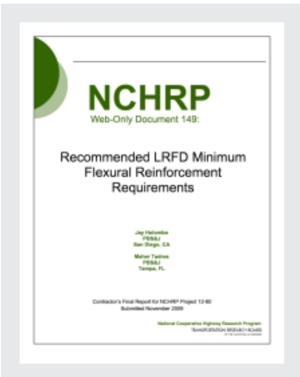

Recommended LRFD Minimum Flexural Reinforcement Requirements

DETAILS

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Jay Holombo, PBS&J, San Diego, CA, was the principal investigator and lead author, and Maher K. Tadros, University of Nebraska, Lincoln (UNL) and formerly with PBS&J, Tampa, FL, was the coauthor of this report. Other research team members include: Paul Morel, PBS&J, San Diego, CA, who developed Design Examples 1, 2, and 5 and performed the parametric study design calculations; Sami Megally, PBS&J, San Diego, CA, who developed Design Examples 3 and 4; Andrzej Nowak, UNL, who assisted with the statistical analysis; Stephen Seguirant, Concrete Technologies, Inc., Tacoma, WA, who provided vital input and feedback throughout the project. Daniel Tassin, International Bridge Technologies, Inc., San Diego, CA; Morad Ghali with PBS&J, Tampa, FL; and Artur Czarnecki of Grontmij, Dublin, Ireland (formerly with PBS&J, Tampa, FL) also provided valuable input on the research efforts.

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ABSTRACT

This report documents and presents the results of a study of minimum reinforcement requirements for the design of concrete bridge structures. This study included a review of U.S. and international practice, test data and research findings related to minimum reinforcement requirements and flexural cracking of concrete structures. A total of 4 representative methods of specifying minimum reinforcement were evaluated and compared by performing design calculations on a wide range of concrete bridge members. The findings of this study suggest that in nearly all cases lightly reinforced concrete members can develop the nominal flexural strength and have significant strength and ductility reserves after cracking has occurred. Also, the modulus of rupture over estimates the flexural cracking stress of concrete bridge members. A rational approach to the specification of minimum reinforcement is proposed, where variables are appropriately factored and includes the maximum rather than nominal strength of the section as a true measure of ductile versus brittle response.

SUMMARY

Introduction

Minimum flexural reinforcement is prescribed in the *AASHTO LRFD Bridge Design Specifications* (also referred to as the "LRFD specifications") for reinforced and prestressed concrete members to reduce the probability of brittle failure (AASHTO, 2007). This minimum reinforcement is based on providing flexural capacity greater than the moment at which cracking of the concrete is anticipated to occur. The intent of providing this additional flexural capacity is to prevent brittle failure without sufficient warning or redistribution of load.

It is recognized that there is significant variability in the cracking moment. Recently, the flexural cracking strength has been increased from $0.24\sqrt{f'c}$ to $0.37\sqrt{f'c}$ (ksi) in the LRFD specifications. This increase is to recognize increasing use of high strength concrete and of the wide range of scatter in modulus of rupture tests. As a result of this recent increase, excessive amounts of reinforcement and corresponding increased cost have been experienced, especially in externally prestressed segmental concrete bridge girders. Design examples have demonstrated that a prestressed concrete member may be considered over-reinforced, which is now defined as compression-controlled in the LRFD specifications, and not satisfy the minimum flexural reinforcement requirement.

The flexural cracking strength in the LRFD specifications is based on modulus of rupture test data, which consists of small-scale flexure capacity tests, where units are 4 or 6 inches deep and most are typically moist cured up to testing. Most of this data is not applicable to concrete bridge members because curing methods to not reflect field conditions and member size effects are not accurately represented. Therefore, test data on the cracking strength of full-size concrete members and small-scale units cured under realistic conditions should form the basis of minimum reinforcement specifications.

The objective of this research is, to develop recommended revisions to the AASHTO LRFD Bridge Design Specifications and Commentary for rational design of minimum reinforcement to prevent brittle failure of concrete sections. This objective is achieved by evaluating the effectiveness of minimum reinforcement provisions on a database of structures that are represented in the LRFD Specifications. A summary of the research is as follows:

1. Review and synthesize U.S. and international practice and research on minimum flexural reinforcement (MFR).

2. Evaluate minimum reinforcement models and select 4-candidates for parametric studies.
3. Develop a database of concrete bridge structures and components where minimum reinforcement provisions apply.
4. Evaluate safety, reliability, and economy by applying minimum reinforcement candidate provisions to the structures listed in the database.
5. Propose revisions to the AASHTO LRFD Bridge Design Specifications.
6. Demonstrate proposed provisions with design examples.

Findings

Tests have shown that lightly reinforced and prestressed concrete members have significant strength and ductility capacity after cracking has occurred, where both the nominal and ultimate flexure capacities (including the effects of strain hardening of the reinforcement, or prestress) were achieved. These tests were conducted with devices that apply increasing displacement increments regardless of whether the loads are increasing or decreasing, which may not be representative of actual bridge loading. If these same tests were conducted by applying increasing load increments without means of stopping displacements after loads decrease, a number of these specimens would fail without warning because the ultimate strength is less than the cracking strength. Therefore, minimum flexural reinforcement should be based on the ultimate strength rather than the nominal strength. It should be noted that the flexural strength of prestressed concrete members is based on the actual strength of the steel at ultimate in the LRFD specifications.

The flexural cracking strength of concrete members is highly variable and is sensitive to the curing methods and the size of concrete units tested. Most of the modulus of rupture test units are moist-cured up to the time of testing and not allowed to surface dry. Results of modulus of rupture tests have demonstrated significant sensitivity to curing, especially for high strength concrete. Carrasquillo, et al. (1981) noted a 26 percent decrease in the 28-day modulus of rupture in high strength concrete when units were allowed to dry after 7-days of moist curing over units that were moist cured until testing. The flexural cracking stress of concrete members has been shown to significantly reduce with increasing member depth. Shioya, et al. (1989) observed that the flexural cracking strength is proportional to $H^{-0.25}$, where H is the overall depth of the flexural member. Based on this observation, a 36.0 in. deep girder should achieve a flexural cracking stress that is 36 percent lower than a 6.0 in. deep modulus of rupture test specimen. The combined result of both effects is that the flexural cracking stress of a concrete bridge member should substantially lower than the flexural cracking stress from a modulus of rupture test made from the same concrete.

A review of US and international practice on specifying minimum reinforcement has shown that all methods investigated are based on a similar premise, which is providing flexural strength in excess of the cracking strength of concrete by an acceptable margin. Some methods further simplify the process, thereby allowing direct calculation of the minimum reinforcement. The method specified in the Eurocode (2006), the Japanese Code (1998), the ACI Code regarding reinforced concrete members and the method developed by Leonhardt are examples of this simplified approach. The LRFD specifications, the ACI Code regarding prestressed concrete members and the Canadian Highway Bridge Design Code (CSA, 2006) require the nominal strength be greater than the cracking moment by a factor of safety. The amount of minimum reinforcement specified varies significantly as reflected in the prescribed flexural cracking stress. The highest cracking stress, for the purposes of checking minimum reinforcement, is specified in the LRFD specifications at $0.37\sqrt{f'c}$ (ksi), and the lowest is in the Canadian Highway Bridge Design Code (CSA, 2006) at $0.15\sqrt{f'c}$ (ksi).

Based on the results of the review of practice and research on minimum reinforcement, the NCHRP 12-80 project team developed a rational method of calculating minimum flexural reinforcement. In this method (referred to as the "Modified LRFD method"), separate factors for flexural cracking and for prestress are used to improve consistency, safety and economy. The method utilizes the maximum strength of the section, which includes the strain hardening of the reinforcement to help achieve consistent safety for all concrete members covered by the provisions in the LRFD specifications.

To evaluate and compare methods of specifying minimum reinforcement, a parametric study was performed on four representative methods investigated as part of this project. These methods included the LRFD Specifications, the Eurocode, the procedure developed by Leonhardt, and the Modified LRFD method. This study required the calculation of minimum reinforcement for a wide variety of concrete member types. Results of the parametric study show that the Modified LRFD method provides the level of safety for all concrete members should be based on the strength at ultimate. This is largely due to the recognition that the ultimate strength of a member, including the effects of strain hardening, is the true measure of whether or not the section is ductile. Also, a rational method of specifying minimum reinforcement, where the flexural cracking and prestress can be factored separately, does not significantly increase the computational complexity from the method currently specified in the LRFD Specifications.

Conclusions

Specifying minimum flexure reinforcement should be based on a rational approach to prevent brittle failures of concrete bridge members. This approach should recognize that lightly reinforced and prestressed concrete members have significant strength and ductility in the post-cracked state. Flexural capacity of concrete bridge sections designed to strength limit state moment demand requirements will be able to resist these design moments in the post-cracked state regardless of whether or not minimum reinforcement requirements are met. Further, lightly reinforced members can achieve the full flexural capacity including the effects of strain hardening. Therefore, specification of minimum reinforcement should be limited to statically determinate bridge members and the positive bending of continuous bridge members if adequate post-crack ductility is demonstrated at or near the supports, where positive bending is defined as moments that cause tension along the bottom fiber at midspan.

The Resistance Factor (ϕ), as defined in the LRFD specifications, is reduced in compression-controlled or transition sections to reduce the probability brittle failure. Specifying minimum reinforcement also increases strength to reduce the probability of brittle failure. Therefore, for the purpose of specifying minimum reinforcement, ϕ should not be reduced in compression-controlled or transition regions because both requirements address the same deficiency that is lack of ductility. Inverted T girders and continually prestressed spliced girders and box sections have been shown to fall into the compression-controlled and or transition regions and not meet minimum reinforcement requirements. Since minimum reinforcement requirements are specified to reduce the probability of non-ductile failure, adding tension reinforcement in these regions would only make the section less ductile. A more logical approach is to increase compression reinforcement.

For the purposes of specifying minimum reinforcement, the flexural cracking strength of concrete members should be based on test data represents actual service condition of concrete bridges. Based on tests of small-scale units subject to realistic curing conditions and large-scale units, the flexural cracking strength of $0.37\sqrt{f'c}$ (ksi) is a reasonable upper bound value with a low probability of being exceeded, and $0.24\sqrt{f'c}$ (ksi) is an appropriate average value. For precast segmental joints, $0.24\sqrt{f'c}$ (ksi) is an appropriate upper bound value. Prestress can be a substantial component of the flexural cracking strength. However, the variability of prestress is far less than variability of the flexural cracking stress, and should be factored accordingly. By factoring prestress and the flexural cracking stress differently, more consistent levels of safety can be prescribed.

Recommendations

The Modified LRFD method is recommended to replace the current minimum reinforcement provisions in the LRFD specifications. This method:

- specifies flexural cracking strengths and appropriate factors that are based on small-scale flexure tests specimens cured under conditions that represent actual concrete bridge girder construction and large scale test specimens.
- factors flexural cracking strength and prestress separately to account for differences in variability.
- recognizes post-cracking strength and ductility capacity of lightly reinforced concrete members, thus, allowing for the elimination of minimum reinforcement provisions in negative bending regions if sufficient ductility capacity is verified.
- eliminates the reduced resistance factor for compression-controlled or transition sections for the purpose of evaluating and specifying minimum reinforcement.

If this method is implemented, specifying excessive reinforcement as a result of minimum reinforcement provisions should be eliminated. In particular, segmental bridges will see substantial reductions in the amount of prestress required to meet minimum reinforcement provisions. The minimum reinforcement provisions in the Modified method provide a more consistent level of safety for all concrete members than the LRFD specifications.

A general lack of understanding of the behavior of lightly reinforced and prestressed concrete members could be the reason for the wide variation in the amounts of reinforcement prescribed in practice. Presentations on the behavior of concrete members with relatively small reinforcement or prestress content are recommended to be given through future technology transfer seminars to reduce this lack of understanding.

CHAPTER 1 INTRODUCTION AND RESEARCH APPROACH

1.1 PROBLEM STATEMENT

Minimum flexural reinforcement is prescribed in the *AASHTO LRFD Bridge Design Specifications* (also referred to as the "LRFD specifications") for reinforced and prestressed concrete members to reduce the probability of brittle failure (AASHTO, 2007). This minimum reinforcement is based on providing flexural capacity greater than the moment at which cracking of the concrete is anticipated to occur. The intent of providing this additional flexural capacity is to prevent brittle failure without sufficient warning or redistribution of load.

It is recognized that there is a wide variability in the cracking moment. Recently, the flexural cracking strength has been increased from $0.24\sqrt{f'c}$ to $0.37\sqrt{f'c}$ (ksi) in the LRFD specifications. This increase is to recognize increasing use of high strength concrete and of the wide range of scatter in modulus-of-rupture tests, as shown in experiments by Mokhtarzadeh and French (2000). This recent increase, combined previously incorporated safety factors, has resulted in excessive amounts of reinforcement, especially in segmental concrete bridge box girders. Design examples have demonstrated that a prestressed concrete member may have an amount of reinforcement so large as to cause the member to fail in a compression-controlled mode, while still not satisfying the minimum flexural reinforcement requirement. This anomaly was obviously not intended by the LRFD specifications.

Tests have shown that lightly reinforced and prestressed concrete members have significant inelastic strength and ductility when tested with displacement-controlled application devices, as shown in Section 2.1. Figure 1 shows a typical moment-rotation relationship of a reinforced concrete member. If a displacement controlled testing is conducted in a laboratory setting, the entire moment-rotation diagram can be generated. The load is introduced in the form of controlled displacement increments and the hydraulic jacking pressure continues to be applied regardless of whether the load drops at any point or not, which may not be representative of actual bridge loading. However, if these same tests were conducted in load-control mode, a number of these specimens would fail without warning if M_o is smaller than M_{cr}, where M_o is the moment corresponding to the ultimate (rather than yield) strength of the reinforcement and M_{cr} is the cracking moment. Based on this observation, minimum flexural reinforcement should be based on the ultimate strength of the reinforcement rather than the yield strength, which corresponds to M_n in Figure 1. This observation also implies that minimum reinforcement provisions in the current LRFD specifications applied inconsistently for reinforced compared to prestressed concrete members. For reinforced concrete, M_n is defined in terms of the yield strength of the mild reinforcement, while for prestressed concrete it is defined in terms of the ultimate strength of the prestressing steel.

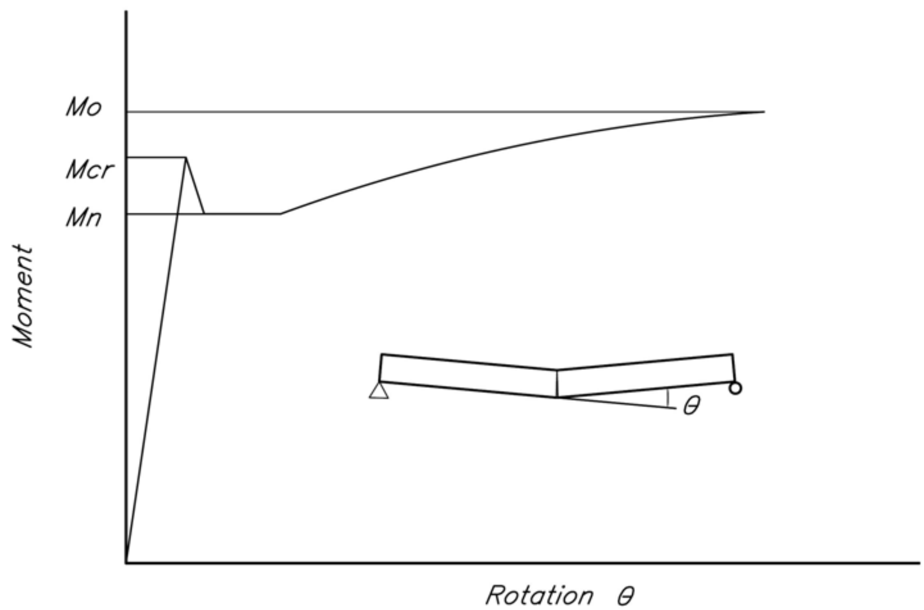

Figure 1. Moment-rotation response of a lightly reinforced concrete member

Statically indeterminate structures deserve special considerations because of the ability to internally redistribute loading effects from negative to positive bending. The LRFD specifications restrictions on where redistribution is allowed are related to the net-tensile strain at ultimate, which implies that the section ductility is inversely proportional to the amount of tensile reinforcement.

As shown in Figure 2, cracking will typically occur under negative moment first and then positive moment. Circumstances where it is permissible to forgo minimum reinforcement requirements in the negative bending regions for continuous bridges is discussed in Section 2.4.7, along with recommended detailing practice to achieve the required ductility capacity that allows for satisfactory redistribution.

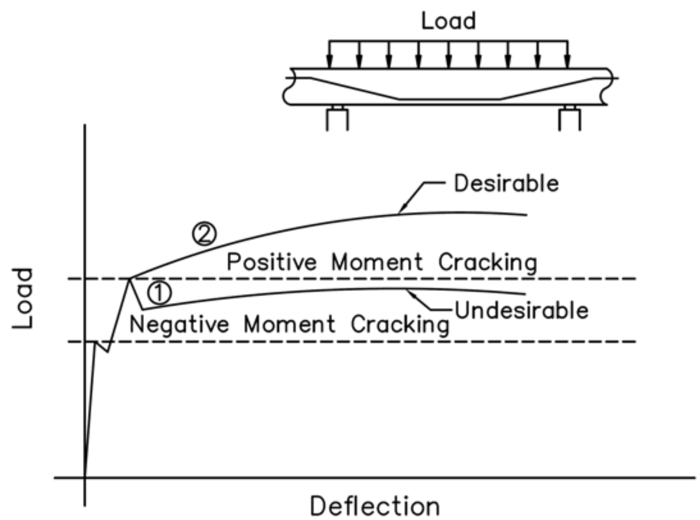

Figure 2. Load-displacement response of an interior span of a continuous member

1.2 RESEARCH OBJECTIVES

The objective of this research is, to develop recommended revisions to the AASHTO LRFD Bridge Design Specifications and Commentary for rational design of minimum reinforcement to prevent brittle failure of concrete sections. This objective is achieved by evaluating the effectiveness of minimum reinforcement provisions on a database of structures that are represented in the LRFD Specifications. A summary of the research is as follows:

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5. Propose revisions to the AASHTO LRFD Bridge Design Specifications.
6. Demonstrate proposed provisions with design examples.

1.3 RESEARCH TASKS

To accomplish these objectives, the following research tasks were performed. These tasks are quoted directly from the NCHRP 12-80 project request for proposals.

Task 1. Review U.S. and international practice, performance data, research findings, specifications, and other information related to minimum reinforcement requirements and flexural cracking of concrete structures. This information shall be assembled from technical literature and from unpublished experiences of engineers, bridge owners, fabricators, and others. Records of brittle flexural failures of laboratory or in-service elements are of particular interest.

Task 2. Identify and compare models to determine minimum flexural reinforcement. Models should not be limited to those used in developing the LRFD specifications. The NCHRP will select the models for use in Task 6.

Task 3. Assemble a database of concrete structures and components to which the LRFD minimum flexural reinforcement (bonded and unbonded) requirements apply. The database shall be populated with sufficient information to permit calculation of all appropriate cross-section loads and resistances.

Task 4. Develop a detailed work plan to use the database structures and components to compare the reinforcement requirements and reliability of not more than three minimum reinforcement models selected by the NCHRP.

Task 5. Submit an interim report within four months of the contract start that documents the findings of Tasks 1 through 4. Include a list of proposed design examples to be submitted in Task 7. The contractor will be expected to meet with the NCHRP approximately one month later. Work may not proceed on subsequent tasks without NCHRP approval of the work plan.

Task 6. Perform the work plan as approved by the NCHRP.

Task 7. Develop specifications with supporting commentary for recommendation to the AASHTO Highway Subcommittee on Bridges and Structures. Provide a minimum of five step-by-step design examples illustrating the application of the specifications. Compare the designs to those produced by the current AASHTO specifications.

Task 8. Revise the specifications, commentary, and design examples in accordance with NCHRP review comments (Draft 2).

Task 9. Submit a final report that documents the entire research effort.

1.4 RESEARCH WORK PLAN

The work plan identified in Task 6 was developed to achieve the objectives of this project after the data collection phase of this project. This work plan consisted of the following items:

1.4.1 Refine the Modified LRFD Method

A new approach to determine minimum reinforcement is proposed to meet the objectives of the NCHRP 12-80 project. As the name suggests, the Modified LRFD method is based on the minimum reinforcement procedure in the LRFD specifications. In this procedure, variables that influence minimum reinforcement are factored separately to account for differences in variability. Development of these factors is the subject of this task. For concrete flexural cracking, the data presented in Section 2 is used determine a factor that is appropriate. The prestress variability effect on the flexural cracking strength is relatively small regarding the flexural cracking strength. Therefore, a reduced factor, compared to the current 1.2 factor, is warranted, as discussed in Section 2.2.

Design methods such as strain compatibility analysis are utilized to develop flexural strength of selected structures within the Concrete Bridge Member Database, to see if any methods in the procedure can be simplified.

1.4.2 Perform the Parametric Study

To evaluate candidate minimum reinforcement methods, design calculations were performed on the bridges within the Concrete Bridge Member Database, as described in Section 3.1.1. Design calculations were performed using state-of-the-practice design tools to develop design forces, moments, and shears.

The preparation of tables of minimum reinforcement along with appropriate graphs compare each method versus such variables as concrete compressive strength, spacing of girders, depth of members and width and thickness of bottom and top flanges. As a result these methods are easily and directly compared for quick evaluation.

1.4.3 Evaluate the Statistical Parameters of Minimum Flexural Reinforcement

To aid in interpretation of applicable test data, a statistical analysis is performed, as described in Section 2.3. The focus of this analysis is on the flexural cracking strength of concrete bridge members.

To evaluate the appropriateness of the statistical parameters, the cumulative distribution function (CDF) of the modulus-of-rupture is plotted on the normal probability paper. Any normal CDF on the normal probability paper is represented by a straight line. The methods used to develop CDF plots are described in such references as Nowak and Collins (2000) and in TRB Circular E-C079.

1.5 KEY DEFINITIONS

For convenience of the reader, the following definitions are given:

- f_{pe} - strand stress due to effective prestress.
- f_{ps} - strand stress at ultimate flexure.
- f_y - stress in mild reinforcement at specified yield strain (0.0021 for grade 60 steel).
- f_u - ultimate (peak) stress in mild reinforcement just before rupture.
- $\mathbf{M_{cr}}$ - theoretical cracking moment.
- $\mathbf{M_o}$ - nominal ultimate moment capacity including the effects of strain hardening, as illustrated in Figure 1.
- M_n - nominal flexural capacity as defined by the LRFD specifications, excluding strain hardening for conventionally reinforced sections with mild steel reinforcement (see Figure 1) and including strain hardening for sections reinforced with prestressing strands.
- M_u- ultimate demand moment (or required strength) due to factored applied loads.
- γ_3 – ratio of yield to ultimate steel stress for non-prestressed steel, (for example, 0.67 for A615 and 0.75 for A706 Grade 60 reinforcement). Note that γ_3 is taken =1.0 for prestressing strands as the codes already utilize the full stress-strain relationship.

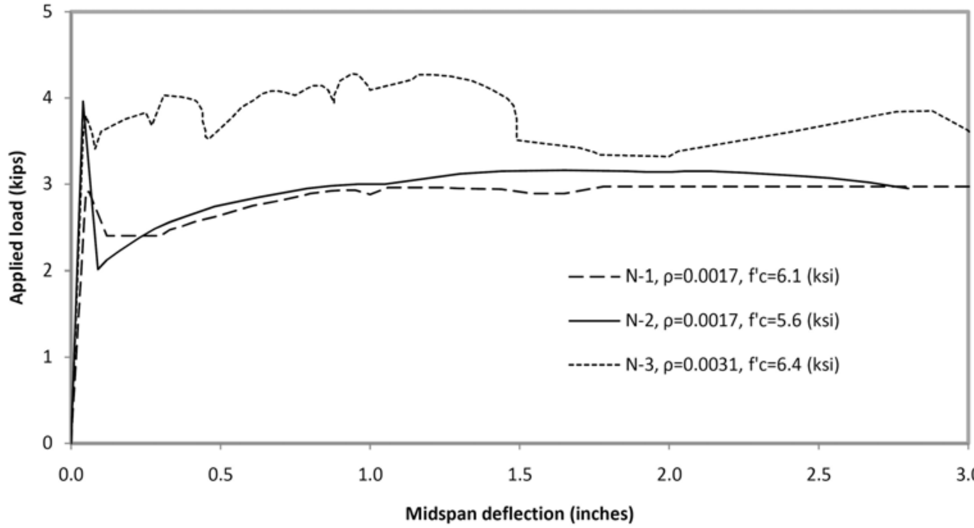
CHAPTER 2 FINDINGS

2.1 OBSERVED RESPONSE OF LIGHTLY REINFORCED CONCRETE AND PRESTRESSED CONCRETE MEMBERS

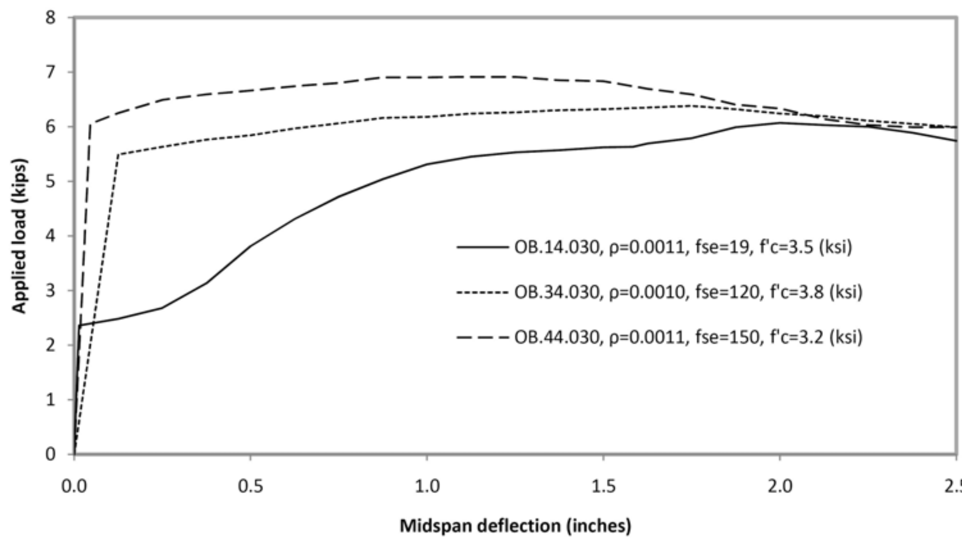
Testing of a large number of lightly reinforced and prestressed concrete beams at the University of Illinois demonstrated that significant inelastic displacements can be achieved, "… and none of the beams tested failed without large warning deflections," as presented in a journal paper by Freyermuth and Aalami (1997). These experiments included lightly reinforced, internally-prestressed and externally-prestressed concrete components.

Test set up consisted of 4-point loaded simply-supported concrete beams measuring 12 in. deep by 6 in. wide. The load-deflection plots of lightly reinforced concrete members, shown in Figure 3(a), indicate that substantial strength and ductility was observed after cracking occurred, and the ultimate strength reflects the strain-hardened resistance developed in the reinforcement rather than yield. The response of lightly prestressed concrete members with internal or bonded tendons in Figure 3(b) shows all units had significant post-cracking strength and ductility. Each unit in this set had nearly identical dimensions and areas with different amounts of prestress applied in each tendon. Although the initial cracking strength varied, all units achieved similar strengths at a displacement between 2.0 and 2.5 inches. The response of lightly prestressed units with external (or unbonded) tendons in Figure 3(c) shows that after a drop in strength due to cracking, resistance increases due to stretching of the external tendon. All units demonstrated significant post-cracking strength and ductility.

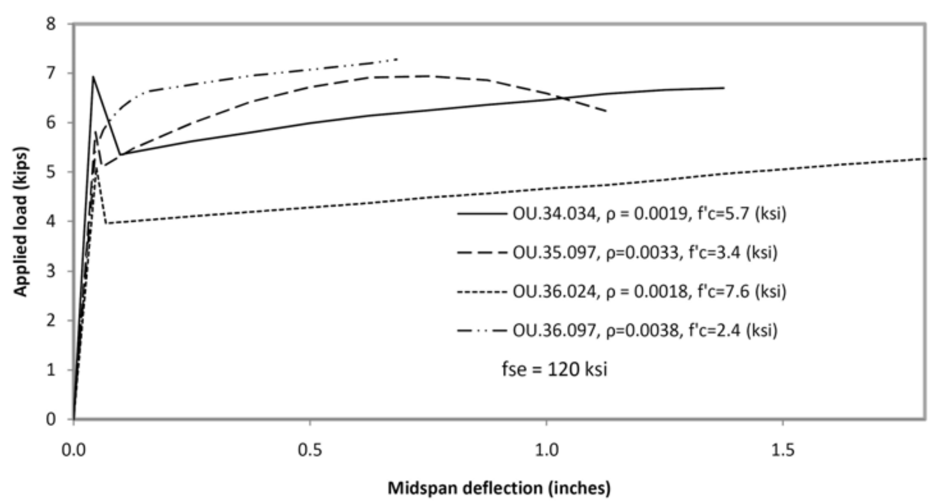
As discussed previously, these tests were conducted with a load-displacement regime may not be representative of actual bridge loading. In this system, loads are introduced in the form of controlled displacement increments, and the hydraulic jacking pressure continues to be applied regardless of whether the load drops at any point or not. If these same experiments were conducted by applying increasing loads without any means of stopping the displacements if the strength drops, a number of the specimens would have failed without warning because the ultimate strength (including the effects of strain hardening in the reinforcement) was less than the cracking strength. Based on this observation, minimum reinforcement requirements should be based on the ultimate strength instead of the yield strength of the reinforcement.



a.) Reinforced concrete members



b.) Prestressed concrete members (bonded)



c.) Prestressed concrete members (unbonded)

Figure 3. Load-deflection response of lightly reinforced and prestressed concrete members from the University of Illinois, (Freyermuth and Aalami, 1997), (Warwaruk, Sozen and Seiss, 1960)

Please note that the ultimate nominal flexural strength in this report refers to the flexural strength of a cross section with the resistance factor taken as unity (thus the word nominal). The corresponding symbol is M_o. The yield nominal flexural strength is based on the yield strength of mild reinforcement and is referred to in the LRFD specifications as M_n. It should be noted that the LRFD specifications refer to M_o for prestressed section as M_n.

For precast segmental construction, cracking generally starts at the joints between precast segments. Research was conducted at the University of California, San Diego (UCSD) on seismic performance of precast segmental bridges. This experimental program was initiated by the American Segmental Bridge Institute (ASBI) and Caltrans and was funded by Caltrans. The experimental program consisted of three phases, in which performance of joints in positive moment regions was investigated in Phase I. A prototype span-by-span structure was designed and used as the basis for design of test units. Details about the experimental program can be found in a research report (Megally et al., 2002) as well as journal paper (Megally et al., 2003).

In Phase I of the experimental program, four 2/3 scale specimens were tested under reversed cyclic loading up to failure. The test variables included internal bonded tendons, external tendons or combination of internal bonded and external tendons. Each test unit consisted of six epoxy-bonded precast segments.

In these experiments, flexure cracks were consistently located immediately adjacent to the match-cast surface, as shown in Figure 4. The researchers concluded that the main reason is the formation of a weak layer of concrete, referred to as a “Laitance Layer”. This so called “Laitance Layer” is composed of more cement and sand and probably few coarse aggregates as a result of its proximity to the end surface of the segment. With few coarse aggregates, the concrete within the laitance layer is weaker than concrete internal to the precast segment itself. As a result, concrete of the laitance layer cracks at a lower flexural cracking stress than what would be expected for concrete within the segments and away from the joints. Based on the experimental values for cracking moment, section properties of test specimens and prestressing forces at time of joint opening, the modulus of rupture was calculated. The calculated modulus of rupture values varies from $3.0\sqrt{f'c}$ to $7.3\sqrt{f'c}$ (psi) indicating that a coefficient of 7.5 may be a reasonable upper bound. Note that the depth of UCSD precast segmental test units is four feet and depth of precast segmental superstructure used for span-by-span construction in the I-4 Crosstown Connector in Tampa, Florida is nine feet.

Figure 4. Test unit with 100% external tendons (Photo by Sami Megally)

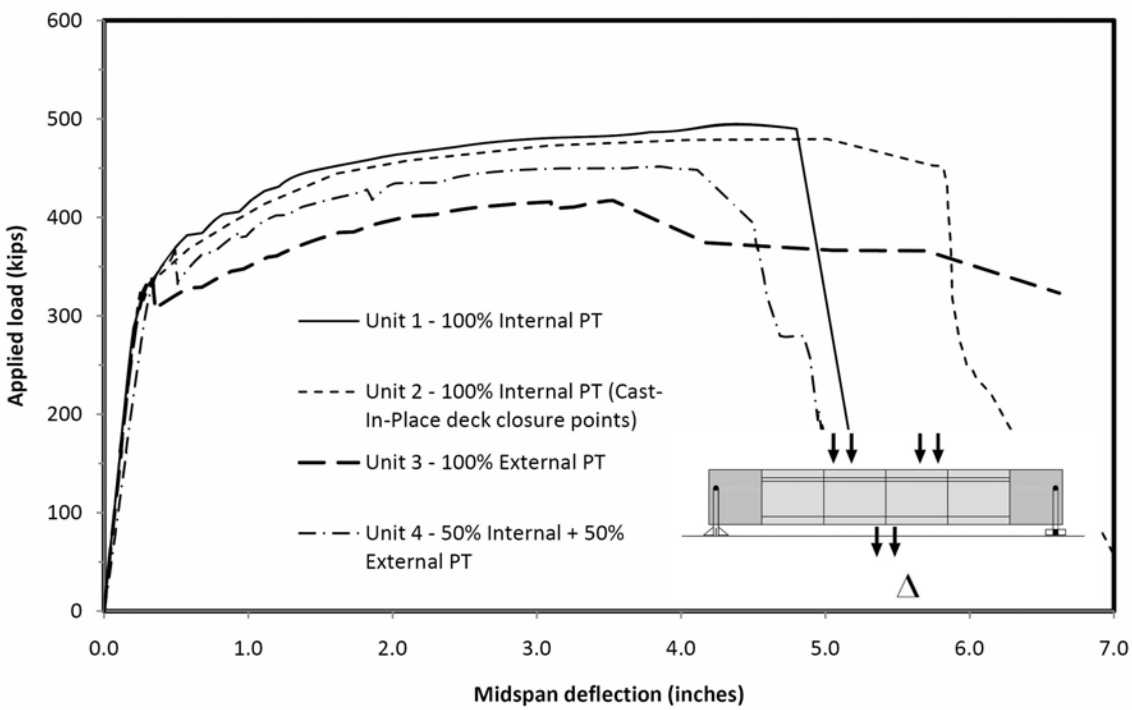


Figure 5. Load-displacement envelopes for segmental bridge specimens (Megally et al., 2003)

The test unit shown in Figure 4 had external tendons only, at the time when the maximum displacement was reached. It clearly demonstrates very large displacement without rupture of the tendon or total collapse, as would be guarded against with the minimum reinforcement limits even when subject to fully reversed cyclic load and displacement cycles. Figure 5 shows the envelope of the load-displacement response of all Phase I units. These tests confirm, as mentioned previously, that in statically determinate bridge members, ultimate moment capacity in excess of the cracking moment will prevent failures from occurring without warning.

2.2 FLEXURAL TENSILE STRENGTH

Flexural tensile strength of concrete bridge members is highly variable and is dependent on many variables including mix design, aggregate size, curing methods, finish, and member dimensions. Since concrete in tension is a brittle material, a small imperfection in the member results in reduced strength. Therefore, increasing the amount of concrete subject to tension increases the possibility of having a flaw that reduces the cracking strength.

Testing of flexural tension strength has been performed using methods such as direct tensile testing on concrete cylinders, split cylinder testing and modulus of rupture tests. Since these tests are somewhat complicated, directly correlating the flexural tensile strength with specified compressive strength is preferred. However, as shown in the following sections, this correlation with real-size concrete bridge members is dependent on many variables.

For evaluating serviceability, and limiting cracking during prestress transfer, a lower bound estimate of the concrete flexural stress is of interest. However, for the purposes of establishing minimum flexural reinforcement, a mean and upper bound estimate of flexural cracking is of particular interest.

2.2.1 Direct Testing of Concrete Fracture in Tension

Testing of concrete in direct tension is challenging and requires specialized equipment, and the results of which are subject to the influence of boundary conditions and accidental eccentricity (Gonnerman and Shuman, 1928). This is largely due to the fact that the stress-strain response of concrete in tension is linear until cracking occurs. Microcracks at the aggregate-paste boundaries initiate at the weakest point and spread until the section is completely cracked making this procedure very sensitive to specimen quality and testing methods.

Split cylinder testing is more commonly used to evaluate the tensile strength of concrete than direct methods. In this procedure, a standard 6x12 cylinder is compressed transversely. The entire section is not subject to tension, and the cylinder is relatively small, as compared to the bottom flange of a bridge girder. However, split-cylinder tests consistently demonstrate concrete tensile strengths that are typically 65% of the flexural tension measured in a modulus-of-rupture test (Neville, 1981).

2.2.2 Modulus of Rupture

Modulus of rupture is measured using the ASTM Designation: C78 – Standard Method for Flexural Strength of Concrete (Using Simple Beam with Third-Point Loading). As shown in Figure 6, the test units are loaded at one-third of the support spacing, and the height of the units is one-third of the beam length. Based on a plane-sections-remain-plane approximation, the modulus-of-rupture is calculated using the following equation:

$$f_r = PL/bd^2 \quad (1)$$

where f_r is the modulus of rupture, b is the member width, d is the specimen height, and P is the load measured from the test machine.

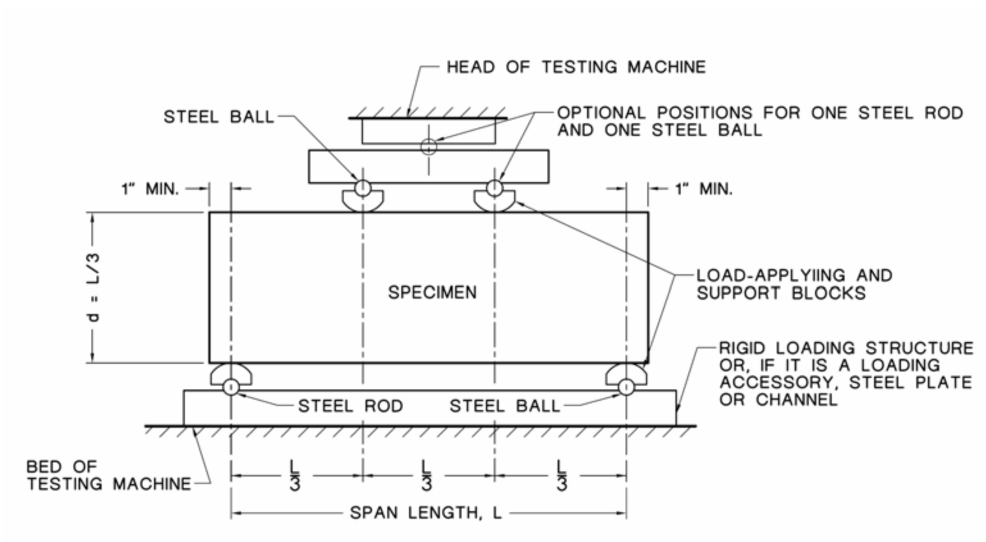
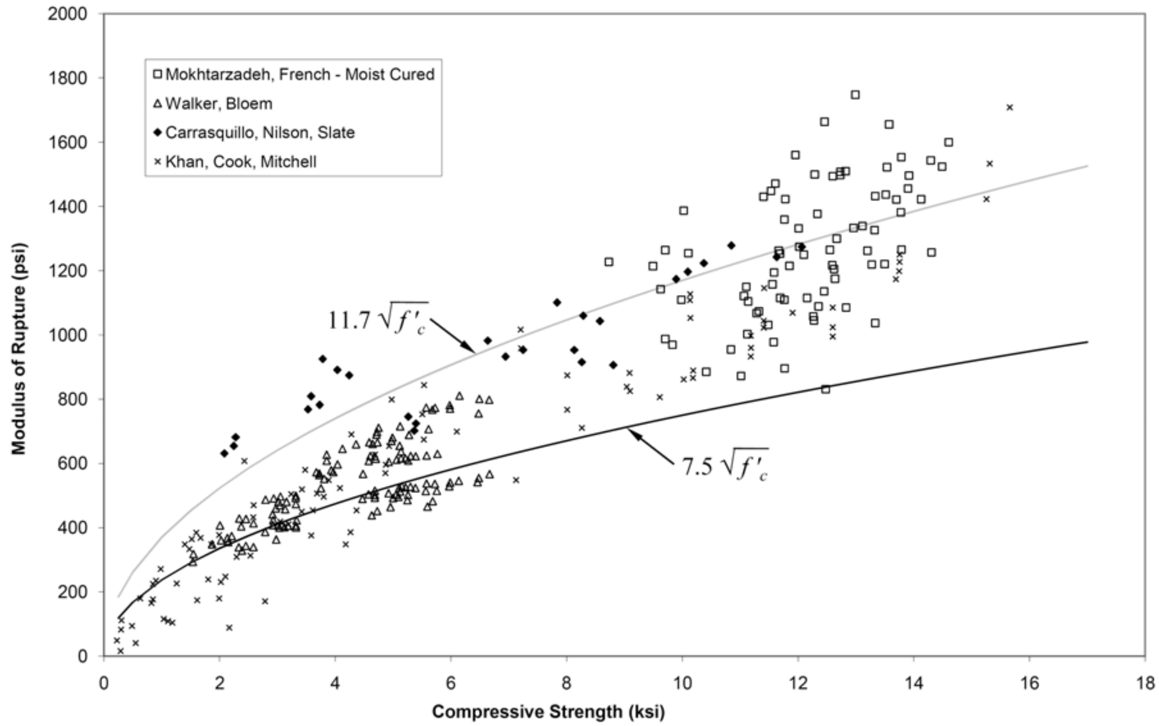
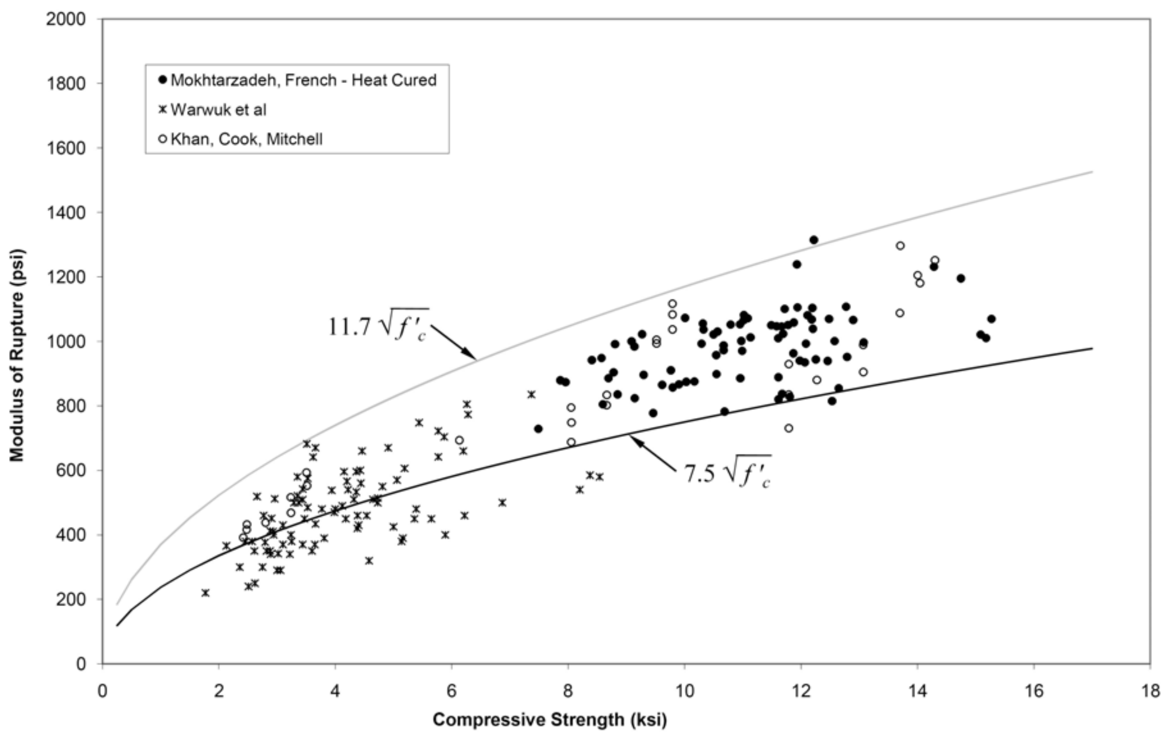


Figure 6. Modulus of rupture loading schematic (ASTM, 2008)

This method has been used in the testing of concrete for the construction of concrete slabs and pavements. Therefore, the specimen sizes are typically six inches deep, and in some cases four inches deep.



a.) Moist-cured units



a.) Non-moist-cured units

Figure 7. Modulus of rupture test data from Warwaruk et al. (1960,) Mokhtarzadeh and French (2000), Walker and Bloem (1960), Khan et al. (1996) and Carasquillo et al. (1981)

Correlation between the modulus-of-rupture and the compressive cylinder strength is challenging because the mechanisms of failure are different. Kaplan (1959) observed a difference of up to 40% in the modulus-of-rupture strength based on the type of aggregate used. This is largely due to the bond between mortar and aggregate, and, therefore, an aggregate that produces a high compressive strength may not give high strengths in tension or flexure.

Another significant factor in the flexural tension strength is methods of curing. Modulus-of-rupture tests are sensitive to curing methods, and this is especially true for high-strength concrete, which has a greater propensity to develop shrinkage cracks. Carrasquillo, et al. (1981), noted a 26% reduction in the 28-day modulus-of-rupture if high-strength units were allowed to dry after 7-days of moist curing over units that were moist cured until testing. These units were 4-inches deep with a 28-day compressive strength of 10,200 psi. Mokhtarzadeh and French noted (2000) that the modulus-of-rupture of moist cured specimens was on average 30% higher than their heat-cured counterparts. It was noted that the heat curing leads to differential shrinkage strains that decrease the apparent flexural strain at rupture.

Based on the observed effect of curing the modulus-of-rupture test data shown in Figure 7 are separated into two separate categories. Moist cured units shown in Figure 7a, indicate that the modulus of rupture can be substantially higher than $11.7\sqrt{f'c}$ (psi) [$0.37\sqrt{f'c}$ (ksi)], as specified in the LRFD specifications for the purpose of checking minimum reinforcement. For non-moist cured units, the average is substantially lower, and more consistent with the $f'_c{}^{0.5}$ trend between higher and lower strength concretes.

2.2.3 Size Effects on the Flexural Cracking Strength

It has been observed that increasing the volume of concrete subject to direct tension lowers the cracking stress. Therefore with deeper beams, it is expected that more concrete is subject to direct tension than with shallower beams immediately prior to cracking. Wright, (1952) has illustrated this with a series of test between three to eight inches deep. These tests indicate a clear drop in flexural cracking strength with depth. One explanation for this phenomenon is that cracking in tension is initiated at imperfections at the aggregate-paste interface, and the more volume of concrete subject to tension the higher the probability of applying tension at an imperfection. In flexure, the highest tension is confined to the extreme tension fiber. This is especially true for relatively shallow sections where, prior to cracking, flexural tension stress is zero a short distance away at the neutral axis. For relatively deep sections, tension stresses in the bottom flange are closer to being uniform prior to cracking.

Therefore, an imperfection that initiates cracking is more likely to be encountered in a deep member because more area is subject to what can be approximated as uniform tension.

In a series of test conducted at the Shimizu Institute of Technology in Japan, similar beams measuring from 6-inches deep to 10-feet deep were tested to evaluate the effect of size on shear. The researchers noted that the flexural tension strength decreases with increasing depth, and proposed the following relation:

$$F_b = F(H^{-1/4}) \quad (2)$$

where F_b is the flexural strength, F is the flexural strength at a reference depth of unity, and H is the section depth. (Shioya, et al., 1989)

A plot of test results on larger-scale units with depths measuring 0.3 ft to 10 ft, including those mentioned previously are shown in Figures 8 and 9. This data is from various experiments, where flexural cracking was not the primary consideration and on a wide variety of shapes including rectangular, T-beams, and AASHTO Standard shapes and Bulb-Tee girders, and in some cases the top flange is subject to flexural tension. Figure 8 shows the flexural cracking strength as a function of corresponding f'c. The trend indicates that the cracking stress increases with $f'_c{}^{0.5}$, as indicated with lines representing $7.5\sqrt{f'c}$ and $11.7\sqrt{f'c}$ (psi). As shown, none of the recorded cracking strengths exceeded $11.7\sqrt{f'c}$ (psi). The flexure cracking strength is on average lower than the modulus-of-rupture. The flexural cracking strength is plotted as a function of depth in Figure 9. As shown, the trend is inversely proportional to the member depth.

It has been observed that increasing the volume of concrete subject to direct tension lowers the cracking stress. Therefore with deeper beams, it is expected that more concrete is subject to direct tension than with shallower beams immediately prior to cracking. Recorded cracking strength of full-depth members is plotted in Figures 8 and 9. This data is from several experiments, where flexural cracking was not the primary consideration, on a wide variety of shapes including rectangular, T-beams, and AASHTO Standard shapes and Bulb-Tee girders.

The recorded flexural cracking stress of concrete members with depths ranging from 0.3 ft to 10 ft is shown in Figure 9 as a function of f'c. The trend indicates that the cracking stress increases with $f'_c{}^{0.5}$, as indicated with lines representing $7.5\sqrt{f'c}$ and $11.7\sqrt{f'c}$ (psi). As shown, none of the recorded cracking strengths exceeded $11.7\sqrt{f'c}$ (psi). The cracking data in Figure 9 is shown as a function of depth and $f_r/(f'_c{}^{0.5})$ representing the horizontal and the vertical axes, respectively. As shown, the member cracking stress decreases with depth.

