FROM THE ARCHIVES

Research Briefings, 1987 (1987)

Pages
21

Size
8.5 x 11

ISBN
0309310954

Research Briefing Panel on Order, Chaos, and Patterns: Aspects of Nonlinearity; Committee on Science, Engineering, and Public Policy; National Academy of Sciences; National Academy of Engineering; Institute of Medicine

Research Briefings 1987

Report of the Research Briefing Panel on Order, Chaos, and Patterns: Aspects of Nonlinearity

for the Office of Science and Technology Policy,
the National Science Foundation,
and Selected Federal Departments and Agencies

Committee on Science, Engineering,
and Public Policy (U.S.). Research Briefing Panel on Order, Chaos, and Patterns.
National Academy of Sciences
National Academy of Engineering
Institute of Medicine

NATIONAL ACADEMY PRESS
Washington, D.C. 1987

National Academy Press 2101 Constitution Avenue, NW Washington, DC 20418

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This work was supported by the National Science Foundation.

Printed in the United States of America

Research Briefing Panel on Order, Chaos, and Patterns: Aspects of Nonlinearity

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1987

1. Order, Chaos, and Patterns: Aspects of Nonlinearity (CPSMR)
2. Biological Control in Managed Ecosystems (CLS)
3. Chemical Processing of Materials and Devices for Information Storage and Handling (CPSMR)
4. High-Temperature Superconductivity (COSEPUP)

Policy Topic

5. Research and Research Funding: Impact, Trends, and Policies (CPSMR)

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2. Decision Making and Problem Solving (CBASSE)
3. Protein Structure and Biological Function (IOM)
4. Prevention and Treatment of Viral Diseases (IOM)

1985

1. Remote Sensing of the Earth (CPSMR)
2. Pain and Pain Management (IOM)
3. Biotechnology in Agriculture (BA)
4. Weather Prediction Technologies (CPSMR)
5. Ceramics and Ceramic Composites (CETS)
6. Scientific Frontiers and the Superconducting Super Collider (CPSMR)
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1984

1. Computer Architecture (CETS)
2. Information Technology in Precollege Education (NAS)
3. Chemical and Process Engineering for Biotechnology (CPSMR)
4. High-Performance Polymer Composites (CPSMR)
5. Biology of Oncogenes (IOM)
6. Interactions Between Blood and Blood Vessels (Including the Biology of Atherosclerosis) (IOM)
7. Biology of Parasitism (IOM)
8. Solar-Terrestrial Plasma Physics (CPSMR)
9. Selected Opportunities in Physics (CPSMR)

1983

1. Selected Opportunities in Chemistry (CPSMR)
2. Cognitive Science and Artificial Intelligence (CBASSE)
3. Immunology (IOM)
4. Solid Earth Sciences (CPSMR)
5. Computers in Design and Manufacturing (CETS)

1982

1. Mathematics (CPSMR)
2. Atmospheric Sciences (CPSMR)
3. Astronomy and Astrophysics (CPSMR)
4. Agricultural Research (BA)
5. Neuroscience (IOM)
6. Materials Science (CETS)
7. Human Health Effects of Hazardous Chemical Exposures (CLS)

*The reports listed here are published in *Research Briefings 1987, Research Briefings 1986,* etc., by the National Academy Press, Washington, D.C.

Preface

Research Briefings 1987 is the sixth volume of research briefing reports published by the Committee on Science, Engineering, and Public Policy (COSEPUP).* It brings to 37 the number of such reports prepared on a broad range of topics since the first volume in 1982 (see the list of topics on page iv). The briefings are prepared at the request of the President's Science Advisor, who also serves as Director of the Office of Science and Technology Policy (OSTP), and the Director of the National Science Foundation (NSF).

Five reports are presented in this collection—the first four on what might be called traditional science and technology topics similar to those covered in earlier years, and the fifth on a policy topic, which is a new departure for the research briefing activity. The policy briefing was undertaken at the specific request of Erich Bloch, Director of the NSF, who encouraged COSEPUP to apply the research briefing approach to a broader set of issues. One of the four traditional briefings (High-Temperature Superconductivity) was also prepared at the specific request of the NSF director after the 1987 briefing activity was already well under way, in response to the exciting new developments in superconductivity in ceramic oxide materials announced earlier this year.

Research briefing topics generally are selected by the OSTP and NSF directors in the late fall in response to suggestions put forward by COSEPUP. COSEPUP's suggestions are selected from a much larger list of suggestions offered by the commissions and boards of the National Research Council (NRC); members of the NAS, NAE, and IOM Councils; members of COSEPUP; as well as officials of the NSF and OSTP. Individual briefings are designed either (1) to assess the status of a field and identify high-leverage research opportunities and barriers to progress in the field (including where appropriate, progress in commercial exploitation), or (2) to identify and illuminate critical aspects of a policy issue related to the health of U.S. science and technology. The briefings are then prepared by panels of experts, usually in the spring, with the day-to-day assistance of NRC staff. This schedule allows time for COSEPUP review in late spring and

*COSEPUP is a joint committee of the National Academy of Sciences (NAS), the National Academy of Engineering (NAE), and the Institute of Medicine (IOM).

Report of the Research Briefing Panel on Order, Chaos, and Patterns: Aspects of Nonlinearity

INTRODUCTION AND BACKGROUND

Linear analysis developed as a formal mathematical discipline during the nineteenth century, and in the intervening years its applications have achieved many spectacular successes throughout science and engineering. But in fact most phenomena observed in nature are nonlinear, and the linear approximations historically used to describe them are too often tacit admissions that the true problems simply cannot be solved. In some instances, including many of technological importance, the effects of nonlinearity can be understood in terms of small perturbations on linear behavior. In other cases, however, incorporation of the true nonlinearities completely changes the qualitative nature of the system's possible behavior. This report focuses on several aspects of these essentially nonlinear phenomena.

