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Optimization of Cooling Systems

David C. Zietlow

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DEDICATION

Dr. Max Wessler, mentor and friend, who provided me a firm foundation in the fundamentals of thermodynamics

ABSTRACT

Most energy systems are suboptimized. Businesses and consumers are so focused on initial costs that they underestimate the effect of operating the energy system over its life. This suboptimization creates a fantastic opportunity to not only make a wise decision financially but also reduces the environmental impact of energy systems. There are three simple tools, known to all mechanical engineers, that when added to traditional thermodynamics enable an engineer to find the true optimum of an energy system. In this concise textbook, you will be equipped with these tools and will understand how they are applied to cooling systems.

The target audiences for this textbook are mechanical engineering students in their first semester of thermodynamics all the way to engineers with up to 20 years of experience. First semester thermodynamic students will benefit the most from Appendices A and C in Chapter 1. The rest of Chapter 1 is written at a level where any undergraduate mechanical engineering student who is taking heat transfer will be able to quickly assimilate the knowledge. The textbook also has the depth to handle the latent load, which will provide the practicing engineer with the tools necessary to handle the complexity of real cooling systems.

KEYWORDS

Optimization, Design, Thermal Systems, Cooling Systems, Systems Approach, Deductive Problem-Solving Strategy, Parameter Optimization, Direct Expansion Air-Conditioning, Chillers, Building Envelope, Insulation Thickness, System Effects, Latent Load, Heat Transfer, Mass Transfer, Economic Analysis, Total Life Cycle Costs, Objective Function, Return on Investment, Environment, Global Warming, Energy Efficiency, Energy Conservation

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PREFACE

While I was in graduate school at the University of Illinois at Urbana-Champaign, I wondered what set the system pressure in the condenser and evaporator of an air-conditioning system. I began to look at system models and evaluate the variables and tried to understand cause and effect. I would end up spinning in circles always seeming to chase my tail as I traveled around the cycle from compressor to condenser to expansion valve to evaporator and back again. It was not until over 10 years later after I had been teaching thermodynamics for 5 years that I understood the system effects.

The conductance form of the heat transfer equation was a key tool in understanding the relationship between the heat transfer surface area in the heat exchanger and its effect on the system. Without the conductance equation, the connection cannot be made. The conductance form of the heat transfer equation is the first of three tools to help you understand this system effect along with how to design and optimize energy systems.

The second tool is engineering economics. Since energy costs occur in the future and future dollars are not as valuable as today's currency, engineering economics provides a way to accurately discount future dollars. This discounting of future currency allows you to directly compare today's expenditures with future expenditures. It is not possible to optimize most energy systems without considering the cost.

The third and final tool is parameter optimization. This third tool is primarily used to connect the design variable to the initial cost. Parameter optimization is also essential in the development of an accurate model based on fundamental principles. The empirical parameters in both the convective heat transfer coefficient correlation and the mass transfer are highly dependent on the geometry of the application. Therefore, these empirical parameters often need to be tuned to the experimental data collected for the energy system.

In addition to the tools for optimization, you will be introduced to a new method to solve open-ended problems. If you are an engineer you are by definition a problem solver. Most problems you face in industry are open-ended, which is different from the problems you solved in your training. The deductive problem-solving strategy provides the structure you need to solve problems when you are not sure what inputs are necessary.

Once an optimum is found, the question becomes how to handle the uncertainty in all the variables that contribute to the solution. A nondimensional sensitivity analysis is developed and demonstrated for each example in each chapter. The nondimensionalization allows you to compare one variable to the next and determine which one has a greater influence on the result. Then you can prioritize which variables need to be more accurate.

ACKNOWLEDGMENTS

The idea for this textbook occurred while writing a proposal to justify a sabbatical. After teaching the principles contained within this textbook for several years, it became evident that these concepts would be useful for a wider audience. I am grateful to Bradley University for allowing me the time in the form of a sabbatical, to initiate this work. Thank you Dr. D. Paul Mehta, Chairman of the Mechanical Engineering Department, for providing me with many opportunities to try the ideas contained in this textbook in the crucible of the classroom.

I owe a special thank you to Dixie, my wife. She gave me 2 years of Saturdays to write, which enabled me to make vital forward progress on this work. Without her gift of time this work would not have been possible.

The input from four different reviewers has been instrumental in communicating the contents of this text in a clear and concise way. Dr. John Meyer, an advanced refrigerant systems specialist at Hanon Systems, Dr. J. Steven Brown, an Associate Dean of Engineering at The Catholic University of America, and Dr. Eric Ratts, an Associate Professor of Mechanical Engineering at the University of Michigan, all provided valuable technical reviews of this work. Dr. Peter Dusenbery, an Associate Professor of English, provided advice on the particular use of words like “this” and “address” and how to use an active rather than passive voice. The final product is much improved as a result of these reviewers’ comments.

Finally, Adam Smetters, Ryan Sell, Michael Gorbach, Jack Stein, Paige Engelhardt, Syed Majeedullah, and Nicholas Deprez volunteered to assist in the polishing of the figures within the textbook along with proofreading. Also I thank Escaline Charlette Aarthi, Austin Williams, and Luke Peterson.

CHAPTER 1

OPTIMIZATION OF COOLING SYSTEMS

INTRODUCTION

This concise textbook will improve the way you think about the design of cooling systems. If energy systems are the forest, then cooling systems are a tree in the forest. The scope of this work is to focus on cooling systems and, in particular, the most common cooling systems based on the vapor-compression cycle. One of the occupational hazards of being an engineer is to get caught up in the branches of the tree through detailed analysis. There are many great textbooks that cover the details of cooling systems. The goal of this work is to show how all the branches fit together, to form the tree, in a systematic way. This systematic approach is missing in traditional thermodynamic textbooks. As a matter of fact, the current textbooks on thermodynamics do not equip the students with the tools they need to optimize, much less design, the vapor-compression cycle. This work bridges the gap between the textbook and reality, in a concise way.

"Design," like the word "love," is overused to the point where it has lost its meaning. When design is used in the context of cooling systems typically what is understood is that the size of the cooling system is to be determined. Sizing the system is an important first step. But design is more, much more, than sizing. When you design a cooling system, you want the best cooling system for the application. This leads you to another overused and misapplied word, "optimization." In any optimization process, one needs to determine the objective function. This is a function that has a minimum or maximum. The best objective function for most designs is the total life-cycle costs. Figure 1.1 demonstrates when initial costs and the present worth of the operating costs are combined to form the

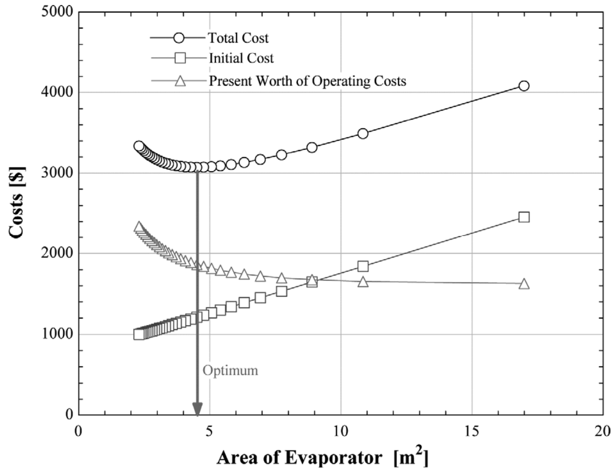

Figure 1.1. Cost as a function of evaporator surface area. Analysis based on an effectiveness-NTU model of direct-expansion air conditioner with a dry evaporator (basic model described in detail in this chapter). The optimum condenser area of 6.5 m^2 and compressor efficiency of 79.5 percent are kept constant as the evaporator area is varied. The air conditioner serves an envelope with an annual cooling load of 20,000 kw-hr in a location where the average temperature is 30°C. The space within the envelope is maintained at 20°C. The average rate of cooling over the season is 10 kW. The remaining input variables are specified in Table 1.2. (EES, 2014)

total life-cycle costs then a distinct minimum occurs. Before these costs can be determined three simple tools need to be added to the traditional presentation of thermodynamics.

The first and most important tool to be added to traditional thermodynamics is the conductance equation, Equation 1.14, for heat exchangers which relates the heat transfer surface area to the difference in temperature between the two fluids. Focusing on the evaporator, the heat transfer rate is known for a given application. It is based on the cooling load, $\dot{Q}_{load}$, at average operating conditions. The load is fixed once the location, orientation, and building envelope have been designed (see Figure 1.2). Assume the overall heat transfer coefficient, U, is fixed. Therefore, the overall conductance (UA) in kilowatt per Kelvin of the heat exchanger is only a function of the heat transfer surface area.

Consequently, the conductance equation provides a relationship between the design variable (heat transfer surface area) and the temperature difference between the refrigerant and the other fluid (water for chillers and air for direct-expansion air conditioners). So as the area increases, the temperature difference will decrease. As the temperature difference decreases, the irreversibility decreases. Less irreversibility translates (covered in greater detail later) to a higher coefficient of performance (COP) which lowers the power consumption for a given application.

The application sets the rate of cooling energy which can be determined using standard heat transfer analysis, which is described well in other air conditioning design textbooks (American Society of Heating Refrigerating and Air-Conditioning Engineers, Inc., 2013; Kuehn, 1998; Stoecker, 1982). So following the progression from the conductance form of the heat transfer equation through the COP, the power requirements in kilowatts can be related to the design variable, heat transfer surface area in meters squared.

Lower power consumption provides lower operating costs. But the operating costs occur in the future. Since future currency is less valuable than today's, you will need the second tool added to traditional thermodynamics. That tool is engineering economics, which discounts future currency from an annual distribution to its present worth. Now you can compare the present worth of the operating costs with the initial costs.

To determine the initial cost, use the third tool added to traditional thermodynamics. The third tool will relate the design variable to the initial cost. This relationship is determined by obtaining quotes from the supplier for heat exchangers of different sizes. From the quotes a parameter optimization, the third tool, can be performed to determine the empirical relationship between the design variable and the initial cost.

In summary, to perform an optimization of a cooling system three tools need to be added to traditional thermodynamics. Putting these three tools together in a systematic way, makes it possible to see how the design variable affects both the initial and operating costs.

The design process for a cooling system contains three main steps. The first step is to determine the cooling load based on the building envelope, its orientation, and location (see Figure 1.2). The cooling load is then used to design the fluid, typically water or air, distribution system (pump/pipe or fan/duct). Finally, the last step is to design the components of the vapor-compression cycle. Chapter One will focus on the design of the vapor-compression cycle. You have been introduced to the design of the evaporator which will be explored in greater detail. Additionally, you will also understand how thermal sciences and economics affect the design of the compressor and condenser.

Before you begin with the optimization though there is a new way to solve problems, you will find useful. Most likely, you were trained in school to solve problems where all the inputs needed to solve the problem were specified. This allowed you to use inductive reasoning, connecting observations with theory, as you match the inputs with some laws or definitions

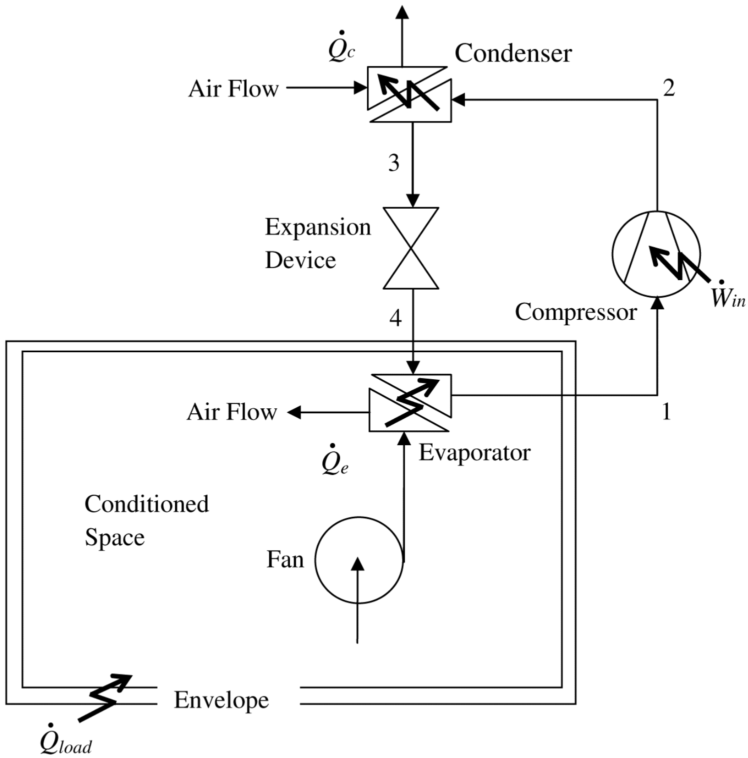

Figure 1.2. Overall schematic of a cooling systems. This schematic presents the key components in the design process for cooling systems. $\dot{Q}_{load}$ is the rate of heat transferred to the conditioned space. Under steady-state conditions the evaporator must remove this same rate of heat transfer $(\dot{Q}_e)$. To remove this heat the compressor requires power $(\dot{W}_{in})$. Finally all the energy transferred to the refrigerant in the evaporator and compressor is rejected in the condenser $(\dot{Q}_e)$.

which were just covered in the previous week. It is quite a shock then when you begin your first employment and your supervisor does not tell you what inputs you need to solve the problem. Often, you are fortunate if the objective is clearly defined. The deductive problem-solving strategy (Zietlow, 2006) will provide you the structure you need to tackle open-ended problems. For this reason, the equations for optimizing the vapor-compression cycle will be developed using the deductive problem-solving strategy. This strategy is designed to guide you from the objective back to the inputs to be specified. Once you master this strategy, you will be able to solve open-ended problems with greater ease.

To illustrate the difference between inductive and deductive reasoning, consider a simple example where you have a rigid container of a gas. Your objective is to find the specific volume, v, of the gas. As a student you would have just covered the ideal gas law, $Pv = RT$, and the textbook problem would specify the type of gas along with its temperature, T, and pressure, P. Armed with these specific observations, as a student you would use your short-term memory to recall the ideal gas law which relates the two known properties with the unknown property. This is inductive reasoning where you go from the specific observations to a general theory which ties the observations together.

Now fast forward to today, you covered the ideal gas law several months or years ago. Your supervisor would like you to determine the specific volume of the gas. Armed with this objective you ask if there are any theories which relate this variable to other variables which could be measured or specified. Using your long-term memory, you think of the ideal gas law. Then you evaluate the ideal gas law deductively by going from the theory to the particulars. You recognize that the pressure and temperature can be measured and if the gas is known, the value for the gas constant, R, can be found in a table.

Many other problem-solving strategies confirm the benefits of solving open-ended problems. The McMaster Problem Solving Program (Woods et al., 1997) found the most effective way to teach problem-solving skills to students is to use a workshop approach. The key components to the workshop are an introduction, pretest, application, and immediate feedback. Students are then asked to reflect on what they learned in a journal. In the McMaster program they had four different workshops. The first two helped the students develop the analytical skills they needed to solve well-defined, typical homework problems. The third workshop concentrated on team problem-solving. The fourth and final workshop dealt with solving open-ended problems. The deductive problem-solving strategy presented here is a tool which is most applicable to this fourth workshop in the solution of open-ended problems.

Suliman (2004) introduces a new format to teaching engineering based on problems as opposed to a lecture format. Small groups of students are given a problem each week. A faculty tutor is assigned to guide the students to identify the key issues related to the problem. Rather than have the faculty provide the facts to the students through a lecture, the students need to learn on their own the content at the appropriate breadth and depth. The deductive strategy will provide the students with the structure needed to succeed with the open-ended nature of this problem-based format.

Problem-solving is a key to a holistic approach presented by Jordan et al. (2000). They present the importance of open-ended problems to assist future teachers in understanding the connection between the principles of science and math and the physical world. The deductive problem-solving strategy is a valuable tool that can be used to systematically solve a wide variety of science and math problems.

Hill (1998) discusses how creativity can be developed through open-ended problem-solving. Unfortunately in these problems both order and disorder coexist. The deductive strategy presented here can provide a framework to channel the disorder associated with open-ended problems.

Systematic Innovative Thinking (SIT) is a method that provides a balance between order and disorder that is essential in fostering creativity. Barak (2002) provides a brief overview of this method and how it was applied to produce valuable product innovation. Unfortunately, current textbook problems are too structured and do not provide the students with the level of disorder they will face in the workplace. The deductive problem-solving strategy helps the student deal with the higher disorder associated with problems where the inputs are not specified. Therefore, open-ended problems expose the students to higher disorder which fosters creativity.

OPTIMIZATION OF THE BASIC VAPOR-COMPRESSION CYCLE

The introduction has provided you with an overview of the tree without getting caught up in the branches. With the background provided you will now be presented with the details of the relationships discussed. To assist you on your exploration of the branches, the instructions are given in bold font while the associated explanations are in normal font. The bold numbers preceding each instruction corresponds to the summary of the deductive problem-solving strategy given in Appendix A. The summary of the deductive problem-solving strategy was derived from the one provided by Zietlow (2006). As discussed previously, the best objective function for the optimization of cooling systems is the total life-cycle costs. The goal is to minimize the total costs as a function of the design variables which in this case are the heat transfer surface areas of the evaporator and condenser and the isentropic efficiency of the compressor.

The first key step, in solving any engineering problem, is to **[1] draw a schematic of the system** as shown in Figure 1.3. In this schematic, the heat exchangers are represented by two right triangles. Each side of the right triangle represents the inlet or outlet for the fluid passing through the heat exchanger. The hypotenuse represents the heat transfer surface area. In the schematic, all energy and fluid flows are identified along with

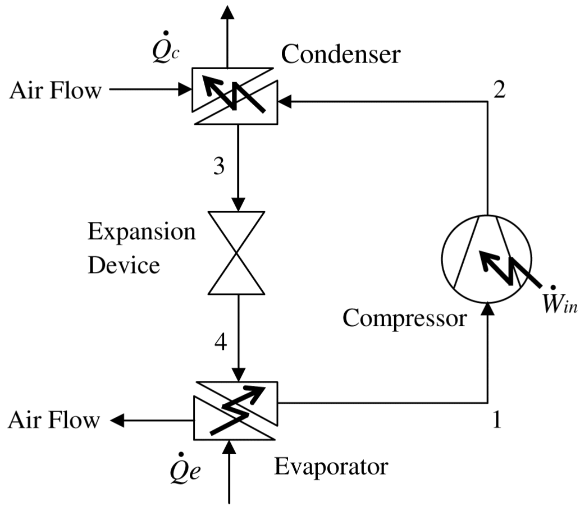

Figure 1.3. Schematic of the vapor-compression cycle. This schematic contains the four major components of the vapor-compression cycle along with any significant energy transfers. The vapor-compression cycle is general enough to represent either direct-expansion air conditioning, refrigeration or an air-to-air heat pump. The expansion device could be a thermal expansion valve, capillary tube or orifice tube. The heat exchangers (i.e., evaporator and condenser) are typically cross-flow in configuration.

the hardware. The numbers identify physical locations between the components of the system which correspond to thermodynamic state points used in the analysis.

Another key step, in the solution of thermodynamic problems, is to **[2] construct a property plot based on refrigerant properties** as seen in Figure 1.4. **[2a] Mark all state points between each of the components.** The numbers in the property plot correspond to the physical location between the components identified in the schematic (Figure 1.3). To understand the influence of the heat exchangers, it is useful to **[2b] superimpose the inlet temperature of the other fluid (e.g., air or water) on this plot**. Having both temperatures on the same plot illuminates the temperature difference between the fluids, which is related to the heat transfer surface area through the conductance form of the heat transfer equation. The pressure versus specific enthalpy diagram is useful in illustrating how each of the components of the system performs. After learning the paths for isothermal processes and isentropic processes it is easy to see how far each of the components performs from its ideal process. The compressor

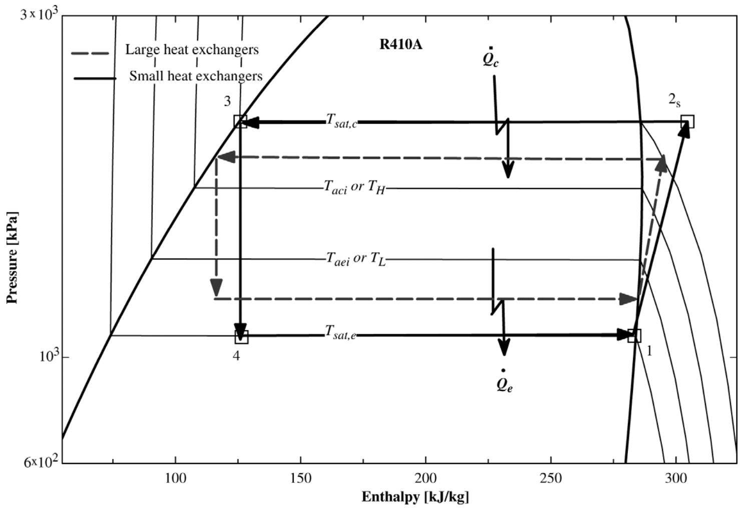

Figure 1.4. Property plot of the vapor-compression cycle (EES, 2014). The processes of the vapor-compression cycle with heat exchangers of two different sizes (i.e., large and small) are displayed on pressure and enthalpy coordinates for the refrigerant (*R*410*a*). Superimposed on the plot are isotherms representing the inlet temperatures (T_{aci} and T_{aei}) of the heat transfer fluids on the other side of the heat exchangers. The temperature differences between the other fluids and the refrigerant in the condensing ($T_{sat,c}$) and evaporating ($T_{sat,e}$) regions, respectively, graphically illustrate the degree of irreversibility in the heat exchangers. As the surface area of the heat exchanger increases, this temperature difference and associated irreversibilites decrease. The directions of heat transfer for both the condenser $(\dot{Q}_c)$ and evaporator $(\dot{Q}_e)$ are also indicated.

performance is referenced to the isentropic process while the heat exchangers are referenced to the temperature of the thermal reservoir which is the inlet temperature of the fluid other than the refrigerant. The closer the refrigerant temperature is to the thermal reservoir provided by the other fluid, the more reversible the heat transfer and the more efficient the cycle.

Now that you have a firm understanding of the vapor-compression cycle by formulating the schematic and property plot, you are ready to begin formulating the solution. **[3]** First, **identify the objective function**. In this case, as discussed previously, the objective function is the total life-cycle cost. **[4a] Formulate the first equation**. When using the deductive strategy, the first equation needs to contain the objective function. Finding the appropriate equation is the most challenging part of the deductive problem-solving strategy. To assist you in finding an equation use one of the five possible sources for equations identified in Table 1.1.

Table 1.1. Possible sources for an equation

1. **[4b.i]** A law of science or math such as conservation of energy, addition, area of a circle, etc.
2. **[4b.ii]** A definition such as efficiency, specific volume, COP, heat exchanger effectiveness, etc.
3. **[4b.iii]** A property relationship using the state postulate. This postulate states; for a simple, compressible substance the state can be determined with two independent, intensive properties. Once the state is known any other intensive thermodynamic property can be found as a function of two intensive properties as long as they are independent.
   Caution: When a refrigerant is in the mixture region, typically at the inlet to the evaporator, temperature and pressure are dependent on one another. Therefore, another independent intensive property is needed to define the state. Typically, specific enthalpy is used in the case of the evaporator inlet.
4. **[4b.iv]** A regression analysis that is typically used for finding the relationship between the initial cost of a component and its relevant design variable.
5. **[4b.v]** Knowledge of the process such as isobaric, isentropic, etc.

[4b.i] Use a law of math, specifically addition, for the first equation. Here the two components {initial cost (*IC*) and present worth of operating cost (PW_{OC})} are summed to form the total life-cycle costs (*TC*) as shown in Equation 1.1. None of these three variables can be specified so you will need to develop at least two more independent equations before you can solve for the total cost. As an aid to solving the problem, **circle each variable that will need an independent equation [4.c.ii]**. Other graphical aids to the deductive approach are a parallelogram for the dependent variable and a rectangle for variables that can be specified. **Draw a parallelogram [4c.iii] around and circle** total cost since it is the dependent variable of the first equation. As you proceed through the equation development, move from left to right through the circled variables. If new variables are circled in future equations, develop those equations first and then return to previous equations after satisfying the latest equation. Having a systematic way to process the circled variables will prevent you from missing any equations or variables.

$$TC = IC + PW_{OC} \tag{1.1}$$

[4d] Moving from left to right in Equation 1.1, the next variable that requires an equation is the initial cost. For the initial cost, **[4b.i] sum the initial costs of each of the components** affected by a design variable {evaporator (evap), condenser (cond) and compressor (comp)} of the vapor-compression cycle as shown in Equation 1.2.

$$IC = IC_{evap} + IC_{cond} + IC_{comp} \tag{1.2}$$

[4c][4d][4b.iv] Use parameter optimization on cost quotes from the evaporator supplier to determine the parameter, cost per unit area (C_{uae}). Then multiply this parameter by the heat transfer surface area (A_e) to determine the initial cost for the evaporator. There are two different areas you need to consider, especially when air is used as one of the heat transfer fluids. First consider, the primary surface area of the channels for the refrigerant and second the fins. For ease of calculation you will use the exterior (or air-side) surface areas. The sum of these areas is the design variable that influences both initial and operating costs. For the time being, assume this area is specified as the design variable. You will see later, from a convergence standpoint, it is better to specify the saturation temperature of the refrigerant as the design variable and then calculate the area. For equation development and system understanding, it is more intuitive to specify the area. Obtain quotes from the evaporator supplier to determine the cost per unit area for the channels and fins. **[4c.i] Place a rectangle around variables you can specify.**

$$IC_{evap} = A_e \times C_{uae} \tag{1.3}$$

Use the same reasoning, **[4.b.iv]** develop an equation to determine the initial cost for the condenser as seen in Equation 1.4.

$$IC_{cond} = A_c \times C_{uac} \tag{1.4}$$

[4.b.iv] Formulate a parameter optimization to relate the isentropic efficiency (η) of the compressor to its initial cost. The design variable for the compressor is the isentropic efficiency. For a particular compressor, the isentropic efficiency is typically provided by the supplier. Price quotes for compressors with different efficiencies enable the development of a parameter optimization as shown in Figure 1.5.

An example of parameter optimization for the initial cost of compressors with power input of 1.0 [kW] is provided in this figure. The solid line is the model equation in the form of Equation 1.5 with a cost coefficient (CC_{comp}) of 30 [$] and an exponent ($e$) of 1.5. The symbols are sample quotes for the compressor at different isentropic efficiencies.

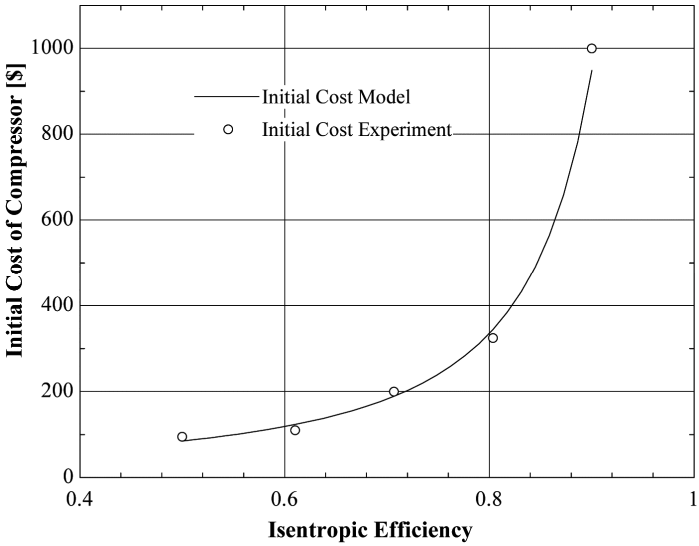

Figure 1.5. Parameter optimization of compressor cost as a function of isentropic efficiency

The parameter optimization provides you with the empirical parameters, that is, cost coefficient (CC_{comp}) and exponent (e), you need to determine the initial cost of the compressor as shown in Equation 1.5. The form of Equation 1.5 was chosen to handle the limiting case of an ideal compressor, for which the isentropic efficiency is 1, where it becomes increasingly more expensive to manufacture.

$$IC_{comp} = \frac{CC_{comp} \times \dot{W}}{(1-\eta)^{e}} \tag{1.5}$$

[4.h.] Check if all the variables circled in Equation 1.2 have equations, which allow these variables to be determined. Once all the design variables and cost coefficients are specified, the initial cost of the heat exchangers can be determined. At this point in the presentation of the deductive problem-solving strategy, it is useful to step back and review our location in the process (Figure 1.6).

Except for the compressor, the initial cost of each heat exchanger can be determined from specified variables **[4.h.]**. Note how the equations associated with the circled variables in Equation 1.2 are all at the same level of indentation **[4.e.]**. Indenting equations is a useful housekeeping tactic to help insure you return to the correct equation after satisfying a variable.

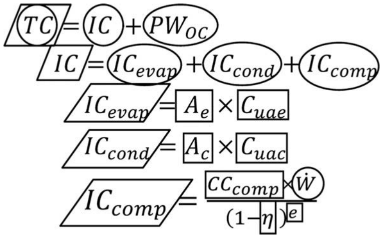

Figure 1.6. Review of equations through to initial costs

Up until now the engineers have been feeling left out of the analysis, but that will end since an equation is needed to find the actual power required by the compressor. Now there are two equations which have the compressor power in them. The definition of isentropic efficiency and conservation of energy applied to the real compressor. Save the conservation of energy to determine the actual enthalpy at the exit of the compressor, if needed. Therefore, **[4.b.ii] apply the definition** of isentropic efficiency $(\boldsymbol{\eta})$ since it relates the ideal rate of work $\left(\dot{W}_{in_s}\right)$, to the actual rate of work $\left(\dot{W}_{in}\right)$ as displayed in Equation 1.6. Important tip for finding the right equation: Use the simplest form of the equation. For example there is a form where the isentropic efficiency is given in terms of enthalpies. To develop this form the conservation of energy is applied to both the ideal and real compressors. Therefore, in the single equation one definition and one law of science are mixed together. The more complicated form of the equation is harder to understand and therefore, for clarity sake, is not recommended.

$$\eta = \frac{\dot{W}_s}{\dot{W}} \tag{1.6}$$

The ideal rate of work is the power required for the ideal compressor, where no disorder or entropy (s) is created in the process. This can only occur in an isentropic process (s = constant) which is frictionless and without the transfer of heat (adiabatic). The isentropic efficiency was specified earlier when determining the initial cost of the compressor and Equation 1.6 will be used to solve for the actual compressor power. This power is the shaft power driving the compressor. This basic model assumes the electric motor driving the shaft is 100 percent efficient. A more accurate model would add the definition of efficiency for an electric motor to account for the losses in the electric motor.

To find the rate of work into the ideal compressor, **[4.b.i] apply the conservation of energy** to it. Before this is done, it is useful to review the most general form of the conservation of energy, in rate form, as shown in Equation 1.7.

$$\Delta\dot{E}_{system} = \dot{E}_{in} - \dot{E}_{out} \tag{1.7}$$

Here it is shown that the change in energy stored within the control volume with respect to time $(\Delta\dot{E}_{system})$ equals the rate of energy transfer entering the control volume $(\dot{E}_{in})$ minus the rate of energy transfer leaving the control volume $(\dot{E}_{out})$. When the system is at "steady-state steady-flow" the energy stored within the system does not change with respect to time. Therefore, the left-hand side of the equation is zero and the conservation of energy becomes the rate of energy into the control volume equals the rate of energy out as shown in Equation 1.8.

$$\dot{E}_{in} = \dot{E}_{out} \tag{1.8}$$

The rate of energy transfer can be in the form of heat $(\dot{Q})$ or work $(\dot{W})$ or the result of mass $(\dot{m})$ crossing the boundary of the control volume. For a mass crossing the boundary, the energy transfer can be accounted by multiplying the mass flow rate $(\dot{m})$ by the sum of the specific enthalpy (***h***), kinetic (***ke***) and potential energies (***pe***) as shown in Equation 1.9. Keep in mind, each mass crossing the boundary needs to be included. For heat and work, the direction shown in the schematic determines whether the energy transfer is entering or leaving the control volume. If the wrong direction is assumed, then the conservation of energy will reveal this by producing a negative number for the heat and/or work.

$$\dot{E} = \dot{m}(h + ke + pe) \tag{1.9}$$

When applying the conservation of energy to a component, in this case the ideal compressor, it is useful to refer to the schematic in Figure 1.3 and the property plot in Figure 1.4. Assuming steady-state steady-flow, apply Equation 1.8 to the ideal compressor. Also, because you have significant power entering the ideal compressor, you are assuming the change in specific kinetic and specific potential energy of the fluid is negligible relative to the change in specific enthalpy of the fluid. Therefore, the kinetic and potential energy terms for the inlet and outlet flows cancel each other.

$$\boxed{\dot{W}_s} + \dot{m}_r \times h_1 = \dot{m}_r \times h_{2s} \tag{1.10}$$

Since you are assuming steady-state, steady-flow conditions, then the conservation of mass yields the change of mass, stored within the control volume, relative to time is zero. Therefore, the rate of mass entering the compressor at the inlet, state-point one, equals the rate of mass leaving the compressor at the outlet, state-point two which you will represent by the symbol $(\dot{m}_r)$. Equation 1.10 will be used to solve for the power required by the ideal compressor.

At this point it is useful to provide a tip on how to **[4.c.] determine if a variable can be specified or if it needs an equation**. One question to ask is, "Can the variable be easily measured?" Now the mass flow rate of refrigerant can be measured, but not easily. It would require the installation of an expensive flow meter in the refrigerant circuit. This is a rare measurement, usually reserved for research applications and prototype validation. Additionally, enthalpy is a derived thermodynamic property based on the internal energy and flow work of a fluid. There is no direct way to measure it. Since the mass flow rate and enthalpies cannot be specified directly, you will need equations for these variables.

