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Modeling and measurement of geometric error of machine tools

Methodology and implementation

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Abstract

One of the main performance criteria for a machine tool is its ability to manufacture dimensionally and geometrically accurate parts. In this context evaluation of the geometric and kinematic accuracy of machine tools is important for achieving a high accuracy of machine parts. Moreover, the potential of predicting the accuracy of the machine tool outcome would benefit by reducing the non-value adding operations of machined parts measurement.

The aim of this thesis is to develop a methodology for modeling geometric errors of machine tools in order to evaluate the geometric and kinematic accuracy and estimate the machined part accuracy by predicting the error motion of the machine tool for a given toolpath.

The thesis consists of the description of the methodology that includes and explains aspects necessary for the development of the machine geometric error model. Additionally, a laser interferometer measurement process of the geometric errors and the various parameters necessary for the model development is presented.

A three axis machine tool is utilized in order to investigate and analyze the model and to measure the geometric errors. An analysis is made for evaluating the accuracy of the machine tool. Finally, a computational implementation of the model is made and two simple toolpaths are generated in order to demonstrate the potential of the model in predicting the geometric errors of the machine.

Sammandrag

En av de viktigaste prestationskriterierna för en verktygsmaskin är dess förmåga att tillverka dimensionellt och geometriskt korrekta delarna. I detta sammanhang är utvärderingen av den geometriska och kinematiska noggrannhet av maskinverktyg är viktigt för att uppnå en hög noggrannhet av maskindelar. Dessutom skulle möjligheten att förutsäga noggrannheten av verktygsmaskiner resultatet gynnas genom att minska icke värdeskapande verksamhet bearbetade delar mätning.

Syftet med denna avhandling är att utveckla en metod för att modellera geometriska fel av verktygsmaskiner för att utvärdera den geometriska och kinematiska noggrannhet och uppskatta den bearbetade komponentens noggrannheten genom att förutsäga felet verktygmaskinens rörelse för en given verktygsbana.

Avhandlingen beskriver en metod som omfattar och förklarar de aspekter som är nödvändiga för utveckling av en geometrisk felmodell för verktygsmaskiner. Dessutom presenteras en mätprocess som, använder laserinterferometer, för identifiering av de geometriska fel och parametrar som är nödvändiga för att utveckla modellen.

En tre-axlig verktygsmaskin används för att undersöka och analysera modellen samt för att mäta dess geometriska fel. En analys görs för att utvärdera verktygsmaskinens noggranhet. Slutligen är denna felmodell beräknad och två enkla verktygsbanor används i syfte att visa modellens potential att förutsäga geometriska fel i maskinen.

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Chapter 1.
Introduction

For the manufacturing industry there has always been an increasing demand for higher dimensional and geometrical accuracy and surface finish of manufactured parts. Among the industries that require tighter tolerances to be achieved, typical examples constitute the automotive, aerospace and medical devices industries, including manufacturing trends like automatic assembly and micro manufacturing. In this regard, realization of the desired part accuracy imposes the criteria in the selection of the manufacturing systems e.g. machine tools.

Part accuracy is closely related to machining system's capability, which in turn is determined by the interaction between machine tool and cutting process. Machine tool capability is commonly reflected on the tool-workpiece interface in terms of kinematic/positioning, thermal, static and dynamic accuracy.

To address the challenge of increasing the performance and improve the capability of machine tools, machine tool builders have introduced new optimized design techniques, use of exotic materials and employed – design and manufacture for precision – philosophy. However the manufacturing costs involved in such an endeavor are large and often the changes machine tools are subjected to (e.g. deformations due to cutting forces or thermal loads) cannot be fully accounted for during the design stage.

Another way of increasing the performance is to continuously improve and develop new identification and evaluation methods of the machine tool's capability [2]. Over the years the fields of machine tool metrology[1] and machine tool testing have gained increased importance for the industry. A wide variety of metrological equipment and test methods have been available for evaluating the performance and capability of machine tools. International test standards (e.g. ISO 230 series) purposed to cover all aspects of machine testing, have managed to raise awareness and meet the requirements to a degree. Testing standards are essential if machine capability is to address the needs of both the machine tool builders and the end users. To meet these needs, the standards are, to a certain extent, a compromise in the sense of economic and quality aspects. Economic considerations mean that machine

[1] Machine tool metrology can be defined as the activity relating to the discipline of measuring the performance characteristics such as geometric errors of a machine tool under static and dynamic conditions [55]

non-productive time must be kept to a minimum, while quality considerations mean that the calibration should be detailed enough to provide meaningful results. To find the optimum between the two factors is a crucial issue when developing or improving test methods.

In this context a robust testing methodology and tools that could be used for successfully evaluating the capability of a machining system to produce parts right the first time are still required. Apart from accuracy issues, the methodology should also be able to address constraints imposed by the industrial environment such as speed and reliability of the methodology, ease of use and economical limitations.

In the development of machining strategies and process plans the advantages of such a framework, would provide the ability of selecting the appropriate machining system according to the desired specifications [54]. In this context it would be of great economic interest the prediction of the resulting accuracy of the machined part prior to the start of production [3]. At the manufacturing level, it should provide the potential of analyzing and controlling the accuracy loss and act as an evaluation aid for machine maintenance. Finally, for machine tool testing it should enable both end-users and machine tool builders to assess the capability of the design and performance of the machine tool.

Such a methodology should take into account various measurement parameters affecting the measurement outcome and be able to get rid of dependencies and redundancies. Moreover, it should be flexible and adaptable for future needs such as estimating the volumetric accuracy of the machine tool as well as be able to assess the capability of the machine tool from a static and dynamic point of view.

1.2 Purpose and motivation

The aim of this thesis is the development of a robust methodology for modeling and measuring the geometric and kinematic errors of machine tools that would fit industrial needs. As part accuracy is interlinked to the accuracy and precision of the machine tool, by controlling or predicting the accuracy of the output of the machine tool will enable to control and predict the accuracy of the machined part. In that way we reduce the need to measure the accuracy of the parts frequently, reducing the non-value adding operations

The purpose is to develop a geometric and kinematic error model that would describe the behavior of the machine tool. Additionally, it focuses in describing a consistent way for measuring geometric errors and documenting the measurement parameters. As an essential part of the modeling and measurement process, emphasis is given in defining the measurement point in which errors should be measured for each single axis. The ultimate goal of the modeling methodology is:

- To be applicable to a variety of machine tools of different kinematic configurations and different number of axes.
- Establish a well-documented measurement practice describing the measurement point in a consistent way.
- Predict kinematic errors of the machine tool given a toolpath

Ultimately, the aim is to establish an understanding of the interlink between the accuracy levels of the machined parts and the source of errors in the machine tool by correlating the geometrical deviations of the part to the predicted accuracy of the machine tool.

1.3 Delimitations

A machining system's capability is dependent on the interaction between machine tool, process and control system. As machine tool is a core part of a machining system, the focus in this thesis is in evaluating and predicting machine tool accuracy.

As part of the measurement and modeling methodology, certain error effects were not measured and thus not included in the modeling approach. As such, thermal induced errors, errors due to static or dynamic effects and multi-axis motion control errors were not considered nor measured. Similarly, dynamic motion characteristics (e.g. acceleration and deceleration) of the slides and guide-ways were not accounted for.

During measurement of the individual error components, all axes not participating in the measurement process were maintained in a fixed documented position, and the measured errors were considered to be independent from the other axes error effects. Additionally, due to lack of proper measurement instrumentation (digital level) the roll errors of two of the three axes could not be measured.

1.4 Outline of the thesis

The work presented in this thesis is mainly focused on developing a geometric error model for a machine tool and communicate the importance of a robust method for measurement of geometric errors that affect the accuracy of machine tools.

In chapter 1 an introduction of the thesis with its scope and delimitations is presented.

Chapter 2 introduces the concepts of machining system and machine tool and the various sources of error that affect their accuracy. Also the concept of machine tool kinematic structure representation is explained and described and the parameters of machine tool geometric errors are presented.

Chapter 3 contains the modeling methodology along with a detailed analysis of the approach. Also, the model developed for a three axis machine tool based on this method is described.

Chapter 4 presents the measurement process approach together with considerations and results.

Chapter 5 introduces the results obtained by the measurements and the computational model developed in chapter 3 and a discussion on the outcome of the model.

Chapter 6 draws conclusions and suggests future work that can be carried out, to further enrich the computational model.

Chapter 2.
State of the art

Manufacturing of high accuracy parts sets the basic requirement for a robust machining system, capable to manufacture to the specified tolerances. Although, a controlled and repeatable process is important for maintaining quality, machining systems' accuracy and precision has a determinant role in the part's accuracy.

The development of more accurate machining systems based on new and optimized design methods, contributed in the pursuit of higher accuracy. However, due to the nature of the error sources and the varying industrial environment, constant effort for improvement have driven the concept of machine tool error modeling, for calibration, error prediction and compensation purposes, as a cost effective way of realizing better performance.

2.1 Machining system and machine tool

A machining system can be defined as a system consisting of several physical modules (e.g. spindle, ballscrews, slides) and processes (e.g. milling and turning) connected to each other. Each module of the system is essential in order to manipulate and eventually produce the desired shapes and features in parts made of metal or other materials [4].These can be categorized as:

- Machine tool
- Cutting tool
- Workpiece
- Fixture
- Cutting process

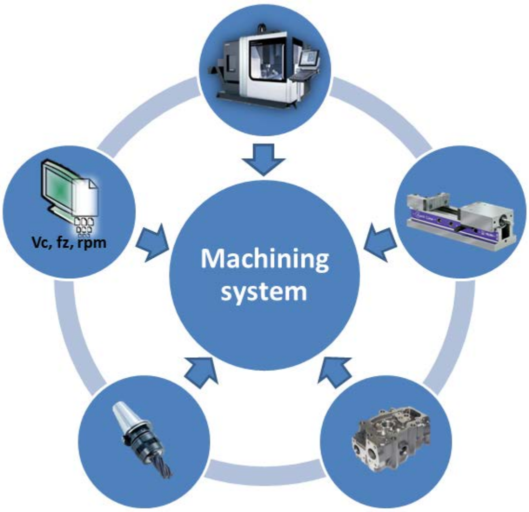


Figure 1. Various modules that constitute a machining system. Starting from top in the clockwise direction they are: machine tool, fixture, workpiece, cutting tool and cutting process.

Every module may be comprised of other parts or sub-systems, each having certain attributes that might specify and affect the outcome during their interaction. Commonly, the main module of a machining system is the machine tool, which acts as the connecting link to the other modules. As one of the fundaments of a machining system, machine tool capability has a determinant role in the ability of the machining system to produce accurate parts within the specified tolerances. As stated before, machine tool precision and accuracy is determined by the relative deviation between the cutting tool and the work piece.

2.2 Literature review

One of the early approaches in machine tool modeling by Leete [5] presented a modeling method in terms of trigonometric equations. Hocken et al. [6] attempted to numerically compensate a three axis measuring machine using an error matrix method of stored error measured values over the range of the work space. Since then, various researchers have developed modeling methods for predicting, calibrating and compensating numerically the errors of machine tools.

Ferreira and Liu [7] developed an analytical quadratic model for the prediction of geometric errors of a machine using rigid body kinematics. The objective was to develop a reference-part based error model to identify the parameters of positioning error of a machine with minimal measurement. Thereby, error measurements obtained either from a workpiece or by a metrology pallet and touch trigger probes were used for estimating the coefficients of the model. Liu and Mou [8]describe a similar approach with quadratic and cubic error models where they investigated the complexity of both models.

Donmez et al. [9] presented a general modeling methodology for prediction and compensation of geometric and thermally induced errors. The model was relating the deviation in the relative position between the cutting tool and the workpiece to the errors of the components of the machine tool structure. Generation of the model was based on the principles of rigid body kinematics and homogeneous transformation matrices (HTM) to describe the spatial relationship between adjacent machine tool components. Measurements have been carried out using laser interferometer and least squares curve fitting was employed to generate models for the parameters of the error components.

Kiridena and Ferreira [10] used the Denavit–Hartenberg (D-H) method to construct a direct kinematic model that showed the effects of the links position independent geometric errors on the accuracy of the machine. Also, in a series of papers model and identify the systematic errors for 3 axis machine tools. Using n-dimensional polynomials they model the error parameters to be identified by measuring the positions of 27 rectangular posts in the machine tool workspace. In a similar approach Kreng et al. [11] designed a metrology pallet to update coefficients of the error model periodically.

Other researchers like Hai et al. [12] and Phank et al. [13] relying on rigid body kinematics principle and HTM's used polynomial functions to model and represent the geometric errors of a three-axis machine tool axis and employed telescoping ball-bar measurement to identify the model coefficients. Lin and Shen [14] proposed a matrix summation approach to simplify the homogeneous matrix transform approach into modeling the geometric errors of five axis machine tools . Mayer et al. [15] demonstrated an approach of decomposing the geometric errors of a five axis machine tool into two categories of independent from the speed of the trajectory geometric errors and the dynamic geometric errors. The method relied on a Jacobian matrix to express the pose of the links. A non-contact in-house developed instrument called CapBall, was used to record the actual tool center point position and compare it with the machine encoder signals to evaluate the total geometric error.

Admittedly, the most widely used modeling approach is based on the rigid body kinematics, in which HTM or D-H transformation matrix is used to represent coordinate transformation between rigid body frames. In addition, numerous mathematical approaches for describing the measured component error, recorded by different measurement methods can be used like e.g. n^{th} order standard or piece-wise polynomial fitting, Legendre polynomials, B-splines or statistical methods.

2.2.1 Direct and indirect measurement

Performing an analysis of the geometric and kinematic errors of a machine and error prediction, the relevant parameters need to be determined and measured. Depending on the machine geometry and the purpose of the modeling process, suitable methods can be utilized. Schwenke et al. [16] reported in a comprehensive overview, methods and techniques available at the time, for machine tool error measurement and identification. They were distinguished into direct and indirect geometric error measurement methods, according to the provided data of different aspects of the machine tool.

"Direct" measurement allows the measurement of single error motions for a single machine component at a time, without any contribution from other axes or components. A classification based on the metrological reference or the principle way of carrying a measurement can be done, identifying material-based methods that use artifacts (e.g. straightedges, step gauges, calibrated scale systems and artifacts), laser-based methods that use a laser beam as a reference (e.g. laser interferometer, autocollimator) and gravity-based methods (e.g. inclinometers, spirit levels etc.). Of the most favorable and widely adopted measuring method is laser interferometer which combined with special optics is suitable for measuring almost every geometric error of a linear axis, with the exception of roll errors (rotation around the axis of motion) that still rely on electronic levels. The high precision and long-coherence length of the laser beam (~20cm for He-Ne laser) allows to be employed in measuring a multitude of short and long axis machines. However, great care has to be taken on environmental or other sources, as any errors that influence the wavelength or frequency of the laser beam can be transferred into errors in the measurement.

