Quality of Instructional Explanation and its Relation to Student Learning in Primary Mathematics

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Abstract

Instructional explanation is deliberately designed for the purpose of teaching, and serves as responses to various queries from teachers, students, or queries about the nature of a domain in a school subject matter. The present study aimed to identify the features of quality instructional explanation that might serve as proximal indices of students' learning outcome. To do so, the study, firstly, developed an instrument to assess the quality of instructional explanations. Secondly, it examined how quality of instructional explanation would affect students' learning outcomes. Finally, the study investigated whether the teachers' backgrounds, such as mathematics knowledge for teaching or years of teaching, might yield high-quality of instructional explanation.

The data source of the current study was from the project "Has curriculum reform made a difference? Looking for change in classroom practice" (Ni, Li, Cai, & Hau, 2009). Thirty-nine teachers (20 teachers using a reform-oriented curriculum vs. 19 teachers utilizing a conventional curriculum) and their 2,239 students were included. All 39 classes had completed videotapes of lessons teaching new knowledge on the content of number. Teachers' instructional explanation was examined in terms of its structural features and quality. Quality of instructional explanation was evaluated on the three dimensions, accuracy (truthfulness), richness, and coherence. Student learning outcomes were measured with regard to cognitive (calculation, simple problem solving, and complex problem solving) and affective learning outcomes (self-reported interest in learning mathematics, classroom participation, views of mathematics, and views of learning mathematics).

The results indicated that the Chinese mathematics teachers scored high on the indices of accuracy (truthfulness) and coherence in instructional explanation. Their scores on the dimension of richness were relatively lower. Moreover, we found that a teacher's performance of instructional explanation was relatively stable across lessons. A few relationships were found between teachers' background factors (i.e., teaching age, educational level, mathematical knowledge for teaching, and belief towards mathematics) and their use of instructional explanation. Curriculum in use affected the quality of teachers' instructional explanation. For example, the teachers who used the reform-oriented curriculum were more likely to encourage students to use mathematical language, use multiple solutions and multiple representations than their counterparts of non-reformed classes. The teachers of non-reform classes were more inclined to use accurate teaching language and make connections with general concepts and principles than their peers of reform classes.

Teachers' instructional explanations were found to affect student learning outcomes. However, the effects differentiated across students' learning outcomes. We observed more effects on students' cognitive achievement than on affective achievement. Moreover, the effect of instructional explanation on student learning was moderated by curriculum in use, students' SES, and prior achievement. The effect appeared more obvious in the nonreform classes than the reform classes.

This study was significant in three respects. It has contributed to an understanding of the instructional features of Chinese mathematics classes and their relationship with student learning. Moreover, it has provided a useful tool for reserach and practice in matheamtics education. Finally, the results have some implications for teacher education.

摘 要

教学性解释指的是课堂对话中，师生为了解决学科疑问而共同建构的一种解释。本研究试图通过教学性解释，识别影响学生学习的敏感性变量。自编工具评价教学性解释的质量。考察教学性解释与学生学业成就的关系，并探讨教师哪些背景因素有助于生成高质量的教学性解释。

本研究数据源于项目“课程改革的成效—教师课堂教学实践变化”(Ni, Li, Cai, & Hau, 2009)。39 名小学五年级数学教师及其 2239 名学生参与研究。其中，20 位教师来自新课程组，19 位教师来自原课程组。对上述教师课堂观察录像，取他们的数论新授课进行分析。考察教师教学性解释的结构特征和质量。并从三个维度评价教学性解释的质量：准确性、丰富性和连贯性。学生学业评估包括两方面：认知成绩（计算、简单问题解决和复杂问题解决）、学生对数学学习的看法和态度（学习兴趣、课堂参与、数学观、数学学习观）。

结果表明，中国小学数学教师课堂教学性解释的准确性及连贯性得分均较高。丰富性得分略低。并且，教师教学性解释的表现相对稳定。研究还发现，教师背景因素（如教龄、学历、数学知识和数学观）与其教学性解释关系甚微。课程影响教学性解释的质量。新课程教师在解释过程中，更多鼓励学生使用数学语言，运用多种策略及多元表征。原课程教师更注重使用准确的教学语言和联系一般性数学概念及原理。

教师教学性解释影响学生学业表现。但其效应因不同学业成就变量而异。我们观察到，教学性解释对学生认知成绩的影响甚于其对学生数学态度的效应。课程、学生 SES 及原有成绩在教学性解释和学生学业表现关系中起调节作用。例如，较之与新课程组，教学性解释在原课程班级出现更多显著效应。本研究有助于我们理解中国数学课堂的教学特征以及教师教学与学生学习之间的关系。此外，它为我们进行数学教育相关研究及实践提供了有用的工具。研究结果也将对教师教育带来一定启示。

谢辞

Dear Dr. Li,

Congratulations on your successful completion of your PhD program. It is a great accomplishment. …… Ni 23th, Aug. 2011

见到导师发来邮件，忍不住眼泪打转。三年来，她始终在旁。给予无限信任；让我负责课题报告，写英文文章，用新方法处理数据，不断激发我的研究潜力。默默表达支持；虽然知道我用英语写论文，会增加工作负担。但一如既往，让我大胆尝试。每次报告前，浏览我的课件，提出宝贵意见。温暖中传递智慧；给我时间，聆听我学业、生活及工作上的困惑。帮助我面对、接受、处理及放下。倪老师，还记得美国开会期间，咱们的谈话吗？要踏踏实实做好手中事。记得"12 点的故事"吗？有的时候，人要善于等待。每次面试前，微笑告诉我，be prepared and be myself，积极思考，得体处事。正是这些日积月累的信任、支持与鼓励，让我不断成长，在学术及生活路上走得更加自信、坦然。

这段旅途中，很幸运得到其他几位师长的指导和无私帮助。萧宁波老师，总是睿智提问，给予充满信任的微笑[1]。侯杰泰老师，鼓励我从错误中学习，热情递上推荐信[2]。黄毅英老师，跟我谈做学问、求职，还谈心[3]。蔡金法老师，亦师亦友。分享求知习惯，评审论文，并送上温暖支持[4]。卢乃桂老师，屡屡给予关爱，让我在为求职而沮丧时倍感力量[5]。还有，我的硕士导师，邹

---

[1] "You're most welcome. I wish you the best in your career". Lingpo

[2] "Hope you can get some papers published. Best wishes". KT

[3] "静观万物皆自得".NY

[4] "Please keep me informed about your job searching and let me know anything I can assist. Take care". Jinfa

[5] "The important thing is to be patient and not rush into a job. It is heart-warming to have great students. Take good care of yourself". Lu

泓。六年来，她的指导与关心，总是如一泓泉水般，滋润我心[6]。

素未谋面、远在河南郑州的58个老师，还有3000多个孩子们。你们好吗？正是由于你们的积极相助，导师的课题以及我的博士论文才得以顺利完成。深圳南山区育才小学的杨凌会、张乾两位老师，也谢谢你们。牺牲周末，帮忙完善施测工具，并让我有机会更多了解一线教师的教学故事。

教育学院各位同学，尤其是106成员。舍友刁妹，你们都是我在CU的families，温暖相伴。

最后的感谢送给我的父母。不管何时何地，你们始终为我提供最有力的支持，让我毫无顾忌地按照内心，做最好、最快乐的自己[7]。

[6]"昨天在办公室几位师弟师妹们还问起你的毕业去向。心里一直惦念着你，好在很快就要答辩了，工作也在寻找中" 邹泓

[7]"宝贝，有我们在。只要你觉得舒服开心，我们就赞成"。Dad & Mom

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Chapter 1 Introduction

For more than a decade, researchers and educators in various regions have been concerned with how classroom discourse affects opportunities to learn, student thinking, and student learning. In school mathematics, influential documents such as the National Council of Teachers of Mathematics's (2000) *Professional Standards for Teaching Mathematics and Principles and Standards for School Mathematics*, and *Decision of the Reform and Development of Basic Education* (2001) issued by the Chinese government, have called for teachers to emphasize communication that allows students to develop conceptual understanding of mathematics. Such communication consists, in large part, of encouraging students to present conjectures; pushing students to both explain and justify their conjectures to their peers, and otherwise promoting debate and discussion of ideas. And these forms of instruction are often characterized in different ways by the key phrases such as "**discourse-intensive**" (Cazden, 2001), "**extended discourse**" (Schleppenbach, Perry, Miller, Sims, & Fang, 2007), "**authentic discussion**" (Hadjioannou, 2003; 2007), and "**authentic instruction**" (Newmann & Wehlage, 1993; Newmann, Marks, & Gamoran, 1996).

Although the communication and instruction as indicated above are well advocated, the researchers have also suggested that the conversations about mathematical ideas (as opposed to the genre of Initiation-Response-Evaluation) provide a necessary, but not sufficient, foundation for high-level talk (Kazemi & Stipek, 2001). The merits of learning-by-talking cannot be taken for granted (Sfrad & Kieran, 2001) because using a particular form of discourse does not necessarily result in a particular kind of instruction and further bring about the desired learning outcomes (Cazden, 2001). Instead, how the discourse is used and constructed, that is, the specific content of the discourse in instruction, may be a more important factor when taking students' academic development into account (Murphy, Wilkinson, Soter, Hennessey, & Alexander, 2009).

In this context, the current study investigated the teacher use of instructional explanation, which was operationally defined as the explanations that are embedded in teachers' instructional talk with students. They are co-constructed by the teacher and students to respond to various queries in primary mathematics. The explanations are presented in the format of extended discourse, in which a teacher poses continued and follow-up questions after a student provides an answer. Instructional explanation was chosen as the focus of the present study for three reasons. Firstly, it is recognized as a legitimate part of the instructional landscape (Leinhardt, 2001). And explanation is ubiquitous in teaching (Chi, Siler, Jeong, Yamauchi, & Hausmann, 2001; Perry, 2000). Secondly, use of instructional explanation can be a critical aspect of the teaching repertoire (Renkl, 2002; Leinhardt & Steele, 2005). And an investigation of instructional explanation with the format of extended discourse helps to better describe and understand a teacher's performance as a whole. As various components contained in the instructional explanation, such as the teacher's questioning and response to student answers. Finally but not the least, such kind of classroom discourse is consistent with the merits of the advocated teaching practices. Therefore, it is important to examine whether use of instructional explanation with identified features, would bring about desired student learning outcomes.

Concerning the construct of instructional explanation (IE), the researcher, firstly, developed an instrument to evaluate its quality. The quality indicators of IE were mostly consistent with the advocated practices, and they were well established in the western literature. The study aimed to identify whether these advocated features of classroom discourse in general and instructional explanation in specific might serve as proximal indices of students' mathematics learning outcome in Chinese classrooms. To do so, the study, firstly, developed an instrument to assess the quality of instructional explanations observed in 110 Chinese elementary mathematical lessons of 39 teachers. Secondly, it examined whether or not the identified features of instructional explanation would affect students' mathematics learning outcomes in Chinese classrooms. And finally, the study investigated the relationship between teacher knowledge, belief and their use of instructional explanation.

Results of the study would make three specific contributions to the literature. Firstly, the findings of the study helped understand the classroom instruction in China. Secondly, the results helped understand the relationship between mathematics teaching and learning in Chinese classrooms. And finally, the developed instrument provided a useful tool for research and practice in mathematics education.

In the chapters that follow, the thesis is organized into four parts. Firstly, the literature of the research on instructional explanation was reviewed. Then, the research methodology was explained. Next, results of the study were displayed and discussed. And finally, general discussions and conclusions were presented.

Chapter 2 Literature Review

This literature review includes five parts. Firstly, the background of the research on instructional explanation was introduced. Then, the significance of instructional explanation was discussed. Next, the criteria for the evaluation of instructional explanation was described and synthesized. After that, the relationship between instructional explanation and teacher knowledge for teaching mathematics and beliefs about mathematics was explicated. And finally, based on these works, the research questions and hypothesis were formulated.

I Background of the Research

For a long history, the topics of educational psychology centered on issued of learning, but did not include teaching (Gage, 1963). Affected by the dominance of behaviorism in research as well as the political stimulation that the U.S. government issued relevant educational policy to improve the quality of school education (e.g., National Defense Education Act, 1958), the study of teaching became a new enterprise within educational psychology (Gage, 1963; Shulman, 1986/1990). The interest was primarily in identifying general characteristics of teacher behavior that were systematically related to significant improvement in student knowledge and understanding. Research of this type on teaching was also known as "**criterion-of-effectiveness**" paradigm (Gage, 1963, 1978), or **teaching effectiveness approach** (Shulman, 1986), or "**process-product" program** (Brophy & Good, 1986; Dunkin & Biddle, 1974). This paradigm dominated the study of teaching, and reached its peak in the late 1960s and early 1970s. Apparently, all these research focus on teaching and learning, rather than the teaching itself.

Following the shift from the focus on learning to teaching on learning, there has been another shift since 1980's being developed in educational psychology, that is, interest is primarily in identifying domain-specific, rather than domain-general, characteristics of effective teaching. The content, or substance, of a subject matter being taught and learned, which lad dormant for a long period and was dubbed as the problem of "the missing paradigm", returned to its proper place in research programs on teaching (Shulman, 1986/1990). This shift was prompted by the consistent finding that high performance of both human and AI systems was constrained by domain specific knowledge (e.g., Chi, Glaser, & Farr, 1988). A series of studies was conducted that documented the ways in which the subject matter taught influenced the kinds of representations teachers used in their teaching. Variations in teachers' knowledge, belief about their subject matter, and how to teach it significantly influenced the quality of pedagogy and further their students' learning (Grossman, 1990; Staub & Stern, 2002; Speer, 2008; Baumert et al., 2010; Shechtman, Roschelle, Haertel, & Knudsen, 2010).

Spoken language is the medium by which much teaching takes place and in which students demonstrate to the teacher much of what they have learned (Cazden, 1986, p.432). To date, **classroom discourse,** defined as the language that teachers and students use to communicate with each other in the classroom (Nuthall, 1998), has become a central topic of research on teaching. Researchers and educators have concerned about how classroom discourse would affect opportunities to learn, student thinking, and consequently student learning. Among the literature on classroom discourse, the most robust form of classroom talk documented has been characterized by that the teacher asks a question, a student responds, and the teacher follows through with a comment, often evaluative in nature. This forms of discourse or stable discourse genre is called **Initiation-Response-Evaluation** (I-R-E) (Mehan, 1979; Cazden, 1988) or **Initiation-Response-Follow up** (I-R-F) (Sinclair & Coulthard, 1975; Wells, 1993) sequence, as well as that labeled as **"triadic dialogue"** (Lemke, 1990). It is believed that IRE/F sequence associates with direct instruction of skills or the elicitation of so-called **known-information** questions, and contributes to more accurate transmission of information (Hicks, 1995; Macbeth, 2003). This discourse genre allows a teacher simultaneously maintain a high degree of control in the classroom, probe students' current conceptual understandings, and orchestrate a description of those concepts using students' as well as the discipline's words to bring them toward grasping a set of clearly specified concepts (Polman, 2004) . Also, it can be seen as a "**cultural tool**" (Polman & Pea, 2001) that is familiar to teachers from their own childhoods and familiar to most students after a couple of years in school. Thus, teachers and many students using IRE/F may understand well the norms of the speech genre (Bakhtin, 1986) which they are using in interaction with one another in classroom. They know what sorts of roles they are expected to play, what steps follow one another, what to say, how to say, and when to say it.

However, along with the emphasis on conceptual understanding, critical thinking, and problem solving complemented to knowledge and skills acquisition in school education, as well as the development and well-acceptance of the **constructivism** and **social cultural** framework, learning is regarded as an active process of knowledge construction and meaning making in the course of interaction, and classroom discourse is a mediator of students' learning. IRE/F is therefore viewed inadequate for achieving the new teaching goals. Other classroom discourse genres, in contrast to IRE/F, therefore, have been advocated. These forms of instruction are often characterized in different ways by the key phrases such as "**discourse-intensive**" (Cazden, 2001), "**extended discourse**" (Schleppenbach, Perry, Miller, Sims, & Fang, 2007), "**authentic discussion**" (Hadjioannou, 2003; 2007), and "**authentic instruction**" (Newmann & Wehlage, 1993; Newmann, Marks, & Gamoran, 1996). Though they differ in some ways; there are key overlapping components which emphasize considerable and substantive interaction about the ideas of a topic, opportunity for students to articulate ideas and opinions, making connections across knowledge as well as connections to the real-life, and deep and conceptual understanding of the knowledge. The teachers' role in the classroom discourse becomes more one of an "**orchestrator**" or **facilitator** of discussion. They deliberately pose more authentic questions, create instances of uptake, withhold or skip their evaluation of students' response, in order to give the floor for a much longer period of time for students to present conjectures, as well as to explain and justify their conjectures or opinions to their peers (Nystrand, 1997; Nystrand, Wu, Gamoran, Zeiser, & Long, 2001; Cazden, 2001). The students become active agents in the process of learning, engaging in verbal conjecturing that is subject to public questioning. It is believed that such kind of classroom discourse allows students more opportunities to engage in the learning process, develop conceptual, or so-called higher level, understanding of the subject matter, and foster students' interest in learning (Hiebert & Wearne, 1993; Ball, 1991; Lampert, 1990; Cobb, Yackel, & McClain, 2000； Walshaw & Anthony, 2008).

It is the latter kinds of classroom discourse that the current study aims to explore. The bulk of the research on discourse in such "**nontraditional lessons**" (Cazden, 2001) has focused on the description of the communication as truly central. However, the studies often involved only one teacher or a few teachers, and few of them examined whether the so-called higher level, nontraditional talk contributes to students' learning outcome in a direct way (e.g., Leinhardt & Steele, 2005; McClain & Cobb, 2001; Schleppenbach et al., 2007). Also, as some researchers pointed out, the merits of learning-by-talking cannot be taken for granted (Sfrad & Kieran, 2001), because using a particular form of discourse does not necessarily result in a particular kind of instruction and further bring about the desired learning outcomes (Cazden, 2001). Instead, how the discourse is used and constructed, that is, the specific content of the discourse in instruction, may be a more important factor when taking students' academic development into account (Murphy, Wilkinson, Soter, Hennessey, & Alexander, 2009).

In this context, the researcher of the present study had made an initial effort to examine some features of classroom discourse and its relationship with student achievement in the Chinese mathematics classrooms. The pilot study involved 45 lessons of 15 teachers and 932 students. According to Perry and colleagues' definition of "extended discourse"- continued questioning and discussion after an answer was provided (Schleppenbach et al., 2007), the researcher firstly identified the episodes of extended discourse. She then applied Leinhardt's framework of **instructional explanation** to analyze specific content of the identified episodes. These include connection with mathematics and the real situation, use of examples, use of multiple representations, new information building, explanation, and errors identification and address (Leinhardt, 2001, 2010; Leinhardt & Steele, 2005). The results turned out that, first, there existed variability in terms of numbers of extended discourse across the lessons and teachers. In addition, seventy-four percent of the identified episodes of extended discourse were associated with explanation, 10.53% with representations, 8.19% with error identification and 7.02% with use of examples. Secondly, number of extended discourse was not shown to be related to students' mathematical achievement in both basic knowledge and skills and affective outcomes. However, the number of explanation in the identified extended discourse was found to associate with most of the dimensions of students' learning outcome.

These results were consistent with the previous finding that using a particular form of discourse did not necessarily result in a particular kind of instruction and further brought about desired learning outcomes (Sfrad & Kieran, 2001; Cazden, 2001).It is the specific content embedded in extended discourse that makes difference in students' learning and development. Due to the largest percentage of explanation in number of identified extended discourse and its effect on student mathematical achievement in the pilot study, the researcher decided to shift the focus from the frequency of extended discourse to instructional explanation which is embedded in extended discourse. Instructional explanation embedded in extended discourse is considered to provide a legitimate and researchable teaching moment as component. Leinhardt put it this way (2001):

> If we choose one instructional moment to explore carefully, the criteria necessary for identifying a researchable teaching moment must be clear. The researchable teaching moment should be commonly recognizable as being a legitimate part of the instructional landscape; it should be a generally agreed-upon critical aspect of the teaching repertoire-that is, doing it well should matter; it should involve all the major actors in the drama of the class-teachers, content, and students; and it should have the potential to be reflective of differences among subject matter areas, reflective of responsiveness to the unique features of a given student group, and reflective of differences in teaching approaches. It should be a commonplace of the instructional landscape (Leinhardt, 2001, p.338).

Instructional explanation embedded in classroom discourse satisfies all the criteria described above, which will be justified further in the sections below. In addition to its theoretical significance, there is a great need for the investigation of the features of instructional explanation in Chinese mathematics classrooms from a practical perspective.

More specifically, in 2001, mainland China embarked on a major reform of the 9-year compulsory education with the publication of the *Guidelines on Curriculum Reform of the 9-year Compulsory Education* (Ministry of Education, 2001). In comparison with the previous curriculum reforms, the latest one calls for a fundamental change, not only in the methods of teaching but also the way in which students learn (Li & Ni, 2011; Ni, Li, Q., Li, X., & Zhang, 2011). In particular, the aim of this reform is to divert the focus away from the transmission of knowledge by teachers and to move towards the construction of knowledge by students. Specifically, it calls for classroom instruction to provide space for more active participation of students in providing explanations, conducting arguments, and reflecting on and clarifying their thinking. Of the advocated forms of instruction, provision of explanation, especially engaging students in the process of explanation, is given unprecedented emphasis and attention in classroom teaching. Consequently, learning about how instructional explanation is carried out and unfolded over classroom period, what the quality of explanation looks like, as well as what is the relationship between quality of explanation and students' learning outcomes become significant for both educational research and practice.

In the current study, we were interested in exploring what were the features of explanation that might bring about positive effect on students' achievement gain in mathematics, in other words, what features of quality instructional explanation might serve as proximal indices of students' learning outcome. In addition, we were also interested in learning about what was the relationship between teachers' background factors, such as years of teaching experience, mathematical knowledge for teaching, and their use of high-quality explanation in classroom instruction.

In the following sections, the literature on instructional explanation was organized into three parts. Firstly, the researcher discussed the significance of instructional explanation. Next, the criteria for evaluating the quality of instructional explanation in mathematics pedagogy were presented. Finally, the discussion centered on the relationship between teachers' knowledge, belief and their use of instructional explanation.

II the Significance of Instructional Explanation for Classroom Teaching and Learning

1. Explanation and Instructional Explanation

Explanations are ubiquitous and diverse in nature (Keil, 2006). For example, within months of uttering their first words, children ask "why", and by 3 years of age, children can provide explanations for things that happen (Wellman, Hickling, & Schult, 1997). As adults, we may frequently seek the explanations of why the prices are going up steadily, how filial piety is formed in China, and why Bruce Lee is immoral in audiences' hearts. Philosophically, explanation has been understood in a number of ways. For example, Aristotle identified four causes, or "modes of explanation", that pick out different aspects of an answer to a why-question: the efficient, final, formal and material causes (Achinstein, 1983; Lombrozo, 2006; also see Table 1 for examples about the four causes). Most of the scholars understand explanation from a functional perspective. Specifically, they regard explanation as a process of unifying disparate phenomena (Kitcher, 1981), identifying the causal or statistical relevance of mechanisms (Hempel & Oppenheim, 1948; Salmon, 1989), and a merely pragmatic virtue offering a narrative designed to account for an effect (van Fraassen, 1980). It is notable that, although explanations seem to be a large and natural part of our cognitive lives, at the moment, there is neither a satisfying formal account of explanation nor agreement about the important informal criteria for good explanation, producing what one review casts as 'an embarrassment for the philosophy of science' (Newton-Smith, 2000; Keil & Wilson, 2000). However, based on various views, explanation can be generally defined as answers to some sort of actual or implied query (Leinhardt, 2001).

Table 1 Aristotle's Four "Causes" or Modes of Explanation (Lombrozo, 2006, p.465)

Cause or mode of explanation	Description	Example
Efficient	The proximal mechanisms of change	A carpenter is an efficient cause of a bookshelf.
Final	The end, function or goal	Holding books is a final cause of a bookshelf.
Formal	The form or properties that make something what it is	Having shelves is a formal cause of a bookshelf
Material	The substance of which something is constituted.	Wood is a material cause of a bookshelf.

Instructional explanation, distinct from other types of explanation, is deliberately designed for the purpose of teaching, and serves as responses to various queries from teachers, students, or texts in a school subject. It is usually embedded in the teachers' verbalized instructional talk with students (Dagher & Cossman, 1992; Stigler & Hiebert, 1999), it can also be presented in the format of written explanations though (Coleman, Brown, & Rivkin, 1997; Wolfe & Goldman, 2005). As a legitimate part of classroom instructional landscape, there are two kinds of way towards instructional explanation. One is that, instructional explanation is conceived of prefabricated information that is provided by teachers or tutors, and learners passively receive without engagement in the activities of explanation (Renkl, 2002; Cromley & Azevedo, 2005; VanLehn, Siler, Murray, Yamauchi, & Baggett, 2003). Conversely, the other scholars view instructional explanation as a useful vehicle for engaging in meaningful learning. The information in instructional explanation is constructed and worked out by the teacher and the students together (Hardy, Jonen, M¨oller, & Stern, 2006; Klahr & Nigam, 2004). Examples are provided below to illustrate the teacher-provided explanation and teacher-student co-constructed explanation.

Example 1 Instructional explanation provided by the teacher

1. Teacher: **Why 0 does not have a reciprocal?** Firstly, 0 can not be a denominator. Secondly, 0 multiplying any number will get 0. Therefore, 0 does not have a reciprocal. Understand?
2. Students (together): Yes.
3. T: Right. So 0 does not have a reciprocal. (Ms Wu and her fifth-grade students in a dialogue on reciprocal, transcription lines 48-50.)

Example 2 Instructional explanation co-constructed by the teacher and students

1. Teacher: I have a question for you: the reciprocal of 0 is 0?
2. Students (together): Wrong.
3. Teacher: **Why do you say it is wrong**? Tell me the reason. Jia, would you like to say *something?*
4. Jia: 0 can not be a numerator as well as a denominator.
5. Teacher: So your conclusion is what?
6. Students (together): Wrong, the sentence is wrong.
7. Teacher: This sentence is wrong. In other words, 0 does not have reciprocal. Or 0 cannot be denominator. Ok, are there any different opinions? Zhang?
8. Zhang: Because 0 does not have a reciprocal, the sentence is incorrect.
9. Teacher: Yes, so I am asking why 0 does not have a reciprocal. Well, you please.
10. Xia: Because by multiplying any number, 0 will not get 1. Therefore, 0 does not have a reciprocal.
11. Teacher: Yes, she did an excellent job. And she gave an answer according to the concept of reciprocal. That is, if one multiplies the other and their product equals 1, then we say one is the other's reciprocal. We know that $0\times0=?$
12. Students (together): 0.
13. Teacher: It does not equal to 1. So 0 does not have a reciprocal. Do you all understand?
14. Students (together): Yes. (Ms Yan and her fifth-grade students in a dialogue on reciprocal, transcription lines 122-137)

In the current study, instructional explanations (IE) are specifically referred to the explanations that are embedded in teachers' instructional talk with students. They are co-constructed by the teacher and students to respond to various queries in primary mathematics. The explanations are presented in the format of extended discourse, in which a teacher poses continued and follow-up questions after a student provides an answer, and the explanations are finally generated based on an open discussion with a relative clear closure (see the second example above).

2. The Significance of Instructional Explanation for Classroom Teaching and Learning

Instructional explanation is significant in two ways. One is that, explanation is ubiquitous in teaching. The other is that explanation is found to make difference in students' learning.

2.1 Explanation is Ubiquitous in Teaching

Explanation is ubiquitous. It is found that, in naturalistic one-to-one tutoring, 53% of tutor statements and 37% of tutee statements were explanations. Moreover, in constrained tutoring, that is, even when the tutors were explicitly asked not to give explanations, 4% of their utterances were providing explanations, and 23.2% of student statements were explanations generated by themselves (Chi, Siler, Jeong, Yamauchi, & Hausmann, 2001). Explanations are also the core of teachers' lectures in classroom teaching. For example, it has shown that in U.S. classrooms, especially in mathematics instruction, almost all new content was introduced through teacher explanation (Barr, 1988).

Explanation is also prevalent in Chinese classroom teaching. According to Perry's study of explanations of mathematical concepts involving 160 teachers (40 Chinese teachers, 40 Japanese teachers, 80 U.S. teachers) (Perry, 2000), the percentage of activities (e.g., question-and-answer, seatwork, evaluation, explanation, choral responses, mental calculation, and teacher gives directions) that included explanation was 52%, 47% and 40% respectively for the Japanese, Chinese and U.S. fifth-grade classrooms. For the Chinese classrooms, most of the explanations were provided by the teachers, and about one forth was co-constructed by the teachers and the students. Li and Ni (2009) investigated 32 primary mathematics teachers' classrooms in a city of the South China, and they found that 67.65% of teacher questions were posed to request for student-generated explanation and analysis in expert teachers' classes, whereas 35.61% in the novice teachers' classrooms. Also, as indicated in the above section, 71.4% of the identified episodes of extended discourse were associated with instructional explanation which was co-constructed by the teacher and students in our pilot study.

2.2 Linking Instructional Explanation to Students' Learning

Explanation poses advantages in learning. In philosophy, accounts of explanation, no matter how different they may be, agree that explanation is intimately related to understanding, and good explanations succeed in producing understanding (Friedman, 1974; Kitcher, 1989; Trout, 2007). Empirical research in education and cognitive science also suggests that explanation, either students' self-explanation or instructional explanation provided by the others as well as that co-constructed by the teacher and students, play a key role in students' learning (Chi, Bassok, Lewis, Reimann, & Glaser, 1989; Renkl, 2002; Leinhardt & Steele, 2005). For the specifics of the reviewed empirical studies, please see Appendix I.

