Development of LRFD Specifications for Horizontally Curved Steel Girder Bridges

DETAILS

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NATIONAL COOPERATIVE HIGHWAY RESEARCH PROGRAM

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- Dr. Chai H. Yoo, Auburn University: Literature search
- Dr. Andrzej S. Nowak, University of Nebraska: Statistical calibration
- Mr. Mike Grubb, Bridge Software Development International, Ltd.: Preparation of the specifications articles.

The research team would like to acknowledge the contributions of many of Modjeski and Masters's staff. Particularly acknowledged are the contributions of Mr. Christopher Smith and Mr. Kevin Johns to the preparation of two curved girders design examples, which are available online at http://www.transportation.org/sites/bridges/docs/Box%20Girder.pdf and http://www.transportation.org/sites/bridges/docs/I-Girder.pdf.

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Dr. Dennis Mertz, University of Delaware, reviewed and commented on the recommended specification articles. His contributions to the work are greatly appreciated.

FOREWORD

By David B. Beal
Staff Officer
Transportation Research Board

This report contains the findings of research performed to develop design specifications for horizontally curved steel girder bridges. The developed specifications have been adopted by AASHTO and are included in the 2005 Interims to the third edition of the *AASHTO LRFD Bridge Design Specifications*. Detailed examples showing the application of the specifications to the design of an I-girder bridge and a box-girder bridge are available from AASHTO. The material in this report will be of immediate interest to bridge designers.

In the late 1980s and early 1990s, AASHTO and the FHWA recognized the need to address problems with design and construction of horizontally curved steel girder highway bridges. AASHTO, through the NCHRP, embarked on an overhaul of the AASHTO *Guide Specifications for Horizontally Curved Highway Bridges*. NCHRP Project 12-38, "Improved Design Specifications for Horizontally Curved Steel Girder Highway Bridges," provided recommended load factor design (LFD) and construction specifications based on the state of the art that addressed many of the problems associated with the design and construction of these structures.

While the LFD specifications were under development, the FHWA and others began conducting significant research that enhanced the understanding of steel bridge behavior in general and horizontally curved bridges in particular. The FHWA research included tests on a full-scale I-girder bridge, and knowledge of the moment and shear capacities of horizontally curved I-girder bridges resulted from this research. Analytical work, also funded by the FHWA, resulted in the unification of the design equations for straight and curved steel girders.

NCHRP Project 12-52 was performed by Modjeski & Masters, Inc., with the assistance of Mike Grubb, Andrzej Nowak, Don White, and Chai Yoo. The report fully documents the effort leading to the specifications and contains an extensive compilation of abstracts of horizontally curved girder bridge research reports. Appendix C, "Calibration of LRFD Design Specifications for Steel Curved Girder Bridges," and Appendix D, "Comparison of Curved Steel I-Girder Bridge Design Specifications," can be downloaded from http://trb.org/news/blurb_detail.asp?id=5965 or from the Project 12-52 website at trb.org/nchrp.

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SUMMARY

Development of LRFD Specifications for Horizontally Curved Steel Girder Bridges

AASHTO's *Guide Specifications for Horizontally Curved Highway Bridges* (hereafter referred to as the "Guide Specifications") was first published in 1980. The specifications were based on work conducted in the late 1960s and early 1970s by a group of researchers called the "Consortium of University Research Teams" (CURT). The research work resulted in guidance on the analysis of curved bridges and equations for determining the strength and checking the stability of curved girders. An updated version of the Guide Specifications was published in 1993. The 1980 Guide Specifications was written in the allowable stress design (ASD) format. The 1993 Guide Specifications was written in both the ASD and the load factor design (LFD) format. As a result of the work on the National Cooperative Highway Research Program (NCHRP) 12-38 project, the Guide Specifications were updated again and the updated version, written in the LFD format, was published in 2003.

In 1999, the NCHRP 12-52 project was initiated to develop design provisions for curved bridges in the AASHTO load and resistance factor design (LRFD) format. These provisions were intended to be incorporated into the specifications to extend the specifications' coverage to curved bridges. Statistically calibrating the curved bridge design provisions was required to ensure smooth merging of these provisions into the then-existing straight girder design provisions.

The original organization of the NCHRP 12-52 project called for a two-phase approach. Phase I was intended to produce curved bridge design provisions that were based on the information available at that time. These specifications were intended to be revised in Phase II based on the results of the then-ongoing research on curved bridges. This research was funded by the Federal Highway Administration (FHWA). Several universities collaborated with the FHWA in conducting this research.

Phase I of the NCHRP 12-52 project produced curved bridge design provisions as planned. It also produced two design examples, one of a box-girder bridge and the other of an I-girder bridge. However, at that time it became clear that the FHWA-sponsored research would produce a new set of design provisions that would be applicable to both straight and curved bridges and that would have some terms of the equations "dropping out" when applied to straight bridges. The new set of provisions was considered to be a significant improvement toward streamlining the design provisions. It was decided not to publish the design specifications developed in Phase I of the project and to develop a new set of specifications and design examples based on the results of the FHWA-sponsored research in Phase II. These provisions were approved by ballot of the AASHTO Highway Subcommittee on Bridges and Structures (HSCOBS) in 2003 and 2004 for straight girders and curved girders, respectively. The straight girder provisions were published in the third edition of AASHTO LRFD specifications in 2004. The curved girder provisions were published in the 2006 interim to the AASHTO LRFD specifications.

In addition to the recommended specifications that were subsequently adopted by AASHTO, the NCHRP 12-52 project resulted in the following:

- **The statistical calibration of the load and resistance factors for curved bridges.** This calibration indicated that the factors developed for straight girders are applicable to curved girders.
- **The comparison of resistance analysis conducted using the AASHTO Guide Specifications for curved bridges to those conducted using the new LRFD-based design provisions.** Twenty-one existing bridges provided by several state DOTs and 11 simulated bridges were used in this comparison. The comparison indicated that member proportions will not be significantly altered in unanticipated ways and that anticipated changes manifested themselves in the example bridges.
- **The updating of the I-girder and box-girder bridge design examples.** These examples, originally produced in the NCHRP 12-38 project, were updated in Phase I of the project and then updated again in Phase II based on the new design provisions developed in Phase II of the project.

The curved bridge design provisions, the statistical calibration work, and the comparison between the existing designs and those conducted using the new provisions are included in this report. The two design examples are available on the AASHTO website at http://www.transportation.org/sites/bridges/docs/Box%20Girder.pdf and http://www.transportation.org/sites/bridges/docs/I-Girder.pdf.

CHAPTER 1

Introduction and Research Approach

1.1 Introduction

The history of specification development related to horizontally curved girder bridges spans more than 30 years. During the late 1960s and early 1970s, a group of researchers called the "Consortium of University Research Teams" (CURT) developed guidance on the analysis of curved girder bridges and characterizations of the strength and stability of curved girders. This work lead to software products and the design provisions that became codified in the 1980 AASHTO *Guide Specifications for Horizontally Curved Highway Bridges* (hereafter referred to as the "Guide Specifications") (*1*). These specifications were initially produced in the allowable stress design (ASD) format. In 1993, an updated version of the 1980 Guide Specifications was released (*2*). The 1993 Guide Specifications was written in both allowable stress design (ASD) and load factor design (LFD) format.

Recognizing the need to update the technology in these earlier design specifications, a major research effort was initiated by the FHWA that involved both experimental and analytic investigations of curved steel bridges. The experimental work undertaken in the FHWA's Turner-Fairbank Highway Research Laboratory involved a three-girder, single-span structure. The experimental program has been widely reported and has been ongoing for over a decade (*3, 4, 5*). The FHWA's experimental program was augmented by tests of single girders at several universities and by large-scale finite element analysis with nonlinear materials, plate out-of-flatness, and nonlinear geometries (*6*). Comparisons were made between the results of tests on the curved girder test frame and analytic results.

One of the pivotal features of the finite element analysis was its ability to accurately represent the capacities due to local flange buckling and lateral torsional buckling, both in terms of the resistance of the cross-section and in terms of the deflected shape. The agreement between the analytical and experimental results was excellent and made it possible to augment a limited number of experimental tests with hundreds of analytic investigations.

An update to the 1993 Guide Specifications was prepared under NCHRP Project 12-38 (*7, 8*) and was published in 2003 (*9*). The state of the art of curved girder specifications and a review of the intervening 10 years' advances in understanding the resistance of curved sections resulted in a more modern specification, which was written in the LFD format. The provisions for I-girders retained some of the features introduced in the 1980 Guide Specifications, but made significant advances in the recognition of the need to directly interrelate the lateral flange bending stress, or "warping" stress, with the vertical bending stress. This interrelation was done by subtracting part or all of the lateral flange bending stress from the resistance of the cross-section. The need for additional stud connectors in composite sections due to the radial component of shear between the deck and girders was also recognized.

Recognizing the need to include curved girder bridges in the *AASHTO LRFD Bridge Design Specifications* (*10*), AASHTO asked the NCHRP to initiate Project 12-52 to develop recommended state-of-the-art design specifications for horizontally curved steel bridges. The recommended design specifications were required to be statistically calibrated and to be written in the load and resistance factor design (LRFD) format. The recommended specifications were also required to incorporate research results accumulated over the years, including the results of the FHWA research. To ensure a smooth transition to the new provisions, the design example for an I-girder bridge and the design example for a box-girder bridge (which were both developed under the NCHRP 12-38 project) would have to be updated to reflect the application of the new provisions. In addition, to further investigate the effect of the new design provisions on the required girder sections, design comparisons were conducted for a large number of existing and simulated bridges.

1.2 Research Objective

The request for proposals (RFP) for the NCHRP 12-52 project stated the research objective as follows:

> The objective of this research is to prepare specifications for the design and construction of horizontally curved steel girder bridges (for both I- and box-girders) in a calibrated load and resistance factor design (LRFD) format that can be recommended to AASHTO for adoption. The specifications shall be based on the *Recommended Specifications for Horizontally Curved Steel Highway Bridges* developed under NCHRP Project 12-38, which are in a load factor design format, supplemented by the results of the FHWA large-scale curved I-girder tests as they become available.

1.3 Scope of the Study

The scope of the study was generally determined by the tasks identified in the RFP as the tasks anticipated to be encompassed by the research. The task description, copied from the RFP, is as follows.

PHASE I

Task 1. Review and evaluate pertinent domestic and international research, on the basis of applicability, conclusiveness of findings, and usefulness for the development of LRFD Specifications for horizontally curved steel girder bridges. The focus of this Task shall be primarily on the review and evaluation of all pertinent material from NCHRP Project 12-38 and the FHWA steel curved girder bridge project.

Task 2. Review the background of the calibration procedures used in NCHRP Project 12-33, "Development of a Comprehensive Bridge Specification and Commentary." Using these procedures, the predictor equations from the NCHRP Project 12-38 specifications, any available test data, and the target reliability index and load factors from the AASHTO *LRFD Bridge Design Specifications* compute resistance factors.

Task 3. Apply the available knowledge from Task 1, the results of Task 2, and the design and construction specifications for horizontally curved steel girder highway bridges developed in NCHRP Project 12-38 to develop a set of draft changes to the AASHTO *LRFD Bridge Design Specifications* and the AASHTO *LRFD Bridge Construction Specifications.* Fold these curved girder provisions into the existing provisions for tangent girder bridges as efficiently as possible. Identify significant differences between the provisions for curved and tangent girders and any gaps in available knowledge for consideration in Phase II. Identify and incorporate provisions in the NCHRP Project 12-38 specification, not specific to curved girders, that should be in the LRFD Specifications and identify provisions (whether or not specific to curved girders) to be deleted.

Task 4. Use the draft specifications developed in Task 3 to evaluate critical sections of at least 10 existing representative structures that capture variations in major geometric parameters (e.g., span, curvature, skew, beam type, location and type of transverse components, and connection details). These same structures should be evaluated with the draft specifications from NCHRP Project 12-38 for comparison.

Task 5. Provide an Interim Report documenting the results of Tasks 1 through 4. A meeting with the Project Panel will be held approximately one month after delivery of this report, which must be submitted before July 1, 2000.

Task 6 and Task 7. Under Task 6, based on the comparisons in Task 4 and on review comments from the Project Panel, revise the draft LRFD design and construction specifications developed in Task 3. Using the revised draft LRFD Specifications, rework the two design examples produced under the NCHRP 12-38 project. Provide a summary of the differences between the existing design examples and those prepared using the proposed LRFD Specifications. Under Task 7, submit the material developed in Task 6 for Panel review recognizing that the recommended specification must be submitted before December 1, 2000.

Task 8. Prepare a final report that documents the entire Phase I research effort and includes the recommended specifications, revised based on the Task 7 reviews. The design examples shall also be included in an appendix.

PHASE II

Task 9 and Task 10. Under Task 9, develop a work plan to resolve differences and to fill in the gaps in knowledge identified in Task 3 of Phase I and submit for Panel review. Items to be considered include, but are not limited to, the effects of bearings and staged construction, and additional design conditions (sublimit states) and load combinations needed for curved girders. The latest information from the FHWA curved girder project should be evaluated in preparing this work plan. This work plan should prioritize activities needed and should include cost and time estimates for completion of each activity. The contractor should anticipate meeting with the Project Panel to discuss the proposed work plan. It is anticipated that not all the activities will be funded. The Project Panel will identify the activities to be funded. Under Task 10, complete the activities approved in Phase II, Task 9, for implementation.

Task 11. Provide further proposed revisions to the draft specification based on the results of Phase II, Task 10. Expand the Phase I calibration to include a greater range of design parameters and to include results from the FHWA project, as they become available. Evaluate critical sections of the 10 structures used in Phase I, Task 4, and at least 30 additional existing representative structures. Use the load factors of the AASHTO *LRFD Bridge Design Specifications* to compute the resistance factors corresponding to target reliabilities from 2.0 to 4.0 by intervals of 0.25.

Task 12. Develop commentary on the selection of the level of analysis necessary for specific design situations and evaluate the need for adjustment factors to be applied when less sophisticated analysis procedures are used. This material should be based on the findings from modeling at least three curved bridges (two I-girder and one box-girder). Analyses should use at least three levels of sophistication: V-load or M/R, grid analysis, and finite element three-dimensional analysis.

Task 13. Submit the recommended specifications and the two design examples modified as necessary. *Note: This material must be submitted before December 1, 2001.*

Task 14. Submit a final report that describes the entire research effort in Phases I and II.

1.4 Research Approach

Phase I was intended to incorporate into the Guide Specifications the basic provisions developed in the NCHRP 12-38 project, which were published as the 2003 Guide Specifications (*9*), with only minor updating. It was anticipated that the FHWA-funded research would result in a new set of resistance equations, and Phase II was intended to incorporate these equations into the specifications.

In Phase I, the following work was completed:

- The literature search conducted under NCHRP Project 12-38 (*11*) was updated. The updated literature search (*12*) is included as Appendix A.
- Initial calibration studies were conducted that determined that the loads and load factors currently in the LRFD specifications could be retained and used with curved systems (*13*). This work is included in Appendix C (which is available online at http://trb.org/news/blurb_detail.asp?id=5965).
- Specification provisions based on the recommendations of the NCHRP 12-38 project were revised to fit within the then-existing edition of the AASHTO LRFD specifications (*14*).
- The two design examples originally prepared under the NCHRP 12-38 project were updated (*15, 16*), one for a curved I-girder bridge and one for a curved box-beam bridge.

The original scope for Project 12-52 envisioned publishing the recommended specifications developed under Phase I of the project for use until such time as the FHWA curved girder project produced sufficient results to write an updated specification under Phase II of the project. In 2001, the NCHRP panel directing Project 12-52 became satisfied that the FHWA project was moving quickly enough that within about 2 years it would be possible to develop provisions based on that work. This work was anticipated to result in updated resistance equations. Rather than publishing design provisions and then superseding them in approximately 2 years, the panel decided not to publish the Phase I provisions and instead work toward a schedule that envisioned the adoption of Phase II provisions in 2004, so that it would be available in ample time for the 2007 FHWA deadline for use of AASHTO LRFD specifications on all federally funded projects.

The decision not to publish Phase I design provisions deemed the work conducted under Phase I, with the exception of the initial calibration work, obsolete. Therefore, the literature search (Appendix A) and the initial calibration work (Appendix C) are the only work from Phase I covered in this report. The remaining work was part of Phase II.

CHAPTER 2

Findings

2.1 Literature Search

The scope and intent of the literature search was to continue the exhaustive literature survey conducted as Phase I of the FHWA-funded project, "Curved Steel Bridge Research Project" (DTFH61-93-C-00136) and published as Interim Report I: Synthesis (*17*). Although the report date of the synthesis is December 1994, the cut-off date of the literature survey activity is assumed to be June 1993. Therefore, the literature collected and included in the updated literature search (*12*) are those published after June 1993 and up to January 2000, the dates the original and updated searches were conducted, respectively. A copy of the literature search report is included as Appendix A.

2.2 Design Specifications

The results of the FHWA-sponsored research indicated that resistance formulations applicable to both straight and curved systems could be developed. This led to a decision by the technical panel directing the work of the NCHRP 12-52 project to archive the curved girder design provisions developed under Phase I of the project and to develop a new set of design provisions based on the FHWA-sponsored work. The latter set of provisions was determined to be applicable to both straight and curved bridges. These provisions were incorporated into the AASHTO LRFD specification by ballot of the AASHTO Highway Subcommittee on Bridges and Structures (HSCOBS) in 2003 for straight girders. These provisions were revised by ballot of HSCOBS in 2004 to incorporate the curved girder requirements. The straight girder provisions were published in the third edition of the AASHTO LRFD specifications in 2004, and the curved bridge requirements were published in AASHTO's 2006 interim specifications.

The theoretical developments behind the design provisions balloted by HSCOBS in 2004 for curved bridges were done by others as documented in the references. The role of the NCHRP 12-52 project was to implement that work in the AASHTO LRFD specifications.

2.3 Calibration

2.3.1 Scope

The objective of this section is to document the calibration of the design code for steel curved girder bridges, consistent with the AASHTO LRFD specifications. It is assumed that load factors for curved girders remain the same as for straight girders. Therefore, the focus is on the determination of resistance factors. The calibration study was conducted by the University of Michigan (UMich) and was supervised by Professor Andrzej Nowak. Following is a brief description of the calibration study. More detailed information is included in Appendix C, which is available online at http://trb.org/news/blurb_detail.asp?id=5965.

The relationship between the resistance factor, ϕ, and reliability index, β, is a complex function that includes nominal (design) values of load and resistance, and statistical parameters of load and resistance such as bias factors, λ, and coefficients of variation, V. The bias factor is defined as the ratio of mean-to-nominal value, and coefficient of variation is the ratio of standard-deviation-to-mean value. The statistical parameters were derived for straight girders (*18, 19*). An important part of this calibration is to determine values of these parameters for curved girders.

The statistical load model developed for straight bridges (*19*) includes the maximum expected effects of dead load and live load and dynamic load. The maximum truck weights corresponding to various periods of time up to 75 years were determined by extrapolation of truck survey results. The multiple-truck presence in a lane and in adjacent lanes was considered based on field observations and by Monte Carlo simulations. The statistical parameters of truck weights, including extrapolations for longer periods of time, do not depend on bridge curvature.

However, the live load effect in a girder (moment and shear) depends on load distribution. In particular, girder distribution factor (GDF) represents the fraction of the lane moment (or shear) per girder. In calibration of the code for straight girders, it was assumed that the bias factor for GDF was 1.0. This means that, on average, the code-specified GDF is equal to the actual GDF. Because of geometry, the load distribution strongly depends on the degree of curvature. Therefore, an important task in this study is calculation of the bias factors and coefficients of variation for the load distribution method used in the design. It is assumed that the design analysis is performed using the commercial program developed by Bridge Software Development International, Ltd. (BSDI). To determine the statistical parameters for load distribution, the results of design analysis are compared with field measurements and with results of an advanced finite element analysis.

The bridge resistance model depends on the statistical parameters of materials and geometry. Therefore, the latest available material test data are reviewed and used in derivation of the bias factors and coefficients of variation for moment and shear capacity of curved girders.

The statistical parameters of load, load distribution, and resistance are derived for selected structures. The structural and reliability analysis is performed for three representative structures:

- Bridge A: Minnesota Bridge No. 27998
- Bridge B: Minnesota Bridge No. 62705
- Bridge C: Fore River Bridge, Portland Maine

The calibration work involved the development of load and resistance models, analysis of reliability, selection of the target reliability index, and calculation of load and resistance factors. Three bridges were analyzed using the finite element method (FEM) as part of the calibration. The analytical boundary conditions in the FEM analysis were calibrated using the actual field test data for one bridge (referred to as Bridge A). The objective of FEM analysis for Bridges B and C was to validate the statistical model developed for Bridge A.

2.3.2 Study Bridges

The basic parameters for the three bridges considered in this study are as follows:

- **Bridge A:**
  - Location: Minnesota
  - Length: 295 feet
  - Number of spans: 2 continuous
  - Radius of curvature: 285 feet
  - Number of girders: 4 spaced at 9 feet
  - Roadway width: 30 feet (two lanes)
- **Bridge B:**
  - Location: Minnesota
  - Length: ~105 feet
  - Number of spans: 1 simply supported
  - Radius of curvature: 106 feet
  - Number of girders: 4 spaced at 8 feet, 4 inches
  - Roadway width: 28 feet (two lanes)
- **Bridge C:**
  - Location: Portland, Maine
  - Name: Fore River Bridge No. 2
  - Length: 273 feet
  - Number of spans: 3 continuous
  - Radius of curvature: 175 feet
  - Number of girders: 4 (spaced at 8 feet)
  - Roadway width: 28 feet (two lanes)

2.3.3 Calibration Procedure

The calibration procedure used in the development of the AASHTO LRFD specifications (*18, 19*) was used to calibrate the curved bridge design provisions. The major steps of the calibration include the following:

1. **Selection of representative structures:** Existing and planned structures were considered. The data on the bridges were provided by various state DOTs. Several parameters—such as span, curvature, number of girders, and spacing between the girders—were considered. From the population of curved girder steel bridges, three representative structures were selected to be used as a reference in this study. Bridge A, which was field tested by the University of Minnesota (UMinn), was included in this set. The bridge provided an opportunity to compare analytical and experimental (test) results.
2. **Identification of the load and resistance parameters and formulation of the limit state functions:** The load parameters include dead load, live load, dynamic load, and load effects (including bending, torsion, and shear). It is important to determine the absolute value of load effects in various combinations. The behavior of a girder was assumed to follow the trends observed during testing of curved girder bridges (*20*).
3. **Development of load and resistance models:** This step involved gathering the available statistical data and calculating missing and/or additional parameters by simulations. For Bridge A, the work on resistance models included the analysis of results of testing conducted by UMinn and advanced FEM computations to develop a reference for comparisons. For Bridges B and C, the FEM analysis was performed by the UMich, and the results were compared

with the analysis carried out by Modjeski and Masters using software marketed by BSDI.

4. **Selection of the reliability analysis procedure:** There are numerical procedures available, but they were developed for specific limit state functions. Thus, it was most efficient to develop a customized procedure for this project. The reliability analysis is performed to determine the reliability indices. As in the original calibration, the procedure was performed for a 75-year service life and involved extrapolation.
5. **Reliability analysis for the selected representative structures:** The reliability indices were calculated for the limit state functions identified in Step 2. The reliability index spectrum was reviewed to identify the trends and discrepancies.
6. **Selection of the target reliability index:** The selected target reliability index is consistent with the AASHTO LRFD specifications for straight bridges, as calculated in the Calibration Report (*19*).
7. **Calculation of resistance factors:** It is assumed that load factors remain the same as in the AASHTO LRFD specifications. However, the load factors for some of the combinations that are specific for curved girders may require some special load combination factors. The resistance factors are determined by trial and error. Various possible resistance factors were tried (each rounded to the nearest 0.05). For each set of factors, the reliability indices were calculated, and the optimum resistance factors correspond to the closest fit to the target reliability index.
8. **Final selection of the resistance factors:** This step involves the verification of the calculated factors by additional reliability analysis, check of special cases (e.g., combinations with dominating dead load), and selection of load and resistance factors consistent with the rest of AASHTO LRFD specifications. Simplicity of the specifications was an important consideration.

2.3.4 Load Models

2.3.4.1 Load Components

The major load components of highway bridges are dead load, live load (both static and dynamic), environmental loads (e.g., temperature, wind, and earthquake) and other loads (e.g., collision and emergency braking). Load components are random variables. Their variation is described by the cumulative distribution function (CDF) and/or by parameters such as the mean value, bias factor (i.e., mean-to-nominal ratio), and coefficient of variation. The relationship among various load parameters is described in terms of the coefficients of correlation.

The basic load combination for highway bridges is a simultaneous occurrence of dead load, live load, and dynamic load. Therefore, these three load components are considered in the present study. It is assumed that the economic service life for newly constructed bridges is 75 years. The extreme values of load are extrapolated from the available database. Nominal (i.e., design) values of load components are calculated according to AASHTO's *Standard Specifications for Highway Bridges* (hereafter referred to as "AASHTO Standard") (*21*) and AASHTO LRFD specifications (*10*).

2.3.4.2 Dead Load

Dead load is the gravity load due to self weight of the structural and nonstructural elements permanently connected to a bridge. Because of different degrees of variation, it is convenient to consider three components of dead load: weight of factory-made elements (i.e., steel, precast concrete members), weight of cast-in-place concrete members, and weight of the wearing surface (i.e., asphalt). All components of dead load are treated as normal random variables. The statistical parameters were derived in conjunction with the development of the *Ontario Highway Bridge Design Code* (OHBDC) (*22*) and AASHTO LRFD specifications (*10*), and they are listed in Table 1. The bias factors are taken as used in the previous bridge code calibration work; however, the coefficients of variation are increased to include human error as recommended (*23*).

In case of steel curved girder bridges, the critical load combination can occur during construction, prior to composite action with the concrete deck slab. The parameters of dead load during construction are also taken as given in Table 1.

The calculation of dead load effects (i.e., moments and shear forces) for curved girders (using girder distribution factors) involves a considerable degree of variation. In this study, the statistical parameters were determined using the field measurements performed by UMinn, finite element analysis performed by UMich, and calculations performed by UMinn using grillage model for Bridge A. The cumulative distribution function of the stress ratio obtained by UMich and UMinn is plotted on normal probability paper in Figure 1. The average value is 0.95, and the coefficient of variation, V, is equal to 0.12. Therefore, in this study, the bias factor for dead load effect, λ, is equal to 1.00, and the coefficient of variation, V, is equal to 0.15.

2.3.4.3 Live Load

Live load covers a range of forces produced by vehicles moving on a bridge. The effect of live load depends on many

Table 1. Statistical parameters of dead load.

Category of component	Bias factor	Coefficient of variation
Factory-made (precast)	1.03	0.08
Cast-in-place	1.05	0.10
Asphalt surface	1.00*	0.25

* Mean thickness equal to 3.5 in. (90 mm).

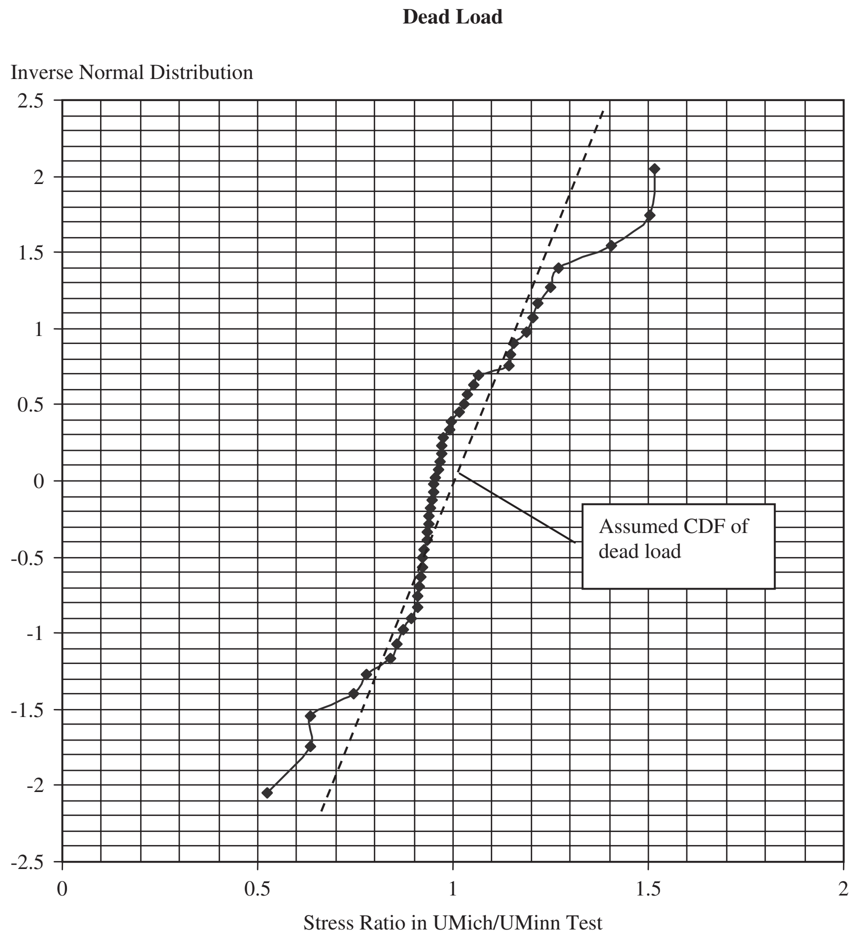


Figure 1. Cumulative distribution function of the stress ratio due to dead load determined for Bridge A.

parameters, including the span length, truck weight, axle loads, axle configuration, position of the vehicle on the bridge (transverse and longitudinal), number of vehicles on the bridge (multiple presence), girder spacing, and stiffness of structural members (slab and girders). The effect of these parameters is considered separately. The live load model was originally determined during the development of the AASHTO LRFD specifications.

For curved girder bridges, the spans are mostly 60 to 150 feet (18 to 45 meters). For this span range, the bias factor is 1.25 to 1.35, with the larger value corresponding to shorter spans. The mean value is the 75-year maximum midspan moment, and nominal value is the AASHTO-specified HL-93 load effect in one lane.

For two lanes, the maximum 75-year live load is the effect of two trucks side by side, with each truck being equal to the heaviest truck predicted in any 2-month period (*19, 24*). The ratio of the mean maximum 2-month moment and the mean maximum 75-year moment is 0.85. Therefore, the bias factor for two lanes is a product of 1.25 to 1.35 and 0.85, resulting in 1.05 to 1.15.

The girder distribution factors (GDFs) used in the analyses were determined using the straight GDF included in the AASHTO LRFD specifications. Both field measurements and FEM analysis were considered when the bias factor for GDFs for curved girders was determined. The CDF of the ratio of stress obtained by UMich and UMinn is shown in Figure 2 on the normal probability paper.

For the most loaded components (i.e., girders), the bias factor for GDF is 0.75, with the coefficient of variation being 0.12. The overall bias factor for live load moment in a curved girder is 0.80 to 0.85, and the coefficient of variation is 0.215, including dynamic load.