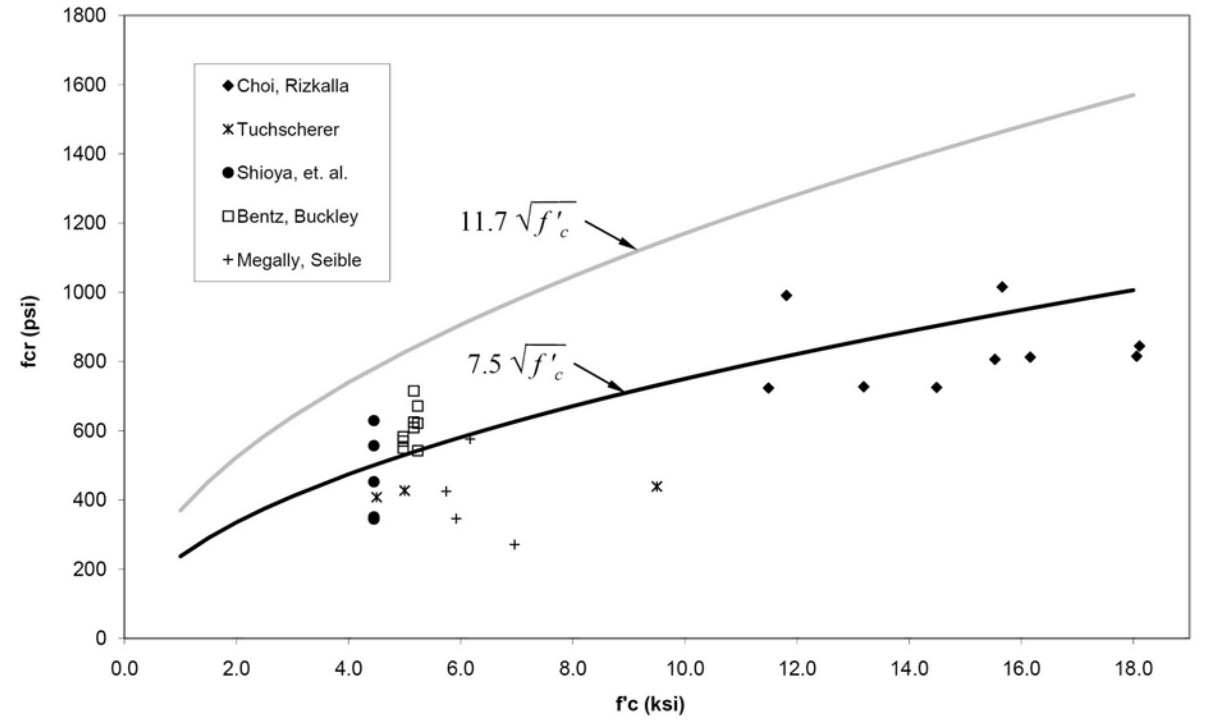


Figure 8. Observed cracking stress of full-depth concrete members versus f'_c

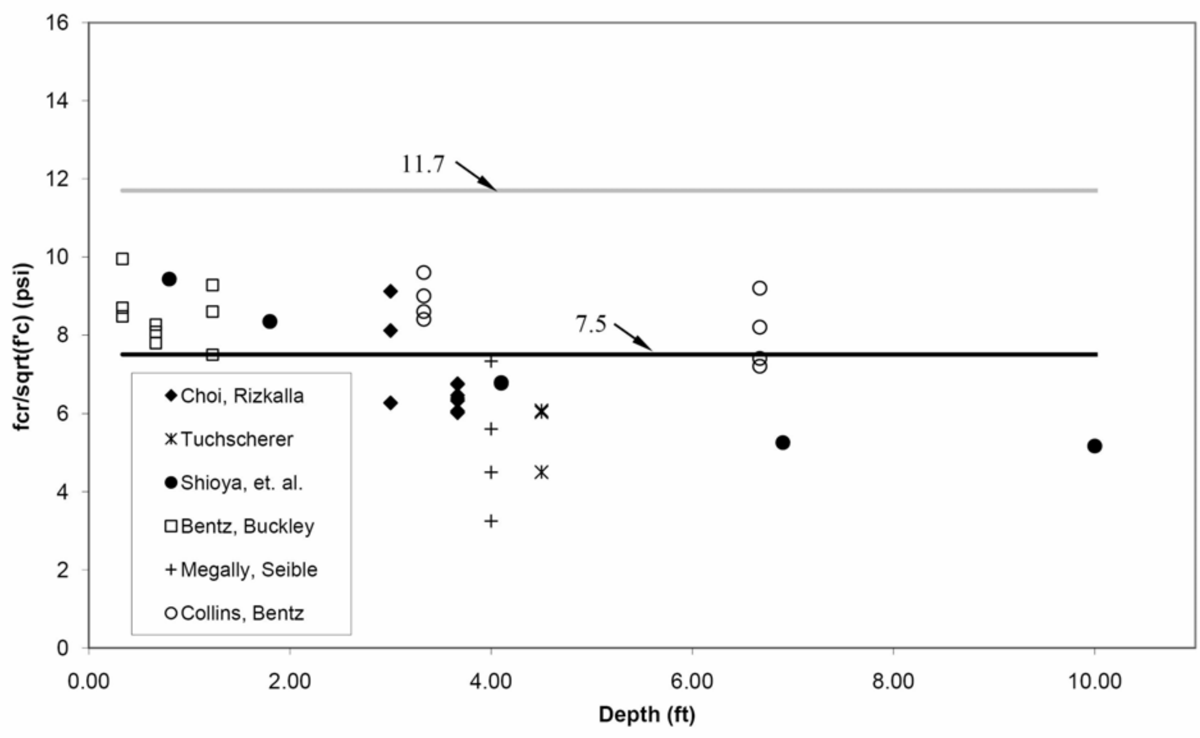


Figure 9. Observed $f_r/(f'c^{0.5})$ test data of full-depth concrete members versus depth

2.3 STATISTICAL ANALYSIS OF CONCRETE FLEXURAL STRENGTH

A statistical analysis of the flexural cracking strength of concrete members has been performed to facilitate interpretation of experimental data. These results aid in evaluating the level-of-safety provided by the minimum reinforcement provisions to prevent brittle flexural response.

This analysis focuses on the flexural tension strength of concrete members, as this parameter has by far the most variability and the most influence on the MFR provisions. Modulus-of-rupture test data per ASTM C78 is abundant, and reporting of recent data from these tests on high-strength concrete was the impetus for increasing the LRFD flexural cracking stress to $0.37\sqrt{f'c}$ (ksi) from $0.24\sqrt{f'c}$ (ksi) in 2005. Since the applicability of this data to deep bridge members is suspect because of the influence of member size on the flexural cracking stress, available data on the observed cracking strength of full-depth bridge members is also analyzed.

Prestress can have a significant effect on the flexural cracking strength of concrete. Therefore, variability of prestress is presented, where prestress losses provide the most significant level of uncertainty. Evaluation of the moment carrying capacity is not a part of this study because uncertainty in material strength and dimensional tolerances are captured in the Resistance Factor (ϕ).

2.3.1 Analysis Methods

To facilitate the interpretation of results, and determination of statistical parameters, the Cumulative Distribution Function (CDF) of relevant data is plotted on normal probability paper. Any normal CDF on normal probability paper is represented by a straight line. The methods for construction and the use of normal probability paper are described in Nowak and Collins (2000) and in TRB Circular E-C079. The intent is to identify trends in the distribution function and determine if the normal distribution assumption is appropriate for the dataset. Based on this distribution, parameters are developed to evaluate the consistency and safety of the minimum reinforcement methods investigated in this research.

2.3.3 Modulus of Rupture

Correlation between the modulus of rupture and the compressive cylinder strength is challenging because the mechanisms of failure are different. As discussed previously, the modulus-of-rupture strength is largely due to the bond between mortar and aggregate, and, therefore, an aggregate that produces a high compressive strength may not give high strengths in tension or flexure. Further, modulus-of-rupture is highly sensitive to curing methods. Moist-cure right up to the time of testing does not represent field conditions.

The Cumulative Distribution Function (CDF) is plotted for the ratio of modulus of rupture (f_r) test data to the corresponding square root of f'c for the combined data in Section 2.2.2 in Figure 10**.** Moist cured units were excluded, because moist cure up to the time of testing is not representative of field conditions. In this plot, the horizontal axis is the $f_r/(f'_c{}^{0.5})$ and the vertical axis represents the number of standard deviations from the mean value. As mentioned previously, normally distributed data will plot as a straight line, and data can be modeled assuming normal distribution.

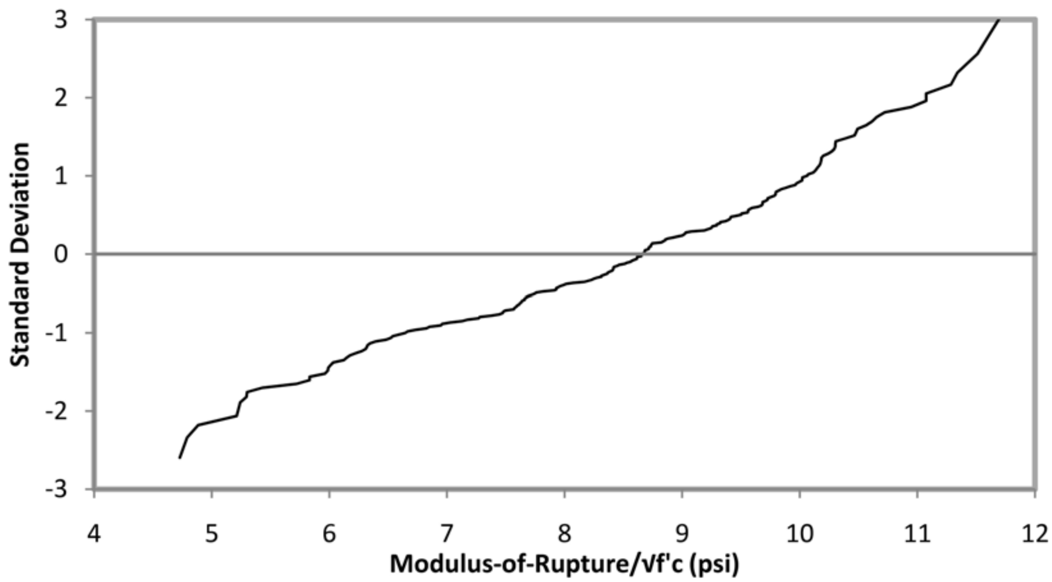


Figure 10. Cumulative distribution function plot of $f_r/(f'c^{0.5})$ test data in psi units (moist-cured data excluded)

Based on the assumption of normal distribution, statistical parameters were developed for all sets of modulus of rupture data presented in Section 2.2.2 for each source (Table 1a) and as a combined dataset (Table 1b) for both most-cured and non moist-cured units. As shown, the data indicates a higher average modulus of rupture for moist-cured units, especially for concrete strengths exceeding 8.0 ksi. As mentioned previously, moist curing until testing does not represent field conditions, where concrete is allowed dry after a short cure period. For the non-moist-cured units, the value currently used in the LRFD Specifications of 11.7√f'c (psi) [0.37√f'c (ksi)] is above two standard deviation value.

Table 1. Statistical parameters of $f_r/(f'_c{}^{0.5})$ (psi) assuming normal distribution

a.) Per reference

$f_r/(f'_c{}^{0.5})$ (psi)	Carrasquillo (1981)	Khan (1996)	Mokhtarzadeh & French (2000)		Walker (1960)	Warwaruk (1960)
Average	12.0	8.33	11.5	9.34	9.10	7.57
Std. Dev.	1.50	2.32	1.65	0.909	0.74	1.56
COV	0.125	0.278	0.143	0.097	0.081	0.21
Range f'_c (ksi)	2.1-12.1	0.2-15.7	8.7-14.6	7.5-15.3	1.5-6.0	1.2-8.3
Size (in.)	4x4x14	4x4x16	6x6x24	6x6x24	6x6x36	6x6x24
Cure Method	Moist	Varies	Moist	Heat	Moist	Not stated

b.) Total for all Data Sets

	$f_r/(f'_c{}^{0.5})$ (psi)	
	Moist cured	Non-moist cured
Average	9.32	8.49
Standard deviation σ	2.43	1.53
Ave. + 2(σ)	14.2	11.6
COV	0.26	0.18
Average f'c (ksi)	6.83	7.58

2.3.3 Full-Size Member Cracking Strength

The (CDF) is plotted for the ratio of the full-depth member cracking stress test data described in Section 2.2.3, to the corresponding square root of f'c in Figure 11. In this plot, the horizontal axis is $f_{cr}/(f'_c{}^{0.5})$ and the vertical axis represents the number of standard deviations from the mean value. The average depth for all members evaluated is 3.0 ft. As mentioned previously, normally distributed data will plot as a straight line, and the plot is essentially straight, which indicates that the data can be modeled assuming normal distribution.

A summary of the statistical parameters for the full-size test data based on a normal distribution is shown in Table 2. The average flexural cracking strength is below $7.5\sqrt{f'c}$ (psi) [$0.24\sqrt{f'c}$ (ksi)], and two standard deviations above the average is well below $11.7\sqrt{f'c}$ (psi) [$0.37\sqrt{f'c}$ (ksi)]. Incorporation of the depth of the member in specifying the flexural cracking stress was considered in the research. As shown in Table 2, the coefficient of variation reduces considerably with the addition of the parameter $H^{-0.2}$. However, there is a tradeoff between ease-of-use and accuracy when developing the strength of the section. Considering the variability of the measured flexural cracking strength, parameter of depth in the minimum reinforcement provisions should not be included.

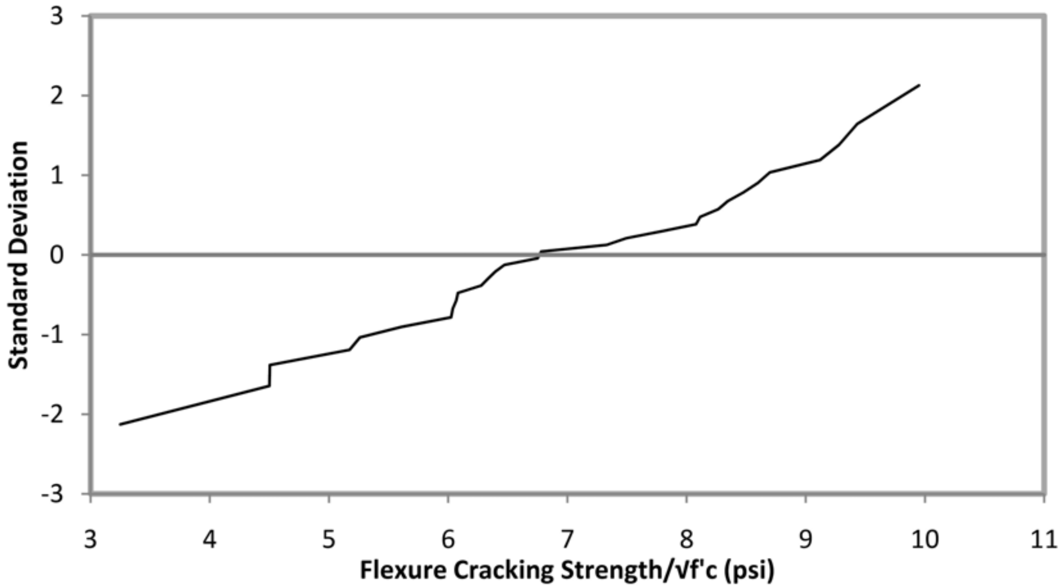


Figure 11. Cumulative distribution function plot of $f_{cr}/(f'c^{0.5})$ test data of full-size units

Table 2. Statistical parameters of full-size concrete member flexural cracking stress assuming normal distribution

	f_{cr} (psi)	$\frac{f_{cr}}{f_c'^{0.5}}$ (psi)	$\frac{f_{cr}}{f_c'^{0.5} H^{-0.2}}$ (psi, ft)
Average	610	7.02	8.07
Standard deviation σ	190	1.65	1.35
Ave. + 2(σ)	990	10.3	10.8
COV	0.31	0.24	0.17

2.3.4 Prestressed Variability

The level of prestress has a significant impact on the flexural cracking strength of concrete members. Methods and research on anticipated prestress and the amount of prestress loss that is anticipated to occur over the life of the bridge are covered in detail in the PCI Bridge Manual (2005) for pretensioned members.

The variability of prestress losses in pretensioned members has been evaluated by Steinberg (1995) and Gilbertson & Ahlborn (2004) and Tadros et al. (2003, 2009). Results of these studies are based on the variability of parameters including jacking force, initial and final concrete strengths, relative humidity, dimensional tolerances, time-of-jacking and others. In both studies Monte Carlo Simulations were used to evaluate overall variability of prestress losses. Gilbertson & Ahlborn (2005) demonstrated prestress losses deviate from nominal by less than 4%

within a confidence interval of 95% for a 70-inch I-girder using the AASHTO LRFD method for calculating prestress losses.

Tadros, et al., demonstrated that long term prestress loss due to creep, shrinkage and relaxation can vary by as much as 30% from the mean value. Considering that the loss is about 17% of the prestress force, the variation in the prestress force can be as much as 0.3*0.17 = 0.05.

2.3.5 Summary of Statistical Analysis of Flexural Cracking Strength

Statistical analysis of concrete member cracking strength demonstrates the following:

- Cumulative distribution function plots show that the ratio of the flexural cracking strength to the square root of the compressive strength indicates that the normal distribution assumption is appropriate for all datasets evaluated.
- The average modulus of rupture for units not subject to moist cure is 8.5√f'c (psi) [0.27√f'c (ksi)] based on test data from test data evaluated in Section 2.2.2. Modulus of rupture is sensitive to curing, and moist curing is not representative actual field conditions.
- For the combined dataset of units not subject to moist cure, the modulus of rupture of 11.6√f'c (psi) [0.37√f'c (ksi)] is 2 standard deviations above the mean implying a 98 percent confidence interval.
- Full size concrete members crack at significantly lower flexural stresses than modulus of rupture specimen, and the data suggests that the cracking stress is inversely proportionality to the section depth.
- Average and plus-two standard deviation cracking stress for full-size members are 7.0√f'c and 10.3√f'c (psi), respectively. Based on this dataset, the value 11.7√f'c (psi) 0.37√f'c (ksi) is 2.85 standard deviations from the mean, which implies a 99.8 percent confidence interval.

2.4 METHODS AND PROCEDURES FOR DEVELOPING MINIMUM REINFORCEMENT

In the U.S., bridge members are generally governed by the AASHTO LRFD Bridge Design Specifications, while building members are generally governed by ACI 318 Building Code

Requirements for Structural Concrete. The LRFD specifications have unified provisions for reinforced, partially prestressed, and fully prestressed concrete (Section 5). ACI 318 has different provisions for reinforced concrete (in Chapter 10) and prestressed concrete (in Chapter 18). There are significant differences between the two documents. There may be justification for some of the differences, primarily due to the different character of the applied loads. Otherwise, the provisions should be very similar or even identical.

The applicability of the LRFD specifications to segmental bridges is a primary question in this research. The reduced cracking strength at the segment joints should be somehow accounted for. Also, external tendons are often used, especially in span-by-span construction, where very low steel stress at the Strength Limit States is generally assumed in design. That stress can be far below the stress that corresponds to rupture of the tendons. The European Code differs in the approach to providing minimum reinforcement, featuring simplified prescriptive equations, which are applicable to both reinforced and prestressed concrete bridge members.

2.4.1 AASHTO LRFD

Minimum flexural reinforcement is evaluated uniformly for all concrete sections with two requirements. Fundamentally, these requirements are:

(a) The flexural design strength of the section being considered should be larger than the cracking moment by an acceptable safety margin, and

(b) If one is assured that the member will be unlikely to crack under a magnified factored load moment, then requirement (a) may be waived. The magnification factor provides an additional safety margin beyond the margin provided by the standard load factors

AASHTO Section 5.7.3.3.2 states that the amount of reinforcement shall be adequate to satisfy at least one of the following conditions:

$$\phi M_n \geq 1.2 M_{cr}, \text{ or} \tag{3}$$

$$\phi M_n \geq 1.33 M_u \tag{4}$$

where ϕM_n, M_{cr} and M_u are the design strength, cracking moment and required strength (factored load moment). The resistance factor, ϕ, in the LRFD Specifications is taken as 1.0 for prestressed concrete and 0.9 for reinforced concrete when a member is designed as tension-controlled, that is the strain in the extreme tension steel layer is not less than 0.005. The tension-controlled resistance factor for segmental bridges is 0.95 for bonded systems and 0.90 for unbonded systems. The cracking moment is derived from the formula:

$$f_{cpe} - \frac{M_{nc}}{S_{nc}} - \frac{(M_{cr} - M_{nc})}{S_c} = -f_r \quad (5)$$

where f_{cpe} is the extreme (precompressed) tension fiber stress due to effective prestress, (M_{nc} / S_{nc}) is the stress due to forces applied before composite action from a concrete topping or deck is affected, and f_r is the modulus of rupture. The formula as written in the Fourth Edition (2007) of the Specifications is shown below:

$$M_{cr} = S_c\left(f_r + f_{cpe}\right) - M_{nc}\left(\frac{S_c}{S_{nc}} - 1\right) \geq S_c f_r \qquad \left[\mathit{AASHTO}\ 5.7.3.3.2 - 1\right]$$

In the AASHTO equation, the term $S_c f_r$ is a lower limit; interestingly, it had been applied as an upper limit in preceding editions. The net effect is that the moment due to non-composite loads, primarily the deck weight, is not allowed to exceed the effect of prestress on the cracking moment. These two respective terms in the equation are: $-M_{dnc}\left(S_c / S_{nc} - 1\right)$ and $S_c\left(f_{cpe}\right)$.

It is not clear why setting a limit of $S_c f_r$, whether as an upper or lower limit, is necessary. Also, it is not clear why there are no explicit provisions for noncomposite members. It is possible that noncomposite sections can suddenly rupture under load, whether that load is an overload on the non-composite section in service or the wet weight of the deck during construction. Provisions for non-composite sections can be included by simply specifying that S_{nc} be substituted for S_c in AASHTO Eq. 5.7.3.3.2-1.

Section 5.7.3.3.2 states that the requirements must be met "at any section of a flexural component." This implies that all sections of any given span must satisfy these requirements. As shown in Figure 12, a pretensioned member with draped strands has to have significant strength demands at sections other than midspan in order to meet the requirements stated previously.

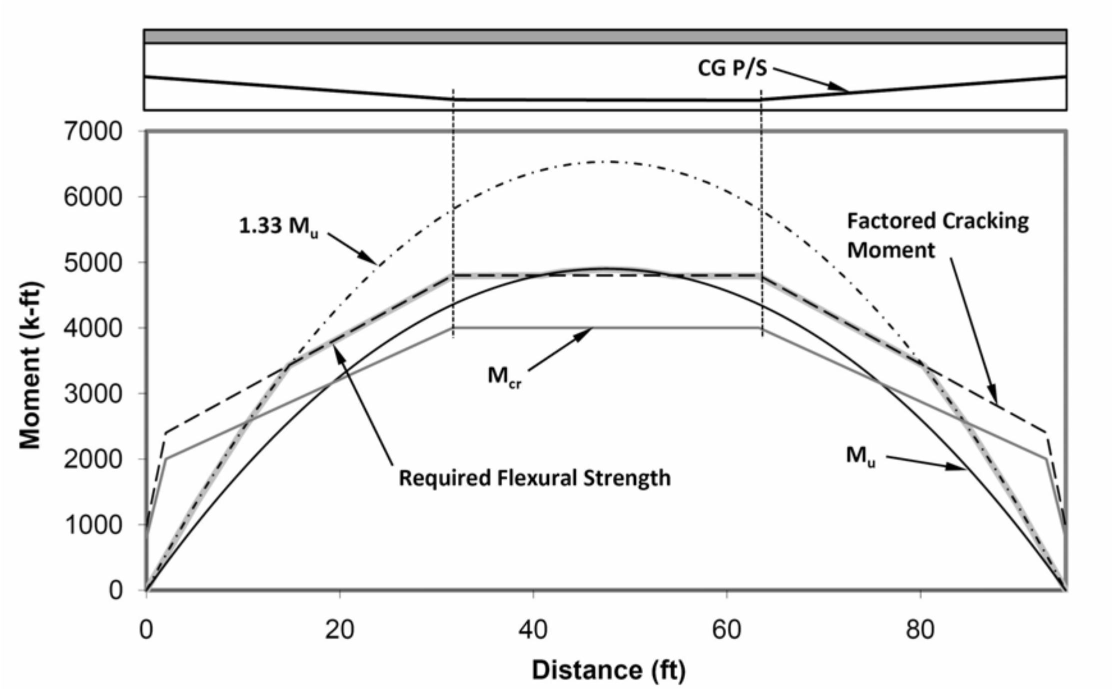


Figure 12. Cracking moment versus factored load moment in a pretensioned member with draped strands

It has been suggested to provide a loading capacity greater than the cracking load for a given span rather than requiring flexural capacity greater than the cracking moment at "any" ("every") section in a span for convenience. However, to ensure that the load capacity is greater than the cracking load, the load envelopes have to be characterized. A uniformly distributed load could be used to represent moving point load envelopes for simple-spans. However, this representation is inadequate for continuous structures and is not recommended.

2.4.1.1 Flexural Cracking Strength

For calculation of M_{cr} in Section 5.7.3.3.2 of LRFD specifications, the modulus of rupture is given as,

$$0.37\sqrt{f_c'}\ \text{(ksi)},\ \text{or}\ 11.7\sqrt{f_c'}\ \text{(psi)} \tag{6}$$

Equation 6 provides an upper bound value of the expected modulus of rupture that would lead to more conservative design compared to earlier LRFD provisions ($7.5\sqrt{f_c'}$ (psi) in 2005 and prior versions of AASHTO). The higher limit was introduced to reflect research results for high strength concrete as endorsed by ACI Committee 363 (ACI, 1992) on high strength concrete. It has been shown that using the higher limit in segmental box girder bridges could result in a 20% to 30% increase in required prestressing and in excessive cambers.

The applicability of the modulus of rupture specified in AASHTO to segmental bridges is questionable because the test results discussed previously indicate that the concrete layer in precast segments in vicinity of the segment-to-segment joint is relatively weak. As discussed in Section 2.1, a value of $7.5\sqrt{f'c}$ (psi) [$0.37\sqrt{f'c}$ (ksi)] should be an upper bound value for the flexural cracking strength of segment to segment joints rather than an average or lower bound value.

2.4.1.2 Flexural Capacity

In its simplest form, the flexural capacity is calculated as:

$$\phi M_n = \phi A_s f_y (d - a/2) \tag{7}$$

for reinforced concrete, and

$$\phi M_n = \phi A_s f_{ps} (d - a/2) \tag{8}$$

for prestressed concrete.

The resistance factor ϕ varies between 0.75 and 1.00 for prestressed concrete and between 0.75 and 0.90 for non-prestressed concrete. Because the issue of minimum reinforcement should relate to members with very little amounts of reinforcement, the upper limits of 0.90 for reinforced concrete and 1.00 for prestressed concrete is of primary concern. In segmental construction an upper value of 0.95 is also used in some situations. In some segmental and spliced I-girder applications, the reinforcement levels are so high as to enforce the compression controlled ϕ of 0.75 and give a false alarm that minimum reinforcement limits are not met. Obviously, this is not the intent of the minimum reinforcement limits.

For the sake of the discussion that follows, assume that $\phi = 1.00$. The second variable to discuss is the lever arm depth between the tensile reinforcement and the compression block. This appears to be straight forward and not subject to much debate. The third and most important variable is the steel stress at ultimate flexure. It has been a customary practice to use the yield strength of mild reinforcement f_y to represent that value, based on the justification that the stain hardening and ultimate steel strength occur beyond the point in which the section is assumed to have practically "failed." The true flexural strength when the steel ruptures should correspond to its ultimate strength f_{su}. Freyermuth and Aalami (1997) show that the ratio $f_{su}/f_y = 1.75$ for grade

40 steel and 1.50 for grade 60 steel. It is possible to expand these ratios to cover steel strengths up to Grade 270 low relaxation strands, which are known to have a yield strength = 0.9 of the ultimate, or f_{pu}/f_{py}= 1.11.

In regard to the flexural strength equation for prestressed concrete, the value f_{ps} is determined on the basis of strain compatibility, following the stress strain diagrams for low relaxation Grade 270 steel, up to a stress of 270 ksi. This is obviously inconsistent with the treatment of conventionally reinforced concrete as has been pointed out by several authors, including Ghosh (1987). Also, Jack Evans and Henry Bollman of FDOT made the same remarks in AASHTO Committee T10 correspondence. This explains in part the call by Washington DOT at T10 to increase the 1.33 factor applied to M_u to a higher value for prestressed concrete in order to have a consistent factor of safety as the 1.33 with reinforced concrete. Ghosh (1987) calls for a factor of 1.6, while Washington DOT has called for a value of 2.0 in some of the early T10 correspondence (in 2004-2005). By considering the ultimate steel stress, rather than the yield stress, for all steel grades in flexural capacity calculations, the discrepancy on this issue disappears.

The calculation of the stress in unbonded and external post-tensioned tendons at ultimate is more complex than in bonded and internal tendons. The LRFD specifications provide the following equation.

$$f_{ps} = f_{pe} + 900\left(\frac{(d_p - c)(2 + N_s)}{2l_i}\right) \leq f_{py} \qquad (9)$$

where f_{pe} is effective prestress, c is neutral axis depth, d_p is steel depth, N_s is number of supports between anchors, and l_i is length between anchors. A first approximation of the stress f_{ps} is the effective prestress plus 15 ksi (or about 165-190 ksi). Although this stress is much lower than the 270 ksi it takes to rupture the tendon, experimental studies and detailed analysis have shown that this equation is accurate, where concrete crushes prior to reaching tendon failure (Tassin, et al., 1996).

2.4.2 AASHTO Segmental Guide Specifications

In the 1989 version of the AASHTO Segmental Guide Specifications, there were no requirements for minimum flexural reinforcement. However, the commentary addressed the issue with the following:

The minimum reinforcement provision of Section 9.18.2.1 of the AASHTO specification was developed to avoid a brittle failure in grossly under-reinforced simple-span precast, prestressed section. Application to segmental concrete bridges results in requirements of more bonded reinforcement for bridges with more conservative (arbitrary) design tensile stress levels, which is contrary to load requirements. Minimum reinforcement requirements are adequately covered by the allowable stress and load factor requirements of these specifications.

Minimum flexural reinforcement provisions were added to the 1999 edition of the Guide Specifications for Design and Construction of Segmental Concrete Bridges to be consistent with the AASHTO specifications. However, this addition is also provided with commentary as follows.

A comprehensive proposal for the revision of the ACI minimum reinforcement requirements, including elimination of the 1.2 times the cracking moment provision, has been published in the ACI Structural Journal.

This section in the commentary is referring to the paper by Freyermuth and Aalami. Clearly, the commentary indicates concerns of the economic impact of specifying minimum flexural reinforcement for segmental bridges.

2.4.3 ACI 318

The ACI 318 Building Code follows essentially the same requirements as followed by the LRFD specifications, with the flexural strength required to be greater than the smaller of a factored cracking moment, M_{cr}, and a magnified factored moment, $1.33M_u$. However, there are distinct differences between ACI and AASHTO in the factors and in the method of application of these two requirements.

For reinforced concrete, ACI covers the minimum reinforcement requirements in Section 10.5. The "cracking moment" requirement is satisfied through a direct minimum steel area formula, as follows:

$$A_{s,min} = \frac{3\sqrt{f'_c}}{f_y} b_w d \quad \text{(psi)} \tag{10}$$

The quantity $3\sqrt{f'_c}$ may not be taken less than 200 psi to comply with requirements in older versions of ACI. Equation 10 and the associated exceptions are intended by ACI to give similar requirements to those given by Equation 3, but in a “simpler” form. Committee 318 has attempted, since the 1963 introduction of the strength design method for conventionally reinforced concrete members, to avoid design calculations involving section properties. In his unpublished study, C. P. Siess recommended that for reinforced concrete, the flexural strength of a section should simply be greater than or equal to the cracking moment. The margin between cracking and failure is provided by strain hardening of the mild reinforcement (a 50% increase in stress for Grade 60 reinforcement) and the strength reduction factor ϕ □= 0.90. Accordingly, ACI 318 performed a parametric study to derive Equation 10 by equating ϕM_n with $1.0M_{cr}$, using $f_r = 7.5\sqrt{f'_c}$, for a wide variety of section shapes and sizes.

The web width, b_w, must be changed for T-sections with the flange in the tension zone, to the lesser of $2b_w$ or the actual flange width. Apparently some judgment was used by Committee 318 to decide that an “effective” flange width of $2b_w$ is adequate for minimum reinforcement determination using the cracking moment criterion. However, Freyermuth and Aalami (1997) have shown that when actual width is used the cracking moment is so large that this criterion will almost always be superseded by the $1.33M_u$ criteria.

ACI Chapter 18 covers provisions unique to prestressed concrete. This is the same strategy followed by the LRFD specifications. The LRFD specifications have a unified treatment of structural concrete, whether fully prestressed, partially prestressed, or conventionally reinforced, similar to the practice in Europe.

Section 18.8 of ACI 318-05 states that “The total amount of prestressed and nonprestressed reinforcement shall be adequate to develop a **factored load** at least 1.2 times the cracking load computed on the basis of modulus of rupture f_r specified in 9.2.3. This requirement shall be permitted to be waived for: (a) two-way, unbonded post-tensioned slabs; and (b) flexural members with shear and flexural strength at least twice that required by 9.2.”

One difference between ACI and AASHTO for prestressed concrete is in the value of ϕ, which is taken = 0.9 in ACI and is given different values in AASHTO depending on the type of member. Other significant differences are: 1) the factored load limit is $1.33M_u$ in AASHTO (at any given section) and 2.0 × the factored load in any given span in ACI, and 2) the modulus of rupture is based on the older coefficient of 0.24 in ksi (7.5 in psi) in ACI as opposed to the larger 0.37 in ksi (11.7 in psi) coefficient in the LRFD specifications.

The 2.0 coefficient was adopted by ACI 318 as a conservative number even though the unpublished study by Professor C.P. Siess recommended a coefficient of 1.67. Ghosh (1987) explained that the 1.33 coefficient for reinforced concrete, when based on a yield strength of grade 60 steel, is in reality a 1.33*(90)/(60) = 2.0 factor when the tensile strength of 90 ksi is used rather than the yield strength of 60 ksi in calculating the flexural strength. Ghosh disagreed with that value and explained that it should be only 1.6. This factor is determined as $1.33*(f_{pu}/f_{py})$ = 1.33*(270)/(0.85*270) for stress relieved strands. Ghosh's suggested modifications were not accepted by ACI 318. They did not cover all types of prestressing reinforcement or the cases where prestressing steel and mild steel existed in the same section. Stress relieved strand is no longer in use by the great majority of users.

Note that for prestressed members ACI 318, unlike AASHTO, requires that the minimum reinforcement criteria relate to loading on a member rather than satisfaction of the minimum reinforcement in ALL sections of the member. That latter requirement is still enforced in Section 10.5 of ACI 318 for reinforced concrete, thus creating an inconsistency within the ACI Code.

Additional provisions are given in Section 18.9 for minimum bonded reinforcement in unbonded post-tensioned members. Except for two-way slabs, the ACI 318-05 requires that

$$A_{s,min} = 0.004A_{ct} \quad (11)$$

at both the positive and negative moment sections of continuous post-tensioned members, where $A_{s,min}$ is the minimum additional bonded reinforcement and A_{ct} is the area of the part of the section between the center of gravity of the gross section and the tension face. For two-way slabs, different minimum amounts are specified for positive and for negative moment sections, depending on the bottom fiber stress at service load conditions. It should be noted that in the 2008 edition of ACI 318, minimum reinforcement is eliminated in unbonded systems.

2.4.4 Freyermuth and Aalami—CEB-FIP

Freyermuth and Aalami (1997) proposed a unified and simplified approach to the requirements of minimum reinforcement in the ACI 318-95 Code. Their approach was a further development of the provisions in the Third and Fourth editions of the European Code known as CEB-FIP Model Code for Concrete Structures published in 1978 and 1990 respectively. The CEB-FIP requirements as quoted by Freyermuth-Aalami follow:

"9.2.2-Beams

9.2.2.1-Longitudinal reinforcement: A minimum area of longitudinal bonded reinforcement should be provided to avoid brittle failure in case of unforeseen loss of concrete tensile strength.

Commentary (Notes): If a specific study is not carried out in this respect, the area of longitudinal tensile bonded reinforcement provided should be at least taken equal to:

0.0015b_td for steel grades S400 (58,000 psi) and S500 (72,500 psi)

0.0025b_td for steel grades S220 (31,900)

where b_t is the average width of the concrete zone in tension. In a T-beam, if the neutral axis in the ULS is located in the flange, the width of the latter is not taken into account in evaluating b_t."

Freyermuth and Aalami, in analyzing a large number of test specimens previously produced by Warwaruk, Sozen, and Siess of the University of Illinois in 1957, 1960, and 1962, found the CEB-FIP provisions to be deficient in some cases. Accordingly, they proposed a 1/3 increase to the first formula which applies to steel grades commonly used in North America. Also, to simplify, they proposed a change from an average width of the tension zone of the section to web width. Thus

$$A_{s,min} = 0.002 b_w d \tag{12}$$

To include concrete and steel strength as variables, Freyermuth and Aalami, converted Equation 10 to two equations, one for use in reinforced concrete members and the other for prestressed members:

$$A_{s,min} = \frac{3.0\sqrt{f'_c}}{f_{su}} b_w d \tag{13}$$

$$A_{s,min} = \frac{9.0\sqrt{f'_c}}{f_{pu}} b_w d \tag{14}$$

It is interesting to note that Equation 13 is almost identical to that in the ACI 318 Code for reinforced concrete except that the ultimate steel strength rather than the yield strength is used. For Grade 60 steel, the ratio is about 1.5. Thus, their formula gives 2/3rds of the ACI Code limit for that grade. Using the ultimate as opposed to the yield strength of steel seems to make more sense in calculating minimum reinforcement limits where steel is expected to go through strain hardening and rupture at member capacity. They cited a previous study which had demonstrated this to be true in cases where steel content was less than 25% of the balanced steel content.

The simplicity of this method is attractive. It may be possible to revert back to the simpler Equation 13 for all cases and to replace the web width with an average width of the tension zone, similar to what is being used at this time for shear design using the Modified Compression Field Theory. The effect of prestressing might be included in a similar manner without significant loss of simplicity.

2.4.5 International Practice

2.4.5.1 Canadian Code (CAN/CSA-S6-06)

The Canadian Highway Bridge Design Code (CAN/CSA-S6-06) has similar provisions for minimum reinforcement as those of the AASHTO LRFD provisions discussed in Section 2.4.1 of this report. The amount of reinforcement shall be adequate so that the factored flexural resistance, M_r, is at least 1.20 times the cracking moment or 1.33 times the factored moment (see Equations 3 and 4).

A major difference between the Canadian Code and the AASHTO LRFD is in the calculation of cracking moment. The Canadian Code adopted the term "cracking strength" instead of "modulus of rupture" to define the stress level at which concrete cracking occurs. Instead of $0.37\sqrt{f_c^{'}}\ [ksi]\ \ (11.7\sqrt{f_c^{'}}\ [psi])$, as specified in the AASHTO LRFD, the Canadian Code specifies a cracking strength of $0.15\sqrt{f_c^{'}}\ [ksi]\ \ (4.8\sqrt{f_c^{'}}\ [psi])$. Thus, the cracking moment according to the Canadian Code for a reinforced concrete section is only 41% percent of the cracking moment calculated according to the AASHTO LRFD provisions.

Adoption of a relatively low value of the concrete cracking strength is based on research results, which suggest that larger concrete sections exhibit more shrinkage cracking than smaller sections and therefore their value of cracking strength is lower than the conventional value of $0.24\sqrt{f_c^{'}}\ [ksi]\ \ (7.5\sqrt{f_c^{'}}\ [psi])$ that has been given in earlier Canadian Design Codes.

2.4.5.2 CEB-FIP

Provisions of the CEB-FIP MC90 have been discussed earlier in Section 2.4.4 (CEB-FIP – Freyermuth and Aalami).

2.4.5.3 Eurocode

The Eurocode 2, Specifications for the Design of Concrete Structures, consists of two parts. Part 1 has general specifications and design specifications for building structures. Part 2 contains design and detailing rules pertaining to bridge structures. This second part is written as a supplement to Part 1, and only specifications that differ from building structures are included.

The minimum longitudinal reinforcement in beams, $A_{s,min}$ according to Part 1 of the Eurocode is given by:

$$A_{s,\min} = 0.26 \frac{f_{cr}}{f_y} b_t\, d \qquad \geq \quad 0.0013\, b_t\, d \tag{15}$$

Where f_{cr} is the flexural cracking strength of concrete, f_y is the elastic limit (yield strength) of reinforcement, b_t is the average width of concrete zone in tension and d is the depth measured from the extreme compression fiber to centroid of tensile steel reinforcement. In a T-beam and when the flange is in compression, the width b_t shall be taken as width of the web.

The concrete tensile strength is determined from a table based on concrete class, or concrete compressive strength. It is interesting to note that the tensile strength of concrete is calculated as,

$$f_{cr} = 0.3(f'_c)^{2/3}\ [\text{MPa}] = 1.58(f'_c)^{2/3}\ [\text{psi}] \tag{16}$$

where f_{ck} is the specified minimum compressive strength of concrete that is similar to f'_c. The limits are concrete strengths up to 50 MPa or 7,250 psi.

For concrete strengths exceeding 7,250 psi, the cracking strength is related to f'_c by a logarithmic function.

$$f_{cr} = 2.12 \ln (1 + f_{cm}/10)\ [\text{MPa}] = 307 \ln (1+f_{cm}/1450)\ [\text{psi}] \tag{17}$$

where f_{cm} is the mean compressive strength given as follows:

$$f_{cm} = f'_c + 8\ [\text{MPa}] = f'_c + 1{,}160\ [\text{psi}] \tag{18}$$

For comparison purposes, the cracking strength for a specified compressive strength, f'_c = 33 MPa = 4785 psi is 500 psi (= 3.5 MPa). This is compared to a modulus of rupture value of 809 psi (=5.6 MPa) according to the LRFD specifications. Thus, for a reinforced concrete with a 5,000 psi compressive strength, the cracking according to the Eurocode is approximately 47 percent of cracking moment calculated based on the AASHTO LRFD specifications. It is interesting to note that the Eurocode gives 5th and 95th percentile values of the cracking stress as 0.7 f_{cr} and 1.3f_{cr}, respectively.

In Part 2 of the Eurocode the minimum reinforcement in prestressed concrete members is addressed directly with the following:

$$A_{s,min} f_y + A_{ps} \Delta\sigma_p \geq M_{rep}/d_e \tag{19}$$

where M_{rep} is the cracking strength of the concrete, assuming that no prestress is applied to the section, d_e is the lever arm to the tension steel and $\Delta\sigma_p$ is the smaller of $0.4f_{pu}$ and 72.5 ksi. This procedure is computationally simpler than the method specified in the LRFD because the cracking moment does not depend on the amount of prestress. For precast segmental structures, the Eurocode recommends that M_{rep} be taken as zero at the joints.

2.4.5.4 Japan Specification for Highway Bridges

The Japan Road Association specifies the following regarding minimum flexural reinforcement. The first requirement in reinforced concrete relates to preventing the propagation of cracks with reinforcement in an amount not less than 0.15% of the cross-sectional area of the member. The second requirement relates to providing a minimum reinforcement for flexural resistance to prevent brittle failure.

> *… The cross sectional area of the main axial tensile reinforcement placed in a reinforced concrete structure shall be determined in accordance with Equation 6.4.1*

1) *Girder:* $A_{st} \geq 0.005 b_w d$ (20)
2) *Members that are so thin in the direction of action of shear forces that diagonal tensile reinforcement cannot be placed*

$$A_{st} \geq 0.01 b_w d \quad (21)$$

A_{st}: *Cross sectional area of the main axial tensile reinforcement*

b_w: *Web thickness of the girder*

d: Effective height

> *However if, reinforcement in a girder is placed in an amount of not less than 4/3 times the required cross sectional area, the provision of 1) need not be referred.*

Requirement 2) is referring to slab construction, where no shear reinforcement is provided. Commentary addresses these specifications with the following:

> *… a member with very little main axial tensile reinforcement could fail abruptly when unexpected bending stress occurs. This clause is to prevent sudden failure of concrete structures. However girders are provided with sufficient main axial reinforcement in general, so it is not always appropriate to stipulate the minimum amount of steel in terms of a ratio to the cross-sectional area of the member. Therefore, a separate exceptional stipulation was established for girders in terms of a ratio to the amount of steel required by stress calculation or the like.*

For prestressed concrete, minimum reinforcement is not evaluated at the strength limit state. Minimum reinforcement is only required when tensile stresses (below specified limits) are expected under service conditions. In these regions, the amount of reinforcement must be sufficient to balance the tension force equivalent to the respective concrete tension assumed in the uncracked state. The code further suggests that for unbonded or externally prestressed members, the live load be increased by 35% to limit cracking that could lead to reduced durability.

The maximum allowed stress in the reinforcement is 26 ksi. Prestressed strand can be used to serve the same purpose, assuming that the gain in stress between the uncracked and cracked states is below the allowable limit for mild reinforcement.

The commentary addresses the advantages of using service evaluation of the minimum reinforcement in-lieu of the strength limit state as:

> *…calculation of the tension reinforcement is generally easier and moreover, is on the safer side.*

It is further mentioned that prestress members at strength limit states should be evaluated for extreme events such as collision or earthquake loads.

2.4.6 Leonhardt's Method

Fritz Leonhardt is one of the fathers of modern prestressed concrete. One of several classical books he wrote was on fundamentals of design of prestressed concrete, which was translated from German to English and published in the U.S. in 1964 (Leonhardt, 1964). Leonhardt provides a method for calculation of minimum flexural reinforcement which is described below, see Figure 13. The stress in the section just before it cracks is represented. The stress f_{ct} is the tensile capacity in the concrete. As a conservative approximation, a simplified stress distribution as shown in Figure 13(c) is assumed, whereby the stress is assumed zero at the centroid of the gross concrete section. According to this approach, the stress resultant F_{ct} must be resisted upon cracking with adequate reinforcement A_{ps}.

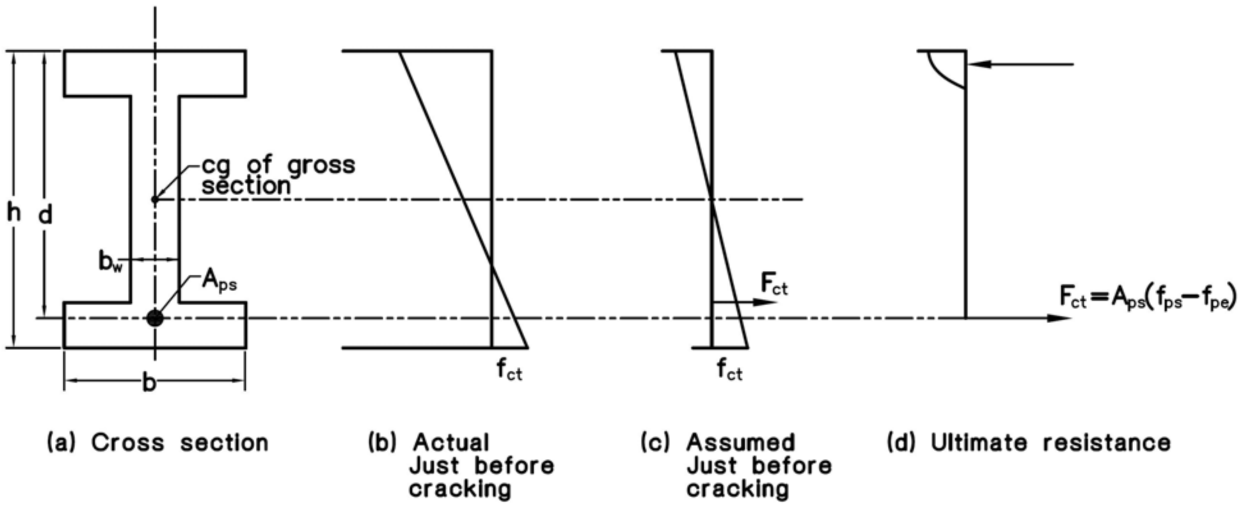


Figure 13. Leonhardt's Minimum Reinforcement Method

The stress in that reinforcement after cracking is assumed to be the incremental stress from effective prestress f_{pe} (existing just before cracking) to the stress f_{ps} at ultimate flexure, or $(f_{ps}-f_{pe})$. Thus,

$$F_{ct} = \kappa(\frac{1}{2} b_w \frac{h}{2}) f_{ct} \tag{22}$$

$$F_{ct} = A_{ps}(f_{ps} - f_{pe}) \tag{23}$$

The coefficient κ is a function of the geometry of the tensioned area of the cross section. It is =1 for rectangular sections where b_w=b.

The American Segmental Bridge Institute (ASBI) in a submittal to T10 in January of 2007, proposed to adopt Leonhardt's method with some modifications as follows:

Revise min. reinforcement formula to:

$$A_{ps}(f_{ps} - f_{pe}) \geq 1.2F_{ct} \tag{24}$$

for prestressed members, and to

$$A_s f_y \geq 1.2F_{ct} \tag{25}$$

for conventionally reinforced members

For prestressed members with both mild reinforcement and prestressing tendons, an equivalent prestressing steel area to the available mild reinforcement equal to $A_s f_y /(f_{ps} - f_{pe})$ *may be assumed to contribute to the min. prestressing reinforcement.*

Upper bound tensile strength of concrete = $0.23\sqrt{f_c^{'}}\,(\text{ksi})$*, or* $7.3\sqrt{f_c^{'}}\,(\text{psi})$ *is proposed. This formula is shown by ASBI, based on research done by members of this team to better represent tensile strength in large structural members, compared to the current formula which was based primarily on lab testing of small, shallow specimens with steep stress gradients.*

Waive minimum reinforcement requirements for externally (unbonded) post-tensioned members; this is consistent with a recent decision by ACI 318 for the 2008 Edition.

For members with a combination of internal (bonded) and external (unbonded) tendons use the majority type of the tendons for min. reinforcement criteria.

For deep members, use the strut and tie method to determine minimum reinforcement.

The elegance of the Leonhardt/ASBI approach is in its simplicity. It eliminates the need for using effective prestress and cracking moments to determine minimum reinforcement and is similar to the method specified in the Eurocode. The only complexity is in determining the F_t value for non-rectangular tension zones. Using the “tensile area of the section” and assuming it to be defined as the area on the tension side of the centroidal axis is already an acceptable concept, used in the shear provisions of AASHTO. This method is somewhat more rigorous than the CEB/FIP method as it accounts for the type of steel and the tensile strength of the concrete. However, it seems to ignore the compression side of the moment resistance in a flexural member. It appears to involve two seemingly offsetting approximations in calculating the cracking resistance and the ultimate resistance. For the cracking resistance, prestressing is a major effect and the cracking moment should include the compression stress resultant and the lever arm between that resultant and the prestressing reinforcement. For the ultimate resistance, the resistance is calculated in a manner that is similar to the Eurocode (i.e., f_{ps}-f_{pe}). This is a simple method to obtain a uniform margin of safety for the minimum reinforcement requirements, without engaging the designer in complex calculations.

A Technical Advisory Committee was retained by ASBI to review this approach. It consisted of leaders in the design/research community, including Breen, Combault, Dolan, Ganz, Goodyear, and Seible. The majority of that committee’s reactions were positive and supportive.

2.4.7 Modified LRFD Method

To meet the objectives of the research project, the NCHRP 12-80 project team developed a method for determining minimum reinforcement that is based on the current method specified in the LRFD specifications. This method, referred to herein as the Modified LRFD Method, is developed to be suitable for all structure types covered in the AASHTO LRFD Bridge Design Specifications and achieve appropriate and consistent safety. As mentioned previously, the each component of the minimum reinforcement requirement is factored separately to account for variability. The method is described as follows:

$$\phi M_n \geq M_{fcr} \tag{26}$$

where M_{fcr} is the factored cracking moment calculated as:

$$M_{fcr} = \gamma_3 \left[(\gamma_1 f_r + \gamma_2 f_{cpe}) S_c - M_{dnc} \left(\frac{S_c}{S_{nc}} - 1 \right) \right] \tag{27}$$

and the requirement can be waived if

$$\phi M_n \geq 1.33 M_u \tag{28}$$

where:

f_r = the flexural cracking stress of concrete taken as $0.24\sqrt{f'c}$ (ksi).

f_{cpe} = compressive stress in concrete due to effective prestress forces only (after allowance for all prestress losses) at extreme fiber of section where tensile stress is caused by externally applied loads (ksi).

M_{dnc} = total unfactored dead load moment acting on the monolithic or noncomposite section (k-ft).

S_c = section modulus for the extreme fiber of the composite section where tensile stress is caused by externally applied loads (in^3).

S_{nc} = section modulus for the extreme fiber of the monolithic or non-composite section where tensile stress is caused by externally applied loads (in^3).

The following factors account for variability in the flexural cracking strength of concrete, variability of prestress and the ratio of nominal yield stress of reinforcement to ultimate.

γ_1 = flexural cracking variability factor (1.6 for concrete bridge members and 1.2 for precast segmental structures).

γ_2 = Prestress variability factor (1.1 for internal [bonded] tendons and 1.0 for external [un-bonded] tendons).

γ_3 = f_y/f_u (0.67 for A615 and 0.75 for A706 Grade 60 Reinforcement). For prestressed concrete structures, use 1.0.

ϕ= Resistance factor (1.0 for prestressed concrete, 0.9 for non-prestressed and externally prestressed segmental bridge girders) constant for the purpose of checking minimum reinforcement.

The true advantage of this method is that the sources of variability in computing the cracking moment and the resistance are appropriately factored. In the case of the moment resistance the maximum, or overstrength moment used, is the true measure of whether or not the section is brittle when subject to force-control loads, as shown in Figure 1. The cracking stress factor is applied to the modulus of rupture, which has a far greater variability than the amount of prestress (f_{cpe}) at the extreme fiber. This cracking stress factor will account for such things as concrete strength gain with time and size effects mentioned previously.

The cracking stress factor of is based on providing a high probability that the factored strength will be greater than actual flexure cracking strength. The value of γ_1 = 1.6 results in a factored stress of $12.0\sqrt{f'c}$ (psi) [$0.38\sqrt{f'c}$ (ksi)] that is greater than 2 standard deviations above the average modulus of rupture (or a 98% probability of not being exceeded), when evaluating non-moist cured units, as discussed in Section 2.3.2. When considering full-size member data, the factored stress is 2.9 standard deviations above the mean.

For precast segmental structures, a reduced value of the flexure cracking strength factor is justified due to the laitance layer effect discussed in Section 2.1. Based on this effect, it is suggested that $7.5\sqrt{f'c}$ (psi) [$0.24\sqrt{f'c}$ (ksi)] is an upper bound flexural cracking strength and the flexural cracking strength factor (γ_1) should be equal to 1.2 to be consistent with the LRFD specifications (prior to the 2005 Interim Revisions).

