Reliability Assessment of Structural Concrete with Special Reference to Shear Resistance

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Declaration

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Abstract

Structural design standards based on the principles of structural reliability are gaining worldwide acceptance and are fast becoming the new basis of structural safety verification. The application of these principles to establish a standardised basis for structural design using partial factor limit states design procedures is done in the European Standard for the Basis of Structural Design EN 1990 from which it is adapted to the South African Basis of Design Standard for Building and Industrial Structures SANS 10160-1. The basis of design requirements stipulated in EN 1990 and SANS 10160-1 apply to all aspects of structural design: This includes reliability levels of structural performance and their differentiation and management; identification of various limit states and design situations; the specification of all the basic variables; separate treatment of actions and material-based resistance. However, application of these requirements is then primarily focused on actions whilst the provision for structural concrete is then left to the materials based design standards.

This two-part thesis describes a systematic assessment of the degree to which the application of the reliability framework presented in the basis of design requirements has been achieved in the present generation of structural concrete design standards. More importantly, attempts are made to identify ways in which the process can be advanced. Special attention is drawn to issues that are specific to South African conditions and practice in structural concrete.

Part One of the thesis focuses on the key elements of the reliability framework presented in EN 1990 and traces to what extent the requirements have been propelled through the design stipulations of the Eurocode Standard for Design of Concrete Structures EN 1992-1-1. The implications of the different reference level of reliability between the Eurocode default value of $\beta = 3.8$ and that characteristic of South African practice $\beta = 3.0$ through various issues are highlighted. The use and advantage of explicit treatment of reliability performance on reliability management related to some aspects of quality control are explored. A critical aspect is the shear prediction model providing unconservative estimates of shear resistance.

Part Two of the thesis focuses on characterising the model factor of the EN 1992-1-1 shear prediction model for members requiring design shear reinforcement. This is done by a comparison to a compiled experimental database with special focus on situations with high reinforcement ratios. The significance of the modelling uncertainty in shear prediction is

verified by this comparison. The use of the more conceptually rational modified compression field theory (MCFT) to improve on the quality of shear predictions is investigated and proves to yield more precise values with lower scatter hence making it a more reliable tool for predicting shear. The MCFT can then be used as reference for the reliability calibration and possible improvement for the Eurocode procedure.

Opsomming

Strukturele ontwerpstandaarde gebaseer op die beginsels van strukturele betroubaarheid verkry wêreldwye aanvaarding en word vinnig die nuwe basis van strukturele veiligheid bevestiging. Die toepassing van hierdie beginsels om ŉ gestandaardiseerde basis vir strukturele ontwerp is bevestig deur gebruik te maak van gedeeltelike-faktorbeperkende stadiums ontwerpprosedures in die Europese Standaard vir die Basis van Strukturele Ontwerp EN 1990 waarvandaan dit herbewerk is na die Suid-Afrikaanse Basis van Ontwerp Standaarde vir Bou en Industriële Strukture SANS 10160-1. Die basis van ontwerpvereistes bepaal in EN 1990 en SANS 10160-1 is van toepassing op alle aspekte van strukturele ontverp: Dit sluit inbetroubaarheidsvlakke van struktureele prestasie en hul diversifikasie en bestuur; identifikasie van verskeie beperkende state en ontwerpsituasies; die spesifikasie van al die basiese veranderlikes; afsonderlike behandeling van aksies en materiaal-gebaseerde weerstand. Desnieteenstaande, die toepassing van hierdie voorwaardes is dan hoofsaaklik gefokus op aksies terwyl die voorsiening van strukturele beton is dan gelaat op die materiaal-gebaseerde ontwerpstandaarde.

Hierdie tweeledige verhandeling beskryf ŉ stelselmatige beoordeling van die graad waartoe die toepassing van die betroubaarheidsraamwerk aangebied word in die basis van ontwerpvereistes bereik in die huidige generasie van strukturele beton-ontwerp standaarde is. Meer belangrik, pogings is aangewend om die maniere hoe die proses bevorder kan word te identifiseer. Spesiale aandag word gevestig op kwessies wat spesifiek op Suid-Afrikaanse toestande en praktyke in strukturele beton toepaslik is.

Deel Een van die verhandeling fokus op die sleutel-dele van die betroubaarheidsraamwerk aangebied in EN 1990 en skets die mate waartoe die vereistes aangespoor word deur die ontwerp voorskrifte van die *Eurocode Standard for the Design of Concrete Structures* EN 1992-1-1. Die implikasie van die verskillende verwysingsvlakke van betroubaarheid tussen die Eurocode standaardwaarde van $\beta = 3.8$ en die eienskap van Suid-Afrikaanse praktyk $\beta = 3.0$ deur verskillende kwessies word uitgelig. Die gebruik en voordeel van spesifieke behandeling van betroubaarheidsuitvoering op betroubaarheidsbestuur verwantskap met sekere aspekte van kwaliteit kontrole word ondersoek. ŉ Kritiese aspek is die model vir die voorspelling van skuif-weerstand wat die onkonserwatiewe beramings vir skuif-weerstand gee.

Deel Twee van die verhandeling fokus op karakterisering die modelfaktor van die EN 1992-1-1 skuif-weerstand voorspellings-model. Dit word gedoen deur ŉ vergelyking na ŉ saamgestelde eksperimentele databasis met spesifieke fokus op situasies met hoe herbevestigingsvergelykings. Die oorheersing van die modellering- onsekerheid in skuif-weerstand voorspelling is bevestig deur hierdie vergelyking. Die gebruik van ŉ meer konseptuele rasionele gemodifiseerde druk-veld teorie (bekend as MCFT) om die kwaliteit van skuif voorspelling te verbeter is ondersoek en verskaf ‘n meer presiese waarde met laer verspreiding wat lei tot ŉ meer betroubaree instrument om skuif mee te voorspel. Die MCFT word dan gebruik as verwysing vir die betroubaarheid-samestelling en moontlike verbetering van die Eurocode prosedures.

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List of Symbols

a	Shear span
a	Half-diameter of longitudinal reinforcement
a_d	Design value of geometrical data
A_{sw}	Area of the 2-legs of a stirrup or shear reinforcement
b_w	Beam width
C	Concrete cover
d	Effective depth of member
e	Diameter of stirrup or link
f_1	Principal tensile stress in cracked reinforced concrete
f_2	Principal compressive stress in cracked reinforced concrete
f_c	Compressive cylinder strength of concrete
f_{ck}	Characteristic compressive cylinder strength of concrete
f_{cr}	Principal tensile stress in reinforced concrete at cracking
f_{yw}	Yield strength of stirrups
h	Height or depth of a member
K_{FI}	Multiplication factor for actions, used as differentiation measure
m	Lever arm multiplier term
n_l	Number of layers of longitudinal reinforcement provided in design
p	Probability or fractile estimate required from a distribution

$P\,(\cdot)$	Probability argument or statement
P_f	Probability of failure
R	Function of resistance determined on the basis of design values of individual material properties and geometrical data
R_d	Function of design resistance determined as for R (above), but divided by γ_{Rd}
s	Spacing of stirrups
s_k	Skewness of a distribution
u'_N	Modified transformed variable of the three-parameter log-normal distribution
u_p	Standardised p-fractile of a distribution
$u_{norm,p}$	Fractile of a standardised random variable with a normal distribution
U	Standardised variable
$V_{Rd,max}$	Upper limit of shear resistance due to crushing of concrete compressive struts
$V_{Rd,s}$	Design ultimate shear resistance of member due, solely, to stirrup resistance
x_i^*	Checking point value of basic variable X at the i^{th} iteration
x_p	p −fractile estimate of the distribution
X_i	Material strengths and/or product properties
$X_{k,i}, X_k$	Characteristic values of material strengths and/or product properties
$X_{d,i}, X_d$	Design value of material strengths and/or product properties
z	Internal lever arm
α	Proportion of variability of concrete strength to that of resistance
α_{cc}	Coefficient accounting of scale effects of concrete

α_X	Direction cosine of sensitivity factor of specified variable X
β	Reliability index
β_T	Target level of reliability
$\beta_{T,R}$	Target level of reliability for resistance
γ_C	Partial safety factor for concrete
$\gamma_{m,i}$	Partial safety factor for material strength or product property
γ_{Rd}	Partial safety factor that covers model uncertainties of resistance and geometrical data variations
γ_S	Partial safety factor for steel
γ_{Sd}	Partial safety factor that caters for model uncertainties in actions and action effects
γ_f	Partial safety factor for actions catering for uncertainty in representative values of actions
γ_m	Partial safety factor for material or product properties catering for uncertainty in their representative values
γ_M	Operational partial factor for material property, assumed as taking some account of model and geometrical uncertainty
γ^*	Partial factor due to increased quality control of material strength
ε_1	Principal tensile strain in cracked reinforced concrete
ε_2	Principal compressive strain in cracked reinforced concrete
ε_x	Longitudinal strain in the web of a beam
ε_y	Transverse strain in the web of a beam
η	General representation of conversion factor taking scale effects into account
θ	Angle of inclination of concrete compressive struts relative to horizontal axis

μ_X	Mean value of the random variable X
ρ_l	Longitudinal reinforcement ratio
ρ_w	Web or shear reinforcement ratio
σ_X	Standard deviation of basic variable X
$\mu_X^N;\ \sigma_X^N$	Transformed normal mean and standard deviation, respectively, of variable X
φ	Probability density
Φ	Probability distribution
$\varphi(\boldsymbol{X})$	Probability density distribution of the vector of basic variables X
ω_l	Longitudinal tension reinforcement index
ω_t	Shear reinforcement index
Ω_f	Coefficient of variation of material strength
Ω_X	Coefficient of variation of random variable or material property X
Ω_X^*	Improved (reduced) coefficient of variation of basic variable X due to quality management measures

PART I: ASSESSMENT OF RELIABILITY IMPLEMENTATION IN THE STRUCTURAL EUROCODES

Chapter 1

INTRODUCTION TO THE RELIABILITY BASIS OF STRUCTURAL DESIGN

1.1 INTRODUCTION

Society expects that the occupants and users of buildings and structures and persons in their vicinity are safe. Further, society anticipates that the failure and adverse performance (extensive cracking, excessive deflections or vibrations) of buildings and structures is extremely rare. This then poses the responsibility to design and construction professionals to produce structures that are safe and durable. In addition, robustness, which has to do with unidentified structural hazards and the use of integral structural design practice to avoid damage disproportionate to the cause, has become a fundamental basis of design requirement. To be able to achieve safe and durable structures, design and construction professionals require a system of verifying adequate structural performance of buildings and structures by applying rational and safe procedures through the different stages of a project: planning, design, analysis, detailing, construction, and maintenance of structures.

1.2 HISTORY AND EVOLUTION OF BASIS OF DESIGN FORMATS

In general, problems of structural design must be resolved in the face of various uncertainties. Uncertainties arise not only in the assessment of actions which the structure has to sustain, and from the occasional lack of control during the production processes of the materials and components required, but also from incomplete knowledge about the mechanical formulations describing the response of the structure and its capacity to sustain those actions. Structural reliability techniques, compared to other basis of design formats, are aimed at rationally quantifying and assessing the effects of uncertainties associated with all aspects of structural design. The uncertainties in the design and construction process are represented by way of mathematical statistics and the assessment of structural performance is conducted through probabilistic concepts and analyses. Such treatment of uncertainties gives a rational scientific (decision tool) approach to the calibration of structural design provisions. A brief background and evolution of bases of design formats is given below.

Past generations of design codes were composed of design philosophies that lacked rational basis. This could result in designs that were highly uneconomical due to treating uncertainties in design to arise principally from the resistance part of mechanical formulations. In reality, uncertainty is however inherent not only from both the action and resistance part of mechanical formulations, but can arise from aspects such as modelling uncertainty and geometrical imperfections. The oldest of the commendable design philosophies, the Permissible Stress Method, was based on elastic material response principles and treated uncertainties to arise solely from the resistance side. Aside from not being able to represent the semi-plastic post peak behaviour of concrete, the Permissible Stress Method only applied a single factor of safety to the resisting stress. The stress due to the actions was based on an assessment composed of characteristic and nominal action and dimension values respectively. Since this method was based on a stress analysis it can only be used as a safety verification technique at local points in a structure and cannot be used to verify the safety of cross sections and members. The subsequent design philosophy, the Total Factor of Safety Method, was very similar to its predecessor save for the difference that it was aimed at safety verification of cross sections and hence loses ability to directly account for the variability of materials. To achieve this, a factor of safety was applied to resisting forces that are determined using the material properties and nominal dimensions of the structure whose material will provide resistance to applied actions. However, both these methods were not fully representative of the problem in nature by not treating the uncertainties from the action side in any rational manner. As a result, there was only a single measure of reliability reflected by the factor of safety applied to the resistance side in both instances.

The later introduced, and currently widely internationally adopted, design philosophy is the Partial Factor Limit States Design Method. This method applies partial factors to increase action values and reduce material property values to generate their design values for use in a limit state assessment. Hence, the introduction of this method achieved significant improvement in the economy of structures. The governing condition of a limit state assessment is that the action effects should be less than the available resistance. In this method, dimensions are generally implemented at nominal values, but in some cases (second-order effects, geometrical imperfections, buckling) can assume design values by applying some tolerance limit. This method can account for the variability of materials by applying partial safety factors to the material properties. Further, it can also be used for safety verification of cross-sections and members since the action effects and resistance force of cross-sections are calculated for use in the limit state verification. Until recently, partial safety factors used in limit state design verifications were derived mainly by expert judgement and by reference to sound traditional designs, thereby lacking the appropriate rational and scientific treatment they require. Structural reliability techniques arise as an attempt or method to represent variability and performance of physical models of structural systems by taking account of the distributions of the basic variables in mechanical formulations used for limit state verifications. Basic variables are the most fundamental quantities the designer has to consider in mechanical formulations. Structural reliability techniques are consistent with the Partial Factor Limit States Design format in the sense that partial factors can be derived from reliability analyses and calibration exercises and then applied in limit state verifications. The application of structural reliability as the theoretical basis for limit states design ensures that improved economic performance is achieved together with improved safety performance across a wide range of practical design situations.

1.3 CURRENT STATE OF STRUCTURAL RELIABILITY IN DESIGN CODES

The probabilistic basis of structural design is developed extensively and continuously updated in the Joint Committee on Structural Safety (JCSS) Probabilistic Model Code (JCSS, 2001). The current collaborators whose efforts support the JCSS are renowned structural engineering bodies; International Association of Bridge and Structural Engineering (IABSE), Conseil International du Bâtiment (CIB), Fédération International du Béton (*fib*), European Convention for Constructional Steelwork (ECCS), Réunion Internationale des Laboratories et Experts des Matériaux (RILEM). Each of these constitutive bodies of the JCSS specialises in an aspect that is key to structural engineering thus the model code contains a wealth of comprehensive information and proposals on various issues in structural design across different materials that require reliability treatment. The systematically presented structural reliability principles in the JCSS model code are laid down in a standardised manner in the international standard General Principles on Reliability for Structures ISO 2394 (ISO, 1998).

The application of these principles to establish a standardised basis for structural design using partial factor limit states design procedures is done in the European Standard for the Basis of Structural Design EN 1990 (EN 1990, 2002) from which it is adapted to the South African Basis of Design Standard for Building and Industrial Structures SANS 10160-1 (SANS 10160-1, 2010).

The basis of design requirements stipulated in EN 1990 and SANS 10160-1 apply to all aspects of structural design: This includes reliability levels of structural performance and their differentiation and management; identification of various limit states and design situations; the specification of all the basic variables; separate treatment of actions and material-based resistance. In the process of converting the principles of structural reliability into deterministic design rules through a method of calibration, the emphasis is placed to a large extent on actions whilst the provision for structural concrete is then left to the materials based design standards. Although a reliability framework is developed also for structural resistance in the form of partial factors, model and resistance factors for respective classes of failure modes, systematic calibration of the materials standards is limited (Holický, Retief & Dunaiski, 2007).

Parametric studies of representative cases of structural resistance are used to derive guidelines for structural resistance performance that could be used as basis-of-structural-design requirements to be considered in the formulation of the materials standards (Holický et al., 2007). Ideally, all aspects of the basis of design requirements should be considered. Such explicit reliability guidelines for the specification of structural resistance should improve the unification between the standards for the basis of structural design, actions and structural materials.

fib Model Codes (initially established as the CEB-FIP Model Codes)

The Model Codes; the first of which was published in 1978, the second in 1990 and the notable 2010 Draft Model Code which was published mid-2010 for review and is still yet to be passed by the general assembly of the *fib*; are advanced state of the art technical documents aimed at synthesising research findings and technical information with a view of translating them into practice. By virtue of their international character, the model codes are more general than most national codes and provide more detail to aid in the process of national code development and drafting which should take into account national conditions and requirements for structural design (CEB-FIP, 1993). The Model Codes have served as fundamental reference and background documents during the entire building process of the structural Eurocodes, since their inception in the mid-seventies to on-going Eurocode writing activities at current, and will surely still be used in future.

The aim of the Model Codes, and of the *fib* in general, is not to repeat the content in subsequent revisions, but to rather collect, record, review and disseminate current state of the art research and technical information to encourage their use in practice. Each subsequent revision of the model code may be used as basis for national code development activities of its era, where such activity is normally centred on the more mature concepts and well understood state of knowledge which has been verified by relevant experience. On the other hand, some of the model codes' innovative contents may, after further calibration, need additional elaboration and this activity continues within the association, with the outcome being appropriately published in a series of technical publications referred to as Bulletins.

Given the importance of the Model Code presented above, its view and treatment of structural reliability is key for consideration by national codes. Volume 1 of 2 of the 1978 Model Code, which was compiled by the JCSS, achieved the goal of unifying the basic data for design rules for different types of construction and civil engineering materials by use of structural reliability techniques (CEB-FIP, 1978). Volume I, after introducing the general reliability framework, then only proceeded to give detail on the implementation for actions but with no specific guidance for resistance. The 1990 Model Code showed no extensions of the reliability framework in its provisions, giving reference to the 1978 Model Code for guidance on reliability with its focus delving more towards the establishment of more general and advanced material characteristics and design models. The 2010 Draft Model Code has shown significant improvements from its predecessor in terms of implementation of the reliability framework, with visible trends of reliability aspects considered through different methods and cases of the entire design process. A review of the state of reliability in the 2010 Draft Model Code is given in Chapter 2.

1.4 RESEARCH MOTIVATION AND SIGNIFICANCE

The European Commission's initiative to harmonise technical barriers between EU member states to allow exchange of information and intensify trade relations has caused each member states' national structural design standards to be replaced by a unified set of Eurocodes. In this transformation, The British Standard The Structural Use of Concrete BS 8110-1 (BS 8110-1, 1997) on which the currently operational South African standard The Structural Use of Concrete SABS 0100-1 (SABS 0100-1, 2000) is based, is being withdrawn and replaced by a new operational Eurocode Standard for Design of Concrete Structures EN 1992-1-1 (EN 1992-1-1, 2004). For the on-going revision of South Africa's standard for the design of concrete structures, which will be newly referred to as SANS 10100-1, the South African Concrete Code Committee has chosen to adopt EN 1992-1-1 as reference.

The Eurocodes can be viewed as a general set of reference standards which need to be made operational as national standards through the selection of Nationally Determined Parameters in National Annexes. A key parameter for which national choice is allowed and has grave effect on matters concerning reliability is the selection of the target level of reliability, of which the Eurocode recommends a value of $\beta = 3.8$ and South Africa uses a value of $\beta = 3.0$. Retief and Dunaiski (2009) propose that the reliability assessment of a future South African concrete standard could therefore consist firstly of reviewing the degree to which EN 1992 complies with and applies reliability principles as set out in EN 1990; and secondly to calibrate it in accordance with SANS 10160-1 requirements, including required levels and classes of reliability for the restricted scope of building structures.

Using EN 1990 and EN 1992-1-1 and their relation as base, this two-part thesis describes a systematic assessment of the degree to which the application of the reliability framework presented in the basis of design requirements has been achieved in the present generation of structural concrete design standards. More importantly, attempts are made to identify ways in which the process can be advanced. Special attention is drawn to issues that are specific to South African conditions and practice in structural concrete.

1.5 OBJECTIVES OF THE STUDY

The general objective of the study is to systematically study and trace the extent that EN 1990 and EN 1992-1-1 are harmonised in terms of the reliability based framework. The reliability framework and its requirements are identified in EN 1990 and its implementation is traced by studying the provisions for structural concrete resistance in EN 1992-1-1. During this process, it is imperative that an assessment is made of the implications for South Africa where national choice is allowed. Further, where abstraction or incompleteness in the implementation of the reliability framework is identified in structural concrete provisions, improvements or suggested actions to harmonise design practice are recommended. Therefore, the specific objectives of the thesis can be outlined as:

1. To map out and study the reliability framework and requirements as presented in EN 1990.

2. To trace the extent to which the reliability framework is implemented in deriving the EN 1992-1-1 design provisions by use of relevant references and background documents.

3. To extend the reliability framework where abstraction or incompleteness is found in provisions for structural concrete resistance. Such efforts are made concerning some quality aspect of reliability management. Requirements of the framework are also exercised through extensive assessment of the model factor and reliability performance of provisions for members requiring design shear reinforcement.

1.6 STRUCTURE OF THE THESIS

The thesis is broken down into two parts. Part One of the thesis focuses on the key elements of the reliability framework presented in EN 1990 and traces to what extent the requirements have been propelled through the design stipulations of the Eurocode Standard for Design of Concrete Structures EN 1992-1-1. Chapter 2 presents the extensive basis of design requirements for all structural materials as set out in EN 1990, with a brief insight into reliability implementation and recommendations of the Draft *fib* 2010 Model Code for concrete structures. Thereafter, an assessment of the reliability-based provisions in EN 1992-1-1 is presented where their adequacy and completeness are critically reviewed. Chapter 3 focuses on resolving inconsistent issues of reliability-base that are identified in the provisions for structural concrete. A general summary to Part One is given in Chapter 4.

Part Two of the thesis focuses on characterising the model factor and reliability performance of the EN 1992-1-1 shear prediction model for members requiring design shear reinforcement. Chapter 5 gives all the necessary background and literature on the shear prediction model, whereas Chapter 6 gives background on the techniques required for statistical and reliability analyses of the shear prediction procedures. Chapter 7 is dedicated to the determination and characterisation of the model factor for the Eurocode shear prediction model for members requiring design shear reinforcement. This chapter culminates at the important step of selecting an appropriate distribution of the model factor to be used in the reliability analysis procedures that form the base of Chapter 8. Chapter 8 is concerned with the reliability analysis of the EN 1992 shear prediction model for members requiring stirrups.

1.7 NOTE ON REFERENCE TO STANDARDS

Repeated reference is made throughout the thesis to EN 1990 and EN 1992-1-1. For ease of reference and for convenience, EC 0 is adopted to refer to EN 1990 and EC 2 is adopted to refer to EN 1992-1-1.

Chapter 2

OVERVIEW OF THE RELIABILITY FRAMEWORK FOR STRUCTURAL RESISTANCE

2.1 INTRODUCTION

This Chapter takes a systematic view at the link between the internationally accepted basis of design requirements and how they are implemented in current provisions for structural concrete design that rely on these concepts. For this assessment, the case of the structural Eurocodes is studied. Thus, EC 0 is used as reference for the basis of design requirements and is further used to map out the reliability framework for structural performance. EC 2 is used as reference to determine to what end the principles of structural reliability, in accordance with the framework from EC 0, have been applied in developing the provisions for structural concrete resistance.

EC 0 describes the Principles and requirements for safety, serviceability and durability of structures. The requirements apply to all aspects of structural design: This includes:

1. Reliability levels of structural performance and their differentiation and management,
2. Identification of the various limit states and design situations,
3. The specification of all the basic variables, and
4. Separate treatment of actions and materials-based resistance.

All Eurocodes rely on EC 0 for the basis of structural design. The provisions of EC 0 are therefore applicable to all materials and as such only the requirements that are independent of material properties are dealt with therein. The design philosophy for safety verification is based on the limit state concept used in conjunction with the partial factor method. In the case of partial factor limit states design, performance requirements are expressed in terms of design situations to be considered, limit state principles and associated reliability levels. A significant part of the basis-of-design procedures stipulates action combination schemes for the various design situations and limit states. Provision for structural resistance is then left to materials-based design standards; with guidance given for the implementation of the principles of structural reliability in the standardisation process provided in annexes and referenced material.

Parametric studies of representative cases of structural resistance are used to derive guidelines for structural resistance performance that could be used as basis-of-structural-design requirements to be considered in the formulation of materials standards (Holický et al., 2007). Holický et al. (2007) suggest that in the calibration of materials standards in accordance with the reliability framework, parameters to be taken into account include the specification of characteristic material properties, partial material, model and resistance factors, and the nature of failure consequences for the various failure modes. However, the general procedures for concrete design entail the way in which provision is made for sufficient reliability of structural resistance. This is typically done through schemes of partial factors and specification of characteristic values for basic variables applicable to design procedures.

2.2 STRUCTURAL RESISTANCE PERFORMANCE (Based on EC 0)

2.2.1 Basis of design requirements

Four basic fundamental basis of design requirements can be summarised from Section 2 of EC 0 (Narayanan & Beeby, 2005). The structure should be designed and executed in such a way that it will:

1. During its intended life, with appropriate degrees of reliability and in an economical way:
    a. Sustain all actions and influences that are likely to occur during execution and use
    b. Remain fit for the intended use

2. Have adequate mechanical resistance, serviceability and durability
3. In the event of fire, have adequate resistance for the required period of fire exposure

4. Not be damaged by accidents (e.g. explosion, impact and consequences of human error) to an extent disproportionate to the original cause.

Section 2.1(6) of EC 0 states that the basic requirements should be met by the choice of suitable materials, by appropriate design and detailing and by specifying control procedures for design, production, execution and use relevant to the particular project. All these factors affect structural resistance and should be considered when specifying the provisions for any materials standard.

2.2.2 Reliability management

Section 2.2 of EC 0 deals with the important issue of reliability management. Reliability management is concerned with factors that affect the reliability performance of structures. Aspects of reliability management form the basis of schemes for reliability differentiation of structures. Reliability management, in simple terms, is concerned with the attainment of appropriate reliability levels for structures and the control of construction works attaining different reliability levels. The different reliability levels are linked to different reliability classes that are differentiated or adjusted in accordance with stipulated risk principles. Quality management is stated, and exercised in informative Annex B in EC 0, as an essential component of reliability management. EC 0 specifies that the choice of the levels of reliability for a particular structure should take account of the relevant factors, including:

1. The possible cause and/or mode of attaining a limit state,
2. The possible consequences of failure in terms of risk to life, injury, potential economic losses,
3. Public aversion to failure, and
4. The expense and procedures necessary to reduce the risk of failure.

Annex B, which is concerned with the management of structural reliability for construction works with regard to ultimate limit states excluding fatigue, is an informative annex that provides additional guidance on reliability management. This annex is also applicable to appropriate clauses in the actions and materials standards, EN 1991 to EN 1999, and is key when considering provisions in the materials standards, in this case EC 2, when reviewing its provisions related to aspects of reliability differentiation. In this annex, a framework is formulated to allow different levels of reliability to be adopted either between different constructions or by classification/differentiation of the components in a single structure. Two procedures are considered effective in controlling attained reliability levels:

1. One allowing for moderate differentiation in the partial factors for actions and resistances,
2. The other allowing for differentiation between various types of construction works in the requirements for quality levels of the design and execution process.

Reliability differentiation is an important issue concerning structural performance and in formulating the provisions for materials standards and is therefore treated separately in Section 2.2.3 below.

2.2.3 Reliability differentiation

EC 0 defines reliability differentiation as measures intended for the socio-economic optimisation of the resources to be used to build construction works, taking into account all the expected consequences of failures and the cost of the construction works. Annex B, which gives the framework for reliability differentiation of structures as introduced in Section 2.2.2 above, is explored in an attempt to assess the requirements that set or affect the reliability performance of structures.

Consequence classes

As a first step into the differentiation process, consequence classes that consider the consequences of failure and malfunction of the structure are established. Consequence classes (CCs) are distinguished based on the importance, in terms of consequences of failure, of the structure or structural member concerned. Table 2.1 gives a description of the criteria associated with each Consequence class and examples of civil engineering works where each class may be applicable.

Table 2.1. Definition of consequences classes (EN 1990, 2002)

Consequence Class	Description	Examples of buildings and civil engineering works
CC3	**High** consequence for loss of human life, or economic, social or environmental consequences **very great**	Grandstands, public buildings where consequences of failure are high (e.g. a concert hall)
CC2	**Medium** consequence for loss of human life, economic, social or environmental consequences **considerable**	Residential and office buildings, public buildings where consequences of failure are medium (e.g. an office building)
CC1	**Low** consequence for loss of human life, and economic, social or environmental consequences **small or negligible**	Agricultural buildings where people do not normally enter (e.g. storage buildings), greenhouses

Differentiation by β values

Reliability classification, and therefore differentiation as well, can be represented by β indexes or reliability indices. Three reliability classes RC1, RC2 and RC3 may be associated with the three consequence classes CC1, CC2 and CC3. This implies that specification of a consequence class is directly linked to a specific reliability class and vice versa is true. Table 2.2 shows the recommended minimum values for the reliability index β for the ultimate limit states with associated consequence classes.

Table 2.2. Recommended minimum values for reliability index β (ultimate limit states)

Reliability Class	**Consequence class**	**Minimum values for β**	
		1 year reference period	**50 years reference period**
RC3	CC3	5.2	4.3
RC2	CC2	4.7	3.8
RC1	CC1	4.2	3.3

Differentiation by measures relating to partial factors

Reliability differentiation can be achieved by distinguishing classes of partial factor for actions, γ_F, to be used in fundamental combinations for persistent design situations. A multiplication factor, K_{FI}, may be applied to the partial factor for actions. Recommendation of the value of K_{FI} to be used for each reliability class are given in Table 2.3. Load combinations and design situations where K_{FI} is applicable to persistent design situations for limit state verifications are discussed and considered in Section 2.2.4.

Table 2.3. K_{FI} factor for actions (EN 1990, 2002)

K_{FI} **factor for actions**	**Reliability class**		
	RC1	RC2	RC3
K_{FI}	0.9	1	1.1

EC 0 states that the multiplication factor, K_{FI}, may be used as a differentiation measure for the same design supervision and inspection levels i.e. design and supervision levels not differentiated but maintained. However, and in contrast, it is stated that accompanying measures such as the level of quality control for the design and execution of the structure, may be associated to classes of the multiplication factor. A three level system for control during design and execution is adopted and is expressed by the specification of design supervision levels and inspection levels associated with reliability classes. The quality management and control measures in design, detailing and execution which are given in Annex B aim to eliminate failures due to gross errors, and ensure resistances assumed in the design. Though not normally used, it is stated that reliability differentiation may also be applied through the partial factors on resistance, γ_M. Reference is made to Annex B6 in EC 0 for further guidance on this issue and is dealt with in the Section *Partial factors for resistance properties* below.

Design supervision differentiation

Design supervision differentiation consists of various organisational quality control measures which can be used together. Three possible design supervision levels can be prescribed. Design supervision levels (DSLs) are linked to reliability classes and are chosen according to the importance of the structure and in accordance with National requirements or the design brief, and implemented through appropriate quality management measures. Table 2.4 presents the different design supervision levels with associated characteristics for each level and the minimum recommended requirements for checking of calculations, drawings and specifications.

Table 2.4. Design supervision levels (DSL) (EN 1990, 2002)

Design Supervision Levels	**Characteristics**	**Minimum recommended requirements for checking of calculations, drawings and specifications**
DSL3 relating to RC3	Extended supervision	Third party checking: Checking performed by an organisation different from that which has prepared the design
DSL2 relating to RC2	Normal supervision	Checking by different persons than those originally responsible and in accordance with the procedure of the organisation.
DSL1 relating to RC1	Normal supervision	Self-checking: Checking performed by the person who has performed the design

Inspection during execution

As with design supervision levels, three inspection levels (ILs) are defined. The characteristics and requirements, together with associated reliability index classification, for inspection levels are given in Table 2.5. In Annex B it is suggested that the inspection levels may be linked to the quality management classes selected and implemented through appropriate quality management measures.

Table 2.5. Inspection levels (IL) (EN 1990, 2002)

Inspection Levels	**Characteristics**	**Requirements**
IL3 relating to RC3	Extended inspection	Third party inspection
IL2 relating to RC2	Normal inspection	Inspection in accordance with the procedures of the organisation
IL1 relating to RC1	Normal inspection	Self inspection

Partial factors for resistance properties

Reduction in the value of partial factors for material, product property or a member resistance is warranted if an inspection class higher than that of standard order required is exercised. For example, partial factor modification may be warranted if a structure is designed to be of RC2 by reliability index classification but IL3 measures are exercised during execution. It is noted in Annex B that such a reduction, which allows for example for model uncertainties and dimensional variation, is not a reliability differentiation measure but merely a compensation measure exercised in order to keep the reliability level dependent on the efficiency of the control measures. Partial factor modification is allowed in an informative annex in EC 2 related to aspects of quality control but the provisions of Annex B in EC 0 are not fully exercised. This issue is developed and extensively treated in Chapter 3 of the thesis.

2.2.4 Limit state design

In this Section the principles of limit state design, basic variables and verification by the partial factor method are studied in terms of their implications for structural resistance and reliability performance. These three issues are dealt with in Sections 3, 4, and 6 of EC 0 respectively. Annex C, an informative annex addressing the basis for partial factor design and reliability analysis in the Eurocodes, provides information and theoretical background to the partial factor method described in section 6 of EC 0, and is thus applicable to all the structural Eurocodes i.e. EN 1991 to EN 1999.

Principles of limit state design

The Eurocodes adopt the partial factor method, or limit states semi-probabilistic method, as the method for verification of structural safety (European Concrete Platform, 2008). A limit state can be defined as a condition beyond which the structure no longer fills the relevant design criteria. Two categories are defined by the consequences associated with the attainment of a limit state: ultimate limit state and serviceability limit state. Ultimate limit states are associated with loss of equilibrium of the whole structure, or failure or excessive deformation of a structural member and they generally concern the safety of people. Table 2.6 shows the ultimate limit state classification according to EC 0.

Serviceability limit states correspond to conditions beyond which specified service requirements for a structure or structural member are no longer met. Exceeding these limits causes limited damage but means that the structures do not meet design requirements: functional requirements (not only of the structure, but also of machines and services), comfort of users, appearance, damage to finishes and non-structural members (European Concrete Platform, 2008). EC 0 identifies three different types of combinations for serviceability limit states verifications:

1. Characteristic combination applicable to the more severe irreversible limit states,
2. Frequent combination applicable to reversible limit states, and
3. Quasi-permanent combination applicable to reversible limit states.

Table 2.6. Ultimate limit state classification

Notation	Definition
EQU	Loss of static equilibrium of the structure or any part of it considered as a rigid body, where: - minor variations in the value or the spatial distribution of actions from a single source are significant (e.g. self-weight variations) - the strengths of construction materials or ground are generally not governing
STR	Internal failure or excessive deformation of the structure or structural members, including footings, piles, basement walls, etc., where the strength of construction materials of the structure governs.
GEO	Failure or excessive deformation of the ground where the strengths of soil or rock are significant in providing resistance
FAT	Fatigue failure of the structure or structural members

Basic variables are to be used in structural and load models that are implemented in limit state verifications at their design values. Design values of basic variables are achieved by the use of partial safety factors applied to representative values of the basic variables adopted in design calculations. The basic variables relevant to resistance are material or product properties and geometrical data. Limit state verifications are required to be carried out for all relevant design situations and load cases. Design situations refer to sets of physical conditions representing the real conditions occurring during a certain time interval for which the design will demonstrate that relevant limit states are not exceeded. In common cases, deign situations are classified as:

1. Persistent design situations, referring to conditions of normal use,
2. Transient situations, referring to temporary conditions of the structure e.g. during construction or repair,
3. Accidental situations, involving exceptional conditions of the structure or its exposure, including fire, explosion, impact etc., and
4. Seismic situations, where the structure is subjected to a seismic event.

Each design situation is characterised by the presence of several actions on the structure. As such, much focus in EC 0 is directed towards the treatment of combinations of actions and relevant partial and combination of actions for every design situation relevant to each limit state. Stipulation of the resistance basis of design requirements are presented at a high level of abstraction, providing for the diverse characteristics of structural material, ranging from structural steel, through concrete, to geotechnical design.

Specification of basic variables and verification by the partial factor method

The European Concrete Platform (2008) presents a concise Table, reproduced as Table 2.7 below, which shows the steps to pass from the characteristic values of individual material strengths or product resistances to the design values of structural resistance. The provisions given in Table 2.7 summarise the requirements from Section 4 and 6 of EC 0 that deal with basic variables and verification by the partial factor method. Several material properties are involved in structural design. The main one is strength and it is represented through a characteristic value. For structural stiffness parameters such as moduli of elasticity, creep coefficient, thermal expansion coefficient etc, the characteristic value is taken as the mean value. EC 0 defines the characteristic value of a property of a material as the 5 % fractile of its statistical distribution where a minimum value of the property is the minimal failure limit and as the 95 % fractile where a maximum value is the limiting value. The minimal failure limit is usually applicable to material or product properties in problems of structural design. Design values of the basic variables are not introduced directly into the design models used for limit state verifications. Characteristic or some other suitable representative value of the basic variable is rather used in combination with the appropriate partial factor to describe the design value of the basic variable in the limit state model.

Table 2.7. Procedure to determine the design values of resistances starting from the characteristic values of strength (European Concrete Platform, 2008)

Expression	Comment
X_i	Material strengths and product resistances involved in the verifications are identified
$X_{k,i}$	Characteristic values of material strengths and product resistances are introduced
$X_{d,i} = \eta\ X_{k,i} / \gamma_{m,i}$	The design value of a material property is determined on the basis of its characteristic value, through the two following operations: a) divided by a partial factor γ_m, to take into account unfavourable uncertainties on the characteristic of this property, as well as any local defaults; b) multiply, if applicable, by a conversion factor η mainly aimed at taking into acount scale effects.
$R(\eta X_{k,i} / \gamma_{m,i}\ ;\ a_d)$	Determine the structural resistance on the basis of design values of individual material properties and of geometrical data
$R_d = 1 / \gamma_{Rd}\ R(\eta X_{k,i} / \gamma_{m,i}\ ;\ a_d)$	Following a procedure similar to the one for calculating the design value of action effects, the design value of structural resistance is determined on the basis of the individual material properties and of geometrical data multiplied by a aprtial factor γ_{Rd} that covers the model uncertainties of resistance and the geometrical data variations, if these are not explicitly taken into account in the model.
$R_d = R(\eta X_{k,i} / \gamma_{M,i}\ ;\ a_d)$	As for the action effects, factor γ_{Rd} is often integrated in the global safety factor $\gamma_{M,i}$, by which the characteristic material strength is divided: $\gamma_{M,i} = f(\gamma_{Rd}, \gamma_{m,i})$

The provision for model uncertainties as part of resistance safety factors is an important development in terms of implications for materials standards that will be based on this concept. In an attempt to explore this requirement, Holický et al. (2007) consider cases in research where the model factors are determined statistically from data sets describing flexural and shear behaviour of reinforced concrete elements. It is shown from the results that different model factors are applicable to the different types of failure modes considered, namely flexure and shear. This difference in model uncertainty influences the value of partial factors that are applicable for flexure and shear limit state verifications. Model uncertainties for other failure modes should be studied and quantified for derivation of adequate safety elements in this regard. Figure 2.1, reproduced from Annex C in EC 0, gives a schematic representation of the factors necessary in calibrating partial factors for action effects, γ_F, and for resistance, γ_M.

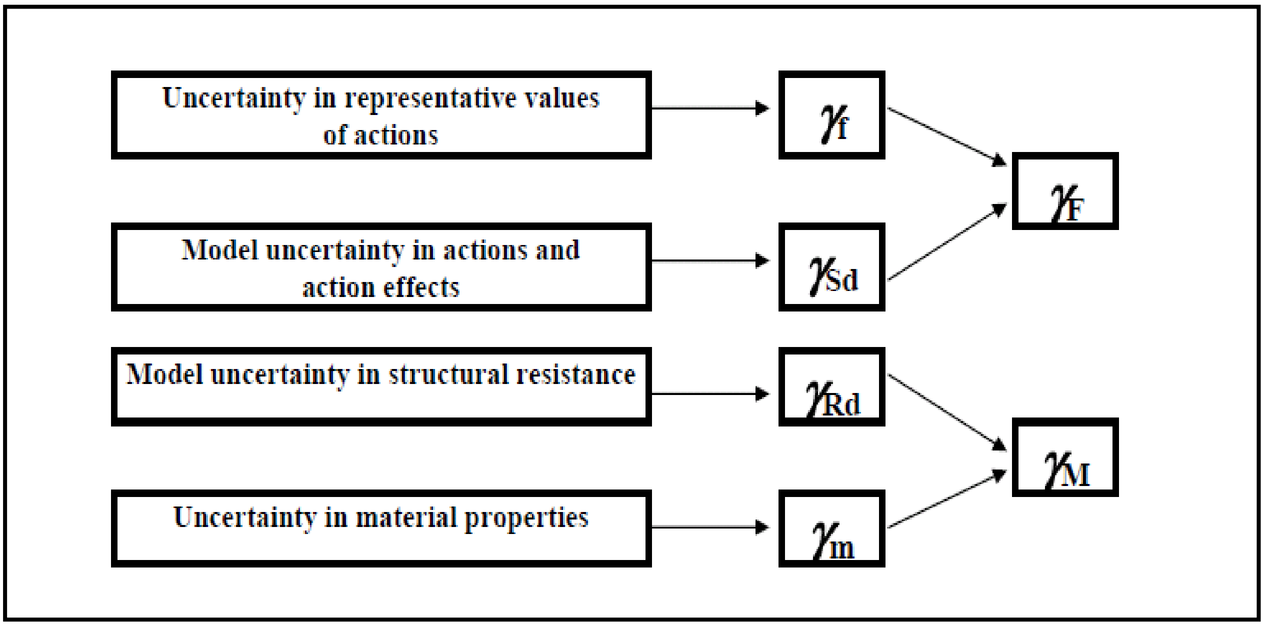


Figure 2.1. Relation between individual partial factors (EN 1990, 2002)

Partial factors for different modes of failure should be derived from representative parametric studies. The First order reliability method (FORM), which is elaborated on and applied later in the thesis for shear resistance, is prescribed for use in assessing the design values of resistance. This method allows reliability analyses and calibration to be separated for actions and resistances. Partial factors are determined as the ratio between characteristic values of resistance (determined by use of characteristic values of basic variables) and design values of resistance (determined from FORM analyses). The specified reliability index, β, is a key parameter in the FORM procedure and therefore affects final values of partial factors derived from the process. Partial factors can be considered as direct measures of reliability that are tied to limit state verifications for different failure modes. The reliability index for members and structures is left open for national choice and selecting a different target value implies that relevant parametric studies should be conducted to derive values for partial factors for resistance as well as action effects. The target values of reliability indexes as recommended by the Eurocodes that are applicable to each of the limit states for specified period is given in Table 2.8. The preceding description of partial factor determination is applicable to the ultimate limit states or failure modes.

Table 2.8. Target reliability index β for Class RC2 structural members

Limit state	**Target reliability levels**	
	1 year	50 years
Ultimate	4.7	3.8
Fatigue		1.5 to 3.8
Serviceability (irreversible)	2.9	1.5

For serviceability limit states the partial factors for the properties of materials, γ_M, should be taken as unity (1.0) except if differently specified in EN 1992 to EN 1999. This implies that for serviceability limit state verifications, resistance is determined at its characteristic value.

2.2.5 Summary of the basis of design stipulations

A summary of the extensive basis of design requirements presented in preceding Sections are summarised in a concise manner in Table 2.9 below, which was also used in a paper by Mensah, Retief and Barnardo (2010).

Although EC 0 allows a simple adjustment of actions through an importance factor K_{FI} the preferred way to adjust for reliability classes is through quality control measures. Target reliability levels as expressed in terms of the reliability index (β_T) are given in informative annexes, firstly for ultimate limit state class RC2 then for differentiated classes RC1 and RC3. Values are implemented nationally by member states. Structural resistance performance is directly linked to reliability through quantitative partial factor limit states procedures only for the ultimate limit state RC2 case. All other design situations and limit states are treated in an indirect manner, with quality control (QC) measures playing a central role.

Table 2.9. Summary of structural performance requirements

Design situations		β_T	**Resistance**
Serviceability Limit State (SLS)			
Long-term			Action values, combinations differentiated -Resistance R_k
Reversible			
Irreversible		1.5	
Ultimate Limit State (ULS) - Equilibrium (EQU); Structural (STR); Geotechnical (GEO); Fatigue (FAT)			
Reference (Persistent)	RC2	3.8	$\beta_{T,R} = 0.8 \times 3.8 = 3.0$
Reliability Differentiation	RC1	3.3	Actions: $\gamma_F \cdot K_{FI}$
	RC3	4.3	Resistance: Preferred - Quality measures
Transient - actions n-adjust			No resistance change
Reliability management			Allow γ_M adjustment
Accidental: action specified		(P_A)	- Resistance R_k - Ductility, ties, etc
Robustness: notional action			

2.3 STRUCTURAL CONCRETE RESISTANCE (Based on EC 2)

This Section gives a brief overview of the state of unification between the basis of design requirements stipulated in EC 0 and their implementation in the treatment of the relevant EC 2 provisions.

2.3.1 Basis of design

Section 2 of EC 2 that deals with the basic requirements of the basis of structural concrete design states that concrete structures should be designed in accordance with the general rules of EC 0 and with actions defined in EN 1991. However, EC 2 has some additional requirements (European Concrete Platform, 2008). As is expressly stated in EC 2, the basic requirements of EC 0 Section 2 are deemed to be satisfied for all concrete structures if:

1. limit state design is carried out with the partial factor method in accordance with EC 0,
2. If actions are defined in accordance with EN 1991,
3. If combinations of actions in accordance with EN 1990, and finally
4. If resistance, durability and serviceability are dealt with in accordance with EN 1992.

Harmonisation between the basis of design, loading and concrete code code is implied by this requirement.

2.3.2 Reliability differentiation

Section 2.1.2 of EC 2 on Reliability management states that a design based on the use of partial factors for resistance in EC 2 and the partial factors given in EC 0 annexes for actions lead to a structure associated with class RC2. Throughout the code, no further guidance is given on the treatment of other reliability classes that are identified in EC 0 other than reference to Annex B and Annex C in EC 0.

2.3.3 Partial factor modification

A single set of partial factors for concrete and steel, $\gamma_C = 1.5$ and $\gamma_S = 1.15$ respectively, which can be viewed to achieve reliability class RC2 when EC 0 and EN 1991 factors are applied to actions, are provided for use for resistance verifications of all limit states and design situations, save for the distinction of accidental situations. For accidental design situations $\gamma_C = 1.2$ and $\gamma_S = 1.0$ are suggested for use. Otherwise, lower values of partial factors for concrete, γ_C, and for steel, γ_S, are allowed for use if justified by measures that reduce the uncertainty in the calculated resistance. These measures are based on the control of geometry, particularly of critical sections, and of concrete strength. Table 2.10 summarises the scheme for which partial material factors can be reduced which is presented in Annex A of EC 2.

A detailed assessment of how the reliability framework may be used to justify these reductions in partial factors for both steel and concrete is given in Chapter 3. Guidance is also given on the implementation of partial factor reduction scheme for South African codification efforts, taking into account the reference levels of reliability set by the South African basis of design code SANS 10160-1.

Table 2.10. Scheme for partial material factor reduction based on control of geometry and control of concrete strength

EC2 clause	Subclause	Heading	EC2 Recommendation	Comment
A.2.1	(1)	Reduction based on quality control and reduced deviations, $\gamma_{S,\ red1}$	$\gamma_{S,\ red1} = 1,1$	Execution must be subject to QC system that reduces unfavourable deviations of cross-section dimensions(see EC 2 Table A1, Annex A).
	(2)	Reduction based on quality control and reduced deviations, $\gamma_{C,\ red1}$	$\gamma_{C,\ red1} = 1,4$	Applicable under condition A.2.1(1) and c.o.v of concrete strength must be ≤ 10 %
A.2.2	(1)	Reduction based on using reduced or measured geometrical data in design, $\gamma_{S,\ red2}$ & $\gamma_{C,\ red2}$	$\gamma_{S,\ red2} = 1,05$; $\gamma_{C,\ red2} = 1,45$	Calculation of design resistance based on critical geometrical data, including effective depth (see Figure A1, Annex A), which are either: (1) reduced by deviations, or (2) measured in the finished structure.
	(2)	Reduction based on using reduced or measured geometrical data in design, $\gamma_{C,\ red3}$	$\gamma_{C,\ red3} = 1,35$	Applicable conditions given in A2.2(1) and c.o.v of concrete strength must be ≤ 10 %
A.2.3	(1)	Reduction based on assessment of concrete strength in the finished structure, η & $\gamma_{C,\ red4}$	$\eta = 0,85$; $\gamma_{C,\ red4} = 1,3$	Recommended values maintained. For concrete strength values based on testing in a finished structure or element, see EN 13791, EN 206-1 and relevant product standards, γ_C may be reduced by the conversion factor η. The value of γ_C to which this reduction is applied may already be reduced according to A.2.1 or A.2.2. However, the resulting value must be ≥ $\gamma_{C,\ red4}$
A.3.2 & A.4	-	Partial factors for materials & Precast elements	-	Reduced partial factors for materials $\gamma_{C,\ pcred}$ and $\gamma_{S,\ pcred}$ may be used in accordance with the rules in A.2, if justified by adequate control procedures. The rules given in A.2 for insitu concrete structures also apply to precast concrete elements as defined in 10.1.1.

2.3.4 Summary of the implementation of reliability framework in EC 2

A minimal system of partial material factors are used to provide for sufficient reliability across a wide range of structural concepts (reinforced, prestressed, lightweight, unreinforced, composites); configurations (beams, slabs, columns walls, foundations); failure modes (flexure, axial, shear, torsion, stability) and their combinations. Numerous assumptions, simplifications, empirical constants and other measures are imbedded in the design procedures, without explicit treatment of reliability. Although provision is made for modelling factors to provide for these effects, this is not applied transparently. Efforts are therefore required to properly extend the reliability framework to all the necessary provisions for structural concrete resistance to achieve unification of materials, actions and basis of design standards.

2.4 REVIEW OF RELIABILITY IN THE DRAFT *fib* 2010 MODEL CODE

The *fib* 2010 Model Code represents a substantial improvement in terms of the implementation of the reliability framework in the preparation of its stipulations, mainly introduced and partially developed in the 1978 CEB-FIP Model Code and had not matured sufficiently in implementation by the time of print of the CEB-FIP Model Code of 1990 hence their subtle and superficial treatment therein. The nature of reliability implementation would depend on what aspect of construction practice is being considered, from design through to construction and maintenance. More explicit treatment of reliability is made visible from the on-set of the basis of design verification procedures, allowing for full probabilistic safety formats, usually preferred in evaluating the residual life of existing structures, as well as reliability or probability based partial factor limit states verifications which are more commonly adopted for newer designs.

Further, the newly incorporated Procedures concerning Robustness can be viewed as related to reliability considerations in the sense that it’s an aspect of design concerned with structural performance and safety. Older designs, due to excessive conservatism inherent in the then adopted design procedures used for most design cases, led to structures that often possessed ample redundancy. Conversely, newer and improved design methods which now achieve improved economic designs (less material used) by the use of rational design procedures for safety verification reduce the robustness enjoyed by older structures (Walraven, 2005). This is particularly the case and of concern for the fast emerging trend of slender and lighter structures mainly owing their existence to the increased production and use of higher strength concretes and other forms of advancing concrete technology. Structural and non-structural measures to ensuring robustness are also cited in the JCSS Probabilistic Model Code (JCSS, 2001).

Any measure or action taken to somewhat enhance structural performance e.g. Increase structural resistance, to reduce actions and action effects, protection by design or through construction methods against structural hazards, or to increase production efficiency and quality etc., can be regarded as done in view of, and to enhance structural reliability performance. Hence, the more detailed performance-based design criteria aim at better defining structural service life can be viewed as major improvements in terms of reliability aspects relating to more accurate assessment of the real structural behaviour.

Previous design philosophies or bases prescribed that designs were carried out regarding two major stages of a structure's performance, namely the Ultimate Limit State (ULS) and the Serviceability limit state (SLS). The ULS was usually checked for static loading whereas to control the SLS only the deflection and crack width were considered. Walraven (2005) declares that experiences with damage to construction works have indicated that there has not been sufficient attention for a number of design criteria. Therefore, in addition to robustness, to better describe the structural performance other explicit design criteria have been established in the 2010 Draft Model Code as (*fib*, 2010):

1. Static and non-static loading (fatigue, impact and explosion, earthquake),
2. Tightness,
3. High and low temperatures (fire and cryogenic),
4. Limit states for durability are established, and
5. Initial ideas are given regarding design for sustainability.

In a nutshell, the most important aspect introduced by the advent of the new *fib* 2010 Draft Model Code is the introduction of *Time* as a design parameter. Due regard of Time provokes the awareness that safety and serviceability are not the only requirements of real structures, but other aspects as sufficient resistance against deterioration should also be considered. In this regard, a strategy for maintenance must be developed at design in addition to the basic activity of ensuring that the intrinsic resistance at the start of the service life satisfies the relevant design criteria. This, coupled with more elaborate performance-based design criteria are aimed at better defining service life of a structure during design. This leads to a more life-cycle oriented design basis.

Section 2.2 in EC 0 on Reliability Management declares that levels of reliability relating to structural resistance and serviceability can be achieved by, amongst a combination of other issues, the accuracy of the design models used. Therefore some differentiation of the design of structures is possible through the type of design models used in design. Generally, more accurate design models are computationally more involving and require more effort to execute. The benefits of more rigorous design analyses are normally compensated for complex structures where such complex analyses can lead to major cost benefits usually tied to the allowance of use of less material in design. Ties to reliability in this regard are visible in the 2010 *fib* Draft Model Code where levels of approximation, I through IV, are suggested for use. The levels differ in the complexity of the applied methods and the accuracy of the results as shown by Bentz (2010) in a follow-up *fib* Bulletin to the Draft of the Model Code. The levels of approximation for design for shear allow scope for applying reliability differentiation specifically to the shear design of reinforced concrete members in practice.

Since the quality of construction is of large significance for the durability of structures, adequate attention is given to a number of important construction aspects in the Draft *fib* 2010 Model Code. The issue of partial factor modification for resistance, first introduced in the CEB-FIP 1990 Model Code and the provisions were repeated unaltered in the *fib* 2010 Draft Model Code, is explored in detail in Chapter 3.

Chapter 3

IMPLEMENTATION OF THE RELIABILITY FRAMEWORK

3.1 INTRODUCTION

Various reliability issues, identified from the review of the basis of design requirements and their implementation in Chapter 2 and from published research, are explored and addressed in this Chapter. Firstly, the influence the reference level of reliability, β, has on structural performance is tackled, with a specific view on how its value affects the choice of operational partial safety factors to be adopted in design codes for structural resistance. The case of different levels of reliability stipulated for the design of structures as set by SANS 10160-1 ($\beta = 3.0$) and the recommended value set by EC 0 ($\beta = 3.8$) is taken as central to this Section. The reliability classes and associated risk criteria adopted for use by SANS 10160-1 are briefly reviewed.

Parametric studies of representative cases of reinforced concrete members should be conducted to identify and characterise the elements that affect the reliability performance across the scope of application of concrete in practice. This would, in effect, form basis of design requirements for structural resistance. These requirements in current design formats present themselves by the use of partial factors and characteristic or sufficiently representative values of basic variables, the application of quality measures and possible limitations concerning the use of any of the aforementioned aspects. Parametric studies performed by Holický, Retief and Wium (2010) which were aimed at assessing partial factor requirements for selected concrete members in accordance with South African conditions are reviewed. Some of the results from the research are summarised but critically reviewed as they provide evidence that provokes further research to be done for other reinforced concrete elements representative of different modes of resistance. In this regard, the model uncertainty for shear resistance of reinforced concrete members requiring stirrups and its effect on reliability performance is later investigated in the thesis. Results from the Review of the work by Holicky et al. (2010) provide not only motivation for the work done on shear in the thesis but also serve as templates to extend research and fully calibrate the shear prediction model in future work. Other research efforts that provided essential background information and motivation for the work carried out in the thesis on shear are outlined.

Finally, the issue of partial factor reduction in EC 2 and its link to the reliability management requirements of resistance presented in EC 0 were explored. Informative Annex A in EC 2 allows partial factor reduction if control of geometry, especially at critical sections, and control of concrete strength are exercised as measures to reduce resistance uncertainty. The essence of this review emanates from the link between the conditions required for partial factor reduction and the effect the application of reliability differentiation has in achieving these conditions.

Reliability can be reflected and managed throughout the design process. In general, any procedure, measure or action aimed at increasing structural performance or increasing the quality of constructed works, whether or not they can be quantified, can be viewed as having reliability implications. It is therefore advised that as South Africa reviews EC 2 as reference for on-going revisions of the concrete code, the various champions in charge of various aspects of codified design as Materials, Structural Mechanics, and Detailing, should be responsible to verify the correctness, rationality, and applicability of the provisions in each of the respective sections. Table 3.1 presents a concise summary of the Sections and issues in EC 2 to be reviewed by the various champions of the South African concrete code review committee responsible for Materials, Structural Mechanics, and Detailing.

Table 3.1. Roles for verification of EC 2 by the South African review committee

Limit State	Failure Mode	Materials	Structural Mechanics		Detailing	
			Analysis	Resistance	General	Elements
Ultimate	Flexure	**SECTION 3:** Characteristic values. Rheological models **SECTION 10, 11, 12** Material classes	**SECTION 5** Modelling uncertainty for action effects; behaviour	**SECTION 6, 7** Valid SM models for resistance; serviceability behaviour	**SECTION 8** Validity of assumptions for resistance. Constructability	**SECTION 9** Ditto for elements. (Importance for seismic resistance – Robustness)
	Shear					
	Torsion					
	Punching					
	Fatigue					
	Durability					
Service	Stress					
	Cracking					
	Detailing					

3.2 REFERENCE LEVEL OF RELIABILITY

3.2.1 Choice of reference level of reliability (β)

Member states of the European community and other countries using the Eurocodes as base for their standards development are required to set reference reliability levels, commonly referred to as target reliability levels β_T, based on structural risk acceptance criteria suitable for national practice. Different levels of risk acceptance criteria correspond to a minimum reliability. Table 3.2 presents the classification of reliability classes as described by SANS 10160-1 with some examples of design situations in which each class may be applicable.

