Studies on Decentralized Supply Chain: Incentives and Coordination

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Supply chain coordination and associated contracts have been an active research area for supply chain management research. Yet, little has been done in addressing robustness matters of design, evaluation, and implementation for these coordination contracts. In chapter 2 and 3, we develop a consistency framework for supply chain contracts and classify a number of well-studied contracts into groups. We demonstrate with examples that coordination contracts can be evaluated by their consistency properties. Based on precise mathematical definitions and subsequently developed structural properties and management insights, we are not only able to measure the goodness of supply contracts but also to reveal the nature of their coordination. Our findings open an avenue for design, evaluation and imple-

mentation of supply chain coordination contracts.

In Chapter 4, we consider a supply chain which consists of a manufacturer, a logistics service provider (LSP) and a retailer. The LSP provides the emergency replenishment, financing and logistics services. The advent of the LSP changes the structure and incentive in the traditional one-supplier-one-retailer channel. We develop a framework of 3-player game to investigate the dynamics and competitive behaviors with multiple decision sequences. We provide the explicit equilibria for different decision sequences and demonstrate the possibility that the LSP and the manufacturer collude to create price increment, which squeezes out the retailer. Whereas, the triple marginalization effect is alleviated.

摘要

供应链整合与相关合同的设计是供应链管理领域中的一个活跃的研究方向。然而，对于促进供应链整合的合同，它们从设计，评估，应用方面对外界条件的适应性议题却是一个研究上的空白。在第二和三章里，我们建立了一个有关各种一致性的研究框架，并将一些领域上熟识的合同进行了探讨和分类。我们发现这些合同在适应性上的表现各异。我们以严格的数学形式的定义对适应性和一致性的概念进行了界定，从而可以对各种合同的优劣进行考量，更加深刻的揭示了这些合同能促成供应链整合的本质因素。我们对一致性要素的发现开辟了一个供应链合同从设计，评价到应用的一个崭新的思路和研究空间。

在第四章中，我们考虑了一个包含生产商，零售商和第三方物流公司的供应链模型。其中，这个第三方物流公司对前两者提供了库存，运输，紧急库存调配和融资的服务。在这种情况下，生产商和零售商的决策受到了影响。我们建立了一个模型来研究在不同决策次序下，这三个独立决策者的各种动机和变化。在达到各种均衡时，我们对供应链的效率进行了比较，并指出当第三方物流公司和生产商串通以后，可以获得强大的议价能力。这时，虽然对零售商的获利造成很大的负面影响，但整体供应链的效率却得到了提升。

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To Liu Xiaodong, if it is not you, this thesis would be ready much sooner.

This work is dedicated to my parents:

Chunjiang Lü and Li Zhang

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Chapter 1

Introduction and Literature Review

1.1. Introduction

1.1.1 Contractual Coordination and its Robustness

Contracts are fundamental business instruments that suppliers and retailers engage in. The presence of double marginalization in vertical supply chains and inventory competition in horizontal supply chains has inspired a considerable interest in design and analysis of coordination contracts that optimize the performance of supply chains. However, most of the existing studies focus on the design of these contracts for specific supply chains, whereas investigations on their robustness are rare. It would also be important to explore properties that facilitate their adoption and implementation.

Businesses execute some contracts repeatedly and others as one-time agreements. In the former, the supplier-retailer relationship can change over time, and therefore the competitive structure may change as well. In the latter, the outcomes are usually random, and therefore the profit division is sample-path dependent. These observations raise questions regarding the applicability of contracts and their desired outcome for supply chain coordination, and underscore the importance of the robustness of contracts. Yet, the supply chain management research has focused mainly on designing coordination contracts for various supply chains. The robustness of contracts is yet to be systematically studied.

In this paper, we set out an agenda to study the robustness of coordination contracts: 1) the robustness of profit allocation with respect to its expected value and along a sample path, 2) the robustness of coordination with respect to different decision sequences, and 3) the robustness of contracting agents with respect to compliance regimes. Specifically, we would like first to understand the interplay between channel coordination and coordination flexibility. Second, we investigate the relationship between contract terms and the notion of channel profit allocation. Third, we study the dependency of coordination contracts on the competitive decision-making process. Last, we scrutinize the forced and voluntary compliance regimes.

For these purposes, we evaluate the robustness of coordination with respect to the flexibility of distributing channel profit and sample path dependency. With a clarification of the allocation function, a measurement used in the coordination literature, and the sharing function, reflecting the contract terms, we consider three cases of supply chain coordination with respect to the robustness in channel profit allocation: (i) the total distributed channel profit equals the centralized profit, denoted as coordination; (ii) the distributed supply chain yields a flexible allocation of the expected optimal channel profit, denoted as *flexible coordination in expectation*; and (iii) the distributed supply chain yields a flexible allocation of the random channel profit, denoted as *flexible coordination in sample-path* or simply *sample-path flexible*, which maintains a consistent channel profit distribution in each random outcome. The flexible sample-path coordination possesses a number of desired properties. For example, it gives the same probability for agents to achieve a specified profit.

We continue to evaluate the robustness of coordination with respect to the structure of the game that agents play in determining their individual decisions and in leading to the division of channel profit. For the game structure, we consider two scenarios that lead to coordination: (i) the game leads to a centralized solution, denoted as *coordination*; (ii) the game achieves coordination that is independent of the game structure, denoted as *sequence-independent coordination.* The sequence independency maintains a consistent outcome regardless of whether the game is simultaneous or Stackelberg.

The last consistency issue is whether a contract and its resulting competitive game lead to an action which is in the best interest of the agents. We say that such a contract leads to a *voluntary compliance coordination.* Furthermore, we develop sufficient conditions for sequence-independent and voluntary compliance coordinations. We demonstrate that a sequence-independent coordination contract requires that the equilibrium solutions of the two Stackelberg games led by agents 1 and 2, respectively, coincide. Likewise, a voluntary compliance coordination contract requires that the solutions of the agent's optimization problems over their corresponding reaction functions coincide.

With the concepts defined above, we examine a number of coordination contracts in the literature. With one dimension measuring coordination flexibility in the sense of coordination, flexible coordination, and sample path coordination and the other dimension measuring coordination contracts in the sense of path dependency, sequence independency, and voluntary compliance, we classify coordination contracts reported in the literature into a 3×3 matrix. Finally, we redesign a coordination contract into one with high levels of consistency.

1.1.2 The Supply Chain with a Logistics Service Provider

Most, if not all, companies encounter the capacity problem and problems of complying with delivery service owing to large fluctuations in demand. Many of them choose to outsource isolated logistics activities such as transportation and warehousing to third party service providers to alleviate the problems. An increasing number of logistics service providers (LSPs) perform some or all of those outsourced functions. Starting from 1980s, some successful multinational LSPs are noticeable, such as DHL/Exel.Kuehne + Nagel, Schenker, Ups, Panalpina, C.H. Robinson, TNT Logistics, Schneider and NYK Logistics.

The academic literature has also recognized the critical impact of the advent of the LSP age. In the logistics management area, there emerges the theoretical background for the development of third party arrangements, including both the transaction cost theory and the network theory. The transaction cost analysis explains the conditions under which third party arrangements become preferable to the firms. The network theory explains the dynamics in third party companies. The development of industrial facilities and theoretical frameworks suggest that the analysis for the LSPs deserve significant attention.

Although the prevalence of the LSPs is becoming widespread, there has been little research studying the impact of the LSP on the inventory management. Beside the traditional logistics functions, the LSP takes positions of inner supply chains, in the sense of organizing the distribution of warehouses and making the inventory decisions for replenishment of retailers. The LSP can provide new options for the retailer to replenish the inventory in the middle of the selling season. In this research, we would like to develop a model to study the incentives in a supply chain consisting of a supplier, a retailer, and a LSP. We begin with the newsvendor setting, and proceed to incorporates the LSP with the corresponding backup inventory into the model. Second, we consider two different game outcomes resulted from two different decision sequences. The difference of these outcomes needs to be understood. Third, we consider the situation of coalition by two of the players. The research question is whether a coalition of two players would hurt the profit of the other.

The starting point for the inventory model is the newsvendor model. A supplier as a brand owner produces products upon the request of a retailer, who is facing a random demand and exogenous retailing price. By introducing the LSP, the shipping, warehousing and financial supporting functions are performed. Within the single selling season, the retailer first makes an order from the supplier, receives the delivered products from the LSP, and then sells to the market. Due to the double marginalization effect, there is a trend of understocking by the retailer. With the LSP, more units are stocked as a backup inventory. Moreover, from the view of the supply chain members, this backup inventory has impact on their profit and their optimal decisions. Consequently, they adjust the strategies. With the analysis of the gaming model, we would like to characterize the new equilibrium of the supply chain, as a decentralized entity.

The LSP signs a contract with the supplier or the retailer to provide an emergent replenishment in the middle of the selling season, which we call the backup agreement. The backup inventory by the third party is of plenty practical evidences, for examples Toyota Motor Corporation, IBM, and HP. In general, the volume of the backup inventory can either be decided by the supplier or the LSP toward his own interest. For the service price charged by the LSP, we consider two scenarios, namely, charging the supplier or the retailer. In the first scenario, the supplier requires the immediate payment to deliver the products. But the retailer does not have enough cash before the demand is realized. The LSP could lend the retailer money and charge a financing rate at the end of the selling season. We call lending money to the downstream retailer is forward financing. This scenario often happens in famous brand owners like HP, who supplies laptops to the retailer all over the world. In the second scenario, the retailer requires to make payment to the supplier after the selling season. However, the supplier would not like to wait because it cannot start new production until it gets the money back. The LSP could pay the supplier when delivery and charge the supplier for a financing rate. We call this backward financing. This is the case in many OEM factories, especially e-component suppliers. As in most industrial companies, let the charge of the LSP be a proportion of the ordering quantity. Therefore, there is a triple marginalization, since one more layer is incorporated into the supply chain. We also assume the information is symmetric to all firms, including the demand distribution information, and cost functions. It is not a restrictive assumption when the LSP establishes a long term cooperative relationship with the members of the supply chain. And the IT capability allows to do so. In the survey by the 14th annual study of third party logistics, 74% shippers (suppliers or retailers) and 77% the LSPs identify that transparency and good communication are the key factors as contributing to successful shipper experiences with the LSPs. The IT integration with the LSPs has been widely used to share the data and commitment.

There are three main findings in this paper. First, as facing a random demand, by adopting the backup agreement, the LSP and the retailer combine their inventory as the newsvendor solution. Unsurprisingly, the supply chain incurs a tripe marginalization. Second, we analyze the different behaviors when the supplier and the LSP are integrated, as well as the retailer and the LSP. For the former case, there exists an incentive for the supplier-LSP coalition to raise wholesale price to the retailer. Whereas the double marginalization effect vanishes, and the total ordering quantity increases; and the profit also increases. For the latter case, the supply chain incurs double-marginalization effect. Third, we characterize the decentralized system, when the supplier offers a different wholesale price w_l and w_r for the LSP and the retailer. It demonstrates that the supplier offers a lower price to the LSP, and a higher price to the retailer.

1.2. Literature Reviews

There is a vast body of literature on supply chain coordination with contracts, such as buy-back contracts (Pasternack, 1985), revenue sharing contracts (Cachon and Lariviere, 2005), target rebate contracts (Taylor, 2002), quantity-discount contracts (Jeuland and Shugan, 1983), and price-discount sharing contracts (Bernstein and Federgruen, 2005). Tsay *et al.* (1998) and Cachon (2003) provide an extensive review and classify coordinations based on contract forms. Cachon (2003) defines two key research questions: Which contracts coordinate what supply chains and how good are the coordination contracts? The former issue is concerned with checking if the supply chain decisions constitute a coordinating Nash equilibrium. The latter judges the goodness of a supply contract according to its flexibility in dividing the channel profit. In this paper, we are interested in the goodness of a coordination contract in terms of its robustness against changes in the business setting, decision making sequence, and desires of fulfilling the agreement.

Jeuland and Shugan (1983) demonstrate that a quantity-discount contract not only coordinates the channel, but also divides the expected channel profit among the members. Cachon and Lariviere (2005) study revenue sharing contracts and articulate their strength of being able to arbitrarily allocate the expected profit. They argue that a particular profit split depends on the respective bargaining powers of the supply chain agents. Cachon (2003) shows that every possible expected profit allocation is feasible with the buy-back contract. Other flexible contracts include channel rebate contracts (Taylor, 2002), wholesale sponsored contracts (Netessine and Rudi, 2004) and quantity-flexibility contracts (Tsay, 1999). The contract enforcement matter is widely studied in economics. For example, Krasa and Villamil (2000) study optimal contracts with contract enforcement as a decision. The enforcement matter is also introduced to the supply chain coordination studies. Without a formal treatment of the enforcement matter, Jeuland and Shugan (1983) argue that quantity-discount contracts are free of enforcement considerations.

Cachon and Lariviere (2001) introduce notions of forced and voluntary compliance to address enforcement. According to them, a forced compliance requires the supplier to be liable for the orders of the retailer and a voluntary compliance represents the opposite. Cachon (2003) shows that buy-back contracts ensure voluntary compliance for the supplier whereas sales rebate contracts do not. A related topic is the robustness of a coordination contract. Cachon (2003) articulates how a voluntary compliance contract improves the robustness of a supply chain against irrational agents. Chen (1999), Porteus (2000) and Watson (2002) study how a coordination contract improves a supply chain's performance against irrational orders of the retailer(s). Other robustness issues studied include coordination concerns regardless of the form of the demand distribution (Bernstein and Fedegruen, 2005 and Netessine and Rudi, 2004). Another line of research involves the decision sequence issues of contracts and games. It is widely assumed in the literature that coordination is achieved with natural and exogenously determined sequences of decisions. To our best knowledge, the study of e-commence retailers who use drop-shipping is the only work that demonstrates that decision sequences matter (Netessine and Rudi 2004).

The sequence issue is important in game theory. Gal-Or(1985) and Dowrick(1986) evaluate first-mover and second-mover advantages in duopoly models, where both agents set quantities or prices. They show that the monotonicity of the best-response functions determines which mover does better. If one's reaction function is an increasing function of the other's decision, then the second mover does better; the price game with a differentiated substitutable product is an example. If one's reaction function is a decreasing function with respect to the other's decision, then the first mover does better; the quantity game with a substitutable product is an example. Hamilton and Slutsky (1990) examine the novel issue of endogenous timing by adding an initial stage at which players simultaneously decide when they choose their actions. Then the basic game is played simultaneously or sequentially. They formally analyze the equilibria of two different games: the extended game with an observable delay and the extended game of action commitment. Robson (1990), Anderson and Engers (1994), Amir and Grilo (1999) and Amir *et al.* (1999) show that an endogenous-timing game yields a simultaneous-move equilibrium in quantity competition and a sequential-move equilibrium in price competition. Amir and Stepanova (2006) show that an agent with a sufficiently large cost advantage over its rival has a first-mover advantage. Their Bertrand duopoly with an endogenous-timing scheme (with risk-dominance equilibrium selection) yields a unique equilibrium outcome: a sequential play with the low-cost agent as the leader. He *et al.* (2007) survey recent applications of Stackelberg differential game models in supply chain management and marketing channels.

For the analysis of the decentralized system, there is a raising trend in the literature of the supply chain management applying the results of game theory to predict the outcomes of the decentralized decision process. Leng and Parlar (2005) present a comprehensive review of game theoretic applications in supply chain management area for the last decades. The non-cooperative and cooperative game structures with simultaneous and sequential decision sequences under complete and incomplete information are considered. The static game approach for the inventory problem has been studied by a number of researchers. Parlar (1988) first studies inventory problem for a single-period context supply chain model with two substitutable products. The paper derives a Nash equilibrium and demonstrates that cooperation between two retailers can increase their profit. Lippman and McCardle (1997) consider a competitive newsboy model. They propose four rules of demand allocation for an initial allocation and show the existence of Nash equilibrium for the general setting. Netessine and Rudi (2003) extend the stage game model to multiple players. They compare the differences between centralized and decentralized supply chain. The dynamic game approach is also applied in some literature. For the wholesale price competition, Lariviere and Porteus (2001) provide a Stackelberg equilibrium for a one-supplier-one-retailer supply chain. As a monopolist, the supplier determines the wholesale price, the retailer makes order accordingly. The outcome of the game reviews that as the variability of demand decreases, the retailer's price sensitivity decreases, the wholesale price increases, the decentralized system becomes more efficient and the monopoly supplier squeezes the retailer's profit. Dong and Rudi (2004) further extend the problem to accommodate multiple retailers. With a sequential game structure, the manufacturer as a wholesale price setter is better with transshipment between the groups of retailers, because the pooling effect makes the retailers less price sensitive.

In practice, the LSP owns the products temporarily during the products are handed over. Common views are that the change of product ownership reduces the default risk if the LSP provides a financial aid to the money constraint retailer. The LSP charges a service rate which includes the interest and handling rate. There is a research area studying inventory/capacity decisions with financial concerns. As Dada and Hu (2008) point out that the capital-constrained newsvendor borrows funds to procure an amount that is less than ideal. Buzacott and Zhang (2004)'s paper emphasizes on the industrial trend of assets-based financing. They use a newsvendor model to exam the motivations between banks and retailers. It appears that the banks are better off using asset-based financing while the retailer's cash return is able to be enhanced. Xu and Birge (2004) study a model that production and financing decisions are jointly made. They show that the production decision is affected by the financial constraint, and find that optimal production decisions are negatively correlated with the optimal debt-to-market value leverage ratio. Li, Shubik and Sobel (2005) consider that the financial and inventory decisions are correlated flows of materials and cash. They develop a dynamic model that inventory and financial decisions are made at the same time and are interacted directly with each other. In particular, they demonstrate that inventory stocking levels are different for dividend-maximization and profit-maximization firms.

The concept of backup inventory appears in early dates. Eppen and Iyer (1997) and Tsay (1999)'s supply chain first models incorporates the backup inventory. Eppen and Iyer (1997) provide a practical instance, in which the principle company and its manufacturer implement the backup agreement in a two-period selling season. The manufacturer holds back a constant fraction of the commitment and delivers the remaining units. After observing early demand, the principle company can order up to the backup quantity and receive quick delivery. They derive the optimal solution for such supply chain with supportive empirical data. It shows that the backup agreement can benefit both firms. In our model, we consider the backup agreement provided by the LSP who is self-interested. The backup inventory is also related to the quantity flexible contract, in which purchase quantities could be within a prespecified quantity window. Tsay (1999) studies the quantity flexible contract (QF) in a decentralized supply chain. It is shown that the QF contract can allocate the costs of market demand uncertainty so as to lead to the systemwide optimal outcome.

□ **End of chapter.**

Chapter 2

Concepts in the Robustness of Contract

2.1. Definitions and Classification

It helps to begin with an example where the coordination contracts are not robust with respect to profit allocation, game structure, and compliance regime.

2.1.1 A Motivating Example

Example 2.1.1. (TRANSSHIPMENT PRICE CONTRACT):
A horizontal supply chain consists of two independently owned retailers in a newsvendor setting. The product cost and the sales price are c and p, respectively. The demand D_i for retailer i has the probability distribution function $F_i(\cdot)$, $i = 1, 2$, known to all. The demand is fulfilled first by each retailer from

its inventory. In the case when one of the retailers has excess inventory and the other is stocked out, a transshipment from the former to the latter takes place with a unit cost c_t. For the centralized solution, retailers jointly choose the order quantities q_1 and q_2 to maximize the expected aggregate profit $\pi(q_1, q_2) = pE\min\{D_1, q_1\}+pE\min\{D_2, q_2\}+(p-c_t)E\min\{(D_1-q_1)^+, (q_2-D_2)^+\} + (p-c_t)E\min\{(D_2-q_2)^+, (q_1-D_1)^+\} - c(q_1+q_2)$. The solution denoted by (q_1^*, q_2^*) is unique.

For a decentralized supply chain, a transshipment price contract (TPC) is denoted as $z = \{t_1, t_2\}$, where t_i is the unit payment from retailer i as the receiver of the transshipped product to retailer $3-i$ as the provider, $i = 1, 2$. When a retailer increases its order quantity, it reduces the potential opportunity of transshipment from its competitor. In general, the sum of the equilibrium order quantities can be larger or smaller than the total ordered quantity in the centralized optimal solution (Eppen 1979). The payoff function for retailer i is

$$\begin{aligned}\pi_i(q_1, q_2, z) &= E\{p\min\{D_i, q_i\} + (p-t_i)\min\{(D_i - q_i)^+, \\ &\quad (q_{3-i} - D_{3-i})^+\} + (t_{3-i} - c_t)\min\{(D_{3-i} \\ &\quad -q_{3-i})^+, (q_i - D_i)^+\}\} - cq_i.\end{aligned} \tag{2.1}$$

The reaction functions of the retailers, as illustrated in Figure

??(a), are

$$\begin{cases} r_1(q_2, z) &= \arg\max_{q_1} \pi_1(q_1, q_2, z), \\ r_2(q_1, z) &= \arg\max_{q_2} \pi_2(q_1, q_2, z). \end{cases} \tag{2.2}$$

The reaction function of retailer i is a decreasing function of q_{3-i} and has a q_i-intercept $r_i(0, z)$, which is the maximum order quantity for retailer i to fulfill two streams of demands. The reaction function of retailer i has an asymptote characterized by $q_i = F^{-1}(\frac{t_i - c}{t_i})$. Hence, the strategy space is $[0, r_1(0, z)] \times [0, r_2(0, z)]$. The outcome of the simultaneous inventory game is the Nash equilibrium N(z), for which Rudi *et al.* (2001) obtain a coordination contract $z^* = \{t_1^*, t_2^*\}$ satisfying the system

$$\begin{cases} q_1^* &= \arg\max_{q_1} r_2(q_2^*, z^*), \\ q_2^* &= \arg\max_{q_2} r_1(q_1^*, z^*). \end{cases} \tag{2.3}$$

The coordination contract z^* determines the distribution of the channel profit in the simultaneous game. With this z^*, the Stackelberg equilibrium S$_1(z^*)$ represents the best outcome for retailer 1 on the reaction curve of retailer 2, as illustrated in Figure ??(a). The equilibrium S$_1(z^*)$ can be deviated from the channel optimum. However, one can revise the TPC to $z' = \{t_1', t_2'\}$ for S$_1(z')$ to achieve the channel coordination, where

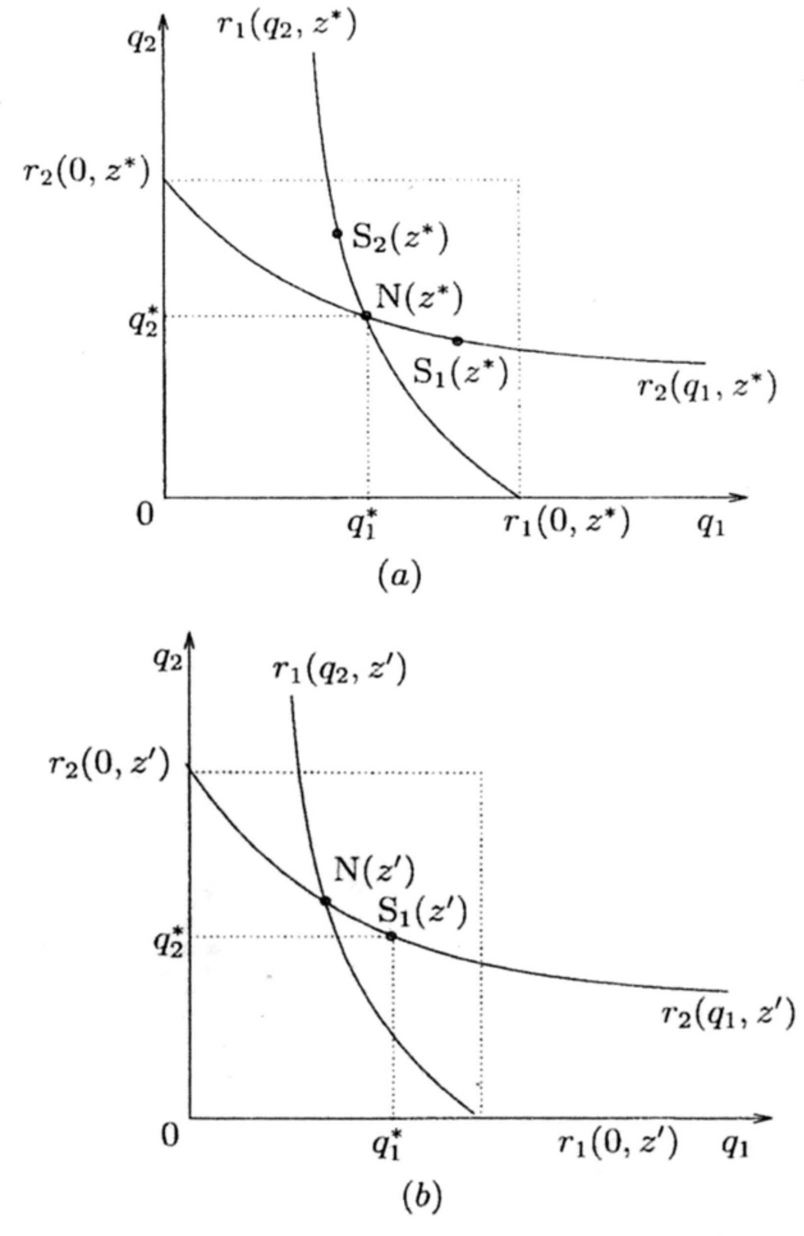

Figure 2.1: Reaction curves for transshipment price contracts

z' satisfies the system

$$\begin{cases} q_1^* = \arg\max_{q_1} \pi_1(q_1^*, r_2(q_1^*, z'), z'), \\ q_2^* = r_2(q_1, z'). \end{cases} \tag{2.4}$$

Equations (??) suggests that the Stackelberg equilibrium coincides with the optimal order quantities (q_1^*, q_2^*), as shown in Figure ??(b). Note that contract z^* and z' have different transshipment prices, i.e., there may not exist a single contract that coordinates the supply chain for the simultaneous and Stackelberg games, simultaneously.

Consider a numerical example, where $p = 10$, $c = 4$, $c_t = 0.1$, D_1 and D_2 are i.i.d. normal with $\mu = 1000$ and $\sigma = 300$. Table ?? lists computational results in three cases. Case 1 represents a simultaneous game with coordinating transhipment prices $t_1^* = t_2^* = 7.11$ resulting in an even split of the channel profit. Case 2 represents the Stackelberg game with retailer 1 as the leader and we see that with the transshipment prices $t_1^* = t_2^* = 7.11$, the Stackelberg equilibrium deviates from the channel optimum. Case 3 represents the same Stackelberg game, where the transshipment prices $t_1' = 7.01$ and $t_2' = 6.85$ are determined from (??) in order to obtain the channel optimal order quantities, and thus yielding the channel optimal profit.

Let us recapitulate a number of observations. First, the TPC coordinates the supply chain for each game structure. However,

	Transshipment Prices	Game Type	Equilibrium (y_1^e, y_2^e)	Profit (π_1, π_2, π)	/
1	$t_1^* = t_2^* = 7.11$	Simultaneous	(1054,1054)	(5181, 5181, 10362)	0.50
2	$t_1^* = t_2^* = 7.11$	Stackelberg	(1107,945)	(5317, 4624, 9941)	0.52
3	$t_1' = 7.01, t_2' = 6.85$	Stackelberg	(1077,1030)	(5245, 5117, 10362)	0.51

Table 2.1: Computational results for TPC with different decision sequences

due to the existence of the first-mover advantage, the distribution of the channel profit depends on the decision sequence of the game played. Second, the TPC provides one specific allocation of the channel profit for each game, and thus, it is not flexible in allocating the channel profit. Third, the TPC is in the execution of a forced compliance regime. Retailer i's profit is increasing in q_i and decreasing in q_{3-i}. Therefore, the leading retailer has an incentive to deviate from the channel optimum in seeking a better profit. This example demonstrates that the TPC coordinates the supply chain under strict conditions. It motivates us to evaluate other contract types in terms of their robustness in distributing the channel profit, the game settings, and the compliance regimes.