"Indirect" methods detect superposed errors associated with simultaneous motion of two or more machine axes. They provide a quick way for assessing the positioning capability and contour accuracy of a machine. As with direct methods a variety of measurement techniques are available. Artifacts based methods use totally or partially uncalibrated artifacts, or calibrated artifacts. Contour measurement methods require simultaneous movement of at least two axes in straight lines or a circular path (e.g. Ballbar, Grid encoder). Equipment from direct methods can be employed to measure in different positions in the work volume of the machine. Spherical interpolation method uses simultaneous linear and rotational movements of the machine axes to maintain the nominal position between the tool and workpiece and measure the relative movements caused by errors in one, two or three coordinates. Typical example of such measurement is the R-Test for measurement of rotational axes. Description and guidelines about various geometric test procedures, either direct or indirect can be found in (Ref ISO203-1)

Classification of measurement methods into direct and indirect can also be done according to the immediacy of the application of the measurement method to the machine tool or object of interest. In this aspect “direct“ methods account for all error measurements methods applied directly on the machine tool, and provide measurement data involving on or more axes of motion. Direct methods can be further distinguished into operational and off-operational depending on the number of axes involved in the measurement process and their cumulative contribution. In this aspect measurement of a single axis constitutes off-operational direct measurement method as it does not represent combined motion of different machine tool components.

“Indirect” methods on the other hand, obtain error measurement results by measuring the output of the system on an interim object. Measurement results provide data for making inferences about the state of the machine and the attributes or errors of interest. A typical example of an indirect measurement according to this interpretation is manufacturing of a test artifact which links part errors to the machine errors enabling the assessment of the machine tool accuracy.

Understanding of the geometric error sources and their effects in machine tools, as well as the effect of the machine tool design (kinematic buildup, utilized components etc.) is necessary. It facilitates the analysis of the effects of error sources, and identification of the modeling parameters. Furthermore, machine tool modeling for accurate simulation requires measurement of the real machining system.

Direct measurement methods, especially laser interferometer, provide high precision in measuring and evaluating the accuracy of machines. Also, since they are focusing on one error component at a time, they allow for better identification of single errors. Due to difficulty and time-demand, along with the requirement for experienced engineers, indirect methods are often preferred as easier and faster to perform. Yet, identifying the different errors is not always feasible, as the measurement result is influenced by the combined effect of various error sources.

In the following sections the topics of geometric error sources, description of the geometric errors and modeling approach are presented.

2.3 Error sources in machine tools

Machine tool performance and accuracy is affected by various error sources that influence the geometry of the components composing the structural loop of the machine, causing distortions in the accuracy of the machined parts. Due to the complex structures of the machine tools, the total accuracy is a result of the interaction of the different components and their deviations. The main error sources that influence accuracy can be categorized in the following parts [16]:

- Kinematic/Geometric errors
- Thermal effects
- Dynamic effects
- Static load effects
- Motion control effects

Furthermore, machine tool errors can be divided into systematic errors (accuracy) and random (precision) errors [22]. Systematic errors can be described and measured, enabling the modeling and prediction, while random errors are hard to predict.

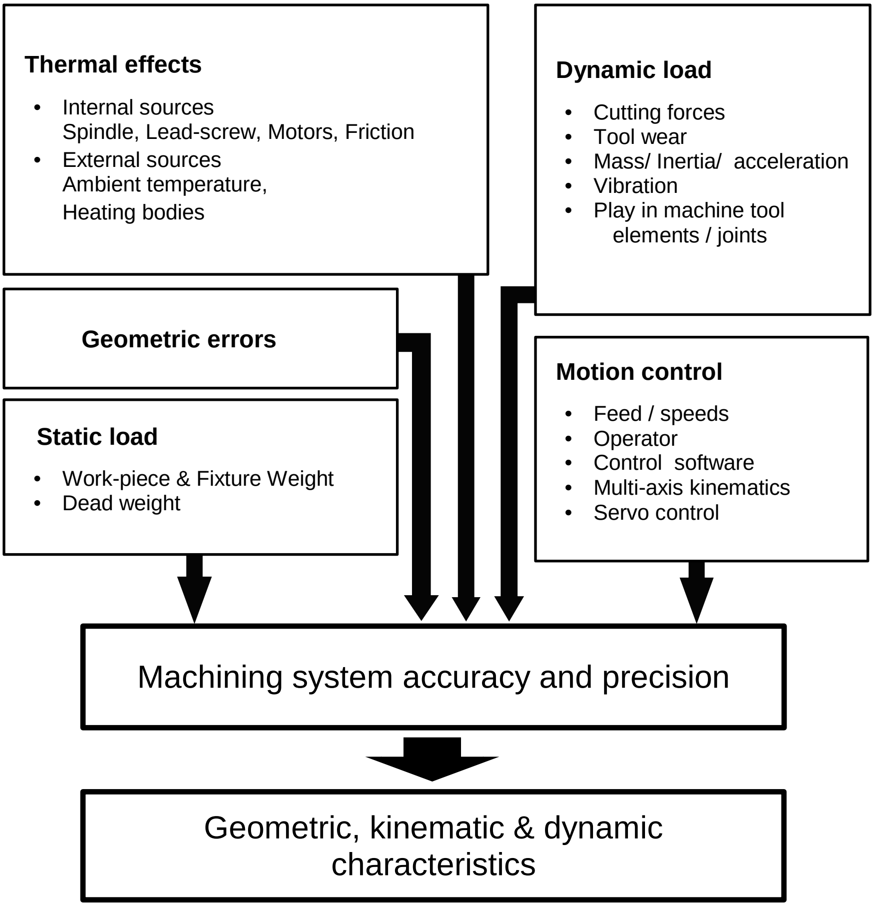


Figure 2 Description of main error sources that affect machining system accuracy

2.3.1 Geometric and Kinematic errors

Geometric errors are the errors due to the imperfect geometry of the machine tool guide-ways and other structural components, such as machine bed, bearings, carriages, lead-screws, rams etc. and their misalignment in the machine structural configuration [9], [19],[23], [24], [25], [26], [27], [28]. They are affected by thermal and dynamic loading and vary slowly with time exhibiting good repeatability. The systematic character of these errors can be changed due to time-to-time collisions, as well as wear of the components, resulting in a change of the geometry of the machine [28]. Geometric errors are difficult and sometimes impossible to be measured individually due to accessibility issues. However, they can be determined by the influence on the kinematic structure of the machine and the resulting error motion of the axes. Usually they are measured by laser interferometry.

Kinematic or motion errors are closely related to geometric errors of the structural components of the machine tool, and can be partially considered as a resulting effect of the geometric errors during the co-ordinate movement of the functional components. As such, motion errors are functions of at least the position of the carrying axis and occur mainly during the execution of interpolation algorithms.

2.3.2 Thermal effects

Thermal effects are among the major inaccuracy contributions in a machining system. Heat can be introduced internally, generated by moving components of the machine tool such as spindle, transmission motors, bearings, lead-screw nuts and by the cutting process. External sources of heat also create temperature gradients that affect the various parts of the system. The temperature gradients induce elastic strains to the system’s structural loop due to the different expansion coefficients of the components, creating distortions that influence their geometry and produce errors in the relative position of the tool tip with respect to the work-piece [21], [22], [29], [38], [30]. Obviously, every machining system is sensitive to different thermal effects, thus making it important to have a clear view of which are the major heat sources and the contribution each of them has on the geometric errors. The total effect on the accuracy of a machine tool due to thermal influence on modules in the machining system may be determined by measurement of the geometric and kinematic behavior [31].

2.3.3 Dynamic effects

The structural loop of a machine tool is usually subject to dynamic effects that influence its dynamic behavior, producing distortions in the tool path trajectory to be realized. Such effects originate from varying forces such as rapidly changing cutting forces during the machining process, inertial forces caused by acceleration or deceleration of axes as well as vibrations induced distortions, due to uneven dynamic characteristics of the structural elements or tool wear [32], [33]. Since the dynamic stiffness of the system's structural loop changes under different machining conditions due to the varying nature of these effects, deformations arising during machining are hard to predict and compensated.

2.3.4 Static load effects

Accuracy of a machining system is affected by quasi-static forces due to limited static stiffness of the structural loop. Deformations are caused by the structures own weight and by the movement of the carriages. Gravity forces introduce loads due to the mass of the workpiece and the fixture that change according to their relative position within the work-envelope. Also, stresses are produced by the assembly forces of the systems components influencing the structures geometry and accuracy. The systematic nature of most of these effects allows the measurement and correction during the design stage, or compensate through software modules.

2.3.5 Multi-axis positioning and control system

The control system that commands the servo drives and ball-screw drives used to position the axes in the programmed position can significantly affect the accuracy of the machining system. Feedback from sensors and encoders providing control of feeds and accelerations, as well as the measuring system that provides feedback of the position of the axes, can introduce errors. In addition to that, the NC program that sends the commands to the controller and the operator can introduce errors to the accuracy of the system.

2.4 Kinematic structure and representation of machine tools

For the purposes of kinematic analysis of machine tools, a valid description of the kinematic structure is required.

Provided a certain level of abstraction the elements of a mechanism can be described as links or rigid bodies connected by joints. A joint reduces the possibilities of movement of two links, defining the motion constraints between the successive rigid bodies according to its type (prismatic or revolute). The representation of a joint (kinematic restriction) between two adjacent links is called kinematic pair and defines the geometric aspect of the motion constraints. Links and joints can be connected in series or parallel, creating mechanical kinematic chains incorporating all the elements of the mechanism. A machine tool can be seen as a mechanical kinematic chain between the tool and the workpiece.

For multi-axis machine tools a serial layout is common, where two kinematic chains between the workpiece and the tool can be identified. The type of a machine's kinematic structure and number and type of axes (linear or rotary) determine the errors presented in a kinematic model of the machine tool.

In a typical three-axis serial kinematics machine tool, the three axes are arranged orthogonal to each other, creating two kinematic chains. In Figure 3 the kinematic geometry and topology of a three-axis machine tool showing the two chains and a model of the simple kinematic structure of the same machine is illustrated. The kinematic chain can be described by an ordered list of the axes, workpiece (w), bed (b), and tool (t) as defined in ISO/DIS 10791-1 (2012). For the machine in Figure 3 the description of the kinematic chain is [t (C) Z Y b X' w]. The prime on the X axis means that the positive direction is defined opposite to the right hand rule convention. The C axis in the parenthesis represents the spindle, which can be omitted from potential calculations due to its high rotation speed compared to the speed of the machine's axes [40].

During analysis of a mechanism, simplifications in the model are common. Of the most common assumptions is that a body or link is completely rigid neglecting any deviations in its shape caused under stress [34]. The spatial relationship and the coordinate transformation between two neighboring rigid body frames can be described by an HTM.

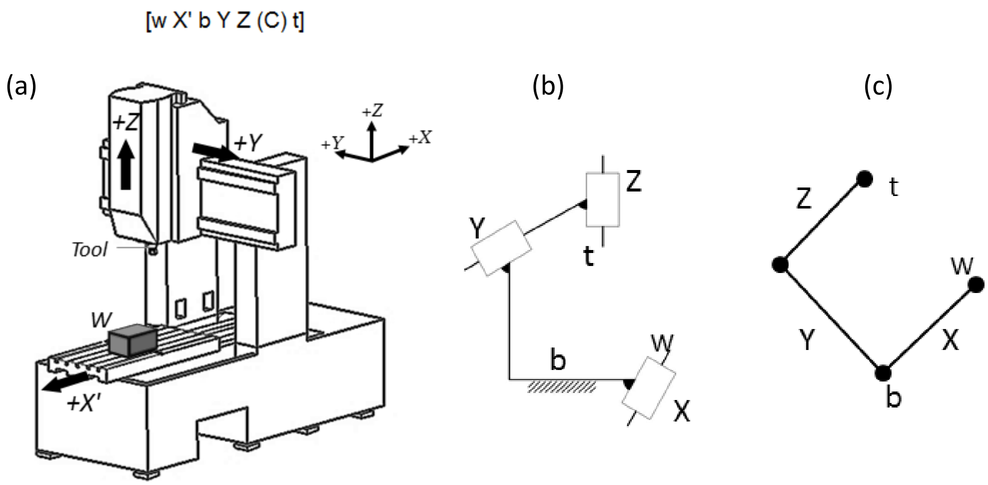


Figure 3. In (a) the kinematic structure of a 3-axis vertical machine tool, with kinematic chain description [wX'bYZ(C)t] is illustrated. In (b) and (c) a schematic representation of the geometry and topology of the same machine is presented. The (C) axis (spindle) of the kinematic chain appearing in (a) is not included as an axis, due to its high rotation speed. In a similar manner a rotary table of a 5-axis machine would have been considered in the geometry and topoligy as its rotation speed is lower.

A homogeneous transformation matrix in three dimensional space is a 4x4 matrix that is used to represent one coordinate frame with respect to another or to a reference frame. Similarly it can be used to express a coordinate vector in one coordinate frame with respect to another. For example, describing the transformation of a position vector $\boldsymbol{p}$ in position and orientation between the i^{th} coordinate system with respect to (i-1^{th}), can be expressed as

$$\begin{bmatrix} \mathbf{p}_{i-1} \\ \cdots \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & \\ 0 & 1 & 0 & \mathbf{r}_{i-1} \\ 0 & 0 & 1 & \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} & & & 0 \\ & \mathbf{R}_{i-1/i} & & 0 \\ & & & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \mathbf{p}_{i} \\ \cdots \\ 1 \end{bmatrix} \tag{1}$$

$$\begin{bmatrix} \mathbf{p}_{i-1} \\ \cdots \\ 1 \end{bmatrix} = \begin{bmatrix} & \mathbf{R}_{i-1/i} & & \mathbf{r}_{i-1} \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \mathbf{p}_{i} \\ \cdots \\ 1 \end{bmatrix} \tag{2}$$

where

$${}^{i-1}\mathbf{T}_i = \begin{bmatrix} \mathbf{R}_{i-1/i} & \mathbf{r}_i \\ \mathbf{0} & 1 \end{bmatrix} \tag{3}$$

is the HTM.

The position vector $\boldsymbol{r}_{i-1}$ locates the origin of frame i in the $(i$-$1)$ coordinate frame and the rotation matrix $\boldsymbol{R}_{i-1/i}$ incorporates the direction cosines, that describe the orientation of frame i with respect to coordinate frame (i-1). Thereby the transformation matrix is an augmentation of the rotation and translation between the two frames. An easier to understand form of the same matrix is

$$
{}^{i-1}\mathbf{T}_i = \left[\begin{array}{ccc:c} o_{1x} & o_{2x} & o_{3x} & p_x \\ o_{1y} & o_{2y} & o_{3y} & p_y \\ o_{1z} & o_{2z} & o_{3z} & p_z \\ \hdashline 0 & 0 & 0 & 1 \end{array}\right] \tag{4}
$$

where the vectors **O**$_1$, **O**$_2$ and **O**$_3$ are the orientation cosines (describe the orientation of a coordinate frame with respect to another) and $\boldsymbol{p}$ the position vector of the origin of the i^{th} frame within the $(i\text{-}1)^{th}$ frame.

