Pricing American Style Employee Stock Options having GARCH Effects

Gbenga Joseph Arotiba

A-mini-thesis submitted in partial fulfillment of the requirement

for

the degree of Master of Science

in the

Department of Mathematics and Applied Mathematics

University of the Western Cape, South Africa.

Supervisor: Prof. Kailash C. Patidar

May 2010

KEYWORDS

Employee Stock Options

Black Scholes Option Pricing Model

American Options

Incentive Stock Options

Non-Qualified Stock Options

International Revenue Code

Financial Accounting Standard Board

(FAS 123): Financial Accounting Standard, Statement No. 123

Binomial Tree Method

(GARCH): Autoregressive Conditional heteroskedasticity and Generalized Autoregressive Conditional Heteroskedasticity

Volatility

ABSTRACT

We investigate some simulation-based approaches for the valuing of the employee stock options. The mathematical models that deal with valuation of such options include the work of Jennergren and Naeslund [L.P Jennergren and B. Naeslund, A comment on valuation of executive stock options and the FASB proposal, Accounting Review 68 (1993) 179-183]. They used the Black and Scholes [F. Black and M. Scholes, The pricing of options and corporate liabilities, Journal of Political Economy 81(1973) 637-659] and extended partial differential equation for an option that includes the early exercise. Some other major relevant works to this mini thesis are Hemmer et al. [T Hemmer, S. Matsunaga and T Shevlin, The influence of risk diversification on the early exercise of employee stock options by executive officers, Journal of Accounting and Economics 21(1) (1996) 45-68] and Baril et al. [C. Baril, L. Betancourt, J. Briggs, Valuing employee stock options under SFAS 123 R using the Black-Scholes-Merton and lattice model approaches, Journal of Accounting Education 25 (1-2) (2007) 88-101]. The underlying assets are studied under the GARCH (generalized autoregressive conditional heteroskedasticity) effects. Particular emphasis is made on the American style employee stock options.

DECLARATION

I declare that *Pricing American Employee Stock Option having GARCH Effects* is my work, that it has not been submitted before for any degree or examination in any other university, and that all the sources I have used or quoted have been indicated and acknowledged by complete references.

Gbenga Arotiba

May 2010

ACKNOWLEDGEMENT

Foremost, I give all glory to God almighty for seeing me through this beautiful time financially, spiritually, health wise and socially.

I am cordially grateful to my supervisor, Professor Kailash C. Patidar, whose accompaniment, motivation, push, encouragement, and understanding from the beginning of my program, made me to have an inclination of my field.

I also appreciate Prof. Danielle Kotze, Prof. Renette Blignaut and Mr. Leslie P. Selbourne for their moral support and encouragement.

Lastly, I extend my outmost regards to my family and friends, Chief and Mrs Arotiba for their financial, parental and moral support, my humble brother, Dr Omotayo Arotiba, my loyal and supportive friends, Onaneye Alfred, Adesola Rotimi, Fadipe Oluwaseun, Dr and Mrs Babajide, Tomiwa, Tosin, Odetola Tayo, I am grateful to all of you.

LIST OF ACRONYMS

ARCH	Autoregressive Conditional Heteroskedasticity
BSMD	Black-Scholes-Merton Differential Equation
BSOPM	Black Scholes Option Pricing Model
CBOE	Chicago Board Options Exchange
CBOT	Chicago Board of Trade
CRR	Cox, Ross and Rubinstein
ESO	Employee Stock Options
EWMA	Exponentially Weighted Moving Average
FAS123	Financial Accounting Standard 123
FASB	Financial Accounting Standard Board
FMV	Fair Market Value
GARCH	General Autoregressive Conditional Heteroskedasticity
GJR GARCH	Glosten-Jagannathan-Runkle Generalized Autoregressive Conditional Heteroskedasticity
IASB	International Accounting Standard Board
IFRS2	International Financial Reporting Standard 2
IRC	International Revenue Code
ISOs	Incentive Stock Options
NGARCH	Nonlinear Generalized Autoregressive Conditional Hetersoskedasticity
NQSOs	Non-Qualified Stock Options
PDE	Partial Differential Equation

QGARCH	Quadratic Generalized Autoregressive Conditional Heteroskedasticity

List of figures

CONTENTS

CHAPTER ONE

1. General Introduction

1.1 History Remarks

An option is a contract between a buyer and seller that grants the holder the right but not the obligation to buy or sell a particular asset at a specific price on or before the options expiration date. Options are derivative securities because their value depends on that of the underlying asset.

The conception of option trading cannot really be pointed out. However, the history is trusted to that of forward contract which began in ancient times and was mainly used by farmers. A forward contract is an agreement to buy something in future.

The official commencement of option trading was in the United States in the year 1848 with the creation of the Chicago Board of Trade. Afterwards, other exchanges opened in the U.S. including the Kansas City Board of Trade, the Minneapolis Grain Exchange, and the New York Cotton Exchange. History has it that pricing options was entirely ad hoc. It was advantageous to traders with instinctive knowing about how other traders would price options, made money and those without it lost money.

Through 1900's, there were numerous formal efforts in creating an equation to value options. Nevertheless, all these equations made unreasonable assumptions that could not be applied in the real world until 1973, when the modern financial options market came into existence through the opening of the Chicago Board Options Exchange (CBOE) by the Chicago Board of Trade (CBOT). Fischer Black and Myron Scholes derived a formula called the Black-Scholes formula for pricing options, and was published in their 1973 paper, "The pricing of Options and Corporate Liabilities" since then, the Black-Scholes formula has been modified in different forms to give a better valuing strategy.

The Black-Scholes model works under ideal conditions which are:

i. The interest rate r is constant and known through the life of the option

ii. Assuming a lognormal stock price, the variance rate of return is constant and follows a stochastic process. The variance rate is also proportional to the square of the stock price.
iii. Dividend is zero, that is, pays no dividend yield.
iv. The option is allowed to be exercised at maturity alone (European) this is one of the reasons why we didn’t use the model. An employee stock option is American style. That is, you can choose to exercise the option before maturity.
v. During the life of the option, we assume no transaction cost.

Jennergren and Näslund (1993) commented on executive stock valuation. In the paper, the Financial Accounting Standard Board (FASB) proposal was discussed extensively. Foster et al. (1991) agrees with the Black Scholes model but with the condition that there must be a constant dividend yield. The value of an Employee Stock Option (ESO) drops if executed before maturity (early exercise reduces the value of the option). Kulatilaka and Marcus (1994) talked about the proposed FASB proposal that will make firms put into account the expense of employee stock options. In the exposure draft released by FASB, they presented reasons and arguments that either the binomial model or the Black Scholes model will be reasonable in valuing ESO.

In 2004, FASB released a share-based statement which demands firms to create invoice for ESO grants. A fair value method was proposed in the statement, and the value of the option must be an expense. León and Vaello-Sebastia (2009) introduced a volatility conditioned under Generalized Autoregressive Conditional Heteroskedasticity (GARCH) settings and applied algorithms in valuing American style employee stock option. In Angel, Léon and Vaello-Sebastia (2009) the GARCH framework for volatility is based on the fact that volatility is not time varying under the Black-Scholes prototype.

The GARCH model used by Engle (2001) was introduced by Bollerslev (1986). This model is a modification of the Autoregressive Conditional Heteroskedasticity (ARCH) model developed by Engle (1982). Taylor (1986) also worked on the modeling of Generalized Autoregressive Conditional Heteroskedasticity (GARCH). The GARCH model works by dealing with past residuals where the conditional variance is a mathematical relation of the past conditional variances. The ARCH/GARCH model is discussed and well explained in Chapter two.

We used two numerical methods in the valuing of employee stock options: the binomial tree method and the finite difference method. This is because of the iteration technique that encourages early execution of the stock before maturity. This process works with time differences(Δt), therefore makes it possible for an employee to exercise the option before maturity at different time intervals if he/she pleases. Emphasis are laid more on the binomial tree method with few examples in illustrating how an employee can make decisions on either to execute before maturity or keep the option for a longer period.

1.2 Option Pricing the Using Black-Scholes Framework

Black Scholes Pricing Formula

For a non-paying dividend stock, the Black Scholes pricing formula for a call option is:

$$V_c = S_0 N(d_1) - K e^{-rt} N(d_2) \tag{1.1}$$

Where,

$$d_1 = \frac{\ln\left(S_0/K\right) + \left(r + \sigma^2/2\right)T}{\sigma\sqrt{T}},$$

$$d_2 = \frac{\ln\left(S_0/K\right) + (r - \sigma^2/2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T},$$

and the value of the put is,

$$V_p = Ke^{-rt}N(-d_2) - S_0N(-d_1) \tag{1.2}$$

$N(d_1)$ and $N(d_2)$ are cumulative probability functions of the standard normal distribution,

K is the strike price,

S_0 is the current stock price,

T is the maturity time,

r is the annual risk free interest rate,

σ is the volatility.

Black-Scholes-Merton Differential Equation (BSMD)

Following a lognormal distribution and a general Weiner process, the BSMD can be derived from the Ito's lemma.

From the generalized Wiener process

$$dS = \mu S dt + \sigma S \epsilon \sqrt{\Delta t}, \tag{1.3}$$

Where $\epsilon\sqrt{\Delta t} = dz$.

From (1.3) we generate a price M which is a function of S and t,

$$\frac{\partial M}{\partial t} + (r-\delta)S\frac{\partial M}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 M}{\partial S^2} = rM, \tag{1.4}$$

The derivation of (1.4) can be seen in Hull (2009).

1.3 Literature Review

Black and Scholes (1973) derived a formula called the Black-Scholes Option Pricing Model (BSOPM) this model has and still is being used for option pricing. There has been much research on how to value options and improve the use of the BSOPM because the formula only works under ideal conditions.

The factors affecting option pricing: current stock price, option price, time to expiration, volatility, risk-free interest rate, dividend value, has given rise to different formulas and adjustment of the Black-Scholes Option Pricing Model. Individually, the factors can generate different formulas for valuing option pricing. For example, considering time T to expiration, problems can arise from expiration date, like for European options, options can only be exercised at maturity, but for American-style option, an investor can choose to exercise before maturity which will make the BSOPM unsuitable for such cases.

In Rodrigo and Mamon (2006) an explicit formula for option pricing on a dividend paying stock with a time dependent Black-Scholes partial differential equation was derived. This is derived by the product of an adjusted Black-Scholes Option Pricing Model of a non-dividend paying stock with constant parameters, the fraction of a time-varying strike price and a constant-parameter strike price, and a discount factor modified with defined parameters that are time-variables. Company et al. (2006) modified the Black-Scholes Option Pricing Model for a dividend paying stock, numerically. They used the delta-defining sequence of the generalized Dirac delta function to obtain an integral formula. Company et al. in (2008) continued with the numerical solution of the Black-Scholes Merton differential equation by applying the semi discretisation technique. In the work of Jódar et al. (2005), the Mellin transformation method was proposed, this was the Mellin method applied by Company et al. (2006).

All this modification, transformation and proposals of new formulas arose from the fact that an American-style option cannot be valued using the same formula as for a European-style options. We considered employee stock options in our work. The employee stock options (ESOs) are American style because it can be exercised before maturity, unlike an

European option that must be exercised at maturity alone. The ability to terminate the contract or an employee forfeiting the option if the employment is terminated is also a factor to be considered. Jennergren and Näslund (1993) discussed the Financial Accounting Standard Board (FASB) proposal where it was stated that ESOs should be valued at fair values, with a lower bound constraint. It was argued by Foster et al. (1991) that the BSOPM should be the prospect model for valuing ESOs but in the case of constant, continuous dividend yield. Kulatilaka and Marcus (1994) also elaborated on the Financial Accounting Standard Board proposal and a conventional option pricing model, but argued from the point that the ESOs are not transferable and therefore, the employee can decide to exercise the option early due to different constraints. He further explained that the conventional pricing model has the possibility of misevaluating ESOs, and suggested a model that considers the price of ESOs devaluating due to the conventional model.

Under ESOs, volatility is time-varying and this leads us to the pricing of ESOs under the generalized autoregressive conditional heteroskedasticity (GARCH) framework. In the work of Duan et al. (2001), generally, a numerical method was used in valuing American options with a GARCH option pricing model. The GARCH model was used because the method is based on the approximation of asset price by a finite state, time-homogenous Markov chain. The Markov Chain under GARCH option pricing framework works well and serves as an alternative to other numerical methods. Medvedev and Scaillet (2010) introduced an approach that is accurate in pricing options with time-to-maturity up to several years. An extension of the stochastic volatility and stochastic interest rates was used by analyzing the impact of volatility mean-reversion and correlations on the American put price.

