Risk Analysis Against Electricity Market Index and Portfolio Optimisation

O S K A R   E R I C S S O N

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Risk Analysis Against Electricity Market Index and Portfolio Optimisation

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Oskar Ericsson, June 10, 2014

Abstract

There has been a lack of a transparent index to compare electricity portfolios against for many years. Most industrial firms hedge the risks for their electricity needs by buying forward contracts which guarantee the price of a certain amount of power for a year or part of a year. The problem is to know if the company has made good deals since the available comparisons are average spot prices. In this thesis the objectives are to construct a relevant index and then evaluate possible portfolios against this index, giving risk measures such as Value-at-Risk and Expected Shortfall. The resulting index buys a small part of the needed power amount to each trading day's closing price of the forward contracts traded by the portfolio. Thus, the index buys the volume *wanted power amount* divided by *number of trading days* of the used forward contracts each trading day the contracts are available. Another objective is to suggest an optimal trading policy that minimise the expected portfolio cost based on historical price data. This is evaluated by constrained optimisation algorithms. Suggestions for the optimal hedge volumes and when to buy the forward contracts are given based on the historical prices. This reveals how expensive different forward contracts are relative to spot prices for the respective period.

Sammanfattning

Det har länge saknats ett transparent index att jämföra elhandelsportföljer med. De flesta industriföretag säkrar priser för sina elektricitetsbehov genom att köpa terminskontrakt som garanterar ett visst pris för ett år eller för delar av år. Detta görs för att inte utsättas för risker med höga spotpriser. Problemet blir för företaget att veta om det har gjort bra affärer eftersom det saknas relevanta jämförelser, till exempel är det missvisande att jämföra mot spotpriser vilka främst påverkas av väderprognoser. Målen med denna uppsats är att skapa ett relevant index, för att sedan jämföra elhandelsportföljer med index genom att ge riskmått som Value-at-Risk och Expected Shortfall. Indexportföljen handlar en liten och lika stor volym till varje handelsdags stängningspris för respektive terminskontrakt. Alltså, index handlar *bestämd effekt* delat med *antal handelsdagar* i varje använt terminskontrakt under varje handelsdag som respektive terminskontrakt finns tillgängligt. Ett ytterligare mål med denna studie är att utvärdera handelsstrategier för dessa kontrakt för att föreslå en optimal strategi som minimerar den förväntade portföljkostnaden utifrån historiska priser. Detta görs genom optimeringsalgoritmer. Förslag till optimala volymer som säkras med terminskontrakt och när kontrakten ska köpas ges utifrån historiska priser. Detta anger hur dyra terminskontrakten är relativt spotpriserna för respektive period.

Contents

1 Introduction

For many years there has been a substantial lack of a good, transparent index to compare electricity portfolios against. While comparable indices exist, they are manufactured by companies operating in the electric market which creates trust issues. Electricity is a special commodity due to its non-storable nature. The electricity market consists of both a spot market, where electricity is delivered instantly, and a forward market, where contracts which settle the price of a given amount of electricity for a certain time period are traded. Most industrial companies secure the price for their energy needs with forwards that guarantee electricity for a year or quarters of a year to reduce the risk of a high spot price. The problem for the companies is to evaluate if they have made good trades since they may have bought several contracts at different prices and thus have no transparent index to compare against. In this thesis the objectives are to construct a useful index and then evaluate possible simulated portfolios against this index, resulting in relevant risk measures. Another objective is to formulate the trading policy in order to minimise the portfolio costs.

Chapter 2 gives a background to the electricity market. Chapter 3 describes the purpose of this thesis along with an example of an electricity portfolio. Chapter 4 gives a mathematical background for the key features used in this thesis. The methods for the analysis are explained in Chapter 5. Chapter 6 displays the results of the investigations along with some discussion. The conclusions from the results are then drawn and discussed further in Chapter 7.

2 Background

2.1 Electricity Market

The Nordic, except Iceland, and Baltic countries share a common electricity market known as Nord Pool. It is owned by the transmission system operators in these countries. [13]

Electricity is a non-storable commodity, i.e. it is not possible to buy electricity at a certain time point and then use it later for a reasonable price. While there exist possibilities to store power, the cost is much too high for an end user. However, in the electricity market there are a number of forward contracts available which guarantee a price of a certain amount of electrical power for some time period. For example, a company may buy a quarterly contract that secures the price of 1 MW during a quarter of a year. This means that a Q1 contract for 1 MW is a contract that guarantees that the buyer buys 1 MW every hour during the whole first quarter of the year at the settled price. There is also a spot market, where day-ahead contracts that secure physical delivery for the different hours the upcoming day. The trading of physical power on the spot market takes place on Nord Pool. [14]

The forward contracts are purely financial. This means that the actual energy consumption is bought on the spot market, but the price difference between the forward price and the spot price is then settled in the clearing system for the company that has bought the forward contract. The trading of financial contracts is done on the commodities market owned by Nasdaq. [13]

Trading on the spot market is done on all days, but trading in the financial forward contracts is only possible on weekdays. While it is now possible to trade yearly forward contracts for ten years ahead, in the years of interest in this thesis (2000-2013) it was only possible to trade for five years [12]. Quarterly contracts begin their trading January 2nd (if it is a weekday) two years before the concerned year. Since all the quarterly contracts for a specific year start their trading on the same day, but end their trading on the last weekday before the settlement period begins, it means that Q4 contracts are traded for nine months longer than Q1 contracts.

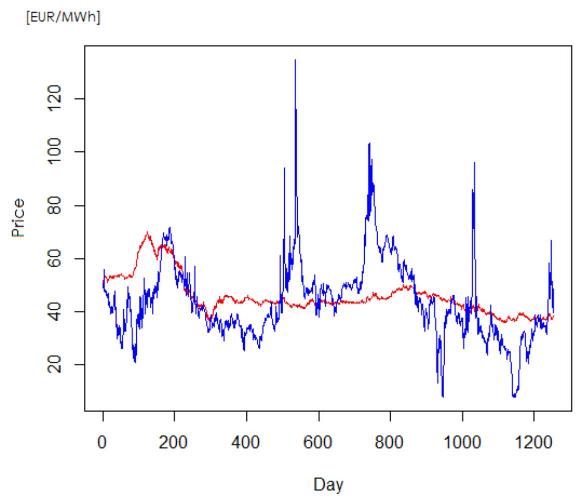

Figure 1: Graph over the price of the yearly forward contract for 2013 (red curve) compared to the spot price during the trading days of the financial contract (blue curve). The spot price is much more fluctuating than the forward price. Trading on the spot market is done on Nord Pool, while the financial contract is traded on the commodities market owned by Nasdaq.

Since November 1st, 2011, Sweden is divided into four different price areas. This is due to the fact that the net capacity to transfer electricity is limited and most of the hydro power is produced in northern Sweden while the consumption is higher in the southern parts of the country. Standard contracts are traded to the system price common to the Nord Pool area. The price levels in the different areas are generally as follows: $SE3$, which is the price area including Stockholm and where most of the population lives, has a "medium" high price. The prices in $SE1$ and $SE2$, which are the northern parts of the country, are usually lower. The prices in $SE4$, which are the southern-most parts of Sweden, are often higher. The price areas have created incentives to increase the transfer capacity between regions to decrease the price differences. When the price areas were introduced the prices in $SE4$ were often substantially higher than the rest of the country, which led to people living in $SE4$ complaining about price discrimination. Nowadays, the price differences are much lesser and the price is equal in all the Swedish areas on many days which is a consequence of increased transfer capacity in the power grid. The monthly average prices are shown in Figure 2. The graph displays that the price areas mostly had different prices during the first year that they existed and that the prices are now as good as equal. A contract which make up the difference between the system price and that of a specific price area was previously called contract-for-difference (Cfd), but is now called EPAD. [14]

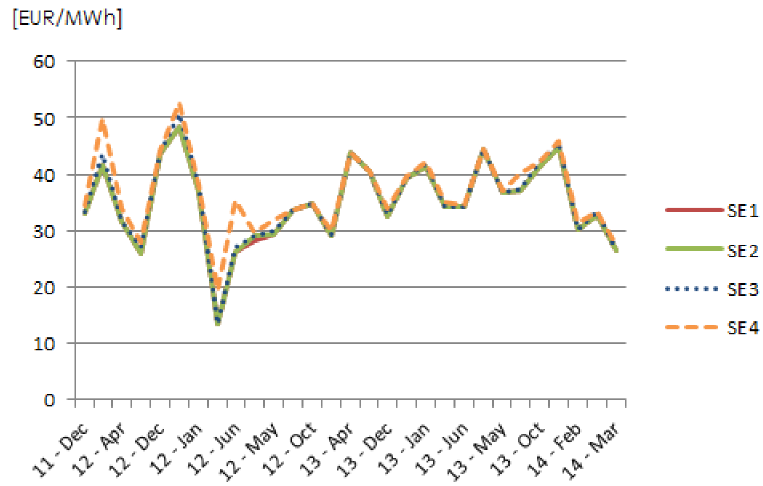

Figure 2: Graph over the monthly spot prices in the different price areas in Sweden.

As we see in Figure 3, the price in the Swedish areas were the same at the moment the picture was collected except for $SE4$ where it was substantially higher. The arrows indicate the direction of the flow of power, which is from north to south in Sweden, as there is more production and less power usage in the northern parts. [15]

From an information provider for the European energy markets known as Montel, it is possible to collect historical prices for different contracts. These prices are crucial for the construction of indices and analysis in this thesis.[11]

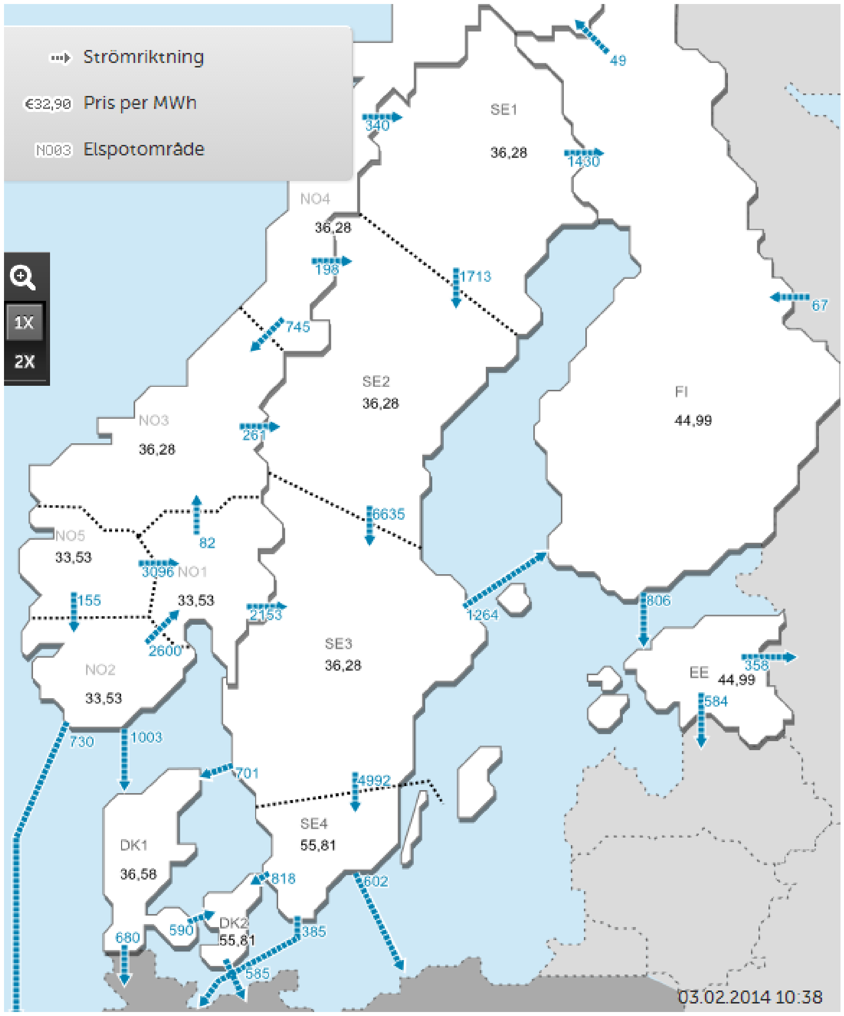

Figure 3: Map over the different price areas in the Nord Pool area. The figure is collected from SvK. [15]

2.2 Factors Affecting the Electricity Price

The price for electricity is influenced by supply and demand, which in turn are influenced by multiple factors. Since about half of Sweden's electricity supply consists of hydro power, the amount of water in the reservoirs has a direct impact on the market price and is the most influential factor over a short time period. Expected and actual availability in the nuclear power plants is also a major influence on the short term price. Over a longer time horizon (several years); other natural resource prices, such as coal and gas, are more significant. [14]

The emission trading system in the EU area creates an incentive to lower emissions by using fees on carbon dioxide and other gases harmful to the climate. The price for emission is regulated by the market. Since the introduction of trades with emission rights, the price for electricity has generally been higher. [14]

Taxes in the electric market and fees in the electric grid also influence the price. Different regions have different taxes and fees so that consumer prices vary locally. The taxation level also differ between production and consumption of electricity and also for different lines of business, so that an industrial company may not pay the same electricity consumption tax as a real estate company. However, the differences in taxation are so small in comparison to the prices that they are neglected in this thesis. [14]

Figure 1 shows that the prices on the spot market fluctuate much more than the prices in the financial forward market where the price structure is rather flat from day to day. This is because the spot prices depend on the actual consumption for each hour, so that prices are generally lower during the nights when there is less consumption. The spot prices are also higher during winter than in the summer, since there is more need of heating when the outside temperature is cooler. Svenska Kraftnät defines the hours 6-22 on weekdays as high load hours (HL) and the rest of the hours are low load (LL). High load hours are more expensive than low load hours. The forward contracts on the other hand settles the price for every hour during a longer period of time which may not even start for several years. [13]

The trading costs on Nord Pool Spot consist of a variable fee of 0.04 EUR/MWh and a settlement fee of 0.005 EUR/MWh, giving a total of 0.045 EUR/MWh. [13] The trading costs for forward contracts are the variable cost of 0.0042 EUR/MWh plus the clearing fee of 0.0089 EUR/Mwh, resulting in a total of 0.0131 EUR/MWh. [12] The amount of power secured by forward contracts also need to be settled in the physical spot market, so the electricity volume that is traded in the financial market is exposed to slightly higher trading costs. Since there are no fees that depend on the number of trading occasions, just fees depending on the traded volumes, the trading costs will be neglected in the analyses in this thesis since the price differences in trading costs between hedged and not hedged volumes are so small in comparison to the prices. In Figure 1, we see that the spot and forward prices are in the range $4 - 135$ EUR/MWh with averages around 45 EUR/MWh which clearly shows that the trading costs are very small in comparison.

2.3 Previous Studies

Most studies presented in related literature try to construct models which describe the price for electricity, see for example *Geman et al.* (2006) [4]. This study emphasises the difference between electricity and other commodities due to its non-storable property. It states that there is no simple relation between spot and forward prices because of this.