2.2.1 Self-explanation

Self-explanation refers to the explanation generated to oneself (Chi et al., 1989). Studies show that, when learners were asked to provide explanations to themselves, they learned more effectively. This phenomenon is known as the **self-explanation effect**, initially discovered by Chi and her colleagues (Chi, et al., 1989). In their experimental study, they analyzed the self-generated explanations from talk-aloud protocols that "good" and "poor" students produced while learning procedural skill from examples provided in a physics text. Eight college students, who first studied the prose sections of an introductory physics text, were then asked to explain (on a voluntary basis) whatever they understood from reading a sequence of action statements from three worked-out solution examples. The result showed that, "good" students (*n*=4) who were subsequently more successful at solving problems at the end of the chapter (averaging 82% correct in the posttest) were the ones who spontaneously generated a greater number of self-explanations while studying the examples (15.3 explanations per example). Moreover, these explanations, which were guided by accurate monitoring of students' own understanding and misunderstanding, were refined and expanded the conditions for the action parts of the example solutions, and were related these actions to principles in the text. However, "poor" students (*n*=4) did not monitor their learning accurately, and averaged 46% correct on the posttest and generated only 2.8 explanations per example. The self-explanation effect has been replicated in other laboratories, all in the domain of learning a procedural skill (e.g., applications of principles of electricity and magnetism to the Aston mass spectrometer, Ferguson-Hessler & de Jong, 1990; algebra word problems, Nathan, Mertz, & Ryan, 1994).

Chi and her colleagues extended that finding, showing that self-explanation could also be facilitative when it was explicitly promoted in the context of learning declarative knowledge from biology text on the human circulatory system (Chi, de Leeuw, Chiu, & LaVancher, 1994). In this study, 14 eight-grade student were merely asked to generate explanation after reading each line of a passage, and 10 students in the control group read the same text twice without the prompt to self-explain. All of the students were tested for their knowledge about circulatory system before and after reading the text. The result indicated that the prompted group had a greater gain from the pretest to the posttest. Moreover, within the prompted students, who generated a large number of self-explanations (the higher explainers), learned with greater understanding than those who generated fewer explanations.

Till now, the self-explanation effect has been documented in a broad range of domains, such as conceptual change in five-year-old children's understanding of number conservation (Siegler, 1995; 2002), 3.5-year-old children's acquisition of false belief understanding (Amsterlaw & Wellman, 2006), and college students' improvements in Chinese reading comprehension (e.g., prose learning, Yang & Weng, 2008; Chinese science explanatory texts, Jing & Lu, 2009). The effect has also been reported under various settings, including regular classroom instruction and laboratory environments, as well as reading from text and computer-assisted learning (Cong, 2007; Williams & Lombrozo, 2010; Aleven & Koedinger, 2002; Chi et al., 1994). It was found that the greatest benefit of self-explanation is in transfer and generalization to problems and inferences that require going beyond the material originally studied (e.g., category learning, college students, Williams & Lombrozo, 2010; mathematics learning, college students, Ren, 2008; mathematical problem-solving, grade 9 students, Cong, 2007). In addition, the results from comparative group studies also suggested the advantage of self-explanation on learning in comparison with alternative strategies, such as thinking aloud, reading materials multiple times, or receiving feedback in the absence of explanations (Chi et al., 1994; Wong, Lawson, & Keeves, 2002; Rittle-Johnson, 2006).

It is notable that, although self-explanation makes difference in students' learning, the literature also indicated that learning through self-explanation was not easy to do. Not all students generated explanation spontaneously. There were considerable individual differences in students' ability to self-explain (Chi et al., 1989; Renkl, 1997, 2002). Some studies indicated that few students were good self-explainers (Renkl, Stark, Gruber, & Mandl, 1998). The challenge therefore is to support students who are not inclined to self-explain or are not good at generating effective self-explanation. For this, some researchers turn to investigate instructional explanation and its relation to students' learning (Große & Renkl, 2006; Stark, Kopp, & Fischer, 2011)

2.2.2 Instructional Explanation

A number of studies examined instructional explanation, which is provided for others, and its relationship with students' learning. For this, there were two lines of studies. One was conducted in laboratories, and instructional explanation was provided either by the experimenter or by the computer (Renkl, 2002; Cong, 2007). The others were carried out in classroom. Instructional explanations in these studies were often co-constructed by the teacher and students (Leinhardt & Steele, 2005; Leinhardt, 1993).

A. Instructional Explanation Provided by the Others

The studies conducted in the laboratories were mostly computer-based, where provision of instructional explanation was in the form of written feedback and presented by default. For example, Alexander Renkl in the University of Freiburg, investigated the effect of instructional explanation on students' computer-supported example-based learning (e.g., mathematics), and searched for methods to improve the effect of instructional explanation on students' learning in his laboratory (Renkl, 2002; Schworm & Renkl, 2006; Berthold & Renkl, 2009, 2010; Wittwer, Nuckles, Landmann, & Renkl, 2010). In one of his studies, Renkl examined the effect of instructional explanation on learning from **worked-out examples** from the domain of probability calculation. By worked-out examples, they consist of a problem formulation, solution steps, and the final solution itself. In the study involving 48 student teachers, the participants were assigned into two groups: the experimental group with instructional explanation and the control group without the provision of instructional explanation. Specifically, after all the participants read the worked-out examples, finished the new examples and were provided the complete and correct solution steps sequentially, there was an "Explanation" button installed in the experimental group. The screen of explanation presented the rationale of the solution, such as "the multiplication principle of probability is used to solve the present problem, that is p (A and B) = p (A)*p (B)". In the program version of the control group, it was not possible to read any explanation or help. The result indicated that the experimental group outperformed the control group in **far transfer** (d= 0.57) and the effect did not depend on prior knowledge. There was no significant difference in terms of **near transfer**. By far transfer, it refers to the test that required the participants to solve problems that were not structurally similar to the worked examples in the learning phase. For near transfer, the problem structural was similar between the test and the worked examples.

Most of these experimental studies were conducted based on a well-structured domain, such as mathematics and science (e.g., physics, chemistry, and medicine). And the participants were mainly consisted of college students. On the basis of 21 such studies regarding instructional explanations in example-based learning on computer, Wittwer and Renkl (2010) carried out a meta analysis, and they found that the benefits of instructional explanations for example-based learning per se were minimal (*d*=0.16), and the effect was affected by the other factors, such as type of outcome measure, learning domain, type of instructional explanation and provision manner of instructional explanation. For example, instructional explanations were found more helpful for acquiring conceptual knowledge (*d*=0.36) than for acquiring procedural knowledge, and yielded a significant effect size in studies with mathematics as a learning domain (*d*=0.22).

B. <u>Instructional Explanation Co-constructed by Teacher and Students</u>

The other researchers investigated instructional explanation and students' learning in regular classroom teaching. Distinguished from the experimental studies in laboratories, most of such studies used a descriptive approach.

Gaea Leinhardt of the University of Pittsburgh, known for her "**expert-novice contrast**" approach for research on teaching, conducted series of studies on instructional explanation in mathematics classrooms (Leinhardt, 1987, 1989). Leinhardt selected the teachers for her study whose students had exceeded expected achievement scores in standardized mathematics test for 3 consecutive years. These would be the "experts" to whom she contrasted the "novice" teachers, usually the student teachers who were in their last semester of study program and were actively engaged in student teaching. Then, she conducted meticulous observations in the classrooms of these teachers, recording everything the teachers said and did over long periods of time, in order to establish the rhythms of routine and repeated strategies they employed in their teaching. Leinhardt also interviewed the teachers before and after each videotaped lesson regarding their mathematical knowledge and observational patterns or routines, using such analytic methods as "**semantic nets**" to plot those patterns. Here, semantic nets contain the presented concepts and connections between them across lessons in teacher statement as well as in students' statement, which allow researchers and others to readily "see" what was cognitively available in the lesson as a whole and how all the ideas were interrelated (Leinhardt, 1987). In this manner, she documented the way in which the expert mathematics teachers differed from the novice teachers—in mathematical knowledge, the quality of their explanations, and other aspects of their competence in mathematical pedagogy. For example, Leinhardt found that when teaching functions and graphing, the expert teaches were able to foster meaningful connections between function and graphing while a novice one tended to miss the opportunities (Stein, Baxter, & Leinhardt, 1990). Also, on a sequence of subtraction lessons, the experts explicitly explained to their students the conditions for the use of the concept of regrouping. Conversely, the novice teachers were not able to provide the explanation adequately.

In a recent study, Leinhardt and Steele (2005) traced a 10-lesson unit on functions and their graph taught by Magdalence Lampert (professor of University of Michigan, and mathematics teacher in primary school) to a 5th-grade classroom. They used this trace to help analyze and systematize the complexity of classroom discourse in general and co-constructed instructional explanations in particular. The analysis showed that Lampert's instructional dialogues served two purposes: they developed co-constructed instructional explanations of the key mathematical concepts (e.g., origin, ordered pair, absolute value), and they allowed the class to navigate a meaningful path through the relevant mathematics. Although they clearly did not have enough data on Lampert's students to link her teaching, they offered four examples of student behavior to suggest that the students were learning important information about mathematics. The examples included the students' ability to remember the main ideas from the previous lessons, and to solve all of the public errors. Students were also able to share their insights, emerging reasoning, and justifications. In addition, the students were not only able to give many different reasons in support of an idea but also to do so in collaboration with one another.

Similar to the findings revealed in Leinhardt's study, the co-constructed instructional explanation was found important in terms of its motivational consequences for student learning. Firstly, the process of inquiry, discussion and meaning making fostered students' interest in learning, irrespective of what was a subject matter (Hickey et al, 2001; Favero, Boscolo, Vidotto, & Vicentini, 2007). In contrast, the instruction, that teachers dominated the classroom discourse, and not allowed for critical and independent opinions, arouse anger and anxiety in children, and hence, undermined intensive academic engagement and learning interest (Assor & Kaplan, 2001; Assor, Kaplan, Kanat-Maymon, & Roth, 2005). Also, the interactions between students and teachers in classroom discourse contributed to students' self-efficacy (Pintrich & Schunk, 2002; Nie & Lau, 2010). It also helped students to experience agency and autonomy in their own learning by providing them with more opportunities to explain and justify their ideas, as well as to make choices (Yackel & Cobb, 1996; Whitenack & Yackel, 2002). Finally, the co-constructed instructional explanations are important because they "carry" the overall pedagogical messages of the classroom through both style and stance and because they contain critical elements of legitimacy, modality, and function from the discipline, and thus shape students' deposition towards a subject matter as well as their view of learning the subject matter (Leinhardt & Steele, 2005; Lampert, 1990).

For example, in mathematics classrooms, the teacher or the text authority contributed to the development of students' view of mathematics as predetermined and unarguable truths, which they either did or did not understand, but not the discipline to which they could possibly contribute or to question (Schoenfeld, 1992; Hamm & Perry, 2002). Likewise, the studies also showed that when the teachers shared responsibility and authority by means of explicit negotiation, adaptation, and intervention (e.g., overlapped their speech with students' response), it helped students to learn what counts as a mathematical explanation, as well as what counts as a different, sophisticated and efficient mathematical solution (Forman, Mccormick, & Donato, 1998; Yackel & Cobb, 1996).

Compared to the explanation generated to oneself – self-explanation, the instructional explanation received from the others (e.g., the computer, the experimenter) as well as co-constructed by the teacher and students, revealed potential advantages. One of the advantages is that the instructional explanation is more likely to display knowledge in a correct, complete and coherent form, which makes difference in students' learning. By contrast, students' self-explanation is not always correct, clear, or complete (Renkl, 1997; Leinhardt, 1993, 2001, 2010). Another advantage lies in the role of instructional explanation played in intervention when learners face impasses and cannot generate explanations autonomously to solve problems. This argument was supported by a series of studies (Sanchez, Garcia-Rodicio, & Acuna, 2009; Schworm & Renkl, 2006). For example, Wittwer and Renkl (2010) found that instructional explanation showed a larger effect size in conceptual knowledge and mathematic learning, as both might be more difficult and induce more uncertainty for learners when they were left alone.

Related to this issue, instructional explanation is also helpful when the learners cannot monitor their own comprehension precisely and are not aware of their needs for explanatory support. For instance, Renkl (2002) reported that there were a number of students in the explanatory support condition who did not request instructional explanation as many times, even though they actually needed them. These students had low prior knowledge, and they rarely used the explanatory support or generate sufficient explanation to themselves, which caused detrimental effect on their learning (Chi et al., 1989). Similar results were also arrived at in the study by Aleven and Koedinger (2000). Additionally, many students were also inclined to be miscalibrated with respect to their understanding; that is, they thought they understood in far more detail than they really did. This bias, the "illusion of understanding" also hindered their learning (Keil, 2006; Chi, de Leeuw, Chiu, & LaVancher, 1994). These studies suggest the deficiency of learner's meta-cognitive skills and the potential benefits of instructional explanation in such situations.

2.2.3 Self-explanation and Instructional Explanation

Some empirical studies compared effects of self-explanation and instructional explanation (Große & Renkl, 2006; Cong, 2007; Sánchez, García-Rodicio, & Acuña, 2009). For example, involving 170 student teachers as the sample, Große and Renkl (2006) found that both self-explanation and instructional explanation enhanced students' procedural knowledge in mathematics learning. And the effect did not differ significantly between students' self-explanation and the instruction explanation. However, the instructional explanation yielded a larger effect in learning conceptual knowledge, which asked the learners to discuss the advantage and disadvantages of solution methods and the like. Likewise, based on 165 grade-9 students working on probability problems, Cong (2007) found that self-explanation facilitated students' near transfer and far transfer, but the condition of self-explanation combined with instructional explanation was found more effective. Here, instructional explanation was presented as feedback after students' generation of incorrect self-explanation. Also, study by Schworm and Renkl (2006) revealed that instructional explanation was perceived more helpful by the participants (student teachers); however, it was the self-explanation led to the best learning outcome. What is more, the instructional explanation reduced student teachers' efforts to generate self-explanation and thereby their performance in instructional design of geometry and physics. These studies verified the benefit of self-explanation and instructional explanation on students' learning. Meanwhile, they suggested the potential advantages and disadvantages of both under certain conditions. To be specific, based on the empirical studies mentioned above, it seems beneficial to join self-explanation and instructional explanation. However, the timing issue, i.e., provision of the instructional explanation before or after students' self-explanation (e.g., Schworm & Renkl, 2006; Cong, 2007), becomes a significant determinant of students' learning outcome.

In order to find ways to join self-explanations and instructional explanations in a way that combines their respective advantages, Renkl (2002) proposed what is called "**SEASITE**" (self-explanation activity supplemented by instructional explanation) principles for the design of instructional explanations, including as much self-explanation as possible, as much instructional explanation as necessary, provision of feedback to reduce the learners' illusions of understanding, provision on a learner's demand, progressive help, and focus on principles in terms of the content of instructional explanation.

3. The Mechanism of the Explanation's Effects on Student Learning

Why is the process of explaining helpful for learning, and especially for deep learning: acquiring knowledge and understanding in a way that leads to its retention and use in future contexts? Researchers, motivated by philosophical theories, cognitive science and learning theories, have generated a number of proposals about the mechanisms that underlie explanation's beneficial effects on learning. These include the meta-cognitive benefits of engaging in explanation (such as prompting learners to identify and fill gaps in their knowledge), explanation's integration of new information with existing knowledge, explanation's constructive nature, and its role in dynamically repairing learners' mental models of particular domains (Chi et al., 1994; Chi, 2000; Siegler, 2002; Crowley & Siegler, 1999). Generating explanations may also guide learners to interpret what they are learning in terms of unifying patterns or regularities and promote the discovery of broad generalizations (Friedman, 1974; Kitcher, 1981; Williams & Lombrozo, 2010). Given the diversity of the processes that can underlie learning, it is likely that explanation influences learning via the multiple mechanisms (Nokes & Ohlsson, 2005).

The works above indicate that explanation, either self-explanation or instructional explanation, contributes to students' learning. However, research also suggests that the quality of explanations, rather than the source of explanation (self vs. other), is the key (Rittle-Johnson, 2006). In the following sections, the focus of the literature review turned to the criteria for a good explanation.

III Criteria for the Evaluation of Instructional Explanation in Teaching Mathematics

In this section, the review consists of two parts. First, we focused on the criteria for assessing quality of instructional explanation based on the literature from the perspectives of scientific philosophy, communication, and pedagogy respectively in the western. And then, we reviewed and summarized the distinctive features of instructional explanation that made effective teaching in Chinese mathematics classroom.

1. The Scientific Philosophy Perspective

Carl G. Hempel and Paul Oppenheim (1948) published their seminal essay "Studies in the Logic of Explanation." Almost everything written on the nature of scientific explanation in the following years derives directly or indirectly from that essay. According to their **deductive-nomological model**, an explanation could be divided into two major constituents, the explanandum and the explanans. The **explanandum** is that which is being explained, and the **explanans** is that which does the explaining. That is, the explanandum is the "X" in "Why X?", and the explanans is the (an) answer to the explanation-seeking why question. If a proposed explanation is to be sound, its constituents have to satisfy certain conditions of adequacy, which may be divided into logical and empirical conditions. The **logical conditions** of adequacy include, first, the explanandum must be a logical consequence of the explanans; second, the explanans must contain **general laws**, and these must actually be required for the derivation of the explanandum; and finally, the explanans must have **empirical content**, i.e., it must be capable, at least in principle, of test by experiment or observation. The **empirical condition** of adequacy refers to that the sentences constituting the explanans must be **true**. The explanans has not only to satisfy some conditions of factual correctness, but also to be highly confirmed by all the relevant evidence available. Although their model creates a lot of controversy in the philosophy of science (Salmon, 1989), the criteria they proposed for a sound explanation, especially for the explanans, are significant, and was echoed from the empirical research of effective instructional explanation.

Researchers also identified other important dimensions to guide the evaluation of explanation, including **simplicity, relevance, consilience and coherence** (Keil, 2006; Lombrozo, 2007; Thagard, 1989)**.** For example, it was found that simplicity is used as a basis for evaluating explanations and for assigning prior probabilities when unambiguous probability information is absent (Lombrozo, 2007). Simplicity here means that the explanans should be as small as possible, and the explanandum should be as big as possible. Relevance could be recognized in their simplest and most straightforward forms. In other words, the speakers should be informative, which corresponds to the relation maxim of Grice's Cooperative Principle (Grice, 1975). Additionally, explanations are more appealing when they use diverse forms of evidence for initial causes as William Whewell (1847) argued for the notion of "**consilience**". **Coherence** speaks of the different elements of an explanation working in concert to achieve an internally consistent package. And it has also been defined in terms of constraint satisfaction (Thagard, 2000; Thagard & Verbeurgt, 1998), which means a set of elements is coherent to the extent that each element in a set positively constraints other ones, or be negatively constraining, that is, they contradict or causally block other elements. What is more, coherence is related to a notion of systematicity, the extent to which elements form a tightly interconnected, mutually supporting relational structural. In Trout's view (2007), coherence consists of three features: completeness, plausibility and consistency.

2. The Communication Perspective

Explanations concern transaction and create trajectories (Keil, 2006). They often take place in and take the form of conversation, even within one mind. If explaining is considered a communicative activity, then "explanations are constrained by general rules of conversation" (Hilton, 1990). Based on the **Cooperative Principle**, that is "Make your conversational contribution such as is required, at the stage at which it occurs, by the accepted purpose or direction of the talk exchange in which you are engaged" (p.45), Grice (1975) proposed four conversational maxims. The first is **quality**. That is not saying things that are untrue; e.g., if I need sugar as an ingredient in the cake you are assisting me to make, I do not expect you to hand me salt. The second is **quantity**, which means not saying more or less than is required; e.g., if you are assisting me to mend a car, when I need four screws, I expect you to hand me four, rather than two or six. The third is **relation,** referring to that one does not say things that are extraneous. The last is **manner,** be brief, be orderly, and avoid obscurity and ambiguity. Though Grice is interested in the informal logic underlying everyday conversations, his framework provides important insights into the quality of classroom discourse in general and instructional explanation in particular (Needels, 1988; Forman & Larreamendy-Joerns, 1998; Rodrigues & Thompson, 2010; Nunn, 2006).

Based on Grice's four criteria, Needles (1988) identified six communicative logic variables to indicate the quality of classroom discourse (Table 2). Involving a sample of 10 sixth-grade science teachers and their students, Needles examined the quality of classroom discourse measured with the criteria and its relationship with students' performance. The results showed negative relationships between violations of the communicative logic and high-aptitude students' percentage-correct response on the targeted learning tasks.

Table 2 the Application of Grice's Criteria in Needles' Study

Grice (1975)		**Needles (1988)**
Criteria	Variables	Definitions
Quality	incorrect use of words	Discourse containing self-contradictions or semantically anomalous statements
	incorrect causal relationships	Discourse that reverses a cause-effect relationship or attributes a cause to the wrong factor
Quantity	omission of necessary definitions	The use of a technical word without providing its definition
	omission of causal factors	Mention of an effect without adequately explaining important causal factors or the direction of the effect
Relation	irrelevancy	The presentation of information unrelated to the content being taught
Manner	confusing syntax	Obscure referents, or presented information in an incomplete manner

Both the perspectives of scientific philosophy and communication proposed criteria for the evaluation of explanation. However, it is notable that most of the criteria had little to do with teaching and learning. As a consequence, it might be worthwhile to try to specify what features seem to be characteristic of particularly effective explanation in pedagogy.

3. The Pedagogy Perspective

Here, the pedagogy perspective is specifically in the case of mathematics. As a legitimate part of pedagogy, the effectiveness of instructional explanation depends not only on the principles of explanation from the perspective of scientific philosophy and communication, but also on their fidelity to the essential features of a subject matter and to the learners (Ball, 1993; Lampert, 2001; Shulman & Quinlan, 1997). A large body of research, theoretically and empirically, touches on the distinguishing characteristics of effective instructional explanation.

For example, Gaea Leinhardt, by conducting comparative studies of explanation in mathematics and history, has identified seven aspects with respect to the instructional explanation (Leinhardt & Steele, 2005; Leinhardt, 1993, 2001). First, there is a significant object of inquiry or problem within the domain. Also, the instructional explanation must contain a useful set of examples and non-examples that exemplify the concept being explored. Third, appropriate representations must be available for both teacher and student to use. New information must be built on prior knowledge, intuitions, and inquiries. Fifth, the core principles of the concept must be clearly marked on completion of the explanation. Sixth, the conditions of use, or boundary conditions, for the concept must be established. Finally, the nature of errors must be resolved. Wittwer and Renkl (2008), by focusing exclusively on learning by receiving instructional explanations from a variety of empirical research, proposed four guidelines for conducting the instructional explanations effectively. They are (a) to be adapted to the learner's knowledge prerequisites; (b) to focus on concepts and principles; (c) to take into account the learners' ongoing cognitive activities but (d) not to replace learners' knowledge-construction activities. In addition, Heather Hill, collaborated with Deborah Ball and other researchers, has developed measures of mathematical knowledge for teaching and the mathematical quality of teaching (Hill, Blunk, Charalambous, Lewis, Phelps, Sleep, & Ball, 2008; Hill, Schilling, & Ball, 2004). As a synthesis of these works, the pedagogical criteria of instructional explanation in teaching mathematics are deliberated below.

3.1 Richness of Mathematics in Instructional Explanation

Lots of research indicated that the substantial content embedded in classroom discourse is a much stronger predictor of students' learning when compared with the format and structural of the interaction (Murphy, Wilkinson, Soter, Hennessey, & Alexander, 2009; Perry, 2000). In mathematics class, richness of mathematics, such as teachers' use of mathematical language, emphasis of general concepts and principles (e.g., additional principle in probability), multiple solution methods and multiple representations, allows students to build a conceptual mathematical base as well as connections within and among different components of mathematics.

Teacher's use of **mathematical language** is regarded as both highly variable and also a key feature of the mathematical quality of classroom instruction (Hill et al., 2008). The amount of day-care teachers' math-related talk was found to significantly related to the growth of preschoolers' conventional mathematical knowledge over the school year (Klibanoff, Levine, Huttenlocher, Vasilyeva, & Hedges, 2006). Language constructs meaning for students as they move toward modes of thinking and reasoning characterized by precision, brevity, and logical coherence in mathematics (Marton & Tsui, 2004). Effective teachers share with their students the conventions and meanings associated with mathematical discourse, representations, and forms of argument, so as to fine-tune students' mathematical thinking and enculturate students into the mathematics (Yackel & Cobb, 1996; Wood, 2002; Khisty & Chval, 2002).

Numbers of studies, which used different terminologies, such as **conceptual explanation**, declarative explanation, rule-based and principle-based explanation, found that explanations, focused on general concepts, rules and principles, helped learners move beyond specific learning tasks and acquire the knowledge that can be flexibly applied to new problem-solving situations and, thus, deepen understanding in a knowledge domain (King, 1994; Hohn & Moraes, 1997/1998; Roscoe & Chi, 2007, 2008). For example, Berthold and Renkl (2010) showed that prompts or training for focused processing regarding the central principles and concepts of explanation were especially effective to foster learning outcomes in computer-mediated instructional communication settings. Likewise, Fuchs and her colleagues conducted a classroom-based experiment to explore methods for helping students generate conceptual mathematical explanations during peer-mediated learning activities. The results revealed that, the group that was trained to offer elaborated and conceptual mathematical explanations, asked more participatory, procedural questions (e.g., "what minus what", "can you do that", "what do you need to do"), and provided more conceptual explanations 10 weeks after the training. Moreover, the students of such group performed much better than their peers who were trained to provide explanations that were elaborated but did not touch on the mathematical concepts (Fuchs, Fuchs, Hamlett, Phillips, Karns, & Dutka, 1997; Fuchs, Fuchs, Kams, Hamlett, Dutka, & Katzaroff, 1996).

Also, comparing, reflecting on, and discussing **multiple solution methods** have been a central tenet of the reform pedagogy in mathematics for the past 20 years (Silver, Ghousseini, Gosen, Charalambous, & Strawhun, 2005). The experimental studies indicated that comparing and contrasting alternative solution methods helped students differentiate the important features such as the shortcut step, the efficiency of methods and, hence, facilitate conceptual and procedural knowledge in solving algebra equation and arithmetical problems (Schwartz & Bransford, 1998； Große & Renkl, 2006; Rittle-Johnson & Star, 2007). Furthermore, cross-cultural comparative studies revealed that teachers in high-performing countries such as Japan and Hong Kong often have students produce and discuss multiple solution methods (Stigler & Hiebert, 1999).

Likewise, appropriate **representations** must be available for both teachers and students to use, especially when teaching and learning mathematics or the hard sciences (Leinhardt, 2001). In those domains, the representations can include operational representations or diagrammatic representations (White, 1993). For appropriateness and effectiveness, representations must connect in relevant and explicit ways to the explanations being developed, because representations in and of themselves speak nothing and they may even cause split-source effect which impairs learning if students do not understand the connections (Berthold & Renkl, 2009; Ainsworth & Van Labeke, 2004; Ainsworth, 2006; Sweller, 2005).

Similarly, instructional explanations must contain a useful set of **examples** and nonexamples, which exemplify the concepts being explored. For learning to occur, the examples need to encapsulate a range of critical features and should be unpacked, with the features that make them an example clearly identified. Studies indicated that examples constitute a fundamental part of effective teaching in well-structurald domains such as mathematics (Große & Renkl, 2007; Renkl, 2005; Paas & van Gog, 2006). Examples connect prior information with new information. They can help to demonstrate the legality of a principle or even to show when a concept does not apply; and they can be used to clarify a core query, that is, to help students see exactly what the question is (Rissland, 1991; Atkinson, Derry, Renkl, & Wortham, 2000).

3.2 Tailoring Instructional Explanation to Students' Needs and Engaging Them into the Process of Explanation

For Confucius, there is a famous saying that "I hear and I forget, I see and I remember, I do and I understand". Although rich mathematics contains great resources for students, this is not sufficient. Instructional explanation must tailor to students' needs and further actively engage students into the process of explanation, because the success of instructional explanations depends not only on the sender and his or her ability to provide good explanations but also on the recipient learners (Cohen, Raudenbush, & Ball, 2003; Berthold & Renkl, 2010).

For example, Webb and colleagues found that level of elaboration in explanations provided to tutees affected their learning outcomes. The highest level of explanation received correlated positively with students' arithmetic and mathematical reasoning skills. However, a follow-up constructive activity that student engaged in, such as explaining how to solve problems using concepts stated or implied in the explanations received, showed a larger effect on learning (Webb, Troper, & Fall, 1995; Webb & Mastergeorge, 2003; Webb, Ing, Kersting, & Nemer, 2006). Furthermore, Neber (1995) found that the quality of the explanation itself was not significantly related to students' knowledge acquisition; but it made difference in students' learning when students integrated the information of the explanation into their ongoing problem-solving activities. Consequently, although instructional explanations provide students with learning opportunities, they result in effective learning more likely only if they encourage students' productive engagement (Wittwer & Renkl, 2008).

On this issue, the researchers stress the importance of adaptive instructional explanation in order to meet the students' needs (Wittwer & Renkl, 2008; Wittwer, Nuckles, Landmann, & Renkl, 2010). For a provided explanation to be adaptive, some studies considered the timing issue (explanations presented on learner demand) or the combination of timing and cognitive dissonance (explanation presented after errors or impasses). For instance, Webb and her colleagues suggested that instructional explanation should be relevant to the learners' particular misunderstanding or misconception (Webb, 1989; Webb & Palinscar, 1996). Only then might explanations close gaps in the learners' understanding, remove misconceptions, address errors, and foster connections between new information and prior knowledge (Webb & Mastergeorge, 2003). Additionally, the errors could trigger reflection, self-explanation, and discussion that, in turn, lead to greater procedural flexibility and a better understanding (VanLehn, 1999; Große & Renkl, 2007; Siegler, 2002). The significance of this issue is also evident in the consistent positive effects of the self-explanation on learning (Rittle-Johnson, 2006; Chi, et al., 2001). Because self-explanations are out of the students' needs and carried out by the students themselves; besides, self-explanations are constructed out of the learner's prior knowledge and much more timing; hence, they are adaptive and effective (Renkl, 2002).

Related to adaptive instructional, many studies also suggested the moderate effect of individual factors on the link between classroom discourse and student learning. According to Lubienski (2000a, 2000b, 2002) and Harris and Williams (2011), higher SES students were more likely to participate in the whole-class discussion actively, they expressed confidence in their abilities to make sense of the mathematical discussion and problems, and believed that the discussions exposed them to different mathematical ideas. However, lower SES students tended to say they were confused by conflicting ideas in the discussion. They desired more specific direction from the teacher, and wished the teacher just tell them "the rules" so they could have more time to practice. Also, more of the low SES students said they were unsure of what they were supposed to be learning in open discussion in classroom.

