Sensitivity Analysis and Material Parameter Estimation using Electromagnetic Modelling

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SENSITIVITY ANALYSIS AND MATERIAL PARAMETER ESTIMATION USING ELECTROMAGNETIC MODELLING
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Abstract

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Estimating parameters is the problem of finding their values from measurements and modelling. Parameters describe properties of a system; material, for instance, are defined by mechanical, electrical, and chemical parameters. Fisher information is an information measure, giving information about how changes in the parameter effect the estimation. The Fisher information includes the physical model of the problem and the statistical model of noise. The Cramér-Rao bound is the inverse of the Fisher information and gives the best possible variance for any unbiased estimator.

This thesis considers aspects of sensitivity analysis in two applied material parameter estimation problems. Sensitivity analysis with the Fisher information and the Cramér-Rao bound is used as a tool for evaluation of measurement feasibilities, comparison of measurement setups, and as a quantitative measure of the trade-off between accuracy and resolution in inverse imaging.

The first application is with estimation of the wood grain angle parameter in trees and logs. The grain angle is the angle between the direction of the wood fibres and the direction of growth; a large grain angle strongly correlates to twist in sawn timber. In the thesis, measurements with microwaves are argued as a fast and robust measurement technique and electromagnetic modelling is applied, exploiting the anisotropic properties of wood. Both two-dimensional and three-dimensional modelling is considered. Mathematical modelling is essential, lowering the complexity and speeding up the computations. According to a sensitivity analysis with the Cramér-Rao bound, estimation of the wood grain angle with microwaves is feasible.

The second application is electrical impedance tomography, where the conductivity of an object is estimated from surface measurements. Electrical impedance tomography has applications in, for example, medical imaging, geological surveillance, and wood evaluation. Different configurations and noise models are evaluated with sensitivity analysis for a two-dimensional electrical impedance tomography problem. The relation between the accuracy and resolution is also analysed using the Fisher information.

To conclude, sensitivity analysis is employed in this thesis, as a method to enhance material parameter estimation. The sensitivity analysis methods are general and applicable also on other parameter estimation problems.

Keywords: sensitivity analysis, material parameter, Fisher information, Cramér-Rao bound, electromagnetic modelling, grain angle, wood, electrical impedance tomography, inverse problems

Sammanfattning

Estimering av parametrar är att finna deras värde utifrån mätningar och modellering. Parametrar beskriver egenskaper hos system och till exempel material kan definieras med mekaniska, elektriska och kemiska parametrar. Fisherinformation är ett informationsmått som ger information om hur ändringar i en parameter påverkar estimeringen. Fisherinformationen ges av en fysikalisk modell av problemet och en statistisk modell av mätbruset. Cramér-Rao-gränsen är inversen av Fisherinformationen och ger den bästa möjliga variansen för alla väntevärdesriktiga estimatorer.

Den här avhandlingen behandlar aspekter av känslighetsanalys i två tillämpade estimeringsproblem för materialparametrar. Känslighetsanalys med Fisherinformation och Cramér-Rao-gränsen används som ett redskap för utvärdering av möjligheten att mäta och för jämförelser av mätuppställningar, samt som ett kvantitativt mått på avvägningen mellan noggrannhet och upplösning för inversa bilder.

Den första tillämpningen är estimering av fibervinkeln hos träd och stockar. Fibervinkeln är vinkeln mellan växtriktningen och riktningen hos träfibern och en stor fibervinkel är relaterad till problem med formstabilitet i färdiga brädor. Mikrovågsmätningar av fibervinkeln presenteras som en snabb och robust mätteknik. I avhandlingen beskrivs två- och tredimensionella elektromagnetiska modeller som utnyttjar anisotropin hos trä. Eftersom matematisk modellering minskar komplexiteten och beräkningstiden är det en viktig del i estimeringen. Enligt känslighetsanalys med Cramér-Rao-gränsen är estimering av fibervinkeln hos trä möjlig.

Den andra tillämpningen är elektrisk impedanstomografi, där ledningsförmågan hos objekt bestäms genom mätningar på ytan. Elektrisk impedanstomografi har tillämpningar inom till exempel medicinska bilder, geologisk övervakning och trämätningar. Olika mätkonfigurationer och brusmodeller utvärderas med känslighetsanalys för ett tvådimensionellt exempel på elektrisk impedanstomografi. Relationen mellan noggrannhet och upplösning analyseras med Fisher information.

För att sammanfatta beskrivs känslighetsanalys som en metod för att förbättra estimeringen av materialparametrar. Metoderna för känslighetsanalys är generella och kan tillämpas också på andra estimeringsproblem för parametrar.

Nyckelord: känslighetsanalys, estimering, materialparameter, Fisherinformation, Cramér-Rao-gränsen, elektromagnetisk modellering, fibervinkel, trä, elektrisk impedanstomografi, inversa problem

Preface

This thesis treats sensitivity analysis of material parameter estimation problems based on electromagnetic modelling. The research work has been carried out at Linnaeus University in Växjö, Sweden, and is presented for the doctoral degree in engineering physics. The thesis is based on the following papers, referred to in the text by their Roman numerals.

Papers included in the thesis

I. T. Sjödén, S. Nordebo, and B. Nilsson, "Microwave Modelling and Measurements for Early Detection of Spiral Grain in Wood", *14th International Symposium on Nondestructive Testing of Wood*, Eberswalde, Germany, May 2005.