In the work of León and Vaello-Sebastia (2009), a simulation-based approach was used for the ESO fair value putting into consideration the departure risk, possibility of early exercise and the vesting period. A GARCH effect was introduced, and analysis with respect to ESO characters, and constant volatility case was performed. The method was compared with the Financial Accounting Standard report 123 method. Duan and Wei

(2005) argued the fact that incentive effects of ESO depends on risk typography. The GARCH option pricing method was used to prove this. According to Huddart (1994), employer's cost valued by numerical methods is much less then the options' Black Scholes value. The research has significance for alteration to the accounting treatment of ESOs circumstance by Financial Accounting Standards Board (FASB).

Because the ESOs are American-style, the use of numerical methods in the pricing it is mandatory for optimum results. Zhao et al. (2007) designed the compact finite difference method to attain quick and accurate solutions to Partial Differential Equation problems. Due to the existence of an optimal exercise boundary for American options, three ways of combining compact difference method were developed: i) the use of implicit condition with a transformed PDE being non-negative to detect the optimal exercise value. ii) an algebraic nonlinear equation with a solution at every time step. iii) a free boundary value method introduced by Barone-Adesi and Lugano is used because of the high accuracy for space. An implicit method was also presented by Zvan et al. (2000) for solving PDE frameworks under algebraic constraints which are barriers, early exercise features.

It is therefore obvious that it remains a concern, the accuracy in valuing ESOs under different constraints. Numerical methods for valuing ESOs has been proved to be the best because it is American style, also, it is effective to value the ESOs under the GARCH framework because it is of the time-varying volatility due to vesting period and long life of the option.

1.4 Outline of the thesis

"In understanding today, we have to search yesterday" In Chapter one, a brief history about the option price and the origin was discussed, the Black Scholes option pricing formula is stated in Chapter one with the general knowledge and understanding of the field research discussed in the literature review.

In Chapter two, the history of ESO is mentioned. Details how an employee stock option can be valued using the proposed method by the Financial Accounting Standards Board is given. We put into consideration the factors that make the employee stock option defer from the standardized traded options. This made the binomial tree method a suitable method for the valuing of employee stock options.

The generalized autoregressive and conditional heteroskedasticity (GARCH) is discussed in Chapter three. GARCH is the model used in estimating a time-varying volatility used in the employee stock option plan. Different modifications of the GARCH model is also mentioned but not thoroughly discussed because we are still working on the advantages and disadvantages of each model over the other.

In Chapter four, two numerical method was implemented in the valuing of an employee stock option. A binomial tree method is used to value employee stock options, and different examples in the case of a dividend yield and a non dividend paying stock are sited. We used the binomial tree in valuing ESOs because the ESO is an American style, and considering the possibility of early exercise, a numerical approach that considers execution at different time intervals should be used. Even, the FASB proposal supports these. The second numerical method is the finite different method. This method is also discussed but yet to be imputed into the computer as a simulation. We are currently working on this and hope to come out with a good result for future work.

CHAPTER TWO

2. Employee Stock Options

Employee stock options (ESO) are call options issued as a form of compensation to shareholders or executives by their company. The reason behind this is to motivate and encourage executives to act in the best interests of the company's shareholders.

2.1 History of ESO

The birth of employee stock option programs can be traced back to 1957. In 1957, there were some young men in the Shockley Semiconductor Laboratory who opted out to start their own business due to poor management. At the initial stage, they were short of funds but later got start-up equity from Fairchild Camera & Instrument, of Syossett, New York. The company gave them a condition for the upfront equity to be a contract (right to buy the company shares if things starts fine). In advantage to the young men, things went well and Fairchild bought enough shares to make them the founder.

Many employees were inspired because of the outturn of Fairchild Semiconductors. This led to many employees leaving their various companies to start a new company in the Silicon Valley. The revolution forced employers to give corresponding employee stock options to keep their employees. However, most of the employers insisted that stock options should not be given to non-executives. This also inspired more employees to leave to start their own companies. During this era, stock options became a shield to top executives because it wasn't taxed. Therefore, more money was generated through the issuing of the Employee stock option. But for Silicon Valley inauguration, they used the stock options in attracting experienced executives without bringing forth expenses which can cause bankruptcy. They became an example to most companies and for this reason, the employee stock options was liberated. In the mid-1970s, stock options was already a noticeable trace of quality in Silicon Valley.

The notice of stock options was indirectly provided by Washington. In 1950, when the top personal income tax rate was 91%, tax legislation in Congress allowed profits from stock options to be taxed at the capital gains rate, then 25%, provided the employee held the stock for at least one year. For those wealthy enough to be able to hold stocks for that period of time, this was an enticing tax loophole. The tax-haven status of stock options was whittled away during the 1960s and 1970s as Washington targeted the shelters of the rich. In 1976 lawmakers did away with the tax-shelter status of stock options altogether. As a result, there was no longer a tax benefit for holding on to shares once they were exercised. With the top tax rate still 70%, stock options were taxed as heavily as regular income.

The spark was provided by the Reagan tax cuts, first in 1981 and then again in 1986, which lowered the top personal income tax rate to 50% and then to 28%. Coincidently, this coincided with beginning of a bull market that saw the Dow Jones Industrial Average rise from 800 in February of 1982 to 11,500 in January of 2000.

Those that had maintained their employee stock options programs during the 1970s as Washington assaulted the tax-haven status were greatly rewarded following the tax cuts. The first company executives to take advantage of this new environment were executives at Toys “R” Us. As Toys “R” Us emerged from bankruptcy in 1978, it set aside a full 15% of its shares for stock options for managers and executives. Within four years, the stock price had risen 20-fold. Following the tax cuts, The Chief Executive Officer, Charles Lazarus cashed in his options for $43 million, while other executives pulled in a respectable $30 million. For executives at other employers, this was impossible not to notice, and the experience at Toy “R” Us was the first of many in the stock option bonanza FASB(1995). While executives at other employers were busy trying to replicate the experience at Toys “R” Us, Lazarus and his associates made the next logical step: options are great – so why not options for everyone. Goldstein, the current Chief Executive Officer and one of the benefactors of the original bonanza, claimed that options were a great motivational tool for employees. Accordingly, stock options began working their way down to non-executives. While they were used only occasionally for most employees, stock options for executives became standard practice in an attempt to solve the principle agent problem.

This led to the popularity of employee stock options in the 1990s and early 2000s. The options are at the money on the date of issue.

In the year 2004, 31st of March, the Financial Accounting Standard Board released a written document titled "Exposure Draft, Share-Based Payment, an Amendment of FASB Statements No. 123 and 95." It is a proposal that companies must give account of the expense Employee Stock Option plans.

The summary of the FASB report: "The exposure draft covers a wide range of equity-based compensation arrangements. Under the Board's proposal, all forms of share-based payments to employees, including employee stock options, would be treated the same as other forms of compensation by recognizing the related cost in the income statement. The expense of the award would generally be measured at fair value at the grant date. Current accounting guidance requires that the expense relating to so-called fixed plan employee stock options only be disclosed in the footnotes to the financial statements."

The reason behind the proposal is due to the fact that valuing Employee Stock Options (ESOs) has been an upshot in a figure of tax cases and disputes within. Historically, there has not been a prolonged intense look into the accuracy of the models used by appraisals.

2.2 Different types of ESOs

There are two types of ESOs,: Incentive Stock options (ISOs) and Non-Qualified Stock Options (NQSOs).

2.2.1 Incentive Stock Options

According to the International Revenue Code of 1976, ISOs were codified. There has been a demonstrating ability of growth in the use of ESOs in the 1990s. According to the European Commission Enterprise and Industry release (August 2004), about 7% out of the entire United State's workforce are currently receiving stock options.

The ISOs are not limited to time when it comes to granting. According to (International Revenue Code, section 422) different criteria are necessary to get ISO treatment:

- The options must be granted pursuant to a written plan that is approved by the shareholders of the company within one year of the plan's adoption by the board of directors.
- The plan must specify the aggregate number of shares that may be issued.
- The plan must indicate the employees, or class of employees, eligible to receive the options.
- The exercise price must be at least equal to the fair market value (FMV) of the underlying shares at the grant date.
- The options must be granted only to employees and must be exercised during employment or within three months of termination of employment.
- The options cannot be exercisable more than ten years after grant.
- Options granted to an employee that are first exercisable in any one year must be capped at $100,000 in underlying share value based on the value of the shares of the date of grant (any options granted in excess of this amount are treated as non-qualified stock options).

The company decides and uses discretion on who gets options and the amount of options to be granted, this is because there is no law in the US discriminating such.

2.2.2 Non-Qualified Stock Options

Just like the Incentive Stock Options (ISOs), there is no limit to term of grant and execution of stock options. Generally, the option term is confined to ten years.

- According to the confined period, the option must be exercised otherwise the employer forfeits the option.
- Through the plan, arrangements are not made for who grants the option. It is stated in the qualification standard plan that the company makes such decision.
- The US does not have any law against or discriminating Non- Qualified Stock Options.
- Any company can grant options under this type of plan.

Also, stock options are not charged to tax at grant and at vesting period.

Below are the differences between ESOs and standardized options:

Employee stock options are non-standardized call options issued as a contract between employee and the employer. Employee stock options have some qualities that make it non-standardized.

The exercise price is not standardized and it is always the current price of the company's stock at the grant date.

The quantity is unspecified unlike the general 100 shares per contract for standardized stock options.

ESOs have a maturity that surpasses the standardized options maturity.

ESOs are mostly not transferable therefore leaving the condition of either being exercised or allow it to expire.

ESOs are contracts between the employee and the employer, making the contract private and not over the counter like the standardized options.

ESOs are tax favoured compared to standardized stock options, this is because the fair market value cannot be determined. Therefore, no tax event occurs at the grant of the option. But there are few exceptions to Non-Qualified Stock Options (NQSOs). Most NQSOs granted to employees are taxed upon exercise but the ISOs are not.

2.3 Valuing ESOs

Due to the sharp growth of ESO in the 1990s, it has been a concern to accounting and financial regulatory bodies to suggest how ESO's fair value can be calculated. The International Financial Reporting Standard 2(IFRS2), share based payments standard published by the International Accounting Standard Board (IASB), this report has been adopted by the European Union since 2005 and obligated by all traded firms.

IFRS2, being the first standard that forced firms to recognize all share-based payments, including ESO's, as an expense.

However, IFRS2 does not decide the pricing models to be used for calculating fair value. It only depicts the factors to be considered when valuing the fair value. The factors to be considered are, the time to maturity, exercise price, stock price, dividend yield, the volatility, and the risk-free interest rate of the option. IFRS2 also advises that the Black and Scholes (1973) volatility and some parameters are not time varying and therefore makes the Black-Scholes Option Pricing Model, defective.

The FASB (2004, pages 43-44) statement states that "Several valuation techniques, including a lattice model (an example of which is a binomial model) and a closed-form model (an example of which is the Black-Scholes-Merton formula) meet the criteria required by this statement for estimating the fair values of employee share options and similar instruments. Those valuation techniques or models, sometimes referred to as option-pricing models, are based on well-established financial economic theory. Those models are used by valuation professionals, dealers of derivative instruments, and other experts to estimate the fair values of options and similar instruments related to equity securities, currencies, interest rates, and commodities. Those models are used to establish trade prices for derivative instruments, to establish fair market values for U.S. tax purposes, and to establish values in adjudications. Both a lattice model and a closed-form model can be adjusted to account for the characteristics of share options and similar instruments granted to employee"

The Financial Accounting Standard 123 (FAS123R) was revised to suit the IFRS2 because the FAS123R does not define a predilection for a particular valuation model in estimating the fair value. But does itemize the factors required in the valuation model according to FAS123R, paragraph A18 León and Vaello-Sebastia (2009).

As widely known and accepted, the Black-Scholes model developed 1973 has been the most used option pricing model. This is because it is straightforward with a request of only five inputs: the current stock price, option strike price, estimated volatility, risk-free interest rate and time till expiration. The Black Scholes Option Pricing Model is not suitable for the ESOs valuation because of the ESO's vesting period, employee turnover and early exercise. This is because the Black Scholes Option Pricing Model assumes the risk-free interest rate and volatility are constant agreed by Sellers et al. (2008).

