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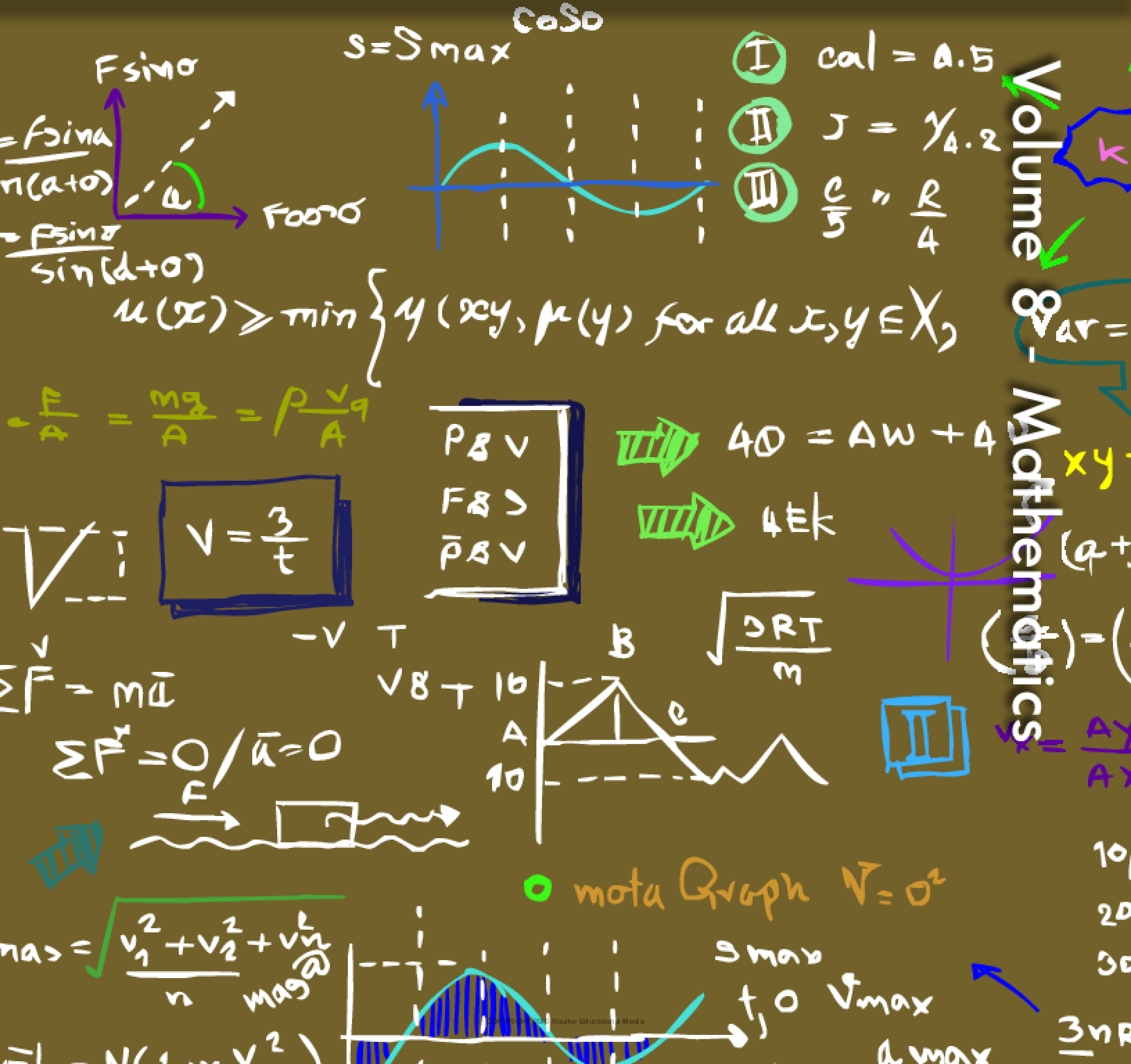
Volume 8 - Mathematics



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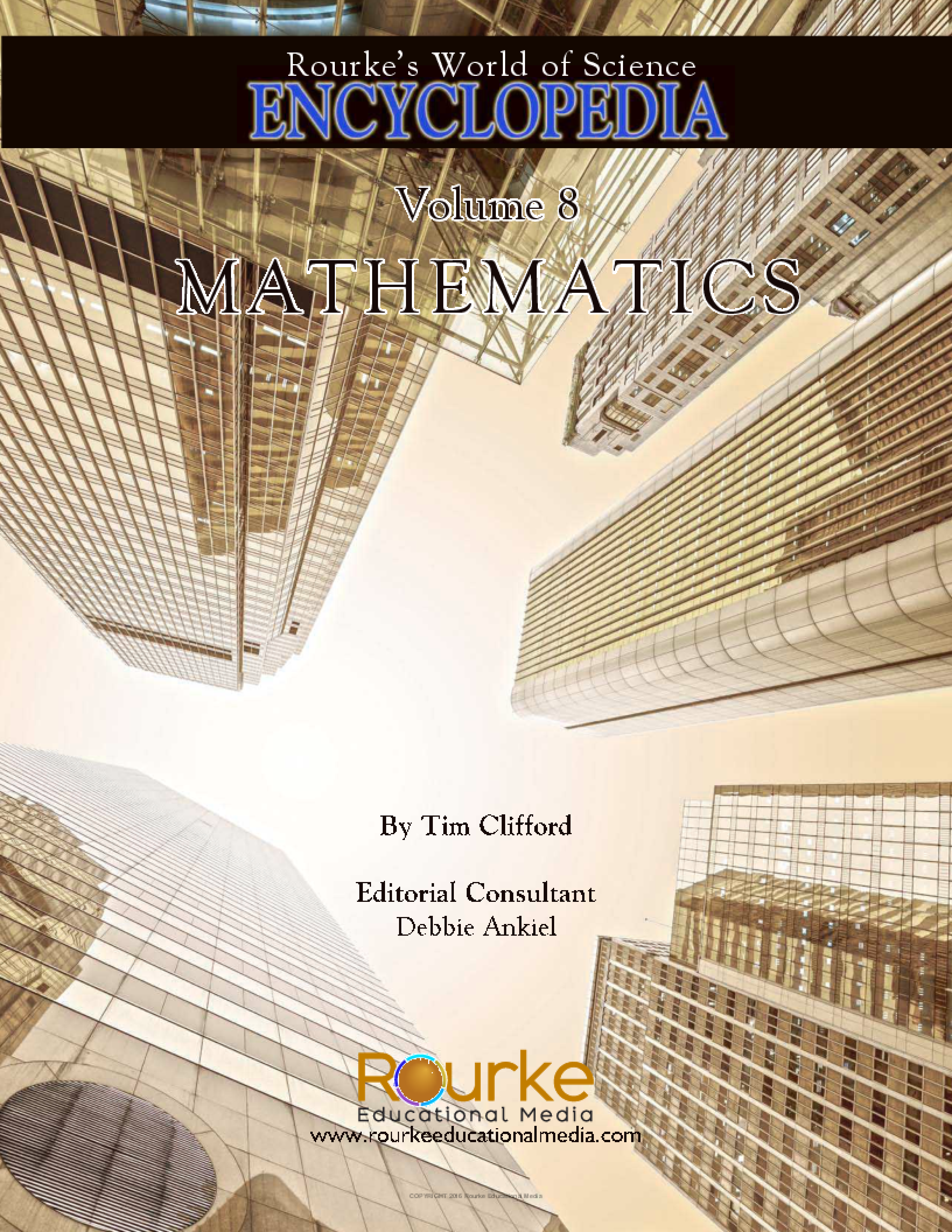
Volume 8

MATHEMATICS



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What Is Mathematics?

When you think of mathematics, you probably don't think of science. But mathematics is a science.

Mathematics is the study of numbers, shapes, change, quantities, and patterns. It also looks at the relationships between them. Math is a short way of saying mathematics.

Not only is mathematics a science, all other scientists use math in their work. Can you think of how a chemist, biologist, or astronomer might use math?

HOW SCIENTISTS USE MATH

Scientists		
Biologists		use math to compare microscopic objects sizes and shapes.
Chemists		use math to measure chemicals for solutions.
Meteorologists		use math to predict weather patterns.
Physicists		use math to estimate lasers power.

Timeline of Mathematics 70,000 to 300 BC

70,000 BC	South Africa - ochre rocks adorned with scratched geometric patterns
35,000 BC to 20,000 BC	Africa and France - earliest known prehistoric attempts to quantify time
20,000 BC	Nile Valley - the Ishango Bone is possibly the earliest reference to prime numbers and Egyptian multiplication
3400 BC	Mesopotamia - the Sumerians invent the first numeral system, and a system of weights and measures
3100 BC	Egypt - earliest known decimal system allows indefinite counting by way of introducing new symbols
2800 BC	Indus Valley Civilization on the Indian subcontinent - earliest use of decimal ratios in a uniform system of ancient weights and measures
2700 BC	Egypt - precision surveying
2600 BC	Indus Valley Civilization - objects, streets, pavements, houses, and multi-storied buildings are constructed at perfect right angles
1000 BC	Egypt - use of common fraction by Egyptians
400 BC	India - mathematicians write a mathematical text which classifies all numbers into three sets: enumerable, innumerable, and infinite
300 BC	Egypt - Euclid proves the infinitude of prime numbers and presents the Euclidean algorithm
300 BC	Mesopotamia - the Babylonians invent the earliest calculator, the abacus
300 BC	India - mathematician Pingala writes the "*Chhandah Shastra*", which contains the first Indian use of zero as a digit (indicated by a dot) and also presents a description of a binary numeral system, along with the first use of Fibonacci numbers and Pascal's triangle

Timeline of Mathematics 260 BC to AD1100s

250 BC	Mexico - the Olmecs people were using a true zero (a shell glyph) several centuries before Ptolemy
240 BC	Greece - Eratosthenes uses his sieve algorithm to quickly isolate prime numbers
140 BC	Greece - Hipparchus develops the bases of trigonometry
50 BC	India - Indian numerals, the first positional notation base-10 numeral system, begins developing in India
AD 250	Greece - Diophantus uses symbols for unknown numbers in terms of the syncopated algebra
AD 300	India - the earliest known use of zero as a decimal digit is introduced by Indian mathematicians
AD 700s	India - Shridhara gives the rule for finding the volume of a sphere and also the formula for solving quadratic equations
AD 750	Europe - Al-Khawarizmi is considered father of modern algebra since he was the first to bring Indian mathematics to Europe
AD 1030	Iran - Ali Ahmad Nasawi divides hours into 60 minutes and minutes into 60 seconds
AD 1100s	India - Indian numerals were modified by Arab mathematicians to form the modern Hindu-Arabic numeral system (used universally in the modern world)
AD 1100s	Europe - the Hindu-Arabic numeral system reaches Europe through the Arabs

Like any science, mathematics must show us things we didn't know. Proofs and theorems are the basis for math. A proof is a mathematical argument that convinces others to believe it is true.

A theorem is a mathematical statement supported by one or more proofs. Mathematicians can prove that theorems are true. There are many proofs and theorems in math.

Mathematics uses reasoning and logic to solve problems. In math, you solve a problem by thinking. Other sciences rely on experiments to solve a problem.

Common Core and STEM Give Meaning to Mathematics

When the "math wars" began in the 1990s, on one side were those who argued for a new focus on concepts and reasoning rather than drilling students on their times-tables. On the other were the traditionalists, who said the progressive approach allowed students to become unmoored from the building blocks of the subject, leaving them unprepared for more advanced mathematics.

The writers of the Common Core math standards have sought a middle ground. By the early 2000s, every state had developed and adopted its own learning standards that outlined what students in grades 3-8 and high school should be able to do. Every state also had its own definition of proficiency, which is the level at which a student is determined to be sufficiently educated at each grade level and upon graduation. This lack of standardization was one reason why states decided to develop the Common Core State Standards in 2009.

The creation of the math standards was in large part an editing process. So the Common Core math standards tackle fewer topics, and also move students more slowly through arithmetic, subtraction, multiplication and the other operations that build up to more complex math, particularly algebra.

Applying mathematics during engineering design challenges can help children develop critical thinking, problem solving, and communication skills.

Understanding involves making connections. Challenging students with important mathematics prepares them to solve problems in class and at home. NCTM's Principles and Standards for School Mathematics (2000) outlines five Process Standards that are essential for developing deep understanding of mathematics:

1. Problem Solving
2. Reasoning and Proof
3. Communication
4. Connections
5. Representation

Integrated engineering units of study allow for the application of mathematics skills in real-world contexts, removing barriers and enhancing the development of NCTM's five Process Standards and the Common Core's eight Standards for Mathematical Practice. The purpose for learning is evident with a coherent, integrated curriculum, freeing students to reason about complex problems, analyze multiple solutions, and communicate ideas and results. They develop habits of mind along with the necessary mathematics skills.

A hairstylist uses math to measure the chemicals to color hair. Businesses use mathematics when dealing with money. Stores add up our bills using math.

Math is also the basic tool used by scientists and engineers. They use it to describe the universe and everything within it.

Math isn't just for school. Math is everywhere and we use it every day!

Number Systems

Humans have used numbers for a very long time. Throughout the history of man, every culture used some form of mathematics. The numbers that we use today have been around for thousands of years.

The Base Ten System

Today, we use a base ten number system. The digits used in base ten are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. Using those 10 digits, we can make any number in the system.

The base ten system was probably created because we have ten fingers to count on. There are other bases as well. Some bases use more than ten numbers. Some use fewer.

What numbers can you make with these digits?

Digits	Whole Numbers
5, 7	57 75
1, 3, 9	139 193 319 391 913 931

The Base Two System (Binary Numbers)

In the base two system, only two digits are used. They are 0 and 1. Most computers use the binary system. Binary numbers are base two because all numbers can be made using zero or one.

Let's look at how the base ten numbers look in binary. Can you figure out the pattern?

The Base Ten Digits	In Base Two There Would Be...
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001

Base two is difficult for humans to use. Larger numbers get very large. The number 100 in base two is 1100100!

Find out more

The Biggest Number is Infinity

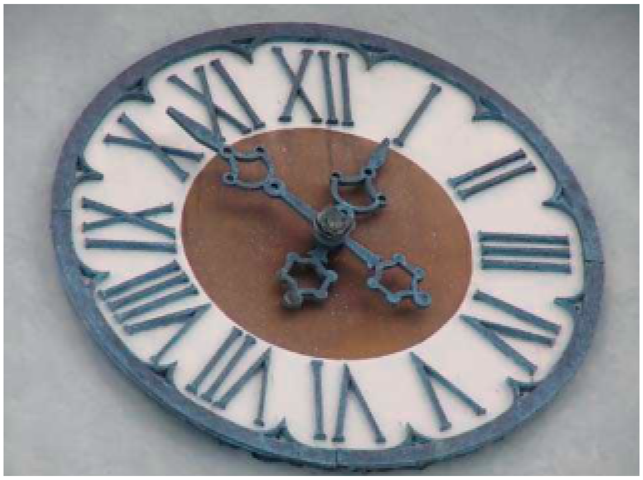
Infinity is used to describe a number that goes on forever. You can never reach infinity no matter how high or low you count!

In 1665, mathematician John Wallis (1616-1703) created a symbol for infinity and we still use his symbol today. The symbol for infinity, ∞, looks like a sideways 8.

Roman Numerals

In ancient Rome, people wrote numbers differently. They used numerals that look interesting, but they are hard to use in mathematics.

Roman Numerals	Arabic Numerals
I	1
II	2
III	3
IV	4
V	5
VI	6
VII	7
VIII	8
IX	9
X	10
L	50
C	100
D	500
M	1000



This clock shows that it is seven minutes to one.

Arabic Numerals

Today, we use Arabic numerals. The digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 are all Arabic numerals. These numerals were developed in the Middle East thousands of years ago.