2.3.4.4 Dynamic Load

The dynamic load model was previously developed and was verified by field measurements (*25, 26, 27*). Dynamic load is a function of three major parameters: road surface roughness, bridge dynamics (i.e., frequency of vibration), and vehicle dynamics (i.e., suspension system). It was observed that dynamic strain and deflection are almost constant and

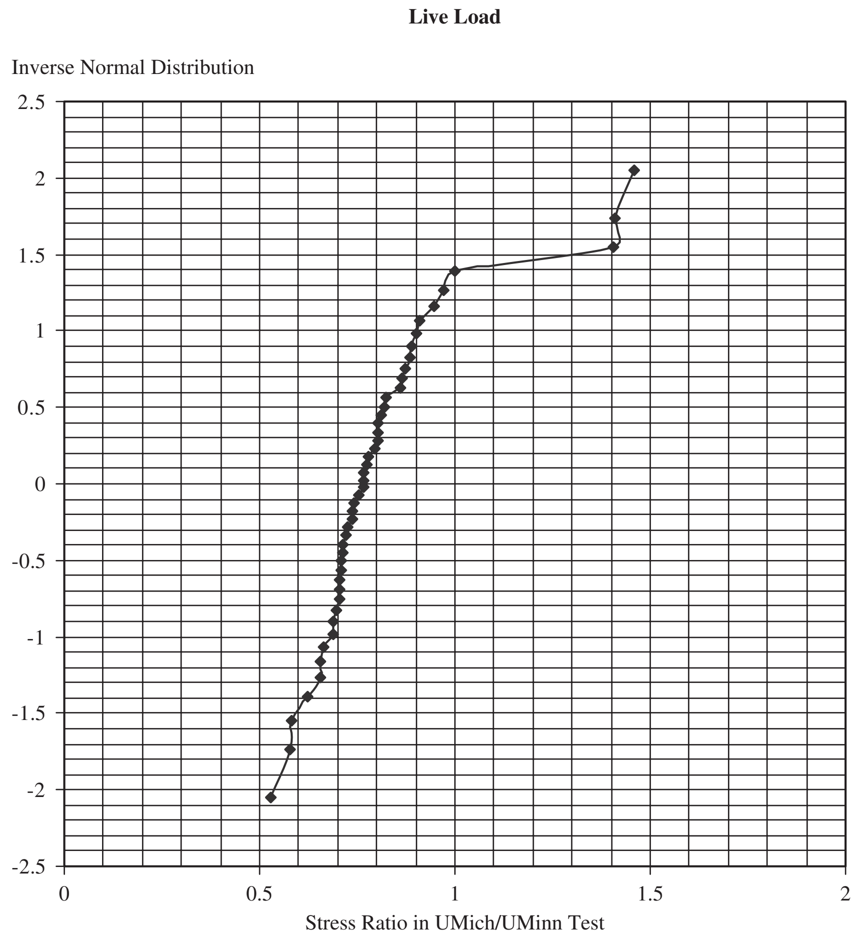


Figure 2. Cumulative distribution function of the stress ratio due to live load determined for Bridge A.

do not depend on truck weight. Therefore, the dynamic load, as a fraction of live load, decreases for heavier trucks.

For the maximum 75-year values, the corresponding dynamic load factor (DLF) does not exceed 0.15 of live load for a single truck and 0.10 of live load for two trucks side by side. The coefficient of variation of dynamic load is about 0.80. The results of the simulations indicate that DLF values are almost equally dependent on road surface roughness, bridge dynamics, and vehicle dynamics. The actual contribution of these three parameters varies from site to site and is very difficult to predict. Therefore, it is recommended to specify DLF as a constant percentage of live load.

2.3.4.5 Load Ratios

In the reliability analysis, the absolute values of load components are not important. However, the relative values of load components affect the statistical parameters of the total load effect. Therefore, load components are expressed in terms of relative values (i.e., load ratios). The ratios of load components are determined for the selected bridges. The calculations were performed for Bridge A, as listed in Table 2. For example, D1 equal to 4 and D2 equal to 9.5 means that the ratio of D1/D2 is equal to 4/9.5. The load ratios are different during construction, and these ratios are also shown in Table 2. Similarly, load ratios are calculated for Bridges B and C, and these load ratios are included in Appendix C. Bridges A, B,

Table 2. Load ratios considered for Bridge A.

Stage	Spans	D_1*	D_2*	D_3*	LL*	IL*
operation	150 ft	4	9.5	0	4.5	0.75
construction	150 ft	4	7.5	0	0.25	0

* D1, D2, D3, LL and IL denote weight of factory-made components, weight of cast-in-place components, weight of wearing surface, live load allowance, and dynamic load allowance, respectively.

and C are considered representative for the current trends in curved girder bridge design.

2.3.5 Resistance Models

For straight girders, the resistance (i.e., load-carrying capacity), R, is considered a product of three factors, M, F, and P:

$$R = M\,F\,P$$

where M represents the material properties (i.e., strength), F represents the dimensions or fabrication (i.e., area, cross-section properties such as section modulus, and moment of inertia), and P represents the professional factor (i.e., analysis). In curved girder bridges, there is an additional factor, S, representing system behavior. The statistical parameters of S are based on field tests and FEM analysis. The statistical parameters for materials, fabrication, and the professional factor are all detailed in Appendix C, which is available online at http://trb.org/news/blurb_detail.asp?id=5965.

2.3.6 Calibration Results

It was assumed that the load factors remain the same as in the AASHTO LRFD specifications. The objective of the calibration was to determine the optimum value of the resistance factor for curved girder bridges. The major difference between a straight bridge and a curved bridge is in the girder distribution factors. This is the only major factor that can affect the reliability.

The reliability analysis was performed to establish the relationship between the resistance factor and reliability index for the three considered bridges. The results are shown in Figures 3, 4, and 5 for Bridges A, B, and C, respectively. They are also summarized in Table 3.

The results indicate that a resistance factor ϕ equal to 1.0, which is the same resistance factor previously specified for straight bridges for the design cases considered, resulted in an adequate reliability factor (3.70 to 4.51). The details of the analysis and tests of the calibration work are detailed in Appendix C, which is available online at http://trb.org/news/blurb_detail.asp?id=5965.

2.4 Design Comparisons

2.4.1 Objective

The purpose of the design comparisons was to perform a comparison among the three most recent curved girder design specifications: the 1993 Guide Specifications (*2*), the 2003 Guide Specifications (*9*), and the 2006 Interim AASHTO LRFD specifications that resulted from NCHRP Project 12-52.

2.4.2 Application of the NCHRP 12-50 Process

The core concepts outlined in NCHRP Project 12-50 were used to formulate a subdomain of curved girder bridges

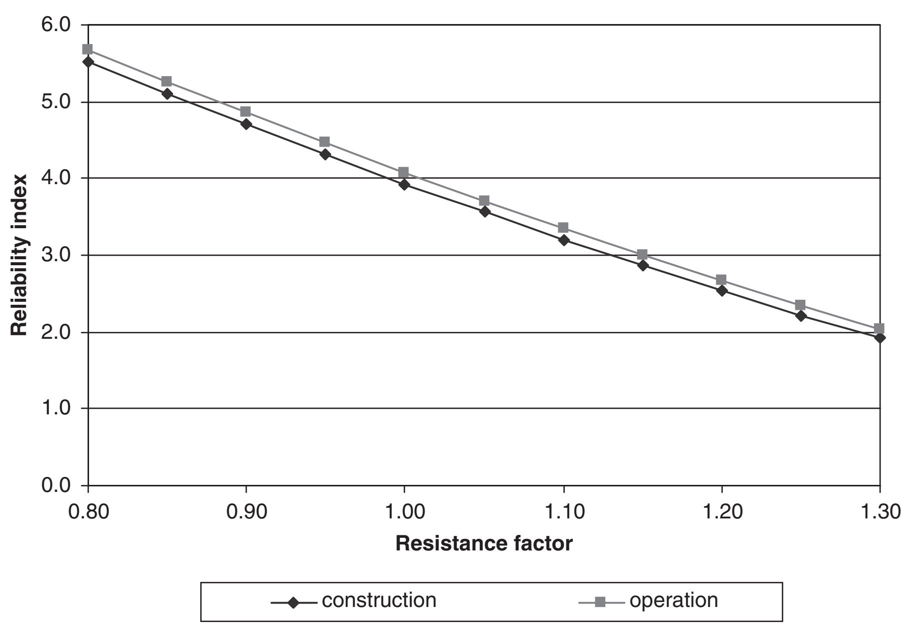


Figure 3. Reliability index as a function of resistance factor for Bridge A.

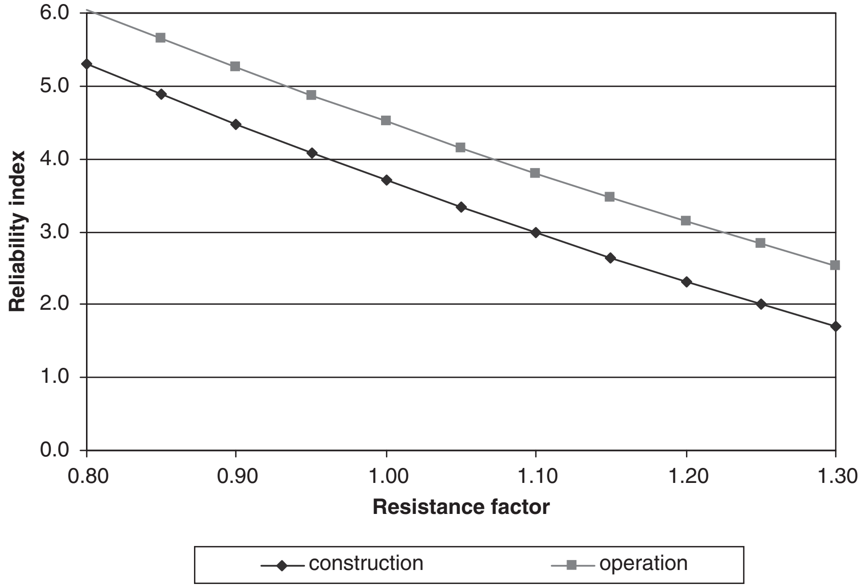


Figure 4. Reliability index as a function of resistance factor for Bridge B.

to be compared as part of the NCHRP 12-52 research. The NCHRP 12-50 methodology relates to the organization and manipulation of bridge data enough that the methodology can be effectively used as a tool for software testing. One of the other NCHRP 12-50 "use cases" that was documented in the 12-50 report was the use of similar techniques for the purpose of comparing the results obtained from different design specifications (or proposed changes to an existing set of design specifications). This particular application of the NCHRP 12-50 methodology fits in well with the curved girder bridge design comparisons outlined herein.

The NCHRP 12-50 research team used spreadsheets and databases as tools for generating, storing, and manipulating the input and output data for the various bridge subdomains

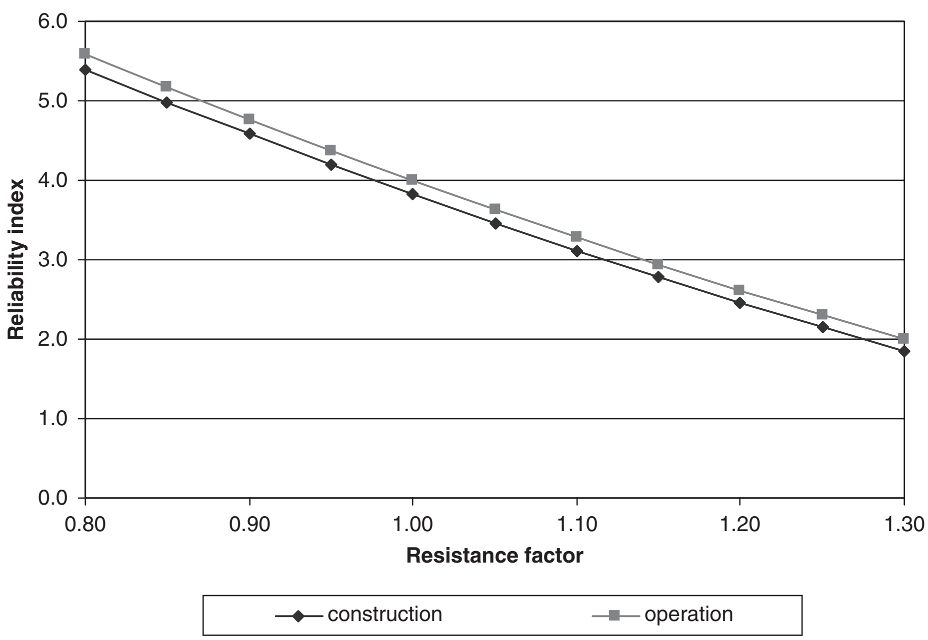


Figure 5. Reliability index as a function of resistance factor for Bridge C.

Table 3. Reliability indices for various values of ϕ for the considered bridges.

Resistance factor, ϕ	Reliability index, β					
	Construction			Operation		
	Bridge A	Bridge B	Bridge C	Bridge A	Bridge B	Bridge C
0.80	5.52	5.31	5.40	5.67	6.04	5.59
0.85	5.10	4.89	4.98	5.26	5.64	5.17
0.90	4.70	4.48	4.58	4.85	5.25	4.77
0.95	4.31	4.08	4.19	4.46	4.87	4.38
1.00	3.93	3.70	3.82	4.08	4.51	4.00
1.05	3.56	3.34	3.46	3.71	4.15	3.64
1.10	3.21	2.98	3.11	3.35	3.80	3.28
1.15	2.87	2.65	2.78	3.00	3.47	2.94
1.20	2.54	2.32	2.46	2.67	3.15	2.61
1.25	2.22	2.01	2.15	2.35	2.83	2.30
1.30	1.92	1.71	1.86	2.03	2.53	1.99
1.35	1.63	1.42	1.58	1.73	2.24	1.70
1.40	1.35	1.14	1.31	1.44	1.96	1.42
1.45	1.08	0.88	1.05	1.17	1.69	1.15
1.50	0.82	0.63	0.80	0.90	1.43	0.89

that were developed under that project. The large volume of data that needed to be manipulated necessitated the use of a large database. In the case of the NCHRP 12-52 research, the amount of data generated was comparatively modest, and it was convenient to use spreadsheets exclusively to accommodate the input/output and comparative data for this project. Although the NCHRP 12-50 project involved subdomains of hundreds of computer-generated bridges, the majority of the bridges that were used as a basis for comparison in the NCHRP 12-52 research were existing bridges obtained from various state DOTs. These bridges were supplemented with variations to several additional bridges to form a set of bridges termed "simulated bridges."

A strict application of the NCHRP 12-50 method would require the assignment of database keys for the results of interest. However, since the data manipulation was limited to spreadsheets and the NCHRP 12-52 database is not being actively maintained, this part of the process was not carried out (the assignment of the database keys would have to be coordinated with the database administrator to avoid conflicts with any existing identifiers). Nonetheless, the input and output data that were generated as part of the NCHRP 12-52 research are tabulated and organized in such a way that does not preclude the addition of this information to an actively maintained NCHRP 12-50 database at a later time.

2.4.3 Methodology

Sample information from 32 bridges was collected from various agencies and compiled by Modjeski and Masters, Inc. Bridges 1 through 21 were submitted by state DOTs and/or design agencies and represent real, in-service bridges using typical modern-day construction. Bridges 22 through 32, the "simulated bridges," are examples that have been modified from existing structures for analysis purposes. The full report for this portion of the project, which contains additional information and further examination of the specifications, is outlined in Appendix D, which is available online at http://trb.org/news/blurb_detail.asp?id=5965.

Basic geometric information about the bridges in the sample (e.g., span length and curvature radius) is included in Table 4. Several of the key aspects of the bridges in this sample are as follows:

- Most of the bridges are multispan, continuous girder bridges, and 4 of the 32 bridges are single span.
- For the greatest portion of the bridges, the critical positive moment section occurred in an end span and the critical negative moment section occurred over the first interior pier.
- The critical shear section generally occurred at or near a pier.
- The vast majority of the bridges used Grade 50 steel exclusively; only two had hybrid sections, both occurring in the negative moment region. Four bridges used Grade 36 steel throughout.
- Two of the bridges had haunched girders.
- Twenty of the 32 bridges in the sample were transversely stiffened, and none had longitudinal stiffeners.
- Among the existing bridges, the longest span of the bridges in the sample was 190 feet, and the minimum radius was 239 feet.

Table 4. General bridge information.

Bridge Number	Submitted by	Location	Type	No. of Girders	Width of Bridge (approx)	Avg. Span (approx)	Unbraced Length (approx)	Radius								Depth of Web
								Girder 1/A	Girder 2/B	Girder 3/C	Girder 4/D	Girder 5/E	Girder 6/F	Girder 7/G	Girder 8/H	
1	PENN DOT (Hard Copy)	Mifflin County (PA) L.R. 470	Three-Span Continuous Composite Plate Girder	5	38 ft	85 ft	16.5 ft	554.25 ft	562.09 ft	569.92 ft	577.75 ft	585.58 ft				48 in
2	PENN DOT (Hard Copy)	Centre County (PA) S.R. 6026 CO5	Three-Span Continuous Composite Plate Girder	4	34 ft	130 ft	17.0 ft	936.31 ft	945.06 ft	953.81 ft	962.56 ft					68 in
3	PENN DOT (Hard Copy)	Chester & Montgomery Counties (PA) S.R. 0202 SEC. 404	One-Span Composite Plate Girder	6	36 ft	150 ft	10.5 ft	293.94 ft	287.81 ft	281.69 ft	275.56 ft	269.44 ft	263.31 ft			68 in
4	Wyoming DOT (Hard Copy)	UPRR Separation STA 7+084.075 Sweetwater County (WY)	Three-Span Continuous Composite Plate Girder	6	40 ft	90 ft	17.0 ft	2608.27 ft	2614.83 ft	2621.39 ft	2627.95 ft	2634.51 ft	2641.08 ft			44 in
5	Wyoming DOT (Hard Copy)	Bridge Over Tongue River Sheridan County (WY)	Three-Span Continuous Composite Wide Flange Girder	4	30 ft	45 ft	12.0 ft	405.51 ft	397.64 ft	389.76 ft	381.89 ft					33 in (W 840 x 176)
6	Wyoming DOT (Hard Copy)	Bridge Over Gunbarrel Creek Park County (WY)	Simple-Span Composite Haunched Plate Girder	5	42 ft	130 ft	14.0 ft	NA	NA	NA	NA	NA				haunched 87 in to 61 in
7	Wyoming DOT (Hard Copy)	Bridge Over North Fork Shoshone River Park County (WY)	Three-Span Continuous Composite Haunched Plate Girder	5	42 ft	165 ft	7.0 ft	1261.81 ft	1270.66 ft	1279.52 ft	1288.39 ft	1297.24 ft				haunched 94 in to 55 in
8	Illinois DOT (Hard Copy)	Illinois Route 96 Over Burton Creek Adams County (IL)	Three-Span Continuous Composite Wide Flange Girder	6	34 ft	90 ft	20.0 ft	1408.79 ft	1403.02 ft	1397.24 ft	1391.47 ft	1385.70 ft	1379.92 ft			36 in (W 920 x 271)
9	Illinois DOT (Hard Copy)	Illinois Route 3 Over Sexton Creek Alexander County (IL)	Three-Span Continuous Composite Plate Girder	5	44 ft	135 ft	22.5 ft	1891.53 ft	1900.69 ft	1909.86 ft	1919.03 ft	1928.19 ft				54 in
10	Illinois DOT (Hard Copy)	Illinois Route 408 Over Napoleon Hollow Draw STA 579+04 Pike County (IL)	Three-Span Continuous Composite Plate Girder	5	44 ft	90 ft	24.5 ft	3850.22 ft	3841.72 ft	3833.22 ft	3824.72 ft	3816.22 ft				42 in
11	Illinois DOT (Hard Copy)	Bowman Ave. Over F.A.I Route 74 Vermilion County (IL)	Two-Span Continuous Composite Plate Girder	8	66 ft	90 ft	12.5 ft	530.87 ft	522.37 ft	513.87 ft	505.37 ft	496.87 ft	488.37 ft	479.87 ft	471.37 ft	34 in
12	HDR Eng./Iowa DOT (Hard Copy)	US 75 Over Ramp 11000 Woodbury County (IA)	Five-Span Continuous Composite Plate Girder	4	44 ft	170 ft	25.0 ft	1181.36 ft	1181.36 ft	1181.36 ft	1181.36 ft					84 in

13	PENN DOT (Hard Copy)	Wabash Tunnel HOV Facility Bridge No. 29 Wabash HOV Ramp Allegheny County (PA)	Two-Span Continuous Plate Girders, One Simple Span Plate Girder, One Simple Span Precast Box Beam	4	36 ft	110 ft	21.0 ft	239.00 ft	229.67 ft	220.33 ft	211.00 ft					54 in
14	TNDOT (Electronic)	Interstate 65 North Bound Ramp "B" Davidson County (TN)	Three-Span Continuous Composite Plate Girder	5	44 ft	185 ft	20.0 ft	1524.89 ft	1533.89 ft	1542.89 ft	1551.89 ft	1560.89 ft				63 in
15	TNDOT (Electronic)	Ramp "D" Over Interstate 40 Davidson County (TN)	Three-Span Continuous Composite Plate Girder	5	44 ft	175 ft	24.0 ft	368.14 ft	377.14 ft	386.14 ft	395.14 ft	404.14 ft				60 in
16	TNDOT (Electronic)	Ramp "M" Over State Route 1 Rutherford County (TN)	Two-Span Continuous Composite Plate Girders	3	32 ft	170 ft	10.0 ft	497.71 ft	486.46 ft	475.21 ft						72 in
17	NCDOT (Hard Copy)	Slater Road Over I-540 Wake-Durham Counties (NC)	Three-Span Continuous Plate Girder	4	38 ft	115 ft	20.0 ft	966.70 ft	976.87 ft	987.04 ft	997.21 ft					74.8 in
18	NCDOT (Electronic)	Fox Road Over I-540 Wake County (NC)	Two-Span Continuous Plate Girder	4?	unknown	120 ft	17.0 ft	2280.44 ft	2280.44 ft	2280.44 ft	2280.44 ft					72 in
19	NYDOT (Hard Copy)	NYS Route 30 Over Schoharie Creek Schoharie County (NY)	Four-Span Continuous Composite Plate Girder	4	30 ft	140 ft	23.0 ft	807.80 ft	816.07 ft	824.35 ft	832.62 ft					56 in
20	Modjeski and Masters, Inc.	Lincoln Highway (SR 3070) in Chester County, PA crossing four sets of Amtrak rails {9}	One-Span Continuous Plate Girder	6	50 ft	150 ft	17.5 ft	520.13 ft	528.38 ft	536.63 ft	544.88 ft	553.13 ft	561.38 ft			varies 62 in to 84.5 in
21	NYSDOT	Cold Springs Road over Erie Barge Canal	Three-Span Continuous Plate Girder	5	36 ft	105 ft	17.5 ft	1894.33 ft	1902.17 ft	1910.00 ft	1917.83 ft	1925.67 ft				51 in
22	BSDI	Example Problem	Three-Span Continuous Plate Girder	4	44 ft	150 ft	27.0 ft	NA								69 in
23	Modjeski and Masters, Inc.	Harper's Ferry Bridge, WV - Simulated Bridge	Three-Span Continuous Plate Girder	5	52 ft	190 ft	20.0 ft	1122.89 ft	1134.04 ft	1145.19 ft	1156.34 ft	1167.49 ft				85 in
24	Modjeski and Masters, Inc.	Fore River Bridges, Portland, ME Beach Street Ramp - Simulated Bridge	Three-Span Continuous Plate Girder	4	32 ft	90 ft	14.0 ft	169.00 ft	177.00 ft	185.00 ft	193.00 ft					48 in
25	Modjeski and Masters, Inc.	Fore River Bridges, Portland, ME Main Span Unit Between Piers 95 and 135 - Simulated Bridge	Four-Span Continuous Plate Girder	6	86 ft	180 ft	23.5 ft	1180.80 ft	1195.89 ft	1210.97 ft	1226.05 ft	1241.14 ft	1256.22 ft			82 in

(continued on next page)

Table 4. General bridge information (Continued).

Bridge Number	Submitted by	Location	Type	No. of Girders	Width of Bridge (approx)	Avg. Span (approx)	Unbraced Length (approx)	Radius								Depth of Web
								Girder 1/A	Girder 2/B	Girder 3/C	Girder 4/D	Girder 5/E	Girder 6/F	Girder 7/G	Girder 8/H	
26	Modjeski and Masters, Inc.	Washington State Bridge No. 530/120 - Simulated Bridge	Three-Span Continuous Plate Girder	4	46 ft	150 ft	14.5 ft	492.00 ft	504.00 ft	516.00 ft	528.00 ft					81 in
27	Modjeski and Masters, Inc.	Washington State SR405/SR167 - Simulated Bridge	Four-Span Continuous Plate Girder	3	28 ft	125 ft	20.0 ft	515.42 ft	524.93 ft	534.44 ft						54 in
28	Modjeski and Masters, Inc.	Washington State SR405 - Simulated Bridge	Four-Span Continuous Plate Girder	3	32 ft	170 ft	15.0 ft	346.00 ft	357.00 ft	368.00 ft						82 in
29	Modjeski and Masters, Inc.	Missouri Bridge A5658 - Unit A1 - Simulated Bridge	Three-Span Continuous Plate Girder	4	30 ft	65 ft	10.5 ft	145.75 ft	153.25 ft	160.75 ft	168.25 ft					48 in
30	Modjeski and Masters, Inc.	Missouri Bridge A5682 - Unit 4 - Simulated Bridge	Four-Span Continuous Plate Girder	4	32 ft	70 ft	15.5 ft	248.98 ft	257.80 ft	266.63 ft	275.45 ft					47 in
31	Modjeski and Masters, Inc.	Minnesota Bridge No. 62707 - Simulated Bridge	Two-Span Continuous Plate Girder	5	32 ft	130 ft	11.0 ft	163.25 ft	169.63 ft	176.00 ft	182.38 ft	188.75 ft				50 in
32	Modjeski and Masters, Inc.	Minnesota Bridge No. 62705 - Simulated Bridge	One-Span Composite Plate Girder	4	32 ft	105 ft	12.0 ft	97.50 ft	105.83 ft	110.00 ft	114.17 ft	122.50 ft				64 in

- Among the simulated bridges, the minimum radius was approximately 120 feet.
- The webs varied in depth from 33 inches to 94 inches.
- Lateral bending stresses were not submitted for some of the bridges in the study. For one of these bridges, the original design ignored the lateral stresses. Another bridge consisted of straight girders with a curved deck. The lateral stresses of the other bridges were estimated using the V-load method.

For each bridge, the critical positive flexure, critical negative flexure, and critical shear locations were either submitted by the agency or determined by Modjeski and Masters, Inc., from the documentation submitted.

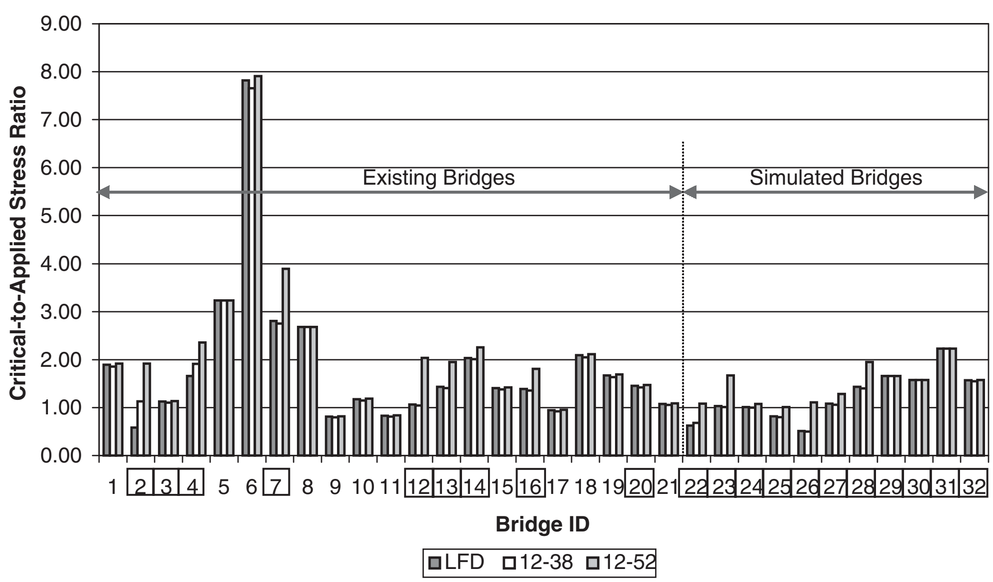
The critical-to-applied stress ratios according to each specification were plotted for each bridge. Submitted load effects (e.g., moments and shears) were assumed to be service loads, and the applied stress calculations used the load factors for the LRFD Strength I combination as a basis for comparison. An attempt to remove the effects of the load factors was made by also plotting the critical stress calculated by each specification normalized to the critical stress calculated by the 1993 Guide Specifications. These comparisons of normalized stress are the primary focus of the analyses outlined in this section. Additional analyses, which are outlined in Appendix D (available online at http://trb.org/news/blurb_detail.asp?id=5965), were performed to compare the LRFD Service II load provisions with the corresponding provisions in both the 1993 Guide Specifications and the 2003 Guide Specifications. It should be noted that when a reference to an equation number is made in this section (i.e., Section 2.4), the equation number is that used in the specifications.

2.4.4 Shear Design

Appendix D contains a series of flowcharts that outline the shear design protocols and variables according to each of the three specifications—the 1993 Guide Specifications, the 2003 Guide Specifications, and the 12-52 recommended specifications (which have subsequently been adopted by AASHTO and published in the 2006 interim specifications). The reader is encouraged to consult the appropriate specification for clarification and further information regarding the design protocols. In general, the shear design protocols for the three analysis specifications are very similar. This is true for both transversely stiffened and unstiffened members. The main differences among the three specifications are as follows:

- The maximum transverse stiffener spacing has been progressively increased from *D,* the depth of the web, in the 1993 Guide Specifications to 3*D* in the 12-52 recommended specifications.
- The 12-52 recommended specifications allow for the consideration of the additional post-buckling strength from tension-field action in the shear critical stress calculations.

Figure 6 shows the shear critical-to-applied stress ratio for the sample of bridges according to the three specifications, while



Note: Stiffened bridges are denoted with a box around the number identifier.

Figure 6. Shear critical-to-applied stress ratio.

Figure 7 shows the shear critical stress normalized to that calculated using the 1993 Guide Specifications. In these figures, as in the shear-related figures in Appendix D, the stiffened bridges are denoted with a box around the number identifier. The figures also differentiate the existing bridges from the simulated bridges. In general, the critical-to-applied stress ratios for the three analysis specifications are relatively consistent among specifications and within the sample.

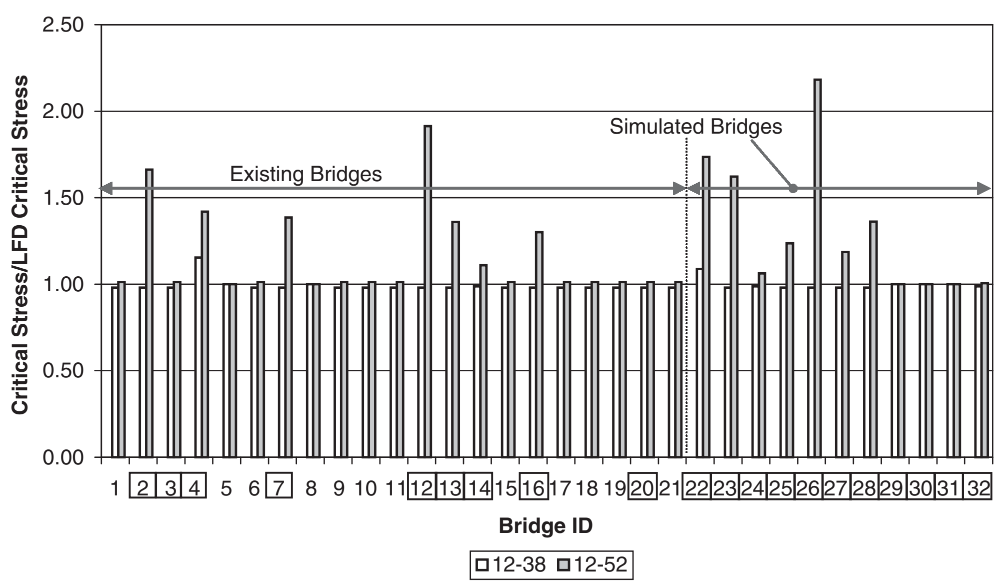
All bridges except for Bridges 4 and 22 exhibited equal or slightly higher shear critical stress according to the 1993 Guide Specifications than according to the 2003 Guide Specifications. The change in maximum transverse stiffener spacing among the three specifications is evident in the critical stress values for Bridges 4 and 22. These two bridges are the only ones that have a transverse stiffener spacing value greater than the depth of the web and are the only ones that exhibit a higher critical stress according to the 2003 Guide Specifications than according to the 1993 Guide Specifications. This is because the sections are analyzed as unstiffened according to the 1993 Guide Specifications when the transverse stiffener spacing exceeds the maximum value of *D*.