II. T. Sjödén, S. Nordebo, and B. Nilsson, "Wood Grain Angle Estimation in Logs with Microwave Modelling", *Second Conference on Mathematical Modeling of Wave Phenomena*, Växjö university, Sweden, 14–19 August, 2005, AIP Conference Proceedings, vol. 834, pp. 278–285, 2006.

III. B. Nilsson, S. Nordebo, and T. Sjödén, "Estimation of Twist in Uniaxial Cylinders with Inverse Electromagnetic Scattering", *IEEE Transactions on Antennas and Propagation*, vol. 57, no. 10, pp. 3264–3273, 2009.

IV. T. Sjödén, B. Nilsson, and S. Nordebo, "Numerical Verification of a Microwave Impedance Model for a Twisted Wooden Cylinder", *3rd Conference on Mathematical Modeling of Wave Phenomena*, Växjö, Sweden, 9 – 13 June 2008. AIP Conference Proceeding, vol 1106, pp. 243–252, 2009.

V. T. Sjödén and S. Nordebo, "Sensitivity Analysis for Measurement Configuration Evaluation in Electrical Impedance Tomography", submitted to Inverse Problems in Science and Engineering, 2012.

VI. T. Sjödén, S. Nordebo, and B. Nilsson, "On the Accuracy and Resolution in Inverse Imaging", submitted to Journal of Engineering Mathematics, 2012.

Related papers, not included in the thesis

- T. Sjödén, S. Nordebo, and B. Nilsson, "Sensitivity analysis for inverse problems in electrical impedance tomography", *6th International Conference on Inverse Problems, Modeling and Simulation*, Antalya, Turkey, 21– 26 May, 2012.
- T. Sjödén, M. Gustafsson, S. Nordebo, and F. Soldovieri, "Fisher information analysis in electrical impedance tomography", *European Geosciences Union General Assembly*, Vienna, Austria, 22 – 27 April, 2012.

- S. Nordebo, M. Gustafsson, T. Sjöden, and F. Soldovieri, "Data fusion for electromagnetic and electrical resistive tomography based on maximum likelihood", *International Journal of Geophysics*, Article ID 617089, pp. 1-11, 2011.
- S. Nordebo, R. Bayford, B. Bengtsson, A. Fhager, M. Gustafsson, P. Hashemzadeh, B. Nilsson, T. Rylander, and T. Sjöden, "Fisher Information Analysis and Preconditioning in Electrical Impedance Tomography", *Journal of Physics: Conference Series, XIV International Conference on Electrical Bioimpedance and 11th Conference on Biomedical Applications of Electrical Impedance Tomography*, Florida, USA, Vol. 224, No. 1, ID 012057, 2010.
- S. Nordebo, R. Bayford, B. Bengtsson, A. Fhager, M. Gustafsson, P. Hashemzadeh, B. Nilsson, T. Rylander, and T. Sjöden, "An Adjoint Field Approach to Fisher Information Based Sensitivity Analysis in Electrical Impedance Tomography", *Inverse Problems*, vol. 26, 125008, 2010.
- S. Nordebo, B. Bengtsson, M. Gustafsson, B. Nilsson, and T. Sjödén, "Fisher information analysis and gradient based optimization for electrical impedance tomography", *5th International Conference on Inverse Problems, Modeling and Simulation*, pp. 65–66, Antalya, Turkey, 24–29 May, 2010.
- T. Sjödén, B. Nilsson, and S. Nordebo, "Determination of wood parameters using electromagnetic measurements", *5th International Conference on Inverse Problems, Modeling and Simulation*, pp. 70–71, Antalya, Turkey, 24–29 May, 2010.
- B. Nilsson, T. Sjödén, S. Nordebo, and H. Säll, "A Method for Under Bark Detection of the Wood Grain Angle Radial Dependence", *Wood Material Science and Engineering*, vol. 2, no. 3-4, 2007.
- B. Nilsson, S. Nordebo, and T. Sjödén, "Inverse Scattering for Determination of Twist in Uniaxial Cylinders", *EMB 07 Proceedings: Swedish National Conference on Computational Electromagnetics*, 177-184, October 18-19, 2007, Lund University.
- T. Sjödén, B. Nilsson, and S. Nordebo, "Microwave Modelling and Signal Estimation for Detection of Wood Properties", *Radiovetenskaplig konferens RVK*, Linköping, Sweden June 14-16 2005.

Acknowledgments

It has been a long way from start to now. Some time ago, it began with the question, can we measure the wood grain angle with microwaves. Around this question, my thesis work has evolved with new questions and become what it is today. I am grateful to my supervisors Börje Nilsson and Sven Nordebo, for all the support during the way. We have had many fruitful discussions and I have been given guidance, inspiration and encouragement.

The applications in my thesis have connections to wood. I would like to thank Doctor Harald Säll for his kind explanations of wood properties and the refining process from harvester to sawmill. I would also like to thank Professor Andrei Khrennikov and the research profile in Mathematical Modelling for support.

The School of Computer Science, Physics and Mathematics is an interdisciplinary environment and there is room for interesting discussions; spanning from mathematics, physics, and editing, to apps, scouting, and knitting. Special thanks to former and present colleagues at the Physics and Electrical science department, solving small and big questions. My fellow phd-students Doctor Jonas Lundbäck, Sara Rydström, Oscar Lindhe, and Stefan Gustafsson have shared different parts of my way and thesis work. I value and appreciate the company.

I have had a great support from my family, Gustavsson and Sjödén, and friends, minding children, taking long walks, fixing things, and so forth. Finally, Daniel, Tova-Li, and Malte, you bring happiness into my life.

