EE-GSEC - An Energy Efficient Diversity Combining Scheme

by

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B.Eng., Mumbai University, 2007

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ABSTRACT

An energy-efficient diversity scheme based on the well researched Generalized-Switch-and-Examine Combining (GSEC) is presented. The presented scheme is more efficient in terms of providing better average combined SNR per active path. This results in considerable processing power savings of the receiver especially compared to the GSC scheme. EE-GSEC performance in terms of the average combined SNR, outage probability and average bit error rate (BER) are comparable to GSEC under certain conditions. EE-GSEC's complexity performance is better than GSC and same as GSEC. This results in a considerable hardware cost savings at the receiver. However, the complexity savings come at the cost of performance when compared to GSC. This is a natural trade-off and needs to be considered when designing a wireless communication system. A thorough statistical analysis of the presented scheme is performed and then used to mathematically formulate the performance and complexity expressions. Using simulations the performance and complexity of EE-GSEC is examined and compared against other competing schemes. An energy efficiency analysis that validates the efficiency claims of the scheme is also performed.

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DEDICATION

To my Parents

Chapter 1

Introduction

1.1 Need for Diversity in Wireless Communication

The performance of a wireless communication channel is severely degraded due to the phenomenon of fading. Fading can occur when the transmitted radio signal experiences

- **Multi-path propagation** due to reflections from terrestrial objects such as buildings.

- **Shadowing** due to interference from terrestrial objects causing random variations in the propagated signal's power.

Fading causes the transmitted signals to undergo divergent attenuation, distortions, delays and phase shifts. Due to this the receiver can experience constructive and destructive construction when processing the multi-path signals. Destructive construction causes significant degradation of the signal power. This further degrades, among other performance criteria, the error probability of the system.

In a wireless propagation environment the probability that a single signal path undergoes fading is very high. On the other hand if the information is transmitted over multiple independent signal paths, the probability that these path will undergo deep fading simultaneously is low. This technique, sending the same information over independent fading paths, is called diversity and it can drastically improve the performance a wireless communication system over fading channels.

1.2 Techniques for Achieving Diversity

Some of the most common methods for achieving diversity in a wireless system are:

1. Transmitting the same narrowband signal at different carrier frequencies. This technique is called **Frequency diversity.** Independent fading of the signal is achieved by separating the carrier frequencies by at least the coherence bandwidth of the channel. However, this technique requires additional transmit power to send the information over multiple frequency bands.

2. Transmitting the desired signal over different periods of time. This technique is called **Time diversity.** Independent fading of the signal is achieved by separating the time durations by at least the coherence time of the channel. This technique does not increase the transmit power. However, since the same information is transmitted in the time slots, instead of using the same time slots to transmit different information, the data rate of the system is reduced.

3. Using antenna array at the transmitter or receiver. This type of diversity is called **Space diversity.** Independent fading of the signal is achieved by spacing the antennas such that different received copies of the signal undergo independent fading. No additional requirement for transmit power and time/bandwidth usage on the system are the biggest advantage of this technique over the above mentioned techniques. However, physical constraints may limit its application as multiple antennas are required.

1.3 Traditional Diversity Combining Schemes

From the above discussion we observe that the goal of diversity is to mitigate the effects of fading by logically combining the multiple independent fading paths. There are multiple techniques developed to achieve the combining process. These techniques vary in their implementation complexity and overall performance. Some of the more traditional techniques include:

1. **Selection Combining (SC)**: In SC the combiner selects the path or antenna branch with the highest SNR. SC eliminates the need for co-phasing since only one branch output is used at a given time. Performance of SC has been a topic of much research. For example refer to [1] and [2].

2. **Switch Combining (SWC)**: In SWC the diversity paths are scanned sequentially and the first path with SNR greater than a pre-determined threshold is selected. This technique is also referred to as threshold combining. If the SNR of the selected branch drops below the threshold, the combiner switches to another branch. Refer to [2] and [3] for performance analysis.