Past years, many models have been formulated to value the fair value of ESOs. In the work of Jennergren and Näslund (1993), the Black and Scholes (1973) model was extended to get the partial differential equation (PDE) for an option where early exercise is considered as a derived stopping time measured by the first step of a Poisson process with constant intensity.

According to Appendix B of FAS 123R, a three step valuation procedure was proposed and discussed:

i. Estimate the expected life of the option.
ii. Black and Scholes (1973) model or Cox, Ross and Rubinstein (1979) binomial tree to value the option, with the expected life as the time to maturity.
iii. Values should be adjusted; this will make room for possibility of an employee leaving the company during the vesting period

The proposed FAS123R to correct the inability of the Black-Scholes model in valuing ESOs:

i. ESO are not marketable, i.e they cannot be traded, according to the Black Scholes framework, ESOs must worth less than a traded option that can be sold or exercised at will. But the FAS123R corrected this by allowing companies to use shorter expected life instead of the actual option term.
ii. ESOs can be forfeited if employee's employment is terminated. The Black-Scholes model assumes that options cannot be forfeited prior to expiration (generally ten years). This was fixed by the FAS123 by proposing that a company can "hair cut" the value.

iii. Normally, ESOs have much longer term than traded options but the Black-Scholes model always assumes that volatility is constant through the entire life of the option. Unfortunately, the FAS123 could not fix this but further propose a lattice Binomial Method which is time varying.

iv. Also, ESOs are can be exercised earlier before the end of term because ESOs are American-style Options. But the Black-Scholes model only assumes European-style (can only be exercise at the end of term).

Although, it was noted that FAS123R original version acknowledge the lattice model as a better model and mandate its use. However, in the final version of the FAS123R, this requirement was dropped on the basis that some ESOs possess few features and therefore can be valued by the Black Scholes option pricing model (BSOPM).

2.3.1 Binomial Lattice Model

Sellers et al. (2008) explained step wisely how the Binomial lattice model works. The model starts with the current stock price and makes use of the expected stock price volatility to envision more than one potential outcomes or future price changes. An example of a two-period binomial lattice model with a volatility of 25%, time step of 0.0417(15.21days), risk free interest rate of 5% stock and exercise price of $30 is illustrated below in Figure 2.1. This example is for an American call option. The figure illustrates two step binomial lattice model with a stock price of $30 and options which will mature at the end of the second period. During the first period, the price is expected to either go up to $31.5706 or down to $28.5025. At the closing price at the second period, the stock prices are expected to range from $33.2235 to $27.0893. If the stock price is $33.2235, and the ESO has not been forfeited or exercised early, we can assume that the ESO will be exercised at maturity. A good explanation for this is by viewing from the perspective of an investor, exercise will occur only at the upper node of Figure 2.1, where the price is $33.2235. Due to the complexity of the exercise rule, one period earlier, that is where the stock price is $31.5706, the investor can decide to exercise and receive a payoff of $31.5706 - $30 = $1.5706 or can decide to hold on the option until the next period. If he decides to hold on, he can get a payoff of $3.2235 (if stock price rises) or zero (if stock price falls to $30). So, the exercise rule at the first period depends on the will of the investor to take a risk.

If probabilities are assigned at each node outcomes, then, the value of the options can be determined at maturity. Generally, 1,000 time steps are used for lattice models with each time steps making up a discrete change in stock price.

The value of options on stock increases as the volatility of the stock price increases. This is due to the greater chance that options will be in-the-money at some point.

At each node:
Upper value = Underlying Asset Price
Lower value = Option Price
Faded value is at the point of exercise.

Strike price = 30
Discount factor per step = 0.9979
Time step, dt = 0.0417 years, 15.20 days
Growth factor per step, a = 1.0021
Probability of up move, p = 0.5077
Up step size, u = 1.0523
Down step size, d = 0.9503

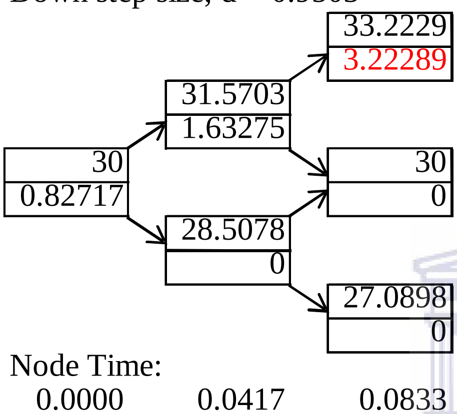

Figure 2.1: Example of a two-step binomial method

In contrary, the Black-Scholes Option Pricing Model assumes the option is European (can only be exercised at maturity). Whereas, the lattice approach make it possible for the option to be exercised at any time (that is American-style). Sellers et al. (2008) gave an example of three decision rules that can reflect the probability at every node: (i) Terminate employment and forfeit an unvested option, (ii) Terminate employment and immediately exercise a vested option or (iii) terminate employment and forfeit a vested option that has no intrinsic value. This can further reflect proposed changes in the risk-free interest rate of return.

An important factor that also reduces the value of an ESO is the vesting period and an expected turnover. This leads to the yielding of a higher option price by the Black-Scholes Option Pricing Model than a binomial tree when the turnover characteristics and vesting are present. The calculation of the up factor u, probability p of an upward movement, down factor d, probability q of a downward movement and a, which is $e^{\sigma\sqrt{\Delta t}}$ will be discussed later in Chapter 4.

CHAPTER THREE

Estimation of Volatility used in the ESO Models

Fisher Black and Myron Scholes achieved a major breakthrough in Black and Scholes (1973), the achievement led to the development of the Black-Scholes Option Pricing Model. Since then, remarkable efforts have been made in extending and affecting positively the option valuation models and computations. This has enabled financial analysts to calculate with accuracy, the value of a stock option.

Stock prices are assumed to follow a Markov process, that is, only the present value of the stock is relevant for prediction. Black-Scholes model proved the possibility of setting a univocal price for an option which price depends on the random price of a traded security.

Volatility

Volatility is used to measure the risk of financial instrument. Volatility can either be measured by using the standard deviation or variance. As this refers to the amount of uncertainty, a higher volatility means that wide spread or a dramatical change of price movement over a short period of time while lower volatility means security price does not vacillate dramatically but changes at a steady pace.

When calculating risk, the levels of volatility and correlations are to be considered over a short period of time but when valuing derivatives, forecast of volatilities and correlations are to be considered over the whole life of the derivative.

Volatility can be expressed mathematically as

$$\sigma_T = \sigma\sqrt{T},$$

where σ_T is the generalized volatility for time horizon in T years, and

$\sigma = \frac{\sigma_{SD}}{\sqrt{P}}$ is the annual volatility.

The annual volatility is σ, the daily logarithmic return of stock have a standard deviation σ_{SD} and P the time period of returns.

The autoregressive conditional heteroskedasticity (ARCH) and the generalized autoregressive conditional heteroskedasticity (GARCH) models are the standard tools for estimating volatility.

ESTIMATING VOLATILITY

3.1 ARCH/GARCH

The generalized autoregressive conditional heteroskedasticity (GARCH) family will be considered as our model for estimating volatility.
The generalized autoregressive conditional heteroskedasticity model will be referred to as GARCH models onwards.

Autoregressive conditional heteroskedasticity (ARCH) was first introduced by Engle (1982). The GARCH model is the modification/generalization of the autoregressive conditional heteroskedasticity (ARCH) model. The GARCH model was introduced by Bollerslev (1986). ARCH was introduced as a new model to econometrics with mean zero, sequentially correlated process with non-constant variances conditional on the past, but constant unconditional variance.

After this discovery, the ARCH model has been very important in the volatility forecast. More work has been done in the generalization and modification of the ARCH model. The general idea is based on analyzing or forecasting with the past information under standard assumptions.

The introduction of the least square model presumes the expected value of error terms is similar at any point, when squared “The assumption is called homoskedasticity, and it is assumed to be the focus of ARCH/GARCH models. Data in which the variances of the error terms are not equal, and may reasonably be expected to be larger for some points or ranges of the data than for others, are said to suffer from heteroskedasticity. The standard warning is that in the presence of heteroskedasticity, the regression coefficients for an ordinary least squares regression is still unbiased, but the standard errors and confidence intervals estimated by conventional procedures will be too narrow, giving a false sense of precision. Instead of considering this as a problem to be corrected, ARCH and GARCH models treat heteroskedasticity as a variance to be modeled" stated by Engle (2001, page 1, paragraph 2).

The ARCH model functions by assuming the variance of future return being equally weighted average of the squared residue of the past. For forecasting effectiveness, the more current events will be more pertinent and therefore higher weights should be assigned. The purported ARCH model by Engle (1982) makes these weights to be parameters and therefore allowing us to estimate them.

One of the advantages of the GARCH model over the ARCH model is that it has a way of correcting weights and those weights never go totally to zero.

Let σ_n be the volatility of the market at day n, ${\sigma_n}^2$ on T day is the variance rate as estimated at the end of day n-1, presuppose the market value at the end of day i is S_i, the variable u_i is the continuously compounded return between day $\mathrm{i}-1$ and day i

$$u_i = \frac{S_i - S_{i-1}}{S_{i-1}}. \qquad (3.1)$$

The variance rate using current observance of u_i is ${\sigma_T}^2$.

$${\sigma_n}^2 = \frac{1}{m}\sum_{i=1}^{m} u_{n-i}^2 , \qquad (3.2)$$

Equation (3.2) assigns equal weights to $u_{n-1}^2, u_{n-2}^2, \dots, u_{n-m}^2$. Giving more weight to recent data, we apply a model such that

$${\sigma_n}^2 = \sum_{i=1}^{m} \alpha_i u_{n-i}^2, \qquad (3.3)$$

where α_i is the weight assigned to observations, i days ago, weights must also sum to 1 i.e

$$\sum_{i=1}^{m} \alpha_i = 1 .$$

If we assume the presence of a long-run average variance and consider the average weight assigned. From Equation (3.3), a new model will be derived leading us to the ARCH (m) model. In this formula, V_L is the long-run variance and a new weight γ assigned to it,

$$\sigma_n{}^2 = \gamma V_L + \sum_{i=1}^{m} \alpha_i u_{n-i}^2 \ , \tag{3.4}$$

Equation (3.4) can also be written as

$$\sigma_n{}^2 = \omega + \sum_{i=1}^{m} \alpha_i u_{n-i}^2,$$

where $\omega = \gamma V_L$.

This process tell us that more preference is giving to recent observations in terms of weight.

GARCH (p,q) model is the general form of ARCH model where p in the parentheses denotes the number of autoregressive lags while q refers to the specified number of moving average lags.

GARCH (p,q) regression model is given by

$$\varepsilon_t | \psi_{t-1} \backsim N(0, \sigma_n) \ , \tag{3.5}$$

$$\sigma_n = \omega + \alpha \sum_{i=1}^{p} u_{n-1}^2 + \sum_{i=1}^{q} \beta \sigma_{n-1}^2, \tag{3.6}$$

.

$$\omega + A(K) u_{n-1}^2 + B(K) \sigma_{n-1}^2$$

For example, GARCH (1,1) is set up for a one period forecast. That is, A (1) and B (1).

GARCH models are mean reverted because the variance rate and a constant unconditional variance. This is one of the advantages of GARCH (1,1) over the exponential weight moving average (EWMA). In the case where ω is 0, GARCH (1,1) reduces to EWMA.
The GARCH (1,1) model of is given by

$$\sigma_n^2 = \omega + \alpha u_{n-1}^2 + \beta \sigma_{n-1}^2. \qquad (3.7)$$

After ω, α and β have been estimated, γ can be calculated as $1 - \alpha - \beta$, while the long term variance is ω/γ.

For GARCH (1,1), $\alpha + \beta < 1$ is required to avoid a negative long-term variance.

That is, the long-term average variance = $\omega / 1 - \alpha - \beta$

In the long-term variance was successfully proved by Bolleslev (1986) from the theory of finite-dimensional ARCH (q) it is to be expected that $\alpha + \beta < 1$ which answers the wide-sense stationarity.