Words to know

digit (DIJ-it): any Arabic numerals from 1 to 9, and sometimes 0

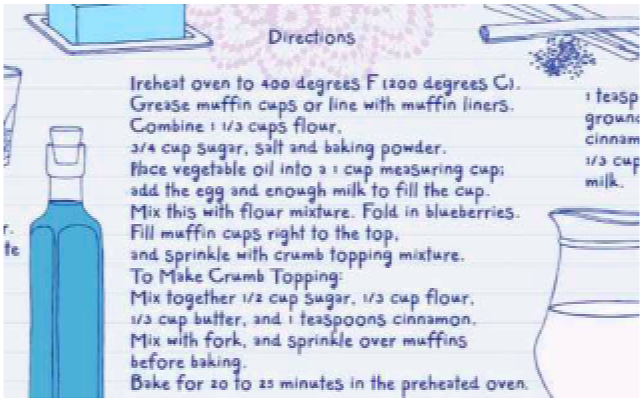
number (NUHM-bur): a word or symbol used for counting and for adding and subtracting

numeral (NOO-mur-uhl): the symbol that represents a number, such as 4 or IV

Numbers

Numbers are everywhere. We use numbers to tell time. They help us count money. We measure food with them. We use numbers in everything we do. Numbers are an important form of communication. Every language on the planet has some form of number system. Look around and see all the numbers in your world.

The Federal Reserve prints a serial number on each bill printed. There are a maximum of 832 bills with the same serial number. Serial numbers help prevent counterfeiting.



Directions

Preheat oven to 400 degrees F (200 degrees C).
Grease muffin cups or line with muffin liners.
Combine 1 1/3 cups flour,
3/4 cup sugar, salt and baking powder.
Place vegetable oil into a 1 cup measuring cup;
add the egg and enough milk to fill the cup.
Mix this with flour mixture. Fold in blueberries.
Fill muffin cups right to the top,
and sprinkle with crumb topping mixture.
To Make Crumb Topping:
Mix together 1/2 cup sugar, 1/3 cup flour,
1/3 cup butter, and 1 teaspoons cinnamon.
Mix with fork, and sprinkle over muffins
before baking.
Bake for 20 to 25 minutes in the preheated oven.

We use numbers in recipes.

Digital clocks display the time using numbers.

There are many different types of numbers. The most common are whole numbers. Whole numbers are the ones we use for counting. They are numbers like 1, 4, 15, 27, and 42. They are also called natural or positive numbers. A positive number is greater than zero.

Sometimes numbers are negative. This means that they are less than zero. A negative number looks like a whole number with a minus sign (-) in front of it.

Integers are all the natural numbers, the negative numbers, and zero combined.

MATHEMATICS

In mathematics, we put numbers together in different combinations using different functions. This helps make new numbers. These new numbers give people important information. Math begins with simple counting. Then comes addition, subtraction, multiplication, and division. Arithmetic is another name for these four operations. There are also other ways to use numbers.

Addition	2 + 2 = 4
Subtraction	4 - 2 = 2
Multiplication	3 x 3 = 9
Division	8 ÷ 4 = 2

Find out more

Zero Means Nothing

The number zero actually means there is no number! It is used as a place holder when other numbers are not being used. In the number 205 the zero holds the "tens place". If we left out the zero we'd have 25. Would you rather get $205 or $25 for your birthday?

The ancient Greeks and Romans did not have a zero in their numbers. This made it difficult for them to write down mathematical problems.

Counting

The simplest way of using numbers is to count them. Counting from 1 to 10 is easy. You can use your ten fingers. One hand can count to 5 by putting up one finger at a time. The other hand can count from 6 to 10 by putting up the rest of the fingers. This is why many number systems are based on 10.

We can also use our fingers to perform simple addition and subtraction problems.

Often people have to count in their heads without using their fingers. This is a very important skill. People who can count in their heads can learn how to do all kinds of mathematics easily. Sometimes, people count on paper by making tally marks. We can also use a device called a counter. Often, computers and calculators are used to count for us.

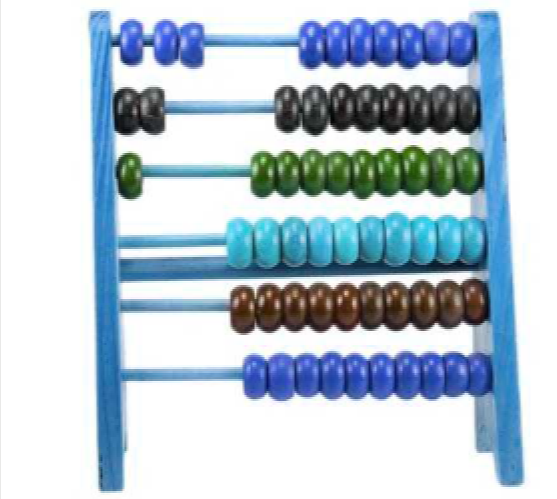
Tally marks

Find out more

You Can Do Math with Beads

Ancient Greeks invented the first calculator. The abacus is a device with beads strung on wires or bars. People can learn to do math very quickly with it. Using an abacus, you can do addition, subtraction, multiplication, and division.

If you travel around the world, you will probably see someone using an abacus in the marketplaces where they sell the goods they make or food they grow in their gardens.



Just about every language on Earth uses the Arabic numbers 0 though 9. Every language has different words for these numbers. Some of the words look the same but sound very different.

	French	Spanish	Japanese	Chinese
0	zéro (ZAY-roh)	cero (SHE-roh)	zero (zeh-doh)	ling (ling)
1	un (uhn)	uno (OO-noh)	ichi (ee-chee)	yi (ee)
2	deux (dyew)	dos (dose)	ni (nee)	er (ar)
3	trois (twah)	tres (trace)	san (sahn)	san (sahn)
4	quatre (kat)	cuatro (KWAH-troh)	shi (shee)	si (ss)
5	cinq (sank)	cinco (SEEN-koh)	go (goh)	wu (ooh)
6	six (seece)	seis (sace)	doku (doh-koo)	liu (lyo)
7	sept (set)	siete (see-EH-tay)	shichi (shee-chee)	qi (chee)
8	huit (weet)	ocho (OH-cho)	hachi (hah-chee)	ba (bah)
9	neuf (nuhf)	nueve (NWEH-vay)	ku (koo)	jiu (jyo)
10	dix (deece)	diez (DEE-ehs)	ju (joo)	shi (shh)

Leonardo Fibonacci (c. 1170-c. 1250)

Getting to know...

Leonardo Fibonacci was born in Italy around 1170. He was from the town of Pisa and is sometimes called Leonardo of Pisa or Leonardo Pisano. In his time, people in Europe still used Roman numerals. He learned to speak and read Arabic in Algeria. Fibonacci realized it was easier to use Arabic numerals for multiplication and division. He wrote a book about solving math problems using Arabic numerals.

Fibonacci found a way to solve equations with two unknown quantities. He also noticed that a series of numbers in which each number is the sum of the two preceding numbers like 0, 1, 1, 2, 3, 5, 8, 13...has many interesting properties. This is now called a Fibonacci series.

Addition

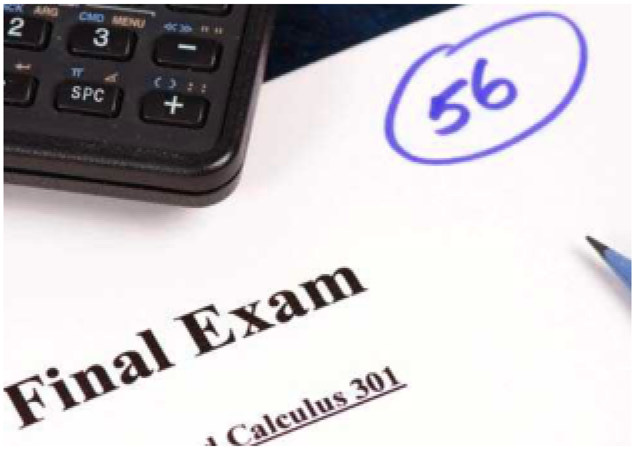
One way of putting numbers together is to add them. Addition is the combining of numbers to make a new number. We can add two or more numbers together.

People use addition all the time. We can add the prices of items we buy to find the total. We can add the points on a test to see the score.

Terms Used in Addition

Addends are the numbers that we add together. The sum is the total of all the added numbers.

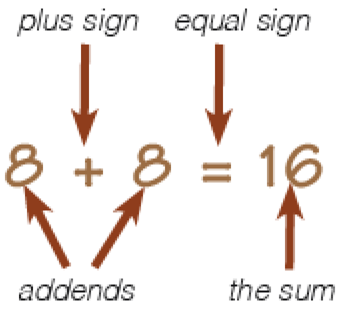
We add prices together to get a total at the store.



The teacher adds the points together for a final score.

Symbols Used in Addition

The plus sign (+) is used to show when numbers should be added. The equal sign (=) is used to separate the two equal sides of a mathematical statement.

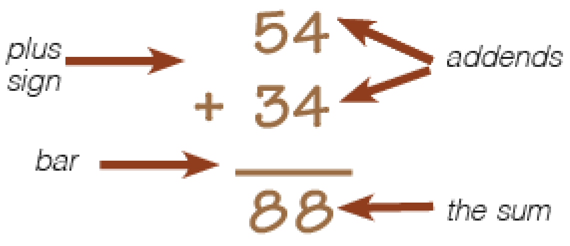


Adding from Left to Right. Adding numbers from left to right is useful for small numbers.

$$6 + 6 = 12$$

Adding by Going Down. It is often easier to add bigger numbers on paper by going down.

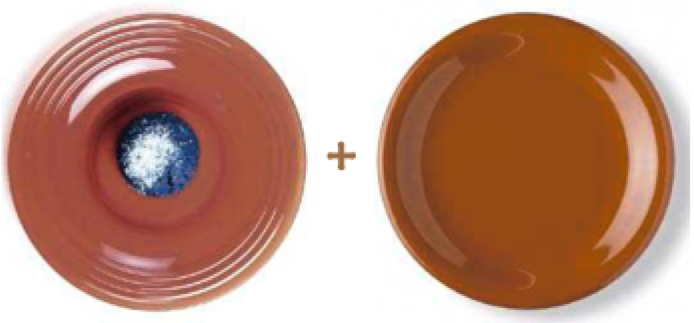
A line called a bar replaces the equal sign to separate the sum from the numbers being added.



Adding Zero. If you add zero to a number, the number does not change.

$$6 + 0 = 6$$

$$12 + 0 = 12$$



1 muffin plus 0 muffins equals 1 muffin

Addition is Commutative

In addition, you can reverse the numbers you add and the sum will not change. We call this the commutative property.

$$102 + 103 = 205$$

$$103 + 102 = 205$$

Subtraction

The opposite of addition is subtraction. Addition puts numbers together, but subtraction takes one number away from the other. In subtraction, you find the difference between two numbers. We can subtract two or more numbers from each other.

People use subtraction all the time in their lives. We can subtract to find out how much money we will have left after we buy something. We can also find out how much time we have left to complete a task with subtraction.

You can use your watch to subtract and make sure you get to school or work on time.

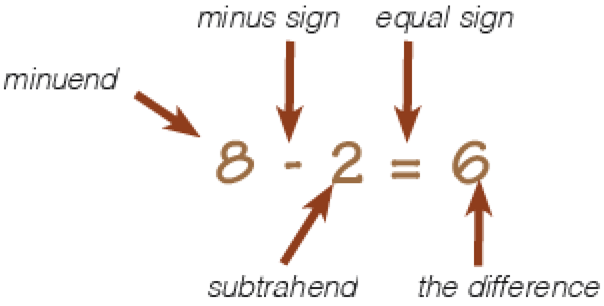
Terms Used in Subtraction

In subtraction, we make a new number. The new number is the difference between the numbers.

If one child is 5'6" and another child is 3'6", you can subtract the two measurements and discover the difference in their heights is two feet.

Symbols Used in Subtraction

The minus sign (-) is used to show when numbers should be subtracted. The equal sign (=) is used to separate the two equal sides of a mathematical statement.



Subtracting from Left to Right. Subtracting numbers from left to right is useful for subtracting small numbers.

$$7 - 3 = 4$$

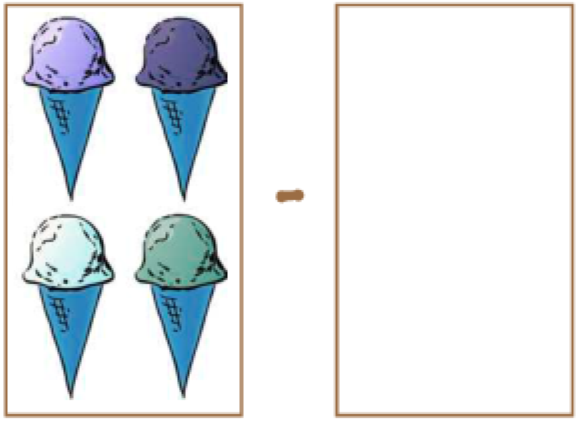
Subtracting by Going Down. It is often easier to subtract bigger numbers on paper by going down.

We use a bar instead of the equal sign to separate the difference from the big numbers being subtracted.

Subtracting Zero. If you subtract zero from a number, the number does not change.

$$6 - 0 = 6$$

$$12 - 0 = 12$$



4 ice cream cones minus zero ice cream cones equals 4 ice creams

Subtraction is Not Commutative

If you reverse the numbers in a subtraction problem, you will usually get the wrong answer. Subtraction does not have commutative properties.

$$5 - 3 = 2$$

$$3 - 5 = -2$$

Multiplication

Sometimes, numbers are added to themselves many times, over and over. A simple way of describing this is multiplication.

For example, you can add the number 6 four times, like this:

$$6 + 6 + 6 + 6 = 24$$

In this problem, the number 6 appears four times. Instead of adding, you can multiply four times six and get the same answer:

$$4 \times 6 = 24$$

Find out more

Arrays

Drawing an array is a way to create a picture of the relationship between addition and multiplication.

There are five rows with four cars in each. You can write the equation 5 x 4 instead of 4 + 4 + 4 + 4 + 4. The answer is still 20.

Terms Used in Multiplication

When we multiply two or more numbers together, we call the answer the product.

$$3 \times 2 = 6 \longleftarrow \text{the product}$$

Symbols Used in Multiplication

Several different symbols indicate we should use multiplication to solve a math problem. In beginning math, we usually use the times symbol (x) to show that numbers are going to be multiplied. Sometimes a dot (•) or an asterisk (*) is used in between numbers that are being multiplied. If you see an equation with parenthesis around groups of numbers, that also means to multiply.

$$\underbrace{(3+2)}_{\downarrow}\underbrace{(6+2)}_{\downarrow} = 40$$

$$5 \times 8 = 40$$

The equal sign (=) is used to separate the two equal sides of a mathematical statement.

Multiplying Across. For smaller numbers, you can multiply on a line going across. You can use the x, the dot, or the asterisk to show multiplication.

$$4 \times 9 = 36$$

$$4 \bullet 9 = 36$$

$$4 * 9 = 36$$

The product of 4 times 9 is 36.

Multiplying by Going Down. It is often easier to multiply bigger numbers on paper by going down.

$$\begin{array}{lr} & 24 \\ & \times\quad 5 \\ \text{bar} \rightarrow & \overline{} \\ & 120 \leftarrow \text{the product} \end{array}$$

The bar is used instead of the equal sign when multiplying this way. The product, or answer, is below the bar.