A master's thesis in economy that investigates the relationship between spot prices and futures is *Hansson* (2007) [5]. This study reaches the conclusion that futures prices seem to be substantially higher than what could be expected based on the spot prices.

Both *Neuman* (2006) [8] and *Finas* (2008) [3] try to model the spot price by using time series. *Finas* (2008) uses supply and demand and underlying factors such as natural resource prices to model the spot price, while *Neuman* (2006) concludes that the price for electricity mostly shows resemblance to an ARMA (auto-regressive moving average) process time series.

All the theses agree that the spot price is hard to predict, but it may simply be explained by stating that the price depends on the most expensive production method needed in order to meet demand (for example burning coal if hydro power is not enough). This results in price dynamics that feature upward jumps when a more expensive production sets in. *Hugmark* (2004) [6] tries to describe the spot price from the hydro reservoir levels.

Eriksson (2002) [2] investigates which risk measures that should be included in and calculated by a computer program for trading on the electricity market called CLICK. The study concludes that Value-at-Risk and Expected Shortfall are best suited, which contributes to the choice of using these risk measures in this thesis.

3 Purpose

The purpose of this thesis is to construct a useful index against which electricity portfolios easily may be compared and also to suggest trading policies for portfolio managers so that the expected cost of the hedge portfolio is minimised. The index problem is described by an example.

3.1 Example of an Electricity Portfolio

Say that a company needs 10 MW for one year and 4 MW extra for the first and fourth quarters of the year. Suppose that it buys 1 MW of the yearly contract at five different time points, then 3 MW later and finally the remaining 2 MW for the year. Further, it buys 1 MW in each of the quarterly contracts at four separate occasions. Thus, the company has bought forward contracts for a yearly contract at seven different time points and each of the quarterly contracts at four other points in time. This results in a portfolio which has traded contracts at fifteen different prices. The company may wonder if it has done good business, but what price should it compare its portfolio against? It is easy to calculate an average price at which it has bought electricity to, but the only transparent price comparison is the rolling spot index which is not a relevant comparison since the company's portfolio only trades in forwards.

The aim here is to use an index which it is easy to use as a benchmark. In this case, the company may want to start its trading four years before the start of delivery. This means that it can buy at about 1 000 trade days. This results in an index that buys $\frac{10}{1000}$ MW = 10 kW of the yearly contract to each trading day's closing price for the yearly contract. The quarterly contracts are only available two years in advance [13], resulting in the index portfolio buying $\frac{4}{500}$ MW = 8 kW for the first and fourth quarters to each trading day's closing price. The choice of a linear trading for the index portfolio gives a transparent and easily understandable index, which were objectives that Sweco opted for.

If a company specifies a certain policy that adds restrictions to when trading is allowed or defines percentages of the electricity consumption that should be secured at a certain time, the index should reflect these criteria. For example, a client may state that trading is not allowed earlier than 3 years before the target year, the hedged volume 2 years before delivery should be 20% of the total volume and that the hedged volume 1 year before delivery should be 50% of the total volume. These restrictions would then result in an index which increases the daily traded volume with 2 years to delivery and then further increases the daily traded volume with 1 year to the target year in order to reflect the policy.

The unrestricted linear approach gives an index which buys the suggested volume for each of the forward contracts at the chosen time period's average price for respective contract. Of course, the number of trading days used in this example is an approximation, and when executing real calculations the number of trading days will differ for different years. Also, while it is typically not possible to buy smaller volumes than 1 MW on the commodities market, the index will trade in this way since the index trades are fictional trades used for comparison. The index trading strategy is a possible strategy under certain circumstances. For example, there exists a possibility to trade smaller volumes for electric portfolios if the volume of the traded contract, which is at least 1 MW, is split between several portfolios. [13]

The average price for the yearly contract YR-15 that the example portfolio has traded to is illustrated in Figure 4 along with the daily settlement prices and the resulting index price for this contract. The bought volume is shown by the yellow triangles and has the unit MW. The first five time points thus illustrate trades of 1 MW, the second-to-last shows a buy of 3 MW and the last trading occasion shows a buy of 2 MW. The resulting average price for this customer (per MWh) is higher than the resulting index price, which is an effect of the higher forward prices when the time to delivery is longer. This effect can be seen as a risk premium which is paid in order to secure a price so that the risk is reduced. The illustration in Figure 4 uses 1 027 trading days.

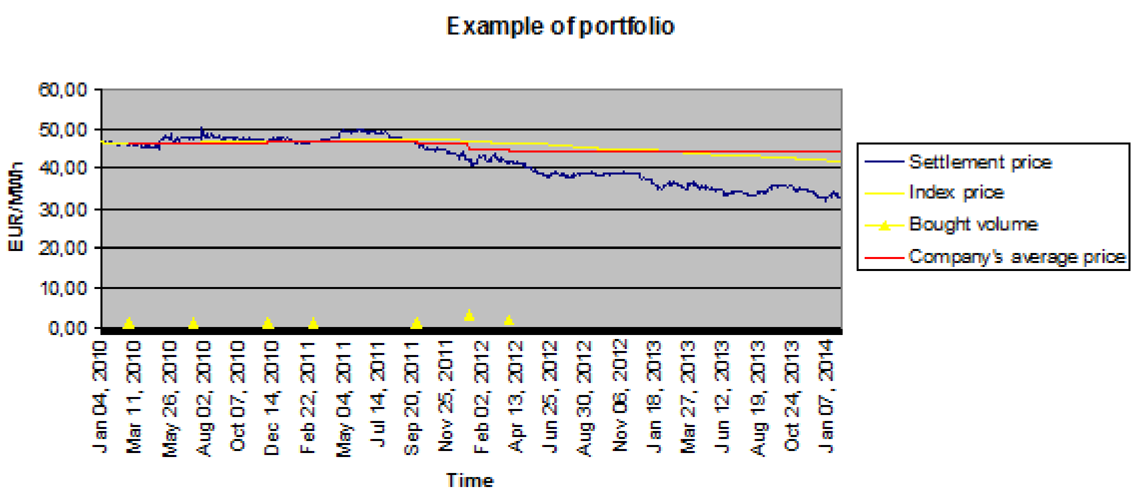

Figure 4: Graph illustrating an example portfolio trading the yearly contract YR-15. The figure is self-produced from closing prices collected from Montel. [11]

In addition to the yearly contract, the cost of the quarterly contracts needs to be taken into consideration. By creating example and index portfolios for the quarterly contracts in the same way as for the yearly contract above and then multiplying the average prices with the corresponding volumes and the number of hours in the period, we get that our example portfolio has a total cost of 4 533 418 EUR and that the index portfolio has a total cost of 4 337 375 EUR. Thus, in this example the company has made a loss of 196 043 EUR compared to the index portfolio.

In reality, the forward portfolio will probably not hedge the consumption perfectly so some portion will need to be bought at spot prices. Since this is a fictional example, the spot cost is not included here. When using real consumption profiles later, the spot cost will be included both for evaluated portfolios and index portfolios.

The risk manager of the company may be interested in the size of the financial risk the company faces by buying contracts in this manner. For similar portfolios it may thus be interesting to study how the number of trading days and at how many different time points contracts are bought influence risk measures such as Value-at-Risk. Since it is normally not possible to buy less than 1 MW at a time, the number of time points at which contracts are bought is limited by the total volume. However, if the portfolio manager is the responsible trader for a large number of portfolios, the power amount that is bought in the market may be split between many portfolios so that in practice a small volume is assigned for a certain portfolio. This enables the portfolio manager to trade according to the index portfolio [17]. The details for the index portfolio costs are given in Section 4.4.

Some companies may be more interested in securing a certain electricity price early in the trading period for the forward contracts (up to a year ahead of the delivery period), rather than getting the lowest possible costs. This is typically the case for real estate companies which can forward much of their costs to the tenants living in their buildings.

3.2 Objectives

The objectives of this project can thus be formulated as follows;

- To create a useful index against which electricity portfolios easily may be compared to. It is desirable to manufacture both a simpler index for private customers and more customised indices suited to companies' specific consumption profiles.
- To suggest trading policies for portfolio managers so that the expected cost of the hedge portfolio is as low as possible while keeping the financial risk down. This may be formulated as an optimisation problem:

  $$\min_{\boldsymbol{\theta} \in \boldsymbol{\Omega}} \quad E[\boldsymbol{C}(\boldsymbol{\theta})]$$

  where $\boldsymbol{C}(\boldsymbol{\theta})$ is the total cost of the portfolio given the parameter vector $\boldsymbol{\theta}$ which must belong to the feasible region $\boldsymbol{\Omega}$ (more on this in Chapter 4.4).
- To study the effect of the number of trading occasions for the forward contracts has on risk measures such as Value-at-Risk and Expected shortfall. The risk measures compare the portfolio costs to the index cost. The cost of the index portfolio may be seen as the cost of the risk free strategy that is achieved by buying the same small volume as the index portfolio at each day's closing price, which is enabled if the portfolio manager can split the minimal trade volume allowed by the market between several portfolios. Also, if trading is done via a broker, the volumes are practically allowed to be as small as desired and the index strategy can thus be achieved (trading via brokers are not considered here and therefore broker fees are not used). The hedged volumes for the index portfolio are chosen to be the largest integer volumes belonging to the feasible region $\boldsymbol{\Omega}$ to enable trading according to this strategy.

4 Mathematical Background

4.1 The Empirical Distribution

When working with unknown distribution functions, it is possible to approximate the distribution from empirical data. [7]

Consider a sample $Z_1, \ldots, Z_N$ of independent and identically distributed random variables or vectors with a common distribution function $F(z) = P(Z \leq z)$, where Z is an independent copy of Z_k and $Z \leq z$ holds component-wise if Z is a vector. The true distribution function $F(z)$ is unknown, but can be modelled by an empirical distribution made from observations $z_1, \ldots, z_N$ of the random variables or vectors $Z_1, \ldots, Z_N$. We approximate the unknown distribution function by assigning probability weights $1/N$ to each observation z_k. This yields

$$F_N(z) = \frac{1}{N} \sum_{k=1}^{N} I(z_k \leq z)$$

where I is the indicator function and $F_N(z)$ is the empirical distribution function from the observations $z_1, \ldots, z_N$. Similarly,

$$F_{N,Z}(z) = \frac{1}{N} \sum_{k=1}^{N} I(Z_k \leq z)$$

is the stochastic counterpart of the empirical distribution for the random samples $Z_1, \ldots, Z_N$.

From the (strong) law of large numbers, we have that

$$\frac{1}{N} \sum_{k=1}^{N} X_k \to E[X]$$

with probability 1 as $N \to \infty$ if the expected value $E[X]$ exists finitely and $X_1, X_2, \ldots, X_N$ is a sequence of independent copies of a random variable X. By choosing $X_k = I(Z_k \leq z)$, we get that $E[X] = P(Z_k \leq z) = F(z)$. The law of large numbers then implies that $\lim_{N\to\infty} F_{N,Z}(z) \to F(z)$. This means that for a sufficiently large sample size, the empirical distribution function $F_{N,Z}(z)$ is a good approximation of the true distribution function.

4.2 Value-at-Risk

The Value-at-Risk is the most popular risk measure and it is defined as follows [7]:

$$VaR_p(X) = min\{m : P(mR_0 + X < 0) \leq p\}; \quad p \in [0,1]$$

where R_0 is the return of the risk-free asset and p is the chosen risk level of the portfolio value, X, at a future time 1.

It is easy to implement the Value-at-Risk of a position with value X at time 1, as the smallest amount of money m that if added to the position now and invested in the risk-free asset ensures that the probability of a strictly negative value at time 1 is $\leq p$.

One often rewrites the $VaR_p(X)$ as follows:

$$\begin{aligned} VaR_p(X) &= min \quad \{m : P(mR_0 + X < 0) \leq p\} \\ &= min \quad \{m : P(-X/R_0 > m) \leq p\} \\ &= min \quad \{m : 1 - P(-X/R_0 \leq m) \leq p\} \\ &= min \quad \{m : P(-X/R_0 \leq m) \geq 1 - p\} \end{aligned}$$

If we then let $L = -X/R_0$, where X is the net gain from the investment, we get that $VaR_p(X)$ is the $(1-p)$-quantile of L. It is therefore possible to write:

$$VaR_p(X) = F_L^{-1}(1-p)$$

When working with empirical distributions, $VaR_p(X)$ is estimated by:

$$\widehat{VaR_p}(X) = L_{[Np]+1,N}$$

where $L_{1,N} \geq \ldots \geq L_{N,N}$ are the sorted losses and $[Np]$ is the integer part of Np.

Value-at-Risk is easily calculated and has a clear interpretation, but is often criticised for not taking the tail beyond level p into account at all. Two different loss distributions may have the same values at level $1-p$ but despite that one has a heavier tail than the other, this extra risk is not reflected in VaR_p. To quantify the risk in the tails, one may instead use Expected shortfall.

4.3 Expected Shortfall

Expected shortfall is, with minor technical modification, called "Average Value-at-Risk". It is defined by [7];

$$ES_p(X) = \frac{1}{p}\int_0^p VaR_u(X)du$$

When using empirical distributions, we get the Expected shortfall by sorting the Value-at-Risk values and use summation. Thus we get;

$$\widehat{ES}_p(X) = \frac{1}{p}\left(\sum_{i=1}^{[Np]} \frac{L_{k,N}}{N} + \left(p - \frac{[Np]}{N}\right) L_{[Np]+1,N}\right)$$

where N is the sample size and p is the chosen risk level.

In this thesis, N is chosen to be large enough so that Np is an integer which means that the last term in $\widehat{ES}_p(X)$ vanishes.

Also, ES has the subadditivity property, i.e.

$$ES_p(X_1 + X_2) \leq ES_p(X_1) + ES_p(X_2)$$

where X_1 and X_2 are two random variables. VaR does not have this property. ES quantifies the risk in the tail and is interpreted as the average loss given that the outcome is worse than at the chosen VaR level.

4.4 Electricity Portfolio Costs

The total cost of a certain electricity consumption during a year is given by:

$$\boldsymbol{C(\theta)} = V_{year} \cdot P_{year} \cdot hours_{year} + \sum_{n=1}^{4} (V_{Q_n} \cdot P_{Q_n} \cdot hours_{Q_n}) + \sum_{h=1}^{hours_{year}} (V_{h,spot} \cdot S_h)$$

where h is the index ranging over the hours of the year, V_{year} is the volume in MW that is hedged by the yearly contract, P_{year} is the price in EUR/MWh of the yearly contract, V_{Q_n} is the volume that is hedged by the forward contract for quarter n, P_{Q_n} is the price of that quarterly contract, $V_{h,spot}$ is the volume that is not hedged for hour h and S_h is the spot price for that hour.