In a rational analysis the target reliability is considered as a control parameter subject to optimisation (JCSS, 2001). However, to date, national reliability levels are set by calibration to a long experience of building tradition. For South African structural design standards, a reference level of reliability of $\beta = 3.0$ of class RC2 is prescribed for use by the national basis of design standard SANS 10160-1. This value of reference reliability was set from extensive calibration of structures designed in accordance with the now outdated national loading and basis of design code SABS 0160-1989. The reference level of reliability set for use in South Africa is also defendable from recommended values suggested for use by the JCSS probabilistic model code and the international standard ISO 2394.

Table 3.2. Classification of SANS 10160-1 reliability classes

Class	**β_T**	**Accidental Consequence Class**	**Seismic Class (Public safety)**	**Geotechnical Category**
RC1	2.5	Single occupancy $\leq$ 3 storeys	Minor (agriculture)	Small structure; no stability or movement
RC2	3.0	Residential, office etc; $\leq$ 4 storeys	Ordinary	Conventional structure / foundation
RC3	3.5	Residential, office etc; 5 - 15 storeys	Important (schools; assembly)	Ground / structure require geotechnical input
RC4	4.0	Public in large #; stadia > 5000	Vital (hospital; fire; power)	Large; unusual; complex; risky;

Table 3.3 presents the schemes of recommended levels of reliability, differentiated by risk criteria, suggested for use by ISO 2394 and the JCSS probabilistic model code. South Africa's target level of reliability, $\beta_T = 3.0$, can generally be viewed as associated with constructions that have moderate consequences of failure, usually determined considering loss of human life and environmental concerns, and moderate costs in terms the safety measures required to reduce risks.

It is observable from Table 3.2 that in addition to adopting a different reference level of reliability from the value suggested for use by EC 0, SANS 10160-1 also specifies four reliability classes, RC1 to RC4, as classifiable for structures. It should be noted that although RC2 serves as reference, with $\beta = 3.0$, RC2 and RC3 in fact represent a division of the Eurocode RC2, generally at building of four and five storeys (Retief & Dunaiski, 2009). Table 3.4 shows the relation between the SANS 10160-1 and EC 0 reliability classes together with their associated risk factors.

Table 3.3. Schemes of recommended levels of reliability by ISO 2394 and JCSS PMC based risk principles

	ISO 2394			
Relative cost of safety measures	**Consequences of failure**			
	Small	Some	Moderate	Great
High	0	1.5	2.3	3.1
Moderate	1.3	2.3	3.1	3.8
Low	2.3	3.1	3.8	4.3

	JCSS Probabilistic Model Code		
Relative cost of safety measures	**Consequences of failure**		
	Minor	Moderate	Large
Large	1.7	2.0	2.6
Normal	2.6	3.2	3.5
Small	3.2	3.5	3.8

Table 3.4. Reliability classes and associated risk elements for SANS 10160-1 and EC 0

	Function of facility, probability or consequence of failure		
Class	***SANS 10160-1***	***EN 1990***	**Class**
RC1	Low loss of life, economic, social ; small environmental	Low for loss of human life, and economic, social or environmental small or negligible	RC1
RC2	Moderate loss of life, economic, social; Considerable environmental	Medium for loss of human life, economic, social or environmental considerable	RC2
RC3	High loss of human life, very great economic, social, environmental		
RC4	Post-disaster function / beyond the boundaries	High for loss of human life, or economic, social or environmental very great	RC3

3.2.2 Influence of level of reliability on structural performance

Relation between β and P_f

The level of reliability, β, is related to the probability of failure, P_f, by the relation:

$$P_f = \Phi(-\beta) \qquad [3.1]$$

Where Φ is the cumulative distribution function of the standardised Normal distribution. Table 3.5 shows some numerical representation of the relation between β and P_f.

Table 3.5. Relation between β and P_f

P_f	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}	10^{-6}	10^{-7}
β	1.28	2.32	3.09	3.72	4.27	4.75	5.20

It is obvious from inspection of Table 3.5 that increased β-levels imply reduced probability of failure. In terms of practical design requirements, the implication is that when β-levels are increased measures that reduce failure of structures should also be increasingly implemented. In the design provisions under review, differentiation according to reliability classification is expressed through different levels of quality management, mainly to reduce gross errors, and partial factor modification aimed at controlling conservatism in design calculations. The following Section treats the probabilistic basis of partial factors for material as described by EC 0 in order to portray the effect β has on the value of partial factors adopted for design.

Probabilistic basis of partial factors for material properties

A concise form of the study herein presented was done as part of an assessment conducted by Holický and Retief (2010). The study is generic and is dealt with elaborately here, giving some relevant statistical and probability background where necessary.

From Table 2.7 in Chapter 2, the following expression holds for describing the design value of any basic resistance variable, $X_{d,i}$:

$$X_{d,i} = \eta \frac{X_{k,i}}{\gamma_{m,i}} \quad [3.2]$$

All symbols used in Equation [3.2] are explained in Chapter 2. Ignoring the conversion factor for scale effects, η, and the use of subscript i, the partial factor for materials, γ_m, may be denoted as:

$$\gamma_m = \frac{X_k}{X_d} \quad [3.3]$$

In accordance with the basis of design requirements, the characateristic value of a material or product property is the 5 % fractile of its statistical distribution. The concept of fractiles is extremely important in understanding how partial factors are derived from probabilistic analyses. In the rest of the discussion, its theory is first briefly introduced and then implemented as necessary into the process of defining material partial factors.

For a given fractile, p, the p-fractile, x_p, denotes such a value of the random variable (material property), X, for which it holds that values less than or equal to x_p occur with probability p (Holický, 2009). It is however more convenient to represent occurrences of random variables in standardised normal form so as to enable the use of commonly available tables in literature to calculate exceedance (-or non-exceedance) probabilities. The standardised variable, U, is derived from the actual variable X using the formula:

$$U = \frac{X - \mu_X}{\sigma_X} \quad [3.4]$$

Where μ_X and σ_X denote the mean and standard deviation, respectively, of the actual variable X. By this transformation the original random variable x_p can be converted to the standardised variable u_p.

Therefore, if $\Phi\ (u)$ is the distribution function of the standardised normal variable U, the probability of non-exceedance of the stipulated value, u_p, can be expressed as:

$$P(U \leq u_p) = \Phi(u_p) = p \tag{3.5}$$

Figure 3.2 illustrates the concept of lower fractile estimates, which are more commonly applicable to resistance properties, on a Probability density plot. The probability density or random variable u, $\varphi(u)$, is derived by integrating the distribution function $\Phi(u)$. Tables of the normal probability density plot are commonly available in literature.

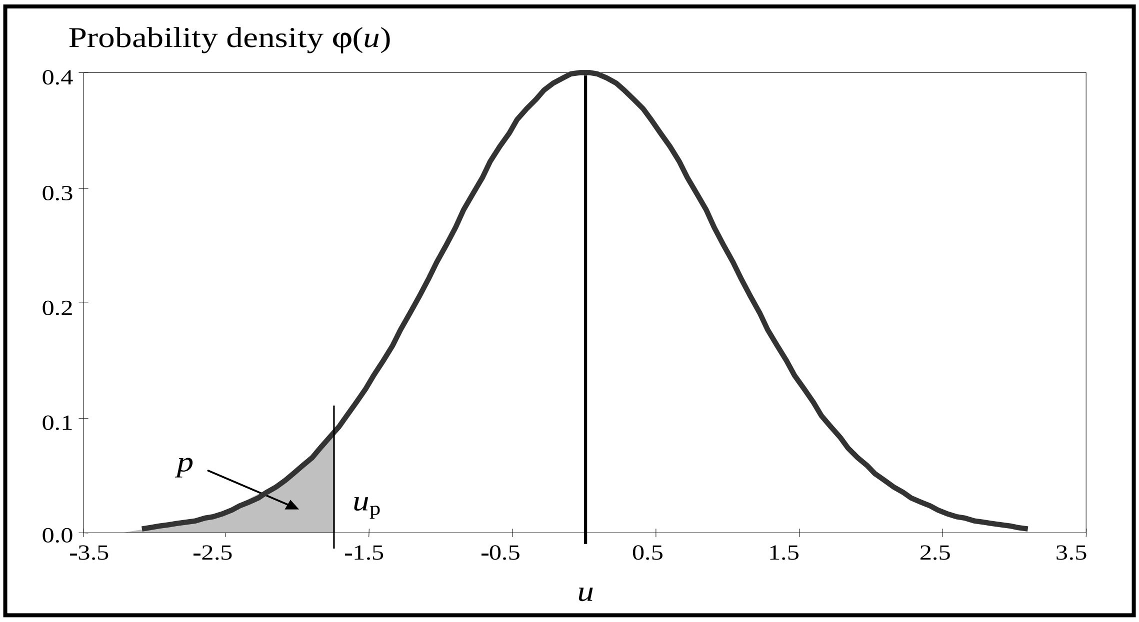


Figure 3.2. Definition of the fractile for a standardised random variable U

Hence, for a specified probability p of 5 % for characteristic values, $u_p = u_{0.05}$. For the standardised normal distribution of U, the fractile $u_{0.05} = -1.645$. Substituting u_p for U and x_p for X into Equation [3.4] and thereafter transposing it to make x_p subject :

$$x_p = \mu_X + u_p\sigma_X = \mu_X(1 + u_p\Omega_X)$$

$$\therefore X_k = x_{0.05} = \mu_X(1 - 1.645 \cdot \Omega_X) \quad [3.6]$$

Where Ω_X denotes the coefficient of variation of random variable, or the material property, X.

Through means of reliability separation for action effects and resistance, EC 0 specifies the design value, R_d, of a resistance, R, using the probabilistic relationship:

$$P(R \leq R_d) = \Phi(-\alpha_R \beta) \quad [3.7]$$

Here Φ denotes the distribution function of the standardised Normal distribution, α_R is the resistance sensitivity factor for which EC 0 allows approximation $\alpha_R = 0.8$ and β is the reliability index as specified for use at a national level. Equation [3.7] is fully applicable in determining the way in which the design value of material properties are determined, and not just resistance as a whole. As such, the design value of material property, X_d, can be expressed as:

$$P(X \leq X_d) = \Phi(-\alpha_R \beta) \quad [3.8]$$

By comparing Equation [3.5] and Equation [3.8] it is clear that the design value of a material property corresponds to some fractile estimate of the distribution. It can further be deduced that $u_p = -\alpha_R \beta$ for the design value of a material property. X_d, as in Equation [3.6] for X_k, can then be expressed as:

$$X_d = x_{-\alpha_R \beta} = \mu_X(1 - \alpha_R \beta \Omega_X) \quad [3.9]$$

Substituting the values of X_k and X_d from Equations [3.6] and [3.9] into Equation [3.3] results in:

$$\gamma_m = \frac{X_k}{X_d} = \frac{1 - 1.645 \cdot \Omega_X}{1 - \alpha_{R\beta} \Omega_X} \quad [3.10]$$

Equation [3.10] is derived based on the assumption that the random variable X is normally distributed. Some resistance properties may be described by the normal distribution.

However, it is generally expected (JCSS, 2001) that the resistance of structural members and their properties may be described by a two-parameter lognormal distribution with the lower bound at the origin. Holický (2009), in accordance with accepted statistical theory, states that it is possible to calculate the fractile for the log-normal distribution from the value of the fractile of a standardised random variable with a normal distribution using the relation:

$$x_p = \frac{\mu_X}{\sqrt{1+\Omega_X^2}} \exp u_{norm,p} \sqrt{\ln(1+\Omega_X^2)} \qquad [3.11]$$

Where $\mu_{norm,p}$ is the fractile of a standardised random variable with a normal distribution. In this instance, the partial safety factor for materials may be summarised and expressed as in Equation [3.12] below:

$$\gamma_m = \frac{X_k}{X_d} = \frac{\frac{1}{\sqrt{1+\Omega_X^2}} \exp\left(u_{0.05}\sqrt{\ln(1+\Omega_X^2)}\right)}{\frac{1}{\sqrt{1+\Omega_X^2}} \exp\left(u_{-\alpha_R\beta}\sqrt{\ln(1+\Omega_X^2)}\right)} \cong \frac{\exp(-1.645\times\Omega_X)}{\exp(-\alpha_R\beta\times\Omega_X)} \qquad [3.12]$$

Note that the approximation indicated in the last expression in Equation [3.12] is fully acceptable for small coefficients of variation, in the order of $\Omega_X < 0.2$.

Equations [3.10] and [3.12] are plotted in Figures 3.3 and 3.4 respectively. The graphs show the trends of the partial material factor with varying coefficient of variation and varied levels of reliability for the separate assumed cases of normal and lognormal distribution of material property. Indications are given in the Figures to show the implication of differently adopted reference β-levels by SANS 10160-1 and EC 0 on partial factor requirements. Figure 3.5 shows for two separate cases, $\beta = 3.0$ and $\beta = 3.8$, illustrating how partial factors for material are dependent on the coefficient of variation as per SANS 10160-1 and EC 0 respectively.

The following observations can be made from the graphs presented in Figure 3.3 and Figure 3.4:

1. In each of the two Figures it can be viewed that partial factors for material properties are dependent on two factors
   a. The reference level of reliability, β.
   b. The coefficient of variation of the material property, Ω_X.

2. Further, by comparison of the two Figures, it can be seen that the values of the partial factors will differ for different distributions (in this case normal and lognormal) given the same reliability level and coefficient of variation.

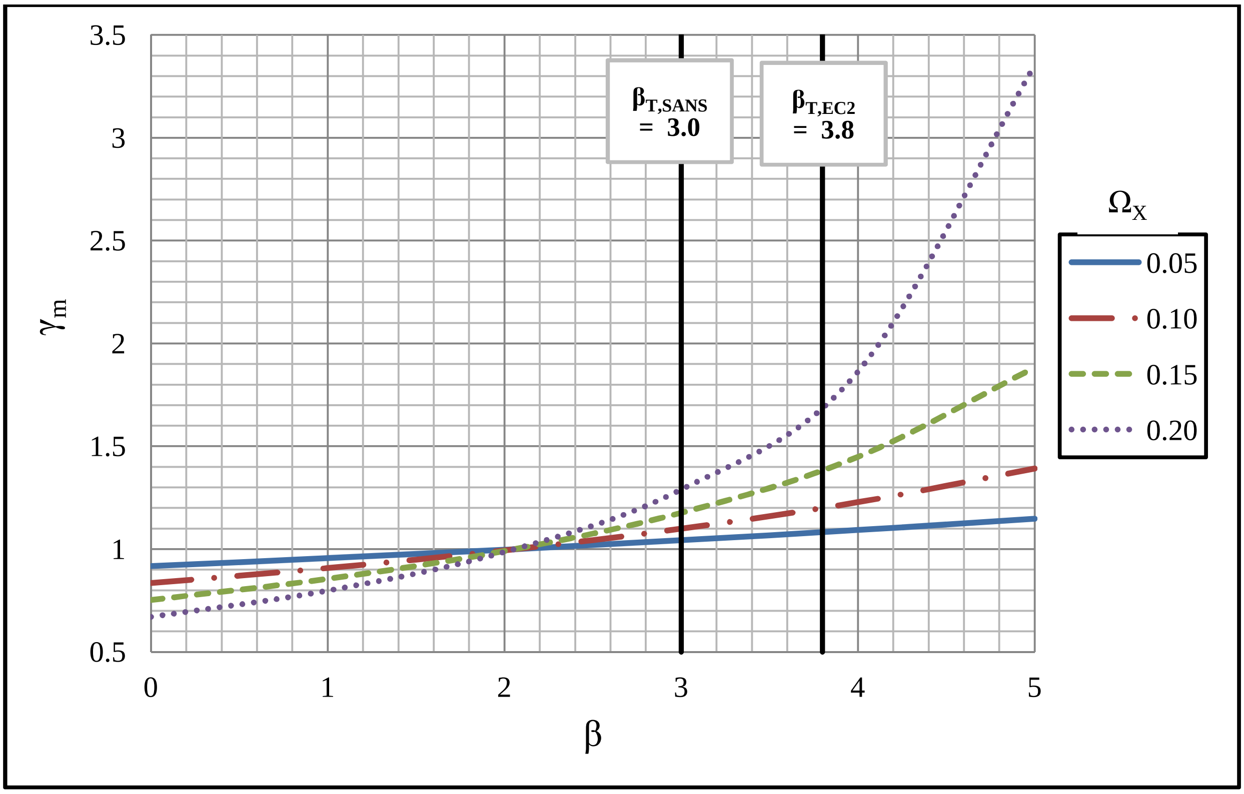


Figure 3.3. Variation of γ_m with β for selected coefficients of variation, Ω_X, and normal distribution of X

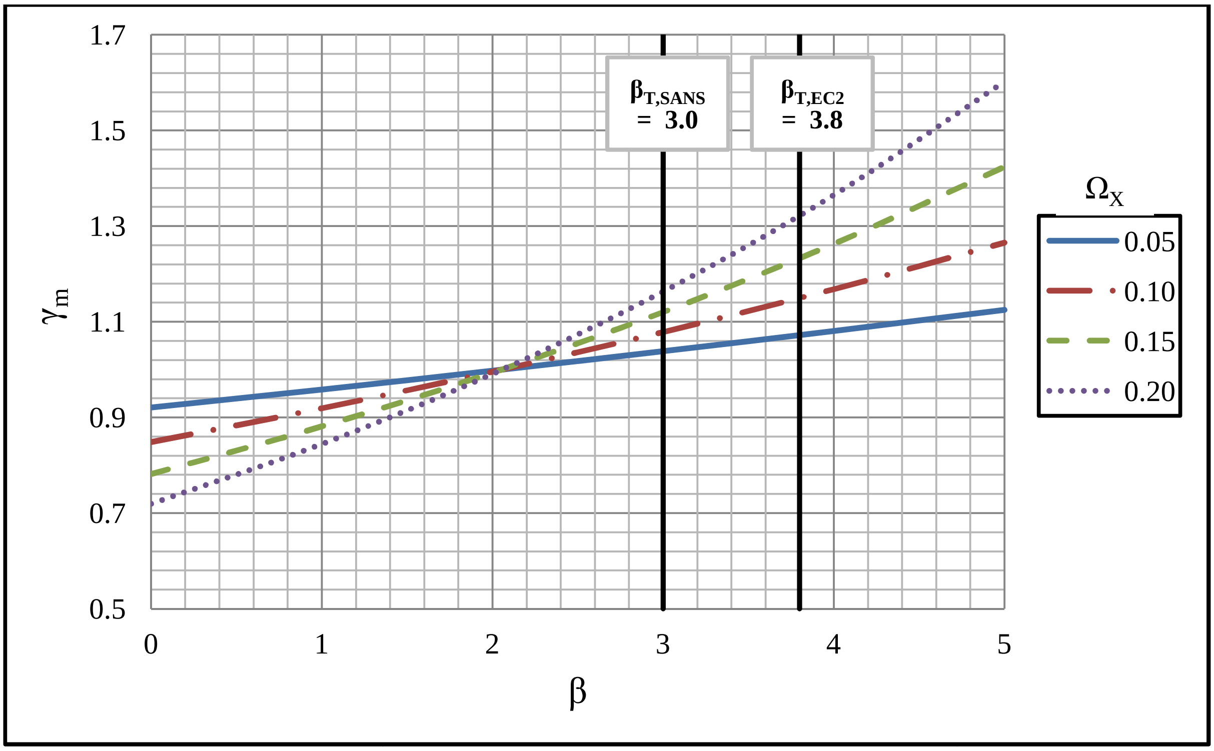


Figure 3.4. Variation of γ_m with β for selected coefficients of variation, Ω_X, and log-normal distribution of X

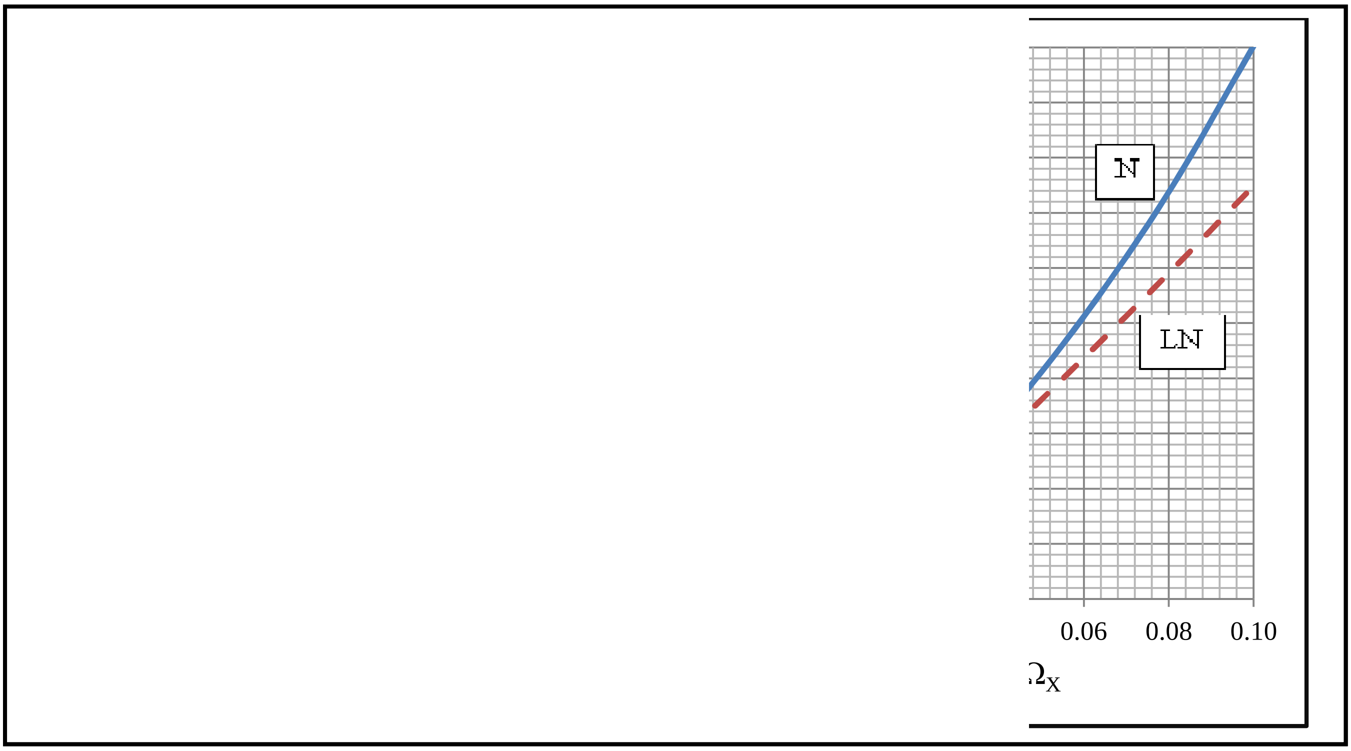


Figure 3.5. Variation of γ_m with β for selected coefficients of variation, Ω_X, and normal and log-normal distribution of X

Based on the observations from Figure 3.3 and Figure 3.4, the following conclusions can be logically deduced:

1. Reliability differentiation through specification of different reliability classes each associated with specific levels of reliability should have an effect on operational partial factors adopted in design

2. Improved quality measures, aimed at reducing uncertainty, can be effective in reducing deviations of a material property by efficient control of the production process. Therefore, quality measures and coefficient of variation are related and the selection of operational partial factors should be dependent to some extent on quality measures.

Theoretical models of random material properties and their statistics affect reliability analyses and the choice of operational partial factors. Models commonly available in literature are specified based on assessments of European production quality. Studies for South African derived models for material properties are encouraged in an effort to draft provisions that are as representative of local conditions as possible.

This Section shows how partial factors are derived to cater for unfavourable variations in the properties of materials. However, as presented in Table 2.7 and schematically shown in Figure 2.1, partial factors for model uncertainties and important geometrical effects should be determined and incorporated into values of global partial factors used in design.

3.3 PARAMETRIC STUDIES AND THE CHARACTERISTICS OF RESISTANCE FOR SHEAR

3.3.1 Parametric study for selected reinforced concrete elements

Holický et al. (2010) investigate partial factors for selected concrete members by considering parametric studies of two different reinforced concrete members, a slab and a short centrically loaded column, as representative examples of flexural and compressive structural members. The study considered South Africa's reference level of reliability, $\beta = 3.0$, as basis of evaluation for adequate partial factor schemes for adoption in South Africa's new design code for concrete structures SANS 10100-1. In addition to some conclusions on appropriate values for the partial factors in accordance with the scheme in present use, namely the use of partial factors γ_S for steel and γ_C for concrete for structural concrete resistance verification, important insights from the reliability analyses were gained into mechanisms and factors having influence on the reliability performance of the considered elements. This is particularly critical in the cases of model uncertainty and reinforcement ratio, ρ. Most of the results from the considered elements affecting reliability performance for slabs and columns give indications that further studies are required for other types of structural elements that are representative of other modes of structural behaviour. Such treatment of reinforced concrete resistance would result in the use of partial factors that achieve sufficient reliability in all fields of application of codified provisions. Some important aspects, including those just mentioned, and issues from work done by Holický et al. (2010) are summarised in bullet form below:

1. Various analyses aimed at understanding the influence model uncertainty has on structural performance were considered. The model factor was found to have significant influence on structural performance with its increasing uncertainty reducing reliability performance. However, the statistics of the model uncertainty used in the study were derived from suggestions in the JCSS working documents and therefore more convincing data are encouraged for use when available. By means of a sensitivity analysis, the study shows that the reliability for columns is dominated by model uncertainty whilst in the case for slabs its effect is second to that of steel depth but still has significant influence on reliability performance. Due to differences in the influence of model uncertainty on reliability performance,

further characterisation of the model uncertainty for different structural elements representative of different modes of structural behaviour is thus warranted.

2. In accordance with the findings of a separate study (Holicky et al., 2007) the reliability of reinforced concrete members was found to be dependent on the reinforcement ratio, ρ. The reliability performance for columns increases with increasing reinforcement ratio. This trend is counter-intuitive for slabs, with dipping levels of reliability as ρ increases. Therefore, the effect of ρ on the reliability performance on other concrete members representative of different modes of structural behaviour should be investigated. The counter-intuitive effect that ρ has on the reliability performance of slabs shows an obvious weakness in the judgment based selection of partial factors which further motivates and re-affirms the adoption of developing design codes based on probabilistic principles.

3. The study conducts extended reliability analyses to determine suitable partial factors that achieve the resistance reliability requirements for the concrete members considered. By way of assessing theoretical partial factors for the slab and the column, a difference is shown in partial factors applicable to the same basic variables in the different formulations for resistance. Thus, calibration for concrete should include studies for other elements to fully classify partial factor trends across the field of application of concrete structures and various structural members to achieve consistent performance. Traditional, and currently in use, limit state design formats implement two partial material factors, γ_S for steel and γ_C for concrete, for resistance verifications. Though the partial factor for concrete strength at the design point (theoretical partial factor) is most significant, it is however shown that partial factors are applicable to other basic variables not explicitly considered when designing according to codes. These variables include section breadth and height as well as model uncertainty. The use of two partial factors for resistance verification in design codes is thus an oversimplification requiring additional conservatism during calibration to provide for the neglected basic variables. Partial factors required for model uncertainty are found to be quite significant compared to other basic variables, including that of steel which is explicitly considered in design codes. This fact, plus other aspects described in Bullet 1 above, gives strong indications that the model uncertainty, which is specific to member type and mode of resistance,

requires further characterisation and should be calibrated as part of operational partial factors used when designing according to codes of practice.

4. By way of Global resistance factor analysis it is shown that the specification of characteristic values of material strengths for steel and concrete, f_{yk} and f_{ck} respectively, play an important role in achieving sufficient reliability. In fact, it is further shown that the specification of characteristic material properties plays a more prominent role than the values of the partial factors in achieving sufficient reliability for both slabs and columns. However, the exact nature of the trends of reliability performance with characteristic values of material properties should be characterised for each mode of structural resistance, as differences are noticed between the trends of the slab and column in this regard.

5. The study clearly demonstrates how partial factors for the resistance cases considered are established to meet reliability performance requirements through a calibration process that is independent of load considerations i.e. by reliability separation for resistance, $\beta_{T,R} = 0.8 \times 3.0 = 2.4$ for South African conditions. Hence, the study provides a good guide for similar reliability-based parametric analyses of other reinforced concrete members.

3.3.2 Characteristics of resistance for members requiring design reinforcement for shear

The review in Section 3.3.1 considers partial factor requirements to achieve sufficient reliability performance for flexural and compressive failure modes of concrete resistance. Stimulation is therein created for efforts to carry out such a study for shear and other modes of concrete resistance.

Cladera and Mari (2007) performed research that was aimed at characterising the quality of shear resistance predictions for members requiring stirrups based on different design models. This was done to compare the prediction quality of other commonly used models and some from literature to the prediction quality of the EC 2 design model. Cladera and Mari show that the EC 2 design procedure may be slightly unconservative for highly shear-reinforced members and may be too conservative for slightly reinforced beams as no concrete contribution is considered. Shear predictions were characterised by model factor trends, where the model factor, MF, is the ratio between the tested failure load, V_{test}, to the failure load predicted by any design model, V_{pred}. Therefore:

$$\left(MF = \frac{V_{test}}{V_{pred}}\right) \quad [3.13]$$

Thus, values of MF greater than 1 indicates the design prediction model is conservative and vice versa is true. The average model factor, MF, obtained by comparison of EC 2 predictions to the total test database, however, exhibits a high conservative bias and a high coefficient of variation as compared to other modes of resistance. Therefore, though there is slight unconservatism for highly shear-reinforced members, the EC 2 shear prediction model is generally safe. The scatter of results, shown in Chapter 7, when predictions are compared to empirical tests indicates that the design procedure neglects some primary variables affecting shear resistance. Nonetheless, the EC 2 model for shear is in current use and the more important issue is therefore how to calibrate the design model, together with its associated uncertainty, to achieve acceptable reliability performance. In terms of reliability, the more critical issue would obviously be to calibrate the model to ensure that sufficient provision is made to cater for the unconservatism in highly-reinforced concrete members. Only then should efforts be directed towards controlling levels of conservative margins for lesser reinforced concrete members. The fact that the EC 2 shear prediction model is already in use should, however, not restrict research on the use of other models that better predict shear resistance of concrete members with stirrups. In fact, such research is encouraged for the use of more accurate and rational-scientific formulations in future codes of practice.

Not only has model uncertainty been proven to have important effects on reliability performance, but more so for shear, the model factor (which describes model uncertainty) has a high coefficient of variation associated with it. Even more critical, is that the predictions are shown to be unconservative for members with high stirrup ratios. Therefore, much work needs to be done to properly calibrate the EC 2 shear prediction model for members requiring stirrups. As a result, Part 2 of the thesis was crafted to first carry out a statistical evaluation of model factor realisations to a compiled database and to characterise it for reliability modelling. Thereafter, some reliability analyses were conducted to assess the reliability performance of the EC 2 shear prediction model. It was found that the model factor dominates the reliability performance of shear resistance. The reliability of EC 2's design method for members with stirrups subjected to shear was found to be of acceptable performance and should lead to safe designs when used in South African practice.

3.4 RELIABILITY MANAGEMENT AND PARTIAL FACTOR MODIFICATION

Annex A in EC 2 allows partial factor reduction provided it can be justified by measures aimed at reducing uncertainty in calculated resistance of members. As previously established, these measures are based on the control of geometry especially at critical sections and of concrete strength. However, no explicit relation is expressed between the modification allowed in EC 2 and guidance given in the requirements for such action in EC 0. Recall from Section 2.2.3 that Annex B in EC 0 states that although reliability differentiation may also be applied through partial factors on resistance, as is recommended for actions, this is not normally used. This implies that the provision has not been fully explored yet. Further guidance in EC 0 states that a partial factor for a material or product property or a member resistance may be reduced if an inspection class higher than that required according to Table 2.5 and/or more requirements are used. However, EC 0 explicitly states that such a reduction, which allows for example for model uncertainty and dimensional variation, is not a reliability differentiation measure but serves merely as a compensatory measure in order to keep the reliability level dependent on the efficiency of the control measures. The effects of quality control are herein modelled to make an assessment of how reliability management could possibly influence the value of operational partial factors used in design.

For guidance to extend the reliability framework to partial factor modification, reference was made to the Draft *fib* 2010 Model Code. It was later discovered that the provision for partial factor modification in the Draft *fib* 2010 Model Code was carried over from its predecessor, the CEB-FIP 1990 version of the Model Code. The repetition of the provision implies that, perhaps to date, sufficient exploration and implementation of the provision has not been fully achieved in codified practice. The common provision from the Model Codes advises that the values of γ_C and γ_S should be increased if standard geometrical tolerances in design are not fulfilled. Conversely, γ_C and γ_S may be reduced by 0.1 and 0.05 respectively, at the maximum, if standard geometrical tolerances can be reduced by 50 % and are strictly controlled. The provision does not, however, favour the reduction of the set of partial factors according to the degree of control of concrete strength. It argues that such action does not seem justified, because the variation of the control of the specimen can more rationally be taken into account by the compliance criteria included in the control itself. It further argues that even if a better quality of concrete, characterised by a lower coefficient of variation of the strength ensured for a given characteristic strength, would not justify reducing the γ_M values, because this would imply also a lower mean strength. The Model Codes prefer that in instances of very good quality management, e.g. for precast concrete, the conversion factor η included in γ_C, which accounts for scale effects, is adjusted to reduce γ_C.

EC 2, on the other hand, allows in informative annex A, the reduction of γ_S if a quality control system that ensures certain geometrical tolerances is implemented. Corresponding reduction of γ_C is applicable only if the conditions required for the reduction of γ_S are enforced, plus the additional requirement that the coefficient of variation of concrete strength should not exceed 10 %. The concise conditions required for partial modification as given in EC 2 are presented in Table 2.10. To the preference of the Model Codes, EC 2 also suggests that the conversion factor η may be adjusted to reduce the value of γ_C. This reduction of γ_C may, however, be applied after initial reduction of the partial factor due to quality management ensuring reduced deviations in geometry and concrete strength.

It is therefore clear that EC 2 allows modification of the partial factors due to quality control of concrete strength variations, and not just by adjusting the value of η as is preferred by the Model Codes. EC 2 procedures seem to give the benefit of reduced deviations of concrete strength to the designer and design process by allowing reduction of the partial factors used in design, whereas the Model Codes seem to give the benefit to the concrete producer or manufacturer by not adjusting partial factors but increasing the quality of concrete above that required by the design. This should, somewhat, increase structural integrity through superior performance of the concrete which boosts the reputation of the concrete producer.

An investigation was therefore conducted to assess the viability of reducing the operational value of partial factors based on control of the deviations of concrete strength.

3.4.1 Investigation of the effect of reduced deviations on partial factors

The Model Codes argued that quality control may not always achieve a better mean strength of concrete, but could rather achieve a better coefficient of variation associated with a given characteristic strength, thus reducing the mean. Therefore, the investigation was conducted to determine the effect of keeping either the characteristic strength constant and varying the mean, or varying the characteristic strength and keeping the mean constant. Equation [3.12], assuming a log-normal distribution, was used to describe the partial factor for materials, in this case concrete. Operational partial factors for the case of the Eurocode or South African reliability requirements could then be read off Figure 3.3 for the appropriate coefficient of variation of the material property or resistance.

The European Concrete Platform (2008) reports that for EC 2 the coefficient of variation for resistance used in determining the partial factors for materials can be thought of as composed of the coefficients of variation for material strength, model uncertainty and geometry. We can then define α as the ratio of concrete strength variation, Ω_C, to the total coefficient of variation of resistance, herein defined as Ω_R, expressed below as:

$$\alpha = \frac{\Omega_C}{\Omega_R} \qquad [3.14]$$

The coefficient of variation of resistance, Ω_R, can be appropriately elaborated as:

$$\Omega_R = \sqrt{\Omega_C^2 + \Omega_{Rest}^2} \qquad [3.15]$$

Where Ω_{Rest} is a representation of the combined coefficient of variation due to model uncertainty and geometry, with the coefficient of variation of concrete strength singled out as Ω_C.

γ^* was then established as the value of the partial factor due to reduced deviations of concrete strength. As previously discussed, two definitions of this partial factor, γ_1^* and γ_2^*, were considered as defined in the cases below.

Case 1 – Keeping the characteristic strength constant and varying the mean

γ^* is, for this case, defined as:

$$\gamma^* = \frac{X_k}{X_d^*} \qquad [3.16]$$

Where X_k remains the same as in Equation [3.12] and:

$$X_d^* = \mu_R^* \exp(-\alpha_R \beta \Omega_R^*) \qquad [3.17]$$

Where X_d^* and μ_R^* are the adjusted design and mean values of concrete strength, respectively, based on the increased quality control. Ω_R^* is the reduced coefficient of variation of concrete strength also due to increased quality control. Since, in this situation, the characteristic strength of the materials is kept constant, the following equation applies:

$$X_k = \mu_R \exp(-1.645\Omega_R) = \mu_R^* \exp(-1.645\Omega_R^*) \qquad [3.18]$$

Equation [3.18] implies that in this case, even when the mean and coefficient of variation of resistance are altered due to quality control, the characteristic value remains the same. Transposing Equation [3.18], μ_R^* can be expressed as:

$$\mu_R^* = \frac{\exp(-1.645\,\Omega_R)}{\exp(-1.645\,\Omega_R^*)} \cdot \mu_R \qquad [3.19]$$

Substituting Equation [3.19] into Equation [3.17], and then substituting the result and $X_k = \mu_R \exp(-1.645\Omega_R)$ into Equation [3.16] and replacing γ^* by γ_1^*, results in:

$$\gamma_1^* = \frac{\exp(-1.645\,\Omega_R^*)}{\exp(-\alpha_R \beta \Omega_R^*)} \qquad [3.20]$$

Case 2 – Keeping the mean strength constant and varying the characteristic value

γ^* is, for this case, defined as:

$$\gamma^* = \frac{X_k^*}{X_d^*} \qquad [3.21]$$

Where $X_K^* = \mu_R \exp(-1.645\Omega_R^*)$ and $X_d^* = \mu_R \exp(-\alpha_R \beta \Omega_R^*)$. It must be noted that the mean value, μ_R, is kept constant and the values of X_K^* and X_d^* are due to the improved coefficient of variation of resistance, Ω_R^*, which is in turn due to improved quality control. Therefore, γ_2^*, or the ratio of X_k^* to X_d^*, can therefore be defined as:

$$\gamma_2^* = \frac{\exp(-1.645\,\Omega_R^*)}{\exp(-\alpha_R \beta \Omega_R^*)} \qquad [3.22]$$

By comparing Equations [3.20] and [3.22], it is evident that $\gamma_1^* = \gamma_2^*$. Hence, it can be concluded that whether the mean or characteristic value of concrete strength are interchangeably kept constant or varied as a result of quality control, the value of the partial factor γ^* remains unaltered.

Given the unified expression of $\gamma^* = \gamma_1^* = \gamma_2^*$, a parametric study was conducted to determine the effect that different levels of quality control of concrete strength would have on the value of operational partial factor γ_C used in design. The study was conducted to determine the scale or order of reduction of partial factors given a specified amount of improvement of variation in concrete strength due to improved quality control measures. The scale of partial factor reduction was monitored by the relation:

$$scale\ of\ partial\ factor\ reduction = \frac{\gamma^*}{\gamma_{spec}} \qquad [3.23]$$

Where γ^* is as previously defined and γ_{spec} is the value of the partial factor for standard control of variation of concrete strength as obtained from Equation [3.12] and/or Figure 3.4, corresponding to the β-level and coefficient of variation required by the assessment. For purposes of this investigation, the total coefficient of variation for resistance, Ω_R, was assumed to be $15\ \%$. This is a reasonable assumption as the European Concrete Platform (2008) suggests a value for variability of concrete resistance of about $16.58\ \%$ for normal quality control of aspects concerning resistance during design and execution.

The procedure for plotting the graph of $\frac{\gamma^*}{\gamma_{spec}}\ vs. \alpha$ is outlined below.

Procedure for plotting graph of $\frac{\gamma^*}{\gamma_{spec}}$ *vs.* α

1. The process was initiated by assuming α, e.g. $\alpha = 0,\ 0.1,\ 0.2,\ 0.3\ etc.$ α is the ratio of concrete strength variation to the total variation of resistance as defined in Equation [3.14].

2. From Equation [3.14], the value of Ω_C was calculated since Ω_R was assumed fixed as 0.15 for the assessment. This value of Ω_C was used to assess partial factor requirements for conditions of normal quality control.

3. From Equation [3.15], Ω_{Rest} was calculated. No improvement due to increased quality control was thereafter carried out on Ω_{Rest}. The idea was to isolate the effect improved quality control had in controlling the variation for concrete strength, and subsequently to determine the effect this would have on partial factors according to Eurocode and SANS requirements.

4. Thereafter, the improved coefficient of variation of concrete strength, Ω_C^*, was improved (reduced) by the following convention:

   $$\Omega_C^* = \left(\frac{100 - \%\ improvement}{100}\right) \times \Omega_C \qquad [3.22]$$

   Where $\%\ improvement$ is equivalent to the percentage improvement of concrete strength variation due to an effective quality management system.

5. Ω_C and Ω_C^* were used to determine γ_{spec} and γ_C^*, respectively, using Equation [3.12] and Equations [3.20] or [3.22] for a given β and $\%\ improvement$ as well as varying α.

6. A new α is selected and bullets 1 to 5 are repeated for a given β and $\%\ improvement$.

The above process produces downward curves, each for a single β and $\%\ improvement$, on the graph axes $\frac{\gamma^*}{\gamma_{spec}}$ $vs.\alpha$. Table 3.6 and Table 3.7 show the values of $\frac{\gamma^*}{\gamma_{spec}}$ at varying α and percentage improvement of deviations of concrete strength for $\beta = 3.0$ and $\beta = 3.8$ respectively. Figure 3.6, based on data from Tables 3.6 and 3.7, is aimed at characterising the trend of $\frac{\gamma^*}{\gamma_{spec}}$ $vs.\alpha$ for South African ($\beta = 3.0$) and European ($\beta = 3.8$) requirements for resistance.

Table 3.6. Values of $\frac{\gamma^*}{\gamma_{spec}}$ for varying α and improvements on deviations of concrete strength for $\beta = 3.0$

	γ^*/γ_{spec}						
α	***at 5 % improved*** $\boldsymbol{\Omega_C^*}$	***at 10 % improved*** $\boldsymbol{\Omega_C^*}$	***at 20 % improved*** $\boldsymbol{\Omega_C^*}$	***at 30 % improved*** $\boldsymbol{\Omega_C^*}$	***at 50 % improved*** $\boldsymbol{\Omega_C^*}$	***at 70 % improved*** $\boldsymbol{\Omega_C^*}$	***at 90 % improved*** $\boldsymbol{\Omega_C^*}$
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.1	0.9999	0.9999	0.9998	0.9997	0.9996	0.9995	0.9994
0.2	0.9998	0.9996	0.9992	0.9988	0.9983	0.9979	0.9977
0.3	0.9995	0.9990	0.9982	0.9974	0.9961	0.9953	0.9948
0.4	0.9991	0.9983	0.9967	0.9953	0.9930	0.9915	0.9907
0.5	0.9986	0.9973	0.9948	0.9926	0.9889	0.9864	0.9851
0.6	0.9980	0.9961	0.9924	0.9891	0.9836	0.9798	0.9778
0.7	0.9973	0.9946	0.9896	0.9849	0.9771	0.9715	0.9685
0.8	0.9964	0.9929	0.9862	0.9799	0.9689	0.9607	0.9563
0.9	0.9954	0.9910	0.9822	0.9739	0.9586	0.9463	0.9391
1	0.9944	0.9887	0.9776	0.9666	0.9449	0.9238	0.9031

Table 3.7. Values of $\frac{\gamma^*}{\gamma_{spec}}$ for varying α and improvements on deviations of concrete strength for $\beta = 3.8$

	γ^*/γ_{spec}						
α	***at 5 % improved*** $\boldsymbol{\Omega_C^*}$	***at 10 % improved*** $\boldsymbol{\Omega_C^*}$	***at 20 % improved*** $\boldsymbol{\Omega_C^*}$	***at 30 % improved*** $\boldsymbol{\Omega_C^*}$	***at 50 % improved*** $\boldsymbol{\Omega_C^*}$	***at 70 % improved*** $\boldsymbol{\Omega_C^*}$	***at 90 % improved*** $\boldsymbol{\Omega_C^*}$
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.1	0.9999	0.9998	0.9996	0.9995	0.9992	0.9990	0.9990
0.2	0.9996	0.9992	0.9985	0.9979	0.9968	0.9962	0.9958
0.3	0.9991	0.9982	0.9966	0.9952	0.9928	0.9913	0.9905
0.4	0.9984	0.9968	0.9939	0.9913	0.9871	0.9843	0.9829
0.5	0.9974	0.9950	0.9904	0.9863	0.9796	0.9750	0.9726
0.6	0.9963	0.9927	0.9861	0.9800	0.9700	0.9630	0.9595
0.7	0.9950	0.9901	0.9808	0.9724	0.9581	0.9479	0.9426
0.8	0.9934	0.9869	0.9746	0.9632	0.9433	0.9286	0.9207
0.9	0.9916	0.9834	0.9674	0.9522	0.9248	0.9031	0.8904
1	0.9896	0.9793	0.9590	0.9391	0.9007	0.8637	0.8283

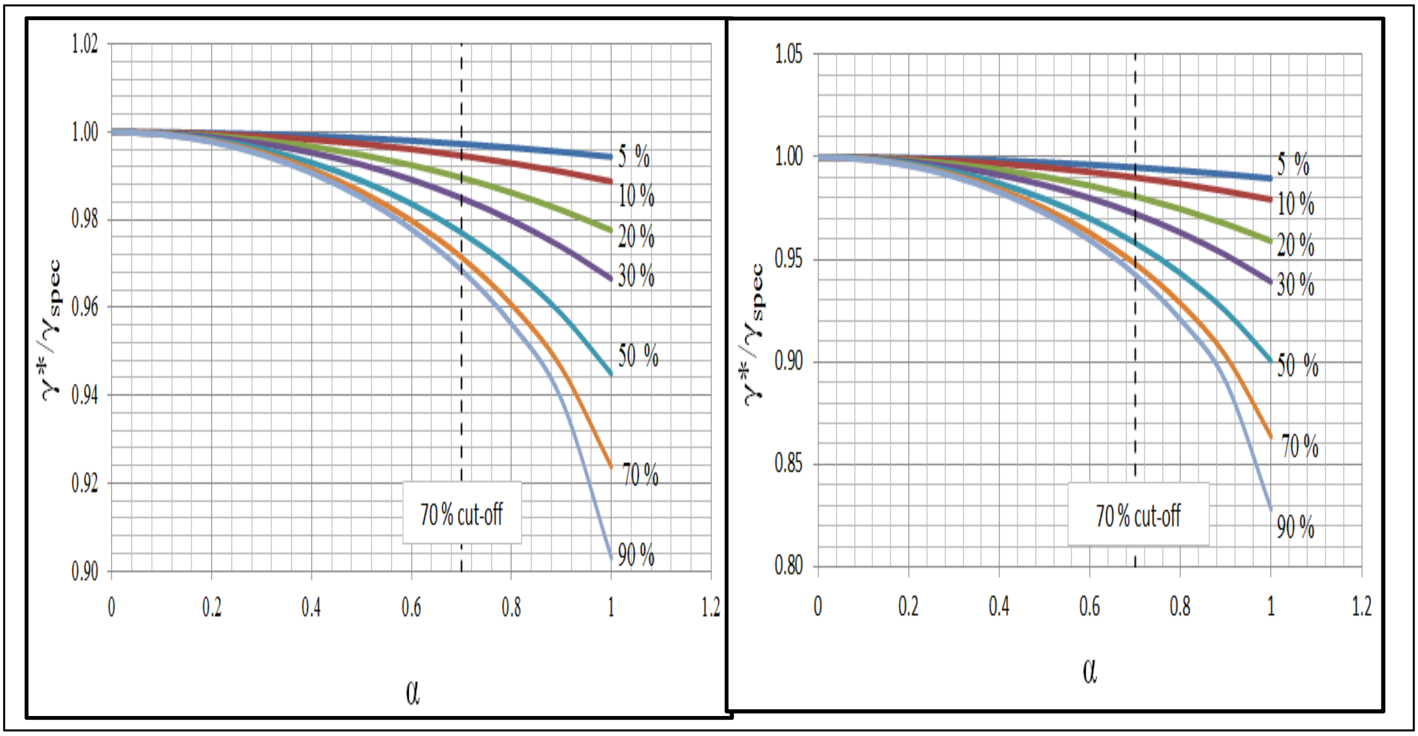


Figure 3.6. Variation of $\frac{\gamma^*}{\gamma_{spec}}$ against α for selected % $improvement$ of concrete strength deviations for $\beta = 3.0$ (***left***) and $\beta = 3.8$ (***right***)

Figure 3.6 shows that the partial factor can be reduced based on reduced deviations of concrete strength and is thus coherent with the suggestions made in EC 2. A judgement call can be made to state that the material property contribution to the total coefficient of variation need not exceed 70 % as modelling uncertainties and geometrical imperfections do contribute as well to the total coefficient of variation. Hence, reduction of partial factors is warranted to the maximum limit of α as 70 %, which is less sensitive as compared to the partial factor reduction experienced in the unrealistic zone of the Graphs when α is greater than 70 %. For % $improvement$ of 90 %, at the 70 % cut-off shown in Figure 3.5, partial factors can be reduced by roughly 3 % following SANS requirements and about 5 % for the case of the Eurocodes.

The results of the investigation demonstrate that the partial factors can be altered based on different levels of quality control. Section 3.4.2 considers a practical assessment, where the effects of differentiated levels of quality control on operational partial factors for Eurocodes are established, and some recommendations are made on the subsequent reduction of partial factors according to the different reliability classes in EC 2, RC1 to RC3. The framework is then extended to SANS 10160-1 requirements where $\beta = 3.0$ and four reliability classes, RC1 to RC4, are established.

3.4.2 Modelling of quality control effects

According to the European Concrete Platform (2008) partial factors in EC 2 were determined from:

$$\gamma_M = \exp\left(\alpha_R \cdot \beta_T \cdot \Omega_R - 1.64 \cdot \Omega_f\right) \qquad [3.23]$$

Equation [3.23] is derived by mathematical simplification of Equation [3.12]. Ω_R is herein defined as:

$$\Omega_R = \sqrt{\Omega_m^2 + \Omega_G^2 + \Omega_f^2} \qquad [3.24]$$

Where Ω_R is the coefficient of variation of resistance, Ω_m the coefficient of variation of model uncertainty, Ω_G the coefficient of variation of geometrical factors and Ω_f the coefficient of variation of material strength. All other variables have been previously defined in this Chapter. Representative models for variability of steel and concrete based on standard European practice, as used by the European Concrete Platform, are described under Normal Quality Control in Table 3.8. Models describing the variability of steel and concrete under improved quality control, derived largely in accordance with Annex A of EC 2 which is summarised in Table 2.10, are also shown in Table 3.8. It should be noted that the coefficient of variation of material strength, Ω_f, for steel does not change with improved quality control. The manufacture of steel is considered to be already highly controlled and therefore taken to be separate from quality control levels on site and at design offices, which are generally of concern here and in EC 0.

Table 3.8. Representative models for variability of steel and concrete

Material	Level of Quality Control	Ω_m	Ω_G	Ω_f	Ω_R
Steel	Normal Quality Control	2.5 %	5 %	4 %	6.87 %
	Improved Quality Control	1.5 %	2.5 %	4 %	4.95 %
Concrete	Normal Quality Control	5 %	5 %	15 %	16.58 %
	Improved Quality Control	4 %	2.5 %	10 %	11.06 %

For concrete, an additional factor of 1.15 is multiplied to the value of the partial factor determined from Equation [3.23]. This factor, suggested for use by the European Concrete Platform is introduced to cover uncertainty arising from the concrete being tested with test specimens specially made and cured in laboratories or controlled conditions rather than from the finished structure. This factor is equivalent to the factor η that takes into account scale effects as presented in Table 2.7. The conversion factor must be determined by measures that are as credible as possible e.g. being deduced from extensive comparative tests. The way in which the values of such coefficients are determined affects the reliability of resistance performance. Figure 3.7 presents graphs based on Equation [3.23] indicating partial factors for concrete, γ_C, and steel, γ_S, as a function of the target reliability for the two cases of standard practice and Improved quality control (QC) measures in accordance with these adjustments. For the design of reference case RC2 structures, EC 2 recommends use of partial factors $\gamma_C = 1.5$ for concrete and $\gamma_C = 1.15$ for steel which are shown in Figure 3.7.

The distinction between reliability classes RC2 and RC3 set by EC 0 is taken as central in describing the effect reliability differentiation of structures may have on the value of operational partial factors. Moving from RC2 to RC3 represents an increase in target reliability of structures from 3.8 to 4.3 for a 50 year reference period. This has practical implications guided by differentiation based on quality measures related to levels of design supervision and inspection of construction works as described in Section 2.2.3. The argument here is that proper implementation of these measures can be effective in achieving the level of control required on the control of geometry and on the control of concrete strength that warrant partial factor reduction as guided by Annex A in EC 2.

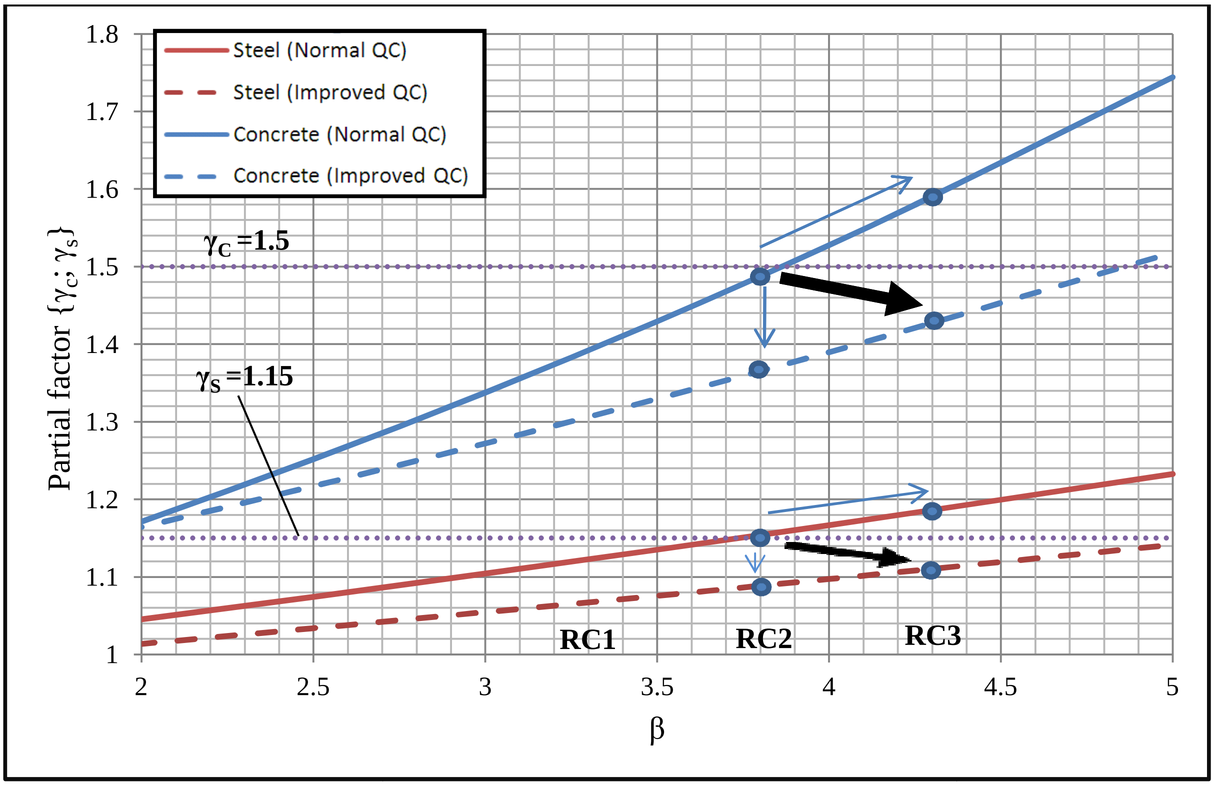


Figure 3.7. Partial factors for concrete and steel based on the European Concrete platform (2008) model; showing three reliability classes RC1, RC2 & RC3 according to EC 0 classification.

The partial factors are shown in Figure 3.5 specifically for:

1. The target levels of reliability for the two reliability classes, RC2 and RC3 from EC 0, and
2. Adjusted for improved quality control as implied by EC 2.

These two sets of partial factors represent the alternative measures available for reliability management. It is clear that improved QC measures, achievable through upper differentiation of structures, are effective in managing reliability. The values γ_C for concrete and γ_S for steel of 1.37 and 1.09, respectively, for improved quality control can be compared to the values 1.35 and 1.05 indicated in Table 2.5 as derived from Annex A in EC 2. The lower set of partial factors adopted for use by EC 2 indicate that substantial credit is given for improved quality control. A more critical issue is the conversion of EC 2 procedures for RC2 to other reliability classes, in particular the important case of RC3 structures. The combined effect of upwards adjustment of partial factors for a higher reliability, together with implementing quality control measures and taking credit for such, as indicated by the bold arrows in Figure 3.7, indicate the effectiveness of the quality control measures in required reduced partial factors 1.43 for concrete (γ_C) and 1.11 for steel (γ_S) even though the reliability class is raised. It should be noted that this adjustment does not merely represent a compensation measure to maintain the reliability level, as stated in Annex B of EC 0. If credit is taken for improved quality control measures, the revised partial factors need to be again adjusted upwards for RC3 structures, as indicated in Figure 3.7. No clear guidance is given in EC 2 on this, and in EC 0 only the principles are given.