The difficulties posed by essential nonlinearity can be illustrated by a familiar example. When water flows through a pipe at low velocity, its motion is laminar and is characteristic of linear behavior: regular, predictable, and describable in simple mathematical terms. However, when the velocity exceeds a critical value, the motion becomes turbulent, with eddies moving in a complicated, irregular, erratic way that typifies nonlinear behavior. Many other nonlinear phenomena exhibit sharp and unstable boundaries, erratic or chaotic motion, and dramatic responses to very small influences. Such properties typically defy full analytical treatment and make even quantitative numerical description a daunting task. And yet, this task must be confronted, for the point where phenomena become nonlinear is often precisely where they become of interest to technology. In applications ranging from laser/plasma interactions in inertial-confinement thermonuclear fusion, to designs for high-performance and fuel-efficient aircraft, to advanced oil recovery, nonlinearity prevails.

Within the past two decades, the systematic, coordinated investigation of nonlinear natural phenomena and their mathematical models has emerged as a powerful and exciting interdisciplinary subject. Studies of nonlinearity seek to understand a variety of complicated, nonlinear problems encountered in nature and to discover their common features. The scientific methodology has depended on the synergetic blending of three distinct approaches:

- "Experimental mathematics," which is the use of cleverly conceived computer-based numerical simulations, typically involving visualization techniques such as interactive, high-quality graphics, to give qualitative insights into and to stimulate conjectures about analytically intractable problems;
- Novel and powerful analytical mathematical methods to solve, for example, certain nonlinear partial differential equations and to analyze nonlinear stability; and
- Experimental observation of similar behavior in natural nonlinear phenomena in many different contexts and the quantification of this similarity by high-precision experiments.

The success of this three-pronged attack is clearly evidenced by the remarkable progress already made toward solving many nonlinear problems long considered intractable. Essential to this progress has been the discovery that distinct nonlinear phenomena from many fields do indeed display common features and yield to common methods of analysis. This commonality has allowed the rapid transfer of progress in one discipline to other fields and confirms the inherently interdisciplinary nature of the subject. Despite these stimulating developments, however, the present-day approach to nonlinear problems is not entirely systematic. Rather it relies on the identification and exploitation of paradigms, namely, unifying concepts and associated methodologies that are broadly applicable in many different fields.

This report focuses on three of the central paradigms of nonlinearity: coherent structures, chaos, and complex configurations and pattern selection. The following sections cover recent progress in research and future opportunities for research and technological applications of these paradigms, the international standing of U.S. work in the field, and administrative strategies for enhancing progress in this important interdisciplinary subject.

PARADIGMS OF NONLINEARITY: DEFINITIONS, OPPORTUNITIES, AND APPLICATIONS

Coherent Structures and Order

From the Red Spot of Jupiter, to clumps of electromagnetic radiation in turbulent plasmas, to microstructures on the atomic scale, long-lived, spatially localized, collective excitations abound in nonlinear systems. These coherent structures show a surprising order in the midst of complex nonlinear behavior and often represent the natural modes for expressing the dynamics. Thus, for example, isolated coherent structures may dominate long-time behavior, and analysis of their interactions may explain the major aspects of the dynamical evolution. Recognition of these possibilities constitutes a fundamental change in the approach to nonlinear systems and has opened up a range of new analytical and computational techniques that yield deep insights into nonlinear natural phenomena.

Although the importance of vortices and eddies in turbulent fluid flows has been appreciated since ancient times, the critical event in the modern concept of coherent structures was the discovery in 1965 of the remarkable "soliton" behavior of localized nonlinear waves governed by the Korteweg–deVries equation, which describes waves in a shallow, narrow channel of water (e.g., a canal) and in many other physical media. Solitons represent coherent structures in the purest sense in that their form is exactly restored after temporary distortion during interactions. Surprisingly, many equations, of wide applicability, have turned out to support solitons, and a major mathematical success has been the revelation that most of these equations can be solved explicitly and systematically by a novel analytical technique known as the inverse spectral transform.

These developments have drawn upon and greatly stimulated several branches of pure mathematics, including infinite-dimensional analysis, algebraic geometry, partial differential equations, and dynamic systems theory. For instance, soliton equations have been shown to correspond to a very special subclass of those nonlinear dynamic systems that have an infinite number of independent parts. Technically, the number of parts is referred to as the number of degrees of freedom or the phase-space dimension. The special characteristic of a soliton equation is that it describes a Hamiltonian dynamic system of infinite phase-space dimension that is, in technical parlance, completely integrable. The Hamiltonian consequently possesses infinitely many independent conservation laws, which determine its behavior. The existence of individual solitons can be understood as a delicate balance between nonlinear focusing and dispersive broadening, while the invariance of solitons under interactions is a consequence of the many conservation laws.

A wide variety of soliton equations has been discovered, allowing a broad range of applications to natural phenomena. In fiber optics, Josephson transmission lines, conducting polymers and other chainlike solids, and plasma "cavitons," the prevailing mathematical models are slight modifications of soliton equations. Thus, with systematic approximations, the behavior of real physical systems can be described quite accurately. An example of potential technological significance can be drawn from nonlinear optics. In this discipline, as the name suggests, nonlinear phenomena, including self-induced transparency, optical phase conjugation, and optical bistability, are dominant. Considerable recent research has investigated the prospect of using solitons to improve long-distance communications in optical fibers. At low intensities, light pulses in optical fibers propagate linearly and tend to disperse, degrading the signal. To compensate for this and reconstruct the pulse, repeaters must be added to the fiber at regular intervals. If the light intensity is increased into the nonlinear regime, soliton pulses can be formed, the nonlinearity compensating for dispersion. In the idealized limit of no dissipative energy loss, the solitons propagate without degradation of shape; they are indeed the natural, stable, localized modes for propagation in the fiber. Further, realistic theoretical estimates suggest that a soliton-based system could have an information rate one order of magnitude greater than that of conventional linear systems. Although detailed questions of practical implementation remain (primarily costs), the prospects for using optical solitons in long-distance communication are real.