2.1.2 Sharing and Allocation Functions

In view of the motivating Example ??, we define performance measures for coordination contracts. We use the concepts developed in the group decision theory that deals with situations in which a group faces intertwined external and internal problems. The external problem involves the choice of an action taken by the group and the internal problem involves the distribution of the group payoff among the members. Raiffa (1970) and LaValle (1978) illustrate these ideas lucidly with a series of examples. Gan *et al.* (2004) use the group decision theory in the context of supply chain coordination with risk-averse agents. The problem of contract design for a supply chain and its execution is obviously a group decision problem. The external problem is concerned with decisions regarding the order/production quantities and item prices. The internal problem is to decide the share of the channel profit by setting the wholesale price, the side payment, and the refund price on the returned units.

Let $(\Omega, \mathcal{F}, P)$ denote the probability space along with E denoting the expectation operator. Let S_i be the external action space of agent i, $i = 1, 2$. For any given external joint action $y_i \in \mathsf{S}_\mathsf{i}$, $i = 1, 2$, and a contract $z \in C$, where C is the set of all contracts of the type under consideration, retailer i's profit is denoted by $\Pi_i(y_1, y_2, z, \omega), i = 1, 2$, and the channel profit $\Pi(y_1, y_2, z, \omega) = \Sigma_i \Pi_i(y_1, y_2, z, \omega)$, $\omega \in \Omega$. The fractional shares of the channel profit, determined by the contract z, are denoted by the sharing functions

$$\theta_i(y_1, y_2, z, \omega) := \frac{\Pi_i(y_1, y_2, z, \omega)}{\Pi(y_1, y_2, z, \omega)}, \quad i = 1, 2, \tag{2.5}$$

which clearly depend on the external decision (y_1, y_2), the contract z, and the sample path ω (LaValle, 1978).

It is important to point out the difference between the sharing functions and the allocation functions used widely in the supply chain literature. The allocation functions determine the division of the *expected* channel profit $\Pi(y_1, y_2, z) = \Sigma_i \Pi_i(y_1, y_2, z)$, where $\pi_i(y_1, y_2, z) = E\Pi_i(y_1, y_2, z, \omega), i = 1, 2$ (Tsay *et al.* 1998 and Cachon 2003). Let $0 \leq \lambda_i \leq 1$ denote the distribution of the expected channel profit, given the external decisions (y_1, y_2) in contract z. Then,

$$\lambda_i(y_1, y_2, z) := \frac{\pi_i(y_1, y_2, z)}{\pi(y_1, y_2, z)} = \frac{E[\theta_i(y_1, y_2, z, \omega)\Pi(y_1, y_2, z, \omega)]}{E\Pi(y_1, y_2, z)}. \tag{2.6}$$

Let us use the buy-back and quantity discount contracts to highlight the similarities and differences between the sharing and the allocation functions. A buy-back contract $z = \{w, b\}$ specifies the trans-payment from the retailer to the supplier as $wq - b(q - D)^+$ for any demand realization $\omega = D$. We obtain the sharing functions for the retailer and the supplier as

$$\begin{cases} \theta_r(q, z, D) = \dfrac{(p - b)\min\{D, q\} - (w - b)q}{p\min\{D, q\} - cq} = \dfrac{p - b}{p} = \lambda, \\ \theta_s(q, z, D) = \dfrac{(w - c)q - b(q - D)^+}{p\min\{D, q\} - cq} = \dfrac{b}{p} = 1 - \lambda. \end{cases} \tag{2.7}$$

The second equality above is due to the fact that the buy-back contract z is designed to be a coordination contract, implying $\frac{p-b}{p} = \frac{w-b}{c} = \lambda$. Similarly, the allocation functions for the retailer and the supplier can be expressed as

$$\begin{cases} \lambda_r(q,z) = \dfrac{E\{(p-b)\min\{D,q\}\} - (w-b)q}{E\{p\min\{D,q\}\} - cq} = \dfrac{p-b}{p} = \lambda, \\ \lambda_s(q,z) = \dfrac{(w-c)q - bE(q-D)^+}{E\{p\min\{D,q\}\} - cq} = \dfrac{b}{p} = 1-\lambda. \end{cases} \quad (2.8)$$

With expressions (??) and (??), it is clear that θ_i and $\lambda_i, i = r, s$, are constant functions with respect to the decision q and the sample path D. Therefore, for the buy-back contract z, the sharing functions and the allocation functions are identical.

For the all-unit quantity-discount contract (Moorthy, 1987), the trans-payment from the retailer to the supplier can be expressed as $w(q)q$, where the unit wholesale price $w(q)$ is a decreasing function of the expected sales $S(q) = E\min\{q, D\}$. Provided that $w(q) = (1-\lambda)p\frac{S(q)}{q} + \lambda c$, where $0 \leq \lambda \leq 1$, the sharing functions for the retailer and the supplier are

$$\begin{cases} \theta_r(q,z,D) = \dfrac{p\min\{q,D\} - w(q)q}{p\min\{q,D\} - cq} \\ \qquad = \dfrac{p\min\{q,D\} - (1-\lambda)pS(q) - \lambda cq}{p\min\{q,D\} - cq}, \\ \theta_s(q,z,D) = \dfrac{(w(q)-c)q}{p\min\{q,D\} - cq} = \dfrac{(1-\lambda)(pS(q) - cq)}{p\min\{q,D\} - cq}. \end{cases} \quad (2.9)$$

From (??), the sharing function $\theta_r(q,z,D)$ depends on the demand realization D. The larger the demand is, the larger the retailers' share of the channel profit is, and the smaller the supplier's share is. On the other hand, the allocation functions are

$$\begin{cases} \lambda_r(q,z) = \dfrac{pE\min\{q,D\} - (1-\lambda)pS(q) + \lambda cq}{pE\min\{q,D\} - cq} = \lambda, \\ \lambda_s(q,z) = \dfrac{(1-\lambda)pS(q) + (1-\lambda)cq}{pE\min\{q,D\} - cq} = 1-\lambda. \end{cases} \tag{2.10}$$

Here, the allocation functions for the retailer and the supplier are constants, and different from the sharing functions (??).

We are now ready to present our first result in defining the degree of coordination and subsequent coordination consistency for coordination contracts.

2.1.3 Consistency in Profit Allocation

Before proceeding to the consistency of the supply chain coordination, we denote $(y_1^*, y_2^*) = \arg\max_{y_1,y_2} \pi(y_1^*, y_2^*)$ as a centralized optimal decision for the supply chain. If it is not unique, then let $Y^* = \{(y_1^*, y_2^*)\}$ be the set of all optimal decisions. Note that some supply chains have a single decision, such as the buy-back contract and the revenue sharing contract. When the supply chain involves multiple agents, much of the literature uses a non-cooperative gaming approach.

We introduce the following notation. In the simultaneous game, we obtain a Nash equilibrium $\mathrm{N}(z) = (y_1^{\mathrm{N}}(z), y_2^{\mathrm{N}}(z))$, by solving

$$\begin{cases} y_1^{\mathrm{N}}(z) = r_1(y_2^{\mathrm{N}}(z), z), \\ y_2^{\mathrm{N}}(z) = r_2(y_1^{\mathrm{N}}(z), z), \end{cases} \tag{2.11}$$

where the reaction functions r_1 and r_2 are defined in (??). In the Stackelberg game with agent 1 as the leader, an equilibrium $\mathrm{S}_1(z) = (y_1^{\mathrm{S}_i}(z), y_2^{\mathrm{S}_i}(z))$ solves the system

$$\begin{cases} y_1^{\mathrm{S}_1}(z) = \arg\max_{y_1} \pi_i(y_1, r_2(y_1, z), z), \\ y_2^{\mathrm{S}_1}(z) = r_2(y_1^{\mathrm{S}_1}(z), z). \end{cases} \tag{2.12}$$

The Stackelberg game $S_2(z)$ can be similarly defined.

In general, Let $(y_1^e(z), y_2^e(z))$ denote the outcome of a game following the contract z. We can now state our first definition toward characterizing the consistency of the coordination contracts.

Definition 2.1.1. (Coordination with Flexibility)

i) *For any contract z, if the decentralized outcome equals to one of the centralized optimal solutions, that is $(y_1^e(z), y_2^e(z)) = (y_1^*, y_2^*)$, then the contract z coordinates the supply chain.*

ii) *Denote C as the set of coordination contracts with same type. For any given $\lambda \in [0, 1]$, if there exists a coordination contract $z \in C$ such that the allocation function $\lambda_1(y_1^e(z), y_2^e(z), z) = \lambda$, then the contract type z coordinates the supply chain with flexibility in expectation.*

iii) *For any coordination contract* z, *if the sharing function is independent of the sample path* ω, *i.e.*, $\theta_1(y_1^e(z), y_2^e(z), z, \omega) = \lambda_1(y_1^e(z), y_2^e(z), z)$ *for each* ω, *then the contract* z *coordinates the supply chain with flexibility in sample-path.*

For the buy-back contract, C is the feasible parameter set $\{(w,b)|c \leq w \leq p,\ b = w - c(p-w)/(p-c)\}$. A contract $z \in C$ specifies a buy-back contract with the wholesale price w and the buy-back price b. It is easily seen that the quantity discount contract coordinates the supply chain with expectation flexibility (see (**??**)), and the buy-back contract coordinates the supply chain with sample-path flexibility (see (**??**)). On the other hand, the TPC achieves coordination without flexibility.

2.1.4 Consistency in Decision Sequences and Compliance Regimes

In this subsection, we introduce the concepts of consistency for the decision sequences and compliance regimes. For the former, we look at the Stackelberg and simultaneous games, and see whether a type under consideration contract leads to the same equilibrium for each game. For the latter, we see whether the outcome is desirable for each participant. We confine our study to three decision making sequences in two-agent supply chains.

Definition 2.1.2. (SEQUENCE CONSISTENCY) *Given a coordination contract $z \in C$ and the Stackelberg equilibrium $S_i(z) = (y_1^{S_i}(z), y_2^{S_i}(z))$, $i = 1, 2$, the contract z is sequence consistent if $S_1(z) = S_2(z)$.*

In a Stackelberg game, the leader chooses the best possible payoff for himself on the reaction function of the follower. Therefore, we can choose the Nash equilibrium $\mathrm{N}(z)$, when the contract is sequence consistent.

Compliance regimes discuss the consequences of fulfilling a given contract. Since contracts should be enforceable, the forced compliance is the default option in the sense that the non-complying agent gets penalized. However, a compliance regime can also be voluntary when the agents adhere to the contract voluntarily.

Definition 2.1.3. (COMPLIANCE CONSISTENCY) *For a coordination contract $z \in C$, if the system optimal (y_1^*, y_2^*) satisfies the equations*

$$\begin{cases} \pi_1(y_1^*, y_2^*, z) = \max_{y_1, y_2} \pi_1(y_1, y_2, z), \\ \pi_2(y_1^*, y_2^*, z) = \max_{y_1, y_2} \pi_2(y_1, y_2, z), \end{cases} \tag{2.13}$$

then the coordination contract z is compliance consistent.

Thus, when a contract is compliance consistent, then there exists a decision pair (y_1^*, y_2^*) which simultaneously maximizes the profit of both agents. It is clear that the decision pair also maximizes the channel profit. Moreover, when the contract is compliance consistent, any deviation from the decision (y_1^*, y_2^*) makes both agents worse off simultaneously.

Example 2.1.2. Consider the target rebate contract (TRC) with $z = \{w, \nu, T\}$, where the supplier charges the retailer the wholesale price w and rebates the retailer ν for each unit of sales exceeding the target T. Taylor (2002) demonstrates that there exists a critical value $\tau_0(\nu)$ such that the supply chain can be coordinated if the sales target T is less than $\tau_0(\nu)$, and w and ν satisfy

$$F(q^*) = \frac{p+\nu-w}{p+\nu} = \frac{p-c}{p},$$

where q^* is the optimal order quantity. With the TRC, the expected payoffs for the supplier and the retailer are

$$\pi_r(q,z) = \begin{cases} pE\{\min(q,D)\} - wq \\ +\nu E\{(\min(q,D)-T)^+\}, & q \geq T, \\ pE\{\min(q,D)\} - wq, & \text{otherwise}; \end{cases} \tag{2.14}$$

$$\pi_s(q,z) = \begin{cases} (w-c)q - \nu E\{(\min(q,D)-T)^+\}, & q \geq T, \\ (w-c)q, & \text{otherwise}, \end{cases}$$

respectively. Taking derivative with respect to ordering quantity q and (??) into consideration, we have

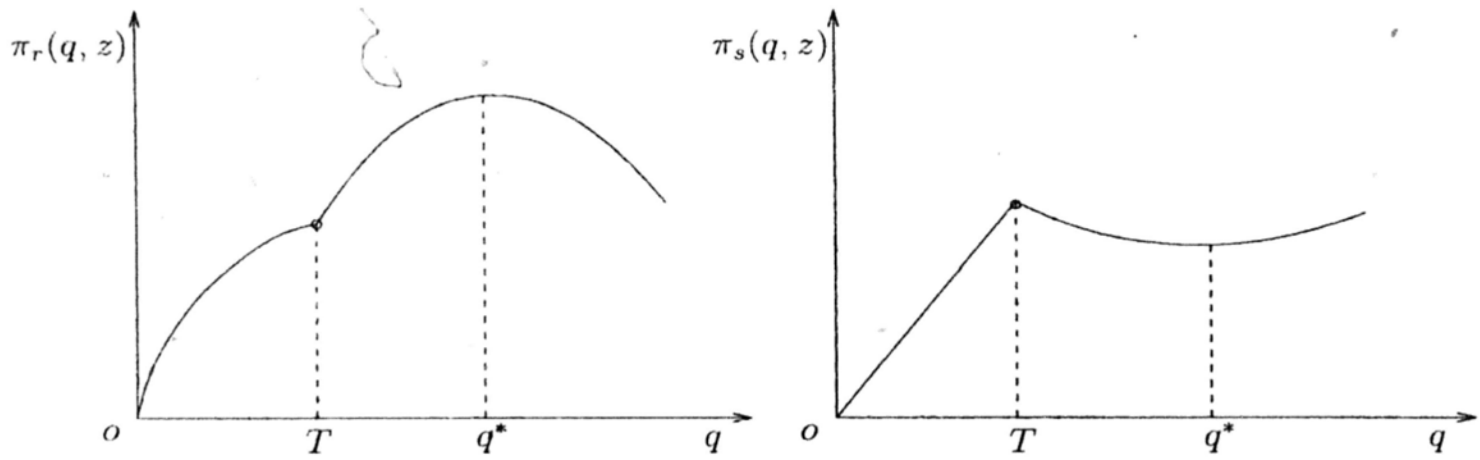

Figure 2.2: The expected profit of the agents in target rebate contract

$$\frac{d\pi_s(q,z)}{dq} = \begin{cases} w - c - \nu + \nu F(q) = \nu(F(q) - F(q^*)), & q \geq T \\ w - c, & \text{otherwise.} \end{cases} \tag{2.15}$$

Expression (??) shows that the profit of the supplier $\pi_s(q,z)$ is increasing in q as long as the order quantity is less than T. When the order quantity q is between T and q^*, the profit $\pi_s(q,z)$ is a decreasing function of q. Therefore, the retailer's order quantity q^* is not desirable from the view point of the supplier. Based on the coordinating contract z, the retailer orders q^* unit from the supplier. But the supplier has an incentive to deliver only T unit. Therefore, the coordination is not voluntary. The supplier has to be forced to deliver the full amount that required by the retailer.

□ **End of chapter.**

Chapter 3

Structural Results and Examples

With definitions and examples above, we are now ready to differentiate coordination contracts in terms of their consistency with respect to the profit allocation, decision making sequence and compliance regime variations. In what follows, we develop the structural results with more examples that demonstrate the properties for different classes of contracts and sufficient conditions that substantiate one class to another.

3.1. Properties of the Sample-path Flexibility

Proposition 3.1.1. *For a sample-path flexible coordination contract $z \in C$, if a realization is favored by one agent, then it is also favored by the other. That is, for any ω and $\omega' \in \Omega$,* *if* $\Pi_i(y_1^*, y_2^*, z, \omega) > \Pi_i(y_1^*, y_2^*, z, \omega')$ *for some* $i \in \{1, 2\}$, *then* $\Pi_{3-i}(y_1^*, y_2^*, z, \omega) > \Pi_{3-i}(y_1^*, y_2^*, z, \omega')$.

PROOF. Contract set C coordinates the supply chain with the sample path flexibility. For any $z \in C$, the sharing rule $\theta_i(y_1^*, y_2^*, z) = \lambda_i(y_1^*, y_2^*, z)$, $i = 1, 2$. Therefore, $\Pi_i(y_1^*, y_2^*, \omega) = \theta_i(y_1^*, y_2^*, z)\Pi(y_1^*, y_2^*, \omega) > \theta_i(y_1^*, y_2^*, z)\Pi(y_1^*, y_2^*, \omega') = \Pi_i(y_1^*, y_2^*, \omega')$. □

A significant advantage of being sample-path flexible is that the random factor affects the profit of the agents in the same manner. Specifically, when the channel profit is large (small), then profits of the agents are large (small).

Proposition 3.1.2. *Assume that a contract set* C *coordinates the supply chain with sample path flexibility. For any given reservation profit target* π_0 *and any allocation factor pair* $a_1 \in [0, 1]$ *and* $a_2 \in [0, 1]$ *such that* $a_1 + a_2 = 1$, *there exists a coordination contract* $z \in C$, *such that the probabilities of reaching the reservation target* π_0 *are equal for agents* 1 *and* 2. *That is,*

$$Pr(\Pi_1(y_1^*, y_2^*, z, \omega) > a_1\pi_0) = Pr(\Pi_2(y_1^*, y_2^*, z, \omega) > a_2\pi_0).$$

PROOF. For the coordinating contract C with sample-path flexibility, for given a_1, a_2, there exists a contract $z \in C$ and its associated sharing rule θ, such that the probabilities of reaching the reservation target z_0 are equal for agent 1 and 2. That is,

$$Pr(\Pi_1(y_1^*, y_2^*, z, \omega) > \lambda_1\pi_0) = Pr(\lambda_1\Pi(y_1^*, y_2^*, z, \omega) > \lambda_1\pi_0)$$
$$= Pr(\Pi(y_1^*, y_2^*, z, \omega) > \pi_0) = Pr(\Pi_2(y_1^*, y_2^*, z, \omega) > \lambda_2\pi_0).$$

□

This proposition is equivalently stated as follows. For any given reservation profits π_{01} and π_{02} for agents 1 and 2, respectively, there exists a coordination contract $z \in C$, such that the probabilities of reaching the reservation profit of each agent are equal. That is

$$Pr(\Pi_1(y_1^*, y_2^*, z, \omega) > \pi_{01}) = Pr(\Pi_2(y_1^*, y_2^*, z, \omega) > \pi_{02}).$$

The proposition demonstrates that a coordination contract with sample-path flexibility is not only able to allocate the expected profit among the agents, but also leaves the same probability for each agent to achieve the reservation payoffs.

3.2. Sufficient Conditions for Sequence Consistency

We first present a lemma which characterizes a relation between the equilibria of the simultaneous and the Stackelberg games.

Lemma 3.2.1. *For a contract $z \in C$, consider the equilibrium $S_i(z)$ for the Stackelberg game led by agent i and the Nash equi-*

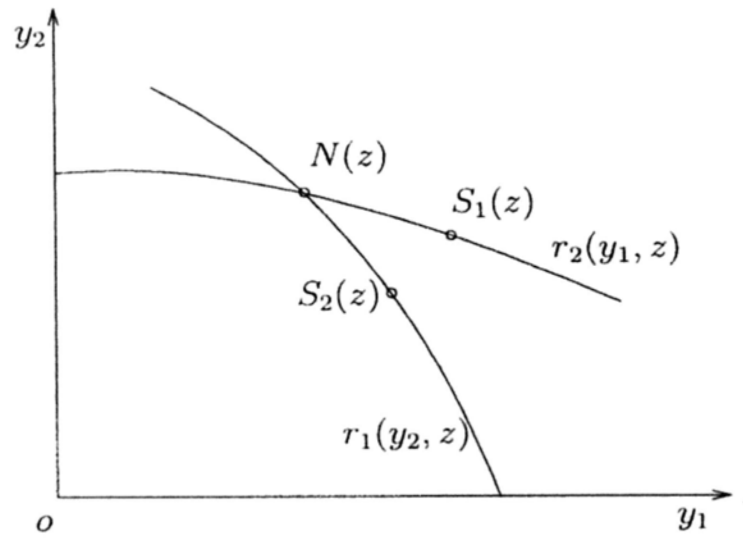

Figure 3.1: Equilibria and reaction functions

librium $N(z)$ for the simultaneous game. If $S_1(z)$ and $S_2(z)$ coincide, then each of them also coincides with the Nash equilibrium $N(z)$.

PROOF. The Nash equilibrium $\mathrm{N}(z)$ is at the intersection of reaction functions of agent 1 and 2. The Stackelberg equilibrium $\mathrm{S}_i(z)$ is on the reaction curve of agent j. Therefore, when $\mathrm{S}_1(z)$ and $\mathrm{S}_2(z)$ coincide, it is on both reaction curves. Hence, it is on the intersection of the two reaction curves, that is $\mathrm{S}_1(z) = \mathrm{S}_2(z) = \mathrm{N}(z)$. □

The equality $\mathrm{S}_i(z) = \mathrm{N}(z)$ indicates that agent i does not have the first-mover advantage. Then, agent 1 cannot improve the performance when given the opportunity to decide before agent 2 does, and vice versa. Note that there exist games in which $S_1(z) = N(z)$ but $S_2(z) \neq N(z)$. Therefore, the equalities $S_1(z) = S_2(z)$ and $S_1(z) = S_2(z) = N(z)$ are equivalent.

Example 3.2.1. Consider the game in the strategic form:

	L	R
U	11,7	12,2
D	2,2	10,10

Agent 1 chooses L or R, and agent 2 chooses U or D. The payoffs are shown in the table, e.g., $\pi_1(L, U, z) = 11$, $\pi_2(L, U, z) = 7$. Here contract z is dummy. Then the reaction functions are $r_1(U, z) = R$, $r_1(D, z) = 10$, $r_2(L, z) = U$ and $r_2(R, z) = D$. The Nash equilibrium $N(z) = \{(R, D)\}$, which coincides with the system optimal $Y^* = \{(R, D)\}$ where $\pi(R, D) = 20$. For the Stackelberg game equilibria, we have

$$S_2(z) = N(z) = \{(R, D)\},\ N(z) \neq S_1(z) = \{(L, U)\}.$$

Next, let us consider models with smooth profit functions. We provide the sufficient conditions for sequence consistency. Rubio (2006) argues that if a game has orthogonal reaction functions, then the first-mover advantage disappears. Then the Stackelberg equilibrium remains unchanged when the leadership is changed.

Proposition 3.2.1. (SUFFICIENT CONDITION I (RUBIO 2006)) *For a coordination contract z, if the payoff functions of the agents are twice differentiable with respect to their decisions and* $\frac{\partial^2 \pi_i(y_1, y_2, z)}{\partial y_i \partial y_{3-i}} = 0$, $i = 1, 2, j \neq i$, *then the contract is sequence consistent.*

Rubio (2006) also develops a condition under which the reaction function of an agent is invariant to the decision of the other. In the discrete-time models, the condition is that the reaction functions are constants and thus the reaction curves are parallel to the axes. Particularly, if the profit functions are separable in y_1 and y_2, i.e., if

$$\begin{cases} \pi_1(y_1, y_2, z) &= G_1(y_1) + G_2(y_2), \\ \pi_2(y_1, y_2, z) &= G_3(y_1) + G_4(y_2), \end{cases} \tag{3.1}$$

then the condition is satisfied.

Proposition 3.2.2. (SUFFICIENT CONDITION II FOR SEQUENCE CONSISTENCY) *For a coordination contract z, if the payoff functions of both agents are differentiable and one of them is independent of the other's decision, then the contract is sequence consistent. That is, if* $\frac{\partial \pi_{3-i}(y_1, y_2, z)}{\partial y_i} = 0$ *for* $i = 1$ *or* 2, *then* $S_1(z) = S_2(z)$.

PROOF. Without loss of generality, we assume that $\frac{\partial \pi_2(y_1, y_2, z)}{\partial y_1} = 0$. That is, the payoff function of agent 2 is independent of y_1.

Therefore, $r_2(y_1, z)$ is a constant. Then, $r_2(y_1) \equiv y_2^N(z)$, for all y_1, where the Nash equilibrium N is denoted by $(y_1^N(z), y_2^N(z))$. Next, let us consider the Stackelberg game where agent 1 is the leader. The objective of agent 1 is $\max_{y_1} \pi_1(y_1, r_2(y_1, z), z) = \max_{y_1} \pi_1(y_1, y_2^N(z), z) = \pi_1(r_1(y_2^N(z), z), y_2^N(z), z) = \pi_1(y_1^N(z), y_2^N(z), z$ The first equality in the above expression relies on the fact that $r_2(y_1, z)$ is a constant, the second equality is based on the definition of the reaction function, and the last equality is the definition of the Nash Equilibrium. Therefore, agent 1 chooses $\arg\max_{y_1} \pi_1(y_1, r_2(y_1, z), z) = y_1^N(z)$, thus $S_1(z) = N(z)$. Next, let us consider the equilibrium, where agent 2 is the leader, clearly that its best strategy is still $y_2 = y_2^N(z)$ which is irrelevant of the choice of y_1. As the follower, agent 1 chooses $r_1(y_2^N(z), z) = y_1^N(z)$, thus $S_2(z) = N(z)$. It proves that the contract is sequence consistent. Similarly, the case for $\dfrac{\partial \pi_1(y_1, y_2, z)}{\partial y_2} = 0$ can be proved. □

As a special case of (??), let $G_3(y_1) = 0$, then clearly $\dfrac{\partial \pi_2(y_1, y_2, z)}{\partial y_1} =$ 0. With the form $\pi_2(y_1, y_2, z) = G_4(y_2)$, agent 1's decision has no impact on agent 2. We know that the best decision of agent 2 is always $y_2^N(z)$ in any decision sequence setting. The decision of agent 1 is determined by its reaction function, which is $y_1(z) = \arg\max_{y_1} \pi_1(y_1, y_2^N(z), z)$, and therefore the decision sequence does not change the outcome of the decentralized decision process.

We now illustrate Proposition ?? by a supply chain selling complementary products with a subsidy contract. Fang and Wang (2010) consider such a supply chain with N complementary products in terms of the package sale. The following example is a special case of their model with two complementary products sold by two agents.

Example 3.2.2. (COMPLETE COMPLEMENTARY PRODUCTS COORDINATION)
Consider a supply chain consisting of two retailers selling two products. The products 1 and 2 are acquired with unit costs c_1 and c_2, respectively. The demand D for the packaged product (1+2), consists of a unit each of products 1 and 2. The retail price for the packaged product is $p = p_1 + p_2$, from which retailers 1 and 2 receive p_1 and p_2, respectively. The leftovers of products 1 and 2 are then sold with individual demands D_1 and D_2 at prices p_1' and p_2' satisfying $p_1' \leq p_1$ and $p_2' \leq p_2$, respectively. The decisions are the ordering quantities (q_1, q_2). Hence, the channel profit is

$$\begin{aligned} \pi(q_1, q_2) &= pE\min\{D, q_1, q_2\} + p_1'E\min\{q_1 - \min\{D, q_1, q_2\}, D_1\} \\ &\quad -c_1q_1 + p_2'E\min\{q_2 - \min\{D, q_1, q_2\}, D_2\} - c_2q_2, \end{aligned}$$

which can be verified as a jointly concave function of (q_1, q_2).