Therese Sjödén
Smulen, Ör, April 2012

Contents

1 Introduction

Estimation of material parameters based on measurements is a general problem appearing in various applications. Parameters describe the characteristics of a material and are included in physical models. This thesis concerns electromagnetic modelling of the relation between parameters and measurements. The models are mathematical and include the parameters. To estimate parameters, it is an advantage to include prior knowledge of the parameters and models of the measurement noise. The measurements are described with a probability density function, dependent on the physical model and the statistics of the noise.

Estimators assign values to parameters with the use of measurements and models; examples are mean values and maximum likelihood estimators. With maximum likelihood (ML) estimation, for example, the parameters are estimated by maximizing the conditional probability density function with respect to the parameters. The variance of an estimator describes the uncertainty in the estimation; an estimator with small variance is more efficient and has better performance than one with larger variance.

The sensitivity of a problem describes how changes in the parameters effect the performance of an estimation. Sensitivity analysis can be used to evaluate parameter estimation problems. The Fisher information is an information measure, coupling the parameter dependence to the variance of the estimate. In this thesis, the Fisher information is applied for sensitivity analysis. Furthermore, the inverse of the Fisher information gives the Cramér-Rao bound (CRB), which is a lower bound for the variance of any unbiased estimator. This bound is asymptotically achieved for a large number of measurements with ML-estimation. Sensitivity analysis is studied and the methods are applied on two relevant material parameter estimation problems.

The sensitivity with different measurement configurations, or variable values, can be compared and set-ups with better variance can be chosen. Hence, the Fisher information can be used for optimization of a measurement set-up. Whether estimation is possible with desired accuracy for a certain configuration or not, can be analysed with the CRB. The CRB indicates, for instance, how many measurements or other improvements of the set-up that is needed. Section 2 introduces notions and theory for estimating material parameters and describes sensitivity analysis with Fisher information and CRB.

In this thesis, sensitivity analysis is used to investigate measurement feasibilities of wood grain angle measurements on trees and logs. The grain angle of a tree is the angle between the direction of growth and the wood fibres. The size of the grain angle is of interest for the wood industry since it strongly correlates to form-stability after drying. Demands from the industry are that the measurements are fast and robust. Measurements of wood parameters are introduced and motivated in section 3.

To meet the need of fast grain angle measurements, microwave measurements are proposed in this thesis. Wood is an anisotropic dielectric material interacting with an electromagnetic wave. Through interaction with wood, the amplitude and the polarization of the electromagnetic wave is changed. Measurements of wood parameters using electromagnetic properties are described in section 4. Modelling using only a few parameters is applied to lower the complexity of the problem and render possible estimation of the grain angle and dielectric properties. Three different cases are considered: plane boards, near-field measurements on a log and far-field measurements on a log. The CRB is used to evaluate the possibility to measure and to indicate the needed signal-to-noise-ratio. According to this sensitivity analysis, measurement of the grain angle is feasible.

In this thesis, sensitivity analysis is also applied on an electric impedance tomography (EIT) measurement. In EIT, the conductivity of a material is estimated through surface measurements. The conductivity is related to the material parameters. Voltage is applied on surface sensors and the resulting currents and potentials are measured. Finding the conductivity from these measurements is an ill-posed problem where a small error in the measurement data can cause a great error in the parameter estimation. EIT measurements are described in section 5.

The use of sensitivity analysis for EIT problems is discussed in the last part of section 5. With sensitivity analysis, the estimation performance is evaluated for different configurations and noise models. The CRB is used as a measure of the trade-off between accuracy and resolution in the reconstructed image. Accuracy is how close the estimator is to the true value of the parameter and resolution is how small objects that can be distinguished.

Section 6 introduces aspects of inverse problems, exemplifying how the application examples in the thesis is ill-posed. Summaries of the included papers are given in section 7 and conclusions in section 8.

2 Parameter Estimation and Sensitivity Analysis

A parameter describes the characteristics of a material or, more general, a property of a system. From measurements, the value of a parameter is estimated or determined. This section introduces some notion and theories concerning estimation of parameters. A parameter θ could be a single value, or a vector. To estimate a parameter, a measurement model is applied

$$x = f(\theta) + w, \tag{2.1}$$

where x is the measurement vector with N elements, f is a model describing the system dependent of the parameter θ, and w is the measurement noise vector [34, 65, 75]. The corresponding probability density function (PDF) of the measurement data x is denoted $p(x;\theta)$. Material parameter estimation based on electromagnetic modelling is considered in this thesis; hence

f is described by electromagnetic modelling. The model mathematically describes the physics, relating the parameters, the noise, and the measurement. The parameters could both be known and unknown. A common and reasonable assumption is that the measurement noise vector contains samples from a white Gaussian stochastic process with zero mean and covariance matrix R [34]. However, the theories are not constrained to this assumption.

To find the estimate $\hat{\theta}$ of the parameter θ, the (inverse) function g

$$\hat{\theta} = g(x), \tag{2.2}$$

interprets the measurements and assigns a corresponding value to the parameter [34, 48, 65, 75]. The expectation value $\mathcal{E}\{\cdot\}$ of an unbiased estimator is equal to the true parameter value

$$\mathcal{E}\{\hat{\theta}\} = \theta. \tag{2.3}$$

Estimators can be biased so that the expectation value deviate from the true value $\mathcal{E}\{\hat{\theta}\} \neq \theta$. The variance $\mathrm{var}\{\cdot\}$ of an estimator indicates its performance and efficiency, hence estimators with small variance are preferable. Mean values, maximum likelihood (ML) and least squares (LS) are examples of estimations. ML estimation maximises the PDF of the measurement with respect to the unknown parameters. LS estimation minimise the squared difference between the measurements and the model. With white Gaussian noise, estimation with ML and LS are equivalent.