A brief assessment of how the developments in this Section would apply to South African conditions is presented below.

Provision for South African conditions

Partial factor modification based on quality measures in accordance with reliability classification of structures from SANS 10160-1 is considered. The most important reliability related adaptation is to maintain the local reference target reliability value of $\beta_T = 3.0$. The required values of partial factors for concrete and steel for the four reliability classes RC1 to RC4 are shown in Figure 3.8.

The overall results show that partial factors of 1.5 for concrete and 1.15 for steel as suggested for use in EC 2 would be conservative, confirming results of the analysis by Holický et al. (2010). This is true even for higher reliability classes, provided that quality control measures are applied; as should be the case for more important structures. Partial factors of $\gamma_C = 1.4$ for concrete and $\gamma_S = 1.1$ for steel should suffice throughout the various reliability classes, provided that improved quality control measures are applied to RC3 and RC4. For RC1 there is clearly an option of reducing partial factors.

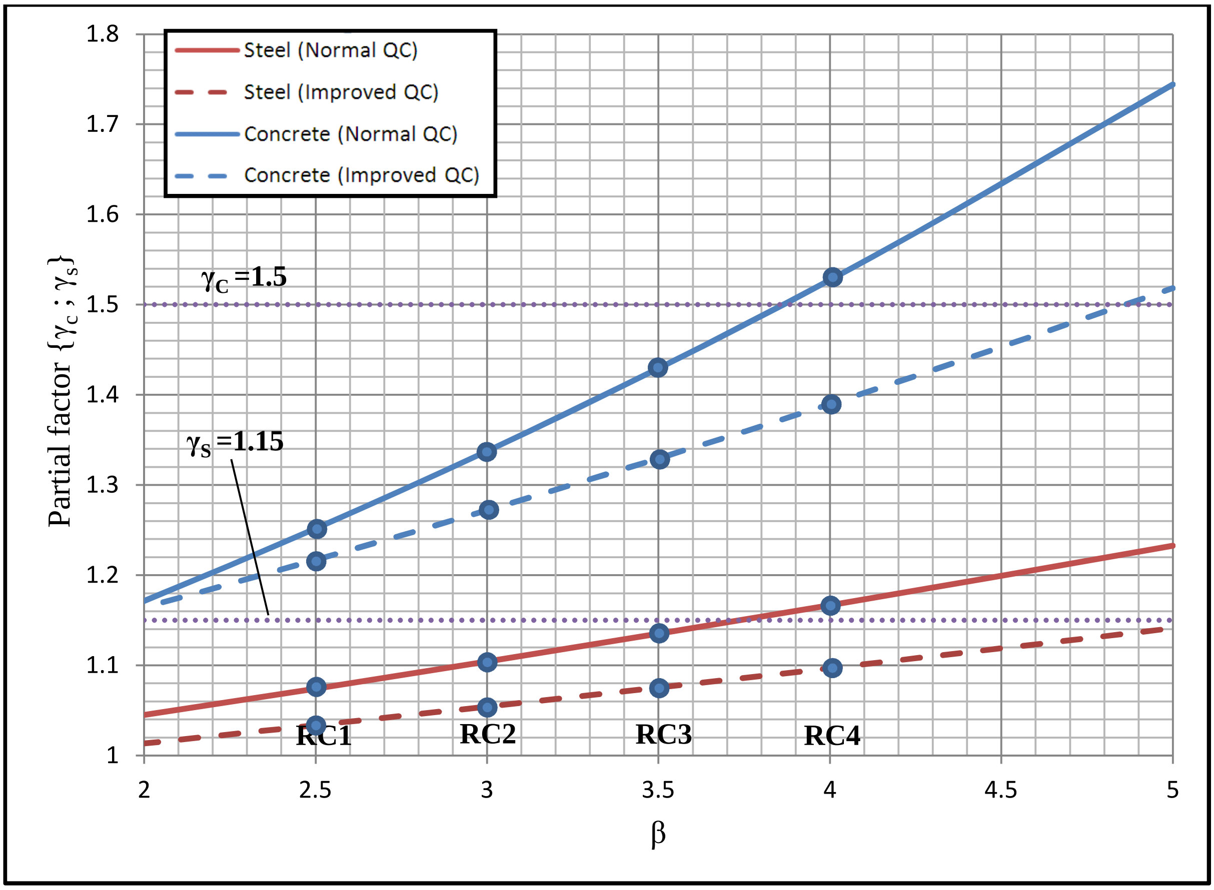


Figure 3.8. Partial factors for steel and concrete for SANS 10160-1 (RC1 to RC4) based on the European Concrete Platform (2008) model

CHAPTER 4

SUMMARY TO PART 1

The basis of design requirements set by EC 0 are concisely presented, with some key issues highlighted, in Chapter 2. These requirements are in line with internationally accepted provisions for the reliability based design of structures and construction works. Following some development in Chapter 2, Chapter 3 dealt with:

1. The choice of the reference level of reliability and the effect its choice has on the value of operational partial factors adopted in design.

2. The importance parametric studies have in specifying basis of design requirements for resistance. This is illuminated by reviewing the results from a study conducted by Holicky et al. (2010). The dominating influence the model uncertainty has on reliability performance is an important finding of this study. This effect does not seem to have been calibrated adequately, as per basis of design requirements, as part of operational factors prescribed for use. The modelling uncertainty associated with the EC 2 shear resistance prediction model for members requiring stirrups is introduced.

3. The link between levels of reliability management presented in EC 0 and partial factor modification allowed by Annex A of EC 2. Though no clear guidance is given in EC 2, levels of reliability management are proven to be effective in managing reliability levels through partial factor modification.

These issues present but a small sample of aspects that require reliability treatment for structural concrete resistance. Much research is still required for reliability aspects related to robustness and serviceability of structures. Model uncertainty across the range of modes of resistance of concrete elements has not yet been fully been incorporated in determining operational partial factors. Part 2 of the thesis is therefore concerned with the statistical and reliability evaluation of shear reinforced concrete members, as a first step in calibrating the EC 2 shear prediction model. An important advancement of this assessment is that an attempt is made to appropriately characterise the model uncertainty of the EC 2 shear prediction model by comparison to a compiled database. The statistics of the model factor are then used for reliability analyses carried out in Part 2 and can further be used to determine suitable partial factors for shear reinforced concrete members. Research conducted by Holický et al. (2010), which is critically summarised in Chapter 2, provides important guidance for South African efforts to calibrate resistance models for structural concrete to suit national conditions and practice. This guidance was useful for the initial calibration of EC 2's shear prediction model for South African practice.

PART II: STATISTICAL AND RELIABILITY EVALUATION OF SHEAR REINFORCED CONCRETE MEMBERS

CHAPTER 5

INTRODUCTION TO PART 2 AND LITERATURE REVIEW

5.1 INTRODUCTION

This Chapter is aimed at providing all the necessary insight and background information on

1. The design theories for shear dealt with in the thesis. The EC 2 shear prediction model, which is based on the lower bound plasticity theory, and the modified compression field theory (MCFT) are the theories herein considered.

2. The statistical and probabilistic techniques that form the base of the theory of structural reliability as well as enable its application.

However, a brief historical background and the focus of the investigation on shear for members requiring stirrups are first established in Section 5.1.1 and 5.1.2 below. This is done so as to first demonstrate that accurate predictions of shear resistance has long been an issue and further, to provide some logic and premise as to why reliability techniques were applied to the EC 2 shear prediction model. In addition, it is also made clear as to why the MCFT was used in the thesis.

5.1.1 Brief historical background of shear design theories for members requiring stirrups

Proper understanding of shear, both with and without shear reinforcement, seems to elude many practitioners and researchers alike. Research efforts on understanding shear date back to as early as 1899 in which Ritter carried out an investigation to determine an effective type of stirrup reinforcement (Balázs, 2010). Inspired by crack patterns typically observed in beam tests, Ritter and later Mörsch in 1902 postulated that after a shear reinforced concrete beam cracks due to diagonal tension stresses, it can ideally be thought of as a parallel chord truss with compression diagonals inclined at 45° with respect to the longitudinal axis of the beam. The stirrups or bent-up bars and the bottom longitudinal reinforcement act as tensile members whilst virtual concrete struts and concrete in the compression zone act as compression members. The truss systems proposed by Mörsch are given in Figure 5.1, where the shaded concrete strips represent the compressed struts. The tensile forces in the shear reinforcement are then obtained by analysis of the truss itself. This model is easy to understand and has ever since been used as the starting point for the development of design models.

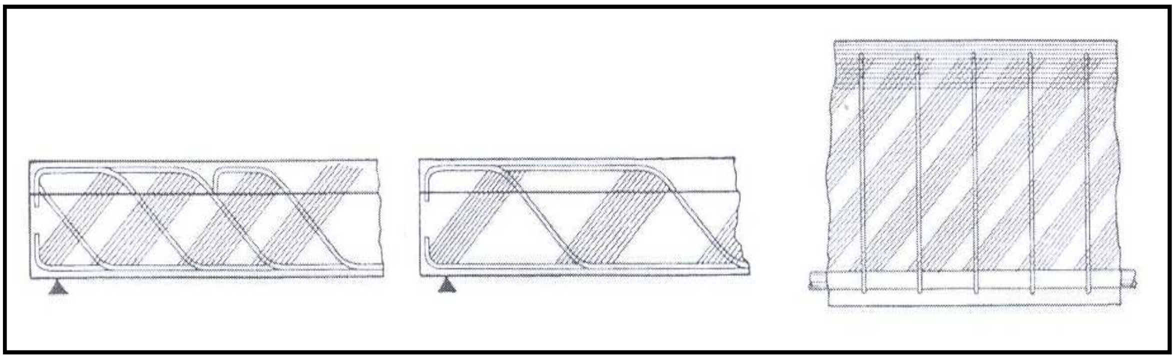


Figure 5.1. Simple or multiple truss systems with bent-up bars and stirrups (Mörsch, 1908, as cited in Balázs, 2010)

Post 1902 to date much research effort has been directed towards the near impossible task of matching shear failure with theoretical models. Fairly recently, in October 2010, a special workshop on shear and punching shear in reinforced and fibre-reinforced concrete members was held in Salò, Italy, to critically discuss and review the latest research and validate shear provisions to be adopted by the new 2010 Model Code. Over the last 50 to 60 years pivotal work by Kupfer, Walther, Kani, Leonhardt and Mönig, Thürlimann et al., Walraven and Collins and Vecchio, amongst others, has contributed much to better understanding of shear in reinforced concrete, including the influence of prestressing on shear behaviour. The important developments concerning design theories that are applicable to non-prestressed members with shear reinforcement are highlighted.

Following his presentation in 1962 at the Shear Colloquium in Stuttgart (Balázs, 2010), Kupfer published a paper in a CEB bulletin in 1964 (ACI-ASCE Committee 445 Report, 2009) that analysed a truss model consisting of linearly elastic members and neglecting the concrete tensile strength, he provided a solution for the inclination of diagonal cracks. Kupfer proposed to select the shear reinforcement with simultaneous yielding of the longitudinal reinforcement and suggested to define the inclination of the strut by using the principle of minimum deformation work. In 1973, Leonhardt and Mönig published a textbook that gave details on the so-called classic truss analogy by Mörsch with 45° struts as well as an improved truss analogy. They gave experimental evidence that illustrated that a lower amount shear reinforcement, expressed as ρ_w, reduces the inclination of cracks in the shear span as shown in Figure 5.2. It can also be observed that more shear reinforcement increases the number of cracks developed before failure, thus promoting more behaviour of structures.

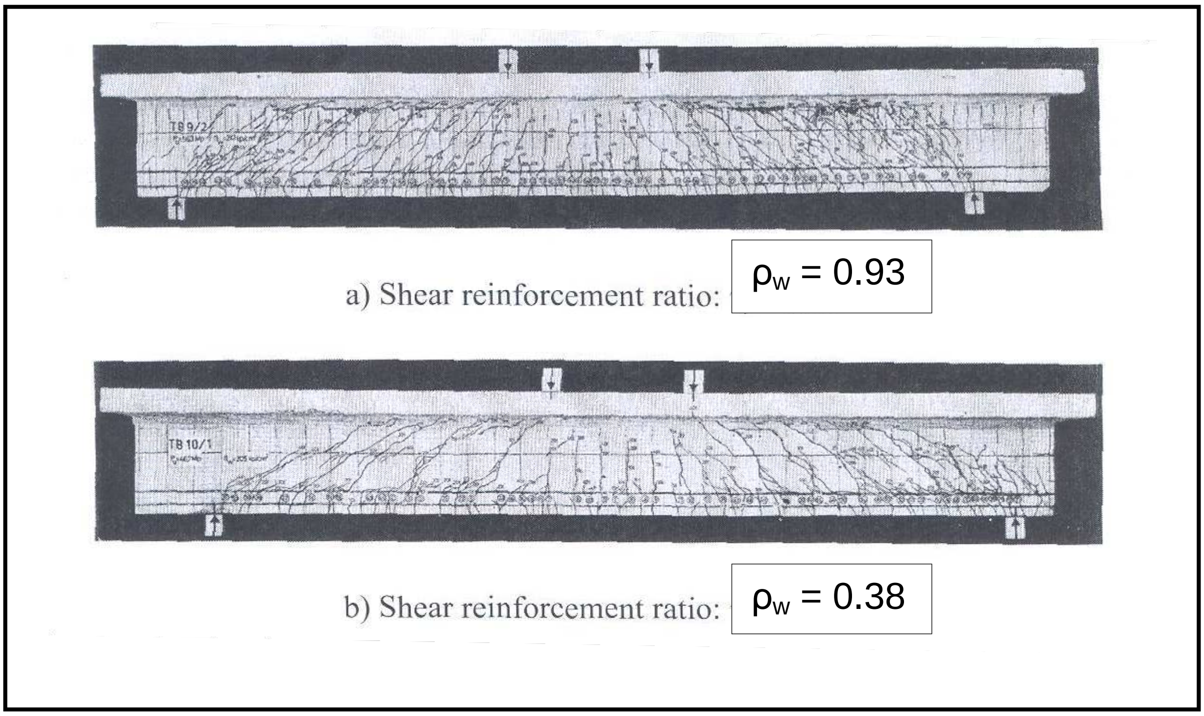


Figure 5.2. Crack patterns of T beams with very different amounts of shear reinforcement (Leonhardt & Mönig, 1973 as cited in Balázs, 2010)

Further, Leonhardt and Mönig paid special attention to the possible failure of concrete struts in I-sections with large flanges and large amounts of web reinforcement as well as relatively thin webs. It was noticed that for such a beam configuration the compressive struts may suddenly fail between inclined cracks, even before the web reinforcement yields as was the case for the I-beam shown in Figure 5.3. It was established therein that web compression failure should give the upper limit of shear resistance.

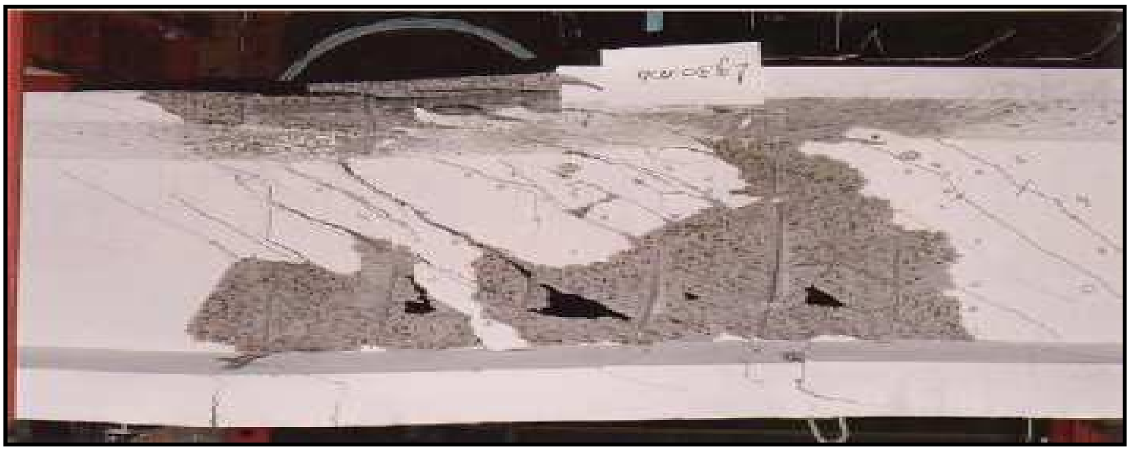


Figure 5.3. Sudden web compression failure due to large amounts of shear reinforcement (Joint Research Centre, 2008)

Further development of plasticity theories by Nielsen and Braestrup in 1975 extended the applicability of the model to non-yielding domains. In 1987, Schlaich, Schäfer and Jennewein extended the truss model for beams with uniformly inclined diagonals. This approach is particularly relevant in regions where the distribution of strains is significantly nonlinear along the depth. Such regions are described as D-regions in Section 5.1.2.

It must be noted that in 1929, the German engineer H.A. Wagner was the first to describe the angle of inclination of diagonal tension whilst solving an analogous problem of shear during the design of a stressed-skin aircraft (ACI-ASCE Committee 445 Report, 2009). Wagner assumed that after the thin metal skin buckled, it could continue to carry shear by a field of diagonal tension, provided that it was stiffened by transverse frames and longitudinal stringers. In an approach later termed the tension field theory and using the deformations of the system, he assumed that the angle of inclination of the diagonal tensile stresses in the buckled thin metal skin would coincide with the angle of inclination of the principal tensile strain as determined from the deformations of the skin, the transverse frames, and the longitudinal stringers. Kupfer’s work, published in 1964, only presented approaches for determining the angle of inclination of the concrete struts assuming that the cracked concrete and reinforcement were linearly elastic. As an improvement, methods for determining the angle of inclination applicable over the full loading range and based on Wagner's developments were established for members in torsion and shear by Collins in 1974 (ACI-ASCE Committee 445 Report, 2009). This procedure became known as the compression field theory (CFT). Vecchio and Collins (1986) developed the modified compression field theory for reinforced concrete elements subjected to shear, which unlike the CFT, accounts for the influence of tensile stresses in the cracked concrete. The MCFT was calibrated to the so-called Toronto large panel tests. Collins et al. (2007), claim that one of the reasons it has taken so long to develop an adequate theory for shear is that the traditional type of shear test in which a simply supported beam is subjected to one or two point loads, while simple to perform, yields results which are difficult to use as the basis of a theoretical model. The modified compression field theory was developed by testing reinforced concrete elements in pure shear using the membrane element tester as shown in Figure 5.4.

Figure 5.4. Membrane element tester (Collins et al., 2007)

5.1.2 Motivation for the investigation on shear

In full view of all the developments presented in Section 5.1.1 above, there is no internationally accepted design theory for shear. Some countries still adopt the Ritter-Mörsch truss models whilst others such as Canada, USA and Spain, have updated their standards to more current and rational models based on the MCFT for shear design.

Through the years, a wealth of investigations have been conducted to determine the accuracy and precision that unfactored design models can achieve in predicting true shear resistance as determined from experiments. This is reflected by the statistics of the model factor, defined in Equation [3.13], which is the ratio of a test result from an experiment to a predicted result by an appropriate model. A model factor greater than 1 implies that the prediction model is conservative and vice versa is true. It has been established from the conclusions of the investigations that, in general, both the accuracy and precision of the model factor associated with shear predictions are not as consistent and reliable as those for other modes of structural resistance, such as flexure and axial deformation. In contrast, the MCFT has been proven to be a powerful tool in predicting shear, even rivalling prediction accuracy obtained from flexural calculations is some instances (Bentz, 2010). This, however, comes as a result of using a model that is relatively complex to understand and computationally more involving than when conventional truss methods are used, usually requiring iterations to converge to the final solution when implemented using a spreadsheet. To this aid, Bentz (2000) as part of his PhD dissertation at University of Toronto, where the MCFT was developed, created a program that is able to predict the full load-deformation response to failure of a reinforced concrete member by sectional analysis. Ultimate shear strength of sections can thus be predicted which is the main objective of design at this point; where model uncertainty is not satisfactory reflecting incomplete understanding of the phenomena. Once adequate prediction of ultimate strength can be made for shear during design, then durability and serviceability aspects can be explored so as to optiminise design with an ultimate view of cutting costs and excess use of material in construction practice. These can be viewed as benefits from scientific and technological advances.

With these uncertainties in shear prediction, the question now is how to proceed to design for shear as it clearly is a phenomena affecting structural performance? The answer is twofold:

1. By the use of structural reliability techniques, to appropriately calibrate shear models with their inherent uncertainties, so as to build in sufficient conservatism into the procedures by the use of partial factors and characteristic values to ensure that safe designs are achieved with regard to shear resistance every time they are used.

2. To apply rational scientific methods such as non-linear analyses through the use of finite elements and use of the MCFT to improve on the model uncertainty inherent in the model itself. In any case, as a requirement of design bases, the model would still have to be calibrated to achieve sufficient conservatism that accounts for other uncertainties.

Narayanan and Beeby (2005) in the Guide to EC 2, state that despite the uncertainties associated with shear prediction models, shear design can be carried out with confidence as the models in EC 2 have been tested against and adjusted to fit a large set of experiments. This is obviously the case for members not requiring shear reinforcement, as EC 2 adopts an empirical formula for design and sufficient background on this work is traceable from research presented by the European Concrete Platform (2008). For the case of members requiring shear reinforcement, the European Concrete Platform (2008) defends the use of the EC 2 prediction model stating that not only does it agree with physical reality, but also because it is a simple equilibrium method, giving transparent view of the flow of forces. Figure 5.13, in Section 5.2.4, gives some view of how the EC 2 variable angle truss method fits with a database consisting of reinforced T- and I-sections with stirrups. No explicit reliability treatment of the model is indicated as was done for members without shear reinforcement.

Interestingly, the EC 2 shear prediction model for shear reinforced members has been shown to yield rather conservative estimates of shear strength for members with low amounts of shear reinforcement whilst unconservative estimates are realised at high amounts of shear reinforcement (Cladera, 2002 and Cladera & Mari, 2007). These findings have been confirmed in Chapter 7 of this thesis to a separately compiled database. Cladera's work which shows some inconsistent trends in modelling uncertainty of the EC 2 shear predicition model compared to other models, has led to Spain adopting an alternative procedure based on the MCFT for the design of shear reinforced members (EHE-08, 2010).

Since the Eurocode prediction model is already in use and a review of the EC 2 model is adequate for the South African adoption of the Eurocodes, Bullet 1 above is taken as central to this thesis. That is, the reliability of the EC 2 shear prediction model is investigated to determine the main parameters that affect the structural performance of shear-reinforced structures. This can be considered as a first step into calibrating the EC 2 shear prediction model. The MCFT is, however, also applied in the thesis to reduce the model uncertainty, through improvement of the statistics (mean and standard deviation) of the model factor, associated with shear prediction. This motivated the use of the MCFT as the general probabilistic model (gpm), or true resistance model, to be used in the reliability analysis of the EC 2 procedure. The concept of the general probabilistic model is explained later in the thesis.

5.1.3 Scope and focus of the investigation on shear for members requiring stirrups

The subject of shear design of reinforced concrete can be divided into two broad categories, known as B and D regions, where B stands for beam or Bernoulli, and D stands for discontinuous or disturbed (ACI-ASCE Committee 445 Report, 2009). In B regions the distribution of strains is linear, whereas the distribution is nonlinear in D regions. This thesis is concerned with B region shear only. Design of B-regions for shear is the dominant design situation in practice. Nonetheless, EC 2 does contain separate provisions for the design of D regions in shear; those are, punching shear and strut and tie models for arch action. A structural concrete member can consist entirely of a D region though it is the more common situation to have B and D regions in the same member or structure. Figure 5.5 below clearly illustrates this concept.

As shown in Figure 5.5, D regions extend a distance equal to the member depth away from any discontinuity, such as change in cross section or the presence of concentrated loads. Examples of B regions in shear include normally reinforced beams, prestressed beams, columns and slabs of constant cross section. Conversely, D-regions encompass regions with discontinuities in cross section, such as beam to column joints, flat slab to column connections, corbels and footings.

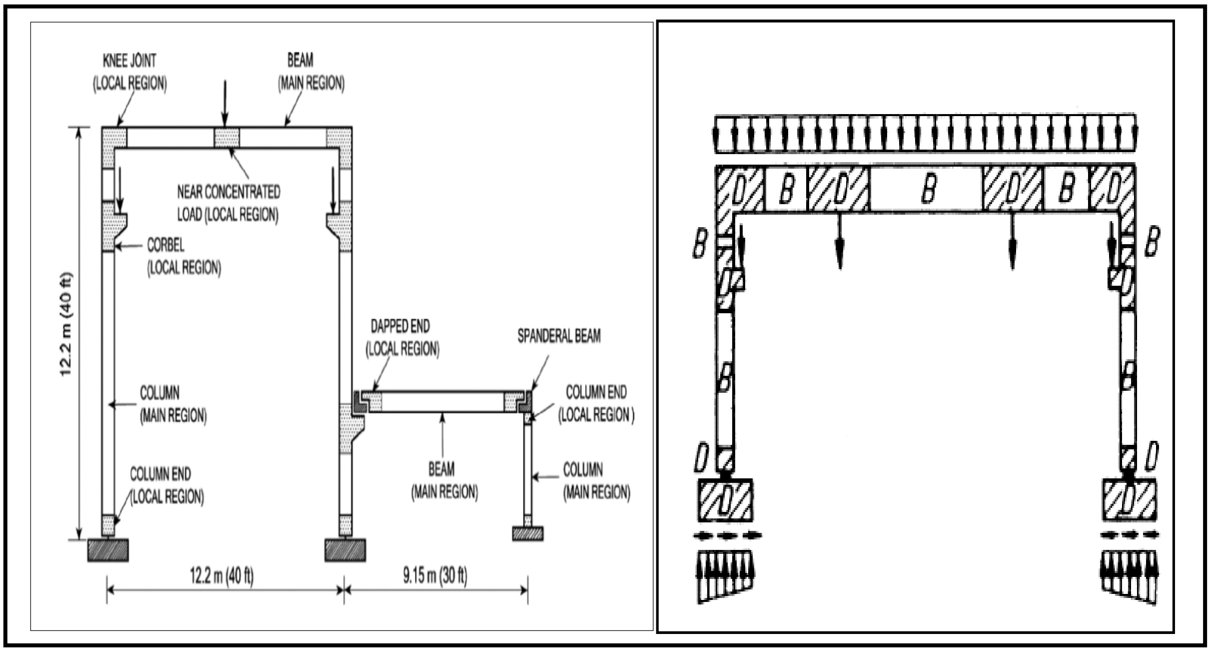


Figure 5.5. Typical frame-type reinforced concrete structures. ***Left***: D regions are shaded portions of the structure and the rest are B-regions (Hsu & Mo, 2010). ***Right***: D and B regions as indicated (ACI-ASCE Committee 445 Report, 2009)

Further, only members requiring vertical shear reinforcement or stirrups are considered in this investigation. That is, members not requiring design shear reinforcement and members designed with bent-up bars for shear resistance are not dealt with in the thesis. In practice, most beams are provided with at least minimum shear reinforcement save for those of minor structural importance such as lintels. The primary focus for members requiring design stirrups was to determine:

1. The statistical properties of the model uncertainty, reflected by the model factor obtained by comparison to a compiled database, of the EC 2 shear prediction model for members requiring stirrups. By use of the Program Response 2000 (Bentz, 2000), the modified compression field theory is applied to the compiled database in order to improve the model uncertainty of the shear prediction model, thus motivating its use as general probabilistic model in reliability analyses of the EC 2 prediction model. This is explained briefly below and elaborated later in the thesis. Bentz (2010) classifies the use of Response-2000 as a Level IV approximation according to the system of the draft of the fib 2010 Model Code (*fib*, 2010). This is the highest level

of approximation possible for shear, achieving the highest precision and accuracy in predictions but at the expense of increased computation time and effort as compared to lower approximation models.

2. To perform preliminary reliability analyses of a representative test case to determine the factors that affect the safety performance of structures in shear.

Though it is the reliability of the EC 2 shear prediction model that is sought, the MCFT is used as the general probabilistic model for shear against which the reliability of the EC 2 prediction model is sought. In principle, the general probabilistic model is a model of true shear resistance. In practice, however, the general probabilistic model can be viewed as a design model with the least uncertainty in its model factor. Various research, including work done by Bentz (2010) and Yoon, Cook and Mitchell (1996) amongst a wealth of others, shows that the MCFT is consistently better at predicting the ultimate shear strengths of concrete members as compared to other design models, mainly those adopted by codes of practice.

As already established in Part 1 of the thesis, a review of EC 2 is an important step for South African efforts in revising the new national code of practice for concrete design SANS 10100-1. The general suite of Eurocodes contains a number of parameters that depend on local conditions including climatic conditions and traditionally used construction materials. Consequently, the values or specification of the parameters might be considerably diverse in different countries. Due to lack of local data the UK National Annex to Eurocode 2 (2005) and the Background Paper to the UK National Annexes to EN 1992-1-1 (2006) are used to extract the UK's nationally determined parameters (NDPs) for use in the investigation on shear. Such dependence on British documents is warranted as the current South African code of practice for concrete structures, SABS 0100-1, is based on the now outdated British code of practice for concrete structures BS 8110-1. However, the use of locally determined parameters is encouraged where possible and research to determine appropriate NDPs for South Africa is a necessary task.

5.2 OVERVIEW AND USE OF SHEAR DESIGN THEORIES FOR MEMBERS REQUIRING STIRRUPS

5.2.1 Mechanism of shear transfer in members with stirrups

Shear reinforcement does not prevent cracks from forming in a member. Its purpose is to ensure that the member will not undergo shear failure before the full bending capacity is reached. When inclined cracks form in a member with stirrups, only the bars that cross the cracks contribute to the shear resistance of the member as shown in Figure 5.6 below.

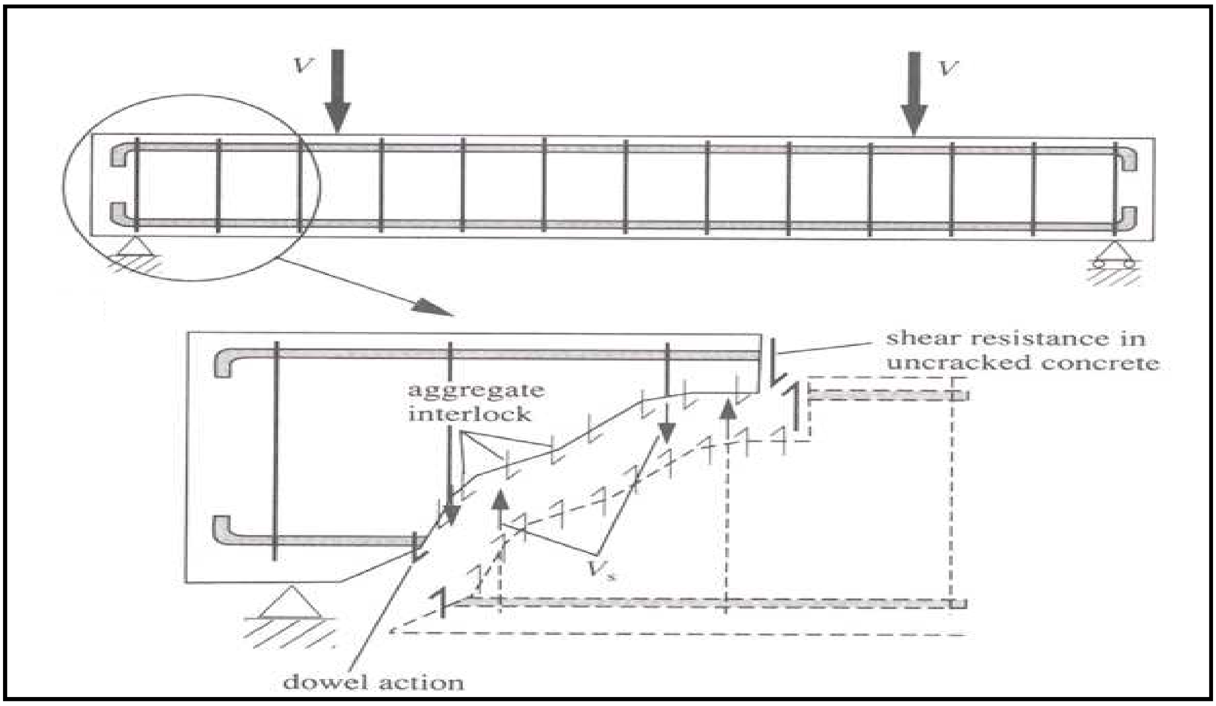


Figure 5.6. Shear resistance of member with stirrups. Elevation showing reinforcement (***top***) and transfer of shear force through the member (***bottom***) . (O'Brien & Dixon, 1995)

The total shear capacity, V_u, provided by the section can be considered as a combination of the capacity of the reinforcement and that of the concrete. Therefore:

$$V_u = V_s + V_c \qquad [5.1]$$

Where V_s is the contribution of the shear reinforcement and V_c, the concrete resistance, is the contribution from dowel action, aggregate interlock and the shear stresses in the uncracked concrete. The dowel action of the reinforcement results from the resistance of the reinforcement to local bending and the resistance of the concrete to localised crushing near the reinforcement. Aggregate interlock occurs in cracked members and results from the forces transmitted across the crack by interlocking pieces of aggregate protruding at a crack. Shear stresses in the uncracked concrete refers to the resistance provided by the portion of the beam where the axial stress is compressive.

If the applied shear force is sufficiently large, the shear reinforcement will reach its yield strength. Beyond this point, the reinforcement behaves plastically and the cracks open more rapidly (O'Brien & Dixon, 1995). As the cracks widen, the proportion of the shear resisted by aggregate interlock is reduced forcing an increase in dowel action and shear stress in the uncracked portion of the section. Failure finally occurs by crushing of the concrete in the compression zone or splitting of the longitudinal tension reinforcement.

5.2.2 Causes and types of shear failure in members without shear reinforcement

Shear reinforcement is provided to control detrimental cracking behaviour that has been experienced in members not reinforced for shear. As such, the cracking or failure of members unreinforced for shear should be well understood as they mostly also affect members with shear reinforcement.

Inclined cracks must develop in a member before complete shear failure can occur (O'Brien & Dixon, 1995). Reinforced concrete structures typically undergo two forms of cracking which are shear-web cracking and shear-flexure cracking. Shear-web cracking occurs at sections where the shear stress predominates throughout the depth and bending moments are negligible or where the web width of the member is small. Such situations are not very commonly encountered in practice but do sometimes occur. On the other hand, shear-flexure cracking occurs in members that are exposed to significant amounts of both bending and shear stresses. The type of shear failure which occurs in a particular member depends on various factors including (ACI-ASCE Committee 445 Report, 2009 and O'Brien & Dixon, 1995):

1. The shape of the cross section
2. The size of the cross section
3. The amount of longitudinal tension reinforcement
4. Maximum moment to shear ratio divided by the effective depth (M/Vd) or alternatively the a/d ratio
5. Axial force

Much of the research into shear behaviour of concrete beams and slabs, including the tests compiled to form the shear database in this thesis, have been carried out using a three-point of four-point bending test in which shear forces and moments are predominant at different locations throughout the length of the beam. Figure 5.7 shows the typical test arrangement and associated bending moment of the four point bending test. The test setup is applicable to members with or without shear reinforcement.

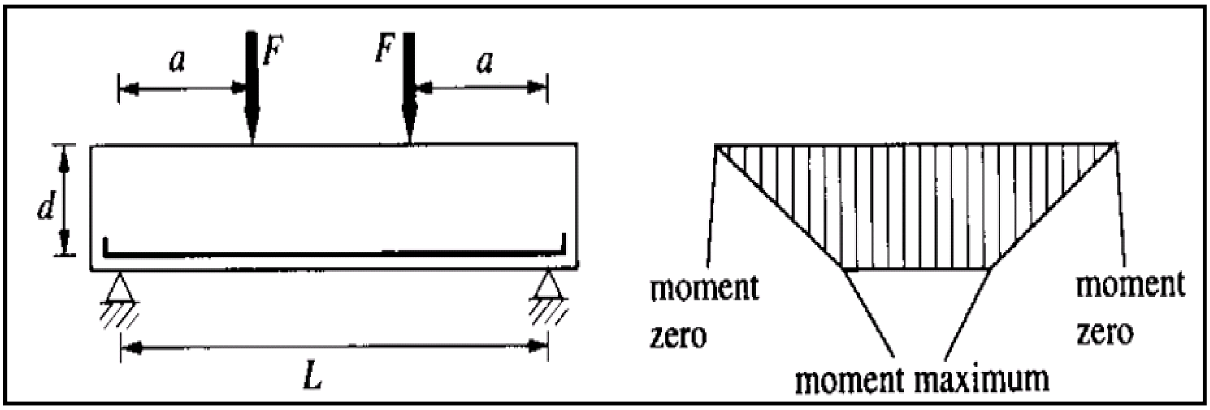


Figure 5.7. Typical test arrangement and bending moment diagram for member without shear reinforcement (O'Brien & Dixon, 1995)

Many empirical design formulae for shear to date have been derived based on the characteristics of shear potrayed in such tests. From extensive testing over the years, it is well established that the shear capacity of a member is strongly dependent on a/d. In general, the shear capacity decreases with increasing a/d. As such, shear failures can be categorised, according to a/d, as illustrated in Table 5.1 (O'Brien & Dixon, 1995). For different members falling into the same category, the sequence of events and the nature of failure are approximately the same.

Table 5.1. Categories of shear failure with associated types of failure based on a/d ratio

Category	a/d	Type of failure
I	$0 < a/d \leq 1$	Deep beam failure
II	$1 < a/d \leq 2.5$	Shear bond failure
III	$2.5 < a/d \leq 6$	Shear compression failure
IV	$6 < a/d$	Diagonal tension failure

Members with very short spans or which have a large effective depth are commonly referred to as deep beams and fall into Category I of Table 5.1. Diagonal shear-web cracks form as the load is almost transferred to the reaction by means of compression. The crack propagates away from the support and toward the applied load. Failure may occur in several ways; by anchorage failure, bearing failure at the support or location of the load, or by cracking failure of the arch. Failure may occur at several times the initial cracking load.

Members that fall into category II behave in a similar manner to Category I members, in that shear-web cracks develop in the region between the loaded sections and the supports. Thereafter, unlike deep beam failure, the crack propagates along the tension reinforcement destroying the bond between reinforcement and concrete in its vicinity. This form of failure is called shear-bond failure. Alternatively, Category II members may fail owing to dowel failure of the longitudinal tension reinforcement at the point of the inclined crack. Further, shear compression failure may also occur which is characterised by crushing failure of the concrete at points of load application.

Category III members are likely to develop flexural cracks before the compressive force is great enough to develop shear-web cracks. The flexural cracks closer to the support, where shear forces are significant compared to moments, develop into inclined shear-flexure cracks and propagate towards the applied loads thus splitting the member in the process. This type of failure is usually referred to as diagonal tension failure. The load at which diagonal tension failure occurs is approximately half that for shear compression/shear bond failure (O'Brien & Dixon, 1995). A majority of beams in practice fall into this Category hence making diagonal tension the most common type of failure.

Category IV members are so slender that shear is hardly ever an issue and as such they tend to fail in pure flexure. This implies that the longitudinal reinforcement yields and the concrete above it crushes before shear cracking occurs.

It should be noted that anchorage failure is not regarded as a shear failure but is rather viewed as a consequence of shearing action in beams. In addition to the types of shear failure considered above, web crushing may occur mostly in members with thin webs that are quite heavily reinforced for shear with wide flanges. This type of failure is more prone to occurring in T- and I-sections as opposed to occurring in beams of uniform rectangular cross-section.

5.2.3 Derivation of the EC 2 variable strut inclination method for design of stirrups

Though not taken into account explicitly, the standard truss model with no concrete contribution is explained by the existence of aggregate interlock and dowel forces in the cracks, which allow a lower inclination of the compression diagonals and the further mobilisation of the stirrup reinforcement (ACI-ASCE Committee 445 Report, 2009). This is shown schematically in Figure 5.6. However, consistent with the principles of truss models first introduced by Ritter and Mörsch, a reinforced concrete beam in shear can be represented by an analogous truss as shown in Figure 5.8. The design equations for shear for members requiring stirrups in EC 2 are derived from the relationships depicted in Figure 5.8 (Mosley et al., 2007).

In the variable strut inclination method all the shear force will be resisted by the provision of stirrups with no direct contribution from the shear capacity of the concrete itself. Using the method of sections it can be seen that, at Section $X - X$ in Figure 5.8, the design force in the vertical link member $V_{Rd,s}$ must equal the design shear force V_{Ed}, that is:

$$V_{Rd,s} = \frac{f_{yk}}{1.15} A_{sw} = f_{ywd} A_{sw} = V_{Ed} \qquad [5.2]$$

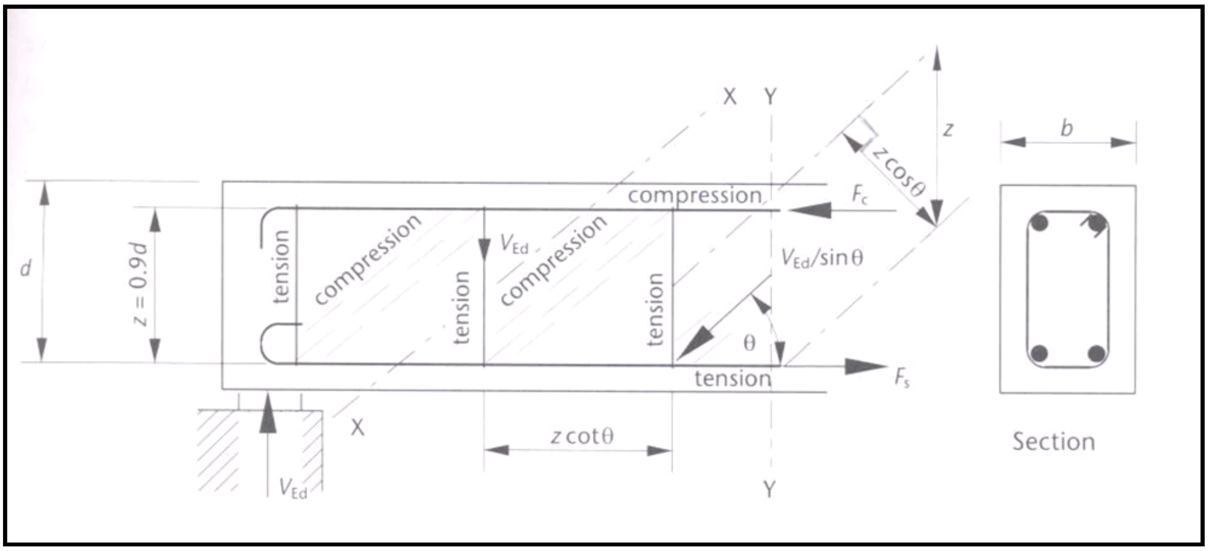


Figure 5.8. Assumed truss model for the variable strut inclination method (Mosley et al., 2007)

If the links are spaced at a distance s apart, then the force in each link is reduced proportionately and is given by:

$$V_{Rd,s}\frac{s}{z\cot\theta} = f_{ywd}A_{sw}$$

$$\therefore\ V_{Rd,s} = V_{Ed} = \frac{A_{sw}}{s} z\, f_{ywd}\, \cot\theta \qquad [5.3]$$

The angle θ increases with the magnitude of the maximum shear force on the beam and hence the compression forces in the diagonal concrete members. EC 2 limits θ to occur between 21.8° ($\cot\theta = 2.5$) and 45° ($\cot\theta = 1$). For most cases of predominately uniformly distributed loading the angle θ will be 21.8° but for heavy and concentrated loads it can be higher in order to resist crushing of the concrete diagonal members (Mosley et al., 2007). The limits placed on θ, which affect the quality and performance of the model's predictions, are set from applying the plasticity theory to the truss model. An appraisal of how the plasticity truss model is implemented in deriving these limits is given in Section 5.2.4.

EC 2 provides an upper limit, $V_{Rd,max}$, on design shear force that is limited by the ultimate crushing strength of the diagonal concrete strut in the analogous truss and its vertical component. As can be seen from Figure 5.8, the effective cross sectional area of concrete acting as the diagonal strut can be taken as $b_w \times z\cos\theta$ and the design concrete stress, f_{cd}, is:

$$f_{cd} = \frac{\alpha_{cc} f_{ck}}{\gamma_C} \qquad [5.4]$$

Where α_{cc} is the coefficient taking account of long term effects on the compressive strength and of unfavourable effects resulting from the way the load is applied. It a parameter open for national choice and the UK adopts a value of 0.85. γ_C is the partial material factor for concrete and is open for national determination. The UK adopts the recommended value of $\gamma_C = 1.5$ for persistent and transient design situations. Therefore:

$$Ultimate\ strength\ of\ the\ strut\ = ultimate\ design\ stress\ \times\ cross - sectional\ area$$

$$= \left(\frac{f_{ck}}{1.5}\right) \times (b_w \times z\cos\theta) \qquad [5.5]$$

$$\therefore \qquad its\ vertical\ component\ = \left[\left(\frac{f_{ck}}{1.5}\right) \times (b_w \times z\cos\theta)\right] \times \sin\theta \qquad [5.6]$$

So that, $V_{Rd,max} = f_{cd} b_w z \cos\theta \sin\theta$ [5.7]

Which by conversion of the trigonometric functions can also be expressed as (Mosley et al., 2007):

$$V_{Rd,max} = f_{cd} b_w z/(\cot\theta + \tan\theta) \qquad [5.8]$$

In EC 2 Equation [5.8] is modified by two factors whose values are subject to national choice: the inclusion of a strength reduction factor, ν_1, taking account of the fact that the beam web, which is transversally in tension, is not as well suited to resist the inclined compression as for cylinder tests and a coefficient accounting of compression in the chord, and α_{cw}, due to prestress. For cases where there is no prestress, the UK adopts the same value of $\alpha_{cw} = 1$ as is recommended in EC 2 and is thus not further considered here. On the other hand, the UK adopts in part the recommended value of ν_1 in EC 2, maintaining:

$$\nu = 0.6\left[1 - \frac{f_{ck}}{250}\right] \qquad [5.9]$$

However, in cases where the design stress of the shear reinforcement is below 80 % of the characteristic yield stress, UK suggests a value of ν_1 different from that recommended in EC 2, which is given in its national annex as:

$$\nu_1 = 0.54(1 - 0.5cos\alpha) \qquad for\ f_{ck} \leq 60\ MPa \qquad [5.10a]$$

$$\nu_1 = (0.84 - f_{ck}/200)(1 - 0.5cos\alpha) > 0.5 \qquad for\ f_{ck} \geq 60\ MPa \qquad [5.11a]$$

For the case of vertical stirrups, $\alpha = 90°$ and the Expressions above reduce to:

$$\nu_1 = 0.54 \qquad for\ f_{ck} \leq 60\ MPa \qquad [5.10b]$$

$$\nu_1 = (0.84 - f_{ck}/200) > 0.5 \qquad for\ f_{ck} \geq 60\ MPa \qquad [5.11b]$$

Equation [5.10] and [5.11] were suggested to be altered for the UK in an investigation conducted and later presented by Jackson & Salim (2006). The motivation to alter EC 2's recommended limits was based on a better fit to experimental data under the 80 % yield rule using the limits presented above than those suggested for use in EC 2.

Section 9.2.2(5) of EC 2 requires that a minimum amount of shear reinforcement, described by the shear reinforcement ratio ρ_w, be provided for all members requiring design shear reinforcement. This is a parameter open for national choice and the UK adopts the value recommended in EC 2. For cases with less than minimum shear reinforcement, the reinforcement provided is ineffective and the shear resistance is best calculated as for a member without shear reinforcement. This, as part of the general relationship between the design shear force and the amount of shear reinforcement, is shown in Figure 5.9. The minimum amount of shear reinforcement, $\rho_{w,min}$, is given in EC 2 as:

$$\rho_{w,min} = (0.08\sqrt{f_{ck}})/f_{yk} \qquad [5.12]$$

An additional requirement for links, as set by EC 2, is that the stirrup spacing must not exceed, in any direction, the lesser of 75 % of the effective member depth, d, and 600 mm.

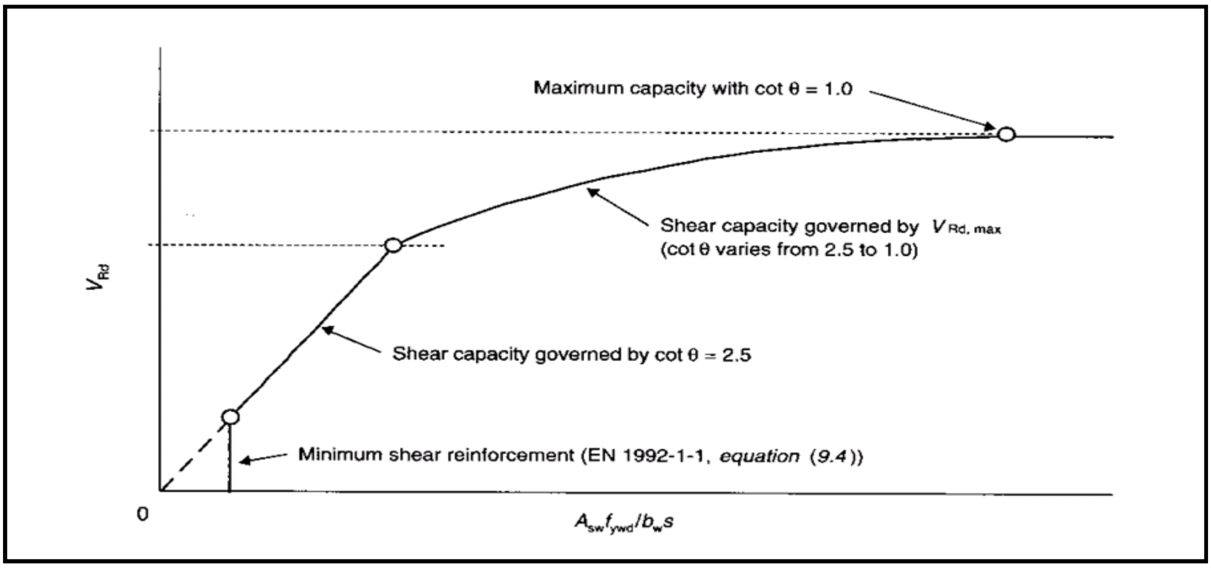


Figure 5.9. Relationship between the design shear force and the amount of shear reinforcement (Narayanan & Beeby, 2005)

It will be noted from Equation [5.3] that the smaller the angle θ, the greater is the shear capacity based on the shear reinforcement. However, the shear capacity based on the crushing strength of the strut, given by Equation [5.8], decreases with decreasing values of θ below 45°. Hence the maximum capacity corresponds to the situation where the capacity based on the shear reinforcement just equals the capacity based on the strength of the strut (Narayanan & Beeby, 2005). According to Narayanan and Beeby (2005), this implies that the actual conditions at failure may be established using Equation [5.13] to estimate the value of θ for which $V_{Rd,s} = V_{Rd,max}$, and then using this value of θ to obtain the required amount of shear reinforcement. Therefore, at failure θ can be found by equating Equation [5.3] and Equation [5.8], thereby yielding (European Concrete Platform, 2008):

$$\theta = sin^{-1}\sqrt{\frac{A_{sw}f_{ywd}}{b_w s v_1 f_{cd}}} \qquad [5.13]$$

Figure 5.10, taken from the European Concrete Platform (2008), depicts the development of $V_{Rd,s}$ and $V_{Rd,max}$ for decreasing angle of strut inclination

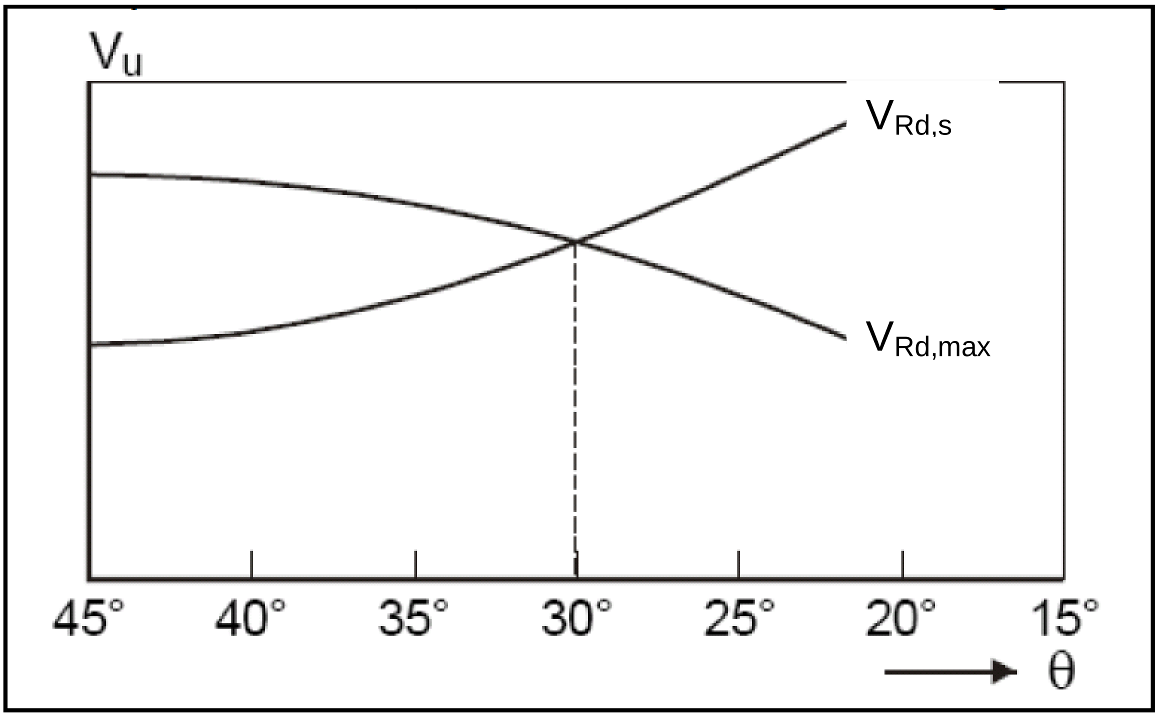


Figure 5.10. Dependence of $V_{Rd,s}$ and $V_{Rd,max}$ on the strut inclination (European Concrete Platform, 2008)

5.2.4 Application of the plasticity theory to the truss model

For members requiring design shear reinforcement, stirrups in this case, EC 2 incorporates a variable angle truss model based on the lower bound theory of plasticity. It should be noted that in this model of shear behaviour all the shear will be resisted by the provision of links with no direct contribution from the shear capacity of the concrete itself (Mosley & Bungey, 2007). Jensen and Lapko (2009) establish that lower bound solutions are applicable when the theory of plasticity is used to find values of the carrying capacity, which are lower than or equal to the yield load by creating stress fields, which fulfill equilibrium conditions and are safe according to the failure criterions of the materials. On the other hand, upper bound solutions are applicable when a load equal to or greater than the yield load is sought by creating failure mechanisms and using the work equation on the mechanisms. An exact solution is found when the highest possible lower bound solution is equal to the lowest possible upper bound solution. In this instance, the yield load or carrying capacity is exactly satisfied. The theory of plasticity assumes the yield plateau as a typical stress-strain curve of a plain concrete specimen under compression instead of strain softening as would be more representative of highly reinforced or unreinforced concrete structures.

The plasticity truss model, as is adopted by EC 2, is based on the assumption that both the longitudinal and the transverse steel must yield before failure (Hsu & Mo, 2010). In order to ensure this mode of failure, the shear elements are divided into two types: under-reinforced and over-reinforced. The application of the theory of plasticity to under-reinforced concrete structures, where failure is governed by yielding of steel reinforcement, seems reasonable. In contrast, it is far less obvious that the theory of plasticity can be applied to over-reinforced or unreinforced concrete structures where the behaviour is governed mainly by the concrete (Ashour & Yang, 2007). This intuitive notion is backed by the results of an investigation presented by Cladera and Mari (2007) that concludes that the unfactored EC 2 prediction model may be slightly unconservative for highly shear-reinforced members and too conservative for slightly reinforced concrete beams as no concrete contribution is considered.

The equations that Ritter and Mörsch derived are applied in the plasticity truss model. Note that the amount of steel in both directions is expressed as a ratio here, and shear stresses are used, not forces.

Shear stress at yielding of stirrups: $\nu_{sw} = \rho_w f_{yw} \cot\theta$ [5.14a]

Shear stress in concrete struts: $\nu_d = f_c \sin\theta \cos\theta$ [5.15]

Shear stress at yield of longitudinal reinforcement: $\nu_{sl} = \rho_l f_{yl} \tan\theta$ [5.16a]

Equilibrium of the beam shear element shown in Figure 5.8 illustrates that the shear force has a component in the longitudinal direction as expressed by Equation [5.16].

From the plasticity theory's assumption that the longitudinal and transverse reinforcement simultaneously yield at failure, then $\nu_{sw} = \nu_d = \nu_{sl}$ at failure. Note that the compression stress in the concrete struts f_c is not equal to the crushing strength of the concrete $f_{c,max}$.

Equation [5.14a] and Equation [5.16a] can be expressed in terms of f_c by substituting Equation [5.15] into each of the two, as follows:

$$f_c \sin\theta \cos\theta = \rho_w f_{yw} \frac{\cos\theta}{\sin\theta} \quad \Longrightarrow \quad f_c \sin^2\theta = \rho_w f_{yw} \qquad [5.14b]$$

$$f_c \sin\theta \cos\theta = \rho_l f_{yl} \frac{\sin\theta}{\cos\theta} \Longrightarrow f_c \cos^2\theta = \rho_l f_{yl} \qquad [5.16b]$$

And:

$$v_d = f_c \sin\theta \cos\theta \qquad [5.15]$$

Now adding the final results from Equation [5.14b] and [5.16b] and utilising the identity $\sin^2\theta + \cos^2\theta = 1$, it follows that:

$$\rho_l f_{yl} + \rho_w f_{yw} = f_c \qquad [5.17]$$

The equation can be normalised in terms of the crushing strength of the diagonal compression struts, $f_{c,max}$:

$$\frac{\rho_l f_{yl}}{f_{c,max}} + \frac{\rho_w f_{yw}}{f_{c,max}} = \frac{f_c}{f_{c,max}} \Longrightarrow \omega_l + \omega_t = \frac{f_c}{f_{c,max}} \qquad [5.18]$$

Where ω_l is known as the longitudinal reinforcement index and ω_t is known as the shear reinforcement index. On this basis, the three failure conditions can be defined as follows:

Under-reinforced elements: $\omega_l + \omega_t < 1$ [5.19]

Over-reinforced elements: $\omega_l + \omega_t > 1$ [5.20]

Balanced condition: $\omega_l + \omega_t = 1$ [5.21]

Over-reinforced elements do not conform to the assumptions of the plasticity truss model and are not considered any further.