For the decentralized channel, the payoff function for retailer

i is

$$\begin{aligned} & \pi_i(q_i, q_{3-i}, z_0) \\ = \ & p_i E \min\{D, q_1, q_2\} + p_i' E \min\{q_i - \min\{D, q_1, q_2\}, D_i\} - c_i q_i \\ = \ & \begin{cases} p_i E \min\{D, q_i\} + p_i' E \min\{q_i - \min\{D, q_i\}, D_i\} - c_i q_i, & q_i < q_{3-i}, \\ p_i E \min\{D, q_{3-i}\} + p_i' E \min\{q_i - \min\{D, q_{3-i}\}, D_i\} - c_i q_i, & q_i \geq q_{3-i}, \end{cases} \end{aligned} \tag{3.2}$$

where z_0 denotes the null contract or no contract. Clearly, $\pi_i(q_1, q_2, z_0)$ is piecewise concave in q_i for $q_i \in [0, q_{3-i}]$ and $q_i \in [q_{3-i}, +\infty), i = 1, 2$. On the line $q_1 = q_2$, it is possible to check that the left derivative of $\pi_i(q_1, q_2, z_0)$ is greater than the right derivative. Therefore, $\pi_i(q_1, q_2, z_0)$ is concave in $q_i \in [0, \infty)$. From (??), we can obtain the optimal order quantity of retailer i as $\gamma_i(z_0)$ when q_{3-i} is infinitely large and as $\zeta_i(q_{3-i}, z_0)$, otherwise, as follows:

$$\begin{cases} \gamma_i(z_0) = \arg\max\limits_{q_i}\{p_i E \min\{D, q_i\} + p_i' E \min\{q_i \\ - \min\{D, q_i\}, D_i\} - c_i q_i\}, \\ \zeta_i(q_{3-i}, z_0) = \arg\max\limits_{q_i}\{p_i' E \min\{q_i - \min\{D, \\ q_{3-i}\}, D_i\} - c_i q_i\}. \end{cases} \tag{3.3}$$

Following this notion and expression (??), the reaction function of retailer i is

$$r_i(q_{3-i}, z_0) = \begin{cases} \zeta_i(q_{3-i}, z_0), & q_{3-i} \leq \gamma_i(z_0), \\ \gamma_i(z_0), & q_{3-i} > \gamma_i(z_0). \end{cases}$$

Without loss of generality, we may assume that $\gamma_1(z_0) \leq \gamma_2(z_0)$. The following considerations help us to draw the reaction curves and delineate the strategy space for contract z_0 in Figure ??(a). If retailer 2 orders nothing, retailer 1 would like to order the optimal quantity to satisfy the individual demand D_1, that is $q_1 = F_1^{-1}(\frac{p'-c_1}{p'})$, denoted as $A_1(z_0)$ in the figure. As q_2 increases from 0 to $\gamma_1(z_0)$, the reaction function $r_1(q_2, z_0) = \zeta_1(q_2, z_0)$, and it increases from $A_1(z_0)$ to $\gamma_1(z_0)$. When q_2 is larger than $\gamma_1(z_0)$, $r_1(q_2, z_0) = \gamma_1(z_0)$ and it is a constant. Similarly, we can depict the reaction function $r_2(q_1, z_0)$ as the curve originating from $A_2(z_0) = q_2 = F_2^{-1}(\frac{p'-c_2}{p'})$, as an increasing function $\zeta_2(q_1, z_0)$ from $A_2(z_0)$ to $\gamma_2(z_0)$, and as the constant $\gamma_2(z_0)$ afterwards. Hence, $r_i(q_{3-i}, z_0)$ is an increasing function for $q_{3-i} \in [0, M]$, $i = 1, 2$, where M is an arbitrarily large number. Consequently, the strategy space $[\gamma_1(z_0), M] \times [\gamma_1(z_0), M]$ is depicted as the shadowed area in Figure ??(a). The Nash equilibrium $\mathrm{N}(z_0) = (q_1^{\mathrm{N}}(z_0), q_2^{\mathrm{N}}(z_0))$ is the intersection point of the agents' reaction curves.

We now consider the channel efficiency. From $\gamma_1(z_0) \leq \gamma_2(z_0)$ follows $q_1^* < q_2^*$. Furthermore, we next show that the channel

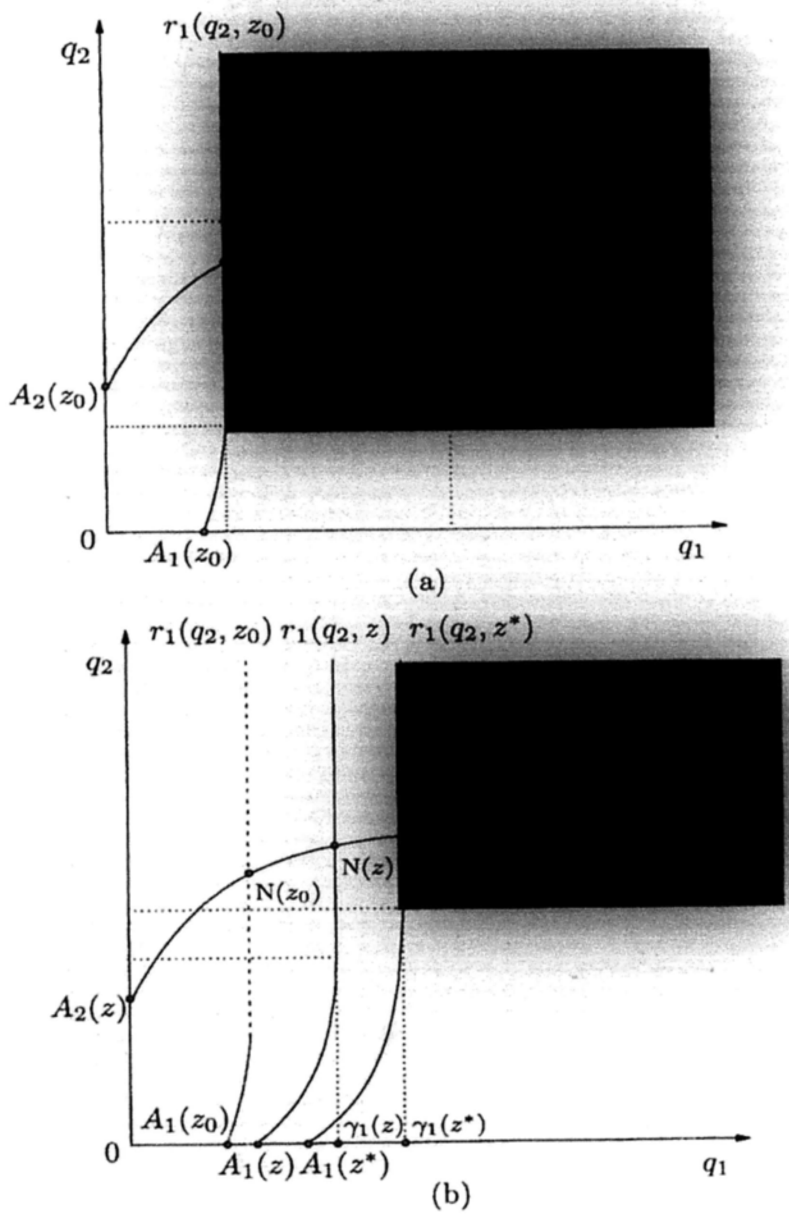

Figure 3.2: The reaction functions of z_0, z and z^*

optimal $y^* = (q_1^*, q_2^*)$ is on the reaction function of retailer 2. The channel optimal order quantity for retailer 2 is

$$\begin{aligned} q_2^* &= \arg\max_{q_2 > q_1^*} \pi(q_1^*, q_2) &(3.4)\\ &= \arg\max_{q_2 > q_1^*} \{p_2' E \min\{q_2 - \min\{D, q_1^*\}, D_2\} - c_2 q_2\} \\ &= \zeta_2(q_1^*, z_0). \end{aligned}$$

Recall that the terms on the right side of (??) for $i = 1$ are independent of q_2 as long as $q_1 \leq q_2$. Thus the optimization of (??) with respect to q_2 reduces to optimizing the second equality in (??), i.e., q_2^* is as given in (??). Therefore, we can conclude that the channel optimal (q_1^*, q_2^*) is on the reaction function of retailer 2 shown as y^* illustrated in Figures ??(a) and ??(b).

For the decentralized supply chain, retailer 1 ignores the potential of increasing package sales for retailer 2, whose reaction function is increasing in q_1. Therefore, the equilibrium $\mathrm{N}(z_0)$ is on the lower left of y^* as shown in Figure ??(a), where $q_1^{\mathrm{N}}(z_0) < q_1^*$ and $q_2^{\mathrm{N}}(z_0) \leq q_2^*$. Clearly, $\mathrm{N}(z_0)$ is inefficient.

We now demonstrate that an incentive contract can be designed to entice retailer 1 to increase his order quantity by subsidizing him for his unsold inventories. Such a contract $z = \{b\}, b \leq c_1$, provides a trans-payment $T(q_1, z) = b(q_1 - \min\{D, q_1\} - D_1)^+$ between the retailers, where b is the unit subsidy for the unsold inventories, and it is similar to the buy-back scheme. Under the contract, we denote the optimal order quantity of retailer i as

$$\begin{cases} \gamma_i(z) = \arg\max_{q_i}\{p_i E\min\{D, q_i\} + p'_i E\min\{q_i - \min\{D, q_i\}, D_i\} \\ \qquad -c_i q_i - (-1)^i \cdot T(q_1, z)\}, \\ \zeta_i(q_{3-i}, z) = \arg\max_{q_i}\{p'_i E\min\{q_{3-i} - \min\{D, q_{3-i}\}, D_i\} \\ \qquad -c_i q_i + (-1)^i \cdot T(q_1, z)\}. \end{cases} \tag{3.5}$$

The reaction function of retailer i is

$$r_i(q_{3-i}, z) = \begin{cases} \zeta_i(q_{3-i}, z), & q_{3-i} \leq \gamma_i(z), \\ \gamma_i(z), & q_{3-i} > \gamma_i(z). \end{cases}$$

We again assume that $\gamma_1(z) \leq \gamma_2(z)$ draw the reaction curves and depict the strategy space for contract z in Figure ??(b). If retailer 2 orders nothing, retailer 1 makes decision in response to demand D_1 by taking the subsidy into consideration, that is $q_1 = F_1^{-1}(\frac{p'-c_1}{p'-b})$ as denoted by $A_1(z)$ in the figure. Clearly, $r_1(q_2, z)$ is an increasing function of b. By (??), the reaction function $r_1(q_2, z)$ equals to $\zeta_1(q_2, z)$, which increases from $A_1(z)$ to $\gamma_1(z)$ when q_2 increases from 0 to $\gamma_1(z)$. The reaction function $r_1(q_2, z)$ becomes a constant when q_2 is larger than $\gamma_1(z)$. With the incentive contract z, the payoff function of retailer 2 is $\pi_2(q_1, q_2, z) = \pi_2(q_1, q_2, z_0) - T(q_1, z)$. Consequently, $\arg\max_{q_2} \pi_2(q_1, q_2, z) = \arg\max_{q_2} \pi_2(q_1, q_2, z_0)$. Therefore, the reaction function of retailer 2 remains unchanged, and is as in Figure ??(a). The Nash equilibrium $\mathrm{N}(z) = (q_1^{\mathrm{N}}(z), q_2^{\mathrm{N}}(z))$ for any constant z can be easily obtained as in Figure **??**(b). Moreover, there exists a coordination contract $z^* = \{b^*\}$, satisfying $\gamma_1(z^*) = q_1^*$, where the equilibrium $\mathrm{N}(z^*) = y^*$ is as illustrated in Figure **??**(b). Also, the strategy space is $[\gamma_1(z^*), M] \times [\gamma_1(z^*), M]$ as shown shadowed in Figure **??**(b).

In the strategy space defined by the contract z^*, the payoff function π_1 is independent of q_2. That is, $\pi_1(q_1, q_2, z^*) = \pi_1(q_1, z^*)$. Thus, by Proposition **??**, we conclude that the coordination contract z^* is sequence consistent.

We can now state a lemma and the third sufficient condition, which it leads to, for sequence consistency.

Lemma 3.2.2. *For a coordination contract $z \in C$ in a simultaneous or a Stackelberg game, if the payoff functions are differentiable, then* $\frac{\partial \pi_i(y_1, y_2, z)}{\partial y_{3-i}}|_{(y_1,y_2)=(y_1^e(z), y_2^e(z))} = 0$, $i = 1, 2$.

PROOF. First, let us consider the simultaneous game setting. The decentralized supply chain leads to Nash equilibrium $(y_1^N(z), y_2^N(z))$. The supply chain coordination ensures that $(q_1^N(z), q_2^N(z)) = (q_1^*, q_2^*)$.

By the definition of the equilibrium and coordination, we have

$$\begin{cases} \left.\frac{\partial \pi_1(y_1, y_2, z)}{\partial y_1}\right|_{(y_1,y_2)=(y_1^N(z), y_2^N(z))} = 0 \\ \left.\frac{\partial \pi_2(y_1, y_2, z)}{\partial y_2}\right|_{(y_1,y_2)=(y_1^N(z), y_2^N(z))} = 0 \end{cases}$$

$$\text{and}\begin{cases}\left[\frac{\partial\pi_1(y_1,y_2,z)}{\partial y_1}+\frac{\partial\pi_2(y_1,y_2,z)}{\partial y_1}\right]\Big|_{(y_1,y_2)=(y_1^N(z),y_2^N(z))}=0\\ \left[\frac{\partial\pi_1(y_1,y_2,z)}{\partial y_2}+\frac{\partial\pi_2(y_1,y_2,z)}{\partial y_2}\right]\Big|_{(y_1,y_2)=(y_1^N(z),y_2^N(z))}=0.\end{cases}\tag{3.6}$$

By simple transformation, it is straight forward to show that the equality $\frac{\partial\pi_i(y_1,y_2,z)}{\partial y_j}|_{(y_1,y_2)=(y_1^c(z),y_2^c(z))}=0$, $i,j=1,2$ holds.

Second, let us consider the dynamic game setting. Without loss of generality, we assume that agent 1 is the Stackelberg game leader. Again the supply chain is coordinated, hence we know the Stackelberg equilibrium is the supply chain optimum, i.e., $(y_1^{S_1}(z),y_2^{S_1}(z))=(y_1^*,y_2^*)$. By the definition of Stackelberg equilibrium, we have

$$\begin{cases}\left[\frac{\partial\pi_1(y_1,r_2(y_1,z),z)}{\partial y_1}+\frac{\partial\pi_1(y_1,r_2(y_1,z),z)}{\partial r_2}\cdot\frac{dr_2(y_1)}{dy_1}\right]\Big|_{y_1=y_1^{S_1}(z)}=0\\ \frac{\partial\pi_2(y_1,y_2,z)}{\partial y_2}\Big|_{(y_1,y_2)=(y_1^{S_1}(z),y_2^{S_1}(z))}=0.\end{cases}\tag{3.7}$$

Since we know $r_2(y_1^{S_1}(z))=y_2^{S_1}(z)$, equation (??) can be rewritten as follows.

$$\begin{cases}\frac{\partial\pi_1(y_1,\ y_2^{S_1}(z),z)}{\partial y_1}\Big|_{y_1=y_1^{S_1}(z)}+\left(\frac{\partial\pi_1(y_1^{S_1}(z),\ y_2,z)}{\partial y_2}\Big|_{y_2=y_2^{S_1}(z)}\right)\cdot\frac{dr_2(y_1)}{dy_1}\Big|_{y_1=y_1^{S_1}(z)}=0,\\ \frac{\partial\pi_2(y_1,\ y_2,z)}{\partial y_2}\Big|_{(y_1,y_2)=(y_1^{S_1}(z),\ y_2^{S_1}(z))}=0.\end{cases}\tag{3.8}$$

By simple transformation of equation (??) and (??), we can derive the equality $\frac{\partial\pi_i(y_1,y_2,z)}{\partial y_j}|_{(y_1,y_2)=(y_1^c(z),y_2^c(z))}=0$, $i,j=1,2$. This completes the proof. □

With Lemma ??, we have the third sufficient condition for contracts with smooth payoff functions.

Proposition 3.2.3. (SUFFICIENT CONDITION III FOR SEQUENCE CONSISTENCY) *For a coordination contract $z \in C$, in a Nash or a Stackelberg setting, if $\pi_1(y_1, r_2(y_1, z), z)$ and $\pi_2(r_1(y_2, z), y_2, z)$ are differentiable and concave with respect to y_1 and y_2, respectively, then z is sequence consistent.*

PROOF. Assume contract z achieves coordination for a given game structure and its associated decision sequence, the equilibrium in the decentralized supply achieves the supply chain optimum, i.e., $(y_1^e(z), y_2^e(z)) = (y_1^*, y_2^*)$. We next show that $S_1(z) = (y_1^{S_1}(z), y_2^{S_2}(z)) = (y_1^e(z), y_2^e(z))$ holds for the given decision sequence.

By the assumption that $\pi_1(y_1, r_2(y_1, z), z)$ is concave in y_1, we need to prove that if agent 1 is the leader, (y_1^*, y_2^*) satisfies the first order condition, as follows.

$$\begin{cases} \left.\frac{\partial \pi_1(y_1,\ y_2^e(z), z)}{\partial y_1}\right|_{y_1=y_1^e(z)} + \left(\left.\frac{\partial \pi_1(y_1^e(z),\ y_2, z)}{\partial y_2}\right|_{y_2=y_2^e(z)} \right) \cdot \left.\frac{dr_2(y_1)}{dy_1}\right|_{y_1=y_1^e(z)} = 0, \\ \left.\frac{\partial \pi_2(y_1,\ y_2, z)}{\partial y_2}\right|_{(y_1,y_2)=(y_1^e(z),y_2^e(z))} = 0. \end{cases}$$

By Lemma **??**, we know that

$$\left.\frac{\partial \pi_1(y_1,\ y_2^e(z), z)}{\partial y_1}\right|_{y_1=y_1^e(z)} = \left.\frac{\partial \pi_1(y_1^e(z),\ y_2, z)}{\partial y_2}\right|_{y_2=y_2^e(z)}$$
$$= \left.\frac{\partial \pi_2(y_1,\ y_2, z)}{\partial y_2}\right|_{(y_1,y_2)=(y_1^e(z),y_2^e(z))} = 0.$$

Therefore, the above first order condition holds at the decision $(y_1^e(z), y_2^e(z))$. That $(y_1^*, y_2^*) = (y_1^e(z), y_2^e(z))$ is the Stackelberg equilibrium.

Similarly, it can be shown that $S_2(z) = (y_1^{S_2}(z), y_2^{S_2}(z)) = (y_1^e(z), y_2^e(z))$. Hence, $S_1(z) = S_2(z)$. The coordination is sequence consistent. □

The incentive alignment problem between a marketing manager and an operation manager, studied by Jerath *et al.* (2007), is a good example to illustrate this sufficient condition.

Example 3.2.3. (MARKETING AND PRODUCTION COORDINATION)
In a company, the marketing manager decides the sales effort x in promoting the product, whereas the production manager decides the production quantity q in targeting to lower the production cost and attempting to match the demand with the supply. The demand $D(x)$ is assumed to be an additive function of the sales effort x and a random factor ε, thus $D(x) = x + \varepsilon$. It is further assumed that the random factor ε is uniformly distributed in $[0, \sigma]$. The inventory acquisition cost is c_b and the penalty and holding costs are c_p and c_h, respectively. Moreover, the marketing manager and the production manager incur the cost of the sales effort, $C_m x^2$, and the cost of production, $C_p q^2$, respectively. The objective for the company is to maximize the expected profit, which can be written as

$$\begin{aligned} \pi(x,q) &= pE\min\{D(x), q\} - c_h E(q - D(x))^+ - c_p E(D(x) - q)^+ \\ &\quad - c_b q - C_m x^2 - C_p q^2. \end{aligned}$$

It can be proved that $\pi(x,q)$ is jointly concave in x and q. Hence, the centralized optimal decision (x^*, q^*) is unique.

The company provides an incentive contract $z = (\alpha_m, \alpha_p, \beta_p)$ to the two managers, by rewarding each manager an amount proportional to the sales revenue and penalizing the production manager for the inventory cost. Under such a contract, the two managers make their distinctive decisions x and q to maximize their individual payoffs, which are written as

$$\left\{\begin{array}{rcl} \pi_m(x,q,z) &=& \alpha_m p E\min\{x+\varepsilon, q\} - C_m x^2, \\ \pi_p(x,q,z) &=& \alpha_p p E\min\{x+\varepsilon, q\} - C_p q^2 \\ && -\beta_p(c_h E(q-x-\varepsilon)^+ - c_p E(x+\varepsilon-q)^+ + c_b q) \end{array}\right. \tag{3.9}$$

for the marketing and production managers, respectively. The partial derivatives with respect to the individual profit functions (??) are

$$\left\{\begin{array}{rcl} \dfrac{\partial \pi_m(q,x,z)}{\partial x} &=& \alpha_m p F(q-x) - 2C_m x, \\ \dfrac{\partial \pi_p(q,x,z)}{\partial q} &=& -(\alpha_p p + \beta_p c_h + \beta_p c_p) F(q-x) + \alpha_p p + \beta_p c_p \\ && -\beta_p c_b - 2C_p q. \end{array}\right.$$

The second-order partial derivatives with respect to the individual decisions can be shown to be negative, as the payoff functions are concave with respect to their individual decisions. We also require the parameters of the contract z to satisfy the inequality $\alpha_p p + \beta_p c_p - \beta_p c_b > 0$. This condition ensures that the produc-

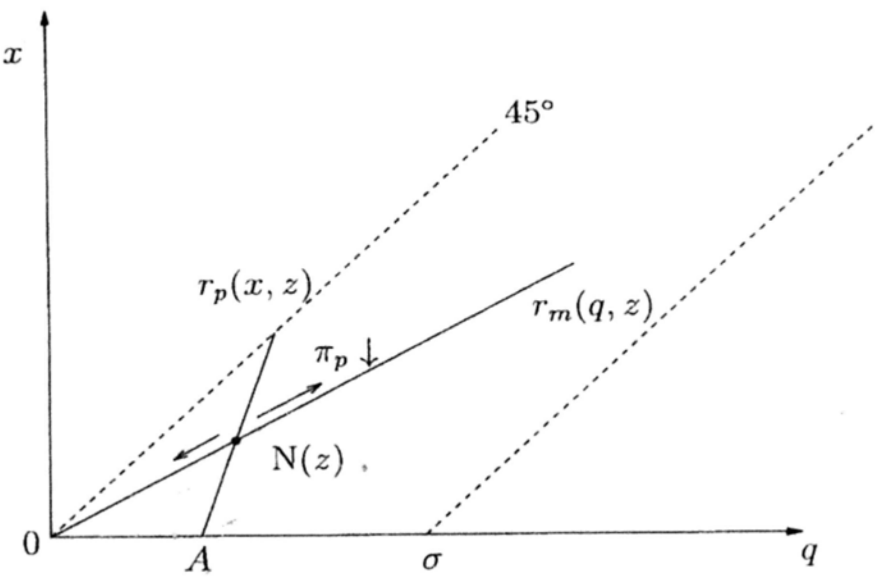

Figure 3.3: The reaction curves for the managers

tion manager has enough incentive to build a positive inventory. Otherwise, the partial derivative of $\pi_p(x, q, z)$ with respect to q is always negative, and the production manager would set $q = 0$.

Because the demand is uniformly distributed over $[x, x + \sigma]$ for any given effort $x \geq 0$, the production manager would limit his production q in this interval. Thus, the strategy spacing is the area defined by $x \leq q \leq x + \sigma$ and $x \geq 0$, shown as the area between the two dashed lines and the z-axis in Figure ??.Within this restricted space, $f(x) = \frac{1}{\sigma}$ and $F(x) = \frac{x}{\sigma}$, and the reaction functions can be derived as follows:

$$\begin{cases} r_m(q, z) = \dfrac{\alpha_m p}{\alpha_m p + 2\sigma C_m} q, \\ r_p(x, z) = \dfrac{(\alpha_p p - \beta_p c_b + \beta_p c_p)\sigma}{\alpha_p p + \beta_p c_h + \beta_p c_p + 2C_p\sigma} + \dfrac{\alpha_p p + \beta_p c_h + \beta_p c_p}{\alpha_p p + \beta_p c_h + \beta_p c_p + 2C_p\sigma} x. \end{cases} \quad (3.10)$$

The reaction curves are linear as shown in Figure ??, and their slopes satisfy $|\frac{dr_m(q,z)}{dx}| < 1$ and $|\frac{dr_p(x,z)}{dq}| < 1$. The reaction curve of the marketing manager passes through the origin. When the production manager produces nothing, the marketing manager would exert zero sales effort. On the other hand, the reaction curve of the production manager passes through point $A > 0$. When the marketing manager spends no effort, there is still a positive demand uniformly distributed over $[0, \sigma]$. Hence, the production manager would order a positive quantity. Hence in the simultaneous decision making situation, the Nash equilibrium $\mathrm{N}(z)$ is unique in the strategy space.

Jerath *et al.* (2007) derive a coordinating contract $z^* = (\alpha_m^*, \alpha_p^*, \beta_p^*)$. We show now that the contract z^* is sequence consistent. We substitute the managers' respective reaction functions for their decisions into their payoff functions. We show that the derived payoff functions $\pi_m(x, r_p(x, z^*), z^*)$ and $\pi_p(r_m(q, z^*), q, z^*)$ are concave in x and q, respectively, by showing that their second-order derivatives are negative. First, let us verify the concavity for the payoff function of the production manager.

The first and the second order derivatives are

$$
\begin{aligned}
\frac{d\pi_p(q, r_m(q, z^*), z^*)}{dq} &= (\alpha_p^* p - \beta_p^* c_b + \beta_p^* c_p) - (\alpha_p^* p + \beta_p^* c_h + \beta_p^* c_p) \\
& F(q - r_m(q, z^*)) \cdot (1 - \frac{dr_m(q, z^*)}{dq}) \\
& -(\alpha_p^* p + \alpha_p^* c_h) c_p (1 - \frac{dr_m(q, z^*)}{dq}) - 2C_p q, \\
\frac{d^2\pi_p(q, r_m(q, z^*), z^*)}{dq^2} &= -(\alpha_p^* p - \beta_p^* c_b + \beta_p^* c_p) f(q - r_m(q, z^*)) \\
& (1 - \frac{dr_m(q, z^*)}{dq})^2 - 2C_p. \qquad (3.11)
\end{aligned}
$$

Because $\alpha_p^* p - \beta_p^* c_b + \beta_p^* c_p \geq 0$, the second-order derivative is negative. For the payoff function of the marketing manager, the first and second derivatives are

$$
\begin{aligned}
\frac{d\pi_m(r_p(x, z^*), x, z^*)}{dx} &= \alpha_m^* p(1 - F(r_p(x, z^*) - x))(\frac{dr_p(x, z^*)}{dx} - 1) \\
& +\alpha_m^* p - 2C_m x, \\
\frac{d^2\pi_m(r_p(x, z^*), x, z^*)}{dx^2} &= -\alpha_m^* p f(r_p(x, z^*) - x)(\frac{dr_p(x, z^*)}{dx} - 1)^2 \\
& -2C_m \leq 0, \qquad (3.12)
\end{aligned}
$$

and the second-order derivative is clearly negative.

Now, by Proposition ??, we know that the payoff of the marketing (production) manager first increases and then decreases along the respective reaction curve, and reaches its maximal value at the Nash equilibrium N(z^*), which is also the channel optimal. Therefore, the equilibria of the Stackelberg games coincide with the Nash equilibrium, and the coordination contract z^* is sequence consistent.

Note that the concavities of $\pi_p(q, r_m(q, z^*), z^*)$ in q and $\pi_m(x, r_p(x, z^*), z^*)$ in x are independent of the density function f (see (??) and (??)). It is the cost structure and the linear profit split that make the contract z^* sequence consistent. Also, we would like to point out that the coordination of the supply chain does require the demand distribution to be uniform.

3.2.1 Sufficient Conditions for Compliance Consistency

In order to develop sufficient conditions for compliance consistency, we introduce the notions of the preference set and the voluntary compliance set. For a given contract z, provided that agent i makes all decisions, we define the *preference* set $V_i(z)$ for agent i as the collection of his optimal decisions:

$$V_i(z) = \{(y_1^*, y_2^*) \mid (y_1^*, y_2^*) = \arg\max_{y_1, y_2} \pi_i(y_1, y_2, z)\}, \ i = 1, 2. \tag{3.13}$$

We denote the *voluntary compliance* set $V(z)$ for the contract z as the intersection of the preference sets $V_1(z)$ and $V_2(z)$, i.e., $V(z) := V_1(z) \cap V_2(z)$. From Definition **??** of voluntary compliance, it is clear that the contract z coordinates the supply chain with compliance consistency if $V(z) \neq \emptyset$. Alternatively, the the preference sets defined in (??) can also be written as

$$\begin{cases} V_1(z) = \{(r_1(y_2^*, z), y_2^*) \mid y_2^* = \arg\max_{y_2} \pi_1(r_1(y_2, z), y_2, z)\}, \\ V_2(z) = \{(y_1^*, r_2(y_1^*, z)) \mid y_1^* = \arg\max_{y_1} \pi_2(y_1, r_2(y_1, z), z)\}. \end{cases}$$

Comparing with the Stackelberg equilibrium S_i which lies on the reaction function of agent $3-i$, the preference set $V_i(z)$ is included in the reaction curve of agent i, $i=1,2$.

