cases where the variances are not equal, the test statistics formula for testing the difference between the two means is different.

Example 4.11

Suppose that two independent random samples are taken from two processes with equal variances and we would like to test the null hypothesis that there is no difference between the means of two processes or the means of the two processes are equal; that is,

$$H_0 : \mu_1 - \mu_2 = 0 \quad \text{or} \quad H_0 : \mu_1 = \mu_2$$

$$H_1 : \mu_1 - \mu_2 \neq 0 \qquad H_1 : \mu_1 \neq \mu_2$$

The data from the two processes are given here:

Sample 1	**Sample 2**
$n_1 = 80$	$n_2 = 70$
$\bar{x}_1 = 104$	$\bar{x}_2 = 106$
$s_1 = 8.4$	$s_2 = 7.6$

$\alpha = 0.05$

Note that when n_1, n_2 are large and σ_1, σ_2 unknown, we use the normal distribution. The test statistic for this problem is

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

Solution: The test can be done using four methods which are explained here.

Method (1): z-Value Approach

Critical values: The critical values and the decision areas based on $\alpha = 0.05$ are shown in Figure 4.10.

Decision Rule:

$$\text{Reject } H_0 \text{ if } z > 1.96$$
$$\text{or if } z < -1.96$$

Test Statistic Value:

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{(104 - 106) - 0}{\sqrt{\frac{(8.4)^2}{80} + \frac{(7.6)^2}{70}}} = -1.53$$

The test statistic value z, $-1.53 > z_{\text{critical}} = -1.96$; do not reject H_0.

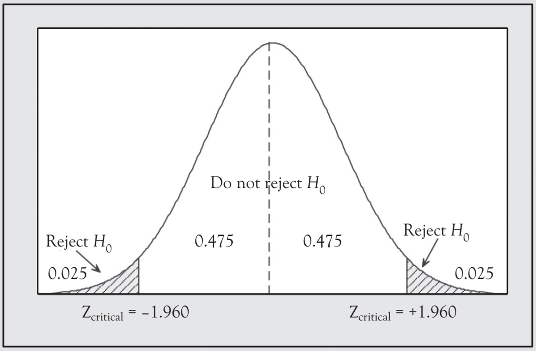


Figure 4.10 Critical values and decision areas

Method (2): p-value Approach

Calculate the *p*-value and compare it to α. Note that in this method we compare a probability to a probability that is, we compare the given level of significance α or the type I probability to the probability obtained from the data (or, the observed level of significance). The decision rule and the procedure is explained here.

Decision Rule: If $p \geq \alpha$; do not reject H_0.
If $p < \alpha$; reject H_0.

Calculating *p*-value: the *p*-value is calculated by converting the test-statistics value into a probability. In method (1), we calculated the test statistic value z. This value was -1.53 or, $z = -1.53$. This test statistics value is converted to a probability (see the Figure 4.11).

In Figure 4.11, $z = 1.53$ is the test statistic value from method 1.

From the standard normal table, $z = 1.53$ corresponds to 0.4370. The *p*-value is calculated as shown here.

Probability of $z > 1.53 = 0.5 - 0.4370 = 0.0630$

Probability of $z < -1.53 = 0.5 - 0.4370 = 0.0630$

In a two-sided test such as, this one the *p*-value is obtained by adding the probabilities in both tails of the distribution. Thus,

$$p = 0.0630 + 0.0630 = 0.1260$$

Since $p(= 0.1260) > \alpha(= 0.05)$; do not reject $H_{0.}$

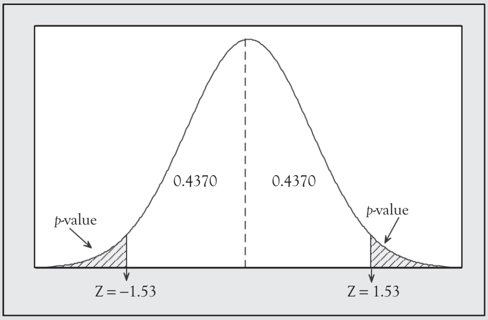


Figure 4.11 p-values

Testing Two Means for Dependent (Related) Populations or Hypothesis Testing for Paired Samples (Paired t-Test)

In some situations, there may be a relationship between the data values of the samples of two populations and the data values or the sample values from one population may not be independent of the sample values from the other population. The two populations may be considered dependent in such cases.

In cases where the populations are considered related, the observations are paired to prevent other factors from inflating the estimate of the variance. This method is used to improve the precision of comparisons between means. The method of testing the difference between the two means when the populations are related is also known as *matched sample* test or *the paired-test*.

We are interested in testing a two-sided or a one-sided hypothesis for the difference between the two population means. The hypotheses can be written as

$H_0 : \mu_d = 0$ $H_1 : \mu_d \neq 0$ *Two-tailed or Two-sided test*	$H_0 : \mu_d \leq 0$ $H_1 : \mu_d > 0$ *Right tailed or Right-sided test*	$H_0 : \mu_d \geq 0$ $H_1 : \mu_d < 0$ *Left-tailed or Left-sided test*

Note: the difference, *d* can be taken in any order; (sample 1 – sample 2) or (sample 2 – sample1).

Test Statistic: If the pairs of data values X_{1n} and X_{2n} are related, and are not independent; the average of the differences ($\bar{d}$) follows a *t*-distribution and the test statistic is given by

$$t_{n-1} = \frac{\bar{d} - \mu_d}{s_d / \sqrt{n}} \tag{4.27}$$

where

$\bar{d}$ = average of the differences = $\frac{\sum d_i}{n}$

S_d = standard deviation of the differences;

$$s_d = \sqrt{\frac{\sum (d_i - \bar{d})^2}{n-1}} = \sqrt{\frac{\sum d_i^2 - \frac{(\sum d_i)^2}{n}}{n-1}}$$

N = number of observations (sample size)

$t_{n-1,\frac{\alpha}{2}}$ = critical t-value from the t-table for $(n - 1)$ degrees of freedom and appropriate α

The confidence interval given here can also be used to test the hypothesis

$$\bar{d} \pm t_{n-1,\frac{\alpha}{2}} \frac{s_d}{\sqrt{n}} \tag{4.28}$$

Summary

In this chapter, we discussed three important topics that are critical to statistical process control. In particular, we studied sampling and sampling distribution, estimation and confidence intervals, and hypothesis testing. Samples are used to make inferences about the population and this can be done through sampling distribution. The probability distribution of a sample statistic is called its *sampling distribution*. We explained the *central limit theorem* and its role in analyzing and solving problems involving control charts. Besides sampling and sampling distribution, other key topics covered in this chapter included point and confidence interval estimates of means and proportions.

Two types of estimates used in inferential statistics were discussed. These are: (a) the *point estimates*, which are single-value estimates of the population parameter, and (b) the *interval estimates* or the *confidence intervals,* which are a range of numbers that contain the parameter with specified degree of confidence known as the *confidence level*. Confidence level is a probability attached to a confidence interval that provides the reliability of the estimate. In the discussion of estimation, we also discussed the *standard error* of the estimates, the *margin of error*, and the *sample size* determination.

We also discussed the concepts of hypothesis testing, which is directly related to the control charts in statistical process control. Hypothesis testing is one of the most useful aspects of statistical inference. We provided several examples on formulating and testing hypothesis about the population mean and population proportion. The tests are used in a number of quality control problems and form the basis of many of the statistical process control techniques to be discussed in the coming chapters.

CHAPTER 5

Process Variation—How It Affects Product Quality

Introduction

Variation reduction is one of the major objectives of a quality program. Therefore, measuring variation is critical to reducing variation in the products and processes. One of the common ways of measuring variation in the data is by calculating the variance and standard deviation. The Greek letter σ^2 (read as *sigma-squared*) represents the variance of a population data and σ represents the standard deviation. The corresponding symbols for the variance and standard deviation of a sample data are s^2 and s. The standard deviation σ is a measure of spread or deviation around the mean. Larger the variation, more spread out the data is. Two or more sets of data may have the same average, but their spread or variability may be different. This is shown in Figure 5.1. It can be seen from this figure that the data sets A, B, and C have the same mean but different variation—curve C has less spread or variability than curve B and Curve A has the maximum variation of all the three curves.

Detecting Variation in the Processes

Before we can analyze and control variation in the processes, it is important to determine the variation and the patterns of variation. One of the tools commonly used to study and describe variation in the processes is to plot the quality characteristic or the measurements of key variable over time or create a *time series plot*. This plot is commonly known as a *run chart*. Run charts are very useful tool in describing and understanding the pattern of variation in processes.

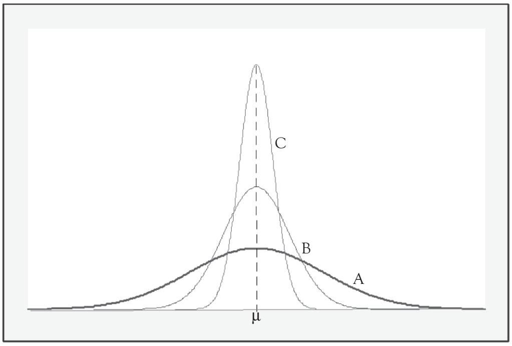


Figure 5.1 Three sets of data with same mean but different variation

Measuring Process Variation: Run Charts

A run chart is used in quality to analyze the data either in the preliminary or development stage of a product. To be able to see the stability and capability of a process, plotting the data over time is very helpful. A run chart is a simple and helpful tool that can visually display stability and variation in the process over some time period. The chart is also used to identify a trend or a shift in the process. The run chart can be converted to a control chart by adding the control limits which we will study later in this chapter.

Before analyzing the variation in the data, it is important to understand the types of variation. There are two typical causes of variations in a process. These are: (1) the *random variation* (also known as the *common* or *natural* causes of variation) and (2) the *assignable* or *special* causes of variation.

The natural variation is always present in a process. This is also known as the *background noise*. If the background noise is relatively small, it is considered an acceptable level of process performance. A system that is operating with only chance or natural causes of variation is said to be in a state of statistical control. The common causes of variation are inherent variations that may be attributed to humans, machines, methodology, and so on.

The second type of variation is known as the *assignable* or *special cause of variation*. The assignable cause means that a cause or causes can be assigned to a specific problem in the process. In manufacturing, the assignable causes may be due to wrong machine adjustments, defective raw material, calibration of measurement equipment, changes in the conditions within the machine, and so on. These can cause a shift in the process (upward or downward). This type of variation is greater than the chance variation or the background noise and may show recognizable patterns or trends in the run chart. This is unacceptable process performance. It is known as the assignable cause of variation because the variation can be related to a specific cause or causes. A process that shows patterns, trends, or shifts is operating in the presence of assignable cause and is usually out of control.

A run chart can be used to determine whether the process is running in a state of control, or whether special or assignable causes are influencing the process, thereby making the process out of control. As indicated earlier, the process is said to be in control if it is operating with only chance or random variations.

Figure 5.2 shows a run chart in which the weights of tea bags are monitored. The weights (in ounce) of the bags are plotted as they are packaged by a machine. A quality technician plots the weights in the order of production. The purpose of the plot is to monitor the variation in the quality characteristic (the weight of the tea bags). Recall that the run chart depicts graphically the variation in the process output over time and is also called a time series plot. A visual examination of the plot

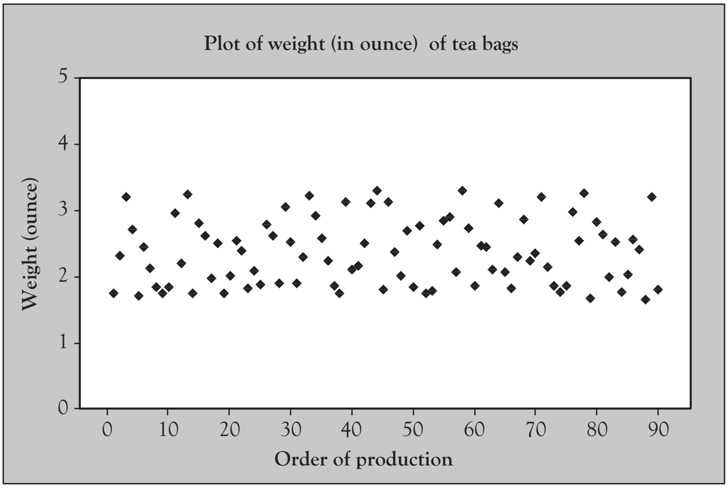


Figure 5.2 Run chart showing the weights of tea bags

in Figure 5.2 gives us some idea about the variation in the weights of the bags. The plot shows the weights of 90 bags that are supposed to be 2.4 ounces. The variation in the weights of the bags is evident from the plot. Although we can clearly see the variation in the weights, it is difficult to see any type of patterns (up or down) or oscillations (high and low).

To be able to see the variation in the weights and also the possible pattern, the plot in Figure 5.2 can be further enhanced by joining the plotted points using straight lines and also by plotting the average or the mean of the weights on the plot. Figure 5.3 shows the resulting plot.

This run chart in Figure 5.3 conveys much more information compared to the plot in Figure 5.2 that just showed the plotted points. We can see that the weights of the bags vary and are plotting up and down the center line, which is the mean weight. This type of variation is acceptable as long as it is within certain specified limit. Later on we will examine this type of plot in more depth and provide in depth analysis by adding other features in the plot that will enable us to monitor and control the weight of the product within acceptable limits.

The plot in Figure 5.3 shows the process variation that may be considered stable as the weights are fluctuating up and down the mean in a random fashion. The run charts can depict other types of patterns such as, an increasing or decreasing trend, a cyclical pattern, a shift in the

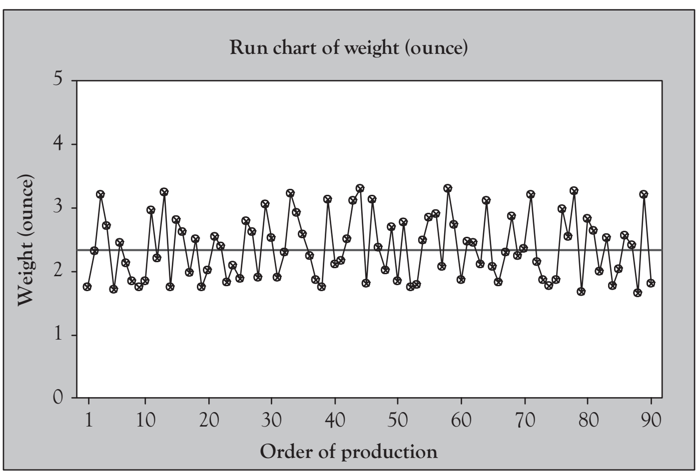


Figure 5.3 Plot created by joining the points using straight lines and plotting the mean line in Figure 5.2

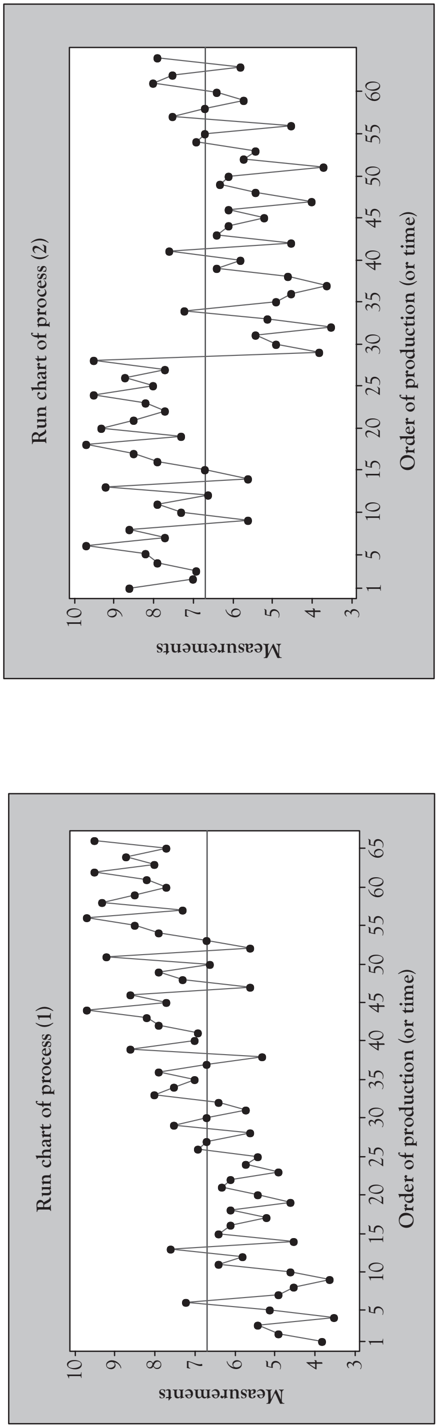


Figure 5.4 (Continued)

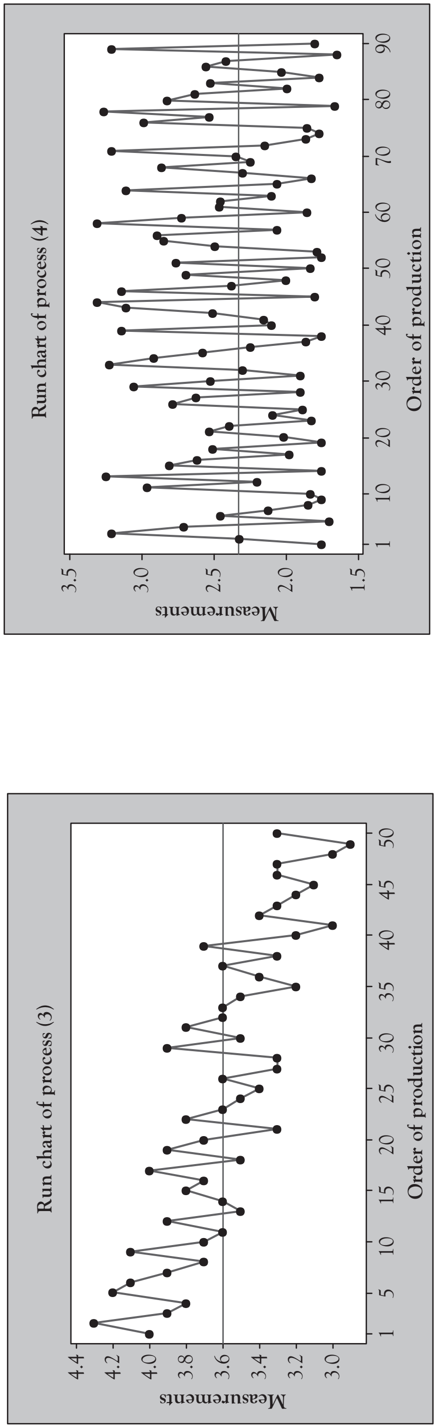


Figure 5.4 (Continued)

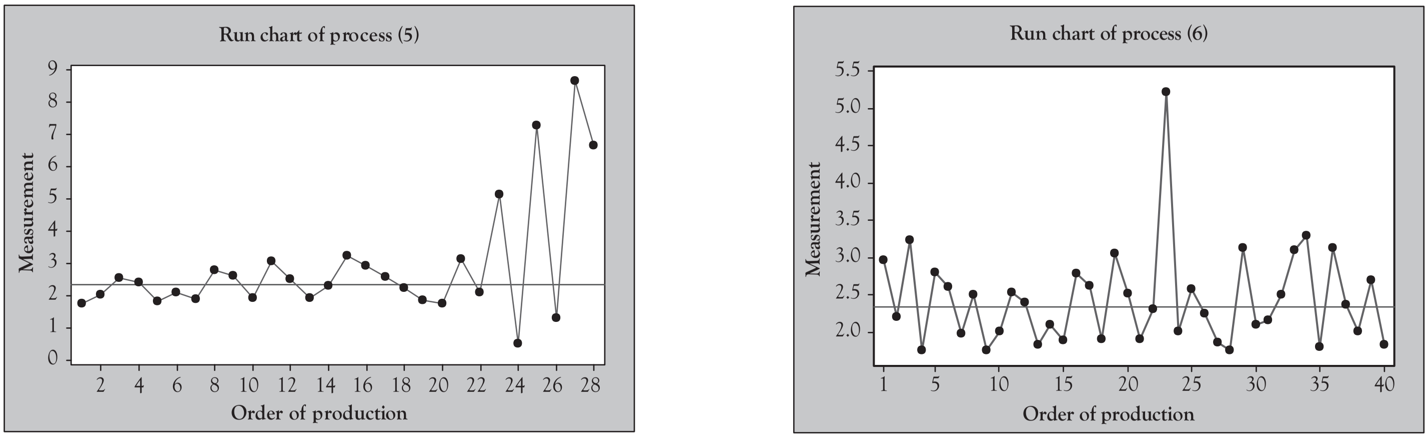


Figure 5.4 Run charts depicting different patterns in processes (a) Upward trend, (b) A shift in the process, (c) Downward trend, (d) A stable process, (e) Increase in variation, and (f) A jump in the process variation

process, or a sudden change in the variation of the process. These patterns provide useful information about the process and the variation in the process. Figures 5.4(a) to (f) shows some possible patterns of variation.

The patterns shown in Figure 5.4(a) through (f) are very helpful in studying and monitoring the process variation.

Characteristics of the Measured Output Variable

The run charts described in the Figure 5.4 plot the key quality characteristic or the output variable that are helpful in understanding the variation in the process. They are also critical in finding the cause or the causes of these variations so that corrective actions can be taken to control the variation. Since the output variable is plotted in a time sequence, the measured variable can be described by a particular *probability distribution* at any given point in time. This probability distribution describes the possible values the variable can assume and the likelihood of their occurrence. Figure 5.5 shows the possible distributions of the measured output variable at different points in time. Note that the distribution of a particular output variable at any given time is generated based on the probability distribution of the first time period.

The distribution of the output variable described in Figure 5.5 can change over time. The change may occur in three different ways. Over time, there may be a change in the mean or the location of the distribution. Also, the variance of the distribution may change resulting into change in the shape of the distribution or there may be a change in both the mean and the variance of the distribution. All these changes

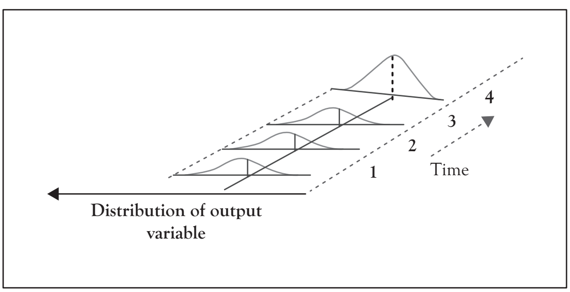
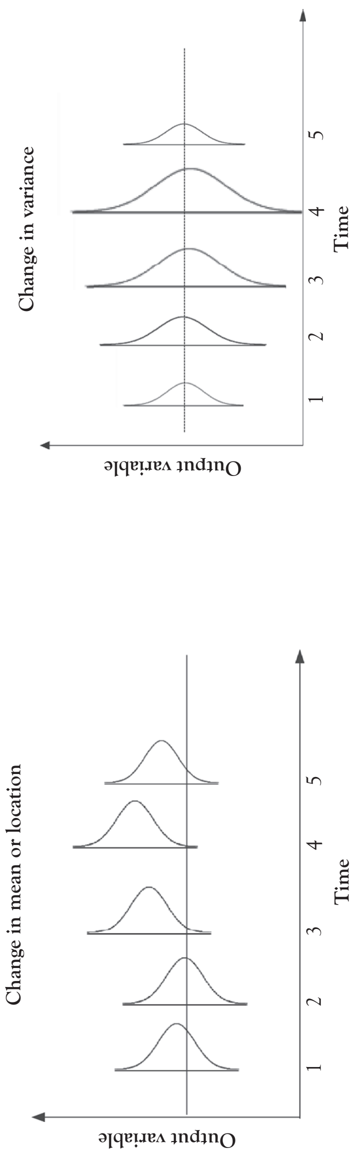


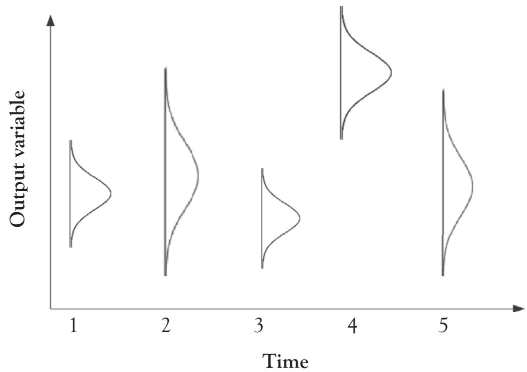
Figure 5.5 Distribution of the output variable at four points in time



(a) Change in the mean of the measured variable

(b) Change in the variance of the measured variable

Figure 5.6 ***(Continued)***



(c) Change in the mean and variance of the measured variable

Figure 5.6 Possible changes over time in the measured output variable (a) Change in the mean of the measured variable, (b) Change in the variance of the measured variable, and (c) Change in the mean and variance of the measured variable

affect the process and must be monitored and controlled. Figure 5.6(a) to (c) describe these changes. A change in the distribution of the output variable is usually referred as a *change in the process*. Thus, a change in the mean of the output variable means a shift in the process mean.

Figure 5.6(a) shows how a change in the mean or location of the process may occur. The horizontal line on the graph is the mean of the process. It seems like there is an upward shift in the process mean. There can also be a downward shift. Figure 5.6(b) shows the change in the variability of the measured output variable. The larger variation at a given time is indicated by the larger spread in the distribution of the output variable being measured. Figure 5.6(c) shows a change in both the mean and the variation in the process output.

Note: In the previous illustrations, the distribution of the output variable is shown to be normally distributed. The distribution may also assume other forms and it may not always be normally distributed.

Describing the Patterns of Run Charts in Figure 5.4(a) through (f) in Terms of Distributions

The run charts in Figure 5.4(a) to (f) showed some of the possible patterns in the process output. These patterns were helpful in understanding the behavior of the quality characteristic being measured. The plots in this figure were time series plots showing the behavior of the output variable over time. Sometimes it may be helpful to see the patterns in terms of the distribution they might represent. We usually don't plot the distribution while studying the patterns of the output variable; we simply create a time series plots as in Figure 5.4(a) to (f). Figure 5.7 shows the run charts of Figure 5.4(a) to (f) in form of distributions. These plots help us understand the possible distributions and the patterns at different points in time.

The change in the behavior of the process output may occur as a result of changing conditions over time. For example, an upward or downward trend (Figure 5.7(a) and (c)) in the measured output is an indication of the change in the process mean. Consider a manufacturing process where a cutting tool is used to machine a rod to certain diameter. If the cutting tool gets dull over time or the set parameters of the machine change over a long period of use, it may affect the output variable resulting into a gradual change.

A shift in the process as shown in Figure 5.7(b) is an indication that the process mean has shifted. This type of shift can be up or down showing a change in the mean but not necessarily a change in the process variance. The shift may be caused due to improper training of the operators performing certain jobs. Other reasons may include a change in the raw material quality, inexperienced operator or equipment.

The process output shown in Figure 5.7(d) shows a relatively constant mean and a constant variance over time. This is an indication of a constant process and this is the most desirable case.

A gradual increase in the process variance as shown in Figure 5.7(e) is usually caused by the operator fatigue in a manufacturing process or a worker performing repetitive job over a long period of time. Examples may be order processing, typing, or preparing orders to customer specifications.

A situation shown in Figure 5.7(f) is an indication of sudden change in the process. This may be caused by a damaged or breakage of the cutting tool in a machining operation of the manufacturing process. Figure 5.7(g) shows a cyclical pattern that may be caused due to continuous upward and downward shift in the process.

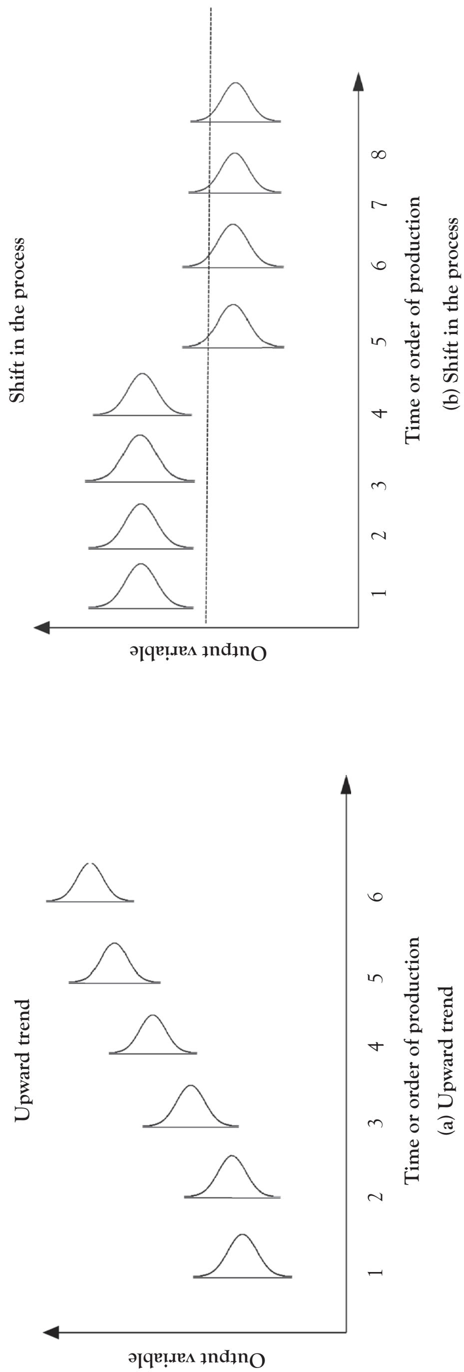


Figure 5.7 (Continued)

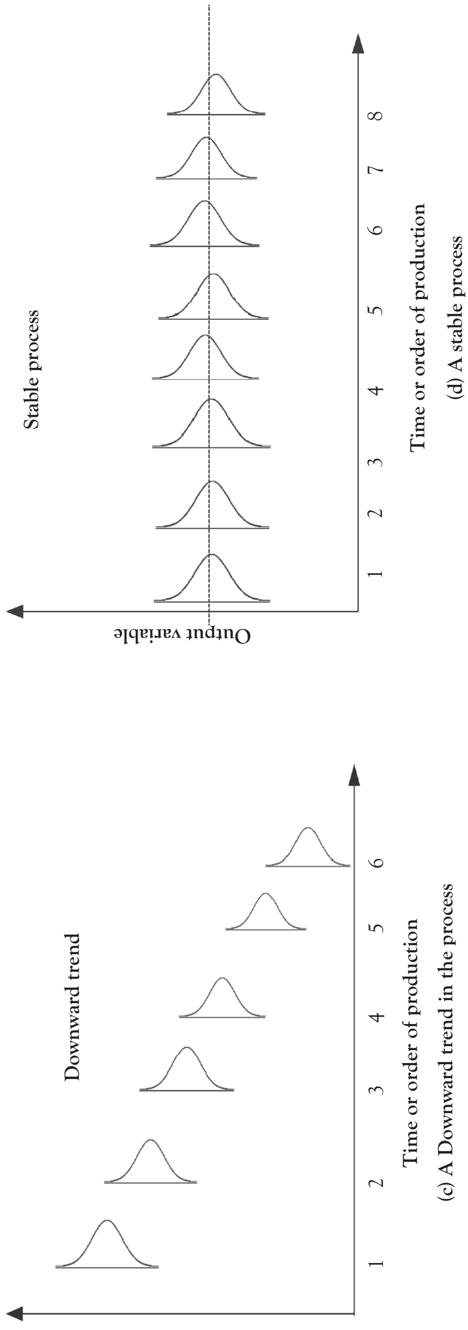
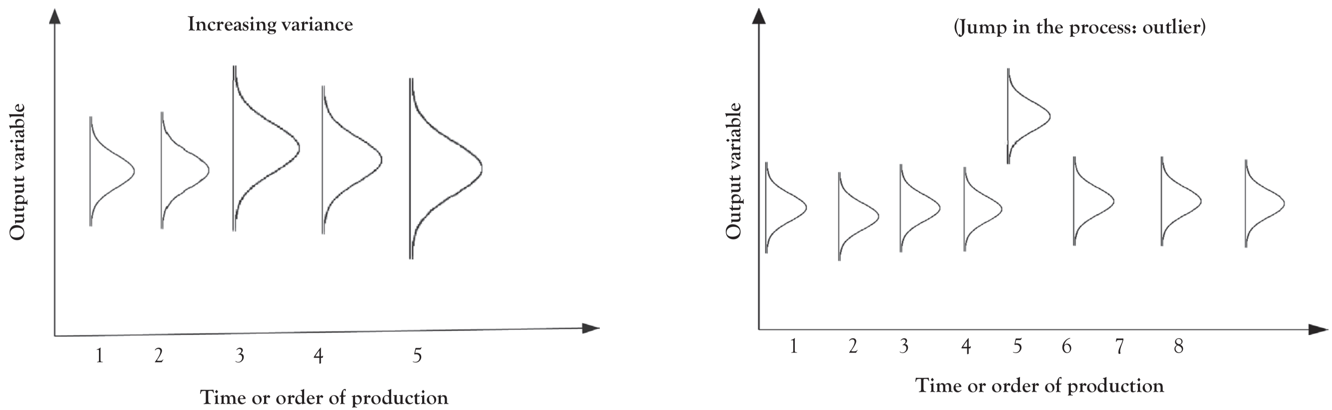


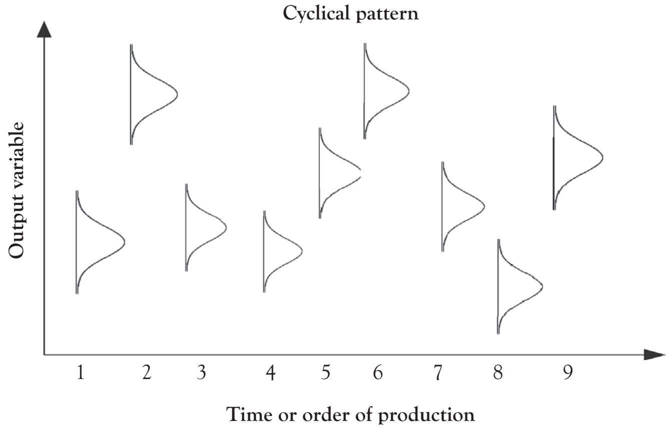
Figure 5.7 (Continued)



(e) A process showing increase in variance

(f) Sudden change in the process output

Figure 5.7 (Continued)



(g) A cyclical pattern

Figure 5.7 Patterns showing variation in the process (a) Upward trend, (b) Shift in the process, (c) A downward trend in the process, (d) A stable process, (e) A process showing increase in variance, (f) Sudden change in the process output, and (g) A cyclical pattern

A change in the process output over time in one or more ways as shown in Figure 5.7 is not desirable. These changes make the process out of control. It is critical to maintain the stability in the process output. For the process to be stable, the distribution of the process output should not change over time. If the output of the process does not change over time we say that the process is stable and is in control. On the other hand, if the process output changes over time, we say that the process is out of control. Figure 5.8(a) shows a stable process or a process that is in statistical control. Figure 5.8(b) shows an out of control process.