The analysis of transversely stiffened members with the 1993 Guide Specifications and the 2003 Guide Specifications is more conservative than with the 12-52 recommended specifications in several instances, as evidenced by the lower normalized critical stress values. These cases are for Bridges 2, 4, 7, 12–14, 16, and 22–28. For these bridges, the 12-52 recommended specifications allow the additional post-buckling strength from tension-field action to be considered. This post-buckling strength is also recognized in the 1993 Guide Specifications and AASHTO LRFD specifications straight girder provisions.

According to the 12-52 recommended specifications, Eq. 6.10.9.2-1, which is identical to Eq. 6-4 in the 2003 Guide Specifications, is used for both stiffened and unstiffened webs to account for the shear-yielding or shear-buckling strength of the web. Note that this equation is unnumbered in the 1993 Guide Specifications.

$$V_{cr} = CV_p \qquad Eq.6.10.9.2\text{-}1$$

The 12-52 recommended specifications use an additional equation, Eq. 6.10.9.3.2-2, to account for the post-buckling strength of the interior panels of stiffened webs that satisfy certain geometric requirements. The use of this equation produces a higher multiplier on V_p for these webs. Eq. 6.10.9.3.2-2 is used in the 12-52 recommended specifications for both straight and curved steel girders and is applicable for Bridges 2, 4, 12–14, 16, and 22–31. Among the bridges with stiffened webs, only Bridge 7 does not meet the geometric requirements for the use of this equation and, subsequently, uses a more conservative variation, identified as Eq. 6.10.9.3.2-8. The flowcharts in Appendix D (available online at http://trb.org/news/blurb_detail.asp?id=5965) outline the number of bridges in the sample that meet the requirements for each classification according to each of the specifications for the stiffened and unstiffened critical stress equations.



Note: Stiffened bridges are denoted with a box around the number identifier.

Figure 7. Shear critical stress divided by 1993 Guide Specifications shear critical stress.

$$V_n = V_p\left[C + \frac{0.87(1-C)}{\sqrt{1+\left(\frac{d_O}{D}\right)^2}}\right] \qquad \text{Eq.6.10.9.3.2-2}$$

$$V_n = V_p\left[C + \frac{0.87(1-C)}{\sqrt{1+\left(\frac{d_O}{D}\right)^2}+\frac{d_O}{D}}\right] \qquad \text{Eq.6.10.9.3.2-8}$$

Bridges 29, 30, and 31 also qualified for the use of the post-buckling strength provision. However, no additional strength is realized for cases when the ratio of the shear-buckling resistance to the shear yield strength, C, is equal to unity; in this situation, Eq. 6.10.9.3.2-2 becomes Eq. 6.10.9.2-1 and only the buckling strength is considered. The equations used to determine C for each of the three specifications are listed in Appendix D and are essentially equivalent.

Although also stiffened, Bridges 3, 20, and 32 were analyzed at the end panels and are therefore not able to rely on the formation of a tension field or to account for any additional post-buckling strength. These observations are evident in Figure 7.

2.4.5 Flexural Design

The flowcharts in Appendix D outline the number of bridges in the sample that met the requirements for each classification according to each of the specifications for the composite positive flexure (C+), composite negative flexure (C–), noncomposite positive flexure (NC+), and noncomposite negative flexure (NC–), respectively. Furthermore, they outline the design protocols and variables used for each of the three specifications. The reader is encouraged to reference the appropriate specification for clarification and further information regarding the design protocols.

Table 5 outlines the width-to-thickness (i.e., slenderness) ratio limits for the various classifications of the flanges according to the three specifications. For a yield strength of 50 ksi, the limits for compact and slender flanges are lower with the 1993 Guide than with the 2003 Guide, but only marginally so. For the 12-52 recommended specifications, the ratios are slightly higher than for the 2003 Guide Specifications.

The following trends, as they relate to the studied bridges, are worth noting from the flowcharts:

- For the 12-52 recommended specifications related to composite sections in positive flexure (Section 6.10.7), there are two bridges in the sample for which the section is regarded as noncompact; it should be noted, however, that because plastic design is not permitted, all composite curved steel girders in positive flexure must be analyzed as noncompact according to 12-52 Section 6.10.6.2.2.
- According to the 12-52 recommended specifications, the majority of the bridges in the sample were classified as compact for flange local buckling (FLB) considerations and as noncompact for lateral-torsional buckling (LTB) considerations for all design conditions.
- For the composite negative flexure and noncomposite section recommended specifications of 12-52 (Section 6.10.8), the case where the compression flange is continuously braced is not represented in the sample because such a case would be atypical. The bottom flange is usually braced at discrete points where the cross-frames exist.
- Generally, the LTB considerations controlled the critical stress of the bridges in the sample, which is logical given the classifications pertaining to their buckling behavior.
- None of the specifications reduce the capacity of a continuously braced flange below the yield strength, F_y.
- In a sense, the magnification factor in the 12-52 recommended specifications, defined in Section 6.10.1.6, replaces the ρ factors from the 1993 Guide Specifications and the 2003 Guide Specifications. A comparison reveals that both factors increase the tendency of the section to deform because of secondary effects.
- Hybrid sections are not allowed by the 2003 Guide Specifications, but are allowed by the 1993 Guide Specifications and the 12-52 recommended specifications, which have similar provisions for the consideration of stresses in hybrid sections. The provisions for hybrid sections in the 1993 Guide Specifications consider only yielding of the tension flange in positive bending regions or only yielding of the compression flange in negative bending regions, while the 12-52

Table 5. Flange classifications by slenderness ratio.

Specifications	Compact Flange	Noncompact Flange	Slender Flange
1993 Guide Specifications	$b_f/t_f \le 14.31$	$14.31 < b_f/t_f \le 19.68$	$b_f/t_f > 19.68$
2003 Guide Specifications	$b_f/t_f \le 18$	$18 < b_f/t_f \le 23$	$b_f/t_f > 23$
12-52 Recommended Specifications	$b_f/t_f \le 18.3$	$18.3 < b_f/t_f \le 24$	$b_f/t_f > 24$

recommended specifications, in Eq. 6.10.1.10.1-1, have been adapted to include all positions of the neutral axis and all combinations of yield strengths for the various portions of the girder.

One noticeable difference among the three specifications is the fact that while the 1993 Guide Specifications and the 2003 Guide Specifications consider the lateral bending stress to reduce the critical stress, the 12-52 recommended specifications consider the lateral bending stress as a load. In order to alleviate this difference, the appropriate portion of the lateral bending stress was deducted from the critical stress calculated according to the 12-52 recommended specifications to obtain the bending resistance, F_{bu}.

$$F_{bu} = \phi_f F_n - \frac{1}{3} f_l$$

The flexural capacity figures in Appendix D (which is available online at http://trb.org/news/blurb_detail.asp?id=5965) use the bending resistance values, with the exception of the critical-to-applied stress plots, which consider the flange critical stress, F_n, as outlined in the recommended specifications directly. Note that the gaps occurring in the negative flexure figures for Bridges 3, 6, 20, and 32 are because those bridges are simple-span structures. Furthermore, Bridges 24, 25, 29, and 30 use Grade 36 steel throughout and Bridges 1 and 22 have hybrid sections in the negative moment regions.

In spite of the differences between the flexural design specifications, the calculated capacity values are, for the most part, very similar. The main causes of major differences in capacity among the three specifications are as follows:

- Changes in the classification of the flange (e.g., compact, noncompact, and slender) among the three specifications;
- Lateral stress consideration (including magnification) according to the 12-52 recommended specifications as compared with the reductions due to the r factors used in the 1993 Guide Specifications and the 2003 Guide Specifications; and
- Noncomposite design as a temporary condition.

Again, specific information regarding the capacity of the bridges for each of the design conditions is outlined in Appendix D.

Although the proportion varies based on the design specification selected, most composite positive flexure sections are controlled by the tension flange, while most negative flexure sections are controlled by the compression flange. For the noncomposite section design checks, the compression flange controls both positive and negative flexural sections. This control indicates that the compression flange is the critical flange in unbraced or discretely braced conditions.

2.4.6 Summary of Comparison Results

Based on the analyses, the following observations and conclusions can be made regarding the shear design protocols in the 1993 Guide Specifications, the 2003 Guide Specifications, and the 12-52 recommended specifications (which have subsequently been adopted by AASHTO and published in the 2006 interim specifications):

Strength I load case, shear design:

- The maximum transverse stiffener spacing has been progressively increased from D, the depth of the web, in the 1993 Guide Specifications to $3D$ in the 12-52 recommended specifications.
- All three specifications use the ratio of the shear-buckling resistance to the shear yield strength, C. A comparison of the equations used to calculate C in each of the specifications reveals that the constants for the equations are equivalent.
- In general, the critical stress values for the three specifications are consistent except that the 12-52 recommended specifications allow for the consideration of the additional post-buckling strength from tension-field action in the shear critical stress calculations. This post-buckling strength is also recognized in the 1993 Guide Specifications and AASHTO LRFD specifications straight girder provisions and results in a higher critical stress for the majority of the stiffened bridges in the sample.

Strength I load case, flexural design:

- Flexural analysis, according to all of the specifications, is divided between composite and noncomposite sections, positive and negative flexure, and compression and tension flanges.
- Only modest changes are made to the slenderness limits for the various classifications of the flanges (e.g., compact, noncompact, and slender) according to the three specifications. These changes, however, are often enough to change the classification of a flange and, therefore, the critical stress of the section.
- All three of the specifications account for an increased tendency of the curved girders to deform due to secondary bending effects. The 1993 Guide Specifications and the 2003 Guide Specifications consider ρ factors, and the 12-52 recommended specifications consider a magnification factor for the lateral bending stress. For the 1993 Guide Specifications and the 2003 Guide Specifications, the ρ factors act to reduce the critical stress of the section based on lateral bending stress and geometry resulting in F_{cr1}. The magni-

fier on the lateral bending stress for the 12-52 recommended specifications focuses on the magnitude of the longitudinal bending stress, resulting in a final combined reduction in the stress limit due to the effects of geometry (i.e., either FLB or LTB considerations) and bending.

- The 1993 Guide Specifications and the 2003 Guide Specifications consider the lateral bending stress to reduce the critical stress, while the 12-52 recommended specifications consider the lateral bending stress as a load. The final effect of including the lateral bending stress in all of the specifications is to reduce the useable stress limit for gravity loads.
- For the bridges considered, the noncomposite design was a temporary condition, and the noncomposite dead loads that were provided for the final structure were used to analyze the noncomposite structure. Information was not provided regarding any temporary support points or the construction sequence for a more inclusive analysis.
- Hybrid sections are not allowed by the 2003 Guide Specifications, but are allowed by the 1993 Guide Specifications and the 12-52 recommended specifications, which have similar provisions for the consideration of stresses in hybrid sections. Minor reductions in the critical stress occur when the yield strength of the web is less than the yield strength of one or both of the flanges.
- In spite of the differences between the flexural design specifications, the calculated critical stress values are largely very similar.

2.5 Design Examples

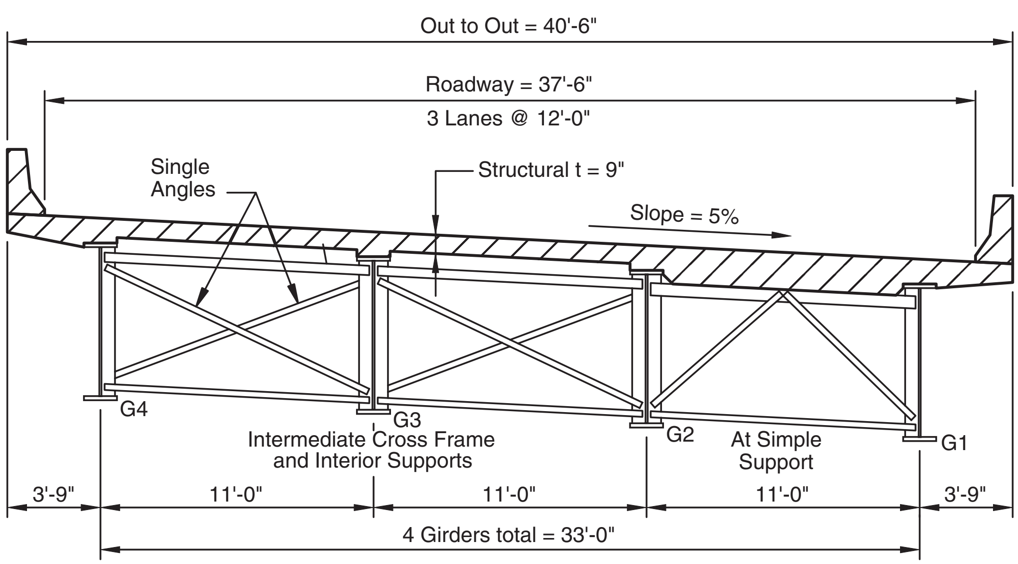
Two design examples, one for an I-girder bridge and one for a box-girder bridge, were developed in the NCHRP 12-38 project. As part of Phase I of the NCHRP 12-52 project, these design examples were updated to conform to Phase I recommended specifications. The work plan for Phase II called for updating the two design examples to conform to the design provisions approved by AASHTO. This work was conducted, and the two design examples are available online at http://www.transportation.org/sites/bridges/docs/Box%20Girder.pdf and http://www.transportation.org/sites/bridges/docs/I-Girder.pdf.

Following is a brief description of the bridges used for the two examples.

I-Girder bridge example:

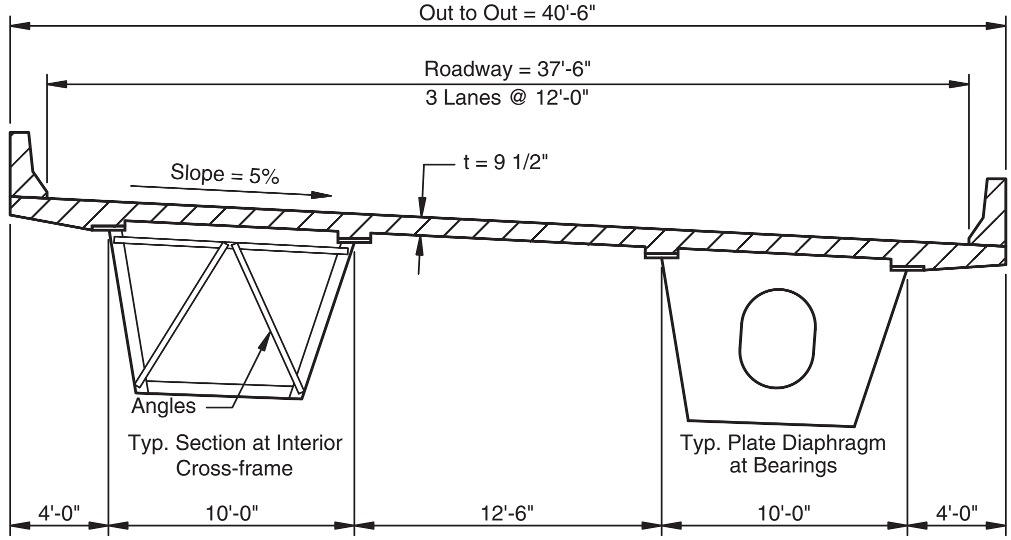
- Three continuous spans: 160 feet, 210 feet, and 160 feet.
- Centerline radius: 700 feet.
- Girder spacing: 11 feet, 0 inches.
- Overhang width: 3 feet, 9 inches.
- Out-to-out deck width: 40 feet, 6 inches.
- Three 12-foot design lanes.
- Total deck thickness: 9.5 inches (includes a 1/2-inch integral wearing thickness).

Figure 8 shows a cross-section of the I-girder example bridge.



Deck concrete – f'_c = 4,000 psi $E = 3.6\text{x}10^6$ psi
Haunch – 20 in. wide, 4 in. deep measured from top of web
Permanent deck forms are present
Total deck thickness = 9.5 in., structural thickness = 9.0 in.

Figure 8. I-girder bridge example, cross-section.



Deck concrete – f'_c = 4,000 psi $E = 3.6x10^6$ psi
Haunch – 20 in. wide, 4 in. deep measured from top of web
Permanent deck forms are present
Total deck thickness = 9.5 in.

Figure 9. Box-girder bridge example, cross-section.

Box-girder bridge example:

- Three continuous spans: 160 feet, 210 feet, and 160 feet.
- Centerline radius: 700 feet.
- Individual tub girder web spacing: 10 feet, 0 inches.
- Interior web spacing: 12 feet, 6 inches.
- Out-to-out deck width: 40 feet, 6 inches.
- Three 12-foot design lanes.
- Total deck thickness: 9.5 inches (no provision for integral wearing thickness).

Figure 9 shows a cross-section of the box-girder example bridge.

CHAPTER 3

Conclusions

Based on the results of the work on this project, the following conclusions may be drawn:

- For the design cases considered in the calibration, using the same load and resistance factors used in the development of AASHTO LRFD specifications resulted in an adequate reliability factor (3.7 to 4.51).
- In spite of the differences between the flexural design provisions in the 1993 Guide Specifications, the 2003 Guide Specifications, and the 12-52 recommended specifications, the three specifications produced very similar flexural resistance.
- The shear resistance values calculated by the three specifications are similar except that the 1993 Guide Specifications and the 12-52 recommended specifications produce higher shear resistance of stiffened girders due to consideration of the post-buckling behavior by these two specifications.

References

1. AASHTO (1980). *Guide Specifications for Horizontally Curved Highway Bridges.* American Association of State Highway and Transportation Officials, Inc., Washington, DC.
2. AASHTO (1993). *Guide Specifications for Horizontally Curved Highway Bridges.* American Association of State Highway and Transportation Officials, Inc., Washington, DC.
3. Duwadi, S. R., Hall, D. H., Yadlosky, J. M., Yoo, C. H., and Zureick, A. H. (1995). "FHWA-CSBRP I-Girder Component Testing." *Proceedings of ASCE Structures Congress XIII,* Boston, MA, April 2–5, 1995, pp. 1695–1698.
4. Grubb, M. A., and Hall, D. H. (2000). *Philosophy and Design of the I-Girder Bending Component Tests.* FHWA Curved Steel Bridge Research Project. Report submitted to the FHWA, Turner-Fairbank Highway Research Center, McLean, VA.
5. White, D. W., Jung, S. K., and Chang, C. J. (2002). "Design of Curved Composite Test Bridge," Report to PSI Inc. and FHWA, December.
6. White, D. W., Zureick, A. H., Phoawanich, N. P., and Jung, S. K. (2001). "Development of Unified Equations for Design of Curved and Straight Steel Bridge I Girders," Final Report to AISI, PSI, Inc. and FHWA, October.
7. Yoo, C. H., Hall, D. H., and Sabol, S. A. (1995). "Improved Design Specifications for Horizontally Curved Steel-Girder Highway Bridges." *Proceedings of ASCE Structures Congress XIII,* Boston, MA, April 2–5, 1995, pp. 1699–1702.
8. Hall, D. H., Grubb, M. A., and Yoo, C. H. (1999). *NCHRP Report 424: Improved Design Specifications for Horizontally Curved Steel Girder Highway Bridges,* Transportation Research Board, Washington, DC.
9. AASHTO (2003). *Guide Specifications for Horizontally Curved Highway Bridges.* American Association of State Highway and Transportation Officials, Inc., Washington, DC.
10. AASHTO (2004). *AASHTO LRFD Bridge Design Specifications.* 3rd ed. American Association of State Highway and Transportation Officials, Inc., Washington, DC.
11. Yoo, C. H. (1996). "Progress Report on FHWA-CSBRP-Task D." FHWA Contract No. DTFH61-92-C-00136, *Auburn University Department of Civil Engineering Interim Report,* submitted to HDR Engineering, Inc., Pittsburgh, Office, Pittsburgh, PA, August 1996.
12. Yoo, C. H., and Choi, B. H. (2000). "Literature Search–Horizontally Curved Steel Girder Bridges," NCHRP 12-52 Quarterly Report, submitted to the Transportation Research Board, Washington, DC, March 2000.
13. Nowak, A. S., Szwed, A., and Galambos, T. V. (2001). "Calibration of LRFD Design Code for Steel Curved Girder Bridges, NCHRP 12-52." University of Michigan.
14. Modjeski and Masters (2001). "Revised Proposed Draft Specifications." NCHRP 12-52 (Phase I) Quarterly Report, submitted to the Transportation Research Board, Washington, DC, December.
15. Modjeski and Masters (2001). "LRFD Design Example–Horizontally Curved Steel I-Girder Bridge." NCHRP 12-52 Quarterly Report, submitted to the Transportation Research Board, Washington, DC, September.
16. Modjeski and Masters (2001). "LRFD Design Example–Horizontally Curved Steel Box Girder Bridges." NCHRP 12-52 Quarterly Report, submitted to the Transportation Research Board, Washington, DC, December.
17. Zureick, A., Naqib, R., and Yadlosky, J. M. (1994). "Curved Steel Bridge Research Project." Interim Report I, "synthesis," FHWA Contract No. DTFH61-93-C-00136, FHWA, McLean, VA, December.
18. Nowak, A. S. (1995). "Calibration of LRFD Bridge Code," *ASCE Journal of Structural Engineering,* Vol. 121, No. 8, pp. 1245–1251.
19. Nowak, A. S. (1999). *NCHRP Report 368: Calibration of LRFD Bridge Design Code,* Transportation Research Board, Washington, DC.
20. White D. W., Zureick, A. H., and Phoawanich, N. (2001). "Development of Unified Equations for Design of Curved and Straight Steel Bridge I-Girders," American Iron and Steel Institute and Federal Highway Administration, presentation.
21. AASHTO (2002). *Standard Specifications for Highway Bridges.* 17th ed. American Association of State Highway and Transportation Officials, Inc., Washington, DC.
22. Ontario Ministry of Transportation and Communications (1991). *Ontario Highway Bridge Design Code.* OHBDC, 3rd Edition Downsview, Ontario.
23. Ellingwood, B., Galambos, T. V., MacGregor, J. G., and Cornell, C. A. (1980). "Development of a Probability Based Load Criterion for American National Standard A58," *NBS Special Publication 577,* National Bureau of Standards, Washington, DC.
24. Nowak, A. S. (1993). "Live Load Model for Highway Bridges," *Journal of Structural Safety,* Vol. 13, Nos. 1+2, December, pp. 53–66.
25. Hwang, E. S., and Nowak, A. S. (1991). "Simulation of Dynamic Load for Bridges," *ASCE Journal of Structural Engineering,* Vol. 117, No. 5, May, pp. 1413–1434.
26. Nassif, H. and Nowak, A. S. (1995). "Dynamic Load Spectra for Girder Bridges," *Transportation Research Record 1476,* Transportation Research Board, pp. 69–83.
27. Kim, S.-J., and Nowak, A. S. (1997). "Load Distribution and Impact Factors for I-Girder Bridges," *ASCE Journal of Bridge Engineering,* Vol. 2, No. 3, August, pp. 97–104.

APPENDIX A

Literature Search

CONTENTS

LIST OF ABBREVIATIONS AND SYMBOLS

a = distance between transverse stiffeners,
b = width of flange,
b_f = width of compression flange,
c = curvature parameter,
C = shear strength constant,
D = depth of the web panel,
D_c = depth of the web panel in compression,
f_b = normal stress,
F_b = allowable bending stress,
f_v = shear stress,
F_v = allowable shear stress,
F_y = minimum specified yield stress,
f_w = warping stress (flange lateral bending stress),
k = elastic shear buckling coefficient,
L = length of girder,
M_u = ultimate vertical bending moment,
R = radius of curvature,
R_d = reduction factor of shear due to initial out of flatness or reduction factor of deflection, as appropriate,
R_s = reduction factor of stress,
t = thickness of flange,
t_f = thickness of flange,
t_w = thickness of the web panel,
V_p = plastic shear capacity,
V_u = ultimate shear capacity,
x = subtended angle between adjacent cross frames,
y = critical moment ratio,
ℓ = unbraced length of compression flange of I-girder,
Ψ = reduction factor of local buckling of compression flange, and
Ψ_w = parameter relating bend-buckling of curved I-girder web.

A1 Introduction

A1.1 General

The scope and intent of this literature search was to continue the exhaustive literature survey that was conducted as Phase I of the FHWA-sponsored project, "Curved Steel Bridge Research Project" (DTFH61-93-C-00136), and published as Interim Report I, "Synthesis" (*1*). Although the report date of the Synthesis is December 1994, the cut-off date of the literature survey activity is assumed to be June 1993. Therefore, the literature collected and included in this report are those published after June 1993 and up to the time this search was conducted in early 2000. References made to articles in AASHTO *LRFD Bridge Design Specifications* and AASHTO *Standard Specifications for Highway Bridges* use the article numbers existing at the time of conducting the literature search.

A1.2 Objective

The objective of this task was to perform a comprehensive up-to-date literature search after June 1993 on publications describing work related to the design and analysis of horizontally curved steel box-girders and I-girders. A brief synopsis of the information in each reference is provided for a speedy review.

A1.3 Research Procedure

In order to facilitate the objective, a comprehensive literature search was conducted using computerized searching, manual paging, and personal contact. The initial search resulted in the collection of approximately 100 references. Each reference was reviewed to determine its appropriateness for the design and analysis of horizontally curved steel girder bridges. References that were obviously not relevant to horizontally curved steel girders were eliminated. General references on analysis methodology that may have applied to both steel and concrete were retained. This process resulted in over 90 references, which were placed in the electronic database.

A1.4 Electronic Database

The information compiled on horizontally curved steel bridge girders was stored in the Microsoft Access database management software, which runs under the Windows operating system. This database program was chosen simply because the Synthesis is already in this software. Because this report is considered to be a continuation of the Synthesis, the database format was made identical to that used in the Synthesis.

The information pertaining to a particular reference was stored in six different fields. The first three fields contained material about the title, author, and year of publication. The fourth field was for the source of publication (journal, conference, publisher, etc.), and the fifth field was for additional information, such as the order of contract number. The sixth field was for a summary or abstract of the publication.

In addition to these six fields, check boxes were added to the database form for further information. These check boxes can be used to distinguish whether a reference is on box-girders or I-girders, static or dynamic, thermal or failure analysis, or other aspects of response.

The information could then be used to sort and filter required publications. As an example, one can request all design work by Hall that was published after 1993, and that information can be sorted according to the year in an ascending order. The result of a filtering and/or sorting process can be on-screen (which is temporary) or can be saved as a query (which can be used at a later time). Additionally, one can also search a certain field for a particular word or name.

A1.5 Synthesis

Because of budget and time constraints, the collection of references could not be exhaustive, and the evaluation of the information described in each reference could not be sufficiently detailed. Nevertheless, the synthesis is very useful for digesting quickly what has been studied after 1993 for further understanding of the complex behavior of horizontally curved steel bridge girders.

A2 Overview of Literature Search

A2.1 General

Ninety-six references were identified, of which 91 had abstracts. Of the 91 references with abstracts, 89 were reviewed. The remaining two references with abstracts could not be acquired. When sorted according to broad subject area, the 91 references reveal the following distribution: analysis, 45; design, 22; progress report, 6; bridge test, 3; synthesis, 3; computer program, 2; and cylindrical shells, rings, and others, 10. All of the authors of the progress reports are associated with two recent multiyear national projects on steel girder curved bridges: FHWA Curved Steel Bridge Research Project (CSBRP) and NCHRP Project 12-38. A vast majority of references deal with the analysis of curved beams. A significant number of these references have an academic, rather than practical, approach. Many references are for the derivation of a new or modified spatial curved beam element or its derivative with six or seven kinematic degrees of freedom. Some of these so-called new curved beam elements include one or more of the following rather special effects: shear deformation, rotatory inertia terms in a consistent mass matrix, and noncoincidental centroid and shear center. These curved beam elements may be incorporated into either 1*D* or 2*D* analysis of curved girders. This literature search yielded a significantly higher proportion of references dealing with design problems (22 out of 91) than did the Synthesis results summarized in *NCHRP Report 424* (*2*) (approximately 50 out of 540). Table A-1 shows the distribution of the 91 references based on a broad classification. Contents of some references (*2, 3, 4, 5, 6, 7*) are so comprehensive that it is inappropriate to classify them as either an analysis type or a design type. Therefore, the total number of references given in each category in Table A-1 will exceed 91.

A2.2 Analysis of Curved Bridges

Hall et al. (*8, 9*) presented three generic methods of curved bridge analysis, 1D line girder analysis, 2D planar grid analysis, and 3D finite element method. An early formulation of the V-load method (*10*) is the basis of a typical line girder analysis method applied to I-girder bridges and M/R method (*11*) is the 1D line girder analysis method applied to horizontally curved box girder bridges. A variety of 2D planar grid analysis methods for horizontally curved bridges are available, including finite strip method, finite element method, and finite difference method. Any curved bridge analysis method that does not recognize the section depth is categorized as 2D planar grid method or 1D line girder analysis method. In 3D finite element method, the section depth is recognized and the forces in cross-frames or diaphragms are evaluated as an integral part of the analysis. A 3D skeletal space frame analysis method is not considered a 3D finite element method because of its inability to rigorously account for the plate/shell action of the girder web and the deck. Any commercially available, general-purpose finite element analysis codes—such as NASTRAN, ABAQUS, ADINA, and ANSYS—can be used to provide full 3D analyses of curved bridges. There are, however, a few special-purpose proprietary codes—such as BSDI 3D System (*12*). Article 4.1 of Ref. (*4*) requires that the analysis be performed using a rational method that accounts for the interaction of the entire superstructure. Inability to recognize the section depth in 1D and 2D analysis methods may lead to a serious compromise in the determination of member forces due to temperature variations, wind loads, skewed support, bearing orientation, centrifugal forces, and prestressing of the deck. Although it generally requires additional efforts, strategies, and time to model the structure and to pre- and post-processing the input and output data, a full 3D finite element analysis of a curved bridge automatically satisfies the requirements of Article 4.1 of Ref. (*4*).

A2.3 Design

References dealing with the design of curved box-girder bridges are fewer than references dealing with the design of curved I-girder bridges. Because of the superior torsional strength of the box-girders, closed box-shape girders are better suited for horizontally curved bridges. However, the addition cost associated with the fabrication of curved box-girders may offset the advantage afforded by the box-girders.

Articles 1.8 and 2.9 of Ref. (*13*) require that the diaphragms or cross-frames be full-depth members designed as the primary load-carrying members and that they be attached closely to the flanges. Article 9.3 of Ref. (*4*) requires that eccentricity between the cross-frame members or diaphragm flanges and the girder flanges be recognized in the analysis, in the design of connection plates, and in their connection to the web and flange. In the design and construction of horizontally curved

Table A-1. Distribution of references.