As a special case, when there is only one decision maker, say agent 2, the expected payoff functions for agents 1 and 2 are $\pi_1(y_2, z)$ and $\pi_2(y_2, z)$, respectively. A compliance consistent contract requires that the optimal decision of agent 2 is also in the best interest of agent 1, that is, $\arg\max_{y_2} \pi_1(y_2, z) = \arg\max_{y_2} \pi_2(y_2, z)$. For example, with the buy-back contract, the preferred inventory levels of the supplier and the retailer are the same as the channel order quantity q^*. Hence, the voluntary compliance set is $\{q^*\}$. In contrast, as discussed immediately after Definition **??**, the preference sets for the retailer and the supplier in Example **??** are $\{q^*\}$ and $\{T\}$, respectively. Therefore, the voluntary compliance set is empty for the target rebate contract.

For contracts with smooth payoff functions, we show that the joint concavity ensures the compliance consistency. To illustrate this, we present Example **??** below, which is a simplified version of Cachon (2003) obtained by omitting the internal market and the uncertainty in the production process.

Example 3.2.4. (WHOLESALE PRICE CONTRACT IN AN ENDOGENOUS CAPACITY NEWSVENDOR MODEL)

A vertical supply chain consists of a manufacturer and a retailer. A single product is produced by the manufacturer and sold through the retailer to the market with a stochastic demand D. The manufacturer makes the capacity decision Q_m and the retailer decides the inventory level q_r. The unit cost of the production capacity is increasing in Q_m. The order q_r is either fully or partially fulfilled depending on the capacity Q_m. It is assumed that the unsold products are simply disposed. The manufacturer incurs an increasing convex cost $c(Q_m)$ for the production capacity Q_m. The manufacturer charges the retailer a wholesale price w. With the wholesale price contract $z = \{w\}$, the expected payoff functions for the manufacturer and the retailer are, respectively,

$$\begin{cases} \pi_m(Q_m, q_r, z) = w \min\{Q_m, q_r\} - c(Q_m), \\ \pi_r(Q_m, q_r, z) = pE \min\{Q_m, q_r, D\} - w \min\{Q_m, q_r\}. \end{cases} \tag{3.14}$$

For the centralized supply chain, let Q denote the capacity and q denote the inventory level. It is clear that $Q = q$. Therefore, for the centralized supply chain, we optimize the payoff function $\pi(Q) = pE \min\{D, Q\} - c(Q)$. Note that $pE \min\{D, Q\}$ is concave and $c(Q)$ is convex in Q. Therefore, the payoff function $\pi(Q)$ is concave in Q. The optimal capacity Q^* is the solution of the first-order-condition

$$p(1 - F(Q)) - \frac{dc(Q)}{dQ} = 0. \tag{3.15}$$

For the decentralized supply chain with the contract $z = \{w\}$, by (??), the payoff functions and the decisions of the manufacturer and the retailer are intertwined. To facilitate our analysis, we define the quantities

$$\begin{cases} \hat{Q}_m(z) = \arg\max_{Q_m}\{wQ_m - c(Q_m)\}, \\ \hat{q}_r(z) = \arg\max_{q_r}\{pE\min\{q_r, D\} - wq_r\}. \end{cases} \tag{3.16}$$

as the solution of the first-order conditions

$$\frac{dc(Q_m)}{dQ_m} - w = 0 \text{ and } p(1 - F(q_r)) - w = 0. \tag{3.17}$$

However, these quantities would be the optimal decision only when feasible, which they are not. In order to obtain the optimal decision, we observe that $\hat{Q}_m(z)$ is an increasing function of w and $\hat{q}_r(z)$ is a decreasing function of w. Therefore, there exists a contract z' that leads to $\hat{Q}_m(z') = \hat{q}_r(z')$, and as shown subsequently, coordinates the channel.

In the decentralized supply chain, the reaction functions of the manufacturer and the retailer are

$$r_m(q_r, z) = \begin{cases} q_r, & q_r < \hat{Q}_m(z), \\ \hat{Q}_m(z), & q_r \geq \hat{Q}_m(z); \end{cases} \text{ and}$$

$$r_r(Q_m, z) = \begin{cases} Q_m, & Q_m < \hat{q}_r(z), \\ \hat{q}_r(z), & Q_m \geq \hat{q}_r(z). \end{cases} \tag{3.18}$$

Without loss of generality, for $z_1 = \{w_1\}$ and $z_2 = \{w_2\}$ such that $w_1 \leq w_2$, we use (??) and (??) to obtain the reaction curves of the manufacturer and the retailer as shown in Figure ?? (a). With the contract z_1, if the retailer orders nothing, the manufacturer sets the capacity to zero. As the retailer increases his order, the manufacture increases his capacity until it reaches $\hat{Q}_m(z_1)$, and then remains there. Therefore, the reaction curve is $0 - B_1 - D_1$ in Figure ?? (a). With the contract z_2, since $\hat{Q}_m(z_1) \leq \hat{Q}_m(z_2)$, the reaction curve for the manufacturer is $0 - B_2 - D_2$. Similarly, the reaction curves of the contracts z_1 and z_2 for the retailer are $0-A_1-C_1$ and $0-A_2-C_2$, respectively.

Since $\hat{Q}_m(z')$ and $\hat{q}_r(z')$ solve (??) for $w = z'$, we have $p(1 - F(Q)) = \frac{dc(Q)}{dQ}$ at $Q = \hat{Q}_m(z') = \hat{q}_r(z')$. Thus, $\hat{Q}_m(z') = \hat{q}_r(z') = Q^*$. With the identification of Q^*, the reaction functions (??) reduce to

$$r_m(q_r, z) = \begin{cases} q_r, & q_r < Q^*, \\ Q^*, & q_r \geq Q^*; \end{cases} \quad \text{and} \quad r_r(Q_m, z) = \begin{cases} Q_m, & Q_m < Q^*, \\ Q^*, & Q_m \geq Q^*, \end{cases}$$

and these are depicted in Figure ?? (b).

The preference sets for the manufacturer and the retailer are $V_m(z') = \{(q_r, Q_m) \mid Q_m = Q^*,\ q_r \geq Q^*\}$ and $V_r(z') = \{(q_r, Q_m) \mid q_r = Q^*,\ Q_m \geq Q^*\}$, respectively, these are $y^* - D$ and $y^* - C$ in Figure ?? (b), respectively. The intersection of these two preference sets $y^* - D$ and $y^* - C$ is the voluntary compliance set $V(z') = V_m(z') \cap V_r(z') = Y^*$. We conclude that the contract z' coordinates the supply chain with compliance con-

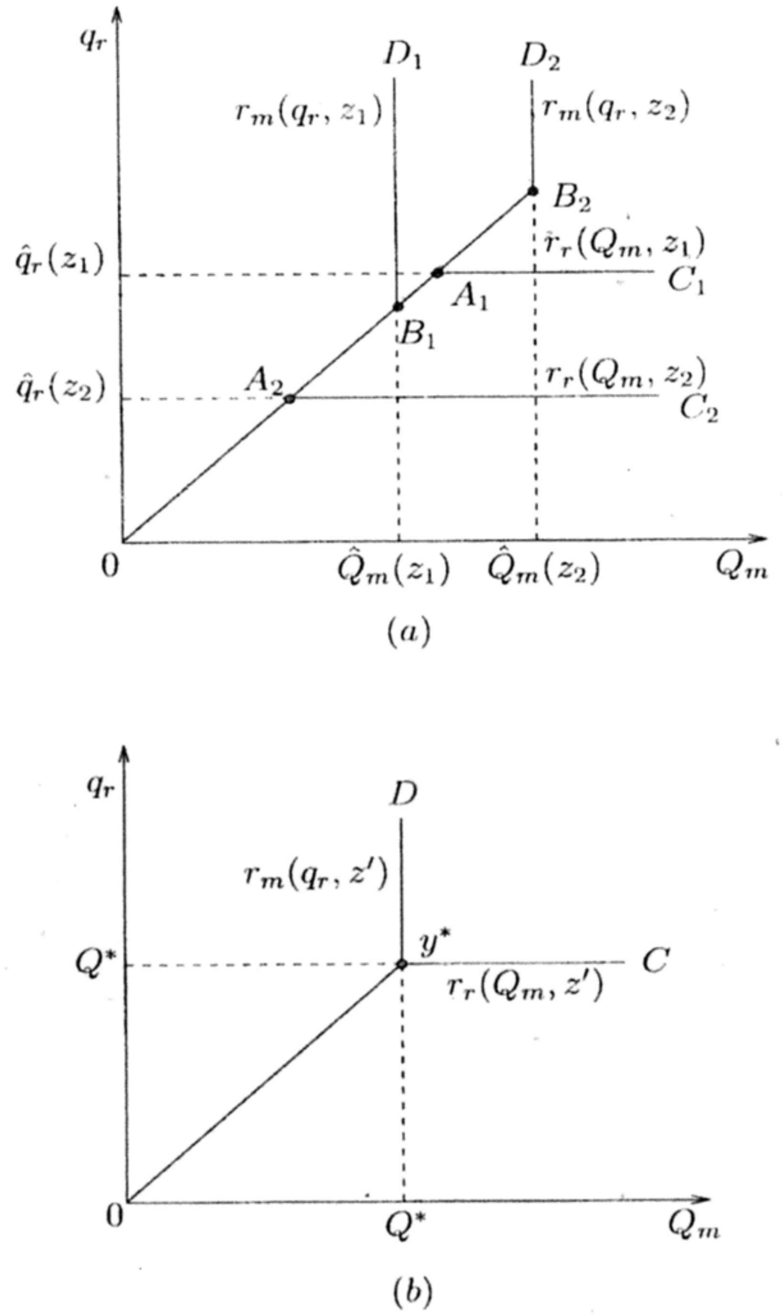

Figure 3.4: Reaction curves of endogenous capacity Newsvendor

sistency. Moreover, Y^* is known as the *Pareto-dominant* Nash equilibrium (Fudenberg and Tirole, 1991), defined as the Pareto optimal decision in the set of Nash equilibria. Fudenberg and Tirole (1991) further argue that the agents are more likely to coordinate on a Pareto-dominant Nash equilibrium if there exist multiple equilibria. Therefore, in the context of this paper, the voluntary compliance set is the subset of the Pareto-dominant Nash equilibria.

Finally, we consider the consistency in allocation of the channel profit. Denote all the coordinating wholesale price contracts as the set C. Note that a contract $z \in C$ coordinates the supply chain, if and only if $z = z'$. Hence, the set C consists of the coordination contract z' only. As a result, the profit allocation is fixed, and therefore not flexible.

Proposition 3.2.4. (SUFFICIENT CONDITION I FOR COMPLIANCE CONSISTENCY) *For a contract $z \in C$ which achieves coordination by either the simultaneous game or a Stackelberg game, if $\pi_1(y_1, y_2, z)$ and $\pi_2(y_1, y_2, z)$, the payoff functions of agents 1 and 2, are twice differentiable and jointly concave with respect to (y_1, y_2) and have a local minimum in $[0, \bar{y}_1] \times [0, \bar{y}_2]$, then the coordination is compliance consistent.*

PROOF. First, we assume that the Stackelberg game led by agent 2 achieves the supply chain coordination. To prove the compliance consistency, for each agent i, we need to verify that the Stackelberg equilibrium $(y_1^{S_2}(z), y_2^{S_2}(z))$ is in its preference set. By the joint concavity of the payoff function, we only need to show that the equilibrium is the local minimal, which satisfies the first order condition. That is $\frac{\partial \pi_i(y_1,y_2,z)}{\partial y_1} = \frac{\partial \pi_i(y_1,y_2,z)}{\partial y_2} = 0$ at $(y_1, y_2) = (y_1^{S_2}(z), y_2^{S_2}(z))$, $i = 1, 2$.

The Stackelberg game equilibrium satisfies the first order conditions:

$$\left\{\frac{\partial \pi_2(y_1, y_2, z)}{\partial y_1} \cdot \frac{\partial r_1(y_2, z)}{\partial y_2} + \frac{\partial \pi_2(y_1, y_2, z)}{\partial y_2}\right\}\Bigg|_{(y_1^{S_2}(z), y_2^{S_2}(z))} = 0,$$
$$\frac{\partial \pi_1(y_1, y_2, z)}{\partial y_1}\Bigg|_{(y_1^{S_2}(z), y_2^{S_2}(z))} = 0.$$

The definition of coordination yields the following first order conditions:

$$\frac{\partial(\pi_1(y_1, y_2, z) + \pi_2(y_1, y_2, z))}{\partial y_1}\Bigg|_{(y_1^{S_2}(z), y_2^{S_2}(z))} = 0$$
$$\frac{\partial(\pi_1(y_1, y_2, z) + \pi_2(y_1, y_2, z))}{\partial y_2}\Bigg|_{(y_1^{S_2}(z), y_2^{S_2}(z))} = 0.$$

From the above equalities, we know

$$\frac{\partial \pi_2(y_1, y_2, z)}{\partial y_1}\Bigg|_{(y_1^{S_2}(z), y_2^{S_2}(z))} = \frac{\partial \pi_2(y_1, y_2, z)}{\partial y_2}\Bigg|_{(y_1^{S_2}(z), y_2^{S_2}(z))} = 0.$$

Since π_2 is jointly concave in y_1 and y_2, we know that $(y_1^{S_2}(z), y_2^{S_2}(z))$ is the global maximum decision for agent 2, which also means the equilibrium is in the preference set of agent 2, namely $(y_1^{S_2}(z), y_2^{S_2}(z)) \in$

$V_2(z)$. Meanwhile, for agent 1, we have

$$\left.\frac{\partial \pi_1(y_1, y_2, z)}{\partial y_1}\right|_{(y_1^{S_2}(z), y_2^{S_2}(z))} = \left.\frac{\partial \pi_1(y_1, y_2, z)}{\partial y_2}\right|_{(y_1^{S_2}(z), y_2^{S_2}(z))} = 0,$$

which indicates that $(y_1^{S_2}(z), y_2^{S_2}(z)) \in V_1(z)$. Therefore, we have shown that the voluntary compliance set $V(z) = \{(y_1^{S_2}(z), y_2^{S_2}(z))\} \neq \emptyset$. The Stackelberg game led by agent 2 is compliance consistent.

Similarly, when coordination is achieved in the Stackelberg game setting where agent 1 is the leader, we can prove that set $V(z)$ is non-empty too. Finally, we prove that set $V(z) \neq \emptyset$ under the static game setting. The Nash equilibrium leads to the following first order conditions

$$\left.\frac{\partial \pi_1(y_1, y_2, z)}{\partial y_1}\right|_{(y_1^N(z), y_2^N(z))} = \left.\frac{\partial \pi_2(y_1, y_2, z)}{\partial y_2}\right|_{(y_1^N(z), y_2^N(z))} = 0.$$

And, the definition of coordination yields the following first order conditions:

$$\begin{aligned}
\left.\frac{\partial(\pi_1(y_1, y_2, z) + \pi_2(y_1, y_2, z))}{\partial y_1}\right|_{(y_1^N(z), y_2^N(z))} &= 0 \\
\left.\frac{\partial(\pi_1(y_1, y_2, z) + \pi_2(y_1, y_2, z))}{\partial y_2}\right|_{(y_1^N(z), y_2^N(z))} &= 0.
\end{aligned}$$

Therefore, we have

$$\begin{aligned}
\left.\frac{\partial \pi_1(y_1, y_2, z)}{\partial y_1}\right|_{(y_1^N(z), y_2^N(z))} = \left.\frac{\partial \pi_1(y_1, y_2, z)}{\partial y_2}\right|_{(y_1^N(z), y_2^N(z))} &= 0 \\
\left.\frac{\partial \pi_2(y_1, y_2, z)}{\partial y_2}\right|_{(y_1^N(z), y_2^N(z))} = \left.\frac{\partial \pi_2(y_1, y_2, z)}{\partial y_1}\right|_{(y_1^N(z), y_2^N(z))} &= 0.
\end{aligned}$$

It indicates that the equilibrium $(y_1^N(z), y_2^N(z))$ maximizes the payoffs of agent 1 and 2 individually. Hence, it is in the voluntary compliance set $V(z)$, which has been proved non-empty.

Therefore, we have proved that the joint concavity of the payoff functions ensures the coordinating contract to be compliance consistent in either Stackelberg or static game settings. □

Next, we illustrate the sufficient condition for the compliance consistency by the internet drop-shipping model. Netessine and Rudi (2004) conduct discussion on the cost sharing contract which coordinates the dropping shipping model. We limit our discuss for a single period setting.

Example 3.2.5. (Cost Sharing Contract for the Drop-shipping Model)
In the drop-shipping model, the retailer sells the product to the customers, who in turn receives the shipment directly from the wholesaler. The decisions of interest are the retail price p, wholesale price w, and the wholesaler's inventory level q obtained/produced at the unit cost c.

Netessine and Rudi (2004) assume that the demand $D(x) = \varsigma(x) + \varepsilon$, where $\varsigma(x)$ is an increasing concave function of the customer acquisition cost (or sales effort) x and ε is a random variable. They show that the supply chain can be coordinated, by the sharing of costs x and c by the wholesaler and the retailer. Specifically, a contract $z = \{w, \frac{w-c}{p-c}, \frac{c(p-w)}{p-c}\}$, where the wholesaler sponsors a proportion $\frac{(w-c)}{p-c}$ of x and the retailer compensates the wholesaler by the fraction $\frac{(p-w)}{p-c}$ of the production cost c for each unsold product.

The expected payoff functions for the retailer and wholesaler are

$$\begin{aligned}
\pi_r(x,q,z) &= E[(p-w)\min(D(x),q) - x + \frac{(w-c)}{p-c}x \\
&\quad -\frac{c(p-w)}{p-c}(\{q-D(x)\}^+)] \\
&= \frac{p-w}{p-c}E[p\min(D(x),q) - x - cq] = \frac{p-w}{p-c}\pi(x,q), \\
\pi_w(x,q,z) &= E[w\min(D(x),q) - cq - \frac{(w-c)}{p-c}x \\
&\quad +\frac{c(p-w)}{p-c}(\{q-D(x)\}^+)] \\
&= \frac{w-c}{p-c}E[p\min(D(x),q) - x - cq] = \frac{w-c}{p-c}\pi(x,q).
\end{aligned}$$

Clearly, the expected payoffs are fractions of the channel payoff. We show that the expected payoff of the supply chain $\pi(q,x)$ is jointly concave in x and q. The diagonal elements of the Hessian matrix of $\pi(q,x)$ are negative and its determinant is positive (see Appendix). This proves that $\pi(x,q)$ is jointly concave in x and q. Moreover, $\pi_r(x,q,z)$ and $\pi_w(x,q,z)$ are jointly concave in x and q. Therefore, the coordination contract z is compliance consistent.

Alternately, the compliance consistency can be shown by the notion of the preference sets. Since $\pi(x,q)$ is jointly concave, there is a unique optimal decision (x^*, q^*) and therefore $V_w(z) = V_r(z) = \{(x^*, q^*)\}$. The voluntary compliance set $V(z) = V_w(z) \cap V_r(z) = \{(x^*, q^*)\}$.

Furthermore, the preference set of each agent is equal to the supply chain optimal decision set Y^*, and therefore the voluntary compliance set is equal to Y^*. Thus, the decision in the voluntary compliance set coordinates the supply chain.

Finally, let the contract set C be the set of all the coordinating cost sharing contracts. That is, $C = \{(w, a_1, a_2) | c \leq w \leq p, a_1(x) = \frac{(w-c)}{p-c}x, a_2(x) = \frac{c(p-w)}{p-c}x\}$. It is clear that the sharing function $\theta(z)$ is constant for each contract $z \in C$. By varying $z \in C$, it is therefore possible to achieve any $\theta(z)$ between 0 and 1. Hence, the cost sharing contract type C is also profit distribution consistent.

For the problem with twice differentiable payoff functions, we can use the second order condition to verify the consistency. We have our last sufficient condition as follows.

Proposition 3.2.5. (SUFFICIENT CONDITION II FOR COMPLIANCE CONSISTENCY) *For a coordination contract $z \in C$ which achieves coordination in either a simultaneous game or a Stackelberg game, if the payoff functions $\pi_1(r_1(y_2, z), y_2, z)$ and $\pi_2(y_1, r_2(y_1, z), z)$ are twice differentiable and concave with respect to y_2 and y_1, respectively, then z is compliance consistent.*

PROOF. To show that the coordinating contract is compliance consistent, we need to verify whether equilibrium is in the voluntary set, i.e., $(y_1^e(z), y_2^e(z)) \in V_i(z)$, $i = 1, 2$.

Note that $(y_1^e(z), y_2^e(z)) \in V_1(z)$ is equivalent to the fact that $y_2^e(z) = \arg\max_{y_2} \pi_1(r_1(y_2, z), y_2, z)$. Consider agent 1, $\pi_1(r_1(y_2, z), y_2, z)$ is concave in y_2, then $y_2^e(z)$ is the minimum if it satisfies the first order condition. By Lemma ??, we have

$$
\begin{aligned}
& \left.\frac{\partial \pi_1(r_1(y_2, z), y_2, z)}{\partial y_2}\right|_{y_2=y_2^e(z)} \\
= & \left.\left\{\frac{\partial \pi_1(r_1, y_2^e(z), z)}{\partial r_1} \cdot \frac{d r_1(y_2, z)}{d y_2} + \frac{\partial \pi_1(y_1^e(z), y_2, z)}{\partial y_2}\right\}\right|_{y_2=y_2^e(z)} \\
= & \left(\left.\frac{\partial \pi_1(y_1, y_2^e(z), z)}{\partial y_1}\right|_{y_1=y_1^e(z)}\right) \cdot \left.\frac{d r_1(y_2, z)}{d y_2}\right|_{y_2=y_2^e(z)} \\
& + \left.\frac{\partial \pi_1(y_1^e(z), y_2, z)}{\partial y_2}\right|_{y_2=y_2^e(z)} \\
= & \; 0 \cdot \left.\frac{d r_1(y_2, z)}{d y_2}\right|_{y_2=y_2^e(z)} + 0 = 0.
\end{aligned}
$$

This completes the proof. □

This proposition can be illustrated by the example of marketing and operation coordination. Back to Example ??, for the coordination problem of the marketing and operation managers, we have shown that the coordinating incentive payment contract is sequentially consistent. We now consider the special case of the incentive payment coordinating contract, where $\beta_p = 0$. It is named the sales incentive contract, and $z' = \{\alpha'_m, \alpha'_p\}$. The payoff functions can be expressed as follows,

$$\begin{cases} \pi_m(x,q,z') &= \alpha'_m p E \min\{x+\varepsilon, q\} - C_m x^2, \\ \pi_p(x,q,z') &= \alpha'_p p E \min\{x+\varepsilon, q\} - C_p q^2. \end{cases}$$

Again we restrict the strategy space to $x \leq q \leq x+\sigma$ and $x \geq 0$, the reaction functions are

$$\begin{cases} r_m(q,z') = \dfrac{\alpha'_m p}{\alpha'_m p + 2C_m\sigma} q, \\ r_p(x,z') = \dfrac{\alpha'_p p}{\alpha'_p p + 2C_p\sigma}(x+\sigma). \end{cases}$$

Provided z' is a coordinating contract, we want to show that z' is compliance consistent. We plug in the reaction function $r_p(x,z')$ and $r_m(q,z')$ into the payoff $\pi_m(q,x,z')$ and $\pi_p(q,x,z')$. We have $\pi_m(q, r_m(q,z'), z')$ and $\pi_p(r_p(x,z'), x, z')$. The first and second order derivatives for the marketing manager are

$$\begin{aligned} \frac{d\pi_m(q, r_m(q,z'), z')}{dq} &= \frac{2\alpha'_m p C_m \sigma}{\alpha' p + 2C_m\sigma} - \frac{4\alpha'_m p C_m^2 \sigma + 2\alpha'^2_m p^2 C_m}{(\alpha'_m p + 2C_m\sigma)^2} q, \\ \frac{d^2\pi_m(q, r_m(q,z'), z')}{dq^2} &= -\frac{4\alpha'_m p C_m^2 \sigma + 2\alpha'^2_m p^2 C_m}{(\alpha'_m p + 2C_m\sigma)^2} < 0. \end{aligned}$$

The first and second order derivatives for the production manager are

$$\begin{aligned} \frac{d\pi_p(r_p(x,z'), x, z')}{dx} &= \alpha'_p p - \frac{4C_p\alpha'^2_p p^2}{(\alpha'_p p + 2C_p\sigma)^2} x, \\ \frac{d^2\pi_p(r_p(x,z'), x, z')}{dx^2} &= -\frac{4C_p\alpha'^2_p p^2}{(\alpha'_p p + 2C_p\sigma)^2} < 0. \end{aligned}$$

Hence, the payoff functions satisfy the sufficient condition II (Proposition ??). The contract is compliance consistent.

Our last result demonstrates the connection of the compliance consistency and the sequence consistency.

Proposition 3.2.6. (Equivalent Condition) *Assume that a coordination contract $z \in C$ is compliance consistent. If both agents have decisions to make, the Stackelberg equilibria* S_1 *and* S_2 *uniquely exist, and the optimal decision of the centralized channel is unique, then the coordination is sequence consistent.*

Proof. Since contract z coordinates the supply chain with compliance consistency, and the centralized optimal decision is unique, the voluntary compliance set is $V(z) = \{(y_1^*, y_2^*)\}$. Next, we need to show $S_1(z) = S_2(z) = \{(y_1^*, y_2^*)\}$.

Since $V(z) = V_1(z) \cap V_2(z)$, the optimal decision $(y_1^*(z), y_2^*(z)) \in V_2(z)$, thus, $\pi_2(y_1^*(z), y_2^*(z)) = \max_{y_1,y_2} \pi_2(y_1, y_2, z)$. Therefore, $y_2^*(z) = \arg\max_{y_2} \pi_2(y_1^*(z), y_2, z)$, which means that

$$r_2(y_1^*(z), z) = y_2^*(z).$$

On the other hand, the optimal decision $(y_1^*(z), y_2^*(z)) \in V_1(z)$, hence, $(y_1^*(z), y_2^*(z)) = \arg\max_{y_1,y_2} \pi_1(y_1, y_2, z)$. Then, we have the inequalities hold as follows.

$$\begin{aligned}
&\pi_1(y_1^*(z), y_2^*(z), z) = \max_{y_1,y_2} \pi_1(y_1, y_2, z) \\
&\geq \max_{y_1} \pi_1(y_1, r_2(y_1, z), z) \geq \pi_1(y_1^*(z), r_2(y_1^*(z), z), z) \quad (3.19) \\
&= \pi_1(y_1^*(z), y_2^*(z), z).
\end{aligned}$$

The first inequality holds because the maximum payoff of agent 1 limited on the reaction curve of agent 2 is less than and at most equal to the maximum payoff achieved globally. The second inequality holds because $(y_1^*(z), r_2(y_1^*(z), z))$ is one decision on the reaction curve of agent 2, the corresponding payoff is not greater than the optimal payoff on that reaction curve. Also, by first and last term of inequality (**??**), we know all the inequalities hold as equalities, especially,

$$\max_{y_1} \pi_1(y_1, r_2(y_1, z), z) = \pi_1(y_1^*(z), r_2(y_1^*(z), z), z).$$

Therefore, we have shown that

$$\begin{cases} y_1^*(z) = \arg\max_{y_1} \pi_1(y_1, r_2(y_1, z), z) \\ y_2^*(z) = r_2(y_1^*(z), z). \end{cases}$$

This proves that $\{(y_1^*(z), y_2^*(z))\} = S_1(z)$. With similar agrement, we can also show that $\{(y_1^*(z), y_2^*(z))\} = S_2(z)$. This completes the proof. □

To complete the comparison of the two concepts, we demonstrate that the compliance consistency is more strict property than the sequence consistency by the following example.