Theorem 3.1 Milhoj (1984)

The GARCH (p,q) process as defined in (3.5) and (3.6) is wide-sense stationary (that is the mean value is independent of time) with $E(u_n)$=0, $var(u_n) = \omega(1 - \alpha - \beta)^{-1}$ $and\ cov(u_n, u_s) = 0$ for

$t \neq s\ if\ and\ only\ if \alpha + \beta < 1$

Proof:

This proof is according to the first theorem in Milhoj (1984). Defined by

$$\varepsilon_t = \eta_t u_t^{0.5}, \text{ where } \eta_t \sim N(0,1). \qquad (3.8)$$

$we\ make\ our\ n = t, (that\ is, time)\ u_n = \eta_t, make\ the\ parameters\ \omega = \alpha_0, \alpha = \alpha_i\ and\ \beta$

Subsequent substitution yields

$$h_t = \alpha_0 + \alpha_i \sum_{i=1}^{p} \eta_{t-i}^2 h_{t-i} + \sum_{i=1}^{q} \beta_i h_{t-i}$$

$$= \alpha_0 + \sum_{j=1}^{p} \eta_{t-j}^2 \, \alpha_j \left(\alpha_0 + \alpha_i \sum_{i=1}^{p} \eta_{t-i}^2 \, h_{t-i-j} + \sum_{i=1}^{q} \beta_i h_{t-i-j} \right)$$

$$+ \sum_{j=1}^{q} \beta_j \left(\alpha_0 + \alpha_i \sum_{i=1}^{p} \eta_{t-i}^2 \, h_{t-i-j} \right. \qquad (3.9)$$

$$\left. + \sum_{i=1}^{q} \beta_i h_{t-i-j} \right) \ldots \quad = \alpha_0 \sum_{j=1}^{q} M(t,k),$$

where $M(t,k)$ involves all the terms of the form

$$\prod_{i=i}^{q} \alpha_j^{a_i} \prod_{j=i}^{p} \beta_j^{b_j} \prod_{l=1}^{n} \eta_{t-s_l}^2$$

For

$$\sum_{i=1}^{q} a_i + \sum_{i=1}^{p} b_j = k \,, \qquad \sum_{i=1}^{p} a_i = n,$$

and $1 \leq S_1 < S_2 < \cdots < S_n \leq max\{kq, (k-1)q + p\}$.

Thus,

$M(t,0) = 1$,

$$M(t,1) = \sum_{i=1}^{q} \alpha_i \eta_{t-i}^2 + \sum_{i=1}^{p} \beta_i$$

$$M(t,2) = \sum_{j=1}^{q} \alpha_j \eta_{t-j}^2 \left(\sum_{i=1}^{q} \alpha_i \eta_{t-i-j}^2 + \sum_{i=1}^{p} \beta_i \right)$$

$$+ \sum_{j=1}^{p} \beta_j \left(\sum_{i=1}^{q} \alpha_i \eta_{t-i-j}^2 + \sum_{i=1}^{p} \beta_i \right).$$

In general

$$M(t, k+1) = \sum_{i=1}^{q} \alpha_i \eta_{t-i}^2 M(t-i, k) + \sum_{i=1}^{p} \beta_i M(t-i, k) \,. \tag{3.10}$$

Since η_t^2 *is* $i.i.d.$, the moments of $M(t, k)$ do not depend on t, and in particular

$$E\big(M(t, k)\big) = E\big(M(s, k)\big) for\ all\ k, t, s, \tag{3.11}$$

From (3.10) and (3.11) we get

$$M(t, k+1) = \left(\sum_{i=1}^{q} \alpha_i + \sum_{i=1}^{p} \beta_j\right) E(M(t, k))$$

$$\vdots$$

$$= \left(\sum_{i=1}^{q} \alpha_i + \sum_{i=1}^{p} \beta_j\right)^{k+1} E\big(M(t, 0)\big) \tag{3.12}$$

$$= \left(\sum_{i=1}^{q} \alpha_i + \sum_{i=1}^{p} \beta_j\right)^{k+1}$$

Finally, by (3.10) (3.11) and (3.12)

$$E(\varepsilon_t^2) = \alpha_0 E\left(\sum_{k=0}^{\infty} M(t, k)\right)$$

$$= \alpha_0 \sum_{k=0}^{\infty} E\big(M(t, k)\big)$$

$$= \alpha_0 \left(1 - \sum_{i=1}^{q} \alpha_i - \sum_{i=1}^{p} \beta_j\right)^{-1}$$

if and only if

$$\sum_{i=1}^{q} \alpha_i + \sum_{i=1}^{p} \beta_j < 1,$$

and ε_t^2 converges almost surely.

$E(\varepsilon_t)$=0, $and\ cov(\varepsilon_t, \varepsilon_s) = 0\ for\ t \neq s.$

The GARCH model has been edited and improved by many scholars in the past couple of decades. Different kinds of models have been produced from the ARCH/GARCH model, some of the models with the formula, and the relevance to volatility forecast are:

3.2 NGARCH

The Nonlinear Generalized Autoregressive Conditional Heteroskedasticity (NGARCH) also known as Nonlinear Asymmetric GARCH (1,1). The NGARCH was introduced by Engle and Ng (1993). In this case, an extra parameter θ is introduced. The parameter θ is normally estimated positive for stock returns and reflects the leverage effect. This denotes that negative returns increase future volatility in a greater measure than positive returns of the same magnitude for stock returns.

$$\sigma_t^2 = \omega + \alpha(\epsilon_{t-1} - \theta\sigma_{t-1})^2 + \beta\sigma_{t-1}^2. \qquad (3.13)$$

where $\alpha, \beta \geq 0$; $\omega > 0$ and $\omega, \beta, \alpha, \epsilon, \theta$ are coefficients.

3.3 EGARCH

Exponential generalized autoregressive conditional heteroskedasticity (EGARCH) was modeled by Nelson (1991) as

$$ln\sigma_t^2 = \omega_t + \sum_{k=1}^{p} \beta_k g(Z_{t-k}) + \sum_{k=1}^{p} \alpha_k ln\sigma_{t-k}^2 \qquad (3.14)$$

where $g(Z_t) = \left(\theta Z_t + \lambda(|Z_t|) - E(|Z_t|)\right)$, σ_t^2 is the conditional variance, $\omega, \beta, \alpha, \theta, \lambda$ are parameters, and Z_t is a standard normal variable.

Since $ln\sigma_t^2$ may be negative, there are restrictions on the parameters.

3.4 QGARCH

Quadratic GARCH (QGARCH) was modeled by Seneta (1995).

In the work of Seneta (1995), "Quadratic ARCH Models" QGARCH nets the standard deviation discussed by Robinson (1991) and asymmetric model in Engle (1990). QGARCH is also used for modeling positive and negative shocks through the asymmetric effect. There are three non-trivial advantages for this nesting (see Seneta (1995) p3 paragraph 2).

The residual model of a GARCH (1,1) σ_t is

$\epsilon_t = \sigma_t Z_t$.

where Z_t is independent and identically distributed, and

$$\sigma_t^2 = K + \alpha\epsilon_{t-1}^2 + \emptyset\epsilon_{t-1} . \tag{3.15}$$

3.5 GJR-GARCH

There are few similarities between the QGARCH and the Glosten-Jagannathan-Runkle GARCH (GJR-GARCH) models. This model was developed by Glosten-Jagannathan-Runkle (1993). One of the similarities is the asymmetry model. The idea is to model $\epsilon_t = \sigma_t Z_t$ where Z_t remains independent and identically distributed (i.i.d) and

$$\sigma_t^2 = K + \delta\sigma_{t-1}^2 + \alpha\epsilon^2{}_{t-1} + \emptyset\epsilon^2{}_{t-1} I_{t-1} \tag{3.16}$$

where $I_{t-1} = 0 \; if \; \epsilon_{t-1} \geq 0, I_{t-1} = 1 \; if \epsilon_{t-1} < 0$.

The GARCH model deals with the continuously compounded daily return which is normally distributed. Data shows that the unconditional distribution of daily returns exhibits some facts like fat tails (extremely large kurtosis) and asymmetries León and Vaello-Sebastia (2009).

Due to the volatility clustering, the GARCH model is used to calculate the volatilities used in Chapter four, because the conditional variance is a function of past conditional variances and past residuals.

CHAPTER FOUR

In this Chapter, we focused on two Numerical methods for valuing Employee Stock Options (ESO). These methods are based on the fact that ESOs can be exercised at any time because it is an American-Style option.

Time discretization is an advantage of a numerical solution. DerivaGem software is used to generate the trees and the graph in the Binomial tree section of this Chapter.

4. Numerical Solution of ESO Models

Numerical methods are an alternative to pricing derivatives when specific formulas are not available. It deals with iterations and therefore gives more accuracy with time.

There are three major numerical procedures for valuing options in the absence of exact formulas like the Black Scholes Pricing Option Model (BSOPM). The tree method, the Monte Carlo simulations mostly used for derivatives of attached to the history of the underlying variables and the finite difference method, which usually used for pricing American Options and other derivatives with the investor's decision prioritized to maturity. This makes the numerical methods an alternative to the BSOPM.

Focus will be placed on valuing employee stock options using the binomial tree method.

4.1 Binomial Tree

The binomial tree method is a diagram mapping different possible paths succeeded by the stock price over the life of an option. Due to the lack of analytic methodology in valuing American options, binomial trees stand to be the most useful approach in the valuing of American option.

The binomial approach requires the breaking down of the option life into a large number of small time intervals of length Δt.

Because the method is stepwise, there is a probability of an upward movement and a probability of a downward movement of each step.

Cox, Ross and Rubinstein (CRR) (1979) proposed the binomial model and assumes that the stock price movements are composed of a large number of small binomial movements.

Considering a stock price S_0, a current option price M of the stock, a total time T throughout the life of the option, a possibility of the stock price moving up by S_0u or down by S_0d from S_0 (where $u > 1$ and $d < 1$). When there is an upward movement to S_0u, we say the payoff from the option is M_u, likewise if there is a downward movement from S_0 to S_0d we say the payoff from the option is M_d.

Generalizing a one-step binomial tree model with no-arbitrage debate, the value of Δ that makes the portfolio riskless.

For an upward movement in the stock price, the portfolio value through the life of the option is

$$S_0u\Delta - M_u.$$

The value for a downward movement in the stock price is :

$$S_0u\Delta - M_d.$$

Equating the two equation gives

$$S_0u\Delta - M_u = S_0u\Delta - M_d,$$

and the portfolio value becomes

$$\Delta = \frac{M_u - M_d}{S_0u - S_0d}. \tag{4.1}$$

Figure 4.1 represents a one-step binomial tree.

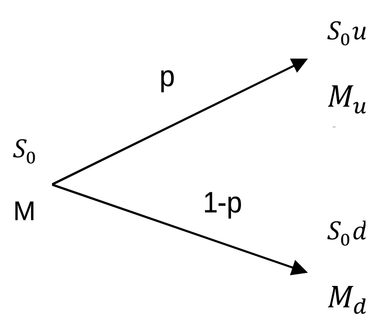

Figure 4.1: One-step binomial tree.

Where, S is the stock and M the option price.

Moving along the nodes at time T and denoting the risk-free interest rate by r, the value of the portfolio becomes,

$$(S_0 u\Delta - M_u)e^{-rT},$$

the portfolio's cost becomes

$$S_0 u\Delta - M.$$

Therefore it follows that

$$S_0 u\Delta - M = (S_0 u\Delta - M_u)e^{-rT}.$$

Solving for M, we have

$$M = S_0\Delta(1 - ue^{-rT}) + M_u e^{-rT}.$$

Substituting equation (4.1) for Δ and simplifying reduces the equation to the option price

$$M = e^{-rT}[pM_u + (1-p)M_d] \tag{4.2}$$

where

$$p = \frac{e^{-rT} - d}{u - d} \tag{4.3}$$

$$a = e^{-rT} \text{ therefore, } \quad p = \frac{a-d}{u-d}$$

According to the CRR approach Cox et al. (1979) we assume $u = {}^{1}/_{d}$ and therefore:

$$u = e^{\sigma\sqrt{\Delta t}},$$

$$d = e^{-\sigma\sqrt{\Delta t}}.$$

The CRR approach is not the only way of building a binomial tree. By setting p as the probability of the upward movement and ensures no arbitrage .