$\boldsymbol{\theta}$ is the parameter vector consisting of the volumes and time points for trading of different contracts, which must belong to the feasible region $\boldsymbol{\Omega}$.

$$\boldsymbol{\theta} = (V_{year}, V_{Q1}, V_{Q2}, V_{Q3}, V_{Q4}, i_{year,1}, \ldots, i_{year,J_{year}}, i_{Q1,1}, \ldots, i_{Q1,J_{Q1}}, \ldots, i_{Q4,1}, \ldots, i_{Q4,J_{Q4}})$$

where $i_{k,x}$ states *when* the xth trading occasion for contract k occurs and J_k is the number of trading occasions for contract k (so that $i_{Q1,J_{Q1}}$ represents the time of the last trading occasion for the forward contract for the first quarter, for example).

We define the volumes $\widehat{V_{Q_n}}$, which are the average consumption volumes per hour for each quarter in the consumption profile, as:

$$\widehat{V_{Q_n}} = \frac{1}{hours_{Q_n}} \sum_{h=1}^{hours_{Q_n}} V_h, \quad 1 \leq n \leq 4$$

where $hours_{Q_n}$ is the total number of hours in quarter n.

The constraints for the feasible region $\mathbf{\Omega}$ are:

- $$V_{year} + V_{h,Q_n} + V_{h,spot} = V_h$$

  where V_h is the total electricity consumption for hour h, i.e. the hedge volumes + the volume not hedged add up to the total volume for each hour. All portfolios under study in this thesis are required to buy the same volume of electricity for each hour, which is given by the used consumption profile. This, of course, means that if the hedge volumes increase, the volumes bought on the spot market decrease by an equal amount.

- $$0 \leq V_{year}$$
  $$0 \leq V_{Q_n}, \quad 1 \leq n \leq 4$$

  meaning that short positions are not allowed in the forward contracts. Short positions are typically not used by portfolio managers whose task is to hedge electricity consumption. Producers of energy of course sell forward contracts, but that is not the focus for this thesis.

- $$\alpha\widehat{V_{Q_n}} \leq (V_{year} + V_{Q_n}) = V^*_{Q_n} \leq \beta\widehat{V_{Q_n}}, \quad 1 \leq n \leq 4$$

  meaning that the given policy formulation states that the hedged volume should be inside a given interval. A typical value for the hedged volume is that it should be $70 - 105\%$ of the total volume, resulting in $\alpha = 0.7$ and $\beta = 1.05$. The volume $\widehat{V_{Q_n}}$ is taken to be deterministic since this thesis focuses on the perspective of a portfolio manager, which bases the trading strategy on the consumption plan given by the customer. If the customer deviates from the consumption plan, the cost does not fall on the portfolio manager.

- $$i_{k,x} \in \tau_k, \quad \forall i_{k,x}$$

  meaning that all trading occasions x for contract k must happen on allowed trading days. For example, the trading in yearly contracts is only allowed on weekdays, starting five years before the start of the target year (this has been the case historically which is why this thesis apply this rule, but trading in yearly forwards is now allowed from ten years prior to the target year, although not many firms engage in trading that far ahead in time) and ending on the last weekday before the beginning of the target year. Thus τ_{year} in this study consists of weekdays five years before the start of the target year, up to the last weekday prior to the target year.

The volume that needs to be bought at spot prices is the volume that is not covered by the hedge contracts:

$$V_{h,spot} = V_h - (V_{year} + V_{Q_n})$$

where V_h is the actual consumption during hour h and n is the quarter that hour h belongs to. Note that $V_{h,spot}$ may be negative if the portfolio is over-hedged, in which case the surplus is sold back to Nord Pool at the spot price for that hour.

P_{year} may be calculated by:

$$P_{year} = \frac{1}{V_{year}} \sum_{x=1}^{J_{year}} V_x \cdot P_x$$

where J_{year} is the number of occasions on which yearly contracts are bought, V_x is the volume bought on the xth occasion and P_x the price paid on the xth trading occasion. Similarly, this can be done for the four quarterly contracts. This means that the total cost for a forward contract (the second quarter is used in the example) is calculated by:

$$C_{Q2} = (30 + 31 + 30) \cdot 24 \cdot V_{Q2} \cdot P_{Q2} = hours_{Q2} \cdot V_{Q2} \cdot P_{Q2}$$

since there are $30 + 31 + 30$ days in the second quarter and 24 hours each day.

The total cost of the index portfolio for the same electricity consumption profile is given by:

$$\boldsymbol{C_I} = V^I_{year} \cdot P^I_{year} \cdot hours_{year} + \sum_{n=1}^{4} (V^I_{Q_n} \cdot P^I_{Q_n} \cdot hours_{Q_n}) + \sum_{h=1}^{hours_{year}} (V^I_{h,spot} \cdot S_h)$$

where h is the index ranging over the hours of the year, V^I_{year} is the volume in MW that is hedged by the yearly contract, P^I_{year} is the index price in EUR/MW of the yearly contract, $V^I_{Q_n}$ is the volume that is hedged by the forward contract for quarter n, $P^I_{Q_n}$ is the index price of that quarterly contract, $V^I_{h,spot}$ is the index volume that is not hedged for hour h and S_h is the spot price for that hour.

P^I_{year} is calculated by:

$$P^I_{year} = \frac{1}{|\tau_{year}|} \sum_{i=1}^{|\tau_{year}|} P_i$$

where $|\tau_{year}|$ is the number of trading days for the yearly contract, resulting in that P^I_{year} is the average price over the trading interval for the yearly contract. The quarterly contract prices are calculated similarly.

The volumes $V^I_{Q_n}$ and V^I_{year} are chosen as integer values belonging to the feasible region $\boldsymbol{\Omega}$ to simplify the index trading strategy by not making the daily traded volumes smaller than necessary. This is done by identifying the minimum of the $\widehat{V_{Q_n}}$ (typically $\widehat{V_{Q2}}$ or $\widehat{V_{Q3}}$), this is denoted by $\widehat{V_{Q_z}}$, and setting the corresponding $V^I_{Q_n} = 1$ so that the index portfolio trades in all of the selected forward contracts with at least 1 MW. Then, V^I_{year} is taken to be the largest integer value allowed by

$$\alpha\widehat{V_{Q_z}} - 1 \leq V^I_{year} \leq \beta\widehat{V_{Q_z}} - 1$$

Further, the rest of the $V^I_{Q_n}$ are set to be the largest integer value allowed by

$$\alpha\widehat{V_{Q_n}} - V^I_{year} \leq V^I_{Q_n} \leq \beta\widehat{V_{Q_n}} - V^I_{year}$$

This means that the parameter set for the index portfolio belongs to the feasible region $\boldsymbol{\Omega}$ and the index portfolio thus constitutes a possible trading strategy for a portfolio manager. The index cost can therefore be used as a benchmark for other electricity portfolios.

4.5 Price Models

The historical simulation approach used here collects prices in vectors and then draws with replacement an integer j from a uniform distribution on the set $\{1, \ldots, N\}$, where N is the length of the price vector. The modelled price is then the price in position j. This procedure can be motivated by noting that the prices are more or less stationary between different years, so that no transformation of the prices is needed.

While it is possible to simulate costs by using historical prices, another approach is to look at the historical prices and estimate their volatility and then produce a fictional simulation for how they might develop.

A suggested stochastic model for the spot price for hour h is:

$$S_h = S_{h-1} \cdot exp(\mu + \sigma W_h), \quad 2 \leq h \leq 8760$$

where μ is a constant representing the drift, σ is the volatility of the spot price and W_h are independent and identically distributed increments distributed as $W_h \sim N(0, 1)$. The starting price is set to be $S_1 = S_{avg}$, i.e. the average of all the used spot prices. The initial idea is to set $\mu = -\sigma^2/2 \approx -0.03$, since the expected value of an exponentially distributed random variable without any trend factor is $e^{\sigma^2/2}$. However, this resulted in a bit too cheap average price. Instead, by setting $\mu = -0.02$ the average spot cost for the stochastic model is about equal to the spot cost of the historical simulation model.

As the spot prices differ so much depending on whether it is winter or summer and whether it is a high or low load hour, the hourly spot prices are divided into four groups (x,y): summer low load (S,LL), summer high load (S,HL), winter low load (W,LL) and winter high load (W,HL). In order to prevent too small prices and prices that "blow up" (for instance, very expensive prices during the summer are unrealistic), the minimum and the maximum of the historical spot prices for the different price groups act as boundaries for the prices. The starting price is set to be the average price, i.e. $S_1 = S_{avg}$.

The suggested model gives a very volatile price that has a tendency to get stuck on the boundaries for several hours, which is not desirable. To get rid of this problem, a trend that works as a mean reversion mechanism is introduced by setting:

$$\mu_h = -0.02 + k \cdot ln\left(\frac{S_{avg}}{S_{h-1}}\right), \quad 2 \leq h \leq 8760$$

where k is a constant that determines the rate of the mean reversion effect.

Testing different values of k gives that $k = 0.2$ results in a decent model that still features price peaks of the same magnitude as the historical maximum prices, which is something that should be included in order to represent the risk of high spot prices.

The stochastic spot model price is thus given by:

$$S_h = S_{h-1} \cdot exp(\mu_h + \sigma W_h), \quad 2 \leq h \leq 8760$$

$$S_{min,x,y} \leq S_h \leq S_{max,x,y}$$

where $S_{min,x,y}$ and $S_{max,x,y}$ are the minimum and maximum prices of price group x, y and the price groups and parameters are as described above. This model will later be referred to as "the price group model".

Another model suggestion for the spot price that is similar to the one described above, is a discretized Ornstein-Uhlenbeck process. The Ornstein-Uhlenbeck process is mean-reverting and described by the following equation:

$$dx_t = \theta(\mu - x_t)\, dt + \sigma\, dW_t$$

where μ is the mean value, σ is the volatility, θ is a positive constant, and W_t denotes the Wiener process. Discretizing this equation and applying it on the spot price for hour h yields:

$$\Delta S_h = \theta(\mu_h - S_h) + \sigma W_h, \quad 1 \leq h \leq 8760$$

where μ_h is the mean price for hour h given by the prices during the years 2006-2013 (leap years are adjusted by removing February 29th), σ is the estimated volatility from how the prices differ from the mean (lower than the volatility in the previous model), and W_h are independent and identically distributed increments distributed as $W_h \sim N(0, 1)$. The starting price for the process is given by the years 2006-2013 average spot price for hour 1. To avoid unrealistic price peaks during the summer, the maximum and minimum prices for the last 30 days are used as boundaries, i.e. the price peaks to the left in Figure 5 given by the blue dashed curve act as an upper bound for 30 days after the peak. Since $1 + \Delta S_h$ is the first order Taylor approximation of the exponential function $e^{\Delta S_h}$, this leads to the suggested modified Ornstein-Uhlenbeck model for the spot prices:

$$S_h = S_{h-1} exp\left(\theta \cdot ln\left(\frac{\mu_h}{S_{h-1}}\right) + \sigma W_h\right), \quad 2 \leq h \leq 8760$$

$$\min_{h-30\cdot 24, h} S_x \leq S_h \leq \max_{h-30\cdot 24, h} S_x$$

where $\min\limits_{h-30\cdot 24,h} S_x$ and $\max\limits_{h-30\cdot 24,h} S_x$ denotes the minimum and maximum of the actual spot prices during 2006-2013 in the last 30 days prior to hour h. $\theta = 0.5$ gives a reasonable mean-reversion effect that still allows price peaks.

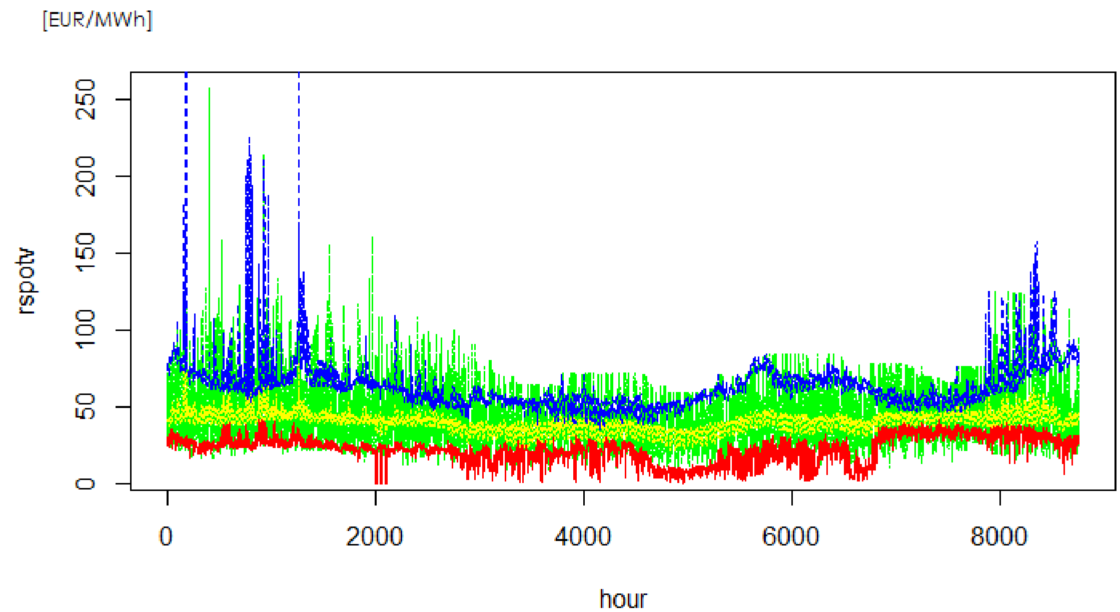

Figure 5: Graph illustrating the spot model prices for the modified Ornstein-Uhlenbeck process with $\theta = 0.5$ (green solid curve) based on the spot prices for the years 2006-2013. The average of those years' (with the exclusion of February 29th) price for each hour, i.e. μ_h in the modified Ornstein-Uhlenbeck process, is shown by yellow dotted curve. The minimum and maximum of those years' (with the exclusion of February 29th) prices for each hour are given by the red solid curve and the blue dashed curve.

Figure 5 displays that the spot model price is very volatile, but seems to fit decently *on average* with the historical data. The modelled spot price is based on μ_h, which is the yellow dotted curve in Figure 5. The correlation between the stochastic spot model price and the average price curve is 32%. To achieve a very high correlation is nigh on impossible since the modelled prices are stochastic. An accurate spot price model is very complex and depend heavily on weather forecasts (Sweco has three full time employees that has worked on a spot price model for two years, which suggests that an accurate model is beyond the scope of this thesis), while the model suggested here is not based on a specific year but rather how the spot prices may be *on average for a general year*. It is clear that the price peaks are much lower during the summer. This was a main objective for the model as the prices should be higher during the winter when the consumption also is higher. This holds for both models.

5 Method and Theory

5.1 Policy Evaluation

In order to evaluate an optimal trading policy for portfolio managers, historical simulation is used. The idea is to use the actual hourly electricity consumption for the property in Sweden owned by a certain real estate company for the year 2013 in order to evaluate different trading policies. The same profile is then used for all the years of interest, so that more data is taken into account which gives more legitimacy to conclusions. Since the plan for electricity usage is supplied by a real estate company, it is plausible to believe that the consumption profile does not change very much from year to year due to shifts in weekday dates and the profile may thus be used for different years. As the profile has an hourly resolution, the cost calculations given in Section 4.4 may be used. The constraints on the parameter vector $\boldsymbol{\theta}$ are also given in that section. A graph displaying the daily consumption for this profile is given in Figure 9.