Sensitivity analysis of an inverse problem aims at quantifying the performance of the estimation. In this thesis, sensitivity analysis is performed with the Fisher information $\mathcal{I}$ and the Cramér-Rao lower bound (CRB). The Fisher information is an information measure corresponding to the sensitivity of the parameter in the model and is given by the expectation value of the second derivate of the PDF $p(x;\theta)$ with respect to the parameter θ,

$$\mathcal{I} = -\mathcal{E}\left\{ \frac{\partial^2 \ln p(x;\theta)}{\partial \theta^2} \right\}. \tag{2.4}$$

The CRB is calculated from the inverse of the Fisher information and gives the best possible variance of any unbiased estimator,

$$\mathrm{var}(\hat{\theta}_i) \geq \mathcal{I}_{ii}^{-1}, \tag{2.5}$$

where i indicates the parameter index and ii the corresponding diagonal element. With ML estimation, the lower bound is achieved asymptotically when the data set is large. The Fisher information and CRB are defined also for complex valued parameters [6, 34]. Mixed complex and real parameters are considered in paper II. Real parameters and complex valued measurement data are applied in paper V and VI.

The CRB and the Fisher information can be used as tools to evaluate a measurement set-up, regarding for instance, bandwidth, frequencies, sensor placement or measurement configurations [8, 41, 43, 53, 70]. The CRB indicates whether or not a parameter can be estimated to the desired accuracy. In the included papers, the CRB is used to evaluate estimation performance in various measurement set-ups and variables. The variables could be adjustable such as frequencies, distances and current configuration schemes, or they could depend on the object such as moisture content or density. These evaluations can be utilised in the development and optimization of the measurement set-up.

Preconditioning with Fisher information, for gradient-based quasi-Newton reconstruction algorithms for inverse problems, is described in [53, 54]. The Fisher information is an estimate of the Hessian [34, 54] and incorporates parameter scaling, compare Jacobi preconditioning [16, 35]. Fisher information can complement other regularization techniques, offering a systematic and quantitative method to gain more information about the problem.

The ML-based singular value decomposition (SVD) is established in [53] and paper VI, incorporating the noise model in the reconstruction algorithm. The CRB is used, in paper VI, to quantify the trade-off between accuracy and resolution in inverse imaging. For a desired resolution, the CRB limits the accuracy that is achievable.

3 Wood Measurements

Wood is a renewable raw material and has a market in different wood products, for example, construction timber, furniture, pulp, and energy. For construction timber wooden logs are processed in a sawmill. During the process, wood parameters such as diameter and length, are measured. The parameters are included in the optimization of the sawing. Today, most sawmills optimize the profit of each log in contrast to the possible optimization toward customers' requirements.

According to [28], building constructors are not satisfied with form-stability in construction timber. The customers' demands are dimensions, form-stability, and strength. The building constructors expectations of form-stability and the corresponding requirements on the products are quantified in [28]. Current standards in Sweden accept much more twist and warp than is accepted in structural building [13]. Dissatisfaction with form-stability could lead to lost market shares to other construction products.

To meet the demands from the customers, measurements, classification, and sorting are needed. Measurements of key parameters, correlating directly to relevant properties are essential [58]. Modulus of elasticity (MOE) predicts the stiffness of sawn timber. Measurement of MOE on standing trees or logs correlates to MOE in finished construction timber [66]. Another parameter effecting the quality of sawn timber is the wood grain angle. The grain angle is the angle between the direction of growth and the wood fibres, see figure 3.1. A large grain angle under bark in Norway spruce

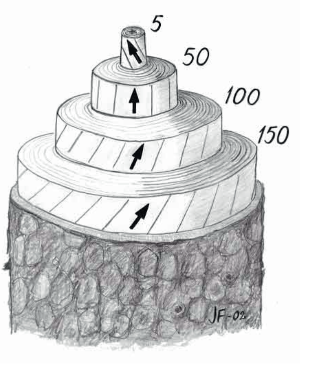

Figure 3.1: Spiral grain orientation in a log at different annual rings (5, 50, 100, 150) in an old spruce tree. The grain angle is the angle between the direction of growth and the wood fibres. (Illustration from Harald Säll, Forestry and Wood Technology, Linnaeus University, Växjö.)

(*Picea abies*) strongly correlates to twist in sawn timber [29, 63]. With the measurement of key parameters early in the refining process, possibly at harvesting, every log could be processed in an efficient way. This is exemplified in figure 3.2, where the boards in the top row have been sawn from a log with large grain angle and possess twist.

Wood is a complex material, where several different material properties collaborate, giving the properties of sawn timber. Hence, also measurement and classification of wood is a complex task. Section 3.1 describes methods for estimation of the material parameter wood grain angle and section 3.2 gives examples of imaging of wood properties. Some of the techniques coincide, but for the grain angle only one parameter is estimated, whereas when imaging more information is searched.

Figure 3.2: Picture of sawn boards from four trees with normal grain angle(six bottom layers) and a tree with large grain angle (top row). (Photo, Harald Säll, Forestry and Wood Technology, Linnaeus University, Växjö.)