Multiplying by Zero. Math is not magical. If you multiply a number by zero, the product is always zero.

$$4 \times 0 = 0$$

$$20 \times 0 = 0$$

$$1{,}000 \times 0 = 0$$

Multiplication is Commutative

You can reverse the numbers you are multiplying. The product will be the same.

$$4 \times 6 = 24$$

$$6 \times 4 = 24$$

Find out more

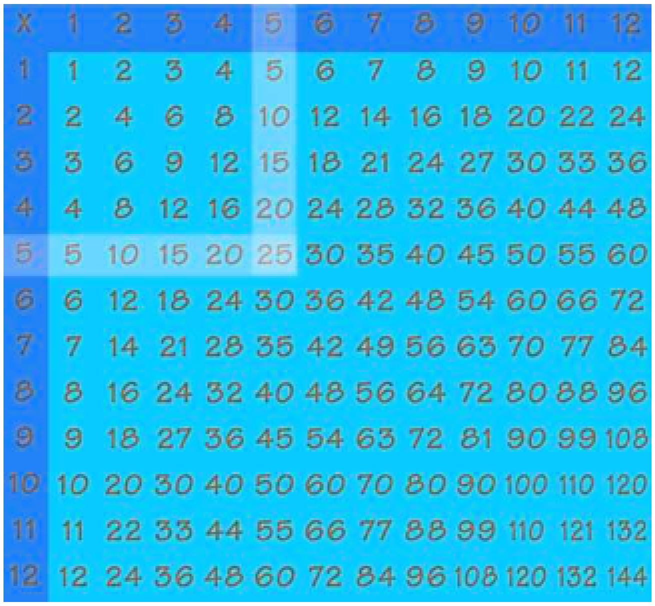
The Multiplication Table

A multiplication table shows how we multiply the numbers 1 through 10. It's helpful to have a multiplication table around when you are memorizing your multiplication facts.

X	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

There are a few ways to use a multiplication table.

- Pick the two numbers you want to multiply
- Find the first number in the top orange row
- Find the second number in the orange column on the left
- Move down the column from the number on the top and across the row from the number on the left
- The solution is the number where they meet



x	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

5 x 5 = 25

Another way to use the table is to move down one column or across one row to see the products of one number. Most people memorize these columns or rows.

Division

PROJECTS
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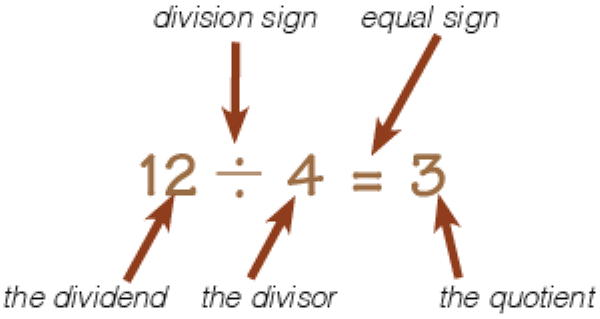
Numbers are broken into equal pieces when they are divided. Division tells us how many of one number it takes to equal another number.

We use division in everyday life. We often divide up our time to see how long we can spend on activities. We divide treats to make sure everyone has an equal amount.

If four people were to divide this pizza to share, each person would get an equal amount. Eight divided by 4 is 2. You can write the problem this way: 8÷4 = 2.

Terms Used in Division

In division, each number has a name. The number that we are dividing is called the dividend. The number that we are using to divide is called the divisor. The quotient is the result, or answer.



When one number is divided by another, the dividend is equal to the quotient itself times the divisor. In this way, division is the opposite of multiplication.

$$3 \times 5 = 15$$

$$15 \div 3 = 5$$

Symbols Used in Division

There are several different symbols used to let you know you need to use division to solve a math equation.

One way to divide numbers is on a line from left to right using the division symbol (÷).

$$12 \div 3 = 4$$

A slash (/) can also be used to show division.

$$12 \ / \ 3 = 4$$

Another way of writing a division problem is to use a corner-like symbol (⟌). The divisor goes outside the symbol on the left. The dividend goes inside the symbol and the answer, or quotient, is written on top of the symbol.

$$\begin{array}{r} 4 \\ 3\overline{)\ 12} \end{array}$$

Dividing by Zero

If you divide a number by zero, the quotient is always zero.

$$12 \div 0 = 0$$

$$7 \div 0 = 0$$

Division is Not Commutative

You can't reverse the numbers you are dividing. The quotient will usually not be the same.

$$15 \div 5 = 3$$

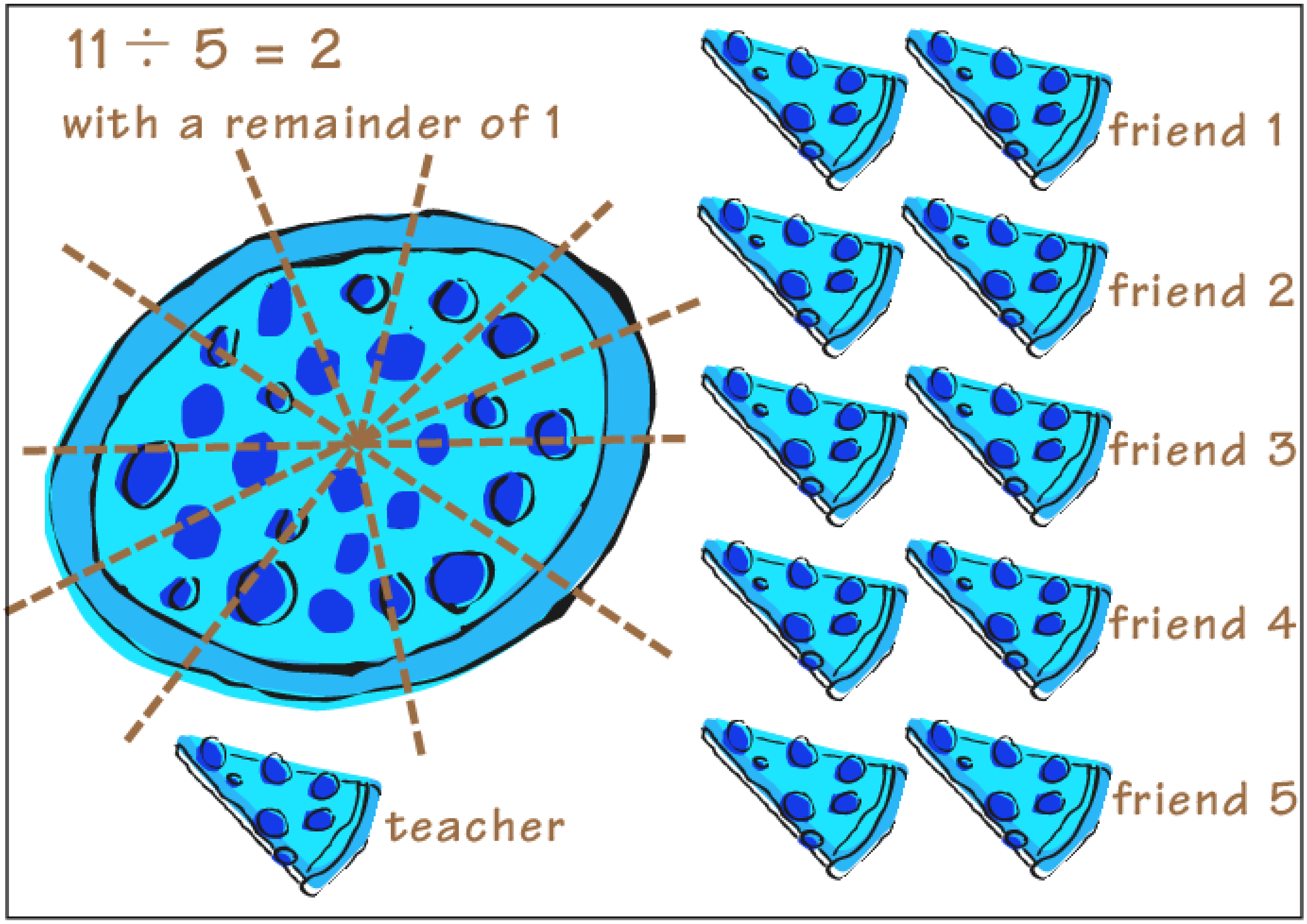
$$5 \div 15 = .333$$

Remainders

Sometimes numbers cannot be divided evenly. The quotient is not a whole number. When this happens, there is a remainder. The remainder is the number left over after the dividend has been evenly divided.

For example, if you have a pizza cut into 11 pieces and you want to share it equally with 5 friends, each friend will get 2 pieces of pizza and you'll have one piece left over for your teacher.

$$\begin{array}{r} 2r1 \\ 5\overline{)\ 11} \\ \underline{-10} \\ 1 \end{array}$$



Long Division

In more advanced division, there is not a traditional remainder. Instead, long division is used to calculate exact quotients by extending the answer using decimals or fractions.

Step 1.
Divide 7 by 6

$$\begin{array}{r} 1 \\ 6\overline{)75} \end{array}$$

Step 2.
Then multiply 6 times 1 and subtract answer from 7

$$\begin{array}{l} 1 \\ 6\overline{)75.0} \\ \underline{-6} \\ 1 \end{array}$$

Step 3.
Now bring down the 5 and divide 15 by 6

$$\begin{array}{l} 1 \\ 6\overline{)75.0} \\ \underline{-6\downarrow} \\ 15 \end{array}$$

Step 4.
Redo all steps until you get to zero

$$\begin{array}{l} 12.5 \\ 6\overline{)75.0} \\ \underline{-6\downarrow} \\ 15 \\ \underline{-12\downarrow} \\ 30 \\ \underline{30} \\ 0 \end{array}$$

Sets

We think of a set as a group of things that go together. A chess set has many different pieces. Some are black and some are white. Some are large and some are small. Some pieces are shaped like horses, and some like castles. Yet, they are all part of the same set.

All pieces in this game are part of the same chess set.

In mathematics, a set is a collection of numbers or things that are grouped together. Each item in a set is called an element, or member, of that set. Sets often have something in common.

Set theory is the study of possible sets and it is the basis of most math. You may not talk about set theory but much of the math that you do each day in school is using set theory. Mathematicians use brackets { } to show what is in a set.

This is the set of even numbers from two to ten.

{2, 4, 6, 8, 10}

This is the set of whole numbers that are greater than nine.

{10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23...}

Georg Cantor (1845-1918)

Getting to know...

Georg Cantor was born March 3, 1845. He is best known as the creator of set theory, which has become a foundational theory in mathematics. In 1904, the Royal Society of London awarded Cantor its Sylvester Medal. Georg Cantor died in 1918.

Finite Sets

When something is finite, it has a limit or an end. The set {2, 4, 6, 8, 10} is a finite set. It contains only five items. If it is possible to count the number of items in a set, it is a finite set.

Infinite Sets

When something is infinite, it has no limit or end. For example, the set of whole numbers is infinite. No matter how large a number is, you can always make it larger by adding 1 to it. The set of whole numbers has no end.

It would be impossible to write an infinite set. You could never stop writing! So, we show that a set is infinite by using dots.

{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11...}

The three dots show that the numbers would go on to infinity.

Subsets and Supersets

In a subset, all the elements of the set are part of a superset. A superset can have many different subsets.

The set of odd whole numbers is a superset.

{1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31...}

Some possible subsets of the set of odd whole numbers are:

- The set of odd whole numbers from 1 through 5

{1,3,5}

- The set of odd whole numbers less than 10

{1,3,5,7,9}

- The set of odd whole numbers greater than 15 and less than 31

{17, 19, 21, 23, 25, 27, 29}

Find out more

Naming Sets

Sets are often named by using capital letters. We might name two sets like this:

$A=\{5,10,15,20,25,30\}$ $B=\{10,20,30\}$

Naming sets makes them easier to talk about. We can say that "Set B is a subset of Set A" or that "Set A is a superset of Set B".

Instead of writing all of that, we can also use symbols. The symbol $\subseteq$ stands for subset. So instead of writing "Set B is a subset of Set A", we can write "$B \subseteq A$".

Empty Sets

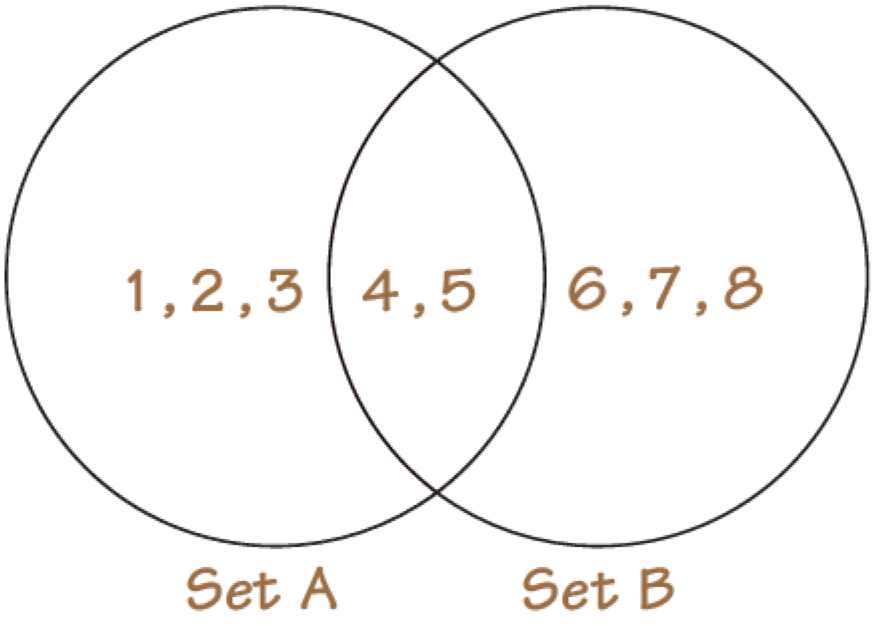
There is a special set called an empty set, or null set. An empty set contains no elements. We show it by using empty brackets { }, or by using the symbol ø. The empty set is a subset of all other sets.

Venn Diagrams

We can use pictures to help us understand sets. These pictures are called Venn diagrams. They show relationships between sets. Let's look at these two sets:

$A=\{1,2,3,4,5\}$ and
$B=\{4,5,6,7,8\}$

Both sets contain the numbers 4 and 5. We can show this easily with a Venn diagram.



Intersection of Sets

The place where the circles overlap is called the intersection of the two sets. The numbers 4 and 5 are there because they are parts of both sets. We use the symbol ∩ to show where the circles intersect.

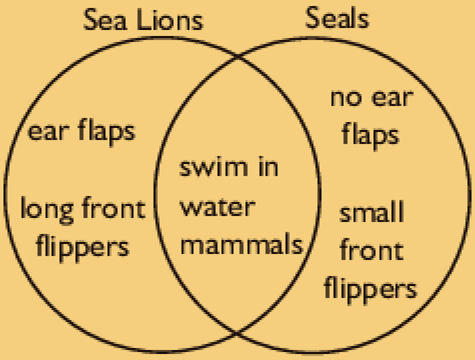
Union of Sets

The union of sets is all the elements of both sets. In the previous Venn diagram, the union of A and B would be {1,2,3,4,5,6,7,8}. We use the symbol ∪ to show a union. We would write that as A ∪ B.

Find out more

Using Venn Diagrams to Compare and Contrast

Venn diagrams can be used in subjects besides math to show how things are the same (compare) and how they are different (contrast). For example, in reading you can show how two characters are alike and different. In science, you can compare different species. In social studies, you can use a Venn diagram to compare and contrast two countries. There's no end to how you can use Venn diagrams!