An appropriate factor is applied to the prestress in the concrete (γ_2) is 1.1 to account for the possibility of concrete stresses due to prestress being higher than specified. This value appears to be too conservative for pretensioned members, for which a value of the factor (γ_2) of 1.05 may be more appropriate, as discussed in Section 2.3.5. However, post-tensioned structures are subject to losses due to friction and anchor set. This is especially true for draped tendons in long, cast-in-place post-tensioned box girder bridge frames. Based on the range of friction coefficients stated in Article 5.9.5.2.2 of the LRFD specification, which is from 0.15 to 0.25, the factor (γ_2) of 1.1 is appropriate.

External (un-bonded) tendons will remain essentially elastic in precast segmental bridge girders loaded to the ultimate flexural strength, because the tendons are allowed to stretch along the entire length after cracking has occurred, as demonstrated in laboratory experiments and analytical studies (Megally 2003 and Tassin 1997). Based on this observation, the difference in the tendon stress between its in-service working state and the ultimate state ($f_s - f_{pe}$) should remain constant regardless of the initial prestress (f_{pe}), within reasonable working limits. If the prestress losses are underestimated, and the actual prestress is 10% higher than assumed, the ultimate strength should increase essentially the same amount as the cracking moment, as shown in Figure 14. Therefore, any unintended increase in prestress cancels out of the minimum reinforcement check, and the prestress variability factor (γ_2) should be 1.0 for externally prestressed concrete bridge girders.

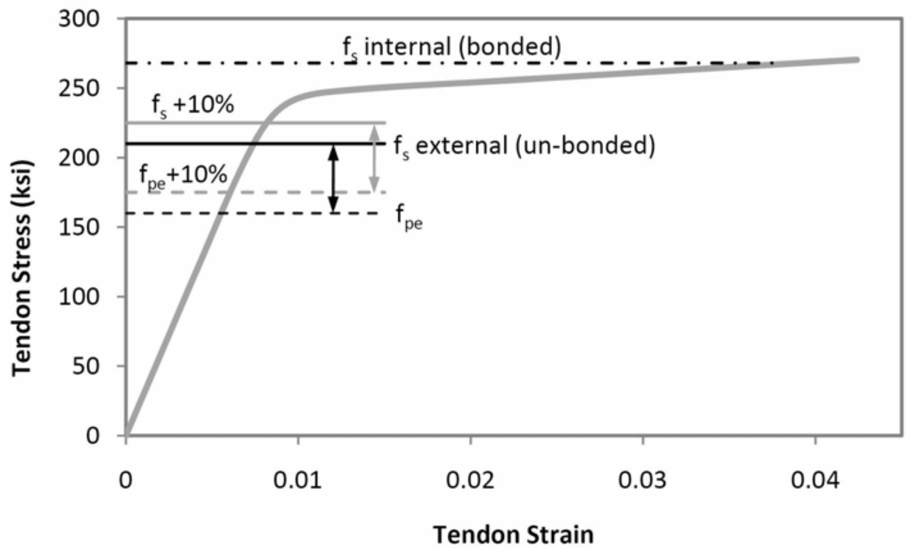


Figure 14. External and internal tendon stress-strain response illustration

Provided that sufficient ductility exists, minimum reinforcement requirements can be waived for negative bending regions of continuous spans. The reason for this exemption is that negative bending regions will crack prior to positive regions. As shown in Section 2.1, lightly reinforced and/or prestressed sections have significant post-cracking strength and ductility. If minimum flexural reinforcement provisions are met within the peak positive bending regions, the structure will perform in a ductile manner and collapse will not occur without large warning deflections. With exception to spans with hinges and cantilevered bridges during construction, negative bending regions are not critical for minimum reinforcement assuming that adequate ductility in the post-cracked state is provided. The LRFD specifications provide guidance on the implied ductility of concrete members in Article 5.7.3.5. This section specifies the permission of moment redistribution if the net tensile strain of the extreme tension reinforcement at ultimate exceeds span then minimum reinforcement provisions need not be checked in the negative bending region.

Minimum reinforcement provisions should not apply to compression-controlled or transition regions because sections in this category require greater strength with the variable Resistance Factor (ϕ) to account for reduced ductility, and an additional factor of safety is not required. Conditions where minimum reinforcement provisions are not satisfied and the net-tensile strain indicates that the section is over-reinforced are inverted t-beams and heavily prestressed box girder sections. Under these conditions, it is more logical to add reinforcement to the compression zone rather than the amount of tension reinforcement that would result in further reduced ductility.

By specifying minimum reinforcement for tension controlled sections only, there is a lack of consistency in the application of minimum reinforcement. A section with a slightly lower net tensile strain ε_t than is required to be tension controlled may have a slightly reduced ϕ without having to meet minimum reinforcement requirements. A remedy to this apparent lack of consistency is to make the ϕ constant regardless of the net tensile strain, as shown in Figure 15. For all other strength limit states, ϕ is reduced for compression controlled and transition regions.

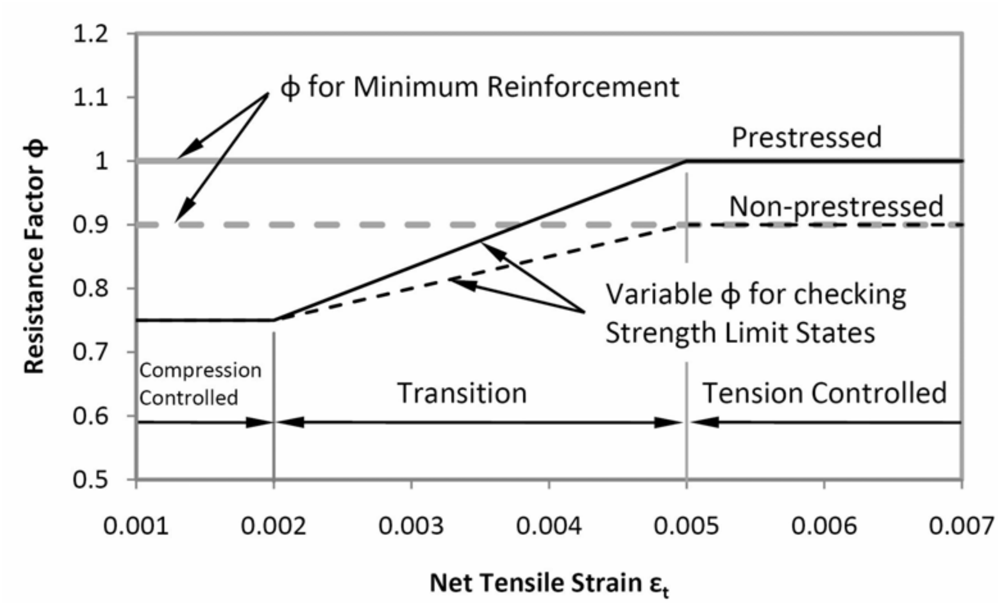


Figure 15. Graphical representation of variable resistance factor

2.4.8 Comparison of Minimum Flexural Reinforcement Provisions

A summary of the minimum flexural reinforcing moment provisions are shown in Table 3. These provisions are shown with the pound-per-square-inch (psi) units for direct comparison. In this table, qualitative remarks regarding applicability and ease-of-use are provided.

It should be noted that the methods investigated as part of this research fall into two separate categories.

(1) *Strength Methods:* The LRFD specifications, the CSA, the ACI 318 (prestressed concrete section) and the Modified LRFD methods are similar in that the minimum reinforcement is specified by requiring that the flexural strength must be greater than cracking by an acceptable safety margin. Minimum prestress in these methods are calculated through trial-and-error.

(2) *Prescribed Area Methods:* The remaining methods are based on providing minimum reinforcement and/or prestress that is greater than the cracking strength by an acceptable safety margin, but the methods are further simplified so the amount of reinforcement and/or prestress is calculated directly. These methods include Leonhardt, Eurocode, JRA (Japan), and the reinforced concrete section of ACI code.

In regard to the strength methods, there is a wide variability in the calculation of flexural cracking strength, which varies from $4.8\sqrt{f'c}$ (psi), in the CSA (Canadian), to $11.7\sqrt{f'c}$ (psi), in the LRFD Specifications.

Table 3. Minimum flexural reinforcement provisions table

Method	Sectional Requirements	Flexural Cracking Strength (psi)	Over-Demand Requirements	Advantages/ Disadvantages
AASHTO LRFD	$\phi M_n \geq 1.2M_{cr}$	$11.7\sqrt{f'c}$	$\phi M_n \geq 1.33M_u$	Universal in application. Highest sectional requirements. Computation of cracking moment is complicated. Often controls negative bending.
ACI 318	$A_s \geq 3\sqrt{f'_c}/f_y b_t d$ Reinforced $\phi M_n \geq 1.2M_{cr}$ Prestress	$7.5\sqrt{f'c}$	$\phi M_n \geq 2.0\ M_u$	For R/C Members – Ease of use. Unconservative sectional requirements flanges in tension. Same requirements as the AASHTO LRFD with lower Modulus-of-rupture (MOR) values.
CSA (Canadian)	$\phi M_n \geq 1.2M_{cr}$	$4.8\sqrt{f'c}$	$\phi M_n \geq 1.33M_u$	Same as AASHTO LRFD with lower MOR.
CEB – FIP	$A_s \geq 0.0015b_t d$ (S400 and S500) $A_s \geq 0.0025b_t d$ (S200)	N/A	N/A	Ease of use. May not applicable to bridge girders with flanges in tension.
JRA	$A_{st} \geq 0.005b_w d$ $A_{st} \geq 0.01b_w d$ (slabs)	N/A	$M_r \geq 1.33M_u$	Applies to reinforced concrete. Minimum reinforcement for P/S at service limit state only.
Leonhardt	$A_s \geq 1.2F_{ct}/f_y$ Reinforced $A_s \geq 1.2F_{ct}/(f_{ps}-f_{pe})$ Prestressed	$7.3\sqrt{f'c}$	N/A	F_{ct} is the strength at cracking of the tension zone assuming a neutral axis depth at the centroid. Ease of use.
Eurocode	$A_s \geq 0.26\ f_{cr}/f_y\ b_t d$ $A_s \geq .0013\ b_t d$ Reinforced $A_s \geq M_{rep}/(d_e\ \Delta\sigma_p)$ Prestressed	$1.58\ f'_c{}^{2/3}$ ($f'c \geq 7{,}250$ psi) $307\ln(1+f_{cm}/1450)$		M_{rep} is the cracking strength assuming no prestress. d_e is the lever arm to the tension steel. $\Delta\sigma_p$ is the smaller of 0.4 f_{pu} and 72.5 ksi. M_{rep} is assumed zero at segmental joints.
Modified LRFD Method	$\phi M_n \geq \gamma_3(\gamma_1 f_r+\gamma_2 f_{cpe})S$ γ_1 = 1.6 Cracking factor γ_2 = 1.1 Prestress factor $\gamma_3 = f_y/f_u$ (1.0 prestress)	$7.5\sqrt{f'c}$	$\phi M_n \geq 1.33\ M_u$	Compares ultimate instead of nominal moment capacity. Separate load factors for cracking and prestress components to reduce “chasing your tail” effect.

CHAPTER 3 INTERPRETATION, APPRAISAL AND APPLICATION

To evaluate the methods of prescribing minimum reinforcement, a parametric study was performed on four representative minimum reinforcement methods of the eight described in Section 2.4. The criterion for this evaluation includes reliability, as defined as providing a consistent level-of-safety for all concrete bridge members covered in the LRFD specifications, economy and ease-of-use, as described in Section 3.1. Based on this evaluation, recommended changes to the LRFD specifications are provided, which is the subject of Section 3.2. And to illustrate how the minimum reinforcement is prescribed using the proposed modifications to the minimum reinforcement requirements, design examples are presented in Section 3.3.

3.1 PARAMETRIC STUDY OF MINIMUM REINFORCEMENT PROVISIONS

Due to budget constraints, not all of the methods investigated in Section 2.4 were evaluated as part of the parametric study, and only four representative methods were selected. Two of the methods are considered Strength Methods – notably the LRFD specifications, and the Modified LRFD methods, and two are Prescriptive Area Methods, including the Leonhardt and Eurocode methods.

For reference purposes, the parametric-study methods are listed in Table 3 and are described below.

- **LRFD** – AASHTO LRFD Bridge Design Specifications (4^{th} Edition) minimum reinforcement provisions. While the requirements are simple, the interpretation can be complex, especially in the negative bending regions of prestressed concrete members. However, the application of the method is consistent for all types of concrete members and provides the highest level of resistance in most cases.
- **Leonhardt** – ASBI proposed minimum reinforcement provisions to AASHTO T10 in January, 2007. Application of this method is relatively simple, and could provide the required safety. Effects of the concrete member on the compression side of the neutral axis and prestress are not considered.
- **Eurocode** – Eurocode 2, Design of Concrete Structures, EN 1992-2-2, 2006. Similar to the Leonhardt method in the calculation of prestress. Overall section is used to compute the cracking force, whereas, the section below the neutral axis is used in Leonhardt.
- **Modified LRFD** – Modified minimum reinforcement provisions to the LRFD specifications, 2009. Developed to provide consistent safety without additional computational complexity that is

prescribed in the current LRFD specifications. Appropriate factors for flexural cracking and prestress improve economy and consistency.

The parametric study includes computing the required minimum reinforcement and/or prestress using the candidate methods, as listed previously on the members listed in the concrete member database. The database includes both the reinforced concrete and prestressed concrete members. The procedure used in the parametric study includes the following steps:

Step 1: Compute the required minimum reinforcement and/or prestress using candidate methods:

For each concrete member in the database, the minimum reinforcement methods are applied to find the minimum area of reinforcement ($A_{s,min}$) or the minimum area of prestress steel ($A_{ps,min}$), as discussed in Section 3.1.2

Step 2: Compute the theoretical cracking moment (M_{cr}):

Based on the computed $A_{s,min}$, or $A_{ps,min}$, M_{cr} is calculated using a single theoretical cracking stress, as discussed in Section 3.1.3. Although M_{cr} is the same for reinforced concrete members, prestress concrete member differ because M_{cr} is dependent on the amount of prestress.

Step 3: Compute the nominal moment at overstrength (M_o):

Using $A_{s,min}$,or $A_{ps,min}$, for each of MFR provisions, M_o is calculated using strain-compatibility, as discussed in Section 3.1.3. The significance of M_o is illustrated in Figure 1.

Step 4: Compare M_o/M_{cr} for each of the candidate provisions.

The M_o/M_{cr} ratio is used to determine how ductile or brittle a member is when subject to flexural loads (i.e., $M_o/M_{cr} < 1$ is brittle), and the level of safety provided. Consistency is also compared between methods for the full range of concrete members.

3.1.1 Concrete Structures Database

The concrete structures database is intended to represent the range of structures commonly used for construction covered by the AASHTO LRFD Bridge design specifications. Of particular interest is evaluating parameters that have a significant effect on the minimum flexural reinforcement provisions.

In developing this database of concrete structures, the range of spans, girder types, spacing, and concrete strengths is based on recommended practice found in the PCI Bridge Design Manual and the ASBI Segmental Box Girder Standards. Further, DOT guidelines including those from Florida (FDOT, 2008), California (Caltrans, 1998), Nebraska (NDOR, 2008) and Washington State (WSDOT, 2008) were evaluated.

Table 4. Concrete member database structural dimension limits

Bridge Types		Span – L (ft)		Depth/Span		Girder Spacing (ft)	
No.		**Min**	**Max**	**Simple**	**Cont**	**Min**	**Max**
Cast-in-place Bridges							
2	Slab	20	45	0.07L	0.06L		
2	Reinforced concrete box	60	120	0.06L	0.055L	6.5	14
2	P/T Slab	40	70	0.03L	0.027L		
2	P/T Conc. Box*	80	250	0.045L	0.040L	6	20
Precast Concrete Bridges							
2	Slabs	20	50	0.03L	0.03L		
2	Double Tees	30	60	0.05L	0.05L	4	4
2	Box beams	50	120	0.033L	0.030L	3	4
2	I-girders	70	200	0.045L	0.040L	6	12
2	U-beams	80	200	0.045L	0.04L	12	26
Segmental Bridges (precast)							
2	Span x span	100	150	0.045L	0.040L	28	45
2	Balanced Cantilever **	100	200	N/A	0.025L	28	45
Concrete Substructure Elements							
2	Footings	12	35				
2	Cap beams	20	60	0.045L	0.04L		
26	**Total**						

*Practical upper limit for prismatic members – haunched members up to 600 ft. **Upper limit noted in ASBI Standards (ASBI, 2000)

Based on this review Table 4 was developed to capture the range of practical applicability regarding structure dimensions. By using these guidelines, the structures database captures the upper and lower bounds of each structure classification.

3.1.1.1 Cast-in-Place Concrete

- *Conventionally Reinforced Slab*

Conventionally reinforced slab bridges are commonly used for low and relatively short spans. The California Department of Transportation has fully-designed and detailed slab bridge standards, and the details for concrete slabs are directly from these standards for span lengths of 16 and 44 feet, the upper and lower limits provided in Table 5, respectively.

It should be noted that the Caltrans permits the use of concrete compressive strength of 3,600 psi for concrete slab bridges.

Table 5. Conventional slab structure dimensions

	CRS1	CRS2	Units
No. of spans	2	2	
Span length	16.0	44.0	ft.
Depth	10.5	21.5	in.
f'c	3.6	3.6	ksi

- *Post-tensioned Slab*

Post tensioned concrete slab bridges are typically limited to applications where the permanent structure depth is limited, and the spans are below 70 feet. The minimum compressive strength is 4,000 psi, as specified in the LRFD Specifications. The slab bridges parameters listed in Table 6 are selected to cover the applicable range of application.

Table 6. Post-tensioned slab structure dimensions

	CPS1	CPS2	Units
No. of spans	3	3	
Span length (max)	40.0	70.0	ft.
Depth	12	24	in.
f'c	4.0	4.0	ksi

- *Cast-in-place Box Girder (Caltrans type)*

For cast-in-place box girder bridges, Caltrans standards are used to determine the concrete dimensions for girder spacing and slab thicknesses. The upper and lower bound ranges are shown in the tables. It should be noted that multiple span bridge will be used to develop design forces for comparison and design of minimum flexural reinforcement. Symbols used to illustrate the dimensions listed in the Tables 7 and 8 are shown in Figure 16.

Table 7. Reinforced concrete box girder structure dimensions

	BRC1	BRC2	Units
No. of spans	3	3	
Span length (max)	60.0	120.0	ft.
Depth	3.2	6.6	ft.
S_G	6.4	13.0	ft.
T_d	7.1	9.5	in.
T_s	6.0	9.3	in.
T_w	8	8	in.
f'_c	3.6	3.6	ksi

Table 8. Prestressed concrete box girder structure dimensions

	BPT1	BPT2	Units
No. of spans	3	3	
Span length (max)	80	250	ft.
Depth	3.2	10	ft.
S_G	6.4	20	ft.
T_d	7.0	10.1	in.
T_s	6.0	10.1	in.
T_w	12	12	in.
f'_c	4.0	4.0	ksi

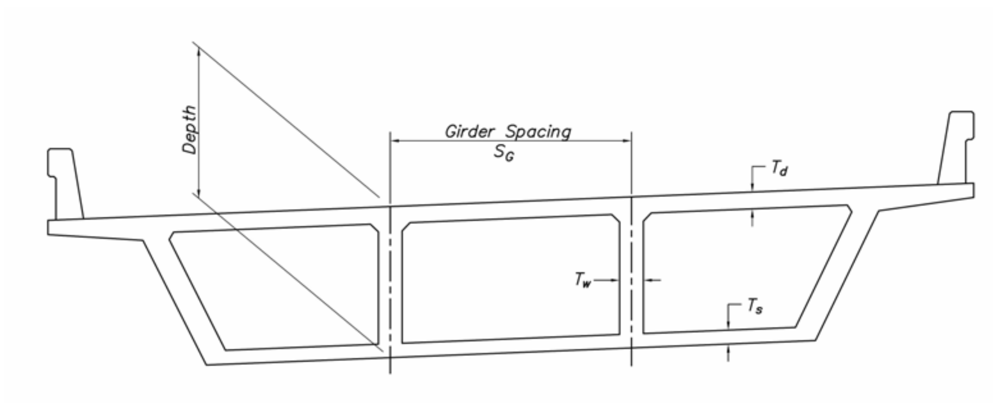


Figure 16. Dimension callouts for cast-in-place box girder examples

- *Cap Beams*

Cantilever cap-beams tend to be heavily reinforced, and as a result, minimum flexural reinforcement provisions rarely control. For integral bridges in California, an additional two-foot of width is required to confine the cap-beam to column joint region. In addition, the flange is also effective prior to cracking, and should be included in calculating the cracking moment. Therefore, minimum reinforcement requirements could control the flexural design of cap-beams in this region. Dimensions of the cap-beam studied are listed in Table 9.

Table 9. Cap beam dimensions

	CAP1	CAP2	Units
No. of spans	1	1	
Span length (max)	20	40	ft.
Depth	4.0	10.0	ft.
Width	6.0	10.0	ft.
Top flange	7.5	10.0	in.
Bottom flange	12	12	in.
f'_c	4.0	4.0	ksi

- *Footings*

Minimum flexural reinforcement provisions typically do not control because these elements are never deepened to meet requirements for architectural purposes. However, footings are covered in the parametric study and the dimensions for these footings are listed in Table 10.

Table 10. Footing dimensions

	F1	F2	Units
Width	14	30	ft.
Depth	5.0	10.0	ft.
Pile/Spread	Spread	Piles	
f'c	4.0	4.0	ksi

3.1.1.2 Precast/Prestressed Concrete

- *I-Girder*

These bridges represented in this portion of the database are intended to represent the full range of precast-pretensioned I-Girder bridge spans. Two separate shapes are utilized, and include an AASHTO beam and an NU Beam.

Table 11. Precast prestressed I-girder dimensions

	PCI1	PCI2	Units
No. of spans	3	3	
Span length (max)	70	200	ft.
Girder depth	3.0	8.0	ft.
Girder spacing	6.0	10.0	ft.
Deck thickness	7.5	8.5	in.
f'c girder	5.0	10.0	ksi
f'c deck	4.0	5.5	ksi

- *Bathtub, U-Beam*

The weight of these units limits the length that can be hauled to the site at about 140 feet. Beyond these limits, splicing is required either continuously over the cap beam or within each span with post-tensioning.

Table 12. Precast prestressed U-beam dimensions

	PUB1	PUB2	Units
No. of spans	3	3	
Span length (max)	80	200	ft.
Girder depth	3.0	8.0	ft.
Girder spacing	10.0	16.0	ft.
Deck thickness	7.0	7.0	in.
f'c girder	7.0	10.0	ksi
f'c deck	4.0	5.5	ksi

- *Box-Beam*

The box-beam in this application includes a five inch thick pour-in-place topping, which provides a uniform driving surface and allows for continuity reinforcement to be placed across continuous supports. This system has been used effectively for bridges with limited temporary and permanent vertical clearance constraints. However, the section is relatively heavy, which makes this section uneconomic for longer spans.

Table 13. Precast prestressed box-beam dimensions

	PBB1	PBB2	Units
No. of spans	3	3	
Span length (max)	60	120	ft.
Girder depth	27	47	in.
Girder width	48	36	in.
CIP topping	5.0	5.0	in.
f'c girder	5.0	5.0	ksi
f'c deck	4.0	4.0	ksi

- *Precast Slabs*

Precast prestressed concrete deck slabs are utilized without a cast-in-place concrete topping, and asphalt is applied to provide the uniform driving surface. It should be noted that the topping can be cast-in-place concrete and the design procedure, as related to minimum reinforcement provisions would be similar to the box beam described in this appendix. Design charts for this bridge type are listed in Caltrans, Bridge Design Aids. (Caltrans, 1989)

Table 14. Precast prestressed slab dimensions

	PPS1	PPS2	Units
No. of spans	3	3	
Span length (max)	20	48	ft.
Girder depth	12	21.5	in.
Girder width	48	36	in.
f'c girder	4.0	4.0	ksi

3.1.1.3 Segmental Concrete Bridges

The segmental concrete bridge shapes used for this project are from the AASHTO-PCI-ASBI Segmental Box Girder Standards (ASBI, 2000), and represent the upper and lower bounds of girder dimensions provided in these standards. Overall dimensions for these sections are listed in Tables 15 and 16.

Table 15. Span-by-span segmental bridge girder dimensions

	SBS1	SBS2	Units
No. of spans	1	1	
Span length (max)	100	150	ft.
Girder depth	6.0	8.0	ft.
Girder width	28.0	45.0	ft
f'c girder	7.0	7.0	ksi

Table 16. Balanced cantilever bridge girder dimensions

	SBC1	SBC2	Units
No. of spans	3	3	
Span length (max)	100	200	ft.
Girder depth	6.0	10.0	ft.
Girder width	28.0	45.0	ft
f'c girder	7.0	7.0	ksi

3.1.2 Minimum Flexural Reinforcement

Minimum flexural reinforcement was calculated using the candidate provisions for all members in the concrete database. The following is a discussion on the interpretation and assumptions used to determine the minimum reinforcement. This is particularly true for prestress concrete members because the cracking moment is directly related to the cracking moment and the post-cracking resistance.

3.1.2.1 Reinforced Concrete Members

In reinforced concrete members, MFR is calculated directly without iteration for all of the candidate provisions. For rectangular beams and slabs, the calculations can be simplified for direct comparison. This comparison is illustrated in Figure 16.

In developing this figure, the following assumptions were made:

1. The section is rectangular with A615 Grade 60 reinforcement.
2. The effective depth d is assumed to be 0.9H, where H is the overall depth of the member.
3. In calculating the moment capacity ($\phi M_n = \phi A_s f_y(d-a/2)$), a/2 is equal to 0.05d.
4. The strength reduction factor $\phi = 0.9$. It should be noted that the Eurocode does not include a resistance factor. Instead, the material strengths are factored.

5. For the Modified LRFD method, the ratio of the maximum strength to the nominal strength $M_o/M_n = f_u/f_y = 1/\gamma_3 = 1.5$.

As shown in Figure 17, the Modified and Eurocode methods provide essentially equivalent levels of reinforcement. However, the two methods diverge beyond f'c = 12 ksi. Also, the form of the equation for minimum reinforcement is essentially the same for all with exception to the Eurocode because of the different relation for the cracking stress. With these simplifying assumptions, Table 19 compares the methods directly.

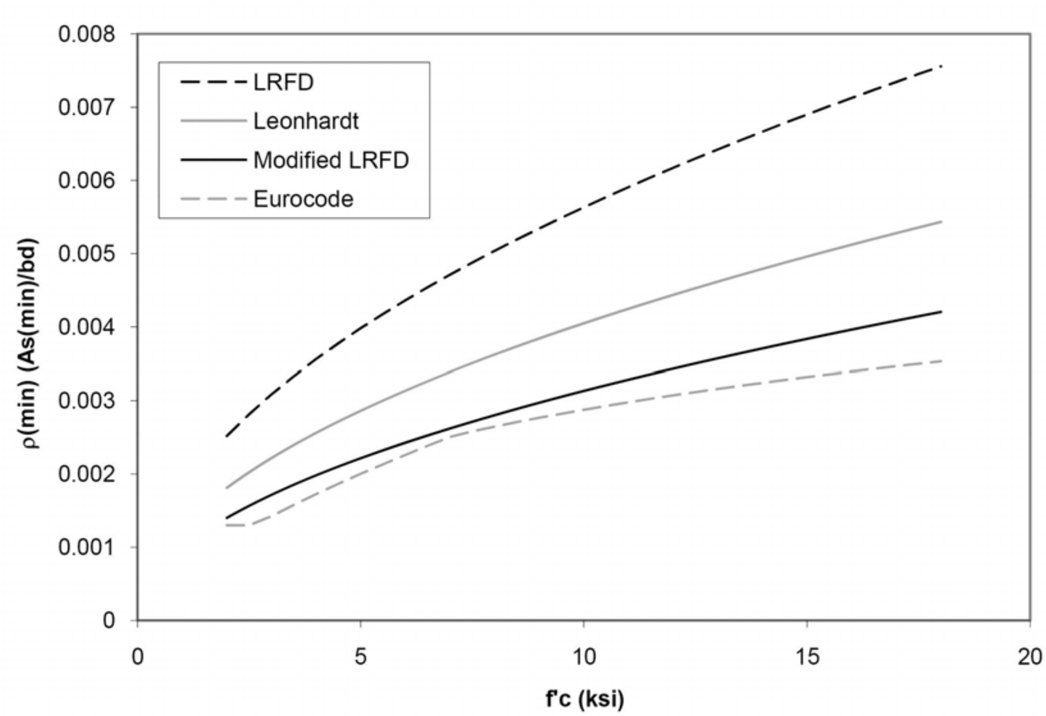


Figure 17. Minimum reinforcement requirements for rectangular reinforced concrete members

For non-rectangular sections, these simplifying assumptions are no longer valid because each method requires a different approach. Both the LRFD and the Modified LRFD provisions are based on calculating the cracking stress using the section modulus of the entire section. In the Leonhardt method, an equivalent cracking force of the section below the neutral axis is resisted by reinforcement. For members with flanges in tension, this is easily calculated by computing the average stress in flange and in the web and multiplying this by the respective area. The Eurocode has a similar approach to the Leonhardt minimum reinforcement provisions, where minimum reinforcement is based on the area of concrete below the neutral axis. However, Eurocode minimum reinforcement provisions further simplify the design computation by specifying uniform stress over this area.

Table 17. Minimum reinforcement ratios for rectangular reinforced concrete sections

Method	$\rho_{(min)}\ f_y/\sqrt{f'_c}$
LRFD	3.38
Leonhardt	2.43
Modified	1.88

3.1.2.2 Prestressed Concrete Members

The difficulty in calculating minimum reinforcement provisions for prestress sections is that the cracking moment and the subsequent post-cracking resistance for the LRFD and Modified methods are dependent on the amount of prestress. Therefore, these MFR methods require iteration.

The following assumptions were used to calculate MFR for all methods.

1. The cracking moment includes the use of composite, transformed section properties and a cracking stress of $7.5\sqrt{f'c}$ (psi).
2. The cracking moment and minimum prestress was determined based on iteration with an assumed prestress loss of 30 ksi to account for anchor set, friction, and long-term prestress losses.
3. For composite sections, an assumed non-composite moment of zero was used. Since the parametric study is intended to represent a wide range of prestressed sections, the zero moment assumption is considered conservative.
4. Nominal moment at overstrength M_o includes strain hardening of the reinforcement corresponding to either rupture of the prestress strand ($\varepsilon_{su} = 0.04$), or a peak compressive strain of 0.003.
5. Sections were analyzed under positive bending only, since negative bending regions will crack prior to positive bending regions in most continuous spans, which allows for redistribution of load. With exception to spans with hinges and cantilevered bridges during construction, negative bending regions are not critical for minimum reinforcement.
6. Compression-controlled or transition sections are not considered as part of this study. Examples of sections that are compression-controlled and do not meet the minimum reinforcement requirements are negative bending regions of continually prestressed bridges with relatively wide top flanges. Sections in this category require greater strength with the variable Resistance Factor (ϕ) to account for reduced ductility. Therefore, an additional factor-of-safety is not required.

7. For the Modified provisions, the minimum reinforcement is calculated based on the following: γ_1 = 1.6 (assuming that $f_{cr} = 7.5\sqrt{f'_c}$ (psi) for the cracking stress) and γ_1 = 1.1 for the effective prestressing. Development of these factors is presented in Section 2.4.7.

3.1.2.2.1 Internal (Bonded) Prestress Concrete Members

The iterative process of calculating MFR is illustrated in Figure 18, for one web and contributory flange of a cast-in-place post-tensioned box girder structure with bonded tendons. In this diagram, the moment capacity ϕM_n is plotted with the required flexural strength using the LRFD and the Modified methods as a function of the area of prestress strand A_{ps}. The minimum reinforcement is found at the intersection of the moment capacity curve and the required respective strength. As shown, the LRFD and Modified methods appear to be similar. However, the amount of minimum reinforcement using the Modified procedure (2.7 sq. in.) is substantially less than the LRFD procedure (4.2 sq. in).

It should be noted that both the Leonhardt and Eurocode procedures do not require iteration. Both methods compare the reserve strength $(f_{su} - f_{ps})A_{ps}$ in the tendon to the cracking strength of the concrete without prestress. Therefore, the calculations are simplified, and the concern over lack-of-convergence is eliminated.

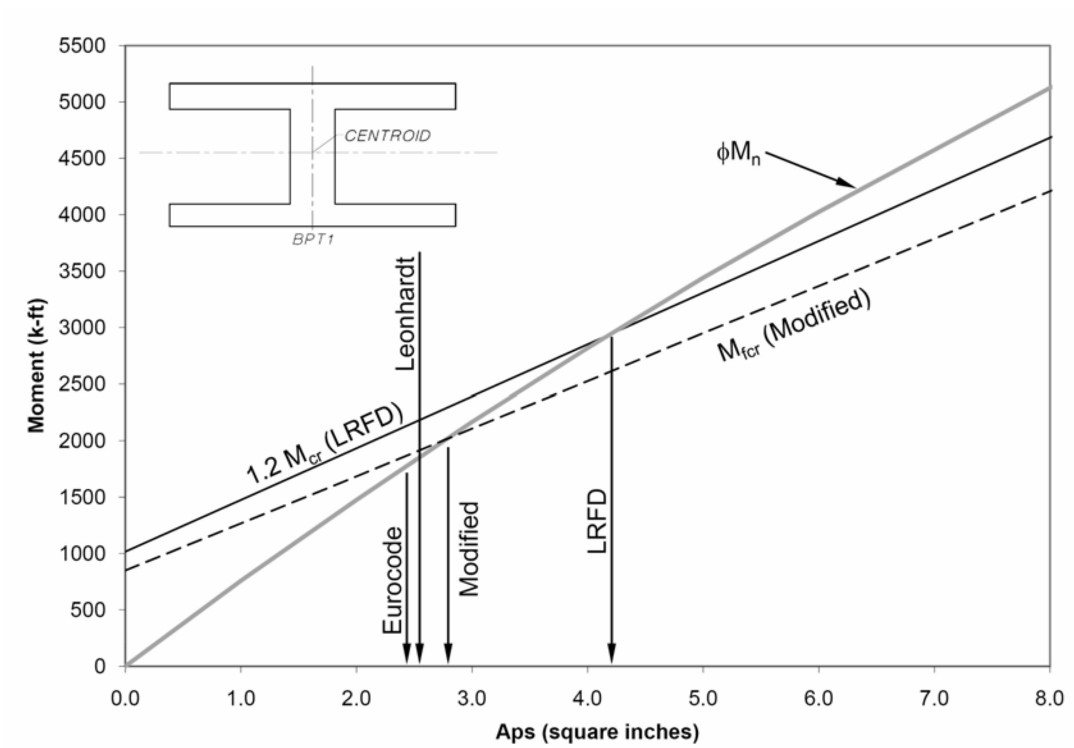


Figure 18. Minimum reinforcement provisions for a prestressed concrete box girder BPT1

3.1.2.2.2 (External) Unbonded Prestress Concrete Members

Sections SBS1 and SBS2 feature unbonded tendons, and the moment capacity is based on the AASHTO equations for tendon prestress at ultimate. Since the unbonded tendon stress cannot be determined from a section analysis, either detailed finite-element analysis, or approximate code equations are required. It has been shown that the AASHTO LRFD equations can adequately approximate the tendon stress at ultimate. Therefore, these equations were used in calculating minimum reinforcement.

As shown in Figure 19, the moment capacity does not intersect the 1.2 M_{cr} line (as required by the LRFD minimum reinforcement provisions), making it impossible to satisfy without the addition of bonded reinforcement. The capacity line crosses the strength requirements for the Modified procedure. However, the required strength is not exceeded by a significant margin, which indicates convergence may not be found on other members. Based on a low probability of achieving cracking strengths exceeding 7.5 $\sqrt{f'c}$ (psi) at segment joints, a reduced $\gamma_1 = 1.2$, as discussed in Section 2.4.7 for precast segmental members, is justified.

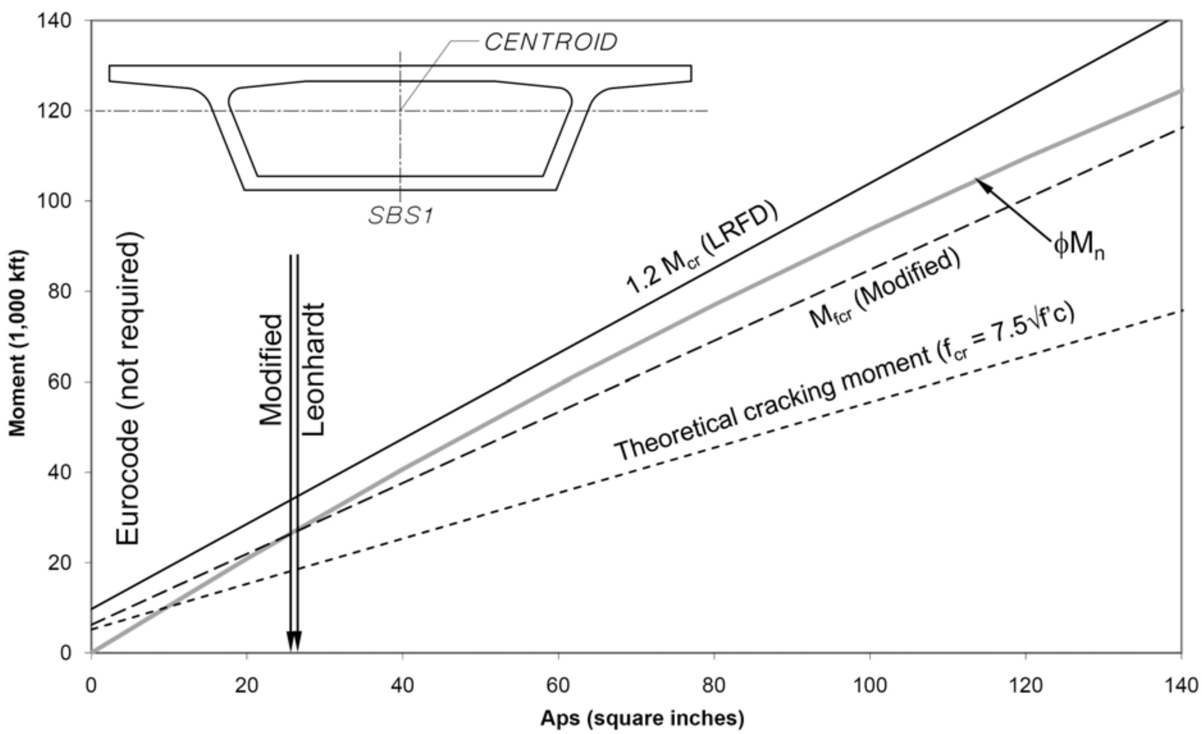


Figure 19. Minimum reinforcement provisions for an unbonded prestressed concrete box girder SBS1

For the Leonhardt method, the amount of prestress is calculated directly, and the strength at ultimate of the tendon is calculated using the AASHTO LRFD equations. The total area of reinforcement is less than half required using the Modified procedure. For comparison purposes, the theoretical cracking moment is plotted in Figure 19, assuming a cracking strength of 7.5 $\sqrt{f'c}$ (psi).

3.1.3 Cracking Moment (M_{cr})

The cracking moment M_{cr} is based on the computed minimum amount of reinforcement (or prestress) and the cracking stress of $7.5\sqrt{f'c}$ (psi). Flexural cracking stress is related to the compressive strength to the one-half power, and the member depth. Figure 9 shows the member flexural strength as a function of member depth based on test results of a wide range of concrete member sizes. As shown in this plot, the assumptions in the Modified and LRFD methods are conservative, especially for members exceeding 3 ft in depth. Introducing another variable as part of the minimum reinforcement calculations is feasible. However, considering the limited number of data points and wide scatter of measured cracking strengths of large-scale specimen, including depth as an additional variable is not justified. In addition to those stated previously in determining the minimum flexural reinforcement, prestress losses of 30 ksi includes all long term and instantaneous losses.

3.1.4 Nominal Moment at Overstrength (M_o)

With minimum amount of reinforcement or prestress, a strain compatibility analysis was performed to evaluate the flexural strength of the concrete member. This analysis was used in-lieu of code equations because the strength is evaluated with the effects of strain hardening. The moment-curvature analysis using strain compatibility was performed using the analysis program “Response 2000” (Bentz, 2005). The moment corresponding to a peak compressive strain of 0.003, or a peak reinforcement strain of 0.15 is M_o for reinforced concrete members. For bonded prestressed concrete, M_o is the moment corresponding to a peak concrete compressive strain of 0.003, or a tension strain in the prestress tendon of 0.04. The M_o for unbonded members corresponds to the tendon force prescribed in the AASHTO LRFD.

3.1.5 Parametric Study Results

The data and calculations developed as part of the parametric study are listed in Appendix A. This includes a ratio of M_o/M_{cr}, which is an indicator of the level of safety provided by each of the methods for each concrete member. A description of the data in the tables is defined under the following headings:

- **Section -** concrete structure from the database listed in the interim report
- **Method** - candidate MFR provision
- **Required Area of Steel -** area of flexural reinforcement required to meet the respective MFR provision
- **M_{cr}** - theoretical cracking moment based on an assumed cracking stress of $7.5\sqrt{f'c}$ (psi)

- **M_o** – nominal moment at overstrength including the effects of strain hardening, as illustrated in Figure 1.
- **M_o/M_{cr}** - ratio of the nominal moment at overstrength to the cracking moment

This last term is the effective factor of safety or brittleness ratio. This brittleness ratio allows for evaluation of the minimum reinforcement methods regarding safety, ease-of-use and economy. The average brittleness ratio (M_o/M_{cr}) results of the 26 concrete members evaluated are plotted in Figure 20. Assuming normal distribution, the Standard Deviation and Coefficient of Variation (COV), which is the ratio of the standard deviation and the average values, are presented in Table 20.

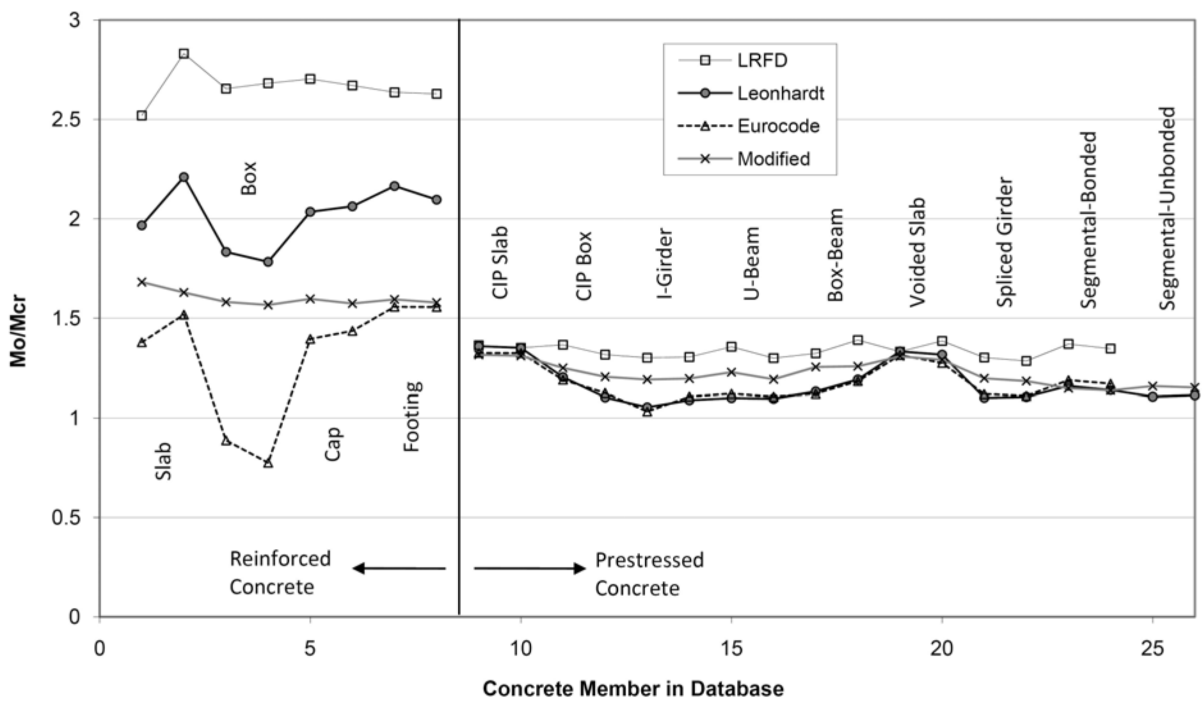


Figure 20. Parametric Study Mo/Mcr Ratios

Reinforced concrete members (total 8) and prestressed concrete members (total 18) are separated in Table 18 to show the variation within each classification. The LRFD method has the highest average M_o/M_n ratio of all the candidate provisions. Within each of the two categories, there is relatively low variation in the results. Most of the variation is a result of the definition of the ultimate strength between prestressed and reinforced concrete, as illustrated in Figure 20. For lightly reinforced concrete members, the ultimate strength of the reinforcement can be achieved prior to reaching the crushing strength of the concrete. Therefore, unless the section is an inverted tee, substantial reserve strength exists in reinforced concrete. By contrast, the nominal strength is by definition based on the tendon strength at ultimate in prestressed concrete members. Therefore, no reserve strength is available beyond the nominal capacity. In this regard, the LRFD method does not provide a consistent level of safety.

Table 18. Combined brittleness ratio M_o/M_{cr} statistical results

	LRFD	Leonhardt	Eurocode	Modified
Reinforced Concrete				
Average	2.67	2.02	1.31	1.60
Standard Deviation	0.09	0.15	0.31	0.04
COV	0.03	0.07	0.23	0.02
Prestressed Concrete*				
Average	1.33	1.18	1.17	1.24
Standard Deviation	0.03	0.11	0.10	0.05
COV	0.03	0.09	0.08	0.04
Combined*				
Average	1.82	1.49	1.23	1.37
Standard Deviation	0.66	0.43	0.20	0.18
COV	0.36	0.29	0.17	0.13

*Excludes Segmental Data

The results listed in Table 18, reveal similar results for the Leonhardt and Eurocode methods. This similarity indicates that the differences in reserve safety provided by the nominal strength of reinforcement have been taken into account. While the Eurocode provides separate methods for prestressed and reinforced concrete members, the method is unconservative for box girder sections. The Modified LRFD method provides a unified approach with the most consistent level of safety of all methods investigated. Data from precast segmental girders is excluded from the statistical analysis because, Eurocode does not require minimum reinforcement for this type of girder, and the convergence was not obtained for the provisions specified in the LRFD specifications.

3.1.6 Recommendations

Based on the results of the parametric study and related documentation, it is recommended to change the minimum reinforcement provisions in the LRFD specifications to the Modified LRFD method, as discussed in Section 2.4.7, and in following section. This change is recommended because the Modified LRFD method provides a consistent level of safety for all components in the database of concrete structures. This consistency is largely due to the recognition that the maximum strength including the effects of strain hardening should be considered when evaluating minimum reinforcement. Also, each component of the minimum reinforcement evaluation is factored appropriately, resulting in uniform reliability in achieving resistance against brittle failure. Finally, the Modified LRFD method offers economy, where compression-controlled and transition-region sections are not subject to minimum reinforcement requirements.

3.2 PROPOSED REVISIONS TO THE AASHTO LRFD SPECIFICATIONS

The following are the changes the project team recommends regarding minimum flexural reinforcement provisions in the LRFD specifications. As shown, the recommended code changes are presented first, and then, changes to the commentary are provided.

Deletions are shown as a single ~~strikethrough~~.

Additions are shown as underlined.

LRFD - 5.4.2.6 Modulus of Rupture

Unless determined by physical tests, the modulus of rupture, f_r, in ksi, for specified concrete strengths up to 15.0 ksi, may be taken as:

- For normal weight concrete: $0.24\sqrt{f'_c}$
- ~~When used to calculate the cracking moment of a member in Articles 5.7.3.4 and 5.7.3.6.2 $0.24\sqrt{f'_c}$~~
  - ~~When used to calculate the cracking moment of a member in Article 5.7.3.3.2 $0.37\sqrt{f'_c}$~~
- For lightweight concrete:
- For sand-lightweight concrete $0.20\sqrt{f'_c}$
- For all lightweight concrete $0.17\sqrt{f'_c}$

When physical tests are used to determine modulus of rupture, the tests shall be performed in accordance with AASHTO T97 and shall be performed on concrete using the same proportions and materials as specified for the structure.

LRFD - C5.4.2.6

~~Data show that most modulus of rupture are between $0.24\sqrt{f'_c}$ and $0.37\sqrt{f'_c}$ (ACI 1992, Walker and Bloem 1960; Khan, Cook and Mitchell 1996). It is appropriate to use the lower bound when considering service load cracking. The purpose of the minimum reinforcement in Article 5.7.3.3.2 is to assure that the nominal moment capacity of the member is at least 20 percent greater than the cracking moment. Since~~

the actual modulus of rupture could be as much as 50% greater than 0.24√f'c, the 20 percent margin of safety could be lost. Using an upper bound is more appropriate in this situation.

Most modulus of rupture test data on normal weight concrete is between 0.24√f'c and 0.37√f'c (ksi) (Walker and Bloem 1960; Khan, Cook and Mitchell 1996). A value of 0.37√f'c has been recommended for the prediction of the tensile strength of high-strength concrete (ACI 1992). However, the modulus of rupture is sensitive to curing methods, and nearly all of the test units in the dataset mentioned previously were moist cured until testing. Carrasquillo, et al. (1981), noted a 26-percent reduction in the 28-day modulus of rupture if high strength units were allowed to dry after 7-days of moist curing over units that were moist cured until testing.

The flexural cracking stress of concrete members has been shown to significantly reduce with increasing member depth. Shioya, et al. (1989) observed that the flexural cracking strength is proportional to $H^{-0.25}$, where H is the overall depth of the flexural member. Based on this observation, a 36.0 in. deep girder should achieve a flexural cracking stress that is 36 percent lower than a 6.0 in. deep modulus of rupture test specimen.

Since modulus of rupture units are either 4 or 6 inches deep and moist cured up to the time of testing, the modulus of rupture should be significantly greater than the flexural cracking strength of an average size bridge member composed of the same concrete. Therefore, 0.24√f'c is appropriate for checking minimum reinforcement in Section 5.7.3.3.2.

The properties of higher strength concretes are particularly sensitive to the constitutive materials. If test results are to be used in design, it is imperative that tests be made using concrete with not only the same mix proportions, but also the same materials as the concrete used in the structure.

The given values may be unconservative for tensile cracking caused by restrained shrinkage, anchor zone splitting and other tensile forces caused by effects other than flexure. The direct tensile strength stress should be used for these cases.

LRFD - 5.7.3.3.2 Minimum Reinforcement

Unless otherwise specified, at any section of a flexural component, the amount of prestressed and non-prestressed tensile reinforcement shall be adequate to develop a factored flexural resistance, at least equal to the lesser of

- The factored cracking moment ~~1.2 times the cracking moment~~, M_{fcr}, determined on the basis of elastic stress distribution and the modulus of rupture, f_r, of the concrete specified in Article 5.4.2.6, where M_{fcr} may be taken as:

~~$$M_{cr} = S_c(f_r + f_{cpe}) - M_{dnc}\left(\frac{S_c}{S_{nc}} - 1\right) \geq S_c f_r$$~~ ~~(5.7.3.3.2-1)~~

$$M_{fcr} = \gamma_3\left[(\gamma_1 f_r + \gamma_2 f_{cpe})S_c - M_{dnc}\left(\frac{S_c}{S_{nc}} - 1\right)\right] \quad (5.7.3.3.2\text{-}1)$$

where:

f_r = modulus of rupture of concrete specified in Article 5.4.2.6.

f_{cpe} = compressive stress in concrete due to effective prestress forces only (after allowance for all prestress losses) at extreme fiber of section where tensile stress is caused by externally applied loads (ksi).

M_{dnc} = total unfactored dead load moment acting on the monolithic or noncomposite section ~~(k-ft)~~ (k-in).

S_c = section modulus for the extreme fiber of the composite section where tensile stress is caused by externally applied loads (in^3).

S_{nc} = section modulus for the extreme fiber of the monolithic or non-composite section where tensile stress is caused by externally applied loads (in^3).

Appropriate values for M_{dnc} and S_{nc} shall be used for any intermediate composite sections. Where the beams are designated for the monolithic or noncomposite section to resist all loads, substitute S_{nc} for S_c in the above equation for the calculation of M_{cr}.