In the more general case, coherent structures interact strongly and do not necessarily maintain their form or even their separate identities for all times. Instabilities generating fluid vortices can lead to vortex pairs, and a pair may merge to form a single coherent structure equivalent to a new and larger vortex. Interactions among shock waves give rise to diffraction patterns of incident, reflected, and transmitted waves. Bubbles and droplets interact through merging and splitting. Significantly, physical examples of these more general coherent structures are nearly universal and, apart from the structures already mentioned, include elastoplastic waves and shear bands, chemical-reaction waves and nonlinear diffusion fronts, phase boundaries, and dislocations in metals. There is a deep mathematical basis to this universality. In a first approximation, these nonlinear wavelike phenomena are subject to conservation laws. In contrast to the soliton case, there are usually only a few conserved quantities (e.g., mass, energy, and momentum). Nonetheless, these few conservation laws strongly restrict the possible behavior of the system. Nonlinearity implies that the speed of a wave depends on the amplitude of the wave itself. As a result, the conservation laws lead to focusing and defocusing of waves. The defocused waves disperse, while the focused waves become coherent structures, the nonlinear modes in which the dynamics is naturally described. They may dominate the long-time behavior of the system, engage in complicated motions and interactions, or organize into complex configurations and patterns.

Fluid vortices—a classic example of which is provided by the Red Spot of Jupiter (Figure 1)—can be used to illustrate the essential role of general coherent structures in nonlinear systems. The existence and stability of the Red Spot of Jupiter have been confirmed since the seventeenth century. A more modern example is the vortex pattern formed in the wake of an airfoil. These vortices are of sufficient size and importance that they govern the allowed spacing between aircraft at landing and thus limit the efficiency of airport utilization. Similarly, the manner in which vortices are shed from the airfoil strongly affects fuel efficiency and is essential in designing high-performance aircraft. Specifically, vortices are microstructures that make up the critical turbulent boundary layer at the wing surface. More generally, an understanding of the highly nonlinear dynamics of vortices is one of the central problems of applied fluid dynamics.

Further examples of dominant coherent structures can be drawn from almost any field of the natural sciences or engineering. Chemical-reaction fronts are important in many situations and, in flame fronts and internal combustion engines, are coupled strongly to fluid modes. Concentration

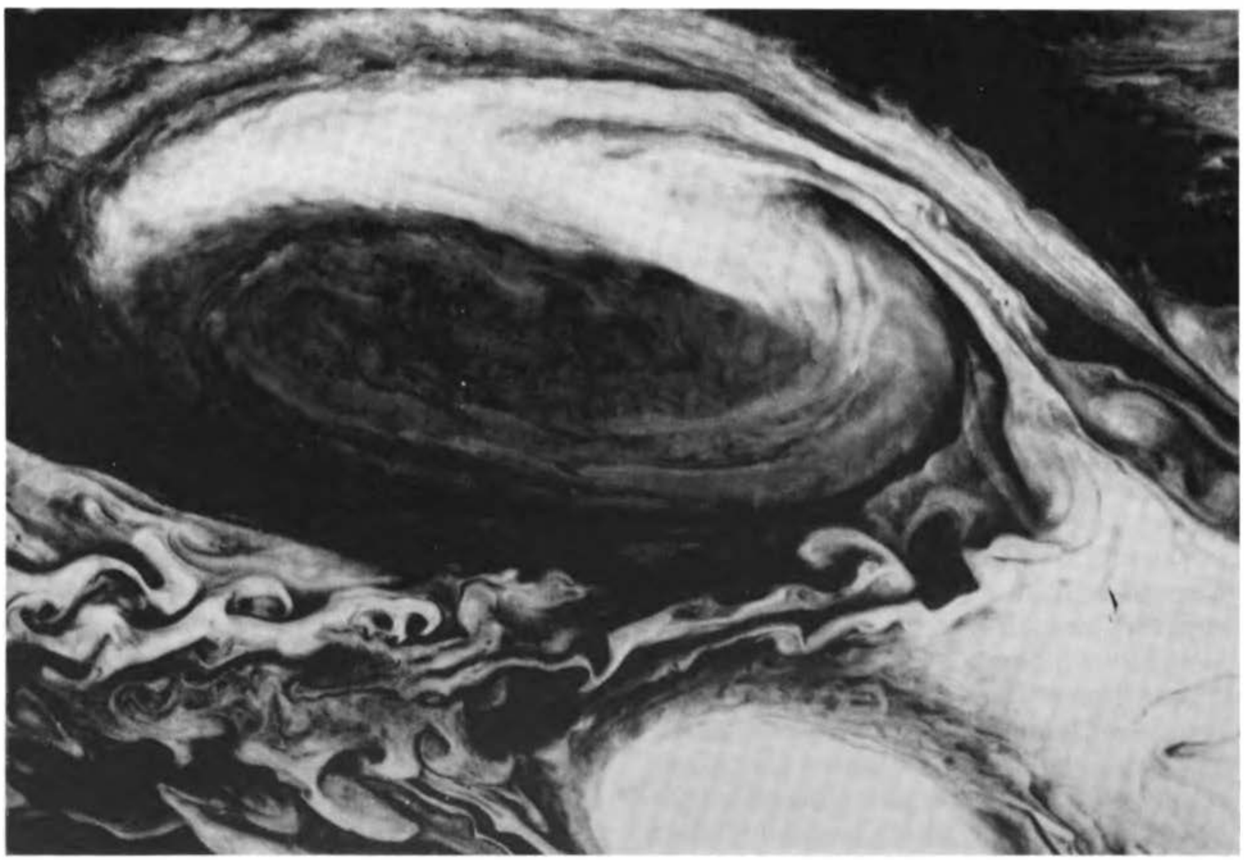

Figure 1 A close-up of the giant Red Spot of Jupiter, a coherent structure that exists in the turbulent shear flow in the Southern Hemisphere. Note the coexistence of this large vortex with smaller eddies on many different scales. Although it is not apparent from this single image, the series of time-lapse photographs taken by the Voyager spacecraft shows that the Red Spot is highly dynamic, spinning rapidly and moving westwardly at 11 km/hr. (Courtesy National Aeronautics and Space Administration, Jet Propulsion Laboratory)