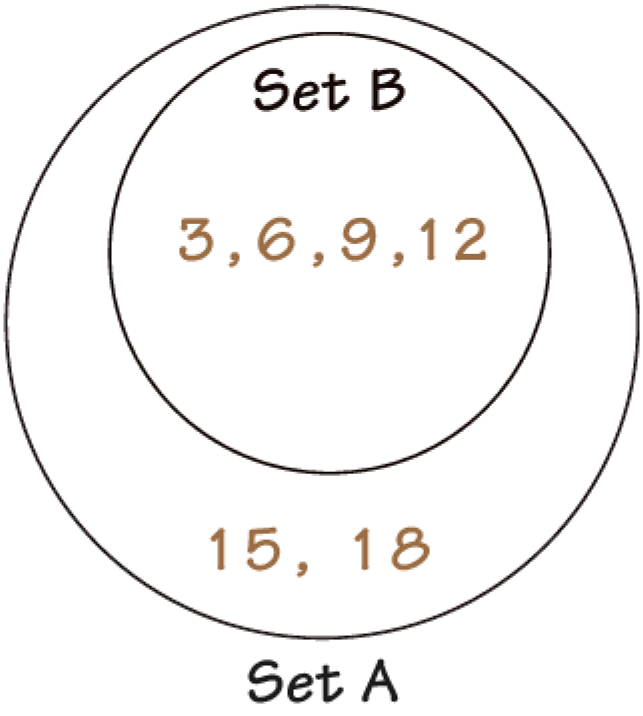


Showing Subsets and Supersets

It's easy to show subsets and supersets using Venn diagrams. Let's use these sets:

A={3,6,9,12,15,18} and B={3,6,9,12}

A is the superset and B is the subset. To show this in a Venn diagram, just put one circle inside the other.



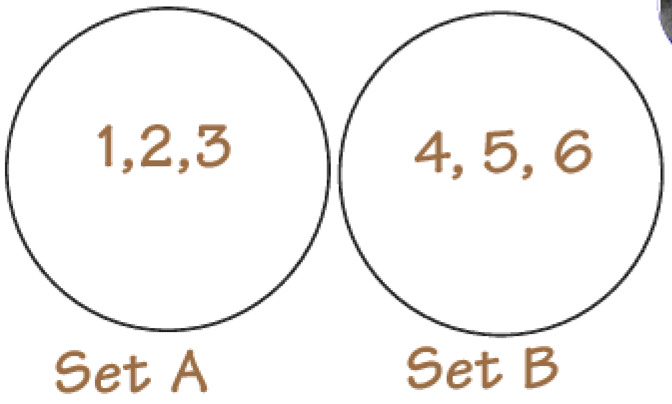
Showing Disjoint Sets

Sets are called disjoint when they have none of the same elements. Let's use these sets:

A={1,2,3} and B={4,5,6}

We would show this on a Venn

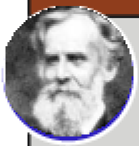
diagram by drawing two circles that do not intersect.



The only subset of these sets is the empty set { }.

$$A \cap B = \varnothing$$

John Venn (1834-1923)



Getting to know...

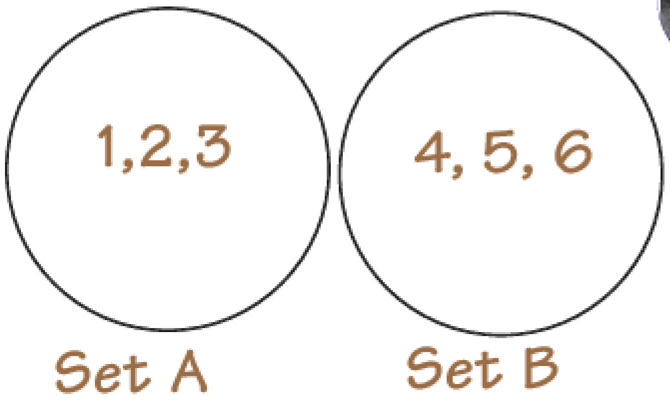
John Venn was born in 1834 in Hull, England. He was educated in London and went to college at Cambridge University. He later became a teacher there.

Venn became interested in logic and wrote several books on the subject. In 1881, he published a book called *Symbolic Logic*, which explained the Venn diagrams that he is now famous for. This work made him a very well known scholar. He worked on improving his diagrams over the years. He died in 1923 at the age of 88.

Ratios

Ratios are comparisons between numbers. We usually show this relationship by using the colon symbol (:). For example, most people have ten fingers. Suppose you show your hands like this:

diagram by drawing two circles that do not intersect.



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$$A \cap B = Ø$$

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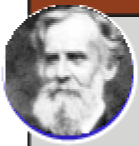

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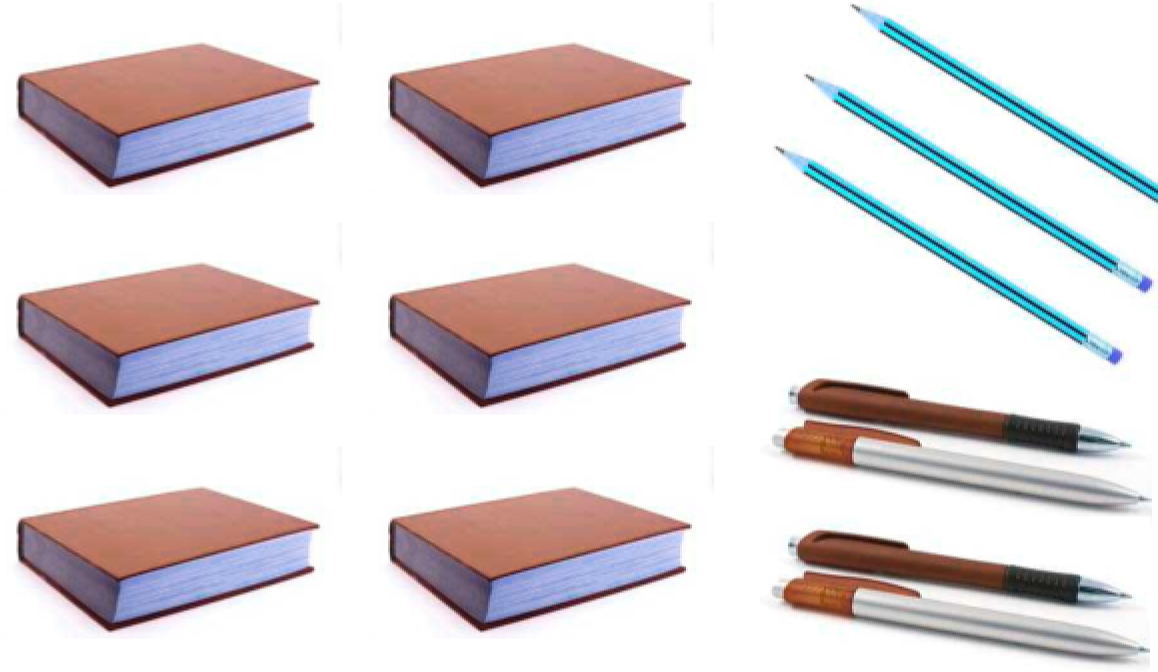
Ratios

Ratios are comparisons between numbers. We usually show this relationship by using the colon symbol (:). For example, most people have ten fingers. Suppose you show your hands like this:

Out of a total of ten fingers, only five are showing. This ratio can be shown as 5:10. To read this ratio you would say “a ratio of 5 to 10.” The comparison is between the total number of fingers showing and the total number of fingers.

Suppose you opened up your book bag. Inside are 6 books, 3 pencils, and 4 pens. The ratio of books to pens is 6:4. The ratio of pencils to the total number of items is the bag is 3:13.



It's simple! A ratio is just a comparison between two different things.

Find out more

Television Ratios

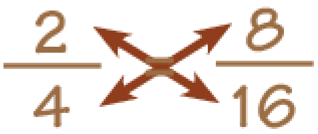
On televisions, all screens used to be a ratio of 4:3. That meant for every 4 inches (10.2 cm) of width, the screen was 3 inches (7.6 cm) high. Today, many televisions and computer monitors come in a ratio of 16:9. For every 16 inches (40.6 cm) wide the screen is, it is 9 inches high. This is wider than the old 4:3 standard. Because of this, TVs with a 16:9 ratio are called widescreen.

Proportions

Proportions are comparisons between two ratios. It is a statement that the ratios are equal. Proportions use an equal sign. They look like this:

$$\frac{2}{3} = \frac{6}{9}$$

To show whether proportions are equal, we cross multiply.



$$8 \times 4 = 32$$

$$2 \times 16 = 32$$

When we cross multiply, the product of each equation is 32. When the products are the same, the proportions are equal.

If you took a test with ten questions, each correct answer might be worth 10 points. The ratio would be 1:10. If you got 7 questions right, your score would be a 70. The proportion would look like this:

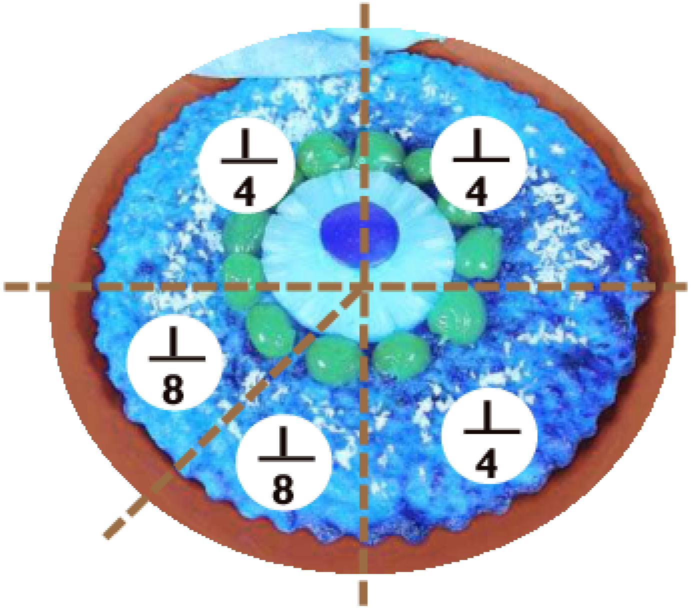
$$\frac{1}{10} = \frac{7}{70}$$

Imagine baking a cake. The cake is made up of ingredients. You might use flour, sugar, and butter. These ingredients have to be mixed together in certain amounts, or proportion. If you use the wrong proportions of ingredients your cake will be ruined. The proper ratio of ingredients will make a great cake. Proportions are very important.

If followed carefully, the recipe will produce a delicious treat!

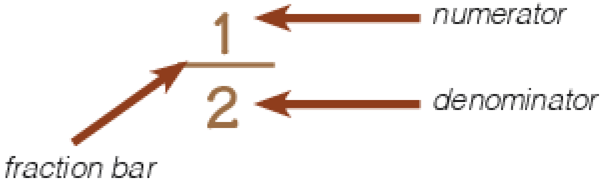
Fractions

One way of describing a ratio is with fractions. A fraction is used to show the ratio between two numbers. It can also show a piece, or portion, of a number.



Parts of a Fraction

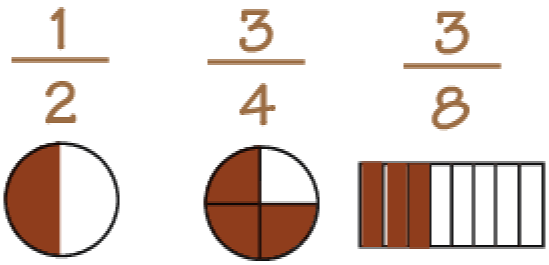
The top part of a fraction is called the numerator. The bottom part of a fraction is called the denominator. The line between them is called the fraction bar.



Types of Fractions

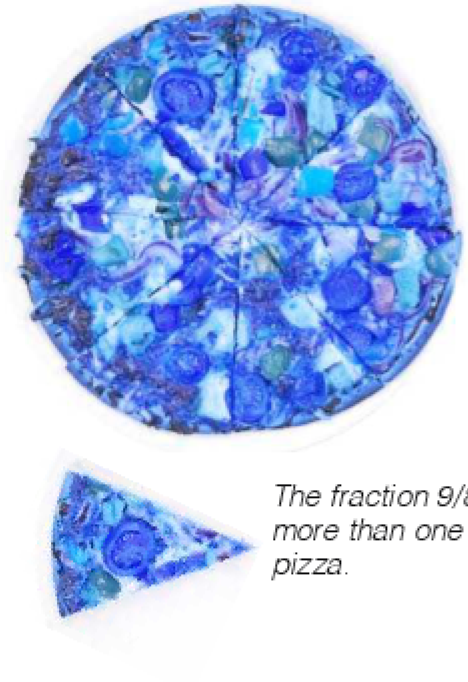
There are many different types of fractions.

In a proper fraction, the numerator is a smaller number than the denominator. These fractions equal an amount that is less than 1 whole.



The fraction 7/8 means less than one whole pizza.

In an improper fraction the numerator is a larger number than the denominator. These fractions equal a number greater than 1 whole.



The fraction 9/8 means more than one whole pizza.

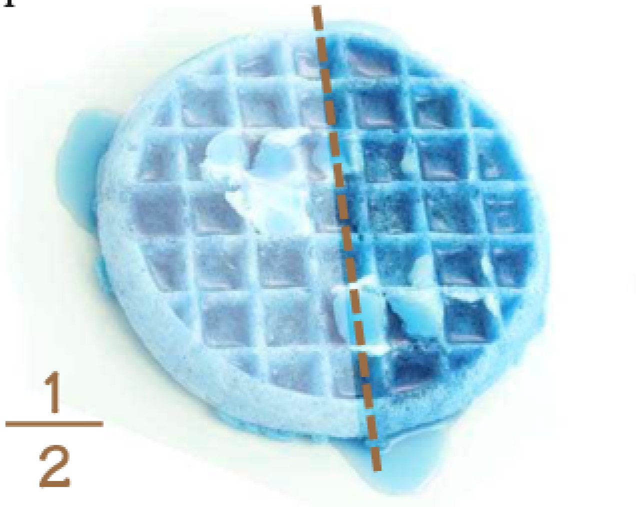
MATHEMATICS

Equivalent Fractions

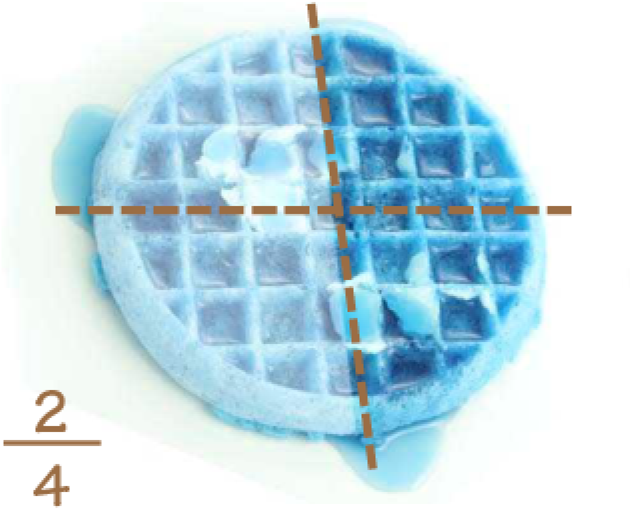
If fractions have the same ratio, they are equivalent. Look at the waffle photos.

$$\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$$

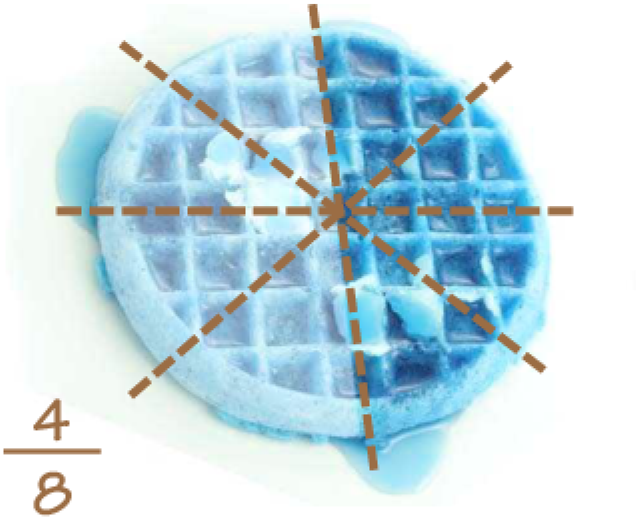
These fractions are all equal to half the waffle.