The following factors account for variability in the flexural cracking strength of concrete, variability of prestress and the ratio of nominal yield stress of reinforcement to ultimate:

γ_1 = 1.6 accounts for the variability of concrete flexure cracking, which can be reduced to 1.2 for precast segmental structures.

γ_2 = 1.1 accounts for the variability of prestress losses.

γ_3 = f_y/f_u (0.67 for A615 Grade 60 and 0.75 for A706 Grade 60 reinforcement). For prestressed concrete structures, use 1.0.

ϕ = 1.0 for prestress concrete and 0.9 for non-prestressed concrete regardless of net tensile strain for the purpose of specifying minimum reinforcement.

- 1.33 times the factored moment required by the applicable strength load combinations specified in Table 3.4.1-1.

The provisions for Article 5.10.8 shall apply.

If adequate ductility is provided in continuous spans to allow for moment redistribution per Article 5.7.3.5, then minimum reinforcement provisions of this article need not apply for negative bending.

LRFD - C5.7.3.3.2

Minimum reinforcement provisions are intended to reduce the probability of brittle failure, by providing flexural capacity greater than the cracking moment. Testing of a large number of lightly reinforced and prestressed concrete members at the University of Illinois demonstrated that significant inelastic displacements can be achieved, and none of the beams tested failed without large warning deflections (Freyermuth & Aalami 1997). If these experiments were conducted in load control, a number of specimens would have failed without warning because the ultimate strength (including the effects of strain hardening) was less than the cracking strength. Based on this observation, the ultimate strength should be used instead of the nominal strength as a true measure of brittle response. The ratio of steel stress at yield to ultimate (γ_3) sufficiently approximates the nominal to ultimate strength for lightly reinforced concrete members. Since the ultimate strength of a prestress tendon is utilized in the calculation of flexural capacity, $\gamma_3 = 1.0$ for internally prestressed concrete members.

The sources of variability in computing the cracking moment and resistance are appropriately factored (Holombo and Tadros, 2009). The factor applied to the modulus of rupture (γ_1) is greater than the factor applied to the amount of prestress (γ_2) to account for greater variability.

For precast segmental construction, cracking generally starts at the segment joints. Research at the University of California, San Diego has shown that flexure cracks occur adjacent to the epoxy-bonded match-cast face, where the accumulation of fines reduces the tensile strength (Megally et al., 2003). Based on this observation, a reduced γ_1 factor of 1.2 is justified. Experimental and analytical studies have shown external prestress tendons are essentially elastic at the ultimate limit state (Tassin & Dodson 1997). Therefore, an increase in prestress and associated cracking moment is offset by a corresponding increase in post-cracking strength. Since the variability of prestress essentially has no effect on minimum reinforcement, γ_2 can be reduced to 1.0 for prestress concrete members with external tendons.

Indeterminate structures typically have redundancy and ductility, inherent to lightly-reinforced concrete members, which allows redistribution of moments. Both the AASHTO LRFD and ACI 318 (2008) require a 50% increase in tension-controlled strain limit from 0.005 to 0.0075 in order to redistribute negative moments. Increasing the quantity of reinforcement to meet minimum reinforcement provisions can adversely affect this ductility. Minimum reinforcement provisions should be confined to positive bending regions if adequate ductility is demonstrated in the negative bending regions.

Specifying minimum reinforcement and reducing the resistance factor (ϕ) for sections that do not qualify as tensioned controlled, as defined in Section 5.5.4.2, accomplish the same objective, that is to provide additional strength to reduce the probability of brittle failure. Examples of sections that could be compression controlled and do not meet minimum reinforcement requirements include negative bending regions of continually prestressed bridges with relatively wide top flanges and inverted T beams. Applying minimum reinforcement provisions to compression-controlled or transition regions is redundant because additional strength is already required for the same deficiency of reduced ductility. Under these conditions, it is more logical to add reinforcement to the compression zone rather than it is to increase the amount tension reinforcement that would result in further reduced ductility.

3.3 DESIGN EXAMPLES

The design examples are intended to illustrate the use of minimum reinforcement provisions on common bridge types that are encountered in practice where minimum flexural reinforcement should be the controlling effect in the flexural design. This is commonly the case for multi-span bridges with widely varying span lengths, where the depth of the bridge is constant for the full length. With exception to the span-by-span segmental bridge example, all bridges have some form of continuity, which has been a particular challenge for design engineers to implement minimum flexural reinforcement provisions.

The following is a brief description of the design examples to be developed using the recommended minimum reinforcement provisions developed as part of the research.

3.3.1 Multi-Span Precast Concrete Girder Made Continuous with Composite Deck

This is one of the most common types of structures used for freeway bridges and overpasses. This three-span precast/prestressed girder example features a single long span in the middle along with two short side spans, as shown in Figure 21 and is the subject of Example B-1. A uniform depth is used to reduce set-up costs and improve aesthetics. It is intended that the side spans are short enough so the minimum flexural provisions control the design in the positive bending regions.

Seventy-two inch bulb-tee girders are featured in this example since the bottom flange tends to be relatively narrow, thus limiting the amount of rotational ductility that can be sustained in the negative bending region. Strength limit states were checked at 10^{th} points within each span in addition to minimum reinforcement provisions using the Modified LRFD method.

Minimum reinforcement controls the number of prestress strands at the point of maximum positive moment in the side spans. It should be noted that it is not necessary to increase the jacking force, and thus the cracking moment. In the negative bending region the net tensile strain exceeds 0.0075, and the minimum reinforcement check is not required. However, the amount of reinforcement provided must satisfy the flexural strength limit states at that section.

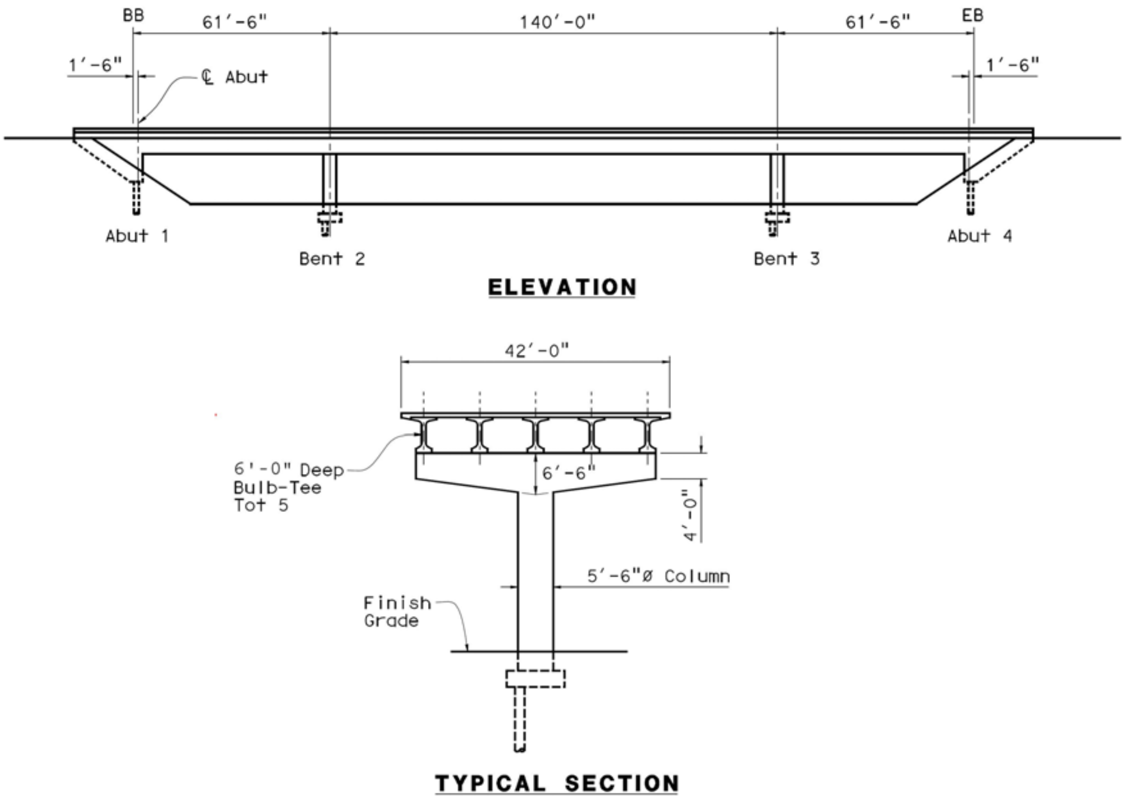


Figure 21. Precast concrete girder made continuous with composite deck example details

3.3.2 Cast-in-Place Concrete Box Girder

A three-span cast-in-place concrete box girder bridge that is commonly built in California and Nevada is the subject of this design example. As with the first example, the side spans are far shorter than the end spans while the depth of the bridge is constant along the entire length, as shown in Figure 22. Because the bridge is monolithic, the bridge resists all loading continuously including any prestress forces. All prestress consists of continuous post-tensioning that runs full length of the bridge. To control camber and reduce friction losses, the post-tensioning tendon midspan eccentricity is reduced in the shorter spans were flexural demands are reduced.

For this type of structure, it is more economical to design the post-tensioning cables for service loads, and add mild reinforcement in localized areas as needed to resist strength limit state loads including minimum reinforcement provisions (Caltrans, 1989). It is anticipated that minimum flexural reinforcement will control the design of this mild reinforcement in these side spans.

The bridge is 42.0 ft wide and 6.5 ft deep. The girders are spaced at 11.0 ft on center and are flared from 12 in. to 18 in. at the abutments and the bents. The soffit is flared to 12 in. at the bents. The columns are circular with a diameter of 6.0 ft. Caltrans has amended the LRFD specifications, so the allowable tension stress is limited to zero tension under permanent loads and $0.19\sqrt{f'c}$ (ksi) under the sum of the permanent and live loads. The jacking force is designed under the Service III limit state and is estimated with the software CT Bridge to be 6,200 kips.

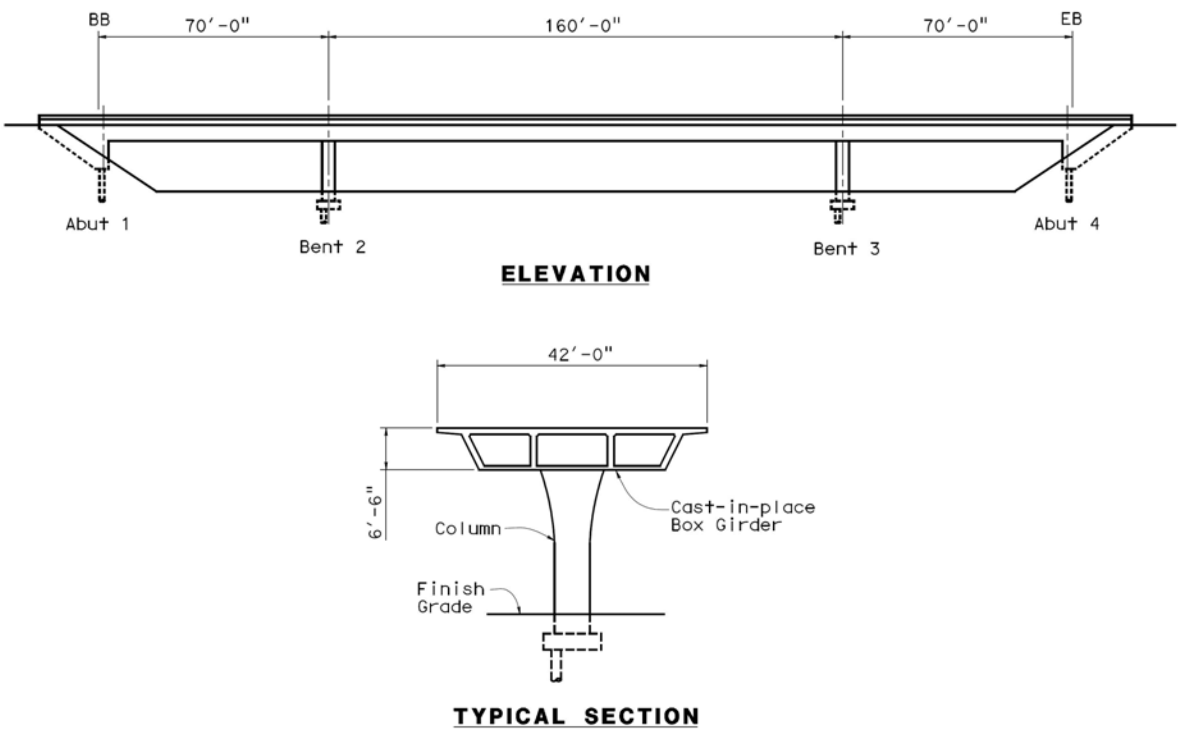


Figure 22. Cast-in-place box girder example details

Minimum flexural moments, Strength I moments, and nominal moment capacities (including the capacity of the post-tensioning tendons only) are plotted in Appendix B. The minimum flexural reinforcement is the controlling load case in Span 1 with the 1.33Mu controlling in the positive bending region. In the negative bending region, the net tensile strain is 0.015, which exceeds the requirement for redistribution. It should be noted that the negative bending capacity should be greater than the Strength limit states assuming full continuity.

3.3.3 Span-by-Span Segmental Bridge with External Tendons

A two-span precast segmental bridge is the subject of this design example. The bridge is built using the span-by-span construction method. The bridge chosen for this example is part of the I-4/Lee Roy Selmon Expressway in Tampa, FL. Each of the two spans in this bridge is simply supported. Only Span 2 of this bridge is the subject of this example. This represents a relatively large depth-to-span ratio bridge in which the minimum flexural reinforcement requirement could control the design. For Span 2, the cross section consists of a single-cell box section with long overhangs as shown in Figure 23. The length of this span is approximately 115'-6" and the bridge is prestressed by means of external unbonded tendons.

For precast segmental bridges with no bonded reinforcement or bonded tendons crossing the joints, no tensile stresses are allowed at all segment-to-segment joints under service loads. Longitudinal analysis and design of this bridge included concrete stresses under service loads, flexural capacity, shear capacity, principal stresses in the box girder webs and minimum flexural reinforcement requirements. Except for the minimum flexural reinforcement requirement, design is satisfactory with the use of four external tendons on each side of the box section; three of these tendons are composed of 19-0.6" ϕ strands, and the fourth tendon is composed of 15-0.6" ϕ strands. Thus, the total number of external unbonded strands in this bridge is 144-0.6" ϕ strands.

Figure 24 shows the bending moments along the length of the single span bridge (Span 2). The figure shows the minimum design moments due to cracking according to the current LRFD specifications and based on the proposed method (Modified LRFD). It is clear that the proposed provisions significantly reduce the minimum required design per the LRFD specifications. The figure also indicates in the middle third of the span length, the $1.33M_u$ controls over the $1.20M_{cr}$ (AASHTO LRFD Specifications) or the cracking moment based on the proposed modified LRFD method. Thus, $1.33M_u$ controls the MFR in this case. Figure 24 also shows the factored flexural moment capacity, which is higher than the ultimate moment, M_u, at all sections. However, in the middle 80 ft of the span length, the minimum flexural reinforcement requirement is not satisfied and the prestressing is controlled by the minimum reinforcement requirement. It should be noted that depth of the box girder is 9 ft, whereas the span length is about 115 ft only. Thus, the superstructure is relatively deep, which results in flexural design controlled by minimum reinforcement requirements. To satisfy minimum reinforcement, as specified in the Modified LRFD method, a total of 160 strands are required, in contrast to the 144 strands required to satisfy all other strength limit states.

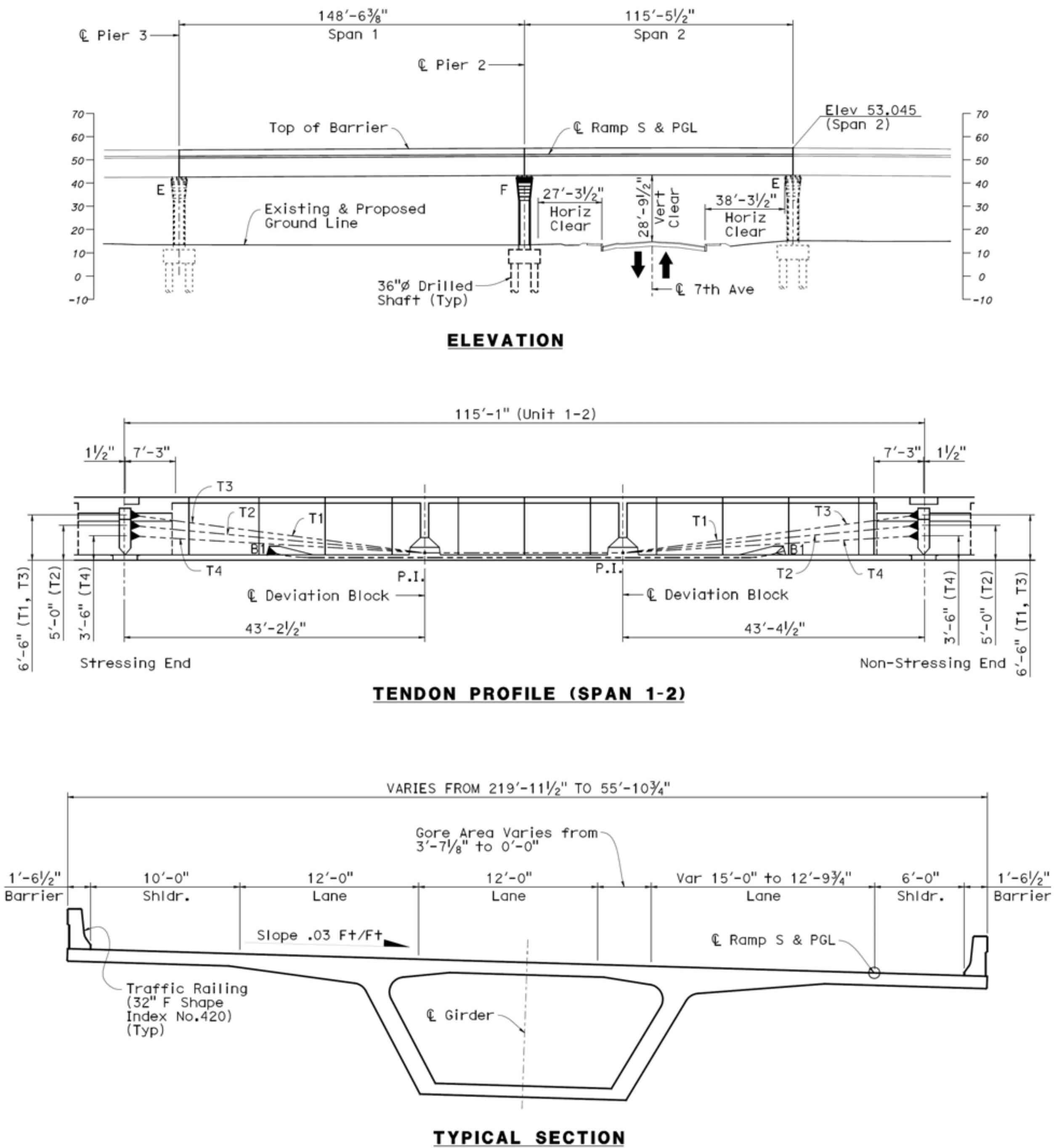


Figure 23. Span-by-span precast segmental bridge example details

Analysis of this bridge was done using LARSA 4D. Construction stages and time-dependent effects were considered in the analysis. Hand calculations for the midspan section at of the bridge are included in Appendix B.

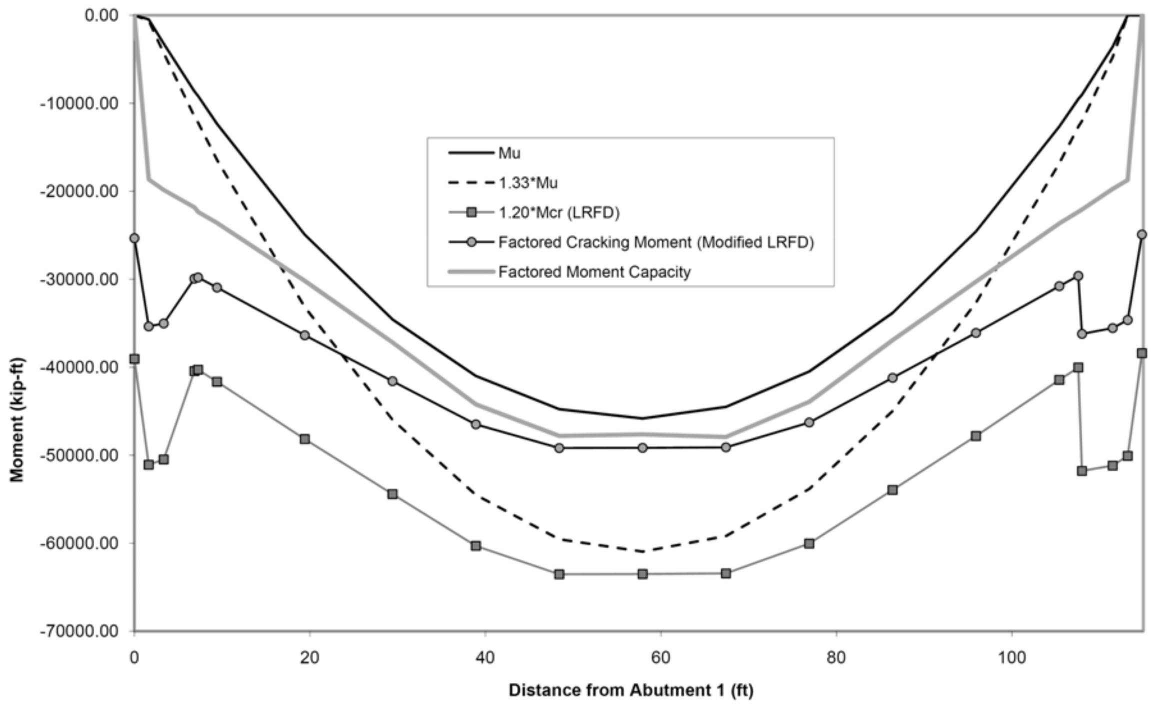


Figure 24. Span-by-span precast segmental bridge strength moment profiles

3.3.4 Balanced Cantilever Bridge with Internal Tendons

A four-span precast segmental bridge using the cantilever construction method is the subject of this design example. It is part of the I-4/Lee Roy Selmon Expressway in Tampa, FL. Elevation view of the bridge is shown in Figure 25. The approximate lengths of spans are 147'-3", 186'-1", 186'-9" and 145'-6" for Spans 1 through 4, respectively, with a total bridge length of 665'-7". The cross section consists of the single-cell box section shown in Figure 25. The deck width is 30'-1" and is constant along the entire length of the bridge.

For precast segmental bridges, no tensile stresses are allowed at all segment-to-segment joints under service loads. Longitudinal analysis and design of this bridge included concrete stresses under service loads, flexural capacity, shear capacity, principal stresses in the box girder webs and minimum flexural reinforcement requirements. At the first segment-to-segment joint next to Pier 8-3 in Span 4 (most critical section for negative moment), there are a total of 254-0.6" ϕ internal (bonded) strands and 114-0.6" ϕ unbonded strands (external tendons). In the positive moment region in Span 4 (most critical section for positive moment), the only prestressing is provided by the continuity external tendons and total number of strands is 114.

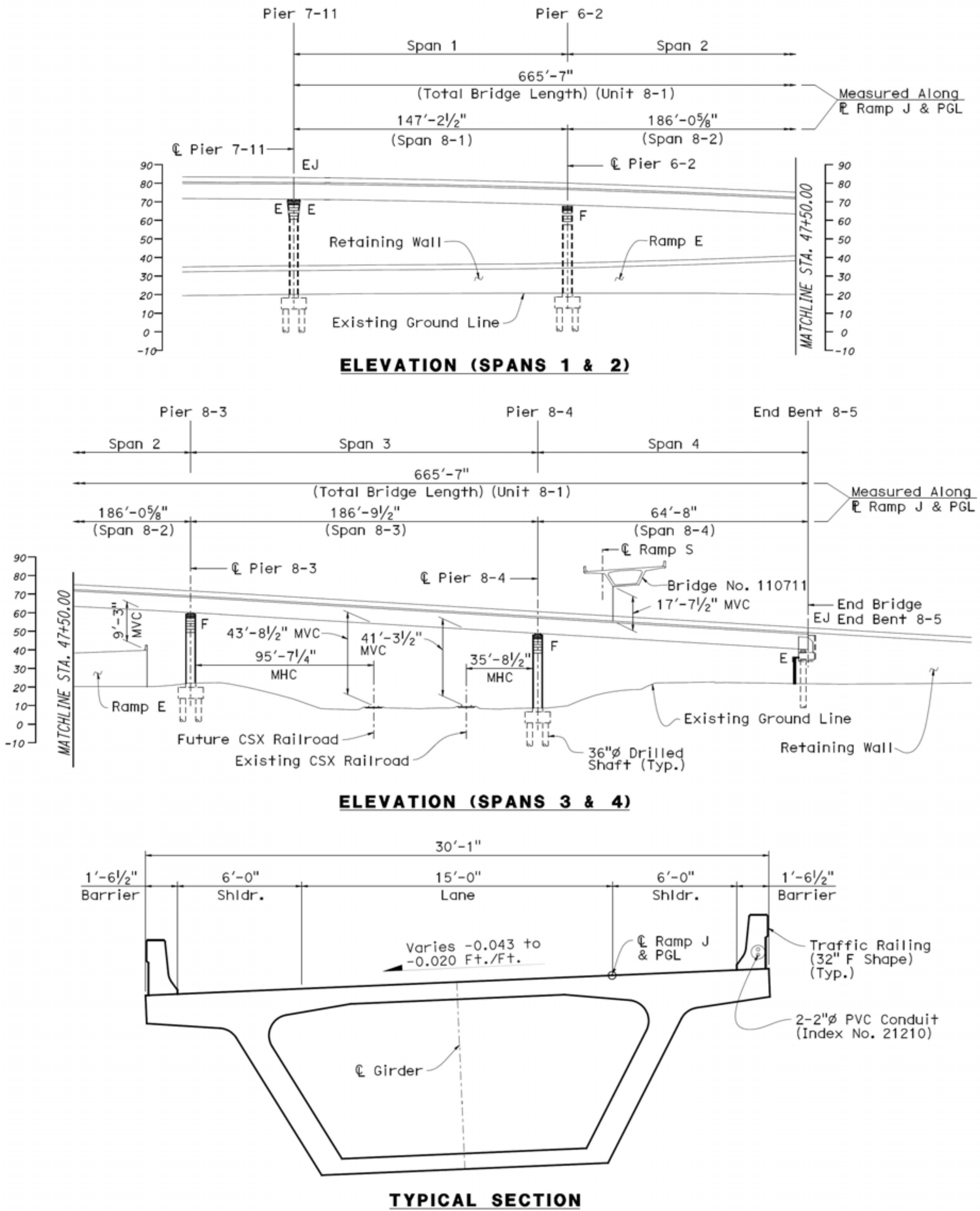


Figure 25. Balanced cantilever precast segmental bridge example details

The bridge is almost symmetric about centerline of Pier 8-3. Moment demand and capacity profiles are shown in Appendix B. These profiles show the factored cracking moments according to the current AASHTO LRFD Specifications and based on the proposed (Modified LRFD) method. It is clear that the proposed provisions considerably reduce the minimum required design moments (MFR). In this example, the conditions for minimum reinforcement have not been met. Therefore, negative bending regions are required to meet minimum reinforcement requirements. $1.33M_u$ controls over the $1.20M_{cr}$ (AASHTOLRFD Specifications) or the cracking moment based on the proposed Modified LRFD method.

Analysis of this bridge was done using LARSA 4D. Construction stages and time-dependent effects were considered in the analysis. Calculations for the section at first segment-to-segment joint in Span 4 (joint at Pier 8-4) as well as maximum positive moments are shown in detail in Appendix B.

3.3.5 Cap Beam

The cap beam selected for this example is typical for precast and steel girder type bridges, as shown in Figure 26. Rectangular in cross-section, the cantilever portions of the cap are tapered. The center span is approximately twice the length of the cantilevers.

Based on the design configuration, $1.33M_u$ controls the center span, while the cantilever spans are controlled by the Modified LRFD cracking moment demands. Moment demands and capacities are plotted in Appendix B.

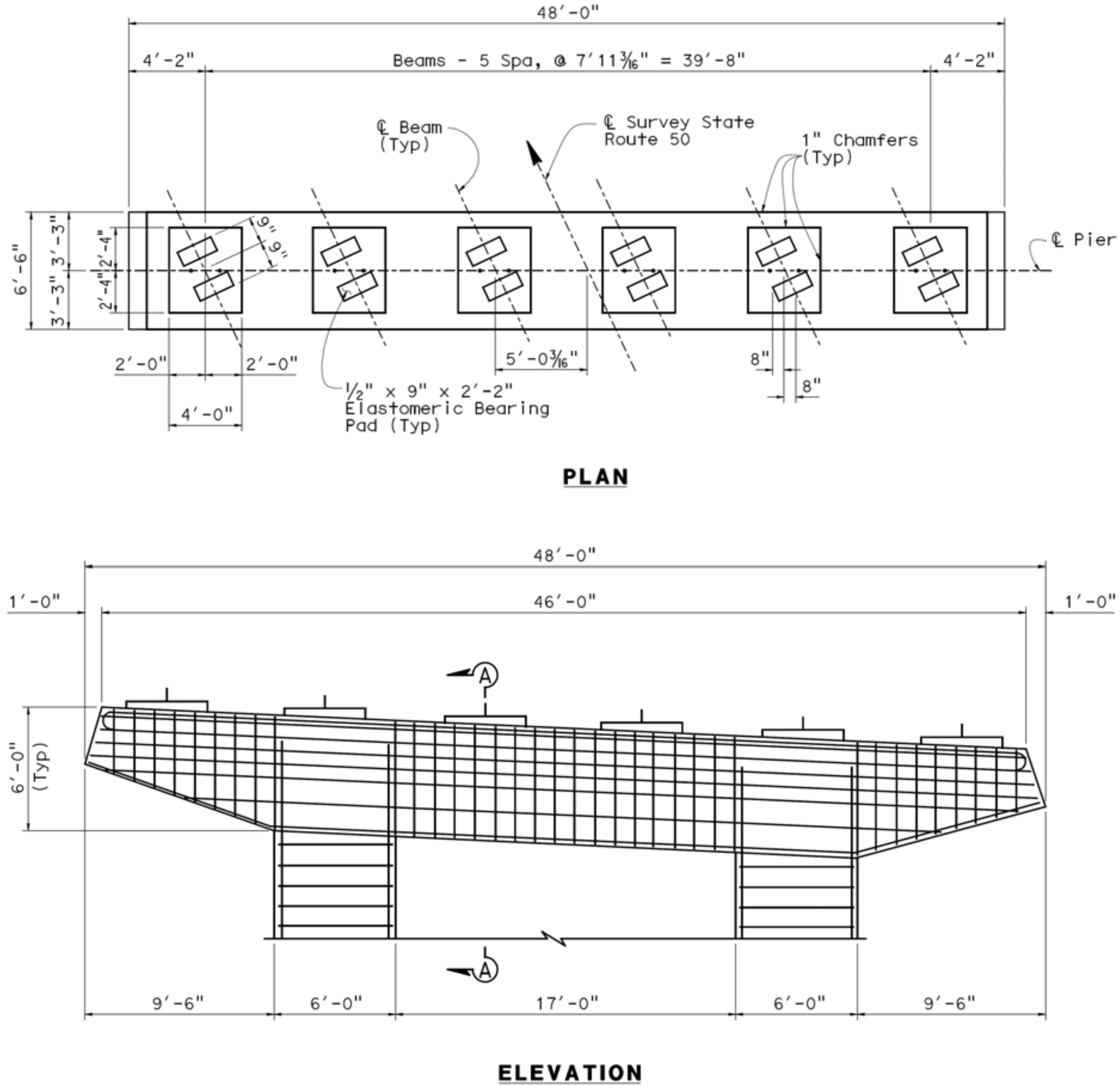


Figure 26. Concrete cap beam example details

CHAPTER 4 CONCLUSIONS AND SUGGESTED RESEARCH

4.1 CONCLUSIONS

Recommended revisions to the LRFD specifications and commentary are proposed for the rational design of minimum reinforcement to prevent brittle failure of concrete members. These revisions are based on the research provided in this report including the observed response of lightly reinforced and prestressed concrete members, and review of methods specified in US and international codes. A parametric study conducted on four representative methods of determining minimum reinforcement demonstrates that recommended revisions provide an appropriate and a consistent level of safety for all structure types and materials covered by the specifications.

4.1.1 Conclusions on the Observed Response of Lightly Reinforced and Prestressed Concrete Members

Based on the observed response of lightly reinforced and prestressed concrete members, the following conclusions are made:

1. Lightly reinforced and prestressed concrete members can develop the full nominal moment after cracking has occurred. Beams tested at the University of Illinois included reinforced concrete, prestressed concrete and externally prestressed concrete members suggest that the full nominal moment capacity is developed after cracking has occurred. An exception could be inverted t-beam structures with relatively wide bottom flanges that can be categorized as both over-reinforced and under-reinforced.

2. Lightly reinforced and prestressed concrete members exhibit ductile response when subject to displacement-controlled load regimes. In the post-cracked state, the neutral axis is relatively close to the extreme compression fiber, and, therefore, significant inelastic rotation is required before crushing strains develop. Although a single crack implies large inelastic strains in the reinforcement, these strains spread into the concrete member adjacent to the crack through localized debonding, also referred to as strain penetration.

3. Flexural cracking strength of the concrete is dependent on many variables including, curing methods, aggregates, compressive strength and the overall member size. Modulus of rupture of high strength concrete units is shown to be particularly sensitive to curing methods.

4. Modulus-of-rupture test units are 4 to 6 inches deep and moist cured until testing. Since curing has a significant influence on the cracking strength of concrete, and it has been repeatedly shown that the deep members crack at a lower stress than their small-scale counterparts, modulus of rupture is an inaccurate representation of the flexural cracking strength of concrete members.
5. Precast segmental bridges exhibit lower flexural cracking strength than conventional concrete structures, as discussed in Section 2.1. Flexural cracks in these structures are typically initiated immediately adjacent to the match-cast joint where an accumulation of fines and course aggregate reduce the tensile strength.
6. Reinforcement of lightly reinforced concrete members typically exhibit strains well into the strain hardening region and the maximum strength is defined by the ultimate strength rather than yield.

4.1.2 Conclusions on the Review of US and International Practice

Based on the review of US and International practice of specifying minimum reinforcement, the following conclusions are made:

The practice of specifying minimum reinforcement and prestress in concrete members fall into two separate categories.

1. *Strength Methods:* The LRFD specifications, the CSA, the ACI 318 (prestressed concrete section) and the Modified LRFD methods are similar in that the minimum reinforcement is specified by requiring that the flexural strength must be greater by an acceptable safety margin. Minimum prestress in these methods are calculated through trial-and-error.
2. *Prescribed Area Methods:* The remainder of the methods are based on providing minimum reinforcement and/or prestress that is greater than the cracking strength by an acceptable safety margin, but the methods are further simplified so the amount of reinforcement and or prestressed is calculated directly. These methods include Leonhardt, Eurocode, JRA (Japan), and the reinforced concrete section of ACI 318.

The variation in the amount of reinforcement specified is considerable. For the strength methods, the flexural cracking strength varies from $0.15\sqrt{f'c}$ in the CSA (Canadian) code to $0.37\sqrt{f'c}$ in the LRFD specifications. For prestressed concrete members, there is considerable variation in approaches from not evaluating minimum reinforcement at the strength-limit-state in the JRA (Japan) code to the trial-and-error methods implied in the ACI and LRFD specifications.

4.1.3 Conclusions on the Parametric Study

Of the four methods evaluated as part of this study two can be considered Strength methods and the other two are Prescribed Area methods. These methods are based on providing nominal strength in excess, by an acceptable margin, of the flexural cracking strength. In Prescribed Area methods, minimum reinforcement is calculated based on simplified equations, as required to meet this basic approach. Based on the parametric study, it has been shown that the Modified LRFD method provides the most consistent level of safety provided for all concrete members in the database. This is largely due to the recognition that the ultimate strength of the member, including the effects of strain hardening, is a true measure of whether or not the section is ductile. Further, the method provides for economic design, and its complexity is similar to the requirements currently prescribed in the LRFD specifications.

4.2 SUGGESTED RESEARCH

Minimum reinforcement provisions recommended in this report specify minimum levels of flexural strength in the post-cracked state to exceed the cracking strength by an appropriate safety margin. It has been shown that full-size members exhibit cracking strengths far below small-scale modulus-of-rupture specimens. However, test data on large-scale flexural members is limited because the flexural cracking strength is typically not the primary focus in laboratory experiments, and the data is highly variable. Testing on both large and small-scale lightly reinforced and prestressed concrete units made from identical concrete mixes could be beneficial to provide further data on influence of size on the flexural cracking strength of concrete members. Further, testing of segmental bridge girders with external prestressing could provide additional data on the strength and ductility of externally prestressed sections and the cracking stress in the concrete layer adjacent to the match-cast joint.

A general lack of understanding on the behavior of lightly reinforced and prestressed concrete members could be the reason for the wide variation in the amounts of minimum reinforcement prescribed in practice. Presentations on behavior of concrete members with relatively small reinforcement content given through future technology transfer seminars may be useful in reducing this lack of understanding.

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Appendix A Parametric Study Results

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SUMMARY - REINFORCED CONCRETE SECTIONS

Section	Method	Required area of steel (in^2)	M_{cr} (kip-ft)	M_o (kip-ft)	M_o / M_{cr}	Notes
CRS1	AASHTO	0.44	8.6	21.6	2.52	Calculation of M_{cr} uses transformed section properties
	LEONHARDT	0.27	8.5	16.7	1.97	
	EUROCODE	0.16	8.4	11.6	1.38	
	MODIFIED	0.25	8.4	14.2	1.68	
CRS2	AASHTO	0.78	36.9	104.6	2.83	Calculation of M_{cr} uses transformed section properties
	LEONHARDT	0.56	36.3	80.2	2.21	
	EUROCODE	0.37	35.7	54.2	1.52	
	MODIFIED	0.44	35.8	58.3	1.63	
BRC1	AASHTO	5.96	545.2	1447.5	2.65	Calculation of M_{cr} uses transformed section properties
	LEONHARDT	3.76	531.0	973.4	1.83	
	EUROCODE	1.67	515.8	458	0.89	
	MODIFIED	3.37	526.8	832.8	1.58	
BRC2	AASHTO	19.40	3857.3	10345.2	2.68	Calculation of M_{cr} uses transformed section properties
	LEONHARDT	12.01	3732.5	6655.1	1.78	
	EUROCODE	5.27	3607.3	2795.9	0.78	
	MODIFIED	11.00	3705.4	5803.2	1.57	
CAP1	AASHTO	16.04	1829.4	4945.9	2.70	Calculation of M_{cr} uses transformed section properties
	LEONHARDT	11.65	1794.7	3651.7	2.03	
	EUROCODE	7.66	1763.5	2460.6	1.40	
	MODIFIED	9.03	1773.4	2831.4	1.60	
CAP2	AASHTO	56.70	16925.6	45203.9	2.67	Calculation of M_{cr} uses transformed section properties
	LEONHARDT	42.33	16608.8	34255.4	2.06	
	EUROCODE	28.46	16298.4	23400.3	1.44	
	MODIFIED	32.12	16366.3	25751	1.57	
F1	AASHTO	30.26	4258.6	11226.4	2.64	Calculation of M_{cr} uses transformed section properties
	LEONHARDT	23.18	4200.9	9093.2	2.16	
	EUROCODE	16.50	4133.9	6436.4	1.56	
	MODIFIED	17.06	4137.1	6594.8	1.59	
F2	AASHTO	129.18	36533.4	96038.2	2.63	Calculation of M_{cr} uses transformed section properties
	LEONHARDT	99.36	35977.9	75412.4	2.10	
	EUROCODE	70.91	35458.3	55156	1.56	
	MODIFIED	72.91	35479.9	56003.9	1.58	

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SUMMARY - PRESTRESSED SECTIONS

Section	Method	Required area of steel (in^2)	M_{cr} (kip-ft)	M_o (kip-ft)	M_o / M_{cr}	Notes
CPS1	AASHTO	0.26	53.5	72.9	1.36	Tendon prestrain is (0.75*270-30) / 28500 = 0.006
	LEONHARDT	0.25	51.7	70.3	1.36	
	EUROCODE	0.20	43.3	57.3	1.32	
	MODIFIED	0.19	42.3	55.6	1.32	
CPS2	AASHTO	0.42	139.1	187.9	1.35	Tendon prestrain is (0.75*270-30) / 28500 = 0.006
	LEONHARDT	0.41	136.3	184	1.35	
	EUROCODE	0.33	113.8	150.6	1.32	
	MODIFIED	0.31	108.3	141.9	1.31	
BPT1	AASHTO	4.17	2161.3	2952	1.37	Tendon prestrain is (0.75*270-30) / 28500 = 0.006
	LEONHARDT	2.54	1532.5	1844	1.20	
	EUROCODE	2.45	1498.0	1782.1	1.19	
	MODIFIED	2.85	1653.7	2066.5	1.25	
BPT2	AASHTO	27.95	47400.9	62391.1	1.32	Tendon prestrain is (0.75*270-30) / 28500 = 0.006
	LEONHARDT	13.98	28983.0	31956.3	1.10	
	EUROCODE	14.87	30176.7	33940.5	1.12	
	MODIFIED	18.88	35472.5	42746.4	1.21	
PCI1	AASHTO	2.61	1644.0	2137.8	1.30	Tendon prestrain is 6 ms; girder concrete prestrain is -0.423 ms at bottom and 0.123 ms at top
	LEONHARDT	1.13	901.2	949.9	1.05	
	EUROCODE	1.07	872.7	900.8	1.03	
	MODIFIED	1.59	1113.4	1326.5	1.19	
PCI2	AASHTO	9.09	13876.8	18096.7	1.30	Tendon prestrain is 6 ms; girder concrete prestrain is -0.333 ms at bottom and 0.059 ms at top
	LEONHARDT	4.76	8787.8	9565.9	1.09	
	EUROCODE	4.99	9055.7	10018.4	1.11	
	MODIFIED	6.20	10375.3	12409.7	1.20	
PUB1	AASHTO	8.77	5012.5	6796.8	1.36	Tendon prestrain is 6 ms; girder concrete prestrain is -0.347 ms at bottom and 0.093 ms at top
	LEONHARDT	3.40	2529.3	2782.1	1.10	
	EUROCODE	3.58	2609.2	2924.1	1.12	
	MODIFIED	4.53	2991.0	3673.1	1.23	
PUB2	AASHTO	12.68	19146.1	24867.1	1.30	Tendon prestrain is 6 ms; girder concrete prestrain is -0.324 ms at bottom and 0.095 ms at top
	LEONHARDT	7.04	12760.4	13983.9	1.10	
	EUROCODE	7.21	12945.6	14308.1	1.11	
	MODIFIED	8.78	14506.7	17296	1.19	

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Section	Method	Required area of steel (in^2)	M_{cr} (kip-ft)	M_o (kip-ft)	M_o / M_{cr}	Notes
PBB1	AASHTO	2.90	1034.7	1368.4	1.32	Tendon prestrain is 6 ms; girder concrete prestrain is -0.331 ms at bottom and 0.112 ms at top
	LEONHARDT	1.41	633.2	716.9	1.13	
	EUROCODE	1.37	622.7	698	1.12	
	MODIFIED	1.86	739.4	927.5	1.25	
PBB2	AASHTO	3.07	1851.9	2572.9	1.39	Tendon prestrain is 6 ms; girder concrete prestrain is -0.268 ms at bottom and 0.091 ms at top
	LEONHARDT	1.49	1116.2	1330.1	1.19	
	EUROCODE	1.46	1103.0	1304.9	1.18	
	MODIFIED	1.68	1183.0	1487.9	1.26	
PPS1	AASHTO	0.84	139.8	186.1	1.33	Tendon prestrain is (0.75*270-30) / 28500 = 0.006
	LEONHARDT	0.81	136.0	181.2	1.33	
	EUROCODE	0.65	112.6	147.9	1.31	
	MODIFIED	0.62	107.8	140.9	1.31	
PPS2	AASHTO	1.20	340.9	472.1	1.39	Tendon prestrain is (0.75*270-30) / 28500 = 0.006
	LEONHARDT	0.91	279.0	367.3	1.32	
	EUROCODE	0.82	261.6	333.4	1.27	
	MODIFIED	0.85	267.5	345.2	1.29	
PSP1	AASHTO	5.91	6852.4	8917.5	1.30	Tendon prestrain is 6 ms; girder concrete prestrain is -0.338 ms at bottom and 0.062 ms at top
	LEONHARDT	3.16	4400.7	4843.7	1.10	
	EUROCODE	3.32	4541.0	5081.7	1.12	
	MODIFIED	3.95	5039.3	6034.3	1.20	
PSP2	AASHTO	10.28	28819.4	37016.5	1.28	Tendon prestrain is 6 ms; girder concrete prestrain is -0.306 ms at bottom and 0.075 ms at top
	LEONHARDT	6.21	20349.8	22480.2	1.10	
	EUROCODE	6.27	20473.6	22716.6	1.11	
	MODIFIED	7.29	22307.8	26403.6	1.18	
SBS1	AASHTO	No convergence				
	LEONHARDT	26.54	26448.9	29248.1	1.11	
	EUROCODE	67.09	59269.5	71748.1	1.21	
	MODIFIED	24.59	24496.0	28373.9	1.16	
SBS2	AASHTO	No convergence				
	LEONHARDT	39.30	52643.2	58620.6	1.11	
	EUROCODE	90.80	111731.3	131159.4	1.17	
	MODIFIED	39.37	51845.0	59697.7	1.15	
SBC1	AASHTO	29.32	29012.7	39740.4	1.37	Tendon prestrain is (0.75*270-30) / 28500 = 0.006
	LEONHARDT	15.31	18146.1	21039.4	1.16	
	EUROCODE	16.62	19159.3	22764.5	1.19	
	MODIFIED	14.78	17727.9	20346	1.15	
SBC2	AASHTO	43.39	73562.0	99053.4	1.35	Tendon prestrain is (0.75*270-30) / 28500 = 0.006
	LEONHARDT	22.55	45663.2	51986	1.14	
	EUROCODE	24.70	48554.0	56838.2	1.17	
	MODIFIED	22.40	45450.3	51666.7	1.14	

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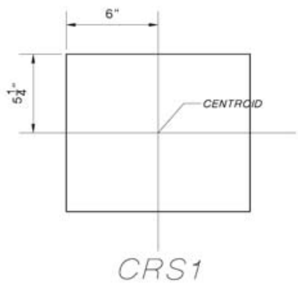
CALCULATION OF MINIMUM REINFORCEMENT - CONVENTIONAL SLAB CRS1

h =	10.50 in
b =	12.00 in
A =	126.00 in2
I =	1157.63 in4
y_b =	5.25 in
f'_c =	3.6 ksi
f_y =	60 ksi
Clearance clr =	2 in



1) AASHTO LRFD 5.7.3.3.2

$\text{Ø} M_n \geq 1.2 M_{cr}$ with Ø = 0.9 (AASHTO 5.5.4.2.1)

$f_r = 0.37 \times f'_c{}^{0.5}$ = 0.702 ksi (AASHTO 5.4.2.6)
$S_c = I / y_b$ = 220.5 in3
$M_{cr} = S_c \times f_r$ = 154.8 kip-in
$\text{Ø} M_n = \text{Ø} \times A_s \times f_y \times (d - a/2) = \text{Ø} \times A_s \times f_y \times (d - A_s \times f_y / 2 / 0.85 / f'_c / b)$ (AASHTO 5.7.3.2.2)
Bar diameter d_b = 0.75 in assuming #6 bar
$d = h - clr - d_b / 2$ = 8.13 in
Solve for A_s = 0.44 in2 -> provide 1-#6 bar

2) LEONHARDT

$A_s \geq 1.2 F_{ct} / f_y$
$f_{ct} = 0.23 \times f'_c{}^{0.5}$ = 0.436 ksi
$F_{ct} = f_{ct} \times y_b \times b / 2$ = 14 kips
A_s = 0.27 in2 -> provide 1-#5 bar

3) EUROCODE

$A_s \geq 0.26 \times f_{ctm} / f_y \times b_t \times d$ (Eq 1) and $A_s \geq 0.0013 \times b_t \times d$ (Eq 2)
$f_{ck} = f'_c$ = 3.6 ksi = 25 Mpa
$f_{ctm} = 0.30 \times f_{ck}{}^{2/3}$ = 2.6 Mpa = 0.372 ksi
Bar diameter d_b = 0.50 in assuming #4 bar
$d = h - clr - d_b / 2$ = 8.25 in
(Eq 1) A_s = 0.16 in2
(Eq 2) A_s = 0.13 in2
As = 0.16 in2 -> provide 1-#4 bar

4) PROPOSED METHOD

$\text{Ø} M_n \geq 0.67 \times 1.6 f_r \times S_c$ with Ø = 0.9

$f_r = 0.237 \times f'_c{}^{0.5}$ = 0.450 ksi
$\text{Ø} M_n = \text{Ø} \times A_s \times f_y \times (d - a/2) = \text{Ø} \times A_s \times f_y \times (d - A_s \times f_y / 2 / 0.85 / f'_c / b)$
Bar diameter d_b = 0.63 in assuming #5 bar
$d = h - clr - d_b / 2$ = 8.19 in
Solve for A_s = 0.25 in2

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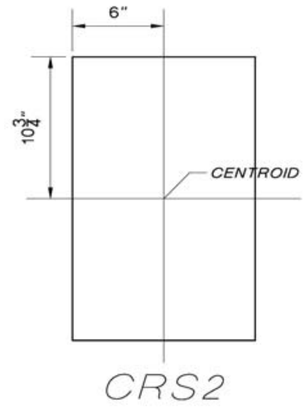
CALCULATION OF MINIMUM REINFORCEMENT - CONVENTIONAL SLAB CRS2

h = 21.50 in
b = 12.00 in
A = 258.00 in2
I = 9938.38 in4
y_b = 10.75 in
f'_c = 3.6 ksi
f_y = 60 ksi
Clearance clr = 2 in



1) AASHTO LRFD 5.7.3.3.2

$Ø M_n \geq 1.2 M_{cr}$ with Ø = 0.9 (AASHTO 5.5.4.2.1)

$f_r = 0.37 \times f'_c{}^{0.5}$ = 0.702 ksi (AASHTO 5.4.2.6)
$S_c = I / y_b$ = 924.5 in3
$M_{cr} = S_c \times f_r$ = 649.0 kip-in
$Ø M_n = Ø \times A_s \times f_y \times (d - a / 2) = Ø \times A_s \times f_y \times (d - A_s \times f_y / 2 / 0.85 / f'_c / b)$ (AASHTO 5.7.3.2.2)
Bar diameter d_b = 0.75 in assuming #6 bar
$d = h - clr - d_b / 2$ = 19.13 in
Solve for A_s = 0.78 in2 -> provide 2-#6 bars

2) LEONHARDT

$A_s \geq 1.2 F_{ct} / f_y$
$f_{ct} = 0.23 \times f'_c{}^{0.5}$ = 0.436 ksi
$F_{ct} = f_{ct} \times y_b \times b / 2$ = 28 kips
A_s = 0.56 in2 -> provide 2-#5 bars

3) EUROCODE

$A_s \geq 0.26 \times f_{ctm} / f_y \times b_t \times d$ (Eq 1) and $A_s \geq 0.0013 \times b_t \times d$ (Eq 2)
$f_{ck} = f'_c$ = 3.6 ksi = 25 Mpa
$f_{ctm} = 0.30 \times f_{ck}{}^{2/3}$ = 2.6 Mpa = 0.372 ksi
Bar diameter d_b = 0.50 in assuming #4 bar
$d = h - clr - d_b / 2$ = 19.25 in
(Eq 1) A_s = 0.37 in2
(Eq 2) A_s = 0.30 in2
As = 0.37 in2 -> provide 2-#4 bars

4) PROPOSED METHOD

$Ø M_n \geq 0.67 \times 1.6 f_r \times S_c$ with Ø = 0.9

$f_r = 0.237 \times f'_c{}^{0.5}$ = 0.450 ksi
$Ø M_n = Ø \times A_s \times f_y \times (d - a / 2) = Ø \times A_s \times f_y \times (d - A_s \times f_y / 2 / 0.85 / f'_c / b)$
Bar diameter d_b = 0.75 in assuming #6 bar
$d = h - clr - d_b / 2$ = 19.13 in
Solve for A_s = 0.44 in2

