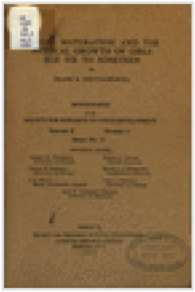

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Sexual Maturation and the Physical Growth of Girls Age Six to Nineteen

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CHAPTER I

METHOLOGICAL PROBLEMS AND SPECIFIC HYPOTHESES

This study is concerned with two major problems. First, what procedures and statistical methods are appropriate for the analysis of longitudinal data, that is, repeated measurements on the same children? The advantages of longitudinal data have long been recognized, but there is still much uncertainty about what should or could be done with such data after it has been collected. The problem is too vast for any single study, but at least a conscious attempt may be made to advance its solution. Second, what are the patterns of physical growth among contrasted groups of girls whose first menstruation occurs at different ages? Since the first menstruation, or menarche, is conditioned by underlying endocrine factors, an association between menarcheal ages and differential patterns of physical growth would suggest that these patterns are also conditioned by endocrine factors. Although the problem of statistical methods supplied the initial impetus for this study and is regarded as of coordinate importance, the text is addressed to the general reader. As far as possible statistical and procedural details have been collected in Appendices A and B or relegated to footnotes.

1. Methodological Problems

The first American study based on repeated measurements of the same individuals was published more than sixty years ago by Bowditch (8, see numbered bibliography following the concluding chapter). For over forty years it has been the settled conviction of students of child growth that only repeated measurements would contribute substantially to our understanding of developmental processes (see Boas, 6). The collection of the necessary longitudinal data has been a very slow process, but, now that over two hundred studies employing such data have been reported, a disturbing phenomenon is apparent. With few exceptions the methods of analysis and the reports of findings proceed as if only cross-sectional data on different groups of children at different ages were available. Students of child growth are thus faced with basic problem of developing better methods for the analysis of longitudinal data.

The problem is more serious and more difficult than is generally realized. Two decades of concentration on the perfection of psychometric and of diagnostic procedures have provided a wealth of measuring instruments. Attention has been gradually shifting to the actual use of these instruments in genetic and developmental studies. Just at this point, however, the difficulties inherent in longitudinal data and the fetters imposed by tradition have conspired to produce only more of the old cross-sectional reports. In at least one notable instance these factors have actually inhibited the analysis of the data even though large sums had been invested in their collection.

The difficult nature of the problem arises from four sources. In the first place the collection of adequate longitudinal data presents many difficulties in addition to those involved in the collection of cross-sectional data. In spite of the most careful planning illnesses or accidents will produce gaps or omissions in the data. If the study begins with 1000 six year old children, it is probable that less than 500 will be available ten years later. Under normal conditions it is exceedingly difficult to measure children at their exact birthdays and difficult enough to measure them at regular intervals. When the seriatim measures of many children are examined, some will appear to be erratic or impossible in the light of the whole series. Longitudinal studies require time. The personnel collecting the data may change and with changes in personnel come unknown and undeterminable changes in the methods of taking the measurements. With the passage of time new and better techniques and measurements are developed. Hence, there arises the dilemma of retaining the old method or of adopting the new. Hence also, a measurement which seemed most desirable at the outset of a study may seem inadequate at the end. At the end it is easy to look back and criticise, yet difficult to begin anew and do better. Gaps in the data, overlapping records, irregular intervals, erratic figures, changing personnel, the possibility of unconscious changes in methods of measurement, and either actual changes or failures to make changes in methods give longitudinal data the appearance of confusion and chaos. The timid soul, fearing any departure from accepted procedures and keeping a weather eye on sharp-shooting critics, can only produce a perfunctory analysis of the cross-sectional aspects of the data. Appendix A of this study is devoted to a consideration of techniques for dealing with these limitations of longitudinal data.

Second, there has been a pernicious tendency, or temptation, to concentrate on the collection of data and to trust to luck that funds might be available for their analysis. Thus, thousands of dollars have been spent on the collection of longitudinal data, hundreds of dollars have been spent on cross-sectional problems, and only dollars have been devoted to the developmental aspects of the data. This peculiar set of behavior patterns is one more illustration of our perverse tendency to import into the field of human biology procedures which are best adapted to the natural sciences. In the natural sciences the creative work is often done prior to the collection of the data and the experimental set-up is the crux of the matter. If the experimental set-up is adequate only the most elementary computations from observed data will often provide an unequivocal yes or no answer to the problem. Studies of child growth, on the contrary, can not be rigidly experimental; only rarely is it possible to provide a clear yes or no solution to a given developmental problem. Understanding of child growth can not be had by the mere collection of data. It can only be had by the persistent and continued study and analysis of collected data. The cross-sectional nature of most reports based on longitudinal data reflect preoccupation with data collection to the neglect of analysis.

The non-experimental nature of studies of child growth suggests a third difficulty. If the developmental data are quantitative and at all extensive then the analysis must be statistical. A host of difficulties arise at this point. On the one hand available statistical methods of analyzing longitudinal data, at least their application to problems of child growth, leave much to be desired. The more familiar routine procedures involving the calculation of averages, standard deviations, and correlations were not intended for and are often inadequate for the analysis of longitudinal data. The uncritical use of these procedures is largely responsible for the anomalous situation that reports based on longitudinal data often yield only more of the old cross-sectional findings. The less familiar procedures involved in curve fitting, which would seem to be particularly well adapted to longitudinal data, also present an anomalous situation in that essentially all of them have been developed in connection with and for the most part applied to averaged data obtained from purely cross-sectional studies of different populations at different ages. In this connection the writer is inclined to believe that only one of the functions which have been served in other fields by curve fitting promise much for a science of child growth. Curve fitting has served in many fields to smooth out the irregularities, assumed to be fortuitous, in the obtained data. As will be amply demonstrated in scores of charts growth curves based on longitudinal data are exceptionally smooth and regular. In physics and engineering an empirical equation often serves the very practical need of exact interpolations between observation points. Patterns of growth in children change too gradually and instruments for measuring these changes are too crude to justify similar attempts at precise interpolation. In physical chemistry curve fitting serves beautifully to express in exact and compact form the law of a chemical process which occurs in time. Studies by Wetzel (37) suggest the great promise of this approach, but it is perhaps too early to rely upon such formulations. In any case better observational data are essential to further progress.

On the other hand, granted adequate statistical methods and a critical imagination in their application, there are still formidable difficulties. Each of the multitude of specialists collecting longitudinal data can not become statistician, nor is it possible for the statistician to be more than reasonably familiar with a multitude of special fields. We have yet to capitalize the possibilities of cooperation. Finally, while there can be no science of human biology without quantitative and statistical methods and in limited areas experimental attacks, it is quite improbable that these alone can provide the necessary understanding for the highly complicated and individual problems involved in child management, treatment, and control.

The more important contributions of this monograph to statistical methods of analyzing longitudinal data are limited to the demonstration in Chapter III of the high degree of reliability with which patterns of physical growth may be determined from longitudinal data, to the development in Appendix A of methods of dealing with certain limitations of longitudinal data, and to the suggestion in Appendix B that the statistical analysis of longitudinal data should be concerned primarily with growth increments and should be carried out separately for small homogeneous groups of cases who are contrasted in respect to some important variable. Graphical methods of analysis are employed extensively in the presentation of the constructive findings of Chapters IV to XIII.

2. Developmental Problems, The Specific Hypotheses

If we are to have a science of human growth it is not enough that techniques be available for dealing with the limitations of longitudinal data and that talent and money be devoted to analysis. All of these are fruitless in the absence of specific problems. Here is the fourth major source of cross-sectional studies of longitudinal data. In the absence of specific problems the student of child growth can no more apply statistical or graphical methods of analysis than the physicist can employ his skill in setting up experiments. There is, of course, a plenitude of development problems, but these have often been formulated in such general or such vague terms as to provide no concrete suggestions of how they might be approached or of what might be done to solve them. In this connection there is a vast gulf between the general theory of relativity and the specific hypothesis deduced from the theory that the rays of light from a distant star are bent in passing near the sun. It is only the specific hypothesis which is amenable to empirical test. Such specific hypotheses pointing directly to the necessary data and appropriate methods of analysis are almost non-existent in the field of longitudinal studies of growing children.

The general theory of this study is that patterns of physical growth during the adolescent period are largely under the control of endocrine factors which presumably begin to operate at an early age in some children and at a late age in others. For convenience of study the early or late advent of the menarche, or first menstruation, is used as a sign or symptom of these underlying factors. Hence, we may formulate the specific hypotheses that during the adolescent period the early or late advent of the menarche is closely associated (a) with physical size, and (b) with the timing of the changes in the contrasting patterns of physical growth. The matter of the close association between the timing of the menarche and the timing of changes in the pattern of physical growth is essential to the general theory that underlying biological, and presumably endocrine, factors are largely responsible for the differential patterns of growth during the adolescent period. Given this general theory and these specific hypotheses, the procedure is to divide a given population of girls into homogeneous groups according to the advent of the menarche and then to study the changing patterns of growth in these contrasted groups.

That there is an association between degree or stage of sexual maturation and physical size has been shown by Baldwin (4), Abernethy (1), Boas (5), Richey (27), and Leal (20). Among thirteen year old girls, for example, those who have already menstruated tend to be taller and heavier than those who have not menstruated. A chart for standing height presented by Boas makes the contrasting growth trends of early and late maturing groups of girls especially clear. So far, however, this association has been merely an association of the same interest as an association between intelligence and the number of erupted teeth. How well this phenomenon is understood may be gathered from a paper by Van Dyke (35) entitled "The *Effect* of the Advent of Puberty on the Growth in Height and Weight of Girls" (Italics supplied). Not only does the very title of this paper suggest a serious misunderstanding, but an egregious blunder in analysis led Van Dyke to conclude that the association between menarcheal ages and height and weight was exactly the opposite of the true relationship (31).

CHAPTER II

THE NATURE AND LIMITATIONS OF THE PHYSICAL MEASUREMENTS OF THE HARVARD GROWTH STUDY

The initial problem of this monograph was suggested by the multitude of reports which, though based on longitudinal data, have yielded only cross-sectional findings. Such reports pointed to the necessity of developing better methods for the analysis of longitudinal data. The most fruitful attack on this problem seemed to require the analysis of actual longitudinal data in an attempt to solve a genuine developmental problem. The physical measurements collected by the Harvard Growth Study supplied the necessary longitudinal data and the availability of menarcheal ages in the Harvard records determined the specific developmental problem.

The Harvard Growth Study was initiated by Professor Walter F. Dearborn in the fall of 1922 with all the children in the first grades of the public schools of the city of "M" in the state of Massachusetts. In the spring and fall of 1923 all the children in the first grades of "R" and "B", also in Massachusetts, were added to the study. These children, totaling about 3,650 cases, were followed with annual measurements as long as they remained in school. The measurements included eleven physical dimensions, roentgenograms of the wrist, dental examinations for the number of erupted permanent teeth, and intelligence and achievement tests. Supplementary data included dates of birth, dates of measurement, indications of ethnic origin, father's occupation, and school progress and school marks. Information concerning the onset of the first menstruation was collected for some of the girls in the cities of "M" and "B".

A verified transcript of the data on 315 girls whose records include information on the first menstruation has been made available to the writer through the courtesy of Professor Dearborn and with the cooperation of his research assistant Dr. J. W. M. Rothney. The available measurements include standing height, sternal height, sitting height, weight, iliac diameter, chest breadth, chest depth, the number of erupted permanent teeth, and skeletal ages as estimated from roentgenograms of the wrist. Measurements of head width and length were not included in the transcript. Indications of ethnic origin include North European, Italian, South European, Negro, and Mixed stock together with a separate category for Jewish cases. Records concerning the first menstruation are in years and months. Chronological ages in years and months were made available, but more exact chronological ages to two decimals have also been calculated.

As a pioneer attempt to collect longitudinal data on a large number of children and over a long period of years the Harvard Growth Study encountered a good many difficulties. Some of these difficulties grew out of the pioneer nature of the enterprise and others were necessarily involved in working with public school populations. Before presenting a systematic statement of the more important features and specific limitations of the data a number of general observations are in order. First, many of the difficulties encountered by the Study are more or less inevitable in any longitudinal study. Only those who have attempted the collection of a considerable body of longitudinal data can appreciate the difficulties which are involved, particularly under public school conditions. Second, some of the limitations of the data are only apparent. These arise from traditional points of view and cross-sectional methods of analysis. Third, if exactly the same data were available on cross-sectional samples of different groups of children at different ages, many of the difficulties would vanish. For example, there would be no gaps in the data, no overlapping records, no hazards from changing personnel and changing methods of measurement, and no figures which seem erratic or impossible in the light of the whole series. That is, the more precise and fuller information provided by longitudinal data introduces complications which are not involved in cross-sectional data. Fourth, in weighing all the difficulties of the data' it must not be forgotten that they consist of measurements repeated annually on the same children in many cases for twelve consecutive years. It is the considered judgment of the writer that the materials of the Harvard Growth Study represent easily the finest collection of longitudinal records available for the study of physical growth during the age period from six to nineteen. Better data, in the sense of more cases and longer records, will probably never be available. Better data, in the sense of half as many cases followed over as long a period together with either more measurements or more accurate measurements or more supplementary data, will not be available for analysis within a period of at least fifteen years. A full discussion of the nature of the data poses a problem in exposition. Certain features of the data require an extended and technical discussion. At this point such features will be merely listed and defined and their fuller discussion will be reserved for Chapter III and Appendices A and B. In the following catalogue the first five items relate more particularly to the accuracy of the data, while the remaining items relate more particularly to the administrative conditions under which the data were collected.

1. Measurements over clothing. The detailed methods of measurement employed by the Harvard Growth Study are not reported here since they will be made available in another monograph to appear later in this series. Methods of measurement conform as closely as possible to standardized and accepted procedures with the important exception that public school conditions necessitated the taking of measurements over clothing. This is a serious limitation for the measurement of weight and particularly of iliac diameter, chest breadth, and chest depth. However, with rare exceptions all measurements were taken during the morning thus introducing a measure of control of several minor disturbing factors which influence weight and standing height.

2. Changing methods of measurement. Although in the collection of the data no conscious changes in methods of measurement were made there was a large and constantly shifting personnel suggesting the possibility of unconscious changes in methods of taking measurements from month to month and from year to year. In the case of chest depth, at least, it is clear from the internal evidence of the data that measurements taken during the fall of 1923 are not comparable with those of earlier and later years.

3. Erratic measurements. In the first two years of the Study physical measurements were taken only once. From the third to the sixth year of the Study all linear measurements were taken in duplicate by two

independent anthropometrists; thereafter, they were taken in triplicate. The duplicate and triplicate measurements were compared in the field and, where they disagreed by more than .5 millimeters in the case of head measurements or more than 1.1 centimeters in the case of other measurements, additional observations were made. The measurements of weight were double-checked every other year. The recorded dimension was based on the duplicate or triplicate measurements which agreed. Even in later years, however, the records for iliac diameter, chest breadth, and chest depth contain a disturbingly large proportion of figures which seem erratic or impossible in the light of the whole series.

4. Unsatisfactory menarcheal data. As already noted menarcheal data are available for only two school systems. The information from city "B" is believed to be accurate. The information from "M", however, was obtained by questionnaires sent to mothers during the eight year of the Study when the subjects ranged in age from eleven to sixteen. Since the questionnaires were sent out before many girls had menstruated, a considerable number of records consist of indefinite notations such as "advent of menarche not by" such and such an age.

5. Unverified birth records. Birth dates were taken from school records and have never been systematically checked against birth certificates except in cases where there was reason for suspecting that the school records might be in error. Experience in systematically verifying such school records leads the writer to believe that this feature constitutes only a technical limitation of the data.

6. Selective factors. The Harvard Growth Study was initiated with approximately 1800 children of each sex. At the end of the Study only one quarter of these children were still in school and available for study. Of the original population of girls the writer has data on only 315 cases and only 248 of these will be subjected to intensive study. Hence, the available cases represent a rather selected group, that is, they have been selected for relatively long attendance in the same schools and for the availability of menarcheal data.

7. Overlapping ages and records. City "M", where initial measurements were made in the fall of 1922, required children to be six years old for admission to the first grade. Further, the first grades in this system contained a considerable number of children who were retarded one and even two years. City "B", where initial measurements were made in the fall of 1923, required children to be only five years old for admission to the first grade. Hence, a wide range of ages is involved for any given year of the Study. During the second year of the Study, for example, the age range is from five to eleven. From this fact and from the fact that a few children were added to the original groups it follows that some of the records begin at age five and some as late as age twelve. Among the available cases one record ends as early as eleven, one as late as age twenty. Data for all 248 cases are not available at any age.

8. Random gaps in the data. Absences from school when the measurements were being taken result in a considerable number of random gaps in the data. In such cases all the physical measurements on a given child for a given year are missing from the record.

9. Systematic gaps in the data. In addition to these random gaps there are systematic gaps in the data. Iliac diameter was measured in the fall of 1922, omitted in the spring and fall of 1923, and reinstated

in the spring of 1924. Chest breadth and chest depth were measured from the fall of 1922 to the fall of 1923, omitted from the spring of 1924 to the spring of 1926, and reinstated in the fall of 1926. Overlapping ages mean that these systematic gaps occur at all ages from seven to fourteen.

10. Absence of measurements on birthdays. Administrative considerations made it impossible to obtain measurements at exact birthdays.

11. Long intervals between measurements. Children in cities "M" and "B" were measured each fall, those in "R" each spring. This insures the elimination of seasonal factors within each population. However, in the age range from ten to fourteen the pattern of physical growth, especially of standing height, changes so rapidly that annual measurements are hardly adequate.

12. Irregular intervals between measurements. A regular program of the staff moving from school to school insured for the most part that the measurements were made almost exactly one year apart. However, the schedule of measurements at "M" in the fall of 1922 was much delayed and many children were not measured until the following January and February.

13. Absence of supplementary data.. There is an almost complete absence of supplementary data which might throw light on the records of particular individuals or which might provide the leverage for special studies. The available supplementary data are limited to indications of ethnic origin and to the menarcheal ages.

In the above catalogue of limitations some difficulties are more serious than others. The really serious problem, however, is created by the fact that all of these difficulties are entangled in the same data. In justification for working with such material it has been urged that many of these difficulties are more or less inherent in longitudinal data, that certain supposed difficulties (for example, overlapping ages and records) are actually sources of strength, that other limitations arise from the availability of more voluminous data that students of child development have been accustomed to handle, and that the data are by far the best that are available for the study of physical growth during the age period from six to nineteen. The supreme strength of the data is that it consists of repeated measurements on the same children. As will be demonstrated in the next chapter repeated measurements have very great advantages over cross-sectional measurements. This demonstration paves the way for the development in Appendix A of methods which overcome or greatly alleviate almost all of the limitations of the data. Students of the Harvard data should note the concluding sections of Appendix B which present summary statements of procedures and methods of analysis. Finally, the internal consistency of the results presented in Chapters IV to XIII provide unmistakable evidence of the soundness of the Harvard data as a whole.

CHAPTER III

THE ADVANTAGES OF LONGITUDINAL DATA

The importance of repeated measurements on the same children has been urged for so long and by so many as to become the first article of faith among students of child development. Nevertheless, the multitude of longitudinal studies which have yielded only cross-sectional findings suggest that this faith has been degenerating to the level of pure dogma. This chapter emphasizes only one point, namely, that repeated measurements will determine the pattern of growth from age to age with a reliability greatly exceeding that of cross-sectional data. This should be obvious as a general proposition, but a full appreciation of the greater reliability of trends established by longitudinal is another matter. Similarly, the statistical principles involved in the analysis are well known to statisticians, but as far as the writer is aware they have never been employed in connection with repeated measurements of growing children.

1. Averages Versus the Pattern of Growth in Ten Cases

For the purpose of illustrating concretely both the common sense belief in and the statistical basis for the greater reliability of longitudinal data a special study of the standing heights of ten cases has been made. The choice of such a small number of cases has been deliberate in order that the individual growth curves and the probable errors of the pattern of growth may be amenable to graphical representation. Studies involving all available cases will be presented in a later section. The particular ten cases selected for study are all the cases in whom the menarche occurred between age 13-0 and 13-3, who were measured within three months of their birthdays, and who were followed with annual measurements from age seven to seventeen.

Figure 1 presents the individual growth curves in standing height for these ten individuals. Two interpolations have been made, one at age sixteen and one at seventeen. The initial measurements of three cases have been adjusted where the interval between the first and second measurements differed more than ten per cent from an exact year. For convenient representation measurements made within three months of birthdays have been plotted to even years. Attention is called to the very striking uniformity of these ten curves with the exception of that for case #2379. Figure 2 has been prepared to illustrate more clearly the uniformity of the patterns generated by these curves. Each individual curve in Figure 1 has been moved upward or downward so that the average height of each case from age seven to seventeen is the same; for example, 9.0 centimeters were added to the stature of case #2492 at each age.

Figure 3 presents the annual increments in standing height of these same ten cases. For convenient representation the vertical scale at the left has been magnified four times in comparison with that for the gross

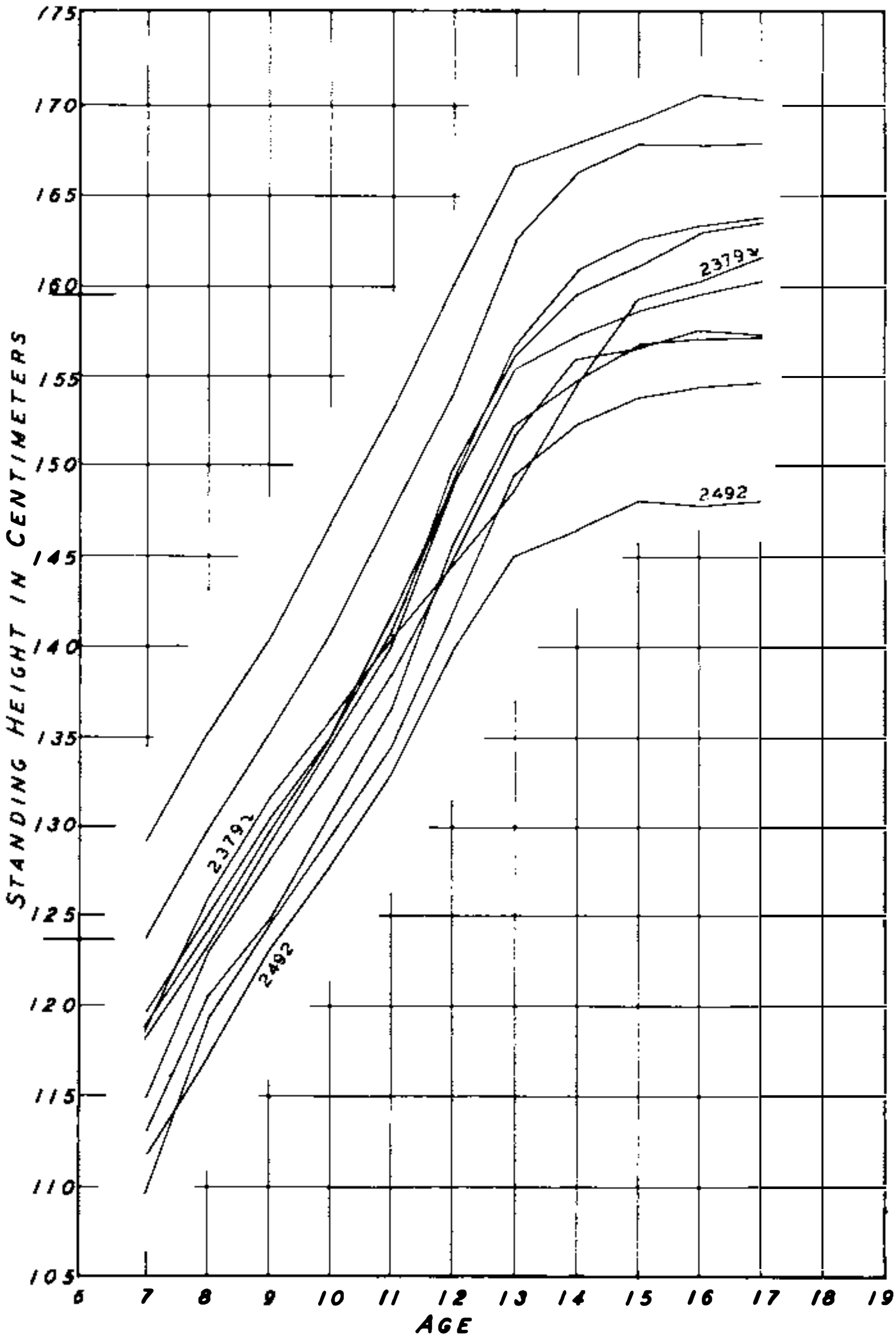

Fig.1.--Individual Growth Curves in Standing Height Plotted to Even Years of Ten Girls Who Were Measured Within Three Months of Birthdays and Who Menstruated Between Age 13-0 and 13-3.

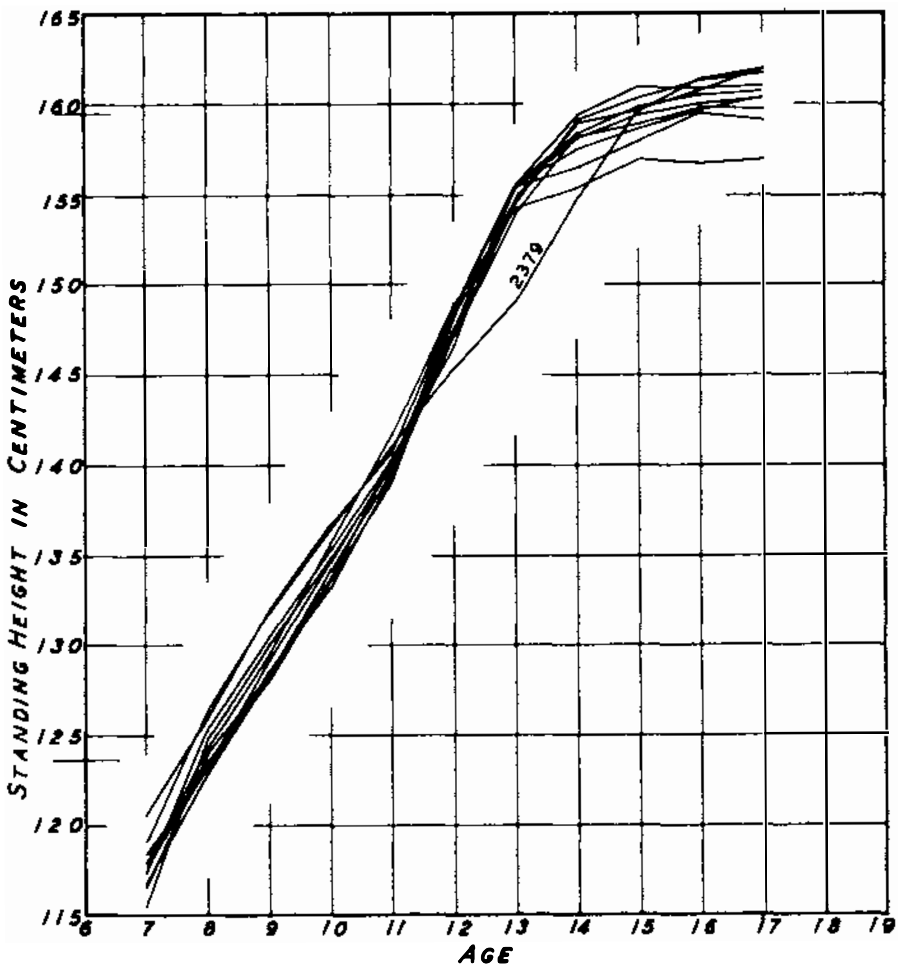

Fig. 2.--Individual Growth Curves in Standing Height Redrawn From Figure 1 to Eliminate Differences in Gross Size While Reproducing Exactly Each Individual Pattern.

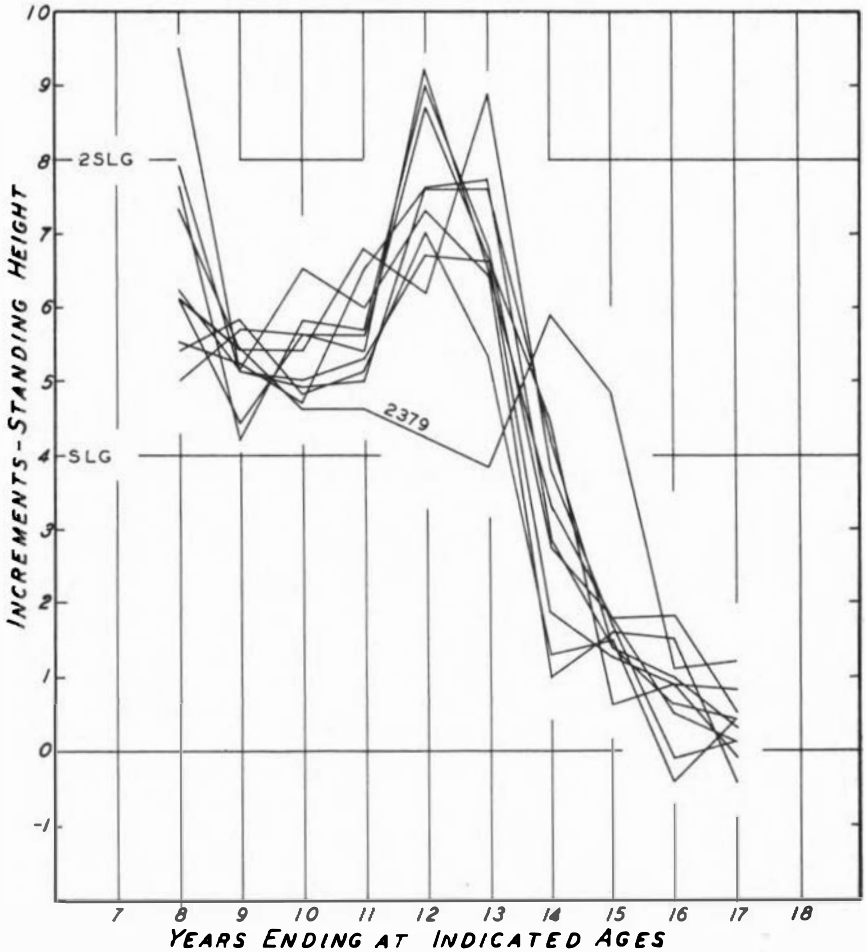

Fig. 3.--Individual Increment Curves in Standing Height for the Same Ten Cases Presented in Figures 1 and 2.

dimensions of Figures 1 and 2[1]. Figure 3 brings out somewhat more clearly the progressively changing pattern and the exceptions to this pattern. For the year ending at age 8.0 growth is rapid, the increments ranging from 5.0 to 9.5 centimeters per year. For the year ending at age 9.0 the increments with two exceptions are smaller. Small annual increments characterize the years ending at age 10.0 and 11.0. With two exceptions the increments for the year ending at 12.0 are larger than in the previous year. Increments for the year ending at 13.0 continue large, but only two are larger than the previous year. The records show that these cases menstruated for the first time between 13-0 and 13-3. The chart shows that the annual increments for the year beginning approximately at the onset of the menarche and ending at age 14.0 are very much smaller, with the exception of case #2379. With two exceptions the increments for the year ending at age 15.0 are still smaller and, again with two exceptions, they are still smaller for the year ending at age 16.0. Attention is called to the very small range of the increments at any age. For the year ending at 9.0, for example, the smallest and largest increments are 4.2 and 5.8 centimeters or a maximum range of only 1.4 centimeters. At age nine the gross heights vary from 122.8 to 140.2 or a maximum range of 17.4 centimeters. It is this phenomenon of longitudinal data which makes it possible to determine the pattern of growth from age to age with a reliability greatly exceeding that of cross-sectional data.

So far no reference has been made to the average height of these ten cases. Obviously, an average calculated from only ten cases must have a rather large probable error. Another group of ten cases who menstruated between 13-0 and 13-3 would probably give different averages. But, just as the pattern of growth in Figures 1 to 3 would not be changed by the elimination of any three or four cases while the average would be greatly changed by the elimination of the same number of tallest cases, so the pattern would not be changed while the averages might be greatly changed by another ten cases. This distinction between the pattern of growth from age to age and the average gross size at a particular age is fundamental. A reliable average height requires a large number of cases. The pattern, however, is unmistakably defined whether five, or ten, or a thousand cases are available.

Before presenting the statistical justification for this distinction

[1] *In figures 1 and 2 note the lines across the vertical scale at the left at 123.6 and 159.5 centimeters. These indicate respectively average standing heights at age eight (7.50 to 8.49) and seventeen as computed directly from all cases available at these ages. Save for a few obvious exceptions growth from age eight to seventeen is represented throughout all charts showing absolute size by the same vertical distance. Hence, all such charts can be superimposed for comparative purposes. All the vertical scales which result from this procedure have been magnified four times to provide scales for the charts of annual increments. Hence, all such charts can also be superimposed. To emphasize this comparability, all charts of annual increments carry three horizontal grid lines, the vertical spacing of which is the same throughout. These three grid lines indicate zero growth per year, one ninth of the average growth from age eight to seventeen (the SLG grid) and two ninths of the average growth from age eight to seventeen (the 2SLG grid). That is, straight-line growth from age eight to seventeen would give increments which would fall on the SLG grid line.*

it will be illuminating to sharpen the contrast between longitudinal and cross-sectional data. For this purpose ten sets of cross-sectional data have been "made up" by selecting ten cases at random at each age from seven to seventeen from among cases measured within three months of their birthdays. The first case selected at random at age seven has a stature of 111.8 centimeters, the first case at age eight a stature of 119.6, etc. The first case selected at random at each age (representing eleven different individuals) has been charted in Figure 4 as if a single individual were involved; similarly for the second case selected at random at each age, etc. In cross-sectional data there is, of course, no connection between a measurement at one age and another measurement a year later so that lines might well have been drawn between all possible pairs of points. Since the negative growth trends of Figure 4 are quite unreal, this chart may be regarded as a caricature of cross-sectional data. Nevertheless, it seems to the writer to present a more correct visual picture for comparison with the longitudinal data of Figure 1 than would a chart which merely spotted the separate observational points at each age. It is no wonder that the trend of averages based on cross-sectional data is often irregular in spite of large numbers of cases. Common sense perceives at once that the longitudinal data of Figure 1 will give a more accurate definition of the pattern of growth than the cross-sectional data of Figure 4 which involves exactly the same number of cases. As far as the writer is aware, no one has called attention to the statistical aspects of this contrast although the essential statistical principles which are involved have been widely applied to other situations and other types of data.

2. Statistical Aspects of Longitudinal and Cross-sectional Data on Only Ten Cases

Statistical aspects of the data on the longitudinal and cross-sectional cases are illustrated in Figure 5 to 10. In Figure 5 the middle curve represents the trend of the average height as determined from repeated measurements on ten cases. The upper and lower curves represent the range of plus and minus four probable errors above and below each mean.[1] It is apparent that another group of ten cases might give very different averages -- conceivably these might be as high or as low as the upper and lower curves of Figure 5.

The continuous curves of Figure 6 recapitulate the data of Figure 5 for the longitudinal data while the dash curves give similar data for the cross-sectional and random samples of ten cases at each age. It is readily apparent that the trend of the averages determined from the cross-sectional data is much more irregular. Nevertheless, the range of plus and minus four probable errors above and below each cross-sectional

[1] *Since these data are merely illustrative to be supplemented in a later section by an analysis of all available cases, the conventional formula for the probable error of the mean has been applied. For discussions of errors involved in small samples see Student (34), Rider (28), and Ezekiel (15).*

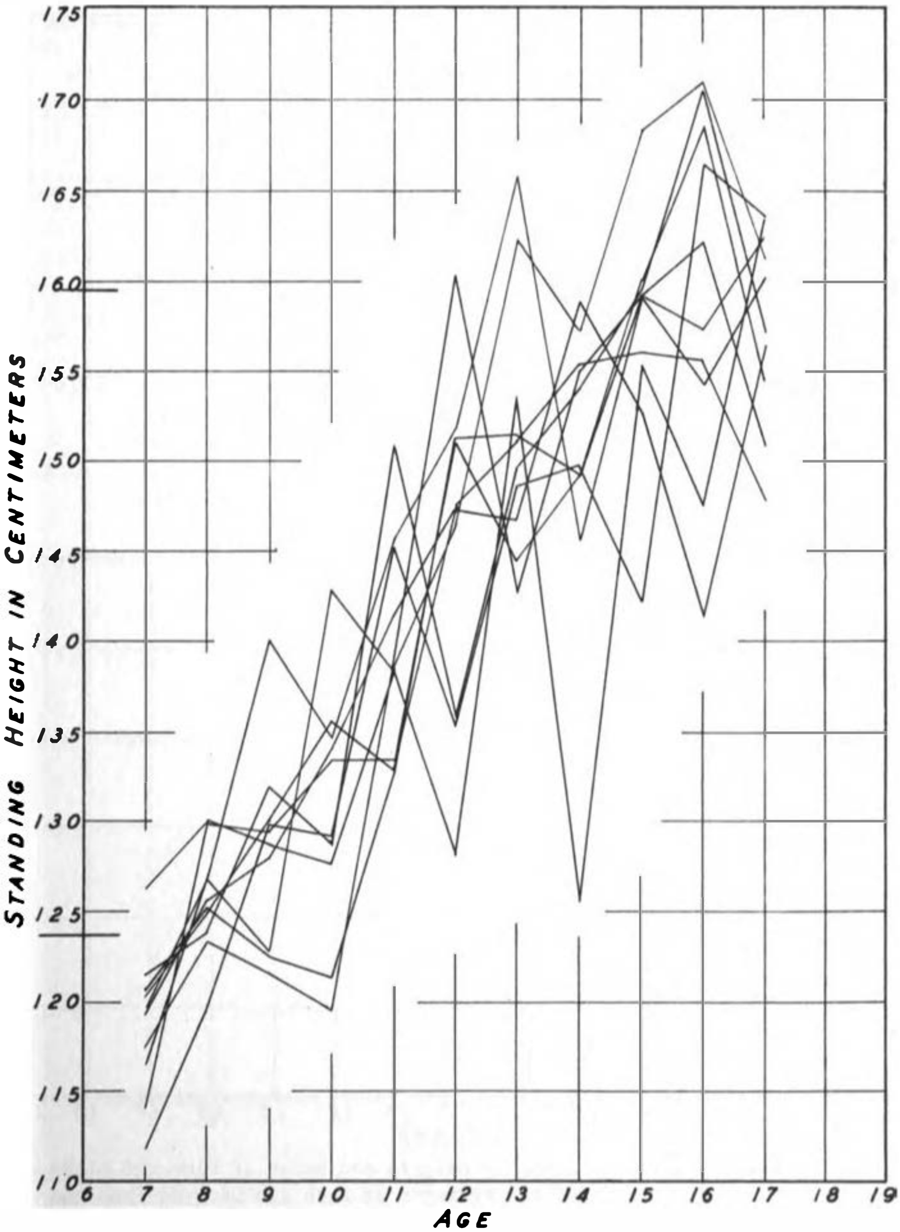

Fig. 4.--Ten "Individual Growth Curves" in Standing Height Made up by Selecting Ten Cases At Random at Each Age; Caricaturing the Irregularity of Cross-sectional Data.

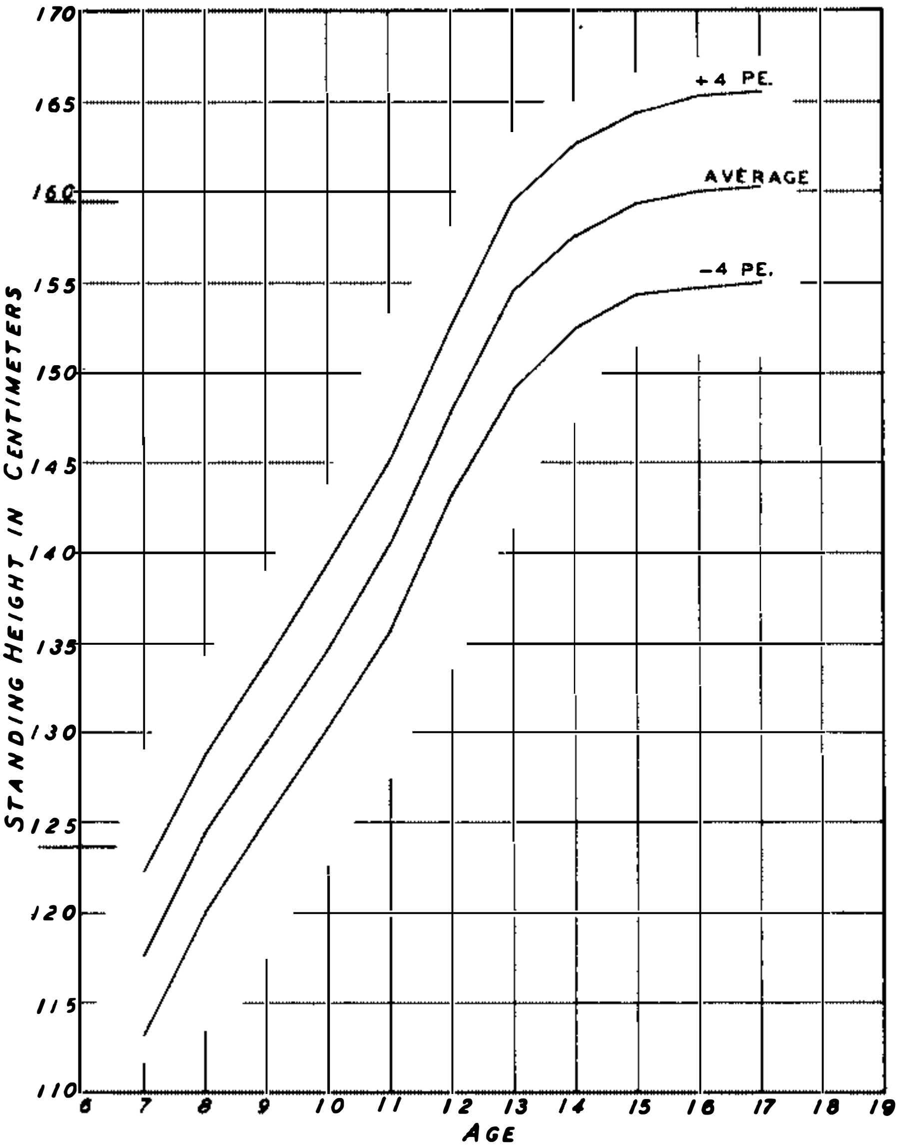

Fig. 5.--Average Standing Heights and Range of Plus and Minus Four Probable Errors of the Averages at Each Age Derived From the Ten Cases of Figure 1.

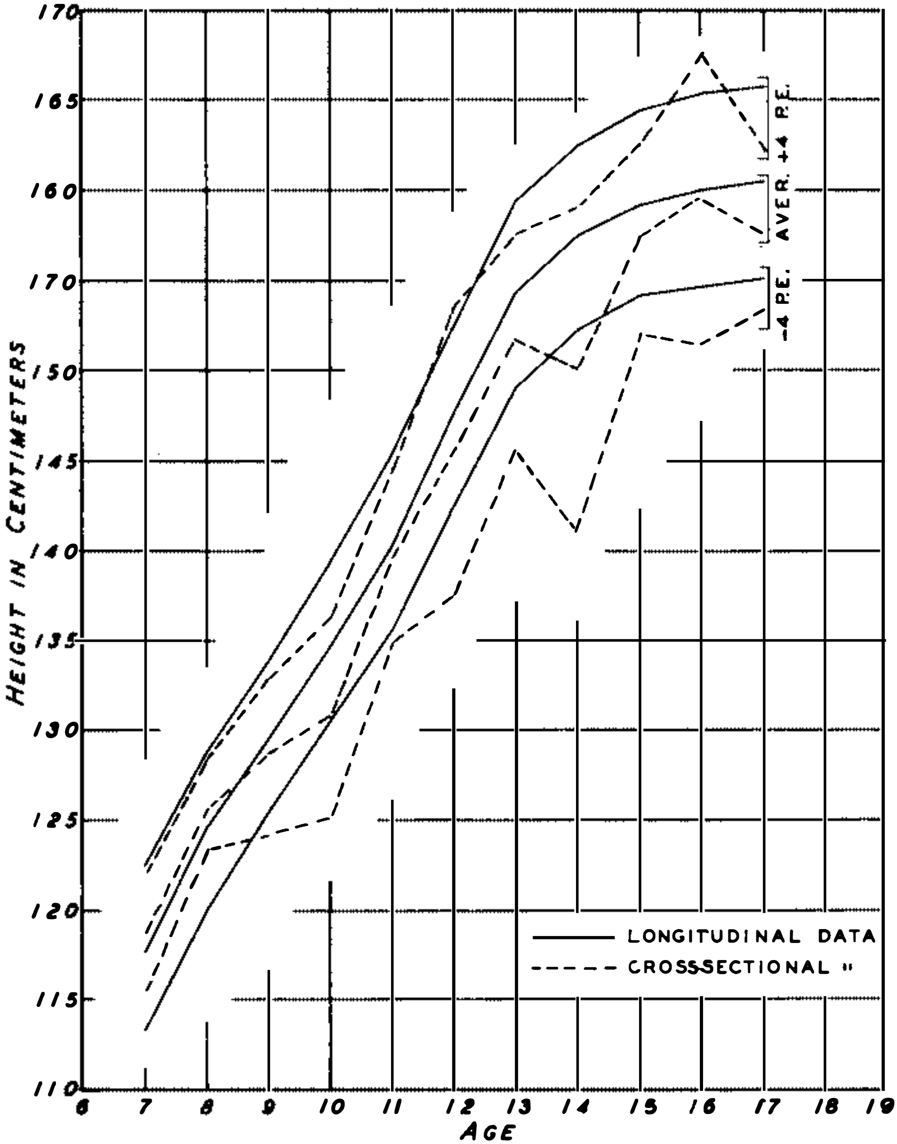

Fig.6.--Average Standing Heights and Range of Plus and Minus Four Probable Errors of the Averages for Longitudinal Data on Ten Cases (Solid Curves) and for Cross-sectional Data on Ten Cases Selected at Random at Each Age (Dash Curves).

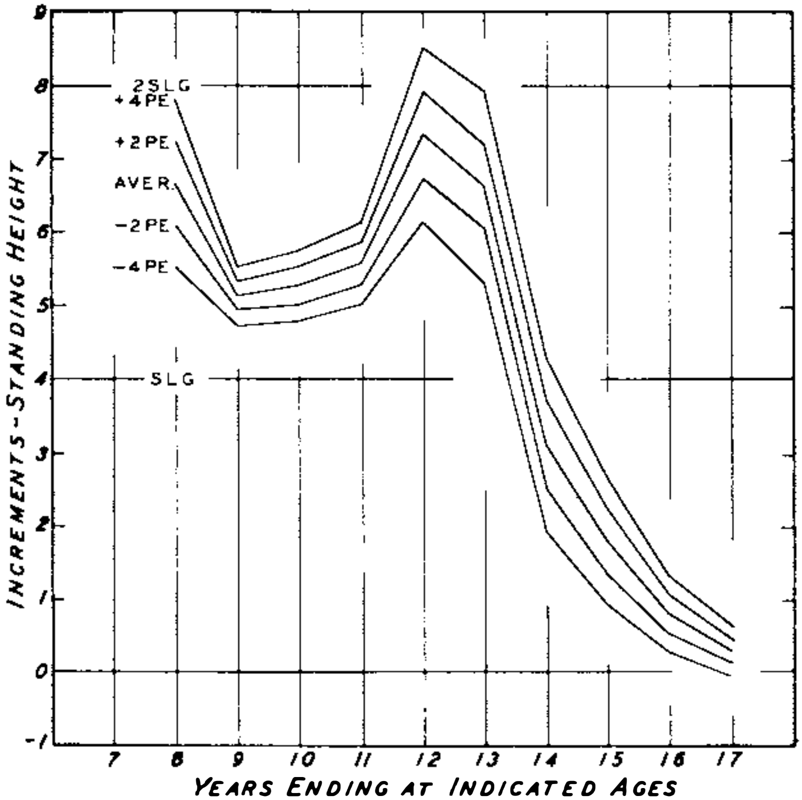

Fig. 7.--Average Annual Increments in Standing Height and Range of Plus and Minus Two and Four Probable Errors of the Averages for Longitudinal Data on Ten Cases Presented on Figure 3.

mean is not much greater on the whole than the similar range above and below each longitudinal mean. That is, while the trend of the longitudinal averages is more regular, the error involved in the separate averages represents no advantage over cross-sectional data. In longitudinal studies just as in cross-sectional studies a reliably determined average height requires large numbers of cases.

We turn now to the annual increments. The individual increment curves for the longitudinal data have already been presented in Figure 3. In Figure 7 the middle curve represents the trend of the average increment. The extreme upper and lower curves represent the range of plus and minus four probable errors above and below each mean increment. Plus and minus two probable errors are also plotted for reasons which will be apparent in a moment. For the year ending at age eight, for example, the average increment of 6.66 centimeters has a probable error of $\pm$.28. These probable errors have been calculated directly from the individual increments. They might, however, have been calculated from

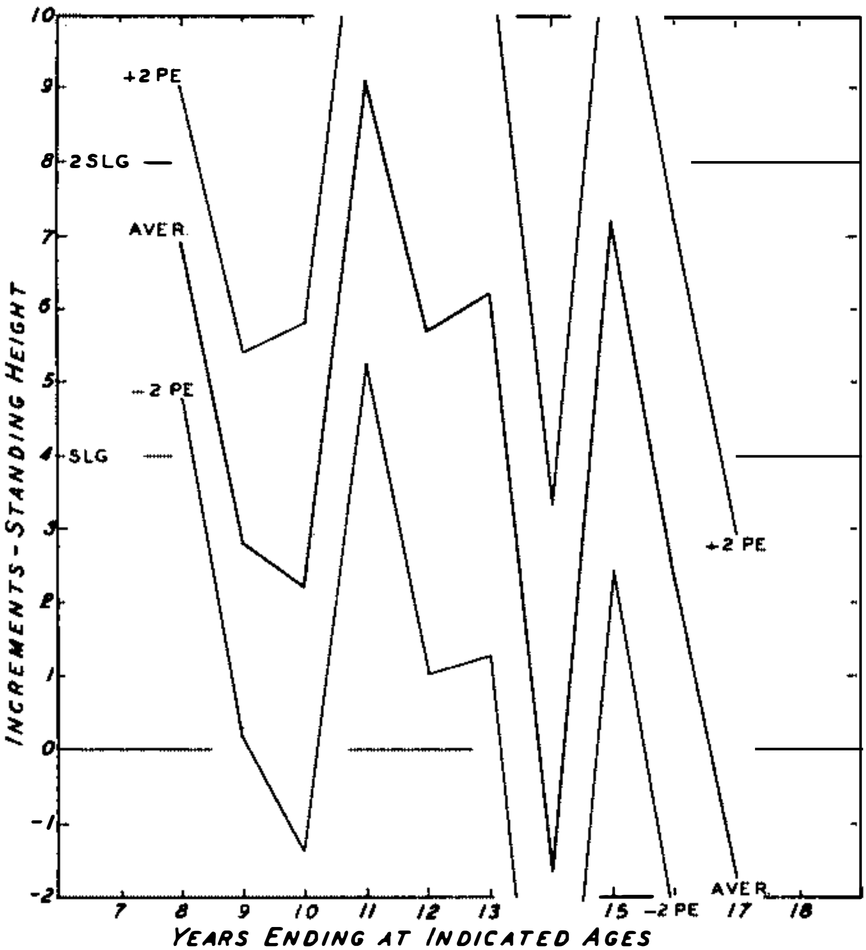

Fig. 8.—Average Annual Increments in Standing Height and Range of Plus and Minus Two Probable Errors of the Averages for Cross-sectional Data on Ten Cases Selected at Random at Each Age.

the gross heights by the formula

$$\text{P.E. difference} = .6745\ \sqrt{\sigma_{E_1}^2 + \sigma_{E_2}^2 - 2r_{12}\sigma_{E_1}\ \sigma_{E_2}}$$

in which σ_{E_1} and σ_{E_2} are the standard errors of the average height at two successive ages and in which r_{12} is the correlation between the heights at one age and heights a year later. Probable errors calculated directly from the increments and indirectly from the above formula give identical results. Attention is called to the fact that the probable errors of the average increment are much smaller than the probable errors of the average height. For the year ending at age eight the probable error of the average increment is $\pm$ .28 whereas the probable error of the average height at age eight is $\pm$ 1.06 or nearly four times as large. This contrast follows from the fact already noted that there are large differences between the tallest and shortest individual at any age whereas there are very small differences between the largest and smallest increment over any interval of a year.

Similarly, Figure 8 presents the average increments and the range of error in the increments as determined from the cross-sectional and random samples of ten cases at each age. Since the averages from year to year are irregular, the average increments fluctuate wildly and it has been impossible within a chart of the same size a Figure 7 to represent the range of even plus and minus two probable errors. In this case, of course, it is not possible to calculate probable errors directly from the individual increments and it has been necessary to use the indirect formula given above. Since in cross-sectional data there is no correlation between statures at successive ages the third term under the radical is zero and this formula reduces to

$$\text{P.E. difference} = .6745 \sqrt{\sigma_{E_1}^2 + \sigma_{E_2}^2}$$

which is the conventional formula for the probable error of the differences between two means. The contrast between Figures 8 and 7 is extreme.

The probable errors of the increase in height from age to age as given in Figures 7 and 8 may obviously be used to determine the probable errors of the slopes of the curves for the gross heights from age to age. Figure 9 presents the trend of the average heights and plus and minus four probable errors of the slope of the curves from age to age for the longitudinal data. It must be remembered, however, that average height considered by itself has a very large probable error. Hence, the scale at the left does not record centimeters of height. Instead, it is noted that each of the indicated units of vertical distance represents five centimeters. That is, we are foreswearing any interest in the average heights and are concentrating our attention on the changing pattern of growth or slope of the curves from age to age. We begin at age 7.0 with an arbitrary point. Moving from this point to age 8.0 the upper and lower limits of the solid black area give the range of plus and minus four probable errors of the slope of the curve between these two ages. Similar data are given for each annual interval to age seventeen. Dotted lines have been added to destroy the illusion that the errore vanish at each age. Comparable data for the cross-sectional and random samples of ten cases at each age are given in Figure 10.

3. Additional Data

Two limitations of the findings just presented require the analysis of additional data. First, the probable errors calculated from only ten cases are themselves subject to comparatively large probable errors. Second, data have been presented for height only and the conclusions for height hold to a considerably lesser degree for other physical dimensions.

For the purpose of these supplementary studies the standard deviations of increments and of gross measurements recorded in Table 29, section six of Appendix B have been employed. The standard deviation of

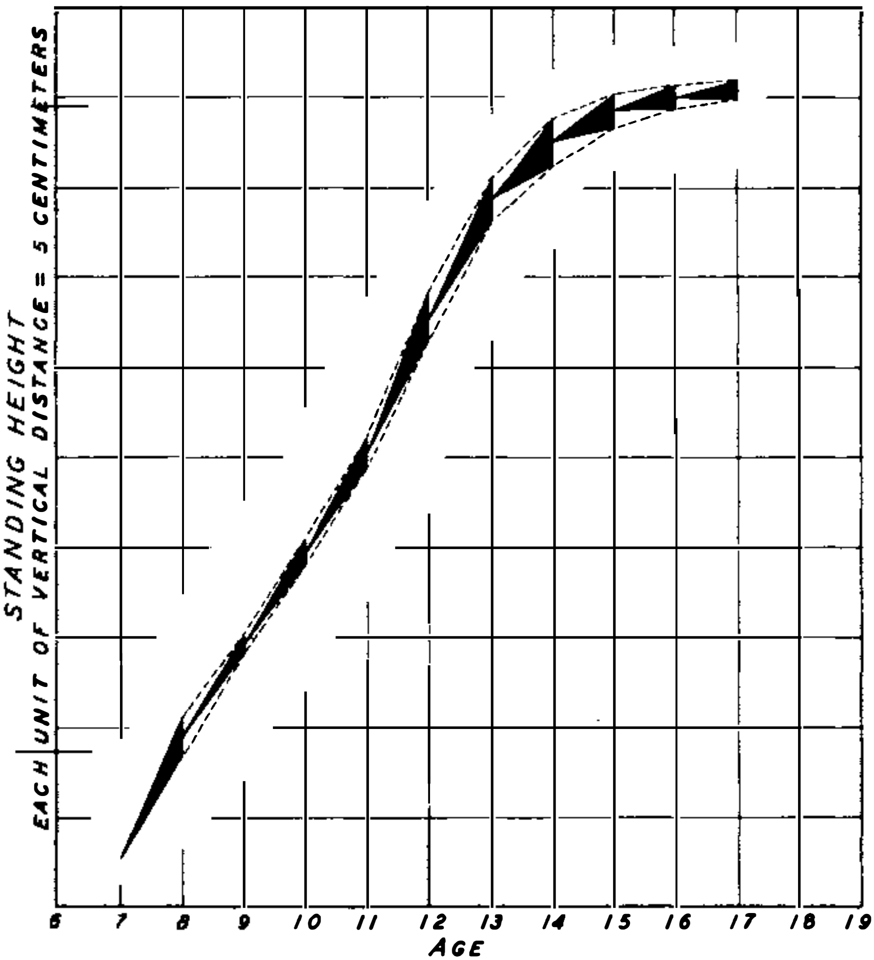

Fig. 9.--Patterns of Growth in Standing Height and Range of Plus and Minus Four Probable Errors of the Slope of the Curve From Age to Age Based on Longitudinal Data on Ten Cases.

the annual increments in standing height for the year ending at age nine (8.75 to 9.74) based on the total of 194 available cases is .95 centimeters. Hence, the probable error of the average increment of the 194 cases is .046. The standard deviations of the gross standing heights of all cases at age eight and nine are 5.82 and 6.40 centimeters. Hence, the probable errors of the average standing heights at age eight and nine are .282 and .310. Proceeding as if only cross-sectional data were available the probable error of the difference or of the average annual increment in standing height is .419. This is more than nine times as large as the probable error based on longitudinal data. To

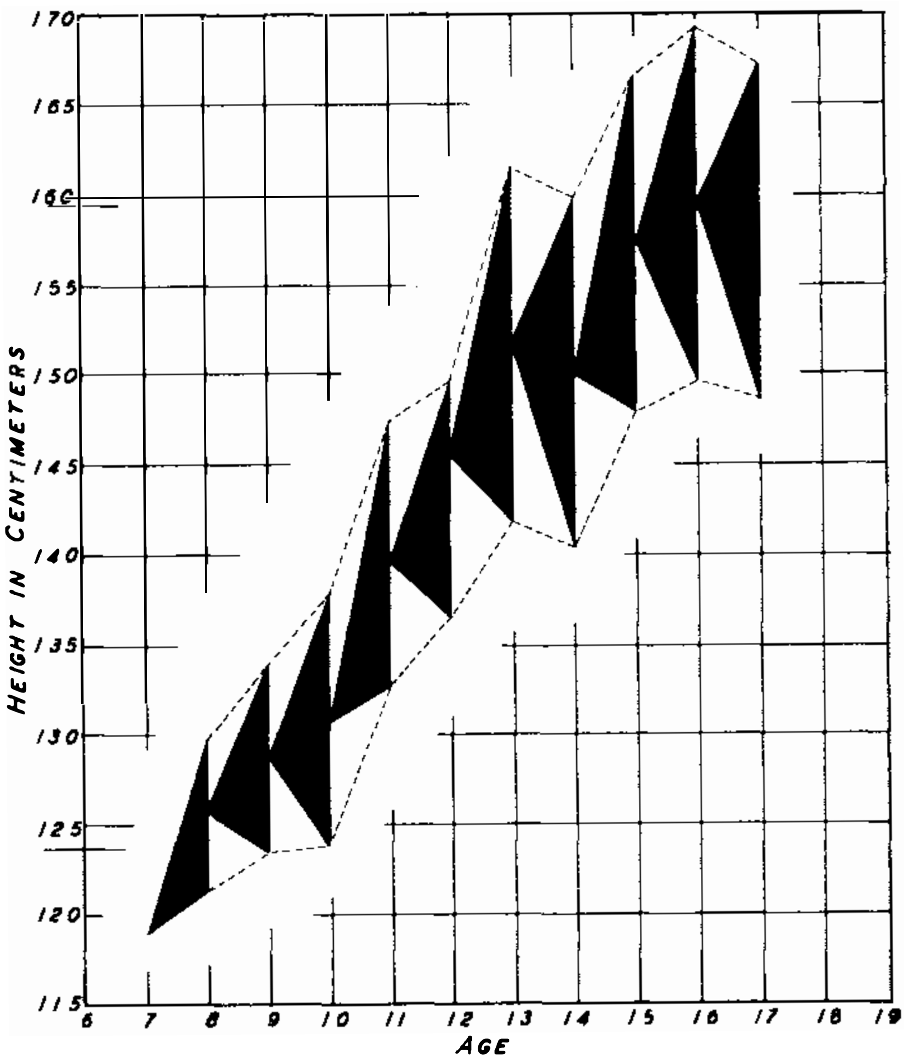

Fig. 10.--"Patterns of Growth" in Standing Height and Range of Plus and Minus Four Probable Errors of the Slope of the Curve From Age to Age Based on Cross-sectional Data on Ten Cases at Each Age.

reduce this probable error of .419 to .046 would require 16,000 cases of cross-sectional data at each age[1] or 82.9 times as many cases at each age. Similarly, at each successive age from ten to sixteen 57.2, 41.9, 42.9, 23.0, 26.8, 55.5, and 154.8 times as many cases of cross-sectional data would be required to give growth trends as reliable as those determined from longitudinal data. Or, on the average, repeated measurements on a given number of cases from age eight to sixteen will serve as well as 60.6 times as many cases at each age with cross-sectional measurements. Since each of the 248 cases were measured for standing height on the average 10.2 times we have the equivalent of cross-sectional measurements on approximately 150,000 cases.

Even these figures understate the reliability of longitudinal data since we are not at all interested in the growth patterns of children in general. We are interested primarily in the growth patterns of homogeneous groups of cases. Returning to age nine and revising the above calculations for homogeneous groups we find that the weighted average of the standard deviations of increments in standing height for each of eight menarcheal groups is .87 instead of .95. This gives a probable error of only .042 instead of .046. To obtain a probable error as small as .042 from cross-sectional data would require 19,300 cases at each age instead of 16,000 or 99.5 instead of 82.9 times as many cases at each age. Similarly, at each successive age from ten to sixteen 93.6, 71.7, 62.2, 40.3, 60.2, 131.3, and 321.1 times as many cases of cross-sectional data would be required to give growth trends for homogeneous groups as reliable as those determined from longitudinal data. Or, on the average, repeated measurements on a given number of cases in a homogeneous group from age nine to sixteen will serve as well as 110 times as many cases at each age of cross-sectional measurements. From this point of view we have in our 248 cases the equivalent of cross-sectional measurements of standing height on approximately 270,000 cases.

Given the foregoing explanation of procedures and discussion of results, the data for other dimensions may be very briefly summarized. In the case of sitting height repeated measurements from age nine to sixteen on a given number of cases in unselected and in homogeneous groups will serve as well as 22.1 and 29.4 times as many cases at each age of cross-sectional measurements. Comparable figures for weight are 21.1 and 28.4 and for iliac diameter 14.2 and 16.7.

4. Summary and Comments

When longitudinal data are available, the position of any average relative to adjacent averages, the average annual increments, the slopes of the curves from one age to the next, and the changing pattern of growth generated by the averages are all very precisely determined. For the purpose of determining the growth trends of homogeneous groups, repeated measurements on the available population of only 248 cases

[1] The formula is $N = \dfrac{.6745^2(\sigma_1^2 + \sigma_2^2)}{PE_{INC}^2} = \dfrac{.6745^2(5.82^2 + 6.40^2)}{.046^2} = 16,091$

in which σ_1 and σ_2 are the standard deviations of the gross dimensions and PE_{inc} is the probable error of the average increment calculated directly from the increments.

represents the equivalent of approximately 40,000 cross-sectional measurements in the case of iliac diameter and the equivalent of approximately 270,000 cross-sectional measurements in the case of standing height.

The greater reliability of longitudinal data for the determination of growth trends has three important implications. First, the limitations of longitudinal data, which have seemed to be insurmountable and fundamentally damaging, become only petty and annoying problems. Second, the data suggest that, in longitudinal studies, the emphasis of the analysis should fall on the growth increments rather than on gross dimensions. Third, the data suggest the desirability of working with homogeneous groups of cases and insures the reliability of trends determined from small numbers of cases. All three implications are essential to the present study: the first in dealing with the somewhat extreme limitations of the Harvard data and the second and third in testing the specific hypothesis that the timing of the advent of the menarche and of changes in the pattern of growth are associated.

It is important that three cautions be kept in mind. First, it must be emphasized that longitudinal data have no advantage over cross-sectional data for the purpose of determining the average size of children in general. Second, it must be noted that the foregoing discussion implies a fairly large rather than a very small population for longitudinal studies. If all children had essentially the same pattern of growth, then repeated measurements on ten or twenty cases would suffice to determine that pattern. But, there are many different patterns and a multitude of factors associated with or determining these patterns. If these different patterns and the associated factors are to be studied, the total population should be large enough to permit division of the cases into very homogeneous groups to the end that the history of different groups may be compared. Finally, it should be observed that the discussion stresses only a limited and restricted aspect of the advantages of longitudinal data.

CHAPTER IV

GROUP PATTERNS OF GROWTH IN STANDING HEIGHT

A discussion of the patterns of growth in standing height quite naturally comes first among the ten measurements to be considered. Growth trends in standing height are far more reliable than for other measurements except possibly leg length. Both the group and the individual patterns of growth, to which separate chapters are devoted, are clearly defined. A full discussion of the results will make it possible in Chapter VI to condense the discussion and to present only a selected number of charts for sternal height, sitting height, and derivative leg lengths. Since a full statement of statistical, technical, and procedural details is available in the appendices, only a minimum of text will be devoted to the most essential aspects of the analysis. In this and succeeding chapters the reader's attention will be directed again and again to the graphical representation of the data. It is essential that the different types of charts be clearly understood since each has been designed to illustrate some feature of the data which is less readily apparent in other forms.

1. Hypotheses

The general theory and specific hypotheses of this study need to be reviewed at this point. It is imagined that the pituitary gland begins to function above a certain critical level as early as age seven or eight and as late as age twelve or thirteen in different individuals. Among other functions it is well known that the secretion of this gland has two properties: a gonad or sex stimulating property and a growth stimulating property. Hence, it is probable that both the process of sexual maturation and the period of accelerated physical growth are initiated at the same time in a given individual. In girls the process of sexual maturation begins with the budding of the breasts, the appearance of pubic hair, and after some time culminates as early as age ten or as late as age sixteen in the first menstruation or menarche. Fertile ovulation and true sexual maturity probably come still later. In the midst of this process or toward its close the developing gonads begin to pour new hormones into the blood which serve to check the growth stimulating factor of the pituitary and to close the epiphyses of long bones. Hence, it is supposed that the endocrine system provides mechanisms which bring the period of accelerated physical growth to a close in some definite time relationship to the advent of the menarche. Since the understanding of these endocrine relationships is still in the formative period, it would be premature to attempt a more precise statement. Recent and authoritative reviews are available in a volume edited by Allen (2) and in a volume sponsored by the American Council on Pharmacy and Chemistry (3).

The implications of these speculations in relation to the available data of the Harvard Growth Study lead to the specific hypotheses that the early or late advent of the menarche is associated (a) with physical size, and (b) with the timing of the changes in the patterns of physical growth.

2. Group Differences in Average Height

The available population has been divided into groups according to the age of first menstruation. In this section attention will be devoted to the differences between these groups in average height (a) at each chronological year of age and (b) at time intervals before and after the menarche. It should be noted at the outset that the data for average height are less reliable, the graphical representations less clear, and the differences between the groups less significant than the data for average increments in height to be presented in later sections.

The discussion of the averages centers around Figure 11. The horizontal scale at the bottom records the mid-points of chronological age intervals. For example, age 6.0 is the mid-point of the age interval from 5.75 to 6.24, age 6.5 is the mid-point of the age interval from 6.25 to 6.74, etc. Attention is called to the lines across the vertical scale at the left at 123.6 and 159.5 centimeters. These indicate respectively the average standing heights at age 8.0 (7.50 to 8.49) and at 17.0 as computed directly from all cases available at these ages. Save for a few obvious exceptions, the difference between the average of all available cases at age 8.0 and 17.0 is represented throughout all charts showing gross size by the same vertical distance. Not only are the charts of gross size comparable whether they concern averages or individuals, they are also comparable from measurement to measurement. For example, the difference between the average of all available cases at 8.0 and 17.0 is 35.9 centimeters in the case of standing height and only 3.9 centimeters in the case of chest depth; nevertheless, the same vertical distance in the charts is used to represent both differences. Hence, a tracing on transparent paper from Figure 11 for standing height can be superimposed for comparison on Figure 68 for chest depth if the age scale at the bottom and the two lines cutting the vertical scale at the left are also superimposed.

Since most charts present eight curves it has proved impractical to distinguish them by solid lines, dash lines, dot-dash lines, etc. Instead in Figure 11 at age 10.0 and 13.5 the column of letters indicates the order of the curves for the eight groups into which the total population is here divided. As noted in the lower right of the chart the available population has been classified into eight groups depending on the age at which the first menstruation or the menarche appeared. Group A includes cases who first menstruated before age 11-6. Groups B, C, D, E, F, and G include cases who first menstruated at succeeding intervals of six months. Group H includes cases who first menstruated after age 14-5. Since the total population includes only 248 cases, the number of individuals in each group is comparatively small, there being only sixteen, eighteen, and nineteen cases in Groups B, H, and G. Further, the number of cases in each group becomes smaller and smaller prior to age nine and after age sixteen. Various combinations as well as sub-divisions of these eight major groups must be noted from chart to chart.

(a). Differences between menarcheal groups in average size at each chronological year of age. Reference to Figure 11 will show that the maximum differentiation of the menarcheal groups occurs at age 12.0.

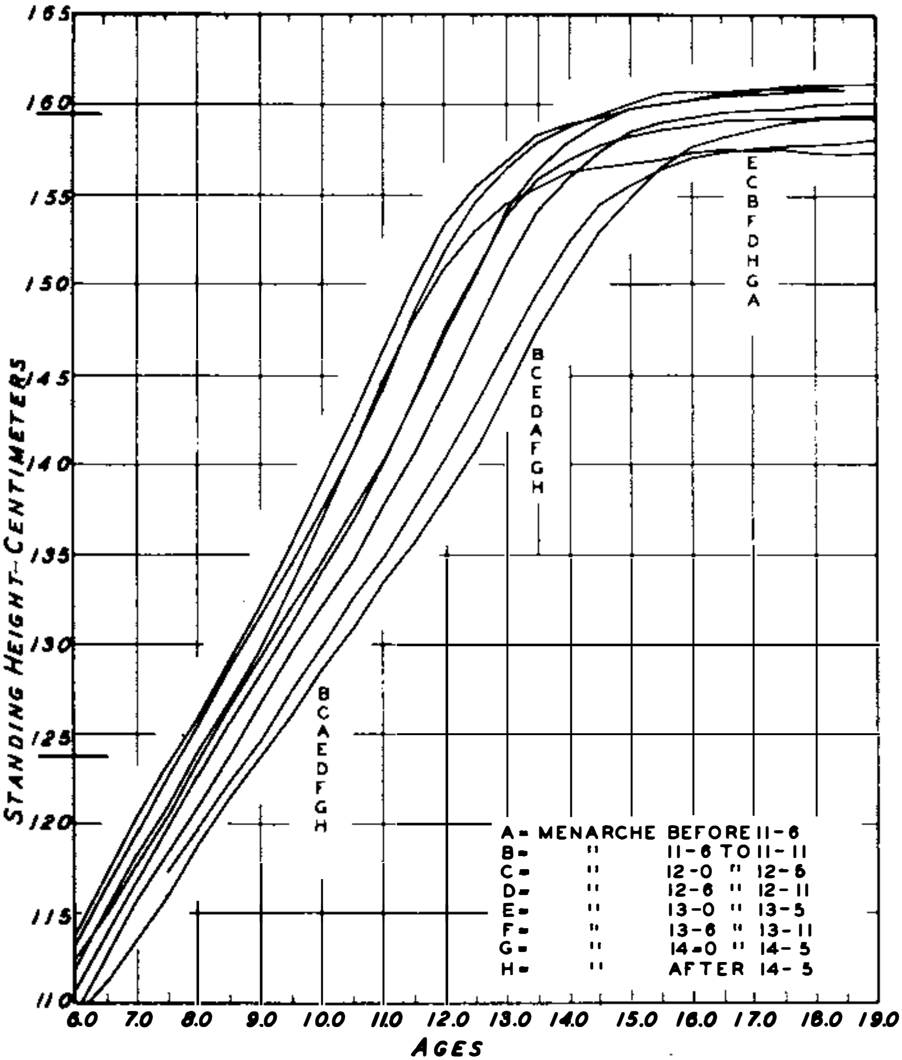

Fig. 11.--Growth Trends in Average Standing Height for Each of Eight Groups of Cases Menstruating at Different Ages.

At this age Group B averages 153.18 centimeters and Group H 138.09 centimeters tall. Although only sixteen and eighteen cases are involved the difference of 15.09 centimeters is many times as large as its probable error. Another way of expressing the magnitude of this difference is to relate it to the average growth of the whole population which takes place during the nine year interval from age 8.0 to 17.0. On this basis, a difference of fifteen centimeters is equivalent to about 3.6 years or 40.6% of the growth occurring on the average between age eight and seventeen. If a horizontal line is drawn at 155 centimeters another plane of maximum differentiation is obtained. Group B attained an average stature of 155.42 centimeters at age 12.0, but Group H did not catch up until two and a half years later. Significant differences occur from measurement to measurement in the maximum differentiation of the menarcheal groups. These will be summarized in Chapter X.

The closest approximation to a regular differentiation of the eight groups occurs at age 11.0. Here, Group B is the tallest, followed by A, C, D, E, F, G, and H. Only Group A is out of order and by a single position. A year earlier at age 10.0, Group A is out of order by two positions and Groups D and E are reversed. The same relative positions are maintained from age six to ten in a remarkably uniform manner. Apparently, the underlying factors which are responsible for the differentiation of these groups begin to operate at a very early age. From age 11.0 to maturity the association becomes less and less close. At age 17.0, for example, Group E is the tallest, followed by C, B, F, D, H, G, and A.

(b). Differences between menarcheal groups in average size at time intervals before and after the menarche. A star on each curve in Figure 11 indicating the average age of onset of the menarche would be desirable, but the close overlapping of the curves makes this impractical. However, close inspection of Figure 11 will show that there is a clearly defined tendency for the early maturing groups to be shorter at the advent of the menarche than are the middle and late maturing groups. The following tabulation gives the detailed figures for average standing height at the menarche for each of the eight groups.

Groups	Averages of Standing Height	Averages or Mid-points of Menarcheal Ages
A	144.52	11.00
B	151.60	11.75
C	153.25	12.25
D	152.33	12.75
E	155.16	13,25
F	154.92	13.75
G	153.20	14.25
H	154.87	15.00

It is important that the above series of average heights be read in connection with the menarcheal ages at the right of the tabulation. At the advent of the menarche Group A (age 11.0) averages 10.35 centimeters shorter than Group H (age 15.0). At age 11.0 Group A (at the menarche) averages 11.26 centimeters taller than Group H (four years before the menarche).

The differences between the groups at the menarche increase at longer time intervals before the menarche and decrease at longer time intervals after the menarche. These trends are shown in Figure 12. Four years before the menarche Group A is 15.26 centimeters shorter than Group H; four years after the menarche Group A is only 2.55 centimeters shorter than Group H. The combined B plus C, the D plus E, and the F plus G groups show the same trends. The visual impression of this chart is in marked contrast to that of Figure 11. Nevertheless, save for combining the data of the middle groups, the two charts present exactly the same original data. The difference between the two charts is due to the fact that in Figure 11 the horizontal scale at the bottom gives chronological ages expressed in terms of deviations from birth, whereas in Figure 12 age is expressed in terms of deviations from the advent of the menarche.

The data of Figure 12 have important implications for the understanding of the patterns of physical growth in terms of underlying endocrine factors. If it is assumed that the endocrine system begins to stimulate physical growth at some more or less definite time interval before the advent of the menarche, then, the individuals in Group A are stimulated when they are comparatively short and far from their mature stature whereas the individuals in Group H are stimulated when they are comparatively tall and much nearer their mature stature. If it is also assumed that the endocrine system provides mechanisms which bring the period of accelerated growth to a close in some time relation to the advent of the menarche, then, the early maturing individuals must grow at a more rapid rate in order to achieve the same mature stature.

3. Contrasting Slopes of Growth Curves

In the foregoing discussion and especially in Chapter III it has been emphasized that an average height at any given age for the small number of available cases must necessarily have a comparatively large probable error. Hence, the almost perfectly regular differentiation of the eight groups in average height at age 11.0 may be in part a matter of good luck. Nevertheless, when longitudinal data are available and averages from age to age are determined for identical cases, then, the position of any average relative to adjacent averages has an exceedingly small probable error. This means that the slopes of the curves from age to age are very precisely determined. However, only a brief discussion of the slopes will be presented because it is much more convenient to present all the discussion in terms of annual increments. Not only are the slopes direct functions of the increments, they are also difficult to represent graphically. In Figure 11, for example, the upward slope of all curves is so great that the contrasting slopes, in which we are primarily interested, are not apparent except to the closest inspection. However, if the reader will apply a straight-edge to Figure 11 and follow directions, quite marked differential and changing slopes can be recognized. At age 8.5 Group A averages 126.65 centimeters tall and three years later 148.01 centimeters. A straight-edge superimposed on these two points makes an angle of 65.4° with the base. Similar application of a straight-edge to the curve for Group H over the same age range gives a decidedly gentler slope, the angle being 54.3°. Now, the slopes generated by the averages from 8.5 to 11.5 for the eight groups

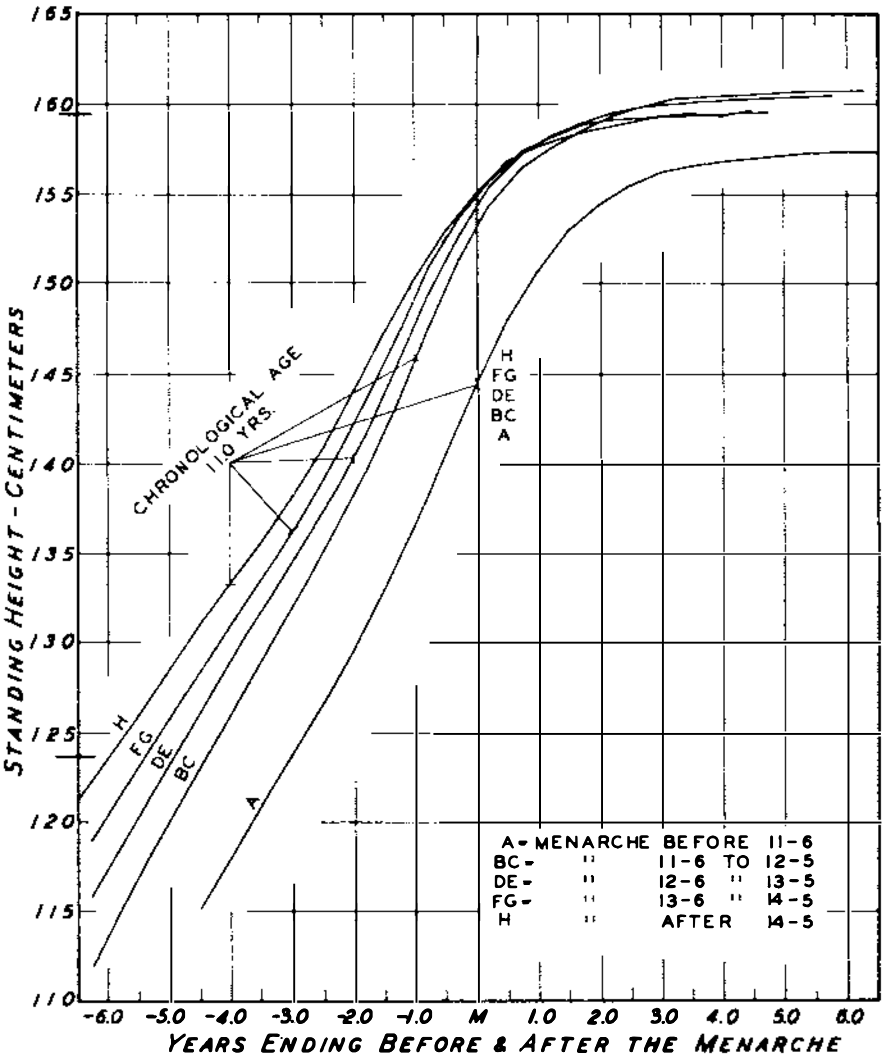

Fig. 12.--Growth Trends in Average Standing Height for Each of Five Groups of Cases Menstruating at Different Ages Arranged so that Points Corresponding to the Advent of the Menarche are in the Same Vertical Line.

fall in a precisely regular order, the angles being 65.4°, 64.1°, 62.4°, 60.4°, 59.8°, 58.6°, 55.8°, and 54.3° respectively for Groups A, B, C, D, E, F, G, and H. Hence, in addition to the almost regular differentiation of the eight groups in average size there is a regular differentiation of the eight groups in the slopes of their growth curves. Application of this method of analysis to the age range from 12.5 to 15.5 reveals another series of differential slopes. Here, it is readily apparent to even casual inspection that the curve for Group A has a very small slope while that for Group H has a comparatively steep slope. Again, the angles are in regular order but exactly the reverse of the situation from age 8.5 to 11.5. The detailed figure are 21.4°, 24.2°, 29.0°, 37.2°, 42.9°, 48.0°, 52.2°, and 56.8°, respectively for Groups A, B, C, D, E, F, G, and H. The regularity of the data can only be described as beautiful.

As an introduction to the next section it only needs to be reemphasized that the slopes of the curves are functions of the increments. Thus, from age 8.5 to 11.5 the three year increments corresponding to the slopes are 22.4, 21.1, 19.7, 18.1, 17.1, 16.9, 15.1, and 14.3 centimeters respectively for groups A, B, C, D, E, F, G, and H. Instead of working with three year increments, however, attention will be centered on the annual increments, that is, on the slopes of the curves over period of one year.

4. Annual Increments in Standing Height

In Chapter III it was emphasized that gains or increments from one age to the next constitute the really fundamental data of this study. In contrast to averages of gross size, the average increments have very small probable errors. In the case of standing height the probable errors of average increments calculated from longitudinal data are so small that approximately one hundred times as many cases of cross-sectional data at each age would be required to give equally reliable gains from year to year. Charts of the average increments clarify, instead of obscure, the changing slopes of the growth curves and the contrasted patterns of growth. Finally, the differentiation of the eight menarcheal groups is considerably more significant in terms of increments than in terms of gross size; that is, the correlation between menarcheal ages and the increments is higher than between menarcheal ages and gross size (see Chapter XI). Hence, from this point on the discussion is directed almost exclusively to the changing and contrasted patterns of growth as revealed by the increments.

Figure 13 will serve to introduce the graphical representation of the increments. The scale at the left is in centimeters and has been magnified four times in comparison with that of Figure 11. It will be recalled from the discussion of Figure 11 that, with a few exceptions, all charts of gross size have been so drawn that growth from age 8.0 to 17.0 is represented by the same vertical distance. Hence, simple multiplication of all such scales by four makes all the charts of the annual increments directly comparable. In order to emphasize this comparability all charts of increments carry three horizontal grid lines indicating respectively zero growth per year, one ninth of the average growth from age 8.0 to 17.0 (the SLG or straight-line-growth grid), and two ninths of the average growth from age 8.0 to 17.0 (the 2SLG or

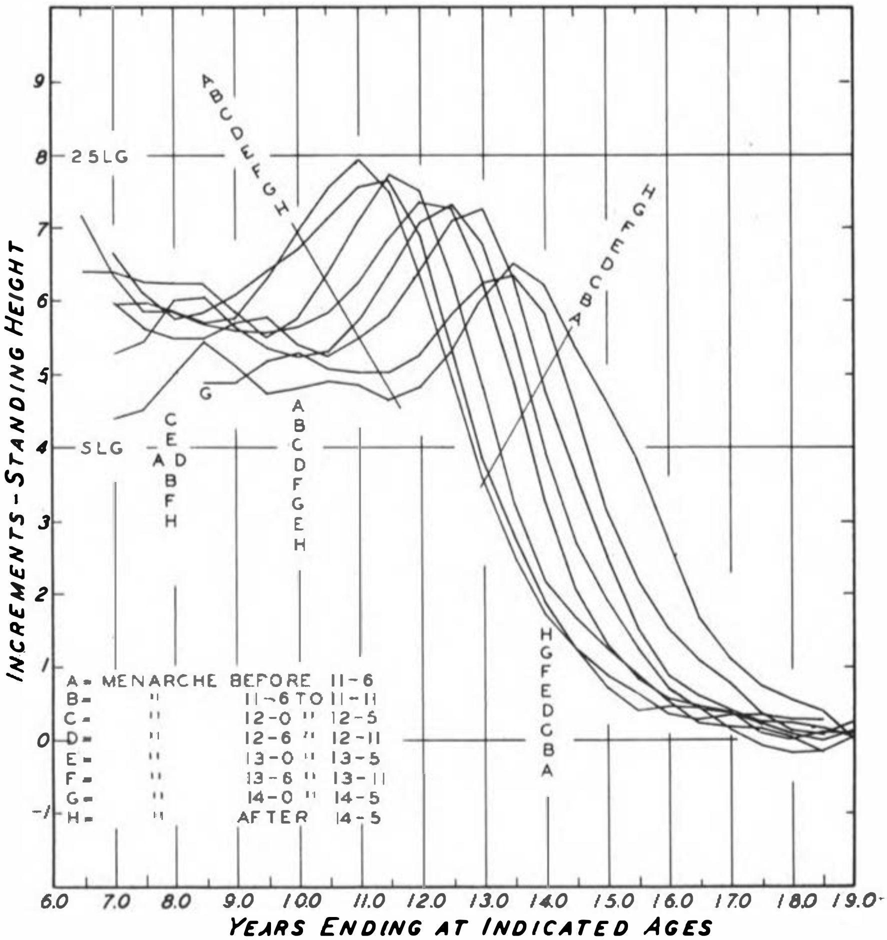

Fig. 13.--Average Annual Increments in Standing Height of Eight Groups of Cases Menstruating at Different ages.

twice-straight-line-growth). In all charts of increments the vertical distance between these three grid lines is the same and represents the same fractional proportion of the growth which takes place on the average from 8.0 to 17.0. In the case of standing height the difference between the averages of all available cases at ages 8.0 and 17.0, or an interval of nine years, is 35.9 centimeters. If growth were evenly distributed over these nine years or if the trend were represented by a straight line, then one year's growth would be one ninth of 35.9 or almost exactly 4.0 centimeters per year. Hence, 4.0 centimeters of growth in a year is equivalent to straight-line growth while 8.0 centimeters of growth in a year is equivalent to two times straight-line growth. In the case of chest depth, only .43 centimeters of growth in a year is equivalent to straight-line growth, etc.

Figure 13 presents in terms of increments the same original data as was presented in Figure 11 in terms of gross size. To emphasize this point it will be convenient to read the two charts together. Group A averages 136.58 and 144.52 centimeters tall at ages 10.0 and 11.0 (Figure 11) giving an increment of 7.94 centimeters for the year ending at age 11.0 (Figure 13). Similarly, Group H averages 128.42 and 133.26 centimeters at ages 10.0 and 11.0 (Figure 11) giving an increment of 4.84 centimeters for the year ending at age 11.0 (Figure 13). For the year ending at age 11.0 the average increments of the eight menarcheal groups are in regular order, the detailed figures being 7.94, 7.56, 7.11, 6.21, 5.73, 5.50, 5.00, and 4.84 centimeters for Groups A, B, C, D, E, F, G, and H respectively (Figure 13). Hence, (Figure 11) the slopes of the growth curves of the eight menarcheal groups from age 10.0 to 11.0 are in regular order. It is important to note that Figure 13 makes this phenomenon unmistakably clear whereas in Figure 11 the nature of the chart plus the sheer mechanical errors involved in even careful plotting make it virtually impossible to distinguish any such phenomenon.

Figure 13 presents the most important data of the entire chapter and a full discussion of its various features is in order. Charts and discussion in subsequent sections are devoted to miscellaneous associated problems and to the clarification of minor points.

(a). Attention has been called to the regular order of the increments for the year ending at age 11.0. A similarly regular but exactly reversed order occurs for the year ending at age 14.0. The average increment for Group H is the largest followed by G, F, E, D, C, B, and A, the detailed figures being 6.19, 5.84, 4.92, 3.95, 3.28, 2.08, 1.90, and 1.69 centimeters respectively.

(b). The regular sequence of periods of maximum growth should be noted. The period of maximum growth for Group A is from age 10.0 to 11.0 or the year ending at age 11.0. For Groups B and C the period of maximum growth is the year ending at 11.5, for Group D the year ending at 12.0, for Group E the year ending at 12.5, for Group F the year ending 13.0, and for Groups G and H the year ending at 13.5.

(c). There is a tendency for the magnitude of the maximum increments to decrease from the early to the late maturing groups. The maximum increment for Group A is the largest (7.94 centimeters) followed in order by Groups C, B, D, E, F, H, and G (6.37 centimeters).

(d). Three phases of the pattern of growth can be distinguished in all eight curves. First, there is a period of slightly but significantly decelerating or less and less rapid growth lasting from two to three and a half years; second, a period of markedly accelerating or faster and faster growth lasting about two and a half years; and third, a period of abruptly decelerating or slower and slower growth. For example, Group A grew 7.18 centimeters for the year ending at 6.5; annual increments are slightly but progressively smaller and smaller for the next two years declining to 5.73 centimeters for the year ending at 8.5; annual increments are progressively and markedly larger and larger for the next two and a half years reaching a peak of 7.94 centimeters at age 11.0; thereafter, the annual increments decrease at first slowly, then precipitously, and then more and more slowly as the individuals attain their mature stature. Similarly, in the case of Group H there is a slight spurt for the year ending at 8.5, slightly slower growth to the year ending at 11.5, rapid acceleration to the year ending at 13.5, and thereafter progressively smaller increments.