3. **Maximal Ratio Combining (MRC)**: MRC utilizes the SC and SWC techniques more efficiently by 'weighing' each diversity path and combining the weighted SNRs. Diversity paths with high SNRs are weighted more than the paths with lower SNRs. The goal is to choose the weights that maximizes the channel gain. Refer to [4] and [5] for detail operation.

4. **Equal Gain Combining (EGC)**: EGC eliminates the need for full channel knowledge of MRC by simply co-phasing all diversity paths and combing them with equal weight. Refer to [2] for more analysis information.

These techniques have been well researched for their behavior, complexity to implement and performance. For example, refer to [4], [2] and [3]. MRC is the optimal combining scheme from a performance perspective. However, its highly complex implementation and sensitivity to channel estimation has been focus of much research. For example, refer to [4] and [5] . Briefly, the main constraints in using MRC as the combining scheme stems from the fact that MRC requires the system to have complete knowledge of the channel condition for each diversity path. In addition, the complexity of implementation of MRC increases as the available number of diversity path increases.

1.4 Advanced Diversity Combining Schemes

In order to reduce the complexity resulting from the aforementioned schemes a number of advanced diversity schemes have been proposed. These schemes are able to reduce the complexity by selecting only a subset of the available diversity paths and combing the selected paths in the MRC fashion [6]. The basic idea behind the implementation of these schemes is to select only the best diversity paths, based on the threshold test, and ignore the weak paths. This leads to a significant reduction in hardware complexity as the number of diversity paths for combining are reduced compared to traditional schemes.

The advanced diversity combining schemes include:

1. **Generalized Selection Combining (GSC)**: GSC is also referred to as hybrid selection/maximum ratio combining (H-S/MRC) scheme. The GSC combiner estimates and rank the SNR of all L available diversity paths. It then selects the L_c strongest paths and combine them in MRC fashion. The complexity savings stem from the fact that only L_c paths need to be processed as compared to L paths in the traditional MRC scheme. A number of variants resulting from further improvements to GSC have been widely studied. For example, refer to [2, 3] and [7–17]

2. **Generalized Switch and Examine Combining (GSEC)**: Although GSC and its variants are able to provide significant complexity savings over the traditional schemes, this complexity is still relatively high. In particular GSC and its variants require full SNR estimation of all the paths, which still entails a certain level of complexity. GSEC was proposed as a scheme for diversity rich environments that offers further reduced complexity as compared to GSC. GSEC implements a SNR threshold check and only those diversity paths whose SNR is greater than the pre-determined threshold are selected. GSEC tries to connect L_c acceptable paths out of available L paths to be combined in the MRC fashion based on a checking process. If enough acceptable paths are not found, GSEC randomly selects the remaining paths [18] and [6].

3. **GSEC with post-examining selection (GSECps)**: GSECps was proposed to offset the degradation of performance that could result from the random selection of unacceptable paths. The situation is magnified during poor channel conditions when the GSEC combiner, not being able to find enough acceptable paths, selects paths randomly that could have very low SNR. GSECps only selects the best unacceptable paths and combine them along with the acceptable paths leading to a better performance than GSEC [19] and [18].

Some examples of systems that implement the above mentioned diversity techniques include wideband code-division multiple-access (WCDMA) systems [7,8], ultra wideband (UWB) systems, millimeter-wave systems [9], and multiple-input/multiple-output (MIMO) systems [20].

1.5 Contribution and Organization of Thesis

The presented EE-GSEC scheme is a variant of the above described GSEC scheme. EE-GSEC's operation is the same as GSEC when there are enough acceptable paths available for combining. However, when enough acceptable paths are not available, EE-GSEC only uses the available acceptable paths and ignores the unacceptable paths altogether. As EE-GSEC only uses the best paths, it is able to offer better energy efficiency as compared to GSEC and even GSC. However, we can expect an efficiency-performance trade-off resulting from the fact that GSEC always uses L_c paths whereas EE-GSEC could use only L_a paths, where $L_a \leq L_c$, for MRC combining.