When a process is in control, it will produce products within the desired limits or in other words, it will produce acceptable products. A controlled process is stable and its behavior can be predicted over time. It means that the future outcomes of the process are expected to repeat the past. In a process that is out of control, the future outcomes of the process are unpredictable. An out of control process will produce products that do not meet the specification requirements. They do not meet the quality standards, and will deliver inferior quality products or services. As indicated earlier, poor quality products may result into high costs and will make it difficult for the company to stay competitive.

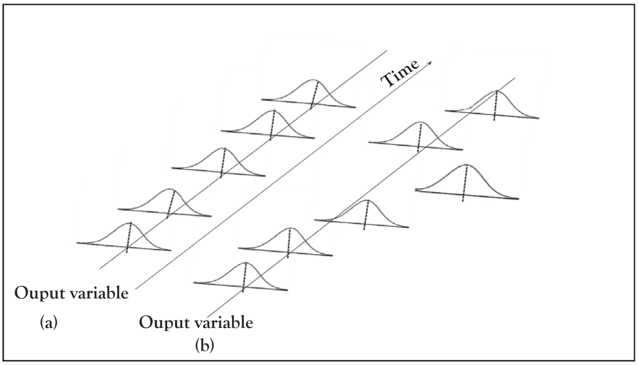


Figure 5.8 (a) A stable process or a process in control, (b) An out-of-control process

The next chapter introduces control charts—a tool that is designed to detect and control the processes by identifying and eliminating the out-of-control conditions by taking appropriate actions. Monitoring the process using control charts and eliminating the cause or causes of problems in the process brings the process under control. This helps to reduce variation resulting into a stable process that is capable of producing quality products. One of the major objectives of the control chart is to reduce the variation in the process. The process of detecting and eliminating the causes of out-of-control conditions and reducing or eliminating the variation using control charts is called *statistical process control.*

Summary

This chapter explained the variation and their effects on the product quality. Variation is inherent part of any process. No two products are exactly the same. There is always some amount of variation present. If the variation is within certain limit, it does not affect the product quality.

We studied two main types of variations in a processes: (1) the random variation (also known as the common or natural causes of variation), and (2) the assignable or special causes of variation. The natural variation is always present in a process. This is also known as the background noise. If the background noise is relatively small, it is considered an acceptable level of process performance. A system that is operating with only chance or natural causes of variation is said to be in a state of statistical control.

The second type of variation is known as the assignable or special cause of variation. The assignable cause means that a cause or causes can be assigned to a specific problem in the process. These are unacceptable process performance. These can cause a shift in the process (upward or downward). This type of variation is greater than the chance variation or the background noise and may show recognizable patterns or trends. A process that shows patterns, trends, or shifts is operating in the presence of assignable cause and is usually out of control. *Run charts* are the tools used to measure variation. They help us see visually the patterns, shifts, or trends in a process. We presented several examples with cases that helped us understand the variation.

When a process is in control (running with only natural variation), it will produce products within the desired limits or in other words, it will produce acceptable products. A controlled process is stable and its behavior can be predicted over time. It means that the future outcomes of the process are expected to repeat the past. In a process that is out of control, the future outcomes of the process are unpredictable. An out of control process will produce products that do not meet the specification requirements. They do not meet the quality standards and will deliver inferior quality products or services. As indicated earlier, poor quality products may result into high costs and will make it difficult for the company to stay competitive.

CHAPTER 6

Control Charts: Fundamentals and Concepts

Introduction

In the previous chapter, we studied the run charts that helped us describe the changes in the process output and also the variation. Just as the run chart is a time series plot that plots the process variable over time, a control chart is a modified form of a run chart with *control limits* added on the run charts. A control chart is one of the tools used to monitor and control a process and systematically reduce process variability. Systematic variation reduction in product and process quality characteristics leads to better product or service performance, better perceived quality by customers, and eventually enhanced competitive position and improved market share. Since the control charts are used to study the variation in a process, an understanding of types of variations is important to understand the control charts. These patterns of variation were described in Figures 5.4 and 5.7 in the previous chapter. As explained earlier, variations can be divided into two broad categories commonly known as the *chance* or *random causes of variation and assignable cause of variation*.

Chance and Assignable Causes of Process Variation

All processes exhibit variation. In any process, a certain amount of inherent or natural variability is always present. There are some variations that we can control (controllable) and others that we cannot control (uncontrollable). The natural variability is known as *chance (or common) causes of variation*. These variations are due to unavoidable causes and they are difficult to detect and usually difficult to control. *A process running* *under only the chance (*or, *common causes of variation) is said to be in state of statistical control.* Thus the variations are identified as:

- *Random variation*: These are chance or uncontrollable variation.
- *Nonrandom variation*: These are assignable causes of variation and have causes that can be identified.

In the presence of large variations in the products, the parts will not fit correctly, products will not function properly, and the product reliability will be affected.

A process with random variation is said to have common causes of variation, whereas a nonrandom variation has special causes of variation that can be attributed to the process. Control charts are designed to signal or detect nonrandom (special causes) of variation.

The random variation is centered around the mean and occurs with consistent amount of dispersion. This type of variation is natural variation and is difficult to control (referred to as *uncontrolled variation*).

Nonrandom (or *special cause*) variations are known as *assignable causes of variation*. The cause may be a shift in the process mean or some unexpected occurrence. Figure 6.1 shows a process that is running with only chance causes of variation.

Figure 6.2 shows a process that is running under assignable causes of variation.

The assignable or nonrandom variation occurs with nonconsistent amount of dispersion and makes the process out of control. In a control chart, control limits are established to control the process. These limits

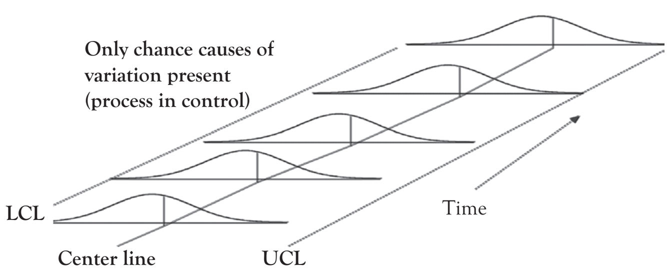


Figure 6.1 A process running with only chance causes of variation

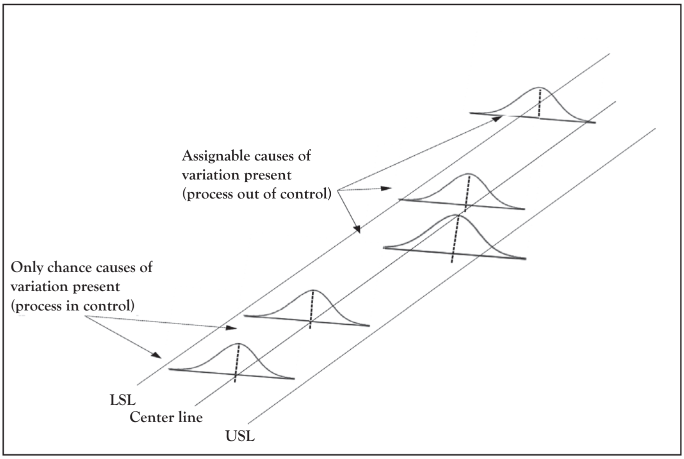


Figure 6.2 A process running with assignable causes of variation

are known as the lower control limit (LCL) and the upper control limit (UCL). The LCL and UCL are a specific distance from a center line (CL). The CL represents the mean of the process or the mean of the quality characteristic being measured. These control limits will be discussed in detail later. The quality characteristic to be controlled may be a variable or an attribute. Some examples of variables are the diameter of rod, length, assembly time, lead time, or other such variable. A quality characteristic, such as, the number of defects produced by a production process is an example of attribute data.

In a control chart, if the quality characteristic of interest plots within the control limits (LCL and UCL) and certain requirements are met, the process producing the product is said to be under statistical control.

Control Charts

A control chart is a graphical display of an output variable or a quality characteristic of interest that plots the variation in that quality characteristic. Such quality characteristic may be the weight of food cans or diameter of shafts that needs to be controlled. Such quality characteristics are known as variables and the chart designed to study the variables is known as variables control chart. If the quality characteristic to be controlled is the mean or the average of the output variable, the chart is known as the control chart for the mean or the *x*-bar chart. In this chart, the mean of the quality characteristic is calculated using a sample of certain size and is plotted on the y-axis, whereas the time or the sample number is plotted on the x-axis. The points in the plot are connected using straight lines that make it easier to visualize the pattern evolving over time. Figure 6.3 shows an example of a typical variable control chart.

The control chart in Figure 6.3 contains a center line (CL) and two horizontal lines labeled UCL and LCL. The CL represents the mean of the quality characteristic being measured. The CL is calculated when the process is in control. The two control limits are equally spaced from the CL and are positioned in such a way that when the process is in control, all the sample points will fall between these two limits. If the sample points (e.g., the average of the quality characteristic calculated using a sample of certain size) fall between the control limits, we say that the process is in control. However, when the sample point plots above or below the control limits, it is an indication of out-of-control condition. In such cases, the process must be investigated to find the cause or causes of out-of-control condition and corrective action be taken to bring the process in control. An out-of-control condition on a control chart is an indication that assignable cause or causes are making the process go out of control.

A control chart displays both the natural and unnatural variations which are known as common and special causes of variation, respectively. The control chart distinguishes between these two variations. *The common causes of variation are inherent variations that occur naturally in the process and are expected*. These variations can be attributed to humans, machines, materials, or methods. Special causes of variation are not inherent to the

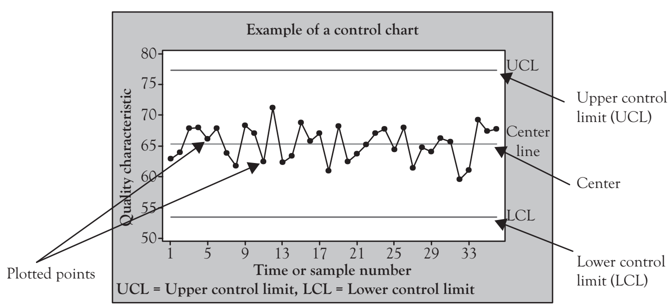
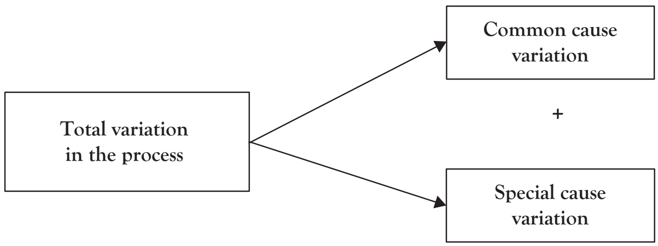


Figure 6.3 An example of a typical control chart

process. These variations can usually be attributed to some cause (assignable cause of variation) and must be corrected once they are identified. Usually, an assignable cause of variation leads to one or more data points falling outside of the control limits on the control chart. These variations reflect a range of unexpected variability. Thus the total variation in a process is the sum of common cause and special cause variation (see the following figure).



If the common cause variation is small, the control chart is used to monitor and improve the process over time. However, if the common cause variation is large, the cause or causes should be investigated to improve the process. Some of the reasons of large common cause variations may be worn out or old machines and equipment that warrants replacement to further improve the process. Figure 6.4 shows the regions of the common and special causes of variation in a control chart.

Statistical Basis of Control Charts

It is important to note that even if all the sample points are plotting within the UCL and LCL, the process still may be out of control. The points on

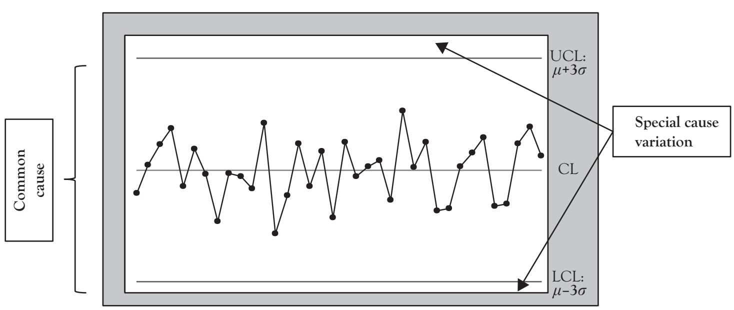


Figure 6.4 A control chart showing common and special causes of variation

the control charts must plot in a random manner and must meet certain conditions before we can say that the process is actually in control. For example, if nine of the plotted points in a row are above the CL but below the UCL or above the LCL, it may be an indication that the process is becoming unstable. Similarly, if 14 of the plotted points alternate up or down the CL, it is an indication that something is wrong. A number of points plotting above or below the CL in a sequence are an indication of nonrandom pattern and out of control condition. Removing these conditions may improve the process performance. We will study more out-of-control conditions in the analysis part of the control charts.

Figure 6.5 shows the statistical basis of control charts and how the control chart works. It shows the importance of sampling and sampling theory. The concepts of sampling, sampling distribution and the central limit theorem play an important role in the design, construction, and analysis of control charts. Note that the variation of individual items is always more than the variation of samples of more than one item. Therefore, we always use a sample of certain size ($n = 5$ or $n = 10$). The averages or the means of these samples are plotted on the control chart if the mean is the quality characteristic of interest (see Figure 6.5).

Three-Sigma Limits in Control Charts

It is a common practice to position the UCL and LCL at three standard deviations from the CL. These limits are commonly known as three-sigma

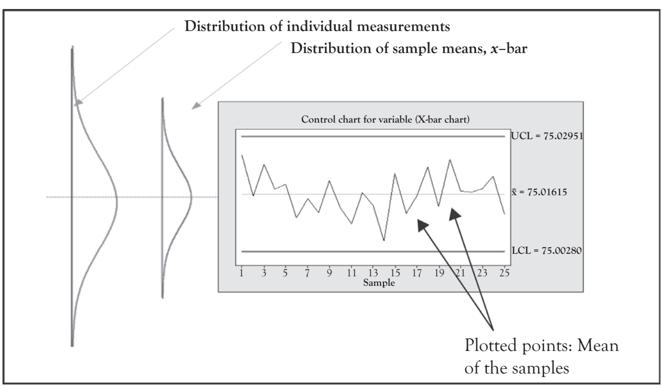


Figure 6.5 Statistical basis of control chart

control limits and are calculated when the process is in control. The basic assumption for the three-sigma limits is that the process follows a *normal distribution*. In a control chart with three-sigma control limits following a normal distribution, the probability of a point falling outside of the control limits is 0.0027 or 0.27 percent or less than three chances in 1,000 (assuming that the process is in control). This can be directly derived from the property of normal distribution and empirical rule we studied earlier. Figure 6.6 shows a control chart with three-sigma control limits.

Figure 6.7 shows the probability of a point falling outside of a control limit. Note that if the process follows a normal distribution then the three-sigma limits contain 99.73 percent of the points falling

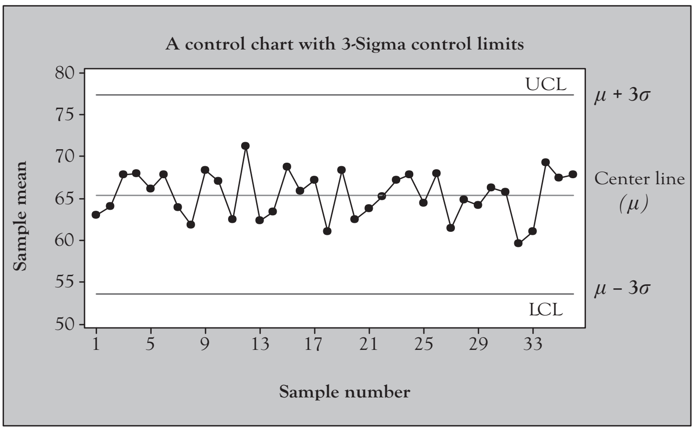


Figure 6.6 A control chart with 3-sigma control limits

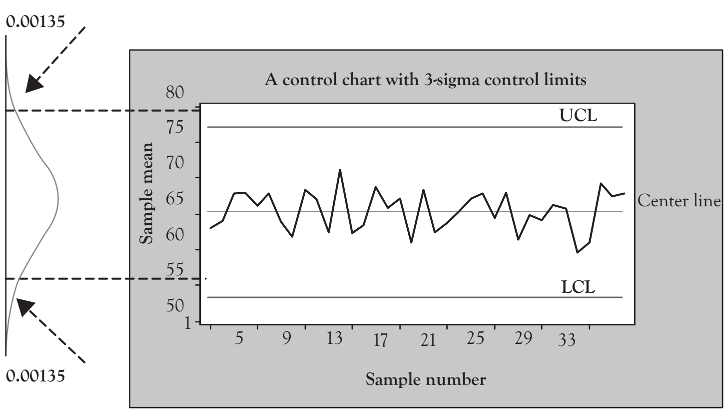


Figure 6.7 Probability of a point falling beyond control limits

within the UCL and LCL and 0.27 percent outside of the control limits. Thus the points falling above and below the control limits are 0.135 percent.

Relationship Between Control Chart and Hypothesis Testing

In Chapter 4, we studied the hypothesis testing procedures where we conducted hypothesis tests to make inference about population parameter or parameters using the information in the sample data. In this section, we will see that there is a close relationship between the hypothesis testing and control charts. Using the control charts, we can make inference about the process. The null and alternate hypothesis for a control chart can be stated in terms of whether the process under investigation is under control or not. The null and alternate hypotheses for a control chart can be written as:

H_0: The process is in control.

H_1: The process is not in control.

These hypotheses can be tested each time we plot a point on the control chart. If a point plots within the control limits, we fail to reject the null hypothesis and conclude that the process is under control. On the other hand, a point plotting outside of the control limits would lead to a conclusion that the process is out of control. The control limits are analogous to the critical values in the hypothesis testing.

Recall that in a hypothesis test, there is always a possibility of making one of the two types of errors—type I and type II errors. Type I error (α) is the probability of rejecting a true null hypothesis, whereas a type II error (β) is failing to reject a null hypothesis when it is false.

In a control chart situation, the probability of a type I error can be defined as concluding that the process is out of control when in fact, it is in control. Similarly, the probability of type II error for the control chart would be to conclude that the process is in control when it is out of control. Earlier we noted that the probability of a point falling outside of the control limits in a control chart with three-sigma control limits is

0.0027 or 0.27 percent. This is the type I error probability for this control chart.

The plot of type II errors for a control chart is known as an *operating characteristic curve* or the OC curve. This plot is helpful in detecting the process shifts of different magnitudes of a control chart.

In control charts, type I error occurs when we conclude that an assignable cause of variation is present when in fact, a chance cause of variation is present. This means that in a control chart with three standard deviations limits (3σ limits), type I error will occur 0.27 percent (or, 3 out of 1000) of the time that is, if a point is outside the control limits and we conclude that the out of control condition is due to assignable cause when it is in fact, due to chance cause will occur 0.27 percent of the time. One of main reasons of using the three-sigma control limits in the control charts is the small type I error associated with the use of three-sigma limits. The other advantage is that the three-sigma limits yields very small false-alarm *signals.*

A type II error occurs when we conclude that a chance cause of variation is present when in fact, an assignable cause is present. This means that a point is inside the control limits and we conclude that it is due to the chance cause when it is due to assignable cause then a type II error is present.

Types of Control Charts

Control charts can be divided into two main categories: (1) control charts for variables and (2) control charts for attributes.

- A variable is a continuous measurement such as weight, height, length, volume, diameter, and so on.
- An attribute is an either-or situation: a product is defective or not. Attributes are also good or bad, pass or fail, and so on.

Table 6.1 shows the major variables and attribute control charts. These charts are described separately in the control phase of Six Sigma.

Table 6.1 Types of variables and attribute control charts

Variables control chart	Attributes control charts
$\bar{x}$ (mean or average)	*p* (proportion defective)
R (range)	*np* (number of defective or nonconforming)
MR (moving range)	*c* (number of nonconforming in a sample space)
s (standard deviation)	*u* (number of defects per unit)
I Chart (control chart for individuals)	

Example 6.1: Control Chart for Individual Measurements

In this example, we demonstrate the construction of a simple control chart. This chart is known as the *chart for individual measurements*. Consider the data in Table 6.2. This data shows 36 measurements on the weight (in ounce) of certain beverage container. A control chart is to be designed to monitor and control the weight of the beverage containers. Using the following data, calculate the mean and standard deviation of the sample of $n = 36$ measurements. Use the calculated statistics to construct the three-sigma control limits and construct the chart for the individual measurements. What conclusion you can draw about this process?

Solution:

Using all the 36 measurements, the descriptive statistics were calculated using a computer package. The values are shown here.

Descriptive Statistics of Weight

Variable	N	N*	Mean	SE Mean	St Dev	Minimum	Q1
Weight	36	0	67.928	0.0836	0.501	67.093	67.608

Median	Q3	Variable	Maximum
67.944	68.262	Weight	69.362

Table 6.2 Weight (in ounce) of 36 beverage containers (in order they were filled)

Order	Weight	Order	Weight	Order	Weight
1	67.7963	13	67.8872	25	67.9746
2	67.2499	14	67.4696	26	68.1972
3	67.6538	15	67.9127	27	68
4	67.2936	16	67.7269	28	68.1909
5	68.2796	17	67.6046	29	67.693
6	69.3617	18	68.1529	30	67.3818
7	68.4466	19	68.7936	31	68.0504
8	68.2075	20	67.1496	32	68.3517
9	68.069	21	68.1597	33	68.3711
10	67.2628	22	67.7104	34	67.1913
11	67.8139	23	68.4184	35	67.0927
12	68.373	24	67.6169	36	68.4856

The mean and the standard deviation of the sample of $n = 36$ from the previous table are:

$$\bar{x} = 67.928$$
$$s = 0.501$$

Taking the earlier sample values as the estimates for the population mean μ and standard deviation σ, the CL and the three-sigma UCL and LCL for the control chart are calculated. The CL is the average of all the 36 measurements which is $\bar{x} = 67.928$. The UCL and LCL for the charts are:

$$\bar{x} + 3s = 67.928 + 3(0.501) = 69.408$$
$$\bar{x} - 3s = 67.928 - 3(0.501) = 66.447$$

Once the CL and the control limits are established, the individual values of 36 sample weights are plotted on the chart. Each small solid circle on the chart represents an individual value. The control chart is shown in Figure 6.8.

The control chart shows that none of points are plotting above or below the UCL and LCL. Also, the plotted points appear to be

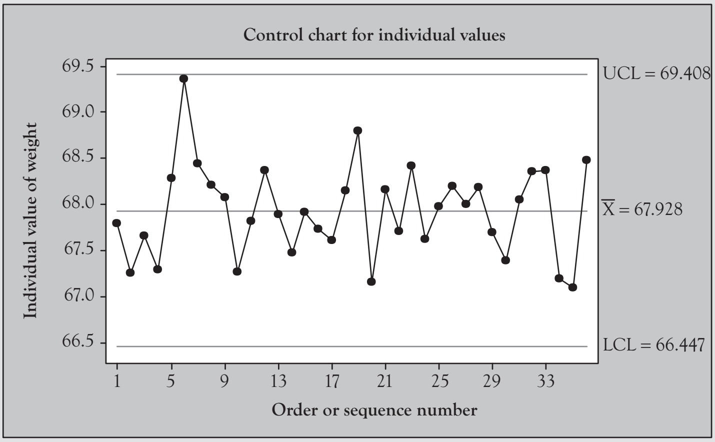


Figure 6.8 Control chart for individual measurements

randomly distributed above and below the CL. There is no obvious visible pattern displayed by the plotted points that is, there is no sign of these points showing a trend, cyclical, or nonrandom pattern. Recall that with each plotted point, the control chart tests the null hypothesis that the process is in control. For our example, we fail to reject the null hypothesis that the process is in control. Therefore; based on this sample data we can conclude the process is in control.

Summary

A control chart is one of the tools used to monitor and control a process and systematically reduce process variability. Systematic variation reduction in product and process quality characteristics leads to better product or service performance, better perceived quality by customers, and eventually enhanced competitive position and improved market share. In this chapter, we provided control chart fundamentals and how the control charts works. We explained the common and special causes of variation related to control charts. The other topics critical to understanding the control charts are the statistical basis behind them and the three-sigma limits on the control charts. These topics were discussed in detail. We also explained the relationship between the control charts and hypothesis testing, and different types of control charts used in monitoring and improving quality.

CHAPTER 7

Control Charts for Variables

Introduction

In this chapter, we discuss a very widely used control chart—the control chart for monitoring the mean and variation of a variable that is to be controlled. This chart is used to study the variations that occur in the mean or central tendency and dispersion in a process data. The control chart for monitoring the mean is referred to as the $\bar{x}$ (*x*-bar) chart and is used to monitor the variation in the process mean by plotting the average value of samples of certain size. Another chart that is used to monitor and study the variation in the process is known as the *R* chart or the control chart for range. In practice, both the $\bar{x}$ and *R* chart are used in conjunction and are the most widely used charts in industry. Used together, the charts are used to monitor and control the process. The $\bar{x}$ chart is used to determine whether a process has gone out of control because of a shift in the process mean. The *R* chart tells us whether the out of control condition has occurred because of the change in the variation in the process. Recall that out of control conditions may occur because of a change in the mean or a change in the variance of the process or both. We will first provide an example of $\bar{x}$ chart. The example is intended to provide an understanding and working of the $\bar{x}$ chart.

Example 7.1: Control Chart for Monitoring the Process Mean—the $\bar{x}$ Chart

In the previous chapter, we discussed the construction and working of individual value control chart. The chart demonstrated the variation in the individual measurements of a variable and also showed if there was a shift in the process mean.

In this section, we explain the $\bar{x}$ chart. As outlined earlier, the $\bar{x}$ chart is used when the average or the mean of a quality characteristic is of interest. The chart is used to monitor and detect whether the mean of the process has shifted. The difference between the individual value chart and the $\bar{x}$ chart is that the $\bar{x}$ chart or chart for the process mean uses the mean or the average of the samples drawn from a process rather than the individual measurements. Thus the $\bar{x}$ chart plots the sample means. In this example we will:

a. Discuss the general structure of the control chart for the mean or $\bar{x}$ chart with the center line (CL) and upper and lower control limits. Show the general form of the $\bar{x}$ chart with the UCL, LCL, and the CL for the chart and explain how the sampling distribution of $\bar{x}$ is used in determining the mean and standard deviation of $\bar{x}$. Explain how the chart of averages differs from the chart of individual values in the previous chapter.
b. Discuss the role of Central Limit Theorem in describing the sampling distribution of the sample mean ($\bar{x}$).
c. Discuss the advantage of taking samples of certain size and plotting the sample average rather than plotting the individual values to monitor the mean of the process.

a. General structure of the control chart for the mean or $\bar{x}$ chart

In a control chart for the mean or the $\bar{x}$ chart, the data contains k samples or subgroups of certain size rather than individual measurements. The mean of each sample is computed and plotted on the chart. These means are computed by averaging the values in each sample. Since the chart plots the means ($\bar{x}$) of each sample, the standard deviation for computing the control limits is not the standard deviation (σ) of individual observation, but it is the standard deviation of $\bar{x}$ referred to as $\sigma_{\bar{x}}$.