In a study of 118 university students, Große and Renkl (2007) found that, a mixture of correct and incorrect solutions in worked examples fostered far transfer performance only for ''good'' learners who had favorable prior knowledge; if learners had poor prior knowledge, providing correct solutions only was more favorable. Kroesbergen and colleagues (2004) also revealed that recent reforms in mathematics instruction requiring students to construct their own knowledge may not be effective for low-achieving students. In their study, they compared the effects of small group constructivist and explicit mathematics instruction in basic multiplication on low-achieving students' performance and motivation (aged 8-11 years). They found that the math performance of students in the explicit instruction condition improved significantly more than that of students in the constructivist condition.

In addition to SES and prior achievement, sex is another key variable. Some scholars argued that girls were particularly likely to benefit from teaching methods that emphasize students' personal construction of mathematical ideas through problem solving and cooperative group work (e.g., Becker, 1995; Boaler, 1997; Isaacson, 1990). However, a study on elementary school children showed that when solving problems, girls were more likely than boys to use the given, standard algorithms as opposed to invented strategies (Fennema, Carpenter, Jacobs, Franke,

& Levi, 1998).

3.3 Effective Teaching in Chinese Mathematics Classroom

Chinese students have excelled in many international assessments of mathematics achievement. For example, the newly released PISA 2009 showed that students in Shanghai and Hong Kong ranked top 1 and 3 respectively in mathematics among 65 countries and regions. A large number of comparative studies attempted to explore the factors that contribute to the high performances of Chinese students, and they identified some distinguished features characterizing classroom discourse in Chinese classrooms. Specifically, in comparison with their counterparts from U.S., Chinese teachers developed a better understanding of subject matter content, and they were more likely to demonstrate their flexible representation of such understanding in their classrooms (e.g., Ma, 1999). In classroom instruction, Chinese mathematics teachers were shown to be more likely to use explanations that were based on mathematical principles (such as place value or de- and recomposing 10) which are generalizable across problems (Perry, 2000). They focused more on mathematical rules, procedures and reasoning in the extended discourse[8] (Schleppenbach,et al. , 2007). They tended to use a variety of well-presented and carefully sequenced examples to help students acquire concepts and to provide immediate feedback to students (Zhang & Zhou, 2003). They requested more mathematical statements and explanations from their students (Miller, Correa, Sims, Noronha, & Fang, 2005), and placed substantial emphasis on students' errors, and used errors to prompt student discussion of mathematical concepts. While U.S.

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[8] Extended discourse is a particular discourse practice, which refers to questioning and discussion led by the teacher after students provides a correct response. It is essentially an I-R-E sequence that the teacher has extended typically by withholding evaluation of an answer and instead asking the student follow-up questions (Schleppenbach et al., 2007, p.382)

teachers chose to give a direct answer or tended to neglect to address the errors (Schleppenbach, et al., 2007; Wang & Murphy, 2004; Stevenson & Stigler, 1992).

Existing studies also revealed that coherence, mostly defined as logical connection or consistency of the structural of instructional content, is a salient characteristic of mathematics classroom instruction in China (Wang & Murphy, 2004; Stigler & Hiebert, 2004; Stigler & Perry, 1988; Lopez-Real, Leung, & Marton, 2004). For example, using the video data from the TIMSS study, Hiebert et al. (2003) and Leung (2005) pointed out that lessons in Hong Kong are more coherent than those in other countries. In particular, Leung indicated that "90% of the Hong Kong lessons are judged to be thematically coherent, with the remaining 10% moderately thematically coherent". Bryan and colleagues (Bryan, Wang, Perry, Wong, & Cai, 2007) compared and contrasted the similarities and differences of teachers' views of effective mathematics teaching and learning from Australia, Mainland China, Hong Kong, and the United States. They found that, of the four groups of teachers interviewed, only teachers from Mainland China (n=9) and Hong Kong (n=12) explicitly addressed the issue of having a well-structurald, coherent lesson for effective teaching. A typical statement for mainland Chinese teachers is that: "An effective lesson should have all the steps [of instruction] closely serving for the essential content points…so that students can actively participate in each step."

In addition to the highlight of the thematic coherence, a few studies also revealed that the Chinese mathematics teachers emphasized coherence in terms of classroom discourse (Wang & Cai, 2007; Chen & Li, 2010). For example, in their interview of 9 Chinese mathematics teachers, Wang and Cai (2007) found that the interviewee believed that the effective lesson should have a coherent structural, which included following consecutive processes: introducing, explaining, questioning, practicing, and summarizing. Likewise, based on four consecutive lessons from a teacher of a key elementary school in the eastern part of China who had over 10 years of teaching experience, Chen and Li (2010) analyzed the teacher's instruction content and process as well as the teacher's use of classroom discourse. The teacher was found to make explicit transitions from one activity to another, made causal links to previous knowledge both within a single lesson and across the sequence of lessons to help students learn fraction division in a logically progressive way.

The previous works help describe the effective teaching in Chinese mathematics classroom. However, there are some caveats that should be noted. Firstly, most of the results were arrived at based on cross-cultural comparative case studies, and thus, the generalization of the conclusions was limited. Secondly, when conducting the research, the researchers did not make a direct link between the teachers' classroom practice and students' learning outcome. Therefore, the conclusions that labeled those practice as features of effective mathematics instruction in mainland China are questionable. More empirical research is needed to verify those conclusions.

Based on the literature reviewed above, there are two insights regarding the criteria to evaluate the quality of instructional explanation. On the one hand, explanation itself as well as its evaluation is complex, as they are described and perceived in the multiple perspectives. On the other hand, there are communalities about what make good explanations across the different perspectives, which is summarized in Table 3 below.

Table 3 Common Criteria for Good Explanation across the Different Perspectives

Criteria		**Truthfulness**	**Richness**	**Coherence**
Perspective	Scientific Philosophy	The explanans must be true (Hempel & Oppenheim, 1948)	The explanans must contain general laws (Hempel & Oppenheim, 1948)	The explanandum must be a logical consequence of the explanans (Hempel & Oppenheim, 1948); Completeness, plausibility and consistency (Trout, 2007)
	Communication/ Speech Act Theory	Quality (Grice, 1975)	Quantity, relation (Grice, 1975)	Manner (Grice, 1975)
	Pedagogy	Errors identification and correction; correct information in explanation (Leinhardt, 2001, 2010; Renkl, 1997; Siegler, 1995)	Mathematical language, general concepts and principles, multiple solutions and representations (Marton & Tsui, 2004; Webb & Mastergeorge, 2003; Silver et al., 2005; Leinhardt, 2001)	Logical connection or consistency of the structural of instructional content and classroom discourse (Stigler & Hiebert, 2004; Zhang & Zhou, 2003; Chen & Li, 2010)

Based on the works, three criteria including truthfulness, richness, and coherence are identified to evaluate the quality of explanation in mathematics classroom teaching. However, it should be noted that, just like two sides of the same coin, overemphasis of the truthfulness, or richness or coherence may produce undesirable effect under some conditions. For example, it is well-accepted that coherence is a distinguished feature of Chinese mathematics classroom instruction and it contributes to Chinese students' high performance in mathematics. However, the profound coherence may also reduce ambiguity tolerance, a condition for creativity (Lubart, 1999; Mills, 1959), in students' style of thinking. Likewise, several studies also indicated that Chinese students sought accuracy and correctness at the expense of development of their sense of adventure, another condition for creativity. For example, Cai found that Chinese students chose to give up when confronting uncertainty in complex problem solving, while their counterparts, U.S. students, tended to write something on the test, even they did not know the answers to the problems (Cai, 1997, 1998). Likewise, the overemphasis of richness of the mathematical content may take cognitive load to the limits of working memory capacity, and thus hinders student learning (Sweller, 2006; Kalyuga, 2010). Therefore, we should bear the possible caveats of the criteria above in mind.

In the previous sections, we have discussed what makes a good and high-quality instructional explanation. Next, we would turn to the discussion centering on the teacher background factors that may relate to their use of high-quality instructional explanation. In particular, we focused on the factors of teacher knowledge and belief and their effects on instructional explanation.

IV Instructional Explanation and Teacher Knowledge and Belief

It has been generally accepted that teachers' belief and knowledge play an important role in their pedagogy. In this section, we review studies of both elementary and secondary school teachers to gain a broader view of how teachers' belief and knowledge may impact on the teachers' use of instructional explanations.

What a teacher knows impacts on classroom instruction. During the past decades, researchers have spent much time defining the necessary components of the required knowledge base for teaching. For this, Lee Shulman conducted series of seminal works (1986, 1987). In one of his paper entitled "Knowledge and teaching: foundations of the new reform"(1987), he proposed that the teacher knowledge included seven categories: content knowledge, general pedagogical knowledge, curriculum knowledge, pedagogical content knowledge, knowledge of learners and their characteristics; knowledge of educational contexts, and knowledge of educational ends, purposes, and values, and their philosophical and historical grounds. Many researchers have followed his framework and conducted a large body of studies on teacher knowledge, and its relation to teaching (e.g., Gossman, 1990; Bromme, 1994; Ball, 1990, 1991).

Many studies have shown that the quality of instructional explanations is related to teacher content knowledge. Ma's pivotal study (1999) of Chinese and US teachers examined the relationship between the teachers' subject-matter knowledge and their instructional practices, including the quality of explanations. It was found that, Chinese teachers had more in-depth understanding of arithmetic concepts and tended to use explanations that include basic principles to help students make connections between different mathematical topics. However, US teachers who had mostly procedural knowledge used procedural and computational practices in the course of instructional explanation. Likewise, due to the different levelS of mathematical knowledge, Chinese teachers were found to be more capable of providing coherent instruction than their peers in other countries based on the video data from the TIMSS study (Stigler & Perry, 1988; Leung, 2005). Teachers' subject matter knowledge was also found to affect other areas in mathematical pedagogy. For example, the missing of key mathematical ideas of functions and graphing was found to relate to a narrowing of a fifth grade teachers' instruction in three ways: the lack of provision of groundwork for future learning in this area, overemphasis of limited truths, and missed opportunities for fostering meaningful connections between key concepts and representations (Stein, Baxter, & Leinhardt, 1990).

To provide effective teaching, it is also crucial that the teacher has deep understanding of the knowledge in pedagogy, especially the ability to assess and diagnose learners' thinking, capabilities, and understanding (Ball, Thames, & Phelps, 2008; Shepard, 2001; Carpenter, Fennerma, & Franke, 1996). There is evidence suggesting that, more successful teachers possess more knowledge about typical misconceptions that students might have in a certain domain and are better able to monitor students' understanding and adapt their goals and practice for diverse students (Hogan, Rabinowitz, & Craven, 2003).They knew when to "step in and out", provided adaptive explanations, responded constructively and patiently to errors, and showed a sensibility for directing the discussion to ensure that important mathematical ideas were being developed (Ball, 1991; Fennema, Franke, Carpenter, & Carey, 1993; Lampert & Blunk, 1998; Hill, Blunk, Charalambous, Lewis, Phelps, Sleep, & Ball, 2008; Baumert, et al., 2009). On the other hand, less effective teachers, even who had high level of domain knowledge, overestimated students' understanding required to solve algebra problems (Nathan & Petrosino, 2003). To be specific, the teachers tended to view symbolic reasoning and mastery of equations as a necessary prerequisite for word equations and story problem solving, which was in contrast with students' actual performance patterns. Also, they used a rule-based explanation, repeating the rule even when students continued to make mistakes (Tirosh, Even, & Robinson, 1998).

The research also showed consistent associations between teacher beliefs and their practices. The findings indicated that, teachers who believed that students learn mathematics by constructing their own understanding in the process of solving problems, assumed a proactive role in classroom discourse. They actively engaged students in activities that assisted them to construct mathematical concepts, requiring reasoning and creativity, gathering and applying information, discovering, and communicating ideas (Wood, Cobb, & Yackel, 1991; Yackel & Cobb, 1996; Lampert, 1991; Thompson, 1992; Peterson, Fennema, Carpenter, & Loef, 1989; Stipek, Givvin, Salmon, & MacGyvers, 2001). In contrast, teachers who viewed the mathematics as collections of static, objective, and well-structurald knowledge, and knowing mathematics means being skilful and efficient in performing procedures and manipulating symbols without necessarily understanding what they represent, tended to transmit the facts, rules and procedures directly (Stigler & Hiebert, 1997; Thompson, 1992; Wood, Cobb, & Yackel, 1991). With respect to the use of explanation, Putnam (1992) found that a teacher with this belief was more likely to explain a problem to the students by telling them what to do, stressing the steps of a procedure, or stating a rule. The teacher also believed that learning ''why'' would only confuse the students. Although most of the studies were not directly related to instructional explanations, these studies offer important insights into understanding the relationship between teacher belief and their use of explanation in classroom teaching.

Several studies examined teachers' beliefs regarding students' abilities to learn mathematics. For example, Ma (1999) showed that even though a teacher may have conceptual understanding of a topic, she might choose to take a procedural direction when teaching if she perceived that the student was not capable of reaching conceptual understanding. Similarly, Levenson, Tsamir, and Tirosh (2010) investigated sixty-one Israeli elementary school teachers' preferences for mathematically based (MB) and practically based (PB) explanations (Figure 1). It was found that although teachers generated more MB explanations than PB explanations on their own, they chose to use mostly PB explanations in their teaching. Because they believed that PB explanations would be most convincing to their students.

MB Explanation: Using the multiplicative identity

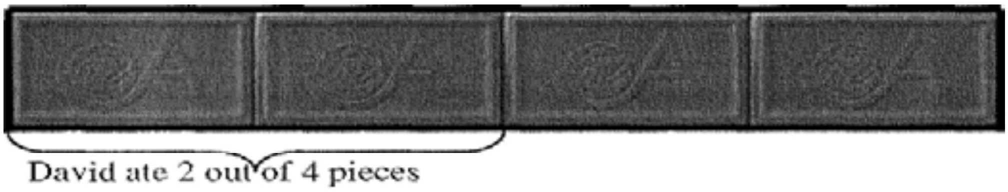
We know that every number times one is equal to itself: 2/4×1 = 2/4. But three-thirds is equal to one whole: 1=3/3. If we multiply 2/4 by 3/3 we get 2/4×3/3 = 6/12

2/4×1 = 2/4 but 2/4×3/3 = 6/12

Therefore, 2/4=6/12

PB Explanation with illustration

Mom gave David and Miri identical chocolate bars. David's bar is divided into 4 equal pieces. David ate 2 pieces.



Miri's bar of chocolate is divided into 12 equal pieces. Miri ate 6 pieces. Each of the children ate the same amount of chocolate. Therefore, 2/4=6/12.

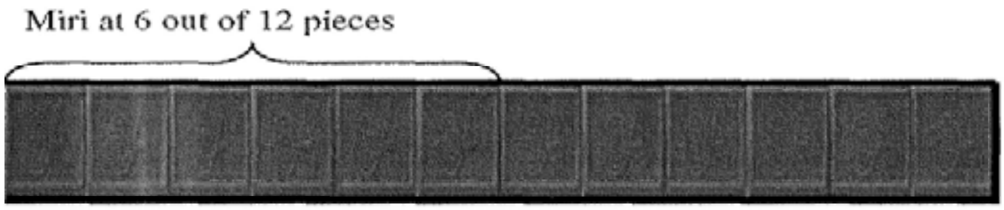


Figure.1 MB and PB Explanations for Why 2/4=6/12 (Levenson, Tsamir, & Tirosh, 2010, p347)

Taken together, the findings suggest that teacher knowledge and beliefs may play an important role in giving effective explanations. Deficits in such pedagogical knowledge and belief might explain why explanations are often not used appropriately and effectively and therefore, hinder students' learning (Chi, et al., 2001; N¨uckles, Wittwer, & Renkl, 2005; Perry, 2000; VanLehn, et al., 2003; Webb & Palinscar, 1996).

The previous works suggest that explanations (self-explanation and instructional explanation) play an important role in students' learning. And researchers have proposed some criteria for the evaluation of good or high-quality of instructional explanation from various perspectives. The findings also indicate that use of instructional explanation is affected by teacher knowledge and belief. These works help understand the instructional explanation's function, its evaluation, as well as the factors influencing its use. However, there are three limitations with this body of research.

Firstly, although explanation is ubiquitous in teaching and it makes difference in students' learning, little is known about the situation of instructional explanation in Chinese mathematics classrooms. For example, what is the feature of instructional explanation in Chinese mathematics classes? How is the quality of instructional explanation? Some cross-cultural comparative studies investigated instructional explanation in Chinese classes. However, most of these studies were based on cases, and the investigation of the explanation focused on the quantity (e.g., frequency, length) or one or two aspects of quality (e.g., connections between knowledge, procedural vs. declarative knowledge) (e.g., Perry, 2000; Ma, 1999; Schleppenbach et al., 2007). More research is needed which should systematically examines both the quantity and quality of instructional explanation in Chinese mathematics classrooms.

Secondly, it is noted that when examining the effect of explanation on learning, most of the empirical studies focused more on the source of explanation (self vs. others vs. co-constructed) than on the quality of explanation (Rittle-Johnson, 2006). Further, when investigating the quality of instructional explanation, the researchers paid much attention to the static and discrete parts of the instructional explanation. In particular, a number of such studies concerned with the effect of the explanation constructed based on procedures or general principles on students' learning (Berthold & Renkl, 2009; King, Staffieri, & Adelgais, 1998). Therefore, more systematic studies are needed to delve more deeply into the interrelations between different aspects of instructional explanation and their relative importance to learning (Wittwer & Renkl, 2008).

Also, it should be noted that most of the experimental studies focused on students' cognitive learning outcomes, and non-cognitive achievement was kept out of the way. This is especially unfortunate because it is clear that both cognitive and motivational factors are relevant to successful learning and thus, research on instructional explanations needs to look into its effect on affective outcome of learning as well (Brown 1992; Berthold & Renkl, 2010).

Furthermore, as mentioned in the previous sections, although the cross-cultural comparative studies found some distinguished features of effective teaching in Chinese mathematics classrooms (Perry; 2000; Schleppenbach et al., 2007). Most of the studies did not make a direct link to students' learning outcome, and thus, future research is needed to fill the gap.

Finally, when exploring the effect of explanation on learning, few studies took the explainers' background information into account, such as knowledge and belief. However, many other studies have shown that previous knowledge not only affected the number of explanations generated, but also the quality of the explanations (e.g., accuracy), which further affected students' performance on their cognitive performance (Chi et al., 1989; Renkl, 2002). Using grade 10 mathematics teachers and students as sample, Baumert and colleagues (2010) also found a substantial positive effect of teachers' pedagogical content knowledge on students' learning gains that was mediated by the provision of cognitive activation and individual learning support. Similarly, the research also indicated that the individual's prior belief had an impact on generating explanations (Lombrozo, 2006). As a consequence, some researchers suggest that the roles of knowledge and belief should not be neglected when examining the relation between explanation and learning (Perry, 2000; VanLehn et al., 2003; Wittwer & Renkl, 2008).

V. Research Questions and Hypothesis

Based on the previous literature review and the noted limitations of the literature, the present study was expected to add to existing research in the following ways. Firstly, it would contribute to the understanding of the feature of instructional explanation in primary mathematics class in mainland China. Secondly, it would provide new evidence concerning the relationship between instructional explanation and students' cognitive and affective learning outcomes in mathematics. Thirdly, it would address the relationship between the quality of instructional explanation and teachers' mathematical knowledge for teaching and their views of mathematics.

The conceptual framework of the study is presented in Figure 2.

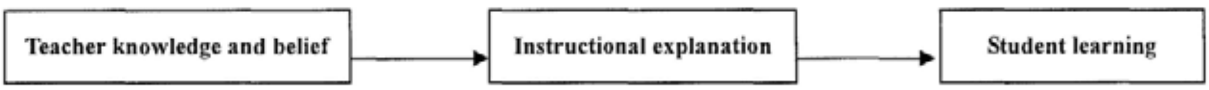


Figure2. Relationship between Teacher Knowledge and Belief, Instructional Explanation and Student Learning

More specifically, the relationship between instructional explanation and student achievement is depicted in Figure 3 below.

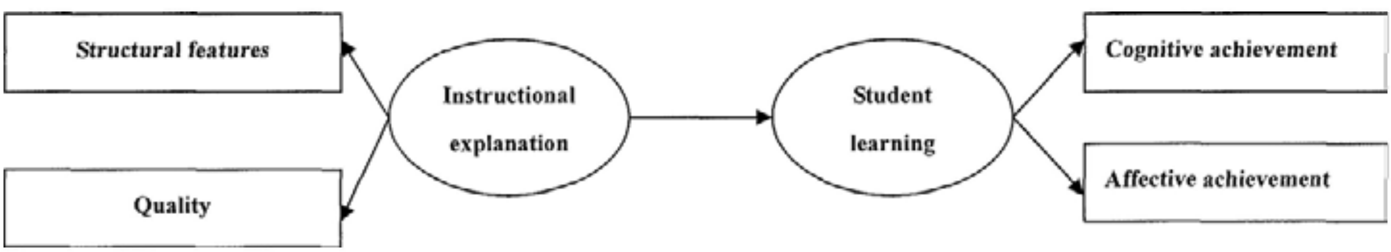


Figure 3. Relationship between Instructional Explanation and Student Learning.

In Figure 3, the structural features of instructional explanation refers to the number of identified episode of IE, the number of turns, and the number of students involved in an episode of IE. Quality of instructional explanation refers to the three evaluation criteria: truthfulness, richness, and coherence of instructional explanation (see Appendix III for the evaluation criteria). Students' cognitive achievement consists of the skills of mathematical calculations, the skills of carrying out mathematical explanations and communications. Affective outcomes include self-reported interest and dispositions towards learning mathematics. The specifics about these variables are explained in the methodology section.

The research questions are as follows:

1. What would be the cognitive features of instructional explanation in the primary mathematics classrooms in mainland China? More specifically, how often would the instructional explanation occur in the primary mathematics classrooms? What would be the structural features of the identified instructional explanation episodes (e.g., the number of turns, students involved in an episode of instructional explanation)? How would the quality of teacher instructional explanation in terms of the evaluation criteria?

2. What would be the relationship between the instructional explanation and students' cognitive and affective learning outcomes in mathematics? To be specific, how would the structural indicators of instructional explanation (e.g., the number of episodes of instructional explanation, the number of turns as well as students contributed to one episode of instructional explanation) affect students' cognitive achievement and attitude towards mathematics? How would the quality of instructional explanation influence students' achievement in mathematics? Would the effects be consistent or differentiated across students'

cognitive and affective learning outcomes? Would the effects be moderated by students' SES and prior achievement?

3. What background factors (e.g., curriculum in use, years of teaching experience, years of implementing the new curriculum, and educational level) might account for the variation in the use of instructional explanation by the teachers? How would the teachers' mathematical knowledge for teaching and their views of mathematics affect their use of instructional explanation in terms of both quantity and quality?

Based on the reviewed literature, the following hypotheses were postulated.

1. The cognitive features of instructional explanation

The features of instructional explanation include two parts: quantity and quality. For the quantity, it refers to the structural features such as the number of identified episodes of IE, the number of turns and the number of students involved in an episode. The number of turn is the unit of a successive series of exchanges. And it refers to the sequence of words or actions by a group member bracketed by the words of another group member(s). Turns unaccompanied by words (e.g., writing "3 × 40" were also counted as speaker turns (Chiu, 2008). Here, both turns and students involved were begun to count as the teacher or the student asked explanation-seeking question (e.g., "why", "how do you think about that"). Quality of IE refers to the three evaluation criteria: truthfulness, richness, and coherence.

Hypothesis 1-1 Empirical studies indicated that, Chinese mathematics teachers developed a better understanding of subject matter content, and they were more capable of organizing instructional content in a coherent manner (Ma, 1999; Stigler

& Hiebert, 2004; Stigler & Perry, 1988). Besides, Chinese teachers placed emphasis on mathematical richness in classroom instruction. They focused on mathematical rules, procedures and reasoning, and were more likely to ask students to produce and discuss multiple solution methods (Schleppenbach,et al. , 2007; Stigler & Hiebert, 1999). However, studies also revealed that some Chinese teachers focused on the diversity of solution methods, but without paying enough attention to the optimization of a solution (Yu, 2003; Huang, 2005). Consequently, it was expected that the Chinese teachers would score higher on the dimension of truthfulness and coherence, compared to their scores on the dimensions of richness. Their performance on the dimension of richness would be more differentiated between the teachers.

Hypothesis 1-2 It is expected that the structural indicators of instructional explanation would correlate to the quality indicators. In particular, numbers of turns and numbers of students contributed to an episode would positively correlate to the dimension of richness and coherence of instructional explanation. For example, the more turns and students involved in an episode, the more likely that teacher would use more mathematical language and pose more responsive follow-up questions.

2. Relationship between instructional explanation and students' learning

Students' learning refers to the cognitive and affective learning outcomes in mathematics. Cognitive learning outcome included calculation, simple problem solving, and complex problem solving. Affective learning outcome contained self-reported learning interest, classroom participation, views of mathematics, and views of learning mathematics.

Hypothesis 2-1 It was expected that the structural features of instructional explanation (e.g., turns, students involved in an episode), would positively correlate to students' cognitive and affective learning outcomes. For example, the number of students involved in the IE presumably would affect students' expressed interest in learning mathematics, classroom participation, views of mathematics and views of learning mathematics.

Hypothesis 2-2 The measured quality of instructional explanation was expected to associate with students' cognitive achievement (Atkinson, 2002; Stark et al., 2011). Additionally, the truthfulness, richness, and coherence of IE was expected to positively correlate to the students' affective outcomes, such as learning interest, classroom participation, views of mathematics, and views of learning mathematics (Leinhardt & Steele, 2005).

Hypothesis 2-3 We assumed that the effect of instructional explanation on student learning would be moderated by students' SES as well as by prior achievement. According to the previous research, high SES students were more likely to participate in the classroom discussion. They benefited more from the discussion of different ideas (Lubienski, 2000; Harris & Williams, 2011). Also, in a study of university students, Große and Renkl (2007) found that, a mixture of correct and incorrect solutions in worked examples fostered far transfer performance only for ''good'' learners who had favorable prior knowledge; if learners had poor prior knowledge, providing correct solutions only was more favorable.

3. Relationship between background factors and instructional explanation

Hypothesis 3-1 Curriculum in use would affect the features of the identified episode of instructional explanation. Specifically, because the new curriculum has encouraged classroom instruction to engage more students in the classroom discourse, the classes using the new curriculum were expected to have more turns and students involved in the instructional explanation. Out of the same reason, since the new curriculum has advocated multiple solutions and multiple representations, the reform classes would score higher on the dimension of richness. However, the curriculum in use would not affect the truthfulness (accuracy) and coherence of instructional explanation. Years of implementing the reform, would correlate positively to the quantity of instructional explanation, such as turns and students involved in an identified episode. Additionally, teachers' experience of implementing the reform would contribute to their quality of instructional explanation. It was because the longer the teachers involved in using the new curriculum, the more likely they would lead an open discussion and engage more students in the construction of the explanation. Years of teaching experience was expected to affect the dimension of truthfulness. The experienced teacher would be more able to anticipate, identify and address the students' errors and misconceptions.

Hypothesis 3-2 Teachers' mathematical knowledge for teaching affected their use of instructional explanation in both structural features and quality. Specifically, the teachers, who had a better understanding of mathematical knowledge, would be more likely to co-construct the explanation with students, elaborate sufficiently and engage more students in explanation for a specific query. They would also score higher on the scale of the truthfulness, richness, and coherence of instructional explanation.

Hypothesis 3-3 Teachers' belief of mathematics influenced the use of instructional explanation. Teachers, who held an open view towards mathematics, would be inclined to generate the explanation with students, would encourage more students' participation in the discussion. In addition, these teachers were more inclined to construct rich and coherent explanations with their students. There was an expectation that teachers' belief of mathematics would not affect the truthfulness of instructional explanation.

Chapter 3 Methodology

1. Study Sample

The data source of the current study was from the project "Has curriculum reform made a difference? Looking for change in classroom practice" (Ni, Li, Cai, & Hau, 2009). The participants of the project included 58 fifth-grade teachers and their 3,415 students from 20 schools, in a city of central China. There were 32 teachers with 1,959 students who used a reformed curriculum, and 26 teachers with 1,456 students who used a conventional curriculum. All the teachers were video taped in three consecutive lessons, except three teachers who had two lessons only, and thus 171 lessons were generated, including 96 lessons with the new curriculum and 75 lessons with the conventional curriculum. The instructional content of the teachers contained both number and geometry.

For the current study, 39 teachers (20 reform teachers VS. 19 non-reform teachers) and their 2,239 students (male, 1,237; female, 1,002), were included. The teachers were selected based on two criteria: firstly, all these teachers had lessons teaching new knowledge involving only with the content of number, such as division of fraction, divisor, multiple, and prime numbers. This helps to minimize the impact of instructional content on the teachers' practice. Secondly, all the teachers had complete videotapes of no less than two lessons, which guaranteed the representativeness of the teachers' performance of the classroom practice. The background information of the 39 teachers was provided in Appendix II.

2. Measures

The measures included three major sections. One was for assessing students' learning outcomes. The second part measured teacher's mathematical knowledge for teaching as well as their belief of mathematics. The third part was for evaluating the quality of instructional explanation. The measures of student learning outcomes and teacher backgrounds were done in the project "Has curriculum reform make a difference" (Ni, et al., 2009). The part for assessing the quality of instructional explanation was carried out in the present study. The measures are described below.

2.1 Measures of Student Learning Outcome

It was conceptualized that student learning outcomes in mathematics as including the cognitive aspects such as the skills of mathematical calculations, the skills of carrying out mathematical explanations and communications, and the affective aspects as interest in and dispositions towards learning mathematics.

It is noteworthy that the measures of students' mathematics achievement were administered to the students on three occasions over a period of 18 months. The first administration took place in the first term of fifth grade, the second at the beginning of sixth grade and the last one at the end of sixth grade. In the current study, the first time achievement were controlled for as the students' prior knowledge, and the third time performance were treated as the dependent variables.