Chapter 2 introduces the presented scheme and describes the mode of operation in detail. A complete statistical analysis is performed to derive the expressions for the cumulative distribution function (CDF), the probability density function (PDF) and the moment generating function (MGF). We perform Monte Carlo simulation to verify and confirm the derived CDF. The statistical analysis are used to examine the performance analysis of the presented scheme in terms of average combined SNR, outage probability and average bit error rate (BER). A mathematical analysis of the complexity for the presented scheme is also performed. To investigate the energy efficiency of EE-GSEC we derive the expression for average number of active paths required by EE-GSEC. This metric is important in comparing the energy efficiency of EE-GSEC over GSC.

In Chapter 3 we perform $MATLAB_{©}$ simulations based on the mathematical formulations of Chapter 2 to further examine the performance and complexity of EE-GSEC. These are compared and evaluated against GSC and GSEC. In particular the performance-complexity trade-off between these schemes is examined. Based on the average combined SNR per active path analysis, EE-GSEC is compared against GSC. The results verify the energy efficiency of EE-GSEC over GSC.

Chapter 4 provides a summary of the work presented in this thesis and also suggests a road map for future research.

Chapter 2

System Model and Mode of Operation of EE-GSEC Scheme

2.1 Introduction

The proposed Energy Efficient Generalized Switch-and-Examine Combining (EE-GSEC) scheme is a variant of the GSEC scheme which, as the name suggests, improves the power savings at the receiver where the scheme is implemented. The power saving is achieved by utilizing only those diversity signal paths, also referred to as acceptable paths, that have their Signal-to-Noise (SNR) above a certain threshold and ignoring the other paths. This compared with GSEC (and even GSECps) that uses the unacceptable paths, paths with SNR less than the threshold, to match the inputs to the MRC combiner. However this entails a performance trade-off between the schemes that will be analyzed in Chapter 2.

We start by defining the system and channel model for the proposed scheme. We then perform statistical analysis for deriving the closed-form expression for the cumulative distribution function (CDF), the probability density function (PDF) and the moment generating function (MGF) of the output SNR of the EE-GSEC combiner. The statistical analysis will enable the performance analysis of the proposed scheme in terms of the outage probability, average combined SNR and average error rate. A complexity analysis is also performed to quantify the average number of path estimations and average number of comparisons required to select the acceptable diversity paths.

2.2 System and Channel Models

We examine the system and channel model for the EE-GSEC scheme. We start with introducing the receiver structure and proceed with examining the channel model.

2.2.1 Receiver Structure

The receiver for EE-GSEC needs to incorporate two basic building blocks for the implementation of the proposed scheme. These blocks are described below:

- **A Switch-and-Examine Combiner**: To examine the instantaneous SNR of each received diversity path and switch between the paths post examining [18].
- **A Maximal Ratio Combiner**: To implement an efficient scheme to coherently combine the independent paths to mitigate the effects of fading.

The two aforementioned blocks are shown in Figure 2.1.