The under-reinforced condition $\omega_l + \omega_t < 1$

Substituting Equations [5.14b] and [5.16b] into Equation [5.15], the shear stress at simultaneous yield of shear and longitudinal reinforcement is then given by:

$$v = \sqrt{(\rho_l f_{yl})(\rho_w f_{yw})}$$

And dividing both sides by $f_{c,max}$:

$$\frac{v}{f_{c,max}} = \sqrt{\omega_l \omega_t} \qquad [5.22]$$

Dividing Equation [5.14b] by Equation [5.16b], θ can be calculated by:

$$\tan\theta = \sqrt{\frac{\omega_t}{\omega_l}} \qquad [5.23]$$

The variable angle truss model as applied in EC 2 is based on one of the cases of the balanced condition presented below.

The balanced condition, $\omega_l + \omega_t = 1$

Three cases are distinguished within the limits of the balanced condition, where:

$Case\ (1): \omega_l = \omega_t = 0.5$

For this case of the balanced condition, the yielding of both the longitudinal and transverse steel occur simultaneously with the crushing of the concrete struts at effective stress. From Equation [5.23], $\theta = 45°$ always for this case.

$Case\ (2){:}\ \omega_t < 0.5$

In this case the transverse steel has yielded and is followed by the yielding of the longitudinal steel with simultaneous crushing of the concrete. The longitudinal reinforcement index becomes $\omega_l = 1 - \omega_t$, hence

$$\frac{v}{f_{c,max}} = \sqrt{\omega_t(1-\omega_t)} \qquad [5.24]$$

$$\tan\theta = \sqrt{\frac{\omega_t}{(1-\omega_t)}} \qquad [5.25]$$

$\tan\theta$ in Equation [5.23] will always be less than 1, since $\omega_t < 0.5$, and thus θ is always less than 45° for this case. EC 2 adopts this Case for use in its variable strut inclination method for shear design.

$Case\ (3){:}\ \omega_l < 0.5$

The longitudinal steel has yielded, but the concrete crushes simultaneously with the yielding of the transverse steel. The transverse steel will then be determined by the balanced condition, $\omega_t = 1 - \omega_l$, therefore:

$$\frac{v}{f_{c,max}} = \sqrt{\omega_l(1-\omega_l)} \qquad [5.26]$$

$$\tan\theta = \sqrt{\frac{(1-\omega_l)}{\omega_l}} \qquad [5.27]$$

$\tan\theta$ in Equation [5.23] will always be greater than 1, since $\omega_l < 0.5$, and thus θ is always greater than 45° for this case.

Conditions for design

The balanced condition can also be expressed graphically by a semicircular curve in a $\frac{v}{f_{c,max}}$ vs. ω_t. Squaring both sides of Equation [5.24] and adding 0.5^2 on both sides

$$\left(\frac{\nu}{f_{c,max}}\right)^2 + (\omega_t - 0.5)^2 = 0.5^2 \qquad [5.28]$$

Equation [5.28] represents a circle with radius 0.5 and centre located on the ω_t axis at $\omega_t = 0.5$. This circle, half of which is shown in Figure 5.11, gives the nondimensional relationship between the shear stress ν and the transverse steel stress, $\rho_w f_{yw}$. The axis pointing to the left, drawn to represent ω_l, represents of appropriate values of longitudinal steel that satisfies the balanced condition.

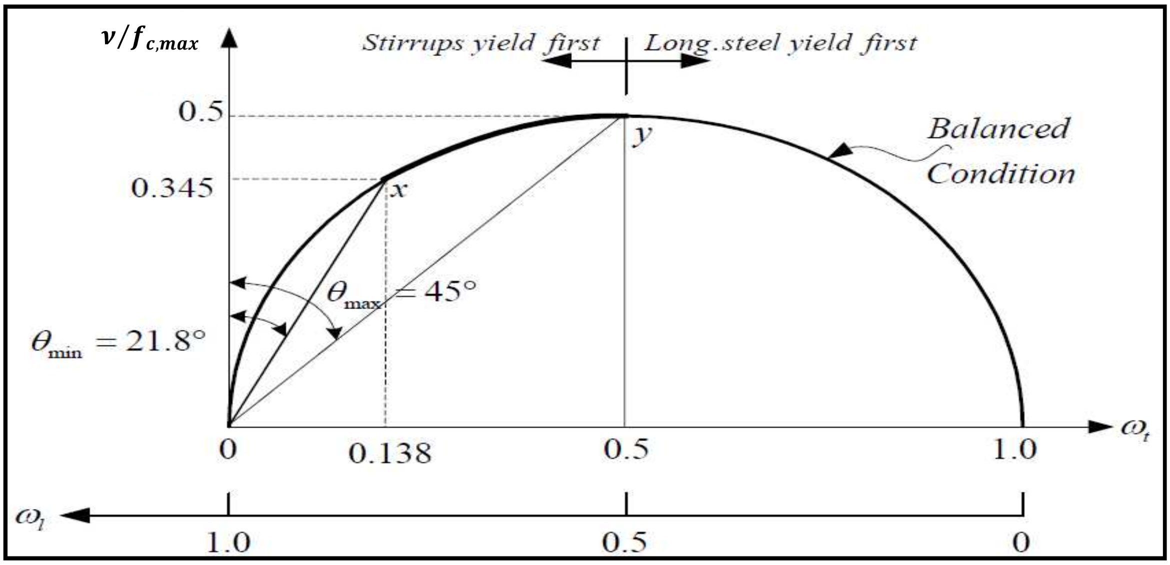


Figure 5.11. Relationship for shear stress ratio *vs.* reinforcement ratios for the balanced condition (Hsu, 1993, as cited in Huber, 2005)

Figure 5.11 shows that, for given amounts of transverse or shear reinforcement, much larger capacities are predicted than those based on the traditional Mörsch model, which assumes a strut inclination of 45°. Further, it can be noticed that for low amounts of shear reinforcement very flat angles of θ are predicted so that mostly lower limits are given to avoid under-reinforced members.

When Equation [5.22] is substituted into Equation [5.23] and ω_l is eliminated, then we get that θ can be calculated from:

$$\tan\theta = \frac{\omega_t}{\nu/f_{d,max}} \qquad [5.29]$$

Design of reinforcement within the semicircle will give an under-reinforced element, while the region outside the semicircle represents over-reinforcement. The EC 2 shear prediction model requires stirrups to be designed in order to satisfy Case 2 of the balanced condition: That is, the case where the shear reinforcement yields before simultaneous crushing of the diagonal concrete struts and yielding of the longitudinal reinforcement.

For Case 2, θ is always less than$45°$. In addition, EC 2 places a lower limit on θ which corresponds to ω_t of 0.138 as calculated from Equation [5.29]. From Figure 5.11 it is evident that the EC 2 shear design procedure is a blend of a constant θ method and a variable angle method. For reinforced concrete members with $\omega_t \leq 0.138$, the normalised shear resistance $\nu/f_{c,max}$ increases linearly with increasing ω_t, due to constant value of θ of $21.8°$. In the instance that ω_t exceeds 0.138, θ gradually increases from $21.8°$ to $45°$ in a non-linear manner. This implies that the shear resistance increases non-linearly with increasing amount of shear reinforcement.

In principle, after initial cracking a redistribution of forces occurs in the webs of shear reinforced concrete beams, resulting in strut inclinations smaller than $45°$. If the shear reinforcement at a crack yields, the truss can, by rotation of the compression struts to a lower inclination, activate more stirrups for the transmission of the shear force and, as such, extend the zone of failure. Due to strut rotation, the stress in the concrete struts increases. Consequently, rotation can only continue until crushing of the concrete occurs. This is schematically shown in Figure 5.12.

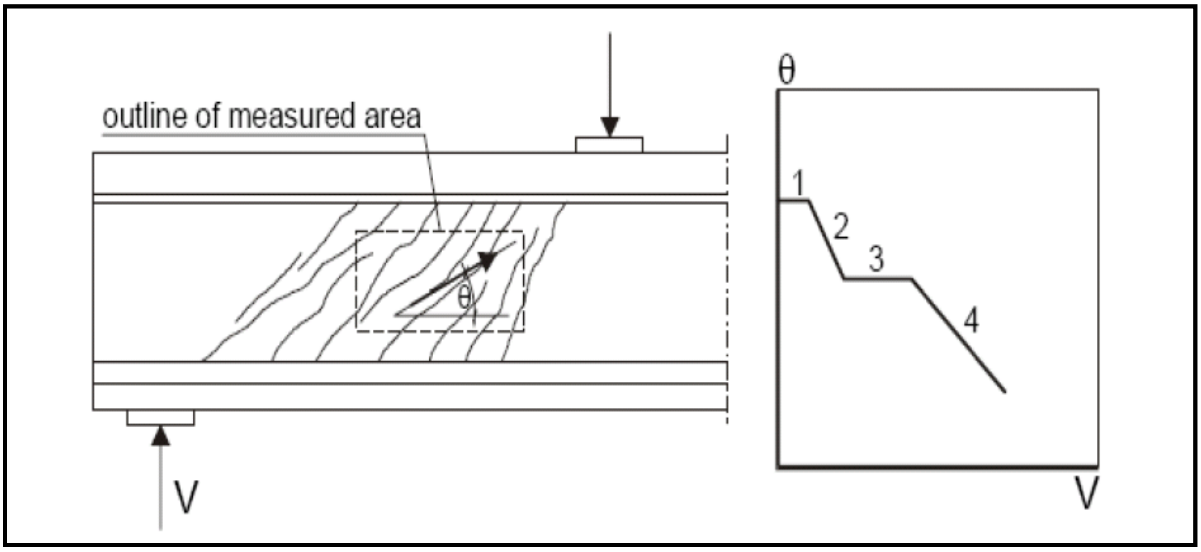


Figure 5.12. Schematic representation of the rotation of the concrete struts as measured on the web of beams with shear reinforcement (European Concrete Platform, 2008)

The four steps of strut rotation, shown in Figure 5.12, are further elaborated. In the beginning of shear loading the beams is uncracked in shear so that the principal strain direction is 45°. At the formation of inclined shear cracks the principal strain direction decreases. After having reached the stabilised inclined crack pattern, a new type of elastic equilibrium is obtained. At this point, the constant principal strain direction is dependent on the stiffness ratio of the concrete member in the cracked state. At yield of the stirrups, through rotation of the struts to a lower inclination in which the beam activates more stirrups to carry the load, the web searches for a new state of equilibrium. Simultaneously, the compressive stress in the concrete struts increases. Stress in the concrete diagonal struts will increase until the crushing strength is attained and the beam fails in shear. Some indication of the accuracy of this prediction method, as is adopted by EC 2, is given in Figure 5.13. More accurately, Figure 5.13 shows a verification of Equation [5.3] and [5.8] with the limit $\cot\theta = 2.5$ to a limited database comprising non-prestressed T- and I - beams with stirrups as presented by the European Concrete Platform (2008). Such limited justification that lacks reliability basis is not adequately sufficient to defend the model as properly calibrated for use in practice or for adoption by design codes. It is therefore the aim of this thesis to initiate, through preliminary reliability analyses done in Chapter 8, a proper calibration of the EC 2 variable strut inclination method.

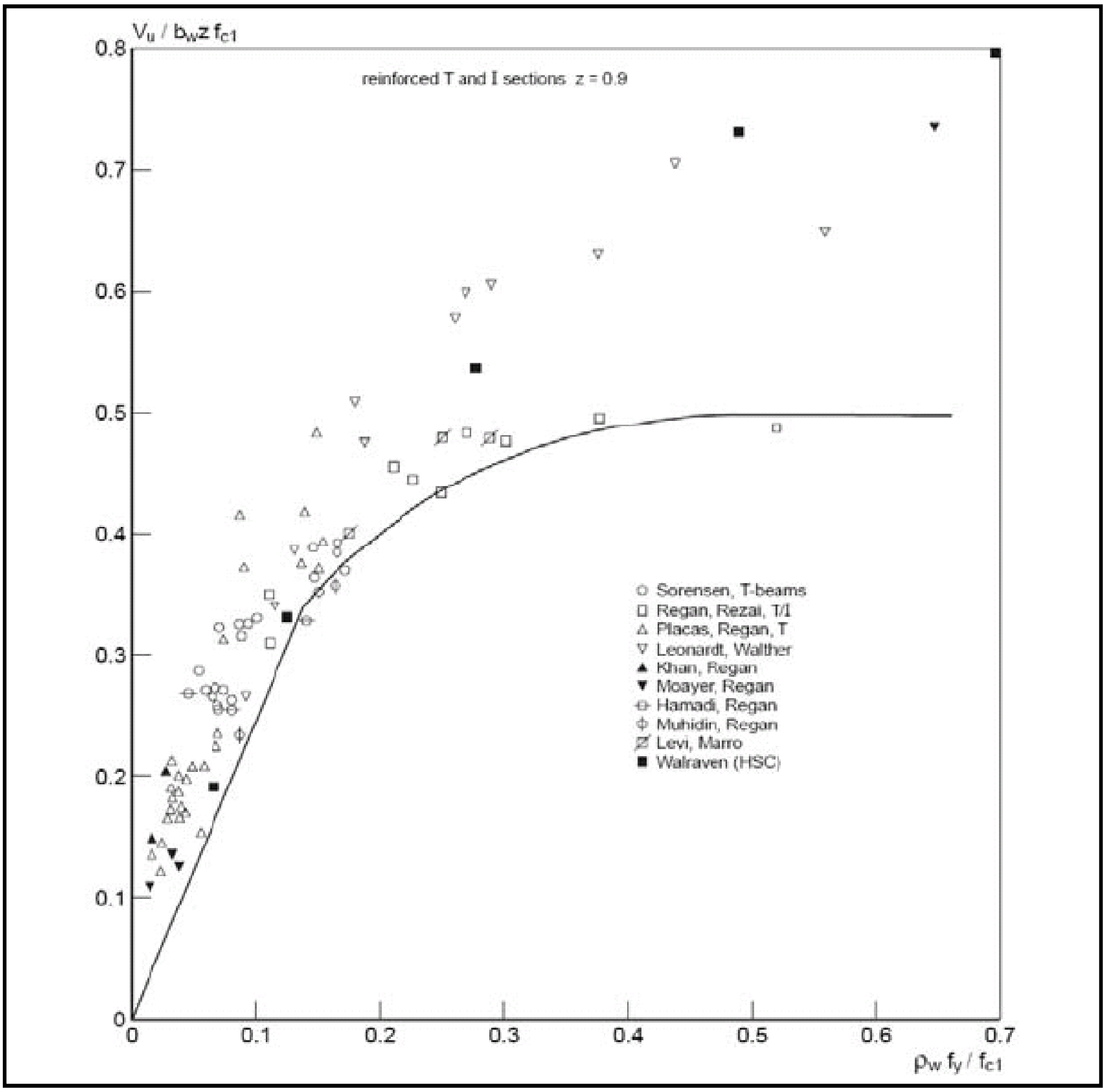


Figure 5.13. Non-prestressed beams with vertical stirrups – relationship between shear strength and stirrup reinforcement (European Concrete Platform, 2008)

5.2.5 Background of the Modified Compression Field Theory (MCFT): Important aspects and derivations

The modified compression field theory (MCFT) was developed from the compression field theory (CFT) for reinforced concrete in torsion and shear. In both models, the cracked concrete is treated as a new material with its own stress-strain characteristics. The CFT and the MCFT both have an extended rational base as compared to conventional truss models that arises from not just considering equilibrium, but additionally treating compatability as well as more general stress-strain relationships of the steel and concrete, all of which are formulated in terms of average stresses and average strains. The angle of inclination of the compressive struts, θ, is determined by considering the cross-sectional dimensions of a member and its deformations, caused by bending moments concomitant with shear at the studied section, of the transverse reinforcement, the longitudinal reinforcement and the diagonally stressed concrete (Cladera & Mari, 2007). With these methods alongside equilibrium conditions, compatability conditions, and stress-strain relationships for both the reinforcement and the diagonally cracked concrete, the load deformation response of a member subjected to shear can be determined. The MCFT is a further development of the CFT that accounts for the influence of tensile stresses in cracked concrete. It recognises that the local stresses in both the concrete and the reinforcement vary from point to point in the cracked concrete, with high reinforcement stresses but low concrete tensile stresses occurring at crack locations.

The MCFT may be explained as a truss model in which the shear strength is the sum of the steel and concrete contribution. As such, it provides itself as a general model for the load-deformation behaviour of two-dimensional cracked reinforced concrete subjected to shear. It models concrete considering concrete stresses in principal directions summed with reinforcing stresses assumed to be only axial. A key assumption used to simplify the development of the MCFT is that the principal strain coincides with the principal stress directions. This assumption is confirmed by experimental measurements, which show that the principal directions of stress and strain are parallel within $\pm 10°$ (Vecchio & Collins, 1986).

The MCFT takes into account tensile stresses in the concrete between the cracks, and employs experimentally verified average stress-strain relationships for the cracked concrete (Vecchio & Collins, 1986). Concrete stress-strain relationships in compression and tension were originally derived from tests performed by Vecchio at the University of Toronto using the membrane element tester shown in Figure 5.4 (Bentz, 2000). The MCFT, unlike the conventional plasticity truss models such as the EC 2 model, accounts directly for the components of shear failure such as aggregate interlock and friction, dowel action and longitudinal steel, and shear carried across uncracked concrete (ACI-ASCE Committee 445 Report, 2009).

The most important assumption in the model is that the cracked concrete in reinforced concrete can be treated as a new material with empirically defined stress-strain behaviour.

This behaviour differs from the traditional stress-strain behaviour as determined from conventional cylinder compressive strength tests. The strains used for these stress-strain relationships are average strains, that is, they lump together the combined effects of local strain at cracks, strains between cracks, bond-slip, and crack slip (Bentz, 2000). According to Bentz (2000), the calculated stresses are also average stresses in that they implicitly include stresses between cracks, stresses at cracks, interface shear transfer on cracks, and dowel action. In contrast, failure of reinforced concrete elements may not be governed by average stresses, but rather by considering the steel stress at the crack and the ability of the crack surface to resist stresses. Therefore, an explicit check must be made to ensure that the average stresses are compatible with the actual cracked condition of the concrete.

In accordance with the conceptual scheme of the MCFT presented above, the Equations of the MCFT are developed below with reference to Figure 5.14, which shows the free body diagrams for stress and strains and their associated Mohr's circles.

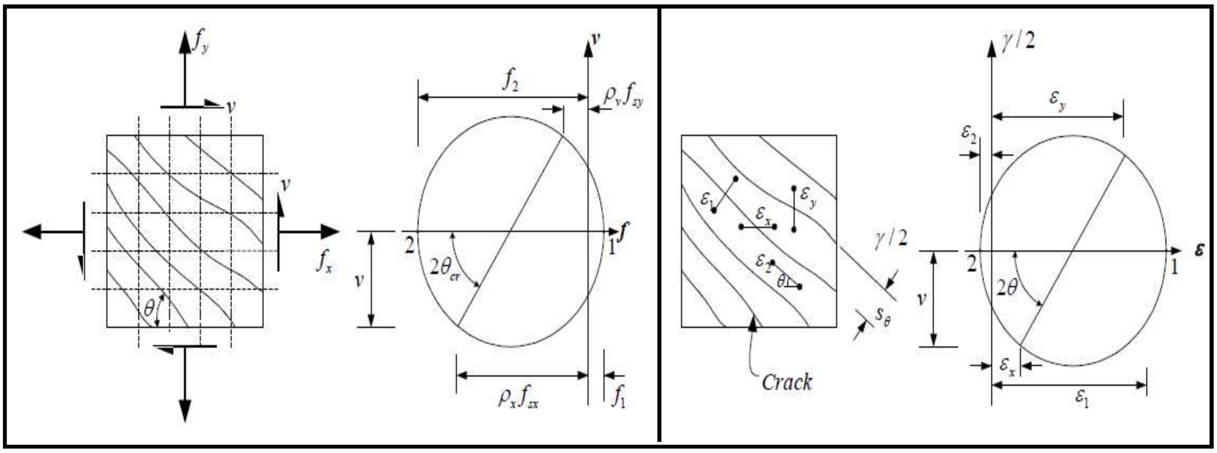


Figure 5.14. Average stress (***left***) and average strain (***right***) free body diagrams with associated mohr's circles in a reinforced concrete element (Vecchio & Collins, 1986)

Equilibrium

By considering force equilibrium, the stress in the transverse and longitudinal reinforcement are related to the shear stress and tensile stresses in the cracked concrete by Mohr's circle for concrete stresses. Therefore, shear in the section is resisted by the diagonal compressive stresses, f_2, together with the diagonal tensile stresses, f_1. The tensile stresses vary from 0 at the cracks to a maximum between cracks. From the left portion of Figure 5.14, the following Equations are derived:

$$\rho_y f_{sy} = f_y + \nu \tan\theta - f_1 \qquad [5.30]$$

$$\rho_x f_{sx} = f_x + \nu \cot\theta - f_1 \qquad [5.31]$$

$$f_2 = \nu(\tan\theta + \cot\theta) - f_1 \qquad [5.32]$$

Where ρ_x and ρ_y are the reinforcement ratios in the longitudinal and transverse directions, f_x and f_y are the stresses in concrete in the x- and y-directions respectively, and ν is the shear stress on the element.

Compatability

Compatability requires that any deformation experienced by the concrete must be matched by an identical deformation of the reinforcement. Thus, it holds that:

For the transverse direction, $\varepsilon_{sy} = \varepsilon_{cy} = \varepsilon_y$ [5.33a]

For the longitudinal direction, $\varepsilon_{sx} = \varepsilon_{cx} = \varepsilon_x$ [5.33b]

From the Mohr's circle on the right of Figure , the strains in the transverse and longitudinal directions (ε_y and ε_x) are related to the principal tensile strain ε_1 and the principal compressive strain ε_2 as follows:

$$\varepsilon_x + \varepsilon_y = \varepsilon_1 + \varepsilon_2 \qquad [5.34]$$

$$\tan^2\theta = \frac{\varepsilon_x - \varepsilon_2}{\varepsilon_y - \varepsilon_2} \qquad [5.35]$$

The stress-strain relationship for steel and cracked concrete are needed to relate the stresses (Equations [5.30] to [5.32]) to the strains (Equations [5.33a] and [5.33b])

Reinforcement stress-strain relationship

The reinforcement stress-strain relationship is a typical bilinear diagram, of which linear pre-yield stresses can be determined from hooke's law as:

$$f_{sx} = E_s \varepsilon_x \leq f_{x,yield} \quad [5.36a]$$

$$f_{sy} = E_s \varepsilon_y \leq f_{y,yield} \quad [5.36b]$$

Where E_s is the modulus of elasticity of the reinforcement, $f_{x,yield}$ and $f_{y,yield}$ are the yield points of the reinforcement in the $x-$ and $y-$directions respectively, and f_{sx} and f_{sy} are the service stresses in the reinforcement in the $x-$ and $y-$directions respectively.

Concrete stress-strain relationship

It can be observed from Figure 5.14 that the concrete web acts not only in compression in direction 2, but also acts in tension in direction 1. The stress-strain relationships for diagonally cracked concrete, applied in the MCFT, are given by the following relationships derived from experiments:

$$\varepsilon_2 = -0.002(1 - \sqrt{1 - f_2/f_{2,max}} \quad [5.37]$$

$$Where,\ \ f_{2,max} = f_c/(0.8 + 170\varepsilon_1) \quad [5.38]$$

$$And,\ \ f_1 = \frac{f_{cr}}{1+\sqrt{500\varepsilon_1}} \quad [5.39]$$

$$But\ with\ the\ limitation\ that,\ \ f_1 \leq \frac{0.18\sqrt{f_{c,max}}\tan\theta}{0.3+\frac{24w}{agg+16}} \quad [5.40]$$

$$Where,\ \ w = \frac{s_x}{\sin\theta}\varepsilon_1 \quad [5.41]$$

f_{cr} is the principal compressive stress at initial cracking taken as $0.33\sqrt{f_{c,max}}$ in MPa units, $f_{2,max}$ is the crushing strength of the diagonally compressed strut, w is the crack width in mm, s_x is the perpendicular spacing of cracks inclined at θ. The concrete and reinforcement stress-strain graphs are presented in Figure 5.15. Figure 5.15 illustrates that the behaviour of cracked concrete subjected to tensile straining differs from that of the cylinder test where no tensile straining takes place. The peak compressive stress is much reduced compared to that of the cylinder test where no tensile straining takes place.

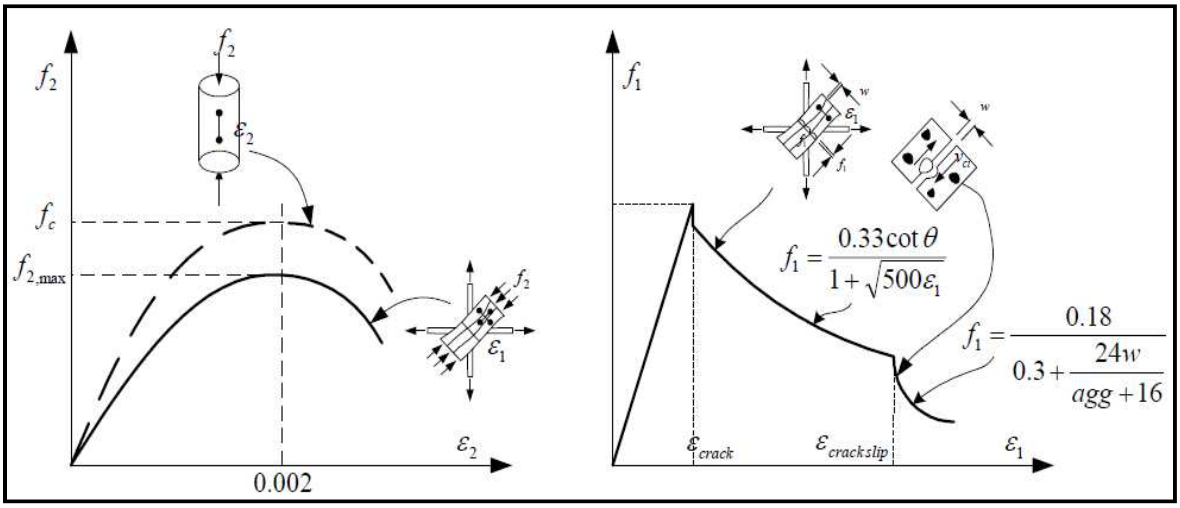


Figure 5.15. Stress strain relationship for cracked concrete in compression (***left***) and tension (***right***). (Collins et al., 1996, as cited in Huber, 2005)

Crack check

In checking the conditions at the crack, the actual complex crack pattern is idealised as a series of parallel cracks, all occurring at an angle θ to the longitudinal reinforcement and spaced a distance s_θ apart. Figure 5.16 depicts, in terms of free body diagram and associated mohr's circle, the equilibrium in terms of local stresses at a crack.

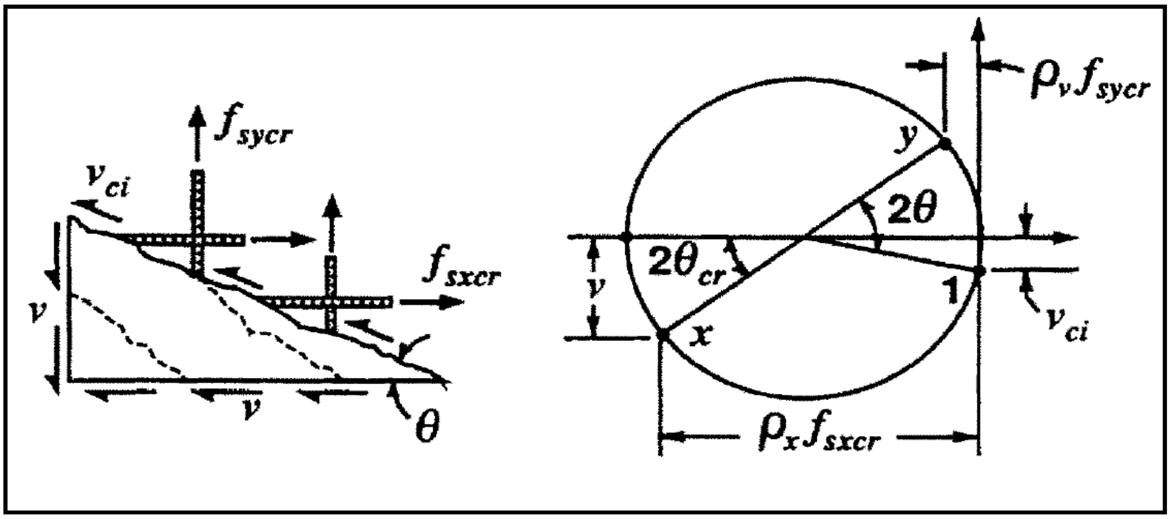


Figure 5.16. Equilibrium in terms of local stresses at a crack (ACI-ASCE Committee 445 Report, 2009)

From Figure 5.16, the reinforcement stresses at a crack can be determined as:

$$\rho_x f_{sxcr} = f_x + v \cot\theta + v_{ci} \cot\theta \qquad [5.42]$$

$$\rho_y f_{sycr} = f_y + v \tan\theta - v_{ci} \tan\theta \qquad [5.43]$$

The ability of the crack interface to transmit the shear stress, v_{ci}, depends on the crack width, w. The limiting value of v_{ci} is given by the inequality (ACI-ASCE Committee 445 Report, 2009):

$$v_{ci} \leq 0.18 \frac{\sqrt{f_{c,max}} \tan\theta}{0.3 + \frac{24w}{agg+16}} \qquad [5.44]$$

5.2.6 Design of members based on the MCFT

The relationships of the MCFT derived above can be used to predict the shear strength of beams. There are a few simplified methods on the MCFT that have been adopted in a number of national design codes. The design procedure elaborated by the ACI-ASCE Committee 445 on shear and torsion (2009) is elaborated in this Section. Assuming that the shear stress in the web is equal to the shear force divided by the effective shear area $b_w d$, and that, at failure, the stirrups will yield, equilibrium Equations [5.30] to [5.32] can be rearranged to give the following expression for the shear strength, V_n (ACI-ASCE Committee 445 Report, 2009):

$$V_n = V_c + V_s + V_p \qquad [5.45]$$

$$V_n = f_1 b_w d \cot\theta + \frac{A_v f_y}{s} d \cot\theta + V_p \qquad [5.46]$$

$$V_n = \beta \sqrt{f_{c,max}} b_w d + \frac{A_v f_y}{s} d \cot\theta + V_p \qquad [5.47]$$

Where V_c is the shear strength provided by the tensile stresses in the cracked concrete, V_s is the shear strength provided by tensile stresses in the stirrups, and V_p is the vertical component of the tension in the inclined prestressing tendon. The effective shear depth, d, is taken as the flexural lever arm and it need not be taken less than $0.9d$. β, in this particular case, is the concrete tensile stress factor indicating the ability of diagonally cracked concrete to resist shear.

The shear stress that the web of a beam can resist is a function of the longitudinal straining in the web. For design calculations, ε_x can be approximated as the strain in the tension chord of the equivalent truss. From this, it therefore holds that:

$$\varepsilon_x = \frac{M_u/d_v + 0.5N_u + 0.5V_u \cot\theta - A_{ps} f_{po}}{E_s A_s + E_p A_{ps}} \leq 0.002 \qquad [5.48]$$

Where M_u, V_u, and N_u can take the value of the factored design forces (moment, shear and axial force respectively) or for shear strength evaluations, can be the unfactored forces themselves. f_{po} is the stress in the tendon when the surrounding concrete is at zero stress, which may be taken as 1.1 times the effective stress in the prestressing f_{se} after all loses. A_s and A_{ps} are the area of nonprestressed and prestressed reinforcement respectively, both placed on the tension side of the member.

If ε_x and the crack spacing are known, the shear capacity corresponding to a given quantity of stirrups can be calculated. This is equivalent to finding the values of β and θ. On the other hand, values of β and θ determined from the modified compression field theory (Vecchio & Collins, 1986) and suitable for members with at least minimum web reinforcement are given in Figure 5.17. Minimum reinforcement ratios can be set by different countries for use, as given in Equation [5.12] for EC 2 and adopted for use in the UK. The following assumptions were made in determining the values presented in Figure 5.17:

1. The amount and spacing of the stirrups would limit the crack spacing to about $300\ mm$.
2. The values of θ in Figure 5.17 ensure that the tensile strain in the stirrups is at least equal to 0.002, and
3. The compressive stress in the concrete does not exceed the crushing strength

The values of θ that satisfy these requirements from Figure 5.17 can be considered to result in close to the smallest amount of total shear reinforcement being required to resist a given shear.

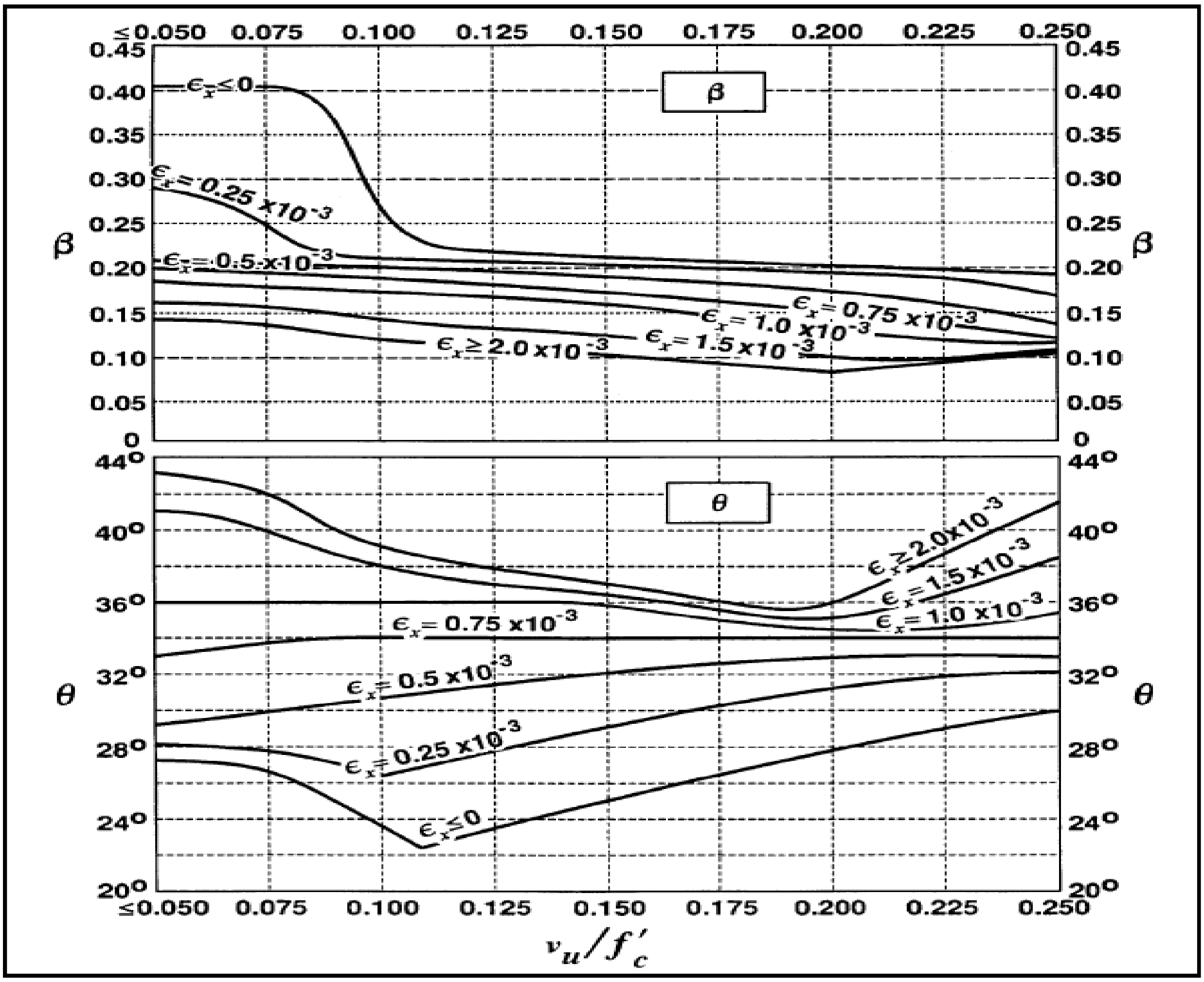


Figure 5.17. Values of β and θ for members containing at least the minimum amount of stirrups (ACI-ASCE Committee 445 Report, 2009)

5.2.7 Modelling of members for shear using Response 2000

Response 2000 is a sectional analysis program that calculates the strength and ductility of a reinforced concrete cross-section subjected to shear, moment and axial load. All the three loads are considered simultaneously to find the full load-deformation response using the latest research based on the MCFT. The Program was developed at the University of Toronto by Evan Bentz in 2000 and is available for free download at: http://www.ecf.utoronto.ca/~bentz/r2k.htm

The following three assumptions are made in Response 2000:

1. The beam theory is applicable to all sections modelled, that is plane sections remain plane even at the ultimate limit state.
2. No significant clamping stress is taken as acting through the depth of the beam. This implies that fixed supports cannot be modelled in Response 2000. If there is transverse clamping, the real strength will be higher than that predicted by the program.
3. The MCFT can be used for biaxial stress-strain behaviour throughout the depth of the beam.

Through the use of the assumptions stated above, the generally well known fibre model of sectional analysis is extended to include the effects of shear. In order to carry out an ultimate shear strength prediction based on the MCFT using Response 2000 the following input parameters have to be specified:

1. Concrete cylinder compressive strength
    a. *Aggregate size*
    b. *Compression softening characteristic equation*
2. Longitudinal steel yield strength
3. Transverse steel yield strength
4. Stirrup spacing
5. Clear cover
6. Section breadth
7. Section height
8. Number of bars and area of top non-prestressed reinforcement
9. Number of bars and area of bottom non-prestressed reinforcement
10. Stirrup type (None, single leg, open stirrup, closed stirrup, hoop, T-headed single leg, interlocking hoops)
11. Loading configuration

Some additional parameters are determined automatically based on specific inputs of the parameters listed above. In most cases the automatically determined values are maintained for the analysis of ultimate shear strength, save for the case of Aggregate size and Compression softening as shown in the bullet list above. Important aspects and modifications of these settings are discussed below. Other unaltered parameters are not discussed here and reference can be made to the program user manual for default settings.

Crack spacing

Response 2000 automatically calculates the crack spacing based on the CEB crack spacing provision that has been adopted from work done by Walraven. However, the program has the flexibility of allowing the user to input crack spacing as desired depending on the aim of the investigation. The recommended CEB crack spacing at a given depth z is given by:

$$Crack\ spacing = 2c + 0.1\, d_b/\rho \qquad [5.49]$$

Where c is the diagonal distance to the nearest reinforcement in the section from current depth, d_b is the diameter of the nearest bar, and ρ is the percentage of steel within a depth of $z \pm 7.5d_b$. The recommended value of the crack spacing is maintained in this investigation, so as to model as best as possible the real behaviour of the section or member.

Definition of concrete cylinder strength

On specifying the concrete cylinder compressive strength, other parameters are automatically determined as shown in Figure 5.18

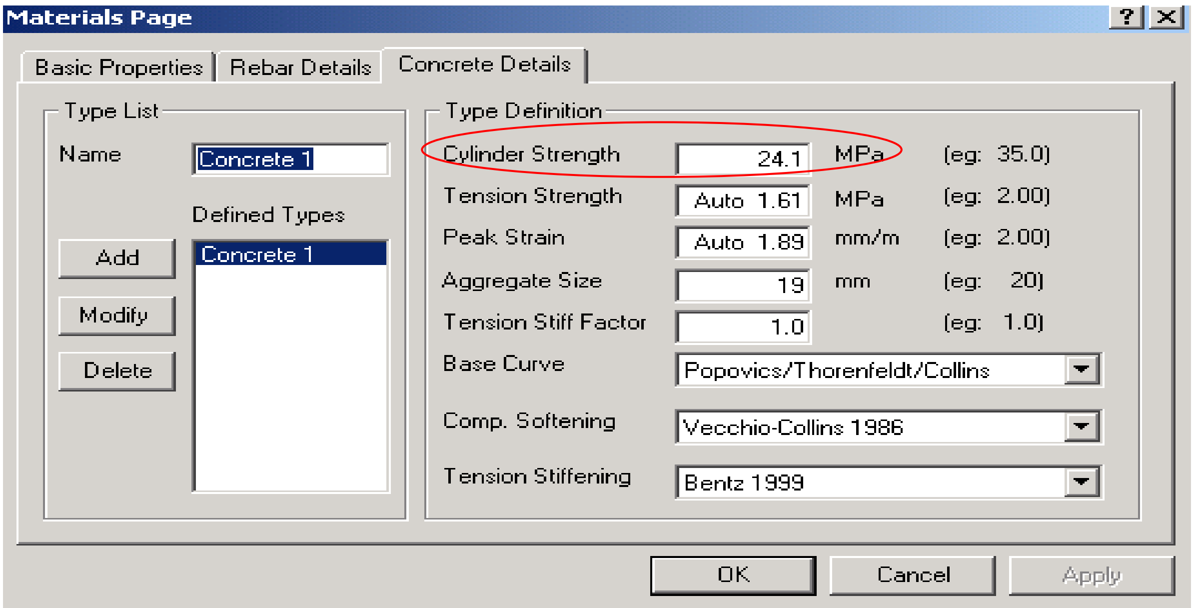


Figure 5.18. Concrete Details tab showing cylinder strength input with associated automatically defined parameters

The Aggregate Size and Compression softening parameters are altered form default specifications. The basis and the way in which these parameters were altered are discussed.

Aggregate size

Appropriate aggregate sizes as reported in the literature sources and publications used to compile the database were input into Response 2000 for ultimate shear strength analysis. Aggregate sizes were not always reported and were reasonably estimated in a select few cases. The estimation of aggregate sizes was centred on the concrete cylinder compressive strength values of the tests. It is common worldwide convention to use larger aggregates for normal strength concretes and smaller aggregates for high strength concretes. For normal strength concretes of about $40\,MPa$ characteristic cylinder strength and less, $19\,mm$ maximum aggregate size is normally used in the mix design. For high and very high strength concretes above $40\,MPa$ cylinder strength, maximum aggregate sizes are commonly specified between 7 and $12\,mm$. This reduction in aggregate size affects the interface shear transfer mechanism as rough cracks no longer form around the aggregates for high strength concrete as in normal concrete. Rather, for high strength concrete, smooth cracks occur through the aggregate. To appropriately model this effect, the maximum aggregate size was linearly reduced to $0\,mm$ in the concrete cylinder strength range between 60 and $80\,MPa$ as suggested by the Response 2000 user manual. Thus, for high strength concretes of 80 MPa and above, $0\,mm$ maximum aggregate size was prescribed for use in the analysis regardless of the aggregate size in the physical member.

Compression softenining

As shown in Figure 5.18, the Vechio-Collins 1986 equation is set as the program default setting to describe compression softening. The Response 2000 user manual, however, suggests the use of the Porasz-Collins 1988 Equation for very high strength concretes above $90\,MPa$. This recommendation was set for use in the investigation for the sections made from high strength concretes.

Definition of reinforcement bar yield strength

On specifying the steel yield strength for the reinforcing bars, both longitudinal and transverse, other parameters as shown in Figure are automatically determined by the Program. No alteration was made to any of the parameters based on reinforcement yield strength.

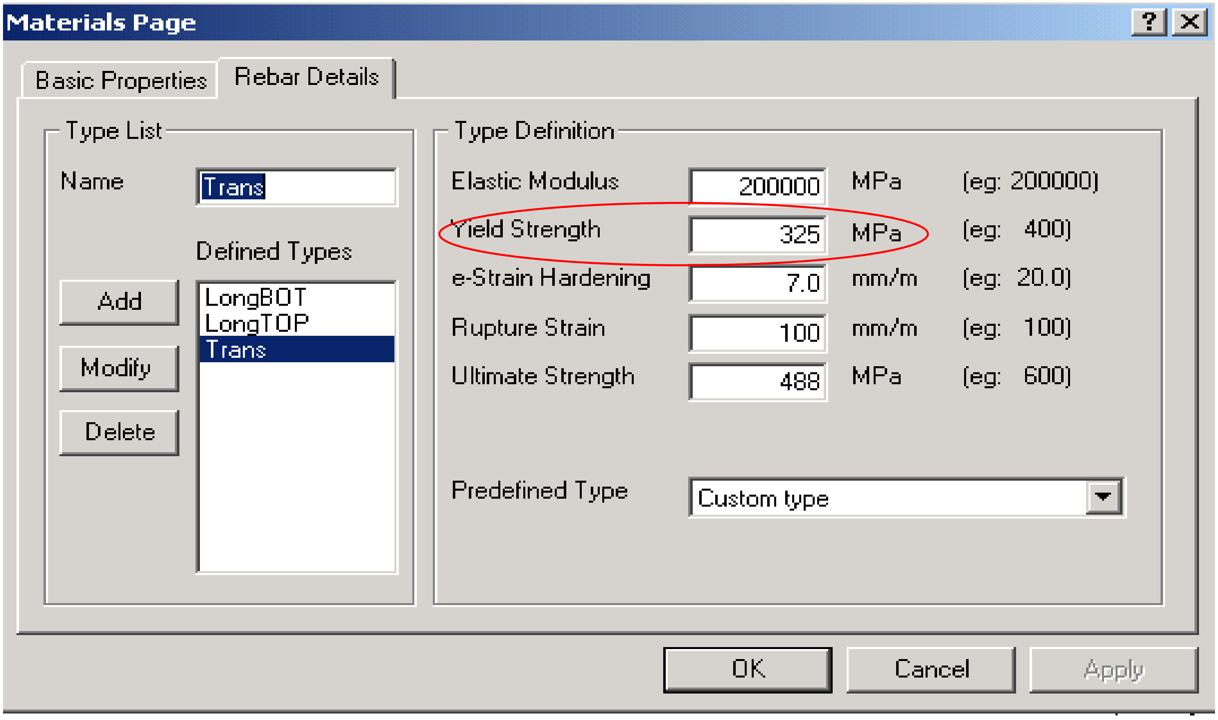


Figure 5.19. Reinforcement bar details showing steel yield strength input and other automatically determined parameters

Loading, geometry and the specification of other member properties

For shear analysis, Response 2000 allows the analysis of one or two point loads between supports. This is typical of test setups used for investigating shear. Figure 5.20 shows the Full Member Properties window from the program where loads, some geometry and other member properties are specified.

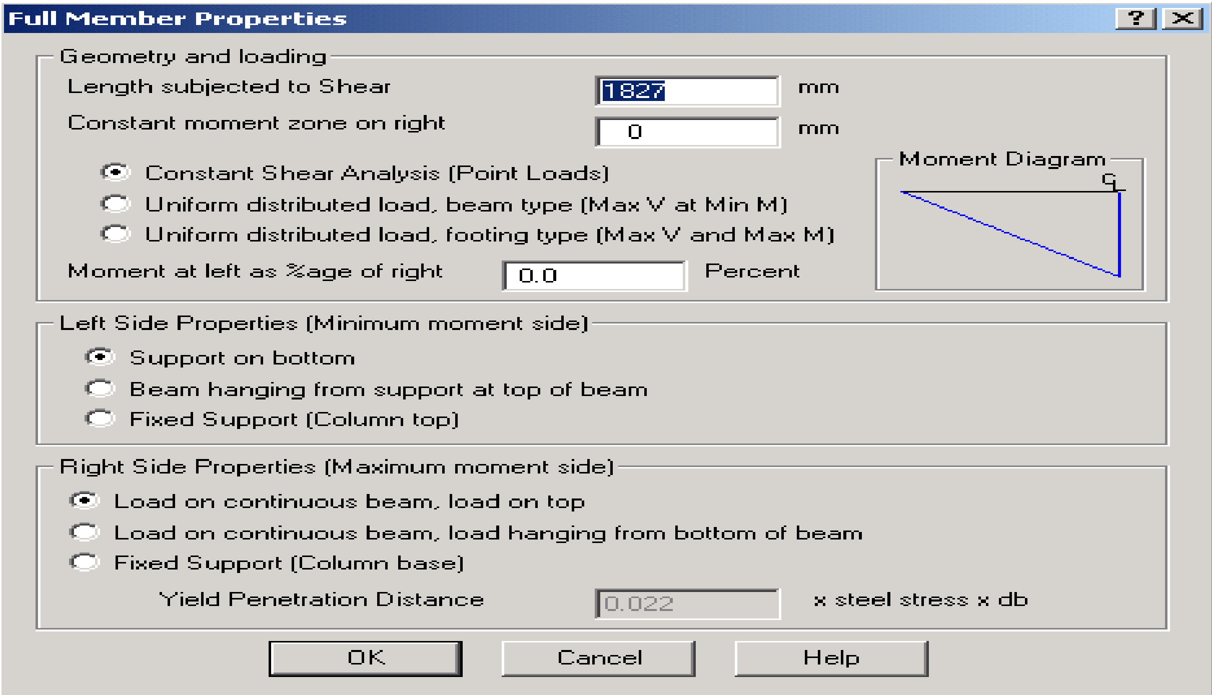


Figure 5.20. Full member properties window showing input of loads and other member properties for analysis

Outputs from Response 2000

Once all the input parameters were adequately specified for analysis, Response 2000 provided a cross-sectional plot, an example of which is shown in Figure 5.21, to give a clear depiction of the section. As a result, any errors between the modelled cross section and the test setup reported in literature were easily recognisable.

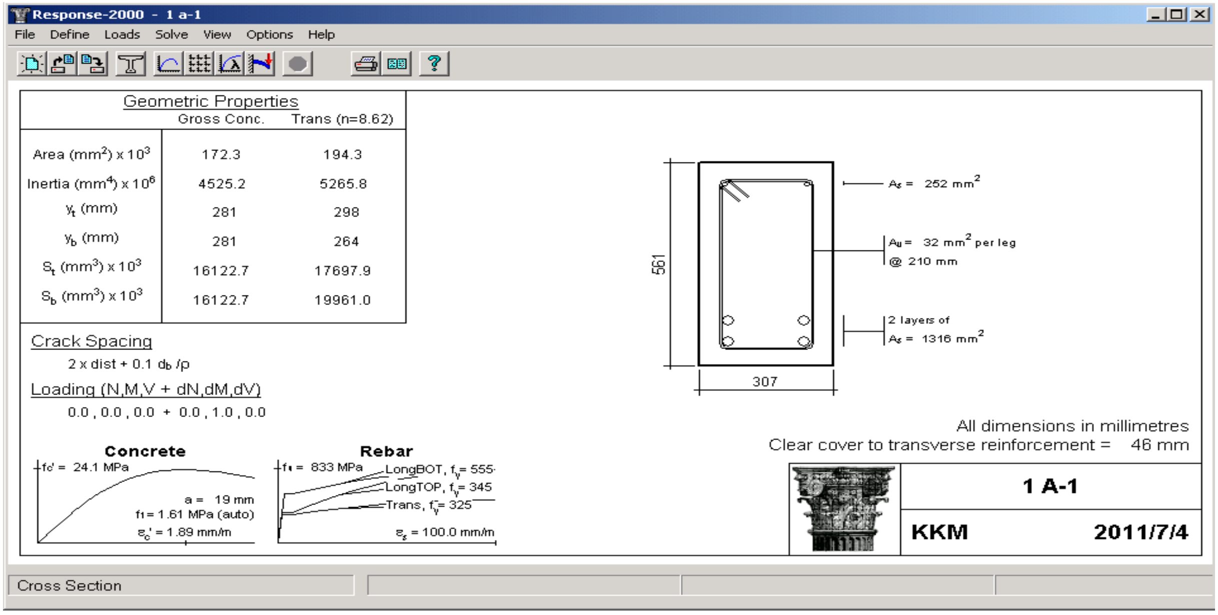


Figure 5.21. Cross-sectional plot and parameter output from Response 2000

Upon satisfactory modelling of the cross-section, a member response analysis is conducted using the cross section details and the member as described in Section. The results of the analysis are given in Figure 5.22.

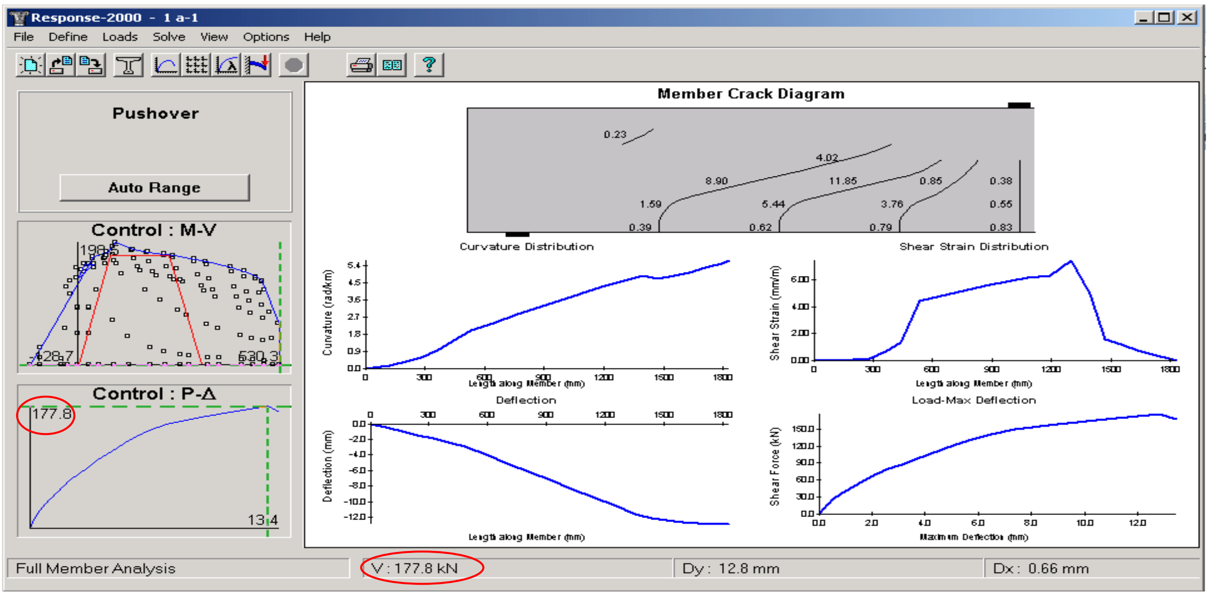


Figure 5.22. Results of the member response analysis with ultimate shear load indicated as the values enclosed by the ovals

CHAPTER 6

BACKGROUND TO THE RELIABILITY ANALYSIS OF THE EC 2 SHEAR DESIGN METHOD

6.1 INTRODUCTION AND OVERVIEW OF STRUCTURAL RELIABILITY THEORY AND RELIABILITY ANALYSIS

Taerwe (1993) states that every structural design problem is generally reduced to the verification of some well-defined limit states which are descriptors of discrete stages in the structural behaviour of a structure. Simply stated, structural design is based on idealised limit state models that are aimed at verifying that structural load, and its effects, do not exceed the available structural resistance. The models for use are more or less accurate descriptors of undesired states of a structure given a set of boundary conditions like actions, strengths, geometrical properties, environmental influence etc. In most cases the reality behind such models is not known in detail; that is, the models are uncertain (König, Hosser & Wittke, 1985). Some uncertainties can never be eliminated absolutely and must therefore be taken into account when designing or verifying construction works (Holický, 2009). The following type of uncertainties can usually be identified (Holický, 2009):

1. Natural randomness of actions, material properties and geometric data,
2. Statistical uncertainties due to limited available data,
3. Uncertainties of theoretical models owing to the simplification of the actual conditions,
4. Vagueness due to inaccurate definitions or performance requirements,
5. Gross errors in design, execution and operation of the structure, and
6. Lack of knowledge of the behaviour of new materials in real conditions.

Holický states that the order in which the uncertainties bulleted above are listed corresponds approximately to the decreasing amount of current knowledge and availability of theoretical tools with which to analyse them and take them into account in design.

It is evident that structural design is a task fraught with uncertainty. For the worst case scenario, incorrect estimation and treatment of uncertainties in design may result in structural load exceeding structural resistance, thus implying system failure. According to Ang and Tang (1984), in order to explicitly represent or reflect the significance of uncertainty, the available supply (structural resistance) and required demand (structural load) may be modelled as random variables. A random variable can be described as a physical quantity or state of nature that cannot be determined with certainty through measurements, experiments, judgement, prediction models, or some other credible method of parameter estimation. Statistical parameters and distributions are used to describe random variables.

Probability theory, by the use of statistical representations of random variables, describes the occurrence of random events. As such, the occurrence of realisations of the structural load relative to the occurrence of realisations of structural resistance is possible by the use of the probability theory. Within the framework of the probability theory, full safety performance assessments can be made of structures in terms of exceedance probabilities. In this light, the probability of failure may be defined as the likelihood that the structural load exceeds the structural resistance. Conversely, the probability of survival or safety may be defined as the likelihood that the structural resistance exceeds the structural load and its effects. In these terms, therefore, the reliability of a system may be more realistically measured in terms of probability. ISO 2394 (1998) defines reliability as the ability of a structure to comply with given requirements under specified conditions during the intended life for which it was designed.