Repeating the generalized formula (4.2) will give

$$M_u = e^{-rT}[pM_{uu} + (1-p)M_{ud}], \tag{4.4}$$

$$M_d = e^{-rT}[pM_{ud} + (1-p)M_{dd}]. \tag{4.5}$$

Substituting (4.4) and (4.5) into (4.2), we arrive at

$$M = e^{-2r\Delta T}[p^2 M_{uu} + 2p(1-p)M_{ud} + (1-p)^2 M_{dd}]. \tag{4.6}$$

For a risk-neutral valuation, the above equation is consistent and the variables $p^2, 2p(1-p), (1-p)^2$ are the probabilities of reaching the upper, middle and lower final nodes respectively.

In Figure 4.2, a binomial model is used to form the complete tree. At different time intervals, the asset price S_0 is known.

At Δt, we have two possible asset prices, S_0u and S_0d and three possible asset values at time $2\Delta t$ and so on.

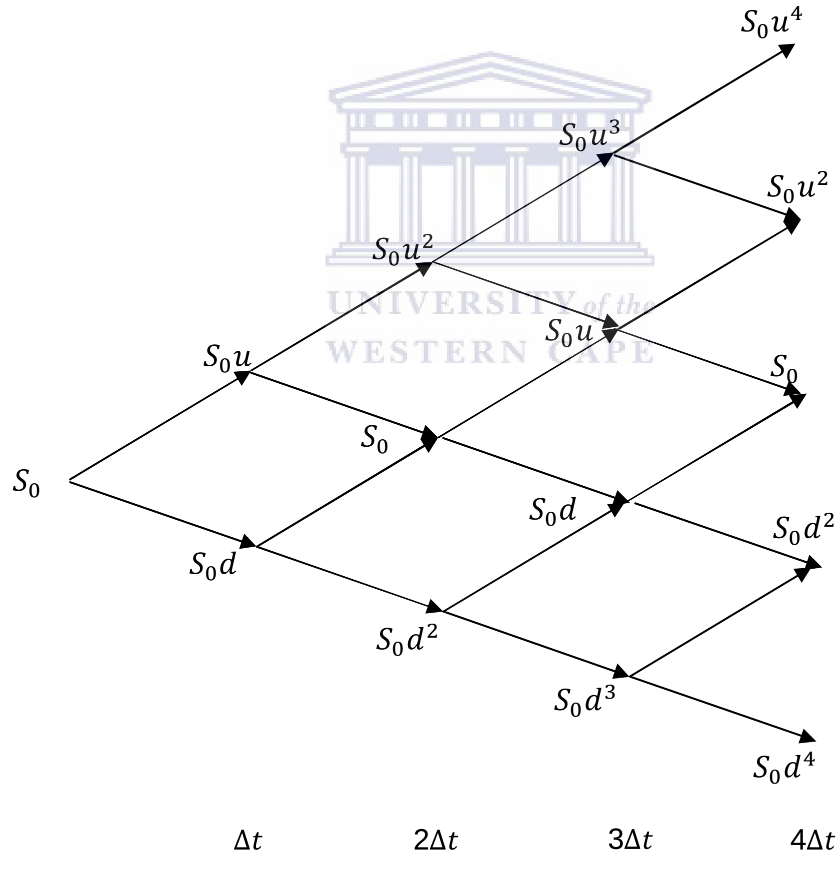

Figure 4.2: Four-step binomial tree

We will consider an example to illustrate the possibility of early exercise. This is because in American options, it is a necessity to check the possibility of early exercise at each node and also to know if it's worth holding on to the option for a longer time.

Viewing this algebraically, an American put option on a non-dividend-paying stock is divided into N subintervals of length Δt. Reference is made to the jth node at time $i\Delta t$ as the (i, j) node, where $0 \leq i \leq N \; and \; 0 \leq j \leq i$. Let's make $M_{i,j}$ the value of the option at node(i, j), the stock price at node $(i, j)$is $S_0 u^j d^{i-j}$ Hull (2009). So far an American put has a value at expiration equal to max$(K - S_T, 0)$. Then,

$M_{i,j} = \max(K - S_0 u^j d^{i-j}, 0) \qquad j = 0, 1, \ldots, N$.

There is a probability p of an upward movement from the (i, j) node at time $i\Delta t$ to the $(i+1, j+1)$ node at time $(i+1)\Delta t$, and a probability q moving from node (i, j) at time $i\Delta t$ to the node $(i+1)\Delta t$. The risk neutral valuation under an assumption that there is no early exercise gives

$M_{i,j} = e^{-r\Delta t}\left[pM_{i+1,j+1} + qM_{i+1,j}\right]$.

With the consideration of an early exercise, the value for $M_{i,j}$ must be compared to the option intrinsic value to give

$M_{i,j} = \max\left\{K - S_0 u^j d^{i-j}, e^{-r\Delta t}\left[pM_{i+1,j+1} + qM_{i+1,j}\right]\right\}$,

for $0 \leq i \leq N \; and \; 0 \leq j \leq i$.

In the binomial tree, calculation is done backwards, that is, starts from T through the intervals of $i\Delta t$ which makes the early exercise possible at every time steps.

Has $\Delta t \to 0$, the American put option price is obtained Hull (2009). In some cases, the CRR approach for finding u becomes invalid when terms of higher order than Δt are ignored. When this happens, we set $p = 0.5$ and $u \; and \; d$ becomes

$$u = e^{\left(r - q - \frac{\sigma^2}{2}\right)\Delta t + \sigma\sqrt{t}} \; and \qquad (4.7)$$

$$d = e^{\left(r - q - \frac{\sigma^2}{2}\right)\Delta t - \sigma\sqrt{t}} . \qquad (4.8)$$

This happens when time steps are large enough for $\sigma < \left|(r - q)\sqrt{\Delta t}\right|$

Example 4.1

Let's consider a non-dividend paying American put option when the stock price =$100, strike price=$100, the risk free interest rate = 10% per annum, and the volatility = 30%, total period of 5 months that is T = 0.4167 using a 5 step binomial model, Δt = 0.0833.

The binomial tree in Figure 4.3 is generated using the DerivaGem Software.

The major reason why the binomial model is preferred to the BSOPM is because of the possibility of exercising before maturity. In Figure 4.3, the option can be exercised at nodes F, C, B, A, respectively starting from the 2nd step. For example, at node I the option is $10.2941 if excercised, but if not excercised, the original value is 10.2941. This is calculated by That means at that stage, it best for the investor to wait further before exercising the option because the payoff at this point is zero.

$$(p \times option\ price\ of\ the\ higher\ node + q \times option\ price\ of\ the\ lower\ node)e^{-r\Delta t}$$

$$(0.5266 \times 5.41477 + 0.4734 \times 15.9041)e^{-.1\times 0.0833} = \$10.2941.$$

Unlike node C, the value of the option if excercised is greater than the value of the option if not exercised. Value if exercised is $22.8808 and the value if not exercised is $22.0512.

$$(0.5266 \times 15.9041 + 0.4734 \times 29.2787)e^{-.1\times 0.0833} = \$22.0512.$$

The payoff for node C is ($22.8808 - $22.0512) = $0.8296

The investor can exercise at this point or still decide to wait further.

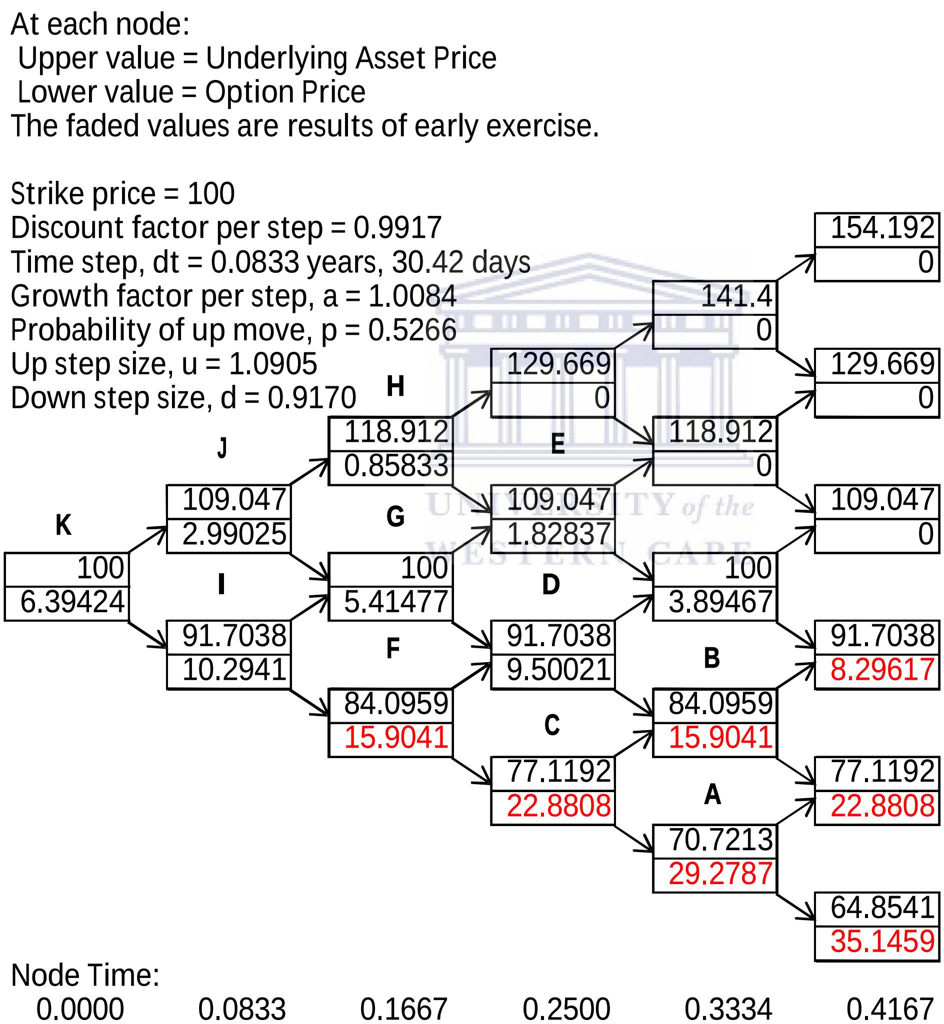

Figure 4.3: American put on non-dividend paying stock generated by DerivaGem

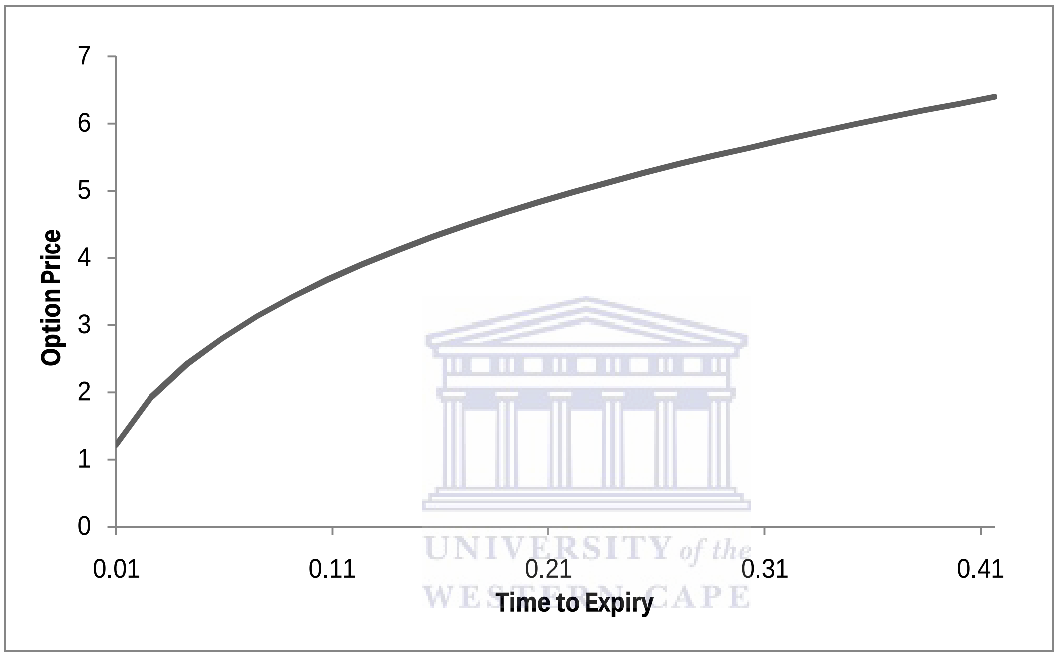

Figure 4.4: Graph of Option price against time of maturity for a non dividend paying stock

Figure 4.4 explains that the increase in option price is directly proportional to the increase in time to expiry. It means that it is not wise and profitable for an investor to exercise early in this scenario.