The rationale for using $\sigma_{\bar{x}}$ in computing the control limits for the $\bar{x}$ chart follows from the properties of sampling and sampling distribution. Note that the control chart for the average tracks the variation in the sample mean ($\bar{x}$). It follows from the sampling theory that the distribution of $\bar{x}$ of the sample means for a sample size of four or more ($n \geq 4$) follows a normal distribution with mean μ

and standard deviation $\sigma_{\bar{x}}$. This is true even if the samples are drawn from a non-normal population. Since $\sigma_{\bar{x}} = \sigma/\sqrt{n}$, the three-sigma UCL and LCL are given using the following expressions:

$$\text{UCL: } \mu + 3\left(\frac{\sigma}{\sqrt{n}}\right) \text{ and}$$

$$\text{LCL: } \mu - 3\left(\frac{\sigma}{\sqrt{n}}\right) \tag{7.1}$$

This is graphically shown in Figure 7.1.

b. Role of Central Limit Theorem in describing the sampling distribution of the sample mean ($\bar{x}$)

The sampling distribution of the sample mean in the $\bar{x}$ chart can be explained using the Central Limit Theorem discussed in Chapter 4. This theorem states that as the sample size increases, the distribution of the sample mean $\bar{x}$ approaches a normal distribution. For a large sample ($n \geq 30$), the sampling distribution of $\bar{x}$ will be approximately normally distributed with mean μ and standard deviation $\sigma_{\bar{x}}$. Even for small sample sizes of four or five, the sampling distribution of the sample mean $\bar{x}$ can be approximated by a normal distribution. This is why we can use the normal distribution in determining the control limits for the $\bar{x}$ chart.

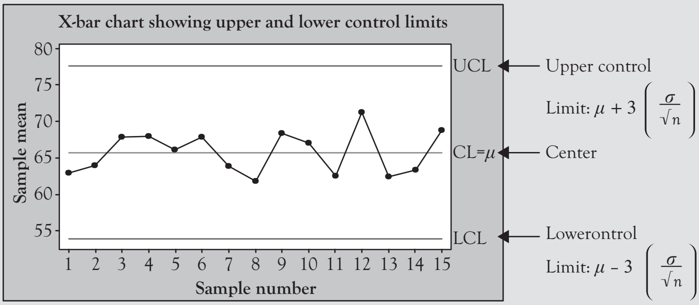


Figure 7.1 The general form of $\bar{x}$ control chart

Note: Each solid circle represents the average of the sample or the sample mean.

c. Plotting the sample average rather than plotting the individual values

Sample averages are used in the $\bar{x}$ control chart rather than individual values because the average values indicate a change in the variation much faster. The chart becomes more sensitive by plotting and tracking the sample means compared to the individual values. Also, with a sample size of two or more ($n \geq 2$), a measure of variation can be obtained for a sample.

Developing Control Charts for Monitoring the Mean and Variation of a Process: The $\bar{x}$ and *R* Chart

This section outlines the procedure and steps to develop the control charts for the mean and dispersion or the variation—the $\bar{x}$ and R chart. These are the control charts for variable. The following are the steps to develop these charts:

1. Select the quality characteristic of interest
2. Select the samples or subgroups and the sample size n for the chart and the frequency with which the samples are to be drawn from the process
3. Collect the data for the charts
4. Establish the CL and the control limits for the charts
5. Implement the chart
6. Revise the CLs and control limits if necessary

Selecting the Quality Characteristic for the Chart

The quality characteristic for the $\bar{x}$ and R charts is a variable that is measurable and can be expressed in numbers. The selected quality characteristic should be the one that directly affects the product quality. In case of manufacturing, the selected quality characteristic may be one or more of the basic units of length, mass, time, temperature, volume, diameter, energy, density, or pressure. The variables are often selected based on the production problems or cost. These variables affect the product quality and contribute to the cost of poor quality. However, one should be careful in selecting the variables to control. Usually, a number of variables are involved in manufacturing a product. In such cases, the selection of the right quality characteristics or variables is critical. It is a waste of effort and uneconomical to control a large number of variables using the $\bar{x}$ and R charts.

Selecting the Sample Size (n) or Subgroups for the Charts

The charts for variables are a function of the sample size. This means that the sensitivity of the chart is affected by the size of the sample. Therefore, selecting the right sample size and also the frequency of the sampling (how often the samples will be drawn from the process, for example every hour, or twice in a shift) is critical.

The samples are groups of items that are plotted on the control chart. These groups or samples are often called *rational subgroups*. Note that the $\bar{x}$ chart monitors the subgroup variability, or variability, over time while the R chart measures the variability within subgroups, or the process variability, at a given time.

In rational subgroups, the size and the frequency of samples are selected in such a way that the samples will have a minimum variation within a subgroup and a maximum variation between subgroups.

In a rational subgroup scheme, the variation within the subgroup is only due to chance causes. This within subgroup variation is used to establish the control limits. The variation between subgroups, on the other hand, is used to determine the long-term stability of the control charts. Two methods are used to select the subgroups:

(a) Instant-Time Method and (b) Period-of-Time Method

In the *instant-time method* the subgroups are selected from the products produced at one instant of time. For example, five consecutive parts produced by a machine, or five parts from a lot of recently produced products. The next subgroup would be five consecutive parts at a different time instant—may be one hour later. *This scheme of subgroup selection will have a minimum variation within a subgroup and a maximum variation between subgroups.*

In the *period-of-time* method, the subgroups are selected from the products produced over a period of time so that they are representative of all the products in that time period. Suppose a quality technician is responsible for selecting subgroups of samples from a production process. He visits the production line every hour. For his first subgroup, he randomly selects a subgroup of five products produced in the previous hour. He then selects another subgroup of five from the products produced between the visits. The period-of-time method of subgroup selection will have a maximum variation within a subgroup and minimum variation between subgroups.

Out of the previous two methods, the instant-time method of subgroup selection is most commonly used. One of the major objectives of $\bar{x}$ chart is to quickly detect the assignable or special causes of variation. The instant-of-time scheme of subgrouping provides a particular time reference for determining the assignable causes of variation. It provides the measures of changes in the process average that is sensitive to the anticipated causes of variation.

Sample Size or Subgroup Size (n)

In selecting the subgroup size, the following should be noted:

a. The sample size is inversely proportional to the width of the control limits in the control chart. This means that as the sample or subgroup size increases, the control limits get closer to the CL. As a result, the control chart becomes more sensitive to small variations in the process average.
b. For $\bar{x}$ and R chart, at least 20 samples or subgroups of size two or more ($n \geq 2$) are needed.
c. Sample size of $n = 4$ to $n = 10$ are usually used in developing the control chart. A sample size of $n = 5$ is very commonly used. Note that the sample size also depends on the type of product.
d. For expensive products or where destructive testing is involved a small sample of size $n = 2$ or $n = 3$ can be used as it will minimize the destruction of expensive products.

e. Larger subgroup sizes make the control chart more sensitive to the small changes in the process average but increases the cost of inspection per subgroup. There has to be a tradeoff between the increased sensitivity and the cost of inspection due to larger subgroup size.
f. For the sample size of four or more ($n \geq 4$), the distribution of the sample means $\bar{x}$ follows a normal distribution. This property is the statistical basis of the control charts and is used in constructing the CL and control limits.
g. For subgroup sizes of more than 10 ($n > 10$), $\bar{x}$ and s charts (control charts for the mean and standard deviation) should be used instead of $\bar{x}$ and R charts (control charts for the mean and range).
h. The $\bar{x}$ chart should be able to detect the shift in the process quickly. The sample size plays a great role in detecting the shift in the process. If the purpose of the chart is to detect moderate-to-large shifts in the process (a shift of 2σ or larger), a sample size of $n = 4$ to $n = 6$ should be used. If it is required to detect smaller shifts in the process, larger sample size ($n = 15$ to $n = 25$) is needed.

Collect the Data

To collect the data for the $\bar{x}$ and R charts, a form similar to the one shown in Table 7.1 can be used. The data consists of k samples or subgroups each of size n. The following form shows that samples of size $n = 5$ are collected. This means that each sample will have five measurements or observations. The data are recorded in the 'Measurements' column of Table 7.1. Since the $\bar{x}$ chart plots the average of the subgroups, the average $\bar{x}$ for each of the k samples is calculated and recorded in the 'Average' column. Also, the range for each sample is calculated and recorded in the 'Range' column. The sample range, R, is used to estimate the standard deviation and to develop the R chart. Additional columns such as, date, time, and a comment column may be added to the data collection form to include additional details.

Table 7.1 Data collection form for the $\bar{x}$ and R chart

Sample no.	Date	Time	Measurements x_1 x_2 x_3 x_4 x_5	Average $\bar{x}$	Range R	Comment
1						
2						
3						
4						
:						
:						
k						

Determine the Control Limits for $\bar{x}$ and R Charts

To develop the control limits for the $\bar{x}$ and R charts, we will use the information in Table 7.2. This table shows the general format of the data for $\bar{x}$ and R charts. Column (1) shows number of samples, 1, 2, … , k for a quality characteristic for which the charts are to be developed. Columns (2) through (6) show the n number of observations for each sample. For example, if each sample is of size five then $n = 5$. Column (7) shows the mean of each sample (or subgroup) and the range for each sample is calculated and stored in column (8).

Centerline and Control Limits for $\bar{x}$ Chart

The centerline and the control limits for the $\bar{x}$ chart are calculated as shown. All calculations refer to Table 7.2.
Centerline: The CL or the mean of the process:

$$\text{CL: } \bar{\bar{x}} = \frac{\sum_{i=1}^{k} \bar{x}_i}{k} = \frac{\bar{x}_1 + \bar{x}_2 + \ldots + \bar{x}_k}{k} \tag{7.2}$$

where $\bar{\bar{x}}$ = the average of sample averages (read as 'x double bar')
$\bar{x}_i$ = average of the ith sample
k = number of samples

Table 7.2 Data format for $\overline{x}$ and R charts

	Measurements						
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Sample number	X_1	X_2	X_3		X_n	**Sample mean** ($\overline{x}_i$)	**Range** (R_i)
1	x_1	x_2	x_3	…	x_n	$\overline{x}_1$	R_1
2	x_1	x_2	x_3	…	x_n	$\overline{x}_2$	R_2
3	x_1	x_2	x_3	…	x_n	$\overline{x}_3$	R_3
4	x_1	x_2	x_3	…	x_n	$\overline{x}_4$	R_4
:	x_1	x_2	x_3	…	x_n	:	:
:	x_1	x_2	x_3	…	x_n	:	:
:	x_1	x_2	x_3	…	x_n	:	:
:	x_1	x_2	x_3	…	x_n	:	:
k	x_1	x_2	x_3	…	x_n	$\overline{x}_k$	R_K
						$\overline{\overline{x}}$	$\overline{R}$

The UCL and LCL are determined using the following expressions:

$$\text{Upper control limit, UCL: } \overline{\overline{x}} + 3\left(\frac{\sigma}{\sqrt{n}}\right) \tag{7.3}$$

$$\text{Lower control limit, LCL: } \overline{\overline{x}} - 3\left(\frac{\sigma}{\sqrt{n}}\right) \tag{7.4}$$

where, σ = process standard deviation, and
n = sample size

In Equations (7.3) and (7.4), the process standard deviation σ is usually unknown and must be estimated before the control limits can be computed. This can be done in several ways including: (1) estimate the standard deviation by calculating the standard deviation of each of the *k* samples and find the average of these *k* standard deviations to get an overall estimate of σ, (2) use the standard deviation calculated from large samples previously generated for this process, that is, use the historical data generated earlier when the process was in control to estimate the standard deviation, and (3) estimate the standard deviation using average of range method or $\overline{R}$ method to estimate the process standard deviation σ. The third approach is the most common and used by the industry. This approach requires calculating the range R (where R = difference between the maximum and minimum values in the sample) of each of the k samples and calculating the average of the ranges of k samples denoted by $\overline{R}$. The calculations to estimate the process standard deviation is as follows:

First, calculate the average of the ranges for the k samples:

$$\overline{R} = \frac{\sum_{i=1}^{k} R_k}{k} = \frac{R_1 + R_2 + \ldots + R_k}{k} \tag{7.5}$$

where R_i = range of sample i,
$\overline{R}$ = average of sample ranges
k = number of samples

It can be shown that dividing $\overline{R}$ in Equation (7.5) by a constant d_2, provides an unbiased estimator for σ. The estimated σ, written as $\hat{\sigma}$ can be given by

$$\hat{\sigma} = \frac{\overline{R}}{d_2}$$

The constant d_2 depends on the sample size n and its value can be obtained for different sample sizes (usually for n = 2 to n = 25) from the table of Constants for Control Chart in the appendix. Substituting the value of $\hat{\sigma}$, in Equations (7.3) and (7.4) we get

$$\text{UCL} = \overline{\overline{x}} + 3\left(\frac{\frac{\overline{R}}{d_2}}{\sqrt{n}}\right) \quad \text{and} \quad \text{LCL} = \overline{\overline{x}} - 3\left(\frac{\frac{\overline{R}}{d_2}}{\sqrt{n}}\right)$$

or,

$$\text{UCL} = \overline{\overline{x}} + 3\left(\frac{\overline{R}}{d_2\sqrt{n}}\right) \quad \text{and} \quad \text{LCL} = \overline{\overline{x}} - \left(\frac{3}{d_2\sqrt{n}}\right)\overline{R}$$

The earlier equations can be written as:

$$\text{UCL} = \bar{\bar{x}} + \left(\frac{3}{d_2\sqrt{n}}\right)\bar{R} \quad \text{and} \quad \text{LCL} = \bar{\bar{x}} - 3\left(\frac{\bar{R}}{d_2\sqrt{n}}\right) \tag{7.6}$$

The factor $3 / d_2\sqrt{n}$ is an estimator of $\sigma_{\bar{x}}$ and is denoted by A_2. Thus

$$A_2 = \frac{3}{d_2\sqrt{n}}$$

Therefore, Equation (7.6) can be written as:

$$\text{UCL} = \bar{\bar{x}} + A_2\bar{R} \quad \text{and} \quad \text{LCL} = \bar{\bar{x}} - A_2\bar{R} \tag{7.7}$$

where the value of A_2 can be obtained from the table of *Constants for Control Chart* in Appendix C.

Summary of the Steps for the $\bar{x}$ Chart

1. Obtain k samples of size two or more ($n \geq 2$) where the number of samples, k is at least 20 ($k \geq 20$).
2. Set up a table similar to Table 7.2 and record your observations. Use this table as a guide for reference.
3. Calculate the mean ($\bar{x}_i$) and the range R_i for each of the samples or subgroups (i = 1, 2, …, k).
4. Calculate the mean of the sample means, $\bar{\bar{x}}$ and the mean of the sample ranges, $\bar{R}$

$$\bar{\bar{x}} = \frac{\sum_{i=1}^{k}\bar{x}_i}{k} = \frac{\bar{x}_1 + \bar{x}_2 + \ldots + \bar{x}_k}{k}$$

and,
$$\bar{R} = \frac{\sum_{i=1}^{k} R_k}{k} = \frac{R_1 + R_2 + \ldots + R_k}{k}$$

where $\bar{\bar{x}}$ = the average of sample averages (read as 'x double bar')
$\bar{x}_i$ = average of the ith sample
k = number of samples

R_i = range of sample i,
$\overline{R}$ = average of sample ranges

5. Determine the UCL and LCL using:

$$\text{UCL} = \overline{\overline{x}} + A_2\overline{R}$$

$$\text{LCL} = \overline{\overline{x}} - A_2\overline{R}$$

where the value of A_2 can be obtained from the table of *Constants for Control Chart* in the appendix.

6. Plot the means of each of the k samples in the order the samples were obtained ($i = 1, 2, \ldots, k$)

Note: In practice, the necessary calculations and the $\overline{x}$ chart are developed using a computer package. We will provide computer details later.

The R-Chart—Control Chart for Monitoring the Variation in a Process

While the $\overline{x}$ chart is used to monitor and detect the shift in the mean of a process, the *R* chart is used to study and detect the variation in the process. The process can be brought under control by detecting and eliminating the causes of shift in the mean and variance of the process. Recall that the control charts are used to detect the assignable cause or causes of variation. The process can get out of control if the mean of the process shifts or the variance changes over time, or both the process mean and variance change.

In this section, we will study the *R* chart—the control chart for monitoring the variation in a process. In practice, the $\overline{x}$ and *R* charts are constructed together and the *R* chart is interpreted before the $\overline{x}$ chart. We will demonstrate the applications of both the charts simultaneously using an example. Before we do that, we will present the underlying principles behind the *R* chart, its general structure, and the control limits.

In the *R* chart, we monitor the variation in the process by plotting the sample ranges. The range for each of the sample is calculated and plotted on the R chart (see Table 7.3, column 8). The other chart used for monitoring the variation in the process is known as the *s-chart* (the control chart for the standard deviation). In case of the s-chart, we calculate the standard deviation of each sample or subgroup and plot the sample standard deviation on the chart. The s-chart is preferred when the subgroup size is 10 or more ($n \geq 10$). For smaller sample size ($n \leq 9$), the R chart is as effective as the s-chart. The R chart is used more widely compared to the s-chart because the sample range is much easier to calculate compared to the sample standard deviation.

Centerline and Control Limits for R Chart

The calculations of the centerline and the control limits for the R chart have the same logic as that of the $\bar{x}$ chart we developed earlier. The three-sigma limits for the $\bar{x}$ were developed by calculating the CL denoted by $\bar{\bar{x}}$ (the mean of the sample means) and the control limits by calculating the standard deviation of $\bar{x}$ or $\sigma_{\bar{x}}$. In a similar way, the CL for the R chart is calculated by taking the average of the ranges denoted by $\bar{R}$ of the samples and the control limits are calculated by calculating the standard deviation of R denoted by σ_R.

The *CL* for the R chart is calculated as shown here (see Table 7.2 for the data format of the chart).

$$\bar{R} = \frac{\sum_{i=1}^{k} R_k}{k} = \frac{R_1 + R_2 + \ldots + R_k}{k}$$

where $R_1, R_2, \ldots, R_k$ are the range of each of the sample $i = 1, 2, \ldots, k$, $\bar{R}$ is the average of sample ranges. Note that $\bar{R}$ is an estimate of μ_R. Figure 7.2 shows the general form of CL and the control limits for the R chart.

Note: Each solid circle represents the range R for a sample or subgroup (see Table 7.2).

In practice, the control limits for the R chart, $\pm 3\sigma_R$ is calculated using an estimator for σ_R. This estimator is given by

$$\hat{\sigma}_R = d_3 \left(\frac{\bar{R}}{d_2} \right) \tag{7.8}$$

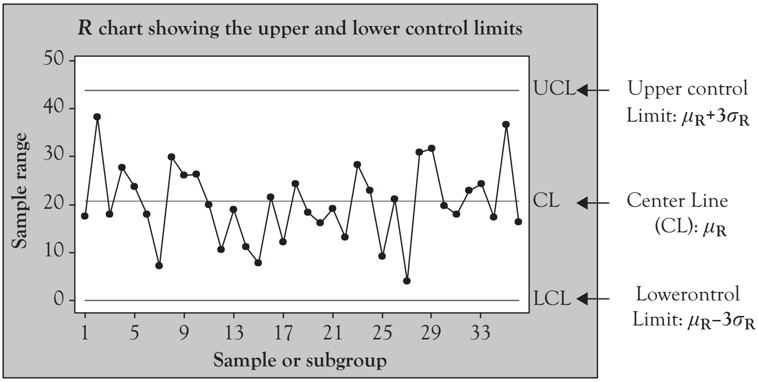


Figure 7.2 The general from of the R chart

where d_2 and d_3 are the constants whose values depend on the size of the sample. These values can be obtained from the table of *Constants for Control Chart* in Appendix C. Using Equation (7.8) the UCL and LCL for the *R* charts can be calculated as:

$$\text{UCL} = \overline{R} + 3\hat{\sigma}_R = \overline{R} + 3d_3\left(\frac{\overline{R}}{d_2}\right) \tag{7.9}$$

$$\text{LCL} = \overline{R} - 3\hat{\sigma}_R = \overline{R} - 3d_3\left(\frac{\overline{R}}{d_2}\right) \tag{7.10}$$

Factoring out $\overline{R}$ from Equation (7.9),

$$\text{UCL} = \overline{R} + 3\hat{\sigma}_R = \overline{R} + 3d_3\left(\frac{\overline{R}}{d_2}\right) = \overline{R}\left(1 + \frac{3d_3}{d_2}\right) = \overline{R}D_4 \tag{7.11}$$

Similarly, Equation (7.10) can be written as

$$\text{LCL} = \overline{R} - 3\hat{\sigma}_R = \overline{R} - 3d_3\left(\frac{\overline{R}}{d_2}\right) = \overline{R}\left(1 - \frac{3d_3}{d_2}\right) = \overline{R}D_3 \tag{7.12}$$

Thus the UCL and LCL for the *R* chart can be given as:

$$\text{UCL} = \overline{R}D_4 \qquad \text{and} \qquad \text{LCL} = \overline{R}D_3 \tag{7.13}$$

In Equations (7.13):

$$D_4 = \left(1 + \frac{3d_3}{d_2}\right) \qquad \text{and} \qquad D_3 = \left(1 - \frac{3d_3}{d_2}\right)$$

The values of D_3 and D_4 can be obtained from the table of *Constants for Control Chart* in the appendix.

Note: The value of D_3 is negative for the samples of size $n = 2$ through $n = 6$. This will result into the LCL value for the R chart to be below zero. Since the sample range cannot take negative values, the negative control limit is meaningless. Therefore, the D_3 values for samples of size $n = 2$ through $n = 6$ in the *Constants for Control Chart* in the appendix are zero. Thus the lower control limit for the R chart for $n \leq 6$ is zero or LCL = 0.

Summary of Steps for R Chart

1. Obtain k samples of size two or more ($n \geq 2$) where the number of samples, k is at least 20 ($k \geq 20$).
2. Set up a table similar to Table 7.2 and record your observations. Refer to Table 7.2 for the explanation of calculations here.
3. Calculate the range R_i for each of the samples or subgroups (i = 1, 2, …, k).
4. Calculate the mean of the sample ranges, $\overline{R}$ using

$$\overline{R} = \frac{\sum_{i=1}^{k} R_k}{k} = \frac{R_1 + R_2 + \ldots + R_k}{k}$$

where k = number of samples
R_i = range of sample i,
$\overline{R}$ = average of sample ranges

5. Determine the UCL and LCL using:

$$\text{CL} = \overline{R}$$

$$\text{UCL} = \overline{R}D_4$$

$$\text{LCL} = \overline{R}D_3$$

The values of D_3 and D_4 can be obtained from the table of *Constants for Control Chart* for a given sample size in the appendix.

6. Plot the ranges of each of the k sample in the order the samples were obtained ($i = 1, 2, \ldots, k$)

Note: In practice, the necessary calculations for the R chart is developed using a computer package. In the next example, we provide manual calculations.

Example 7.2: Constructing and Analyzing Control Charts for Variables ($\bar{x}$ and R charts)

In this example, we will demonstrate the construction and application of $\bar{x}$ and R charts. Consider the data in Table 7.3. This data shows 20 samples or subgroups each of size five drawn from a process that manufactures shafts. The quality inspector visits the process and takes a sample of five consecutive cans. He then visits the process every hour thereafter and takes a sample of five cans till he has 20 samples. The samples show the weight of the cans in ounces. The variable of interest is the average amount of fill weight. Table 7.3 column (1) shows the sample number. Columns (2) through (6) show the five measurements for each sample (each sample is of size five that is, $n = 5$). Column (7) shows the mean of each of the 20 samples and the range for each sample is calculated and recorded in column (8).

a. Construct $\bar{x}$ chart for the data in Table 7.3.
b. Construct R chart for the data in Table 7.3.
c. What conclusions can be drawn from the $\bar{x}$ and R charts?

Constructing the $\bar{x}$ chart requires that we calculate the means of each of the 20 samples in Table 7.3. Then we calculate the mean of these sample means ($\bar{\bar{x}}$) which is the CL for the $\bar{x}$ chart. Finally, we calculate the UCL and LCL. All these calculations can be easily performed using a computer. Here we provide the manual calculations that will help us understand the calculations in the chart and also to interpret the computer generated control charts.

Refer to Table 7.3 that shows 20 samples or subgroups (k = 20 subgroups) each of size five or ($n = 5$). That is, each sample contains

Table 7.3 Twenty samples each of size n = 5 (measured in mm)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Sample number						**Sample mean ($\bar{x}_i$)**	**Range (R_i)**
1	75.045	75.017	75.034	75.007	75.023	75.0252	0.038
2	75.010	75.007	75.016	75.026	75.019	75.0156	0.019
3	75.003	75.039	75.036	75.020	75.017	75.0230	0.036
4	75.017	75.011	75.008	75.030	75.020	75.0172	0.022
5	75.007	75.022	75.030	75.004	75.029	75.0184	0.026
6	75.024	75.009	75.012	75.000	75.008	75.0106	0.024
7	75.010	75.021	75.009	75.015	75.020	75.0150	0.012
8	75.000	75.018	75.008	75.030	75.003	75.0118	0.030
9	75.023	75.010	75.024	75.020	75.019	75.0192	0.014
10	75.013	75.015	75.005	75.022	75.010	75.0130	0.017
11	75.009	75.013	75.009	75.010	75.005	75.0092	0.008
12	75.019	75.015	75.022	75.015	75.011	75.0164	0.011
13	74.998	75.017	75.013	75.012	75.027	75.0134	0.029
14	75.021	74.982	75.009	75.015	74.999	75.0052	0.039
15	75.027	75.029	75.013	75.014	75.022	75.0210	0.016
16	75.015	74.999	75.02	75.013	75.011	75.0116	0.021
17	75.009	75.027	75.001	75.020	75.022	75.0158	0.026
18	75.021	75.025	75.033	75.018	75.015	75.0224	0.018
19	74.999	75.017	75.018	75.020	75.012	75.0132	0.021
20	75.015	75.025	75.028	75.035	75.018	75.0242	0.020
						$\bar{\bar{x}} = 75.0161$	$\bar{R} = 0.02235$

the measurements of finished products. The samples are shown in columns (2) through (6) of Table 7.3. Once the data are obtained, the next step is to calculate the mean of each of the 20 sample means and sample ranges. The mean and the range for the first sample can be calculated as shown here.

$$\bar{x}_1 = \frac{75.045 + 75.017 + 75.034 + 75.007 + 75.023}{5} = 75.0252$$

$$R_1 = 75.045 - 75.007 = 0.038$$

The mean and the range of remaining samples are calculated in a similar way. The means ($\bar{x}_i$) and ranges (R_i) of the rest of the samples are shown in columns (7) and (8). Next, we calculated the mean of the sample means ($\bar{\bar{x}}$) and the mean of the sample ranges ($\bar{R}$). These calculations are demonstrated here.

$$\bar{\bar{x}} = \frac{75.0252 + 75.0156 + \ldots + 75.0242}{20} = 75.0161$$

$$\bar{R} = \frac{0.038 + 0.019 + \ldots + 0.020}{20} = 0.02235$$

The above values are shown in the last row of Table 7.3.

Centerline and Control Limits for $\bar{x}$ Chart

The CL of the chart is given by $\bar{\bar{x}}$. The control limits for the $\bar{x}$ chart are given by Equation (7.7). This equation requires the value of the constant A_2 that can be obtained from the table of *Constants for Control Chart* in the appendix.

For $n = 5$, the value of A_2 is 0.577 or $A_2 = 0.577$. Using this value the UCL and LCL are

$$UCL = \bar{\bar{x}} + A_2\bar{R} = 75.0161 + 0.577(0.02235) = 75.0289$$

$$LCL = \bar{\bar{x}} - A_2\bar{R} = 75.0161 - 0.577(0.02235) = 75.0032$$

The $\bar{x}$ control chart is shown in Figure 7.3.

Constructing the R chart requires that we calculate the range of each of the 20 samples in Table 7.3. Then we calculate the mean of these sample ranges ($\bar{R}$), which is the CL for the R chart. Finally, we calculate the UCL and LCL. The manual calculations for the centerline and control limits are provided here.

Refer to Table 7.3 containing 20 samples or subgroups (k = 20 subgroups) each of size five or ($n = 5$). The range for the first sample is

$$R_1 = 75.045 - 75.007 = 0.038$$

The range of remaining samples is calculated in a similar way. The ranges (R_i) of the rest of the samples is shown in column (8)

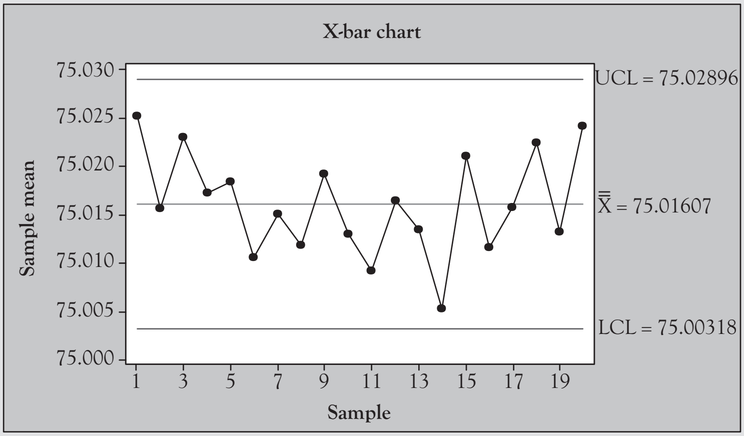


Figure 7.3 The $\bar{x}$ control chart of the data in Table 7.3

of Table 7.3. Next, we calculate the mean of the sample ranges ($\bar{R}$) as:

$$\bar{R} = \frac{0.038 + 0.019 + \ldots + 0.020}{20} = 0.02235$$

Centerline and Control Limits for *R* Chart

Determine the UCL and LCL using:

$$\text{UCL} = \bar{R}D_4$$

$$\text{LCL} = \bar{R}D_3$$

The values of D_3 and D_4 can be obtained from the table of *Constants for Control Chart* in the appendix. For $n = 5$, these values are: $D_3 = 0.000$ and $D_4 = 2.114$. Using $\bar{R} = 0.02235$ and the values of D_3 and D_4

$$\text{UCL} = \bar{R}D_4 = 0.02235(2.114) = 0.0472$$

$$\text{LCL} = \bar{R}D_3 = 0.02235(0.000) = 0$$

The *R* chart is shown in Figure 7.4.

In practice, both the $\bar{x}$ and *R* control charts are constructed together and the *R* chart is interpreted first. Recall that the *R* control

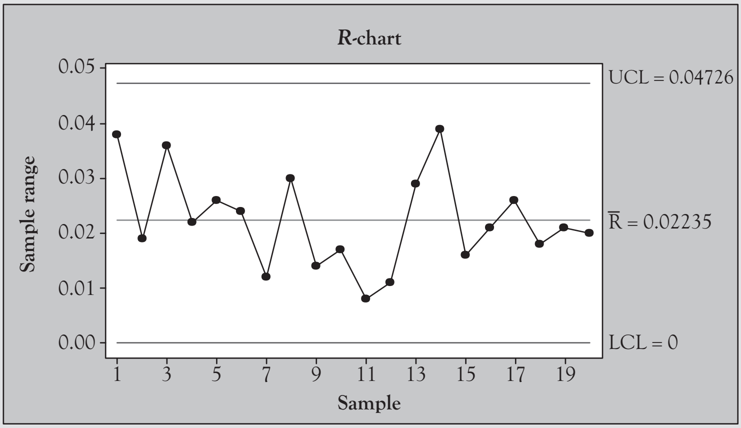


Figure 7.4 The R control chart of the data in Table 7.3

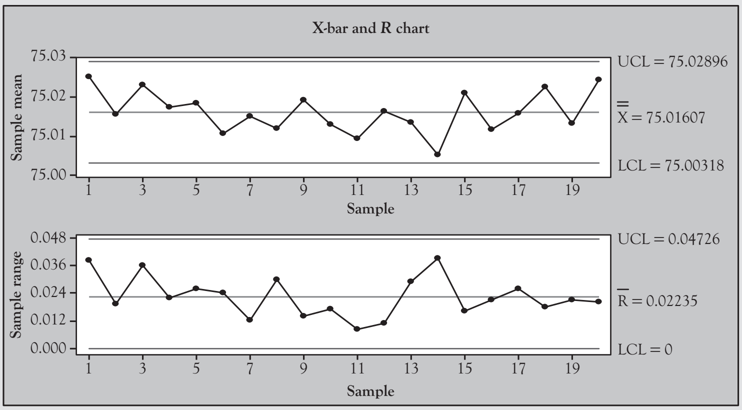


Figure 7.5 The $\bar{x}$ and R control chart of the data in Table 7.3

chart is used to monitor the variability in the process and the $\bar{x}$ chart should be constructed only when the process variation is stable. If the process variation is not stable and the R chart is not in control, the control limits of the $\bar{x}$ chart are meaningless. This is because the limits of the $\bar{x}$ are the functions of the process variation. Figure 7.5 shows the $\bar{x}$ and R control charts together.