$\frac{1}{2}$



$\frac{2}{4}$



$\frac{4}{8}$

You can find equivalent fractions yourself. Multiply the numerator and denominator by the same number.

$$\frac{1}{2} \times \frac{2}{2} = \frac{2}{4}$$

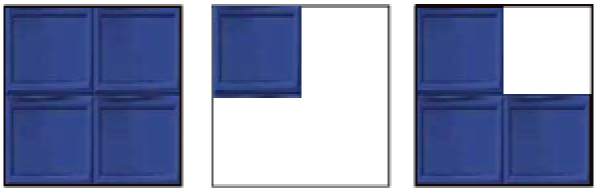
$$\frac{1}{2} \times \frac{4}{4} = \frac{4}{8}$$

Adding and Subtracting Fractions

To add or subtract fractions, they must have the same denominator. If they do, just add or subtract the numerators.

$$\frac{1}{5} + \frac{2}{5} = \frac{3}{5}$$

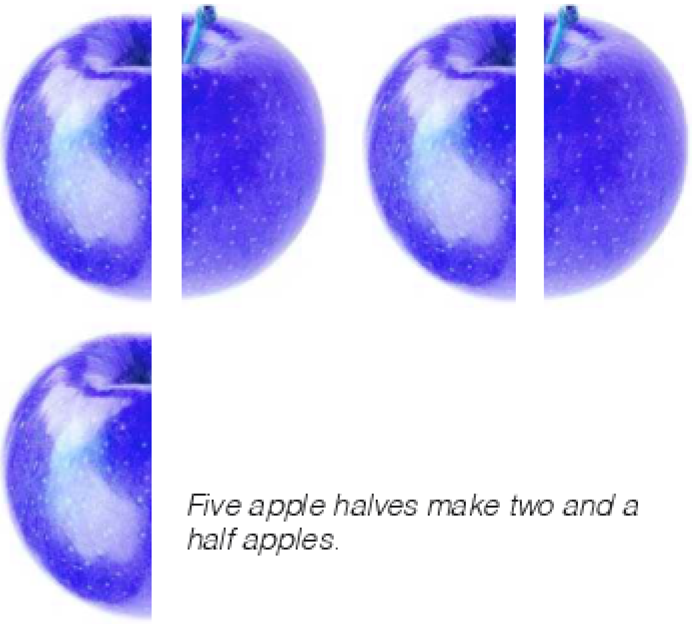
$$\frac{5}{7} - \frac{2}{7} = \frac{3}{7}$$



$$\frac{4}{4} - \frac{1}{4} = \frac{3}{4}$$

Improper fractions can be made into a separate whole number followed by a proper fraction. These are called mixed fractions. You can change an improper fraction into a mixed fraction by dividing the denominator into the numerator.

$$\frac{5}{2} = 2\frac{1}{2}$$



Five apple halves make two and a half apples.

Finding a Common Denominator

You can't add or subtract fractions unless they have the same denominator. If their denominators are different, you must find a common denominator for them.

A common denominator for the fraction 2/3 and 1/2 must be found in order to add or subtract them. To find a common denominator, list the multiples of 3 and the multiples of 2. Some of your numbers will appear in both lists. One common denominator is 6.

2 4 ⑥ 8 10
3 ⑥ 9 12 15

An equivalent fraction for 2/3 is 4/6. An equivalent fraction for 1/2 is 3/6. Now, these fractions can be added.

$$\frac{3}{6} + \frac{2}{6} = \frac{5}{6}$$

These fractions can also be subtracted.

$$\frac{3}{6} - \frac{2}{6} = \frac{1}{6}$$

Multiplying Fractions

When you multiply fractions, just multiply straight across. Multiply the numerators. Then multiply the denominators. The result is your new fraction.

$$\frac{5}{7} \times \frac{2}{3} = \frac{5 \times 2}{7 \times 3} = \frac{10}{21}$$

You can also multiply whole numbers and fractions. Just put a 1 under the whole number to make it a fraction, and multiply.

$$2 \times \frac{3}{7} = \frac{2}{1} \times \frac{3}{7} = \frac{6}{7}$$

Dividing Fractions

It's almost as easy to divide fractions as it is to multiply them. Just take the second fraction (the one after the division symbol), flip it upside down, and then multiply. That "flipped" fraction is called a reciprocal.

Take a problem like 1/8 ÷ 1/3. Multiply 1/8 by the reciprocal of 1/3, or 3/1.

$$\frac{1}{8} \div \frac{1}{3} = \frac{1 \times 3}{8 \times 1} = \frac{3}{8}$$

You can also divide whole numbers and fractions. Just put a 1 under the whole number to make it a fraction, and divide by flipping the fraction and multiplying.

$$5 \div \frac{1}{4} =$$

$$\frac{5}{1} \div \frac{1}{4} =$$

$$\frac{5 \times 4}{1 \times 1} = \frac{20}{1}$$

$$= 20$$

Decimals

What is a Decimal?

A number can be divided forever. It can be divided into smaller and smaller parts. One way to show these small parts is with a fraction. Another way is with a point called a decimal. A decimal point exists after every whole number, even if it is not written there.

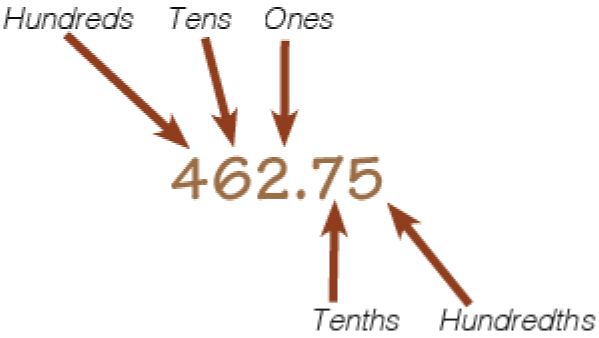
$$3 = 3.$$

$$415 = 415.$$

The fraction 1/2 is equal to 0.5. Decimals can be very important in calculating money, scientific numbers, and test grades. Another way of writing 50 cents is to write $0.50.

How Decimals Work

Any numbers to the left of a decimal point are whole numbers. Any numbers to the right of a decimal point show a fraction of a whole number. The places have different names as you move to the left or the right of the decimal point. To the left are the ones, tens, hundreds, thousands, ten thousands, and so on. To the right are the tenths, hundredths, thousandths, ten thousandths, and so on. We write 4 hundreds, 6 tens, 2 ones, 7 tenths, and 5 hundredths as 462.75.



Terminating and Repeating Decimals

Some decimals just end. They have a specific number of decimal places. These are called terminating decimals. An example is 462.75. It ends after the 5. Sometimes a decimal never ends. The final number is repeated over and over in a regular pattern. This is called a repeating decimal. The fraction 1/3 is equal to 0.33333.... The dots show that the 3 repeats forever. You can also write a bar over the repeating part. If you write 0.3 and draw a line over the 3, you show that the 3 repeats forever.

$$0.333\ldots = 0.\overline{3}$$

Rounding Off Decimals

Sometimes, a calculation can lead to a decimal that never ends and never repeats. You can't write a decimal that never ends. Instead, you can round it off. A number is rounded up if the next decimal is 5 or greater. The number stays the same if the next decimal is 4 or less. The number 0.0236 can be rounded up to 0.024. Money is usually rounded off to the nearest penny.

Statistics

Mathematics can be used to predict things. We can figure out when things may happen. This type of mathematics is called statistics.

Statistics can be used to calculate many things. You can figure out the odds of winning the lottery. You might find the number of kids who like to eat strawberry ice cream.

The odds of winning most state lotteries are more than 18 million to one. If you save one dollar a month for the next 20 years you will have saved $240. However, if you add the interest you will get it will be worth more than you originally had.

Statistics are used everywhere. All the sciences use them. Sports teams use statistics to see how each player is doing. Teachers and schools use statistics to analyze how well students are learning. You may use statistics when you figure out how long it takes to do your homework.

Find out more

Percentages

The word "percentage" means "per 100." A percentage is the ratio of a number per 100. The percent symbol (%) shows that a number is a percentage. Twenty-five percent can be written as 25% and is equal to the ratio 25/100. An equivalent fraction to 25/100 is 1/4, or one-fourth.

It is easy to write percentages with decimals. Just move the decimal point two places to the left. Twenty-five percent can also be written as the decimal 0.25. One hundred percent is simply equal to the number 1.

Percentages are used in many different areas. A store may have a sale where everything is 10% off. This means that 10 cents will be deducted for every dollar spent. People often leave a tip of 15% when they eat at a restaurant. This means that they will leave 15 cents for every dollar that the food cost them. Use the example below to practice calculating a tip.
Cost of dinner: $20. Tip: 15%

$$20 \times 0.15 = 3$$

The tip would be $3. The total bill would be $23.

Taxes are also measured in percentages.

Grades in school are sometimes measured in percentages. Scoring 100% on a test means you got all the answers right!

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Averages

One form of statistics is called averages. To find an average, add up a group of numbers. Then divide the sum by the amount of numbers. We can figure out the average test scores in a school by adding up all the scores and then dividing that total by the number of people who took the test.

An average is sometimes called the mean. This is not the same as the median. In a list where all the numbers are written in order from lowest to highest, the median is the middle number. On the list, the median has as many numbers lower than itself as it has numbers higher than itself. The median and the mean are usually different numbers.

Student	1	2	3	4	5

Score $20 + 30 + \underset{\text{median}}{\underset{\nearrow}{35}} + 40 + 45 = 170 \div 5 = \underset{\text{mean}}{\underset{\uparrow}{34}}$

34 is the average test score

Find out more

Baseball Stats

Baseball fans love to keep track of how baseball players are doing. Managers, coaches, and fans all use statistics, or stats. You can find these stats in the sports section of your newspaper. Some baseball stats are simple totals of home runs, stolen bases, and runs batted in, or RBIs. Other stats are ratios or percentages. A pitcher has an earned run average, or ERA. This is the average number of runs that the pitcher gives up in nine innings.

A batting average tells how well a baseball player hits. It is calculated using the number of times the player has come to bat and the number of times he has gotten a hit. A player who hits safely every time would be hitting 1.000. We would say, “He is batting a thousand.” Most hitters have a batting average between .200 (two hundred) and .300 (three hundred). This means that they get 2 or 3 hits for every 10 times they come to bat. Great hitters have a batting average of over .300. In baseball, a player who gets out 7 out of 10 at bats is considered a good hitter.

Probability

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Probability tells us how likely something is to happen. It is sometimes called chance. Another name for probability is odds.

Probability can be measured in the real world. Start flipping a coin in the air. Flip it 100 times. The odds are that it will land on heads about 50 times, and tails about 50 times. The chances are equal. We call this a 50-50 chance.

You can figure out the chance of just about anything. Rolling a 6-sided die will give you a 1 in 6 chance of rolling a 5. If you do this thousands of times, the odds remain the same. The average ratio of rolling a 5 will be 1 in 6, or 1/6. The chances of rolling two 5's in a row are much smaller.

Playing games with dice make them exciting because you are never sure what number you will roll.

Find out more

The Lottery Has Bad Odds

In a lottery, people pick numbers they hope will match the official numbers chosen. The more numbers you can pick from, the lower the probability that the numbers will match. Most people who play do not win. You have a better chance of being struck by lightning than of winning millions of dollars in a lottery!

Algebra

Figuring out a math problem when you know all the numbers is usually easy. But what if one of the numbers is unknown? The problem would be more difficult. You would need to use algebra to figure it out. Algebra is the branch of mathematics that deals with unknown quantities. The principles of algebra are used to describe everything from exchanges of money to lines on a graph. Algebra uses all the rules of arithmetic. It also creates new rules.

Variables

In algebra, an unknown number is called a variable. We use letters to stand for the unknown numbers. Often, variables are letters such as a, b, and c or x, y, and z. Variables can stand for many different values or a range of values.

Variables can be used to solve problems even if all the values are not known. Imagine that a passenger freight train is 200 meters long. If the freight train is 30 meters less than twice the length of a passenger train, how long is the passenger train?

You can use algebra to determine the length of this train.

You can write this word problem as an equation. Let the variable x represent the length of the passenger train.

Twice the length of the passenger train would be 2x, or two times x. The length of the freight train is 30 meters less than that.

$$200 = 2X - 30$$

To solve this equation, the x must be by itself. To do this, add 30 to both sides of the equation.

$$30+200=2X-30+30$$

or

$$230=2X$$

Then divide both sides of the equation by 2.

$$230 \div 2=2X\div2$$

or

$$115 = X$$

The passenger train is 115 meters long.

Exponents

Mathematicians often multiply numbers by themselves. An easy way to show this is to use an exponent. An exponent is a little number written above and to the right of a number or variable. It tells how many times the number or variable should be multiplied by itself. For example:

$$5^3 = 5 \times 5 \times 5 = 125$$

Numbers or variables with an exponent of 2 are called squared numbers. We say the variable x^2 as "x squared". It is the same as x times x. Numbers or variables with an exponent of 3 are called cubed. We say the variable y^3 as "y cubed". It is the same as y times y times y.

Exponents are sometimes called the powers of a variable. We would say 5^3 as "5 to the third power".

Find out more

Scientific Notation

Many numbers are either really big or really small. One way to show these numbers is to simply write out all that make them up. You can also show big numbers or numbers much less than 1 by using scientific notation. Scientific notation is a way of writing a number as the product of a number between 1 and 10 multiplied by the number 10 raised to a power. The number is big when the exponent on the 10 is positive. The number is very small when the exponent on the 10 is negative.

It is usually easier to show very big or very small numbers using scientific notation than it is to write out the entire number. The average distance from the Earth to the Sun is about 149,000,000,000 meters. It is much easier to write that distance from the Earth to the Sun as 1.49×10^{11} meters. The mass of a single neutron inside of an atom is really, really small. Written out, it would be 0.00000000000000000000000000167 kilograms. In scientific notation, this number is written 1.67×10^{-27} kilograms, with a minus sign next to the exponent.

Roots

The root is a number multiplied by itself. When a number is squared, it is multiplied by itself. An example would be 3 x 3. In this example, the root number is 3. The number 3 squared, or 3 x 3, is equal to 9.

Square Root

The square root shows what quantity multiplied by itself will be equal to a specific number. For example, the square root of 25 is 5, because 5 x 5 =25.

The symbol for a square root ($\sqrt{}$) goes around the number. Since the square root of 25 is 5, we would write it like this:

$$\sqrt{25}=5$$

Cube Root

The cube root shows what quantity multiplied by itself twice will be equal to a specific number. The cube root of 8 is 2, or

$$\sqrt[3]{8}=2$$

because

$$8 = 2 \times 2 \times 2$$

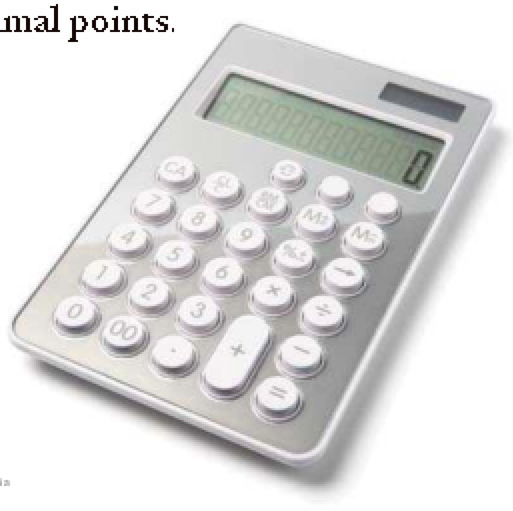
Roots are the opposite of exponents. The square root of the square of any number is equal to that number. The square root of $x^2 = x$.