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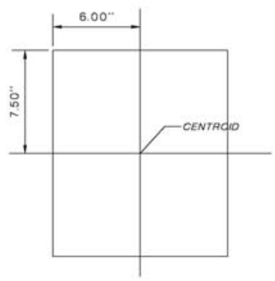
CALCULATION OF MINIMUM REINFORCEMENT - POST-TENSIONED SLAB CPS1

$h =$ 15.00 in
$b =$ 12.00 in
$A =$ 180.00 in2
$I =$ 3375.00 in4
$y_b =$ 7.50 in
$f'_c =$ 4.0 ksi
$f_y =$ 60 ksi
$f_{pu} =$ 270 ksi



CPS1

1) AASHTO LRFD 5.7.3.3.2

$Ø M_n \geq 1.2 M_{cr}$ with $Ø =$ 1.0 (AASHTO 5.5.4.2.1)

Given $d_p =$ 13.50 in
Assume min $A_{ps} =$ 0.26 in2 (for trial and error purpose)
$P_f = A_{ps} \times (0.75 \times f_{pu} - 30 \text{ ksi}) =$ 45.3 kips assuming 30 ksi losses
$f_{cpe} = P_f / A + P_f \times (d_p - y_t) \times y_b / I =$ 0.855 ksi
$f_r = 0.37 \times f'_c{}^{0.5} =$ 0.740 ksi (AASHTO 5.4.2.6)
$S_c = I / y_b =$ 450.0 in3
$1.2 M_{cr} = 1.2 \times S_c \times (f_r + f_{cpe}) =$ 861 kip-in

$Ø M_n = Ø \times A_{ps} \times f_{ps} \times (d_p - a / 2)$ (AASHTO 5.7.3.2.2)
$c = (A_{ps} \times f_{pu}) / (0.85 \times f'_c \times 0.85 \times b + 0.28 \times A_{ps} \times f_{pu} / d_p) =$ 1.96 in
$f_{ps} = f_{pu} \times (1 - 0.28 \times c / d_p) =$ 259 ksi
$Ø \times A_{ps} \times f_{ps} \times (d_p - a / 2) =$ 861 kip-in matches $1.2 M_{cr}$

2) LEONHARDT

$A_{ps} \geq 1.2 F_{ct} / (f_{pu} - f_{pe})$
$f_{ct} = 0.23 \times f'_c{}^{0.5} =$ 0.460 ksi
$F_{ct} = f_{ct} \times y_b \times b / 2 =$ 21 kips
$f_{pe} = 0.75 \times f_{pu} - 30 \text{ ksi} =$ 172.5 ksi assuming 30 ksi losses
$A_{ps} =$ 0.25 in2

3) EUROCODE

$A_p \geq M_{rep} / (z \times \Delta\sigma_p)$

$f_{ck} = f'_c =$ 4.0 ksi = 28 Mpa
$f_{ctm} = 0.30 \times f_{ck}{}^{2/3} =$ 2.8 Mpa = 0.401 ksi
$M_{rep} = S_c \times f_{ctm} =$ 180.5 kip-in
$\Delta\sigma_p = \text{Min}(0.4 \times 270, 72.5) =$ 72.5 ksi
$z = 0.9 \times d =$ 12.15 in
$A_p =$ 0.20 in2

4) PROPOSED METHOD

$Ø M_n \geq 1.0 \times (1.6 f_r + 1.1 f_{pe}) \times S_c$ with $Ø =$ 1.0

Assume min $A_{ps} =$ 0.19 in2 (for trial and error purpose)
$P_f = A_{ps} \times (0.75 \times f_{pu} - 30 \text{ ksi}) =$ 33.5 kips assuming 30 ksi losses
$f_{pe} = P_f / A + P_f \times (d_p - y_t) \times y_b / I =$ 0.632 ksi
$f_r = 0.237 \times f'_c{}^{0.5} =$ 0.474 ksi
$1.0 \times (1.6 f_r + 1.1 f_{pe}) \times S_c =$ 654 kip-in

$Ø M_n = Ø \times A_{ps} \times f_{ps} \times (d_p - a / 2)$
$c = (A_{ps} \times f_{pu}) / (0.85 \times f'_c \times 0.85 \times b + 0.28 \times A_{ps} \times f_{pu} / d_p) =$ 1.46 in
$f_{ps} = f_{pu} \times (1 - 0.28 \times c / d_p) =$ 261.8 ksi

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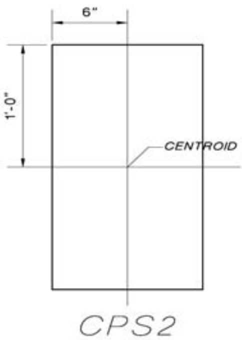
CALCULATION OF MINIMUM REINFORCEMENT - POST-TENSIONED SLAB CPS2

h = 24.00 in
b = 12.00 in
A = 288.00 in2
I = 13824.00 in4
y_b = 12.00 in
f'_c = 4.0 ksi
f_y = 60 ksi
f_{pu} = 270 ksi



1) AASHTO LRFD 5.7.3.3.2

$\text{Ø}\, M_n \geq 1.2\, M_{cr}$ with Ø = 1.0 (AASHTO 5.5.4.2.1)

Given d_p = 21.60 in
Assume min A_{ps} = 0.42 in2 (for trial and error purpose)
$P_f = A_{ps} \times (0.75 \times f_{pu} - 30\ \text{ksi})$ = 72.5 kips assuming 30 ksi losses
$f_{cpe} = P_f / A + P_f \times (d_p - y_t) \times y_b / I$ = 0.855 ksi
$f_r = 0.37 \times f'_c{}^{0.5}$ = 0.740 ksi (AASHTO 5.4.2.6)
$S_c = I / y_b$ = 1152.0 in3
$1.2\, M_{cr} = 1.2 \times S_c \times (f_r + f_{cpe})$ = 2205 kip-in

$\text{Ø}\, M_n = \text{Ø} \times A_{ps} \times f_{ps} \times (d_p - a / 2)$ (AASHTO 5.7.3.2.2)
$c = (A_{ps} \times f_{pu}) / (0.85 \times f'_c \times 0.85 \times b + 0.28 \times A_{ps} \times f_{pu} / d_p)$ = 3.14 in
$f_{ps} = f_{pu} \times (1 - 0.28 \times c / d_p)$ = 259 ksi
$\text{Ø} \times A_{ps} \times f_{ps} \times (d_p - a / 2)$ = 2205 kip-in matches 1.2 M_{cr}

2) LEONHARDT

$A_{ps} \geq 1.2\, F_{ct} / (f_{pu} - f_{pe})$
$f_{ct} = 0.23 \times f'_c{}^{0.5}$ = 0.460 ksi
$F_{ct} = f_{ct} \times y_b \times b / 2$ = 33 kips
$f_{pe} = 0.75 \times f_{pu} - 30$ ksi = 172.5 ksi assuming 30 ksi losses
A_{ps} = 0.41 in2

3) EUROCODE

$A_p \geq M_{rep} / (z \times \Delta\sigma_p)$

$f_{ck} = f'_c$ = 4.0 ksi = 28 Mpa
$f_{ctm} = 0.30 \times f_{ck}{}^{2/3}$ = 2.8 Mpa = 0.401 ksi
$M_{rep} = S_c \times f_{ctm}$ = 462.2 kip-in
$\Delta\sigma_p$ = Min(0.4 x 270, 72.5) = 72.5 ksi
z = 0.9 x d = 19.44 in
A_p = 0.33 in2

4) PROPOSED METHOD

$\text{Ø}\, M_n \geq 1.0 \times (1.6\, f_r + 1.1\, f_{pe}) \times S_c$ with Ø = 1.0

Assume min A_{ps} = 0.31 in2 (for trial and error purpose)
$P_f = A_{ps} \times (0.75 \times f_{pu} - 30\ \text{ksi})$ = 53.6 kips assuming 30 ksi losses
$f_{pe} = P_f / A + P_f \times (d_p - y_t) \times y_b / I$ = 0.632 ksi
$f_r = 0.237 \times f'_c{}^{0.5}$ = 0.474 ksi
$1.0 \times (1.6\, f_r + 1.1\, f_{pe}) \times S_c$ = 1675 kip-in

$\text{Ø}\, M_n = \text{Ø} \times A_{ps} \times f_{ps} \times (d_p - a / 2)$
$c = (A_{ps} \times f_{pu}) / (0.85 \times f'_c \times 0.85 \times b + 0.28 \times A_{ps} \times f_{pu} / d_p)$ = 2.34 in
$f_{ps} = f_{pu} \times (1 - 0.28 \times c / d_p)$ = 261.8 ksi
$\text{Ø} \times A_{ps} \times f_{ps} \times (d_p - a / 2)$ = 1675 kip-in matches $(1.6\, f_r + 1.1\, f_{pe}) \times S_c$

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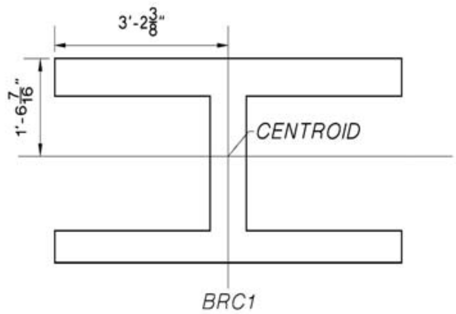
CALCULATION OF MINIMUM REINFORCEMENT - REINFORCED CONCRETE BOX GIRDER BRC1

h =	38.40 in
b_f =	76.80 in
b_w =	8.00 in
t_d =	7.10 in
t_s =	6.00 in
A =	1208.48 in2
I =	268255.86 in4
y_b =	19.96 in
f'_c =	3.6 ksi
f_y =	60 ksi
Clearance clr =	2 in



1) AASHTO LRFD 5.7.3.3.2

$\varnothing M_n \geq 1.2 M_{cr}$ with Ø = 0.9 (AASHTO 5.5.4.2.1)

$f_r = 0.37 \times f'_c{}^{0.5}$ = 0.702 ksi (AASHTO 5.4.2.6)
$S_c = I / y_b$ = 13438.0 in3
$M_{cr} = S_c \times f_r$ = 9433.8 kip-in
$\varnothing M_n = \varnothing \times A_s \times f_y \times (d - a / 2) = \varnothing \times A_s \times f_y \times (d - A_s \times f_y / 2 / 0.85 / f'_c / b_f)$ (AASHTO 5.7.3.2.2)
Bar diameter d_b = 0.88 in assuming #7 bar
$d = h - clr - d_b / 2$ = 35.96 in
Solve for A_s = 5.96 in2 -> provide 10-#7 bars

2) LEONHARDT

$A_s \geq 1.2 F_{ct} / f_y$
$f_{ct} = 0.23 \times f'_c{}^{0.5}$ = 0.436 ksi
$F_{ct} = f_{ct} \times ((y_b - t_s)^2 \times b_w / (2 \times y_b) + (2 \times y_b - t_s) \times t_s \times b_f / (2 \times y_b))$ = 188 kips
A_s = 3.76 in2 -> provide 9-#6 bars

3) EUROCODE

$A_s \geq 0.26 \times f_{ctm} / f_y \times b_t \times d$ (Eq 1) and $A_s \geq 0.0013 \times b_t \times d$ (Eq 2)
$f_{ck} = f'_c$ = 3.6 ksi = 25 Mpa
$f_{ctm} = 0.30 \times f_{ck}{}^{2/3}$ = 2.6 Mpa = 0.372 ksi
Bar diameter d_b = 0.50 in assuming #4 bar
$d = h - clr - d_b / 2$ = 36.15 in
$b_t = ((y_b - t_s) \times b_w + t_s \times b_f) / y_b$ = 28.7 in
(Eq 1) A_s = 1.67 in2
(Eq 2) A_s = 1.35 in2
As = 1.67 in2 -> provide 9-#4 bars

4) PROPOSED METHOD

$\varnothing M_n \geq 0.67 \times 1.6 f_r \times S_c$ with Ø = 0.9

$f_r = 0.237 \times f'_c{}^{0.5}$ = 0.450 ksi
$\varnothing M_n = \varnothing \times A_s \times f_y \times (d - a / 2) = \varnothing \times A_s \times f_y \times (d - A_s \times f_y / 2 / 0.85 / f'_c / b)$
Bar diameter d_b = 0.75 in assuming #6 bar
$d = h - clr - d_b / 2$ = 36.03 in
Solve for A_s = 3.37 in2

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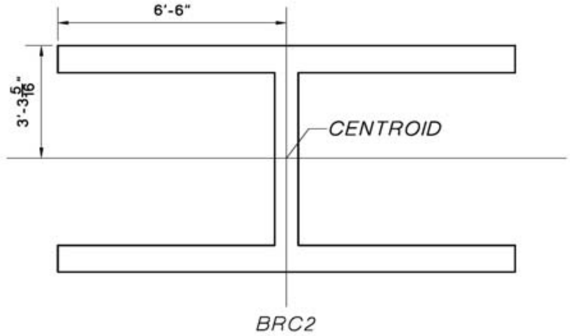
CALCULATION OF MINIMUM REINFORCEMENT - REINFORCED CONCRETE BOX GIRDER BRC2

h = 79.20 in
b_f = 156.00 in
b_w = 8.00 in
t_d = 9.50 in
t_s = 9.30 in
A = 3416.00 in2
I = 3740350.86 in4
y_b = 39.89 in
f'_c = 3.6 ksi
f_y = 60 ksi
Clearance clr = 2 in



1) AASHTO LRFD 5.7.3.3.2

$\varnothing M_n \geq 1.2 M_{cr}$ with Ø = 0.9 (AASHTO 5.5.4.2.1)

$f_r = 0.37 \times f'_c{}^{0.5}$ = 0.702 ksi (AASHTO 5.4.2.6)
$S_c = I / \underline{y}$ = 93772.5 in3
$M_{cr} = S_c \times f_r$ = 65830.7 kip-in
$\varnothing M_n = \varnothing \times A_s \times f_y \times (d - a / 2) = \varnothing \times A_s \times f_y \times (d - A_s \times f_y / 2 / 0.85 / f'_c / b_f)$ (AASHTO 5.7.3.2.2)
Bar diameter d_b = 1.13 in assuming #9 bar
$d = h - clr - d_b / 2$ = 76.64 in
Solve for A_s = 19.40 in2 -> provide 20-#9 bars

2) LEONHARDT

$A_s \geq 1.2 F_{ct} / f_y$
$f_{ct} = 0.23 \times f'_c{}^{0.5}$ = 0.436 ksi
$F_{ct} = f_{ct} \times ((y_b - t_s)^2 \times b_w / (2 \times y_b) + (2 \times y_b - t_s) \times t_s \times b_f / (2 \times y_b))$ = 600 kips
A_s = 12.01 in2 -> provide 21-#7 bars

3) EUROCODE

$A_s \geq 0.26 \times f_{ctm} / f_y \times b_t \times d$ (Eq 1) and $A_s \geq 0.0013 \times b_t \times d$ (Eq 2)
$f_{ck} = f'_c$ = 3.6 ksi = 25 Mpa
$f_{ctm} = 0.30 \times f_{ck}^{2/3}$ = 2.6 Mpa = 0.372 ksi
Bar diameter d_b = 0.63 in assuming #5 bar
$d = h - clr - d_b / 2$ = 76.89 in
$b_t = ((y_b - t_s) \times b_w + t_s \times b_f) / y_b$ = 42.5 in
(Eq 1) A_s = 5.27 in2
(Eq 2) A_s = 4.25 in2
As = 5.27 in2 -> provide 17-#5 bars

4) PROPOSED METHOD

$\varnothing M_n \geq 0.67 \times 1.6 f_r \times S_c$ with Ø = 0.9

$f_r = 0.237 \times f'_c{}^{0.5}$ = 0.450 ksi
$\varnothing M_n = \varnothing \times A_s \times f_y \times (d - a / 2) = \varnothing \times A_s \times f_y \times (d - A_s \times f_y / 2 / 0.85 / f'_c / b)$
Bar diameter d_b = 0.88 in assuming #7 bar
$d = h - clr - d_b / 2$ = 76.76 in
Solve for A_s = 11.00 in2

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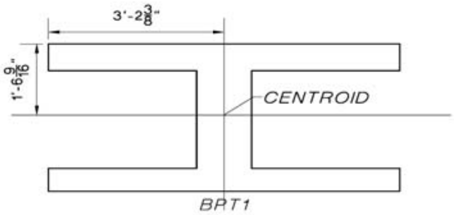
CALCULATION OF MINIMUM REINFORCEMENT - PRESTRESSED CONCRETE BOX GIRDER BPT1

h = 38.40 in
b_f = 76.80 in
b_w = 12.00 in
t_d = 7.00 in
t_s = 6.00 in
A = 1303.20 in2
I = 272966.55 in4
y_b = 19.84 in
f'_c = 4.0 ksi
f_y = 60 ksi
f_{pu} = 270 ksi



1) AASHTO LRFD 5.7.3.3.2

$Ø M_n \geq 1.2 M_{cr}$ with Ø = 1.0 (AASHTO 5.5.4.2.1)

Given d_p = 34.56 in
Assume min A_{ps} = 4.17 in2 (for trial and error purpose)
$P_f = A_{ps} \times (0.75 \times f_{pu} - 30 \text{ ksi})$ = 719.9 kips assuming 30 ksi losses
$f_{cpe} = P_f / A + P_f \times (d_p - y_t) \times y_b / I$ = 1.389 ksi
$f_r = 0.37 \times f'^{0.5}_c$ = 0.740 ksi (AASHTO 5.4.2.6)
$S_c = I / y_b$ = 13760.1 in3
$1.2 M_{cr} = 1.2 \times S_c \times (f_r + f_{cpe})$ = 35160 kip-in

$Ø M_n = Ø \times A_{ps} \times f_{ps} \times (d_p - a / 2)$ (AASHTO 5.7.3.2.2)
$c = (A_{ps} \times f_{pu}) / (0.85 \times f'_c \times 0.85 \times b + 0.28 \times A_{ps} \times f_{pu} / d_p)$ = 4.88 in
$f_{ps} = f_{pu} \times (1 - 0.28 \times c / d_p)$ = 259 ksi
$Ø \times A_{ps} \times f_{ps} \times (d_p - a / 2)$ = 35160 kip-in matches $1.2 M_{cr}$

2) LEONHARDT

$A_{ps} \geq 1.2 F_{ct} / (f_{pu} - f_{pe})$
$f_{ct} = 0.23 \times f'^{0.5}_c$ = 0.460 ksi
$F_{ct} = f_{ct} \times ((y_b - t_s)^2 \times b_w / (2 \times y_b) + (2 \times y_b - t_s) \times t_s \times b_f / (2 \times y_b))$ = 207 kips
$f_{pe} = 0.75 \times f_{pu} - 30 \text{ ksi}$ = 172.5 ksi assuming 30 ksi losses
A_{ps} = 2.54 in2

3) EUROCODE

$A_p \geq M_{rep} / (z \times \Delta\sigma_p)$

$f_{ck} = f'_c$ = 4.0 ksi = 28 Mpa
$f_{ctm} = 0.30 \times f_{ck}^{2/3}$ = 2.8 Mpa = 0.401 ksi
$M_{rep} = S_c \times f_{ctm}$ = 5520.8 kip-in
$\Delta\sigma_p$ = Min(0.4 x 270, 72.5) = 72.5 ksi
$z = 0.9 \times d$ = 31.10 in
A_p = 2.45 in2

4) PROPOSED METHOD

$Ø M_n \geq 1.0 \times (1.6 f_r + 1.1 f_{pe}) \times S_c$ with Ø = 1.0

Assume min A_{ps} = 2.85 in2 (for trial and error purpose)
$P_f = A_{ps} \times (0.75 \times f_{pu} - 30 \text{ ksi})$ = 491.7 kips assuming 30 ksi losses
$f_{pe} = P_f / A + P_f \times (d_p - y_t) \times y_b / I$ = 0.949 ksi
$f_r = 0.237 \times f'^{0.5}_c$ = 0.474 ksi
$1.0 \times (1.6 f_r + 1.1 f_{pe}) \times S_c$ = 24799 kip-in

$Ø M_n = Ø \times A_{ps} \times f_{ps} \times (d_p - a / 2)$
$c = (A_{ps} \times f_{pu}) / (0.85 \times f'_c \times 0.85 \times b + 0.28 \times A_{ps} \times f_{pu} / d_p)$ = 3.37 in
$f_{ps} = f_{pu} \times (1 - 0.28 \times c / d_p)$ = 262.6 ksi
$Ø \times A_{ps} \times f_{ps} \times (d_p - a / 2)$ = 24799 kip-in matches $(1.6 f_r + 1.1 f_{pe}) \times S_c$

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CALCULATION OF MINIMUM REINFORCEMENT - PRESTRESSED CONCRETE BOX GIRDER BPT2

h = 120.00 in

b_f = 240.00 in

b_w = 12.00 in

t_d = 10.10 in

t_s = 10.10 in

A = 6045.60 in2

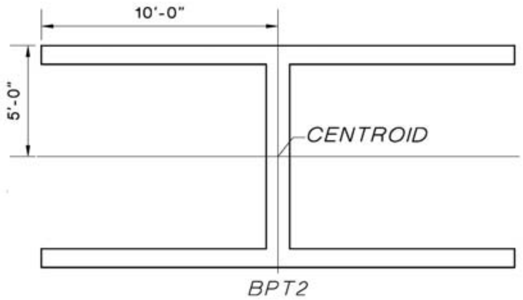
I = 15673772.18 in4

y_b = 60.00 in

f'_c = 4.0 ksi

f_y = 60 ksi

f_{pu} = 270 ksi



1) AASHTO LRFD 5.7.3.3.2

$\varnothing M_n \geq 1.2 M_{cr}$ with $\varnothing$ = 1.0 (AASHTO 5.5.4.2.1)

Given d_p = 108.00 in

Assume min A_{ps} = 27.95 in2 (for trial and error purpose)

$P_f = A_{ps} \times (0.75 \times f_{pu} - 30 \text{ ksi})$ = 4822.0 kips assuming 30 ksi losses

$f_{cpe} = P_f / A + P_f \times (d_p - y_t) \times y_b / I$ = 1.684 ksi

$f_r = 0.37 \times f'_c{}^{0.5}$ = 0.740 ksi (AASHTO 5.4.2.6)

$S_c = I / y_b$ = 261229.5 in3

$1.2 M_{cr} = 1.2 \times S_c \times (f_r + f_{cpe})$ = 759754 kip-in

$\varnothing M_n = \varnothing \times A_{ps} \times f_{ps} \times (d_p - a / 2)$ (AASHTO 5.7.3.2.2)

$c = (A_{ps} \times f_{pu}) / (0.85 \times f'_c \times 0.85 \times b + 0.28 \times A_{ps} \times f_{pu} / d_p)$ = 10.58 in

$f_{ps} = f_{pu} \times (1 - 0.28 \times c / d_p)$ = 263 ksi

$\varnothing \times A_{ps} \times f_{ps} \times (d_p - a / 2)$ = 759754 kip-in matches $1.2 M_{cr}$

2) LEONHARDT

$A_{ps} \geq 1.2 F_{ct} / (f_{pu} - f_{pe})$

$f_{ct} = 0.23 \times f'_c{}^{0.5}$ = 0.460 ksi

$F_{ct} = f_{ct} \times ((y_b - t_s)^2 \times b_w / (2 \times y_b) + (2 \times y_b - t_s) \times t_s \times b_f / (2 \times y_b))$ = 1136 kips

$f_{pe} = 0.75 \times f_{pu} - 30 \text{ ksi}$ = 172.5 ksi assuming 30 ksi losses

A_{ps} = 13.98 in2

3) EUROCODE

$A_p \geq M_{rep} / (z \times \Delta\sigma_p)$

$f_{ck} = f'_c$ = 4.0 ksi = 28 Mpa

$f_{ctm} = 0.30 \times f_{ck}^{2/3}$ = 2.8 Mpa = 0.401 ksi

$M_{rep} = S_c \times f_{ctm}$ = 104810.2 kip-in

$\Delta\sigma_p = \text{Min}(0.4 \times 270, 72.5)$ = 72.5 ksi

$z = 0.9 \times d$ = 97.20 in

A_p = 14.87 in2

4) PROPOSED METHOD

$\varnothing M_n \geq 1.0 \times (1.6 f_r + 1.1 f_{pe}) \times S_c$ with $\varnothing$ = 1.0

Assume min A_{ps} = 18.88 in2 (for trial and error purpose)

$P_f = A_{ps} \times (0.75 \times f_{pu} - 30 \text{ ksi})$ = 3256.0 kips assuming 30 ksi losses

$f_{pe} = P_f / A + P_f \times (d_p - y_t) \times y_b / I$ = 1.137 ksi

$f_r = 0.237 \times f'_c{}^{0.5}$ = 0.474 ksi

$1.0 \times (1.6 f_r + 1.1 f_{pe}) \times S_c$ = 524796 kip-in

$\varnothing M_n = \varnothing \times A_{ps} \times f_{ps} \times (d_p - a / 2)$

$c = (A_{ps} \times f_{pu}) / (0.85 \times f'_c \times 0.85 \times b + 0.28 \times A_{ps} \times f_{pu} / d_p)$ = 7.21 in

$f_{ps} = f_{pu} \times (1 - 0.28 \times c / d_p)$ = 265.0 ksi

$\varnothing \times A_{ps} \times f_{ps} \times (d_p - a / 2)$ = 524796 kip-in matches $(1.6 f_r + 1.1 f_{pe}) \times S_c$

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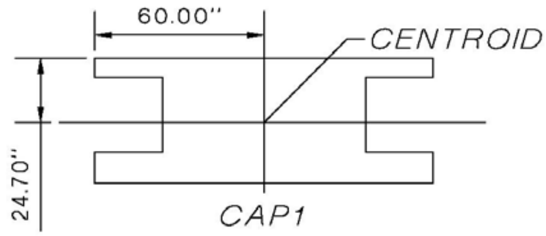
CALCULATION OF MINIMUM REINFORCEMENT - CAP BEAM CAP1

h = 48.00 in
b_f = 120.00 in
b_w = 72.00 in
t_d = 7.50 in
t_s = 12.00 in
A = 4392.00 in2
I = 1004240.88 in4
y_b = 23.30 in
f'_c = 4.0 ksi
f_y = 60 ksi
Clearance clr = 2 in



1) AASHTO LRFD 5.7.3.3.2

$\varnothing M_n \geq 1.2 M_{cr}$ with Ø = 0.9 (AASHTO 5.5.4.2.1)

$f_r = 0.37 \times f'_c{}^{0.5}$ = 0.740 ksi (AASHTO 5.4.2.6)
$S_c = I / y$ = 43100.5 in3
$M_{cr} = S_c \times f_r$ = 31894.3 kip-in
$\varnothing M_n = \varnothing \times A_s \times f_y \times (d - a/2) = \varnothing \times A_s \times f_y \times (d - A_s \times f_y / 2 / 0.85 / f'_c / b_f)$ (AASHTO 5.7.3.2.2)
Bar diameter d_b = 1.27 in assuming #10 bar
$d = h - clr - d_b / 2$ = 45.37 in
Solve for A_s = 16.04 in2 -> provide 13-#10 bars

2) LEONHARDT

$A_s \geq 1.2 F_{ct} / f_y$
$f_{ct} = 0.23 \times f'_c{}^{0.5}$ = 0.460 ksi
$F_{ct} = f_{ct} \times ((y_b - t_s)^2 \times b_w / (2 \times y_b) + (2 \times y_b - t_s) \times t_s \times b_f / (2 \times y_b))$ = 583 kips
A_s = 11.65 in2 -> provide 15-#8 bars

3) EUROCODE

$A_s \geq 0.26 \times f_{ctm} / f_y \times b_t \times d$ (Eq 1) and $A_s \geq 0.0013 \times b_t \times d$ (Eq 2)
$f_{ck} = f'_c$ = 4.0 ksi = 28 Mpa
$f_{ctm} = 0.30 \times f_{ck}{}^{2/3}$ = 2.8 Mpa = 0.401 ksi
Bar diameter d_b = 0.88 in assuming #7 bar
$d = h - clr - d_b / 2$ = 45.56 in
$b_t = ((y_b - t_s) \times b_w + t_s \times b_f) / y_b$ = 96.7 in
(Eq 1) A_s = 7.66 in2
(Eq 2) A_s = 5.73 in2
As = 7.66 in2 -> provide 13-#7 bars

4) PROPOSED METHOD

$\varnothing M_n \geq 0.67 \times 1.6 f_r \times S_c$ with Ø = 0.9

$f_r = 0.237 \times f'_c{}^{0.5}$ = 0.474 ksi
$\varnothing M_n = \varnothing \times A_s \times f_y \times (d - a/2) = \varnothing \times A_s \times f_y \times (d - A_s \times f_y / 2 / 0.85 / f'_c / b)$
Bar diameter d_b = 0.88 in assuming #7 bar
$d = h - clr - d_b / 2$ = 45.56 in
Solve for A_s = 9.03 in2

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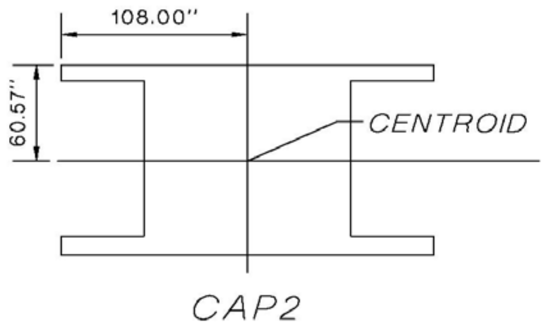
CALCULATION OF MINIMUM REINFORCEMENT - CAP BEAM CAP2

h =	120.00 in
b_f =	216.00 in
b_w =	120.00 in
t_d =	10.00 in
t_s =	12.00 in
A =	16512.00 in2
I =	23559695.63 in4
y_b =	59.43 in
f'_c =	4.0 ksi
f_y =	60 ksi
Clearance clr =	2 in



1) AASHTO LRFD 5.7.3.3.2

$Ø M_n \geq 1.2 M_{cr}$ with Ø = 0.9 (AASHTO 5.5.4.2.1)

$f_r = 0.37 \times f'_c{}^{0.5}$ = 0.740 ksi (AASHTO 5.4.2.6)

$S_c = I / \underline{y}$ = 396427.7 in3

$M_{cr} = S_c \times f_r$ = 293356.5 kip-in

$Ø M_n = Ø \times A_s \times f_y \times (d - a / 2) = Ø \times A_s \times f_y \times (d - A_s \times f_y / 2 / 0.85 / f'_c / b_f)$ (AASHTO 5.7.3.2.2)

Bar diameter d_b = 1.41 in assuming #11 bar

$d = h - clr - d_b / 2$ = 117.30 in

Solve for A_s = 56.70 in2 -> provide 37-#11 bars

2) LEONHARDT

$A_s \geq 1.2 F_{ct} / f_y$

$f_{ct} = 0.23 \times f'_c{}^{0.5}$ = 0.460 ksi

$F_{ct} = f_{ct} \times ((y_b - t_s)^2 \times b_w / (2 \times y_b) + (2 \times y_b - t_s) \times t_s \times b_f / (2 \times y_b))$ = 2117 kips

A_s = 42.33 in2 -> provide 34-#10 bars

3) EUROCODE

$A_s \geq 0.26 \times f_{ctm} / f_y \times b_t \times d$ (Eq 1) and $A_s \geq 0.0013 \times b_t \times d$ (Eq 2)

$f_{ck} = f'_c$ = 4.0 ksi = 28 Mpa

$f_{ctm} = 0.30 \times f_{ck}{}^{2/3}$ = 2.8 Mpa = 0.401 ksi

Bar diameter d_b = 1.13 in assuming #9 bar

$d = h - clr - d_b / 2$ = 117.44 in

$b_t = ((y_b - t_s) \times b_w + t_s \times b_f) / y_b$ = 139.4 in

(Eq 1) A_s = 28.46 in2

(Eq 2) A_s = 21.28 in2

As = 28.46 in2 -> provide 29-#9 bars

4) PROPOSED METHOD

$Ø M_n \geq 0.67 \times 1.6 f_r \times S_c$ with Ø = 0.9

$f_r = 0.237 \times f'_c{}^{0.5}$ = 0.474 ksi

$Ø M_n = Ø \times A_s \times f_y \times (d - a / 2) = Ø \times A_s \times f_y \times (d - A_s \times f_y / 2 / 0.85 / f'_c / b)$

Bar diameter d_b = 1.13 in assuming #9 bar

$d = h - clr - d_b / 2$ = 117.44 in

Solve for A_s = 32.12 in2

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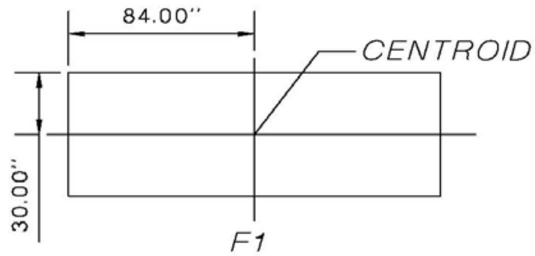
CALCULATION OF MINIMUM REINFORCEMENT - FOOTING F1

h =	60.00 in
b =	168.00 in
A =	10080.00 in2
I =	3024000 in4
y_b =	30.00 in
f'_c =	4.0 ksi
f_y =	60 ksi
Clearance clr =	3 in



1) AASHTO LRFD 5.7.3.3.2

$Ø M_n \geq 1.2 M_{cr}$ with Ø = 0.9 (AASHTO 5.5.4.2.1)

$f_r = 0.37 \times f'_c{}^{0.5}$ = 0.740 ksi (AASHTO 5.4.2.6)
$S_c = I / y_b$ = 100800.0 in3
$M_{cr} = S_c \times f_r$ = 74592.0 kip-in
$Ø M_n = Ø \times A_s \times f_y \times (d - a / 2) = Ø \times A_s \times f_y \times (d - A_s \times f_y / 2 / 0.85 / f'_c / b)$ (AASHTO 5.7.3.2.2)
Bar diameter d_b = 1.27 in assuming #10 bar
$d = h - clr - d_b / 2$ = 56.37 in
Solve for A_s = 30.26 in2 -> provide 24-#10 bars

2) LEONHARDT

$A_s \geq 1.2 F_{ct} / f_y$
$f_{ct} = 0.23 \times f'_c{}^{0.5}$ = 0.460 ksi
$F_{ct} = f_{ct} \times y_b \times b / 2$ = 1159 kips
A_s = 23.18 in2 -> provide 24-#9 bars

3) EUROCODE

$A_s \geq 0.26 \times f_{ctm} / f_y \times b_t \times d$ (Eq 1) and $A_s \geq 0.0013 \times b_t \times d$ (Eq 2)
$f_{ck} = f'_c$ = 4.0 ksi = 28 Mpa
$f_{ctm} = 0.30 \times f_{ck}{}^{2/3}$ = 2.8 Mpa = 0.401 ksi
Bar diameter d_b = 1.00 in assuming #8 bar
$d = h - clr - d_b / 2$ = 56.50 in
(Eq 1) A_s = 16.50 in2
(Eq 2) A_s = 12.34 in2

As = 16.50 in2 -> provide 21-#8 bars

4) PROPOSED METHOD

$Ø M_n \geq 0.67 \times 1.6 f_r \times S_c$ with Ø = 0.9

$f_r = 0.237 \times f'_c{}^{0.5}$ = 0.474 ksi
$Ø M_n = Ø \times A_s \times f_y \times (d - a / 2) = Ø \times A_s \times f_y \times (d - A_s \times f_y / 2 / 0.85 / f'_c / b)$
Bar diameter d_b = 1.00 in assuming #8 bar
$d = h - clr - d_b / 2$ = 56.50 in
Solve for A_s = 17.06 in2

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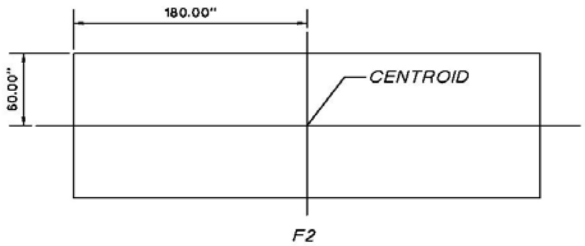
CALCULATION OF MINIMUM REINFORCEMENT - FOOTING F2

$h =$ 120.00 in
$b =$ 360.00 in
$A =$ 43200.00 in2
$I =$ 51840000 in4
$y_b =$ 60.00 in
$f'_c =$ 4.0 ksi
$f_y =$ 60 ksi
Clearance clr = 6 in



1) AASHTO LRFD 5.7.3.3.2

$Ø M_n \geq 1.2 M_{cr}$ with Ø = 0.9 (AASHTO 5.5.4.2.1)

$f_r = 0.37 \times f'_c{}^{0.5} =$ 0.740 ksi (AASHTO 5.4.2.6)
$S_c = I / y_b =$ 864000.0 in3
$M_{cr} = S_c \times f_r =$ 639360.0 kip-in
$Ø M_n = Ø \times A_s \times f_y \times (d - a / 2) = Ø \times A_s \times f_y \times (d - A_s \times f_y / 2 / 0.85 / f'_c / b)$ (AASHTO 5.7.3.2.2)
Bar diameter $d_b =$ 1.69 in assuming #14 bar
$d = h - clr - d_b / 2 =$ 113.15 in
Solve for $A_s =$ 129.18 in2 -> provide 58-#14 bars

2) LEONHARDT

$A_s \geq 1.2 F_{ct} / f_y$
$f_{ct} = 0.23 \times f'_c{}^{0.5} =$ 0.460 ksi
$F_{ct} = f_{ct} \times y_b \times b / 2 =$ 4968 kips
$A_s =$ 99.36 in2 -> provide 64-#11 bars

3) EUROCODE

$A_s \geq 0.26 \times f_{ctm} / f_y \times b_t \times d$ (Eq 1) and $A_s \geq 0.0013 \times b_t \times d$ (Eq 2)
$f_{ck} = f'_c =$ 4.0 ksi = 28 Mpa
$f_{ctm} = 0.30 \times f_{ck}{}^{2/3} =$ 2.8 Mpa = 0.401 ksi
Bar diameter $d_b =$ 1.41 in assuming #11 bar
$d = h - clr - d_b / 2 =$ 113.30 in
(Eq 1) $A_s =$ 70.91 in2
(Eq 2) $A_s =$ 53.02 in2

As = 70.91 in2 -> provide 46-#11 bars

4) PROPOSED METHOD

$Ø M_n \geq 0.67 \times 1.6 f_r \times S_c$ with Ø = 0.9

$f_r = 0.237 \times f'_c{}^{0.5} =$ 0.474 ksi
$Ø M_n = Ø \times A_s \times f_y \times (d - a / 2) = Ø \times A_s \times f_y \times (d - A_s \times f_y / 2 / 0.85 / f'_c / b)$
Bar diameter $d_b =$ 1.41 in assuming #11 bar
$d = h - clr - d_b / 2 =$ 113.30 in
Solve for $A_s =$ 72.91 in2

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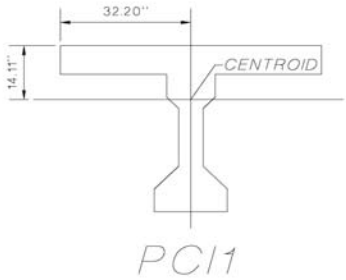
CALCULATION OF MINIMUM REINFORCEMENT - PRECAST PRESTRESSED I-GIRDER PCI1

$h =$ 43.50 in
$b_f =$ 64.40 in
$t_d =$ 7.50 in
$A =$ 852.00 in2
$I =$ 172939.74 in4
$y_b =$ 29.39 in
$f'^{g}_{c} =$ 5.0 ksi
$f'^{d}_{c} =$ 4.0 ksi
$f_y =$ 60 ksi
$f_{pu} =$ 270 ksi



1) AASHTO LRFD 5.7.3.3.2

$\varnothing M_n \geq 1.2 M_{cr}$ with $\varnothing =$ 1.0 (AASHTO 5.5.4.2.1)

Given $d_p =$ 39.15 in
Assume min $A_{ps} =$ 2.61 in2 (for trial and error purpose)
$P_f = A_{ps} \times (0.75 \times f_{pu} - 30\text{ ksi}) =$ 450.7 kips assuming 30 ksi losses
$f_{cpe} = P_f / A + P_f \times (d_p - y_t) \times y_b / I =$ 2.828 ksi
$f_r = 0.37 \times f'^{g\,0.5}_{c} =$ 0.827 ksi (AASHTO 5.4.2.6)
$S_c = I / y_b =$ 5884.3 in3
$1.2 M_{cr} = 1.2 \times S_c \times (f_r + f_{cpe}) =$ 25813 kip-in

$\varnothing M_n = \varnothing \times A_{ps} \times f_{ps} \times (d_p - a / 2)$ (AASHTO 5.7.3.2.2)
$c = (A_{ps} \times f_{pu}) / (0.85 \times f'^{d}_{c} \times 0.85 \times b + 0.28 \times A_{ps} \times f_{pu} / d_p) =$ 3.69 in
$f_{ps} = f_{pu} \times (1 - 0.28 \times c / d_p) =$ 263 ksi
$\varnothing \times A_{ps} \times f_{ps} \times (d_p - a / 2) =$ 25813 kip-in matches $1.2 M_{cr}$

2) LEONHARDT

$A_{ps} \geq 1.2 F_{ct} / (f_{pu} - f_{pe})$
$f_{ct} = 0.23 \times f'^{g\,0.5}_{c} =$ 0.514 ksi
$F_{ct} =$ 91.5 kips (see hand calculations)
$f_{pe} = 0.75 \times f_{pu} - 30\text{ ksi} =$ 172.5 ksi assuming 30 ksi losses
$A_{ps} =$ 1.13 in2

3) EUROCODE

$A_p \geq M_{rep} / (z \times \Delta\sigma_p)$

$f_{ck} = f'^{g}_{c} =$ 5.0 ksi = 35 Mpa
$f_{ctm} = 0.30 \times f_{ck}^{2/3} =$ 3.2 Mpa = 0.466 ksi
$M_{rep} = S_c \times f_{ctm} =$ 2739.6 kip-in
$\Delta\sigma_p = \text{Min}(0.4 \times 270, 72.5) =$ 72.5 ksi
$z = 0.9 \times d =$ 35.24 in
$A_p =$ 1.07 in2

4) PROPOSED METHOD

$\varnothing M_n \geq 1.0 \times (1.6 f_r + 1.1 f_{pe}) \times S_c$ with $\varnothing =$ 1.0

Assume min $A_{ps} =$ 1.59 in2 (for trial and error purpose)
$P_f = A_{ps} \times (0.75 \times f_{pu} - 30\text{ ksi}) =$ 274.4 kips assuming 30 ksi losses
$f_{pe} = P_f / A + P_f \times (d_p - y_t) \times y_b / I =$ 1.722 ksi
$f_r = 0.237 \times f'^{g\,0.5}_{c} =$ 0.530 ksi
$1.0 \times (1.6 f_r + 1.1 f_{pe}) \times S_c =$ 16135 kip-in

$\varnothing M_n = \varnothing \times A_{ps} \times f_{ps} \times (d_p - a / 2)$
$c = (A_{ps} \times f_{pu}) / (0.85 \times f'^{d}_{c} \times 0.85 \times b + 0.28 \times A_{ps} \times f_{pu} / d_p) =$ 2.27 in
$f_{ps} = f_{pu} \times (1 - 0.28 \times c / d_p) =$ 265.6 ksi
$\varnothing \times A_{ps} \times f_{ps} \times (d_p - a / 2) =$ 16135 kip-in matches $(1.6 f_r + 1.1 f_{pe}) \times S_c$

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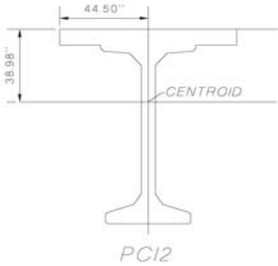
CALCULATION OF MINIMUM REINFORCEMENT - PRECAST PRESTRESSED I-GIRDER PCI2

$h =$ 104.50 in
$b_f =$ 88.99 in
$t_d =$ 8.50 in
$A =$ 1953.70 in2
$I =$ 3000090.93 in4
$y_b =$ 65.52 in
$f'^g_c =$ 10.0 ksi
$f'^d_c =$ 5.5 ksi
$f_y =$ 60 ksi
$f_{pu} =$ 270 ksi



1) AASHTO LRFD 5.7.3.3.2

$\varnothing M_n \geq 1.2 M_{cr}$ with $\varnothing =$ 1.0 (AASHTO 5.5.4.2.1)

Given $d_p =$ 94.05 in
Assume min $A_{ps} =$ 9.09 in2 (for trial and error purpose)
$P_f = A_{ps} \times (0.75 \times f_{pu} - 30 \text{ ksi}) =$ 1567.8 kips assuming 30 ksi losses
$f_{cpe} = P_f / A + P_f \times (d_p - y_t) \times y_b / I =$ 2.811 ksi
$f_r = 0.37 \times f'^{g\,0.5}_c =$ 1.170 ksi (AASHTO 5.4.2.6)
$S_c = I / y_b =$ 45788.9 in3
$1.2 M_{cr} = 1.2 \times S_c \times (f_r + f_{cpe}) =$ 218758 kip-in

$\varnothing M_n = \varnothing \times A_{ps} \times f_{ps} \times (d_p - a / 2)$ (AASHTO 5.7.3.2.2)
$c = (A_{ps} \times f_{pu}) / (0.85 \times f'^d_c \times 0.775 \times b + 0.28 \times A_{ps} \times f_{pu} / d_p) =$ 7.44 in
$f_{ps} = f_{pu} \times (1 - 0.28 \times c / d_p) =$ 264 ksi
$\varnothing \times A_{ps} \times f_{ps} \times (d_p - a / 2) =$ 218758 kip-in matches $1.2 M_{cr}$

2) LEONHARDT

$A_{ps} \geq 1.2 F_{ct} / (f_{pu} - f_{pe})$
$f_{ct} = 0.23 \times f'^{g\,0.5}_c =$ 0.727 ksi
$F_{ct} =$ 387.1 kips (see hand calculations)
$f_{pe} = 0.75 \times f_{pu} - 30 \text{ ksi} =$ 172.5 ksi assuming 30 ksi losses
$A_{ps} =$ 4.76 in2

3) EUROCODE

$A_p \geq M_{rep} / (z \times \Delta\sigma_p)$

$f_{ck} = f'^g_c =$ 10.0 ksi = 70 Mpa
$f_{ctm} = 2.12 \times \ln(1 + (f_{ck} + 8) / 10) =$ 4.6 Mpa = 0.669 ksi
$M_{rep} = S_c \times f_{ctm} =$ 30619.2 kip-in
$\Delta\sigma_p = \text{Min}(0.4 \times 270, 72.5) =$ 72.5 ksi
$z = 0.9 \times d =$ 84.65 in
$A_p =$ 4.99 in2

4) PROPOSED METHOD

$\varnothing M_n \geq 1.0 \times (1.6 f_r + 1.1 f_{pe}) \times S_c$ with $\varnothing =$ 1.0

Assume min $A_{ps} =$ 6.20 in2 (for trial and error purpose)
$P_f = A_{ps} \times (0.75 \times f_{pu} - 30 \text{ ksi}) =$ 1070.3 kips assuming 30 ksi losses
$f_{pe} = P_f / A + P_f \times (d_p - y_t) \times y_b / I =$ 1.919 ksi
$f_r = 0.237 \times f'^{g\,0.5}_c =$ 0.749 ksi
$1.0 \times (1.6 f_r + 1.1 f_{pe}) \times S_c =$ 151576 kip-in

$\varnothing M_n = \varnothing \times A_{ps} \times f_{ps} \times (d_p - a / 2)$
$c = (A_{ps} \times f_{pu}) / (0.85 \times f'^d_c \times 0.775 \times b + 0.28 \times A_{ps} \times f_{pu} / d_p) =$ 5.12 in
$f_{ps} = f_{pu} \times (1 - 0.28 \times c / d_p) =$ 265.9 ksi
$\varnothing \times A_{ps} \times f_{ps} \times (d_p - a / 2) =$ 151576 kip-in matches $(1.6 f_r + 1.1 f_{pe}) \times S_c$

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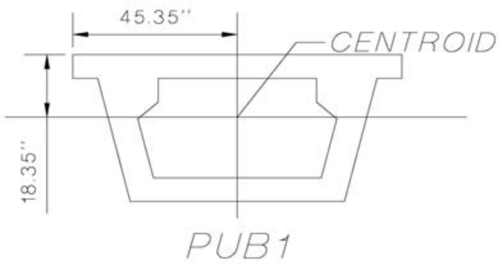
CALCULATION OF MINIMUM REINFORCEMENT - PRECAST PRESTRESSED U-BEAM PUB1

h = 43.00 in
b_f = 90.71 in
t_d = 7.00 in
A = 1667.03 in2
I = 377841.47 in4
y_b = 24.65 in
f'^g_c = 7.0 ksi
f'^d_c = 4.0 ksi
f_y = 60 ksi
f_{pu} = 270 ksi
b_w = 33.46 in



1) AASHTO LRFD 5.7.3.3.2

$\text{Ø } M_n \geq 1.2\ M_{cr}$ with Ø = 1.0 (AASHTO 5.5.4.2.1)

Given d_p = 38.70 in
Assume min A_{ps} = 8.77 in2 (for trial and error purpose)
$P_f = A_{ps} \times (0.75 \times f_{pu} - 30\text{ ksi})$ = 1512.8 kips assuming 30 ksi losses
$f_{cpe} = P_f / A + P_f \times (d_p - y_t) \times y_b / I$ = 3.232 ksi
$f_r = 0.37 \times f'^{g\ 0.5}_c$ = 0.979 ksi (AASHTO 5.4.2.6)
$S_c = I / y_b$ = 15328.3 in3
$1.2\ M_{cr} = 1.2 \times S_c \times (f_r + f_{cpe})$ = 77462 kip-in

$\text{Ø } M_n = \text{Ø} \times A_{ps} \times f_{ps} \times (d_p - a / 2)$ (AASHTO 5.7.3.2.2)
$c = (A_{ps} \times f_{pu} - 0.85 \times f'_c \times (b - b_w) \times h_f) / (0.85 \times f'^d_c \times 0.85 \times b_w + 0.28 \times A_{ps} \times f_{pu} / d_p)$ = 8.83 in
$f_{ps} = f_{pu} \times (1 - 0.28 \times c / d_p)$ = 253 ksi
$\text{Ø} \times A_{ps} \times f_{ps} \times (d_p - a / 2)$ = 77462 kip-in matches $1.2\ M_{cr}$

2) LEONHARDT

$A_{ps} \geq 1.2\ F_{ct} / (f_{pu} - f_{pe})$
$f_{ct} = 0.23 \times f'^{g\ 0.5}_c$ = 0.609 ksi
F_{ct} = 276.5 kips (see hand calculations)
$f_{pe} = 0.75 \times f_{pu} - 30\text{ ksi}$ = 172.5 ksi assuming 30 ksi losses
A_{ps} = 3.40 in2

3) EUROCODE

$A_p \geq M_{rep} / (z \times \Delta\sigma_p)$

$f_{ck} = f'^g_c$ = 7.0 ksi = 50 Mpa
$f_{ctm} = 0.30 \times f_{ck}^{2/3}$ = 4.1 Mpa = 0.591 ksi
$M_{rep} = S_c \times f_{ctm}$ = 9052.1 kip-in
$\Delta\sigma_p = \text{Min}(0.4 \times 270, 72.5)$ = 72.5 ksi
$z = 0.9 \times d$ = 34.83 in
A_p = 3.58 in2

4) PROPOSED METHOD

$\text{Ø } M_n \geq 1.0 \times (1.6\ f_r + 1.1\ f_{pe}) \times S_c$ with Ø = 1.0

Assume min A_{ps} = 4.53 in2 (for trial and error purpose)
$P_f = A_{ps} \times (0.75 \times f_{pu} - 30\text{ ksi})$ = 782.1 kips assuming 30 ksi losses
$f_{pe} = P_f / A + P_f \times (d_p - y_t) \times y_b / I$ = 1.671 ksi
$f_r = 0.237 \times f'^{g\ 0.5}_c$ = 0.627 ksi
$1.0 \times (1.6\ f_r + 1.1\ f_{pe}) \times S_c$ = 43555 kip-in

$\text{Ø } M_n = \text{Ø} \times A_{ps} \times f_{ps} \times (d_p - a / 2)$
$c = (A_{ps} \times f_{pu}) / (0.85 \times f'^d_c \times 0.85 \times b + 0.28 \times A_{ps} \times f_{pu} / d_p)$ = 4.52 in
$f_{ps} = f_{pu} \times (1 - 0.28 \times c / d_p)$ = 261.2 ksi
$\text{Ø} \times A_{ps} \times f_{ps} \times (d_p - a / 2)$ = 43555 kip-in matches $(1.6\ f_r + 1.1\ f_{pe}) \times S_c$