fronts arise in the leaching of minerals from ore beds. Fronts between infected and uninfected individuals can be identified in the epidemiology of diseases such as rabies. In geology, elastoplastic waves are important in the slow, long-time deformation of structures. For example, salt domes are formed by a gravitational instability in which the flow of rock layers occurs on geological time scales. Understanding the development of such geological formations is important both theoretically and in the evaluation of potential oil reservoirs. Finally, at the microscopic level the nonlinear dynamics of dislocations may lead to novel effects crucial for interpreting the behavior of materials subjected to high strain rates, and transport phenomena in certain classes of quasi-one-dimensional materials may be controlled by the nonlinear coherent structures they support.

A final example with potential major technological implications is the recent identification of new types of coherent structures and interactions in wave phenomena in oil reservoirs. The essential discovery is that when the speeds of two families of nonlinear waves coincide, a type of nonlinear resonance may give rise to a surprising range of novel wave phenomena. It has recently been shown that nonlinear resonance of this type must occur in three-phase flow in oil reservoirs, and a systematic program is well under way to identify and classify all possible types of nonlinear wave interaction and to assess their importance for oil recovery methods.

Given the ubiquity and importance of coherent structures in nonlinear phenomena, it is gratifying that recent years have witnessed remarkable progress in studying them and that there is great promise for still deeper insights. Significantly, this progress has been achieved by precisely the synergy among computation, theory, and experiment that characterizes nonlinear science. In particular, experimental mathematics has been essential to the understanding of coherent structures and their interactions. Typically, the forms of the coherent structures are not immediately obvious from the underlying nonlinear equations. Hence visualizations of flow patterns and dynamics using interactive graphics will play an increasingly important role.

In summary, coherent structures reflect an essential paradigm of nonlinear science, providing a unifying concept and an associated methodology at the theoretical, computational, and experimental levels. Their importance for technological applications, as well as their inherent interest for fundamental science, guarantees their central role in all future research in this subject.

Chaos

The appearance of irregular, aperiodic, intricately detailed, unpredictable motion in deterministic systems is a truly nonlinear effect. Loosely termed chaos, it is remote from linear phenomena. Although chaotic motion is observed, the processes are strictly deterministic: sufficiently accurate knowledge of an initial state allows arbitrarily accurate predictions—but only over a limited interval of time. In particular, it is not necessary to drive a process randomly to observe motion of a stochastic character. Indeed, attempting to model "deterministically chaotic" systems as responding to random forces fails to capture their true behavior.

While the mathematical seeds had already been planted by Poincaré at the turn of the century, they have germinated only in the past three decades, with the advances in interactive computation that we have termed experimental mathematics playing an essential role. One striking recent development has been the recognition that certain chaotic motions unfold themselves with a total lack of regard for the specific mechanisms at work: objects exhibiting certain complex motions follow similar destinies independent of whether their microscopic behavior is governed by equations derived from the theory of chemical interactions, or fluids, or electromagnetism. The discovery of this universality and its application to experiments on the transition to turbulence is one of the triumphs of nonlinear science.

The field of chaotic dynamics continues to undergo explosive growth, with many advances and applications being made across a broad spectrum of disciplines, including physics, chemistry, engineering, fluid mechanics, ecology, and economics. Chaotic systems can be observed in both experimental data and numerical models. Examples include the weather, chemical systems, and beating chicken hearts. The dripping of household faucets can be chaotically irregular, while it has been argued that the satellite Hyperion of Saturn tumbles chaotically in its eccentric elliptical orbit, having no fixed axis because it is constantly kicked by the varying tidal pulls of Saturn.

Medical research has revealed that many physiological parameters vary chaotically in the healthy individual, while more regularity can be a sign of pathology. For example, the familiar pattern of the beating heart is subtly irregular under close examination, and the absence of chaotic components seems to occur in pathological conditions. Similarly, the normally chaotic oscillations of red and white blood cell densities become periodic in some leukemias and anemias. There are many similar examples including periodic catatonias and manic-depressive disorders.

Recent research suggests possible applications to realistic economic models. General equilibrium-theory models have been constructed that are chaotic, but with parameter values that do not mesh well enough with empirical studies to be persuasive. On the other hand economists, motivated by the ideas of chaotic dynamics, have developed new and powerful statistical tests for analyzing time series, which may be useful in other areas of nonlinear science.

As this brief listing suggests, deterministic chaos is essential to the understanding of many real-world nonlinear phenomena. To indicate further aspects of our present understanding, more technical detail is necessary. The concept of the phase-space dimension of a dynamic system was discussed earlier. For a complex object, this dimension is a priori quite high; for a continuous system, such as a fluid, it is in fact infinite. However, if many parts are effectively locked together, as in a coherent structure like a fluid vortex, the effective dimension is reduced, perhaps drastically. This general phenomenon is referred to as mode reduction. As the character of the system's motion changes, so will the number of reduced modes and hence the effective dimension. In the example of pipe flow quoted in the introduction, as velocity increases, the fluid motion becomes suddenly more complex. Such sudden transitions to qualitatively new motions are related to the mathematical phenomenon of bifurcations. Recent advances in the study of bifurcations provide an understanding of the mechanism leading from ordered to chaotic behavior. More specifically, transitions in the behavior of physical systems can arise through an infinite cascade of bifurcations, the best known and first isolated of which is period doubling. This period-doubling cascade is controlled by a special behavior (with certain scaling properties) just at the point of transition, which fully organizes both the orderliness prior to transition and the chaotic behavior after it. Significantly, theory shows that this behavior is correctly expressed by a very low-dimensional, mode-reduced dynamics, independent of the original phase-space dimension of the system. Even more important, the behavior is universal: whatever the system, the properties exhibited are identical. Recent experimental confirmation of these theoretical predictions in systems from convecting fluids to nonlinear electronic circuits is one of the triumphs of nonlinear science.