In the general case of a mechanical chain of N rigid bodies connected in series as illustrated in Figure 4, at a nominal configuration (without taking into account any errors), the position and orientation of the end effector, can be expressed with respect to the reference frame by successive multiplication of the HTM's, starting at the end of the chain and moving towards the reference frame [22].

$$
{}^{R}\mathbf{T}_N = {}^{F}\mathbf{T}_1\, {}^{1}\mathbf{T}_2\, {}^{2}\mathbf{T}_3 \ldots {}^{i-1}\mathbf{T}_i \ldots {}^{N-1}\mathbf{T}_N \tag{5}
$$

$$
{}^{R}\mathbf{T}_N = \prod_{i=1}^{N} {}^{i-1}\mathbf{T}_i = {}^{F}\mathbf{T}_1\, {}^{1}\mathbf{T}_2\, {}^{2}\mathbf{T}_3 \ldots \tag{6}
$$

Machine tools constitute a special case of mechanical kinematic structures. Thereby, utilizing HTMs, the kinematic structure can be decomposed into a series of coordinate transformations, describing the spatial relationships of each of the structural elements of the machine. Under the assumption of rigid body kinematics and by assigning a coordinate frame to each machine axis, it is possible to describe its motion with respect to a reference coordinate system (axis or reference).

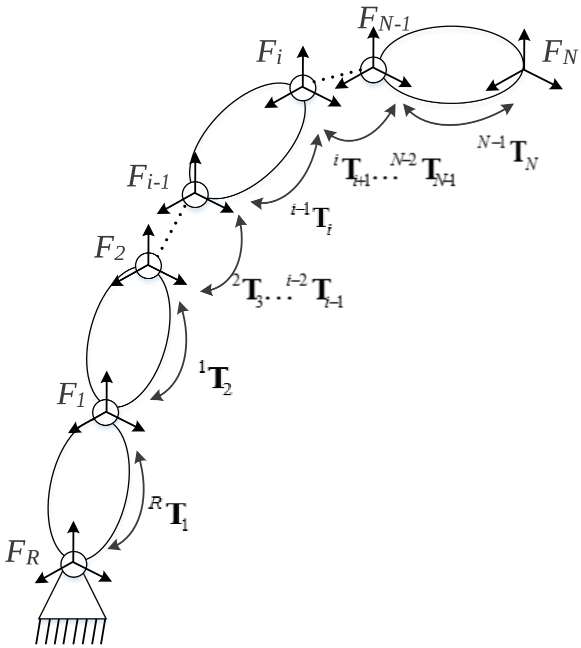


Figure 4. Schematic of *N*-DoF kinematic chain. Each frame F represents a joint (axis) assigned a coordinate frame. An HTM denoted as ${}^{i-1}T_i$ describes the spatial relationship between two consecutive frames. The kinematic structure can be decomposed into a series of transformations expressing the entire mechanism.

2.5 Geometric and Kinematic errors description

A rigid body has 6 degrees of freedom in space describing its position and orientation in a three-axis Cartesian coordinate system.

In the case of a machine tool, the motion of each linear axis is constrained by guide-ways in the direction specified by the joints in the kinematic structure, allowing for only one degree of freedom of nominal movement. So, ideally the description of the slide's position and movement with respect to the reference frame or preceding axis's frame could be expressed by pure translation affecting only the last column of the HTM.

For example, considering the motion of a linear X axis with respect to a reference frame and assuming there are no offsets between the reference and the axis coordinate system origin, the movement of the slide can be described by the transformation matrix

$$ {}^{R}\mathbf{T}_{X} = \left[\begin{array}{ccc:c} 1 & 0 & 0 & x \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \hdashline 0 & 0 & 0 & 1 \end{array} \right] \quad (7) $$

by changing only the coordinate that specifies the axis's position. Similarly, for Y and Z axes the coordinate transformation will express the relative motion of Y and Z with respect to the preceding frame (reference or carrying axis's).

However due to geometric imperfections, their movement and position can be described by six degrees of freedom in the form of geometric and kinematic errors.

The geometric errors of a linear axis refer to the error of each axis individually and the errors between axes. They are divided into component errors (geometry related) and location errors (kinematic). Component errors address deviations with six measure values that are dependent on axis of motion. For a linear axis, 3 linear and 3 rotation error components associated with its nominal movement are identified. Linear errors can be further analyzed into a linear positioning error and two straightness errors (vertical and horizontal) and rotation errors into two tilt errors motions (pitch and yaw) and roll error motion [16]. In Figure 5 an example of the six component errors of a horizontal linear carriage along its X axis nominal movement is illustrated. For a 3-axis machine tool there are in total 18 component errors and 3 location errors, as presented in Table 1.

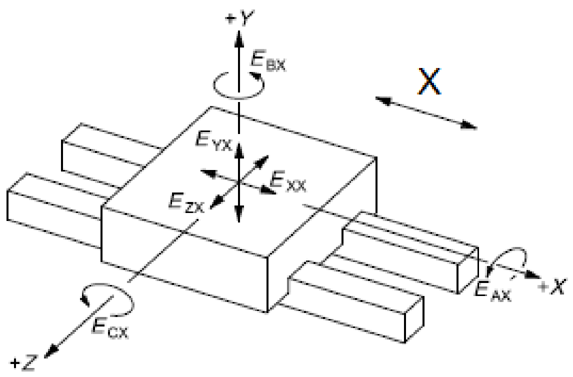


Figure 5. Component errors of a linear X axis [ISO 230-1 (2012)]

- E_{XX} (linear positioning error): can be defined as the translational error motion of a machines moving component along its axis of travel.
- E_{YX}, E_{ZX} (straightness errors): translational error of a machine component that can occur in either of the two directions orthogonal to the axis of motion
- E_{AX}, E_{BX}, E_{CX} (angular errors): rotation errors of the machine's moving components (slides) around the axis of movement (roll) or around the two axes orthogonal to the axis of nominal movement (pitch and yaw).

Table 1. Component and location errors of a three axis machine tool

AXIS	Linear Errors Positioning	Straightness		Rotation errors (Roll, Pitch, Yaw)		
X	E_{XX}	E_{YX}	E_{ZX}	E_{AX}	E_{BX}	E_{CX}
Y	E_{YY}	E_{XY}	E_{ZY}	E_{BY}	E_{AY}	E_{CY}
Z	E_{ZZ}	E_{XZ}	E_{YZ}	E_{CZ}	E_{BZ}	E_{AZ}
Location errors	E_{A0Z}, E_{C0Y}, E_{B0X}					

The naming convention is adopted from ISO 230-1 [39] and is based on a combination of three letters for describing the errors. The letter E comes from the word error, it is followed by the letter of the axis corresponding to the direction of motion and the second letter corresponds to the name of the axis of motion.

Component errors can be incorporated into an HTM, describing the effect of the errors on the carriage motion. For a linear axis, the HTM that describes the effects of the errors on the slides position with respect to the axis reference coordinate frame, and by assuming small angle approximation, can be expressed as:

$$\mathbf{E}_{error} = \begin{bmatrix} 1 & -E_C & E_B & E_X \\ E_C & 1 & -E_A & E_Y \\ -E_B & E_A & 1 & E_Z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (8)$$

The previous matrix represents the general form of the error matrix in terms of individual error components of a machine's linear axis. The last letter for each error component is omitted as the form of the error matrix in Eq.8 is universal independent of the axis name or direction of motion.

2.5.1 Location errors

Location errors address deviations in the position and orientation of an axis of motion with respect to the nominal position and orientation of the axis in the axis's or machine's reference frame. They are position independent as they are considered not to be affected by local disturbances. Simply put, location error can be defined as the average line of the axis of motion [34].

Orientation errors can be described as the relative orientation between the reference straight line associated with the motion trajectory of a linear axis and the reference straight line associated with the motion trajectory of another linear axis. Particularly, one can consider squareness as the property of two straight lines where the angle between the two is 90° and parallelism as the property of two straight lines that have the same angle of inclination to the abscissa of a common coordinate plane [ISO 230-1]. The error in squareness between two linear axes can be described as the deviation from the nominal 90° between the reference straight lines of the trajectory of a point on the two axes.

For a three-axis machine tool, three location errors can be identified for each axis of linear motion, two orientation and one position error, resulting in 9 location error parameters for the machine tool (without considering spindle errors).

However, all axes deviations from zero position (position errors) can be neglected or be set to zero by proper selection of reference coordinate frame, eliminating redundant measurement and facilitating the modeling process.

In a similar approach as with component errors, location errors can be represented by an HTM. In the following matrix (see Eq.9) an example of the squareness errors of X axis to Y and Z axes, of a three-axis machine tool are illustrated. The location errors regarding the zero position of an axis would be assigned in the last column of the matrix. In this case the position vector is being set equal to zero making the model formulation more understandable.

$$\mathbf{S} = \left[\begin{array}{ccc|c} 1 & -E_{C0X} & E_{B0Z} & 0 \\ E_{C0X} & 1 & 0 & 0 \\ -E_{B0Z} & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \tag{9}$$

The notation is also adopted by ISO 230-1 [39] where the first character after E (for error) corresponds to the direction of deviation, the second is the numeral zero and the last letter is the axis of concern.

2.5.2 Functional point

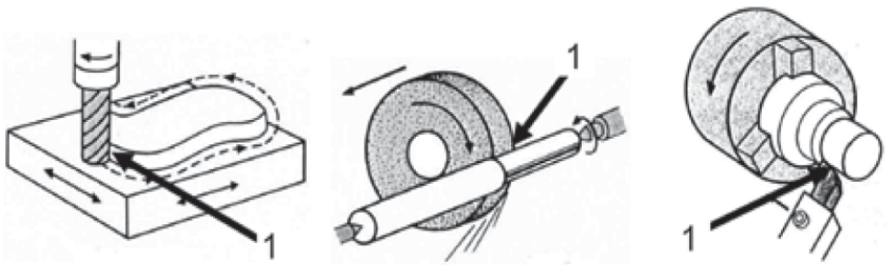
The *functional point* is defined in ISO 230-1 (2012) as “cutting tool center point or point associated with a component on the machine tool where cutting tool would contact the part for the purposes of material removal”. This is a single point that is rigidly attached to a machine tool component that can move within the machine tool’s work envelope (see Figure 6).

The error motion of the individual machine tool components and the effect of the component errors due to their configuration in the structural loop results in the actual relative position between the tool and the workpiece—the functional point. The error motion of components needs to be known at the trajectory of the functional point to enable prediction of its contribution to workpiece geometrical accuracy.

Although, measurement of linear errors can be done anywhere in the workspace, it is considered good practice that the measured point (MP) coincide with the functional point, as it relates the measured error motion directly to the geometric characteristics of the machine tool.

Additionally, the distance between the functional point and the measurement point has a magnifying effect to the uncertainty of the predicted accuracy, due a lever effect from the angular errors. Thus, by minimizing the distance between the two points, minimizes the contribution to the uncertainty.

Applying measurement setups that represent the cutting tool's trajectory allow the measured errors to include the contribution of angular error (roll, pitch and yaw) of the axes.



[ISO 230-1, 2012]

Figure 6. Examples of functional points (1) [ISO 230-1, 2012]

Chapter 3.
Geometric error modeling

In developing a machine tool kinematic model to describe the geometric and kinematic errors of the structural components, some aspects of the model need to be determined prior the analysis of the model. Machine tool kinematic structure and coordinate frames location need to be specified. This will facilitate the construction of the model, in describing the position and orientation of the axes of motion and relating the measured errors to the axis reference frame, as well as the measurement process. Furthermore, modeling of the component's parametric errors needs to be addressed and considerations or assumptions regarding the model explained.

In this thesis the machine tool utilized to investigate and analyze the model and to measure the component errors is a Mazak AJV 25-405 three-axis portal type vertical machine tool (Machining center ISO 10791-2 type V10 [42]). In the following sections a description of establishing a reference coordinate frame and axes coordinate frames, the HTM representation of the machine's structure as well as the general error model of the machine tool will be presented together with considerations and aspects deemed important during the modeling process.

3.1 Coordinate frames location

Assigning a global reference coordinate system and coordinate frames to the axes of the machine can be considered of the most important steps in building the machine tool model. Selecting the location of the coordinate frames should serve in defining the relative location and motion errors of the various components and facilitating the measurement process. In many studies on geometric and kinematic modeling [8], [10], [42] the reference coordinate frame with respect to which all the errors are calculated by the successive transformations, is attached to a fixed point on the bed or base structure of the machine or outside the machine work envelope using metrological frames [6]. Also, it is customary the coordinate frames of the machine elements to be located at the center of the rigid body (axis or carriage). A motivation for locating the reference coordinate system in the bed is that it offers a unique fixed and "tangible" point, reliable enough to use as origin of the global frame. That perspective is relied on the fact that machine tool base frame undertakes very slow changes over time, thus appear to be good method to position the global reference frame. Following a similar thinking, the coordinate frames of the various axes are attached to the machine's elements in line with the axis's scale or in the guideways; thus attempting to eliminate any rotation error effects due to offsets from the measuring scale during measurement.

However this approach proves to be problematic in some aspects. Several machine tool parameters need to be determined prior to modeling and error measurement (e.g. offsets between axes coordinate frames, location of global reference frame etc.). Restricted access to the bed, carriages and axes makes it difficult or infeasible to measure and usually time consuming and makes the model machine type dependent and demanding to evaluate.

For the purpose of the model developed in this thesis, the global reference coordinate system is chosen to be coincidental with the machine home position. It is a position easy to locate and understand. This approach allows the modeling methodology to be machine type independent as it can encompass different machine and mechanism structures and definition of the relative errors. Moreover it allows for easier alignment of the measurement devices, facilitating the measurement setup.

According to the proposed methodology in ISO 230-1 (2012), a reference coordinate system can be defined arbitrarily anywhere in the workspace best suited to the needs of the process. For the methodology developed in this study, the reference coordinate frame has been chosen to be located at the spindle tip, when the axes are set to home position. Alternatively, if it is deemed more appropriate the workpiece coordinate system can be defined as the global reference frame.

By utilizing the reference straight lines of the machine's axes of motion, the position and orientation of the coordinate frame can be specified. The primary axis is chosen to be Z axis, thus its reference straight line coincides with the Z axis of the reference coordinate frame. This way the two orientation parameters of squareness can be set to zero.

The Y axis is chosen as the secondary axis so as its reference straight line projection to the plane normal to the Z axis defines the Y axis of the reference frame. Finally, X axis and the origin of the reference coordinate system are defined following the right hand rule (see Figure 7).