3.1 Measuring the Wood Grain Angle

There are several possible methods or physical phenomena that can be used for grain angle measurement [7, 71]: ultrasonic, microwaves, laser, visible light, infrared light, ionizing radiation, nuclear magnetic resonance, and neutron radiography. Ultrasonic measurements cannot be used for remote sensing of wood, since the transmitter requires physical contact with the log due to the great difference in density between air and wood. Great costs are an obstacle for ionizing radiation, nuclear magnetic resonance and neutron radiography. Laser enables a fast, simple and cheap estimation of the wood grain angle, but the bark needs to be removed [55]; a few sawmills use laser measurements today. The Slope-of-grain indicator [47] performs a capacitive measurement of the grain angle, however, slowed down by the need of mechanical rotation. Other methods, for example, X-ray [67] and infrared light give much information, demanding extensive data processing or image analysis.

A simple and robust measurement technique suitable for in-field use, with bark present, is still needed. The approach should be non-destructive and remote. It is an advantage if only the grain angle needs to be determined. Measurements with microwaves could meet the need of fast, in-expensive and non-destructive testing [7, 64]. Microwave measurements of the wood grain angle are described in section 4. The papers I, II and III consider modelling for measurements of the wood grain angle with microwaves. Great complexity and variations of parameters of wood imply the need of theoretically and technically advanced solutions [5]. However, for industrial application, the measurement techniques also need to be fast and robust.

3.2 Imaging of Wood

High-resolution images of wood, finding the interior properties, can be constructed with different techniques. Bucur [7] mentions ionizing radiation, microwave, and ultrasonic imaging. The choice of technique depends on the phenomenon of interest. Microwave tomography [31] produce high-resolution images of knots, but the measurement and calculation time is long. If detection of decay is the objective, simpler measurements and other algorithms can perform well enough.

Three techniques for detecting decay in trees; electric tomography, ultra sonic (33 kHz), and georadar tomography (1-1.5 GHz) are compared in [51], and all the methods can find the presence of decay. Finding colour differences in beech trees are described in [79, 80] with resistivity tomography and measurements of the complex resistivity of oak is presented in [46]. Electric resistivity tomography is used for estimation of sapwood and heartwood relations in Scots pine (*Pinus sylvestris L.*) [4].

The advantage of combining results from sonic and electrical impedance tomography (EIT) for non-destructive testing of trees is presented in [61]. Sonic tomography gives information about the bio-mechanical properties of the tree. The sound velocities are measured and compared but different de-

Figure 3.3: Measurement of the interior of a beech tree, searching for red coloured heartwood with EIT. (Photo and experimental measurement equipment, Bengt Bengtsson, Romele Elektronik AB.)

fects in trees can result in the same velocities and reconstructed image. EIT images the chemical properties and for example cavities (low impedance) and decay (high impedance) are possible to distinguish [17]. The proposed combination is used in the PiCUS: Treetronic system [59].

The EIT problem considered in the papers V and VI is motivated by the application with determination of wood properties, see figure 3.3. The analysis is, however, not restricted to that problem, but have a wider interpretation, aiming at the sensitivity analysis and its possibilities, see section 2.

4 Microwave Measurements of the Wood Grain Angle

Considering the previous description of wood measurements and demands from industry and customers in section 3, electromagnetic measurements of the wood grain angle are described in this section. Electromagnetic fields are described by the Maxwell equations. The Maxwell equations are a mathematical model of the dynamics of the fields [26]. The constitutive relations describe material properties and contain the dielectric properties of the material. The fields and the flux densities within the material are coupled with the constitutive relations. Simple isotropic materials have the same properties in all directions, while more complex materials can have different properties in different directions [10, 40]. Materials with different properties in different directions are called anisotropic.

Wood could be described as an anisotropic material with different electric permittivity along and perpendicular to the wood fibres [74]. The dielectric properties of wood depend on different material properties such as density, temperature, moisture content, and species. There is also frequency dependence. An electromagnetic wave in the microwave region, 0.3-300 GHz [60], passing through wood is attenuated and its polarization is changed. Polarization is an orientation property of the electromagnetic field and changes due to the anisotropy. The attenuation depends mainly on the water, that is, the moisture content, since water absorbs electromagnetic microwaves efficiently. While attenuation mainly affects the part of the wave travelling in wood, the effect of the change in polarization is also present in the reflected part of the wave. Dielectric properties of wood are investigated in [14, 50, 56] and tabulated in [74].

Different electromagnetic methods using microwaves, have been developed for estimating wood parameters [37, 45], where one of the parameters is the grain direction. A microwave tomography of logs using a frequency range of 2-18 GHz is presented in [30, 31], where the reflected field is detected and the internal properties have been found through image analysis. Measuring and characterizing the properties of a wooden board are exemplified in [12, 27, 44, 45, 68]. To estimate the grain angle in a board from a field transmitted through wood, simultaneous estimation of moisture content is needed in the measurement method described in [68]. Reflection of an electromagnetic field from a plane wooden board is modelled in paper I. A linearly polarized electromagnetic wave is incident and the cross polarization is measured in the reflected field. The cross polarization is introduced with the anisotropic properties and it has a minimum when the linearly polarized field is aligned with the grain. Hence, the wood grain angle can be found in the reflected field, regardless of moisture content, density or temperature.

In paper I and II, incident plane waves towards the log is considered. The electromagnetic modelling is described in [52]. The electromagnetic field is measured with sensors placed close to the log and two-dimensional modelling applies. The estimator for the grain angle is found from a relation between the total field and the components introduced by the anisotropy.