Once it is recognized that the original equations contain superfluous information because of mode reduction, it becomes important to deduce the actual number of effective equations—that is, the dimension of the reduced system—and then to determine the form of the equations. The first part of this program has been well implemented in the last few years by so-called phase-space reconstruction techniques. Provided that the data support a dimension of below, say, 10, that number can be extracted reliably. Indeed, ideas from thermodynamics provide a graphic depiction that can quickly illuminate some details of the nature of the excitations as well as the dimension. These methods, however, must be refined.

The second part of this program has rarely been accomplished and then only on a case-by-case basis. In some instances, assumed forms can be fit to the data. At this point an easily simulated simple set of equations completely replaces the original ones. For example, three first-order ordinary differential equations exactly replace the full fluid equations throughout a certain regime of motion. Now a real payoff accrues: the model system can easily be time-dependently forced, in contrast to an actual experimental fluid with its physically imposed exigencies, such as boundaries. This can lead to insights of profound technological importance. A recent Soviet effort has apparently succeeded by just this program in forestalling the onset of turbulence in a nozzle flow by imposing periodic stress; clearly such suppression (or enhancement) of turbulence could have many vital applications. More generally, away from transition regions, the specific forms of the mode-reduced equations may play a role. In this regard, an important and generally open problem is to establish the relation, if any, between coherent structures observed in a given motion and the reduced modes that in principle characterize the motion. In certain specific problems, notably perturbed soliton equations and models for chemical-diffusion fronts, progress has been made, but much further research is required.

To delve still deeper into current progress and to indicate what may lie ahead, it is necessary to introduce some additional terminology. For dissipative systems (e.g., those with friction) a wide class of initial motions may in the long-time limit approach some set of phase-space points, which is then called an attractor. Very commonly an attractor is a single point or a closed curve. However, sometimes the attracting set is much more irregular, and for a "strange attractor" the dimension need not even be an integer. This concept of fractional dimension, related to mathematical work begun in the 1920s, has recently become more widely appreciated through the development and application of the theory of such "fractal" objects. Knowledge of fractals is essential to understanding modern nonlinear dynamic systems theory. For example, in a deterministically chaotic system, the attracting set can be a chaotic strange attractor, on which two initially very close points begin to separate exponentially fast. This yields an exquisite sensitivity to initial conditions, for tiny initial uncertainties later produce profound ones. In general, a complicated physical system may contain several attractors, each with its own basin of attraction. A subtle further consequence of nonlinear dynamics is that the boundaries between these basins of attraction can themselves be extraordinarily complex and, in fact, fractal. These fractal basin boundaries mean that totally different long-time behavior can result from indistinguishably close initial configurations.

An illustration of these concepts is provided by weather forecasting. A chaotic dynamic model, based on a crude approximation of atmospheric fluid flow, explains why weather prediction works only for short periods of time. Since small uncertainties grow so rapidly, there is a limit on how far ahead one can predict whether it will rain on a given day, no matter how large and fast the computer that is used to forecast. At the same time, specific familiar local weather patterns—for example, summer thundershowers in the mountains—can be understood in terms of attractors in local models of weather.

Figure 2 depicts a strange attractor found in a model simulation of the behavior of an optical switch. The sequence shown reveals the persistence of the attractor's convoluted structure at successively greater magnifications. This nontrivial structure appearing on all scales correctly suggests that the object does not fill the two-dimensional surface on which it lies, but rather is a fractal with dimension between 1 (a smooth curve) and 2 (a smooth surface). In fact it has dimension 1.7. An unmistakable property of the sequence of Figure 2 is that the very small details are reminiscent of the entire object. This property is called scaling, the formal theory of which allows the construction of fine detail from crude features. Thus, a conceptually new means of describing complicated objects has emerged from these studies. The systematic classification of the strange sets that arise in low-dimensional chaotic motions remains one of the challenges of current studies in nonlinear dynamics.

The impact of deterministic chaos is only now beginning to be felt throughout science. The recognition that even simple systems can exhibit incredibly complicated behavior and that this behavior can be quantified is now widely appreciated and is being applied in many fields. Given the generality of mode reduction and the universality of certain aspects of chaos, the scientific applicability of the concepts of chaotic motion will grow significantly with each step in unraveling these matters.

Complex Configurations and Pattern Selection

When an extended nonlinear system is driven far from equilibrium, the many localized coherent structures that typically appear in it can organize into an enormous range of spatial patterns, regular or random. This process is familiar in turbulent fluid flows (note the complex pattern surrounding the Red Spot in Figure 1) in which temporal behavior is chaotic, but it also occurs in many other phenomena, ranging from mesoscale textures in metallurgy to markings on seashells. The resulting problem of complex configurations and pattern selection represents a third paradigm of nonlinearity.