Example 4.2

Lets consider a business with the following employee stock option plan:

Market price of the ESO = \$50

Exercise price = \$50

Time duration = 10 years

Risk-free interest rate = 5%

Volatility = 25%

Vesting period of 4 years

No dividend.

The employee is not allowed to exercise the option within the first 4 years. The time step (Δt) is 2 years, therefore, the employee can only exercise in one of the nodes starting from G to A.

Under normal standard option rules, it is acceptable to start exercise the option in Figure 4.5 starting from the second step, but it’s not possible to do so because of the vesting period, which is part of the differences between a standard option and an ESO.

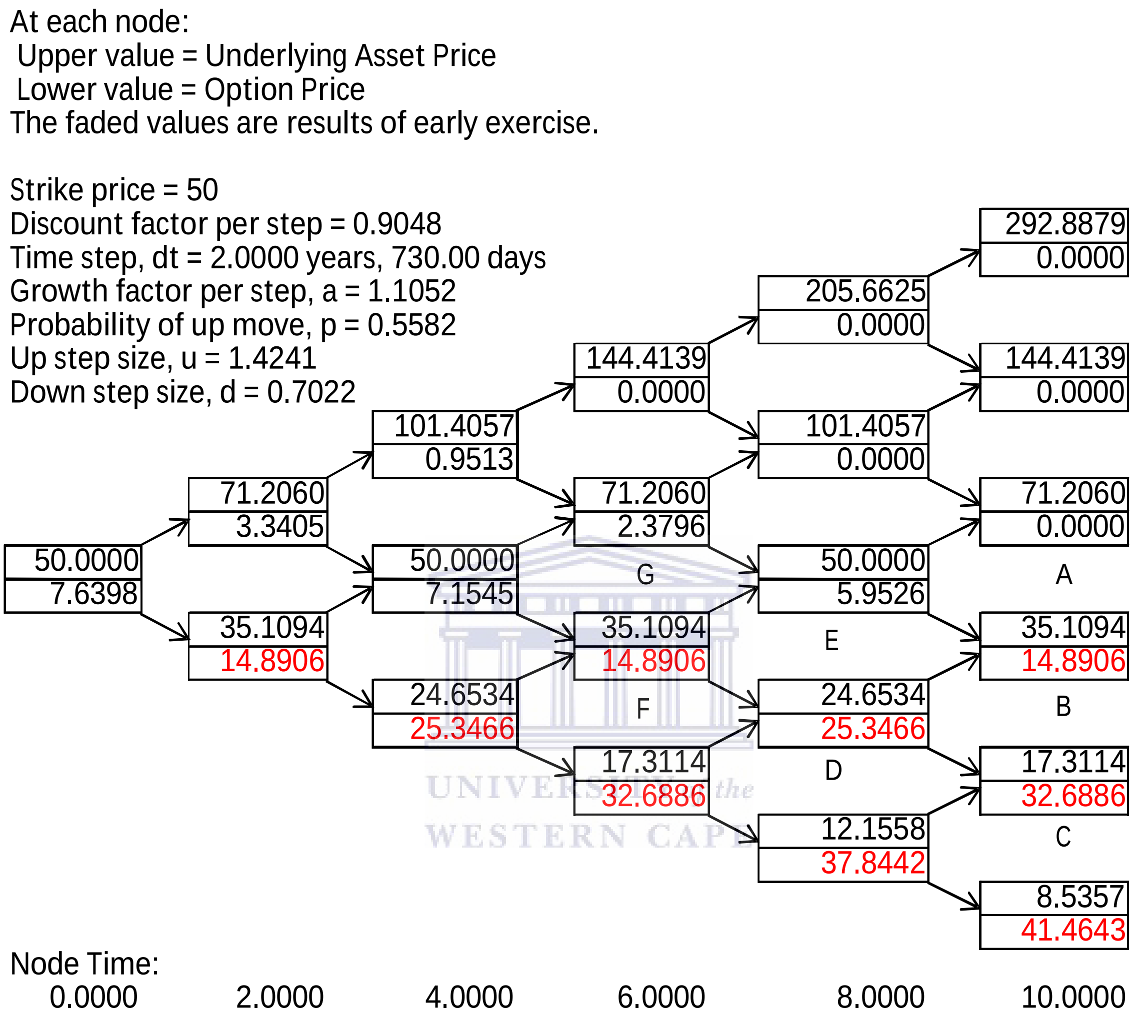

Figure 4.5: Example of a five-step tree with dividend yields

Binomial Tree for a Known Dividend Paying Stock

For a dividend paying stock, having the dividend as a percentage of the stock price known, the tree adopts a different form but still examined carefully and methodically the way the tree with non-dividend paying stock is examined.

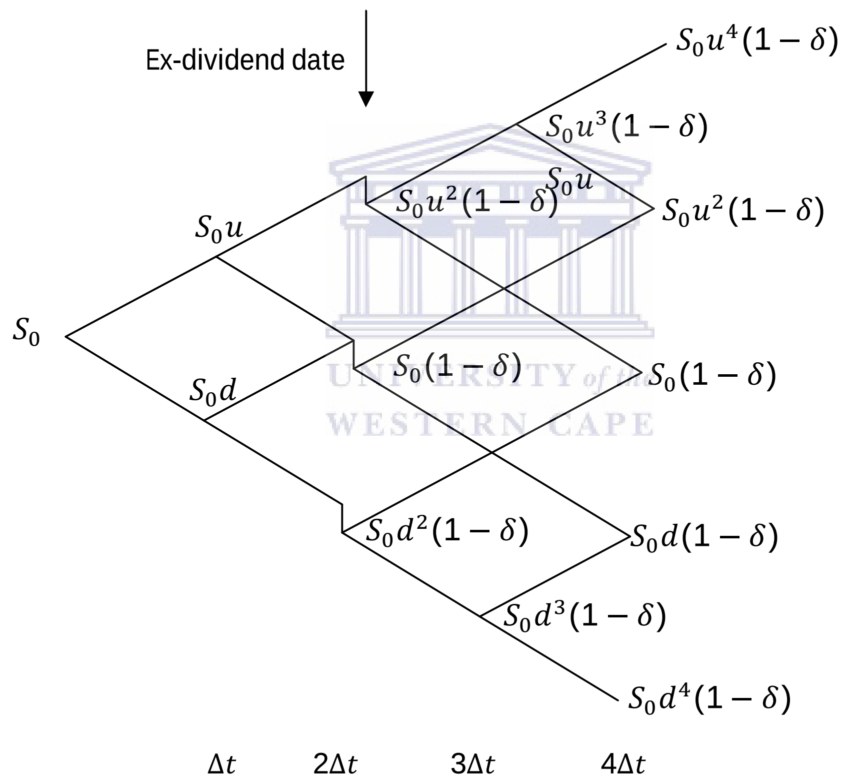

Figure 4.6: Binomial tree of Stock paying with a known dividend yield.

If the $i\Delta t$ is earlier in time to the stock becoming ex-dividend, as the nodes on the tree is equivalent to the stock prices

$S_0u^jd^{i-j}, j = 0,1, \dots, i.$

But if the time $i\Delta t$ is subsequent to stock ex-dividend, the nodes, then the nodes correspond to stock prices,

$S_0(1-\delta)u^j d^{i-j}, j = 0,1, \dots, i$ where δ is the dividend yield.

Example 4.3

Considering an Employee Stock Option plan of :

Market price of the ESO = $110

Exercise price = $100

Time duration = 6 years

Risk-free interest rate = 10%

Volatility = 30%

Vesting period of 3 years

Ex-dividend date = 6 months

Dividend = $15

This tree in Figure. 4.7 explains an option where the present value of the dividend is added at each node. A vesting period of 3 years signifies that the employee can only start excecution after the third step.

In the case of dividend paying stock, when an option lasts longer than expected, the volatility forecast can be high and thus makes the assumption of a known dividend yield more suitable than a known cash dividend yield. This is because the assumption will be invalid to be the same for all the stock prices. In the work of Hull and White (1998), a control variate technique was introduced to ameliorate the accuracy of pricing American option.

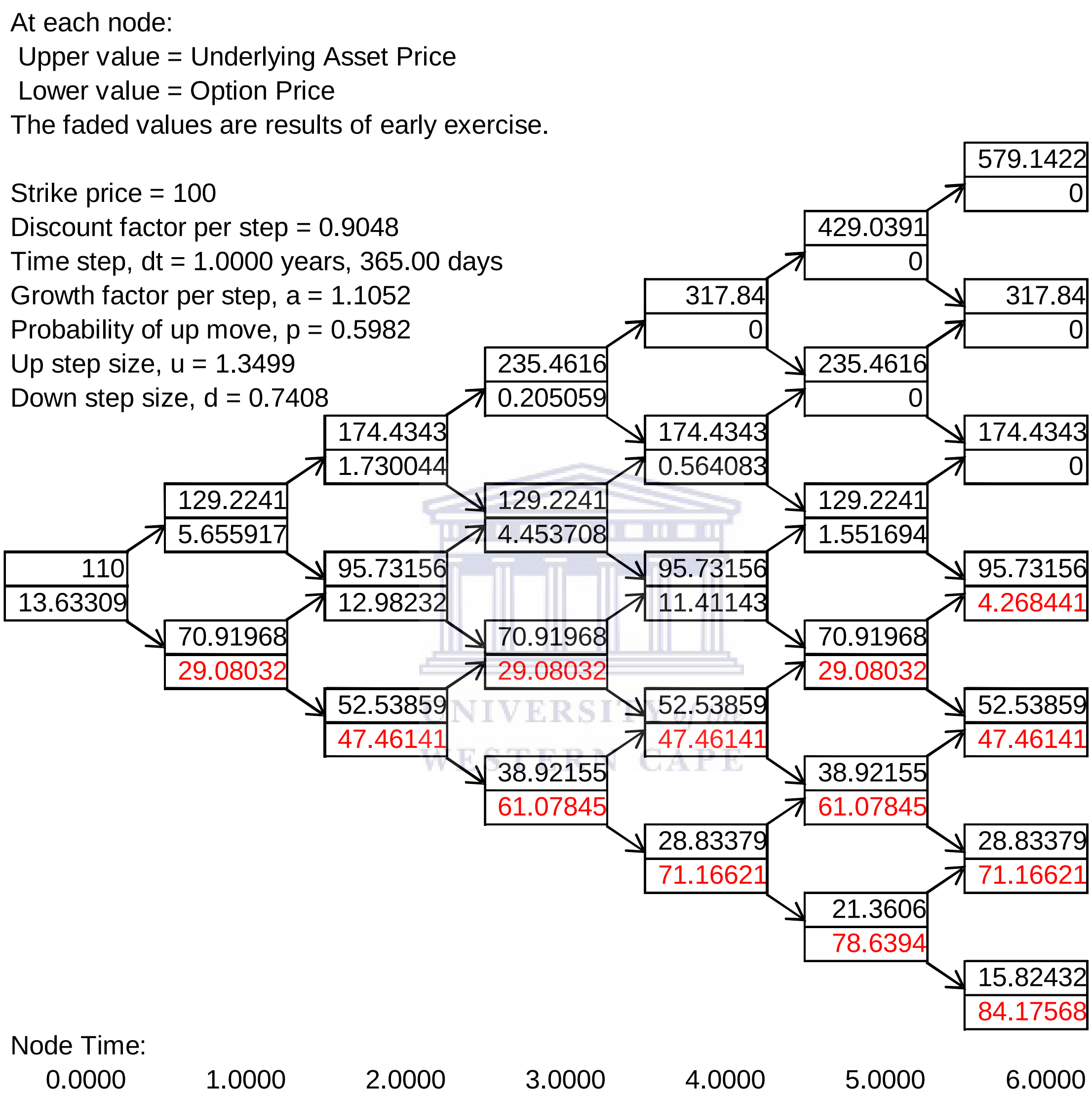

Figure 4.7: Example of a six-step binomial tree for a dividend paying

Employee Stock Option Plan

An employee stock option plan is always written by a lawyer and sincerely, it can be very hard to interpret. The ESO plan is a contract binding the employee and the employer (the company) together. An example of an Incentive Stock Option written by CROCS, INC. under the 2007 Equity Incentive Plan to an employee is given below,from the source (http://contracts.onecle.com).