Example 3.2.6. Consider the following game with the random profit functions

$$\begin{aligned} \Pi_1(y_1, y_2, z, \omega) &= \lambda\omega - 2y_1^2 + y_2^2, \\ \Pi_2(y_1, y_2, z, \omega) &= (1-\lambda)\omega - 2y_2^2 + y_1^2. \end{aligned}$$

ω is a realization of the random factor with expectation μ. $\lambda \in [0,1]$ is the contract parameter. The strategy space is $(y_1, y_2) \in [0,3] \times [0,3]$. It is clear that the payoff functions satisfy the sufficient condition II for sequence consistency. Hence, $N(z) = S_1(z) = S_2(z) = \{(0,0)\}$.

Since the expected profit of the supply chain is $\pi(y_1, y_2) = \mu - y_1^2 - y_2^2$, we know that $(0,0)$ is the optimal decision for the supply chain. Therefore, we demonstrate that the contract is coordinating and sequence consistent.

Meanwhile, we can see $V_1(z) = \{(0,3)\}$ and $V_2(z) = \{(3,0)\}$. Hence, the voluntary compliance set $V(z) = \emptyset$. Therefore the coordination is not compliance consistent.

Before concluding this section, we would like to revisit the transshipment price contract (TPC) that we discussed in Example ??. As we have pointed earlier, the TPC coordinates the supply chain but lacks of consistency properties. Now, with a newly designed transshipment profit sharing contract (TPSC), we would like to show that the TPSC yields the desired consistency properties.

Example 3.2.7. (TRANSSHIPMENT PROFIT SHARING CONTRACT)
We concluded in Example ?? that the TPC is sequence inconsistent and inflexible in profit allocation. To fix these inconsisten-

cies, we design a transshipment profit sharing contract (TPSC), where the transshipment revenue (generated from entering into the contract) is allocated by the transshipment prices and the cost of entering into the TPSC is shared by trans-payment. With z_0 denoting the null contract, retailer i obtains his payoff from his baseline demand D_i by maximizing his profit function $\Pi_i(D_i, q_i, z_0) = p\min\{q_i, D_i\} - cq_i$, and thereby finding his optimal order quantity q_i^0. On the other hand, ordering a quantity q_i under the TPSC, retailer i incurs the opportunity cost

$$L_i(q_i, D_i) = p\min\{q_i^0, D_i\} - cq_i^0 - p\min\{q_i, D_i\} + cq_i, \;\; i = 1, 2.$$

With the TPSC $z'' = \{t_1, t_2, \lambda\}$ where t_i and λ are the transshipment price and the fraction of the trans-payment, respectively, retailers i and $(3-i)$ pay $(1-\lambda)L_{3-i}(q_{3-i}, D_{3-i})$ and $\lambda L_i(q_i, D_i)$ to retailer $(3-i)$ and i, respectively. Thus, the profit of retailer i is

$$\begin{aligned}\Pi_i(q_1, q_2, D_1, D_2, z'') \;=\;& p\min\{q_i, D_i\} - cq_i + (p - t_i)\min\{(D_i - q_i)^+, \\ & (q_{3-i} - D_{3-i})^+\} \\ & +(t_{3-i} - c_t)\min\{(D_{3-i} - q_{3-i})^+, (q_i - D_i)^+\} \\ & +(1-\lambda)L_i(q_i, D_i) - \lambda L_{3-i}(q_{3-i}, D_{3-i}).\end{aligned}$$

If we specialize the transshipment price and the fraction of the trans-payment as

$$\begin{cases} t_1 = (1-\lambda)p + \lambda c_t, \\ t_2 = \lambda p + (1-\lambda)c_t, \end{cases} \tag{3.20}$$

then the payoff function of retailer i can be written as a linear transformation of the channel profit, that is,

$$\begin{aligned} \pi_i(q_1, q_2, z'') &= E(\lambda[\Pi(q_1, q_2, D_1, D_2) - p\min\{q_i^0, D_i\} + cq_i^0 \\ &\quad - p\min\{q_{3-i}^0, D_{3-i}\} + cq_{3-i}^0] + p\min\{q_i^0, D_i\} - cq_i^0) \\ &= \lambda[\pi(q_1, q_2) - \pi_i(q_i^0, z_0) - \pi_{3-i}(q_{3-i}^0, z_0)] + \pi_i(q_i^0, z_0). \end{aligned}$$

Hence, the reaction function of retailer i is

$$r_i(q_{3-i}, z'') = \arg\max_{q_i} \pi_i(q_i, q_{3-i}, z'') = \arg\max_{q_i} \pi(q_1, q_2).$$

Since the supply chain optimal (q_1^*, q_2^*) is the only solution of the equations

$$\begin{cases} q_1 = \arg\max_{q_1} \pi(q_1, q_2), \\ q_2 = \arg\max_{q_2} \pi(q_1, q_2), \end{cases}$$

it is clear that $(q_1^{\mathrm{N}}(z''), q_2^{\mathrm{N}}(z'')) = (q_1^*, q_2^*)$. Thus, z'' is a coordination contract where retailers make their inventory decisions simultaneously.

Next, let us consider the Stackelberg game with retailer 1 as the leader. His problem is to maximize $\pi_1(q_1, r_2(q_1, z''), z'')$.

Since $\pi(q_1, q_2)$ is concave,

$$\begin{aligned}\pi(q_1^*, q_2^*) &= \max_{q_1,q_2} \pi(q_1, q_2) = \max_{q_1}[\max_{q_2} \pi(q_1, q_2)] \\ &= \max_{q_1} \pi(q_1, \arg\max_{q_2} \pi(q_1, q_2)) \\ &= \max_{q_1} \pi_1(q_1, r_2(q_1, z''), z'').\end{aligned}$$

Thus, $S_1(z'') = N(z'') = (q_1^*, q_2^*)$. Similarly, $S_2(z'') = N(z'')$. Just as we sketched the retailers' reaction curves for the TPC in in Figure ?? (a), we depict the reaction curves for the TPSC in Figure ?? (b). Each curve has a finite q_i intercept $r_i(0, z'')$, $i = 1, 2$. Hence, the strategy space is $[0, r_1(0, z'')] \times [0, r_2(0, z'')]$. The simultaneous and the Stackelberg equilibria coincide for the TSPC as seen in Figure ?? (b).

We now show that the contract z'' is also consistent in the profit allocation, the decision sequence, and the compliance regime. First, by collecting terms, we rewrite the profit function with the demand realization as (d_1, d_2) as

$$\begin{aligned}\Pi_i(q_1, q_2, d_1, d_2, z'') - \Pi_i(q_i, d_i, z_0) &= \lambda[\Pi(q_1, q_2, d_1, d_2) - \Pi_i(q_i, d_i, z_0) \\ &\quad -\Pi_{3-i}(q_{3-i}, d_{3-i}, z_0)],\end{aligned}$$

where the left-side is the contractual profit for retailer i and the right-side is the percent of the contractual channel profit. Note that the trans-payment only applies to the profit generated by entering into the TPSC. Therefore, by Definition ??, the contractual profit with the TPSC is shared between the retailers in the proportions λ and $1-\lambda$, regardless of the demand realization. The allocation and sharing functions $\lambda(q_1, q_2, z)$ and $\theta(q_1, q_2, d_1, d_2, z)$ are constants. Furthermore, since $S_1(z'') = S_2(z'')$, the coordination is sequence consistent. Moreover, payoff function of retailer i, $\pi_i(q_1, q_2, z''), i = 1, 2$, is jointly concave with respect to q_1 and q_2, and therefore by Proposition ??, the contract z'' is consistent in compliance regime as well.

Finally, let us revisit the numerical example: $p = 10$, $c = 4$, $c_t = 0.1$, D_1 and D_2 are i.i.d. normal with $\mu = 1000$ and $\sigma = 300$. The profit from the local sales for each retailer is $\pi_i(q_i^0, z_0) = 4420$, and the expected contractual profit $\pi^C(q_1^*, q_2^*) = 1322$. By changing the contract parameters (t_1, t_2, λ) in a way so as to satisfy $t_1 + t_2 = 10$ and $t_1 = 10\lambda$, the outcome channel profits of the games remain channel optimal. Table ?? lists three cases: Case 1 represents the situation in which the transshipment prices $t_1 = t_2 = 5$ lead to a coordinated supply chain, with the channel profit split evenly between the agents and Cases 2 and 3 represent outcomes with different splits of the channel profit.

□ **End of chapter.**

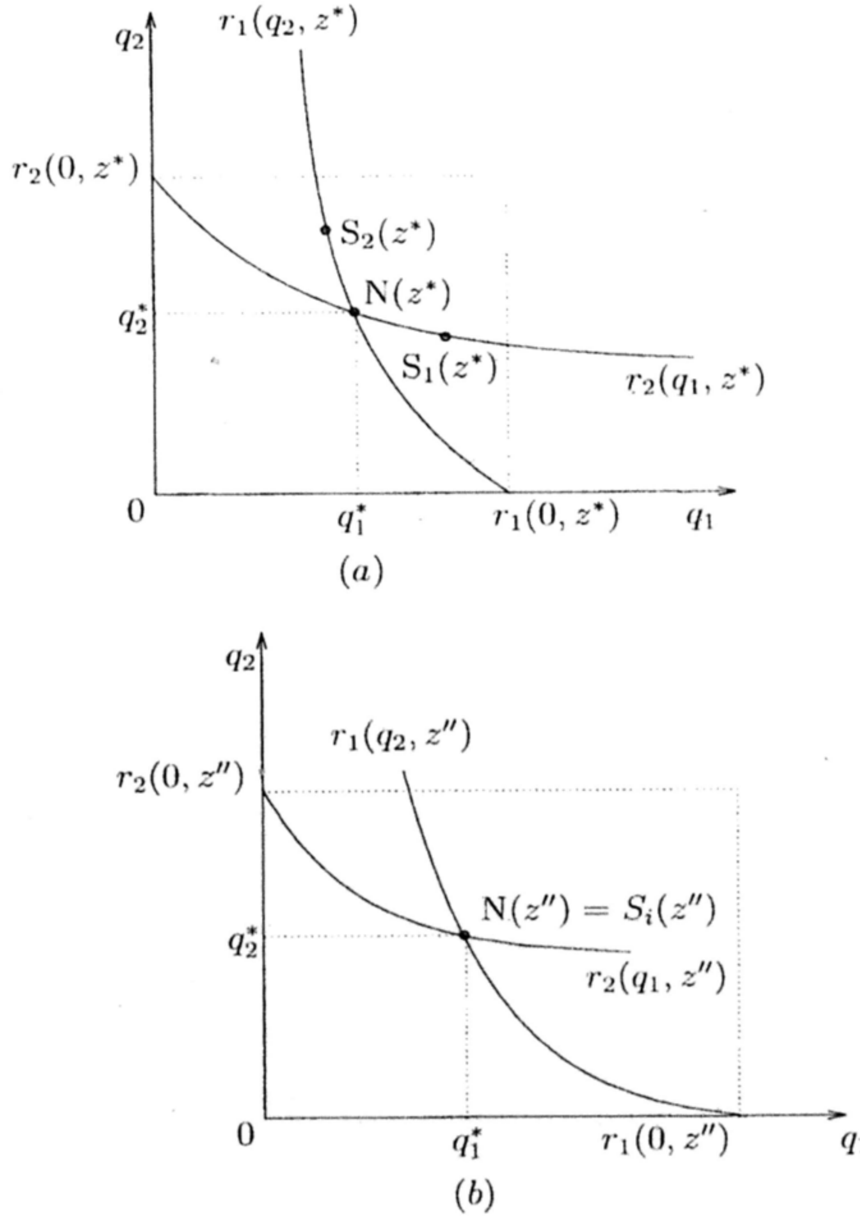

Figure 3.5: Reaction curves in TPC (z^*) and TPSC (z'')

Cases	Contract z (t_1, t_2, λ)	Game Type	Profit (π_1, π_2, π)	Split of π^C	Fraction 1 for π^C
Case 1	$(5.0, 5.0, 0.50)$	Simultaneous	(5181,5181,10362)	(661, 661)	0.50
Case 2	$(5.2, 4.8, 0.52)$	Stackelberg	(5107,5055,10362)	(687, 635)	0.52
Case 3	$(5.1, 4.9, 0.51)$	Stackelberg	(5094,5068,10362)	(674, 648)	0.51

Table 3.1: Computational results for TPSC with different parameters

Chapter 4

Decentralized Supply Chain with LSP

4.1. A Completely Decentralized Model

As a building block of the inventory theory, the standard newsvendor model deals with the perishable product, and only allows one ordering opportunity. The retailer makes the order decision and the supplier just produces to this order quantity. We consider the supply chain with a supplier, a retailer and a logistics service provider (LSP). By holding a backup inventory, the LSP provides another ordering opportunity for the retailer up to a pre-purchased quantity q_l. When the retailer runs out of inventory, then the LSP is able to tranship the backup inventory to satisfy those surplus customers. The LSP charges a replenishment price p_l to compensate for the stocking risk. The events are as follows. First, the supplier decides the wholesale prices

w_r for the retailer and w_l for the LSP. Then the retailer makes order q_r from the supplier, and the LSP order q_l. The supplier produces the amount, and delivers to the retailer and the LSP. After that, the demand realizes, and the inventory is consumed. Finally, the leftovers are salvaged.

For the backup inventory, we assume that the LSP has to bear the risk of stocking. It can neither force the retailer to buy out those inventory nor find other retailing channel to sell those inventory to the market. Hence, the backup inventory can only be set up if it is profitable to the LSP. Moreover, the LSP charges the retailer the service rate α. The random demand D follows distribution function F with density f. Without loss of generality, we assume that F is a strictly increasing function and $F(x) = 0$, if $x \leq 0$. The production cost c and the retailer price p are exogenous. The salvage value is normalized to zero.

In our model, the three firms are independent decision makers. Their objectives are to maximize the expected profit. Next, the firms' expected profit are written as follows. The retailer's payoff is

$$\begin{aligned}\pi_r(q_l, q_r, w_r) =& pE\min\{D, q_r\} + (p - p_l)E\min\{(D - q_r)^+, q_l\} \\ &- (1 + \alpha)w_r q_r.\end{aligned} \tag{4.1}$$

The LSP's payoff is

$$\pi_l(q_l, q_r, w_l, w_r) = p_l E \min\{(D - q_r)^+, q_l\} - w_l q_l + \alpha w_r q_r. \tag{4.2}$$

The supplier's payoff is

$$\pi_s(q_l, q_r, w_l, w_r) = (w_l - c)q_l + (w_r - c)q_r. \tag{4.3}$$

In next section, we characterize the outcome of the decision game assuming that the supplier offers a uniform wholesale price, namely $w_l = w_r = w$.

4.1.1 The Ordering Game Between the Retailer and the LSP

Applying the backward induction, we start from stage two. Given the wholesale price w, the LSP and the retailer make their individual orders. The replenishment price p_l and the charge rate α is given. The decision to make is the ordering quantities (q_l, q_r) with the payoffs (??) and (??). For the given retailer's decision q_r, the LSP's best response is denoted by $r_l(q_r, w)$.

Proposition 4.1.1. *the LSP's best response r_l to the retailer's order quantity q_r is*

$$r_l(q_r, w) = \max\{0, \quad F^{-1}(\frac{p_l - w}{p_l}) - q_r\}. \tag{4.4}$$

PROOF. The LSP's expected profit function is transformed in the following way.

$$
\begin{aligned}
\pi_l(q_l, q_r, w) &= p_l E \min\{(D-q_r)^+, q_l\} - wq_l + \alpha w q_r \\
&= p_l[\int_0^{q_r} \min\{0, q_l\} f(\xi)d\xi + \int_{q_r}^{q_r+q_l} \min\{\xi - q_r, q_l\} f(\xi)d\xi \\
&\quad + \int_{q_r+q_l}^{+\infty} \min\{\xi - q_r, q_l\} f(\xi)d\xi] - wq_l + \alpha w q_r \\
&= p_l[0 + \int_{q_r}^{q_r+q_l} (\xi - q_r) f(\xi)d\xi + \int_{q_r+q_l}^{+\infty} q_l f(\xi)d\xi] \\
&\quad - wq_l + \alpha w q_r \\
&= p_l[(\xi - q_r)F(\xi)|_{q_r}^{q_r+q_l} - \int_{q_r}^{q_r+q_l} F(\xi)d\xi + q_l F(\xi)|_{q_r+q_l}^{+\infty}] \\
&\quad - wq_l + \alpha w q_r \\
&= p_l[q_l F(q_r+q_l) - \int_{q_r}^{q_r+q_l} F(\xi)d\xi + q_l - q_l F(q_r+q_l)] \\
&\quad - wq_l + \alpha w q_r \\
&= p_l[q_l - \int_{q_r}^{q_r+q_l} F(\xi)d\xi] - wq_l + \alpha w q_r.
\end{aligned}
$$

We take the derivative with respect to q_l,

$$\frac{\partial \pi_l(q_l, q_r, w)}{\partial q_l} = p_l[1 - F(q_r + q_l)] - w.$$

And the second order derivative is

$$\frac{\partial^2 \pi_l(q_l, q_r, w)}{\partial q_l^2} = -p_l f(q_r + q_l) < 0.$$

The profit function is concave in q_l within $[0, +\infty)$. Hence, the best response decision $r_l(q_r, w)$ satisfies the first order condition, or equals 0 when the maximum is attained at negative quantity. That proves $r_l(q_r, w) = \max\{0, \quad F^{-1}(\frac{p_l - w}{p_l}) - q_r\}$.

□

Noted that the above first order condition can also be written as

$$F(q_r + q_l) = \frac{p_l - w}{p_l}. \tag{4.5}$$

This expression has a better indication on the "backup"'s meaning. As we see, the LSP's best strategy is to order up to the level that makes the aggregate inventory $q_r + q_l$ reach the service level $(p_l - w)/p_l$. The service charge rate α also has no impact on the LSP's decision strategy. However as we will see the retailer's order q_r decreases when α increases.

Next we develop the retailer's response function $r_r(q_l, w)$ with respect to the quantity q_l.

Proposition 4.1.2. *The reaction function $r_r(q_l, w)$ satisfies*

$$(1 - \frac{p_l}{p})F(r_r(q_l, w) + q_l) + \frac{p_l}{p}F(r_r(q_l, w)) = \frac{p - (1 + \alpha)w}{p}. \tag{4.6}$$

PROOF. The retailer's expected profit function is

$$\begin{aligned}
\pi_r(q_l, q_r, w) &= pE\min\{D, q_r\} + (p - p_l)E\min\{(D - q_r)^+, q_l\} \\
&\quad -(1 + \alpha)wq_r \\
&= p[q_r - \int_0^{q_r} F(\xi)d\xi] + (p - p_l)[q_l - \int_{q_r}^{q_r + q_l} F(\xi)d\xi] \\
&\quad -(1 + \alpha)wq_r. \qquad (4.7)
\end{aligned}$$

We take the derivative with respect to q_r and have

$$\frac{\partial \pi_r(q_l, q_r, w)}{\partial q_r} = p - pF(q_r) - (p - p_l)[F(q_r + q_l) - F(q_r)] - (1+\alpha)w.$$

And the second order derivative are

$$\begin{aligned}\frac{\partial^2 \pi_r(q_l, q_r, w)}{\partial q_r^2} &= -pf(q_r) - (p - p_l)[f(q_r + q_l) - f(q_r)] \\ &= -p_l f(q_r) - (p - p_l) f(q_r + q_l) < 0.\end{aligned}$$

Hence π_r is concave in q_r, and the first order condition $\frac{\partial \pi_r(q_l, r_r(q_l,w),w)}{\partial q_r} = 0$ gives rise to

$$\frac{p - p_l}{p} F(r_r(q_l, w) + q_l) + \frac{p_l}{p} F(r_r(q_l, w)) = \frac{p - (1+\alpha)w}{p}.$$

As we assume that $p_l > (1+\alpha)w$, the derivative is positive at $q_r = 0$ for any given $q_l \in [0, +\infty)$. Hence the optimality of the concave function is achieved at a positive value. Thus, $r_r > 0$ and it satisfies the first order condition. □

4.1.2 Simultaneous Game outcomes

After the wholesale price w is given, we assume that the LSP and retailer place orders simultaneously to the supplier. There is a static game of ordering between the retailer and the LSP. The solution is the Nash equilibrium, which both agents can not be better off by unilaterally deviating from. The Nash equilibrium is the intersection of the reaction curves. To this end, we have the following theory.

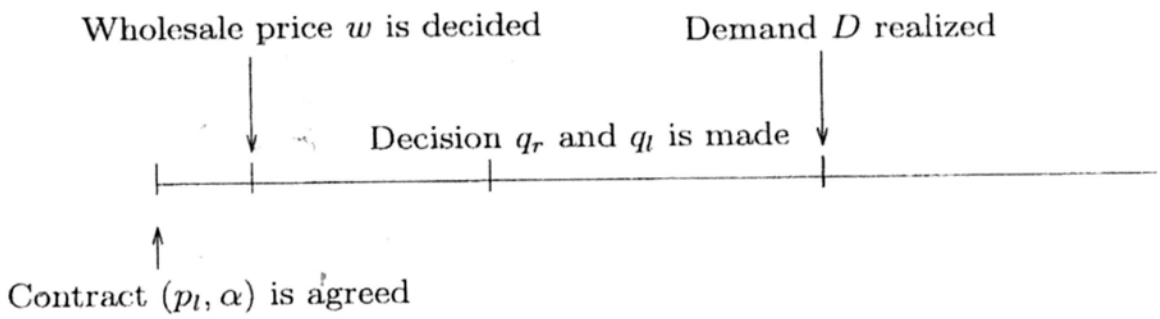

Figure 4.1: The time line of simultaneous inventory decision game

Theorem 4.1.1. *There exists a unique and globally stable Nash equilibrium for the static ordering game for the retailer and LSP. If*

$$p_l \geq p/(1+\alpha) \tag{4.8}$$

holds, the LSP builds up a positive backup inventory, that is $q_l^N > 0$ and the Nash equilibrium is

$$\begin{cases} q_r^N(w) &= F^{-1}\left(\frac{p_l-(1+\alpha)w}{p_l} + \frac{w(p-p_l)}{p_l^2}\right) \\ q_l^N(w) &= F^{-1}\left(\frac{p_l-w}{p_l}\right) - F^{-1}\left(\frac{p_l-(1+\alpha)w}{p_l} + \frac{w(p-p_l)}{p_l^2}\right). \end{cases} \tag{4.9}$$

Otherwise, the Nash equilibrium is degenerated to

$$\begin{cases} q_r^N(w) &= F^{-1}\left(\frac{p-(1+\alpha)w}{p}\right) \\ q_l^N(w) &= 0. \end{cases}$$

PROOF.

First we assume $p_l \geq p/(1+\alpha)$. Consider the slope of the reaction curves. Curve $\widehat{ac}$ is the reaction function $r_l(q_r, w)$, and

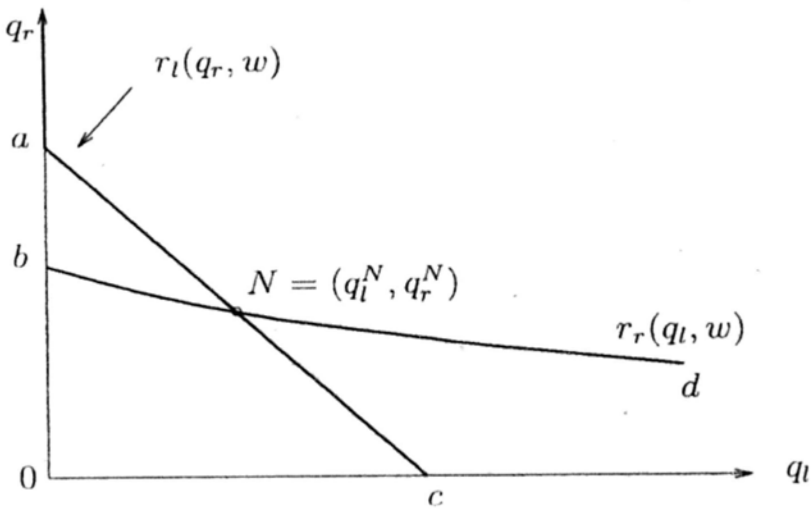

Figure 4.2: The equilibrium in the simultaneous game

curve $\widehat{bd}$ is the reaction function $r_r(q_l, w)$. The intercepts a and c are of value $F^{-1}(\frac{p_l - w}{p_l})$, b is of value $F^{-1}(\frac{p-(1+\alpha)w}{p})$. The slope of curve $\widehat{bd}$ is less than 1, since

$$\left|\frac{\partial r_r(q_l, w)}{\partial q_l}\right| = \left| - \frac{(p - p_l)f(q_r + q_l)}{(p - p_l)f(q_r + q_l) + p_l f(q_r)}\right| < 1.$$

Therefore the two response curves always have intersection and intersected both on positive values if and only if the intercept a is of higher value than b is, namely

$$\frac{p - (1 + \alpha)w}{p} \leq \frac{p_l - w}{p_l}.$$

Or equivalently $p_l \geq p/(1+\alpha)$. On the other hand, $|\frac{\partial r_l(q_r, w)}{\partial q_l}| = 1$. Therefore, $|\frac{\partial r_r(q_l, w)}{\partial q_l}| \cdot |\frac{\partial r_l(q_r, w)}{\partial q_l}| < 1$. From Fudenberg and Tirole (1991), we know that there exists a unique and globally stable Nash equilibrium, which can be derived by solving the equations of the reaction functions jointly.

$$\begin{cases} r_r(q_l, w) = & q_r \\ r_l(q_r, w) = & q_l. \end{cases}$$

That is

$$\begin{cases} (1 - \frac{p_l}{p})F(q_r + q_l) + \frac{p_l}{p}F(q_r) = \frac{p-(1+\alpha)w}{p}. \\ F^{-1}(\frac{p_l - w}{p_l}) - q_r = q_l. \end{cases}$$

Collecting terms, the equilibrium is in the form of expression (??).

When $p_l < p/(1+\alpha)$, the reaction of the LSP is $r_l(q_r, w) = 0$. Then retailer faces a convention newsvendor problem and has the optimal ordering quantity equal to

$$q_r^N(w) = F^{-1}(\frac{p-(1+\alpha)w}{p}),$$

where $q_l^N(w) = 0$. □

It can be seen that $(p - (1+\alpha)w)/p$ is the service rate that the retailer prefers for given wholesale price w, and $(p_l - w)/p_l$ is the best service rate which the LSP prefers. The inequality (??) is the condition under which the LSP prefers a better service rate than the retailer does. Only then the LSP builds up a positive backup inventory to pursuit the service rate. We denote $k = \frac{p-p_l}{p_l}$. By the assumption $p \le (1+\alpha)p_l$, it is clear that $0 \le k \le \frac{(1+\alpha)p_l - p_l}{p_l} = \alpha$. Then the Nash equilibrium can be

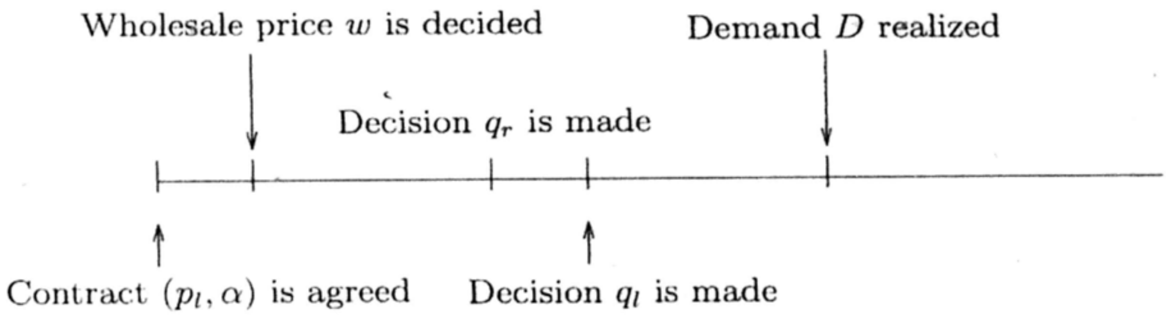

Figure 4.3: The time line of the sequential inventory decision game

written as

$$\begin{cases} q_r^N(w) = & F^{-1}(\frac{p_l - (1+\alpha-k)w}{p_l}) \\ q_l^N(w) = & F^{-1}(\frac{p_l - w}{p_l}) - F^{-1}(\frac{p_l - (1+\alpha-k)w}{p_l}). \end{cases} \tag{4.10}$$

Sequential Inventory Decision

Next, we consider the sequential decision sequence, in which the LSP places order q_l after the observation of q_r. The order game is also regarded as Stackelberg game between the LSP and the retailer. The sequential setting makes more sense in the practical operations. Because the LSP ships the retailer's order, hence it can observe the quantity q_r. But the retailer may not know the volume of the backup inventory q_l. We assume that the retailer also recognizes that the LSP makes decision afterward. Then the retailer is the leader of the Stackelberg game. Compared with equation (??), the retailer's payoff is changed to

$$
\begin{aligned}
&\pi_r(r_l(q_r, w), q_r, w) \\
= \;& pE\min\{D, q_r\} + (p - p_l)E\min\{(D - q_r)^+, r_l(q_r, w)\} - (1+\alpha)wq_r \\
= \;& p[q_r - \int_0^{q_r} F(\xi)d\xi] + (p - p_l)[r_l(q_r, w) - \int_{q_r}^{q_r + r_l(q_r, w)} F(\xi)d\xi] \\
& -(1+\alpha)wq_r.
\end{aligned}
$$

Plug in the reaction function of the LSP, equation (??), we have the first order derivative:

$$
\frac{\partial \pi_r(r_l(q_r, w), q_r, w)}{\partial q_r} = \begin{cases} p_l(1 - F(q_r)) - (1+\alpha)w & q_r < F^{-1}(\frac{p_l - w}{p_l}) \\ p(1 - F(q_r)) - (1+\alpha)w & q_r \geq F^{-1}(\frac{p_l - w}{p_l}). \end{cases} \tag{4.11}
$$

The second order derivative:

$$
\frac{\partial^2 \pi_r(r_l(q_r, w), q_r, w)}{\partial q_r^2} = \begin{cases} -p_l f(q_r) & q_r < F^{-1}(\frac{p_l - w}{p_l}) \\ -pf(q_r) & q_r \geq F^{-1}(\frac{p_l - w}{p_l}). \end{cases}
$$

The profit function $\pi_r(r_l(q_r), q_r, w)$ is concave in q_r on interval $[0, F^{-1}(\frac{p_l - w}{p_l})]$ and $(F^{-1}(\frac{p_l - w}{p_l}), +\infty)$ respectively. However, on $q_r = F^{-1}(\frac{p_l - w}{p_l})$, the left derivative is not less than the right derivative, hence the function is not concave on $[0, +\infty)$.