At present, this paradigm is being investi-

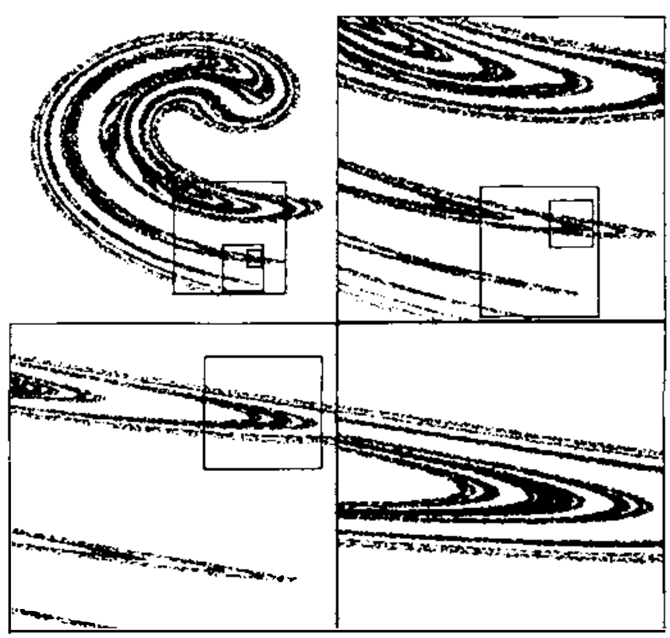

Figure 2 The trajectory traced out by the time evolution of a nonlinear dynamic system modeling the behavior of an optical switch. The complicated path never closes on itself and hence the motion never exactly repeats: the trajectory is a "strange attractor." As the three successive magnifications (top right, lower left, lower right) suggest, the intricate detail persists, in slightly modified form, on all length scales. (Courtesy Institute for Physical Science and Technology, University of Maryland)

gated on two levels. The first level is the experimental-mathematical search for complicated, anisotropic configurations that go beyond the highly symmetric patterns that have been accessible via traditional closed form, pencil-and-paper calculations. The second level is the attempt (as in various experimental studies of fluid flows) to determine how they arise dynamically. Nonlinear competitions can determine which particular pattern emerges from the bewildering array typically explored by the chaotic interaction of the individual components.

An increasingly tractable instance of pattern selection is provided by the behavior of unstable fluid interfaces, where instabilities can give rise to entrainment and to a chaotic mixing layer. There are many examples of this phenomenon. An interface separating fluids moving at different velocities is subject to shear instabilities and, through a process known as roll-up, leads to wound-up vortices along the surface. The original boundary becomes fully entangled by coherent structures (vortices) in the final state. Figure 3 illustrates the complex patterns formed by this shear instability in a case of particular technological importance that was mentioned earlier, namely, the vortices that occur in the wake of an aircraft. Recently, multiple-scale analytic techniques have been applied to derive approximate phase and amplitude equations which, in some fairly simple circumstances, can describe the evolution of these patterns. Another important instance of interfacial instability, with potential technological implications for metallurgical processes and crystal growth problems, occurs in phase transitions in supersaturated or metastable media. Here nonuniform growth of the stable phase produces fingers, known as dendrites, which compete, grow irregu-

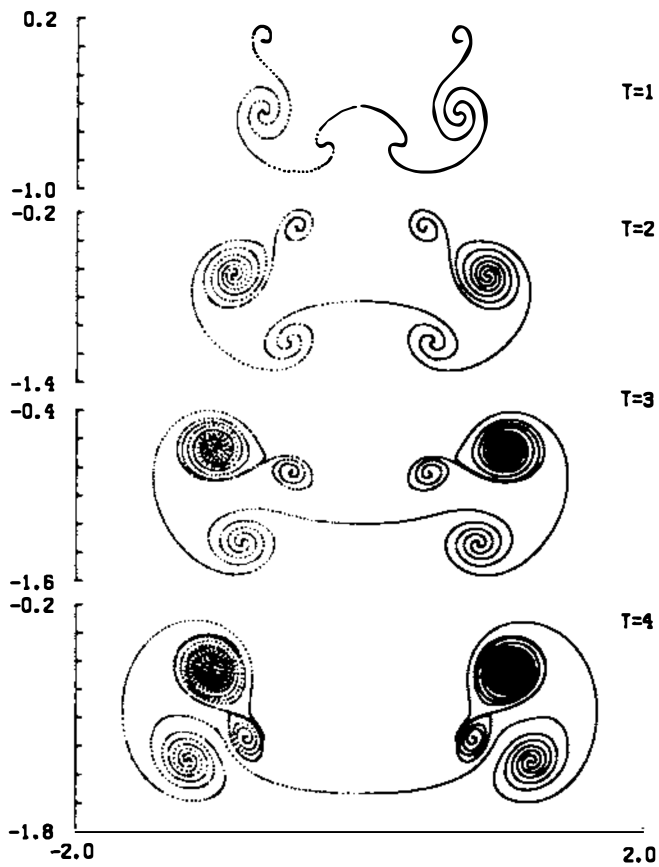

Figure 3 Results of a numerical simulation of vortex sheet model for the shear layer that forms in an aircraft wake. The aircraft is flying perpendicular to and into the plane of the figure. The wake is shown at four positions downstream from the wing's trailing edge. Computational points are drawn on the left, and an interpolating curve is drawn on the right. Initially, the vortex sheet is the straight line segment $-1 \leq x \leq 1$, $y = 0$, corresponding to the wing's trailing edge. Single-branched wingtip vortices form at the sheet's end points. Double-branched spirals form further inboard due to the effects of deployed flaps and the fuselage. The vortices' roll-up and interaction are strongly nonlinear. (Courtesy Robert Krasny, Courant Institute, "Computation of Vortex Sheet Roll-up in the Trefftz Plane," *Journal of Fluid Mechanics*, in press)

larly, and produce still more complex configurations and patterns, such as found in snowflakes.

To illustrate this interfacial instability in a technologically vital context, we note that the displacement of oil by water in an oil reservoir sometimes leads to an unstable interface. This Saffman–Taylor instability and the resulting viscous fingering are critical to efficient oil recovery; consequently geologists, petroleum engineers, theoretical physicists, applied mathematicians, computer scientists, and experts from other disciplines have focused intensely on this problem. The specific technical issue is that almost half of the oil deposited in limestone or other porous media is typically unrecovered during ordinary oil extraction because it remains stuck in the pores. To recover this oil, a technique called water flooding is used, in which water is injected into the field to force out the oil. The viscous fingering phenomenon often means that nothing is recovered but the injected water, slightly polluted by traces of oil. Clearly a full understanding of this effect and ways to control it are of great importance.