"CROCS, INC.

2007 EQUITY INCENTIVE PLAN

Incentive Stock Option Agreement

Name of Participant:

Number of Shares Covered: Grant Date:

Exercise Price Per Share: Expiration Date:

Exercise Schedule (Cumulative):

This is an Incentive Stock Option Agreement ("Agreement") between Crocs, Inc., a Delaware corporation (the "Company"), and you, the Participant identified above, effective as of the Grant Date specified above.

Recitals*

A. The Company maintains the Crocs, Inc. 2007 Equity Incentive Plan (the "Plan"); and

B. Pursuant to the Plan, the Compensation Committee of the Board of Directors (the "Committee") administers the Plan, has the authority to determine the awards to be granted under the Plan and, subject to certain limitations contained in the Plan, has the authority to delegate such authority to persons who are not Non-Employee Directors of the Company; and

C. The Committee, either acting on its own or through certain of its authorized delegates, has determined that you are eligible to receive an award under the Plan in the form of an Incentive Stock Option (the "Option");

NOW, THEREFORE, the Company hereby grants this Option to you subject to the following terms and conditions:

Terms and Conditions

1. **Grant**. You are granted this Option to purchase the number of Shares specified at the beginning of this Agreement.

2. **Exercise Price**. The purchase price to you of each Share subject to this Option shall be the exercise price specified at the beginning of this Agreement, which price shall not be less than the Fair Market Value of a Share as of the Grant Date.

3. **Non-Statutory Stock Option**. This Option is intended to be an "incentive stock option" within the meaning of Section 422 of the Code, and shall not be an "incentive stock option" to the extent it does not so qualify. The terms of this Agreement and the Plan shall be interpreted and administered so as to satisfy the requirements of the Code.

4. **Exercise Schedule**. This Option shall vest and become exercisable in accordance with the schedule specified at the beginning of this Agreement. The exercise schedule is cumulative, meaning that to the extent this Option has not been exercised and has not expired, terminated or been cancelled, it may be exercised with respect to any or all of the Shares as to which this Option has vested and become exercisable.

The vesting of this Option may be accelerated and it may be exercised in full under the circumstances described in Section 8 of this Agreement if it has not expired prior thereto.

Unless the context clearly indicates otherwise, any capitalized term that is not defined in this Agreement shall have the meaning set forth in the Plan as it currently exists or as it is amended in the future

5. **Expiration**. This Option shall expire at 5:00 p.m. Mountain Time on the earliest of: (a) The Expiration Date specified at the beginning of this Agreement, which shall not be later than ten years after the Grant Date;

(b) The last day of the period following the termination of your employment during which this Option can be exercised (as specified in Section 7 of this Agreement);

(c) The date your employment is terminated through discharge for Cause; or

(d) The date (if any) fixed for cancellation pursuant to Section 8 of this Agreement.

No one may exercise this Option, in whole or in part, after it has expired, notwithstanding any other provision of this Agreement.

6. **Procedure to Exercise Option**.

Notice of Exercise. This Option may be exercised by delivering advance written notice of exercise to the Company at its headquarters in the form attached to this Agreement or in such other form as may be approved by the Company from time to time or by notifying the Company's outside Plan administrator of your intent to exercise and complying with all requirements set forth by such outside Plan administrator. If the person exercising this Option is not you, he/she also must submit appropriate proof of his/her right to exercise this Option.

Tender of Payment. Upon giving notice of any exercise hereunder, you shall provide for payment of the purchase price of the Shares being purchased through one or a combination of the following methods:

(a) Cash (including check, bank draft or money order);

(b) To the extent permitted by law, through a broker-assisted cashless exercise in which you irrevocably instruct a broker to deliver proceeds of a sale of all or a portion of the Shares to be issued pursuant to the exercise to the Company in payment of the purchase price of such Shares;

(c) By delivery to the Company of unencumbered Shares having an aggregate Fair Market Value on the date of exercise equal to the purchase price of such Shares (or in lieu of such delivery, by tender through attestation of such Shares in accordance with such procedures as the Committee may permit); or

(d) By authorizing the Company to retain from the total number of Shares as to which the Option is exercised that number of Shares having an aggregate Fair Market Value on the date of exercise equal to the purchase price of such Shares.

Notwithstanding the foregoing, you shall not be permitted to pay any portion of the purchase price with Shares if the Committee, in its sole discretion, determines that payment in such manner is undesirable.

Delivery of Shares. As soon as practicable after the Company receives the notice of exercise and payment provided for above, it shall deliver to the person exercising the Option, in the name of such person, a certificate or certificates representing the Shares being purchased (net of the number of Shares sold or withheld, if any, to pay the exercise price). The Company may alternatively satisfy this obligation to deliver Shares by a book entry made in the records of the Company's transfer agent or by electronically transferring such Shares to an account designated by the person exercising the Option. The Company shall pay any original issue or transfer taxes with respect to the issue or transfer of the Shares and all fees and expenses incurred by it in connection therewith. All Shares so issued shall be fully paid and nonassessable. Notwithstanding anything to the contrary in this Agreement, the Company shall not be required to issue or deliver any Shares unless issuance and delivery complies with all applicable legal requirements, including compliance with applicable state and federal securities laws.

7. **Employment Requirement**. This Option may be exercised only while you remain employed with the Company or a parent or subsidiary thereof, and only if you have been continuously so employed since the date of this Agreement; provided that:

(a) This Option may be exercised for a period of one year after the date your employment ends if your employment ends because of death or Disability, but only to the extent it was exercisable immediately prior to the end of your employment.

(b) If your employment terminates after a declaration made pursuant to Section 8 of this Agreement and Section 18 of the Plan in connection with a Fundamental Change, this Option may be exercised at any time during the period permitted by such declaration.

(c) If your employment is terminated by the Company for Cause, this Option shall expire, and all rights to purchase Shares hereunder shall terminate immediately upon such termination.

(d) If your employment terminates for any reason other than as provided above, this Option may be exercised at any time within three months after the time of such termination of employment, but only to the extent that it was exercisable immediately prior to such termination of employment.

Notwithstanding any provision of this Agreement, this Option may not be exercised after it has expired as provided in Section 5 of this Agreement.

8. **Acceleration of Option**. If a Fundamental Change shall occur, then the Committee may, but shall not be obligated to, take the actions described in Section 18 of the Plan.

9. **Limitation on Transfer**. During your lifetime, only you (or your Successor) may exercise the Option. You may not assign or transfer the Option except to a Successor in the event of your death.

10. **Market Standoff Agreement**. If requested by the Company and an underwriter of Shares (or other securities) of the Company, you agree not to sell or otherwise transfer or dispose of any Shares (or other securities) of the Company held by you during the period requested by the managing underwriter following the effective date of a registration statement of the Company filed under the Securities Act of 1933, provided that all officers and directors of the Company are required to enter into similar agreements. Such agreement shall be in writing in a form satisfactory to the Company and such underwriter. The Company may impose stop-transfer instructions with respect to the Shares (or other securities) subject to the foregoing restriction until the end of such period.

11. **No Stockholder Rights Before Exercise**. No person shall have any of the rights of a stockholder of the Company with respect to any Share subject to this Option until the Share actually is issued to him/her upon exercise of this Option.

12. **Adjustments for Changes in Capitalization**. This Option shall be subject to adjustments for changes in the Company's capitalization as provided in Section 17 of the Plan.

13. **Interpretation of This Agreement**. All decisions and interpretations made by the Committee with regard to any question arising under this Agreement or the Plan shall be binding and conclusive upon the Company and you. This Agreement is subject to and shall be construed in accordance with the terms of the Plan. If there is any inconsistency between the provisions of this Agreement and the Plan, the provisions of the Plan shall govern.

14. **Discontinuance of Employment**. This Agreement shall not give you a right to continued employment with the Company or any Affiliate of the Company, and the Company or any such Affiliate
employing you may terminate your employment and otherwise deal with you without regard to the effect it may have upon you under this Agreement.

15. **Tax Withholding**. Delivery of Shares upon exercise of this Option shall be subject to any required withholding taxes. As a condition precedent to receiving Shares upon exercise of this Option, you shall be required to pay to the Company, in accordance with the provisions of Section 15 of the Plan, an amount equal to the amount of any required tax withholdings.

16. **Option Subject to Plan, Certificate of Incorporation, and Bylaws.** You acknowledge that this Option and the exercise thereof is subject to the Plan, the Certificate of Incorporation, as amended from time to time, and Bylaws, as amended from time to time, of the Company, and any applicable federal or state laws, rules, or regulations.

17. **Obligation to Reserve Sufficient Shares**. The Company shall at all times during the term of this Option reserve and keep available a sufficient number of Shares to satisfy this Agreement.

18. **Binding Effect**. This Agreement shall be binding in all respects on your heirs, representatives, successors, and assigns.

19. **Choice of Law**. This Agreement is entered into under the laws of the State of Delaware and shall be construed and interpreted thereunder (without regard to its conflict of law principles).

IN WITNESS WHEREOF, you and the Company have executed this Agreement effective as of the   day of …… , 200 …".

Example 4.4

We consider a 10 years Employee stock option plan of a dividend yield \$5 (12.5% dividend rate) during the life of the option. The initial stock price is \$40, strike price is \$40 (same fair market value signifying an Incentive Stock Option), a risk free interest rate of 5% per annum, volatility of 25% and the ex-dividend date of 3 years and a vesting period of 4 years.

In Figure 4.8, the upper value denotes the current employee stock price, the lower node denotes the option price (that is, what we are actually pricing). The faded values tells us that early exercise took place at such nodes. Also, because of the vesting period of 4 years, the employee cannot start execution until the node 6 and above. This gives the employee an option to exercise or keep the option for a longer period.Though, any exercise at this period will be profitable. Considering node 11, the original value of the option if not exercised will be

$$(p \times option\ price\ of\ the\ higher\ node + q \times option\ price\ of\ the\ lower\ node)e^{-r\Delta t}$$

Where p is the probability of the upward movement and $q = 1 - p$ is the probability of the downward movement.

$$(0.5393 \times 12.19957 + 0.4607 \times 23.13819)e^{-.05\times 1} = \$16.3982$$

The value of the option if exercised is \$18.349, therefore, it is all right if the employee decides to exercise because there is a payoff of \$18.349 – \$16.3982 = \$1.951

This is the major advantage of binomial tree model over the BSOPM (ability to exercise at anytime).