Real Numbers and Imaginary Numbers

In general, two different types of numbers are the result of roots. Most of the numbers we use every day are called real numbers. The square root of a positive number will give a real number. Sometimes, mathematics can be used to make imaginary numbers. The square root of a negative number will give an imaginary number. We do not use imaginary numbers to count objects or measure distance. They are part of a complex number system that aid in understanding of things that are described by ordinary numbers. When determining the strength of an electromagnetic field, scientists use imaginary numbers.

How to Find Square Roots

Calculators and computers are often used to find the square roots of numbers. The square root of a number may result in a number with many decimal points.

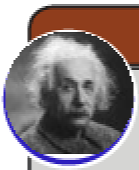


Roots

You can also find root numbers, square roots, squares and cubes using a chart like the one below. Because some square roots have many numbers to the right of the decimal, they are rounded here to the nearest thousandth.

Root Number	Squared	Cubed	Square Root
1	1	1	1.000
2	4	8	1.414
3	9	27	1.732
4	15	64	2.000
5	25	125	2.236
6	36	216	2.449
7	49	343	2.646
8	64	512	2.828
9	81	729	3.000
10	100	1,000	3.162
11	121	1,331	3.317
12	144	1,728	3.464
13	169	2,197	3.606
14	196	2,744	3.742
15	225	3,375	3.873
16	256	4,096	4.000
17	289	4,913	4.123
18	324	5,832	4.243
19	361	6,859	4.359
20	400	8,000	4.472

Albert Einstein (1879-1955)



Getting to know...

Albert Einstein was born in 1879 in Germany. He is considered one of the great thinkers of all time. People even use the word “Einstein” to mean a genius!

One of Einstein’s greatest contributions was a famous equation, $E=MC^2$. This equation shows the relationship between mass and energy.

Equations

An equation is a statement in mathematics that two things are equal. An equation always has two sides that are separated by an equal sign. Equations can be very simple, such as:

$$2 + 2 = 4$$

The amounts on both sides of the equal sign are the same.

Equations with Variables

A more complex equation may have a variable in it:

$$9 + X = 15$$

Equations may have many variables in them.

Polynomial Equations

An algebraic expression with two or more powers of a variable or variables is called a polynomial equation. An example of a polynomial equation is

$$y = 3X^3 + 5X^2 - 6$$

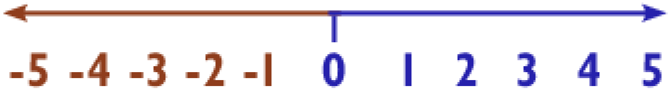
Solving Equations

The trick to solving the equation is to get the variable by itself and away from all the other numbers. Arithmetic is often used to solve equations. Whatever is done to one side of an equation has to be done to the other side.

Coordinates and Grids

Number Lines

Number lines are pictures in which numbers are shown along marks on a line. Some number lines have both positive and negative numbers on them with the number 0 in the middle. The 0 on a number line is known as the origin. They are called real number lines. Numbers can be plotted, or drawn, along the number line. We call the points on a number line coordinates.



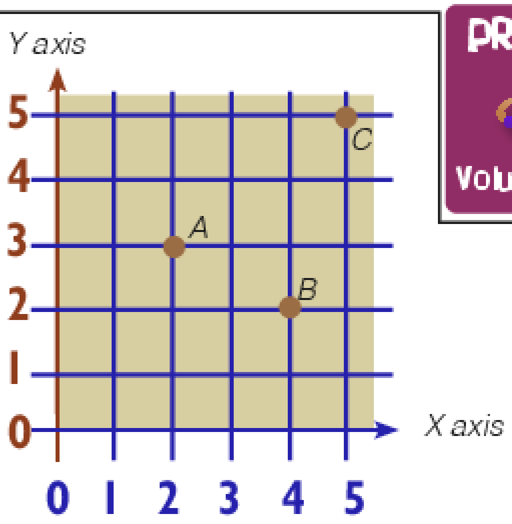
Coordinate Grids

Sometimes, two real number lines are put together. This is called a coordinate grid. The number line going from left to right is called the x-axis. The line going up and down is called the y-axis. The two lines cross where the zero is located on each line. This is called the origin. This sort of grid is sometimes called a coordinate plane.

Graphing Coordinates

Coordinates name a point on a graph. When writing coordinates, use the x-axis coordinate first, followed by the y-axis coordinate.

Point A: (2, 3)
Point B: (4, 2)
Point C: (5, 5)



Coordinates

Numbers called coordinates can be shown on the plane. The process of drawing the position of numbers is called graphing. Coordinates in a plane are often represented by the variables x and y. A point on the plane can be written with the coordinates (x,y). Numbers can be put in place of the variables. Then, a dot is drawn on the graph where those two coordinates meet. A line can be drawn between two points that are graphed. Algebraic equations can also be graphed.

Words to know

axis (AK-siss): a line at the side or bottom of a graph

grid (grid): a set of straight lines that cross each other to form a regular pattern of squares

plane (plane): a flat surface

Emmy Noether (1882-1935)

Getting to know...

Emmy Noether was born in Germany in 1882. She studied to be a teacher but then turned to mathematics. Women could not enroll at German universities. Noether was allowed to attend classes and earned a Ph.D. She worked at the local university without pay because she was a woman. The famous mathematician David Hilbert invited her to the University of Göttingen. Noether taught classes listed under Hilbert's name.

Noether made contributions to the field of abstract algebra. She also worked on the mathematics behind Albert Einstein's theory of relativity. In 1933, the Nazis took power in Germany and Noether was fired because she was Jewish. She moved to the United States but soon died following an operation.

Geometry

Geometry is the branch of mathematics that studies objects in a defined space. Geometry is all about shapes and how to describe them.

Geometry studies mathematical designs. They are all around us. Lines, triangles, squares, rectangles, circles, and ellipses are found everywhere in nature. The shape of a snail shell is spiral. Lines are everywhere you look.

A snail shell is an example of a Fibonacci Spiral. It follows the Fibonacci numbers or pattern. The pattern continues by adding the two previous numbers to get the next number in the sequence 0, 1, 1, 2, 3, 5, 8, 13....

Basic Geometry

Basic geometry deals with two-dimensional shapes. Anything that is flat is a two-dimensional surface. In geometry, this kind of surface is called a plane.

Advanced Geometry

More advanced geometry involves three-dimensional shapes. These shapes are objects that exist in space. Three-dimensional objects can be made by rotating or combining two-dimensional objects. The world around you is three-dimensional. But many things in the world can be viewed more easily by looking at them in two dimensions.

Sphere

Cube

Pyramid

Cylinder

Words to know

dimension (duh-MEN-shuhn): the dimensions of an object are the measurements of its length, width, and height

ellipse (i-LIPS): an oval shape

rotate (ROH-tate): to turn around and around like a wheel

Euclid (birth c. 300BC-death unknown)

Getting to know...

Euclid was probably born in Greece between 335 and 300 BC. He attended a famous school called the Academy. Ptolemy, the ruler of Egypt, invited Euclid to teach at the Museum of Alexandria. Euclid created his own school of mathematics and wrote mathematical papers.

His most important book was called *The Elements*. It arranged the principles of geometry in a logical way. Euclid was so important that all ideas in geometry different from his are called non-Euclidean.

Lines

Lines

A line is said to be infinite, or without end. It stretches forever in both directions. A point is an exact location on a line. There are an infinite number of points on any line. We name a line by naming any of its two points.

There are two points on this line, point x and point y.

Rays

A ray is a line that starts at one point. It then goes on forever in one direction. It is named for its endpoint and one other point found on the ray.

We would call the beginning of the ray point A.

Line Segments

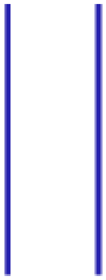
A line segment is part of a line. It starts at one point and ends at another point. It is named for its two endpoints.

This line segment is known as line segment AB.

Parallel Lines

Two straight lines put side by side can be parallel. Parallel lines always remain the same distance apart from each other. The lines could go on forever and never cross.

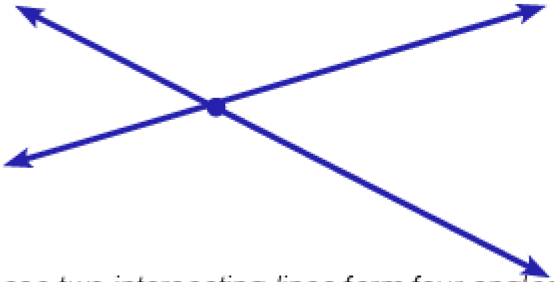
As long as the two lines could never meet, they will be considered parallel.



Train tracks must remain parallel for the train to move and stay on the track.

Intersecting Lines

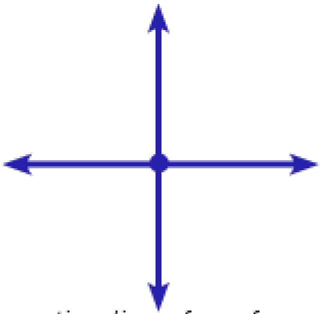
Lines that cross are said to intersect each other. They intersect at one common point, forming four angles. Intersecting lines cannot be parallel. Lines that look like they are very close to each other form two very small angles and two very wide angles where they intersect.



These two intersecting lines form four angles, two obtuse angles greater than 90°, and two acute angles, less than 90°.

Perpendicular Lines

Lines that make a perfect cross are said to be perpendicular. Perpendicular lines are the opposite of parallel lines. They form four right angles when they cross.



These two intersecting lines form four angles. All are right angles, or exactly 90°. These lines are perpendicular to one another.

Angles

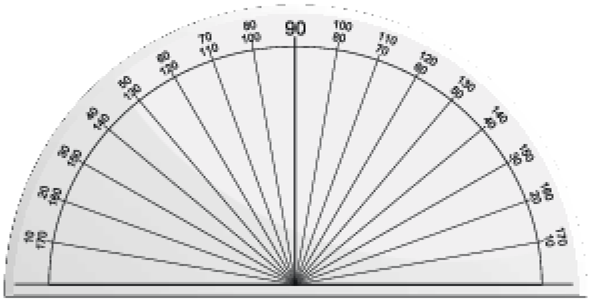
An angle is formed when two lines meet each other at a point. The point where the two lines meet is the vertex of the angle.

Measuring Angles

The common unit of measure for an angle is called degree. The degree symbol (°) always follows a number.

A line with one end held in place can be rotated a total of 360° around. A whole circle has 360° in it. You have rotated 360° when you spin completely around. Half of a circle has 180° in it.

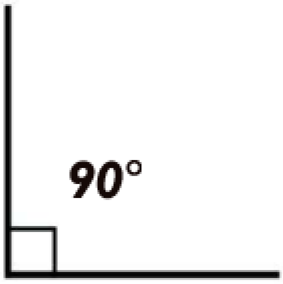
A device called a protractor can be used to measure angles.



A protractor

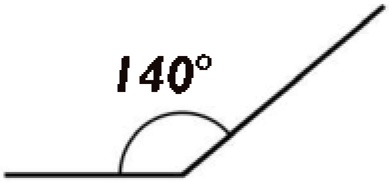
Right Angles

Perpendicular lines have an angle of 90°. This is often called a right angle.



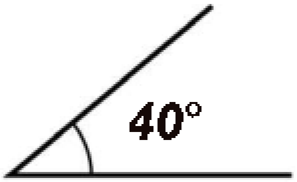
Obtuse Angles

Angles greater than 90° are called obtuse angles.



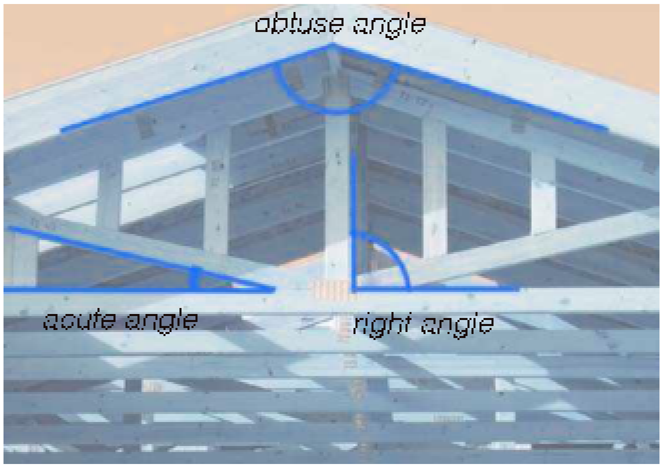
Acute Angles

Angles smaller than 90° are called acute angles.



How We Use Angles

Angles are very important in many different fields. Houses are built using very precise angles. Most houses have walls that are perpendicular to the floor. The roofs of some houses rise up at an angle to help water and snow run off. Angles are important in art and many of the sciences.

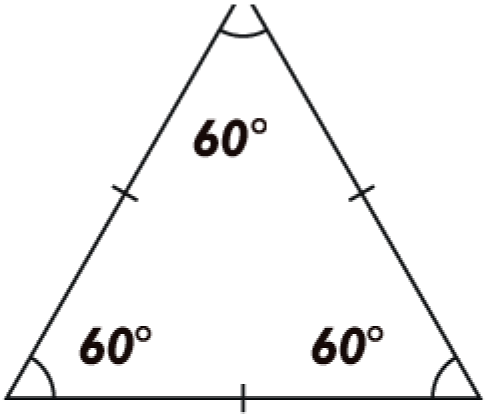


Triangles

Triangles are shapes that have three sides and three angles. The word "triangle" even means "three angles." The sum of all the angles in a triangle is always 180°. There are several different types of triangles.

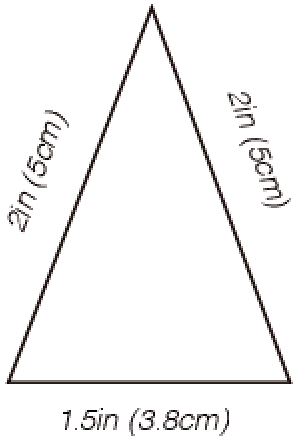
Equilateral Triangles

A triangle whose sides are all the same length is called an equilateral triangle. All three of its angles are 60°.



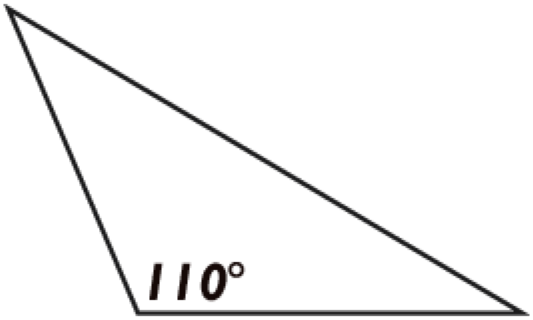
Isosceles Triangles

A triangle with two sides that are the same length is called an isosceles triangle.



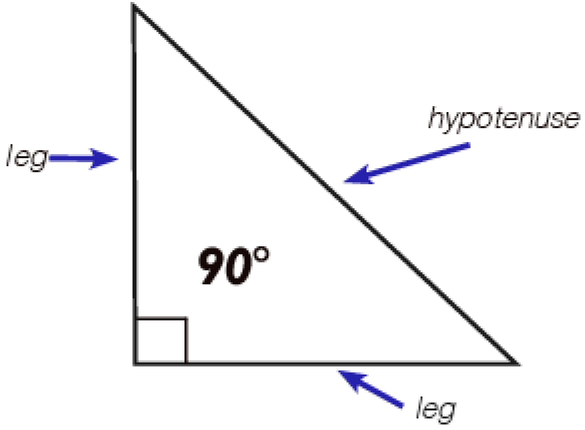
Obtuse Triangles

A triangle with one angle bigger than 90° is called an obtuse triangle.