6.1.1 Reliability analysis of the EC 2 shear design method

In accordance with Holický (2009), the fundamental task of the theory of structural reliability is the analysis of a simple requirement that the action effect, E, is smaller than the structural resistance, R. It therefore holds that:

$$E < R \qquad [6.1]$$

The reliability of any mode of structural resistance, in this case shear of members with stirrups, derives from the difference between the expected values of the resistance and that of the applied loads (Huber, 2005). Equation [6.1] enables the description of a safe state, limit state and failure state of any structural component. The limit state forms the distinction between the safe and failure states of a structure and is defined by the limit state function as:

$$R - E = 0 \qquad [6.2]$$

To reflect uncertainties inherent in design, the resistance and load effect are subject to statistical distributions that reflect the central tendencies of occurrence of their mean values relative to one another and the dispersion of the respective mean values. Of particular interest is the region of expected failure at which the higher end tails of load models or distributions overlap with the lower end tails of the resistance models. This is illustrated in Figure 6.1. It can be deduced from Figure 6.1 that the probability of failure denoted as P_f, can be reduced if the difference in the central tendency of occurrence of the mean values of E and R were increased or if the spread of realisations around the mean is reduced in either or both of E and R. Further, the concept of the reliability index is illustrated for a case where both E and R are assumed to be normally distributed. This example is applicable to very simple cases in reality and is only provided here to aid in defining the reliability index, β. In general, the reliability index is the distance, expressed in terms of standard deviations, σ_{R-E}, from the mean value of the safety margin, $\mu_R - \mu_E$, from the limit state condition or failure point $G = R - E = 0$.

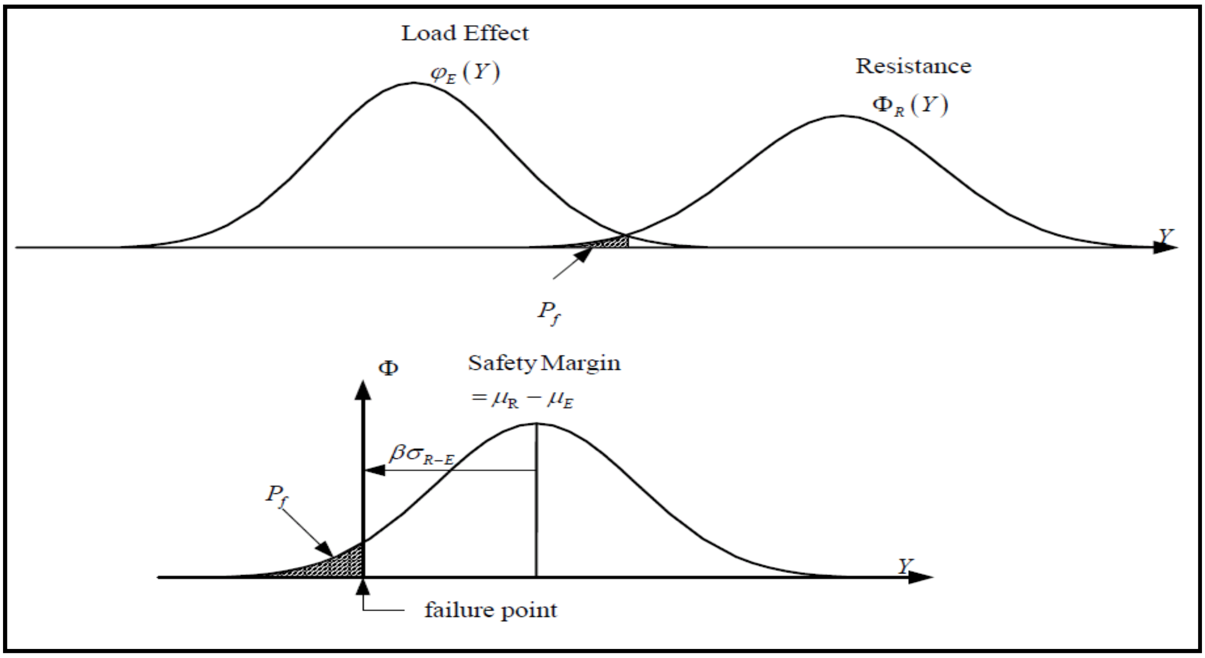


Figure 6.1. Probability Density Function of the Limit State function (Huber, 2005)

A general performance function for any mode of resistance can be defined by:

$$g(\boldsymbol{X}) = R - E \qquad [6.3]$$

Where $\boldsymbol{X} = (X_1, X_2, \dots, X_n)$ is vector of basic state (or design) variables on which both the load and resistance are dependent. A structure is safe so long as $g(\boldsymbol{X}) > 0$ and has failed to meet its intended function if $g(\boldsymbol{X}) \leq 0$. In this general case, the probability of failure can be given by:

$$P_f = Probability\ (g(X) \leq 0) = \int_{g(\boldsymbol{X})\leq 0} \varphi_g(\boldsymbol{X})\, dX \qquad [6.4]$$

Where $\varphi_g(\boldsymbol{X})$ denotes the joint probability density distribution of the vector of basic variables X. In practice, evaluation of the joint probability density functions of actions and resistances is a formidable task as information is often unavailable or difficult to obtain for reasons of insufficient data (Ang & Tang, 1984). Furthermore, even in cases where the required distributions are specified, the determination of Equation [6.4], which requires numerical integration, may be impractical as much effort is required. As such, the First

Order Reliability Method (FORM) method is implemented as a practical tool of conducting reliability analyses. An added advantage of the form method is that the reliability index, β, may be measured entirely as a function of the mean and standard deviation of basic variables when there is no information on the probability distributions. The FORM procedure is outlined in Section 6.1.2. The relation between β and P_f was established earlier in Equation [3.1] and is repeated here for convenience:

$$P_f = \Phi(-\beta) \qquad [6.5]$$

Where Φ is the cumulative distribution function of the standardised Normal distribution.

Through means of reliability separation, it may be assumed that the overall reliability index β, may be split into the resistance part, expressed by the resistance index $\beta_R = \alpha_R\beta$, and the load effects part, expressed by the load effect index $\beta_E = -\alpha_E\beta$. α_R and α_E are the FORM sensitivity factors and were established in Chapter 2 as $\alpha_R = 0.8$ and $\alpha_E = -0.7$ as recommended in EC 0. The most important feature of reliability separation is that material codes, or resistance performance, can be calibrated independent of loading codes and loading considerations in general. The mutual relationship between the resistance failure probability, P_R, and the resistance index, β_R, is given as (Holicky et al., 2010):

$$P_R = \Phi\ (-\beta_R) = P\{R(\boldsymbol{X}) < R_d(\boldsymbol{X_k}, \boldsymbol{\gamma})\} \qquad [6.6]$$

Where $R(\boldsymbol{X})$ is a general probabilistic model (gpm), for shear in this case, representing the resistance side of the limit state equation, similar to R in Equation [6.3]. $R_d(\boldsymbol{X_k}, \boldsymbol{\gamma})$ is the deterministic code design shear resistance for a specific case for which the reliability is to be determined. Therefore, $\boldsymbol{X}$ denotes the vector of basic variables, $\boldsymbol{X_k}$ represents the vector of their characteristic values, and $\boldsymbol{\gamma}$ the vector of relevant partial factors. The Modified Compression Field theory (MCFT) is adopted for use as the general probabilistic model in Chapter 8 of the thesis. In effect, R_d is the shear resistance of a specific case determined using the variable strut inclination method for shear in EC 2 incorporating all necessary safety elements for design incorporated in design e.g. partials factors, characteristic values, conversion factors etc. By FORM analysis, the resistance reliability index, β_R, may be determined through the assessment of the performance function:

$$g(\boldsymbol{X}) = R(\boldsymbol{X}) - R_d(\boldsymbol{X_k}, \boldsymbol{\gamma}) \quad [6.7]$$

König et al. (1985) establish that highly sophisticated models, such as general probabilistic models, can be used as models of true shear resistance in the absence of valid tests and measurements from practice. The MCFT has proven to be probably the best design model available for making accurate predictions of shear resistance and can thus be employed as the general probabilistic model.

6.1.2 First Order Reliability Method (FORM)

Coupled with easier assessment of the probability of failure through the determination of β, the FORM method is also an effective decision tool as it provides more insight into the reliability process than numerical integration or simulation methods. By the use of numerical examples, various reliability elements such as partial factors and direction cosines of basic variables can be investigated to determine which basic variables most affect the reliability performance for shear or any other mode of structural resistance. Holicky et al. (2010) establish that effective management of reliability performance, in terms of attaining specified reliability levels, can be achieved by applying partial factors to the variables that most affect reliability performance. The relative importance of basic variables is reflected by the relative values of their direction cosines; that is, the greater the value of the direction cosine, the greater the influence of the parameter on reliability performance of the structure considered.

It should be emphasised that the FORM method is consistent with the equivalent normal representation of non-normal distributions. Only the mean and standard deviation of the normal distribution are required for FORM analyses, hence non-normal distributions only need to be transformed to normal distributions at specific points where the performance is being assessed. Hence, an essential step in using the FORM method is first transforming the basic variables of various distributions into a transformed space of normal variables. The algorithm outlined in Holický (2009) was implemented in a spreadsheet to conduct the reliability analysis of the EC 2 shear prediction model. The steps of the algotirhm presented by Holický (2009) and used in the thesis are presented below with necessary inclusions.

1. Formulate the of the limit state function $g(\boldsymbol{X}) = 0$ and specify theoretical models for basic variables $\boldsymbol{X} = \{\boldsymbol{X_1}, \boldsymbol{X_2}, \ldots, \boldsymbol{X_n}\}$. ∴

$$g(\boldsymbol{X}) = R(\boldsymbol{X}) - R_d(\boldsymbol{X_k}, \boldsymbol{\gamma}) = 0 \qquad [6.8]$$

More guidance on the specification and treatment of basic variables is given in Section 6.2 as the way in which they are specified and treated greatly influence the validity of results obtained from reliability analyses.

2. Estimate the design point $\boldsymbol{x}^* = \{x_1^*, x_2^*, \ldots, x_n^*\}$, for example using the means of $n - 1$ basic variables, and the value of the last basic variable is calculated from $g(\boldsymbol{x}^*) = 0$.

3. Evaluate the equivalent normal distribution for all basic variables at $\boldsymbol{x}^*$. For an individual variate, the equivalent normal dsitribution for a nonnormal variate may be obtained such that the cumulative probability as well as the probability density ordinate of the equivalent normal distribution are equal to those of the correspnding nonnormal distribution at the appropriate point, x_i^*, on the failure surface (Ang & Tang, 1984). Hence, equating the cumulative probabilities, we have:

$$\Phi\left(\frac{x_i^* - \mu_{X_i}^N}{\sigma_{X_i}^N}\right) = F_{X_i}(x_i^*) \qquad [6.9]$$

Where $\mu_{X_i}^N$ *and* $\sigma_{X_i}^N$ are the mean value and standard deviation, respectively, of the equivalent normal distribution for X_i. $F_{X_i}(x_i^*)$ is the original Cumulative Distribution Function (CDF) of X_i evaluated at x_i^*

Transposing Equation [6.9] and making the equivalent normal mean subject yields:

$$\mu_{X_i}^N = x_i^* - \sigma_{X_i}^N \Phi^{-1}\left[F_{X_i}(x_i^*)\right] \qquad [6.10]$$

Furthermore, equating the corresponding probability density ordinates at x_i^* means yields,

$$\frac{1}{\sigma_{X_i}^N}\varphi\left(\frac{x_i^*-\mu_{X_i}^N}{\sigma_{X_i}^N}\right)=f_{X_i}(x_i^*) \qquad [6.11]$$

Transposing Equation [6.11] and making the equivalent normal standard deviation subject:

$$\sigma_{X_i}^N=\frac{\varphi\left\{\Phi^{-1}\left[F_{X_i}(x_i^*)\right]\right\}}{f_{X_i}(x_i^*)} \qquad [6.12]$$

Equations [6.9] to [6.12] only give superficial treatment of how basic variables are transformed and the specific cases of non-normal distributions adopted in Chapter 8 are elaborated on in Section 6.2 when the theoretical models of basic variables are dealt with.

4. Transformed design point $\boldsymbol{u^*} = \{u_1^*, u_2^*, ...u_n^*\}$ of standardized normal variables $\boldsymbol{U} = \{U_1, U_2, ...U_n\}$ corresponding to $\boldsymbol{x^*} = \{x_1^*, x_2^*, ...x_n^*\}$ is determined using Equation:

$$\boldsymbol{u_i^*}=\frac{x_i^*-\mu_X^N}{\sigma_X^N} \qquad [6.13]$$

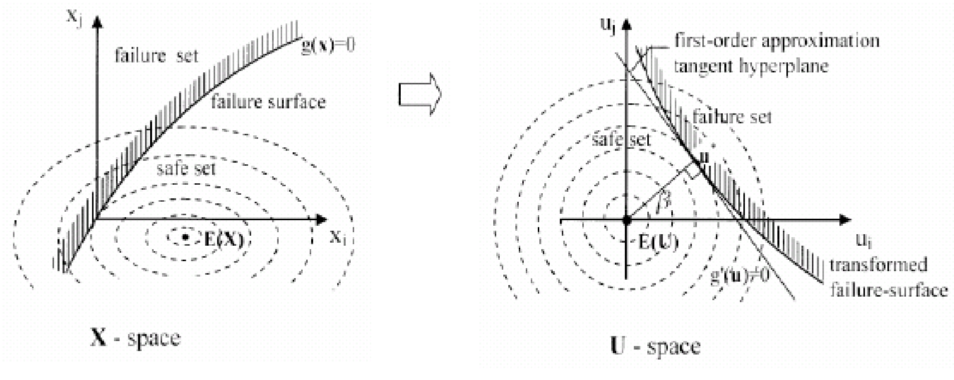


Figure 6.2. Schematic representation of the standard normal transformation process (Taken from Dithinde, 2007)

5. Partial derivatives of the limit state function with respect to $\boldsymbol{U} = \{U_1, U_2, \ldots, U_n\}$ are determined at the design point and denoted as the vector $\boldsymbol{D} = \{D_1, D_2, \ldots, D_n\}$ and:

$$D_i = \frac{\partial G}{\partial U_i} = \frac{\partial G}{\partial X_i}\frac{\partial X_i}{\partial U_i} = \frac{\partial G}{\partial X_i} \cdot \sigma_X^N \qquad [6.14]$$

6. The reliability index β is estimated as:

$$\beta = -\frac{\{D\}^T\{u^*\}}{\sqrt{\{D\}^T\{D\}}} \qquad [6.15]$$

For non-linear performance functions, as is the case for shear resistance shown in Section 5.2, there is no unique distance from the failure surface to the origin of the reduced varaites but rather a linear approxiamtion by the use of a tangent plane to the failure surface at $\{x_1^*, x_2^*, \ldots, x_n^*\}$ to approximate the failure surface.

7. The vector of sensitivity factors is determined as:

$$\alpha = \frac{\{D\}}{\sqrt{\{D\}^T\{D\}}} \quad or \quad \alpha_i^* = \frac{\left(\frac{\partial g}{\partial X_i}\right)_*}{\sqrt{\sum_i \left(\frac{\partial g}{\partial X_i}\right)_*^2}} \qquad [6.16]$$

8. A new design point for *n*-1 standardized variables and original variables are given as:

$$u_i^* = \alpha_i \beta_i \qquad [6.17]$$

$$x_i^* = \mu_{Xi}^N - u_i^* \sigma_{Xi}^N \qquad [6.18]$$

9. A new design point of the remaining variables follows from $g(\boldsymbol{x}^*) = 0$.

10. Steps 3 to 9 are repeated until β and design point $\{\boldsymbol{x*}\}$ are specified with a suitable accuracy.

6.2 THEORETICAL MODELS OF BASIC VARIABLES

The proposed models for the basic random variables in the thesis are prescribed considering middle values of action variances, common structural conditions and normal quality control of material properties (Holický, 2009). Holický (2009) presents a compilation of several theoretical models that can be used for the representation of basic random variables in structural reliability assessments. This is considered a sound compilation as it is extracted from renowned sources as the JCSS probabilistic model code, CIB reports, the SAKO report and other significant contributions. Direct cross reference is also made to the JCSS probabilistic model code (2001) and to the SAKO report (1999). The models prescribed by Holický (2009) are given in Table 6.1 below. For the distributions, the symbol N denotes the normal distribution, GU the gumbel distribution, LN the Log-normal distribution, and BET the beta distribution. Though not shown in the Table, some basic variables that have negligible uncertainty or effect on reliability performance may be treated as deterministic values. Deterministic values are defined by a single value with no associated uncertainty of statistical distribution to reflect such.

Table 6.1. Conventional models of basic variables for time-invariant reliability analyses (Holický, 2009)

No.	Category of variables	Name of basic variables	Sym. X	Dim-ension	Dis-trib.	Mean μ_X	St. dev. σ_X	Prob. $\Phi_X(X_k)$
1	Actions	Permanent	G	kN/m^2	N	G_k	0,03-0,10μ_X	0,5
2		Imposed-5 years	Q	kN/m^2	GU	0,2Q_k	1,1μ_X	0,995
3		Imposed-50 y.	Q	kN/m^2	GU	0,6Q_k	0,35μ_X	0,953
4		Wind-1 year	W	kN/m^2	GU	0,3W_k	0,5μ_X	0,999
5		Wind-50 years	W	kN/m^2	GU	0,7W_k	0,35μ_X	0,890
6		Snow - 1 year	S	kN/m^2	GU	0,35 S_k	0,70 μ_X	0,998
7		Snow -50 year	S	kN/m^2	GU	1,1 S_k	0,30 μ_X	0,437
8	Material	Steel yield point	f_y	MPa	LN	$f_{yk}+2\sigma$	0,07-0,10μ_X	0,02
9		Steel strength	f_u	MPa	LN	$\kappa\, \mu_{fy}$	0,05μ_X	-
10	strengths	Concrete	f_c	MPa	LN	$f_{ck}+2\sigma$	0,10-0,18μ_X	0,02
11		Reinforcement	f_y	MPa	LN	$f_{yk}+2\sigma$	30 MPa	0,02
12	Geometry	IPE profiles	A, W, I	m^{2,3,4}	N	0,99X_{nom}	0,01-0,04 μ_X	≅0,73
13	steel sect.	L-section, rods	A, W, I	m^{2,3,4}	N	1,02X_{nom}	0,01-0,02 μ_X	≅0,16
14	Geometry	Cross-section	b, h	m	N	b_k, h_k	0,005-0,01	0,5
15	concrete	Cover of reinf.	a	m	BET	a_k	0,005-0,015	0,5
16	cross-sect.	Additional ecc.	e	m	N	0	0,003-0,01	-
17	Model un-	Load effect factor	θ_E	-	N	1	0,05-0.10	-
18	certainties	Resistance factor	θ_R	-	N	1-1,25	0,05-0,20	-

Once the distributions of each of the basic variables have been specified, the next task is to understand how they will be transformed into equivalent normal form to be able to be used in reliability analysis. The Sections below treat the non-normal distributions used in Chapter 8 of the thesis with a view of how they are transformed to equivalent normals. It is shown in Chapter 7 and 8 that the model uncertainty is an important parameter affecting structural performance in shear for members requiring stirrups. To this effect, model uncertainties are treated conceptually as well in the Sections to follow.

6.2.1 Background of the general three parameter log-normal distribution

In general, a random variable X has a three-parameter log-normal distribution if the transformed variable, Y, has a normal distribution:

$$Y = \ln|X - x_0| \qquad [6.19]$$

In this relation x_0 denotes the lower or upper bound of the variable X, which depends on the skewness, s_k. If the variable has a mean, μ_X, and a standard deviation, σ_X, then the lower or upper bound can be expressed as:

$$x_0 = \mu_X - \frac{\sigma_X}{c} \qquad [6.20]$$

Note that for negative skewness, c, is a negative quantity and in that case x_0 is indicative of the upper bound of the distribution. The coefficient, c, is given by the value of the skewness according to the relation:

$$s_k = c^3 + 3c \qquad [6.21]$$

From which follows an explicit relation for c:

$$c = \left[\left(\sqrt{s_k^2 + 4} + s_k\right)^{1/3} - \left(\sqrt{s_k^2 + 4} - s_k\right)^{1/3}\right] 2^{-1/3} \qquad [6.22]$$

Thus, when specifying a theoretical model, it is therefore possible to consider the skewness s_k or alternatively the lower or upper bound of the distribution x_0 (besides the mean μ_X and standard deviation σ_X). In general, the skewness provides better characteristic of the overall distribution of the population (particularly of large populations) than the lower or upper bounds (Holický, 2009).

The probability density function and distribution function of the general three-parameter log-normal distribution may be obtained from the well-known normal distribution using a modified (transformed) standardised variable u'_N obtained from the original standardised random variable $u' = (x - \mu_X)/\sigma_X$ as:

$$u'_N = \frac{\ln\left(\left|u' + \frac{1}{c}\right|\right) + \ln\left(|c|\sqrt{1+c^2}\right)}{\sqrt{\ln(1+c^2)}} sign(s_k) \qquad [6.23]$$

Where $sign(s_k)$ equals $+1$ for $s_k > 0$ and -1 for $s_k < 0$. The probability density function $\varphi_{LN,U}(u)$ and the distribution function $\Phi_{LN,U}(u) = \Phi_{LN,X}(x)$ of the log-normal distribution are given as:

$$\varphi_{LN,U}(u) = \frac{\varphi(u'_N)}{\sigma_U\left(\left|u' + \frac{1}{c}\right|\right)\sqrt{\ln(1+c^2)}} \qquad [6.24]$$

$$\Phi_{LN,X}(x) = \Phi_{LN,U}(u) = \Phi(u'_N) \qquad [6.25]$$

Where $\varphi(u'_N)$ and $\Phi(u'_N)$ denote the probability density and distribution function of the standardised normal variable.

A special case of the three-parameter log-normal distribution is the log-normal distribution with the lower bound at zero ($x_0 = 0$). This distribution depends on two parameters only – the mean, μ_X, and standard deviation, σ_X. In such a case the coefficient, c, is equal to the coefficient of variation, Ω_X. It further follows from Equation [6.21] that:

$$s_k = \Omega_X^3 + 3\Omega_X \qquad [6.26]$$

Thus, the log-normal distribution with the lower bound at zero always has a positive skewness.

6.2.2 Background of the two-parameter log-normal distribution

Ang and Tang (1975) give the density function, $f_x(x)$, and distribution function between limits a and b of an arbitrary random variable, X, that follows the two parameter log-normal distribution as:

$$f_x(x) = \frac{1}{\sqrt{2\pi}\zeta x} exp\left[-\frac{1}{2}\left(\frac{\ln x - \lambda}{\zeta}\right)^2\right] \qquad 0 \leq x \leq \infty \qquad [6.27]$$

$$P(a < X \leq b) = \frac{1}{\sigma\sqrt{2\pi}}\int_a^b \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right] dx \qquad [6.28]$$

Where $\lambda = E(\ln X)$ and $\zeta = \sqrt{Var(\ln X)}$ are, respectively, the mean and standard deviation of $\ln X$, and are the parameters of the distribution. The mean, λ, and the standard deviation, ζ, of $\ln X$ are defined as:

$$\lambda = \ln \mu_X - \frac{1}{2}\zeta^2 \qquad [6.29]$$

$$\zeta = \sqrt{\ln\left(1 + \frac{\sigma_X^2}{\mu_X^2}\right)} = \sqrt{\ln(1 + \Omega_X^2)} \qquad [6.30]$$

Note that ζ will have to be determined before the solution to Equation [6.29] is sought.

The density function and probability generating function of the two-parameter Lognormal distribution can be found by use of the standardised normal distribution:

Let $s = \frac{\ln x - \lambda}{\zeta}$ [6.31]

Then $dx = x\zeta ds$, and:

Then $f_x(x) = \frac{1}{\sqrt{2\pi}\zeta x} exp\left[-\frac{1}{2}\left(\frac{\ln x - \lambda}{\zeta}\right)^2\right] \qquad 0 \le x \le \infty$ [6.32]

$$P(a < X \le b) = \frac{1}{\sqrt{2\pi}}\int_{(\ln a-\lambda)/\zeta}^{(\ln b-\lambda)/\zeta} e^{-(1/2)s^2}\, ds = \Phi\left(\frac{\ln b-\lambda}{\zeta}\right) - \Phi\left(\frac{\ln a-\lambda}{\zeta}\right) \qquad [6.33]$$

Where $\Phi(-)$ denotes the cumulative distribution function of the standard normal distribution.

Ang and Tang (1984) then proceed to represent the density function and distribution function of the log-normal distribution in the form of the standardised normal distribution, similar to the end results Equation [6.10] and [6.12], as:

$$\Phi_{LN,X}(x_i^*) = \Phi\left(\frac{\ln x^* - \lambda_X}{\zeta_X}\right) \qquad [6.34]$$

$$\varphi_{LN,X}(x^*) = \frac{1}{x^*\zeta_X}\varphi\left(\frac{\ln x^* - \lambda_X}{\zeta_X}\right) \qquad [6.35]$$

Where $\Phi_{LN,X}(x_i^*)\ and\ \varphi_{LN,X}(x_i^*)$ denote the cumulative distribution and probability density function of the lognormal distribution of the original variate at the checking point x_i^*. Therefore, for the two-parameter log-normal distribution:

$$\sigma_{X_i}^N = \frac{\varphi\left\{\Phi^{-1}\left[\Phi\left(\frac{\ln x^*-\lambda_X}{\zeta_X}\right)\right]\right\}}{\frac{1}{x^*\zeta_X}\varphi\left(\frac{\ln x^*-\lambda_X}{\zeta_X}\right)} = \frac{x^*\zeta_X\left[\varphi\left(\frac{\ln x^*-\lambda_X}{\zeta_X}\right)\right]}{\varphi\left(\frac{\ln x^*-\lambda_X}{\zeta_X}\right)} = x^*\zeta_X \qquad [6.36]$$

$$\mu_{X_i}^N = x^* - x^*\zeta_X\left(\frac{\ln x^*-\lambda_X}{\zeta_X}\right) = x^* - x^*(\ln x^* - \lambda_X) = x^*(1 - \ln x^* + \lambda_X) \qquad [6.37]$$

6.2.3 Upper triangular distribution

Some basic variables in design are often expressed in terms of lower and upper limit values. This is usually the case when the values of variables are fixed on a judgement basis. Nonetheless, the variable still requires some statistical parameters to be able to be used in the FORM procedure. Ang and Tang (1984) describe that if there is bias associated with the realisation of a given variable within the lower limit, x_l and x_u, some form of skewed distribution should be adopted. In this regard, if it is judged or postulated that there is bias toward the higher values within the specified range, then the upper triangular distribution, as shown in Figure 6.3 would be appropriate.

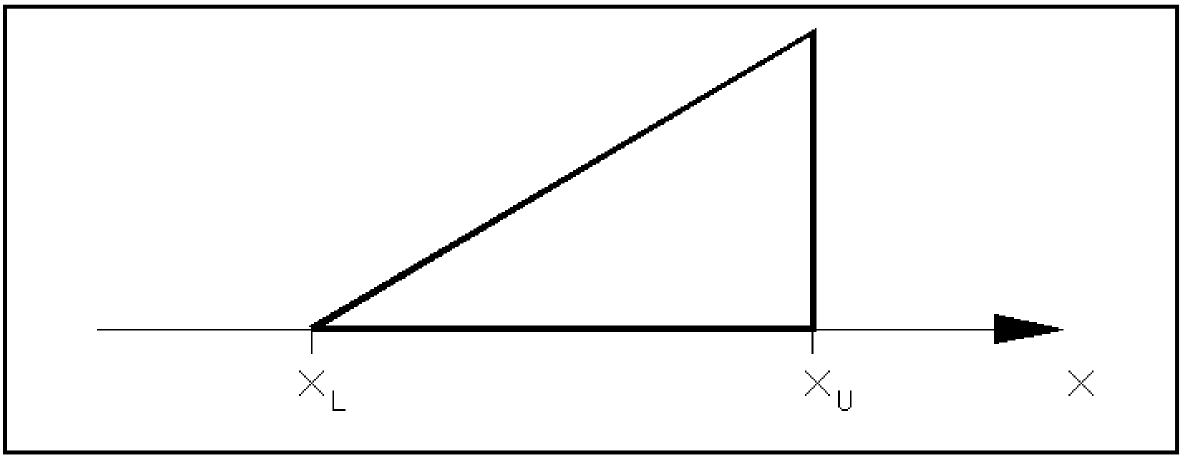


Figure 6.3. Upper triangular Probability density function between x_L and x_U

For an arbitrary random variable x, the mean value, μ_X, can be determined as :

$$\mu_X = \frac{1}{3}(x_L + 2x_U) \qquad [6.38]$$

And the corresponding coefficient of variation, Ω_X, is:

$$\Omega_X = \frac{1}{\sqrt{2}}\left(\frac{x_U - x_L}{2x_U + x_L}\right) \qquad [6.39]$$

Since the probability density and cumulative distribution function are not clearly defined the mean and standard deviation from the upper triangular distribution can be approximated to equal that of a distribution that can take account of the inherent skewness as can be seen from Figure 6.3. In the thesis the three-parameter log-normal distribution was used for this process and once the mean and variation were determined by equation [6.38] and [6.39], Equation [6.20] to [6.25] can then be applied to treat the variable as log-normally distributed with appropriate skewness.

6.2.4 Model uncertainties

Since in developing a resistance model certain influences are either consciously or unconsciously neglected, deviations between analysis and tests are to be expected. An analysis and assessment of uncertainty must include the uncertainties in the design variables as well as in the prediction models (Ang & Tang, 1984). For good models the model factor is approximately equal to one. Since, however, conservative models are used it often results in model factor's greater than one. For good models, such as bending resistance of concrete and even steel, the coefficient of variation is just a few percent, whereas for poor models such as shear and punching shear values in the region of 10 to 20 % are typical (Schneider, 2006). Taerwe (1993) states that special calibration of the model uncertainty as part of the global resistance factor is warranted for coefficients of variation of 20 % and above. For smaller coefficients of variation it could be tentatively suggested that the model uncertainties don't require an additional safety factor if safe side models are used.

Figure 6.4 gives a schematic overview of how model uncertainties can be treated. It is illustrated in Figure 6.4 that model uncertainties can be treated at different levels of refinement ranging from the top of the chart with experience or judgement based treatment going down to the more rigorous and rational methods of full probabilistic analysis. The way in which model uncertainties are dealt during calibration exercises is largely dependent on whether or not statistical information on the model uncertainty is available, and if thereafter, severe trends exist between the model factor and any of the shear parameters. In the event that sufficient information is available for a statistical evaluation of model uncertainty, as is the case for shear, and it is found to be dominating or important to structural performance, it should be sufficiently accounted for and properly calibrated into the resistance model or the global partial safety factor.

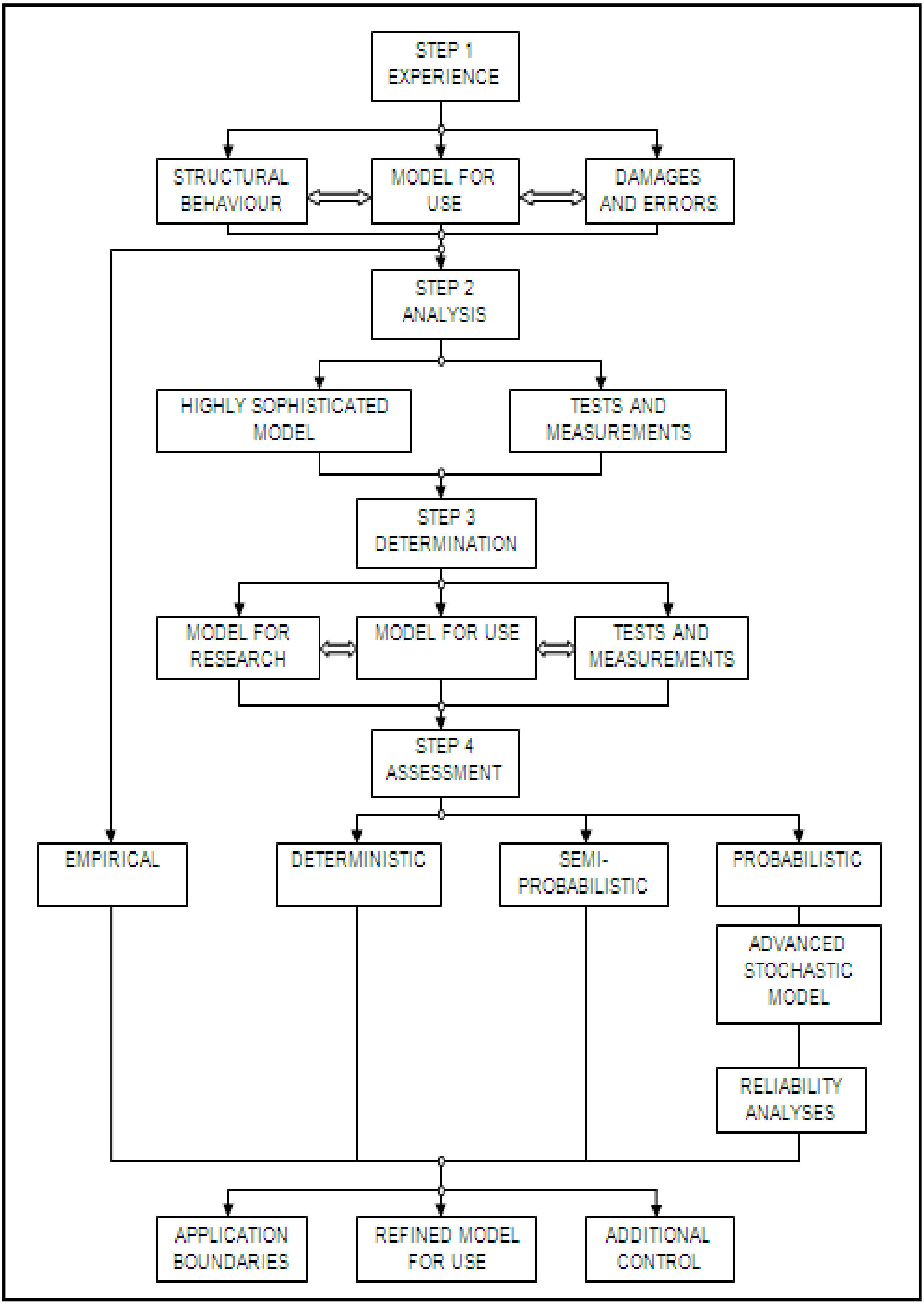


Figure 6.4. Flow chart for treatment of model uncertainties (König et al, 1985)

CHAPTER 7

DETERMINATION OF THE MODEL FACTOR FOR THE EC 2 AND MCFT SHEAR PREDICTION MODELS

7.1 INTRODUCTION

The central focus of this Chapter is to determine the statistics of the model uncertainty, reflected by the Model Factor (MF) and its standard deviation, associated with the prediction models for shear resistance for members requiring stirrups. In effect, the statistics of two separate cases of the model factor are herein characterised; one case for the EC 2 variable strut inclination method and the other case for the MCFT. Characterisation of the model uncertainty was achieved by comparing ultimate shear strengths obtained from physical tests on representative members to the unfactored predictions of each of the design models.

The term ‘Unfactored design model’ refers to the instance when the design model is used to make a prediction of true shear resistance by eliminating all conservative bias, done through the neglect of safety elements incorporated for design purposes such as partial factors and characteristic values. However, even the unfactored design model cannot accurately predict the failure shear strength of beams determined from tests and experimental databases. This implies that uncertainties are not only inherent in the design basic variables, but also arise due to some uncertainty associated with the prediction model itself. It should be noted that although tests from experiments are taken to represent true shear resistance, they too have some experimental error and uncertainty inhibiting them from being truly representative of the actual situation or condition in the structure. To cater for this, conversion factors are sometimes applied to experimentally determined parameters in codified design models.

7.2 THE EXPERIMENTAL DATABASE

The experimental database used in determining the model factor was compiled from research published in most part by the American Concrete Institute (ACI) Structural Journal over the past 50 years. The different tests, conducted by different researchers, were aimed and hence designed to reflect different aspects of performance of shear reinforced concrete beams. The references of the of the published papers used in compiling the database are included after the main references at the end of the thesis. Many of the experimental studies on shear from the ACI Structural Journal have resulted in the use of different design rules and approaches worldwide. As a result, the database does provide some representation of beams in practice. Detailed statistical properties of the database are given in Section 7.2.2 and 7.2.3. The database, as well all predictions made against it, is provided on the CD attached at the end of the thesis.

Shear strength predictions were provided by the use of the variable strut inclination method for the case of EC 2 comparison whilst MCFT predictions were taken from the results of the sections from the test database being modelled in Response-2000. Response-2000 predictions could not be made against every test result in the compiled experimental database owing to the fact that a wealth of data is required for its use, some of which was not reported in a number of cases. Therefore, two subsets of the data were created; the complete database to which EC 2 predictions are made, and the other the subset of tests which can be modelled in Response-2000. Statistical comparisons of these subsets are given throughout the following discussions.

7.2.1 Composition of the database

The entire database consisted of 222 reinforced concrete beams with stirrups which were collected from published experimental data from some 24 literature sources. The 222 tests in the database represents a shortlist value from a greater database that was filtered to contain members adhering to the following criteria:

1. All beams had to have at least the minimum amount of stirrups as set by EC 2. This is a parameter left open for national choice and the UK adopts the value given in

Equation [5.12]. This relates to the links providing a minimum force governed by the following expression:

$$F_{w,min} = A_{sw} f_{yw} = 0.08\sqrt{f_c} \cdot b_w \cdot s \qquad [7.1]$$

Where $F_{w,min}$ is the resistance force provided by the minimum links. It should be noted in Equation [7.1] that the steel yield strength, f_{yw}, and concrete compressive strength, f_c, are taken at their mean reported values in literature.

2. The maximum spacing limit for stirrups of $0.75\ d$ or $600\ mm$ as set by the Eurocode and maintained for use in the UK was adhered to.

3. All beams in the compiled database were simply supported and had predominantly 1 or 2 gravity point loads between supports, although 4 gravity point loads were placed between the supports in four tests. The tests with four point loads could not be modeled in Response-2000 as it models a symmetric half-portion of the beam, allowing a single load input with the length of the constant moment zone on the right of the load also required as input. This implies a maximum of two point loads between supports for a beam can be modeled in Response 2000.

4. Most beams in the database were in the range $2.5 \leq \mathrm{a}/d \leq 6.0$. Only four beams, that had $\mathrm{a}/d = 2.49$, fell outside the aforementioned range. Beyond an a/d of 6.0, the members are governed by flexural failure and an a/d less than 2.5 represents deep beams which are characterised under D-region shear, where other methods are preferred for their design like the strut and tie approach.

5. For all beams in the database it was ensured, according to the EC 2 truss model, that the ultimate crushing strength of the concrete struts, $V_{Rd,max}$, was greater than the ultimate strength in the stirrups, $V_{Rd,s}$, at predicted failure. This check promotes ductile behaviour by trying to avoid situations where concrete crushes prematurely, in this case due to web crushing.

6. In so far as was noticeable, possible bending and anchorage failures were eliminated from the experimental database. Qualitative assessments of patterns of shear failure made during the tests and reported in literature were reviewed and filtered for suspect cases of flexural, deep beam, or anchorage failure.

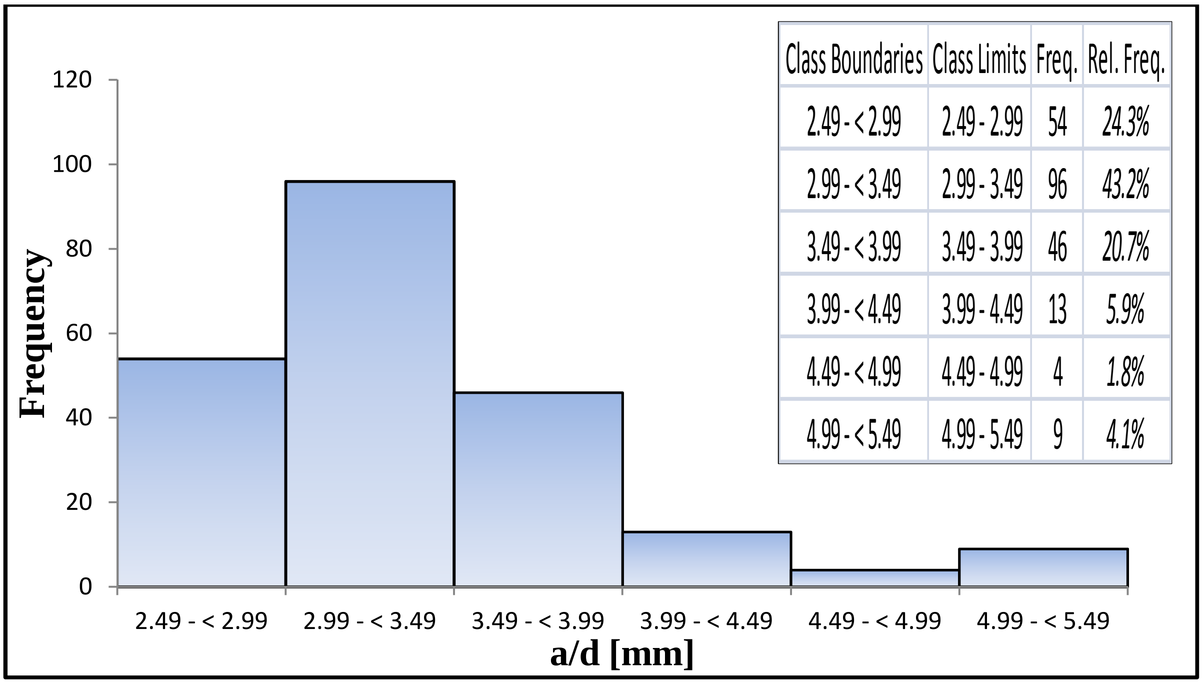
As hinted in Bullet 3 above, for some reason or the other, some beams could not be modelled in Response-2000. As can be noticed from Section 5.2.7, Response-2000 requires a wealth of input data before an ultimate shear strength prediction is made. A variable such as aggregate size was probably not well reported in earlier tests as it was not always identified as important for shear resistance. The implication is that Response-2000 could only be used to model a select number of cases where the appropriate data or information for its use was deducible. Thus, two sets of data were created; the first being the complete database of 222 tests against which the EC 2 model factor is characterised, and a subset of 116 tests against which Response-2000 can be modelled and MCFT can be characterised.

7.2.2 Range and distribution of the shear parameters of the database

For the model factors obtained and characterised in this Chapter to be of any real value, they ought to have been obtained from a database that is at least representative of beams constructed in practice or representative of situations in which the code can be applied. In terms of the shear resistance of members with stirrups, the main parameters that are known to affect shear strength are investigated in terms of their frequency of occurrence grouped by classes of occurrence. The considered shear parameters are:

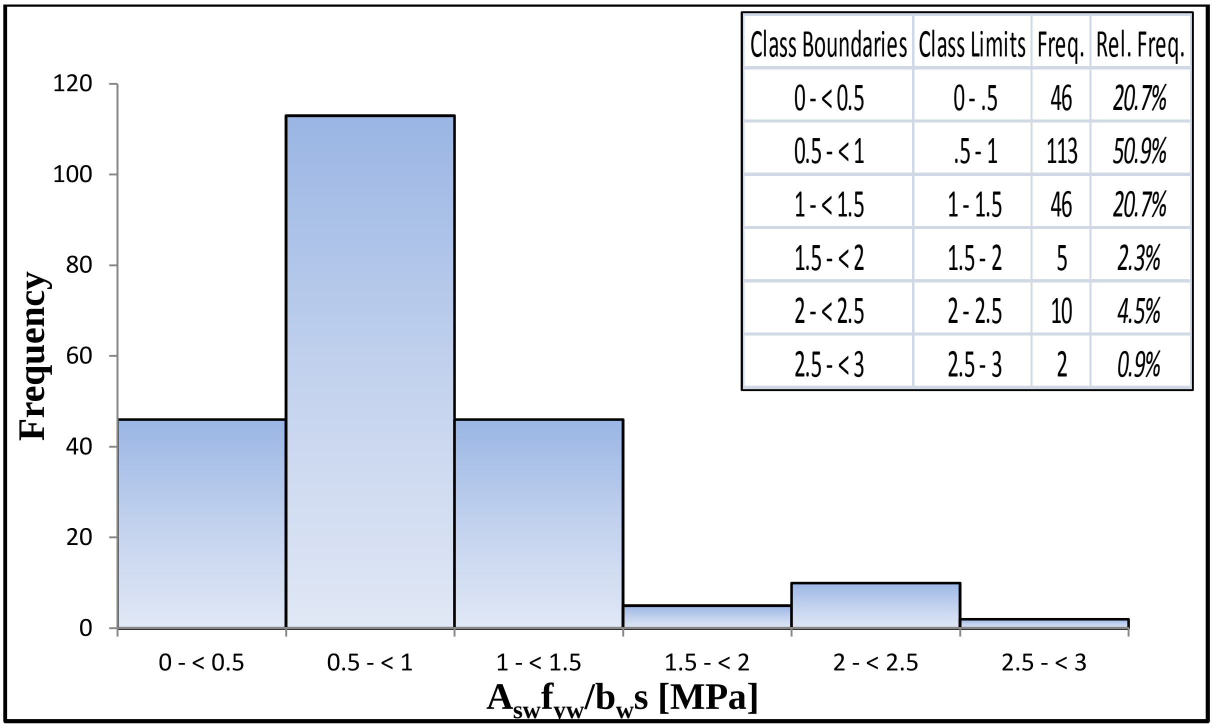
1. Shear span to depth ratio, a$/d$
2. Amount of vertical shear reinforcement, expressed as $A_v f_{yv}/b_w s$ in MPa
3. Percentage of longitudinal tension reinforcement, $\rho_l = 100A_s/bd$
4. Breadth of the member, b_w
5. Member effective depth, d
6. Concrete cylinder compressive strength, f_c

Separate Histogram plots for each of the shear parameters against the entire database of 222 tests are given in Figure 7.1 to Figure 7.6 below. The frequencies and relative frequencies of occurrence of each grouped class of data for each shear parameter are also given in the respective Figures.



Class Boundaries	Class Limits	Freq.	Rel. Freq.
2.49 - < 2.99	2.49 - 2.99	54	24.3%
2.99 - < 3.49	2.99 - 3.49	96	43.2%
3.49 - < 3.99	3.49 - 3.99	46	20.7%
3.99 - < 4.49	3.99 - 4.49	13	5.9%
4.49 - < 4.99	4.49 - 4.99	4	1.8%
4.99 - < 5.49	4.99 - 5.49	9	4.1%

Figure 7.1. Distribution of a/d for the entire database of 222 tests



Class Boundaries	Class Limits	Freq.	Rel. Freq.
0 - < 0.5	0 - .5	46	20.7%
0.5 - < 1	.5 - 1	113	50.9%
1 - < 1.5	1 - 1.5	46	20.7%
1.5 - < 2	1.5 - 2	5	2.3%
2 - < 2.5	2 - 2.5	10	4.5%
2.5 - < 3	2.5 - 3	2	0.9%

Figure 7.2. Distribution of $A_{sw}f_{yw}/b_{w}s$ for the entire database of 222 tests

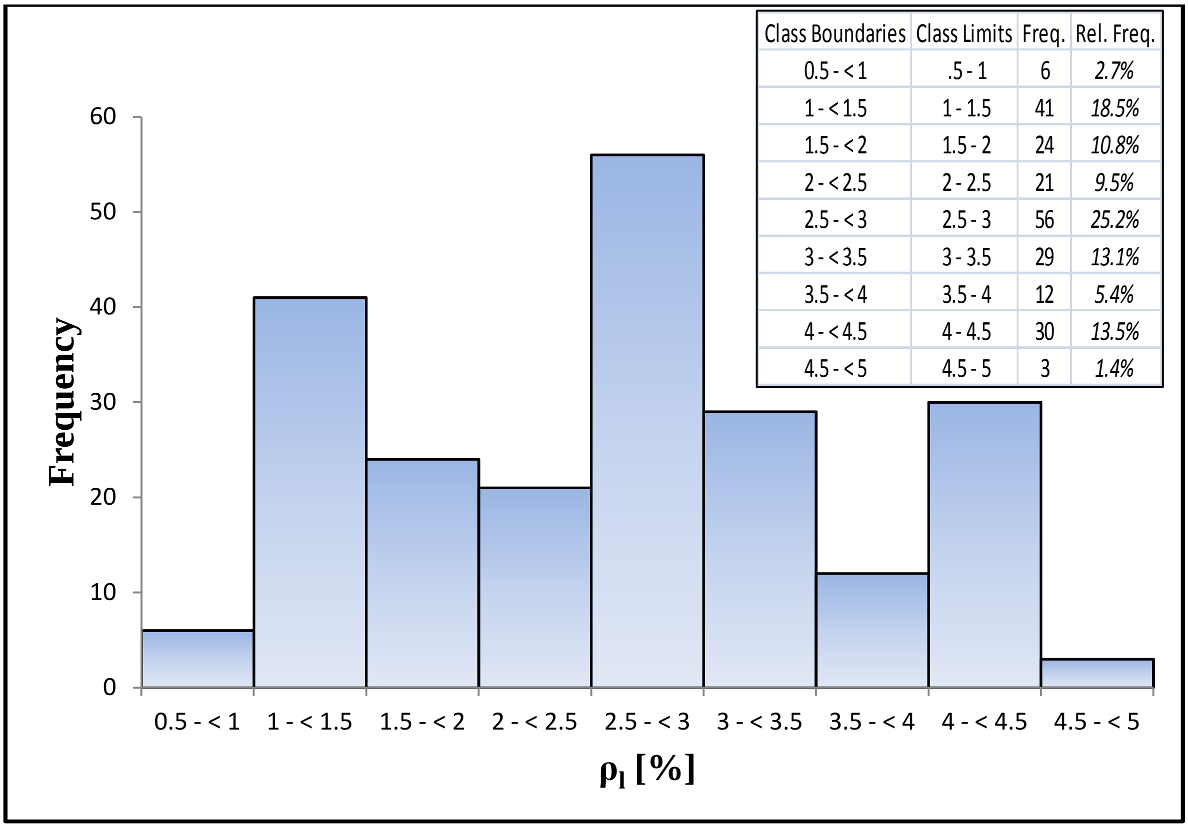


Figure 7.3. Distribution of ρ_l for the entire database of 222 tests

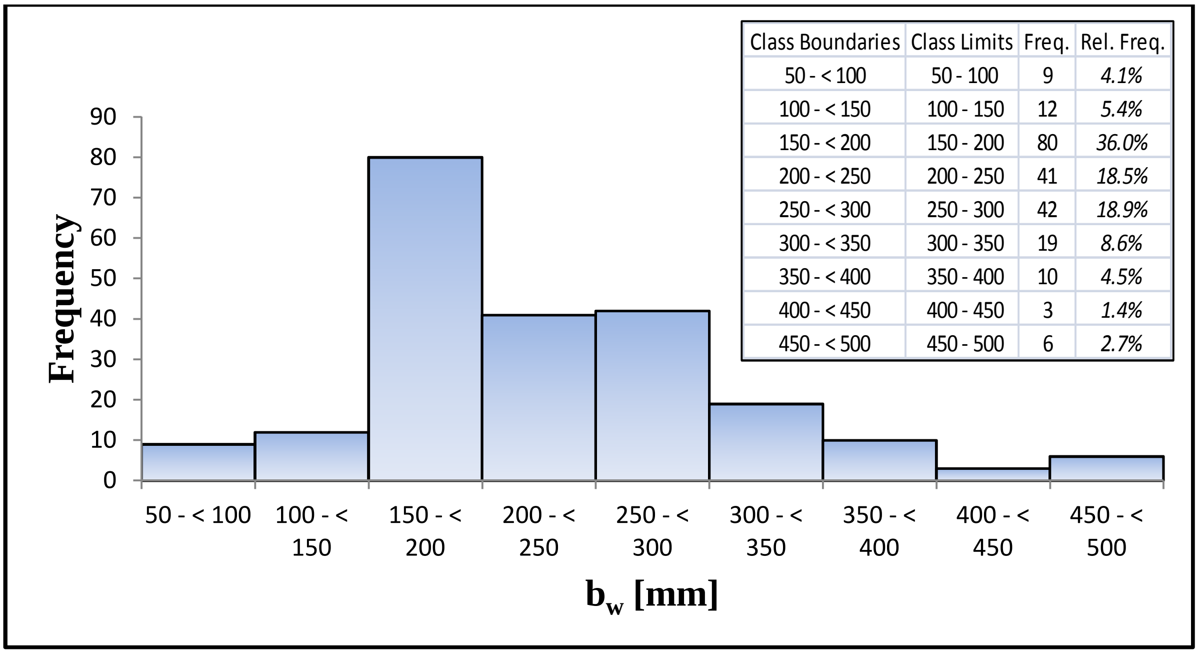
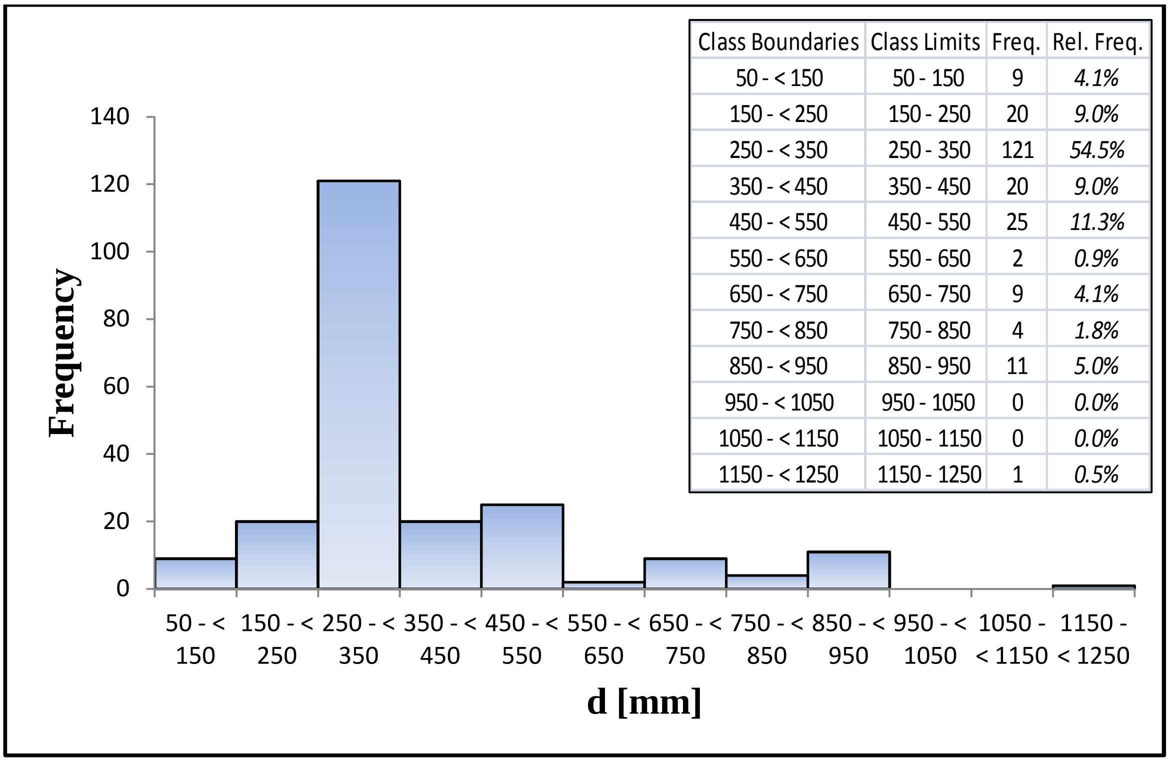
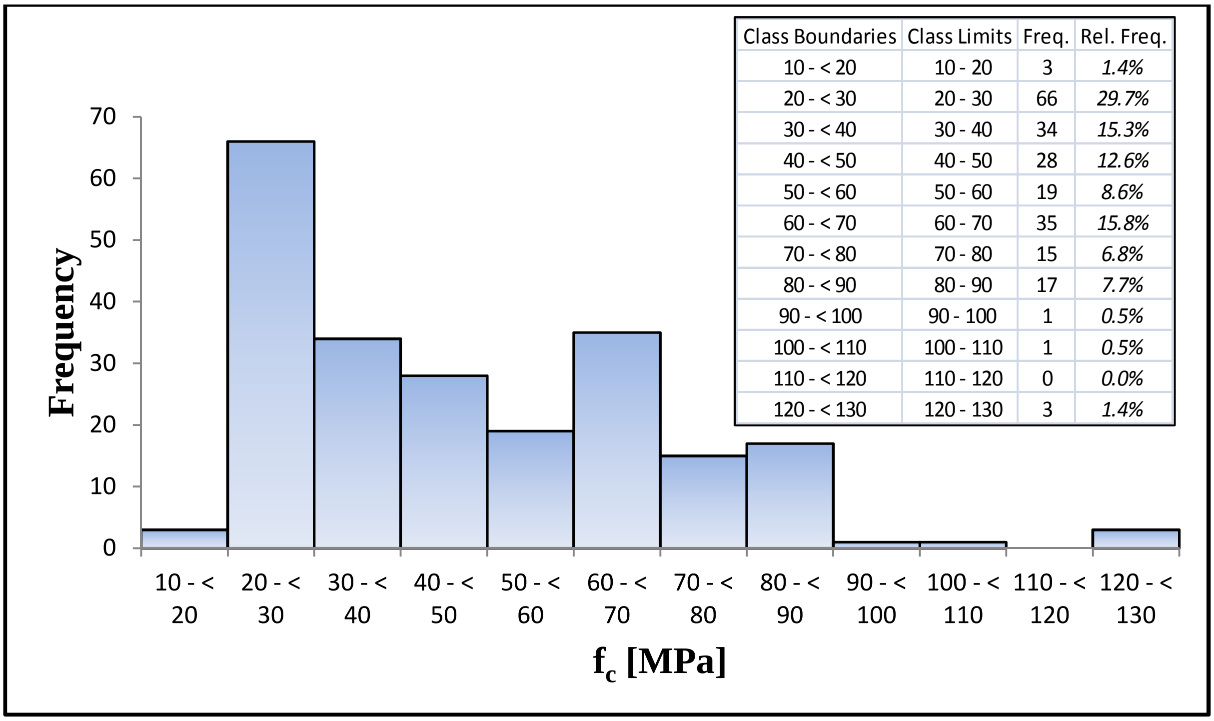


Figure 7.4. Distribution of b_w for the entire database of 222 tests



Class Boundaries	Class Limits	Freq.	Rel. Freq.
50 - < 150	50 - 150	9	*4.1%*
150 - < 250	150 - 250	20	*9.0%*
250 - < 350	250 - 350	121	*54.5%*
350 - < 450	350 - 450	20	*9.0%*
450 - < 550	450 - 550	25	*11.3%*
550 - < 650	550 - 650	2	*0.9%*
650 - < 750	650 - 750	9	*4.1%*
750 - < 850	750 - 850	4	*1.8%*
850 - < 950	850 - 950	11	*5.0%*
950 - < 1050	950 - 1050	0	*0.0%*
1050 - < 1150	1050 - 1150	0	*0.0%*
1150 - < 1250	1150 - 1250	1	*0.5%*

Figure 7.5. Distribution of d for the entire database of 222 tests



Class Boundaries	Class Limits	Freq.	Rel. Freq.
10 - < 20	10 - 20	3	*1.4%*
20 - < 30	20 - 30	66	*29.7%*
30 - < 40	30 - 40	34	*15.3%*
40 - < 50	40 - 50	28	*12.6%*
50 - < 60	50 - 60	19	*8.6%*
60 - < 70	60 - 70	35	*15.8%*
70 - < 80	70 - 80	15	*6.8%*
80 - < 90	80 - 90	17	*7.7%*
90 - < 100	90 - 100	1	*0.5%*
100 - < 110	100 - 110	1	*0.5%*
110 - < 120	110 - 120	0	*0.0%*
120 - < 130	120 - 130	3	*1.4%*

Figure 7.6. Distribution of f_c for the entire database of 222 tests

Recall that MCFT predictions could not be made for the entire database of 222 tests, but were rather made to the 116 tests that could be modelled in Response-2000. Therefore, Figures 7.1 to 7.6 are not applicable to the subset of 116 tests. Table 7.1 and 7.2 are presented below to show how the maximum and minimum values of some critical parameters related to shear varied between the entire database of 222 tests and the subset of 116 tests. The standard deviation of the shear parameters within each of the data sets are also indicated on the respective Tables.