To provide an equation to determine the mass flow rate refer back to Figure 1.3, and consider how cooling systems are designed. For a given application, the cooling load is fixed once the building's envelope, orientation, and location are specified. At steady-state conditions the cooling load is equal to rate of energy the evaporator $\left(\dot{Q}_e\right)$ needs to remove from the space. Since this heat transfer rate is known, it would be useful to have an equation with both heat transfer rate and mass transfer rate. **Use a law of science [4.b.i], more specifically, the conservation of energy applied to the evaporator** as seen in Equation 1.11. The conservation of energy may be applied once for each component, real or ideal, and still be an independent equation. Again you may assume steady-state steady flow with negligible changes in kinetic and potential energies.

$$\boxed{\dot{Q}_e} + \boxed{\dot{m}_r} \times \textcircled{h_4} = \dot{m}_r \times h_1 \qquad (1.11)$$

The specific enthalpy at state-point four (h_4) is difficult to find using property relationships because pressure and temperature, the least expensive measurements to make, are not independent in the mixture region (see Figure 1.4). Therefore, **[4.b.i] use the conservation of energy** around the expansion device to find the specific enthalpy at state-point four (h_4) as a function of the specific enthalpy at state-point three (h_3). As a matter of fact, they are equal to each other as long as the expansion device is adiabatic and there is no work crossing its control volume.

$$\boldsymbol{h_4} = \boldsymbol{h_3} \tag{1.12}$$

Since the specific enthalpy cannot be measured directly **[4.b.iii.] use a property relationship** to find the specific enthalpy at state-point three. Assuming the refrigerant has a single chemical composition; it can be treated as a simple substance. For a simple, compressible (where *Pdv* work is the only relevant work mode) substance, the state postulate concludes: two independent, intensive properties will define the state. The quantity of refrigerant in the vapor-compression system determines the degree of subcooling at the entrance to the expansion valve. To minimize complexity, assume the system contains the amount of refrigerant to provide a saturated liquid condition at the inlet to the expansion device. One of the independent, intensive properties is the temperature at state-point three ($T_{sat,c}$). The other property is quality or vapor mass fraction (x) since it is assumed the liquid is saturated. Consequently, Equation 1.13 is the result of applying the state postulate and its associated property relationship to determine the specific enthalpy at state-point three.

$$\boldsymbol{h_3} = \boldsymbol{f}\left(\boldsymbol{R}410\boldsymbol{a}, \boldsymbol{T_{sat,c}}, \boldsymbol{x} = 0\right) \tag{1.13}$$

The condenser, shown in Figure 1.3, can be divided into two different sections; de-superheating and condensing. Each of these will be treated as a separate heat exchanger. The inlet temperature to the de-superheating section is set by the outlet condition of the compressor through the conservation of energy. On the other hand, the inlet pressure is set by the size of the condenser. Keep in mind, the pressure drop due to friction is not addressed in this work. Therefore, the pressure is related to the saturation temperature of the refrigerant as it condenses. The conductance form of the heat exchanger equation shows how the saturation temperature is influenced by the area of the condenser. Focus on the condensing region of the condenser since the majority of the heat transfer surface area is used for condensing. In other words, assume the influence of the saturation temperature on the heat transfer surface area used to de-superheat the refrigerant is negligible.

Before advancing to the details of the heat exchanger analysis, look at an *important relationship* that is typically overlooked in three traditional courses, namely thermodynamics, heat transfer, and advanced HVAC design. This interesting relationship occurs *between the saturation temperature of the refrigerant and the size of the heat exchanger* or heat transfer surface area. In traditional thermodynamics (Cengel and Boles, 2015), the heat transfer rate is either specified or determined from the conservation of energy. In traditional heat transfer (Holman, 2009), the heat exchanger is considered as a stand-alone component, separated from the system which it serves. Many of the HVAC design textbooks (Kuehn, 1998) follow this same pattern of isolating the heat exchanger from the system. Therefore, either the area or the temperature difference is specified for a given heat exchanger which then overly constrains the design to one particular size of heat exchanger. To avoid overly constraining the problem, it is important to understand how the surface area affects the saturation temperature.

The conductance form of the heat exchanger equation (Equation 1.14) will help you understand the relationship between area and temperature. It is easiest to understand the relationship when the conductance form of the equation is applied to the evaporator. For the evaporator, the heat transfer rate ($\dot{Q}_e$) and inlet temperature of air (T_{aei}) are fixed. The conductance form of the heat exchanger equation provides a key insight into this often overlooked *system effect*. As the area increases, so does the saturation temperature of the refrigerant. Assume proper changes are made to the fluid flow rate and geometry of the heat exchanger so both the heat transfer rate and overall heat transfer coefficient (U) are kept constant.

$$\boxed{\dot{Q}_e} = \boxed{U}\ \overset{\uparrow}{A}\ \left(\boxed{T_{aei}} - \overset{\uparrow}{T}_{sat,e}\right) \tag{1.14}$$

This is significant in the performance of the vapor-compression cycle because the saturation temperature in the evaporator sets the "apparent" temperature of the low temperature reservoir $(\boldsymbol{T_L})$. The actual temperature of the low temperature thermal reservoir is the inlet temperature of the air to the evaporator. This equivalency between the two temperatures can be seen by looking at Figure 1.4. In the case of reversible heat transfer, the saturation temperature approaches this inlet temperature as the area increases. In the figure the only difference between the two process plots are the sizes of the heat exchangers. The saturation temperatures are closer to the inlet air temperatures for the system with large heat exchangers. Since the limited heat transfer surface area sets the saturation temperature, treating this temperature as the "apparent" temperature of the thermal reservoir then accounts for the irreversibility of the heat transfer.

Recall for the Carnot cycle the ideal COP (COP_s) is the temperature of the low temperature reservoir (T_L) divided by the temperature difference between the high and low temperature reservoirs.

$$\overset{\uparrow}{\overline{\boldsymbol{COP_s}}} = \frac{\overset{\uparrow}{\overline{\boldsymbol{T_L}}}}{\boldsymbol{T_H} - \underset{\uparrow}{\underline{\boldsymbol{T_L}}}} \tag{1.15}$$

Therefore, as the saturation temperature in the evaporator rises from an increase in heat transfer surface area the value of the denominator in Equation 1.15 decreases. Also the numerator increases both of these contribute in the same direction to improve the ideal COP_s. Based on the definition of *COP* (see Equation 1.16), as the COP increases, the rate of work into the compressor $\left(\dot{W}_{in}\right)$ decreases for a given application. Since for a given application the amount of heat to be removed by the evaporator $(\dot{Q}_e)$ is fixed.

Earlier, in Equation 1.14, the inlet temperature of the air for the evaporator was fixed. Once the application of the cooling system is determined, the temperature set point of the control system dictates the inlet temperature of the air for the evaporator. The last variable to consider in Equation 1.14 is the overall heat transfer coefficient (*U*). Its value is determined by the velocity of the fluids on both sides of the heat exchanger, the fluids themselves, and the dimensions and materials for the heat exchanger channel and fins. In a comprehensive optimization, all of these would be design variables. In order to avoid getting tangled up in the branches, the optimization of these variables will be presented in a future work. Presently assume these design variables are specified to keep both the heat transfer rate and overall heat transfer coefficient constant as area is varied. The constant heat transfer rate assumes the envelope surrounding the conditioned space has already been optimized based on minimum total life cycle costs (see Chapter Two).

$$\overset{\uparrow}{\overline{\boldsymbol{COP}}} = \frac{\boxed{\boldsymbol{\dot{Q}_e}}}{\underset{\downarrow}{\underline{\boldsymbol{\dot{W}_{in}}}}} \tag{1.16}$$

For the conductance form of the equation to accurately quantify the heat transfer rate, assume the mass flow rate of the air $(\dot{m}_a)$, which is typically the fluid on the other side of the evaporator, is high. With a high flow rate, the air side of the evaporator becomes a thermal reservoir. Remember the definition of a thermal reservoir is where you can transfer a finite quantity of heat without affecting a substance's temperature. At a high mass flow rate, the conservation of energy applied to the air side of the evaporator (Equation 1.17) will support this conclusion. As the mass flow rate of air increases the temperature of the air at the outlet (T_{aeo}) approaches that of the inlet (T_{aei}).

$$\boxed{\dot{Q}_e} = \overset{\uparrow}{\dot{m}_a} \boxed{c_p} \left(\boxed{T_{aei}} - \overset{\uparrow}{T_{aeo}} \right) \tag{1.17}$$

Now that you have an overall appreciation for the system effects **[4.d.] return to searching for an equation that will provide the saturation temperature in the condenser** ($\boldsymbol{T_{sat,c}}$). There are two common methods to analyze the performance of heat exchangers; the log mean temperature difference and the effectiveness-NTU method. Since the effectiveness-NTU method provides more robust convergence in numerical solutions, use it in this analysis. Recall the definition of effectiveness is the actual heat transfer rate divided by the maximum possible heat transfer rate which is based on the maximum temperature difference $\left(\Delta T_{max,c}\right)$ in the heat exchanger, in this case, the condensing region of the condenser.

[4.b.i] Use the math law of subtraction to determine this temperature difference as seen in Equation 1.18.

$$\Delta \boldsymbol{T_{max,c}} = \boxed{\boldsymbol{T_{sat,c}}} - \boxed{\boldsymbol{T_{aci}}} \tag{1.18}$$

Where, T_{aci} is the temperature of the air at the condenser inlet. Next **multiply** the minimum heat capacitance fluid $\left(C_{min,c}\right)$ by this maximum temperature difference to find the maximum heat transfer rate $\left(\dot{Q}_{max,c}\right)$.

$$\boldsymbol{\dot{Q}_{max,c}} = \boldsymbol{C_{min,c}} \times \boxed{\Delta \boldsymbol{T_{max,c}}} \tag{1.19}$$

The maximum heat transfer rate $(\dot{Q}_{max,c})$ can be determined from the **definition of heat exchanger effectiveness** for the condensing section of the condenser (ε_c) as seen in Equation 1.20.

$$\boldsymbol{\varepsilon_c} = \frac{\boldsymbol{\dot{Q}_c}}{\boxed{\boldsymbol{\dot{Q}_{max,c}}}} \tag{1.20}$$

For all heat exchangers where the rate of minimum heat capacitance to maximum heat capacitance is zero Equation 1.21 may be used to calculate the effectiveness of the heat exchanger. In condensers and evaporators the maximum heat capacitance is infinity, because the heat capacitance is the product of the specific energy C_p and the mass flow rate and the specific energy is infinite. The definition of specific energy is the change in specific enthalpy divided by the change in temperature. When condensing or evaporating, there is a finite change in specific enthalpy with no change in temperature assuming the pressure is constant in the heat exchanger.

$$\varepsilon_c = 1 - e^{-NTU_c} \tag{1.21}$$

Equation 1.21 is a result of integrating the conductance form of the heat exchanger equation along the length of the heat exchanger channels and is contained in any traditional heat transfer textbook (Holman, 2009). The integration is required for improved accuracy because the temperature difference varies along the length of the flow path when the air flow rate is at a practical level. NTU_c is the number of transfer units for the condenser. Using its definition yields Equation 1.22.

$$NTU_c = \frac{U_c \times A_c}{C_{min,c}} \tag{1.22}$$

This is a place in the analysis where it is easy to get caught up in the branches of the analysis. Hold the overall heat transfer coefficient (U_c) constant, in order minimize complexity and to focus on the system analysis of the vapor-compression cycle. Recall, the product of overall heat transfer coefficient and the condenser heat transfer surface area (A_c) is called the conductance of the heat exchanger. The conductance of the heat exchanger is one over the sum of the thermal resistances from transferring heat from the refrigerant to the air. The derivation of thermal resistances result from Ohm's analogy and the basic constitutive equations; Newton's Law of Cooling and Fourier's Law of Conduction. Refer to any of the traditional heat transfer textbooks (Bergman, 2011; Cengel and Ghajar, 2015; Holman, 2009) for the derivation of the thermal resistances.

At this point you may be asking yourself how Equations 1.18 to 1.21 relate to the conductance form of the heat exchanger equation. To answer this question there are two ways to approach it. One way would be to go back to the differential element of the heat exchanger and integrate the conductance equation along the length of the heat exchanger. This is covered well in existing textbooks and is rather involved. A simpler way to answer this question is to substitute the definition of maximum heat transfer rate, Equation 1.19, into effectiveness, Equation 1.20. Then this result is substituted along with Equation 1.22 for NTUs into Equation 1.21. Solving for the heat transfer rate in the condenser $(\dot{Q}_c)$, results in Equation 1.23.

$$\dot{Q}_c = \left[1 - e^{-\left(\frac{U_c \times A_c}{C_{min,c}}\right)} \right] \times C_{min,c} \times \Delta T_{max,c} \tag{1.23}$$

Equation 1.23 illustrates the relationship, like the conductance form of the heat exchanger equation, between the heat transfer surface area (A_c) and the temperature difference $(\Delta T_{max,c})$ of the fluids on each side of the heat exchanger.

For the area of the condenser you can choose any relevant area used in the calculation of the overall heat transfer coefficient. For convenience, use the sum of the exterior surface area of the channels and fins.

The surface areas of the channels and fins are already specified; therefore the total area of the condenser can be calculated. This in turn can be used in Equation 1.22 to find the number of transfer units. But most of you are saying, wait a minute; I cannot calculate the number of transfer units without the minimum heat capacitance. Here is where you need to trust the deductive problem-solving method. Refer to Figure 1.7. The minimum heat capacitance was circled in Equation 1.19 and you will develop the equation for it when you return Equation 1.19 after finding the maximum heat transfer rate for the condenser. For now you will assume you have that equation and can calculate the minimum heat capacitance.

You have everything you need to determine the effectiveness for the condenser (ε_c) even though you have yet to calculate the minimum heat capacitance ($C_{min,c}$). In the deductive problem-solving strategy you circle a variable requiring an equation only once. The equation for minimum heat capacitance will be provided later in Equation 1.26 when you return to Equation 1.19 to handle the next circled variable.

Once the number of transfer units is known, use Equation 1.21 to solve for the effectiveness of the condenser. This then leads, using step **[4.g]**, to the actual heat transfer rate in the condenser $(\dot{Q}_c)$. To find this heat transfer rate, **apply the conservation of energy to the condenser**. You have not used this law yet on this component, therefore it is an independent equation. Using the rate form of the conservation of energy applied

$$\dot{Q}_{max,c} = C_{min,c} \times \Delta T_{max,c} \quad 1.19$$

$$\varepsilon_c = \dot{Q}_c / \dot{Q}_{max,c} \quad 1.20$$

$$\varepsilon_c = 1 - e^{-NTU_c} \quad 1.21$$

$$NTU_c = \frac{U_c \times A_c}{C_{min,c}} \quad 1.22$$

Figure 1.7. Review of equations 1.19 to 1.22

to the control volume of the refrigerant side of the condenser yields Equation 1.24. In the property plot (Figure 1.4) this equation takes you from a saturated vapor at the entrance of the condenser to state-point three at the exit of the condenser.

$$\dot{m}_r \times h_{g,c} = \dot{Q}_c + \dot{m}_r \times h_3 \tag{1.24}$$

A previous equation, Equation 1.11, provides the mass flow rate. Next **use a property relation** to find the specific enthalpy of a saturated vapor at the inlet to the condensing section $(h_{g,c})$. The two independent intensive properties known at this point are the saturation temperature in the condenser $(T_{sat,c})$ from Equation 1.18 and the vapor mass fraction (x), which is one, since the refrigerant is a saturated vapor.

$$h_{g,c} = f\left(R410a, T_{sat,c}, x = 1\right) \tag{1.25}$$

This then completes all the equations you need to satisfy the unknown variables in Equation 1.24, the conservation of energy for the condenser, which allows you to calculate the heat transfer rate in the condensing section of the condenser. Now Equation 1.20, the definition of effectiveness, has all it needs to determine the maximum possible heat transfer rate in the condensing section of the condenser. The next variable that needs an equation is the minimum heat capacitance $(C_{min,c})$ which is circled in Equation 1.19, the definition of maximum rate of heat transfer. **Use the definition of heat capacitance** which is the product of the specific energy $(C_{p,ac})$ and the mass flow rate $(\dot{m}_{ac})$ for the minimum heat capacitance fluid, which is air in this case. Note: "Specific energy" is used in place of the more common designation of "specific heat" because heat is not a property of a substance.

$$C_{min,c} = C_{p,ac} \times \dot{m}_{ac} \tag{1.26}$$

Assume the water vapor in air does not change phase when passing through the condenser, so treat the fluid as dry air. Dry air is a simple, compressible substance. Therefore, only two independent, intensive properties are needed to define the state. The temperature will be determined by the average climate for the location of the condenser. For more robust convergence evaluate the specific energy (C_p) at the inlet temperature (T_{aci}). For improved accuracy this can be replaced with the average temperature of the air passing through the condenser. The second independent, intensive property is the atmospheric pressure (P_{aci}).

$$\boldsymbol{C_{p,ac}} = \boldsymbol{f}(\boldsymbol{air}, \boxed{\boldsymbol{T_{aci}}}, \boxed{\boldsymbol{P_{aci}}}) \tag{1.27}$$

For this next section, you may find it helpful to refer to Appendix B where the entire set of equations for the basic model is contained. With the minimum heat capacity, the maximum temperature difference can be calculated using Equation 1.19. With the maximum temperature difference, the saturation temperature in the condenser can be calculated using Equation 1.18. The saturation temperature allows you to calculate the specific enthalpy at state-point three.

With the specific enthalpy at state-point three, you know the specific enthalpy at state-point four based on the conservation of energy applied to the expansion valve shown in Equation 1.12. The specific enthalpy at four, allows you to determine the mass flow rate of refrigerant using the conservation of energy applied to the evaporator in Equation 1.11. Wait, what about the specific enthalpy at state-point one? An equation has not been developed to determine this quantity yet. This is a valid point and you need to make sure you have been applying the deductive problem-solving strategy and each variable requiring an equation is circled once and only once. The specific enthalpy at state-point one was circled in Equation 1.10 when the conservation of energy was applied to the ideal compressor. Therefore, since an equation will be developed for the specific enthalpy at state-point one once you reach that circled variable, you will assume you have its value for any subsequent equations.

Now that you have the mass flow rate of refrigerant using the conservation of energy applied to the evaporator shown in Equation 1.11 you are ready to return to Equation 1.10. The mass flow rate was the first of three variables requiring an equation when the conservation of energy was applied to the ideal compressor. The next variable which requires an equation is your beloved specific enthalpy at state-point one (h_1), which you were so concerned about just one paragraph ago. You realize you have used all the possible applications of the conservation of energy where the specific enthalpy at state-point one could be determined, namely; the evaporator, and the ideal compressor. The conservation of energy for the actual compressor is reserved to find the actual enthalpy at the compressor outlet. You may be tempted to use the definition of enthalpy ($h = u + Pv$) at this point but that would defeat the purpose of why the property enthalpy was developed in the first place. Therefore, **use a property relationship** since one has not yet been applied to determine this specific enthalpy. ***CAUTION:*** Keep in mind, only one property relationship can be used for each independent property, at each state point, to maintain independent equations. Otherwise, a dependent equation would be formed if the same three variables were simply rearranged. For example *h = f(T,x) and T = f(h,x)* use the same equations and therefore are not independent.

$$\boldsymbol{h_1} = \boldsymbol{f}(\boldsymbol{R}410\boldsymbol{a}, \boldsymbol{T_{sat,e}}, \boldsymbol{x} = 1) \tag{1.28}$$

In order to keep this first system analysis simple, assume saturated vapor exits the evaporator. A practical example of this occurs in systems where the expansion device is an orifice or capillary tube in combination with an accumulator. Therefore, the temperature at state-point one is the saturation temperature in the evaporator. This saturation temperature, like that in the condenser, is primarily a function of the size of the evaporator in terms of heat transfer surface area.

Also, to keep this first system analysis simple, assume the temperature of the exterior surface of the evaporator is above the dew point of air. Therefore, the mass of water vapor in the air remains constant throughout the cooling process. This limits the applicability of the model but it will greatly simplify the model. Since sensible heat transfer is the only type occurring you can use the effectiveness-NTU method to model the evaporator. In the next section of this chapter you will account for the latent heat transfer. Assuming the pressure is constant along the direction of refrigerant flow the saturation temperature is also constant as shown in the property plot in Figure 1.4. Therefore, use the maximum temperature difference $(\Delta T_{max,e})$ to find the saturation temperature of the refrigerant $(T_{sat,e})$ in the evaporator.

$$\boldsymbol{\Delta T_{max,e}} = \boldsymbol{T_{sat,e}} - \boldsymbol{T_{aei}} \tag{1.29}$$

To find the maximum temperature difference, apply the definition of maximum heat transfer rate $(\dot{Q}_{max,e})$ to the evaporator, as shown in Equation 1.30.

$$\boldsymbol{\dot{Q}_{max,e}} = \boldsymbol{C_{min,e}} \times \boldsymbol{\Delta T_{max,e}} \tag{1.30}$$

Next, use the definition of effectiveness (ε_e) to find the maximum heat transfer rate.

$$\varepsilon_e = \frac{\dot{Q}_e}{\dot{Q}_{max,e}} \tag{1.31}$$

To find the effectiveness, the conductance form of the heat transfer rate is integrated along the refrigerant flow path to yield Equation 1.32.

$$\varepsilon_e = 1 - e^{-NTU_e} \tag{1.32}$$

The number of transfer units (NTU_e) by definition is the overall conductance of the heat exchanger divided by the minimum heat capacitance fluid $(C_{min,e})$.

$$NTU_e = \frac{U_e \times A_e}{C_{min,e}} \tag{1.33}$$

The overall conductance is the product of the overall heat transfer coefficient and the relevant heat transfer surface area $(U_e \times A_e)$; the exterior surface of the channels and fins. Just as with the condenser the overall heat transfer coefficient is assumed to be fixed. With the NTUs you can find the effectiveness and the maximum heat transfer rate.

Next the minimum heat capacitance $(C_{min,e})$ is needed before the maximum temperature difference is calculated. By definition it is the product of the mass flow rate $(\dot{m}_{ae})$ and the specific energy for the fluid with the minimum heat capacitance. Since the specific energy (C_p) of the refrigerant is infinity as the refrigerant passes from the inlet to the outlet of the evaporator, the air becomes the fluid with the minimum heat capacitance.

$$C_{min,e} = C_{p,aei} \times \dot{m}_{ae} \tag{1.34}$$

Determine the specific energy of the air entering the evaporator $(C_{p,aei})$ with a property relationship. Since you are assuming the water vapor does not change state from inlet to outlet in the direction of air flow, air can be treated as a simple, compressible substance. Therefore, based on the state postulate only two independent, intensive properties are required to define the state and determine any other thermodynamic property. Here, use temperature (T_{aei}) and pressure (P_{aei}).

$$C_{p,aei} = f(air, T_{aei}, P_{aei}) \tag{1.35}$$

This allows you to find the minimum heat capacitance which was needed to find the maximum temperature difference in the evaporator. With the temperature difference you are able to use Equation 1.29 to determine the saturation temperature which is the same as the temperature at the outlet of the evaporator. This gives you the information needed to find the specific enthalpy at state-point one using Equation 1.28. This specific enthalpy was circled in Equation 1.10 where the conservation of energy was applied to the ideal compressor. You may find it helpful to refer back to Appendix B to follow this progression of the deductive strategy.

The next variable which requires an equation is the specific enthalpy at state-point two assuming an ideal compressor $(h_{s,2})$. Use a property relationship (Equation 1.36) to find the specific enthalpy leaving the ideal compressor. Here it is useful to refer back to the property plot in Figure 1.4 to select the two independent, intensive properties. The first property is the pressure (P_2) since the exiting pressure is the same between the real and ideal compressors. The second comes from the fact that the compressor is an ideal compressor. An ideal compressor is reversible and adiabatic and therefore isentropic. Since you know the thermodynamic state at state-point one, the inlet to the compressor, you can find the specific entropy at state-point one. Since the process is isentropic that means the specific entropy at state-point two is known as well. Therefore, the second independent, intensive property for determining the specific enthalpy is the specific entropy $(s_{s,2})$.

$$\boldsymbol{h_{s,2}} = \boldsymbol{f}(\boldsymbol{R}410\boldsymbol{a}, \boldsymbol{P_2}, \boldsymbol{s_{s,2}}) \tag{1.36}$$

The pressure at state-point two (P_2) is the same as the pressure at state-point three (P_3), which is the saturation pressure in the condenser $(P_{sat,c})$, since you are assuming constant pressure heat rejection in the condenser **[4.b.v.]**. The saturation pressure can be found using a property relationship as a function of the saturation temperature $(T_{sat,c})$.

$$\boldsymbol{P_2} = \boldsymbol{P_{sat,c}}\left(\boldsymbol{R}410\boldsymbol{a}, \boldsymbol{T_{sat,c}}\right) \tag{1.37}$$

As discussed previously, the specific entropy at state-point two for the ideal compressor $(s_{s,2})$ is equal to the specific entropy at state-point one (s_1) since the ideal compression process is isentropic **[4.b.v.]**.

$$\boldsymbol{s_{s,2}} = \boldsymbol{s_1} \tag{1.38}$$

To find the specific entropy at state-point one (s_1) Equation 1.39 uses a property relationship **[4.b.iii.]** based on the two independent, intensive properties known at state-point one, namely, temperature and quality.

$$s_1 = f(R410a, T_{sat,e}, x = 1) \tag{1.39}$$

You may find it helpful to refer to Appendix B to follow the next chain of events. With the pressure and specific entropy at ideal state-point two you are able to use a property relationship to find the ideal specific enthalpy. Now you have everything you need to calculate the ideal rate of work into the compressor using Equation 1.10. Next, calculate the actual rate of work into the compressor using the definition of isentropic efficiency given in Equation 1.6. The deductive strategy takes you back to Equation 1.5 where you calculate the initial cost of the compressor. Once that is calculated you are able to finish calculating the initial cost of the system using Equation 1.2.

[4.g.] Return to Equation 1.1 and identify the next circled variable, in this case, the present worth of the operating cost.

To better understand the relationship between initial cost and operating costs recall a useful schematic from microeconomics; the cash flow diagram as shown in Figure 1.8.

The annual operating costs are assumed to be uniformly dispensed throughout the lifetime (n) of the vapor-compression cycle. Engineering economics are used to determine the equivalence of the uniform series in today's dollars or the present worth of the operating costs.

From the cash flow diagram, you observe that the energy costs associated with the cooling system occur in the future. Conservatively, it is assumed these costs are uniform over the life of the system. In actuality, these costs are most likely to increase due to fuel price escalation, heat exchanger fouling and compressor deterioration. **[4.b.i] Determine the present worth of a uniform series, in this case, the annual operating costs (PW_{oc}), by multiplying the annual operating costs (OC) by the series present worth factor (PA).**

$$PW_{oc} = OC \times PA \tag{1.40}$$

[4.b.i.] Multiply the annual energy consumption with the cost per unit of energy $(Cost_{elec})$, in this case electricity, to determine the annual operating costs (OC).

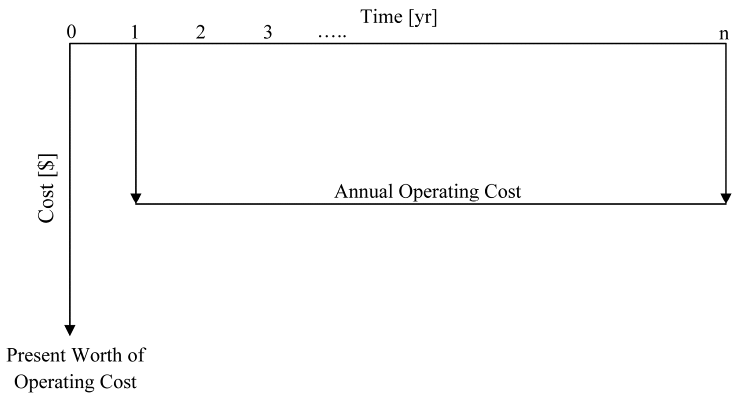

Figure 1.8. Cash flow diagram for a uniform series

$$OC = AE \times C_{ue} \tag{1.41}$$

Calculate the annual energy consumption by multiplying the power into the compressor $(\dot{W}_{in})$ with its operating time (OT).

$$AE = \dot{W}_{in} \times OT \tag{1.42}$$

Determine the operating time (OT) of the compressor from the annual cooling load (*CL*) and the definition [4.b.ii.] of heat transfer rate $(\dot{Q}_e)$ at the average operating conditions specified in the analysis. Equation 1.43 assumes you have already determined the optimum envelope for the conditioned space. Optimizing the envelope is the topic of Chapter Two in this work. Energy simulation, either an hour by hour or bin analysis, may be used to determine the annual cooling load of the optimized envelope.

$$\dot{Q}_e = \frac{CL}{OT} \tag{1.43}$$

With the operating time, the annual operating costs can be calculated which takes you back to Equation 1.40 for the present worth of the operating costs. The next variable which needs an equation is the discount factor for a uniform series or series present worth factor (PA). Equation 1.44 comes from any microeconomics textbook (Blank and Tarquin, 2013) for a uniform series of expenditures which need to be brought to its present worth, where (*marr*) is the client's minimum acceptable rate of return.

$$PA = \frac{\left(1+marr\right)^{n}-1}{marr\times\left(1+marr\right)^{n}} \tag{1.44}$$

You now have a complete set of equations **[4.h.]** to determine the total life cycle costs for a simple direct-expansion cooling system for a dry evaporator. To do this, it required 36 equations for the 36 variables with 17 inputs requiring specification and 3 design variables. The data required for specification does not require any measurements of variables within the refrigerant loop. Additionally, the values for the specified variables can be determined prior to fabricating the actual cooling system.

Since you have a set of independent algebraic equations, an equation solver can be used to determine the solution to this set of equations. You have developed this set of equations assuming the heat transfer surface areas are specified. There are two practical drawbacks to specifying the areas: first, 19 of the 36 equations will need to be solved simultaneously and second there are significant difficulties with convergence. To overcome both these issues, specify the saturation temperatures of the refrigerant in the condenser and evaporator as the independent design variable in place of the areas. This will minimize the effects of both of the previously mentioned difficulties.

Fortunately, specifying saturation temperatures does not present an insurmountable obstacle to the solution of the set of equations. As you have seen there is an exclusive relationship between the area of the heat exchangers and the corresponding saturation temperature of the refrigerant. So if the saturation temperature of the refrigerant is specified, there is only one value of heat transfer surface area corresponding to that temperature. Therefore, when a saturation temperature is specified, you are essentially specifying the design variable, heat transfer surface area.

The current set of equations is useful for accurately modeling a direct-expansion air conditioning cycle with sensible cooling only. However, when you assume the air flow rates are high a simpler set of equations emerge which allow you to better perceive the system effects of the design variables, especially the area of the heat exchangers. This simplest system model was developed by Zietlow (2014) is presented in Appendix C. It contains only 23 equations with 13 specified variables along with the three design variables. The high air flow rates allow you to replace the effectiveness-NTU heat exchanger analysis with the conductance form of the heat transfer equation.

Now that the total life cycle cost can be determined for any set of isentropic efficiencies, and heat transfer surface areas for the evaporator and condenser the next step is to find the optimum. There are several multivariate optimization algorithms that will work for this set of equations and it is beyond the scope of this text to cover the details of these algorithms. Basically, the optimization algorithm will start with an initial guess for the independent (design) variables, calculate the value for the objective function (total cost), then generate new guesses for the design variables in an effort to lower the total cost. It will continue to iterate until a minimum is found.

The values for the specified variables for a base case for cooling a residence are given in Table 1.2. Ideally, the most likely values for these variables at the average operating condition over a year are selected for the base case. The sensitivity analysis will then show which of these variables require the most attention in regards to accuracy.

For the base case the optimum values for the design variables are 4.5 m^2 for the evaporator, 6.6 m^2 for the condenser and an isentropic efficiency of 79.5 percent for the compressor. This yields a total cost of $3,070 in today's dollars.

Table 1.2. Specified variables for air conditioning system analysis at base case conditions

Variable Name	Value	Units	Source
CC_{comp}	10	$	Compressor Supplier Quotes
C_{ue}	0.1	$/kW-hr	Energy Supplier Rates
C_{uac}	80	$/m^2	Condenser Supplier Quotes
C_{uae}	100	$/m^2	Evaporator Supplier Quotes
CL	20,000	kW-hr	Based on Annual Cooling Load
e	2	-	Compressor Supplier Quotes
$marr$	0.2	-	Client or Owner
$\dot{m}_{ac}$	2.0	kg/s	Mass flow rate measurement
$\dot{m}_{ae}$	1.0	kg/s	Mass flow rate measurement
n	15	yrs	ASHRAE Handbook
P_{aci}	100.0	kPa	Air Pressure Reading at Inlet
P_{aei}	100.0	kPa	Air Pressure Reading at Inlet

(*Continued*)

Variable Name	Value	Units	Source
$\dot{Q}_e$	10	kW	Annual Energy Analysis
T_{aci}	30	°C	Thermocouple in Inlet Airstream
T_{aei}	20	°C	Thermocouple in Inlet Airstream
U_c	0.20	kW/m^2-°C	Thermal Resistance Analysis
U_e	0.30	kW/m^2-°C	Thermal Resistance Analysis

One of the challenges for engineers is the uncertainty of all the specified variables given for the base case in Table 1.2. For example, no one knows what is going to happen to energy prices or weather in the future. There is also uncertainty in the actual thermal performance of the building envelope. One way to handle these uncertainties is to perform a sensitivity analysis. In order to compare the importance of each variable it is useful to normalize the independent variable by calculating the percent change from the base case using Equation 1.45.

$$\%\boldsymbol{variable} = \frac{(\boldsymbol{variable} - \boldsymbol{variable}_b)}{\boldsymbol{variable}_b} \times 100\% \tag{1.45}$$

Vary each of the independent variables, over the range from 50 percent below their base case values to 40 percent above. For each value of the independent variable, determine the minimum total life cycle cost. This parametric analysis provides a relationship showing how the independent variable affects the dependent variable. Since the independent variables are normalized they can be presented on the same plot. The top six variables producing the greatest slope on this plot (Figure 1.9) are the ones which require extra effort to secure accurate information. Therefore, for this example you will want to make sure the temperature of the air at the inlet to the condenser, the cost per unit of electricity, the cooling load, the minimal acceptable rate of return, the heat transfer rate to the evaporator $\left(\dot{Q}_e\right)$ and the temperature of the air at the evaporator inlet are accurate because they have the greatest influence on the result. The remaining input variables located in Table 1.2 have less influence and therefore are not included on the plot for sake of clarity.