(e). Just as the magnitude of the maximum increment decreases from the early to the late maturing groups (7.94 to 6.50 for Groups A and H) so both the magnitude of the increments for the first phase (7.18 to 5.46 for Groups A and H) and the magnitude of the minimum increment or inflection point between the first and second phase (5.73 to 4.63 for Groups A and H) also decrease from the early to the late maturing groups.

(f). It follows that the degree of acceleration during the second growth phase is similar for all eight groups. In Group A growth accelerates from 5.73 to 7.94 centimeters per year or an acceleration of 2.21 centimeters. These accelerations vary from 1.87 to 2.24 centimeters with the exception of Group G where the acceleration is only 1.47 centimeters.

(g). The maximum increments or inflection points between the second and third phase are intimately timed with the advent of the menarche. In Group A the maximum increment is for the year ending at 11.0 and the menarche occurs on the average at 11.0. That is, age 10.0 to 11.0 is the year of maximum growth and the menarche coincides on the average with the close of that year. In Group B the maximum increment is for the year ending at 11.5 and the menarche occurs on the average at 11.75 or three months after the close of the year of maximum growth. In Groups C, D, E, F, and G comprising three-fourths of the whole population the maximum increments are for the years ending respectively at ages 11.5, 12.0, 12.5, 13.0, and 13.5 and in every case the menarche occurs nine months later at average ages of 12.25, 12.75, 13.25, 13.75, and 14.25 respectively. In Group H the maximum increment is for the year ending at 13.5 and the menarche occurs on the average at 15.0 or eighteen months later. In addition to this intimate timing between age of maximum growth and the advent of the menarche there is also a progressive shift in this timing, i. e., in Group A the menarche occurs coincident with the close of the year of maximum growth while in Group H the menarche occurs eighteen months after the close of the year of maximum growth.

(h). Likewise, the initiation of accelerated growth is intimately timed with the advent of the menarche and shows a similar progressive shift in this timing. The minimum increment or inflection point between the first and second phase for Group A is the year ending at 8.5 of 5.73 centimeters. Although this point is determined by the very slight margin of only .04 centimeters, it may nevertheless be taken as the best approximation to the age at which the acceleration is initiated. On this basis the menarche occurs on the average 2.5 years after the initiation of accelerated growth. Comparable figures for Groups B, C, D, E, F, G, and H are 3.75, 2.75, 3.25, 3.25, 3.25, 3.25, (or 2.75) and 3.50 years respectively. Considering the uncertainties in determining the point at which accelerated growth is initiated, these figures are unexpectedly uniform.

(i). While all eight menarcheal groups show essentially the same pattern of slightly decelerating, markedly accelerating, and sharply decelerating growth, there are nevertheless two consistent differences between the early and late maturing groups. In the first place the growth of the early maturing groups is more intense or the increments are larger when comparisons are made at similar points on the pattern of growth. Averaging the ten increments over the period three and a half years prior to and one year after the maximum increment separately for each group gives the following average increments at corresponding points on the pattern: 6.58, 6.54, 6.47, 6.23, 6.03, 5.96, 5.43, and 5.34 centimeters for Groups A, B, C, D, E, F, G, and H respectively. In the second place the first or slightly decelerating phase occupies a progressively longer and longer period of time from early to late maturing groups. The exact difference can not be computed because the curves are not reliably determined prior to age eight. It is clear, however, that this slightly decelerating phase is arrested at age 8.5 in Group A whereas in Group H it continues to age 11.5. These phenomena are directly related to the fact, so strikingly portrayed in Figure 12, that the early maturing groups are shorter than the late maturing groups at given time intervals before the menarche.

5. Methods of Analysis and Reliabilities

Three minor digressions from the main course of the discussion may be most conveniently inserted in this and in sections six and seven. With section eight and Figure 19 we will return to the discussion of Figure 13.

Figures 14 and 15 present graphically the original basic data for standing height from which Figures 11, 12, 13, 19, 20, 21, 23, and 24 are derived. These data are inserted at this point for the purpose of illustrating methods of analysis and for the contribution which they may make to the question of the reliability of patterns of growth. In the analysis of the data the available population of 248 cases was initially divided into two groups: cases measured for the most part within three months of their birthdays (sub-population X, Figure 14) and cases measured for the most part within three months of the half-yearly anniversaries of birthdays (sub-population Y, Figure 15). These two sub-populations were further divided into eight groups according to advent of the menarche and all of the data have been worked out separately for the resulting sixteen sub-groups. It is to be noted that some of the sub-groups contain very small numbers of cases. In Figure 14 sub-groups XB and XG consist of only seven and four cases. Further, the number of cases becomes smaller and smaller prior to age nine and after sixteen. The largest sub-group, consisting of 35 cases, has only 11 measurements at age 6.5, only 20 at age 7.5, 29 at age 17.5, and only 6 at age 18.5. That is, the individual records begin at different ages and end at different ages. This overlapping of the records makes it impossible to obtain true growth curves by simply averaging the available measurements at successive ages since identical cases are not always available at successive ages. Hence, all the computations were carried out in terms of the annual increments giving basic data for each dimension and for each of sixteen sub-groups as illustrated in Figures 14 and 15. Analysis of such basic data gives Figures 11, 12, 13, 19, 20, 21, 23, and 24. For example, the curves of Figure 13 are the simple unweighted averages of the corresponding pairs of curves in Figures 14 and 15. Thus, the average increment of Group A in Figure 13 for the year ending at age 11.0 of 7.94 centimeters is the average of 8.21 centimeters from Figure 14 at age 11.0 and of 7.66 from Figure 22 midway between points at age 10.5 and 11.5. The combined data give observation points at intervals of half a year although the original measurements were taken at annual intervals. Growth trends in average size were obtained by calculating one average for each sub-group at an age having

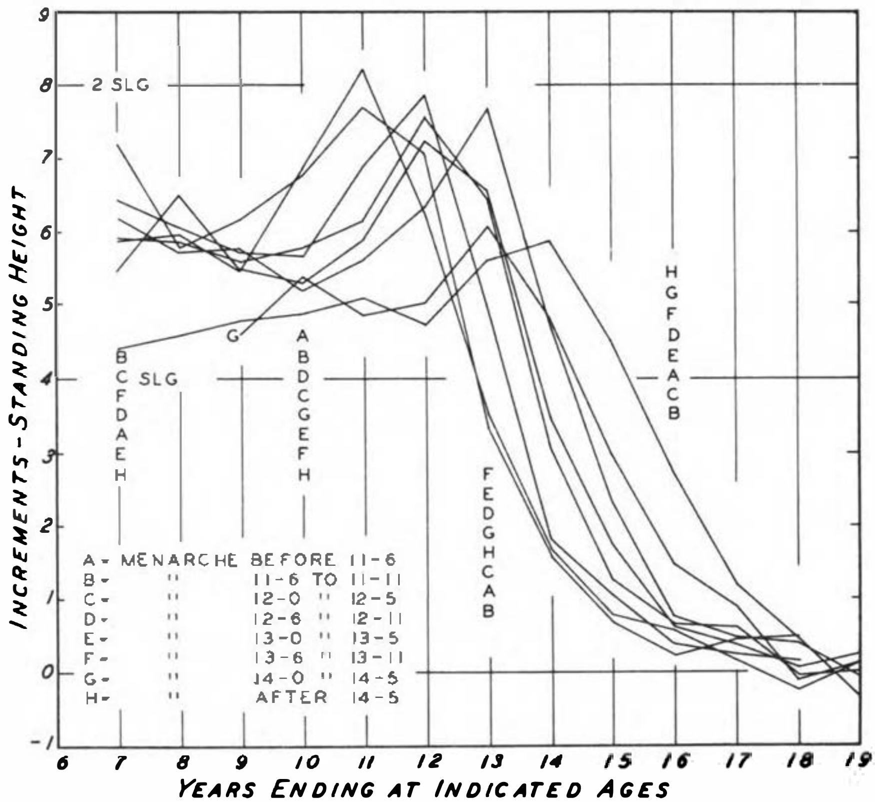

Fig. 14.--Average Annual Increments in Standing Height for each of Eight Groups of Cases Menstruating at Different Ages and who were Measured Within Three Months of Birthdays.

measurements for all cases and by adding and subtracting the average annual increments from this base figure. Accordingly the data of Figure 11 were derived from Figure 13 or the reverse of the usual procedure.

The critical reader may make his own comparisons between Figures 14 and 15 for the purpose of testing the reliability and consistency of the patterns in the eight menarcheal groups. It is only necessary to note that some of the differences between the two charts are genuine and should not be attributed to chance factors. For example, in Figure 14 Group XA shows a very sharp maximum increment of 8.21 centimeters for the year ending at age 11.0, whereas in Figure 15 Group YA shows a flat topped curve with increments of 7.59 and 7.74 for the years ending at 10.5 and 11.5. This is a significant difference. If Group YA had been measured at birthdays instead of at the half-yearly anniversaries of birthdays they would probably have shown a sharp maximum increment as in Figure 14 instead of the flat topped curve of Figure 15. That is, the data are sensitive to even slight changes of six months in the timing of measurements in relation to the timing of the menarche. This, indeed, is the major reason for carrying out the whole of the analysis separately for cases measured approximately at birthdays and for cases measured approximately at the half-yearly anniversaries of birthdays

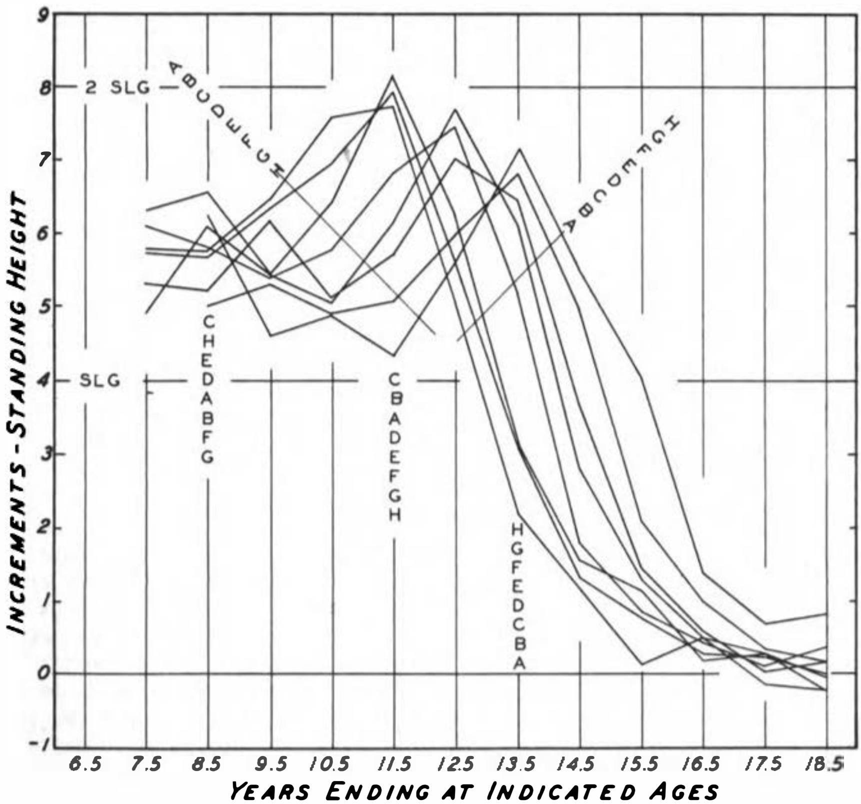

Fig. 15.--Average Annual Increments in Standing Height for each of Eight Groups of Cases Menstruating at Different Ages and who were Measured Within Three Months of the Half-yearly Anniversaries of Birthdays.

and for the further sub-division of each of these groups into eight menarcheal groups.

6. Tall Versus Short Children

It has been known for forty years that short children tend to grow slowly and to reach their mature stature at a comparatively late age while tall children tend to grow more rapidly and to reach their mature stature at a comparatively early age (Boas, 6). Since short children in the age range from seven to ten will include a large proportion with late menarcheal ages while the tall children will include a large proportion with early menarcheal ages, it is apparent that the data of this study provide an explanation of the contrasted patterns of tall and short children.

It has already been suggested in the discussion of Figure 1 and 2 that, within a given group of cases having the same menarcheal age, the pattern for tall and short children is the same. Figures 16 and 17 present a more thorough test of this point. Cases menstruating between 13-0 and 13-5 and measured within three months of the half-yearly anniversaries of birthdays (sub-group YE) have been selected for this purpose. Three cases with short records and one case with doubtful menarcheal age have been discarded (see Appendix B, section three). Heights for each individual from age 9.5 to 15.5 were averaged and the resulting values used to divide the group into the ten tallest, a middle group of eleven, and the ten shortest. Figure 16 shows the average gross heights of these three groups. Over the whole age range, the tallest ten cases average 13.02 centimeters taller than the ten shortest cases. Considering the great differences in gross stature and the probable errors of trends based on only ten cases, the similarity of the pattern of annual increments shown in Figure 17 is remarkable. The increments of the tallest ten cases and of the shortest ten cases differ on the average by only .51 centimeters.

7. Comparative Data

Comparative data are always of considerable interest but only one body of published data is comparable and this is limited to standing height. Reference has already been made to the very striking chart of Boas (5) showing growth trends in standing height for groups of cases menstruating at each age from ten to fifteen. His report gives two sets of detailed numerical data from which annual increments for different menarcheal groups may be determined. First, he presents in his Table 9 the average heights at each age for each of eight menarcheal groups. Using these average heights to determine annual increments gives extremely erratic patterns for the reason that the averages were calculated without regard to the identity of the cases from age to age. Second, he presents in his Table 12 the detailed distributions of annual increments at each age for six groups menstruating at age ten, eleven, twelve, thirteen, fourteen and fifteen, but no summary of these data in terms of average increments is given. Although the grouping is coarse, average increments have been computed from these distributions. The resulting trends are presented in Figure 18. Some confirmation of the Harvard data is provided by this chart in that the annual increments of the six menarcheal groups for the years ending at ages 10.0 and 14.0 are in almost regular order as in Figures 13, 14, and 15. The curves of Figure 18, however, are more irregular, the maximum increments less marked, and the accelerations more feeble. In part these differences are due to errors involved in computing average increments from coarsely grouped distributions. In part also they are due to the fact that Boas interpolated the individual records in order to obtain estimated heights at exact birthdays and apparently applied straight-line interpolation in the calculation of these estimated figures.

8. Growth Patterns Before and After the Period of Maximum Growth

Figure 19 is a simple redrawing of Figure 13 with the maximum increments arranged in the same vertical line. This is designed to illustrate more clearly the similarity of the pattern of growth in each of the eight menarcheal groups and the tendency of the early maturing groups to have

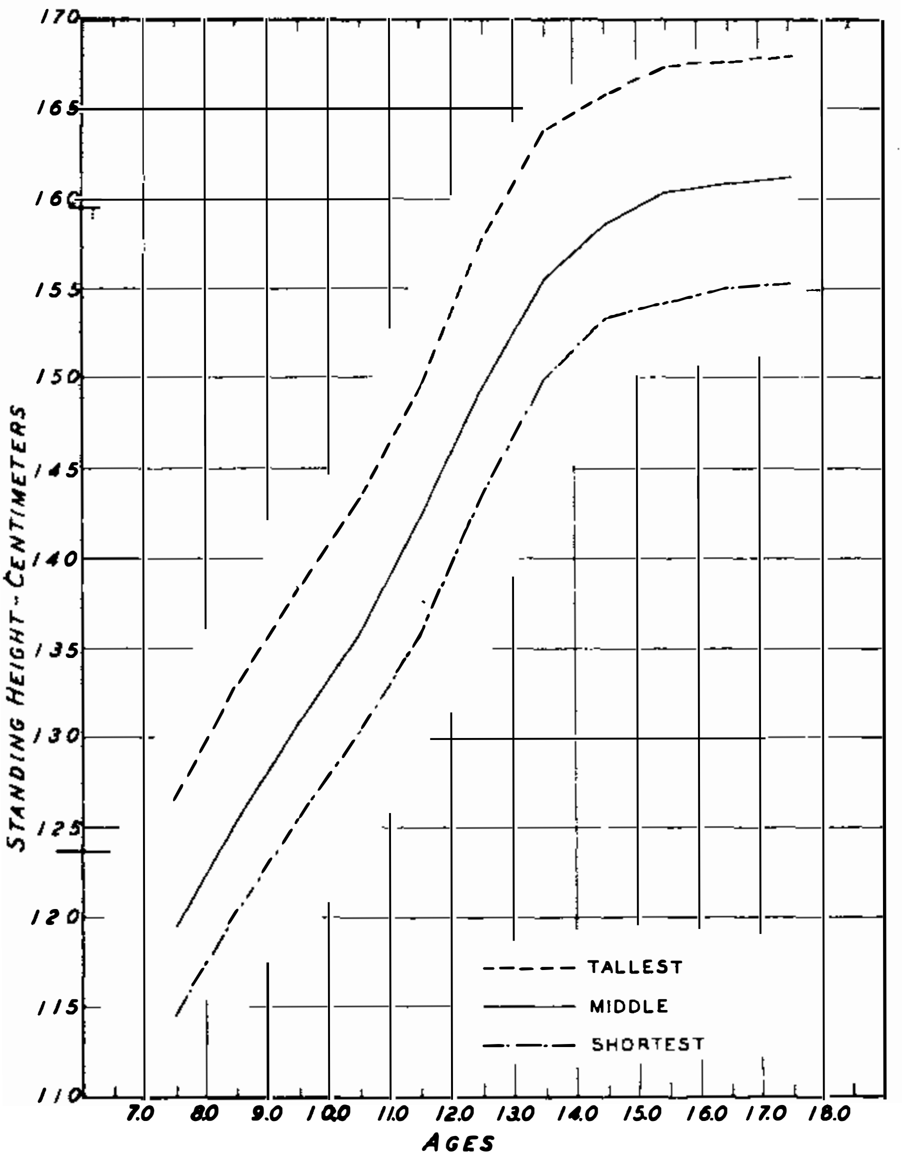

Fig. 16.--Growth Trends in Average Standing Height of Cases With the Same Menarcheal Age, 13-0 to 13-5, Divided Into Tallest, Middle, and Shortest Cases.

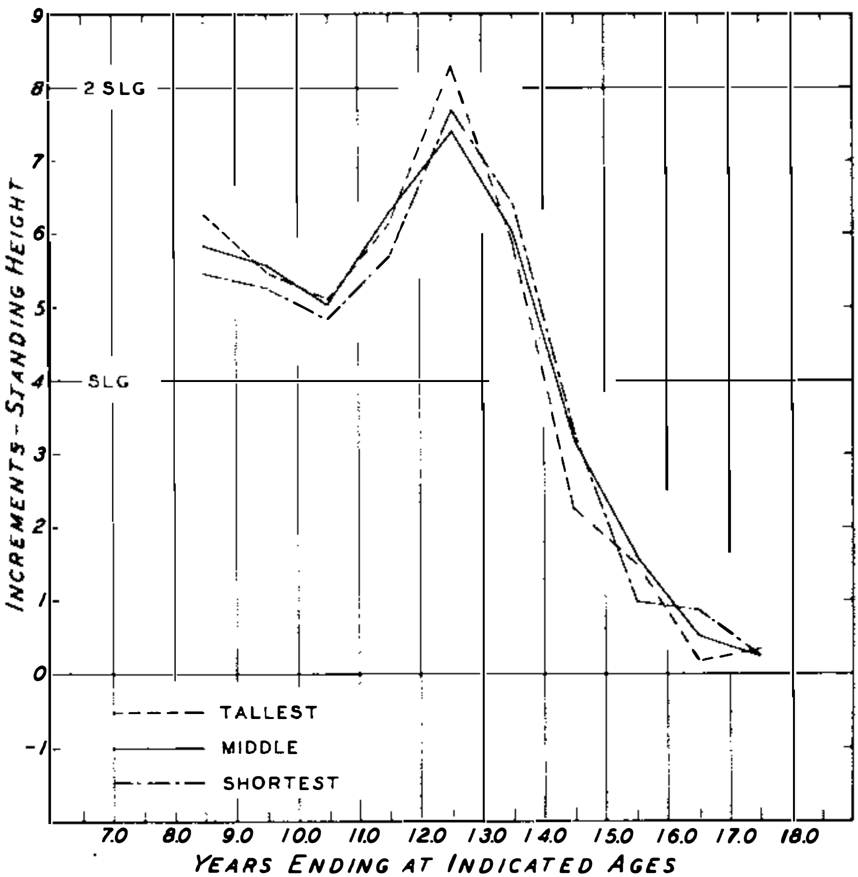

Fig. 17.--Average Annual Increments in Standing Height of Cases with the Same Menarcheal Age, 13-0 to 13-5, Divided Into Tallest, Middle, and Shortest Cases.

larger increments when comparisons are made at similar points of the pattern. It is to be noted that the maximum increments for Groups B, D, and G are not clearly defined. Hence, the true position of these three curves might be shifted to the right or left by half a year. Considering the small number of cases in each group the similarity of the eight patterns is very striking. Only the curve for Group B from two to four years before the maximum increment and the curve for Group H two years after the maximum increment depart significantly from the general pattern. Four years before the maximum increment the eight groups are arranged in nearly regular order. Similarly, at the maximum increment

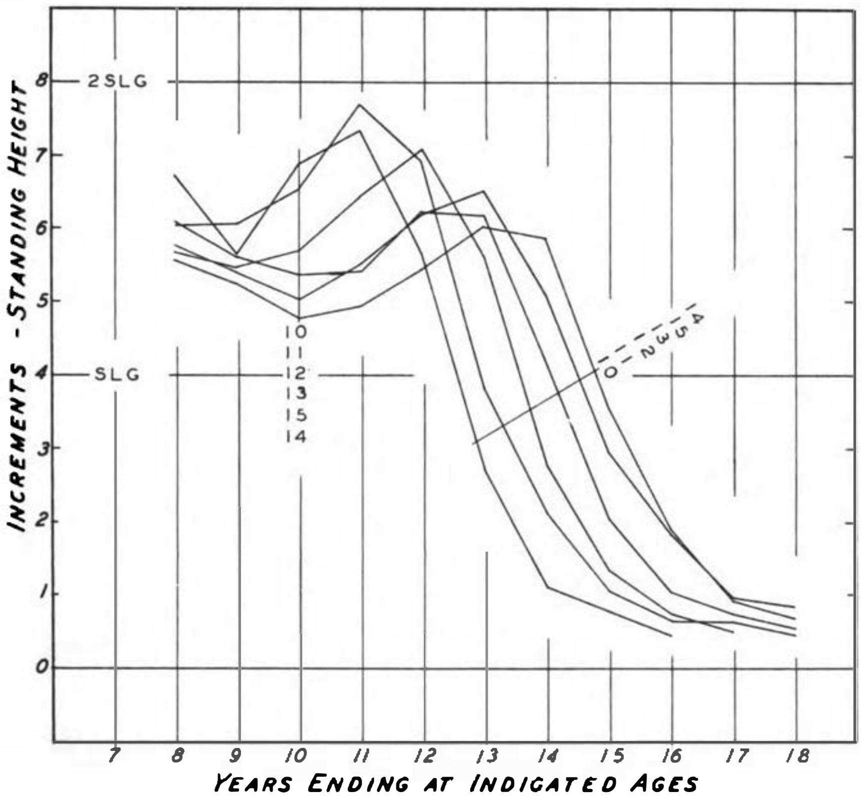

Fig. 18.--Average Annual Increments in Standing Height of Six Groups of Cases Who First Menstruated at Age 10 (10-0 to 10-11), 11, 12, 13, 14, and 15; Derived from Data of Boas.

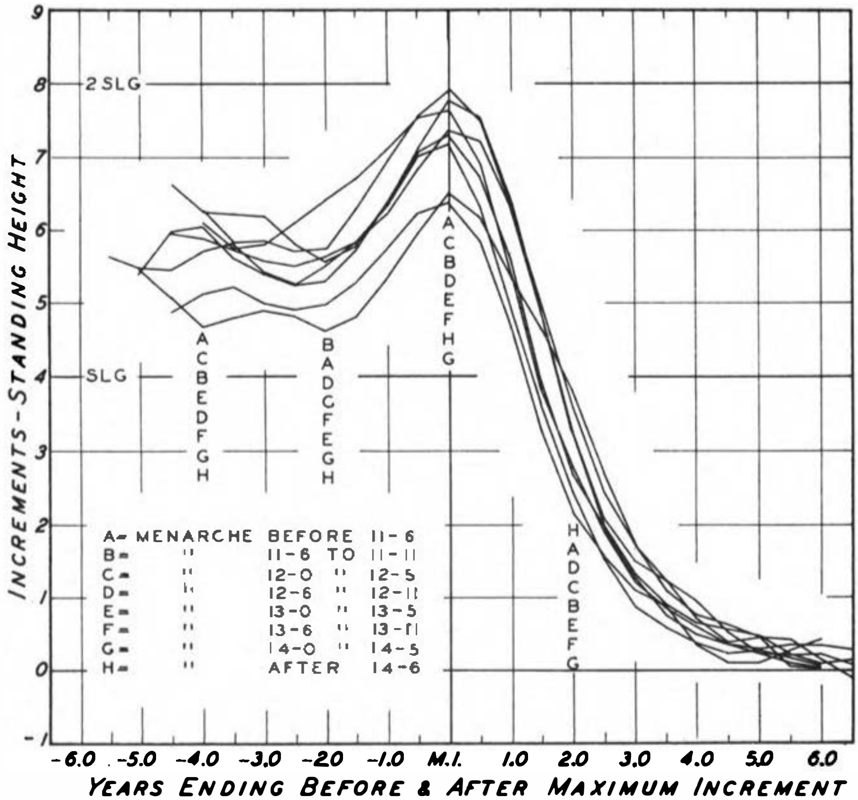

Fig. 19.--Average Annual Increments in Standing Height of Eight Groups of Cases Menstruating at Different Ages Arranged so that the Maximum Increments of All Curves are in the Same Vertical Line.

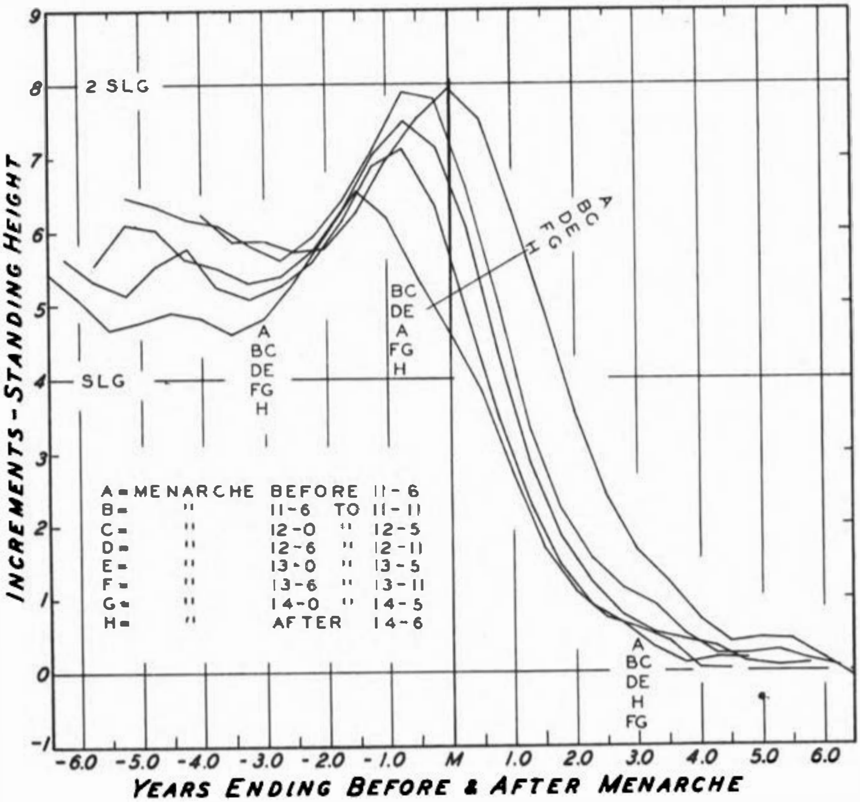

Fig. 20.—Average Annual Increments in Standing Height of Five Groups of Cases Menstruating at Different Ages Arranged so that Points on Their Curves Corresponding to the Advent of the Menarche are in the Same Vertical Line.

only Groups B and G are out of order by a single position.

9. The Pattern of Growth Before and After the Menarche

In the preceding section the curves of Figure 13 were arranged with the maximum increments in the same vertical line. The curves may also be arranged so that the points corresponding to the menarche are in the same vertical line. Figure 20 presents such an arrangement in which the data for Groups B and C, D and E, and F and G have been combined. Although the method of combining the groups is a little technical, it should be noted that the B and C curves of Figure 13 were not combined directly. Rather, the XB curve (Fig. 14) and the YC curve (Fig. 15) were first combined at identical deviation ages from the menarche and the resulting XBYC data combined with a similar composite of the XC curve (Fig. 14) and the YB curve (Fig. 15). This merely insures that the timing of the pattern in relation to the advent of the menarche is as precise as possible. The data here presented in terms of increments are identical with the data presented in Figure 12 in terms of gross size.

The discussion of Figure 13 concerning the precise timing of the maximum increment in relation to the advent of the menarche should be recalled at this point. On the average in Group A, Group B, Groups C to G, and Group H the menarche occurs respectively exactly at, three months after, nine months after, and eighteen months after the close of the year of maximum growth. It is this progressive shift which Figure 20 is particularly designed to illustrate.

10. The Pattern of Annual, Semi-annual, and Bimonthly Increments Before and After the Menarche

The extra labor involved in computing the much more reliable pattern of increments represented by the BC, DE, and FG groups was undertaken in part for the purpose of exploring the trend of semi-annual or six-month increments. These are given in Figure 21 in which the centimeter scale at the left has been magnified by two in order that the data may be directly comparable with other charts. That is, an increment of two centimeters in six months is represented by the same vertical distance as an increment of four centimeters in a year and both are equivalent to straight line growth. The chart is an approximation to the results which would be found if children were measured at six month intervals and the resulting increments analyzed according to their time relationships to the advent of the menarche.

This chart and Figure 20 should be read together. According to Figure 20 the maximum annual increment of Group BC is 7.91 centimeters for the year ending at deviation age -.75 (nine months before the menarche) but this point is not clearly defined since the increment for the year ending at -.25 is 7.81 centimeters. Now, the annual increment of 7.91 centimeters is composed of two six-month increments of 3.76 and 4.15 centimeters and the second of these six-month increments (4.15) plus a third six month increment of 3.66 give the annual increment of 7.81. Hence, the series 3.76, 4.15 and 3.66 gives the BC curve in Figure 21 a very sharp peak while exactly the same data (3.76 plus 4.15 and 4.15 plus 3.66) give the BC curve in Figure 20 a flat topped peak.

Having established or explained the identity of the data in the two charts four features of Figure 21 which are in contrast with those of Figure 20 may be noted. The maximum increments of Figure 21 are distinctly higher and the minimum or inflection points between the first and second phases are distinctly lower than those of Figure 20. For example, the maximum six-month increment for the BC group is 4.15 centimeters or at the rate of 8.30 centimeters per year whereas the maximum annual increment is only 7.91 centimeters. It follows that the six-month increments indicate a sharper acceleration, sharper peak increments, and a more precipitous slowing of growth. The inflection points marking the transition from the first to the second phase of growth and the inflection points marking the transition from the second to the third phase are placed at an earlier time interval in relation to the advent of the menarche.

Figure 22 represents an attempt to explore this problem still further using only cases who menstruated between age 12-6 and 13-5[1]. Eighty-six cases were divided into six groups according to the timing of measurements in relation to the advent of the menarche and the trends of these six groups combined to give observation points at intervals of two months. The solid curve represents the trend of the annual increments while the dash curve represents the trend of the two-month increments multiplied by six to make the data comparable with the annual increments.

While the data of Figures 21 and 22 are somewhat speculative, they nevertheless suggest quite forcefully the importance of semi-annual or quarterly or even monthly measurements of standing height during the period of transition from faster and faster to slower and slower growth.

11. Growth Rates

Students who have attempted to construct mental growth curves have been plagued by the indefiniteness of their units of measurement and have been inclined to look with envy on those who work with such unequivocal units as centimeters and kilograms. While a centimeter is a centimeter, a centimeter of standing height is not comparable with a centimeter of head breadth. In the case of chest depth, indeed, that centimeter may represent nothing but error. While a kilogram of body weight is a kilogram of body weight, nevertheless it may represent very different proportions of bone, muscle, and fat from individual to individual and from age to age, to say nothing of varying amounts of clothing (sic), and empty or full stomach or bladder. Finally, two increments in a year of ten kilograms from initial weights of twenty and fifty kilograms represent the most radically different phenomena. Graphical representation of these varying and incomparable units pre-

[1]*Cases with interpolated figures from age 10.0 to 13.0, with suspected menarcheal ages, and with abnormal patterns of growth in standing height were excluded. Deviation ages from the menarche were computed for each case and these were used to divide the cases into six groups: those measured approximately at the menarche, those measured ten months before and two months after the menarche, those measured eight months before and four months after, six months before and six months after, four months before and eight months after, and two months before and ten months after. These six groups were then combined by an extension of the procedure described in appendix B, section four, giving observation points at intervals of two months.*

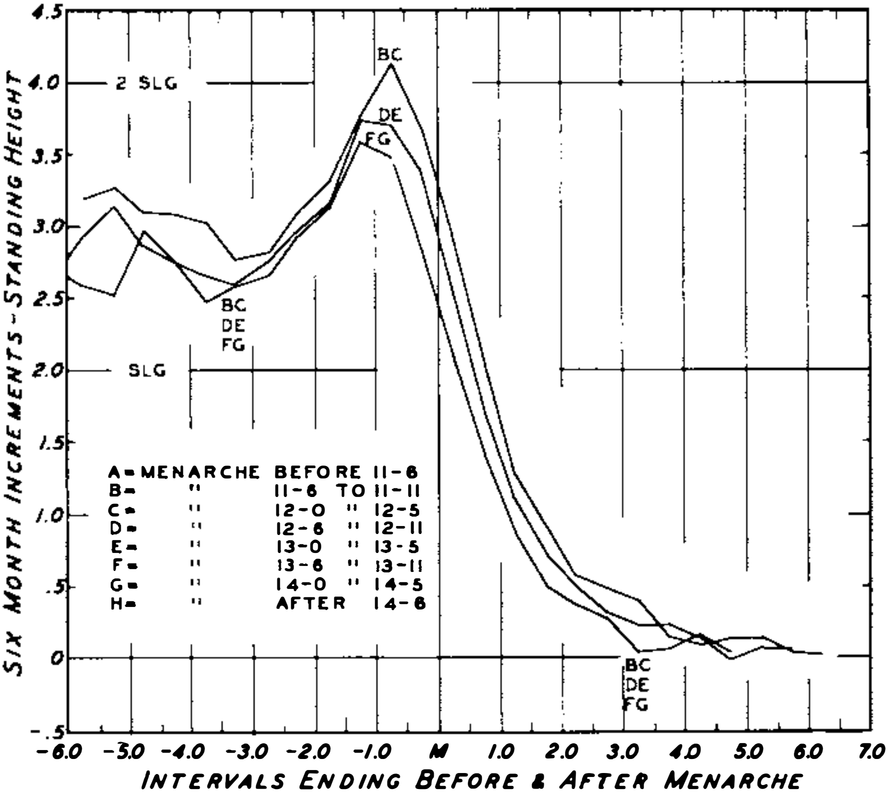

Fig. 21.--Six Month Increments in Standing Height of Three Groups of Cases Menstruating at Different Ages Arranged so that Points Corresponding to the Advent of the Menarche are in the Same Vertical Line.

sents a problem which is not amenable to any certain solution. Here it is sufficient to note that two methods have been employed to translate such diverse units into comparable graphical representations. The first procedure, employed exclusively up to this point, sets up a chart in which the vertical distance from the zero to the SLG grid line is the same throughout all charts of absolute increments and always represents straight line growth or one ninth of the growth attained on the average from eight to seventeen. The second procedure, illustrated by Figures 23 and 24, translates the absolute increments into relative or percentage rates of growth.

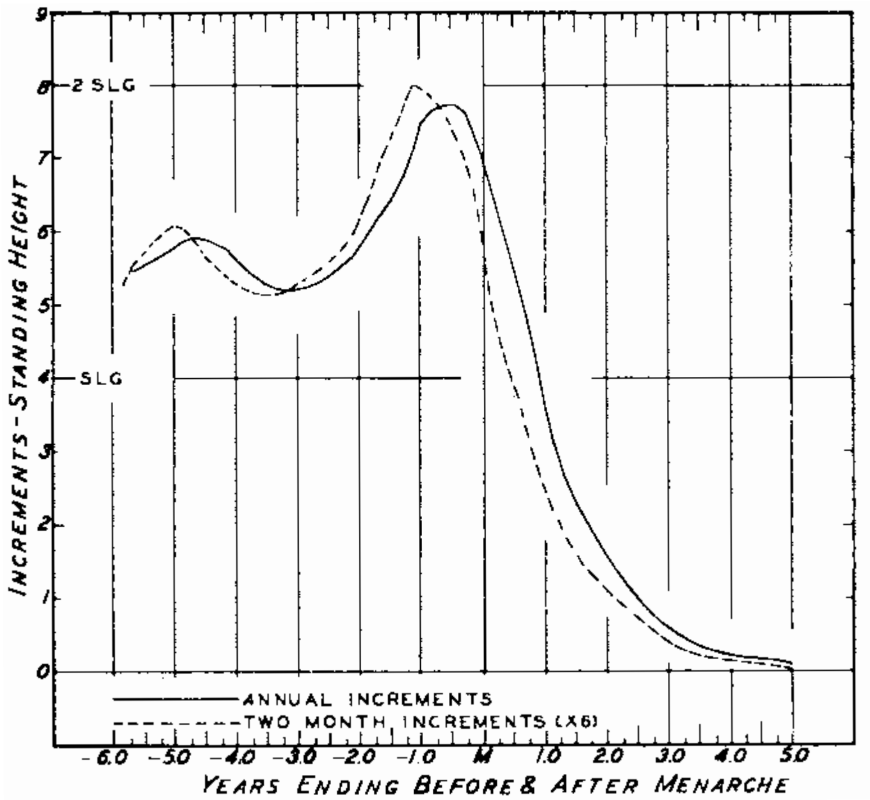

Fig. 22.--Annual Increments and Two Month Increments in Standing Height of Cases Menstruating between Age 12-6 and 13-5.

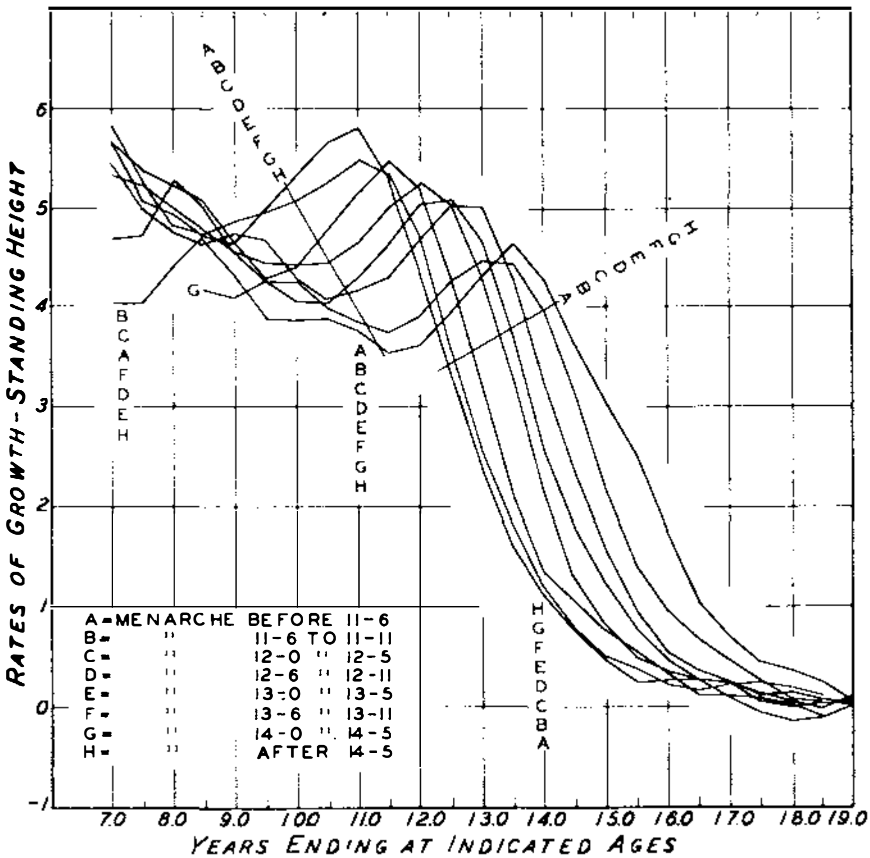

Fig 23.--Percentage Rates of Growth Per Year in Standing Height of Eight Groups of Cases Menstruating at Different Ages.

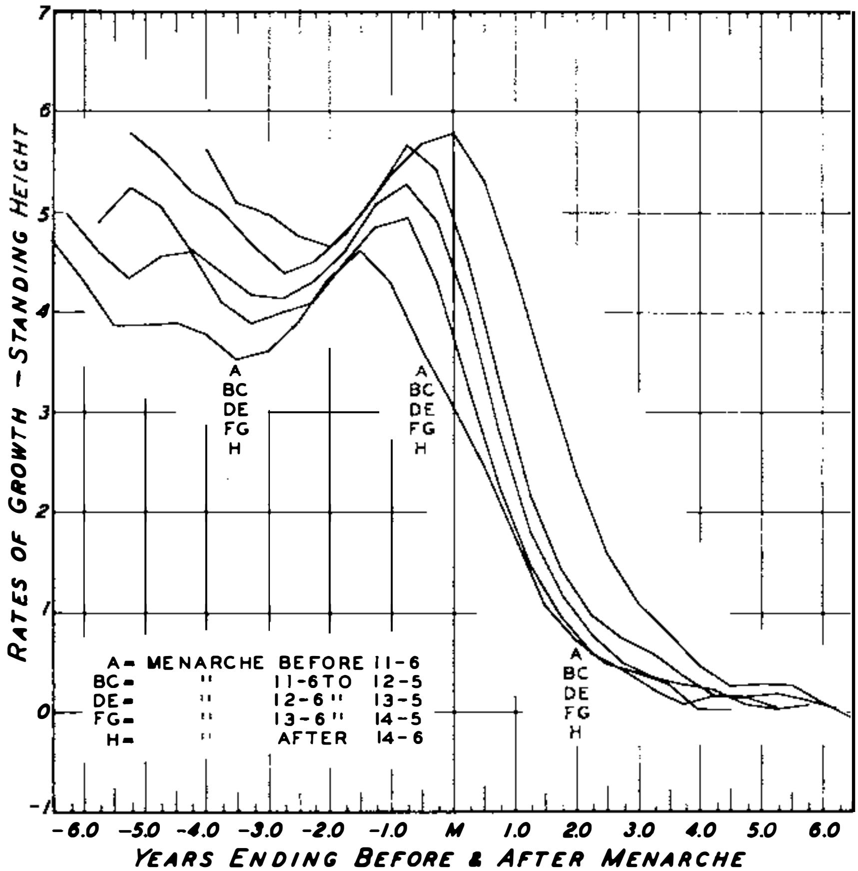

Fig. 24.--Percentage Rates of Growth Per Year in Standing Height of Five Groups of Cases Arranged so that Points Corresponding to the Advent of the Menarche are in the Same Vertical Line.

The rates of growth presented for each of the eight menarcheal groups in Figure 23 should be compared with the corresponding absolute increments presented in Figure 13. In Figure 13 the maximum increment of Group A is 7.94 centimeters representing the increase in stature from 136.60 centimeters at age 10.0 to 144.52 centimeters at age 11.0. The increment of 7.94 divided by initial stature at age 10.0 indicates an increase of 5.81% for the year ending at age 11.0 as charted in Figure 23. In order to make Figure 23 as comparable as possible with Figure 13 the vertical distance representing the 5.81% maximum rate for Group A has been set equal to the vertical distance representing the maximum increment of 7.94 centimeters.

The features of Figure 23 which distinguish it from Figure 13 are as follows. Rates of growth during the early years are greatly accentuated in comparison with the corresponding increments, so much so that in six of the eight groups these rates exceed the peak rates which correspond to the maximum increments. That is, relative growth in terms of percentages tends to be more intense during the early years although the actual increments in centimeters are smaller than during the middle range of years. The downward slope of the rates during the first or decelerating phase of growth is much more striking than the downward trend of the corresponding increments. Conversely, the acceleration of the relative growth rates during the second phase is less marked. Similar alterations may be noted in Figure 24 which should be compared with Figure 20.

CHAPTER V

INDIVIDUAL PATTERNS OF GROWTH IN STANDING HEIGHT

Individual patterns of growth necessarily conform more or less to the group patterns, but the individual exceptions to general trends are so numerous that the data of this chapter are in some contrast to the data of the preceding chapter. The contrast between individual and group patterns may be attributed to four factors. First, the limitations of the Harvard data, which tend to cancel out in group trends, are accentuated in the individual records. Measurements at annual intervals instead of at semi-annual or quarterly intervals, the necessity for interpolating for random gaps, erratic measurements which might have been eliminated by more rigorous editing, and suspected menarcheal age records are important factors restricting the study of individual records. Second, even though contrasting group trends are regular and consistent, a wide range of genuine individual differences must be expected within each group. This is particularly true since the sixteen sub-groups are not clearly distinguished. Group XD, for example, does not consist of cases all measured exactly at birthdays and all menstruating exactly at age 12.75. Instead, it consists of cases measured for the most part within three months of birthdays who were reported by their mothers to have menstruated between 12-6 and 12-11. Third, statistical procedures for summarizing group trends artificially simplify the complexity of the data. This is particularly true when a series of average increments is presented in the form of a curve. Each average increment may be a good representation of the central trend of the data from which it is derived, but a series of such average increments generates a pattern which is still further removed from the original complexity of the individual patterns. The greater the departure of the group pattern from the typical individual pattern, the more forceful the suggestion that the original grouping of the cases is not fine enough. Finally, a closer study of individual patterns suggests that the analysis of the data in terms of menarcheal age alone is not enough. Analysis of the data in terms of both menarcheal age and age of maximum growth will provide a much more realistic and generalized description of the actual pattern of individual growth. The need for such an analysis is reinforced by the above remarks concerning the pattern generated by a series of average increments.

The data of this chapter fall into three parts. Sections one and two are concerned with individual differences or variabilities in gross heights and in annual increments. Sections three and four present individual growth curves and individual increment curves. Sections five and six present an analysis of the data in terms of both menarcheal age and age of maximum growth.

1. Variabilities in Standing Height

The extent, nature, causes, and consequences of variability constitute one of the central problems of human biology. It is a curious fact that the literature presents hundreds and hundreds of charts contrasting the growth trends in average size of boys and girls, but charts contrasting their variabilities are almost non-existent. Even frankly cross-sectional studies often fail to report any measure of variability. The problem is entitled to intensive study on its own account and for this reason only sample charts based on the data of Table 29 of Appendix B will be presented. While Figures 25 to 29 are the only charts which bear specifically and directly on the question of variabilities, it should be noted that essentially all charts make substantial indirect contributions to the understanding of changing variabilities.

Figure 25 presents the trends of average heights, the range of plus and minus one probable error of average heights, and the range of plus and minus one standard deviation of the distribution of heights for Groups A and H at each age for which at least thirteen cases are available. As noted in the legend Group A is represented by a vertical-horizontal cross-hatch while Group H is represented by a diagonal-diagonal cross-hatch. The light cross-hatch represents the range of plus and minus one standard deviation of the distribution while the heavy cross-hatch represents the range of plus and minus one probable error above and below respective means. The standard deviations range from 4.94 centimeters at age 8.5 for Group A to 8.04 centimeters at age 16.5 for Group H. That is, the vertical distance including approximately 68% of the cases of each group ranges from ten to sixteen centimeters. With from thirteen to twenty-two cases the probable errors of the means range from .75 centimeters at age 12.5 for Group A to 1.45 centimeters at age 16.5 for Group H. The means are significantly differentiated from age 9.0 to 14.0, but it is readily apparent that in terms of standard deviations there is considerable overlapping of the individuals in the two groups at all ages.

The limitation of a chart of the type of Figure 25 is that only two groups can be conveniently contrasted. Selecting only ages 9.0, 11.5, 14.0 and 16.5, Figure 26 makes comparisons between all eight menarcheal groups and the total population. At each of the selected ages the eight groups are arranged from left to right beginning with Group A at the extreme left and followed by Groups B, C, D, E, F, G, and H with the total population at the extreme right. The mid-point of each vertical line indicates the average standing height, the points just above and below indicate the range of plus and minus one probable error of the average standing height, and the total length of each line indicates the range of two standard deviations of the distribution of gross heights or of plus and minus one standard deviation above and below each average. It should be noted that each group retains its characteristic standard deviation through each of the four age levels, but too much significance should not be attributed to the differences among the standard deviations of the eight groups since the number of cases is comparatively small. The extreme overlapping of the individuals in the eight groups, as indicated by their standard deviations, is clearly portrayed.

2. Variabilities in Slopes or Annual Increments of Height

It is to be emphasized in both Figures 25 and 26 that the standard deviations refer to the gross heights at each age and that the probable errors relate to the averages of gross heights at each age. That is, these charts say nothing whatever about either the variability of the slopes of individual curves or the probable errors of the slopes

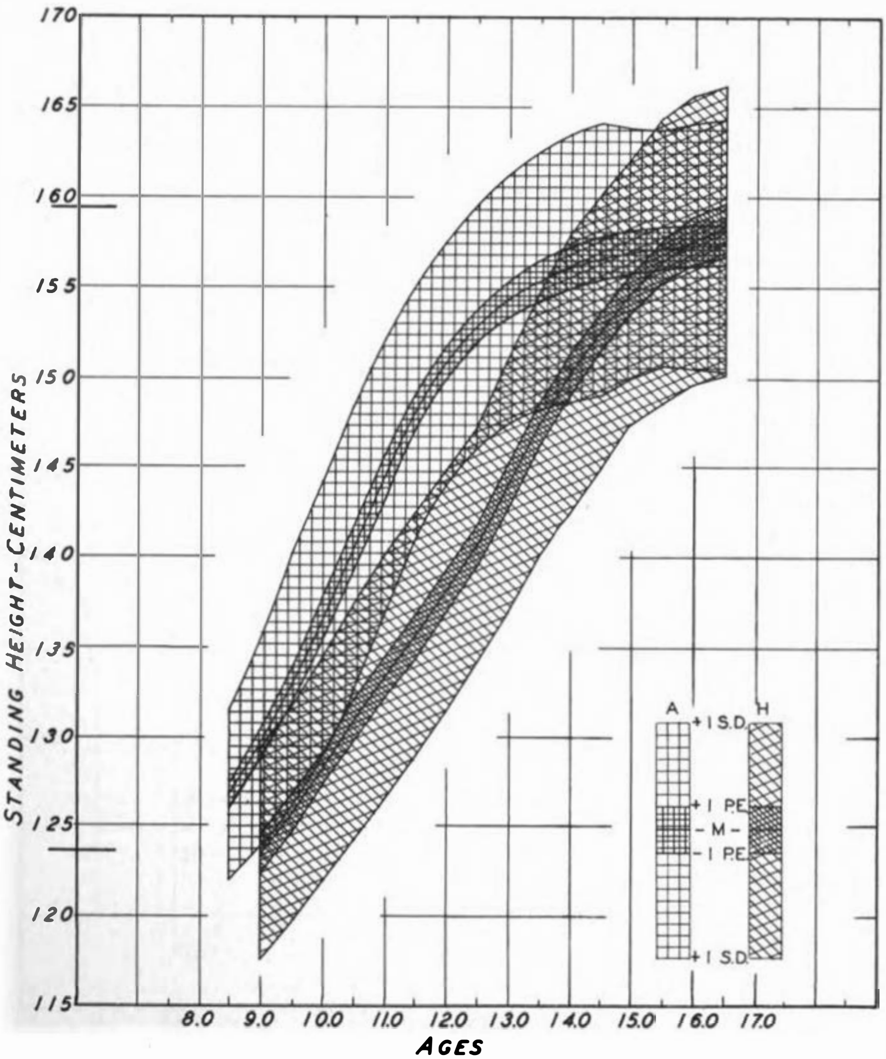

Fig. 25.--Growth Trends in Average Standing Height, Range of Plus and Minus One Probable Error of the Averages, and Range of Plus and Minus One Standard Deviation of the Distribution of Standing Heights for Early and Late Maturing Groups.

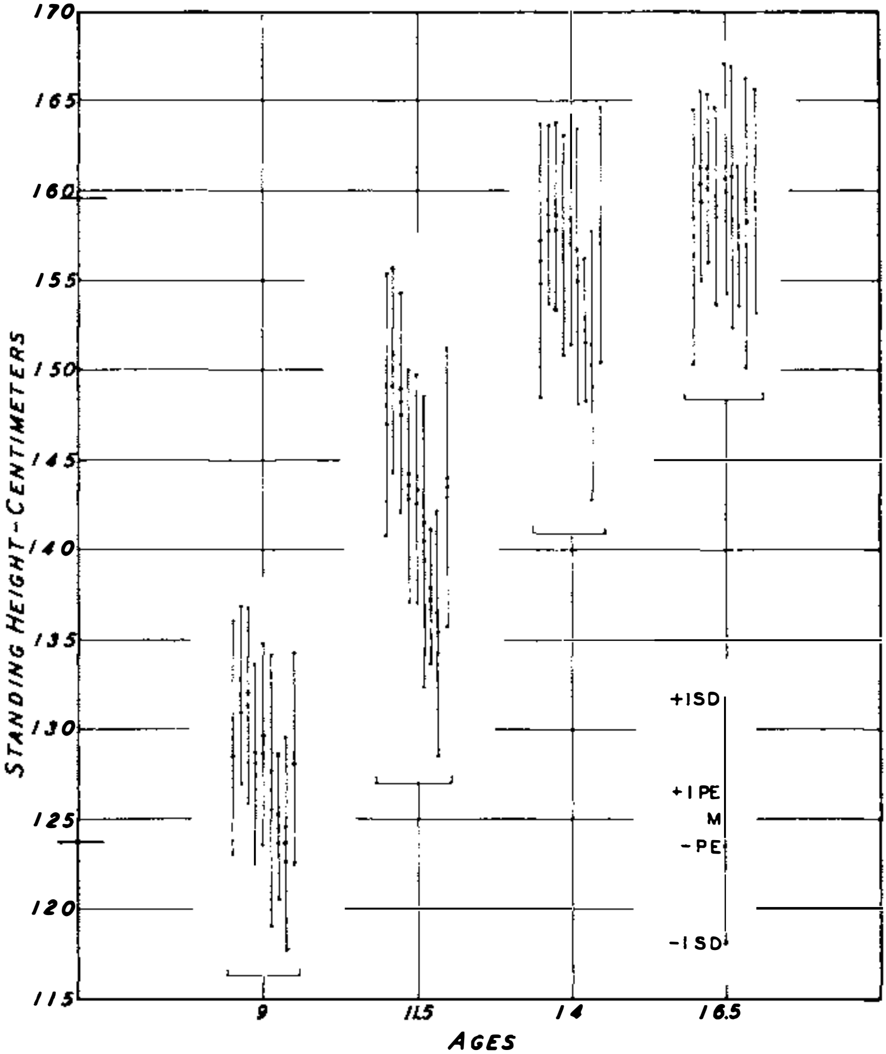

Fig. 26.--Average Standing Heights, Range of Plus and Minus One Probable Error of the Averages and Range of Plus and Minus One Standard Deviation for Each of Eight Groups of Cases Menstruating at Different Ages and for the Total Population at Selected Ages.

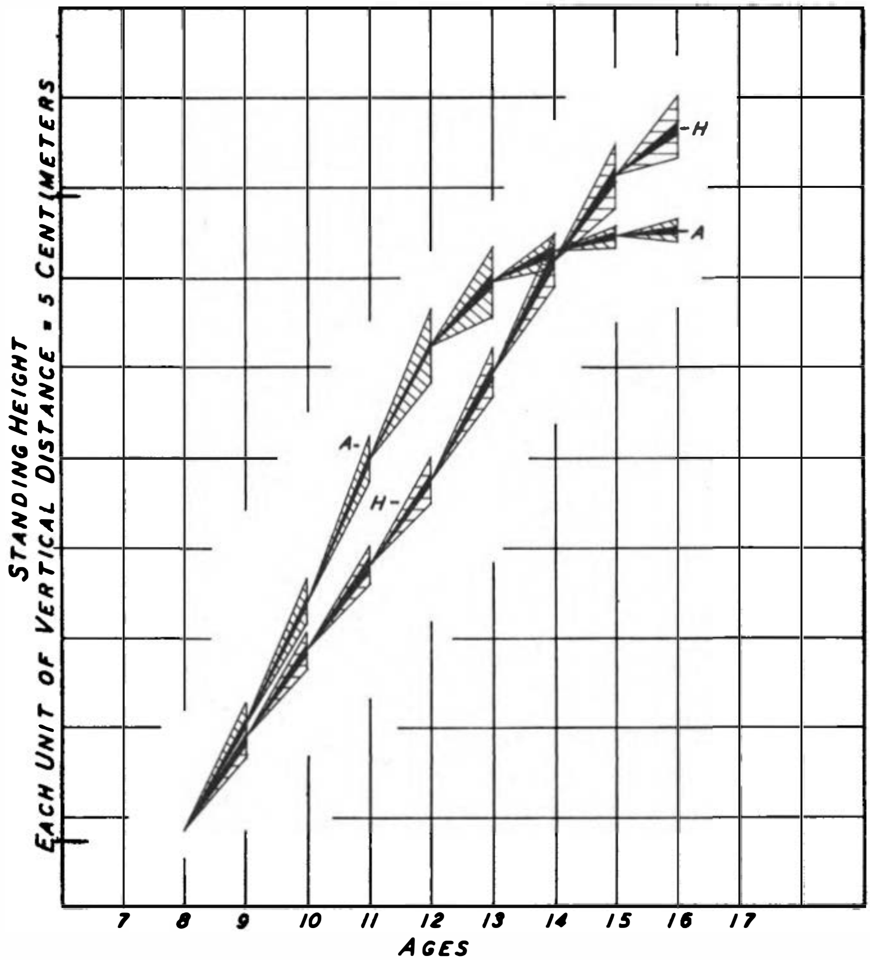

Fig. 27.—Slopes from Age to Age Generated by Average Standing Heights of Early and Late Maturing Groups of Cases Together With the Probable Errors of These Slopes and the Standard Deviations of Individual Slopes.

generated by a series of averages based on identical cases from age to age. Figure 27 presents a redrawing of Figure 25 for the purpose of clarifying this contrast. Here we are not interested in averages of gross size, but only in the position of each average relative to adjacent averages. Hence, the centimeter scale at the left has been omitted and, instead, it is indicated that each unit of vertical distance equals five centimeters. To further emphasize the unimportance of the average size the curves for both groups are started from the same arbitrary point at age 8.0. That is, an initial difference of more than five

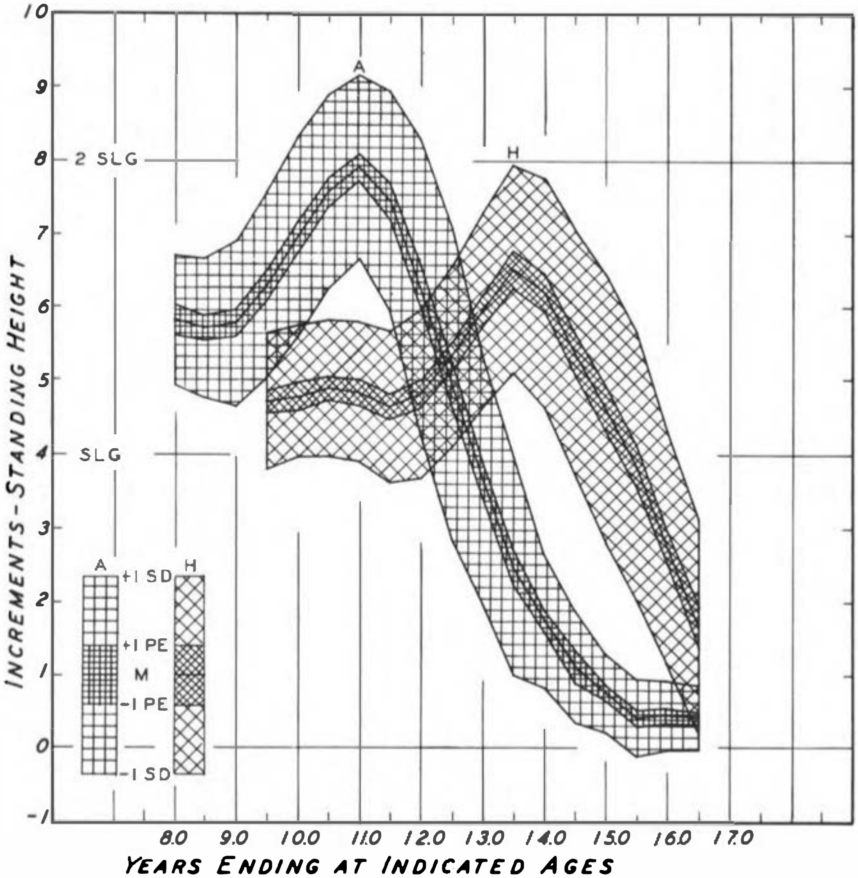

Fig. 28.--Average Annual Increments in Standing Height, Probable Errors of Means, and Standard Deviations of Distributions of Increments for Early and Late Maturing Groups of Cases.

centimeters at age 8.0 is being discarded or ignored. Moving from this point to the right for successive years the solid black wedge-like areas indicate the range of plus and minus one probable error of the slope of the curve or of the average increment while the cross-hatched wedge-like areas indicate the range of plus and minus one standard deviation of the distribution of the individual slopes or increments. It is apparent that the two curves have very different slopes. The slope for Group A from age 10.0 to 11.0 is much steeper giving an average increment of 7.94 $\pm$ .18 centimeters and approximately 68% of the individual curves give increments of from 6.67 to 9.17 centimeters; whereas, for Group H the much gentler slope gives an increment of only 4.84 $\pm$ .15 centimeters and approximately 68% of the individual curves give increments of from 3.92 to 5.78 centimeters. This chart should be compared with Figure 9 showing the range of plus and minus four probable errors of the slopes

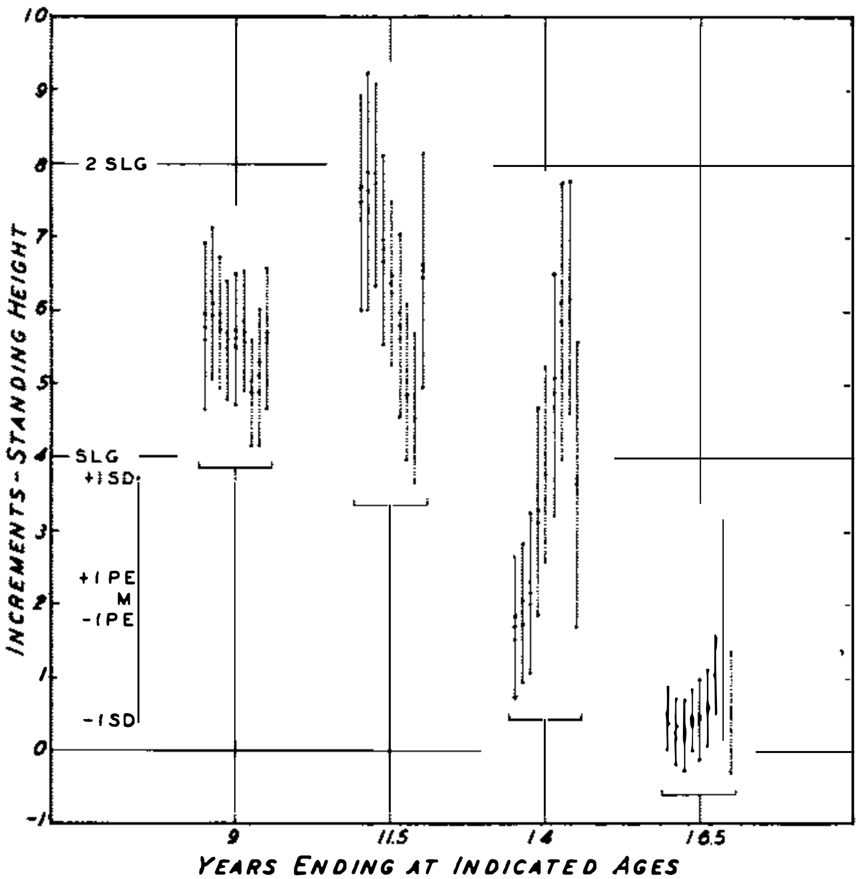

Fig. 29.—Average Annual Increments in Standing Height, Probable Errors of Means, and Standard Deviations of Distributions of Increments at Selected Ages for Each of Eight Groups of Cases and for the Total Population.

from age to age based on ten cases menstruating from age 13-0 to 13-3.

Magnifying the scale at the left of Figure 27 by four, expressing its data in terms of annual increments, and adding data for age 9.5 to 10.5, etc., gives Figure 28. In contrast with Figure 25 which shows some overlapping of the standard deviations of gross size for the two groups at all ages, Figure 28 shows no overlapping of the standard deviations of the annual increments from age 10.5 to 11.5 and from 13.5 to 16.0.

Whereas Figure 28 gives the variability in the increments of two groups at all ages, Figure 29 gives the variabilities in the increments for all groups and for the total population at selected ages. The standard deviations of the distribution of annual increments for the year ending at age 9.0 show almost complete overlapping. For the year ending at age 11.5, however, Groups A, B, and C are completely differentiated from Group H and almost completely differentiated from Group G. Similarly, for the year ending at 14.0 Groups A and B are completely differentiated from Groups F, G, and H. The comparatively small degree of overlapping of the standard deviations of distributions of increments shown by Figure 29 should be contrasted with the large degree of overlapping of comparable data for gross size shown by Figure 26.

3. Individual Growth Curves in Standing Height

For the purpose of this and the following section five sets of ten cases have been selected with menarcheal ages approximating as closely as possible ages 11.0, 12.0, 13.0, 14.0, and 15.0. That is, the five sets of cases were selected respectively from Group A, from Groups B and C, from Groups D and E, from Groups F and G, and from Group H. Preference was given to the selection of cases with a long series of measurements and with at least two measurements before the advent of the menarche. These specifications yield as nearly homogeneous groups as possible and insure the elimination of bias in their selection. However, five cases with doubtful menarcheal ages have been excluded from these selections.

Figure 30 presents the individual growth curves of ten cases menstruating approximately at 11.0 (10-7 to 11-3, dash curves) and of ten cases menstruating approximately at age 15.0 (14-5 to 15-2, continuous curves). The visual impression of the individual curves of this chart is in extreme contrast with that created by the group curves of Figure 11. Nevertheless, some semblance of order may be discerned. Although the ten individuals maturing approximately at age 11.0 average somewhat too short to be representative of early maturing cases, they are generally taller, especially at age 12.0. The clearest differentiation of the two groups is in the slopes of their curves. From age 9.0 to 12.0 individuals in the early maturing group show a decidely more rapid rate of growth than those of the late maturing group; whereas, from 12.0 to 16.0 this differentiation is reversed.

Figure 30 should be compared with Figure 1 which gives the individual growth curves of ten cases menstruating between 13-0 and 13-3. It should be noted, however, that all the cases of Figure 1 were measured within three months of birthdays and that their curves are plotted to even years; whereas, in Figure 30 cases were selected regardless of the timing of measurements relative to birthdays and their curves are plotted to exact ages. Figure 1 includes only cases measured over the same age range, while Figure 30 includes cases selected regardless of their age ranges. While Figure 1 is designed to clarify the consistency of individual growth patterns, Figure 30 presents a more realistic picture of the original raw data and tends to obscure the consistency of the individual patterns within each group.

Figure 30 presents three distinct phenomena: first, the extreme overlapping of the records in gross size at each age; second, the generally greater stature of the early maturing group; and third, the differential slopes of the two sets of curves. So many features in one chart, plus the idiosyncracies of each of the twenty individuals, tend to obscure any one feature and, more particularly, the first feature obscures the third. Figure 31 is designed to eliminate the first feature while retaining all the individual details of the third. The same twenty cases are involved. Each individual curve has been moved upward or downward to conform to the average stature of its group, but the details of each individual pattern are exactly reproduced. Figure 2 presents a similar redrawing of the individual curves of Figure 1. In

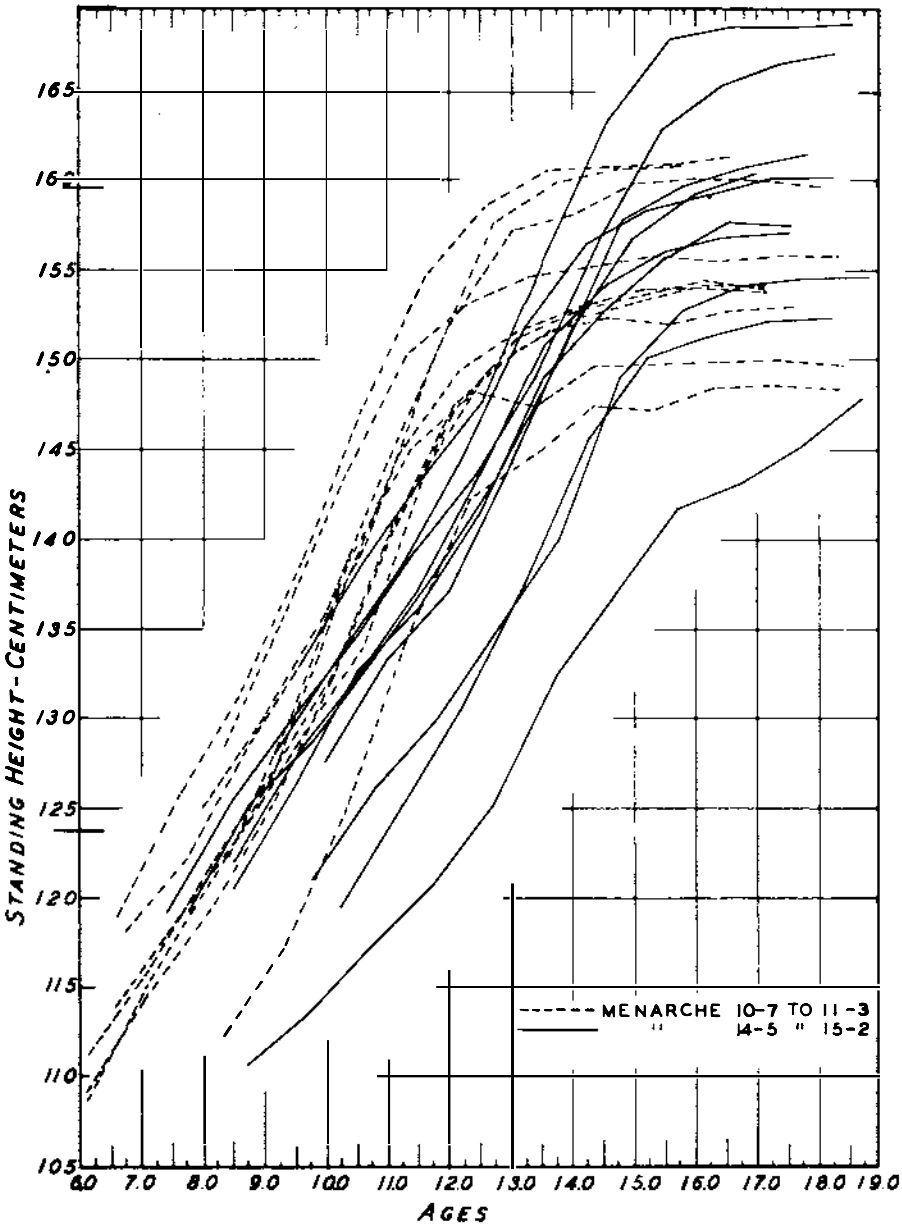

Fig. 30.--Individual Growth Curves in Standing Height Plotted to Exact Ages of Ten Cases Menstruating Between Age 10-7 and 11-3 (Dash Lines) and of Ten Cases Menstruating Between 14-5 and 15-2 (Continuous Lines).

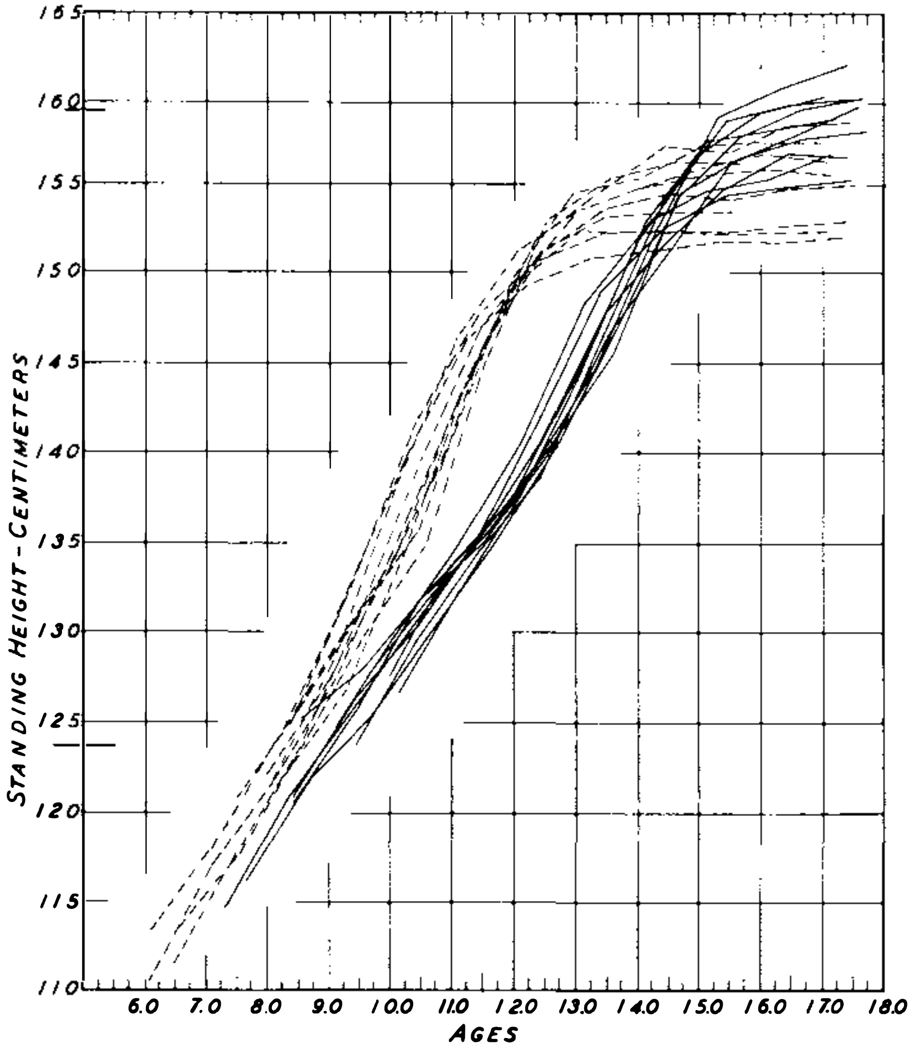

Fig. 31.--Individual Growth Curves of Figure 30 Redrawn to Eliminate Differences in Average Stature Within Each Group While Reproducing Exactly the Details of Each Individual's Pattern of Growth.

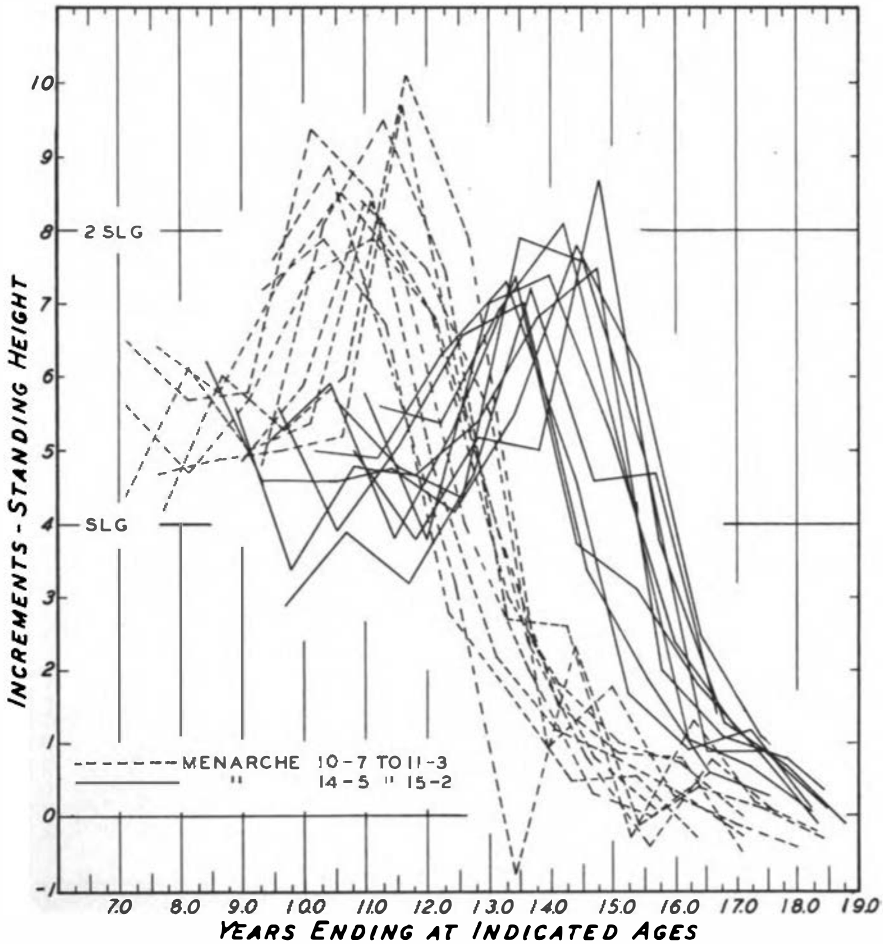

Fig. 32.--Individual Increment Curves in Standing Height Plotted to Exact Ages of Ten Cases Menstruating Between Age 10-7 and 11-3 (Dash Lines) and of Ten Cases Menstruating Between 14-5 and 15-2 (Solid Lines).

Figure 31 and Figure 2 the consistency of the individual slopes within each of the three groups and the markedly different slopes of the three groups are readily apparent. Since the early and late maturing cases of Figures 30 and 31 are selected from Groups A and H these charts should be compared with Figures 26 and 27.

4. Individual Increment Curves in Standing Height

Using the cases whose individual growth curves in gross size were presented in Figures 30 and 31 and charting the annual increments instead for each of the twenty individuals gives Figure 32. Figures 1, 2,

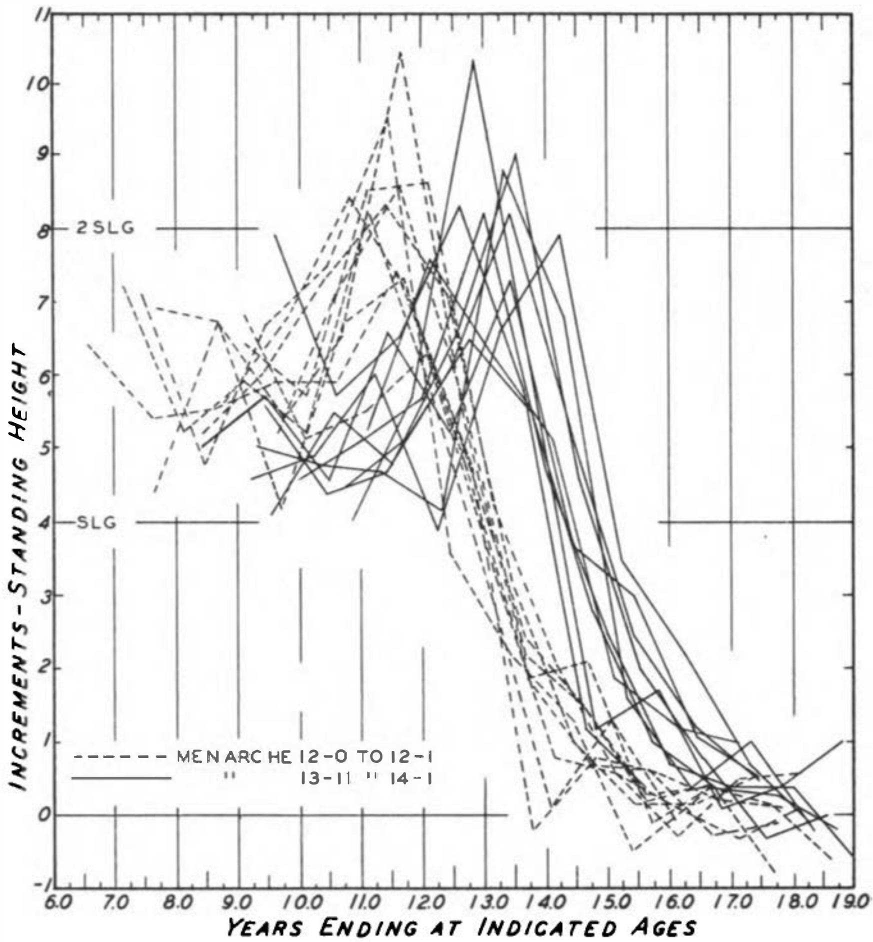

Fig. 33.--Individual Increment Curves in Standing Height Plotted to Exact Ages of Ten Cases Menstruating Between 12-0 and 12-1 (Dash Lines) and of Ten Cases Menstruating Between 13-11 and 14-1 (Solid Lines).

and 3 form a comparable series. Figure 33 presents similar individual increment curves for ten cases menstruating at approximately age 12.0 (12-0 to 12-1, dash curves) and at approximately age 14.0 (13-11 to 14-1, continuous curves). Figure 35 repeats the curves for the cases menstruating approximately at age 12.0, this time for comparison with ten cases menstruating at age 13.0 (all at 13-0, continuous curves).

Figures 32, 33, and 34 may be considered as a unit. In each of these charts the dash curves indicate the earlier maturing and the continuous curves the later maturing group of ten cases. Three types of comparisons need to be made. First, in each of the three charts the

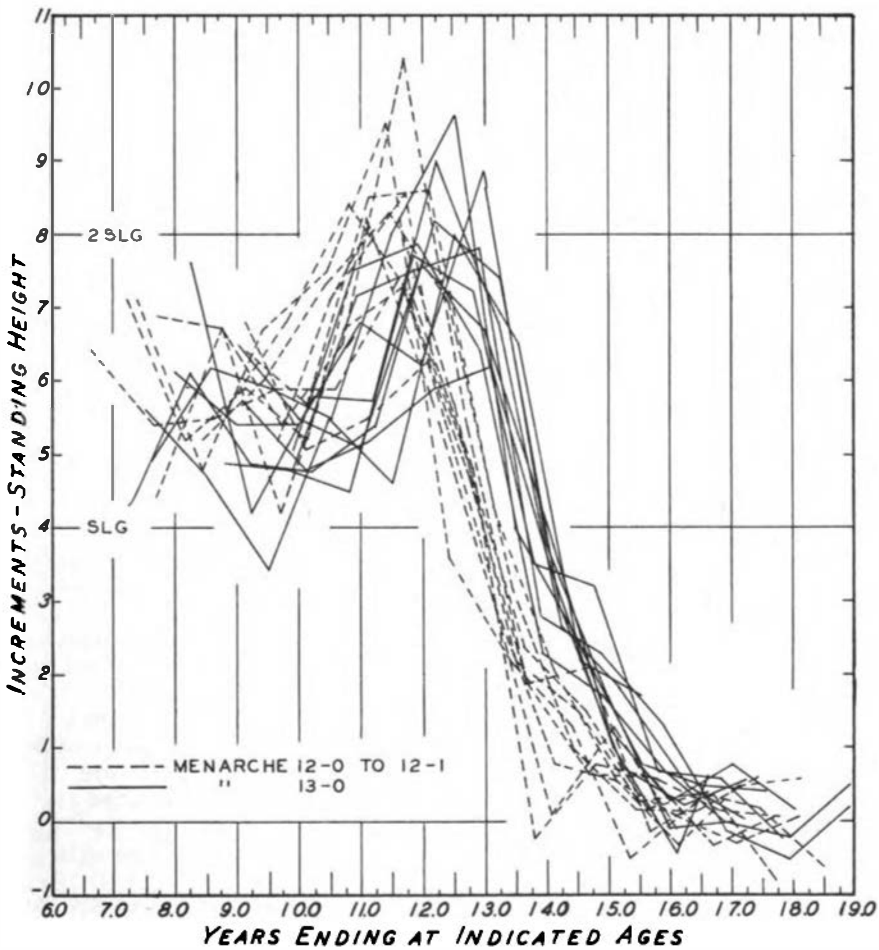

Fig. 34.--Individual Increment Curves in Standing Height Plotted to Exact Ages of Ten Cases Menstruating Between 12-0 and 12-1 (Dash Lines) and of Ten Cases Menstruating at Age 13-0 (Solid Lines).

earlier and later maturing cases should be contrasted in respect to the timing of various phases of the growth pattern and the magnitude of the increments with special attention to the extent to which the curves overlap or are completely differentiated. Second, comparisons should be made between the three charts. That is, the reader should compare the earlier maturing group of Figure 32 (menarche 10-7 to 11-3) with the later maturing group of Figure 34 (menarche 13-0), etc. Finally, the individual patterns of growth generated by the five sets of ten cases should be compared with the group patterns generated by averaging the increments of the eight menarcheal groups which are presented in Figure 13.

A more detailed statement of the similarities and contrasts of individual and group patterns of growth will be presented later. Here it is sufficient to note only the one most striking contrast, i.e., the comparative sharpness of the maximum increments of the individual curves. In part this contrast is due to the fact that measurements were taken at annual instead of at semi-annual or quarterly intervals. That is, if annual increments based on quarterly measurements were plotted for each individual, the resulting curves at the inflection points would be much less sharp. In part, also, the contrast is one of different emphases. In Figure 32, for example, the peak increments of ten cases selected from Group A average 8.87 centimeters for the year ending on the average at age 10.8. These figures emphasize the maximum increments and subordinate the exact ages. In Figure 13 the average increments of the twenty-two individuals in Group A show a maximum of 7.94 centimeters for the year ending at age 11.0. These figures subordinate the magnitude of the increments and emphasize the exact age. While the second emphasis is the conventional one, it is no more valid than the first.

5. Analysis by Menarcheal Age and Age of Maximum Growth

This section presents an analysis of the data for standing height by menarcheal age and by age of maximum growth in standing height. That is, each of the eight menarcheal groups have been divided into still smaller sub-groups according to the age period at which maximum growth occurred. For the purpose of this analysis only cases whose maximum increment in standing height is clearly defined have been included. Excluded cases are as follows: eight cases with unusual or abnormal patterns of growth in height, nine cases whose records cover too limited a range of years to clearly define the maximum increment, seventeen cases where the maximum increment is determined by an interpolated figure, and forty cases where the maximum increment in height does not exceed adjacent increments by half a centimeter. However, cases with doubtful menarcheal ages have been included.

Data from Group E, cases who menstruated between 13-0 and 13-5, which are presented in Figure 35, will serve to illustrate the nature of the raw data resulting from this method of analysis. When the 41 cases in this group are divided according to the half-yearly interval in which the maximum increment falls, seven sub-groups result. The maximum increment of one case is for the year ending at age 11.0 (mid-point of the interval 10.75-11.24), of three cases for the year ending at age 11.5, of nine cases for the year ending at age 12.0, etc., as indicated by the numbers just above each peak increment. It is to be noted particularly that these group patterns present the same sharp peaks as the individual patterns of Figures 32, 33 and 34. Now, these raw data may be treated in two ways. First, exact ages may be emphasized and the points on the seven curves may be averaged regardless of their magnitude. For example, for the year ending at age 11.0 the points on the curves are 9.10, 7.47, 5.48, 5.34, 5.28, 5.07, and 4.60 centimeters. The weighted average of these is 5.57 centimeters. Proceeding in this fashion at each age gives the broken curve of Figure 36. This, in a round about laborious way, is almost exactly the procedure employed in Chapter IV. To emphasize this point the dot-dash curve of Figure 36 reproduces from Figure 13 the trends for all 59 cases of Group E. Although the dash-dash curve is based on only 41 cases while the dot-dash

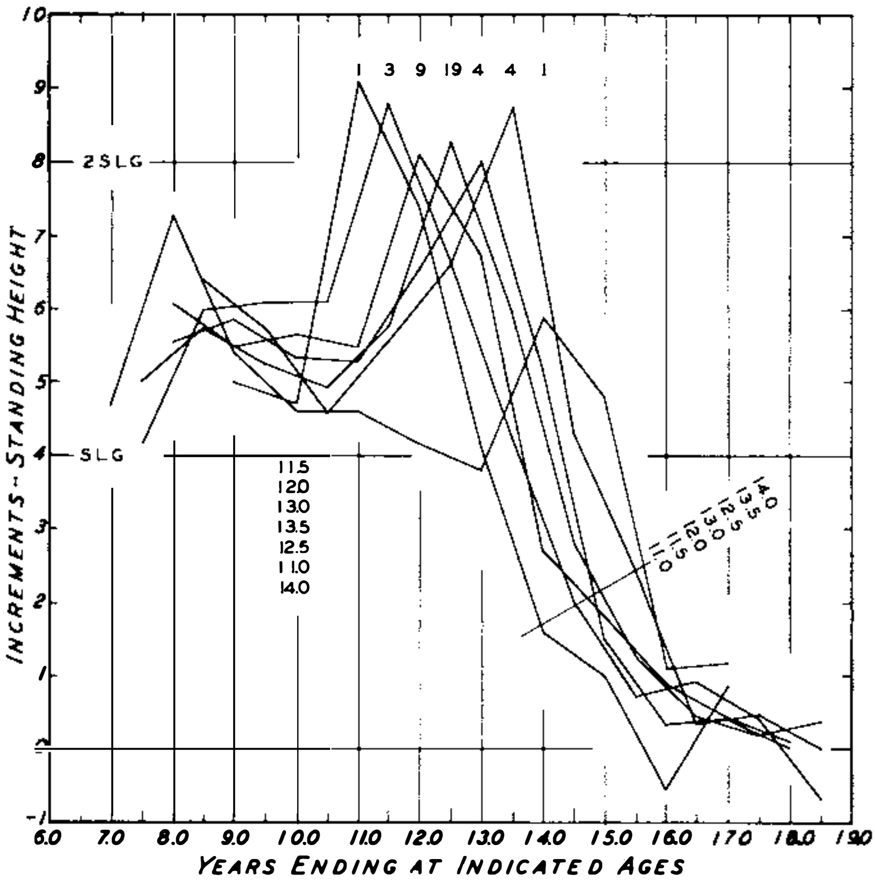

Fig. 35.--Annual Increments in Standing Height of 41 Cases Menstruating Between 13-0 and 13-5 when Sub-divided According to Age of Maximum Growth.

curve is based on 59 cases, the trends are almost identical. Second, corresponding points on the patterns of the seven curves of Figure 35 may be emphasized and these corresponding points may be averaged regardless of exact ages. For example, the successive peak increments are 9.10, 8.80, 8.11, 8.30, 8.02, 8.75, and 5.90 centimeters and the weighted average of these is 8.27 centimeters. Similarly, the weighted average of corresponding points one, two, three, etc., years before and after the peak increment may be computed. Finally, in order to place the resulting data on a time scale the ages at the close of the year of maximum growth for each of the 41 cases may be averaged. This average is 12.48. The resulting trend is represented by the continuous curve of Figure 36.