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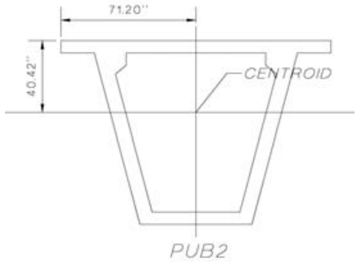
CALCULATION OF MINIMUM REINFORCEMENT - PRECAST PRESTRESSED U-BEAM PUB2

h = 103.00 in
b_f = 142.39 in
t_d = 7.00 in
A = 3002.75 in2
I = 4081912.23 in4
y_b = 62.58 in
f'^g_c = 10.0 ksi
f'^d_c = 5.5 ksi
f_y = 60 ksi
f_{pu} = 270 ksi



1) AASHTO LRFD 5.7.3.3.2

$Ø M_n \geq 1.2 M_{cr}$ with Ø = 1.0 (AASHTO 5.5.4.2.1)

Given d_p = 92.70 in
Assume min A_{ps} = 12.68 in2 (for trial and error purpose)
$P_f = A_{ps} \times (0.75 \times f_{pu} - 30 \text{ ksi})$ = 2187.4 kips assuming 30 ksi losses
$f_{cpe} = P_f / A + P_f \times (d_p - y_t) \times y_b / I$ = 2.697 ksi
$f_r = 0.37 \times f'^{g\ 0.5}_c$ = 1.170 ksi (AASHTO 5.4.2.6)
$S_c = I / y_b$ = 65227.1 in3
$1.2 M_{cr} = 1.2 \times S_c \times (f_r + f_{cpe})$ = 302689 kip-in

$Ø M_n = Ø \times A_{ps} \times f_{ps} \times (d_p - a / 2)$ (AASHTO 5.7.3.2.2)
$c = (A_{ps} \times f_{pu}) / (0.85 \times f'^d_c \times 0.775 \times b + 0.28 \times A_{ps} \times f_{pu} / d_p)$ = 6.51 in
$f_{ps} = f_{pu} \times (1 - 0.28 \times c / d_p)$ = 265 ksi
$Ø \times A_{ps} \times f_{ps} \times (d_p - a / 2)$ = 302689 kip-in matches $1.2 M_{cr}$

2) LEONHARDT

$A_{ps} \geq 1.2 F_{ct} / (f_{pu} - f_{pe})$
$f_{ct} = 0.23 \times f'^{g\ 0.5}_c$ = 0.727 ksi
F_{ct} = 572.2 kips (see hand calculations)
$f_{pe} = 0.75 \times f_{pu} - 30$ ksi = 172.5 ksi assuming 30 ksi losses
A_{ps} = 7.04 in2

3) EUROCODE

$A_p \geq M_{rep} / (z \times \Delta\sigma_p)$

$f_{ck} = f'^g_c$ = 10.0 ksi = 70 Mpa
$f_{ctm} = 2.12 \times \ln(1 + (f_{ck} + 8) / 10)$ = 4.6 Mpa = 0.669 ksi
$M_{rep} = S_c \times f_{ctm}$ = 43617.6 kip-in
$\Delta\sigma_p = \text{Min}(0.4 \times 270, 72.5)$ = 72.5 ksi
$z = 0.9 \times d$ = 83.43 in
A_p = 7.21 in2

4) PROPOSED METHOD

$Ø M_n \geq 1.0 \times (1.6 f_r + 1.1 f_{pe}) \times S_c$ with Ø = 1.0

Assume min A_{ps} = 8.78 in2 (for trial and error purpose)
$P_f = A_{ps} \times (0.75 \times f_{pu} - 30 \text{ ksi})$ = 1513.7 kips assuming 30 ksi losses
$f_{pe} = P_f / A + P_f \times (d_p - y_t) \times y_b / I$ = 1.866 ksi
$f_r = 0.237 \times f'^{g\ 0.5}_c$ = 0.749 ksi
$1.0 \times (1.6 f_r + 1.1 f_{pe}) \times S_c$ = 212126 kip-in

$Ø M_n = Ø \times A_{ps} \times f_{ps} \times (d_p - a / 2)$
$c = (A_{ps} \times f_{pu}) / (0.85 \times f'^d_c \times 0.775 \times b + 0.28 \times A_{ps} \times f_{pu} / d_p)$ = 4.53 in
$f_{ps} = f_{pu} \times (1 - 0.28 \times c / d_p)$ = 266.3 ksi
$Ø \times A_{ps} \times f_{ps} \times (d_p - a / 2)$ = 212126 kip-in matches $(1.6 f_r + 1.1 f_{pe}) \times S_c$

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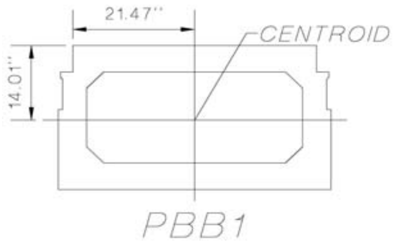
CALCULATION OF MINIMUM REINFORCEMENT - PRECAST PRESTRESSED BOX-BEAM PBB1

h = 27.00 in
b_f = 42.93 in
t_d = 5.00 in
A = 633.04 in2
I = 60659.69 in4
y_b = 12.99 in
f'^g_c = 5.0 ksi
f'^d_c = 4.0 ksi
f_y = 60 ksi
f_{pu} = 270 ksi
b_w = 15.26 in



1) AASHTO LRFD 5.7.3.3.2

$Ø M_n \geq 1.2 M_{cr}$ with Ø = 1.0 (AASHTO 5.5.4.2.1)

Given d_p = 24.30 in
Assume min A_{ps} = 2.90 in2 (for trial and error purpose)
$P_f = A_{ps} \times (0.75 \times f_{pu} - 30\text{ ksi})$ = 500.3 kips assuming 30 ksi losses
$f_{cpe} = P_f / A + P_f \times (d_p - y_t) \times y_b / I$ = 2.099 ksi
$f_r = 0.37 \times f'^{g\,0.5}_c$ = 0.827 ksi (AASHTO 5.4.2.6)
$S_c = I / y_b$ = 4669.7 in3
$1.2 M_{cr} = 1.2 \times S_c \times (f_r + f_{cpe})$ = 16399 kip-in

$Ø M_n = Ø \times A_{ps} \times f_{ps} \times (d_p - a / 2)$ (AASHTO 5.7.3.2.2)
$c = (A_{ps} \times f_{pu} - 0.85 \times f'_c \times (b - b_w) \times h_f) / (0.85 \times f'^d_c \times 0.85 \times b_w + 0.28 \times A_{ps} \times f_{pu} / d_p)$ = 5.88 in
$f_{ps} = f_{pu} \times (1 - 0.28 \times c / d_p)$ = 252 ksi
$Ø \times A_{ps} \times f_{ps} \times (d_p - a / 2)$ = 15912 kip-in matches $1.2 M_{cr}$

2) LEONHARDT

$A_{ps} \geq 1.2 F_{ct} / (f_{pu} - f_{pe})$
$f_{ct} = 0.23 \times f'^{g\,0.5}_c$ = 0.514 ksi
F_{ct} = 114.6 kips (see hand calculations)
$f_{pe} = 0.75 \times f_{pu} - 30\text{ ksi}$ = 172.5 ksi assuming 30 ksi losses
A_{ps} = 1.41 in2

3) EUROCODE

$A_p \geq M_{rep} / (z \times \Delta\sigma_p)$

$f_{ck} = f'^g_c$ = 5.0 ksi = 35 Mpa
$f_{ctm} = 0.30 \times f_{ck}^{2/3}$ = 3.2 Mpa = 0.466 ksi
$M_{rep} = S_c \times f_{ctm}$ = 2174.1 kip-in
$\Delta\sigma_p = \text{Min}(0.4 \times 270, 72.5)$ = 72.5 ksi
$z = 0.9 \times d$ = 21.87 in
A_p = 1.37 in2

4) PROPOSED METHOD

$Ø M_n \geq 1.0 \times (1.6 f_r + 1.1 f_{pe}) \times S_c$ with Ø = 1.0

Assume min A_{ps} = 1.86 in2 (for trial and error purpose)
$P_f = A_{ps} \times (0.75 \times f_{pu} - 30\text{ ksi})$ = 321.4 kips assuming 30 ksi losses
$f_{pe} = P_f / A + P_f \times (d_p - y_t) \times y_b / I$ = 1.349 ksi
$f_r = 0.237 \times f'^{g\,0.5}_c$ = 0.530 ksi
$1.0 \times (1.6 f_r + 1.1 f_{pe}) \times S_c$ = 10887 kip-in

$Ø M_n = Ø \times A_{ps} \times f_{ps} \times (d_p - a / 2)$
$c = (A_{ps} \times f_{pu}) / (0.85 \times f'^d_c \times 0.85 \times b + 0.28 \times A_{ps} \times f_{pu} / d_p)$ = 3.87 in
$f_{ps} = f_{pu} \times (1 - 0.28 \times c / d_p)$ = 258.0 ksi
$Ø \times A_{ps} \times f_{ps} \times (d_p - a / 2)$ = 10887 kip-in matches $(1.6 f_r + 1.1 f_{pe}) \times S_c$

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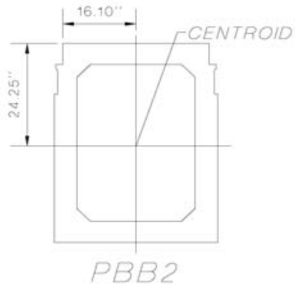
CALCULATION OF MINIMUM REINFORCEMENT - PRECAST PRESTRESSED BOX-BEAM PBB2

h = 47.00 in
b_f = 32.20 in
t_d = 5.00 in
A = 719.39 in2
I = 196297.71 in4
y_b = 22.75 in
f'^{g}_{c} = 5.0 ksi
f'^{d}_{c} = 4.0 ksi
f_y = 60 ksi
f_{pu} = 270 ksi
b_w = 15.26 in



1) AASHTO LRFD 5.7.3.3.2

$Ø M_n \geq 1.2 M_{cr}$ with Ø = 1.0 (AASHTO 5.5.4.2.1)

Given d_p = 42.30 in
Assume min A_{ps} = 3.07 in2 (for trial and error purpose)
$P_f = A_{ps} \times (0.75 \times f_{pu} - 30 \text{ ksi})$ = 529.6 kips assuming 30 ksi losses
$f_{cpe} = P_f / A + P_f \times (d_p - y_t) \times y_b / I$ = 1.999 ksi
$f_r = 0.37 \times f'^{g\,0.5}_{c}$ = 0.827 ksi (AASHTO 5.4.2.6)
$S_c = I / y_b$ = 8628.5 in3
$1.2 M_{cr} = 1.2 \times S_c \times (f_r + f_{cpe})$ = 29268 kip-in

$Ø M_n = Ø \times A_{ps} \times f_{ps} \times (d_p - a / 2)$ (AASHTO 5.7.3.2.2)
$c = (A_{ps} \times f_{pu} - 0.85 \times f'_c \times (b - b_w) \times h_f) / (0.85 \times f'^{d}_{c} \times 0.85 \times b_w + 0.28 \times A_{ps} \times f_{pu} / d_p)$ = 10.91 in
$f_{ps} = f_{pu} \times (1 - 0.28 \times c / d_p)$ = 251 ksi
$Ø \times A_{ps} \times f_{ps} \times (d_p - a / 2)$ = 28965 kip-in matches $1.2 M_{cr}$

2) LEONHARDT

$A_{ps} \geq 1.2 F_{ct} / (f_{pu} - f_{pe})$
$f_{ct} = 0.23 \times f'^{g\,0.5}_{c}$ = 0.514 ksi
F_{ct} = 121.3 kips (see hand calculations)
$f_{pe} = 0.75 \times f_{pu} - 30 \text{ ksi}$ = 172.5 ksi assuming 30 ksi losses
A_{ps} = 1.49 in2

3) EUROCODE

$A_p \geq M_{rep} / (z \times \Delta\sigma_p)$

$f_{ck} = f'^{g}_{c}$ = 5.0 ksi = 35 Mpa
$f_{ctm} = 0.30 \times f_{ck}^{2/3}$ = 3.2 Mpa = 0.466 ksi
$M_{rep} = S_c \times f_{ctm}$ = 4017.2 kip-in
$\Delta\sigma_p = \text{Min}(0.4 \times 270, 72.5)$ = 72.5 ksi
$z = 0.9 \times d$ = 38.07 in
A_p = 1.46 in2

4) PROPOSED METHOD

$Ø M_n \geq 1.0 \times (1.6 f_r + 1.1 f_{pe}) \times S_c$ with Ø = 1.0

Assume min A_{ps} = 1.68 in2 (for trial and error purpose)
$P_f = A_{ps} \times (0.75 \times f_{pu} - 30 \text{ ksi})$ = 289.5 kips assuming 30 ksi losses
$f_{pe} = P_f / A + P_f \times (d_p - y_t) \times y_b / I$ = 1.093 ksi
$f_r = 0.237 \times f'^{g\,0.5}_{c}$ = 0.530 ksi
$1.0 \times (1.6 f_r + 1.1 f_{pe}) \times S_c$ = 17691 kip-in

$Ø M_n = Ø \times A_{ps} \times f_{ps} \times (d_p - a / 2)$
$c = (A_{ps} \times f_{pu}) / (0.85 \times f'^{d}_{c} \times 0.85 \times b + 0.28 \times A_{ps} \times f_{pu} / d_p)$ = 4.72 in
$f_{ps} = f_{pu} \times (1 - 0.28 \times c / d_p)$ = 261.6 ksi
$Ø \times A_{ps} \times f_{ps} \times (d_p - a / 2)$ = 17691 kip-in matches $(1.6 f_r + 1.1 f_{pe}) \times S_c$

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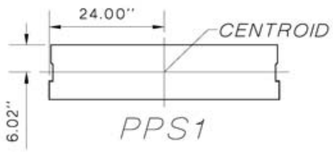
CALCULATION OF MINIMUM REINFORCEMENT - PRECAST PRESTRESSED SLAB PPS1

h = 12.00 in
b = 48.00 in
A = 567.14 in2
I = 6852.25 in4
y_b = 5.98 in
f'_c = 4.0 ksi
f_y = 60 ksi
f_{pu} = 270 ksi



1) AASHTO LRFD 5.7.3.3.2

$\varnothing M_n \geq 1.2 M_{cr}$ with Ø = 1.0 (AASHTO 5.5.4.2.1)

Given d_p = 10.80 in
Assume min A_{ps} = 0.84 in2 (for trial and error purpose)
$P_f = A_{ps} \times (0.75 \times f_{pu} - 30 \text{ ksi})$ = 144.0 kips assuming 30 ksi losses
$f_{cpe} = P_f / A + P_f \times (d_p - y_t) \times y_b / I$ = 0.855 ksi
$f_r = 0.37 \times f'^{0.5}_c$ = 0.740 ksi (AASHTO 5.4.2.6)
$S_c = I / y_b$ = 1145.9 in3
$1.2 M_{cr} = 1.2 \times S_c \times (f_r + f_{cpe})$ = 2193 kip-in

$\varnothing M_n = \varnothing \times A_{ps} \times f_{ps} \times (d_p - a / 2)$ (AASHTO 5.7.3.2.2)
$c = (A_{ps} \times f_{pu}) / (0.85 \times f'_c \times 0.85 \times b + 0.28 \times A_{ps} \times f_{pu} / d_p)$ = 1.56 in
$f_{ps} = f_{pu} \times (1 - 0.28 \times c / d_p)$ = 259 ksi
$\varnothing \times A_{ps} \times f_{ps} \times (d_p - a / 2)$ = 2193 kip-in matches $1.2 M_{cr}$

2) LEONHARDT

$A_{ps} \geq 1.2 F_{ct} / (f_{pu} - f_{pe})$
$f_{ct} = 0.23 \times f'^{0.5}_c$ = 0.460 ksi
F_{ct} = 65.8 kips (see hand calculations)
$f_{pe} = 0.75 \times f_{pu} - 30$ ksi = 172.5 ksi assuming 30 ksi losses
A_{ps} = 0.81 in2

3) EUROCODE

$A_p \geq M_{rep} / (z \times \Delta\sigma_p)$

$f_{ck} = f'_c$ = 4.0 ksi = 28 Mpa
$f_{ctm} = 0.30 \times f_{ck}^{2/3}$ = 2.8 Mpa = 0.401 ksi
$M_{rep} = S_c \times f_{ctm}$ = 459.7 kip-in
$\Delta\sigma_p$ = Min(0.4 x 270, 72.5) = 72.5 ksi
z = 0.9 x d = 9.72 in
A_p = 0.65 in2

4) PROPOSED METHOD

$\varnothing M_n \geq 1.0 \times (1.6 f_r + 1.1 f_{pe}) \times S_c$ with Ø = 1.0

Assume min A_{ps} = 0.62 in2 (for trial and error purpose)
$P_f = A_{ps} \times (0.75 \times f_{pu} - 30 \text{ ksi})$ = 106.4 kips assuming 30 ksi losses
$f_{pe} = P_f / A + P_f \times (d_p - y_t) \times y_b / I$ = 0.632 ksi
$f_r = 0.237 \times f'^{0.5}_c$ = 0.474 ksi
$1.0 \times (1.6 f_r + 1.1 f_{pe}) \times S_c$ = 1665 kip-in

$\varnothing M_n = \varnothing \times A_{ps} \times f_{ps} \times (d_p - a / 2)$
$c = (A_{ps} \times f_{pu}) / (0.85 \times f'_c \times 0.85 \times b + 0.28 \times A_{ps} \times f_{pu} / d_p)$ = 1.16 in
$f_{ps} = f_{pu} \times (1 - 0.28 \times c / d_p)$ = 261.8 ksi
$\varnothing \times A_{ps} \times f_{ps} \times (d_p - a / 2)$ = 1665 kip-in matches $(1.6 f_r + 1.1 f_{pe}) \times S_c$

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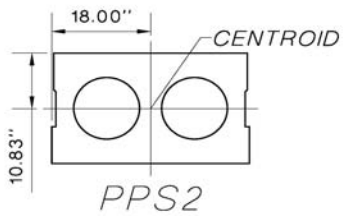
CALCULATION OF MINIMUM REINFORCEMENT - PRECAST PRESTRESSED SLAB PPS2

h = 21.50 in
b = 36.00 in
A = 531.82 in2
I = 27433.25 in4
y_b = 10.67 in
f'_c = 4.0 ksi
f_y = 60 ksi
f_{pu} = 270 ksi



1) AASHTO LRFD 5.7.3.3.2

$\varnothing M_n \geq 1.2 M_{cr}$ with Ø = 1.0 (AASHTO 5.5.4.2.1)

Given d_p = 19.35 in
Assume min A_{ps} = 1.20 in2 (for trial and error purpose)
$P_f = A_{ps} \times (0.75 \times f_{pu} - 30\text{ ksi})$ = 206.2 kips assuming 30 ksi losses
$f_{cpe} = P_f / A + P_f \times (d_p - y_t) \times y_b / I$ = 1.071 ksi
$f_r = 0.37 \times f'_c{}^{0.5}$ = 0.740 ksi (AASHTO 5.4.2.6)
$S_c = I / y_b$ = 2571.1 in3
$1.2 M_{cr} = 1.2 \times S_c \times (f_r + f_{cpe})$ = 5588 kip-in

$\varnothing M_n = \varnothing \times A_{ps} \times f_{ps} \times (d_p - a / 2)$ (AASHTO 5.7.3.2.2)
$c = (A_{ps} \times f_{pu}) / (0.85 \times f'_c \times 0.85 \times b + 0.28 \times A_{ps} \times f_{pu} / d_p)$ = 2.97 in
$f_{ps} = f_{pu} \times (1 - 0.28 \times c / d_p)$ = 258 ksi
$\varnothing \times A_{ps} \times f_{ps} \times (d_p - a / 2)$ = 5588 kip-in matches $1.2 M_{cr}$

2) LEONHARDT

$A_{ps} \geq 1.2 F_{ct} / (f_{pu} - f_{pe})$
$f_{ct} = 0.23 \times f'_c{}^{0.5}$ = 0.460 ksi
F_{ct} = 73.9 kips (see hand calculations)
$f_{pe} = 0.75 \times f_{pu} - 30\text{ ksi}$ = 172.5 ksi assuming 30 ksi losses
A_{ps} = 0.91 in2

3) EUROCODE

$A_p \geq M_{rep} / (z \times \Delta\sigma_p)$

$f_{ck} = f'_c$ = 4.0 ksi = 28 Mpa
$f_{ctm} = 0.30 \times f_{ck}{}^{2/3}$ = 2.8 Mpa = 0.401 ksi
$M_{rep} = S_c \times f_{ctm}$ = 1031.6 kip-in
$\Delta\sigma_p = \text{Min}(0.4 \times 270, 72.5)$ = 72.5 ksi
$z = 0.9 \times d$ = 17.42 in
A_p = 0.82 in2

4) PROPOSED METHOD

$\varnothing M_n \geq 1.0 \times (1.6 f_r + 1.1 f_{pe}) \times S_c$ with Ø = 1.0

Assume min A_{ps} = 0.85 in2 (for trial and error purpose)
$P_f = A_{ps} \times (0.75 \times f_{pu} - 30\text{ ksi})$ = 146.8 kips assuming 30 ksi losses
$f_{pe} = P_f / A + P_f \times (d_p - y_t) \times y_b / I$ = 0.762 ksi
$f_r = 0.237 \times f'_c{}^{0.5}$ = 0.474 ksi
$1.0 \times (1.6 f_r + 1.1 f_{pe}) \times S_c$ = 4106 kip-in

$\varnothing M_n = \varnothing \times A_{ps} \times f_{ps} \times (d_p - a / 2)$
$c = (A_{ps} \times f_{pu}) / (0.85 \times f'_c \times 0.85 \times b + 0.28 \times A_{ps} \times f_{pu} / d_p)$ = 2.14 in
$f_{ps} = f_{pu} \times (1 - 0.28 \times c / d_p)$ = 261.6 ksi
$\varnothing \times A_{ps} \times f_{ps} \times (d_p - a / 2)$ = 4106 kip-in matches $(1.6 f_r + 1.1 f_{pe}) \times S_c$

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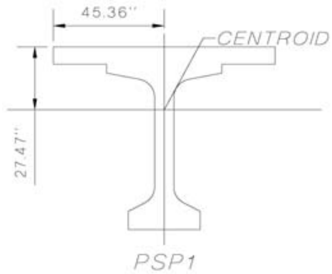
CALCULATION OF MINIMUM REINFORCEMENT - SPLICED PRECAST PRESTRESSED GIRDER PSP1

h = 79.50 in
b_f = 90.71 in
t_d = 7.50 in
A = 1737.88 in2
I = 1365254.77 in4
y_b = 52.03 in
f'^{g}_{c} = 7.0 ksi
f'^{d}_{c} = 4.0 ksi
f_y = 60 ksi
f_{pu} = 270 ksi



1) AASHTO LRFD 5.7.3.3.2

$Ø M_n \geq 1.2 M_{cr}$ with Ø = 1.0 (AASHTO 5.5.4.2.1)

Given d_p = 71.55 in
Assume min A_{ps} = 5.91 in2 (for trial and error purpose)
$P_f = A_{ps} \times (0.75 \times f_{pu} - 30\ \text{ksi})$ = 1019.1 kips assuming 30 ksi losses
$f_{cpe} = P_f / A + P_f \times (d_p - y_t) \times y_b / I$ = 2.436 ksi
$f_r = 0.37 \times f'^{g\ 0.5}_{c}$ = 0.979 ksi (AASHTO 5.4.2.6)
$S_c = I / y_b$ = 26239.8 in3
$1.2 M_{cr} = 1.2 \times S_c \times (f_r + f_{cpe})$ = 107540 kip-in

$Ø M_n = Ø \times A_{ps} \times f_{ps} \times (d_p - a / 2)$ (AASHTO 5.7.3.2.2)
$c = (A_{ps} \times f_{pu}) / (0.85 \times f'^{d}_{c} \times 0.85 \times b + 0.28 \times A_{ps} \times f_{pu} / d_p)$ = 5.94 in
$f_{ps} = f_{pu} \times (1 - 0.28 \times c / d_p)$ = 264 ksi
$Ø \times A_{ps} \times f_{ps} \times (d_p - a / 2)$ = 107540 kip-in matches $1.2 M_{cr}$

2) LEONHARDT

$A_{ps} \geq 1.2 F_{ct} / (f_{pu} - f_{pe})$
$f_{ct} = 0.23 \times f'^{g\ 0.5}_{c}$ = 0.609 ksi
F_{ct} = 256.6 kips (see hand calculations)
$f_{pe} = 0.75 \times f_{pu} - 30\ \text{ksi}$ = 172.5 ksi assuming 30 ksi losses
A_{ps} = 3.16 in2

3) EUROCODE

$A_p \geq M_{rep} / (z \times \Delta\sigma_p)$

$f_{ck} = f'^{g}_{c}$ = 7.0 ksi = 50 Mpa
$f_{ctm} = 0.30 \times f_{ck}^{2/3}$ = 4.1 Mpa = 0.591 ksi
$M_{rep} = S_c \times f_{ctm}$ = 15495.9 kip-in
$\Delta\sigma_p$ = Min(0.4 x 270, 72.5) = 72.5 ksi
$z = 0.9 \times d$ = 64.40 in
A_p = 3.32 in2

4) PROPOSED METHOD

$Ø M_n \geq 1.0 \times (1.6 f_r + 1.1 f_{pe}) \times S_c$ with Ø = 1.0

Assume min A_{ps} = 3.95 in2 (for trial and error purpose)
$P_f = A_{ps} \times (0.75 \times f_{pu} - 30\ \text{ksi})$ = 681.9 kips assuming 30 ksi losses
$f_{pe} = P_f / A + P_f \times (d_p - y_t) \times y_b / I$ = 1.630 ksi
$f_r = 0.237 \times f'^{g\ 0.5}_{c}$ = 0.627 ksi
$1.0 \times (1.6 f_r + 1.1 f_{pe}) \times S_c$ = 73379 kip-in

$Ø M_n = Ø \times A_{ps} \times f_{ps} \times (d_p - a / 2)$
$c = (A_{ps} \times f_{pu}) / (0.85 \times f'^{d}_{c} \times 0.85 \times b + 0.28 \times A_{ps} \times f_{pu} / d_p)$ = 4.01 in
$f_{ps} = f_{pu} \times (1 - 0.28 \times c / d_p)$ = 265.8 ksi
$Ø \times A_{ps} \times f_{ps} \times (d_p - a / 2)$ = 73379 kip-in matches $(1.6 f_r + 1.1 f_{pe}) \times S_c$

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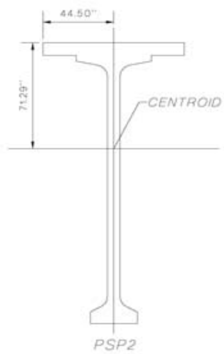
CALCULATION OF MINIMUM REINFORCEMENT - SPLICED PRECAST PRESTRESSED GIRDER PSP2

$h =$ 188.50 in
$b_f =$ 88.99 in
$t_d =$ 8.50 in
$A =$ 2664.36 in2
$I =$ 12160103.94 in4
$y_b =$ 117.21 in
$f'^{g}_{c} =$ 10.0 ksi
$f'^{d}_{c} =$ 5.5 ksi
$f_y =$ 60 ksi
$f_{pu} =$ 270 ksi



1) AASHTO LRFD 5.7.3.3.2

$\varnothing M_n \geq 1.2 M_{cr}$ with $\varnothing =$ 1.0 (AASHTO 5.5.4.2.1)

Given $d_p =$ 169.65 in
Assume min $A_{ps} =$ 10.28 in2 (for trial and error purpose)
$P_f = A_{ps} \times (0.75 \times f_{pu} - 30 \text{ ksi}) =$ 1772.7 kips assuming 30 ksi losses
$f_{cpe} = P_f / A + P_f \times (d_p - y_t) \times y_b / I =$ 2.486 ksi
$f_r = 0.37 \times f'^{g\,0.5}_{c} =$ 1.170 ksi (AASHTO 5.4.2.6)
$S_c = I / y_b =$ 103746.3 in3
$1.2 M_{cr} = 1.2 \times S_c \times (f_r + f_{cpe}) =$ 455129 kip-in

$\varnothing M_n = \varnothing \times A_{ps} \times f_{ps} \times (d_p - a / 2)$ (AASHTO 5.7.3.2.2)
$c = (A_{ps} \times f_{pu}) / (0.85 \times f'^{d}_{c} \times 0.775 \times b + 0.28 \times A_{ps} \times f_{pu} / d_p) =$ 8.48 in
$f_{ps} = f_{pu} \times (1 - 0.28 \times c / d_p) =$ 266 ksi
$\varnothing \times A_{ps} \times f_{ps} \times (d_p - a / 2) =$ 455129 kip-in matches $1.2 M_{cr}$

2) LEONHARDT

$A_{ps} \geq 1.2 F_{ct} / (f_{pu} - f_{pe})$
$f_{ct} = 0.23 \times f'^{g\,0.5}_{c} =$ 0.727 ksi
$F_{ct} =$ 504.4 kips (see hand calculations)
$f_{pe} = 0.75 \times f_{pu} - 30 \text{ ksi} =$ 172.5 ksi assuming 30 ksi losses
$A_{ps} =$ 6.21 in2

3) EUROCODE

$A_p \geq M_{rep} / (z \times \Delta\sigma_p)$

$f_{ck} = f'^{g}_{c} =$ 10.0 ksi = 70 Mpa
$f_{ctm} = 2.12 \times \ln(1 + (f_{ck} + 8) / 10) =$ 4.6 Mpa = 0.669 ksi
$M_{rep} = S_c \times f_{ctm} =$ 69375.5 kip-in
$\Delta\sigma_p = \text{Min}(0.4 \times 270, 72.5) =$ 72.5 ksi
$z = 0.9 \times d =$ 152.69 in
$A_p =$ 6.27 in2

4) PROPOSED METHOD

$\varnothing M_n \geq 1.0 \times (1.6 f_r + 1.1 f_{pe}) \times S_c$ with $\varnothing =$ 1.0

Assume min $A_{ps} =$ 7.29 in2 (for trial and error purpose)
$P_f = A_{ps} \times (0.75 \times f_{pu} - 30 \text{ ksi}) =$ 1258.1 kips assuming 30 ksi losses
$f_{pe} = P_f / A + P_f \times (d_p - y_t) \times y_b / I =$ 1.764 ksi
$f_r = 0.237 \times f'^{g\,0.5}_{c} =$ 0.749 ksi
$1.0 \times (1.6 f_r + 1.1 f_{pe}) \times S_c =$ 325739 kip-in

$\varnothing M_n = \varnothing \times A_{ps} \times f_{ps} \times (d_p - a / 2)$
$c = (A_{ps} \times f_{pu}) / (0.85 \times f'^{d}_{c} \times 0.775 \times b + 0.28 \times A_{ps} \times f_{pu} / d_p) =$ 6.05 in
$f_{ps} = f_{pu} \times (1 - 0.28 \times c / d_p) =$ 267.3 ksi
$\varnothing \times A_{ps} \times f_{ps} \times (d_p - a / 2) =$ 325739 kip-in matches $(1.6 f_r + 1.1 f_{pe}) \times S_c$

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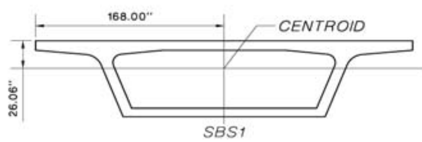
CALCULATION OF MINIMUM REINFORCEMENT - SPAN-BY-SPAN SEGMENTAL BRIDGE GIRDER SBS1

h = 72.00 in
b_f = 336.00 in
A = 6259.16 in2
I = 4565870.34 in4
y_b = 45.94 in
f'_c = 7.0 ksi
f_y = 60 ksi
f_{pu} = 270 ksi



1) AASHTO LRFD 5.7.3.3.2

$Ø M_n \geq 1.2 M_{cr}$ with Ø = 0.9 (AASHTO 5.5.4.2.2)

Given d_p = 64.80 in
Assume min A_{ps} = 80.00 in2 (for trial and error purpose)
$P_f = A_{ps} \times (0.75 \times f_{pu} - 30 \text{ ksi})$ = 13800.0 kips assuming 30 ksi losses
$f_{cpe} = P_f / A + P_f \times (d_p - y_t) \times y_b / I$ = 7.584 ksi
$f_r = 0.37 \times f'_c{}^{0.5}$ = 0.979 ksi (AASHTO 5.4.2.6)
$S_c = I / y_b$ = 99387.7 in3
$1.2 M_{cr} = 1.2 \times S_c \times (f_r + f_{cpe})$ = 1021239 kip-in

$Ø M_n = Ø \times A_{ps} \times f_{ps} \times (d_p - a / 2)$ (AASHTO 5.7.3.2.2)
$f_{pe} = 0.75 \times f_{pu} - 30$ ksi = 172.5 ksi
l_i = 1200 in
N_s = 0
$l_e = 2 l_i / (2 + N_s)$ = 1200 in
Assume f_{ps} = 212.0 ksi
$c = (A_{ps} \times f_{ps}) / (0.85 \times f'_c \times 0.70 \times b)$ = 12.12 in
$f_{ps} = \text{Min}(f_{pe} + 900 \times (d_p - c) / l_e \,;\, 243.5)$ = 212.0 ksi
$Ø \times A_{ps} \times f_{ps} \times (d_p - a / 2)$ = 924408 kip-in NO CONVERGENCE ACHIEVED

2) LEONHARDT

$A_{ps} \geq 1.2 F_{ct} / (f_{ps} - f_{pe})$
$f_{ct} = 0.23 \times f'_c{}^{0.5}$ = 0.609 ksi
F_{ct} = 1006.2 kips (see hand calculations)
$f_{pe} = 0.75 \times f_{pu} - 30$ ksi = 172.5 ksi assuming 30 ksi losses
Assume f_{ps} = 218.0 ksi (for trial and error purpose)
A_{ps} = 26.54 in2
$c = (A_{ps} \times f_{ps}) / (0.85 \times f'_c \times 0.70 \times b)$ = 4.13 in
$f_{ps} = \text{Min}(f_{pe} + 900 \times (d_p - c) / l_e \,;\, 243.5)$ = 218.0 ksi matches assumed f_{ps}

3) EUROCODE

$A_p \geq M_{rep} / (z \times \Delta\sigma_p)$

$f_{ck} = f'_c$ = 7.0 ksi = 50 Mpa
$f_{ctm} = 0.30 \times f_{ck}{}^{2/3}$ = 4.1 Mpa = 0.591 ksi
$M_{rep} = S_c \times f_{ctm}$ = 58693.3 kip-in
$\Delta\sigma_p$ = 15.0 ksi
$z = 0.9 \times d$ = 58.32 in
A_p = 67.09 in2

4) PROPOSED METHOD

$Ø M_n \geq 1.0 \times (1.2 f_r + 1.0 f_{pe}) \times S_c$ with Ø = 0.9

Assume min A_{ps} = 24.59 in2 (for trial and error purpose)
$P_f = A_{ps} \times (0.75 \times f_{pu} - 30 \text{ ksi})$ = 4241.3 kips assuming 30 ksi losses
$f_{pe} = P_f / A + P_f \times (d_p - y_t) \times y_b / I$ = 2.331 ksi
$f_r = 0.237 \times f'_c{}^{0.5}$ = 0.627 ksi
$1.0 \times (1.2 f_r + 1.0 f_{pe}) \times S_c$ = 306438 kip-in

$Ø M_n = Ø \times A_{ps} \times f_{ps} \times (d_p - a / 2)$
Assume f_{ps} = 218.2 ksi
$c = (A_{ps} \times f_{ps}) / (0.85 \times f'_c \times 0.70 \times b)$ = 3.83 in
$f_{ps} = \text{Min}(f_{pe} + 900 \times (d_p - c) / l_e \,;\, 243.5)$ = 218.2 ksi
$Ø \times A_{ps} \times f_{ps} \times (d_p - a / 2)$ = 306438 kip-in matches $(1.2 f_r + 1.0 f_{pe}) \times S_c$

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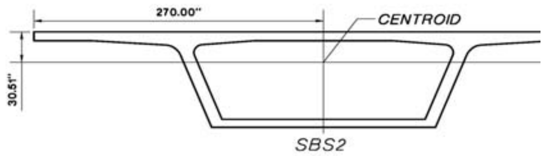
CALCULATION OF MINIMUM REINFORCEMENT - SPAN-BY-SPAN SEGMENTAL BRIDGE GIRDER SBS2

h = 96.00 in
b_f = 540.00 in
A = 9354.51 in2
I = 11744489.52 in4
y_b = 65.49 in
f'_c = 7.0 ksi
f_y = 60 ksi
f_{pu} = 270 ksi



1) AASHTO LRFD 5.7.3.3.2

$\text{Ø} M_n \geq 1.2\ M_{cr}$ with Ø = 0.9 (AASHTO 5.5.4.2.2)

Given d_p = 86.40 in
Assume min A_{ps} = 140.00 in2 (for trial and error purpose)
$P_f = A_{ps} \times (0.75 \times f_{pu} - 30\text{ ksi})$ = 24150.0 kips assuming 30 ksi losses
$f_{cpe} = P_f / A + P_f \times (d_p - y_t) \times y_b / I$ = 10.108 ksi
$f_r = 0.37 \times f'_c{}^{0.5}$ = 0.979 ksi (AASHTO 5.4.2.6)
$S_c = I / y_b$ = 179332.6 in3
$1.2\ M_{cr} = 1.2 \times S_c \times (f_r + f_{cpe})$ = 2385924 kip-in

$\text{Ø} M_n = \text{Ø} \times A_{ps} \times f_{ps} \times (d_p - a / 2)$ (AASHTO 5.7.3.2.2)
$f_{pe} = 0.75 \times f_{pu} - 30\text{ ksi}$ = 172.5 ksi
l_i = 1800 in
N_s = 0
$l_e = 2\ l_i / (2 + N_s)$ = 1800 in
Assume f_{ps} = 209.2 ksi
$c = (A_{ps} \times f_{ps}) / (0.85 \times f'_c \times 0.70 \times b)$ = 13.02 in
$f_{ps} = \text{Min}(f_{pe} + 900 \times (d_p - c) / l_e\ ;\ 243.5)$ = 209.2 ksi
$\text{Ø} \times A_{ps} \times f_{ps} \times (d_p - a / 2)$ = 2157183 kip-in NO CONVERGENCE ACHIEVED

2) LEONHARDT

$A_{ps} \geq 1.2\ F_{ct} / (f_{ps} - f_{pe})$
$f_{ct} = 0.23 \times f'_c{}^{0.5}$ = 0.609 ksi
F_{ct} = 1352.7 kips (see hand calculations)
$f_{pe} = 0.75 \times f_{pu} - 30\text{ ksi}$ = 172.5 ksi assuming 30 ksi losses
Assume f_{ps} = 213.8 ksi (for trial and error purpose)
A_{ps} = 39.30 in2
$c = (A_{ps} \times f_{ps}) / (0.85 \times f'_c \times 0.70 \times b)$ = 3.74 in
$f_{ps} = \text{Min}(f_{pe} + 900 \times (d_p - c) / l_e\ ;\ 243.5)$ = 213.8 ksi matches assumed f_{ps}

3) EUROCODE

$A_p \geq M_{rep} / (z \times \Delta\sigma_p)$

$f_{ck} = f'_c$ = 7.0 ksi = 50 Mpa
$f_{ctm} = 0.30 \times f_{ck}{}^{2/3}$ = 4.1 Mpa = 0.591 ksi
$M_{rep} = S_c \times f_{ctm}$ = 105904.6 kip-in
$\Delta\sigma_p$ = 15.0 ksi
$z = 0.9 \times d$ = 77.76 in
A_p = 90.80 in2

4) PROPOSED METHOD

$\text{Ø} M_n \geq 1.0 \times (1.2\ f_r + 1.0\ f_{pe}) \times S_c$ with Ø = 0.9

Assume min A_{ps} = 39.37 in2 (for trial and error purpose)
$P_f = A_{ps} \times (0.75 \times f_{pu} - 30\text{ ksi})$ = 6791.8 kips assuming 30 ksi losses
$f_{pe} = P_f / A + P_f \times (d_p - y_t) \times y_b / I$ = 2.843 ksi
$f_r = 0.237 \times f'_c{}^{0.5}$ = 0.627 ksi
$1.0 \times (1.2\ f_r + 1.0\ f_{pe}) \times S_c$ = 644736 kip-in

$\text{Ø} M_n = \text{Ø} \times A_{ps} \times f_{ps} \times (d_p - a / 2)$
Assume f_{ps} = 213.8 ksi
$c = (A_{ps} \times f_{ps}) / (0.85 \times f'_c \times 0.70 \times b)$ = 3.74 in
$f_{ps} = \text{Min}(f_{pe} + 900 \times (d_p - c) / l_e\ ;\ 243.5)$ = 213.8 ksi
$\text{Ø} \times A_{ps} \times f_{ps} \times (d_p - a / 2)$ = 644736 kip-in matches $(1.2\ f_r + 1.0\ f_{pe}) \times S_c$

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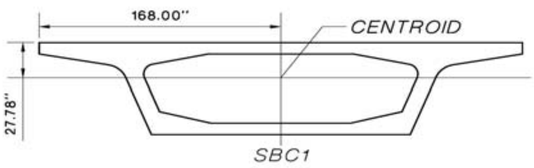
CALCULATION OF MINIMUM REINFORCEMENT - BALANCED CANTILEVER BRIDGE GIRDER SBC1

h = 72.00 in
b_f = 336.00 in
A = 7471.00 in2
I = 5269583.65 in4
y_b = 44.29 in
f'_c = 7.0 ksi
f_y = 60 ksi
f_{pu} = 270 ksi



1) AASHTO LRFD 5.7.3.3.2

$Ø M_n \geq 1.2 M_{cr}$ with Ø = 0.95 (AASHTO 5.5.4.2.2)

Given d_p = 64.80 in
Assume min A_{ps} = 29.32 in2 (for trial and error purpose)
$P_f = A_{ps} \times (0.75 \times f_{pu} - 30\text{ ksi})$ = 5058.2 kips assuming 30 ksi losses
$f_{cpe} = P_f / A + P_f \times (d_p - y_t) \times y_b / I$ = 2.254 ksi
$f_r = 0.37 \times f'^{0.5}_c$ = 0.979 ksi (AASHTO 5.4.2.6)
$S_c = I / y_b$ = 118979.1 in3
$1.2 M_{cr} = 1.2 \times S_c \times (f_r + f_{cpe})$ = 461563 kip-in

$Ø M_n = Ø \times A_{ps} \times f_{ps} \times (d_p - a / 2)$ (AASHTO 5.7.3.2.2)
$c = (A_{ps} \times f_{pu}) / (0.85 \times f'_c \times 0.70 \times b + 0.28 \times A_{ps} \times f_{pu} / d_p)$ = 5.52 in
$f_{ps} = f_{pu} \times (1 - 0.28 \times c / d_p)$ = 264 ksi
$Ø \times A_{ps} \times f_{ps} \times (d_p - a / 2)$ = 461563 kip-in matches $1.2 M_{cr}$

2) LEONHARDT

$A_{ps} \geq 1.2 F_{ct} / (f_{pu} - f_{pe})$
$f_{ct} = 0.23 \times f'^{0.5}_c$ = 0.609 ksi
F_{ct} = 1244.2 kips (see hand calculations)
$f_{pe} = 0.75 \times f_{pu} - 30$ ksi = 172.5 ksi assuming 30 ksi losses
A_{ps} = 15.31 in2

3) EUROCODE

$A_p \geq M_{rep} / (z \times \Delta\sigma_p)$

$f_{ck} = f'_c$ = 7.0 ksi = 50 Mpa
$f_{ctm} = 0.30 \times f_{ck}^{2/3}$ = 4.1 Mpa = 0.591 ksi
$M_{rep} = S_c \times f_{ctm}$ = 70262.9 kip-in
$\Delta\sigma_p$ = Min(0.4 x 270, 72.5) = 72.5 ksi
$z = 0.9 \times d$ = 58.32 in
A_p = 16.62 in2

4) PROPOSED METHOD

$Ø M_n \geq 1.0 \times (1.2 f_r + 1.1 f_{pe}) \times S_c$ with Ø = 0.95

Assume min A_{ps} = 14.78 in2 (for trial and error purpose)
$P_f = A_{ps} \times (0.75 \times f_{pu} - 30\text{ ksi})$ = 2550.2 kips assuming 30 ksi losses
$f_{pe} = P_f / A + P_f \times (d_p - y_t) \times y_b / I$ = 1.136 ksi
$f_r = 0.237 \times f'^{0.5}_c$ = 0.627 ksi
$1.0 \times (1.2 f_r + 1.1 f_{pe}) \times S_c$ = 238244 kip-in

$Ø M_n = Ø \times A_{ps} \times f_{ps} \times (d_p - a / 2)$
$c = (A_{ps} \times f_{pu}) / (0.85 \times f'_c \times 0.70 \times b + 0.28 \times A_{ps} \times f_{pu} / d_p)$ = 2.82 in
$f_{ps} = f_{pu} \times (1 - 0.28 \times c / d_p)$ = 266.7 ksi
$Ø \times A_{ps} \times f_{ps} \times (d_p - a / 2)$ = 238244 kip-in matches $(1.2 f_r + 1.1 f_{pe}) \times S_c$

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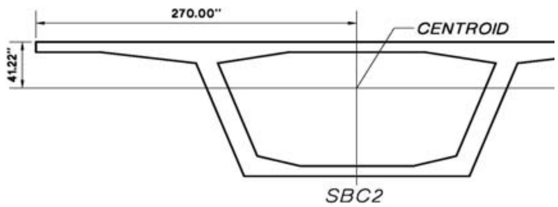
CALCULATION OF MINIMUM REINFORCEMENT - BALANCED CANTILEVER BRIDGE GIRDER SBC2

h =	120.00 in
b_f =	540.00 in
A =	12020.08 in2
I =	23216472.58 in4
y_b =	78.78 in
f'_c =	7.0 ksi
f_y =	60 ksi
f_{pu} =	270 ksi



1) AASHTO LRFD 5.7.3.3.2

$\varnothing M_n \geq 1.2 M_{cr}$ with $\varnothing$ = 0.95 (AASHTO 5.5.4.2.2)

Given d_p = 108.00 in
Assume min A_{ps} = 43.39 in2 (for trial and error purpose)
$P_f = A_{ps} \times (0.75 \times f_{pu} - 30\ ksi)$ = 7484.2 kips assuming 30 ksi losses
$f_{cpe} = P_f / A + P_f \times (d_p - y_t) \times y_b / I$ = 2.319 ksi
$f_r = 0.37 \times f'^{0.5}_c$ = 0.979 ksi (AASHTO 5.4.2.6)
$S_c = I / y_b$ = 294700.1 in3
$1.2 M_{cr} = 1.2 \times S_c \times (f_r + f_{cpe})$ = 1166130 kip-in

$\varnothing M_n = \varnothing \times A_{ps} \times f_{ps} \times (d_p - a / 2)$ (AASHTO 5.7.3.2.2)
$c = (A_{ps} \times f_{pu}) / (0.85 \times f'_c \times 0.70 \times b + 0.28 \times A_{ps} \times f_{pu} / d_p)$ = 5.14 in
$f_{ps} = f_{pu} \times (1 - 0.28 \times c / d_p)$ = 266 ksi
$\varnothing \times A_{ps} \times f_{ps} \times (d_p - a / 2)$ = 1166130 kip-in matches $1.2 M_{cr}$

2) LEONHARDT

$A_{ps} \geq 1.2 F_{ct} / (f_{pu} - f_{pe})$
$f_{ct} = 0.23 \times f'^{0.5}_c$ = 0.609 ksi
F_{ct} = 1832.4 kips (see hand calculations)
$f_{pe} = 0.75 \times f_{pu} - 30$ ksi = 172.5 ksi assuming 30 ksi losses
A_{ps} = 22.55 in2

3) EUROCODE

$A_p \geq M_{rep} / (z \times \Delta\sigma_p)$

$f_{ck} = f'_c$ = 7.0 ksi = 50 Mpa
$f_{ctm} = 0.30 \times f_{ck}^{2/3}$ = 4.1 Mpa = 0.591 ksi
$M_{rep} = S_c \times f_{ctm}$ = 174034.8 kip-in
$\Delta\sigma_p$ = Min(0.4 x 270, 72.5) = 72.5 ksi
$z = 0.9 \times d$ = 97.20 in
A_p = 24.70 in2

4) PROPOSED METHOD

$\varnothing M_n \geq 1.0 \times (1.2 f_r + 1.1 f_{pe}) \times S_c$ with $\varnothing$ = 0.95

Assume min A_{ps} = 22.40 in2 (for trial and error purpose)
$P_f = A_{ps} \times (0.75 \times f_{pu} - 30\ ksi)$ = 3864.8 kips assuming 30 ksi losses
$f_{pe} = P_f / A + P_f \times (d_p - y_t) \times y_b / I$ = 1.197 ksi
$f_r = 0.237 \times f'^{0.5}_c$ = 0.627 ksi
$1.0 \times (1.2 f_r + 1.1 f_{pe}) \times S_c$ = 609881 kip-in

$\varnothing M_n = \varnothing \times A_{ps} \times f_{ps} \times (d_p - a / 2)$
$c = (A_{ps} \times f_{pu}) / (0.85 \times f'_c \times 0.70 \times b + 0.28 \times A_{ps} \times f_{pu} / d_p)$ = 2.67 in
$f_{ps} = f_{pu} \times (1 - 0.28 \times c / d_p)$ = 268.1 ksi
$\varnothing \times A_{ps} \times f_{ps} \times (d_p - a / 2)$ = 609881 kip-in matches $(1.2 f_r + 1.1 f_{pe}) \times S_c$

Appendix B Design Examples

B.1 MULTI-SPAN PRECAST CONCRETE GIRDER MADE CONTINUOUS WITH COMPOSITE DECK

This is one of the most common types of structures used for freeway bridges and overpasses. This three-span precast/prestressed girder example features a single long span in the middle along with two short side spans, as shown in Figure B-1. A uniform depth is used to reduce set-up costs and improve aesthetics. It is intended that the side spans are short enough so the minimum flexural provisions control the design in the positive bending regions.

Seventy-two inch bulb-tee girders are featured in this example since the bottom flange tends to be relatively narrow, thus limiting the amount of rotational ductility that can be sustained in the negative bending region.

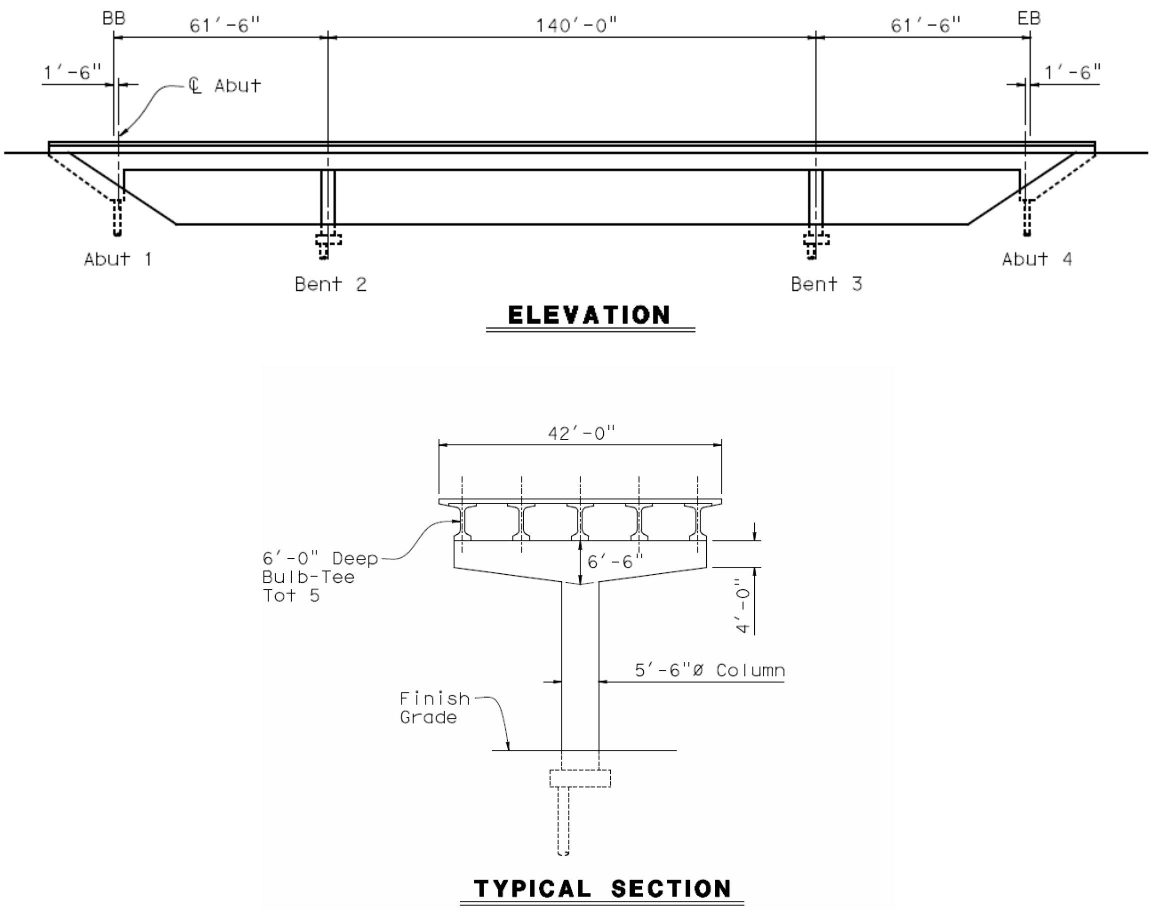


Figure B-1. Precast Concrete Girder Made Continuous with a Composite Deck

B.1.1 Description of Bridge

Bridge dimensions

The bridge is 42.0 ft wide and 6.83 ft deep at the supports. The 6.0 ft deep bulb-tee girders are spaced at 9.0 ft on center. The deck is 8.0 in. thick. The columns are circular with a diameter of 5.5 ft.