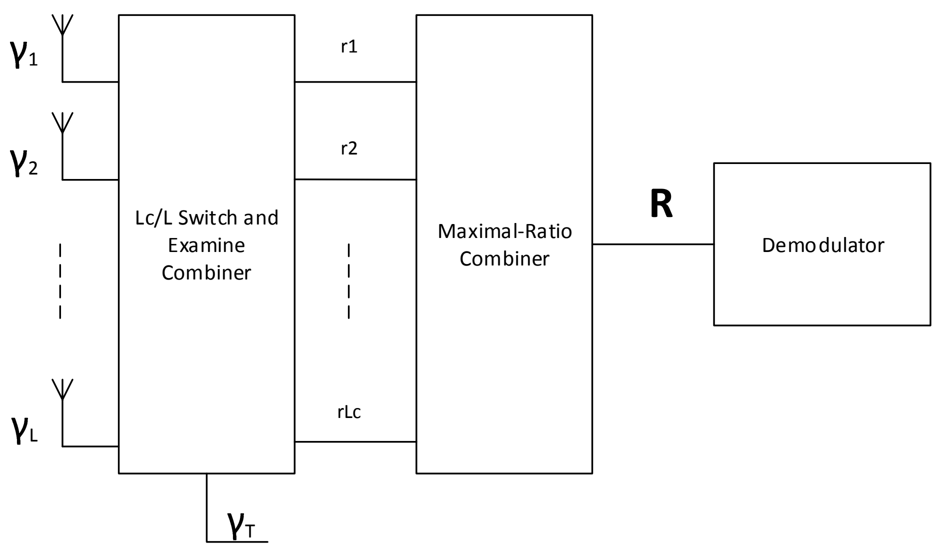

Figure 2.1: EE-GSEC Combiner Structure

The switch-and-examine combiner receives independent and identically distributed (i.i.d) diversity paths. In this case the combiner is receiving a total of L diversity paths with instantaneous signal-to-noise ratio (SNR) of $\gamma_i, i = 1, \ldots, L$. Depending on the number of branches of the MRC the combiner tries to select those paths whose instantaneous SNR is above a certain fixed pre-determined threshold, γ_T. In this case the combiner examines and tries to select L_c acceptable paths from the available

L diversity paths. Hence, the combiner can be designated as a L-input/L_c-output switch-and-examine combiner (SEC) (L_c/L SEC). Section 2.2.3 will describe the steps to update the paths of the L_c/L SEC.

The output of the L_c/L SEC combiner are up to L_c selected paths and are fed to the MRC to be combined as per MRC rule. The instantaneous SNR for these paths are represented as $\gamma_j, j = 1, \ldots, L_c$. Hence, the resulting SNR of the overall combiner (L_c/L SEC + MRC), γ_c, can be calculated as

$$\gamma_c = \sum_{j=1}^{L_c} \gamma_j \tag{2.1}$$

If the number of acceptable paths,L_c,is determined to be equal to the number of received diversity paths L i.e. $L_c = L$ then (2.1) can be modified as (2.2) which is the overall SNR of a traditional MRC combiner [4, 5].

$$\gamma_c = \sum_{j=1}^{L} \gamma_j \tag{2.2}$$

We note that when there is only one acceptable path i.e. $L_c = 1$ the overall combiner is reduced to a traditional Switch-and-Examine (SEC) combiner [21].

The receiver performs path selections systematically and in conjunction with the channel coherence time which is defined as the time during which the channel response remains constant. This can be achieved by inserting a short guard period every channel coherence time. During this short guard period the receiver performs channel estimations, comparison for selecting acceptable paths and subsequently performing MRC on the selected paths. The channel coherence time is a predefined system specification and is based on the channel under consideration.

2.2.2 Channel Model

A block fading model, where the fading of each path is assumed to be constant over a channel coherence time and to vary independently after one coherence time, is considered. The block fading channel is commonly used to mathematically develop a wireless communication channel. The model is able to represent the essential features of a practical transmission scheme over fading channels. In addition to the block fading channel model, it is assumed that the multi-path envelope of each path follows independent and identical Rayleigh fading model. To implement the EE-GSEC scheme, as stated earlier, implementation of a guard period every channel coherence time is required. Figure 2.2 shows the illustration of a block fading channel model with the proposed guard period.