The objective is to use a policy that solves the optimisation problem:

$$\min_{\boldsymbol{\theta} \in \boldsymbol{\Omega}} \quad E[\boldsymbol{C}(\boldsymbol{\theta})]$$

$$\boldsymbol{\theta} = (V_{year}, V_{Q1}, V_{Q2}, V_{Q3}, V_{Q4}, i_{year,1}, \ldots, i_{year,J_{year}}, i_{Q1,1}, \ldots, i_{Q1,J_{Q1}}, \ldots, i_{Q4,1}, \ldots, i_{Q4,J_{Q4}})$$

$$\begin{aligned} \boldsymbol{\Omega}: \quad & V_{year} + V_{h,Q_n} + V_{h,spot} = V_h \\ & 0 \leq V_{year} \\ & 0 \leq V_{Q_n}, \quad 1 \leq n \leq 4 \\ \alpha\widehat{V_{Q_n}} \leq (V_{year} + V_{Q_n}) = & \, V^*_{Q_n} \leq \beta\widehat{V_{Q_n}}, \quad 1 \leq n \leq 4 \\ & i_{k,x} \in \tau_k, \quad \forall i_{k,x} \end{aligned}$$

where $\boldsymbol{C}(\boldsymbol{\theta})$ is the total cost of the portfolio using the consumption profile given by the real estate company and the constraints are those that are described in Section 4.4. The dominating perspective in this thesis is that of a portfolio manager, not that of the company that orders the task of keeping the electricity costs down. This is because the portfolio managers are the ones doing the trading and the aim of the policy formulation is to buy fixed volumes in an optimal way (maybe under some restrictions), rather than limit the total power usage. The volumes are supposed to be small enough so that the trades do not influence the market prices, but the portfolio manager is large enough so that the index price corresponds to a possible trading strategy where the traded volumes (which need to be of a certain quantity to be allowed in the market) can be split between several portfolios to correspond to the small volumes used by the index portfolio.

Suppose there is an industrial company that orders portfolio management and that the policy is flexible (to some degree) with the hedge volumes. This allows for slight alterations with the volumes so that the effect of the hedge sizes is evaluated as well as when the contracts are traded. Here we take a flexible view on policy formulation, which allows changes in both the hedge volumes and the time points when contracts are bought. The consumption profile used in this analysis is that of the Swedish real estate company for 2013, as stated earlier.

The parameters that may be changed in the policy formulation are the different hedge volumes for different contracts, how large hedge volumes are bought on each occasion and when contracts should be bought. When the delivery period of a forward contract has started, there is no more trading in that contract.

The difference between the hedged volume and the actual power usage is bought or sold at spot prices. The index will use the same total volume, but always hedge the maximal integer volume in MW without over-hedging (which is when the hedged volume is larger than the consumed). The index also buys the difference between the hedged volume and the total volume to spot prices.

In this thesis, the hedge may be constructed by quarterly contracts and yearly contracts. By buying 1 MW of each of the four quarterly contracts, the hedge is equal to that of 1 MW in a yearly contract. By constructing hedge portfolios with either more quarterly contracts or more yearly contracts, it is possible to evaluate which of these hedge approaches is cheaper.

The initial hypothesis is that; if the forward prices are higher than the spot prices, as suggested by *Hansson* (2007) [5], it will be profitable to hedge the lowest volume that is allowed by the policy.

As previously mentioned, the total volumes have to be settled in the physical market where trading costs only depend on the volumes and since the total volumes are not varied, the trading costs are equal for a given consumption profile. The trading fees in the financial market are small enough to be neglected. Currency effects are also neglected since the study focuses on electricity costs, not exchange rates. Additionally, the study uses system prices without including contract-for-difference costs for different price areas in order to be generally applicable and not specific for a certain area.

The aim here is to vary how large the hedge volumes should be and how long before delivery that contracts are bought in order to minimise the expected portfolio cost. This is done by using the different simulation techniques described in the next section.

5.2 Simulation with Price Models

By collecting daily spot prices from January 1st 1999 up to March 25th 2014 and gathering them according to month [11], the 95%-quantile price for each month may be estimated from the historical data in order to represent the risk of expensive spot prices. These rather high prices will then represent the spot prices for the respective months in some of the simulations (whereas the actual hourly spot prices are used in other simulations, resulting in higher spot costs where the estimated prices are used).

For the spot price models in Section 4.5, the hourly spot prices for January 1st 2006 - December 31st 2013 are used, as the models work better if the prices are more or less stationary which they seem to be during this period (the prices show periodicity and fluctuate, but show no clear trend between different years).

Since the forward price curves are rather flat (meaning that no transformation of the prices is needed), the simulation of prices at which contracts are bought are randomly drawn with replacement among all available daily settlement prices. For example: a portfolio that consists of a total volume of 4 MW of a yearly contract bought at 4 different occasions is simulated by randomly drawing 4 prices from around 1 250 different prices (the yearly contract is available for 5 years, with around 250 trading days each year). This procedure can be motivated by noting that the contract may be bought on any of the 1 250 trading days that the contracts is available, unless prohibited by the trading policy. The actual portfolios that are evaluated will consist of different contracts, not just a yearly forward contract, and will also consist of larger volumes.

In order to evaluate different policies, the volumes that should be bought during specific time periods are given and then prices are simulated by drawing from the prices corresponding to these periods, rather than buying at prices randomly selected from all available prices. The unconstrained simulation may for example correspond to a manager that buys the full hedge volume during a few days with several years to delivery, which can be argued to be unrealistic.

While it is possible to simulate policies for different years by using the prices for that year, a model that is relevant for an unknown future year may also be wanted. This can be done by pooling together forward contracts for a number of years. The prices for the years 2008-2016 are used here, since the price indices for these years seem stationary (see Figure 11). The spot prices for January 1st 2006 - March 25th 2014 are then divided into the price groups suggested in Section 4.5 and may be used either for the "price group model" given in that section or for historical simulation. The modified Ornstein-Uhlenbeck model uses the hourly spot prices for January 1st 2006 - December 31st 2013. Altogether, the simulations in this thesis are done in three different ways:

- Method 1: Historical simulation of forward prices for a specific year with actual spot prices for that year. This method allows to simulate prices for the forward contracts from different time periods of the trading period according to a certain policy formulation, corresponding to the trader buying hedge volumes in predefined periods. This clear interpretation is something that the method where prices from several years are used for simulation lacks. The downside is that since the prices are known, it is easy to identify when contracts should be entered in order to give a low cost which is not something that is possible to do in reality. To evaluate policies properly, this procedure should be repeated for several different years and the average costs may then be compared. This method is used to determine how to divide the trading intervals of the different contracts, i.e. stating *when* to buy forward contracts.

- Method 2: Historical simulation of forward prices for the years 2008-2016 and historical simulation of spot prices using the spot prices for January 1st 2006 - March 24th 2014, as described above. This model seems reasonable, but is not very useful in evaluating different trading strategies describing *when* to enter forward contracts. It is more suited for comparing if bigger or smaller *volumes* should be hedged.

- Method 3: Historical simulation of forward prices for the years 2008-2016 with either of the spot models given in Section 4.5. This method is similar to the previous method and is thus suited for suggesting the *volumes* that should be hedged. The spot model will be slightly more expensive, on average, than the historical simulation of spot prices as the spot models have larger upsides than downsides in absolute terms. This is because of the relative price development that gives larger absolute change for higher prices. This increased risk of high spot prices is precisely what industrial firms want to hedge against. Methods 2 and 3 are used to determine the hedge volumes for different contracts.

6 Results

6.1 Index for Regular Households

A standard electric consumption used in price comparisons in newspapers is 20 000 kWh for a Swedish house that uses electricity for heating. For a typical small apartment without electricity-based heating, the yearly power consumption is around 2 000 kWh [10].

A regular year consists of $24 \cdot 365 = 8760$ hours. The usual consumption profile is higher during the winter, meaning that more power is needed during the first and fourth quarters of a year. The consumption profiles are for a typical small apartment and an electricity-heated house in the Stockholm area [18]. The low consumption for the apartment in January and also in August-September could be due to the fact that the tenant was away for some time. This is not crucial for the analysis since the same profile is used for all years and for both fixed and flexible prices.

The consumption profiles used here are shown in Table 1.

Table 1: Table showing power consumption profiles for a regular house and regular apartment each month as percentages of the yearly power usage [18].

Month	Part of house consumption [%]	Part of apartment consumption [%]
January	13	8
February	13	12
March	8	14
April	8	12
May	7	8
June	6	8
July	5	5
August	5	3
September	7	3
October	7	6
November	8	8
December	13	13

The indices for regular households use the monthly average of all reported prices to the Swedish Energy Markets Inspectorate. The indices only use prices in area $SE3$ since most of the Swedish population lives there. Also, the option to select electronic invoicing is used when collecting the index costs so that no unnecessary billing costs are included in the price comparisons [10].

This means that the index costs for regular households were constructed by multiplying the average consumer price for an apartment or house, which is given in öre/kWh, by the yearly consumption volume and the monthly percentage given in Table 1. The resulting yearly costs, given in kronor, are shown in Figures 6 and 7.

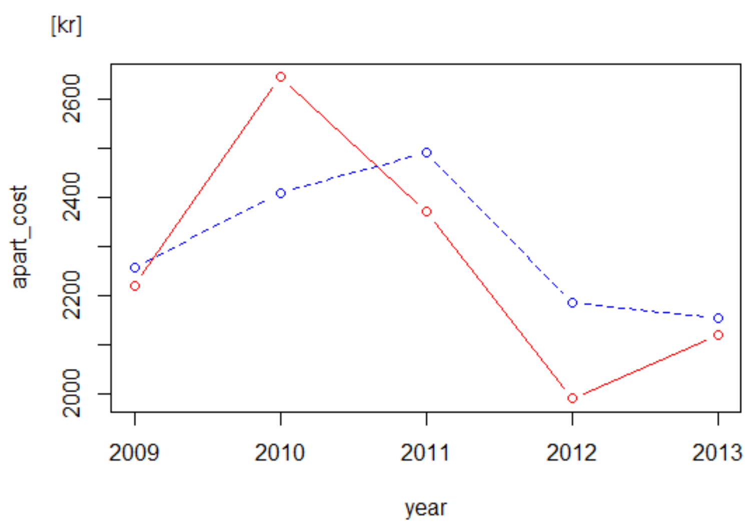

Figure 6: Graph illustrating the yearly index costs in Swedish kronor for an apartment which consumes 2 000 kWh each year, with a flexible price (red solid curve) and bound 1-year price (blue dashed curve).

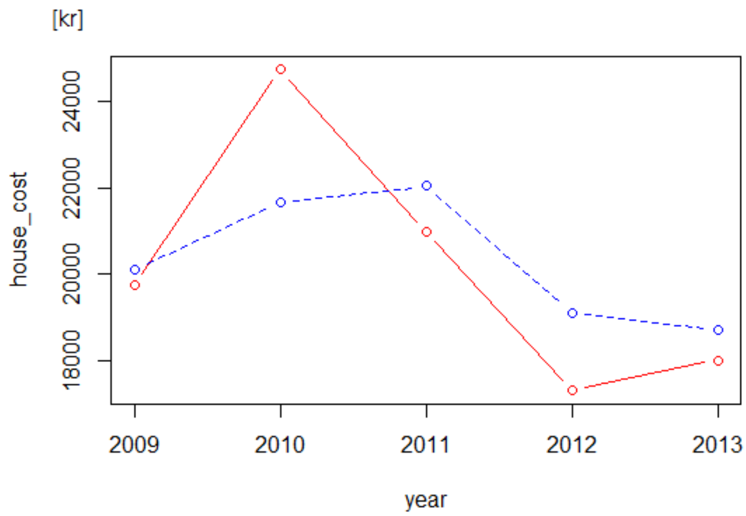

Figure 7: Graph illustrating the yearly index costs in Swedish kronor for a house which consumes 20 000 kWh each year, flexible price (red solid curve) and bound 1-year price (blue dashed curve).

The graphs are similar for the apartment index and the house index. The year 2010 was the most expensive for both and 2012 was cheapest. Of course, the differences for different years are much larger for the house since its power usage is ten times that of the consumption of the apartment but the house also has larger differences between different months so that a larger part of the yearly consumption is used during the winter when the price for electricity is higher. We also see that the total house costs are not ten times the total apartment costs which shows that house owners pay less than apartment owners per kWh. The flexible price curves are more volatile and cheaper for most years than the bound 1-year prices, as expected. The only year where the bound prices were cheaper is in 2010, when the flexible prices was at its most expensive. This is also reflected in the fact that the bound 1-year prices are most expensive for 2011, which is the year after the flexible price peak so that price fixings made during 2010 are more expensive than deals made during other years.

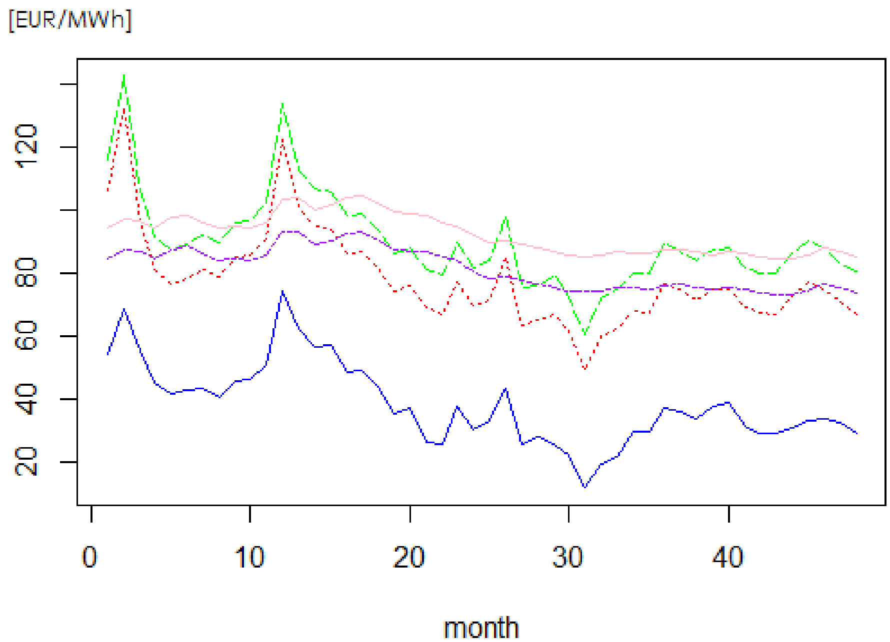

Figure 8: Graph displaying the average of the consumer prices reported to the Swedish Energy Markets Inspectorate for each month for the years 2010-2013, recalculated to EUR/MWh and adjusted for Value-added tax (VAT). The different curves display the spot price on Nord Pool (blue solid curve), flexible price for houses (red dotted curve), flexible price for apartments (green dashed curve), bound 1-year price for houses (purple double-dashed curve) and bound 1-year price for apartments (pink solid curve).