Table 7.1. Descriptive statistics of shear parameters for complete database of 222 tests

Descriptive statistics	**b_w [mm]**	**d [mm]**	**f_c [mm]**	**ρ_l [%]**	**$A_{sw}f_{yw}/b_ws$ [MPa]**	**a/d**
Minimum	76.20	95.00	12.00	0.50	0.21	2.49
Maximum	457.20	1200.00	125.40	4.54	2.62	5.40
Standard Deviation	81.33	184.73	22.57	1.02	0.48	0.62

Table 7.2. Descriptive statistics of shear parameters for subset of 116 tests

Descriptive statistics	**b_w [mm]**	**d [mm]**	**f_c [mm]**	**ρ_l [%]**	**$A_{sw}f_{yw}/b_ws$ [MPa]**	**a/d**
Minimum	76.20	95.00	20.70	0.50	0.33	2.49
Maximum	457.20	925.00	125.40	4.54	2.62	5.00
Standard Deviation	96.06	209.07	23.27	0.91	0.48	0.64

In terms of extreme values, the subset of 116 tests compares well with the entire database. However, obvious changes in the standard deviation are observed from the values presented in Table 7.1 and 7.2. The model factor associated with the EC 2 prediction model will later be determined to the subset of 116 tests to determine the effect the reduced database size has on the character of the model factor.

7.3 CALCULATION OF THE MODEL FACTOR

The model factor associated with a single experiment is determined as the ratio of the test measured shear strength of a single test to the value of the unfactored shear strength provided either by predictions using either the EC 2 model or Response 2000. In accordance with equation [3.13], the model factor is given as:

$$MF = \frac{V_{test}}{V_{pred}} \qquad [7.2]$$

A value of $MF > 1$ implies that the prediction model yields a lower value of ultimate shear strength and is thus conservative, whilst a value of $MF < 1$ implies that the prediction model yields higher ultimate strength than is actually available in the structure and is thus unconservative.

7.3.1 Calculation of the model factor for the EC 2 shear prediction model

The unfactored shear resistance according to the EC 2 variable strut inclination method may be expressed as:

$$V_{Rd,s} = \frac{A_{sw}}{s} z f_{yw,measured} \cot\theta$$

Where, $\theta = \sin^{-1}\sqrt{\frac{A_{sw} f_{yw,measured}}{b_w s v_1 f_{c,measured}}}$ [7.3]

$f_{yw,measured}$ and $f_{c,measured}$ are the mean measured strengths of the steel yield and concrete compressive cylinder ($150 \times 300\ mm$) strength respectively. In cases where cube strength tests for concrete compressive strength were reported in literature, the necessary conversions to equivalent compressive cylinder strengths were made. Further, the strengths of non-conventional cylinder sizes (say, $100 \times 200\ mm$) used in determining compressive strength were converted to represent the strength of the conventional $150\ x\ 300\ mm$ cylinder. These adjustments were carried out with guidance from research by French and Mokhtarzadeh(1993); Aïtcin, Miao, Cook and Mitchell (1994); Carrasquillo and Carrasquillo (1998); and Mphonde and Frantz (1985).

No partial safety factors are applied to either the steel or concrete strength. The coefficient ν_1 taking account of the reduced strength of concrete cracked in shear is determined as described in Section 5.2.3 with the concrete strength threshold condition implemented as $f_{c,measured}$ when applicable.

7.3.2 Calculation of the model factor for the MCFT prediction model

Predictions of ultimate shear resistance according to the MCFT were determined as a result of the test section being modeled in Response-2000 and subjected to a constant shear analysis. The detailed procedure on the use of Response-2000 to obtain ultimate shear strength predictions is given in Section 5.2.7. To obtain unfactored estimates of the ultimate shear strength in Response-2000, mean estimates from samples or unbiased estimates of the input parameters outlined in Section 5.2.7 were used in the Program.

7.4 STATISTICAL PROPERTIES OF THE MODEL FACTOR

7.4.1 Basic statistical properties

The main statistical properties used to characterise model uncertainty are the mean or expected value of the model factor, determined by averaging all realisations of the model factor for each test in the database, and its standard deviation. The coefficient of variation is an alternative measure of dispersion and is also included in the results. Three cases of the model factor are reported in this Section. The two main cases are:

1. The statistics of the model factor associated with the EC 2 shear prediction model as determined from comparison to the entire database of 222 tests

2. The statistics of the model factor associated with the MCFT predictions from Response 2000 from comparison to subset of 116 tests

The third, somewhat less important case, is the statistics of the model factor associated with the EC 2 prediction model as determined from comparison to the subset of 116 tests. This case was considered to serve as comparison of how the statistics of the model factor associated with the EC 2 shear prediction model would change if compared to the same subset of data to which predictions according to the MCFT were made. Table 7.3 gives the basic descriptive statistical parameters of the model factor associated with the MCFT and EC 2 prediction models.

Table 7.3. Results of model factors associated with EC 2 and MCFT prediction models

	116 tests		**222 tests**
Statistics of the Model Factor	**MCFT**	**EC 2**	**EC 2**
Average	1.14	1.70	1.65
Std. Dev.	0.20	0.53	0.51
C.O.V	0.18	0.32	0.31
Minimum	0.72	0.59	0.59
Maximum	1.76	3.21	3.21

From the results presented in Table 7.3, it can be viewed that all models generally provide conservative predictions of shear resistance. The average or mean value of the EC 2 model factor and its standard deviation as determined against the entire database and the subset of 116 tests do not differ significantly. It should be noted that from here on, the model factor for EC 2 based on the entire database of 222 tests will be used in all discussions. The EC 2 model factor was only characterised to the 116 tests to determine whether there were serious deficiencies in the reduced database so as to significantly change the character of the model factor. No significant changes were detected.

The EC 2 model factor, together with its large conservative bias of 65 %, also has a large standard deviation of 0.51 associated with it. Such a large deviation implies a large spread of values around the mean, implying that great differences in some cases of data may exhibit model factor far from 1.65. The critical cases are, of course, the events in which the model factor deviates significantly below 1.65 and close to or even less than 1, and the design situations in which they occur. Correlation and regression analyses were done to determine the trends of the model factor with the shear parameters to assess the relation of the model uncertainty to the variation in a given shear parameter.

The MCFT is clearly the more accurate and precise prediction model for determining the shear resistance of members with stirrups. Not only is it characterised by a lower mean conservative bias of 14 %, but it also has a lower standard deviation of 0.20 associated with it. This implies that less variation in the mean can be expected than for the EC 2 shear prediction model across the different design situations reflected through a variation in the shear parameters.

7.4.2 Correlation and regression analysis

Scatter plots of the model factor versus the different shear parameters can be made to determine what the general trend a given shear parameter has on the model factor. A trend in the model factor with a shear parameter indicates that the model does not fully take account of the effect that the parameter has on the shear resistance of tested beams. If a trend is strong enough, some control on how the model is used in regions of unsafe performance should be exercised. The control may present itself either by limiting the range of applicability of the model in such situations, or perhaps specific calibration measures to incorporate adequate safety elements in the model that lead to conservative designs even when the model is used in such situations. This basically implies that the model factor be appropriately incorporated in partial factor determination to cater for adverse situations.

Trends in the model factor of the EC 2 design model or the MCFT can be investigated by determining their correlation with the shear parameters. The Pearson's correlation coefficient, r, lends itself as a descriptor of the correlation between two variables and is defined as:

$$r = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum(x_i - \bar{x})^2}\sqrt{\sum(y_i - \bar{y})^2}} \qquad -1 \leq r \leq 1 \qquad [7.4]$$

Where x_i and y_i are the i^{th} experimental observation of the x and y parameters. $\bar{x}$ and $\bar{y}$ are the mean values of all the experimental observations of the x and y parameters respectively.

Two random variables that have a non-zero correlation are said to be correlated. The Pearson's correlation coefficient measures the strength of the relationship between two variables. An r of -1 indicates a perfect negative linear relationship between variables, an r of 0 indicates no linear relationship between variables, and an r of 1 indicates a positive perfect linear relationship between variables. The coefficient of determination, R^2, is used to judge the adequacy of the regression model. R^2 can be closely approximated as the square of the correlation coefficient of a given parameter. Linear regression models were fitted to scatter plots of the model factor for each of the design models versus the individual shear parameters. Montgomery and Runger (2007) advise that the R^2 statistic be used with caution, because it is always possible to make R^2 close to unity by simply adding enough terms to the model. For example, polynomial and power functions can fit complex patterns of scatter plots but are analytically more complex to deal with than simple linear regression or other models of fit.

A combination of a relatively strong correlation coefficient and a relatively strong coefficient of determination indicate that a strong linear relationship exists between the model factor and the shear parameter (Huber, 2005). If the correlation coefficient is strong but R^2 is relatively weak this indicates that the relationship between the model factor and the shear parameter is not linear. A non-linear regression may be a better fit as would be characterised by a stronger R^2 for such a fit.

Table 7.4 shows the correlation data of the MCFT with the shear parameters according to the subset of 116 tests as well as the correlation the correlation data of the EC 2 shear prediction model with the entire shear database of 222 tests. The correlation data of the EC 2 shear prediction model with the shear parameters according to the subset of 116 tests are also included in Table 7.4. Further, Table 7.5 gives a summary of the regression coefficients of the regression lines that were fit to the scatter plots of the model factor associated with each prediction model against the various shear parameters. Figure 7.7 and Figure 7.8 show the scatter plots and hence illustrate the trends of the EC 2 and MCFT prediction models, respectively, against the various shear parameters.

By visual inspection of Tables 7.4 and 7.5, the amount of shear reinforcement, $A_{sw}f_{yw}/b_w s$, clearly has the strongest relationship with the model factor, with an r of -0.68 and an R^2 of 0.4559 for the EC 2 shear prediction model. An r of -0.24 and an R^2 of 0.059 describe the relationship between the amount of reinforcement and the model factor of the MCFT prediction model. It should be noted that the EC 2 prediction model has correlation of r of -0.69 with the amount of shear reinforcement when characterised to the subset of 116 tests. This corresponds to an R^2 value of about 0.4761. Thus, even for the reduced database size of 116 tests, the data is sufficiently representative to capture the general correlation trend between the model factor and the amount of shear reinforcement.

Table 7.4. Correlation data of the shear parameters with the EC 2 and MCFT prediction models

	116 tests		**222 tests**
Variable	**Correlation coefficient, r (MCFT)**	**Correlation coefficient, r (EC 2)**	**Correlation coefficient, r (EC 2)**
a/d	-0.12	0.14	0.11
$A_{sw}f_{yw}/b_ws$	-0.24	-0.69	-0.68
ρ_l	-0.25	0.06	0.05
b_w	0.25	0.13	0.09
d	0.07	0.08	-0.05
f_c	-0.02	0.20	0.13

Table 7.5. Summary of the linear regression data for the EC 2 and MCFT prediction models

	MCFT predictions to 116 tests			**EC 2 predictions to 222 tests**		
Variable	**gradient**	**intercept**	**R^2**	**gradient**	**intercept**	**R^2**
a/d	-0.0369	1.2599	0.0137	0.0928	1.3374	0.0128
$A_{sw}f_{yw}/b_ws$	-0.1019	1.2282	0.059	-0.7074	2.2431	0.4559
ρ_l	-0.0559	1.3003	0.0636	0.0239	1.582	0.0023
b_w	0.0005	1.0161	0.0636	0.0006	1.5189	0.0089
d	0.00007	1.1142	0.055	-0.0001	1.6974	0.0027
f_c	-0.0002	1.1513	0.0003	0.003	1.5018	0.0178

By considering the regression data presented in Table 7.5, it can be observed that the gradient values of the regression lines are generally very shallow, of decimal significance and much less than 1, implying that no distinct relationship exists between the model uncertainty and most of the shear parameters. However, quite a steep gradient coefficient of -0.7074 is realised for the regression line fit to the scatter plot between the model factor and the amount of shear reinforcement for the EC 2 prediction model. This fact, coupled with a higher correlation coefficient with the model factor as well as a relatively high coefficient of determination provide evidence that there is a trend of some significance between the amount of shear reinforcement and the EC 2 prediction model. The implication is that full account of $A_{sw}f_{yw}/b_w s$ its effects are not adequately modelled by the EC 2 variable strut inclination method for shear. It should be noted that no alarming trend of results are observed between the MCFT shear prediction model and $A_{sw}f_{yw}/b_w s$, as it is characterised by a lower gradient of the regression line of -0.1019 and r and R^2 as reported above. Nonetheless, regarding the statistics presented in Tables 7.4 and 7.5 and from Figure 7.8, it can be observed that even for the MCFT, which is across board a better predictor of shear, the amount of shear reinforcement still most affects the performance relative to other shear parameters.

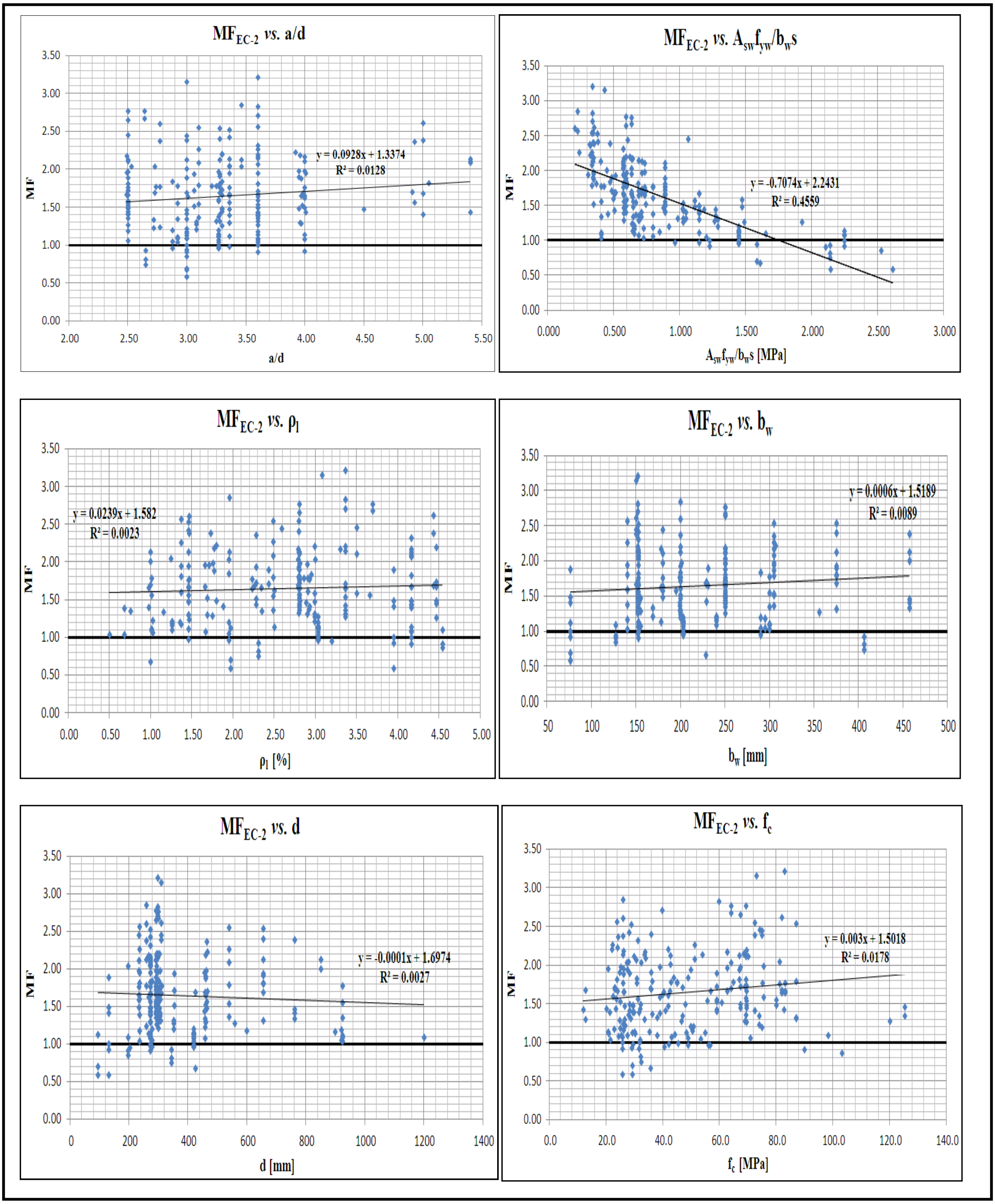


Figure 7.7. Trends of model factor with shear parameters (a/d, $\frac{A_{sw}f_{yw}}{b_w s}$, ρ_l, b_w, d, and f_c) comparing the EC 2 prediction model to the complete database

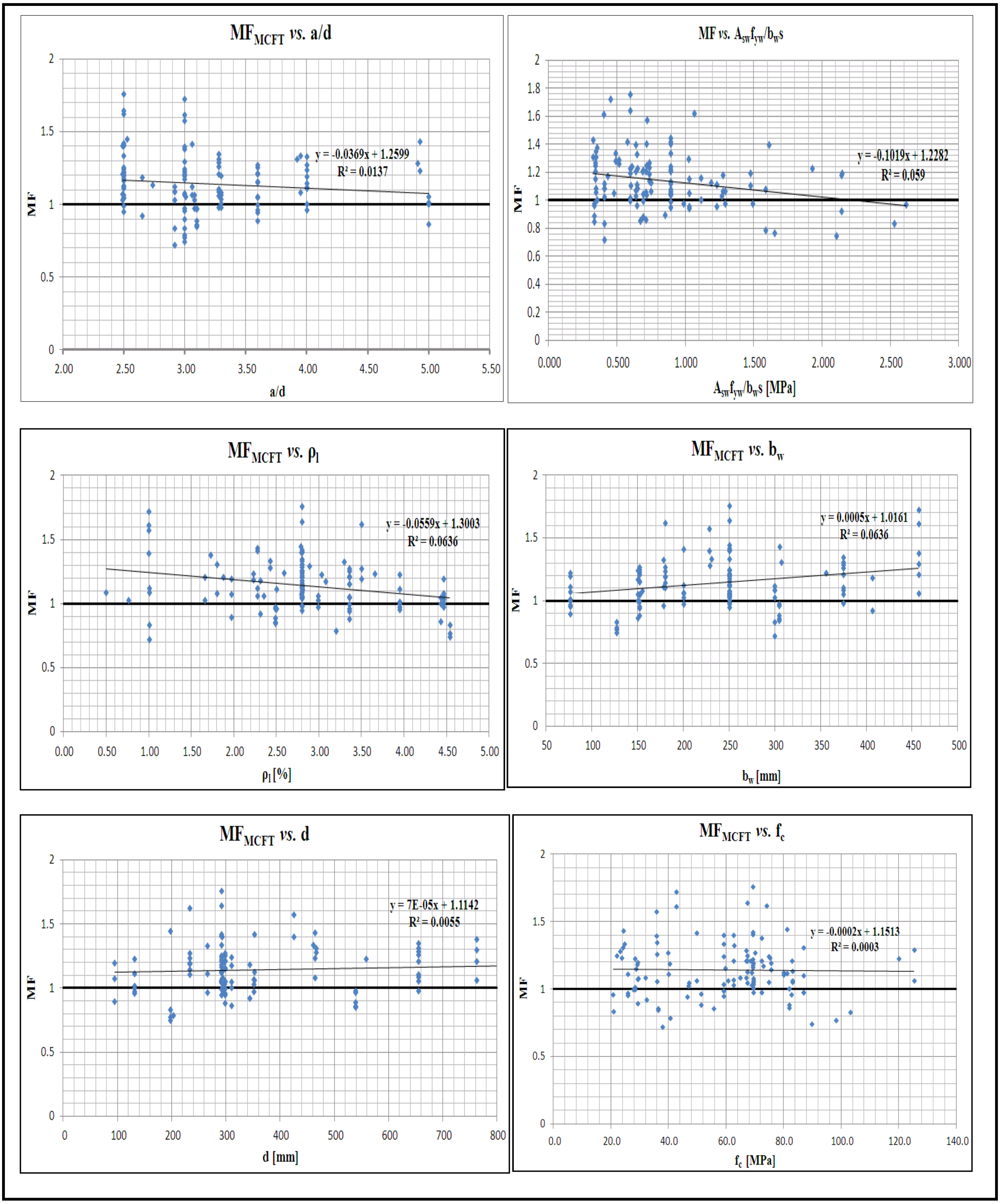


Figure 7.8. Trends of model factor with shear parameters (a/d, $\frac{A_{sw}f_{yw}}{b_w s}$, ρ_l, b_w, d, and f_c) comparing the MCFT to the database of 116 tests

Trends of the model factor with the different shear parameters are discussed systematically below.

Trends with a$/d$*,* ρ_l*, and* f_c

It can be observed from Figure 7.7 that there is a net increasing bias of the EC 2 model factor with an increase in a$/d$, ρ_l, and f_c. All the three shear parameters have positive correlations with the EC 2 model factor, with r of 0.11 for a/d, 0.05for ρ_l and 0.13 for f_c. The model uncertainty can be viewed as conservative throughout all situations of each of the shear parameters considered, with no distinct linear relationship between the model factor and the shear parameters as they are characterised by low gradients of their respective regression lines together with low coefficients of determination.

Opposite trends in the model factor of the MCFT prediction model for shear were realised with a$/d$, ρ_l, and f_c than for the case of the EC 2 model factor. It can be observed from Figure 7.8 that there is a decreasing bias in the model factor with an increase in each of the three parameters. a$/d$ has an r of -0.12, ρ_l an r of -0.2,5 and f_c an r of -0.02 when analysed against the MCFT model factor. For the range of the shear parameters considered, no unconservative values of the model factor were realised. However, adequate conservative bias can be viewed as necessary for design models through all design situations hence making any decrease in conservative bias of particular interest. From a practical viewpoint, the situations in which the model factor approaches 1 occur less frequently, for example ρ_l gets closest to 1 at a value of 4.5 % whilst most practical design situations are characterised by the use of ρ_l of 0.5 – 1.5 %. As judged from the relative values of the correlation coefficients, even less severe trends occur for a$/d$ as well as f_c. More data points or physical tests should be fit to the MCFT prediction model and characterised against the shear parameters to intensify and better characterise any underlying trends with the model factor.

Trends with the amount of shear reinforcement, $A_{sw}f_{yw}/b_w\, s$

From Figure 7.7,considering only the EC 2 model factor, there are clearly much more data points for lower amounts of shear reinforcement of less than 1.25 or 1 MPa than for higher values. As such, some non-linear regression would definitely better suit the scatter plot, especially at providing better representation of the data in regions where more statistical data are available. The model factor decreases with increasing amount of shear reinforcement and starts to become unconservative at $A_{sw}f_{yw}/b_w\,s$ of about 1.7 MPa. Given in Figure 7.9 below, is a logarithmic trendline or regression fit that better suits the data as suggested by the higher coefficient of determination of 0.5011, as opposed to 0.4559 when the linear fit is used.. Better fits, as characterised by higher R^2 values, to the data are possible. For example, better fits were obtained when the power or polynomial regression fits were applied to the scatter plot. However, such models are more complex than the logarithmic fit and in exchange for increased complexity, do not prove to result in much increase of the accuracy of the regression fit.

The relation between the amount of shear reinforcement and the MCFT model factor can be viewed in Figure 7.8. The model factor also decreases with an increasing amount of shear reinforcement, but starts to become unconservative at a value of about 2.1 MPa. It would be desirable to see whether this value would increase or decrease if more data points or test results were available for consideration. The net non-linear trend is still observable from the scatter plot, but obviously with less strength than for the EC 2 model factor owing to the fewer data points used in characterising the MCFT model factor. A logarithmic regression fit of the scatter plot of the MCFT model factor versus the $A_{sw}f_{yw}/b_w\,s$ is given in Figure 7.10.

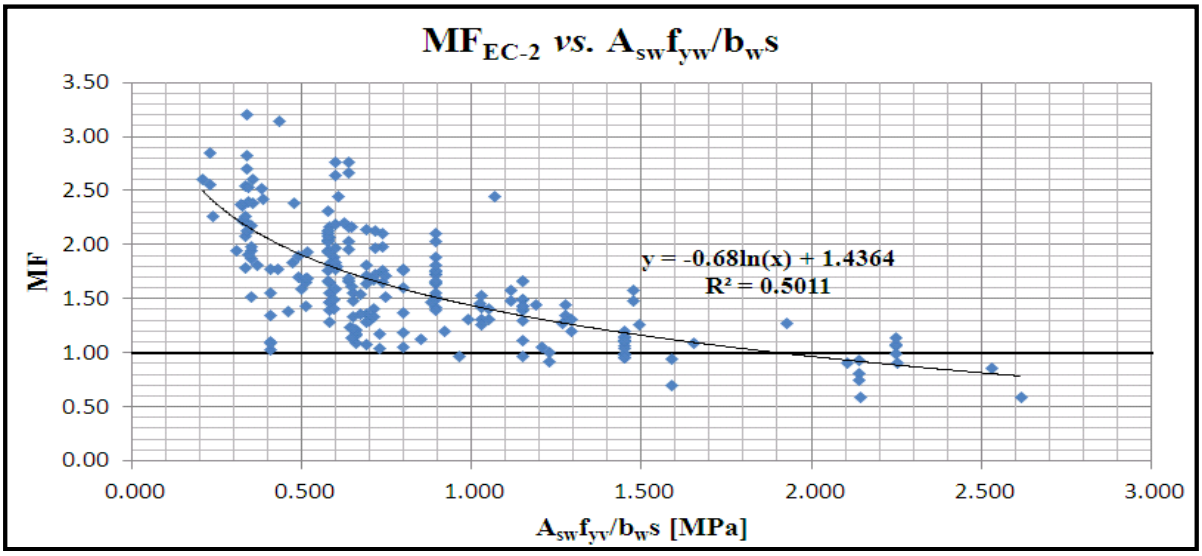


Figure 7.9. Logarithmic regression trendline fit to the scatter plot of the EC 2 model factor against the amount of shear reinforcement

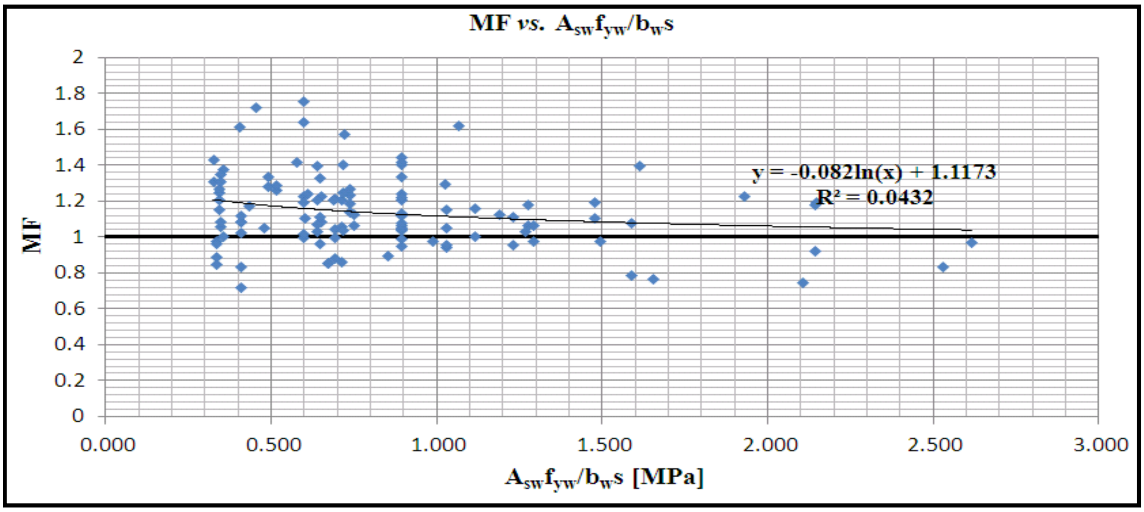


Figure 7.10. Logarithmic regression trendline fit to the scatter plot of the MCFT model factor against the amount of shear reinforcement

For both cases of the model factor the logarithmic fit improves the limits at which the models become unconservative. However, the logarithmic is not a better regression fit to the data of 116 tests, shown in Figure 7.10 with an R^2 of 0.0432, as opposed to the corresponding graph from Figure 7.8 which has an $R^2$of 0.059. Thus, the linear trendline fit of data should be used when the MF is characterised against $A_{sw}f_{yw}/b_w\,s$ for the data set of 116 tests. On the other hand, the logarithmic fit provides better characterisation of the MF trend with $A_{sw}f_{yw}/b_w\,s$ for the full database of 222 tests. From Figure 7.9 it can be observed that the EC 2 model factor now becomes unconservative at a slightly increased value of about 1.9 MPa whereas no unconservative estimates of the model factor are realised for the range of data considered in Figure 7.10 for the case of the MCFT, though the general trend of decreasing conservative bias with increasing amounts of shear reinforcement is maintained.

Trends with b_w

There is an increase in the conservative bias of the model factor with an increase b_w for both the EC 2 and MCFT prediction models. However the correlation of b_w with the model factor is stronger for the MCFT, with an r of 0.25, than for the EC 2 prediction model, which has a r of 0.09. For both cases, no unconservative values of the model factor were realised over the design situations considered. For the case of the MCFT, as can be seen from Figure 7.8, marginal conservative bias is witnessed for low values of b_w approaching $50\,mm$. Such small widths of beams are not anticipated and rarely occur in practice, owing to the fact that their construction would be quite difficult, especially in terms of placing reinforcement and achieving adequate cover and can be viewed as an impractical range of the data. Thus, satisfactory performance is expected through all situations of the model factor with variations in b_w.

Trends with d

In general, for both the EC 2 and MCFT prediction models and the test data considered in each case, adequate account seems to have been taken of the effective depth in regard to model uncertainty, or simply the size effect in shear. The size effect in shear is the phenomena where the ultimate shear strength of a member decreases with its increasing size and is more pronounced in members without shear reinforcement, where the spacing and width of the cracks significantly affects the shear resistance of the member. The trends, however, between d and the model factors are somewhat opposite; the conservative bias decreases for the EC 2 model factor with increasing values of d, and vice versa holds for the relation of the MCFT with d. The EC 2 model factor has an r of -0.05 with d, whilst the MCFT has an r of 0.07 with d. No alarming cases or trends of the relationship between the model factor and d are observed for both the prediction models.

7.5 MODELLING OF THE MODEL FACTOR FOR THE LIMIT STATE EQUATION

In this Section, appropriate continuous statistical distributions are sought to represent the model factors for the EC 2 and MCFT prediction models. The respective distributions will be used to represent the model factors as basic variables during reliability modelling carried out in Chapter 8. Renowned structural engineering documents such as the JCSS Probabilistic Model Code (JCSS, 2001) usually provide guidance that log-normal or normal distributions be assumed for model factors in situations where no explicit information on the model factor is available. Holický (2009), whose proposals are given in presented in Table 6.1, recommends that the normal distribution be used for representing the model factor. For the purposes of reliability modelling using the FORM method, the normal distribution is the easiest to manipulate as no transformation of the distribution will have to be undertaken.

The choice of the normal distribution can be used with confidence where very little skewness in the data is observed as zero skewness is reflected when the normal distribution is used. Skewness of a distribution is a dimensionless quantity that describes the asymmetry associated with a distribution or its bias toward generating higher or lower values of the variable. In this regard, the normal distribution is symmetric and is hence not capable of reflecting asymmetry. The skewness, s_k, from a sample of data can be calculated from:

$$s_k = \frac{n}{(n-1)(n-2)} \sum \left(\frac{x_i - \bar{x}}{\sigma}\right)^3 \quad [7.5]$$

Where n is the number of tests or data points in the database under evaluation and σ is the unbiased estimator of the sample standard deviation as obtained from the test results. Holický (2009) advises that a skewness of 0.5 can be taken as a relatively high value, thus implying its significance should be taken into account in random variables that have an equal or higher skewness than 0.5. The value of s_k of the EC 2 model factor was determined to be 0.482 and that of the MCFT was found to be 0.612. The value of 0.482 was taken roughly equal to 0.5 and as such distributions other than the normal distribution were sought to appropriately represent the model factor realisations.

Easy-Fit software was used to carry out statistical goodness of fit tests to determine which distribution would best represent the model factor realistaions for associated with each of the Predicition models. The trial version of the software is available for free download at: http://www.mathwave.com/ . Easy-Fit is a distribution fitting software program has the 61 continuous distributions, each of which were tested against input data of the model factor realisations to determine the distribution of best fit to the data. The Kolmogorov-Smirnov goodness of fit test is the goodness of fit test preferred by the program, although other alternatives as the Anderson-Darling and Chi-squared goodness of fit tests can be opted for use by the program. The Kolmogorov-Smirnov test is adopted for use in the thesis. Table 7.6 and Table 7.7 show the ranking of suitable distributions that can be used to model the EC 2 and MCFT model factors respectively. It is deducible through inspection of the Tables that the lower the test statistic of a model according to the Kolmogorov-Smirnov test, the better the fit of the continuous distribution considered to the model factor realisations.

Table 7.6. Results of the Kolmogorov-Smirnov test from Easy-Fit for the EC 2 model factor

	Result of Kolmogorov-Smirnov test	
Distribution	**Statistic**	**Rank**
General 4-parameter Gamma	0.02813	1
3-Parameter Log-normal	0.041	9
2-Parameter Log-normal	0.05517	25
Normal	0.05768	28

Table 7.7. Results of the Kolmogorov-Smirnov test from Easy-Fit for the MCFT model factor

	Result of Kolmogorov-Smirnov test	
Distribution	**Statistic**	**Rank**
4-Parameter Burr	0.02813	1
2-Parameter Log-normal	0.041	4
3-Parameter Log-normal	0.05517	5
Normal	0.05768	29

Selected models of the model factor for reliability analysis

The 3-Parameter log-normal distribution will be used to model the model factor associated with EC 2's variable strut inclination method for members requiring stirrups. On the other hand, the 2-Parameter log-normal distribution will be adopted for use in describing the occurrence of realisations of the MCFT shear prediction model during reliability analyses in Chapter 8. Problems of structural engineering commonly adopt the normal and log-normal distributions to describe basic random variables, especially those concerned with describing resistance variables. Thus, the log-normal distributions are preferred for use here. It should be noted that better fits are available, as can be seen from the rankings in Table 7.6 and Table 7.7, but are more complex to deal with especially in having to transform them to an equivalent normal space as is required when the FORM method is used for reliability analyses. Figure 7.11 and Figure 7.12 show histograms from Easy-Fit with the normal and log-normal distributions super-imposed to reflect the different trends of the distributions in describing model factor realisations of the EC 2 and MCFT prediction models respectively.

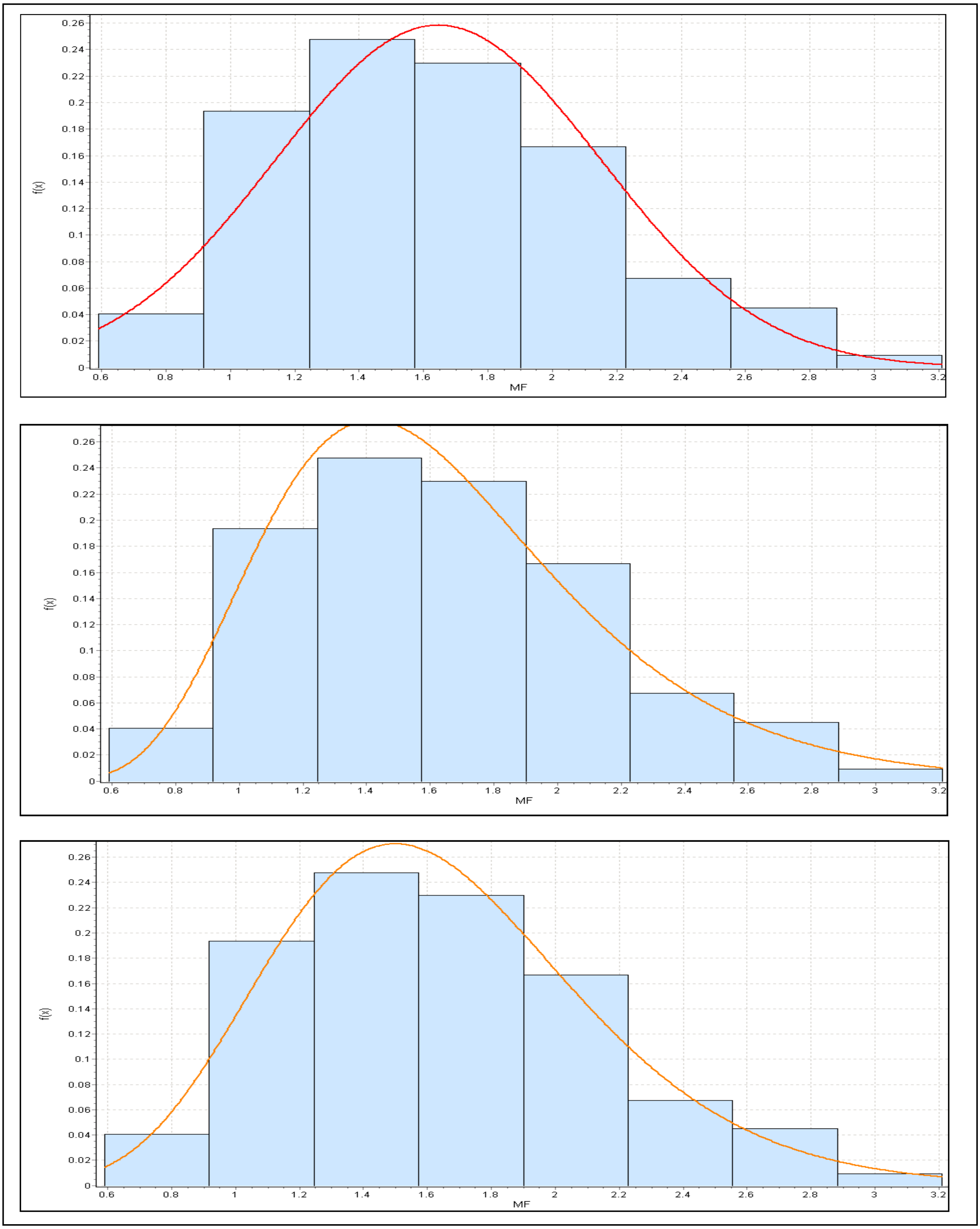


Figure 7.11. Normal (***top***), 2-parameter log-normal (***middle***), and the 3-parameter log-normal distribution (***bottom***) of the EC 2 model factor

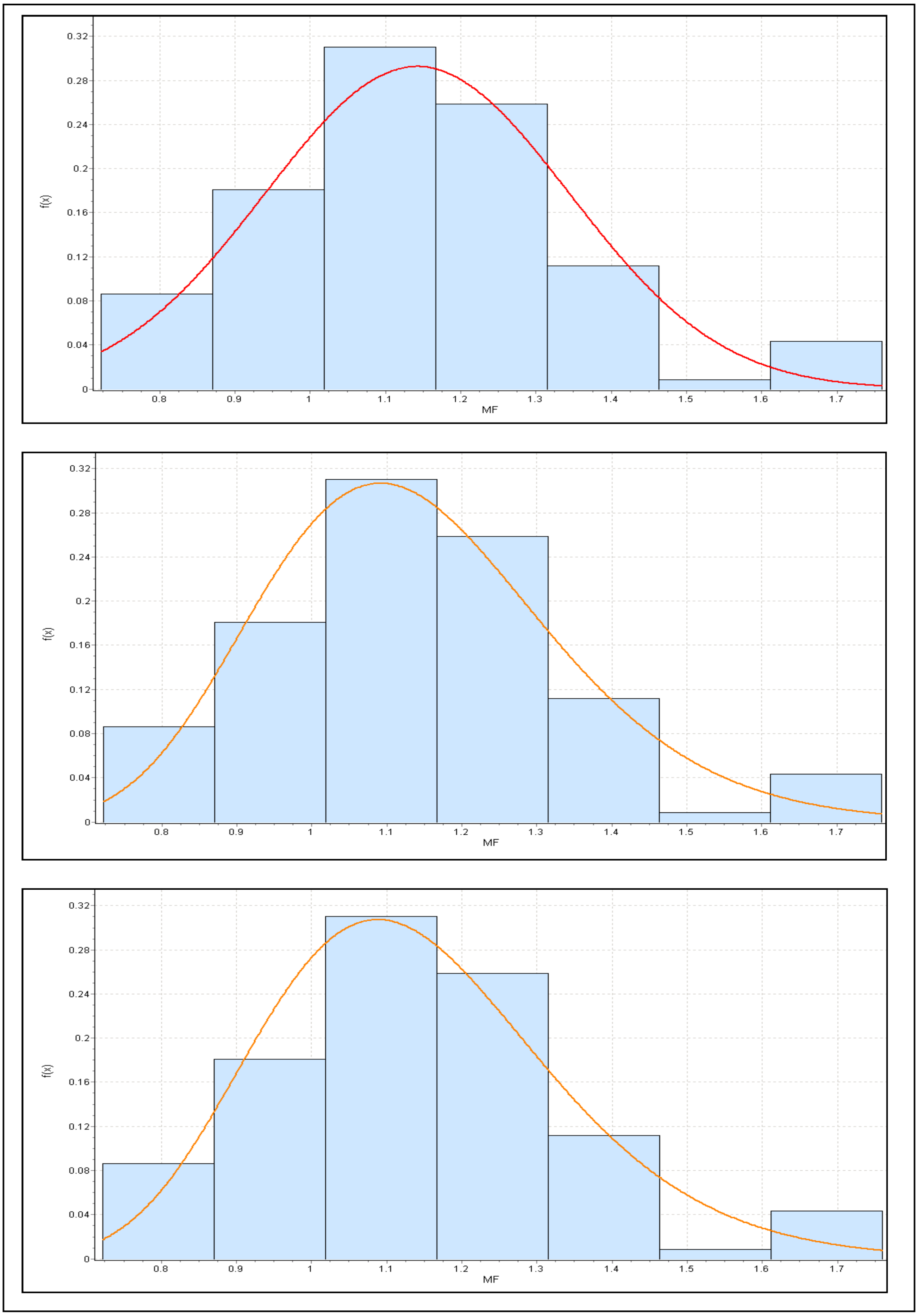


Figure 7.12. Normal (***top***), 2-parameter log-normal (***middle***), and the 3-parameter log-normal distribution (***bottom***) of the MCFT model factor

CHAPTER 8

RELIABILITY ANALYSIS OF THE EC 2 SHEAR PREDICTION MODEL FOR MEMBERS REQUIRING STIRRUPS

8.1 INTRODUCTION

A full investigation aimed at establishing the appropriate calculation steps that enable the adequate reliability assessment of the EC 2 variable strut inclination method are presented in this Chapter. The FORM method of reliability analysis was used to determine the reliability of the EC 2 shear design method in two slightly different procedures, briefly described as:

1. One in which the EC 2 shear prediction model is converted for use as the general probabilistic model, against which the reliability of the EC 2 design procedure is sought. The EC 2 prediction method is converted to a general probabilistic model (gpm) when unbiased values of all the basic variables, all of which were initially represented as random variables with statistical distributions, are used in the model to make a prediction, alongside neglecting the use of any partial safety factors that are incorporated for use when the design model is applied in practice. And,

2. The other in which the more rational-scientific MCFT is adopted for use as the general probabilistic model in the reliability analysis. MCFT predictions of true shear resistance are obtained by entering unbiased and unfactored values of the relevant basic variables, most of which were initially represented as random variables with statistical distributions, into the analysis Program Response-2000.

It can be concluded from the results of the model factor presented in Chapter 7 that the MCFT is a better predictor of shear resistance for members requiring stirrups than the EC 2 variable strut inclination method. The model factor associated with the MCFT is characterised by a lower mean bias of 1.14, as opposed to EC 2’s value of 1.65, as well as a lower spread or standard deviation of 0.20, as opposed to EC 2’s value of 0.51. Furthermore, the EC 2 model is sensitive to the amount of shear reinforcement, as shown in Figure 7.7 and Figure 7.9. It is therefore obvious that the MCFT provides more precise and consistent predictions of shear resistance than the EC 2 variable strut inclination method and in this regard, strictly speaking, the MCFT should automatically be adopted as the general probabilistic model for shear resistance. However, and in general, any model for shear can serve as the general probabilistic model in reliability analyses so long as its model factor can be established with reasonable accuracy coupled with it being expressed in its unbiased and unfactored form.

A limited parametric study, conducted considering two representative design situations referred to as test cases, was conducted to establish the relative contributions toward the reliability performance due to the identified shear parameters. A further, and arguably more important, objective of this Chapter was to determine the reliability indices ($\beta's$) associated with each of the design situations considered; this can be viewed as a rational assessment that is in line with modern basis of design requirements and was aimed at analysing how safe the shear design provisions are when implemented in practice. It was found that the EC 2 variable strut inclination method is adequately safe for the design situations considered, but with some concern about its performance in design situations where large amounts of shear reinforcement are provided in design. These issues are discussed later in this Chapter.

The reliability index, β, associated with both test cases was determined in two separate analyses; one where the EC 2 prediction method is converted to serve as the gpm in the probabilisitic model for shear and the other where the MCFT was adopted as the gpm. For both test cases, one of which was representative of a design situation with low amounts of shear reinforcement and the other with high amounts of shear reinforcement, it is shown that the model factor (MF) has dominating influence on the performance of members designed according to EC 2 . In fact, the model factor was found to completely dominate shear reliability performance for members with stirrups in the reliability analyses where the EC 2 prediction model was converted for use as the gpm. For such instances, this warranted the development of a simplified reliability model, in which the model factor is the only basic random variable considered in the general probabilistic model and therefore completely influences the distribution for the shear of members requiring stirrups, for quick and meaningful analyses into the reliability performance of the EC 2 design method.

When the MCFT was used as the gpm, some noticeable contribution toward reliability performance was exhibited by the basic random variables for concrete cover (C), the yield strength of the stirrups (f_{yw}) and the concrete strength (f_c). Naturally, the MCFT requires more basic input data to make a prediction of ultimate shear strength than the EC 2 prediction model for shear. Therefore, in these instances, the simplified reliability model represented the basic variables MF, C, f_{yw}, and f_c as the only random variables in the gpm and neglected the distributions of the other basic variables, thus treating them deterministically.

As a result of the full investigation, where two possible general probabilistic models are considered for use in reliability modelling, some differences in the results of the reliability analysis of the test cases are demonstrated between the two separate analyses. In general, higher β-estimates are derived for the test cases when the MCFT is used as the gpm as compared to corresponding analyses of the test cases when the EC 2 prediction model is employed as the gpm in the reliability analysis. Furthermore, an investigation aimed at determining the influence the shear-span-to-depth ratio, a/d, has on shear strength predictions showed that the MCFT as is implemented in Response-2000 takes due account of changes in ultimate shear resistance for members with varying influences from shear-flexure interactions; predicting higher shear resistance for shear-critical members (members loaded significantly in shear where a/d is closer to 2.5) to members less heavily loaded in shear where even flexure may dominate as the primary load carrying mechanism (higher a/d ratios, closer to 6).

After an a/d of 4.0, Response-2000 predicts much reduced values of ultimate shear strength for the test case herein considered as representative of a design situation with relatively high amounts of shear reinforcement ($A_{sw}f_{yw}/b_w s \approx 1.8\ MPa$). This strong trend of results was not realised for the test case that was representative of a design situation with low amounts of shear reinforcement ($A_{sw}f_{yw}/b_w s \approx 0.4\ MPa$) where only slight and gradual decrease was experienced in the ultimate shear strength predictions as a/d increased.

The EC 2 prediction model is inert to changes in a/d and is applied in practice for the design of shear-critical regions or regions some distance from the support or load points and fails at characterising shear behaviour along the full member length. This finding does not discredit the EC 2 prediction model due to the fact that it does not lend itself to describing the full shear behaviour of members, but is rather a sectional design method applied to critical shear locations that are usually some distance from the support. Further, by viewing the appropriate graph in Figure 7.7, it can be observed that EC 2's variable strut inclination method portrays adequate conservative bias at the different a/d considered. The design for shear from the critical or design section is usually applied throughout the design at all other necessary locations in a member. The basis of EC 2's design method is therefore consistent with the design approach or philosophy. In a real beam within the confines of $2.5 \leq a/d \leq 6.0$, the shear-flexure interactions that cause diagonal web cracks in a single beam differ along the shear span to the extent that the ultimate shear strength is determined by the weakest location in shear i.e. the first point along the shear span to fail in shear. Regions in a member prone to shear failure occur where either the design of the section lacks integrity when loaded in shear or due to high influence of shear in the moment-shear interaction, a situation that usually occurs closer to support points of the member.

In this investigation, the general approach was that a full probabilistic model was first developed to establish the relative contribution the different basic variables that contribute to shear resistance have on its reliability performance before the dominating variable(s) is/are isolated to represent the resistance probability model. The inverse process is fully applicable and sometimes preferred; building the reliability model by first representing the true shear resistance distribution by parameters expected to dominate its performance as reported in literature or found from preliminary analyses. Whichever method is used, the achievement of proper basic reliability modelling tools paves the way for full parametric analyses to be conducted for a number of design situations which can be chosen to represent different situations in practice.

The flowchart of the complete reliability assessment for the two design situations considered in this investigation is presented in Figure 8.1.

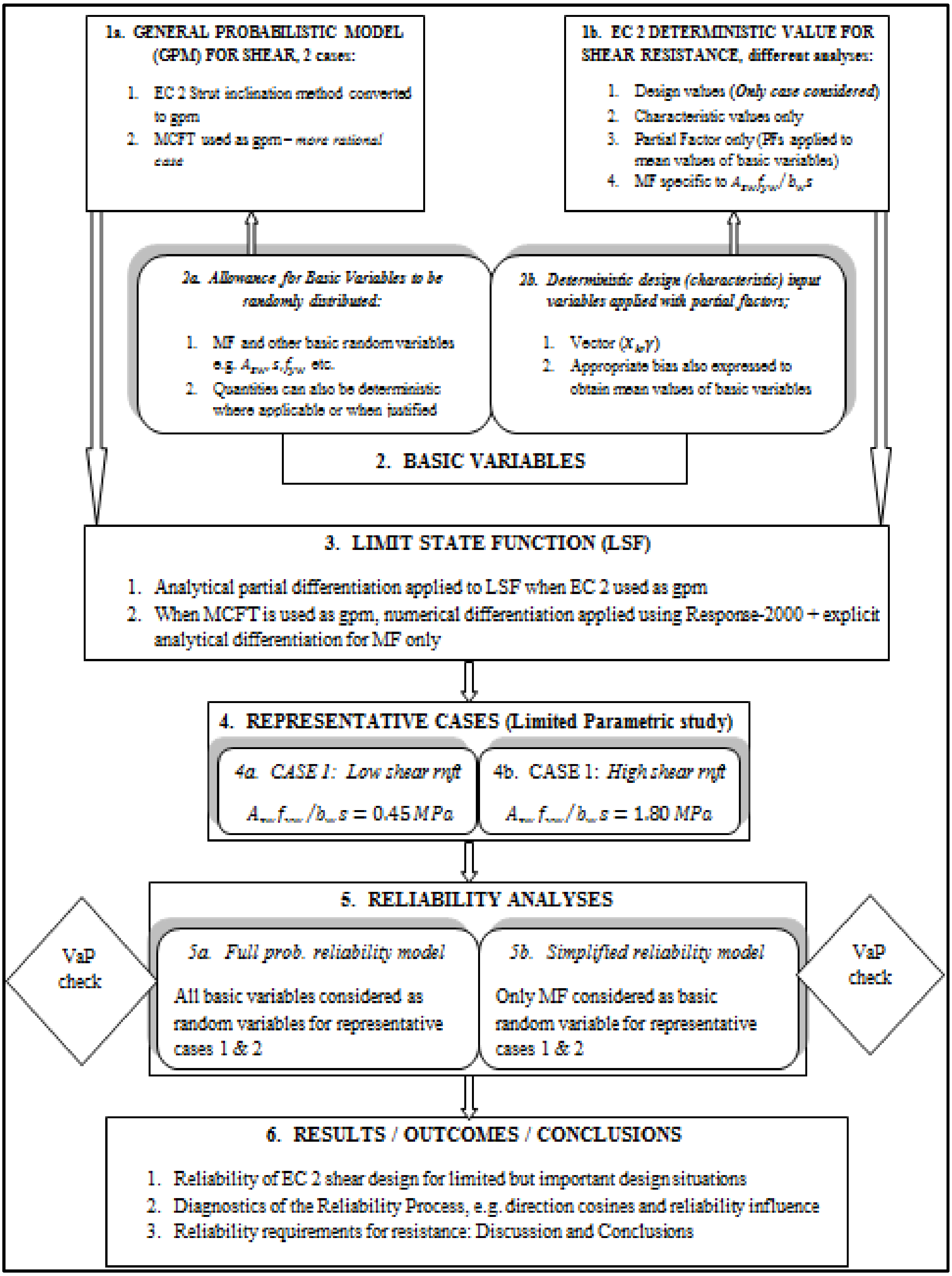


Figure 8.1. Flowchart of the parametric reliability investigation for members requiring stirrups designed using EC 2's variable strut inclination method.

8.2 RELIABILITY ANALYSIS USING THE EC 2 PREDICTION MODEL AS THE GENERAL PROBABILISTIC MODEL (GPM)

In this Section, the EC 2 variable strut inclination method is converted for use as the gpm in the development of the performance and limit state function for the shear resistance of members requiring stirrups. The gpm is a descriptor of true shear resistance in nature and should thus be modelled by the most accurate prediction method. The results from Chapter 7, as well as general literature on the MCFT, prove that the MCFT is a better predictor of shear resistance than most available shear resistance models including the variable strut inclination method on which EC 2 is based. However, any model can be used as the gpm should it be expressed in its unbiased and unfactored form, alongside including the statistics of its model factor as a basic random variable in the formulation. The EC 2 variable strut inclination method was treated in this manner to enable its use as the gpm in the performance function for shear.

8.2.1 Development of the performance function for members requiring stirrups

Defining the Performance Function for the shear of members requiring stirrups

Recall form Chapter 5 that shear design using the variable strut inclination method is based on the contribution of the steel shear reinforcement only, where crushing of the inclined concrete struts is checked to avoid situations where premature web crushing may occur. In design situations where the web crushing strength is predicted to be lower than the yield strength of the stirrups, the width of the section is normally increased to an extent that the web crushing strength exceeds or at least equals the yield strength of the stirrups. The performance function for the reliability analysis is, therefore, based on the steel contribution of the stirrups provided during the design of a section or member. Consistent with the background of reliability analysis presented in Chapter 6, the performance function can then be defined as:

$$g(\boldsymbol{X}) = V_{EC2_{gpm}}(\boldsymbol{X}) - V_{EC2_{design}}(\boldsymbol{X_k}, \boldsymbol{\gamma})$$

$$\therefore g(\boldsymbol{X}) = MF \cdot \left[\frac{A_{sw}}{s} z f_{yw} cot\theta\right] - V_{EC2_{design}}(\boldsymbol{X_k}, \boldsymbol{\gamma}) \qquad [8.1]$$

Where $V_{EC2_{gpm}}(\boldsymbol{X})$ represents the distribution of 'true' shear resistance, based on EC 2's variable strut inclination method for shear, determined using unbiased values of the basic deterministic and random variables and neglecting the use of partial factors in the resistance model. $V_{EC2_{design}}(\boldsymbol{X_k}, \boldsymbol{\gamma})$ is the single deterministic value of shear resistance as would be determined for a practical design situation in accordance with the stipulations in EC 2. The vectors $\boldsymbol{X_k}$ and $\boldsymbol{\gamma}$ imply that the single deterministic value of shear resistance is calculated using appropriate characteristic values of all the basic variables, which are all treated deterministically when the code method is applied for design, as well as the appropriate partial safety factors. Figure 8.2 shows schematically the probabilistic representation of the performance function.