2.1.1 Cognitive Measures of Mathematics Achievement.

The cognitive measures of mathematics achievement contained the three parts: calculation, simple problem solving and complex problem solving. The first two parts, containing all MC questions, were developed based on the four cognitive processes identified by Mayer (1987; 2003) that are involved in solving math word problems. These are: translation, converting word sentence into a numerical representation of the described situation; integration, selecting and combining information into a coherent representation of the given problem; planning, breaking down the problem to be solved by steps; and execution, carrying out mathematical operations. There were a total of 32 multiple-choice questions, six items for each of the first three dimensions and 14 items for the execution (calculation) dimension. The items of the first three dimensions were grouped as routine or simple problem solving and those of the last dimension as calculation. The classification was done based on the understanding that the items for execution merely required calculations, whereas the items for the other three dimensions called for interpreting the problems or indicating steps for getting answers to the problems but no any calculation was required. These items were intended to assess the Two Basics, that is, basic mathematical concepts and basic mathematical calculations.

Three separate common factor analyses were conducted on the student responses from three administrations of the measures to examine the assumed dimensions (Ni, et al. 2011). Except for the dimension of planning whose items showed low loadings on any factors, a relative clear structural was shown for the other items. Across the three data points (the students were assessed for three times), the total variances explained by the 4-6 factors ranged from 40-48%, and communities from 10% to 12%, which was reasonable. Based on the results, all the items were retained except for the planning items for the present study. The retained items were grouped into the two categories, calculation and routine problem solving as explained above.

For the part of complex problem solving, a total of 12 open-ended tasks were developed. In responding to the open-ended tasks, students were required to show their solution processes and provide justifications for their answers. An example of open-ended question is as this "Ming and Fang, high school students, take a part-time job. Ming earns 15 RMB per day and Fang earns 10 RMB per day. 1) How many days do Ming and Fang have to work respectively so that they will earn the same amount of money? Show how you found your answer. 2) This problem has more than one answer. Find another answer and explain." Detailed description of the open-ended problems can be found in Cai (1995, 2000).

The responses to the open-ended questions were evaluated with the scoring rubric of 0-4 point scale, not acceptable (0), minimal (1), satisfactory (2), good (3) and excellent (4). The raters included elementary school mathematics teachers and graduate students in educational psychology or in mathematics. Two raters independently scored five percents of the student responses. The inter-rater agreements were 0.876, 0.891, and 0.880 respectively for the three administrations of the open-ended questions (Ni, et al., 2011).

The correlations between the three measures of mathematics achievement, calculation, simple problem solving, and complex problem solving, ranged from 0.42 to 0.46 for the first time, and from 0.40 to 0.43 for the third time. The correlations between the three times of assessment were 0.31-0.34 for the measure of computation, 0.32-0.37 for simple problem solving, and 0.50-0.58 for complex problem solving (Ni, et al., 2011).

2.1.2 Affective Measures of Mathematics Achievement.

Four facets of affective mathematics achievement (Li, 2004) included 1) students' perceived interest in mathematics (e.g. "Mathematics interests me because I find it stimulating to solve a math problem."), 2) students' perceived participation in math classroom (e.g., "I feel anxious when I sit in a math class."), 3) students' ideas about what mathematics is about (e.g., "Mathematics is about numbers and their computations."), and 4) students' views of learning mathematics as a process of reasoning and reflection (e.g., "I sometimes stick to my own thoughts even though my thoughts were wrong because of the fear to reveal my weakness."). The questionnaire included 35 items to measure the four aspects. It used a likert scale of five levels, "strongly disagree," disagree," "not sure," "agree," and strongly agree." For the three administrations of the questionnaire with the sample of students, Alpha for the scale of interest ranged from 0.87 to 0.92; for classroom participation from 0.80-0.85; for views of mathematics from 0.65-0.72; and for views of learning mathematics from 0.61 to 0.70. The test-retest correlations for the four scales were 0.56-0.70 for the scale of interest, 0.57-0.68 for classroom participation, 0.48-0.59 for views of mathematics, and 0.48-0.58 for views of learning mathematics (Ni, et al., 2011). The correlations between the four scales of mathematics achievement, interest, classroom participation, views of mathematics, and views of learning mathematics ranged from 0.31 to 0.59 for the first time and from 0.48 to 0.62 for the third time.

2.1.3 Measure of Student Family Socioeconomic Status.

The students' parents were asked to complete a questionnaire which required information about the family's social economic status. The indicators of the social economic status included family income, father and mother's educational levels, and father and mother's occupations. The index values for the indicated occupations by the parents were calculated according to the *Standard International Socio-economic Index of Occupational Status* (Ganzeboom, De Graaf, Treiman, & De Leeuw, 1992). Through a factor analysis, the parents' income, occupations, education levels were aggregated into the latent variable, SES, which was standardized to be used in the data analyses (Ni, et al., 2011).

2.2 Measures of Teachers' Knowledge and Belief

The teacher measures were consisted of three parts, that is, teachers' mathematical knowledge for teaching, teachers' belief of mathematics, and their demographic information.

2.2.1 Measure of Teacher Mathematical Knowledge for Teaching

The teachers were administrated the Measures of Teachers' Mathematics Knowledge for Teaching, which was originally developed by Hill, Schilling and Ball (2004). There are 14 multiple choice items related to mathematics knowledge and pedagogy. The Alphas for the two aspects were ranged from 0.71 to 0.84 (Ni, et al., 2009).

2.2.2 Measure of Teacher Belief towards Mathematics

Four items were used to measure how the teachers viewed about mathematics. They are 1) "The domain of mathematics has been changing and developing", 2) "The mathematical knowledge is always decided, and the answers to the question are predetermined", 3) "Mathematics is calculation and computation with a set of number and symbol", and 4) "Doing mathematics is carried out step by step based on the logic". The Alpha coefficient is 0.68. All the items used a 5-point Likert scale ranging from 1(strongly disagree) to 5 (strongly agree).

2.2.3 Measure of Teacher Background Information

Six items were employed to examine the teacher's demographic information, including sex, years of teaching, years of implementing the new curriculum, years of teaching mathematics, educational level, and the major of an undergraduate of associate degree.

2.3. Measures of Quality of Instructional Explanation

2.3.1. Identifying Episodes of Instructional Explanation

A. <u>How to Define an Episode of Instructional Explanation</u>

Instructional explanations occur in instructional episodes. Technically, it is necessary to identify episodes of instruction in order to locate instructional explanation in time and space. We called this as episodes of IE.

We referred to any exchanges on an identified content involving instructional explanation as an *episode*, which should not be confused with the term "*turns*" that we will explain later. To qualify as an episode of IE, a teaching episode must satisfy two conditions. First, the episode must have begun with a teacher-initiated question or with a student-initiated question that is accepted by the class for a continued discussion, and the episode focuses on explanation-seeking for the question by the teacher and the students. Second, the episode is presented with the format of extended discourse--continued questioning and discussion after an initial answer is provided. More specifically, a student's answer to the question may serve as the beginning to a large discussion about specific mathematical concepts, algorithms, principles, rules, and reasoning needed to the initial question (Schleppenbach et al., 2007). Therefore, an episode must contain at least two substantive student responses to the initial explanation-seeking question.

Take three episodes for example below to illustrate what counted as an episode of instructional explanation in the present study. In Example 1, an explanation was requested for providing the rationale for an answer. The teacher and her students were discussing the least common multiple. Student 32 answered that "the least common multiple of 4 and 5 was 5". The teacher withheld evaluation and instead asked the follow-up questions to work out an explanation. In Example 2, the explanation focused on understanding concepts, terms, or theorems. Here, the teacher and the students were trying to understand "what does it mean by "一律八折"? Like Example 1, the teacher raised a question, followed up, and finally generated an explanation with her students together. In both examples, there is a focus query, and the query is unfolded in the format of extended discourse. Therefore, example 1 and 2 were counted as an episode of instructional explanation. In contrast, in Example 3, the teacher initiated the question—"why the reciprocal of 1 is 1?". However, the discourse was not substantially extended after the student Cai provided an explanation. The condition of the extended discourse was not satisfied, that is, to contain at least two substantive student responses, and thus this was not counted as an episode of IE.

Example 1 Providing the Rationale for an Answer

1. Teacher: What is the least common multiple of 4 and 5?
2. Student 32: Their least common multiple is 5.
3. Teacher: **Could you explain that**?
4. Student 32: Oh, I made a mistake. It should be 20. Because they are relatively-prime relationship. Their product is 20, so it should be 20.
5. Teacher: What is the relationship between these two numbers?
6. Students (together): Relatively-prime.
7. Teacher: Relatively-prime relationship. So their least common multiple equals to these two numbers' ……?
8. Students (together): Product.
9. Teacher: 4 multiplied 5 equals to what?
10. Students (together): 20. (Ms Li and her fifth graders in a dialogue on least common multiple number, line 174-183)

Example 2 Understanding Concepts, Terms, or Theorems

1. Teacher: Ok, the textbook tells us that all have been marked down by 20% (一律八折). **What does it mean by "一律八折"?** Wang?
2. Wang: It means all clothes were 20 percent off.
3. Teacher: You please?
4. Student 9: It means that the goods, which originally worth 10RMB, is 8RMB now after the 20 percent off.
5. Teacher: 20 percent off from what?
6. Student 9: It was 20 percent off the original price.
7. Teacher: How can we express 20 percent off either? Can you use another way?
8. Student 9: We can also use 8/10.
9. Teacher: 8/10 of what?
10. Student 9: The current price is 8/10 of the original price.
11. Teacher: So 20 percent off is based on what?
12. Student 9: Based on the original price.
13. Teacher: Right. The current price is based on 8/10 of the original price. (Ms Wang and her fifth-grade student in a dialogue on fraction division, transcription lines 79-91)

Counter-example 3

1. Teacher: Why the reciprocal of 1 is 1? Give a reason please, Cai?
2. Cai：Because the reciprocal of 1 is 1/1，1/1=1.
3. Teacher: Yes, all understand?
4. Student (together): Yes. (Ms Yan and her fifth-grade students in a dialogue on reciprocal, transcription lines 117-121.)

In the current study, instructional explanation included giving mathematical meaning to terms, ideas or procedures, meaning of steps, or solution methods, such as how a term be understood, why a procedure works, why a solution method makes sense, and why an answer is true or false. In most cases, the explanation-seeking questions were posed, as in, "Can you explain that?" or "Why do you do that?" or "What do you think?" or "Tell me the reason please." or "How could you explain it?" or "Why you think it is false?" or "What do you mean?" or " What does it mean?". The mere descriptions of steps (first I did x, then I did y) or simply providing definitions without attached meaning were not be counted as episodes of instructional explanation.

B. How to Determine the Beginning and the Ending of an Episode

The end of an episode may be marked differently, depending on an initial question asked as well as the specific purpose of a context. Mostly, an episode was indicated in end when the aim of the explanation had been realized and the teacher moved to another topic. In general, an episode of IE began only after a student gave a complete answer, which was followed with the explanation-seeking question. The criteria for identifying an episode of instructional explanation were designed to capture the extended discussion about explanation to one question, not all interactions between the teacher and students.

It is important to note that the kind of classroom discourse examined in this study was mainly verbal, spoken discourse. However, students' written answers and the teacher's written responses on the black-board would also be included in the study of IE, since the frequency of that situation was high in Chinese mathematics classes. Moreover, the students' written answers on the board made an important part to know about students' mathematical thinking, and provided a good learning opportunity for teachers and students to discuss. Also, the teacher's written responses explicitly reinforced the "important", "difficult", and "hinge" of the teaching materials.

2.3.2. Developing an Instrument to Evaluate the Quality of the Instructional Explanation

Three dimensions were identified for the evaluation of the quality of instructional explanation. They were truthfulness, richness, and coherence. Their definitions and intentions are provided below respectively. Subsequently, the criteria and the items for each dimension have been developed that are explained afterwards.

A. Items Development

We started our work for high-quality instructional explanation in primary mathematics classroom in China. For this purpose, three sources had been depended on when we were developing the items, including the existing literature, the existing scales or observational protocol of similar nature, and classroom transcription materials from the videotaped lessons of the 39 teachers.

As discussed in the previous sections, a converging set of criteria has been derived from the perspective of scientific philosophy, communication, and pedagogy. We have particularly referred to a number of literatures on instructional explanations in teaching and learning mathematics (Leinhardt & Steele, 2005; Stein & Kucan, 2010; Wittwer & Renkle, 2008), quality of instruction (Hill, et al., 2008), effective teaching (Muijs & Reynolds, 2001), and productive discussion in the subject matter of mathematics (Stein, Engle, Smith, & Hughes, 2008), as well as authentic instruction in general (Newmann & Wehlage, 1993; Newmann, et al., 1996). The purpose was to identify the research-based indices for the scale of truthfulness, richness, and coherence to evaluate IE in mathematics. Also, as education is embedded in the culture, we also referred to the literature with respect to effective teaching in the Chinese mathematics classrooms. Most of the reviewed studies were conducted under the cross-cultural settings. The results helped us to capture the unique features of effective pedagogy in mathematics teaching and learning in China.

We have also searched several standardized protocols for observing the characteristics of instruction (or videotaped records of instruction). Most of them were designed for the mathematics and science classes. For example, the Coding Rubric for Measuring the Mathematics Quality of Instruction (Hill, Ball, Bass, & Schilling, 2006; Hill, 2010), the Reformed Teaching Observation Protocol (RTOP; Sawada & Pilburn, 2000), Inside the Classroom Observation and Analytic Protocol (Horizon Research, 2000), MCC Classroom Observation Protocol for Mathematics, the Classroom Assessment Scoring System for Secondary Settings (CLASS-S), and Protocol for Language Arts Teaching Observation (PLATO). These instruments, in complementary to the literature as described above, have helped us to cross-validate which key index makes a high-quality classroom instruction in general and instructional explanation in particular. Notably, of these instruments, the work of Hill and her colleagues seemed much more plausible for our reference in two ways (Hill et al., 2008). First, the measure has been designed especially for the mathematics quality of instruction, which is closer to our purpose in the current study. And secondly, the covered content, as well as the definition of each indicator, are more specific and detailed, which helps to decrease the bias when coding segments of the transcription materials. For example, the measure includes some of the items about richness of mathematics, working with students and mathematics, errors and imprecision, as well as student participating in meaning-making and reasoning. Under each dimension, the intention is specified, and items are developed accordingly with well-defined criteria for rating "low", "mid", and "high".

However, the instrument for the current study is distinguished from Hill's works in some ways. More specifically, in this study, the instrument was developed to describe and evaluate the quality of instructional explanation. However, the Mathematical Quality of Instruction (MQI, Hill et al., 2006) instrument is intended to capture a range of teacher work with mathematical content, curriculum materials, and students. MQI consists of five sections, including instructional formats and content, knowledge of mathematical terrain of enacted lesson, use of mathematics with students, mathematical features of the curriculum and the teacher's guide, and use of mathematics to teach equitably. Therefore, the content of MQI is much broad, when compared to the specific focus on IE of the scale developed and used in the current study. Further, in choosing the five sections, Hill and colleagues aimed to not only evaluate the mathematical quality of a lesson, but also to provide information on the factors that might affect mathematical quality, including particularly the mathematical content and curriculum materials with which teacher were working. Obviously, for the present study, the latter purpose of Hill and her teams was not in the consideration in developing the instrument. Accordingly, the related dimensions and items were excluded from the scale for the present study.

Finally, we also studied the transcription materials from the videotapes of the teachers' lessons carefully and purposely. The materials not only contributed to revealing whether the indices from the literature were appropriate and effective to reflect and distinguish levels of the quality of instructional explanation among the teachers, but also pointed to some important issues to be resolved, concerning the content and validity of the present scale.

Built on these works, 11 items were being developed for evaluating the quality of instructional explanation. Specifically, 3 items were identified for the scale of truthfulness, 4 items for that of richness, and 4 items for that of coherence. It is worthwhile to point out again that most of these items were well established from the existing literature regarding effective mathematics teaching and learning, and they were applicable to most of mathematics classrooms.

Following that, our focus then turned to specifying the rating criteria for each item. When developing the rating criteria, several considerations were taken into account. First, the rating criteria were expected to capture the difference among individual teachers to the largest extent. Second, the differences captured by the rating criteria were expected to be significant for mathematical teaching and learning. For example, whether a teacher uses various mathematical languages for conveying content, and whether a teacher engages the students in the use of mathematical language, should make difference in student learning. And third, the rating criteria across the items were expected to be consistent. All the items use a 4-point Likert scale ranging from 1 (low) to 4 (high). In general, 1 point implies that the teacher does not perform well on his/her own, and he/she also does not engage students into the concerned learning process. A 2-point indicates that the teacher does either aspect. A 3-point implies that the teacher does both. And a 4-point refers to high performance in both aspects. The instrument is attached in Appendix III.

B. <u>Pilot Study</u>

We were interested in identifying the proximal factors that would affect students' mathematics achievement, particularly mathematics problem solving. Accordingly, based on the change in their students' performance in solving open-ended problems and the affective measures from the first to the third time of the assessment, 8 teachers were selected for a pilot study. Notably, all the students of the 8 classes changed positively on the measure of complex problem solving. In particular, the improvement of 4 classes was above the average level of the 29 classes, and the other 4 classes were below the average level. Among the former, two of the classes showed improvement on the affective measures, whereas the other two had decreased scores on the measures. The pattern was the same for the other group of the 4 classes. Thus, 8 classes were divided into 4 groups accordingly. By contrasting the quality of instructional explanations from the 8 teachers, we piloted the instrument to see how it worked.

The quality of the instrument was examined. We requested 4 experts in mathematics education for comments on the instrument. Inter-rater agreement was obtained on identifying IE episodes and ratings the quality of these episodes respectively. To begin, the researcher identified all the episodes of IE. Another independent coder then examined 25% of these episodes. In this case, the independent coder examined 87 of the 346 episodes. Simple agreement between coders for the identification of episodes was .96.

To determine the reliability of ratings on the quality of the instructional explanation episodes, the researcher invited 3 fifth-grade mathematics teachers and 1 Ph. D student of educational psychology. They participated in a full-day workshop for 3 consecutive Saturdays in which the researcher trained them to use the scale. At the last day of the workshop, the researcher and the teachers coded 48 episodes independently. In order to avoid the sequential effect, the episodes were arranged in different orders for the raters. The inter-rater agreements were 70.83%-100% for the ratings of the episodes on the 11 items of the instrument. Intra-rater reliability was also computed after 1 week. It was conducted on the basis of 87 episodes, including 47 episodes from the reform classroom and 40 episodes from the non-reform classroom. The episodes were selected randomly from both groups, and arranged in different orders. The intra-rater agreements were 90%-99%[9].

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[9] Notably, we made revisions in the 4 items for the dimension of richness in the final coding. Specifically, each item was divided into two sub-items, which aims to explicitly examine 1) whether the teacher **him/herself** is able to use mathematical language, to make connections with general concepts and principles, to use multiple solutions as well as multiple representations; and 2) whether the teacher is able to **engage the students** in the four indicators of richness. However, the results showed that use of multiple solutions and multiple representations were constructed by the teacher and students together for most of the time. Therefore, the correlation between the sub-items was 0.96 and 0.94 respectively for these two indicators, multiple solutions and multiple representations. Therefore, we combined the scores of the sub-items for the two indicators.

3. Data Analysis

To test the hypotheses posed by the current study, descriptive statistics and multi-level regression analysis would be used. And we conducted the data analysis with statistical packages of SPSS 18.0 and HLM 6.02. The procedures of the analysis are explained in the chapters on results of the study.

Chapter 4 The Cognitive Features of Instructional Explanation in the Chinese Mathematics Classrooms

The present study aimed to identify the features of quality of IE that might serve as proximal indices of student learning outcome. To do so, the study, firstly, investigated the cognitive features of IE in the Chinese mathematics classrooms. This chapter first describes the observation of the structural features and the quality of IE in the mathematics classrooms. It then presents the results about relationship of the teacher background factors and their use of IE for another chapter.

As a caveat, the researcher reminds the readers that the development of the scale and the statistics presented in this thesis are exploratory in the sense that they are presented as a basis for the discussion of one of the critical incidences of classroom teaching-instructional explanation, of a particular discourse structural (i.e., extended discourse-initiation-response-follow-up……). But by no means can these statistics results be generalized to Chinese lessons or teachers in general, because there were only 110 elementary mathematical lessons with the instructional content of number from 39 teachers in the current study. The content of the lessons were confined to the number, such as division of fraction, least common multiple, and divisor and multiple.

1. Results

1.1 The Cognitive Features of Instructional Explanation

In all, we identified 346 episodes, with 181 occurring in the reform classes and 165 in the non-reform classes. To obtain a general picture of instructional explanation used in the lessons, we measured the quantitative and qualitative aspects of instructional explanation. For the quantitative aspect, we examined the structural features such as the number of episode of instructional explanation, the turns, and the number of different students / whole class involved in an episode. The measured quality aspects refer to the truthfulness, richness and coherence of instructional explanation.

We began our analysis by an examination of an individual teacher's scores on the variables across lessons. The results did not show a significant difference between the lessons within teacher in most of the variables. This implies that a teacher's performance in terms of the measured aspects of IE was relatively stable across lessons. However, the range of the identified numbers of episodes (4 to15) was large among the teachers. We selected 9 longest episodes for each teacher, which is the medium of the distribution. This helped to minimize the errors from the numbers of episodes. Finally, we had 304 episodes, with 160 occurring in the reform classes and 144 in the non-reform classes.

Notably, we conducted descriptive statistics on the basis of 346 and 304 episodes, respectively. The results were similar to a large extent. In the following section, the researcher would present the results based on the 304 episodes.

Table 4 Descriptive Results of the Cognitive Features of Instructional Explanation

Variables		Mean	SD	Min	Max
Quantitative Indicator					
Structural features	Number of episodes [a]	8.87	2.79	4	15
	Turns	13.60	3.71	7.40	23.33
	Individual students involved	2.30	1.03	.60	5.67
	Whole class involved	3.26	1.20	1.25	6.33
Qualitative Indicator					
Truthfulness	Accurate mathematical knowledge	3.94	.16	3.25	4.00
	Accurate teaching language[10]	3.91	.13	3.50	4.00
	Student error identification and address [b]	3.37	.60	1.00	4.00
Richness	Mathematical language[11] used by the teacher	2.41	.35	1.50	3.11
	Mathematical language encouraged for students to use	2.54	.34	1.83	3.20
	General concepts and principles used by the teacher	2.63	.50	1.00	3.22
	General concepts and principles encouraged for students to use	2.80	.42	1.40	3.33
	Use of multiple solutions	1.48	.49	1.00	2.56
	Use of multiple representations	1.48	.42	1.00	2.71
Coherence	Relevant and well-organized topic	3.99	.03	3.83	4.00
	Clear teaching language	3.84	.17	3.33	4.00
	Follow-up question	2.93	.26	2.33	3.40
	Response to student answers	3.18	.58	1.75	4.00

Notes: a. the number of episodes was calculated based on 346 episodes. b. the statistics was based on 22% of the 304 episodes, since 78% was coded as not applicable.

As shown in table 4, on average, each teacher had about 9 IE episodes. The variability between the teachers was large, ranged from 4 to 15. When the teachers used instructional explanation, they would encourage around two individual students and the whole class students engaging in the generation of IE.

With respect to the quality of instructional explanation, the results turned out as expected. The Chinese teachers had extremely high performance in the indices of truthfulness and coherence. The variance was quite small, which could be seen from

[10] Teaching language refers to all of the verbal and written language a teacher uses in classroom instruction.
[11] Mathematical language specifically refers to mathematical terminology, theorems, formulas and the like.

the minimum and maximum of the variables. Specifically, when constructing instructional explanations, they were able to provide accurate mathematics knowledge, to present with precise mathematical language, and to identify and address student errors appropriately. They were also able to organize the explanation relevantly and clearly, as well as to interact with the students coherently. For example, the teachers responded to student answers positively and consistently. The teachers had relatively lower scores on the dimension of richness, especially for the use of multiple solutions and multiple representations. We observed that the teachers rarely made comparisons or connections between multiple solutions purposefully and explicitly. The variability on the measures between the teachers was also relatively larger.

Table 5 Correlation Matrix of the Indicators of Quality of Instructional Explanation

	1	2	3	4	5	6	7	8	9	10	11	12	13
1. Accurate mathematics knowledge	1												
2. Accurate teaching language	.05	1											
3. Student error identification and address	.57**	.23	1										
4. Mathematical language used by the teacher	.17	.26	.42*	1									
5. Mathematical language encouraged for students to use	.08	.09	.12	.28	1								
6. General concepts and principles used by the teacher	.15	.38*	.46*	.67**	.04	1							
7. General concepts and principles encouraged for students to use	.05	.32	.26	.43**	.11	.83**	1						
8. Use of multiple solutions	-.01	-.26	.14	.10	.52**	.07	.03	1					
9. Use of multiple representations	.15	-.19	-.15	-.06	.31	-.29	-.28	.15	1				
10. Relevant and well-organized topic	.40*	.26	-.27	-.03	.09	-.02	-.09	-.16	.20	1			
11. Clear teaching language	.01	.25	-.09	-.06	-.20	-.13	.01	-.33*	-.13	-.10	1		
12. Follow-up question	.34*	.10	.08	.24	.54**	-.01	-.03	.26	.30	.30	.13	1	
13. Response to student answers	.10	.22	.31	.64**	.35*	.54**	.29	.37*	-.01	.12	-.20	.36*	1

Note: *p<.05 **p<.01 ***p<.001

Table 5 displays the correlations among the scores on the 13 items assessing quality of IE. As can be seen in Table 5, for the dimension of truthfulness, when generating instructional explanation, teachers' providing accurate mathematics knowledge was positively correlated with their identification and address of student errors. And teacher's accuracy in teaching language was not necessarily related to their mathematics knowledge and identification and address of student errors. This could be true in classroom instruction. For instance, in Example 1 below, the teacher's teaching language was accurate. However, she did not address the student's error (0 in itself is 1) appropriately and thoroughly. Also, the mathematics knowledge provided was not true at the end of the dialogue, saying that 0 could not be in a fraction. Because 0 could make the fraction like 0/2, it just could not be denominator. And the teacher did not detect and address the errors. Conversely, in Example 2, the teacher provided accurate mathematics knowledge and the students did not make any errors. However, there were trivial errors in teaching language. It should be "dividing 1 into 6 equal sections" instead of "dividing 1 into 6 sections" .

Example 1 Inappropriate Address of Student Errors in IE

1. Student 9: A whole number, if divided by another number, than the result equals to this whole number multiplies its reciprocal. However, the zero is not the case.
2. Teacher: Why the zero is not the case?
3. Students (together): (the students made quick response) Because zero did not have reciprocal.
4. Teacher: Well, go on please.
5. Student 9: ***Because 0 in itself is 1.***
6. Students (together): （the students refuted the studnet9's answer）No, it is wrong.
7. Teacher: So zero did not ……?
8. Students (together): Zero did not have reciprocal.
9. Teacher: ***And it could not be***……?
10. Students (together):***Fraction***.(Ms Yang and her fifth-grade students in a dialogue on division of fraction, transcription lines 50-59)

Example 2 Inaccurate Teaching Language in IE

1. Teacher: Look at these two numbers, 1/6 and 1/5. Which one is bigger? And why?
2. Student 9: 1/5 is bigger. Because 1/5 means dividing 1 into 5 sections and we took one of the sections. However, 1/6 means that 1 is divided into 6 sections. Therefore, 1/5 is bigger.
3. Teacher: Can anybody give a better explanation? It sounds not so complete.
4. Student 10: 1/5 means that 1 is divided into 5 sections and 1/6 is divided into 6. In other words, the sections of 1/5 are less than that of 1/6. So its size is bigger.
5. Teacher: I see. You means that ***1/6 is divided 1 into 6 sections***, and ***1/5 is divided 1 into 5 sections***. If we took one of the sections, 1/6 is smaller than 1/5. (Mr Zhang and his fifth-grade students in a dialogue on least common multiple, transcription line 25-29)

The distinction of accuracy of mathematical knowledge and teaching language is illustrated in the following examples. In Example 3, the teacher provided inaccurate mathematical knowledge, saying that "0 does not have a reciprocal, so it can not be a numerator". The error was not detected and corrected. Also, she did not recognize the error of student 14 in the discourse. Both helped to point out the gap in mathematical knowledge of the teacher. The teaching language was clear of the errors. Conversely, in Example 4, the teacher's mathematical knowledge was accurate based on the discourse. However, an error occurred in the blackboard writing and the teacher detected and corrected the error immediately. It was considered as the error of teaching language.

Example 3 Inaccurate Mathematical Knowledge in IE

1. Teacher: Dividing a number equals to multiplying this number's reciprocal. Just now she said zero should be excluded. Why zero should be excluded? You please.
2. Student 12：Because zero does not have a reciprocal.
3. Teacher: Zero does not have a reciprocal. Right? Ok, you please.
4. Student 13: Because 1×0 equals to 0, and no matter which number multiplying 0 equals to 0.
5. Teacher: No matter which number multiplying 0 equals to 0, so 0 does not have a ......?
6. Students (together): Reciprocal!
7. Teacher: You please.
8. Student 14: ***Because 0 does not have a reciprocal. Any number multiplying 0 equals to 0.*** Teacher: Is it right?
9. Students (together): Yes!
10. Teacher: ***Because 0 does not have a reciprocal, it can not be a numerator***. (Ms Yang and her fifth-grade students in a dialogue on division of fraction, line 102-112)

Example 4 Inaccurate Teaching Language in IE

1. Teacher: Just now some students tell me that they have already known what is reducing fraction. Could you tell me what is simple fraction in your own words?
2. Student 15: I think if the greatest common divisor of the numerator and denominator is 1, then the fraction is a simple fraction.
3. Teacher: Oh, this is his understanding. What about you?
4. Student 16: Both the numerator and the denominator of a simple fraction are relatively prime numbers.
5. Teacher: Ok, good, he said much better.
6. Student 17: The fraction whose numerator and denominator are relatively prime numbers is identified as a simple fraction.
7. Teacher: Great, anything else? Can any one give an example to illustrate what is a simple fraction? You please.
8. Student 18: 1/2 is a simple fraction.
9. Student 19: 3/4.
10. Student 20: 1/3.
11. Teacher: Does the fractions of this kind have a great number?
12. Students (together): Yes!
13. Teacher: What is the characteristic of these fractions? 1/2、3/4、1/3 and the like. What is the characteristic of these numbers?
14. Student 21: Their numerators and denominators are relatively prime number.
15. Teacher: Then it means that their common divisor is what?
16. Students (together):1!
17. （Board writing: ***In the simple fraction, the numerator and denominator is 1***） The fractions of this kind are identified as the simple fractions. Read after me please. The numerator and denominator of the fraction, begin.
18. （Students read the board writing aloud together）
19. Teacher: ***It should be the common divisor 1. (Adding the "the common divisor of" to the sentence)*** Ok, one more again please!
20. Students (together): The common divisor of the numerator and denominator is 1.
21. Teacher: The fractions ……!
22. Students (together): The fractions are identified as the simple fractions. (Mr Zhang and his fifth-grade students in a dialogue on reducing fraction, line 25-46)

On the dimension of richness, teacher using more mathematical language was related to making more connection with general concepts and principles both for teachers and for students. Students encouraged to use more mathematical language by a teacher, was positively correlated with the use of multiple solutions. And there was a high correlation between the teacher's and students' making connection with general concepts and principles, suggesting that many of the connections were made by the teacher and students together. Use of multiple solutions was unrelated to use of multiple representations.