From the $\bar{x}$ and R control charts, we can see that all the plotted points on both the charts are within the UCL and LCL. Therefore, we could conclude that the process is under control. However, more extensive tests are required before we can conclude with certainty that the process is under control. These tests determine the presence of nonrandom variation in the chart.

A process may be out-of-control even when all the points plot within the control limits. This may be due to the presence of unnatural runs of variation present in the process that make it out-of-control. Specific tests are performed to detect this pattern of unnatural variation. These tests are explained in the next section under *Interpreting the Control Charts*.

Interpreting the Control Charts

The control charts are used to determine whether the process is in control or out of control. Before we draw any conclusion whether the process is in control or not, we need to understand what is meant by *state of control*; that is, what conditions must be met before we can conclude that the process is control. Similarly, what conditions lead us to conclude that the process is out of control? We discuss here the conditions required for the processes to be in control and out of control.

Process in Control

The following conditions are necessary before we can conclude that the process is in a state of control:

1. When all the plotted points in the control chart are within the control limits, it usually is an indication that the assignable causes have been eliminated from the process and the process is in the state of control.
2. When the process is in control, there exists a natural pattern of variation in the plotted points on the chart. This natural pattern of variation can be identified by these three conditions: (1) approximately 34 percent of the plotted points lie between one standard deviation on both sides of the CL, (2) approximately 13.5 percent of the plotted points lie between one and two standard deviations on either side of the CL, and (3) approximately 2.5 percent of the plotted

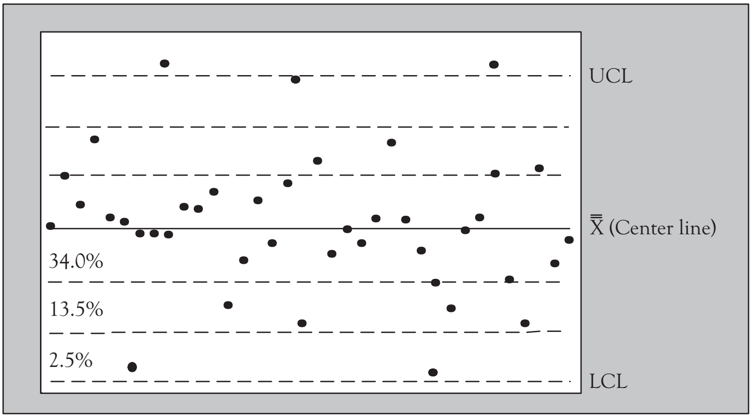


Figure 7.6 Pattern of variation in control chart when the process is in control

points lie between two and three standard deviations on both sides of the CL.* The previous pattern of variation is illustrated using the control chart in Figure 7.6.

When the process is in control, only chance causes of variation are present, and the assignable causes of variation have been eliminated from the process.

Why It Is Important to Have a Process in Control?

Process Out-of-Control

A process is considered out-of-control when one or more points (the average of the sample values) plots outside of the control limits. An out-of-control condition is an indication that the assignable cause is present indicating a change in the process. An out-of-control process is not stable. The unstable and assignable causes of variation make it difficult to predict the future behavior of the process. Most out-of-control processes affect the quality of the products and services. When the cause or the causes of assignable causes are found and eliminated, a process attains a stable state.

* The percentage refers to the imaginary band. In practice, the bands are usually not plotted.

Usually an out-of-control process is identified by the points plotting outside of the control limits, but a process can also be considered out-of-control even when all the points fall inside the control limits. In cases where all the points are plotting inside the control limits, there may be unnatural runs of variation present in the process that make it out-of-control.

The runs of variation of the plotted points on the control chart provide information about the process being in control or out-of-control. To identify the runs and interpret the control chart to determine whether the process is out-of-control, it is usually helpful to divide the chart in six zones. This is shown in Figure 7.7. The figure shows a $\bar{x}$ control chart divided into six zones where each zone is one standard deviation wide. These zones are labeled A, B, and C zones. The regions within one standard deviation of CL ($\bar{\bar{x}}$) are called Zone C; the regions between one and two standard deviations from the CL are called Zone B; whereas, the regions between two and three standard deviations from the CL are called Zone A.

Figure 7.8 shows six tests—tests (1) to (6) check various out-of-control conditions in control charts. Figure 7.8, test (1) shows a point plotting above the UCL and a point plotting below the LCL. These are indication of the presence of assignable causes that must be corrected. In the control charts labeled tests (2) through test (5) of Figure 7.8, the plotted points

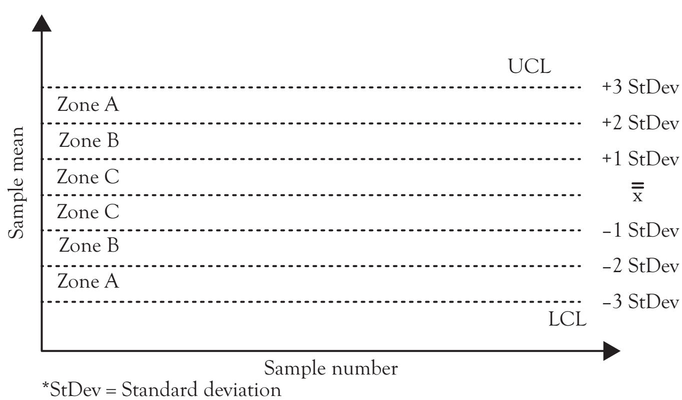
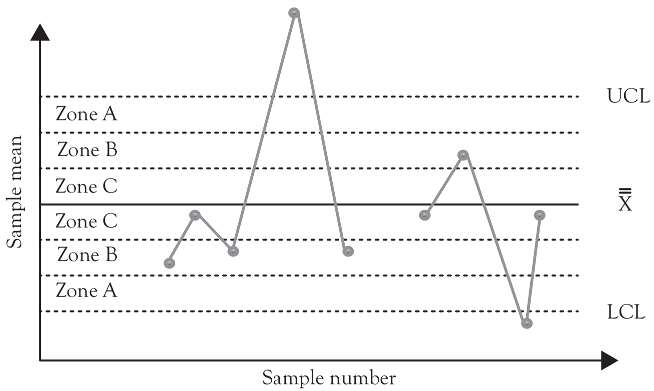
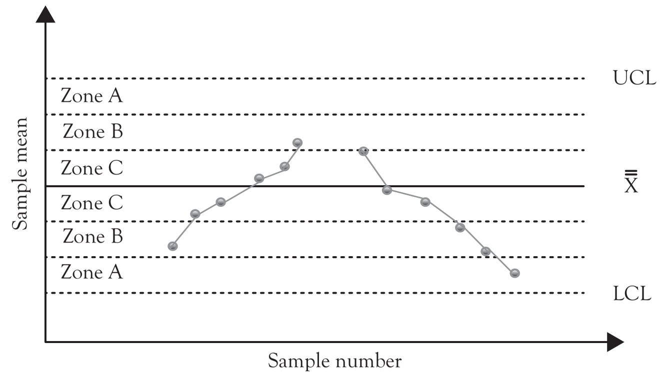


Figure 7.7 The $\bar{x}$ control chart divided into six zones (StDev = Standard deviation)

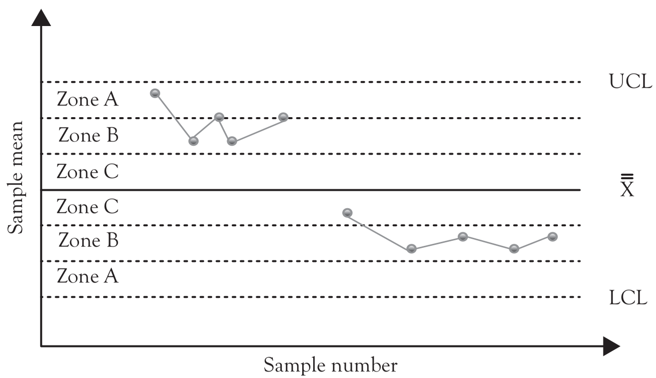


Test 1: One point beyond zone A

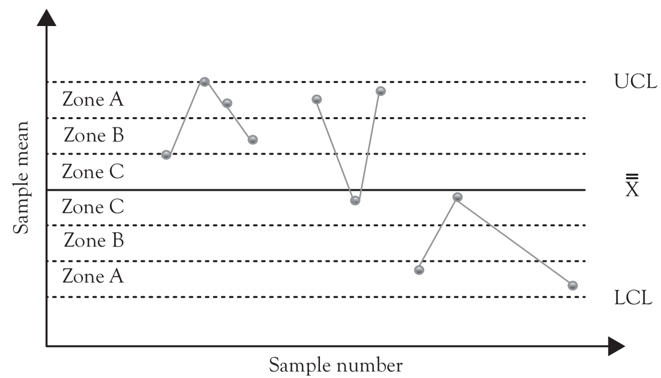


Test 2: Six Points in a row steadily increasing or decreasing

Figure 7.8 (Continued)



Test 3: Four out of five points in a row in zone B or beyond



Test 4: Two out of three points in a row in zone A

Figure 7.8 (Continued)

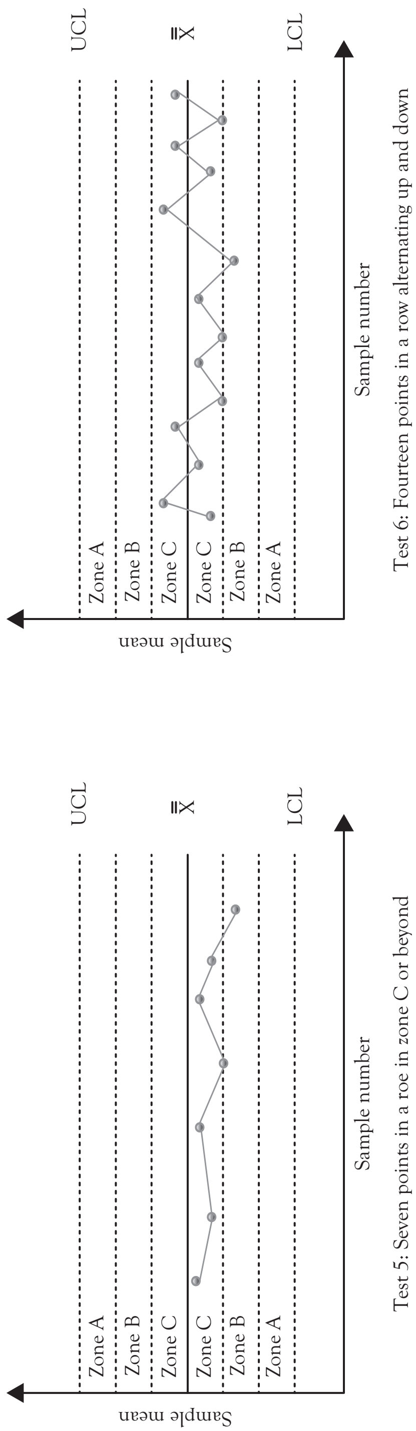


Figure 7.8 Processes showing unnatural runs—process out-of-control

are inside the control limits but these processes are still out-of-control. The reason or reasons these processes are considered out-of-control is that the plotted points show the presence of unnatural or nonrandom pattern of variation that have not gone beyond the control limits.

The plots in Figure 7.8 refer to the *pattern-analysis of control charts*. The patterns are summarized in Table 7.4.

Computer programs such as, MINITAB has the option of automatically performing the pattern-analysis in control charts. Table 7.5 summarizes the pattern analysis performed by MINITAB. It shows the tests performed by the software for special causes in a control chart.

Table 7.4 Summary of the patterns in Figure 7.8

Test (1)	One point beyond zone (A) or outside the upper or lower control limit
Test (2)	Six points in a row steadily increasing or decreasing
Test (3)	Four out of five points in zone B or beyond
Test (4)	Two out of three points in a row in zone A
Test (5)	Seven points in a row in zone C or beyond
Test (6)	Fourteen points in a row alternating up and down.

Table 7.5 MINITAB* tests for special causes in a control chart

Test for special cause		K
1.	1 point > K standard deviations from CL	3
2.	K points in a row on same side of CL	9
3.	K points in a row, all increasing or all decreasing	6
4.	K points in a row, alternating up or down	14
5.	K out of k + 1 points > 2 standard deviations from the CL (same side)	2
6.	K out of k + 1 points > 1 standard deviation from the CL (same side)	4
7.	K points in a row within one standard deviation of CL (same side)	15
8.	K points in a row within one standard deviation from CL (either side)	8

* MINITAB Statistical Software.

Note that the pattern analysis or the tests for special causes performed by MINITAB are easier to understand and implement because the tests do not require the chart to be divided into different zones as shown in Figure 7.8.

Example 7.3: Constructing and Analyzing a Control Chart

The data in Table 7.6 shows 25 samples each of size five from a manufacturing process that produces a certain type of shaft. The quality characteristic of interest is the diameter of the machined shaft.

a. Construct a $\bar{x}$ chart for the data in Table 7.6.
b. Construct a R chart for the data in Table 7.6.
c. Construct the $\bar{x}$ and R charts using one plot. Why is it important to analyze the R chart before the $\bar{x}$ chart?
d. From the $\bar{x}$ and R chart, can we conclude that the process is stable and in control?

Solution: Note the following for the data in Table 7.6:

- The quality characteristic (diameter) is a variable
- Each row shows a subgroup of size five ($n = 5$)
- There are 25 samples each of size five
- To calculate the CL and the UCL and LCL for the $\bar{x}$ control chart, the mean of each sample was calculated and recorded in column (7) of Table 7.6.
- For the R chart, the range of each sample is calculated. These are shown in column (8) of Table 7.6.
- The mean of all sample means ($\bar{\bar{x}} = 80.0256$) is the CL and
- the average of the ranges is $\bar{R} = 0.02252$ (see Table 7.6).

Before we construct the $\bar{x}$ chart, we explain the calculations of the centerline and the control limits for this chart.

Centerline and Control Limits for $\bar{x}$ Chart

The CL of the chart is given by $\bar{\bar{x}}$. The calculations for the control limits for the $\bar{x}$ chart are described here. This equation requires the value of constant A_2 that can be obtained from the table of *Constants for Control Chart* in the appendix.

For $n = 5$, the value of A_2 is 0.577 or $A_2 = 0.577$. Also,

$$\bar{\bar{x}} = \frac{\sum \bar{x}_i}{25} = 80.0256 \text{ and } \bar{\mathrm{R}} = 0.02252$$

Using this value the UCL and LCL are

$$UCL = \bar{\bar{x}} + A_2\bar{R} = 80.0256 + 0.577(0.02252) = 80.03859$$

$$LCL = \bar{\bar{x}} - A_2\bar{R} = 80.0256 - 0.577(0.02252) = 80.01260$$

The $\bar{x}$ control chart is shown in Figure 7.9.

Centerline and Control Limits for R Chart

The CL for the *R* chart is given by $\bar{R}$. For our example, $\bar{R} = 0.02252$ Determine the UCL and LCL using:

$$\mathrm{UCL} = \bar{R}D_4$$

$$\mathrm{LCL} = \bar{R}D_3$$

The values of D_3 and D_4 can be obtained from the table of *Constants for Control Chart* in the appendix. For $n = 5$, these values are: $D_3 = 0.000$ and $D_4 = 2.114$. Using $\bar{R} = 0.02252$ and the values of D_3 and D_4

$$\mathrm{UCL} = \bar{R}D_4 = 0.02252(2.114) = 0.0476$$

$$\mathrm{LCL} = \bar{R}D_3 = 0.02235(0.000) = 0$$

Table 7.6 Twenty five samples of size n = 5 from a shaft manufacturing process

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Sample number						Sample mean ($\bar{x}_i$)	Range (R_i)
1	80.056	80.028	80.045	80.018	80.034	80.036	0.038
2	80.011	80.018	80.017	80.017	80.020	80.017	0.009
3	80.014	80.050	80.047	80.031	80.028	80.034	0.036
4	80.028	80.022	80.019	80.041	80.031	80.028	0.022
5	80.018	80.033	80.041	80.015	80.040	80.029	0.026
6	80.035	80.020	80.023	80.011	80.019	80.022	0.024
7	80.021	80.032	80.020	80.026	80.031	80.026	0.012
8	80.011	80.029	80.019	80.041	80.014	80.023	0.030
9	80.034	80.021	80.035	80.031	80.030	80.030	0.014
10	80.024	80.026	80.016	80.033	80.021	80.024	0.017
11	80.020	80.024	80.020	80.021	80.016	80.020	0.008
12	80.030	80.026	80.033	80.026	80.022	80.027	0.011
13	80.009	80.028	80.024	80.023	80.038	80.024	0.029
14	80.032	79.993	80.020	80.026	80.010	80.016	0.039
15	80.038	80.040	80.024	80.025	80.033	80.032	0.016
16	80.026	80.010	80.031	80.024	80.022	80.023	0.021
17	80.020	80.038	80.012	80.031	80.033	80.027	0.026
18	80.032	80.036	80.044	80.029	80.026	80.033	0.018
19	80.010	80.028	80.029	80.031	80.023	80.024	0.021
20	80.026	80.036	80.039	80.046	80.029	80.035	0.020
21	80.014	80.017	80.015	80.021	80.012	80.016	0.009
22	80.020	80.025	80.016	80.012	80.035	80.022	0.023
23	80.016	80.005	80.006	80.035	80.030	80.018	0.030
24	80.041	80.034	80.019	80.026	80.036	80.031	0.022
25	80.008	80.001	80.021	80.043	80.039	80.022	0.042
						$\bar{\bar{x}} = 80.0256$	$\bar{R} = 0.02252$

The R chart is shown in Figure 7.10.

Figure 7.11 shows both the $\bar{x}$ and R charts in one plot. In practice, both the $\bar{x}$ and R control charts are constructed together and the R

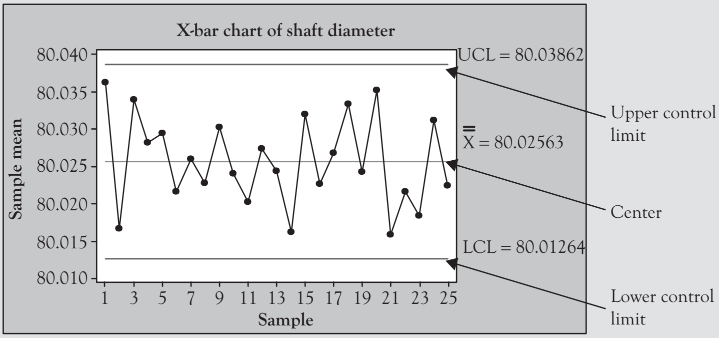


Figure 7.9 The $\bar{x}$ control chart of the shaft manufacturing process

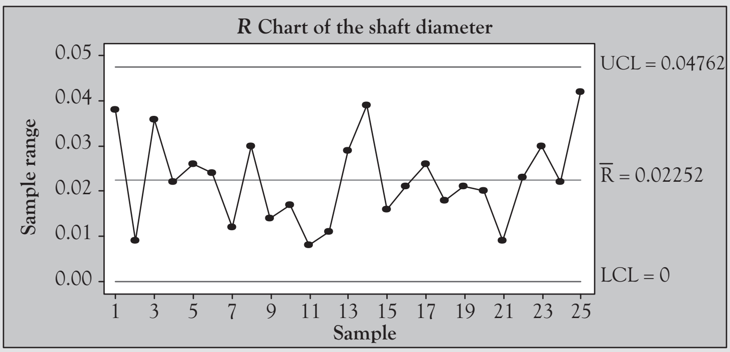


Figure 7.10 The R control chart of the shaft diameter

chart is interpreted first. Since the R control chart is used to monitor the variability in the process, the $\bar{x}$ chart should be constructed only when the process variation is stable. If the process variation is not stable and the R chart is not in control, the control limits of the $\bar{x}$ chart have no meaning because the limits of the $\bar{x}$ chart are the functions of the process variation.

The $\bar{x}$ and R charts in Figure 7.11 show that all the points plot within the control limits indicating no out of control conditions. As indicated earlier, it is not enough for the sample points to plot within the control limits for the process to be in control. Even if all the points fall within the control limits, there may be nonrandom pattern of variation that must be identified before we can conclude that the process in fact, is in control.

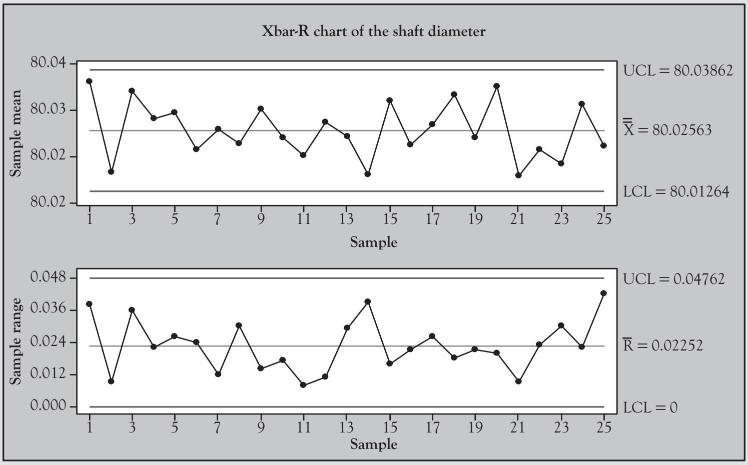


Figure 7.11 X-bar and R chart of the shaft diameter

To identify these nonrandom patterns of variation, tests 1 through 6 in Figure 7.8 are applied. Referring to Figure 7.8, we see that to apply these rules, we need to divide the control chart in A, B, and C zones where each zone is one standard deviation wide and are as labeled A, B, and C zones. The regions within one standard deviation of CL ($\bar{\bar{x}}$) are called Zone C; the regions between 1 and 2 standard deviations from the CL are called Zone B, whereas the regions between 2 and 3 standard deviations from the CL are called Zone A (see Figure 7.8). The zone boundaries can be calculated using the expression here:

$$\bar{\bar{x}} \pm k\left(\frac{\frac{\bar{R}}{d_2}}{\sqrt{n}}\right)$$

where $\bar{\bar{x}} = 80.0256$, $\bar{\text{R}} = 0.02252$, d_2 is a constant that depends on the sample size. For our example, $d_2 = 2.326$ for $n = 5$ from the table of *Constants for Control Chart* in the appendix. In the previous expression, k is the standard deviation. Setting $k = 1, 2, 3$ the zone boundaries that are 1, 2, and 3 standard deviations from the CL can be calculated. Statistical software such as MINITAB can be used to calculate the zone boundaries. Figure 7.12 shows the $\bar{x}$ chart with the zone boundaries.

The $\bar{x}$ chart with the zone boundaries in Figure 7.12 was compared to the six patterns in Figure 7.8. A careful examination revealed no out-of-control conditions. All the points are within the control limits and there appears to be no nonrandom variation in the control chart. Therefore, we can conclude that this process is in control.

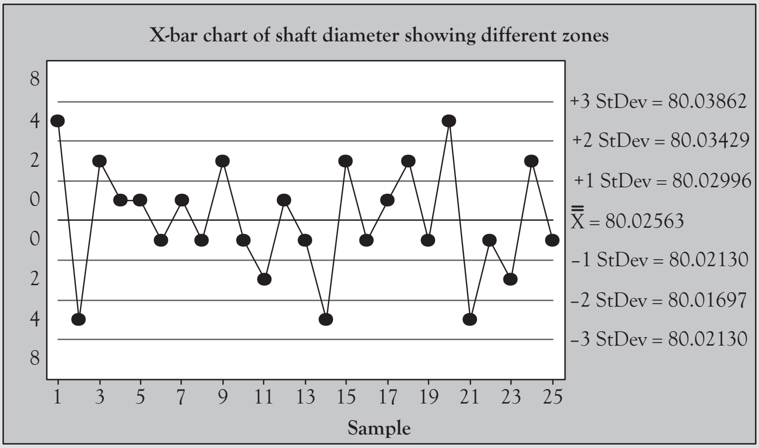


Figure 7.12 The $\bar{x}$ chart of the shaft diameter with zone boundaries

Example 7.4: Monitoring the Continued Process

The process in the Example 7.3 continued and 15 new samples of size $n = 5$ were collected. Table 7.7 shows the additional samples along with the sample means and ranges. Recall that the quality characteristic of interest is the diameter of a machined shaft.

a. Construct a R chart for the data in Table 7.7.
b. Construct a $\bar{x}$ chart for the data in Table 7.7.
c. Construct the $\bar{x}$ and R charts using one plot. Why is it important to analyze the R chart before the $\bar{x}$ chart?

d. From the, $\bar{x}$ and R chart, can we conclude that the process is still stable and in control?

Table 7.7 Additional samples from the process

Sample						Sample mean ($\bar{x}_i$)	Range (R_i)
1	80.056	80.028	80.045	80.018	80.034	80.036	0.038
2	80.011	80.018	80.017	80.017	80.02	80.017	0.009
3	80.014	80.05	80.047	80.031	80.028	80.034	0.036
4	80.028	80.022	80.019	80.041	80.031	80.028	0.022
5	80.018	80.033	80.041	80.015	80.04	80.029	0.026
6	80.035	80.02	80.023	80.011	80.019	80.022	0.024
7	80.021	80.032	80.02	80.026	80.031	80.026	0.012
8	80.011	80.029	80.019	80.041	80.014	80.023	0.030
9	80.034	80.021	80.035	80.031	80.03	80.030	0.014
10	80.024	80.026	80.016	80.033	80.021	80.024	0.017
11	80.02	80.024	80.02	80.021	80.016	80.020	0.008
12	80.03	80.026	80.033	80.026	80.022	80.027	0.011
13	80.009	80.028	80.024	80.023	80.038	80.024	0.029
14	80.032	79.993	80.02	80.026	80.01	80.016	0.039
15	80.038	80.04	80.024	80.025	80.033	80.032	0.016
16	80.026	80.01	80.031	80.024	80.022	80.023	0.021
17	80.02	80.038	80.012	80.031	80.033	80.027	0.026
18	80.032	80.036	80.044	80.029	80.026	80.033	0.018
19	80.01	80.028	80.029	80.031	80.023	80.024	0.021
20	80.026	80.036	80.039	80.046	80.029	80.035	0.020
21	80.014	80.017	80.015	80.021	80.012	80.016	0.009
22	80.02	80.025	80.016	80.012	80.035	80.022	0.023
23	80.016	80.005	80.006	80.035	80.03	80.018	0.030
24	80.041	80.034	80.019	80.026	80.036	80.031	0.022
25	80.008	80.001	80.021	80.043	80.039	80.022	0.042
26	80.038	80.041	80.056	80.012	80.026	80.035	0.044
27	80.021	80.036	80.016	80.041	80.027	80.028	0.025
28	80.013	80.025	80.011	80.026	80.016	80.018	0.015

29	80.034	80.036	80.029	80.017	80.032	80.030	0.019
30	80.029	80.026	80.027	80.012	80.023	80.023	0.017
31	80.02	80.029	80.041	80.046	80.03	80.033	0.026
32	80.034	80.028	80.044	80.021	80.031	80.032	0.023
33	80.027	80.03	80.016	80.022	80.024	80.024	0.014
34	80.041	80.026	80.042	80.051	80.026	80.037	0.025
35	80.056	80.031	80.026	80.042	80.038	80.039	0.030
36	80.027	80.016	80.021	80.036	80.05	80.030	0.034
37	80.041	80.046	80.046	80.031	80.045	80.042	0.015
38	80.061	80.036	80.038	80.041	80.052	80.046	0.025
39	80.043	80.039	80.062	80.051	80.052	80.049	0.023
40	80.036	80.031	80.055	80.026	80.046	80.039	0.029
						$\bar{\bar{x}} = 80.0286$	$\bar{R} = 0.02318$

Solution:

(a) Figure 7.13 shows the *R* chart. From the chart it is evident that there are no out-of-control points. The tests for out-of-control conditions show that the *R* chart is in control. Therefore, we construct the $\bar{x}$ chart and analyze it (see part (b)).

(b) Figure 7.14 shows the $\bar{x}$ chart. The chart also shows different zones that make it easy to check for the out-of-control

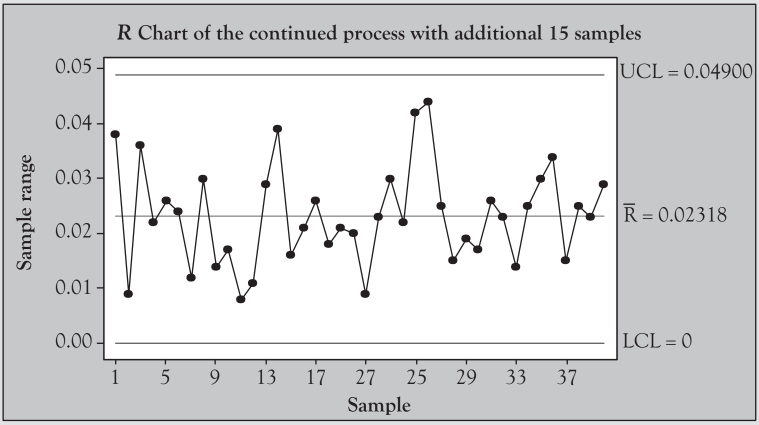


Figure 7.13 R chart for the data in Table 7.7

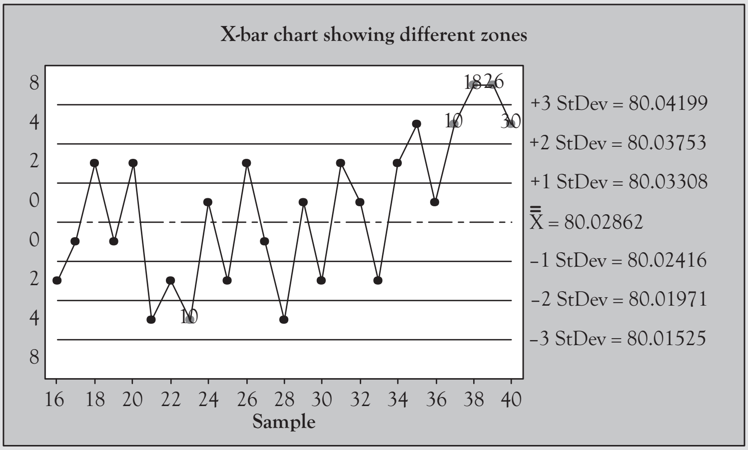


Figure 7.14 The $\bar{x}$ chart showing the out-of-control conditions

Table 7.8 Tests for out-of-control conditions

Test results for zone chart TEST. Cumulative score greater than or equal to zone 4 score. **Test failed at points: 14, 23, 37, 38, 39, 40** * WARNING * If graph is updated with new data, the results above may no longer be correct.