Right Triangles

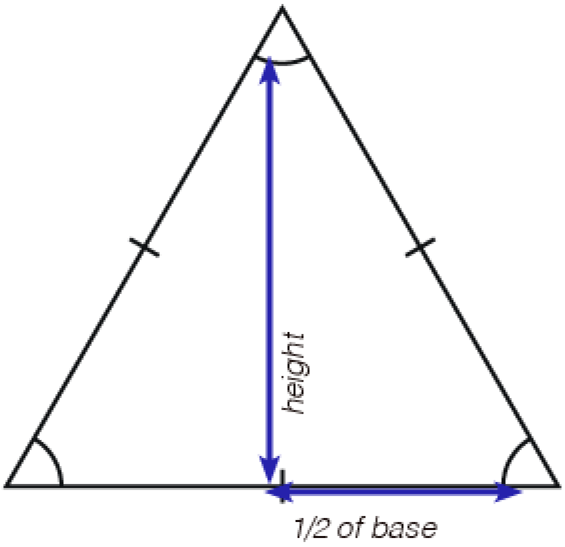
A right triangle has a 90° angle in it. The short sides of a right triangle are called legs. The long side that is opposite the right angle is called the hypotenuse.



The Area of a Triangle

The area, or space inside, of a triangle is 1/2 of the total base width times the total height. The formula looks like this:

$$A = \frac{1}{2} B*H$$



Find out more

The Pythagorean Theorem

Right triangles have many special properties. Thousands of years ago, people figured out that the squares of the sides of any right triangle are related to each other. The sum of the squares of the legs of a right triangle is equal to the square of its hypotenuse. This is called the Pythagorean Theorem.

The Pythagorean Theorem is often written as an equation:

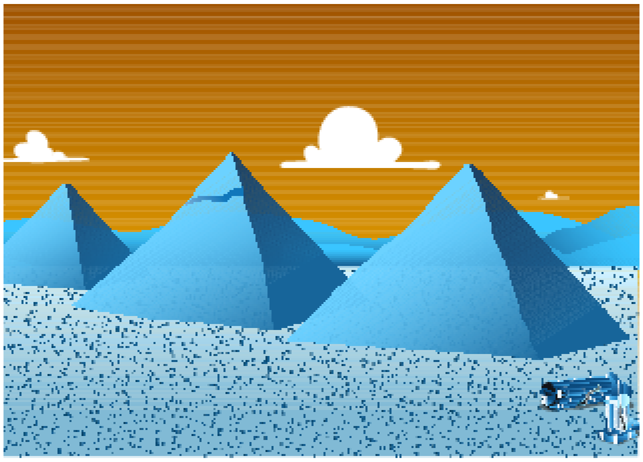
$$a^2 = b^2 = c^2$$

In this equation, a and b are the lengths of the two legs of the triangle and c is the length of the hypotenuse. The length of one side of a right triangle can be figured out using basic algebra if the other two sides are known.

Pythagoras was a mathematician from ancient Greece. The Pythagorean Theorem is named after him because he was the first person to prove the theorem. Ancient Egyptians used the theorem to help them build the Great Pyramids. People use the Pythagorean Theorem today to figure out distances between points.

Pyramids

A three-dimensional object with a single base and sides made out of triangles is a pyramid.



Ancient Egyptians built pyramids using the Pythagorean Theorem.

Trigonometry

There is a form of mathematics called trigonometry. It uses the properties of triangles to solve mathematical problems. Trigonometry can be used to figure out angles. Much of trigonometry involves measurements using right triangles.

Rectangles and Squares

Rectangles

Rectangles are shapes that have four sides and four right angles.

The opposite sides of a rectangle are always parallel to each other.

The two longer sides are parallel and the two shorter sides are parallel.

Some rectangles are long and thin. Others are short and squat.

Carl Friedrich Gauss
(1777-1855)

Getting to know...

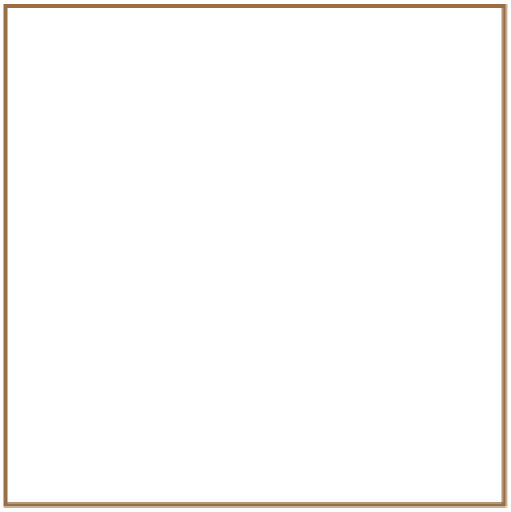
Carl Friedrich Gauss was born in 1777 in what is now Germany. He was a child prodigy. This means that he showed great talent at a very young age. His family was poor, but a rich Duke supported him. Gauss became a professor of astronomy when the Duke died in a battle.

Gauss loved mathematics. He discovered how to build a polygon with seventeen sides. He found a general rule for which polygons could be made and how to build them. Gauss wrote a book on number theory that made him famous. He also revolutionized mathematics with his work in geometry and algebra. He helped build the foundations for statistics. Gauss was called the Prince of Mathematics.

Doors and windows often have rectangular shapes. The area of a rectangle can be found by multiplying the width times the height.

Squares

Squares are rectangles whose sides are all the same length.



All four sides are equal in length and form right angles.

Parallelograms

Both rectangles and squares are special types of parallelograms. Parallelograms are four-sided shapes that have two pairs of parallel sides. They do not always have right angles.

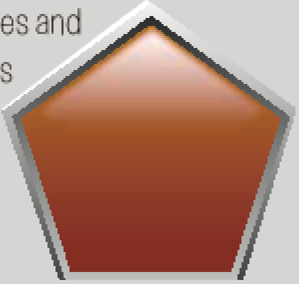
The longer sides are parallel and the shorter sides are parallel as well. The longer and shorter sides are not perpendicular because they do not form a right angle when they meet.

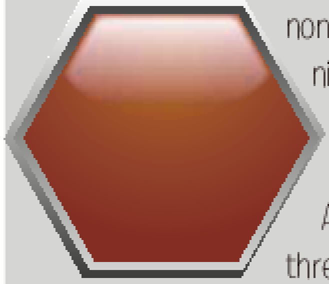
Find out more

Polygons and Polyhedrons

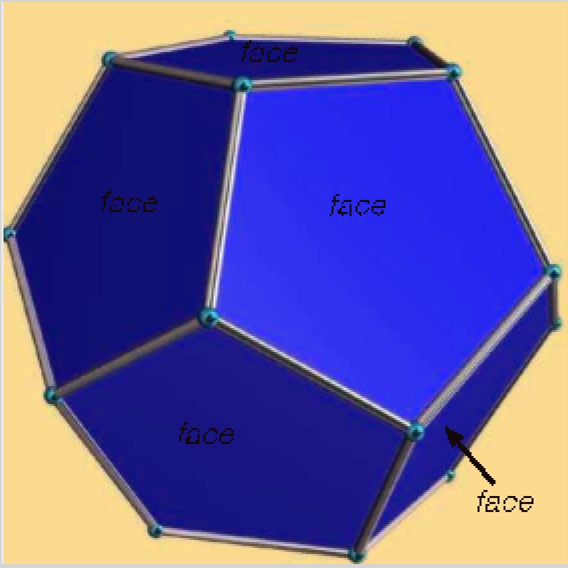
A polygon is a two-dimensional shape made of straight lines. Usually, the word "polygon" refers to shapes with sides that are equal in length. The names of different polygons use special prefixes to tell you how many sides they have. A pentagon has five sides and five angles. The headquarters of the U.S. Department of Defense got the name Pentagon because the building has five sides.

A hexagon has six sides and six angles. A heptagon has seven sides and seven angles. An octagon has eight sides and eight angles. A stop sign is an octagon. A nonagon has nine sides and nine angles. A decagon has ten sides and ten angles.

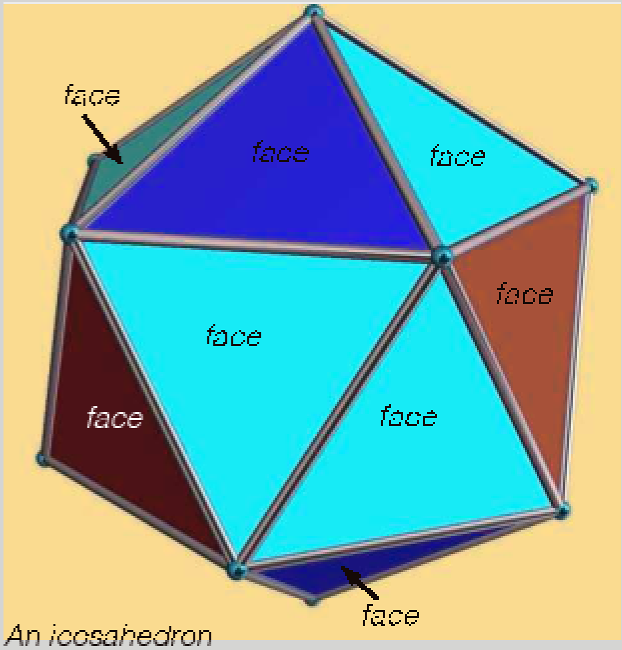




A polyhedron is a three-dimensional solid made of two-dimensional planes. The names of different polyhedrons tell you how many faces they have. The ancient Greeks found five polyhedrons that have all sides, angles, and faces equal. They called them regular solids or Platonic solids. The tetrahedron has four faces. The cube has six faces. The octahedron has eight faces. The dodecahedron has twelve faces. The icosahedron has twenty faces.



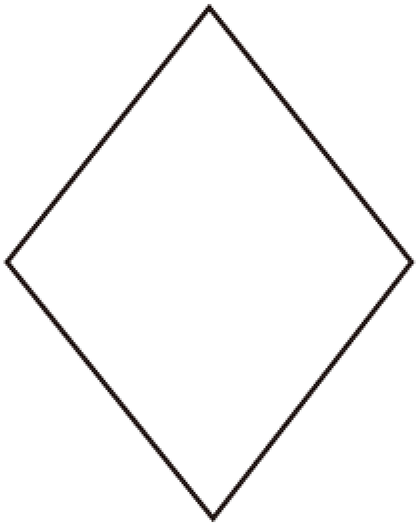
An dodecahedron



An icosahedron

Rhombuses

A rhombus has equal sides like a square but is tilted so that is does not have right angles.

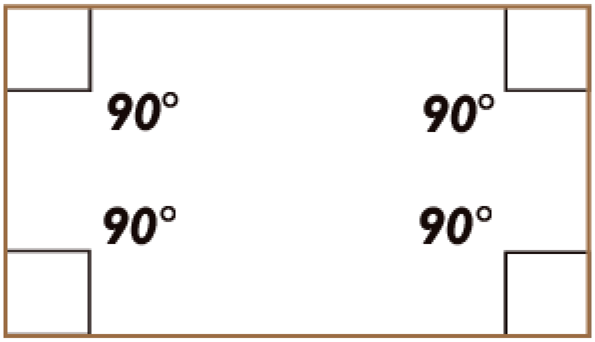


Sometimes, we use the word diamond when referring to a rhombus.

Quadrilaterals

All four-sided shapes made up of straight lines are called quadrilaterals. These include the parallelogram, the rhombus, the square, the rectangle, and the trapezoid.

The sum of all the angles in a quadrilateral is always 360°.



90° + 90° + 90° + 90° = 360°

Cubes

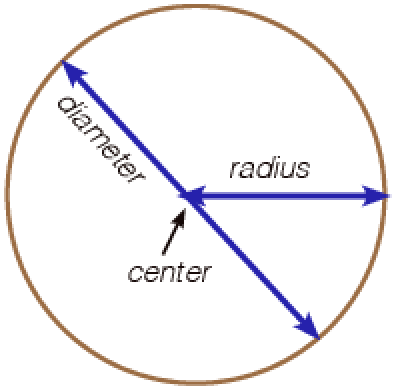
A three-dimensional shape that is made out of six squares is called a cube.

Circles

A circle is a shape made by drawing a curve completely around a point. Everywhere on the curve is the same distance from the point.

Diameter and Radius

The distance all the way across the center of a circle is called the diameter. The distance from the center to the edge is the radius.



Circumference

The distance all the way around a circle is called the circumference.

Pi

The ratio of the circumference to the diameter of a circle is a special number known as pi. This number is the same for every circle. Pi is equal to the never ending decimal number 3.141596.... It is usually rounded off to 3.14. It is often written as the Greek symbol π.

Finding the Circumference of a Circle. The circumference of a circle is equal to the diameter times π. Since the diameter is twice the radius, the circumference is equal to 2 times the radius multiplied by π. The equation to find the circumference of a circle is:

$$c=2\pi r$$

In this equation, c stands for circumference and r stands for radius. A circle with a radius of 3 centimeters has a circumference equal to 2 x π x 3. This number is equal to:

$$2 \times 3.14 \times 3$$

or

$$18.84 \text{ centimeters}$$

Archimedes
(287 BC- death unknown)

Getting to know...

The ancient Greek scientist, Archimedes was born in Syracuse, Sicily. His exact birth year is uncertain but was around 287 BC. He probably studied at the Library and Museum of Alexandria.

Archimedes was one of the greatest mathematicians in history. He calculated the value of the number pi (π). He also found the volume and surface area of a sphere. Archimedes invented many military machines and a water pump called Archimedes' screw. He is famous for shouting "Eureka!" (which means "I have found it!") when he discovered the idea of buoyancy while he was taking a bath.

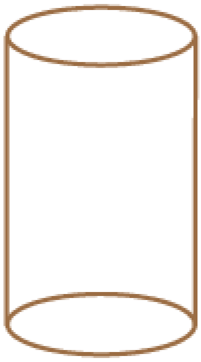
Finding the Area of a Circle. The area of a circle is equal to π times the radius squared. The equation to find the area of a circle is:

$$A=\pi r^2$$

In this equation, A stands for area and r stands for radius. A circle with a radius of 2 centimeters has an area equal to π x 2^2. This number is equal to 3.14 x 4, or 12.56 square centimeters (cm^2).

Circles in Three Dimensions

A circle in three dimensions makes a ball, or sphere. Circles can also form a tube called a cylinder. How many spheres and cylinders can you find in your classroom or your home? Do you think you'd find more cylinders or more spheres in your kitchen?

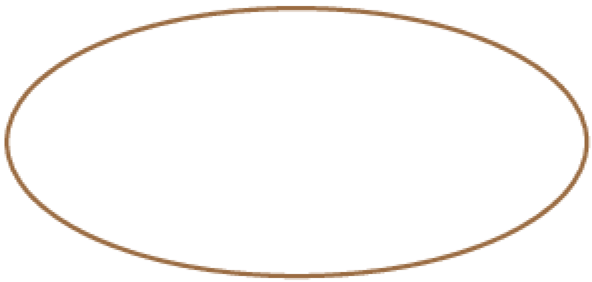


A cylinder

A sphere

Ellipses

An ellipse is an oval. Ellipses do not have a constant radius. Instead, they have two foci (fixed points). The sum of the distance to each focus is the same for every point on an ellipse.

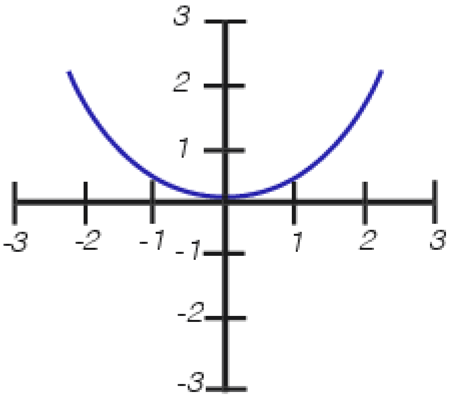


An ellipse.