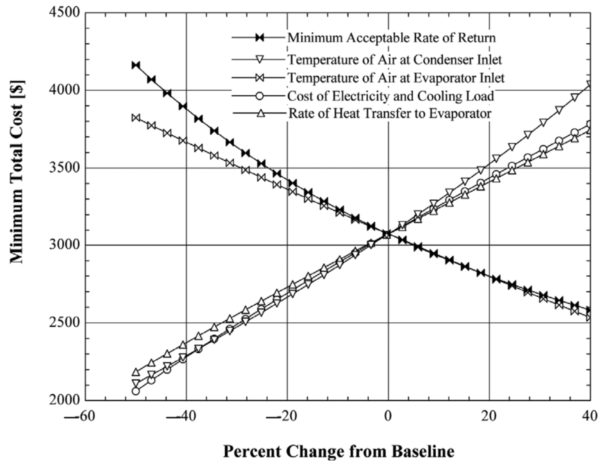

Figure 1.9. Sensitivity analysis. Plot based on output from a basic model (effectiveness-NTU) of a direct-expansion air conditioning system assuming a dry evaporator. The base case was determined from the values of the variables listed in Table 1.2. (EES, 2014)

ACCOUNTING FOR CONDENSATION OF WATER VAPOR

With this introduction to modeling the air conditioning cycle you can accurately determine the optimum heat transfer surface areas of the heat exchangers and the isentropic efficiency of the compressor. The most limiting assumption, from a practical standpoint, for the basic system model is no water vapor has condensed from the air stream onto the outside surface of the evaporator. For most applications the latent load is between 5 and 35 percent of the total load. This section will present a method to account for this latent load on the evaporator. You need to be warned however you will need to venture into the woods. You will risk entanglement in the branches as you properly account for the latent load. First, go back to the basic system model of a dry evaporator starting with Equation 1.31 or the definition of evaporator effectiveness.

When water vapor condenses, there are two different mechanisms for transferring heat. The first is called sensible heat transfer rate $\left(\dot{Q}_{e,s}\right)$ where the temperature is the driving potential. This is the heat transfer rate that was calculated in the basic system model. The definition of effectiveness $\left(\varepsilon_e\right)$ is then used to find the maximum sensible heat transfer rate $(\dot{Q}_{max,e})$.

$$\varepsilon_e = \frac{\dot{Q}_{e,s}}{\dot{Q}_{max,e}} \tag{1.46}$$

Please note that Equation 1.46 only calculates the sensible portion of the heat transferred in the evaporator therefore you need to circle the heat transfer rate and find an equation to relate it to the total heat transfer rate. But first you need to find the effectiveness. This is the same as with the dry coil where the conductance form of the heat transfer rate is integrated along the refrigerant flow path to yield Equation 1.47 which is the same as Equation 1.32.

$$\varepsilon_e = 1 - e^{-NTU_e} \tag{1.47}$$

The number of transfer units (NTU_e) by definition is the overall conductance of the heat exchanger divided by the minimum heat capacitance fluid as shown in Equation 1.48 which is the same as Equation 1.33 but is repeated here for continuity.

$$NTU_e = \frac{U_e \times A_e}{C_{min,e}} \tag{1.48}$$

The overall conductance is the product of the overall heat transfer coefficient and the relevant heat transfer surface area, the exterior surface of the channels and fins. The overall heat transfer coefficient is assumed to be fixed. For the wet evaporator though the overall heat transfer coefficient needs to account for the effect of the liquid water which has now condensed on the surface. The effect of water will be covered in the next section. The minimum heat capacitance was circled in Equation 1.30 and will be determined at the appropriate place in the analysis. With the NTUs you can find the effectiveness.

Next, the analysis deviates significantly from the dry coil analysis. First, find the relationship between the sensible heat transfer rate and the total heat transfer rate. This comes from the fact; heat is transferred when energy is removed from some of the water vapor in the air stream causing it to condense. This is called latent heat transfer and the physical mechanism differs significantly from the sensible heat transfer since the temperature does not change during the condensation process. Equation 1.49 shows the total heat transfer rate ($\dot{Q}_e$) is the sum of the latent ($\dot{Q}_{e,l}$) and sensible heat transfer rates.

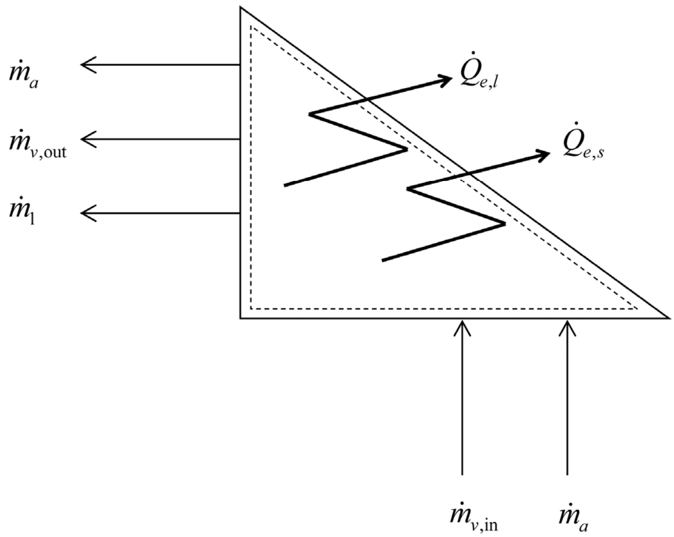

Figure 1.10. Control volume for a wet evaporator—air side only. Both the sensible $(\dot{Q}_{e,s})$ and latent $(\dot{Q}_{e,l})$ rates of cooling are represented as the moist (dry air $\dot{m}_a$ plus water vapor $\dot{m}_{v,\text{in}}$) air entering the evaporator loses water vapor to condensation. The condensation results in liquid water $(\dot{m}_l)$ leaving the control volume.

$$\dot{Q}_e = \dot{Q}_{e,s} + \dot{Q}_{e,l} \tag{1.49}$$

The total heat transfer rate is specified so you need to determine the latent heat transfer rate next using the conservation of energy for the condensing water vapor as it travels through the evaporator as given in Equation 1.50. Figure 1.10 shows the new control volume for the air side of the evaporator including the water vapor and liquid.

$$\dot{m}_l \times h_{g,v} = \dot{Q}_{e,l} + \dot{m}_l \times h_{f,l} \tag{1.50}$$

The driving potential for the latent heat transfer is the difference between the humidity ratio of air at the inlet (ω_{aei}) and the humidity ratio at saturated conditions (ω_{sat}) evaluated at the saturation temperature of the refrigerant. Since mass transfer is analogous to convective heat transfer substitute the mass transfer rate $(\dot{m}_l)$ for the heat transfer rate, the convective mass transfer coefficient (λ_l) for the convective heat transfer coefficient and the difference in humidity ratio for the temperature difference in Newton's law of cooling to obtain Equation 1.51.

$$\dot{m}_l = \lambda_l A_e \rho_{aei} (\omega_{aei} - \omega_{sat}) \tag{1.51}$$

The mass transfer coefficient can be determined using an empirical correlation similar to the convective heat transfer coefficient where the thermal conductivity is replaced with the mass diffusivity (Di_{ae}), and the Prandtl number is replaced by the Schmidt number (Sc_{ae}). The characteristic length is the hydraulic diameter of the spacing between the fins, which are assumed to be infinite plates (2 x Sp_{efi}). This yields Equation 1.52.

$$\lambda_l = \left(\frac{Di_{ae}}{2 \times Sp_{efi}} \right) C_{mt_l} (Re_{ae})^{e_{Re_{mt_l}}} (Sc_{ae})^{e_{Sc_{mt_l}}} \tag{1.52}$$

The mass diffusivity (Di_{ae}) is determined from a property relationship for a mixture of air and water vapor as shown in Equation 1.53 evaluated at its inlet state entering the evaporator (T_{aei}). Once convergence is obtained the mean temperature of the air can be determined and used in place of the inlet temperature for improved accuracy. The mass diffusivity can be obtained from the *Journal of Physical Chemistry* (Marrero, 1972).

$$\ln(Di_{ae}) = \ln(A) + S \times \ln(T_{aei}) \tag{1.53}$$

Where, A = 0.00000187 and S = 2.072 for gaseous air and water for temperature between 287 K and 450 K. Equation 1.53 yields a mass diffusivity in cm/s when given a temperature in Kelvin.

The spacing of the evaporator fin is determined by Equation 1.54.

$$Sp_{efi} = \frac{1}{\rho_{efi}} - th_{efi} \tag{1.54}$$

The Reynolds number (Re_{ae}), which is the dimensionless ratio of the momentum forces to the viscous forces, is given in Equation 1.55. It is the product of the density of air (ρ_{aei}), velocity (V_{ae}) and characteristic length divided by the dynamic viscosity of the air (μ_{aei}). Evaluate the fluid properties at the inlet state of the air entering the evaporator.

$$Re_{ae} = \frac{\rho_{aei} \times 2 \times Sp_{efi} \times V_{ae}}{\mu_{aei}} \tag{1.55}$$

Use the definition of volumetric flow rate $(\dot{V}_{aei})$ in Equation 1.56 to determine the velocity (V_{ae}). The free flow area (A_{ffe}), needs to account for the obstruction from the liquid water which has condensed onto the surfaces of the evaporator.

$$\dot{V}_{aei} = V_{ae} \times A_{ffe} \tag{1.56}$$

In the basic model, it was assumed the mass flow rate was measured. So to convert mass flow to volumetric flow rate use the definition of density as shown in Equation 1.57. Since it is less expensive to measure volumetric flow rate, the definition for density would typically be used the other way around, to convert the volumetric flow rate to mass flow rate.

$$\rho_{aei} = \frac{\dot{m}_{ae}}{\dot{V}_{aei}} \tag{1.57}$$

It is important to note, all the measurements for finding the density need to be made at the same location in the flow stream where the volumetric flow rate is required, in this case the inlet to the evaporator. The easiest way to calculate the free flow area (A_{ffe}) for the wet evaporator as shown in Equation 1.58, is to subtract three cross-sectional areas normal to the air flow from the face area (A_{ef}):

1. Refrigerant channels or tubes (A_{ec}),
2. Fins (A_{efi}), and
3. Liquid water on the surfaces (A_l).

$$A_{ffe} = A_{ef} - A_{ec} - A_{efi} - A_l \tag{1.58}$$

The evaporator face area (A_{ef}) is found using Equation 1.59 by multiplying the height of the evaporator (H_e) by its width (Wi_e) as shown in Figure 1.11. The end tanks or manifolds and return bends have been omitted from Figure 1.11 since they are not typically exposed to the air flow.

$$A_{ef} = H_e \times Wi_e \tag{1.59}$$

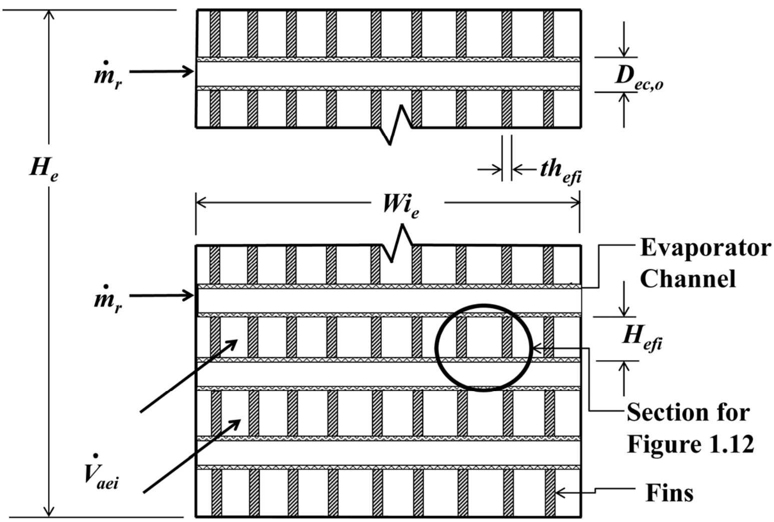

Figure 1.11. Section view of a cross-flow evaporator normal to air flow

View of a column of evaporator channels and associated fins where the cutting plane slices through the center of the channels. The overall height (H_e) is indicated as well as the overall width (Wi_e) of the active heat transfer surface. The mass flow rate of refrigerant $(\dot{m}_r)$ is in the horizontal direction while the volumetric flow of air $(\dot{V}_{aei})$ is perpendicular to it, also in the horizontal direction into the plane of the section view. The channels are characterized by their outside diameter ($D_{ec,o}$) while the fins by their height (H_{efi}) and thickness (th_{efi}).

The cross-sectional area of the refrigerant channels (A_{ec}) is found in Equation 1.60 by multiplying the outside diameter (or height for noncircular channels) of an individual channel $(D_{ec,o})$ by the width of the evaporator (Wi_e), assuming the channels are oriented in the horizontal direction, and then by the number of rows of evaporator channels (N_{ecr}) in the same vertical plane. This assumes for multiple rows that the refrigerant channels are aligned with one another in the direction of flow. If they are in a staggered arrangement then the minimum free flow area may occur in the diagonal between channels of adjacent rows. Most standard heat transfer textbooks show how to account for the minimum flow area for staggered channels.

$$A_{ec} = (D_{ec,o} \times Wi_e) \times N_{ecr} \tag{1.60}$$

Equation 1.61 is used to determine the cross-sectional area of the fins normal to the air flow (A_{efi}) by multiplying the height of the fin between the refrigerant channels (H_{efi}) by the thickness of the fin (th_{efi}) by the fin density (ρ_{efi}) by the width of the evaporator (Wi_e) by the number of rows of channels (N_{ecr}) plus one. The one is added since there typically is one more row of fins than refrigerant channels so all the channels have fins on both sides.

$$\boldsymbol{A_{efi}} = (\boldsymbol{H_{efi}} \times \boldsymbol{th_{efi}}) \times \boldsymbol{\rho_{efi}} \times \boldsymbol{Wi_e} \times (\boldsymbol{N_{ecr}} + 1) \tag{1.61}$$

The final area (A_l) that needs to be subtracted from the face area to determine the free flow area is based on the thickness of the liquid water on the fins as shown in Equation 1.62. This is similar to the equation for fin area with one exception. The fin thickness is replaced by two times the liquid film thickness since there will be liquid water on both sides of the fin.

$$\boldsymbol{A_l} = \boldsymbol{H_{efi}} \times 2 \times \boldsymbol{th_l} \times \boldsymbol{\rho_{efi}} \times \boldsymbol{Wi_e} \times (\boldsymbol{N_{ecr}} + 1) \tag{1.62}$$

There are several forces which affect the thickness of the liquid film as shown in Figure 1.12. In the vertical direction surface tension forces tend to increase the thickness of the water film while gravitational forces decrease the thickness of the film.

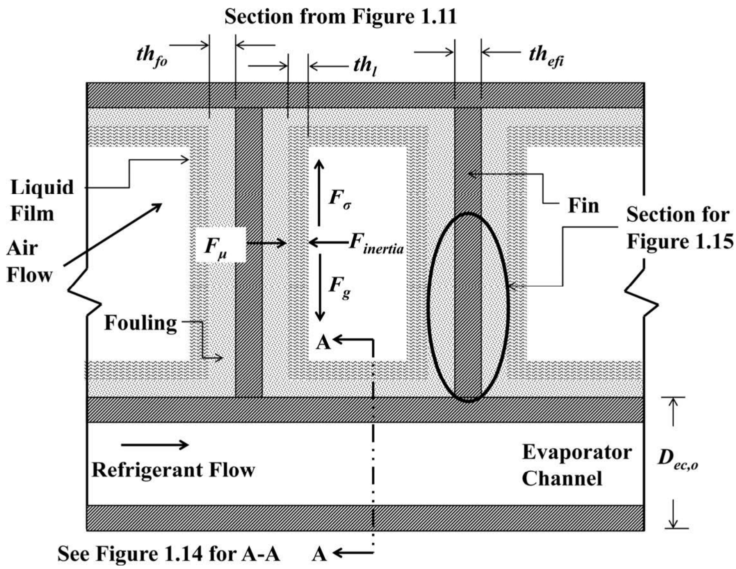

Figure 1.12. Relevant forces affecting the thickness of liquid water on the air side of cross-flow evaporators

Figure 1.12 illustrates the four forces [inertial ($F_{inertial}$), viscous (F_{μ}), surface tension (F_{σ}), and gravity (F_g)] which influence the thickness of the liquid film (th_l). Characteristics of the fin (th_{efi}) and channel ($D_{ec,o}$) along with fouling (th_{fo}) of the surface are also displayed.

The dimensionless Bond number (Bo_l) can be used to account for the trade-off between surface tension and gravitational forces. The Bond number is defined as the ratio of gravitational to surface tension forces. In the flow direction, the inertial forces of the air traveling between the fins will decrease the thickness of the film while the viscosity of the water will tend to increase the thickness of the film. The dimensionless Reynolds number (Re_l) is used to account for trade-off between inertial and viscous forces. Combining these dimensionless numbers into an empirical relationship yields Equation 1.63 where the forces that increase the film thickness (surface tension and viscosity) are in the numerator and the forces that decrease the film thickness (gravity and inertia) are in the denominator. Therefore the dimensionless numbers need to be in the denominator. Note: For the sake of clarity the boxes around the exponents were omitted.

$$th_l = \frac{C_{th_l}}{Bo_l^{e_{Bo_{th_l}}} \times Re_l^{e_{Re_{th_l}}}} \tag{1.63}$$

Determine the constant (C_{th_l}) and exponents $(e_{Bo_{th_l}})$ and $(e_{Re_{th_l}})$ from experimental data. The definition of Bond number is used in Equation 1.64. It is the density of the water (ρ_l) multiplied by the acceleration due to gravity (g) multiplied by the height of the fin (H_{efi}) squared divided by the surface tension of water (σ_l) and the constant of proportionality from Newton's second law (g_c).

$$Bo_l = \frac{\rho_l \times g \times H_{efi}^2}{\sigma_l \times g_c} \tag{1.64}$$

Evaluate the density (ρ_l) and the surface tension (σ_l) of liquid water at the saturation temperature of the refrigerant $(T_{sat,e})$ in Equations 1.65 and 1.66, respectively.

$$\rho_l = f(water, T_{sat,e}, x = 0) \tag{1.65}$$

$$\sigma_l = f(water, T_{sat,e}, x = 0) \tag{1.66}$$

The Reynold's number (Re_l) needs the inertial properties of air and the viscous properties of liquid water as shown in Equation 1.67. The reason for this it is the inertia of the air which thins the liquid but it is the viscosity of the liquid that resists the thinning.

$$Re_l = \frac{\rho_{aei} \times 2 \times Sp_{efi} \times V_{ae}}{\mu_l} \tag{1.67}$$

The inertial properties of air were determined previously while the dynamic viscosity of water (μ_l) is evaluated at the saturation temperature of refrigerant as given in Equation 1.68.

$$\mu_l = f(water,\ T_{sat,e}, x = 0) \tag{1.68}$$

Now the thickness of the liquid can be calculated using Equation 1.63, which in turn allows for the free flow area of the evaporator to be calculated, using Equation 1.58, which in turn yields the velocity of air traveling between the fins in Equation 1.56. Now in order to calculate the Reynolds number in Equation 1.55 the next variable you need is the dynamic viscosity of air (μ_{aei}) which is determined using a property relationship. It is evaluated at the inlet temperature, pressure and relative humidity of the air as shown in Equation 1.69.

$$\mu_{aei} = f(T_{aei}, P_{aei}, RH_{aei}) \tag{1.69}$$

With the calculation of Reynolds number you are led back to empirical correlation for the convective mass transfer coefficient or Equation 1.52 where you need the Schmidt number (Sc_{ae})

$$Sc_{ae} = \frac{\nu_{aei}}{Di_{ae}} \tag{1.70}$$

The momentum diffusivity (ν_{aei}) is the ratio of dynamic viscosity (μ_{aei}) to density for the liquid water (ρ_{aei}) as shown in Equation 1.71.

$$\nu_{aei} = \frac{\mu_{aei}}{\rho_{aei}} \tag{1.71}$$

Now you have everything you need to calculate the mass transfer coefficient given in Equation 1.52. Next, Equation 1.51 shows you will need to calculate the density of air at the evaporator inlet.

For the basic system model you assumed the air is dry. Here you account for the changes in water vapor as the air as it passes through the evaporator. Therefore, you need an additional property, specifically relative humidity, to define the state. As a result, the density is a function of temperature (T_{aei}), pressure (P_{aei}) and relative humidity (RH_{aei}) as shown Equation 1.72.

$$\boldsymbol{\rho_{aei}} = \boldsymbol{f}(\boldsymbol{T_{aei}}, \boldsymbol{P_{aei}}, \boldsymbol{RH_{aei}}) \tag{1.72}$$

The next unknown is the humidity ratio of air entering the evaporator (ω_{aei}). This is determined using the properties of moist water in Equation 1.73.

$$\boldsymbol{\omega_{aei}} = \boldsymbol{f}(\boldsymbol{air_{H_2O}}, \boldsymbol{T_{aei}}, \boldsymbol{P_{aei}}, \boldsymbol{RH_{aei}}) \tag{1.73}$$

Again going back to Equation 1.51, you see you need to determine the humidity ratio of air at saturation conditions (ω_{sat}). You will do this using a property relationship for moist air evaluated at the saturation temperature of the refrigerant in the evaporator as shown in Equation 1.74.

$$\boldsymbol{\omega_{sat}} = \boldsymbol{f}(\boldsymbol{air_{H_2O}}, \boldsymbol{T_{sat,e}}, \boldsymbol{P_{aei}}, \boldsymbol{RH} = 1.0) \tag{1.74}$$

Typically in mass transfer, the concentration gradient is used as the driving potential. The next two equations show how the humidity ratio provides this information starting with its definition as shown in Equation 1.75.

$$\boldsymbol{\omega} = \frac{\boldsymbol{m_v}}{\boldsymbol{m_a}} \tag{1.75}$$

Where, (m_v) is the mass of water vapor and (m_a) is the mass of dry air. Assume an ideal gas and know the sum of the partial pressures of dry air and water vapor equal the total pressure. Derive the relationship shown in Equation 1.76 from the definition of humidity ratio given in Equation 1.75. This derivation is presented in most thermodynamic textbooks. Finally, you know that for an ideal gas the partial pressure is directly proportional to the concentration of a gas, in this case water vapor. Since Equation 1.76 shows that the humidity ratio is a function of the partial pressure of water vapor in dry air then it shows that the humidity ratio is also a function of concentration.

$$\omega = 0.622\left(\frac{P_v}{P - P_v}\right) \tag{1.76}$$

Figure 1.13 demonstrates how the difference in humidity ratio drives the mass transfer of water vapor condensing on the cold surface.

The driving potential (ω_{aei}-ω_{sat}) for mass transfer is illustrated on this property plot for moist air. Warm moist air with a dry bulb temperature of 35°C and relative humidity of 37 percent comes in contact with a surface whose temperature is 0.2°C.

With Equation 1.51 satisfied you are ready to return back to the conservation of energy for the condensing water given in Equation 1.50. The saturation enthalpies for vapor $(h_{g,v})$ and liquid $(h_{f,l})$ need to be determined in Equations 1.77 and 1.78 using property relations for water evaluated at the inlet temperature of air and saturation temperature of refrigerant, respectively.

$$h_{g,v} = f(water, T_{aei}, x = 1) \tag{1.77}$$

$$h_{f,l} = f(water, T_{sat,e}, x = 0) \tag{1.78}$$

Now that you have the latent rate of heat transfer, you are able to pick up where you left off in the basic system model by finding the minimum heat capacitance using Equation 1.30 so the maximum temperature difference can be found.

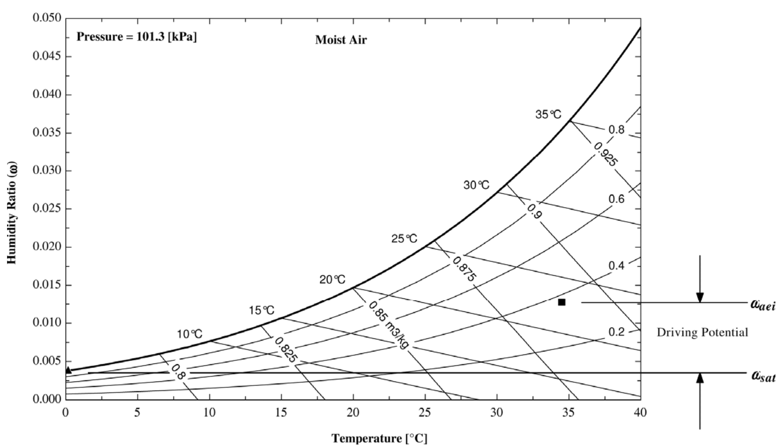

Figure 1.13. Psychrometric chart-driving potential for condensation of water vapor (EES, 2014)

To upgrade the basic, dry evaporator model with the new wet evaporator model, replace Equations 1.31 through 1.33 with Equations 1.46 through 1.78. Now you have a more complex system but a much more practical model since you are able to account for the water vapor that condenses on the cold surfaces of the evaporator. Perform a sensitivity analysis on this new system model to determine how the variables related to the condensation of water influence the optimum.

DETERMINATION OF THE OVERALL HEAT TRANSFER COEFFICIENT (*U*)

Both the simple and basic models allow you to determine the optimum heat transfer surface area. To keep these system models as simple as possible the overall heat transfer coefficient was assumed to be fixed and specified as an input. This section is designed to allow you to determine the overall heat transfer coefficient as a function of geometry and flow. These details will allow you to accurately determine the reduction in flow rate required to keep the heat transfer rate constant. Making the connection between flow and overall heat transfer coefficient will allow you to also take the next step toward finding the optimum velocities of the fluids, diameter and spacing of flow channels and fins.

Up to this point in your understanding of the system model keeping U constant has kept you from getting tangled in the branches of the analysis. In order to relate the value for this coefficient to the geometry of the heat exchanger and velocities you will need to venture into the branches of the tree. First, assume one-dimensional heat transfer between the refrigerant and the other heat transfer fluid. Then continue the analysis with the most general form of the overall conductance equation including a liquid film of water condensing on the outside surface and fouling of the heat transfer surfaces. Assuming a cylindrical channel for the refrigerant flow the cross-section of a typical refrigerant channel is shown in Figure 1.14.

With the typical cross-section there are six thermal resistances. Starting with the refrigerant, the first resistance is a convective resistance $\left(R_r\right)$ between the flow of the refrigerant and the inside surface of the fouled refrigerant channel. Second, is the conductive resistance $\left(R_{fo,i}\right)$ of any fouling of the inside surface of the channel. Third, is the conductive resistance $\left(R_{ec}\right)$ of the channel material. Fourth, is the conductive resistance $\left(R_{fo,o}\right)$ of the

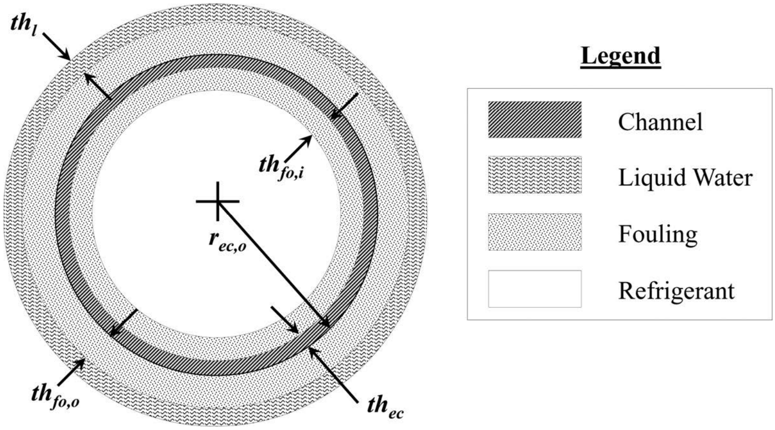

Figure 1.14. Typical cross-section of a refrigerant channel for an air-to-refrigerant evaporator
Illustrates the cross-section of a circular evaporator channel with refrigerant flowing on the inside and moist air in cross-flow on the outside. Characteristics of the interior fouling ($th_{fo,i}$), channel material thickness (th_{ec}) and radius ($r_{ec,o}$), outside fouling ($th_{fo,o}$), and liquid condensate (th_l) are identified.

fouling on the outside surface. Fifth, is the conductive resistance (R_l) of the liquid film of water which will condense in an evaporator when the surface temperature is below the dew point temperature of the air. Last, is the convective resistance (R_a) of the other heat transfer fluid whether it is air, water, a glycol mixture, etc.

Use the deductive problem-solving strategy to find the overall heat transfer coefficient (U_e). Equation 1.79 originates from the definition of overall conductance for a series of thermal resistances.

$$U_e \times A_e = \frac{1}{R_r + R_{fo,i} + R_{ec} + R_{fo,o} + R_l + R_a} \tag{1.79}$$

You have already specified the outside heat transfer surface area (A_e) since this is the easiest area to calculate given the geometrical data for the heat exchanger. Equations will be needed for each of the individual resistances. Use the constitutive relationship based on the mode of heat transfer to find the thermal resistance. So for conduction, use Fourier's law of conduction and for convection use Newton's law of cooling to construct the thermal resistance. Starting from left to right, using Equation 1.79 as your guide, apply the appropriate thermal resistance relationship to find each term. The first variable is the convective resistance on the inside of the heat transfer channel (R_r) shown in Equation 1.80 where two phases (liquid and vapor) of refrigerant exist.

$$R_r = \frac{1}{\alpha_r \times A_{ecs,i}} \tag{1.80}$$

Technically, you need to circle the inside surface area of the evaporator channels $\left(A_{ecs,i}\right)$ but since this is a thermal science textbook and not a geometry textbook, it is assumed you can determine the inside surface area given the exterior geometry of the heat exchanger. To find the convective heat transfer coefficient (α_r) use the definition of Nusselt number (Nu_r) as displayed in Equation 1.81.

$$Nu_r = \frac{\alpha_r \times D_{ec,i}}{k_{r,l}} \tag{1.81}$$

The Nusselt number can then be found using an empirical correlation for the geometry and refrigerant for the given application. Please note that if the geometry or fluid varies from those under which the correlation was developed then there could be significant errors in determining the Nusselt number and consequently the convective heat transfer coefficient. There is, however, a parameter optimization technique covered in the next section of this chapter which can be used to adapt any model to a specific application. Use this technique with experimental data or manufacturer's data at different refrigerant flow rates or cooling loads. Convective heat transfer, in two-phase flow, is a challenging topic because of the interaction between the liquid and vapor. Typically, in a direct-expansion evaporator, the inlet quality or vapor mass fraction is high enough so the predominate flow regime throughout the evaporation process is annular flow. Also, the liquid velocity is high enough to suppress nucleate boiling, so the primary mechanism for heat transfer is the convective transfer as opposed to nucleate boiling transfer. Therefore, use the convective transfer term from the Chen (1966) correlation as given in Equation 1.82 (Kuehn, 1998). Although the exponents in this correlation are specified they have not been boxed for the sake of readability.

$$Nu_r = C_{Nu_r} \times Re_r^{e_{ReNu_r}} \times Pr_r^{e_{PrNu_r}} \times \left(1 + X_{tt}^{e_{XttNu_r}}\right)^{e_{1XttNu_r}} \times \left(\frac{Pr_r + 1}{2}\right)^{e_{Pr1Nu_r}} \tag{1.82}$$

For two-phase flow, Chen uses a different formulation of the definition of Reynolds number as shown in Equation 1.83. In place of the product of density and velocity, the mass flux is used.

$$Re_r = \frac{D_{ec,i} \times G_r \times (1 - x_e)}{\mu_{r,l}} \tag{1.83}$$

If there are more than one circuit (the number of parallel refrigerant channels) in the evaporator, then the total mass flow rate of the refrigerant needs to be divided by the number of circuits. The total mass flux is multiplied by the quantity one minus the vapor mass fraction (x_e). The product of these two terms then provides the liquid mass flux. So in essence the Reynolds number is based on the liquid mass flux.