Repeating this process for each of the eight menarcheal groups gives Figure 37. Here we have eight generalized group patterns each of which

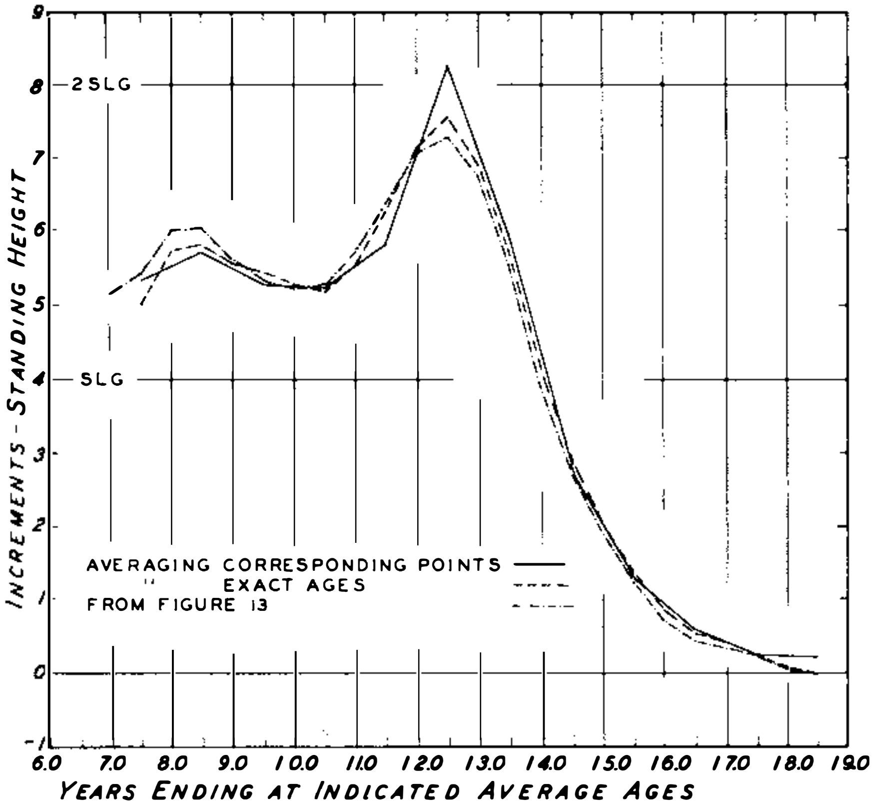

Fig. 36.--Annual Increments in Standing Height of Cases Menstruating Between 13-0 and 13-5; Illustrating Differences from two Methods of Combining Individual Increments.

is a very excellent representation of the typical pattern of individual growth. Just as Figure 20 is the central chart of Chapter IV, so Figure 37 is the most important chart of this chapter. The different methods employed in obtaining the basic data of these charts, as explained in connection with Figures 35 and 36, must be understood. In Figure 13 each curve represents an emphasis on averaging increments at identical ages regardless of the magnitude of these increments or of their position on the pattern of growth. This emphasis plus the availability of observation points at intervals of six months tend to flatten and round the peak increments. In Figure 37 each curve represents an emphasis on averaging increments at corresponding points on the pattern regardless of chronological ages. This emphasis plus the fact that observation points are available only at intervals of a year tend to heighten and sharpen the peak increments. Figure 13 is a true picture of the growth patterns of the eight menarcheal groups from age to age, but it does not provide good models of the individual patterns. The individual patterns as illustrated in Figures 32, 33, and 34 show generally larger and

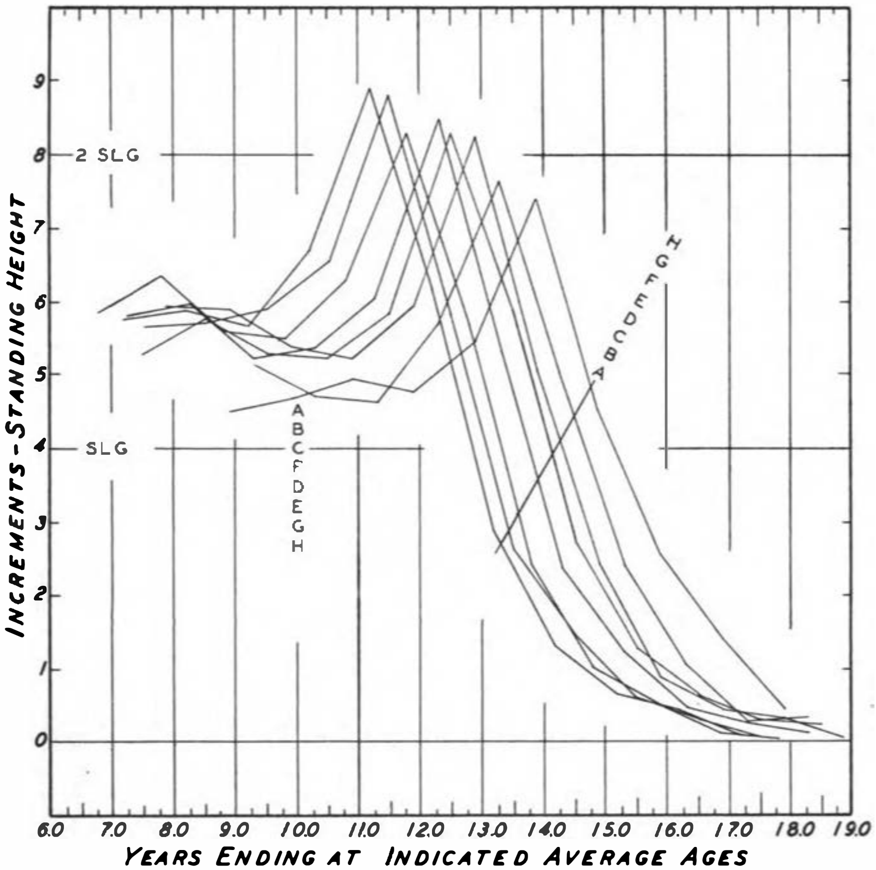

Fig. 37.--Annual Increments in Standing Height of Eight Groups of Cases Menstruating at Different Ages; Individual Increment Curves of Each Group Combined to Emphasize the Averaging of Corresponding Points of the Pattern.

sharper peak increments than the group patterns of Figure 13. Figure 37 is also a true picture of the growth patterns of the eight menarcheal groups and provides very excellent representations of the typical individual pattern, but it tends to blur the timing of the pattern in relation to chronological age. Further, if measurements on each individual were available at quarterly intervals, the peak increments of Figure 37 would be very slightly higher and much more rounded.

The data of Figure 37 may be analyzed just as the data of Figure 13 were analyzed. Thus, in Figure 38 the curves are arranged with the maximum increments in the same vertical line. This chart should be compared with the corresponding arrangement in Figure 19. Figure 39 is designed to facilitate this comparison. Here, the broken curve is the weighted average of the curves of Figure 19 and the continuous curve is

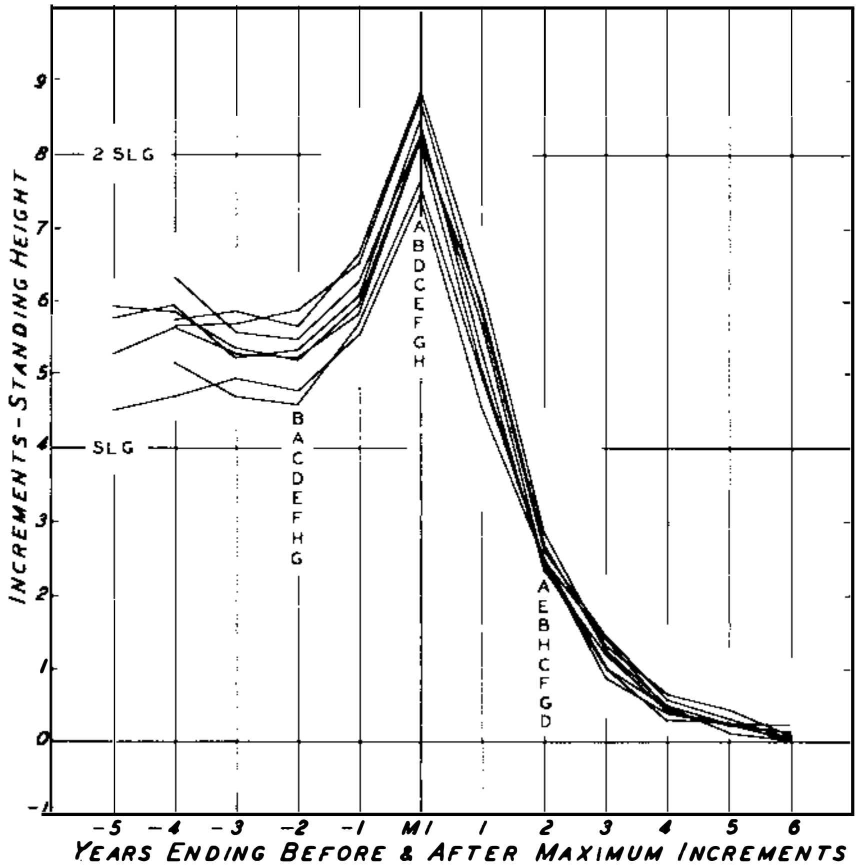

Fig. 38.--Annual Increments in Standing Height of Eight Groups of Cases Menstruating at Different Ages Redrawn from Figure 37 and Arranged so that Maximum Increments are in the Same Vertical Line.

the weighted average of the curves of Figure 38. The inset curve in Figure 39 is an attempt to represent the pattern of annual increments based on quarterly measurements and analysis according to menarcheal age and age of maximum growth. Continuing the analysis of the data of Figure 37 the curves may be arranged so that points corresponding to the advent of the menarche are in the same vertical line. Figure 40 presents this arrangement which should be compared with the corresponding arrangement in Figure 20.

We are now ready for a detailed reading of Figure 37. To facilitate comparison with the reading of Figure 13 the various features are catalogued in the same order and lettered to correspond, but only distinguishing features will be discussed.

(a). Note the regularity of annual increments for the years ending at ages 11.0 and 14.0.

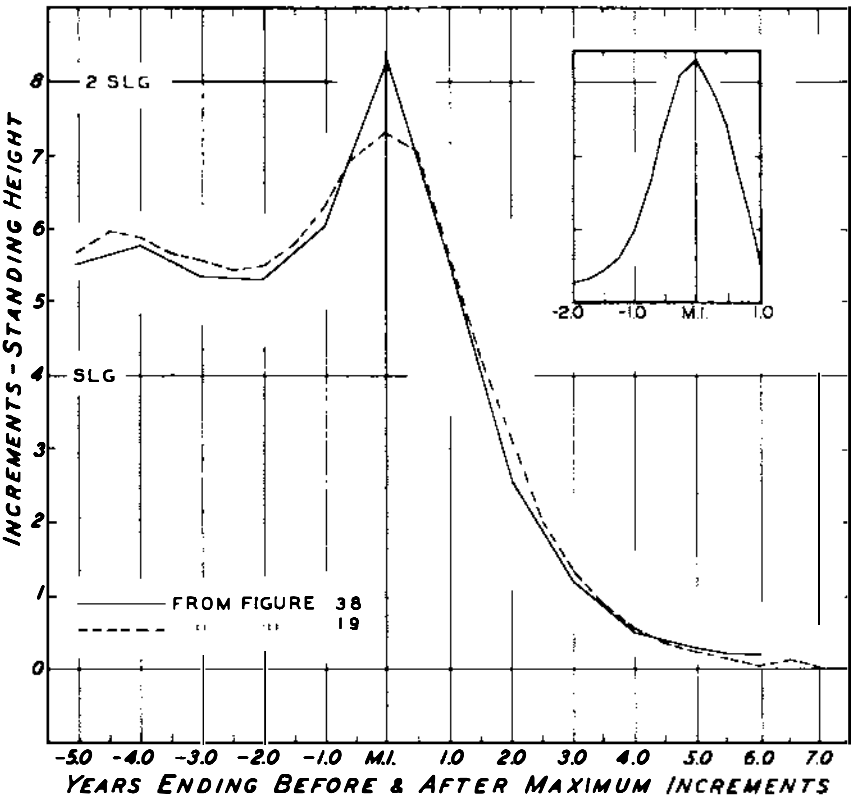

Fig. 39.--The Pattern of Annual Increments in Standing Height Before and After the Maximum Increment; Weighted Averages of the Eight Curves of Figures 19 and 38.

(b). The maximum increments are in regular order. Average ages at the close of the year of maximum growth are 11.21, 11.47, 11.75, 12.31, 12.48, 12.90, 13.33, and 13.89 for Groups A, B, C, D, E, F, G, and H respectively. These points are much more reliably determined than the corresponding points of Figure 13.

(c). Note declining trend of peak increments.

(d). Three phases of growth may be distinguished in all eight curves. These are clearly illustrated in Figure 38 and summarized in Figure 39 for comparison with the similar phases shown by the curves of Figures 13 and 19. In Figure 37 the three phases of growth for each of the eight groups are much more clearly defined than in Figure 13.

(e). The magnitude of the increments during the first and second phase decrease in order from the early to the late maturing groups. See especially Figure 38.

(f). The degree of acceleration is similar in all eight groups. For the year ending two years before the maximum increment the average

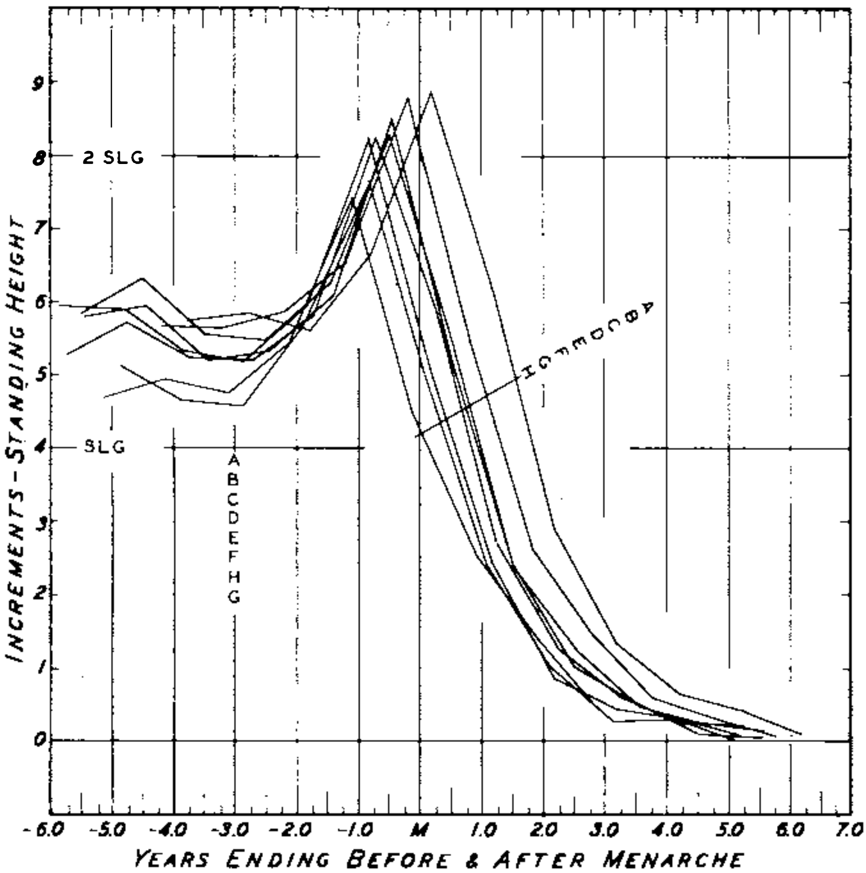

Fig. 40.--Annual Increments in Standing Height of Eight Groups of Cases Menstruating at Different Ages; Curves Arranged so that Points Corresponding to the Advent of the Menarche Age are in the Same Vertical Line.

growth is 5.29 centimeters while the average of the peak increments is 8.30 centimeters giving an average acceleration of 3.01 centimeters. The acceleration in five of the eight groups is almost identical ranging from 2.90 to 3.12 centimeters. The extreme accelerations range from 2.63 centimeters for Group H and 2.79 centimeters for Group C to 3.25 centimeters for Group A. These accelerations are much larger than those of Figure 31 and represent a truer picture of the typical individual acceleration.

(g). The maximum increments are intimately timed with the advent of the menarche. In Group A the close of the year of maximum growth occurs on the average .21 years after the menarche. In Groups B, C, D, E, F, G, and H the close of the year of maximum growth comes .22, .50, .44, .77, .85, .92, and 1.11 years before the advent of the menarche. This

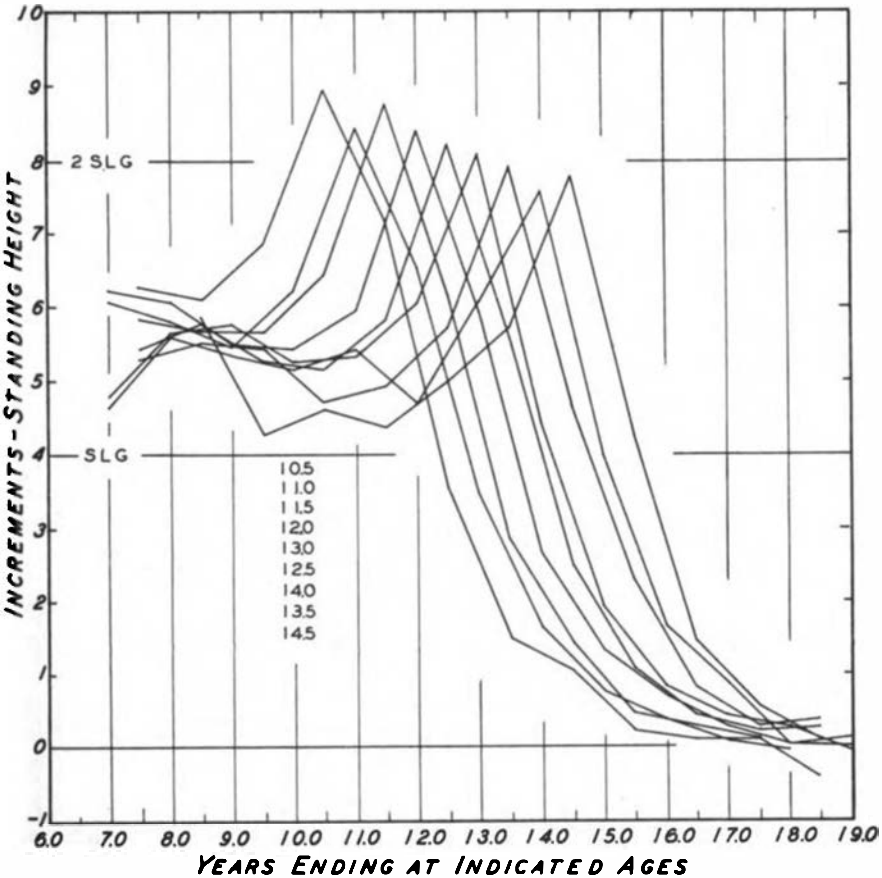

Fig. 41.--Annual Increments in Standing Height of Nine Groups of Cases with Different Periods of Maximum Growth.

progression is more regular than in the curves of Figure 13. See Figure 40.

(h). Similarly, the decided acceleration of growth in Groups A, B, C, D, E, F, G, and H begins 2.79, 3.22, 3.50, 3.44, 3.77, 3.85, 3.92, and 4.11 years before the advent of the menarche. See Figure 40.

(i). The larger increments of the early maturing groups during the first and second phases of growth are directly related to the fact (see Figure 12) that the early maturing groups are shorter at given time intervals before the menarche.

6. Analysis by Age of Maximum Growth

A by-product of the foregoing treatment of the data is an analysis of trends in terms of age of maximum growth alone. When the 174 cases are classified according to this one factor, nine groups result. The

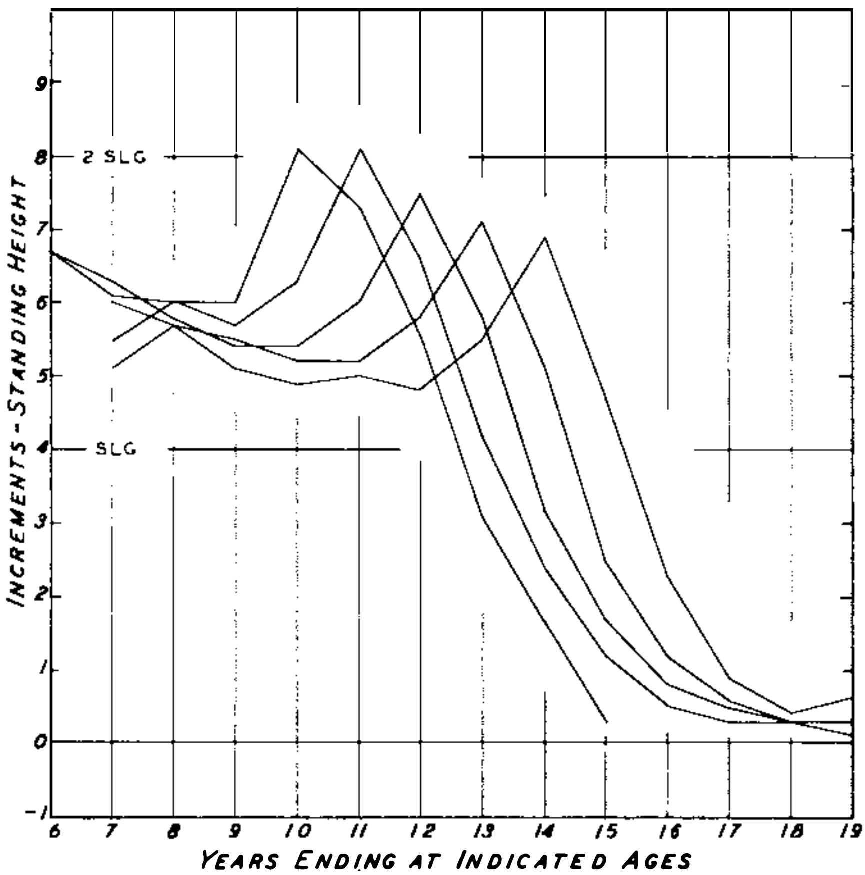

Fig. 42.—Annual Increments in Standing Height of Five Groups of Cases with Different Periods of Maximum Growth; Horace Mann Non-Hebrew Girls, Data of Boas.

maximum increment of six cases is for the year ending at age 10.5 (mid-point of 10.25 to 10.74). Similarly, the maximum increments of 13, 26, 34, 37, 28, 18, 7, and 5 cases are for the years ending respectively at ages 11.0, 11.5, 12.0, 12.5, 13.0, 13.5, and 14.5. These data are presented in Figure 41. The curves of this figure are strikingly similar to those of Figure 37, but the nine curves of Figure 41 necessarily show greater differentiation because no complicating regression effect of menarcheal age classification is involved.

Figure 42 presents comparative curves for non-Hebrew Horace Mann girls based on Table 15 of Boas (5). It has already been noted that Boas interpolated all measurements, probably by straight-line methods, to exact birthdays. Hence, his classification by age of maximum growth yields five groups with maximm increments for the years ending at ages 10.0, 11.0, 12.0, 13.0, and 14.0. That is, comparisons of Figures 41

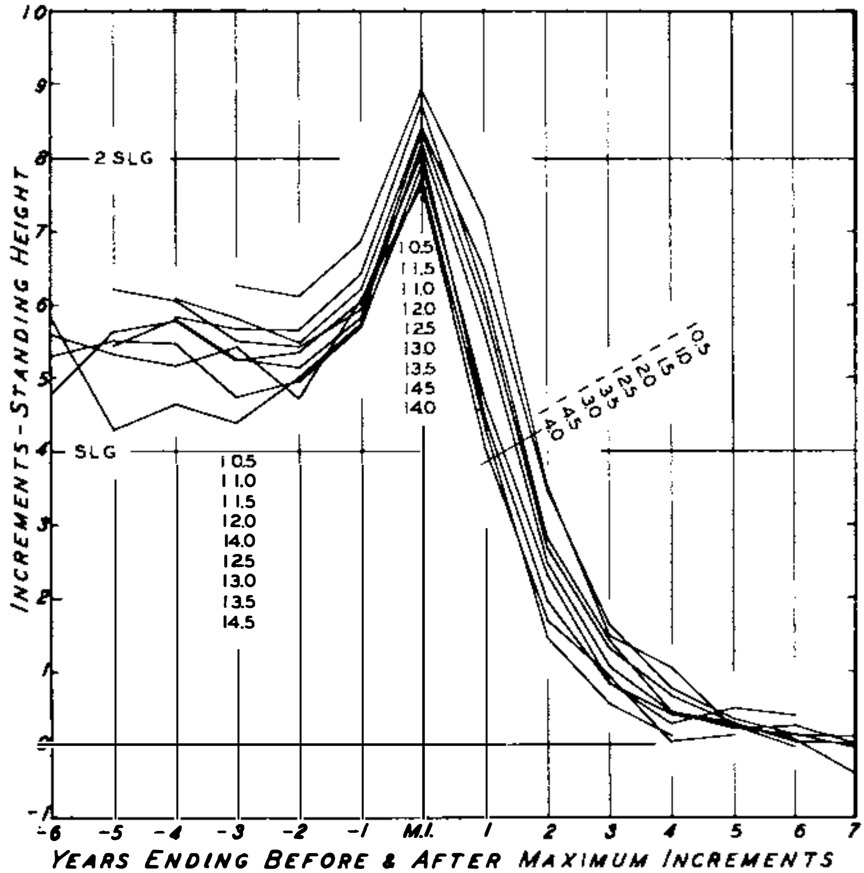

Fig. 43.--Annual Increments in Standing Height of Nine Groups of Cases with Different Periods of Maximum Growth; Redrawn From Figure 41 and Arranged so that Maximum Increments are in the Same Vertical Line.

and 42 should concentrate on the four groups with maximum increments for the years ending at ages 11.0, 12.0, 13.0, and 14.0. Keeping these differences in mind, the two charts agree very closely except that the degree of acceleration and the magnitude of the peak increments are decidedly greater in Figure 41. The flatter curves of Boas are doubtless due to the tendency of interpolation procedures to smooth out the sharp changes in the pattern of individual growth[1].

In Figure 43 the curves of Figure 41 are rearranged so that the maximum increments are in the same vertical line. A feature of this

[1]The critical student should also note that Figure 42 does not resemble Figure 3 of Boas although both curves are based on the same data. Figure 3 of Boas is incorrectly drawn.

chart in contrast to Figures 19 and 38 is the larger increments of the groups with early maximum increments throughout the whole range of ages. Even four years after the maximum increment the nine groups are in almost regular order. The explanation of the generally larger increments of the groups with early periods of maximum growth goes back once more to Figure 12. This point is so important that it needs the reemphasis provided by Figure 44. Only five groups with maximum increments for the years ending at ages 10.5, 11.5, 12.5, 13.5, and 14.5 are charted and these are arranged with the period of maximum growth in the same vertical line. Chronological age 11.0 and average menarcheal ages are also spotted. The close correspondence between the period of maximum growth and the average age at which the menarche appears should be noted. In all five groups the decided acceleration of growth begins two years before the close of the period of maximum growth. If it is assumed that this acceleration of growth is due to endocrine stimulation, then the individuals whose maximum growth period is from 9.5 to 10.5 are stimulated at age 8.5 when they average only 130.18 centimeters tall; whereas, the individuals whose maximum growth period is from 13.5 to 14.5 are stimulated at age 12.5 when they average 140.92 centimeters tall. The early stimulation of short statures is compensated by much more rapid growth so that all groups attain much the same mature stature.

Figure 45 completes the analysis of the curves of Figure 41. Here, the five groups with maximum increments for the years ending at 10.5, 11.5, 12.5, 13.5, and 14.5 are arranged so that points corresponding to the advent of the menarche are in the same vertical line. The contrast between this chart and Figure 40, which gives comparable data for the eight menarcheal groups, illustrates the regression effect of one variable on another. The average menarcheal age of the 174 cases is 12.94 and the average age at the close of the period of maximum growth is 12.39. Standard deviations are 1.10 and .95 years for the menarcheal ages and maximum increment ages. These differences together with the regression effect are responsible for the apparent reversal of the curves in Figure 40 and 45. When the 174 cases are classified according to menarcheal age as in Figure 40, the maximum increments of Groups A and H come on the average .21 years after and 1.10 years before the advent of the menarche. When the same cases are classified according to maximum growth periods as in Figure 45, maximum increments of groups for the years ending at 10.5 and 14.5 come on the average .77 and .15 years before the menarche. A much clearer representation of these relationships is presented in Figure 112 of Chapter XII in connection with a correlational analysis of the data.

7. Summary of Chapters IV and V

The data of Chapters IV and V constitute a rather complete verification of the original hypotheses that the menarche is associated with (a) physical size and (b) the timing of changes in the differential patterns of physical growth. During the early teens there is a fairly close association between physical size and the menarche, the earlier maturing groups being taller than the later maturing groups (Figure 11). The generalized pattern of growth generated by the annual increments shows three clearly defined phases (Figure 39): (a) a slight decelerating phase lasting about two years (b) a sharply accelerating phase

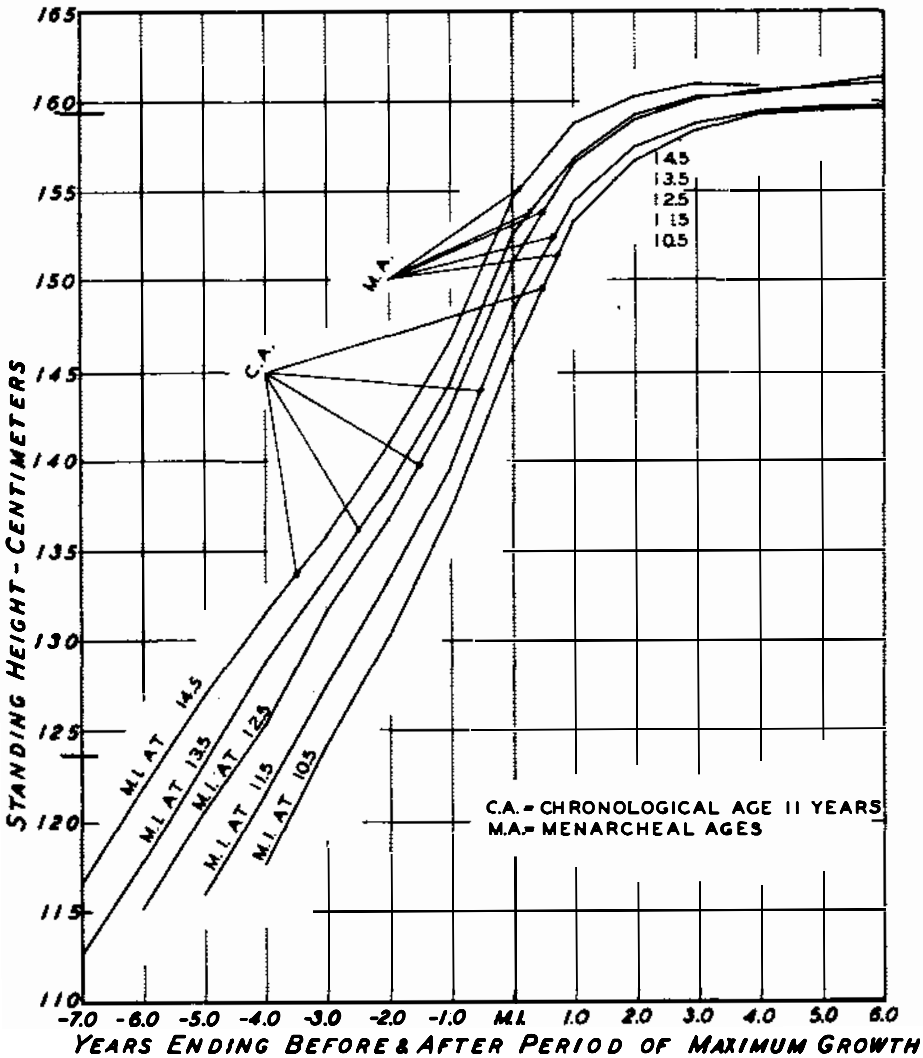

Fig. 44.--Growth Trends in Average Standing Height of Five Groups of Cases With Maximum Periods of Growth for the Years Ending at Age 10.5, 11.5, 12.5, 13.5, and 14.5 Plotted so that Periods of Maximum Growth are in the Same Vertical Line.

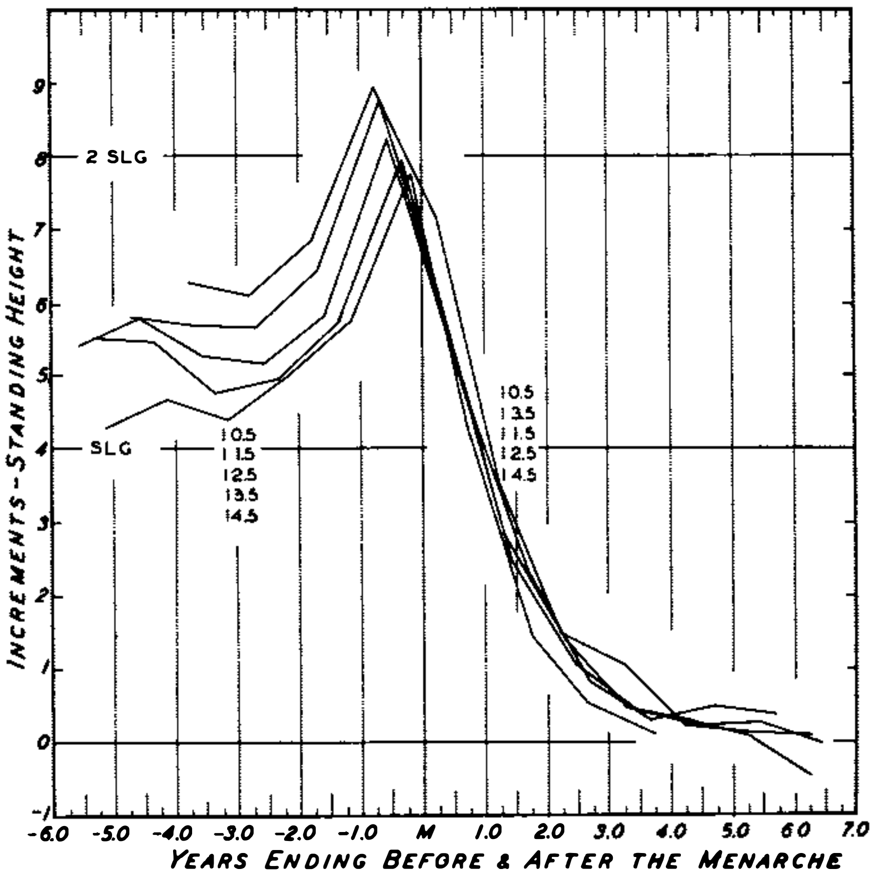

Fig. 45.--Annual Increments in Standing Height of Five Groups of Cases with Different Periods of Maximum Growth Arranged so that Points Corresponding to the Average Menarcheal Ages are in the Same Vertical Line.

lasting about two years and (c) a very sharply decelerating phase lasting about four or five years. The three phases of growth occur at very different chronological ages when the eight menarcheal groups are considered (Figures 13, 37, and 41), but are strikingly similar when corresponding points on the pattern are super-imposed (Figures 19, 38 and 43). The inflection points or transition periods between three phases are intimately related to the advent of the menarche (Figures 12, 13, 20, 21, 22, 23, 32, 33, 34, 37, 40, 41, 44, and 45). Evidence to be presented in Chapter XII shows that the age period during which maximum growth in standing height occurs is more closely associated with the menarche than other aspects of physical growth. The data are in entire accord with the suggestion that changing patterns of physical growth during the adolescent period are under the control of endocrine factors.

They are also in accord with the more specific suggestions that the process of sexual maturation and of accelerated physical growth are initiated at the same time presumably by the pituitary and that the endocrine mechanisms which bring the period of sexual maturation to a close in the advent of the menarche also operate to bring the period of accelerated growth to a close at approximately the same time.

The interrelations of assumed endocrine stimulation and observed patterns of physical growth, however, are not rigidly mechanical and uniform for all eight menarcheal groups. The initiation of accelerated physical growth comes when the early maturing individuals are comparatively young, comparatively short, and far from their mature stature whereas the late maturing individuals are older, taller, and nearer their mature stature (Figures 12 and 44). Under these conditions rigidly uniform patterns would result in great differences in mature stature between the groups. Instead, the patterns may be described as strikingly similar yet significantly and progressively different from group to group (Figures 19, 38, and 43). The difference consists in the fact that during the period of most intense growth the early maturing individuals grow more rapidly than the late maturing individuals. That is, the early maturing individuals anticipate and compensate by more intense growth for their comparative shortness at the initiation of accelerated growth or of assumed pituitary stimulation, while the late maturing individuals anticipate and compensate by less intense growth for their comparative tallness at the initiation of accelerated growth. This compensatory mechanism appears to be the working compromise of two forces: one in the endocrine system determining the timing of accelerated growth and one in the hereditary endowment determining mature stature. The idea of a compensatory or compromise mechanism would seem to be important because the underlying endocrine forces must work themselves out not only in different ways for early and late maturing groups but in different ways for each measurement.

Some obvious cautions must also be recorded. First, the findings which show an intimate timing of physical growth patterns and the advent of the menarche stand on one level, while the interpretations which are in terms of endocrine factors stand on another. The future may greatly modify interpretations without altering the findings. Second, assuming that the endocrine interpretation is sound, it is not to be inferred that these factors are the sole causal agents in determining physical growth. Endocrine factors, or factors associated with the menarche, determine very largely the early or late appearance of the characteristically changing patterns of growth, but they are negligibly associated with mature stature. Further, there is a significant association between menarcheal ages and stature prior to the assumed initiation of endocrine stimulation. This would suggest the operation of other antecedent factors possibly in the hereditary endowment of the individual.

CHAPTER VI

SITTING HEIGHT AND LEG LENGTH

The very complete discussion of the data for standing height makes it possible in this chapter to greatly condense the discussion of the data for sternal height, sitting height, and leg length (standing height minus sitting height). The measurements might be analyzed to give trends for trunk length (sternal height minus sitting height) and for sternal-vertex height (standing height minus sternal height) but no data for these dimensions and very little for sternal height will be presented. Similarly, the data might be employed to give a large number of ratios, but only two such ratios will be considered. Since the distinguishing patterns of growth in the eight menarcheal groups have been so thoroughly explored for standing height, the emphasis of the chapter shifts to the differences in the patterns of growth in sitting height and leg length. Indeed, succeeding chapters will devote less and less attention to the eight menarcheal groups and more and more attention to the differences between measurements. Since the elucidation of these differences presupposes comparable graphical representations, some space must be devoted to this problem. Alternative graphical methods will be illustrated incidentally to the material of Chapter XIII which attempts to strike a balance between the contrasts of early versus late maturing groups and the contrasts of each measurement versus every other.

1. Growth Trends in Average Size

Figure 46 presents growth trends in average sternal height for each of eight groups of girls menstruating at different ages. Comparable data for sitting height and for leg length are presented in Figures 47 and 48. Comparable data for standing height were presented in Figure 11. All four charts are drawn so that growth from age eight to seventeen, as determined by averaging the measurements of all available cases at these ages, is represented by the same vertical distance. Average sternal heights at age eight and seventeen, for example, are indicated in Figure 46 by the two horizontal lines across the vertical scale at 98.6 and 130.3 centimeters. Similar points for sitting height are at 67.3 and 84.9 centimeters and for leg length at 56.3 and 74.6 centimeters. That is, a tracing on transparent paper of any one of these charts can be superimposed on any other by superimposing the age scales at the bottom and the points on the vertical scale corresponding to average size at ages eight and seventeen. Figure 49 presents a partial superimposing of Figures 47 and 48 for early and late maturing groups of girls. In this chart the centimeter scales at the left have been translated into percentages of the average growth between age eight and seventeen. Since the total gain between age eight and seventeen is 17.6 centimeters in the case of sitting height and 18.3 centimeters in the case of leg length, the units of vertical distance representing a given number of centimeters are not very different. It is important that superimposed curves as in Figure 49 should not be misinterpreted. Curves showing the sitting height of early and late maturing groups are

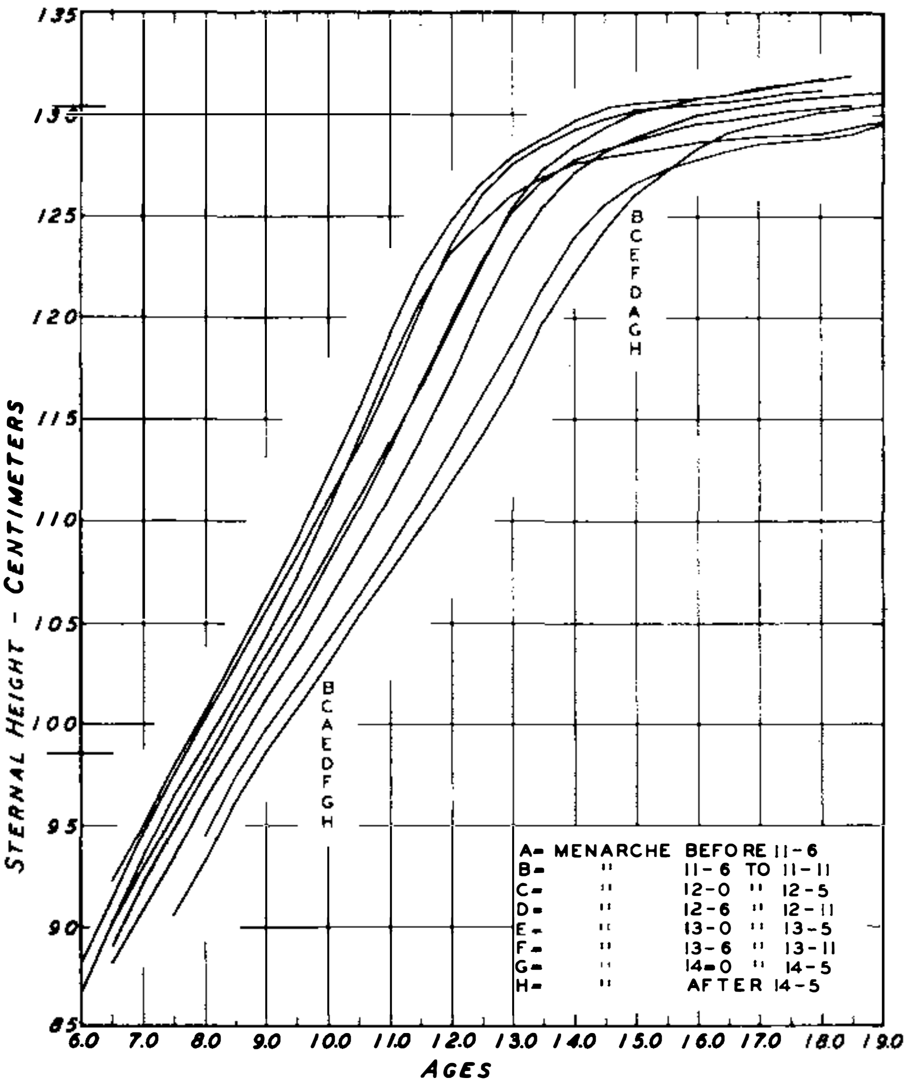

Fig. 46.--Growth Trends in Average Sternal Height of Eight Groups of Cases Menstruating at Different Ages.

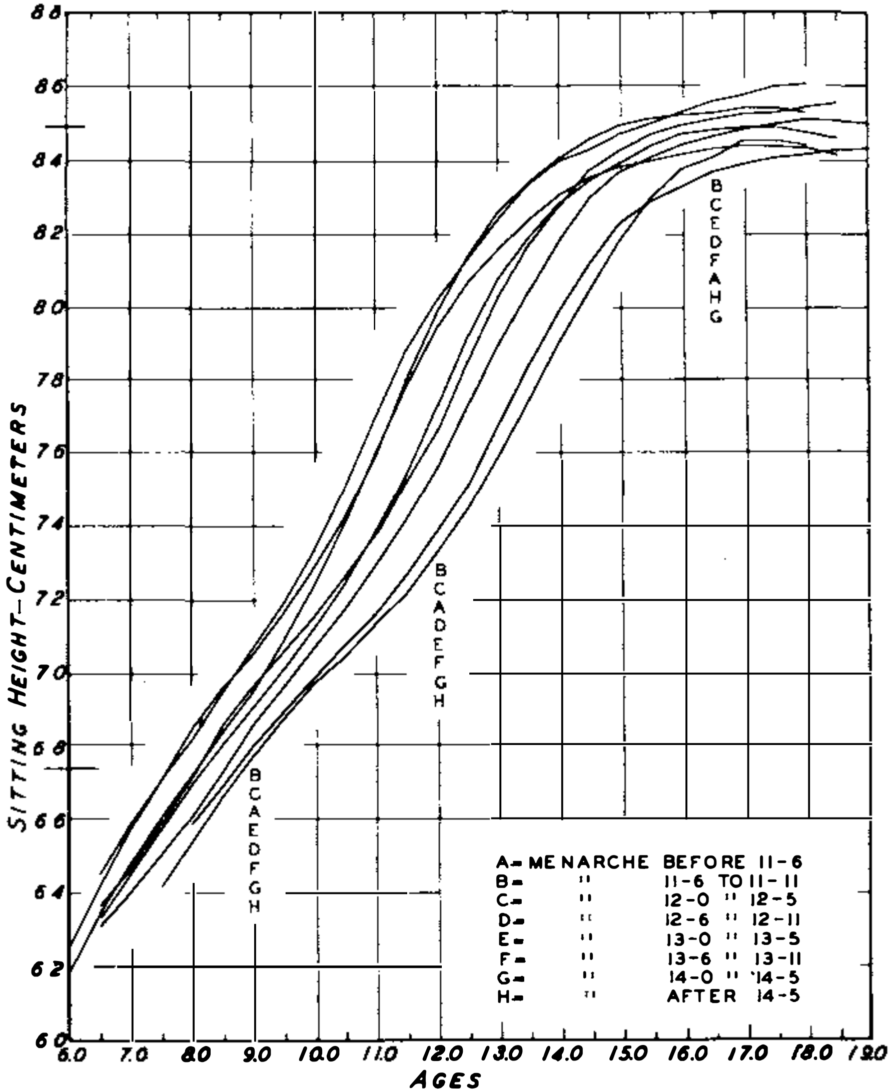

Fig. 47.--Growth Trends in Average Sitting Height of Eight Groups of Cases Menstruating at Different Ages.

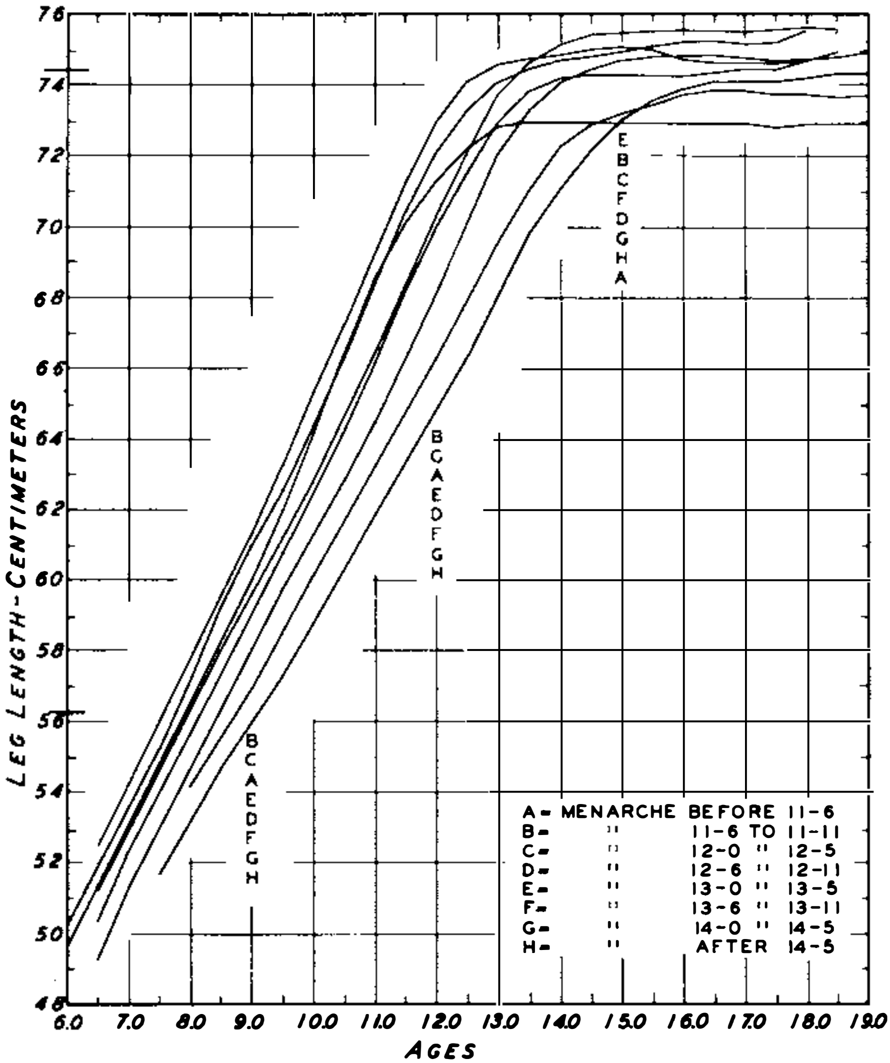

Fig. 48.--Growth Trends in Average Leg Length of Eight Groups of Cases Menstruating at Different Ages.

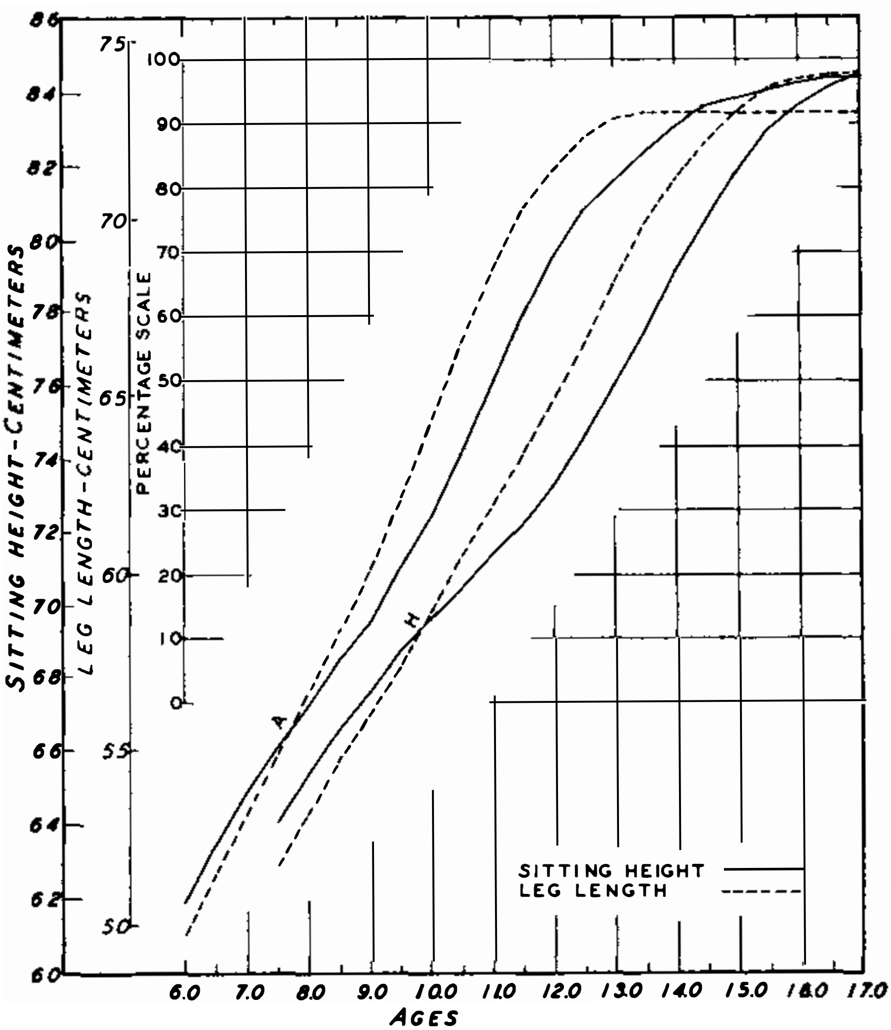

Fig. 49.—Growth Trends in Average Sitting Height and Leg Length of Early and Late Maturing Groups of Girls.

directly comparable both as to gross size and slopes and similarly for leg length curves, but curves for sitting height and leg length are comparable only in respect to their slopes. In order to make this point unmistakably clear Figure 50 reproduces the data of Figure 49 on the same arithmetic scale. Such a chart has the advantage of showing that sitting height is a much larger dimension than leg length, but it has the disadvantage that curves for the same cases are so far apart as to make comparisons difficult.

Since a very complete discussion of comparable data for standing

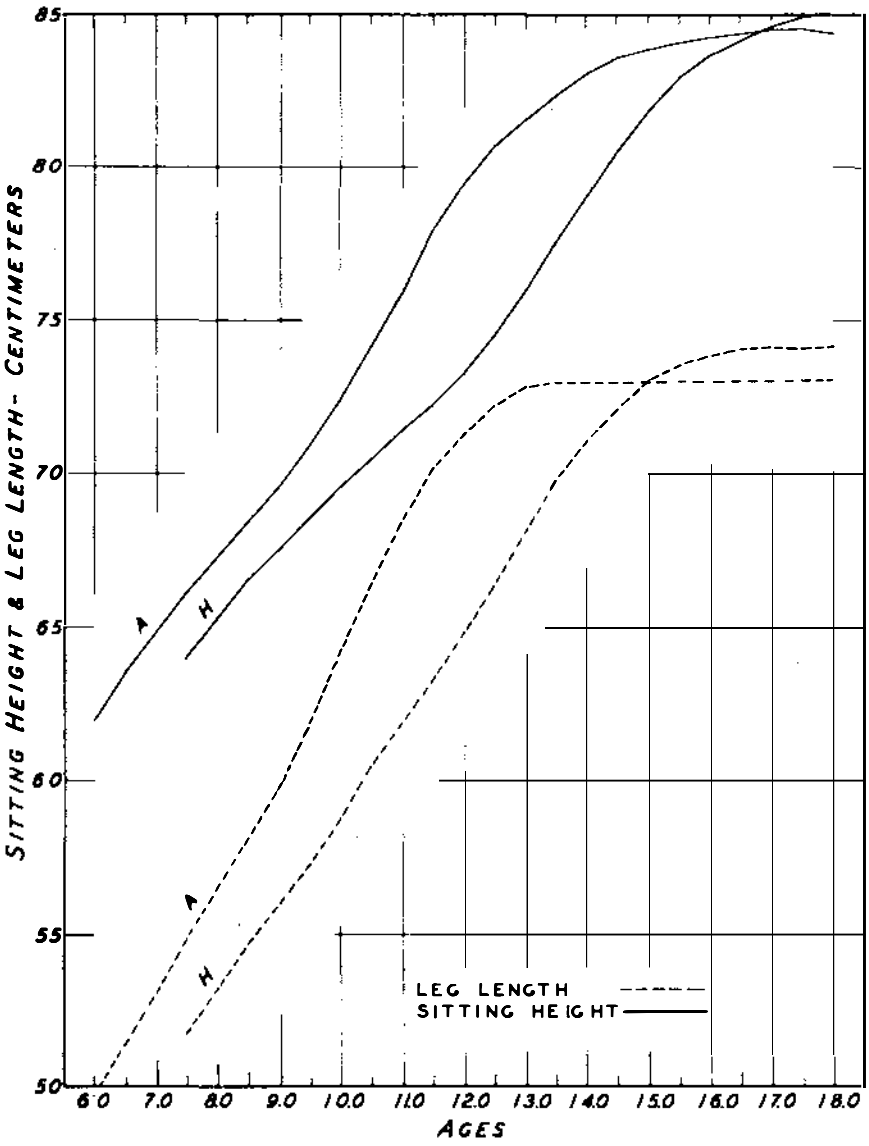

Fig. 50.--Growth Trends in Average Sitting Height and Leg Length of Early and Late Maturing Groups of Girls Plotted to the Same Arithmetic Scale.

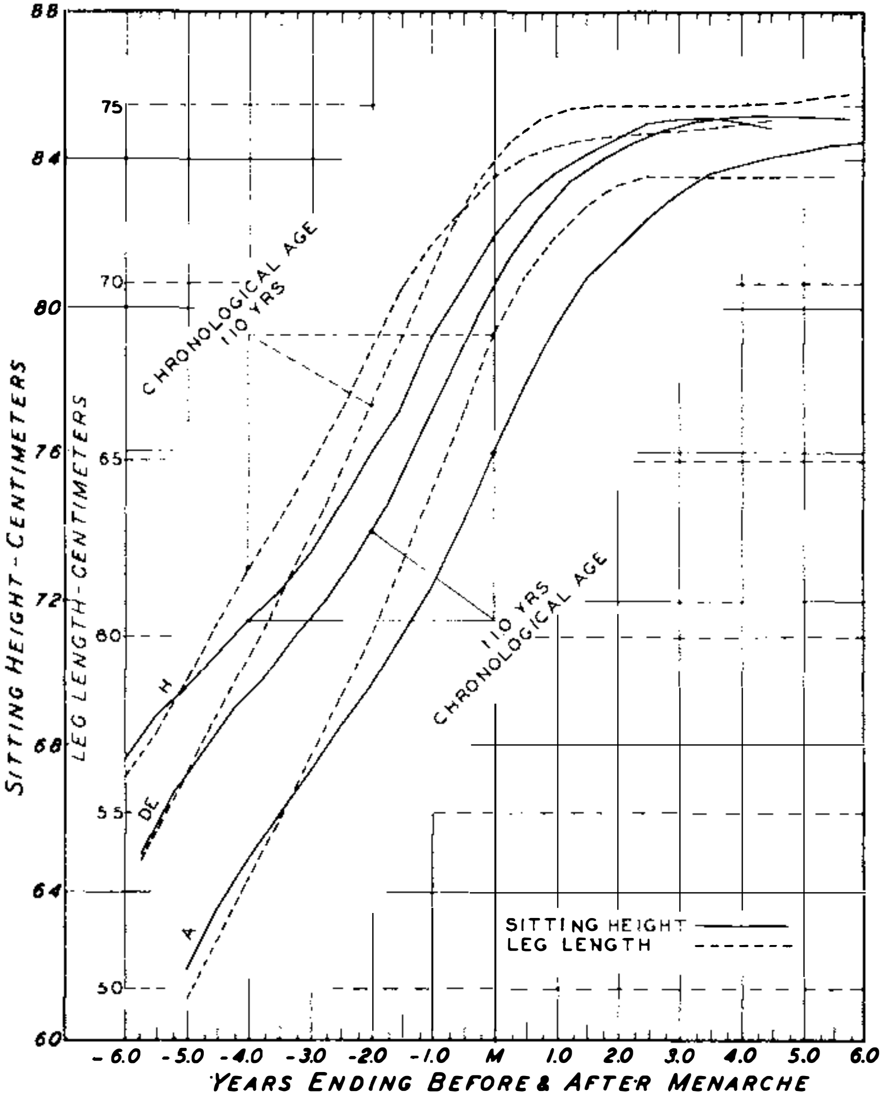

Fig. 51.--Growth Trends in Average Sitting Height and Leg Length of Three Groups of Girls Menstruating at Different Ages Arranged so that Points Corresponding to the Advent of the Menarche are in the Same Vertical Line.

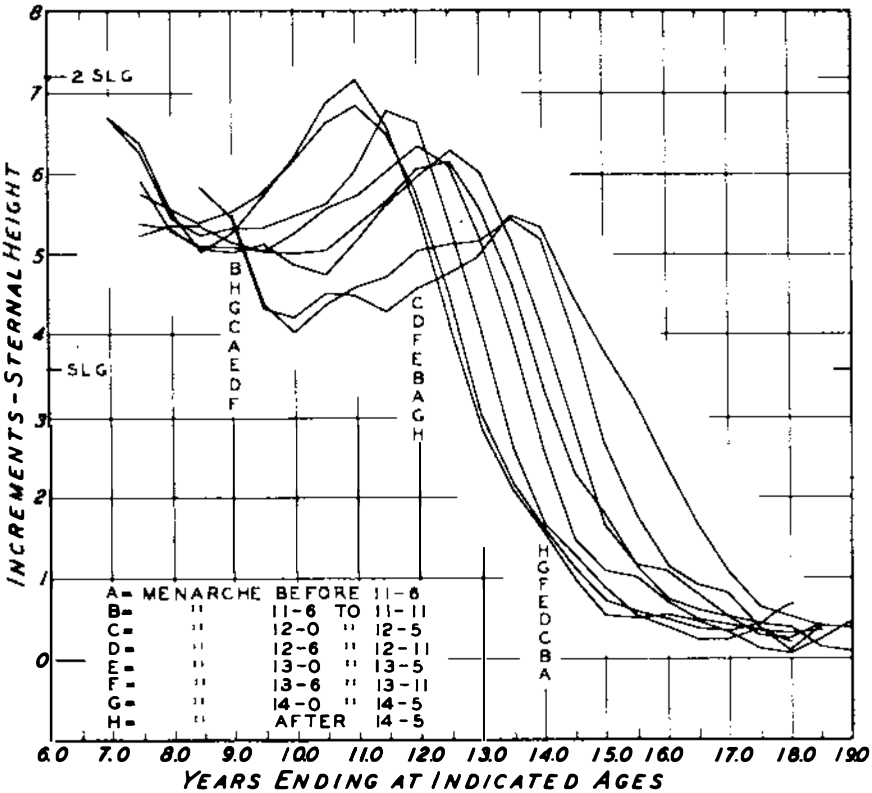

Fig. 52.--Average Annual Increments in Sternal Height of Eight Groups of Girls Menstruating at Different Ages.

height was given in sections two and three of Chapter IV, only the most summary comments must suffice for Figures 46 to 50. The growth trends for sternal height are almost indistinguishable from those for standing height, but trends for sitting height are in marked contrast with those for leg length. These contrasts are most clearly illustrated in Figures 49 and 50 in which the continuous curves represent sitting height and the dash curves leg length. At age 8.0 the individuals in Group H (menstruating at 14-6 or later) average 65.31 centimeters in sitting height and 53.21 centimeters in leg length (Figure 50). This is considerably below the average of all eight year olds (Figure 49). From age 8.0 to 12.0 the curve of Group H for sitting height rises much more slowly than that for leg length and this marked divergence is much more clearly illustrated in Figure 49 than in Figure 50. At age 12.0 Group H has attained 33.6% of the growth in sitting height and 46.6% of the growth in leg length which takes place on the average for children in general between age eight and seventeen (Figure 49). From age 12.0 to 13.5 the two curves for Group H are more nearly parallel and thereafter they converge to age 17.5. Similar contrasts are apparent in the pair of curves

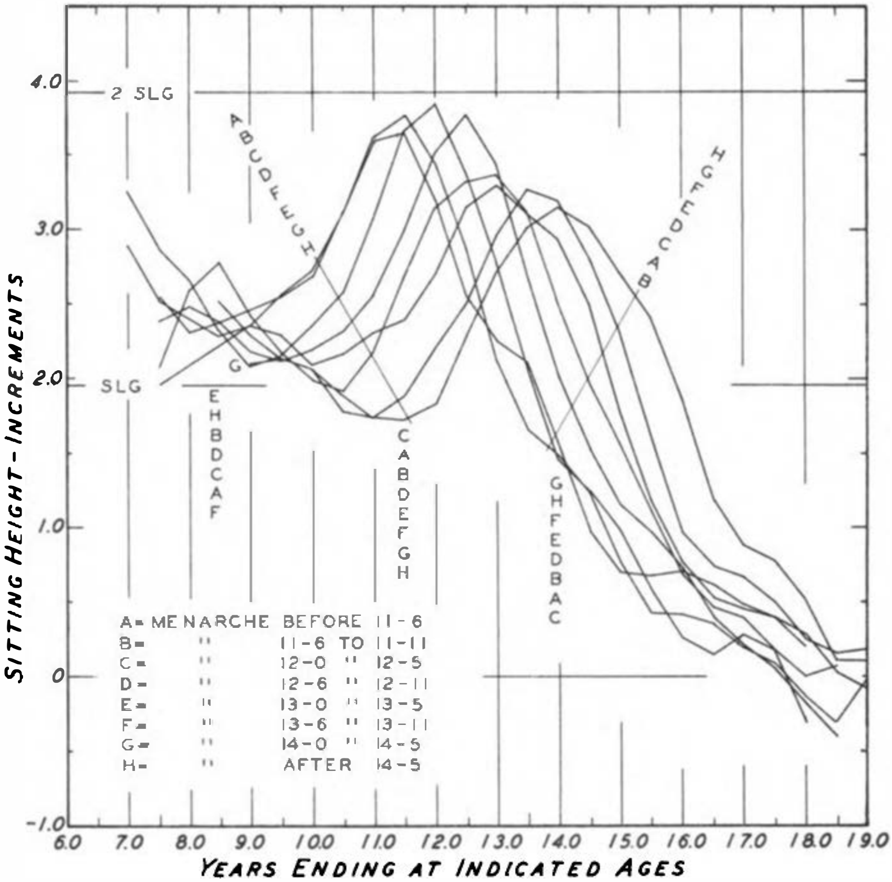

Fig. 53.—Average Annual Increments in Sitting Height of Eight Groups of Girls Menstruating at Different Ages.

for Group A. Close inspection of Figures 47 and 48 or the superimposing of these charts will show similar contrasts for each of the eight menarcheal groups. In general the growth curves for leg length are close approximations to straight lines with very abrupt transitions to mature status whereas the curves for sitting height show succeeding phases of rapid growth, slower growth, very rapid growth, and a more gradual transition to mature status.

In Figure 51 the chronological age scale has been replaced to show time in terms of years before and after the advent of the menarche, and trends for early, middle, and late maturing groups in sitting height and leg length are presented. Comparable data for standing height were presented in Figure 12. At the advent of the menarche the early maturing individuals of Group A average much below the late maturing individuals of Group H and this difference becomes greater for longer time

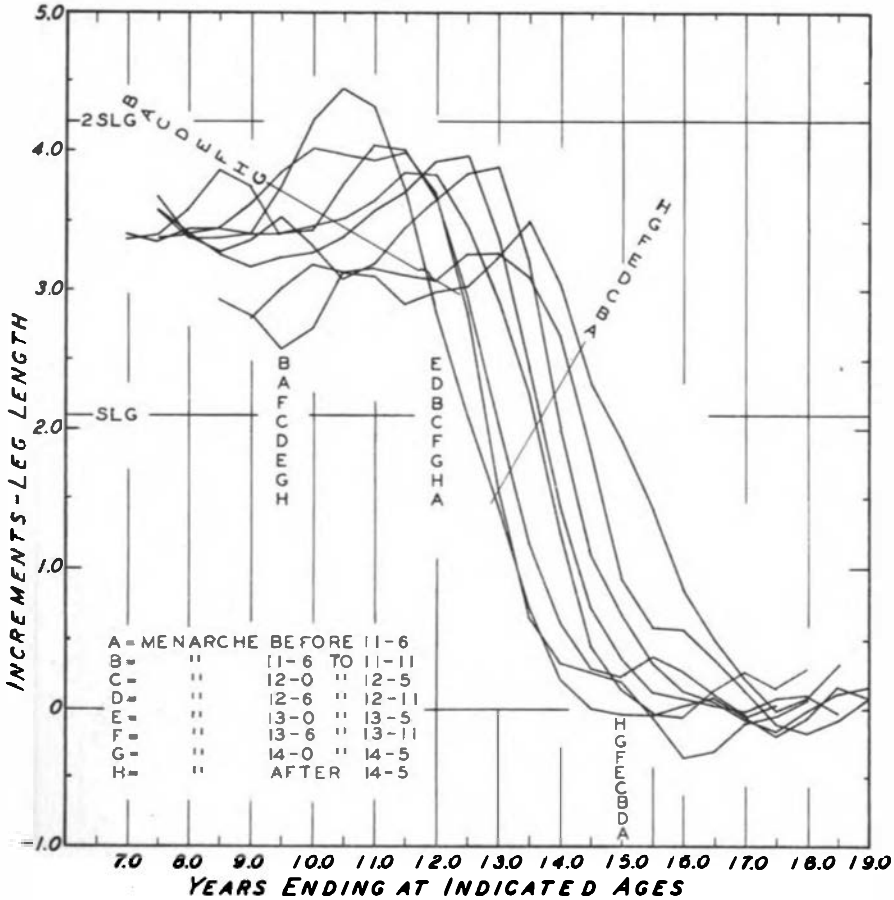

Fig. 54.--Average Annual Increments in Leg Length of Eight Groups of Girls Menstruating At Different Ages.

intervals before the advent of the menarche and less for longer time intervals after the menarche.

2. Annual Increments in Sitting Height and Leg Length

Charts of annual increments present much more convenient representations of the changing pattern of growth. Throughout, such charts carry three horizontal grid lines indicating respectively zero growth, one-ninth of the average growth between age eight and seventeen (the SLG or straight-line-growth grid), and the two-ninths of the average growth between age eight and seventeen (the 2SLG or twice-straight-line-growth grid) and throughout the vertical distance between these three lines is the same. Figures 52, 53, and 54 present annual increments in sternal height, sitting height, and leg length for each of eight groups of girls

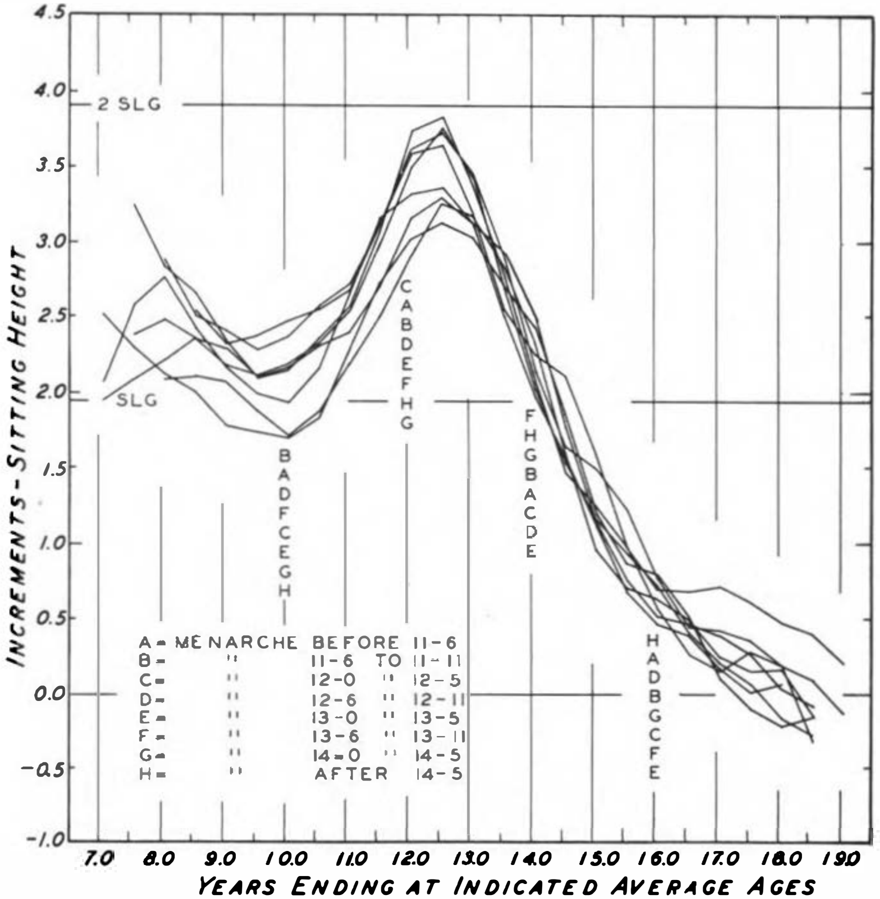

Fig. 55.--Average Annual Increments in Sitting Height of Eight Groups of Girls Menstruating at Different Ages Arranged so that the Maximum Increments in Sitting Height are in the same Vertical Line.

with different menarcheal ages. These charts are comparable to Figure 13 for standing height and should be read in the light of the full discussion of section four of Chapter IV.

Figures 55 and 56 superimpose the curves of the eight menarcheal groups separately for sitting height and for leg length so that corresponding points on the pattern of growth are as nearly as possible in the same vertical line. These charts are comparable to Figure 19 for standing height. In the case of standing height and sitting height the maximum increments have been arranged in the same vertical line, but there are only two clearly defined maximum increments for leg length and it has been necessary to arrange these curves in the light of the whole pattern. Hence, the time scales in Figures 55 and 56 are in terms of years ending at the indicated average ages. In all three charts the

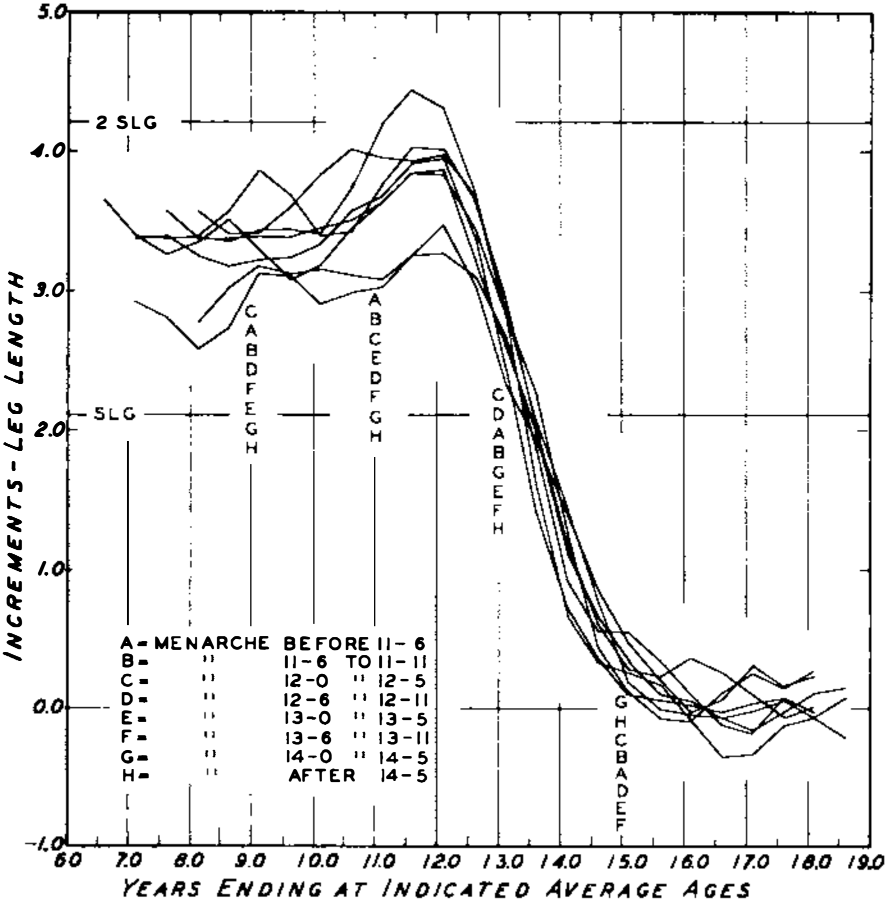

Fig. 56.—Average Annual Increments in Leg Length of Eight Groups of Girls Menstruating at Different Ages Arranged so that Corresponding Points on the Pattern of Growth are in the Same Vertical Line.

similarity of the patterns for each of the eight menarcheal groups is striking. Similarly, all three charts show that the earlier maturing groups tend to have larger increments than the later maturing groups.

Figure 57 presents generalized curves for sitting height and leg length obtained by calculating the weighted averages of the eight curves in Figures 55 and 56. The annual increments in sitting height are initially rather large averaging 2.65 centimeters for the year ending on the average at age 7.55. During the next two years growth in sitting height proceeds at a slower and slower pace, the annual increments dropping to 2.08 and 2.11 centimeters for the years ending at average ages of 9.55 and 10.05. Two and a half years of markedly accelerated growth carry the average increments up to a peak of 3.56

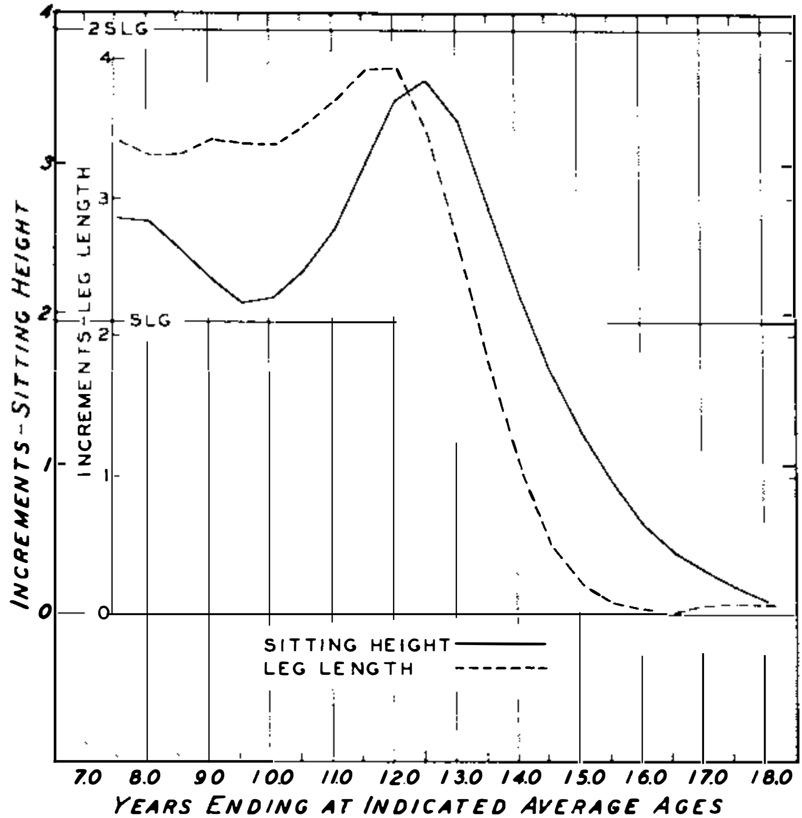

Fig. 57.--Generalized Patterns of Annual Increments in Sitting Height and Leg Length Obtained by Weighting and Combining the Curves of Figures 55 and 56.

centimeters and thereafter there is a gradual deceleration to age eighteen. The pattern for leg length is in marked contrast. The initial increments for a period of two and a half years range between 3.30 and 3.43 centimeters or almost exactly straight-line growth. A slight acceleration begins on the average at age 10.65 corresponding to the initiation of the acceleration in sitting height. Whereas the acceleration of sitting height is very marked and runs for two and a half years, that for leg length is slight and lasts for only a year and a half, the maximum increments being 3.91 and 3.92 centimeters at average ages of 11.65 and 12.15. Thereafter the deceleration is marked and growth in leg length practically ceases on the average at age 15.65. Attention is called particularly to the fact that the maximum increment in leg length occurs on the average at age 12.15 (or 11.65) while the maximum increment in sitting height occurs on the average at age 12.55 and that the menarche appears on the average at age 12.97. There are, of course, as pointed out in Chapter V, many individual exceptions to such generalized trends. Since the available cases were measured only once a year, their records

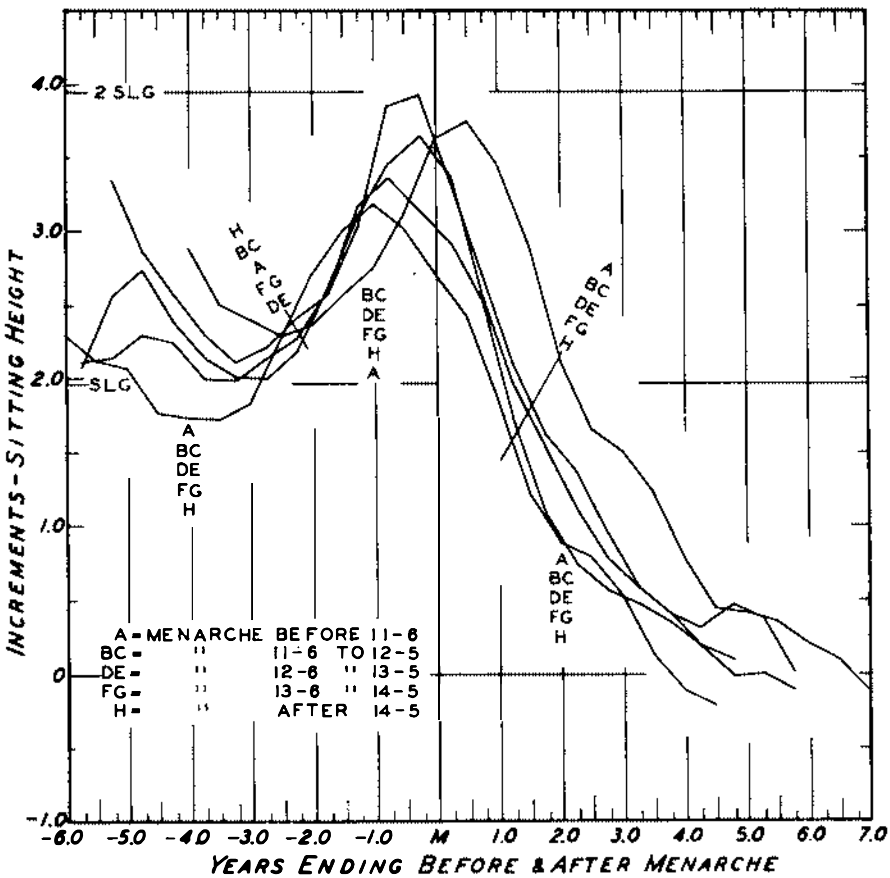

Fig. 58.--Average Annual Increments in Sitting Height of Five Groups of Girls Menstruating at Different Ages Arranged so that Points Corresponding to the Advent of the Menarche are in the Same Vertical Line.

are not particularly suitable for determining the frequency with which the maximum increment in leg length precedes that in sitting height. Eliminating cases who show two maximum increments in sitting height, cases whose records cover too limited a range of years to determine a maximum increment, etc., there remain 232 cases for a frequency count. Confining attention to only the three increments in leg length which occur one year before, coincident with, and one year after the maximum increment in sitting height, we have the following results: in 136 instances or 58.6% the maximum increment in leg length precedes that in sitting height, in 71 instances or 30.6% the maximum increment in leg length coincides with that in sitting height, and in 16 instances or 6.9% the maximum increment in leg length follows that in sitting height. Nine cases or 3.9% fail to show a maximum increment in leg length.

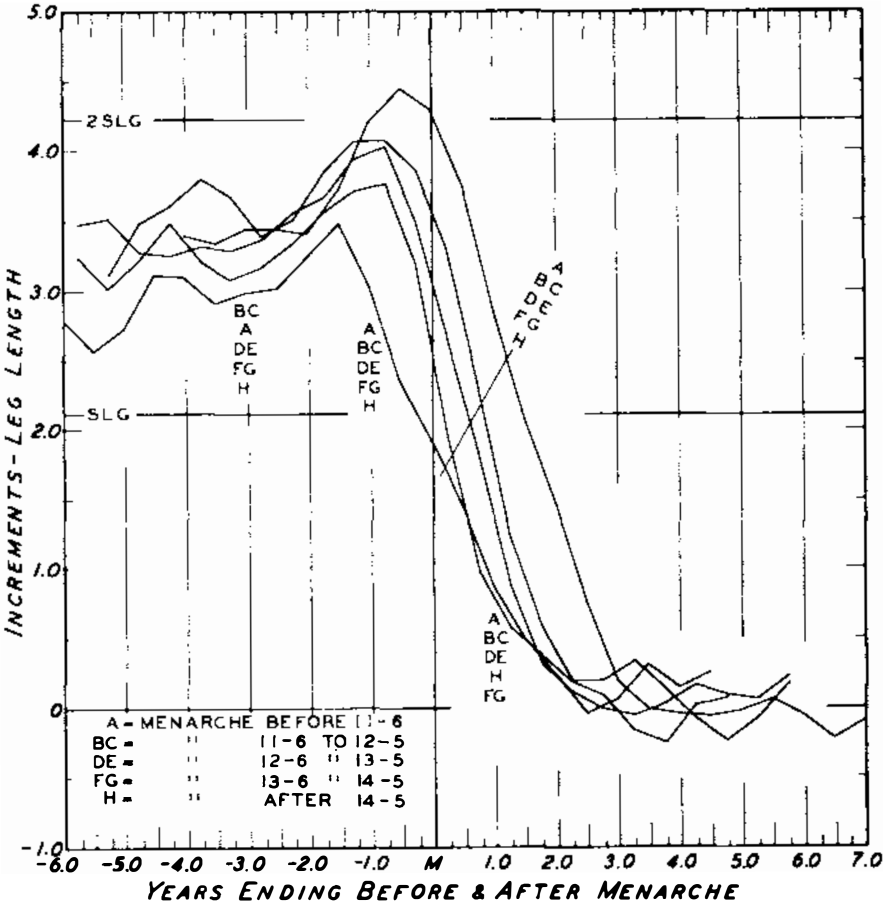

Fig. 59.—Average Annual Increments in Leg Length of Five Groups of Girls Menstruating at Different Ages Arranged so that Points Corresponding to the Advent of the Menarche are in the Same Vertical Line.

These figures rather clearly establish the tendency of accelerated growth in sitting height to continue for some time after growth in leg length has begun to slow down.

Figure 58 and 59 present annual increments in sitting height and in leg length for five groups of girls so arranged that points corresponding to the advent of the menarche are in the same vertical line. Figure 60 super-imposes these two charts. Comparable data for standing

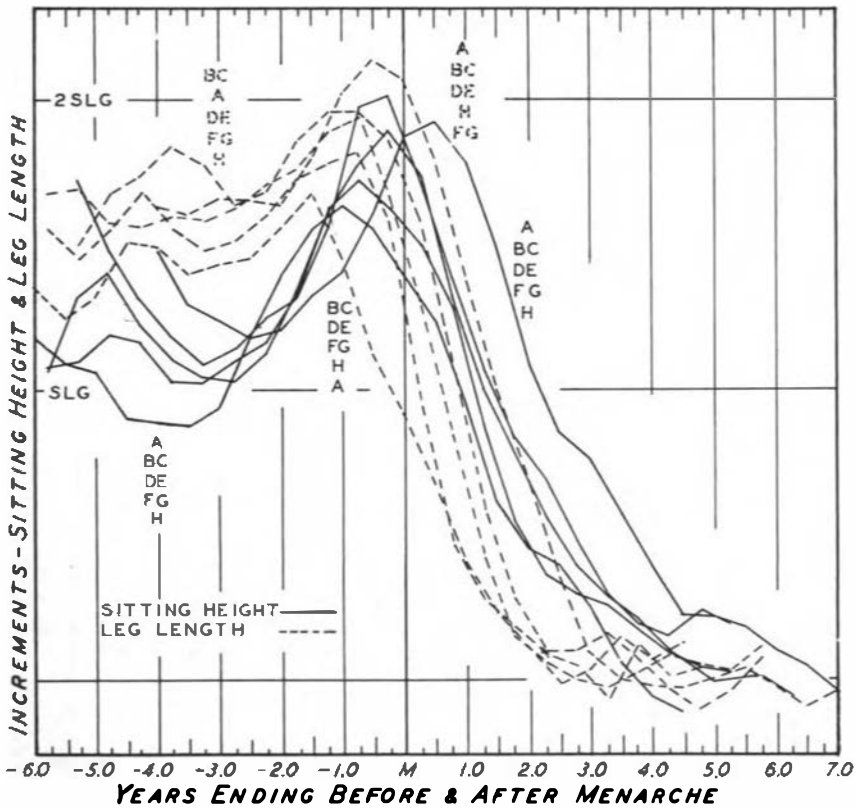

Fig. 60.—Average Annual Increments in Sitting Height and Leg Length of Five Groups of Girls Menstruating at Different Ages Arranged so that Points Corresponding to the Advent of the Menarche are in the same Vertical Line.

height were presented in Figure 20.[1] Having discussed the contrasts between the generalized patterns of sitting height and of leg length as illustrated in Figure 57, Figure 60 offers a convenient picture for the elucidation of contrasts between early and late maturing groups of girls. Reference will also be made to Figures 53 and 54. The individuals in Group A, menstruating before 11-6 or on the average at 11.0,

[1]*It should be recalled at this point that Groups BC, DE, and FG do not represent the simple combination of Groups B and C, D and E, and F and G. Instead, the original sub-groups XB and YC were weighted and combined at identical deviation ages from the menarche and the resulting XBYC data combined with a similar composite of the XC and YB sub-groups, etc., insuring that changes in the pattern of growth are precisely orientated in relation to the advent of menarche.*

show their most intense growth in sitting height for the year ending at age 11.5 (Figure 53) or six months after the menarche (Figures 58 and 60). Their maximum increment averages 3.74 centimeters which is smaller than the maximum increment of Groups B, C, and D, but considerably larger than the maximum increment of Groups E, F, G, and H. Accelerated growth in sitting height for Group A begins during the year ending at age 9.0 or during the year ending two years before the menarche and lasts for two and a half years. The total acceleration is from 2.29 centimeters per year to 3.74 centimeters or 1.45 centimeters. In comparison, the individuals in Group H, menstruating after 14-5, or on the average at age 15.0, show their most intense growth in sitting height for the year ending at age 14.0 or twelve months before the menarche. Their maximum increment averages only 3.16 centimeters which is less than that of any other group. Accelerated growth begins during the year ending at 12.0 or during the year ending three years before the menarche and lasts two years. The total acceleration is from 1.72 centimeters to 3.16 centimeters or 1.44 centimeters. That is, the patterns of growth in sitting height for Groups A and H are strikingly similar save that growth in Group A is more intense (Figure 55). While the patterns of the two groups are similar, the timing of this pattern in relation both to chronolcgical age and the advent of the menarche is radically different. The B, C, D, E, F, and G groups, of course, tend to be intermediate to Groups A and H in the timing of their patterns in relation to chronological age and the menarche. A similar reading of the charts and similar comments apply to the pattern of growth in leg length.