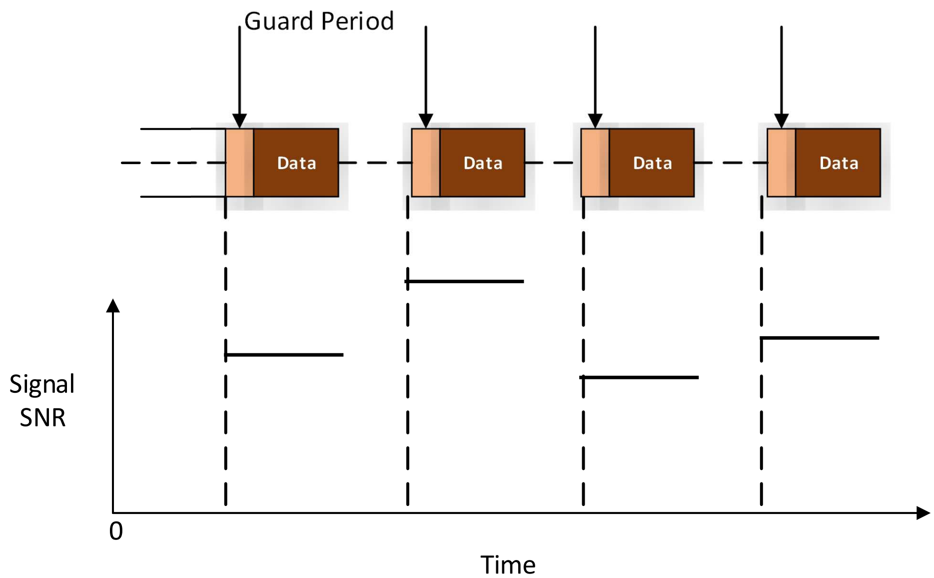

Figure 2.2: Block Fading Channel Model

The instantaneous SNR, γ, of the received diversity path is an exponential random variable whose cumulative distribution function (CDF), $P_\gamma(x)$, is given by

$$P_\gamma(x) = 1 - e^{-\frac{x}{\bar{\gamma}}} \tag{2.3}$$

where $\bar{\gamma}$ is the average SNR per path. Based on (2.3) the probability density function (pdf) $p_\gamma(x)$, and moment generating function (MGF) $M_\gamma(t)$ of the received SNR γ per path can be expressed as

$$p_\gamma(x) = \frac{1}{\bar{\gamma}} e^{-\frac{x}{\bar{\gamma}}} \tag{2.4}$$

and

$$M_\gamma(t) = \int_0^\infty e^{tx} p_\gamma(x) dx = \frac{1}{(1 - t\bar{\gamma})} \tag{2.5}$$

respectively.

2.2.3 Mode of Operation

We now examine the path selection process of the L_c/L SEC combiner. The EE-GSEC combiner, L_c/L SEC, selects acceptable diversity paths for MRC by examining and comparing as many paths as necessary and as possible. The paths can be examined in any order without preference as the diversity paths are assumed to undergo independent and identical fading. However, to save power and avoid redundancy, the combiner starts with the examination of paths to which the MRC inputs are currently connected. The combiner compares the instantaneous SNR of each diversity path against a pre-determined SNR, γ_T that can also be referred to as the fixed switching threshold.

If the path's instantaneous SNR, γ, is less than γ_T the receiver moves to examine the next available $L - L_c$ diversity paths. If an acceptable path is found i.e. path with $\gamma \geq \gamma_T$, the combiner connects that path to the MRC input and moves to the next available path. The combiner, in search for the next available path, starts from the next remaining available path in succession. There are three scenarios to be considered here:

1. **If the combiner is able to find at least L_c acceptable paths**: The receiver will combine all or some ,depending on number of inputs to MRC, in the MRC fashion.

2. **If the combiner is not able to find at least L_c acceptable paths**: The receiver in this case will combine all the L_a, where $L_a < L_c$, acceptable paths.

3. **If the combiner is not able find any acceptable path i.e.** $L_c = 0$: The receiver will continue to examine the paths every channel coherence time till at least one acceptable path is found.

The operation can be explained more distinctly with the help of a sample operation.

Figure 2.3 shows a sample path-selection process of a EE-GSEC combiner during the proposed guard period. There are five available diversity paths each having instantaneous SNR γ_1, γ_2,γ_3, γ_4, and γ_5. Let us assume that only a single path has instantaneous SNR greater than the fixed switching threshold γ_T i.e the instantaneous SNR of the diversity paths satisfy $\gamma_1 < \gamma_T$, $\gamma_2 < \gamma_T$, $\gamma_3 < \gamma_T$, $\gamma_4 > \gamma_T$, and $\gamma_5 < \gamma_T$. Let us assume that the receiver is using the second and third diversity path before