On the dimension of coherence, it was found that the higher level of a teacher's follow-up questions was related to the teacher's better response to student answers, indicating the consistency of teacher's interaction with their students in the process of instructional explanation. There was no significant correlation between a teacher's interaction with the students and his/her complete and clear teaching language as well as with the teacher's well-organized teaching content.

In addition to the intra-correlations between the items within each dimension of evaluation criteria, the results also showed that a teacher's provision of accurate mathematics knowledge was positively associated with his/her well-organized teaching content and level of their follow-up questions. And a teacher's accurate teaching language was positively correlated with his/her making connection with general concepts and principles. Also, a teacher's better identification and address of student errors was related to his/her more use of mathematics language and of connection with general concepts and principles. A teacher's use of mathematical language was positively associated with his/her response to student answers. A teacher's follow-up questions and response to student answers helped to lead students' use of more mathematical language. Moreover, a teacher's making connection with general concepts and principles was positively correlated with their response to student answers. A teacher's use of multiple solutions was negatively correlated with his/her complete and clear teaching language. And it was positively associated with the teacher's response to student answers.

Table 6 Correlations between the Measures of Quantity and Quality of Instructional Explanation (*n*=39)

	Number of episodes	Turns	Individual students involved	Whole class involved
1. Accurate mathematics knowledge	.17	.06	-.09	.09
2.Accurate teaching language	.12	.07	-.01	.12
3.Student error identification and address	.28	.24	.11	.20
4.Mathematical language used by the teacher	.33*	.51**	.22	.48**
5.Mathematical language encouraged for students to use	.21	.46**	.71**	-.15
6.General concepts and principles used by the teacher	.44**	.33*	.14	.37*
7.General concepts and principles encouraged for students to use	.31	.22	.16	.16
8.Use of multiple solutions	.16	.50**	.73**	.03
9.Use of multiple representations	.07	.15	.14	-.02
10.Relevant and well-organized topic	.08	.05	.04	-.05
11.Clear teaching language	-.20	-.13	-.35*	.11
12.Follow-up question	.16	.33*	.40*	-.11
13.Response to student answers	.18	.46**	.45**	.12

Note: *$p<.05$. **$p<.01$. ***$p<.001$

Associations between the structural features and quality of IE were also examined (see Table 6). The results indicated that, the structural features of instructional explanation were not associated with the indices of the truthfulness. However, there was a significant correlation between the structural features and the indices of richness and coherence. Specifically, the more frequent instructional explanations occurred, the more likely that a teacher used mathematical language and made connections with general concepts and principles. The turns of an episode was positively correlated with the use of mathematical language both for the teacher and students, teacher's making connection with general concepts and principles, use of multiple solutions, the level of teacher's follow-up questions, teacher's response to student answers. Besides, the number of individual students contributing to an episode was positively associated with student use mathematical language, use of multiple solutions, the level of teacher's follow-up question and teacher's response to student answers. And the number of individual students involved in an episode was negatively correlated with a teacher's complete and clear teaching language. Moreover, the whole class involved in an episode was positively associated with a teacher's use of mathematical language and making connection with general concepts and principles.

1. 2 Teachers' Background Factors and Use of Instructional Explanation

In order to examine the association between the variables of IE and those of teachers' demographic factors, the Pearson correlation was conducted. As shown in Table 7, there was no significant relationship between the two sets of variables in general. However, the teacher's educational level was positively correlated with their provision of accurate mathematics knowledge, and it was negatively correlated with use of multiple solutions. Moreover, teacher belief about mathematics was negatively associated with the number of episodes, that is, a teacher, who viewed mathematics as static and fixed, was less likely to generate instructional explanation with the extended form. The results also indicated that years of teaching experience were negatively correlated with the use of clear and complete teaching language.

Table 7 Correlations between Teachers' Demographic Factors and the Measures of Instructional Explanation (*n*=39)

Variable	Teaching age	Reform age [a]	Educational level	Teacher knowledge	Teacher belief
Quantitative Indicator					
Number of episodes [b]	-.02	-.08	.17	.24	-.44*
Turns	-.04	-.21	.05	.19	-.26
Individual students involved	.04	.12	-.13	.13	-.10
Whole class involved	-.03	-.27	.01	.02	-.10
Qualitative Indicator					
Accurate mathematics knowledge	.03	-.03	**.37***	-.16	-.06
Accurate teaching language	-.27	.42	**.19**	.04	.17
Student error identification and address	.11	.24	.09	.07	-.33
Mathematical language used by the teacher	-.10	.04	.25	.18	-.10
Mathematical language encouraged for students to use	-.07	.30	.07	.00	-.19
General concepts and principles used by the teacher	-.21	.14	.20	.19	-.17
General concepts and principles encouraged for students to use	-.25	.25	.19	-.01	-.07
Use of multiple solutions	-.04	-.08	-.36*	.07	-.08
Use of multiple representations	.05	-.25	-.01	.08	-.26
Relevant and well-organized topic	.07	-.25	.29	-.19	.17
Clear teaching language	-.40*	.25	.29	-.06	.18
Follow-up question	-.14	0	.32	.24	-.11
Response to student answers	-.13	-.07	.22	.20	.04

Notes: a. the sample of the correlation between reform age and feature of IE is conducted within the reform group. b. the correlation between number of episode and feature of IE is conducted with 346 episodes.

To assess the effect of curriculum used in the classrooms on the teachers' IE, one-way ANOVA was conducted. As can be seen in Table 8, there was no significant difference in the indices of quantity of IE between the reform and non-reform teachers. On the indices of quality, the reform teachers had significantly higher scores than non-reform teachers on leading students to use mathematical language, use of multiple solutions, and use of multiple representations. This finding was consistent with the previous analysis with 58 teachers using another coding framework. While the non-reform teachers had a significantly higher scores than the reform teachers on accurate use of teaching language, as well as making connection with general concepts and principles both for the teacher and for students. Moreover, we found no significant difference between the two groups in the indices of coherence.

Table 8 Means and Standard Deviations of Reform and Non-reform Teachers on the Measures of Instructional Explanation

Variables	Reform teachers (n = 20)		Non-reform teachers (n =19)		F
	Mean	SD	Mean	SD	
Quantitative Indicator					
Number of episodes [a]	9.05	2.50	8.68	3.13	.16
Turns	13.89	3.35	13.29	4.12	.25
Individual students involved	2.57	.77	2.03	1.21	2.78
Whole class involved	.19	.04	.17	.06	.31
Qualitative Indicator					
Accurate mathematics knowledge	3.93	.20	3.95	.13	.13
Accurate teaching language	3.85	.14	3.97	.06	11.75**
Student error identification and address	2.99	1.24	2.36	1.67	1.84
Mathematical language used by the teacher	2.38	.39	2.44	.30	.29
Mathematical language encouraged for students to use	2.68	.32	2.40	.31	7.59**
General concepts and principles used by the teacher	2.44	.61	2.83	.24	6.77*
General concepts and principles encouraged for students to use	2.63	.51	2.98	.14	8.31**
Use of multiple solutions	1.65	.53	1.29	.38	6.10*
Use of multiple representations	1.63	.34	1.33	.45	5.53*
Relevant and well-organized topic	3.99	.04	4.00	.00	1.91
Clear teaching language	3.81	.16	3.88	.17	1.88
Follow-up question	2.99	.27	2.86	.23	2.89
Response to student answers	3.10	.63	3.27	.53	.85

Notes: a. the comparison of number of episode is based on 346 episodes.

*p<.05. **p<.01. ***p<.001

2. Discussion

The results reported in this chapter are concerned with the two research questions. The discussion is therefore organized in the way accordingly. Firstly, we discuss the cognitive features of IE displayed in the mathematics classrooms, as well as the relationship between the teachers' background factors and their use of instructional explanation. We then address the issues about the structure of the scale for the evaluation of quality of instructional explanation.

2.1 The Cognitive Features of Instructional Explanation in the Mathematics Classrooms

2.1.1 Structural Features of Instructional Explanation in the Classrooms

The features of instructional explanation in the classrooms were examined in terms of its structural features and quality. For the structural features, the descriptive results showed a large variance of the number of episodes as well as the average turns per episode among the 39 teachers. About 2 different individual students engaged in the process of IE per identified episode, which was consistent with the finding of the previous study (Schleppenbach et al., 2007). The teacher also involved the whole class in the generation of the IE, and the frequency was 3 per episode on average. The statement of the whole class involved in the explanation was rich, such as computation (e.g., that equals 4), simple answer (e.g., true or false), indication of understanding and /or agreement (e.g., yeah), reasoning (e.g., 0 multiplying any number will get 0, so 0 does not have a reciprocal), and rule / term recall (e.g., it is based on the rationale of integer division) (Schleppenbach et al., 2007).

2.1.2 Quality of Instructional Explanation in the Mathematics Classrooms

Consistent with the hypothesis, it was found that the Chinese mathematics teachers scored high on the dimension of truthfulness and coherence. In the process of instructional explanation, they were able to provide accurate mathematical knowledge, use concise and accurate mathematical language, identify and address student errors correctly, organize the explanation in a good manner, as well as lead a coherent interaction with the students. The findings go beyond the previous research that mainly relied on a few cases to provide evidence about the feature of Chinese mathematics teachers' instructional practices.

Two reasons, among others, presumably account for the results. Firstly, the Chinese teachers' good performance in terms of truthfulness and coherence were presumably affected by their pre-service experiences. Mathematics, as a main school subject in the basic education system, has been attached with great importance in mainland China (Li, 2008). For example, mathematics is one of the compulsory subjects in the National Matriculation Test. And there was a widespread saying that "math, physics and chemistry are the strongest power to the world" (*xuehao shulihua, zoubian tianxia dou bupa*) in the 20th century. These factors presumably motivated the teachers to build solid mathematics knowledge before they began their teaching careers. Likewise, the teachers' views of a good or effective mathematics lesson had been shaped largely by their learning experiences (Correa, Perry, Sims, Miller, & Fang, 2008; Wilson, 1990). As Lortie (1975) suggested that, teachers might unintentionally acquire culturally shared beliefs about teaching and learning in childhood, when potential teachers were students and participated in an "apprenticeship of observation."

Secondly, the organizational features in China also highly support the teachers in their classroom instruction. Here, the supportive factors were discussed from three levels, including the level of nation, school, as well as individual teachers. Notably, all these levels were inter-related and connected, rather than independent and separated.

At the national level, the well-organized and detailed curriculum materials, including the textbooks, teacher manuals and exercise book, play an important role in teachers learning about subject matter as well as instruction (Ball & Cohen, 1996; Lloyd & Frykholm, 2000). In addition to the curriculum materials, there are also other accompanied resources, such as national teaching journals which teachers can contribute articles to share their experience, and exemplary lesson (i.e., a lesson plan with a videotape), which help the teachers to improve their teaching (Huang & Li, 2009; Wang & Cai, 2007).

At the school level, the teaching research group and lesson preparation group, also contribute to the quality of teachers' instructional practices. Teachers are organized to discuss general issues of teaching, observe and analyze instruction for one another in teaching research group. As a sub-organization of the teaching research group, the lesson preparation group provides an opportunity for teachers to study the curriculum materials, plan lessons and units together, and share teaching experiences on a regular basis (Paine & Ma, 1993; Wang & Paine, 2003).

At the level of individual teacher, two factors were associated with teachers' performance in classroom teaching. Firstly, we argue that it is the human nature that the individuals want to do a good job on his or her position. Based on informal communication with the frontier teachers, it was found that almost all the teachers desired to improve their teaching and further facilitated students' learning. Secondly, the quality of their instructional practice also affected their promotion and development. There are municipal and national teaching skill competitions in China (teaching a lesson and explaining the design of the lesson). Teachers' participation in this kind of activities would be helpful for their promotion (Huang, Peng, Wang, & Li, 2009; Huang & Li, 2009).

Compared with the high scores on the dimensions of truthfulness and coherence, the scores on the dimension of richness were a little lower. And the result revealed that the teachers actively engaged the students into the use of mathematical language and connection with general concepts and principles. The findings concurred with the earlier work that indicated that there were some student-centered features within a tightly teacher-controlled class, and students' participation was valued and a lot of encouragement was given as well in Chinese classes (e.g., Lopez-Real, Mok, Leung, & Marton, 2004; Mok, 2006, 2009). However, we also observed that, in the teachers' discourse with students, they paid more attention to social scaffolding[12] than to analytic scaffolding (Williams & Baxter, 1996; Baxter & Williams, 2010). The teachers focused on engaging more students to participate in the classroom discourse and on providing more opportunities for students to speak in the classes. Their scaffolding of mathematical ideas was relatively insufficient and thus the discourse was not extended substantially to some extent. For instance, when using multiple solutions and representations in IE, the teachers rarely made comparisons or connections between the solutions and representations explicitly and purposefully. Take the following episode for example, a teacher was discussing why "4/7÷3 =

[12] Social scaffolding refers to the scaffolding of norms for social behavior and expectations regarding discourse. And analytic scaffolding means the scaffolding of mathematical ideas for students.

4/7×1/3" with her students. In the dialogue, she actively invited more students to engage in the discussion by repeatedly asking "you please" (你来说一说). Although there were some follow up questions to respond to student answers, it is obvious that this teacher put more attention to the students' participation than to the mathematical content. After their discussions, the teacher did not end up with a summary or highlight of the various explanations for "4/7÷3=4/7×1/3", which was significant for students' understanding.

Example Follow-up Questions to Engage More Students in IE.

1. Teacher: I found that XX's solution is different. Could you share with us? Your arithmetic formula is different from the others.
2. Student 19：Not much difference, my formula is 4/7÷3=4/7×1/3=4/21.
3. Teacher: 4/7 is divided by 3, **why do you multiply 1/3?**
4. Student 19：As shown above, 4/7÷3 means dividing 4/7 into 3 sections and we took one of the sections. Also, 4/7 multiplied 1/3 means dividing 4/7 into 3 sections and we took one of them. So their answers were the same.
5. Teacher: **Great，who can say again? You please?**
6. Student 20: I have another solution. I am thinking that why we need to multiply the denominator. Because when we were learning multiplication of fractions, we multiplied the numerator by the integer. Now this is division of fractions. It means dividing the denominator into several sections, so we multiply the denominator.
7. Teacher: Ok, **this is your understanding. You please.**
8. Student 21：I am trying to make sense what he said. The denominator multiplied 3 means it was reduced by 3 times.
9. Teacher: The denominator multiplied 3. What reduced 3 times?
10. Student 21：The result was reduced by 3 times.
11. Teacher: Did you multiply 7 by 3?
12. Student 21：Yes, if the denominator 7 multiplied 3, than the result was reduced by 3 times.
13. Teacher: So again, it was dividing 4/7 into 3 sections equally. **We have one more explanation now. Well, you please.**
14. Student 22：I found a pattern. In all these arithmetic, if it was division, then we used the number to multiply the denominator, and we got 21 in this task. The numerator 4 in 4/7 is not needed.
15. Teacher: The numerator 4 is not needed when multiplying. We multiplied by 7. Your opinion is similar to that of student 20, right? Ok, why the denominator should multiply 3? Why did you do that? Who have figured out? **Would you like to say something?** (continued)

16. Student 23：As said by student 21, 4/7 divided by 3 means that 4/7 was reduced by 3 times. If the denominator multiplied 3, then the result would be reduced by 3 times accordingly.
17. Teacher: Right. Actually it was reduced 4/7 by 3 times. How many sections are there if we divided 4/7 equally?
18. Student：3！
19. Teacher: 3. OK, **XX, can you explain one more time please? 4/7÷3=4/7×1/3，why are they equal to each other? Could you explain by your figure?**
20. Student 24：4/7 multiplied 1/3 means 1/3 of 4/7. And 4/7÷3 means divided 4/7 into 3 sections, and we got one of the sections. So they are the same.
21. Teacher: Good. **Who can say once again?** XX.
22. Student 25：Our explanation is the same with the student 24.
23. Teacher: **OK, who would like to say a little more about this question. XX.**
24. Student 26：4/7×1/3.
25. Teacher: What does it mean?
26. Student 26: It means 1/3 of 4/7.
27. Teacher: That is right.
28. Student 26: I still feel different. 4/7×1/3, 1×4 equals to 4. Therefore, 3×7 is ok.
29. Teacher: En, like the method of XX. You got it, too. Well, **you please.**
30. Student 27: I think that 3 can be seen as 3/1.
31. Teacher: 3 can be seen as 3/1, right?
32. Student 27：And then we multiplied its reciprocal. 4/7×1/3 equals to 4/21。
33. Teacher: Multiplied 1/3. 3 is seen as 3/1. So the result would be 4/21. XX found that divided a number equals to the number multiplied its reciprocal. Is this finding right or not? Ok, we have known that dividing 4/7 into 3 sections equally，means what is 1/3 of 4/7？So they are equal to each other.

(Miss Wang and her fifth-grade students in a dialogue on division of fraction, line110-142)

2.2 The Relation of Teachers' Background Factors with Instructional Explanation

Comparable to many studies that have found little correlation between teacher demographic variables and teacher mathematical knowledge, the use of the features of linguistic pedagogy (i.e., specialized and general academic vocabulary, discourse structural, mathematical examples, and evaluative comments), as well as classroom quality (i.e., instructional support like content understanding, analysis and problem solving, and quality of feedback; classroom organization) (Hill et al. 2005; Izsák, Orrill, Cohen & Brown, 2010; Bailey, Chang, Heritage, & Huang, 2010; Malmberg, Hagger, Burn, Mutton, & Colls, 2010) , we also found relatively few teacher background characteristics that were related to the teacher's use of IE in terms of structural features and quality. Also, the teachers' mathematical knowledge for teaching and teachers' views of mathematics were not shown to be associated with their use of IE statistically, which was out of our expectation. One exception was that the more teachers viewed mathematics as fixed and static, the less likely that they generated instructional explanation with their students.

However, the results revealed the effect of curriculum in use on the quality of instructional explanation. Specifically, the reform teachers were more likely to lead students to use mathematical language, multiple solutions and multiple representations. These results were attributed to the impact of the new curriculum reform in mainland China, which advocated students' more engagement in classroom discourse. And the new curriculum supported more use of multiple solutions and multiple representations (Li & Chang, 2007; Li, et al., 2011). When using multiple solutions, we discovered that both groups focused on the solutions' diversity, without paying enough attention to the optimization of a solution. This finding extended the previous research that revealed that some of the reform teachers overly pursued an active atmosphere in lessons and did not further discuss the efficiency, advantage, as well as disadvantage for multiple solutions (Yu, 2003; Huang, 2005). We also observed that, in the classes using the new curriculum, the teachers tended to encourage the students to select a solution that they liked to solve a problem, which was not the case for the non-reform classes. The results also showed that the non-reform teachers were more inclined to use accurate teaching language, as well as to make connections with general concepts and principles. The reason presumably is because the classes implementing the conventional curriculum paid more attention to mathematical rigor. Larger sample size is needed in future studies to valid the conclusions.

2.3 The Structure of Quality of Instructional Explanation

As stated by Douglas (2009): "Classroom instruction is a complex enterprise that occurs at the intersection of teachers, students, and texts within the surrounding classroom, school, and community environments. Progress in studying the complexity of classroom instruction on a large scale relies on our ability to pose research questions at the appropriate levels of analysis and to attempt to answer the questions using rigorous methods" (p.518).

In the current study, we did the exploratory work to look into the quality of instructional explanation in the Chinese mathematics classes. For this, we developed the instrument to evaluate the quality of instructional explanation. The instrument contained the three dimensions, truthfulness, richness, and coherence, based on the literature of what constitutes good explanation. And the indices for the dimensions were mostly generated from the literature of effective teaching in general and effective mathematics teaching in particular. Here, quality is more about the content of an explanation. The evaluation was based on the teacher's overall performance in the process of generating instructional explanation, rather than a specific and discrete words or sentence. Also, we focused the evaluation on teachers, rather than on students.

2.3.1 Correlations between the Items within a Dimension

It was of interest to note that not all of the indices showed significant correlations with one another. Specifically, the teacher's provision of accurate mathematics knowledge was found to be positively associated with their identification and address of student errors. However, both of them were unrelated to the teacher's accuracy in teaching language. Presumably, the first two were mostly constrained by the teacher's subject matter knowledge and pedagogical knowledge, but the latter was more fluctuant and affected by factors like task conditions, individual teacher's instructional habits and dispositions. This result was corroborated with the finding that a teacher's provision of accurate mathematics knowledge was positively associated with his/her well-organized instructional content and level of follow-up questions.

For the dimension of richness, we did not find a significant correlation between the use of multiple solutions and multiple representations, which was consistent with the result of our previous analysis with 171 lessons of 58 teachers (Ni, et al., 2009). In this study, the use of multiple solutions exclusively focused on specific content. However, by multiple representations, it was considered as carriers of knowledge and thinking tools (Cai & Lester Jr., 2005). It concerns more the forms to present the content. Take two examples for instance, in Example 1, the teachers led the students to use two different solution strategies to answer whether 54 could be divided by 3 exactly. In Example 2, the teacher and her students used two kinds of representation, including mathematical example as well as life example, in order to illustrate what means by "reciprocal". Notably, confined to the instructional content of the number, in the current study, most of the representations were presented in the forms of the words and number, rather than figures, hands-on manipulatives, and the like.

Example 1 Use of Multiple Solution Strategies in IE

1. Teacher：Could 54 be divided into 3 with no remainder?
2. Students (together):：Yes!
3. Teacher：Why? Zhang please!
4. Student Zhang: ***Because 54 divided by 3 equals 18, and there is no remainder，so it could be divided by 3 exactly***. (solution strategy 1)
5. Teacher：Could you make use of the features of the number that could be divided into 3 with no remainder to explain the question? Li?
6. Student Li: ***5 adding 4 equals 9, and 9 could be divided by 3 exactly, so 54 could be divided into 3 with no remainder***. (solution strategy 2)
7. Teacher：Is it Right?
8. Students (together):Yes! (Ms Shui and her fifth-grade students in a dialogue on features of the number that could be divided into 3 with no remainder, transcription line 109-116)

Example 2 Use of Multiple Representations in IE

1. Teacher：Well, sit down please. Let me tell you what is reciprocal. We called two numbers whose product equals 1 as reciprocals. Could anybody say a little bit about what means by "reciprocal", Zhou?
2. Student Zhou: Mutual, each other is reciprocal.
3. Teacher：Mutual, each other is reciprocal. ***Who can give an example***, Chen?
4. Student Chen: For example, 2/3 is the reciprocal of 3/2, and 3/2 is the reciprocal of 2/3.
5. Teacher：Chen used ***the data*** to show his understanding. Let's use a ***life example*** to illustrate the meaning of "mutual".
6. Student 15: Just say I and Xu are desk mates. I cannot say I am the desk mate. It should say that I am the desk mate of Xu.
7. Teacher：Um, what about Xu?
8. Student Xu: She is my desk mate too.
9. Teacher：So you are ……?
10. Students (together): Desk mates for each other (*huwei tongzhuo*).
11. Teacher：Is it right?
12. Students (together): Yes. (Ms Li and her fifth-grade students in a dialogue on reciprocal, transcription line 88-100)

In order to better capture the differences in the use of IE among the teachers, the indicators on the dimension of richness were sub-divided into two aspects: a teacher his/herself using rich mathematics and the teacher encouraging his/her students to use rich mathematics. The results indicated that the two aspects were highly correlated on the scale of making connections with general concepts and principles, use of multiple solutions, and multiple representations. However, it was not the case for the scale of the use of mathematical language. This suggests that a teacher's use of mathematical language was independent of his/her leading students' use of mathematical language. Two inter-related reasons, among others, might account for the results. Firstly, compared to the use of mathematical language, making connection with general concepts and principles, the use of multiple solutions and multiple representations were of higher cognitive demanding for the students. This may allow more probability for the teacher to scaffold (Nathan & Kim, 2009). Secondly, the curricular objectives also motivated the teachers to engage the students when making connections with general concepts and principles, using of multiple solutions and multiple representations, because most of these issues were explicitly identified as "important", "difficult", and "hinge" in the teaching manuals (Paine & Fang, 2006; Li, 2005).

Existing studies on instructional coherence tended to be concerned with whether or not the teaching content in a lesson being connected or integrated, such as the studies of TIMSS (Stigler & Hiebert, 2004; Leung, 2005). According to Stein and Glenn (1982), a good mathematics lessons is like a story. It is more than just a sequence of events. Each event must be organized and interconnected such that the story has a beginning, middle, and end, as well as a consistent theme that runs throughout the lesson with a clear scheme. Besides, the characters of the story, the teacher and their students, interacted in a coherent way so as to promote the development of the story. Therefore, the instructional coherence examined in the studies not only refers to the particular instructional content, but the dynamic classroom discourse. A few studies had done the seminal works in this respect (e.g., Chen & Li, 2010; Seidel, Rimmele, & Prenzel, 2005). In the current study, the coherence was measured in terms of its content and the teacher's interaction with students. The latter aspect included the teacher's follow-up question and response to students' answers. A Moderate correlation was found between these two aspects. However, there was no significant correlation between the well-organized and relevant content and the quality of teacher's interaction with the students. In other words, they were relatively independent aspects of teachers' instructional practices, each contributing their own unique role. More research is needed that will explore the relation between these two aspects in future.

2.3.2 Correlations of the Items between the Dimensions

The results showed a moderate and high correlation among some of the indices of the evaluation criteria. And there was some overlap among the dimensions of truthfulness, richness, and coherence. For example, the teacher's provision of more accurate mathematics knowledge was related to their well-organized instructional content, and higher level of follow-up questions. Also, when the teachers identified and addressed student errors, they were more likely to use more mathematical language, as well as to make connections with general concepts and principles. Moreover, the more responsive to students' answers, the more likely that the teachers used mathematical language, made connections with general concepts and principles, as well as multiple solutions.

2.3.3 Issues Raised by the Correlation Results

Taken together, the results revealed that there was some overlap among the indices of three dimensions of the evaluation criteria. Within dimension, some indices were independent of each other. The results have two implications. On the one hand, they contribute to the understanding of the relationship among the components of IE. On the other hand, as a scale, a lot of works are still needed in future research. Ideally, most of the indices should be correlated to each other. Moreover, the intra-correlation of the items within a dimension should be larger than the inter-correlations across the items of various dimensions.

Confronted with such a complex construct, this is a challenging work, including the development of the items and the rating criteria as well as the conduction of rating. Take the dimension of richness for example, it included the use of mathematical language, connection making with general concepts and principles, the use of multiple solutions and multiple representations. If we look at the rubric, all of them were related to richness of mathematics, and there was one common factor across these four items. However, they were also measuring the different but key aspects of mathematics teaching and learning. The aspects could be independent of one another, and their effects on student learning probably would vary (also see the results of the followed study in the next chapter). It was also not easy to define the indicators of the instrument and use the rubrics. For example, what counts as mathematical language, general concepts and principles, multiple solutions and representations? Was there a general criterion to evaluate the quality of a teacher's use of mathematical language, general concepts and principles, multiple solutions and multiple representations? Do the criteria make sense in both theory and practice? How differentiated were the scales ranged from 1 to 4, especially the zone between 2 and 3? Could the scale apply to each identified episode of IE fairly? To what degree that different individuals viewed the same thing when using the rubric? All these questions need to be dealt with when developing and applying the instrument. Many rounds of revision will be needed to make the instrument applicable for most mathematics classrooms. And meanwhile, the revision should be based on the theory as well as empirical studies.

Chapter 5 The Relationship of Instructional Explanation to Student Learning Outcomes

The present study aimed to identify the features of quality of IE that might serve as proximal indices of student learning outcome. In this chapter, the procedure of data analysis is explained firstly. Then, study results are presented. And followed is a discussion of the results.

1. Procedure of the Data Analysis

Separate analyses were performed to examine the relation of student learning outcomes to the quantitative and to the qualitative indicators of IE respectively. The description of the analysis procedure below applies to the two sets of analysis. However, the description involves only the qualitative indicators of IE as the predictors of students' achievement gain as an illustration.

Because the student data were embedded in the data of classroom level, the analysis was conducted with a multilevel model of 2, 239 students nested within 39 classes to decompose the variance in student outcome measures into student and class level. Before the regression analysis, an unconditional model was conducted to compute the variance components across the two levels. The general equations for the two-level models are as follows, and the meaning of the equations is explained in the paragraph followed.

Level-1 model at student level

$Y_{ij}=\beta_{oj} + r_{ij}$

Level-2 model at class level

$\beta_{oj}= \gamma_{oo} +\mu_{0j}$

In the level-1 model, Y_{ij} is the achievement gain for a student i in class j. β_{oj} is the average score of class j. r_{ij} is the residual variance of student i in class j. In the level-2 model, γ_{oo} represents the mean achievement of all the students. μ_{0j} is the residual variance of class j.

In the analysis, the variables of SES, sex, and prior achievement were entered as controls at the student level and the variable of curriculum at the class level. However, at the class level, we did not put teacher's background factors, such as years of teaching experience, years of implementing the reform, teacher mathematical knowledge, and teacher belief in the equation. There were two reasons for this. Firstly, the current study did not aim to explore the effect of teachers' background qualifications on student learning. Secondly, we considered that these background factors affected students' learning via teachers' instructional practices.

With these controls in place, we then examined the classroom variables of interest. The variables of quality of IE, including truthfulness, richness, and coherence, were investigated to test the relationship between the quality of instructional explanation and student learning. A preliminary analysis showed interaction effects between curriculum and teachers' use of instructional explanation. The interaction terms were consequently added to the equations. The equations for the two-level models are as follows.