Theorem 4.1.2. *The unique Stackelberg equilibrium* (q_l^{Sr}, q_r^{Sr}) *exists. There exists* $\eta \leq p/(1+\alpha)$*, such that if* $p_l \geq p/(1+\alpha)$*, then*

$$
\begin{cases} q_r^{Sr}(w) = F^{-1}(\frac{p_l - (1+\alpha)w}{p_l}) \\ q_l^{Sr}(w) = F^{-1}(\frac{p_l - w}{p_l}) - F^{-1}(\frac{p_l - (1+\alpha)w}{p_l}), \end{cases} \tag{4.12}
$$

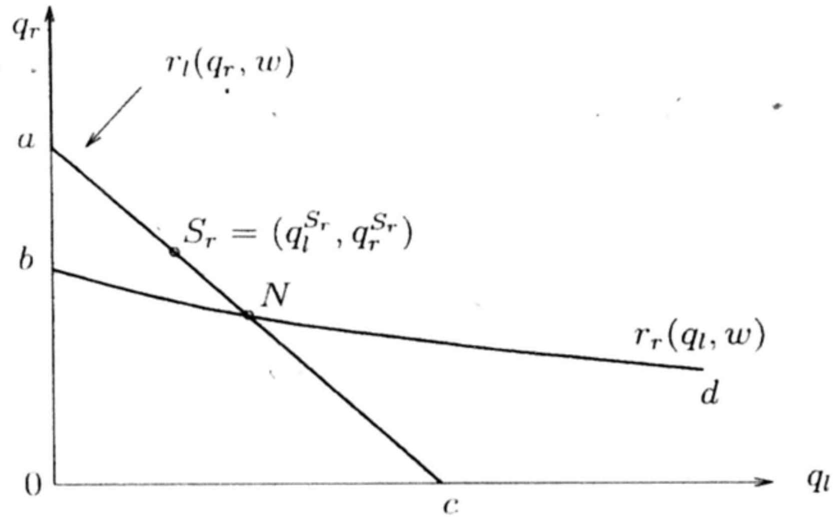

Figure 4.4: The equilibrium in the Stackelberg game

and if $p_l < \eta$, *then*

$$\begin{cases} q_r^{Sr}(w) &= F^{-1}(\frac{p-(1+\alpha)w}{p}) \\ q_l^{Sr}(w) &= 0. \end{cases} \tag{4.13}$$

PROOF. Since, the profit function $\pi_r(r_l(q_r, w), q_r; w)$ is concave in interval $[0, F^{-1}(\frac{p_l-w}{p_l})]$ and $(F^{-1}(\frac{p_l-w}{p_l}), +\infty)$ separately. The maximal profit can be achieved in either $[0, F^{-1}(\frac{p_l-w}{p_l})]$ or $(F^{-1}(\frac{p_l-w}{p_l}), +\infty)$. There are different cases.

Case 1, $\frac{p-(1+\alpha)w}{p} < \frac{p_l-w}{p_l}$. $\pi_r(r_l(q_r, w), q_r, w)$ is decreasing in $(F^{-1}(\frac{p_l-w}{p_l}), +\infty)$, and the local minimum in $[0, F^{-1}(\frac{p_l-w}{p_l})]$ is $F^{-1}(\frac{p_l-(1+\alpha)w}{p_l})$ which is also the global minimum. Hence the

Stackelberg equilibrium is in $[0, F^{-1}(\frac{p_l-w}{p_l})]$, namely,

$$\begin{cases} q_r^{Sr}(w) = & F^{-1}(\frac{p_l-(1+\alpha)w}{p_l}) \\ q_l^{Sr}(w) = & F^{-1}(\frac{p_l-w}{p_l}) - F^{-1}(\frac{p_l-(1+\alpha)w}{p_l}). \end{cases}$$

Case 2, $\frac{p_l-w}{p_l} \leq \frac{p-(1+\alpha)w}{p}$. The profit $\pi_r(r_l(q_r,w),q_r,w)$ is first increasing then decreasing in $(F^{-1}(\frac{p_l-w}{p_l}),+\infty)$, hence it has a local maximum $F^{-1}(\frac{p-(1+\alpha)w}{p})$. But it is not necessary to have the global optimal. We need to compare the two local optimal quantity. In the interval $[0, F^{-1}(\frac{p_l-w}{p_l})]$,

$$\begin{aligned} & \max_{q_r} \pi_r(r_l(q_r,w),q_r,w) \\ = \; & p[F^{-1}(\frac{p_l-(1+\alpha)w}{p_l}) - \int_0^{F^{-1}(\frac{p_l-(1+\alpha)w}{p_l})} F(\xi)d\xi] \\ & +(p-p_l)[F^{-1}(\frac{p_l-w}{p_l}) - F^{-1}(\frac{p_l-(1+\alpha)w}{p_l}) - \\ & \int_{F^{-1}(\frac{p_l-(1+\alpha)w}{p_l})}^{F^{-1}(\frac{p_l-w}{p_l})} F(\xi)d\xi] - (1+\alpha)wF^{-1}(\frac{p_l-(1+\alpha)w}{p_l}). \end{aligned}$$

The profit is a function of p_l, where $p_l \geq w$. When p_l is reduced to w, $q_r = q_l = 0$, and $\pi_r = 0$. In $(F^{-1}(\frac{p_l-w}{p_l}),+\infty)$,

$$\begin{aligned} \pi_r(r_l(q_r,w),q_r,w) = \; & p[F^{-1}(\frac{p-(1+\alpha)w}{p}) - \int_0^{F^{-1}(\frac{p-(1+\alpha)w}{p})} F(\xi)d\xi] \\ & -(1+\alpha)wF^{-1}(\frac{p_l-(1+\alpha)w}{p_l}). \end{aligned}$$

The profit is not a function of p_l. Hence, there exists a value $w \leq \eta \leq p/(1+\alpha)$ of p_l, for all $p_l < \eta$, the local minimum in

$(F^{-1}(\frac{p_l-w}{p_l}), +\infty)$ is the global optimal, where the LSP does not hold backup inventory. Therefore,when $p_l \geq \frac{p}{1+\alpha}$, the LSP holds positive inventory. When $p_l < \eta$, the LSP does not hold backup inventory. □

Comparing the outcomes of the two different sequences of decision making, from the expression (??) and (??), we know that for $p_l \geq p/(1+\alpha)$, it always holds that $q_r^N \geq q_r^{Sr}$ and $q_l^N \leq q_l^{Sr}$. And the profit $\pi_r(q_l^N, q_r^N, w) \leq \pi_r(q_l^{Sr}, q_r^{Sr}, w)$ and $\pi_l(q_l^N, q_r^N, w) \geq \pi_l(q_l^{Sr}, q_r^{Sr}, w)$. In other words, the retailer has first mover advantage, if $p_l \geq p/(1+\alpha)$. The retailer has motivation to announce its ordering quantity q_r to the LSP, before the supplier starting the production.

For given wholesale price, we denote the outcome of the decision game as $(q_l^e(w), q_r^e(w))$. e stands for Nash or Stackelberg. From either of these equilibrium outcome, we do not see any benefit for the supply chain to incorporating the LSP. The service level is lowered to $F(\frac{p_l-w}{w})$. As the LSP demands a portion of profit, there is a curse of triple marginalization, which results in an under stocking problem.

4.1.3 The Stackelberg Game on Stage One

In stage one, the supplier offers a wholesale price w. Then the LSP and the retailer make orders $(q_r^e(w), q_l^e(w))$ from the supplier based on the wholesale price w. By equation (??), the profit of the supplier only relies on the sum $q_r^e(w) + q_l^e(w)$.

By Theorem ?? and ??, we know that if $(1+\alpha)p_l \geq p$, the LSP holds a non-negative backup inventory. And the total production quantity as the solution of the ordering game satisfies

$$F(q_r^e(w) + q_l^e(w)) = \frac{p_l - w}{p_l}.$$

If $p_l < \eta$, the LSP does not hold backup inventory. The total production quantity is q_r, which is the solution of the newsvendor problem, namely,

$$F(q_r^e(w) + q_l^e(w)) = F(q_r) = \frac{p - (1+\alpha)w}{p}.$$

Then we substitute into the expression (??) and have completed the proof. Therefore, the supplier's profit is

$$\pi_s(w) = \begin{cases} (w-c)F^{-1}(\frac{p_l - w}{p_l}) & p_l \geq p/(1+\alpha) \\ (w-c)F^{-1}(\frac{p-(1+\alpha)w}{p}) & p_l < \eta. \end{cases} \tag{4.14}$$

Since the production quantity depends on the higher coverage preferred either by the retailer or by the LSP. The supplier sets wholesale price w to maximize $\pi_s(w)$. Unfortunately, the profit function is not concave in general. We denote the generalized failure rate by $g(x) = xf(x)/(1-F(x))$ which gives the percentage decreasing in the probability of a stock put from increasing the stocking quantity by 1%. A distribution has an increasing generalized failure rate (IGFR) if $g(x)$ is weakly increasing for all x. The IGFR assumption is not restrictive because it captures most common distributions, e.g., the normal, the uniform, and the gamma.

As Lariviere and Porteus (2001), a parallel argument shows the unimodal property of the profit function. We omit the proof.

Theorem 4.1.3. *If the demand distribution F is IGFR, the profit function $\pi_s(w)$ is unimodal on $[0, +\infty)$ for both cases in expression (??).*

One could apply the first order analysis to find the optimal value of w under the IGFR distribution. IGFR is not restrictive. All functions with an increasing failure rate (IFR) are IGFR. But the inverse is not true.

If the LSP sets the transfer price p_l greater than $p/(1+\alpha)$, it has the incentive to hold a positive backup inventory which raised the sales revenue of the supplier. As we discussed, the positive quantity of backup inventory provides the retailer a chance to reduce the probability of stock out and bring in extra revenue as well.

4.2. A Coalition Model

In this section, we investigate the behavior of the channel members when the coalition is formed between the supplier and the

LSP, and the retailer and the LSP in the sense that two of the agents maximize their aggregate profit.

4.2.1 The Integrated Supplier and the LSP v.s. the Retailer

We consider the situation when the supplier and the LSP form a coalition. Since we are only interested in the non-degenerated ordering game, we assume that the inequality $p_l \geq p/(1+\alpha)$ holds throughout this discussion. The players in the decision game now become the supplier-LSP coalition and the retailer. We denote the payoff of the coalition as π_{sl}. The backup inventory decision q_l is now made by the supplier-LSP coalition with the payoff function as

$$\begin{aligned}\pi_{sl}(q_r, q_l, w) &= p_l E\min\{(D-q_r)^+, q_l\} - cq_l + ((1+\alpha)w - c)q_r \\ &= p_l[q_l - \int_{q_r}^{q_r+q_l} F(\xi)d\xi] - cq_l + ((1+\alpha)w - c)q_r.\end{aligned} \tag{4.15}$$

Next, we consider two different decision sequences.

Stackelberg Ordering Subgame

The time sequence is as follows. The supplier-LSP coalition first announces the wholesale price w to the retailer. Then the retailer makes an order q_r. The supplier-LSP coalition adjusts the quantity to $q_r + q_l$ and starts production. Then the products are shipped to the LSP's warehouse and the selling season begins.

With the backup inventory decision, the problem for the supplier-LSP coalition is to maximize expression (??). The reaction function can be written as

$$r_{sl}(q_r) = F^{-1}(\frac{p_l - c}{p_l}) - q_r.$$

Compared with (??), the coalition's optimal backup inventory level has a wider coverage, due to the elimination of double marginalization between the supplier and the LSP. For given w, the problem for the retailer is to maximize the following objective,

$$\begin{aligned}\pi_r(q_r, r_{sl}(q_r), w) &= pE\min\{D, q_r\} + (p - p_l)E\min\{(D - q_r)^+, r_{sl}(q_r)\} \\ &\quad -(1+\alpha)wq_r.\end{aligned}$$

A parallel result is derived as in the three-agent setting. Plug in the reaction function r_{sl}, we have the first order derivative:

$$\frac{\partial \pi_r(r_{sl}(q_r), q_r, w)}{\partial q_r} = \begin{cases} p_l(1 - F(q_r)) - (1+\alpha)w & q_r < F^{-1}(\frac{p_l - c}{p_l}) \\ p(1 - F(q_r)) - (1+\alpha)w & q_r \geq F^{-1}(\frac{p_l - c}{p_l}). \end{cases} \tag{4.16}$$

The second order derivative

$$\frac{\partial^2 \pi_r(r_{sl}(q_r), q_r, w)}{\partial q_r^2} = \begin{cases} -p_l f(q_r) & q_r < F^{-1}(\frac{p_l - c}{p_l}) \\ -pf(q_r) & q_r \geq F^{-1}(\frac{p_l - c}{p_l}). \end{cases}$$

The profit function $\pi_r(r_l^2(q_r), q_r, c)$ is concave in q_r on interval $[0, F^{-1}(\frac{p_l-c}{p_l})]$ and $(F^{-1}(\frac{p_l-c}{p_l}), +\infty)$ respectively. Since $p_l \geq p/(1+\alpha)$, the Stackelberg equilibrium is

$$\begin{cases} q_r^{Sr}(w) &= F^{-1}(\dfrac{p_l-(1+\alpha)w}{p_l}) \\ q_l^{Sr}(w) &= F^{-1}(\dfrac{p_l-c}{p_l}) - F^{-1}(\dfrac{p_l-(1+\alpha)w}{p_l}). \end{cases} \tag{4.17}$$

The supplier-LSP coalition determines the wholesale price w to maximize their payoff.

$$\begin{aligned} & \pi_{sl}(q_r^{Sr}(w), q_l^{Sr}(w), w) \\ = \ & p_l E \min\{(D - q_r^{Sr}(w))^+, q_l^{Sr}(w)\} - c q_l^{Sr}(w) + ((1+\alpha)w - c) q_r^{Sr}(w) \\ = \ & p_l[q_l^{Sr}(w) - \int_{q_r^{Sr}(w)}^{q_r^{Sr}(w)+q_l^{Sr}(w)} F(\xi)d\xi] - c q_l^{Sr}(w) + ((1+\alpha)w - c) q_r^{Sr}(w). \end{aligned}$$

Theorem 4.2.1. *The joint profit of the supplier-LSP coalition is an increasing function of the wholesale price to the retailer. In fact, the collusion of the supplier and the LSP provide them incentives to squeeze out the profit share of the retailer.*

PROOF.

$$\begin{aligned} & \frac{\partial \pi_{sl}(q_r^{Sr}(w), q_l^{Sr}(w), w)}{\partial w} \\ = \ & p_l \frac{dq_l^{Sr}(w)}{dw} + p_l F(q_r^{Sr}(w)) \frac{dq_r^{Sr}(w)}{dw} - c \frac{dq_l^{Sr}(w)}{dw} \\ & + (1+\alpha) q_r^{Sr}(w) + ((1+\alpha)w - c) \frac{dq_r^{Sr}(w)}{dw}. \end{aligned}$$

And from expression (??), we know $F(q_r^{Sr}(w)) = \frac{p_l-(1+\alpha)w}{p_l}$ and

$\frac{d(q_r^{Sr}(w)+q_l^{Sr}(w))}{dw} = \frac{d[(p_l-c)/p_l]}{dw} = 0$. Therefore, we have

$$\begin{aligned}
&\frac{\partial \pi_{sl}(q_r^{Sr}(w), q_l^{Sr}(w), w)}{\partial w} \\
&= p_l \frac{dq_l^{Sr}(w)}{dw} + (p_l - (1+\alpha)w)\frac{dq_r^{Sr}(w)}{dw} - c\frac{dq_l^{Sr}(w)}{dw} \\
&\quad +(1+\alpha)q_r^{Sr}(w) + ((1+\alpha)w - c)\frac{dq_r^{Sr}(w)}{dw} \\
&= (p_l - c)(\frac{dq_r^{Sr}(w)}{dw} + \frac{dq_l^{Sr}(w)}{dw}) + (1+\alpha)q_r^{Sr}(w) \\
&= (p_l - c)(\frac{d(q_r^{Sr}(w) + q_l^{Sr}(w))}{dw}) + (1+\alpha)q_r^{Sr}(w) \\
&= (1+\alpha)q_r^{Sr}(w) > 0.
\end{aligned}$$

Therefore, the payoff function is an increasing function of w.

□

Note that when the supplier and the LSP form a coalition, the double margins of the backup inventory vanishes. The coverage of the total production quantity $q_l + q_r$ is maximal. When p_l is close to the retailing price, the coverage is close to the centralized optimal coverage $\frac{p-c}{p}$. Secondly, assume p_l is close enough to the retailing price p in the sense that the margin $p - p_l$ could be ignored. Then by setting the wholesale price w equal to $p/(1+\alpha)$, the supplier-LSP coalition can obtain all the channel profit, leaving nothing to the retailer. In that case, the retailer dose not hold inventory, $q_r = 0$. Due to the elimination of the double marginalization effect, the channel efficiency is guaranteed, or say the system is coordinated. Holding other parameter unchanged by varying w from c to $p/(1+\alpha)$, there is a continuum of profit allocation between the retailer and the supplier-LSP coalition while remaining the total efficiency.

Static Ordering Subgame

In this model, we assume that the supplier-LSP coalition announces the wholesale price w at the beginning. Then the coalition makes the backup inventory decision and the retailer makes the order at the same time. After that, the production and delivery take place, and the selling season begins. This situation happens when the retailer does not know that the supplier and the LSP are united. Therefore, the retailer may model its counterpart as the LSP with the payoff function π_l. Since the retailer's payoff does not change, its strategy is not changing. For the decision process, there are two stages. Again we apply the backward induction to analyze the outcome of the game.

In stage 2, the wholesale price w is taken as given. The supplier-LSP coalition and the retailer optimize the following payoffs jointly:

$$\begin{cases} \pi_{sl}(q_r, q_l, w) &= p_l E\min\{(D-q_r)^+, q_l\} - cq_l + ((1+\alpha)w - c)q_r \\ \pi_r(q_r, q_l, w) &= pE\min\{D, q_r\} + (p-p_l)E\min\{(D-q_r)^+, q_l\} \\ & \quad -(1+\alpha)wq_r. \end{cases}$$

Proposition 4.2.1. *Given that the wholesale price w and in-*

equality $p_l \geq p/(1+\alpha)$, there exists a unique Nash equilibrium for the second stage static game, that is,

$$\begin{cases} q_r^N(w) &= F^{-1}(\frac{p_l-(1+\alpha-k)w}{p_l}) \\ q_l^N(w) &= F^{-1}(\frac{p_l-c}{p_l}) - F^{-1}(\frac{p_l-(1+\alpha-k)w}{p_l}). \end{cases}$$

PROOF. Given that $p_l \geq p/(1+\alpha)$, the supplier-LSP coalition's reaction function is

$$r_{sl}(q_r) = F^{-1}(\frac{p_l-c}{p_l}) - q_r.$$

And the retailer's reaction function is the same as in the decision subgame versus the LSP. The reaction is implicitly expressed as equation (??). We jointly solve the equation system and obtain the above solution. □

In stage 1, the supplier-LSP coalition incurs a different objective against the situation in sequence No.1. The objective is written as

$$\begin{aligned} & \pi_{sl}(q_r^N(w), q_l^N(w), w) \\ = \; & p_l E\min\{(D-q_r^N(w))^+, q_l^N(w)\} - cq_l^N(w) + ((1+\alpha)w-c)q_r^N(w) \\ = \; & p_l[q_l^N(w) - \int_{q_r^N(w)}^{q_r^N(w)+q_l^N(w)} F(\xi)d\xi] - cq_l^N(w) + ((1+\alpha)w-c)q_r^N(w). \end{aligned}$$

Theorem 4.2.2. *If the demand distribution F is IGFR, the profit function $\pi_{sl}(q_r^N(w), q_l^N(w), w)$ is a unimodal function of w.*

PROOF. Since $q_r^N(w) = F^{-1}(\frac{p_l-(1+\alpha-k)w}{p_l})$ is the strictly decreasing function of the wholesale price w, we consider an equivalent problem with decision q_r^N. We change the expression to

$$q_{sl}^N(q_r^N) = F^{-1}(\frac{p_l - c}{p_l}) - q_r^N, \text{ and}$$
$$w(q_r^N) = \frac{p_l(1 - F(q_r^N))}{1 + \alpha - k}.$$

Therefore the objective becomes

$$\begin{aligned}
& \pi_{sl}(q_r^N, q_l^N(q_r^N), w(q_r^N)) \\
= \; & p_l[q_l^N(q_r^N) - \int_{q_r^N}^{q_r^N + q_l^N(q_r^N)} F(\xi)d\xi] - cq_l^N(q_r^N) + ((1+\alpha)w(q_r^N) - c)q_r^N \\
= \; & p_l[F^{-1}(\frac{p_l - c}{p_l}) - q_r^N - \int_{q_r^N}^{F^{-1}(\frac{p_l-c}{p_l})} F(\xi)d\xi] - cF^{-1}(\frac{p_l - c}{p_l}) \\
& + \frac{(1+\alpha)p_l}{1+\alpha-k}(1 - F(q_r^N))q_r^N.
\end{aligned}$$

We take the derivative

$$\begin{aligned}
& \frac{d\pi_{sl}(q_r^N, q_l^N(q_r^N), w(q_r^N))}{dq_r^N} \\
= \; & \frac{d}{dq_r^N}[p_l[F^{-1}(\frac{p_l - c}{p_l}) - q_r^N - \int_{q_r^N}^{F^{-1}(\frac{p_l-c}{p_l})} F(\xi)d\xi] - cF^{-1}(\frac{p_l - c}{p_l}) \\
& + \frac{(1+\alpha)p_l}{1+\alpha-k}(1 - F(q_r^N))q_r^N] \\
= \; & -p_l(1 - F(q_r^N)) + \frac{(1+\alpha)p_l}{1+\alpha-k}[1 - F(q_r^N) - f(q_r^N)q_r^N] \\
= \; & \frac{kp_l}{1+\alpha-k}(1 - F(q_r^N)) - \frac{(1+\alpha)p_l}{1+\alpha-k}f(q_r^N)q_r^N.
\end{aligned}$$

It is easy to see that $\frac{d\pi_{sl}(q_r^N, q_l^N(q_r^N), w(q_r^N))}{dq_r^N} \geq (<) 0$ if and only if the generalized failure rate $g(q_r^N) = \frac{f(q_r^N)q_r^N}{1-F(q_r^N)} \leq (>) \frac{k}{1+\alpha}$. F is IGFR, so that g is an increasing function. Therefore, the profit function $\pi_{sl}(q_r^N, q_l^N(q_r^N), w(q_r^N))$ is unimodal in q_r^N. As a result, the profit function $\pi_{sl}(q_r^N(w), q_l^N(w), w)$ is unimodal in w. □

The numerical results in the later section indicate that the supplier is biased on raising the wholesale price and seizes on a majority of the channel profit.

4.2.2 The Integrated Retailer and the LSP v.s. the Supplier

We consider the retailer and the LSP collude in this section. They jointly make decisions q_r and q_l to maximize the joint profit π_{rl} which is equal to $\pi_r + \pi_l$. The decision game now has two stages. The suppler acts as a Stackelberg leader, who determines the wholesale price. The retailer-the LSP coalition makes orders from the supplier.

$$\begin{aligned}\pi_{rl}(q_r, q_l, w) &= pE\min\{D, q_r\} + pE\min\{(D-q_r)^+, q_l\} - w(q_r+q_l) \\ &= pE\min\{D, q_r+q_l\} - w(q_r+q_l).\end{aligned}$$

It straight forward that the problem is equivalent to a convention newsvendor problem for the retailer-the LSP coalition. The external decision that affects the coalition's payoff, is the total inventory $q_r + q_l$. Therefore the optimal decision for given w is $q_r + q_l = F^{-1}(\frac{p-w}{p})$. The double margins of the backup inventory between the retailer and the LSP are eliminated, hence the overall production quantity $q_r + q_l$ has a greater coverage than that in the three firms competition, which is $\frac{p_l-w}{p_l}$. Consider the situation at which the supplier and the LSP form a coalition, the coverage is $\frac{p_l-c}{p_l}$. It is not so straight forward to see which situation has more coverage, thus, achieves better system efficiency. we will compare the outcomes of the two cases in numerical examples in later section.

4.3. Model with Strategically Wholesale Price Discrimination

In this section, we assume that the supplier, the retailer and the LSP make decision separately. The supplier offers differentiated wholesale price w_l and w_r simultaneously to the LSP and the retailer. Then the LSP decides the replenishment price p_l. And then the retailer places an order q_r and the LSP places an order q_l before (after) observing q_r. Then, the supplier starts production. Finally, the goods q_l+q_r are shipped by the LSP to its warehouse, and the selling season begins.

We first consider the second stage ordering subgame. Assume that the LSP sets the replenishment price p_l satisfying $p_l \geq p/(1+\alpha)$. And the retailer accepts the replenishment, if $p-p_l \geq 0$. We also assume $w_l \leq w_r$, since the LSP is not going to accept a wholesale price that is even higher than the market wholesale price. With these assumption, we know that the supplier creates a source of income from the wholesale price discrimination. For any given w_r, w_l, we have the follow results.

Theorem 4.3.1. *The Stackelberg equilibrium is as follows:*

$$\begin{cases} q_r^{Sr}(w_l, w_r) &= F^{-1}(\frac{p_l - (1+\alpha)w_r}{p_l}) \\ q_l^{Sr}(w_l, w_r) &= F^{-1}(\frac{p_l - w_l}{p_l}) - F^{-1}(\frac{p_l - (1+\alpha)w_r}{p_l}). \end{cases} \tag{4.18}$$

The Nash equilibrium can be explicitly expressed as follows.

$$\begin{cases} q_r^{N}(w_l, w_r) = & F^{-1}(\frac{p_l-(1+\alpha-k)w_r}{p_l}) \\ q_l^{N}(w_l, w_r) = & F^{-1}(\frac{p_l-w_l}{p_l}) - F^{-1}(\frac{p_l-(1+\alpha-k)w_r}{p_l}). \end{cases} \tag{4.19}$$

In the first stage, the supplier makes decision. The supplier needs first to decide whether it should offer the products to the LSP for the purpose of backup inventory, to choose proper w_r and w_l to maximize its profit π_s.