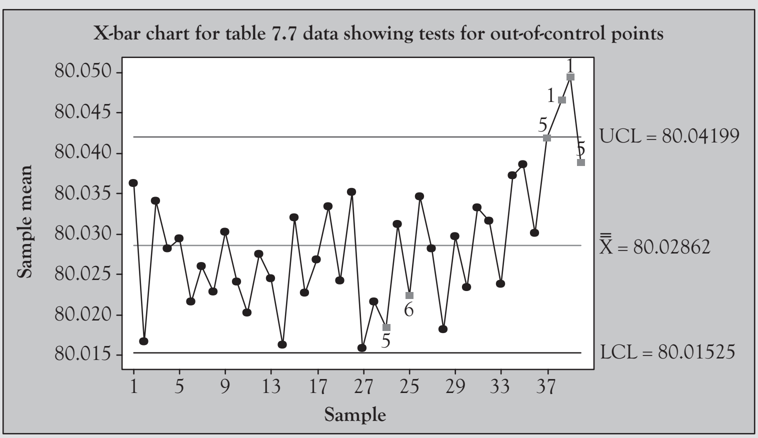


Figure 7.15 The $\bar{x}$ control chart with out-of-control points

conditions. We used MINITAB to check for the out-of-control conditions. Table 7.8 shows the results of out-of-control conditions and indicates where the tests failed.

Figure 7.15 shows the $\bar{x}$ chart with out-of-control points. We applied the automatic test options to check for the out-of control conditions and also for the check of nonrandom patterns in the chart. The results are shown in Table 7.9.

From the previous plots and the test results it is evident that the process is out-of-control. Corrective actions are needed to bring the process in control.

Table 7.9 Test results for out-of control points for Xbar chart

Test results for out-of control points for Xbar chart
TEST 1. One point more than 3.00 standard deviations from CL.
Test Failed at points: 38, 39
TEST 5. 2 out of 3 points more than 2 standard deviations from CL (on one side of CL).
Test Failed at points: 23, 37, 38, 39, 40
TEST 6. 4 out of 5 points more than 1 standard deviation from CL (on one side of CL).
Test Failed at points: 25, 38, 39, 40
* WARNING * If graph is updated with new data, the results above may no
* longer be correct.

Revising the Centerline and Control Limits if Necessary

The process in Example 7.2 continued and 25 samples of size five ($n = 5$) were collected. The new samples were used to construct the $\bar{x}$ and R control charts. Both the charts showed out-of-control points. At this point it is necessary to analyze the control charts and find the cause or causes of out-of-control conditions. If there are convincing reasons to believe that the conditions are because of the assignable causes, then they should be corrected, the out-of-control points should be discarded, and the revised control limits should be calculated. The next example demonstrates how to calculate the revised control limits.

Example 7.5: Revising the Control Limits of Control Charts ($\bar{x}$ and R charts)

Table 7.10 shows 25 new samples of the process explained in Example 7.2.

a. Construct the $\bar{x}$ and R control charts of the samples. Analyze the control charts for out of control conditions and revise the control limits if necessary.
b. If you revised the control limits, construct the $\bar{x}$ and R control charts using the revised limits and check for the out of control conditions.

Table 7.10 Additional samples for Example 7.2

Sample						Sample mean $\bar{x}_i$	Range: R_i
1	75.045	75.017	75.034	75.007	75.023	75.0252	0.0380
2	75.010	75.007	75.016	75.026	75.019	75.0156	0.0190
3	75.003	75.039	75.036	75.020	75.017	75.0230	0.0360
4	75.017	75.011	75.008	75.030	75.020	75.0172	0.0220
5	75.007	75.070	75.030	75.054	75.046	75.0414	0.0630
6	75.024	75.009	75.012	75.000	75.008	75.0106	0.0240
7	75.010	75.021	75.009	75.015	75.020	75.0150	0.0120
8	75.000	75.018	75.008	75.030	75.003	75.0118	0.0300
9	75.023	75.010	75.024	75.020	75.019	75.0192	0.0140
10	75.013	75.015	75.005	75.022	75.010	75.0130	0.0170
11	75.009	75.013	75.009	75.010	75.005	75.0092	0.0080
12	75.019	75.015	75.022	75.015	75.011	75.0164	0.0110
13	74.998	75.017	75.013	75.012	75.027	75.0134	0.0290
14	75.021	74.982	75.009	75.015	74.999	75.0052	0.0390
15	75.002	74.909	74.993	75.004	75.000	74.9816	0.0950
16	75.015	74.999	75.020	75.013	75.011	75.0116	0.0210
17	75.009	75.027	75.001	75.020	75.022	75.0158	0.0260
18	75.021	75.025	75.033	75.018	75.015	75.0224	0.0180
19	74.999	75.017	75.018	75.020	75.012	75.0132	0.0210
20	75.015	75.025	75.028	75.035	75.018	75.0242	0.0200
21	75.013	75.016	75.024	75.020	75.011	75.0168	0.0130
22	75.019	75.014	75.005	75.021	75.024	75.0166	0.0190
23	75.070	75.064	75.005	75.048	75.099	75.0572	0.0940
24	75.030	75.023	75.008	75.015	75.025	75.0202	0.0220
25	74.997	74.990	75.010	75.032	75.028	75.0114	0.0420
						$\bar{\bar{x}} = 75.0171$	$\bar{R} = 0.03012$

Solution:

The $\bar{x}$ and R control charts for the 25 samples in Table 7.10 are shown in Figures 7.16 and 7.17. As can be clearly seen from the charts, the process is not in control and there are out-of-control points on both the $\bar{x}$ and R control charts. The subgroups or samples 5, 15, and 23 are out-of-control on the $\bar{x}$ chart, and subgroups 15 and 23 on the R chart are plotting above the UCL. It can also be seen from these charts that a large number of points are plotting below the CL which is due to the influence of the large values of the out-of-control points on both the $\bar{x}$ and R control charts.

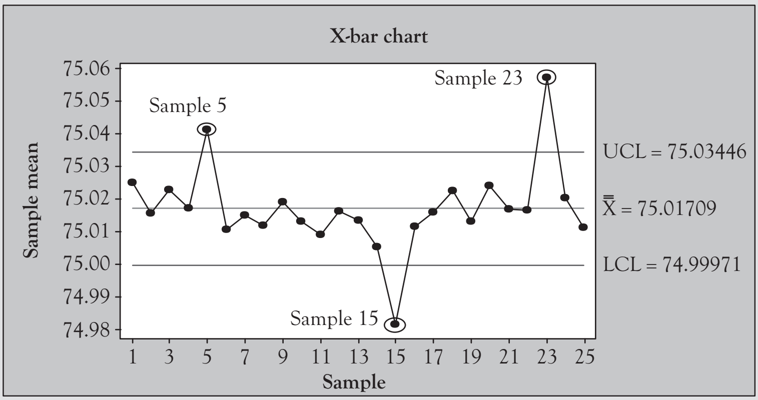


Figure 7.16 The $\bar{x}$ chart showing out-of-control points

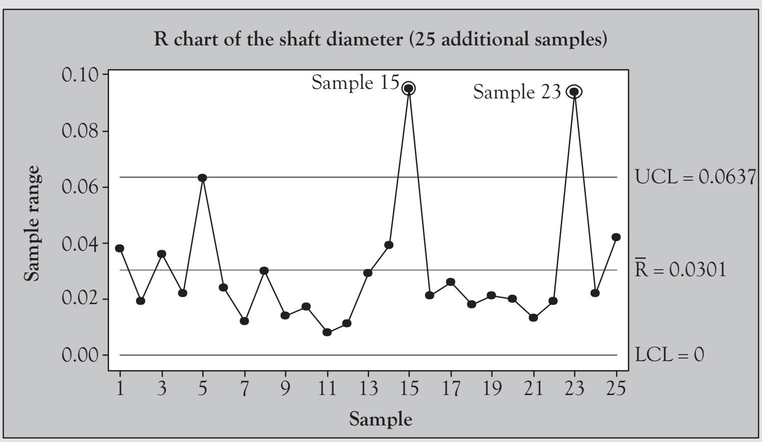


Figure 7.17 The R chart showing out-of-control points

First, we analyze the R chart. From this chart, it is evident that the process is not stable because of out-of-control points (subgroup 15 and 23 are out-of-control). An investigation of out-of-control conditions for these points found assignable causes that can be related to the machine. This could be the reason for the out-of-control conditions. The conditions identified were dull cutting tool and tool breakage. Therefore, the subgroups 15 and 23 can be discarded and appropriate corrective actions be taken.

A similar investigation and analysis of $\bar{x}$ control chart revealed that the out-of-control conditions for the subgroups 5, 15, and 23 were also due to assignable causes. Therefore, these points are discarded. Note that these points are not part of the natural variation while the other points on the chart indicate a stable process.

Revising the Control Limits

Once it is determined that the out-of-control conditions were caused due to the assignable causes, the control limits can be revised by discarding the out-of-control points. Since the $\bar{x}$ and R control charts are constructed simultaneously and the R chart is analyzed before the $\bar{x}$ chart, the following two rules are used to discard the subgroups or the samples:

1. If the out-of-control point on either $\bar{x}$ and R control charts is displayed and has assignable cause, then the point from one chart and the corresponding point on the other chart is also discarded.
2. Only the out-of-control point of the subgroup is discarded.

In this text, we will follow rule (1) mentioned earlier. This means that if an out-of-control point on the $\bar{x}$ chart is discarded, the corresponding point on the R chart is also discarded and vice versa.

$\bar{x}$ Chart

$$\bar{\bar{X}}_{new} = \frac{\sum \bar{x} - \bar{x}_d}{k - k_d}$$

R Chart

$$\bar{\bar{R}}_{new} = \frac{\sum R - R_d}{k - k_d}$$

After discarding the data point or points, the revised CL and control limits are calculated as shown here.

where $\overline{\overline{X}}_{new}$ = revised CL for the $\bar{x}$ chart

$\bar{x}_d$ = discarded subgroup averages

k_d = number of discarded subgroups

$\overline{\overline{R}}_{new}$ = revised CL for R chart

R_d = discarded subgroup ranges

Calculation of $\overline{\overline{X}}_{new}$ after discarding subgroups 5, 15, and 23 (see Table 7.10)

$$\overline{\overline{X}}_{new} = \frac{\sum \bar{x} - \bar{x}_d}{k - k_d} = \frac{1875.43 - 75.0414 - 74.9816 - 75.0572}{25 - 3} = 75.0158$$

$$\overline{\overline{R}}_{new} = \frac{\sum R - R_d}{k - k_d} = \frac{0.753 - 0.0630 - 0.0950 - 0.0940}{25 - 3} = 0.0228$$

The $\bar{x}$ and R control charts with revised control limits are shown in Figures 7.18 and 7.19. We used MINITAB to construct these charts. The control charts passed all the tests for out-of-control conditions. Thus we can conclude that the process is stable and in-control.

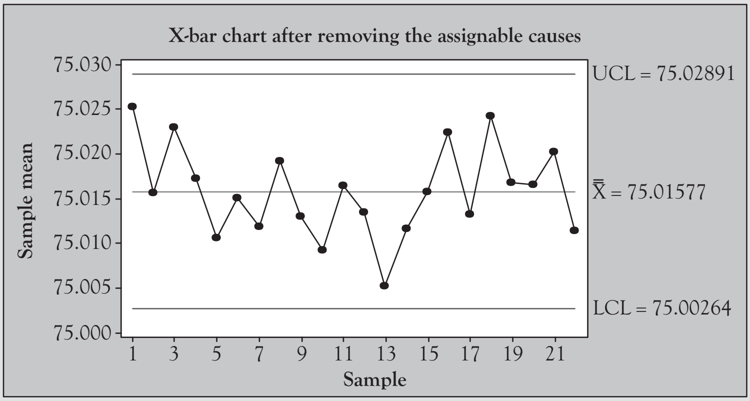


Figure 7.18** **The revised $\bar{x}$ chart (after discarding out-of-control points)

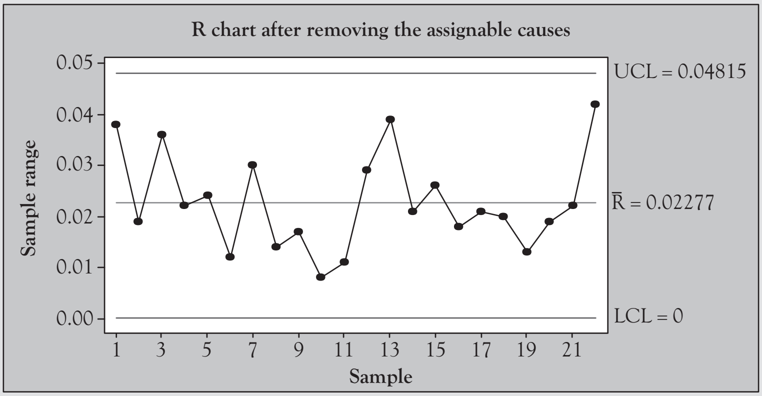


Figure 7.19 The revised R chart (after discarding out-of-control points)

Summary

In this chapter, we studied very widely used control charts—the control charts for monitoring the mean and variation of a process. These are control charts for variables commonly known as the $\bar{x}$ (x-bar) and R chart. The major steps in developing these charts including the selection of the quality characteristic, the sample size (n) or subgroups requirement, the data collection process, and calculation of the control limits were discussed in detail. A major part of the chapter was devoted in explaining and interpreting the control charts. The patterns of variation in the control charts were explained in detail. The tests for out of control conditions using computer software were emphasized. The chapter presented a number of examples with detailed calculations and explained how to interpret the control charts for variables, and how to deal with out of control conditions.

CHAPTER 8

Control Charts for Attributes

Introduction

Control charts are mainly of two kinds: (1) Control Charts for Variables and (2) Control Charts for Attributes

- A variable is a continuous measurement such as weight, height, length, volume, diameter, and so on. In the previous chapter we discussed the control charts for variables—the $\bar{x}$ and R control charts. These charts are most widely used in monitoring the quantitative variables.
- In this section, we discuss the charts for monitoring the qualitative variables known as the control charts for *attributes*. The term attribute refers to those quality characteristics that conform or do not conform to specifications. An *attribute* is an either-or situation: a product is defective or not. Attributes are also good or bad, pass or fail, and so on. Attribute control charts are used to monitor categorical variables that are measured on a nominal scale.
- One of the most commonly used attribute chart is the p-chart. *This control chart is used to monitor the proportion of defective units in a process.* Some examples where the p-chart are used include the proportion (percentage) of defective motors used in a food processor, proportion of defective micron chips used in a computer, the proportion of transaction errors made by banks, and the proportion of billing errors made by a company providing Internet service.

The charts for attributes are not as powerful as the chart for variables as the variables are numerical measurements and have more information compared to attributes where the items are classified as conforming or nonconforming. The attribute charts have more application in service industries where many quality characteristics are difficult to measure on a numerical basic but can be measured easily as attributes.

Types of Attribute Control Charts

The p-Chart

The *p*-chart is similar to a variable control charts in structure, except that *p*-charts plot statistics from count data rather than measurements that are variables.

The Rationale Behind the p-Chart

The *p*-chart can be used in cases where a quality characteristic of interest cannot be represented numerically. In such cases, we classify the inspected item as either *conforming* or *nonconforming*. These are commonly known as the item being *defective* or *nondefective*.

The *p*-chart is the chart for fraction nonconforming or fraction of defective products produced by a process and is commonly known as the control chart for fraction nonconforming. The *p*-chart represents the fraction nonconforming (expressed in decimal) but is also commonly expressed as percent of nonconforming or proportion of defective items. *Thus a p-chart is the chart for fraction nonconforming or a chart for proportion defective.*

Development of a *p*-Chart

Just as in the $\bar{x}$ and R control charts or charts for variables, the mean and variation of the process change over time and need to be controlled, the process proportion can also change over time and therefore, the proportion of defective or nonconforming products produced by a process needs to be controlled.

The proportion nonconforming is usually denoted by p and is defined as the number of nonconforming or defective items in a population divided by the population size.

The p-chart is constructed in the same way as the $\bar{x}$ and R charts. To construct this chart, at least 20 samples are obtained when the process is believed to be in control. Let x be the number of nonconforming units in the sample, and n is the sample size then the *sample proportion nonconforming* is given by

$$\hat{p}_i = \frac{\text{Number of nonconforming or defective items in the sample}}{\text{Sample size}} = \frac{x_i}{n}$$

The distribution of random variable $\hat{p}_i$ can be obtained from the binomial distribution and therefore, the mean and standard deviation of $\hat{p}_i$ are

$$\mu_{\hat{p}} = p$$
$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

For large samples, the distribution of $\hat{p}$ is approximately normally distributed. Since the chart monitors the process fraction nonconforming p, it is known as the p chart.

Center Line and Control Limits for the p-Chart

Suppose the true fraction nonconforming p in a process is known or is a specified value that can be used as a standard value, then the center line (CL) and the upper and lower control limits UCL and LCL for the p chart can be given by

$$\begin{aligned} \text{UCL} &= p + 3\sqrt{\frac{p(1-p)}{n}} \\ \text{CL} &= p \\ \text{LCL} &= p - 3\sqrt{\frac{p(1-p)}{n}} \end{aligned} \tag{8.1}$$

Since the true fraction nonconforming p is unknown and therefore, must be estimated using the sample data. The approximate estimator of p is $\bar{p}$. The $\bar{p}$ is the overall proportion of nonconforming (or, defective) items in the nk units sampled and is given by:

$$\bar{p} = \frac{\text{Total number of nonconforming or defective items in all } m \text{ samples}}{\text{Total number of items sampled}}$$

The CL and the control limits for the p chart can now be calculated by substituting $\bar{p}$ in place of p in Equation (8.1). Figure 8.1 shows the general structure of the p chart with the CL, UCL and LCL.

Sample Size for *p*-Chart

The sample size for the p-chart is usually much larger than the sample size used for the $\bar{x}$ and R charts. The reasons behind using the large sample is that most processes monitored using the p chart in industry have relatively small proportion of nonconforming (5 percent or less). In such cases, a small sample size would not contain any nonconforming output and the proportion nonconforming would be zero.

The following formula provides a rule of thumb (suggested by Montgomery) to avoid the problem mentioned earlier and also to avoid getting a negative LCL in situations when both p and n are small.

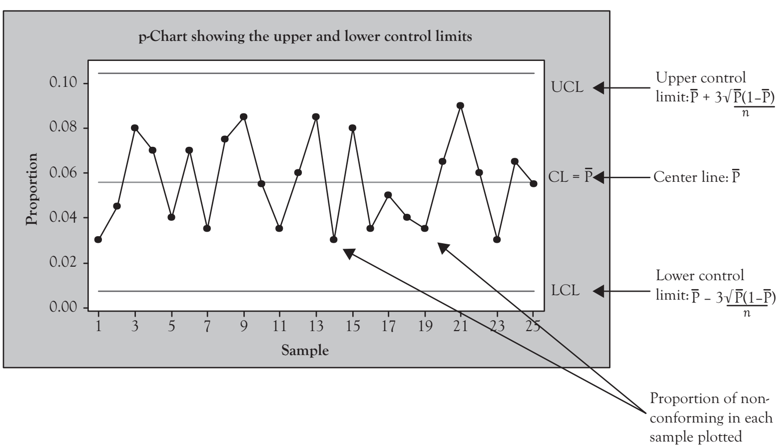


Figure 8.1 The general structure of p chart

Sample Size for *p* Chart

Select the sample size, n such that $n > \frac{9(1-p_0)}{p_0}$

where n = sample size, and p_0 = an estimate of the process proportion

Note that smaller is the process proportion p_0 larger is the sample size. For example, if $p_0 = 0.04$ then the sample size

$$n > \frac{9(1-0.04)}{0.04} = 216$$

Thus, a sample size of at least 216 should be used.

Steps for Constructing a *p*-Chart

1. Select the quality characteristic(s) for which the chart is to be developed.

   A *p*-chart is used to control the proportion of nonconforming. It can be used to control the proportion of nonconforming of a single quality characteristic, a group of quality characteristics, a single product, or a group of products. Once it is determined whether the chart is going to be used for a single quality characteristic or a group of quality characteristics, then the data for a single characteristic can be used for other quality characteristics.
2. Determine the sample size or the subgroup size using the following strategy:

   Select the sample size, n such that $n > \frac{9(1-p_0)}{p_0}$

   where n = sample size, and p_0 = an estimate of the process proportion

   The value of p_0 can be determined from the sample data (based on the calculation of $\hat{p}$)or can be pre-established based on the knowledge of the process.

   Note: The size of the sample is a function of the proportion of nonconforming. For example, if it is known that the proportion nonconforming (p) in a product is 0.0015 (0.15

percent) and a sample size, n of 1000 is selected then the average number of nonconforming in the sample would be 1.5 [$np = 1000(0.0015) = 1.5$]. This is very small and is not sufficient to construct an efficient chart. On the other hand if the proportion nonconforming is known to be 0.12 and a sample of 100 is taken, it would make a good chart. Therefore, it is important to select the appropriate sample size that will provide adequate number of nonconforming for the chart. The minimum suggested sample size for a p-chart is 50.

3. Collect the sample data. The number of samples should be at least 25.
4. Calculate the proportion of nonconforming for each sample using the following formula:

$$\hat{p}_i = \frac{\text{Number of nonconforming or defective items in the sample}}{\text{Sample size}} = \frac{x_i}{n}$$

5. Establish the CL and control limits:

$$\text{CL: } \bar{p} = \frac{\text{Total number of nonconforming or defective items in all } m \text{ samples}}{\text{Total number of items sampled}}$$

$$\text{UCL: } \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$\text{LCL: } \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

In the above equations, m = number of samples, n = sample size, and $\hat{p}$ is the overall proportion of nonconforming in the mn items sampled.

6. Plot the CL, the control limits, and the sample proportions from each of the m samples on the control chart.
7. Revise the CL and the control limits by discarding the out of control points if necessary.

Example 8.1: Construction and Application of *p* Chart

Table 8.1 shows the number of defective motors produced by a manufacturing process that produces mini motors for the cooling fans used in computers. To monitor the proportion of nonconforming (or the proportion of defective motors) a *p*-chart is to be developed for which 25 samples of size 200 were collected from the production process. The number of defectives in each sample is shown in Table 8.1.

a. Calculate the proportion defective for each sample.
b. Calculate $\bar{p}$ or the overall proportion defective in 25 samples each of size 200.
c. Calculate the CL and the UCL and LCL.
d. Show the control limits and the sample proportions on a *p*-chart.

Solution:

a. Table 8.2 shows 25 samples each of size 200 from the manufacturing process that produces motors. The numbers of defectives are shown in column (3) and the proportion of defective for each sample is shown in column (4) of Table 8.2. The proportion defective for the first sample is calculated as follows:

$$\hat{p}_i = \frac{\text{Number of nonconforming or defective items in the sample}}{\text{Sample size}} = \frac{x_i}{n} = \frac{14}{200} = 0.07$$

The proportion defective for each of the samples is calculated in a similar way and recorded in column (4) of the table here.

Table 8.1 Number of defective motors

No. of defectives												
14	8	13	4	15	20	14	8	12	15	14	22	18
21	10	5	13	16	7	14	20	14	15	6	8	

Table 8.2 Twenty five samples each of size 25

Sample (1)	Sample size (2)	No. of defective (3)	Sample proportion (4)
1	200	14	0.070
2	200	8	0.040
3	200	13	0.065
4	200	4	0.020
5	200	15	0.075
6	200	20	0.100
7	200	14	0.070
8	200	8	0.040
9	200	12	0.060
10	200	15	0.075
11	200	14	0.070
12	200	22	0.110
13	200	18	0.090
14	200	21	0.105
15	200	10	0.050
16	200	5	0.025
17	200	13	0.065
18	200	16	0.080
19	200	7	0.035
20	200	14	0.070
21	200	20	0.100
22	200	14	0.070
23	200	15	0.075
24	200	6	0.030
25	200	8	0.040
Total	5000	326	

b. The overall proportion defective in 25 samples each of size 200 is calculated as:

$$\bar{p} = \frac{\text{Total number of nonconforming or defective items in all } m \text{ samples}}{\text{Total number of items sampled}} = \frac{326}{5000} = 0.0652$$

c. The CL and the UCL and LCL:

The CL for the p chart is $\overline{p}$ which was calculated in part (b). The UCL and LCL are calculated using the $\overline{p}$ and are shown here.

$$UCL = \overline{p} + 3\sqrt{\frac{\overline{p}(1-\overline{p})}{n}} = 0.0652 + 3\sqrt{\frac{0.0652(1-0.0652)}{200}} = 0.1176$$

$$LCL = \overline{p} - 3\sqrt{\frac{\overline{p}(1-\overline{p})}{n}} = 0.0652 - 3\sqrt{\frac{0.0652(1-0.0652)}{200}} = 0.0128$$

d. The p-chart along with the control limits and the sample proportion for each sample (shown in column (4) of Table 8.2) is shown in Figure 8.2.

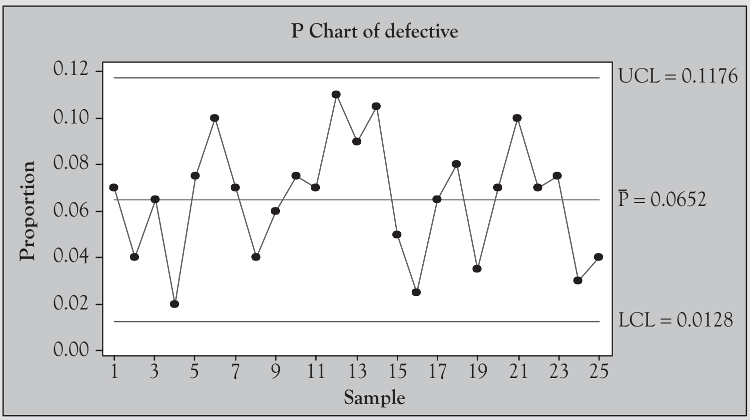


Figure 8.2 P-chart for the data in Table 8.2

Analyzing and Interpreting the *p*-Chart

1. *Process Out-of-control*: A process is considered out of control if one or more sample proportion points fall outside of the control limits. A process can also be out of control if one or more of the conditions in Table 8.3 for special causes are detected:

   The conditions of special causes below can be automatically checked using computer packages such as MINITAB.

Table 8.3 Tests for special causes in p-charts

Test for special cause		k
1.	1 point > k standard deviations from CL	3
2.	K points in a row on same side of CL	9
3.	K points in a row, all increasing or all decreasing	6
4.	K points in a row, alternating up or down	14

2. *Process in-control*: If none of the above out-of-control conditions are present then we conclude that the process is in control. A process that is in-control should be left alone. Sometimes, the process may be in-control but may show a large pattern of variation depicted by the plotted sample proportion points. It may be an indication of the presence of common cause variation. These common cause variations should be investigated and eliminated or minimized to further improve the process.

Example 8.2: Construction and Application of *p* Chart

A manufacturing process produces microchips used in automobile computers. Recently, a new process to manufacture these chips was implemented that led to an increased percent of defective chips. To stabilize the chip manufacturing process, the production manager instructed the quality technician to collect a sample of 100 chips every day for the next 25 days. The chips were analyzed for defects. Table 8.4 shows the number of defective chips found in each sample of size 100.

a. Calculate the proportion defective for each sample.
b. Calculate $\bar{p}$ or the overall proportion defective in 25 samples each of size 200.

Table 8.4 Number of defective microchips

Number of defective										
13	8	16	11	16	8	11	7	14	18	14
12	23	7	14	18	23	13	9	5	9	12
9	3	6								

c. Construct a *p*-chart to monitor the process.
d. Is the process under control? If not, use the tests for detecting special causes of variation.
e. If out-of-control conditions are detected, investigate the causes and suggest ways to stabilize and bring the process under control.

Solution:

a. Table 8.5 shows 25 samples each of size 100. The numbers of defectives are shown in column (3) and the proportion of defective for each sample is shown in column (4) of Table 8.5. The proportion defective for the first sample is calculated as follows:

$$\hat{p}_i = \frac{\text{Number of nonconforming or defective items in the sample}}{\text{Sample size}} = \frac{x_i}{n} = \frac{13}{100} = 0.13$$

The rest of the proportions are calculated in the similar way.

b. The overall proportion defective $\bar{p}$

$$\bar{p} = \frac{\text{Total number of nonconforming or defective items in all } m \text{ samples}}{\text{Total number of items sampled}} = \frac{299}{2500} = 0.1196$$

c. The UCL and LCL are calculated and shown here. The *p*-chart is shown in Figure 8.3. Table 8.6 shows the results of the tests for special causes.

$$\text{UCL} = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.1196 + 3\sqrt{\frac{0.1196(1-0.1196)}{100}} = 0.2169$$

$$\text{LCL} = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.1196 - 3\sqrt{\frac{0.1196(1-0.1196)}{100}} = 0.0223$$

d. From the *p*-chart in Figure 8.3 we can see that the process is not in control. The proportion defective for samples 13 and 17 are out of control. The test results for special causes in Table 8.6 reports that the test failed at points 13 and 17. These points are more than 3.00 standard deviations from the CL plotting above

Table 8.5 Number of defective chips

Sample (1)	Sample size (2)	No. of defective (3)	Proportion defective (4)
1	100	13	0.13
2	100	8	0.08
3	100	16	0.16
4	100	11	0.11
5	100	16	0.16
6	100	8	0.08
7	100	11	0.11
8	100	7	0.07
9	100	14	0.14
10	100	18	0.18
11	100	14	0.14
12	100	12	0.12
13	100	23	0.23
14	100	7	0.07
15	100	14	0.14
16	100	18	0.18
17	100	23	0.23
18	100	13	0.13
19	100	9	0.09
20	100	5	0.05
21	100	9	0.09
22	100	12	0.12
23	100	9	0.09
24	100	3	0.03
25	100	6	0.06
Total	2500	299	

the UCL. The process proportion defective appears to be much higher at these sample points.

e. Since the process is out-of-control, the control chart should not be continued to monitor the process until the reasons for out of control conditions are investigated. Since the points 13

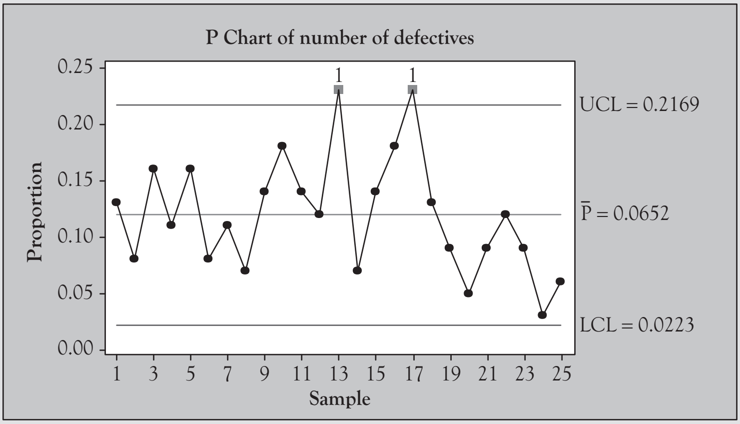


Figure 8.3 P-chart for the chip manufacturing process

Table 8.6 Tests for special causes

Test results for *p* chart of No. nonconforming
TEST 1. One point more than 3.00 standard deviations from CL.
Test Failed at points: 13, 17
* WARNING * If graph is updated with new data, the earlier results may no longer be correct.

and 17 are out-of-control, the CL and the control limits are not representative of the process.