Category	References
Progress Report	14–19
Bridge Test	20–22
Computer Program	12, 23
Synthesis	1, 24, 25
Others	26–35
Analysis, I-Girder (Static)	6, 7, 36–60
Analysis, I-Girder (Buckling)	6, 7, 38, 61–64
Analysis, I-Girder (Dynamic)	6, 65–74
Analysis, Box-Girder (Static)	6, 75–77
Analysis, Box-Girder (Dynamic)	6, 78, 79
Design, I-Girder	2–8, 38, 80–90
Design, Box-Girder	2, 4–6, 9, 91–94

highway bridges in the early 1960s, these provisions were not followed closely. As a result, a large number of bridges were retrofit later, and, in some instances, inadequate cross-framing was a contributing factor for bridges to be classified as structurally deficient.

A3 Curved I-Girder Bridges

A3.1 Analysis

Hall et al. (*8*) presented interesting results of a series of comparative analyses made on an example I-girder bridge. It is a three-span (160 feet, 210 feet, 160 feet), four-girder (11-foot girder spacing) structure with a centerline curvature radius of 700 feet. The bridge was analyzed by a 3D finite element analysis (*12*), a 2D grid analysis (MSC/NASTRAN), and a 1D V-load analysis. There exists a close correlation in the dead load analysis results among all three analysis methods. Fairly close correlation existed between the 3D finite element analysis and the 2D grid analysis results for live load. However, the 1D V-load analysis produced significantly different vertical bending moments in the live load analysis. The discrepancy between the V-load and the finite element analysis results was up to 70%. Much of this discrepancy was probably due to the wheel load distribution factors (AASHTO Article 3.23) used to determine the primary vertical bending moments in the V-load analysis. This discrepancy in the live load analysis results will likely be improved if more accurate wheel load distribution factors are used in the V-load analysis.

A3.2 Design

Hall et al. (*8*) present a comprehensive curved I-girder bridge design example. Ref. (*8*) is 243 pages long and includes essentially all major design procedures that reflect the provisions of the "Recommended Specifications for Steel Curved-Girder Bridges" (*4*) (hereafter referred to as the 12-38 recommended specifications).

Some of the recent research results that can be incorporated into design specifications are highlighted below.

A3.2.1 Nominal Bending Strength

The nominal bending strength predictor equations in the Guide Specifications appear to be quite conservative, as shown in Table A-2. Hall et al. (*2*) demonstrate that the lateral flange bending stress (f_w) due to curvature in the McManus-Culver predictor equation in the Guide Specifications is double-counted. Therefore, the lateral flange bending stress due to curvature at the critical cross-frame location must be set equal to zero. This fact and an alternate critical stress expression based on yielding of the flange due to combined lateral bending and vertical bending are implemented in the 12-38 recommended specifications.

An interaction equation based on full plastification of a compact I-section has been introduced in 1988 by Nakai and Yoo (*6*). Schilling (*87*) extended this concept to flange-compact sections and noncompact sections in 1996. These interaction equations are, however, limited to doubly symmetric I-shaped sections. The ultimate strength of the section is controlled by the combined action of vertical bending and lateral flange bending. The magnitude of the lateral bending stress is affected by the radius of curvature and the cross-frame spacing. Therefore, these equations are not well suited for use as ultimate strength predictors.

Since the combination of the vertical and lateral flange bending moments depends on the unbraced length (i.e., cross-frame spacing) of the compression flange for a given loading and the radius of curvature, spacing of the cross-frame is an important parameter in the development of an ultimate strength predictor equation. Yoo et al. (*94*) presented yield interaction equations for nominal bending strength of curved I-girders. There are 17 interaction equations encompassing composite sections that are doubly symmetric, singly symmetric, I-shaped compact, flange-compact, noncompact, cracked, and uncracked. It is assumed that only singly symmetric compact sections can be made hybrid. For compact-flange and noncompact sections, however, a homogeneous section is assumed because hybrid construction will not yield significantly higher moment capacities. The radius of curvature, the cross-frame spacing, the material properties, and the cross-section geometry are included as variables of the interaction equations. Although these interaction equations can be evaluated in an iterative, fast-converging manner, they are programmed to use on a PC. Comparison of selected test results from the literature versus analytically predicted vertical moment capacities is given in Table A-2. The values in the column under the heading "Interaction" were evaluated from the interaction equations.

The values under the heading of "L-T Bklg" were computed by the regression formula derived by Yoo et al. (*64*) for slender sections,

$$y = \left(1 - \gamma x^{\beta}\right)^{\alpha} \tag{1}$$

where $\alpha = 2.152$, $\beta = 2.129$, $\gamma = 0.1058$, x = subtended angle between the adjacent cross-frames in radian, and y = critical vertical moment ratio = $M_{xcr,cv}/M_{xcr,st}$. The subscript cv stands for curved girder, and st stands for straight girder. It appears feasible that the nominal bending strength limit state of curved girders can be assessed in a manner similar to AASHTO LRFD Articles 6.10.5 and 6.10.6 because the strength predictor equations are available for compact, noncompact, and slender sections.

Table A-2. Comparison of bending test results with six strength predictor equations M_u in k-in., Ratio = Analytical (DEN = 12) M_u/Test M_u.

Tested by	Specimen ID	Test Results M_u (k-in.)	Guide Spec.		Yoo et al.				Nakai (*6*)		Fukumoto		Hanshin (*5*)	
			M_u (k-in.)	Ratio	L-T Bklg. (*64*)		Interaction (*90*)		M_u (k-in.)	Ratio	M_u (k-in.)	Ratio	M_u (k-in.)	Ratio
					M_u (k-in.)	Ratio	M_u (k-in.)	Ratio						
Culver	L1-A	1830	1716	0.94	1865	1.02	1701	0.93	1999	1.09	1504	0.82	1107	0.61
	L2-A	1830	1749	0.96	1888	1.03	1726	0.94	1993	1.09	1533	0.84	1136	0.62
	GI-5	1377	*	*	1279	0.93	1232	0.89	1279	0.93	989	0.72	652	0.47
	GO-8	2120	1992	0.94	2062	0.97	2156	0.89	2213	1.04	1912	0.90	1164	0.55
Nakai	M1	8098	6965	0.86	6965	0.86	7972	0.98	5657	0.70	8007	0.99	7509	0.93
	M2	7754	3708	0.48	6986	0.90	7058	0.91	6508	0.84	6187	0.80	4713	0.61
	M3	6131	2627	0.43	5985	0.98	6031	0.98	5356	0.87	4803	0.78	3504	0.57
	M4	7203	*	*	6958	0.97	5498	0.76	7972	1.11	4317	0.60	2654	0.37
	M5	5902	*	*	5766	0.98	4535	0.77	6790	1.15	3592	0.61	2298	0.39
	M6	6287	*	*	5665	0.90	4482	0.71	6649	1.06	3503	0.56	2277	0.36
	M7	6547	*	*	6148	0.94	4819	0.74	7176	1.10	3818	0.58	2371	0.36
	M8	2935	*	*	2979	1.01	1654	0.56	1451	0.49	1054	0.36	*	*
	M9	5548	*	*	5951	1.07	4253	0.77	6934	1.25	3073	0.55	*	*

* The central angle for this girder exceeds the limitation of the procedure.

A3.2.2 Curvature Effects on Elastic Lateral-Torsional Buckling

Although the classical bifurcation type lateral-torsional buckling may not be observed in horizontally curved girders, the system eigenvalue indicates a critical elastic lateral-torsional buckling moment. The results of a series of theoretical and numerical analyses detailed in Ref. (*64*) on a number of hypothetical curved girders have been quantified in a regression formula (*23*). Additional study results are presented in Refs. (*38*) and (*7*).

A3.2.3 Cross-Frame Spacing and Lateral Bracing Effects

Davidson et al. (*3*) presented a regression formula for the cross-frame spacing resulting from a number of three-dimensional finite element analyses of a large number of hypothetical curved bridges:

$$\ell = L\left[-\ell n\left(\frac{(f_w/f_b)R\,b_f}{6.108L^2}\right)\right]^{-1.52} \quad (2)$$

where f_w/f_b = warping-to-bending stress ratio, L = girder span length in feet, R = radius of curvature in feet, and b_f = compression flange width in inches. The desired cross-frame spacing can also be extracted from the V-load analysis concept (*7*) as

$$\ell = \left[\frac{5}{3}\left(\frac{f_w}{f_b}\right)R\,b_f\right]^{0.5} \quad (3)$$

where units for R and b_f need to be consistent (either inches or feet). The placement of lateral bracing on top and bottom flanges makes the I-girders behave as pseudo-box-girders. Depending on the radius of curvature and the subtended angles of the girders, the addition of lateral bracing reduces the vertical deflection, flange bending stress, and warping stress significantly.

A3.2.4 Local Buckling of Curved I-Girder Flanges

Davidson and Yoo (*80*) presented a simple yet conservative regression formula for the limiting width-to-thickness ratio of curved I-girder compression flange after a series of numerical investigations on curved I-girders of practical design proportions.

$$\left(\frac{b}{t}\right)_{cv} \le \left(\frac{b}{t}\right)_{st}\sqrt{\psi} \quad \text{where } \psi = \left(1.05 - \frac{\ell^2}{4R\,b_f}\right) \le 1.0 \quad (4)$$

A3.2.5 Strength of Curved I-Girder Web Panels under Pure Shear

Lee and Yoo (*86*) conducted a three-dimensional finite element investigation on a number of hypothetical curved I-girder web panels encompassing a wide variety of practical design parameters. The aspect ratio, a/D, of transversely stiffened web panels was varied from 0.5 to 2.0, and the web slenderness ratio, D/t_w, was varied from 90 to 300. In practical designs of curved bridge plate girders, the value of the curvature parameter, $c = (a^2)/(8Rt_w)$, has been shown to be less than 1.0 (6). The curvature parameter varied from 0.5 to 1.0 inch. The shear strength is computed by

$$V_u = R_d V_p (0.6C + 0.4) \quad (5)$$

where

$$V_p = 0.58F_y D t_w \quad (6)$$

The constant C is to be determined by

$$C = 1.0 \qquad \text{for} \quad \frac{D}{t_w} < \frac{6{,}000\sqrt{k}}{\sqrt{F_y}} \quad (7)$$

$$C = \frac{6{,}000\sqrt{k}}{(D/t_w)\sqrt{F_y}} \qquad \text{for} \quad \frac{6{,}000\sqrt{k}}{\sqrt{F_y}} \le \frac{D}{t_w} \le \frac{7{,}500\sqrt{k}}{\sqrt{F_y}} \quad (8)$$

$$C = \frac{4.5\times10^7 k}{(D/t_w)^2 F_y} \qquad \text{for} \quad \frac{D}{t_w} > \frac{7{,}500\sqrt{k}}{\sqrt{F_y}} \quad (9)$$

and the strength reduction factor, R_d, due to the initial out-of-flatness, D/120, permitted by the *Bridge Welding Code* (*95*) is given by

$$R_d = 0.8 \qquad \text{for} \quad \frac{D}{t_w} < \frac{6{,}000\sqrt{k}}{\sqrt{F_y}} \quad (10)$$

$$R_d = 0.8 + 0.2\left(\frac{(D/t_w)\sqrt{F_y}/\sqrt{k} - 6{,}000}{6{,}000}\right)$$

$$\text{for} \quad \frac{6{,}000\sqrt{k}}{\sqrt{F_y}} \le \frac{D}{t_w} \le \frac{12{,}000\sqrt{k}}{\sqrt{F_y}} \quad (11)$$

$$R_d = 1.0 \qquad \text{for} \quad \frac{D}{t_w} > \frac{12{,}000\sqrt{k}}{\sqrt{F_y}} \quad (12)$$

A3.2.6 Curved I-Girder Web Panels Subjected to Bending

Davidson et al. (*84, 85*) examined the limiting values of the web slenderness ratio of curved I-girders based on the control of excessive bulging and the amplified web stresses. Based on a lateral pressure analogy developed in this study and a series of nonlinear finite element analyses, formulations for the reduction in allowable web slenderness due to curvature effects were presented based upon both a limit on allowable bulging transverse displacement and based upon maximum allowable stress.

$$\left(\frac{D}{t_w}\right)_{cv} \leq \left(\frac{D}{t_w}\right)_{st} R_d \tag{13}$$

where

$$R_d = 0.185\sqrt{\frac{R}{D}} \leq 1.0 \tag{14}$$

$$\left(\frac{D}{t_w}\right)_{cv} \leq \left(\frac{D}{t_w}\right)_{st} R_s \tag{15}$$

where

$$R_s = \sqrt{\frac{1}{\psi_w}} \tag{16}$$

$$\text{and } \psi_w = \sqrt{1 + 0.161\left(\frac{D_c}{t_w}\right)\left(\frac{D_c}{R}\right) + 0.128\left(\frac{D_c}{t_w}\right)^2\left(\frac{D_c}{R}\right)^2} \left[1 + 1.5\left(\frac{D_c}{R}\right)^{0.5}\right] \tag{17}$$

where the subscripts in reduction factors shown in Equations 14 and 16 stand for deflection and stress, respectively.

A3.2.7 I-Girder Webs Subjected to Combined Bending and Shear

Davidson et al. (*82, 83*) discovered from the results of a large number of displacement finite element analyses that the vertical moment capacities of curved I-girders are reduced somewhat less than 5% when the girders are simultaneously subjected to shearing forces of 60% of their shear capacities. An interaction graph showing these small adjustments is given in Refs. (*7*), (*38*), (*82*), and (*83*). Effects of longitudinal stiffeners on curved I-girder webs are presented in Refs. (*7*), (*38*), and (*83*).

A3.2.8 Lifting of Slender Curved I-Girders

Davidson (*38*) and Yoo (*7*) present the results of a series of finite element analyses to determine the critical lifting points under three variations of lifting schemes of a long slender curved girder during handling, transporting, and erection. The stresses that are induced by the self-weight of the girder during lifting are likely to be substantially less than the specified minimum yield stress of the girder. However, the excessive displacements and rotations during lifting can be problematic.

A3.2.9 Constructibility Limit State

The past performance record of horizontally curved highway bridges has been excellent. Most problems associated with curved bridges occurred during construction. The constructibility issue of curved bridges, along with the deck casting sequence, has become onerous. Article 2.5 of Ref. (*4*) mandates that the constructibility limit state be checked at each critical stage during construction. Hall et al. (*8*) demonstrate how this constructibility limit state check is made.

A4 Curved Box-Girder Bridges

A4.1 Analysis

Razaqpur and Li (*75, 76*) present a new thin-walled, multicell box-girder finite element that can model extension, flexure, torsion, warping, distortion, and shear lag effects. The element is one-dimensional. The element is based on the generalized Vlasov's thin-walled beam theory and the finite element technique. Interaction between the longitudinal and transverse deformations of the box-girder is accounted for by using a series of parameters. The cross-sections need to be prismatic, and, because the deck is represented by a sector plate, the supports need to be in the radial direction. Yabuki et al. (*77*) present an incremental nonlinear analysis technique on an essentially one-dimensional curved box-girder model, including local buckling and initial stress effects on the ultimate strength of the girder. After comparing analytically predicted values with the test results of two scaled model specimens, Yabuki et al. concluded that the degree of correlation was satisfactory. Galdos et al. (*78*) present a methodology for determining impact factor for curved box-girder bridges. Their recommendation has been implemented in the recommended specifications. Generally, impact factors for box-girders are higher than for I-girders. Using the finite element method, Sennah and Kennedy (*79*) present the results of a parametric study conducted on 120 simply supported, curved, composite, multicell bridges for the natural frequencies and mode shapes. The analytically obtained values were compared with results from tests on four 1/12 scaled model bridges.

Hall et al. (*9*) presented interesting results of a series of comparative analyses made on an example box-girder bridge. The bridge is a three-span (160 feet, 210 feet, 160 feet), two-tub-girder (22 feet, 6 inches girder spacing) structure with a centerline radius of curvature of 700 feet. The bridge was analyzed by a 3D finite element analysis (*12*), a 2D grid analysis (MSC/NASTRAN), and a 1D M/R analysis. There was a close correlation of the vertical bending moments obtained from all three analysis methods in all cases considered. Good correlation of these values was expected because the geometry and the relatively high torsional rigidity of the box-girders in this particular example minimize the increase in the vertical bending moments due to curvature. As the spacing between centers of the two box-girders exceeds 14 feet, the wheel load distribution was determined according to the computation of simple beam reactions (footnote "f" of AASHTO Table 3.23.1). Therefore, the total number of truck wheel loads was identical in all three analysis methods. The concrete deck was modeled as a continuum in the 3D finite element model. In both the grid analysis method and the M/R method, girders are represented as one-dimensional elements and the rigid concrete deck cannot be represented as a continuum. The 3D finite element analysis also properly recognizes the physical location of the two bearings at each support point. Two bearings at each support point cannot be physically represented in the grid model, or in the M/R method, due to the limitations imposed by one-dimensional modeling of the girders. Although the vertical bending moments compared well among the three analysis methods, the difference in torsional moments and shears between the 3D finite element analysis and the other analysis methods is more significant. The grid and M/R methods tended to underestimate the torques in each girder at the supports and overestimate the shear in the diaphragms between the boxes at each support.

A4.2 Design

Hall et al. (*9*) presented a comprehensive curved box-girder (tub-girder) design example. Ref. (*9*) is 152 pages long and includes essentially all major design procedures reflecting the provisions of the "Recommended Specifications for Steel Curved-Girder Bridges" (4).

Cheung and Foo (*91*) presented the results of a parametric study, using the finite strip method, on three simply supported concrete-steel composite box-girder bridge models, including single-, double-, and triple box-cross sections. Internal forces were evaluated using a 20-term Fourier series. The models do not include diaphragms. Vertical bending moment ratios were presented for a variety of loading cases and other design parameters, such as span lengths, subtended angles, and span-to-depth ratios.

Sennah and Kennedy (*92, 93*) presented the results of a parametric study, using the finite element method, on 120 simply supported curved box-girder bridge models. The analytically obtained results are compared with test results on four 1/12 scaled model bridges. The parametric study and tests were conducted on concrete-steel composite box-girder bridges. A series of regression equations were given for vertical bending moments, deflections, and maximum axial forces in bracings.

The Hanshin guidelines (*5*) include the following interaction equation for the ultimate strength of a box-girder as

$$\left(\frac{f_b}{F_b}\right)^2 + \left(\frac{f_v}{F_v}\right)^2 \leq 1.2 \qquad (18)$$

where

f_b = total normal stress, including stresses due to vertical bending, lateral flange bending, and cross-sectional distortion;
f_v = shear stress;
F_b = allowable bending stress; and
F_v = allowable shear stress.

Although both (f_b/F_b) and (f_v/F_v) are not permitted to be greater than 1.0, the sum of the square of these ratios may be taken as high as 1.2. The Hanshin guidelines do not use composite girders, but do use the allowable stress design method.

The practice of curved box-girder design in North America heavily relies on composite tub-girder construction. Top flanges of tub-girders are designed according to the provisions of open cross-section, I-girder designs for factored dead loads and construction loads prior to hardening of the deck. Top flanges of horizontally curved tub-girders are also braced laterally to resist torsional shear flow. However, these lateral bracing members are not subjected to any additional loading after the deck is hardened. Yoo et al. (*94*) presented a simplified design method using top flange lateral bracing members by transforming a lattice wall into a pseudo-box wall.

A5 Conclusions and Recommendations for Further Study

This task resulted in 91 references published from June 1993 after the "Synthesis" (*1*) pertaining to the analysis, design, and experimental investigations of horizontally curved I-girder and box-girder bridges. These references were reviewed briefly and are placed in an electronic database that is easy to access, query, and update as additional research is completed. Some recommendations deserving immediate attention are summarized below.

A5.1 Analysis Methods

Horizontally curved girders are one of the least understood structural elements in common use today. Designers, owners, and contractors will undoubtedly benefit from research leading to improved fundamental understanding of curved girder bridge behavior. New analysis tools are available, including finite element computer programs that permit detailed analyses of both structural elements and entire structures. These analysis tools have not been adequately applied to horizontally curved girder bridges. The Guide Specifications require that the entire superstructure be considered in the analysis. However, approximate methods that do not have well-defined limitations are permitted. Even the refined methods have limitations and assumptions that remain unexplored. There is a great need to quantify the reliability of various analysis methods and to explore how and when each method might be best applied. It is well known that all V-load analyses are not the same. It is less well known that all 3D finite element analyses do not provide the same results. Application of the methods needs to be studied and compared with field measurements. The failure mechanism of a curved bridge needs to be defined. Tests of full-scale bridges to failure are needed. Any refined analysis methods need to be correlated with the experimentally obtained data.

A5.2 I-Girders

Multiple I-girders attached to a common composite reinforced concrete deck should be investigated because the shear center of the bridge superstructure shifts above the deck after the deck has hardened. Tests on web stiffening should be conducted to evaluate the shear strength, the bend-buckling strength, and the effectiveness of various details. Tests of shear connectors that are subjected to large lateral forces near cross-frames in fatigue are needed. Evaluation of the effectiveness of bottom flange bracing at various stages of bridge construction and service is needed.

A5.3 Box-Girders

There is a need to investigate an interaction equation for the ultimate strength of box-girders under the combined action of flexure and torsion. The effectiveness of internal cross-bracing in reducing cross-section distortion needs to be investigated. Although Ref. (*96*) is widely used to check the distortion-induced stresses in curved box-girders, the adequacy of its use for other-than-straight box-girders is not well defined. Design rules for internal diaphragms need to be advanced. Design rules of longitudinal stiffeners in curved compression flanges are urgently needed.

A5.4 Constructibility

During lifting and when the girders are temporarily unbraced, lateral deflections and twists are large. Research is needed to define which conditions limit the applicability of small deflection theory. Research is also needed to assess the lateral deflection and twist limitations during construction to ensure that stresses and deflections will not exceed those permitted.

A5.5 Extreme Event Limit State

Design rules of horizontally curved girder bridges under extreme event limit state are urgently needed. No simple equation is available to determine the natural frequency of horizontally curved bridges.

References

1. Zureick, A., Naqib, R., and Yadlosky, J. M. (1994). "Curved steel bridge research project." Interim Report I, "Synthesis," FHWA Contract No. DT FH61-93-C-00136, FHWA, McLean, VA, December.
2. Hall, D. H., Grubb, M. A., and Yoo, C. H. (1999). *NCHRP Report 424: Improved Design Specifications for Horizontally Curved Steel Girder Highway Bridges.* Transportation Research Board, National Research Council, Washington, D.C.
3. Davidson J. S., Keller, M. A., and Yoo, C. H. (1996). "Cross-frame spacing and parametric effects in horizontally curved I-girder bridges." *Journal of Structural Engineering,* ASCE, Vol. 122, No. 9, September.
4. Hall, D. H., and Yoo, C. H. (1998). "Recommended specifications for steel curved-girder bridges." Appendix D of the final report submitted to National Cooperative Highway Research Program, Project 12-38, Transportation Research Board, National Research Council, Washington, D.C., December.
5. Hanshin Express Public Corporation and Steel Structure Study Committee (1988). "Guidelines for the Design of Horizontally Curved Girder Bridges (Draft)." Hanshin Expressway Public Corporation, October.
6. Nakai, H., and Yoo, C. H. (1988). *Analysis and Design of Horizontally Curved Steel Bridges.* McGraw-Hill Book Company, New York.
7. Yoo, C. H. (1996). "Progress Report on FHWA-CSBRP-Task D." FHWA Contract No. DTFH61-92-C-00136, Auburn University, Department of Civil Engineering Interim Report submitted to HDR Engineering, Inc., Pittsburgh Office, Pittsburgh, PA, August.
8. Hall, D. H., Grubb, M. A., and Yoo, C. H. (1999). "Design example, horizontally curved steel I girder bridge." Appendix E of the final report submitted to National Cooperative Highway Research Program, Project 12-38, Transportation Research Board, National Research Council, Washington, D.C., May.
9. Hall, D. H., Lawin, A. R., and Yoo, C. H. (1999). "Design example, horizontally curved steel box girder bridge." Appendix F of the final report submitted to National Cooperative Highway Research Program, Project 12-38, Transportation Research Board, National Research Council, Washington, D.C., July.
10. Richardson, Gordon and Associates, "Analysis and Design of Horizontally Curved Steel Bridge Girders," United States Steel Structural Report, ADUCO 91063, May 1963.
11. Tung and Foundation, "Approximate Torsional Analysis of Curved Box Girders by the M/R Method," *Engineering Journal,* AISC, Vol. 7, No. 3, July 1970.
12. Hall, D. H. (1994). "BSDI 3D System." Internal Document, Bridge Software Development International, Ltd., Coopersburg, PA.
13. AASHTO (1993). *Guide Specifications for Horizontally Curved Highway Bridges.* American Association of State Highway and Transportation Officials, Inc., Washington, D.C.
14. Duwadi, S. R., Grubb, M. A., Yoo, C. H., and Hartmann, J. (2000). "Federal highway administration's horizontally curved steel bridge research project." *Proceedings of the 5th International Bridge Conference,* Tampa, FL, April 3–5.
15. Duwadi, S. R., Hall, D. H., Yadlosky, J. M., Yoo, C. H., and Zureick, A. (1995). "FHWA-CSBRP I-girder component testing." *Proceedings of the Structures Congress 13,* ASCE, Vol. 2, pp. 1695–1698, Boston, MA, April 24–28.
16. Duwadi, S. R., Yadlosky, J. M., and Yoo, C. H. (1994). "Horizontally curved steel bridge research-update 2." *Proceedings of the Structures Congress 12,* ASCE, Vol. 2, pp. 1071–1076, Atlanta, GA, April 24–28.
17. Hall, D. H. (1997). "Proposed curved girder provisions for AASHTO." *Building to Last Structures Congress—Proceedings,* Vol. 1, ASCE, New York, NY, pp. 151–154.
18. Yoo, C. H. (1998). "Recommended specifications for horizontally curved steel-girder highway bridges." *Proceedings of the 1998 Pacific Rim Steel Structures Conference,* Seoul, Korea, October 13–16. pp. 37–40.
19. Yoo, C. H., Hall, D. H., and Sabol, S. A. (1995). "Improved design specifications for horizontally curved steel-girder highway bridges." *Proceedings of the Structures Congress 13,* ASCE, Vol. 2, pp. 1699–1702, Boston, MA, April 2–5.
20. Galambos, T. V., Hajjar, J. F., Leon, R. T., Huang, W. H., Pulver, B. E., and Rudie, B. J. (1996). "Stresses in steel curved girder bridges." Minnesota Department of Transportation, Office of Research Administration, Transportation Building, 395 John Ireland Boulevard, St. Paul, Minnesota 55155-1899.
21. Richardson, J. A., and Douglas, B. M. (1993). "Results from field testing a curved box-girder bridge using simulated earthquake loads." *Earthquake Engineering and Structural Dynamics,* Vol. 22, No. 10.
22. Utah State University. (2000). "Bridge Test Report."
23. Arman EPST. (1994). "A finite element program for automatic static and dynamic analysis and design of horizontally curved I-girder bridges: CIG4BR (Curved I-Girder 4 degrees of freedom bridge)." AKMAN Engineering Production Software Technologies, 9350 Washington Blvd., Lanham, Maryland 20706-3119, USA.
24. Kitada, T., Nakai, H., and Murayama, Y. (1993). "State-of-the art on research, design and construction of horizontally curved bridges in Japan." *Proceedings of the SSRC Annual Technical Session,* Milwaukee, WI.