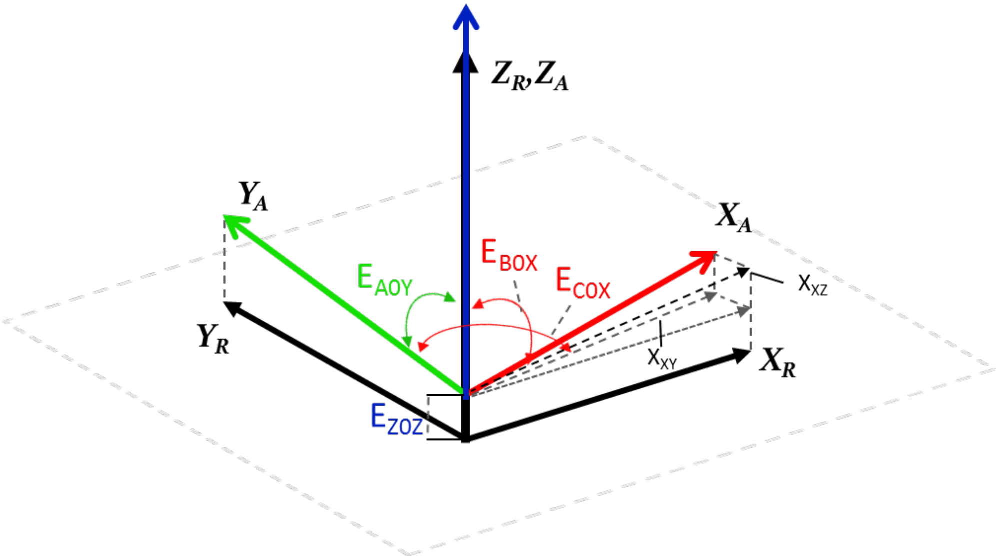


Figure 7. Defining reference coordinate frame where X_R, Y_R, Z_R are the reference frame's axes and the X_A, Y_A and Z_A the reference straight lines of the axes of motion. Squareness errors (E_{C0X} E_{B0X} and E_{A0Y}) between the axes due to the setup are visible. Reference Z axis is defined as primary axis and coincides with machine's Z physical axis, eliminating any squareness errors associated with it. Y axis is defined as secondary reference axis and has one location error, the squareness E_{A0Y}. For X axis two squareness errors of X to Y and to Z (E_{C0X} E_{B0X}) are present. In the figure X_{XY} and X_{XZ} are the X_A axis orthogonal projections to XZ and XY planes respectively, with respect which the squareness errors are calculated. The faded plane visible is the reference XY plane

3.1.1 Axes reference frames and local coordinate frames

After establishing a global reference coordinate frame, the two kinematic chains (tool chain and workpiece chain) that form the machine tools structural configuration can be described. To model the joints and the axes movement, axis reference frames are assigned. Their location is chosen so as to facilitate the measurement and modeling procedure and eliminate any offsets between the different axes frames that need identification and measurement.

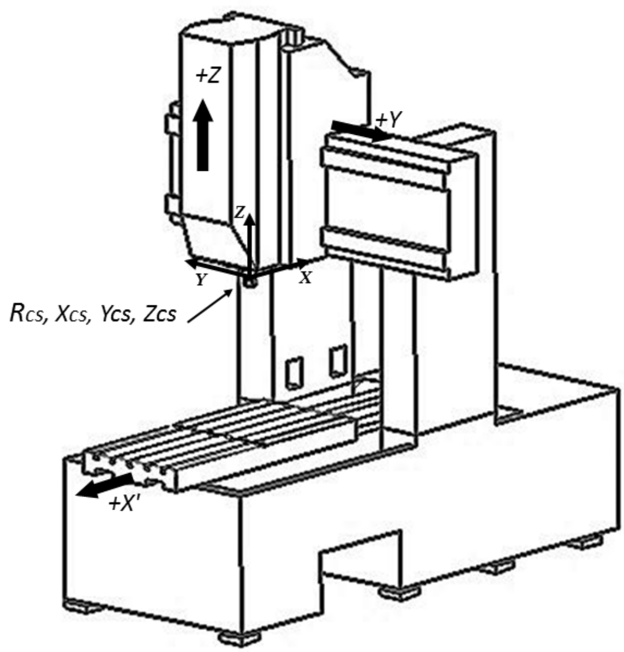


Figure 8. Coordinate frames assignment on the 3-axis vertical machine tool. Rcs corresponds to the reference coordinate frame and Xcs, Ycs, Zcs to the X, Y and Z axes frames respectively (figure adopted and modified from [51])

For each axis, a local coordinate system rigidly attached to the carriage and which follows the movement of the axis is specified. Within the space of this coordinate frame a point of interest can be defined, as a measured point or functional point. Another coordinate frame that does not follow the axis and in which the movement of the respective axis and the errors of the axis are defined, is positioned to coincide with the global reference frame or the local coordinate frame of the carrying axis. This allows for individual measurement of the axis errors by locating the measured point in the local coordinate frame, decoupled from any contributions of the other axes motion. Following the ISO 10303-105 [52] nomenclature the local coordinate frame assigned to follow each axis is called second pair frame and the axis reference frame in which the motion and direction of the axis is described is named first pair frame.

The aforementioned coordinate frames positions are illustrated in Figures 9 and 10 for Y and Z axes. In the left image of Figure 9, the first pair frame of Y axis while in home position it coincides with the global reference coordinate frame. When the machine is commanded to move to a new position the first pair frame is considered to maintain its position, whereas the second pair frame moves along with the axis. The measured point or functional point is defined within the second pair frame's

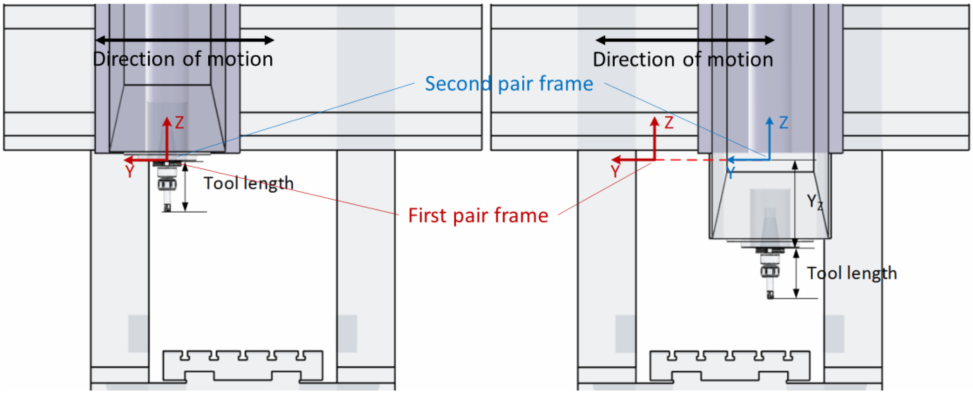


Figure 9. Position of the first and second pair frames of Y axis. In the left instance the carriage is in home position where both frames coincide with each other and with the global reference frame. On the right a random position of the axis is illustrated where the two frames are visible. Measured point is defined within the local coordinate frame or second pair frame of the axis.

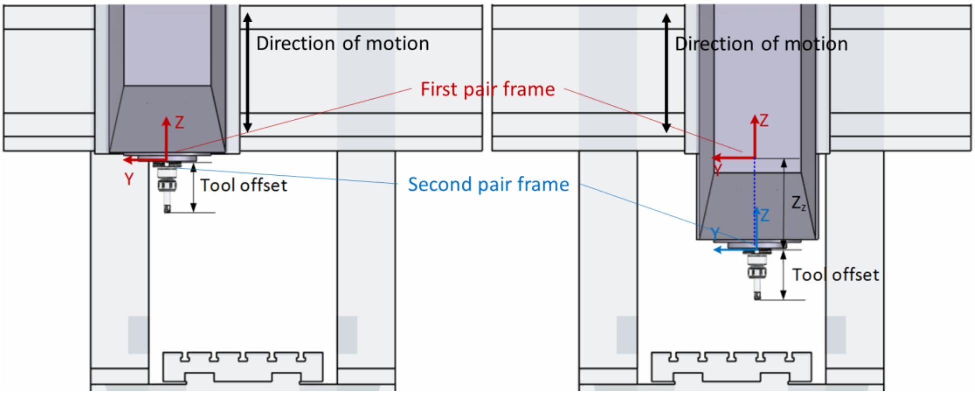


Figure 10. Position of the two coordinate frames defined for the Z axis. On the left the two frames coincide with each other, with the Y axis frames and with the global reference frame. In the right is illustrated a random position of the Z axis. It is visible that the first pair frame is attached to the second pair frame of Y axis. The second pair frame follows the motion of Z axis and within its space the functional point of the axis is defined.

coordinate space. As one can observe the measured point in the Y axis is dependent on the commanded position of the Z axis and the length of the tool without though the measured errors being affected by any possible influences of the Z axis.

Identifying the coordinate frames of Z axis (see Figure 10) the first pair frame coincides with the Y axis's second pair frame and follows the nominal path during the axis movement. When all the machine axes are set to home position, both frames coincide with each other and with the global reference frame. In a random instance as depicted in figure 10, the second pair frame's position is defined by the commanded axis z motion. Measuring point as well as functional point is dependent on the tool offset in x, y and z direction. Depending on the machine tools kinematic configuration and purpose of the model each of the axes local coordinate frames can be defined accordingly.

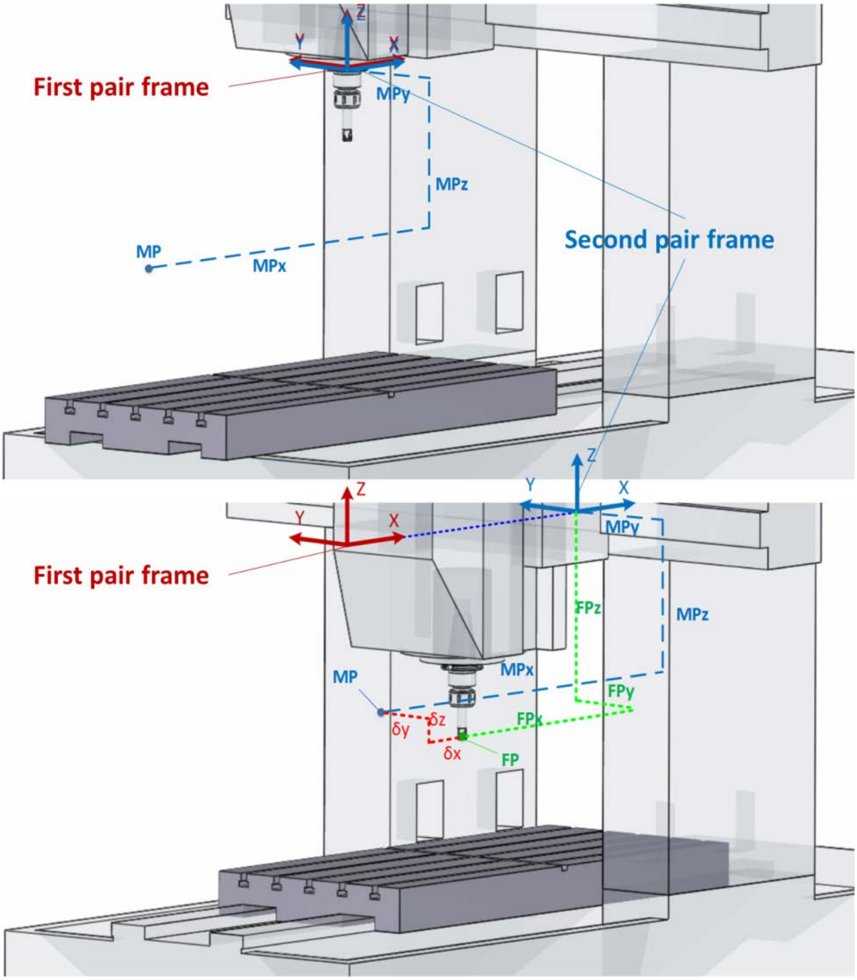


Figure 11. X axis coordinate frames locations. First pair frame is located at the tip of the spindle when in home position and coincides with the global reference frame. The second pair frame follows the motion of the axis. In the second pair frame measured point (MP) and functional point (FP) are defined. The offsets between the two are marked as δx, δy and δz.

3.1.2 Angular errors effect and functional point

One of the significant issues that need to be addressed is the influence the angular error motion of the slides has on linear displacement errors due to a lever effect. According to Bryan's [44] revision on Abbé's principle, angular error motions need to be measured for calculating the effect on the linear errors, in case an offset from the functional point exists.

A common approach for measuring the errors of a linear axis as reported by Kiridena [10] states that linear errors must be measured along the ideal lines representing the axes in the machine's kinematic model, otherwise a linear component due to the angular errors have to be included in the calculations. The problem with this approach is that when considering Abbe or Bryan principles we need a fixed pivot or rotation point. This means that linear positioning errors need to be measured in line with the axes scale, which requires the axis reference coordinate frame to be located in the guideways, where also the pivot point will be. However, there is no knowledge of where the guideways are located and even if there was there is the question of where should the frame be positioned. Moreover, there is no guarantee that the pivot point would lie in line with the scale or might change depending on the axis position.

As stated previously, measurement of the linear displacement errors need to be carried at the trajectory of the functional point. Defining a measurement point to coincide with the functional point in the workspace ensures that the effect from angular errors will be incorporated in the measured errors. In other words no lever effect due to offset between functional point and measurement point will exist. If the measured point does not coincide with the functional point but its position is well documented within the axis local coordinate frame, the effects of the angular errors can be calculated in the local frame.

For the purpose of the thesis, in each axis has been chosen an individual measuring point (MP) that all linear positioning errors would be measured. This also facilitates the measurement as each point would be chosen based on the ease of positioning and setting up the measurement devices. Angular errors on the other hand, suffer no ambiguity in their definition since they are unaffected by other errors and are not dependent on the position of the measuring point

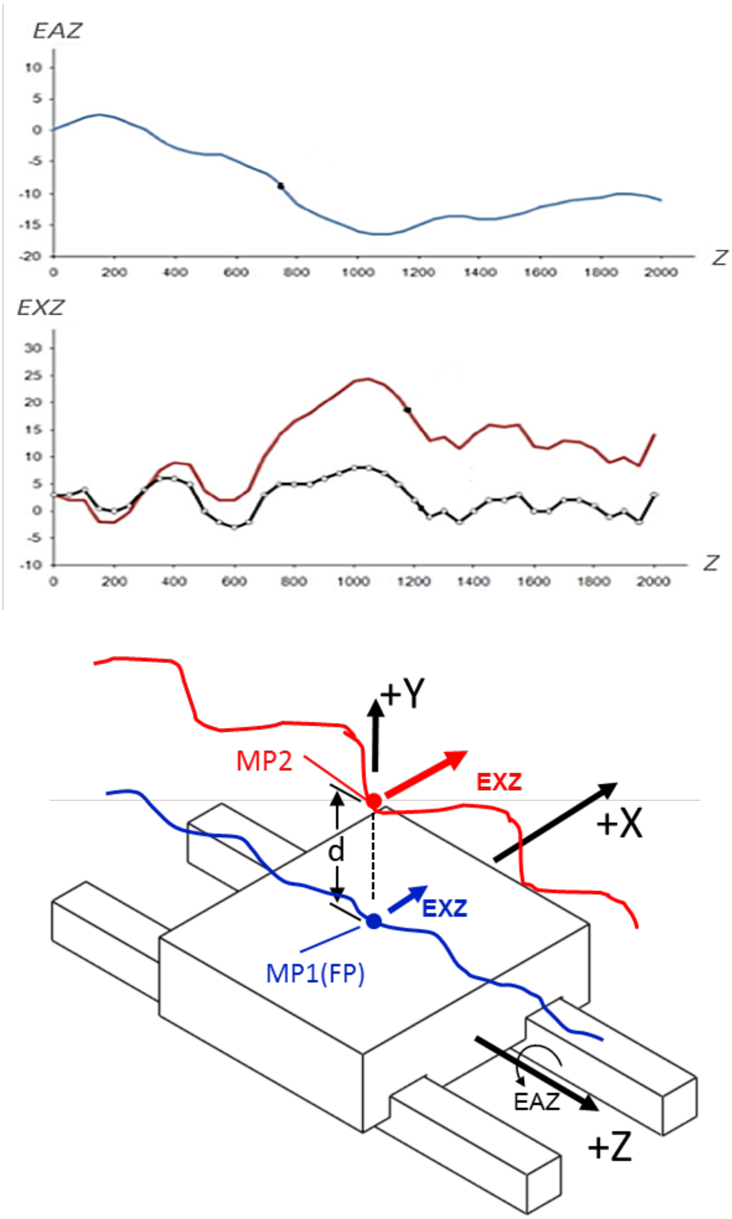


Figure 12. Effect of the functional point and the effect of the roll error (EAZ) in measuring the straightness deviation at the same Z axis position but at different y coordinates (source [45], 39])

A representative example of how measuring at a measurement point (*MP2*) different from the functional point (*MP1*) can be seen in Figure 12. The effect of the angular errors to the magnitude of the measured straightness error at different trajectories due to the offset which creates a lever effect is illustrated. By considering *MP2* to be the center of rotation or the pivot point, and selecting a point *FP1* everywhere within the slide's work space, the influence from the angular error motion can be estimated by knowing the offset distances from the point.