Material properties

$f'_c = 7.5 ksi$ (girders)
$f'_{ci} = 5.5 ksi$ (girders)
$f'_c = 4.5 ksi$ (deck)
$f_y = 60 ksi$
$E_s = 29{,}000 ksi$
$f_{pu} = 270 ksi$
$E_{ps} = 28{,}500 ksi$

Prestress force

The working prestressing force is designed with the software Conspan and is estimated to be 325 kips for an interior girder in Spans 1 and 3 and 1,146 kips for an interior girder in Span 2.

B.1.2 Minimum Flexural Reinforcement – Modified LRFD Method

At the outside face of support (negative moment):

Design moments (per interior girder):

$M_{SW} = 0 k - ft$
$M_{DL-PC} = 0 k - ft$
$M_{deck} = 0 k - ft$
$M_{ADL-DC} = -248 k - ft$
$M_{ADL-DW} = -387 k - ft$
$M_{HL-93} = -1{,}694 k - ft$

$$M_u^{StrengthI} = 1.25 M_{ADL-DC} + 1.5 M_{ADL-DW} + 1.75 M_{HL-93}$$
$$M_u^{StrengthI} = 1.25 \times (-248) + 1.50 \times (-387) + 1.75 \times (-1{,}694)$$
$$M_u^{StrengthI} = -3{,}855 k - ft$$

Section properties:

$h_{nc} = 6 ft = 72 in$ (non-composite section)
$h_c = 6.71 ft = 80.5 in$ (composite section)
$b_f = 26 in$ (girder bottom flange width)

$h_f = 6in$ (compression flange)

$I_{nc} = 26.33 ft^4$ (non-composite section)

$I_c = 54.16 ft^4$ (composite section)

$A_{nc} = 5.33 ft^2$ (non-composite section)

$A_c = 10.09 ft^2$ (composite section)

$\bar{y}_b^{nc} = 36.6in$ (distance from section CG to bottom fiber – non-composite section)

$\bar{y}_b^{c} = 55.38in$ (distance from section CG to bottom fiber – composite section)

Required flexural reinforcement:

The prestressing capacity is neglected for the negative moment capacity.

$$\phi M_n = \phi A_s f_y (d - \frac{a}{2}) = \phi A_s f_y (d - \frac{A_s f_y}{2 \times 0.85 f'_c b})$$

The section is tension-controlled and $\phi = 0.90$

$d = 76.80in$ assuming #11 mild steel reinforcement

$$3{,}855 \times 12 = 0.90 \times A_s \times 60 \times (76.80 - \frac{A_s \times 60}{2 \times 0.85 \times 7.5 \times 26})$$

Solve the quadratic equation for $A_s = 11.46in^2$.

The net tensile strain is:

$$\varepsilon_s = 0.003\left(\frac{d-c}{c}\right) \text{ where } c = \left(\frac{A_s f_y}{0.85 \beta f'_c b}\right) = \left(\frac{11.46 \times 60}{0.85^2 \times 7.5 \times 26}\right) = 4.88in$$

Therefore, $\varepsilon_s = 0.003\left(\frac{76.8 - 4.88}{4.88}\right) = 0.044$

T*he net tensile strain is greater than 0.0075, which satisfies Article 5.7.3.5 for redistribution. Therefore, per the proposed revised Article 5.7.3.3.2, minimum reinforcement is not required for negative bending.*

Summary:

$P_f = 325kip$

$A_s = 11.46in^2$ mild steel reinforcement in deck.

<u>**At 0.5 Span 1 (positive moment):**</u>

Design moments (per interior girder):

$$M_{SW} = 354k - ft$$

$M_{DL-PC} = 27k - ft$
$M_{deck} = 408k - ft$
$M_{ADL-DC} = -38k - ft$
$M_{ADL-DW} = -60k - ft$
$M_{HL-93} = 1{,}036k - ft$
$M_u^{StrengthI} = 1.25(M_{SW} + M_{DL-PC} + M_{deck}) + 0.9M_{ADL-DC} + 0.65M_{ADL-DW} + 1.75M_{HL-93}$
$M_u^{StrengthI} = 1.25 \times (354 + 27 + 408) + 0.9 \times (-38) + 0.65 \times (-60) + 1.75 \times 1{,}036$
$M_u^{StrengthI} = 2{,}724k - ft$

Section properties:

$h_{nc} = 6ft = 72in$ (non-composite section)
$h_c = 6.71ft = 80.5in$ (composite section)
$b_f = 9ft$ (deck effective width for an interior girder)
$h_f = 8in$ (compression flange)
$I_{nc} = 26.33ft^4$ (non-composite section)
$I_c = 54.16ft^4$ (composite section)
$A_{nc} = 5.33ft^2$ (non-composite section)
$A_c = 10.09ft^2$ (composite section)
$\overline{y}_b^{nc} = 36.6in$ (distance from section CG to bottom fiber – non-composite section)
$\overline{y}_b^{c} = 55.38in$ (distance from section CG to bottom fiber – composite section)

Minimum reinforcement by the proposed method:

$\phi M_n \geq M_{fcr}$ where $M_{fcr} = \gamma_3\left[(\gamma_1 f_r + \gamma_2 f_{cpe})S_c - M_{dnc}(\frac{S_c}{S_{nc}} - 1)\right]$ and $\phi M_n \geq 1.33M_u$
$\gamma_3 = 1.0$ for prestressed structures
$\gamma_1 = 1.6$
$f_r = 0.237\sqrt{f'_c} = 0.237 \times \sqrt{7.5} = 0.649ksi$
$\gamma_2 = 1.1$
$d_p = 78in$ (distance from P/S CG to top deck)
$e_{nc} = \overline{y}_b^{nc} - (h_c - d_p) = 36.6 - (80.5 - 78) = 34.10in$ (prestressing eccentricity)
$P_f = 325kips$ (prestressing force after all losses per Conspan analysis)

$$f_{cpe} = \frac{P_f}{A_{nc}} + \frac{P_f e_{nc} \bar{y}_b^{\,nc}}{I_{nc}} = \frac{325}{5.33\times 12^2} + \frac{325\times 34.10\times 36.6}{26.33\times 12^4} = 1.166ksi$$

$$S_c = \frac{I_c}{\bar{y}_b^{\,c}} = \frac{54.16\times 12^4}{55.38} = 20{,}279in^3$$

$$M_{ndc} = M_{SW} + M_{DL-PC} + M_{deck} = 354 + 27 + 408 = 789k-ft$$

$$S_{nc} = \frac{I_{nc}}{\bar{y}_b^{\,nc}} = \frac{26.33\times 12^4}{36.6} = 14{,}917in^3$$

$$M_{fcr} = 1.0\times(1.6\times 0.649 + 1.1\times 1.166)\times\frac{20{,}279}{12} - 789\times(\frac{20{,}279}{14{,}917} - 1) = 3{,}638k-ft$$

$$M_{fcr} = 3{,}638k-ft \ge M_u^{StrengthI} = 2{,}724k-ft$$

$M_{fcr} = 3{,}638k-ft \ge 1.33M_u^{StrengthI} = 1.33\times 2{,}724 = 3{,}623k-ft$ so $1.33M_u^{StrengthI} = 3{,}623k-ft$ controls the design.

Calculation of ϕM_n^{ps} from prestressing:

Calculate ϕM_n^{ps} per the simplified method.

$$A_{ps} = \frac{P_i}{0.75\times f_{pu}} = \frac{351.5}{0.75\times 270} = 1.736in^2 \text{ (8-0.6in } \phi \text{ strands)}$$

$$\beta_1 = 0.85 - 0.05\times(f'_c - 4) = 0.85 - 0.05\times(4.5-4) = 0.825$$

$$k = 0.28$$

Assume rectangular section behavior:

$$c = \frac{A_{ps}f_{pu}}{0.85f'_c\beta_1 b_f + kA_{ps}\frac{f_{pu}}{d_p}} = \frac{1.736\times 270}{0.85\times 4.5\times 0.825\times 9\times 12 + 0.28\times 1.736\times\frac{270}{78}} = 1.37in$$

$$a = \beta_1 c = 0.825\times 1.37 = 1.13in \le h_f = 8in$$

Therefore, the section behavior is rectangular.

$$f_{ps} = f_{pu}(1 - k\frac{c}{d_p}) = 270\times(1 - 0.28\times\frac{1.37}{78}) = 268.7ksi$$

Check the force equilibrium:

Flange force $C_f = 0.85f'_c ab_f = 0.85\times 4.5\times 1.13\times 9\times 12 = 466.8kip$

Prestressing force $T_{ps} = A_{ps}f_{ps} = 1.736\times 268.7 = 466.5kip$ within 0.07% of $C_f \rightarrow$ equilibrium is satisfied.

$$M_n^{ps} = A_{ps} f_{ps} d_p - \frac{0.85 f'_c a^2 b_f}{2} = (1.736 \times 268.7 \times 78 - \frac{0.85 \times 4.5 \times 1.13^2 \times 9 \times 12}{2}) \div 12 = 3{,}010 k - ft$$

The section is tension-controlled and $\phi = 1.00$

$\phi M_n^{ps} = 1.00 \times 3{,}010 = 3{,}010 k - ft$

$\phi M_n^{ps} = 3{,}010 k - ft \le 1.33 M_u^{StrengthI} = 3{,}623 k - ft$ so additional strands are required.

Try 2 additional strands with the same eccentricity at mid-span.

$A_{ps} = 1.736 + 2 \times 0.217 = 2.17 in^2$ (10-0.6in ϕ strands)

$P_f \approx 325 \times \frac{2.17}{1.736} = 406 kip$ (prestressing force after all losses)

Since $1.33 M_u^{StrengthI}$ controls the design, increasing the amount of prestressing strands does not increase the minimum reinforcement demand.

Assume a rectangular section behavior:

$$c = \frac{A_{ps} f_{pu}}{0.85 f'_c \beta_1 b_f + k A_{ps} \frac{f_{pu}}{d_p}} = \frac{2.17 \times 270}{0.85 \times 4.5 \times 0.825 \times 9 \times 12 + 0.28 \times 2.17 \times \frac{270}{78}} = 1.71 in$$

$a = \beta_1 c = 0.825 \times 1.71 = 1.41 in \le h_f = 8 in$ therefore the section behavior is rectangular.

$$f_{ps} = f_{pu}(1 - k\frac{c}{d_p}) = 270 \times (1 - 0.28 \times \frac{1.71}{78}) = 268.4 ksi$$

Check the force equilibrium:

Flange force $C_f = 0.85 f'_c a b_f = 0.85 \times 4.5 \times 1.41 \times 9 \times 12 = 582.5 kip$

Prestressing force $T_{ps} = A_{ps} f_{ps} = 2.17 \times 268.4 = 582.5 kip$ equal to $C_f \rightarrow$ equilibrium is satisfied.

$$M_n^{ps} = A_{ps} f_{ps} d_p - \frac{0.85 f'_c a^2 b_f}{2} = (2.17 \times 268.4 \times 78 - \frac{0.85 \times 4.5 \times 1.41^2 \times 9 \times 12}{2}) \div 12 = 3{,}751 k - ft$$

The section is tension-controlled and $\phi = 1.00$

$\phi M_n^{ps} = 1.00 \times 3{,}751 = 3{,}751 k - ft \ge 1.33 M_u^{StrengthI} = 3{,}623 k - ftt$

Summary:

$P_f = 325 kip$

$A_{ps} = 2 \times 0.217 = 0.434 in^2$ of additional prestress strand area required to meet the Modified Method Minimum Reinforcement provisions.

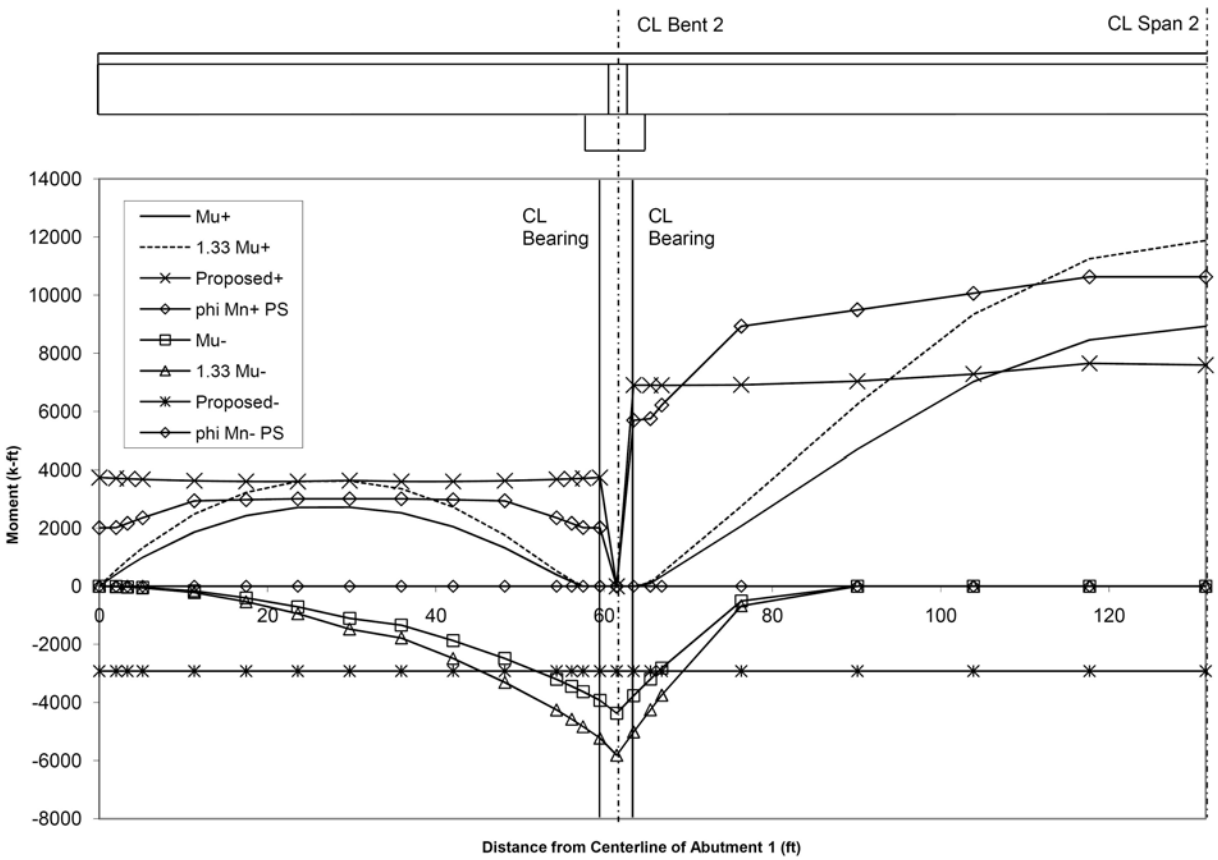


Figure B-2. Moment Profiles for the Precast Girder Example

B.2 CAST-IN-PLACE CONCRETE BOX GIRDER

A three-span cast-in-place concrete box girder bridge that is commonly built in California and Nevada is the subject of this design example. As with the first example, the side spans are far shorter than the end spans while the depth of the bridge is constant for along the entire length. Because the bridge is monolithic, the bridge resists all loading continuously including any prestress forces. All prestress consists of continuous post-tensioning that runs full length of the bridge. To control camber and reduce friction losses, the post-tensioning tendon midspan eccentricity is reduced in the shorter spans were flexural demands are reduced.

For this type of structure, it is more economical to design the post-tensioning cables for service loads, and add mild reinforcement in localized areas as needed to resist strength limit state loads including minimum reinforcement provisions. It is anticipated that minimum flexural reinforcement will control the design of this mild reinforcement in these side spans.

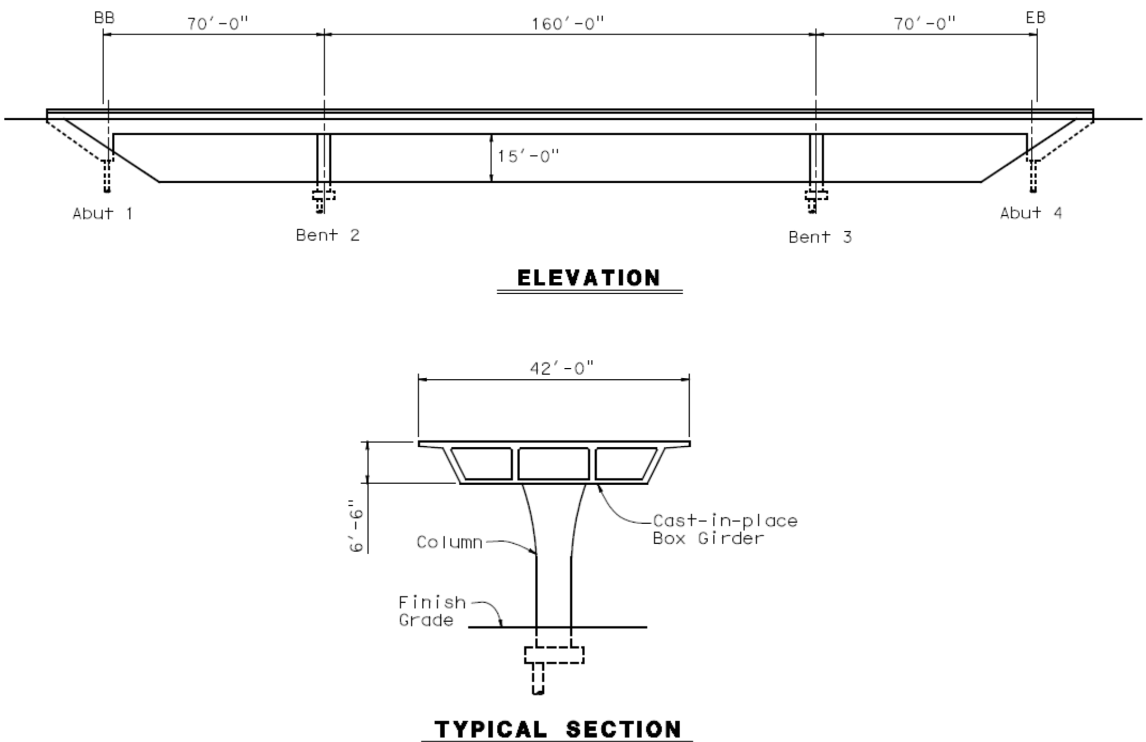


Figure D-3. Cast-in-Place Box Girder

B 2.1 Bridge Layout

The bridge is 42.0 ft wide and 6.5 ft deep. The girders are spaced at 11.0 ft on center and are flared from 12 in. to 18 in. at the abutments and the bents. The soffit is flared to 12 in. at the bents. The columns are circular with a diameter of 6.0 ft.

Material Properties

$$f'_c = 4ksi$$
$$E_c = 3{,}644ksi$$
$$f_y = 60ksi$$
$$E_s = 29{,}000ksi$$
$$f_{pu} = 270ksi$$
$$E_{ps} = 28{,}500ksi$$

Prestress Forces

The allowable tension stress is limited to 0 under permanent loads and $0.19\sqrt{f'c}$ (ksi) under the sum of the permanent and live loads. The jacking force is designed under the Service III limit state and is estimated with the software CT Bridge to be 6,200 kips.

B2.2 Minimum Flexural Reinforcement – Modified LRFD Method

At the outside face of support (negative moment):

Design moments:

$$M_{DC} = -15{,}653k - ft$$
$$M_{DW} = -1{,}821k - ft$$
$$M_{SecP/S} = -568k - ft$$
$$M_{HL-93} = -8{,}836k - ft$$
$$M_{P-15} = -15{,}529k - ft$$

$$M_u^{StrengthI} = 1.25M_{DC} + 1.50M_{DW} + 1.00M_{SecP/S} + 1.75M_{HL-93}$$
$$M_u^{StrengthI} = 1.25\times(-15{,}653) + 1.50\times(-1{,}821) + 1.00\times(-568) + 1.75\times(-8{,}836)$$
$$M_u^{StrengthI} = -38{,}329k - ft$$

$$M_u^{StrengthII} = 1.25M_{DC} + 1.50M_{DW} + 1.00M_{SecP/S} + 1.35M_{P-15}$$
$$M_u^{StrengthII} = 1.25\times(-15{,}653) + 1.50\times(-1{,}821) + 1.00\times(-568) + 1.35\times(-15{,}529)$$
$M_u^{StrengthII} = -43{,}830k - ft$ controls the design.

Section properties:

$$h = 6.5ft = 78in$$
$h_f = 12in$ (compression flange)
$b_f = 28.5ft = 342in$ (compression flange)

$b_w = 72in$
$I = 538.03 ft^4$
$A = 90.78 ft^2$
$\bar{y}_b = 40.96in$ (distance from section CG to bottom fiber)
$\bar{y}_t = 37.04in$ (distance from section CG to top fiber)

Calculation of ϕM_n^{ps} from prestressing:

The calculation of ϕM_n^{ps} per the strain-compatibility method is an iterative process.

$$A_{ps} = \frac{P_{jack}}{0.75 \times f_{pu}} = \frac{6,200}{0.75 \times 270} = 30.62in^2$$

$d_p = 62.90in$ (distance from P/S CG to bottom fiber)
$e = d_p - \bar{y}_b = 62.90 - 40.96 = 21.94in$ (prestressing eccentricity)
$P_f = 5,031.9kips$ (prestressing force after all losses per CT Bridge analysis)
$\beta_1 = 0.85$

Assume $c = 8.30in$
$a = \beta_1 c = 0.85 \times 8.30 = 7.05in \le h_f = 12in$ therefore the section behavior is rectangular.
The strain of the prestress tendons consists of the following:
Effective prestress at service load: $\varepsilon_{ps1} = \frac{P_f}{E_{ps} A_{ps}} = \frac{5,031.9}{28,500 \times 30.62} = 0.00576$

At decompression: $\varepsilon_{ps2} = \frac{P_f}{E_c A_c}(1 + \frac{e^2}{r^2}) = \frac{5,031.9}{3,644 \times 90.78 \times 12^2}(1 + \frac{21.94^2}{(\frac{538.03 \times 12^4}{90.78 \times 12^2})}) = 0.00017$

At limit state:

$$\varepsilon_{ps3} = \varepsilon_c \frac{(d_p - c)}{c} = 0.003 \times \frac{(62.90 - 8.30)}{8.30} = 0.01973$$

Thus $\varepsilon_{ps} = \varepsilon_{ps1} + \varepsilon_{ps2} + \varepsilon_{ps3} = 0.00576 + 0.00017 + 0.01973 = 0.0257 \ge 0.0086$

$$f_{ps} = f_{pu} - \frac{0.04}{\varepsilon_{ps} - 0.007} = 270 - \frac{0.04}{0.0257 - 0.007} = 267.9ksi$$

Check the force equilibrium:
Flange force $C_f = 0.85 f'_c ab_f = 0.85 \times 4 \times 7.05 \times 342 = 8,203.5kip$
Prestressing force $T_{ps} = A_{ps} f_{ps} = 30.62 \times 267.9 = 8,203.1kip$ equal to $C_f \rightarrow$ equilibrium is satisfied.

$$M_n^{ps} = A_{ps} f_{ps} d_p - \frac{0.85 f'_c a^2 b_f}{2} = (30.62 \times 267.9 \times 62.90 - \frac{0.85 \times 4 \times 7.05^2 \times 342}{2}) \div 12 = 40{,}586 k-ft$$

$\varepsilon_{ps3} = 0.01973 \geq 0.005 \rightarrow$ the section is tension-controlled and $\phi = 0.95$

$\phi M_n^{ps} = 0.95 \times 40{,}586 = 38{,}557 k-ft \leq M_u^{StrengthII} = 43{,}830 k-ft$ so mild steel is required.

Required flexural reinforcement:

The calculation of ϕM_n is an iterative process.
$d_s = 74.68 in$ assuming #6 mild steel reinforcement
Try $A_s = 16.67 in^2$ and assume $c = 9.30 in$
$a = \beta_1 c = 0.85 \times 9.30 = 7.91 in \leq h_f = 12 in$ therefore the section behavior is rectangular.
The strain of the prestress tendons consists of the following:
Effective prestress at service load: $\varepsilon_{ps1} = 0.00576$
At decompression: $\varepsilon_{ps2} = 0.00017$
At limit state:

$$\varepsilon_{ps3} = \varepsilon_c \frac{(d_p - c)}{c} = 0.003 \times \frac{(62.90 - 9.30)}{9.30} = 0.0173$$

Thus $\varepsilon_{ps} = \varepsilon_{ps1} + \varepsilon_{ps2} + \varepsilon_{ps3} = 0.00576 + 0.00017 + 0.0173 = 0.0232 \geq 0.0086$

$$f_{ps} = f_{pu} - \frac{0.04}{\varepsilon_{ps} - 0.007} = 270 - \frac{0.04}{0.0232 - 0.007} = 267.5 ksi$$

$$\varepsilon_t = \varepsilon_c \frac{(d_s - c)}{c} = 0.003 \times \frac{(74.68 - 9.30)}{9.30} = 0.0211 \geq \frac{f_y}{E_s} = \frac{60}{29{,}000} = 0.00207 \text{ so } f_s = f_y = 60 ksi$$

Check the force equilibrium:
Flange force $C_f = 0.85 f'_c a b_f = 0.85 \times 4 \times 7.91 \times 342 = 9{,}191.9 kip$
Prestressing force $T_{ps} = A_{ps} f_{ps} = 30.62 \times 267.5 = 8{,}191.9 kip$
Mild steel force $T_{mild} = A_s f_y = 16.67 \times 60 = 1{,}000.2 kip$
Tension $T_{ps} + T_{mild} = 8{,}191.9 + 1{,}000.2 = 9{,}192.1 kip$ within 0.002% of compression $C_f \rightarrow$ equilibrium is satisfied.

$$M_n = A_{ps} f_{ps} d_p + A_s f_y d_s - \frac{0.85 f'_c a^2 b_f}{2}$$

$$M_n = (30.62 \times 267.5 \times 62.90 + 16.67 \times 60 \times 74.68 - \frac{0.85 \times 4 \times 7.91^2 \times 342}{2}) \div 12 = 46{,}137 k-ft$$

$\varepsilon_t = 0.0211 \geq 0.005 \rightarrow$ the section is tension-controlled and $\phi = 0.95$

Also, the net tensile strain is greater than 0.0075, which satisfies Article 5.7.3.5 for redistribution. Therefore, per the proposed revised Article 5.7.3.3.2, minimum reinforcement is not required to be checked for negative bending.

$\phi M_n = 0.95 \times 46{,}137 = 43{,}830 k - ft$ matches $M_u^{StrengthII}$

Summary:

$P_{jack} = 6{,}200 kip$

$A_s = 16.67 in^2$ mild steel reinforcement

<u>At 0.7 Span 1 (positive moment):</u>

Design moments:

$$M_{DC} = -6{,}148 k - ft$$
$$M_{DW} = -730 k - ft$$
$$M_{SecP/S} = 25 k - ft$$
$$M_{HL-93} = 3{,}916 k - ft$$
$$M_{P-15} = 5{,}242 k - ft$$

$$M_u^{StrengthI} = 0.9 M_{DC} + 0.65 M_{DW} + 1.00 M_{SecP/S} + 1.75 M_{HL-93}$$
$$M_u^{StrengthI} = 0.9 \times (-6{,}148) + 0.65 \times (-730) + 1.00 \times 25 + 1.75 \times 3{,}916$$
$$M_u^{StrengthI} = 871 k - ft$$

$$M_u^{StrengthII} = 0.9 M_{DC} + 0.65 M_{DW} + 1.00 M_{SecP/S} + 1.35 M_{P-15}$$
$$M_u^{StrengthII} = 0.9 \times (-6{,}148) + 0.65 \times (-730) + 1.00 \times 25 + 1.35 \times 5{,}242$$
$$M_u^{StrengthII} = 1{,}094 k - ft$$

Section properties:

$h = 6.5 ft = 78 in$

$h_f = 8.63 in$ (compression flange)

$b_f = 42 ft = 504 in$ (compression flange)

$b_w = 48 in$

$I = 446.45 ft^4$

$A = 71.24 ft^2$

$\overline{y}_b = 45.11 in$ (distance from section CG to bottom fiber)

$\overline{y}_t = 32.89in$ (distance from section CG to top fiber)

Minimum reinforcement by the Modified LRFD Method:

$\phi M_n \geq M_{fcr}$ where $M_{fcr} = \gamma_3 \left[(\gamma_1 f_r + \gamma_2 f_{cpe}) S \right]$ and $\phi M_n \geq 1.33 M_u$

$\gamma_3 = 1.0$ for prestressed structures

$\gamma_1 = 1.6$

$f_r = 0.237\sqrt{f'_c} = 0.237 \times \sqrt{4} = 0.474ksi$

$\gamma_2 = 1.1$

$d_p = 28.15in$ (distance from P/S CG to top fiber)

$e = d_p - \overline{y}_t = 28.15 - 32.89 = -4.74in$ (prestressing eccentricity)

$P_f = 4{,}932.9kips$ (prestressing force after all losses per CT Bridge analysis)

$$f_{cpe} = \frac{P_f}{A} + \frac{(P_f e - M_{SecP/S})\overline{y}_b}{I} = \frac{4{,}932.9}{71.24 \times 12^2} + \frac{(4{,}932.9 \times (-4.74) - 25 \times 12) \times 45.11}{446.45 \times 12^4} = 0.365ksi$$

$$S = \frac{I}{\overline{y}_b} = \frac{446.45 \times 12^4}{45.11} = 205{,}223in^3$$

$$M_{fcr} = 1.0 \times (1.6 \times 0.474 + 1.1 \times 0.365) \times \frac{205{,}223}{12} = 19{,}836k - ft \geq M_u^{StrengthII} = 1{,}094k - ft$$

$M_{fcr} = 19{,}836k - ft \geq 1.33 M_u^{StrengthII} = 1.33 \times 1{,}094 = 1{,}455k - ft$ so

$1.33 M_u^{StrengthII} = 1{,}455k - ft$ controls the design.

Calculation of ϕM_n^{ps} from prestressing:

The calculation of ϕM_n^{ps} per the strain-compatibility method is an iterative process.

$$A_{ps} = \frac{P_{jack}}{0.75 \times f_{pu}} = \frac{6{,}200}{0.75 \times 270} = 30.62in^2$$

$\beta_1 = 0.85$

Assume $c = 5.60in$

$a = \beta_1 c = 0.85 \times 5.60 = 4.76in \leq h_f = 8.63in$ therefore the section behavior is rectangular.

The strain of the prestress tendons consists of the following:

Effective prestress at service load: $\varepsilon_{ps1} = \dfrac{P_f}{E_{ps} A_{ps}} = \dfrac{4{,}932.9}{28{,}500 \times 30.62} = 0.00565$

At decompression:

$$\varepsilon_{ps2} = \frac{P_f}{E_c A_c}(1+\frac{e^2}{r^2}) = \frac{4{,}932.9}{3{,}644\times 71.24\times 12^2}(1+\frac{(-4.74)^2}{(\frac{446.45\times 12^4}{71.24\times 12^2})}) = 0.00014$$

At limit state:

$$\varepsilon_{ps3} = \varepsilon_c \frac{(d_p - c)}{c} = 0.003\times\frac{(28.15-5.60)}{5.60} = 0.01208$$

Thus $\varepsilon_{ps} = \varepsilon_{ps1} + \varepsilon_{ps2} + \varepsilon_{ps3} = 0.00565 + 0.00014 + 0.01208 = 0.0179 \geq 0.0086$

$$f_{ps} = f_{pu} - \frac{0.04}{\varepsilon_{ps} - 0.007} = 270 - \frac{0.04}{0.0179 - 0.007} = 266.3 ksi$$

Check the force equilibrium:

Flange force $C_f = 0.85 f'_c \, a b_f = 0.85\times 4\times 4.76\times 504 = 8{,}155.3 kip$

Prestressing force $T_{ps} = A_{ps} f_{ps} = 30.62\times 266.3 = 8{,}154.7 kip$ within 0.007% of $C_f \rightarrow$ equilibrium is satisfied.

$$M_n^{ps} = A_{ps} f_{ps} d_p - \frac{0.85 f'_c a^2 b_f}{2} = (30.62\times 266.3\times 28.15 - \frac{0.85\times 4\times 4.76^2\times 504}{2}) \div 12 = 17{,}512 k-ft$$

$\varepsilon_{ps3} = 0.01208 \geq 0.005 \rightarrow$ the section is tension-controlled and $\phi = 0.95$.

$\phi M_n^{ps} = 0.95\times 17{,}512 = 16{,}637 k-ft \geq 1.33 M_u^{StrengthII} = 1{,}455 k-ft$ so no additional mild steel reinforcement is required.

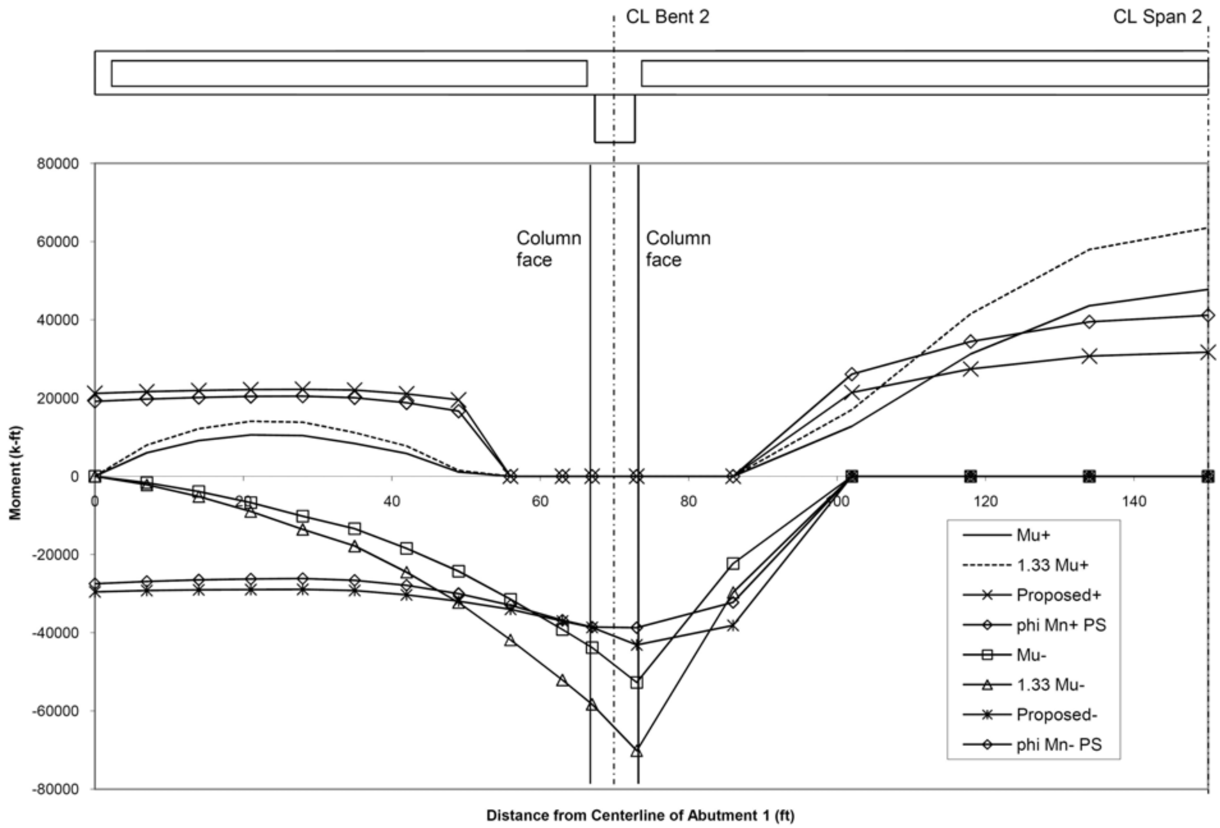


Figure B-4. CIP Box Girder Moment Profiles

B.3 SPAN-BY-SPAN SEGMENTAL BRIDGE WITH EXTERNAL TENDONS

INTRODUCTION

A two-span precast segmental bridge is the subject of this design example. The bridge is built using the span-by-span construction method. The bridge chosen for this example is part of the I-4/Lee Roy Selmon Expressway in Tampa, FL.

Each of the two spans in this bridge is simply supported. Only Span 2 of this bridge is the subject of this example. This represents a relatively large depth-to-span ratio bridge in which the minimum flexural reinforcement requirement could control the design. An elevation view of this bridge is shown in Figure B-5.

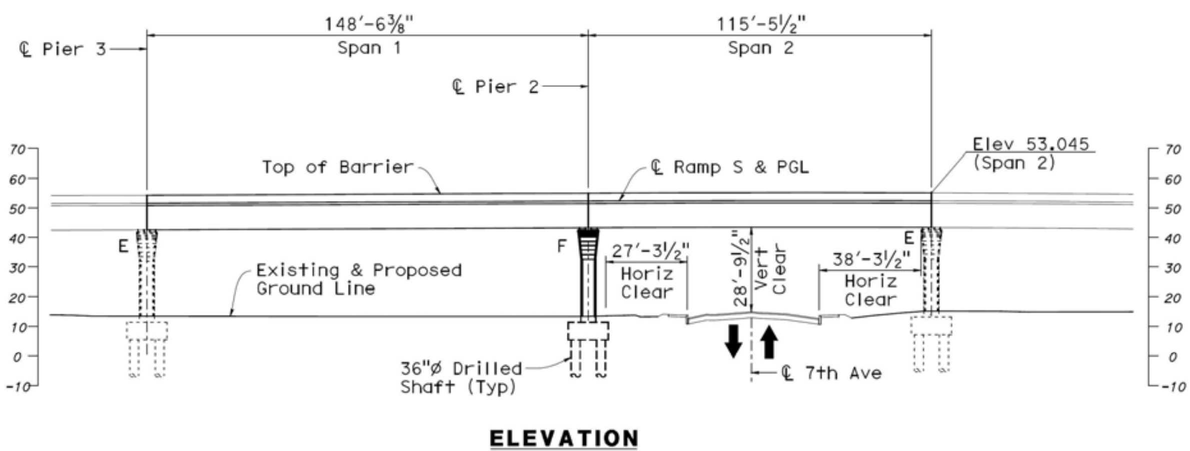


Figure B-5. Precast Segmental Span-By-Span Bridge Design Example

For Span 2, the cross section consists of a single-cell box section with long overhangs as shown in Figure B-6. The deck width is variable as indicated in Figure B-5. The length of Span 2 is approximately 115'-6" and the bridge is prestressed by means of external unbonded tendons as shown in the tendon layout in Figure B-7.

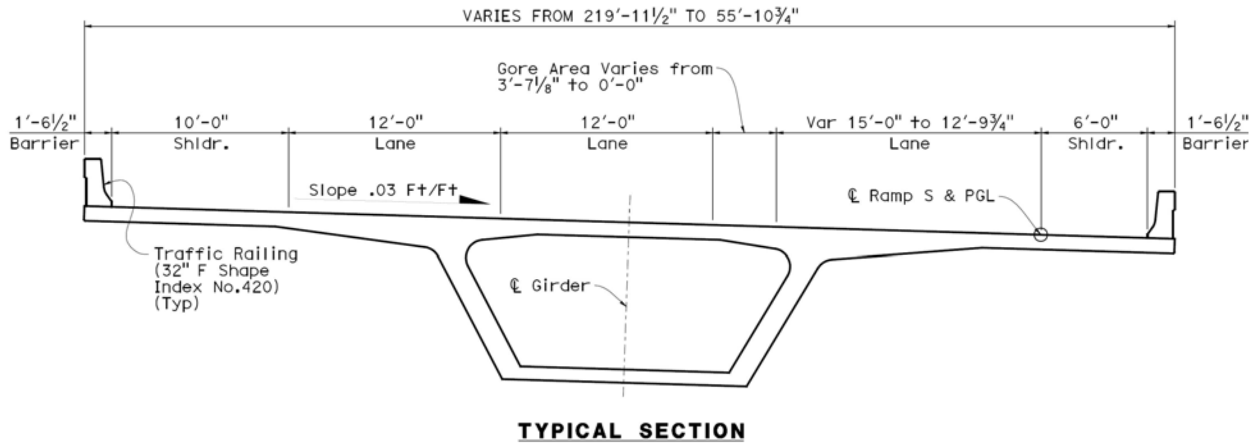


Figure B-6. Cross section (Span 2)

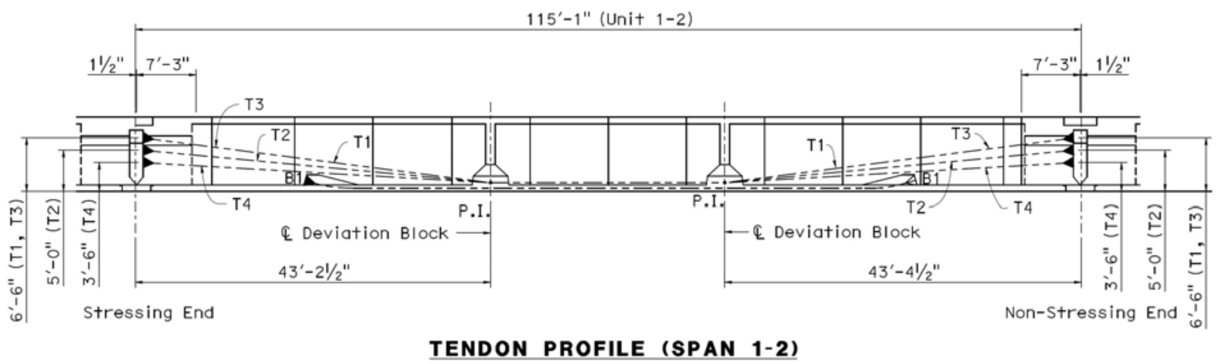


Figure B-7. Prestressing Tendon Layout (Span 2)

SPECIFICATIONS

This example is designed based on the *AASHTO LRFD Bridge Design Specifications 4th Edition, 2007.*

MATERIAL PROPERTIES

$f'_c = 6.5\ ksi$

$E_c = 4{,}888\ ksi$

$f_y = 60\ ksi$

$E_s = 29{,}000\ ksi$

$f_{pu} = 270\ ksi$

$E_{ps} = 28{,}500\ ksi$

PRESTRESS DESIGN

For precast segmental bridges with no bonded reinforcement or bonded tendons crossing the joints, no tensile stresses are allowed at all segment-to-segment joints under service loads. Longitudinal analysis and design of this bridge included concrete stresses under service loads, flexural capacity, shear capacity, principal stresses in the box girder webs and minimum flexural reinforcement requirements. Except for the minimum flexural reinforcement requirement, design is satisfactory with the use of four external tendons on each side of the box section; three of these tendons are composed of 19-0.6" ϕ strands and the fourth tendon is composed of 15-0.6" ϕ strands. Thus, the total number of external unbonded strands in this bridge is 144-0.6" ϕ.

MOMENT DIAGRAMS

Figure B-8 shows the bending moments along the length of the single span bridge (Span 2). The figure shows the minimum design moments due to cracking according to the current AASHTO LRFD Specifications and based on the proposed method (Modified LRFD). It is clear that the proposed provisions significantly reduce the minimum required design moments (MFR). The figure also indicates that along the entire span length, the $1.33M_u$ controls over the $1.20M_{cr}$ (AASHTO LRFD Specifications). However, in the middle third of the span length, the cracking moment based on the proposed modified LRFD method controls over $1.33M_u$.

Figure B-8 also shows the factored flexural moment capacity, which is higher than the factored moment, M_u, at all sections. However, in the middle 80 ft of the span length, the minimum flexural reinforcement requirement is not satisfied and the prestressing will be controlled by the MFR requirement. It should be noted that depth of the box girder is 9 ft, whereas the span length is about 115 ft only. Thus, the superstructure is relatively deep, which could result in the flexural design being controlled by the MFR requirement. Alternative design to satisfy the MFR requirement will be discussed in the following sections.

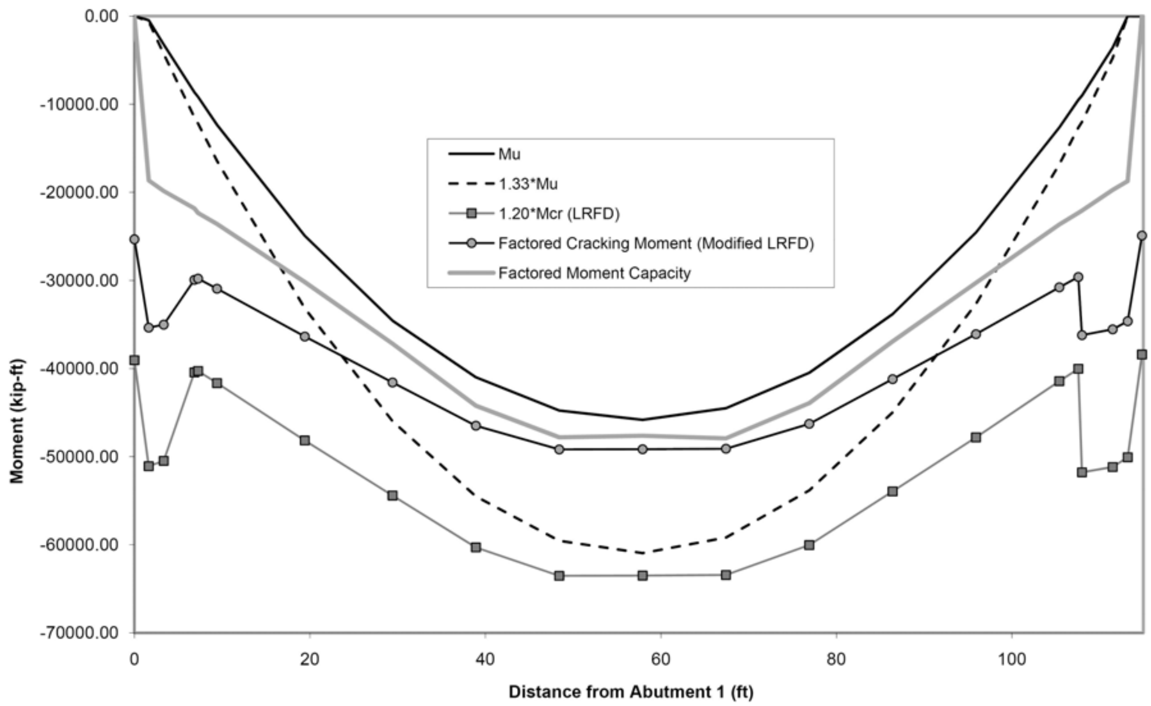


Figure B-8. Cracking Moment, Factored Moment and Flexural Capacity of a Precast Segmental Span-By-Span Bridge Example

Analysis of this bridge was done using LARSA 4D. Construction stages and time-dependent effects were considered in the analysis. Below are hand calculations for the midspan section of the bridge.

Design moments:

Sign convention is positive for moment resulting in tensile stress at bottom surface (opposite to the sign shown in Figure B-8).

$M_{DC} = 24{,}677\ k-ft$ Self wt, ½" sacrificial wearing surface, diaphragms & barriers

$M_{DW} = 0\ k-ft$ No utilities or future wearing surface

$M_{SecP/S} = 0\ k-ft$ No secondary effects from prestressing for a single span bridge

$M_{TU} = 0\ k-ft$ No moments from uniform temperature rise for a single span bridge

$M_{TG} = 0\ k-ft$ No moments from temperature gradient for a single span bridge

$M_{HL-93+I} = 8{,}560\ k-ft$

$$M_u = M_u^{StrengthI} = 1.25M_{DC} + 1.50M_{DW} + 1.00M_{SecP/S} + 1.75M_{HL-93+I} + 0.50M_{TU} + 0.50M_{TG}$$
$$M_u = 1.25\times(24{,}677) + 1.50\times(0) + 1.00\times(0) + 1.75\times(8{,}560) + 0.50\times(0) + 0.50\times(0)$$
$$M_u = 45{,}826\ k-ft$$

Section properties:

$h = 9\ ft = 108\ in$ The sacrificial surface is included as external load only

$h_f = 9.5\ in$ (minimum thickness of compression flange)

$b_f = 58.625\ ft = 703.5\ in$ (compression flange width)

$b_w = 30\ in$

$I = 819.97\ ft^4 = 17{,}002{,}898\ in^4$

$A = 91.02\ ft^2 = 13{,}106.88\ in^2$

$\bar{y}_b = 78.84\ in$ (distance from section CG to bottom fiber)

$\bar{y}_t = 29.16\ in$ (distance from section CG to top fiber)

Calculation of ϕM_n from prestressing:

In Figure B-8, the moment capacity is calculated using LARSA 4D. At the midspan section, the factored flexural moment capacity is 47,631 kip-ft. The flexural capacity for the midspan section is calculated below using the AASHTO LRFD equations, which may result in slightly different values from those calculated by LARSA 4D.

$A_{ps} = 144\times 0.217\ in^2 = 31.248\ in^2$ Total of 144-0.6" ϕ strands (external unbonded)

$d_p = 94.75\ in$ (distance from P/S CG to top fiber)

$e = d_p - \bar{y}_t = 94.75 - 29.16 = 65.59\ in$ (tendon eccentricity)

$\beta_1 = 0.85 - 0.05\,(f_c' - 4) = 0.725$

Effective prestressing force in external tendons (from LARSA 4D):

$P_f = 5{,}247\ kips$

$$f_{pe} = \frac{P_f}{A_{ps}} = \frac{5{,}247\ kips}{31.248\ in^2} = 167.9\ ksi$$

Length of external tendon (approximate): $l_i = 114.83\ ft$

Number of support hinges crossed by external tendons (single span): $N_i = 0$

Effective length of external tendons:

$$l_e = 2\,\frac{l_i}{(2+N_i)} = 114.83\ ft$$

Depth of compression zone: Assume $f_{ps} = 228\ ksi < f_{py} = 243\ ksi$

$$c = \frac{A_{ps}\ f_{ps}}{0.85\ f_c^{'}\ \beta_1\ b_f} = 2.53\ in$$

Depth of neutral axis is smaller than deck thickness. Thus, use of equations for rectangular sections is justified.

Stress in external tendons at ultimate moment:

$$f_{ps} = f_{pe} + 900\frac{(d_p - c)}{l_e} = 228\ ksi\ < f_{py} = 243\ ksi$$

This stress is the same as assumed above. Thus, no iterations are needed.

Tensile force at ultimate moment: $T = A_{ps}\ f_{ps} = 7{,}124.5\ kips$

Depth of equivalent rectangular stress block: $a = \beta_1\ c = 1.83\ in$

Resistance factor: $\phi = 0.90$ (segmental bridges with unbonded tendons)

Factored flexural moment capacity:

$$\phi\ M_n = \phi\ A_{ps}\ f_{ps}\ (d_p - \frac{a}{2}) = 50{,}139\ kip - ft$$

LARSA 4D calculated the factored moment capacity as 47,633 kip-ft (about 5% difference). It should be noted that the above-calculated factored moment capacity does not take into account the reduction in moment arm of the external tendons due to deflection of the superstructure. Thus, the predicted flexural capacity will be less than 50,139 kip-ft.