Proposition 4.3.1. *If the supplier increases the value of w_r, the retailer will decrease the order quantity and the LSP will increase the backup inventory. The total ordering quantity remains. If the supplier increases the value of w_l, the retailer will keep the order quantity and the LSP will decrease the backup inventory. Therefore the total ordering quantity is cut down.*

$w_r \nearrow$, $w_l \rightarrow$	$q_r^{Sr} \searrow$, $q_l^{Sr} \nearrow$, $q_r^{Sr} + q_l^{Sr} \rightarrow$
$w_r \rightarrow$, $w_l \nearrow$	$q_r^{Sr} \rightarrow$, $q_l^{Sr} \searrow$, $q_r^{Sr} + q_l^{Sr} \searrow$

The supplier's profit function is

$$\pi_s(q_l^{Sr}, q_r^{Sr}, w_r, w_l) = (w_r - w_l)F^{-1}(\frac{p_l - (1+\alpha)w_r}{p_l}) + (w_l - c)F^{-1}(\frac{p_l - w_l}{p_l}).$$

Let $w_l = w$, $w_r > w$. We can see that the second term is the profit gained in the uniform wholesale price setting. The first part is the increment of profit. Therefore, the wholesale of the backup inventory becomes another source of the supplier's income.

Next result shows the relationship between the optimal wholesale prices in the uniform and differentiated settings.

Theorem 4.3.2. *Given the transfer price p_l, the service charge rate α, and the Stackelberg sub-ordering-game. We have $w_r^* > w^*/(1+\alpha)$ and $w_l^* < w^*$, where w^* is the optimal wholesale price with the uniform wholesale price setting, w_r^*, w_l^* are the optimal wholesale prices with the differentiated wholesale prices setting.*

PROOF. We plug in the expression of the equilibrium into the supplier's profit function, then the latter is the function of the wholesale price (w_r, w_l) only.

$$\pi_s(w) = (w - c)F^{-1}(\frac{p_l - w}{p_l})$$

$$\pi_s(w_r, w_l) = (w_r - w_l)F^{-1}(\frac{p_l - (1+\alpha)w_r}{p_l}) + (w_l - c)F^{-1}(\frac{p_l - w_l}{p_l}).$$

We denote $w^* = \arg\max_w \pi_s(w)$, $(w_r^*, w_l^*) = \arg\max_{w_r, w_l} \Pi_s(w_r, w_l)$ and $\tilde{w}_r = (1+\alpha)w_r$ for the convince of argument.

$$\begin{aligned}
\pi_s(\tilde{w}_r, w_l) &= (\frac{\tilde{w}_r}{1+\alpha} - w_l)F^{-1}(\frac{p_l - \tilde{w}_r}{p_l}) + (w_l - c)F^{-1}(\frac{p_l - w_l}{p_l}) \\
&= (\frac{\tilde{w}_r}{1+\alpha} - \frac{c}{1+\alpha} + \frac{c}{1+\alpha} - w_l)F^{-1}(\frac{p_l - \tilde{w}}{p_l}) \\
&\quad +(w_l - c)F^{-1}(\frac{p_l - w_l}{p_l}) \\
&= \frac{1}{1+\alpha}(\tilde{w}_r - c)F^{-1}(\frac{p_l - \tilde{w}_r}{p_l}) + (w_l - c)F^{-1}(\frac{p_l - w_l}{p_l}) \\
&\quad -(w_l - \frac{c}{1+\alpha})F^{-1}(\frac{p_l - \tilde{w}_r}{p_l}) \\
&= \frac{1}{1+\alpha}\pi_s(\tilde{w}_r) + \pi_s(w_l) - (w_l - \frac{c}{1+\alpha})F^{-1}(\frac{p_l - \tilde{w}_r}{p_l}).
\end{aligned}$$

Now, assume that $\tilde{w}_r^* \le w^*$, we want to show that this assumption leads to a contradiction. Therefore, $w_r^* > w^*/(1+\alpha)$ gets proved. Consider the partial derivative:

$$\frac{\partial \pi_s(\tilde{w}_r, w_l)}{\partial \tilde{w}_r} = \frac{1}{1+\alpha}\frac{d\pi_s(\tilde{w}_r)}{d\tilde{w}_r} - (w_l - \frac{c}{1+\alpha})\frac{\partial F^{-1}((p_l - \tilde{w}_r)/p_l)}{\partial \tilde{w}_r}.$$

Because w^* is the maximal point of the unimodal function $\pi_s(w)$, $\pi_s(w)$ is increasing when $w \in [0, w^*]$. As assumed that $\tilde{w}_r^* \in [0, w^*]$, we know the term $\frac{d\pi_s(\tilde{w}_r)}{d\tilde{w}_r} \ge 0$ when $\tilde{w}_r = \tilde{w}_r^*$. For the second term, since $\frac{\partial F^{-1}((p_l - \tilde{w}_r)/p_l)}{\partial \tilde{w}_r} < 0$, it is strictly positive. Therefore, $\frac{\partial \pi_s(\tilde{w}_r, w_l)}{\partial \tilde{w}_r} > 0$ when $\tilde{w}_r = \tilde{w}_r^*$. It is a contradiction to the optimality of $\tilde{w}_r^*$.

Similarly, we assume that $w_l^* \ge w^*$. Consider the partial derivative:

$$\frac{\partial \pi_s(\tilde{w}_r, w_l)}{\partial w_l} = \frac{d\pi_s(w_l)}{dw_l} - F^{-1}(\frac{p_l - \tilde{w}_r}{p_l}).$$

Because w^* is the maximal point of the unimodal function $\pi_s(w)$, $\pi_s(w)$ is decreasing when $w \in [w^*, p]$. As assumed that $\tilde{w}_l^* \in [w^*, p]$, we know the term $\frac{d\pi_s(w_l)}{dw_l} \leq 0$ when $w_l = w_l^*$. Therefore, $\frac{\partial \pi_s(\tilde{w}_r, w_l)}{\partial w_l} < 0$ when $w_l = w_l^*$. This contradicts to optimality of w_l^*. That proves $w_l^* < w^*$. □

4.4. Numerical Analysis

In this section, we present numerical analysis to corroborate and supplement the previous results. Throughout the analysis, we consider the demand D which is normal with mean 500 and variance 300. The retailing price $p = 100$ and the production cost $c = 30$. The centralized optimal production quantity is 657 and the associated profit is 25164.0.

In Table 1, we demonstrate the supply chain performance in decentralized supply chain with 3 independent firms. We denote s-l-r(SN) as the sequence where supplier makes decision first, then the LSP and retailer make their decision simultaneously. And s-l-r(SS) as the sequence where supplier makes decision first, the LSP makes decision second and the retailer makes de-

Table 4.1: Supply chain performance (Three independent firms)

Type	α	p_l	w^*	q_r^e	q_l^e	π_s/π	π_l/π	π_r/π	efficiency
s-l-r(SN)	0.05	99.0	69.3	318	25	0.71	0.06	0.23	75%
s-l-r(SN)	0.05	96.0	67.6	334	5	0.68	0.06	0.26	74%
s-l-r(SN)	0.05	90.0	67.1	339	0	0.67	0.06	0.27	74%
s-l-r(SN)	0.10	99.0	69.3	285	57	0.71	0.11	0.17	75%
s-l-r(SN)	0.10	92.0	65.2	326	8	0.63	0.12	0.25	74%
s-l-r(SN)	0.10	90.0	64.6	334	0	0.62	0.12	0.26	74%
s-l-r(SN)	0.20	99.0	69.3	210	132	0.71	0.20	0.08	75%
s-l-r(SN)	0.20	82.0	59.4	334	13	0.53	0.22	0.25	71%
s-l-r(SN)	0.20	75.0	60.2	323	0	0.54	0.22	0.24	72%
s-l-r(SS)	0.05	99.0	69.3	312	31	0.71	0.06	0.23	75%
s-l-r(SS)	0.05	96.0	67.6	308	31	0.68	0.06	0.26	74%
s-l-r(SS)	0.05	90.0	67.1	339	0	0.67	0.06	0.27	74%
s-l-r(SS)	0.10	99.0	69.3	278	64	0.71	0.11	0.17	75%
s-l-r(SS)	0.10	92.0	65.2	269	66	0.64	0.11	0.26	74%
s-l-r(SS)	0.10	90.0	64.6	334	0	0.62	0.12	0.26	74%
s-l-r(SS)	0.20	99.0	69.3	202	141	0.71	0.20	0.08	75%
s-l-r(SS)	0.20	82.0	59.4	163	158	0.53	0.16	0.31	71%
s-l-r(SS)	0.20	75.0	60.2	323	0	0.54	0.22	0.24	72%

Table 4.2: Supply chain performance (The supplier-LSP coalition v.s. the retailer)

Type	α	p_l	w or w^*	q_r^e	q_l^e	π_{sl}/π	π_r/π	efficiency
sl-r(SS)	0.05	99	40	557	97	0.28	0.71	99%
sl-r(SS)	0.05	99	60	395	259	0.69	0.31	99%
sl-r(SS)	0.05	99	70	305	350	0.83	0.17	99%
sl-r(SS)	0.05	96	40	547	99	0.28	0.72	98%
sl-r(SS)	0.05	96	60	379	267	0.67	0.32	98%
sl-r(SS)	0.05	96	70	283	364	0.81	0.19	98%
sl-r(SS)	0.10	96	40	531	115	0.33	0.67	98%
sl-r(SS)	0.10	96	60	353	293	0.72	0.28	98%
sl-r(SS)	0.10	96	70	245	401	0.85	0.15	98%
sl-r(SS)	0.10	92	40	516	119	0.32	0.68	97%
sl-r(SS)	0.10	92	60	327	308	0.69	0.31	97%
sl-r(SS)	0.10	92	70	205	430	0.81	0.19	97%
sl-r(SS)	0.20	90	40	475	154	0.39	0.61	96%
sl-r(SS)	0.20	90	60	248	382	0.75	0.25	96%
sl-r(SS)	0.20	90	70	50	580	0.82	0.18	96%
sl-r(SS)	0.20	82	40	435	168	0.37	0.63	93%
sl-r(SS)	0.20	82	60	150	452	0.67	0.33	93%
sl-r(SS)	0.20	82	65	3	600	0.69	0.31	93%
sl-r(SN)	0.05	99	89	25	629	0.98	0.02	99%
sl-r(SN)	0.10	92	82	120	515	0.87	0.13	97%
sl-r(SN)	0.20	82	71	195	408	0.77	0.23	93%

Table 4.3: Supply chain performance (The supplier v.s. the LSP-retailer coalition)

Type	α	p_l	w^*	$q_r^e + q_l^e$	π_s/π	π_{lr}/π	efficiency
s-lr(S)	-	-	60.2	422	0.59	0.41	86%

Table 4.4: Supply chain performance (Differentiated wholesale price)

α	p_l	w_r^*	w_l^*	q_r^e	q_l^e	π_s/π	π_l/π	π_r/π	efficiency
0.05	99.0	69	37	315	282	0.79	0.05	0.16	99%
0.05	97.5	68	35	314	294	0.78	0.05	0.17	99%
0.05	96.0	68	35	314	294	0.77	0.05	0.18	98%
0.10	99.0	69	37	281	314	0.78	0.11	0.11	99%
0.10	95.0	67	34	273	337	0.71	0.11	0.18	98%
0.10	92.0	64	31	283	343	0.70	0.12	0.18	95%
0.20	99.0	69	37	206	189	0.81	0.15	0.05	99%
0.20	90.0	64	35	184	399	0.73	0.19	0.08	95%
0.20	82.0	61	31	174	429	0.67	0.20	0.13	93%

cision third. w^* is the Stackelberg equilibrium for the supplier, that is the optimal wholesale price, given the outcome (q_r^e, q_l^e) of the ordering subgame. w^* depends on p_l when $p_l \geq p/(1+\alpha)$, and α when $p_l < p/(1+\alpha)$. The decentralized supply chain efficiency depends on the overall production quantity. Therefore, the efficiency depends on p_l when $p_l \geq p/(1+\alpha)$, and α when $p_l < p/(1+\alpha)$. In Table 2, we consider the colluded supplier and the LSP. We only consider one decision sequence: the supplier-LSP coalition presents the wholesale price w, then the retailer makes orders, and the coalition adjusts the final production quantity. In Table 3, we calculate the result of the decision process as a single Stackelberg game. The supplier is the leader, presenting the wholesale price. Then the retailer-the LSP coalition makes the inventory decisions, including the ordering quantity q_r and the backup quantity q_l jointly. In Table 4, we consider the three independent decision makers provided that the supplier offers differentiated wholesale prices w_l and w_r. The supplier adjusts the two prices to optimize its profit. For the sequence of decisions, we only consider the Stackelberg subgame, where the supplier chooses the wholesale price, then the retailer makes order, finally the LSP decides the quantity of backup inventory.

Figure 1 compares the channel performance and the profit allocation for various settings. We fix the transfer price $p_l =$

92 and the service rate $\alpha = 0.1$. In the second column, we assume the wholesale price $w = 70$, instead of p, which leaves the retailer a reservation profit. In all the other columns, we apply the equilibrium decisions as in the Table 1-4. The supply chain efficiency in those settings, from the best to the worst, is $sl - r(SS)$, $s - l - r(SS)(w_r, w_l)$, $s - lr(SS)$ and $s - l - r(SS)$.

The inefficiency of the supply chain comes from the double marginalization effect between the LSP and the supplier, and between the supplier and the retailer. One of the remedies is to combine the parties into one. From the numerical results, we can see that by combining the retailer and the LSP, the system profit increases by approximately 11%, while by combining the supplier and the LSP, the system profit increases by approximately 25% and approaches the centralized optimal profit. The benefit is doubled in supplier-LSP coalition. Therefore, the supplier-retailer double marginalization effect is the main source of the low channel efficiency. Compared with the complete decentralized system $s - l - r(SS)$, the coalition of the supplier and the LSP makes the retailer less power. Hence less profit is allocated to the retailer. However, when the LSP and the retailer form a coalition, the two parties get a better profit in total while the supplier's profit does not shrink.

Interestingly, when the supplier is allowed to present different

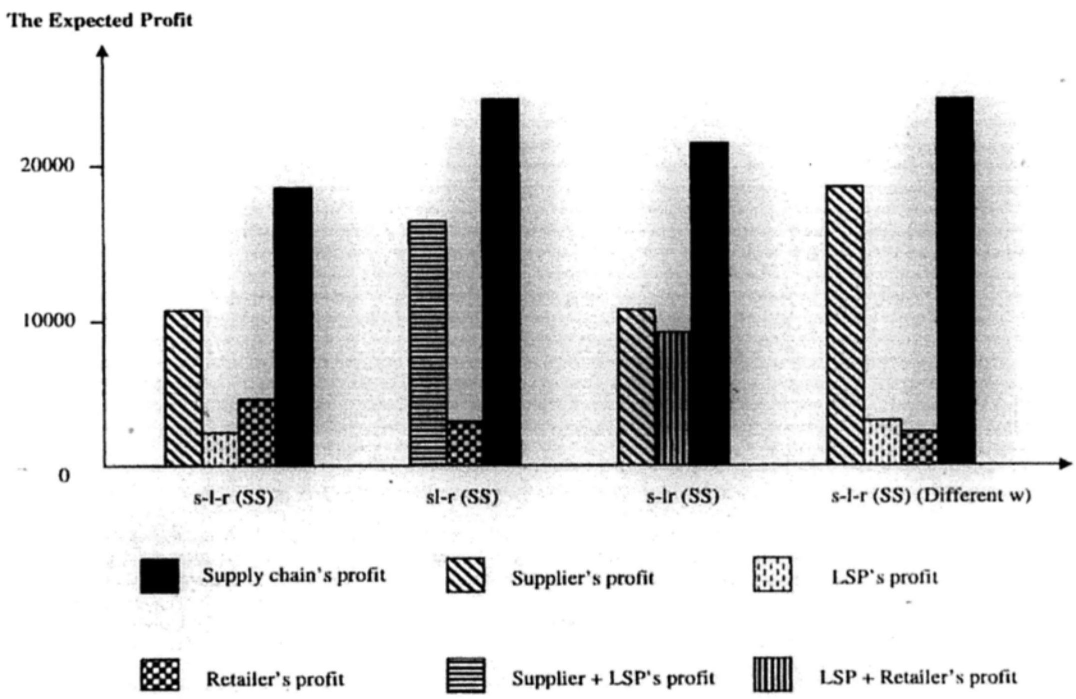

Figure 4.5: Performance with different settings

wholesale prices to the procurement of the traditional retailing inventory and the backup inventory, the supplier has incentive to settle the two prices with a huge gap in between. Compared with the uniform wholesale price model, price w_r is raised, while price w_l is cut in our numerical results. The efficiency is higher than both collusion situations in Table 2 and 3. The under stocking problem, recognized as the double marginalization effect, is solved by allowing the entrance of backup inventory with very low acquisition price. The channel efficiency is increased by 20% to the complete decentralized channel, and by 8% to the retailer-the LSP coalition situation. The supplier enjoys the increment of the over all profit while taking a greater part of the channel profit.

4.5. Concluding Remarks

The rapid emergence of third-party LSPs results in significant change in the supply chain management. Many emerging questions are not solved, and concerns are not fully understood. This paper attempts to tackle one aspect of the inventory management issue for a supply chain model. Our basic standpoint is that the supply chain could be a benefit for the incorporation of LSPs, even though the LSP is self-interested behaved. First, as facing a random demand, by adopting the backup agreement,

Chapter 5

Conclusion

The presence of the vast supply chain coordination literature motivates us to study the robustness of the coordination contracts. We believe that the robustness of a contract can be characterized in terms of how consistent it is in distributing the channel profit distribution, in the agents' desires to fulfill the contract, and in the face of changing game structures. In this paper, we have attempted to establish a framework of the consistency of supply chain coordination contracts. We focus on three aspects: consistency in the channel profit distribution, consistency with respect to the decision making sequences, and consistency with respect to the compliance regimes.

In the supply chain coordination literature, the ability of a contact to flexibly distribute the channel profit has been considered a desirable property. A flexible contract permits the allocation of the expected channel profit to the supply chain agents in any predetermined proportion. We find that a number of contracts exhibit a stronger flexibility in the sense that they are able to arbitrarily allocate the channel profit in any realization of the random demand. Thus, we define two notions of flexibility: the expectation flexibility and the sample-path flexibility. Consequently, we distinguish between an *allocation* function and a *sharing* function. As a performance indicator, the former is widely used in the literature as terms of payment and the latter is usually included as a part of a supply contract. We denote the ability of being able to maintain the same predefined ratio in the distribution of the channel profit along any sample path as the consistency in the channel profit distribution. We show that this consistency leads to equal probabilities for reaching a reservation payoff target and equal preferences on the parts of the supply chain agents to the outcome of random factors.

In the supply chain coordination literature, the bargaining power of an agent is not explicitly evaluated. For supply chains with agents possessing equal powers, the competition is resolved by a simultaneous game. On the other hand, if there is a strong agent, then the competition is resolved by a Stackelberg game lead by the strong agent. In reality, the relative strength of the agents may change over the time. Therefore, we introduce the notion of the consistency of coordination contracts in the decision-making sequence. For contracts that are sequence conthe LSP and retailer combine their inventory as the newsvendor solution. Unsurprisingly, the supply chain incurs a tripe marginalization. Second, we analyze the different behaviors when the supplier and the LSP are integrated as well as the retailer and the LSP. For the former case, there exists an incentive for the supplier-LSP coalition to raise wholesale price to the retailer. Whereas the double marginalization effect vanishes, and the total ordering quantity is raised, and the profit is also enhanced. For the latter case, the supply chain incurs double-marginalization effect. Third, we characterize the decentralized system when the supplier offers a different wholesale price w_l and w_r for the LSP and the retailer. It demonstrates that the supplier offers a lower price to the LSP, and a higher price to the retailer, to compare with the same wholesale price situation.

□ **End of chapter.**

sistent, the supply chain is coordinated regardless of the competitive game structure.

Our third and last concept is that of the compliance consistency of a contract, which ensures that it is in the interest of each agent to fulfill the contract. We explain that the sequence consistency obtains when the outcome of optimizing the payoff of one agent over the reaction curve of the other coincides with the Nash equilibrium. Furthermore, we show that the compliance consistency also occurs when the outcome of optimizing by any agent on its own reaction curve coincides with the Nash equilibrium. It is also revealed that the compliance consistency implies the sequence consistency. Finally, we develop a number of sufficient conditions for obtaining sequence and compliance consistencies. These turn out to be natural conditions; e.g., the concavity of the payoff functions with respect to each decision ensures the sequence consistency whereas their joint concavity in all decisions ensures the compliance consistency.

We provide examples to illustrate the concepts and the results developed in the paper. For this, we have revisited a number of well-known coordination contracts and classified them in a 3x3 matrix providing a taxonomy of the coordination contracts (See Table ??).

We would like to point out that some coordination contracts

Table 5.1: Taxonomy of Coordination Contracts

Consistencies/	Consistency in Profit Distribution	Coordination in Expectation	Coordination for Fixed Profit Distribution
Compliance Consistent	TPS contract in two-retailer system (Example ??) Cost-sharing contract in drop-shipping model (Netessine and Rudi 2004) (Example ??) Quantity discount contract in pricing competition model (Jeuland and Shugan 1983) PDS contract in pricing newsvendor (Bernstein and Federgruen 2005) Revenue sharing contract in newsvendor model (Cachon and Lariviere 2005) Buy-back in newsvendor model (Pasternack 1985)	Quantity discount contract in newsvendor model (Cachon 2003) Quantity-flexible contract in newsvendor model (Tsay 1999)	Buy-back contract in complimentary Newsvendor (Fang and Wang 2010) (Example ??)
Consistency in Sequential Decisions	Example ??	Sales based incentive (Jerath *et al.* 2007) (Example ??)	Example ?? Wholesale price in newsvendor with endogenous capacity (Cachon 2003) (Example ??)
Sequence Dependent		Target-rebate contract in newsvendor model (Taylor 2002) (Example ??) Revenue sharing contract in quantity competing (Cachon and Lariviere 2005)	Transhipment price contract (Rudi *et al.* 2003) (Example ??) PDS contract with competing retailers (Bernstein and Federgruen 2005)

exhibit consistency properties in other dimensions too. Take the buy-back contract again for an example, in addition to the consistency in distribution of channel profit at the system optimal point, the distribution of channel profit is consistent with the channel efficiency as well. We have shown that $\theta_i(y_1, y_2, z, \omega) = E\theta_i(y_1, y_2, z, \omega), i = 1, 2$. The consistency in channel efficiency can be expressed as $\theta_i(y_1, y_2, z, \omega) = \theta_i(z)$, $i = 1, 2$. In other words, the sharing rule of the buy-back contract is independent of decisions. Expressions (??) and (??) reveal that $\lambda_1(q, z) = \theta_1(q, z, D) = \lambda$ and $\lambda_2(q, z) = \theta_2(q, z, D) = 1 - \lambda$. Therefore, the buy-back contract is consistent for distribution of channel profit regardless of the channel efficiency. We would like also to point out that it is possible to develop other sufficient conditions for consistency matters of supply contracts. For example, if reaction curves of contract z are all slope down or slope up, it can been shown that contract z is compliance consistency as well. The reaction curve exhibits slope down or slope up when the competitive strategy is a substitute or complimentary one. For the substitute strategy, the reaction decreases with respect to the decision of the counterpart. Such a strategy occurs in the inventory-quantity competition with substitutable products. For the complimentary strategy, the reaction increases with respect to the decision of the counterpart. Such a strategy occurs in the price competition.

In this paper, we have limited our analysis in a two-agent supply chain. However, concepts developed in this paper, such as the allocation and sharing rules, consistency in channel profit distribution, decision sequences and compliance regimes, can be extended to n agents. Moreover, we have focused for single-period contracts with the inventory quantity or capacity decisions for a fixed sale price or cost parameters. The price is definitely an important factor in decision making for contracts. In addition, we have also assumed that the contracts are with invariant demand distributions. There are robustness issues for supply chains with pricing and demand information. It has been shown that coordination contracts some times are contingent to the distribution of random factors. These are the potential areas for further investigation of consistency issues over coordination contracts.

□ **End of chapter.**

Appendix A

Equation Derivation

Example ??

For the static game settings,

$$\begin{aligned}\Pi_i(t, q_i, D_i) &= p\min\{D_i, q_i\} + (p-t)\min\{(D_i - q_i)^+, (q_j - D_j)^+\} \\ &+ t\min\{(D_j - q_j)^+, (q_i - D_i)^+\} - cq_i.\end{aligned}$$

$\Pi_1(t, q_1, D_1) + \Pi_2(t, q_2, D_2) = p\min\{D_1 + D_2, q_1 + q_2\} - c(q_1 + q_2)$. So the system optimal ordering quantity is the solution of this newsvendor problem. $q_1 + q_2 = F^{-1}(\frac{p-c}{p})$, when $p = \$10, c = \$4, D_i \sim N(1000, 300)$, $D_1 + D_2 \sim N(2000, 424)$, optimal $q_1 + q_2 = 2107$, channel profit is $p\int_0^{q_1+q_2} \bar{F}(x)dx - cq = \$10362$. Next we calculate the value of t which leads the Nash equilibrium to the system optimal. The Nash equilibrium is the intersaction of the two response functions, $r_2(q_1)$ and $r_1(q_2)$, that is $(q_1, q_2) = (q_1, r_2(q_1)) = (r_1(q_2), q_2)$.

$$\begin{cases} \frac{\partial\pi_1(q_1,q_2)}{\partial q_1} &= 0 \\ \frac{\partial\pi_2(q_2,q_1)}{\partial q_2} &= 0. \end{cases}$$

By pluging in the optimal ordering quantity $(1054, 1054)$ to one of the equation above. We can calculate

$$t = \frac{p\bar{F}_1(q_1) - c + p[E\min\{(D_i - q_i)^+, (q_j - D_j)^+\}]'_{q_1}}{[E\min\{(D_i - q_i)^+, (q_j - D_j)^+\}]'_{q_1} - [E\min\{(D_j - q_j)^+, (q_i - D_i)^+\}]'_{q_2}}$$

which is the coordinating transshipment price. When the retailer 1 announces the ordering quantity q_1 before the retailer 2 places order, the game is dynamic. Keeping the coordinating transshipment price unchanged, we calculate the best order quantity by induction: $q_1 = \arg\max_{q_1} \pi(t, q_1, r_2(q_1))$, or $\arg\max_{q_1} r_1(r_2(q_1))$.

Example ??