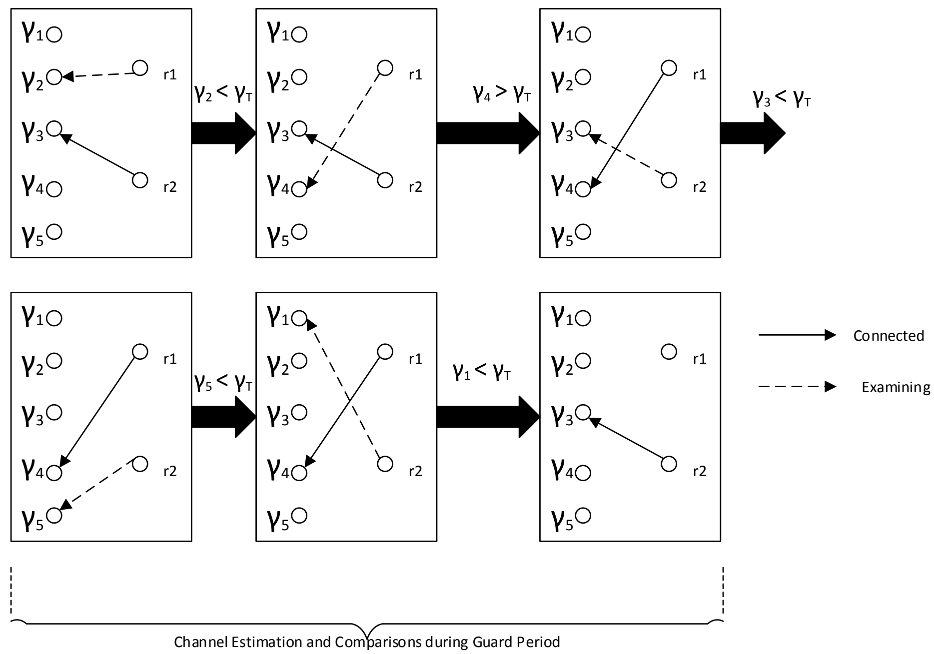

Figure 2.3: Sample operation of the L_c/L SEC

the start of the guard period as a result of previous path selection. Hence, we have $r_1 = \gamma_2$ and $r_2 = \gamma_3$. As stated earlier the combiner starts with the examination of already connected paths therefore it starts with examining γ_2. As $\gamma_2 < \gamma_T$ the combiner moves to the next path in succession which is γ_4. Note that the combiner did not check γ_3 as it is already connected to the MRC. Since $\gamma_4 > \gamma_T$ the combiner selects this path i.e. it sets $r_1 = \gamma_4$. The combiner having selected the first acceptable diversity path now switches to examining γ_3. As $\gamma_3 < \gamma_T$, the combiner switches to examine γ_5 which is also unacceptable as $\gamma_5 < \gamma_T$. The combiner now switches to examining γ_1 and finds it is, too, unacceptable. The combiner has now examined all available diversity paths and has selected γ_4 as the acceptable path for the current MRC input.

From the above mode of operation we can deduce that the number of comparisons required by an L_c/L EE-GSEC combiner in the guard period is at most L comparisons. Comparing this to the number of comparisons for a L_c/L GSC combiner which is $L_c \times L$ it is clear that EE-GSEC, on an average, requires fewer channel comparisons than GSC. The number of path estimations, however, remain same as the GSEC scheme as we shall see in Section 2.5.

It should be noted here that when there are enough acceptable diversity paths. i.e., $L_a > L_c$, EE-GSEC scheme operates in the same manner as GSEC and GSECps.

2.3 EE-GSEC Statistics

In order to analyze the performance of EE-GSEC in fading environments, it is important to constitute the statistical characterization of its combined SNR. The cumulative distribution function (CDF) of the combined SNR is derived first. Based on the CDF, the exact analytical expression of the probability density function (PDF) and moment generating function (MGF) is then derived.