We see in Figure 8 that the flexible price curves are more or less the spot curve with a large parallel shift upwards to cover the energy company's costs. The bound 1-year prices are more expensive, on average, than the flexible prices, but vary less which is of course the purpose of bound prices. The Value-added tax (VAT) has been excluded.

6.2 Customised Index for Companies

For a larger customer it may be needed to be able to customise the index and comparative calculations according to a certain policy. Instead of trading an equal amount each day in the trading interval, a client may state that trading may start at a certain time or that some percentages of the total volume should be hedged in different periods. This results in indices that change the daily traded volumes during the trading interval. In the analysis conducted here, the index portfolios trade according to the unrestricted linear strategy.

A Swedish real estate company supplied information about their monthly consumption prognosis for 2014-2018 along with actual hourly power usage for 2013. Based on this information, it is possible to suggest hedge volumes for this portfolio. The real estate company owns houses in all Swedish price areas, but here we use the total consumption and suggest that contracts are traded at system prices in order to be able to use the available price history.

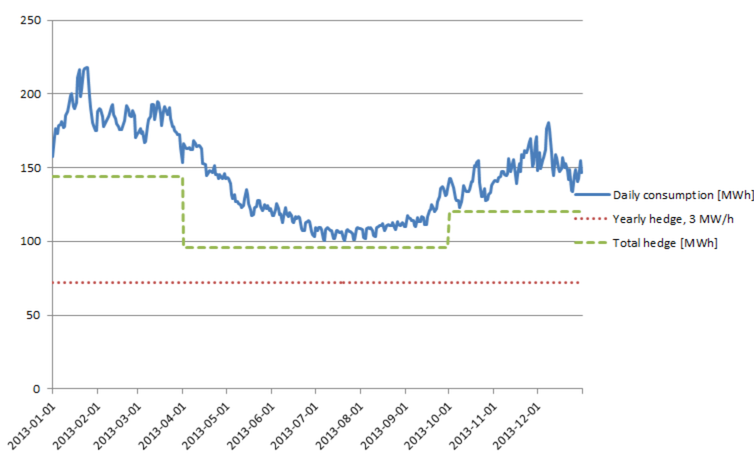

Figure 9: Graph illustrating the daily consumption for the previously mentioned real estate company for 2013 along with a suggested hedge portfolio, which buys at least 1 MW for each quarterly contract and more for Q1 and Q4 without overhedging for any day.

Figure 9 shows the consumption profile that is used in most of the analyses in this thesis. The consumption profile has an hourly resolution, but has been summed up to the daily resolution in the graph above to give a better overview. The hedge portfolio suggested here makes use of all the quarterly contracts by securing the price for 1 MW for Q2 and Q3 when the power usage is low, 3 MW for Q1, 2 MW for Q4 and 3 MW in the yearly contract. This gives the maximal hedge without overhedging the consumption for any day.

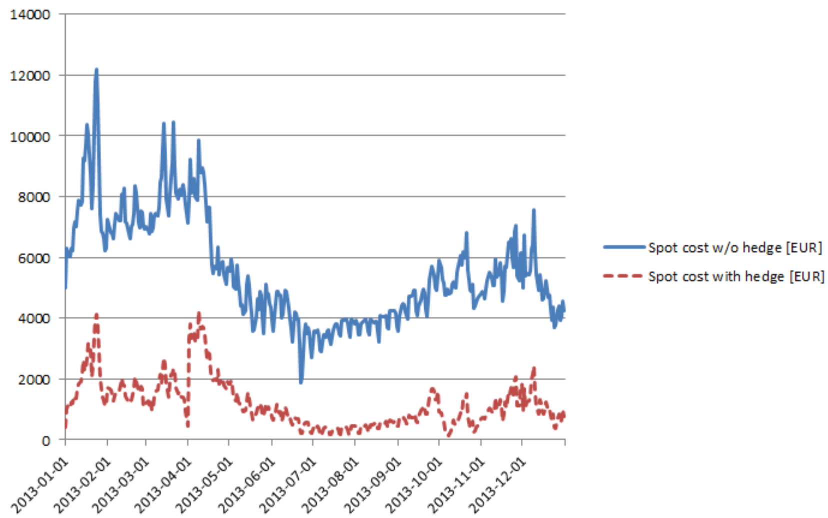

Figure 10: Graph illustrating the daily consumption for the real estate company multiplied by the daily settlement spot prices for 2013. The upper solid curve shows the daily costs when the whole volume is bought at spot prices and the lower dashed curve displays the spot cost when the volume suggested in Figure 9 is hedged, so that only the volume that is not hedged is bought at spot prices. The actual hedge cost is unknown but may be simulated by the historical simulation approach, so that the total costs may be compared.

Figure 10 creates a clear picture over how much the spot costs are lowered for the given consumption profile by using the hedge profile given in Figure 9. By comparing this graph to the previous one, we notice that while the smallest of the daily consumption values is about half the size of the largest value, the smallest cost on the unhedged curve is only about 1/6 times the biggest cost. This suggests that the spot price per kWh on the most expensive day must have been around three times the price per kWh of the cheapest day. The costs are given for each day, while most other analyses in this study compare total costs for whole years.

Figure 11: Graph showing the yearly index costs for the real estate company's consumption profile for 2013 moved in time, given in EUR. The red solid curve display the costs for the suggested hedge profile from Figure 9 and the blue dashed curve display the costs if the whole consumption is bought at spot prices for the respective years. The prices for the forward contracts for the years up to and including 2005 were given in NOK/MWh, where the exchange rates NOK/EUR for each trading day were used to calculate the prices in EUR.

As we see in Figure 11, there is no apparent evidence suggesting that the forward prices are overpriced since the average costs during this period (2000-2013) for the hedged portfolio is about equal to the portfolio with no forward contracts. In fact, the hedged portfolio has a slightly lower average cost for the whole period, but the spot portfolio is a bit cheaper during the more recent period 2006-2013. The overall trend of the portfolio prices is rising, which could in part be explained by an increase in the consumer price level and the introduction of emission trading rights.

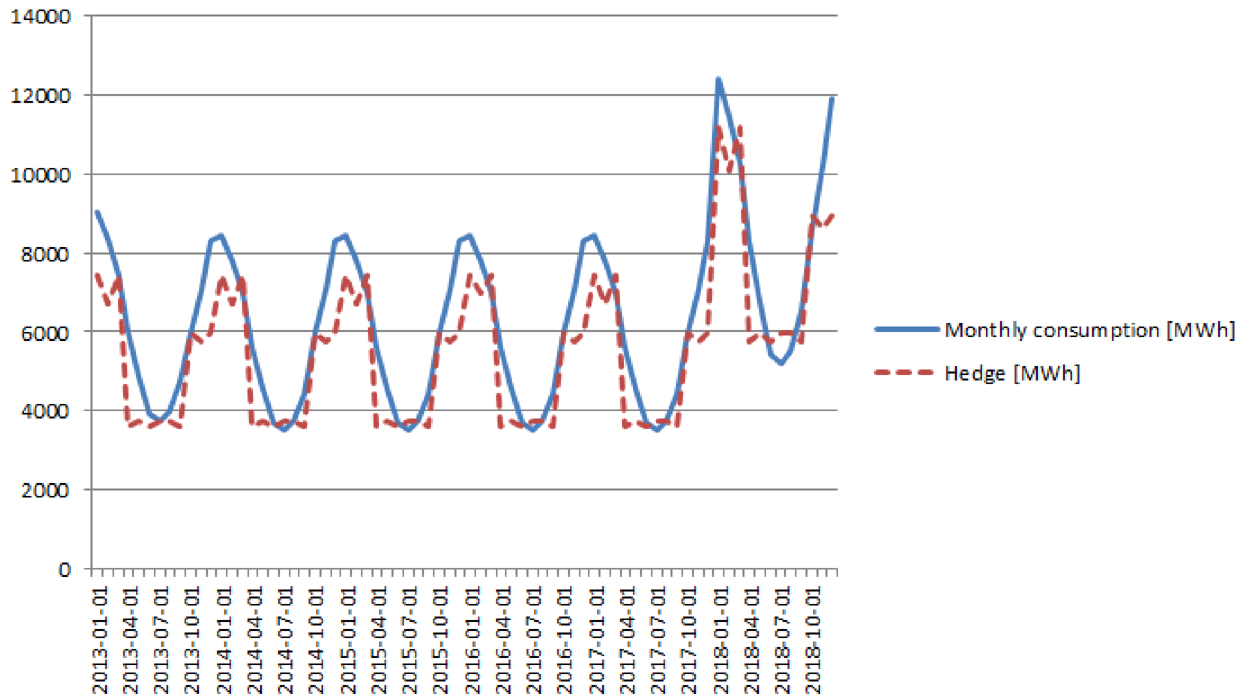

Figure 12: Graph illustrating the monthly total consumption plan for the previously mentioned Swedish real estate company 2014-2018 with actual consumption for 2013 along with the suggested hedge portfolio. They obviously plan to increase their property in 2018, which can be seen by the increased power usage. The hedge portfolio is constructed by yearly and quarterly contracts bought at system prices.

6.3 Risk Measures for Simulated Portfolios

Simulations based on historical data where contracts are bought according to an unrestricted random draw with replacement from all available prices resulted in the histogram shown in Figure 13. 1 MW was bought on each trading occasion and the sample size was 10 000. The volumes were decided from the consumption plan for 2015 supplied by a Swedish real estate company, for which the suggested hedge buys 4 MW for the yearly contract, 6 MW for Q1, 1 MW for Q2, 1 MW for Q3 and 4 MW for Q4. This is shown in Figure 12.

The average cost of the simulations was 2 438 305 EUR. The corresponding index cost was 2 439 448 EUR. The average cost and the index cost are very similar, but the average cost is slightly smaller which is likely an effect of that more than half of the available prices are cheaper than the average price since the forward prices drop as the time to delivery decreases. The average price of the yearly forward contract for 2015 is shown in Figure 14.

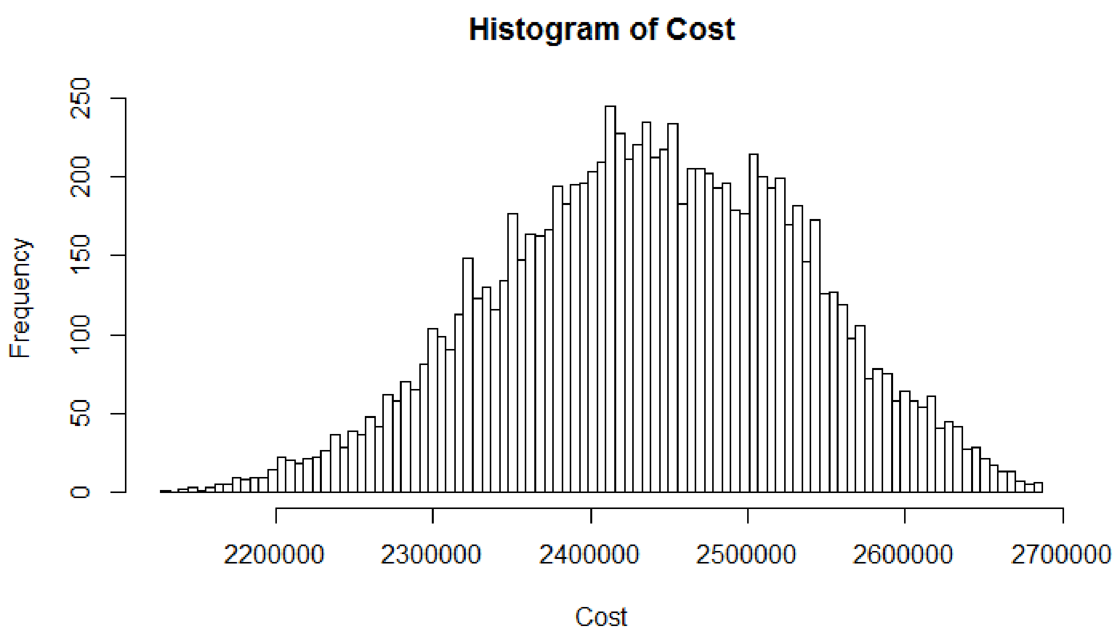

Figure 13: Histogram showing the simulated costs for 4 MW for the yearly contract, 6 MW for Q1, 1 MW for Q2, 1 MW for Q3 and 4 MW for Q4. 1 MW is bought at each simulated price. The sample size was 10 000.

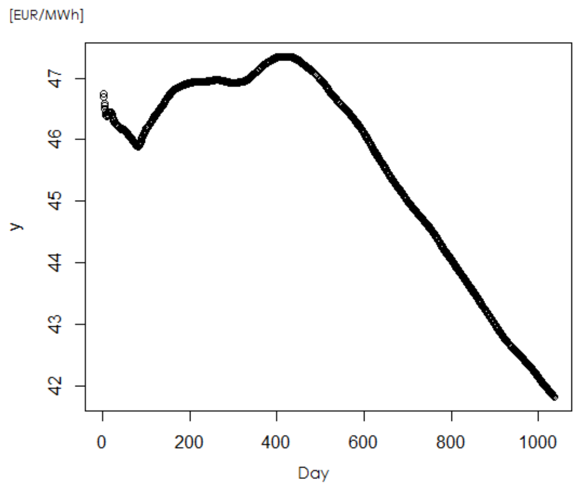

Figure 14: Graph displaying how the average price for the trading interval up to a certain day of the forward contract YR-15 has developed during the period of 1 027 trading days.

By comparing the simulated portfolio costs to the index portfolio cost, we define the risk measures as:

$$\rho(-\boldsymbol{C}(\boldsymbol{\theta}) + \boldsymbol{C_I})$$

since lower costs are preferable to higher. $\boldsymbol{C}(\boldsymbol{\theta})$ is the portfolio cost given the parameter vector $\boldsymbol{\theta}$ and $\boldsymbol{C_I}$ is the cost of the index portfolio.

Using 4, 6, 1, 1 and 4 trading occasions for the yearly, Q1, Q2, Q3 and Q4 contracts respectively, the resulting risk measures were:

$VaR_{0.01} = 231\,117$ EUR
$ES_{0.01} = 252\,499$ EUR
$VaR_{0.05} = 164\,438$ EUR
$ES_{0.05} = 203\,129$ EUR

This means that 95 % of the simulations made a smaller loss than 164 438 EUR compared to the index portfolio, as given by the VaR value.