From the formulation of the model, we have the payoff functions as follows,

$$\begin{cases} \pi_m(q, \phi, z^*) &= w_m^* + \alpha_m^* pE\min\{D(\phi), q\} - C(\phi), \\ \pi_o(q, \phi, z^*) &= w_o^* - \alpha_o^*(c_h E(q - D(\phi))^+ - c_p E(D(\phi) - q)^+ - c_b q) \\ & \quad +\beta_o^* pE\min\{D(\phi), q\} - B(q). \end{cases}$$

We reorganize the terms,

$$
\begin{aligned}
\pi_m(q,\phi,z^*) &= w_m^* + \alpha_m^* pE\min\{D(\phi),q\} - C(\phi)\\
&= w_m^* + \alpha_m^* pE\min\{\phi+\mu+\varepsilon,q\} - C(\phi)\\
&= w_m^* + \alpha_m^* pE\min\{\varepsilon,q-\phi-\mu\} + \alpha_m^* p(\phi+\mu) - C(\phi);\\
\pi_o(q,\phi,z^*) &= w_o^* - \alpha_o^*(c_h E(q-D(\phi))^+ + c_p E(D(\phi)-q)^+ + c_b q)\\
&\quad +\beta_o^* pE\min\{D(\phi),q\} - B(q)\\
&= w_o^* + (\beta_o^* p + \alpha_o^* c_h + \alpha_o^* c_p)E\min\{q,D(\phi)\}\\
&\quad -\alpha_o^*(c_h+c_b)q - \alpha_o^* c_p(\mu+\phi) - B(q)\\
&= w_o^* + (\beta_o^* p + \alpha_o^* c_h + \alpha_o^* c_p)E\min\{\varepsilon,q-\phi-\mu\}\\
&\quad -\alpha_o^*(c_h+c_b)q + (\beta_o^* p + \alpha_o^* c_h)c_p(\mu+\phi) - B(q).
\end{aligned}
$$

The first derivatives are

$$
\begin{aligned}
\frac{\partial \pi_m(q,\phi,z^*)}{\partial \phi} &= \alpha_m^* p(1-F(q-\phi-\mu))(-1) + \alpha_m^* p - \frac{dC(\phi)}{d\phi}\\
&= \alpha_m^* pF(q-\phi) - \frac{dC(\phi)}{d\phi},\\
\frac{\partial \pi_o(q,\phi,z^*)}{\partial q} &= (\beta_o^* p + \alpha_o^* c_h + \alpha_o^* c_p)(1-F(q-\phi-\mu))\\
&\quad -\alpha_o^*(c_h+c_b) - \frac{dB(q)}{dq}\\
&= (\beta_o^* p - \alpha_o^* c_b + \alpha_o^* c_p) - (\beta_o^* p + \alpha_o^* c_h + \alpha_o^* c_p)\\
&\quad F(q-\phi-\mu) - \frac{dB(q)}{dq}.
\end{aligned}
$$

The derivative of the response function with respect to the other retailer's decision can be written as follows (Fudenberg and Tirole (1991)).

$$\begin{aligned}\frac{dr_m(q,z^*)}{dq} &= -\frac{\frac{\partial^2\pi_m(q,\phi,z)}{\partial\phi\partial q}}{\frac{\partial^2\pi_m(q,\phi,z)}{\partial\phi^2}} = -\frac{\frac{\partial\{\alpha_m^* pF(q-\phi-\mu)-\frac{dC(\phi)}{d\phi}\}}{\partial q}}{\frac{\partial\{\alpha_m^* pF(q-\phi-\mu)-\frac{dC(\phi)}{d\phi}\}}{\partial\phi}} \\ &= \frac{\alpha_m^* pf(q-\phi-\mu)}{\alpha_m^* pf(q-\phi-\mu)+\frac{d^2C(\phi)}{d\phi^2}} = \frac{\alpha_m^* p}{\alpha_m^* p + 2\sigma\frac{d^2C(\phi)}{d\phi^2}}.\end{aligned}$$

The last equality is due to the uniformly distributed demand. It can be seen that $\frac{dr_m(q,z)}{dq}$ is a constant between 0 and 1. Therefore, $\frac{d^2r_m(q,z)}{dq^2} = 0$. Next, we verify that $\frac{d^2\pi_o(q,r_m(q,z),z)}{dq^2} \leq 0$. The first and second derivatives are

$$\begin{aligned}&\frac{d\pi_o(q,r_m(q,z^*),z^*)}{dq} \\ = \ &(\beta_o^* p - \alpha_o^* c_b + \alpha_o^* c_p) - (\beta_o^* p + \alpha_o^* c_h + \alpha_o^* c_p)F(q - r_m(q,z^*) - \mu)\cdot \\ &(1-\frac{dr_m(q,z^*)}{dq}) - (\beta_o^* p + \alpha_o^* c_h)c_p(1-\frac{dr_m(q,z^*)}{dq}) - \frac{dB(q)}{dq}, \\ &\frac{d^2\pi_o(q,r_m(q,z^*),z^*)}{dq^2} \\ = \ &-(\beta_o^* p - \alpha_o^* c_b + \alpha_o^* c_p)f(q - r_m(q,z^*) - \mu)(1-\frac{dr_m(q,z^*)}{dq})^2 \\ &-\frac{d^2B(q)}{dq^2}.\end{aligned}$$

Since $B(q)$ is quadratic increasing function, $\frac{d^2B(q)}{dq^2} \geq 0$. Therefore, it is clear the above second order derivative is non-positive, thus, $\pi_o(q,r_m(q,z^*))$ is concave in q.

We also show that $\pi_m(r_o(\phi, z^*), \phi, z^*)$ is concave in ϕ.

$$
\begin{aligned}
\frac{dr_o(\phi, z^*)}{d\phi} &= -\frac{\frac{\partial^2 \pi_o(q,\phi,z^*)}{\partial\phi\partial q}}{\frac{\partial^2 \pi_o(q,\phi,z^*)}{\partial q^2}} \\
&= -\frac{\frac{\partial\{(\beta_o^* p - \alpha_o^* c_b + \alpha_o^* c_p) - (\beta_o^* p + \alpha_o^* c_h + \alpha_o^* c_p)F(q-\phi-\mu) - \frac{dB(q)}{dq}\}}{\partial\phi}}{\frac{(\beta_o^* p - \alpha_o^* c_b + \alpha_o^* c_p) - (\beta_o^* p + \alpha_o^* c_h + \alpha_o^* c_p)F(q-\phi-\mu) - \frac{dB(q)}{dq}\}}{\partial q}} \\
&= \frac{(\beta_o^* p - \alpha_o^* c_b + \alpha_o^* c_p) f(q-\phi-\mu)}{(\beta_o^* p - \alpha_o^* c_b + \alpha_o^* c_p) f(q-\phi) + \frac{d^2 B(q)}{dq^2}}.
\end{aligned}
$$

It can be seen that $\frac{dr_o(\phi,z^*)}{d\phi}$ is a constant between 0 and 1. Therefore, $\frac{d^2 r_o(\phi,z^*)}{d\phi^2} = 0$. Next, we verify that $\frac{d^2\pi_m(r_o(\phi,z^*),\phi,z^*)}{d\phi^2} \leq 0$. The first and second derivatives are

$$
\begin{aligned}
\frac{d\pi_m(r_o(\phi, z^*), \phi, z^*)}{d\phi} &= \alpha_m^* p(1 - F(r_o(\phi, z^*) - \phi))(\frac{dr_o(\phi, z^*)}{d\phi} - 1) \\
&\quad + \alpha_m^* p - \frac{dC(\phi)}{d\phi}, \\
\frac{d^2\pi_m(r_o(\phi, z^*), \phi, z^*)}{d\phi^2} &= -\alpha_m^* p f(r_o(\phi, z^*) - \phi)(\frac{dr_o(\phi, z^*)}{d\phi} - 1)^2 \\
&\quad - \frac{d^2 C(\phi)}{d\phi^2} \leq 0.
\end{aligned}
$$

This completes the proof.

Example ??

The profit function for the wholesaler is

$$
\begin{aligned}
\pi_w(A, q, z) &= E[w \min(D(A), q) - cq - a_1(A) + a_2(\{q - D(A)\}^+)]. \\
&= E[(w - a_2)\min(D(A), q) - (c - a_2)q - a_1 A].
\end{aligned}
$$

When $a_1(A) = \frac{(w-c)A}{p-c}$, $a_2(Q) = c(1-a_1)Q$. Therefore,

$$\begin{aligned}\pi_w(A,q,z) &= E[\frac{p(w-c)}{p-c}\min(D(A),q) - \frac{c(w-c)}{p-c}q - \frac{w-c}{p-c}A] \\ &= \frac{w-c}{p-c}E[p\min(D(A),q) - cq - A].\end{aligned}$$

Take the first derivatives:

$$\begin{aligned}\frac{\partial\pi_w(A,q,z)}{\partial q} &= pPr(D(A) > q) - c \\ \frac{\partial\pi_w(A,q,z)}{\partial A} &= pPr(D(A) < q)\frac{dED(A)}{dA} - 1.\end{aligned}$$

The second derivatives are:

$$\begin{aligned}\frac{\partial^2\pi_w(A,q,z)}{\partial q^2} &= -pf_{D(A)}(q) < 0 \\ \frac{\partial^2\pi_w(A,q,z)}{\partial A^2} &= -pf_{D(A)}(q)(\frac{dED(A)}{dA})^2 \\ &\quad +pPr(D(A) < q)\frac{d^2ED(A)}{dA^2} < 0 \\ \frac{\partial^2\pi_w(A,q,z)}{\partial q\partial A} &= pf_{D(A)}(q)\frac{dED(A)}{dA} > 0.\end{aligned}$$

The determinant of the Hessian is

$$\begin{aligned}&\frac{\partial^2\pi_w(A,q,z)}{\partial A^2}\cdot\frac{\partial^2\pi_w(A,q,z)}{\partial q^2} - \left(\frac{\partial^2\pi_w(A,q,z)}{\partial q\partial A}\right)^2 \\ = &-p^2 f_{D(A)}(q)Pr(D(A) < q)\frac{d^2ED(A)}{dA^2} > 0.\end{aligned}$$

This is true, because $\frac{d^2ED(A)}{dA^2} < 0$.

Example ?? second part

The payoff functions are

$$\begin{aligned}
\pi_m(q,\phi,z') &= w'_m + \alpha'_m pE\min\{D(\phi),q\} - C(\phi)\\
&= w'_m + \alpha'_m pE\min\{\varepsilon, q-\phi-\mu\} + \beta'_m p(\phi+\mu) - C(\phi),\\
\pi_o(q,\phi,z') &= w'_o + \beta'_o pE\min\{D(\phi),q\} - B(q)\\
&= w'_o + \beta'_o pE\min\{\varepsilon, q-\phi-\mu\} + \beta'_o p(\phi+\mu) - B(q).
\end{aligned}$$

As in Example ??, the derivatives of the reaction functions $\frac{dr_0(q,z')}{dq}$ and $\frac{dr_m(\phi,z')}{d\phi}$ are constant between 0 and 1. We first show

$$\frac{d^2\pi_m(q,r_o(q,z'),z')}{dq^2} \leq 0.$$

The first and second derivatives are

$$\begin{aligned}
\frac{d\pi_m(q,r_o(q,z'),z')}{dq} &= \alpha'_m p(1-F(q-r_o(q,z'))) + \alpha_m p\cdot\frac{dr_o(q,z')}{dq}\\
&\quad -\frac{dC(r_o(q,z'))}{dr_o}\cdot\frac{dr_o(q,z')}{dq},\\
\frac{d^2\pi_m(q,r_o(q,z'),z')}{dq^2} &= -\alpha'_m pf(q-r_0(q,z'))(1-\frac{dr_o(q,z')}{dq})\\
&\quad +\alpha'_m p\cdot\frac{d^2r_o(q,z')}{dq^2} - \frac{d^2C(r_o(q,z'))}{dr_o^2}\cdot(\frac{dr_o(q,z')}{dq})^2\\
&\quad -\frac{dC(r_o(q,z'))}{dr_o}\cdot\frac{d^2r_o(q,z')}{dq^2}\\
&= -\alpha'_m pf(q-r_0(q,z'))(1-\frac{dr_o(q,z')}{dq})\\
&\quad -\frac{d^2C(r_o(q,z'))}{dr_o^2}\cdot(\frac{dr_o(q,z')}{dq})^2 \leq 0.
\end{aligned}$$

Second, by calculating the derivatives, we show

$$\frac{d^2\pi_o(r_m(\phi,z'),\phi,z')}{d\phi^2} \leq 0.$$

The first and second derivatives are

$$
\begin{aligned}
\frac{d\pi_o(r_m(\phi,z'),\phi,z')}{d\phi} &= -\beta_o'p(1-F(r_m(\phi,z')-\phi))+\beta_o'p \\
&\quad -\frac{dB(r_m(\phi,z'))}{dr_m}\cdot\frac{dr_m(\phi,z')}{d\phi}, \\
\frac{d^2\pi_o(r_m(\phi,z'),\phi,z')}{d\phi^2} &= -\beta_o'pf(q-r_0(q,z'))(1-\frac{dr_m(\phi,z')}{d\phi}) \\
&\quad -\frac{d^2B(r_m(\phi,z'))}{dr_m^2}\cdot(\frac{dr_m(\phi,z')}{d\phi})^2 \\
&\quad -\frac{dB(r_m(\phi,z'))}{d\phi}\cdot\frac{d^2r_m(\phi,z')}{d\phi^2} \\
&= -\beta_o'pf(q-r_0(q,z'))(1-\frac{dr_m(\phi,z')}{d\phi}) \\
&\quad -\frac{d^2B(r_m(\phi,z'))}{dr_m^2}\cdot(\frac{dr_m(\phi,z')}{d\phi})^2 \le 0.
\end{aligned}
$$

This completes the proof.

Example ??

We directly show that the TPSC is sequence consistent. We start with a static game setting. The response function for retailer i is to maximize the expected payoff with respect to decision of retailer j, that is

$$
\begin{aligned}
r_i(q_j,z) &= \arg\max_{q_i}\pi_i(q_i,q_j,z) = \arg\max_{q_i} E\{\Pi_i^D(q_i^0,D_i) \\
&\quad +\lambda(\Pi_i^T(q_i,q_j,D_i,D_j)+\Pi_j^T(q_i,q_j,D_i,D_j)-L_i(q_i,D_i) \\
&\quad -L_j(q_j,D_j))\} \\
&= \arg\max_{q_i} E(\Pi_i^T(q_i,q_j,D_i,D_j)+\Pi_j^T(q_i,q_j,D_i,D_j) \\
&\quad -L_i(q_i,D_i)-L_j(q_j,D_j)). \qquad \text{(A.1)}
\end{aligned}
$$

The last equality holds, due to the fact that $E\Pi_i^D(q_i^0, D_i)$ and λ are constant. Similarly, the response function of retail j can be written as

$$\begin{aligned} r_j(q_i) &= \arg\max_{q_j} E(\Pi_i^T(q_i, q_j, D_i, D_j) + \Pi_j^T(q_i, q_j, D_i, D_j) \\ &\quad -L_i(q_i, D_i) - L_j(q_j, D_j)). \end{aligned} \tag{A.2}$$

The Nash equilibrium can be either derived by jointly solving (??) and (??), or found by definition which gives rise to the maximizer (q_1, q_2) of $E(\Pi_i^T(q_i, q_j, D_i, D_j) + \Pi_j^T(q_i, q_j, D_i, D_j) - L_i(q_i, D_i) - L_j(q_j, D_j))$. Now we proceed with a Stackelberg game by assuming retailer i is the leader of the game. Obviously, retailer j still plays its best response to decision of retailer i, that is $q_j = r_j(q_i)$. The optimization problem for retailer i is

$$\begin{aligned} & \max_{q_i} E\Pi_i(\lambda, q_i, r_j(q_i), D_i, D_j) \\ = & \max_{q_i} E\{\Pi_i^D(q_i^0, D_i) + \lambda(\Pi_i^T(q_i, r_j(q_i), D_i, D_j) \\ & +\Pi_j^T(q_i, r_j(q_i), D_i, D_j) - L_i(q_i, D_i) - L_j(r_j(q_i), D_j))\} \\ = & E\Pi_i^D(q_i^0, D_i) + \lambda \max_{q_i} E\{\Pi_i^T(q_i, r_j(q_i), D_i, D_j) \\ & +\Pi_j^T(q_i, r_j(q_i), D_i, D_j) - L_i(q_i, D_i) - L_j(r_j(q_i), D_j)\}. \end{aligned} \tag{A.3}$$

Since $E\Pi_i^D(q_i^0, D_i)$ and λ are constant, therefore, the last equality holds. Note that

$$E\Pi_j(\lambda, q_i, r_j(q_i), D_1, D_2) = \max_{q_j} E\Pi_j(\lambda, q_i, q_j, D_1, D_2),$$

therefore, by the payoff function, the above equality can be rewritten as

$$\begin{aligned}
& E\Pi_j^D(q_j^0, D_j) + (1-\lambda)E\{\Pi_i^T(q_i, r_j(q_i), D_i, D_j) \\
& +\Pi_j^T(q_i, r_j(q_i), D_i, D_j) - L_i(q_i, D_i) - L_j(r_j(q_i), D_j)\} \\
= \; & E\Pi_j^D(q_j^0, D_j) + (1-\lambda)\max_{q_j} E\{\Pi_i^T(q_i, q_j, D_i, D_j) \\
& +\Pi_j^T(q_i, q_j, D_i, D_j) - L_i(q_i, D_i) - L_j(q_j, D_j)\},
\end{aligned}$$

thus, we have

$$\begin{aligned}
& E\{\Pi_i^T(q_i, r_j(q_i), D_i, D_j) + \Pi_j^T(q_i, r_j(q_i), D_i, D_j) \\
& -L_i(q_i, D_i) - L_j(r_j(q_i), D_j)\} \\
= \; & \max_{q_j} E\{\Pi_i^T(q_i, q_j, D_i, D_j) + \Pi_j^T(q_i, q_j, D_i, D_j) \\
& -L_i(q_i, D_i) - L_j(q_j, D_j)\}.
\end{aligned}$$

Substitute the right hand side equation to (??), we have

$$\begin{aligned}
& \max_{q_i} E\Pi_i(\lambda, q_i, r_j(q_i), D_i, D_j) \qquad (A.4) \\
= \; & E\Pi_i^D(q_i^0, D_i) + \lambda \max_{q_i}\{\max_{q_j} E\{\Pi_i^T(q_i, q_j, D_i, D_j) \\
& +\Pi_j^T(q_i, q_j, D_i, D_j) - L_i(q_i, D_i) - L_j(q_j, D_j)\}\} \\
= \; & E\Pi_i^D(q_i^0, D_i) + \lambda \max_{(q_i, q_j)} E\{\Pi_i^T(q_i, q_j, D_i, D_j) \\
& +\Pi_j^T(q_i, q_j, D_i, D_j) - L_i(q_i, D_i) - L_j(q_j, D_j)\}.
\end{aligned}$$

By (??), the solution of the Stackelberg game is the maximizer of $E(\Pi_i^T(q_i, q_j, D_i, D_j) + \Pi_j^T(q_i, q_j, D_i, D_j) - L_i(q_i, D_i) - L_j(q_j, D_j))$. It means that the outcomes are the same for both static and dynamic game.

□ **End of chapter.**

Appendix B

Backward Financing Model for LSP

In this section, we consider the backward financing scenario. The retailer has comparatively strong power and requires to pay the purchasing cost after the selling season. The supplier need financial support from the LSP, and the LSP charges the supplier for an interest rate β. We write the individual profit function as follows.

The retailer's expected profit is

$$\pi_r(q_l, q_r, w) = pE\min\{D, q_r\} + (p - p_l)E\min\{(D - q_r)^+, q_l\} - wq_r. \tag{B.1}$$

The expected profit for the LSP is

$$\pi_l(q_l, q_r, w) = p_l E\min\{(D - q_r)^+, q_r\} - wq_l + \beta w(q_r + q_l). \tag{B.2}$$

And the supplier's expected profit changes to

$$\pi_s(q_l, q_r, w) = [(1 - \beta)w - c](q_r + q_l). \tag{B.3}$$

We assume that $p_l \leq p$, β are fixed, the supplier's wholesale price must be set less than p_l. Now we consider the response function of the LSP.

Proposition B.0.1. *the LSP's best response r_l to the retailer's order quantity q_r is*

$$r_l(q_r, w) = \max\{0, \quad F^{-1}(\frac{p_l - (1-\beta)w}{p_l}) - q_r\}. \qquad \text{(B.4)}$$

PROOF. the LSP's expected profit function is transformed in the following way.

$$\begin{aligned}
\pi_l(q_l, q_r, w) &= p_l E \min\{(D - q_r)^+, q_l\} - (1-\beta)wq_l + \beta w q_r \\
&= p_l[\int_0^{q_r} \min\{0, q_l\} f(\xi)d\xi + \int_{q_r}^{q_r+q_l} \min\{\xi - q_r, q_l\} f(\xi)d\xi \\
&\quad + \int_{q_r+q_l}^{+\infty} \min\{\xi - q_r, q_l\} f(\xi)d\xi] - (1-\beta)wq_l + \beta w q_r \\
&= p_l[0 + \int_{q_r}^{q_r+q_l} (\xi - q_r) f(\xi)d\xi + \int_{q_r+q_l}^{+\infty} q_l f(\xi)d\xi] \\
&\quad -(1-\beta)wq_l + \beta w q_r \\
&= p_l[(\xi - q_r)F(\xi)|_{q_r}^{q_r+q_l} - \int_{q_r}^{q_r+q_l} F(\xi)d\xi + q_l F(\xi)|_{q_r+q_l}^{+\infty}] \\
&\quad -(1-\beta)wq_l + \beta w q_r \\
&= p_l[q_l F(q_r + q_l) - \int_{q_r}^{q_r+q_l} F(\xi)d\xi + q_l - q_l F(q_r + q_l)] \\
&\quad -(1-\beta)wq_l + \beta w q_r \\
&= p_l[q_l - \int_{q_r}^{q_r+q_l} F(\xi)d\xi] - (1-\beta)wq_l + \beta w q_r.
\end{aligned}$$

We take the derivative with respect to q_l,

$$\frac{\partial \pi_l(q_l, q_r, w)}{\partial q_l} = p_l[1 - F(q_r + q_l)] - (1-\beta)w.$$

And the second order derivative is

$$\frac{\partial^2 \pi_l(q_l, q_r, w)}{\partial q_l^2} = -p_l f(q_r + q_l) < 0.$$

The profit function is concave in q_l within $[0, +\infty)$. Mathematically, the optimal value of q_l maximizes π_l or say, the best response decision $r_l(q_r, w)$ satisfies the first order condition or equals 0 when the maximum is attained at negative quantity. That proves

$$r_l(q_r, w) = \max\{0, \quad F^{-1}(\frac{p_l - (1-\beta)w}{p_l}) - q_r\}.$$

□

The above first order condition can also be written as

$$F(q_r + q_l) = \frac{p_l - (1-\beta)w}{p_l}. \tag{B.5}$$

The LSP's best strategy is to order up to the level that makes the total inventory $q_r + q_l$ attain the service level $(p_l - (1-\beta)w)/p_l$. The financial service charge rate β in this setting has impact on the LSP's decision. Next we develop the retailer's response function.

Proposition B.0.2. $r_r(q_l, w)$ *is the response function of the retailer if and only if it satisfies*

$$\frac{p-p_l}{p} F(r_r(q_l, w) + q_l) + (1 - \frac{p-p_l}{p}) F(r_r(q_l, w)) = \frac{p-w}{p}. \tag{B.6}$$

PROOF.

The retailer's expected profit function

$$\begin{aligned}\pi_r(q_l, q_r, w) =& pE\min\{D, q_r\} + (p - p_l)E\min\{(D - q_r)^+, q_l\} - wq_r \\ =& p[q_r - \int_0^{q_r} F(\xi)d\xi] + (p - p_l)[q_l - \int_{q_r}^{q_r+q_l} F(\xi)d\xi] - wq_r.\end{aligned}$$

We take the derivative with respect to q_r,

$$\frac{\partial \pi_r(q_l, q_r, w)}{\partial q_r} = p - pF(q_r) - (p - p_l)[F(q_r + q_l) - F(q_r)] - w.$$

And the second order derivative is

$$\begin{aligned}\frac{\partial^2 \pi_r(q_l, q_r, w)}{\partial q_r^2} =& - pf(q_r) - (p - p_l)[f(q_r + q_l) - f(q_r)] \\ =& - p_l f(q_r) - (p - p_l)f(q_r + q_l) < 0.\end{aligned}$$

So the profit function is concave in q_r, and the first order condition is written as

$$\frac{p - p_l}{p}F(r_r(q_l, w) + q_l) + (1 - \frac{p - p_l}{p})F(r_r(q_l, w)) = \frac{p - w}{p}.$$

As we assume that $p_l > w$, the derivative is positive at $q_r = 0$ for any given $q_l \in [0, +\infty)$. So the optimality of the concave function is achieved at positive value. Thus, $r_r(q_l, w) > 0$ and it satisfies the first order condition. □

Again, the Nash equilibrium is the intersection of the two response curves.

Theorem B.0.1. *There exists a unique and globally stable Nash equilibrium. If*

$$(1 - \beta)p \leq p_l \tag{B.7}$$

holds, the LSP builds up a positive backup inventory.

PROOF.

Curve $\widehat{ac}$ is the response function $r_l(q_r, w)$, and curve $\widehat{bd}$ is the response function $r_r(q_l, w)$. The intercepts a and c are of value $F^{-1}(\frac{p_l-(1-\beta)w}{p_l})$, b is of value $F^{-1}(\frac{p-w}{p})$. The slope of curve $\widehat{bd}$ is less than 1 since

$$|\frac{\partial r_r(q_l, w)}{\partial q_l}| = | - \frac{(p-p_l)f(q_r+q_l)}{(p-p_l)f(q_r+q_l)+p_l f(q_r)}| < 1.$$

So the two response curves always have intersection and intersected both on positive values if and only if the intercept a is of higher value than b, namely

$$\frac{p-w}{p} \le \frac{p_l-(1-\beta)w}{p_l}.$$

We then rewrite the expression to $(1-\beta)p \le p_l$. On the other hand, $|\frac{\partial r_l(q_r,w)}{\partial q_l}| = 1$. So $|\frac{\partial r_r(q_l,w)}{\partial q_l}| \cdot |\frac{\partial r_l(q_r,w)}{\partial q_l}| < 1$. By Fudenberg and Tirole (1991), we know that there exists a unique and globally stable Nash equilibrium. Also it can be explicitly expressed as follows.

$$\begin{cases} q_r^N(w) &= F^{-1}[(\frac{p-w}{p} - \frac{p-p_l}{p} \cdot \frac{p_l-(1-\beta)w}{p_l})\frac{p}{p_l}] \\ q_l^N(w) &= F^{-1}(\frac{p_l-(1-\beta)w}{p_l}) - F^{-1}[(\frac{p-w}{p} - \frac{p-p_l}{p} \cdot \\ & \quad \frac{p_l-(1-\beta)w}{p_l})\frac{p}{p_l}]. \end{cases} \tag{B.8}$$

□

$(p-w)/p$ is the cycle service rate that the retailer wants to achieve. $(p_l-(1-\beta)w)/p_l$ is the cycle service rate which the LSP prefers to bring the industry inventory to. The inequality (??) is the condition under which the LSP wants to achieve a higher cycle service rate. Only then the LSP builds up a positive backup inventory.

Similarly, we consider the dynamic decision sequence with the Stackelberg equilibrium $(q_l^{Sr}(w), q_r^{Sr}(w))$.

In stage one, the supplier offers a wholesale price w, then the LSP and the retailer make orders $(q_r^e(w), q_l^e(w))$ from the supplier based on the wholesale price w. By equation (??), the profit of the supplier only relies on the sum $q_r^e(w)+q_l^e(w)$, not on the individual value. To consider the expression (??), we have the following,

Proposition B.0.3. *The supplier's profit is*

$$\pi_s(w) = \begin{cases} ((1-\beta)w-c)F^{-1}(\frac{p_l-(1-\beta)w}{p_l}) & p_l \geq (1-\beta)p \\ ((1-\beta)w-c)F^{-1}(\frac{p-w}{p}) & p_l < (1-\beta)p. \end{cases} \tag{B.9}$$

PROOF. By Theorem ??, we know that if $p_l \geq (1-\beta)p$, the LSP holds a non-negative backup inventory. And the total inventory as the outcome of the ordering game satisfies

$$F(q_r^e(w) + q_l^e(w)) = \frac{p_l-(1-\beta)w}{p_l}.$$

Otherwise, the LSP does not hold backup inventory. The total inventory is q_r, satisfying

$$F(q_r^e(w) + q_l^e(w)) = F(q_r) = \frac{p - w}{p}.$$

Then we substitute into the expression (??) and have completed the proof. □

In this section, we do not consider the other decision sequences, which are elaborated in the forward financing section, because all the structures and results are mathematically equivalent.

Now, we remark on the key differences between the backward and forward financing models. First, assume that $q_l^e(w) = 0$, namely the LSP does not hold backup inventory, then the choice of inventory $q_r(w) = \frac{p-(1+\alpha)w}{p}$ in the forward financing model, and $q_r(w) = \frac{p-w}{p}$ in the backward financing model. Hence, the forward financing setting aggravates the double marginalization effect, given the same wholesale price. Second, assume $q_l^e(w) > 0$, the LSP holds positive inventory, then $q_r^e(w) + q_r^e(w) = F^{-1}(\frac{p_l-w}{p_l})$ in the forward financing model and $q_r^e(w) + q_r^e(w) = F^{-1}(\frac{p_l-(1-\beta)w}{p_l})$ in the backward financing model.

The system service rate dose not change with different decision sequences. However, it dose change with the LSP's financial service charge, when the LSP holds zero inventory in the forward financing model and positive inventory in the backward financing model. However, neither case demonstrates a positive effect of the incorporation of the LSP from the supply chain point of view.

□ **End of chapter.**

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