Recent work of a combined experimental, theoretical, and computational nature has led to a semiquantitative understanding of several specific aspects of this problem. First, laboratory experiments have established, under controlled conditions, the nature of the complex configurations that arise in certain parameter ranges of viscous fingering. Figure 4 shows an image of one such configuration in a flat, effectively two-dimensional cylindrical cell. This branched, complex configuration is a fractal. To estimate the fractal dimension, imagine covering the image of the viscous fingering with square cells of side l and calculating, for a given l, the number of cells required to cover the object entirely. As the length of the side l goes to zero, the number of cells required grows as $1/l^d$, where d is the fractal dimension. Performing this calulation for the viscous finger in Figure 3 gives $d = 1.70 \pm 0.05$.

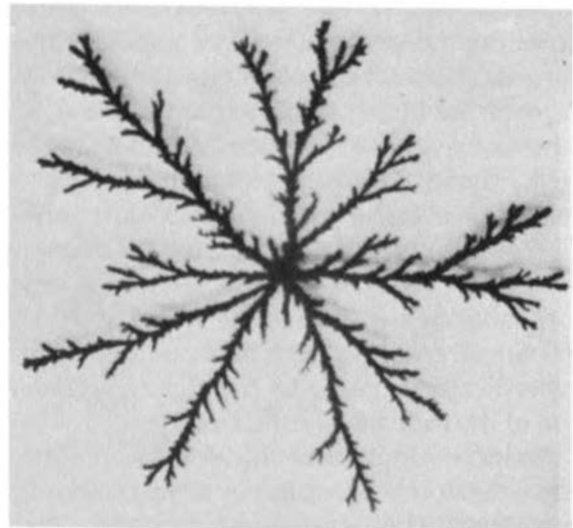

Figure 4 A viscous fingering effect observed when water (black) is forced through a circular inlet in the center of a flat, cylindrical Hele Shaw cell originally filled with high-viscosity fluid. The pattern has a reproducible numerical value, measured by several methods, including the one described in the text, for the fractal dimension of 1.70 ± 0.05. (Courtesy G. Daccord, J. Nittmann, and H. E. Stanley, *Physical Review Letters*, 56:336, 1986)

Hence, this object possesses a fractional dimension closer to that of a plane surface ($d = 2$) than to that of a straight line ($d = 1$). Second, in both the viscous fingering and dendritic growth problems, analytic studies identified an intriguing nonuniqueness to certain features of the pattern selection in the simplest models. Additional physical effects, such as the inclusion of surface tension, were then shown to remove at least in part this nonuniqueness. Although the resulting pattern selection problem has not yet been fully solved, exciting recent progress includes an analytic treatment of effects beyond all orders in perturbation theory. Third, computational simulations have suggested a number of different models and approaches to the problem. Much further research is required, but an accurate and practical procedure for modeling realistic problems now seems possible.

Fractals play an essential role in several other areas of practical application of the paradigm of complex configurations. In an effort to make ceramics tougher—that is, able to contain a few large flaws without failing—much interest has focused on fractal crack patterns. These arise primarily from two sources: the voids that develop during the sintering process, and the materials harder than ceramics—for example, diamond—normally used to machine them. Instead of moving straight along the surface of the ceramic in a planar path, the propagating crack takes a more tortuous route if it interacts with some microscopic feature of the ceramic—for instance, a second material added to the primary constituent to enhance its toughness. Since the crack will expend more energy in moving out of the plane than it would in propagating unimpeded, it will do less damage to the overall ceramic. Interestingly, the fractal dimension of the crack appears to be related to the fracture toughness of the ceramic. Electron micrographs of cracks put into silica-nitride ceramics, one of the new high-performance materials being considered for high-temperature, high-stress applications such as engine parts, were used to determine the fractal dimension of the cracks. The higher the fractal dimension, the tougher the ceramic.

In certain surface processes, such as roughening, fractal patterns also are observed. For these surface fractals, the lower limit of the fractal dimension is 2, characteristic of a perfectly smooth surface, and the upper limit is 3, a surface so rough and convoluted that it has become a three-dimensional object. The complex configuration of these fractal surfaces can be very important, particularly for processes such as chemical catalysis, where in many cases the higher the fractal dimension of the surface, the greater the catalytic effect.

Many further interesting and relevant illustrations of complex configurations and patterns can be found in nonlinear phenomena from virtually all disciplines. In the biological sphere, the richness of pattern formation is particularly evident, from tigers' stripes to human digits. Certain features of the problem of morphogenesis can already be understood from plausible nonlinear mathematical models. The development of convection rolls during the transition to turbulence in a fluid heated from below has been extensively studied experimentally and successfully modeled using a combination of computational and analytic techniques. On the other hand, understanding the pattern formation seen in fully developed, three-dimensional turbulence remains one of the most challenging problems of modern science.

Finally, a fascinating class of discrete nonlinear dynamic systems, known as cellular automata, exhibit remarkable pattern formation properties and are currently being subjected to rigorous mathematical scrutiny. At a more speculative level, these highly discrete systems have suggested novel computational algorithms—often called lattice-gas models—for solving certain continuum nonlinear partial differential equations. These algorithms may prove especially valuable for computers based on massively parallel architectures, although both their virtues and their limitations require further study.

This section has focused only on those paradigms of nonlinear science that have been most thoroughly developed and explored, but there are clear indications of many other emerging paradigms. Two are particularly exciting. The concept of adaptation refers to nonlinear dynamic systems that adapt or evolve in response to changes in their environment. Here one crucial aspect is that the nonlinear equations describing the system can themselves be modified on a slow time scale. Among the initial tentative applications of this concept are models for the human immune system and for autocatalytic networks of proteins. A related but somewhat distinct concept is often termed connectionism and reflects the appealing idea that many simple structures connected together can exhibit complex behavior collectively because of the connections. Recent specific instances of this approach include mathematical models called neural networks. Typically only loosely patterned after true neurological systems, these models are remarkable in their promise for being able to learn behavior from experience. The concepts of familiar dynamic systems—such as basins of attraction and coexistence of multiple stable patterns—have already played a crucial role in interpreting the behavior of these more complex systems.