The total mass flux (G_r) is defined as the mass flow rate of refrigerant $(\dot{m}_r)$ flowing through the channel divided by the cross-sectional area $(A_{ecs,i})$ of the refrigerant channel as seen in Equation 1.84. Assume the area is specified in order to focus on the thermal science development.

$$G_r = \frac{\dot{m}_r}{A_{ecs,i}} \tag{1.84}$$

The vapor mass fraction is the average between the inlet and outlet of the evaporator as given in Equation 1.85. The vapor mass fraction of the outlet is 1.

$$x_e = (x_4 + 1)/2 \tag{1.85}$$

The vapor mass fraction at the inlet can be determined using a property relationship as a function of the specific enthalpy and temperature as shown in Equation 1.86.

$$x_4 = f\left(R410a,\ T_{sat,e}, h_4\right) \tag{1.86}$$

Equation 1.87 shows the dynamic viscosity $(\mu_{r,l})$ is evaluated at the saturated liquid properties using a property relationship.

$$\mu_{r,l} = f(R410a, T_{sat,e}, x = 0) \tag{1.87}$$

Referring back to the empirical relationship for Nusselt number (Nu_r) given in Equation 1.82, the Prandlt number (Pr_r) is determined based on its definition as shown in Equation 1.88. Again, as with the Reynolds number, all properties are evaluated for a saturated liquid at the saturation temperature of the refrigerant in the evaporator.

$$Pr_r = \frac{C_{p,r,l} \times \mu_{r,l}}{k_{r,l}} \tag{1.88}$$

The only property left to find for the Prandlt number is the specific energy of the refrigerant liquid ($C_{p,r,l}$). Equation 1.89 uses a property relationship to find this value.

$$C_{p,r,l} = f(R410a, T_{sat,e}, x = 0) \tag{1.89}$$

The next variable in Equation 1.82, or the empirical correlation for the Nusselt number (Nu_r), is the Lockhardt–Martinelli parameter (X_{tt}) which is defined in Equation 1.90. This parameter attempts to account for the differences between single- and two-phase flow by using three components: the ratio of liquid to vapor on the basis of mass, the ratio of vapor density to liquid density, and the ratio of liquid dynamic viscosity to vapor dynamic viscosity. Since the Chen correlation has a negative exponent for the Lockhardt–Martinelli parameter, the physical meaning of these components are inverted. Therefore, the convective heat transfer coefficient will increase as the mass of the vapor increases, as the density of the liquid relative to the vapor increases and as the viscosity of the vapor relative to the liquid increases.

$$X_{tt} = \left(\frac{1 - x_e}{x_e}\right)^{e_{xe}} \times \left(\frac{\rho_{r,v}}{\rho_{r,l}}\right)^{e_\rho} \times \left(\frac{\mu_{r,l}}{\mu_{r,v}}\right)^{e_\mu} \tag{1.90}$$

Determine the exponents to each term from experimental data and parameter optimization. Use property relationships for the liquid and vapor densities ($\rho_{r,l}$, $\rho_{r,v}$) and the vapor viscosity of refrigerant ($\mu_{r,v}$) as show in Equations 1.91, 1.92, and 1.93, respectively.

$$\rho_{r,l} = f(R410a, T_{sat,e}, x = 0) \tag{1.91}$$

$$\rho_{r,v} = f(R410a, T_{sat,e}, x = 1) \tag{1.92}$$

$$\mu_{r,v} = f(R410a, T_{sat,e}, x = 1) \tag{1.93}$$

Now you have all the variables to satisfy Equation 1.82. This takes you back to Equation 1.81 where you need the thermal conductivity of refrigerant liquid. Use the property relationship in Equation 1.94 to determine the conductivity.

$$k_{r,l} = f(R410a, T_{sat,e}, x = 0) \tag{1.94}$$

The next thermal resistance in Equation 1.79 is due to fouling of the inside surface of the refrigerant channel $(R_{fo,i})$. Assuming the fouling is uniformly distributed throughout the inside surface of the channel model it as a conduction resistance in the same way the channel wall is modeled. Integrate Fourier's law assuming cylindrical coordinates which yields Equation 1.95. For noncircular cross-sections use the hydraulic radius in place of the radius. The inside radius of the fouling material is determined by subtracting the thickness of this material $(th_{fo,i})$ from the inside radius of the channel $(r_{ec,i})$.

$$R_{fo,i} = \frac{\ln\left(\frac{r_{ec,i}}{r_{ec,i} - th_{fo,i}}\right)}{2 \times \pi \times Wi_e \times N_{ec} \times k_{fo,i}} \tag{1.95}$$

The inside radius of the channel $(r_{ec,i})$ is calculated in Equation 1.96 by subtracting the thickness of the channel wall (th_{ec}) from the outside radius $(r_{ec,o})$.

$$r_{ec,i} = r_{ec,o} - th_{ec} \tag{1.96}$$

Find the thermal conductivity of the fouling material $(k_{fo,i})$ by using a property relationship for the material, limestone in this case. Evaluate the thermal conductivity at the temperature of the refrigerant as shown in Equation 1.97.

$$k_{fo,i} = f('Rock - limestone', T_{sat,e}) \tag{1.97}$$

For the channel wall Fourier's law of conduction is integrated from the inside surface to the outside surface assuming a cylinder to yield Equation 1.98 for the thermal resistance of the channel wall (R_{ec}).

$$R_{ec} = \frac{\ln\left(\frac{r_{ec,o}}{r_{ec,i}}\right)}{2 \times \pi \times Wi_e \times N_{ec} \times k_{ec}} \tag{1.98}$$

The thermal conductivity of the channel (k_{ec}) is provided in Equation 1.99 by a property relationship for the channel material evaluated at the temperature of the refrigerant.

$$k_{ec} = f('Aluminum', T_{sat,e}) \quad (1.99)$$

Next, account for the exterior fouling of the channel $(R_{fo,o})$. The exterior radius of the fouling material is found by adding the thickness of this material $(th_{fo,o})$ to the exterior radius of the channel. This will be handled in a similar manner as the interior fouling as given in Equation 1.95.

$$R_{fo,o} = \frac{\ln\left(\frac{r_{ec,o} + th_{fo,o}}{r_{ec,o}}\right)}{2 \times \pi \times Wi_e \times N_{ec} \times k_{fo,o}} \quad (1.100)$$

Use a property routine, Equation 1.101 to determine the thermal conductivity of the exterior fouling material $(k_{fo,o})$ at the temperature of the refrigerant.

$$k_{fo,o} = f('Rock - limestone', T_{sat,e}) \quad (1.101)$$

On the outside of the fouling material, the water vapor in the air will condense if the surface temperature is below the dew point temperature of the air. When condensation occurs, a liquid film will form on the outside surface. Find the exterior radius of the liquid film by adding the thickness of the water (th_l), which was found in Equation 1.63, to the exterior radius of the fouling material. This thermal resistance will be treated as a conduction resistance (R_l) as shown in Equation 1.102.

$$R_l = \frac{\ln\left(\frac{r_{ec,o} + th_{fo,o} + th_l}{r_{ec,o} + th_{fo,o}}\right)}{2 \times \pi \times Wi_e \times N_{ec} \times k_l} \quad (1.102)$$

Use a property routine, Equation 1.103 to determine the thermal conductivity of the water (k_l) at the temperature of the refrigerant.

$$k_l = f(water, T_{sat,e}, x = 0) \quad (1.103)$$

The final thermal resistance is the convective resistance on the air side of the direct-expansion evaporator. Use Newton's law of cooling to determine this resistance. Wait a minute, not so fast you say. What about the fins? If you are going to apply Newton's law of cooling and you include the area of the fins, then you need to assume that the fins are at the base temperature along their entire length. You know this is not true for fins of any practical length because the fin's temperature approaches that of the air as heat is transferred to the fin.

A great way to account for this reduction in heat transfer is to use the definition of fin efficiency. Fin efficiency is the actual heat transfer rate divided by the heat transfer rate if the fin were at its base temperature along its entire length. So you can modify the fin area $(A_{efi,hts})$ by multiplying it by the fin efficiency (η_{efi}) yielding an equivalent area which can be used directly in the thermal resistance equation as given in Equation 1.104.

$$\boldsymbol{R_a} = \frac{1}{\boldsymbol{\alpha_a} \times (\boldsymbol{A_{ec}} + \boldsymbol{\eta_{efi}} \times \boldsymbol{A_{efi}})} \tag{1.104}$$

Use the definition of Nusselt number to find the convective heat transfer coefficient (α_a) as shown in Equation 1.105. Assuming fins are infinite plates, use two times the spacing between the fins (Sp_{efi}) for the hydraulic diameter.

$$\boldsymbol{Nu_a} = \frac{\boldsymbol{\alpha_a} \times 2 \times \boldsymbol{Sp_{efi}}}{\boldsymbol{k_a}} \tag{1.105}$$

Provide an empirical correlation, Equation 1.106 in this case Dittus–Boelter, to determine the Nusselt number (Nu_a). Choose Dittus–Boelter because of its simple form and the fact the fins form a channel.

$$\boldsymbol{Nu_a} = \boldsymbol{C_{Nu_a}} \times \boldsymbol{Re_{ae}}^{e_{ReNu_a}} \times \boldsymbol{Pr_a}^{e_{PrNu_a}} \tag{1.106}$$

The constant and two exponents are empirical parameters which are found using experimental data and parameter optimization for the geometry and fluids similar to the application. They are assumed to be known for this model and are therefore enclosed with a rectangle. The Reynolds number for the air flow (Re_{ae}) is already determined in Equation 1.55. Please note this is where the overall heat transfer coefficient (U) is connected to the air flow. This is the level of detail required to accurately account for the influence of air flow on overall heat transfer coefficient. So when the area of the heat exchanger changes you can determine more accurately how much the saturation temperature is affected.

Next use the definition of Prandlt number (Pr_a) in Equation 1.107 to determine this value for air. Evaluate all the properties at the temperature of the air at the evaporator inlet.

$$Pr_a = \frac{C_{p,aei} \times \mu_{aei}}{k_a} \quad (1.107)$$

The dynamic viscosity of the air (μ_{aei}) is already determined in Equation 1.69. The thermal conductivity of the air (k_a) will be determined when you come back to the definition of Nusselt number (Nu_a) in Equation 1.105. Therefore, the only variable you need an equation for at this time is the specific energy of air $(C_{p,aei})$. Equation 1.108 uses a property relationship evaluated at the temperature of the air at the inlet to the evaporator.

$$C_{p,aei} = f(AirH20, T_{aei}, RH_{aei}, P_{aei}) \quad (1.108)$$

Returning to the definition of Nusselt number in Equation 1.105 leads you to find the thermal conductivity of air. Use a property relationship for air Equation 1.109 to find this conductivity at the temperature of air at the evaporator inlet.

$$k_a = f(Air, T_{aei}, P_{aei}) \quad (1.109)$$

The deductive strategy takes you back to Equation 1.104 for the thermal resistance on the air side where the fin efficiency is needed. You have already accounted for the fouling on the outside of the channel but you need to account for the fouling on the outside of the fins as well. Currently, no other air conditioning textbook addresses fouling of the fins. Figure 1.15 illustrates the key variables which influence the heat transfer through the fouling layer.

Illustrated in Figure 1.15 is an isometric drawing of a plane fin with fouling. A control volume of length Δx is identified showing relevant heat transfer rates. Heat enters the control volume through the fouling material $(\dot{Q}_{conv})$ by means of convection and exits through the fin material $(\dot{Q}_{fin})$ and associated fouling $(\dot{Q}_{foul})$ by means of conduction. Characteristics of the fin width (Wi_{efi}) and thickness (th_{efi}) along with the fouling thickness (th_{fo}) are identified.

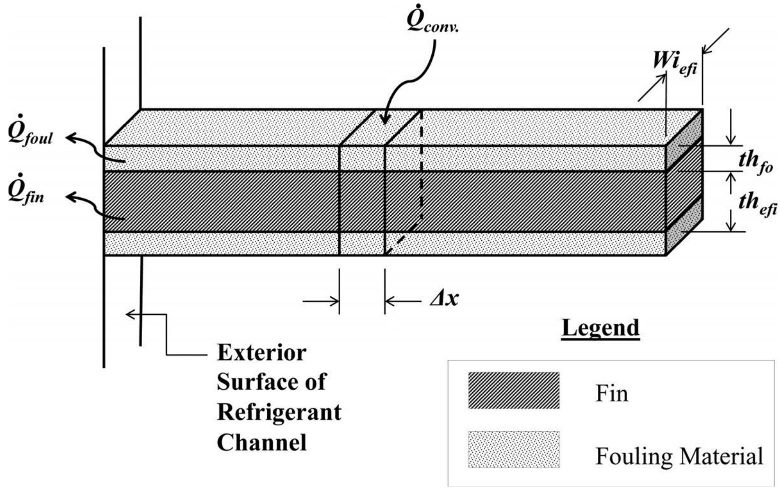

Figure 1.15. Fin with fouling

The expression for fin efficiency (η_{efi}) given in Equation 1.110, which is developed in heat transfer textbooks, assumes one-dimensional heat transfer along the length of the fin $(H_{efi}/2)$. This formulation of fin efficiency assumes a rectangular fin attached to a plane wall. This would effectively model a flat refrigerant flow channel with fins. However, there will be some error using Equation 1.108 for a tube-and-fin geometry. A two-dimensional heat transfer analysis would be necessary to quantify the error.

$$\eta_{efi} = \frac{\tanh\left(m \times H_{efi}/2\right)}{m \times H_{efi}/2} \tag{1.110}$$

The intermediate variable *m* is defined in Equation 1.111.

$$m = \sqrt{\frac{\alpha_a \times Per_{fo,efi}}{k_{fo,efi} \times A_{fo,efi}}} \tag{1.111}$$

You have already determined the convective heat transfer coefficient. The perimeter $(Pe_{fo,efi})$ needs to be modified to include the dimensions of the fouling material as shown in Equation 1.112.

$$Per_{fo,efi} = 2 \times (th_{efi} + 2 \times th_{fo,o} + Wi_{efi}) \tag{1.112}$$

Use Fourier's Law of Conduction and set the rate of heat entering the refrigerant channel equal to the sum of rate of heat leaving the fouling and fin materials. Assuming one dimensional heat transfer the temperature gradient $(\Delta T/\Delta X)$ cancels from each term and you are left with Equation 1.113.

$$\boldsymbol{A_{fo,efi}} \boldsymbol{k_{fo,efi}} = \boldsymbol{k_{fo,o}} \times \boldsymbol{A_{fo}} + \boldsymbol{k_{fi}} \times \boldsymbol{A_{efi}} \tag{1.113}$$

Where, $\boldsymbol{A_{fo,\,efi}}$ is the cross sectional area normal to heat transfer of the fin and fouling material. $\boldsymbol{k_{fo,\,efi}}$ is the equivalent thermal conductivity. The conductivity of the fin (k_{fi}) can be found using a property relationship, evaluated at the temperature of the air at the evaporator inlet, as specified in Equation 1.114.

$$\boldsymbol{k_{fi}} = \boldsymbol{f}\left(\textbf{'Aluminum'}, \boldsymbol{T_{aei}}\right) \tag{1.114}$$

Equation 1.115 shows the cross-section area is the sum of the fin and fouling areas normal to the flow of heat.

$$\boldsymbol{A_{fo,efi}} = \boldsymbol{A_{efi}} + \boldsymbol{A_{fo}} \tag{1.115}$$

This completes the set of equations needed to find the overall heat transfer coefficient which allows you to determine the optimum heat transfer surface area given the geometry of the evaporator along with its inlet states and flow rates for the refrigerant and air. This is done by adding 37 equations (Equations 1.79 to 1.115) and 37 unknowns (circled variables) to the system model previously developed.

Next, determine the overall heat transfer coefficient for the condenser. This is less complicated than the evaporator since you do not have a liquid film of water condensing on the surface. The thermal resistance related to the liquid film is eliminated, so you are left with five thermal resistances as shown in Equation 1.116.

$$\boldsymbol{U_c} * \boldsymbol{A_c} = \frac{1}{\boldsymbol{R_{r,c}} + \boldsymbol{R_{fo,i,c}} + \boldsymbol{R_{cc}} + \boldsymbol{R_{fo,o,c}} + \boldsymbol{R_{a,c}}} \tag{1.116}$$

The developments of these thermal resistances are similar to that of the evaporator except you are using the geometry of the condenser. You have just completed what is considered by industry as a theoretical model of a direct-expansion vapor-compression cycle. Nothing but the inlet conditions of the two air streams entering the evaporator and condenser, the geometric details of each heat exchanger, and the isentropic efficiency of the compressor are needed to determine the performance of the system. With the proper economic data, the minimum total life cycle costs can be found as a function of the heat transfer surface areas and the efficiency of the compressor. Test the robustness of the results of the optimization of this theoretical model with a sensitivity analysis.

EMPIRICAL PARAMETER OPTIMIZATION

Even though you have what is considered a theoretical model in industry, the equations for convective heat and mass transfer coefficients as well as the liquid film thickness rely on empirical correlations. Whenever the geometry or operating conditions are outside the range of the experiments for which the empirical parameters were determined the empirical correlations lose accuracy. For this reason you may be required to determine the empirical parameters using experimental data and optimization techniques. The objective function for parameter optimization is to minimize the error between the model and the experiment as shown in Figure 1.16.

The flow chart in Figure 1.16 shows the key steps in optimizing any empirical correlation. The dependent variable could be convective heat transfer coefficient, convective mass transfer coefficient, liquid film thickness, or initial cost of a component. Conventional optimization routines are used to generate new guesses for the parameters. A multivariate optimization routine is needed if more than one parameter is being determined.

The empirical correlations that may require parameter optimization include the following:

- Convective heat transfer coefficients
    - Air side of the evaporator and condenser
    - Refrigerant side of the evaporator and condenser
- Convective mass transfer coefficient for water vapor condensing on outside surface of the evaporator
- Thickness of the film of liquid water on the outside surface of the evaporator
- Cost coefficients for the initial cost of the evaporator, condenser, or compressor

Each of these correlations will influence the following variables, respectively, in the system:

- Heat transfer surface areas for the evaporator and condenser. The air-side coefficient will have a greater influence since air has a higher thermal resistance.
- Latent rate of heat transfer
- Thermal resistance of the liquid film on the air side of the evaporator
- Initial cost of the system

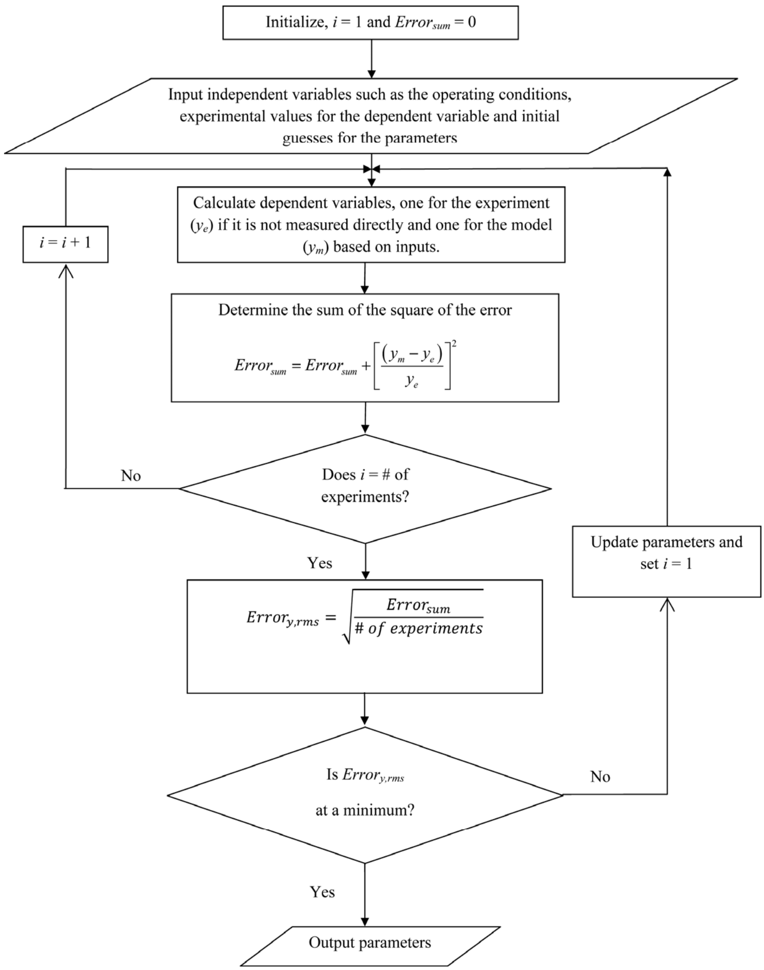

Figure 1.16. Flow chart for optimizing parameters

CHAPTER SUMMARY

In this chapter you were able to see how all the individual tools you learned in your undergraduate mechanical engineering program can be used in a systematic way to optimize direct-expansion vapor-compression cycles. You were first exposed to some simple models where the system effects could easily be observed. Then you were equipped with details that allow you to model realistic systems. Some of the key concepts of this chapter are as follows:

- The proper objective function for optimizing the vapor-compression cycle is total life-cycle costs. The optimum occurs at the minimum total costs.
- For a given application when you increase the surface area of the heat exchanger the saturation temperature of the refrigerant approaches the inlet temperature of the heat transfer fluid on the other side of the heat exchanger. This increases the COP which lowers the power required to cool the space. Superimposing the inlet temperature of the other fluids on the pressure-specific enthalpy diagram for the refrigerant provides a useful visual of this important relationship.
- Deductive problem-solving provides a structured approach to handling the open-ended problems you face once you leave the classroom. Appendix A contains a summary of the deductive problem-solving strategy. Appendix B contains a concise formulation of the basic system model of the vapor-compression cycle with a dry evaporator. An even simpler model (Zietlow, 2014), which assumes high air flow rates through the heat exchangers, can be found in Appendix C.
- Since water vapor condenses on the surface of most direct-expansion evaporators you were provided with a physics-based method to account for the latent energy transfer. The latent load was calculated based on the convective mass transfer coefficient with a driving potential based on the concentration gradient of the water vapor. This is analogous to the sensible heat transfer rate using Newton's law of cooling.
- The thickness of the liquid water film on the evaporator surfaces is predicted based on relevant forces acting on the film. The surface tension and viscosity of the liquid increase this thickness while the inertia of the air and gravity decrease the thickness.

These forces are accounted for using the dimensionless Bond and Reynolds numbers as independent variables in an empirical correlation.

- Since scale builds up on most heat transfer surfaces over the life of the system, a means to account for this fouling on the channels and fins is provided.
- Sensitivity analysis gives you insight into which variables have the greatest influence on the optimum. More time should be spent on decreasing the uncertainty in the variables which produce the greatest slopes, thus making the optimum design more certain.

PROBLEMS

Use the deductive problem-solving strategy for each of the following problems.

1.1 Determine the mass of air in a room 3 m × 10 m × 20 m with a temperature of 20°C, pressure of 100 kPa.

1.2 Determine how long it would take to fill a cylindrical fuel tank with a diameter of 1 m and length of 3 m. The fuel pump produces a flow rate of 4 liters per minute.

1.3 Determine the mass flow rate of air through a condenser that rejects 15 kW to the environment. The air at the inlet has a temperature of 30°C and a pressure of 100 kPa. At the outlet the temperature of the air rises to 40°C. What is the volumetric flow rate assuming it is measured at the inlet? What would the volumetric flow rate be at the outlet?

1.4 Determine the actual power into the compressor of an air conditioning system where 1.0 kg/s of R134a enters the compressor as a saturated vapor at 10°C. The isentropic efficiency of the compressor is 80 percent. How much does the power change if the refrigerant enters with 10°C of superheat? Tip: Use Engineering Equation Solver (EES, 2014) or a textbook on thermodynamics, for example (Cengel and Boles, 2015), to determine the refrigerant properties.

1.5 Determine the mass flow rate of refrigerant R134a for an evaporator with a heat transfer rate of 100 kW. The refrigerant leaves the condenser as a saturated liquid at 50°C and leaves the evaporator as a saturated vapor at 5°C. Tip: Use Engineering Equation Solver

(EES, 2014) or a textbook on thermodynamics, for example (Cengel and Boles, 2015), to determine the refrigerant properties.

1.6 Repeat the previous problem except the condenser has 5°C of subcooling and the evaporator has 5°C superheat at their outlets. Determine the error in the flow rate if the subcooling and superheat are neglected?

1.7 Determine the heat transfer surface area for the condensing region of a condenser. The saturation temperature of R134a is 45°C. Air enters the condenser at 40°C and 100 kPa and leaves at 44°C with a flow rate of 2.0 kg/s passing through the condensing region. The overall heat transfer coefficient in the condensing region is 0.3 kW/m^2-°C. Assume a cross-flow heat exchanger.

1.8 Determine the heat transfer surface area to remove the superheat from the refrigerant for the condenser in the previous problem. A compressor with an isentropic efficiency of 70 percent supplies refrigerant to the inlet of the condenser where the saturation temperature is 45°C. The refrigerant enters the compressor at 20°C with a corresponding saturation temperature of 10°C. Assume the same ratio of air flow rate to the heat transfer surface area as in problem 1.3. Assume a cross-flow heat exchanger where the minimum heat capacitance fluid is mixed and the other fluid is unmixed. Tip: Use a textbook on heat transfer, for example (Cengel and Ghajar, 2015), to select the appropriate effectiveness-NTU relationship.

1.9 Determine the heat transfer surface area of the subcooling region for the condenser operating under the same conditions as the previous problem except the refrigerant has 10°C of subcooling at the exit and the overall heat transfer coefficient is 0.1 kW/m^2-°C. Compare the subcooling area to the area required to condense the refrigerant. Assume the same ratio of air flow rate to the heat transfer surface area as in problem 1.3. Assume a cross-flow heat exchanger where the minimum heat capacitance fluid is mixed and the other fluid is unmixed. Tip: Use a textbook on heat transfer, for example (Cengel and Ghajar, 2015), to select the appropriate effectiveness-NTU relationship.

1.10 Use the deductive strategy to rearrange the equations for the simple (UA) model assuming the saturation temperatures of evaporator and condenser are specified rather than the heat transfer areas.

1.11 Use the deductive strategy to rearrange the equations for the basic (effectiveness-NTU) model assuming the saturation temperatures of evaporator and condenser are specified rather than the heat transfer areas.

1.12 Determine the optimum surface area for heat transfer for a dry evaporator serving a vapor-compression cycle (simple system model) where the saturation temperature of the refrigerant in the condenser is 50°C and the compressor has an isentropic efficiency of 75 percent. The remaining operating and cost data are supplied in Table 1.3. Plot the processes for a vapor-compression cycle with a small, optimum, and large evaporator on a pressure-enthalpy diagram. Plot the initial, operating costs (in today's currency) and total life-cycle costs as a function of heat transfer surface area. Perform a sensitivity analysis with respect to all specified variables and determine the top five variables which have the greatest effect on the optimum. Explain any differences from the sensitivity analysis given in the text EES tips.

1. Tip 1: Use the minimum–maximum function in Engineering Equation Solver (EES, 2014).
2. Tip 2: In EES, provide a subscript for the enthalpies and pressures using square brackets, [ ], for example h[1] and P[1]. Overlay these state points on the property plot. Include the isotherm for the inlet air temperature to the evaporator.
3. Tip 3: Use the parametric table in EES to generate the data for the cost and sensitivity analysis plots.

1.13 For the evaporator of the previous problem plot both COP and temperature difference (T_aei-T_sat_e) as a function of heat transfer surface area. Use these plots to explain why COP and irreversibility are not realistic objective functions for optimizing the evaporator. Back up your explanation with equations.

1.14 Determine the optimum surface area for heat transfer for a condenser serving a vapor-compression cycle (simple system model with dry evaporator) where the saturation temperature of the refrigerant in the evaporator is 10°C and the isentropic efficiency of the compressor is 75 percent. The remaining cost and operating data are contained in Table 1.3. Plot the processes for a vapor-compression cycle with a small, optimum and large condenser on a pressure-enthalpy diagram. Include the isotherm for the inlet air temperature to the condenser. Plot the initial, operating costs (in today's currency) and total life cycle costs as a function of heat transfer surface area. Perform a sensitivity analysis with respect to all specified variables and determine the five variables which have the greatest effect on the optimum. Explain any differences from the sensitivity analysis given in the text. Tip: See EES tips in problem 1.12

Table 1.3. Specified variables for air conditioning system problems

Variable Name	Value	Units	Source
CC_{comp}	15	\$	Compressor Supplier Quotes
C_{ue}	0.07	\$/kW-hr	Energy Supplier Rates
C_{uac}	90	\$/m^2	Condenser Supplier Quotes
C_{uae}	120	\$/m^2	Evaporator Supplier Quotes
CL	5,000	kW-hr/yr	Based on Annual Cooling Load
e	2	-	Compressor Supplier Quotes
$marr$	0.05	-	Client or Owner
$\dot{m}_{ac}$	2	kg/s	Mass flow rate measurement
$\dot{m}_{ae}$	1	kg/s	Mass flow rate measurement
n	15	yrs	ASHRAE Handbook
P_{aci}	100	kPa	Air Pressure Reading at Inlet
P_{aei}	100	kPa	Air Pressure Reading at Inlet
$\dot{Q}_e$	10	kW	Annual Energy Analysis
T_{aci}	40	°C	Thermocouple in Inlet Airstream
T_{aei}	20	°C	Thermocouple in Inlet Airstream
U_c	0.25	kW/m^2-°C	Thermal Resistance Analysis
U_e	0.30	kW/m^2-°C	Thermal Resistance Analysis

1.15 For the condenser of the previous problem plot both COP and temperature difference (T_sat_c-T_aci) as a function of heat transfer surface area. Use these plots to explain why COP and irreversibility are not realistic objective functions for optimizing the condenser. Back up your explanation with equations.

1.16 Determine the optimum isentropic efficiency for a compressor serving a vapor-compression cycle (simple system model with a dry evaporator) where the saturation temperature of the refrigerant is 10°C in the evaporator and 50°C in the condenser. Plot the processes for a vapor-compression cycle with a low, optimum and high efficiency compressor on a pressure-enthalpy diagram. Plot the initial, operating costs (in today's currency) and total life-cycle costs as a function of heat transfer surface area. Perform a sensitivity analysis with respect to all specified variables and determine the top five variables which have the greatest effect on the optimum. Explain any differences from the sensitivity analysis given in the text. Tip: See EES tips in problem 1.12

1.17 Determine the optimum heat transfer surface areas for the evaporator and condenser and the optimum isentropic efficiency for the compressor for the vapor-compression system (simple system model with a dry evaporator) given in Table 1.3. Plot the processes for a vapor-compression cycle on a pressure-enthalpy diagram. On three separate plots show how the initial, operating costs (in today's currency) and total life-cycle costs vary as a function of heat transfer surface area of the evaporator (plot 1), condenser (plot 2) and isentropic efficiency of the compressor. Perform a sensitivity analysis with respect to all specified variables and determine the five variables which have the greatest effect on the optimum. Explain any differences from the sensitivity analysis given in the text. Tip: See EES tips in problem 1.12

1.18 Repeat previous problem using the basic (effectiveness-NTU) model with a dry evaporator. Determine the error in optimum surface areas of the heat exchangers and isentropic efficiency of the compressor from using the simple (UA) model.

1.19 Given the following cost quotes for evaporators determine the cost parameter(s) using regression analysis. Plot both the results of the regression equation and the quote data as a function of heat transfer surface area. Also, plot the difference between the regression equation and the quote data as a function of heat transfer surface area.

Heat Transfer Surface Area (m^2)	Cost ($)
5	575
10	1,200
15	1,850

1.20 Given the following cost quotes for condensers determine the cost parameter(s) using regression analysis. Plot both the results of the regression equation and the quote data as a function of heat transfer surface area. Also, plot the difference between the regression equation and the quote data as a function of heat transfer surface area.

Heat Transfer Surface Area (m^2)	Cost ($)
10	900
25	2,200
40	3,650

1.21 Given the following cost quotes for compressors determine the cost parameter(s) using regression analysis. Plot both the results of the regression equation and the quote data as a function of isentropic efficiency. Also, plot the difference between the regression equation and the quote data as a function of isentropic efficiency.

Isentropic Efficiency (%)	Cost ($)
60	95
75	250
90	1,450

1.22 Find the overall heat transfer coefficient for a tube-and-fin evaporator with the following geometric and operating data.

Variable	Value with units	Description
$\dot{Q}_e$	10.5 kW	Rate of heat transfer
T_{aei}	24°C	Temperature of air at the evaporator inlet
D_o	0.008 m	Outside diameter of cylindrical channels

Variable	Value with units	Description
Wi_e	0.6 m	Width of evaporator
N_{cpc}	14 [-]	Number of channels per circuit
N_c	4 [-]	Number of circuits
$C_{Nu,o}$	0.023 [-]	Multiplier for Dittus–Boulter correlation
$e_{Re,o}$	0.8 [-]	Empirical exponent for Dittus–Boulter correlation
$e_{Pr,o}$	0.3 [-]	Empirical exponent for Dittus–Boelter correlation for cooling of the fluid
V_o	3 m/s	Velocity of outside fluid
$fluid_o$	Air	Outside fluid specification
P_{aei}	101 kPa	Pressure of air at the evaporator inlet
rho_{fi}	472 1/m	Fin density
th_{fi}	0.0001 m	Fin thickness
H_e	0.72 m	Height of the evaporator
fin material	Aluminum	
α_r	50000 W/m^2-°C	Convective heat transfer coefficient for evaporating refrigerant
th_{ec}	0.002 m	Thickness of channel
channel material	Copper	

1.23 Determine the empirical parameter (multiplier) for the convective heat transfer coefficient correlation for a heat exchanger under the following set of operating conditions. Assume the temperature is constant throughout the six experiments. Tip: Use the minimum–maximum function in Engineering Equation Solver (EES, 2014) to minimize the RMS error between the model and the experiment. Plot Nu vs. Re and also plot error vs. Re.