The two out-of-control points were investigated and it was found that a cause can be assigned for the out-of-control conditions. It was found that a new technician who was still undergoing training was working during that shift when the samples were collected. The out-of-control points resulted because of the wrong judgment and measurement on the part of the new technician. The two out-of-control points were discarded from the data set after the cause for out-of-control condition was identified and a new *p*-chart was created with modified CL and control limits. The chart was tested again for special causes and no special causes or nonrandom variation was detected. The modified *p*-chart is shown in Figure 8.4 with revised CL and control limits.

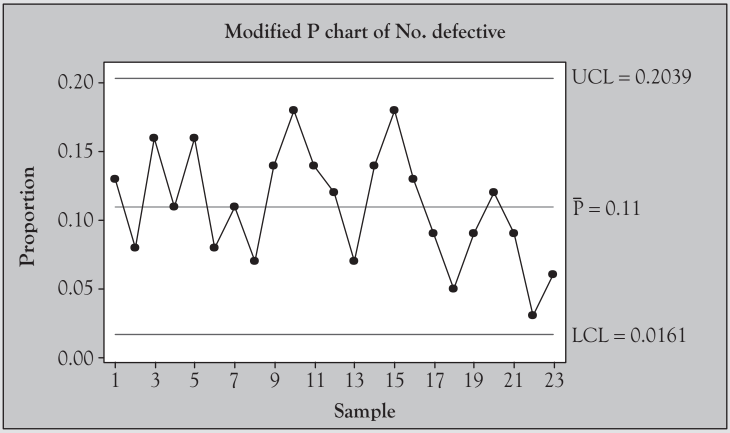


Figure 8.4 The modified p-chart

P-Chart for Variable Subgroup Size

Usually a *p*-chart is developed and used with a constant subgroup size. In some cases, it may be necessary to have varying sample size. This situation occurs when the *p*-chart is to be used for 100 percent inspection of products the output of which varies from day to day. In such cases, subgroups of different sizes may be selected from the day's production. If the subgroup size varies, the control limits will vary with the subgroup size. This is because the control limits are a function of the sample size. The next example illustrates the construction and application of a *p*-chart for a variable subgroup size.

Example 8.3: Construction and Application of *p* Chart for Variable Subgroup Size

A *p*-chart for variable subgroup size is to be established for the barcode scanners manufactured by an electronic manufacturer. Because of the increased customer complains, it was decided to perform 100 percent inspection of the day's production. Table 8.7 shows the subgroup size that varies from day to day, the number of nonconforming or defective scanners, and also the fraction nonconforming. Since the subgroup size varies, the control limits also varies as the control limits are a function of subgroup size. For this variable sample size data:

Table 8.7 Varying samples of barcode scanners for the past 25 days

Sample no. (1)	Sample size (2)	No. of nonconforming (3)	Fraction nonconforming (4)	LCL (5)	UCL (6)
1	2400	83	0.035	0.023	0.045
2	1466	50	0.034	0.020	0.048
3	1950	50	0.026	0.021	0.046
4	2465	61	0.025	0.023	0.045
5	2012	77	0.038	0.022	0.046
6	2183	62	0.028	0.022	0.045
7	1956	63	0.032	0.021	0.046
8	1977	55	0.028	0.022	0.046
9	2259	60	0.027	0.022	0.045
10	1253	91	0.073	0.018	0.049
11	2304	80	0.035	0.022	0.045
12	1479	40	0.027	0.020	0.048
13	2076	74	0.036	0.022	0.046
14	1682	73	0.043	0.021	0.047
15	2365	62	0.026	0.023	0.045
16	2369	98	0.041	0.023	0.045
17	1524	51	0.033	0.020	0.048
18	2205	82	0.037	0.022	0.045
19	2693	68	0.025	0.023	0.044
20	2267	55	0.024	0.022	0.045
21	1656	90	0.054	0.020	0.047
22	1797	102	0.057	0.021	0.046
23	2008	65	0.032	0.022	0.046
24	2397	56	0.023	0.023	0.045
25	2147	68	0.032	0.022	0.045
Total	50890	1716			

a. Calculate the proportion defective for each sample.
b. Calculate $\overline{p}$ or the overall proportion defective in 25 samples.
c. Calculate the UCL and LCL for each subgroup. Construct a p-chart with varying subgroup size.

d. Is the process under control? If not, use the tests for detecting special causes of variation.
e. If out-of-control conditions are detected, investigate the causes and suggest ways to stabilize and bring the process under control.

Solution:

a. The proportion defective for each of the subgroup are calculated and shown in column (4) of Table 8.7. The proportion defective for the first sample is calculated as follows:

$$\hat{p}_i = \frac{\text{Number of nonconforming or defective items in the sample}}{\text{Sample size}} = \frac{x_i}{n} = \frac{83}{2400} = 0.035$$

The rest of the proportions are calculated in the similar way.

b. The overall proportion defective $\overline{p}$ (see the last row of Table 8.7 for totals)

$$\overline{p} = \frac{\text{Total number of nonconforming or defective items in all } m \text{ samples}}{\text{Total number of items sampled}} = \frac{1716}{50890} = 0.0337$$

c. The UCL and LCL for the first subgroup:
Using the value of $\overline{p}$ calculated in part (b), the UCL and LCL for each subgroup can be calculated as shown here:
UCL and LCL for the first subgroup (Sample No. 1):

$$\text{UCL} = \overline{p} + 3\sqrt{\frac{\overline{p}(1-\overline{p})}{n}} = 0.0337 + 3\sqrt{\frac{0.0337(1-0.0337)}{2400}} = 0.045$$

$$\text{LCL} = \overline{p} - 3\sqrt{\frac{\overline{p}(1-\overline{p})}{n}} = 0.0337 - 3\sqrt{\frac{0.0337(1-0.0337)}{2400}} = 0.023$$

These LCL and UCL values are shown under sample 1 and columns (5) and (6) of Table 8.7.

Similarly, UCL and LCL for the sample or subgroup 10:

$$\text{UCL} = \overline{p} + 3\sqrt{\frac{\overline{p}(1-\overline{p})}{n}} = 0.0337 + 3\sqrt{\frac{0.0337(1-0.0337)}{1253}} = 0.049$$

$$\text{LCL} = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.0337 - 3\sqrt{\frac{0.0337(1-0.0337)}{1253}} = 0.018$$

These LCL and UCL values are shown under sample 10 and columns (5) and (6) of Table 8.7. The rest of the proportions are calculated in the similar way.

The control chart with varying subgroup is shown in Figure 8.5. The chart was constructed using MINITAB.

d. As can be seen from Figure 8.5, the process is out of control. The results for the tests for out-of-control conditions are shown in Table 8.8.

e. An investigation of out-of-control conditions indicated that these were due to assignable causes. Therefore, it was decided to discard the out-of-control points. After removing the subgroups 10, 21,

Table 8.8 Test results for special causes

Test results for p chart of NCON

TEST 1. One point more than 3.00 standard deviations from center line.

Test Failed at points: 10, 21, 22

* WARNING * If graph is updated with new data, the results above may no longer be correct.

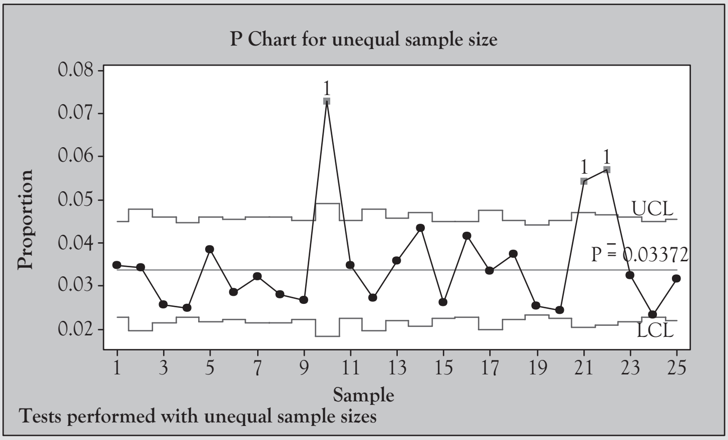


Figure 8.5 P-chart with variable subgroup size (points 10, 21, and 22 are out-of-control)

22 that were responsible for out-of-control points, the process was found to be in-control (see Figure 8.6).

Notes on Implementation of p-chart: The control chart with variable sample size can be created and implemented using many of the popular quality control software, it may be difficult at times for the quality personnel to understand and interpret these charts. The problem of variable subgroup size may be overcome by using an average sample size.

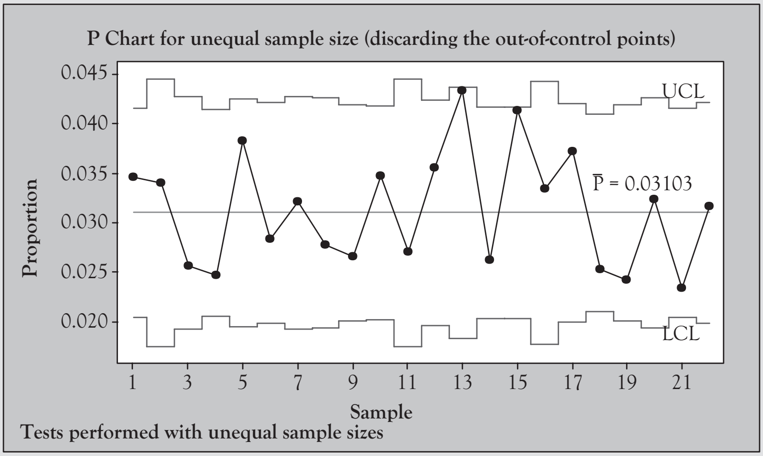


Figure 8.6 P-chart with variable subgroup size (Out-of-control points discarded)

The np Control Chart or the Control Chart for Number Nonconforming (Number of Defects)

Sometimes it may be desirable to base a control chart on *number of nonconforming* rather than the fraction of nonconforming. The *number of nonconforming chart* is known as the *np* chart and is almost identical to the *p*-chart. One of the advantages of the *np* chart is that it is easier to understand than the *p*-chart. For the *np* chart, the subgroup size is constant. The parameters of the *np* chart are determined as follows:

Upper Control limit:	$UCL = np + 3\sqrt{np(1-p)}$
Center Line:	$CL = np$
Lower control Limit:	$LCL = np - 3\sqrt{np(1-p)}$

Example 8.4: Construction and Application of *np* Chart

The quality department of a company that manufactures bar code scanners selects 200 scanners from its production every three days and inspects them for number of defects (number of nonconformities). Inspection results for the constant sample size of 200 and the number of defective scanners found was recorded by the quality control inspector and are shown in Table 8.9. The table also shows the proportion defective for each sample calculated by dividing the number of defective by the sample size. Using the information in the table, determine the CL and the UCL and LCL for the *np* chart and construct the chart.

From the data in Table 8.9

$$\bar{p} = 0.0668, \qquad n = 200$$

and,

$$n\bar{p} = 200(0.0668) = 13.36$$

Using the earlier values, the CL and the UCL and LCL for the *np* chart can be calculated as:

$$\text{UCL} = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})} = 200(0.0668) + 3\sqrt{200(0.0668)(1-0.0668)} = 23.95$$

$$CL = n\bar{p} = 200(0.0668) = 13.36$$

$$\text{LCL} = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})} = 200(0.0668) - 3\sqrt{200(0.0668)(1-0.0668)} = 2.77$$

The chart is shown in Figure 8.7.

Note: The number of nonconforming is a whole number and therefore, the UCL and LCL should also be whole numbers;

Table 8.9 Data for np chart

Sample no.	Sample size	No. of defective	Proportion defective
1	200	14	0.070
2	200	8	0.040
3	200	22	0.110
4	200	4	0.020
5	200	15	0.075
6	200	20	0.100
7	200	14	0.070
8	200	8	0.040
9	200	12	0.060
10	200	15	0.075
11	200	14	0.070
12	200	22	0.110
13	200	10	0.050
14	200	21	0.105
15	200	10	0.050
16	200	5	0.025
17	200	13	0.065
18	200	16	0.080
19	200	7	0.035
20	200	14	0.070
21	200	20	0.100
22	200	14	0.070
23	200	15	0.075 cont…
24	200	6	0.030
25	200	15	0.075
			$\bar{p} = 0.0668$

however, they can be left as fractions. Leaving the control limit values as fractions prevents a plotted point from falling on control limits.

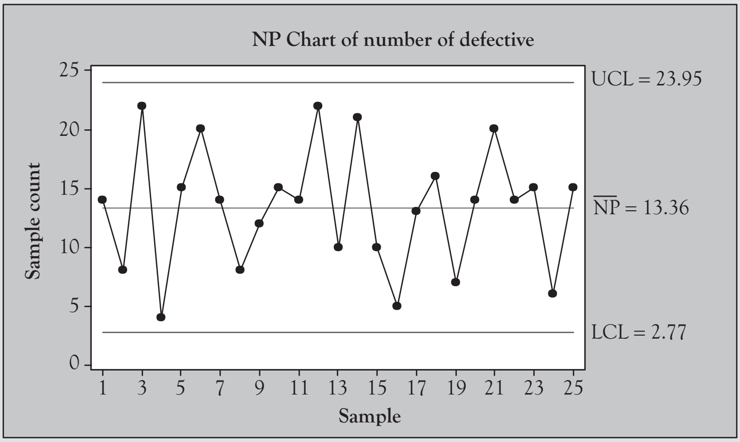


Figure 8.7 NP-chart for the data in Table 8.9

The c-Chart or the Control Chart for Count of Nonconformities

The *p-chart* and the *np-chart* we discussed earlier are used to *monitor the proportion of nonconforming units* and the *number of nonconforming unit*. It is quite possible for one *unit* of product to have more than one nonconformity. In such cases, we consider *the number of nonconformities per unit*. The chart for the number of nonconformities per unit is known as the *c-chart*. A *c*-chart can be developed either for *the total number of nonconformities* in a unit or the *average number of nonconformities per unit*.

Some of the examples where the *c*-chart can be used are number of defective welded joints in every 50 feet of a structure, number of surface finish defects in an aircraft, number of blemishes on a tire, and the number of surface irregularities or dents on the painted surface of a car. In these examples the number of nonconformities is defined per unit where a unit is 50 feet of structure, an aircraft, one tire, and the painted surface of a car.

The *c*-chart is based on the assumption that the occurrences of nonconformities in a unit (or a sample of constant size) follow a Poisson distribution. This also implies that the number of opportunities of the nonconformities is infinitely large and the probability of occurrence of nonconformities at any point is small and constant. The other requirement for the c-chart is that the inspection unit must be the same for each sample. This way the area of opportunity for the occurrence of nonconformities is the same from unit to unit.

The Poisson distribution is described by the average number of occurrence and one important characteristic of this distribution is that the mean and variance of the distribution are equal. The control chart for the number of nonconformities per unit (denoted by c) is very easy to construct. We explain this in the next section.

The conditions of Poisson distribution as applied to the c-chart may not hold exactly but slight deviations are tolerable and will still make the chart work well.

Development of a c-Chart

The c-chart is used for the number of nonconformities in a single unit of inspected product. Note that the inspection unit may be defined as one unit or a group of 5 or 10 units of a product considered as one unit. If the number of defects or nonconformities in the inspection unit occur according to the Poisson distribution with c as the parameter of the distribution then

$$p(x) = \frac{c^x e^{-c}}{x!} \text{ where, } x = 0,1,2,..$$

Where x is the number of nonconformities and c is the parameter of the Poisson distribution which is the mean and the variance of the Poisson distribution where $c > 0$

The parameter c is usually estimated as the average number of nonconformities, $\bar{c}$ from the sample of inspected units.

Steps for Constructing a c-Chart

1. Collect m samples where the number of samples m is at least 20 units. Since, the c-chart monitors the *number of nonconformities per unit; therefore, each sample is obtained by observing a single unit.*

2. Determine the number of nonconformities for each unit. Suppose the number of nonconformities in unit i is c_i.
3. Calculate the average number of nonconformities $\bar{c}$ using:

$$\bar{c} = \frac{\sum c_i}{m}$$

where, m is the number of samples.

4. Establish the control limits as shown:

$$\text{UCL} = \bar{c} + 3\sqrt{\bar{c}}$$
$$\text{CL} = \bar{c}$$
$$\text{LCL} = \bar{c} - 3\sqrt{\bar{c}}$$

Note: If the calculation for LCL yields a negative value, set LCL = 0.

5. Construct the chart by plotting the CL, the control limits, and the number of nonconformities in unit i, c_i.
6. Revise the CL and the control limits by discarding the out of control points if necessary.

Example 8.5: Construction and Application of c Chart

A *c*-chart is to be developed to monitor and control the number of nonconformities (paint blemishes) on the outer surface of the hood of a new automobile model. Table 8.10 shows the number of paint blemishes from a sample of 30 successive samples of 50 hoods. This means that the inspection unit is defined as 50 hoods of the cars.

a. Using the data in Table 8.10, calculate the CL and control limits for the *c*-chart.
b. Construct the *c*-chart by plotting the CL, the control limits, and the number of nonconformities in unit i, c_i.
c. Revise the CL and the control limits by discarding the out of control points if necessary.

Table 8.10 Data for c-chart

Sample no.	No. of nonconformities (c_i)	Sample no.	No. of nonconformities (c_i)
1	21	16	13
2	30	17	22
3	16	18	18
4	10	19	40
5	25	20	30
6	5	21	24
7	28	22	16
8	20	23	28
9	31	24	17
10	25	25	10
11	15	26	28
12	24	27	32
13	19	28	18
14	10	29	27
15	17	30	11

Solution:

a. From Table 8.10, the sum of the nonconformities in the 30 samples are:

$$\sum c_i = 630$$

Therefore, the CL $\bar{c} = \frac{\sum c_i}{m} = \frac{630}{30} = 21$

The UCL and LCL are

$$\text{UCL} = \bar{c} + 3\sqrt{\bar{c}} = 21 + 3\sqrt{21} = 34.75$$

$$\text{LCL} = \bar{c} - 3\sqrt{\bar{c}} = 21 - 3\sqrt{21} = 7.25$$

b. The *c*-chart is shown in Figure 8.8.

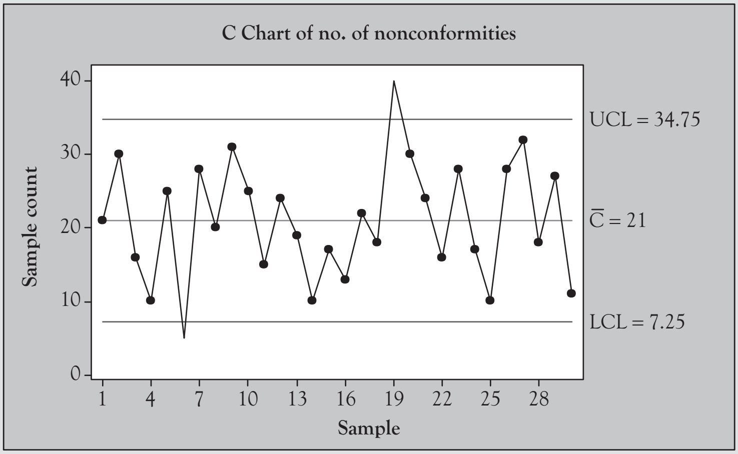


Figure 8.8 The c-chart for the number of nonconformities data in Table 8.10

Table 8.11 Tests results for assignable or out-of-control conditions in the c-chart

Test results for c chart of no. of nonconformities
TEST 1. One point more than 3.00 standard deviations from center line.
Test failed at points: 6, 19
* WARNING * If graph is updated with new data, the results above may no longer be correct.

The plotted points on the *c*-chart in Figure 8.8 are the number of nonconformities (c_i). From the plot, we can see that samples 6 and 19 are plotting outside the control limits. The test results for the out-of-control points conducted using MINITAB are shown in Table 8.11. These points should be investigated further to determine the causes for the out-of-control conditions.

c. Investigation of out-of-control conditions for points 6 and 19 revealed problems due to the operator and the drying time. An inexperienced operator could not classify the blemishes correctly that resulted in low number of blemishes in sample 6. A high number of blemishes were recorded in sample 19 because of the drying time in the furnace exceeded the set time. It seemed reasonable to discard both the samples. Figure 8.9 shows the modified control chart after removing the out-of-control points.

Figure 8.9 shows that the process is in control. A test for out-of-control conditions did not indicate any out-of-control condition. It is important to note here that although the process is in control, the number of nonconformities is still high; therefore, an improvement in the process is warranted.

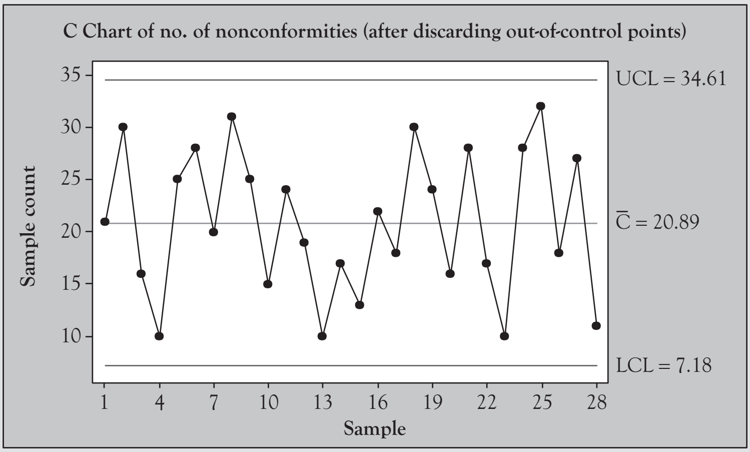


Figure 8.9 Modified c-chart for the data in Table 8.10

Summary

This chapter presented another class of control charts—the control charts for monitoring the qualitative variables known as the control charts for *attributes*. The term attribute refers to those quality characteristics that conform or do not conform to specifications. An *attribute* is an either-or situation: a product is defective or not. Attributes are also good or bad, pass or fail, and so on. Attribute control charts are used to monitor categorical variables that are measured on a nominal scale. A number of attribute control charts were discussed with their applications. The most commonly used attribute charts used in monitoring the product and service quality include the *p*-chart, *np*-chart and *c*-chart. The *p*-chart is used to monitor the proportion of defective units in a process. This chart is usually developed and used with a constant subgroup size. Sometimes, it may be desirable to have varying sample size on a *p*-chart. This situation occurs when the *p*-chart is to be used for 100 percent inspection of products the output of which varies from day to day. A detailed example with varying subgroup size was presented with its advantages and limitations. The other chart presented was the control chart for *number of nonconforming* known as the *np* chart and is almost identical to the *p*-chart. Finally, the chart for the number of nonconformities per unit known as the *c*-chart was discussed. A *c*-chart can be developed either for the total number of nonconformities in a unit or the average number of nonconformities per unit. It is important to note that the basis of the *p*-chart and *c*-chart are the binomial and Poisson distributions respectively.

CHAPTER 9

Process Capability Analysis

Introduction

Process capability analysis is one of the important aspects of overall quality improvement. In this chapter we explain different methods of assessing the process capability of products and services. We also explain the specification limits, and control limits and how they are related to the process capability analysis.

Process capability is the ability of the process to meet specifications. The ability of the process to meet the specifications is related to the *variability* in the process. This variability can be (1) *natural or inherent variability at a given time* or (2) *variability over time*. We may think of this as short-term and long-term variability. Thus, the process capability may be viewed as the *short-term process capability* or *long-term process capability*.

Control Limits

In previous chapters, we studied the control limits in relation to the control charts. It is a common practice to position the upper and lower control limits at three standard deviations from the center line. These limits are commonly known as three sigma control limits and are calculated from the process data when the process is in control. The basic assumption for the three sigma limits is that the process follows a normal distribution. In a control chart with three sigma control limits following a normal distribution, the probability of a point falling outside of the control limits is 0.0027 or 0.27 percent or less than three chances in 1000 (assuming that the process is in control). This can be directly derived from the property of normal distribution.

The control charts are a continuous improvement tool and are designed to measure the variation in the data from the measurements of quality characteristics and differentiate the *assignable* (or *special*) causes of variation from *common* causes. *Common* cause variations are natural variations that are inherent to the process and are expected part of the process. These variations are of much less concern to the manufacturer (as long as they are within certain limits) than *assignable* causes. The control charts are designed to detect the assignable causes quickly. They also tell us whether a shift in the process has occurred and when an adjustment in the process is required.

Specification Limits

Specification limits are the boundary points that are usually derived from customer requirements and should link to what the customer really wants. These limits define the acceptable level for a quality characteristic or an output variable critical to the customer. The specification limits are determined by the customer, design engineers, product designers, or may be set by the management. *These limits are independent of the control limits*. The control limits are designed to monitor and control a process and show whether or not a process is in statistical control.

More specifically, the specification limits are allowable deviation from the target or the nominal value of a product output. A target is what we are trying to aim for, whereas; the nominal value is what would be an ideal value. The target and the nominal values are usually the same but this is not the case always. For example, consider the weight of the fluid content printed on the beverage cans. In filling the 16 oz. beverage cans, the nominal value is the weight indicated on the cans. But as we know that almost all processes show variation and because of this, the weight in the filled cans has equal chance of going up and down the 16 oz. of nominal value. This may result into cans that are below the nominal value of 16 oz. In such cases, we would set the target value higher than the nominal value so that we minimize or eliminate the cans that are below 16 oz.

Control limits should not be confused with the specification limits or the *tolerance limits* as they are sometimes referred to. Control limits are independent of the specification limits and they are determined differently. *Control limits determine what a process is capable of producing. These limits are sometimes referred to as the "voice of the process." The specification limits* *or tolerance limits describe how the product should be manufactured to meet the customer's needs and expectations. Therefore, the specification limits are also referred to as the "voice of the customer."*

Process Capability Applications

Process capability analysis is a statistical technique used in:

- Assessing process variability
- Establishing specification limits (or, setting up realistic tolerances)
- Determining how well the process will hold the tolerances (the difference between specifications).
- Analyzing the process variability relative to the specifications
- Reducing or eliminating the variability to a great extent in products and processes

What Does Process Capability Analysis Tell Us?

The capability of a process should be constantly measured and analyzed. Capability analysis can help us answer the following questions:

- Is the process meeting customer specifications?
- How will the process perform in the future?
- Are improvements needed in the process?
- Have we sustained these improvements, or has the process regressed to its previous unimproved state?

We analyze process capability with capability indexes such as Cp, Pp, Cpk, and Ppk. These are explained in more detail in subsequent sections.

How Is Process Capability Stated?

Process capability may be stated in the form of a probability distribution; for example, a normal distribution with specified mean (μ) and standard deviation (σ).

Process capability is also estimated as a percent outside of specifications. Normal distribution is widely used to assess the process capability (whenever it is reasonable to show or assume that the quality characteristic in question follows a normal distribution). *The six sigma spread (three sigma on each side of the mean) in the normal distribution is considered the measure of process capability (Figure 9.1).*

Assessing Process Capability

The following points should be noted before conducting a process capability analysis:

- Process capability should be assessed once the process has attained statistical control. This means that the special causes of variation have been identified and eliminated. Once the process is stable, the process average and standard deviation should be calculated. For a process that is out of control, the estimated process average and standard deviation are not reliable.
- In calculating process capability, in most cases, the specification limits are required. Since the process capability determines the ability of the process to meet the specifications, it is very important to determine the

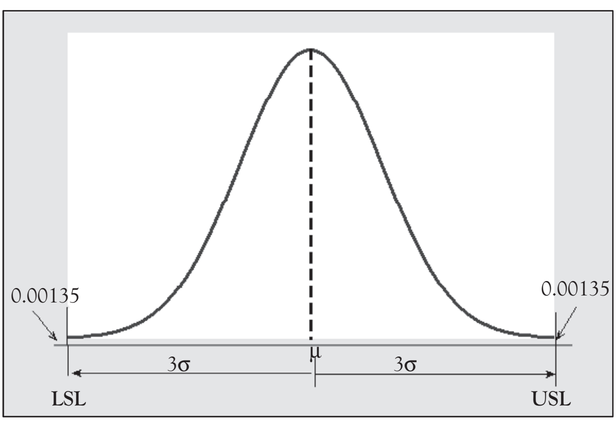


Figure 9.1 Six sigma spread as the process capability

specification limits accurately. Unrealistic or inaccurate specification limits may not provide correct process capability.

- Process capability analysis using a histogram or a control chart is based on the assumption that the process characteristics follow a normal distribution. While the assumption of normality holds in many situations, there are cases where the processes do not follow a normal distribution. Extreme care should be exercised where normality does not hold. In cases where the data are not normal, it is important to determine the appropriate distribution to perform process capability analysis. In case of non-normal data, appropriate data transformation techniques may be used to bring the data to normality before assessing process capability.

In studying the control charts and assessing the process capability, two concepts are important—the *control limits* and the *specifications limits*. A process may be under control but may not be *capable* of meeting the specifications. We first describe the difference between the control limits and the specification limits before discussing the process capability.

Assessing Process Capability Graphically

As mentioned earlier, a process maybe in control but not be capable of meeting the specifications. Figure 9.2 shows various plots of a quality characteristic (the output of a ring diameter being controlled) with the lower and the upper specification limits (LSL and USL) plotted on each plot.

The plots in Figure 9.2 can be used to assess the process capability of the process visually. One way of determining the capability of a process is to construct a frequency histogram when the process is believed to be in control. Such histogram should be created using a large sample of individual measurements preferably, 50 or more. Next, the specification limits and the target value of the quality characteristic being studied are plotted on the histogram. The plots in Figure 9.2 (a) through (g) are explained subsequently.