Aiming for a handheld, or harvester mounted, grain angle measurement device collocating the transmitting and receiving antennas, three-dimensional modelling of electromagnetic reflections from an anisotropic cylinder is described in paper III. A handheld sensor for measurement of decay in wooden cross-arms is described in [36], however, more information from the field is needed to find polarization differences. Contrary to [30, 31] where a full image of the interior of the log is calculated, only the grain angle is searched for here, rendering possible faster calculation methods. To simplify, a surface impedance model is introduced and far-field asymptotic expressions derived, ending in two-dimensional complexity and faster calculation. The surface impedance model is verified in paper IV, where the introduced model error is calculated through comparison between the complete model with a penetrable cylinder and the impedance model.

Sensitivity analysis is applied to evaluate the feasibility of microwave wood grain angle measurements. In the papers I, II, and III, the Cramér-Rao bound is calculated for different measurement configuration variables. The bound gives the best possible variance and is asymptotically achievable

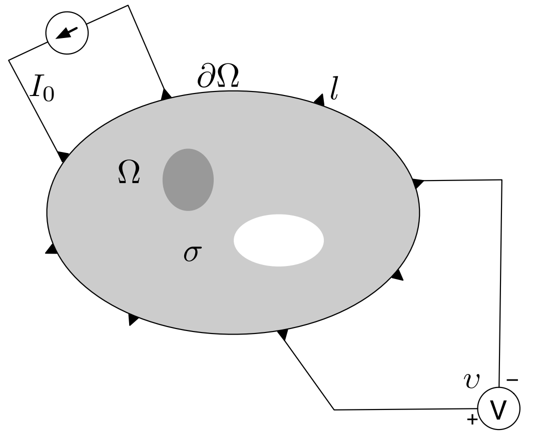

Figure 5.1: EIT measurement set-up. The domain is Ω, $\delta\Omega$ the boundary, l sensor label, I_0 the exciting current and v measured potential differences. Varying conductivity σ is indicated with colour changes.

with Maximum likelihood estimation. According to the evaluations, the effect of grain direction is detectable and hence, the wood grain angle can be measured.

5 Electrical Impedance Tomography

This section gives an introduction to electrical impedance tomography (EIT) and some aspects of modelling and image reconstruction. Sensitivity analysis, described in section 2, is exemplified for EIT at the end of the section.

Measurement of material parameters of an object with electrical surface measurements is called electrical impedance tomography. Other notions and related techniques are bio-impedance imaging, bio-impedance tomography, potential difference tomography and electrical resistance tomography [2, 9, 19, 22]. Figure 5.1 shows an EIT measurement set-up. The spatially varying conductivity σ of the domain Ω with boundary $\partial\Omega$ depends on the material parameters. Measurement sensors $l = 1, \ldots, L$ are attached to the boundary at $\partial\Omega_l \subset \partial\Omega$, where L denotes the total number of sensors. Currents I_0 are injected and the resulting voltages v are measured at the sensors giving an impedance profile of the domain. The measurements are repeated for different, times, frequencies, or configurations. Determination of the material parameters from the measurements is an ill-posed inverse problem [38].

There are many different applications of electrical impedance tomography [22, 78]. Medical applications are for example lung function investigation, detection of breast cancer and stroke assessment [2]. Figure 5.2 shows a system for human head imaging [82]. Within geological science, electric impedance tomography or electric resistivity measurements are for instance

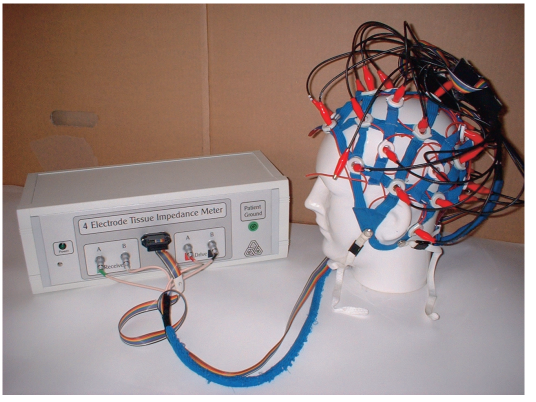

Figure 5.2: The UCLH Mk2 system for human head imaging with EIT. Richard Bayford, School of Health and Social sciences, Middlesex University, London, [82].

used for detection of tunnels [49]. Non-destructive testing on concrete constructions is another application [24, 33]. EIT is also used for measurements of interior properties of wood, see section 3.

The forward problem is formulated with Maxwell equations, harmonic time dependence and quasi-static approximation [22, 26, 72], giving the differential equation

$$\begin{cases} -\nabla \cdot (\sigma \nabla \phi) = 0 & \boldsymbol{r} \in \Omega \\ \sigma \nabla \phi \cdot \hat{\boldsymbol{n}} = g & \boldsymbol{r} \in \partial\Omega, \end{cases} \tag{5.1}$$

where ϕ denotes the resulting voltage potential and $g = \boldsymbol{J}_s \cdot (-\hat{\boldsymbol{n}})$ the Neumann boundary conditions corresponding to the current density at the boundary $\partial\Omega$. The inverse problem is to determine the conductivity from potential measurements on the surface [38]. The inverse problem is hard for several reasons: a local change in the conductivity affects all measurements, there is a finite number of measurements for a possibly infinite number of conductivity parameters, and only a part of the boundary is known. The ill-posed ness of the inverse problem decreases with a priori information [22].