By ignoring the constraint that at least 1 MW should be traded on each occasion (which can easily be done by portfolio managers if they add portfolios together and then divide them), we use 8, 12, 2, 2 and 8 trading occasions for the yearly, Q1, Q2, Q3 and Q4 contracts respectively. This resulted in the risk measures:

$VaR_{0.01} = 159\,351$ EUR
$ES_{0.01} = 182\,826$ EUR
$VaR_{0.05} = 114\,899$ EUR
$ES_{0.05} = 142\,608$ EUR

The average cost of these simulations was 2 439 833 EUR, which is slightly higher than when half the number of trading occasions was used. As could be expected, buying contracts on more occasions (so that smaller volumes are bought on each occasion) centres the outcomes around the index cost. This also lowers the risk measures for the simulations.

When evaluating policies by using the estimated 95%-quantile spot prices the hedging strategy is to hedge as much as possible, which could be expected since these spot prices represent a high spot price risk.

6.4 Policy Evaluation for Simulated Portfolios

Optimisation algorithms based on the methods described in Section 5.2, where 1000 simulations were made for each parameter set, were used to suggest strategies for portfolio managers. Methods 2 and 3 were used to find optimal hedge volumes and method 1 used these volumes and evaluated how to divide the trading interval.

Method 3 with the "price group model" was applied on the parameters

- V_{year}
- $V_{Q_n}, \quad 1 \leq n \leq 4$

with the constraints

- $V_{year} + V_{h,Q_n} + V_{h,spot} = V_h$
- $0 \leq V_{year}, \quad 0 \leq V_{Q_n}, \quad 1 \leq n \leq 4$
- $\alpha\widehat{V_{Q_n}} \leq (V_{year} + V_{Q_n}) = V^*_{Q_n} \leq \beta\widehat{V_{Q_n}}$, with $\alpha = 0.7$ and $\beta = 1.05$.
  The average volumes per hour for each quarter of the used consumption profile were:
  $\widehat{V_{Q_1}} = 7.70, \quad \widehat{V_{Q_2}} = 5.50, \quad \widehat{V_{Q_3}} = 4.66, \quad \widehat{V_{Q_4}} = 6.12 \quad$ [MW]

This yielded the volumes:
$V_{year} = 2.51, \quad V_{Q_1} = 5.45, \quad V_{Q_2} = 1.48, \quad V_{Q_3} = 0.92, \quad V_{Q_4} = 1.80 \quad$ [MW]

This results in the total hedge volumes (in MW) for each quarter (V_{year} added to each quarterly contract):

$$V^*_{Q_1} = 7.96 = 1.03 \cdot \widehat{V_{Q_1}}, \quad V^*_{Q_2} = 3.99 = 0.73 \cdot \widehat{V_{Q_2}},$$
$$V^*_{Q_3} = 3.43 = 0.74 \cdot \widehat{V_{Q_3}}, \quad V^*_{Q_4} = 4.31 = 0.70 \cdot \widehat{V_{Q_4}}$$

By using the modified Ornstein-Uhlenbeck model, the optimiser suggested the volumes:
$V_{year} = 3.17, \quad V_{Q_1} = 4.92, \quad V_{Q_2} = 0.94, \quad V_{Q_3} = 0.89, \quad V_{Q_4} = 1.14 \quad$ [MW]

This results in the total hedge volumes for each quarter (volumes in MW):

$$V^*_{Q_1} = 8.09 = 1.05 \cdot \widehat{V_{Q_1}}, \quad V^*_{Q_2} = 4.11 = 0.75 \cdot \widehat{V_{Q_2}}$$
$$V^*_{Q_3} = 4.06 = 0.87 \cdot \widehat{V_{Q_3}}, \quad V^*_{Q_4} = 4.31 = 0.70 \cdot \widehat{V_{Q_4}}$$

Method 2 was applied on the same parameters with the same constraints as above, which resulted in:
$V_{year} = 2.99, \quad V_{Q_1} = 5.09, \quad V_{Q_2} = 0.94, \quad V_{Q_3} = 0.97, \quad V_{Q_4} = 1.32 \quad$ [MW]

This results in the total hedge volumes for each quarter (volumes in MW):

$$V_{Q_1}^* = 8.08 = 1.05 \cdot \widehat{V_{Q_1}}, \quad V_{Q_2}^* = 3.93 = 0.71 \cdot \widehat{V_{Q_2}}$$
$$V_{Q_3}^* = 3.96 = 0.85 \cdot \widehat{V_{Q_3}}, \quad V_{Q_4}^* = 4.31 = 0.70 \cdot \widehat{V_{Q_4}}$$

The results suggest that the historical forward prices for the first quarter are cheap compared to the historical spot prices for that quarter since the optimiser hedges the largest allowed volume for that quarter. On the other hand, the forward contracts for the second and fourth quarters seem to be relatively expensive compared to the spot prices during these periods as the optimiser suggests hedging as little as possible for these quarters. The hedge for the third quarter is positioned in the middle of the allowed interval, which indicates that the forward prices are similar to the spot prices for that quarter.

The results for the "price group model" do not differ very much from the results of the simulations with the modified Ornstein-Uhlenbeck model, which suggests that they are consistent with each other. As the results for method 3 are consistent with those for method 2, it seems as the spot models given in Section 4.5 are consistent with the historical prices. This is not surprising since the spot models are based on the historical prices.

Method 1 was used to suggest *when* to buy hedge contracts. This was done by dividing the trading interval for forward contract k into n_k parts, where n_k is the number of trading occasions for contract k. The optimisation algorithm will thus suggest $n_k + 1$ parameter values for the respective contract so that a contract that is traded on 1 occasion anywhere on the allowed interval will have the parameter values $[0, 1]$. The fixed parameter values were:

- $V_{year} = 2.51, \quad V_{Q_1} = 5.45, \quad V_{Q_2} = 1.48, \quad V_{Q_3} = 0.92, \quad V_{Q_4} = 1.80, \quad$ [MW]
  i.e. the hedge volumes suggested by the first run with the optimiser with method 3.
- $n_{year} = 3, \quad n_{Q_1} = 6, \quad n_{Q_2} = 2, \quad n_{Q_3} = 1, \quad n_{Q_4} = 2$
  i.e. the respective volume rounded up to the next integer value as the number of trading occasions must have discrete integer values.

The constraints are:

- $i_{k,x-1} < i_{k,x}, \quad \forall k, \quad 1 \leq x \leq n_k$
  meaning that trading occasion x in contract k will happen on the interval $[i_{k,x-1}, i_{k,x}]$ of the total allowed trading interval for contract k.
- $0 \leq i_{k,0} \leq 0.5, \quad \forall k$
  so that no contract may be traded before the allowed trading interval begins. At most, the first half of the trading interval for each contract may be ignored.
- $0.5 \leq i_{k,n_k} \leq 1, \quad \forall k$
  so that no contract may be traded after the allowed trading interval ends and that the trading may not be completed in the first half of the interval.

The simulations were repeated for each of the years 2006-2013. These years were chosen since they were used earlier in the study and the prices are quite stationary which brings consistency to the analysis. Also, the collected prices for the forward contracts are given directly in EUR/MWh for these years, whereas for earlier years the prices are in NOK/MWh so that prices need to be recalculated. The length of the trading intervals for these contracts are 5 years for the yearly contracts, 2 years for the Q1 contracts, 2.25 years for the Q2 contracts, 2.5 years for the Q3 contracts and 2.75 years for the Q4 contracts. The suggested trading intervals are shown in Tables 2 - 3, where the parameter values are multiplied by the allowed trading interval for the respective contract.

Table 2: Table with suggested trading intervals for yearly and Q1 contracts for the years 2006-2013.

Year	$i_{year,0}$	$i_{year,1}$	$i_{year,2}$	$i_{year,3}$	$i_{Q1,0}$	$i_{Q1,1}$	$i_{Q1,2}$	$i_{Q1,3}$	$i_{Q1,4}$	$i_{Q1,5}$	$i_{Q1,6}$
2006	0.014	0.018	0.022	0.985	0.144	0.150	0.513	0.514	0.552	0.556	0.956
2007	0.342	0.344	0.361	0.501	0.064	0.080	0.096	0.358	0.678	0.993	0.996
2008	0.009	0.010	0.021	0.928	0.041	0.048	0.592	0.593	0.597	0.846	0.852
2009	0.381	0.382	0.384	0.998	0.044	0.091	0.316	0.586	0.633	0.988	0.990
2010	0.187	0.218	0.932	0.935	0.105	0.156	0.580	0.587	0.588	0.871	0.886
2011	0.131	0.722	0.722	0.764	0.082	0.083	0.370	0.378	0.779	0.780	0.923
2012	0.199	0.545	0.548	0.985	0.112	0.118	0.356	0.367	0.998	0.999	1
2013	0.414	0.451	0.947	0.950	0.032	0.068	0.711	0.712	0.722	0.726	0.998
average	0.210	0.336	0.492	0.881	0.078	0.099	0.442	0.512	0.693	0.845	0.950
all years	0.232	0.240	0.241	0.995	0.066	0.086	0.531	0.532	0.536	0.987	0.991

Table 3: Table with suggested trading intervals for Q2, Q3 and Q4 contracts for the years 2006-2013.

Year	$i_{Q2,0}$	$i_{Q2,1}$	$i_{Q2,2}$	$i_{Q3,0}$	$i_{Q3,1}$	$i_{Q4,0}$	$i_{Q4,1}$	$i_{Q4,2}$
2006	0.487	0.488	0.501	0.017	0.505	0.372	0.373	0.501
2007	0.009	0.019	0.993	0.058	0.502	0.007	0.051	0.502
2008	0.009	0.498	0.997	0.402	0.500	0.368	0.370	0.501
2009	0.055	0.060	0.999	0.010	0.999	0.493	0.996	0.998
2010	0.434	0.769	0.771	0.459	0.500	0.426	0.427	0.706
2011	0.073	0.074	0.542	0.001	0.502	0.061	0.062	0.503
2012	0.029	0.991	0.993	0.020	0.996	0.495	0.992	0.995
2013	0.499	0.773	0.774	0.498	0.762	0.492	0.941	0.942
average	0.199	0.459	0.821	0.183	0.656	0.339	0.527	0.706
all years	0.038	0.043	0.997	0.005	0.526	0.031	0.032	0.804

By looking at the given *average* time points in Tables 2-3, we note that it is best in general to not start the trading in the yearly contracts during the first year that the contract is available ($i_{year,0} = 0.210 \cdot five\ years \approx one\ year$). The same conclusion is drawn from the optimisation run for *all years* 2006-2013 to see which division of the trading interval that minimises the sum of the costs for these years. Similarly, it looks as it is wise to wait around 2 months before buying any Q1 contracts, at least 1 month for Q2 contracts, start buying Q3 contracts early and wait 1 month for Q4 contracts. These are the best strategies for minimisation of the sum of costs for these years.

The trends for the different forward contracts differ much between different years, but in general the safest approach seems to be to buy most of the hedge volumes somewhere in the middle of the trading intervals as we have noted before that many contracts are relatively expensive when they are new on the market. Many of the contracts show a rising price trend over the intervals which give support to strategies which buy early. The third quarter is the hardest to draw conclusions for, as only 1 trading occasion is used for each of the Q3 contracts. These optimal divisions of the trading intervals also hold if the volumes are changed, as long as the number of trading occasions are fixed.

6.5 Analysis of Price Data

By carrying out analyses of how the price data is distributed, for example using time series, a deeper understanding may be gained.

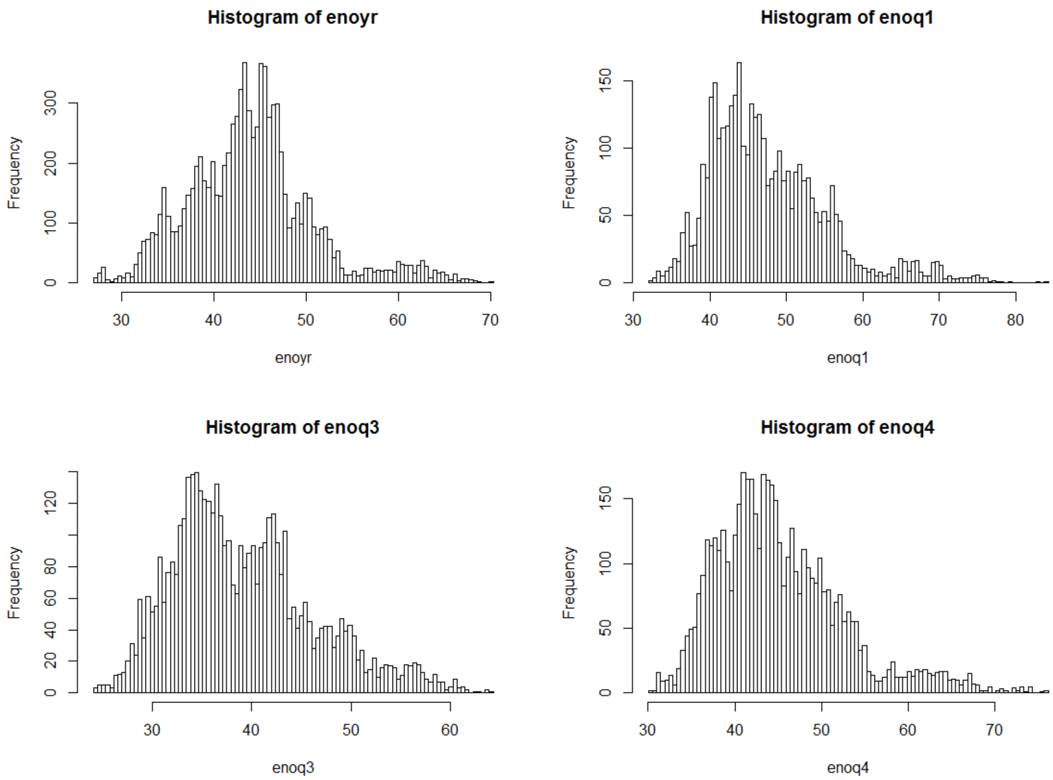

Figure 15: Histograms showing the historical price data for forward contracts for the years 2008-2016. Yearly contract prices (upper left), Q1 prices (upper right), Q3 prices (lower left) and Q4 prices (lower right) are displayed. The distributions are heavy-tailed with a skewness to the right.

Figure 15 shows empirical distributions for forward contracts applying for the years 2008-2016. The distributions are right-skewed, stating that the prices may increase more than they can decrease. As stated before, contracts for the first and fourth quarters are more expensive than for the second and third quarters which can be seen in the histograms (the distribution for the second quarter is similar to that of the third quarter).

The average prices for the respective contracts are:
$\widehat{P_{Q_1}} = 47.67, \quad \widehat{P_{Q_2}} = 40.14, \quad \widehat{P_{Q_3}} = 39.17, \quad \widehat{P_{Q_4}} = 45.14, \quad \widehat{P_{year}} = 44.03$ [EUR/MWh]

For comparison, the average spot prices during the respective period for the years 2006-2013 are:
$\widehat{S_{Q_1}} = 44.25, \quad \widehat{S_{Q_2}} = 37.47, \quad \widehat{S_{Q_3}} = 38.10, \quad \widehat{S_{Q_4}} = 43.03, \quad \widehat{S_{year}} = 40.71$ [EUR/MWh]

The distribution for the third quarter's contracts has the smallest tails and also the lowest average price, but the average spot price for the third quarter is more expensive than the average spot price for the second quarter. This indicates, as seen earlier, that the forward contracts for the third quarter are relatively cheaper compared to the spot prices than the forward contracts for the second quarter. The difference between the average price for a quarterly contract and the average spot prices for the respective quarter is largest for the first quarter, but since the optimisation algorithms suggested a large hedge for this quarter in Section 6.4 the conclusion is that it is desirable to minimise the risk of very high spot prices which are more frequent in the first quarter.