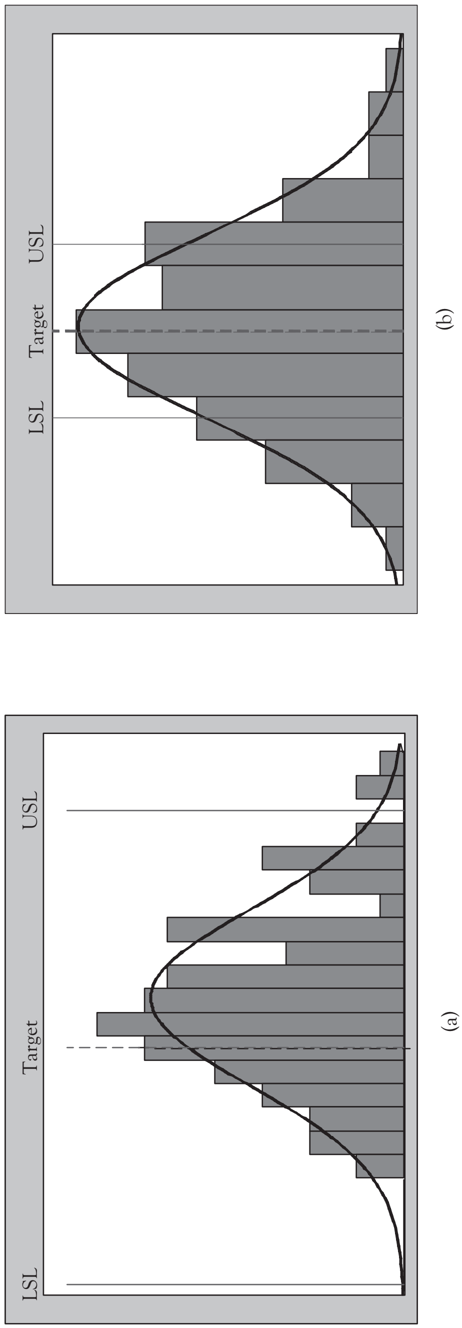


Figure 9.2 (Continued)

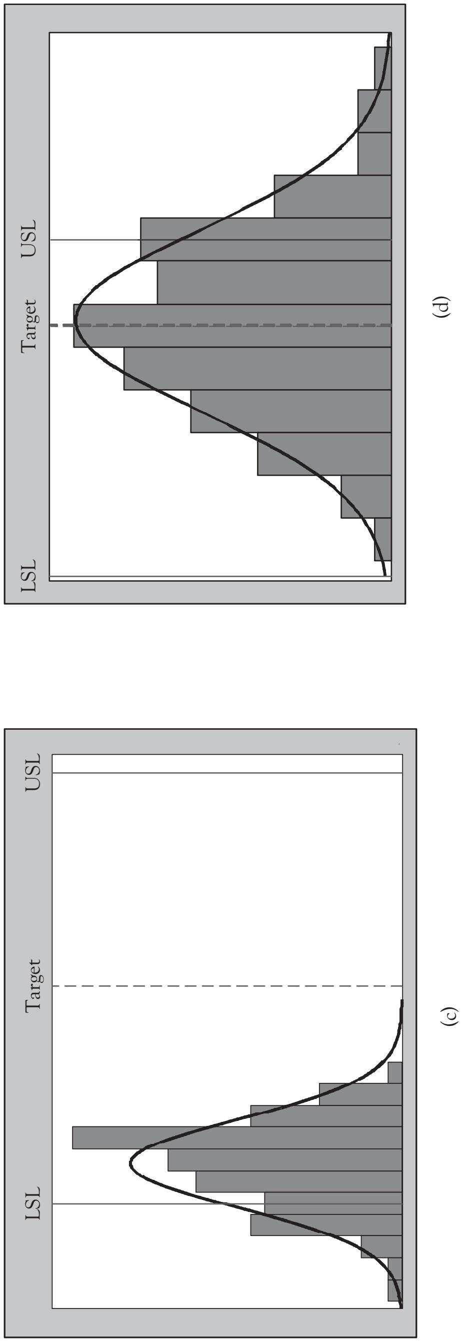


Figure 9.2 (Continued)

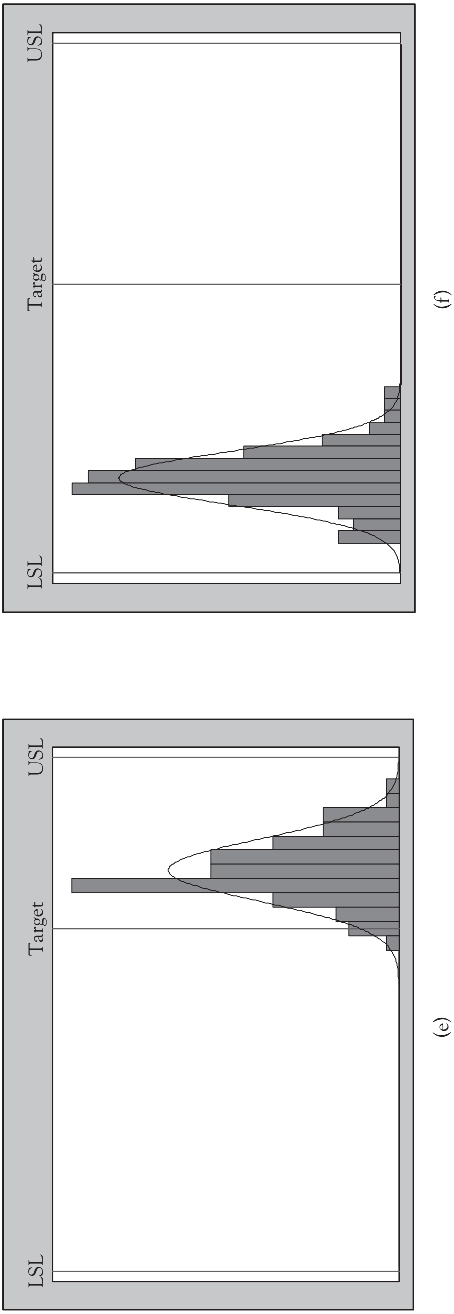
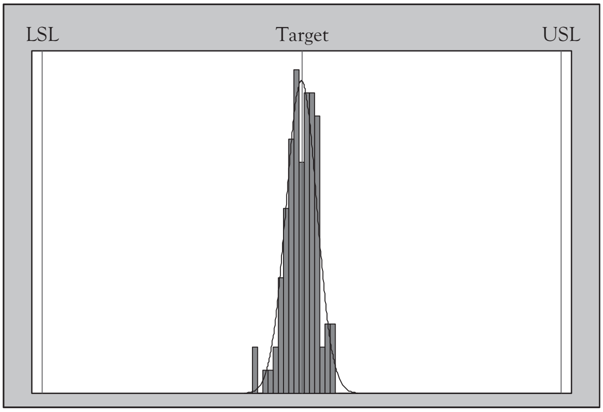


Figure 9.2 *(Continued)*



(g)

Figure 9.2(a)-(g) Graphical display of process capability of different processes in control with LSL and USL plotted on the graphs

Figure (a), (b), (c), and (d) in Figure 9.2 show that a significant percentage of products are outside the upper or lower specification limit. In Figure 9.2(a), the process is not centered meaning it is off target and the process variation is large. Due to this, a large percentage of the products are outside the upper specification limit (USL). In Figure 9.2(b), the process is centered but the process variation is large resulting into a large percent of the products above and below the specification limits. None of these processes is capable as they do not meet the specification requirements. Figure 9.2(c) shows that the process variation is small but is shifted to the left. As a result, a large percentage is below the lower specification limit (LSL). Figure 9.2(d) shows that the process is off centered and has a large process variation. The process is not meeting the USL.

In Figure 9.2 (e) and (f), the process variation is small but off-centered or shifted from the target. These processes are within the specification limits and are meeting the customer requirements. The processes are capable but any change in the process over time or specification requirements will make the process incapable resulting into process improvement initiatives to restore the process.

The process in Figure 9.2(g) is centered with much less variation. The process is meeting the specification limits and is capable of satisfying the customer requirements. This is the most desirable of all the cases described earlier.

Numerical Measure of Process Capability

There are several methods of quantifying and determining the process capability. We discuss the following methods:

1. Process capability using a histogram and normal distribution—by finding the number or percentage of the products outside of the specification limits.
2. Process capability using control charts
3. Process capability using capability indexes
4. Process capability using a statistical package

Process Capability Using a Histogram

One way of expressing process capability is to determine the percentage of the products outside of the specification limits or the percent of nonconforming products. In cases where the process data follow a normal distribution, the nonconformance percentage can be estimated even if the specification limits are not known. In the example presented here, we will use a histogram to estimate the nonconformance rate. When using a histogram to assess the nonconformance rate, it is suggested that at least 100 observations are available and the process is stable so that a reasonable estimate of process capability can be obtained.

Example 9.1: Calculating Process Capability Using Histograms

We will consider the length of 150 measurements (in cm) of a machined part. Using the distribution of the length data, we will determine the process capability. Suppose that the specification limits on the length are 6.00 ± 0.05. We would like to determine the percentage of the parts outside of the specification limits. Since the measurements are very close to normal, we can use the normal distribution to calculate the nonconforming percentage. Figure 9.3 shows the histogram of the length data with the target value and specification limits. Figure 9.4 shows that the data are normally distributed and the process is operating very close to the target.

From Figures 9.3 and 9.4, it is evident that the process is producing a small percentage of nonconforming products above the upper and below the lower specification limits (see calculations below). The percentage above and below the specification limits using normal distribution can be calculated as

$$Z_1 = \frac{x-\mu}{\sigma} = \frac{5.95-5.999}{0.0199} = -2.50 \qquad Z_2 = \frac{x-\mu}{\sigma} = \frac{6.05-5.999}{0.0199} = 2.56$$

(Note that μ and σ are estimated from the data. The estimated values are shown in Figures 9.3 and 9.4). From the standard Normal Table in Appendix A, $Z_1 = -2.50$ corresponds to 0.4938, and $Z_2 = 2.56$ corresponds to 0.4948

Therefore, the percentage of products within the specification limits

$$= 0.4938 + 0.4948$$
$$= 0.9886 \text{ or } 98.86 \text{ percent}$$

The percentage falling outside of the specification limits is (1 − 0.9886) or, 0.0114 (1.14 percent). In parts per million (PPM)

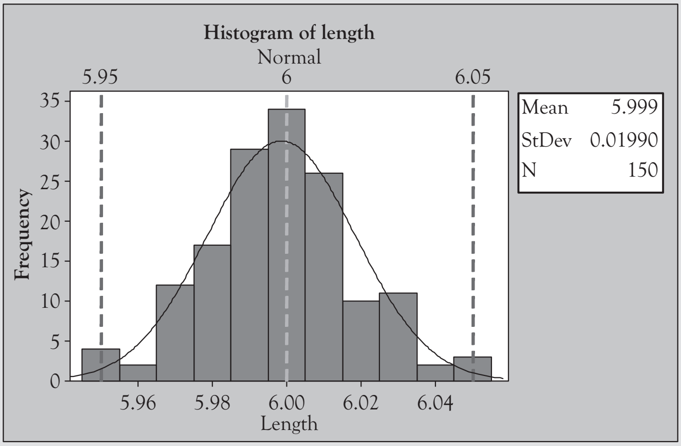


Figure 9.3 Histogram of the length data with specification limits and target

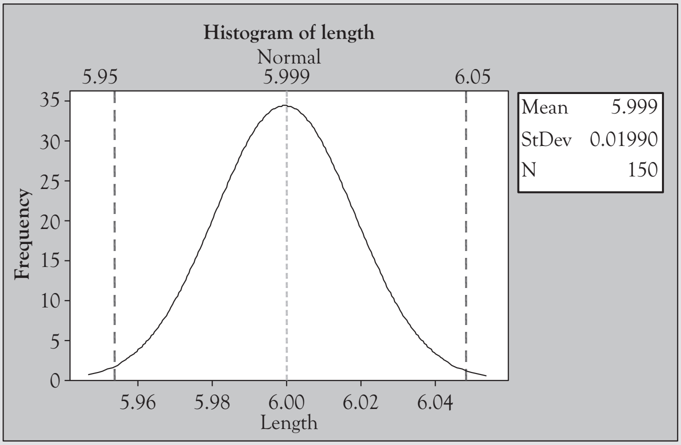


Figure 9.4 Fitted normal curve with reference line for the length data

it translates to 0.0114×10^6 or 11,400 parts outside the specification limits.

Process Capability Using Control Charts

It is a common practice to take the Six Sigma spread of a process's inherent variation as a measure of process capability when the process is stable. Thus, the process capability is the process spread which is equal to Six Sigma. This concept is illustrated using an example here.

Example 9.2: Calculating Process Capability using Control Charts

A chemical company manufactures and markets 50 lb. nitrogen fertilizer for the lawns. Due to some recent problems in their production process, overfilling and underfilling of the bags of fertilizer have been reported. The problem was investigated and appropriate adjustments were made to the machines that were used to fill the fertilizer bags. When the process was believed to be stable, the quality supervisor collected 30 samples each of size five. The control charts for $\bar{x}$ and R were constructed and the tests were conducted for the special or assignable causes. The process was found to be in control and no assignable causes were present. The $\bar{x}$ and R control charts for the process are shown in Figure 9.5. Determine the process capability for this process based on the average of range value, or $\bar{R}$ reported on the R chart. Note that $\bar{R}$—the average of all subgroup range can be used to estimate the process standard deviation.

Solution:

(a) First, estimate the standard deviation, σ from the given information using:

$$\hat{\sigma} = \frac{\bar{R}}{d_2} = \frac{1.958}{2.326} = 0.842$$

Note: $\hat{\sigma}$ is the estimate of σ. The value of $\bar{R}$ is reported in the chart for range in Figure 9.5 and d_2 is obtained from the table—'constants for control charts' in Appendix C. The value of d_2 from the table for a subgroup size of five ($n = 5$) is 2.326. The process capability,

$$6\hat{\sigma} = 6(0.842) = 5.052$$

The process capability here is a function of the estimated standard deviation. A reduction in the value of process capability means reduced process variability and improved capability.

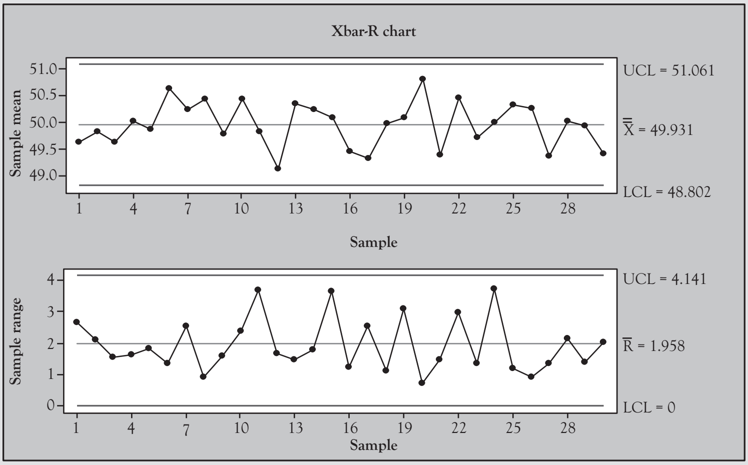


Figure 9.5 The $\bar{x}$ and R control charts for Example 9.2

The process capability obtained here is more meaningful when compared to the ongoing process at a later stage.

(b) The process capability can also be determined by estimating the $\hat{\sigma}$ using the average of standard deviations, $\bar{s}$ of the subgroups instead of average of the subgroup range. Here we demonstrate the calculation of process capability using the standard deviation. Figure 9.6 shows the $\bar{x}-S$—the control charts for the average and standard deviation.

Determine the process capability using the value of $\bar{s}$ in the control chart.

First, estimate the standard deviation, σ from the given information using:

$$\hat{\sigma} = \frac{\bar{s}}{c_4} = \frac{0.791}{0.9400} = 0.841$$

Note: $\hat{\sigma}$ is the estimate of σ. The value of $\bar{s}$ is reported in the chart for standard deviation (s)—the bottom chart in Figure 9.5 and c_4 is obtained from the table—'constants for control charts' in Appendix C. The value of c_4 from the table for a subgroup size of five ($n = 5$) is 0.9400.

The process capability: $6\hat{\sigma} = 6(0.841) = 5.046$

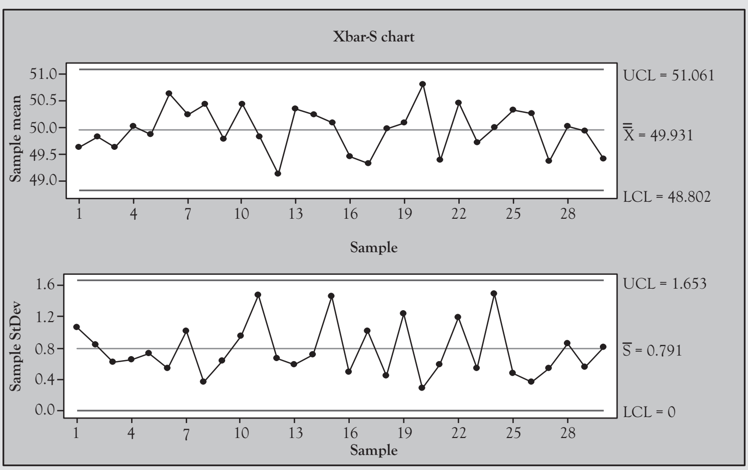


Figure 9.6 The $\bar{x}$ and S-control charts

Example 9.3: Improvement in Process Capability Using Control Charts

In an effort to continuously improve and refine the fertilizer bag filling process in the previous example, the process was further refined and precise adjustments were made in the machines used to fill the fertilizer bags. This led to further reduction in the process variation and the Six Sigma team was able to reduce the process standard deviation. The $\bar{x} - R$ charts for the improved process are shown in Figure 9.7. The process was stable and the tests for the special and assignable causes showed no problem. Calculate the standard deviation using the R-chart and calculate the process capability for the improved process. Determine any improvement in the process capability.

First, estimate the standard deviation, σ from the information in the R-chart in Figure 9.7:

$$\hat{\sigma} = \frac{\bar{R}}{d_2} = \frac{1.659}{2.326} = 0.713$$

Note: $\hat{\sigma}$ is the estimate of σ. The value of $\bar{R}$ is reported in the chart for range in Figure 9.7 and d_2 is obtained from the table—'constants for control charts' in Appendix C. The value of d_2 from the table for a subgroup size of five ($n = 5$) is 2.326.

The process capability,

$$6\hat{\sigma} = 6(0.713) = 4.278$$

Comparing this process capability to the process capability of the initial process in Example 9.2, we find approximately 15 percent improvement in the process capability. Note that the process capability of the initial process in Example 9.2 is 5.052 and the process capability of the improved process in Example 9.3 is 4.278.

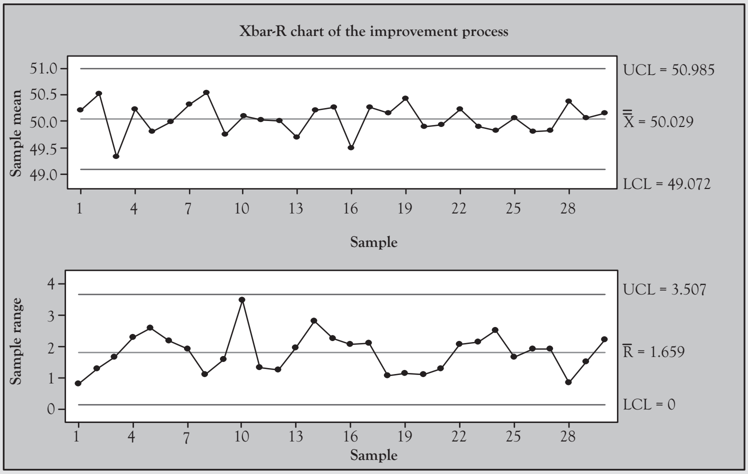


Figure: 9.7 The $\bar{x}$ and R control charts for the improved process

Assessing Process Capability Using Capability Indexes

In this section, we will demonstrate how the process capability and the specification limits (or the tolerance) are combined to provide the capability of the process.

The capability indexes and the specification limits are combined to construct capability indexes that are used to assess the process capability.

In this section, we will consider *capability indexes*. The commonly used capability indexes defined are: Cp, Cpl, Cpu, Cpk, and Ccpk.

The capability indexes are based on the assumption that the process measurements follow a normal or approximately a normal distribution.

The index Cp (also called process capability ratio) is the ratio of allowable spread to the actual spread of the process. The allowable spread is the difference between the USL and LSL, or the tolerance. This is also known as the specification width or the process width. The actual spread of the process is 6σ for a normally distributed process. Thus,

$$C_p = \frac{USL - LSL}{6\sigma} \tag{9.1}$$

Usually, the process standard deviation σ is unknown and must be estimated. The earlier expression is written more appropriately as

$$C_p = \frac{USL - LSL}{6\hat{\sigma}} \tag{9.2}$$

or,

$$C_p = \frac{\text{Specification Width}}{\text{Process Width}}$$

A graphical representation of Cp or the process capability ratio is shown in Figure 9.8.

The process capability ratio, Cp is interpreted in the following way:

Cp < 1.0	Process is not capable of meeting the specifications
Cp = 1.0	Process is marginally capable of meeting specifications
Cp > 1.0	Process is capable of meeting the specification limits

The above three cases are shown in Figure 9.9. The interpretation of Cp earlier is based on the following assumptions:

- The underlying process is stable and in statistical control,
- The measurements are normally distributed, and
- The measurement errors are negligible.

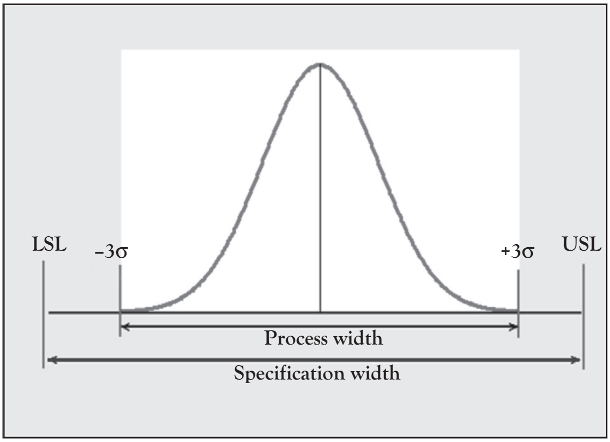


Figure 9.8 Cp or process capability ratio

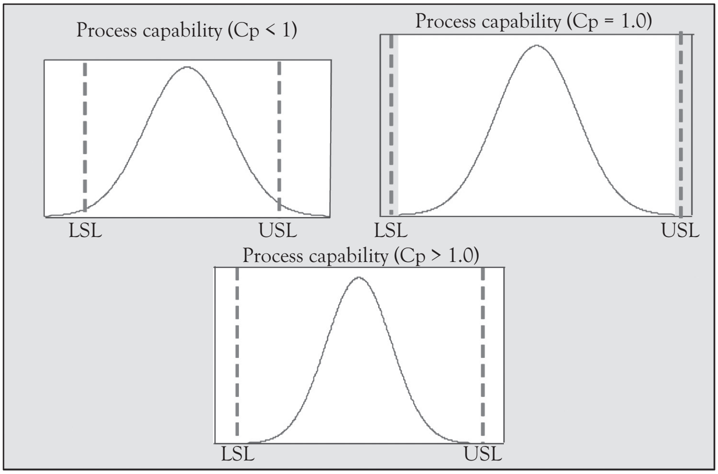


Figure 9.9 Interpretation of process capability for Cp < 1.0, Cp = 1.0 and Cp > 1.0

Table 9.1 Capability indexes for process capability

$C_p = \dfrac{USL - LSL}{6\hat{\sigma}_{within}}$	USL = upper specification limit LSL = lower specification limit $\hat{\sigma}_{within}$ = estimate of within subgroup standard deviation
$C_{PL} = \dfrac{\bar{x} - LSL}{3\hat{\sigma}_{within}}$	Ratio of the difference between process mean and LSL to one-sided process spread $\bar{x}$ = process mean
$C_{PU} = \dfrac{USL - \bar{x}}{3\hat{\sigma}_{within}}$	Ratio of the difference between USL to one-sided process spread
$C_{PK} = Min.\{C_{PU}, C_{PL}\}$	Takes into account the shift in the process. The measure of C_{PK} relative to C_p is a measure of how off-center the process is. If $C_P = C_{PK}$ the process is centered; if $C_{PK} < C_P$ the process is off-center.

The calculation of Cp in Equation (9.2) assumes that the process has both upper and lower specifications. For a one-sided specification limit, the Cp is calculated as Cpl and Cpu (for LSL and USL). From these measures, the indexes Cpk, and CCpk are calculated.

Note that all of the indexes (Cp, Cpl, Cpu, Cpk, and CCpk) use an estimate of the process standard deviation, and the results obtained by these indexes are very sensitive to the estimated value of the standard deviation. The standard deviation is estimated using several different formulas depending upon the form of data (data with subgroup of size one or more).

The estimate of the standard deviation also differs depending upon the long-term or short-term process capability being assessed. Using the appropriate estimate of standard deviation is critical to assessing the correct process capability.

The process capability indexes are summarized in Table 9.1.

Example 9.4

Calculate the capability indexes—Cp, Cpl, Cpu, and Cpk for the process for which the data are given here. Interpret their meaning. Explain the difference between Cp and Cpk.

USL = 10.050, LSL = 9.950, $\hat{\mu} = 9.999$ and $\hat{\sigma}$ =0.0165 as estimates of

μ and σ ($\hat{\mu}$ is the same as $\bar{x}$).

Solution:

$$C_p = \frac{\text{USL} - \text{LSL}}{6\sigma} = \frac{10.050 - 9.950}{6(0.0165)} = 1.01$$

(Note; $\hat{\sigma}$ is the estimate of standard deviation).

$$C_{PL} = \frac{\bar{x} - \text{LSL}}{3\hat{\sigma}} = \frac{9.999 - 9.950}{3(0.0165)} = 0.99$$

$$C_{PU} = \frac{\text{USL} - \bar{x}}{3\hat{\sigma}} = \frac{10.050 - 9.999}{3(0.0165)} = 1.03$$

$$C_{PK} = \{C_{PU}, C_{PL}\} = Min.\{1.03, 0.99\} = 0.99$$

Cp = 1.01 means that the process is marginally capable (just able to meet the specifications). Cp = Cpk means that the process is centered. For this process, these values are not equal; therefore, the process is slightly off-centered.

Difference between Cp and Cpk: The process capability ratio or Cp does not take into account the shift in the process mean. It does not consider where the mean is relative to the specifications. Cp measures only the spread of the specifications relative to the six sigma spread or the process spread. Cpk on the other hand, takes into account the shift in the process mean.

Example 9.5

a. Given $\bar{x} = 70$ and $\hat{\sigma} = 2$ (as estimates of μ and σ), LSL = 58, USL = 82. Calculate the process capability indexes: Cp, Cpl, Cpu, and Cpk.

Solution: The problem is visually shown in Figure 9.10.

$$C_p = \frac{\text{USL} - \text{LSL}}{6\sigma} = \frac{82 - 58}{6(2)} = 2.0$$

$$C_{PU} = \frac{\text{USL} - \bar{x}}{3\sigma} = \frac{82 - 70}{3(2)} = 2.0 \qquad C_{PL} = \frac{\bar{x} - \text{LSL}}{3\sigma} = \frac{70 - 58}{3(2)} = 2.0$$

$$C_{PK} = Min.\{C_{PU}, C_{PL}\} = Min\{2.0, 2.0\} = 2.0$$

b. Calculate the capability indexes for the data in part (a) if the mean has shifted from 70 to 73 (all the other values are same as in part (a)).

Solution: Figure 9.11 shows the original mean and the shift

$$C_p = \frac{\text{USL} - \text{LSL}}{6\hat{\sigma}} = \frac{82 - 58}{6(2)} = 2.0$$

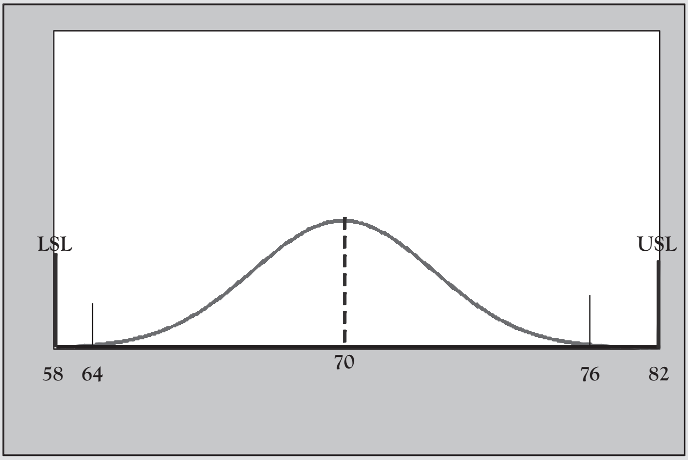


Figure 9.10 LSL and USL for Example 9.5

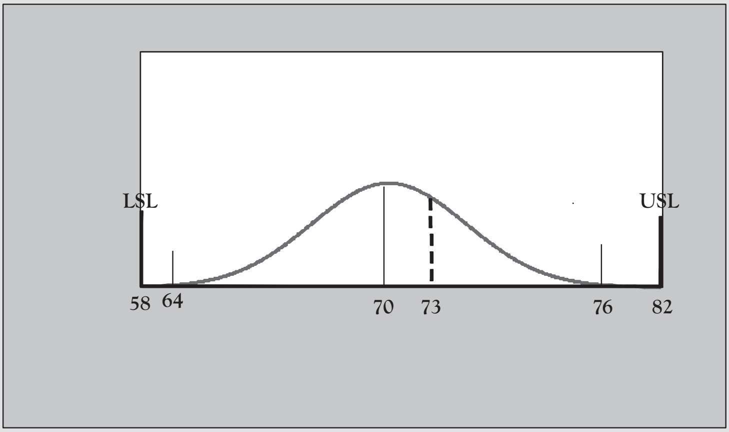


Figure 9.11 Shift in the mean from 70 to 73

$$C_{PL} = \frac{\bar{x} - \text{LSL}}{3\hat{\sigma}} = \frac{73 - 58}{3(2)} = 2.5 \qquad C_{PU} = \frac{\text{USL} - \bar{x}}{3\hat{\sigma}} = \frac{82 - 73}{3(2)} = 1.5$$

$$C_{PK} = Min.\{C_{PU}, C_{PL}\} = Min\{1.5, 2.5\} = 1.5$$

Process Capability Using a Statistical Package: Process Capability for Normally Distributed Data

Process capability can be assessed easily using computer packages. Here we present an example of assessing the process capability using MINITAB statistical package. MINITAB uses both the graphical and numerical approach to process capability. The example demonstrates the capability of a production process that produces certain PVC pipe. The diameter of the pipe is of concern. The specification limits on the pipes are 7.000 ± 0.025 cm. There has been a consistent problem with meeting the specification limits, and the process produces a high percentage of rejects. The data on the diameter of the pipes were collected to determine the process capability of the current process. This would also provide an idea about the improvement for the future production runs. A random sample of 150 pipes was selected and the diameters measured. A process capability report shown in Figure 9.12 was generated using MINITAB.

The process capability report in Figure 9.12 shows that the process producing the pipes is stable. The histogram of the data shows that the measurements follow a normal distribution. Since the process is stable and the measurements are normally distributed, the normal distribution option of process capability analysis can be used to assess the process capability.

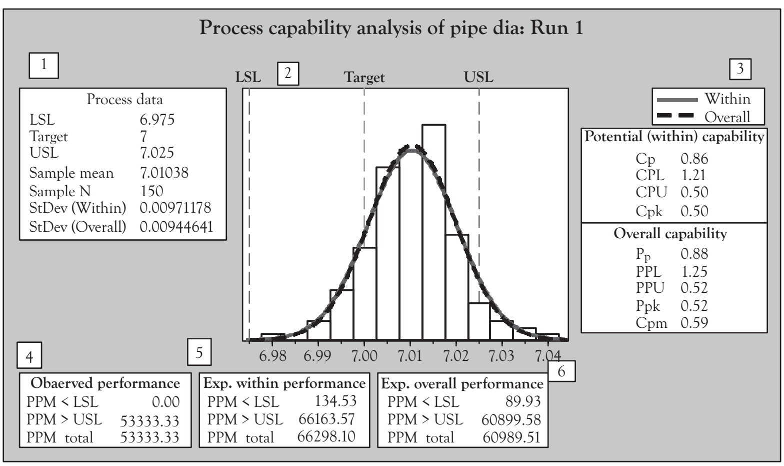


Figure 9.12 Process capability report of pipe diameter: Run 1

Interpreting the Results

Refer to the process capability report in Figure 9.12. This figure provides a detailed process capability of the pipe manufacturing process divided into different sections using boxes. We have labeled the boxes using numbers. The entries in box are explained in the following:

1. The upper left box (*Box 1*) reports the process data including the LSL, target, and the USL. These values are input to the program and are reported as a part of the capability report. Based on the sample data, the program calculates the sample mean and the estimates of *within* and *overall standard deviations*.