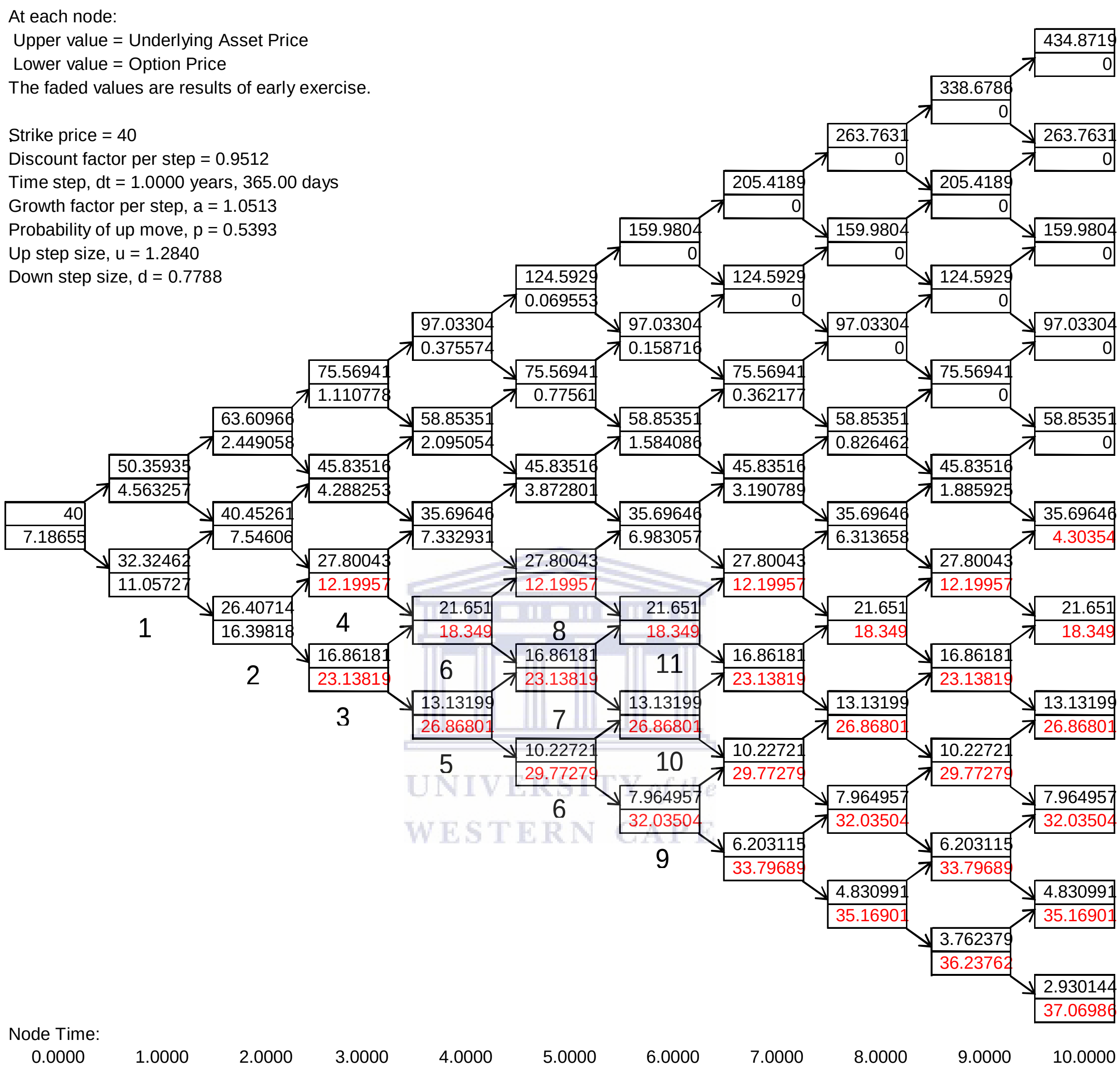

Figure 4.8: Example of a ten-step binomial tree for a dividend paying stock

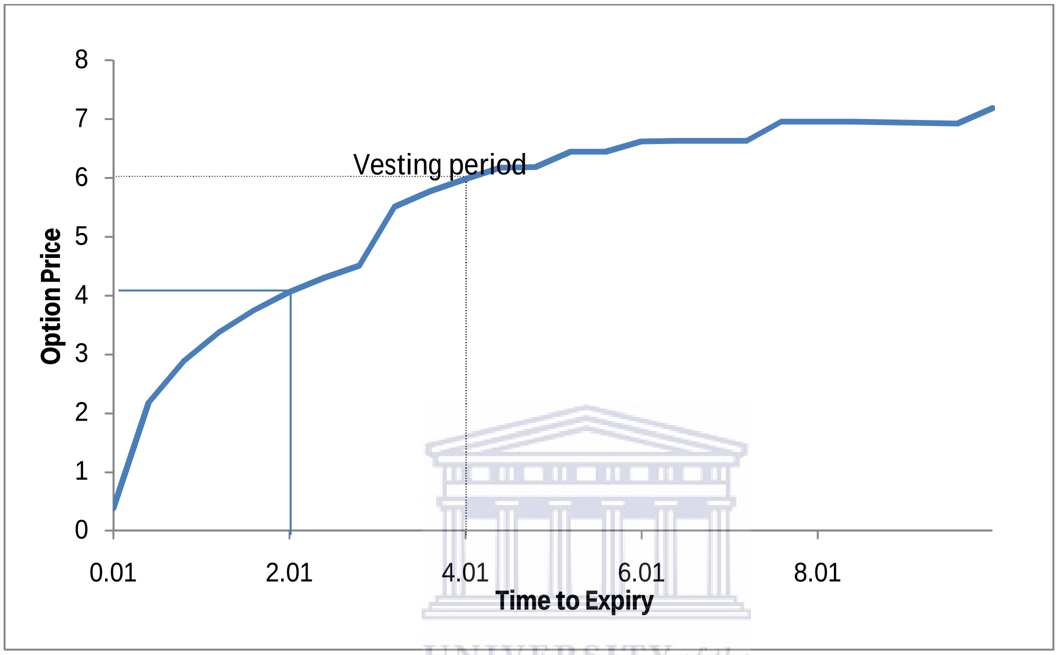

Figure 4.9: Graph of option price against time for

In Figure 4.9, the employee can start making a decision for early exercise or to keep the stock for a longer period after four years.

Above it is shown that the stock actually starts making profit after 2 years of grant, but because of the 4 years vesting period, the employee must wait for 4 years before making decision. The graph also proves that the longer an employee keeps the stock the better for the value of the option. That is, early exercise of ESOs reduces value.

4.2 Finite Difference Method

The finite difference method is a numerical approach for valuing derivatives. This is done by solving the differential equation statisfied by the derivative. This approach acts upon the conversion of the differential equation into a set of difference equations and then solved iteratively.

To explain the procedure, we consider the estimation of an American put option. Where there is a dividend yield of δ.

The Black Scholes-Merton differential equation must be satisfied

$$\frac{\partial M}{\partial t} + (r-\delta)S\frac{\partial M}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 M}{\partial S^2} = rM. \tag{4.9}$$

If the life of the option is T, to get time intervals, we divide T to N equally spaced intervals.

$\Delta t = {}^{T}/_{U}$. Max(U+1) times are considered. We have time intervals 0,$\Delta t, 2\Delta t, \dots, T$

If we assume S_{max} to be the maximum price to be reached to make the put valueless,

$\Delta S = {}^{S_{max}}/_{V}$ giving a total of equally spaced $(V+1)$ stock prices.

The idea behind this is to produce a grid of the form$(U+1)(V+1)$ with points (i,j) on the grids corresponding to time $i\Delta t$ and stock price $j\Delta S$.

Let $M_{i,j}$ be the option value at the points (i,j).

This is a similarity to the tree approach, computations are worked back from the end of life of the option/derivative to the beginning.

The implicit and the explicit method are the two different types of finite difference method. We will discuss and illustrate with the use of implicit finite difference method which is known to be more suitable than the explicit methods.

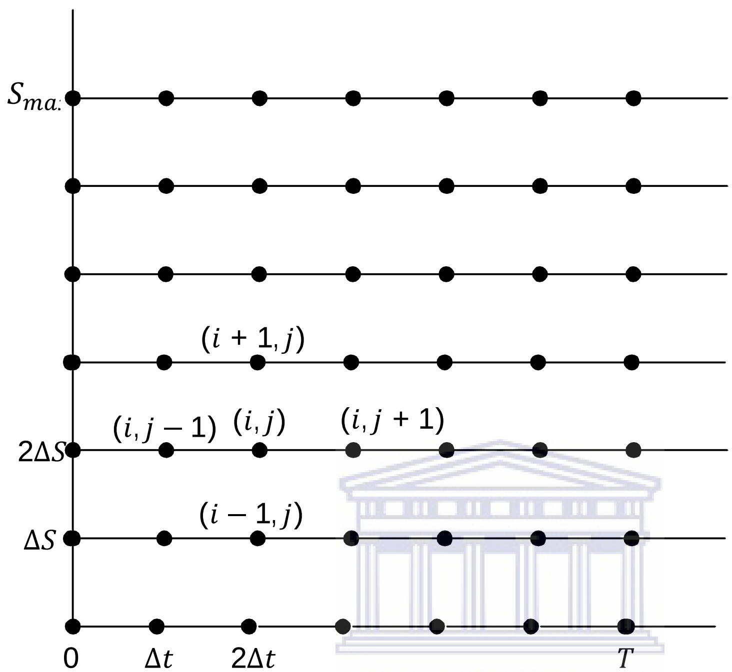

Figure 4.10: A typical grid used in Finite Difference approach

Implicit Finite Difference Method

From the points (i, j) on the grid, in Figure 4.10

$$\frac{\partial M}{\partial S} = \frac{M_{i,j+1} - M_{i,j-1}}{2\Delta S}. \tag{4.10}$$

Equation (4.10) is the average or symmetric approach of the forward difference approximation and the backward difference approximation.

$$\frac{\partial M}{\partial t} = \frac{M_{i+1,J} - M_{i,j}}{\Delta t}, \tag{4.11}$$

$$\frac{\partial^2 M}{\partial S^2} = \frac{M_{i,j+1} - M_{i,j-1} - 2M_{i,j}}{\Delta S^2}. \tag{4.12}$$

Substituting (4.10), (4.11) and (4.12) into Equation (4.9) and also making $S = j\Delta S$

We have

$$\frac{M_{i+1,J} - M_{i,j}}{\Delta t} + (r-\delta)j\Delta S \frac{M_{i,j+1} - M_{i,j-1}}{2\Delta S} + \frac{1}{2}\sigma^2 j^2 S^2 \frac{M_{i,j+1} - M_{i,j-1} - 2M_{i,j}}{\Delta S^2} = rM_{i,j}$$

For $j = 1,2,\dots, U-1$ and $i = 0,1,2,\dots, V-1$.

Therefore, the rearrangement becomes

$$a_j M_{i,j-1} + b_j M_{i,j} + c_j M_{i,j+1} = M_{i+1,J}, \tag{4.13}$$

where

$$a_j = \frac{1}{2}(r-\delta)j\Delta t - \frac{1}{2}\sigma^2 j^2 \Delta t \,,$$

$$b_j = 1 + \sigma^2 j^2 \Delta t + r\Delta t,$$

$$c_j = -\frac{1}{2}(r-\delta)j\Delta t - \frac{1}{2}\sigma^2 j^2 \Delta t.$$

The put value at the maximum time T is $\max(K - S_T, 0)$ hence,

$$M_{v,j} = \max(K - j\Delta S, 0), \tag{4.14}$$

When $i = V - 1$,

$$a_j M_{V-1,j-1} + b_j M_{V-1,j} + c_j M_{V-1,j+1} = M_{N,J}. \tag{4.15}$$

Equation (4.15) is a simultaneous equation of $U - 1$ unknowns.

To know when early exercise is optimal, a comparison has to be done between $M_{V-1,j}$ $and\ K - j\Delta S.\ if\ M_{V-1,j} < K - j\Delta S$ then there is optimum at $T - \Delta t\ and\ M_{V-1,j} = K - j\Delta S.$

The above finite difference method can be implemented on a computer and can be solved. We are currently doing this.

CHAPTER FIVE

Concluding Remarks and Scope for Future Work

Employee stock options (ESOs) differ from the traded options in many ways. The ability for an employee to exercise the option before maturity, vesting period, time varying volatility, etc. makes the Black-Scholes pricing formula ineffective for the American-style employee stock option. The only case where the Black-Scholes formula can be used is when we assume the employee stock option should not be exercised until the expiration date.

The GARCH model explained is actually used in the forecast of the time varying volatility. The assumed volatility used in the examples is based on the GARCH model which extends to the different schemes for the same employee stock option holder based on varied conditions. GARCH model being a short-memory volatility model is used, although, other modified GARCH models where mentioned. Like the CGARCH model is a long-memory volatility model.

The binomial tree method illustrates the activities possible at each node of the tree, considering the vesting period of the ESO examples and the ex-dividend date for the examples with dividend yield.

The finite difference method can also be used in valuing ESOs. This is more suitable for derivatives when an investor must make a decision earlier in time before maturity.
We discussed how the implicit finite difference method can be used as iteration for valuing the ESO. The explicit method has a disadvantage over the implicit method that it converges to the solution of the differential equation only when the change in price and time interval tends to zero. The simulation of the implicit method and the indepth illustration of the explicit method are being worked on currently.

An ESO follows a random walk process and therefore loses value with increasing time. Furthermore, the inability to trade and transfer options adds to the reduction and makes ESO liquid.

According to the revised FASB statement published in 2004 (FAS123R) which was revised to be suitable with the International Financial Report Standard 2 (IFRS2) the method required in valuing ESO was not specified but the factors needed to calculate the minimum value was stated. We would think that a proposal that FASB should come out with a formula or pricing model that will put all the characteristics and parameters into distinctive consideration in valuing ESOs. However, currently we are looking at this aspect in a greater detail.

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