3.2 Machine tool geometric error model

The general modeling methodology is described in Figure 13. In order to implement the geometric error model, machine tool specific data are required to populate the model. Ideal machining data (e.g. toolpath, workpiece origin coordinates in the workspace and tool length) combined with the kinematic structure of the machine provide the axes trajectories in the reference and in the local coordinates frames. Error measurement data are required to realistically calculate and represent the machine tool behavior (laser interferometer). After calculating the geometric errors based on the provided data for each axis individually, the actual toolpath estimation can be produced. Also, having the machine tool errors, a prediction of the manufactured part accuracy can be made.

Before proceeding with the description of the model of the machine, a quick description of the machine's kinematic structure is necessary. The structure consists of a moving table connected to the bed of the machine and modeled as a prismatic joint and represents the X axis of the machine (see Figure 8). On the two columns there is a cross slide which represents the Y axis, on the cross slide is connected the ram which is the Z axis and the spindle is mounted on the Z axis. Both Y and Z axis are modeled as prismatic joints. Finally, a cutting tool is connected to the spindle and a workpiece could be clamped on the table of the machine.

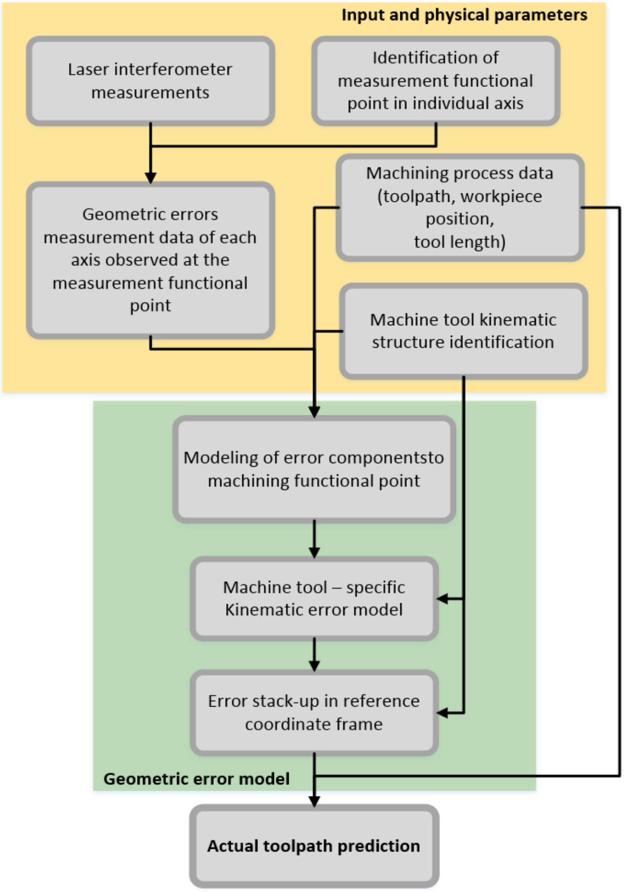


Figure 13. Graph of the proposed methodology for modeling and prediction of the machine tool errors

The global reference frame is located at tip of the spindle when the machine axes are set to home position. For the machine table there is one degree of freedom in the x direction and the HTM that describes the ideal motion is

$$ {}^{R}\mathbf{T}_{X} = \begin{bmatrix} 1 & 0 & 0 & x \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{10} $$

where x is the controlled position of the motion along X axis. For the cross slide or Y axis the degree of freedom is in the y direction and the HTM is

$$ {}^{R}\mathbf{T}_{Y} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{11} $$

and for the z axis or ram the HTM is

$$ {}^{Y}\mathbf{T}_{Z} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & z \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{12} $$

In each axis an individual measuring point is defined in the local coordinate frame in which all measurement will be carried. Selection of that point should correspond in facilitating the measurement process and not to relate to the functional point of any application. For calculating the effects of the angular errors in any other point within the axis's reference frame (different functional point) a matrix is used to represent the difference (offset) between the two. Additionally, an HTM that will express the location of the measured point within the local frame of each axis transforms from the measured point and the error estimation to the axis reference frame. There are no offsets between the axes frames and the preceding frame (either the global reference or the carrying axis frame). Last the HTM that describes the location errors of the axis is calculated.

For the X axis this formulates a matrix $\boldsymbol{H_X}$ that express the actual position of a point in the axis coordinate space as

$$ \mathbf{H}_{X} = \mathbf{S}_{x} \cdot {}^{R}\mathbf{T}_{x} \cdot \mathbf{E}_{Xerror} \cdot \delta\mathbf{P}_{X_{FP}} \tag{13} $$

where $\boldsymbol{S_X}$ is the location errors matrix with respect to Y and Z axis

$$ \mathbf{S}_{x} = \begin{bmatrix} 1 & -E_{C0X} & E_{B0X} & 0 \\ E_{C0X} & 1 & 0 & 0 \\ -E_{B0X} & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{14} $$

$\boldsymbol{E_{Xerror}}$ is the component error matrix and

$$\mathbf{E}_{Xerror} = \begin{bmatrix} 1 & -E_{CX} & E_{BX} & E_{XX} \\ E_{CX} & 1 & -E_{AX} & E_{YX} \\ -E_{BX} & E_{AX} & 1 & E_{ZX} \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{15}$$

and

$$\delta\mathbf{P}_{X_{FP}} = \begin{bmatrix} 1 & 0 & 0 & X_{\delta x} \\ 0 & 1 & 0 & X_{\delta y} \\ 0 & 0 & 1 & X_{\delta z} \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{16}$$

the difference between the functional point or point of interest and the measured point within the axis local coordinate frame.

For the Y axis the matrices that represent the actual position given the component errors and the location errors can be obtained as

$$\mathbf{H}_Y = \mathbf{S}_Y \cdot {}^R\mathbf{T}_Y \cdot \mathbf{E}_{Yerror} \cdot \delta\mathbf{P}_{Y_{FP}} \tag{17}$$

where $\boldsymbol{S}_Y$ is the location error matrix of Y axis with respect to Z axis

$$\mathbf{S}_Y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -E_{A0Y} & 0 \\ 0 & E_{A0Y} & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{18}$$

the component error matrix $\boldsymbol{E}_{Yerror}$

$$\mathbf{E}_{Yerror} = \begin{bmatrix} 1 & -E_{CY} & E_{BY} & E_{XY} \\ E_{CY} & 1 & -E_{AY} & E_{YY} \\ -E_{BY} & E_{AY} & 1 & E_{ZY} \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{19}$$

and the matrix with the offsets between the measured point and the functional point or point of interest.

$$\delta \mathbf{P}_{Y_{FP}} = \begin{bmatrix} 1 & 0 & 0 & Y_{\delta x} \\ 0 & 1 & 0 & Y_{\delta y} \\ 0 & 0 & 1 & Y_{\delta z} \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{20}$$

It is important to mention that despite all three coordinates appear in the δP_{YFP} and δP_{XFP} matrix it does not necessarily mean that there actually exist an offset in that direction.

Finally for Z axis the transformation that describes the actual position represented in HTM matrices is described as

$$\mathbf{H}_Z = {}^{Y}\mathbf{T}_Z \cdot \mathbf{E}_{Zerror} \cdot \delta \mathbf{P}_{Z_{FP}} \tag{21}$$

Here the matrix representing the axis's location errors is not present. As explained earlier the way the global reference frame was defined, Z axis local coordinate system coincides with global reference frame (primary axis) eliminating any potential position and orientation errors. As with the other axes, E_{Zerror} is the error component matrix of the Z axis and $\delta P_{Z\delta FP}$ the offset matrix between the measured point in the Z axis local coordinate system and the machining functional point.

$$\mathbf{E}_{Zerror} = \begin{bmatrix} 1 & -E_{CZ} & E_{BZ} & E_{XZ} \\ E_{CZ} & 1 & -E_{AZ} & E_{YZ} \\ -E_{BZ} & E_{AZ} & 1 & E_{ZZ} \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{22}$$

$$\delta \mathbf{P}_{Z_{FP}} = \begin{bmatrix} 1 & 0 & 0 & Z_{\delta x} \\ 0 & 1 & 0 & Z_{\delta y} \\ 0 & 0 & 1 & Z_{\delta z} \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{23}$$

The matrix H_i of each axis provides the actual position of the machining functional point in the global reference coordinate system estimated for each axis. However in order to obtain only the relative error between the nominal position and the actual position for each axis, we can subtract from $\boldsymbol{H_i}$ the commanded nominal position of the axis plus the offset matrix for the functional point yielding $\boldsymbol{P_{err}}$.

$$\mathbf{P}_{nom} = \mathbf{P}_{X\delta_{FP}} + \begin{bmatrix} 0 & 0 & 0 & x \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & X_{\delta x} + x \\ 0 & 1 & 0 & X_{\delta y} \\ 0 & 0 & 1 & X_{\delta z} \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{24}$$

$$\mathbf{E}_{err} = \mathbf{H}_i - \mathbf{P}_{nom} = \begin{bmatrix} 1 & 0 & 0 & \varepsilon_x \\ 0 & 1 & 0 & \varepsilon_y \\ 0 & 0 & 1 & \varepsilon_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{25}$$

This produces an error matrix $\boldsymbol{E_{err}}$ whose last column values represent a vector that includes only the deviation from the nominal position. Thus adding the errors obtained for each axis separately yields the deviation of the tool tip position in the global reference frame (Eq. 26).

$$\mathbf{E}_{tot} = \mathbf{E}_{X\,err} + \mathbf{E}_{Y\,err} + \mathbf{E}_{Z\,err} \tag{26}$$

As discussed in Section 2.4, the machine's structure can be decomposed into two kinematic chains, one starting at the tool tip (tool chain) and the other at the tool contact point on the workpiece (workpiece chain) (see Figure 14). Obtaining each chain's total kinematic error and by adding the estimated errors of the tool chain and the workpiece chain, the total deviation of the tool in the global reference frame can be estimated giving the total error E illustrated in Figure 14(a).

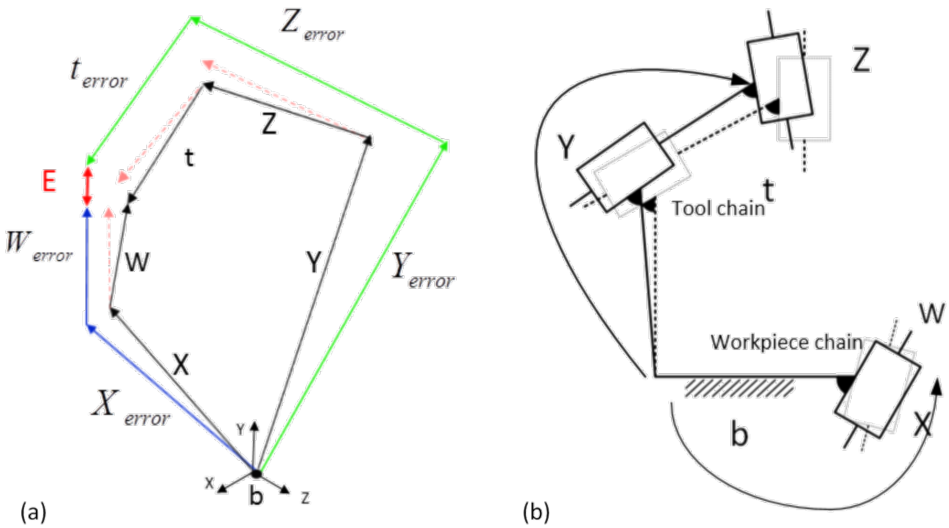


Figure 14. In (a) a representation of the machine tool error vectors illustrating the propagation of the individual axes errors. In (b) an illustration of the nominal (dashed) and actual (solid line) kinematic geometry of the machine tool with the two kinematic chains.

The measured point within each axis local coordinate frame is defined separately and by up to three coordinates x, y and z that specify its position. The equivalent coordinates of the functional point or point of interest are extracted by the trajectory the machine axes are programmed to follow realizing the toolpath.

3.2.1 Modeling of the component errors

Component errors are motion errors of the machine component themselves. Their characteristic is that they are not usually a simple form function of the position of the axis or carriage. Several mathematical tools can be used to describe and model them varying from simple linear functions to polynomial functions, B-Splines, Fourier series, Lagrange polynomials or statistical distributions. Yet, the complexity of these models might become high when calculating the coefficients, or produce high order terms that will overestimate the intermediate error value.

For simplicity, in describing the component errors linear piecewise interpolation has been chosen. As the errors have been measured at specific target positions an interpolant across each interval between two consecutive breakpoints is obtained. For n measuring target points, $n-1$ interpolants will be obtained.

Assuming we have datapoints of the form (x_i, y_i) where x_i, are the target positions corresponding to axis positions and y_i, are the measured values or the error the interpolant is of the form

$$L_i(x) = y_i + (x - x_i)\frac{(y_{i+1} - y_i)}{(x_{i+1} - x_i)} = y_i + s\delta_i, \qquad i = 0,1,2,\ldots,n \tag{27}$$

where

$$s = (x - x_i) \tag{28}$$

is the local variable and

$$\delta_i = \frac{(y_{i+1} - y_i)}{(x_{i+1} - x_i)} \tag{29}$$

the first divided difference *(δ)*.

For finding an error value corresponding to a position between two target points, the interpolant is evaluated at the desired position producing a value. However, since there is always an approximation error, this can be reduced by choosing smaller intervals between the target positions during measurement and consequently between the breakpoints for the interpolation method.

Chapter 4.
Error measurement methodology

In the previous chapter the resulting displacement error between the tip of the tool and the workpiece or machining feature was modeled as a combination of the individual error components of the machine tool structural components. The next step is to measure all these error components in order to populate the model for estimating the error at a position in the machine work envelope.

Laser interferometry was selected as a common measurement method for measuring the individual errors. Due to the working principle of the laser interferometer, direct measurement of each geometric error is performed, minimizing the influence of the other error motions of the machine's components. The long coherence length of the laser beam and its ability to measure with high precision and low uncertainty makes it a popular choice in characterizing the accuracy and repeatability of machine tools positioning capability.