Modelling of the electrodes is important to get the boundary conditions correct, especially within the medical applications [2, 23, 57, 76]. Two common electrode models are the shunt model and the complete electrode model [72, 76], where the complete electrode model is more accurate. Point electrode models can be used when the electrodes are small compared to the domain [20], as in geoscience or wood applications. Four-electrode measurements decrease the influence of faults in the sensor modelling [15, 22]. With four-electrode measurements voltages are never measured at the sensors where currents are excited.

Currents can be excited with different current configurations or excitation schemes. Different excitations are preferable for different applications. Trigonometric and pseudo polar excitation pattern seems to have better performance especially for brain EIT [1, 3, 11, 69, 81]. In the evaluation [39], an adjacent current configuration is less sensitive to modelling errors than a trigonometric configuration. Hence, when comparing configurations, the application and objectives need to be defined. Adjacent and polar current configurations and two noise models are evaluated with sensitivity analysis, for a two-dimensional circular domain and 16 measurement sensors in paper V, using Fisher information and singular value decomposition (SVD).

Estimating, or reconstructing, the conductivity for the electric impedance tomography problem results in an image. To find the image from measurement data the ill-posed problem needs regularization [22, 32, 38, 42, 77]. Singular value decomposition [73] and truncated SVD [21] are applied with a one-step-Newton algorithm for reconstructions in paper V and VI. The performance of the EIT system can be rated with distinguishability of conductivities [1, 25]. The trade-off between accuracy and resolution in an EIT problem is quantified with the Cramér-Rao bound in paper VI.

6 Inverse Problems

The theory of inverse problems is well established, within both mathematics and applied sciences. It has been a mutual development between applications needing solutions and theory establishment for solution strategies. In addition, general solutions are not available for inverse problems, hence knowledge about the application is essential in finding a solution. The more information that is available about the inverse problem, the better possibility to find a stable solution. An inverse problem is described by the opposite of the direct, or forward, problem. The direct problem is often more known and foremost it is well-posed, which correspond to the following properties [18, 38]:

1. There exists a solution of the problem (existence)
2. There is at most one solution of the problem (uniqueness)
3. The solution depends continuously on the data (stability)

Inverse problems are usually ill-posed, that is lacking at least one of the properties above [38]. From the third statement above, a small error in the data can cause a large error in the solution of an ill-posed problem and a regularization strategy is needed to find a stable solution. With regularization, a priori information is incorporated rendering possible a solution to the problem. The a priori information is specific for the application. Deterministic examples of regularization are Tikhonov and the pseudo-inverse based on singular value decomposition [38, 77]. Whereas, statistically based regularization methods are described in, for example [32], with probabilistic considerations of the measurement noise and the modelling errors.

Another type of regularization is modelling of the problem [62]. Then the problem is simplified through the determination of fewer parameters.

Estimating the wood grain angle in logs, see section 4, is an inverse problem. The direct problem would be to calculate the reflected fields when all wood properties and the incident fields are known. The inverse problem is here regularized with modelling of few parameters, determining only the grain angle on the surface, contrary to finding the whole distribution of grain angle and dielectric parameters in the tree.

Electrical impedance tomography (EIT), described in section 5, is another example of an inverse imaging problem. The corresponding direct problem is to calculate the potentials when the conductivity and exciting currents are known. The stability criterion above causes most problems for the inverse problem, where small errors in the measurement data can result in large errors in the conductivity. The instability is reduced with a priori information, such as if the conductivity have only small variations to some known background. In the papers, a truncated pseudo-inverse based on SVD is applied for the reconstructions. Then, the truncation is the regularization.

Also in EIT modelling of fewer parameters can be employed when less information is required for the detection of a phenomenon [42]. For instance, a method for finding the presence of decay in trees could be restricted to few parameters such as healthy and two degrees of decay.

7 Summaries of the Included Papers

The first four papers deal with estimation of grain angle in wooden logs. Three different aspects are considered: plane boards, plane incident waves with measurement sensors placed close to the cylindrical log, and a cylindrical model of twisted wood with the generating and measuring antenna away from the cylindrical log, taking three-dimensional effects into consideration.

Application of sensitivity analysis on electromagnetic impedance tomography measurements is considered in the last two papers. Reconstructions of the conductivity is presented and compared to singular value decomposition and the Cramér-Rao lower bound. Methods for Cramér-Rao lower bound and sensitivity analysis in inverse imaging are described.

Paper I - Microwave Modelling and Measurements for Early Detection of Spiral Grain in Wood

A microwave method is presented for determining the grain angle in boards. Cross polarization measurements of a reflected field does not require a priori information of moisture content, temperature or density. Experiments on room tempered and frozen wood verifies the theory.

A method for measurements of spiral grain in logs is outlined in the paper. The scattering of plane waves of an anisotropic cylinder, measured at points close to the cylinder is considered [52]. Moisture content and frequency dependence for the Cramér-Rao lower bound is calculated for the estimation of the grain angle for room tempered and frozen wood.

Paper II - Wood Grain Angle Estimation in Logs with Microwave Modelling

Electromagnetic properties of wood and scattering from cylindrical wood are described. The model take advantage of that the grain angle is small. A measurement strategy is proposed and the estimation of the grain angle is evaluated with the Cramér-Rao lower bound. The Cramér-Rao lower bounds are compared for the estimation of the grain angle and two complex dielectric parameters for different moisture content. It is concluded that estimating the grain angle is possible with the model assumptions.