25. Zureick, A., and Naqib, R. (1999). "Horizontally curved steel I-girders state-of-the-art analysis methods." *Journal of Bridge Engineering,* ASCE, Vol. 4, No. 1, February.
26. Bouabdallah, M. S., and Batoz, J. L. (1996). "Formulation and evaluation of a finite element model for the linear analysis of stiffened composite cylindrical panels." *Finite Elements in Analysis and Design,* Vol. 21, No. 4, April.
27. Bozhevolnaya, E., and Kildegaard, A. (1997). "Experimental study of a uniformly loaded curved sandwich beam." *Composite Structures,* Vol. 40, No. 2, December.
28. Cleghorn, W. L., Tabarrok, B., and Lee, T. W. (1993). "Vibration of rings with unsymmetrical cross-sections: A finite element approach." *Journal of Sound and Vibration,* Vol. 168, No. 1, November.
29. Dhondt, G., and Kohl, M. (1999). "Effect of the geometry and the load level on the dynamic failure of rotating disks." *International Journal of Solids and Structures,* Vol. 36, No. 6, February.
30. Grubb, M. A., Yadlosky, J. M., and Herrmann, A. W. (1993). "Behavior of horizontally curved steel highway bridges." *Structural Engineering in Natural Hazards Mitigation,* Proceedings of the Symposium on Structural Engineering in Natural Hazards Mitigation. ASCE, New York, NY.
31. Meyerpiening, H. R., and Anderegg, R. (1995). "Buckling and Post-buckling investigations of imperfect curved stringer-stiffened composite shell. A. Experimental investigation and effective width evaluation." *Thin-Walled Structures,* Vol. 23, No. 1–4.
32. Moas, E., Boitnott, R. L., and Hayden, G. O. (1994). "Analytical and experimental investigation of the response of the curved, composite frame/skin specimens." *Journal of the American Helicopter Society,* Vol. 39, No. 3, July.
33. Shanmugam, N. E., Thevendran, V., Richard Liew, J. Y., and Tan, L. O. (1995). "Experimental study on steel beams curved in plan." *Journal of Structural Engineering,* ASCE, Vol. 121, No. 2, February.
34. Shim, V. P. W., and Quah, S. E. (1998). "Solution of impact-induced flexural waves in a circular ring by the method of characteristics." *Journal of Applied Mechanics-Transactions of the ASME,* Vol. 65, No. 3, September.
35. Stiftinger, M. A., Skrnajakl, I. C., and Rammerstorfer, F. G. (1995). "Buckling and postbuckling investigations of imperfect curved stringer-stiffened composite shell, part B. Computational investigations." *Thin-Walled Structures,* Vol. 23, No. 1–4.
36. Choi, J. K., and Lim, J. K. (1993). "Simple curved shear beam elements." *Communications in Numerical Methods in Engineering,* Vol. 9, No. 8, August.
37. Choi, J. K., and Lim, J. K. (1995). "General curved beam elements based on the assumed strain fields." *Computers and Structures,* Vol. 55, No. 3, May.
38. Davidson, J. S. (1996). "Nominal bending and shear strength of horizontally curved steel I-girder bridges." Dissertation submitted to the Graduate Faculty of Auburn University in partial fulfillment of the requirements for the degree of Doctor of Philosophy.
39. Dorfi, H. R., and Busby, H. R. (1994). "Effective curved composite beam finite element based on the hybrid-mixed formulation." *Computers and Structures,* Vol. 53, No. 1, October.
40. Fu, C. C., and Hsu, Y. T. (1995). "Development of an improved curvilinear thin-walled Vlasov element." *Computers and Structures,* Vol. 54, No. 1, January.
41. Hall, D. H. (1996). "Curved girders are special." *Engineering Structures,* Vol. 18, No. 10, October.
42. Ibrahimbegovic, A. (1995). "On finite element implementation of geometrically nonlinear Reissner's beam theory: Three-dimensional curved beam elements." *Computer Methods in Applied Mechanics and Engineering,* Vol. 122, No. 1–2, April.
43. Kang, Y. J., and Yoo, C. H. (1994). "Thin-walled curved beams. II: analytical solutions for buckling of arches." *Journal of Engineering Mechanics,* ASCE, Vol. 120, No. 10, October.
44. Kim, J. G., and Kim, Y. Y. (1998). "New higher-order hybrid-mixed curved beam element." *International Journal for Numerical Methods in Engineering,* Vol. 43, No. 5, November.
45. Koziey, B. L., and Mirza, F. A. (1994). "Consistent curved beam element." *Computers and Structures,* Vol. 51, No. 6, June.
46. Lee, S. S., Koo, J. S., and Choi, J. M. (1996). "Development of a new curved beam element with shear effect." *Engineering Computations* (Swansea, Wales), Vol. 13, No. 2–4.
47. Lim, C. W., Wang, C. M., and Kitipornchai, S. (1997). "Timoshenko curved beam bending solutions in terms of Euler-Bernoulli solutions." *Archive of Applied Mechanics,* Vol. 67, No. 3, February.
48. Lin, S. M. (1998). "Exact solutions for extensible circular curved Timoshenko beams with nonhomogeneous elastic boundary conditions." *Acta Mechanica,* Vol. 130, No. 1–2.
49. Litewka, P., and Rakowski, J. (1997). "Efficient curved beam finite element." *International Journal for Numerical Methods in Engineering,* Vol. 40, No. 14, July.
50. Mentrasti, L. (1996). "Shear-torsion of large curvature beams—Part II. Applications to thin bodies." *International Journal of Mechanical Sciences,* Vol. 38, No. 7, July.
51. Orth, F. J., and Surana, K. S. (1994). "P-version two-dimensional beam element for geometrically nonlinear analysis." *Computers and Structures,* Vol. 50, No. 3, February.
52. Paavola, J., and Salonen, E. M. (1999). "Strain and stress analysis of a curved tapered beam model." *Computers and Structures,* Vol. 72, No. 4–5.
53. Pi, Y. L., and Trahair, N. S. (1996). "Nonlinear elastic behaviour of I-beams curved in plan." Research Report—University of Sydney, School of Civil and Mining Engineering, No. 734, December, pp. 1–45.
54. Raveendranath, P., Singh, G., and Pradhan, B. (1999). "Two-noded locking-free shear flexible curved beam element." *International Journal for Numerical Methods in Engineering,* Vol. 44, No. 2, January.
55. Pak, R. Y. S., and Stauffer, E. J. (1994). "Nonlinear finite deformation analysis of beams and columns." *Journal of Engineering Mechanics,* ASCE, Vol. 120, No. 10, October.
56. Ryu, H. S., and Sin, H. C. (1996). "Curved beam elements based on strain fields." *Communications in Numerical Methods in Engineering,* Vol. 12, No. 11, November.
57. Sengupta, D., and Dasgupta, S. (1997). "Static and dynamic applications of a five noded horizontally curved beam element with shear deformation." *International Journal for Numerical Methods in Engineering,* Vol. 40, No. 10, May.
58. Simpson, M. D., and Birkemoe, P. C. (1997). "Nonlinear finite element modeling of curved girder experiments." *Structures—Design, Concrete and Reinforced Concrete Structures, Bridges Proceedings,* Annual Conference, Canadian Society for Civil Engineering, Vol. 7, Canadian Society for Civil Engineering, Montreal, Quebec, Canada.
59. Thevendran, V., Chen, S., Shanmugam, N. E., and Liew, J. Y. R. (1999). "Nonlinear analysis of steel-concrete composite beams curved in plan." *Finite Elements in Analysis and Design,* Vol. 32, No. 3.
60. Yang, S. Y., and Sin, H. C. (1995). "Curvature-based beam elements for the analysis of Timoshenko and shear-deformable curved beams." *Journal of Sound and Vibration,* Vol. 187, No. 4, November.
61. Hu, N., Hu, B., Yan, B., Fukunaga, H., and Sekine, H. (1999). "Two kinds of CO-type elements for buckling analysis of thin-walled

curved beams." *Computer Methods in Applied Mechanics and Engineering,* Vol. 171, No. 1.
62. Kang, Y. J., and Yoo, C. H. (1994). "Thin-walled curved beams. I: Formulation of nonlinear equations." *Journal of Engineering Mechanics,* ASCE, Vol. 120, No. 10, October.
63. Kang, Y. J., Yoo, C. H., Yu., C., and Lee, H. E. (1993). "Lateral buckling behavior of thin-walled arches." *Proceedings of the Fourth East Asia-Pacific Conference on Structural Engineering and Construction,* pp. 401–406, Seoul, Korea, September 20–22.
64. Yoo, C. H., Kang, Y. J., and Davidson, J. S. (1996). "Buckling analysis of curved beams by finite-element discretization." *Journal of Engineering Mechanics,* ASCE, Vol. 122, No. 8, August.
65. Gendy, A. S., and Saleeb, A. F. (1994). "Vibration analysis of coupled extensional/flexural/torsional modes of curved beams with arbitrary thin-walled sections." *Journal of Sound and Vibration,* Vol., 174, No. 2, July.
66. Howson, W. P., and Jemah, A. K. (1999). "Exact out-of-plane natural frequencies of curved Timoshenko beams." *Journal of Engineering Mechanics,* ASCE, Vol. 125, No. 1, January.
67. Howson, W. P., Jemah, A. K., and Zhou, J. Q. (1995). "Exact natural frequencies for out-of-plane motion of plane structures composed of curved beam members." *Computers and Structures,* Vol. 55, No. 6, June.
68. Hsiao, K. M., and Yang, R. T. (1995). "Co-rotational formulation for nonlinear dynamic analysis of curved Euler beam." *Computers and Structures,* Vol. 54, No. 6, March.
69. Huang, D., Wang, T. L., and Shahawy, M. (1995). "Dynamic behavior of horizontally curved I-girder bridges." *Computers and Structures,* Vol. 57, No. 4, November.
70. Jiang, J., and Olson, M. D. (1994). "Vibration analysis of orthogonally stiffened cylindrical shells using super finite elements." *Journal of Sound and Vibration,* Vol. 173, No. 1, May.
71. Kang, K. J., Bert, C. W., and Striz, A. G (1996). "Vibration analysis of horizontally curved beams with warping using DQM." *Journal of Structural Engineering,* ASCE, Vol. 122, No. 6, June.
72. Kawakami, M., Sakiyama, T., Matsuda, H., and Morita, C. (1995). "In-plane and out-of-plane free vibrations of curved beams with variable sections." *Journal of Sound and Vibration,* Vol. 187, No. 3, November.
73. Wang, R. T., and Sang, Y. L. (1999). "Out-of-plane vibration of multi-span curved beam due to moving loads." *Structural Engineering and Mechanics,* Vol. 7, No. 4.
74. Yildirim, V. (1999). "In-plane free vibration of symmetric cross-ply laminated circular bars." *Journal of Engineering Mechanics,* ASCE, Vol. 125, No. 6, June.
75. Razaqpur, A. G., and Li, H. G. (1994). "Refined analysis of curved thin-walled multicell box girders." *Computers and Structures,* Vol. 53, No. 1.
76. Razaqpur, A. G., and Li, H. G. (1997). "Analysis of curved multicell box girder assemblages." *Structural Engineering and Mechanics,* Vol. 5, No. 1.
77. Yabuki, T., Arizumi, Y., and Vinnakota, S. (1995). "Strength of thin-walled box girders curved in plan." *Journal of Structural Engineering,* ASCE, Vol. 121, No. 5.
78. Galdos, N. H., Schelling, D. R., and Sahin, M. A. (1993). "Methodology for impact factor of horizontally curved box bridges." *Journal of Structural Engineering,* ASCE, Vol. 119, No. 6.
79. Sennah, K. M., and Kennedy, J. B. (1997). "Dynamic characteristics of simply supported curved composite multi-cell bridges." *Canadian Journal of Civil Engineering,* No.4, August.
80. Davidson, J. S., and Yoo, C. H. (1996). "Local Buckling of Curved I-Girder Flanges." *Journal of Structural Engineering,* ASCE, Vol. 122, No. 8, August.
81. Davidson, J. S., and Yoo, C. H. (2000). "Evaluation of strength formulations for horizontally curved flexural members." *Journal of Bridge Engineering,* ASCE, Vol. 5, No. 3, August.
82. Davidson, J. S., Ballance, S. R., and Yoo, C. H. (2000). "Behavior of curved I-girder webs subjected to combined bending and shear." *Journal of Bridge Engineering,* ASCE, Vol. 5, No. 2, May.
83. Davidson, J. S., Ballance, S. R., and Yoo, C. H. (2000). "Effects of longitudinal stiffeners on curved I-girder webs." *Journal of Bridge Engineering,* ASCE, Vol. 5, No. 2, May.
84. Davidson, J. S., Ballance, S. R., and Yoo, C. H. (1999). "Finite displacement behavior of curved I-girder webs subjected to bending." *Journal of Bridge Engineering,* ASCE, Vol.4, No. 3, August.
85. Davidson, J. S., Ballance, S. R., and Yoo, C. H. (1999). "Analytical model of curved I-girder web panels subjected to bending." *Journal of Bridge Engineering,* ASCE, Vol. 4, No. 3, August.
86. Lee, S. C., and Yoo, C. H. (1999). "Strength of curved I-girder web panels under pure shear." *Journal of Structural Engineering,* ASCE, Vol. 125, No. 8.
87. Schilling, C. G. (1996). "Yield-interaction relationships for curved I-girders." *Journal of Bridge Engineering,* ASCE, Vol. 1, No. 1, February.
88. Weaver, DL. (1996). "Steel girder bridges." *Construction Specifier,* Vol. 49, No. 5, May.
89. Yoo, C. H. (1993). "Some Considerations in the design and construction of horizontally curved highway bridges." *Proceedings of the Fourth East Asia-Pacific Conference on Structural Engineering and Construction,* p. 55–60, Seoul, Korea, September 20–22.
90. Yoo, C. H., and Davidson, J. S. (1997). "Yield interaction equations for nominal bending strength of curved I-girders." *Journal of Bridge Engineering,* ASCE, Vol. 2, No. 2, May.
91. Cheung, M. S., and Foo, S. H. C. (1995). "Design of horizontally curved composite box-girder bridges—A simplified approach." *Canadian Journal of Civil Engineering,* Vol. 22, No. 1.
92. Sennah, K., and Kennedy, J. B. (1998). "Shear distribution in simply-supported curved composite cellular bridges." *Journal of Bridge Engineering,* ASCE, Vol. 3, No. 2.
93. Sennah, K., and Kennedy, J. B. (1999). "Simply supported curved cellular bridges: Simplified design method." *Journal of Bridge Engineering,* ASCE, Vol. 4, No. 2.
94. Yoo, C. H., Davidson, J. S., and Zhang, J. (1998). "Top flange diagonal bracing of horizontally curved box girders." *Proceedings of the Engineering Mechanics Conference: A Force for the 21st Century,* La Jolla, CA, May 18–20.
95. ANSI/AASHTO/AWS D1.5-96 (1996), *Bridge Welding Code*.
96. Heins and Hall (1981). *Designer's Guide to Steel Box Girder Bridges,* Bethlehem Steel Corporation (Booklet No. 3500).

Abstracts

AUTHOR: Zureick, A., Naqib, R., and Yadlosky, J. M. (1994)

TITLE: Curved steel bridge research project

INFO: Interim Report I, "Synthesis," FHWA Contract No. DT FH61-93-C-00136, FHWA, McLean, VA, December.

ABSTRACT: A comprehensive literature search on horizontally curved girder bridges up to 1993.

AUTHOR: Hall, D. H., Grubb, M. A., and Yoo, C. H. (1999)

TITLE: NCHRP Report 424: Improved Design Specifications for Horizontally Curved Steel Girder Highway Bridges

INFO: Transportation Research Board, National Research Council, Washington, D.C.

ABSTRACT: The primary objective of NCHRP Project 12-38 was to develop revised Guide Specifications for Horizontally Curved Highway Bridges, based on current practice and technology, that could be recommended to AASHTO for possible adoption. The revised Guide Specifications were to be applicable to the design, fabrication, and erection of horizontally curved steel I-girder and box-girder bridges. The research team was to complete ten tasks in this project. Chapter 1 provides a detailed description of each task. Six appendixes were also prepared and submitted: Appendix A: I-Girder Curvature Study; Appendix B: Curved Girder Design and Construction, Current Practice; Appendix C: A Unified Approach for Designing and Constructing Horizontally Curved Girder Bridges, Proposed AASHTO Specifications (Highlights of Major Changes); Appendix D: Recommended Specifications for Steel Curved-Girder Bridges and Commentary; Appendix E: Design Example, Horizontally Curved Steel I-Girder Bridge; and Appendix F: Design Example, Horizontally Curved Steel Box-Girder Bridge. This report summarizes the development of the revised Guide Specifications and the Appendixes. Appendixes D through F are not published in Report 424.

AUTHOR: Davidson J. S., Keller, M. A., and Yoo, C. H. (1996)

TITLE: Cross-frame spacing and parametric effects in horizontally curved I-girder bridges

INFO: Journal of Structural Engineering, ASCE, Vol. 122, No. 9, September.

ABSTRACT: The finite element method was used to create detailed models of horizontally curved steel I-girder bridges connected by cross-frames. The effects of a number of parameters on the behavior of the curved girder system were established and compared to the effect of these parameters in straight girder systems. The parameters that most significantly affect the behavior of the systems were determined to be the degree of curvature, span length, and flange width. An equation was developed based on a nonlinear statistical regression to provide a preliminary design limit for the cross-frame spacing interval.

AUTHOR: Hall, D. H., and Yoo, C. H. (1998)

TITLE: Recommended specifications for steel curved-girder bridges

INFO: Appendix D of the final report submitted to National Cooperative Highway Research Pro-

gram, Project 12-38, Transportation Research Board, National Research Council, Washington, D.C., December.

ABSTRACT: Recommended specifications and commentary.

AUTHOR: Hanshin Express Public Corporation and Steel Structure Study Committee (1988)

TITLE: Guidelines for the Design of Horizontally Curved Girder Bridges (Draft)

INFO: Hanshin Expressway Public Corporation, October.

ABSTRACT: This is the only semi-official design guide of horizontally curved girder bridges in the world today other than AASHTO Guide Specifications. The document has not been officially adopted by the Japanese specification-issuing body. The guide includes an I-girder design example and a box-girder design example.

AUTHOR: Nakai, H., and Yoo, C. H. (1988)

TITLE: Analysis and Design of Horizontally Curved Steel Bridges

INFO: McGraw-Hill Book Company, New York.

ABSTRACT: The one and only comprehensive technical sequence book on the subject in the world today. The book presents fundamentals of thin-walled curved girder behavior and introduces Japanese curved bridge design and construction practice. The book explains the background research for most provisions of the Hanshin Guidelines.

AUTHOR: Yoo, C. H. (1996)

TITLE: Progress Report on FHWA-CSBRP-Task D

INFO: FHWA Contract No. DTFH61-92-C-00136, Auburn University, Department of Civil Engineering Interim Report submitted to HDR Engineering, Inc., Pittsburgh Office, Pittsburgh, PA, August.

ABSTRACT: Task D of CSBRP is to develop analytical predictor equations for the nominal bending strength and shear strength of I-girders. Included in the interim report, in addition to the nominal bending strength and shear strength of I-girders, are the design of transverse stiffeners and longitudinal stiffeners of I-girders; a guide for an optimum cross-frame spacing; and an independent analysis (NASTRAN) of the component test frame, the local buckling of curved I-girder compression flanges, and the lifting and transporting of slender curved I-girders.

AUTHOR: Hall, D. H., Grubb, M. A., and Yoo, C. H. (1999)

TITLE: Design example, horizontally curved steel I girder bridge

INFO: Appendix E of the final report submitted to National Cooperative Highway Research Program, Project 12-38, Transportation Research Board, National Research Council, Washington, D.C., May.

ABSTRACT: I-girder bridge design example highlighting the design procedure following the revised provisions.

AUTHOR: Hall, D. H., Lawin, A. R., and Yoo, C. H. (1999)

TITLE: Design example, horizontally curved steel box girder bridge

INFO: Appendix F of the final report submitted to National Cooperative Highway Research Program, Project 12-38, Transportation Research Board, National Research Council, Washington, D.C., July.

ABSTRACT: Box-girder bridge design example highlighting the design procedure following the revised provisions.

AUTHOR: Hall, D. H. (1994)

TITLE: BSDI 3D System

INFO: Internal Document, Bridge Software Development International, Ltd., Coopersburg, PA.

ABSTRACT: A special-purpose 3D finite element program package tailored to analyze highway bridge superstructures under dead load and any design live loads, including impact and centrifugal load. Temperature and wind load analyses can readily be conducted. The eccentric load effects resulting from bearing forces on skewed bridges and curved bridges can be examined routinely.

AUTHOR: Duwadi, S. R., Grubb, M. A., Yoo, C. H. and Hartmann, J. (2000)

TITLE: Federal highway administration's horizontally curved steel bridge research project

INFO: Proceedings of the 5th International Bridge Conference, Tampa, FL, April 3–5.

ABSTRACT: Since 1992, the Federal Highway Administration (FHWA) has had a major concentrated research project in the area of horizontally curved steel bridges, the objective of which is to conduct research to better define the fundamental behavior of such bridges. This project involves theoretical work leading to the development of refined predictor equations, and verification of those equations through linear and non-linear analysis and experimental testing of I-girder components. The overall experimental program involves testing of a series of full-scale bending and shear curved steel I-girder components, and subsequently, a full-size bridge. The objectives of this paper are to summarize the development and refinement of predictor equations and to describe the work leading to the first series of experimental tests, which involve testing of full-scale bending components. The research team consists of HDR Engineering, Inc.; Auburn University; Georgia Institute of Technology; BSDI, Ltd.; and FHWA structures and support staff.

AUTHOR: Duwadi, S. R., Hall, D. H., Yadlosky, J. M., Yoo, C. H., and Zureick, A. (1995)

TITLE: FHWA-CSBRP I-girder component testing

INFO: Proceedings of the Structures Congress 13, ASCE, Vol. 2, Boston, MA, April 24–28.

ABSTRACT: This paper is an update on the continuing research ongoing under the FHWA Curved Steel Bridge Research Program (CSBRP). An overview of this comprehensive program was presented at the 1994 ASCE Structures Congress (Duwadi et al. 1994). A brief highlight of all the tasks involved is presented. The major focus of this paper is on the proposed experimental testing of the I-beam components at the FHWA laboratory at the Turner Fairbank Highway Research Center (FHWA 1993).

AUTHOR: Duwadi, S. R., Yadlosky, J. M., and Yoo, C. H. (1994)

TITLE: Horizontally curved steel bridge research—update 2

INFO: Proceedings of the Structures Congress 12, ASCE, Vol. 2, Atlanta, GA, April 24–28.

ABSTRACT: The AASHTO Guide Specifications for Horizontally Curved Highway Bridges was initially issued in 1980 and is based on research performed by the CURT project in the early 1970s. They cover both I-girders and box-girders. AISI later financed the development of related provisions for load factor design of curved girders. These were later adopted by AASHTO as part of the same curved girder guide specifications. More than 12 years of design and construction experience with the guide specifications has uncovered a number of major deficiencies. As a result of this, there exists an urgent need to conduct a series of research studies so that improved recommended guide specifications can be presented to AASHTO for consideration for adoption. To address these needs, the Federal Highway Administration and the AASHTO-sponsored National Cooperative Highway Research Program are jointly administrating a coordinated program of research intended to provide a revised recommended ASD and LRFD design specification and eventually a recommended LRFD-based design specification. This paper presents an overview of these two major research projects.

AUTHOR: Hall, D. H. (1997)

TITLE: Proposed curved girder provisions for AASHTO

INFO: Building to Last Structures Congress—Proceedings, Vol. 1, ASCE, New York, NY.

ABSTRACT: This paper discusses the draft proposed specification for design of horizontally curved steel highway bridges. A draft of the provisions has been submitted to AASHTO for consideration of adoption. An overview of the provisions is provided, including highlights of modifications to the existing AASHTO Guide Specifications for horizontally curved girders.

AUTHOR: Yoo, C. H. (1998)

TITLE: Recommended specifications for horizontally curved steel-girder highway bridges

INFO: Proceedings of the 1998 Pacific Rim Steel Structures Conference, Seoul, Korea, October 13–16.

ABSTRACT: An overview of the Recommended Specifications for Horizontally Curved Steel-Girder Highway Bridges is presented. The research project was funded by NCHRP Project 12-38.

AUTHOR: Yoo, C. H., Hall, D. H., and Sabol, S. A. (1995)

TITLE: Improved design specifications for horizontally curved steel-girder highway bridges

INFO: Proceedings of the Structures Congress 13, ASCE, Vol. 2, Boston, MA, April 2–5.

ABSTRACT: A research program focused on the development of improved design specifications for horizontally curved steel girder highway bridges has been underway for the past two years. The objective of the research is to prepare improved guide specifications and commentary based on current technology and available information. A thorough literature search and evaluation and survey of current practice throughout the world have been reflected in the development of the specifications and commentary.

AUTHOR: Galambos, T. V., Hajjar, J. F., Leon, R. T., Huang, W. H., Pulver, B. E., and Rudie, B. J. (1996)

TITLE: Stresses in steel curved girder bridges

INFO: Minnesota Department of Transportation, Office of Research Administration, Transportation Building, 395 John Ireland Boulevard, St. Paul, Minnesota 55155-1899.

ABSTRACT: A steel curved I-girder bridge system may be more susceptible to instability during construction than bridges constructed of straight I-girders. The primary goal of this project is to study the behavior of the steel superstructure of curved steel I-girder bridge systems during all phases of construction and to ascertain whether the linear elastic analysis software used by Mn/DOT during the design process represents well the actual stresses in the bridge. Sixty vibrating wire strain gages were applied to a two-span, four-girder bridge, and the resulting stresses and deflections were compared with computational results for the full construction sequence of the bridge. The computational results from the Mn/DOT analysis software were first shown to compare well with results from a program developed specifically for this project (called the “UM program”) since the latter permits more detailed specification of actual loading conditions on the bridge during construction. The UM program, in turn, correlated well with the field measurements, especially for the primary flexural stresses. Warping stresses induced in the girders, and the stresses in the crossframes, were more erratic, but showed reasonable correlation. It is concluded that Mn/DOT’s analysis software captures the behavior well for these types of curved girder bridge systems and that the stresses in these bridges may be relatively low if their design is controlled largely by stiffness.

AUTHOR: Richardson, J. A., and Douglas, B. M. (1993)

TITLE: Results from field testing a curved box-girder bridge using simulated earthquake loads

INFO: Earthquake Engineering and Structural Dynamics, Vol. 22, No. 10.

ABSTRACT: This paper presents the results of a unique field test on a curved highway overpass. In the test, large horizontal loads were applied to the superstructure of the bridge and quickly released, causing the bridge to vibrate. The resulting large-amplitude vibrations were intended to be similar to the vibrations caused by earthquakes (horizontal accelerations of up to 25 percent of gravity were measured on the bridge deck). Well-defined lateral, longitudinal, vertical and torsional vibration modes were identified from the test data. The vibration modes were used to verify an analytical model of the bridge’s dynamic response. For this paper, the model was verified using only the fundamental vibration mode, which was primarily a horizontal vibration mode. Using a system identification procedure, the dynamic response model was adjusted until its frequency and mode shape matched the measured frequency and mode shape. Parameters in the verified model were compared with the same parameters calculated from information in the structural drawings. Because the fundamental mode represents a horizontal mode, the bridge parameters identified in this paper were parameters that strongly influence the horizontal response of the bridge.

AUTHOR: Utah State University (2000)

TITLE: Bridge Test Report

INFO: Utah DOT Report UT-03.02

ABSTRACT: A series of static live load tests were conducted on NB I-15 to WB I-215 Connector Ramp, which was to be replaced due to its low structural evaluation rating (structurally deficient). The bridge is a three-span continuous structure (41 feet 6 inches, 69 feet 3 inches, and 41 feet 6 inches) with five I-girders spaced at 8 feet 10 inches on a radius of 480 feet. The girders are noncomposite and prismatic. There are no cross frames. Only small channel section diaphragms are connected to connection plates/transverse stiffeners. Utah State University is the principal investigator with the instrumentation support provided by Bridge Diagnostic Inc. under the general oversight by FHWA/Utah DOT.

AUTHOR: Arman EPST. (1994)

TITLE: A finite element program for automatic static and dynamic analysis and design of horizontally curved I-girder bridges: CIG4BR (Curved I-Girder 4 Degrees of Freedom Bridge)

INFO: AKMAN Engineering Production Software Technologies, 9350 Washington Blvd., Lanham, Maryland 20706-3119, USA.

ABSTRACT: This is a user documentation for CIG4BR (Curved I-Girder 4 Degrees of Freedom Bridge). The main program was developed based on two-dimensional modeling of the bridge superstructure, including warping and the effect of cross frames and diaphragms. There are four worked out examples included.

AUTHOR: Kitada, T., Nakai, H., and Murayama, Y. (1993)

TITLE: State-of-the art on research, design and construction of horizontally curved bridges in Japan

INFO: Proceedings of the SSRC Annual Technical Session, Milwaukee, WI.

ABSTRACT: This paper emphasizes the necessity of research study on the ultimate strength concerning the horizontally curved plate and box-girders through a survey of references published in Japan since 1977. Then the outline of a latest draft design method for curved plate girders is presented. Next, the applicable ranges of parameters on buckling stability of the curved box-girders are shown on the basis of a questionnaire for about 260 curved box-girder bridges. Finally, a special curved continuous spiral girder bridge under construction is also introduced together with a few numerical results based on the elasto-plastic and finite displacement analysis for this bridge under seismic load.

AUTHOR: Zureick, A., and Naqib, R. (1999)

TITLE: Horizontally curved steel I-girders state-of-the-art analysis methods

INFO: Journal of Bridge Engineering, ASCE, Vol. 4, No. 1, February.

ABSTRACT: Current AASHTO specifications pertaining to the analysis and design of horizontally curved bridges are based upon research work conducted prior to 1978. Since then, a significant amount of work has been conducted to further advance analysis methods and to better understand the behavior of these complex structural systems. Unfortunately, the results of these various research efforts are scattered and, in some cases, unevaluated. This paper complements and updates survey articles published in 1968 and 1978 and presents highlights of the analytical work conducted on horizontally curved steel I-girder bridges.

AUTHOR: Bouabdallah, M. S., and Batoz, J. L. (1996)

TITLE: Formulation and evaluation of a finite element model for the linear analysis of stiffened composite cylindrical panels

INFO: Finite Elements in Analysis and Design, Vol. 21, No. 4, April.

ABSTRACT: A finite element model for linear static and free vibration analysis of composite cylindrical panels with composite stiffeners is presented. The proposed model is based on a cylindrical shell finite element, which uses a first-order shear deformation theory. The stiffeners are curved beam elements based on Timoshenko and Saint-Venant assumptions for bending and torsion respectively. The two elements are developed in a cylindrical coordinate system and their stiffness matrices result from a hybrid-mixed formulation where the element assumed stress field is such that exact equilibrium equations are satisfied. The elements are free of membrane and shear locking with correct satisfaction of rigid body motions. Several examples dealing with stiffened isotropic and laminated plates and shells with eccentric as well as concentric stiffeners are analyzed showing the validity of the models.

AUTHOR: Bozhevolnaya, E., and Kildegaard, A. (1997)

TITLE: Experimental study of a uniformly loaded curved sandwich beam

INFO: Composite Structures, Vol. 40, No. 2, December.

ABSTRACT: A sandwich curved beam subjected to a uniform loading is experimentally investigated. Load-deflection and thrust-deflection dependencies are shown to be nonlinear, while load-deformation dependencies for sandwich faces are found to be linear. A technical solution for implementation of the simple support is realized. An actual stiffness of the beam tie is measured experimentally and estimated theoretically.

AUTHOR: Cleghorn, W. L., Tabarrok, B., and Lee, T. W. (1993)

TITLE: Vibration of rings with unsymmetrical cross-sections: A finite element approach

INFO: Journal of Sound and Vibration, Vol. 168, No. 1, November.

ABSTRACT: A finite element model is developed for free, coupled in-plane and out-of-plane, vibration of a curved beam with an arbitrary unsymmetrical cross-section. Solutions of the governing differential equations of static equilibrium are used as shape functions for deriving the element stiffness and mass matrices. The performance of the element developed is assessed by comparing results obtained with those found experimentally, and comparing those obtained using a commercially available finite element package.

AUTHOR: Dhondt, G., and Kohl, M. (1999)

TITLE: Effect of the geometry and the load level on the dynamic failure of rotating disks

INFO: International Journal of Solids and Structures, Vol. 36, No. 6, February.

ABSTRACT: The simplified and the higher-order curved beam theory published previously [International Journal of Solids and Structures 30(1), 137–149 (1993) and International Journal of Solids and Structures 31(14), 1949–1965 (1994)] are applied to rotating disks with rectangular cross-section subject to an instantaneous radial failure. The dependence of the angular size of the debris on the inner to outer radius ratio and on the load level is examined and compared with semi-empirical and experimental results in the literature. Finite element calculations confirm the results obtained with the higher-order beam theory for moderate to high inner to outer radius ratios. It is shown that agreement with the experimental results can be further improved by a proper choice of the boundary conditions.

AUTHOR: Grubb, M. A., Yadlosky, J. M., and Herrmann, A. W. (1993)

TITLE: Behavior of horizontally curved steel highway bridges

INFO: Structural Engineering in Natural Hazards Mitigation, Proceedings of the Symposium on Structural Engineering in Natural Hazards Mitigation. ASCE, New York, NY.

ABSTRACT: Horizontally curved highway bridges represent up to 25 percent of the market for new steel bridge construction each year. However, unanswered questions remain concerning the fundamental behavior of horizontally curved steel girders that may not be adequately addressed in current design specifications. While there has been some isolated research, a coordinated large-scale effort to study curved girder behavior has not been undertaken in over 20 years. The FHWA and the NCHRP are launching significant research programs on curved steel bridges in late 1992 and early 1993, respectively. However, additional research is recommended to supplement these programs to ensure coverage of areas these programs cannot address. One particular area of interest that may not be addressed is field instrumentation and testing of actual in-service curved steel bridges. Field measurements can provide valuable data to complement and verify previous research. The entire curved-girder research effort must be coordinated from the start to avoid needless repetition of effort. All projects must be overseen by technically competent individuals who are willing to devote the time needed to successfully overcome all barriers to success.

AUTHOR: Meyerpiening, H. R., and Anderegg, R. (1995)

TITLE: Buckling and post-buckling investigations of imperfect curved stringer-stiffened composite shell. A. Experimental investigation and effective width evaluation

INFO: Thin-Walled Structures, Vol. 23, No. 1–4.

ABSTRACT: A box-like stringer-stiffened thin-walled CFRP structure was subjected to load cases well beyond the limits of local buckling. The development of the deflection pattern was recorded via optical means and analyzed numerically. In addition, the structure was modeled and analyzed using the MARC FE program in the nonlinear deflection range. The geometrical imperfections of the test structure were recorded by mechanical scanning and optical methods and introduced into the mathematical model. For the perfect ("ideal") and the geometrically imperfect ("real") structural model, the results of the FE analyses were compared and used to judge the effect of geometric imperfections on the post-buckling behavior of the structure. The effective axial stiffness for the various post-buckling states was evaluated and related to analytical estimates of effective width values for orthotropic sheet-like panels.

AUTHOR: Moas, E., Boitnott, R. L., and Hayden, G. O. (1994)

TITLE: Analytical and experimental investigation of the response of the curved, composite frame/skin specimens

INFO: Journal of the American Helicopter Society, Vol. 39, No. 3, July.