3. Percentage Rates of Growth Per Year

In the preceding section the annual increments in sitting height and leg length were contrasted. From age 10.5 to 11.5 the individuals in Group A gained on the average 3.74 centimeters in both sitting height and leg length. This happy coincidence of two identical increments will serve to illustrate the point that identical things are not always identical. In the first place the annual increment of 3.74 centimeters in sitting height represents the culmination of an accelerating phase of growth; whereas, the same increment in leg length represents the initiation of a decelerating phase. In the second place the increment of 3.74 centimeters in sitting height constitutes 21.2% (3.74/17.6) of the growth which occurs on the average between age eight and age seventeen; whereas, the same increment in leg length constitutes only 20.4%. Hence, when these increments are superimposed as in Figure 60 six months after the menarche the point for sitting height is above that for leg length. In the third place the increment of 3.74 centimeters in sitting height represents an increase from 74.10 to 77.84 centimeters or 5.05%; whereas, the same increment in leg length represents an increase from 66.43 to 70.17 centimeters or 5.63%.

The transformation of the curves which occurs when percentage rates rather than increments are considered can be most clearly illustrated by reproducing the two generalized increment curves of Figure 57 and superimposing on them rate curves based on the same original data. This is done in Figure 61 in which the heavy lines represent rates of growth and the light lines represent increments. In both sitting height and leg length the rate curves emphasize the slowing of growth during the

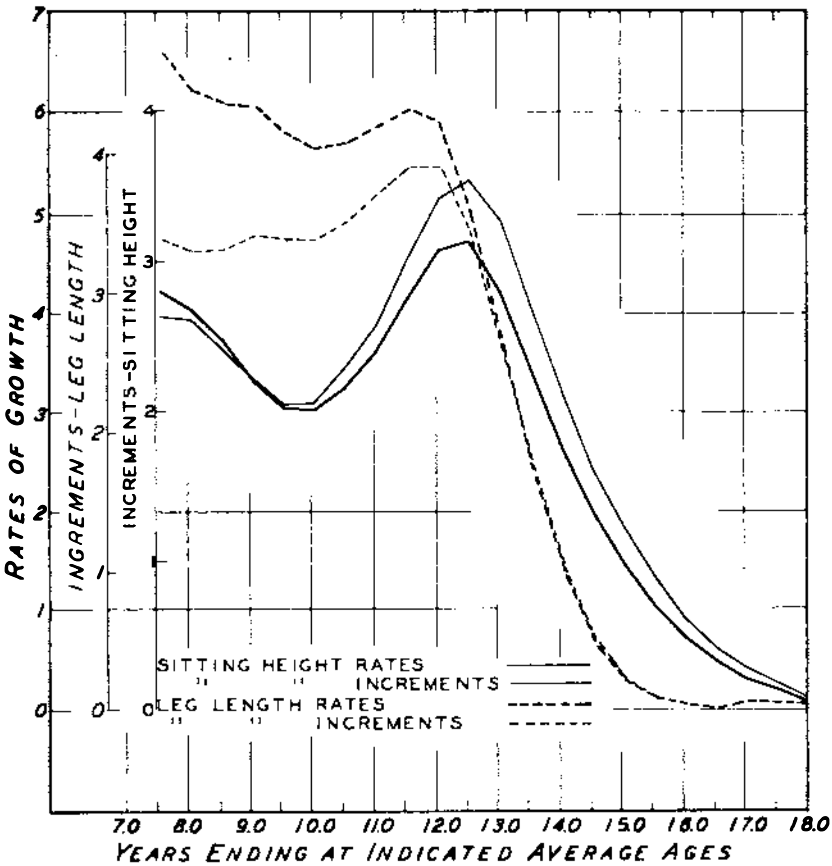

Fig. 61.--Generalized Patterns of Annual Increments and of Percentage Rates of Growth Per Year in Sitting Height and Leg Length.

two and a half years ending on the average at age ten and lessen the apparent magnitude of the accelerating phase.

Rates of growth per year for each of the eight menarcheal groups in both sitting height and leg length are shown in Figure 62 plotted against chronological age. Comparable data for five groups plotted in relation to the advent of the menarche are shown in Figure 63. Comparable data for standing height and employing the same vertical scale at the left were presented in Figures 23 and 24.

4. Percentage of Standing Height to Sitting Height

At birth total body length consists of nearly three-fourths sitting height and only one-fourth leg length. Througnout childhood the legs grow faster than the rest of the body until in the early teens sitting height constitutes less than 53% of standing height. Thereafter sitting

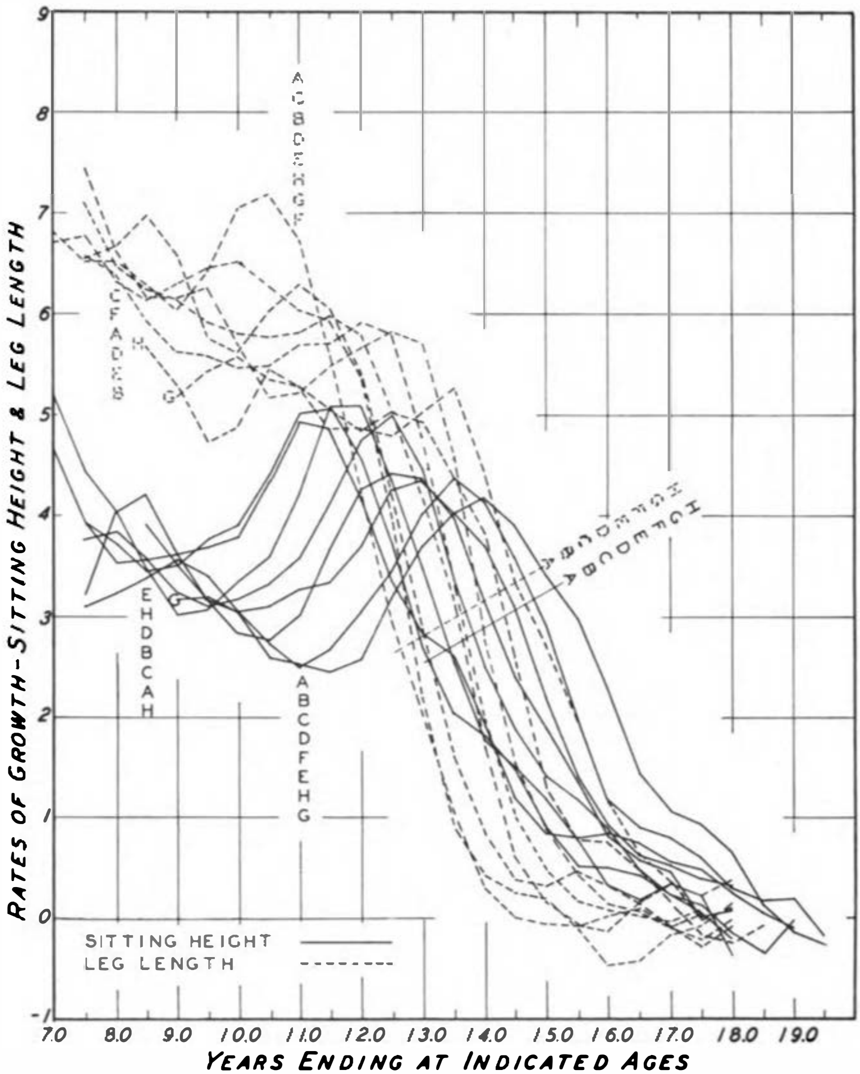

Fig. 62.--Percentage Rates of Growth Per Year in Sitting Height and Leg Length of Eight Groups of Girls Menstruating At Different Ages.

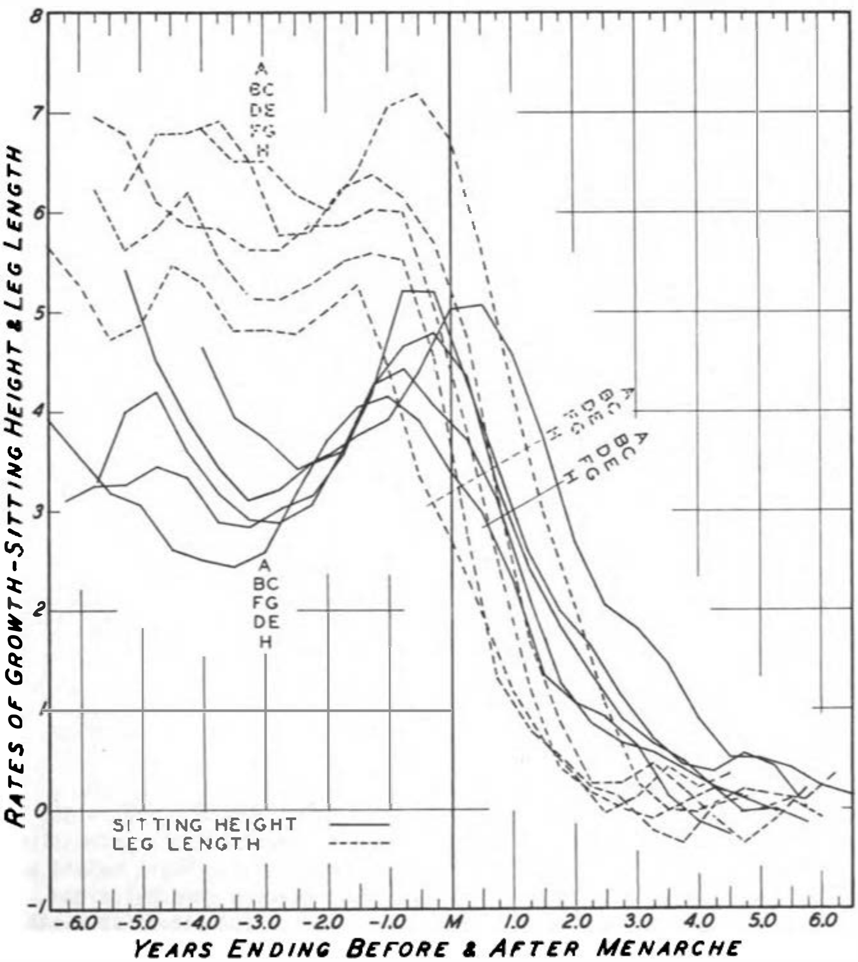

Fig. 63.--Percentage Rates of Growth Per Year in Sitting Height and Leg Length of Five Groups of Girls Menstruating at Different Ages Arranged so that Points Corresponding to the Advent of the Menarche are in the same Vertical Line.

height increases slightly more than leg length. These trends from age 6.0 to 19.0 for Groups A and H are presented in Figure 64. In Group A sitting height falls from 55.5% of standing height at age 6.0 to 52.6% at ages 11.0 and 11.5 and then rises slightly to 53.7%. While the low point of the percentage of standing height to sitting height is reached at ages 11.0 and 11.5 in the case of Group A, the corresponding low point for Group H is not reached until 13.5.

An altogether different picture results if attention is focused on the increments instead of the gross measures. While sitting height constitutes from 52 to 55% of standing height between age six and nineteen, the annual increments of sitting height may constitute any where from

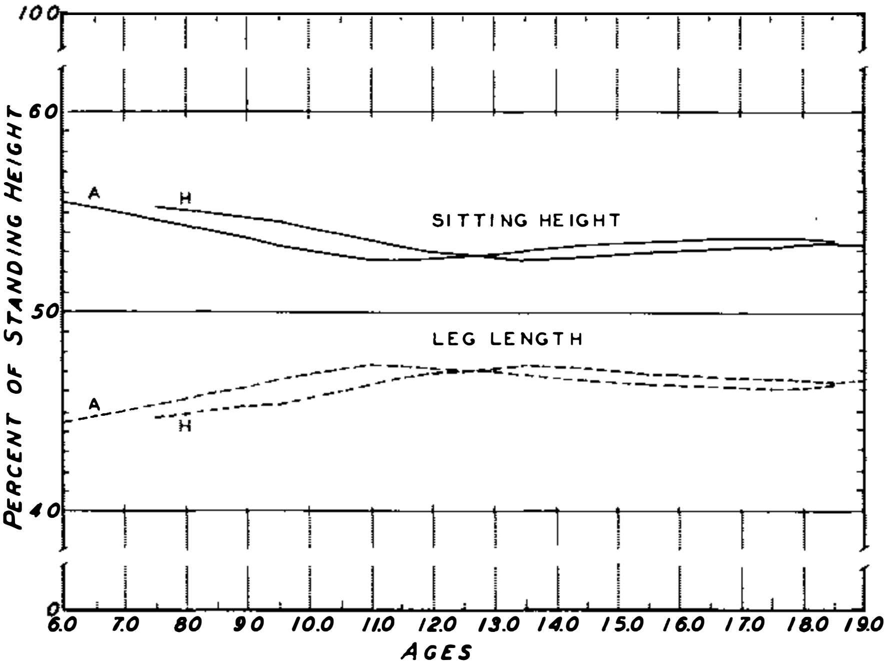

Fig. 64.--Percent of Total Standing Height which is Sitting Height and Leg Length for Early and Late Maturing Groups of Girls.

35% to 100% of the annual increments in standing height. Figure 65 presents data of this type for a selection of four groups menstruating at different ages. The ages at which the curves for sitting height and leg length cross at the 50% mark are in very close sequence for the eight menarcheal groups, being 11.5, 12.6, 11.8, 12.3, 13.0, 13.5, 13.3, and 13.8 years respectively for Groups A, B, C, D, E, F, G, and H. It should be noted that the percentage scale at the left has been reduced by half in comparison with Figure 64 so that the true contrast between Figures 64 and 65 is twice as great as would be suggested by casual inspection.

5. Summary of Findings for Sitting Height and Leg Length

Almost the entire summary of the data for standing height might, with slight changes in wording, be applied to the findings for sitting height and leg length. During the early teens there is the same close association between physical size and the advent of the menarche. The generalized patterns of growth generated by the annual increments show the same three phases: (a) a decelerating phase which is marked in the case of sitting height and very slight or absent in the case of leg length, (b) an accelerating phase which is marked in the case of sitting height and less marked in the case of leg length, and (c) a sharply decelerating phase which is less abrupt in the case of sitting height

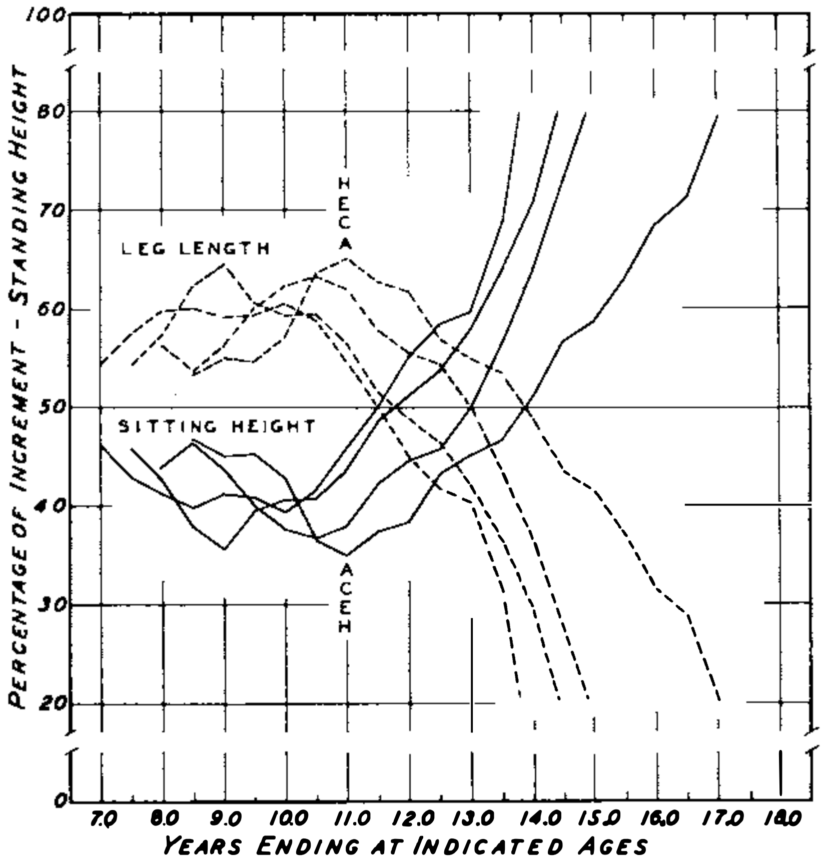

Fig. 65.—Percent of the Annual Increments in Standing Height which is Sitting Height and Leg Length for Four Groups of Girls Menstruating at Different Ages.

and more abrupt in the case of leg length. When the eight menarcheal groups are considered these three phases show the same contrasts in respect to the chronological ages at which they appear, but when chronological age is ignored and corresponding points on the pattern of growth are superimposed the same striking similarity reappears. The inflection points or transition periods between these three phases show the same intimate timing in relation to the advent of the menarche, the only difference being that the maximum increments in sitting height come about a year later than those in leg length. Just as in the case of standing height, the pattern of increments in the early maturing groups is more intense than in the late maturing groups. The findings are in accord with the theory that the associations between menarcheal age and the timing of changes in the patterns of physical growth should be attributed to underlying endocrine factors. Interpretation of the findings in terms of endocrine factors also offers the most plausible explanation of the contrasts between the patterns of sitting height and leg length. Growth in leg length is determined by the growth of long bones which in turn is determined by the age at which the epiphyses close or unite with the shafts. It is probable that these closures are determined by endocrine factors. Hence, the final decelerating phase of growth in leg length is very abrupt. The final decelerating phase of growth in sitting height on the other hand is much less abrupt because no long bones are involved.

CHAPTER VII

ILIAC DIAMETER, CHEST BREADTH, AND CHEST DEPTH

The two chapters on standing height were devoted to a very full discussion and graphical representation of the growth patterns of the eight menarcheal groups emphasizing particularly the great contrasts between these patterns at each chronological age, the striking similarity of these patterns when chronological age is ignored, and the intimate timing of these patterns in relation to the advent of the menarche. Comparable phenomena in the growth patterns of the eight menarcheal groups in sitting height and leg length were presented in Chapter VI with the emphasis of the discussion shifting to the marked contrast between the patterns of the two vertical dimensions. Patterns of growth in iliac diameter, chest breadth, and chest depth show much less contrast while tending to repeat the characteristically differential patterns of the eight menarcheal groups. Since the data are repetitious and somewhat irregular, only a brief discussion and a selected number of charts are presented.

1. Limitations of the Data

A feature of the data which should be especially noted is their confirmation of the general theory and specific hypotheses of this study in spite of their very serious limitations. Even under the most favorable conditions all three dimensions, and particularly chest breadth and depth, are difficult to measure with precision. Individual differences are comparatively small and growth in absolute size is very slow. In addition to these limitations, inherent in the nature of the measurements, there is the complicating factor that all were measured over clothing. It results that the growth records of individuals are very irregular and frequently contain figures which seem impossible in the light of the whole series. As if these difficulties were not enough, there is the further complication that the Harvard Growth Study collected no data for iliac diameter during the second autumn and no data for chest breadth and depth during the third and fourth autumns of the Study.

Procedures employed to overcome these limitations as far as possible are described in detail in sections one, two, and seven of Appendix A. The briefest summary of these procedures must suffice at this point. Measurements which seem erratic or impossible in the light of the whole series have been eliminated. Total eliminations constitute 1.1%, 5.4%, and 5.5% of the number of original figures respectively for iliac diameter, chest breadth, and chest depth. It is only necessary to emphasize that this editing of the data was applied routinely to all cases irrespective of menarcheal ages. All the gaps in the data whether due to absences from school, editorial eliminations, or systematic omissions during the early years of the Study have been filled by interpolations, but in no instance has straight-line interpolation been applied blindly. Interpolated figures constitute 7.0%, 22.5%, and 23.2% of the total number of available figures respectively for iliac diameter, chest breadth, and chest depth (See Table 10 in Appendix A). Most of the interpolations, however, are for the period prior to age eleven. The large number of interpolations tends to iron out chance irregularities in the curves, but no amount of interpolating can create patterns of growth which are not inherent in the original data.

2. Growth Trends in Average Size

Figures 66, 67, and 68 present growth trends in average size for each of the dimensions and for each of the eight menarcheal groups. It should be recalled that average growth from age eight to seventeen is represented by the same vertical distance in these charts. The almost perfectly regular differentiation of the eight menarcheal groups in average size from approximately age ten to fourteen is readily apparent. In order that the curves may be observed in their proper absolute relation to each other Figure 69 presents the trends for early and late maturing groups of girls on the same arithmetic scale.

Growth trends in average iliac diameter, chest breadth, and chest depth give contrasting pictures depending on whether they are plotted to chronological age or to time intervals before and after the menarche just as do the vertical dimensions. Figure 70 presents such data for early and late maturing groups for the three dimensions. This chart should be compared with Figures 12, 44, and 51 which present comparable phenomena for the vertical dimensions. When early and late maturing groups are compared all six linear measurements show increasing differences in average size for longer time intervals before the menarche and decreasing differences in average size for longer time intervals after the menarche. That is, the curves of the early maturing groups of girls are much steeper than the curves of the late maturing groups. Incidentally, Figure 70 illustrates the similarity of the patterns of the three horizontal dimensions when comparisons are confined to the same menarcheal group.

3. Annual Increments and Rates of Growth

Figures 71, 72, and 73 present the annual increments. It should be recalled that the vertical distances between the three horizontal grid lines are the same. That is, an annual increment in chest depth of .43 centimeters is represented by the same vertical distance as an increment in standing height of 4.0 centimeters. The extreme magnification of the scales is such as to reveal minor irregularities of a few hundredths of a centimeter. While the visual impression from these charts is one of great irregularity, close inspection will show that the eight menarcheal groups tend to repeat the characteristic patterns shown by the vertical dimensions. For example, annual increments for the year ending at age 11.5 show the following. The largest and smallest increments in iliac diameter are 1.40 and .75 centimeters, the eight groups standing in the following order: B, A, C, E, D, F, G, and H. The largest and smallest increments in chest breadth are 1.66 and .66 centimeters, the eight groups standing in the following order: A, B, C, D, G, E, F, and H. The largest and smallest increments in chest depth are .98 and .53 centimeters, the eight groups standing in the following order: A, C, B, F, D, G, H, and E. Considering the fact that

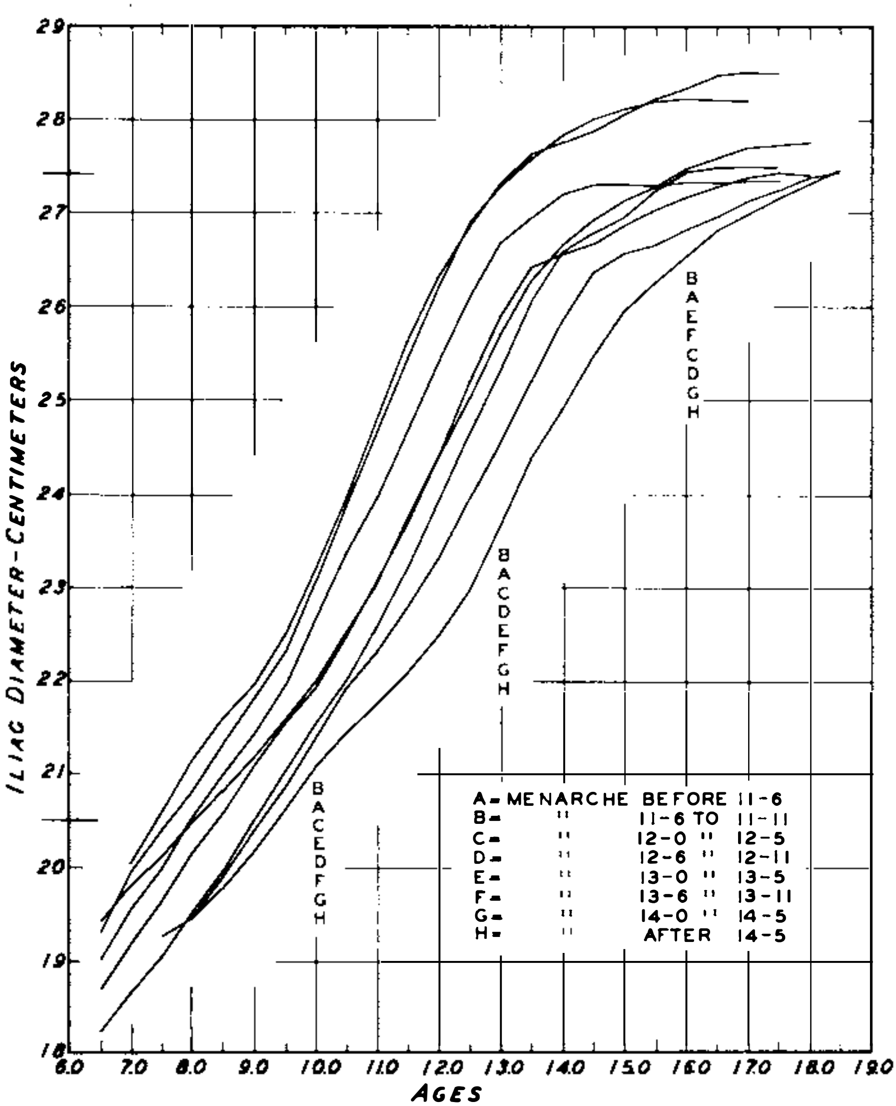

Fig. 66.-Growth Trends in Average Iliac Diameter of Eight Groups of Girls Menstruating At Different Ages.

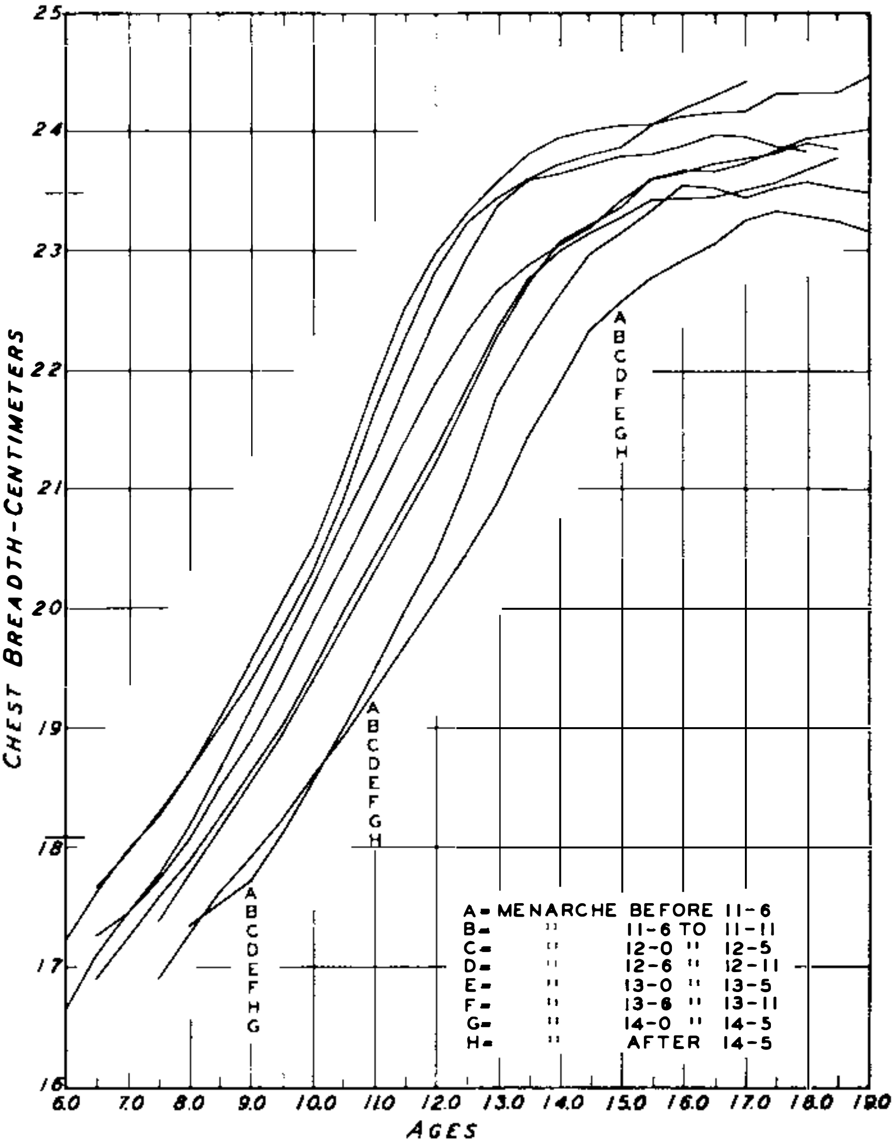

Fig. 67.--Growth Trends in Average Chest Breadth of Eight Groups of Girls Menstruating At Different Ages.

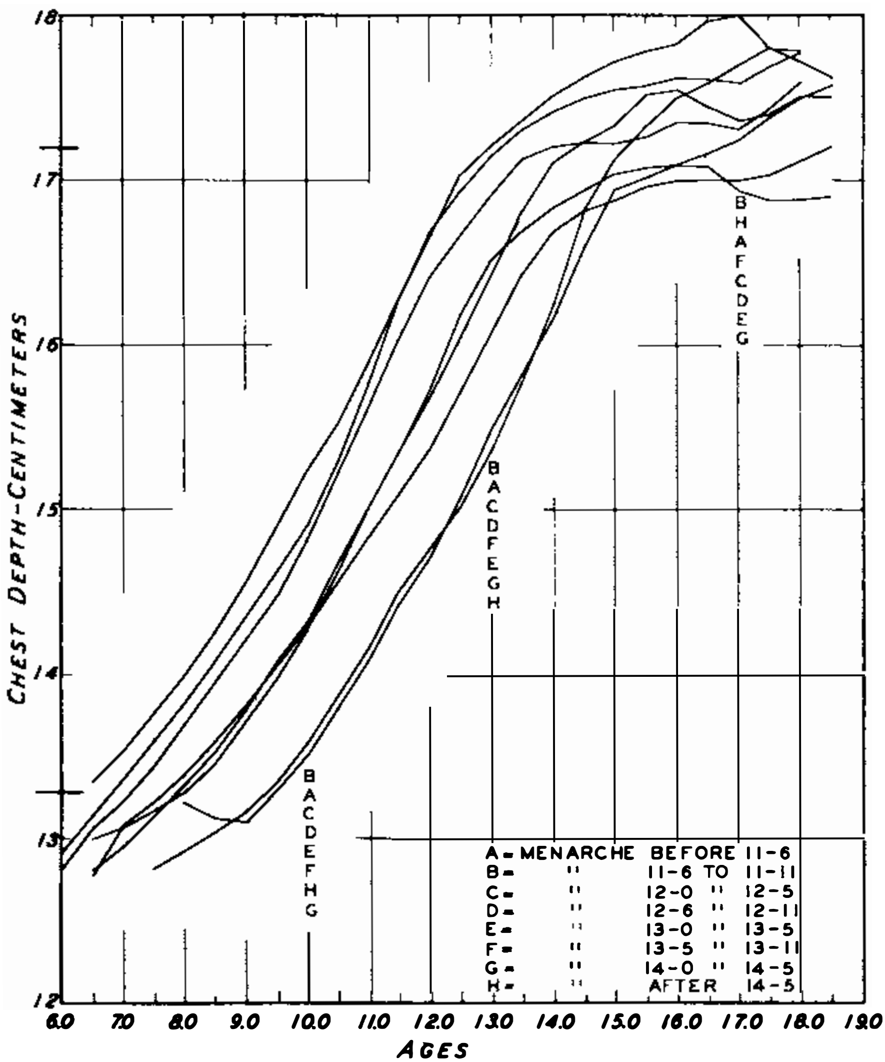

Fig. 68.--Growth Trends in Average Chest Depth of Eight Groups of Girls Menstruating at Different Ages.

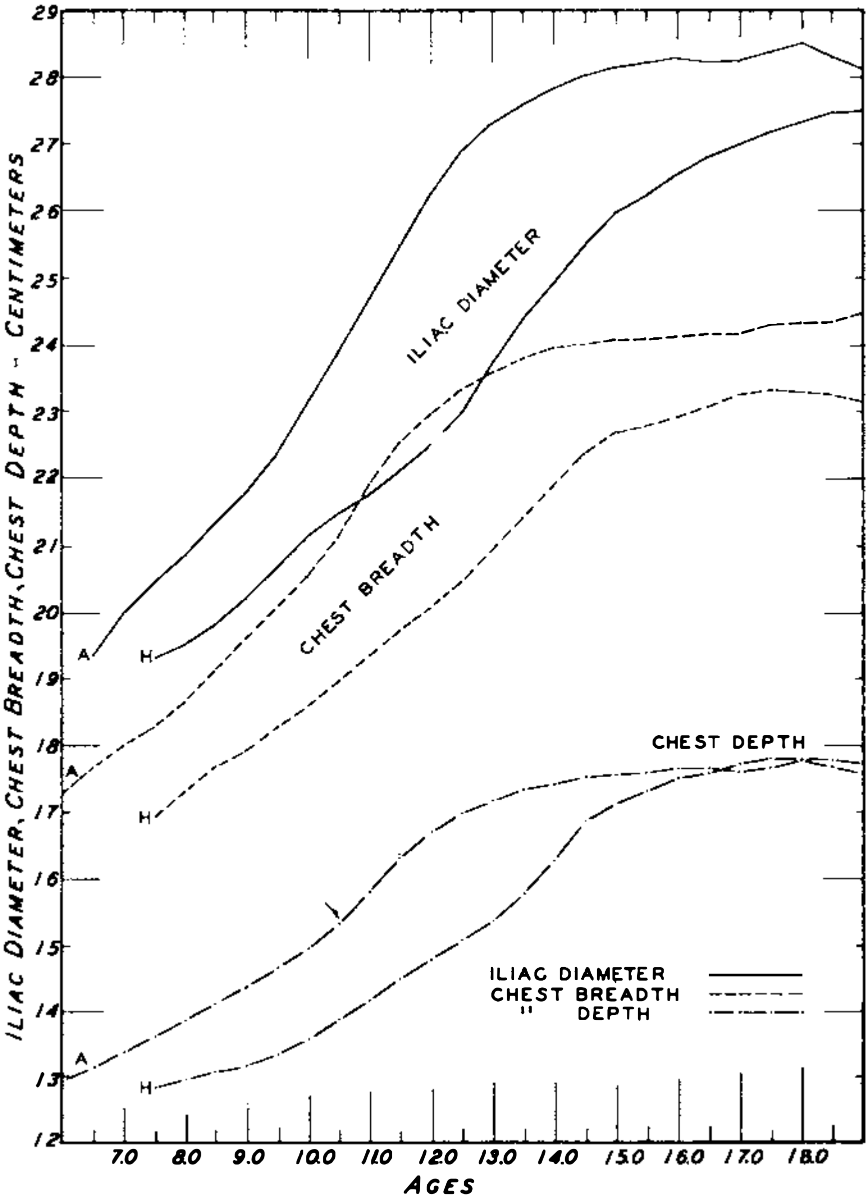

Fig. 69.--Growth Trends in Average Iliac Diameter, Chest Breadth, and Chest Depth of Early and Late Maturing Groups of Girls Plotted to the Same Arithmetic Scale.

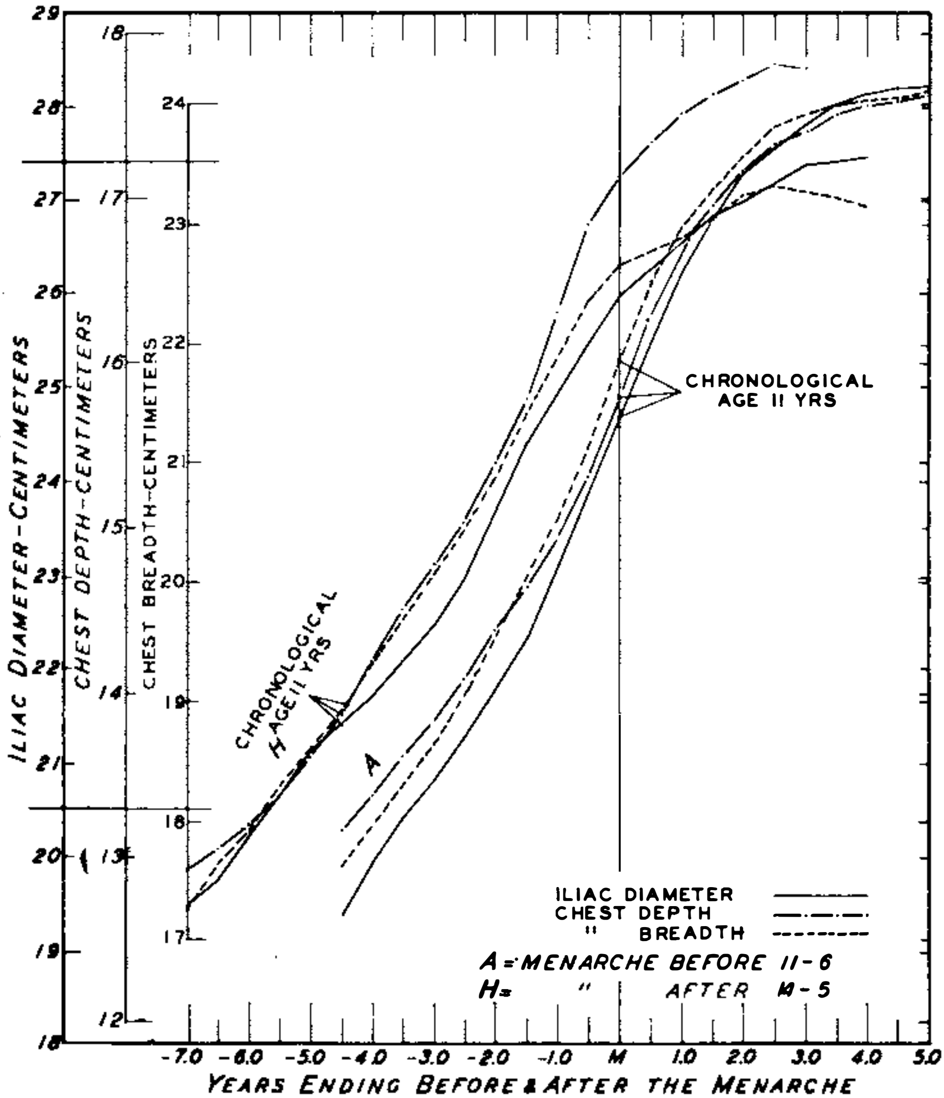

Fig. 70.--Growth Trends in Average Iliac Diameter, Chest Breadth, and Chest Depth of Early and Late Maturing Groups of Girls Arranged so that Points on Their Growth Curves Corresponding to the Advent of the Menarche are in the Same Vertical Line.

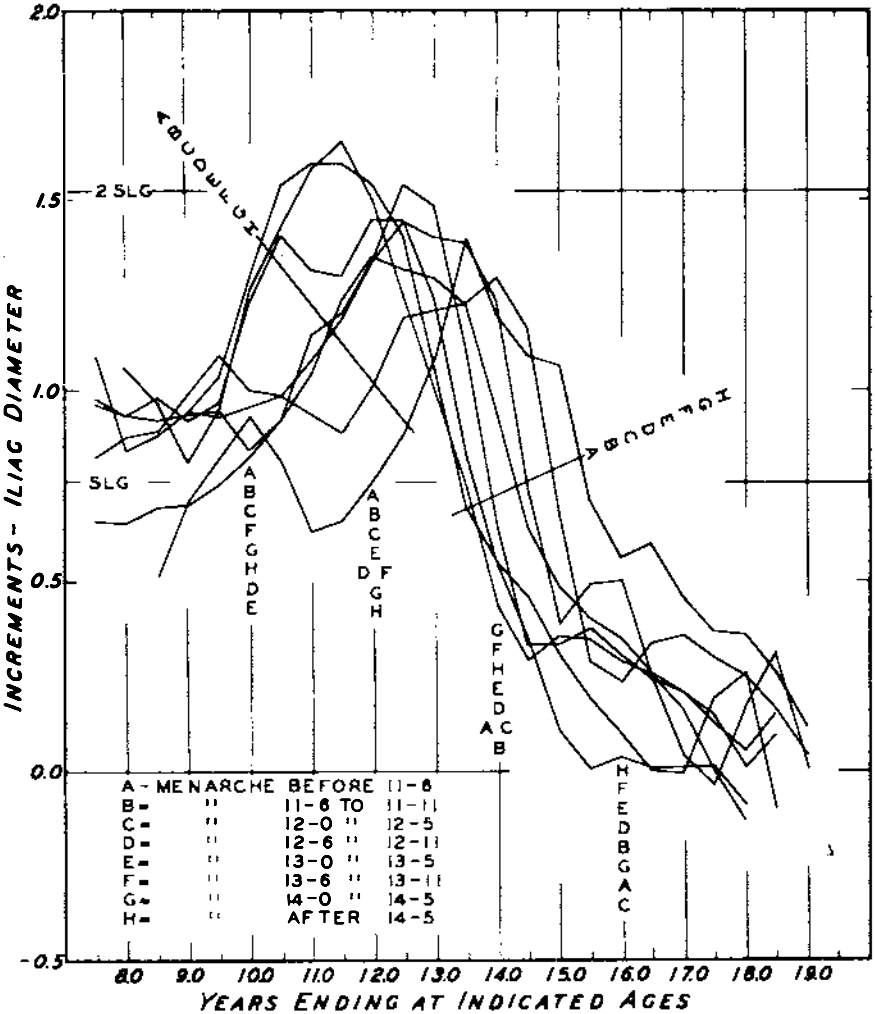

Fig. 71.—Average Annual Increments in Iliac Diameter of Eight Groups of Girls Menstruating at Different Ages.

the greatest difference is only a centimeter and that the exact order of the groups is frequently determined by only hundredths of a centimeter the differential patterns are unexpectedly clear and regular. Increments for the years ending at 13.5 and 14.0 of the eight groups are in a similarly regular but exactly reversed order. Likewise, the diagonal columns of letters at the right show that the decelerating phase of growth in the eight menarcheal groups occurs in almost perfectly regular order. Since no charts are presented with the curves arranged so that points corresponding to the advent of the menarche are in the same vertical line, the reader must perform this type of analysis in imagination.

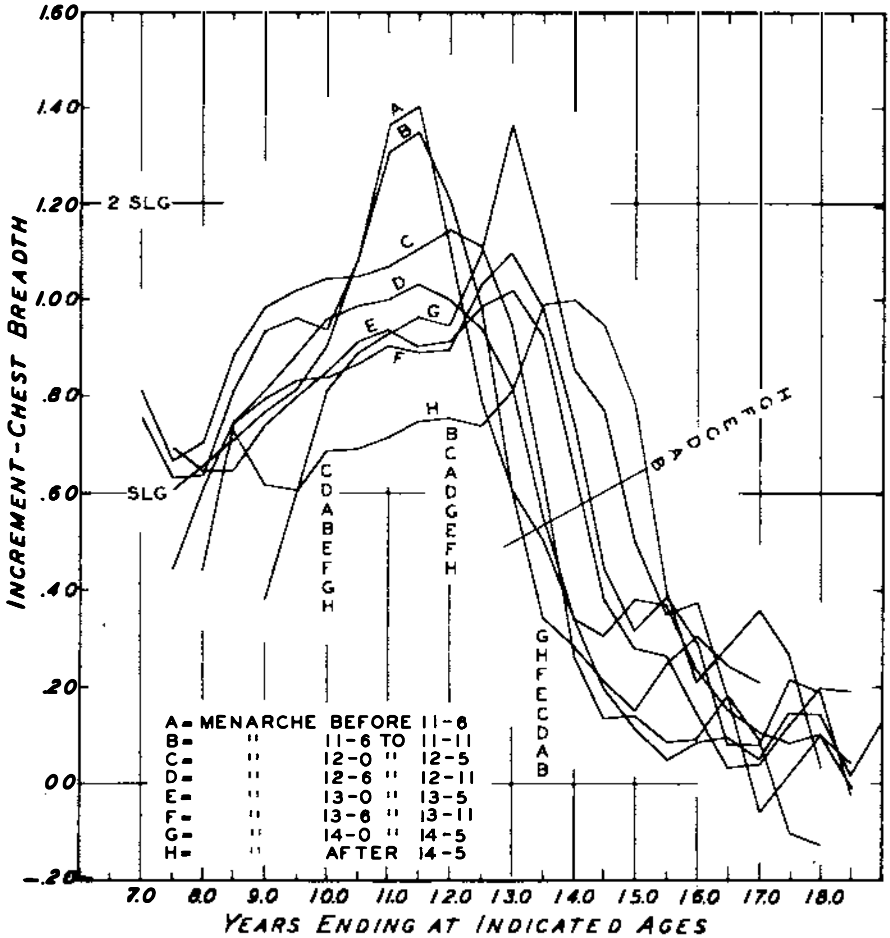

Fig. 72.--Average Annual Increments in Chest Breadth of Eight Groups of Girls Menstruating at Different Ages.

Figures 74, 75, and 76 superimpose the increment curves so that corresponding points on the patterns of growth are in the same vertical line. Weighting and averaging the curves of these charts to give three generalized patterns and superimposing the results together with comparable data for sitting height and leg length yields the curves of Figure 77. Since this chapter concludes the presentation of data for the linear measurements a chart of annual increments plotted to the same arithmetic scale may be introduced so that the absolute increments may be compared. Such a chart is Figure 78. While this chart emphasizes the almost microscopic character of the annual increments in chest depth, the rate curves of Figure 79 emphasize the fact that relative growth in chest depth is almost as great as that of other linear measurements.

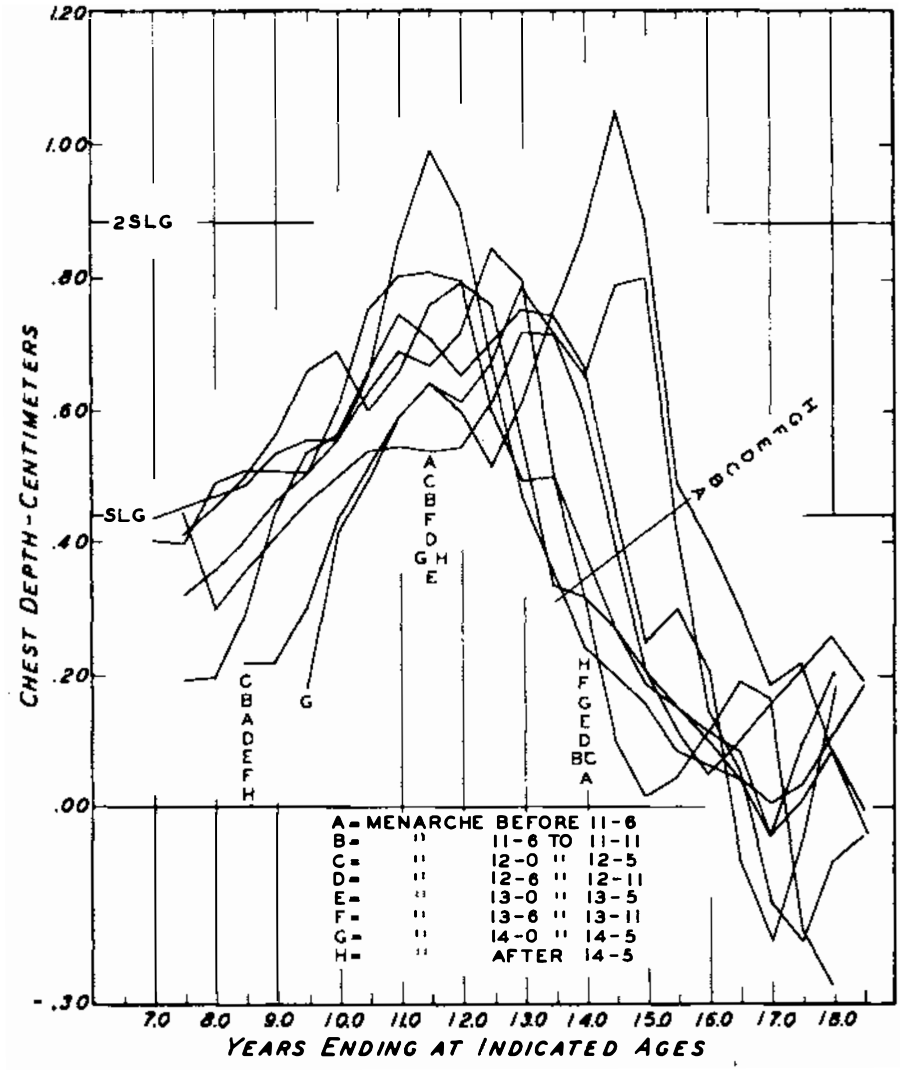

Fig. 73.--Average Annual Increments in Chest Depth of Eight Groups of Girls Menstruating at Different Ages.

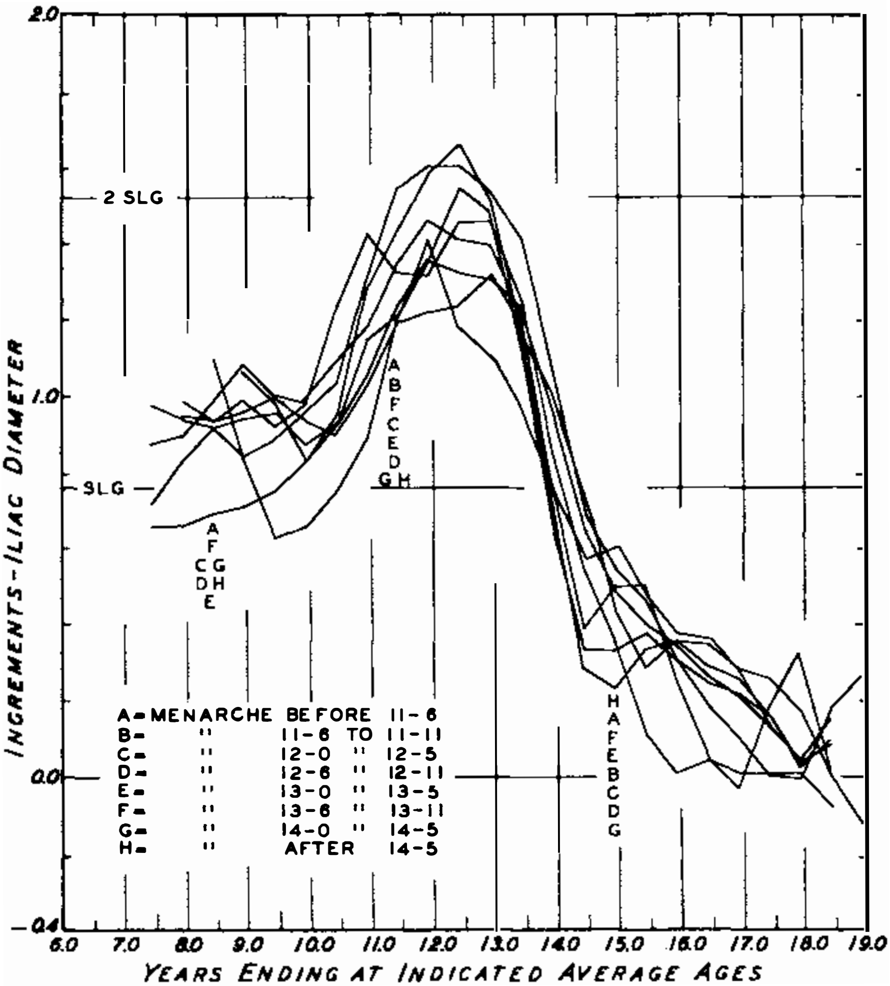

Fig. 74.--Average Annual Increments in Iliac Diameter of Eight Groups of Girls Menstruating at Different Ages Arranged so that Corresponding Points on the Pattern of Growth are in the Same Vertical Line.

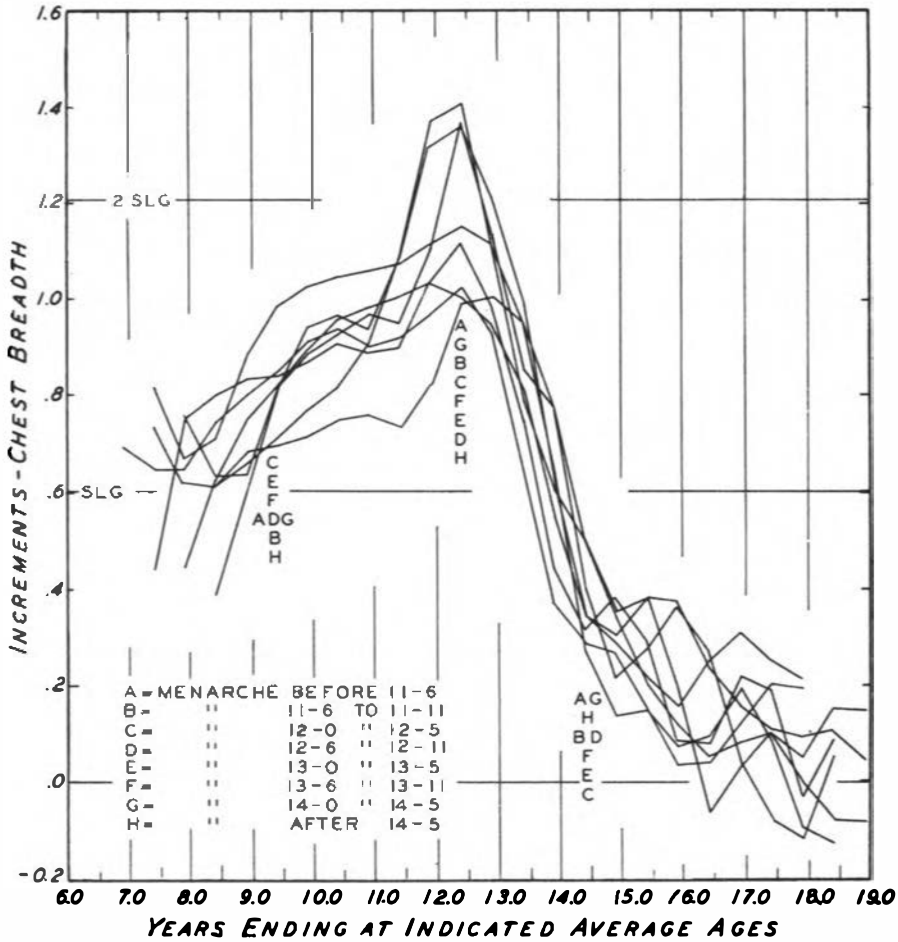

Fig. 75.—Average Annual Increments in Chest Breadth of Eight Groups of Girls Menstruating at Different Ages Arranged so that Corresponding Points on the Pattern of Growth are in the Same Vertical Line.

Both similarities and differences among these patterns should be observed. Of particular interest is the fact that the accelerating phase of growth in iliac diameter is initiated at approximately the same average age as the accelerating phase of sitting height and leg length. The patterns of chest breadth and chest depth resemble each other more closely than other measurements both in terms of increments and rates of growth. Both patterns show a slight arresting of accelerated growth about a year and a half before the peak of the curve is reached and this is followed by about a year of very intense acceleration. This curious coincidence, however, is probably due to chance

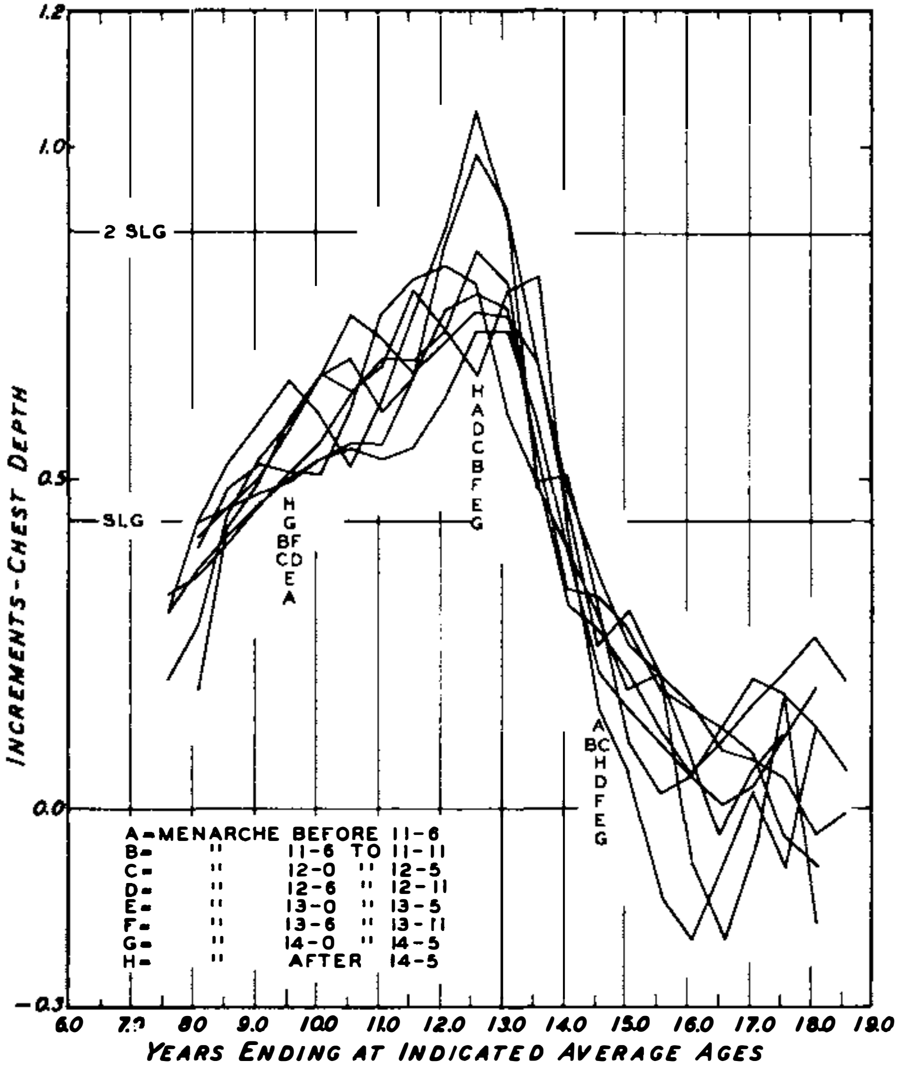

Fig. 76.--Average Annual Increments in Chest Depth of Eight Groups of Girls Menstruating at Different Ages Arranged so that Corresponding Points on the Pattern of Growth are in the Same Vertical Line.

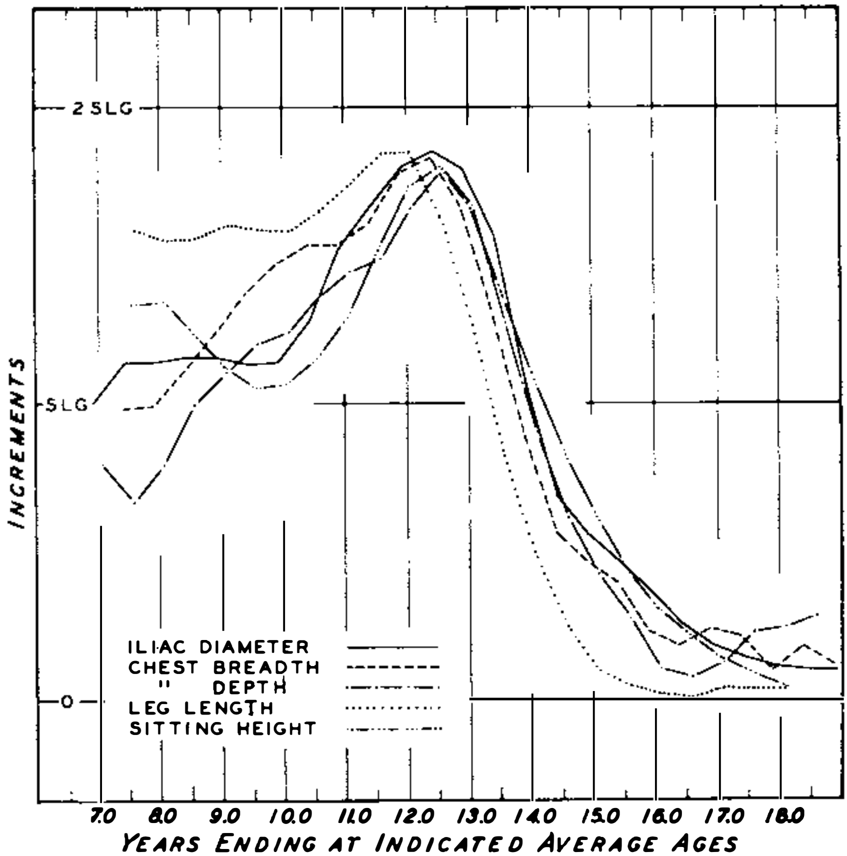

Fig. 77.—Generalized Patterns of Annual Increments in Sitting Height, Leg Length, Iliac Diameter, Chest Breadth and Chest Depth.

factors since it is determined by fluctuations of less than five hundredths of a centimeter.

Beyond these similarities it is readily apparent that each dimension has its own characteristic pattern of growth. In addition to these characteristic differential patterns, there is a significantly differential timing of the periods of maximum growth. Anticipating data for body weight the maximum increments occur for the years ending on the average at age 12.15 (or 11.65) for leg length, at 12.17 for standing height, at 12.40 for chest breadth, at 12.43 for iliac diameter, at 12.55 for sitting height, at 12.62 for weight and at 12.64 for chest depth. It should be recalled that the menarche occurs on the average at 12.97. While the method of computing these values is rather unreliable considering the very small differences involved, the general trend is nevertheless unquestionably significant. The most convincing demonstration of a significant sequence is given by an examination of each

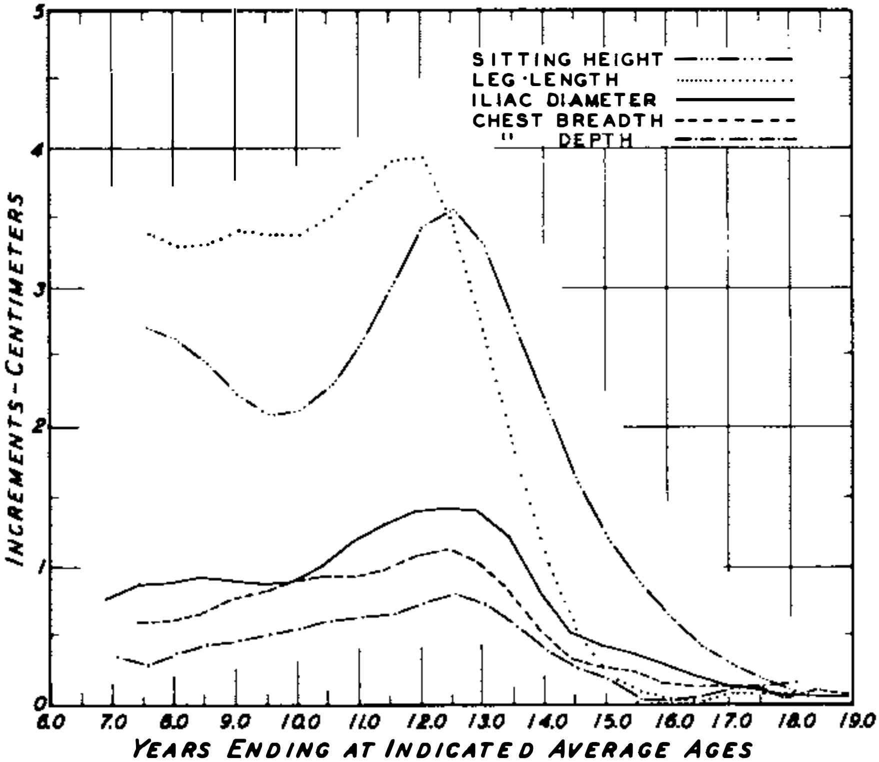

Fig. 78.—Generalized Patterns of Annual Increments in Sitting Height, Leg Length, Iliac Diameter, Chest Breadth and Chest Depth Plotted to the Same Arithmetic Scale.

individual record to determine whether the maximum increments of the various measurements come one year before, coincident with, or one year after the maximum increment in sitting height. Using the 232 cases employed in a similar analysis for Chapter VI and arranging the measurements in their apparent sequence gives the following tabular data.

	Leg Length	Standing Height	Chest Breadth	Iliac Diameter	Chest Depth	Weight
Year before	61.0%	30.2%	30.8%	29.2%	21.3%	16.7%
Coincident	31.8%	65.8%	49.8%	40.2%	44.2%	50.0%
Year After	7.2%	4.0%	19.4%	30.6%	34.5%	33.3%
Total %	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
Total N	223	228	201	209	197	228
Uncertain	9	4	31	23	35	4
Grand Total	232	232	232	232	232	232
Average Age	12.01	12.29	12.44	12.54	12.68	12.72

The probable errors of the percentages range from .9% to 2.4%. The

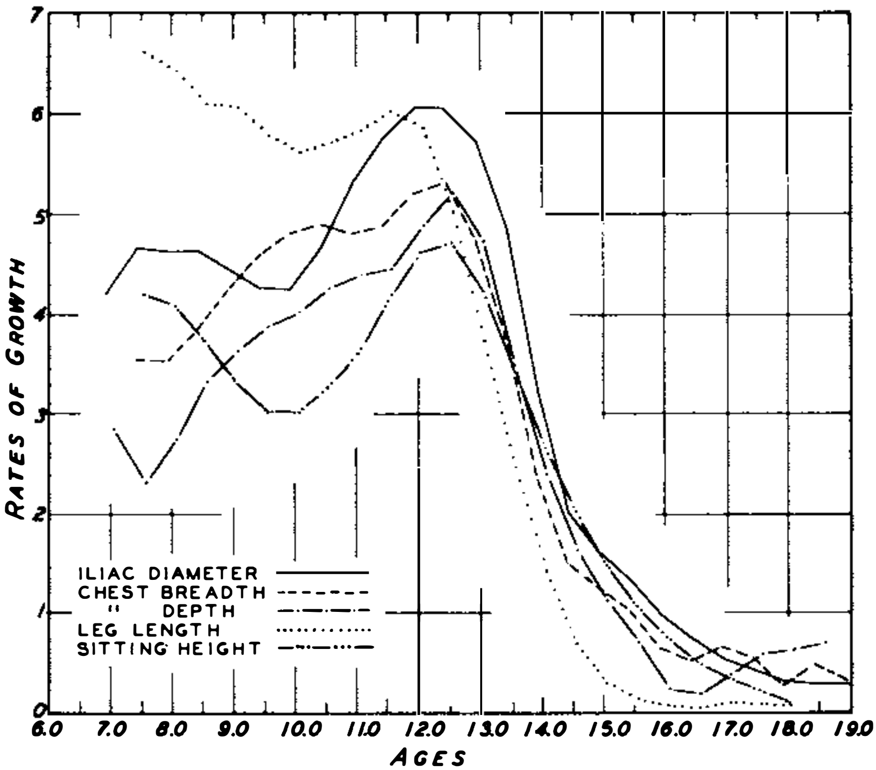

Fig.79.—Percentage Rates of Growth Per Year of Generalized Patterns of Sitting Height, Leg Length, Iliac Diameter, Chest Breadth and Chest Depth.

percentages for leg length are significantly different (four or more probable errors) from those for standing height and, of course, significantly different from all other measurements. Similarly, the percentages for standing height are significantly different from all other measurements; the percentages for chest breadth are significantly different from all other measurements except iliac diameter (3.9 probable errors); the percentages for iliac diameter are significantly different from all other measurements except chest breadth and chest depth; the percentages for chest depth are significantly different from all other measurements except iliac diameter and weight. Considering the fact that only annual measurements are available, the reliability of the differential sequence is unexpectedly clear. Taking 12.55 as the average marking the maximum increment in sitting height and computing intervals before or after this point from the percentages gives the new series of average ages recorded in the tabular data. With the exception of reversing the position of chest depth and weight, these data based on an analysis of individual records arrange the maximum increments of the seven measurements in the same order as do the generalized patterns.

4. Summary of Findings for Iliac Diameter, Chest Breadth, and Chest Depth

In spite of very serious limitations in the original data the findings for iliac diameter, chest breadth, and chest depth confirm the specific hypotheses of this study. The summaries for standing height, sitting height, and leg length might be repeated almost word for word in the case of iliac diameter, the chief differences being as follows: the initial decelerating phase is only apparent in the rates of growth and the transition from the second phase of accelerating growth to the third phase of decelerating growth is very gradual. The patterns for chest breadth and depth, however, show only two phases of acceleration and deceleration. While their patterns are similar, the maximum increments in chest breadth occur about a year earlier than those of chest depth.

All the linear measurements consistently show the following. (a) Each measurement has its characteristic pattern. (b) Each pattern shows three phases: slow or decelerating growth, accelerating growth, and decelerating growth. (c) The inflection points, particularly between the second and third phase, are timed in relation to the advent of the menarche. (d) The patterns of the eight menarcheal groups in relation to chronological age are sharply differentiated. (e) When chronological age is ignored and corresponding points of each pattern are superimposed, the patterns of the eight menarcheal groups are very similar. (f) The growth of early maturing groups is distinctly more rapid and intense and the first or decelerating phase occupies a shorter period of time.

CHAPTER VIII

BODY WEIGHT

So much attention has been devoted to the growth in body weight of rats and man that this measurement has achieved a status and prestige of its own. The prestige of weight as the measurement par excellence in studies of growing organisms is well founded. It is the most practical and objective measurement and in many cases it is the only practical and objective measurement. Its importance in relation to physical well-being is conceded. It is almost exclusively used in experimental studies of diet and nutritional problems. It is more sensitive than any other measurement to environmental influences. For the purpose of the present study, however, the very sensitivity of weight to external influences makes it a less suitable measurement for the reason that attention in this study is focused on the role of internal biological factors.

1. Individual Patterns of Growth in Weight

In the chapter on iliac diameter, chest breadth, and chest depth some space was devoted to the limitations of the data. The irregularities of the individual records were attributed to measurements over clothing, to the inherent difficulties of objective measurement, to the small range of individual differences, and to the small amount of growth per year. The individual growth curves in body weight are also irregular but this irregularity can hardly be attributed to any of the above factors. With rare exceptions weights were taken in the morning session of the school day thus eliminating diurnal factors. While weights were taken over clothing this does not necessarily introduce a variable factor although it does introduce a systematic error. While weights were not systematically taken in duplicate and triplicate, the scales were regularly checked so that the only errors in the records must be outright mistakes of five or ten kilograms in reading the scales. A systematic search has been made for erratic figures which might be attributed to such mistakes, but only four measurements out of over two thousand were rejected.

Partly because individual growth records are of considerable interest and partly for the purpose of illustrating the irregularities of the individual records, Figures 80, 81, and 82 present trends in gross size and in annual increments for the twenty cases whose individual records in standing height were charted in Figures 30, 31, 32. Ten of these cases menstruated approximately at age 11.0 (10-7 to 11-3, dash lines) and the other ten approximately at age 15.0 (14-5 to 15-2, solid lines).

Figure 80 presents the individual growth curves of these early and late maturing groups of cases. Three features of this chart should be noted and compared with the like features of Figure 30. There is, first, a clear differentiation in average size in spite of much overlapping of individual records. This differentiation is rather constant throughout the age range; whereas, in the case of standing height the

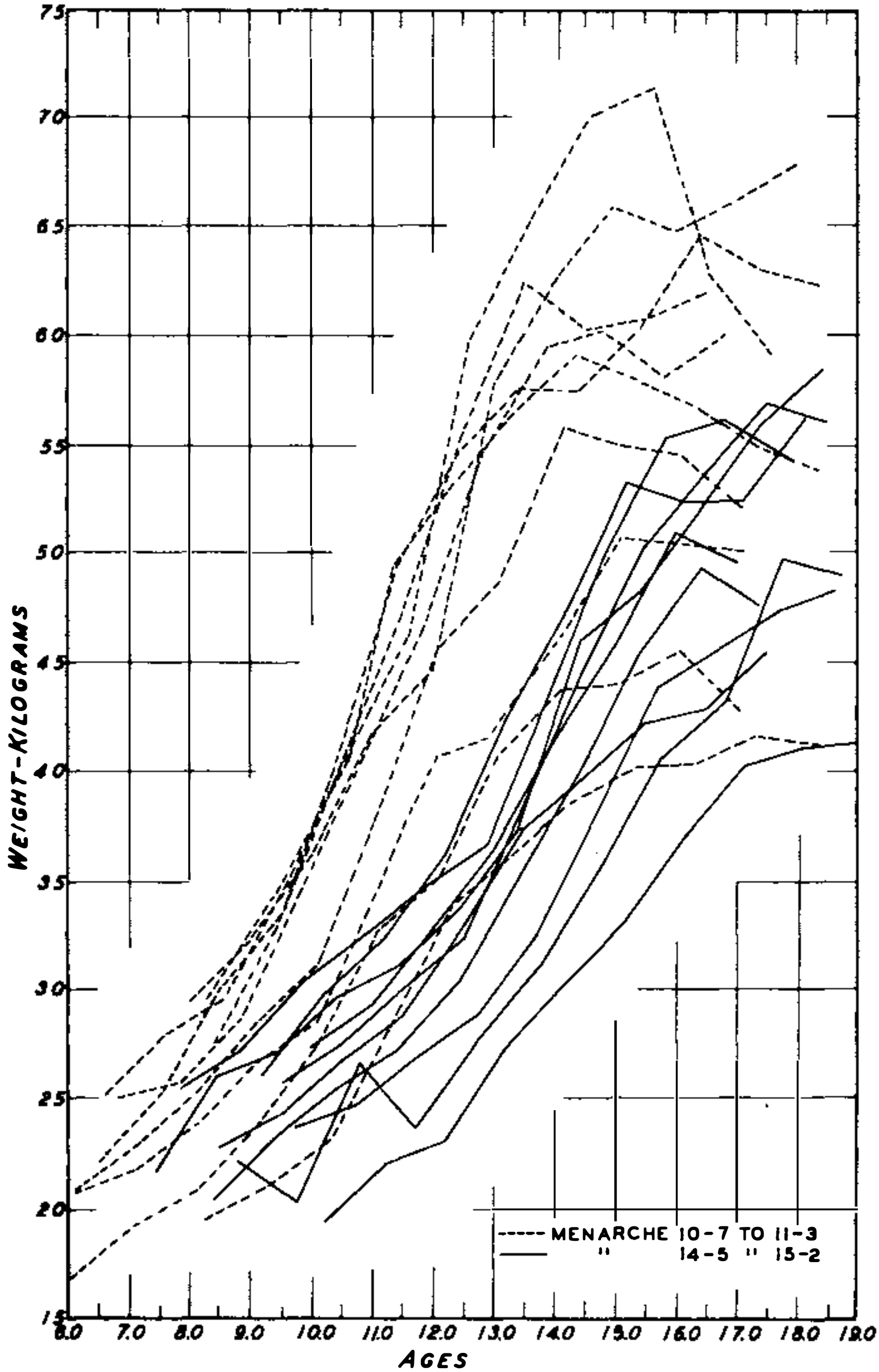

Fig. 80.—Individual Growth Curves in Weight Plotted to Exact Ages of Ten Cases Menstruating Between 10-7 and 11-3 (Dash Lines) and of Ten Cases Menstruating Between 14-5 and 15-2 (Solid Lines).

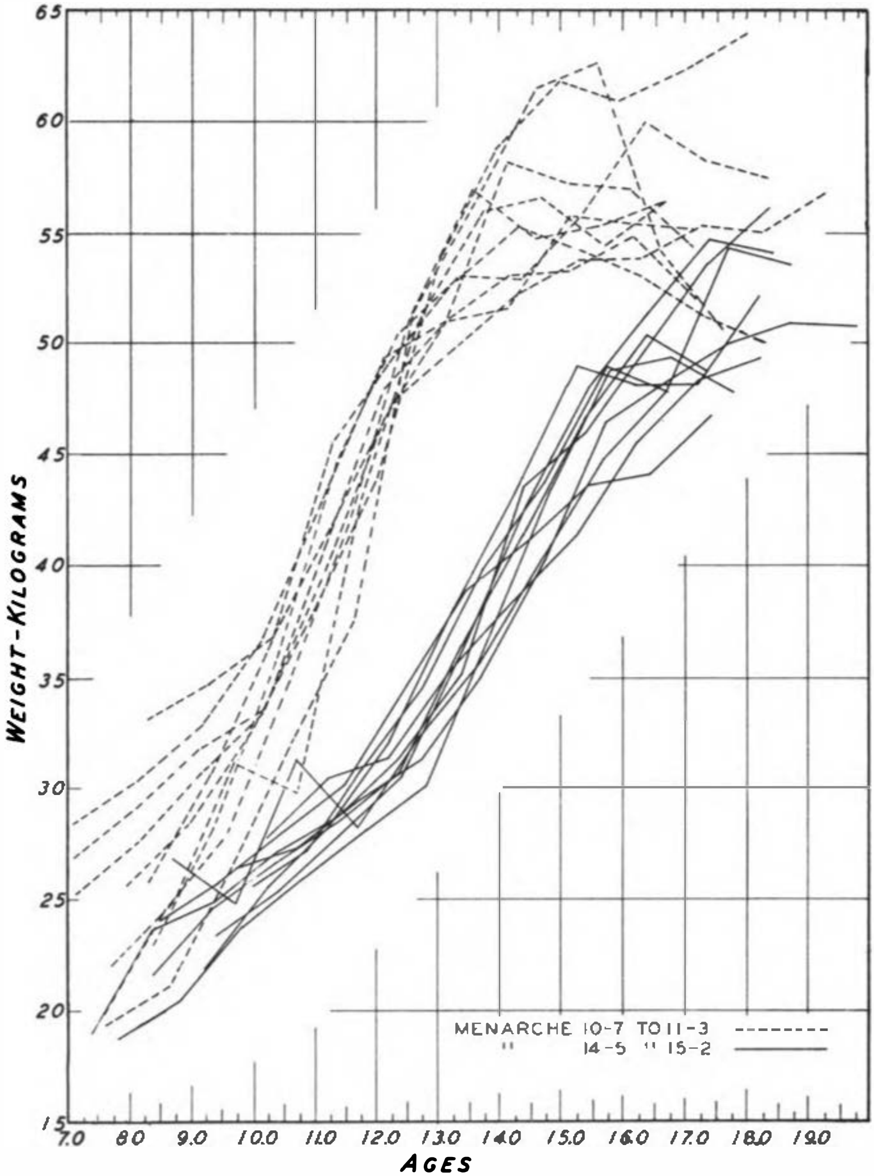

Fig. 81.--Individual Growth Curves in Weight of Figure 80 Redrawn to Eliminate Differences in Average Weight Within Each Group While Reproducing Exactly the Details of Each Individual's Pattern of Growth.

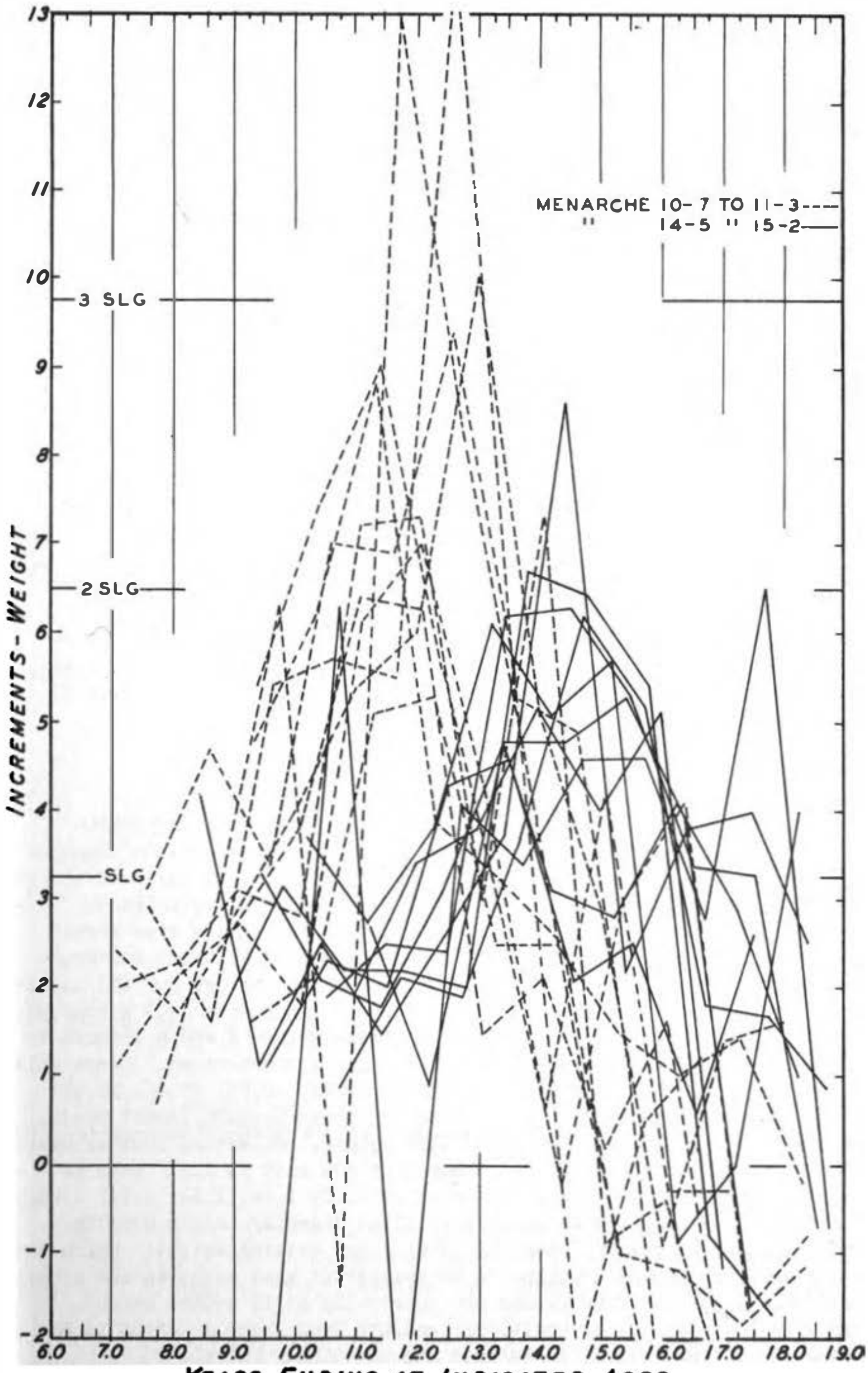

Fig. 82.--Individual Increment Curves in Weight of Ten Cases Menstruating Between 10-7 and 11-3 and of Ten Cases Menstruating Between 14-5 and 15-2.

differentiation is readily apparent only during the middle range of ages. Second, from age nine to thirteen the individual curves of the early maturing group show much steeper upward trends than those of the later maturing group; whereas, from age thirteen to seventeen this situation is reversed. Comparable phenomena appear in Figure 30. Third, Figure 80 presents a graphic picture of the idiosyncracies and irregularities of each individual growth record. These irregularities are much more evident than in the case of standing height.

Figure 81 presents a redrawing of the curves of Figure 80 which eliminates the differences in weight within each group while reproducing exactly the details of each individual curve. In contrast to Figure 31 it is readily apparent that the individual growth curves in weight conform much less closely to the group patterns. It should be obvious that a mathematical equation with a limited number of constants will rarely give a good fit to any individual weight curve. The contrary, however, tends to be true of the individual curves in standing height.

The irregularity of the patterns generated by the individual weight records in contrast to the regularities of those in standing height is most clearly illustrated by the increment curves of Figures 82 and 32. Even Figure 82 does not give a true picture since two increments exceed 13.0 kilograms and seven fall below -2.0 kilograms. The total range of the increments is from 13.6 to -8.5 kilograms which would require for full representation a 50% greater vertical distance on the chart. Given these fantastically irregular trends, the extraordinary feature of Figure 82 is that there is nevertheless a complete absence of overlapping increments of early and late maturing groups from age 11.0 to 12.3 and, with one exception, from age 15.5 to 15.9.

2. Growth Trends in Average Weight

Growth trends in average body weight for each of the eight menarcheal groups are presented in Figure 83. There is a curious tendency for the curves of Groups A, B, and C and of Groups D, E, and F to follow the same general trend and this tendency reappears in the chart of annual increments. The very large differences in average size should be noted. The greatest difference occurs at age 13.0, Group A averaging 18.29 kilograms heavier than Group H. On the average for all available cases growth from age eight to seventeen amounts to 29.0 kilograms. On this basis the greatest difference between Groups A and H amounts to 63.0% of the average growth between age eight and seventeen. Comparable figures for other measurements are 55.5%, 52.9%, 49.5%, 37.4%, 35.4% and 34.8% respectively for iliac diameter, chest breadth, chest depth, leg length, sitting height and standing height. Selecting another basis for comparison, it appears that Group A at the most is 1.51 times as heavy as Group H. Group A at the most is 1.16, 1.14, 1.13, 1.11, 1.09, and 1.08 times bigger than Group H in iliac diameter, chest breadth, chest depth, leg length, standing height, and sitting height. While the early maturing groups continue to be heavier at ages eighteen and nineteen, the later maturing groups are apparently still adding weight. Since the overlapping of individual weights from group to group is extreme the differences at these ages are not highly reliable.

In Figure 84 these curves are rearranged so that points corresponding to the advent of the menarche are in the same vertical line.

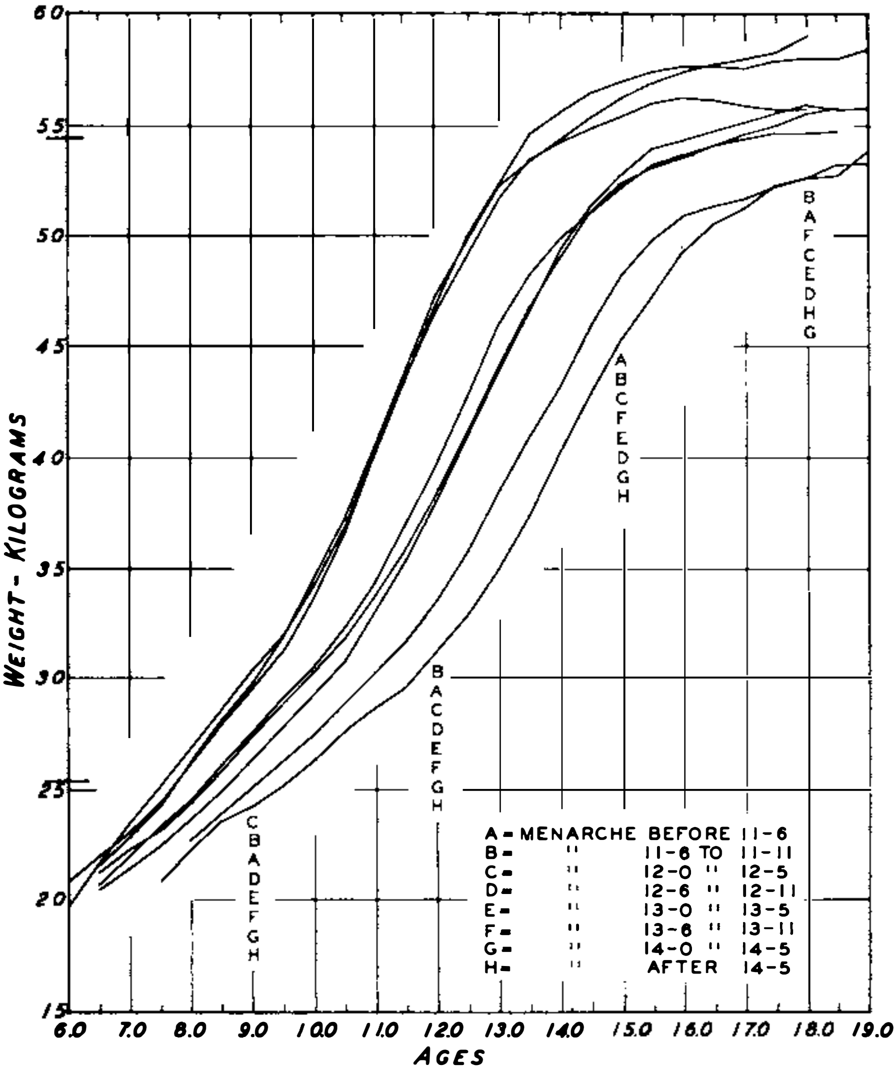

Fig. 83.--Growth Trends in Average Weight of Eight Groups of Girls Menstruating at Different Ages.