   The *StDev(Within)* is the standard deviation within the subgroup. The potential or within capability indexes (explained later) are calculated based on the estimate of $\hat{\sigma}_{within}$ or the variation within each subgroup. If the data are in one column and the subgroup size is one (like in our example where a sample of 150 diameters is entered in one column), this standard deviation is calculated based on the moving range (the adjacent observations are treated as subgroups). If the subgroup size is greater than one, the within standard deviation is calculated using the range or standard deviation control chart. (In MINITAB, you can specify the method you want.)

   The other standard deviation—*StDev(Overall),* or the overall variation, which is the variation of the entire data in the study. The overall capability indexes are calculated based on this estimate. These indexes are explained later.
2. *Box 2* in Figure 9.12 shows the histogram of the data along with two normal curves overlaid on the histogram. One normal curve (with a solid line) is generated using the mean and the estimate of within subgroup standard deviation while the other normal curve (the one with a dotted line) is plotted using the mean and overall estimate of the standard deviation.

   The histogram and the normal curves can be used to check visually whether the process data are normally distributed. To interpret the process capability, the normality assumption must hold. The histogram and the normal curves show that the process is normally distributed.

There is a deviation of the process mean (7.010) from the target value of 7.000. Since the process mean is greater than the target value, the pipes produced by this process exceed the USL. A significant percentage of the pipes are outside of the USL.

3. *Box 3* reports the *potential* or *within process capability* and the *overall capability* of the process (see the right hand side of Figure 9.12).

The potential capability of the process tells what the process would be capable of producing if the process did not have shifts and drifts; or how the process could perform relative to the specification limits (if the shifts in the process mean could be eliminated).

The overall capability of the process tells how the process is actually performing relative to the specification limits.

If there is a substantial difference between within and overall variation, it may be an indication that the process is out of control, or that the other sources of variation are not estimated by within capability (MINITAB).

In Box 3, The value of Cp = 0.86 indicates that the process is not capable (Cp < 1). Also, Cpk = 0.50 is less than Cp = 0.86. This means that the process is off-centered. Note that when Cpk = Cp, then the process is centered midway between the specification limits. The Cpk index provides information about how close the process is to the specification limits.

The value Cpk = 0.50 (less than 1) is an indication that an improvement in the process is warranted. The process can be improved by centering the process and by reducing the variation.

In Box 3, the *overall capability indexes* or the process performance indexes Pp, PPL, PPU, Ppk, and Cpm are also calculated and reported in the capability report (Figure 9.12). Note that these indexes are based on the estimate of overall standard deviation and they determine the overall or long-term capability of the process. Note that Ppk is the index for the whole process.

Pp and Ppk have similar interpretation as Cp and Cpk. For this example, Cp and Cpk values (0.86 and 0.50 respectively) are very close to Pp and Ppk (0.88 and 0.51). When Cpk equals Ppk then the within subgroup standard deviation is the same as that of the overall process standard deviation. For this process, the within and overall standard deviations are close.

The index Cpm is calculated for the specified target value. If no target value is specified, Cpm is not calculated. The Cpm tells whether the process is off-center or deviates from the target. A high Cpm value index indicates a better process.

The process is centered if Cpm, Ppk, and Pp values are the same. For this process, Pp = 0.88, Ppk = 0.51, and Cpm = 0.59. A comparison of these values indicates that the process is off-center.

4. The bottom three boxes (*Boxes 4, 5,* and *6*) in Figure 9.12 report the observed performance, expected within performance, and expected overall process performance in PPM. The *observed performance (Box 4)* in Figure 9.12 shows the following values:

Observed Performance	
PPM < LSL	0.00
PPM > USL	53333.33
PPM Total	53333.33

This means that the number of pipes below the LSL is zero; that is, the process is able to meet the LSL. The number of pipes (out of a million) above the USL is 53333.33. The total number of nonconforming products produced by this process is 53,333 out of a million. These are actual process performances.

5. The values in *expected* within *performance* (Box 5 in Figure 9.12) are based on the estimate of within subgroup standard deviation. These are the average number of parts below and above the specification limits in PPM. For this process, the *Expected within Performance* measures are:

Exp. Within Performance	
PPM < LSL	134.53
PPM > USL	66163.57
PPM Total	66298.10

The earlier values show the average or the expected performance based on the estimate of the within process standard deviation. These may be interpreted as short-term process performance.

6. *Box 6*—The Expected Overall Performance is calculated using similar formulas used in within performance except the estimate of standard deviation is based on overall data. For this process, the *Expected Overall Performance* measures are

Exp. Overall Performance	
PPM < LSL	89.93
PPM > USL	60899.58
PPM Total	60989.51

These values are based on the estimate of overall standard deviation and may be interpreted as the long-term performance of the process.

As can be seen from the earlier analysis, the process is producing a large number of products that do not meet the specifications. An improvement in the process is warranted to reduce the nonconforming products.

Summary

Process Capability is the ability of the process to meet specifications and is one of the important aspects of overall quality improvement. To attain superior quality, the capability of a process should be constantly measured and analyzed. The process capability tells us: (a) whether the process is meeting customer specifications, (b) how will the process perform in the future, (c) whether the process needs improvement, and (d) if we have sustained these improvements, or has the process regressed to its previous unimproved state? The process capability gives us an overall state of the quality by telling us the number of products in a million that do not conform to the specifications. The chapter explained the specification limits and control limits and how they are related to the process capability analysis. We demonstrated different ways of assessing process capability: (1) graphical method, (2) capability using histograms and normal distribution, (3) process capability using control charts, (4) process capability using capability indexes, and (5) computer method—assessing process capability using MINITAB computer software. Examples were presented in all these cases. A detailed process capability report using MINITAB was presented with a case.

CHAPTER 10

Summary, Applications, and Computer Implementation

Introduction

In this book, we discussed statistical techniques and problem solving methods to improve the quality of products and services. The term statistical process control is often used to cover all uses of statistical techniques for the control of product quality. We presented a number of methods and tools to demonstrate how quality improvement methods can be applied to several areas of a company. The quality tools discussed can be applied to a number of areas including manufacturing, process development, engineering design, production and operations, and service industries. The technical tools to achieve quality improvement can also be applied to other areas of a company. One of the important concepts of process control and improvement is the understanding of variability. The companies find it difficult to provide their customers with products that meet all the quality requirements and are flawless. The reason for this is variability. No two products are identical, and there is always a certain amount of variability in every product. If this variation is large, the product becomes unacceptable. It is important to understand the variability and also the sources of variability so that it can be reduced or eliminated if possible. In this text, we learned the sources of variability and the ways of controlling these using methods of statistical process control.

In this chapter, we provide an overview of different control charts, which are major tools of process control. We also discuss the applications of some other control charts (not discussed in Chapters 7 and 8). These charts are used widely in solving quality problems. Finally, the computer implementation of the quality tools and control charts are discussed.

Review and Application Areas of Different Control Charts

We discussed a number of control charts both for variables and attributes with examples. Here we provide a review of the control charts with their application areas. We also discuss some other process control charts and their applications, which we did not discuss in the earlier chapters.

The $\bar{x}$ and R Chart (Variables Control Chart)

- The $\bar{x}$ chart is used to monitor the average of the characteristic being measured.
- The R chart is used to monitor the dispersion of the process. It is used in conjunction with the $\bar{x}$ chart when the process characteristic is a variable.
- When interpreting the $\bar{x}$ chart, the causes for out of control conditions must be examined. MINITAB can perform the tests for out of control conditions.

X and Moving Range Charts for Population Data (Control Chart for Individual Values and Moving Range)

When it is not possible to draw frequent samples because the process is so slow that only one or two products are produced in a day, X and moving range (MR) chart can be used to monitor a variable measurement.

Median Control Charts

Sometimes there may be concerns about the accuracy of the computed subgroup means in the $\bar{x}$ chart. In these cases, median charts can be used. Note that if odd sample size is used, median can be detected easily from the subgroups (use a sample size of 3, 5, or 7). Note that larger the sample size, better is the sensitivity of the chart in terms of detecting the special causes of variation. This is also true for the $\bar{x}$ chart.

$\bar{x}$ and s Chart (Control Chart for the Mean and Standard Deviation)

When the variation or the dispersion of the process characteristic is of major concern, use the $\bar{x}$ chart in conjunction with the s (standard deviation) chart.

The standard deviation chart is used where variation in the process is small and it is needed to detect small shifts in the process (e.g., production of silicon chips for computers). The R chart is not efficient in detecting small variations.

Moving Average Chart

This chart is used for monitoring variables and measurements on a continuous scale. The chart uses past information to predict what the next process outcome will be. This chart can be used to adjust a process in advance before it goes out of control.

Cusum Chart

The cumulative sum or cusum chart is used to identify small shifts in the process where there is no independence between observations.

Attribute Charts: p Charts for Proportion Defective

The p-chart is used to graph the proportion of items defective (nonconforming to specifications) in a sample. These charts are used to determine whether there has been a shift in the proportion defective for a product or service (e.g., wrong orders, late deliveries, accounting errors, defective parts). Subgroup size is usually 50 and 100 units.

np Charts

This is the chart of number of defective (or nonconforming units) in a subgroup. The np chart requires that the sample size of each subgroup be the same each time a sample is drawn.

When subgroup sizes are equal, either the p or np chart can be used. They are essentially the same charts. Some prefer np chart over the p chart as it reflects integers rather than proportions. The applications of both the charts are the same.

c Chart

The c chart is a chart of number of defects (nonconformities) per unit. The requirement is that the units must be of the same sample space: this includes size, height, length, and so on.

The area of opportunity for finding the defects must be the same for each unit. Several individual units can make a sample, but they are grouped as if they are one unit of a larger size. Some examples are: number of flaws in an auto finish for a particular model, number of errors in a form, or number of scratches in glass surface of a product.

u Chart

This is the chart of the average number of defects per unit.

It is different from a c chart that shows the actual number of defects per unit. The number of units sampled in a u chart can be of different size. This chart has the same application as the c chart.

Statistical Process Control Using Computer

Often, in real-world applications, we encounter massive amounts of data that cannot be analyzed easily using manual methods. In such cases, computers becomes indispensable in constructing and analyzing these control charts. Computers are almost always used in modeling, and solving quality problems. All the control charts and the process capability analysis reports in this text were generated using a computer package.

Computer Applications

There are a number of widely used computer packages used in statistics, data analysis, and quality. These include MINITAB, Excel, Stat-graphics, SAS, SPSS, and others. In earlier chapters, we provided a number of computer applications using MINITAB. This is one of the popular and widely used software for Quality and Six Sigma. Figures 10.1 and 10.2 show a screen shot from MINITAB showing different types of variable and attribute control charts available. Using these options, the control charts can be created very easily and can be analyzed for out of control conditions.

Some Examples of Statistical Analysis Using Computer

In Chapter 3, we provided an overview of statistical techniques including the descriptive and inferential statistics, review of probability and probability distributions useful in modeling and solving quality problems. All these techniques can be easily implemented using computer

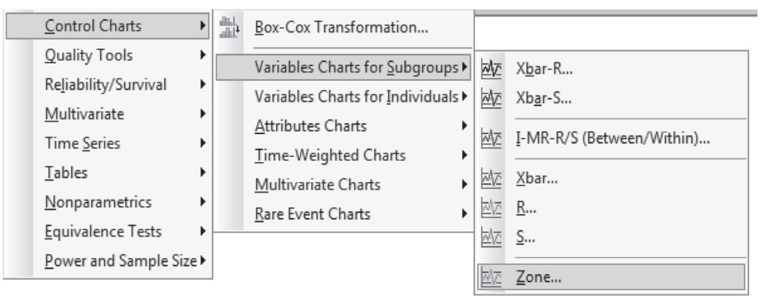


Figure 10.1 Variable control charts in MINITAB

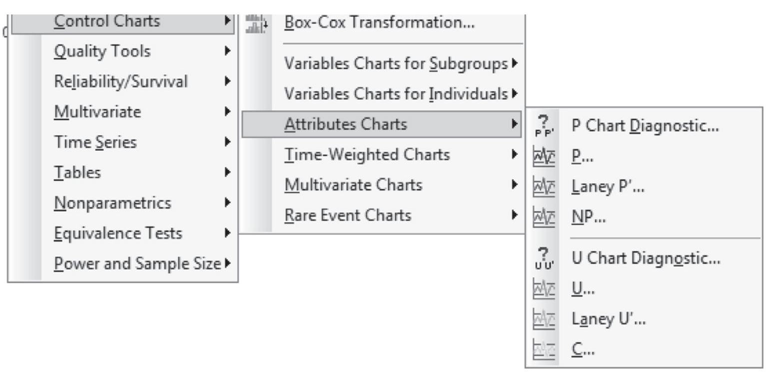


Figure 10.2 Attribute control charts in MINITAB

packages. In Chapter 3, we used a data set from a manufacturer of televisions who was interested in the survival time of one of its components. A sample of 200 television components was tested. The results were shown in Table 3.1. The table showed the life (in hours) of these components rounded to the nearest hours. We used MINITAB to produce a descriptive statistics and graphical summary of the data. These are shown in Figure 10.3. The results are very useful in data analysis.

Example of Quality Tools Using MINITAB

We discussed a number of graphical techniques including histograms, stem-and-leaf-plots, and box plots. There is another set of tools known as *quality tools*. These are a set of graphical tools very widely used in solving quality problems and can be created using quality software. Figure 10.4 shows a commonly used quality tool known as a cause-and-effect diagram widely used in quality projects to find the possible causes of a problem. The figure shows the possible causes of poor quality. MINITAB provides a number of quality tools that are very useful in solving quality problems.

Pareto Chart: A Pareto chart is very similar to a bar chart where the bars are arranged by categories from largest to smallest with a line that

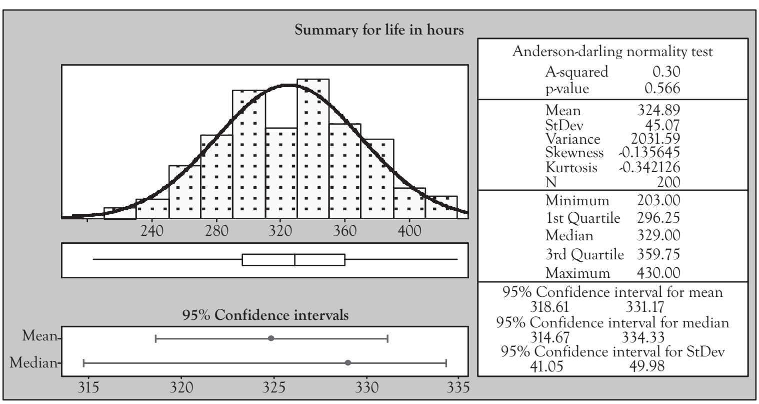


Figure 10.3 Descriptive statistics and graphical summary of television component life

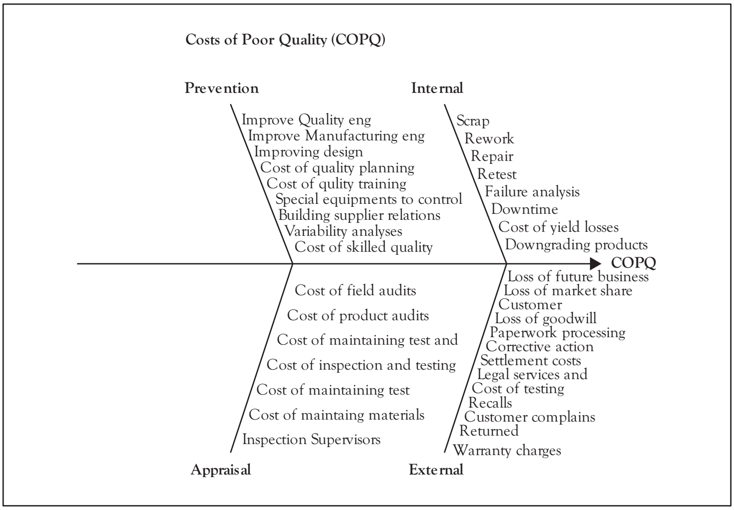


Figure 10.4 Cause-and-effect diagram

shows the cumulative percentage and count of the bars. This chart is widely used in quality improvement projects. Through this chart, the problem that occurs most frequently can be identified quickly and easily. This helps to focus the improvement efforts on the problem having the largest frequency of occurrence. A Pareto chart does not identify the most important category; it identifies the categories that occur most frequently. Pareto charts are also used widely in nonmanufacturing applications.

Figure 10.5 shows a Pareto charts depicting the causes of rejection in a manufactured part. In this chart, the rejection categories are arranged from the largest to the smallest. The Pareto chart in Figure 10.6 shows the same graph with cumulative percentages and counts. The bottom of the chart shows the count, percentage, and cumulative percentage for the categories. The Pareto charts are very useful in isolating and studying the major problem categories.

The tools mentioned earlier are extremely useful in different phases of quality improvement. They are easy to learn and very useful in drawing meaningful conclusions and solving quality problems.

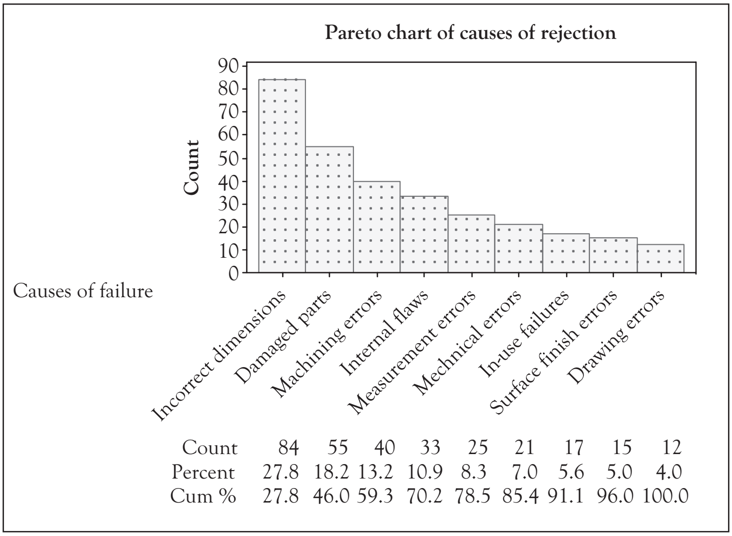


Figure 10.5 Pareto chart of causes of rejection

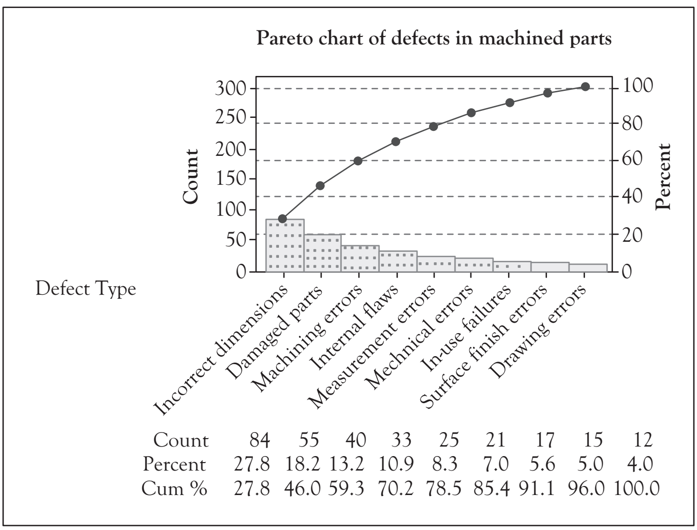


Figure 10.6 Pareto chart of causes of rejection with cumulative percentage

Notes on Implementation

In implementing the control charts, the following guidelines should be followed:

- Understand the process for implementing the process charts
- Know how to interpret the charts
- Know when different process charts are used
- Know how to compute the limits for different charts and the statistical basis behind them.
- Know how to implement the control charts using computer packages.
- Identify critical operations in the process where inspection might be needed
- Identify critical product (quality) characteristics: the characteristics that will have effect on the functioning of the product
- Determine if the product or quality characteristic is a variable or an attribute
- Select appropriate process control chart
- Establish the control limits and use the chart to monitor and improve the process
- Revise the limits when changes are made to the process

Summary

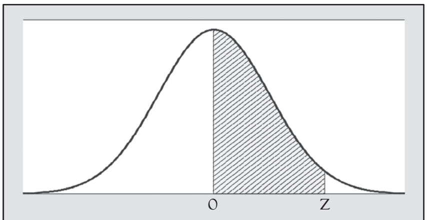
This book provided the following key concepts related to quality and statistical process control:

- Use and importance of quality as a field
- A brief history of quality as a field
- The statistical techniques useful in quality control or process control

- Inferential tools necessary for making inference about process quality
- The methods and philosophy of control charts and why control charts work
- The control charts for variables $\bar{x}$ and R charts, individual value charts and $\bar{x}$ and s charts, $\bar{x}$ charts for variables (MR charts, etc.)
- Control charts for attributes (p-chart, np-chart, c-chart, u-chart)
- Use of computer in solving quality related problems and implementing control charts
- Understand the concepts of process capability analysis and techniques of process capability
- Realize that the statistical process control is integral part of overall quality program—Six Sigma and the control charts are the tools used in control phase of Six Sigma quality

APPENDIX A

Standard Normal Distribution Table



Z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.00	0.004	0.008	0.012	0.016	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.091	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.148	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.17	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.195	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.219	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.258	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.291	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.334	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.377	0.379	0.381	0.383
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.398	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.437	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.475	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.483	0.4834	0.4838	0.4842	0.4846	0.485	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.489
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.492	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.494	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.496	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.497	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.498	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.499	0.499

APPENDIX B

Partial t-Distribution Table

Degrees of Freedom (Df)	Area in Uppet Tail α 0.100	0.050	0.025	0.010	0.005
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.290	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.131
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750
31	1.309	1.696	2.040	2.453	2.744
32	1.309	1.694	2.037	2.449	2.738
33	1.308	1.692	2.035	2.445	2.733
34	1.307	1.691	2.032	2.441	2.728
35	1.306	1.690	2.030	2.438	2.724
:					
:					
infinity	1.282	1.645	1.960	2.327	2.576

APPENDIX C

Table of Control Chart Constants

Constants for X-bar chart			Constants for R chart			Constants for S chart	
Sample Size = n	A_2	A_3	d_2	D_3	D_4	B_3	B_4
2	1.880	2.659	1.128	—	3.267	0	3.267
3	1.023	1.954	1.693	—	2.574	0	2.568
4	0.729	1.628	2.059	—	2.282	0	2.266
5	0.577	1.427	2.326	—	2.114	0	2.089
6	0.483	1.287	2.534	—	2.004	0.030	1.970
7	0.419	1.182	2.704	0.076	1.924	0.118	1.882
8	0.373	1.099	2.847	0.136	1.864	0.185	1.815
9	0.337	1.032	2.970	0.184	1.816	0.239	1.761
10	0.308	0.975	3.078	0.223	1.777	0.284	1.716
11	0.285	0.927	3.173	0.256	1.744	0.321	1.679
12	0.266	0.886	3.258	0.283	1.717	0.354	1.646
13	0.249	0.850	3.336	0.307	1.693	0.382	1.618
14	0.235	0.817	3.407	0.328	1.672	0.406	1.594
15	0.223	0.789	3.472	0.347	1.653	0.428	1.572
16	0.212	0.763	3.532	0.363	1.637	0.448	1.552
17	0.203	0.739	3.588	0.378	1.622	0.466	1.534
18	0.194	0.718	3.640	0.391	1.608	0.482	1.518
19	0.187	0.698	3.689	0.403	1.597	0.497	1.503
20	0.180	0.680	3.735	0.415	1.585	0.510	1.490
21	0.173	0.663	3.778	0.425	1.575	0.523	1.477
22	0.167	0.647	3.819	0.434	1.566	0.534	1.466
23	0.162	0.633	3.858	0.443	1.557	0.545	1.455
24	0.157	0.619	3.895	0.451	1.548	0.555	1.445
25	0.153	0.606	3.931	0.459	1.541	0.565	1.435

Bibliography

Bauer, J.E., G.L. Duffy, and R.T Westcott. 2006. *The Quality Improvement Handbook*. Milwaukee, WI: ASQ Quality Press.

Besterfield, D.H. 2001. *Quality Control*. 6th ed. Upper Saddle River, NJ: Prentice Hall.

Chowdhury, S. 2001. *The Power of Six Sigma*. London, United Kingdom: FT-Prentice-Hall.

Creveling, C.M., J.L. Slutsky, and D. Antis, Jr. 2003. *Design for Six Sigma in Technology and Product Development*. Upper Saddle River, NJ: Prentice Hall.

Crosby, P. 1980. *Quality is Free: The Art of Making Quality Certain*. New York, NY New American Library.

Deming, W.E. 1986. *Out of Crisis*. Cambridge, MA: MIT Center for Advanced Engineering Study.

Feigenbaum, A.V. 2004. *Total Quality Control*. 4th ed. Princeton, NJ: McGraw Hill.

Garvin, D. 1984. "What Does 'Product Quality' Really Means?" *Sloan Management Review* 26, no. 1, pp. 25–43.

Griffith, G.K. 2000. *The Quality Technician's Handbook*. 4th ed. Upper Saddle River, NJ: Prentice Hall.

Juran, J.M., and F.M. Gryna, Jr. 1993. *Quality Planning Analysis*. 3rd ed. New York, NY: McGraw Hill.

McClave, J.T., P.G. Benson, and T. Sincich. 2011. *Statistics for Business and Economic*s. 11th ed. New York, NY: Pearson Education, Inc.

Miller, D. 2005. *Going Lean in Health Care*. Cambridge, MA: Institute for Healthcare Improvement.

Minitab. (n.d.). "Minitab Release 17, Software Documentation." www.minitab.com/en-us/products/minitab/documentation.aspx

Montgomery, D.C. 2004. *Introduction to Statistical Quality Control*. 5th ed. New York, NY: Wiley.

Ostle, B., and R.W. Mensing. 1979. *Statistics in Research*, 76. 3rd ed. Ames, IA: The Iowa State University Press.

Peplinski, J. 2004. *The Importance of Modeling and Simulation in DFSS*, 1–11. Statistical Design Institute, LLC.

Sahay, A. 2014. *Six Sigma Quality: Concepts and Cases*. Vol. 1. 2nd ed., Salt Lake City UT: AMT Digital Publishing.

Index

OTHER TITLES IN OUR SUPPLY AND OPERATIONS MANAGEMENT COLLECTION

Johnny Rungtusanatham, The Ohio State University and
Joy M. Field, Boston College, Editors

- *Understanding the Dynamics of the Value Chain* by William Presutti and John Mawhinney
- *An Introduction to Supply Chain Management: A Global Supply Chain Support Perspective* by Edmund Prater and Kim Whitehead
- *Production Line Efficiency: A Comprehensive Guide for Managers, Second Edition* by Sabry Shaaban and Sarah Hudson
- *Sourcing to Support the Green Initiative* by Lisa Ellram and Wendy Tate
- *Designing Supply Chains for New Product Development* by Antonio Arreola-Risa and Barry Keys
- *Metric Dashboards for Operations and Supply Chain Excellence* by Jaideep Motwani and Rob Ptacek
- *Statistical Process Control for Managers* by Victor E. Sower
- *Supply Chain Information Technology, Second Edition* by David L. Olson
- *Supply Chain Risk Management: Tools for Analysis, Second Edition* by David L. Olson
- *International Operations: How Multiple International Environments Impact Productivity and Location Decisions* by Harm-Jan Steenhuis
- *Better Business Decisions Using Cost Modeling, Second Edition* by Victor Sower and Christopher Sower
- *Improving Business Performance with Lean, Second Edition* by James R. Bradley
- *Lean Communication: Applications for Continuous Process Improvement* by Sam Yankelevitch and Claire F. Kuhl
- *Leading and Managing Lean* by Gene Fliedner
- *Mapping Workflows and Managing Knowledge: Using Formal and Tacit Knowledge to Improve Organizational Performance* by John L. Kmetz

THE BUSINESS EXPERT PRESS DIGITAL LIBRARIES

EBOOKS FOR BUSINESS STUDENTS

Curriculum-oriented, born-digital books for advanced business students, written by academic thought leaders who translate real-world business experience into course readings and reference materials for students expecting to tackle management and leadership challenges during their professional careers.

POLICIES BUILT BY LIBRARIANS

- *Unlimited simultaneous usage*
- *Unrestricted downloading and printing*
- *Perpetual access for a one-time fee*
- *No platform or maintenance fees*
- *Free MARC records*
- *No license to execute*

The Digital Libraries are a comprehensive, cost-effective way to deliver practical treatments of important business issues to every student and faculty member.

For further information, a free trial, or to order, contact:
sales@businessexpertpress.com
www.businessexpertpress.com/librarians

Managing and Improving Quality

Integrating Quality, Statistical Methods and Process Control

Amar Sahay

Quality is a discipline that focuses on product and service excellence. This book is about improving the quality of products and services. The improved quality and reliability lead to higher perceived value and increased market share for a company, thereby increasing revenue and profitability.

The book discusses the concepts and dimensions of quality, costs of poor quality, the importance of quality in this highly competitive global economy, and quality programs—Six Sigma and Lean Six Sigma that focus on improving quality in industries. The text integrates quality concepts, statistical methods, and one of the major tools of quality—Statistical Process Control (SPC)—a critical part of Six Sigma control phase. A significant part of the book is devoted to process control and the tools of SPC—control charts—used for monitoring, controlling, and improving the processes by identifying the causes of process variation. The fundamentals of control charts, along with SPC techniques for variables and attributes, and process capability analysis and their computer applications are discussed in detail. This book fills a gap in this area by showing the readers comprehensive solutions to model and solve quality problems using computers.

Dr. Amar Sahay is engaged in teaching, research, consulting, and training. He has taught/teaching at several Utah institutions, including the University of Utah, Salt Lake Community College, Weber State University, and Westminster College. He has a BS in production engineering (Birla Institute of Technology, India), an MS in industrial engineering and a PhD in mechanical engineering (both from University of Utah, USA). Amar is a certified Six Sigma Master Black Belt and holds an expert level certification in lean manufacturing/lean management. He is the founder of QMS Global LLC; a company engaged in research, training, consulting in the areas of lean six sigma, data visualization, business analytics, manufacturing, and services. Amar is a senior member of Institute of Industrial Engineers and American Society for Quality (ASQ).

Supply and Operations Management Collection

M. Johnny Rungtusanatham and Joy M. Field, *Editors*

ISBN: 978-1-63157-341-5
90000
9 781631 573415

BUSINESS EXPERT PRESS