ABSTRACT: Six-foot-diameter, semicircular graphite/epoxy specimens representative of generic aircraft frames were loaded quasi-statically to determine their load response and failure mechanisms for large deflections that occur in airplane crashes. These frame/skin specimens consisted of a cylindrical skin section co-cured with a semicircular I-frame. The skin provided the necessary lateral stiffness to keep deformations in the plane of the frame in order to realistically represent deformations as they occur in actual fuselage structures. Various frame laminate stacking sequences and geometries were evaluated by statically loading the specimen until multiple failures occurred. Two analytical methods were compared for modeling the frame/skin specimens: a two-dimensional shell finite element analysis and a one-dimensional, closed-form, curved beam solution derived using an energy method. Reduced flange effectiveness was included in the beam analysis to account for the curling phenomenon that occurs in thin flanges of curved beams. Good correlation was obtained between experimental results and the analytical predictions of the linear response of the frames prior to the initial failure. The specimens were found to be useful for evaluating composite frame designs.

AUTHOR: Shanmugam, N. E., Thevendran, V., Richard Liew, J. Y., and Tan, L. O. (1995)

TITLE: Experimental study on steel beams curved in plan

INFO: Journal of Structural Engineering, ASCE, Vol. 121, No. 2, February.

ABSTRACT: This paper is concerned with the ultimate load behavior of I-beams curved in plan. Results obtained from experiments on two sets of I-beams (one comprising rolled sections and the other built-up sections) are presented. The test results for deformations and ultimate strength are found to be in good agreement with the corresponding values predicted by using the elasto-plastic finite element analysis. The effects of residual stresses and radius of curvature to span-length ratio (R/L) on ultimate strength are considered. Each beam was subjected to a concentrated load applied at an intermediate point where the beam was laterally restrained. Test results indicate that the load-carrying capacity decreases with the decrease in the R/L ratio.

AUTHOR: Shim, V. P. W., and Quah, S. E. (1998)

TITLE: Solution of impact-induced flexural waves in a circular ring by the method of characteristics

INFO: Journal of Applied Mechanics-Transactions of the ASME, Vol. 65, No. 3, September.

ABSTRACT: A study of elastic wave propagation in a curved beam (circular ring) is presented. The governing equations of motion are formulated in two forms based on Timoshenko beam theory. Solutions are obtained using the method of characteristics, whereby a numerical scheme employing higher-order interpolation is proposed for the finite difference equations. Results obtained are verified by experiments; it is found that use of the higher-order numerical scheme improves correlation with experimental results. Comparison of the relative accuracy between the two mathematical formulations—one in terms of generalized forces and velocities and the other in terms of generalized displacements—shows the former to be mathematically simpler and to yield more accurate results.

AUTHOR: Stiftinger, M. A., Skrnajakl, I. C., and Rammerstorfer, F. G. (1995)

TITLE: Buckling and postbuckling investigations of imperfect curved stringer-stiffened composite shell, part B. Computational investigations

INFO: Thin-Walled Structures, Vol. 23, No. 1–4.

ABSTRACT: Computational investigations of the buckling and post-buckling behavior of a stringer-stiffened composite wing torsion box employing the finite element method are presented. Perfect and imperfect configurations—considering geometrical imperfections as well as initial stresses—are discussed. Two different load cases are investigated: pure axial loading and axial loading with a superimposed constant torsion moment. The buckling behavior is determined by tracing the load-deflection curves using Riks' path-following method. Additional investigations, such as accompanying eigenvalue analyses and first-ply failure calculations, are performed for the first load case. Special attention is put on the modeling of the stiffened regions, where eccentrically placed layers have to be taken into account due to the bonding of the stiffeners to the inner surface of the box. The results show interesting phenomena, such as additional buckling points in the post-buckling region, which, however, can hardly be detected by simply considering the load-axial displacement path.

AUTHOR: Choi, J. K., and Lim, J. K. (1993)

TITLE: Simple curved shear beam elements

INFO: Communications in Numerical Methods in Engineering, Vol. 9, No. 8, August.

ABSTRACT: We have developed two curved beam elements with two nodes, CSCC and CSLC, based on Timoshenko's beam theory and the curvilinear co-ordinate system. These curved beam elements have been modified from the conventional strain element, which has been applied only to Euler beam analysis. Therefore, they do not have any spurious constraints and locking characteristics. They also show rapid and stable convergence on the wide ranges of beam length to height ratio for linear and non-linear analyses.

AUTHOR: Choi, J. K., and Lim, J. K. (1995)

TITLE: General curved beam elements based on the assumed strain fields

INFO: Computers and Structures, Vol. 55, No. 3, May.

ABSTRACT: Simple two-noded and three-noded general curved beam elements have been formulated on the basis of assumed strain fields and Timoshenko's beam theory. The two-noded element is formulated from constant strain fields and the three-noded one is from linear strain fields. These curved beam elements, designed in a local curvilinear coordinate system, are transformed into a global Cartesian coordinate system in order to analyze effectively the general curved beam structures located arbitrarily in space. Since strain functions are assumed independently, these elements are free from any locking phenomena. Through various numerical tests, it is shown that the suggested general curved beam elements give better convergent characteristics than the modified isoparametric curved beam elements that have been shown in existing studies. These elements are also easy to formulate due to their consistent form of strain fields.

AUTHOR: Davidson J. S. (1996)

TITLE: Nominal bending and shear strength of horizontally curved steel I-girder bridges

INFO: Dissertation submitted to the Graduate Faculty of Auburn University in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

ABSTRACT: The currently used AASHTO Guide Specifications for Horizontally Curved Highway Bridges is primarily based on research performed as part of the CURT project during the early 1970s. Since that time, numerous problems with the Guide Specifications have been revealed. The Guide Specifications in its original form is disjointed and difficult to use. There is significant discontinuity in the compressive strength formulation between compact and noncompact sections, and the strength predicted by the formulations does not approach that predicted by the formulations for straight girders as the radius of the curved girder approaches infinity. For these reasons, among others, it has never been adopted as an integral part of the AASHTO Standard Specifications for Highway Bridges. The present research addresses many of the strength-related issues in the design of horizontally curved I-girder bridges, including 1) the overall lateral-torsional buckling and large displacement behavior including the design spacing of cross-frames and diaphragms; 2) the local buckling behavior of the curved compression flanges; 3) the buckling and finite displacement behavior of the curved web panels under bending, shear, and combined bending and shear; and 4) construction issues involving the large displacement behavior of single long slender I-girder during lifting and transporting. Theoretical and analytical results are presented on the behavior of such systems along with design recommendations. Also, in all applicable areas, an in-depth review and comparison of existing Japanese research results is presented.

AUTHOR: Dorfi, H. R., and Busby, H. R. (1994)

TITLE: Effective curved composite beam finite element based on the hybrid-mixed formulation

INFO: Computers and Structures, Vol. 53, No. 1, October.

ABSTRACT: A laminated curved-beam finite element with six displacement degrees of freedom and three stress parameters is derived and evaluated. Both thermal and hygrothermal effects are included. The element is based on the Hellinger-Reissner principle and the hybrid-mixed formulation. The Timoshenko beam theory and classical lamination theory are employed in the finite element description. Within an element linear displacement interpolation is used; the generalized stresses are interpolated by either stress functions based on the equilibrium equations (P1) or constant stress approximation (P2). The beam element stiffness is obtained explicitly and numerical results show very good displacement prediction compared with analytical solutions. Generalized stresses are predicted accurately at the mid-point of the finite element only for constant stress interpolation. The P1-type element yields more accurate displacement and stress prediction.

AUTHOR: Fu, C. C., and Hsu, Y. T. (1995)

TITLE: Development of an improved curvilinear thin-walled Vlasov element

INFO: Computers and Structures, Vol. 54, No. 1, January.

ABSTRACT: The purpose of this study is to develop a more exact horizontally curved beam finite element in a cylindrical coordinate system. The variation method is used to formulate the stiffness matrix, and the results of the solution are compared with another study using a closed form solution. The stiffness matrix for a curved beam element can be used in analyzing the behavior of horizontally curved bridges for either open or closed sections.

AUTHOR: Hall, D. H. (1996)

TITLE: Curved girders are special

INFO: Engineering Structures, Vol. 18, No. 10, October.

ABSTRACT: Horizontally curved I-girders have been used in highway bridges for over 30 years. Their structural behavior is known to be quite different from straight girders because of the always present nonuniform torsion. Early studies of these members were based on a strength of materials approach. Modifications of these results were made to account for distortion and amplification effects. More recent investigations in Japan have involved inelastic finite element studies. From these studies, various modifications of straight I-girder bending strength equations have been presented. Bending tests of curved beams are compared with these equations. It is suggested that current research may lead to the thought that curved girders are the general case, and straight girders may be considered a special case.

AUTHOR: Ibrahimbegovic, A. (1995)

TITLE: On finite element implementation of geometrically nonlinear Reissner's beam theory: Three-dimensional curved beam elements

INFO: Computer Methods in Applied Mechanics and Engineering, Vol. 122, No. 1–2, April.

ABSTRACT: Finite element implementation of the three-dimensional finite-strain beam theory of Reissner is considered in this work. In contrast with some earlier works on the subject, discussed here are the beam elements whose reference axes are arbitrary space-curved lines. We have shown that an improved representation of curved reference geometry significantly increases the accuracy of the results. However, it also makes the choice of non-locking finite element interpolations somewhat more delicate. A hierarchical displacement interpolation proposed here is proved to be capable of eliminating both shear and membrane locking phenomena. A very satisfying non-locking performance is demonstrated for a set of problems in nonlinear elasto-statics.

AUTHOR: Kang, Y. J., and Yoo, C. H. (1994)

TITLE: Thin-walled curved beams. II: analytical solutions for buckling of arches

INFO: Journal of Engineering Mechanics, ASCE, Vol. 120, No. 10, October.

ABSTRACT: In recent years, after Yoo queried the validity of Vlasov's equations regarding the lateral buckling of a circular arch, papers have been published with a certain degree of disagreement among various researchers. Based on a comprehensive and consistent formulation of curved beam theory presented in the previous paper, closed-form solutions thus obtained are used for comparison of the present theoretical study to others. In the present paper, major potential sources of discrepancies are traced, and the rigor and validity of the present formulation is thereby demonstrated. Also, the study on the buckling modes of arches subjected to the condition of uniform bending gives an insight for better understanding of lateral buckling characteristics of arches.

AUTHOR: Kim, J. G., and Kim, Y. Y. (1998)

TITLE: New higher-order hybrid-mixed curved beam element

INFO: International Journal for Numerical Methods in Engineering, Vol. 43, No. 5, November.

ABSTRACT: The purpose of this work is to show the successful use of nodeless degrees of freedom in developing a highly accurate, locking free hybrid-mixed CO curved beam element. In the performance evaluation process of the present field-consistent higher-order element, the effect of field consistency and the role of higher-order interpolation on both displacement-type and hybrid-mixed-type elements are carefully examined. Several benchmark tests confirm the superior behavior of the present element.

AUTHOR: Koziey, B. L., and Mirza, F. A. (1994)

TITLE: Consistent curved beam element

INFO: Computers and Structures, Vol. 51, No. 6, June.

ABSTRACT: Curved beam finite elements with shear deformation have required the use of reduced integration to provide improved results for thin beams and arches due to the presence of a spurious shear strain mode. It has been found that the spurious shear strain mode results from an inconsistency in the displacement fields used in the formulation of these elements. A new curved beam element has been formulated. By providing a cubic polynomial for approximation of displacements, and a quadratic polynomial for approximation of rotations a consistent formulation is ensured thereby eliminating the spurious mode. A rotational degree of freedom which varies quadratically through the thickness of the element is included. This allows for a parabolic variation of the shear strain and hence eliminates the need for use of the shear correction factor kappa as required by the Timoshenko beam theory. This rotational degree of freedom also provides a cubic variation of displacements through the depth of the element. Thus, the normal to the centroidal axis is neither straight nor normal after shearing and bending allowing for warping of the cross section. Material nonlinearities are also incorporated, along with the modified Newton-Raphson method for nonlinear analysis. Comparisons are made with the available elasticity solutions and those predicted by the quadratic isoparametric beam element. The results indicate that the consistent beam element provides excellent predictions of the displacements, stresses and plastic zones for both thin and thick beams and arches.

AUTHOR: Lee, S. S., Koo, J. S., and Choi, J. M. (1996)

TITLE: Development of a new curved beam element with shear effect

INFO: Engineering Computations (Swansea, Wales), Vol. 13, No. 2–4, pp. 9–25.

ABSTRACT: Two-noded curved beam elements, CMLC and 1MLC, are developed on the basis of Timoshenko's beam theory and curvilinear coordinates. These elements are developed by the separation of the radial displacement into the bending and the shear deflection and the projection of the shear deflection into bending deflection. In the CMLC element, field-consistent membrane strain interpolation is adapted for removing the membrane locking. The CMLC element shows the rapid and stable convergence on the wide range of radius, thickness and length of the curved beam. The field-consistent membrane strain and the separation of radial displacement produce the most efficient linear element possible.

AUTHOR: Lim, C. W., Wang, C. M., and Kitipornchai, S. (1997)

TITLE: Timoshenko curved beam bending solutions in terms of Euler-Bernoulli solutions

INFO: Archive of Applied Mechanics, Vol. 67, No. 3, February, pp. 179–190.

ABSTRACT: This paper presents the exact relationships between the deflections and stress resultants of Timoshenko curved beams and that of the corresponding Euler-Bernoulli curved beams. The curved beams considered are of rectangular cross sections and constant radius of curvature. They may have any combinations of classical boundary conditions, and are subjected to any loading distribution that acts normal to the curved beam centerline. These relationships allow engineering designers to directly obtain the bending solutions of Timoshenko curved beams from the familiar Euler-Bernoulli solutions without having to perform the more complicated shear deformation analysis.

AUTHOR: Lin, S. M. (1998)

TITLE: Exact solutions for extensible circular curved Timoshenko beams with nonhomogeneous elastic boundary conditions

INFO: Acta Mechanica, Vol. 130, No. 1–2, pp. 67–79.

ABSTRACT: A generalized Green function of nth-order ordinary differential equation with forcing function composed of the delta function and its derivatives is obtained. The generalized Green function can be easily and effectively applied to both the boundary value problems and the initial value problems. The generalized Green function is expressed in terms of n linearly independent normalized homogeneous solutions. It is the generalization of those given by Pan and Hohenstein, and Kanwal. Accordingly, the exact solution for static analysis of an extensible circular curved Timoshenko beam with general nonhomogeneous elastic boundary conditions, subjected to any transverse, tangential and moment loads is obtained. The three coupled governing differential equations are uncoupled into one complete sixth-order ordinary differential characteristic equation in the tangential displacement. The explicit relations between the angle of rotation due to bending, the transverse displacement and the tangential displacement are obtained.

AUTHOR: Litewka, P., and Rakowski, J. (1997)

TITLE: Efficient curved beam finite element

INFO: International Journal for Numerical Methods in Engineering, Vol. 40, No. 14, July, pp. 2629–2652.

ABSTRACT: The plane two-node curved beam finite element with six degrees of freedom is considered. Knowing the set of 18 exact shape functions their approximation is derived using the expansion of the trigonometric functions in the power series. Unlike the ones commonly used in the FEM analysis the functions suggested by the authors have the coefficients dependent on the geometrical and physical properties of the element. From the strain energy formula the stiffness matrix of the element is determined. It is very simple and can be split into components responsible for bending, shear and axial forces influences on the displacements. The proposed element is totally free of the shear and membrane locking effects. It can be referred to the shear-flexible (parameter d) and compressible (parameter e) systems. Neglecting d or e yields the finite elements in all necessary combinations, i.e., curved Euler-Bernoulli beam or curved Timoshenko beam with or without the membrane effect. Applying the elaborated element in the calculations a very good convergence to the analytical results can be obtained even with a very coarse mesh without the commonly adopted corrections as reduced or selective integration or introduction of the stabilization matrices, additional constraints, etc., for the small depth-length ratio.

AUTHOR: Mentrasti, L. (1996)

TITLE: Shear-torsion of large curvature beams—Part II. Applications to thin bodies

INFO: International Journal of Mechanical Sciences, Vol. 38, No. 7, July, pp. 723–733.

ABSTRACT: The shear-torsion state of stress in a curved beam, whose cross section is a thin rectangle with sides not parallel to the plane of the beam, is determined in closed form. The maximum value of the shear stress is attained at the concave boundary of the beam. The shear-torsion moment of inertia J_{st} in multi-connected thin-walled cross sections is evaluated. Several examples of cross sections are discussed.

AUTHOR: Orth, F. J., and Surana, K. S. (1994)

TITLE: P-version two-dimensional beam element for geometrically nonlinear analysis

INFO: Computers and Structures, Vol. 50, No. 3, February, pp. 383–392.

ABSTRACT: This paper presents a p-version geometrically nonlinear formulation (GNL) based on the total Lagrangian approach for a three-node two-dimensional curved beam element. The hierarchical element approximation functions and the corresponding nodal variables are derived directly from the Lagrange family of interpolation functions. The resulting element displacement approximation is hierarchical and can be of arbitrary and different polynomial orders in the longitudinal and the transverse directions of the beam element and ensures CO continuity. The element geometry is described by the coordinates of the nodes located on the axis of the beam (middle surface) and the nodal vectors describing the top and bottom surfaces of the element. The element properties are established using the principle of virtual work and the hierarchical element displacement approximation. In formulating the properties of the element complete two-dimensional stresses and strains are considered; hence, the element is equally effective for very slender as well as extremely deep beams. Incremental equations of equilibrium are derived and solved using the standard Newton-Raphson method. The total load is divided into increments, and for each increment of load, equilibrium iterations are performed until each component of the residuals is within a present tolerance. Numerical examples are presented to show the accuracy, efficiency and advantages of the present formulation. The results obtained from the present formulation are compared with those reported in the literature. The formulation presented here removes virtually all of the drawbacks present in existing GNL beam finite element formulations and has many additional benefits. First, the currently available GNL beam formulations are based on fixed order of approximation for the displacements and thus are not hierarchical and have no provision for changing the order of approximation for the displacements u and v. Secondly, the element displacement approximations in the existing formulations are either based on a linearized displacement field for which a true Lagrangian formulation is not possible and the incremental load step size is severely limited or are based on nonlinear nodal rotation function approach in which case even though the description of the displacement field for the element is exact additional complications arise due to the non-cummutative nature of the nonlinear nodal rotation functions. The p-version displacement approximation used here does not involve the traditional nodal rotations that have been used in the existing beam formulations, thus the difficulties associated with their use are not present in this formulation.

AUTHOR: Paavola, J., and Salonen, E. M. (1999)

TITLE: Strain and stress analysis of a curved tapered beam model

INFO: Computers and Structures, Vol. 72, No. 4–5, pp. 565–577.

ABSTRACT: The strain and stress resultant expressions for a tapered curved beam model are derived. The model is a one-dimensional version of the finite element–based shell theory model of Irons and it may be considered as a generalization of the well-known Timoshenko beam model. To gain insight into the properties of the model, the expressions needed are developed analytically without use of finite elements. An effort is made to clarify a statement given previously, which is needed in the application of the theory and apt to lead to some confusion in its interpretation. Small displacement theory is applied. The stress resultant expressions are derived using the principle of virtual work and in addition directly from the stresses acting on the beam cross-section. The stresses obtained by the model in the isotropic elastic case are compared in two simple example problems with those by the Timoshenko model and with the exact values.

AUTHOR: Pi, Y. L., and Trahair, N. S. (1996)

TITLE: Nonlinear elastic behaviour of I-beams curved in plan

INFO: Research Report—University of Sydney, School of Civil and Mining Engineering, No. 734, December, pp. 1–45.

ABSTRACT: The vertical deflections perpendicular to the plane of a horizontal beam curved in plan are coupled with its twist rotations, and its axial deflections are coupled with its horizontal radial deflections. Because of the first of these couplings, a horizontally curved beam subjected to vertical loading has both primary bending and torsion actions. In the nonlinear range, second-order couplings between the vertical and horizontal deflections and the twist rotations are developed, and the nonlinear behavior of the curved beam becomes more complicated. This paper studies the linear, neutral, and nonlinear equilibrium of elastic horizontally curved I-beams under vertical loading, and develops a curved finite element model for their analysis. It is found that when the initial curvature of a curved beam is small, the primary coupling is also small and bending is the major action. In this case, the nonlinear behavior is similar to the elastic flexural-torsional buckling of a straight beam. However, if the initial curvature of the curved beam is not small, the primary coupling becomes significant and both torsion and bending are major actions. In this case nonlinear behavior develops very early and is quite different from rite flexural-torsional buckling behavior of a straight beam.

AUTHOR: Raveendranath, P., Singh, G., and Pradhan, B. (1999)

TITLE: Two-noded locking-free shear flexible curved beam element

INFO: International Journal for Numerical Methods in Engineering, Vol. 44, No. 2, January, pp. 265–280.

ABSTRACT: A new two-noded shear flexible curved beam element which is impervious to membrane and shear locking is proposed herein. The element with three degrees of freedom at each node is based on curvilinear deep shell theory. Starting with a cubic polynomial representation for radial displacement (w), the displacement field for tangential displacement (u) and section rotation (theta) are determined by employing force-moment and moment-shear equilibrium equations. This results in a polynomial displacement field whose coefficients are coupled by generalized degrees of freedom and material and geometric properties of the element. The procedure facilitates quadratic polynomial representation for both u and theta for curved element configurations, which reduces to linear and quadratic polynomials for u and theta, respectively, for straight element configuration. These coupled polynomial coefficients do not give rise to any spurious constraints even in the extreme thin regimes, in which case, the present element exhibits excellent convergence to the classical thin beam solutions. This simple C degree element is validated for beams having straight/curved geometries over a wide range of slenderness ratios. The results indicate that performance of the element is much superior to other elements of the same class.

AUTHOR: Pak, R. Y. S., and Stauffer, E. J. (1994)

TITLE: Nonlinear finite deformation analysis of beams and columns

INFO: Journal of Engineering Mechanics, ASCE, Vol. 120, No. 10, October.

ABSTRACT: A method for solving the finite-displacement problem of a curved elastic beam with axial, shear, and flexural deformation subject to distributed and point loads is presented. Within the context of the kinematic assumptions of the Timoshenko theory, a Lagrangian formulation of the problem is enveloped. In terms of three cross-sectional stress resultants, three Euler equations of equilibrium for the beam are derived with the aid of a variational principle for finite deformation. Upon linearization to small strains and the adoption of a linear elastic constitutive relation between the stress and strain tensors, it is shown that the problem is reducible to a single second-order nonlinear ordinary differential equation. Subject to appropriate boundary conditions, the resulting two-point boundary-value problem is solved by a finite-element method. By virtue of a continuation algorithm, accurate solutions of the system of nonlinear equations can be obtained for a variety of bifurcation and buckling problems. Comprehensive results are presented for two cantilever beams as illustrations.

AUTHOR: Ryu, H. S., and Sin, H. C. (1996)

TITLE: Curved beam elements based on strain fields

INFO: Communications in Numerical Methods in Engineering, Vol. 12, No. 11, November.

ABSTRACT: Two curved beam elements with two nodes and three nodes are designed based on strain fields. At the element level, curvature and membrane strain fields are approximated independently and shear strain fields are incorporated into the formulation by the equilibrium equations. The displacement fields are obtained by integrating the assumed strain fields. Two examples are given to verify the formulations and demonstrate the numerical performance of the two curved beam elements. Analysis results obtained reveal that the elements describe the curved beam behavior correctly and show exceptional accuracy throughout a wide slenderness range.

AUTHOR: Sengupta, D., and Dasgupta, S. (1997)

TITLE: Static and dynamic applications of a five noded horizontally curved beam element with shear deformation

INFO: International Journal for Numerical Methods in Engineering, Vol. 40, No. 10, May.

ABSTRACT: A five-noded thirteen DOF horizontally curved beam element with or without an elastic base is presented. One set of fourth-degree Lagrangian polynomials in natural co-ordinates is used for interpolation of beam geometry and vertical displacement while the angles of transverse rotation and twist are interpolated by another set of third-degree polynomials. For elastic subgrade, the reactive forces offered at any point are assumed to be proportional to the corresponding displacements at that point. The effect of shear deformation is accounted for in the stiffness matrix. In mass matrix evaluation, for dynamic problems, translational as well as rotary inertias have been considered and studied separately. For numerical integration of the stiffness matrix, a four-point Gaussian scheme has been found to be adequate. Numerical results for a number of sample problems and their comparison with analytical solutions have been presented for circular as well as for non-circular curved beams. Displacements, bending moment and torque for static loading with or without elastic foundation, as well as natural frequencies and mode shapes, are computed for different cases. Examples include the problem of a cantilever beam of spiral geometry with different parametric values of the spiral and the agreement with the analytical results establishes the efficacy of the element. The performance of the element has been found to be excellent in both static and dynamic conditions. Sufficient details are presented so that the formulation may be readily used. It is hoped that the large number of numerical illustrations will elucidate the validity and the range of applicability of the element and will also serve as a benchmark for future researchers.

AUTHOR: Simpson, M. D., and Birkemoe, P. C. (1997)

TITLE: Nonlinear finite element modeling of curved girder experiments

INFO: Structures—Design, Concrete and Reinforced Concrete Structures, Bridges Proceedings, Annual Conference—Canadian Society for Civil Engineering, Vol. 7, Canadian Society for Civil Engineering, Montreal, Quebec, Canada.

ABSTRACT: Research involving the ultimate strength characteristics of curved steel girder components began in North America in the early 1970s under the direction of the Consortium of University Research Teams (CURT). Both analytical and experimental programs attempted to isolate the parameters that affected the flexural and shear strength of curved steel girders. Much of the understanding gained from the CURT project is still reflected in the current Guide Specification for Horizontally Curved Highway Bridges. Additional experimental and analytical studies were performed in Japan during the 1980s to further advance the state-of-the-art understanding of curved girder behavior. Although some researchers have used analytical techniques to investigate curved girder behavior, few have employed the flexibility and generality of the finite element method. Thus, the purpose of this paper is to demonstrate the use of the finite element method as applied to the analysis of curved steel girders. In particular, a select number of ultimate strength experiments are modeled using inelastic, large deformation finite element solutions. Residual stresses are also included, and their effect on behavior is discussed.

AUTHOR: Thevendran, V., Chen, S., Shanmugam, N. E., and Liew, J. Y. R. (1999)

TITLE: Nonlinear analysis of steel-concrete composite beams curved in plan

INFO: Finite Elements in Analysis and Design, Vol. 32, No. 3, pp. 125–139.

ABSTRACT: This paper deals with the behavior of structural steel-concrete composite beams curved in plan. The finite element package ABAQUS has been used to study the nonlinear behavior and ultimate load-carrying capacity of such beams. A three-dimensional finite element model has been adopted. Shell elements have been used to simulate the behavior of concrete slab and steel girder, and rigid beam elements to simulate the behavior of shear studs. The proposed finite element model has been validated by comparing the computed values with available experimental results. An acceptable correlation has been observed between the computed and experimental results obtained for beams of realistic proportion.

AUTHOR: Yang, S. Y., and Sin, H. C. (1995)

TITLE: Curvature-based beam elements for the analysis of Timoshenko and shear-deformable curved beams

INFO: Journal of Sound and Vibration, Vol. 187, No. 4, November.

ABSTRACT: Curved beam elements with six degrees of freedom and two-, three-, four- and five-node Timoshenko straight beam elements with four, five, six and seven degrees of freedom, respectively, are proposed to eliminate stiffness locking when applied to the dynamic analysis of the beams. The elements are based on curvature assumptions so that they can represent the bending energy accurately, and the shear strain energy is incorporated into the formulation by way of the equilibrium equation. Eigenvalue problems of Timoshenko and shear-deformable curved beams are analyzed by using the elements. The results of the eigenanalysis show that the curvature-based beam elements are free of locking and are efficient.

AUTHOR: Hu, N., Hu, B., Yan, B., Fukunaga, H., and Sekine, H. (1999)

TITLE: Two kinds of CO-type elements for buckling analysis of thin-walled curved beams

INFO: Computer Methods in Applied Mechanics and Engineering, Vol. 171, No. 1.

ABSTRACT: This paper deals with the spatial buckling analysis of curved beams. First, a second-order expansion for the finite rigid-rotations in nonlinear strain expressions is derived and employed to produce the geometric stiffness matrix. This second-order accurate geometric stiffness matrix can ensure that all significant instability modes can be predicted. Furthermore, Timoshenko's and Vlasov's beam theories are combined to develop two kinds of the CO-type finite element formulations for arbitrary cross-section thin-walled curved beams, which include the isoparametric curved beam element and the strain curved beam element. These two kinds of elements include both shear and warping deformations caused by bending moments and bimoments. In numerical examples, the effect of the second-order terms in the nonlinear strains on the buckling load is investigated. Furthermore, efficiencies of the proposed two kinds of elements are studied in the buckling analysis of curved beam structures.

AUTHOR: Kang, Y. J., and Yoo, C. H. (1994)

TITLE: Thin-walled curved beams. I: Formulation of nonlinear equations

INFO: Journal of Engineering Mechanics, ASCE, Vol. 120, No. 10, October.

ABSTRACT: An extensive investigation on the buckling and large displacement behavior of thin-walled circular beams has been conducted theoretically. Equilibrium equations governing the linear, the bifurcation buckling, and the large displacement behavior have been derived using the principle of minimum total potential energy. An explicit and clear approximation of the curvature effect is made in the derivation process. The paper concludes with a series of fundamental nonlinear equations that describe the elastic behavior of thin-walled curved beams. A companion paper examines closed-form solutions for arch-buckling problems based on the formulations presented in this paper and demonstrates the rigor and the validity of the present formulation.

AUTHOR: Kang, Y. J., Yoo, C. H., Yu, C., and Lee, H. E. (1993)

TITLE: Lateral buckling behavior of thin-walled arches

INFO: Proceedings of the Fourth East Asia-Pacific Conference on Structural Engineering and Construction, Seoul, Korea, September 20–22.

ABSTRACT: Abstract could not be located.

AUTHOR: Yoo, C. H., Kang, Y. J., and Davidson, J. S. (1996)

TITLE: Buckling analysis of curved beams by finite-element discretization

INFO: Journal of Engineering Mechanics, ASCE, Vol. 122, No. 8, August.

ABSTRACT: Recently, an extensive theoretical investigation on the buckling and large-displacement behavior of thin-walled circular beams was reported in a two-paper series. Equilibrium equations governing the linear, bifurcation buckling, and large-displacement behaviors were derived using the principle of minimum total potential energy. This paper first presents the transformation process for finite-element stiffness relationships for a spatial curved beam element with a total of 14 degrees of freedom. It then presents numerical data demonstrating the applicability of the method for the lateral buckling of arches and the lateral-torsional buckling of horizontally curved beams. A numerical comparison between the present formulations and those presented by others is made, along with a comparison to results obtained using three-dimensional finite-element models. Based on results from the lateral bifurcation buckling of horizontally curved beams, a regression equation is formulated representing the reduction in critical moment due to the simple addition of curvature. A comparison of results using this regression equation with results from ultimate strength experimental testing of horizontally curved girders by others resulted in an unexpected excellent correlation.

AUTHOR: Gendy, A. S., and Saleeb, A. F. (1994)

TITLE: Vibration analysis of coupled extensional/flexural/torsional modes of curved beams with arbitrary thin-walled sections

INFO: Journal of Sound and Vibration, Vol. 174, No. 2, July.