Experiment	Re [-]	Nu [-]
1	15000	78
2	16000	79
3	18000	88
4	20500	99
5	28000	105
6	28500	110

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APPENDIX A

SUMMARY OF THE DEDUCTIVE PROBLEM-SOLVING STRATEGY

This strategy starts with the objective of the problem and then works its way to the inputs needed to solve the problem. The key question to ask is which relationship will provide the bridge between what is needed (the dependent variable) and what can be specified or measured (the independent variables). The deductive approach will assist you when you face problems that are not well defined such as the ones you will face in industry. If you discipline yourself to follow all these steps, even for easy problems, you will have a tool to help you when you face a problem where you have no idea where to begin.

1. **Draw a system schematic.** The schematic is essential to the definition of the problem. In the schematic, identify the system boundaries, all energy interactions (i.e., heat and work) which cross the boundaries, identify any relevant reference frames (e.g., position) and the positive direction and any other significant variable.
   a. **Use descriptive variable names** (e.g., P_i rather than x_1 for initial pressure).
   b. **Provide references or sources for all equations** (e.g., conservation of energy, definition of efficiency).
   c. **Summarize any assumptions used when applying the equations. Justify the relevance of the assumptions to the problem** (e.g., ideal gas because the compressibility factor (Z) is near 1).

2. **Construct property plots (e.g., P vs. v, P vs. h, T vs. length,** etc.).
   a. **Mark all state points between each component.**
   b. **For heat exchangers, superimpose the inlet temperature of the other fluid.**
3. **Find: Determine the dependent variable(s). Start with the objective or goal of the problem.** For optimization problems identify the objective function (a dependent function that has a maximum or minimum as the independent variable(s) are varied over their range(s)). One useful objective function for design problems is the total life-cycle cost or present worth of a system. Another objective for comparisons between alternatives is percent difference.
4. **Solution:**
   a. **Determine a relationship (or equation) which contains the dependent variable.** The relationship needs to move the solutions toward something that can be measured or specified. Variables which can often be measured directly with instruments include length, time, flow rate, temperature, force, and pressure. Variables which can be specified often include the isentropic efficiency or heat exchanger geometry.
   b. **Choose from one of the following five basic types of relationships**. Tip: Choose the simplest form of a relationship. For example, the simplest form of the definition of isentropic efficiency for a compressor is the ratio of ideal power input to actual power input. Some authors combine the conservation of energy with the definition to express the isentropic efficiency in terms of enthalpies. The enthalpy form of the equation is less straightforward and therefore should be avoided.
      i. The first relationship is **a law of science or math**. For thermal science problems the following laws may be relevant to the problem: conservation of mass, momentum, and energy. For heat transfer the following laws may be relevant: Fourier's law for conduction, Newton's law of cooling for convection, and Stefan–Boltzmann law for radiation. For convection, empirical relationships are used to determine the heat transfer coefficient. For radiation, view factor relationships may be helpful (e.g., reciprocity, summation, superposition, and symmetry).
      ii. The second relationship is **a definition.** Definitions of efficiency, specific volume, coefficient of performance, is-

entropic efficiency, and heat exchanger effectiveness are useful for energy systems.

iii. The third relationship comes from thermodynamics; **property relationships.** Often these will be used when a property which cannot be measured directly is needed. Examples of these properties which are difficult to measure include specific internal energy, specific enthalpy, or specific entropy. The property relationship connects the unknown property to ones which can be measured or are known from process information (e.g., isentropic). Here it is useful to refer to the property plot to determine which independent, intensive properties are the best.

iv. The fourth is **a parameter optimization.** These are most commonly used to relate initial cost data to the design variable(s).

v. The fifth and last type of relationship is **knowledge about the process.** Examples: constant volume, pressure, or entropy processes.

c. **Evaluate each of the variables in the equation.**
   i. Draw a box around all the variables in the equation that can either be specified or measured.
   ii. Circle all new unknowns that have not already been circled.
   iii. Place a parallelogram around the dependent variable.

d. Moving left to right, for the next circled variable, determine a relationship (equation), from the previous five categories in 4.b, which can be used to solve for it.

e. Indent one level and enter the equation. Note: Every circled variable from the same equation will have the same level of indentation.

f. Repeat steps (c) through (e) until the circled variable of the most recent equation can be determined from specified information.

g. Return to the nearest equation containing a circled variable that does not yet have an equation and repeat steps (d) through (f).

h. Continue adding equations until you have an independent equation for each unknown variable. At this point the solution is complete.

5. Create Summaries.
   a. Summarize all the variables, along with their values and associated units, which need to be specified or measured. Identify the source of the data (e.g., handbook, measurement, etc.).
   b. Create a nomenclature section to define all variables and their units.
6. Perform reality checks.
   a. Check equations for dimensional homogeneity.
   b. For computer solutions, perform sample calculations at one set of conditions.

APPENDIX B

OPTIMIZATION OF A VAPOR-COMPRESSION CYCLE—BASIC MODEL

Equations	**Eq. #**	**Var.**
$TC = IC + PW_{OC}$	1	3
$IC = IC_{evap} + IC_{cond} + IC_{comp}$	2	6
$IC_{evap} = A_e \times C_{uae}$	3	6
$IC_{cond} = A_c \times C_{uac}$	4	6
$IC_{comp} = \dfrac{CC_{comp} \times \dot{W}}{(1-\eta)^e}$	5	7
$\eta = \dot{W}_s / \dot{W}$	6	8
$\dot{W}_s + \dot{m}_r \times h_1 = \dot{m}_r \times h_{2s}$	7	11
$\dot{Q}_e + \dot{m}_r \times h_4 = \dot{m}_r \times h_1$	8	12
$h_4 = h_3$	9	13
$h_3 = f\left(R410a, T_{sat,c}, x = 0\right)$	10	14
$\Delta T_{max,c} = T_{sat,c} - T_{aci}$	11	15
$\dot{Q}_{max,c} = C_{min,c} \times \Delta T_{max,c}$	12	17

Equations	**Eq. #**	**Var.**
$\varepsilon_c = \dot{Q}_c / \dot{Q}_{max,c}$	13	19
$\varepsilon_c = 1 - e^{-NTU_c}$	14	20
$NTU_c = \frac{U_c \times A_c}{C_{min,c}}$	15	20
$\dot{m}_r \times h_{g,c} = \dot{Q}_c + \dot{m}_r \times h_3$	16	21
$h_{g,c} = f\left(R410a, T_{sat,c}, x = 1\right)$	17	21
$C_{min,c} = C_{p,ac} \times \dot{m}_{ac}$	18	22
$C_{p,ac} = f(air, T_{aci}, P_{aci})$	19	22
$h_1 = f(R410a, T_{sat,e}, x = 1)$	20	23
$\Delta T_{max,e} = T_{sat,e} - T_{aei}$	21	24
$\dot{Q}_{max,e} = C_{min,e} \times \Delta T_{max,e}$	22	26
$\varepsilon_e = \dot{Q}_e / \dot{Q}_{max,e}$	23	27
$\varepsilon_e = 1 - e^{-NTU_e}$	24	28
$NTU_e = \frac{U_e \times A_e}{C_{min,e}}$	25	28
$C_{min,e} = C_{p,ae} \times \dot{m}_{ae}$	26	29
$C_{p,ae} = f(air, T_{aei}, P_{aei})$	27	29
$h_{s,2} = f(R410a, P_2, s_{s,2})$	28	31
$P_2 = P_{sat,c}\left(R410a, T_{sat,c}\right)$	29	31
$s_{s,2} = s_1$	30	32
$s_1 = f(R410a, T_{sat,e}, x = 1)$	31	32

Equations	**Eq. #**	**Var.**
$PW_{oc} = OC \times PA$	32	34
$OC = AE \times C_{ue}$	33	35
$\dot{W} = AE / OT$	34	36
$\dot{Q}_e = CL / OT$	35	36
$PA = \frac{(1+marr)^n - 1}{marr \times (1+marr)^n}$	36	36

APPENDIX C

OPTIMIZATION OF A VAPOR-COMPRESSION CYCLE—SIMPLE MODEL

Equations	**Equation Number**
$TC = IC + PW_{OC}$	1
$IC = IC_{evap} + IC_{cond} + IC_{comp}$	2
$IC_{evap} = A_e \times C_{uae}$	3
$IC_{cond} = A_c \times C_{uac}$	4
$IC_{comp} = \dfrac{CC_{comp}}{(1-\eta_s)^e}$	5
$PW_{oc} = OC \times PA$	6
$OC = \dot{W} \times OT \times C_{ue}$	7
$\eta = \dot{W}_s / \dot{W}$	8
$\dot{W}_s + \dot{m}_r \times h_1 = \dot{m}_r \times h_{2s}$	9
$\dot{Q}_e + \dot{m}_r \times h_4 = \dot{m}_r \times h_1$	10
$h_4 = h_3$	11
$h_3 = f\left(R410a, T_{sat,c}, x = 0\right)$	12

Equations	**Eq. #**	**Var.**
$\dot{Q}_c = U_c A_c \left(T_{sat,c} - T_{aci}\right)$	13	
$\dot{m}_r \times h_{g,c} = \dot{Q}_c + \dot{m}_r \times h_3$	14	
$h_{g,c} = f\left(R410a, T_{sat,c}, x = 1\right)$	15	
$h_1 = f(R134a, T_{sat,e}, x = 1)$	16	
$\dot{Q}_e = U_e A_e \left(T_{aei} - T_{sat,e}\right)$	17	
$h_{s,2} = f(R410a, P_2, s_{s,2})$	18	
$P_2 = P_{sat,c}\left(R410a, T_{sat,c}\right)$	19	
$s_{s,2} = s_1$	20	
$s_1 = f(R410a, T_{sat,e},\ x = 1)$	21	
$\dot{\mathrm{Q}}_{\mathrm{e}} = \mathrm{CL} / \mathrm{OT}$	22	
$PA = \dfrac{(1 + marr)^n - 1}{marr \times (1 + marr)^n}$	23	

NOMENCLATURE

Primary variables

A is area [m^2]
air_{H_2O} is moist air
α is convective heat transfer coefficient [W/m^2-°C]
Bo is Bond number [-]
C is empirical multiplier or cost
CL is annual cooling load [kW-hr/yr]
C_{min} is the minimum heat capacity rate one of two fluids in a heat exchanger [kW/°C]
C_p is specific energy at constant pressure of the fluid [kJ/kg-°C]
COP is the coefficient of performance for the cooling system [-]
CC is cost coefficient [$]
D is diameter [m]
Δ is change or difference
$\Delta\dot{E}$ is change of energy stored within the system with respect to time
Di is mass diffusivity [m/s]
e is empirical exponent [-]
$\dot{E}$ is the rate of energy transferred to or from the control volume
ε is the effectiveness of the heat exchanger [kW_{actual}/kW_{max}]
η is the isentropic efficiency of the compressor [-]
F is force [N]
g is acceleration due to gravity [m/s^2]
g_c is the constant of proportionality for Newton's second law of motion [$kg\text{-}m/N\text{-}s^2$]
G is mass flux [$kg/(s\text{-}m^2)$]
H is height [m]
H_2O is water vapor treated as an ideal gas
h is specific enthalpy [kJ/kg]
IC is the initial cost [$]
k is thermal conductivity [W/m-°C]

L is length [m]
λ is the convective mass transfer coefficient [m/s]
m is an intermediate variable used to calculate fin efficiency [1/m]
marr is the client's minimum acceptable rate of return [-]
$\dot{m}$ is mass flow rate [kg/s]
μ is the dynamic viscosity of the fluid [kg/m-s]
N is number [-]
n is the expected life of the system [years]
NTU is the number of heat transfer units [-]
v is kinematic viscosity [m^2/s]
Nu is Nusselt number [-]
OC is the annual operating cost [$/yr]
ω absolute humidity ratio [kg_v/kg_a]
OT is the annual operating time [hrs/yr]
P is pressure [kPa]
PA is the discount factor for bringing a uniform series of annual expenditures to present worth [yrs]
pe is specific potential energy [kJ/kg]
Per is perimeter [m]
Pr is Prandtl number [-]
PW is the present worth [$]
$\dot{Q}$ is the rate of heat transfer [kW]
r is radius [m]
*R*134*a* is refrigerant
*R*410*a* is refrigerant
R is thermal resistance [°C/kW]
Re is Reynolds number [-]
RH is relative humidity [-]
ρ is density [kg/m^3] for fluids or [1/m] for fins
Rock-limestone is the material used to account for the effects of fouling
s is specific entropy [kJ/kg-K]
Sc is Schmidt number [-]
σ is surface tension [N/m]
Sp is spacing [m]
T is temperature [°C]
TC is the total cost in today's dollars of the air conditioning system over its lifetime [$]
th is thickness [m]
U is the overall heat transfer coefficient of the heat exchanger [kW/m^2-°C]
V is velocity [m/s]

$\dot{V}$ is the volumetric flow rate [m^3/s]
$\dot{W}$ is the rate of work or power into the compressor [kW]
Wi is width [m]
X_{tt} is Lockhardt–Martinelli parameter [-]
x is the quality or vapor mass fraction of a two-phase fluid [$kg_v/(kg_l + kg_v)$]

Subcripts

1 is state-point one at inlet to compressor
2 is state-point two at inlet to condenser
3 is state-point three at inlet to expansion device
4 is state-point four at inlet to evaporator
a is air
aci is air at condenser inlet
ac is air at the condenser
ae is air at the evaporator
aei is air at evaporator inlet
aeo is air at evaporator outlet
b is the value of a variable for the base case
Bo is Bond number
c or *cond* is condenser
cc is condenser channel
comp is compressor
e or *evap* is the evaporator
ec is evaporator channel
ecr is evaporator channels per row
ecs is evaporator channel cross-section
ef is evaporator face
efi is evaporator fin
elec is electricity
exp is experimental
f is saturated liquid
fc is fin cross-section
ffe is free flow for evaporator
fi is fin
fo is fouling
g is saturated vapor
H is high temperature thermal reservoir
i is inside
in is either input or inlet

l is liquid
L is low temperature thermal reservoir
ll is latent load
max is maximum
min is minimum
mt is mass transfer
μ is exponent identifier for viscosity ratio in X_{tt} [Equation 1.90]
o is outside
OC is annual operating costs
out is out of system
Pr is Prandtl number
Pr1 is for exponent for the Pr + 1 term in Equation 1.82
r is refrigerant
ρ is exponent identifier for density ratio in X_{tt} (Equation 1.90)
Re is Reynolds number
s is for the ideal or isentropic process (S = constant)
sat is saturation
Sc is Schmidt number
sl is sensible load
system is system or component containing the refrigerant
uac is per unit area of condenser
uae is per unit area of evaporator
ue per unit of electricity
v is vapor
vei is vapor of water at the evaporator inlet
$1X_{tt}$ is exponent for the $1+X_{tt}$ terms in Equation 1.82
xe is exponent for mass fraction ratio in X_{tt} (Equation 1.90)

CHAPTER 2

OPTIMIZATION OF THE ENVELOPE AND EQUIPMENT

INTRODUCTION

In the last chapter, you approached the design of the cooling system from the perspective of the original equipment manufacturer. In Chapter 2, your focus will be from the perspective of the owner of the conditioned space. First, you will learn how to optimize the envelope surrounding the conditioned space, whether the space is a building or vehicle, as well as selecting the best cooling and heating equipment. You will understand how to take the analysis tools from traditional air-conditioning design courses and develop a system of equations which allow you to optimize the thickness of insulation in the envelope. Although you will focus on a wall for a building, the same technique can be applied to windows, doors, and ceilings with minimal modification. Second, you will see how to take what the original equipment manufacturers have to offer and select the best cooling system for the application.

As discussed in the first chapter, the first step in the design process of a cooling system you determine the cooling load. For most applications, this cooling load is a strong function of the envelope surrounding the conditioned space. Before designing the components of the cooling system, it is helpful to know the cooling load. A key design variable for the envelope is the thickness of the insulation. Too little insulation and your energy costs are too high. Too much insulation, you cannot afford to build the envelope.

The scope of Chapter 2 is limited to the optimization of insulation in a wall. Chapter 2 does not provide a comprehensive way to calculate the cooling load. The details of cooling load calculations are covered sufficiently in other textbooks such as Stoecker (1982, Chapter 4) and

Kuehn et al. (1998) as well as handbooks such as American Society of Heating, Refrigerating and Air Conditioning Engineers (ASHRAE, 2013). The same principles used to optimize the wall insulation can be easily translated to other components of the envelope such as windows, doors, and roofs. Additionally, the scope is limited to the conductance through the wall as a result of the temperature difference between the outdoor air temperature and the balance point temperature of the conditioned space.

Typically, when the word optimization is used for an envelope most people will assume you would want to minimize the heat transfer rate. The heat transfer rate is an unrealistic objective function, since as you add additional insulation to the envelope the heat transfer rate through the envelope will decrease. The minimum heat transfer will occur when the insulation thickness reaches infinity.

Another objective function which some people like to use is to minimize the initial cost. To minimize the initial cost you would eliminate the insulation. No insulation will cause the energy costs to be excessive over the life of the structure. The best way to minimize the initial cost is not to build the structure in the first place. This eliminates not only the initial cost but the operating cost as well. It is a win–win situation, unless, of course, you, like most people, value comfort and productivity.

How can the cooling load in kilowatt be compared to insulation thickness in millimeter? You will need to convert both to cost to address this question. Convert the cooling load to operating cost and insulation thickness to initial cost. The insulation thickness not only affects the initial cost of the insulation but also the initial cost of the cooling system, heating system, and structural materials to support the insulation. You will use three tools from heat transfer, regression analysis, and engineering economics to do the necessary conversions. Then you will be able to determine the total life-cycle costs for the system as a function of the design variables (i.e., thickness of insulation, efficiency of furnace and COP of air conditioners). As you will see later, the relationship between the total costs and each of the design variables provides an effective objective function for optimization.

To avoid getting tangled in the branches, limit the analysis to a seasonal performance of the heating and cooling systems. Also you may be tempted to use an hour-by-hour simulation of the system to obtain the best accuracy. At this point, the details of the hourly analysis would detract from your primary focus which is the optimization of the envelope.

OPTIMIZATION OF INSULATION FOR AN ENVELOPE

Develop the equations needed for the optimization using the deductive problem-solving strategy (Zietlow, 2006). First, **determine the primary objective of the problem**. The objective function needs to be clearly defined and quantifiable. For the envelope of a climate-controlled space, the primary objective is to determine the optimum insulation thickness for a wall. For an optimum to occur there needs to be two forces that compete with each other. To discover these forces further problem definition is needed.

Create a system schematic, as shown in Figure 2.1, to assist in the formulation of the objective function for the problem. The system schematic typically evolves as you develop the solution. However, the more thorough and complete the schematic at the onset, the sooner a solution will be obtained. **Identify all key variables on the schematic**. Also, identify all energy flows.

Top view of the walls at a height of four feet during the heating season reveal the thickness of the insulation and wood support members (th) as well as the sheetrock on the inside ($th_{s,o}$) and siding on the outside ($th_{s,i}$) of the structure. Identify the width of each section (W) along with the width of the insulation (W_{in}) and the width of the structural support member, in this case, wood (W_w). The furnace with an efficiency of η consumes energy ($E_{purchased}$) to produce enough heat (Q_{out}) to offset the heat losses through the wood ($\dot{Q}_{l,w}$) and insulation ($\dot{Q}_{l,i}$) to maintain an indoor temperature of T_i when the outdoor temperature is T_o.

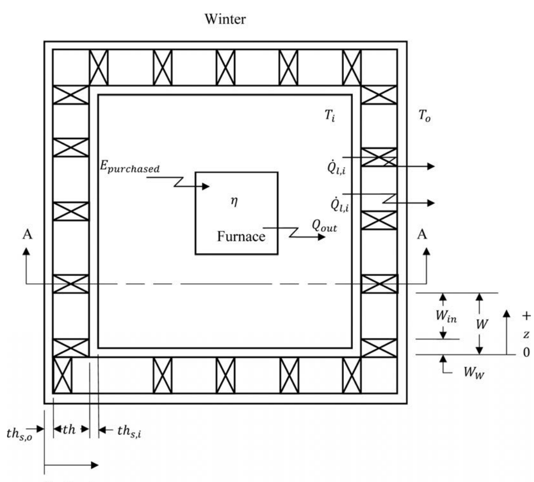

Figure 2.1a. System schematic of envelope during heating season

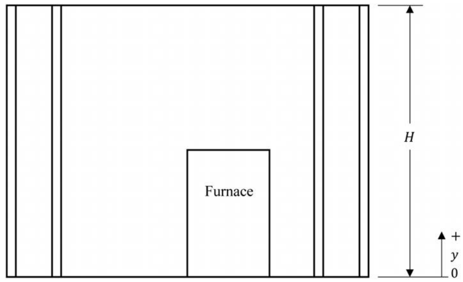

Figure 2.1b. Section View A-A of envelope during heating. The section view illustrates the height (H) of the wall.

In order for the system schematic to be useful, **identify the key features of the system**. To be effective ask yourself some basic questions. What is the function of wall insulation? How does the insulation influence the initial cost of the system? How does it influence the operating cost of the system? The function of insulation is to resist the transfer of heat through the envelope between a conditioned space and the outdoors. As the thickness of the insulation increases its initial cost also increases. Additionally, a structure is needed to hold the insulation in place so as the thickness of the insulation increases so does the thickness and cost of the structure. The heat which passes through the wall needs to be replaced in order to maintain a constant temperature within the conditioned space. A furnace replaces the heat in the winter, and a cooling system removes the heat in the summer. As the insulation increases the heat transfer rate decreases. Therefore, the size of the furnace and cooling system can be reduced along with its initial cost. Finally, as the heat transfer rate drops the amount of energy needed throughout the year decreases along with its associated operating costs.

Therefore, the two opposing forces are the costs which increase with increasing insulation thickness [the initial cost of the insulation (IC_{in}) and wall (IC_w)] versus the costs which decrease with increasing insulation thickness [the initial cost of the furnace (IC_f), the initial cost of the cooling system (IC_c) and operating cost (OC) for maintaining comfortable conditions for the occupants of the building]. See Figure 2.3 for a visual representation of these opposing forces. The operating costs occur in the future over the life of the structure; therefore, these future costs must be converted to their present worth. Now you are able to directly compare the present worth of the operating costs (PW_{OC}) with the initial costs. These costs are combined in the total costs (TC) of the system which is the objective function for this problem as shown in Equation 2.1.

$$TC = IC_{in} + IC_w + IC_f + IC_c + PW_{OC} \tag{2.1}$$

Once you define the objective function, **evaluate each unknown** of this function systematically. You have already evaluated, on a qualitative basis, the relationship between each of these unknowns and the independent design variable in the formulation of the system schematic and objective function. To quantify these relationships, ask yourself how to go from each of the unknowns in the objective function to a measurable quantity. Systematically move from left to right in Equation 2.1. Evaluate the first unknown or the initial cost of the insulation. To be systematic it is important to follow each unknown until all the variables needed to determine the unknown can be measured or specified. Equation 2.2 shows how the dimensions and unit cost of insulation are used to calculate the initial cost of insulation.

$$IC_{in} = H \times W_{in} \times th \times N_s \times Ciu \tag{2.2}$$

where,
H is the height of the wall [m]
W_{in} is the width of the insulation [m]
th is the thickness of the insulation [m]
N_s is the number of sections of wall in the building [#]
Ciu is the cost of insulation per unit volume [$/m^3]

Keep track of the number of equations and unknowns. Tracking will help you determine if any additional equations need to be applied to the problem. An unknown is a variable that cannot be measured or specified directly. In Equation 2.1 there are six unknowns with one equation. Therefore, at least five more equations are needed to solve the problem. Equation 2.2 provides one of these equations. All the new variables introduced in Equation 2.2 are measurable therefore no further equations are needed to calculate the initial cost of the insulation. An alternative to circling unknowns and boxing known variables, as you did in Chapter 1, is to create a table like Table 2.1. Creating a table to track the unknowns and equations was introduced by Zietlow (2006). It demonstrates a helpful way to track this information and determine if you have a solution to the problem. A solution is possible once there is an independent equation for each unknown.

Move to the next variable from Equation 2.1 and create Equation 2.3 for calculating the initial cost of the wall.

Table 2.1. Accounting of equations and unknowns

Equation	Unknown(s)	Measurable or Specified Variables [Known]	Minimum Remaining Equations (Unk − Eq)
2.1	$TC, IC_{in}, IC_w, IC_f, IC_c, PW_{OC}$		6 − 1 = 5
2.2		H, W_{in}, th, N_s, Ciu	6 − 2 = 4
2.3		W_w, Cwu	6 − 3 = 3
2.4	$\dot{Q}_{\max,h}$	Cfu, e_f, η	7 − 4 = 3
2.5	$R_i, R_{s,i}, R_{eq}, R_{s,o}, R_o$	$T_{bp}, T_{o,min}$	12 − 5 = 7
2.6	A	h_i	13 − 6 = 7
2.7	W		14 − 7 = 7
2.8			14 − 8 = 6
2.9		$th_{s,i}, k_{s,i}$	14 − 9 = 5
2.10	R_w, R_{in}		16 − 10 = 6
2.11	A_w	k_w	17 − 11 = 6
2.12			17 − 12 = 5
2.13	A_{in}	k_{in}	18 − 13 = 5
2.14			18 − 14 = 4
2.15		$th_{s,o}, k_{s,o}$	18 − 15 = 3
2.16		h_o	18 − 16 = 2
2.17	$\dot{Q}_{max,c}$, COP_i	Cua, COP, e_a	20 − 17 = 3
2.18		$T_{o,max}$	20 − 18 = 2
2.19		T_{inside}	20 − 19 = 1

Equation	Unknown(s)	Measurable or Specified Variables [Known]	Minimum Remaining Equations (Unk − Eq)
2.20	OC, PA		$22 - 20 = 2$
2.21	OC_h, OC_c		$24 - 21 = 3$
2.22	E_p	Ceu_h	$25 - 22 = 3$
2.23	Q_o		$26 - 23 = 3$
2.24	Q_l		$27 - 24 = 3$
2.25		DD_h	$27 - 25 = 2$
2.26	$Work$	Ceu_c	$28 - 26 = 2$
2.27	Q_c		$29 - 27 = 2$
2.28	Q_g		$30 - 28 = 2$
2.29		DD_c	$30 - 29 = 1$
2.30		i,n	$30 - 30 = 0$

$$IC_w = H \times W_w \times th \times N_s \times Cwu \tag{2.3}$$

where,
W_w is the width of the wood [m]
Cwu is the cost of the wall per unit volume [\$/m^3]

Up to this point anyone with a high school education can develop the equations. However, the next term in Equation 2.1 will require knowledge of parameter optimization. The initial cost of the furnace is determined from Equation 2.4, which accounts for both the size of the furnace and its efficiency. The form of Equation 2.4 was selected so that as the efficiency approaches 100 percent the initial cost goes to infinity. The multiplier (Cfu) and exponent (e_f) are parameters which can be optimized by minimizing the error between the output of Equation 2.4 and supplier cost data. The procedure for empirical parameter optimization is given in Chapter 1 within the section with the same title. An example of how this parameter optimization is performed for the initial cost of a furnace is given later in this chapter.

$$IC_f = \left(\dot{Q}_{max,h} \times Cfu\right) / (1-\eta)^{e_f} \tag{2.4}$$

where,
$\dot{Q}_{max,h}$ is the maximum rate of heat loss for heating [kW]
Cfu is the unit cost for the furnace [$/kW]
η is the efficiency of the furnace in fraction form in contrast to percentage [-]
(e_f) is the exponent determined by applying parameter optimization to supplier data [-]

In Equation 2.4, the heat transfer rate cannot be measured directly, so add it to the list of unknowns. Apply a thermal resistance network to the wall (see Figure 2.2) to determine the rate of heat transfer under design conditions as shown in Equation 2.5. One-dimensional, steady-state conditions are assumed with heat flow in the direction perpendicular to the wall.

$$\dot{Q}_{max,h} = \frac{T_{bp} - T_{o,min}}{R_i + R_{s,i} + R_{eq} + R_{s,o} + R_o} \tag{2.5}$$

where,
T_{bp} is the balance point temperature [°C]
$T_{o,min}$ is the minimum outdoor temperature under design conditions [°C]
R_i is the convective thermal resistance of the air on the inside of the wall [°C/kW]
$R_{s,i}$ is the conductive thermal resistance of the sheetrock on the inside of the wall [°C/kW]
R_{eq} is the equivalent conductive thermal resistance of the insulation and wood [°C/kW]

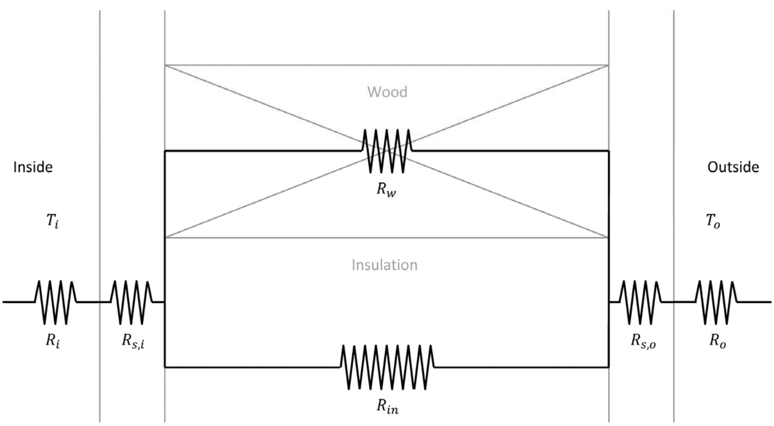

Figure 2.2. Thermal resistance network through a composite wall

$R_{s,o}$ is the conductive thermal resistance of the siding on the outside of the wall [°C/kW]
R_o is the convective thermal resistance of the air on the outside of the wall [°C/kW]

The balance point temperature is the outdoor air temperature where the heat loss through the envelope equals the heat gain (e.g., lights, equipment, solar radiation, and people) within the structure. Find the balance point temperature by repurposing Equation 2.5. Replace the numerator with the temperature difference between the thermostat set point and the outdoor air temperature. Set the heat transfer rate equal to the rate of heat gains in the conditioned space. Solve for the outdoor air temperature to find the balance point.

Next, returning back to the original Equation 2.5, find the minimum outdoor air temperature in the *ASHRAE Handbook of Fundamentals*. For the heating season there are two different values which depend on owner's tolerance for maintaining the temperature within the structure. If the interior temperature needs to be maintained 99.6 percent of the time, then use the lower outdoor temperature. Otherwise, the higher outdoor temperature will maintain the interior temperature 99.0 percent of the time.

Equation 2.5 introduces five new unknowns with the thermal resistances. In Table 2.1, there are now twelve unknowns at this point with only five equations. At least seven more equations are needed to determine the total life-cycle cost. Systematically, moving from left to right in Equation 2.5, Newton's law of cooling and Ohm's analogy will provide one of the equations for the inside thermal resistance of the air as shown in Equation 2.6. Cengel (2007, pp. 133–134) provides an excellent review of the application of Ohm's analogy to Newton's law of cooling.

$$R_i = \frac{1}{h_i A} \tag{2.6}$$

where,
h_i is the convective heat transfer coefficient on the inside surface of the wall [kW/m²-°C]
A is the total area the wall normal to the direction of heat transfer [m²]

For Equation 2.6 the area is not directly measured, therefore, count it as an unknown and write an additional equation using geometry as shown in Equation 2.7. The convective heat transfer coefficient was not counted as an unknown since the ASHRAE *Handbook of Fundamentals* (ASHRAE, 2001, p. 25.2 [Table 1: Surface Conductances and Resistances for Air]) has a table which covers this geometry and operating conditions.

You may be tempted to use an empirical correlation for the convective heat transfer coefficient to make the heat transfer model more general. But you will find when the results of the sensitivity analysis are presented in Figure 2.9 that this will add unnecessary complexity to the model with negligible improvement in accuracy. Looking at it from a theoretical standpoint, the thermal resistance of the insulation is much greater than the convective resistances on either side of the wall. Therefore, the convection resistances will have little impact on the overall heat transfer rate and in turn the operating costs.