The average spot prices are lower than the average prices for the forward prices, but the fact that spot prices are higher when the consumption is higher needs to be remembered. Therefore, the forward prices that seem expensive are probably cheaper than the spot prices during the high load hours. Thus it is wrong to jump to the conclusion that it is cheaper not to hedge the electricity cost by forward contracts just by looking at the average prices.

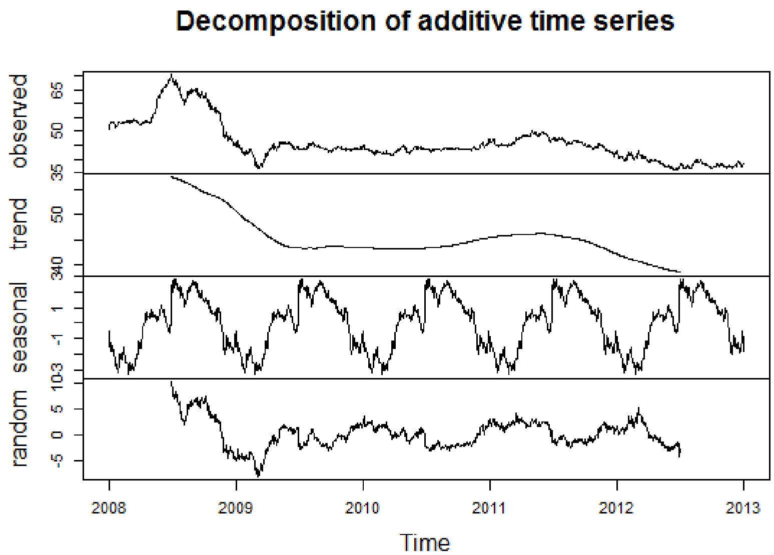

Figure 16: Decomposition of the time series for the price of the yearly contract for 2013. Trading in this contract was allowed during the years 2008-2012.

By analysing the price data for different forward contracts, it can be seen that they are not stationary. Figure 16 shows the decomposition of the time series for the price of the yearly contract for 2013. The optimal trading strategy given in Table 2 suggested that it was best to start buying in this contract after approximately two years into the trading interval, which is seen to be good advice in the observed data. The trend is mostly sloping downwards. The seasonal component is not very large compared to the prices, only ranging from -3 to 3, but it suggests that it was best to buy the yearly contract for 2013 during the first quarter of the years in the trading interval. This was the case for several years (2008, 2009, 2012), while for other years (2010, 2011) the contracts were cheaper during the middle of the year so no general conclusion can be drawn from the seasonal component.

By differencing the time series once, stationarity is achieved. Thus the forward price data may be described by ARIMA$(p, 1, q)$ (auto-regressive integrated moving average) models. The auto-correlation functions and partial auto-correlation functions for the stationary time series can give clues to the values of the parameters (p, q).

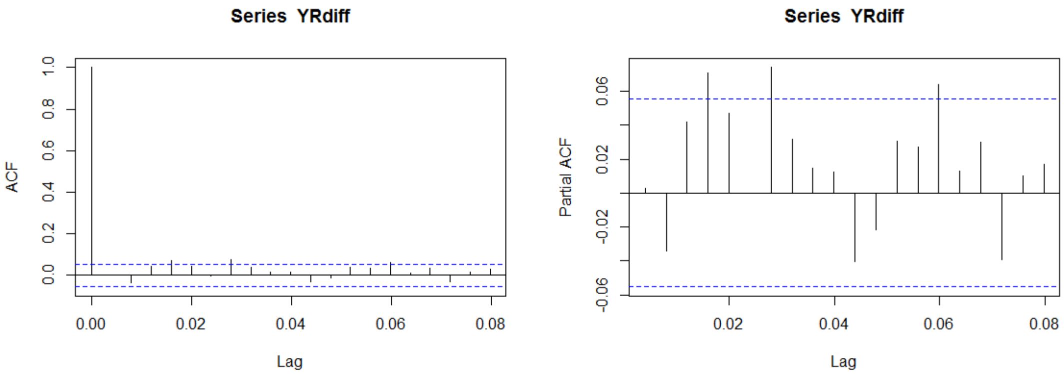

Figure 17: Auto-correlation function and partial auto-correlation function for the differenced time series for the yearly contract for 2013.

The left plot in Figure 17 does not give obvious values for the parameters (p, q) to the time series describing the price of the yearly contract for 2013. Candidate models seem to be ARIMA$(4, 1, 4)$ or ARIMA$(7, 1, 7)$ as lag 4 and lag 7 are above the significance threshold in the ACF and PACF. However, as too many parameters in the model is undesirable, the Akaike Information Criterion with correction for finite sample sizes (AICc) [1] suggests an ARIMA$(1, 1, 1)$ model for this contract. The coefficients were $ar1 = 0.96$ and $ma1 = -0.94$. By investigating other contracts, the AICc is minimised by an ARIMA$(2, 1, 2)$ model for most yearly contracts.

Neuman (2006) [8] described the spot price as an ARMA model. By analysing the hourly spot prices for the years 2006-2013 the result is that an ARIMA$(2, 1, 2)$ model minimises the AICc value for most years' spot prices and thus seems to be the best fit.

Figure 18 shows that the hourly spot prices for January 1st 2006 - March 25th 2014 are not normally distributed. While the medium high prices follow the normal quantiles, the tails are much heavier. In particular, high spot prices are much more common in the data than for a normal distribution.

A log-normal distribution is more heavy-tailed than a normal distribution and is thus a better fit for the spot price distribution. An estimation of the parameters in the log-normal distribution by maximum likelihood yielded $\mu = 3.63$ and $\sigma = 0.413$. Figure 19 and the right plot in Figure 18 shows that the log-normal distribution with these parameters is a decent fit for the distribution of the spot prices, although the distribution of the spot prices is more heavy-tailed.

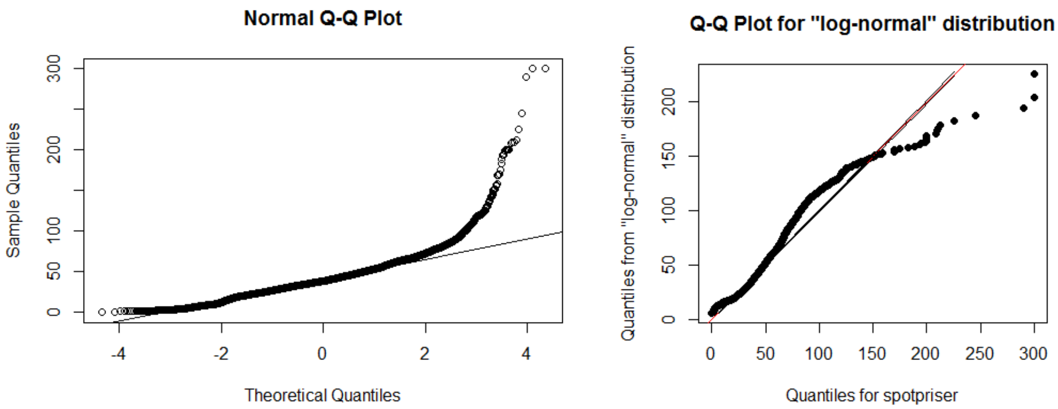

Figure 18: Quantile-quantile plots relating the hourly spot prices for January 1st 2006 - March 25th 2014 to the standard normal distribution (left plot) and to the fitted log-normal distribution.

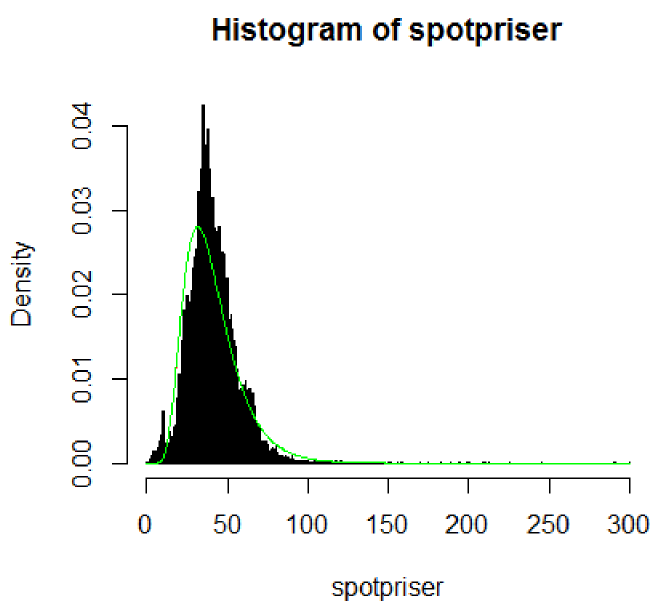

Figure 19: The empirical distribution of the spot prices (black area) compared to the fitted log-normal distribution (green solid line).

6.6 Correlation of Prices between Forward Contracts and Natural Resources

Policy formulations will probably differ for clients engaging in different business segments and also between different roles within the same segment. Two industrial companies that depend on the price of a certain natural resource, where one company earns more if the price is high and the other earns more with a low price, will most likely give different policies to portfolio managers. A positive correlation between the natural resource price and the forward contracts for electricity should result in a large hedge for the firm with higher earnings with higher resource prices and a smaller hedge for the other company, according to known hedging strategies [7].

Figure 20 shows the price indices for steel and yearly forward contracts. The correlation between these two indices is as high as 82.7%, suggesting that companies selling steel products should have large hedge volumes in forward contracts to lower their risk. This may mean that clients whose products show positive correlation between its prices and electricity forward contract prices will set a higher value of the parameter α, described in Section 4.4, than what is used in this study.

While forward prices does not depend on steel prices in the way that they depend on coal prices, it is likely that both forward and steel prices depend on many of the same factors. As we see a very large drop in the steel price index at the time of the financial crisis that started in 2008, a natural conclusion is that the steel price has a heavy dependence on the macroeconomic state. It is a known fact that natural resource prices were very high in early 2008 before the financial crisis [16], which is also shown in Figure 20.

This correlation between the price indices for a certain natural resource and the forward electricity contract indicates that it is likely that an industrial firm employing a portfolio manager would demand a higher degree of hedging in forward contracts than a real estate company whose income is not based on the price of the natural resource. The client whose product shows this kind of correlation with the forward prices is likely to set a higher value for the parameter α than 0.7 which was suggested as a typical value.

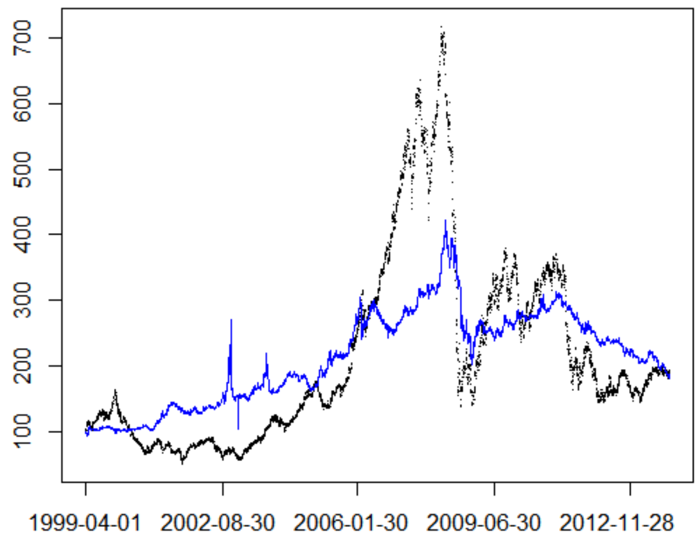

Figure 20: Graph showing how the steel price index (black curve) and the index for yearly forward contracts (blue curve) has developed from April 1st 1999 to April 1st 2014 (only days when both indices are valid are included).

The index for the yearly forward contracts is taken to be the average price of all the yearly contracts traded during a specific day and then normalised so that the average price on April 1st 1999 has the value 100. For example, the yearly forward index on a trading day during the year 2007 is the average price of the forward contracts for the years 2004-2008 divided by the average price of the forward contracts for the years 2000-2002 traded on April 1st 1999 (yearly contracts were only traded three years ahead of settlement at that time).

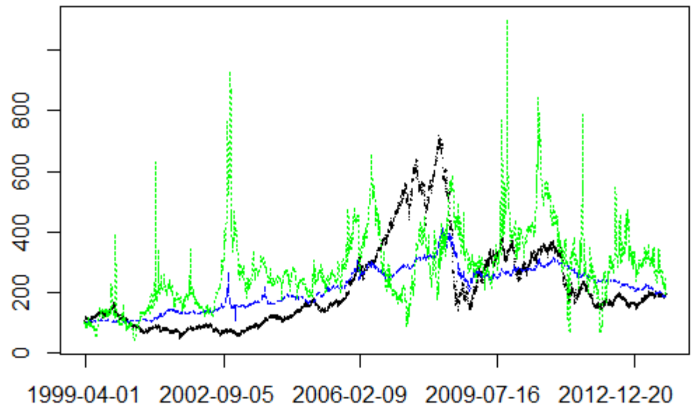

Figure 21: Graph showing how the steel price index (black curve), the index for yearly forward contracts (blue dashed curve) and the spot price index (green dotted curve) has developed from April 1st 1999 to April 1st 2014 (only days when the forward index is valid are included).

Figure 21 shows that the spot price is not as strongly correlated to the steel price index as the yearly forward prices. This indicates that the spot price depends on several other factors, such as precipitation, that the steel price and forward prices does not depend on, suggesting that a higher degree of hedging lowers the risk for firms selling products with high correlation to forward prices.

6.7 Parameter Changes' Effects on Cost Distributions

Figure 22 shows the cost distributions for four different parameter sets belonging to the feasible region. The simulations were done for the year 2010, when the spot prices were expensive resulting in that portfolios with larger hedge volumes are cheaper than portfolios with smaller use of forward contracts.

Figure 22 reveals very different shapes of the distributions for the different strategies. All the shown strategies and also the index portfolio have average costs of just over 2.5 million euros, except for the one shown in the lower left plot which had larger hedge volumes and bought them according to the policy with optimal division of the allowed trading intervals. This shows that large amounts of money may be saved by using a clever strategy. It is of course hard to determine an optimal trading strategy beforehand, but the suggestions made by the optimisation algorithms in this study give the best results for the years 2006-2013. The distributions will differ for different years, but the conclusion that more trading occasions centres the distribution around the index cost is still valid. By comparing the distributions to the right in Figure 22, it is seen that when 0.5 MW is bought on each occasion the extreme scenarios are less extreme than when 1 MW was bought at each time. This concludes that the risk is lowered by trading more similarly to the index strategy.