ABSTRACT: A three-dimensional, two-field, variational formulation is employed to derive the differential equations governing the dynamics of stretching, shearing, bending and twisting, as well as warping modes of deformations in a spatially curved beam with arbitrary cross-section. Correspondingly, the finite element discretization was developed for free vibration analysis based on a Timoshenko-Vlasov thin-walled theory, including the effects of flexural-torsional coupling, shear deformations due to flexure as well as torsional warping, and rotary inertia. Attention was given to the significant curvature effects on the results in cases involving unsymmetrical cross-sections of the thin-walled type. Several numerical examples are given to demonstrate the high accuracy and effectiveness of the element developed.

AUTHOR: Howson, W. P., and Jemah, A. K. (1999)

TITLE: Exact out-of-plane natural frequencies of curved Timoshenko beams

INFO: Journal of Engineering Mechanics, ASCE, Vol. 125, No. 1, January.

ABSTRACT: A powerful and efficient method is presented for finding exact out-of-plane natural frequencies of plane structures composed of curved Timoshenko beams. Initially, exact dynamic stiffness is derived from the governing differential equations of motion in a form that can be used directly in the stiffness method of analysis. This enables any appropriate structure to be modeled according to standard techniques, which, in this case, yield a transcendental eigenvalue problem. Then, it is shown how any desired natural frequency may be obtained with certainty by employing a modification to a well-established algorithm, which ensures that no natural frequencies can be missed and avoids the usual approximations associated with traditional finite elements. Finally, comparisons are made with published results, and an example shows how the natural frequencies of a continuous curved beam are altered when the effects of shear deflection and rotary inertia are considered.

AUTHOR: Howson, W. P., Jemah, A. K., and Zhou, J. Q. (1995)

TITLE: Exact natural frequencies for out-of-plane motion of plane structures composed of curved beam members

INFO: Computers and Structures, Vol. 55, No. 6, June.

ABSTRACT: Exact finite elements form the basis of a new and convenient procedure for converging with certainty upon any required natural frequency of out-of-plane motion of any plane structure composed of slender elastic curved members. Solution of the inherent transcendental eigenvalue problem is achieved through a variation on the powerful Wittrick-Williams algorithm. Two illustrative examples are included.

AUTHOR: Hsiao, K. M., and Yang, R. T. (1995)

TITLE: Co-rotational formulation for nonlinear dynamic analysis of curved Euler beam

INFO: Computers and Structures, Vol. 54, No. 6, March.

ABSTRACT: A co-rotational finite element formulation for the dynamic analysis of a planar curved Euler beam is presented. The Euler-Bernoulli hypothesis and the initial curvature are properly considered for the kinematics of a curved beam. Both the deformational nodal forces and the inertial nodal forces of the beam element are systematically derived by consistent linearization of the fully geometrically nonlinear beam theory in element coordinates, which are constructed at the current configuration of the corresponding beam element. An incremental-iterative method based on the Newmark direct integration method and the Newton-Raphson method is employed here for the solution of the nonlinear dynamic equilibrium equations. Numerical examples are presented to demonstrate the effectiveness of the proposed element and to investigate the effect of the initial curvature on the dynamic response of the curved beam structures.

AUTHOR: Huang, D., Wang, T. L., and Shahawy, M. (1995)

TITLE: Dynamic behavior of horizontally curved I-girder bridges

INFO: Computers and Structures, Vol. 57, No. 4, November.

ABSTRACT: The purpose of this paper is to investigate the dynamic behavior of horizontally curved I-girder bridges due to one or two trucks (side by side) moving across rough bridge decks. The bridge is modeled as a planar grillage beam system composed of horizontally curved beam elements and straight beam elements. Warping torsion is taken into consideration in the analysis. The analytical vehicle is simulated as a nonlinear vehicle model with 11 independent degrees of freedom according to the HS2O-44 truck design loading contained in the American Association of State Highway and Transportation Officials (AASHTO) specifications. The study used four different classes of road surface roughness generated from power spectral density for very good, good, average, and poor roads. The analytical results are very significant and show that the dynamic behavior of curved I-girder bridges is quite different from that of straight girder bridges. The impact factors of bending and shear for inside girders of curved I-girder bridges are significantly smaller than those for outside girders.

AUTHOR: Jiang, J., and Olson, M. D. (1994)

TITLE: Vibration analysis of orthogonally stiffened cylindrical shells using super finite elements

INFO: Journal of Sound and Vibration, Vol. 173, No. 1, May.

ABSTRACT: An efficient numerical technique for the free vibration analysis of orthogonally stiffened cylindrical shell structures is presented. The new technique is developed within the framework of a super finite element method and involves formulation of special cylindrical shell and curved beam elements. The displacement functions used in the present formulation combine the usual polynomials with some analytical functions carefully chosen to provide a good approximation of basic vibration modes in conjunction with coarse grid modeling strategy. Detailed results of vibration application of these elements are presented.

AUTHOR: Kang, K. J., Bert, C. W., and Striz, A. G. (1996)

TITLE: Vibration analysis of horizontally curved beams with warping using DQM

INFO: Journal of Structural Engineering, ASCE, Vol. 122, No. 6, June.

ABSTRACT: The differential quadrature method (DQM) is applied to computation of the eigenvalues of small-amplitude free vibration for horizontally curved beams, including a warping contribution. Natural frequencies are calculated for single-span, curved, wide-flange uniform beams having a range of nondimensional parameters representing variations in warping stiffness, torsional stiffness, radius of curvature, included angle of the curve, polar mass moment of inertia, and various end conditions. Results are compared with existing exact and numerical solutions by other methods for cases in which they are available. The DQM provides accuracy even when only a limited number of grid points are used. In addition, results are given for a cantilever that has one clamped end and one free end; no previous results are known for this case. Finally, parametric results are presented in dimensionless form.

AUTHOR: Kawakami, M., Sakiyama, T., Matsuda, H., and Morita, C. (1995)

TITLE: In-plane and out-of-plane free vibrations of curved beams with variable sections

INFO: Journal of Sound and Vibration, Vol. 187, No. 3, November.

ABSTRACT: Abstract could not be located.

AUTHOR: Wang, R. T., and Sang, Y. L. (1999)

TITLE: Out-of-plane vibration of multi-span curved beam due to moving loads

INFO: Structural Engineering and Mechanics, Vol. 7, No. 4.

ABSTRACT: This paper presents an analytic method of examining the out-of-plane vibration of continuous curved beam on periodical supports. The orthogonality of two distinct sets of mode shape functions is derived. The forced vibration of beam due to moving loads is examined. Two types of moving loads, which are concentrated load and uniformly distributed load, are considered. The response characteristics of beams induced by these loads are investigated as well.

AUTHOR: Yildirim, V. (1999)

TITLE: In-plane free vibration of symmetric cross-ply laminated circular bars

INFO: Journal of Engineering Mechanics, ASCE, Vol. 125, No. 6, June.

ABSTRACT: A parametric study is performed to investigate influences of the opening angles, the slenderness ratios, the material types, the boundary conditions, and the thickness-to-width ratios of the cross section on the in-plane natural frequencies of symmetric cross-ply laminated circular composite beams. Governing equations are obtained based on the classical beam theory. The transfer matrix method is successfully applied to calculate exact natural frequencies with the help of an effective numerical algorithm, which was previously used for isotropic materials. The effects of the shear deformation, the axial deformation, and the rotary inertia are included in the formulation based on the first-order shear deformation theory. The physical system is considered a continuous system. To verify the present theory, two examples are worked out for straight beams. An agreement is presented with the reported results.

AUTHOR: Razaqpur, A. G. and Li, H. G. (1994)

TITLE: Refined analysis of curved thin-walled multicell box girders

INFO: Computers and Structures, Vol. 53, No. 1.

ABSTRACT: The generalized Vlasov's thin-walled beam theory was combined with the finite element technique to develop a new curved thin-walled multicell box girder finite element that can model extension, flexure, torsion, torsional warping, distortion, distortional warping and shear lag effects. For multicell box girders, several distortional modes were introduced to describe the complete distortional behavior of the cross-section. Interaction between the longitudinal and transverse deformations of the box girder was also accounted for. The element is one-dimensional. It has three nodes and employs the conventional polynomial shape functions. For modeling flexure, Timoshenko beam theory was used to take account of shear deformations. A computer program was developed based on the proposed element for the analysis of single and multicell curved box girder bridges. Numerical examples are presented to demonstrate the accuracy and efficiency of the proposed element. Compared with the standard finite element method, the proposed method needs substantially less computer time, memory and input data. The output is also in a form that can be used in design without further manipulation.

AUTHOR: Razaqpur, A. G., and Li, H. G. (1997)

TITLE: Analysis of curved multicell box girder assemblages

INFO: Structural Engineering and Mechanics, Vol. 5, No. 1.

ABSTRACT: A method of analysis is proposed for curved multicell box girder grillages. The method can be used to analyze box girder grillages comprising straight and/or curved segments. Each segment can be modeled by a number of beam elements. Each element has three nodes, and the nodal degrees of freedom (DOFs) consist of the six DOFs for a conventional beam plus DOFs to account for torsional warping, distortion, distortional warping, and shear lag. This element is an extension of a straight element that was developed earlier. For a more realistic analysis of the intersection regions of non-collinear box girder segments, the concept of a rigid connector is introduced, and the compatibility requirements between adjoining elements in those regions are discussed. The results of the analysis showed good agreement with the shell finite element results, but the proposed method of analysis needs a fraction of the time and effort that the shell finite element analysis needs.

AUTHOR: Yabuki, T., Arizumi, Y., and Vinnakota, S. (1995)

TITLE: Strength of thin-walled box girders curved in plan

INFO: Journal of Structural Engineering, ASCE, Vol. 121, No. 5.

ABSTRACT: This paper presents a numerical method for predicting the influence of local buckling in component plates and distortional phenomenon on the ultimate strength of thin-walled, welded steel box girders curved in plan. A modified stress-strain curve allowing for local buckling of plate components is proposed. The effect of distortion is considered by incorporating additional strain in the reference stage of the incremental process in the nonlinear flexure analysis. Theoretical predictions obtained using the proposed method are compared with experimental test results. Nonlinear behavior of the test girders is illustrated, and reasonable agreement between tests and theory is observed.

AUTHOR: Galdos, N. H., Schelling, D. R., and Sahin, M. A. (1993)

TITLE: Methodology for impact factor of horizontally curved box bridges

INFO: Journal of Structural Engineering, ASCE, Vol. 119, No. 6.

ABSTRACT: This paper presents a method for determining the dynamic impact factor of horizontally curved steel box-girder bridges under vehicle loadings. The two-dimensional planar grid analogy is used to model box bridges. The vehicle is idealized as a pair of concentrated forces, with no mass traveling on circumferential paths with constant velocity. Analysis of multigirder and continuous span bridges indicates a tendency for frequency clustering. Static solutions for these bridges are compared with mode superposition and direct integration. The bridge behavior was studied under several truck loading paths. Static and dynamic bridge behavior was observed under these loadings. A rational methodology for determining the impact factor is developed, and alternate impact-factor criteria are proposed to replace the current American Association of State Highway and Transportation Officials guide specification.

AUTHOR: Sennah, K. M., and Kennedy, J. B. (1997)

TITLE: Dynamic characteristics of simply supported curved composite multi-cell bridges

INFO: Canadian Journal of Civil Engineering, No. 4, August.

ABSTRACT: The use of horizontally curved composite bridges in the interchange facilities of modern highway systems has become increasingly popular for economic as well as aesthetic considerations. Cellular steel composite sections with a concrete deck are quite suitable in resisting torsional and warping effects induced by highway curvature. However, this type of structure has inherently created design problems for the designer in estimating the level of dynamic response when subjected to moving vehicles, wind, or seismic conditions. This paper presents a summary of an extensive parametric study, using the finite-element method, in which 120 simply supported, curved composite bridge prototypes are analyzed to evaluate their natural frequencies and mode shapes. The parameters considered in the study are end-diaphragm thickness, cross-bracing system, aspect ratio, span-to-depth ratio, degree of curvature, number of lanes, and number of cells. Results from tests on four 1/12 linear-scale simply supported composite three-cell bridge models of different curvatures are used to substantiate the analytical modeling. The stipulation made in bridge codes for treating a curved bridge as a straight one is examined. Based on the data generated from the parametric study, expressions for the first flexural frequency and hence the dynamic load allowance are deduced. Recommendations for enhancing the torsional resistance to dynamic forces on curved bridges are presented. Two design examples are illustrated.

AUTHOR: Davidson, J. S., and Yoo, C. H. (1996)

TITLE: Local buckling of curved I-girder flanges

INFO: Journal of Structural Engineering, ASCE, Vol. 122, No. 8, August.

ABSTRACT: Two analytical approaches are used to investigate the effect of curvature on the elastic local buckling behavior of horizontally curved I-girder compression flanges. First, a governing differential equation for the isolated curved flange plate elements under various loading and boundary conditions is derived in polar coordinates, and the equation is solved using the finite-difference technique. Numerical results for various curvatures and aspect ratios are generated. Second, the finite-element method using MSCINASTRAN is used where the entire cross section of the curved I-girder is modeled. A comparison of the results from the two approaches is made and differences are explained. The effect of curvature on the elastic buckling of the flange plates is observed and a simple but conservative formulation representing this effect is derived for design use.

AUTHOR: Davidson, J. S., and Yoo, C. H. (2000)

TITLE: Evaluation of strength formulations for horizontally curved flexural members

INFO: Journal of Bridge Engineering, ASCE, Vol. 5, No. 3, August.

ABSTRACT: In 1992 the Federal Highway Administration (FHWA), along with 13 states, began a project to study the behavior of horizontally curved steel bridges. The project is referred to as the Curved Steel Bridge Research Project (CSBRP) and involves studying the behavior of curved members through theoretical, analytical, and experimental research. In this paper, detailed finite element models representing a curved three-girder test frame that is planned under the CSBRP experimental phase are used to evaluate the effects of curvature on the bending strength of curved I-girders. Linear-elastic static, buckling, and combined material and geometric nonlinear analyses are conducted using models that represent the test frame and component test specimens that will be inserted into it. The results are compared with various predictor equations developed from analytical work by the writers and with related work by other researchers, including Japanese research that is not readily available in the U.S. It is demonstrated that the predictor equations developed by the writers are accurate in representing the behavior of the system. Limitations and needed improvements are described as well.

AUTHOR: Davidson, J. S., Balance, S. R., and Yoo, C. H. (2000)

TITLE: Behavior of curved I-girder webs subjected to combined bending and shear

INFO: Journal of Bridge Engineering, ASCE, Vol. 5, No. 2, May.

ABSTRACT: Two previous papers by the authors described the buckling and finite displacement behavior of curved I-girder web panels subjected to pure bending. The papers presented a theoretically pure analytical model and presented equations that describe the reduction in strength due to curvature. This paper describes the buckling and finite displacement behavior of curved web panels under combined bending and shear. Unlike in straight girder web panels, the addition of shear in curved panels is shown to increase the transverse "bulging" displacement of the web prior to buckling. The accompanying decrease in moment carrying capacity is analyzed in a manner similar to that used for the combined bending and shear nominal strength interaction for straight girder design. Preliminary recommendations are made towards forming design criteria for curved webs.

AUTHOR: Davidson, J. S., Ballance, S. R., and Yoo, C. H. (2000)

TITLE: Effects of longitudinal stiffeners on curved I-girder webs

INFO: Journal of Bridge Engineering, ASCE, Vol. 5, No. 2, May.

ABSTRACT: Two previous papers by the authors described the buckling and finite displacement behavior of curved I-girder web panels without longitudinal stiffeners subjected to pure bending. The papers presented a theoretically pure analytical model and presented equations that describe the reduction in strength due to curvature. This paper describes the optimum location and strength effects of one and two longitudinal stiffeners attached to curved I-shaped plate girders. Using lateral load and lateral pressure analogies similar to that described in the earlier papers, strength reduction equations are formulated for curved plate girders with longitudinal stiffeners. A comprehensive comparison is made between the equations developed in this investigation and design equations used in Japanese and American design guides. The applicability and superiority of the method is demonstrated.

AUTHOR: Davidson, J. S., Ballance, S. R., and Yoo, C. H. (1999)

TITLE: Finite displacement behavior of curved I-girder webs subjected to bending

INFO: Journal of Bridge Engineering, ASCE, Vol. 4, No. 3, August.

ABSTRACT: Curvature greatly complicates the behavior of curved plate girders used in bridges. The out-of-plane "bulging" displacement of the curved web results in an increase in stress, which must be considered in the design of plate girders with significant curvature. This paper presents results from geometric nonlinear finite-element analyses used to evaluate the finite displacement behavior of such panels and to formulate deflection amplification factors that can be applied to analytical models to get conservative values for predicting the maximum displacements and stresses of the curved panel. Equations are developed that represent the reduction in nominal strength of the curved web due to the effects of curvature. The applicability of the method is demonstrated, and a comprehensive comparison is made between the equations developed in this investigation and design equations used in Japanese and American design guides.

AUTHOR: Davidson, J. S., Ballance, S. R., and Yoo, C. H. (1999)

TITLE: Analytical model of curved I-girder web panels subjected to bending

INFO: Journal of Bridge Engineering, ASCE, Vol. 4, No. 3, August.

ABSTRACT: Curvature greatly complicates the behavior of curved plate girders used in bridges. The out-of-plane "bulging" displacement of the curved web results in an increase in stress, which must be considered in the design of plate girders with significant curvature. The currently used Guide Specifications for Horizontally Curved Bridges provides web slenderness reduction equations that account for curvature effects, but these equations were based on a regression of limited data from experimental and unit-strip analyses conducted in the early 1970s. This paper presents a theoretically pure analytical model that can be used to predict the transverse displacement and induced plate bending stresses of curved I-shaped plate girder web panels subjected to bending. Several boundary conditions are demonstrated and compared, and the finite-element method is used to verify the closed-form solutions. The effects of curvature on the elastic buckling behavior of curved web panels is also presented. Furthermore, a comprehensive literature review is presented, including numerous Japanese publications not readily available to American researchers.

AUTHOR: Lee, S. C., and Yoo, C. H. (1999)

TITLE: Strength of curved I-girder web panels under pure shear

INFO: Journal of Structural Engineering, ASCE, Vol. 125, No. 8.

ABSTRACT: The bifurcation buckling and the ultimate strength of curved web panels subjected to pure shear are investigated by the finite-element method. To evaluate the ultimate strength of curved web panels subjected to pure shear to the point of failure, both geometric and material nonlinearities are considered. From the nonlinear analysis it is shown that curved web panels are capable of developing considerable post-buckling strength after the bifurcation point. Results of the present study are compared with shear strength of straight girder web panels subjected to pure shear. Comparisons indicate that the straight girder equations can effectively predict the shear strength of curved web panels subjected to pure shear. The deflection curves that are due to a unit-generalized displacement at nodal coordinate and the exact element stiffness matrix are derived based on the solution for the general system. A finite element method can be developed based on the results for the dynamic analysis. Meanwhile, the stiffness locking phenomena accompanied in some other curved beam element methods do not exist in the proposed method.

AUTHOR: Schilling, C. G. (1996)

TITLE: Yield-interaction relationships for curved I-girders

INFO: Journal of Bridge Engineering, ASCE, Vol. 1, No. 1, February.

ABSTRACT: Yield-interaction relationships are developed for compact, compact-flange, and noncompact sections of curved girders under combined vertical and lateral (warping torsion) moments. These relationships define the bending strength of such compact, compact-flange, and noncompact sections if suitable web-slenderness, compression-flange slenderness, and compression-flange bracing limits are satisfied. The most convenient form of these relationships is equations defining reduced flange widths as a function of lateral moment. The correct vertical-bending strength can be calculated from a reduced steel section consisting of the actual web and reduced widths of flanges. Compact flange sections have a compact compression flange and a noncompact web. For straight girders, there is no advantage in using a compact compression flange with a noncompact web. For curved girders, however, the compact flange permits a larger lateral moment to be carried in combination with a given vertical moment.

AUTHOR: Weaver, D. L. (1996)

TITLE: Steel girder bridges

INFO: Construction Specifier, Vol. 49, No. 5, May.

ABSTRACT: The majority of steel bridges constructed today consist of straight or curved girder superstructures with maximum spans of 60 m or less. Although the basic concept of steel girder bridges has remained unchanged since the 1920s, advances in materials and design practice have allowed them to remain one of the most cost-effective and popular types of bridge superstructure systems. Some of the refinements in steel girder bridge design and construction are presented.

AUTHOR: Yoo, C. H. (1993)

TITLE: Some considerations in the design and construction of horizontally curved highway bridges

INFO: Proceedings of the Fourth East Asia-Pacific Conference on Structural Engineering and Construction, Seoul, Korea, September 20–22 (Invited paper).

AUTHOR: Horizontally curved bridges are much more complex than bridges with girders having initial curvature in the horizontal plane. Presented in the paper are some of the major design considerations that are normally not considered in the straight bridge design. Likewise, erection of horizontally curved girders requires careful precautions that are normally extended in the erection of straight girders. A simple span curved I-girder is unstable. Depending upon the severity of curvature, the site condition, and the availability of heavy equipment, an erection scheme can be devised taking the maximum advantage of the situation presented.

AUTHOR: Yoo, C. H., and Davidson, J. S. (1997)

TITLE: Yield interaction equations for nominal bending strength of curved I-girders

INFO: Journal of Bridge Engineering, ASCE, Vol. 2, No. 2, May.

ABSTRACT: Horizontally curved I-girders are subjected to combined vertical bending and torsion under gravity loading alone. The torsional behavior of open I-shaped girders is commonly and conveniently equated to self-equilibrating lateral bending moments in the flanges. The interaction of vertical bending and this lateral flange bending effectively reduces the vertical moment carrying capacity of the section. Yield interaction equations for predicting the nominal bending strength of horizontally curved steel I-girders subjected to vertical moment and torsion are derived. Singly symmetric composite and noncomposite I-shaped cross sections in both the positive and negative moment zones are considered. Strength criteria considered are: (1) complete plastification of the cross section for compact sections; (2) partial yield penetration for compact-flange sections; and (3) initial yielding at the extreme flange tip for noncompact sections. A total of 17 interaction cases are ultimately considered. These strength criteria are based purely on the static equilibrium of the cross section with no secondary amplification considered. The limitations and applicability of the derived equations toward design use are demonstrated and analyzed.

AUTHOR: Cheung, M. S., and Foo, S. H. C. (1995)

TITLE: Design of horizontally curved composite box-girder bridges—A simplified approach

INFO: Canadian Journal of Civil Engineering, Vol. 22, No. 1.

ABSTRACT: Because of their excellent torsional capacity, box girders are used extensively in modern bridge construction having curved alignments. Applications of most design codes have been limited to bridges where the radius of curvature is much greater than the span length and cross-sectional dimensions. To meet the practical requirements arising during the design process, simple design methods are needed for curved bridges. This paper presents the results of a parametric study on the relative behavior of curved and straight box-girder bridges and on the development of a simplified design method for the combined longitudinal moment of curved bridges. The combined moment includes the effects of flexure, torsion, and distortion. Three simply supported concrete-steel composite bridge models—including single-cell, twin-cell, and three-cell box girders—that were subjected to loadings as specified in the Ontario Highway Bridge Design Code were analyzed using the finite strip method. The parameters considered in the study include types of cross section; types, locations, and magnitudes of loads; span lengths; and radius of curvature. Preliminary analysis of the results suggests that the behavior of horizontally curved box-girder bridges is dependent on a variety of parameters, but most importantly on the span-to-radius ratio. Empirical relationships for combined longitudinal moment between curved and straight box-girder bridges are also proposed.

AUTHOR: Sennah, K., and Kennedy, J. B. (1998)

TITLE: Shear distribution in simply-supported curved composite cellular bridges

INFO: Journal of Bridge Engineering, ASCE, Vol. 3, No. 2.

ABSTRACT: Composite steel-concrete multicell box girder bridges are quite often used in modern bridge superstructures with curved alignments. They provide excellent torsional resistance as well as elegant appearance. While current design practices in North America recommend few analytical methods for the design of curved multicell box bridges, practical requirements in the design process require a need for a simplified design method. This paper summarizes the results from an extensive parametric study, using a finite-element model, in which 120 simply-supported curved bridge prototypes are analyzed to evaluate the shear distribution in the webs due to truck loading as well as dead load. Results from tests on four 1/12 linear-scale simply-supported curved composite concrete deck-steel three-cell bridge models are used to substantiate the analytical modeling. The parameters considered in the study are: cross-bracing system, aspect ratio, number of lanes, number of cells, and degree of curvature. Based on the data generated from the parametric study, expressions for the shear distribution factors for truck loadings as well as dead load are deduced. An illustrative design example is presented.

AUTHOR: Sennah, K., and Kennedy, J. B. (1999)

TITLE: Simply supported curved cellular bridges: Simplified design method

INFO: Journal of Bridge Engineering, ASCE, Vol. 4, No. 2.

ABSTRACT: The use of curved composite bridges in interchanges of modern highway systems has become increasingly popular for economic and aesthetic considerations. Bridges with a concrete deck composite with a steel multicell section can adequately resist torsional and warping effects induced by high curvature. Although current design practices in North America recommend few analytical methods for the design of curved multicell box girder bridges, economical requirements in the design process point to a need for a simplified design method. This paper summarizes the results from an extensive parametric study, using the finite-element method, in which simply supported curved composite multicell bridge prototypes are analyzed to evaluate the moment and deflection distributions between girders, as well as the axial forces expected in the bracing system, due to truck loading as well as dead load. Results from tests on four, 1/12 linear-scale, simply supported curved composite concrete deck-steel multicell bridge models are used to substantiate and verify the analytical modeling. The parameters considered in the study are cross-bracing system, aspect ratio, number of lanes, number of cells, and degree of curvature. Based on the data generated from the parametric study, expressions for moment and deflection distribution factors are deduced. Expressions for the maximum axial force in bracing members are also derived. An illustrative design example is presented.

AUTHOR: Yoo, C. H., Davidson, J. S., and Zhang, J. (1998)

TITLE: Top flange diagonal bracing of horizontally curved box girders

INFO: Proceedings of the Engineering Mechanics Conference: A Force for the 21st Century, La Jolla, CA, May 18–20.

ABSTRACT: Steel/concrete composite box girders with inclined webs, otherwise known as tub girders, have been successfully designed and built in the U.S. As modern advancement in welding has permitted box girder fabrication with relative ease, it is expected that the use of composite box girders will grow in urban interchanges. The superior torsional stiffness of box girders, however, cannot be realized until the composite concrete deck has been hardened. For non-composite dead loads, top flange diagonal bracing in horizontally curved girders acts as primary load carrying members. This paper presents a rational design guide for a diagonal tub flange bracing member highlighting several important design considerations. Supporting data used for computation were generated by a three-dimensional finite element analysis on a hypothetical horizontally curved box girder bridge.

Background Research Pertaining to Updated AASHTO LRFD Specifications for Steel Structures, Third Edition

Prepared by Donald W. White, Georgia Tech

Aydemir, M., White, D. W., and Jung, S. K. (2004). "Shear Strength and Moment Shear Interaction in HPS Hybrid I-Girders," Structural Engineering, Mechanics and Materials Report No. 25, School of Civil and Environmental Engineering, Georgia Institute of Technology, Atlanta, GA, 203 pp.

This research examines the maximum strength behavior of transversely stiffened prismatic straight hybrid I-girders having Grade 50 steel webs and Grade 70 steel flanges. A parametric suite of test specimens is designed, and full nonlinear analyses of these girders are conducted using shell finite elements. The primary focus of this work is the evaluation of the postbuckling shear strength as well as the moment-shear interaction characteristics of these types of girders. The members considered include 18 different cross-sections and are subjected to various ratios of the maximum moment-to-shear within the critical test panel. The key cross-section parameters varied within the study are the web slenderness D/t_w, the panel aspect ratio d_o/D, and the ratio of the depth of the web in compression to the total web depth D_c/D. A total of 147 combinations of geometry and loading conditions are studied.

Barth, K. E., White, D. W., Righman, J. E., and Yang, L. (2005). "Evaluation of Web Compactness Limits for Singly and Doubly Symmetric Steel I-Girders," ***Journal of Constructional Steel Research,*** **61(10), 1411–1454.**

This paper presents the results of research aimed at evaluating the web compactness limit for steel I-girders. Specifically, the paper tests the implications of a new web compactness limit equation provided in AASHTO (2003) versus the web compactness limit in the 2001 AASHTO LRFD Specifications. In both Specifications, these limits are required for the nominal moment capacity to equal the plastic moment capacity of the girder, provided other requirements are also satisfied. The 2001 AASHTO LRFD web compactness limit is the same as the limit in the AISC LRFD (1999) Specifications, except the AASHTO provisions place an additional restriction on the web slenderness if the flange slenderness exceeds 75% of the corresponding flange compactness limit, via an interaction equation involving the flange and the web slenderness values.

The origin of the AASHTO (2003) and (2001) web compactness limits is presented along with a performance evaluation of these equations. Specifically, resulting moment capacities from a comprehensive suite of finite element analyses are compared to the capacities that the respective limits are intended to provide. Results indicate there are some limitations in the AASHTO (2001) and AISC (1999) web compactness limits (particularly for singly symmetric cross-sections) that are removed by the AASHTO (2003) web compactness provisions.

Barth, K. E., Hartnagel, B. A., White, D. W., and Barker, M. G. (2004). "Recommended Procedures for Simplified Inelastic Design of Steel I-Girder Bridges," ***Journal of Bridge Engineering,*** **ASCE, 9(3), 230–242.**

This paper presents summary recommendations pertaining to new AASHTO procedures for simplified inelastic design of steel I-girder bridges. First, key developments are summarized that lead to the proposed inelastic design approach. The paper then outlines a set of equations that provide an improved characterization of the inelastic moment-rotation response for a wide range of I-beams and plate girders. Effective plastic moment predictions based on these equations are combined with the recently proposed design method, resulting in greater accuracy and simplicity of the proposed approach. The ease of use of the resulting procedure is illustrated by a design example.

Barth, K. E., and White, D. W. (1997). "Finite Element Evaluation of Pier Moment-Rotation Characteristics in Continuous-Span Steel I-Girders." ***Engineering Structures,*** **20(8), 761–778.**

This paper summarizes the implementation and execution of a reasonably comprehensive set of finite element parametric studies to fill in knowledge gaps in the available experimental data pertaining to the hogging moment-plastic rotation behavior of steel and composite steel-concrete bridge girders. The paper highlights key requirements for proper finite element modeling of the behavior, outlines the design of the finite element parametric studies, and presents the important "numerical test" results. Based on the finite element predictions, simple moment-rotation relationships are developed for use in design and rating of bridge structures.

Beshah, F. (2005). "Moment-Shear Test Series," Volume 3, Curved Steel Bridge Research Project, Federal Highway Administration, Curved Steel Bridge Research Project, Federal Highway Administration, December.

As part of the Federal Highway Administration's curved steel bridge research project (CSBRP), to further perform experimental and analytical study in the fundamental behavior of horizontally curved steel bridges, static load tests on four full-scale curved I-girders were performed under high moment and low shear loading. Each of the four girders had 7.74 m centerline length measured along arc length, 63.63 m radius of curvature, 4.77 m unbraced length, and 8 by 1,219 mm web plate. All girders were fabricated from AASHTO M270 Grade 345 steel. The varied dimensional parameters include the width to thickness ratio of the flange, and the transverse stiffener spacing. Strain gages, potentiometers, load-cells, and tilt meters were placed on all test girders to study their responses.

This report, Volume 3 of the final report, presents detailed descriptions and results from these static load tests. Measured data are compared with nonlinear finite element analysis prediction. Furthermore, the measured data are also used to evaluate design specifications in current bridge codes.