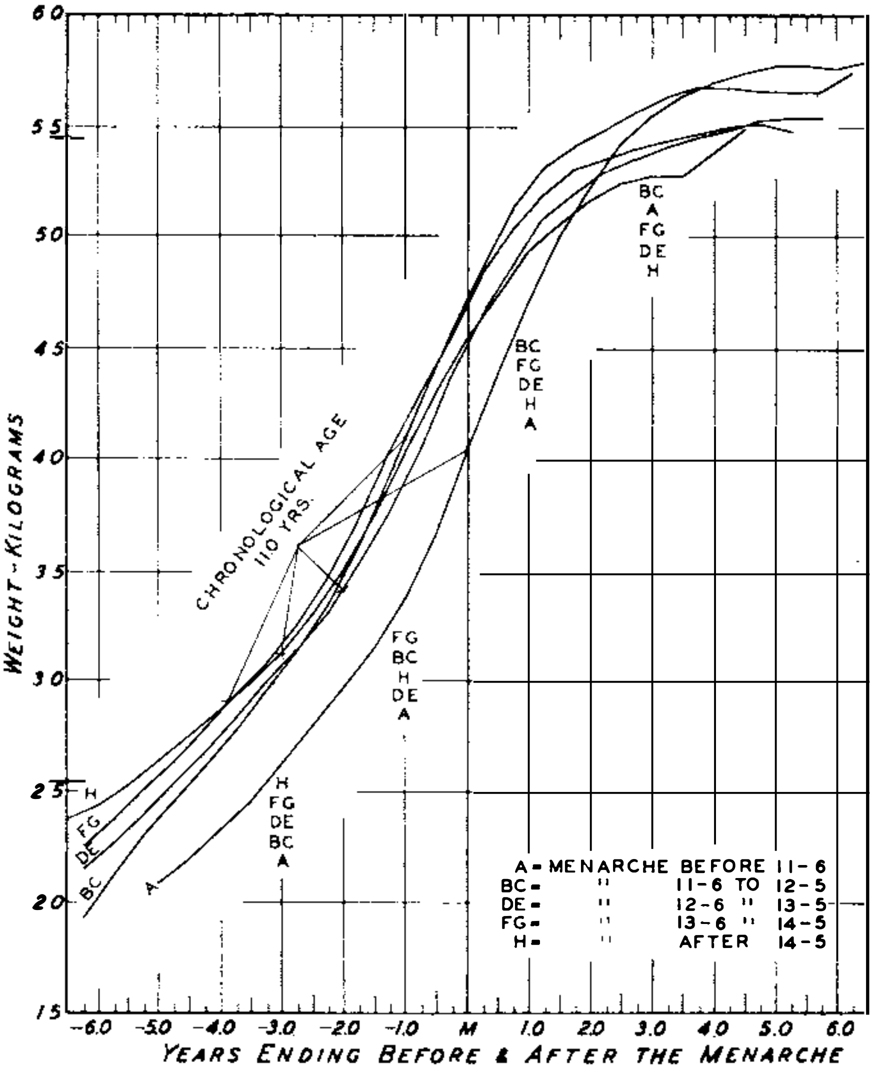

Fig. 84.--Growth Trends in Average Weight of Five Groups of Girls Menstruating at Different Ages Arranged so that Points Corresponding to the Advent of the Menarche are in the Same Vertical Line.

This chart seems to present features not found in comparable charts of other measurements, but first impressions are illusory. It is true that the tendency for the differences to increase for longer time intervals before the menarche and to decrease for longer time intervals after the menarche is absent. The more fundamental differential slopes, however, are strongly in evidence. Three years before the menarche the heaviest group is H, followed by FG, DE, BC, and A in the usual order, the only difference being that the contrasts are not as great as in the case of other measurements. Four years after the menarche this order is almost exactly reversed. Such a reversal of position is not found in other measurements and is to be attributed to the fact already noted in Figure 83 that the differentiation of the groups is maintained even to chronological ages eighteen and nineteen. It follows from this reversal of position that the slope of the curve of the early maturing group is much steeper than that of the late maturing group just as it is in other measurements. If it is assumed that the endocrine stimulation begins to operated three years before the advent of the menarche, then the individuals of Group A are stimulated on the average at age 8.0 when they weigh on the average only 27.94 kilograms while the individuals of Group H are stimulated on the average at age 12.0 when they weigh on the average 31.27 kilograms. Hence, to attain the same or a greater mature weight the individuals of Group A must grow at a faster rate.

3. Annual Increments and Rates of Growth

Figure 85 presents the annual increments of the eight menarcheal groups. While these curves are not as smooth as might be desired, for the most part the eight groups fall in the usual order. Figure 86 superimposes these curves so that corresponding points on the pattern of growth come as nearly as possible in the same vertical line. Attention is called to the fact that the increments of the early maturing groups are generally larger than those of the late maturing groups. Weighting and averaging these curves and superimposing the corresponding generalized patterns for leg length, sitting height, and iliac diameter give Figure 87. The pattern of increments for the years ending on the average from age 10.0 to maturity are strikingly similar for sitting height, iliac diameter, and weight save that the decelerating phase of growth in weight is not as sharp. Prior to this period the curve of weight increments shows a definite and rather steady acceleration. In contrast, the trends for leg length and iliac diameter are horizontal or neutral and that for sitting height is downward. There is much less resemblance between the generalized pattern of weight and of chest breadth and depth. (See Figure 77).

The fact that weight is an over-all or volume measurement instead of a one dimensional linear measurement results in a very great magnification of the percentage rates of growth per year. At the most the linear variables show an increase of about five or six per cent per year, but any volume measurement must show much greater percentage rates. For example, if the sides of a cube are increased five per cent, the volume is increased 15.8%. Hence, Figure 88, which involves the same data as Figure 87 expressed in terms of percentage rates of growth per year, gives an altogether different picture. The percentage rates of growth in weight per year are so large that it has been necessary to

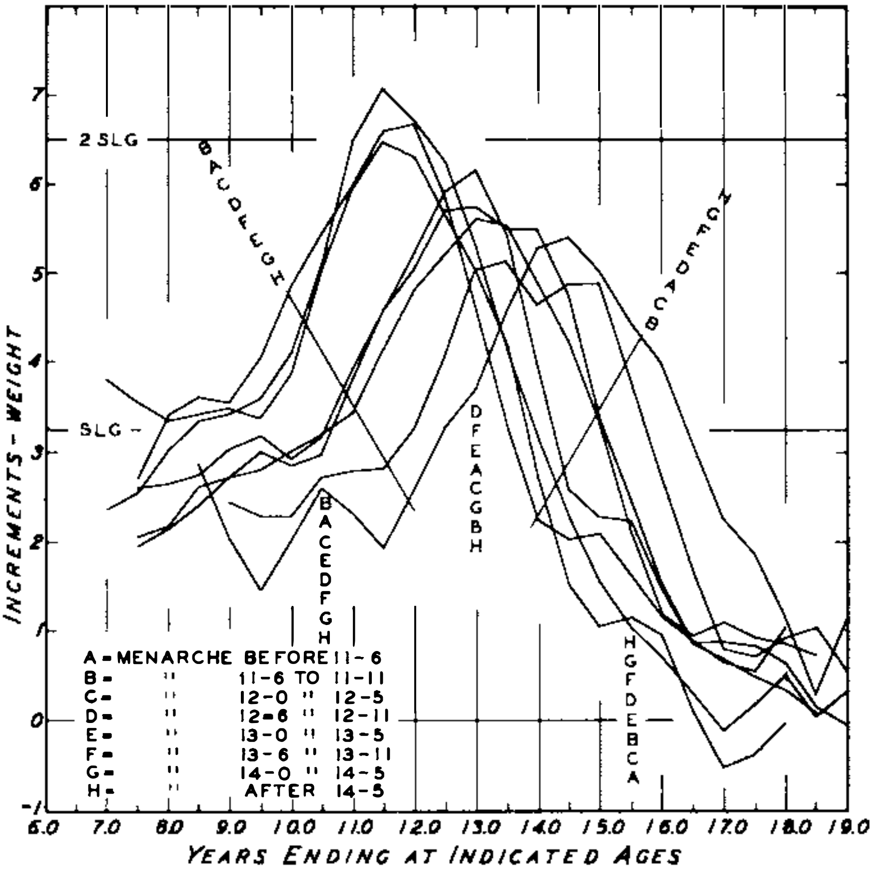

Fig. 85.--Average Annual Increments in Weight of Eight Groups of Girls Menstruating at Different Ages.

reduce the scale at the left to one third of that in Figure 79. Figure 88 is of interest because it reveals a sharp change from constant or slightly declining rates of growth in body weight to sharply accelerating rates. The similarity of this curve to the generalized pattern of increments in standing height (dash curve of Figure 39) may not be very significant but is too striking to pass by without notice. Figure 89 presents percentage rates of growth for each of the eight groups.

Figures 90 and 91 arrange the curves of weight increments and rates of growth so that points corresponding to the advent of the menarche are in the same vertical line. These charts show that the periods of greatest acceleration in weight, just as in other measurements, are closely related to the advent of the menarche.

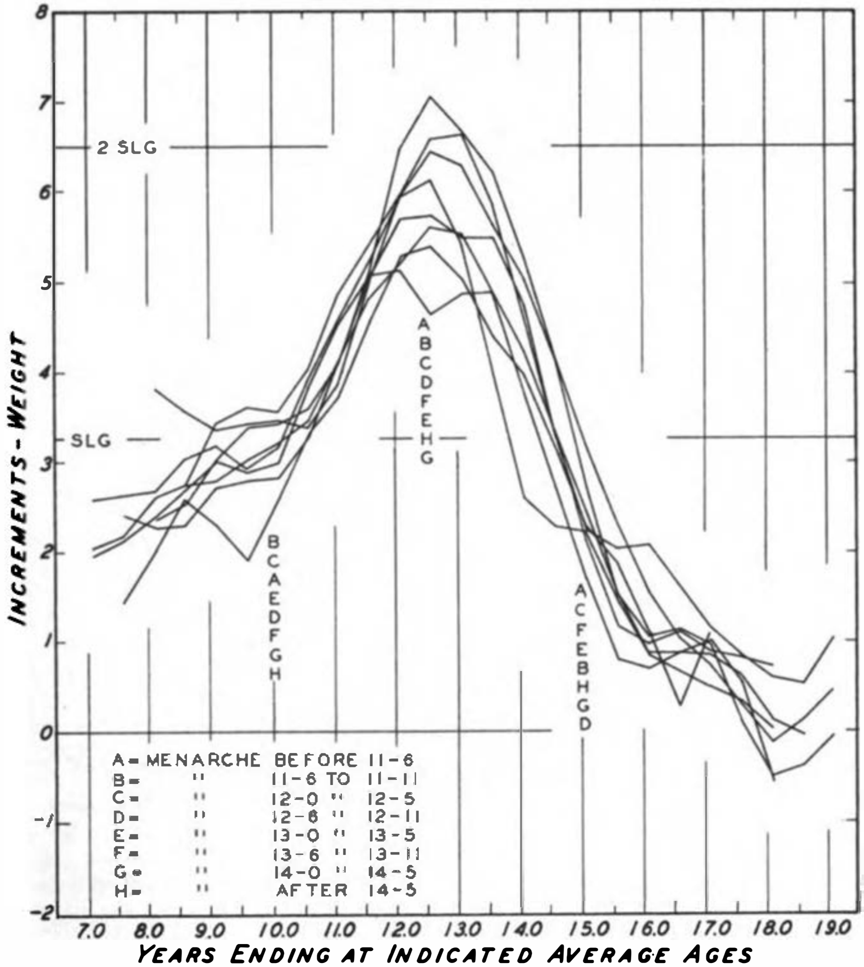

Fig. 86.—Average Annual Increments in Weight of Eight Groups of Girls Menstruating at Different Ages Arranged so that Corresponding Points on the Pattern of Growth are in the Same Vertical Line.

4. Weight in Relation to Other Measurements

The relation of body weight to other measurements and particularly to skeletal measurements has constituted one of the most persistent foci of interest. The literature is full of indices of weight relative to standing or sitting height or to the square or cube of these measurements. In large part this interest arises from the conviction, formalized in height-weight for age standards, that such indices will provide clues to physical fitness or nutritional status. Data on the eight

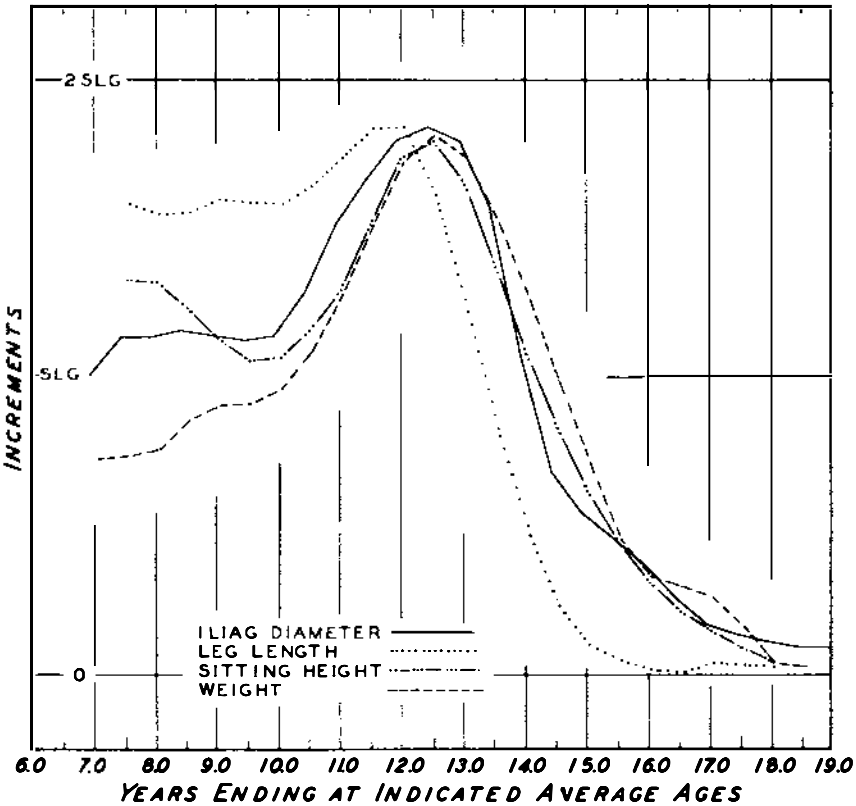

Fig. 87.--Generalized Patterns of Annual Increments in Weight, Sitting Height, Leg Length and Iliac Diameter.

menarcheal groups who are clearly distinguished in average size reveal some unexpected aspects of this general problem.

Attention has already been called to the fact that the differences in average weight of early and late maturing groups are greater than in other measurements. It follows that the early maturing groups are heavier per unit of linear measurement than the late maturing groups. Figure 92 presents sample trends resulting from relating weight to sitting height, leg length and "thickness" (the sum of averages for iliac diameter, chest breadth and chest depth). At age 13.0 Groups A and H weigh .64 and .46 kilograms per centimeter of sitting height, .72 and .51 kilograms per centimeter of leg length, and .77 and .58 kilograms per centimeter of thickness. That is, at age 13.0 Group A averages from 33% to 40% heavier per unit of linear measurement. From age 7.0 to 17.0 body weight per unit of linear measurement is approximately doubled. While the general similarity of these curves to those of Figure 83 is apparent, it should be noted that the vertical scale at the left in Figure 92 has been magnified about ten times.

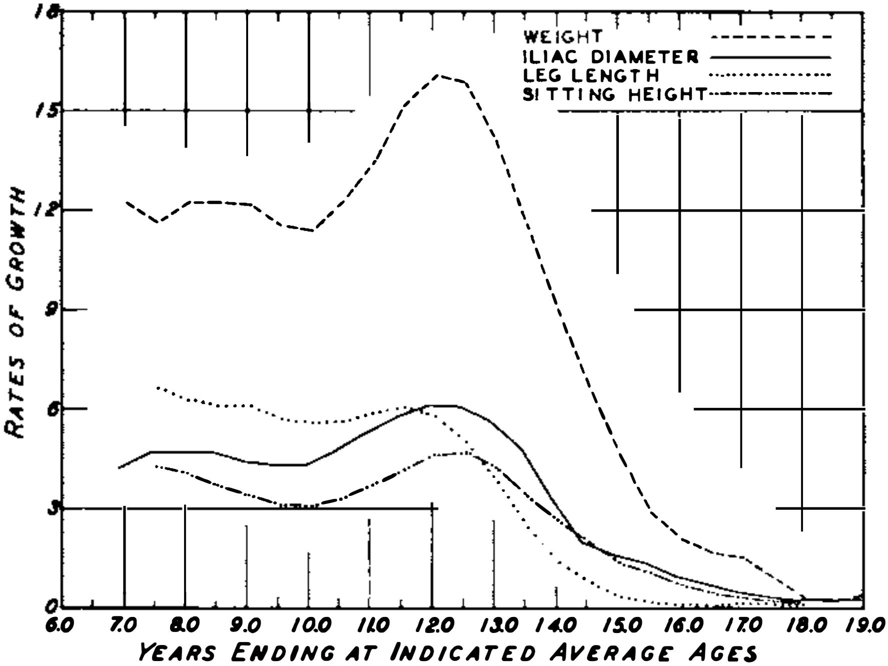

Fig. 88.--Percentage Rates of Growth Per Year of Generalized Patterns in Weight, Sitting Height, Leg Length, and Iliac Diameter.

When weight is related to a linear measurement it is inevitable that the relation will change in a systematic way from age to age for the simple reason that weight is a volume measurement. For example, a cubic container with a side of 100 centimeters has a volume of 1,000,000 cubic centimeters and will hold 100 liters of water weighing 100 kilograms or one kilogram of water per centimeter of linear measurement. If the cubic container is increased so that its three dimensions are 150 centimeters then it will hold 2.25 kilograms of water per centimeter of linear measurement. Under these conditions changing ratios are artifacts of numerical relationships rather than representations of growth phenomena. Accordingly, the only meaningful ratio involving weight must employ a denominator which is expressed in terms of volume, namely, weight relative to the cube of standing height, or weight relative to the cube of sitting height, or weight relative to standing height times chest breadth times chest depth. An even better index is weight relative to volume computed as follows:

$$\frac{\text{Stand.Ht. plus Sitting Ht.}}{2} \times \frac{\text{Chest Breadth plus Iliac Dia.}}{2} \times \text{Chest Depth}$$

Averaging standing and sitting height in effect gives twice as much weight to sitting height as to leg length. Pointing off four places expresses the volume in liters (10,000 cubic centimeters equals a liter and a liter of water weighs a kilogram). The above formula is entirely empirical and represents merely a guess at the most effective method of

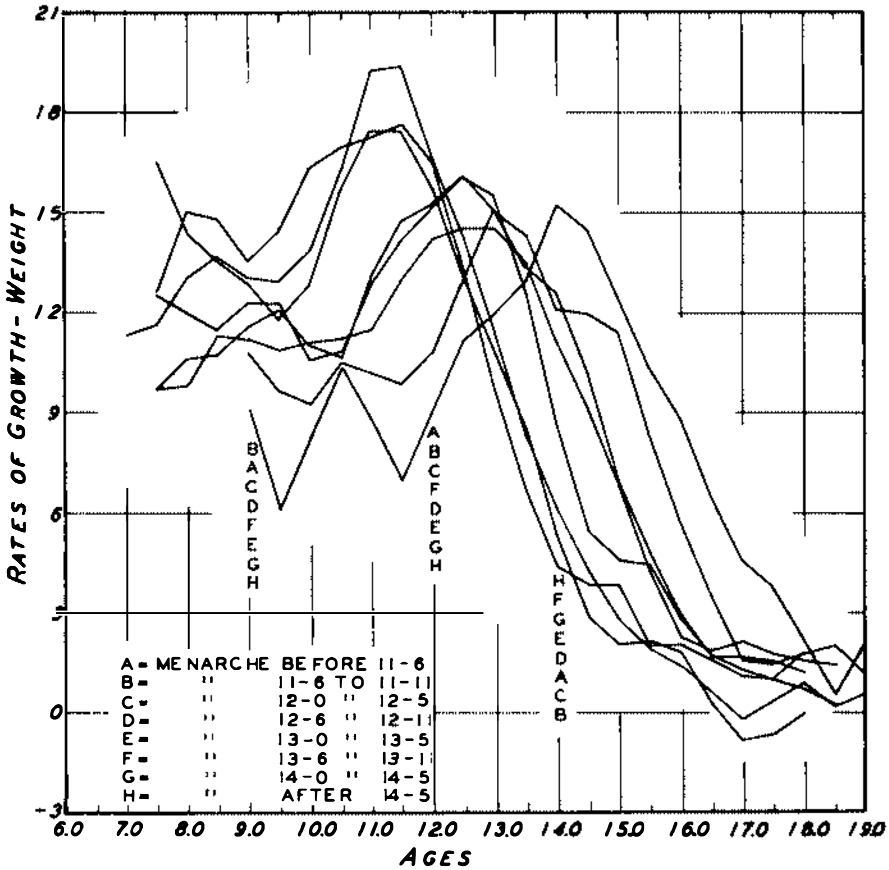

Fig. 89.--Percentage Rates of Growth Per Year in Weight of Eight Groups of Girls Menstruating at Different Ages.

combining the available data. The original intention was to multiply the result by a constant so that on the average body "volume" in liters or over all skeletal size as thus computed would equal weight but it has not been necessary to do this. The resulting indices showing kilograms of body weight per liters of body "volume" for Groups A, D, F, and H are presented in Figure 93. All four curves show an irregularly downward trend to a point corresponding roughly to the advent of the menarche and thereafter an irregularly rising trend. That is, prior to the menarche growth in overall skeletal size is more rapid than growth in body weight while after the menarche the tendency is to add weight more rapidly. These trends, however, are to be attributed in part to the inclusion of leg length in the formula for computing volume.

Figures 83, 92, and 93 should be compared. At the most at age 13.0 Group A is 1.51 times as heavy as Group H (Figure 83). At the most at age 13.0 Group A is 1.40 times as heavy per unit of linear measurement as Group A (Figure 93). At the most at age 14.5 Group A is 1.13 times

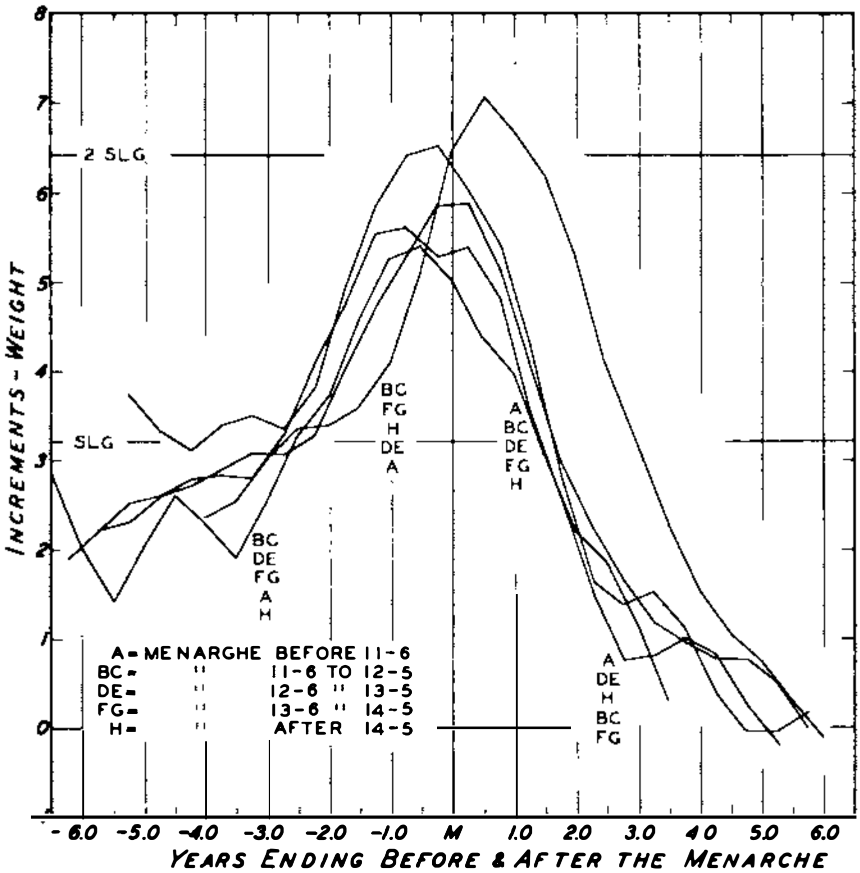

Fig. 90.--Average Annual Increments in Weight of Five Groups of Girls Menstruating at Different Ages Arranged so that Points Corresponding to the Advent of the Menarche are in the Same Vertical Line.

as heavy per liter of body "volume" as Group H (Figure 93). That is, overall skeletal size measured in terms of body "volume" accounts for most of the group differences in average weight.

It is worth noting incidentally that these data add one more item in criticism of height-weight for age standards. Such standards applied to girls during the early teens have a decided tendency to rate late maturing girls as underweight and early maturing girls as over-weight. At the same time the weight relative to volume data suggest either (a) that somewhat similar but very much less serious errors may be involved in standards of normal weight for four skeletal dimensions such as those suggested by Franzen (17) and McCloy (22) or (b) that the late maturing girls actually have a slight tendency to be underweight relative to their skeletal size.

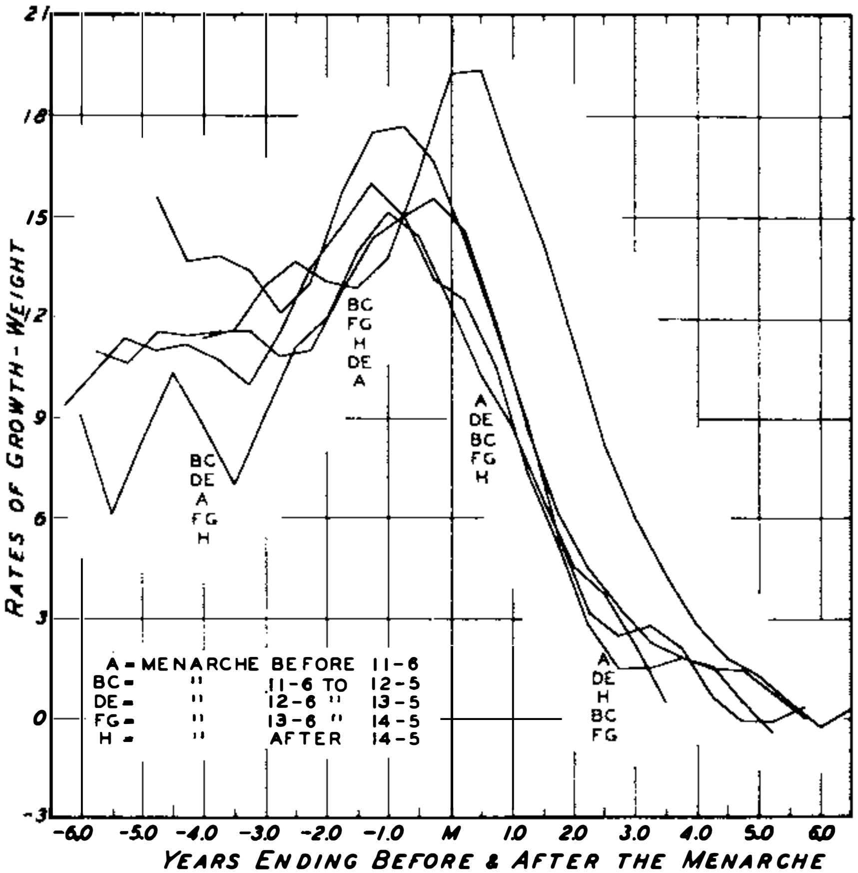

Fig. 91.—Percentage Rates of Growth Per Year in Weight of Five Groups of Girls Menstruating at Different Ages Arranged so that Points Corresponding to the Advent of the Menarche are in the Same Vertical Line.

5. Summary of Findings for Weight

Once more it is true that summaries for standing height or for sitting height and leg length or for iliac diameter, chest breadth and chest depth might be applied almost word for word to growth in body weight. The same three phases of growth are clearly distinguishable: initially, there is a period of slightly accelerating increments and of slightly decelerating relative growth which is followed by a typically accelerating phase and a typically decelerating phase. The inflection points between the second and third phase are timed in relation to the menarche. The patterns of the eight menarcheal groups in relation to chronological age are sharply differentiated. When chronological age is ignored and corresponding points of each group pattern are superimposed,

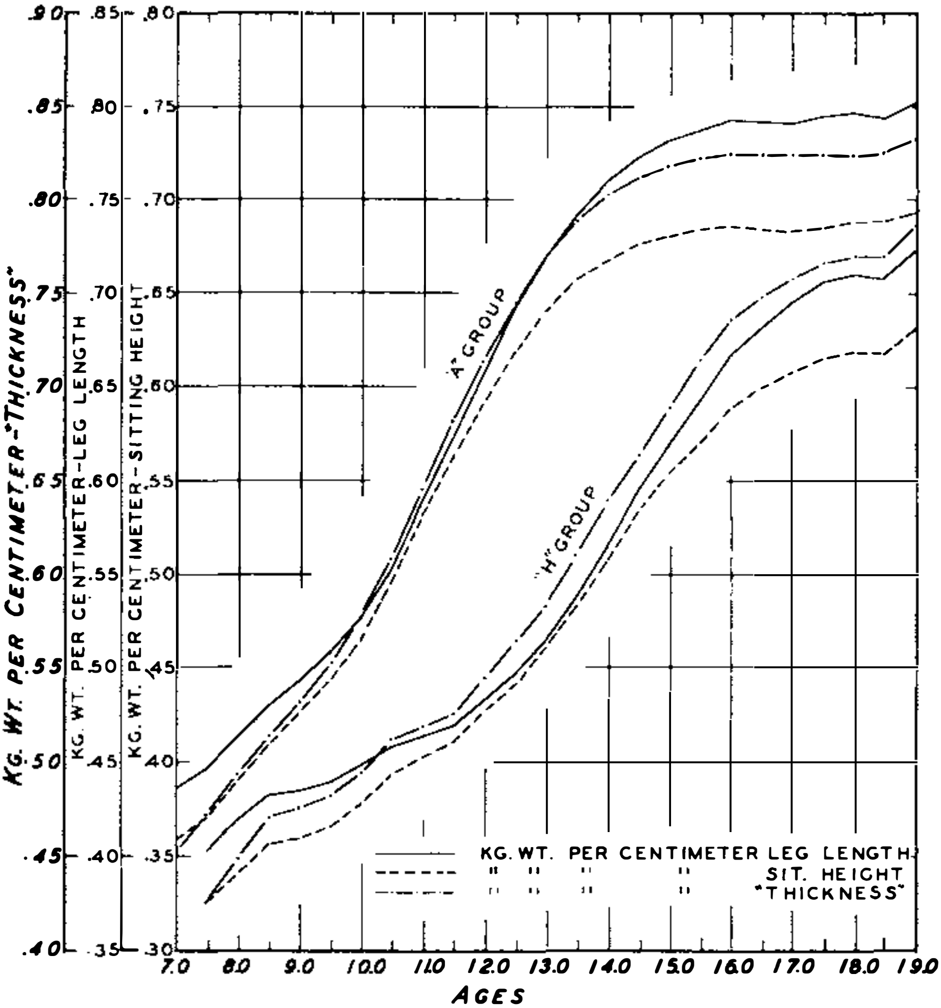

Fig. 92.—Kilograms of Body Weight Per Centimeter of Sitting Height, Leg Length, and "Thickness" of Early and Late Maturing Groups of Girls.

it is apparent that the eight patterns are very similar except that the growth of the early maturing groups is decidedly more rapid and intense.

Growth in body weight just as in all other measurements has its own characteristic pattern. The individual growth curves are distinctly more irregular than those for standing height, reflecting probably the greater influence of environmental factors and the lesser influence of internal biological factors. The most distinguishing feature of body weight is that it is an over-all or volume measurement. It results that the contrasts between early and late maturing groups are greater in the

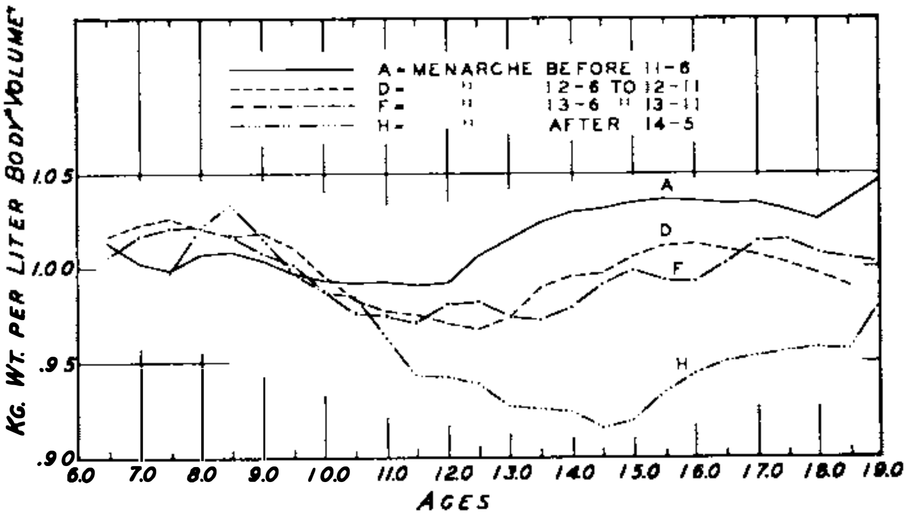

Fig. 93.--Kilograms of Body Weight Per Liter of Body "Volume" of Four Groups of Girls Menstruating at Different Ages.

case of weight. At the age of greatest differentiation Group A averages about 12% bigger in linear measurements than Group H, whereas in the case of weight Group A averages more than 50% heavier than Group H. Similarly, at the age of greatest differentiation, Group A averages about 38% heavier per unit of linear size than Group H. However, when weight is related to body "volume" most of these contrasts tend to disappear, i.e., at the most Group A is only 13% heavier in relation to body volume than is Group H. It is possible that even this difference might disappear with better methods of combining the linear measurements into an estimate of over-all skeletal size.

CHAPTER IX

SKELETAL AGE

The records showing skeletal development in roentgenograms of the wrist and part of the hand are of particular interest because they represent growth in the sense of development as well as growth in the sense of mere increase in magnitude. This very fact, however, introduces difficulties of quantification and measurement. A thorough study of the data might require a series of monographs. Unfortunately, only estimated skeletal ages are available. These were determined by Dr. E. D. West by comparing each X-ray with the standards of T. Wingate Todd (unpublished). The Todd standards were compiled separately for boys and for girls by selecting a typical or median X-ray at each quarterly interval of chronological age from among children of distinctly superior economic backgrounds. An individual skeletal age of 120 months means simply that the individual's X-ray conforms more closely to that of the typical girl age 120 months than to the typical girl age 117 or 123 months. Since skeletal development as shown by X-rays of the wrist and hand is complete in the typical girl of 197 months, the maximum skeletal age is 197 months. It follows from this method of quantification that a growth curve based on the averages of all girls will be a straight line from birth to 197 months at which point the curve will flatten out abruptly to a horizontal trend. Such a curve by its very nature can not represent true accelerations or decelerations of growth. At a given age it can only indicate whether one group is more advanced than another. Over a given age period it can only indicate whether one group is advancing faster than another.

Figure 94 presents the growth trends in average skeletal age of the eight menarcheal groups. The differences at each age in average skeletal development are unusually clear and regular. Figure 95 presents the annual increments. Since the average annual increments of girls in general are arbitrarily defined as increments of twelve months from birth to age 197 months it follows that the increments of the separate groups must conform closely to twelve months per year. Hence, the curves approximate straight lines. Nevertheless, the differential patterns of the eight groups can be observed. For the years ending at ages 10.0 to 12.0 the groups are in almost regular order. For the year ending at 16.0 the increments of the groups are in regular but exactly opposite order. Since these increments are merely relative from group to group at a given age, it follows that increments at one age period are not comparable to increments at another age period. Hence, there is no point in subjecting the data to further analysis.

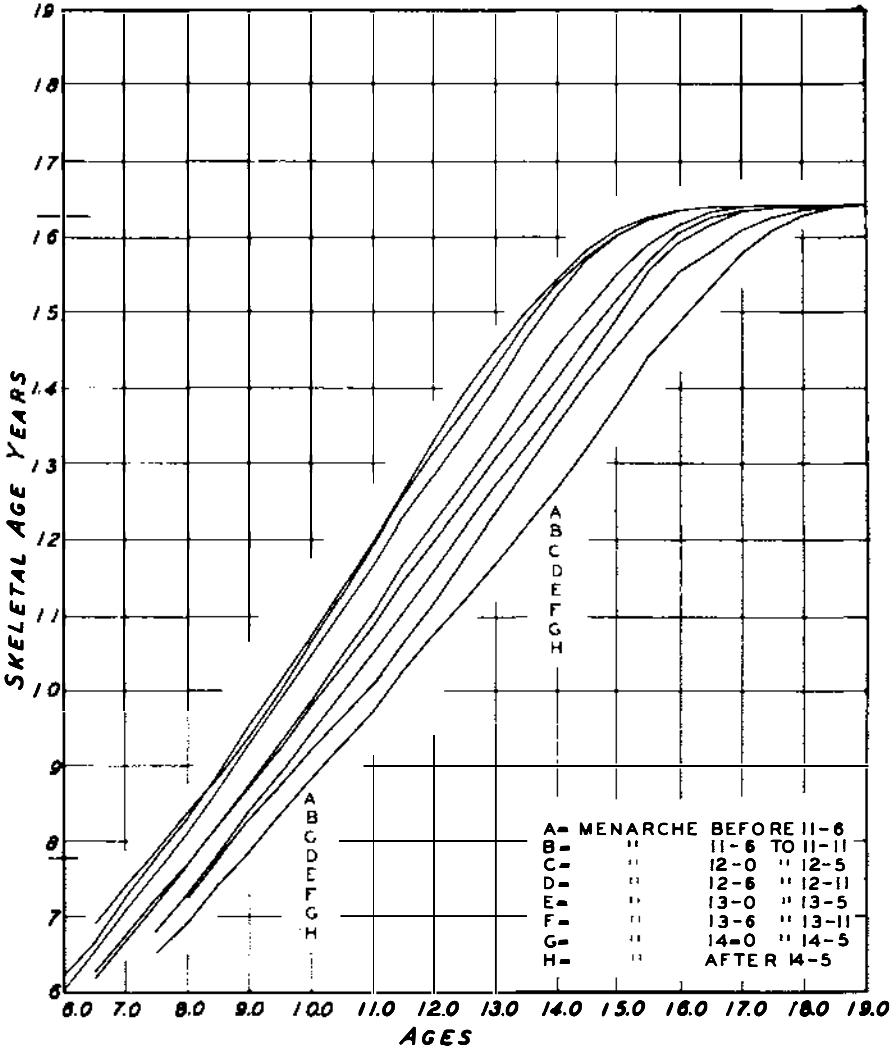

Fig. 94.--Growth Trends in Average Skeletal Age of Eight Groups of Girls Menstruating at Different Ages.

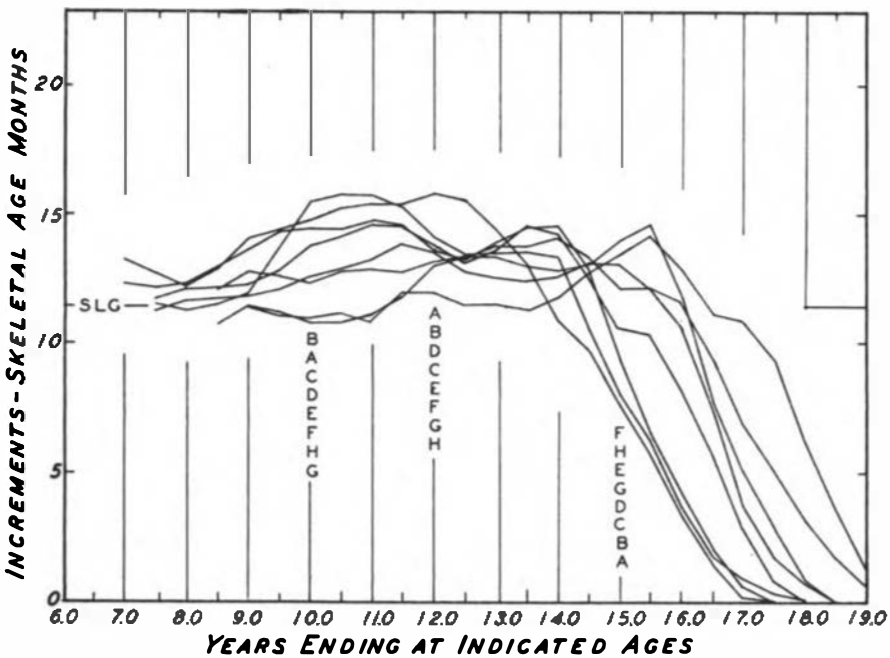

Fig. 95.--Average Annual Increments in Skeletal Age of Eight Groups of Girls Menstruating at Different Ages.

CHAPTER X

ERUPTED TEETH

Each year of the Harvard Growth Study a count was made of the number of permanent teeth fully erupted, half way erupted, and partially erupted. Since the most intense period of growth occurs prior to the visible eruption of teeth, these distinctions have been abandoned and the analysis has been confined to records showing the total number of erupted permanent teeth regardless of the degree of eruption. The records purport to include as erupted those permanent teeth which have been extracted, but records for the older ages show irregularities which suggest that the counts of erupted teeth failed occasionally to include extracted teeth.

Figure 96 shows the growth trends in the average number of erupted teeth at each chronological age and for each of the eight groups of cases. In this chart there is only slightest suggestion of any differentiation of the eight menarcheal groups. Throughout the age range Groups A, B, and C consisting of individuals menstruating before age 12-6 average one or two more teeth than the five groups menstruating after age 12-6. Selecting for special study ages 9.0 and 9.5, where the greatest differentiation seems to occur, gives the following tabular data for cases menstruating prior to and after age 12-6.

	No. Cases	Mean No. Teeth	Standard Deviation
Menarche prior to 12-6	70	15.56	4.06
Menarche after 12-6	158	14.28	3.21

The early maturing groups are more advanced by more than one tooth and more variable. The differences, however, are not statistically reliable, the difference between the means being only 3.5 times as large as its probable error and the difference between the standard deviations being only 3.1 times as large as its probable error.

Since the curves of Figure 96 show similar slopes for each of the eight menarcheal groups, it can be anticipated that increment curves of Figure 97 will fail to show differences among the groups. Indeed, there is not the slightest suggestion of any differentiation. Quite obviously, the underlying factors which condition the development of teeth are radically different from those determining all the other measurements which have been considered.

Explanations of this situation are naturally hazardous. The writer suspects that the explanation is to be found in the fact that the most intensive development of teeth occurs prior to the onset of the endocrine stimulation which, by hypothesis, is responsible for the timing of the menarche and of patterns of growth in all other available measurements. The period of most active eruption of permanent teeth is from age 9.5 to 10.5 whereas the period of most active growth in other available measurements occurs approximately two years later.

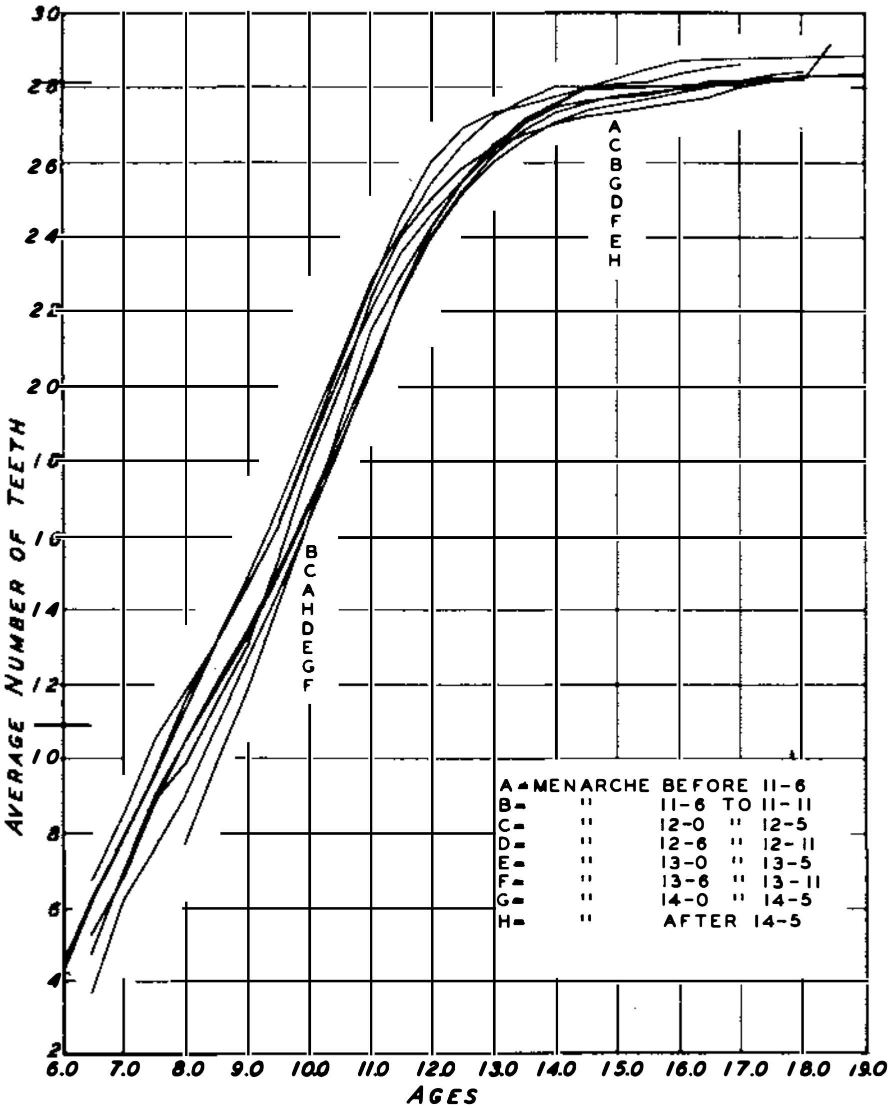

Fig. 96.—Growth Trends in the Average Number of Erupted Permanent Teeth of Eight Groups of Girls Menstruating at Different Ages.

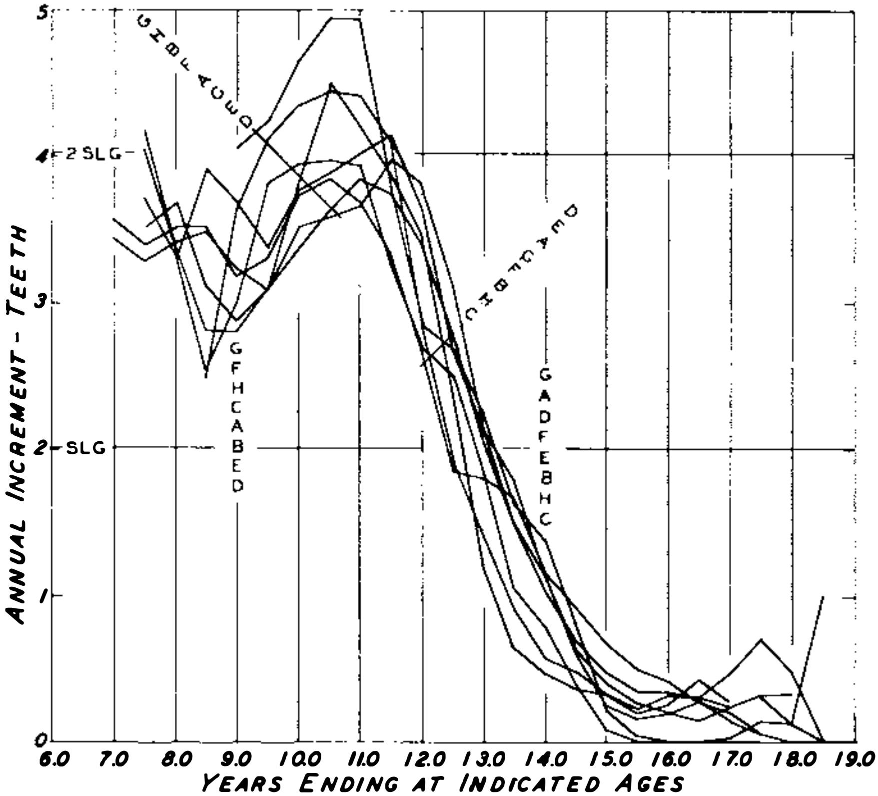

Fig. 97.--Average Annual Increments in the Number of Permanent Erupted Teeth of Eight Groups of Girls Menstruating at Different Ages.

CHAPTER XI

RACIAL SIMILARITIES IN PATTERNS OF GROWTH

Throughout this study the emphasis has fallen on a phenomenon which we have elected to call the "pattern of growth". The pattern of growth is the succession of accelerating and decelerating phases considered as a unit which results from analyzing the growth increments of small and very homogeneous groups of cases. It has been shown that each dimension has its own characteristic pattern. Further, excepting only erupted teeth, the patterns of the eight menarcheal groups are clearly distinguished in that they occur at successive chronological ages and are progressively less intense from early to late maturing groups. Now, this developmental method of analyzing and describing growth processes is the antithesis of the cross-sectional method of merely computing averages for large heterogeneous groups of different children at different ages. Hence, the problem of racial similarities or differences in the pattern of growth is to be sharply distinguished from the conventional problem of racial differences in average size. Here we are interested in determining whether the succession of accelerating and decelerating growth phases is the same or different in two racial groups.

Since the very phrase "racial similarities" may raise some protest, it should be stated at the outset that no denial of racial differences is here made. Indeed, a very significant racial difference is presented in section nine of Appendix A. Data are reported for standing height and weight on 676 girls of North European stock and on 244 girls of Italian stock measured at age eight (7-6 to 8-5) by the Harvard Growth Study. The standing heights of those of North European and of Italian stock average 123.1 and 117.9 centimeters, a difference more than seventeen times as large as its probable error. The North Europeans are distinctly heavier (3.9 times the probable error of the difference), slightly more variable in standing height, and slightly less variable in weight. However, the presence of differences in average size at any age can not decide the issue of racial differences or similarities in the patterns of growth. It should not be necessary to urge the importance of this problem. The literature of anthropology and of child development is notable for the large number of studies of racial differences in average size, and conspicuous for the absence of studies of the developmental patterns of racial groups. The problem is of some little importance for this study since several critics have suggested that the differential patterns of the eight menarcheal groups could be accounted for in part by racial differences.

1. Procedures

The racial composition of the available 248 cases is reported in section nine of Appendix A. There are 193 cases of North European stock, 33 cases of Italian stock, seven cases of South European stock, seven cases classified as Jewish, three Negroes, and five cases of Mixed stock. Given these cases the only feasible comparison is between the 193 North Europeans and the 33 Italians. These cases have been divided into early, middle, and late maturing groups who menstruated prior to age 12-6, between 12-6 and 13-5, and after age 13-5. The early, middle, and late maturing groups of North Europeans consist of 54, 87, and 52 cases while the early, middle, and late maturing groups of Italians consist of 9, 14, and 10 cases. The small number of Italians and the fact that they are distributed among early, middle and late maturing groups in the same manner as the North Europeans should make it evident that racial differences can not account for the differential patterns of the eight menarcheal groups. Each of these groups has been further subdivided into cases measured approximately at birthdays and approximately at the half-yearly anniversaries of birthdays and the resulting data combined as described in section three of Appendix B. The combined data for standing height, sitting height, iliac diameter and weight are reported in Tables 33, 34, and 35 of Appendix B.

2. Growth Trends in Average Size

Figures 98, 99, 100, 101, and 102 present growth trends in average size of girls of North European and of Italian stock classified into early, middle, and late maturing groups. Figure 98 show that girls of North European stock are consistently taller throughout the age range. At age 8.0 the differences range from 2.82 to 4.03 centimeters while at age 17.0 the differences are distinctly larger ranging from 4.23 to 7.77 centimeters. The contrasts in sitting height presented in Figure 99 are much smaller. Girls of North European stock in the three groups at age 8.0 average from .36 to .98 centimeters taller and at age 17.0 average from 1.45 to 1.99 centimeters taller. Figure 100 shows that the larger proportion of the differences in standing height are due to differences in leg length. Girls of North European stock in the three groups at age 8.0 average from 1.86 to 3.67 centimeters bigger and at age 17.0 from 2.24 to 6.22 centimeters bigger. On the whole differences in leg length are about three times as great as differences in sitting height. Data for iliac diameter displayed in Figure 101 are inconsistent. Early and middle maturing groups of North European stock are smaller than the corresponding groups of Italian stock while the late maturing group of North Europeans are larger. Combining the averages at each age from 10.0 to 15.0 where relatively complete data are available shows that the Italian group average only .15 centimeters larger in iliac diameter. Since sitting height and iliac diameter are more important factors determining weight than leg length, it can be anticipated that Figure 102 will show comparatively small differences in weight.

3. Patterns of Growth Generated by the Increments

Figures 103, 104, 105, 106, and 107 present the corresponding annual increments. Figure 103 displays the increments in standing height. There seems to be a consistent tendency for the accelerating phase of the Italian curves to begin about six months earlier and for their decelerating phase to be slightly sharper. Considering the small number of cases, however, greater similarities could hardly be expected. Increments in sitting height presented in Figure 104 also show negligible

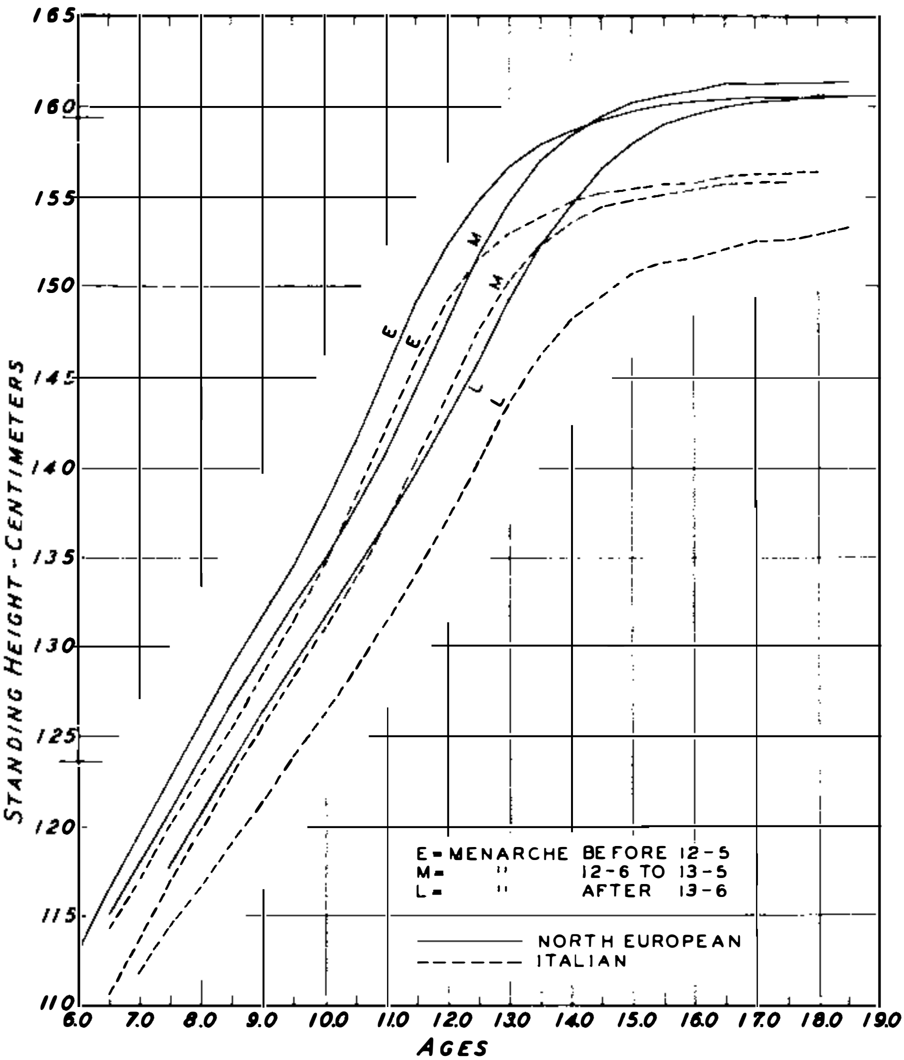

Fig. 98.--Growth Trends in Average Standing Height of Early, Middle and Late Maturing Groups of Girls of North European and Italian Stock.

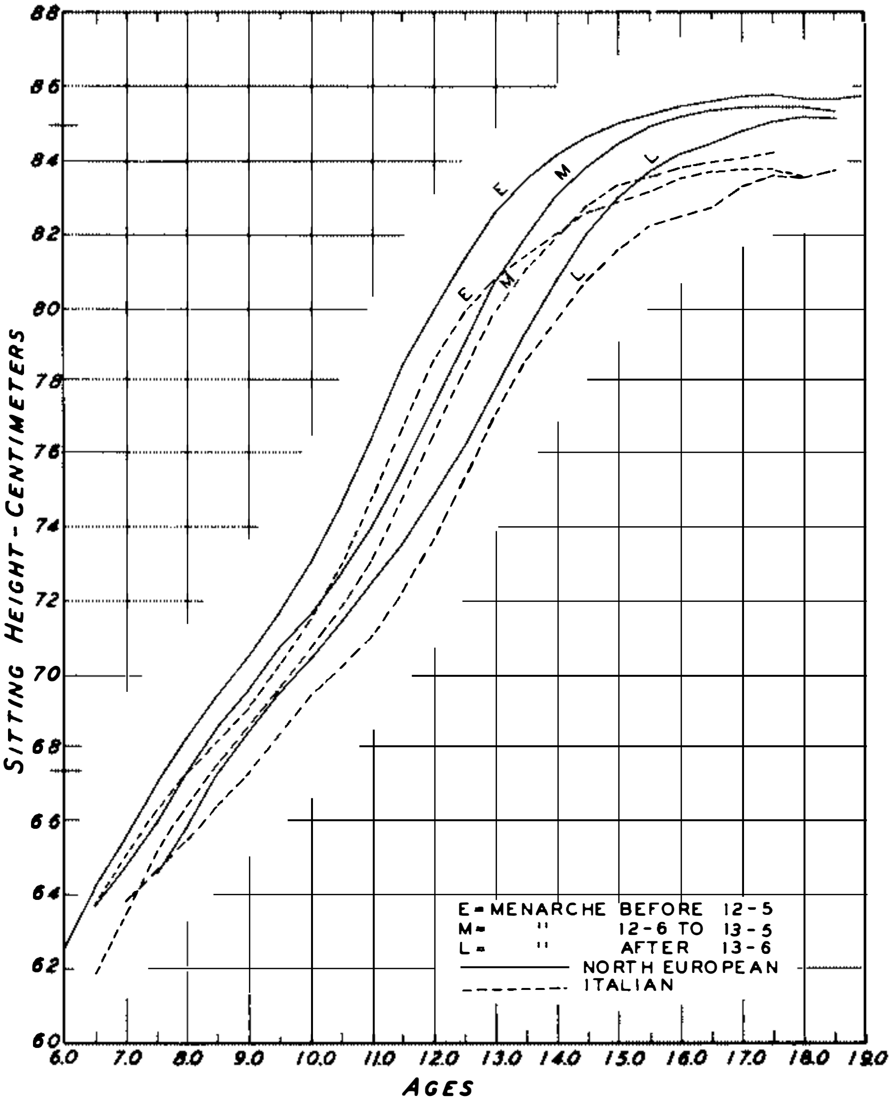

Fig. 99.--Growth Trends in Average Sitting Height of Early, Middle and Late Maturing Groups of Girls of North European and Italian Stock.

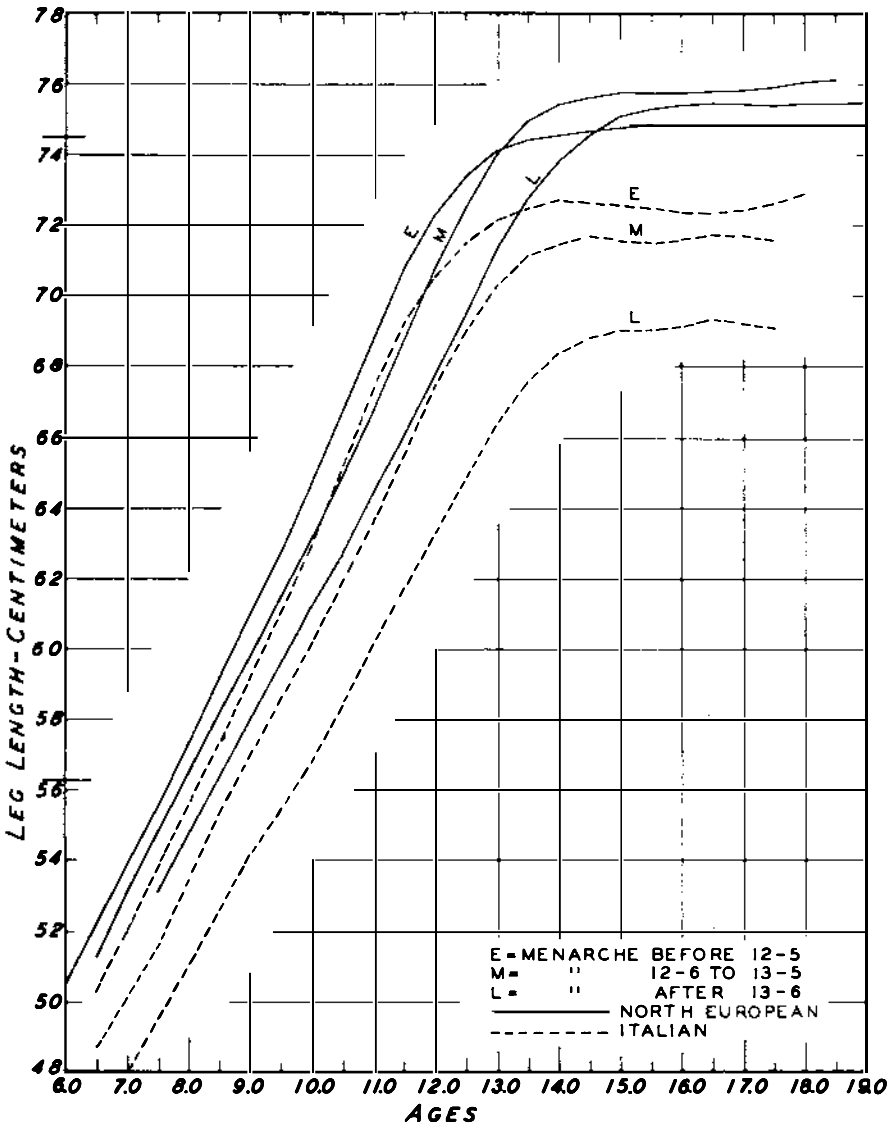

Fig. 100.—Growth Trends in Average Leg Length of Early, Middle and Late Maturing Groups of Girls of North European and Italian Stock.

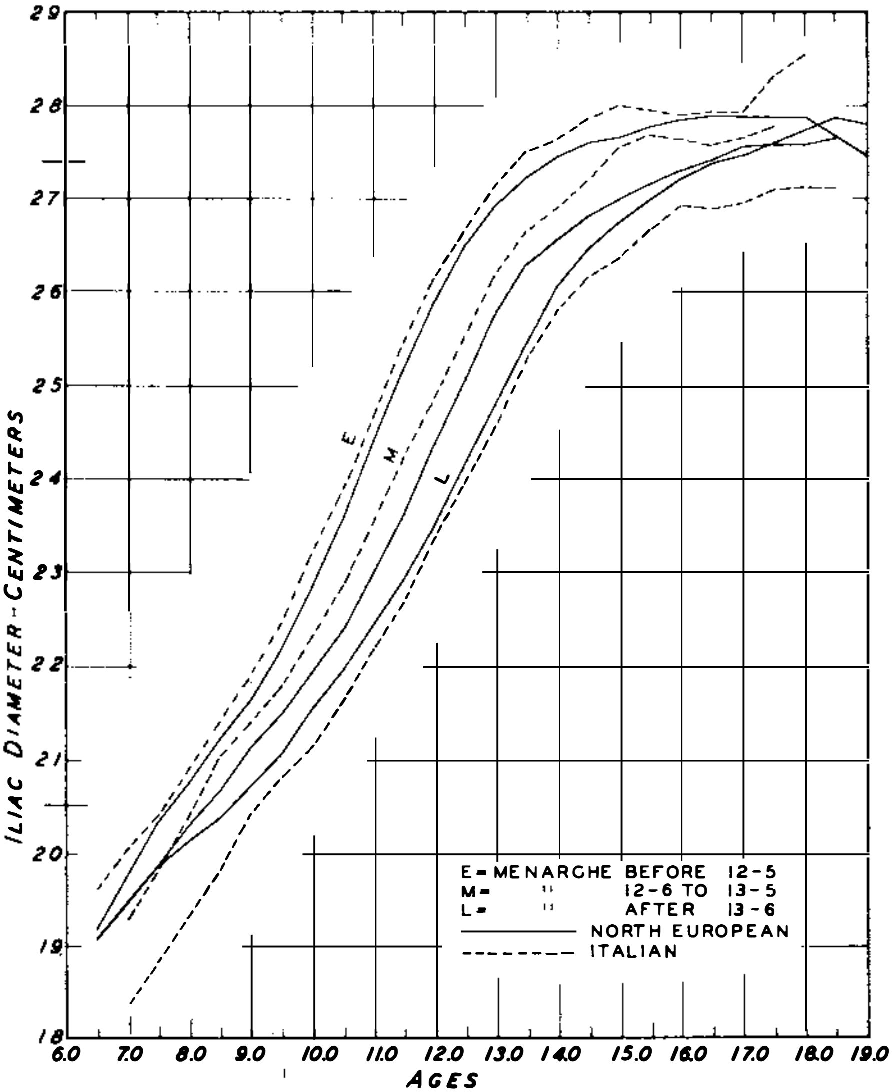

Fig. 101.--Growth Trends in Average Iliac Diameter of Early, Middle and Late Maturing Groups of Girls of North European and Italian Stock.

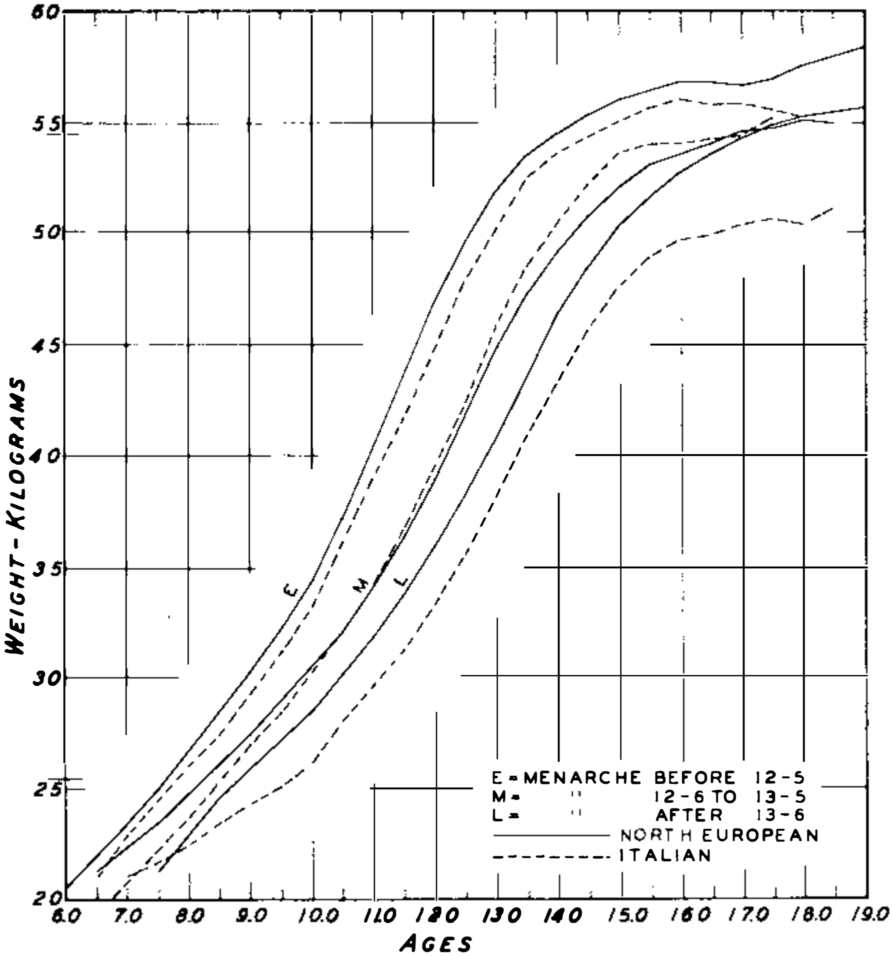

Fig. 102.--Growth Tends in Average Weight of Early, Middle and Late Maturing Groups of Girls of North European and Italian Stock.

racial differences with the exception that the late maturing Italian group reaches its maximum increment six months earlier than the late maturing North European group. Data for leg length, iliac diameter and weight in Figures 105, 106, and 107 are too irregular to warrant detailed discussion, but contrasts between early and late maturing groups are unmistakable, while all apparent racial differences may be attributed to fluctuations involved in the small number of cases.

Curves for early, middle, and late maturing groups have been combined to give generalized curves by superimposing corresponding points on the patterns of growth and averaging. The points superimposed are as follows.

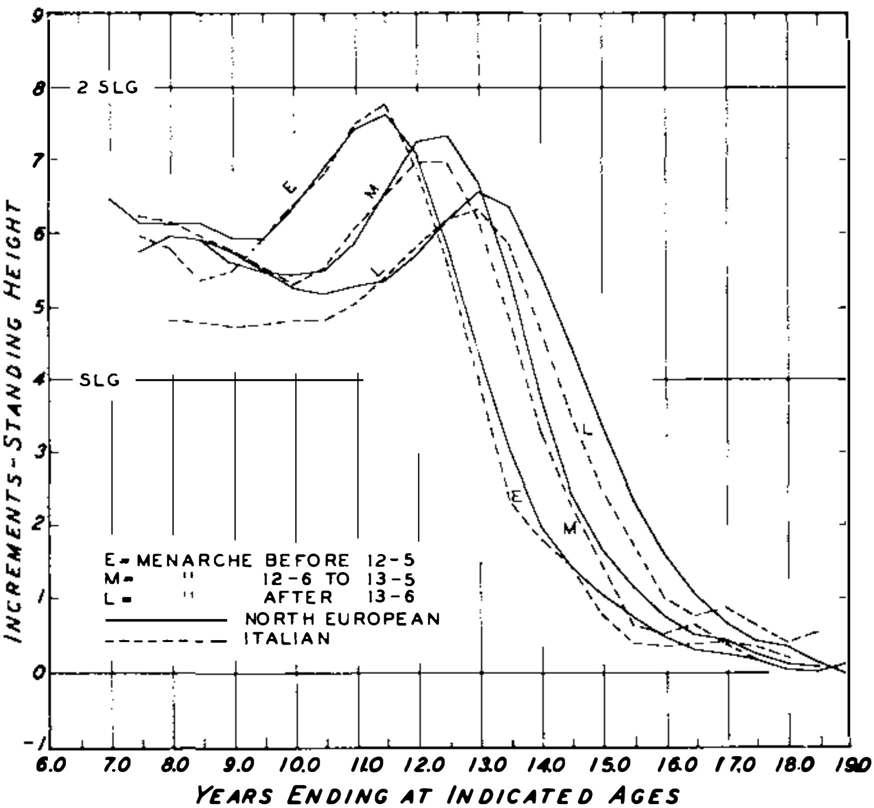

Fig. 103.--Average Annual Increments in Standing Height of Early, Middle and Late Maturing Groups of Girls of North European and Italian Stock.

	Group E.	Group M.	Group L.
Standing Height	11.5	12.5	13.0
Sitting Height	11.5	12.5	13.5
Leg Length'	11.0	12.0	13.0
Iliac Diameter	12.0	13.0	14.0
Weight	11.5	13.0	14.0

With one exception (weight) all curves for Group E have been moved one year forward or to the right to correspond with those of Group M and with one exception (standing height) all curves for Group L have been moved one year back or to the left to correspond with those of Group M. Average ages have also been computed to give the resulting data a time reference.

The resulting five sets of curves are assembled in Figures 108 and 109. It is apparent from Figure 108 that the patterns of growth in standing height, sitting height, and leg length of girls of North

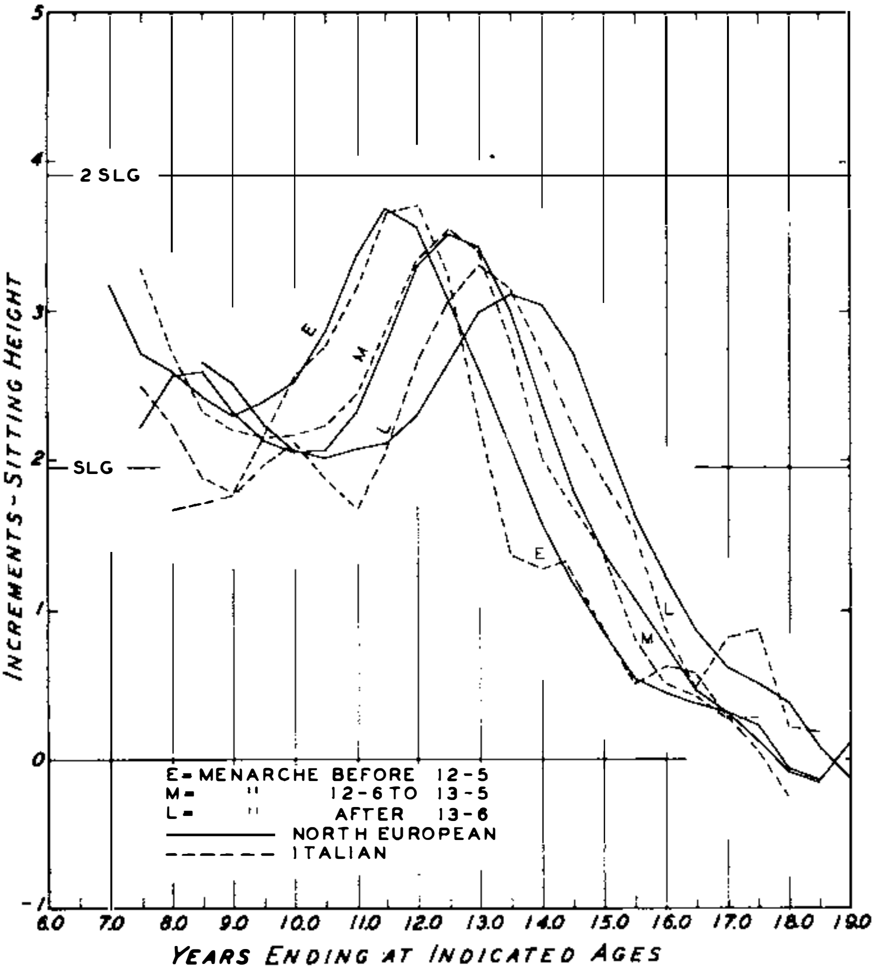

Fig. 104.--Average Annual Increments in Sitting Height of Early, Middle and Late Maturing Groups of Girls of North European and Italian Stock.

European and of Italian stock are very much alike. The most significant difference is that growth in leg length among Italian girls is not as intense although the general pattern is the same. Generalized patterns of growth in iliac diameter seem to show an earlier acceleration on the part of girls of Italian stock. Greater correspondence between the two racial stocks in patterns of growth in weight could hardly be expected.

4. Summary

A comparison of the growth trends of 193 girls of North European

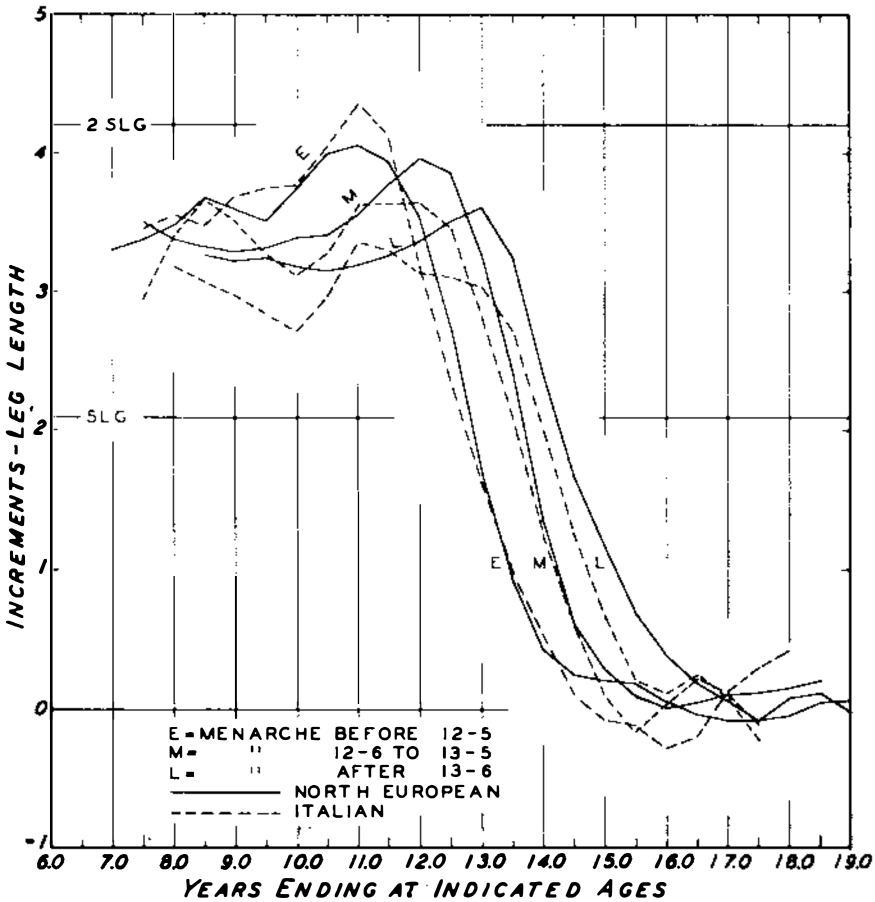

Fig. 105.--Average Annual Increments in Leg Length of Early, Middle and Late Maturing Groups of Girls of North European and Italian Stock.

stock and of 33 girls of Italian stock shows that the North Europeans are consistently taller and that the differences in stature are due largely to differences in leg length. Differences in average sitting height, iliac diameter, and weight are negligible. Considering the small number of Italians the patterns generated by the annual increments of the two groups could hardly be more alike. The only consistent difference is that the pattern of growth in leg length among Italians is less intense.

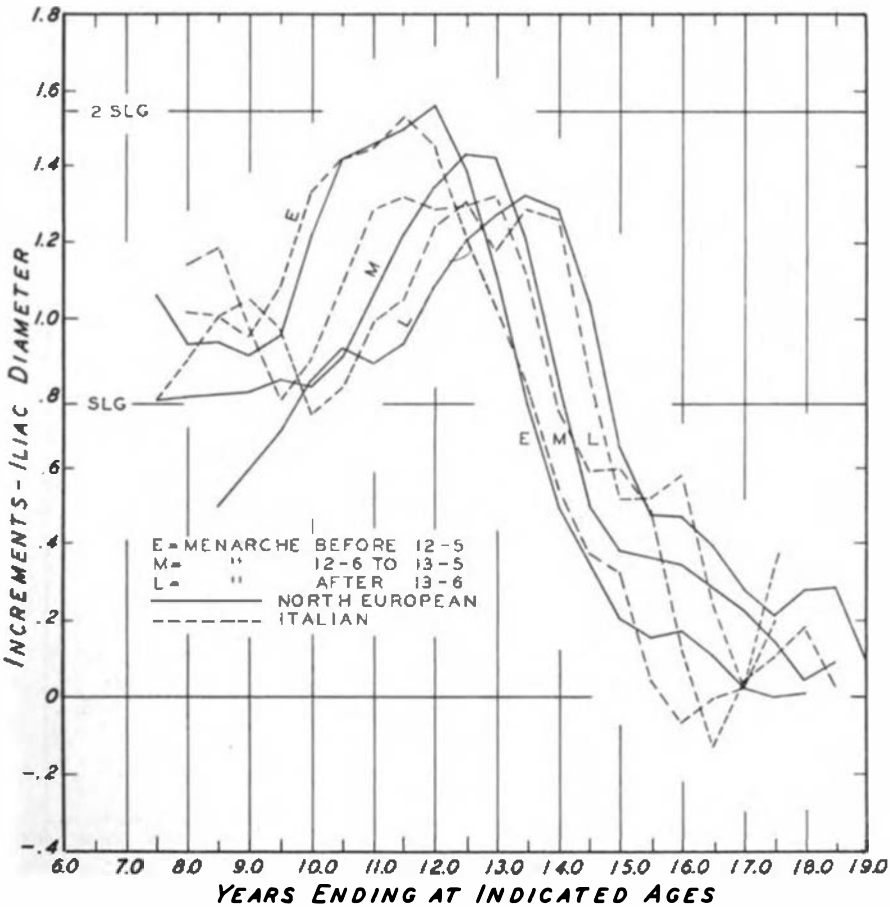

Fig. 106.—Average Annual Increments in Iliac Diameter of Early, Middle and Late Maturing Groups of Girls of North European and Italian Stock.

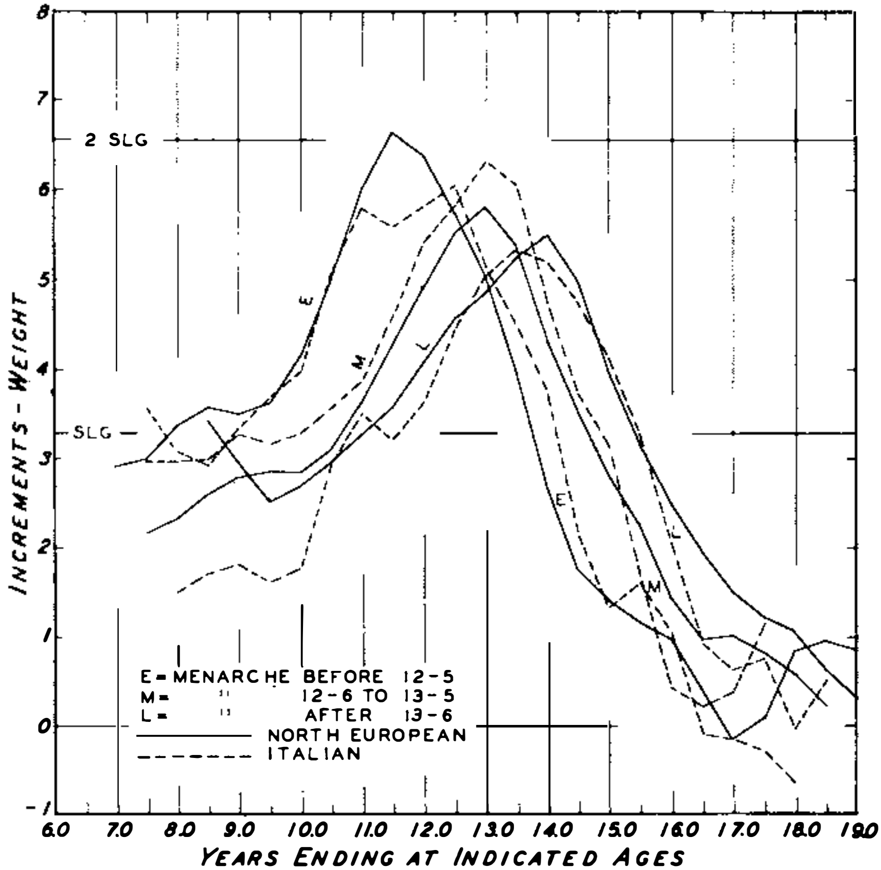

Fig. 107.--Average Annual Increments in Weight of Early, Middle and Late Maturing Groups of Girls of North European and Italian Stock.

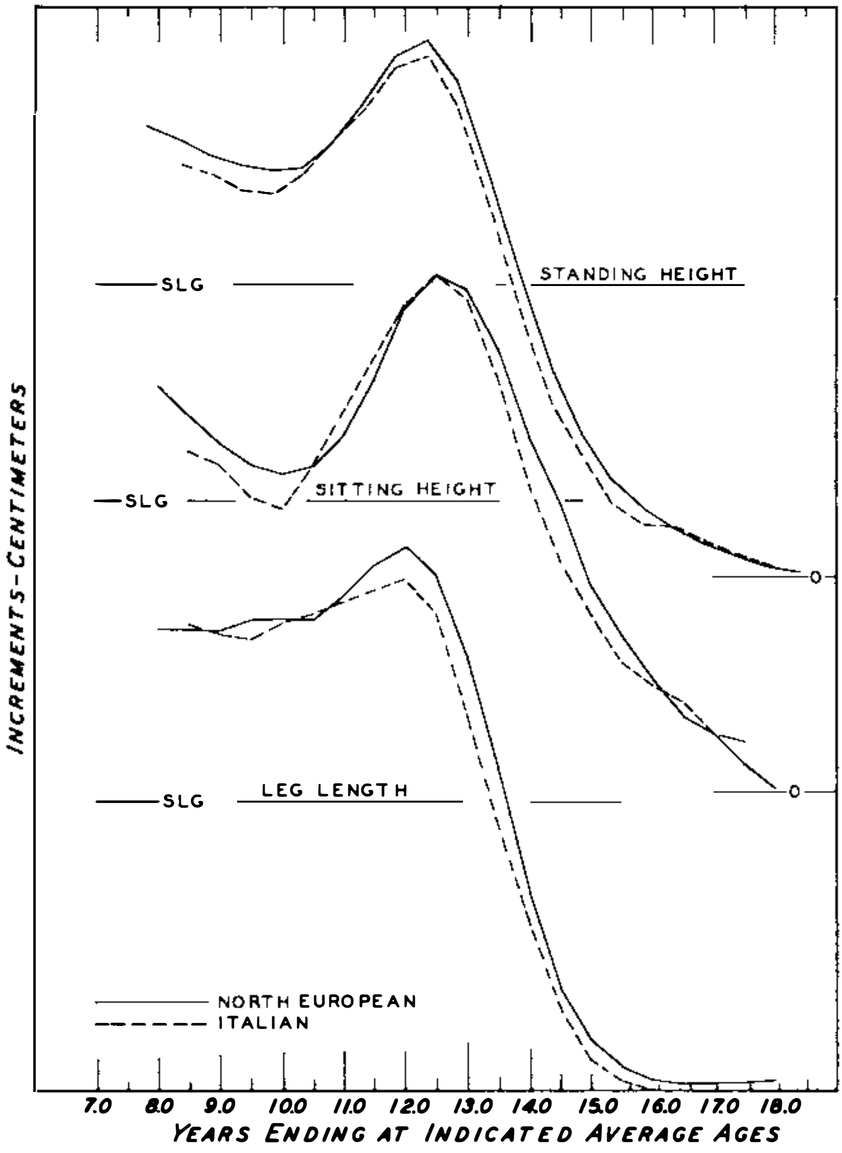

Fig. 108.--Generalized Patterns of Annual Increments in Standing Height, Sitting Height, and Leg Length of Girls of North European and Italian Stock.

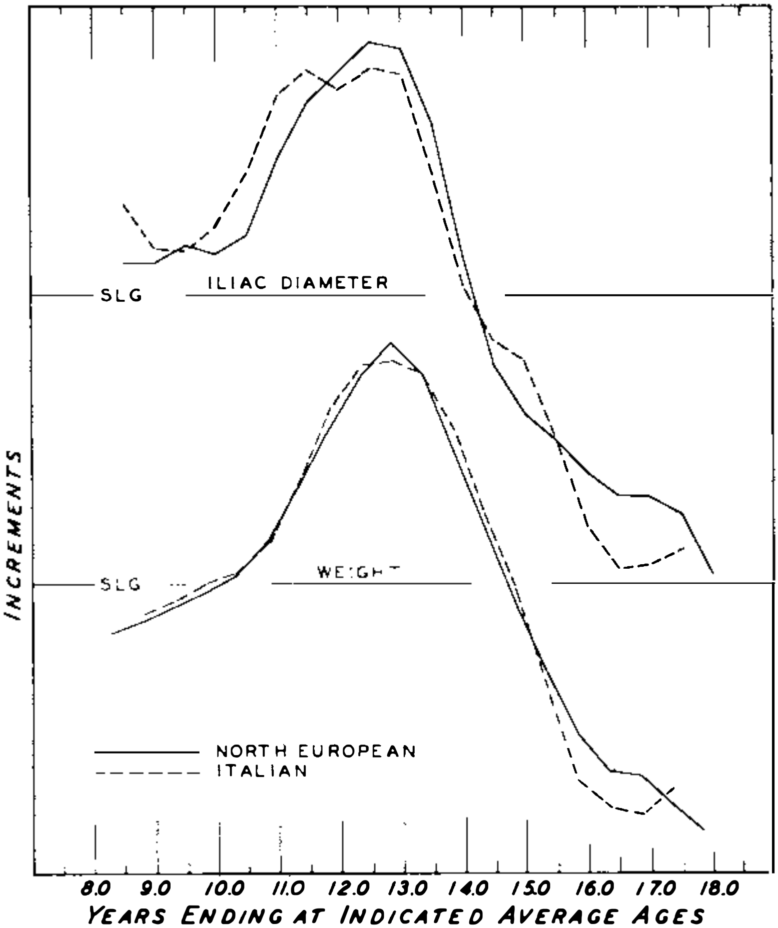

Fig. 109.--Generalized Patterns of Annual Increments in Iliac Diameter and Weight of Girls of North European and Italian Stock.