Great care must be taken during measurement in recording the environmental conditions. Thermal drifts, pressure and relative humidity variations might change the refraction index of the surrounding air affecting the laser beam wavelength, as well as expansion of the machine tool structural components due to thermal changes. Also, correct alignment at setup between the laser beam and the measured axis and alignment of the optics (reflectors, interferometer etc.) to avoid any alignment errors such as cosine error and minimize the uncertainty, is essential.

As component errors are functions of the axis's position, five parametric errors (3 linear displacement and 2 rotation errors) of each are to be measured independently. For straightness and rotation errors different interferometer optics are required. Roll errors cannot be measured with a standard laser interferometer, thus roll angular error of the machine table (X axis) will be measured with precision levels.

4.1 Laser interferometer system and measurement setup

The machine tool basic axes characteristics under study can be seen in Table 2. Although the reported axes travel-ranges from the manufacturer are different than the one stated here, the effective axis travel-range represent the real condition of the machine tool.

Table 2 Machine tool axes specifications

Machine tool specifications (mm)	
Axis effective travel range	
X	990
Y	500
Z	460
Resolution	0.001
Machine Table dimensions	
X	1240
Y	525
Distance from spindle tip to table surface	660

The XL laser measurement system from Renishaw which includes the XL-80 He-Ne single frequency stabilized laser, was utilized in measuring the linear displacement, angular and squareness errors of the machine axes. An environmental compensation unit (XC-80) provides information about the ambient temperature and pressure of the surrounding environment. Different optics for linear positioning, straightness, angular and squareness error measurements were used accordingly. In Table 3 the characteristics of the laser system and performance specifications for different measurement optics are presented as reported by the manufacturer [47].

Although laser interferometers provide great precision in measuring positioning accuracy and repeatability of machine tools, it is a time consuming and sometimes challenging process that requires a skilled operator. Well documenting the location of the measuring optics during setup and the laser beam with respect to the table or the spindle is an important parameter for ensuring the repeatability of the measurement. Also, as positioning and straightness accuracy measurements are affected by angular errors of the axis, knowledge of the trajectory of the measuring point will allow calculating their contribution in case offsets between the measuring point and a point of interest exist.

A simple proposed method in maintaining the same measuring point during the setups (when this is feasible), is to use the laser beam as a reference. This would potentially increase the difficulty in mounting the interferometer optics in the correct position and align them to avoid as much setup and alignment uncertainty in the measurement as possible. Also it might require adopting a practical measurement sequence according to which the different error measurements will be performed.

Table 3 XL Laser measurement system characteristics [46]

XL-80 Laser measurement system characteristics	
Nominal wavelength	633 nm*
Laser frequency accuracy	± 0.1 ppm
Linear measurement	
Axial Range	0-80 m
Accuracy	3 ppm†
Resolution	0.001 µm
Straightness measurement	
Axial Range	0.1–4 m
Straightness range	±2.5 mm
Accuracy	±0.5% ±0.5 ±0.15M^2 µm
Resolution	0.01 µm
Angular measurement	
Axial range	0-15 m
Angular range	±175 mm/m
Accuracy	±0.2%A ±0.5 ±0.1M µm/m
Resolution	0.1 µm/m
Squareness measurement	
Range	±3/M mm/m
Accuracy	±0.5% ±2.5 ±0.8M µm/m
Resolution	0.01 µm/m
Optical square prism error	-0.09 arc-seconds

Where:

M**:** measurement distance in meters of the longest axis. For the angular measurement accuracy M is the distance between the angular interferometer and the reflector in meters.

A**:** angle measured in µrad

% → percentage of displayed value

* Nominal vacuum wavelength 632.990577 nm

† Accuracy with environmental compensation is 0.5ppm

Starting by performing straightness measurements of a linear axis should give the possibility for better alignment of the laser beam with the axis's line of motion and as close to the center line of the table as possible [42].This can be achieved by performing a manual removal of the slope error by aligning the optical axis of the straightness reflector with the machine's axis of travel. The optics are mounted with magnetic bases on the machine table and the spindle. In case the spindle can be locked it is possible the use of a tool holder in combination with extension rods to mount the interferometer or the stationary reflector directly to the spindle. For linear positioning error measurement, the same setup can be used by just exchanging the straightness optics with the linear interferometer and reflector optics without repositioning the magnetic bases or the mounting pillars. Although a bit cumbersome in practice, it ensures that the measurements will be carried at similar if not the same trajectory with the straightness measurements.

4.1.1 Measurement setup parameters and considerations

For each axis, adaptation of the ISO 230-1 [39] guidelines for the selection of measuring intervals was attempted. This means that measuring intervals shall be no longer than 25 mm for axes of 250 mm or less, whereas for longer axes the interval shall be no longer than 1/10 of the axis length. At each target position the machine was programed to halt for 4 seconds in order to come at rest and capture the measurement. Although it is recommended to run a warm up cycle prior to measuring it has been chosen to do the measurements under cold start conditions. Care to set the correct direction of the measuring system to be the same as the axis under test should be taken. Similarly, a correct sign convention (positive or negative) of the error direction should be defined to match the right hand rule for a Cartesian coordinate system or alternatively to match the sign convention used for the axis under test.

For every target position five sets of bidirectional measurements were performed. The traverse speed of the axis under test has been set to 1000 mm/min. In all linear measurements care was taken in minimizing the dead-path error by keeping the separation between the interferometer and the reflector at the datum position under 10 mm. All the measurements initiated with the axis positioned at the furthermost possible position. Although it is clearly appointed that measurement along the entire axis travel range is necessary to fully characterize an axis positioning accuracy, in all measurements spatial constraints or some cases by choice the measurements had to be restricted to a narrower travel distance.

Data capture is performed by a computer equipped with software provided with the interferometer system. It offers various options depending on the measurement. Different facilities and configurations offer enhanced flexibility and usability. Communication between the laser unit and the computer is established by a USB cable. For analysis of the measured data an analysis software tool is available. Options to run analysis and plot the captured data according to different international standards are offered.

4.1.2 Axes setup and measurement

Measurement of the errors of X axis (machine table) was done in the range of -50 to -800 mm with an interval of 50 mm between target positions. The laser head was mounted on a tripod with an adjustable stage and had to be place on the side of the machine. This was also the only time that a cover of the machine had to be removed to gain direct optical access to the working area. For all the performed measurements (positioning, straightness and angular error) the reflector was mounted on the moving table while the interferometer (or Wollaston prism for straightness measurement) was rigidly attached to the stationary spindle, see Figure 15.

For the Y axis measurements, the travel distance of the axis ranged from -25 to -400 mm. However, for the straightness measurement it was shifted by 100 mm from -125 to -500 mm because of the risk of collision due to the position of the straightness optics. An interval of 25 mm was selected between the target positions. Locating the laser and the tripod in front of the machine made alignment of the laser beam easier compared to the X axis setup (Figure 16, Figure 17). Again, after measurement of the straightness errors the beam can be used as a reference. However, there is always difficulty in positioning the optics exactly at the correct location. Therefor fine adjustment of the position of the laser head might facilitate in the final steps of the alignment process.

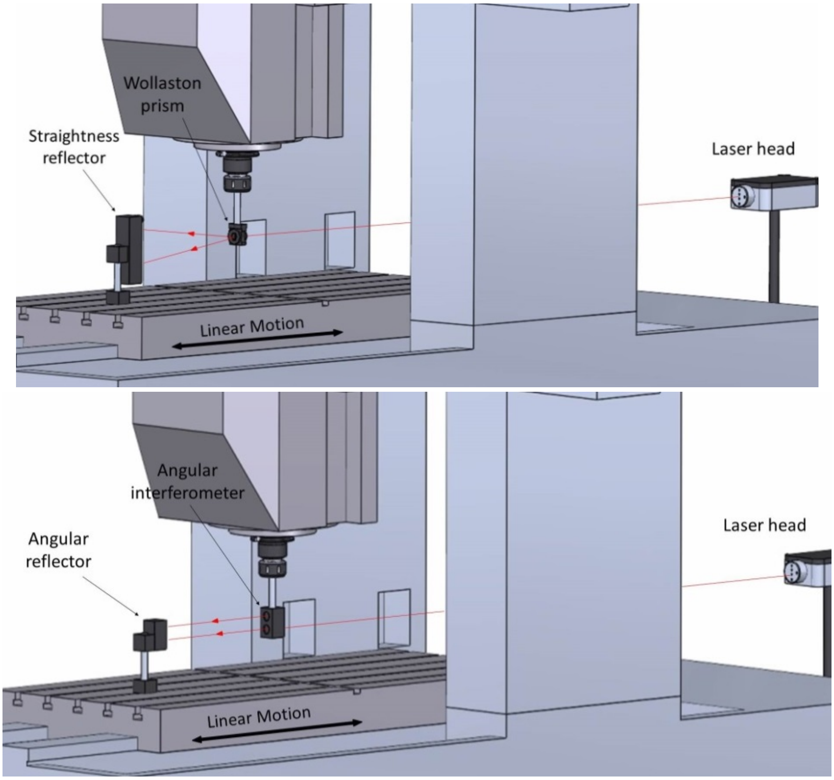


Figure 15. Schematic illustrating measurement setup of vertical straightness (top) and pitch angular error (down) of the X axis. Position (x and y) of the straightness and angular reflector is roughly maintained in the same location on the table. The same applies for the height from table surface. The laser beam was used as reference for realigning the optics between different setups.

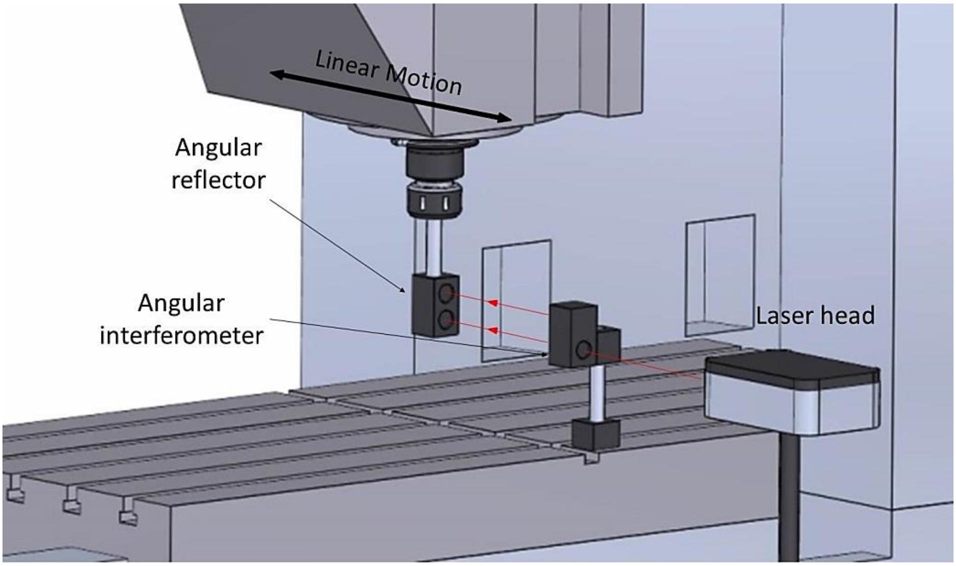


Figure 16. Schematic illustrating measurement setup of pitch angular error motion of the Y axis.

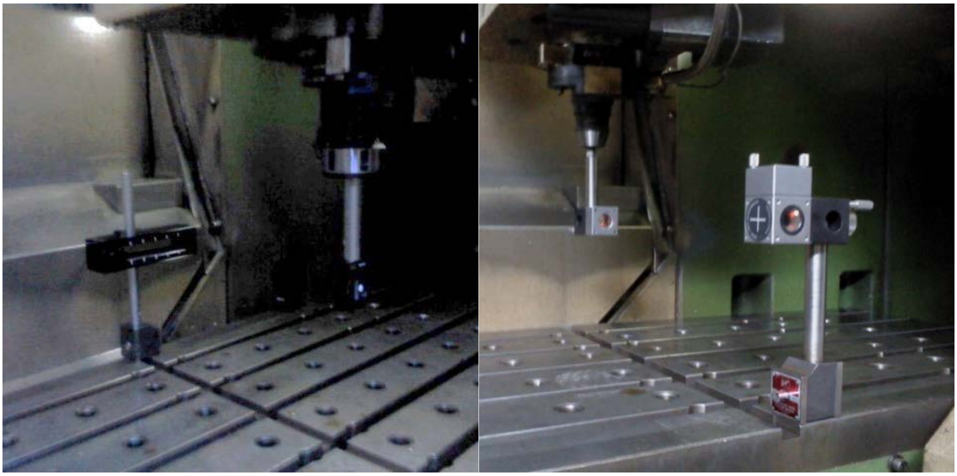


Figure 17. Measurement of the horizontal straightness of the Y axis (left). Straightness reflector was located on the middle line of the machine table and the position was roughly recorded. On the right, the linear positioning measurement of the Y axis can be seen.

Measurements of the Z axis motion errors were performed applying similar rules. Since only the axis's errors where of interest and under measurement, the table (X axis) and the cross carriage (Y axis) were positioned as to locate the stationary optics as close to the center of the table as possible (Figure 18). The travel range of the axis was specified between -25 to -300 *mm* with a 25 *mm* interval between the target positions.

Mounting of the optics on the table was mainly achieved by utilizing magnetic bases. However in cases where this imposed limitations into the total axis travel range or didn't allow for rigid mounting as in Figure 18, clamping jaws were used to fix them in position. Care was taken not to apply excess clamping force which could lead in deforming of the mounting base and probably the optical elements.

Although measurement of the angular errors, as previously stated, suffer no ambiguity due to offset between different measurement trajectories as linear and straightness errors do, locating and positioning of the angular reflector and interferometer followed the same setup approach.