Beshah, F. (2006). "Girder Pair Test," Volume 5, Curved Steel Bridge Research Project, Federal Highway Administration, May.

As part of the Federal Highway Administration's curved steel bridge research project (CSBRP), to further perform experimental and analytical study on the fundamental behavior of horizontally curved steel bridges, a static load test on a full-scale girder pair was performed. The test evaluated the effect of lateral bracing.

This report, Volume 5 of the final report, presents a detailed description and results from this static load test. Measured data are presented and compared with nonlinear finite element analysis prediction. The measured data are also used to evaluate design specifications in current highway bridge codes. Analytical studies are also included to investigate the effect of lateral brace size.

Beshah, F. (2006). "Testing of Composite Bridge," Volume 9, Curved Steel Bridge Research Project, Federal Highway Administration, November.

As part of the Federal Highway Administration's curved steel bridge research project (CSBRP), to further perform experimental and analytical study on the fundamental behavior of horizontally curved steel bridges, static load tests on a full-scale composite curved steel I-girder bridge were conducted. The tests determined the behavior under a number of loading stages up to failure. The bridge was a simple span consisting of three I-girders spaced at 2,667 mm. A 203-mm-thick concrete deck, consisting of radial and longitudinal reinforcement, was composite with the steel girders. The bridge was instrumented with strain gages, potentiometers, load-cells, linear voltage displacements, transducers, and tilt meters.

This volume, Volume 9 of the final report, presents detailed descriptions and results from these tests. Measured data are presented, and comparison to nonlinear finite element analysis prediction is presented in Volume 6 of the final report.

Chang, C.-J., and White, D. W. (2006). "Construction Simulation of Curved Steel I-Girder Bridges," Volume 7, Curved Steel Bridge Research Project, Federal Highway Administration, May.

This study addresses the development of a prototype software system for analysis of horizontally curved steel I-girder bridges, with emphasis on construction simulation. The approach adopted in this research is based on the use of open-walled section beam theory for the I-girders and a beam grillage representation for the composite bridge slab. An approximate approach is targeted for capturing the influence of girder web distortions on composite I-girder responses. Also, recommendations are provided for the use of grillage idealizations in analyzing curved I-girder bridge structural systems. A key focus of the research is on simulating steel erection and staged slab casting processes. The resulting capabilities allow engineers to check deflections, reactions and/or stresses at different stages of the steel erection or concrete casting and determine required crane capacities, tie-down, jacking or come-along forces, incremental displacements due to removal of temporary supports, etc. Also, the capabilities can be used to determine the influence of different steel detailing methods on the bridge geometry, such as the web-plumbness under the steel or total dead load, as well as the implications of geometric tolerances on the structural performance. The fundamental requirements necessary to ensure accuracy of the analysis results are addressed.

Chang, C.-J., White, D. W., Beshah, F., and Wright, W. (2005). "Design Analysis of Curved I-Girder Bridge Systems—An Assessment of Modeling Strategies," Annual Proceedings, Structural Stability Research Council, 349–369.

This paper investigates the qualities and limitations of a number of modeling strategies for design analysis of curved I-girder bridge systems, ranging from modified line-girder analyses to finite element approaches. A representative full-scale composite curved I-girder bridge, tested at the FHWA Turner-Fairbank Highway Research Center (TFHRC), is utilized for assessment of the different approaches. The predictions of key lateral and vertical displacements, reactions, cross-frame forces, and major-axis and flange lateral bending stresses are evaluated with respect to the test bridge. The paper highlights a number of subtle but critical blunders that can occur in the context of the different methods. Engineers need to be aware of these pitfalls in order to avoid potential problems due to inaccurate analysis predictions. The paper closes with a discussion of the efficacy of the various approaches.

Jung, S.-K., and White, D. W. (2006). "Inelastic Behavior of Horizontally Curved Composite I-Girder Bridge Structural Systems," Volume 6, Curved Steel Bridge Research Project, Federal Highway Administration, May.

This study addresses the design and analysis of a full-scale horizontally curved composite test bridge selected to examine the system and component responses of a representative curved composite structure under all loading stages: non-composite dead load, composite service live load, and ultimate loading. The curved composite test bridge is designed at or above a number of maximum limits in AASHTO (2003 and 2004) design provisions. Refined three-dimensional finite element analysis (FEA) models are developed and applied in this research for both the elastic design-analysis as well as linear elastic, geometric nonlinear and full nonlinear FEA simulation of the test bridge system. The full nonlinear FEA model involves the simulation of staged construction, followed consecutively by applied loads on the composite structure in a single continuous process. The composite test bridge was built and tested at the FHWA Turner-Fairbank Highway Research Center for all loading stages. This study provides the synthesis and assessment of the experimental results and corresponding full nonlinear FEA predictions for the test bridge. Furthermore, parametric FEA studies are performed to investigate the responses for a wider range of bridges, with various attributes including skewed supports, integral abutments and different cross-frame detailing methods. Based on the system and component responses of the test bridge as well as the additional FEA parametric studies, this research investigates the implications of the use of enhanced flexural resistance equations that account for flange lateral bending effects from any source, specified in AASHTO (2004) for straight bridge I-girders, in the context of curved I-girder bridges.

Jung, S.-K., and White, D. W. (2006). "Shear Strength of Horizontally Curved Steel I-Girders—Finite Element Studies," *Journal of Constructional Steel Research,* 62(4), 329–342.

This paper presents the results of finite element analysis (FEA) studies of four curved steel I-girder shear components tested experimentally in previous research, as well as parametric extensions of these tests. These studies focus on the influence of horizontal curvature on the maximum strength of transversely stiffened members with web slenderness D/t_w approximately equal to the largest value permitted in AASHTO (2004), and with panel aspect ratios of $d_o/D = 1.5$ and 3.0. These ratios are larger than previously considered in experimental tests of curved I-girders with similar or larger slenderness. The girders studied have subtended angles between their bracing locations of $L_b/R = 0.05$ and 0.10, and web panel d_o/R values ranging from 0.0287 to 0.10. The FEA models incorporate the measured material stress-strain relationships and section dimensions from the physical tests, detailed modeling of the test boundary conditions, residual stresses due to flame cutting and welding, and initial geometric imperfections in the form of buckling mode shapes. The load transfer mechanisms of the test girders are investigated via elastic buckling and full nonlinear analyses. The parametric studies are performed to investigate the effects of residual stresses and geometric imperfections, the behavior of equivalent straight girders, and the influence of reduced flange size on the peak shear capacity and moment-shear interaction.

Jung, S.-K., and White, D. W. (2006). "Strength Behavior of Horizontally Curved Composite I-Girder Bridge Structural Systems," Annual Proceedings, Structural Stability Research Council, to appear, 20 pp.

This paper discusses the strength behavior of a representative horizontally curved composite (steel and concrete) I-girder bridge system that was designed near the limits of the AASHTO (2004) bridge design specifications and tested at the Federal Highway Administration (FHWA) laboratory for its ultimate loading capacity. Particular focus is placed on the force transfer mechanisms within the bridge system as a whole as the test bridge approaches the strength limit. The results clearly indicate that there is a high degree of interaction among the bridge girders. However, despite the complexities associated with the interconnectedness of the bridge girders, all the bridge component and system responses are accurately predicted by linear elastic analysis up to the strength limit based on the provisions of Article 6.10.7.1 in AASHTO (2004) Specifications. Article 6.10.7.1 uses the composite section plastic moment, reduced by the flange lateral bending effects due to torsion, as the base flexural resistance for the strength limit states. AASHTO (2004) presently does not allow the use of Article 6.10.7.1 for the strength limit state checks in horizontally curved I-girder bridges, although these provisions are applicable to general straight I-girder bridges. It limits the base resistance for the strength checks to the member yield moment M_y, rather than M_p, for these bridge types (via Article 6.10.7.2). The findings from this and other research support the potential liberalization of the AASHTO strength design provisions for horizontally curved I-girder bridges.

Jung, S.-K., White, D. W., Beshah, F., and Wright, W. (2005). "Ultimate Strength of Horizontally-Curved Composite I-Girder Bridge Structural Systems," Annual Proceedings, Structural Stability Research Council, 327–347.

This paper provides an overview of a full-scale composite curved I-girder bridge tested at the FHWA Turner-Fairbank Highway Research Center. The paper presents a summary of the broad research plan for the composite bridge test. This is followed by a detailed discussion of key aspects related to the strength design and behavior of the composite test bridge structural system: (1) a brief introduction to the unified flexural resistance equations for curved and straight I-girder design recently implemented in AASHTO (2004), (2) current design restrictions in AASHTO (2004) and why, (3) potential liberalization of these restrictions, (4) refined finite element analysis (FEA) modeling, (5) design attributes of the test bridge pertaining to AASHTO (2004), and (6) preliminary results from refined full nonlinear FEA and from physical testing of the test bridge system.

Kim, Y. D., Jung, S. K., and White, D. W. (2006). "Transverse Stiffener Requirements in Straight and Horizontally Curved Steel I-Girders," *Journal of Bridge Engineering,* ASCE, to appear.

A number of prior research studies have demonstrated that transverse stiffeners in straight I-girders are loaded predominantly by bending induced by their restraint of web lateral deflections at the shear strength limit state, not by in-plane tension field forces. This is at odds with present specification approaches for the design of these components. Furthermore, recent studies have confirmed that curved I-girders are capable of developing substantial shear postbuckling resistance due to tension field action and have demonstrated that the AASHTO LRFD equations for the tension field resistance in straight I-girders may be applied to curved I-girders within specific limits. However, the corresponding demands on transverse stiffeners in curved I-girders are still largely unknown. In this paper, the behavior of one- and two-sided transverse stiffeners in straight and horizontally curved steel I-girders is investigated by refined full nonlinear finite element analysis. New recommendations are developed for design of transverse stiffeners in straight and curved I-girders based on the combined solutions from this research and from prior research studies.

White, D. W., and Chang, C.-J. (2006). "Improved Flexural Stability Design of I-Section Members in AISC (2005)—A Case Study Comparison to AISC (1989) ASD," *Engineering Journal,* AISC, to appear.

The AISC (2005) provisions for the flexural stability design of steel I-section members have been updated relative to previous specifications to simplify their logic, organization and application, while also improving their accuracy and generality. This paper gives a brief overview of the updated provisions and compares and contrasts their flexural resistance calculations with the corresponding calculations from the previous AISC (1989) ASD specification. The relative simplicity and accuracy of the AISC (2005) equations is highlighted.

White, D. W., and Grubb, M. A. (2005). "Unified Resistance Equations for Design of Curved and Tangent Steel Bridge I-Girders," *Proceedings of the 2005 TRB Bridge Engineering Conference,* Transportation Research Board, Washington, DC, July, 121–128.

The provisions of the 2004 AASHTO Load and Resistance Factor Design (LRFD) Specifications for steel I- and box-girder bridge design have been updated relative to previous specifications to simplify their logic, organization and application while also improving their accuracy and generality. These provisions provide a unified approach for the flexural design of both tangent and curved I- and box-girder bridges. Updated resistance equations are a key component of this unified approach. This paper provides an overview of the updated resistance equations for I-section members. The primary focus is on the handling of coupled major-axis bending, minor-axis bending and torsion from any source in both straight and horizontally curved I-section members.

White, D. W. (2004). "Unified Flexural Resistance Equations for Stability Design of Steel I-Section Members—Overview," Structural Engineering, Mechanics and Materials Report No. 24a, School of Civil and Environmental Engineering, Georgia Institute of Technology, Atlanta, GA, 42 pp.

The AASHTO (2004) and AISC (2005) provisions for the flexural design of steel I-section members have been revised in their entirety relative to previous specifications to simplify their logic, organization and application, while also improving their accuracy and generality. This report provides a technical overview of these developments with respect to the stability limit states. The updated AISC and AASHTO flexural resistances are, with minor exceptions, fundamentally the same. However, the organization and format in AASHTO (2004) emphasizes the streamlined design of various I-section member types common for bridge spans of 30 m (100 ft) or larger, while AISC (2005) emphasizes the streamlined design of compact doubly-symmetric I-section members. This report presents the fundamental logic and the associated calculations of these new provisions as a single set of flowcharts for design of all types of steel I-section members, with the exception of composite members in positive bending. That is, all types of composite I-section members in negative bending and all types of noncomposite I-section members are addressed, including hybrid members with different yield strengths for the tension flange, compression flange and/or web, longitudinally stiffened members, members with channel caps on the compression flange or cover plates on one or both flanges, and members with section transitions and/or variable web depth. The flexural resistance equations are presented in a unified format applicable for all of the above types of I-section members. In each of the sections of the report, the presentations start with an emphasis on the "how to" of the resistance calculations. This is followed by more in-depth discussions of the background to the calculation procedures. Various improvements relative to the prior AISC and AASHTO specifications are highlighted.

A paper version of the above report is under review for potential publication in the Journal of Structural Engineering, ASCE.

White, D. W., and Jung, S.-K. (2004). "Unified Flexural Resistance Equations for Stability Design of I-Shaped Members—Uniform Bending Tests," Structural Engineering, Mechanics and Materials Report No. 28, School of Civil and Environmental Engineering, Georgia Institute of Technology, Atlanta, GA, 128 pp.

The AASHTO (2004) and AISC (2005) provisions for flexural design of steel I-section members have been revised in their entirety relative to previous specifications to simplify their logic, organization and application, while also improving their accuracy and generality. This report evaluates the lateral-torsional and flange local buckling (LTB and FLB) resistance predictions from these and previous specifications versus uniform bending experimental test results. A total of 154 rolled and 123 welded I-section member LTB tests, and 11 rolled and 36 welded I-section member FLB tests, are considered. Reliability indices are estimated for Load and Resistance Factor Design (LRFD) of buildings based on the test statistics combined with established statistics for material and fabrication bias factors and the ASCE 7 load model. The notional reliability for LTB is found to be reasonably constant and consistent with the targeted level in the first LRFD specification of 1986. The unified resistance equations, combined with a simple design-oriented procedure for calculation of elastic LTB K factors, are shown to capture the test results accurately throughout the inelastic and elastic LTB ranges, leading to substantial liberalization of the calculated resistances in certain cases. The mean resistances for inelastic LTB and FLB are captured accurately by a linear equation in the corresponding slenderness parameters. The reliability for FLB is found to be slightly higher than that for LTB.

A paper version of the above report is under review for potential publication in the Journal of Structural Engineering, ASCE.

White, D. W., and Kim, Y. D. (2004). "Unified Flexural Resistance Equations for Stability Design of I-Shaped Members—Moment Gradient Tests," Structural Engineering, Mechanics and Materials Report No. 29, School of Civil and Environmental Engineering, Georgia Institute of Technology, Atlanta, GA, 149 pp.

The AASHTO (2004) and AISC (2005) provisions for flexural design of steel I-section members have been revised in their entirety relative to previous specifications to simplify their logic, organization and application, while also improving their accuracy and generality. This report evaluates the lateral-torsional and flange local buckling (LTB and FLB) resistance predictions from these specifications versus moment gradient experimental test results. Uniform bending experimental tests are addressed in a companion report. Two types of moment gradient tests are considered: (1) tests in which the moment varies linearly within the critical unbraced length and (2) tests in which the member is subjected to concentrated transverse load at a specified height relative to the depth of the cross-section, resulting in a multi-linear moment diagram within the critical unbraced length. A total of 27 welded and 10 rolled I-section member FLB tests of Type (1), 73 rolled and 93 welded member LTB tests of Type (1), and 129 rolled and 111 welded member LTB tests of Type (2) are considered. Reliability indices are estimated for Load and Resistance Factor Design (LRFD) of buildings based on the statistics from these tests combined with established statistics for material and fabrication bias factors and the ASCE 7 load model. The report demonstrates that in certain cases, the reliability index is substantially larger for moment gradient loading compared to that for uniform bending. However, for other cases, the notional reliability for members subjected to moment gradient is essentially the same as that estimated in the companion report for uniform moment.

A paper version of the above report is under review for potential publication in the Journal of Structural Engineering, ASCE.

White, D. W., and Barker, M. (2004). "Shear Resistance of Transversely-Stiffened Steel I-Girders," Structural Engineering, Mechanics and Materials Report No. 26, School of Civil and Environmental Engineering, Georgia Institute of Technology, Atlanta, GA, 105 pp.

This report evaluates the accuracy and ease of use of 12 of the most promising models for the shear resistance of transversely stiffened steel I-girders. Several models that are well established in civil engineering practice as well as a number of other recently proposed models are considered. Since the model developed in Basler's seminal research is the method of choice in current American practice, the report focuses on the merits and limitations of the alternative models relative to Basler's. Statistical analyses are conducted on the predictions by the various models using an updated data set from 129 experimental shear tests, including 30 hybrid and 11 horizontally curved I-girders. The results support the conclusion that the form of Basler's model implemented in AASHTO (2004) and AISC (2005) gives the best combination of accuracy and simplicity for calculation of the shear resistance of transversely stiffened I-girders.

A paper version of the above report is under review for potential publication in the Journal of Structural Engineering, ASCE.

White, D. W., Barker, M., and Azizinamini, A. (2004). "Shear Strength and Moment-Shear Interaction in Transversely-Stiffened Steel I-Girders," Structural Engineering, Mechanics and Materials Report No. 27, School of Civil and Environmental Engineering, Georgia Institute of Technology, Atlanta, GA, 82 pp.

With the advent of HPS495W steel, hybrid I-girder designs have become advantageous in bridge design. One limit on hybrid I-girder designs, which decreases their beneficial aspects, is that the use of tension field action is not permitted in determining the shear resistance. This is a significant penalty for hybrid I-girders. Also, the checking of moment-shear strength interaction is a significant complicating factor in the design and capacity rating of I-girders that use tension field action. The requirements for the shear design of hybrid I-girders and the equations for moment-shear strength interaction in AISC (1999) and AASHTO (1998) were developed originally without the benefit of a large body of experimental tests and refined finite element solutions. This report presents the results from the collection and analysis of the data from a total of 186 high-shear low-moment, high-shear high-moment, and high-moment high-shear experimental I-girder tests. References to corroborating refined finite element studies are provided. Particular emphasis is placed on the extent to which web shear postbuckling (tension-field action) strength is developed in hybrid I-girders, as well as on the interaction between the flexural and shear resistances in hybrid and non-hybrid I-section members. The results of the study indicate that within certain constraints that address the effects of small flange size, Basler's shear resistance model can be used with the unified flexural resistance provisions in AASHTO (2004) and AISC (2005) without the need for consideration of M-V strength interaction. Also, the report shows that a form of the Cardiff model can also be used with the unified flexural resistance provisions without the need to consider M-V strength interaction. These conclusions apply to both non-hybrid and hybrid I-girder designs.

A paper version of the above report is under review for potential publication in the Journal of Structural Engineering, ASCE.

White, D. W., and Jung, S.-K. (2003). "Simplified Lateral-Torsional Buckling Equations for I- and Channel-Section Members," Structural Engineering, Mechanics and Materials Report No. 24a, School of Civil and Environmental Engineering, Georgia Institute of Technology, Atlanta, GA, 23 pp.

This report presents a recommended simplified form of the fundamental beam-theory LTB equations for doubly symmetric members, specialized to the elastic LTB resistance of I- and channel-section members. This recommended form is exact for doubly symmetric I-section members and has the advantage of improved accuracy relative to the corresponding AASHTO (1998) equations when applied to singly symmetric I-shapes, including composite I-section members in negative bending. Also, it avoids significantly conservative errors that occur within the double-formula approach for a wide range of rolled wide-flange shapes. Furthermore, the physical significance of each of the terms within the simplified equations is easy to understand. These equations are utilized as the base elastic LTB expressions within the AASHTO (2004) and AISC (2005) specifications.

White, D. W., and Jung, S.-K. (2003). "Simplified Lateral-Torsional Buckling Equations for Singly-Symmetric I-Section Members," Structural Engineering, Mechanics and Materials Report No. 24b, School of Civil and Environmental Engineering, Georgia Institute of Technology, Atlanta, GA, 29 pp.

In the companion report, the authors develop and discuss the advantages of a recommended set of equations for the elastic lateral-torsional buckling (LTB) resistance of I- and channel-section members. These equations are utilized as the base elastic LTB expressions within the AASHTO (2004) and AISC (2005) specifications. The companion report focuses on the characteristics of the recommended equations pertaining to doubly symmetric I-section members and channels. This report extends these developments to singly symmetric I-section members, including composite I-sections in negative bending and channel-capped I-sections. Two approaches are highlighted for calculation of the elastic LTB resistance of these general member types:

1. Ad hoc application of the doubly symmetric equations recommended in a companion report, which is similar to the approach taken in AASHTO (1998) using an alternative set of LTB equations for doubly symmetric I-section members.
2. A simplified form of the rigorous equations obtained from open-walled section beam theory.

A key advantage of the first approach is that it leads to a single set of equations for all I-section members and channels. Also, these equations are simpler to apply than the rigorous beam theory equations for general I-shapes. The recommended doubly symmetric equations give an improved approximation of the rigorous beam theory solution for singly symmetric I-section members compared to the AASHTO (1998) equations. Also, these equations can be applied as a conservative but typically adequate approximation of the elastic LTB resistance for composite I-section members in negative bending. The main disadvantage of the first approach is that it is not rigorous. Therefore, the behavior of the equations must be studied parametrically to ensure that they predict the physical strengths adequately for all practical singly symmetric geometries. Any ranges of parameters that produce unacceptable error must be disallowed. The key advantage of the second approach is that an exact or a highly accurate approximation of the beam theory LTB resistance is obtained. The primary disadvantage of this approach is that the equations are more complex. Also, their extension to the handling of composite I-girders in negative bending is not as straightforward.

White, D. W., and Jung, S.-K. (2003). "Effect of Web Distortion on the Buckling Strength of Noncomposite Discretely-Braced I-Beams," Report No. 24c, School of Civil and Environmental Engineering, Georgia Institute of Technology, Atlanta, GA, 34 pp.

The influence of the web distortional flexibility typically is not addressed explicitly in standards for flexural design of steel I-section members. This is due in part to the fact that there are no simple closed-form solutions that account for these effects, whereas closed-form solutions are well established for elastic lateral-torsional buckling using open-walled section beam theory. Open-walled section beam theory is of course based on the assumption that the cross-section profile does not deform. Also, strength predictions are generally good for a wide range of experimental tests without explicit accounting for web distortion in the lateral-torsional buckling calculations. Nevertheless, it is clear from prior research studies that the web distortional flexibility can lead to a substantial reduction relative to the beam theory lateral-torsional buckling resistance for I-sections with torsionally stiff flanges and relatively thin webs. AASHTO (2004) and AISC (2005) give specific limits on the cross-section geometry and yield strengths intended to control the unconservative errors associated with the neglect of web distortion effects. This report evaluates the effectiveness of these limits. Potential future directions for improved calculation of I-section member flexural resistances are suggested for cases where the influence of web distortion is significant.

A paper version of the above report is under review for potential publication in the Journal of Engineering Structures.

White, D. W. (2002). "LRFD Article 6.10 Draft # 3, Sample Case Studies," Report to AISC-AASHTO Task Committee for Updating of AASHTO LRFD Article 6.10, October, 84 pp.

The goal of this report is to provide engineers with an overall conceptual understanding of how the nominal strengths defined by the proposed resistance equations in Draft # 2 of AASHTO Article 6.10 relate to the nominal strengths defined within the current AASHTO LRFD (2001) Specifications, the AISC LRFD (1999) Specifications, the AASHTO Guide Specifications for Curved Steel Bridge Design (2002), and several additional references and analytical solutions of important note. This is achieved by focusing on specific example case studies. The report addresses the following specific limit states:

1. Generic influence of flange and web slenderness in straight I-girders,
2. Lateral-torsional buckling (LTB) for various key categories of I-shaped member geometries in which the flanges are compact,
3. Strengths as a function of the unbraced length for girders where the maximum resistance is influenced by compression flange local buckling (FLB),
4. Strength and ductility of composite I-girders in positive bending, and
5. Strength under combined major-axis flexure (vertical bending) and flange lateral bending, using members from the second and third of the above groups.

A total of 13 different cases are considered pertaining to straight I-girders with zero lateral bending. Three specific examples and results from a focused parametric study are presented for composite I-girders in positive bending. Four cases are considered with respect to strength under combined vertical and flange lateral bending. Except where noted otherwise, all of the studies are focused on homogeneous members with $F_y = 50$ ksi.

White, D. W., Zureick, A. H., Phoawanich, N., and Jung, S.-K. (2001). "Development of Unified Equations for Design of Curved and Straight Steel Bridge I-Girders," Final Report to American Iron and Steel Institute Transportation and Infrastructure Committee, Professional Services Industries, Inc., and Federal Highway Administration, October, 551 pp.

This research examines analytically and computationally the maximum strength behavior of curved and straight steel I-girders subjected to uniform vertical bending, high shear and low vertical bending moment, and high-shear and high-vertical bending moment, combined with lateral bending due to torsion and/or applied design loads. The theoretical and practical background, qualities, and limitations of existing design predictor equations are outlined. Key existing equations and new predictor equations developed as part of this research are evaluated based on the results of a reasonably comprehensive finite element parametric study, as well as the data from and finite element analyses of prior experimental tests. Based on consideration of the maximum strength predictions as well as pre- and post-peak load-deflection results, a unified set of strength equations is recommended that can be applied to both curved and straight I-girders for all loading conditions, including lateral bending and torsion.

White, D. W., and Barth, K. E. (1998). "Strength and Ductility of Compact-Flange I Girders in Negative Bending," *Journal of Constructional Steel Research,* 45(3), 241–280.

This paper reviews available experimental and finite element test data pertaining to the negative moment-plastic rotation behavior of continuous-span steel I-girders with compact or ultra-compact flanges. Current American specification formulas for the pier-section strength of these types of members and a moment-plastic rotation model recently developed by the authors are examined against the experimental and finite element test results. Several weaknesses in current specification provisions are observed. The new M-θ_p model avoids these weaknesses and provides a lower-bound approximation of the complete moment-plastic rotation response at the pier section.

White, D. W., Ramirez, J. A., and Barth, K. E. (1997). "Moment Rotation Relationships for Unified Autostress Designs of Continuous-Span Bridge Beams and Girders." *Final Report,* Joint Transportation Research Program, West Lafayette, IN, 117 pp.

This report summarizes the development and trial application of simplified moment-rotation relationships for inelastic design of continuous-span beam and girder bridges. The research described within involves the execution of a reasonably comprehensive set of finite element parametric studies to fill in knowledge gaps in the available experimental data pertaining to the hogging moment-plastic rotation behavior of steel and composite steel-concrete bridge girders. Based on these studies, relationships have been developed for the moment-plastic rotation behavior at the pier sections in these types of bridges. The moment-rotation model is validated against available experimental data, several focused new experimental tests, and current American specification strength formulas. This study concludes with a detailed trial inelastic design of a three-span continuous plate-girder bridge using suggested new inelastic design procedures. The characteristics of the calculations and the resulting proportions are compared with those of an elastic design of the same bridge by current AASHTO LRFD procedures. The elastic design is a modified version of a three-span continuous plate-girder bridge example recently published by the American Iron and Steel Institute for a Highway Structures Design Handbook.

Wright, D. W., and Beshah, F. (2006). "Construction of Test Bridge," Volume 8, Curved Steel Bridge Research Project, Federal Highway Administration, November.

As part of the Federal Highway Administration's curved steel bridge research project (CSBRP), to further perform experimental and analytical study in the fundamental behavior of horizontally curved steel bridges, response of the superstructure of full-scale simple-span composite curved steel I-girder bridge during all phases of construction sequence was monitored. The bridge was a simple span consisting of three I-girders spaced at 2,667 mm. A 203-mm-thick concrete deck, consisting of radial and longitudinal reinforcement, was composite with the steel girders. The bridge was instrumented with strain gages, potentiometers, load-cells, linear voltage displacements transducers, and tilt meters.

This volume, Volume 8 of the final report, presents the response of the superstructure during all phases of the construction sequence.

Zureick, A. H., White, D. W., Phoawanich, N., and Park, J. (2002). "Shear Strength of Horizontally Curved Steel I Girders—Experimental Tests," Volume 4, Curved Steel Bridge Research Project, Federal Highway Administration, March, 157 pp.

This report presents the results of four full-scale curved steel I-girder component tests conducted to examine their shear behavior and to determine their maximum shear strengths. The girders were made of AASHTO M270 Grade 345 steel and had a nominal web depth and web thickness of 1,219 mm (48 in) and 8 mm (5/16 in), respectively. The resulting nominal web slenderness ratio D/t_w was 154. Two of the girders, labeled as S1 and S1-S, had a nominal radius $R = 63{,}630$ mm (208.75 ft) and a transverse stiffener spacing such that the ratio d_o/D was 3 for S1 and 1.5 for S1-S (producing $d_o/R = 0.0575$ and 0.0287). The other two test components, labeled as S2 and S2-S, were identical to S1 and S1-S except that their radii were 36,580 mm (120 ft), resulting in $d_o/R = 0.10$ and 0.050. All of the girders were braced against radial deflections at intervals of 3,658 mm (12 ft) along the girder arc. Therefore, the ratio L_b/R was equal to 0.0575 for S1 and S1-S and 0.10 for S2 and S2-S, where L_b is the distance between the bracing systems along the girder arc. All the test girders were instrumented to determine their maximum shear resistance as well as the mechanisms associated with the development of their shear strengths. Of particular interest was the extent to which the curved webs were capable of developing postbuckling strength, and the influence of the horizontal curvature and panel aspect ratio on the development of this strength.

APPENDIXES B–D

Related Materials

The following appendixes are not published herein:

- **Appendix B: Design Specifications** was balloted by the AASHTO Highway Subcommittee on Bridges and Structures (HSCOBS) and adopted as specifications.
- **Appendix C: Calibration of LRFD Design Specifications for Steel Curved Girder Bridges** is available at http://trb.org/news/blurb_detail.asp?id=5965.
- **Appendix D: Comparison of Curved Steel I-Girder Bridge Design Specifications** is available at http://trb.org/news/blurb_detail.asp?id=5965.

Abbreviations used without definitions in TRB publications:

AASHO	American Association of State Highway Officials
AASHTO	American Association of State Highway and Transportation Officials
ADA	Americans with Disabilities Act
APTA	American Public Transportation Association
ASCE	American Society of Civil Engineers
ASME	American Society of Mechanical Engineers
ASTM	American Society for Testing and Materials
ATA	American Trucking Associations
CTAA	Community Transportation Association of America
CTBSSP	Commercial Truck and Bus Safety Synthesis Program
DHS	Department of Homeland Security
DOE	Department of Energy
EPA	Environmental Protection Agency
FAA	Federal Aviation Administration
FHWA	Federal Highway Administration
FMCSA	Federal Motor Carrier Safety Administration
FRA	Federal Railroad Administration
FTA	Federal Transit Administration
IEEE	Institute of Electrical and Electronics Engineers
ISTEA	Intermodal Surface Transportation Efficiency Act of 1991
ITE	Institute of Transportation Engineers
NASA	National Aeronautics and Space Administration
NCHRP	National Cooperative Highway Research Program
NCTRP	National Cooperative Transit Research and Development Program
NHTSA	National Highway Traffic Safety Administration
NTSB	National Transportation Safety Board
SAE	Society of Automotive Engineers
SAFETEA-LU	Safe, Accountable, Flexible, Efficient Transportation Equity Act: A Legacy for Users (2005)
TCRP	Transit Cooperative Research Program
TEA-21	Transportation Equity Act for the 21st Century (1998)
TRB	Transportation Research Board
TSA	Transportation Security Administration
U.S.DOT	United States Department of Transportation