Level-1 model at student level:

$Y_{ij}=\beta_{0j}+\beta_{1j}\,(\text{sex})+\beta_{2j}\,(\text{SES})+\beta_{3j}\,(\text{prior achievement})+r_{ij}$

Level-2 model at class level:

$\beta_{0j}=\gamma_{00}+\gamma_{01}$（curriculum） $+\gamma_{02}$（truthfulness） $+\gamma_{03}$（richness） $+\gamma_{04}$（coherence） $+\gamma_{05}$（curriculum * truthfulness） $+\gamma_{06}$（curriculum * richness） $+\gamma_{07}$（curriculum * coherence） $+\mu_{0j}$

Moreover, to capture the possible moderate effect of students' SES and prior achievements on the effect of instructional explanation on the students achievement gain, the two-level models were finally as follows.

Level-1 model at student level:

$Y_{ij}=\beta_{0j}+\beta_{1j}$ (sex) + β_{2j} (SES) + β_{3j} (prior achievement) + r_{ij}

Level-2 model at class level:

$\beta_{0j}=\gamma_{00}+\gamma_{01}$ (curriculum) $+\gamma_{02}$ (truthfulness) $+\gamma_{03}$ (richness) $+\gamma_{04}$ (coherence) $+\gamma_{05}$ (curriculum * truthfulness) $+\gamma_{06}$ (curriculum * richness) $+\gamma_{07}$ (curriculum * coherence) $+\mu_{0j}$

$\beta_{1j}=\gamma_{10}+\mu_{1j}$

$\beta_{2j}=\gamma_{20}+\gamma_{21}$ (truthfulness) $+\gamma_{22}$ (richness) $+\gamma_{23}$ (coherence) $+\gamma_{24}$ (curriculum * truthfulness) $+\gamma_{25}$ (curriculum * richness) $+\gamma_{26}$ (curriculum * coherence) $+\mu_{2j}$

$\beta_{3j}=\gamma_{30}+\gamma_{31}$ (truthfulness) $+\gamma_{32}$ (richness) $+\gamma_{33}$ (coherence) $+\gamma_{34}$ (curriculum * truthfulness) $+\gamma_{35}$ (curriculum * richness) $+\gamma_{36}$ (curriculum * coherence) $+\mu_{3j}$

In the level-1 model, Y_{ij} is the achievement gain for a student i in class j. β_{0j} is the average score of class j adjusted for the students' sex, SES, and prior achievement. β_{1j}, β_{2j}, and β_{3j} is the slope of student's sex, SES, and prior achievement respectively in class j. r_{ij} is the residual variance of student i in class j. The student variables of sex and SES were entered into the model uncentered, the variable of prior achievement was entered using its group mean centered.

In the level-2 model, γ_{00} represents the mean achievement of all the students. γ_{01}, γ_{02}, γ_{03}, γ_{04}, γ_{05}, γ_{06}, and γ_{07} is the slope of curriculum, truthfulness, richness, coherence, curriculum* truthfulness, curriculum* richness, and curriculum* coherence, respectively. μ_{0j} is the residual variance of class j. γ_{10}, γ_{20}, and γ_{30} are the overall mean slopes of the classes. γ_{21} (truthfulness) and γ_{31} (truthfulness) represent the hypothesized effect of truthfulness on the relationship between SES, prior achievement and students' learning outcomes.

At Level 2, the teacher on the class level, β_{0j} was modeled with the inclusion of dummy variable indicating whether the class used reform-based or conventional curriculum, as well as the variables of the quality of IE—truthfulness, richness, and coherence. The intercept was allowed to vary randomly across level two where μ_{0j} was a random classroom or teacher effect—the deviation of classroom or teacher j's score from the overall mean, assumed to be normally distributed with the mean as 0 and the variance as τ_{β}. All other level-1 variables were fixed and not permitted to vary randomly across classrooms, as the effects of the student demographics variables were modeled as being constant across the classrooms.

It is notable that because of the small sample size at the class level (n=39), the variables of the structural features and quality of IE were analyzed separately. The equations above would be applied to the variables of structural features of instructional explanation as indicated at the beginning of the chapter.

The statistical package HLM 6.02 was used to conduct the analyses. Missing values for the 7 outcome variables were 3, 12, 159, 121, 0, 0, 2 for the first assessment; and 11, 2, 11, 2, 11, 2, 16 for the third assessment. All the missing values were replaced with the average score of each time for the group that the individuals belonged to.

2. Results

The descriptive statistics of the outcome measures are displayed in Table 9. Table 10 presents the variances of the outcome measures located at the class level, indicating that differences in mathematics achievement between the classes were very small, 4.08%-16.39% for the cognitive outcomes and 2.1%-8.1% for the affective outcomes. We further computed the variances separately for the reform and non-reform classes. The results showed that the variances of the outcome measures located at the level-2 differed between the two groups, which might imply an interaction effect between curriculum and the indicators of instructional explanation on the learning outcomes. For example, the variance in the scores of the calculation was larger between the reform classes than that between the non-reform classes. Conversely, the variance in simple problem solving and complex problem solving were much smaller between the reform classes than that between the non-reform classes.

Table 9 Descriptive Results of the Student Learning Outcomes

		Total (n=2,239)		Reform Group (n= 1,197)		Nonreform Group (n=1,042)	
Variables	Max	Mean	SD	Mean	SD	Mean	SD
SES	2.48	0	1	0.22	0.98	-0.25	0.97
Learning Outcomes							
1st Assessment							
Calculation	14	10.84	2.22	11.17	2.32	10.47	2.03
Simple Problem Solving	12	9.26	2.43	9.30	2.42	9.21	2.43
Complex Problem Solving	24	15.45	5.11	15.66	5.11	15.21	5.11
Cognitive Achievement [a]	50	35.56	7.90	36.14	7.92	34.89	7.83
Interest in Learning Mathematics	5	4.17	0.69	4.26	0.65	4.08	0.72
Classroom Participation	5	3.71	0.89	3.72	0.87	3.71	0.92
Views of Mathematics	5	3.68	0.57	3.66	0.58	3.70	0.57
Views of Learning Mathematics	5	3.95	0.61	3.98	0.61	3.92	0.62
Affective Achievement [b]	20	15.52	2.15	15.61	2.11	15.41	2.19
3rd Assessment							
Calculation	14	12.48	1.87	12.04	2.05	12.98	1.51
Simple Problem Solving	12	10.32	1.76	10.25	1.80	10.40	1.70
Complex Problem Solving	24	19.10	4.56	19.28	4.41	18.90	4.72
Cognitive Achievement	50	41.90	6.52	41.57	6.61	42.28	6.41
Interest in Learning Mathematics	5	4.01	0.84	4.08	0.80	3.93	0.89
Classroom Participation	5	3.65	0.93	3.65	0.91	3.64	0.96
Views of Mathematics	5	3.84	0.61	3.85	0.61	3.84	0.60
Views of Learning Mathematics	5	3.95	0.65	3.96	0.66	3.94	0.65
Affective Achievement	20	15.45	2.50	15.54	2.47	15.35	2.53

Note: a. cognitive achievement equals to the sum of the measured calculation, simple problem solving and complex problem solving;

b. affective achievement equals to the sum of the measured interest, classroom participation, views of mathematics, and view of learning mathematics.

Table 10 Results of Variance Component Analysis at the Class Level

Dependent Variables	Variance at Class Level
Total (*n*=39)	
Cognitive Achievement	6.16%
Calculation	16.39%
Simple Problem Solving	5.04%
Complex Problem Solving	4.08%
Affective Achievement	4.9%
Interest in Learning Mathematics	8.10%
Classroom Participation	2.1%
Views of Mathematics	3.3%
Views of Learning Mathematics	3.25%
Reform Classes (*n*=20)	
Cognitive Achievement	5.1%
Calculation	14.82%
Simple Problem Solving	3.1%
Complex Problem Solving	2.4%
Affective Achievement	7.0%
Interest in Learning Mathematics	10.5%
Classroom Participation	2.5%
Views of Mathematics	4.5%
Views of Learning Mathematics	5.4%
Non-reform Classes (*n*=19)	
Cognitive Achievement	7.2%
Calculation	3.43%
Simple Problem Solving	7.3%
Complex Problem Solving	5.6%
Affective Achievement	2.7%
Interest in Learning Mathematics	4.8%
Classroom Participation	1.8%
Views of Mathematics	2.2%
Views of Learning Mathematics	1.2%

The measures of quality of IE include three dimensions containing 11 items. The first dimension, truthfulness, consists of measures of teachers' accuracy in provision of mathematical knowledge, teaching language, and identification and address of students' errors. The second dimension, rich mathematics, focuses on teachers' mathematics richness in terms of the use of mathematical language, connections with general concepts and principles, the use of multiple solutions, and the use of multiple representations. And the third is the coherence of IE in terms of content organization and teachers' interaction with the students.

Our preliminary analysis indicated that, the effects of instructional explanation on student learning were not consistent among the indices within a specific dimension. Therefore, we did not conduct the analysis on the basis of the average scores of the three dimensions: truthfulness, richness, and coherence. Instead, we tested for the 11 specific measures of quality of IE concerning their influence on students' learning outcomes. Separate analyses were conducted with 2,239 students in total, 1,197 students from the reform classes, and 1,042 students from the non-reform classes, respectively. The results indicated that the model was not convergent involving the variables of truthfulness for the non-reform sample. Then, we eventually used 7 indicators of quality of instructional explanation with larger variance to the equations for the analyses. The 7 indicators included 5 items on the dimension of richness and 2 items on coherence. One item on richness, the use of multiple representations, was not selected, because it was negatively associated with the other indices of richness.

For a convenient reading, the results are presented in order for the cognitive outcomes and affective outcomes below. And we presented the results based on 2,239 students, 1,197 students from the reform classes, and 1, 042 students from the nonreform classes, respectively.

2.1 Effects of Instructional Explanation on Cognitive Learning Outcomes

Results of Table 11 based on the whole sample indicate that, students' SES and prior achievement had a positive effect on their cognitive learning outcomes. The students in the non-reform classes scored higher in general cognitive achievement, calculation, and simple problem solving than that of the students in the reform classes.

Both the main effect and interaction effect of the structural features with the factor of curriculum were significant. And it was found that the structural features of IE appeared more obvious effects on students' calculation and simple problem solving. The frequency of a teacher's use of IE only showed the significant effect on students' complex problem solving. The turns per IE episode had a negative association with students' general cognitive achievement (effect size=-0.40, $p<.01$), calculation (effect size=-0.21, $p<.01$), and simple problem solving (effect size=-0.09, $p<.01$). The number of individual students involved in the instructional explanation had a positive effect on students' general cognitive achievement (effect size=1.12, $p<.01$), calculation (effect size=0.61, $p<.01$), and simple problem solving (effect size=0.26, $p<.01$). The frequency of whole class students involved in the generation of IE also contributed to students' calculation (effect size=0.33, $p<.01$).

If we look into the results for the reform and non-reform classes separately (Table 12 and 13), it was found that the effects of structural features of IE occurred only in the reform classes (see Table 12). Specifically, the number of IE episodes had a positive association with students' complex problem solving. The number of individual students involved in the IE positively affected students' general cognitive achievement, calculation, and simple problem solving. Whole class students involved in the explanation positively affected students' calculation.

Notably, thought all of the effects were insignificant statistically in the nonreform classes, the pattern of the effects of structural features on student cognitive learning outcome were consistent across the two groups to a large extent, except for the effect of individual students involved in the generation of instructional explanation.

Table 11 Results of Two-level Analysis of the Cognitive Learning Outcomes in Relation to the Structural Features of Instructional Explanation (*n*=2,239)

	Cognitive Achievement		Calculation		Simple Problem Solving		Complex Problem Solving	
Variable	Coefficient	s.e.	Coefficient	s.e.	Coefficient	s.e.	Coefficient	s.e.
Level 1: Students (*n*=2,239)								
Sex	-0.34	0.20	-0.17	0.07*	0.06	0.07	-0.19	0.15
SES	0.27	0.13*	0.11	0.04*	0.14	0.04***	0.24	0.10*
Prior Achievement	0.54	0.02***	0.28	0.02***	0.28	0.02***	0.52	0.02***
Level 2: Classes (*n*=39)								
Reform	-1.88	0.37***	-1.34	0.16***	-0.33	0.10**	-0.09	0.27
Number of episode	0.13	0.08	0.03	0.04	0.03	0.02	0.11	0.05*
Turns	-0.40	0.11**	-0.21	0.06**	-0.09	0.03**	-0.10	0.08
Individual students involved	1.12	0.38**	0.61	0.16**	0.26	0.09**	0.17	0.23
Whole class involved	0.45	0.25	0.33	0.10**	0.07	0.06	-0.02	0.18
Curriculum*Number of episode	0.40	0.23	0.14	0.10	0.05	0.05	0.24	0.13
Curriculum*Turns	-1.05	0.42*	-0.80	0.22**	-0.06	0.10	-0.15	0.30
Curriculum*Individual students	1.13	0.40**	0.62	0.17**	0.13	0.09	0.26	0.25
Curriculum*Whole class involved	0.29	0.30	0.29	0.13*	-0.00	0.07	-0.03	0.22

Table 12 Results of Two-level Analysis of the Cognitive Learning Outcomes in Relation to the Structural Features of Instructional Explanation (reform group, *n*= 1,197)

	Cognitive Achievement		Calculation		Simple Problem Solving		Complex Problem Solving	
Variable	Coefficient	s.e.	Coefficient	s.e.	Coefficient	s.e.	Coefficient	s.e.
Level 1: Students								
Sex	-0.12	0.30	-0.17	0.09	0.07	0.09	-0.07	0.20
SES	0.35	0.15*	0.19	0.06**	0.23	0.04***	0.17	0.11*
Prior Achievement	0.54	0.02***	0.32	0.03***	0.28	0.02***	0.49	0.03***
Level 2: Classes (*n*=20)								
Number of episode	0.25	0.14	0.08	0.07	0.04	0.03	0.19	0.07*
Turns	-0.67	0.16**	-0.42	0.11**	-0.11	0.04*	-0.12	0.11
Individual students	2.16	0.61**	1.22	0.32**	0.39	0.14*	0.35	0.32
Whole class involved	0.67	0.34	0.57	0.19*	0.07	0.08	-0.07	0.24

Table 13 Results of Two-level Analysis of the Cognitive Learning Outcomes in Relation to the Structural Features of Instructional Explanation (non-reform group, *n*= 1, 042)

	Cognitive Achievement		Calculation		Simple Problem Solving		Complex Problem Solving	
Variable	Coefficient	s.e.	Coefficient	s.e.	Coefficient	s.e.	Coefficient	s.e.
Level 1: Students (*n*=1,042)								
Sex	-0.60	0.27*	-0.15	0.10	0.05	0.09	-0.38	0.22
SES	0.18	0.22	-0.01	0.06	0.02	0.06	0.38	0.16*
Prior Achievement	0.55	0.03***	0.23	0.02***	0.29	0.03***	0.56	0.03***
Level 2: Classes (*n*=19)								
Number of episode	0.01	0.08	-0.01	0.02	0.02	0.02	0.01	0.05
Turns	-0.10	0.15	0.01	0.04	-0.07	0.04	-0.05	0.11
Individual students	-0.01	0.47	-0.00	0.11	0.12	0.12	-0.11	0.35
Whole class involved	0.17	0.37	0.05	0.09	0.04	0.09	0.03	0.28

Concerning the effects of qualitative indicators on the students' cognitive outcomes, the results showed a significant main effect of IE on students' general cognitive achievement, simple problem solving and complex problem solving. Nevertheless, no effect was found on students' calculation. Specifically, teachers' appropriate response to student answers in explanation, had a positive effect on students' general cognitive achievement (effect size=0.87, $p<.05$) and simple problem solving (effect size=0.23, $p<.05$). Mathematical language encouraged for students to use had a positive effect on students' simple problem solving (effect size=0.53, $p<.01$) and complex problem solving (effect size=1.23, $p<.01$).

Out of expectation, however, the results also showed that the use of multiple solutions had a negative effect on students' general cognitive achievement (effect size=-1.05, $p<.05$), simple problem solving (effect size=-0.34, $p<.01$), and complex problem solving (effect size=-0.89, $p<.01$). Likewise, teachers' level of follow up question in IE negatively affected students' simple problem solving (effect size=-0.49, $p<.05$). Teachers' use of mathematical language had a negative association with students' simple problem solving (effect size=-0.50, $p<.01$) and complex problem solving (effect size=-0.89, $p<.05$).

We also found significant interaction effects between curriculum and the qualitative indicators of IE (see Table 14). The results indicated that qualitative indicators were more sensitive to student learning in the nonreform classes than reform classes (see Table 15 and 16). More significant effects were observed in the nonreform classes. Notably, although most of the effects were statistically insignificant in the reform classes, the patterns of the effects were consistent between the two groups to a large extent, such as mathematical language used by the teachers and students, the use of multiple solutions, level of follow-up questions, and teachers' response to student answers. Moreover, in the nonreform classes, teachers' use of general concepts and principles had a positive effect on students' complex problem solving (effect size=1.66, $p<.05$), while general concepts and principles encouraged for students to use had a negative effect on students' general cognitive achievement (effect size=-4.78, $p<.01$), simple problem solving (effect size=-1.66, $p<.05$) and complex problem solving (effect size=-3.95, $p<.001$). However, the pattern of the effect was the opposite in the reform classes though statistically insignificant.

Table 14 Results of Two-level Analysis of the Cognitive Learning Outcomes in Relation to the Quality of Instructional Explanation (*n*=2,239)

	Cognitive Achievement		Calculation		Simple Problem Solving		Complex Problem Solving	
Variable	Coefficient	s.e.	Coefficient	s.e.	Coefficient	s.e.	Coefficient	s.e.
Level 1: Students (*n*=2,239)								
Sex	-0.34	0.21	-0.17	0.07*	0.06	0.06	-0.19	0.15
SES	0.26	0.13	0.11	0.05*	0.14	0.04**	0.24	0.10*
Prior Achievement	0.54	0.02***	0.28	0.02***	0.28	0.02***	0.52	0.02***
Level 2: Classes (*n*=39)								
Reform	-1.47	0.51**	-0.78	0.19**	-0.34	0.18	-0.33	0.32
Mathematical language used by the teacher	-0.99	0.51	-0.10	0.24	-0.50	0.15**	-0.89	0.34*
Mathematical language encouraged for students to use	1.27	0.80	0.07	0.43	0.53	0.18**	1.23	0.38**
General concepts and principles used by the teacher	0.96	0.99	0.46	0.44	0.11	0.28	0.39	0.55
General concepts and principles encouraged for students to use	-1.53	1.00	0.39	0.48	-0.50	0.28	-1.51	0.50**
Use of multiple solutions	-1.05	0.41*	0.02	0.17	-0.34	0.11**	-0.89	0.29**
Follow-up question	-1.31	0.79	-0.62	0.47	-0.49	0.19*	-0.58	0.36
Response to student answers	0.87	0.41*	0.25	0.20	0.23	0.09*	0.45	0.27
Curriculum*Mathematics language used by the teacher	0.32	0.18	-0.02	0.08	0.05	0.05	0.34	0.12*
Curriculum*Mathematics language for the students	-0.14	0.26	0.22	0.15	-0.08	0.06	-0.27	0.12*
Curriculum*General concepts and principles for the teacher	-0.73	0.50	0.02	0.22	-0.06	0.14	-0.63	0.27*
Curriculum*General concepts and principles encouraged students to use	1.25	0.40**	-0.20	0.19	0.32	0.12*	1.19	0.22***
Curriculum*Multiple solutions	0.37	0.190	-0.22	0.07**	0.10	0.05	0.56	0.14**
Curriculum*Follow-up question	0.05	0.20	0.03	0.12	0.05	0.05	-0.10	0.10
Curriculum*Response to student answers	0.33	0.24	0.04	0.11	0.13	0.06*	0.15	0.15

Table 15 Results of Two-level Analysis of the Cognitive Learning Outcomes in Relation to the Quality of Instructional Explanation (reform group, *n*= 1,197)

Variable	Cognitive Achievement		Calculation		Simple Problem Solving		Complex Problem Solving	
	Coefficient	s.e.	Coefficient	s.e.	Coefficient	s.e.	Coefficient	s.e.
Level 1: Students								
Sex	-0.12	0.30	-0.17	0.09	0.07	0.09	-0.07	0.20
SES	0.32	0.15*	0.19	0.06**	0.23	0.04***	0.17	0.11*
Prior Achievement	0.54	0.02***	0.32	0.03***	0.28	0.02***	0.49	0.03***
Level 2: Classes (*n*=20)								
Mathematical language used by the teacher	-0.11	0.72	-0.12	0.42	-0.36	0.15*	0.04	0.42
Mathematical language encouraged for students to use	0.88	1.36	0.63	0.79	0.27	0.26	0.49	0.56
General concepts and principles used by the teacher	-0.44	1.51	0.48	0.84	-0.04	0.27	-0.81	0.78
General concepts and principles encouraged for students to use	1.34	1.40	-0.08	0.79	0.26	0.24	1.22	0.74
Use of multiple solutions	-0.34	0.60	-0.42	0.27	-0.14	0.08	0.17	0.39
Follow-up question	-1.09	1.41	-0.38	0.89	-0.23	0.31	-0.99	0.62
Response to student answers	1.40	0.70	0.27	0.40	0.43	0.11**	0.71	0.40

Table 16 Results of Two-level Analysis of the Cognitive Learning Outcomes in Relation to the Quality of Instructional Explanation (non-reform group, *n*= 1, 042)

Variable	Cognitive Achievement		Calculation		Simple Problem Solving		Complex Problem Solving	
	Coefficient	s.e.	Coefficient	s.e.	Coefficient	s.e.	Coefficient	s.e.
Level 1: Students (*n*=1,042)								
Sex	-0.60	0.27*	-0.15	0.10	0.05	0.09	-0.38	0.22
SES	0.18	0.22	-0.01	0.06	0.02	0.06	0.38	0.16*
Prior Achievement	0.55	0.03***	0.23	0.02***	0.29	0.03***	0.56	0.03***
Level 2: Classes (*n*=19)								
Mathematical language used by the teacher	-1.88	0.78*	-0.14	0.26	-0.69	0.26*	-1.67	0.59*
Mathematical language encouraged for students to use	1.78	0.85	-0.26	0.35	1.01	0.28**	1.66	0.53*
General concepts and principles used by the teacher	2.35	1.30	0.36	0.35	0.23	0.51	1.66	0.74*
General concepts and principles encouraged for students to use	-4.78	1.46**	0.43	0.53	-1.66	0.53*	-3.95	0.73***
Use of multiple solutions	-1.86	0.59*	0.29	0.19	-0.68	0.21**	-1.85	0.45**
Follow-up question	-1.49	0.70	-0.71	0.27*	-0.66	0.25*	-0.16	0.42
Response to student answers	0.31	0.40	0.18	0.15	-0.03	0.16	0.19	0.30

2.2 Effects of Instructional Explanation on the Affective Learning Outcomes

The results of the HLM analysis with the affective learning outcomes—general affective achievement, expressed interest in learning mathematics, mathematics classroom participation, views of mathematics, and views of learning mathematics-are presented in Table 17, 18, 19. The results are based on 2,239 students of the whole sample, 1,197 reform group students, and 1,042 nonreform group students, respectively.

Students' prior achievement showed a positive effect on students' affective learning outcomes. Individual SES positively affected students' classroom participation in mathematics classes. And curriculum in use did not show significant effect on students' affective learning outcomes.

The main effects of structural features of instructional explanation on students' affective learning outcome were insignificant. However, a significant interaction effect was found between curriculum and the structural features of IE. In contrast to the results concerning the cognitive learning outcomes, most of the main effects on the affective outcomes occurred in the nonreform classes. And the results revealed an opposite pattern for the effects of the structural features on students' affective learning outcomes between the reform and nonreform classes. More specifically, in the reform classes, the number of turns had a negative association with students' general affective achievement (effect size=-0.18, $p<.05$), expressed interest in learning mathematics (effect size=-0.06, $p<.05$), views of mathematics (effect size=-0.05, $p<.05$), and views of learning mathematics (effect size=-0.05, $p<.05$). However, the turns had a positive association with general affective achievement (effect size=0.10, $p<.01$), expressed interest in learning mathematics (effect size=0.04, $p<.01$) and classroom participations (effect size=0.04, $p<.01$) in the nonreform classes. This pattern also occurred in the relationship between individual students / whole class students involved in instructional explanation and students' affective outcomes. In the nonreform classes, individual students as well as whole class students involved in an episode of IE were negatively associated with students' affective learning outcomes. However, the associations of these two sets of variables were positive in the reform classes, although they were statistically insignificant. These results suggest that the extended discourse in instructional explanation benefited more for the students of nonreform classes than those of the reform classes on the affective learning outcomes. Out of the expectation, the qualitative indices of IE did not show effect on the students' affective learning outcomes.

Table 17 Results of Two-level Analysis of the Affective Learning Outcomes in Relation to the Structural Features of Instructional Explanation (*n*=2,239)

	Affective Achievement		Interest		Classroom Participation		Views of Mathematics		Views of Learning Mathematics	
Variable	Coefficient	s.e.	Coefficient	s.e.	Coefficient	s.e.	Coefficient	s.e.	Coefficient	s.e.
Level 1: Students (*n*=2,239)										
Sex	0.14	0.09	0.13	0.03***	0.11	0.04**	-0.05	0.02*	-0.04	0.03
SES	0.10	0.06	0.03	0.02	0.09	0.02***	0.02	0.01	0.02	0.02
Prior Achievement	0.67	0.02***	0.59	0.03***	0.55	0.02***	0.47	0.02***	0.46	0.02***
Level 2: Classes (*n*=39)										
Reform	0.08	0.15	0.06	0.05	-0.02	0.05	0.02	0.04	-0.01	0.04
Number of episode	0.02	0.04	0.02	0.01	-0.00	0.01	0.01	0.01	0.02	0.01
Turns	-0.04	0.04	-0.01	0.01	0.00	0.01	-0.02	0.01	-0.02	0.01
Individual students	-0.06	0.14	-0.06	0.05	-0.04	0.04	0.04	0.04	0.02	0.04
Whole class involved	-0.02	0.09	-0.02	0.03	-0.03	0.03	0.01	0.02	0.01	0.02
Curriculum* Number of episode	-0.06	0.10	-0.02	0.03	-0.05	0.03	0.02	0.03	0.02	0.03
Curriculum* Turns	-0.50	0.15**	-0.17	0.05**	-0.13	0.04**	-0.11	0.04**	-0.10	0.04*
Curriculum* Individual students	0.27	0.14	0.07	0.05	0.10	0.03*	0.05	0.04	0.03	0.04
Curriculum* Whole class involved	0.39	0.11**	0.13	0.04**	0.11	0.03**	0.07	0.03*	0.06	0.03

Table 18 Results of Two-level Analysis of the Affective Learning Outcomes in Relation to the Structural Features of Instructional Explanation (reform group, *n*= 1,197)

	Affective Achievement		Interest		Classroom Participation		Views of Mathematics		Views of Learning Mathematics	
Variable	Coefficient	s.e.	Coefficient	s.e.	Coefficient	s.e.	Coefficient	s.e.	Coefficient	s.e.
Level 1: Students (*n*=1,197)										
Sex	0.09	0.12	0.11	0.04**	0.13	0.04**	-0.08	0.03**	-0.07	0.03*
SES	0.17	0.07*	0.04	0.02*	0.10	0.03**	0.05	0.01**	0.03	0.02
Prior Achievement	0.67	0.03***	0.59	0.04***	0.56	0.01***	0.51	0.04***	0.48	0.03**
Level 2: Classes (*n*=20)										
Number of episode	0.00	0.07	0.01	0.02	-0.02	0.02	0.01	0.02	0.02	0.02
Turns	-0.18	0.08*	-0.06	0.02*	-0.03	0.02	-0.05	0.02*	-0.05	0.02*
Individual students	0.21	0.25	0.01	0.08	0.06	0.07	0.10	0.08	0.05	0.07
Whole class involved	0.30	0.18	0.08	0.06	0.06	0.05	0.07	0.05	0.06	0.04

Table 19 Results of Two-level Analysis of the Affective Learning Outcomes in Relation to the Structural Features of Instructional Explanation (non-reform group, *n*= 1, 042)

	Affective Achievement		Interest		Classroom Participation		Views of Mathematics		Views of Learning Mathematics	
Variable	Coefficient	s.e.	Coefficient	s.e.	Coefficient	s.e.	Coefficient	s.e.	Coefficient	s.e.
Level 1: Students (*n*=1,042)										
Sex	0.19	0.15	0.14	0.05**	0.08	0.06	-0.00	0.03	-0.01	0.04
SES	0.03	0.09	0.01	0.03	0.07	0.03*	-0.01	0.02	0.01	0.01
Prior Achievement	0.65	0.03***	0.58	0.04***	0.54	0.03***	0.43	0.03***	0.43	0.03***
Level 2: Classes (*n*=19)										
Number of episode	0.05	0.02	0.03	0.01	0.01	0.01	0.00	0.01	0.01	0.00
Turns	0.10	0.03**	0.04	0.01**	0.04	0.01**	0.01	0.01	0.01	0.01
Individual students	-0.33	0.09**	-0.12	0.04**	-0.14	0.03***	-0.01	0.02	-0.02	0.02
Whole class involved	-0.36	0.06***	-0.13	0.03**	-0.13	0.02***	-0.05	0.01**	-0.04	0.01*

Table 20 Results of Two-level Analysis of the Affective Learning Outcomes in Relation to the Quality of Instructional Explanation (*n*=2,239)

	Affective Achievement		Interest		Classroom Participation		Views of Mathematics		Views of Learning Mathematics	
Variable	Coefficient	s.e.	Coefficient	s.e.	Coefficient	s.e.	Coefficient	s.e.	Coefficient	s.e.
Level 1: Students (*n*=2,239)										
Sex	0.14	0.09	0.13	0.03***	0.11	0.04 **	-0.05	0.02*	-0.04	0.03
SES	0.10	0.06	0.03	0.02	0.08	0.02**	0.02	0.01	0.02	0.02
Prior Achievement	0.67	0.02***	0.58	0.03***	0.55	0.02***	0.47	0.02***	0.46	0.02**
Level 2: Classes (*n*=39)										
Reform	0.08	0.28	0.13	0.11	-0.02	0.10	0.02	0.05	-0.08	0.06
Mathematical language used by the teacher	0.00	0.30	-0.14	0.12	0.07	0.09	0.03	0.07	-0.03	0.06
Mathematical language encouraged for students to use	0.01	0.40	-0.03	0.15	-0.07	0.12	-0.02	0.09	0.121	0.09
General concepts and principles used by the teacher	-0.09	0.50	0.14	0.19	-0.12	0.15	-0.02	0.12	0.05	0.11
General concepts and principles encouraged for students to use	-0.05	0.67	-0.02	0.27	0.10	0.21	0.01	0.12	-0.22	0.12
Use of multiple solutions	-0.14	0.23	-0.03	0.09	-0.02	0.07	0.05	0.05	-0.07	0.05
Follow-up question	-0.30	0.39	-0.15	0.14	0.02	0.11	-0.02	0.09	-0.09	0.09
Response to student answers	0.00	0.26	0.05	0.10	-0.02	0.08	0.01	0.06	-0.05	0.05
Curriculum *Mathematics language used by the teacher	0.04	0.11	-0.00	0.04	0.04	0.03	0.01	0.03	-0.01	0.02
Curriculum *Mathematics language for the students	-0.11	0.14	-0.02	0.05	-0.05	0.04	-0.01	0.03	-0.05	0.03
Curriculum *General concepts and principles for	0.01	0.25	0.01	0.10	-0.07	0.08	0.01	0.06	0.06	0.06

the teacher										
Curriculum *General concepts and principles encouraged students to use	-0.13	0.29	-0.10	0.12	-0.09	0.09	-0.02	0.05	0.02	0.05
Curriculum *Multiple solutions	0.07	0.11	0.02	0.04	0.02	0.03	-0.01	0.02	0.01	0.02
Curriculum *Follow-up question	-0.04	0.10	-0.04	0.04	0.00	0.03	-0.01	0.02	0.02	0.02
Curriculum *Response to student answers	0.08	0.15	0.01	0.06	0.07	0.05	-0.01	0.03	0.01	0.03

Because the proportion of the variance in student achievement gains at the classroom level was small in comparison to the proportion at the student level, we also examined the effect of IE on student learning outcomes with multiple regression. Results of multiple regression were consistent with the results of the HLM analyses.