Paper III - Estimation of Twist in Uniaxial Cylinders with Inverse Electromagnetic Scattering

In this paper, it is assumed that the transmitting and receiving antennas are collocated, requiring three-dimensional modelling for grain angle measurements. Microwave scattering from a uniaxial cylinder is modelled and asymptotic expressions developed. The cylinder is modelled with a surface impedance model. Error bounds of the estimated twist are calculated with the Cramér-Rao lower bound.

Paper IV - Numerical Verification of a Microwave Impedance Model for a Twisted Wooden Cylinder

To render possible fast calculations of twist in wooden cylinders, the cylinder is modelled with a normal surface impedance in paper III rather than a penetrable cylinder. The approximation is verified in this paper with two-dimensional comparisons and calculations of the accuracy. It is possible to find a frequency region for which the errors are in the same order as the measurement noise.

Paper V - Sensitivity Analysis for Measurement Configuration Evaluation in Electrical Impedance Tomography

Fisher information and sensitivity analysis are used to evaluate current configurations in electrical impedance tomography. Adjacent and polar current configurations are compared for two different noise models. Gradient methods, connected to Fisher information and singular value decomposition, are used for reconstructions. Numerical reconstructions of an object with conductivity deviating from the background agree with the sensitivity analysis.

Paper VI - On the Accuracy and Resolution in Inverse Imaging

A quantitative analysis is introduced based on Cramér-Rao bound for the optimal accuracy and resolution in inverse imaging. The Fisher information, singular value decomposition and maximum likelihood criterion is connected. The eigenspaces of the Fisher information is used to quantify the trade-off between the accuracy and the resolution. Cramér-Rao bound provides a lower bound for the error in the estimation of a parameter and it is complemented by deterministic upper bounds. An electrical impedance tomography problem is analysed with the proposed methods. Model errors are considered and numerical reconstructions calculated.

8 Conclusion

Sensitivity analysis has been introduced and applied on different material parameter estimation problems. The problems have been analysed from sensitivity aspects. For these problems conclusions were drawn of measurement possibilities. Furthermore, properties of the problems and the choice of configurations were evaluated. To this end, sensitivity analysis is motivated as a tool to enhance estimation of material parameters.

The Cramér-Rao lower bound (CRB) has been used to evaluate measurement possibilities for wood grain angle. The grain angle is a parameter of wooden logs and quantifies the angle between the wood grain direction and the direction of growth. A measurement of the grain angle of a log predicts occurrence of twist in sawn timber. Different techniques are available for measurement of the wood grain angle, here microwave measurements were proposed as a fast and robust measurement possibility. Estimation of the grain angle demands modelling of electromagnetic fields. The anisotropic properties of wood are imposed to find the grain angle. According to the analysis with the CRB, using available material data, the estimation is possible for varying temperature, moisture content, and frequencies.

Electromagnetic modelling for the estimation of the grain angle has been presented for three different cases: plane boards, near-field and far-field configurations. For plane boards, the model was verified experimentally. Near-field measurements were evaluated with CRB. With the present model assumptions, it is possible to estimate the grain angle. Estimating the grain angle is easier when the dielectric properties of wood are known, but possible also when the dielectric properties need to be estimated in the same measurement. Far-field measurements demand modelling of three-dimensional effects and a surface impedance model was applied to lower the complexity. The impedance model introduces model errors, but they are smaller than the measurement noise (at some dB signal-to-noise ratio). With requirements of a fast measurement an asymptotic version of the model was developed and evaluated. To evaluate the estimation feasibility, the CRB was calculated for some variable configuration parameters. According to the CRB the configuration variables can be chosen to render possible measurement of the grain angle. For estimating the wood grain angle, mathematical modelling of the electromagnetic field is essential.

Electric impedance tomography (EIT) have been analysed from a sensitivity view. A two-dimensional circular model with 16 sensors on the boundary of an object was presented and evaluated with Fisher information, CRB and singular value decomposition (SVD). Adjacent and polar measurement configurations were compared as well as two different noise models, corresponding to different possibilities in the measurement device. According to the reconstructions and the sensitivity analysis, the adjacent measurement configuration performs better than the polar for the present formulation. Noise modelling is a way to enhance reconstruction in EIT, in combination with other reconstruction or regularization methods.

Sensitivity analysis with Fisher information was proposed as a quantitative method for the evaluation of the trade-off between accuracy and resolution in inverse imaging. For this purpose, a connection was established between the Fisher information and the SVD based on maximum likelihood (ML) estimation. The proposed analysis was exemplified with calculation on an EIT problem. Reconstructions with the ML-based pseudo-inverse were presented and the choice of regularization parameter compared with the statistical CRB and the deterministic upper bounds. For a desired resolution, the CRB limits the achievable accuracy, which was also seen in the reconstructions.

To conclude, several aspects of sensitivity analysis have been discussed for material parameter estimation problems. Sensitivity analysis with Fisher information and CRB was presented as a tool for evaluation of measurement set-up, interpretation of accuracy and resolution, as well as gaining more knowledge about the problem. For estimation of the wood grain angle, the electromagnetic modelling was a central part in the estimation and the sensitivity analysis was applied to investigate estimation possibilities. Whereas for the EIT measurement, sensitivity analysis was exemplified through comparisons of current configurations and noise models, as well as quantifying accuracy and resolution.

Sensitivity analysis is argued as a tool in the design and regularization process. Similar analysis can be applied on other parameter estimation problems such as microwave tomography, antenna configuration, or transmission cables.

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