CHAPTER XII

A CORRELATIONAL ANALYSIS OF PHYSICAL DATA AND MENARCHEAL AGES

The central problem of this chapter is to measure statistically the closeness of the association between physical data and the advent of the menarche. We might dispense with such a problem in this study were it not for the fact that the differences between the eight menarcheal groups in respect to average size and average increments convey only the vaguest notion of the closeness of the association when individuals are considered. For example, average annual increments in standing height for the year ending at age 11.0 of 7.94, 7.56, 7.11, 6.21, 5.73, 5.50, 5.00 and 4.84 centimeters respectively for Groups A, B, C, D, E, F, G, and H doubtless suggest opposite conclusions. From the perfect correlation between average increments and average menarcheal ages, the unsophisticated may conclude that the correlation between individual increments and individual menarcheal ages is also perfect, while the skeptical may imagine that the correlation is only .20 or .30. It should be obvious from the theoretical background and specific hypotheses that any association between physical data and menarcheal ages does not suggest causal action of one variable on the other. If anything, such associations suggest prior common causal factors presumably in the endocrine organization of the individual.

A plethora of physical variables is available for a correlational study. In the case of standing height, for example, there are available gross measurements, annual and biennial increments, and rates of growth at each chronological age and at each age interval before and after the advent of the menarche -- a total of nearly a hundred possible variables. Similarly, there are ages at which accelerated growth is initiated, ages at which maximum growth occurs, ages at which mature stature is attained, ages at which certain arbitrary statures are attained, ages at which certain arbitrary proportions of mature stature are attained, etc., etc. A little ingenuity plus separate analysis for cases measured approximately at birthdays and for cases measured approximately at the half-yearly anniversaries of birthdays would easily yield a grand total of many hundred zero order correlations between menarcheal ages and variables derived from standing height alone. It is worth noting that even the most extensive correlational analysis would fail to give the same meaningful and concise results as Figures 11 and 12. On the other hand a few correlations subordinated to the more important data of earlier chapters should serve to define more precisely the exact nature of relationships which in a general way are already obvious.

1. Zero Order Correlations

Tables 1 and 2 present ten correlations between menarcheal ages and each of seven physical measurements. Throughout, the analysis has been carried out separately for cases measured approximately at birthdays and approximately at the half-yearly anniversaries of birthdays. That is, all ages in these tables are the mid-points of intervals of approximately half a year. Interpolated figures are involved and estimated

TABLE 1

ZERO ORDER CORRELATIONS BETWEEN MENARCHEAL AGES AND PHYSICAL SIZE

Variables	At Ages 8.0 and 8.5			At Ages 11.0 to 13.0			At Ages 17.5 and 18.0		
	Age	N	r	Age	N	r	Age	N	r
Standing Height	8.0	91	-.308	11.5	124	-.522	17.5	89	.136
Standing Height	8.5	109	-.299	12.0	123	-.500	18.0	56	.148
Sitting Height	8.0	91	-.281	11.5	124	-.539	17.5	89	.121
Sitting Height	8.5	109	-.251	12.0	123	-.498	18.0	55	.185
Iliac Diameter	8.0	63	-.324	12.5	124	-.608	17.5	88	-.260
Iliac Diameter	8.5	66	-.284	13.0	123	-.495	18.0	56	.012
Chest Breadth	8.0	84	-.187	12.0	117	-.495	17.5	81	-.164
Chest Breadth	8.5	99	-.311	12.5	123	-.585	18.0	44	-.268
Chest Depth	8.0	83	.009	12.0	117	-.424	17.5	83	-.016
Chest Depth	8.5	96	-.351	12.5	122	-.387	18.0	55	.090
Weight	8.0	90	-.210	12.5	124	-.613	17.5	89	-.175
Weight	8.5	108	-.327	13.0	123	-.496	18.0	56	-.029
Skeletal Age*	8.0	100	-.421	12.5	123	-.722	----	--	----
Skeletal Age*	8.5	114	-.429	13.0	123	-.713	----	--	----

* Correlations at 17.5 and 18.0 were not computed because individual differences in skeletal age vanish at these levels.

TABLE 2

ZERO ORDER CORRELATIONS BETWEEN MENARCHEAL AGES AND ANNUAL INCREMENTS

Variables	Years Ending at Ages 11.0 to 12.5			Years Ending at Ages 13.5 to 16.0		
	Age	N	r	Age	N	r
Standing Height	11.0	118	-.595	14.0	123	.679
Standing Height	11.5	123	-.630	14.5	124	.700
Sitting Height	11.0	118	-.489	14.5	124	.528
Sitting Height	11.5	123	-.572	15.0	121	.466
Iliac Diameter	11.0	124	-.325	13.5	124	.352
Iliac Diameter	11.5	121	-.452	14.0	123	.396
Chest Breadth	11.5	111	-.262	14.0	123	.312
Chest Breadth	12.0	110	-.257	14.5	122	.309
Chest Depth	12.0	112	-.215	14.5	124	.342
Chest Depth	12.5	117	-.152	15.0	121	.348
Weight	11.0	117	-.500	14.0	123	.336
Weight	11.5	123	-.603	14.5	124	.425
Skeletal Age	11.0	121	-.322	15.5	116	.453
Skeletal Age	11.5	121	-.362	16.0	110	.553

menarcheal ages have been supplied for ten cases whose records show the advent of the menarche not by age 14-5 or later.

Zero order correlations between menarcheal ages and physical size are reported in Table 1. The first column gives correlations at age 8.0 (7.75 to 8.24) and at age 8.5 (8.25 to 8.74). This age level was selected as the earliest age at which a representative distribution of menarcheal ages is available. With one exception the values are negative and generally fall between -.20 and -.40. That is, the earlier the menarche the bigger the individual. Considering the great differences among these seven measurements the correlations are remarkably uniform except that the values for skeletal age are distinctly higher. The middle columns of Table 1 display comparable correlations three and four years later. Here the attempt has been made to select the particular ages which would show the maximum correlation. Once more the correlations are unexpectedly uniform, ranging from .40 to .60, save that those for skeletal age are distinctly higher. The columns at the right give correlations at age 17.5 and 18.0. None of these correlations are significantly different from zero even when data for age 17.5 and 18.0 are combined. However, the slightly positive correlations with standing height and the slightly negative correlations with weight are in accord with coefficients reported by Abernethy (1), Viteles (36), and Stone and Barker (33).

Correlations between annual increments and menarcheal ages are reported in Table 2. Here again the attempt has been made to select the particular age periods which yield the maximum correlations. A correlation of -.596 between menarcheal ages and increments in standing height for the year ending at age 11.0 means that the early maturing individuals make larger gains in standing height between age 10.0 and 11.0. Increments for the years ending at ages 11.0 to 12.5 show decidedly negative correlations with menarcheal ages, the coefficients varying from -.15 to -.63. Three to five years later all of these decidedly negative correlations have changed to decidedly positive. It is to be particularly noted that in some instances these correlations are higher than those between menarcheal ages and physical size.

To supplement the foregoing analysis a more intensive study of menarcheal ages in relation to standing height has been undertaken. Standing heights at each age level as 7.0, 7.5, 8.0, 8.5, etc., have been plotted against menarcheal ages and the scatter-diagrams for ages 7.0 and 7.5, for ages 7.5 and 8.0, etc., have been superimposed and the correlations determined. Similarly, annual increments in standing height for the years ending at 8.0, 8.5, 9.0, 9.5 etc., have been plotted against menarcheal ages and the scatter-diagrams for increments ending at 8.0 and 8.5 and at 8.5 and 9.0 etc., have been superimposed and the correlations determined. This procedure combines all the available data into a single figure at intervals of half a year. The trends of the resulting correlations from age to age are displayed in Figure 110. The correlations of standing height with menarcheal age are given by the dash-dot curve. At ages 7.0 and 7.5 the correlation is -.334. The coefficients remain fairly constant to ages 9.5 and 10.0. Thereafter the values becomes more and more negative dropping to -.497 at ages 11.5 and 12.0. That is, the taller the individual, the earlier the menarche. From here the correlations become progressively less and less negative, crossing the zero line at age 16.5 and ending at

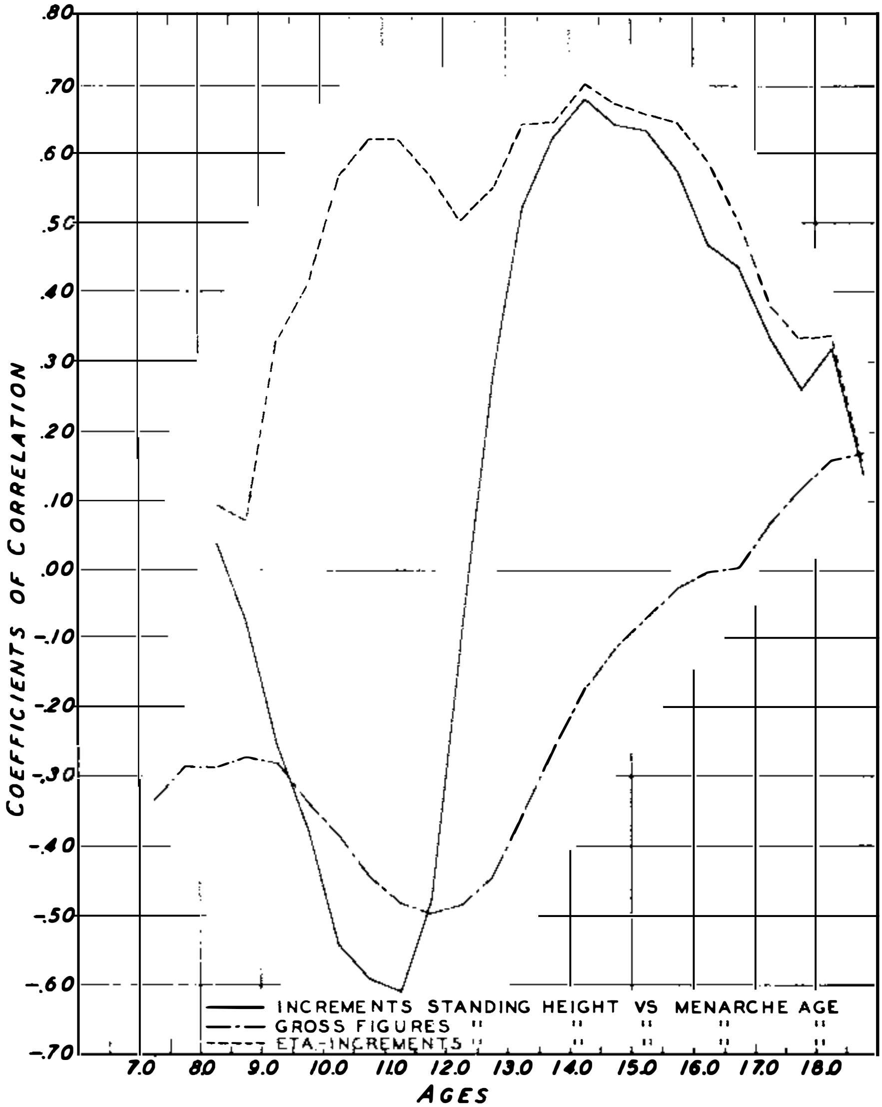

Fig. 110.--Trends with Age of Correlations Between Menarcheal Age and Standing Height and Between Menarcheal Age and Increments in Standing Height.

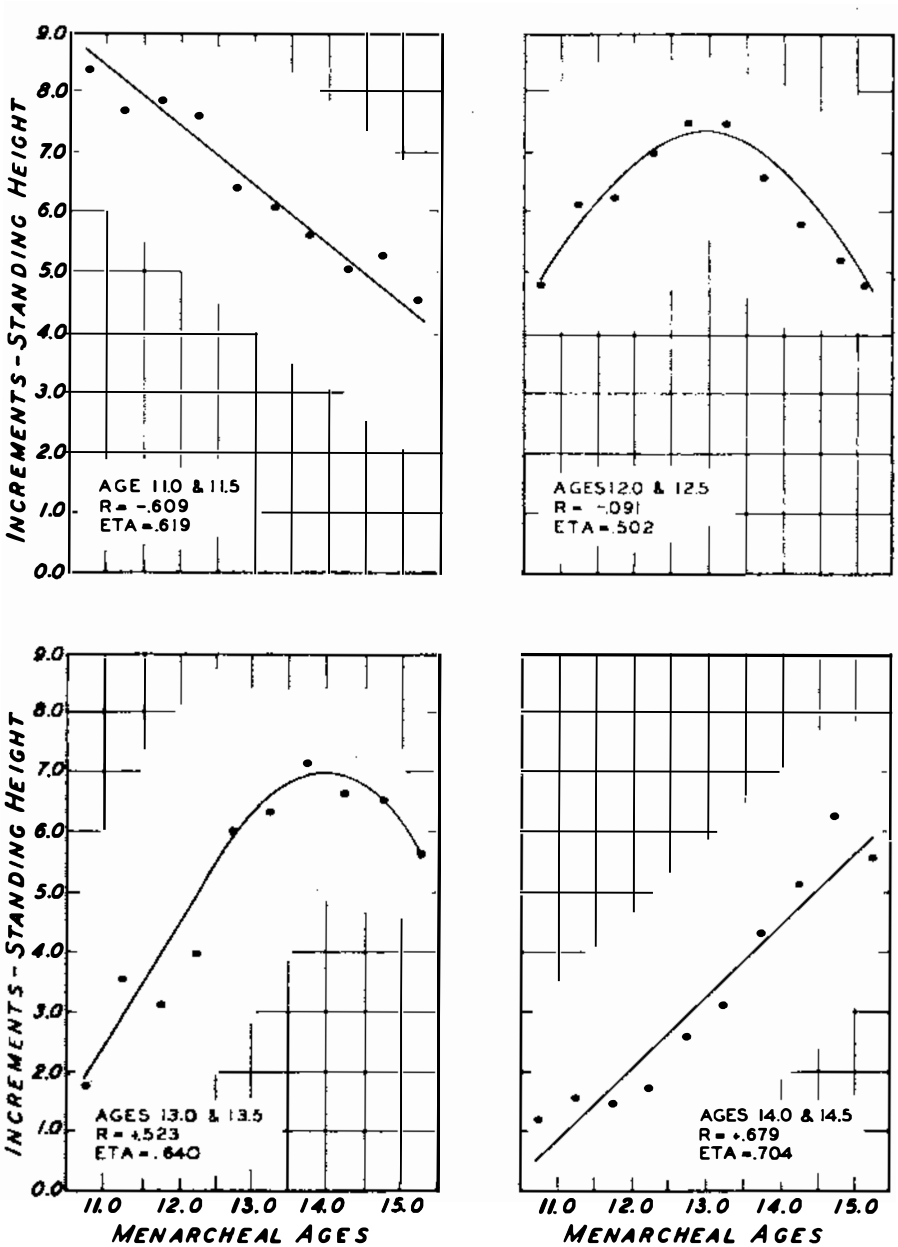

Fig. 111.--Regression Curves of Increments in Standing Height on Menarcheal Ages at Selected Age Levels.

.173 at ages 18.5 and 19.0. Much more striking trends are shown by the correlations between annual increments and menarcheal ages represented by the continuous curve. Initially, increments for the year ending at ages 8.0 and 8.5 show essentially zero correlations with menarcheal ages. A definite trend toward negative correlations appears immediately and the correlations become more and more negative, reaching an extreme negative value of -.609 for annual increments ending at ages 11.0 and 11.5. From this point the correlations change abruptly and within a period of three years have shifted from -.609 to .679. A more spectacular change in the relationship between two variables could hardly be imagined. It is to be particularly noted, however, that a Pearson coefficient of correlation is not adequate to represent the relationship between annual increments and menarcheal ages between ages eleven and fourteen because the regression lines are not linear. Hence, correlation ratios have also been calculated (and corrected for fine grouping). The trend of these ratios, which are necessarily positive but analogous to the more familiar Pearson correlation, is given by the broken curve of Figure 110. Initially, these give approximately the same results as the correlations save that the sign is changed. But, whereas a Pearson correlation gives essentially zero values between menarcheal ages and increments ending at ages 12.0 and 12.5, the correlation ratio is .502. The changing form of the regression curves of increments on menarcheal ages is illustrated in Figure 111. The heavy dots indicate the mean increments corresponding to intervals of menarcheal age. The straight lines of the upper left and lower right sections of the chart are the regression lines of increments on menarcheal age computed from the correlations while the curves of the upper right and lower left sections are free hand estimates of the same relationship. The data of Figure 110 measure the closeness of the association between menarcheal ages and standing height and increments in standing height when individuals are considered and should be compared with Figures 11 and 13 which show the same associations in terms of groups and averages. Figure 111 on the other hand should be recognized as a redrawing of portions of Figure 13.

Further exploration of the data shows that the biennial increments consistently show closer associations with menarcheal ages. It has not seemed worth while to analyze all of the data systematically in terms of biennial increments, but the following sample results will serve to illustrate the point. The most extreme negative correlations between menarcheal ages and annual increments in standing height are -.595 and -.630; the corresponding values involving biennial increments are -.587 and -.695. The most extreme positive correlations between menarcheal ages and annual increments in standing height are .679 and .700; the corresponding values involving biennial increments are .742 and .753. The most extreme negative correlations between menarcheal ages and annual increments in weight are -.500 and -.603; the corresponding values involving biennial increments are -.560 and -.618. The most extreme positive correlations between menarcheal ages and annual increments in skeletal age are .453 and .553; the corresponding values involving biennial increments are .573 and .613

In addition to the foregoing correlations we have also determined the relation between menarcheal age and age at the close of the year of maximum growth in standing height. For the 174 cases with clearly

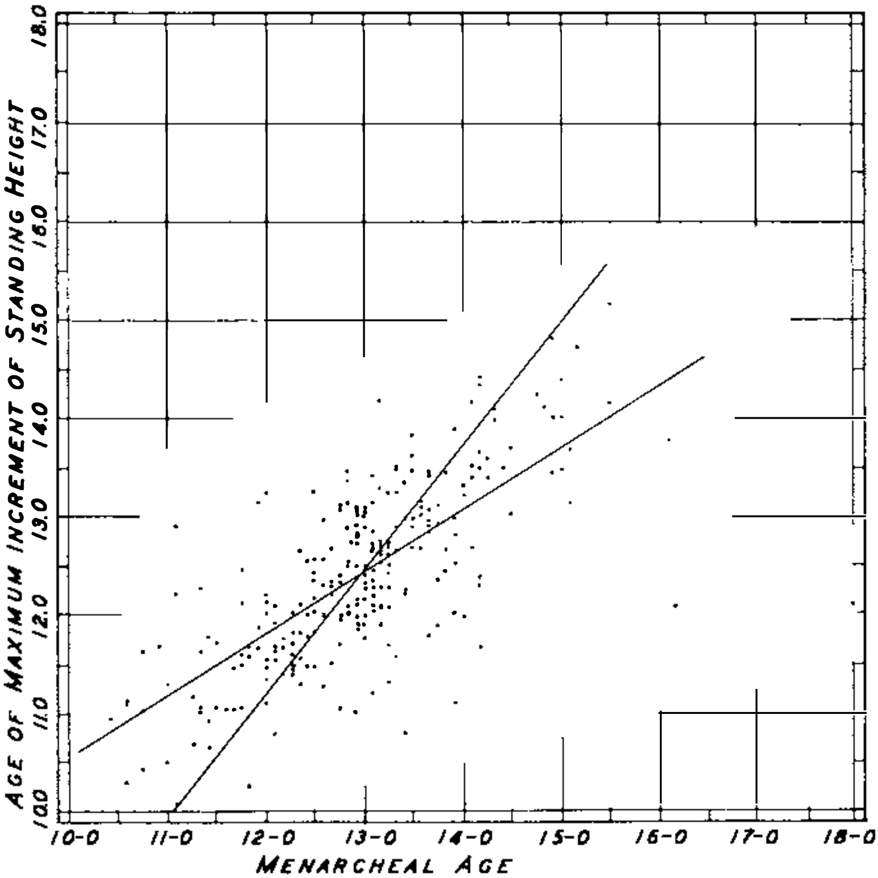

Fig. 112.--Scatter Diagram of Menarcheal Ages Plotted Against Age of Maximum Growth in Standing Height.

defined maximum increments which were employed in sections five and six of Chapter V the correlation is .754. Adding cases where the maximum increment is determined by an interpolated figure and making the best possible estimates for other cases the correlation involving 246 cases is .706. Since this relationship is of considerable interest for the contrasting findings of Chapters IV and V, Figure 112 presents the detailed scatter-diagram and the two regression lines. It will be recalled from Chapter IV that, when cases were classified according to the advent of the menarche, the following relationships appear: Group A menstruating on the average at 11.0 shows its maximum growth for the year ending at age 11.0 while Group H menstruating on the average at 15.0 shows its maximum growth for the year ending at 13.5. In Figure 112 this relationship is represented by the more horizontal of the two regression lines (read up from the bottom of the chart to this line). In section six of Chapter V cases were classified according to age at the close of the year of maximum growth and the following relationships appear: cases whose maximum growth is during the year ending at 10.5 menstruate on the average at 11.27, while cases whose maximum growth is during the year ending at age 14.5 menstruate on the average at 14.65. In figure 112 this relationship is represented by the more perpendicular of the two regression lines. Figure 112 shows very clearly that these apparently contradictory trends represent merely the regression effects which are necessarily involved whenever two variables are not perfectly correlated.

2. Multiple Correlations

Eight variables known to correlate highly with menarcheal ages have been selected for a multiple correlation study. In the previous section all the correlations were originally plotted and determined separately for cases who were measured approximately at birthdays and at the half-yearly anniversaries of birthdays. This insistence on very narrow age ranges seemed necessary in the analysis of the increments because the correlations change with great abruptness from strongly negative to strongly positive in only a few years. However, when scatter diagrams were superimposed very little seemed to be lost by employing less precise age periods. Hence, in this section data for cases measured approximately at birthdays and at the half-yearly anniversaries of birthdays have been combined and will be reported for the population as a whole. Table 3 lists the variables in the order of the magnitude of their correlation with menarcheal age and reports their means and standard deviations while Table 4 displays the intercorrelations. It has been necessary to omit two individuals with very incomplete data, hence, the number of cases is 246.

Since the eight variables showing the apparently highest correlations with menarcheal ages have been selected the list is rather heterogeneous. Three measures of size are included (variables 7, 8, and 9). These are iliac diameter at ages 11.0 and 11.5, weight at ages 12.5 and 13.0, and standing height at ages 11.5 and 12.0. It is of some interest that these measures of bigness stand at the bottom of the list in order of the magnitude of correlations with menarcheal age (-.477 to -.532) and that their intercorrelations are comparatively high (.706 to .804) although different age levels are involved. The multiple correlation of these three measures of physical size with menarcheal ages is only .560. Hence, these variables can make very little independent contribution to the multiple correlation and have been dropped from further analysis. Beginning at the top of the list the variable most closely associated with menarcheal ages is the age period in which the maximum increment in standing height occurs, the correlation being .702. It is noteworthy that this variable correlates only -.466, -.488 and -.490 with measures of size. The next variable is the skeletal ages correlating -.687 with the menarcheal records. Supposedly, this variable measures growth in the sense of development and stage of differentiation rather than growth in the sense of increase in magnitude, but it correlates rather highly (.630, .673, and .684) with measures of size. Next in order are three increment variables. Neglecting signs, these variables correlate on the average .604 with the menarcheal ages, on the average only .495 with each other, and on the average only .495 with measures

TABLE 3

VARIABLES EMPLOYED IN MULTIPLE CORRELATION PROBLEM
MEANS AND STANDARD DEVIATIONS

	Variables	Mean	Standard Deviation
1.	Menarcheal ages -- (months)	155.60	12.96
2.	Age at the close of the year of maximum growth in standing height -- (years)	12.41	.93
3.	Skeletal Ages at 12.5 and 13.0 -- (months)	155.75	13.04
4.	Increments in standing height for the year ending at 14.0 and 14.5 -- (centimeters)	3.09	1.90
5.	Increments in standing height for the year ending at 10.5 and 11.0 -- (centimeters)	6.03	1.40
6.	Increments in weight for the year ending at 11.0 and 11.5 -- (kilograms)	4.56	2.48
7.	Iliac diameter at ages 11.0 and 11.5 -- (centimeters)	25.49	1.86
8.	Weight at ages 12.5 and 13.0 (kilograms)	44.22	8.71
9.	Standing height at ages 11.5 and 12.0 -- (centimeters)	145.26	7.35

TABLE 4

INTERCORRELATIONS OF MENARCHEAL AGES AND PHYSICAL VARIABLES

	Variables*	2	3	4	5	6	7	8	9
1.	Menarcheal Ages	.706	-.687	.680	-.590	-.542	-.532	-.515	-.477
2.	Age M. I. StaH.		-.703	.734	-.646	-.529	-.490	-.466	-.488
3.	Skeletal Age			-.659	.646	.563	.673	.684	.630
4.	Inc. StaH. 14.0 and 14.5				-.497	-.461	-.462	-.434	-.451
5.	Inc. StaH. 11.0 and 11.5					.527	.457	.423	.528
6.	Inc. Wt. 11.0 and 11.5						.535	.665	.508
7.	Iliac. Dia. 11.0 and 11.5							.804	.706
8.	Weight 12.5 and 13.0								.724
9.	StaH. 11.5 and 12.0								

* See Table 3.

of size.

The multiple correlation between menarcheal ages and variables two to six is .785. This, of course, is probably well below the true value which would be obtained if theoretically true menarcheal ages and theoretically true physical measurements were available, but the true correlation can not be estimated with confidence since the reliabilities of the several variables are unknown. Nevertheless, some clear inferences can be drawn. The menarcheal records were obtained under unsatisfactory conditions and can hardly be assumed to be highly reliable. From the multiple correlation, however, it is clear that this reliability must be at least .80 and probably nearer .85 or .90. Assuming that the reliabilities of the menarcheal records and of the combined physical data are .90 and .98 then the theroretically true multiple correlation is approximately .83. Or, assuming reliabilities of .85 and .95, the theoretically true multiple correlation is approximately .87.

Throughout it has been emphasized that no causal action of physical size or of increments on the advent of the menarche or vice versa is involved. Rather the correlation between physical factors and menarcheal ages is assumed to be due entirely to antecedent factors or events presumably in the endocrine organization of the individual. Let it be assumed that science will some day be able to determine the age at which this supposed endocrine stimulation begins to operate. Then, it will be of some interest to determine the correlation between age at the advent of endocrine stimulation and age at the advent of the menarche and between age at the advent of endocrine stimulation and a composite measure of the timing of patterns of physical growth. Theoretically, these correlations should be in the neighborhood of the square root of the multiple correlation, i.e., from about .85 to .90 if uncorrected for attenuation or from about .90 to .95 if corrected for attenuation. Granting that all of this is extremely speculative, it should be clear that the interrelationships of assumed endocrine stimulation, advent of menarche, and the timing patterns of growth are exceptionally close.

3. Summary

At ages 8.0 and 8.5 the correlations between menarcheal ages and such variables as standing height, sitting height, iliac diameter, chest breadth, chest depth, weight, and skeletal age are, with one exception, slightly negative ranging from -.20 to -.40. Three to five years later the comparable correlations are decidedly negative ranging from -.40 to -.70. Still later at ages 17.5 and 18.0 the correlations are essentially zero save that there is probably a significant though very slight positive correlation with standing height and a very slight negative correlation with weight. Omitting data for skeletal age, the three measurements of size which correlate most highly with menarcheal ages yield a multiple correlation of only .560 for all the 248 cases with relatively complete data.

Annual increments show much more striking relationships with menarcheal ages. Increments for the years ending at ages 11.0 to 12.5 correlate from -.20 to -.60 with the advent of the menarche. Three to five years later all of these decidedly negative correlations have changed to decidedly positive, the coefficients ranging from .30 to .70. Four variables involving increments, ages at maximum growth, and measurements of growth in the sense of development rather than in the sense of increase in magnitude yield a multiple correlation with menarcheal ages of .785. That is, the correlation between menarcheal ages and various measures of the timing of the changing pattern of growth is considerably higher than the correlation between menarcheal ages and measures of mere bigness.

Correlations between menarcheal ages and physical data do not imply causal interactions. Rather they point to antecedent factors presemably in the endocrine organization of the individual. If so, the correlation between age at the onset of assumed endocrine stimulation and menarcheal age and between age at the onset of assumed endocrine stimulation and a composite measure of the timing of changes in the pattern of physical growth should be in the neighborhood of .85 to .90 or, if corrected for errors of measurement, in the neighborhood of .90 to .95. Such an inference is highly speculative, but it should be clear that the interrelations of assumed endocrine stimulation, advent of menarche, and the timing of patterns of growth are very high.

CHAPTER XIII

CONTRASTING PATTERNS OF THE MEASUREMENTS AND OF THE EARLY AND LATE MATURING GROUPS

We began in Chapters IV and V by concentrating all our attention on the contrasting patterns of growth in standing height shown by groups of girls menstruating at different ages. In Chapter VI the emphasis was shifted to the contrasts between patterns of growth in sitting height and leg length while not neglecting the contrasts between the menarcheal groups. This shifting emphasis continued and most of the discussion of Chapter VIII was concerned with the contrasts between growth in weight and in linear measurements. In this chapter we attempt to strike a balance between contrasts among measurements and contrasts among groups. Given comparable graphical representations the data suggest the unexpected conclusion that the contrasts between early and late maturing groups are greater than the contrasts between measurements which show accelerated growth during the adolescent period. Data for erupted teeth and skeletal age are excluded from the discussion. Incidentally, the demonstration of the generally greater contrasts between groups than between measurements offers a convenient opportunity to present alternative graphical representations of the data.

1. Contrasting Patterns in Average Size of Measurements and Groups

Four types of charts, only three of which are comparable, will be employed to illustrate contrasting growth trends in average size. The first and simplest type of chart presents two or more dimensions on the same arithmetic scale as in Figures 50 and 69. Such charts, of course, must be limited to dimensions expressed in terms of a common unit of measurement. When so limited the curves for different measurements are comparable only in that a common arithmetic scale is employed. If a large and a small dimension, as standing height and chest depth, are superimposed on such a chart, the pattern of the smaller dimension is obscured. The inadequacies of this type of graphical representation, except for the purpose of illustrating differences in magnitude, as in Figures 50 and 69, are generally conceded.

In a second type of chart the scales may be so constructed that the same vertical distance represents for each measurement the total growth increment between the initiation of accelerated growth and the attainment of maturity. We do not have unambiguous evidence on these points; hence, in this study the total growth increment between age eight and age seventeen for each measurement has been represented by the same vertical distance. Figure 49 presents such a chart for sitting height and leg length plotted against chronological age. It is clear from this chart that the patterns generated by the measurements are different. Similarly, the patterns generated by the early and late maturing Groups A and H are different. Finally, the contrasts between Groups A and H are clearly greater than the contrasts between measurements. Figure 113 presents similar data for standing height, iliac diameter, chest breadth, and weight. Again the contrasts between Groups A and H are

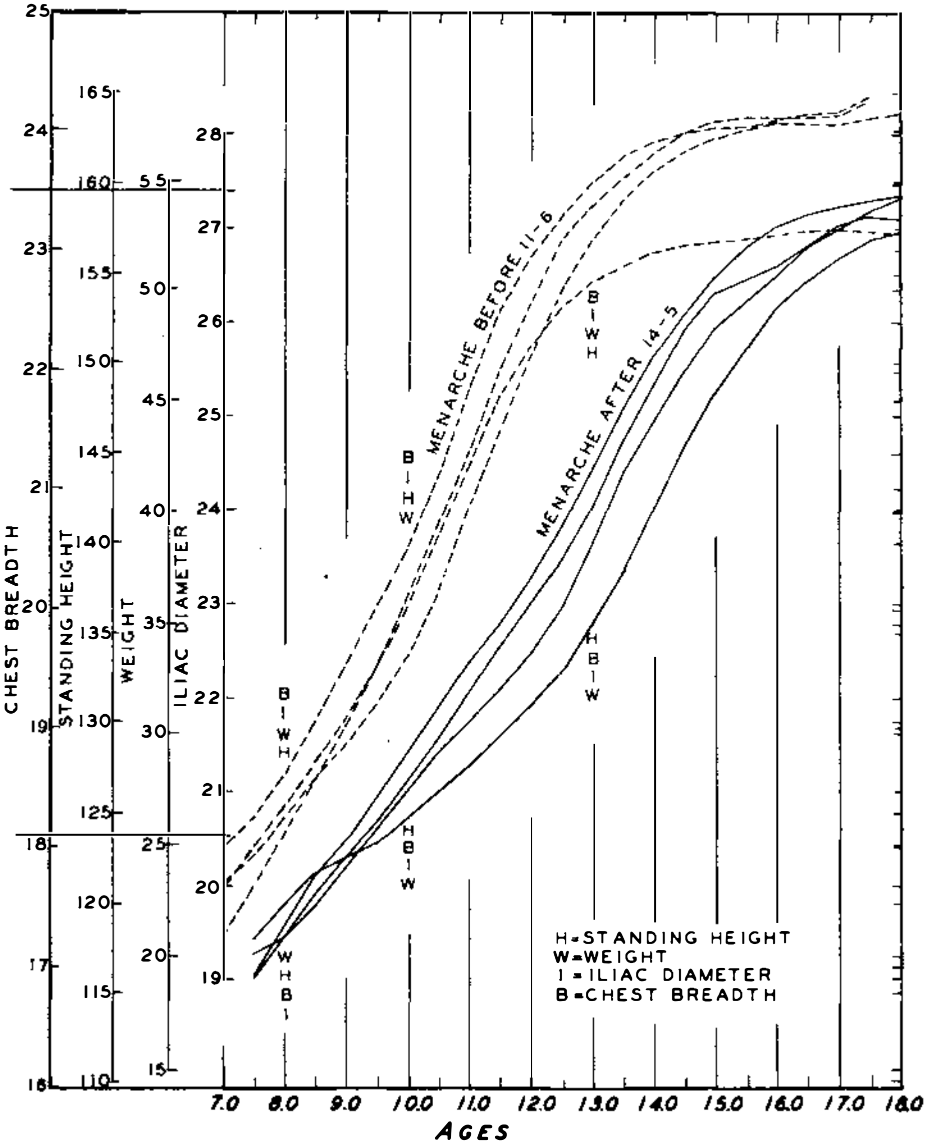

Fig. 113.--Growth Trends in Average Standing Height, Iliac Diameter, Chest Breadth, and Weight of Early and Late Maturing Groups of Girls.

clearly greater than the contrasts between measurements.

Continuing with this type of chart these contrasts may be observed when the curves are arranged so that points corresponding to the advent of the menarche are in the same vertical line. Figure 51 presents such an arrangement for sitting height and leg length and Figure 70 gives the same representation for iliac diameter, chest breadth and chest depth. It is clear from these charts that the contrasts between Groups A and H are greater than the contrasts between measurements. This test might also be applied to Figure 113 by moving the curves for Group A four years forward or to the right. If this were done the same observation would still hold save that the curve of Group A in standing height after age twelve tends to conform more closely to the four curves of Group H than to the other three curves of Group A.

Pushing the analysis still further the contrasts may be observed when the curves are arranged so that corresponding points on the pattern of growth are as nearly as possible in the same vertical line. From Table 28 of Appendix B it appears that the curves of Group A must be moved on the average two and a half years forward or to the right in order that their pattern of increments may conform as closely as possible to pattern of increments in Group H. Rearranging the curves of Figure 113 in this manner gives Figure 114. Again it is readily apparent that the contrasts between the early and late maturing groups are greater than the contrasts between the measurements, save that the pattern of standing height of Group A after age twelve conforms more closely to the patterns of Group H than to the other patterns of Group A.

A third and more common type of chart expresses all measurements as percentages of maturity. That is, the scale ranges from 0% to 100% representing growth from conception to maturity. Since a clear definition of maturity is not available in the data, charts have been constructed in which the scale from 0% to 100% represents growth from conception to age seventeen. Two difficulties with this type of chart should be noted at the outset. In the first place the pattern of growth on such a chart must represent the whole of growth from conception to maturity; hence, the pattern from age six to nineteen is more largely determined by growth differences between conception and age six than by growth differences between age six and nineteen. In the second place such charts do not give comparable visual representations of linear measurements in comparison with weight for the reason that weight is a volume or three dimensional variable. Assume, for example, that the side of one cube is 50% of another, then the volume of the first cube is only 12.5% of the latter. Hence, when linear growth has attained about 50% of maturity, weight will have attained only about 12.5% of maturity.

In order to compare weight and linear measurements on this type of chart, weight should be compared with a volume measurement such as the one described in section four of Chapter VIII. Figure 115 presents such a chart for weight and body "volume". It is apparent that curves for weight and linear-measurements-expressed-in-terms-of-volume are very much alike whereas in respect to both weight and volume the curves for early and late maturing groups are widely contrasted. That is, the contrasts between groups are very much greater than the contrasts between weight and linear-measurements-expressed-in-terms-of-volume. Curves for standing height have been added for the purpose of illustrating the

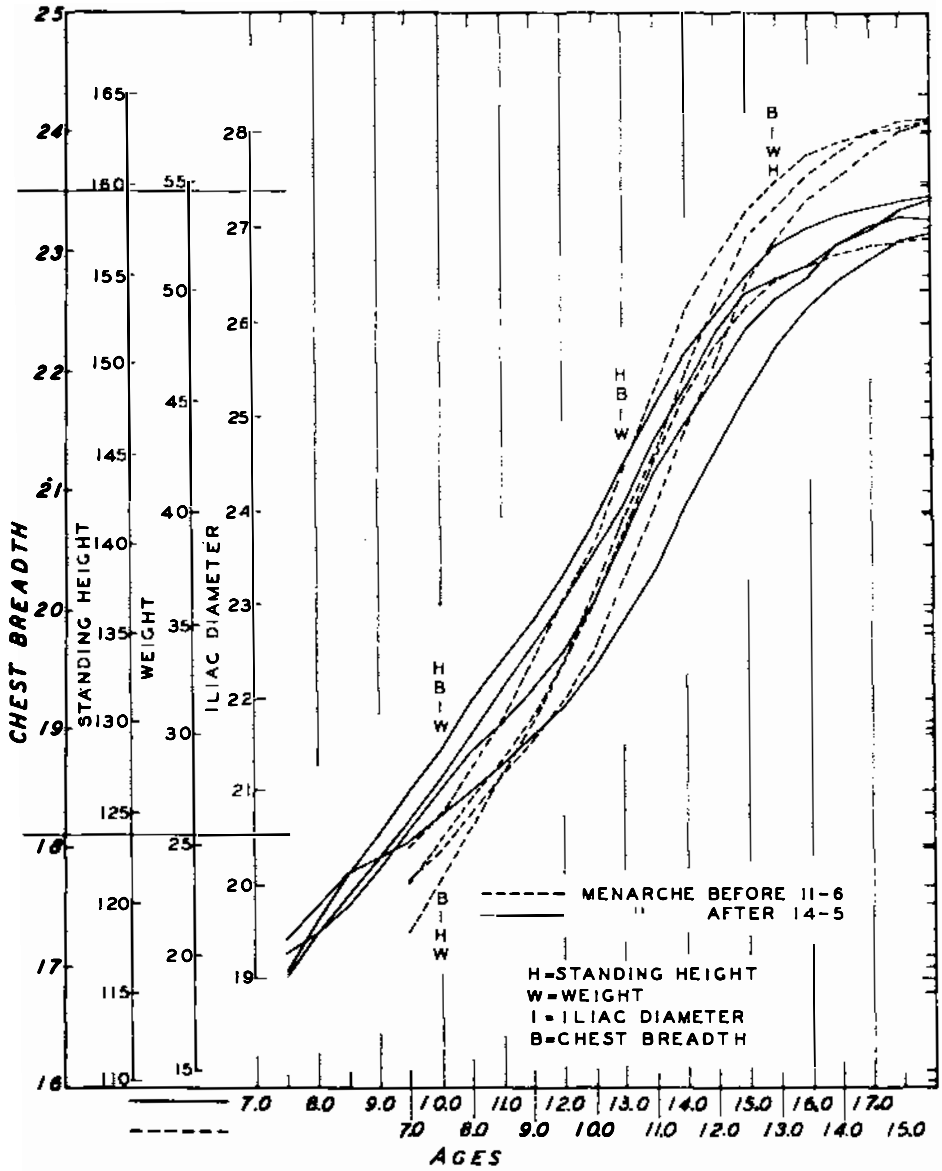

Fig. 114.--Growth Trends in Average Standing Height, Iliac Diameter, Chest Depth and Weight of Early and Late Maturing Girls Arranged so that Corresponding Points on the Patterns of Growth are in the Same Vertical Line.

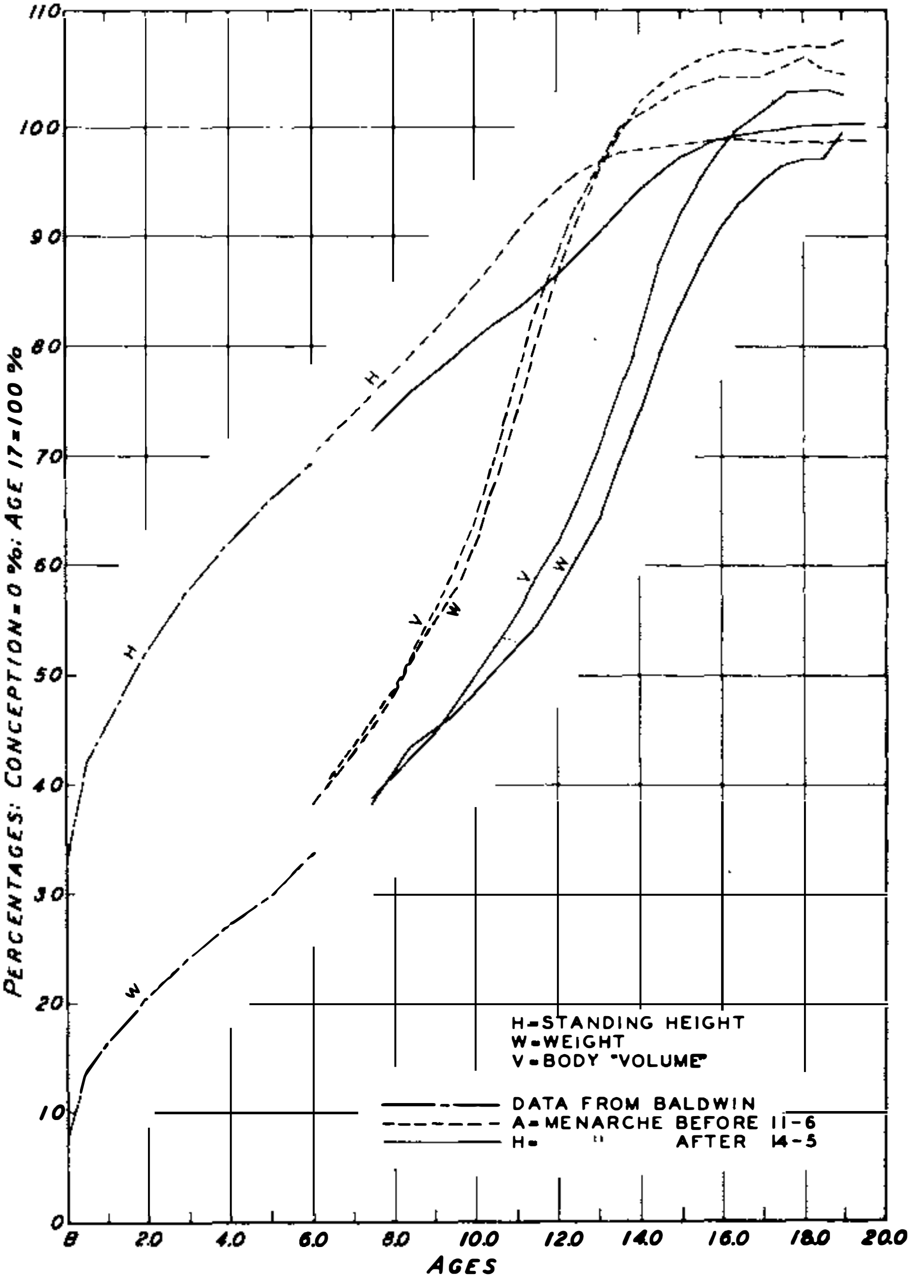

Fig. 115.—Growth Trends in Average Standing Height, Weight, and Body "Volume" of Early and Late Maturing Groups of Girls Expressed as Percentages of Average Size at Age Seventeen.

illusory contrasts when weight and a linear measurement are plotted in such a chart. Curves from birth to age six for height and weight have also been included for their general interest. These are based on data of Baldwin (4).

Magnifying the upper third of Figure 115 and assemblying the linear measurements give Figure 116. Again, the greater contrasts between groups than between measurements is apparent. If the dash curves of the early maturing Group A were moved four years to the right so that points corresponding to the menarche were in the same vertical line, the same observation would hold. If, instead, the curves of Group A were moved two and a half years to the right so that corresponding points on the pattern were in the same vertical line, the contrasts between the measurements would be as great as the contrasts between groups.

In a fourth type of chart the curves are plotted on a logarithmic scale as in Figure 117. In such a chart the slopes of the curves indicate directly percentage rates of growth rather than increase in magnitude. The large contrasts between the early and late maturing Groups A and H are readily apparent, but it is difficult to judge whether contrasts between measurements are greater or less than between groups because of the different vertical positions. Eliminating these differences in vertical position, greatly magnifying the scale at the left of Figure 117, and assemblying the linear measurements give Figure 118. Once more, the contrasts between the growth patterns of early and late maturing groups are greater than contrasts between the growth patterns of different measurements. If the curves of Group A were moved four years to the right, this observation would still hold. If, instead, the curves of Group A were moved two and a half years to the right, the contrasts between the measurements would be as great as the contrasts between groups.

2. Contrasting Patterns in Annual Increments and Rates of Growth of Measurements and Groups

Continuing the testing of whether measurements or groups show greater contrasts Figure 119 presents annual increments of early and late maturing groups for all measurements while Figure 120 presents annual percentage rates of growth for all measurements except weight. Rates of growth in weight are not included in Figure 120 for reasons discussed in section three of Chapter VIII. It is readily apparent that the contrasts between early and late maturing groups are greater than between different measurements. If the curves of Group A were moved four years to the right so that points corresponding to the menarche were in the same vertical line, this observation would still hold. If, instead, the curves of Group A were moved two and a half years to the right so that corresponding points of the pattern of growth were in the same vertical line, the contrasts among measurements would be as great as the contrasts among groups. Figure 121 and 122 in terms of increments present the details of this last test. Figure 121 shows annual increments of early and late maturing groups in standing height, sitting height, and leg length. Considering only these three measurements the usual observation is verified for the first two phases but not for the third or decelerating phase of growth. Figure 122 presents annual increments of early and late maturing groups in iliac diameter, chest breadth, and

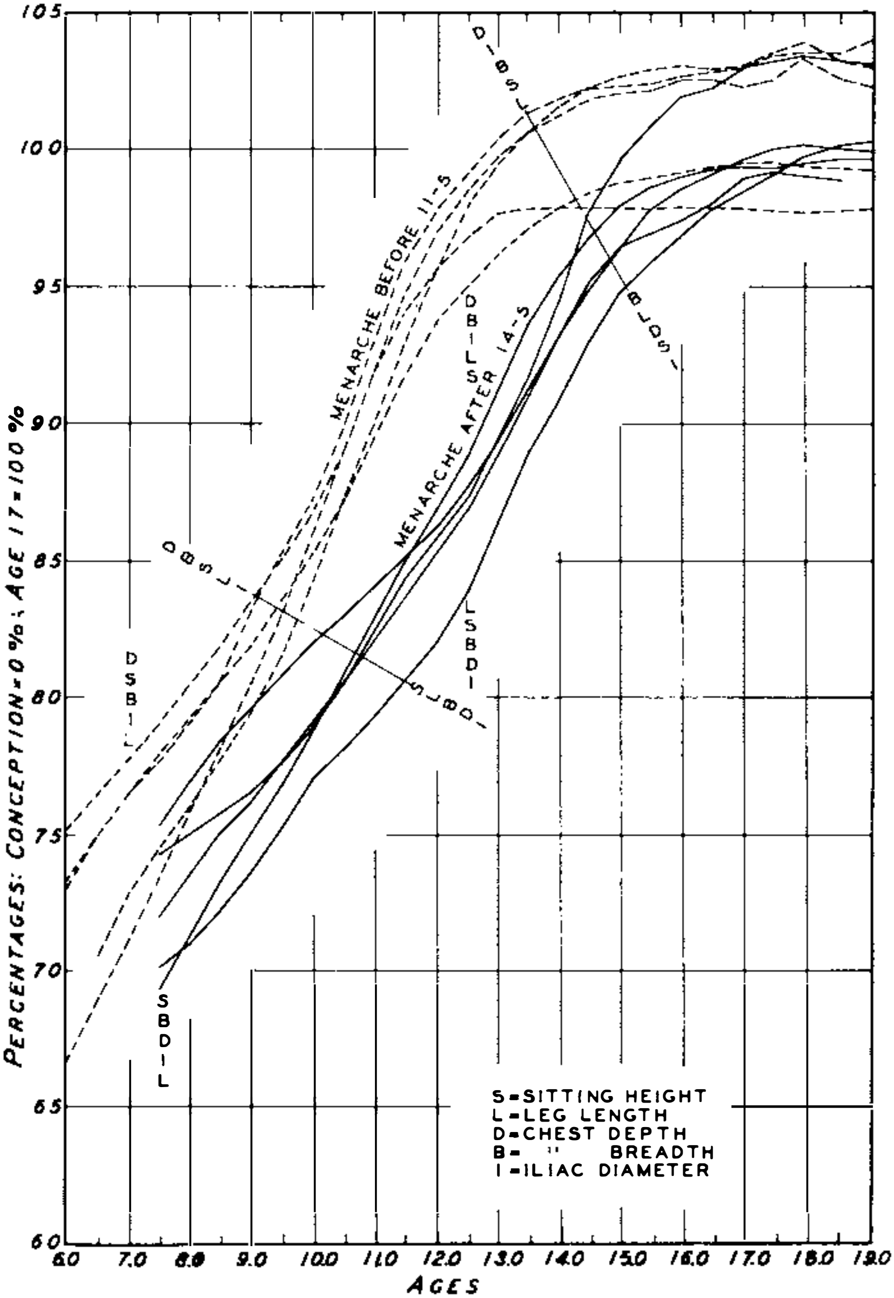

Fig. 116.--Growth Trends in Average Sitting Height, Leg Length, Iliac Diameter, Chest Breadth, and Chest Depth of Early and Late Maturing Groups of Girls Expressed as Percentages of Average Size at Age Seventeen.

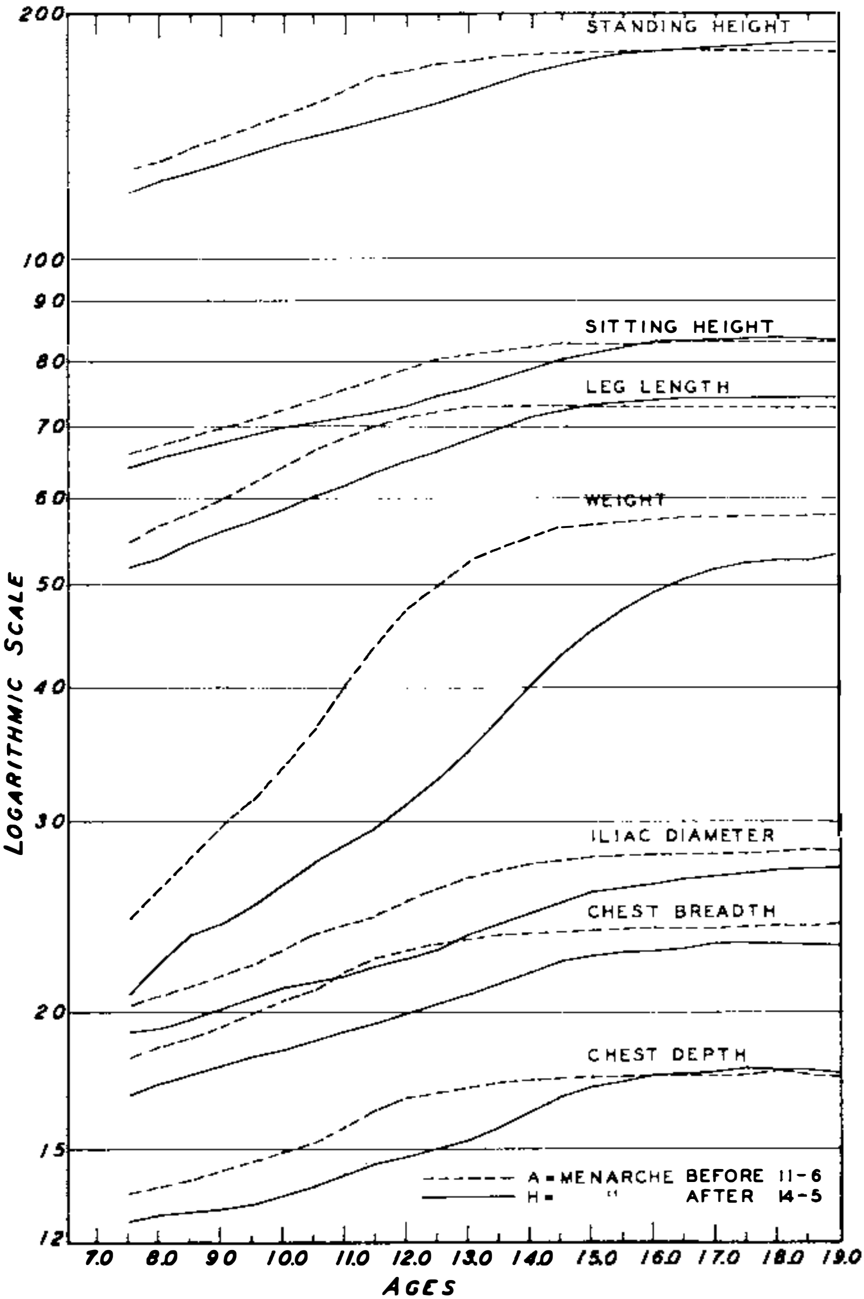

Fig. 117.—Growth Trends in Average Standing Height, Leg Length, Sitting Height, Weight, Iliac Diameter, Chest Breadth, and Chest Depth of Early and Late Maturing Groups of Girls Plotted To a Logarithmic or Ratio Scale.

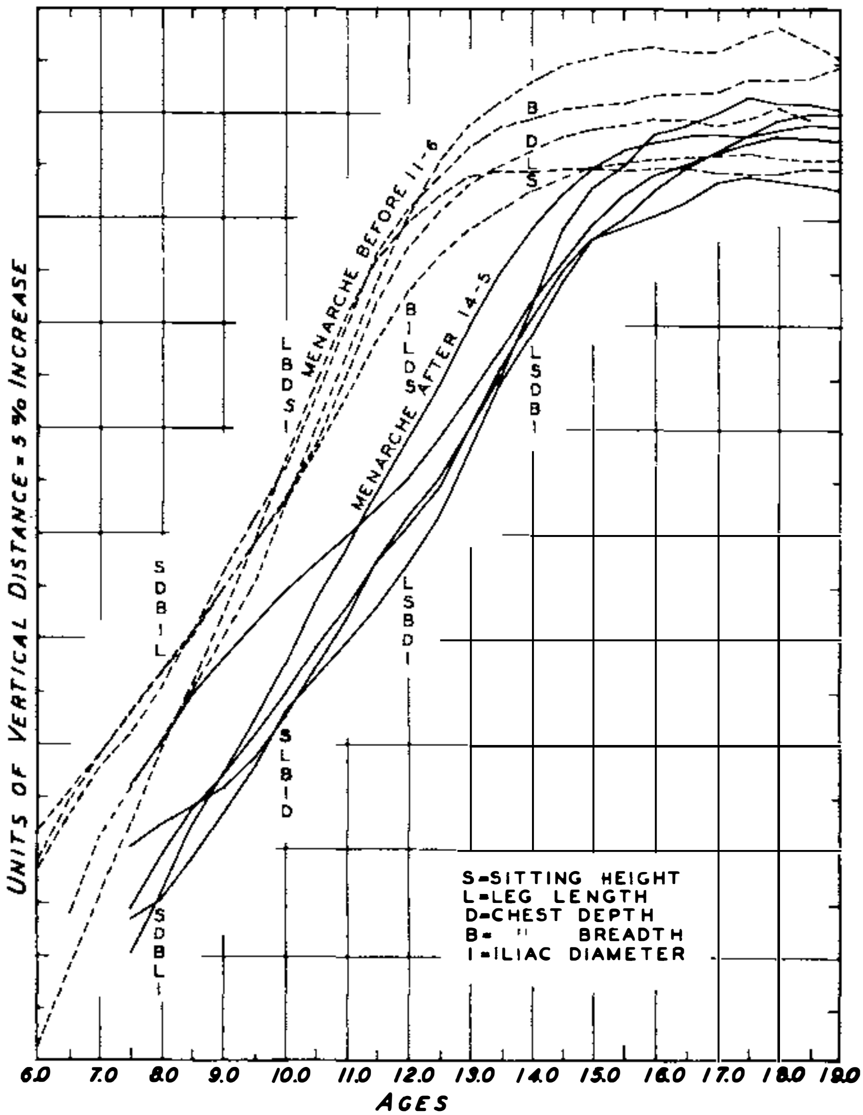

Fig. 118.—Growth Trends in Average Sitting Height, Leg Length, Iliac Diameter, Chest Breadth, and Chest Depth Plotted to a Logarithmic Scale.

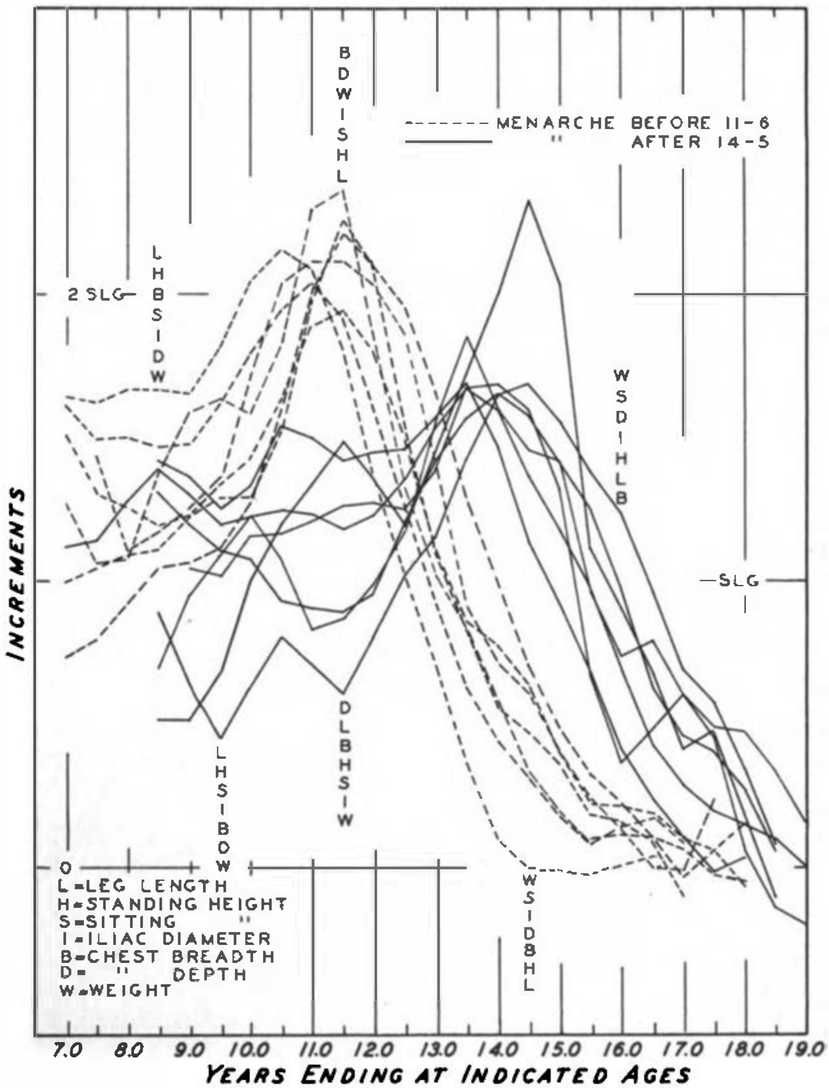

Fig. 119.--Average Annual Increments of Early and Late Maturing Groups in all Measurements.

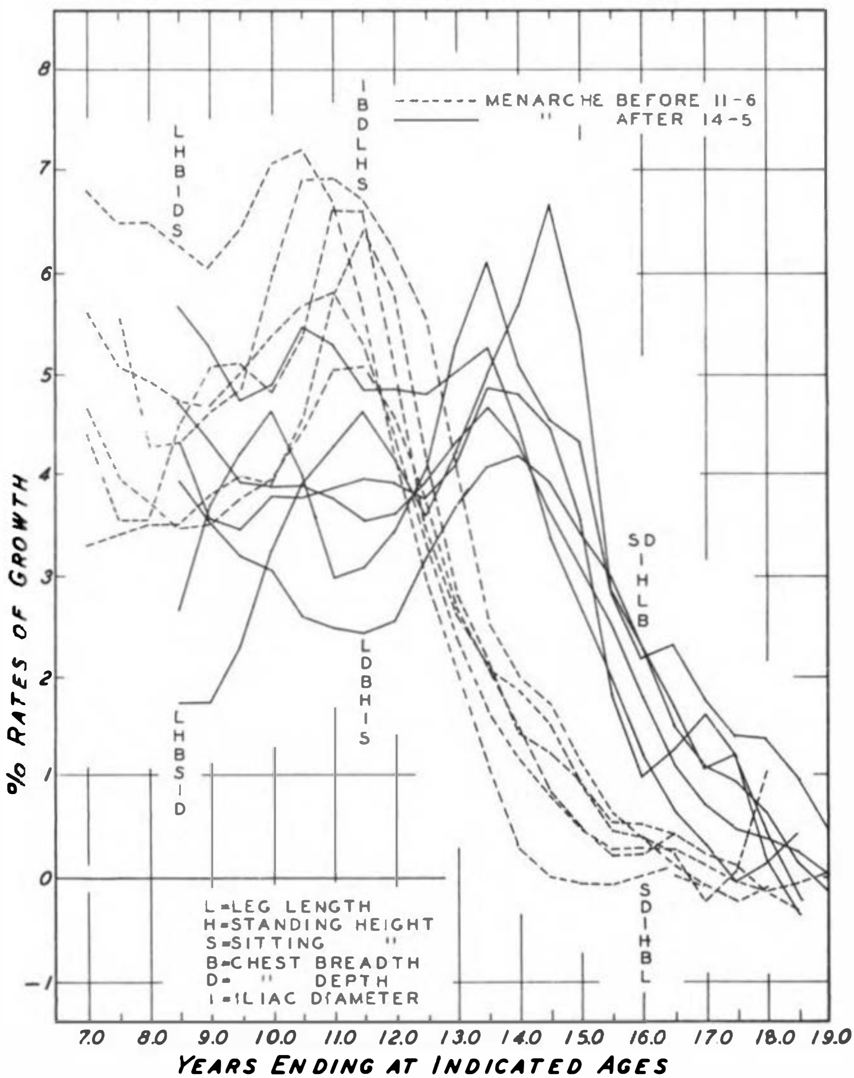

Fig. 120.--Percentage Rates of Growth of Early and Late Maturing Groups in all Measurements Except Weight.

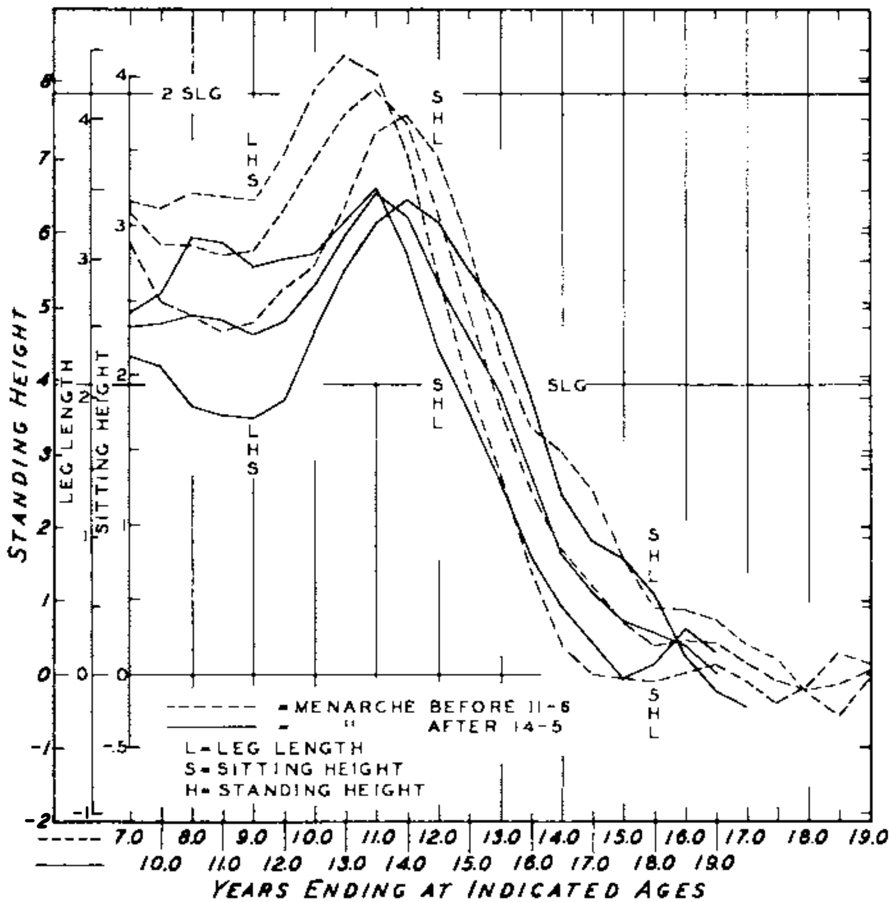

Fig. 121.—Average Annual Increments in Standing Height, Sitting Height, and Leg Length of Early and Late Maturing Groups Arranged so that Corresponding Points on the Patterns of Growth are in the Same Vertical Line.

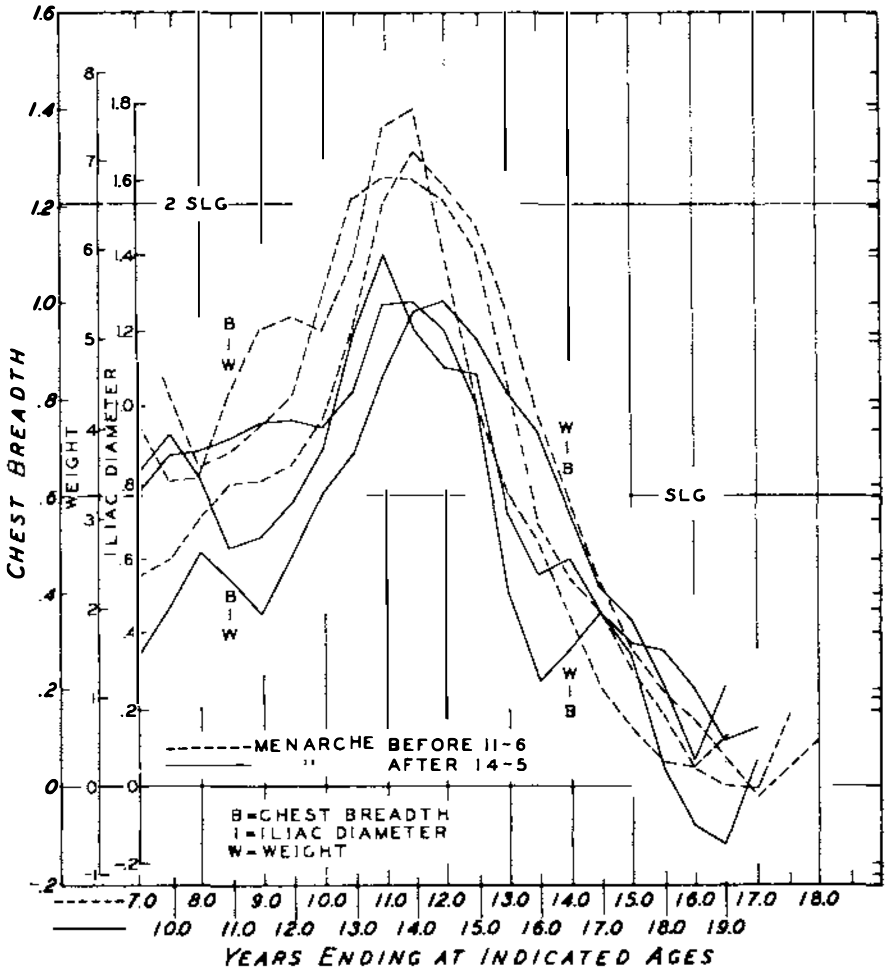

Fig. 122.--Average Annual Increments in Iliac Diameter, Chest Breadth, and Weight of Early and Late Maturing Groups Arranged so that Corresponding Points on the Patterns of Growth are in the Same Vertical Line.

weight. The contrast between groups is greater than between measurements for the second or accelerating phase and for part of the third or decelerating phase of growth.

3. Summary

Chapters IV and V began by emphasizing the contrasting growth patterns of early versus late maturing groups of girls. The generalized curves of Chapters VI to VIII tended to emphasize the contrasting growth patterns of each measurement versus every other measurement. When chronological age is considered the contrasting growth patterns of early versus late maturing groups of girls are clearly greater than the contrasting growth patterns of each measurement versus every other. Six types of graphical representation tell the same story. Even when the curves of the early maturing group are moved forward or to the right two and a half years so that corresponding points on the patterns of growth are in the same vertical line, the contrasts between the growth patterns of early versus late maturing groups are still as large as the contrasts between the growth patterns of each measurement versus every other measurement. These conclusions, of course, hold only for the available measurements which show accelerated growth during the adolescent period.

CHAPTER XIV

SUMMARY AND IMPLICATIONS

1. The Problem

The initial impetus for this study was supplied by the many reports which, though based on longitudinal data, have yielded only cross-sectional findings. The attack upon this problem was founded on the belief that better methods of analysis could be developed only in the process of attempting to solve genuine developmental problems with actual longitudinal data. A fortunate combination of circumstances directed the proposed study to the extensive collection of repeated measurements in the records of the Harvard Growth Study which had been initiated and directed by Professor Walter F. Dearborn during the years 1922 to 1934. The Harvard Study had started with approximately 3600 children in the first grades of three public schools near Boston and had followed them with annual measurements as long as they remained in the same schools. The measurements include eleven physical dimensions, roentgenograms of the wrist and hand, counts of erupted teeth, and intelligence and achievement tests. Professor Dearborn and Dr. J. W. M. Rothney have very generously made available a verified transcript of these data, except head dimensions and intelligence and achievement tests, on 248 girls who were measured annually over periods of from five to twelve years and whose records include information concerning the first menstruation. Given this setting the major problems of this study may be defined as follows. First, what are the most appropriate statistical methods to be employed in the analysis of longitudinal data? Incident to this problem it has been necessary to appraise the value of longitudinal data in comparison with cross-sectional data and to develop methods of overcoming certain limitations of the Harvard Growth data. Second, what are the patterns of physical growth among girls age six to nineteen and what is the association between these patterns and the advent of the first menstruation?

2. Methods of Analyzing Longitudinal Data

The contributions of this study to methods of analyzing longitudinal data may be conveniently reviewed under five headings. First, there is the demonstration in Chapter III that longitudinal data have very great advantages over cross-sectional data. It is true that this proposition has been the first article of faith among students of the physical growth of children for forty years. Nevertheless, there has been a general failure to appreciate the elementary statistical properties of longitudinal data and hence a conspicuous failure in the choice of statistical procedures to be employed in the analysis of longitudinal data. The most essential points can be compressed within three propositions. First, longitudinal data have no advantage whatever over cross-sectional data for the purpose of determining the average size of children in general at any given age. Second, longitudinal data have very considerable advantages over cross-sectional data for the purpose of determining the growth trends in average size of children in general from age to age. In terms of the number of cases required to give equally reliable trends the available longitudinal data for standing height on only 248 cases represents the equivalent of cross-sectional measurements on approximately 150,000 cases. Third, longitudinal data have very great advantages over cross-sectional data for the purpose of determining the growth trends in average size of homogeneous groups of children from age to age. Here the available longitudinal data for standing height represents the equivalent of cross-sectional measurements on approximately 270,000 cases. That is, longitudinal data make it possible to define the patterns of growth of small homogeneous groups of cases with an exceedingly high degree of reliability.

Procedures for dealing with certain limitations of the Harvard Growth data as described in Appendix A constitute a second type of contribution to methods of analysis. Procedures were developed for locating and rejecting erratic and discordant observations; for testing the possibility of unconscious changes in methods of measurement between successive years of the Harvard Growth Study; for marking certain of the reported menarcheal ages as of doubtful accuracy; for overcoming some of the difficulties due to the absence of measurements on birthdays and to long intervals between measurements; for correcting for irregular intervals between measurements; and for dealing with gaps in the records. None of these procedures is essentially novel but taken together they replace seeming confusion with uniform and ordered data which is ready for analysis. Since most of the limitations of the Harvard data are more or less inherent in longitudinal data of any sort, these procedures should be of continuing value. Confidence in the value of longitudinal data and methods of dealing with the special difficulties of such data constitute essential first steps.

A third point of importance is that the analysis be carried out in terms of the individual growth increments. The original data consist of observations at successive ages, but the basic and fundamental data, at least for this study, consist of the measured changes or increments from age to age. Throughout, all the computations have been carried out in the first instance in terms of increments and all growth trends in average size constitute secondary and derived data. Failure to proceed in this manner or to carry out the computations for identical cases from age to age amounts to throwing away the most valuable part of the data. Gross size at a given age should be regarded as merely the cumulative sum of all earlier increments and should be treated as an important datum only where the detailed data on the earlier increments are not available.

A fourth feature of statistical procedures has been called a process of analysis and synthesis whereby the available cases are divided and sub-divided into small homogeneous groups and later combined in different ways to reveal trends which would otherwise be obscure. In this study the major classification of the cases has been according to menarcheal age. Eight such groups were formed and each was further sub-divided into cases measured approximately at birthdays and cases measured approximately at the half-yearly anniversaries of birthdays. All of the computations for all measurements have been carried out separately for each of these sixteen groups. In the case of standing height each of these groups was still further sub-divided according to the age of maximum growth in standing height. The available cases have also been divided into tall versus short and into cases of North European versus those of Italian stock. These specific methods of classification by no means exhaust the possibilities. Analysis in terms of homogeneous groups is fundamental if developmental studies are to advance beyond the stage of describing the growth of children in general. Complementary to this analytic phase is a synthetic phase which combines the growth trends of several small homogeneous groups in order to clarify phenomena which are otherwise obscure. Thus, the original sixteen sub-groups were combined into eight groups giving observation points at semi-annual instead of annual intervals, they were combined into five groups at age intervals before and after the menarche instead of at chronological age intervals, and they were combined to give a single generalized pattern. Similarly, the large number of sub-groups resulting from classification according to both menarcheal age and age of maximum growth in standing height were combined in two distinct ways to give Figures 37 and 41 and a third synthesis would give only a slight variation of Figure 13.

Finally, adequate charts are more than a convenient medium for the presentation and illustration of findings. When, for example, one chart presents the increments of the eight menarcheal groups in relation to chronological age and next gives the same curves superimposed according to comparable points on the patterns of growth, then we may properly speak of graphical analysis which becomes an integral part of the statistical analysis.

3. Patterns of Growth in Relation to the Menarche

A summary of the constructive findings concerning the growth patterns of girls age six to nineteen in relation to the advent of the menarche presents difficulties since a mere verbal statement of the results is necessarily most inadequate. From time to time throughout this study brief summaries of the most essential findings have been presented[1]. While these summaries may seem too brief it should be noted that essentially all of the details are included in the charts and that the text of Chapters IV to XIII and particularly of Chapters VI to XIII is already in summary form. There is little profit in merely recapitulating summaries of summaries. Hence, the discussion brings together only the more important findings in a perspective which emphasizes the uniformity of the trends shown by all dimensions to the neglect of trends shown by each dimension. In compensation for the feebleness of mere verbal summary and for the slighting of details and exceptions, references to the more important charts are inserted.

Omitting for the moment consideration of skeletal age and erupted teeth, the more important findings may be formulated into eight major generalizations. First, each dimension has its own characteristic pattern (especially Figures 57, 61, 77, 79, 87, and 88 and also Figures 49, 50, 51, 60, 62, 63, 69, 70, 108, 109, 113, 115, 116, and 118). Even with charts which make all the curves as comparable as possible, both the growth trends in average sitting height and the series of annual increments in sitting height generate a characteristic pattern which is

[1] *See especially the sections beginning on pages 23, 31, 64, 74, 98, 117, 132, 149, 164, 178, 245, and 246.*

markedly and unmistakably different from the characteristic pattern of growth in leg length (Figures 49, 50, 57, and 61) and these in turn are different from the patterns of other measurements. Extreme contrasts are those between leg length and body weight. Similar patterns are those of chest breadth and depth, but the most rapid growth in chest depth occurs distinctly later than in chest breadth (Figures 77 and 79).

Second, while each dimension has its characteristic pattern, nevertheless, there are similarities in that all show at least two and, more often, three distinct phases (Figures 39, 57, 61, 77, 79, 87, and 88). The first phase is typically decelerating. In the case of sitting height the deceleration is striking even in the increments and more so in the relative growth rates. In the case of standing height, sternal height, and leg length the decelerating phase is less apparent. In the case of iliac diameter and weight the increments show a slight acceleration and only the relative growth rates reveal a decelerating phase. In terms of averages this phase lasts about two and a half or three years. This first or decelerating phase is absent in both chest breadth and chest depth. The second phase is one of decidedly accelerated growth in all dimensions. In the case of sitting height, iliac diameter, and weight the acceleration is marked and lasts for three years while in the case of leg length the acceleration is much less marked and lasts for only two years. The third phase, shown by all dimensions, is one of sharply decelerating growth which is very abrupt in the case of leg length and less abrupt in the case of sitting height, iliac diameter, and weight.

Third, while the characteristic patterns of the several dimensions and the three phases of growth are most clearly portrayed in the generalized curves of annual increments and relative growth rates, they are also apparent in the patterns of each of the eight menarcheal groups. Hence, except for a few minor but significant differences to be noted later, the patterns of the eight menarcheal groups are very much alike. Charts in which chronological age is ignored and corresponding points on the eight increment curves are arranged in the same vertical line are illustrative (Figures 19, 39, 43, 55, 56, 74, 75, 76, and 86).

Fourth, while the patterns generated by the increments of the eight menarcheal groups are very much alike, they occur at very different chronological ages (Figures 13, 23, 37, 52, 53, 54, 71, 73, 85, and 90). That is, the pattern of Group A (menstruating before age 11-6) is similar to that of Group H (menstruating after age 14-5), but the two patterns are separated on the average by an interval of two and a half years of chronological age. The patterns of Groups B, C, D, E, F, and G (menstruating at successive intervals of six months) fall in serial order between Groups A and H. It results at age eleven or twelve that the maximum increments of Group A which mark the climax of the second or accelerating phase tend to coincide with the trough of the curve of Group H just before the initiation of the accelerating phase. Here the increments of Group A tend to be the largest followed in generally regular order by Groups B, C, D, E, F, G and H. Or, in terms of correlations the associations between menarcheal ages and annual increments are negative, averaging -.33 (Table 2, Chapter XII). Three years later this situation is completely reversed Group A having almost ceased to grow while Group H is attaining the climax of the accelerating phase. Here the increments of Group H tend to be the largest followed in generally regular order by Groups G, F, E, D, C, B, and A. Or, in terms of correlations the associations between menarcheal ages and annual increments are positive, averaging .44.

Fifth, since the accelerations and decelerations of the eight menarcheal groups occur at different chronological ages, it follows that growth trends in average size of the eight groups are clearly differentiated (especially Figures 11, 46, 47, 48, 66, 67, 68, and 83 and also Figures 49, 50, 69, 92, 98, 99, 100, 101, 102, and 117). Indeed, the contrasts in the growth trends in average size between early and late maturing groups of girls are much greater than the contrasts between the characteristic patterns of the several dimensions (Figures 113, 115, 116, and 128). At ages six to nine these differences are comparatively small, but the earlier maturing groups tend consistently to be bigger than the later maturing groups. These contrasts become progressively greater up to age twelve or thirteen. Here Group A tends consistently to be the biggest in average size followed in a generally regular order by Groups B, C, D, E, F, G, and H. At this point of greatest contrast Group A averages about 12% bigger in linear measurements, 50% heavier, and about 38% heavier per unit of linear size than Group H. As maturity is attained these differences vanish or are slightly reversed. In terms of correlations the associations between menarcheal ages and physical size average -.28 at age eight, -.54 at ages twelve or thirteen and .01 at ages seventeen and eighteen (Table 1 of Chapter XII and Figure 110).

Sixth, since the decelerating, accelerating, and decelerating phases of the eight menarcheal groups occur in serial order at successive chronological ages, it also follows that the inflection points between these phases are timed in relation to the advent of the menarche. This generalization is important for the endocrine interpretation of the data and the two inflection points need to be considered separately.

The inflection points between the first or decelerating phase and the second or accelerating phase as shown by the generalized patterns of annual increments (Figures 39 and 57) occur on the average at ages 9.67, 9.55, and 9.65 (or 10.15) in the case of standing height, sitting height, and leg length or on the average a full three years before the advent of the menarche. The inflection points for the relative growth rates tend to occur slightly later on the average at ages 9.67, 9.55 (or 10.15), 10.15, 9.93, and 10.12 (Figures 61, 79, and 88) respectively for standing height, sitting height, leg length, iliac diameter and weight. While these ages are "on the average" and hence would hold only for Groups D and E, nevertheless, the indicated interval of three or three and a half years to the advent of the menarche holds very well indeed for all eight groups. In the case of standing height Groups A, B, C, D, E, F, G, and H menstruating on the average at ages 11.0, 11.75, 12.25, 12.75, 13.25, 13.75, 14.25, and 15.0 show their first inflection points at ages 8.5, 8.0, 9.5, 9.5, 10.0, 10.5, 11.0 (or 11.5), and at 11.5 respectively or 2.5, 3.75, 2.75, 3.25, 3.25, 3.25, 3.25 (or 2.75) and 3.5 years before the menarche (Figure 13). Considering the fact that these inflection points are determined by differences averaging only a tenth of a centimeter, such a series is extraordinarily uniform. Comparable data for sitting height shows that the first inflection points in Groups A, B, C, D, E, F, G and H come 2.5, 3.75, 3.25, 3.25, 2.75, 3.75, 3.25, and 3.50 years before the menarche (Figure 53). A comparable

TABLE 5

AVERAGE AGES AT THE SECOND INFLECTION POINT

Dimensions	Generalized Patterns of Increments - Figs. 39, 57, 77, 87	Generalized Patterns of Relative Growth Rates - Figs. 61, 79, 88	Maximum Increment of Individuals Relative to Sitting Height, Chapter VII, Section 3
Leg Length	12.15 (or 11.65)	11.65	12.01
Standing Height	12.17	12.17	12.29
Chest Breadth	12.40	12.40	12.44
Iliac Diameter	12.43	12.43	12.54
Sitting Height	12.55	12.55	12.55
Weight	12.62	12.12	12.72
Chest Depth	12.64	12.64	12.68

TABLE 6

INTERVALS BETWEEN SECOND INFLECTION POINT AND AVERAGE MENARCHEAL AGE FOR EACH OF EIGHT GROUPS

Dimensions	A	B	C	D	E	F	G	H
Leg Length	.00	-.25	-.75	-.75	-.75	-.75	-1.25	-1.50
Standing Height	.00	-.25	-.75	-.75	-.75	-.75	-.75	-1.50
Chest Breadth	.50	-.25	-.25	-.75	-.25	-.75	-1.25	-1.50
Iliac Diameter	.50	-.25	-.25	-.25	-.75	-.75	-.75	-1.00
Sitting Height	.50	-.25	-.25	-.25	-.25	-.75	-.75	-1.00
Weight	.50	-.25	-.75	.25	-.25	-.75	-.25	-.50
Chest Depth	.50	.25	-.25	-.25	-.25	-.75	-.25	-.50
Average	.35	-.17	-.46	-.39	-.46	-.75	-.75	-1.07
Standing Height	.21	-.28	-.50	-.44	-.77	-.85	-.92	-1.10

series for leg length is as follows: 2.0, 3.75, 2.75, 4.25, 3.25, 2.25, and 3.50 years before the menarche (Figure 54). A comparable series for relative growth rates in body weight is as follows: 1.5, 2.75, 2.75, 2.75, 3.75, 3.25, 2.75, and 3.50 years before the menarche (Figure 90).

The second inflection point, that between the accelerating and decelerating phase, is also timed in relation to the menarche. While the first inflection point marking the initiation of accelerated growth occurs at approximately the same time in all dimensions (save chest breadth and depth where it is absent), the second inflection point marking the climax of the accelerating phase shows a serial order for the several dimensions. Table 5 lists the dimensions in the apparent order of their inflection point and records three sets of average ages. Again these ages are "on the average" and would hold true only for Groups D and E. Nevertheless, the general indications that the second inflection points occur from three months to a year before the menarche holds very well for all eight groups. Using the pattern of each group as a whole to determine the inflection points, as recorded in Table 28 of Appendix B, instead of taking the actual maximum increments, Table 6 gives the intervals before and after the menarche. Since the determination of the inflection points is not accurate to less than half a year, all of the intervals are subject to errors of .5 years. Minus signs indicate that the second inflection point occurs prior to the menarche. Averages for the seven dimensions are recorded and to these have been added further data for standing height resulting from the analysis according to both menarcheal age and age of maximum growth in standing height (Figure 37). Groups A, B, C, D, E, F, G and H, who menstruated on the average at ages 11.0, 11.75, 12.25, 12.75, 13.25, 13.25, 13.75, 14.25, and 15.0 respectively, reach the climax of their accelerating phases on the average at ages 11.35, 11.58, 11.79, 12.36, 12.79, 13.00, 13.50, and 13.93 or .35 years after and .17, .46, .39, .46, .75, .75, and 1.07 years before the menarche. In this case, however, "on the average" is merely a convenient way of reading the whole of the table at once. Examination of the details will show that these trends hold very closely for each dimension save for the fact, already noted, that the peak of the accelerating phase in leg length is consistently earlier than in chest depth. There is also a consistent tendency for the inflection points of Group A to occur after and for those of Group H to occur a year before the menarche. In part this due to the regression effect necessarily involved when two factors are not perfectly correlated and to the fact that the standard deviation of menarcheal ages is greater than that of inflection points (Chapter XII, Section 1, Figure 112). Further evidence of the intimate timing of inflection points and menarcheal age is available for standing height. When 174 individuals are classified according to age at the peak of their accelerating phase, the patterns of the resulting groups are timed in relation to the menarche (Figures 41, 44, and 45). Individuals who reach the peak of their accelerating phase at ages 10.5, 11.5, 12.5, 13.5 and 14.5 menstruate on the average at ages 11.27, 12.20, 13.07, 13.84, and 14.65. The correlation between menarcheal ages and age at the second inflection point for these 174 cases with clearly defined peak increments is .754 and for 246 cases is .706 (Figure 112).

So far it has been emphasized that the patterns of decelerating, accelerating, and decelerating growth among the eight menarcheal groups are very similar (Figures 19, 39, 43, 55, 56, 74, 75, 76, and 86). A seventh large generalization is that there are nevertheless consistent and highly significant differences in detail. It has already been noted that the second inflection points of Group A occur at or after the menarche while those of Group H occur before the menarche. While this difference is very consistent for all dimensions, its significance, as already noted, is blurred by regression effects. A much more significant and also very consistent difference is that the intensity of the pattern decreases in generally regular order from Group A through Groups B, C, D, E, F, and G to H. That is, when corresponding points on the pattern are considered, the increments of the early maturing groups are consistently larger than the increments of the late maturing groups. Averaging the increments and relative growth rates over the period of three and a half years before and one year after the peak of the accelerating phase for each group and each dimension gives the data of Table 7. With the exception of chest breadth and chest depth there is a consistent tendency for the magnitude of the increments and of the relative growth rates to decrease progressively from early to late maturing groups. A third significant and consistent difference is that the first or

TABLE 7

AVERAGE INCREMENTS AND RELATIVE GROWTH RATES FOR EACH OF EIGHT GROUPS OVER CORRESPONDING PORTIONS OF THE PATTERN OF GROWTH

Dimensions	Average Increments (Centimeters and Kilogram)							
	A	B	C	D	E	F	G	H
Standing Height	6.58	6.54	6.47	6.23	6.03	5.96	5.43	5.34
Sitting Height	2.92	2.86	2.84	2.78	2.64	2.64	2.45	2.40
Leg Length	3.70	3.68	3.64	3.49	3.40	3.31	3.09	3.00
Iliac Diameter	1.27	1.25	1.23	1.15	1.07	1.21	1.09	.98
Chest Breadth	1.00	.98	1.04	.92	.89	.91	.96	.81
Chest Depth	.67	.65	.64	.64	.58	.67	.67	.70
Weight	4.91	5.01	4.69	4.45	4.26	4.48	4.04	3.86
	Average Rates of Growth Per Year (Percentages)							
Standing Height	5.09	4.88	4.85	4.69	4.42	4.37	4.00	3.95
Sitting Height	4.11	3.98	3.88	3.81	3.57	3.62	3.36	3.25
Leg Length	6.25	5.93	5.96	5.68	5.34	5.23	4.98	4.81
Iliac Diameter	5.63	5.52	5.45	5.12	4.72	5.38	4.81	4.28
Chest Breadth	5.02	4.92	5.20	4.68	4.42	4.53	4.94	4.13
Chest Depth	4.53	4.31	4.33	4.34	3.89	4.53	4.59	4.64
Weight	15.21	15.58	14.45	12.85	12.52	13.39	11.88	11.40

typically decelerating phase occupies a progressively longer and longer period of time from early to late maturing groups. Since the patterns of increments are not reliably established prior to age eight, the exact time periods can not be computed. It is unmistakably clear, however, that this first decelerating phase comes to an end on the average two and a half years later in Group H than in Group A. These three features distinguishing the patterns of early and late maturing groups are doubtless related to the fact that early maturing groups are smaller than late maturing groups two and three years before the advent of the menarche or two and three years before the climax of the accelerating phase of growth (Figures 12, 44, 51, 70, and 94). Three years before the advent of the menarche Group A, age 8.0, averages 12.24 centimeters shorter than Group H, age 12.0 (Figure 12). Three years before the climax of the accelerating phase Group A, age 8.0, averages 7.05 centimeters shorter than Group H, age 10.5. Since the two groups attain almost the same mature stature and since both the accelerating and final decelerating phases occupy the same time period, some compensating adjustments are necessary. The three distinguishing features of the patterns of early and late maturing groups represent different aspects of this adjustment.