The measurement functional point was defined within each axis individually in the local coordinate system. For the X axis the position in the local coordinate system was (x_{fpX}, y_{fpX}, z_{fpX}): (-800, -256, -280) *mm* with the stick-out of the optics used

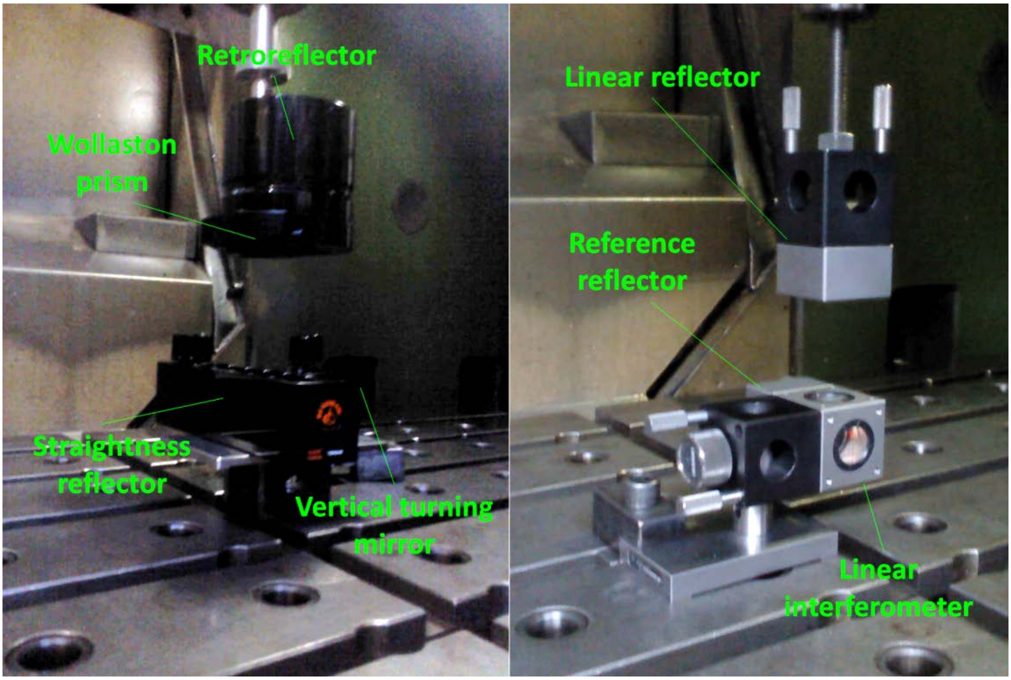


Figure 18. Left: Measurement of the straightness deviations of the Z axis requires a large retroreflector on which the Wollaston prism is attached. A turning mirror is used to divert the laser beam towards the vertical direction of motion. Right: For positioning measurement deviations the linear interferometer has the capability to divert the beam if oriented into the proper way.

during measurement to be 198 mm. For Y axis the measurement functional point in the local coordinate frame has only one relevant coordinate in z direction which is z_{fpY}: -280 mm, as the y coordinate is constantly varying during measurement and x coordinate is fixed with respect to the local frame and is zero. The stick-out of the optics used and which contribute to the position of the functional point is 220 mm. Lastly, for the Z axis the only relevant parameter that contributes to the position of the functional point in the local coordinate frame is the tool length or tool offset. Due to particularity of the measurement process the tool offset varied with respect to the other axis. However, in most of the measurements the value was close to 130 mm.

Chapter 5.
Results

In this chapter the results from the measurement of the geometric errors of the machine obtained with the laser interferometer and the outcome of the machine geometric error model are presented and discussed. Also, a brief analysis and evaluation of the measurement uncertainty is carried out.

5.1 Measurement results

Measurement data obtained from the laser interferometer measurements for each axis were stored and analyzed by the provided analysis software. Depending on the measured error being analyzed a different facility of the software provided a variety of methods according to national and international standards for expressing the results. In Figure 19 the average data of five measurements for the angular errors of X axis are illustrated. The waviness in pitch angular errors might be caused by the ballscrew or drive system or unevenness of the surface of the guideways.

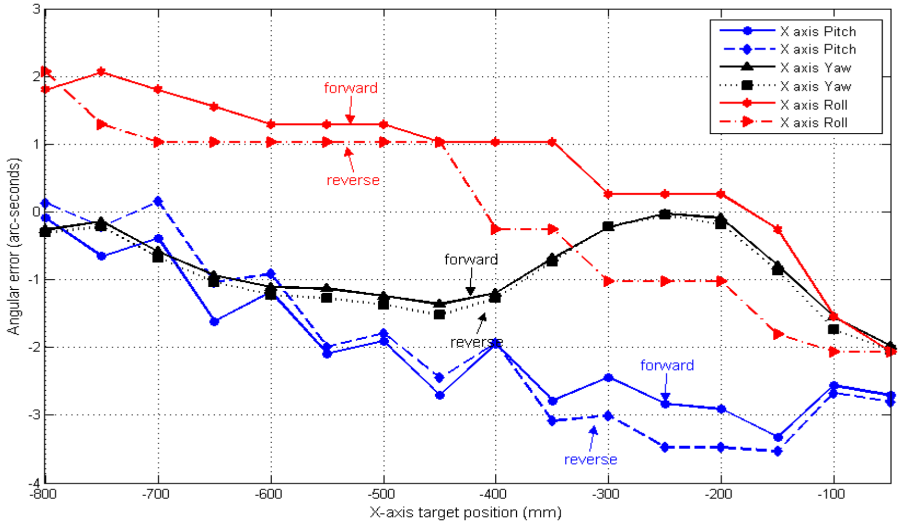


Figure 19. Angular errors of X axis. The data in the figure are the average of 5 measurements in forward and reverse direction of motion

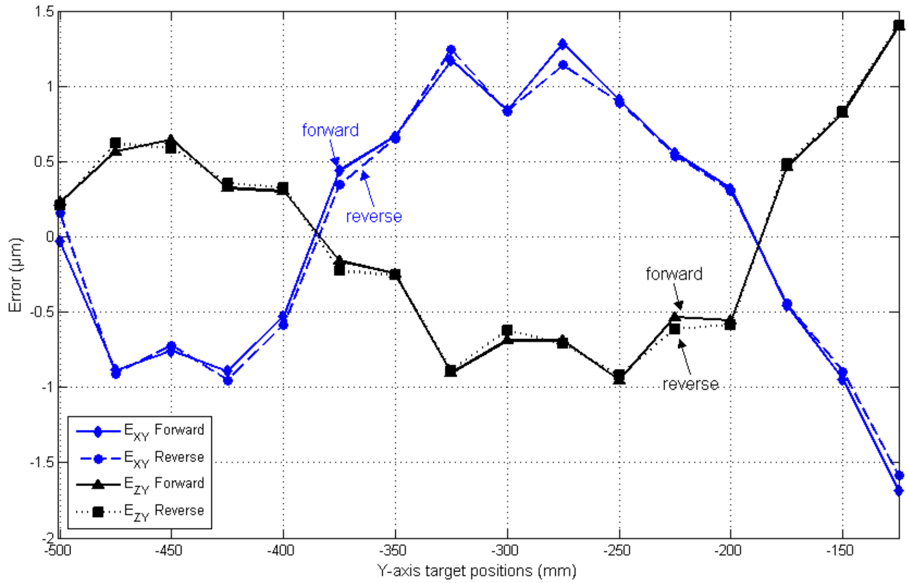


Figure 20. Measured straightness deviation (vertical and horizontal) of Y axis. The data in the figure are the average of 5 measurements in forward and reverse direction of motion

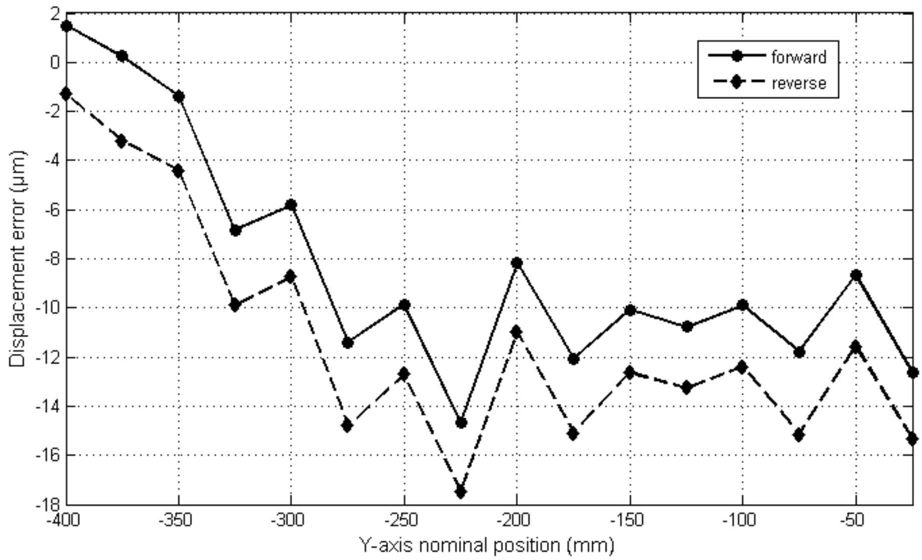


Figure 21. Average of five measurements for the Y axis linear positioning errors in forward and reverse direction of motion.

In Figure 20 and Figure 21 an example of the results of the Y axis straightness and linear positioning errors are illustrated. Each data set (forward and reverse) is the average of five measurement runs in both motion directions. The measurement results presented were calculated according to [35], for the linear and rotation errors, whereas for straightness deviations a least square fit preceded the analysis, in order to remove any residual slope error in the data.

The most important parameters of the reported results are the accuracy, repeatability and positional deviation of the axes as presented in Tables 4, 5 and 6 for the X, Y and Z axis respectively.

Table 4. X axis calculated measurement parameters

X axis	E_{XX} (μm)	E_{YX}/E_{ZX} (μm)	E_{BX} / E_{CX} (μm/mm)[2]
Accuracy **A**	18.752	7.62 / 10.96	0.0212 / 0.0133
Repeatability **R**	4.566	3.92 / 8.88	0.0057 / 0.005
Mean Reversal	2.367	0.16 / 0.82	0 / 0.0005
Mean Deviation **M**	14.620	5.62 / 4.51	0.0167 / 0.0097
Reversal **B**	2.720		0.0031 / 0.001
Sys. Deviation **E**	17.100		0.0179 / 0.0100

Table 5. Y axis calculated measurement parameters

Y axis	E_{YY} (μm)	E_{XY}/E_{ZY} (μm)	E_{AY}/E_{CY} (μm/mm)
Accuracy **A**	19.580	21.98 / 4.79	0.0374 / 0.0455
Repeatability **R**	5.174	19.82 / 3.85	0.0153 / 0.0298
Mean Reversal	2.909	0.06 / 0.25	0.0002 / -0.0021
Mean Deviation **M**	16.170	3.04 / 2.45	0.025 / 0.0212
Reversal **B**	3.440		0.0009 / 0.0038
Sys. Deviation **E**	18.960		0.0252 / 0.0232

Table 6. Z axis calculated measurement parameters

Z axis	E_{ZZ} (μm)	E_{XZ}/E_{YZ} (μm)	Tilt 1/Tilt 2 (μm/mm)
Accuracy **A**	21.835	13.82 / 1.68	0.0251/ 0.0553
Repeatability **R**	16.135	13.82 / 1.45	0.0133 / 0.0314
Mean Reversal	15.553	-0.44 / -0.10	-0.0007/ -0.0016
Mean Deviation **M**	6.090	1.25 / 0.72	0.0129/ 0.0259
Reversal **B**	15.553		0.0014 / 0.0037
Sys. Deviation **E**	21.500		0.0138 / 0.0280

Accuracy as described by ISO 230-4 [36] is "*the maximum translational error in the distance between any two target points along the axis of motion*" and is calculated as follows

[2] The units *μm/mm* can be expressed as *mrad* (milliradians).For converting to arc-seconds the approximate value is 1 *μm/mm* ~ 206.2648 *arc-sec*. Angular errors can also be expressed in μrad (10^{-3} *mrad*)

$$A = \max\left[\bar{x}_i \uparrow +2s_i \uparrow; \bar{x}_i \downarrow +2s_i \downarrow\right] - \min\left[\bar{x}_i \uparrow -2s_i \uparrow; \bar{x}_i \downarrow -2s_i \downarrow\right] \quad (30)$$

where $\bar{x}_i$ is the arithmetic mean of the positional deviations and s_i the standard deviation of the measurement values at target position *i*.

Repeatability is defined as "the maximum value of spread in positioning error at any target point along the range of motion when the system is moved in both the directions multiple times under similar pre-specified conditions" [35] and is calculated as the maximum of the bidirectional repeatability values R_i which are described by

$$R = \max\left[2s_i \uparrow +2s_i \downarrow +|B_i|; R_i \uparrow; R_i \downarrow\right] \quad (31)$$

where B_i is the reversal value and $R_i\uparrow$ the unidirectional repeatability at target position i and equals $R_i \uparrow = 4 \cdot s_i \uparrow$

Bidirectional systematic positional deviation E is defined as the difference between the maximum and minimum of the mean unidirectional positional deviations for both directions at any target position i and is defined as

$$E = \max\left[\bar{x}_i \uparrow ; \bar{x}_i \downarrow\right] - \min\left[\bar{x}_i \uparrow ; \bar{x}_i \downarrow\right] \quad (32)$$

where

$$\bar{x}_i \uparrow = \frac{1}{n}\sum_{j=1}^{n} x_{ij} \uparrow \qquad \bar{x}_i \downarrow = \frac{1}{n}\sum_{j=1}^{n} x_{ij} \downarrow \quad (33)$$

the mean unidirectional positional deviation

Squareness errors between the axes were measured and calculated for the three axis of the machine. For all calculations for the reference straight line was done by least square fitting method. Squareness of X axis relative to Y axis and Z axis calculation yielded: E_{C0Y} 14.569 $\mu m/m$ (~3 *arcs*) and E_{B0X}: -24.362 $\mu m/m$ (~ -5.03 *arcs*) respectively. Squareness of Y axis relative to Z axis is E_{A0Z}: 13.757 $\mu m/m$ (~. 66.697 *arcs*). Since Z axis was the primary axis of the global reference system no location errors were accounted for Z. Similarly, as Y axis is defined as secondary axis, it only has one squareness error calculated.

5.1.2 Evaluation of the measurement uncertainties

There are various error sources that contribute to the measurement uncertainty [37], [48], [49], [50]. Certain assumptions have been made about the conditions of the environment and the measurement device that might affect the measurement result. Thereby, the uncertainty values for the laser interferometer are taken as stated by the manufacturer. Also, it is assumed that the alignment and the setup of the measuring device are done in order to provide sufficient beam intensity, assuming a maximum misalignment angle of 5 $mrad$. The variation in the measurement environment since no drift test was performed for assessment and assuming that the environment inhomogeneity is less than 0.3° C per minute, was approximately considered to be 1.5 μm. The ambient temperature during measurement was on average 20 $\pm$ 1 $°C$. After each setup the measuring system is allowed to stand for a few minutes in order to reach a stable temperature and eliminate any influences from temperature changes due to handling.

Each error source contributing to the uncertainty of the measurement was assumed to be uncorrelated and have a rectangular distribution function. That means that each source is estimated to have an error value that lies in the possible range of (a^+ - a^-). Assuming a 95% level of confidence in the measurement the expanded uncertainty was estimated as U=2u_c (coverage factor k=2), where u_c is the combined standard uncertainty and is calculated according to

$$u_c = \sqrt{\sum_i u_i^2} \tag{34}$$

assuming that the standard uncertainties u_i are of uncorrelated contributors.

The uncertainties due to the individual error sources assuming a rectangular distribution as stated earlier is given according to

$$u_i = \frac{a^+ - a^-}{2\sqrt{3}} \tag{35}$$

Uncertainty of the performance parameters

The uncertainties of the performance parameters calculated from the measured values are estimated as they are important in characterizing the test uncertainty of the measurement.

The uncertainty for the bidirectional repeatability in positioning is described by

$$u(R) = \sqrt{u(B)^2 + u(R\uparrow, R\downarrow)^2} \qquad (36)$$

where $u(B)$ is the uncertainty in the reversal value and $u(R\uparrow, R\downarrow)$ the uncertainty of the unidirectional repeatability and are described by

$$u(B) = 2\sqrt{\frac{u_{EVE}^2}{N} + u_{SETUP}^2} \qquad (37)$$

and

$$u(R\uparrow, R\downarrow) = 4u(s\uparrow) = 4\sqrt{\frac{1}{N-1}} \cdot u_{EVE} \qquad (38)$$

where u_{EVE} is the uncertainty due to environmental variation error and u_{SETUP} the uncertainty due to the repeatability of the measurement setup.