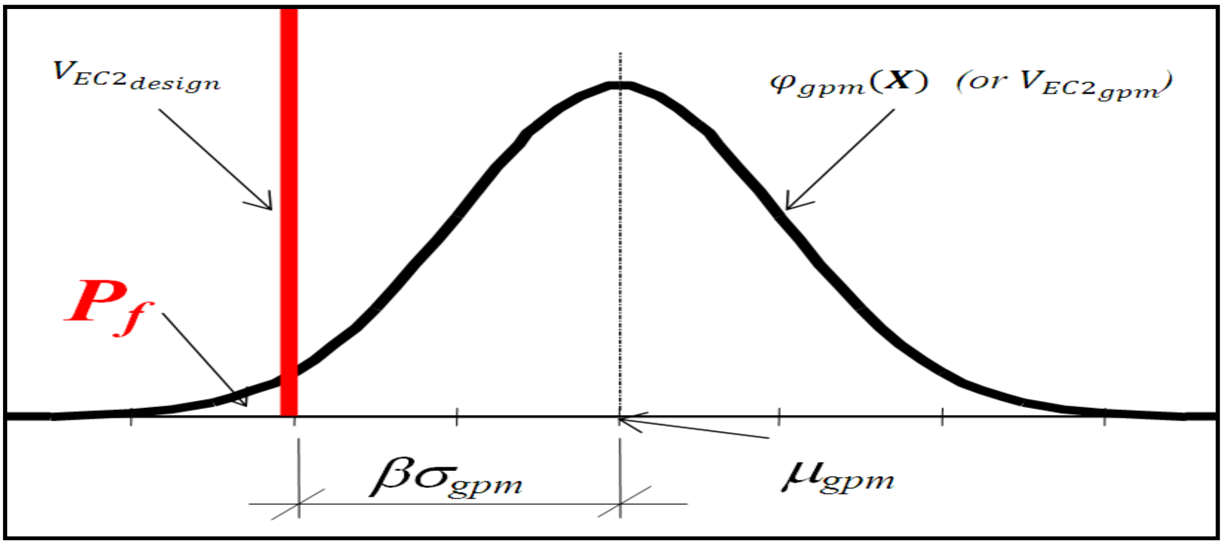


Figure 8.2. Probabilistic representation of the performance function

In simple terms, the performance function represents a problem of supply versus demand; where the true shear resistance, which is represented by the general probabilistic model, forms the *supply* of available strength and the *demand* is represented by the code determined value of ultimate shear resistance. The true shear resistance which truly varies in nature, and is represented by the general probabilistic model, is a distribution because it is a function of basic variables which are allowed to assume random statistical distributions as expressed in the gpm portion of Equation [8.9].

Naturally, the demand side of the performance function is usually represented by a combination of actions or action effects, usually determined from critical load cases, to be exposed to a particular structure or construction during its lifetime of service. The value of the actions or action effects would have been determined in accordance with national codified loading provisions, requirements or considerations. If the code for actions is established on probabilistic bases as modern design formats require, as has been achieved in EC 0 and SANS 10160-1, adequate representation of variability of actions can be assumed. In such cases, the reliability analysis would in effect determine the safety performance of a member or design section of certain physical parameters which are allowed to assume random statistical distributions against the determined action effects introduced into the process as a deterministic design value.

However, through reliability separation materials-based resistance standards can be calibrated independent of loading considerations. In this scenario, as applied for all investigations in this thesis, the ultimate shear resistance determined by the materials-based-resistance codified procedure is used as the demand side of the performance function. This is in effect a representation of the maximum load the member with stirrups is permitted to carry in shear by the code, caused by whatever combination of actions i.e. ultimate limit state for shear in that design situation. The reliability thus characterises the performance of the ultimate limit state condition.

Similar to the definition of characteristic values from the viewpoint of fractiles of statistical distributions presented in Chapter 3, V_{design} is a discrete force as shown in Figure 8.2 and can be considered a fractile of the true shear resistance representing some level of probability of exceedance (or non-exceedance) of the density distribution, $\varphi_{gpm}(x)$, of the general probabilistic model. In the thesis, the extent of exceedance of the mean of true shear resistance from the point of ultimate load or resistance determined by the code is determined through FORM analysis, in which β is the measure of safety performance determined (the number of standard deviations from the mean true resistance to ultimate code determined resistance) and P_f can be determined from Equation [3.1], repeated here as:

$$P_f = \Phi(-\beta) \qquad [8.2]$$

Different values of $V_{EC\ 2_{design}}$ as shown in Figure 8.2 can be considered in independent reliability investigations to determine the separate effect on reliability performance caused by codified procedures such as the use of characteristic values or the effect of using partial factors in determining design resistance. When no alteration is made to the codified procedure, the result of the reliability analysis is a reflection of how much conservative bias, created by the combined use of characteristic values and partial factors in the design model as well as bias inherent in the prediction model itself, exists between the point of true shear resistance in physical reality and that predicted by the code model. Applying this concept, the demand side of the performance function could as well be represented by either:

1. The characteristic shear resistance where partial factors are neglected in determining shear resistance but the characteristic values of the basic variables are used or

2. By the application of partial factors only, in a situation where the characteristic bias of the basic variables is neglected. In these events, the bias inherent in the combination of characteristic or mean values against the point of true shear resistance is assessed in the reliability analysis.

Development of the general probabilistic model for the shear resistance of members requiring stirrups

The vector of basic variables $\boldsymbol{X}$ attached to the general performance function in Equation [8.1] represents the most basic quantities that are required in evaluating the value of the function. That is, terms as z and θ in the performance function, which can be considered as secondary variables, should be expressed in their simplest form.

Design codes usually prescribe that z be implemented in design as some fixed ratio of the effective depth, d, of the section. EC 2 follows suite and prescribes a value of $z = 0.9\,d$ under the conditions that the member depth is constant and the member is not under any axial stress. On the contrary, there is design evidence that suggests that the ratio of z to d can be as low as 0.75 (Jackson & Salim, 2006), for heavily reinforced concrete beams without significant flanges. The fact that EC 2 prescribes the approximation $z = 0.9\,d$ indicates that the code will perceivably rarely ever be applied in situations of design of heavily reinforced concrete beams. On the other hand, the general probabilistic model should be all embracing and take account of all possible design situations. To cater for this purpose, z can reasonably be postulated to be constituted of a general multiplier term, say m, and the effective depth, d, such that the following general expression holds:

$$z = m \cdot d \quad [8.3]$$

The statistics of d are not explicitly reported in literature and the JCSS Probabilistic Model Code (2001) only provides guidance on the mean and standard deviation of the difference (or constructional tolerance) between the constructed and nominal value of d. No direct distribution characteristics or statistics of d as a variable are reported in the JCSS model code. For purposes of probabilistic representation, d is then modelled as the difference between the height of the section, h, less the summation of; the concrete cover, C, from the soffit of the beam to the seal of the bottom-most bar of reinforcement, the diameter of the stirrup, e, and *some multiple* of half the diameter of the longitudinal tension reinforcement, denoted as a. The multiple, herein defined as n_l, related to a is concerned with the number of layers of longitudinal reinforcement provided in the design. This concept is illustrated in Figure 8.3.

By viewing Figure 8.3 and taking cognizance that layers of longitudinal bars are usually spaced transversely at intervals equivalent to the bar diameter of the reinforcement, it can be observed that for a single layer arrangement of longitudinal tension reinforcement $n_l = 1$, whilst for a double layer arrangement of reinforcement $n_l = 3$.

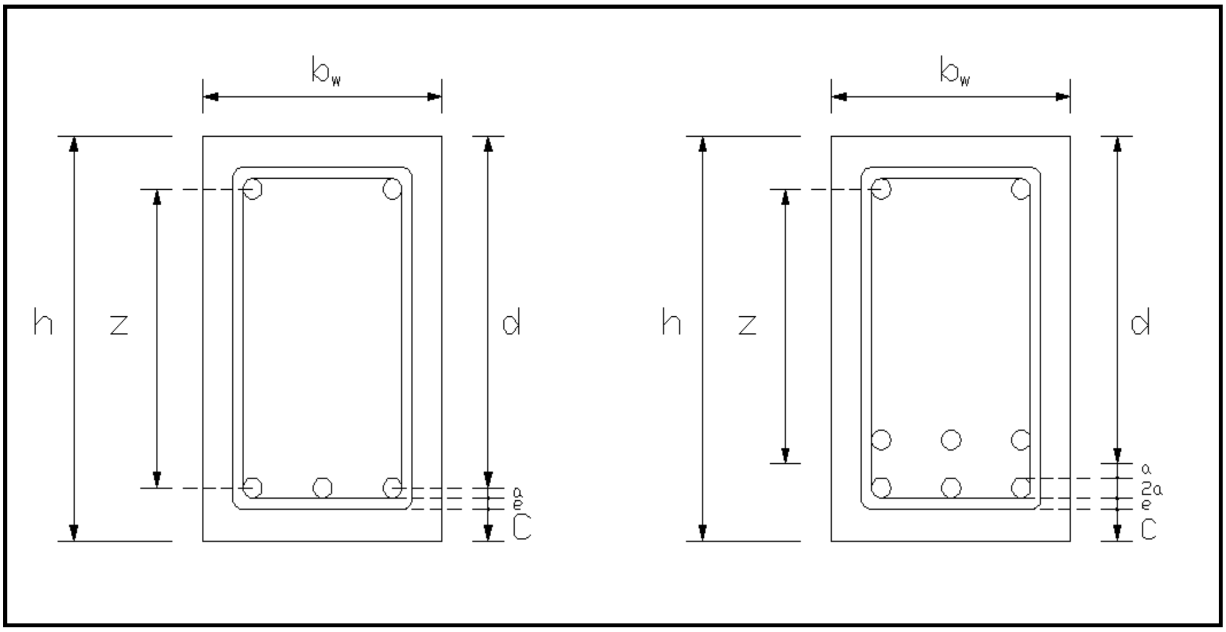


Figure 8.3. Schematic of the cross-section parameters used for probabilistic representation for single (***left***) and double (***right***) layer arrangement of longitudinal reinforcement

Taking this into account, the depth of the Section can be expressed as:

$$d = h - (C + n_l \cdot a + e) = h - C - n_l \cdot a - e \qquad [8.4]$$

Substituting Equation [8.4] into Equation [8.3] results in the following equation for z:

$$z \ = \ m \ \cdot (h - \ C - n_l \cdot a - e) \qquad [8.5]$$

For the general probabilistic model based on EC 2's variable strut inclination method, all conservative bias in the basic variables and partial safety factors were neglected for use in determining the angle of strut inclination, thereby resulting in:

$$\theta_{gpm} = sin^{-1}\sqrt{\frac{A_{sw}f_{yw}}{b_w s v_1 \alpha_{cc} f_c}} \qquad [8.6]$$

Where θ_{gpm} represents the angle of inclination of concrete compressive struts associated with the general probabilistic model. In contrast to the code design method, θ_{gpm} is allowed to assume any value determined by the use of Equation [8.6] and is not subject to any constraints. Further, it should be noted that the coefficient, α_{cc}, that takes account of long term effects on the concrete compressive strength and of unfavourable effects resulting from the way the load is applied, is maintained for use in the general probabilistic model. α_{cc} is not viewed as a safety element for design but rather a conversion factor applied to the design process to better reflect the concrete strength of real structures.

Development of the code determined single deterministic value of shear resistance

In EC 2, the design value of the angle of inclination, herein denoted as θ_d, of the concrete compressive struts is established in Equation [5.13] and is appropriately repeated here, and extended by using Equation [5.4], as:

$$\theta_d = sin^{-1}\sqrt{\frac{A_{sw}f_{ywd}}{b_w s v_1 f_{cd}}} = \sqrt{\frac{A_{sw}\left(f_{ywk}/_{1.15}\right)}{b_w s v_1\left(\alpha_{cc} f_{ck}/_{1.5}\right)}} \qquad [8.7]$$

Where θ_d is subject to the constraints $1 \leq \cot\theta_d \leq 2.5$ or $21.8° \leq \theta_d \leq 45°$. When θ_d is determined to be less than the lower limit of 21.8°, the value of 21.8° should be used in design so as not to give too much credit to the steel contribution to shear resistance. Values of θ_d greater than the upper limit of 45° imply that significant stresses occur in the concrete with the risk of crushing of the concrete web. In this instance, the dimensions of the member, particularly the web width, should be increased until the angle θ_d falls into its code stipulated confines.

$V_{EC2_{design}}(\boldsymbol{X_k}, \boldsymbol{\gamma})$ is a single deterministic value of shear resistance for members with stirrups, which is determined according to EC 2's variable strut inclination method. The code design procedure incorporates the appropriate characteristic values of the basic variables, represented by the vector $\boldsymbol{X_k}$ in Equation [8.8] below, as well as the partial safety material factors for steel and concrete resistance, γ_S and γ_C, which are represented in the same Equation by the vector $\boldsymbol{\gamma}$**.** Therefore:

$$V_{EC2_{design}}(\boldsymbol{X_k}, \boldsymbol{\gamma}) = \frac{A_{sw}}{s} z \frac{f_{ywk}}{1.15} \cot \theta_d \qquad [8.8]$$

Instances where partial safety factors and characteristic values of the basic variables were applied in determining the deterministic resistance, as is proposed by the EC 2 design method, can be considered as the base case used for the reliability assessments.

Statement of the full performance function

Incorporating the developments from Equation [8.3] to [8.8] and substituting them into Equation [8.1], the performance function can be expressed as:

$$g(\boldsymbol{X}) = MF \cdot \left[\frac{A_{sw}}{s} m(h - C - n_l \cdot a - e) f_{yw} \cot\left(sin^{-1}\sqrt{\frac{A_{sw} f_{yw}}{b_w s v_1 \alpha_{cc} f_c}}\right)\right] - V_{EC2_{design}}(\boldsymbol{X_k}, \boldsymbol{\gamma}) \qquad [8.9]$$

8.2.2 Theoretical models of basic variables

The theoretical models of the basic variables for the general probabilistic model of the performance function are presented in Table 8.1 below. The symbols 3-P LN and 2-P LN under the column heading Distribution Type are used to denote the three-parameter and two-parameter log-normal distribution respectively. Both the statistics of the MF associated with the EC 2 ($MF_{EC\ 2}$) and MCFT (MF_{MCFT}) prediction models are reported in Table 8.1.

Table 8.1. Theoretical models of the basic variables for the general probabilistic model

			Basic Statistics				
Var. X_i	**Unit**	**Distn. type**	**Mean, μ_X**	**C.O.V., Ω_X**	**Std. Dev., σ_X**	**Bias**	**Source**
$MF_{EC\ 2}$	-	3-P LN	1.65	-	0.51	-	*Thesis*
MF_{MCFT}	-	2-P LN	1.14	-	0.20	-	*Thesis*
A_{sw}	mm^2	N	nom. A_{sw}	0.02	-	1	JCSS PMC (2001)
s	mm	N	nom. s	0.03	-	1	Mirza & MacGregor (1979) as cited in Huber (2005)
m	-	3-P LN	0.85	0.0416	-	-	*Thesis*
h	mm	N	nom. h	0.01	-	1	Holický (2009)
C	mm	2-P LN	30	-	9	1	Holický et al. (2010) / *Thesis*
a	mm^2	N	nom. a	0.02	-	1	JCSS PMC (2001)
e	mm^2	N	nom. e	0.02	-	1	JCSS PMC (2001)
f_{yw}	MPa	3-P LN	f_{ywk} + 1.645σ	-	30	1 / (1 – 1.645Ω_X)	Holický (2009)
b_w	mm	N	nom. b_w	0.01	-	1	Holický (2009)
α_{cc}	-	2-P LN	0.85	0.1	-	1	Holický et al. (2010)
f_c	MPa	3-P LN	Bias * f_{ck}	0.18	-	1 / (1 – 1.645Ω_X)	Holický (2009)

From inspection of Table 8.1, it can be seen that the mean values of all the parameters related to the geometry of the section e.g. A_{sw}, s, h etc., are taken as the nominal quantities of the respective variables. This is due to the fact that in practical design situations the nominal values of dimensions and geometrical parameters specified in design drawings are normally used as the characteristic or design representative values of the basic variables when used in codified equations during the verification of structural resistance.

Most of the models presented in Table 8.1 are conventional models that are derived for use from recognised sources of literature concerned with the specification of basic variables that enable probabilistic assessment of structures. The basic variables a and e, shown in the Table, are concerned with the area of reinforcement and are therefore prescribed the same distribution as A_{sw}.

However, under the Source column heading in Table 8.1, the models for the basic variables MF, m and C have been indicated to have been developed in the thesis. The statistics of the model factor, as well as its suitable theoretical distribution, were established in Chapter 7 for the EC 2 and MCFT shear prediction models. Thus, only the development of the theoretical models for m and C are dealt with in detail. Comments are also made on the choice of some of the statistical moment parameters for the steel yield strength of the stirrups, f_{yw}, as well as the concrete compressive strength, f_c.

Lever arm multiplier term (m)

The general concept of this basic variable was introduced in the previous Section. From the background information contained therein, it can reasonably be postulated that m is a random variable bound between lower and upper limits of $m_l = 0.75$ and $m_u = 0.90$, respectively. It is important that the distribution of the random variable in the postulate exhibits a higher likelihood of generating values of m closer to 0.9 than 0.75. Ang and Tang (1984) propose the use of an upper triangular probability density function (PDF), which is introduced in Chapter 6, to describe a random variable where the range of values are known to possess a tendency (bias) of generating values closer to the upper limit of the distribution as shown in Figure 6.3. A distribution function that has a bias towards the upper limit has a negative skewness or is skew to the left. Consistent with the procedure described in Chapter 6, the characteristics of the skewed upper triangular PDF are determined as follows (Ang & Tang, 1984):

$$\bar{m} = \mu_m = \frac{1}{3}\big(0.75 + 2(0.90)\big) = 0.85 \qquad [8.10]$$

And,

$$\delta_m = \Omega_m = \frac{1}{\sqrt{2}}\left(\frac{0.90-0.75}{2(0.90)+0.75}\right) = 0.041594516 \qquad [8.11]$$

For the purpose of reliability modelling, a more defined statistical or theoretical distribution of the basic variables is required (e.g. Normal, Lognormal, Beta, Gamma distributions etc). To reflect the negative skewness (skew to the left) in the distribution of the random variable, the moments of the distribution derived from the upper triangular postulate are taken to equal the first and second moments of the general three parameter lognormal distribution with the upper bound at 0.9.

Concrete cover (C)

A study on Partial factors for reinforced concrete members (Holický et al., 2010) uses the moments $\mu_C = 30\,mm$ and $\sigma_C = 9\,mm$. Therein, the Gamma distribution is used to describe the concrete cover as a random variable. The same moments are used for the concrete cover in this study. However, attempts to utilise the Gamma distribution in describing the realisations of C were futile. The Gamma distribution was implemented as a special case of the three-parameter log-normal distribution by establishing skewness as $s_{k,Cover} = \frac{2\sigma_{Cover}}{\mu_{Cover}}$ (Holický, 2009). The use of the Gamma distribution created an irrational model for C that predicted the lower bound of the distribution to be located at about $-16\,mm$, hence implying that negative cover exists in practice. To correct this nonsensical result, using the moments aforementioned in this Section, the two-parameter log-normal distribution was used to represent the distribution of C. The two-parameter log-normal distribution has its lower bound at zero as well as a high positive skewness associated with it, which has the tendency of underestimating the occurrence of negative and overestimating the occurrence of positive deviations from the mean (Holický, 2009) and this effect more rationally represents the distribution of concrete cover in practice. Given the statistical moments of C, its skewness according to Gamma distribution is 0.6 and that according to the two-parameter log-normal distribution is 0.927.

The yield strength of the vertical link/stirrup reinforcement (f_{yw})

Recall form Chapter 3 that the common characteristic value of a material property for resistance is expressed as:

$$X_k = x_{0.05} = \mu_X(1 - 1.645 \cdot \Omega_X) \qquad [8.12]$$

Where, for a five percent characteristic strength or 0.05 fractile, $u_p = -1.645$. However, Holický (2009), as indicated in Table 6.1, reports that tests conducted on manufactured steel specimens indicate that steel strengths in practice are generally more conservative than the five-percent characteristic value and can be seen to conform to the relation:

$$f_{ywk} = f_{yw,5\%} = \mu_{f_{yw}}(1 - 2\Omega_{f_{yw}}) \qquad [8.13]$$

In this study, credit is not taken of the generally more conservative situation in practice and the value of the standardised random variable, $u_p = -1.645$, is used. This is a conservative choice as compared to what exits in practice as it reduces the bias between the characteristic value that is adopted as a representative value in design and the actual or most expected (mean) value in practice. By transposing Equation [8.13], the bias in the random basic variable f_{yw} can be expressed as:

$$\frac{\mu_{f_{yw}}}{f_{ywk}} = \frac{1}{1-1.645\Omega_{f_{yw}}} \qquad [8.14]$$

Holický (2009) gives two feasible options of log-normal distributions that can be applied to describe the basic random variable, f_y. The symbol f_y is used to denote the steel yield point under the category of Material Properties in one instance, and is then used to denote the reinforcement strength under the category of Strength. The distribution of f_y in both cases is log-normal and the mean, in both cases, of the random variable, f_y, is specified as:

$$\mu_{f_y} = f_{yk} + 2\sigma_{f_{yw}} \qquad [8.15]$$

The two definitions of the basic random variable differ in their specification of the coefficient of variation for the distribution. When f_y is considered as a material property the coefficient of variation, Ω_{f_y}, can be considered to occur between the range 0.07 to 0.10 depending on the control of the production/manufacturing procedure. The lower coefficient of variation applies under conditions of high quality control and vice versa is true. In contrast, when f_y is considered as a strength parameter for reinforcement a deterministic quantity of $30\ MPa$ is recommended for use as the standard deviation. The coefficient of variation is then evaluated according to the mean value of the strength of reinforcement used in the design. A deterministic quantity of the standard deviation is applicable for reinforcement since it is produced under workshop conditions that are strictly controlled and viewed as constant in terms of production quality. In this study, the basic random variable f_{yw} denotes the yield strength of reinforcement and can therefore also be viewed as having a fixed standard deviation of $30\ MPa$.

The general three-parameter log-normal distribution was used to model f_{yw} as a two-parameter log-normal distribution when it is implemented under the special condition that its skewness, $\alpha_{f_{yw}}$, equal to (Holický, 2009):

$$s_{k,f_{yw}} = \frac{3\sigma_{f_{yw}}}{\mu_{f_{yw}}} \quad [8.16]$$

Compressive strength of concrete (f_c)

Equations [8.12] to [8.16] are all applicable to f_c as they are to f_{yw}. The main difference in the way the two variables are defined is that for concrete a coefficient of variation is chosen from a range of possible values, and not a single deterministic value of the standard deviation as is utilised for the case of f_{yw}, to reflect the effect the level of quality control and workmanship has on the precision and consistency of concrete produced on site.

Concrete production in practice is widely varied, ranging from precast concrete produced in controlled plants to in-situ concrete produced on site by sometimes low-skilled labour. For this reason, a deterministic quantity of the standard deviation is not used for concrete compressive strength, f_c. Rather, the coefficient of variation is chosen from the range of 0.05

to 0.18 depending on the control of the production procedure. For this study, the coefficient of variation of f_c is chosen as 0.18 to reflect poor workmanship and production control. This is a conservative assumption and it can be varied to assess the effect quality control may have on reliability performance of structures.

8.2.3 Limit state Equation for reliability analysis

The limit state Equation is defined when the performance function is set equal to zero. The limit state function is then given by:

$$g\ (\boldsymbol{X}) =$$

$$MF \cdot \left[\frac{A_{sw}}{s} m(h - C - n_l \cdot a - e) f_{yw} \cdot \cot\left(sin^{-1}\sqrt{\frac{A_{sw} f_{yw}}{b_w s v_1 \alpha_{cc} f_c}}\right)\right] - V_{EC2_{design}}(\boldsymbol{X_k}, \boldsymbol{\gamma}) = 0$$

[8.17]

The partial derivatives of the limit state function, $g\ (\boldsymbol{X}) = 0$, required for the FORM method of reliability analysis were determined using MATLAB. It should be noted that $V_{EC2_{design}}(\boldsymbol{X_k}, \boldsymbol{\gamma})$ is a single deterministic value that is not described by random variables, and as such, falls away with differentiation. MATLAB converts the trigonometric operators in the limit state Equation to an equivalent fraction of the appropriate random variables. Adequate checks were performed to ensure that the MATLAB simplified form of the equation is approximately equivalent to the original form of the equation presented as Equation [8.17]. Applying the format from MATLAB, the limit state function and its partial derivatives at the checking point can be expressed as:

$$g(\boldsymbol{X}) = MF\frac{A_{sw}}{s} m(h - C - n_l \cdot a - e) f_{yw} \frac{\sqrt{1 - \frac{A_{sw} f_{yw}}{b_w s v_1 \alpha_{cc} f_c}}}{\sqrt{\frac{A_{sw} f_{yw}}{b_w s v_1 \alpha_{cc} f_c}}} - V_{EC2_{design}}(\boldsymbol{X_k}, \boldsymbol{\gamma}) = 0 \qquad [8.18]$$

$$\left(\frac{\partial g(\boldsymbol{X})}{\partial MF}\right)^* = \frac{A_{sw}}{s} m(h - C - n_l \cdot a - e) f_{yw} \frac{\sqrt{1 - \frac{A_{sw} f_{yw}}{b_w s v_1 \alpha_{cc} f_c}}}{\sqrt{\frac{A_{sw} f_{yw}}{b_w s v_1 \alpha_{cc} f_c}}} \cdot \sigma_{MF}^N \qquad [8.19a]$$

$$\left(\frac{\partial g(\boldsymbol{X})}{\partial A_{sw}}\right)^{*} = \frac{\frac{1}{2} MF\ m\ (h-C-n_l \cdot a-e) f_{yw} (b_w s v_1 \alpha_{cc} f_c - 2A_{sw} f_{yw})}{b_w\ s^2 v_1 \alpha_{cc} f_c \sqrt{\frac{A_{sw} f_{yw}}{b_w s v_1 \alpha_{cc} f_c}} \sqrt{\frac{b_w s v_1 \alpha_{cc} f_c - A_{sw} f_{yw}}{b_w s v_1 \alpha_{cc} f_c}}} \cdot \sigma_{A_{sw}}^{N} \qquad [8.19b]$$

$$\left(\frac{\partial g(\boldsymbol{X})}{\partial s}\right)^{*} = -\frac{\frac{1}{2} MF\ A_{sw}\ m\ (h-C-n_l \cdot a-e) f_{yw} (b_w s v_1 \alpha_{cc} f_c - 2A_{sw} f_{yw})}{b_w\ s^3 v_1 \alpha_{cc} f_c \sqrt{\frac{A_{sw} f_{yw}}{b_w s v_1 \alpha_{cc} f_c}} \sqrt{\frac{b_w s v_1 \alpha_{cc} f_c - A_{sw} f_{yw}}{b_w s v_1 \alpha_{cc} f_c}}} \cdot \sigma_{s}^{N} \qquad [8.19c]$$

$$\left(\frac{\partial g(\boldsymbol{X})}{\partial m}\right)^{*} = MF\ \frac{A_{sw}}{s} (h - C - n_l \cdot a - e) f_{yw} \frac{\sqrt{1-\frac{A_{sw} f_{yw}}{b_w s v_1 \alpha_{cc} f_c}}}{\sqrt{\frac{A_{sw} f_{yw}}{b_w s v_1 \alpha_{cc} f_c}}} \cdot \sigma_{m}^{N} \qquad [8.19d]$$

$$\left(\frac{\partial g(\boldsymbol{X})}{\partial h}\right)^{*} = MF\ \frac{A_{sw}}{s}\ m\ f_{yw} \frac{\sqrt{1-\frac{A_{sw} f_{yw}}{b_w s v_1 \alpha_{cc} f_c}}}{\sqrt{\frac{A_{sw} f_{yw}}{b_w s v_1 \alpha_{cc} f_c}}} \cdot \sigma_{h}^{N} \qquad [8.19e]$$

$$\left(\frac{\partial g(\boldsymbol{X})}{\partial C}\right)^{*} = -\ MF\ \frac{A_{sw}}{s}\ m\ f_{yw} \frac{\sqrt{1-\frac{A_{sw} f_{yw}}{b_w s v_1 \alpha_{cc} f_c}}}{\sqrt{\frac{A_{sw} f_{yw}}{b_w s v_1 \alpha_{cc} f_c}}} \cdot \sigma_{C}^{N} \qquad [8.19f]$$

$$\left(\frac{\partial g(\boldsymbol{X})}{\partial a}\right)^{*} = -\ MF\ \frac{A_{sw}}{s}\ m \cdot n_l \cdot\ f_{yw} \frac{\sqrt{1-\frac{A_{sw} f_{yw}}{b_w s v_1 \alpha_{cc} f_c}}}{\sqrt{\frac{A_{sw} f_{yw}}{b_w s v_1 \alpha_{cc} f_c}}} \cdot \sigma_{a}^{N} \qquad [8.19g]$$

$$\left(\frac{\partial g(\boldsymbol{X})}{\partial e}\right)^{*} = -\ MF\ \frac{A_{sw}}{s}\ m \cdot n_l \cdot\ f_{yw} \frac{\sqrt{1-\frac{A_{sw} f_{yw}}{b_w s v_1 \alpha_{cc} f_c}}}{\sqrt{\frac{A_{sw} f_{yw}}{b_w s v_1 \alpha_{cc} f_c}}} \cdot \sigma_{e}^{N} \qquad [8.19h]$$

$$\left(\frac{\partial g(\boldsymbol{X})}{\partial f_{yw}}\right)^{*} = \frac{\frac{1}{2} MF\ A_{sw}\ m\ (h-C-n_l \cdot a-e)(b_w s v_1 \alpha_{cc} f_c - 2A_{sw} f_{yw})}{b_w\ s^2 v_1 \alpha_{cc} f_c \sqrt{\frac{A_{sw} f_{yw}}{b_w s v_1 \alpha_{cc} f_c}} \sqrt{\frac{b_w s v_1 \alpha_{cc} f_c - A_{sw} f_{yw}}{b_w s v_1 \alpha_{cc} f_c}}} \cdot \sigma_{f_{yw}}^{N} \qquad [8.19i]$$

$$\left(\frac{\partial g(\boldsymbol{X})}{\partial b_w}\right)^{*} = \frac{\frac{1}{2} MF\ A_{sw}\ m\ (h-C-n_l \cdot a-e) f_{yw}}{b_w\ s \sqrt{\frac{A_{sw} f_{yw}}{b_w s v_1 \alpha_{cc} f_c}} \sqrt{\frac{b_w s v_1 \alpha_{cc} f_c - A_{sw} f_{yw}}{b_w s v_1 \alpha_{cc} f_c}}} \cdot \sigma_{b_w}^{N} \qquad [8.19j]$$

$$\left(\frac{\partial g(\boldsymbol{X})}{\partial \alpha_{cc}}\right)^{*} = \frac{\frac{1}{2} MF\ A_{sw}\ m\ (h-C-n_l \cdot a-e) f_{yw}}{\alpha_{cc}\ s \sqrt{\frac{A_{sw} f_{yw}}{b_w s v_1 \alpha_{cc} f_c}} \sqrt{\frac{b_w s v_1 \alpha_{cc} f_c - A_{sw} f_{yw}}{b_w s v_1 \alpha_{cc} f_c}}} \cdot \sigma_{\alpha_{cc}}^{N} \qquad [8.19k]$$

$$\left(\frac{\partial g(\boldsymbol{X})}{\partial f_c}\right)^{*} = \frac{\frac{1}{2} MF\ A_{sw}\ m\ (h-C-n_l \cdot a-e) f_{yw}}{f_c\ s \sqrt{\frac{A_{sw} f_{yw}}{b_w s v_1 \alpha_{cc} f_c}} \sqrt{\frac{b_w s v_1 \alpha_{cc} f_c - A_{sw} f_{yw}}{b_w s v_1 \alpha_{cc} f_c}}} \cdot \sigma_{f_c}^{N} \qquad [8.19l]$$

8.2.4 Reliability analysis of representative design situations

The reliability analysis of two representative design situations is presented in this Section. The purpose of the analyses conducted was to determine the effects of the relative influences of uncertainty that each of the different basic variables has on the reliability performance for members subjected to shear with stirrups. The choice of the test cases for the analyses was guided by some of the results presented in Chapter 7. It is therein shown that the main uncertainty in the EC 2 and MCFT prediction models alike, is mainly due to the models' inadequacy of making consistent predictions of shear strength at varying amounts of shear reinforcement. Though the statistics related to the bias of the EC 2 prediction model are generally on the conservative side, with a mean value of $\mu_{MF} = 1.65$, it is associated with a high standard deviation which implies that significant variations in MF estimates are expected, and are observed in the shear database provided on the CD, for some test samples in the shear database.

From Figure 7.9, for the EC 2 prediction model, it can be observed that the MFcan be as high as 2.5 at $A_{sw}f_{yw}/b_w s \approx 0.21\ MPa$, progressively decreasing logarithmically to values as low as 0.8 at $A_{sw}f_{yw}/b_w s \approx 2.6\ MPa$. Further, the MF equals 1 at about 1.9 MPa, and progressively falls below 1 with increasing amounts of shear reinforcement. Design situations with relatively high amounts of shear reinforcement are clearly the more critical region of shear performance as conservatism in the EC 2's shear predictions reduces with increasing amounts of shear reinforcement. Hence, of the two cases considered in the thesis, the design situation with high shear reinforcement had a lower β-index than the situation with low amounts of shear reinforcement. The characteristics of shear reliability performance were considered for the following two design situations:

1. A case where the MF is very conservative ($MF \approx 2.0$ from Figure 7.9) i.e. design situation with relatively low amount of shear reinforcement, fixed as $A_{sw}f_{yw}/b_w s = 0.45\ MPa$ using unbiased estimates of the basic variables. EC 2's minimum amount of shear reinforcement to be provided in design was adhered to. Using 25 MPa characteristic concrete strength (f_{ck}), as is done for both representative design situations, Equation [5.12] yields that a minimum of $A_{sw}f_{ywk}/b_w s = 0.4$ be provided

in design. For this design situation $f_{ywk} = 250\,MPa$, implying that $A_{sw}f_{ywk}/b_w s = 0.37 \approx 0.4\,MPa$.

2. A case where the MF is expected close to losing conservatism or conservative bias with its mean value at approximately 1.04 from Figure 7.9 i.e. design situations with relatively high amounts of shear reinforcement, chosen as 1.8 MPa when unbiased and unfactored values of the basic variables are used.

A Double layer arrangement of longitudinal tension reinforcement was assumed for the both design situations; an assumption that affected the value of the effective depth, d, calculated for the section which in turn affects the value of z used in Equations [8.8] and [8.9]. Table 8.2 presents the design or characteristic quantities of the basic variables for both test cases used in the reliability investigation.

Table 8.2. Design specifications of the basic variables for the test case

Variable X_i	**Characteristic quantities *Test case 1***	**Characteristic quantities *Test Case 2***	**Unit / Dimension**	**Bias**
A_{sw}	157.1	157.1	mm^2	1
s	300	125	mm	1
m	0.9	0.9	–	0.94
h	500	500	mm	1
C	30	30	mm	1
a	16	16	mm	1
e	10	10	mm	1
f_{ywk}	250	450	MPa	$1 + (49.35/f_{ywk})$
b_w	350	350	mm	1
α_{cck}	0.85	0.85	–	1
f_{ck}	25	25	MPa	1.42

Procedure of the reliability analysis

The general framework of the FORM method of reliability analysis, used in the thesis, was addressed in Chapter 6. Consistent with this general approach, some few important aspects regarding the implementation of the procedure to the specific case of shear are briefly outlined.

To initiate the reliability evaluation or assessment process, characteristic quantities for the test case were specified as would be done for a conventional design situation in practice. The bias associated with these characteristic values specified for the design is given in Table 8.2, and are equivalent to the ratio of the mean or expected value of a given basic variable against its respective characteristic values adopted in a practical design situation. The mean estimates of the basic variables are used as the initial checking point of failure for the general probabilistic model against the design resistance for the test case which is determined using characteristic values of the variables and partial safety factors. The bias shown in Table 8.2 is therefore equivalent to the ratio of the initial checking point values of the basic variables, $\boldsymbol{x}_{\boldsymbol{i}}^*$, presented in Table 8.3, against each of their respective characteristic values, vector $\boldsymbol{X}_{\boldsymbol{k}}$, given in Table 8.2 and 8.3.

It should be noted that it was the reliability of the limit state function that was sought in the reliability assessment process and as such, the values of the checking point variables in their original untransformed space, $\boldsymbol{x}_{\boldsymbol{i}}^*$, were ensured at the beginning of each iteration to set the performance function equal to the limit state condition of zero. To achieve such objective, and supposing that the performance function is described by n basic variables, the checking point values of $n-1$ variables were estimated at each iterative step, with the value of the n^{th} variable determined so as to set the performance function to zero. To this effect, it can be observed that no characteristic value is prescribed for the MF in the first iteration shown in Table 8.3. The MF was treated as the n^{th} basic variable in the process where its value is chosen to obey the limit state condition.

Once appropriate values of the checking point are established, they are transformed to their equivalent normal statistics, μ_X^N and σ_X^N, and their direction cosines, α_X, are determined. The FORM procedures performed for the thesis were conducted using an MS Excel spreadsheet which is provided on a CD attached at the end of the thesis.

Results of the reliability analysis

Results of the 1^{st} iterations of the FORM analysis process applied to test case 1 and 2 in the problem of shear reliability performance of members requiring stirrups are presented in Table 8.3 and 8.4. Only the 1^{st} iteration for each test case was presented because the performance function for shear is highly non-linear and the limit state surface $g\ (\boldsymbol{X}) = 0$ is an n-dimensional failure surface that would most probably require several iterations before convergence of the final result is achieved (Ang & Tang, 1984). For each of the two test cases, some thirteen iterations of the FORM procedure were conducted without reaching stable convergence of the β-estimate, though the values of β obtained throughout were close to that of the first iterations as indicated in Table 8.3 and 8.4. Ang and Tang (1984) state that unlike the linear case, there is no unique distance (β) from the failure surface to the origin of the reduced variates for non-linear limit state surfaces. Therefore, increased non-linearity will only further complicate the process of locating the design point.

An additional MS Excel analysis tool was created in an attempt to reduce some of the non-linearity associated with the performance function. The action of developing a simplified analysis tools was further motivated by the fact that the model factor dominates shear reliability performance of the EC 2's variable strut inclination method. Further, to validate the results from both developed tools, the VaP analysis package was also used to implement the FORM process for the test cases. Acceptable agreement between the solutions from VaP and those from the developed Excel tools is demonstrated in the relevant Sections to follow.

Table 8.3. Data acquired from the 1^{st} iteration of the FORM analysis for test case 1

β = 3.293								
X_i	**X_k**	**x_i^***	**σ_X**	**Distn**	**μ_X^N**	**σ_X^N**	**$\partial g/\partial X$**	**α_X**
MF	-	0.33	0.51	3-P LN	1.30	0.29	93802.18	0.9954
A_{sw}	157.1	157.1	3.14	N	157.10	3.14	1024.05	0.0109
s	300	300	9	N	300.00	9.00	-1536.07	-0.0163
m	0.9	0.85	0.04	3-P LN	0.86	0.03	3953.08	0.0419
h	500	500	5	N	500.00	5.00	1280.80	0.0136
C	30	30	9	2-P LN	28.71	8.81	-2255.95	-0.0239
a	16	16	0.32	N	16.00	0.32	-245.91	-0.0026
e	10	10	0.2	N	10.00	0.20	-51.23	-0.0005
f_{ywk}	250	299.35	30	3-P LN	297.86	29.93	5118.61	0.0543
b_w	350	350	3.5	N	350.00	3.50	543.35	0.0058
α_{cck}	0.85	0.85	0.085	2-P LN	0.85	0.08	5420.04	0.0575
f_{ck}	25	35.5	6.39	3-P LN	35.49	0.08	129.08	0.0014
							$\sqrt{\sum(\alpha_X^2)}$ =	1.0000

Table 8.4. Data acquired from the 1^{st} iteration of the FORM analysis for test case 2

β = 2.347								
X_i	**X_k**	**x_i^***	**σ_X**	**Distn**	**μ_X^N**	**σ_X^N**	**∂g/∂X**	**α_X**
MF	-	0.63	0.51	3-P LN	1.44	0.34	208080.48	0.9897
A_{sw}	157.1	157.1	3.14	N	157.1	3.14	3342.72	0.0159
s	125	125	3.75	N	125	3.75	-5014.08	-0.0238
m	0.9	0.85	0.04	3-P LN	0.86	0.03	14399.72	0.0685
h	500	500	5	N	500	5	4665.51	0.0222
C	30	30	9	2-P LN	28.71	8.81	-8217.65	-0.0391
a	6	16	0.12	N	16	0.32	-895.78	-0.0043
e	10	10	0.2	N	10	0.2	-186.62	-0.0009
f_{ywk}	450	499.35	30	3-P LN	498.45	29.97	10032.19	0.0477
b_w	350	350	3.5	N	350	3.5	2173.02	0.0103
α_{cck}	0.85	0.85	0.085	2-P LN	0.85	0.08	21676.18	0.1031
f_{ck}	25	35.5	6.39	3-P LN	35.49	0.08	516.24	0.0025
							$\sqrt{\sum(\alpha_X^2)}$ =	1.0000

By comparing the relative magnitudes of the direction cosines, α_X,Table 8.3 and 8.4 show that shear reliability performance for members requiring stirrups is completely dominated by the MF with $\alpha_X = 0.9954$ for test case 1 and $\alpha_X = 0.9897$ for test case 2. Recall that the general probabilistic model is a distribution because it is composed of and described by basic random variables. The significance of the dominance of the MF implies that the distribution of shear resistance will be heavily influenced by the statistics and distribution of the MF. A direction cosine of more than 0.98 in both test cases indicates that the MF completely dominates shear reliability performance in all considered design situations. As such, a simplified reliability model that considered the MF as the only basic random variable in the performance function for shear was created and used to further characterise the reliability performance of members subjected to shear and requiring design stirrups. Considering only one basic random variable in the performance function also reduces the complexity of the limit state surface which leads to fewer iterations of the checking process until acceptable convergence of the β-estimate is achieved. At the design point, convergence should also be realised in the values of the basic random variables and not just in the value of the β-estimate.

8.2.5 Simplified reliability analysis tool

The fact that the MF completely dominates the shear performance of members requiring stirrups provided strong reason to represent the general probabilistic model solely on the variability of the MF. To verify and confirm the action of neglecting the contribution of the distributions of the other basic variables in the gpm, an additional investigation was carried out and is presented below.

When the EC 2 variable strut inclination method is converted for use as the gpm in the performance function for shear, the limits placed on $\cot\theta$ from the codified procedure are lifted and θ assumes any value as determined by θ_{gpm} defined in Equation [8.6]. It should be noted that θ can be considered a secondary variable in the analysis process as it is calculated based on the quantities of other basic variables, as can be seen in Equation [8.6]. A check on how θ varied at each checking point or FORM iteration when the full probabilistic model was implemented in the reliability analysis was conducted. Its purpose was to verify that calculating θ based on deterministic mean value estimates of the basic variables at their first iteration values did not severely alter the true character of θ when allowed to vary in the gpm of the full probabilistic model in Section 8.2.3.

Furthermore, it was envisaged that the trigonometric expression in the limit state Equation [8.18] may lead to excessive non-linearity in the performance function, thus affecting linearisation estimates of β at the checking point during implementation of the FORM method. A replacement constant, denoted as K, was suggested for use to replace the trigonometric expression defining θ, $\cot\left(\sin^{-1}\sqrt{\frac{A_{sw}f_{yw}}{b_w s v_1 \alpha_{cc} f_c}}\right)$. Such a replacement was justified by two closely related facts that were observed from the full investigation:

1. All the basic variables affected by the trigonometric operators in the performance function had low sensitivities, judged by assessing the magnitude of the direction cosines, implying that minimal changes were experienced in their checking point values between iterations. This motivated the basic variables a, e, b_w, α_{cc} and f_c to be considered as deterministic quantities during the simplified FORM analysis.

2. As a result of Bullet 1, the value of the θ_{gpm} did not change more than 1° through the 13 iterations considered for the full investigation. This realisation confirmed the use of a constant value of θ_{gpm}, which was calculated using checking point quantities of the first iteration, through all iterations.

The simplified limit state function was established as:

$$g(\boldsymbol{X}) = MF \cdot \left[\frac{A^*_{sw,1}}{s^*_1} m^*_1 (h^*_1 - C^*_1 - n_l \cdot a^*_1 - e^*_1) f^*_{yw,1} \cdot K\right] - V_{EC2_{design}}(\boldsymbol{X_k}, \boldsymbol{\gamma}) = 0 \qquad [8.20]$$

Where K is a once-off calculated quantity defined as:

$$K = \cot\left(\sin^{-1}\sqrt{\frac{A^*_{sw,1} f^*_{yw,1}}{b^*_{w,1} s^*_1 v^*_1 \alpha^*_{cc,1} f^*_{c,1}}}\right) \qquad [8.21]$$

$A^*_{sw,1}, s^*_1, m^*_1, f^*_{yw,1}, h^*_1, C^*_1, a^*_1, e^*_1, b^*_{w,1}, s^*_1, v^*_1, \alpha^*_{cc,1}, f^*_{c,1}$ represent the checking point values of the basic variables established for the first iteration of FORM analysis which were set as equal to the mean value estimate of each of the variables. The MF is the only random variable in Equation [8.20] and thus the only partial derivative of the of the simplified performance function was defined as:

$$\left(\frac{\partial g(\boldsymbol{X})}{\partial MF}\right)^* = \frac{A^*_{sw,1}}{s^*_1} m^*_1 (h^*_1 - C^*_1 - n_l \cdot a^*_1 - e^*_1)\, f^*_{yw,1} \cdot K \cdot \sigma^N_{MF} \qquad [8.22]$$

The same design examples used for the full investigation, and their procedure for analysis as described in Section 8.2.4, were adopted for use with the simplified performance function.

Quicker convergence of the β-solution was achieved for both test cases using the simplified tool, with a stable β-result yielded in the two iterations performed per test case.

Table 8.5. Data acquired from the 1^{st} iteration of the FORM analysis for test case 1

β = 3.379							
X_i	X_k	x_i^*	σ_X	**Distn**	μ_X^N	σ_X^N	α_X
MF	0.307	0.51	3-P LN	1.29	0.29	92597.93	1
A_{sw}	157.1	3.142	DET	-	-	0	0
s	300	9	DET	-	-	0	0
m	0.85	0.035	DET	-	-	0	0
h	500	5	DET	-	-	0	0
C	30	9	DET	-	-	0	0
a	16	-	DET	-	-	0	0
e	10	-	DET	-	-	0	0
f_{yw}	299.35	30	DET	-	-	0	0

Table 8.6. Data acquired from the 1st iteration of the FORM analysis for test case 2

β = 2.485							
X_i	X_k	x_i^*	σ_X	**Distn**	μ_X^N	σ_X^N	α_X
MF	0.586	0.51	3-P LN	1.42	0.33	203693.84	1
A_{sw}	157.1	3.142	DET	-	-	0	0
s	125	9	DET	-	-	0	0
m	0.85	0.035	DET	-	-	0	0
h	500	5	DET	-	-	0	0
C	30	9	DET	-	-	0	0
a	16	-	DET	-	-	0	0
e	10	-	DET	-	-	0	0
f_{yw}	499.35	30	DET	-	-	0	0

8.2.6 VaP validation of MS Excel tools

A VaP analysis (Schneider, 2006) was conducted to validate the results of the reliability analysis obtained from the use of the MS Excel tools that were developed in Section 8.2.4 and 8.2.5. When the EC 2 variable strut inclination method was adopted for use as the gpm, VaP could be properly used to check the correctness of the calculations as opposed to the case when the MCFT is employed for use as the gpm. In instances where the MCFT was used as the gpm, as was done in Section 8.3, numerical differentiation using Response-2000 was required in determining direction cosines of the basic variables and were not determined through analytical partial differentiation of the limit state function, hence VaP could not be used to verify the correctness of the execution of calculations of the FORM procedure in Section 8.3. However, the verification of acceptable calculations in this Section implied that the general calculation setup and execution of the FORM analysis procedure for the thesis was done satisfactorily, even in cases where the MCFT was adopted for use as the gpm.

As a first attempt to use VaP to validate the MS Excel tools developed for the reliability analysis, it was intended/desired to fully represent the distributions of the basic variables in the performance function for shear when EC 2 is used as gpm. Some difficulties, that led to slight differences in the values of the β-estimate and design point quantities of the basic variables, phased out this process. Firstly, VaP could not implement the 3P-LN distribution with negative skewness as it only allowed the use of the 3P-LN distribution with positive skewness. As such, the basic random variable m could not be appropriately represented, and for initial purposes of verifying the calculation procedure of the FORM analysis m was represented by the 2P-LN distribution in both the Excel and VaP analysis tools. Even after this action, there was still some discrepancy between the values of the β-estimate and of the design point quantities of the basic variables obtained between Excel and VaP tools.

Further investigation showed that the differences in the β-estimate as well as checking point values of the basic variables stem mainly from some slight approximation error in the implementation of the 2P-LN and 3P-LN distributions. The results of the investigation are presented in Table 8.7. In the full probabilistic representation of the performance function, the 3P-LN distribution with positive skewness was appropriately used to represent the model factor associated with the EC 2 prediction model ($MF_{EC\ 2}$ in Table 8.1), f_{yw}, and f_c whilst the 2P-LN distribution was used to model basic variables C and α_{cc}.

To prove the slight differences in the distributions, simplified VaP analyses were conducted that kept the MF as a random variable but represented all other basic variables as deterministic quantities. In one instance, applied to both test cases or design situations, the $MF$was represented by the 3P-LN distribution in VaP and this can be compared to the results in Table 8.5 and Table 8.6 for the simplified analysis of test case 1 and 2 respectively. In the other, also applied to both test cases, the MF was represented by the 2P-LN distribution; this can be considered as an action taken to verify that when the MF was represented by the 2P-LN distribution, some differences in the β-estimate or values of the design point quantities of the basic variables were still realised. This implied that, indeed, the differences in the results of the FORM analysis between VaP and Excel tools could be ascribed to minor differences in implementation of the 2P-LN and 3P-LN distributions. To verify this finding a mean value analysis was conducted to ensure that at least the moments of the distributions were correctly input hence confirming the differences in the results to truly arise from slight approximation error in the lognormal distributions between the developed Excel tools and VaP.

Table 8.7. Results of the FORM analysis for test case 1 and 2, using VaP and Excel tools and considering the MF as a 2P-LN and 3P-LN distribution in separate analyses

		Results from VaP			***Results from Excel tools***		
***MF* distn.**	**Design situations**	**β**	**Design value**	**α_{MF}**	**β**	**Design value**	**α_{MF}**
MF as 3P-LN	**Test case 1**	3.28	0.331	1	3.38	0.307	1
	Test case 2	2.34	0.632	1	2.49	0.586	1
MF as 2P-LN	**Test case 1**	5.17	0.331	1	5.41	0.307	1
	Test case 2	3.03	0.632	1	3.28	0.586	1

The results of Table 8.7 confirm that there were slight differences in the β-estimate of comparable test cases when either the 2P-LN or 3P-LN distribution are isolated to represent the MF as the only basic variable in separate analyses. Slight differences, between comparable cases, were also observed in the design point value of the MF between VaP and the Excel analysis tools. Figure 8.4 and Figure 8.5 show the performance functions for shear implemented in VaP for test case 1 and 2 respectively, in which the results of the expected or mean value analysis of the performance function, $G(E[X])$, are reported in Newtons. These values match those found from the mean or expected value analysis from the Excel tools. Using the MS Excel tools, for test case 1 the result of the mean value analysis of the performance function was 420166.2 Newtons whilst for test case 2 the analysis yielded 619435.1 Newtons. The very slight differences in the mean value analyses can be attributed to some rounding error when specifying the quantities of basic variables in VaP. This implies that no differences in the results of the FORM analysis can be attributed to wrong input of the mean values of the basic variables, and any differences can be attributed to slight approximation error between the two distributions and perhaps some small rounding error.

The values enclosed by ovals in Figure 8.4 and Figure 8.5 are equivalent to the code determined design values of shear resistance in Newtons, $V_{EC2_{design}}(\boldsymbol{X_k}, \boldsymbol{\gamma})$, which were calculated using the data specified for the test cases in Table 8.2. The same values were implemented when the MS Excel tools were used for corresponding analyses. The symbols γ

and z shown in the Figures represent the basic variable $\alpha_{cc}$and f_c respectively. All other symbols, some of which are presented without their subscripts, remain as originally defined.

```
G = (M*(A/s)*m*(h-C-n*a-e)*f*(SQRT(1-((A*f)/(b*s*v*y*z)))/SQRT((A*f)/(b*s*v*y*z))))-105537.7464

G(E[X]) for G:

      G(E[X]) = 420175
```

Figure 8.4. Expected value analysis for test case 1

```
Limit State Function G :

    G = (M*(A/s)*m*(h-C-n*a-e)*f*(SQRT(1-((A*f)/(b*s*v*y*z)))/SQRT((A*f)/(b*s*v*y*z))))-384437.9814

G(E[X]) for G:

    G(E[X]) = 619452
```

Figure 8.5. Expected value analysis for test case 2

8.2.7 Reliability requirements for resistance

From the principle of reliability separation promoted in modern basis of design formats, as in EC 0 and the now fully operational SANS 10160-1, specify that the target β for resistance in the structural Eurocodes should be $\beta_{T,R} = 0.8 \times 3.8 = 3.04$ and that for South African practice should be $\beta_{T,R} = 0.8 \times 3.0 = 2.4$. Hence, For SANS adequate reliability is achieved for the design situations represented by test case 1 ($\beta = 3.379$) and test case 2 ($\beta = 2.485$). In the case of the Eurocode requirements, test case 2 which is representative of a design situation with relatively high amounts of shear reinforcement, sufficient reliability is not achieved as it has a β below 3.04. It should be noted that the EC 2 variable strut inclination method for members with stirrups can be regarded as adequate for South African reliability requirements provided that characteristic values ($\boldsymbol{X_k}$), and more critically, the partial material safety factors ($\boldsymbol{\gamma_C}$ and $\boldsymbol{\gamma_S}$) stipulated for use by EC 0, EC 2 and the relevant British annexes are implemented in determining the deterministic design resistance, $V_{EC2_{design}}(\boldsymbol{X_k}, \boldsymbol{\gamma})$. Any alterations to any of the aforementioned aspects that affect the value of the deterministic design resistance would warrant an investigation aimed at determining their effect on shear reliability performance for members designed with stirrups. Naturally, any action taken to increase the prediction of the design resistance when the codified method is applied in practice would reduce the reliability of the procedures and vice versa is true.

8.3 RELIABILITY ANALYSIS USING THE MCFT AS THE GENERAL PROBABILISTIC MODEL

The application of the more rational scientific and more accurate MCFT as compared to EC 2's variable strut inclination method, as established in Chapter 5 and 7 respectively, as the gpm in the performance function for shear can be viewed as the proper manner in which the performance or limit state function should be formulated. This Section outlines the way in which the MCFT, whose predictions of ultimate shear resistance were done using Response-2000, was used in investigating the reliability performance of EC 2's variable strut inclination method. In this Section, the statistics of the MF specific to the MCFT prediction model, as established in Chapter 7 and reported in Table 8.1 as MF_{MCFT}, were used in the reliability analysis.

8.3.1 The Performance function

In cases where the MCFT was used as the general probabilistic model, the performance function for shear of members with stirrups was defined by:

$$g(\boldsymbol{X}) = MF \cdot (V_{MCFT}) - V_{EC2_{design}}(\boldsymbol{X_k}, \boldsymbol{\gamma}) \qquad [8.23]$$

Where $V_{MCFT}$is the value of shear resistance determined from the Test Section being modelled in Response-2000. V_{MCFT} serves as the descriptor of true shear resistance in Equation [8.23], making it a distribution and can be viewed in this context as replacing $V_{EC\,2_{gpm}}$ in Figure 8.2. $V_{EC2_{design}}(\boldsymbol{X_k}, \boldsymbol{\gamma})$ remains as defined in Section 8.2. The basic input variables that enable V_{MCFT} predictions are allowed to assume random statistical distributions with their partial derivatives evaluated using the procedure outlined in Section 8.3.2. The procedure of implementing the FORM algorithm is essentially the same as that presented in Section 8.2.4, with the main difference arising from the way in which the partial derivatives of the basic variables were determined.

8.3.2 Reliability analysis of the representative design situations

The same representative design situations, test case 1 and 2, as used for the reliability analysis in Section 8.2 were employed for use in this Section. As elaborated in Chapter 5, Section 5.2.7, numerous basic variables are required to enable adequate modelling of sections or members in Response-2000. The input data, which were all considered as basic variables of which some were treated as random variables, for the design situations are shown in Table 8.8 below. Figure 8.6 shows the typical cross-section associated with the design examples with some common dimensional parameters illustrated on the diagram. The beams were assumed to be simply supported, with a single gravity point load placed between supports, and have a double layer arrangement of longitudinal tension reinforcement.