ISSUES, RECOMMENDATIONS, AND CONCLUSIONS

INTERNATIONAL STANDING OF U.S. WORK

Researchers in the United States have played a significant but not dominant role in the recent achievements of nonlinear science. In particular, they have made uniquely important contributions to the experimental mathematics aspect and also provided substantial insights into the experimental and analytic aspects. Nonetheless, the reception of this work within the U.S. scientific community has not been comparable to that seen elsewhere, especially in the Soviet Union and France. In both those countries longstanding traditions in mathematical physics and applied mathematics have helped to stimulate interest in nonlinear phenomena, and the high level of importance that many leading scientists attach to this enterprise is readily noticed in their public comments and in their contributions to the field, particularly in the analytic and experimental areas. Since connections with the active French groups are fairly well established, special emphasis should be placed on strengthening ties with the Soviet efforts, for the United States stands to gain considerably from increased interaction with Russian researchers in this area. From the Soviet perspective, the U.S. leadership in experimental mathematics provides a natural quid pro quo.

In many other countries, work of the highest caliber has been accomplished, and in some there has already been an institutional response, with centers focusing on nonlinear problems established at several universities. It is worth noting that the European Science Foundation recently hosted a meeting on nonlinear science at which the creation of a major European institute on the subject was discussed.

In summary, U.S. research in nonlinear science is of high quality and is widely recognized internationally. Ironically, recognition of this subject within the American scientific community is less developed. Indeed, the interdisciplinary character of the field appears to be problematic for U.S. institutions and agencies. In particular, typical U.S. universities, having departmental structures fairly rigidly defined along traditional disciplines, appear to lack the flexibility to respond adequately to this subject. Students, while interested, seem worried (for good reason) about future positions. In general, while there is strong individual motivation, one hardly senses a more communal national one.

PERSONNEL

Given the interdisciplinary character of nonlinear science, we expect that most of the successful long-term research efforts in this subject will typically result from experts in widely different fields pooling their intellectual resources. Accordingly, agencies and academic administrators should consider both the support of loosely coordinated research networks and the creation of more focused centers in this area. At the same time, however, since many outstanding contributions can be traced to scientists working essentially alone, it is vital to foster and reward high-quality individual research. In particular, the needs of younger scientists eager to become involved but anxious about the lack of a disciplinary base must be confronted. Increased support for junior faculty, postdoctoral fellows, and advanced predoctoral students working on nonlinear problems from an interdisciplinary perspective is clearly necessary. But it is also necessary to ensure the continued input of experts from the traditional disciplines so that studies of nonlinear phenomena address significant and relevant problems.

Several changes in standard university curricula should be contemplated to bring the excitement of this field to still younger students and to train a cadre of potential researchers. With closed-form analyses of interesting nonlinear phenomena infrequent and inadequate, an increased comprehension of the schemes of analysis and calculation is required and the general level of mathematical and computer literacy of all natural-science students should be raised. Coursework in differential equations should include more modern dynamic-systems ideas; calculus should more regularly be followed by deeper courses in analysis; mechanics courses should stress the limitations of perturbation theory and the omnipresence of nonintegrability. A course in numerical methods that leads to intuitive algorithm development based on deep understanding could prepare a researcher to perform meaningful experimental mathematics. Greater exposure should be given to topics such as modern asymptotic and multiple-scale methods, phase and amplitude equations derived from fluids, specific examples of solvable soliton equations, and methods of numerical analysis. Fluids and continuum mechanics should be given higher profiles in physics curricula, and introductory courses in the qualitative phenomenology of chaos and solitons and other nonlinear waves should be generally available. Further, summer institutes focused on specific aspects of nonlinear science should be supported.

Facilities

With respect to facilities, one of the major administrative opportunities is the creation of research centers of excellence, either in institutions with preexisting efforts or in response to new proposals. Crucial to this approach is the provision of block or umbrella funding for the interdisciplinary research, rather than balkanization of the research by dividing support among specific disciplines. Again, however, we stress that grants supporting fundamental research by outstanding individuals in this area should be available. These grants should have one or more natural homes within the organizational structures of the federal funding agencies, and special care should be taken that they are not endangered by their interdisciplinary content. On a much grander scale, perhaps one of the proposed National Science Foundation science and technology centers could be devoted to this subject; given its interdisciplinary nature and broad applicability, this may be an attractive prospect.

The central role of computation in nonlinear science clearly suggests that increased access to supercomputers—at the National Science Foundation centers, the National Center for Atmospheric Research, the National Aeronautics and Space Administration, the Department of Energy laboratories, and elsewhere—is vital for continued progress. In particular, interagency cooperation in enhancing supercomputer access is essential. But apart from supercomputer access, individual researchers must be given high-powered scientific work stations with interactive graphics capabilities and a more truly interactive environment. In this matter theorists doing experimental mathematics really do need to be regarded as experimentalists and supported accordingly with the appropriate hardware. Although funding agency awareness of this situation has grown dramatically over the past five years, still greater support is needed.

Conclusions

As a consequence of its fundamental intellectual appeal and potential technological applications, nonlinear science is currently experiencing a phase of very rapid growth. During this critical period, science administrators in government, education, and industry can play essential roles in further stimulating and guiding this growth. In particular, they can marshal the resources necessary to respond to the challenging research opportunities. In any effort to guide this research, however, it is imperative that nonlinear science be recognized for what it is: an inherently interdisciplinary effort not suited to confinement within any single conventional discipline or department. Hence the administrative structure of research in this area is likely to remain more fragile, and in greater need of attention, than traditional subjects with their natural constituencies. Accompanying this fragility, however, is a remarkable breadth of application and the potential to influence both our basic understanding of the world and our daily life.