From geometry, in particular the area of a rectangle, determine the cross-sectional area of the structure's envelope.

$$A = H \times W \times N_s \tag{2.7}$$

where,
W is the width of each section which is composed of wood and insulation as given in Equation 2.8.

$$W = W_w + W_{in} \tag{2.8}$$

Now with Equation 2.8, all the variables for the first unknown from Equation 2.5 have been specified. Next evaluate the second unknown ($R_{s,i}$). Fourier's law of conduction along with Ohm's analogy provides the bridge between variables which are able to be measured or specified and this unknown as shown in Equation 2.9. Again, Cengel (2007, pp. 133–134) provides an excellent review of applying Ohm's analogy to Fourier's law of conduction.

$$R_{s,i} = \frac{th_{s,i}}{k_{s,i}A} \tag{2.9}$$

where,
$th_{s,i}$ is the thickness of sheetrock on the inside surface [m]
$k_{s,i}$ is the thermal conductivity of sheetrock on the inside surface [kW/m-°C]

For the third unknown from Equation 2.5 there are two different materials, insulation and wood, through which the heat is being transferred in parallel. The electrical resistance analogy for two resistances in parallel provides an equation to account for this case as shown in Equation 2.10. For a review of the derivation of Equation 2.10 please refer to Cengel (2007, p. 147).

$$R_{eq} = \frac{R_w R_{in}}{R_w + R_{in}} \quad (2.10)$$

where,
R_w is the thermal resistance of the wood studs [°C/kW]
R_{in} is the thermal resistance of the insulation [°C/kW]

Since both of these resistances are conduction resistances reapply Ohm's law to Fourier's law of conduction. This produces Equations 2.11 and 2.13.

$$R_w = \frac{th}{k_w A_w} \quad (2.11)$$

where,
k_w is the thermal conductivity of the wood [kW/m-°C]
A_w is the area of the wood normal to the direction of heat flow [m^2]

Use geometry to calculate the area as seen in Equation 2.12.

$$A_w = H \times W_w \times N_s \quad (2.12)$$

$$R_{in} = \frac{th}{k_{in} A_{in}} \quad (2.13)$$

where,
k_{in} is the thermal conductivity of the insulation [kW/m-°C]
A_{in} is the area of the insulation normal to the direction of heat flow [m^2]

Use geometry to determine the area of insulation as shown in Equation 2.14.

$$A_{in} = H \times W_{in} \times N_s \quad (2.14)$$

The next unknown in Equation 2.5, $R_{s,o}$, can now be evaluated since all the variables related to the equivalent resistance, R_{eq}, can be measured or specified. Just as with $R_{s,i}$, apply Fourier's law of conduction to the outside layer of siding.

$$R_{s,o} = \frac{th_{s,o}}{k_{s,o} A} \quad (2.15)$$

where,
$th_{s,o}$ is the thickness of siding on the outside [m]
$k_{s,o}$ is the thermal conductivity of siding on the outside [kW/m-°C]

To complete Equation 2.5, evaluate the last unknown, the convective thermal resistance of the outdoor air, (R_o), using Newton's law of cooling and Ohm's analogy.

$$R_o = \frac{1}{h_o A} \tag{2.16}$$

where,
h_o is the convective heat transfer coefficient on the outside surface of the wall [kW/m^2-°C]

With the last variable from Equation 2.5 specified, calculate the maximum heat transfer rate for heating using Equations 2.5 to 2.16. This in turn allows you to evaluate the initial cost of the furnace using Equation 2.4. Return to Equation 2.1 where the next variable is the initial cost of the air-conditioning system. The initial cost (IC_c) is a function of the size, or cooling capacity, of the cooling system and its coefficient of performance (COP). As the COP approaches the ideal or Carnot COP (COP_i) the initial cost approaches infinity. The form of the equation which captures both the effect of the capacity and COP on initial cost is given in Equation 2.17. For this empirical relationship, determine the multiplier (Cua) and exponent (e_a) using parameter optimization techniques on supplier's data. This process is similar to the one used to determine the empirical parameters for Equation 2.4 where the initial cost of the furnace is calculated. Assume steady-state conditions so that the rate of heat transferred to the evaporator of the air conditioner equals the maximum rate of heat gain through the envelope to determine the size of the air conditioner.

$$IC_c = \dot{Q}_{max,c} \frac{Cua}{\left[COP_i - COP\right]^{e_a}} \tag{2.17}$$

where,
$\dot{Q}_{max,c}$ is the maximum rate of heat gain through the envelope [kW]
Cua is the empirical multiplier relating capacity to initial cost [$/kW]
COP_i is the ideal or Carnot COP
COP is the coefficient of performance of the air-conditioning system [-]
e_a is the empirical exponent relating performance to initial cost [-]

Determine the maximum rate of heat transfer through the walls by using Ohm's analogy for a design day from the cooling season. This is similar to determining the size of the furnace, Equation 2.5, except the heat is flowing from the outside to the inside. For simplicity, assume moisture transfer, and air exchanges are negligible. Refer to Stoecker (1982, Chapter 4), Kuehn (1998) or ASHRAE (2013) for details on how to perform a complete cooling load analysis.

$$\dot{Q}_{max,c} = \frac{T_{o,max} - T_{bp}}{R_i + R_{s,i} + R_{eq} + R_{s,o} + R_o} \tag{2.18}$$

where,

$T_{o,max}$ is the maximum outdoor air temperature at design conditions $[°C]$

The relationship between the Carnot COP and the temperatures of the thermal reservoirs was developed using a practical approach by Zietlow (2014) and is given in Equation 2.19.

$$COP_i = \frac{T_{inside} + 273[°C]}{T_{o,\max} - T_{inside}} \tag{2.19}$$

The temperatures of the thermal reservoirs are the indoor temperature of the air which is set by the control system (i.e., thermostat) and the outdoor temperature of the air at design conditions (i.e., maximum outdoor temperature). The Handbook of Fundamentals by ASHRAE (2009, Appendix to Chapter 14 entitled, "Design Conditions for Selected Locations") has values for the design temperature for locations throughout the world. The Handbook reports three values depending on the performance requirements of the air-conditioning system during hot weather. The best performing system will not meet the cooling requirements only 0.4 percent of the time. The next best system will not meet cooling requirements 1.0 percent of the time. Finally, the lowest performing system will not meet requirements 2.0 percent of the time.

This leaves the last unknown in Equation 2.1, the present worth of operating cost. Multiply the annual operating cost (*OC*) by the discount factor (*PA*) as shown in Equation 2.20. The discount factor takes a uniform distribution of expenditures and brings them up to today's value (see Figure 1.8 in Chapter 1). This allows direct comparison of the operating costs with the initial cost. Otherwise, the operating cost cannot be accurately compared because typically future money is less valuable than today's money. For the distribution of expenditures to be uniform, you need to assume fuel prices do not escalate; the heat exchanger surfaces remain free of scale build up, and the efficiencies of the fans and compressor stay constant. Since all of these assumptions are unlikely you may want to increase energy costs above the current prices to account for these factors. Later in the chapter you will perform a sensitivity analysis to find how sensitive the optimum insulation thickness is to energy costs.

$$PW_{OC} = OC \times PA \tag{2.20}$$

where,
OC is the annual operating costs [$/yr]
PA is the discount factor for a uniform distribution of expenditures [yr]

In Equation 2.20 you assume the lives of the furnace and air conditioner are the same. Typically, the life of an air-conditioning system is 15 years and a furnace is 18 years (ASHRAE Applications Handbook). Therefore, truncate the analysis to the shorter life, in this case 15 years. This will keep the analysis relatively simple without sacrificing much accuracy since the discount factor (*PA*) varies by only 7 percent between year 15 and 18 with a time value of money equal to 10 percent. For other optimization problems you may find the typical service life of other pieces of heating, ventilating, and air-conditioning (HVAC) equipment in the same table of the *ASHRAE Applications Handbook* (ASHRAE, 2007, p. 36.3 [Table 4: Comparison of Service Life Estimates]).

Last, assume maintenance costs are not a function of the thickness of the insulation. Since most maintenance for walls are performed at the surfaces of the wall, which are independent of the insulation, it is reasonable to assume changes in maintenance costs are negligible with respect to insulation thickness.

Sum the energy costs for heating (OC_h) and cooling (OC_c) as shown in Equation 2.21 to find the annual operating costs. You need to assume again the lives of both systems are the same to use Equation 2.21.

$$OC = OC_h + OC_c \tag{2.21}$$

Multiply the total energy purchased for the furnace with the cost of energy on a unit basis for the heating fuel to obtain the operating costs for the furnace as shown in Equation 2.22.

$$OC_h = E_p \times Ceu_h \tag{2.22}$$

where,
E_p is the energy purchased for the furnace over the heating season [kW-hr/yr]
Ceu_h is the unit cost of fuel [$/kWh]

Use the definition of efficiency, useful result over required input, to determine the energy purchased.

$$\eta = \frac{Q_o}{E_p} \tag{2.23}$$

where,
η is the efficiency of the heating system [kW_{out}/kW_{in}]
Q_o is the heat output of the furnace over a typical heating season [kW-hr/yr]

Next, find the heat output of the furnace using the conservation of energy applied to the conditioned space. The heat output of the furnace is lost through the wall (Q_l) over the heating season. The change in energy stored is zero since the conditioned space is maintained at a constant temperature.

$$Q_o = Q_l \tag{2.24}$$

Apply a thermal resistance network to the wall to determine the heat transferred through the wall over the heating season as it was in Equation 2.5. To account for the annual energy consumption the temperature difference, ($T_{bp} - \overline{T}_o$) for each day of the heating season is summed through the year to yield degree-days (DD_h). Where $\overline{T}_o$ is the average outdoor temperature for the day. Multiply temperature difference by time, in this case one day, so the result is units of energy rather than rate of energy.

$$Q_l = \frac{DD_h \times 24 \frac{hr}{day}}{R_i + R_{s,i} + R_{eq} + R_{s,o} + R_o} \tag{2.25}$$

where,
DD_h is the number of degree-days during the heating season [°C-day/yr]
Q_l is the energy loss through the walls over the heating season [kW-hr/yr]

For the air-conditioning system, the work ($Work$) into the compressor typically requires electrical energy. Take the product of the work and cost per unit of electrical energy (Ceu_c) to determine the operating costs for cooling.

$$OC_c = Work \times Ceu_c \tag{2.26}$$

where,
$Work$ is the electrical energy into the compressor over the cooling season [kW-hr/yr]
Ceu_c is the cost per unit of electrical energy [$/kW-hr]

Use the definition of COP to find $Work$ as shown in Equation 2.27. In general, the COP is the useful effect divided by the required input. In this case, the COP is the cooling energy produced by the air-conditioning system divided by the $Work$ done to the compressor. Of course anyone with a high COP will have to work less to accomplish the same results, this is especially useful for a great work–life balance. On the other hand, in a competitive job market the higher your COP the more you can accomplish for the same amount of work.

$$COP = \frac{Q_c}{Work} \tag{2.27}$$

Use the conservation of energy for the conditioned space to find the cooling energy (Q_c). Since the temperature of the space is held constant the change in stored energy within the space is zero. Therefore, the cooling energy equals the energy gain (Q_g) through the walls over the entire cooling season as shown in Equation 2.28.

$$Q_c = Q_g \tag{2.28}$$

Determine the energy gain in a way similar to heat loss in Equation 2.25 except the direction of heat transfer is into the conditioned space. From daily weather data, sum the driving potential ($\overline{T}_o - T_{bp}$) over the cooling season to determine the cooling degree-days (DD_c). To keep the analysis simple consider only conduction heat transfer so the thermal resistances are the same as they were in the heating analysis. Solar radiation heat gains should be included in the calculation of the balance point temperature. Equation 2.29 is the result.

$$Q_g = \frac{DD_c \times 24 \frac{hr}{day}}{R_i + R_{s,i} + R_{eq} + R_{s,o} + R_o} \tag{2.29}$$

where,
Q_g is the heat gain into the conditioned space over the cooling season in (kW-hr/yr).
DD_c are the cooling degree-days summed over the cooling season (C-day/yr)

Use the discount factor (*PA*) for an annual series from Engineering Economics as shown in Equation 2.30.

$$PA = \frac{[1+i]^n - 1}{i \times [1+i]^n} \tag{2.30}$$

where,
i is the time value of money [$\$_{\text{interest}}/\$_{\text{principal}}$]
n is the number of annual payments over the life of the insulation [#]

You can now solve the problem since each unknown has an independent equation associated with it as shown in Table 2.1. You are able to measure or specify all the remaining variables. In summary, for the deductive problem-solving strategy, start with the objective and develop the equations while systematically analyzing each of the unknowns until all the independent equations have been identified and all the remaining variables can be measured or specified. Distinguishing the unknowns from variables which can be specified or measured as displayed in Table 2.1 is an important aspect of the method. You need experience and common sense to be able to separate these variables. Knowledge of instrumentation and sources of reference information are key components you need to distinguish between known and unknown variables.

Calculate the first solution of these equations or the base case. The base case is the most probable case, using the data and assumptions at the time of the solution. Table 2.2 gives the values and sources for all the measured or specified variables for the example problem. The only variable excluded from this table is the thickness of the insulation, *th*, which is the design variable which is specified over a range sufficient to contain the minimum total life-cycle costs.

Select a building with a design cooling load of 10.6 kW located in Chicago, IL, for the base case. Determine the energy costs based on data collected from residential energy bills in the Chicago area in 2014. Collect the initial costs for materials from suppliers. Base the initial costs for the furnace and air conditioner on data from websites. Refer to the section of this chapter entitled “Parameter Optimization for Initial Costs” to see how these cost data were incorporated into the analysis.

Table 2.2. Specified variables for envelope and equipment analysis base case conditions

Variable	Base Value	Units	Source	Sensitivity [%]	Rank
$Ceu_{c,b}$	0.123	[$/kW-hr]	Supplier	2.8	19
$Ceu_{h,b}$	0.03	[$/kW-hr]	Supplier	24.7	8
Cfu_b	14.6	[$/kW]	Parameter optimization	1.6	21
Ciu_b	23	[$/m^3]	Supplier	12.6	11
COP_b	4.0	[kW_{cool}/kW_{in}]	AC optimization	3.2	17
Cua_b	4,650	[$/(Btu/hr)]	Supplier	2.8	19
Cwu_b	305	[$/m^3]	Supplier	10.2	12
DDc_b	468	[°C-day/yr]	Weather data	2.8	19
DDh_b	3,506	[°C-day/yr]	Weather data	24.7	8
e_{ab}	4.06	[-]	Parameter optimization	482.0	1
e_{fb}	0.5533	[-]	Parameter optimization	2.8	19
η_b	0.9	[kW_{out}/kW_{in}]	Furnace optimization	36.6	5
H_b	3	[m]	Drawings	50.0	4
$h_{i,b}$	0.005	[kW/m^2-°C]	Handbook	3.4	16
$h_{o,b}$	0.01	[kW/m^2-°C]	Handbook	1.7	20
i_b	0.1	[$\$_{interest}/\$_{principal}$]	Stock market	33.0	6
$k_{in,b}$	0.000034	[kW/m-°C]	Handbook	9.0	13
$k_{si,b}$	0.00017	[kW/m-°C]	Handbook	6.1	15
$k_{so,b}$	0.00017	[kW/m-°C]	Handbook	6.1	15

Variable	Base Value	Units	Source	Sensitivity [%]	Rank
$k_{w,b}$	0.0008	[kW/m-°C]	Handbook	13.8	10
n_b	40	[#]	Handbook	6.6	14
$N_{s,b}$	1,888	[#]	Drawings	50.0	4
$T_{bp,b}$	18.33	[°C]	Energy analysis	104.0	3
$th_{si,b}$	0.06	[m]	Drawings	3.0	18
$th_{so,b}$	0.06	[m]	Drawings	3.0	18
$T_{inside,b}$	18.33	[°C]	Thermostat setpoint	104.0	3
$T_{o,max,b}$	33.3	[°C]	Weather data	378.4	2
$T_{o,min,b}$	−20	[°C]	Weather data	0.8	22
$W_{in,b}$	0.4	[m]	Drawings	29.6	7
$W_{w,b}$	0.025	[m]	Drawings	23.2	9

Vary the design variable, insulation thickness, and solve the set of equations. Use the plot shown in Figure 2.3 to locate the minimum total life-cycle costs of the entire system. For the base case, the optimum thickness for the insulation is 0.19 meters or 7.5 inches. Numerically, the optimum is found by using a univariant optimization routine, specifically golden section, which searches for the minimum present worth of the total costs by varying the thickness of the insulation. An astute observer realizes there is a significant amount of uncertainty in each of the variables which are specified in Table 2.2. For example it is impossible to predict what fuel costs will be for the next forty years. Or for that matter, will the furnace and insulation for any particular building endure for that length of time? Therefore, another useful tool for the design engineer is the sensitivity analysis.

Hold all the specified variables at a constant value except one to conduct sensitivity analysis. Exercise the independent variable over a range to determine how sensitive the result is to the variable. The result in this analysis is the minimum total life-cycle costs. It is particularly useful if you present the independent variable in a nondimensional form. You can

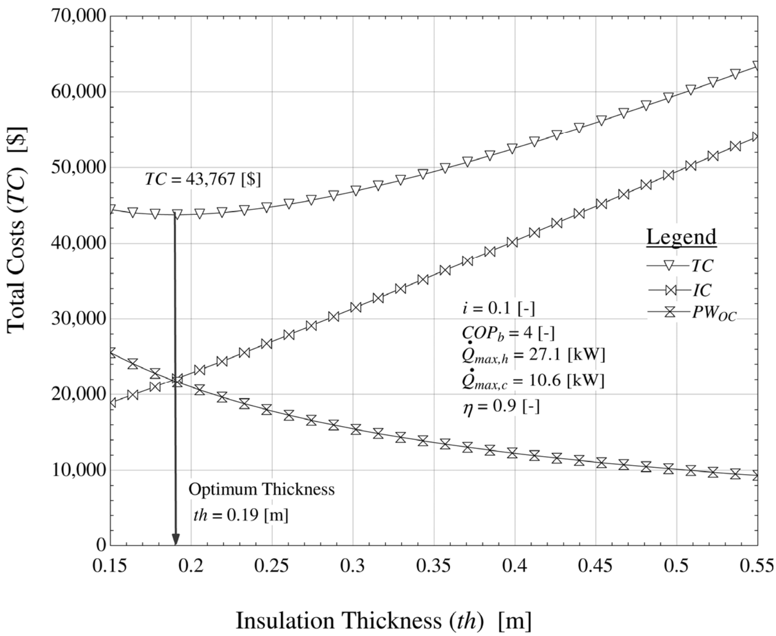

Figure 2.3. Detailed costs, in today's dollars, as a function of insulation thickness (EES, 2014)

then assess the relative sensitivity between the different variables. One way to nondimensionalize is to calculate the value of the variable as a percent difference from the base case as shown in Equation 2.31.

$$P = \frac{(SV - SV_b) \times 100}{SV_b} \tag{2.31}$$

where,
SV is the specified variable (e.g., *Ceu, n*, etc.)
SV_b is the value of the specified variable at the base case as given in Table 2.2
P is the percentage change from the base case [%]

Figures 2.4 through 2.9 contain the results of the sensitivity analysis for the specified variables in this problem. As the specified variable changes, the optimum (minimum) total life-cycle costs are determined. The steeper the slope of the lines, the more sensitive the total costs and the corresponding insulation thickness is to the specified variable. Please note: From one figure to the next, in sequential order, the range of the scale decreases. Each progressive figure displays the trends of variables that are less sensitive than the previous figure. The range of the scale is reduced so it is easy to distinguish the effect of one variable from another.

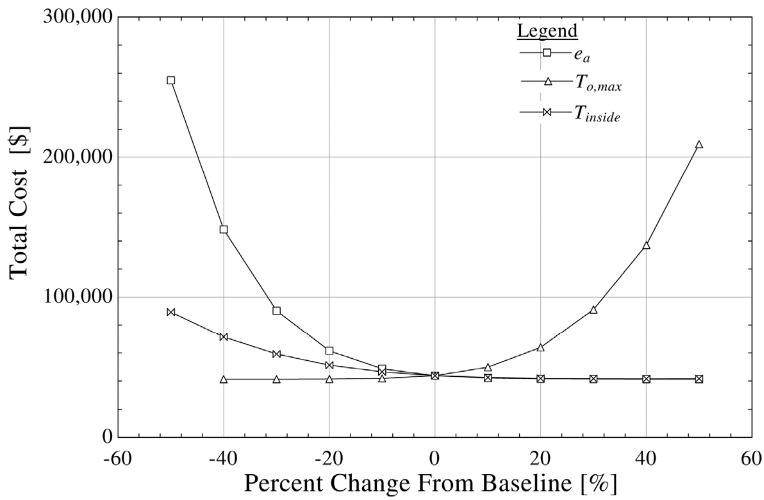

Figure 2.4. Nondimensional sensitivity analysis results—maximum (EES, 2014)

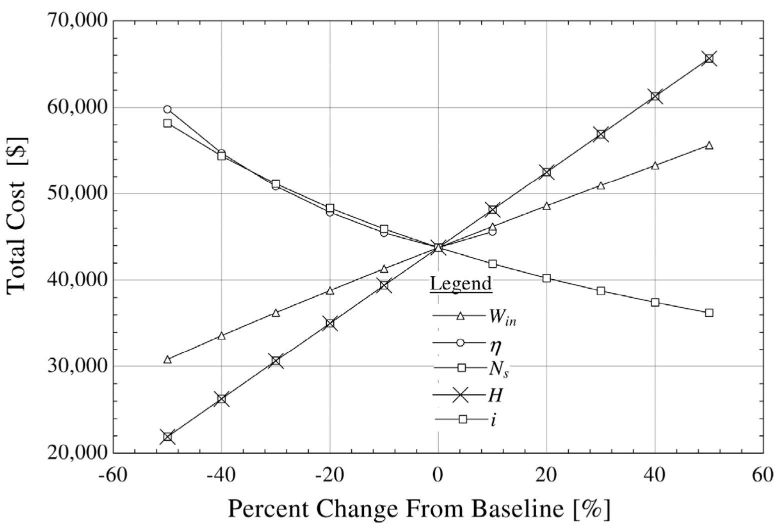

Figure 2.5. Nondimensional sensitivity analysis results—high (EES, 2014)

From Figure 2.4 it is clear the optimum insulation thickness is most sensitive to changes in exponent for the initial cost of the air conditioner (e_a) and the maximum outside air temperature ($T_{o,max}$). For a fifty percent change from the baseline case each of these variables produced a 480 and 380 percent increase, respectively, in the total cost at the optimum thickness. In contrast the cost coefficient of the furnace (*Cfu*) and the outdoor convective heat transfer coefficient (h_o) had little influence on the optimum insulation thickness and the associated cost as seen in Figure 2.9.

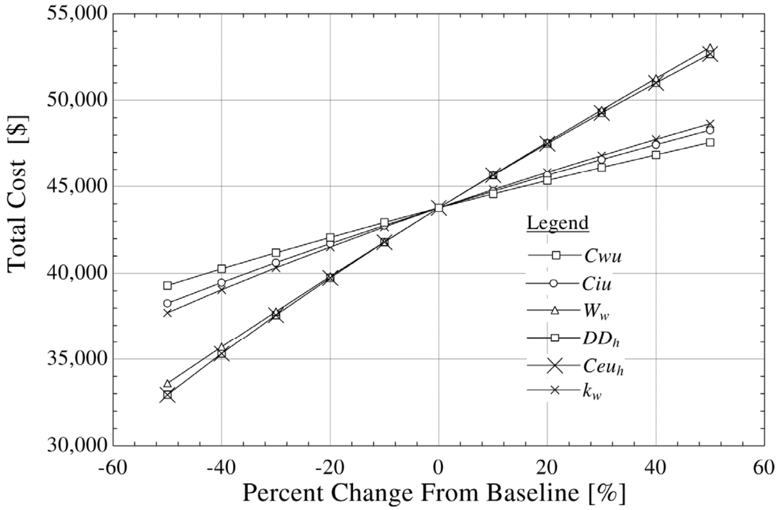

Figure 2.6. Nondimensional sensitivity analysis results—moderately high (EES, 2014)

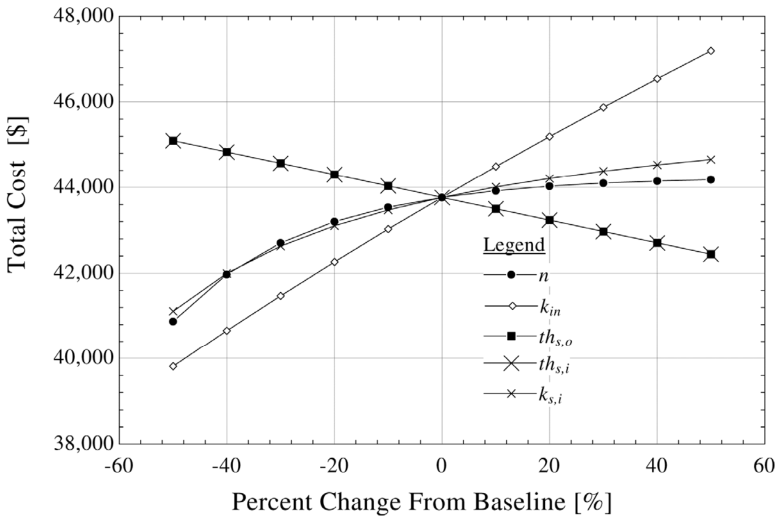

Figure 2.7. Nondimensional sensitivity analysis results—moderately low (EES, 2014)

The last column of Table 2.2 contains the rank of the sensitivity of the total cost to the specified variable. The key benefit of this rank is it provides a priority for how much effort to spend in improving the accuracy of the input value. You will need to spend significantly more effort on the specified variables with a high sensitivity (rank = 1) than those with a low (rank = 22).

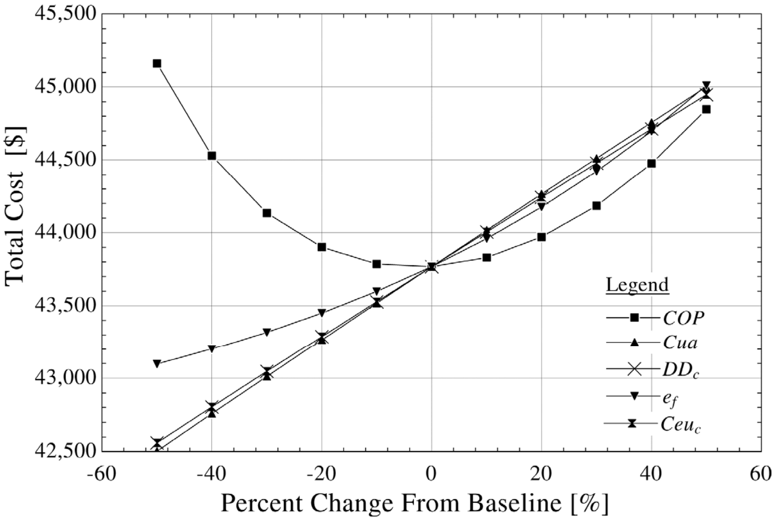

Figure 2.8. Nondimensional sensitivity analysis results—low (EES, 2014)

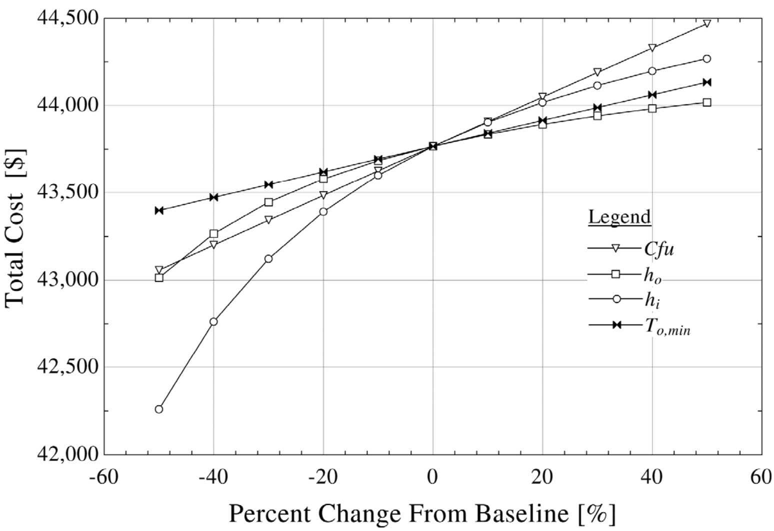

Figure 2.9. Nondimensional sensitivity analysis results—minimum (EES, 2014)

You have seen the benefits of sensitivity analysis and how to interpret the results. You have seen how converting the independent variable to nondimensional form allows you to compare one variable to another. Use caution when applying the results of the sensitivity analysis to other applications because the results are unique to the example presented. Therefore, the rank of the variables may change for different envelopes in different locations.

PARAMETER OPTIMIZATION FOR INITIAL COST

Earlier, Equations 2.4 and 2.17 were presented to determine the initial cost of either the furnace or the air conditioner, respectively, as a function of size and performance. In this section you will see an example of how to conduct an optimization of the parameters (a multiplier and an exponent) for these initial cost equations. First, collect data from the supplier, this is the experimental data. Table 2.3 contains the sample of initial cost data for the furnace used.

Table 2.3. Initial cost data for a furnace

Experiment [#]	Capacity [kW]	Efficiency [-]	Initial Cost [$]
1	11.72	0.80	535
2	17.58	0.80	650
3	23.45	0.80	850
4	29.31	0.80	975
5	35.17	0.80	1,100
6	11.72	0.95	1,150
7	17.58	0.95	1,250
8	23.45	0.95	1,748
9	29.31	0.95	2,150
10	35.17	0.95	2,650

(http://www.furnacepriceguides.com/gas-furnace/. Accessed 1/17/14)

To perform a parameter optimization, enter an initial guess for the empirical parameters (C and e) and specify the total number of experimental data points (n) as shown in Figure 2.10. Set the index to one, and then input the experimental data you collected from the supplier(s). For each set of data, calculate a corresponding value of the initial cost using the model in Equation 2.4 or 2.17 for the furnace or air conditioner, respectively. Determine how close the model is to the experiment by

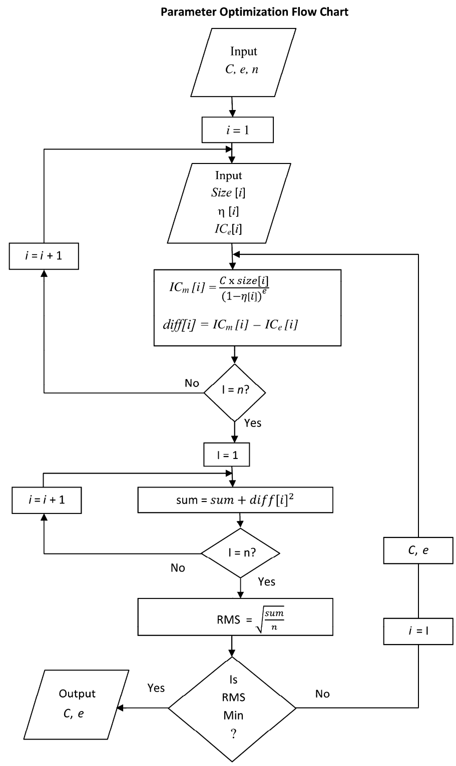

Figure 2.10. Parameter optimization flow chart

calculating the difference. Sum all the squares of the difference, divide by the number of data points and compute the square root. This yields the root mean square (rms) error, which is the objective function of the parameter optimization. If the rms error is not at a minimum, generate new empirical parameters using an optimization algorithm and repeat the process until the rms error is at a minimum.

where,
n is the number of data points
i is the index tracking each data point
Size is the extent of the equipment measured in rate of heat transfer [kW]
η is the efficiency of the equipment [-]
IC_e is the experimental data of initial cost [$]
IC_m is the model predicting initial cost [$]
C is the empirical multiplier
e is the empirical exponent
diff is the difference between the model compared to the experiment
RMS is the root mean square error between the model and the experiment

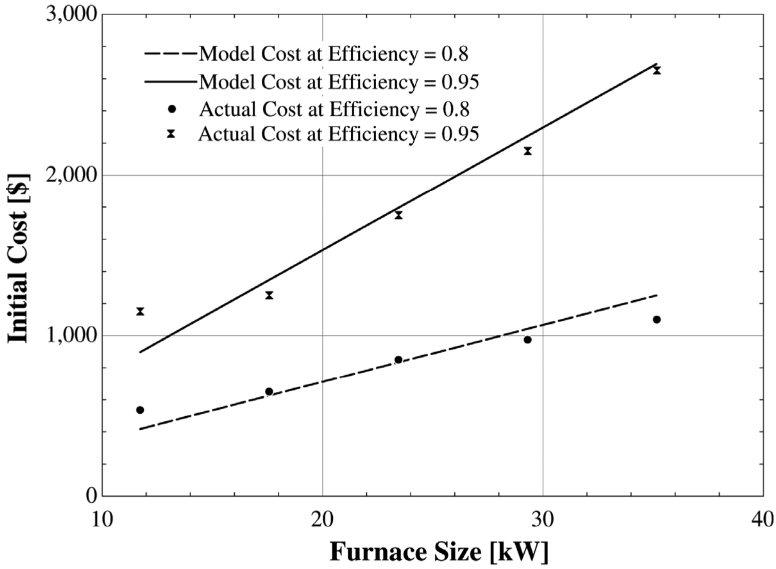

Figure 2.11. Initial cost of furnaces: model and experiment (EES, 2014)

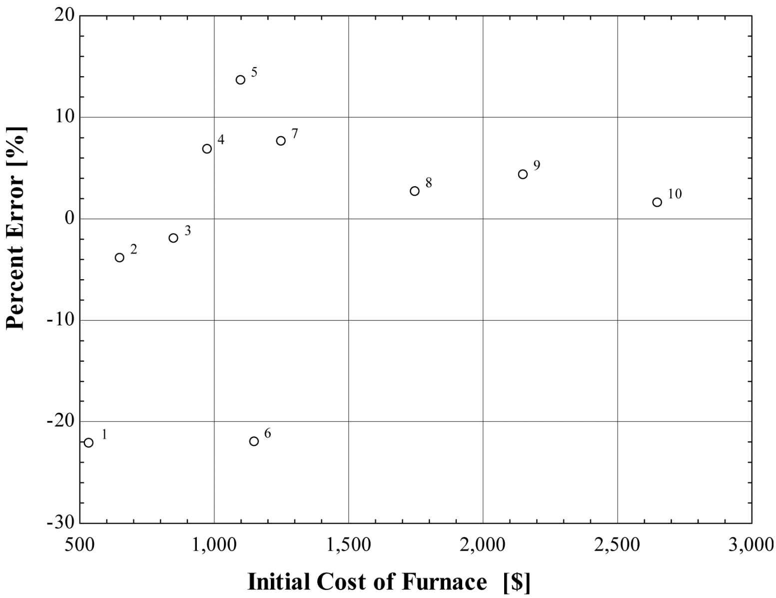

Figure 2.12. Percent error in model of initial cost for furnace (EES, 2014)

Plot the difference of initial cost between the model and experiment, as shown in Figure 2.11, as a function of the experimental initial cost to determine if the differences are within an acceptable range. If not, you may need to either collect more experimental data, in this case supplier quotes, or develop another model form. Another useful way to present the error data is shown in Figure 2.12.