Table 8.8. Design specifications of the basic variables for the test cases

Variable X_i	**Characteristic quantities *Test case 1***	**Characteristic quantities *Test case 2***	**Unit / Dimension**	**Bias**
MF_{MCFT}	1.18	1.18	-	-
A_{sw}	157.1	157.1	mm^2	1
s	300	125	mm	1
m	0.9	0.9	–	0.94
h	500	500	mm	1
C	30	30	mm	1
a	16	16	mm	1
e	10	10	mm	1
f_{ywk}	250	450	MPa	$1 + (49.35/f_{ywk})$
b_w	350	350	mm	1
f_{ck}	25	25	MPa	1.42
Aggregate size (agg)	19	19	mm	1
A_s *(1 bar)*	804.25	804.25	mm^2	1
a/d	2.5	2.5	-	-

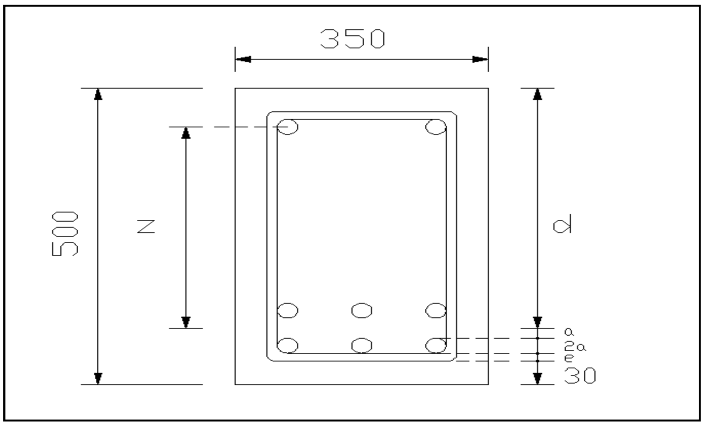


Figure 8.6. Schematic representation of cross-section modelled in Response-2000

The user has no control over the analysis process as Response-2000 carries out the analysis for ultimate shear strength predictions according to the MCFT. As a result, partial derivatives, and therefore direction cosines as well, of the limit state function cannot be determined analytically, except for the MF which is explicitly included in Equation [8.23], but are rather determined by the relatively approximate method of numerical differentiation. To obtain partial derivatives for the test case, each of the basic input variables, $X_{original}$, were increased in turn by 2 %, $X_{2\%\,increase}$, and the change in the ultimate prediction of shear strength associated with an increase of the respective parameter was noted. Thus, for any given basic variable from the vector $\boldsymbol{X}$, the partial derivative at each checking point was determined as:

$$\left(\frac{\partial g(\boldsymbol{X})}{\partial X}\right)^* = \left(\frac{V^*_{MCFT,2\%\,inc} - V^*_{MCFT,orig}}{X^*_{2\%\,increase} - X^*_{original}}\right) \cdot \sigma_X^N \qquad [8.24]$$

The partial derivative of the MF can be determined analytically from Equation [8.23] as:

$$\left(\frac{\partial g(\boldsymbol{X})}{\partial MF}\right)^* = V^*_{MCFT} \cdot \sigma_{MF}^N \qquad [8.25]$$

Where V^*_{MCFT} represents the value of ultimate shear resistance determined by feeding the unbiased checking point values of selected basic variables at each iteration into the Response-2000 program.

Significance of the basic variables

A full investigation was conducted to determine which basic variables dominate the shear reliability performance for members requiring stirrups. Owing to the different conceptual basis and theoretical development of the prediction models, the basic variables that affect the reliability performance when the EC 2 design model is converted for use as gpm in Section 8.2 need not be the same as those that affect the reliability performance when the MCFT is used as the gpm. In addition, Respone-2000 provides the ultimate shear strength for simply supported members and is thus sensitive to a/d. In fact, the value of shear resistance varies significantly with changes in a/d at relatively high amounts of shear reinforcement. The shear span, a, was not treated as a basic variable and therefore did not have its partial derivative determined according to Equation [8.23], but was maintained as its original value determined from the a/d from the characteristic geometrical quantities of the assumed load arrangement. However, the two test cases were investigated at three different a/d ratios per test case; 2.5, 4, and 6. Except for the basic variable agg, all other basic variables presented in Table 8.6 were treated as random variables.

Assuming agg as a deterministic quantity was supported by the fact that design procedures for structural resistance requiring aggregate size usually specify the use of the *maximum* coarse aggregate size. Secondly, the Portland Cement Association (2002) emphasise that variations in coarse aggregate grading and therefore distribution are difficult to anticipate, often making it more economical to maintain uniformity in manufacturing and handling coarse aggregates to reduce variations in gradations. It is therefore promoted to control the production quality of aggregates as opposed to intensive statistical modelling of its highly variable particle size distribution. Further, the influence of aggregates on shear resistance after initial diagonal cracking in members with shear reinforcement becomes secondary due to the fact that the action of stirrups is activated rather than the resistance at the crack being solely dependent on the interface shear characteristics.

The sensitivities or direction cosines, $\alpha_{\boldsymbol{X}}$, of all basic variables were determined at each a/d ratio per test case so as to capture the relative influence of the basic variables toward reliability performance under the given conditions. The results of the first iteration of the FORM analyses performed at each a/d ratio per test case are presented in the following six Tables, Table 8.9 to 8.14. Only the first iteration of the FORM analyses are shown to highlight the relative influence that the different basic variables have on the reliability performance when the MCFT is used as gpm or as the descriptor of true shear resistance. Note that $A_{s,tot}$ indicates the total amount of longitudinal tension reinforcement provided for the design and f_{yl} denotes the yield strength of the longitudinal tension reinforcement. All other basic variables remain as previously defined.

Basic random variables with low direction cosines or sensitivities from the first iterations presented in Tables 8.9 to 8.14 were treated deterministically in the subsequent iterations so as to reduce the number of iterations till convergence, reduce analysis time, and enable a directed and meaningful investigation. Tables 8.9 through to 8.14 can be viewed as summarised results from a preliminary analysis aimed at centering the investigation around critical basic variables that affect reliability performance for members with stirrups.

Table 8.9. First iteration of the FORM analysis for test case 1 when $a/d = 2.5$

β = 3.346								
X_i	X_k	x_i^*	σ_X	***Distn***	$\mu_X{}^N$	$\sigma_X{}^N$	$\partial g/\partial X$	α_X
MF	-	0.417	0.2	2-P LN	0.83	0.073	18546.08	0.5874
A_{sw}	157.1	157.1	3.142	N	157.1	3.142	700	0.0222
s	300	300	9	N	300	9	-2250	-0.0713
h	500	500	5	N	500	5	-2650	-0.0839
C (cover)	30	30	9	2-P LN	28.71	8.81	4403.41	0.1394
f_{yw}	250	299.35	30	3-P LN	297.86	29.93	-22492.92	-0.7124
f_{yl}	450	499.35	30	3-P LN	498.45	29.97	0	0
b_w	350	350	3.5	N	350	3.5	-3200	-0.1014
f_c	25	35.5	6.39	3-P LN	34.94	6.340	-6250.873	-0.1980
$A_{s,tot}$	4825.49	4825.49	96.51	N	4825.49	96.51	8100.02	0.2566
							$\sqrt{\sum(\alpha_X^2)}=$	1.0000

Table 8.10. First iteration of the FORM analysis for test case 1 when $a/d = 4.0$

β = 3.370								
X_i	X_k	x_i^*	σ_X	***Distn***	μ_X^N	σ_X^N	$\partial g/\partial X$	α_X
MF	-	0.419	0.2	2-P LN	0.83	0.074	18546.08	0.6488
A_{sw}	157.1	157.1	3.142	N	157.1	3.142	1400	0.0490
s	300	300	9	N	300	9	-3150	-0.1102
h	500	500	5	N	500	5	-2250	-0.0787
C (cover)	30	30	9	2-P LN	28.71	8.81	-10274.61	-0.3594
f_{yw}	250	299.35	30	3-P LN	297.86	29.93	-18494.17	-0.6469
f_{yl}	450	499.35	30	3-P LN	498.45	29.97	0	0
b_w	350	350	3.5	N	350	3.5	-2800	-0.0980
f_c	25	35.5	6.39	3-P LN	34.94	6.34	892.98	0.0312
$A_{s,tot}$	4825.49	4825.49	96.51	N	4825.49	96.51	300.00	0.0105
							$\sqrt{\sum(\alpha_X^2)}$=	1.0000

Table 8.11. First iteration of the FORM analysis for test case 1 when $a/d = 6.0$

β = 3.251								
X_i	X_k	x_i^*	σ_X	***Distn***	μ_X^N	σ_X^N	$\partial g/\partial X$	α_X
MF	-	0.444	0.2	2-P LN	0.856	0.078	18546.08	0.6156
A_{sw}	157.1	157.1	3.142	N	157.1	3.142	2400	0.0797
s	300	300	9	N	300	9	-2550	-0.0846
h	500	500	5	N	500	5	3250	0.1079
C (cover)	30	30	9	2-P LN	28.71	8.81	-11742.42	-0.3898
f_{yw}	250	299.35	30	3-P LN	297.86	29.93	3998.74	0.1327
f_{yl}	450	499.35	30	3-P LN	498.45	29.97	0	0
b_w	350	350	3.5	N	350	3.5	1050	0.0349
f_c	25	35.5	6.39	3-P LN	34.94	6.34	19645.60	0.6521
$A_{s,tot}$	4825.49	4825.49	96.51	N	4825.49	96.51	100.00	0.0033
							$\sqrt{\sum(\alpha_X^2)}=$	1.0000

Table 8.12. First iteration of the FORM analysis for test case 2 when $a/d = 2.5$

β = 2.435								
X_i	X_k	x_i^*	σ_X	***Distn***	μ_X^N	σ_X^N	$\partial g/\partial X$	α_X
MF	-	0.65	0.2	2-P LN	1.01	0.11	67557.04	0.7831
A_{sw}	157.1	157.1	3.142	N	157.1	3.142	4000	0.0463
s	125	125	3.75	N	125	3.75	-8700	-0.1009
h	500	500	5	N	500	5	6000	0.0696
C (cover)	30	30	9	2-P LN	28.71	8.81	-24952.63	-0.2893
f_{yw}	450	499.35	30	3-P LN	498.45	29.97	16806.76	0.1948
f_{yl}	450	499.35	30	3-P LN	498.45	29.97	600.24	0.0070
b_w	350	350	3.5	N	350	3.5	2500	0.0290
f_c	25	35.5	6.39	3-P LN	34.94	6.34	42863.13	0.4969
$A_{s,tot}$	4825.49	4825.49	96.51	N	4825.49	96.51	400.00	0.0046
							$\sqrt{\sum(\alpha_X^2)}=$	1.0000

Table 8.13. First iteration of the FORM analysis for test case 2when $a/d = 4.0$

β = 0.995								
X_i	X_k	x_i^*	σ_X	***Distn***	μ_X^N	σ_X^N	$\partial g/\partial X$	α_X
MF	-	0.91	0.2	2-P LN	1.10	0.16	67557.035	0.8204
A_{sw}	157.1	157.1	3.142	N	157.1	3.142	0	0
s	125	125	3.75	N	125	3.75	0	0
h	500	500	5	N	500	5	7850	0.0953
C (cover)	30	30	9	2-P LN	28.71	8.81	-24952.63	-0.3030
f_{yw}	450	499.35	30	3-P LN	498.45	29.97	0	0
f_{yl}	450	499.35	30	3-P LN	498.45	29.97	6602.65	0.0801
b_w	350	350	3.5	N	350	3.5	2650	0.0322
f_c	25	35.5	6.39	3-P LN	34.94	6.34	38398.22	0.4663
$A_{s,tot}$	4825.49	4825.49	96.51	N	4825.49	96.51	2700.01	0.0328
							$\sqrt{\sum(\alpha_X^2)}=$	1.0000

Table 8.14. First iteration of the FORM analysis for test case 2 when $\mathrm{a}/d = 6.0$

β = -0.997								
X_i	X_k	x_i^*	σ_X	***Distn***	μ_X^N	σ_X^N	$\partial g/\partial X$	α_X
MF	-	1.36	0.2	2-P LN	1.10	0.240	67557.035	0.9106
A_{sw}	157.1	157.1	3.142	N	157.1	3.142	0	0
s	125	125	3.75	N	125	3.75	0	0
h	500	500	5	N	500	5	5250	0.0708
C (cover)	30	30	9	2-P LN	28.71	8.81	-16145.82	-0.2176
f_{yw}	450	499.35	30	3-P LN	498.45	29.97	0	0
f_{yl}	450	499.35	30	3-P LN	498.45	29.97	4501.81	0.0607
b_w	350	350	3.5	N	350	3.5	1750	0.0236
f_c	25	35.5	6.39	3-P LN	34.94	6.34	25003.49	0.3370
$A_{s,tot}$	4825.49	4825.49	96.51	N	4825.49	96.51	1800.00	0.0243
							$\sqrt{\sum(\alpha_X^2)}=$	1.0000

Results of the preliminary reliability analysis

The results from the Tables presented above indicate that the MF generally dominates the reliability performance for members in shear that require stirrups. There is however more significant dominance of the MF over the influence of other basic variables at high amounts of shear reinforcement (test case 2). At low reinforcement ratio (test case 1), there is noticeable influence of the yield strength of the stirrups, f_{yw}, even slightly dominating when $\mathrm{a}/d = 2.5$ and equaling dominance of the MF when $\mathrm{a}/d = 4.0$. f_{yw} has no observable influence on the reliability performance for members with high amounts of shear reinforcement (test case 2). When $\mathrm{a}/d = 6.0$ for test case 1, as shown in Table 8.11, the concrete compressive cylinder strength f_c slighlty dominates the reliability performance over the influence of the MF. f_c continues to have noticeable influence, second to that of the MF, at all a/d for test case 2, the representative design situation for members with high amounts of shear reinforcement.

Further, for both test cases and at all a/d ratios, there is some significance portrayed by the concrete cover, C. Although the influence of the concrete cover never dominates reliability performance over the other basic variables, it has unwavering influence at all amounts of shear reinforcement and at the different shear spans considered per test case. Hence it should be included in any simplified model of the performance function for shear. The concrete cover, C, can be considered as having a fully dependent relationship with the effective member depth, d, hence its noticeable influence on shear resistance.

From test case 1, the basic variables taken as important were MF, f_{yw} and C whilst from test case 2 MF, f_c and C were considered as important for reliability performance. As a result, four basic variables MF, f_{yw}, f_c and C were maintained for use as basic random variables in the performance function for shear, whilst all other basic variables were treated deterministically.

For test case 1, which is representative of a design situation with low amounts of shear reinforcement, some consistency is shown in the initial β-estimates for the design situation at the different a/d ratios considered. At all a/d ratios for test case 1, the β-estimate was approximately 3.3. Such consistency in the β-estimate at different a/d ratios was not realised from the results of test case 2, the design example that is representative of a design situation with high amounts of shear reinforcement. As expected, lower reliability indices at specific a/d were found for test case 2 as opposed to test case 1. However, within test case 2 at different a/d, the initial reliability indice β varied significantly with, $\beta = 2.435$ at $\mathrm{a}/d = 2.5$, $\beta = 0.995$ at $\mathrm{a}/d = 4.0$, and $\beta = -0.997$ at $\mathrm{a}/d = 6.0$. This implies that for test case 2 or design situations with large amounts of shear reinforcement, as the a/d increases the mean estimate of shear resistance, indicated as μ_{gpm} in Figure 8.7 below, provided by the MCFT reduces until the mean prediction is less than the deterministic design value that is determined according to the EC 2 design procedure.

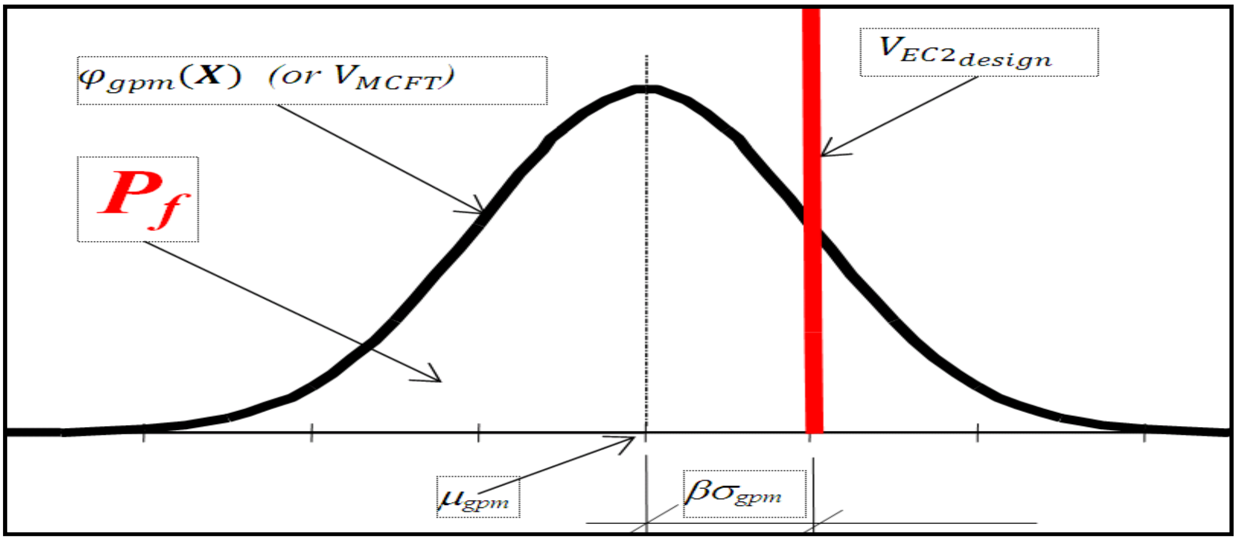


Figure 8.7. Probabilistic representation of the performance function when $\mu_{gpm} < V_{EC\,2_{design}}$

Similar results, predicting a reduction in shear strength with increasing a/d were presented by Sang-Yeol (1999). Sang-Yeol's findings indicate that the ultimate shear resistance predicted by the MCFT reduces slightly at a/d less than 4.0, with a distinct radical decrease in ultimate shear strength after a/d of 4.0. The section modelled was a T-Beam with $A_{sw}f_{yw}/b_w\, s = 1.74\, MPa$, an amount of reinforcement similar to that chosen for test case 2 which is representative of a design situation with high amounts of shear reinforcement. The ultimate shear strength prediction provided by the MCFT is indicated as $V_n(MCFT)$ in Figure 8.8, whereas the steel contribution to shear resistance determined by the MCFT is denoted as $V_s(MCFT)$ and the concrete contribution determined by the MCFT denoted as $V_c(MCFT)$. It is observable from Figure 8.8 that the kink in the ultimate shear resistance trend, $V_n(MCFT)$, at $a/d = 4.0$ is due mainly to a reduction in the steel contribution although there is some slight decrease as well in the concrete contribution after an a/d of about 3.5. Sang-Yeol did not proceed to give reasons for the reported trend of results but just presented them as part of an extensive parametric investigation comparing the ACI and MCFT prediction models for shear resistance.

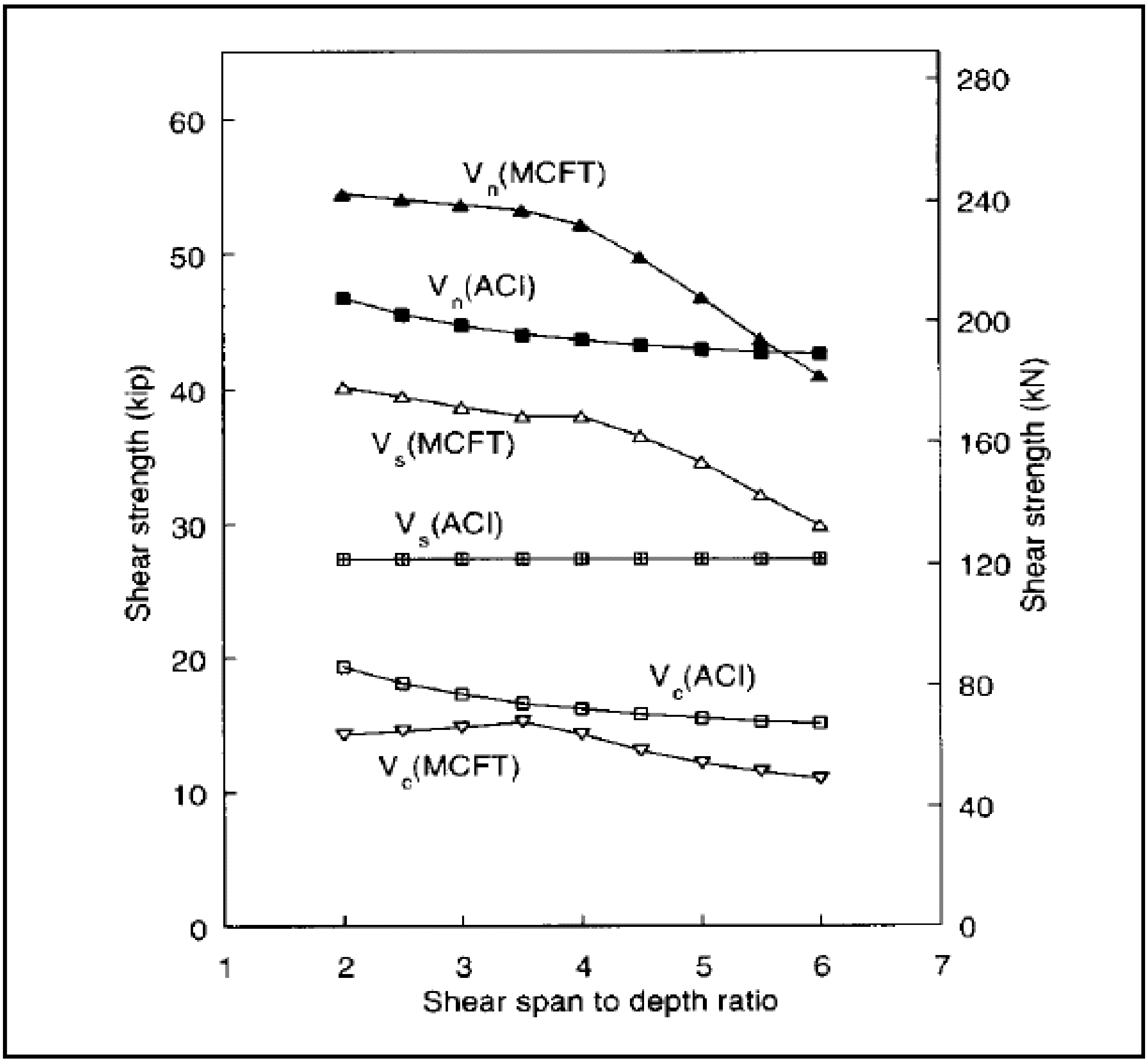


Figure 8.8. Effect of shear span to depth ratio on the shear strength of members with stirrups (Sang-Yeol, 1999)

Evidence presented in this Section clearly suggests that there is some relationship between members with relatively high amounts of shear reinforcement, a/d, and the ultimate shear resistance of members with stirrups. It is well known that as a/d increases, the shear-flexure interaction changes where flexural resistance begins to dominate and the influence of shear resistance subsides. In fact, after a value of a/d of 6.0 as shown in Figure 8.9, common literature reports that flexural failures occur with no critical influence of shear resistance on the members performance. Interestingly, the MCFT by use of Response-2000 also indicates that, not only does the shear-flexure interaction change, but the ultimate shear resistance reduces as well as a/d increases. The following questions arise:

1. Could this be attributed to the MCFT prediction method, which predicts shear of 2-D elements subject to biaxial stress, or to the development theory used to develop Response-2000?

2. Why is the radical decrease in ultimate shear resistance prediction, particularly after a/d of 4.0, not realised for members with low amounts of shear reinforcement?

3. What is it about members with relatively high amounts of shear reinforcement that causes a decrease in the shear resistance prediction, specifically after a/d of 4.0 as illustrated in Figure 8.6, and at what amount of shear reinforcement provided in design does this trend become noticeable?

These remain as open issues as it was not the objective of the thesis to find resolve, though it is encouraged to characterise this relationship in future work. According to well known facts about shear behavior of reinforced concrete members, post a/d of 2.5, significant decrease in shear capacity can expected as shown in Figure 8.9 but with no distinct downward kink in ultimate shear resistance experienced between an a/d of 2.5 and 6.0.

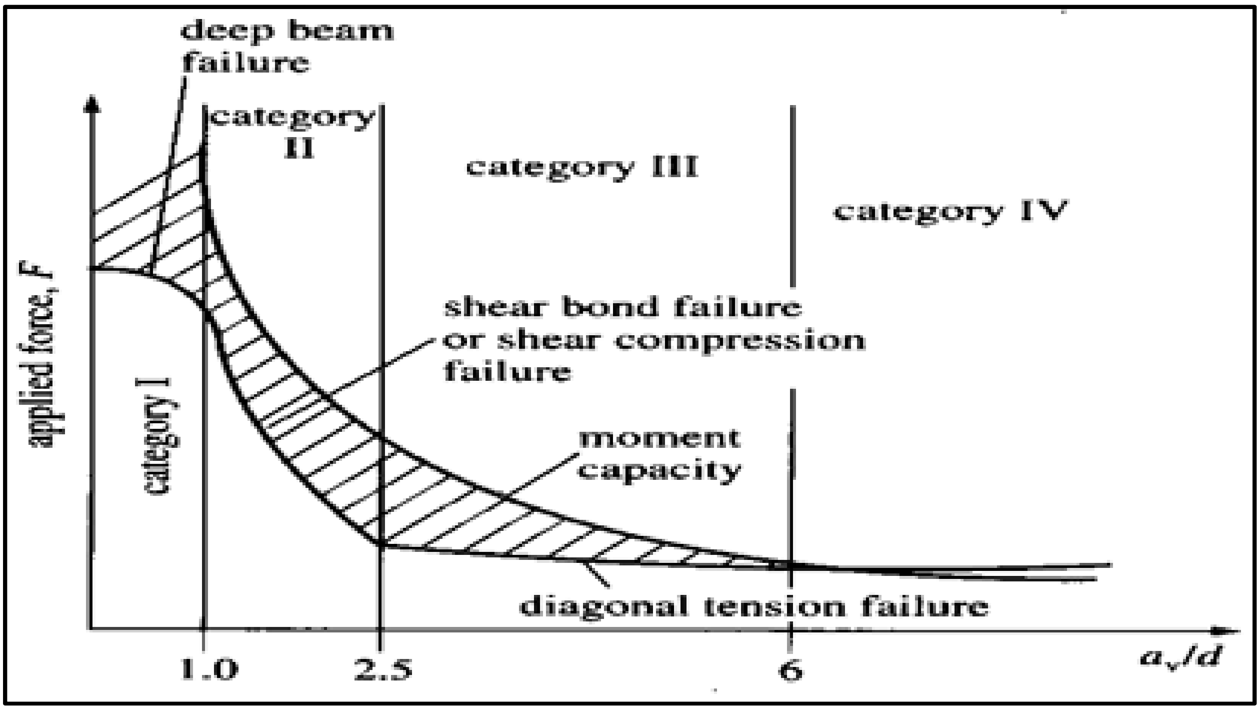


Figure 8.9. Variation is shear capacity with a/d for rectangular beams (after McGregor, 1992, as cited in O'Brien & Dixon, 1995)

Diagonal tension failures, which are characteristic for members with $2.5 < \mathrm{a}/d < 6.0$, represent the most prevalent types of shear-influenced failures that occur in concrete structures. EC 2's variable strut inclination method is intended for the shear design of a member that falls in the just mentioned confines for a/d. Alternative procedures are provided in EC 2 for the design for shear in D- or discontinuous regions such as punching shear and deep beam shear. A decision was made to base the reliability analysis for members with stirrups at an a/d of 2.5 when the MCFT, implemented through the use of Response-2000, is used as the gpm.

The EC 2 prediction model can be considered purely as a sectional design model and does not attempt to describe the full shear-flexure behaviour in member. It is rather applied at a shear critical location, in this case the region of the member subject diagonal tension stresses, along the member length, usually d or $0.9d$ from critical shear zones as applied forces and reactions at supports, for which the resulting design is applied throughout the member. Figure 7.8 in Chapter 7 shows that there is a trend of decreasing bias in MF with increasing a/d, thus implying that the MCFT does attempt to reflect the true behaviour of members with stirrups with respect to a/d. In contrast, for EC 2's variable strut inclination method, Figure 7.7 shows a large conservative bias in MF with a/d for the EC 2 prediction model. A slight and steady increase in prediction bias is observed as a/d increases. This implies that the EC 2 prediction model is not sensitive to a/d but is rather calibrated to be safe when applied at all perceivable a/d encountered in design.

Further, the developments presented above imply that the MCFT does provide rational prediction of possible true shear resistance and its trend with a/d, though Figure 7.8 indicates the bias associated with MF decreases with increasing a/d where it appears to lose conservatism with MF estimates below 1 anticipated after a/d of 6.0.

8.3.3 Simplified reliability analysis

As already established, the reliability analysis was conducted for two test cases. For both test cases, the reliability analysis was conducted at an a/d of 2.5. The basic variables MF, f_{yw}, f_c and C were considered as random in the simplified reliability model used for analysis when the MCFT was used as the gpm. Twenty iterations of the FORM analysis algorithm were completed before convergence of the β-estimate and the design point values of the basic variables were reached for test case 1. Eight iterations were performed to reach convergence of the β-estimate and design point values of the basic variables for test case 2. Tables 8.15 and 8.16 show the summary of the data acquired from the final iteration of the reliability analysis for each of the test cases, test case 1 and 2 respectively. The full procedure and information regarding the FORM iteration process from initial checking point to convergence is included on the CD attached at the end of the thesis.

Table 8.15. Results of the 20th and final iteration for test case 1

β =5.308								
X_i	X_k	x_i^*	σ_X	*Distn*	$\mu_X{}^N$	$\sigma_X{}^N$	$\partial g/\partial X$	α_X
MF	-	0.53	0.2	2-P LN	0.93	0.09	18546.08	0.80
A_{sw}	157.1	157.1	3.142	DET	-	-	0.00	0.00
s	125	300	3.75	DET	-	-	0.00	0.00
h	500	500	5	DET	-	-	0.00	0.00
C (cover)	30	42.73	9	2-P LN	25.78	12.54	-5871.21	-0.25
f_{yw}	250	253.52	30	3-P LN	294.40	25.36	7001.99	0.30
f_{yl}	450	499.35	30	DET	-	-	0.00	0.00
b_w	350	350	3.5	DET	-	-	0.00	0.00
f_c	25	23.25	6.39	3-P LN	32.75	4.18	10774.85	0.46
$A_{s,tot}$	4825.49	4825.49	96.51	DET	-	-	0.00	0.00
							$\sqrt{\sum(\alpha_X^2)}$=	1.0000

Table 8.16. Results of the 8th and final iteration for test case 2

β = 2.876								
X_i	X_k	x_i^*	σ_X	***Distn***	$\mu_X{}^N$	$\sigma_X{}^N$	$\partial g/\partial X$	α_X
MF	-	0.74	0.2	2-P LN	1.05	0.13	67557.04	0.84
A_{sw}	157.1	157.1	3.142	DET	-	-	0.00	0.00
s	125	125	3.75	DET	-	-	0.00	0.00
h	500	500	5	DET	-	-	0.00	0.00
C (cover)	30	36.17	9	2-P LN	27.85	10.62	-22017.03	-0.27
f_{yw}	450	483.95	30	3-P LN	498.24	29.05	13806.08	0.17
f_{yl}	450	499.35	30	DET	-	-	0.00	0.00
b_w	350	350	3.5	DET	-	-	0.00	0.00
f_c	25	27.81	6.39	3-P LN	34.17	4.98	35822.56	0.44
$A_{s,tot}$	4825.49	4825.49	96.51	DET	-	-	0.00	0.00
							$\sqrt{\sum(\alpha_X^2)}$=	1.0000

Discussion of the results of the reliability analysis

By observing the direction cosines, α_X or α_{MF}, of the MF presented in Tables 8.15 and 8.16, 0.80 and 0.84 respectively, it is clear that the MF dominates shear reliability performance for members with stirrups. However, unlike the instance when the EC 2 variable strut inclination method is converted for use as the gpm in Section 8.2, other basic variables are found to be of some significance to reliability performance. For test case 1, f_c, f_{yw}, and C are found, in decreasing order, to have significant influence on shear reliability performance for members with stirrups. For test case 2, the same basic variables were found to be of significance ($\alpha_X > |0.1|$) to shear reliability performance, though the influence of C precedes that of f_{yw} in order of decreasing significance as compared to test case 1.

As anticipated, the design point value of the MF was higher for test case 2 (0.74) than for test case 1 (0.52). Recalling that $MF = \frac{V_{test}}{V_{pred}}$, it implies that, indeed, the EC 2 prediction model is more conservative for design situations with low amounts of shear reinforcement (test case 1) hence the higher β-estimate and vice versa is true. As reported in Tables 8.13 and 8.15, a β-index of 5.308 was found for test case 1 whilst a β-index of 2.876 was found for test case 2.

For comparable test cases, higher β-estimates were found when the MCFT was used as the gpm in the performance function as opposed to when the EC 2 prediction model was converted for use as the gpm. This could most reasonably be ascribed to the fact that the MCFT is a better predictor of ultimate shear resistance for reinforced concrete members with stirrups. However, as can be observed from Table 8.17 below, more noticeable differences in the β-estimate were realised for test case 1. Characteristic values of the MF associated with the amount of shear reinforcement, as read from the appropriate graph in Figure 7.8 for the MCFT and Figure 7.9 for the EC 2 prediction model, are also reported in the Table.

Table 8.17. β-estimate of test cases 1 and 2 determined when MCFT and EC 2 prediction models are used as gpm

	EC 2 as gpm		**MCFT as gpm**	
Test Case	***β-estimate***	***Value of MF***	***β-estimate***	***Value of MF***
Test case 1	3.379	2.00	5.308	1.18
Test case 2	2.485	1.04	2.876	1.04

Recall from Section 8.2 that the choice of design parameters for the test cases was based on the trend of the model factor associated with EC 2's variable prediction model for shear, $MF_{EC\,2}$, with the amount of shear reinforcement, $A_{sw}f_{yw}/b_w s$. This action was taken because the MF was found to be critically sensitive to $A_{sw}f_{yw}/b_w s$. Test case 1 was designed to represent a design situation with low amounts of shear reinforcement whilst test case 2 was designed to represent a design situation with high amounts of shear reinforcement$A_{sw}f_{yw}/b_w s$. The mean values of the MF for each of the test cases, according to the appropriate regression models for the EC 2 and MCFT prediction models, are shown in Table 8.17. Figure 7.9 was used to deduce the mean values of the MF, with respect to

$A_{sw}f_{yw}/b_w s$, for the test cases according to EC 2's prediction model whereas the appropriate graph from Figure 7.8 was used to deduce the mean values of the MF for the test case according to the MCFT.

It is evident form Table 8.17 that the EC 2 and MCFT prediction models have similar bias in their prediction of ultimate shear resistance for test case 2, the section that is representative of a design situation with high amounts of shear reinforcement. Conversely, the same cannot be said for test case 1, as it can observed form Table 8.17 that EC 2's prediction model yields a more conservative result of the MF ($MF = 2.00$) than the MCFT ($MF = 1.18$). This difference in the character of the MF, shown between the two prediction models at low amounts of shear reinforcement, is responsible for the rather considerable difference in the β-estimates determined for test case 1 when either the EC 2 or MCFT prediction models are used as the gpm in the performance function for shear. For both the EC 2 and MCFT prediction models, the general tendency is to underestimate the ultimate shear resistance for members with low amounts of shear reinforcement. However, the MCFT is less conservative implying that it generally predicts higher values of ultimate shear resistance than the EC 2 prediction model for members with low amounts of shear reinforcement. This implies that the distance between the mean of the distribution describing true shear resistance, when MCFT is used as the gpm, and the deterministic design determined according to EC 2's stipulated design procedure (characteristic values + partial factors) is greater in terms of the number of standard deviations than the instance when the EC 2 prediction model is used to describe the distribution of true shear resistance. Figure 8.2 shows schematically the probabilistic representation of the performance function for shear, where β is defined as the number of standard deviations between the mean of the distribution of true shear resistance and the design point of shear resistance.

The central location parameters or mean moment statistics of the $MF$are important in establishing how close a member's predictions are to reality, thus quantifying how accurate the model is or how well a descriptor of the physical process it is. However, the spread of the distribution for the MF, relative to the central location or mean of the MF, is equally or even more important in determining how safe the model is. Safety, in this context, refers to the the number of standard deviations achievable away from the mean before the models' predictions of ultimate shear resistance become unconservative (MF equal to or less than 1.0). The EC 2 prediction model has a higher coefficient of variation, reported in Table 7.3 as 0.31, as opposed to 0.18 for the MCFT. The EC 2 prediction model, due to higher variability, will more readily yield undesirable estimates of ultimate shear resistance than the MCFT. The MCFT can therefore be deemed not only the better predictior of true shear resistance in physical reality, by virtue of having a lower mean bias associated with its MF than for the EC 2 prediction model, but can also be viewed as the safer model in its predictions due to the lower variability associated with the distribution of its model factor.

8.3.4 Reliability requirements for resistance

As established in Section 8.2.7, the target reliability for resistance, $\beta_{T,R}$, in the structural Eurocodes is 3.04 whilst that for for South African practice as set by SANS 10160-1 is 2.4. A β of 5.308 was found for test case 1 whilst a β-value of 2.876 was found for test case 2. Therefore, acceptable reliability, against SANS 10160-1 requirements for the reliability of resistance procedures, was observed for test cases 1 and 2. With a β of 2.876, test case 2 yielded a β below that stipulated by EC 0 and EC 2 for structural concrete resistance. Further, on the basis of these results and those presented in Section 8.2.7, it is comforting to note that the results of the β-estimate are less conservative, or more severe, when the more common situation of converting the EC 2 shear prediction model for use as the gpm was adopted, as opposed to when the MCFT was used as the gpm in describing the true shear resistance for members with stirrups. In normal situations of reliability analysis, the prediction model on which the calibrated design method is based is converted for use as the general probabilistic model.

CHAPTER 9

SUMMARY TO PART 2

Part One of the thesis was aimed at investigating the current state of implementation of reliability principles in the provisions for structural concrete in EC 2 and, upon identifying some incompleteness, advanced the implementation of reliability differentiation to partial factor modification as suggested by informative Annex A in EC 2. Initial ideas about model uncertainties and their significance on reliability performance were introduced with some indication from published research warranting further investigation for shear.

Part Two of the thesis, Chapters 5 to 8, was composed and structured so as to achieve the end goal of characterising the reliability performance of reinforced concrete members with stirrups subjected to shear; a common design situation in practice. Throughout Part 2, special attention is given to model uncertainties to reflect the influence they have on the accuracy of a model's predictions and therefore have impact on safety of designs in practice. The MF was therefore included as a basic variable that influences model's predictions of true shear resistance.

Shear behaviour of reinforced concrete members, both with and without design shear reinforcement, has long been problematic in achieving accurate modeling of its load deformation behaviour as well as ultimate shear resistance predictions. Due to lack of proper conceptual and theoretical understanding of the behaviour of members in shear, prediction models are known at times to be safe or unsafe, both situations of which the design method should be calibrated against.

However, the more prominent design situation in practice is that members are provided with shear reinforcement to limit shear cracks and other deformations resulting from excessive shear related stresses of the concrete, to allow the member to achieve full flexural capacity; a more ductile failure mode that provides sufficient warning (cracking + excessive deflections) before failure.

With the prediction of shear being known as problematic and being carried on through the years as inconsistent, the method of calibration of the design model then becomes important to ensure safe and economic design of structures in practice. Reliability techniques for structural design, to ensure safety of design provisions, have been implemented and recommended for use by the structural Eurocodes. South Africa has chosen to follow suit. Eurocodes are based on general requirements that are made operational in member states by specifying and implementing the Nationally Determined Parameters (NDPs). Therefore, reliability-based assessments are necessary to determine the suitability of the design models in EC 2 for South African practice.

The MCFT has proven to be a better descriptor of true ultimate shear resistance than the EC 2 and other common prediction models. The MCFT can therefore be more rationally applied for theoretical purposes where accurate representation of shear in physical reality is required such as being used as the general probabilistic model in reliability modelling. Unfortunately, the MCFT is computationally involving and time consuming, making it unsuitable for routine and conventional designs. The analysis Program Response-2000 can be viewed as an effort to make use of the MCFT routine in design. It is, however, not the objective of the thesis to promote the use of the MCFT as design method in practice but to use it to reflect the quality of results of the reliability analysis process when different models describing true shear resistance are used in the process. When either the MCFT or EC 2 prediction model were used as gpms, the model factor for shear, MF, was found to dominate reliability performance for members with stirrups subjected to shear. Acceptable reliability was achieved for both design situations considered in the investigation according to SANS 10160-1 reliability requirements for resistance. On the contrary, the test case representative of a member with relatively high amounts of shear reinforcement did not satisfy EC 0's target reliability for resistance of 3.04. The EC 2 design model, provided that $\gamma_C = 1.5$ and $\gamma_S = 1.15$ are used in design, is therefore even better suited to South African conditions than in Europe.

Given the highlights of Part Two of the thesis presented above, a brief breakdown of the Chapters 5 to 8 is given in the discussions below.

Chapter 5 introduced all the necessary background and insight into the conceptual and theoretical formulation of the models (MCFT and EC 2 prediction model) for shear. This background was proved vital in understanding the trends of the MF against the shear parameters. Such analyses could lead to theoretical and conceptual improvements and development for models for shear.

Chapter 6 provided a general overview of reliability principles for structures, emphasising the use of the First Order Reliability Method (FORM) as was applied to the EC 2 design method for shear in the thesis. The EC 2 design method for members with stirrups is based on the variable strut inclination method for shear. Important background was given on the non-normal distributions used in the thesis: two-parameter log-normal distribution (2P-LN) and three-parameter log-normal distribution (3P-LN) distribution. Information on their transformation to equivalent normal statistics was relevant as it enabled use of the FORM method of reliability analysis. It was established in Chapter 7 to represent the MF associated with the variable strut inclination method for shear used as basis for EC 2's design method by the 3P-LN distribution, whilst to represent that associated with the MCFT by the 2P-LN distribution.

Chapter 7 characterised the model factors associated with the EC 2 and MCFT prediction models. Descriptions were therein given of the databases to which comparisons were done to obtain model factor statistics for each of the prediction models.

Chapter 8 was concerned with the actual reliability analysis of the EC 2 shear prediction model for members requiring stirrups. Some interesting and important developments were generated in this Chapter. They are outlined briefly in bullet form below.

1. The development of statistical models describing the internal lever arm, z, and modelling of the concrete cover, C.

2. The MF was shown to dominate reliability performance for the shear resistance of members with stirrups.

3. Acceptable reliability performance was found for both design situations yielded in analysis according to SANS 10160-1 reliability requirements for resistance. The target reliability was however not achieved according to EC 0 requirements when the EC 2 design method was used to design test case 2, though not greatly below $\beta_{T,R}$ of 3.04. The EC 2 variable strut inclination design method is therefore adequate for South African practice, given that partial factors $\gamma_C = 1.5$ and $\gamma_S = 1.15$ are used and appropriate characteristic values of the basic variables are used. Any alternative combinations of partial factors and characteristic values would warrant an appropriate investigation to determine effect on reliability performance.

4. Different gpm's were used in reliability analyses to establish how the choice of the gpm may influence the results of the assessment. Disparities occurred in the results of the analysis due to different prediction models being used. This affects the probability of exceedance of mean shear resistance, μ_{gpm}, as compared to $V_{EC\ 2\ design}$

5. Though not well understood or defined, the MCFT has shown some interesting decrease in ultimate shear resistance of members with relatively high amounts of shear reinforcement post a$/d$ of 2.5. In fact, it predicts a distinct downward kink in ultimate shear resistance predictions after a$/d$ of 4.0. This seems to be a sensible trend as unbiased MCFT ultimate shear strength predictions agree quite well with experimental data as is showcased in Figure 7.8.

CHAPTER 10

CONCLUSIONS AND RECOMMENDATIONS

10.1 CONCLUSIONS

Structural design is a practice which, unfortunately, must be conducted with uncertainties. Uncertainties are inherent in the entire life-cycle of a project from design and detailing through to construction, and now even extending to maintenance and demolition in modern and more advanced design recommendations. The reasonable action thus suffices as to carry out the project process ensuring some rational safety verification of the process. This is a natural requirement, considering that structures, in their failure, pose obvious and serious risks to human life and have adverse economic and sometimes political and social implications.

In order to reflect the requirement of the need of safe and dependable structures, modern international trends of codified design for structures have chosen to adopt reliability-based formats to verify safety. In this format, actions and resistances are expressed as random variables, by way of mathematical statistics, to reflect their randomness in nature and also to enable a probabilistic assessment of the modelled physical processes at large. Probability statements about load and resistance are then used to quantify the probability or likelihood that the structural load will exceed the structural resistance; an event affectionately referred to in the fraternity as the probability of failure, denoted as P_f. Due to uncertainties in both loads and actions, and economic constraint amongst others, the probability of failure can't be considered as zero. This would be an overly conservative assumption as in reality structural failures do occur. Structural uncertainty can be attributed to inadequate modelling of the physical process showing incomplete understanding of the structure and its behaviour, or construction error, or design error, which all affect how the final designed structure would perform in service. The reliability index, β, is therefore introduced into the process of reliability assessment. It is an indirect measure of the probability of failure, P_f, of structures. β is incorporated in the calibration to reflect some acceptably low failure occurrence of structures based on socio-economic risk principles, when considering the drafting of provisions for all aspects of structural design. This implies that cognizance of the possibility of structural failure is taken into account when considering the entirety of the design provisions for structural concrete resistance, hence encouraging cautious and conscience treatment of safety when drafting design recommendations and provisions.

The EU has chosen to base their structural standards, the suite of structural Eurocodes, on reliability principles, which includes their concrete code, EC 2. In an effort to update and review their structural standards, South Africa has chosen to follow suit and base the revision to the new national concrete code SANS 10100-1 on EC 2. The Eurocodes are based on a general set of conditions that aim to cater for the diverse conditions shared amongst EU member states. These varying conditions could be due to different climates, soils, level of workmanship as well as different economic and legal requirements for structures in the different member states. Therefore, before the Eurocodes are made operational in any member state, they should be supported by relevant annexures that provide country-specific guidance on the Nationally Determined Parameters (NDPs). As South Africa has chosen to adopt EC 2 as reference for the national concrete code SANS 10100-1, reliability assessments are necessary to determine the suitability of the adoption of EC 2 for South African practice.

Much work on extending the well-documented reliability framework for structures has at current been achieved for actions, with the extension of the framework being nominally treated for resistance in the materials-based resistance standards. Actions for various specified design situations, together with an associated set of elaborate partial factors and action combination scheme, are specified for use. On the other, a single set of partial factors are used for resistance with its basic variables applied at their characteristic values in design formulations to cater for a wide range of design situations and material characteristics. Evidently, the implementation of the reliability framework for resistance is not as extensive as that for actions and much work still needs to be done to bring the reliability basis of resistance to the same level as that for actions, and further, to continue to apply new trends of reliability verification formats, to both actions and resistances in future.

This thesis can be considered as an effort to review what the reliability requirements for resistance are, and further, to determine to what extent the reliability requirements for resistance are applied in current international standards, specifically the Eurocodes. The basis of design requirements are dealt with in EC 0 with some attention therein given to action combination schemes for various identified design situations. EC 0 can thus be viewed as a material independent standard, but however contains the general requirements for reliability of resistance which can be applied across the various materials-based resistance standards.

This first section of this two-part thesis was structured to:

1. Firstly, present the elements of the reliability framework presented in EC 0, with reference to essential background documents such as the JCSS Probabilistic Model Code, CEB bulletins, and the CEB-FIP and *fib* Model Codes.

2. In the latter of Part One, present the first part of elaboration and implementation of the reliability framework comprising discussions on:

    a. The influence the reference level of reliability, β, has on structural performance with a study given to provide detailed view of how choice of its value affects the quantities of operational partial safety factors to be adopted in design codes of practice, particularly the case of $\beta = 3.0$ for SANS 10160-1 requirements and $\beta = 3.8$ according to EC 0 requirements. In this light, the partial factor set of γ_C and γ_S of 1.5 and 1.15, respectively, as recommended for use in the Eurocodes can be viewed as conservative for South African conditions. Partial factors of 1.4 and 1.1 for γ_C and γ_S, respectively, prove sufficient for flexural and compression resistance in the work considered by Holický et al. (2010) according to SANS 10160-1 requirements. These partial factors have not been verified as sufficient for shear according to SANS requirements is a feasible issue for immediate investigation.

    b. A Critical review of a key study by Holický et al. (2010) as it provides guidance and serves as template for reliability assessment and subsequent calibration of partial factors for other modes of resistance than those considered in the study. The suggestions of this study could be viewed as providing some motivation to characterise the model uncertainty associated with shear as well as its effects on reliability performance.

c. The issue of partial factor modification, particularly reduction of operational partial factors, in EC 2 informative Annex A and its link to the reliability management requirements of structural resistance presented in EC 0 were explored. It is made clear that improved QC measures, achievable through upper differentiation of structures, are effective in managing reliability. The result is possible set of partial safety factors applicable at differentiated levels of quality control. In general, stricter control allows larger reduction of partial factors from operational set in Eurocodes of 1.5 and 1.15. Guidance on how to adjust these values, through a differentiated framework, is given in Chapter 3.

Initial motivation was given in the first part of the thesis to investigate model uncertainty and consequently, the reliability performance, for different modes of resistance. The EC 2 shear design method is derived from the variable strut inclination method for shear which is based on lower bound theory of plasticity. The neglect of concrete contribution to shear resistance, a very important factor when the steel contribution is low, by the variable strut inclination method is responsible for the conservative predictions of shear resistance when model is used to predict shear in members that are slightly shear reinforced. On the other hand, the slightly unconservative results for highly reinforced concrete beams are due to the assumption that the EC 2 procedure that the angle of the concrete struts can be as low as $\cot\theta = 2.5$; meanwhile for highly reinforced beams $\cot\theta$ may only reach values around $1.10 - 1.30$ according to models based on equilibrium and compatability (Cladera and Mari, 2007). This gives rise to modelling uncertainty associated with the prediction model, greater than that associated with other modes of structural resistance. This proved a ripe issue for reliability investigation of the shear performance of members with stirrups.

First, in Chapter 5 a detailed background of the conceptual and theoretical development of the EC 2 and MCFT prediction models for shear were given followed by the background to the techniques required to conduct a reliability assessment of EC 2's design procedure for members requiring design shear reinforcement presented in Chapter 6. The MCFT was considered in this thesis, not as a design method, but as a rational-scientific model that better predicts the true shear resistance of members with stirrups than the EC 2 and other prediction models. This motivated its use as the general probabilistic model describing true shear resistance during the exercise of reliability modelling. However, it is more common to convert the design prediction model for use as the gpm in reliability modelling and this was done first, before the use of the MCFT as the gpm, in Chapter 8.

From Chapter 7, the model factor (MF) associated with the MCFT was characterised by a lower mean bias of 1.14, as opposed to EC 2's value of 1.65, as well as a lower spread or standard deviation of 0.20, as opposed to EC 2's value of 0.51. The MCFT was used in a separate reliability assessment in order to reflect the effect that the use of different predictions would have on the results of the process, mainly the achieved β-estimate and the sensitivity associated with the model factor, MF.

Chapter 8 was concerned with investigating the reliability performance of members with stirrups in shear. Two design situations, referred to as test cases, were taken as central to the reliability investigation. Based on the MF trend with the amount of shear reinforcement for the EC 2 prediction model, test case 1 was representative of design situation with low amounts of shear reinforcement, $A_{sw}f_{yw}/b_w s \approx 0.45\, MPa$, whilst test case 2 was representative of a design situation with relatively high amounts of shear reinforcement. Chapter 8 can be viewed as important for assessing the adequacy of the design method particularly at high amounts of shear reinforcement where the MF was shown to be losing conservatism, implying that the EC 2 prediction model has the tendency to over-predict the strengths of members in such situations. The EC 2 model was found to be, on average, acceptable for the design of members subjected to shear and requiring design shear reinforcement. A summary of the results for the two test cases against SANS 10160-1 and EC 0 requirements is given in Table 10.1.

Table 10.1. Summary of the results of the reliability analysis

Test case No.	**Amount of shear reinforcement, $A_{sw}f_{yw}/b_w s$**	**β-estimate**	
		MCFT as gpm	**EC 2 as gpm**
Test case 1	low: 0.45 MPa	5.308	3.379
Test Case 2	high: 1.8 MPa	2.876	2.485
Reliability requirements for resistance: SANS 10160-1: $\beta_{T,R}$= 2.4 EC 0: $\beta_{T,R}$= 3.04			

The results presented in Table 10.1 show that the calibrated design method for shear in EC 2 achieved acceptable reliability performance according to SANS 10160-1 and marginally to EC 0 requirements, particularly for test case 2; the design situation representative of a member with high amounts of shear reinforcement. Adequate performance is particularly essential at high amounts of shear reinforcement where the MF is found systematically to be marginally conservative-to-unconservative as reported in Chapter 7. By observing Table 10.1, practical validity of EC 2's shear design method for members with stirrups is ensured provided EC 0's recommended partial factors, $\gamma_C = 1.5$ and $\gamma_S = 1.15$, and appropriate characteristic values of the basic variables are used in design.

The β-estimates obtained for test case 2 were, in both cases when the EC 2 prediction model was used as the gpm as well as in the case when MCFT was adopted, found to fall below the threshold value of β of 3.04 as stipulated in EC 0. When the EC 2 prediction model was used as the gpm β was estimated as 2.485, and when the MCFT was used as gpm, β was estimated as 2.876. The severity of this marginal insufficiency of β for test case 2 could possibly be of serious concern. Traditionally, members are designed to fail in flexure; a ductile response that provides enough warning about impending failure. When low amounts of shear reinforcement are provided in design, it could be assumed that the shear stresses in the member are not found to be critical compared to flexure. In this situation, shear reinforcement is still provided in design so as to limit shear stresses to allow the member to achieve full flexural capacity and not to suffer any major shear stresses. On the other hand, when large amounts of shear reinforcement are provided in design, shear stresses are usually found to be high and adequate shear reinforcement must be provided in design to resist the shear-moment interaction of which the influence of shear would dominate. Shear failures are brittle and are therefore extremely undesirable occurrences as no safe evacuation time of the structure is allowed before failure. The reliability requirements for such situations should therefore be strictly adhered to, with even some slight excceedance of minimum reliability requirements desirable. It is therefore suggested that some upper limit be placed to the amount of shear reinforcement to which the EC 2 design method is applied in Europe or a different model of justified acceptable performance be applied in practice when designing in situations or members requiring high amounts of shear reinforcement.

Further, Table 10.1 illustrates that the results of the reliability assessment are dependent on the choice of probabilistic model chosen to represent true shear resistance. It was comforting to find that more severe β-estimates were found for the more commonly applied case in practice where the design prediction model was converted for use as the gpm, as compared to when the more rational scientific MCFT was adopted for use as the gpm. The Model Factor, MF, was found to dominate the shear reliability performance for members with stirrups, regardless of whether the MCFT or EC 2 prediction model were used as the gpm in the reliability assessment. That is, accurate modelling of shear, or lack thereof, poses the biggest uncertainty about the safety performance of design provisions and consequently structures as well. On this note, more accurate modelling of shear is called for and considered still an open area for research.

10.2 RECOMMENDATIONS

The recommendations for the thesis are discussed below as separate bulleted arguments.

1. The data collection from standard tests that characterise the occurrence or realisations of the basic variables that are commonly adopted in the design process such as strength, cover, construction tolerances and so on, is encouraged for South African conditions and practice. Quite a wealth of European-based references were used in describing the theoretical distributions of the basic variables used in reliability modelling in this thesis due lack of availability of South African databases concerning basic variables commonly used in design.

2. Templates for the advancement of reliability in structural design are pretty well documented in recognised publications as the JCSS Probabilistic Model Code, CEB Bulletins, and the Model Codes but with not very convincing, or perhaps lack of transparent, implementation of reliability in creating the design provisions for national codes of practice especially concerning resistance. The implementation of the principles of reliability is therefore an open and on-going process. More effort is required to fully

integrate reliability techniques in design provisions for structural concrete and structures in general. Ways to advance reliability implementation must be continuously identified and advanced. Some improvements can already be seen in some of the stipulations presented in the Draft 2010 *fib* Model Code compared to its predecessor. Some of these include:

a. More defined performance based concepts for design
b. More refined service life concepts centred around maintenance to increase reliable service of structures during use
c. Levels of design for shear, Level I to Level IV, that vary increasingly in computation time and effort, as well as accuracy could, in future, possibly be applied through a differentiation framework to implement as design provisions, or at least assess the viability for such action.
d. Reliability framework was introduced mostly in CEB-FIP 1978 Model Code and its aspects of reliability are increasingly being used as basis for current design recommendations as is done in the Draft *fib* 2010 Model Code.

3. A Limited parametric study, using two test cases each being reprentative of design situation with either low or high amount of shear reinforcement was conducted in the thesis. There is still need for a more extensive and general study of design situations encountered in practice, not only to assess and achieve adequate safety, but to also reduce over conservatism so as to promote economic designs.

4. Theoretically, partial factors are applicable to all basic variables considered in a reliability assessment but only the conventional set of γ_C and γ_S for concrete and steel, respectively, are applied in practice. A single set of partial factorsγ_C and γ_S are applied across wide range of structural concepts, design situations, etc. Hence, extensive calibration of the partial factors is necessary to include the effects of the other variables into the operational set of γ_C and γ_S applied in standards or codes of practice. This should not only be done for shear, but for all other modes of structural resistance. Holický et al. (2010) conduct a reliability based assessment of partial factor requirements for slabs (flexure) and columns (compression) according to South African reliability

requirements and as part of their conclusions, state that $\gamma_S = 1.10$ and $\gamma_C = 1.4$ are sufficient for the modes of resistance considered, including effects of modelling uncertainty and geometry across the operational range of longitudinal tension reinforcement, ρ, provided in design. There is therefore adequate guidance and need for similar assessment to calibrate EC 2's model for shear in accordance with South African reliability requirements. This thesis can be viewed as the initial steps in calibrating the EC 2 design model for members with stirrups to comply with South African conditions and requirements.

5. Taerwe (1993) states that special calibration of the model uncertainty as part of the global resistance factor is warranted for coefficients of variation of 0.2 and above. A coefficient of variation of 0.31 was reported in Chapter 7 to be associated with the EC 2 prediction model for members with stirrups. According to Taerwe, there is, therefore, a need to explicitly consider the effect of model uncertainties when calibrating the partial factor shear.

6. This Thesis showcases the systematic treatment of model uncertainties for structural design and can be used as future reference for efforts in treating other modes of resistance in this way, where model uncertainties are investigated and modelled in reliability assessments. The steps followed in treating the model uncertainty for shear could also be useful in characterising model uncertainties associated with new materials as well.

7. This is perhaps not a critical issue, but the sensitivity of members with relatively high amounts of shear reinforcement tend to show distinct downward trend of results after a/d of 4.0 as presented in the latter parts of Chapter 8. No direct reason was quoted for this finding as it was not the objective of the thesis to investigate the trend. It would, however, be interesting to relate trend to the conceptual development of the MCFT and understand why this is so.

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