For a year with expensive spot prices like in 2010, it is likely that use of larger hedge volumes is profitable. This means that a higher value of the parameter α, as suggested in Section 6.6, will result in cheaper portfolios for years with expensive spot prices. On the contrary, when the spot prices are cheap a higher α value gives more expensive costs.

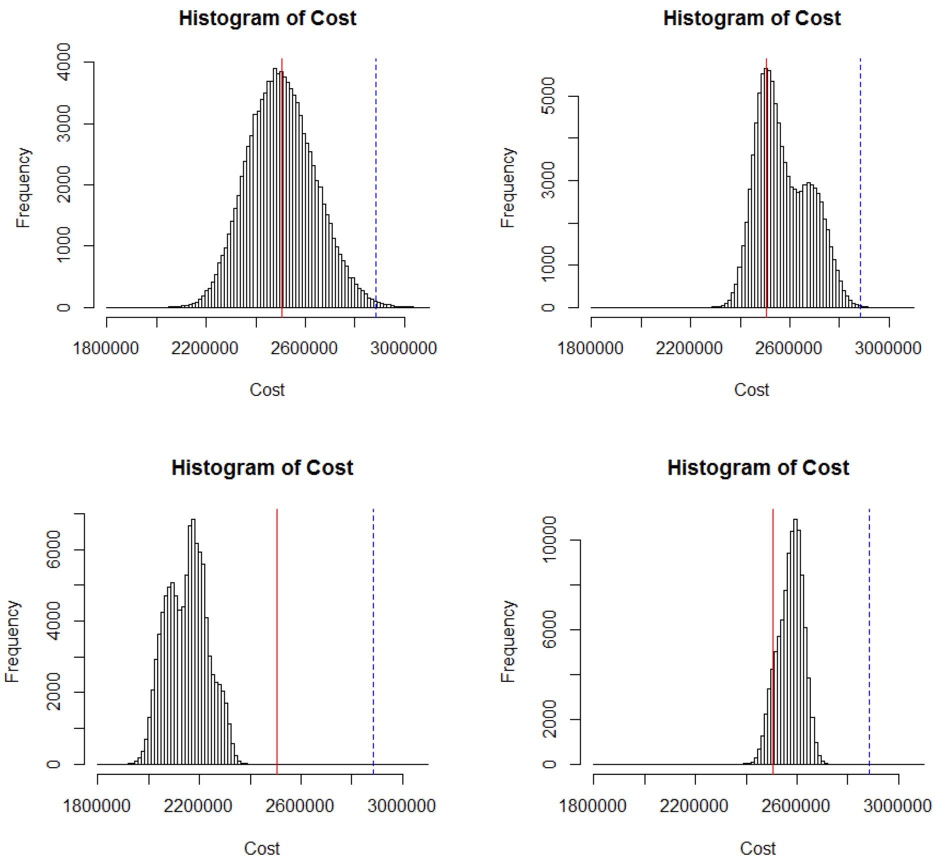

Figure 22: Cost distributions for electricity portfolios for the year 2010, which is the most expensive year yet. The upper left plot shows the result of portfolios with hedge profile (3,4,1,1,2) where the numbers indicate $(V_{year}, V_{Q1}, V_{Q2}, V_{Q3}, V_{Q4})$, i.e. the hedged volume in MW for the yearly forward contract, Q1 contract, Q2 contract, Q3 contract and Q4 contract, with 1 MW bought on each trading occasion and with no restriction on when to buy forward contracts. The upper right plot shows the costs of portfolios with hedge profile (2,4,1,1,2) with the trading intervals divided evenly so that 1 MW is bought in each fraction (1 MW is bought in each quarter of the trading interval for the Q1 contract, for example). The lower left plot displays the costs of portfolios with hedge profile (3,5,2,1,3), traded according to the policy with optimal division of the allowed trading intervals. The lower right histogram gives the distribution of portfolios with hedge profile (2,4,1,1,2), traded according to the even division policy but with only 0.5 MW bought on each occasion. The red solid line shows the cost of the index portfolio and the blue dashed line gives the cost with the whole volume bought on the spot market.

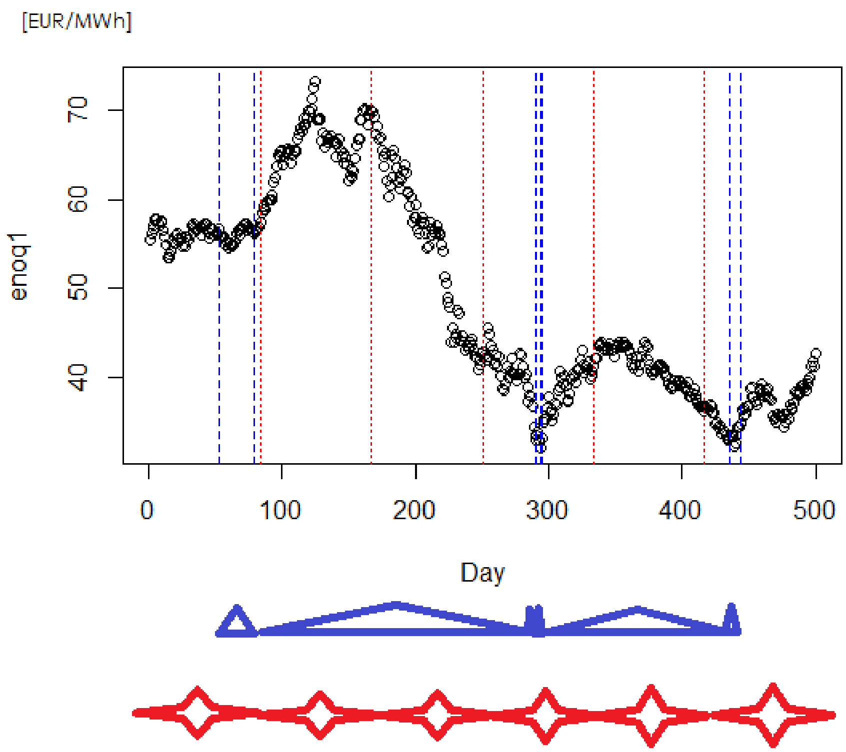

Figure 23: Figure showing the price development of the Q1 2010 contract over the trading interval. The blue dashed lines represent the suggested division of the trading interval so that 1/6 of the total volume for this quarter is bought between two adjacent lines (shown by the blue triangles below the graph where each represents one trading occasion, i.e. a wider triangle allow the trading occasion to happen at some point in a longer time interval than a thinner triangle). The red dotted lines show the even division of the trading interval so that each red star represents the interval in which 1/6 of the Q1 volume may be bought.

Figure 23 shows that the optimal trading policy has a narrower trading interval than the even division policy. It is clear that a strategy that buys a larger volume when the price is low, as the one suggested by the optimisation algorithm, results in a lower cost than a strategy that buys hedge volumes more evenly over the interval.

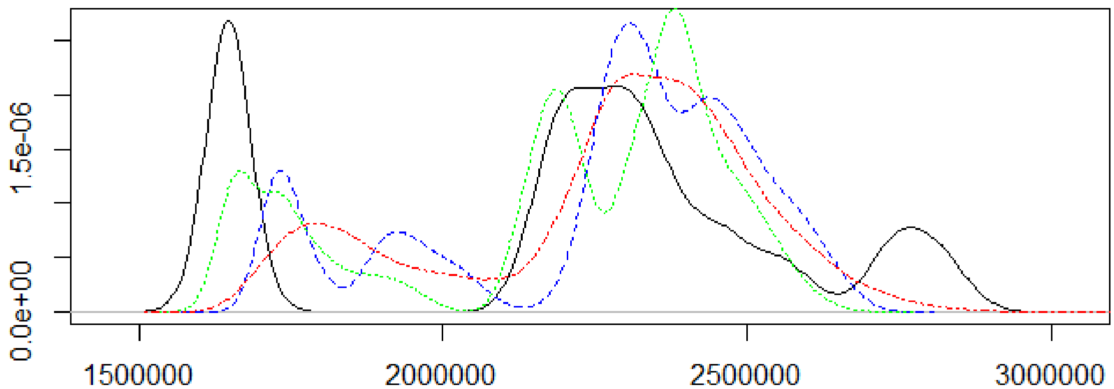

Figure 24: Figure showing densities for the empirical cost distributions for four different trading strategies for the years 2006-2013. A policy that divides the trading interval evenly but finishes the trading 1.5 years before the target year is shown by the black solid line. An unrestricted trading policy is given by the red dot-dashed line and the strategies that are suggested by the *average* and *all years* time points in Tables 2-3 are displayed by the blue dashed curve and the green dotted curve. 10 000 simulations were made for each strategy each year.

As can be seen in Figure 24, the trading strategy suggested by the optimiser for *all years* in Tables 2-3 gives the most narrow cost distribution in order to minimise the average of the costs for the years 2006-2013. This is because the price trends differ so much for these different years so that a trading strategy that gives the lowest average cost cannot have extremely high costs for any year. The trading strategy that finishes the hedge trading early (1.5 years in advance) is also good for these years as it avoids the high prices that feature at the end of the trading intervals for several of these forward contracts, but this strategy also has the heaviest tail which means that this is the most expensive strategy for the most expensive year of the studied period.

The resulting average costs, Value-at-Risk and Expected shortfall for these trading strategies for the years 2006-2013 are given in Table 4. Value-at-Risk and Expected shortfall are compared to the index cost. The strategies that are suggested by the *average* and *all years* time points in Tables 2-3 are denoted by *Average* and *All years* and the policy that divides the trading interval evenly but finishes the trading 1.5 years before the target year is denoted by *Secure early* in Table 4. The total yearly consumption for the profile supplied by the real estate company is 52 447 MWh, which is used to calculate the average costs per MWh.

Table 4: Table giving average costs, Value-at-Risk and Expected shortfall for different trading strategies for the years 2006-2013 with the previously used consumption profile. All values are given in EUR, except the last column which is in EUR/MWh.

Strategy	Average cost	$VaR_{0.05}$	$ES_{0.05}$	Average cost per MWh
Index	2 232 161	NA	NA	42.56
Average	2 262 890	436 712	455 657	43.15
All years	2 188 018	372 992	398 511	41.72
Secure early	2 204 090	628 254	646 060	42.02
Unrestricted	2 263 762	532 101	584 876	43.16

By adding the $VaR_{0.05}$ values to the average costs, the results are the 95% quantile values for the cost distributions. In particular, the unrestricted policy is taken to represent the 95% quantile value for an unknown trader that buys electricity for this consumption profile. This means that it is unlikely that this power consumption costs more than 2 795 863 EUR, which corresponds to the price 53.31 EUR/MWh.

7 Conclusions and Discussion

For private consumers it is often best with flexible price deals on electricity contracts. The fixed price deals are especially expensive the year after a year with high spot prices, so the recommendation is to stay cool and use the flexible prices which were cheaper for most of the studied years and also rather much cheaper on average. This is shown in Figures 6-8.

For larger clients, the analyses point toward the conclusion that forward contracts for the first quarter are relatively cheap compared to the historical and simulated spot prices for this quarter. Quarterly contracts for Q2 and Q4 seem a bit overpriced. These results suggest that it is profitable to hedge as much as possible for Q1 while hedging as little as allowed by the policy for Q2 and Q4. The hedge for the third quarter should be in the middle of the allowed interval to keep the risk down. The results for the yearly contracts vary, but Figure 11 indicates that the prices for the forward contracts seem consistent with the spot prices.

The price analysis also indicates that the contracts for the second quarter are overpriced since the average spot price for this quarter is much lower than the average price of the quarterly contract. This is also the case for the first quarter but since the risk of a very high spot price when the consumption also is high is larger for this quarter, the optimisers suggest a large hedging degree for the first quarter. The reason why Q2 contracts are overpriced could be due to that people are used to the high prices during the first quarter and that the forward prices do not drop as quickly as the spot prices for the second quarter. Fear of cold temperatures that would lead to high spot prices could influence why the Q4 contracts are relatively expensive compared to actual spot prices, since the spot prices are lower than what is believed.

The 95% quantile value for an unknown trader that buys electricity for the supplied consumption profile for the real estate company corresponds to the price 53.31 EUR/MWh, which means that it is unlikely that a similar power usage will pay more than this price on average for a year. This price is also higher than most of the forward prices so that a lower cost than this is easy to obtain. Taxes and EPAD prices also need to be added to this price to get the real price, since this thesis only focuses on system prices.

The main objective for many companies is to secure a guaranteed price several months before the target years starts for budget reasons, rather than trying to get a price as low as possible and risk a higher cost. For example, a real estate company must be able to announce electricity costs to its tenants several months in advance but since the cost mostly is forwarded to the tenants it is less important to get a cheap price. The 95% quantile value corresponds to a price around 50 öre/kWh, but since a private customer cannot trade directly on the Nord Pool market the price is higher for private consumers.

It seems best to wait for a year before starting to buy yearly contracts in order to get a low cost. For the quarterly contracts, the results indicate that it is best *on average* not to trade in quarterly forwards during the first month that they are available. Also, for the fourth quarter it looks as it is good to finish the trading six months before the delivery period.

The suggestions for policy formulation and trading strategies given by the simulations in this thesis are based on fictional scenarios with historical prices. The modelled prices are not based on weather forecasts or natural resource prices, which actual prices have a large dependence of.

The price analysis indicates that while the time series for electricity prices *are not* stationary, the differenced time series *are* stationary. Both the spot prices and yearly forward contract prices are best represented by ARIMA$(2,1,2)$ models for most years, which is suggested by the AICc criterion.

With a product whose prices show strong correlation with forward contract prices, which is the case for steel as shown in Figure 20, a client is likely to use a policy that hedges a larger volume of the electricity consumption with forward contracts to minimise the risks. For years with high spot prices, as in 2010, this is a successful strategy while on the other hand this could lead to unnecessarily high costs for years with low spot prices. But since the correlation is high, the company's profits are higher when the forward prices are higher. As a consequence of this, the company can more easily cover the electricity costs for the forward contracts than for the spot prices which do not show the same strong correlation with the commodity in the example. By choosing a lower hedge, the company is more exposed to weather effects such as dryness which gives a higher spot price. Because of the lower correlation level between the commodity and the spot price, there is a higher risk for expensive electricity costs without an increase in revenue with a smaller hedge.

The contracts in focus for this thesis are yearly and quarterly forward contracts. A topic of interest in further studies could be to look at how monthly or weekly forward contracts are priced compared to the spot prices. Forwards for shorter time periods are probably more fluctuating and dependent on recent weather effects, which could make such an analysis less conclusive.

The perspective of this thesis is that of a portfolio manager and not that of an industrial company. The portfolio manager can only base it's price deals on the given consumption plan, while in reality the actual consumption may differ from the plan due to unforeseen events. This affects the company that has ordered the portfolio management but cannot put any cost on the manager unless any fault has been committed. The analysis of volume risks could be an interesting topic for further studies.

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