Out of ten data points, all but two data points are within ±15 percent of the experimental data. These two points occur when the capacity of the furnace drops below 15 kW. When you drop these two points the model is reasonably accurate.

Initial cost data were also collected for a range of cooling systems as shown in Table 2.4. These data were then used to perform parameter optimization with Equation 2.17. The results of the model, Equation 2.17 with optimized parameters, were compared to the experimental data from Table 2.4 in Figure 2.8.

Table 2.4. Initial cost data for cooling systems

Experiment [#]	Capacity [BTU/hr]	SEER [BTU/W-hr]	Cost [$]
1	24,000	13	2,229
2	24,000	18	2,254
3	36,000	13	2,529
4	36,000	13	2,299
5	36,000	18	3,500
6	36,000	18	4,068
7	48,000	13	3,100
8	48,000	13	3,146
9	48,000	13	2,899
10	48,000	18	4,575

Figure 2.13 shows how the supplier data contained in Table 2.4 compares with the model. Out of the ten cost data points the model predicts nine initial costs within fifteen percent error as shown in Figure 2.14. The one outlier is for the air conditioner with the lowest capacity and lowest COP.

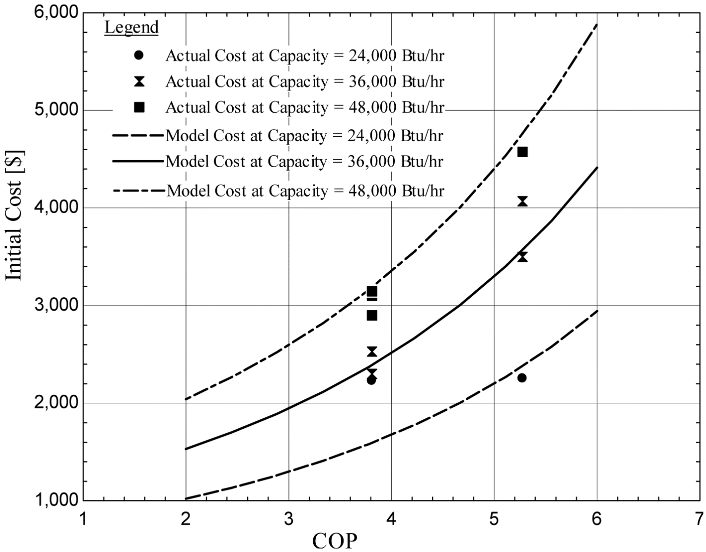

Figure 2.13. Initial cost of cooling systems: model and experiment (EES, 2014)

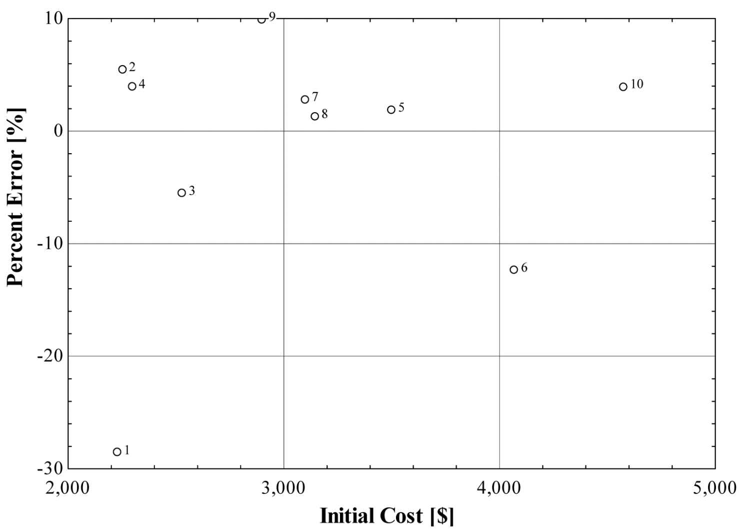

Figure 2.14. Percent error for model of initial cost for cooling systems (EES, 2014)

OPTIMUM COOLING SYSTEM

So far in Chapter Two you have assumed the COP is constant at a baseline value of 4.0 as you varied the insulation thickness. Another valuable analysis is to vary the COP keeping the insulation thickness constant.

Figure 2.15 shows the result of this variation of COP on the relevant costs. This type of analysis is valuable for the owner of the conditioned space. It differs from the original equipment manufacturers' analysis presented in Chapter One since the design variables, which cause the change in COP, are not directly connected to the optimization.

In Chicago, IL there are seven and one half times more heating degree-days than cooling. Using the same baseline holding the insulation thickness constant the COP was varied to find the minimum total cost.

Compared to the optimization of insulation thickness in Figure 2.3 the COP dropped from a baseline of 4.0 to the optimum of 3.886. The total cost (TC) dropped from $43,764 by three dollars. The next step would be to take this new optimum COP of 3.886 and rerun the insulation thickness optimum. It turns out for this case the optimum insulation thickness remains unchanged at 0.19 m. This analysis assumes the cooling system lasts as long as the insulation or 40 years. If the cooling system only lasted 15 years then the optimum COP would be 3.59. The reason the optimum COP drops with the shorter life is because it needs to be replaced sooner. More frequent replacement places a higher weight on the initial cost of the system compared to the operating cost. The optimum insulation thickness remains at 0.19 m even when the COP of 3.59, based on the 15 year life, is used. Practically, you will need to replace the cooling system twice during the life of the building envelope.

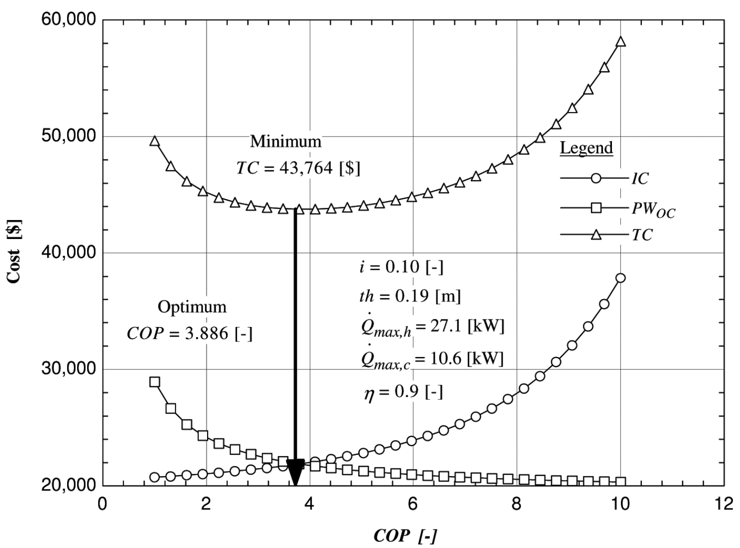

Figure 2.15. Optimum cooling system for Chicago, IL (EES, 2014)

In an application with a larger cooling load along with a larger difference between the baseline and optimum COP you may need to perform a few iterations using this sequential approach. Using a multivariate optimization routine will enable you to converge on not only the optimum insulation thickness and COP but also the optimum furnace efficiency.

Using a multivariate optimization routine changes the furnace efficiency from a baseline of 90 to 93.5 percent for a 15-year life for the furnace. This change in furnace efficiency was not enough to affect the optimum insulation thickness or the COP of the cooling system. The new furnace efficiency reduces the total costs down by $370 to $43,394. Now you, the owner of the conditioned space, are ready to specify the key energy related targets for the bid documents. The competitive bidding process will help to bring the initial cost down without losing ground on the operating costs.

CHAPTER SUMMARY

In this chapter you have applied optimization techniques to the envelope of a structure whose purpose is to maintain a conditioned environment. The analysis was restricted to a wall of a structure. Your selection of the objective function required the use of tools you learned not only in heat transfer but also in engineering economics. Using the total life-cycle costs enabled you to account for the trade-offs between the initial and operating costs. Both costs are affected by the design variable which is the thickness of the insulation. You found the insulation thickness not only affected the initial cost of the insulation but also the initial cost of the structure, furnace, and air conditioner.

After performing a parameter optimization of the initial costs of the heating and cooling systems as both a function of size and efficiency you now have the capability to expand your optimization to the COP and efficiency of these systems. This analysis is useful at the owner's level of analysis. Hopefully, the original equipment manufacturer did their homework and applied the principles of optimization given in Chapter One. If so, then the owner should have available to them the equipment at the optimum level of efficiency.

You have applied the deductive problem-solving strategy, which provides structure to open-ended problems, to another component of the design process. The optimization of wall insulation is the type of open-ended problem engineers encounter in the workplace. Unfortunately, open-ended problems are more challenging to solve than typical textbook problems where all the necessary inputs are provided. The deductive approach is a useful tool to aid you in achieving a successful solution and developing your creativity in the process since open-ended problems promote creativity.

The deductive method is summarized in Appendix A of Chapter 1 and is applied to the optimization of insulation. Instead of boxing and circling variables to track the unknowns a table was used to account for the number of unknowns and associated equations. The extra work involved in tracking variables using the table illustrates the value of boxing and circling variables.

You will find the deductive approach a useful tool to solve open-ended problems. The majority of students who have learned this method recognize the benefits of this systematic approach. These benefits include providing a starting point for the solution of a complex problem, assistance in breaking the problem down into manageable parts, and leading one to the next step in the solution. Many students report that they use this method to assist them in solving problems in other classes.

If you are an educator, consider introducing more open-ended problems in the undergraduate experience. Provide the students with instruction on the deductive approach to equip them to solve these challenging problems. If you teach heat transfer or air-conditioning classes use this sample problem in the class. For other classes the textbook problems can be adapted for this strategy by removing the specified inputs. Finally, if you design buildings be sure to spend adequate time securing accurate (<10 percent uncertainty) data for the exponent parameter of the initial cost function for the cooling system the maximum outdoor air temperature and the indoor air temperature since the optimum is most sensitive to these inputs.

Once a solution is achieved, most design engineers realize that the specified variables have uncertainty. Uncertainty is higher when you introduce economics into the solution. It is important to assess the impact of this uncertainty on the solution. Sensitivity analysis is one way to compare how each of the variables influences the solution. It allows the specified variables to be prioritized. Then you can allocate the time necessary to achieve the accuracy required to gain confidence in the solution. For the more volatile variables such as cost and time value of money, predict the average value over the life of the system.

PROBLEMS

2.1 Determine the optimum COP of the cooling system, efficiency of the furnace, and thickness of the insulation for the walls of a conditioned space located in Tucson, AZ. Except for the design temperatures and degree-days assume all the other inputs are the same as the ones used in the analysis performed in Chapter 2. Plot the initial, present worth of the operating and total costs as a function of each of the three design variables, insulation thickness, cooling system COP, and heating system efficiency. How do the results compare with the same conditioned space in Chicago, IL? Why are the optimums different?

2.2 Perform a sensitivity analysis for each of the independent variables used in the optimization for the conditioned space located in Tucson, AZ. Compared to Chicago does the rank order of the significance of the variables change? If so, what causes the order to change?

2.3 The initial costs for the heating and cooling systems just doubled. Redo the parameter optimization for the initial cost functions for these systems. How does this affect the analysis for Chicago? How does it affect the analysis for Tucson?

2.4 Your building envelop has significant internal heat gains which drops the balance point temperature down to 10°C. After adjusting the degree-days for the new balance point how is the analysis in Chicago affected where the cooling degree-days (CDD) = 1,408 and the heating degree-days (HDD) = 1,477? How does it affect the analysis in Tucson where the adjusted CDD = 3,940 and HDD = 231?

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APPENDIX A

SYSTEM OF EQUATIONS TO OPTIMIZE THE ENVELOPE OF A STRUCTURE

Equation	**Equation Number**
$TC = IC_{in} + IC_w + IC_f + IC_c + PW_{OC}$	1
$IC_{in} = H \times W_{in} \times th \times N_s \times Ciu$	2
$IC_w = H \times W_w \times th \times N_s \times Cwu$	3
$IC_f = \left(\dot{Q}_{max,h} \times Cfu\right) \big/ (1-\eta)^{e_f}$	4
$\dot{Q}_{max,h} = \dfrac{T_{bp} - T_{o,min}}{R_i + R_{s,i} + R_{eq} + R_{s,o} + R_o}$	5
$R_i = \dfrac{1}{h_i A}$	6
$A = H \times W \times N_s$	7
$W = W_w + W_{in}$	8
$R_{s,i} = \dfrac{th_{s,i}}{k_{s,i} A}$	9
$R_{eq} = \dfrac{R_w \times R_{in}}{R_w + R_{in}}$	10

$$R_w = \frac{th}{k_w A_w} \quad 11$$

$$A_w = H \times W_w \times N_s \quad 12$$

$$R_{in} = \frac{th}{k_{in} A_{in}} \quad 13$$

$$A_{in} = H \times W_{in} \times N_s \quad 14$$

$$R_{s,o} = \frac{th_{s,o}}{k_{s,o} A} \quad 15$$

$$R_o = \frac{1}{h_o A} \quad 16$$

$$IC_c = \dot{Q}_{\max,c} \frac{Cua}{[COP_i - COP]^{e_a}} \quad 17$$

$$\dot{Q}_{\max,c} = \frac{T_{o,max} - T_{bp}}{R_i + R_{s,i} + R_{eq} + R_{s,o} + R_o} \quad 18$$

$$COP_i = \frac{T_{inside} + 273[°C]}{T_{o,max} - T_{inside}} \quad 19$$

$$PW_{OC} = OC \times PA \quad 20$$

$$OC = OC_h + OC_c \quad 21$$

$$OC_h = E_p \times Ceu_h \quad 22$$

$$\eta = \frac{Q_o}{E_p} \quad 23$$

$$Q_o = Q_l \quad 24$$

$$Q_l = \frac{DD_h \times 24 \frac{hr}{day}}{R_i + R_{s,i} + R_{eq} + R_{s,o} + R_o} \quad 25$$

$$OC_c = Work \times Ceu_c \quad 26$$

$$COP = \frac{Q_c}{Work} \qquad 27$$

$$Q_c = Q_g \qquad 28$$

$$Q_g = \frac{DD_c \times 24\,\frac{hr}{day}}{R_i + R_{s,i} + R_{eq} + R_{s,o} + R_o} \qquad 29$$

$$PA = \frac{[1+i]^n - 1}{i \times [1+i]^n} \qquad 30$$

Nomenclature

Primary variables

A is area
Ceu_c is the cost per unit of electrical energy
Ceu_h is the unit cost of fuel
Cfu is the unit cost for the furnace
Ciu is the cost of insulation per unit volume
COP is the coefficient of performance of the air-conditioning system
COP_i is the ideal or Carnot COP
Cua is the empirical multiplier relating capacity to initial cost for the air conditioner
Cwu is the cost of the wall per unit volume
DD is the number of degree-days
e_a is the empirical exponent relating performance to initial cost for the air conditioner
e_f is the exponent determined by applying parameter optimization to supplier data for the furnace
E_p is the energy purchased for the furnace over the heating season
η is the efficiency of the furnace
h is the convective heat transfer coefficient
H is the height of the wall
i is the time value of money
IC is the initial cost
k is thermal conductivity
n is the number of annual payments over the life of the insulation
N_s is the number of sections of wall in the building
OC is the operating cost
PA is the discount factor for a uniform distribution of expenditures
PW_{OC} is the present worth of the operating costs
Q_l is the energy loss through the walls over the heating season

Q_o is the heat output of the furnace over a typical heating season

$\dot{Q}_{max,h}$ is the maximum rate of heat loss for heating

$\dot{Q}_{max,c}$ is the maximum rate of heat gain through the envelope

Q_g is the heat gain into the conditioned space over the cooling season

R is thermal resistance

T is temperature

TC is the total cost of the air conditioner, furnace, and envelope over their lifetime

th is thickness

W is the width

$Work$ is the electrical energy into the compressor over the cooling season

Subscripts

bp is the balance point

c is cooling

eq is equivalent

f is furnace

g is gain

h is heating

i is inside

in is insulation

l is loss

max is maximum

min is minimum

o is outdoor

s is siding

w is wood

CHAPTER 3

OPTIMIZATION OF CHILLERS

INTRODUCTION

A chiller is a vapor-compression system which cools water instead of air. While water improves the heat transfer in the evaporator, overall performance is reduced with the addition of an air-to-water heat exchanger and a pump. A chiller is composed of all the components shown in Figure 3.1. The heat transferred from the air is typically done in one of two ways. The most common way is to circulate the chilled water through a tube-and-fin heat exchanger. Another way heat is transferred from the air is to move the chilled water through channels within the structure itself. Either way will require a temperature difference to accomplish the heat transfer.

Starting with the analysis of an air conditioning system in Chapter 1, add the additional air-to-water heat exchanger or the chilled water heat exchanger. A schematic (Figure 3.1) of the system will help you organize the analysis. Organization becomes more important as the system increases in complexity.

The pressure versus enthalpy (P-h) diagram, based on the refrigerant, shown in Figure 3.2 also helps you organize the analysis. Superimpose, on the P-h diagram, the inlet temperatures of the air entering the chilled water heat exchanger and the condenser. The processes of three different size and efficiency chiller systems are plotted on the pressure–enthalpy diagram.

The pressure–enthalpy diagram provides useful insights into the performance of the chiller. The greater the difference between the saturation temperature of the refrigerant and the inlet temperature of air the greater the irreversibility in heat transfer. The less steep the slope for the compression process, the lower the isentropic efficiency of the compressor. Both of these trends lead to a lower coefficient of performance (COP) and greater compressor power consumption for the same application. To reverse these trends larger heat exchangers and higher efficiency compressors are needed. Both increase the initial cost of the system. As long as the

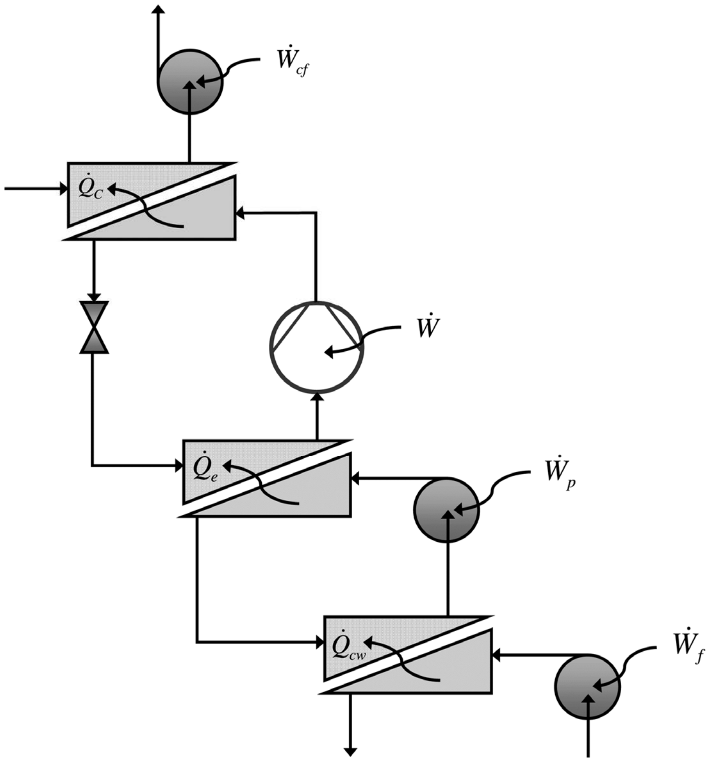

Figure 3.1. Schematic of chilled water system. Starting at the bottom of the figure a fan consuming power $(\dot{W}_f)$ is used to circulate air from the conditioned space through the air side of the chilled water heat exchanger. The chilled water heat exchanger transfers heat $(\dot{Q}_{cw})$ from the air to the chilled water. The water is circulated with a pump consuming power $(\dot{W}_P)$. After leaving the pump the water enters the water side of the evaporator where heat $(\dot{Q}_e)$ is rejected to the refrigerant. The refrigerant is circulated with a compressor which consumes power $(\dot{W})$. After leaving the compressor, the heat $(\dot{Q}_c)$ is rejected from the condenser. The refrigerant is then expanded with an expansion valve before entering the evaporator. On the air side of the condenser a fan consuming power $(\dot{W}_{cf})$ is used to circulate air through the condenser.

savings in operating costs produce an adequate return on investment, larger heat exchangers and more efficient compressors should be used. Please note there are significant changes in the size of the heat exchangers (see Table 3.1), when moving from the optimum system to the large. In contrast, the changes on the pressure–enthalpy diagram (Figure 3.2) are negligible.

On the pressure–enthalpy diagram the cycles are plotted for three different chillers. All three chillers provide the same amount of cooling at 1,187 kW. This size chiller can cool approximately 9,500 m^2 of office space designed to platinum standards established by Leadership in Energy and Environmental Design (LEED). The relevant specifications for these chillers are contained in Table 3.1.

Table 3.1. Specifications for a small, optimum, and large chiller

Chiller	Area of Chilled Water Heat Exchanger [m²]	Area of Evaporator [m²]	Area of Condenser [m²]	Isentropic Efficiency of Compressor [-]	Compressor Power [kW]	Coefficient of Performance [-]
Small	987	87	123	0.70	665	1.78
Optimum	2,604	588	2,180	0.83	295	4.02
Large	10,803	5,205	192,971	0.99	182	6.52

From Table 3.1 observe the significant increase, one to two orders of magnitude, in heat transfer surface areas required to obtain the 38 percent reduction in compressor power when moving from the optimum system to the large. The initial cost for the large system is estimated to be over $12 million. The optimum system has an initial cost of nearly $150,000. With an annual savings of $11,000 it would take over 1,000 years to recover the investment in the large system. Conversely, the small system only costs $56,000 initially. In this case though, the added investment in the optimum system provides an effective return of over 38 percent with an annual savings of nearly $36,000. It is a challenge to find that rate of return on any investment. You will find details on the chiller model and cost data later in this chapter.

As noted earlier, the inlet temperatures of air entering the chilled water heat exchanger and condenser represent the temperatures of thermal reservoirs. For a thermal reservoir a finite amount of heat can be transferred with minimal effect on the temperature of the fluid. The thermostat setting of the conditioned space sets the temperature of the air at the inlet to the chilled water heat exchanger. Finding the optimum thermostat setting for a given application has many factors which affect the occupants' productivity and are covered in other works. The outdoor climate determines the temperature of the air entering the condenser.

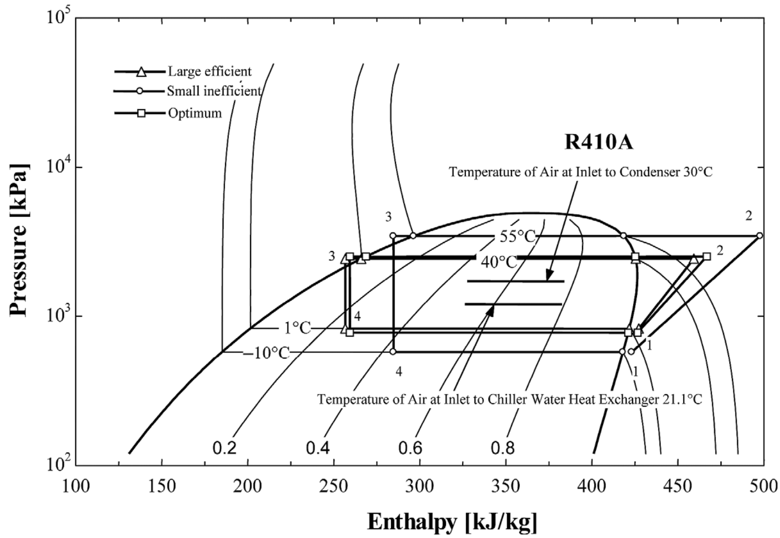

Figure 3.2. Pressure–enthalpy diagram for three different chillers. State-point one is the inlet to the compressor, two is the inlet to the condenser, three is the inlet to the expansion valve, and four is the inlet to the evaporator (EES, 2014).

Focus on the heat transfer surface area as your design variable for the heat exchangers. As the area of the chilled water heat exchanger increases so does the temperature of the water as it approaches the temperature of the air. The increase in water temperature lowers the temperature difference for transferring the fixed amount of heat from the air. A lower temperature difference reduces the irreversibility of the heat transfer. Any reduction in irreversibility will increase the performance of the cycle. Better performance reduces the power consumption for a fixed amount of cooling. The connection between heat transfer surface area and compressor power is explained more fully in Chapter One with Equations 1.14 through 1.16.

The hydraulic diameters of the flow channels on both sides of the air-to-water heat exchanger are assumed constant. In the analysis presented in Chapter 1 the hydraulic diameter was allowed to vary in order to fix the overall heat transfer coefficient (U). Keeping the hydraulic diameter constant, a more practical constraint when designing a heat exchanger, requires a reduction in flow rate as the area is increased in order to keep the heat transfer rate constant. A constant hydraulic diameter complicates the analysis since the reduction in flow rate will reduce the convective heat transfer coefficient as shown in Figure 3.3.

As the convective heat transfer coefficient drops so does the overall heat transfer coefficient. Figure 3.4 demonstrates the impact of increasing the heat transfer surface area on both the temperature difference and the

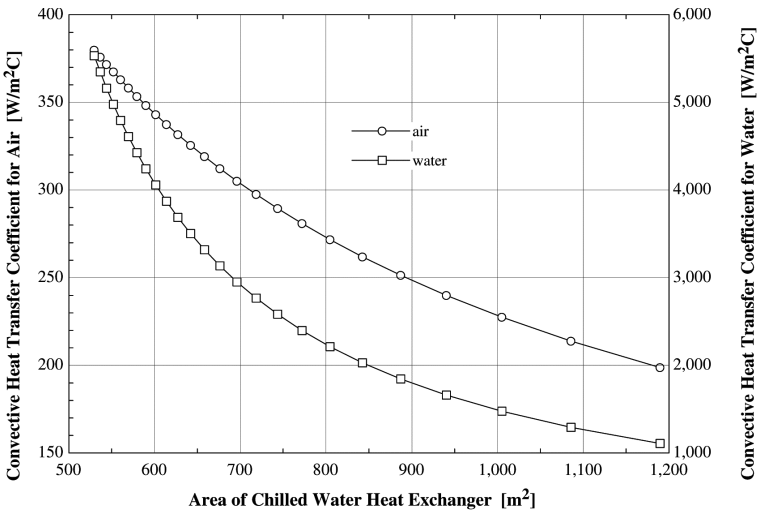

Figure 3.3. Relationship between convective heat transfer coefficients and area in order to maintain a constant heat transfer rate in the chilled water heat exchanger with constant hydraulic diameters (EES, 2014)

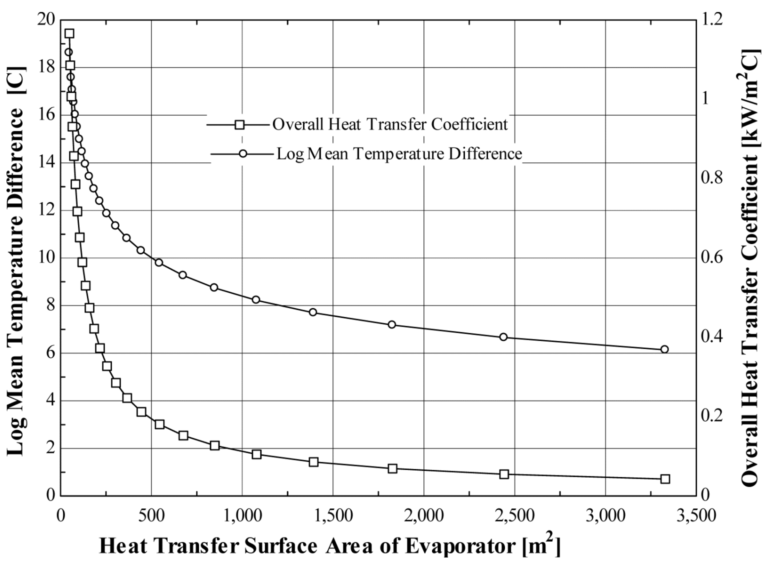

Figure 3.4. Effect of surface area on temperature difference and overall heat transfer coefficient for constant heat transfer rate and hydraulic diameters (EES, 2014)

overall heat transfer coefficient while maintaining the cooling rate and the hydraulic diameters of the fluid flow paths constant. The decrease in temperature difference will reduce the irreversibilities in heat transfer thereby increasing the COP of the cycle. You can accurately model the drop in the overall heat transfer coefficient using a simple convective heat transfer coefficient correlation, as long as it has been validated over a range of operating conditions in a similar geometry.

Currently, you are focused on the heat transfer surface area as your design variable. In a heat exchanger though there are many more design variables (e.g., fin spacing, tube spacing, tube diameter, etc.) which are beyond the scope of this work. To account for these additional variables requires the conservation of momentum. Simplifying the analysis by keeping the hydraulic diameter constant you are essentially keeping these other design variables constant since the hydraulic diameter is determined by the fin spacing, tube spacing, and tube diameter.

The analysis of the evaporator itself is simpler than it was in the practical air conditioning model that was developed in Chapter One, since the moist air is replaced with water which eliminates the latent load associated with moist air and the film thickness that goes with it. Therefore, the simple air conditioning model with a dry evaporator can be quickly adapted for water by replacing air properties with water properties on the nonrefrigerant side of the evaporator. With this adapted model, you are now able to optimize each of the components of the vapor-compression system. But first, the temperature of the chilled water at the inlet to the evaporator is needed. This inlet temperature is an output of the optimization of the chilled water heat exchanger.

Well, your victory over the latent load and its associated complexity has been short lived because unfortunately, the latent load has been shifted to the chilled water heat exchanger. Therefore, the air side of the chilled water heat exchanger will need to include the effects of the latent load. The latent load was accounted for in the section entitled "Accounting for Condensation of Water Vapor" in Chapter One. The analysis of the air-to-water heat exchanger will be the same except for two details. On the waterside for any finite flow rate the temperature of the water will increase as it passes through the heat exchanger. This differs from the refrigerant where the temperature was assumed constant throughout the length of the heat exchanger. Therefore, the integration of the heat transfer equation will produce a different effectiveness-NTU relationship (Kays and London, 1984) as shown in Equation 3.1. The equation assumes the heat exchanger is arranged in a single-pass cross-flow configuration. The minimum heat capacitance (c_{min}) fluid, typically air, is mixed, and the maximum heat capacitance (c_{max}) fluid is unmixed, typically water.

$$\varepsilon = 1 - e^{\left\{-\frac{c_{max}}{c_{min}}\left[1-e^{\left(\frac{-c_{min}\times NTU}{c_{max}}\right)}\right]\right\}} \tag{3.1}$$

The second detail relates to the heat transfer coefficient correlation on the liquid side of the heat exchanger. For refrigerant, you used a two-phase correlation in Equation 1.82. For water, use a single-phase correlation similar to Equation 1.106 since the water does not change phase during the heat exchange process. With Equation 1.106, simply replace air properties with water properties and the hydraulic diameter of the air channel with that of the water.

For the chilled water system you are now equipped to optimize four design variables, namely: the heat transfer surface areas for the condenser, evaporator, and chilled water heat exchanger along with the compressor efficiency. To optimize the fans and pump will require accounting for the frictional losses of the fluids using the conservation of momentum. A future textbook will disclose the optimization of the fluid distribution systems including fans, pumps, hydraulic diameters of heat exchangers, and diameters of ducts and pipes. In the meantime, with three simple tools (heat transfer, economics, and parameter optimization) the value of traditional thermodynamics has been extended to optimize four design variables.

MODEL VALIDATION

Any computer simulation needs to be validated with experimental data before it can be used reliably. The source of "experimental" data for this analysis was provided by the manufacturer of the chiller in the form of catalog data. A parameter optimization of the empirical constants used in the convective heat transfer coefficient correlation was used to minimize the error between the model and the manufacturer's data for the heat transfer rates. For the compressor power the isentropic efficiency was varied until the error in compressor power was minimized.

When working with a Newton–Raphson type equation solver there are three challenges you need to overcome in order to obtain convergence. The first challenge occurs if the geometry is specified and the fluid temperatures are output. The solution is more robust when the fluid temperatures are specified and one of the geometric variables, in this case the width, for each heat exchanger is output. For the condenser and evaporator specify the saturation temperature of the refrigerant and for the chilled water heat exchanger specify the temperature of the water at its inlet.

The second challenge occurs when the geometry of the heat exchangers is not specified by the manufacturer. This lack of detail is common practice among the manufacturers. The interaction between the geometry, the velocities, and the heat transfer create a highly unstable situation. The challenge is overcome by setting the convective heat transfer coefficient to a constant value. You will be able to find the geometric inputs (e.g., tube diameters, fin spacing, and tube spacing) which yield reasonable velocities (1/2 to 2 m/s). The 2012 Handbook on Systems by the American Society of Heating, Refrigerating and Air Conditioning Engineers (ASHRAE, 2012) provides reasonable ranges for the geometry of and velocities through the heat exchangers. Once the geometry and velocities are reasonable, introduce the convective heat transfer coefficient correlations for each of the fluids.

The last challenge occurs when the fluid temperatures are fixed over a range of conditions. The fluid temperatures were fixed in order to solve the first problem of convergence. To keep both the heat transfer rate constant along with the corresponding fluid temperature for a given application will require changing the size of the heat exchangers, which is impractical. To keep the size of the heat exchanger constant, use parameter optimization. In the parameter optimization minimize the error between the model and the experiment for the width of the heat exchanger as a function of the corresponding fluid temperature. For example, minimize the error in the width of the condenser as a function of the corresponding saturation temperature of the refrigerant.

A chiller with a cooling capacity of 1075.1 kW at conditions determined by Air Conditioning, Heating and Refrigeration Institute (AHRI) standard 550/590 is selected to validate the model. You can cool approximately 9,500 m^2 of office floor space with this size chiller. The model is validated using the manufacturer's performance data for the ranges of conditions provided in Table 3.2.