A final generalization is that the patterns of growth shown by tall versus short children and by children of North European versus Italian stock are very similar (Figures 16, 17, and 98 to 108). Children of North European stock are taller and become progressively taller with age, the stature differences being due largely to differences in leg length. The patterns of growth, however, could hardly be more alike save that the pattern of growth in leg length among children of Italian stock is less intense.

All of these eight major generalizations probably apply to growth and development as revealed by roentgenograms of the hand and wrist, but the arbitrary and artificial nature of the data which is in terms of estimated skeletal ages obscures the true growth accelerations and decelerations (Figures 94 and 95). Of these eight generalizations only the first and third apply to the data for permanent erupted teeth (Figures 96 and 97).

Excepting estimated skeletal ages and erupted teeth, the whole of the data may be adequately summarized as follows. Each dimension has its own characteristic pattern of growth and each shows three phases of slow or decelerating growth, of decidedly accelerating growth, and of sharply decelerating growth. The inflection points between these three phases are intimately timed in relation to the advent of the menarche in such a way that Group A, menstruating before 11-6, reaches the climax of the accelerating phase on the average at age 11.35 while Group H, menstruating after age 14-5, reaches the climax of the accelerating phase on the average at age 13.93. The patterns of growth in the eight menarcheal groups are very similar save that the patterns of the later maturing groups show a progressive lengthening of the first or typically decelerating phase and a progressive decrease in intensity.

4. Implications

The process of sexual maturation in girls begins with the budding of the breasts, the appearance of pubic hair, and after some time culminates in the first menstruation or menarche. The process is undoubtedly initiated and controlled by endocrine factors. Hence, the intimate timing of menarcheal ages and inflection points on the pattern of physical growth suggests that the serial order of accelerating and decelerating phases of growth are also initiated and controlled by endocrine factors. The patterns and associations are ordered as if the following were true. The pituitary begins to function above a certain critical level as early as age eight or as late as eleven in different individuals, initiating at the same time the process of sexual maturation and the second or accelerating phase of physical growth. After this phase has progressed for some time new endocrine factors begin to function above their critical levels and in serial order operate to close abruptly the epipheses of long bones, to check the growth stimulating factors of the pituitary, and to bring the process of sexual maturation to a close in the advent of the menarche as early as age eleven or as late as age fifteen in different individuals. It would be rash to assert that this is the true serial order of events but it is clearly a plausible interpretation of the data. Given the same sequence of underlying endocrine factors determining physical growth, it follows that the patterns of early and late maturing groups must be very similar and yet different. Individuals who are initially stimulated at age eight, when they are comparatively small and far from their mature size, have their initial decelerating phase cut short, exhibit a more rapid and intense growth pattern, and reach the climax of their accelerating phase at or after the menarche. Individuals who are initially stimulated at age twelve, when they are comparatively big and not far from their mature size, have their initial decelerating phase prolonged, exhibit a less rapid and intense growth pattern, and reach the climax of their accelerating phase six months or a year before the advent of the menarche.

The implications of the so-called "adolescent spurt" in growth for physical care, diet, and mental hygiene have been elaborated in many text books but upon an insecure foundation of fact derived from cross-sectional studies. The adolescent spurt turns out to be a pre-adolescent phenomenon which reaches its climax not at any one age but as early as age ten or as late as age fourteen. Further, the accelerating phase for all dimensions is concentrated within a period of two or three years rather than distributed over four or five years. The suggestion of greater dietary needs during this period is far more forceful than cross-sectional studies would indicate. The individual growth curves and charts showing growth trends in average size provide new material with which to answer some of the persistent questions of the adolescent, i.e., "Am I normal?", "Will I ever stop growing?", etc. The development of methods of predicting the advent of the menarche from physical data falls outside the scope of this study, but it is clear that fairly accurate predictions beginning as early as age nine could be constructed and would be of considerable clinical value.

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APPENDIX A

METHODS OF DEALING WITH CERTAIN LIMITATIONS OF LONGITUDINAL DATA

Since the initial problem out of which this study has grown was the development of better methods of analyzing longitudinal data, no apology need be offered for the inclusion of two appendices on statistical methods. In Chapter III it was emphasized that an appreciation of the advantages of longitudinal data provides the key to methods of dealing with limitations of such data which were outlined in Chapter II. The difficulties which are involved in working with such data are automatically reduced to the role of minor and technical problems. They remain, however, very real problems. A straight forward exposition directed solely to methods of dealing with these difficulties would be comparatively simple. In the records of the Harvard Growth Study, however, there is the complicating factor that all the typical limitations of longitudinal data in somewhat extreme form, together with some additional limitations, are inextricably entangled in the same data. In addition we have the task of laying the foundation for a constructive attack on a specific developmental problem. For the purpose of convenient discussion this appendix is devoted to methods of dealing with the limitations of the data of the Harvard Growth Study. Appendix B is devoted to methods of analysis and to a presentation of the basic numerical data which lead directly to the constructive findings which have been presented in the body of the monograph. Summaries of these appendices appear in sections eleven and twelve of Appendix B.

1. Erratic Measurements and Editorial Methods

A feature of the Harvard records which has operated to discourage analysis is the presence of a considerable number of figures, which seem, in the light of all the data, to be discordant, erratic, or impossible. To cite the most extreme example, the original transcript of the records contained the following series of annual measurements of standing height on case #918 in which the underlined figure is clearly impossible: 134.5, 139.3, <u>134.5</u>, 149.0, and 156.8 centimeters. In addition to such erratic figures in the records of individuals, an irregular trend in averages may also be cited. A group of 54 cases all of whom were measured during the fall of 1922, 1923, 1926 show the following averages for chest depth: 13.41, 13.17, and 15.37 centimeters. Separate sections are devoted to the two problems which are illustrated by these examples.

The large proportion of questionable figures especially for chest breadth and chest depth is due to a number of factors. First, the measurements were taken over clothing (Clark, 11). Second, the chest measurements are inherently difficult. Third, the measurements were taken by a large and constantly changing personnel. Fourth, systematic checks were not employed during the first two years. Fifth, it is possible that our standards of accurate measurement are spuriously high in that they are based largely on agreements between two independent measurements made at the same time (Boyd, 9). Sixth, and this is important though somewhat paradoxical, the large proportion of apparently erratic figures is due in large part to the availability of longitudinal data. That is, if only cross-sectional data were available, it would be quite impossible from the data to determine whether a particular individual was really 140 or 175 centimeters tall. Instead of being disturbed by the presence of apparently erratic figures, the point to be emphasized is that longitudinal data reveal erratic figures which otherwise would not even be suspected.

If this study were concerned merely with the growth trends of children in general, the problem created by erratic measurements might be dismissed on the ground that the errors would cancel out. Since the measurements beginning in the fall of 1924 were taken in duplicate and later in triplicate, it might also be urged that the data, even from a cross-sectional point of view, are superior to the general run of cross-sectional measurements. In the light, however, of the necessity of dividing the population into a large number of sub-groups (see section one of Appendix B) some method of dealing with these erratic figures seemed imperative. Since there is a total of 2232 series of records and of over 21,000 measurements, a not too laborious method was an important consideration.

The problem raised by discordant observations is an old one in the history of science. Smoothing operations involving moving averages, graduation formulas, and curve fitting have been widely used especially in actuarial science to eliminate irregularities in observed data (See Reitz, 26; Lidstone, 21; and Steffenson, 32). The typical situation for which a smoothing operation is appropriate consists of census or mortality data where there is a tendency to report ages in multiples of five years. In such cases the smoothing operation redistributes the excess frequencies over adjacent ages. Application of a smoothing operation to the available data, however, would amount in many cases to a redistribution of errors. That is, the available data call for the rejection rather than the smoothing of discordant observations. Since the outright rejection of observations is not commonly practiced, some distinctions need to be noted. The original observer is privileged to reject any measurements which in his judgment were made under unusual conditions, while the statistician who has available only the bare observations is in a different position. In general rejections which might favor an announced theses are improper, while those which merely smooth the data have a different status. Finally, grounds for rejection should be distinguished. The classical problem involving rejection on the ground of statistical improbability consisted of the fifteen observations of the vertical semi-diameter of Venus (Pierce, 25). Francis Galton's individual difference problem also stimulated a series of papers (Irwin, 19). It is now recognized, however, that such rejection is somewhat uncertain and arbitrary (Brunt, 10; and Rider, 29). The Harvard data provide a very excellent illustration of both the logic and the uncertainties of this procedure. At age seventeen (16.75 to 17.24) there are 102 records of estimated skeletal age. Ninety-three of these skeletal ages fall between 194 and 197 months, six are distributed between 184 and 193 months, two fall far outside this normal range with skeletal ages of only 179 months, and one record, case

#1272, falls very far outside even these extreme limits with a skeletal age of only 147 months. If only cross-sectional data were available, all probability standards would reject the extreme value of 147 months. However when the whole record of this case is examined, there is nothing to cast suspicion on the accuracy of the estimated skeletal age of 147 months. That is, the entire record of this individual might be rejected on the ground of extreme and abnormally retarded development, but neither the skeletal age of 147 months nor any other measurement can be rejected on the ground that it falls so far outside the normal distribution as to be improbable. However, in no instance has the entire record of an individual been rejected on the ground of abnormality.

A third ground for rejection is present in contradictory observations or observations known to be contrary to fact. Repeated physical measurements are particularly well suited to the application of such a criterion because negative differences or increments of growth, at least of skeletal measurements, should not exceed diurnal fluctuations and the errors involved in measurement. Rejection on the ground of negative increments suggest the systematic calculation of all first and second differences since this is a procedure which has been widely used to spot erratic figures among observations which represent a function of some variable (Scarborough, 30). Since diurnal fluctuations are not involved, it has been assumed as a first approximation that negative differences or increments greater than half a centimeter probably fall outside the limits of error to be expected from the unreliability of the measurements. This seems to be the standard of accuracy which the Harvard Study attempted to achieve when it obtained additional measurements. Such a standard can also be supported from studies of the discrepancies between physical measurements made by two independent observers (Dahlberg, 13). Two variations of the same essential principles have been employed.

The first and less laborious procedure was applied to all measurements over the whole age range and to all 315 cases prior to the availability of a verified transcript of the records. The procedure may be illustrated by the data of Figure 123 which gives the scatter-diagram of all standing heights at all ages plotted against standing heights a year later. All points to the left of the continuous zigzag line represent instances in which height at one age is less than a year earlier. There are only 22 such points in Figure 123 although the total number of negative increments is much larger. For the most part these negative increments occur at ages at which growth has ceased and only twenty-three are greater than half a centimeter. Given such a scatter-diagram we might determine the two regression curves, make distributions of differences between predicted and actual values, and reject all differences which fall beyond certain defined limits. In the present case, however, simple inspection is adequate to reveal the points which deviate most from the central trend. Attention is called to the two most extreme deviates which are underlined. Characteristically, both deviates are from a single case, #918, which was cited at the beginning of this section. The original transcript gave the following series of annual measurements in which the underlined figure is doubtful or impossible: 134.5, 139.3, <u>134.5</u>, 149.0, and 156.8 centimeters. This case provides an excellent illustration of criteria for the rejection of erratic measurements. First, the two pairs of measurements 139.3-134.5

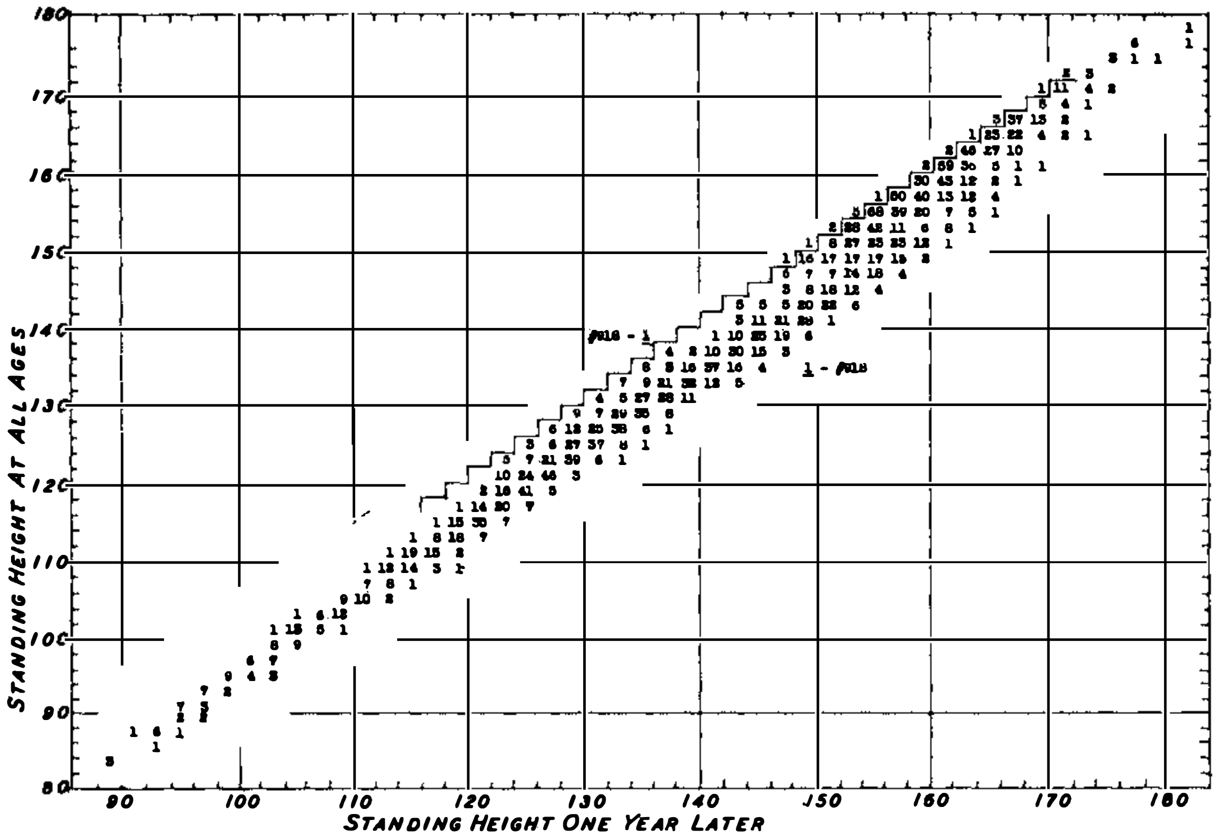

Fig. 123.—Scatter diagram of all Standing Heights at All Ages Plotted Against Standing Heights a Year Later.

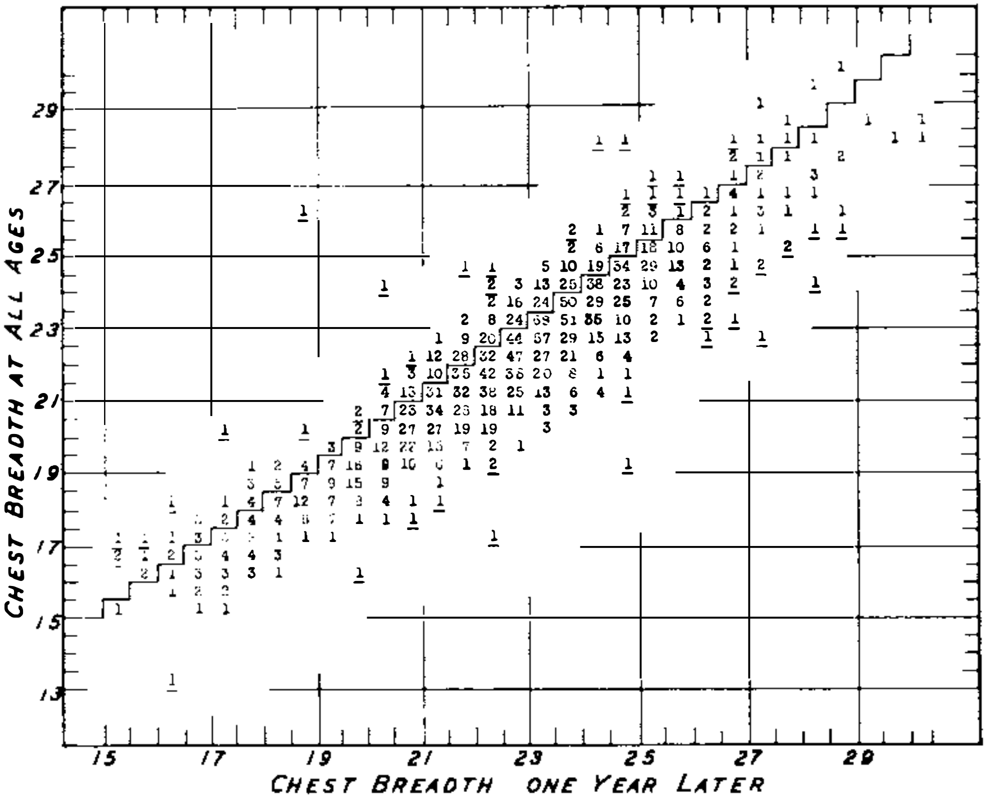

Fig. 124.--Scatter-diagram of all Chest Breadths at all Ages Plotted against Chest Breadths a Year Later, Illustrating Procedure Employed to Spot Erratic Measurements.

and 134.5-149.0, when plotted on a scatter-diagram as in Figure 123, both deviate abnormally from the central trend and in opposite directions. Second, the difference between 139.3 and 134.5 centimeters a year later gives a negative increment of -4.8 which greatly exceeds the limits of error involved in taking the measurement. Likewise, the difference between 134.5 and 149.0 a year later gives a positive increment of 14.5 centimeters which falls so far beyond the extreme range of other increments as to be highly improbable. Finally, these two differences are in opposite directions so that the second order difference (-4.8 minus 14.5) falls still further beyond the extreme range of the second differences. The procedure is not laborious and is obviously objective. Further, in this particular case the procedure proved to be valid since the verified transcript corrected the record of this case. The verified record gives the series 134.5, 139.3, 143.5, 149.0 and 156.8 centimeters. Figure 124 presents a second illustration of this procedure for chest breadth. Here 19% of the plotted points fall to the left of the zigzag line indicating instances in which there is a decline in the measurement of chest breath from one age to the next. There is also a much larger proportion of points which deviate abnormally from the central trend than in the case of standing height. The underlined points indicate those which were marked as erratic on the original transcript of the data. With the exception of iliac diameter, chest breadth, and chest depth, the procedure just illustrated was the method used to spot discordant figures. Only one measurement for sternal height, four for sitting height, four for weight, and none for standing height and skeletal age were rejected as erratic. In the case of erupted teeth 31 figures were discarded. Erratic figures for the number of erupted permanent teeth seem to be due to failure to count extracted teeth as erupted.

The more refined and laborious procedure applied to measurements of iliac diameter, chest breadth, and chest depth involved the calculation of first and second differences. Annual increments were computed throughout the age range and distributed for each age interval as six to seven, seven to eight, etc., without regard to menarcheal ages. The standard deviation of each distribution of increments was determined and all positive increments exceeding the average by two standard deviations were marked as doubtful. All negative increments greater than half a centimeter were also marked as doubtful. The series of gross measurements and of increments for each individual were then examined giving special attention to instances in which two consequtive increments were marked as doubtful. In such cases second differences were also computed and examined. The selection of particular measurements for editorial rejection was made on the basis of the trend established by the whole series of measurements on each individual. Typical editorial eliminations are illustrated by the following cases in which the underlined figures for chest breadth have been discarded. Case #732, beginning at age 11-11: 22.3, 23.2, <u>26.0</u>, 25.4, 25.2, 25.0, 24.7, and 24.9 centimeters. In this case <u>26.0</u> is clearly erratic. The downward trend from 25.4 to 24.7 is also questionable but not sufficiently so to warrant the rejection of either. Case #71, beginning at age 11-3: 19.7, 19.2, <u>24.6</u>, 22.4, 21.9, 22.4, 21.7, and 21.8 centimeters. Here, <u>24.6</u> is clearly erratic, but the trend established by the remaining measures is too uncertain to warrant additional rejections. Although a fairly objective procedure may be set up for the purpose of locating discordant figures, the selection of particular measurements for editorial elimination is not amenable to strictly mechanical routine. The total numbers of editorial eliminations of iliac diameter, chest breadth, and chest depth are 28, 108, and 112 respectively or 1.1%, 5.4%, and 5.5% respectively of the number of original figures.

2. Systematic Changes from One Year to the Next in Methods of Measurement

The average chest depth for a group of 54 cases, all of whom were measured during the fall of 1922, the fall of 1923, and the fall of 1926, at average ages of approximately 7.0, 8.0, and 11.0 are 13.41, 13.17, and 15.37 centimeters. In cross-sectional studies the decline of .24 centimeters between age 7.0 and 8.0 would be attributed to sampling errors, but here it at once suggests the occurance of some systematic change or variation in methods of measurement. This trend may be compared with data collected by the U. S. Public Health Service (Collins, 12). They report averages of chest depth at the nearest birthday on over fifteen hundred girls at each age. Using the annual increments derived from both studies we have the following tabular data.

	Harvard Study	U. S. Public Health
Increment age 7.0 to 8.0	-.24	.30
Average increment per year, age 7.0 to 11.0	.49	.42
Average increment per year, age 8.0 to 11.0	.73	.47

Since the Harvard increment from age 7.0 to 8.0 is abnormally negative while that from age 8.0 to 11.0 is abnormally large, it would appear from these figures that the measurements of chest depth taken during the fall of 1923 are in error. In the light of the data of the U. S. Public Health Service they are on the average .54 centimeters too low. Further evidence of some abnormality in these data is provided by the decidedly bimodal distribution of the increments and the large proportion of negative increments in chest depth between the first and second year of the Study. This distribution is as follows: in three cases the increments are 1.5 centimeters or more, in 19 cases from .5 to 1.4 centimeters, in 6 cases from -.5 to .4 centimeters, in 19 cases from -1.5 to -.6 centimeters, and in 7 cases from -3.0 to -1.6 centimeters. The evidence is unmistakable that some marked departure from the normal method of measuring chest depth is involved on the part of one or more anthropometrists. Given access to the details of the original data which indicate the persons taking each measurement it might be possible to locate the specific causes and make corrections, but this has not been attempted. Instead, these data have been edited by methods described in the preceding section on erratic measurements. However, mere editorial rejection of erratic figures does not solve the problem suggested by this instance. It is important to determine whether there are other instances of apparently systematic changes from one year to the next in methods of measurement.

The effectiveness of examining the individual increments from one year of the Study to the next for bimodal distributions and for excessive proportions of negative increments has already been illustrated. These procedures were applied to chest breadth, chest depth, and iliac diameter using the final selection of 248 cases prior to the editing of the data. With the exception of the one bimodal distribution already described nothing approximating a bimodal distribution by simple inspection was found among twenty-nine which were prepared. While each year of the Study shows a proportion of negative increments, in no one year, save between the first and second in the case of chest depth is this proportion excessive in comparison with other years. These results, however, represent the absence of evidence of systematic changes from year to year in methods of measurement rather than evidence that no changes took place.

The second, and also indecisive, procedure applied to a preliminary selection of 264 cases may be illustrated by the following tabulation which shows the increments in standing height between age eleven and twelve as determined for different groups of identical cases measured during different years of the Study.

Years of Study	Number of Cases	Increments in Height	Departures from Normal Increment
3rd to 4th	7	6.5	-.2
4th to 5th	26	6.6	-.1
5th to 6th	101	6.3	-.4
6th to 7th	87	7.2	.5
7th to 8th	25	6.9	.2
8th to 9th	3	4.3	-2.4
Total	249	6.7	

This tabulation is read as follows. Seven cases age eleven during the third year of the Study who were remeasured at age twelve during the fourth year of the Study showed an average increment of 6.5 centimeters in standing height. On the average 249 identical cases measured during various years of the Study gained 6.7 centimeters between age eleven and twelve. Using this gain of 6.7 centimeters as a standard, the gain between the third and fourth year is -.2 centimeters below normal. Thirteen such tables were prepared. The deviations from normal increments were weighted by the number of cases and averaged. The results give apparently significant fluctuations from year to year. From the sixth to the seventh year of the Study (fall of 1927 to fall of 1928) the weighted deviation from normal is .9 centimeters whereas from the ninth to the tenth year (fall of 1930 to fall of 1931) the weighted deviation from normal is -.7 centimeters. Since measurements in triplicate were taken during these years and since the measurement of standing height is so objective and well standardized as to be hardly subject to systematic changes in methods of measurement, it would appear that the interval from the fall of 1927 to the fall of 1928 was a good growing year whereas the interval from the fall of 1930 to 1931 was not (Palmer, 23). The extreme variations in chest depth, chest breadth, and iliac diameter are approximately half of the extreme variation in standing height which has been cited. Again, however, such data merely fail to provide direct evidence of changes or variations in methods of measurement.

The chief antidote for changes from year to year in methods of measurement, if there are any in addition to the one clear case, is the fact, already noted in the illustrative data cited above for standing height, that growth from one age to the next is determined by measurements spread over a seven year period. Under these conditions the type or error which we have been discussing tends to cancel out.

3. Questionable Menarcheal Age Records

The available data on the advent of the menarche have serious limitations. Only the data from the city of "B" can be regarded as really trustworthy. In this school the physical education instructor, a woman, interviewed each girl over a period of years for the purpose of collecting these data. While these records are believed to be accurate their collection was discontinued when the girls in this school averaged about thirteen years of age. Hence, there is only one case with a recorded menarcheal age of fourteen or later. Only 32 of the 248 cases are from this school.

The questionnaire data obtained in the city of "M" are unsatisfactory from several points of view. The questionnaires were distributed to mothers when the subjects ranged in age from eleven to sixteen. It results that many of the records carry indefinite notations as "advent of menarche not by" such and such an age. Sixty-seven such cases among the 315 originally made available have been discarded for the purposes of this study. Ten such cases where the notation is "advent of the menarche not by" age 14-5 or later have been included among the 248 cases selected for intensive study. No systematic check of the questionnaire returns against other data was attempted nor was the questionnaire procedure repeated. However, in connection with the collection of other data thirty-five cases were rechecked by a social worker in 1934-35. One considerable discrepancy was reported, but it is uncertain whether the original return or that of the social worker in this case, is in error.

The process of verifying the transcript of the data accentuated these uncertainties. It will be recalled that two independent transcripts of the data were made available and that all the discrepancies between these two transcripts were rechecked against the original records. Out of a total of 21,391 physical measurements only 317 or less than 1.5% showed discrepancies, but out of a total of 248 menarcheal ages 53 or 21.3% showed discrepancies between the two transcripts.

In addition to these obvious uncertainties the internal evidence of the data casts doubt on the accuracy of particular menarcheal records. For example, in Figures 1, 2, and 3 attention was called to the markedly exceptional trends shown by case #2379. At various stages of this study several attempts have been made to capitalize this internal evidence for the purpose of marking particular cases as of questionable accuracy. The problem is important for the formulation of a more precise judgment concerning the accuracy of these data as a whole. It was also felt that a list of most doubtful cases might prove valuable to other students. Specifically, such a list might encourage the rechecking of these records by personal interviews with the subjects. Or, studies of the relation between first menstruation and mental growth curves or changing body builds might be facilitated by the outright rejection of the most doubtful records. It is to be noted, however, that with three exceptions[1] the rejection of doubtful records is not involved in this study since this would amount to favoring a thesis already announced. Even when all cases are included in the analysis, the differential patterns are unexpectedly clear and regular. If it is assumed that some of the menarcheal records are in error, then it must be concluded that the true differential patterns are still more clearly defined.

The general procedure employed to mark particular cases as of doubtful accuracy involves four steps. First, a prediction of menarcheal ages from physical measurements is built up from a multiple regression equation. Second, differences between predicted and reported menarcheal ages are computed. Third, it is argued that the most extreme of these differences can be used to mark certain records as of doubtful accuracy. Fourth, this argument is tested in terms of the data from city "B"

[1] *The exceptions are as follows: section six of Chapter IV in which one case was rejected; section nine of Chapter IV in which nine cases were rejected, and section three of Chapter V where five cases were rejected.*

and "M".

The multiple correlation between five physical measurements and menarcheal ages for 246 cases with relatively complete physical data as reported in Chapter XII is .785. The magnitude of this correlation is taken as prima facie evidence that the menarcheal records on the whole are substantially correct. From among the five physical variables four have been selected for the purpose of predicting reported menarcheal ages. The regression equation is

$$\bar{X}_1 = 3.6709x_2 - 1.6484x_3 - .7207x_4 - .2451x_5 - 146.2917$$

in which $\bar{X}_1$ is the predicted menarcheal age in months, x_2 is chronological age in years at the close of the year of maximum growth in standing height, x_3 is the increments in standing height in centimeters for the year ending at ages 14.0 and 14.5, x_4 is the increments in weight in kilograms for the year ending at ages 12.0 and 12.5, and x_5 is skeletal age in months at ages 12.5 and 13.0. The theoretical multiple correlation involving these variables is .782 and the correlation between the computed predictions and reported menarcheal ages is .755.

We now have three distributions: one of the reported menarcheal ages, one of predicted menarcheal ages and one of deviations obtained by subtracting predicted from reported menarcheal ages. Certain constants need to be recorded.

Variables	Mean	S.D.	S.D.^{2}	B_1	B_2
1. Reported M.A.	155.60	12.96	167.96	.286	4.92
2. Predicted M.A.	155.50	9.90	98.01	.036	2.68
3. Deviations	-.1	8.34	69.55	1.251	8.46

It will be noted that the distributions do not test strictly normal. The distribution of reported menarcheal ages is slightly skewed (B_1 being .286) and more definitely leptokurtic or peaked (B_2 being 4.92). The distribution of predicted menarcheal ages is very symmetrical (B_1 being .036) and slightly platikurtic or flat (B_2 being 2.68). The distribution of deviations is markedly skewed (B_1 being 1.251) and markedly leptokurtic (B_2 being 8.46).

To account for the distribution of deviations it is necessary first to analyze the distributions of reported and predicted menarcheal ages. The distribution of reported menarcheal ages may be attributed to three factors: true individual differences, random errors in the records, and gross inaccuracies such as would result from correct information concerning the month and day of first menstruation but incorrect information as to the year. Or, we may write

$$\sigma_{RM}^2 = \sigma_{MA}^2 + \sigma_{MB}^2 + \sigma_{ME}^2 + \sigma_{MG}^2 = 12.96^2 = 167.96$$

in which σ_{RM}^2 is the variance of reported menarcheal ages, σ_{MA}^2 is the variance of that part of the true individual differences in menarcheal ages common to the predicted menarcheal ages, σ_{MB}^2 is the true variance not common to the prediction, σ_{ME}^2 is the variance of random errors, and σ_{MG}^2 is the variance of the gross inaccuracies. Similarly, the predicted menarcheal ages may be defined as

$$\sigma_{PM}^2 = \sigma_{PA}^2 + \sigma_{PB}^2 + \sigma_{PE}^2 = 9.90^2 = 98.01$$

in which σ_{PM}^2 is the variance of the predicted menarcheal ages, σ_{PA}^2 and σ_{PB}^2 are the true individual differences in common with and not in common with reported menarcheal ages, and σ_{PE}^2 is the variance of random errors of measurement. Now, letting σ_D^2 be the variance of the deviations $\sigma_D^2 = \sigma_{RM}^2 - \sigma_{PM}^2$ or 167.96 - 98.01 = 69.95 which checks with the directly calculated value of 69.55. Or

$$\sigma_D^2 = \left(\sigma_{MA}^2 - \sigma_{PA}^2\right) + \left(\sigma_{MB}^2 - \sigma_{PB}^2\right) + \left(\sigma_{ME}^2 - \sigma_{PE}^2\right) + \sigma_{MG}^2 = 69.55$$

It should be apparent that the terms in parentheses tend to cancel out, while gross errors in the reported menarcheal ages, σ_{MG}^2 , do not. That is, if σ_{MG}^2 constitutes, for example, 10% of the original variance in σ_{RM}^2, then it constitutes 24% of the variance in the deviations. The decidedly asymmetrical and leptokurtic nature of the distribution of deviations may be attributed to the larger role assumed by gross inaccuracies. Further, in the light of this logic, it may be urged on sheer grounds of probability that the most extreme of the deviations represent instances in which the reported menarcheal age is of doubtful accuracy.

The difficulty with the argument of statistical improbability is that it is just the improbable which often actually materializes. Further, this criterion does not say whether five or fifty cases should be marked as doubtful. We have, however, a small group of 30 cases from city "B" in which the reported menarcheal ages are believed to be entirely accurate. Whereas, the correlation between reported and predicted menarcheal ages is .755 for all 246 cases, it is .743 for city "M" and .790 for city "B". In city "B", as already noted, the range of reported menarcheal ages is decidedly smaller than in the whole population. When correction is made for this factor and for the restricted range of predicted menarcheal ages the correlation is .889[1]. Whereas the standard deviation of the deviations for all cases is 8.90 it is only 5.10 in city "B". This standard deviation of 5.10, however, can not be taken as a criterion of the true distribution of deviations to be expected from accurate records because it is influenced by the curtailment of reported menarcheal ages. A better criterion is the standard error of estimate obtained by applying the corrected correlation of .898 to the standard deviation of the reported menarcheal ages of 12.96. This is 5.70 months.

For illustrative purposes Figure 125 presents the obtained distribution of deviations (continuous line; S.D. = 8.34 months) and the theoretical normal distribution to be expected if the information from city "B" is taken as a criterion of what should be expected from data free of gross inaccuracies (broken curve; S.D. = 5.70 months). It is readily apparent that the theoretical distribution fits the obtained distribution unusually well considering the apparently great difference in their standard deviations. That is, the magnitude of the standard deviation of the obtained deviations (8.34) is due largely to a few extreme values. According to the criterion we would expect to find among 246 cases only one deviation falling outside the limits of plus and minus 16.36 months. Actually, fourteen cases fall outside these limits. Eliminating these fourteen cases, the remaining 232 deviations have a

[1]*Pearson, Karl, On the influence of double selection on the variation and correlation of two characters. Biometrika, 1908, 6, 111-112. Since correction for restricted range is often erroneously calculated,, it should be noted that* Σ_1 *and* Σ_2 *from Pearson's formulas (i) and (ii) should be substituted in his formula (iii) and solved for* r_{12}.

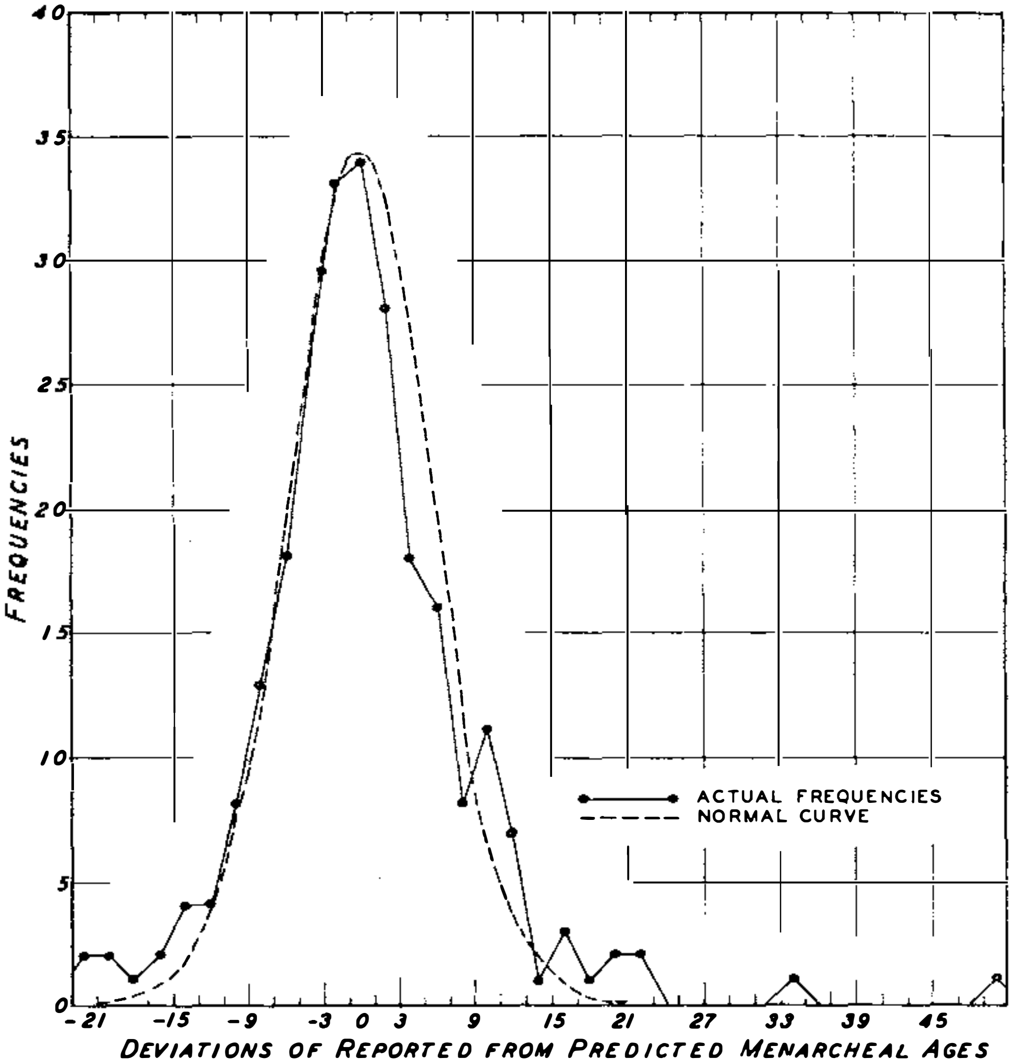

Fig. 125.--Distribution of Differences Between Reported and Predicted Menarcheal Ages in Comparison with Distribution of Differences to be Expected From a Normal Curve.

standard deviation of 6.16 months, B_1 is .011 indicating a symmetrical distribution, and B_2 is 2.94 indicating a closely mesokurtic or normal distribution. These results tend to confirm the validity of the criterion based on the data of city "B".

The fourteen cases which are marked as of doubtful accuracy by this analysis are listed in the following tabulation in the order of the magnitude of their deviations.

	Reported Menarcheal Age		Predicted Menarcheal Age		Deviations	
Case #	Years	Months	Years	Months	Years	Months
1272	18-0	216	13.91	166.9	4.09	49.1
598	16-2	194	13.28	159.3	2.89	34.7
621	11-1	133	12.95	155.4	-1.87	-22.4
346	13-11	167	12.12	145.4	1.80	21.6
697	14-2	170	12.38	148.6	1.78	21.4
1286	11-4	136	13.09	157.1	-1.76	-21.1
680	14-0	168	12.27	147.2	1.73	20.8
1832	11-0	132	12.71	152.5	-1.71	-20.5
43	12-0	144	13.68	164.2	-1.68	-20.2
1695	15-1	181	13.42	161.0	1.67	20.0
1334	13-9	165	12.30	147.6	1.45	17.4
1716	10-9	129	12.18	146.1	-1.42	-17.1
220	11-9	141	13.15	157.8	-1.40	-16.8
2991	16-1	193	14.71	176.5	1.38	16.5

4. Unusual and Abnormal Patterns of Growth

In addition to the list of cases with doubtful menarcheal records it may be of some value to other students to have readily at hand a list of cases with unusual or abnormal patterns of growth in standing height. The spotting of these cases was incidental to a problem which involved the charting of the annual increments in standing height of all individuals. The following tabulation gives the case number, the reported menarcheal age, a statement of the nature of the unusual pattern, and such other comment as seems pertinent.

#1060, advent of menarche not by 15-2, irregular increments for the years ending at age 15.16 and 16.16. Standing height at age 15.16 falls about two centimeters short of expectation and is possibly erratic.

#1272, advent of menarche not by 18-0, an extremely retarded individual, skeletal development about five years retarded.

#2904, menarche at 14-2, irregular increments for the years ending at 12.30, 13.31, and 14.33. Standing height at age 12.30 exceeds expectations and at 13.31 falls short of expectations; possibly these values are erratic.

#1041, menarche at 13-1, abnormally large increment for the year ending at age 8.72 and absence of defined maximum increment.

#2355, menarche at 12-10, irregular increments for the years ending at ages 11.43, 12.38, and 13.38.

#1440, menarche at 13-5, marked absence of defined maximum increment.

5. The Absence of Measurements on Birthdays and Long Intervals Between Measurements

The convenient administration of the Harvard Growth Study required that all measurements be made in the fall or spring of succeeding years. It follows, first, that few measurements were taken approximately at birthdays. This feature of the data has created perplexities for many students. The usual procedure has been to make straight-line interpolations of the individual records to exact ages, -- a procedure which obviously does violence to the data. This amounts to putting a moving average of two years through the individual records, flattening curves, and obscuring the changing pattern of growth. The problem is an imaginary one. Measurements at exact birthdays are wholly immaterial for the purposes of this study or for any study involving continuous records. A more important point is that measurements should be taken at regular and exact intervals.

It follows, second, that measurements were taken at intervals of a year. The limitations of annual measurements may be illustrated by reference to Figures 1 and 2. It is apparent that measurements at annual intervals are too infrequent in the light of the sharp change in trend which occurs at age 13.0. In the case at least of standing and sitting height the data point to the desirability of measurements at semi-annual or even quarterly intervals during the period of two years prior to and a year after the menarche in order that the transition point between accelerating and decelerating growth may be more clearly marked.

These two difficulties have been greatly alleviated by the simple device of dividing the whole number of 248 cases into two sub-populations; those measured for the most part within three months of exact birthdays and those measured for the most part within three months of the half-yearly anniversaries of birthdays. For the purposes of convenient reference cases measured within three months of exact birthdays will be referred to as populations X and Y. There are, of course, a few cases who are difficult of classification. For example, case #1243 was measured at ages 11.26, 12.22, 13.25, 14.26, 15.23, 16.24, 17.20, 18.22, 19.22, and 20.21. This case is classified in the X population (measured for the most part within three months of birthdays) although the first and fourth sets of measurements were taken nearer the half-yearly anniversaries of birthdays. There are 24 borderline cases of this type and 69 sets of measurements on these cases, or 2.7% of the total, which fall outside the exact limits of the particular sub-population in which they are classified. Throughout, all references to the ages of these two populations are the mid-points of age intervals of approximately six months. Thus, population X was measured at age 6.0, 7.0, etc., population Y at age 6.5, 7.5, etc.

The advantages of the procedure of dividing the total population into cases measured approximately at birthdays and cases measured approximately at the half-yearly anniversaries of birthdays are as follows: It avoids the extremely laborious process of estimating dimensions at exact birthdays for each individual. It avoids the systematic errors of linear interpolation to exact birthdays. It cuts in half the errors involved in the common practice of grouping all measurements made within an interval of twelve months. It automatically provides an empirical check on the reliability of the obtained data. Finally, this procedure paves the way for much more flexible and effective attacks on specific problems.

6. Irregular Intervals

It has been remarked that under normal conditions it is exceedingly difficult to obtain measurements at exact birthdays and difficult enough to obtain measurements at exact intervals. In the preceding section it was emphasized that it is the exact intervals rather than measurements on exact birthdays which are important. However, it is possible to overdo even the demand for exact intervals because if this is done accidents to administrative plans and absences from school multiply the gaps in the data. No longitudinal study can escape from the inherent difficulties which arise from the mutual interplay of such considerations.

In Chapter II it was noted that the measurements of the Harvard Growth Study were delayed during the first year of the study. Instead of the measurements being completed on schedule in the fall of 1922 many cases were not measured until the following January and even February of 1923. Table 8 records the nature of the intervals from year to year for a preliminary selection of 264 cases. Between the first and second measurements the average interval, for 94 cases measured both times, is 298 days or 67 days short of an exact year. In 84.0% of these cases the intervals fall short of an exact year by 10% or more. Similarly, the intervals between the second and third autumn average 15 days in excess of an exact year, but only 1.9% of the intervals exceed a year by 10% or more. Again, between the eleventh and twelfth autumn the intervals average 11 days in excess of an exact year, but only 2.4% of the intervals exceed a year by 10% or more. It will be noted from the summary figures that 94.5% of the 2378 intervals considered fall within a range of 10% of an exact year. Hence, it appears that the Harvard Growth Study was quite successful in obtaining measurements at intervals closely approximating exact years.

A number of considerations indicate that it is not necessary to make adjustments for the irregularities in the intervals. In the first place, the greatest irregularity between the first and second year of the study differs from an exact year by only 18.3% on the average. A measured increment of 2.0 kilograms between the first and second year when adjusted would be increased to only 2.4 kilograms. In the second place, measurements during the first year of the study were made on children ranging in age from five to ten, hence the error is spread over a period of five years. In the third place, the error affects only 35% of the total number of cases, since only this proportion was measured during the first autumn of the study. Finally, certain measurements are not sufficiently reliable to promise much gain from the extra refinement. However, measurements of standing height, sternal height, sitting height, and weight made during the first year of the study have been adjusted where the interval to the second measurement falls short of an exact year by 10% or more. In the light of the above discussion

TABLE 8

AVERAGE INTERVALS FROM YEAR TO YEAR, DIFFERENCES FROM AN EXACT YEAR, AND PERCENTAGES OF INTERVALS WITHIN TEN PER CENT OF AN EXACT YEAR

Years of Harvard Study	Number of Identical Cases	Actual Average Interval in Days	Difference from Exact Year	Percentage of Intervals Differing from an exact year. Short 10% or more	Less than 10%	Long 10% or more
1st v. 2nd	94	298	-67	84.0	16.0	0.0
2nd v. 3rd	216	380	15	0.0	98.1	1.9
3rd.v. 4th	249	357	-8	.4	99.6	0.0
4th v. 5th	242	366	1	0.0	100.0	0.0
5th v. 6th	241	363	-2	0.0	100.0	0.0
6th v. 7th	255	364	-1	0.0	100.0	0.0
7th v. 8th	254	362	-3	0.0	100.0	0.0
8th v. 9th	243	366	1	0.0	99.2	.8
9th v. 10th	222	372	6	.5	97.7	1.8
10th v. 11th	198	359	-6	2.1	97.9	0.0
11th v. 12th	148	376	11	0.0	97.6	2.4
12th v. 13th	16	381	5	0.0	93.2	6.2
Total	2378			3.6	94.5	1.9

TABLE 9

NUMBER OF GAPS OF EACH KIND AND NUMBER OF ORIGINAL FIGURES FOR EACH OF NINE DIMENSIONS

Dimensions	Random gaps	Systematic gaps	Editorial gaps	Total gaps	Total original figures.
Standing height	75	0	0	75	2545
Sternal height	75	0	1	76	2541
Sitting height	78	0	4	82	2536
Weight	75	0	4	79	2536
Skeletal age	130	0	0	130	2548
Erupted teeth	80	0	31	111	2294
Iliac diameter	60	84	28	172	2308
Chest depth	51	394	112	557	1914
Chest breadth	56	409	108	573	1891
Total	680	887	288	1855	21,103

an attempt to adjust for irregularities in the intervals between the second and third autumn and between the eleventh and twelfth autumn would probably introduce as much error as would be eliminated.

7. Methods of Dealing With Gaps in the Data

In previous pages three types of gaps in the Harvard Growth Study data have been mentioned. There are random gaps due to accidents or to absence from school when the measurements were being taken. For the most part in such cases all the measurements on a given child are missing for a given year. There are systematic gaps due to the omission of iliac diameter during the fall of 1923 and to the omission of chest breadth and depth during the fall of 1924 and 1925. (However, if the initial measurements on a given child were not made until the fall of 1923, then, the absence of data for iliac diameter for such year is not counted as a gap). Finally, there are editorial gaps due to the discarding of apparently erratic figures. The total number of gaps is 1,855 leaving a total of 21,103 original figures. Table 9 gives the number of gaps of each kind and the number of original figures for each of the nine measurements.

Two different procedures, each yielding much the same results, may be used in dealing with these gaps. One procedure, to be described in greater detail in section two of Appendix B and employed consistantly in dealing with the problem created by overlapping records, consists of carrying out all of the analysis in terms of annual increments and of splicing the resulting average increments together to some base year. This procedure would completely avoid interpolation and is more straight forward and less laborious than interpolation. Its limitation is that it automatically discards the initial and final measurements of a series in which gaps occur just after the initial measurement or just before the final measurement. Under this procedure, it is impossible to bridge the gaps created by the systematic omission of measurements of iliac diameter, chest breadth, and chest depth during the early years of the Study.

The limitations of the procedure just described has led to the systematic interpolation of all the gaps in the records. In no case, however, has straightline or linear interpolation been applied blindly. As will be explained later the 248 cases were divided into sixteen sub-groups. Interpolations for random and editorial gaps have been based on the trend of the data established by the sub-group to which the individual belonged. Where less than ten cases were available for this purpose data from adjacent sub-groups were combined to establish trends. The two year increment shown by the individual record has been apportioned between the first and second year in the same proportions as established by cases for whom complete data are available. For example, if the trend in standing height for a given group at ages 12.0, 13.0 and 14.0 is 150, 157 and 160 centimeters, then, in interpolating for a gap at age thirteen 70% of the two year increment of the individual is apportioned to the second year.

Systematic gaps have also been filled by interpolations. In the case of iliac diameter the trends for identical cases measured at ages six and seven, ages seven and eight, ages eight and nine, ages six and eight, and ages seven and nine without regard to menarcheal groups have

TABLE 10

TOTAL NUMBER OF ORIGINAL AND INTERPOLATED FIGURES AND PROPORTION OF INTERPOLATED FIGURES AT EACH AGE FOR ILIAC DIAMETER, CHEST DEPTH, AND CHEST BREADTH

Ages*	Iliac Diameter		Chest Depth		Chest Breadth	
	Total Figures	%Inter-polations	Total Figures	% Inter-polations	Total Figures	% Inter-polations
5.75-6.74	47	0.0	65	3.1	64	3.1
6.75-7.74	94	47.9	118	29.7	117	23.9
7.75-8.74	134	21.6	179	52.5	183	49.7
8.75-9.74	209	7.2	198	78.8	201	78.1
9.75-10.74	234	6.4	205	55.6	211	56.4
10.75-11.74	246	4.1	230	20.4	230	17.0
11.75-12.74	247	3.6	239	6.3	240	11.2
12.75-13.74	247	3.6	247	8.9	247	7.3
13.75-14.74	247	2.8	247	7.7	245	9.8
14.75-15.74	238	4.2	238	8.0	236	10.2
15.75-16.74	221	3.2	216	6.9	216	9.7
16.75-17.74	191	6.3	178	8.4	175	9.1
17.75-18.74	90	4.4	86	4.6	75	8.0
18.75-19.74	22	0.0	22	0.0	21	4.8
19.75-20.74	3	0.0	3	0.0	3	0.0
All ages	2470	7.0	2471	22.5	2464	23.2

*The unusual age ranges are due to the procedure, described in section five, of dividing the whole number of cases into two populations. Counts of interpolations were made separately for these two populations and combined.

been studied. These indicate that the annual increments in iliac diameter are very much alike. Hence, straightline interpolation has been employed. A count of the number of original and of interpolated figures for iliac diameter has been made at each age and is recorded in Table 10. The data show at age six that 47 original figures are available and no interpolations, at age seven 94 original and interpolated figures are available of which 47.9% are interpolations, at age eight 134 original and interpolated figures are available of which 21.6% are interpolations, at age nine 209 original and interpolated figures are available of which 7.2% are interpolations, etc. The small proportion of interpolations at any age is due in part to the fact that many children were not measured during the fall of 1922 and in part to the fact that interpolations for the omitted data of the fall of 1923 are spread over a five year age range.

Measurements of chest depth and breadth omitted during the fall of 1924 and 1925 have been interpolated on the basis of positively accelerated trends established by over 1500 cases at each age as measured by the U. S. Public Health Service. Since two consecutive years are missing the proportion of such interpolations is much larger than in the case of iliac diameter rising to almost 80% at age nine. Table 10 shows in detail the proportion of interpolations at each age for iliac diameter, chest depth and chest breadth.

Interpolated figures have been compared with the editorially rejected figures which they replace for the purpose of testing the impartiality of the editorial procedures. In the case of iliac diameter 20 rejected figures average 2.04 centimeters higher and eight average 2.87

centimeters lower than the corresponding interpolated values. In the case of chest breadth 50 rejected figures average 1.10 centimeters higher and 58 average 1.22 centimeters lower than the corresponding interpolated values. In the case of chest depth 55 rejected figures average 1.51 centimeters higher and 57 average 1.27 centimeters lower than the corresponding interpolated figures. That is, the editorial procedures rejected almost exactly the same number of abnormally high and of abnormally low figures.

8. Overlapping Records and Completeness of the Data

In longitudinal studies it almost never happens that identical cases are available over a considerable range of ages. Some records will begin at one age and others at another. Some records will end at one age and others at another. This section presents a more precise definition of the completeness and of the degree of overlapping in the available data from the Harvard records. The procedure employed in dealing with the problem created by such records will be described in Appendix B.

Among the 248 cases involved in this study 113 records include at the most twelve annual measurements. The phrase "at the most" is used advisedly since in a few instances X-rays of the wrist were obtained for two years before other measurements were obtained and in other instances were continued for a year longer than other measurements, and vice versa. Even with records covering twelve consecutive years there is considerable overlapping of records. Thirty-eight of these long records begin at age six (5-6 to 6-5), fifty-three begin at age seven, seventeen begin at age eight, and five at age nine. Hence, these records cover the age range from six to twenty, equivalent to fifteen annual measurements. But, data on all 113 cases are available for only nine consecutive years. This discussion indicates that there is no escape from overlapping records unless the study is to be confined to a small number of cases or to a limited age range. When all the 248 cases are considered the overlapping of the records is extreme. One record begins as early as age 5-7, one as late as 12-6. One record ends as early as 11-5, one as late as 20-3. Since one record ends at 11-5 and one does not begin until 12-6 it follows that records for all 248 cases are not available at any age.

Table 11 sets forth the overlapping of the records as a whole. Column one gives the number of cases measured initially at each age: the initial measurements on 48 cases were made between 5-6 and 6-5, etc. Column two gives the number of cases whose records end at each age: one record ended between 10-6 and 11-5, etc. Column three gives the cumulative number available at each age. Attention has already been called to the fact that the total age range covers a period equivalent to fifteen annual measurements and that records for all 248 cases are not available at any age. The overlapping of the records is such that from age seven to eighteen, equivalent to twelve annual measurements, the data at the most are 87.9% complete.

The records for particular measurements necessarily show more overlapping. Nor does the foregoing analysis take account of gaps and omissions within the records. The data for chest depth from age seven to eighteen are 79.8% complete without taking account of gaps and, if these are counted, the records are only 60.8% complete. Completeness,

TABLE 11

NUMBER OF INITIAL AND FINAL MEASUREMENTS AT EACH AGE AND CUMULATIVE NUMBER AVAILABLE AT EACH AGE, THE RECORDS AS A WHOLE

Ages	No. Initial Measurements at Each Age	No. Final Measurements at Each Age	Cumulative No. Available at Each Age
5-6 to 6-5	48	-	48
6-6 to 7-5	92	-	140
7-6 to 8-5	67	-	207
8-6 to 9-5	26	-	233
9-6 to 10-5	10	-	243
10-6 to 11-5	4	1	247
11-6 to 12-5	-	-	246
12-6 to 13-5	1	-	247
13-6 to 14-5	-	2	247
14-6 to 15-5	-	17	245
15-6 to 16-5	-	22	228
16-6 to 17-5	-	81	206
17-6 to 18-5	-	86	125
18-6 to 19-5	-	32	39
19-6 to 20-5	-	7	7

however, is relative to the age range against which the data are tested. Hence, a more meaningful test of the completeness of the Harvard data is a comparison with other longitudinal studies. The most extensive collection of repeated measurements which permit comparison is that of Fleming (16). Considering only 1340 girls with repeated measurements Fleming's observations over the twelve year age range from five to sixteen are only 35.0% complete and, if gaps are counted, only about 32.3% complete. Selecting 279 girls with the longest series of repeated measurements Fleming's observations are only 61.3% complete, and if gaps are counted, only about 56.5% complete.

9. Selective Factors and Nature of Selected Population

Selective factors are involved in any study of children. Cross-sectional studies may attempt to eliminate them by rigidly defining the population, by measuring children over a wide geographical area, and by piling up tens of thousands of cases. Nevertheless, some selective factors are involved and, when school children of different ages are measured, it is a certainty that these factors change from age to age. Longitudinal studies involve smaller numbers of cases and the total population is necessarily less representative of any defined group. Nevertheless, when identical cases are followed from age to age it is a certainty that these selective factors do not change from age to age.

The available 248 cases represent a group selected for long attendance in the same schools and for the availability of relatively precise menarcheal ages. The abnormal conditions surrounding the collection of the information concerning the advent of the menarche at once suggest that the distribution of menarcheal ages may be far from representative. More particularly it is highly probable that the average menarcheal age of the 248 cases is less than that of all the girls in the two school

TABLE 12

PERCENTAGE DISTRIBUTION OF MENARCHEAL AGES; COMPARATIVE DATA

Menarcheal Age Intervals in Years and months*	Harvard Records	Abernethy's Data Chicago	Engle's Data Hebrew	Data of Boas Horace Mann Hebrew	Data of Boas Horace Mann Non-Hebrew	Data of Boas Hebrew Orphan Asylum
18-0 and over	.4	---	---	---	---	---
17-0 to 17-11	---	.2	---	---	---	---
16-0 to 16-11	.8	1.6	1.2	.9	1.7	---
15-0 to 15-11	3.2	6.0	6.8	5.2	6.8	7.6
14-0 to 14-11	10.5	25.3	26.8	17.2	12.7	27.0
13-0 to 13-11	36.3	35.3	34.4	27.6	30.9	31.9
12-0 to 12-11	33.5	23.0	22.4	31.9	31.8	22.7
11-0 to 11-11	12.1	7.6	8.4	13.8	12.7	9.7
10-0 to 10-11	3.2	1.0	---	3.4	3.4	1.1
Total %	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
Total Number	248	487	250	116	236	185
Average Age, Years	13.0	13.5	13.5	13.1	13.1	13.5
Stand. Dev.	1.1	1.1	1.1	1.2	1.2	1.1

* The intervals employed by different students differ slightly.
Abernethy, Ethel F., Correlations in physical and mental growth. Jour. Educ. Psychol., 1925, 16, 539-546. Intervals are not defined.
Engle, E. T., and Shelesnyak, M. C. First menstruation and subsequent menstrual cycles of pubertal girls. Human Biology, 1934, 6, 431-453. Intervals are 10.00 to 10.99, etc.
Boas, Franz, Studies in growth. Human Biology, 1932, 4, 307-350. Intervals are in years and months.

systems from which they were selected. Nevertheless, the comparative data assembled in Table 12 suggest that these selective factors are not serious. While the average menarcheal age of the Harvard cases is lower than any of the five comparative groups the standard deviations are unexpectedly uniform. Further, it is worth emphasizing that, with the exception of Chapter XII, this study might proceed with confidence even though the distribution of menarcheal ages were very far from representative. Since this study is concerned almost entirely with the growth trends of groups of girls menstruating at different ages, it is only necessary that each of these groups be representative.

For the purpose of more clearly defining the nature of the selected population, Tables 13 and 14 present additional comparative data on over a thousand girls measured at age eight (7-6 to 8-5) in the three school systems employed by the Harvard Growth Study. Considering first the racial distributions presented in Table 13, it is apparent that the selected population consists more largely of children of North European stock than the total population available in the Harvard records. Similarly, Table 14 shows that the selected population is definitely taller and heavier. The differences, however, are to be attributed in part to the fact that the total population includes three school systems while the selected population has been drawn from only two of these school systems.

TABLE 13

PERCENTAGE DISTRIBUTION BY RACIAL STOCK; COMPARATIVE DATA

Racial Groups	Selected population All 248 Cases.	Selected Population 173 Cases at Age Eight	Total Population Available in Harvard Records, 1071 Cases at Age Eight
North European	77.9%	80.3%	63.2%
Italian Stock	13.3%	11.0%	22.8%
Jewish	2.8%	2.9%	8.1%
South European	2.8%	2.9%	3.7%
Negro	1.2%	1.2%	1.0%
Mixed	2.0%	1.7%	1.2%
Total	100.0%	100.0%	100.0%

TABLE 14

STANDING HEIGHTS AND WEIGHTS; COMPARATIVE DATA

Racial Groups	Selected Population at Age Eight		Total Population Available in Harvard Records at Age Eight	
	Height	Weight	Height	Weight
North European				
Number:	139	139	676	676
Mean:	124.0	25.4	123.1	23.8
Standard Deviation	5.56	4.64	5.52	3.72
Italian Stock				
Number:	19	19	244	241
Mean:	121.3	25.4	117.9	23.1
Standard Deviation	4.24	3.64	6.20	3.50
All Racial Stocks				
Number:	173	173	1071	1064
Mean:	123.6	25.3	121.5	23.7
Standard Deviation	5.38	4.44	6.02	4.04

APPENDIX B

METHODS OF ANALYSIS AND BASIC DATA

1. Classification of Cases

As already explained in the text the population has been divided into eight groups according to the advent of the menarche. Each of these groups was further divided into two sub-groups depending on whether measurements were made within three months of birthdays or within three months of the half-yearly anniversaries of birthdays. These sixteen sub-groups will be referred to as XA, YA, XB, YB, etc. The XA, XB, XC, etc., sub-groups will be referred to collectively as sub-population X and includes cases measured for the most part within three months of exact birthdays. The YA, YB, YC, etc., sub-groups will be referred to collectively as sub-population Y and includes cases measured for the most part within three months of the half-yearly anniversaries of birthdays. The number of cases in each of the sixteen sub-groups is given in Table 15. As already noted cases whose records show the advent of the menarche "not by" 14-5 or later have been included. There are five such cases both in groups XH and YH. One sub-group consists of only four cases, one of only six, one of only seven, and one of only nine cases. The combined X plus Y groups consist of as few as sixteen, eighteen, and nineteen cases.

As this study nears completion arrangements have been made for the publication in extenso of all the data collected by the Harvard Growth Study on a selected group of cases with relatively long records. This extenso publication of original data will urge all students to specify by number the particular cases selected for their studies in order that others may check computations and verify conclusions. Accordingly, Table 15 sets forth the identification numbers of the 248 cases involved in this study classified according to menarcheal age and the timing of measurements relative to birthdays.

2. Overlapping Records and Derivation of Basic Growth Data

A feature of the Harvard data which is doubtless common to all bodies of longitudinal data is that of overlapping records. Instead of having available the same cases at each age, the records begin at all ages from 5-7 to 12-6 and end at all ages from 11-5 to 20-6. For example, in sub-group XA four cases are available at age 6.0, six cases at 7.0, seven cases at 8.0, nine at 9.0, ten at 10.0, eleven at 11.0 to 14.0, ten cases at 15.0, nine at 16.0 and 17.0, five at 18.0, and one at 19.0. Considering our well established habits built up in the analysis of cross-sectional data, the natural and usual procedure in this situation has been to determine averages at each age for the available cases without regard to the identity of the cases from age to age. In the light of the discussion of Chapter III this procedure clearly distorts the longitudinal nature of the data. The proper procedure is to work with the increments or to determine averages at successive ages for identical cases. When this has been done the resulting increments

TABLE 15

DISTRIBUTION OF THE TOTAL POPULATION AMONG SIXTEEN SUB-GROUPS AND CASE NUMBERS OF INDIVIDUALS CLASSIFIED ACCORDING TO THE ADVENT OF THE MENARCHE AND THE TIMING OF MEASUREMENTS RELATIVE TO BIRTHDAYS

Menarcheal Groups	Menarcheal Age Ranges		Number of Cases in Each Group X Population Measurements at birthdays	Y Population Measurements at half-yearly anniversaries	X plus Y Populations
A	11-5 or earlier		11	11	22
B	11-6 to 11-11		7	9	16
C	12-0 to 12-5		16	18	34
D	12-6 to 12-11		30	19	49
E	13-0 to 13-5		24	35	59
F	13-6 to 13-11		19	12	31
G	14-0 to 14-5		4	15	19
H	14-6 or later		12	6	18
		Total	123	125	248

Sub-population X, Measured within Three Months of Birthdays.
XA. 11 cases: 382; 549; 621; 1243; 1297; 1384; 2167; 2345; 2429; 2610.
XB. 7 cases: 386; 558; 1008; 3043; 3164; 3239; 3322; 1635.
XC. 16 cases: 226; 392; 510; 1007; 1675; 1803; 1973; 2151; 2266; 2497; 2522; 2795; 2933; 3046; 3060; 3226.
XD. 30 cases: 272; 313; 411; 984; 991; 1017; 1038; 1114; 1158; 1198; 1372; 1535; 1735; 1789; 1880; 1978; 2050; 2139; 2169; 2280; 2354; 2382; 2520; 2670; 2886; 3132; 3136; 3216; 3319; 3334.
XE. 24 cases: 106; 115; 179; 333; 361; 732; 1026; 1308; 1309; 1339; 1440; 1504; 1537; 1795; 1869; 2035; 2145; 2278; 2305; 2379; 2471; 2492; 3248; 3262.
XF. 19 cases: 54; 71; 219; 346; 909; 1334; 1489; 1654; 1797; 1899; 1989; 1990; 2213; 2459; 2498; 2533; 2561; 3094; 3230.
XG. 4 cases: 367; 680; 1377; 1849.
XH. 12 cases: 589; 614; 686; 1044; 1060; 1272; 1695; 1875; 1913; 2485; 2991; 3169.

Sub-population Y, Measured within Three Months of Half-yearly Anniversaries.
YA. 11 cases: 344; 842; 1286; 1590; 1663; 1716; 1832; 2114; 2290; 2484; 3321.
YB. 9 cases: 220; 309; 912; 1087; 1719; 2248; 2966; 3162; 3211.
YC. 18 cases: 18; 43; 101; 135; 270; 838; 1126; 1511; 1615; 1689; 1877; 2314; 2538; 2707; 2853; 2914; 2965; 3338.
YD. 19 cases: 11; 62; 147; 258; 754; 1023; 1263; 1374; 1633; 1638; 1655; 2134; 2176; 2355; 2443; 2536; 2544; 3176; 3247.
YE. 35 cases: 61; 171; 292; 334; 494; 557; 892; 940; 947; 1036; 1041; 1100; 1125; 1191; 1259; 1287; 1320; 1352; 1433; 1491; 1498; 1359; 1542; 1572; 1853; 1883; 1916; 1931; 2047; 2252; 2463; 2765; 2710; 3304; 3306.
YF. 12 cases: 383; 506; 689; 1268; 1330; 1528; 1630; 2269; 2403; 2404; 2949; 3307.
YG. 15 cases: 6; 228; 260; 287; 697; 700; 896; 939; 1502; 1759; 1780; 1800; 1903; 1923; 2904.
YH. 6 cases: 120; 186; 415; 918; 1074; 1228.

should be spliced together to give a continuous growth curve.

The contrast in results obtained by the usual method of analysis and the method employed in this study deserves concrete illustration. For this purpose the eleven cases from Group YA from age 5.5 to 8.5 and from age 15.5 to 19.5 will be used. The detailed original data for standing height and the usual cross-sectional analysis are illustrated in the following tabulation. Data from age 9.5 to 14.5 are omitted because complete data for all cases are available at these ages.

Case #. Standing Heights of Individuals at Indicated Ages

Case #.	5.5	6.5	7.5	8.5	15.5	16.5	17.5	18.5	19.5
1286	-----	-----	121.5	126.7	164.3	165.4	165.3	-----	-----
344	119.3	126.9	134.3	141.5	170.2	170.3	-----	-----	-----
2290	-----	121.0	127.0	132.2	160.4	161.1	-----	-----	-----
1663	-----	124.4	130.5	136.5	165.4	165.2	-----	-----	-----
1832	-----	-----	-----	112.2	147.0	148.3	148.5	148.3	148.5
2484	-----	119.0	125.4	131.2	160.8	161.4	-----	-----	-----
2114	-----	118.0	122.2	128.2	160.8	-----	-----	-----	-----
1590	-----	-----	-----	128.4	155.8	155.5	155.8	155.7	-----
1716	-----	114.0	118.7	123.6	151.9	152.7	152.8	-----	-----
842	-----	-----	-----	122.3	149.5	149.9	150.0	149.7	-----
3221	-----	-----	-----	130.8	161.5	162.0	160.4	-----	-----
Aver.	119.3	120.55	125.66	128.51	158.87	159.18	155.47	151.23	148.5
Incr.		1.25	5.11	2.85		.31	-3.71	-4.24	-2.73

The average of all available cases at age 5.5 is 119.30 centimeters, at 6.5 is 120.55, at 7.5 is 125.66, etc. The annual increments derived from these data are 1.25 centimeters between 5.5 and 6.5, 5.11 centimeters between 6.5 and 7.5, etc. When the problem is thus stated in terms of a very small number of cases, the errors are self-evident. The average of 119.30 centimeters at age 5.5 is not comparable with that of 120.55 at 6.5 because different cases are involved. It is even more apparent that the increment of 1.25 centimeters between 5.5 and 6.5 is determined by changes in the available cases rather than by any true growth increment. When only cross-sectional data are available, no other procedure is possible; hence, in cross-sectional studies the attempt is made to collect a large number of observations at each age in the hope that the errors inherent in the procedure may cancel out. Application of this method of analysis to longitudinal data amounts to throwing away the most valuable part of the data. Studies by Richey (27), and by Boas (5), and Martin's analysis of Fleming's data (16) involve these errors.

In contrast, the procedure employed by this study is based on the increments as illustrated by the following tabular data involving exactly the same cases and the same original data.

Case #. Annual Increments in Standing Height of Individuals for the Years Ending at the Indicated Age

Case #	6.5	7.5	8.5	16.5	17.5	18.5	19.5
1286							
1286	—	—	5.2	1.1	-.1	—	—
344	7.6	7.4	7.2	.1	—	—	—
2290	—	6.0	5.2	.7	—	—	—
1663	—	6.1	6.0	-.2	—	—	—
1632	—	—	—	1.3	.2	-.2	.2
2484	—	6.4	5.8	.6	—	—	—
2114	—	4.2	6.0	—	—	—	—
1590	—	—	—	-.3	.3	-.1	—
1716	—	4.7	4.9	.8	.1	—	—
842	—	—	—	.4	.1	.3	—
3321	—	—	—	.5	-1.6	-.6	—
Aver.	7.6	5.80	5.76	.50	-.17	-.20	.2

The contrasting results of these two procedures, presented in Figure 126, do not require comment.

Continuous growth curves showing average size have been computed by splicing the true increments to a base average which involves all the available cases. For example, the average standing height at age 15.5 is 158.87 centimeters; adding the true increment of .50 gives 159.37 at age 16.5, adding -.17 gives 159.20 at age 17.5, adding -.2 gives 159.00 at age 18.5 and adding .20 gives 159.20 at age 19.5. Similarly, the true increments are successively subtracted from the average of all cases at age 8.5 to give age 7.5, 6.5, and 5.5. Growth trends in average size, accordingly, constitute secondary data. The increments constitute the primary data. Figure 127 illustrates the contrasts between results obtained from the usual cross-sectional and from the proper method in terms of average size at successive ages. Incidentally, in these computations it is essential that the original raw figures be summed and divided by N. The reliability of the trends established by longitudinal data is so high that the common practice of computing averages from distributions of grouped data may introduce appreciable errors.

Tables 16 to 24 set forth the detailed numerical data showing growth trends for each of the sixteen sub-groups of cases and for each of the nine measurements. The ages recorded in the left hand column are the mid-points of class intervals of six months. The number of cases recorded for each average includes interpolated figures. Data are given in full even though in some instances the trends are established by only one case. For computational convenience all of the results are reported to two decimals. This does not imply that second decimals are significant. The important point is that the increments from which these data were derived are often significant to the second decimal. The standard deviations in Table 29 should enable the critical student to estimate the errors involved in the case of standing height, sitting height, weight and iliac diameter.

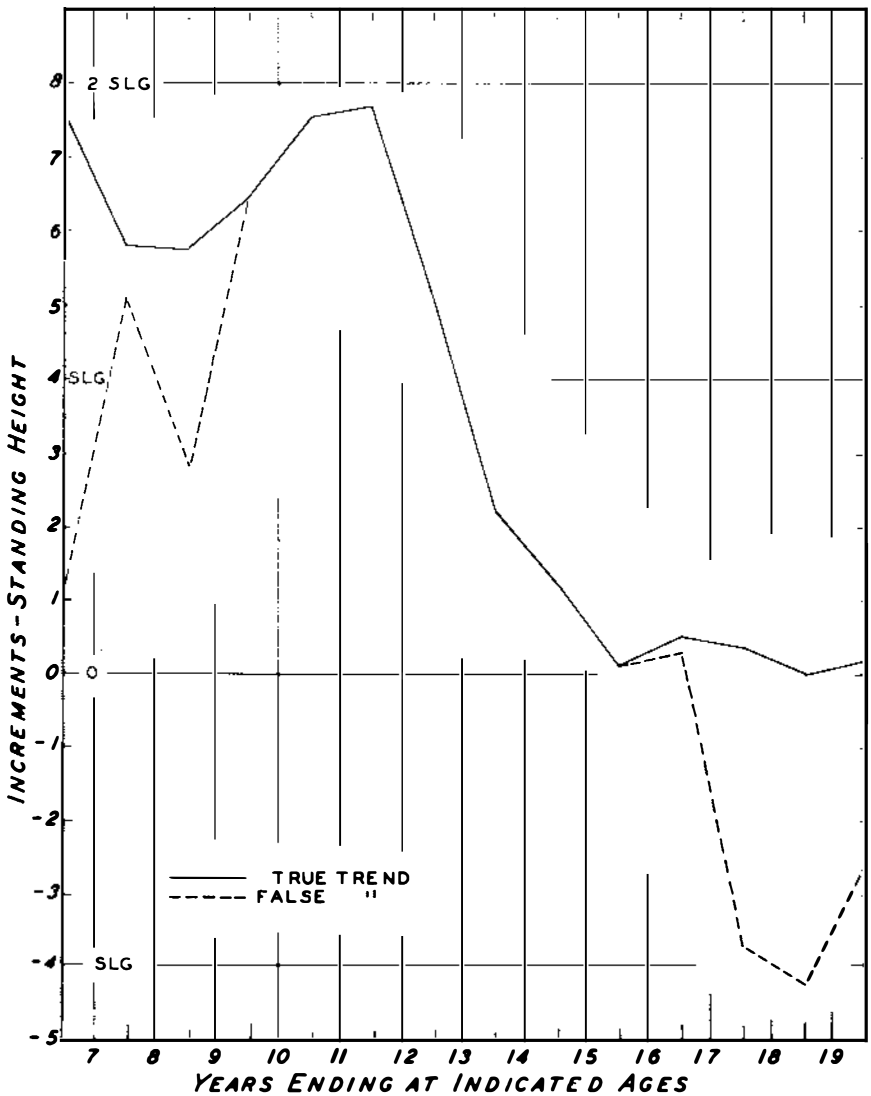

Fig. 126.--True and False Trend of Increments in Standing Height Illustrating Correct and Incorrect Methods of Computing Increments when Longitudinal Data are Available.

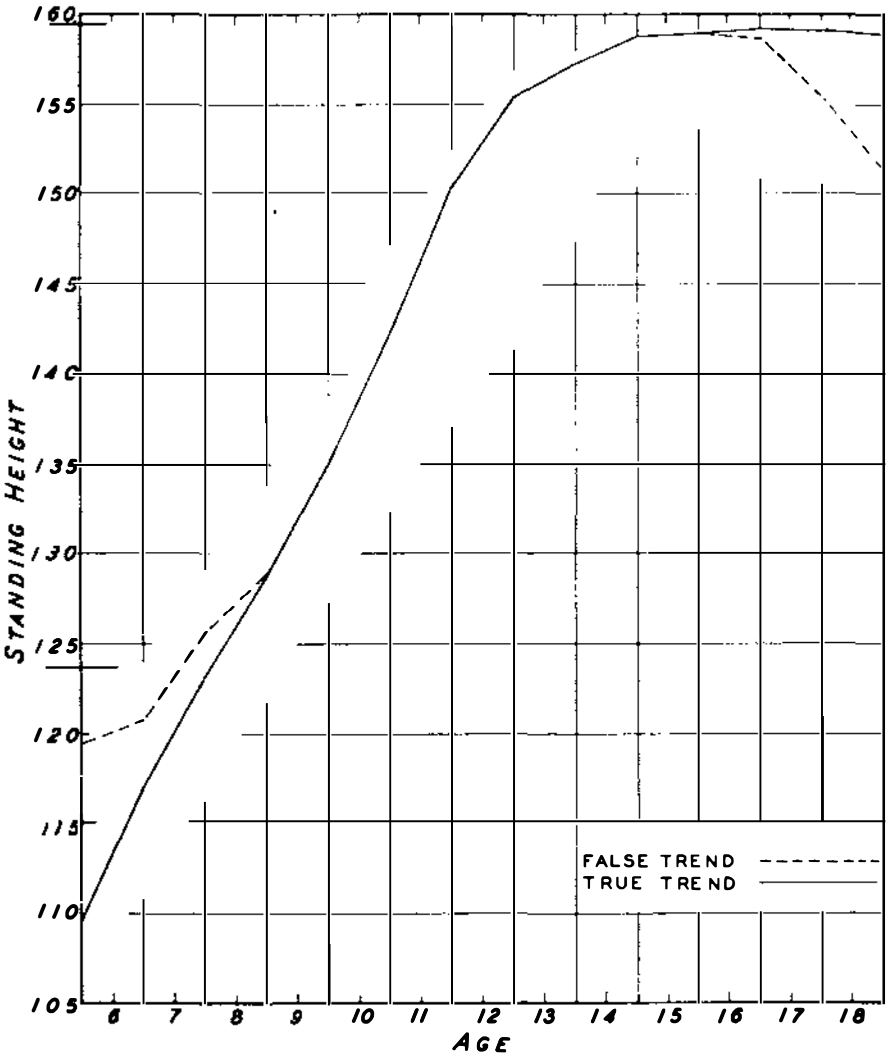

Fig. 127.--True and False Growth Trends in Average Size Illustrating Correct and Incorrect Methods when Longitudinal Data are Available.