Minimum reinforcement by the proposed method (Modified LRFD):

$\phi M_n \geq M_{fcr}$ or $\phi M_n \geq 1.33 M_u$; where $M_{fcr} = \gamma_3\ (\gamma_1 f_{cr} + \gamma_2 f_{cpe}) S$

$\gamma_1 = 1.20$ (proposed for precast segmental bridges)

$f_{cr} = 0.24\sqrt{f'_c} = 0.24 \times \sqrt{6.5} = 0.612\ ksi$

$\gamma_2 = 1.00$ (proposed for bridges with only unbounded tendons)

$\gamma_3 = 1.00$ (tensile resistance is provided by prestressing steel)

Concrete compressive stress at bottom fiber due to prestressing (after losses):

$$f_{cpe} = \frac{P_f}{A} + \frac{P_f\ e\ \bar{y}_b}{I} = \frac{5{,}247}{13{,}106.88} + \frac{5{,}247 \times 65.59 \times 78.84}{17{,}002{,}898} = 2.000\ ksi$$

$$S = \frac{I}{\bar{y}_b} = \frac{17{,}002{,}898}{78.84} = 215{,}663\ in^3$$

$$\gamma_3\ (\gamma_1 f_{cr} + \gamma_2 f_{cpe})S = 1.0 \times (1.20 \times 0.612 + 1.1 \times 2.000)x\frac{215{,}663}{12} = 49{,}142\ k - ft > M_u = 45{,}826\ k - ft$$

$$\gamma_3\ (\gamma_1 f_{cr} + \gamma_2 f_{cpe})S = 49{,}142\ k - ft < 1.33M_u = 1.33 \times 45{,}826 = 60{,}949\ k - ft$$

$\gamma_3\ (\gamma_1 f_{cr} + \gamma_2 f_{cpe})S = 49{,}142\ k - ft$ controls the design.

$\phi M_n \geq \gamma_3\ (\gamma_1 f_{cr} + \gamma_2 f_{cpe})S$ MFR Requirement

$$\phi M_n = 47{,}633\ k - ft < \gamma_3\ (\gamma_1 f_{cr} + \gamma_2 f_{cpe})S = 49{,}142\ k - ft$$

The factored flexural moment capacity calculated by LARSA 4D (used for the plot in Figure B-8) is smaller than the 50,139 kip-ft factored moment capacity calculated above. However, the actual factored moment capacity should be less than 50,139 k-ft as a result of the reduction in the internal moment arm of the section due to vertical downward deflection of the girder at midspan. The flexural moment capacity calculated by LARSA 4D is used in this example. Thus, the minimum flexural reinforcement requirement is not satisfied. However, the factored flexural moment capacity is about 3 percent below the flexural capacity required by the proposed MFR requirements.

Re-Design of Prestressing Steel:

Figure B-9 shows variation of cracking moment, factored moment and factored moment capacity for the midspan section of this bridge as a function of the number of strands in the external tendons. The moment capacities represented in Figure B-9 are based on LARSA 4D calculations. The figure indicates that with increasing the number of 0.6" ϕ strands from 144 to 160, the proposed MRF requirements will be satisfied.

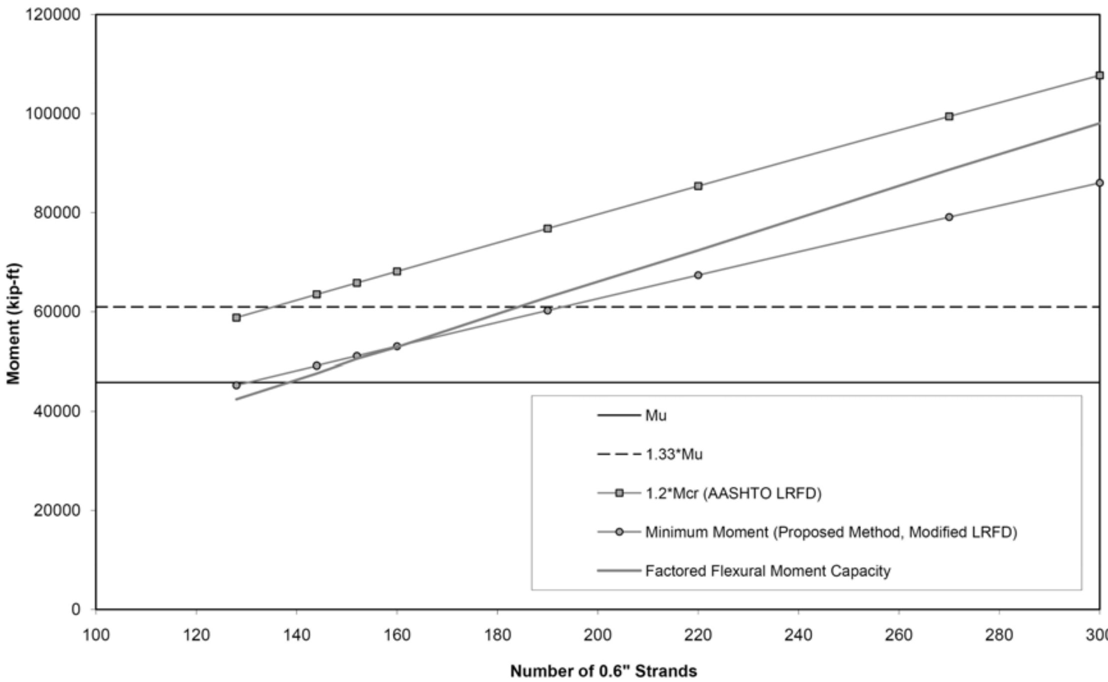


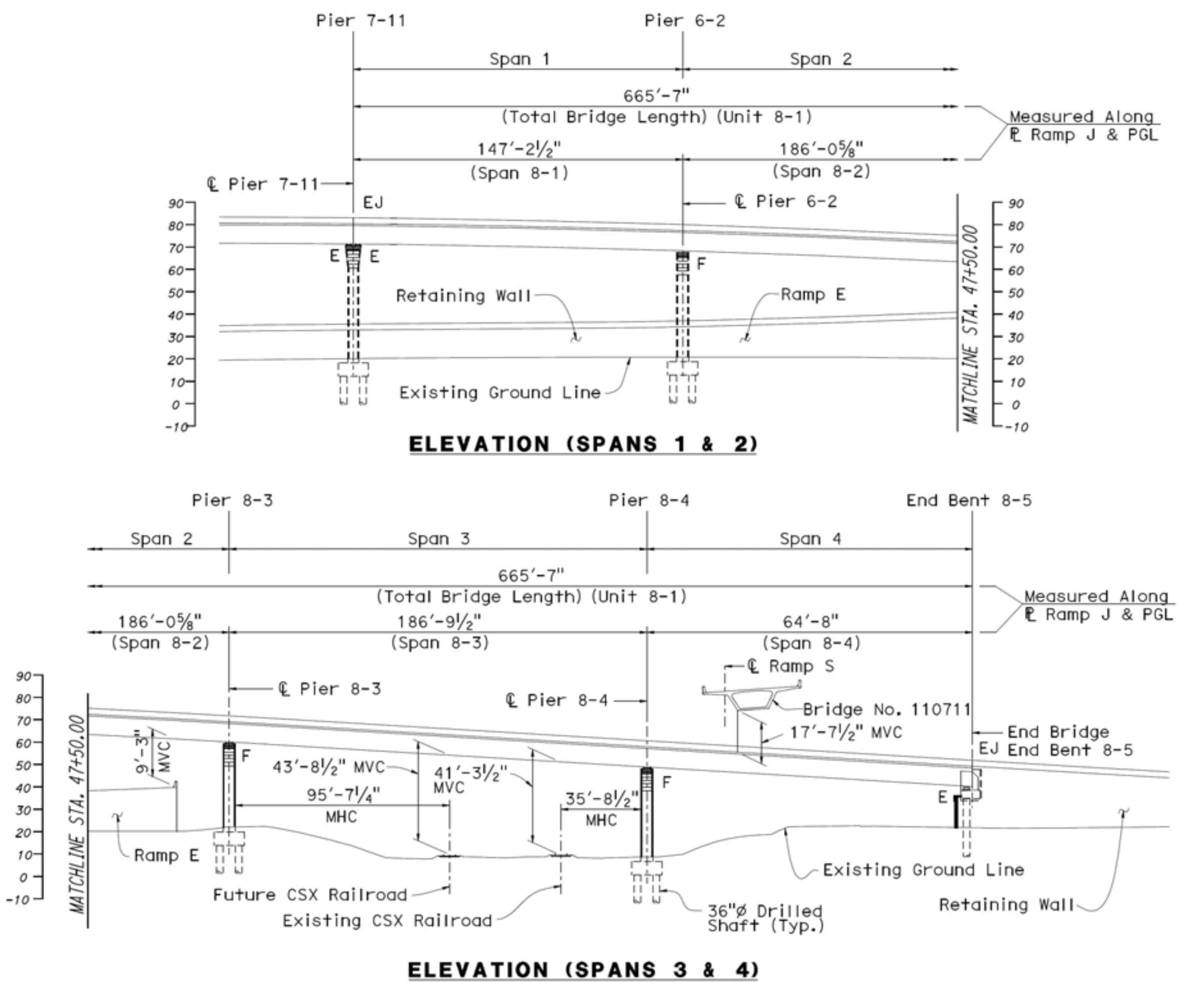
Figure B-9. Variation of Cracking Moment, Factored Moment and Moment Capacity with Number of Strands in External Tendons

It is interesting to note that with the current AASHTO LRFD MFR requirements, the curve representing minimum design moment in Figure B-9 does not intersect with the curve representing the moment capacity, which indicates that no convergence may be obtained to satisfy the MFR by increasing the number of strands (unless $1.33M_u$ controls the MFR requirement). Figure B-9 indicates that such convergence is possible with the use of the proposed equation.

B.4 BALANCED CANTILEVER BRIDGE WITH INTERNAL TENDONS

INTRODUCTION

A four-span precast segmental bridge is the subject of this design example. The bridge is built using the cantilever construction method. The bridge chosen for this example is part of the I-4/Lee Roy Selmon Expressway in Tampa, FL. Elevation view of the bridge is shown in Figure B-10. The approximate lengths of spans are 147'-3", 186'-1", 186'-9" and 145'-6" for Spans 1 through 4, respectively, with a total bridge length of 665'-7".



B-10. Precast Segmental Cantilever Bridge Design Example

The cross section consists of the single-cell box section shown in Figure B-11. The deck width is 30'-1" and is constant along the entire length of the bridge.

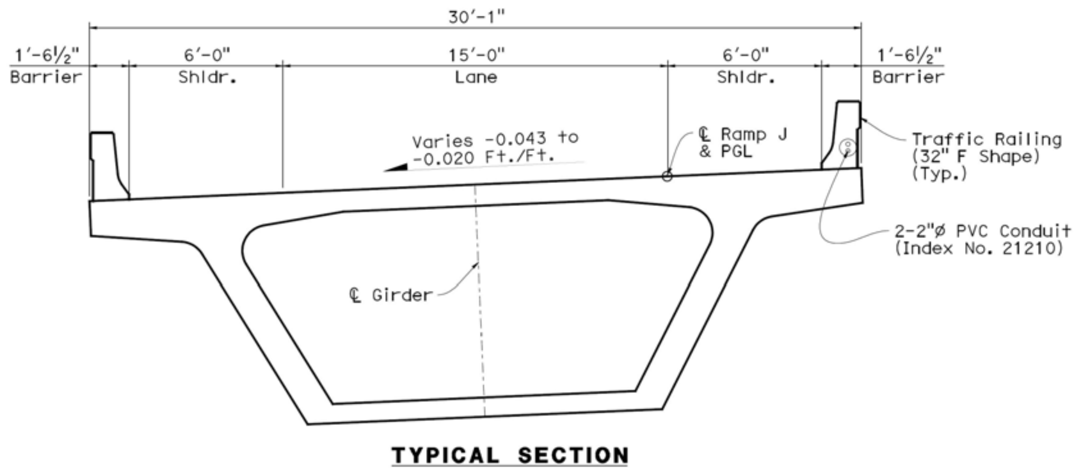


Figure B-11. Cross section (Span 2)

The prestressing steel consists of typical internal (bonded) tendons in the deck slab. Continuity prestressing steel consists of external (unbounded) tendons as shown in Figures B-12 and B-13. There are a total of three external tendons next to each of the two webs (Tendons T3, T4 & T5 in Figures B-12 and B-13).

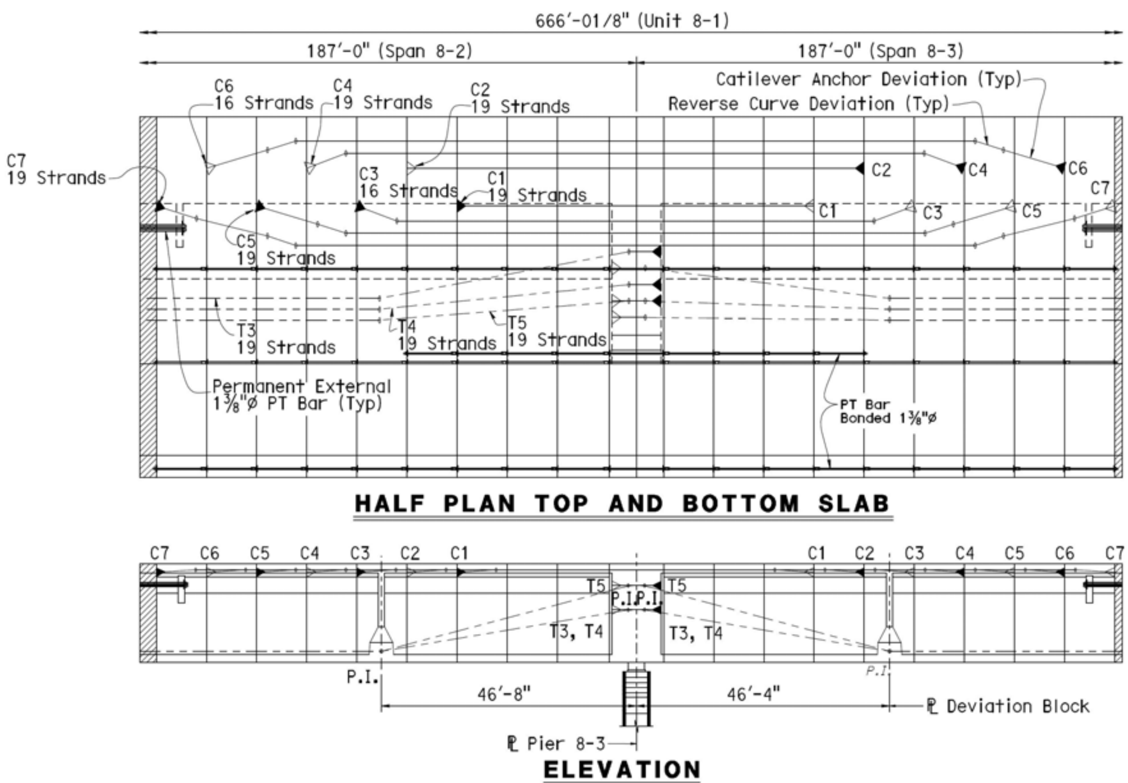


Figure B-12. Tendon Layout for the Precast Segmental Cantilever Bridge Design Example

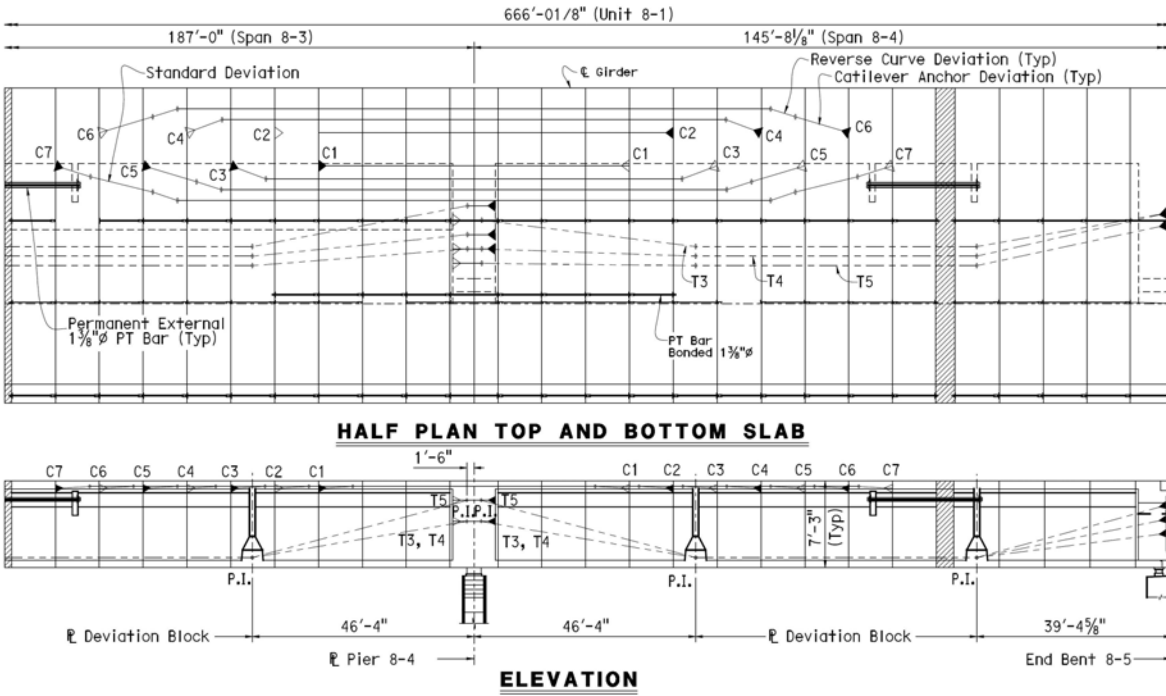


Figure B-13. Tendon Layout for the Precast Segmental Cantilever Bridge Design Example

SPECIFICATIONS

This example is designed based on the *AASHTO LRFD Bridge Design Specifications 4th Edition, 2007.*

MATERIAL PROPERTIES

$f'_c = 8.5\ ksi$

$E_c = 5{,}589\ ksi$

$f_y = 60\ ksi$

$E_s = 29{,}000\ ksi$

$f_{pu} = 270\ ksi$

$E_{ps} = 28{,}500\ ksi$

PRESTRESS DESIGN

For precast segmental bridges, no tensile stresses are allowed at all segment-to-segment joints under service loads. Longitudinal analysis and design of this bridge included concrete stresses under service loads, flexural capacity, shear capacity, principal stresses in the box girder webs and minimum flexural reinforcement requirements. At the first segment-to-segment joint next to Pier 8-3 in Span 4 (most critical section for negative moment), there are a total of 254-0.6" ϕ internal (bonded) strands and 114"-0.6 ϕ unbonded strands (external tendons). In the positive moment region in Span 4 (most critical section for positive moment), the only prestressing is provided by the continuity external tendons and the total number of strands is 114.

MOMENT DIAGRAMS

The bridge is almost symmetric about centerline of Pier 8-3. Figure B-14 shows the negative bending moments along the length of Spans 3 & 4 (from Pier 8-3 to End Bent 8-5). Negative moment results in tensile stresses at top surface of the superstructure. The figure shows the minimum design moments due to cracking according to the current AASHTO LRFD Specifications and based on the proposed method (Modified LRFD). It is clear that the proposed provisions considerably reduce the minimum required design moments (MFR). The figure also indicates that the $1.33M_u$ controls over the $1.20M_{cr}$ (AASHTO LRFD Specifications) or the cracking moment based on the proposed Modified LRFD method. Thus, $1.33M_u$ controls the MFR in this case. Figure B-14 also shows the factored flexural moment capacity, which is higher than $1.33M_u$ at all sections.

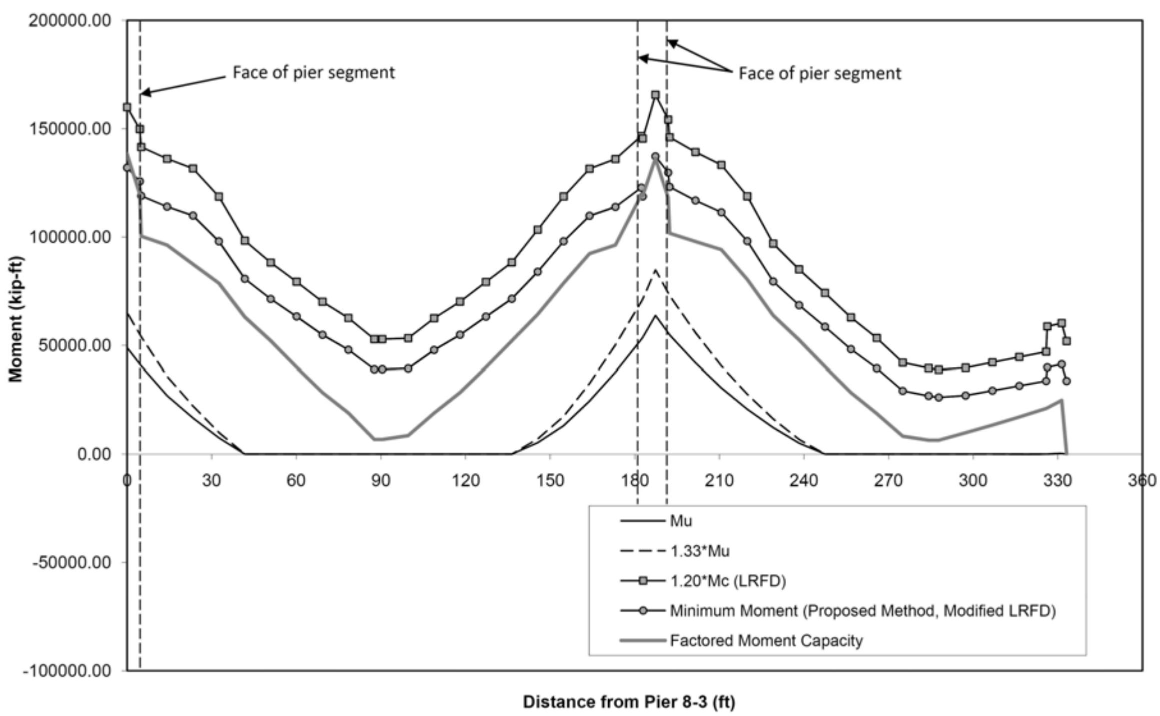


Figure B-14. Cracking Moment, Factored Moment and Flexural Capacity of a Precast Segmental Cantilever Bridge Example (Negative Moments)

Figure B-15 is similar to Figure B-14, but it shows variation of the positive bending moments (bending moments resulting in tensile stresses at bottom surface of the superstructure). Again, use of the Modified LRFD method significantly reduces the required MFR design moment compared to the current AASHTO LRFD provisions. For sections away from the supports, the minimum design moment according to the Modified LRFD method controls over 1.33M_u, whereas 1.33M_u controls MFR for sections near the supports. In all sections, the factored flexural moment capacities exceed the demand moment including the MFR requirements.

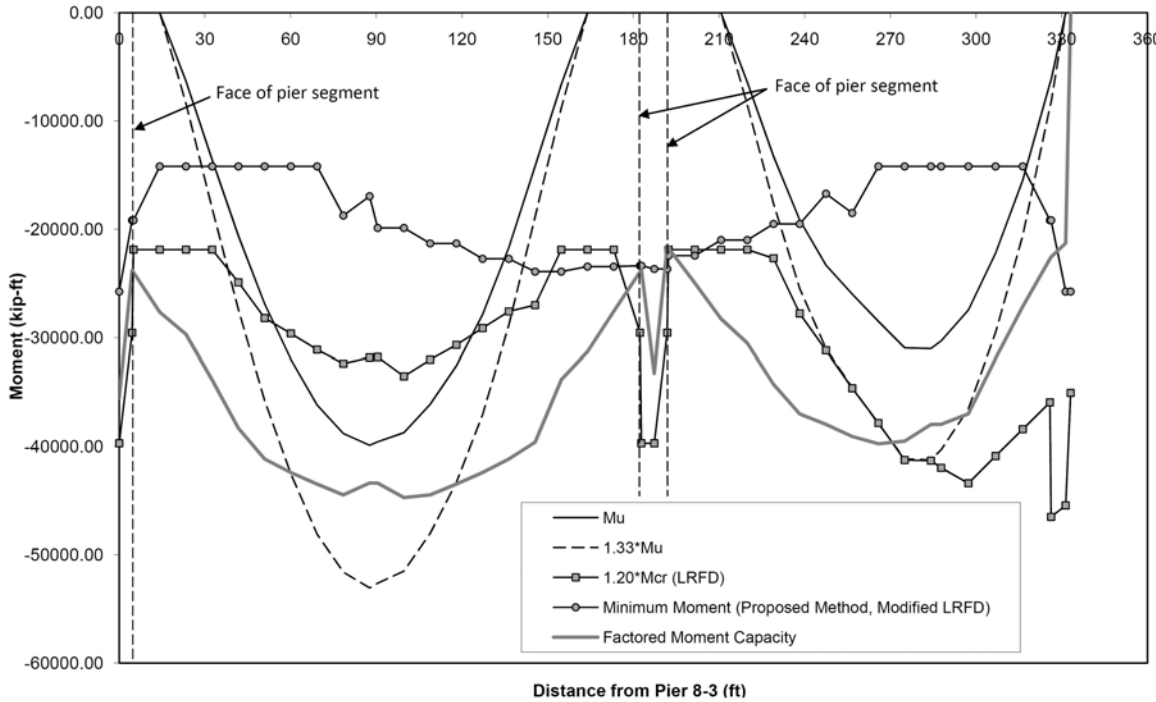


Figure B-15. Cracking Moment, Factored Moment and Flexural Capacity of a Precast Segmental Cantilever Bridge Example (Positive Moments)

Analysis of this bridge was done using LARSA 4D. Construction stages and time-dependent effects were considered in the analysis. Below are hand calculations for the section at first segment-to-segment joint in Span 4 (joint at Pier 8-4) as well as maximum positive moment section in Span 4 of the bridge.

Design moments:

Sign convention is positive for moment resulting in tensile stress at bottom surface (opposite to the sign shown in Figures B-14 and B-15).

Section A: Section at First Joint (Pier Segment) in Span 4:

$M_{DC} = -53{,}167\ k-ft$ Self wt, ½" sacrificial wearing surface, diaphragms & barriers

$M_{DW} = 0\ k-ft$ No utilities or future wearing surface

$M_{LT} = 1{,}842\ k-ft$ Long-term effects (concrete creep & shrinkage and relaxation of prestressing steel)
$M_{SecP/S} = 25{,}000\ k-ft$ Secondary effects from prestressing
$M_{TU} = -2\ k-ft$ Uniform temperature rise
$M_{TG} = -1{,}628\ k-ft$ Temperature gradient
$M_{HL-93+I} = -7{,}727\ k-ft$

$$M_u = M_u^{StrengthI} = 1.25M_{DC} + 1.50M_{DW} + 0.50M_{LT} + 1.00M_{SecP/S} + 1.75M_{HL-93+I} + 0.50M_{TU} + 0.50M_{TG}$$

$$M_u = 1.25\times(-53{,}167) + 1.50\times(0) + 0.50\times(1{,}842) + 1.00\times(25{,}000) + 1.75\times(-7{,}727) + 0.50\times(-2) + 0.50\times(-1{,}628)$$

$$M_u = 54{,}875\ k-ft$$

<u>Section B: Section at Location of Maximum Positive Moment in Span 4:</u>

$M_{DC} = 6{,}792\ k-ft$ Self wt, ½" sacrificial wearing surface, diaphragms & barriers
$M_{DW} = 0\ k-ft$ No utilities or future wearing surface
$M_{LT} = 628\ k-ft$ Long-term effects (concrete creep & shrinkage and relaxation of prestressing steel)
$M_{SecP/S} = 8{,}667\ k-ft$ Secondary effects from prestressing
$M_{TU} = -1\ k-ft$ Uniform temperature rise
$M_{TG} = 1{,}842\ k-ft$ Temperature gradient
$M_{HL-93+I} = 7{,}209\ k-ft$

$$M_u = M_u^{StrengthI} = 1.25M_{DC} + 1.50M_{DW} + 0.50M_{LT} + 1.00M_{SecP/S} + 1.75M_{HL-93+I} + 0.50M_{TU} + 0.50M_{TG}$$

$$M_u = 1.25\times(6{,}792) + 1.50\times(0) + 0.50\times(628) + 1.00\times(8{,}667) + 1.75\times(7{,}209) + 0.50\times(-1) + 0.50\times(1{,}842)$$

$$M_u = 31{,}008\ k-ft$$

Section properties:

Section properties for both Sections A & B are similar.

$h = 9\ ft = 108\ in$ The sacrificial surface is included as external load only

$h_f = 9.0\ in$ (minimum thickness of compression flange for negative moment section)

$h_f = 9.5\ in$ (minimum thickness of compression flange for positive moment section)

$b_f = 13.833\ ft = 166\ in$ (compression flange width for negative moment section)

$b_f = 30.083\ ft = 361\ in$ (compression flange width for positive moment section)

$b_w = 30\ in$

$I = 684.20\ ft^4 = 14{,}187{,}571\ in^4$

$A = 63.33\ ft^2 = 9{,}119.52\ in^2$

$\bar{y}_b = 69.96\ in$ (distance from section CG to bottom fiber)

$\bar{y}_t = 38.04\ in$ (distance from section CG to top fiber)

Calculation of ϕM_n from prestressing:

In Figure B-14 & Figure B-15, the moment capacity is calculated using LARSA 4D. The factored flexural moment capacities are 101,729 kip-ft and 37,958 kip-ft at Sections A & B, respectively. The flexural capacities for both sections are calculated below using the AASHTO LRFD equations, which may result in slightly different values from those calculated by LARSA 4D.

<u>Section A: Section at First Joint (Pier Segment) in Span 4:</u>

$A_{ps1} = 254 \times 0.217\ in^2 = 55.118\ in^2$ Total of 254-0.6" ϕ strands (internal bonded)

$A_{ps2} = 114 \times 0.217\ in^2 = 24.738\ in^2$ Total of 114-0.6" ϕ strands (external unbonded)

$A_s = 3 \times 1.58\ in^2 = 4.74\ in^2$ 3-1.58" ϕ strands high-strength bars in the deck slab

Yield strength for high-strength bars: $f_y = 120\ ksi$

$d_{p1} = 100.50\ in$ (distance from bottom fiber to C.G. of cantilever tendons)

$d_{p2} = 66.83\ in$ (distance from bottom fiber to C.G. of external tendons)

$d_s = 102\ in$ (distance from bottom fiber to C.G. of high strength bars)

$\beta_1 = 0.85 - 0.05\,(f_c^{'} - 4) = 0.625$

Effective prestressing force in external tendons (from LARSA 4D):

$P_{f2} = 4{,}427\ kips$

$$f_{pe2} = \frac{P_{f2}}{A_{ps2}} = \frac{4{,}427\ kips}{24.738\ in^2} = 178.9\ ksi$$

Length of external tendon (approximate): $l_i = 150.50\ ft$

Number of support hinges crossed by external tendons (end span): $N_i = 1$

Effective length of external tendons:

$$l_e = 2\frac{l_i}{(2+N_i)} = 100.33\ ft$$

Depth of compression zone: Assume $f_{ps} = 182\ ksi < f_{py} = 243\ ksi$

$$c = \frac{A_{ps1}\ f_{pu} + A_{ps2}\ f_{ps2} + A_s\ f_y}{0.85\ f_c^{'}\ \beta_1\ b_f + k\ A_{ps1}\ \dfrac{f_{pu}}{d_{p1}}} = 25.22\ in \qquad \text{(k = 0.28 for low-relaxation strands)}$$

Depth of neutral axis is greater than deck thickness. Thus, use of equations for flanged sections should be used.

$$c = \frac{A_{ps1}\ f_{pu} + A_{ps2}\ f_{ps2} + A_s\ f_y - 0.85 f_c^{'}(b_f - b_w)h_f}{0.85\ f_c^{'}\ \beta_1\ b_w + k\ A_{ps1}\ \dfrac{f_{pu}}{d_{p1}}} = 62.79\ in$$

Stress in bonded tendons at ultimate moment:

$$f_{ps1} = f_{pu}\ (1 - k\frac{c}{d_{p1}}) = 223\ ksi$$

Stress in external tendons at ultimate moment:

$$f_{ps} = f_{pe} + 900\frac{(d_{p2} - c)}{l_e} = 182\ ksi\ < f_{py} = 243\ ksi$$

This stress is the same as assumed above. Thus, no iterations are needed.

Tensile force at ultimate moment: $T = A_{ps1}\ f_{ps1} + A_{ps2}\ f_{ps2} + A_s\ f_y = 17{,}362.4\ kips$

Depth of equivalent rectangular stress block: $a = \beta_1\ c = 39.24\ in$

Resistance factor: $\phi = 0.95$

(segmental bridges with bonded tendons providing most of the prestressing)

Factored flexural moment capacity:

$$\phi\ M_n = \phi\ A_{ps1}\ f_{ps1}\ (d_{p1} - \frac{a}{2}) + \phi\ A_{ps2}\ f_{ps2}\ (d_{p2} - \frac{a}{2}) + \phi\ A_s\ f_y\ (d_s - \frac{a}{2}) = 99{,}238\ kip - ft$$

The factored moment capacity calculated by LARSA 4D is 101,729 kip-ft (less than 3% difference).

The factored moment capacity is significantly larger than 1.33Mu (= 72,984 kip-ft). Also, Figure B-14 clearly shows that the factored moment capacity is much larger than 1.33Mu (which controls the MFR requirements for this section). Thus, hand calculations demonstrating the proposed MFR procedure will not be shown for this section, but it will be shown for the positive moment section (Section B).

<u>Section B: Section at Maximum Positive Moment in Span 4:</u>

Prestressing tendons at this section is composed of external (unbonded) tendons only.

$A_{ps} = 114 \times 0.217\ in^2 \quad = 24.738\ in^2$ Total of 114-0.6" ϕ strands (external unbonded)

$d_p = 86.49\ in$ (distance from P/S CG to top fiber)

$e = d_p - \overline{y}_t = 86.49 - 38.04 = 48.45\ in$ (tendon eccentricity)

$\beta_1 = 0.85 - 0.05\,(f_c^{'} - 4) = 0.625$

Effective prestressing force in external tendons (from LARSA 4D):

$P_f = 4{,}427\ kips$

$$f_{pe} = \frac{P_f}{A_{ps}} = \frac{4{,}427\ kips}{24.738\ in^2} = 178.9\ ksi$$

Length of external tendon (approximate): $l_i = 150.50\ ft$

Number of support hinges crossed by external tendons (end span): $N_i = 1$

Effective length of external tendons:

$$l_e = 2\,\frac{l_i}{(2 + N_i)} = 100.33\ ft$$

Depth of compression zone: Assume $f_{ps} = f_{py} = 243\ ksi$

$$c = \frac{A_{ps}\ f_{ps}}{0.85\ f_c^{'}\ \beta_1\ b_f} = 3.69\ in$$

Depth of neutral axis is smaller than deck thickness. Thus, use of equations for rectangular sections is justified.

Stress in external tendons at ultimate moment:

$$f_{ps} = f_{pe} + 900\,\frac{(d_p - c)}{l_e} = 241\ ksi\ > f_{py} = 243\ ksi \qquad \text{Use}\quad f_{ps} = 243\ ksi$$

This stress is the same as assumed above. Thus, no iterations are needed.

Tensile force at ultimate moment: $T = A_{ps}\ f_{ps} = 6{,}011.3\ kips$

Depth of equivalent rectangular stress block: $a = \beta_1\ c = 2.31\ in$

Resistance factor: $\phi = 0.90$ Segmental bridges with unbonded tendons

Factored flexural moment capacity:

$$\phi\ M_n = \phi\ A_{ps}\ f_{ps}\ (d_p - \frac{a}{2}) = 38{,}473\ kip - ft$$

LARSA 4D calculated the factored moment capacity as 37,958 kip-ft (less than 2% difference). It should be noted that the above-calculated factored moment capacity does not take into account the reduction in moment arm of the external tendons due to deflection of the superstructure. Thus, the predicted flexural capacity will be slightly less than 38,473 kip-ft.

Minimum reinforcement by the proposed method (Modified LRFD):

<u>Section B: Section at Maximum Positive Moment in Span 4:</u>

$\phi M_n \geq M_{fcr}$ or $\phi M_n \geq 1.33 M_u$; where $M_{fcr} = \gamma_3 \, (\gamma_1 f_{cr} + \gamma_2 f_{cpe}) S$

$\gamma_1 = 1.20$ (proposed for precast segmental bridges)

$f_{cr} = 0.24\sqrt{f'_c} = 0.24 \times \sqrt{8.5} = 0.700 \; ksi$

$\gamma_2 = 1.00$ (proposed for bridges with only unbounded tendons; positive moment capacity at Section B is provided by only unbounded tendons)

$\gamma_3 = 1.00$ (tensile resistance is provided by prestressing steel)

Secondary moment from prestressing:

$M_{SecP/S} = 8{,}667 \; k - ft$

Concrete compressive stress at bottom fiber due to prestressing (after losses):

$$f_{cpe} = \frac{P_f}{A} + \frac{P_f \; e \; \bar{y}_b}{I} = \frac{4{,}247}{9{,}119.52} + \frac{4{,}247 \times 48.45 \times 69.96}{14{,}187{,}571} - \frac{8{,}667 \times 12 \times 69.96}{14{,}187{,}571} = 0.968 \; ksi$$

$$S = \frac{I}{\bar{y}_b} = \frac{14{,}187{,}571}{69.96} = 202{,}795 \; in^3$$

$$\gamma_3 \, (\gamma_1 f_{cr} + \gamma_2 f_{cpe}) S = 1.0 \times (1.20 \times 0.700 + 1.00 \times 0.968) \times \frac{202{,}795}{12} = 30{,}554 \; k - ft > M_u = 31{,}008 \; k - ft$$

$\gamma_3 \, (\gamma_1 f_{cr} + \gamma_2 f_{cpe}) S = 30{,}554 \; k - ft \leq 1.33 M_u = 1.33 \times 31{,}008 = 41{,}241 \; k - ft$

$\gamma_3 \, (\gamma_1 f_{cr} + \gamma_2 f_{cpe}) S = 30{,}554 \; k - ft$ controls the design.

$\phi M_n \geq \gamma_3 \, (\gamma_1 f_{cr} + \gamma_2 f_{cpe}) S$ MFR Requirement

$\phi M_n = 38{,}473 \; k - ft > \gamma_3 \, (\gamma_1 f_{cr} + \gamma_2 f_{cpe}) S = 30{,}554 \; k - ft$

Thus, the minimum flexural reinforcement requirement is satisfied.

B.5 CAP BEAM

DESCRIPTION OF CAP

The cap beam has a main span of 23 ft and 2 cantilever spans of 12.5 ft each. The cap is 6.5 ft wide and 6 ft deep. The columns are square 6 ft x 6 ft.

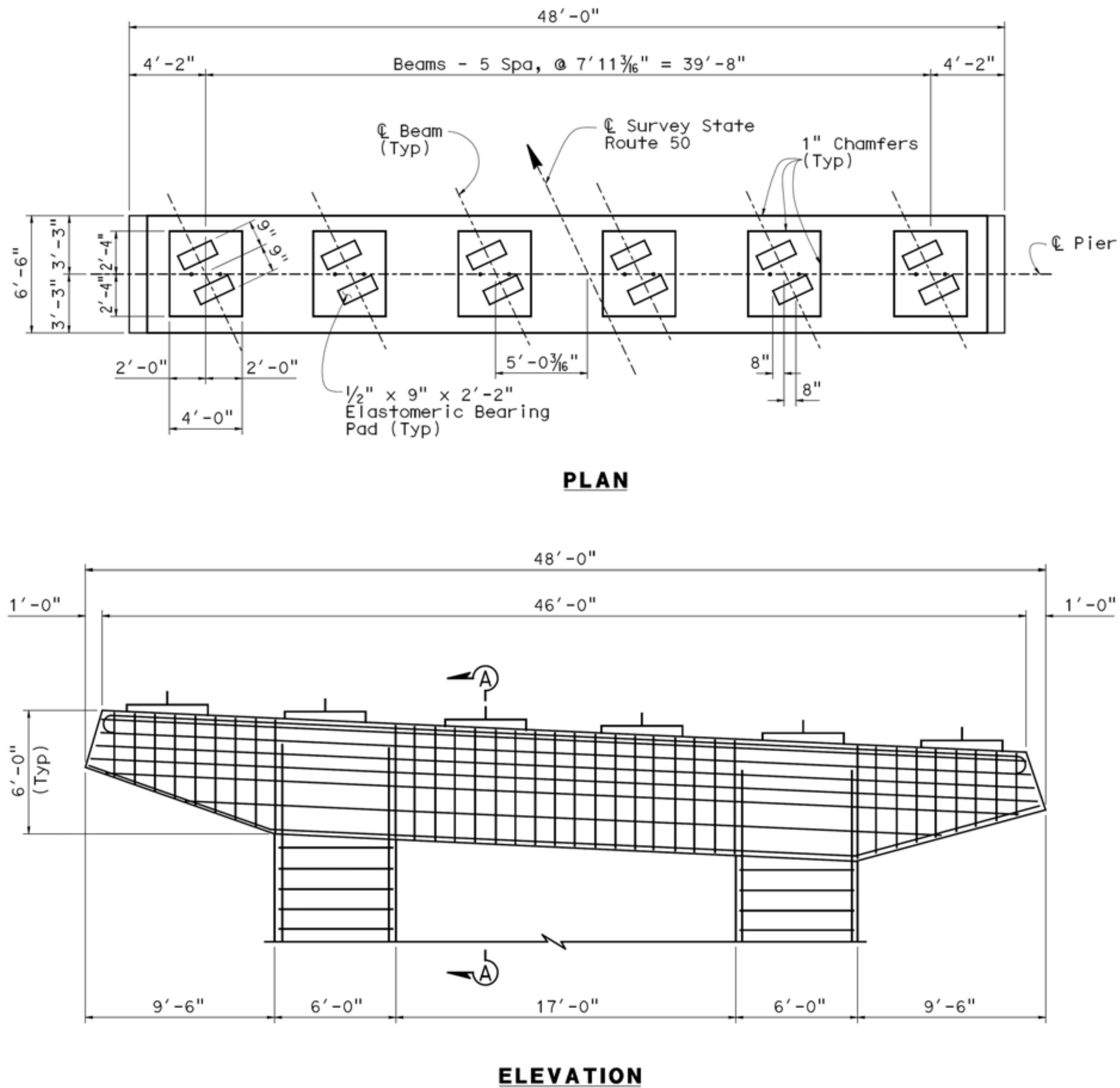


Figure B-16. Cap Beam Design Example Schematics

MATERIAL PROPERTIES

$f'_c = 4ksi$

$E_c = 3{,}644 ksi$
$f_y = 60 ksi$
$E_s = 29{,}000 ksi$

MOMENT DIAGRAMS

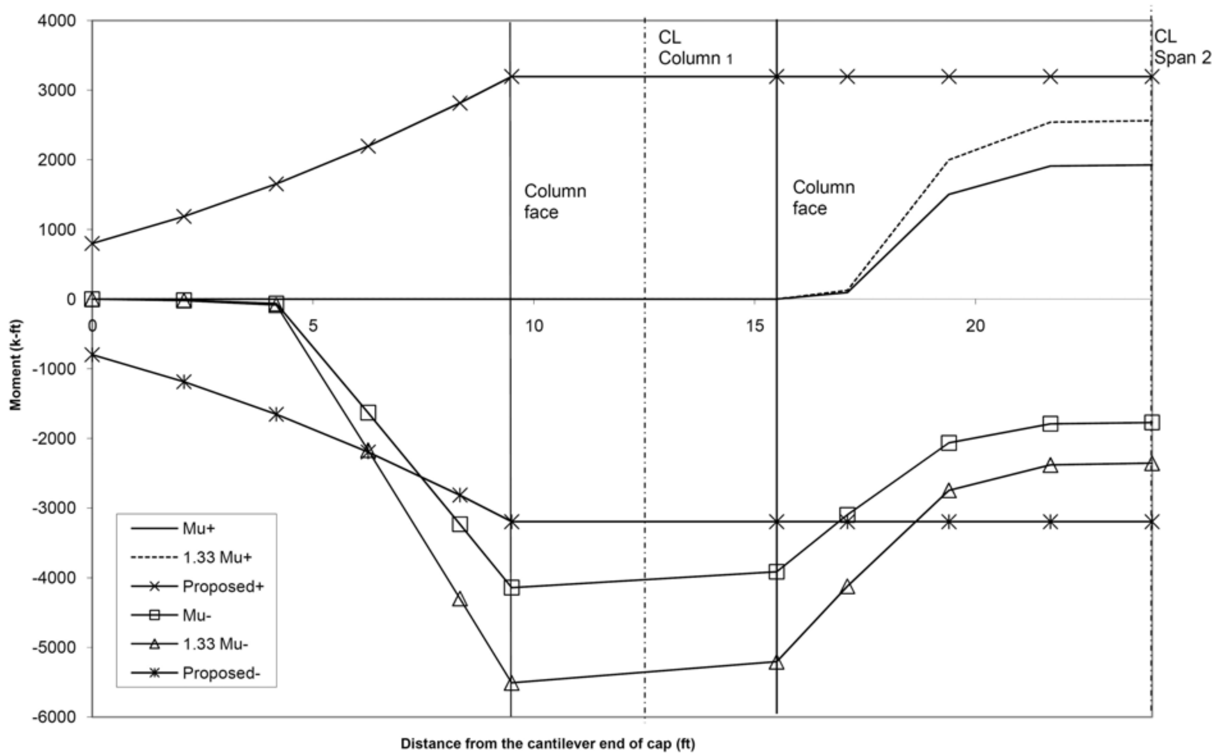


Figure B-17. Cap Beam Design Example Strength limit bending moments

At the inside face of support (negative moment):

Design moments:

$M_{DC} = -1{,}381 k - ft$
$M_{DW} = -183 k - ft$
$M_{HL-93} = -1{,}093 k - ft$

$M_u^{StrengthI} = 1.25 M_{DC} + 1.50 M_{DW} + 1.75 M_{HL-93}$
$M_u^{StrengthI} = 1.25 \times (-1{,}381) + 1.50 \times (-183) + 1.75 \times (-1{,}093)$
$M_u^{StrengthI} = -3{,}914 k - ft$

Section properties:

$h = 6ft = 72in$
$b \ = 6.5ft = 78in$
$I \ = 117ft^4$
$A = 39ft^2$
$\overline{y}_t = 36in$ (distance from section CG to top fiber)

Required flexural reinforcement:

$$\phi M_n = \phi A_s f_y (d - \frac{a}{2}) = \phi A_s f_y (d - \frac{A_s f_y}{2 \times 0.85 f'_c b})$$

The section is tension-controlled and $\phi = 0.90$
$d = 68.7in$ assuming #11 mild steel reinforcement

$$3{,}914 \times 12 = 0.90 \times A_s \times 60 \times (68.7 - \frac{A_s \times 60}{2 \times 0.85 \times 4 \times 78})$$

Solve the quadratic equation for $A_s = 12.94in^2$.

The net tensile strain is:

$$\varepsilon_s = 0.003\left(\frac{d-c}{c}\right) \text{ where } c = \left(\frac{A_s f_y}{0.85\beta f'_c b}\right) = \left(\frac{12.94 \times 60}{0.85^2 \times 4 \times 78}\right) = 3.44in$$

Therefore, $\varepsilon_s = 0.003\left(\frac{68.7 - 3.44}{3.44}\right) = 0.057$, which is greater than 0.0075. *Hence, requirements of Section 5.7.3.5 are met for redistribution, and minimum flexure reinforcement per proposed revised Article 5.7.3.3.2 is not required for negative bending between the columns.*

Summary:
$A_s = 12.94in^2$ mild steel reinforcement is required at the top of cap.

<u>At 0.5 Span 2 (positive moment):</u>

Design moments:

$M_{DC} = -59k - ft$
$M_{DW} = -20k - ft$
$M_{HL-93} = 1{,}138k - ft$

$$M_u^{StrengthI} = 0.9M_{DC} + 0.65M_{DW} + 1.75M_{HL-93}$$
$$M_u^{StrengthI} = 0.9 \times (-59) + 0.65 \times (-20) + 1.75 \times (-1{,}138)$$
$$M_u^{StrengthI} = 1{,}925k - ft$$

Section properties:

The section properties are similar to the ones at the face of support.

Minimum reinforcement by the proposed method:

$\phi M_n \geq M_{fcr}$ where $M_{fcr} = \gamma_3 \gamma_1 f_r S$ and $\phi M_n \geq 1.33 M_u$

$\gamma_3 = 0.75$ for A706 Grade 60 reinforcement, assumed for this example.

$\gamma_1 = 1.6$

$$f_r = 0.237\sqrt{f'_c} = 0.237 \times \sqrt{4} = 0.474ksi$$

$$S = \frac{I}{\overline{y}_b} = \frac{117 \times 12^4}{36} = 67{,}392in^3$$

$$M_{fcr} = 0.75 \times 1.6 \times 0.474 \times \frac{67{,}392}{12} = 3{,}194k - ft \geq M_u^{StrengthI} = 1{,}925k - ft$$

$M_{fcr} = 3{,}194k - ft \geq 1.33M_u^{StrengthI} = 1.33 \times 1{,}925 = 2{,}560k - ft$ so

$1.33M_u^{StrengthI} = 2{,}560k - ft$ controls the design.

$$\phi M_n = \phi A_s f_y (d - \frac{a}{2}) = \phi A_s f_y (d - \frac{A_s f_y}{2 \times 0.85 f'_c b})$$

The section is tension-controlled and $\phi = 0.90$

$d = 68.7in$ assuming #11 mild steel reinforcement

$$2{,}560 \times 12 = 0.90 \times A_s \times 60 \times (68.7 - \frac{A_s \times 60}{2 \times 0.85 \times 4 \times 78})$$

Solve the quadratic equation for $A_s = 8.40in^2$.

Summary:

$A_s = 8.40in^2$ mild steel reinforcement is required at mid-span.

<u>At 0.50 Span 1 (negative moment):</u>

Design moments:

$$M_{DC} = -651k - ft$$
$$M_{DW} = -83k - ft$$
$$M_{HL-93} = -395k - ft$$

$$M_u^{StrengthI} = 1.25M_{DC} + 1.50M_{DW} + 1.75M_{HL-93}$$
$$M_u^{StrengthI} = 1.25 \times (-651) + 1.50 \times (-83) + 1.75 \times (-395)$$
$$M_u^{StrengthI} = -1{,}630k - ft$$

Section properties:

$h = 4.97 ft = 59.7 in$

$b = 6.5 ft = 78 in$

$I = 66.50 ft^4$

$A = 32.31 ft^2$

$\bar{y}_t = 29.85in$ (distance from section CG to top fiber)

Minimum reinforcement by the proposed method:

$\phi M_n \geq M_{fcr}$ where $M_{fcr} = \gamma_3 \gamma_1 f_r S$ and $\phi M_n \geq 1.33 M_u$

$\gamma_3 = 0.75$ for A706 Grade 60 reinforcement, assumed for this example.

$\gamma_1 = 1.6$

$f_r = 0.237\sqrt{f'_c} = 0.237 \times \sqrt{4} = 0.474ksi$

$$S = \frac{I}{\bar{y}_t} = \frac{66.50 \times 12^4}{29.85} = 46{,}196 in^3$$

$$M_{fcr} = 0.75 \times 1.6 \times 0.474 \times \frac{46{,}196}{12} = 2{,}190k - ft \geq M_u^{StrengthI} = 1{,}630k - ft$$

$M_{fcr} = 2{,}190k - ft \geq 1.33M_u^{StrengthI} = 1.33 \times 1{,}630 = 2{,}168k - ft$ so

$1.33M_u^{StrengthI} = 2{,}168k - ft$ controls the design.

$$\phi M_n = \phi A_s f_y (d - \frac{a}{2}) = \phi A_s f_y (d - \frac{A_s f_y}{2 \times 0.85 f'_c b})$$

The section is tension-controlled and $\phi = 0.90$

$d = 56.4in$ assuming #11 mild steel reinforcement

$$2{,}168 \times 12 = 0.90 \times A_s \times 60 \times (56.4 - \frac{A_s \times 60}{2 \times 0.85 \times 4 \times 78})$$

Solve the quadratic equation for $A_s = 8.69in^2$.

Summary:

$A_s = 8.69in^2$ mild steel reinforcement is required in the cantilever span.