In sum, we observed the significant effect of IE on student learning. In particular, more effects were found on the students' cognitive learning outcomes (e.g., general cognitive achievement, calculation, and simple problem solving) than on the affective learning outcomes. Moreover, curriculum was an important moderator between instructional explanation and student learning. The results revealed that the indicators of IE were found more sensitive to student learning in the non-reform classes than to that of the reform classes.

2.3 Moderate Effects of Instructional Explanation on Student Learning Outcomes

As indicated above, this study also aimed to explore the effect of students' individual factors on the link between IE and student learning outcomes. Since the variables of instructional explanation were various, here, we selected the use of multiple solutions as an example to illustrate the relationship between students' SES, prior achievement, teachers' IE and student learning outcomes. The reason that we chose the variable of use of multiple solutions was because a few researchers had done some seminal works (e.g., Große & Renkl, 2007; Lubienski, 2002), and these works would help us to make some connections and better understand the relationship between the variables. As the sequence of analysis above, the researcher firstly conducted the analysis with 2,239 students. If an interaction effect between curriculum and the use of multiple solutions was found, then the analysis would be conducted further with 1,197 reform students and 1,042 non-reform students respectively. The equations for the two-level models are as follows.

Level-1 model at student level:

$Y_{ij}=\beta_{0j}+\beta_{1j}(\text{sex})+\beta_{2j}(\text{SES})+\beta_{3j}(\text{prior achievement})+r_{ij}$

Level-2 model at class level:

$\beta_{0j}=\gamma_{00}+\gamma_{01}(\text{curriculum})+\gamma_{02}(\text{richness})+\gamma_{03}(\text{coherence})+\gamma_{04}(\text{curriculum * richness})+\gamma_{05}(\text{curriculum * coherence})+\mu_{0j}$

$\beta_{1j}=\gamma_{10}+\mu_{1j}$

$\beta_{2j}=\gamma_{20}+\gamma_{21}(\text{use of multiple solutions})+\gamma_{22}(\text{curriculum * use of multiple solutions})+\mu_{2j}$

$\beta_{3j}=\gamma_{30}+\gamma_{31}(\text{use of multiple solutions})+\gamma_{32}(\text{curriculum * use of multiple solutions})+\mu_{3j}$

Results of the analysis revealed a moderate effect of student's SES on the relationship between the use of multiple solutions and students' general affective achievements (effect size=0.32, $p<.01$), views of mathematics (effect size=0.07, $p<.01$), and views of learning mathematics (effect size=0.09, $p<.01$). The use of multiple solutions caused more negative effects to high SES students than low SES students. And the effect was consistent in the reform and non-reform classes.

We found that the relationship between the use of multiple solutions and students' performance on simple problem solving and general cognitive achievement was affected by students' prior achievement, but not for the other cognitive outcomes and affective learning outcomes. And the moderate effect of prior achievement was only found in the non-reform classes. Specifically, students' prior achievement increased the effects of the use of multiple solutions on student general cognitive achievement (effect size=0.13, $p<.01$), and simple problem solving (effect size=0.18, $p<.01$). The use of multiple solutions was more likely to affect high-achieving students' general cognitive achievement, and simple problem solving.

3. Discussion

The current study examined one of the critical incidents in classroom discourse—instructional explanation, and attempted to identify the features of IE that might served as proximal indices of student learning. For this purpose, the IE was investigated in terms of its structural features and quality. With respect to the quality, its indicators were mainly arrived at based on the well-established research findings of the empirical studies. Notably, as one of the parts in classroom discourse, the effect of IE on student learning was affected by many other factors, such as curriculum in use and students' prior achievement. Teachers' use of IE was influenced by numbers of factors, too. Followed is a discussion of the results.

3.1 Effects of Instructional Explanation on Cognitive Learning Outcomes

As it was hypothesized, the results showed that the structural features of instructional explanation affected students' cognitive learning outcomes. However, the effects varied. More effects were found on calculation and simple problems solving. Specifically, the more turns per episode of IE, the more negative effects it had on students' general cognitive achievement, calculation, and simple problem solving. The number of individual students as well as the frequency of the whole class students involved in the generation of IE positively affected students' cognitive learning outcomes. The number of identified episode of IE showed no effect on students' cognitive learning, except for the complex problem solving. The results suggest the impact of the structure of IE on student learning, especially for the application of basis mathematics knowledge and skills. However, it did not necessarily result in desired outcomes for student learning. It was the students' engagement that mattered to student learning, no matter whether engagement in the form of individual or in whole class (Cazden, 2001).

Consistent with the hypotheses, the qualitative indicators of IE showed a significant effect on students' cognitive learning outcome, especially on simple problem solving and complex problem solving, which required the skills of carrying out mathematical explanations and communications. This result was corroborated with the previous study, which revealed that reform-oriented instruction showed stronger relationships with open-ended measures than with multiple-choice tests than with procedural skills in mathematics (Le, Lockwood, Hamilton, & Martinez, 2009). We found that teachers' recognition and expansion of students' contribution as well as teacher's encouraging students to use mathematical language had a positive effect on students' cognitive learning outcomes. However, teachers' use of mathematical language, the use of multiple solutions, and teachers' follow-up question had negative effects on students' cognitive outcomes, which were out of expectation. Three reasons presumably accounted for the results.

Firstly, the quality of implementing these "effective" teaching practices is important. According to the descriptive results, we observed that the teachers scored ranged from 1 to 2.56 on the variable of the use of multiple solution methods, which meant when teachers used multiple solutions, they rarely made comparisons or connections between the solutions in their instructional explanation. They focused on the solutions' diversity, without paying enough attention to the optimization of a solution. The findings were also reported by other researchers (e.g., Yu, 2003; Huang, 2005). In this case, the learners might be urged to put more cognitive effort into integrating many sources of information, which was detrimental to their learning (Sweller, 2005; Rittle-Johnson & Star, 2009). Some empirical studies supported this argument. For example, Jitendra and colleagues compared the effect of single and multiple strategy instruction on third-grade students' mathematical problem solving. They found that, the schema-based instruction (a single strategy) was more effective than multiple strategy instruction[13] in enhancing students' mathematical word problem-solving skills at posttest and maintenance (Jitendra, Griffin, Haria, Adams, & Kaduvettoor, 2007).

Likewise, although follow-up questions were posed in teachers' instructional explanation, most of them were not connected between one another. And they did not facilitate students thinking substantially (mean=2.93, range=1~4). The results were also found in our previous analysis with 58 teachers (Ni, et al., 2009). From the

[13] Multiple strategy instruction refers to the provision of different strategies without further connection, such as using objects, drawing a diagram, and using data from a graph.

transcription materials (also see the example in last section), we observed that lots of teachers focused on engaging more students into the discussions by asking the open questions like “Anything else?”, “What about the other students’ opinions?”, “What do you think?”, and “Who would like to say a little more about this question?”. Or the teachers paid their attention to looking for the right answers through continued questions, without dealing with the students’ errors (see the example below). These questions were not followed up with increasing cognitive demands, such as asking students to differentiate between responses or helping students to clarify the misunderstandings. Hence, like the use of multiple solutions discussed above, teachers’ follow-up questions seemed to be ended in themselves and turned out to be formality.

Example Follow-up Questions to Look for the Right Answers in IE.

1. Students (together): Xiao Lin saved 7 tons water in 3.5 months. How many tons did he save for every month on average?
2. Teacher: He saved 7 tons water in 3.5 months， how many tons did he save for every month on average? Liu Yuan.
3. Student (Liu): Divided 3.5 by 7.
4. Teacher: **Divided 3.5 by 7?** We are saying every month on average. Think about it again. **Who have different idea? Well, you please.**
5. Student 11: Multiplied 3.5 by 7.
6. Teacher: Oh （^-^）**, multiplied 3.5 by 7? Think about it further. Qu.**
7. Student (Qu): Divided 7 by 3.5.
8. Teacher: Divided 7 by 3.5. **You please.**
9. Student 12: Divided 7 by 3.5.
10. Teacher: **Why divided 7 by 3.5?** Ok, who can explain why we divided 7 by 3.5? Wang Hao, **could you say something?**
11. Student (Wang): Because Xiao Lin saved 7 tons water every month……
12. Teacher: Did he save 7 tons water every month?
13. Student (Wang): Oh， he saved 7 tons water in 3.5 month, so the water he saved every month is 7……
14. Teacher: **Ok, Chen Yan.**
15. Student (Chen): Because it was divided a number into several sections, and we got one of them.
16. Teacher: Oh, it was divided a number into several sections equally, and we got……
17. Students (together): one of the sections.
18. Teacher: So we use division. Now, understand?
19. Students (together): Yes. (Miss Li and her fifth graders on a dialogue with division of fraction, line 31-49)

The issues discussed above might also explain why the turns of IE episodes had a negative effect on students' cognitive learning outcomes as well as on affective learning outcomes (though statistically insignificant). Although the teachers used follow-up questions and the discourse was extended, most of them were not substantial in mathematics and did not facilitate the students' understanding sufficiently. Hence, the turns of episodes did not enhance desired learning outcomes in students.

Secondly, the other factors, such as students' individual factors, might affect the relationship between the uses of IE and student learning. For example, our analysis showed that, the use of multiple solutions without comparison and connections caused more negative effect to high-achieving students, which might be related to expertise-reversal effect (also the so-called redundancy effect) (Kalyuga, Ayres, Chandler, & Sweller, 2003; Sweller, 2005; Sweller et al., 1998). The effect can be explained that learners who have already constructed problem-solving schemata might not need instructional guidance any more. Hence, when continuously presented with multiple solutions, learners have to devote their attention to redundant information, which might result in suboptimal learning processes. However, for the low-achieving students, there might be another story. The use of multiple solutions presumably was cognitive demanding, and thus providing one solution was more favorable (Große & Renkl, 2007).

Finally but not the least, a more deeper reason presumably was related to Chinese classroom cultures. In this study, the criteria for good IE did not yield the expected outcomes. Our previous analysis with 58 teachers and their 3,184 students also showed that, some reform-oriented practices, such as probing and use of peer assessment, had a negative effect on students' affective learning outcomes (Ni, et al., 2009). It seems that these advocated instructional practices in western literature might not be necessarily effective in Chinese classes.

In Chinese culture, there is a famous saying that "think more, do more, and talk less". The learners are used to listening and thinking quietly and implicitly in classes. Hence, the discussion-oriented instruction might make them feel pressed and thus turned out to be counterproductive. For example, our findings revealed that, although some of the identified criteria showed netative effects on student learning in general, their effect appeared more obvious in the nonreform classes than the reform classes. Also, we observed that the nonoreform students benefited from the practices of IE where the teacher played most of the parts. For them, the less teacher asked individual students and whole class students to involve in the IE, the better they expressed interest in learning mathematics and classroom participation. Moreover, when making connections to general concepts and principles in mathematics, it showed to be more beneficial for the non-reform students when the connections were made by the teachers. The connection encouraged students to make turned out to be negatively related to the students' general cognitive achievement, simple problem solving and complex problem solving. The results, in part, were because the criteria were more aligned with the reform-oriented pracitce, but the students of the non-reform classes did not receive any training for these practices.

The findings have also raised a few other questions. We observed that the variables of the similar nature did not necessarily result in consistent effects on student learning. Based on the correlation analysis, there was a moderate correlation between teachers' use of mathematical language and their making connection with general concepts and principles in mahtematics (r=0.67). Likewise, the correlation between teachers' making connection with general concepts and students encouraged to make connections was high (r=0.83) . However, the two set variables revealed an opposite effect on students' cognitive learning outcomes. For example, students benefited from their use of mathematical language and teachers' connection making with general concepts and principles in classes. While teachers' use of mathematical language and the making connection with general concepts encouraged for students, showed a negative effect on students' learning. Concerning the results, it seemed more easier for students to use mathematical language than to make connections with general concepts and principles in terms of cognitive demand. But still, why the effect appeared more salient in the non-reform classes? And it was also unclear why teachers' use of mathematical language showed a negative effect both in the reform and the nonrefrom classes. Answers to these questions were not clear. Future research is needed to address more definitively.

3.2 Effects of Instructional Explanation on Affective Learning Outcomes

Out of the expectation, we found no effect of the qualitative indices of IE on students' affective learning outcomes. Two inter-related reasons presumably accounts for the results. Firstly, the indicators of the instructional explanation in the current study concerned more about the cognitive features. And all these indicators were identified as proximal variables of students' learning based on measures of cognitive learning outcomes in the previous studies (Roscoe & Chi, 2007, 2008; Rittle-Johnson & Star, 2007; Leinhardt, 2001). Furthermore, explanations seem to be more associated with our cognitive lives. For example, according to Keil and Wilson (2000), there is a sense both that a given, successful explanation satisfies a cognitive need, and that a questionable or dubious explanation does not. However, there may be also possible that such affective effect was difficult to measure, as affective impact may be less accumulative.

3.3 Discussing the Effects on the Context of Curriculum

Initially, the research proposal only included 29 reform teachers. It aimed to make clear when investigating the relationship between instructional explanation and student learning. However, according to Cohen, Raudenbush, and Ball (2003), "Teaching is what teachers do, say, and think with learners, concerning content, in particular organizations and other environments, in time"(p.124). The conceptualization pushes researchers to design studies capable of providing good evidence about interactions among students, teachers, and content in the broader context of classrooms and school policies, processes, and composition. Additionally, the previous studies showed the important role of curriculum in the link between instructional practices and student learning outcomes (Ni, et al., 2009; McCaffrey, Hamilton, Stecher, Klein, Bugliari, & Robyn, 2001; Tarr, Reys, Reys, Chavez, Shih, & Osterlind, 2008). For example, McCaffrey and colleagues (2001) found that, use of standards-based or reform practices was positively related to achievement on both the multiple-choice and open-ended questions of the Stanford achievement tests for tenth-grade students in reform-oriented courses, whereas use of reform practices was unrelated to achievement in the more traditional courses. This result was consistent with the findings reported by Tarr and colleagues (2008). The study showed that students were positively impacted on the Balanced Assessment in Mathematics by NSF-funded curricula when coupled with either Moderate or High levels of the Standards-Based Learning Environment (SBLE)[14]. There was no statistically significant impact of NSF-funded curricula on students in classrooms with a Low level of SBLE, and the relationship between publisher-developed textbooks and SBLE was not statistically significant.

Based on these works, we finally decided the research design involved 39 teachers, including 20 teachers who implemented the reform-oriented curriculum and 19 teachers who used the conventional curriculum. All the classes were having new lessons about the number. The results indicated that, the identified indicators of

[14] SBLE refers to the environment that is consistent with the recommendations of NCTM, such as providing opportunities for students to make conjectures about mathematical ideas, asking students to explain their responses or solution strategies, encouraging multiple strategies/perspectives, and valuing students' statements about mathematics and using them to build discussion.

quality of IE showed significant effects on students' general cognitive achievement, simple problem solving and complex problem solving. And the effects appeared more obvious in the classes that using the conventional curriculum than the classes that using the reform-oriented curriculum. Consistent with the previous studies, our findings suggest the interaction between curriculum, instructional practices, and student learning. However, the results were out of the expectation in two respects. On the one hand, the identified indicators of quality of IE, which were more aligned with the reform-oriented practices, did not show a positive effect on students' learning outcomes; on the other, the indicators showed more significant effects in the non-reform classes, and few effects were found in the reform classes. As discussed above, two reasons presumably accounted for the results. Firstly, the low quality of the implementation of instructional practices might not enhance desired learning processes and outcomes in students. In order to facilitate student learning, there was a benchmark for teachers' practices, such as Moderate or High levels of the Standards-Based Learning Environment as reported by Tarr's study (Tarr, et al., 2008). Secondly, lack of training background might make students from the non-reform classes more fragile under the context of reform-oriented practices.

Notably, our study also found that, curriculum type was a significant predictor of students' cognitive learning outcomes, which was not observed in Tarr's study (2008). In addition, the results showed the effect of curriculum on teachers' use of IE. As a whole, these results suggest that when discussing the relation of instructional explanation to student learning, we should not out of the context of the curriculum. Otherwise, the conclusion might be misleading.

Chapter 6 General Discussion and Conclusions

The present study aimed to identify the features of quality of IE that might serve as proximal indices of student learning outcome. In this chapter, a general discussion of the findings is organized into four parts. Firstly, the main findings of the study are presented. Then, the contributions of the study are summarized. Next, questions remaining are proposed. And finally, future directions are discussed.

1. Main Findings of the Study

This study explored the three research quesions, including 1) what were the cognitive features of teachers' instructional explanation in the Chinese primary mathematics classrooms; 2) what was the relationship between teachers' background factors and their use of instructional explanation; and 3) what was the relationship between teachers' instructional explanation and student learning outcomes. In response to these research questions and based on the results from the data analyses, we draw the following conclusions.

1.1 The Cognitive Features of Instructional Explanation in the Chinese Classrooms

In the study, the examination of instructional explanation included its structural features (e.g., the number of identified episodes of IE, the number of turns and students involved in the episode) and quality (e.g., truthfulness, richness, and coherence). For the structural features of Chinese teachers' instructional explanation, the results revealed that the variability in the number of identified episodes of IE among 39 teachers was not small. The range was 9 (4-15). On average, there was 14 turns and 2 individual students involved per episode. And teachers would engage the whole class students about three times in the generation of an instructional explanation.

With respect to the quality of instructional explanation, we found that the Chinese mathematics teachers had extremely high performance on the indices of truthfulness and coherence. More specifically, when constructing an instructional explanation, they were able to provide accurate mathematical knowledge, to use accurate teaching language, and to identify and address student errors appropriately. Moreover, they provided students with relevant and well-organized topic, raised follow-up questions that related to student thinking, and recognized and expanded students' contributions. The teachers had relatively lower scores on the dimension of richness, but still satisfactory. They used a variety of mathematical language and made connections to general concepts and principles when generating instructional explanation. Meanwhile, they encouraged the students to use mathematical language and to make connections with general mathematics concepts. The teachers also used multiple solutions and multiple represenstations in explanation. However, we observed that they rarely made comparisons or connections between multiple solutions and representations purposefully and explicitly.

Lastly, we found that a teacher's performance in generating instructional explanation, including its structural features and quality, was relatively stable across lessons.

1.2 Relationship between the Teachers' Background Factors and Instructional Explanation

In general, we found few relations between teachers' background factors (i.e., years of teaching experience, educational level, mathematical knowledge for teaching, and teachers' belief towards mathematics) and their use of instructional explanation. However, curriculum in use made a difference in the quality of teachers' instructional explanation. The teachers who used the reform-oriented curriculum were more likely to encourage students to use mathematical language, multiple solutions and multiple representations than their counterparts of the non-reform classes. On the other hand, the teachers of non-reform classes were more inclined to use accurate teaching language and make connections with general concepts and principles than their peers of the reform classes. Nevertheless, curriculum in use had no effect on the structural features of instructional explanation.

1.3 Relationship between Instructional Explanation and Student Learning

Teachers' instructional explanation affected student learning outcomes. However, the effect differentiated across students' learning outcomes. It showed more effects on students' cognitive achievement than on the affective achievement. Specifically, the structural features and quality of IE had a significant assocation with students' general cognitive achievement, simple problem solving, and complex problems solving. The qualitative indicators of IE had no effect on students' calculation. Neither the structural features nor quality of IE showed significant main effects on students' affective learning outcomes.

Moreover, the effect of instructional explanation on student learning was moderated by the factor of curriculum. Specifically, instructional exlanation showed more significant effects in the nonreform classes than in the reform classes. Students' SES and prior achievement also moderated the effect of instructional explanation on student learning. Take the use of multiple solutions in instructional explanation as example, it was found that instructional explanation had stronger effects on high SES and high-achieving students. When the teachers used multiple solutions in instructional explanation, it caused more negative effects on high SES students' general affective achievement, including their views of mathematics, and views of learning mathematics. Likewise, the results indicated that the use of multiple solutions caused more negative effects on high-achieving students' general cognitive achievement and simple problem solving.

2. Contributions of the Study

Results of the study are considered to make three specific contributions that are discussed in the following sections.

2.1 Understanding the Classroom Instruction in China

Based on the observation of 110 lessons of 39 teachers, this study systematically described the structural features and quality of instructional explanation in the Chinese mathematics classrooms. The findings provide the meaningful information for knowing and understanding Chinese classroom instruction. They will be discussed in terms of the strength and limitations of the classroom instruction in the following sections.

Firstly, the strength of the Chinese classroom instruction was embodied in its quality teaching and homogeneity between the classes. The results indicated that the Chinese mathematics teachers scored extremely high in accuracy and coherence when using IE. They were able to provide accurate mathematical knowledge, to present with precise mathematical language, and to identify and address student errors appropriately. They were also able to organize IE in a relevant and clear way, as well as to interact with the students coherently. The teachers' performance was stable across lessons. In addition, it was found that the variance between the teachers was very small, which helped make sure that most of the students had the access to the quality teaching. Moreover, the results showed that the teachers' demographic factors (e.g., years of teaching, educational level) had no or low correlations with their accuracy and coherence of IE, which again suggests the homogeneity of the teachers' classroom instruction in general and use of IE in specific. These findings complemented to the previous study, which indicated that all Chinese lesson plans were very similar with details in teaching contents and procedure, in comparison to the extreme variance of their U.S. counterparts (Cai & Wang, 2006). The factors, as indicated above, made two strengths of the Chinese classroom instruction.

Secondly, the findings of the study also reveal the limitations of the instructional practice in Chinese classrooms. Like their peers from the other countries, the Chinese teachers were also confronted with the challenges under the context of curriculum reform. On the one hand, they were promoted to implement more reform-oriented practices, such as inquiry, discourse-intense and the like. Notably, most of these practices were not familiar to the teachers. On the other, they were required to finish the instructional content within the limited time. And they should make sure that most of the students acquired the required knowledge and skills. In front of this dilemma, most of the teachers showed some shortcomings in their classroom teaching. For example, their discourse was less substantial in mathematics on some occasions. As we know, most of the reform-oriented practices were proposed and advocated from the western countries. However, classroom teaching, as part of education, is embedded in a specific culture. How to make use of the Chinese effective teaching approaches as well as the practices advocated in the western world is a key issue for the policy makers, educational researchers and the frontier teachers.

2.2 Understanding the Curriculum Reform

The current study was conducted in a curricular context. The findings also have implications for understanding and evaluating a curriculum reform.

Firstly, the findings of the study highlighted the significance of **the match between curriculum in use and teachers' instructional practices** in curriculum reform. The results showed that, curriculum in use was the only factor, out of the other teachers' background factors (e.g., years of teaching experience and educational level), that significantly affected teachers' use of IE. The teachers, who used new curriculum, adopted more practices that are advocated by the reform, such as the use of multiple solutions and multiple representations in IE. Additionally, we found that the negative effects of IE were more obvious in the non-reform classes than in the reform classrooms. The reason, in part, may be due to the mismatch between curriculum and teachers' use of IE, because most of the indicators of IE measured in the study were reform-oriented. Based on these results, it is reasonable to propose that, when implementing and evaluating a curriculum reform, curriculum and teacher practice should be taken into account simultaneously.

Secondly, the findings revealed **the status of teacher practices** under the context of curriculum reform. On the one hand, it was observed that the teachers, influenced by the merits of the reform, actively and purposefully provided more opportunities for the students to voice their opinions. On the other hand, we observed that because of the teachers' focus on student participation, the instructional explanations provided sometimes did not extend substantially in mathematics. For example, teachers used the strategies of repeatedly asking students' opinions (e.g., "you say it", "what do you think", "why do you do that") and "accepting all answers" as ways of simply encouraging students' participation. They did not differentiate between responses and made appropriate connections. Also, when reading the transcription materials of the lessons, we observed that some teachers asked the students to discuss without giving specific instruction explicitly. The purpose was not clear, and the discussion was ended in itself. Similar findings were also reported by other researchers (e.g., Baxter & Williams, 2010; McClain & Cobb, 2001). And these illustrated the limitations of the teachers when changing their ways of teaching to respond to the curriculum reform. The findings also reveal the professional needs for the teachers, which can help them to develop a more flexible and appropriate ways of teaching.

Finally but not the least, the findings of the study raised the issue of **students' adjustment** in curriculum reform. The negative effect of IE on students' cognitive outcomes was more obvious in the non-reform classes than in the reform classrooms. And the investigation of IE and students' affective achievement indicated that, students of the non-reform classrooms expressed more learning interest and classroom participation, as well as more positive views of mathematics and learning mathematics, when the teachers played more of the role in IE. Under the context of national curriculum reform, the non-reform teachers were inevitably affected by the reform. This could be seen from the result that the teachers, irrespective of the reform classes or non-reform classes, actively encouraged the students to engage in the generation of IE. Confronted with the teachers' reform-oriented (i.e., discourse-intensive) practices, the students might feel the pressure and might not know how to react appropriately. This finding was also reported in our previous analysis with a sample of reform group students, as well as in the other studies (e.g., Bicknell, 1998; Anthony & Walshaw, 2002). For example, in an investigation into elementary school students' response to tasks requiring explanations, Anthony and Walshaw (2002) noted that many students did not know how to explain their mathematical ideas; indeed, several students were ill at ease with the proposition that they share their thinking with others. These findings suggest that, in addition to the curriculum development and teacher training, it is also important to pay more attention to students' adjustment in curriculum reform. For example, the educators should provide explicit guidance for students to learn when and how to contribute to mathematical discussions and what to do as a listener (Walshaw & Anthony, 2008).

2.3 Providing a Useful Tool for Research and Practice in Mathematics Education

The developed instrument to evaluate the quality of IE would provide a useful tool for research and practice in mathematics education. Specifically, the instrument could be used for the researchers to describe and evaluate the quality of mathematics classroom discourse in general and instructional explanation in particular. We argue that instructional explanations make a large part of the classroom discourse, and most of the classroom instruction is carried out by IE. Therefore, the criteria for the evaluation of instructional explanation, including truthfulness, richness, and coherence, would be also appropriate and effective for the evaluation of classroom discourse. Also, the instrument would also serve for mathematics teachers. It would help the teachers know more about his or her quality of instructional explanation, and thus improve their teaching.

3. Questions Remaining

There were some caveats that should be noted in interpreting the results of the study, including the issues about the scale, sample size, and the measurement.

In the current study, we got some unexpected outcomes in terms of the correlation matrix of the scale as well as the relationship between IE and student learning outcomes. To be specific, firstly, the correlations between the items within a dimension were insignificant. For example, when generating instructional explanation, teachers' well-orgainized content was not associated with their coherent interaction with the students. Secondly, some correlations of the items between the dimensions were out of expectation. For instance, a teacher's use of multiple representations was negatively associated with his/her accurate use of teaching language. Thirdly, the indicators within a dimension did not work in the same way when making a link to student learning outcomes. And thus it was not appropriate to combine the indicators together. These results, on the one hand, witness the complexity of teaching in general and instructional explanation in particular; on the other, they suggest some problems of the current scale to evaluate the quality of instructional explanation. Three issues need to be noted that are discussed below.

The first issue is about the structure of the instrument. The current scale was less satisfactory in terms of its structure, which was reflected from the correlation matrix and regression result of the IE and student learning outcome. Also, it is obvious to see that some of the indicators were not differentiated, based on the descriptive result of the cognitive features of IE. The standard deviations were very small. Specifically, most of the teachers scored high on the variables of accuracy (truthfulness) and coherence, whereas they scored low on the variables of use of multiple representations overall. Although the result reflects the truth in Chinese mathematics classrooms, we still need to consider this issue when the instrument mainly serves as a reseach tool.

The second issue is about the definitions of the criteria. It was not easy to define the indicators and the rating criteria. For example, what are the general concepts and principles in mathematics, especially when teaching content is about numbers and their operation, such as division of fraction, least common multiple, and divisor and multiple? How to define whether a teacher's follow-up questions lead student thinking or not? What counts as an appropriate use of multiple solutions? Though we compiled the definitions based on various curriculum and teaching resources, such as teacher guide books, teaching plan, teacher transcription materials, and literature of effective teaching, there were still some limitations.