TABLE 16

GROWTH TRENDS IN AVERAGE STANDING HEIGHT, CENTIMETERS, OF GROUPS OF GIRLS MENSTRUATING AT DIFFERENT AGES AND OF SUB-POPULATIONS MEASURED APPROXIMATELY AT BIRTHDAYS AND THE HALF-YEARLY ANNIVERSARIES OF BIRTHDAYS

Exact Ages	Group A Menarche Before 11-6		Group B Menarche 11-6 to 11-11		Group C Menarche 12-0 to 12-5		Group D Menarche 12-6 to 12-11		Group E Menarche 13-0 to 13-5		Group F Menarche 13-6 to 13-11		Group G Menarche 14-0 to 14-5		Group H Menarche After 14-5	
	N	M	N	M	N	M	N	M	N	M	N	M	N	M	N	M
Cases Measured Approximately at Birthdays, Population X.																
6.0	4	110.22	3	114.48	5	112.89	9	110.80	3	111.99	1	107.66			1	109.76
7.0	6	116.10	5	121.68	9	119.33	19	116.72	13	117.42	9	113.86			1	114.16
8.0	7	122.07	7	127.46	14	125.38	26	122.53	20	123.92	12	119.57	1	119.85	4	118.76
9.0	9	127.51	7	133.66	16	131.06	28	128.11	24	129.37	14	125.34	2	124.45	8	123.56
10.0	10	134.39	7	140.43	16	136.68	29	133.84	24	134.64	17	130.53	3	129.80	12	128.43
11.0	11	142.60	7	148.10	16	143.58	30	139.99	24	140.50	19	136.13	4	134.63	12	133.52
12.0	11	148.85	7	155.14	16	151.44	30	147.56	24	147.73	19	142.49	4	139.63	12	138.24
13.0	11	152.40	7	158.51	16	156.41	30	154.02	24	154.29	19	150.15	4	145.70	12	143.82
14.0	11	154.06	7	160.11	16	158.23	30	157.07	24	157.74	19	154.92	4	151.50	12	149.89
15.0	10	154.85	7	160.79	15	159.30	30	158.34	24	159.49	19	157.28	4	154.48	12	154.39
16.0	9	155.42	7	161.00	15	159.69	26	158.98	22	160.12	18	158.05	4	155.95	11	157.12
17.0	9	155.56	4	161.45	14	159.91	25	159.59	22	160.48	17	158.53	4	156.85	8	158.32
18.0	5	155.34	1	161.95	5	160.07	10	159.54	14	160.56	11	158.93	4	156.73	6	158.77
19.0	1	155.44					2	159.54	3	160.80	4	158.86	3	156.83	4	158.39
Cases Measured Approximately at Half-yearly Anniversaries of Birthdays, Population Y.																
5.5	1	109.36			1	108.95			11	115.28	1	114.32				
6.5	6	116.96	5	115.79	9	116.25	10	113.74	20	120.18	1	119.62	1	117.47	1	115.19
7.5	7	122.76	8	121.53	14	122.52	13	119.82	30	126.26	8	124.82	10	122.47	5	121.39
8.5	11	128.51	9	127.21	18	129.07	18	125.61	34	131.68	11	130.97	15	127.78	6	125.97
9.5	11	134.97	9	133.54	18	134.51	18	130.96	35	136.72	11	136.06	15	132.68	6	130.83
10.5	11	142.56	9	140.49	18	140.95	18	136.70	35	142.88	11	141.75	15	137.76	6	135.18
11.5	11	150.30	9	148.41	18	149.13	19	143.52	35	150.58	12	148.76	15	143.75	6	140.70
12.5	11	155.34	9	154.01	17	155.39	19	150.96	35	156.70	12	155.19	15	150.55	6	147.86
13.5	11	157.55	9	157.07	17	158.49	19	156.14	35	159.52	12	158.87	15	155.51	6	153.32
14.5	11	158.76	9	158.41	17	160.07	19	157.95	33	160.84	11	160.32	15	157.62	6	157.37
15.5	11	158.87	9	159.19	14	161.24	19	158.80	30	161.34	9	160.87	15	158.64	6	158.78
16.5	10	159.37	8	159.46	13	161.43	18	159.26	29	161.62	7	160.91	13	158.99	6	159.48
17.5	6	159.21	4	159.71	10	161.70	14	159.39	6	161.57	4	161.11	9	159.19	4	160.31
18.5	3	159.01	2	159.71	2	161.50	3	159.75			2	161.31	1	159.79	1	160.41
19.5	1	159.21														

TABLE 17

GROWTH TRENDS IN AVERAGE SITTING HEIGHT, CENTIMETERS, OF GROUPS OF GIRLS MENSTRUATING AT DIFFERENT AGES AND OF SUB-POPULATIONS MEASURED APPROXIMATELY AT BIRTHDAYS AND THE HALF-YEARLY ANNIVERSARIES OF BIRTHDAYS

Exact Ages	Group A Menarche Before 11-6		Group B Menarche 11-6 to 11-11		Group C Menarche 12-0 to 12-5		Group D Menarche 12-6 to 12-11		Group E Menarche 13-0 to 13-5		Group F Menarche 13-6 to 13-11		Group G Menarche 14-0 to 14-5		Group H Menarche After 14-5	
	N	**M**	**N**	**M**	**N**	**M**	**N**	**M**	**N**	**M**	**N**	**M**	**N**	**M**	**N**	**M**
Cases Measured Approximately at Birthdays, Population X.																
6.0	3	61.76	3	63.15	5	62.04	9	62.21	3	61.79	1	61.68			1	62.63
7.0	6	64.20	5	66.52	9	65.62	19	64.36	13	64.09	9	63.38			1	63.23
8.0	7	66.58	7	68.94	14	68.63	26	66.89	20	67.15	12	65.52	1	66.28	4	65.23
9.0	9	68.78	7	71.43	16	70.66	28	68.99	24	69.51	14	67.99	2	68.38	8	67.33
10.0	10	71.40	7	74.01	16	73.08	29	71.16	24	71.49	17	69.93	3	70.48	12	69.75
11.0	11	75.23	7	77.88	16	75.88	30	73.65	24	73.55	19	72.42	4	72.08	12	71.70
12.0	11	78.73	7	81.23	16	79.89	30	77.21	24	76.84	19	75.10	4	74.30	12	73.25
13.0	11	80.78	7	83.31	16	82.80	30	80.81	24	80.19	19	78.63	4	77.28	12	75.78
14.0	11	82.44	7	84.80	16	84.12	30	82.73	24	82.48	19	81.73	4	80.50	12	78.82
15.0	10	83.29	7	85.40	15	85.22	30	83.73	24	83.96	19	83.44	4	82.90	12	81.29
16.0	9	83.93	7	86.06	15	85.55	26	84.47	22	84.45	18	84.18	4	83.55	11	83.20
17.0	9	84.13	4	86.29	14	85.88	25	84.64	22	84.87	17	84.72	4	84.18	8	84.02
18.0	5	84.13	1	86.39	5	85.30	9	84.49	14	84.72	11	85.14	4	84.35	6	84.64
19.0	1	84.23					2	84.49	3	84.95	4	84.99	3	84.68	4	84.40
Cases Measured Approximately at Half-yearly Anniversaries of Birthdays, Population Y.																
5.5	1	60.14			1	61.24										
6.5	6	64.24	5	64.07	9	64.64	10	63.37	11	64.31	1	63.88				
7.5	7	66.84	8	66.25	14	67.07	13	65.82	20	65.75	1	65.88	1	65.88	1	63.89
8.5	11	69.14	9	68.58	18	69.24	18	68.22	30	68.60	8	67.98	10	66.23	5	66.89
9.5	11	71.86	9	71.17	18	71.30	18	70.31	34	70.75	11	70.36	15	68.38	6	68.87
10.5	11	74.88	9	74.17	18	73.84	18	72.64	35	72.61	11	72.46	15	70.27	6	70.25
11.5	11	78.70	9	77.87	18	77.95	19	75.59	35	75.27	11	74.66	15	72.15	6	71.95
12.5	11	81.71	9	80.27	17	81.25	19	79.54	35	78.61	12	77.87	15	74.59	6	74.50
13.5	11	83.15	9	82.70	17	83.26	19	82.20	35	82.05	12	80.80	15	78.03	6	77.75
14.5	11	84.33	9	83.61	17	84.52	19	83.81	35	84.11	12	83.39	15	80.85	6	81.05
15.5	11	84.48	9	84.36	14	84.97	19	84.85	33	85.35	11	84.52	15	82.54	6	83.67
16.5	10	84.78	8	85.16	13	84.94	18	85.86	30	85.86	9	84.96	15	83.38	6	84.70
17.5	6	84.88	4	85.81	10	85.47	14	85.34	29	86.11	7	85.29	13	83.97	6	85.57
18.5	3	84.25	2	85.81	2	84.82	3	84.87	6	86.23	4	85.25	9	84.05	4	85.59
19.5	1	84.65									2	85.25	1	84.05	1	85.59

TABLE 18

GROWTH TRENDS IN AVERAGE STERNAL HEIGHT, CENTIMETERS, OF GROUPS OF GIRLS MENSTRUATING AT DIFFERENT AGES AND OF SUB-POPULATIONS MEASURED APPROXIMATELY AT BIRTHDAYS AND THE HALF-YEARLY ANNIVERSARIES OF BIRTHDAYS

Exact Ages	Group A Menarche Before 11-6		Group B Menarche 11-6 to 11-11		Group C Menarche 12-0 to 12-5		Group D Menarche 12-6 to 12-11		Group E Menarche 13-0 to 13-5		Group F Menarche 13-6 to 13-11		Group G Menarche 14-0 to 14-5		Group H Menarche After 14-5	
	N	M	N	MM	N	M	N	M	N	M	N	M	N	M	N	M
Cases Measured Approximately at Birthdays, Population X.																
6.0	4	85.24	3	90.99	5	87.55	9	86.21	3	87.43	1	83.43			1	83.90
7.0	6	92.02	5	96.52	9	94.81	19	92.57	12	93.06	9	89.13			1	89.80
8.0	7	97.35	7	101.96	14	99.89	26	97.44	20	98.62	12	94.94	1	94.91	4	93.40
9.0	9	102.53	7	107.57	16	105.14	28	102.64	24	103.54	14	100.06	2	100.41	8	98.90
10.0	10	108.54	7	113.84	16	110.49	29	107.92	24	108.65	17	104.91	3	103.91	12	102.99
11.0	11	116.02	7	120.79	16	116.77	30	113.58	24	114.09	19	110.08	4	108.08	12	107.58
12.0	11	121.62	7	126.47	16	123.14	30	120.18	24	120.26	19	115.91	4	113.30	12	112.17
13.0	11	124.21	7	129.03	16	127.28	30	125.43	24	125.55	19	122.15	4	118.12	12	116.53
14.0	11	125.87	7	130.53	16	128.69	30	128.06	24	128.56	19	126.27	4	123.52	12	122.01
15.0	10	126.25	7	131.04	15	129.65	30	129.14	24	130.06	19	128.11	4	125.92	12	125.77
16.0	9	126.99	7	131.53	15	130.21	26	129.74	22	130.68	18	128.77	4	126.82	11	128.02
17.0	9	127.37	4	131.99	14	130.31	25	130.14	22	131.32	17	129.29	4	127.88	8	129.12
18.0	5	127.23	1	132.89	5	130.51	10	130.20	14	131.64	11	130.03	4	127.60	6	129.80
19.0	1	127.63					2	130.45	3	132.27	4	130.15	3	128.30	4	130.23
Cases Measured Approximately at Half-yearly Anniversaries of Birthdays, Population Y.																
5.5	1	85.07			1	85.57			10	90.39	1	90.08				
6.5	6	91.57	5	90.89	9	91.47	10	89.01	20	95.22	1	95.08	1	90.92	1	89.57
7.5	7	98.32	8	96.69	14	97.97	13	95.17	30	100.65	8	99.68	10	97.52	5	96.67
8.5	11	103.21	9	101.84	18	103.27	18	100.23	34	105.72	11	104.83	15	101.77	5	100.53
9.5	11	108.96	9	107.42	18	108.58	18	105.03	35	110.51	11	109.30	15	106.61	6	105.27
10.5	11	116.03	9	114.12	18	114.28	18	110.57	35	116.03	11	114.94	15	111.39	6	109.25
11.5	11	122.64	9	120.79	18	121.66	19	116.47	35	122.62	12	121.60	15	116.66	6	114.30
12.5	11	126.83	9	125.74	17	126.86	19	122.78	35	127.58	12	126.71	15	122.41	6	120.33
13.5	11	138.80	9	127.98	17	129.37	19	126.83	35	129.91	12	129.69	15	126.51	6	124.68
14.5	11	129.68	9	129.16	17	130.78	19	128.49	33	131.01	11	130.79	15	128.39	6	128.08
15.5	11	130.15	9	129.92	14	131.17	19	129.17	30	131.62	9	131.56	15	129.27	6	129.78
16.5	10	130.58	8	130.29	13	131.31	17	129.69	29	132.06	7	131.94	13	129.63	6	130.22
17.5	6	130.61	4	130.59	10	131.91	14	130.13	6	132.46	4	131.91	9	130.21	4	130.54
18.5	3	130.91	2	131.39	2	131.81	3	130.76			2	132.31	1	131.61	1	131.04
19.5	1	131.61														

TABLE 19

GROWTH TRENDS IN AVERAGE WEIGHT, KILOGRAMS, OF GROUPS OF GIRLS MENSTRUATING AT DIFFERENT AGES AND OF SUB-POPULATIONS MEASURED APPROXIMATELY AT BIRTHDAYS AND THE HALF-YEARLY ANNIVERSARIES OF BIRTHDAYS

Exact Ages	Group A Menarche Before 11-6		Group B Menarche 11-6 to 11-11		Group C Menarche 12-0 to 12-5		Group D Menarche 12-6 to 12-11		Group E Menarche 13-0 to 13-5		Group F Menarche 13-6 to 13-11		Group G Menarche 14-0 to 14-5		Group H Menarche After 14-5	
	N	M	N	M	N	M	N	M	N	M	N	M	N	M	N	M
Cases Measured Approximately at Birthdays, Population X.																
6.0	4	20.67	3	20.49	5	19.94	3	19.94	3	19.99	1	18.07			1	19.57
7.0	6	22.57	5	22.76	9	23.11	19	22.11	13	22.92	9	20.37			1	21.57
8.0	7	25.17	7	26.68	13	26.38	26	24.75	20	25.23	12	22.94	1	22.51	4	23.37
9.0	9	27.97	7	30.21	15	30.02	28	27.80	24	28.11	14	25.61	2	25.61	8	24.69
10.0	10	31.41	7	35.49	15	33.53	29	30.50	24	31.36	17	29.17	3	27.71	12	26.69
11.0	11	37.66	7	41.52	16	39.36	30	34.76	24	34.82	19	33.37	4	30.41	12	29.29
12.0	11	43.60	7	48.50	16	45.86	30	40.16	24	40.05	19	38.11	4	33.63	12	32.09
13.0	11	48.79	7	52.84	16	51.00	30	46.84	24	45.94	19	43.90	4	38.95	12	35.35
14.0	11	52.07	7	54.55	16	53.59	30	51.07	24	50.59	19	49.69	4	43.10	12	40.82
15.0	10	53.75	7	56.91	15	54.48	30	53.24	24	53.94	19	52.39	4	48.38	12	46.21
16.0	9	54.72	7	57.91	15	55.81	26	54.36	22	54.93	18	53.87	4	51.76	11	50.59
17.0	9	54.48	4	58.23	14	55.04	25	55.19	21	56.51	17	55.15	4	51.38	8	52.18
18.0	5	55.96	1	58.53	5	55.16	10	55.71	14	57.42	11	56.10	4	52.53	6	52.96
19.0	1	56.16					2	55.36	3	57.55	4	55.60	3	53.46	4	52.58
Cases Measured Approximately at Half-yearly Anniversaries of Birthdays, Population Y.																
5.5	1	19.63			1	18.27										
6.5	6	22.43	5	21.67	9	22.17	10	20.54	11	21.01	1	21.34				
7.5	7	25.30	8	24.03	14	25.73	12	23.32	20	22.47	1	23.34	1	22.14	1	19.17
8.5	11	29.30	9	27.57	18	29.14	18	25.82	30	25.10	8	25.34	10	23.74	5	23.37
9.5	11	33.40	9	31.22	18	32.32	18	29.33	34	27.63	11	28.23	15	25.73	6	24.59
10.5	11	38.81	9	36.42	18	37.67	18	32.17	35	30.68	11	30.49	15	28.81	6	27.49
11.5	11	46.87	9	43.12	18	44.43	19	36.48	35	34.56	11	35.12	15	31.53	6	28.67
12.5	11	53.76	9	49.18	17	49.94	19	42.29	35	39.42	12	41.26	15	35.47	6	32.22
13.5	11	57.80	9	52.75	17	54.37	19	47.70	35	45.23	12	46.49	15	41.01	6	37.04
14.5	11	59.91	9	54.81	17	55.69	19	49.71	35	49.68	12	51.71	15	45.80	6	42.42
15.5	11	60.66	9	56.37	14	56.84	19	52.53	33	51.79	11	54.54	15	49.03	6	46.34
16.5	10	60,93	8	57.51	13	56.80	18	53.26	30	52.42	9	54.91	15	50.96	6	49.57
17.5	6	60.66	4	58.29	10	56.35	14	53.63	29	53.03	7	55.52	13	52.01	6	52.17
18.5	3	59.93	2	61.04	2	56.40	3	53.69	6	54.01	4	55.60	9	53.27	4	52.55
19.5	1	61.63									2	56.35	1	52.37	1	57.35

TABLE 20

GROWTH TRENDS IN AVERAGE ILIAC DIAMETER, CENTIMETERS, OF GROUPS OF GIRLS MENSTRUATING AT DIFFERENT AGES AND OF SUB-POPULATIONS MEASURED APPROXIMATELY AT BIRTHDAYS AND THE HALF-YEARLY ANNIVERSARIES OF BIRTHDAYS

Exact Ages	Group A Menarche Before 11-6		Group B Menarche 11-6 to 11-11		Group C Menarche 12-0 to 12-5		Group D Menarche 12-6 to 12-11		Group E Menarche 13-0 to 13-5		Group F Menarche 13-6 to 13-11		Group G Menarche 14-0 to 14-5		Group H Menarche After 14-5	
	N	M	N	M	N	M	N	M	N	M	N	M	N	M	N	M
Cases Measured Approximately at Birthdays, Population X.																
6.0	3	18.04			3	18.58	7	18.35	3	19.07	1	17.89			1	17.41
7.0	5	19.50	3	20.47	5	19.72	13	19.37	8	19.87	9	18.69			1	18.21
8.0	6	20.24	5	21.60	10	20.56	22	20.25	11	20.61	10	19.60	1	19.23	3	18.91
9.0	7	21.23	7	22.34	16	21.39	26	21.07	23	21.32	14	20.48	1	20.33	4	20.08
10.0	10	22.51	7	23.79	16	22.56	28	21.84	24	22.18	16	21.35	3	21.43	9	21.41
11.0	11	23.96	7	25.36	16	23.77	30	23.15	24	23.21	19	22.49	4	22.50	12	22.00
12.0	11	25.38	7	26.80	16	25.32	30	24.44	24	24.70	19	23.87	4	23.55	12	22.71
13.0	11	26.40	7	27.71	16	26.70	30	26.03	24	25.98	19	25.21	4	24.80	12	23.75
14.0	11	26.95	7	28.01	16	27.20	30	26.63	24	26.86	19	26.60	4	26.12	12	24.83
15.0	10	27.24	7	28.37	15	27.22	29	26.98	24	27.30	19	26.71	4	26.68	12	26.24
16.0	9	27.40	7	28.53	15	27.35	26	27.23	22	27.67	18	27.35	4	26.88	11	26.73
17.0	9	27.32	4	28.65	14	27.37	25	27.39	22	27.87	17	27.50	4	27.25	8	27.09
18.0	5	27.76	1	28.45	5	27.35	10	27.31	14	27.84	11	27.69	4	27.48	6	27.48
19.0	1	26.96					2	27.86	3	28.27	4	27.72	3	27.35	4	27.51
Cases Measured Approximately at Half-yearly Anniversaries of Birthdays, Population Y.																
5.5																
6.5	5	19.92	3	19.17	6	18.89	6	18.51	8	19.45	1	18.17				
7.5	6	20.96	8	20.14	12	19.87	9	19.49	12	19.99	1	18.97	1	19.29	1	19.97
8.5	8	21.86	8	21.15	13	20.98	12	20.51	20	20.65	2	19.87	1	20.09	2	20.07
9.5	11	22.79	9	21.98	18	21.92	13	21.63	30	21.38	7	21.17	12	20.89	6	20.47
10.5	11	24.51	9	23.32	18	23.55	18	22.44	34	22.28	10	22.13	15	21.78	6	21.15
11.5	11	26.27	9	25.14	18	24.79	19	23.57	35	23.49	10	23.20	15	22.52	6	21.83
12.5	11	27.86	9	26.47	17	26.23	19	25.21	35	24.73	12	24.75	15	23.74	6	22.72
13.5	11	28.45	9	27.33	17	26.93	19	26.35	35	26.12	12	26.15	15	24.92	6	24.48
14.5	11	28.95	9	27.58	17	27.43	19	26.54	35	26.77	12	26.88	15	26.33	6	25.43
15.5	11	29.09	9	27.98	14	27.37	19	26.98	33	27.15	11	27.48	15	26.51	6	25.92
16.5	10	29.04	8	28.37	13	27.32	18	27.25	30	27.39	9	27.59	15	26.87	6	26.68
17.5	6	29.22	4	28.41	10	27.36	14	27.51	29	27.57	6	27.34	13	27.15	6	27.05
18.5	3	29.19	2	28.26	2	27.06	3	27.45	6	27.67	4	27.87	9	27.43	4	27.37
19.5	1	29.19									2	27.32	1	27.53	1	27.47

TABLE 21

GROWTH TRENDS IN AVERAGE CHEST DEPTH, CENTIMETERS, OF GROUPS OF GIRLS MENSTRUATING AT DIFFERENT AGES AND OF SUB-POPULATIONS MEASURED APPROXIMATELY AT BIRTHDAYS AND THE HALF-YEARLY ANNIVERSARIES OF BIRTHDAYS

Exact Ages	Group A Menarche Before 11-6		Group B Menarche 11-6 to 11-11		Group C Menarche 12-0 to 12-5		Group D Menarche 12-6 to 12-11		Group E Menarche 13-0 to 13-5		Group F Menarche 13-6 to 13-11		Group G. Menarche 14-0 to 14-5		Group H Menarche After 14-5	
	N	M	N	M	N	M	N	M	N	M	N	M	N	M	N	M
Cases Measured Approximately at Birthdays, Population X.																
6.0	4	12.55	3	13.11	5	12.92	9	12.88	3	12.32	1	12.99			1	12.80
7.0	6	12.93	4	13.44	9	13.26	19	13.05	12	13.28	9	13.09			1	13.10
8.0	7	13.40	5	13.86	12	13.77	25	13.34	17	13.57	12	13.28	1	12.92	4	13.30
9.0	8	13.94	6	14.44	12	14.31	27	13.81	18	13.95	13	13.79	2	12.42	7	13.42
10.0	8	14.50	6	15.28	13	14.87	28	14.40	18	14.42	13	14.34	3	12.82	9	13.79
11.0	9	15.43	7	15.96	16	15.62	30	15.14	24	14.93	14	15.09	3	13.45	9	14.25
12.0	10	16.43	7	16.58	16	16.52	30	15.81	24	15.42	16	15.63	3	14.22	11	14.83
13.0	11	16.81	7	17.06	16	16.86	30	16.76	24	16.14	19	16.37	4	15.35	12	15.47
14.0	11	16.94	7	17.43	16	17.12	30	17.00	24	16.72	19	17.08	4	16.12	12	16.24
15.0	10	17.17	7	17.51	15	17.15	30	17.18	24	16.86	19	17.12	4	17.32	12	17.36
16.0	9	17.32	7	17.54	14	17.37	25	17.12	22	16.95	18	17.44	4	17.35	11	17.92
17.0	8	17.33	4	17.94	13	17.28	22	17.27	19	16.96	17	17.26	4	16.98	8	17.89
18.0	5	17.71	1	17.34	5	17.82	10	17.56	13	17.09	11	17.58	4	16.78	6	17.81
19.0	1	17.01					2	17.51	3	17.33	4	17.31	3	16.78	4	17.74
Cases Measured Approximately at Half-yearly Anniversaries of Birthdays, Population Y.																
5.5	1	13.06			1	12.41										
6.5	6	13.56	5	13.43	7	13.01	9	12.68	11	12.77	1	12.96				
7.5	6	14.04	7	13.87	9	13.37	13	13.08	20	13.02	1	13.16	1	13.47	1	12.43
8.5	9	14.49	8	14.38	14	13.88	17	13.51	29	13.39	7	13.36	8	13.57	4	12.73
9.5	9	15.05	9	14.97	16	14.35	17	13.99	31	13.89	8	13.89	10	13.98	4	13.07
10.5	10	15.64	9	15.41	16	15.20	17	14.58	32	14.46	9	14.57	10	14.45	4	13.70
11.5	11	16.63	9	16.28	16	16.02	18	15.23	33	15.02	10	15.34	14	15.03	6	14.47
12.5	11	17.28	9	17.27	17	16.59	18	16.10	34	15.67	12	16.11	15	15.41	6	14.90
13.5	11	17.74	9	17.50	17	17.30	19	16.50	35	16.46	12	16.88	15	15.91	6	15.68
14.5	11	17.97	9	17.81	17	17.34	19	16.84	35	16.86	12	17.37	15	16.49	6	16.85
15.5	11	17.93	9	18.05	14	17.30	19	17.02	32	17.04	11	17.79	15	16.71	6	16.95
16.5	9	17.94	7	18.21	13	17.39	18	17.16	29	17.06	9	17.56	15	16.99	6	17.28
17.5	6	17.77	4	17.95	7	17.35	14	17.36	27	17.06	7	17.37	12	16.89	6	17.75
18.5	3	17.90	2	18.35	2	17.40	3	17.62	6	17.23	4	17.60	8	17.05	3	17.75
19.5	1	18.40									2	18.15	1	16.55	1	17.65

TABLE 22

GROWTH TRENDS IN AVERAGE CHEST BREADTH, CENTIMETERS, OF GROUPS OF GIRLS MENSTRUATING AT DIFFERENT AGES AND OF SUB-POPULATIONS MEASURED APPROXIMATELY AT BIRTHDAYS AND THE HALF-YEARLY ANNIVERSARIES OF BIRTHDAYS

Exact Ages	Group A Menarche Before 11-6		Group B Menarche 11-6 to 11-11		Group C Menarche 12-0 to 12-5		Group D Menarche 12-6 to 12-11		Group E Menarche 13-0 to 13-5		Group F Menarche 13-6 to 13-11		Group G Menarche 14-0 to 14-5		Group H Menarche After 14-5	
	N	M	N	M	N	M	N	M	N	M	N	M	N	M	N	M
Cases Measured Approximately at Birthdays, Population X.																
6.0	4	16.88	3	17.93	5	16.75	8	17.31	3	16.90	1	17.36				
7.0	6	17.84	4	18.36	9	17.57	17	17.56	13	17.47	8	17.16			1	17.03
8.0	7	18.42	6	18.94	12	18.21	23	18.20	19	18.05	12	17.82	1	16.84	4	17.53
9.0	8	19.32	7	19.71	12	19.16	25	18.98	20	18.81	13	18.58	2	17.04	7	17.96
10.0	8	20.14	7	20.60	13	20.17	27	19.94	20	19.65	13	19.41	3	17.99	9	18.66
11.0	9	21.57	7	22.01	16	21.21	29	20.92	23	20.67	14	20.36	3	19.09	9	19.39
12.0	10	22.61	7	23.14	16	22.34	30	21.92	24	21.63	16	21.20	3	20.29	11	20.14
13.0	11	23.13	7	23.70	16	23.28	30	22.83	24	22.68	19	22.34	4	22.25	12	20.82
14.0	11	23.39	7	23.98	16	23.40	30	23.16	24	23.31	19	23.09	4	23.08	12	21.78
15.0	9	23.51	7	24.00	15	23.65	30	23.63	23	23.53	19	23.26	4	23.48	12	22.78
16.0	9	23.58	7	24.41	14	23.72	25	23.89	21	23.68	17	23.59	4	23.92	11	22.96
17.0	9	23.51	4	24.71	9	23.86	24	24.08	21	23.72	16	23.61	4	23.60	7	23.33
18.0	3	23.91			3	23.73	8	24.35	12	23.97	9	23.86	3	23.80	6	23.22
19.0	1	24.21					2	24.25	3	24.07	4	23.58	3	23.80	4	23.10
Cases Measured Approximately at Half-yearly Anniversaries of Birthdays, Population Y.																
5.5	1	17.27			1	16.04										
6.5	6	17.87	5	17.19	8	17.04	9	17.12	11	16.62	1	17.69				
7.5	6	18.37	7	17.91	11	17.65	13	17.58	19	17.42	1	17.29	1	17.59	1	16.52
8.5	9	19.27	8	18.67	15	18.62	18	18.36	28	18.05	8	18.09	9	18.09	4	17.52
9.5	9	20.34	9	19.48	16	19.67	18	19.28	30	18.85	10	18.94	11	18.72	4	18.16
10.5	10	21.38	9	20.50	17	20.76	18	20.28	32	19.75	10	19.80	11	19.46	4	18.84
11.5	11	22.94	9	21.93	17	21.88	18	21.33	35	20.59	10	20.68	15	20.25	5	19.59
12.5	11	23.73	9	23.06	17	23.09	18	22.28	35	21.52	12	21.74	15	20.87	6	20.35
13.5	11	24.35	9	23.31	17	23.81	19	22.77	35	22.54	12	22.76	15	21.73	6	21.52
14.5	10	24.56	9	23.61	16	23.92	19	22.99	35	22.87	12	23.21	15	22.66	6	22.45
15.5	10	24.56	9	23.90	14	23.90	19	23.35	32	23.22	11	23.72	15	22.95	6	22.65
16.5	10	24.76	8	24.03	13	24.15	18	23.45	30	23.17	9	23.70	14	23.25	6	22.93
17.5	6	24.89	4	24.20	8	23.94	13	23.38	26	23.29	5	24.00	13	23.36	6	23.35
18.5	2	24.59	2	24.50	2	23.89	3	23.38	6	23.51	3	23.97	9	23.24	4	23.30
19.5	1	24.79									1	24.87	1	23.04	1	23.10

TABLE 23

GROWTH TRENDS IN AVERAGE SKELETAL AGE, IN MONTHS, OF GROUPS OF GIRLS MENSTRUATING AT DIFFERENT AGES AND OF SUB-POPULATIONS MEASURED APPROXIMATELY AT BIRTHDAYS AND THE HALF-YEARLY ANNIVERSARIES OF BIRTHDAYS

Exact Ages	Group A Menarche Before 11-6		Group B Menarche 11-6 to 11-11		Group C Menarche 12-0 to 12-5		Group D Menarche 12-6 to 12-11		Group E Menarche 13-0 to 13-5		Group F Menarche 13-6 to 13-11		Group G Menarche 14-0 to 14-5		Group H Menarche After 14-5	
	N	M	N	M	N	M	N	M	N	M	N	M	N	M	N	M
Cases Measured Approximately at Birthdays, Population X.																
6.0	1	75.83	5	80.10	5	72.70	9	67.76	4	71.94					1	61.71
7.0	6	88.83	6	92.90	11	86.50	18	78.65	14	81.94	11	77.57			1	73.71
8.0	9	100.33	7	103.57	15	99.14	28	90.93	21	93.37	12	88.02	1	91.17	7	82.71
9.0	10	114.77	7	114.28	16	112.81	29	103.11	24	105.04	15	100.77	2	102.17	11	94.71
10.0	11	129.27	7	130.14	16	127.31	29	117.49	24	117.92	19	112.84	3	111.67	12	105.25
11.0	11	144.27	7	145.28	16	141.69	30	131.80	24	131.17	19	126.10	4	121.00	12	116.08
12.0	11	159.45	7	158.57	16	154.75	30	145.00	24	144.96	19	140.05	4	134.25	12	128.42
13.0	11	173.27	7	172.71	16	169.00	30	158.40	24	157.92	19	153.89	4	149.00	12	139.83
14.0	11	183.73	7	188.28	16	183.81	30	174.00	24	170.38	19	165.95	4	164.00	12	150.83
15.0	10	194.33	7	195.14	15	192.81	30	186.03	24	183.62	19	178.79	4	175.25	12	162.83
16.0	9	197.89	6	196.81	15	196.15	26	194.34	22	194.35	18	190.68	4	188.25	10	175.73
17.0	9	198.22	4	196.81	13	196.77	25	197.26	22	196.99	17	196.09	4	194.75	8	188.23
18.0	5	198.22	1	196.81	6	196.77	11	197.35	15	197.59	13	196.94	4	197.00	6	195.40
19.0	1	198.22					2	197.35	2	197.59	4	196.94	4	197.00	4	198.90
Cases Measured Approximately at Half-yearly Anniversaries of Birthdays, Population Y.																
5.5	1	65.39			1	67.71			10	73.48						
6.5	6	79.39	6	79.08	8	77.71	9	75.28	28	85.38	6	81.05	11	78.12	4	78.22
7.5	9	92.56	8	90.41	15	89.08	16	87.17	32	97.16	10	93.89	13	90.03	5	88.97
8.5	11	105.45	9	102.78	17	101.82	17	99.41	35	109.28	11	106.59	15	101.87	6	100.17
9.5	11	119.73	9	116.89	17	116.47	17	111.82	35	122.08	11	119.59	15	114.93	6	111.00
10.5	11	135.36	9	133.11	17	130.82	17	125.70	35	136.31	11	131.50	15	127.33	6	123.00
11.5	11	151.00	9	149.44	17	146.41	18	141.17	35	148.71	12	144.50	15	140.60	6	134.50
12.5	11	167.36	9	162.89	17	159.42	18	154.22	35	161.00	12	157.58	15	153.27	6	146.17
13.5	11	181.18	9	175.33	17	173.88	18	168.78	35	174.28	12	171.17	15	167.00	6	160.00
14.5	11	190.27	9	185.33	17	188.00	18	180.22	32	186.75	11	187.99	15	179.73	6	176.00
15.5	11	194.91	9	193.78	14	195.22	18	190.78	30	195.01	9	194.99	15	188.87	6	185.83
16.5	10	197.71	8	196.66	13	198.22	18	196.72	29	196.70	7	197.56	14	194.73	6	194.83
17.5	6	197.54	4	197.66	10	199.22	16	197.03	7	196.70	4	197.56	9	197.06	5	196.83
18.5	3	197.54	2	198.16	2	199.22	4	197.03			2	197.56	1	199.06	1	197.83
19.5	1	197.54														

TABLE 24

GROWTH TRENDS IN THE AVERAGE NUMBER OF ERUPTED PERMANENT TEETH, OF GROUPS OF GIRLS MENSTRUATING AT DIFFERENT AGES AND OF SUB-POPULATIONS MEASURED APPROXIMATELY AT BIRTHDAYS AND THE HALF-YEARLY ANNIVERSARIES OF BIRTHDAYS

Exact Ages	Group A Menarche Before 11-6		Group B Menarche 11-6 to 11-11		Group C Menarche 12-0 to 12-5		Group D Menarche 12-6 to 12-11		Group E Menarche 13-0 to 13-5		Group F Menarche 13-6 to 13-11		Group G Menarche 14-0 to 14-5		Group H Menarche After 14-5	
	N	M	N	M	N	M	N	M	N	M	N	M	N	M	N	M
Cases Measured Approximately at Birthdays, Population X.																
6.0	3	5.04	2	6.68	4	4.00	6	2.50	3	5.07					1	1.30
7.0	6	8.37	5	9.18	9	7.25	17	7.33	11	7.07	8	5.88			1	5.30
8.0	7	11.54	7	12.58	14	10.69	26	10.62	19	10.80	11	8.13	1	5.75	4	9.30
9.0	9	13.97	7	15.44	16	14.05	28	13.50	24	13.38	14	11.67	2	9.75	7	15.30
10.0	10	17.75	7	19.15	16	17.93	29	17.43	24	16.80	17	16.03	3	14.75	12	20.59
11.0	11	21.45	7	23.29	16	21.93	30	20.77	24	20.92	19	20.38	4	20.75	12	24.67
12.0	11	24.54	7	25.43	16	25.81	30	24.50	24	24.42	19	24.48	4	23.25	12	26.50
13.0	11	26.63	7	27.14	16	26.81	30	26.27	24	26.21	19	26.53	4	25.75	12	27.67
14.0	11	27.45	6	28.14	16	27.37	30	27.67	24	27.00	19	27.27	4	27.75	12	27.92
15.0	10	27.95	5	28.14	15	27.84	29	27.88	24	27.38	19	27.64	4	28.00	12	28.00
16.0	9	28.39	2	28.14	15	27.97	26	28.03	19	27.80	18	27.81	4	28.00	11	28.27
17.0	4	28.64	1	28.14	8	28.22	11	28.21	12	27.97	17	28.05	4	28.00	7	28.41
18.0	3	28.64					1	28.21	2	28.47	4	28.05	4	28.25	5	28.81
19.0	1	28.64									2	28.05	1	32.25	1	28.81
Cases Measured Approximately at Half-yearly Anniversaries of Birthdays, Population Y.																
5.5	1	1.63			1	2.76										
6.5	6	5.63	6	5.57	10	6.76	10	4.75	9	4.39	1	3.25				
7.5	7	9.13	7	10.07	13	10.06	12	9.15	19	8.61	1	7.25	1	7.86	1	10.40
8.5	11	13.27	8	12.07	17	13.68	18	11.48	29	11.66	5	12.25	10	11.86	5	10.40
9.5	11	16.82	9	16.45	18	16.33	18	14.26	34	14.90	10	15.05	15	15.86	6	13.00
10.5	11	20.82	9	20.45	18	21.39	18	17.82	35	18.46	10	18.45	15	20.33	6	17.17
11.5	11	24.09	9	23.78	18	25.17	19	22.21	35	22.12	10	22.55	15	23.66	6	22.50
12.5	11	25.18	9	26.89	17	27.46	19	25.74	35	25.06	12	24.75	15	26.46	6	24.67
13.5	11	27.09	9	27.67	17	27.99	18	27.18	35	26.75	12	27.00	15	27.53	6	25.67
14.5	11	28.27	9	28.00	17	28.17	18	27.51	34	27.54	12	27.67	15	28.00	6	26.50
15.5	10	28.77	8	28.00	14	28.24	17	27.80	32	27.88	11	27.94	15	28.00	6	26.83
16.5	9	28.99	6	28.00	12	28.91	17	28.21	27	28.13	9	28.05	14	28.00	6	27.16
17.5	4	28.99	2	28.00	4	28.91	5	28.21	13	28.51	6	28.55	12	28.17	6	28.33
18.5	1	28.99							1	28.51	1	28.55	1	28.17	1	28.33
19.5											1	28.55				

3. Analysis of Basic Data by Chronological Age

It has been noted that the detailed numerical data of Tables 16 to 24 constitute the basic growth materials of this study. The numerical details, however, are meaningless as they stand and much further analysis is necessary to make clear the various aspects of the data. For convenient exposition three major types of analysis, each with its distinctive variations, may be distinguished. This section presents methods of analysis in terms of chronological age. Section four presents methods of analysis and tables of derived data in terms of time intervals before and after the menarche. Section five presents methods of analysis and derived data in terms of time intervals corresponding to comparable points on the pattern of growth. The same types of analysis have been applied to the basic data for each of the nine measurements. However, for illustrative purposes the discussion of this and the following two sections will be largely in terms of the data for standing height.

The XA and YA, the XB and YB, the XC and YC, etc., sub-groups have been combined to give growth trends in average standing height at intervals of six months. The combined XA plus YA data will be referred to as Group A, etc. The method of making this combination may be most conveniently described by reference to Figure 128 which gives the data separately for sub-group XA (the lower of the three curves with observation points at age 6.0, 7.0, 8.0, etc.), for sub-group YA (the upper of the three curves with observation points at age 6.5, 7.5, 8.5, etc.) and the combined data for Group A (the middle curve). The middle curve representing the combined data is obtained by averaging the points on the upper and lower curves at six month intervals. For example, the spotted point at age 6.0 for cases measured approximately at birthdays is 110.22 centimeters, the corresponding point on the upper curve is 113.16 (the average of 109.36 and of 116.96 at the age 5.5 and 6.5), the simple or unweighted average of 110.22 and 113.16 gives the point spotted for the combined data. This operation, may of course, be carried out in terms of the increment as illustrated in Figure 129. The two procedures give identical results. Incidentally, the almost perfect agreement of the two basic curves in Figure 129, each of which is based on only eleven cases, should be contrasted with the false and true trends of Figure 126. The slight disagreement of these two curves for the year ending at the age of 11.0 is not to be attributed to errors or unreliability. Cases in Group YA, who were measured at the half-yearly anniversaries of birthdays, do not show a sharp peak increment because the timing of the measurements is such as to obscure the maximum increments. While this procedure may seem surreptitiously to reintroduce straight-line interpolation into the data, it is to be emphasized that this procedure retains all of the original observation points whereas straight-line interpolation of the individual records to exact birthdays automatically discards all but a tiny fraction of the original observation points. A more refined procedure for combining sub-groups, which might have been applied and which avoids all semblance of straight-line interpolation, will be described in the next section. The small number of cases in some of the sub-groups and the extra labor involved have limited the application of the more refined procedure to the special situation described in the next section.

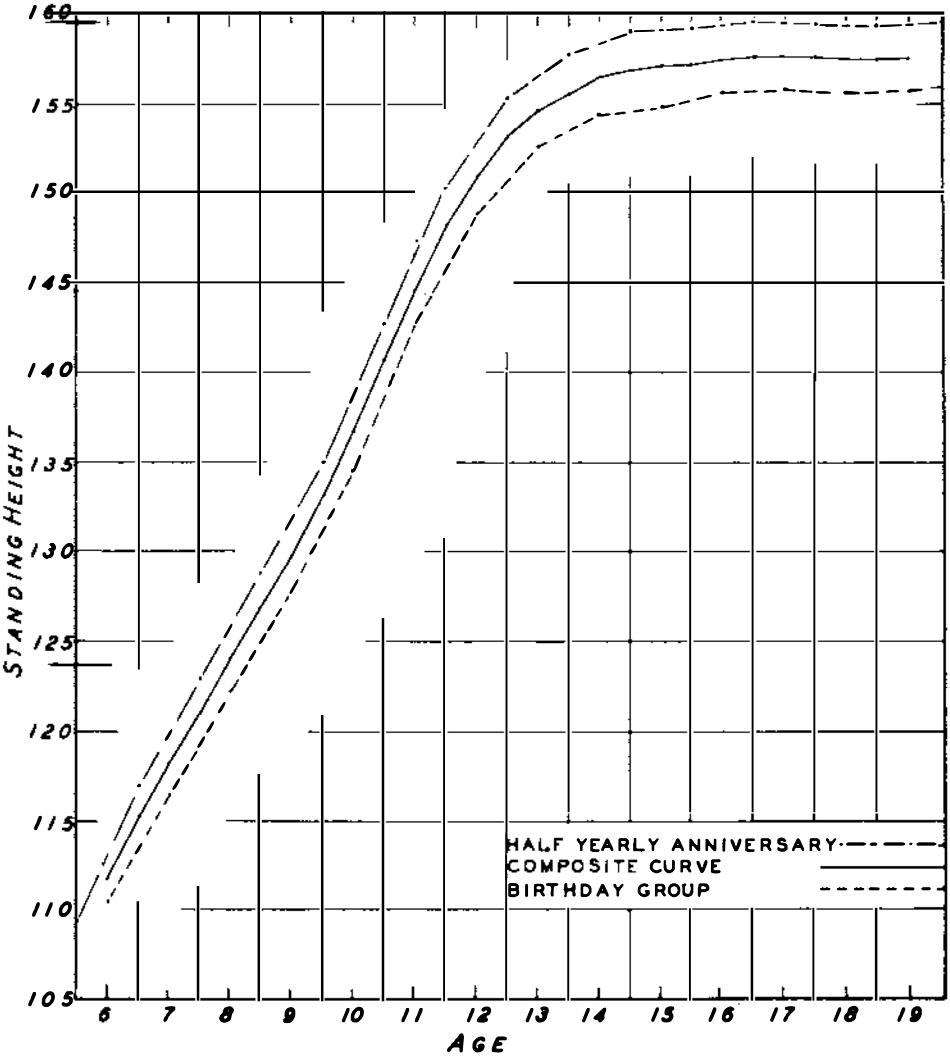

Fig. 128.—Illustrating Method of Combining Data from Cases Measured Within Three Months of Birthdays and From Cases Measured Within Three Months of the Half-Yearly Anniversaries of Birthdays to Give a Single Composite Curve with Observation Points at Six-Month Intervals.

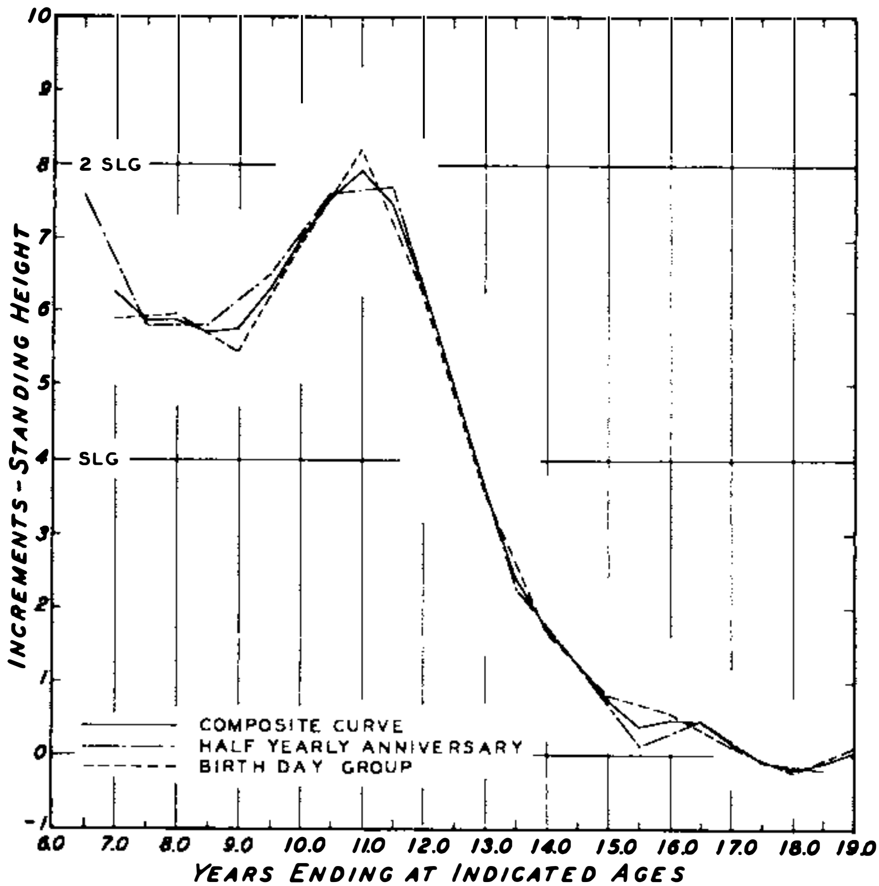

Fig. 129.—Illustrating Method of Combining Data; Redrawing of Figure 128 in Terms of Increments.

4. Analysis of Basic Data by Menarcheal Age

Chronological ages have been expressed as deviations from the advent of the menarche and the sixteen sub-groups have been combined on this basis. The average ages at onset of the menarche in Groups A, B, C, D, E, F, G, and H have been taken as 11.00, 11.75, 12.25, 12.75, 13.25, 13.75, 14.25, and 15.00 respectively. For Groups B to G these are the approximate mid-points of the intervals. The actual average menarcheal age of Group A is 11.03. In the case of Group H it has been necessary to estimate probable menarcheal ages for cases with indefinite records. The median of the probable and reported menarcheal ages for this group is 15.00. It should be noted that Group A and particularly Group H represent rather heterogeneous groups in respect to the advent of the menarche. Subtraction of these average menarcheal ages for the groups from their respective chronological ages gives what may be called deviation ages or physiological ages. For example, at chronological age 12.0 Group XA has a deviation age of -1.00, Group YB has a deviation age of -.25, Group XC has a deviation age of .25, etc. Attention is called to the fact that sub-groups XB, YC, XD, YE, XF and YG have the following series of deviation ages -3.25, -2.25, -1.25, -.25, .75, 1.75, etc.; that sub-groups YB, XC, YD, XE, YF, and XG have the following series -3.75, -2.75, -1.75, -.75, .25, 1.25, etc.; that sub-groups XA and XH have the following series -3.0, -2.0, -1.0, 0.0, 1.0, etc.; and that sub-groups YA and YH have the following series -2.5, -1.5, -.5, .5, 1.5, etc. These deviation ages determine the method of combining the sub-groups. In the case of Group A (XA plus YA) and Group H (XH and YH) the combined data as described in the preceding section are employed and it is only necessary to translate the chronological ages into deviation ages. The remaining twelve groups have been combined into three groups which will be referred to as Groups BC, DE, and FG. The details are as follows. The original summations of raw data were summed and averaged for the following pairs of sub-groups: XB and YC, XC and YB, XD and YE, XE and YD, XF and YG, and XG and YF. The resulting XBYC and XCYB groups, the XDYE and XEYD groups, and the XFYG and XGYF groups were then combined by the method described in section three. At this point a refinement in the method of combining the data which avoids any semblance of straight-line interpolation has been included. The details of this procedure for a portion of the XBYC and XCYB data for standing height is illustrated in the following tabulation:

Deviation Ages (1)	XBYC Data (2)	XCYB Data (3)	Combined Data (4)	XBYC Data (5)	XCYB Data (6)	Combined Data (7)
-1.75	140.80	139.01*	139.90	140.80	138.82*	139.81
-1.25	144.82*	142.47	143.64	144.69*	142.47	143.58
-.75	148.84	146.41*	147.62	148.84	146.61*	147.72
-.25	152.09*	150.35	151.22	152.42*	150.35	151.38
.25	155.33	152.95*	154.14	155.33	153.38*	154.36
.75	156.97*	155.55	156.24	157.26*	155.55	156.40

In columns two and three the starred figures represent straight-line interpolation. The combined data of column four are the unweighted averages of columns two and three. The procedure to this point is the one described in section three. In columns five and six the data of column four have been used to provide the more exact interpolations indicated by the starred figures. For example, according to column four 48.4% of the increase from 139.90 to 147.62 occurred in the first six months of the period of a year from deviation age -1.75 to -.75. Applying this percentage to the original figures gives the more precisely interpolated figure of 144.69 instead of 144.82. In the case of weight, iliac diameter, chest breadth, chest depth, and skeletal age a moving average of three percentages centered was employed to give the more precisely interpolated figures. Column seven is the unweighted average of columns five and six. While column four and seven may seem indistinguishable, calculation of the annual increments will show that something is gained by this procedure.

Detailed numerical data derived in this manner for the BC, the DE,

TABLE 25

GROWTH TRENDS IN AVERAGE STANDING HEIGHT, SITTING HEIGHT, WEIGHT, AND SKELETAL AGE FOR THREE GROUPS OF GIRLS IN TERMS OF AGE DEVIATIONS FROM THE MENARCHE

Devia. from	Group BC Menarche 11-6 to 12-5					Group DE Menarche 12-6 to 13-5					Group FG Menarche 13-6 to 14-5				
Menar.	N	StaH	SitH	Wt	SkeA	N	StaH	SitH	Wt	SkeA	N	StaH	SitH	Wt	SkeA
-7.25											2	113.30	63.42	20.54	
-6.75						23	113.16	62.98	20.44	72.50	11	116.21	64.32	21.50	
-6.25	6	112.09	62.02	19.22	71.64	43	115.80	63.97	21.63	77.76	12	118.94	65.38	22.46	83.13
-5.75	17	115.29	63.82	21.20	77.44	62	118.73	65.06	22.67	83.56	24	121.55	66.42	23.72	89.00
-5.25	26	118.56	65.37	22.96	84.01	71	121.88	66.53	23.94	89.49	32	124.10	67.52	25.00	95.26
-4.75	33	121.66	66.70	24.48	90.05	88	124.75	67.80	25.29	95.46	39	127.08	68.72	26.34	101.39
-4.25	40	124.74	67.94	26.09	96.07	98	127.52	68.92	26.67	101.46	43	129.85	69.77	27.80	107.56
-3.75	46	127.77	69.00	27.88	102.38	104	130.21	69.94	28.20	107.66	46	132.31	70.72	29.16	113.76
-3.25	49	130.55	70.06	29.58	109.06	104	132.83	70.94	29.76	114.20	47	134.92	71.76	30.58	120.02
-2.75	49	133.39	71.22	31.26	116.04	104	135.60	71.94	31.26	120.97	49	137.60	72.86	32.50	126.30
-2.25	49	136.48	72.48	33.42	123.34	106	138.57	73.12	33.04	127.97	49	140.51	74.02	34.68	132.78
-1.75	49	139.81	73.78	36.20	130.84	107	141.86	74.54	35.26	135.21	49	143.76	75.46	37.28	139.57
-1.25	50	143.58	75.51	39.28	138.53	108	145.62	76.22	37.78	142.03	50	147.37	77.17	40.23	146.30
-.75	50	147.72	77.62	42.62	145.97	108	149.36	78.00	40.56	148.44	50	150.88	78.82	42.90	153.03
-.25	50	151.38	79.44	45.80	152.76	108	152.75	79.87	43.64	154.82	50	153.68	80.31	45.52	159.62
.25	49	154.36	80.96	48.70	159.48	108	155.39	81.37	46.45	161.38	50	155.76	81.74	48.28	166.00
.75	49	156.40	82.18	51.22	166.55	108	157.13	82.46	48.77	168.37	50	157.16	82.82	50.32	172.48
1.25	49	157.69	83.06	53.04	173.76	108	158.25	83.34	50.52	175.14	50	158.05	83.45	51.83	179.89
1.75	49	158.63	83.82	54.04	181.11	108	158.99	83.99	51.76	181.40	49	158.60	83.88	52.89	186.62
2.25	49	159.25	84.42	54.69	187.09	106	159.47	84.45	52.77	187.25	48	158.97	84.18	53.30	190.96
2.75	48	159.78	84.77	55.42	191.21	104	159.78	84.78	53.44	192.12	46	159.26	84.44	53.66	194.08
3.25	45	160.17	85.01	56.25	194.28	97	160.00	85.03	53.96	195.10	43	159.30	84.66	54.11	195.97
3.75	45	160.32	85.16	56.56	196.14	95	160.23	85.19	54.41	196.65	41	159.36	84.79	54.66	196.90
4.25	44	160.41	85.32	56.62	197.10	93	160.36	85.22	54.76	197.13	31	159.52	84.85	54.94	197.10
4.75	42	160.56	85.63	56.54	197.34	82	160.35	85.21	55.20	197.20	27	159.56	84.90	54.89	197.20
5.25	36	160.70	85.68	56.59	197.46	44	160.42	85.22	55.26	197.20	12	159.62	84.84	54.74	197.30
5.75	23	160.74	85.65	56.72	197.55	22	160.49	85.12	55.20	197.20					
6.25	12	160.76	85.62	57.44	197.68										

TABLE 26

GROWTH TRENDS IN AVERAGE ILIAC DIAMETER, CHEST DEPTH, AND CHEST BREADTH FOR THREE GROUPS OF GIRLS IN TERMS OF AGE DEVIATIONS FROM THE MENARCHE

Devia. from Menar.	Group BC Menarche 11-6 to 12-5						Group DE Menarche 12-6 to 13-5						Group FG Menarche 13-6 to 14-5					
	Iliac D		Chest D		Chest B		Iliac D		Chest D		Chest B		Iliac D		Chest D		Chest B	
	N	M	N	M	N	M	N	M	N	M	N	M	N	M	N	M	N	M
-7.25													2	18.36	2	13.14	2	17.60
-6.75							18	18.92	23	12.69	22	16.97	11	18.75	11	13.20	10	17.52
-6.25			6	12.78	6	16.69	29	19.30	41	13.00	41	17.26	12	19.18	12	13.30	11	17.40
-5.75	9	18.74	15	13.03	16	17.17	39	19.70	60	13.12	58	17.53	13	19.64	22	13.32	23	17.76
-5.25	14	19.28	24	13.19	25	17.50	45	20.10	69	13.28	68	17.82	14	20.12	29	13.34	31	18.09
-4.75	23	19.74	27	13.40	29	17.80	62	20.51	84	13.46	83	18.16	29	20.65	32	13.61	34	18.48
-4.25	33	20.23	32	13.64	34	18.17	77	20.92	89	13.68	89	18.56	36	21.18	35	13.88	37	18.90
-3.75	36	20.74	38	13.90	40	18.62	91	21.34	93	13.93	93	18.98	41	21.65	35	14.17	37	19.33
-3.25	42	21.30	39	14.16	41	19.08	98	21.78	93	14.20	93	19.43	45	22.12	35	14.52	37	19.78
-2.75	49	21.92	42	14.42	43	19.56	104	22.22	95	14.48	97	19.92	48	22.62	40	14.88	42	20.23
-2.25	50	22.58	44	14.76	45	20.04	104	22.76	101	14.77	100	20.40	48	23.21	41	15.19	42	20.65
-1.75	50	23.30	44	15.14	46	20.56	107	23.42	104	15.07	105	20.88	48	23.92	44	15.52	44	21.14
-1.25	50	23.94	47	15.50	49	21.12	108	24.08	105	15.36	106	21.36	50	24.60	47	15.88	47	21.72
-.75	50	24.69	48	15.92	49	21.75	108	24.76	106	15.74	107	21.86	50	25.28	50	16.24	50	22.24
-.25	50	25.48	48	16.29	49	22.38	108	25.52	106	16.17	107	22.38	50	26.02	50	16.60	50	22.70
.25	49	26.20	49	16.58	49	22.94	108	26.15	107	16.52	107	22.76	50	26.52	50	16.95	50	23.00
.75	49	26.73	49	16.90	49	23.38	108	26.54	108	16.71	108	22.96	50	26.69	50	17.12	50	23.15
1.25	49	27.10	49	17.12	49	23.54	108	26.76	108	16.84	108	23.10	50	26.85	50	17.26	50	23.35
1.75	49	27.35	49	17.20	49	23.63	108	26.93	108	16.94	108	23.26	49	27.10	49	17.38	49	23.52
2.25	49	27.45	49	17.26	48	23.74	105	27.12	105	17.02	105	23.42	48	27.28	48	17.35	46	23.60
2.75	48	27.49	48	17.28	47	23.80	103	27.28	103	17.04	102	23.50	46	27.39	46	17.31	44	23.57
3.25	45	27.57	45	17.34	45	23.84	97	27.41	95	17.04	95	23.54	43	27.42	42	17.18	42	23.66
3.75	45	27.63	44	17.40	44	23.96	96	27.51	91	17.08	94	23.58	40	27.46	40	17.20	37	23.74
4.25	44	27.69	43	17.42	43	24.06	94	27.57	86	17.12	89	23.65	30	27.56	30	17.31	26	23.75
4.75	42	27.73	40	17.44	37	24.08	82	27.60	76	17.24	75	23.72	27	27.65	26	17.34	24	23.66
5.25	36	27.75	31	17.52	29	24.08	44	27.60	43	17.39	39	23.80	12	27.58	12	17.32	11	23.74
5.75	23	27.70	20	17.56	19	24.08	22	27.82	22	17.41	20	23.80						
6.25	12	27.58	12	17.58	9	24.14												

TABLE 27

GROWTH TRENDS IN AVERAGE SIZE; WEIGHTED COMPOSITE OF EIGHT GROUPS SUPERIMPOSED ACCORDING TO TIMING OF PATTERNS

Stand. Ht.		Sit. Ht.		Leg Length		Iliac Dia.		Ch. Breadth		Ch. Depth		Weight	
Age	M	Age	M	Age	M	Age	M	Age	M	Age	M	Age	M
6.17	112.29	6.05	62.00	6.15	49.59	6.43	18.85	6.40	17.04	6.14	12.76	6.12	19.82
6.67	115.04	6.55	63.28	6.65	51.35	6.93	19.29	6.90	17.36	6.64	12.96	6.62	21.09
7.17	118.00	7.05	64.58	7.15	53.01	7.43	19.73	7.40	17.64	7.14	13.12	7.12	22.23
7.67	121.02	7.55	65.93	7.65	54.74	7.93	20.18	7.90	17.97	7.64	13.26	7.62	23.55
8.17	123.92	8.05	67.21	8.15	56.31	8.43	20.64	8.40	18.31	8.14	13.48	8.12	24.95
8.67	126.75	8.55	68.38	8.65	58.06	8.93	21.08	8.90	18.73	8.64	13.70	8.62	26.43
9.17	129.52	9.05	69.45	9.15	59.72	9.43	21.52	9.40	19.15	9.14	13.97	9.12	27.98
9.67	132.22	9.55	70.46	9.65	61.43	9.93	21.97	9.90	19.63	9.64	14.23	9.62	29.49
10.17	135.03	10.05	71.56	10.15	63.09	10.43	22.52	10.40	20.09	10.14	14.53	10.12	31.19
10.67	138.05	10.55	72.75	10.65	64.94	10.93	23.14	10.90	20.57	10.64	14.84	10.62	33.11
11.17	141.39	11.05	74.13	11.15	66.79	11.43	23.82	11.40	21.07	11.14	15.17	11.12	35.43
11.67	145.07	11.55	75.75	11.65	68.85	11.93	24.54	11.90	21.64	11.64	15.50	11.62	38.14
12.17	148.77	12.05	77.56	12.15	70.71	12.43	25.26	12.40	22.19	12.14	15.91	12.12	41.15
12.67	152.01	12.55	79.31	12.65	72.30	12.93	25.94	12.90	22.66	12.64	16.31	12.62	44.19
13.17	154.56	13.05	80.83	13.15	73.35	13.43	26.48	13.40	23.00	13.14	16.66	13.12	46.94
13.67	156,37	13.55	82.03	13.65	74.08	13.93	26.78	13.90	23.20	13.64	16.88	13.62	49.29
14.17	157.58	14.05	82.97	14.15	74.37	14.43	27.01	14.40	23.34	14.14	17.06	14.12	51.16
14.67	158.37	14.55	83.67	14.65	74.58	14.93	27.21	14.90	23.48	14.64	17.15	14.62	52.60
15.17	158.91	15.05	84.19	15.15	74.58	15.43	27.37	15.40	23.58	15.14	17.25	15.12	53.56
15.67	159.29	15.55	84.56	15.65	74.67	15.93	27.48	15.90	23.63	15.64	17.27	15.62	54.14
16.17	159.54	16.05	84.81	16.15	74.61	16.43	27.57	16.40	23.70	16.14	17.29	16.12	54.65
16.67	159.71	16.55	84.98	16.65	74.68	16.93	27.62	16.90	23.78	16.64	17.30	16.62	55.11
17.17	159.86	17.05	85.09	17.15	74.68	17.43	27.86	17.40	23.83	17.14	17.35	17.12	55.51
17.67	159.92	17.55	85.16	17.65	74.75	17.93	27.70	17.90	23.84	17.64	17.40	17.62	55.65
18.17	160.00	18.05	85.17	18.15	74.76	18.43	27.75	18.40	23.94	18.14	17.46	18.12	55.65
18.67	159.99					18.93	27.77	18.90	23.91	18.64	17.52	18.62	55.76
												19.12	55.80

and FG groups for standing height, sitting height, weight, and skeletal age are reported in Table 25 while comparable data for iliac diameter chest breadth, and chest depth are reported in Table 26. This operation was not applied to the data for sternal height and erupted teeth. There are such very small differences in the number of cases from measurement to measurement that only one set of figures showing the minimum number of cases has been reported.

5. Analysis of Basic Data According to Patterns of Growth

The most striking feature of the data is the similarity of the patterns generated by the annual increments of each of the eight menarcheal groups. Figures 19, 38, 45, 55, and 56 are excellent illustrations of this similarity. In these charts both chronological age and the advent of the menarche are ignored and, instead, corresponding points on the pattern of growth are arranged in the same vertical line. A single curve generalizing all the data of such charts was desired for the purpose of making comparisons between measurements as in Figure 57. Such data for each of seven measurements are reported in Table 27.

The first step was to superimpose the curves of the eight groups so that corresponding points on the pattern were arranged in the same vertical line. In the case of standing height and sitting height the maximum increments have been aligned. In the case of other measurements which lack clearly defined maximum increments the whole pattern has been considered. Table 28 sets forth the points on the patterns which were superimposed for the purpose of these computations. In the case of standing height, for example, data for Group A at age 11.0 were combined with data for Group B at age 11.5, with Group C at age 11.5, with Group D at age 12.0, etc. The weighted average of corresponding points on the pattern of growth was then computed, the weights being the number of cases in the smaller sub-group. In order to give the resulting data a time reference, weighted average ages have been computed in the same manner. Although all charts employing such generalized data are in terms of increments or rates of growth, the derived data for each measurement as reported in Table 27 are in terms of gross size. Since all measurements on all 248 cases are summarized in such data the trends are exceptionally smooth even for such measurements as chest breadth and chest depth. It is suggested that specialists in the art of curve fitting should begin with these data.

6. Standard Deviations

Table 29 presents standard deviations for each of the eight menarcheal groups and for the total population for standing height, sitting height, weight, and iliac diameter. Standard deviations are given separately for measures of gross size and for annual increments. The distributions from which these standard deviations were calculated were originally prepared separately for each of the sixteen sub-groups of cases at each age level. The XA and YA distributions, the XB and YB distributions, etc., were then combined giving age intervals such as 7.75, to 8.74, 8.75 to 9.74, etc., the mid-points of which are 8.25, 9.25, etc. Standard deviations of gross measurements opposite a given age are for measurements taken at that age, while standard deviations of annual increments opposite a given age are for the increments ending

TABLE 28

AGES WHICH WERE SUPERIMPOSED IN COMPUTING GENERALIZED PATTERNS OF GROWTH

Groups:	A	B	C	D	E	F	G	H
Standing Height	11.0	11.5	11.5	12.0	12.5	13.0	13.5	13.5
Sitting Height	11.5	11.5	12.0	12.5	13.0	13.0	13.5	14.0
Leg Length	11.0	11.5	11.5	12.0	12.5	13.0	13.0	13.5
Iliac Diameter	11.5	11.5	12.0	12.5	12.5	13.0	13.5	14.0
Chest Breadth	11.5	11.5	12.0	12.0	13.0	13.0	13.0	13.5
Chest Depth	11.5	12.0	12.0	12.5	13.0	13.0	14.0	14.5
Weight	11.5	11.5	11.5	13.0	13.0	13.0	14.0	14.5

at that age. Interpolated figures have been included in these calculations. Values are reported only where ten or more cases are available. Since the data of this table are distinctly subordinate to those of Tables 16 to 24, neither the number of cases nor the averages incident to the calculations are reported. The number of cases may be obtained from the basic tables.

These data have been employed in section three of Chapter III, in the preparation of Figures 25 to 29 of Chapter V and in the selection ofvariables for the correlational study of Chapter XII. While the central problem of this study has necessitated the subordination of the analysis of variabilities, it should be understood that the problem of changing variabilities is entitled to intensive study on its own account.

7. Growth Trends of Cases Classified by Menarcheal Age and by Age of Maximum Growth

Tables 30 and 31 present additional basic data for standing height resulting from an analysis in terms of both menarcheal age and age of maximum growth in standing height. Since sections five and six of Chapter V describe the methods of analysis, it is only necessary here to present the numerical data and to discuss certain technical details.

It should be noted particularly that classification of cases according to the age period of maximum growth must be limited to such highly reliable measurements as standing height (or sitting height) which show clearly defined maximum increments. That is, classification of cases according to the period of maximum growth in weight or chest depth would involve grouping of many cases according to sheer errors of measurement. Hence, this procedure has not been extended to other measurements. On the other hand, much might be learned by the classification of cases according to maximum growth periods in standing height and then studying the growth patterns of the resulting groups in respect to other measurements. Considerable work of this type has been done, but none of the findings are presented since the resulting patterns tend to repeat those obtained from classification according to menarcheal age.

The exclusion of 74 cases from this analysis also requires a few comments. Some of the excluded eight cases with abnormal or unusual patterns of growth might well have been included if the data had been more rigorously edited at the outset (see sections one and four of

TABLE 29

STANDARD DEVIATIONS OF GROSS MEASUREMENTS (GM) AND OF ANNUAL INCREMENTS (AI) FOR EACH OF EIGHT GROUPS OF GIRLS MENSTRUATING AT DIFFERENT AGES AND FOR THE TOTAL POPULATION IN STANDING HEIGHT, SITTING HEIGHT, WEIGHT, AND ILIAC DIAMETER

Age	Group A		Group B		Group C		Group D		Group E		Group F		Group G		Group H		Total	
Mid-Pt.	GM	AI	GM	AI	GM	AI	GM	AI	GM	AI	GM	AI	GM	AI	GM	AI	GM	AI
Standing Height Centimeters.																		
8.25	6.50	.85	4.84	.78	4.98	.73	5.72	.74	5.14	1.20	7.48	1.10	3.70				6.08	.96
9.25	7.00	1.25	5.00	1.13	5.58	.89	5.64	.79	6.00	.80	7.80	.88	3.98	.70	6.08	.92	6.40	.96
10.25	7.80	1.35	4.98	1.52	5.94	.90	5.90	1.12	5.96	.69	7.48	.83	3.68	.86	6.66	.92	6.84	1.22
11.25	7.62	1.22	5.70	1.57	6.33	1.37	6.30	1.26	6.27	1.14	8.04	1.26	3.78	1.05	6.69	.94	7.62	1.58
12.25	6.51	2.29	5.37	1.84	5.43	1.38	6.78	1.34	6.42	1.09	8.43	1.30	3.81	1.22	6.54	1.22	7.68	1.65
13.25	6.88	1.78	5.00	1.48	5.00	1.84	6.24	2.02	6.20	1.49	8.24	1.54	4.20	1.40	7.12	1.34	7.16	2.18
14.25	7.80	.80	4.96	.77	5.16	.85	6.00	1.20	6.32	1.27	7.48	1.73	4.04	2.06	7.72	1.64	6.64	1.89
15.25	6.84	.66	5.56	.76	4.68	.56	6.00	1.03	6.48	.86	6.92	.94	3.92	1.20	7.68	1.92	6.60	1.38
16.25	7.08	.44	5.37	.42	4.80	.46	5.70	.48	6.48	.57	7.44	.55	3.75	.50	8.04	1.48	6.30	
Sitting Height, Centimeters.																		
8.25	2.75	.49	2.21	.61	2.22	.92	2.88	.55	2.50	1.46	3.55	1.10	2.03				2.79	.99
9.25	2.82	.68	2.12	.90	2.38	.70	2.96	.54	2.48	.68	3.32	.58	1.86	.40	2.89		2.80	.60
10.25	3.39	.87	2.46	1.10	2.65	.76	2.97	.73	2.60	.58	3.25	.60	1.62	.67	3.77	1.06	3.10	.83
11.25	3.21	.75	2.49	1.08	3.06	1.21	3.39	1.08	2.79	.86	3.38	.93	2.08	.78	3.64	1.01	3.53	1.18
12.25	2.76	1.26	2.82	1.04	2.86	.92	3.76	.96	2.88	.94	3.54	.96	2.56	.70	3.82	1.03	3.80	1.12
13.25	2.88	1.08	2.44	.96	2.48	1.31	3.36	1.30	2.94	.78	3.28	1.02	2.54	.72	3.92	.74	3.52	1.18
14.25	3.48	.90	2.40	.94	2.24	.83	3.22	1.01	3.00	.98	3.12	1.12	2.20	1.40	4.60	1.16	3.34	1.21
15.25	2.88	.86	2.46	.78	2.32	.68	3.12	.78	2.84	.84	3.18	.89	2.00	1.11	4.64	1.29	3.14	1.02
16.25	2.88	.66	2.24	.90	2.32	.67	2.90	.80	2.88	.72	2.92	.90	2.02	.72	4.70	.92	3.04	.85

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TABLE 29 (Continued)

STANDARD DEVIATIONS OF GROSS MEASUREMENTS (GM) AND OF ANNUAL INCREMENTS (AI)
FOR EACH OF EIGHT GROUPS OF GIRLS MENSTRUATING AT DIFFERENT AGES AND FOR THE TOTAL
POPULATION IN STANDING HEIGHT, SITTING HEIGHT, WEIGHT, AND ILIAC DIAMETER

Weight, Kilograms

Age	Group A		Group B		Group C		Group D		Group E		Group F		Group G		Group H		Total	
Mid-Pt.	GM	AI	GM	AI	GM	AI	GM	AI	GM	AI	GM	AI	GM	AI	GM	AI	GM	AI
8.25	5.22		3.57		6.57	2.24	4.14	1.21	2.88	1.02	4.38		3.57				10.14	1.60
9.25	5.70	1.38	4.11	1.02	7.98	2.45	4.62	1.47	3.00	.95	5.85	1.96	3.72	.85	2.82		5.34	1.63
10.25	7.23	2.80	4.56	1.39	9.09	1.93	4.89	1.41	3.66	1.34	6.00	1.16	4.38	1.19	3.81		6.27	1.82
11.25	9.51	2.61	5.46	2.53	9.48	2.20	5.79	2.45	3.93	1.31	7.50	1.82	5.34	1.39	3.90	1.77	7.62	2.54
12.25	9.75	2.97	6.36	2.78	6.48	2.37	7.32	2.38	4.83	1.85	9.21	2.42	6.00	1.56	4.35	1.55	8.37	2.43
13.25	10.44	2.88	7.32	1.58	6.63	3.12	8.01	2.76	5.16	2.20	9.06	1.89	6.00	1.99	6.00	2.04	8.73	2.54
14.25	10.92	2.81	6.63	2.48	6.18	2.35	7.26	2.67	6.27	2.59	10.62	2.59	6.69	3.44	7.29	2.33	8.58	2.93
15.25	11.37	1.81	7.38	2.76	5.22	2.04	8.64	2.89	6.57	2.34	11.37	1.93	6.84	2.40	7.86	2.28	8.76	2.59
16.25	11.55	2.64	7.14	1.05	5.55	3.03	8.46	1.98	5.73	1.77	11.40	1.61	8.01	2.06	8.04	2.85	8.58	2.36

Iliac Diameter, Centimeters

Age	Group A		Group B		Group C		Group D		Group E		Group F		Group G		Group H		Total	
Mid-Pt.	GM	AI	GM	AI	GM	AI	GM	AI	GM	AI	GM	AI	GM	AI	GM	AI	GM	AI
8.25	1.61	.42	.96	.31	1.44	.54	1.22	.32	.99	.34	1.38	.80					1.36	.48
9.25	1.44	.41	1.05	.28	1.66	.49	1.46	.48	.91	.47	1.49	.71	1.04		.81		1.40	.49
10.25	2.15	.58	1.31	.65	1.62	.52	1.46	.51	.94	.53	1.34	.47	1.30	.38	.96	.63	1.54	.59
11.25	2.40	.62	1.34	.57	1.61	.62	1.58	.58	1.01	.52	1.62	.51	1.26	.57	.89	.38	1.74	.61
12.25	2.53	.76	1.50	.56	1.35	.68	1.59	.58	1.16	.56	1.78	.74	1.22	.44	1.48	.34	1.83	.63
13.25	2.19	.63	1.46	.74	1.37	.80	1.69	.64	1.20	.59	1.72	.48	1.22	.42	1.26	.63	1.76	.66
14.25	2.20	.67	1.57	.56	1.28	.48	1.47	.75	1.18	.65	1.82	.71	1.51	.81	1.26	.61	1.66	.75
15.25	2.24	.58	1.59	.50	1.28	.68	1.48	.61	1.17	.53	1.88	.78	1.60	.79	1.61	.83	1.64	.70
16.25	2.13	.54	1.68	.44	1.23	.51	1.45	.47	1.19	.52	1.78	.65	1.77	.64	1.53	.73	1.63	.57

TABLE 30

GROWTH TRENDS IN AVERAGE STANDING HEIGHT OF EIGHT GROUPS OF CASES MENSTRUATING AT DIFFERENT AGES; ANALYSIS BY MENARCHEAL AGE AND AGE OF MAXIMUM INCREMENTS

Group A Menarche Before 11-6			Group B Menarche 11-6 to 11-11			Group C Menarche 12-0 to 12-5			Group D Menarche 12-6 to 12-11		
Aver. Age	N	M	Aver. Age	N	M	Aver. Age	N	M	Aver. Age	N	M
			5.47	1	109.65	5.75	4	113.06	5.31	1	108.15
6.21	8	113.69	6.47	5	117.45	6.75	11	118.91	6.31	18	112.95
7.21	13	119.44	7.47	8	123.11	7.75	16	125.26	7.31	26	118.75
8.21	15	125.31	8.47	11	128.80	8.75	20	130.85	8.31	34	124.72
9.21	18	130.95	9.47	11	134.70	9.75	20	136.34	9.31	35	130.05
10.21	18	137.62	10.47	11	141.26	10.75	20	142.61	10.31	35	135.40
11.21	18	146.51	11.47	11	150.06	11.75	20	150.89	11.31	35	141.47
12.21	18	152.69	12.47	11	155.88	12.75	20	156.58	12.31	35	149.94
13.21	18	155.57	13.47	11	158.51	13.75	20	159.00	13.31	35	155.26
14.21	17	156.91	14.47	11	159.99	14.75	19	160.01	14.31	35	157.64
15.21	15	157.56	15.47	11	160.59	15.75	18	160.53	15.31	33	158.86
16.21	12	157.99	16.47	10	160.93	16.75	18	160.65	16.31	30	159.33
17.21	5	158.08	17.47	6	160.98	17.75	9	160.68	17.31	26	159.57
18.21	3	157.98	18.47	2	161.13	18.75	1	160.08	18.31	8	159.71
19.21									19.31	1	159.71

Group E Menarche 13-0 to 13-5			Group F Menarche 13-6 to 13-11			Group G Menarche 14-0 to 14-5			Group H Menarche After 14-5		
Aver. Age	N	M	Aver. Age	N	M	Aver. Age	N	M	Aver. Age	N	M
5.48	1	108.26	5.90	2	109.53						
6.48	11	115.56	6.90	7	114.13				6.89	2	117.78
7.48	26	120.83	7.90	13	120.09	7.33	2	116.63	7.89	4	117.68
8.48	37	126.57	8.90	20	125.99	8.33	10	121.43	8.89	9	122.20
9.48	40	131.84	9.90	22	131.37	9.33	11	126.56	9.89	11	126.89
10.48	41	137.07	10.90	24	136.58	10.33	12	131.26	10.89	11	131.85
11.48	41	142.88	11.90	24	142.53	11.33	13	135.81	11.89	12	136.64
12.48	41	151.15	12.90	24	150.78	12.33	13	141.55	12.89	12	142.10
13.48	41	157.08	13.90	24	155.83	13.33	13	149.18	13.89	12	149.52
14.49	41	159.80	14.90	24	158.28	14.33	13	154.19	14.89	12	154.05
15.48	40	161.09	15.90	21	159.16	15.33	13	156.60	15.89	10	156.62
16.48	37	161.70	16.90	20	159.59	16.33	13	157.65	16.89	10	158.05
17.48	34	161.98	17.90	15	159.92	17.33	12	157.94	17.89	8	158.50
18.49	13	162.23	18.90	7	159.96	18.33	9	158.26	18.89	4	159.28
19.48	1	161.53	19.90	1	159.96	19.33	3	158.26			

Appendix A). Similarly, the excluded nine cases whose records cover too limited a range of years to define the maximum increment and the excluded seventeen cases where the maximum increment is determined by an interpolated figure might have been included if the classification had considered the pattern as a whole rather than the maximum increment alone. The forty cases in which the maximum increment does not exceed adjacent increments by half a centimeter represent, in the judgment of the writer, instances in which the accidental timing of annual measurements obscures the maximum increment. That is, if these cases had been

TABLE 31

GROWTH TRENDS IN AVERAGE STANDING HEIGHT OF NINE GROUPS OF CASES WITH MAXIMUM INCREMENTS IN STANDING HEIGHT AT DIFFERENT AGES

A	M.I.		M.I. Age 11.08		M.I. Age 12.02		M.I. Age 12.99		M.I. Age 14.06	
Age	N	M	N	M	N	M	N	M	N	M
Cases Measured at Birthdays, Population X										
6.0			5	113.42	10	111.96	1	110.75	1	106.55
7.0			8	119.50	22	118.21	13	115.55	2	111.25
8.0			12	125.31	30	124.27	20	121.19	4	116.85
9.0			13	130.80	31	129.78	26	126.98	6	122.20
10.0			13	137.02	34	135.23	27	132.26	7	127.38
11.0			13	145.48	34	141.18	28	137.61	7	132.81
12.0			13	152.02	34	149.61	28	143.69	7	137.52
13.0			13	155.53	34	155.58	28	151.79	7	143.59
14.0			13	157.18	34	158.28	28	156.24	7	151.19
15.0			12	157.96	33	159.63	28	158.22	7	155.20
16.0			11	158.35	32	160.23	24	159.06	6	156.90
17.0			9	158.47	31	160.61	22	159.49	6	157.80
18.0			2	158.42	12	160.63	17	159.75	5	157.82
19.0					1	160.63	4	159.70	3	157.95
Cases Measured at Half-yearly Anniversaries of Birthdays, Population Y										
	Age 10.50		Age 11.55		Age 12.50		Age 13.43		Age 14.50	
6.5	2	117.78	14	115.98	14	115.12	1	112.54		
7.5	3	124.07	20	121.81	20	120.57	3	117.84	1	116.74
8.5	6	130.18	24	127.50	33	126.37	16	123.37	2	122.58
9.5	6	137.04	26	133.17	37	131.66	18	128.84	4	126.88
10.5	6	145.97	26	139.61	37	136.83	18	133.56	5	131.52
11.5	6	153.15	26	148.37	37	142.64	18	138.51	5	135.92
12.5	6	156.71	26	154.57	37	150.87	18	144.25	5	140.92
13.5	6	158.22	26	157.42	37	156.49	18	152.21	5	146.68
14.5	6	159.27	26	158.85	37	159.00	18	156.89	5	154.46
15.5	6	159.50	25	159.31	36	160.09	18	159.26	5	158.76
16.5	6	159.63	23	159.64	32	160.55	17	160.10	4	160.24
17.5	2	159.74	17	159.74	30	160.77	16	160.42	4	160.80
18.5	2	159.34	4	159.32	9	161.04	9	160.91	3	160.92
19.5			1	159.52	1	161.04	1	161.31	1	161.52

measured at semi-annual intervals, individual curves of annual increments would have shown clearly defined peak increments in the great majority of cases. Arranging the increments of these cases so that the two largest increments are in the same vertical line and averaging increments on corresponding points of the resulting pattern gives the following series of increments: 6.07, 5.93, 5.31, 5.74, 7.27, 7.36, 3.86, 1.59, .87, .34, .21, and .14 for years ending at average ages of approximately 8.0, 9.0, etc. to age 19.0. A chart of these figures superimposed on Figure 36 agrees exceptionally well with points on the solid curve at corresponding ages.

TABLE 32

AVERAGE STANDING HEIGHTS OF SHORTEST, MIDDLE, AND TALLEST GIRLS SELECTED FROM AMONG CASES MENSTRUATING BETWEEN 13-0 AND 13-6

	Shortest Cases		Middle Cases		Tallest Cases	
Age	N	M	N	M	N	M
6.5	3	109.82	2	114.35	5	121.53
7.5	4	114.72	6	119.35	7	126.53
8.5	7	120.15	10	125.18	10	132.80
9.5	10	125.39	11	130.76	10	138.25
10.5	10	130.21	11	135.83	10	143.38
11.5	10	135.89	11	142.11	10	149.53
12.5	10	143.58	11	149.50	10	157.82
13.5	10	149.98	11	155.49	10	163.71
14.5	10	153.20	11	158.66	10	165.97
15.5	10	154.18	11	160.27	10	167.42
16.5	7	155.05	11	160.78	10	167.61
17.5	7	155.30	11	161.02	9	167.94
18.5	2	155.15	3	160.85	1	168.24

TABLE 33

GROWTH TRENDS IN AVERAGE STANDING HEIGHT, SITTING HEIGHT, ILIAC DIAMETER AND WEIGHT OF ITALIAN AND NORTH EUROPEAN GIRLS MENSTRUATING BEFORE AGE 12-6

	Italian Girls					North European Girls				
	N	StaH.	SitH.	Ild.	Wt.	N	StaH.	SitH.	IlD.	Wt.
6.0						12	113.06	62.48	18.58	20.47
6.5	4	114.12	63.76	19.61	20.99	27	116.44	64.23	19.22	21.96
7.0	5	117.04	65.06	20.04	22.94	33	119.56	65.65	19.81	23.39
7.5	6	120.08	66.26	20.40	24.54	40	122.56	66.96	20.28	24.98
8.0	7	122.84	67.30	20.92	26.02	46	125.66	68.26	20.74	26.76
8.5	8	125.44	68.14	21.40	27.46	50	128.68	69.40	21.22	28.55
9.0	9	128.32	69.09	21.87	29.34	53	131.57	70.56	21.65	30.28
9.5	9	131.35	70.30	22.47	31.12	53	134.59	71.79	22.18	32.18
10.0	9	134.65	71.66	23.20	33.29	53	137.86	73.10	22.87	34.44
10.5	9	138.18	73.08	23.89	36.18	53	141.46	74.65	23.60	37.20
11.0	9	142.16	74.80	24.65	39.08	54	145.30	76.47	24.33	40.47
11.5	9	145.97	76.74	25.42	41.73	54	149.10	78.34	25.10	43.83
12.0	9	149.06	78.51	26.10	44.86	53	152.40	80.02	25.84	46.84
12.5	9	151.54	79.95	26.66	47.76	53	154.92	81.44	26.48	49.57
13.0	9	152.96	80.76	27.12	49.98	53	156.74	82.62	26.96	51.90
13.5	9	153.87	81.31	27.49	52.20	53	157.94	83.53	27.25	53.55
14.0	9	154.78	82.04	27.67	53.68	53	158.76	84.20	27.46	54.58
14.5	8	155.32	82.65	27.86	54.37	53	159.37	84.71	27.60	55.34
15.0	7	155.54	82.89	27.99	55.02	52	159.81	85.04	27.67	55.99
15.5	6	155.72	83.16	27.95	55.98	50	160.11	85.26	27.76	56.50
16.0	5	155.89	83.52	27.92	56.06	50	160.30	85.48	27.84	56.91
16.5	5	156.11	83.75	27.94	55.87	48	160.43	85.62	27.87	56.90
17.0	5	156.31	83.81	27.94	55.90	45	160.54	85.80	27.86	56.75
17.5	4	156.46	83.80	28.28	55.56	34	160.58	85.85	27.87	57.02
18.0	2	156.49	83.55	28.55	55.28	19	160.56	85.74	27.87	57.60
18.5						11	160.58	85.73	27.66	57.98
						5	160.66	85.86	27.46	58.46

TABLE 34

GROWTH TRENDS IN AVERAGE STANDING HEIGHT, SITTING HEIGHT, ILIAC DIAMETER AND WEIGHT OF ITALIAN AND NORTH EUROPEAN GIRLS MENSTRUATING BETWEEN 12-6 AND 13-5

	Italian Girls					North European Girls				
	N	StaH.	SitH.	IlD.	Wt.	N	StaH.	SitH.	IlD.	Wt.
6.0						11	112.18	62.69	18.60	20.13
6.5	2	110.61	61.89		19.36	27	115.08	63.74	19.07	21.33
7.0	6	113.71	63.62	19.28	20.69	33	117.92	64.77	19.48	22.42
7.5	7	116.83	65.16	19.84	22.32	43	120.83	65.97	19.86	23.51
8.0	8	119.85	66.36	20.42	23.65	66	123.87	67.34	20.27	24.76
8.5	10	122.82	67.49	21.02	25.30	77	126.76	68.57	20.66	26.12
9.0	13	125.59	68.56	21.40	26.92	80	129.48	69.66	21.08	27.56
9.5	14	128.26	69.64	21.81	28.48	83	132.22	70.70	21.50	28.98
10.0	14	130.88	70.73	22.29	30.20	84	134.93	71.71	21.91	30.40
10.5	14	133.78	71.88	22.89	32.03	85	137.72	72.78	22.40	32.08
11.0	14	136.96	73.18	23.56	34.08	86	140.81	74.03	22.98	34.04
11.5	14	140.29	74.75	24.21	36.62	87	144.29	75.57	23.62	36.35
12.0	14	143.95	76.52	24.84	39.48	87	148.08	77.32	24.32	38.94
12.5	14	147.30	78.29	25.50	42.44	87	151.66	79.08	25.05	41.82
13.0	14	150.17	79.92	26.16	45.78	87	154.76	80.74	25.74	44.74
13.5	14	152.18	81.08	26.63	48.48	87	157.06	82.08	26.26	47.20
14.0	14	153.45	81.94	26.92	50.47	87	158.51	83.11	26.56	49.20
14.5	14	154.48	82.77	27.22	52.20	87	159.50	83.89	26.76	50.74
15.0	14	154.94	83.32	27.52	53.59	85	160.19	84.48	26.94	52.02
15.5	11	155.12	83.57	27.70	53.90	82	160.68	84.96	27.12	53.01
16.0	10	155.48	83.82	27.64	54.00	79	160.95	85.23	27.28	53.48
16.5	10	155.75	83.99	27.56	54.10	78	161.19	85.42	27.40	53.98
17.0	10	155.86	84.10	27.68	54.38	74	161.38	85.55	27.51	54.51
17.5	9	155.92	84.26	27.77	55.28	52	161.44	85.54	27.55	54.79
18.0						28	161.47	85.47	27.55	55.10
18.5						12	161.50	85.39	27.64	55.00

8. Tall Versus Short Children

Table 32 presents the numerical data incident to the discussion of the growth patterns of tall versus short children presented in section six of Chapter IV. The data are self-explanatory. This type of analysis should be extended to each of the sixteen sub-groups and the data recombined after the manner of the methods suggested in section seven. Extension of this type of analysis not only to all cases but to other measurements and to indices of body build would add greatly to our understanding of how children grow.

9. Racial Differences

The selection of cases and procedures for the study of racial differences are given in full in Chapter XI. It is only necessary at this point to present the numerical data which are reported in Tables 33, 34, and 35.

TABLE 35

GROWTH TRENDS IN AVERAGE STANDING HEIGHT, SITTING HEIGHT, ILIAC DIAMETER AND WEIGHT OF ITALIAN AND NORTH EUROPEAN GIRLS MENSTRUATING AFTER AGE 13-5

	Italian Girls					North European Girls				
	N	StaH.	SitH.	Ild.	Wt.	N	StaH.	SitH.	Ild.	Wt.
7.0	2	111.76	63.84	18.32	21.02	9	114.80	63.45	19.43	19.91
7.5	3	114.21	64.66	18.82	21.74	10	117.66	64.55	19.86	21.22
8.0	3	116.61	65.50	19.33	22.54	16	120.64	65.86	20.10	22.92
8.5	6	119.01	66.38	19.82	23.46	31	123.59	67.21	20.36	24.63
9.0	8	121.36	67.26	20.38	24.36	36	126.39	68.38	20.70	25.86
9.5	9	123.81	68.34	20.79	25.07	44	129.08	69.45	21.06	27.76
10.0	9	126.21	69.40	21.13	26.14	49	131.64	70.44	21.54	28.58
10.5	9	128.66	70.22	21.62	28.00	49	134.26	71.47	21.98	30.12
11.0	10	131.25	71.08	22.12	29.62	51	136.92	72.52	22.42	31.84
11.5	10	134.05	72.30	22.67	31.20	51	139.63	73.58	22.91	33.71
12.0	10	137.06	73.74	23.36	33.24	51	142.60	74.82	23.50	35.92
12.5	10	140.22	75.36	23.98	35.62	52	145.82	76.24	24.12	38.28
13.0	10	143.40	77.04	24.54	38.26	52	149.19	77.80	24.77	40.76
13.5	10	146.07	78.50	25.25	40.92	52	152.18	79.35	25.44	43.50
14.0	10	148.12	79.75	25.80	43.44	52	154.65	80.84	26.05	46.22
14.5	10	149.56	80.72	26.10	45.62	52	156.64	82.09	26.48	48.47
15.0	9	150.65	81.61	26.32	47.56	52	158.02	83.01	26.71	50.20
15.5	8	151.32	82.27	26.62	48.89	51	158.99	83.72	26.96	51.62
16.0	7	151.63	82.48	26.90	49.65	50	159.62	84.23	27.18	52.72
16.5	7	152.06	82.73	26.86	49.82	46	160.04	84.59	27.35	53.57
17.0	6	152.52	83.30	26.92	50.26	44	160.29	84.85	27.46	54.24
17.5	5	152.69	83.60	27.06	50.57	38	160.45	85.09	27.57	54.80
18.0	4	152.91	83.52	27.10	50.24	31	160.62	85.22	27.74	55.32
18.5	3	153.22	83.79	27.09	51.07	22	160.58	85.16	27.86	55.42
19.0						11	160.56	85.09	27.83	55.62

10. Graphic Methods

Suitable graphic methods are peculiarly important for developmental studies of physical growth since the bare numerical data are almost literally unintelligible. The fact that the reproduction of charts by photolithoprinting process is actually less expensive than special typing and reproduction of text has led to the preparation of a large number of charts which under other circumstances might seem extravagant. Methods of chart construction are explained at the outset of Chapter IV. Table 36 records the numerical details which are essential to this method of construction. It is of some interest to note that these data plus the data of Table 3 constitute the only averages for children in general which it has seemed necessary to report.

For the most part the charts speak for themselves and only a few comments are necessary. The critical student will note that selection of ages eight and seventeen as basic points for the purpose of chart construction involves a choice of points which fall outside the age range of changing growth intensities and within the age range for which substantial numbers of cases are available. No one unvarying method is satisfactory, hence the alternative charts of Chapter XIII. The accuracy of all charts, involving over 18,000 plotted points, has been checked and the more serious errors have been corrected. A systematic

TABLE 36

AVERAGE SIZE AT AGE EIGHT (7-6 TO 8-5) AND SEVENTEEN OF ALL AVAILABLE CASES

Measurements	Age Eight N	Age Eight M	Age Seventeen N	Age Seventeen M
Standing Height	173	123.6	192	159.5
Sternal Height	173	98.6	192	130.3
Sitting Height	173	67.3	192	84.9
Iliac Diameter	89	20.5	188	27.4
Chest Breadth	103	18.1	188	23.5
Chest Depth	102	13.3	187	17.2
Weight	173	25.3	191	54.3
Skeletal Age	193	92.9	190	195.4
Erupted Teeth	163	10.9	136	28.2

count of points which were not corrected shows a total of only 38 which are incorrect by one and a half millimeters or more as originally drawn. Since the charts have been reduced about 40% in reproduction this means that only one fifth of one per cent of all points as reproduced are in error by more than a millimeter. It is to be emphasized that no smoothing whatever has been applied in any case. In a few charts, however, psychological considerations have suggested features which are not strictly correct. It is specifically noted in the text that Figure 4 is a caricature. In Figures 7 and 9 broken lines have been added to destroy the illusion that the range of error in the slopes of curves alternately increases and vanishes, but these have been omitted from Figure 27. Figures 25 to 29 involve certain approximations due to the necessary conflicts between methods of computing growth trends and of computing standard deviations. Freedom has also been exercised in plotting or omitting points in the age range from 6.0 to 8.0 and from 17.0 to 19.0 where trends are often determined by a very few cases.

11. Summary of Appendices A and B; The Limitations of the Harvard Data

The catalogue of limitations of the data of the Harvard Growth Study presented in Chapter II needs to be revised at this point in the light both of the procedures employed to overcome these limitations and of the findings presented in the text. Overlapping ages and records (item 7) are actually sources of strength since they alleviate the difficulties due to random gaps (item 8), to systematic gaps (item 9), and to possible changes in methods of measurement (item 2). Selective factors (item 6), random and systematic gaps (items 8 and 9), the absence of measurements on birthdays (item 10), and irregular intervals between measurements (item 12) have almost vanished as limitations. Even the limitation imposed by inaccurate or erratic measurements (item 3) has been partially overcome. The absence of supplementary data (item 13) makes it impossible to carry out studies of vital importance, but the absence of such data has not been a handicap for the specific purposes of this particular study. Similarly, measurements at annual instead of at semi-annual or quarterly intervals (item 11) precludes the determination of certain facts. The combination of systematic gaps (item 9), erratic measurements (item 3), and measurements over clothing (item 1), plus the difficulties of accurate measurement and the small changes which take place during the age range involved conspire to seriously limit the value of the data for chest depth and chest breadth. Even so, the eight menarcheal groups are surprisingly well differentiated by these measurements. Unverified birth-dates (item 5) constitute a limitation which is believed to be technical only, but there are clearly no errors in the time intervals between measurements. Attention should also be called to the fact that analysis of the data in terms of deviation ages before and after the menarche, in terms of corresponding points on the pattern of growth, and in terms of maximum increments in standing height is not influenced by possible errors in the birth records. The most serious limitation of the data is the unsatisfactory methods employed in collecting the menarcheal age records (item 4). Again, however, this limitation almost completely vanishes when the data are analyzed in terms of both menarcheal age and age of maximum growth in standing height. The writer suspects that certain menarcheal records are in error, but the very high multiple correlation between physical data and menarcheal ages reported in Chapter XII should be convincing evidence that even these records are substantially correct.

12. Summary of Appendices A and B; Methods of Analysis

The major features of the analysis which seem important are so intimately interlocked and so mutually interdependent that it is difficult to separate and label them for the purpose of convenient summary. In the following discussion only two features have been identified but the attempt has been made to indicate ramifications of each.

The first and most important point, at least for this study, is the central role of the growth increments. The original data consist of measurements of gross size at successive ages, but the basic and fundamental data consist of the measured changes or increments from age to age. The availability of measured increments for each individual constitutes the most distinguishing feature of longitudinal data and determine methods of analysis in nearly all directions. A rapid review of the entire study from this point of view will be illuminating. While the discussion of Chapter III was formulated in terms of the advantages of longitudinal over cross-sectional data, the real point is that directly measured increments for each individual are far superior to "increments" for children in general which are only inferential and which are necessarily distorted by sampling errors. Negative increments exceeding errors of measurement provide the most convincing ground for the rejection of erratic figures (Appendix A, section one). It is the increments which indicate that the data for chest depth during the second autumn are not comparable with earlier and later years (Appendix A, section two). The sensitivity of the increments to elapsed time makes regular intervals between measurements important and determines the method of adjustment for irregular intervals (Appendix A, section six). In making interpolations for gaps in the data the two year increment shown by the individual record has been apportioned between the first and second year in accordance with the annual increments of cases with complete data (Appendix A, section seven). Analysis of the data in terms of increments is essential in dealing with overlapping records

(Appendix A, section eight; Appendix B, section two). The high reliability of the patterns generated by the increments of very small numbers of cases makes it possible to divide and sub-divide the population into small homogeneous groups (Appendix B, sections one, seven, eight and nine). Charts of increments are much more precise and meaningful and reveal contrasted patterns of growth much more clearly than charts of average size (Chapter IV, sections two, three and four). The age period in which the maximum increment falls and the magnitude of the increments at selected ages are much more closely associated with the advent of the menarche than is mere size at any age (Chapter XII). However, it should not be necessary to urge the primary role of the increments. Size at a given age should be regarded as merely the cumulative sum of all earlier increments and should be treated as an important datum only where the detailed data on the earlier increments are not available[1].

The second feature of the statistical procedures employed may be described as a process of analysis and synthesis in which both the analytic and synthetic phases are mutually necessary and complimentary. The analytic phase consists of the division and subdivision of the available population into a large number of homogeneous groups. Thus the cases have been divided according to the timing of measurements relative to birthdays, according to menarcheal age, according to the age periods in which maximum growth in standing height occurred, according to tallness versus shortness, and according to race. This subdivision into groups each of which contains a very small number of cases relies on the fact that growth trends are reliably determined by comparatively small numbers when longitudinal data are available. The synthetic phase does not merely recombine these small groups into larger groups, the process brings out some feature which was not apparent in the original data. For example, after dividing all cases into two groups depending on the timing of measurements relative to birthdays, the cases are not combined in such a way as to wipe out the original division; instead, they are combined in such a way as to give observation points at intervals of six months (Appendix B, section 3). In this instance the data are still expressed in chronological age, but in other instances chronological age is ignored and the data are combined in terms of time intervals before or after the menarche or in terms of corresponding points on the pattern of growth (Appendix B, sections four, five and seven). Thus, while the analytic phase of the procedure pays strict attention to chronological age, the synthetic phase may arrange the data in terms of time units which are more meaningful than chronological age.

[1] *Incidentally, where growth data are available, there are good mathematical grounds for fitting curves to the differential rather than the integral form of the data. See Hotelling, (18).*

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