The uncertainty for the accuracy measurement is given by

$$u(A) = \sqrt{u(E)^2 + u(R\uparrow, R\downarrow)^2} \qquad (39)$$

with $u(E)$ being the uncertainty due to the bidirectional systematic deviation that is determined by

$$u(E) = \sqrt{u_{DEVICE}^2 + u_{MISALIGNMENT}^2 + u_{SETUP}^2 + \frac{u_{EVE}^2}{N}} \qquad (40)$$

Table 7. Uncertainty estimates for the linear positioning errors of the three axes where u(i) is the resulting combined standard uncertainty and U(i) the expanded uncertainty (units in *μm*)

Parameter	X		Y		Z	
	u(i)	U(i) (k=2)	*u*(i)	U(i) (k=2)	*u*(i)	U(i) (k=2)
R_i	1.2	2.4	1.2	2.4	1.2	2.4
B	0.6	1.2	1.0	2.0	1.2	2.4
E	3.6	6.2	2.4	4.8	2.2	4.0
R	1.3	2.6	1.6	3.2	1.7	3.4
A	3.8	7.6	2.7	5.4	2.5	5

Contributors to uncertainty

Most of the single contributors to the uncertainty are estimated as ranges and as described previously a rectangular distribution is assumed. All contributors are considered to be uncorrelated thus the combined uncertainty when required is calculated as the square root of the sum of squares of the standard uncertainties of the contributors. Some contributors have a dependency on the measurement length that results in differing effect to the uncertainty contribution.

Most notable sources of uncertainty contribution are the measurement device uncertainty, the uncertainty due to the misalignment between the beam of the laser system and the measured axis, the setup uncertainty and the uncertainty of the environmental variation.

The device uncertainty is described by

$$u_{DEVICE} = \sqrt{u_{\lambda}^2 + u_{M,n}^2 + u_{DP}^2 + u_{DR}^2} \quad (41)$$

where u_{λ} is the uncertainty due to laser wavelength accuracy, $u_{M,n}$ is the uncertainty due to changes in the refractive index of air, u_{DP} due to the dead-path error which is assumed zero and u_{DR} the uncertainty due to the device resolution. In table 4 the estimated combined uncertainty for the device together with the units are stated.

The uncertainty due to the misalignment between the laser beam and the axis is described by

$$u_{MISALIGNMENT} = L \cdot \frac{(1 - \cos\gamma)}{2\sqrt{3}} \quad (42)$$

with the misalignment angle γ to be 5 $mrad$. As there is a dependency on the measuring length for each axis, the magnitude of the uncertainty will vary according to the axis under test.

The measurement setup uncertainty depends on the maximum angular error (pitch and yaw) of the axis under test and on the Abbe offset between two possible lines of measurement. It is safe to assume a maximum angular error (pitch and yaw) to be approximately 0.06 $\mu m/mm$ if no measurement results have been obtained. For the Abbe offset although in the current measurement there has not been two different lines of measurement a typical value can be of the order of 25 mm. The setup uncertainty contribution can thus be described by

$$u_{SETUP} = \frac{\sqrt{2} \cdot O_{ABBE} \cdot D_{ANGLE}}{2\sqrt{3}} \quad (43)$$

Finally the uncertainty due to the environmental variation error caused by possible drifts to the measurement setup during the time the measurement is performed is taken by

$$u_{EVE} = \frac{E_{VE}}{2\sqrt{3}} \quad (44)$$

However, since no test was performed prior to the measurement to check for possible drifts due to the environmental variation the error is assumed to be E_{VE} = 1.5 μm.

In Table 8 a summary of the uncertainty contributors for the linear positioning measurements is presented. For straightness calculations the uncertainty of the contributors is illustrated in Table 9. The required data for the device uncertainty were calculated according to the equation in Table 3, under the straightness section. The combined standard uncertainty and the expanded uncertainty (k=2) are 3.4 and 6.7 μm respectively. The estimation is applicable for all axes straightness measurements and calculation of the performance parameters.

Table 8 List of uncertainty contributors for linear positioning errors measurement

Contributor	Uncertainty	units
u_{λ}	0.06	μm
$u_{M,n}$	0.9	μm
u_{DR}	0.002	μm
u_{DEVICE}	1.0	μm
$u_{MISALIGNMENT}$	$3.608 \times 10^{-6} L$	μm
u_{SETUP}	1.2	μm
u_{EVE}	0.87	μm

Table 9. Uncertainty contributors for straightness measurements

Contributor	Uncertainty	units
u_{DEVICE}	0.67	*μm*
$u_{MISALIGNMENT}$	3.0	*μm*
u_{SETUP}	1.1	*μm*
u_{EVE}	0.9	*μm*
$u_c(i)$	**3.4**	*μm*
U(i) (k=2)	**6.7**	*μm*

5.2 Model results

The machine tool geometric error model described in chapter 3 was implemented in MATLAB in order to test its functionality. A main script including functions that calculate the geometric error of each axis was implemented and the result is presented in a visual and analytical way. Two toolpath cases to test the functionality of the algorithm were created. The test toolpath 1 (see Figure 22) is a rectangular shape in the range of X: (-350 *mm* ,-650 *mm*) and Y: (-150 *mm*,-350 *mm*) with Z fixed at -280 *mm*. Tool length for both cases was specified at 200*mm*. Test toolpath 2 (see Figure 23) apart from straight line motion includes two arcs of 180^o degrees. General direction of the straight lines is rotated by 45^o degrees with respect to X axis in order to achieve a simultaneous axis movement. Range of the axes motion is for X axis from about -260 to -740 *mm* in machine workspace coordinates and for Y axis between -50 to -450 *mm*. Z axis was fixed at -200 *mm*. As the algorithm functionality is based on coordinate points in the machine tool workspace, the toolpaths needed to be divided into individual points in order to calculate the errors. The fragmentation step was 5 *mm*, as it allows for less computational load. In cases were higher detail is required the step can be set at the desired size. In Table 10 the maximum deviations in x, y and z directions can be seen for both toolpaths.

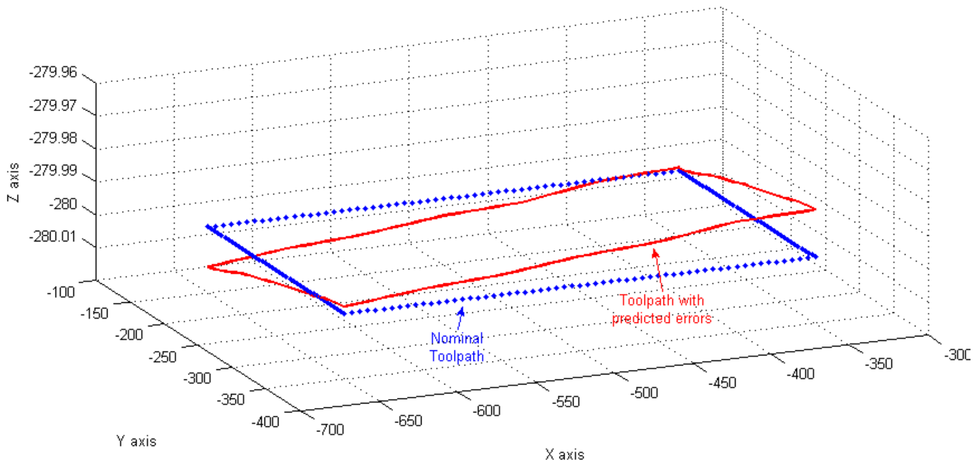


Figure 22. Toolpath test 1. The blue dots represent the nominal toolpath and the red line the calculated toolpath with the estimetaed geometric errors. It is important to note the Z axis is not in scale, hence the disproportion in the figure.

Table 10. Maximum deviations of the toolpaths for each direction of motion.

Maximum deviation in µm	X	Y	Z
Toolpath 1	8.5	24.3	26.7
Toolpath 2	12.7	24.4	27.6

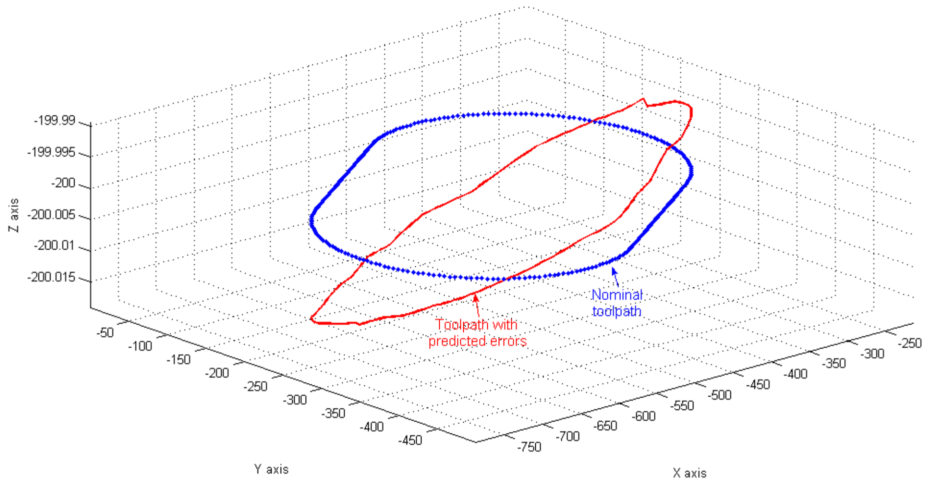


Figure 23. Test toolpath 2. The blue dots represent the nominal toolpath and the red line the calculated toolpath with the estimetaed geometric errors. It is important to note the Z axis is not in scale, hence the disproportion in the figure.

In comparison to the measurement results presented in section 5.1 for the three axes, the maximum predicted deviations of the simulated toolpath do not deviate by much. As the major contributing error for all axes is the linear positioning error, comparing the result to the systematic deviation for each axis it is found in good agreement.

In a rough estimation, it is possible to assess the overall accuracy of the machine by taking the root mean square value of the accuracy of the three axes, always assuming that the measurement results are not correlated. The result can be obtained by the formula

$$A_{total} = \sqrt{A_X^2 + A_Y^2 + A_Z^2} \tag{45}$$

which yields an overall accuracy of $A_{total} = 34.9\ \mu m$.

It should be noted that this value is a rough estimation and does not represent that actual accuracy of the machine. The reason is that it relies on the fact that error measurements are accurate and it only incorporates systematic errors without taking into account any random errors that might amplify or cancel some of the inaccuracies.

Chapter 6.
Discussion, conclusion and future work

The work in this thesis deals with the development of a methodology for measuring and modeling machine tool geometric and kinematic errors. The proposed method describes a general geometric error modeling method which is applicable in different types of machine tools and measurement approach using laser interferometer. The model is applied for of a three axis machine tool in order to predict the resulting error between the cutting tool and the workpiece in an implementation of the model in MATLAB.

6.1 Measurement methodology

Laser interferometry is a popular 'direct' measurement method of very high accuracy and precision, used for measurement of geometric errors of machine tools for assessment and calibration purposes. One of the downsides of the method is that it is very time consuming and requires experience for successful application. Additionally, its sensitivity in environmental variations makes it impractical for frequent use in industrial environments. In chapter 4 a detailed discussion regarding the measurement approach is carried out. The importance of well documenting and maintaining the position of the measurement functional point in each axis coordinate frame throughout the measurement is pointed out.

The proposed measurement approach is purposed in establishing a consistent way for improved communication of the measured performance between users, manufacturers and metrologists.

Results of the performance parameters of the machine tool axes show that the main contributing error of the machine's inaccuracy is the linear positioning errors of the axes. Also it is observed that the Y and Z axis accuracy suffer in the x direction straightness. This can be attributed either to the configuration of the machine axes or to error during measurement setup and alignment of the optics. As Y and Z axis are positioned on the cross-slide between the columns of the machine tool, the weight of the axis might bend the guideways causing a bowing effect causing straightness error to the x direction.

Another important aspect that can significantly decrease the accuracy in predicting the geometric error of the machine is the contribution of the angular errors through a lever effect. Measuring as close to the machining functional point as possible as highlighted in section 2.5.2 is important as the resulting error contribution and the uncertainty in the predicted result can be high proportional to the magnitude of the offset.

Last but not least, due to the inherent nature of measurement method the results do not include any working load. Hence, the assessment of the accuracy might not reflect the actual operating performance of the machine tool. It is possible that during machining there might be a cancellation of some of the geometric errors of the machine due to effects of the cutting process or amplification due to deflection.

6.2 Machine tool geometric error model

In chapter 2 the general approach for representing the kinematic structure of a machine tool have been presented. The various sources of error that affect the accuracy and precision of machine tools have been briefly described in section 2.3 and the geometric and kinematic errors have been analyzed.

The general methodology for developing the machine tool geometric error model has been described including details regarding the formulation of the model. Several steps necessary for implementing the HTM method where explained. The most important include:

- Establish proper local coordinate frames of the axes for describing the geometric errors and global coordinate frame
- Define the measurement functional point within each axis coordinate frame
- Formulate the kinematic model of the machine based on the machine's axes configuration.
- Model the measured component errors

The implementation of the model and its functionality was explained in section 5.2. An illustration of two test toolpath created for demonstrating the predicted actual toolpath and the range of the estimated deviations were presented.

It has been shown that it is possible to develop a kinematic model of a machine tool in order to assess its accuracy and precision by utilizing proper measurement methods and mathematical models.

In this thesis, several modeling related issues were considered. One of the important concepts in the model is the functional point. Despite being defined in ISO230-1 [39] it still lacks in clarity when it comes to implementing it in measurement. An important question related to it is:

- How accurately should the position of the functional point be defined in order to produce reliable results?

In this regard, considering the need for quick assessment methods for machine tools, defining the position of the functional point accurately might prove problematic, if time constrain is in prospect. Also, in case of repetition of a measurement, it needs to be assessed how much the new functional point is acceptable to deviate from the initial one without affecting the measurement results.

Several implementation issues emerged during formulation of the model in MATLAB. Although the theoretical model is straight forward once described, several parameters need to be captured correctly in order to function in the desired way. As the model is dependent on the machine tool axes configuration, realizing a toolpath might be achieved by utilization of the axes in more than one way.

6.3 Future Work

The main focus of the thesis is on geometric and kinematic error modeling for predicting the toolpath accuracy in order to assess machined part's accuracy.

However, for complete assessment of a machine tool and machining system's capability, static stiffness and dynamic stability of the machine tool must be considered. Also, as new machine tool designs enable high transfer and cutting feeds and increased acceleration and deceleration, instabilities from inertial effects might cause higher errors than geometric errors.

Regarding the measurement methodology, despite laser interferometry being a preferred measurement method producing accurate results, the time requirements and skill needed have a deterrent effect for industrial applications.

As the long-term goal is to exploit or develop tools to assess the capability of machine tools suggestions for further development of the current model and future research are:

- Further development of the modeling method, in order to incorporate the ability to manage multi-axis machines (4 and 5 axis)
- Extend the error modeling functionality to take into account static stiffness and deflections from cutting process.
- Examine other mathematical methods for describing the component errors (section 3.2.1)
- Improve the measurement process so that it can be embedded in an industrial environment for easy assessment.
- Develop a generic modular computational tool that can utilize results of different measurement methods for capability assessment and maintenance.

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