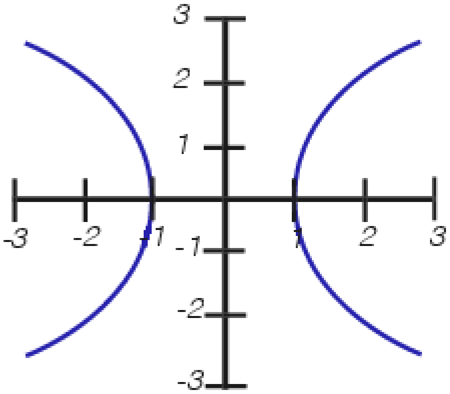
Curves

Sometimes, it is easier to understand an equation when it is represented by a picture. Algebraic equations can be charted on a graph to make geometric curves with many different shapes. Describing algebraic equations with geometric curves is called analytic geometry.

The graph of the equation $y = x^2$ is a parabola. It has a shape like a U.

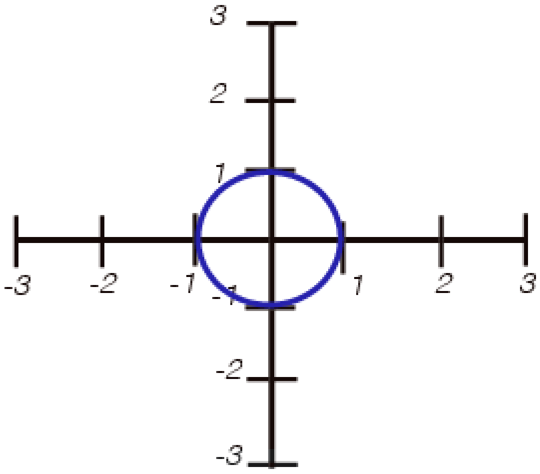


The graph of the equation $x^2 - y^2 = 1$ is a hyperbola.



The graph of the equation $x^2 + y^2 = 1$ is a circle with a radius of 1.

Many other geometric shapes can be made by graphing equations.



Measurement

Have you ever stood on a scale to weigh yourself? Can you estimate how tall you are?

When we measure things, we can figure out its weight, its length, or how much of something it can hold. We can measure time with a clock. We can tell the temperature by reading a thermometer.

If you live in the United States, you probably use the customary (Imperial) system of measurement. When you use words like inches and feet, quarts and gallons, and ounces and pounds, you are using the customary system.

Most countries use the metric system for measurement. Multiples of ten are the base of the metric system. When you want to change from one unit of length to another, you can simply multiply or divide by 10, 100, or 1,000 to form a larger or smaller number. If you use words like meter, liter, and gram, you are using the metric system.

Doctors and scientists use the metric system of measurement.

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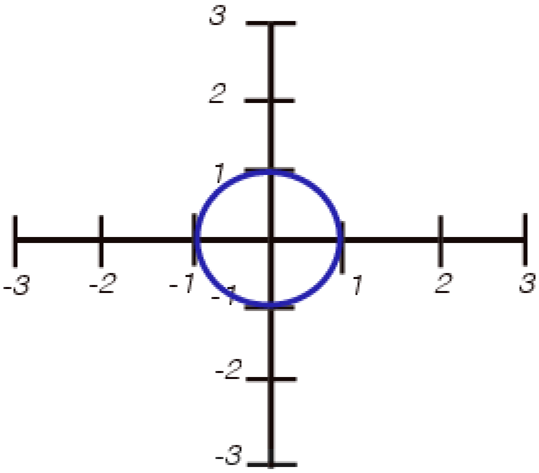

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Doctors and scientists use the metric system of measurement.

Length

Length tells us how long something is from one end to the other. We measure how far we can jump or throw a ball, and we measure how far it is from one city to another.

We would measure the distance this boy threw the ball in feet or meters.

In the customary system, long distances are measured in miles, and smaller things are measured in inches, feet, and yards. If you want to measure something smaller than an inch, you would measure it to the nearest half inch, quarter inch, or eighth of an inch.

In the metric system, long distances are measured in kilometers. Smaller things are usually measured in meters, centimeters, and millimeters.

If you live in the United States, you would probably say that this plant is 3 inches tall.

Customary	Metric
1 ft = 12 in	1 cm = 10 mm
1 yd = 3 ft	1 dm = 10 cm
1 mile = 5,280 ft	1 meter = 10 dm
1 mile = 1,760 yds	1 km = 1000 m

A friend who lives in another country would measure the plant in centimeters. To describe your plant to your friend, you would have to convert inches to centimeters.

1 inch = 2.54 centimeters

3 inches = 3 x 2.54 cm, which equals 7.62 cm.

Your friend, who uses the metric system, can picture your plant more easily when you tell him that it is 7.62 centimeters tall.

Other lengths can be converted from one system to another by multiplying.

Customary to Metric:

1 inch = 2.54 cm

1 yard = 0.9 meters

1 mile = 1.6 kilometers

Metric to Customary:

1 cm = 0.4 inches

1 meter = 3.3 feet

1 km = 0.6 miles

"I weigh 60 pounds."

Mass and Weight

When we talk about how heavy something is, we usually use the word weight. Scientists like to use the term mass instead of weight. Mass is a measure of the amount of matter something contains. It is measured with the metric unit of grams, usually on a balance. Your mass will not change, even if you travel to the moon.

Weight is the measure of the amount of gravity pulling on an object. Your weight would change when you travel to the moon, because the pull of gravity is not as strong .

"I weigh 10 pounds."

"My weight changed, but my mass didn't change. My body still contains all the same stuff it contained when I was on Earth."

An object's mass doesn't change when its location changes. That's why scientists and mathematicians use the term mass instead of weight. On Earth, your mass and weight are considered to be about the same.

Measuring Weight

We can use a scale to measure the weight of an object. When you were a baby, you were probably weighed on a special scale that held you while you were weighed. As you got older, you could stand on the tall scale in the doctor's office. At home, you might have a small bathroom scale. If not, you might use the big one at the grocery store where you shop. These scales tell us how many pounds or kilograms we weigh.

In the customary system, we measure weight in ounces, pounds, and tons. Look around your kitchen, and you will find many foods in small jars and boxes that are measured in ounces. Heavier items, such as sugar and flour, are sold by the pound. A large bag of sugar weighs 5 pounds.

You won't find any food sold by the ton. You'd never be able to carry it home! One ton is the same as 2000 pounds (907 kg).

In the metric system, mass is measured in grams, milligrams, and kilograms. One paperclip weighs about 1 gram (.04 oz). A U.S. nickel weighs 5 grams (.18 oz). A kilogram is much heavier. One thousand grams equal 1 kilogram. We use kilograms when we weigh large objects such as pianos and people. To measure very small amounts, we use milligrams. A milligram is 1/1000 of a gram.


This scale is used to measure the weight of fruits and vegetables in a grocery store.

Customary	Metric
1 pound = 16 ounces	1 g = 1,000 mg
1 ton = 2,000 pounds	1 kg = 1,000 g

Look at the food in your kitchen. Many of the packages show both metric and customary weights.

Your 5 pound bag of sugar is 2.26 kilograms. A small bottle of spice weighs 25 grams (0.9 ounces). Sixteen ounces of rice is 454 grams. You can convert from one system to another by multiplying.

Customary to Metric:

1 ounce = 28 grams

A cookie weighs three ounces. How many grams is that?

3 X 28 = 84

The cookie weighs about 84 grams.

1 pound = .45 kilograms

A box of candy weighs two pounds. How many kilograms is that?

2 X .45 = .9

The box of candy weighs about .9 kilograms

Metric to Customary:

1 gram = 0.035 ounces

A soda weighs about 453 grams. How many ounces is that?

453 X .035 = 16

The soda weighs about 16 ounces.

1 pound = .45 kilograms

A box of candy weighs two pounds. How many kilograms is that?

2 X .45 = .9

The box of candy weighs about .9 kilograms

Volume

When we measure volume, we are finding out how much something will hold. If you are buying orange soda for a group, you might buy it in a 2 liter bottle. A smaller bottle or can might contain 12 to 16 ounces.

One gallon of liquid is the same as four quarts of liquid.

In the customary system, volume is measured in cups, pints, quarts, and gallons. A family that drinks a lot of milk would buy it by the gallon.

Many recipes use one or two cups of milk. For smaller amounts, you can measure fractions of one cup, such as 1/4, 1/3, or 1/2 cup of the liquid. In a recipe, you would also use your measuring cups for dry ingredients such as flour
and sugar.

Sometimes you only want to add a little bit of something to a recipe. You can measure smaller amounts with a set of measuring spoons. If you want to add a very little amount of salt to a recipe, you would use the smallest measuring spoon, 1/8 of a teaspoon. Measuring spoons include many sizes. The largest of all, the tablespoon, is equal to 3 teaspoons.

One tablespoon is equal to three teaspoons.

The metric system measures volume in milliliters and liters. One liter (1 l) is very close to the quart used in the United States. You can find 1 liter bottles of soda and water on the grocery store shelves. You are probably more familiar with 2 liter (2 l) bottles of soda. Two 2 liter bottles are just a little more than one gallon in the U.S. customary system.

Measuring spoons are used for smaller volumes. In a set of metric measuring spoons the smallest measures 1 milliliter (1 ml), and the largest measures 25 milliliters (25 ml).

Customary	Metric
1 pint = 16 ounces	1 l = 1,000 ml
1 pint = 2 cups	
1 quart = 2 pints	
1 gallon = 4 quarts	

Measuring cups often show both metric and customary units of measurement.

Sometimes we want to use a recipe from another country. A measuring cup that shows cups and milliliters makes things easier. But what if we do not have one? It is helpful to be able to convert liters to cups on our own. We can convert from one system to another by multiplying.

Customary to Metric:

1 cup = 0.24 liters

1 quart = 0.95 liters

1 gallon = 3.79 liters

Metric to Customary:

1 liter = 4.2 cups

1 liter = 1.1 quarts

1 liter = 0.26 gallons

Find out more

The Metric System

Numbers in the metric conversion chart are powers of ten (10, 100, 1,000). There is only one unit for length (the meter), one for mass (the gram), and one for volume (the liter). The metric system was developed in the late 1700s to make it easier to understand and convert from one measurement to another.

We know that 1 meter equals 100 centimeters. How many centimeters are there in 5 meters? Just multiply. 1m = 100 cm, so 5 m = 5 x 100 cm, or 500 cm.

The prefixes used for measurement are the same for meters, liters, and grams. The meter is the basic unit of length. The prefix kilo- means 1000. So, a kilometer is 1000 meters. A kiloliter is 1000 liters, and a kilogram is 1000 grams.

meter (m), liter (l), gram (g)
milli- (m) 1/1,000
centi- (c) 1/100
deci- (d) 1/10
deca (da) 10
hecto (h) 100
kilo (k) 1,000

There is a relationship between the different units. The metric system was designed so that 1 cm^3 = 1 ml.
One ml of water weighs 1 gram.
One liter of water weighs 1 kilogram.

Time

How long can you hold your breath? When will dinner be ready? How long until my plant produces watermelons? Time is measured in many ways. We divide a minute into 60 seconds, and count how long we can hold our breath. We set the kitchen timer for 8 minutes to prepare our favorite pasta, but the roast in the oven will bake for at least an hour. It will take days for a plant to grow from a seed, and months until it produces fruit.

Time is a way for us to measure the span between events. We can keep track of time by watching the seasons of the year, noting the rising and setting of the Sun, keeping track on a calender by reading the time on a clock.

Time Conversions

60 seconds = 1 minute
60 minutes = 1 hour
24 hours = 1 day
7 days = 1 week
4 weeks = 1 month (approximately)
12 months = 1 year
365 days = 1 year

Most months last for 30 or 31 days. February is the only month that is exactly four weeks long. But that changes every fourth year. In a leap year, we add a day to February, giving it 29 days in all. This changes the year from

365 to 366 days. We do this because it takes a little more than 365 days for the Earth to orbit the Sun. It really takes 365 days, plus about 6 hours. If we didn't adjust the calendar every four years, our seasons would become confused.

When we want to know the exact time, we use clocks. Digital clocks show the time in numerals. Analog clocks show the time with hour and minute hands on a numbered circular face. The minute hand makes a complete circle every hour, while the hour hand moves slowly from one number to the next. Some analog clocks have a second hand, which circles the clock every minute.

Find out more

A.M. and P.M.

Suppose you receive a note from a friend asking you to call him at 8:00 the next day. Would you call at 8:00 in the morning or 8:00 in the evening? To be clear, your friend should have added A.M. or P.M. to his note. A.M. (from the Latin ante meridiem) means before noon, and P.M. (from the Latin post meridiem) means after noon. If your friend had asked you to call at 8:00 A.M., you would know he wanted you to call him in the morning.

Many decisions we make every day depend on temperature. Our summer clothes are very different from our winter clothes. We plan our activities based on the temperature. Meat is cooked to a certain temperature before we serve it for dinner. Temperature is the degree of heat or cold in something.

We measure temperature with a thermometer. Some thermometers measure the air temperature, others are used in baking. When we are not feeling well, there are different types of thermometers for measuring body temperature.

In the United States, temperature is measured on the Fahrenheit scale. The Celsius scale is used by all scientists and in most other countries.

To convert Fahrenheit temperatures to Celsius, subtract 32 from the Fahrenheit temperature, then multiply by 5/9. A cool fall day might be 59° Fahrenheit or 15° Celsius.

Temperature Scales

Fahrenheit Scale

Water freezes into ice at 32° F
Water boils at 212° F

Celsius Scale

Water freezes into ice at 0° C
Water boils at 100° C

Advanced Math

Some math problems can't be solved with arithmetic. Even algebra and geometry alone can't solve them. Scientists have looked for other types of mathematics to help them solve these problems.

Advanced math can help. Some types of advanced math help us understand our own world. Sometimes, they help us understand the universe. Math can be used to see the universe in new ways.

All the rules of basic mathematics apply to the more advanced forms of mathematics. In this way, the language of mathematics is consistent.

Calculus

Finding the area of a square is easy. A square doesn't get larger or smaller. But some things do change. Calculus helps us to figure out how physical quantities change.

Calculus was created in the seventeenth century by Isaac Newton and Gottfried Wilhelm Leibniz. They developed it to solve mathematical problems that regular algebra and geometry could not solve.

Many discoveries were made with calculus. It is the main mathematical language used in science and technology. It was used to define many of the basic principles of physics. Engineers use calculus to show how something works without having to build it.

Chaos Theory

Many things in nature seem like they are random. Blades of grass blowing in the wind or the shapes of rain clouds are tricky to show using ordinary math. These types of movements and shapes are not entirely random. A branch of mathematics that can explain these things is called chaos theory. This is the study of results that seem unpredictable given some sort of starting point.

Chaos theory, discovered in 1961 by a meteorologist named Edward Lorenz, explains how even the smallest change to the initial conditions of a system can drastically change its behavior forever.

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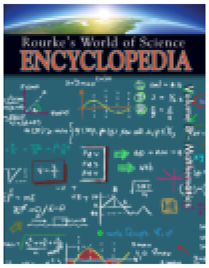

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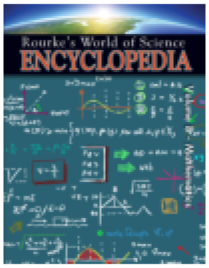

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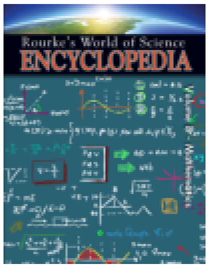

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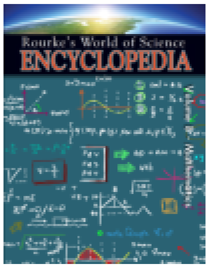

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