Table 3.2. Range of operating conditions for chiller model validation

Variable	Lower Value	Upper Value	Units
Temperature of water at chilled water heat exchanger inlet	5	10	°C
Flow rate of water at chilled water heat exchanger inlet	34.7	54.6	Liter/s
Temperature of air at condenser inlet	30	50	°C

Seven variables are kept constant during the validation. The temperature of the air for the chilled water heat exchanger inlet is held constant at 21.1°C. The volumetric flow rate of air at the condenser inlet is set at 117,042 liters/s based on the manufacturer's data. The condenser has a constant speed fan. The volumetric flow rate of air for the chilled water heat exchanger inlet is half the value of that passing through the condenser at 58,521 liters/s. The air-handler also has a constant speed fan. To model a variable speed fan would require either a bin analysis of an hour-by-hour analysis of the system to provide sufficient accuracy. The pressure drop of all the fluids (e.g., air, water, and refrigerant) passing through the heat exchangers is kept at a minimum at 1 kPa. For the optimum system the pressure rise in the compressor is 1,731 kPa so a pressure drop of 1 kPa is negligible in comparison. The convective heat transfer coefficient for the refrigerant passing through the condenser and evaporator are 16,600 and 16,500 W/m^2-C respectively. The thickness of liquid water on the air side of the chilled water heat exchanger is set to zero. This assumes the decrease in heat transfer due to the conduction resistance of the water is offset by the increase in heat transfer due to the increased turbulence in the liquid to air interface.

To justify the use of a constant heat transfer coefficient for the two-phase regions of the evaporator and condenser the two-phase correlation from Chapter One is used to find the minimum and maximum values of the heat transfer coefficients over the entire range of operating conditions given in Table 3.2. First, the two-phase correlation was tuned to the manufacturer's data using parameter optimization. Then the low, mean, and high values for the heat transfer coefficients were entered into the chiller model. Table 3.3 contains the maximum errors that occur in the overall heat transfer coefficient and heat transfer surface area for both the optimum evaporator and condenser.

Table 3.3. Maximum errors in overall heat transfer coefficients and heat transfer surface area when assuming a constant heat transfer coefficient on the refrigerant side of the heat exchanger

	Overall Heat Transfer Coefficient [%]		**Heat Transfer Surface Area [%]**	
Condition	Evaporator	Condenser	Evaporator	Condenser
Low	−0.235	−4.108	+0.289	+3.853
High	+0.4	+5.458	−0.289	−4.495

Figures 3.5 and 3.6 demonstrate the model agrees with the manufacturer's data with a high degree of accuracy (all errors less than 0.3 percent). The agreement was accomplished by minimizing the error through adjusting

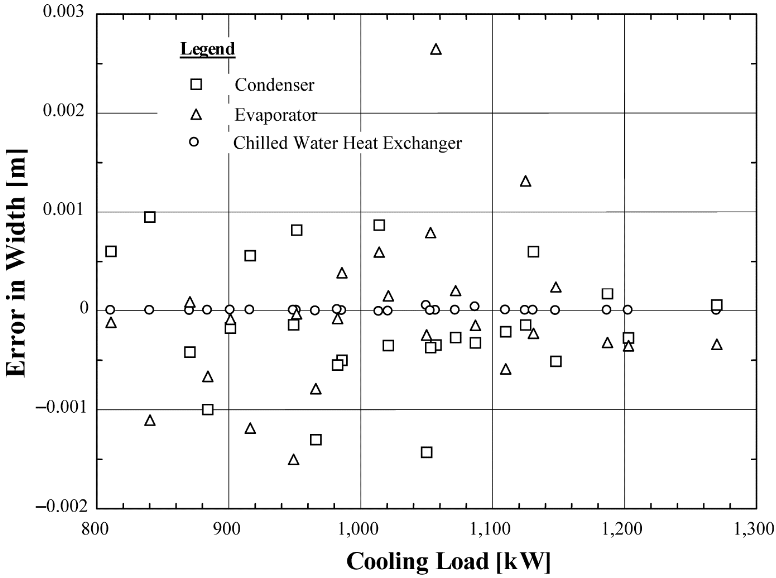

Figure 3.5. Error in width of all three heat exchangers (condenser, evaporator, and chilled water heat exchanger) for the chiller. Over 90 percent of errors are within ±1 mm for heat exchangers with widths greater than 1 m (EES, 2014).

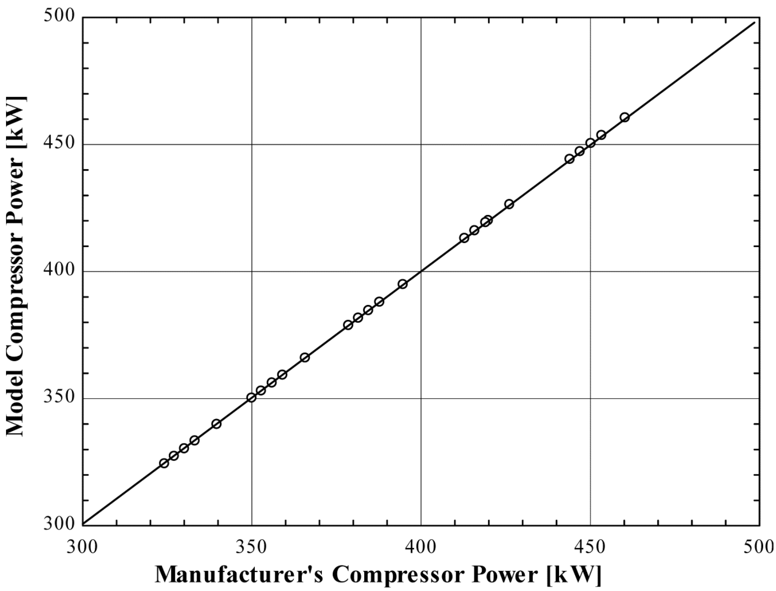

Figure 3.6. Validation of compressor power. The compressor power from the model agrees with the manufacturer's data to at least five significant figures (EES, 2014).

the empirical multipliers for the Dittus–Boelter correlation (Dittus and Boelter as presented by McAdams, 1942; see Equation 1.106) for the water and air flows and the isentropic efficiency of the compressor. The Dittus–Boelter correlation is selected for the model because it is the simplest and most compact of the convective heat transfer coefficient correlations. It, like the other correlations, assumes fully developed turbulent flow throughout the heat exchanger. In reality, a significant portion of the heat exchanger has developing flow. Since an excellent fit is made between the manufacturer's data and the model using the Dittus–Boelter correlation a more complex correlation is not justified. Please note that the exponents to the Reynolds and Prandtl numbers are not varied in the parameter optimization. The only parameter which is optimized is the empirical constant which is multiplied by the Reynolds and Prandtl numbers. Also note true experimental data has higher uncertainties than the agreement obtained here. Therefore, the manufacturer's catalog data is most likely generated from a model that is tuned to experimental data.

OPTIMIZATION OF DESIGN VARIABLES

Now that you have a validated model of the chiller, you are nearly ready to determine the optimum heat transfer surface area for each heat exchanger and the optimum isentropic efficiency of the compressor. The last engineering task you have is to determine the average operating conditions for the chiller system. Assume the envelope you are cooling is located near LaGuardia Airport in New York City where there are 672 cooling degree days (CDDs) [C-day/yr]. The balance point temperature of 18.33°C upon which the CDD are based assumes minimal internal heat gains. Assume the average outdoor temperature is 30°C. Divide the CDD by the difference in temperature between the outdoors and balance point temperature to obtain the annual operating time, of 57.6 days per year, for the chiller. Keep in mind, the method you used here is greatly simplified compared to the actual design of a chiller. In the actual design process you will need to find the annual cooling load using either an hour-by-hour or bin analysis. The annual cooling load is then divided by the average compressor power at the average operating conditions to determine the operating time.

Recall, the definition of balance point temperature. It is the average outdoor environmental temperature where neither heating nor cooling are required. The internal heat gains present in any environmentally controlled space cause the balance point temperature to be lower than the internal temperature of the space. The higher the internal heat gains the lower the balance point temperature. As the balance point drops, the operating time of the chiller increases not only in proportion to the increased temperature difference but also the expanded number of hours the outdoor climate is above the balance point temperature.

Establish a base case by selecting an average operating point from the manufacturer's data which meets the 30°C outdoor air temperature constraint. At this operating point, the temperature of the chilled water is assumed to be 8°C at the inlet to the chilled water heat exchanger, the cooling capacity is 1,187 kW with a chilled water flow rate of 51 liters/s. Next, tune the cost parameters to match the manufacturer's cost quote. The initial cost equations for each of the components are the same as in the basic model of the vapor-compression cycle presented in Chapter One with one exception. The basic model did not have a chilled water heat exchanger. Add an additional equation to account for the initial cost of the chilled water heat exchanger using a linear relationship between the area and the initial cost. The linear relationship is also used in chapter One for the initial costs of the evaporator and condenser.

In summary, you have established the base case, or most likely set of conditions, for the chiller. The base case is presented in Table 3.4.

Table 3.4. Base case for chiller

Variable	Most Likely Value	Units
Cost per unit of electricity	0.07	$/kW-hr
Empirical exponent for regression equation to determine the initial cost of the compressor	2	-
Empirical multiplier for regression equation to determine the initial cost of:		
Chilled water heat exchanger	8	$/m^2
Compressor	5	$/kW
Condenser	10	$/m^2
Evaporator	15	$/m^2
Heat transfer rate of the evaporator	1,187	kW
Interest rate required by the owner of the chiller	0.05	-

Variable	Most Likely Value	Units
Life of the chiller	30	yrs
Operating time	1,383	hrs/yr
Pressure of air at chilled water heat exchanger inlet	100	kPa
Pressure of air at condenser inlet	100	kPa
Relative humidity inside conditioned space	65	%
Temperature of air at chilled water heat exchanger inlet	21.1	C
Temperature of air at condenser inlet	30	C
Volumetric flow rate of chilled water at inlet	51	Liters/s
Volumetric flow rate of air at chilled water heat exchanger inlet	117,042	Liters/s
Volumetric flow rate of air at condenser inlet	58,521	Liters/s

Now, you are ready to determine the optimum areas of the three heat exchangers and isentropic efficiency of the compressor. Your objective is to minimize the total costs as a function of the saturation temperatures in the condenser and evaporator, the temperature of the chilled water entering the chilled water heat exchanger and the isentropic efficiency of the compressor. Remember the temperatures are varied in place of the corresponding area of the heat exchangers to improve the robustness of convergence. Figures 3.7 to 3.10 demonstrate the optimum values for each of the design variables. In these four figures, only one design variable is changed at a time. The remaining design variables are set at their optimum values.

Although the relationship between the heat transfer surface area and initial cost is linear, the cost graphs do not show a linear relationship. As the heat transfer surface area decreases, the irreversibility in heat transfer increases. This increase in irreversibility decreases the COP. For a given application (fixed cooling capacity) as the COP decreases the compressor power increases thereby increasing the initial cost of the compressor. The nonlinearity in initial costs is most evident in Figure 3.8 as the area of the condenser is decreased.

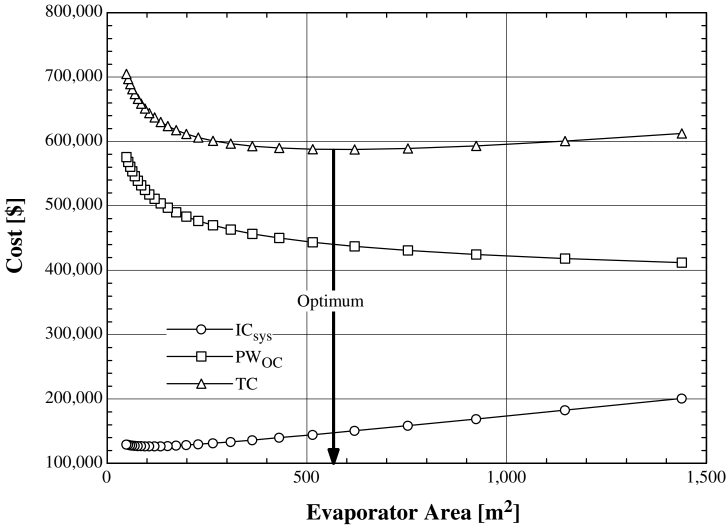

Figure 3.7. Optimum evaporator area (588.2 m^2) based on minimum total costs (*TC*). The initial cost of the system (IC_{sys}) is composed of the cost for the chilled water heat exchanger, evaporator, compressor, and condenser. The present worth of the operating costs (PW_{OC}) accounts for the time value of money ($i = 5$ percent) for the annual energy costs at ($0.07/kW-hr) over the life of the chiller (30 years). The total costs are the sum of the initial and operating costs. The remaining design variables are at their optimum values (EES, 2014).

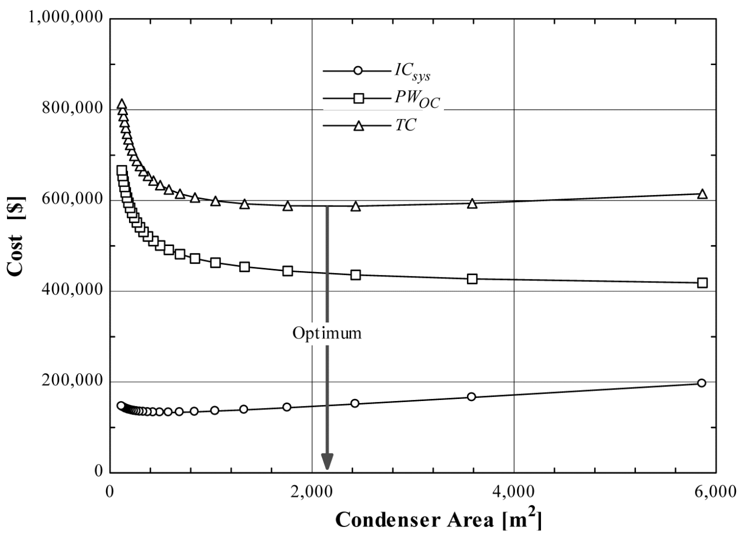

Figure 3.8. Optimum condenser area (2,180 m^2) based on minimum total costs (EES, 2014)

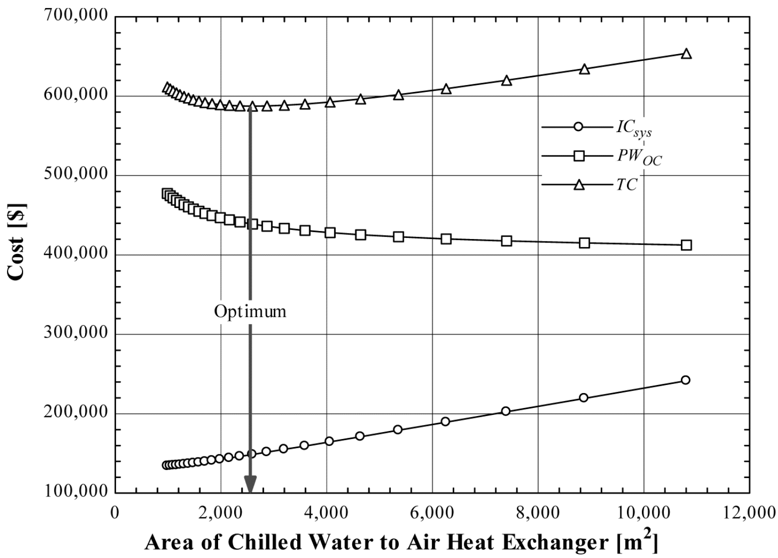

Figure 3.9. Optimum chilled water heat exchanger area (2,604 m^2) based on minimum total costs

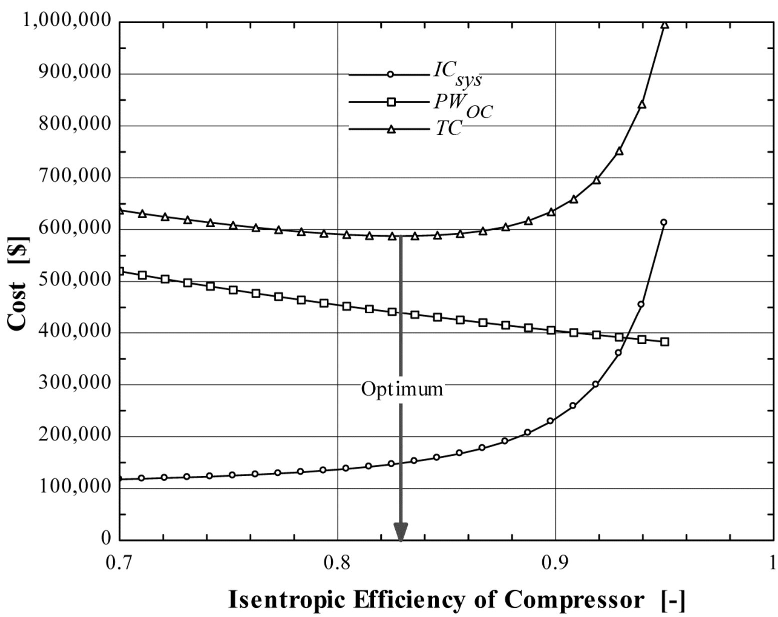

Figure 3.10. Optimum isentropic efficiency (82.9 percent) based on minimum total costs (EES, 2014)

You are quite proud of finding the optimum heat transfer surface areas and isentropic efficiency of the compressor. Congratulations! It usually takes ten to twenty years of experience to understand the relationship between the design variables and their effect on costs. Although it is quite an accomplishment to reach this level of understanding, you need to keep in mind what the Greek philosopher, Heraclitus, said. "The only thing constant is change." There are over twenty variables that can change at any time. How will this affect the optimum? Which variables cause the greatest change in the optimum? The purpose of sensitivity analysis is to help answer these questions.

SENSITIVITY ANALYSIS

Benjamin Franklin wrote in a letter to a friend, "…but, in this world, nothing is certain except death and taxes." There is another certainty that I would add to this quote but it does not apply to the optimization of a chiller. All the variables affecting the optimization of a chiller are uncertain. No one knows what electricity prices will do in the future or how long a chiller will last. Uncertainty is one of the more troubling aspects of life and one of the more difficult realities which engineers face. Sensitivity analysis is a useful tool to minimize the effects of uncertainty.

To perform a sensitivity analysis you need to determine which variables will affect the results of your analysis. For the optimization of a chiller system, at least twenty two variables can influence the total cost of the chiller system. Take each variable one at the time and normalize its difference from the base case by the value of the variable at the base case conditions as shown in Equation 1.45. Keep all the other variables constant as you find the minimum total cost for the new set of conditions for each level of the independent variable. Since the independent variable is normalized then you can compare the effect of one variable to that of another. You will want to pay the most attention to the variable that causes the most change in the total costs. In other words, you will need to spend more time reducing the uncertainty in the variables which have the highest impact on the analysis.

Figures 3.11 to 3.13 contain the results of the sensitivity analysis. The top eight variables are contained in Figure 3.11. These eight variables have the largest impact on the optimization of the chiller presented in this work. The variable that produces the greatest slope, and hence has the greatest impact on the optimum, is the amount of heat which needs to be removed from the conditioned space. Therefore, it is important to predict the cooling load with a high degree of accuracy. Next, the optimum is sensitive to the temperature of the air entering the condenser at least from −15 percent and higher. Therefore, accurate climate data is your second priority. Accurate climate data also helps you with your first priority which is to accurately model the cooling load. The third variable which has the next highest effect on the optimum is the temperature of the air at the inlet to the chilled water heat exchanger. Compared to the first two most influential variables this air temperature has relatively low uncertainty. It is determined by the use of the conditioned space and is controlled by the set point of the thermostat.

The fourth and fifth most influential variables have the same impact on the optimum. The cost for electricity has a high degree of uncertainty especially when you consider the potential fluctuations over the life of the chiller. Often analysts will include fuel price escalation in the economics but this complicates the analysis and introduces more uncertainity. From 2008 to 2014 electricity prices for the commercial sector were essentially flat (U.S. Energy Information Administration, 2015, Table 5.3). Therefore, including fuel price escalation in a chiller optimization would have over predicted the cost of electricity. To accurately determine the operating time requires a precise prediction of the cooling load over time. Since three of the top five variables are related to the cooling load then it is critical to provide an accurate value for this variable.

You see now how to interpret the sensitivity analysis and its usefulness. The order of the variables can change depending on the application. Note how all the variables in Figure 3.11 all affect the present worth of the operating cost of the chiller and none are related to the initial cost of the system. For the application, the present worth of the operating costs is greater than the initial costs as seen in Figures 3.7 to 3.10. Therefore, you should not be surprised if the variables impacting the operating costs have a greater influence on the optimum than the variables affecting the initial costs. If the office is moved to a different location or the relative costs of metal versus energy changes, the order of variables in the sensitivity analysis would change.

Typical engineering analysis allows for ±10 percent uncertainty in a variable. Therefore, the rank of the variables was determined based on the change in minimum total cost over the range of ±10 percent. However, climate, use of a conditioned space and energy costs can vary much more than 10 percent over the life of a chiller. For this reason the sensitivity analysis was plotted over a range of ±50 percent for as many variables as possible. Keep in mind, the rank of the variables can change, as you change

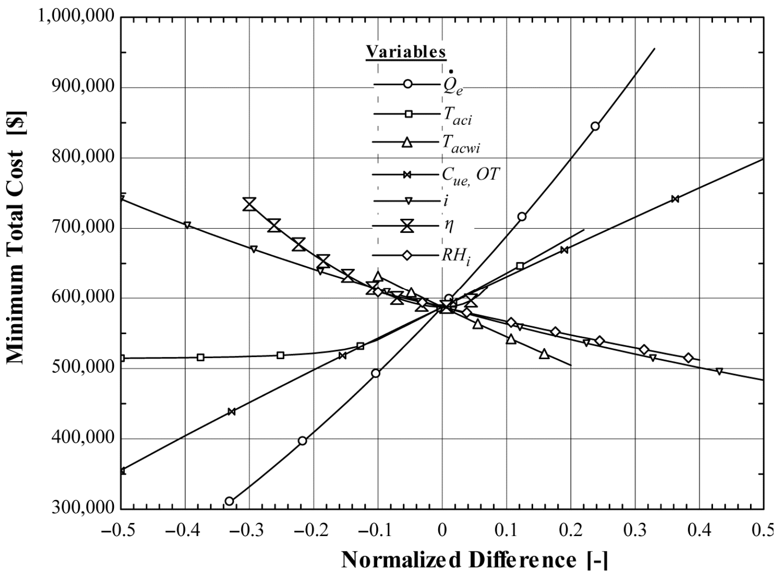

Figure 3.11. High sensitivity analysis of chiller. Top eight variables which influence optimization of the chiller. At a difference of ±10 percent the optimization is most sensitive to the average rate of heat transferred in the evaporator ($\dot{Q}_e$) followed by the temperature of air at the condenser's inlet (T_{aci}), then the temperature of the air at the chilled water heat exchanger inlet (T_{acwi}), then the cost per unit of electricity (C_{ue}) and operating time (OT), then the interest rate (i) required by the owner for an investment, then the isentropic efficiency of the compressor (η) and last the average relative humidity of the air in the conditioned space (RH_i) (EES, 2014).

the range of the independent variables. For example, if the range of uncertainty is ±20 percent then the total cost is more sensitive the efficiency of compressor than the interest rate. At a range of ±10 percent the total cost was more sensitive to the interest rate.

Another advantage of sensitivity analysis is that it expands the applicability of the optimization. For example take the operating time of the chiller system. The current optimization is limited to New York. If you take the same office building and place it in Timisoara, Romania where the CDDs are 625 [C-day/yr], the operating time of the chiller will be at 50 percent below the baseline analysis. The total cost of the optimum chiller drops from $580,000 to $350,000. On the other hand, if the office building were located in Brisbane, Australia, with 1844 CDDs, the operating time would be 50 percent above the baseline. In Brisbane the optimum chiller system will cost $800,000 over its life.

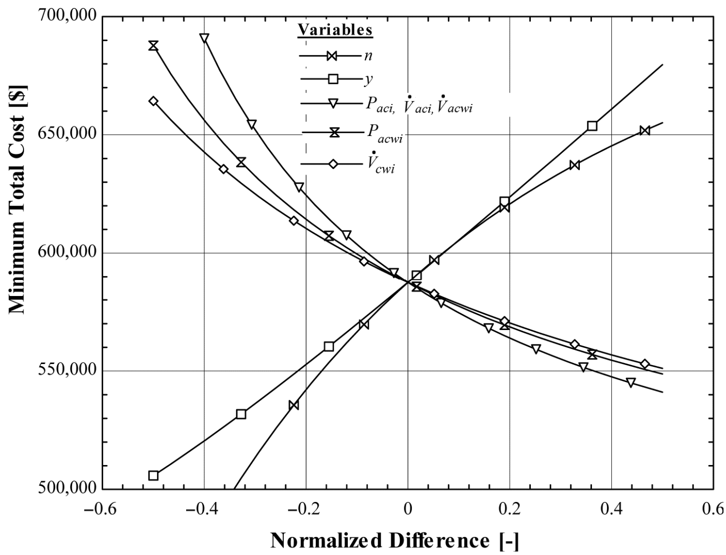

Figure 3.12. Medium sensitivity analysis of chiller. The next seven variables which influence the optimization of the chiller. At a difference of ±10 percent the optimization is moderately sensitive to the life of the chiller (*n*) followed by the empirical exponent to the regression equation for the initial cost of the compressor (*y*), then the pressure of the air at the condenser inlet (P_{aci}), the volumetric flow rate of air at the condenser inlet ($\dot{V}_{aci}$) and volumetric flow rate of air at the chilled water heat exchanger inlet ($\dot{V}_{acwi}$), then the pressure of air at the chilled water heat exchanger inlet (P_{acwi}), and last the volumetric flow rate of chilled water at the chilled water heat exchanger inlet ($\dot{V}_{cwi}$) (EES, 2014).

The next seven variables affecting the optimization are contained in Figure 3.12. Please note the range of the minimum total cost has dropped by over 70 percent from $700,000 to 200,000. None of these seven variables affect the optimization as much as the first eight variables presented in Figure 3.11. The reduced range allows you to see more clearly the differences between the seven variables of medium sensitivity.

The sensitivity of the last seven variables is displayed in Figure 3.13. Notice how the range for the minimum total costs has decreased again. This time it has dropped by 70 percent to 60,000. The optimization is least sensitive to the variables contain in Figure 3.13.

ENVIRONMENTAL ISSUES

Whether you are concerned about global warming, acid rain, smog, ozone or other pollutants, if you can reduce the power consumption, you will reduce the impact of your cooling system on the environment. Fortunately, in

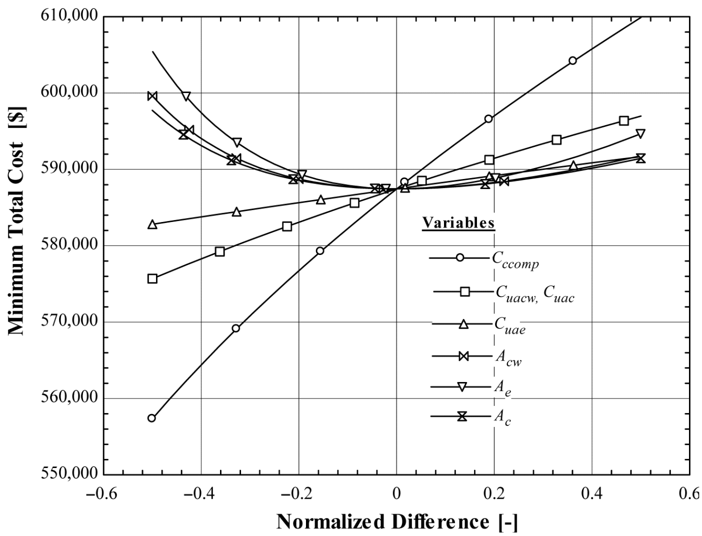

Figure 3.13. Low sensitivity analysis for the optimum chiller. Last seven variables which influence the optimization of the chiller. At a difference of ±10 percent the optimization is most sensitive to empirical cost coefficient for the compressor's initial cost (C_{ccomp}) followed by the empirical multiplier to the regression equation for the initial cost of the chilled water heat exchanger (C_{uacw}) and condenser (C_{uac}), then the empirical multiplier to the regression equation for the initial cost of the evaporator (C_{uae}), the heat transfer surface area of the chilled water heat exchanger (A_{cw}), the area of the evaporator (A_e), and last the area of the condenser (A_c) (EES, 2014).

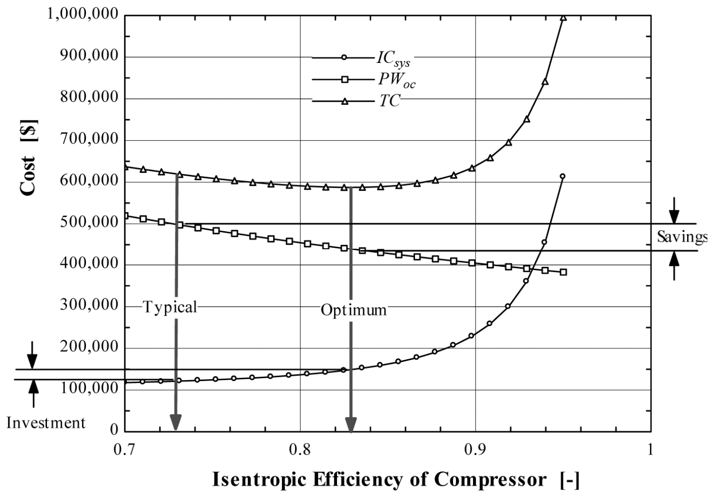

Figure 3.14. How an investment in energy efficiency makes good business sense (EES, 2014)

many cases, you can reduce power consumption and receive a return on your investment. Figure 3.14 illustrates how investing in efficiency makes sense from a business standpoint. If you assume the typical compressor (see vertical line labeled "Typical") has an average isentropic efficiency of 73 percent then improve that efficiency to the optimum (see vertical line labeled "Optimum") at nearly 83 percent it would cost an additional $27,000 (see dimension lines labeled "Investment"). Over a 30-year life with a time value of money of 5 percent, the investment would return more than twice the investment or $57,000 in energy cost savings (see dimension lines labeled "Savings"). Another way to view the economics is the annual savings for the high efficiency compressor is $3,500 which produces a return on investment of over 12 percent.

CHAPTER SUMMARY

With the more complicated chiller system you see the value of a schematic and property plot. The pressure–enthalpy property plot is useful for identifying irreversibilities in the cycle once the inlet temperatures of the air are superimposed on the refrigerant plot. In Chapter One the overall heat transfer coefficient is assumed constant in the basic model. In Chapter Three you use the heat transfer coefficient correlations in the chiller model to connect flow rate and overall heat transfer coefficient.

Adding the first of three tools to traditional thermodynamics, the heat exchanger equation, allows you to match the model to the manufacturer's performance data with a high degree of accuracy. The effectiveness-NTU heat transfer equations lead to the use of heat transfer coefficient correlations to provide a fundamental principles model. The simplest form of the correlation developed by Dittus and Boelter as presented by McAdams(1942) is adequate to provide a high degree of accuracy once the empirical multiplier is determined through parameter optimization. The addition of the heat transfer equation allows you to relate the size of the heat exchangers to the temperatures of the fluids.

With the addition of two more tools, you are able to unleash the power of thermodynamics to optimize the design of chillers. Parameter optimization allows you to relate the design variables to the initial cost of the component and engineering economics allow you to bring future expenditures to the present so they can be compared with the initial costs. For a base case of an office building located in New York, the areas of the chilled water heat exchanger, evaporator and condenser, and the isentropic efficiency of the compressor are determined to provide the lowest total cost.

Sensitivity analysis is a useful technique to mitigate the uncertainty of any optimization. The variables which have the highest impact on the results can be identified. More time should be spent on these high priority variables in an effort to minimize their uncertainty. This prioritization will increase the robustness and reliability of the optimization.

Since many energy systems are suboptimized it makes good business sense to invest in energy efficiency. Any improvement in efficiency results in a reduction in power consumption for a given application. The reduced power consumption reduces the pollutants discharged into our environment.

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Optimization of Cooling Systems

David C. Zietlow

Most energy systems are suboptimized. Businesses and consumers are so focused on initial costs that they underestimate the effect of operating the energy system over its life. This suboptimization creates a fantastic opportunity to not only make a wise decision financially but also reduce the environmental impact of energy systems. There are three simple tools, known to all mechanical engineers, that when added to traditional thermodynamics, enable an engineer to find the true optimum of an energy system. In this concise book, you will be equipped with these tools and will understand how they are applied to cooling systems.

The target audiences for this book are mechanical engineering students in their first semester of thermodynamics through engineers with 20+ years of experience in the design of cooling systems. First semester thermodynamic students will benefit the most from Appendixes A and C in Chapter 1. The rest of Chapter 1 is written at a level where any undergraduate mechanical engineering student who is taking heat transfer will be able to quickly assimilate the knowledge. This book also has the depth to handle the latent load, which will provide the practicing engineer with the tools necessary to handle the complexity of real cooling systems.

Dr. David C. Zietlow, PhD, is a professor of mechanical engineering at Bradley University in Peoria, IL. He has been teaching thermodynamics since 2000. From 2005 to the present, he has equipped his first semester thermodynamic students with the tools they need to optimize energy systems. He has 12 years of industrial experience with Sargent and Lundy, Central Illinois Light Company, Ford Motor Company, and Visteon. He has published 11 journal articles and 14 refereed conference papers in the thermal science field. He is a member of the American Society of Heating, Refrigerating, and Air Conditioning Engineers (ASHRAE) and the American Society of Engineering Educators.

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