In addition, the rating scale seems nonlinear for the indicators of richness. Take the use of multiple solutions for example, the scale measures two aspects of the practice—presence and appropriateness. From 1-point to 3-points, it concerns the presence of the use of multiple solutions. That is, whether a teacher uses multiple solutions in IE, and whether he/she engages the students in the use of multiple solutions. For the 4-points of the scale, the focus turns to whether a teacher uses the multiple solutions appropriately and effectively, such as making connections and comparisons between the different solution methods. Consequently, the scale is nonlinear from 3-points to 4-points. In this study, the teachers scored ranging from 1 to 2.56 on the variable of the use of multiple solutions. And the result showed that the use of multiple solutions had a negative effect on student learning. It was speculated that if the teacher could use the strategy appropriately (i.e., score over 3-points), then the direction of the relation might turn out to be the other way. This kind of problem also appeared in the scale of making connections of general concepts and principles and the use of multiple representations (see Appendix III), and thus further revision of the scale, as well as the research that addresses the relationship between IE and student learning more definitively, are needed in future.

The third issue concerns the implementation of the criteria. Even their definitions were relatively clear, it was still not easy when using the criteria. For example, how to evaluate a teacher's performance, when he/she asks several consecutive follow-up questions in an episode of IE? How to rate whether a teacher discusses the connections between multiple representations thoroughly or not? It is more cognitive demanding when doing the rating between 3-points and 4-points, which may increase measurement errors. In addition, we suspect that coders must possess high levels of mathematical knowledge, and knowledge of mathematics for teaching, in order to evaluate the quality of IE accurately. Confined to the researcher's background in the current study, bias might occur in the process of using the criteria.

In addition to the problems of the scale for IE, another two caveats also should be noted. One limitation of the current study stems from the small sample size. There were only 39 teachers in this study. However, many variables were examined, and thus errors were likely to increase due to the complexity of the HLM models. Also, the content of the lessons were confined to the numbers, such as division of fraction, least common multiple, and divisor and multiple. The results may not be generalized to Chinese lessons or teachers in general. Another limitation is about the instruments to measure mathematics knowledge for teaching and teacher beliefs about mathematics. Both measures were not shown to be sensitive to differentiate the teachers. The measurement of Chinese teachers' mathematical knowledge for teaching may need to concentrate on teachers' pedagogical knowledge in teaching mathematics in future studies. Likewise, the measure of teacher belief should also consider how teachers view students' ability and how teachers view learning mathematics in their instructional practice. For the student measures, there was a lack of alignment between the content of the achivement tests and the curriculum delivered in the classrooms, which probably undermined the the teacher-qualtiy effect size. Consequently, the measures relevant to the specific content of the local curricula could be more appropriate in future research.

4. Future Directions

4.1 The Scale to Evaluate the Quality of Instructional Explanation

As discussed in the previous section, there were several limitations in the current scale to evaluate the quality of IE. Accordingly, several issues need to be considered in future studies.

Firstly, with respect to the structure of the scale, more revisions are needed for each indicator. Specifically, how to make sure that the items under a specific dimension work in the same level and in the same direction? For the same level, it means that the indicators of IE should be parallel to each other, such as use of multiple solutions and multiple representations. With respect to the same direction, it refers to the effect of the indicators on student learning outcomes.

Secondly, the definitions of the criteria need to be more clear and differentiated. For example, how to decide that whether a teacher discusses the connection of multiple representations thoroughly or not? How to make a decision that whether a teacher's follow-up questions facilitate the students' thinking or not? When reading the teachers' transcription materials, we observed that lots of reform teachers asked open questions like "Anything else?", "What about the other students' opinions?", "What do you think?", and "Who would like to say a little more about this question?". These questions appeared to lead the students' thinking, but actually did not turn out to be good to student learning specific content and skills in the classes. The teacher did not synthesize or highlight the key points of these questions. In the non-reform classrooms, although the teachers asked many closed questions, such as request for the short answer or brief reasoning. The questions were unfolded step by step, which helped to hold the students' attentions. We did not differentiate both types of questions in the current scale. More revisions are needed in this regard.

Also, since some of the scales measured both the presence and appropriateness of the practice simultaneously, it is suggested that the two aspects should be separated. But following this, some questions need to be answered -- how to define the level of appropriateness of teacher practices. Should we make the decision according to the students' needs or the teaching goals of a specific lesson? Also, some practices (e.g., follow-up questions, a teacher's response to student answers) occured several times in an IE episode, based on what criteria should we refer to when rating the appropriateness? It is much more difficult to rate the appropriateness than the presence for a researcher. Additionally, it seems that the double criteria of presence and appropriateness are not applicable for some of the items, such as well-organized instructional content and clear teaching language on the dimension of coherence. Therefore, how to deal with the issues is worthwhile to consider in future studies.

4.2 The Relationship between Curriculum, Instructional Explanation, and Student Learning Outcomes

Consistent with the previous studies, our findings suggest the interaction between curriculum, instructional practices, and student learning. However, the results were out of the expectation in two respects. On the one hand, some of the identified indicators of quality of IE did not show positive effects on students' learning outcomes; on the other, the indicators showed more significant effects in the non-reform classes. Few effects were found in the reform classes. Although we speculated that one of the reasons might attribute to the low quality of implementing the instructional practices, lack of training backgrounds for the non-reform students under the context of these reform-oriented practices, as well as the problems of the scale. Still, some questions need to be answered in future studies.

For example, the previous studies suggest that the match between curriculum and instructional practices/learning environment enhance desired learning outcomes in students (McCaffrey et al., 2001; Tarr, et al., 2008). Specifically, reform practices may be more effective when they are used in the context of a course that is designed to be consistent with the principles of standards-based reform. In the current study, although we found that the identified indicators of quality of IE, which were aligned with the reform-oriented practices, showed less negative effects on student learning in the reform classes than the non-reform classes. We are still not sure whether or not the reform teachers used the practices appropriately, then whether or not the results would turn out to be beneficial to student learning. According to our exploratory analysis, it was found that a teacher's recognition and expansion of students' contributions had a positive effect on students' general cognitive achievement, irrespective of the students from the reform or non-reform classes. However, the teachers' appropriate response to student contributions showed the positive effect on students' simple problem solving only in the reform classes. More research is needed that investigates the relationship between curriculum in use, instructional practices, and student learning in future.

Appendix I Empirical Studies of Self-explanation and Instructional Explanation with Student Learning

Reference	Types of explanation	Format and setting	Participants	Learning domain	Results
Self-explanation (SE)					
Williams and ›mbrozo (2010)	Verbalized SE	Comparative group describing items, thinking aloud, free study, and prompt SE	Undergraduates	Category learning artificial categories of alien robots	Explanation promoted the inductio of a broad generalization underlyin category membership, relative to other strategies
Berthold and ›nkl (2009)	Written SE	prompt and response (principle-based SE, rationale-based SE, incorrect SE), computer-based learning environment, comparative group	170 high school students	Mathematics (probability)	self-explanation had effects on the learning outcomes conceptual understanding was improved by eliciting elaborations directed to domains' principles, incorrect elaborations hindered the acquisition of procedural knowledg
Jing and Lu)09)	Verbalized SE	Comparative group with vs without SE	66 college students	Chinese reading comprehension (science explanatory text)	SE facilitated participants' understanding of the texts
Ren (2008)	Written SE	Comparative group SE vs without SE, With vs without instruction	86 college students	Mathematics (concepts and problem solving)	SE promoted near-transfer and far-transfer, and facilitated the grasp of mathematics concepts The SE effe could be extended to 10 days later In particular, the SE under instructi led to the best learning outcome
Yang and ›ng (2008)	Verbalized SE	Comparative group with vs without SE, with vs without feedback in SE	45 college students	Chinese reading comprehension (prose study)	SE demonstrated improvements in prose reading and understanding Feedback during the SE process helped and enhanced the accuracy and effect of SE
Große and ›nkl (2007)	Written SE	prompt, comparative group	118 College students	Mathematics (probability)	Correct SE correlated positively wi learning outcomes, whereas not answering the SE prompts correlate negatively with learning outcomes
Schworm and ›nkl (2007)	Written SE	Written prompt exemplifying-domain prompts, learning-domain prompts, mixed prompts, no prompts Comparative study	72 student teachers	Argumentation skills	Prompts are a suitable means to enhance the quality of SE Learning with SE is a promising method of enhancing skills in ill-structured domains such as argumentation
Rittle-Johnson)06)	Verbalized SE (asked by the experimenter)	Pretest, intervention, immediate posttest, and delayed posttest (2 weeks later) Comparative group direct instruction vs discovery learning, SE vs no SE	85 third- through fifth-grade children	Mathematics (equivalence) procedural learning, procedural transfer, and conceptual knowledge	Both SE and instruction helped children learn and remember a correct procedure immediately or over a delay SE promoted procedu transfer regardless of instructional condition SE did not lead to greater increase i conceptual knowledge
Wong, ›wson, and ›eves (2002)	Verbalized SE	Comparative group, training session—use of SE questions, think-aloud instructions	47 Grade 9 students	Mathematics (geometry)	The SE group showed more freque management of their study processing, and they were more active in drawing upon related geometry knowledge and in generating linking with this related knowledge The SE group achieved a statistical significantly higher score on a posttest
0 Aleven and ›edinger)02)	Written SE	Regular classroom computer-based instruction, comparative group SE group vs problem solving group	41 high school students	Mathematics (geometry)	The explainers achieved better than their peers who did not explain step in regular items and reasoning item Also, they better explained their

					solutions and were more successful on transfer problems
1 Siegler 995)	Verbalized SE	Comparative group, training intervention (no more than 2 weeks) feedback only, feedback plus explain-own-reasoning, feedback plus explain-experimenter's- reasoning Classic Piagetian format	97 5-year-old children	Number conservation	Being asked to explain the experimenter's reasoning produced considerably more learning than either of the other two procedures
2 Chi, de euw, Chiu, La ncher (1994)	Verbalized SE	Comparative group read the same text twice vs prompted SE	28 eighth-grade students	Biology (conceptual knowledge)	The SE group had a greater gain from the pretest to the posttest The higher explainers learned with greater understanding than low explainers
3 Chi, Bassok, wis, Reimann, d Glaser 989)	Verbalized SE	Pretest-knowledge acquisition (learning with example)-problem solving-posttest	10 college students	Physics (procedural knowledge)	Good students generated many explanations, related to principles n the text, guided by accurate monitoring of their understanding Poor students did not generate sufficient SE, monitored inaccurate
Instructional Explanation (IE) Provided by the Others					
4 Stark, Kopp, d Fischer 011)	Written IE	Lab, web-based learning environment, comparative group example format (erroneous examples vs correct examples), feedback format (elaborated feedback vs knowledge of result feedback)	College students	Medicine (domain-specific conceptual knowledge, conditional knowledge	Elaborated feedback supported conditional knowledge (knowledge about the conditions of application conceptual and strategic knowledge and also knowledge about the rationale behind the selection of decisions and procedures) across learning domain
5 Berthold d Renkl (2010)	Written IE	asynchronous computer-mediated instructional communication settings, generic training intervention focused processing of explanations, comparative group	40 high school students	Mathematics (probability theory)	The training group more often elaborated on domain principles, generated fewer incorrect statement and showed a better global quality regarding their focused processing the explanations The training group outperformed their counterparts on procedural an conceptual knowledge
6 Ismail and exander (2005)	Verbalized IE co-constructed by the tutor and tutee	Classroom-based experiment, training for 3 weeks, mutual one-to-one tutorial, comparative groups sequence-questioning-explanation (SQE, students received scripts and question stems), questioning and explanation (QE, stems without script), and questioning (Q, neither stems nor scripts)	48 Grade 10 students	Physics (teacher / researcher-designed comprehension test)	SQE comprehension scores were significantly higher than that of QE and Q students The effect of SQE and QE extended to 4 weeks later The SQE and QE groups also had better performance in interaction in terms of question type and response
7 Atkinson 002)	Written and Verbalized IE provided by the computer	Computer-based learning environment incorporating an animated pedagogical agent, comparative group voice plus agent, text plus agent, voice only, text only, and control	Undergraduate students	Mathematics (proportion) Near transfer, far transfer, affective score	Learners presented with an agent delivering explanations aurally (voice plus agent) outperformed the control peers on measures of transf Learners in the voice-plus-agent condition also outperformed their peers with textual explanations on affective measures (e g, interesting and understandable) and practice problems, near transfer and far transfer
8 King, affieri, and lelgais (1998)	Verbalized IE co-constructed by the tutor and tutee	Classroom-based experiment, training explanation skills and questioning, mutual peer-tutoring, comparative group explanation only (E), inquiry with explanation (IE), and sequenced inquiry with explanation (SIE)	Grade 7 students	Science (human physiology) Teacher / researcher-designed written test (cognitive, meta-cognitive and attitude measures)	Students using the SIE and IE mod of peer tutoring were able to perfor better on inference and integration tasks than those of E, and the effect lasted 8 weeks later The SIE students perceived themselves as gaining significant skill over time in their ability to ask question appropriately when assum the tutor role

					The SIE and IE students also viewe their tutors much more helpful and indicated the satisfaction with the learning partners' tutoring
9 Hohn and orаcs 997/1998)	Written IE	Comparative groups conventional instruction, use of worked examples, worked examples and rule-based elaborations	102 undergraduates	Programming, proficiency test, categorization task, grouping task	The use of rule-based elaborations was more successful than either the use of worked examples or conventional instruction in promoting the learning outcome
0 Fuchs, ichs, Hamlett, illips, Karns, d Dutka (1997)	IE provided mostly by the tutors	Classroom-based experiment, training for 18 weeks, comparative group peer-mediated instruction (PMI)-elaborated help, PMI with training in elaborated help and in methods for providing conceptual mathematical explanations, control group (no PMI) Unit of analysis is class, not students One dyad per classroom (LD and AA students) AA-tutors, LD-tutees	Grades 2-4 students	Mathematics (comprehensive mathematics test)	PMI-Elaborated + Conceptual tutor asked more participatory, procedur questions and provided more conceptual explanations Moreover the achievement of PMI- Elaborate + Conceptual students was higher than that of PMI-Elaborated studen which in turn surpassed that of the contrast group
1 Fuchs, ichs, Karns, imlett, Dutka, d Katzaroff 996)	IE provided mostly by the tutors	Classroom-based experiment Training for 23 weeks One dyad for each class high-achieving or average-achieving as tutor, LD-as tutee,	60 grade 2-4 children	mathematics	High-achieving tutors' explanations were rated higher on conceptual, procedural, and overall quality, incorporated a greater variety of explanatory strategies, earned high conceptual orientation scores, and resulted in better performances among tutees
Instructional Explanation Co-constructed by the Teacher and Students					
2 Leinhardt d Steele (2005)	IE co-constructed by the teacher and students	Regular class teaching, case study, 10 consecutive lessons	Fifth-grade students	Mathematics (function)	Students got the main idea, shared the explanation of a procedure, shared their insights and emerging reasoning as well as justifications, and moved in the mathematics spac
3 Forman, ccormick, and onato, (1998)	IE co-constructed by the teacher and students	Regular class teaching, case study, 9 consecutive lessons	Sixth grade students	Mathematics (perimeter problem)	The teacher tried to share responsibility and authority for explaining and evaluating mathematical problems with her students She also used overlapping speech when students offered an alternative solution strategy However, the student who used the teacher's explanation from the beginning did not encounter teache overlapping speech Eventually all students employed the teachers' explanation
Self-explanation and Instructional Explanation					
4 Cong (2007)	Written SE and IE (feedback when incorrect SE occurs)	Comparative group with vs without SE, SE with IE vs SE without IE	165 grade 9 students	Mathematics (probability)	SE facilitated near transfer and far transfer However, the condition of SE with IE as feedback was found more effective in near transfer and far transfer than that without IE
5 Große and nkl (2006)	Written SE, IE	IE provided by default, comparative group	170 student teachers (Mean age 21 74 years)	Mathematics (procedural skills and conceptual knowledge)	No significant difference when learning procedural skills with multiple solutions, IE was better th SE when learning conceptual knowledge
6 Schworm d Renkl (2006)	Written IE, written and	Computer-based learning environment, comparative group	80 student teachers	Instructional design (geometry and	The condition "SE only" leaded to the best learning outcomes, the

	vocalized SE			physics)	condition "IE only" is best evaluate by the learners IE reduced SE effo and reduced learning outcomes The vocalized SE did not added explanatory value with respect to learning outcomes
7 Gerjets, heiter, and itrambone 006)	Written IE, written SE	Computer-based learning environment, comparative groups	College students	Mathematics (probability)	IE had no effect on students' learni in the modular-example conditions SE prompt did not have any impact on performance for isomorphic problems in the condition of modul examples, however, the percentage correctly generated SE correlated positively with performance in the condition with molar examples
8 Webb and astergeorge 003)	Explanation provided by the tutor and tutee respectively	Classroom-based training, Heterogeneous small group, Level of help received, level of immediate responses to help received,	Grade 7 students	Mathematics (decimal numbers)	The help-seeking behaviors were important determinants of successf posttest performance asking for specific explanations instead of calculations or answers or general admissions of confusion, persistenc in seeking explanations and modification of help-seeking strategies, and application of the he received to the problem at hand Important help-giving behaviors included providing explanations wi verbally labeled numbers and continued explaining instead of resorting to descriptions of numeric procedures
9 Webb, oper, and Fall 995)	Explanation provided by the tutor and tutee respectively	Classroom-based training, Heterogeneous small group, level of help received and subsequently carrying out constructive activity (level of constructive activity on current problem, level of constructive activity on the next problem)	Grade 7 students	Mathematics (decimal numbers, fraction)	Level of constructive activity was t strongest predictor of achievement The level of help that students received predicted level of constructive activity but did not predict achievement directly

Appendix II Demographic Information of the Participating Teachers (n=39)

School ID	Teacher ID	Teaching age	Reform age	Educational Level [a]	Knowledge[c]	Belief[d]	Number of Video- taped lesson
2	3	9	1.92	4	17	2.50	3
2	20	4	3.83	4	15	3	3
2	24	3.83	5.83	3	21	3	3
3	10	21	3	4	18	2.50	3
3	15	18	1.92	3	17	1.50	3
3	33	14	5.42	3	20	2.50	3
4	36	8	2	-- [b]	--	--	3
4	46	14	5.92	4	13	3.25	3
5	34	11	3.83	3	18	2.25	3
5	52	2	2.92	3	16	2.75	3
5	58	13	2.17	3	17	2.50	3
8	35	15	1	--	--	--	3
8	50	23	1	--	--	--	3
9	17	27	1	--	--	--	3
9	42	21	2	3	17	3	3
9	45	6	1	3	17	3	2
10	44	13	1	4	18	2.50	3
15	18	13	5.92	4	15	4	3
15	51	10	5.92	4	18	3	3
15	54	10	1.83	4	17	3	3
12	11	15	0	3	17	4	3
12	19	3	0	3	18	4.25	2
12	1	4	0	4	17	3	3
13	47	13	0	4	18	3	3
13	23	4	0	--	--	--	3
14	55	6	0	4	19	2.75	3
16	30	11	0	--	--	--	3
16	13	9	0	4	16	3	3
16	40	6	0	4	22	3.5	3

17	49	16	0	3	11	3.75	2
17	32	13	0	4	15	2.75	3
17	27	15	0	3	11	3.25	2
18	39	18	0	3	17	3	3
18	37	4	0	4	15	3	3
18	5	11	0	4	16	3.25	2
18	7	4	0	4	12	2.75	2
19	2	5	0	4	18	2	3
20	22	16	0	3	16	3.25	3
20	6	5	0	4	16	3	2

Note: a. Educational level: 3=bachelor, 4=graduate.

b. "-" = missing value.

c. The maximum of the teacher knowledge score was 26.

d. the score of teacher belief was reverse counted, and the range of the teacher belief score was from 1 to 5.

III Criteria for the Evaluation of Quality of Instructional Explanation in Teaching Mathematics

Truthfulness			
The dimension of truthfulness is assumed to capture the **accuracy** of the instructional explanation.			
1. Teacher-provided mathematics knowledge is accurate.			
The item is designed to capture any errors and imprecision of teacher-provided mathematics knowledge in classroom instruction (e.g., errors and imprecision in use of mathematical language, notation, concepts, rules, theorems, and causal relationships). **Note:** when coding, it should be taken the specific context into account. For example, whether it is the issue of mathematics knowledge or teaching language. Moreover, in some cases, the teacher intentionally follows up students' wrong response in order to show the students the consequences of that thinking. All these cases are not counted when coding the errors			
1	2	3	4
A, teacher-provided mathematics knowledge is not accurate in **most of the segment**, or the teacher makes **major errors** in key point of the content. ***And,*** B, the teacher does not detect and address the errors	A, teacher-provided mathematics knowledge is not accurate in **large part of the segment**. ***And,*** B, the teacher does not detect and address the errors	A, teacher-provided mathematics knowledge is not accurate in **part of the segment**. ***And,*** B, the teacher does not detect and address the errors	A, teacher-provided mathematics knowledge is accurate and **clean of errors. *Or,*** B, errors that occur are captured and corrected immediately or later within the segment.
2. The teacher's teaching language is accurate.			

The focus of this item is on whether the teacher's teaching language is accurate.			
1	2	3	4
A, the teacher's teaching language is not accurate in **most of the segment.** ***And,*** B, the teacher does not detect and address the errors	A, the teacher's teaching language is not accurate in **large part of the segment**. ***And,*** B, the teacher does not detect and address the errors	A, the teacher's teaching language is not accurate in **part of the segment**. ***And,*** B, the teacher does not detect and address the errors	A, the teacher's teaching language is accurate and **clean of errors.** ***Or,*** B, errors that occur are captured and corrected immediately or later within the segment.

3. The teacher is able to identify and address students' errors and misconceptions correctly.

This item attempts to capture whether the teacher is able to accurately diagnose the learner' **thinking, understanding and misconception**s, and then address the errors and misconceptions explicitly and thoroughly.

Notes: 1. there are three situations that we code with "**NA** (not applicable)"

1) there are no student errors or misconceptions,
2) the errors are presented as the learning tasks for discussion,
3) it is occasional speech errors and does not cause confusion and misunderstanding; or the student gets the point but is not able to express in a accurate and clear manner

2. Pay attention to the teacher's behavior in the contexts of yes-no question and multiple choices question.

1	2	3	4
The students make errors. And the teacher **does not detect and address** the errors.	The students make errors. And the teacher **detects** the errors. However, the teacher **does not help the students to find the correct solution.**	A, The students make errors. And the teacher **detects and helps** the student to find the correct solution. However, the teacher does not deal with or address the errors directly with the students. ***Or,*** B, the teacher	A, The students make errors. And the teacher **detects and helps** the student to find the correct solution. Moreover, the teacher deals with or addresses the errors directly with the students. ***Or,*** B, the teacher anticipates

		anticipates common student errors and provides instruction. However, the instruction is not thorough to help the student avoid the error.	common student errors and provides thorough instruction that helps avoid the errors.

Richness

This category attempts to capture the depth and range of the mathematics offered to students.

Note: all the coding should be based on the content of extended discourse—dialogue after the first round of Q & A.

1. The teacher exposes students to various mathematical languages.

This item is intended to capture whether the teacher is able to use numerous and various mathematical language; and further, whether the teacher is able to intentionally and explicitly engage students into the use of mathematical language.

1.1 The teacher is able to use various mathematical languages.

1	2	3	4
The teacher does not use mathematical language.	The teacher uses **few** mathematical languages.	The teacher uses **many** mathematical languages.	The teacher uses **lots of** mathematical languages, and emphasizes key mathematical terms.

1.2 The teacher leads the students to use mathematical languages.

1	2	3	4
The teacher does not lead the students to use mathematical language. (e.g., ask the students to give a simple answer, such as "true or false", or indication	The teacher leads the students to use mathematical language, but not intentionally and explicitly. (e.g., ask the students to fill the	The teacher intentionally and explicitly **leads** some students to use mathematical language. (e.g., ask open questions for students to use mathematical	The teacher intentionally and explicitly **presses and encourages** more students to use mathematical language. (e.g., ask students to repeat the mathematical

of understanding and /or agreement, such as "yes or no")	gap, such as "equals to......")	language)	language, practice, read, take notes etc)

2. Appropriate connections are made to general concepts and principles in mathematics.

This item attempts to capture whether the teacher is able to make connections with the general concepts, formula, principles and rules in mathematics; and further, whether the teacher is able to intentionally and explicitly lead students make connections with the general concepts, principles and rules in mathematics.

Note: If the content of the episode is about general concepts and principles, then we rate 3 points; if the teacher further encourages the students to apply these concepts and principles, then we give 4 points.

2.1 The teacher makes connection with general concepts and principles.

1	2	3	4
The teacher does not make connection with general concepts and principles.	The teacher connects general concepts and principles. However, the connection is not discussed with the specific learning tasks.	The teacher connects and discusses general concepts and principles with the specific learning tasks.	The teacher not only connects and discusses the concepts and principles with the current learning tasks, but applies the concepts in new situations.

2.2 The teacher encourages the students to make connections with general concepts and principles.

1	2	3	4
The teacher does not lead the students to connect general concepts and principles.	The teacher engages the students into the connection making with general concepts and principles, but not intentionally and substantially.	The teacher intentionally and explicitly engages the students into the connection with general concepts and principles. (e.g., the teacher asks and probes the	The teacher intentionally and explicitly press and encourage the students to connect the general concepts and principles. Moreover, the teacher asks the

	(e.g., ask for repeat or indication of understanding and /or agreement, such as "yes or no"、"right")	students to make connection with general concepts)	students to apply the concepts and principles in new situations.

3. Multiple solution methods or procedures are explored and valued.

It refers to multiple solution methods or procedures for a single problem or a given problem type.

Notes: it should be noted that the focus of this item is on the **content** of the explanation. A solution does not have to be complete or correct; however, the solution methods or procedures should differ essentially from one another.

1	2	3	4
A, the teacher does not propose multiple solutions. ***And,*** B, the teacher does not explicitly lead the students to generate multiple solutions.	The teacher proposes multiple solutions, without intentionally and substantially engaging the students into the generation of the multiple solutions. (e.g., ask for repeat or indication of understanding and /or agreement, such as "yes or no"、"right")	The teacher intentionally and explicitly leads the students to generate multiple solutions. However, the solutions are listed briefly, without further connection and comparison.	The teacher intentionally and explicitly leads and encourages the students to generate multiple solutions, and further discuss the solutions thoroughly or make connections and comparisons.

4. The teacher uses or requests for multiple representations to represent the same mathematical content.

By multiple representations, it is referred to different ways to present ideas. For example,

1) graphs, pictures, figures, and tables,
2) algebra, equations, numeric procedures, and words,

3) models, material object. Notes: the focus of this item is on the **presentation** of the content.
Key sentences: "can you see it from the figure, would you explain that according to the figure, explain the result with the manipulation please, look at this model, can you give an example for that".

1	2	3	4
A, the teacher does not use multiple representations. ***And,*** B, the teacher does not explicitly lead the students to use multiple representations.	The teacher uses multiple representations, without intentionally and substantially engaging the students into the process. (e.g., ask for repeat or indication of understanding and /or agreement, such as "yes or no"、"right")	The teacher uses multiple representations and intentionally engages the students into the process. However, multiple representations are described briefly, without thorough discussion.	The teacher encourages the students to use multiple representations, and further discusses the relationship between multiple representations.

Coherence

This category captures whether the teacher herself or himself can organize the explanation in a complete, clear, and consistent manner; and whether the teacher can work with students in a coherent manner.
1. The explanation is unfolded around the same or related question/ topic.
This item attempts to examine whether the information contained in the explanation relates to each other to a high degree.

1	2	3	4
The explanation is interrupted by irrelevant topics for most of the segment, such as classroom management.	The explanation is interrupted by irrelevant topics for **large part** of the segment.	The explanation is interrupted by irrelevant topics for **portion** of the segment.	The components of explanation form a tightly interconnected and mutually supporting relational structure.

2. The teacher's presentation is in a complete and clear manner.
This item is intended to examine whether the teacher's presentation is complete and clear, which avoids obscurity, ambiguity and misunderstanding.

Note: due to the time limit, sometimes the teacher ends the discussion incompletely. However, if she / he explicitly points out that the unfinished discussion will be continued in the next lesson, or it is evident that the following lesson begins with the unfinished discussion, then we do not count it as incomplete cases

1	2	3	4
Teacher's presentation is unclear, vague or incomplete for *most* of the segment (e.g., overuse of pronouns (it, they, that, here, there, this).	Teacher's presentation is not clear for *large part* of the segment.	Teacher's presentation is not clear for *portions* of the segment.	Teacher's presentation is clear, complete and unambiguous.

3. The teacher poses guiding questions that relate to and lead student thinking.
This item is intended to capture whether the teacher's follow-up questions relate to the previous students' answer, and further their thinking as to the target question. Note: 1) the focus of this item is on the teacher's **follow-up questions** **(questions after the first "why")** with respect to the key point of the instructional explanation. 2) The rating should be based on the teacher's best performance in the episode.
1. No connection：the teacher seems not "heard" the students' answers **2. Not enlighten**：request for simple repeat, clarification, or **simple answer**（e.g., Would you repeat again? Right? Does it make sense to you? Do you all understand?） **3. Less enlighten**：ask for filling the gap, elaboration or proposing different ideas（e.g., equals to ……，are there any different opinions?）

1	2	3	4
No evidence shows the connection between the students' answers and the teachers' follow-up questions.	The teacher's questions relate to the students' answers, but do not facilitate student's thinking.	The teacher's questions **relate to** the students' answers. However, the follow up questions did not lead student thinking sufficiently.	The teacher's questions **relate to** the students' answers closely. And the follow up question lead student thinking step by step.

4. The teacher recognizes and expands students' contributions.			
This item is intended to capture whether the teacher makes response and helps students to work out an appropriate answer. The teacher's responses may include ignorance or dismiss, simple acknowledgement, repeat or restatement, and expand students' contribution. Note: 1) the focus of this item is on the teacher's **<u>response</u>** (the assertive sentience after the first "why" question) to the key point in the students' answer. Writings on the lack board is also included as the teacher's response. 2) The rating should be based on the teacher's best performance in the episode.			
1	2	3	4
A, the teacher does not recognize or respond to students' contributions. ***Or,*** B, the teacher's response does not relate to the previous student's answer.	The teacher recognizes students' contributions with simple acknowledgement, such as "good, oh, right".	The teacher recognizes students' contributions by repeating or restating all or part of the student's answers.	The teacher recognizes students' contributions by elaborating and expanding the students' answers.

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