2.3.1 CDF of Combiner Output SNR

From the mode of operation of EE-GSEC combiner, L/L_c SEC, it is observed that the instantaneous SNRs, r_j, of selected diversity paths in (2.1) that are greater or equal to the switching threshold γ_T can only take values between 0 and L_c. As these are mutually exclusive and disjoint events, the total probability theorem can be applied to state the CDF of the combiner SNR, γ_c, as

$$\begin{aligned} P_{\gamma_c}(x) &= Pr[\gamma_c < x] \\ &= \pi_{L_c} \times Pr[\gamma_c < x \mid L_a \geq L_c] + \sum_{i=1}^{L_c-1} Pr[L_a = i] \times Pr[\gamma_c < x \mid L_a = i] \\ &\quad + Pr[L_a = 0] \end{aligned} \tag{2.6}$$

where π_{L_c} is the probability that there are at least L_c available paths and for Rayleigh i.i.d can be derived as [18] [Eq. 4]:

$$\begin{aligned} \pi_{L_c} &= \sum_{j=L_c}^{L} \binom{L}{j} [P_\gamma(\gamma_T)]^{L-j}[1 - P_\gamma(\gamma_T)]^j \\ &= \sum_{j=L_c}^{L} \binom{L}{j} \left(1 - e^{-\frac{\gamma_T}{\bar{\gamma}}}\right)^{L-j} e^{-\frac{j\gamma_T}{\bar{\gamma}}} \end{aligned} \tag{2.7}$$

and $Pr[\gamma_c < x \mid L_a \geq L_c]$ is the conditional probability of the event $\gamma_c < x$ given that there are at least L_c acceptable paths. When $L_a \geq L_c$, the EE-GSEC combines L_c acceptable paths. It implies that γ_c is the sum of L_c path SNRs that are greater than γ_T. Hence, $Pr[\gamma_c < x \mid L_a \geq L_c]$ can be expressed as [18] [Eq. 38]:

$$Pr[\gamma_c < x \mid L_a \geq L_c] = 1 - e^{-\frac{x-(L_c\gamma_T)}{\bar{\gamma}}}\left\{\sum_{k=0}^{L_c-1}\frac{1}{k!}\Big(\frac{x-L_c\gamma_T}{\bar{\gamma}}\Big)^k\right\}, x \geq L_c\gamma_T \tag{2.8}$$

$Pr[L_a = i]$ is the probability that there are $i, i < L_c$, acceptable paths which in (2.6) can be calculated as

$$\begin{aligned} Pr[L_a = i] &= \binom{L}{i}[P_\gamma(\gamma_T)]^{L-i}[1 - P_\gamma(\gamma_T)]^i \\ &= \binom{L}{i}\big(1 - e^{-\frac{\gamma_T}{\bar{\gamma}}}\big)^{L-i}e^{-\frac{i\gamma_T}{\bar{\gamma}}} \end{aligned} \tag{2.9}$$

$Pr[\gamma_c < x \mid L_a = i]$ is the probability of the event $\gamma_c < x$ given $L_c = i$. When $L_a < L_c$ the receiver will combine the available L_a acceptable paths. In this case the output SNR of the combiner, γ_c, will be equal to the sum of the SNRs of the available acceptable paths and can be expressed as, $\gamma_c = \sum_{j=1}^{i}\gamma_j^+$, where γ_j^+ denotes the truncated path SNR for j_{th} acceptable path. It follows that

$$P_\gamma\Big[\gamma_c = \sum_{j=1}^{i}\gamma_j^+ < x; L_a = i\Big] = 1 - e^{-}\frac{x-(i\gamma_T)}{\bar{\gamma}}\left\{\sum_{K=0}^{i-1}\frac{1}{K!}\Big(\frac{x-i\gamma_T}{\bar{\gamma}}\Big)^K\right\}, x \geq i\gamma_T \tag{2.10}$$

$Pr[L_a = 0]$ is the probability that there are no acceptable paths available. It can be easily derived from (2.9) by substituting $i = 0$

$$Pr[L_a = 0] = (1 - e^{-\frac{\gamma_T}{\bar{\gamma}}})^L \tag{2.11}$$

Substituting (2.7), (2.8), (2.9), (2.10) and (2.11) in (2.6), the CDF of the combiner SNR, γ_c, $P_{\gamma_c}(x)$ can be expressed in the closed form as:

$$
\begin{aligned}
P_{\gamma_c}(x) = &\left[\sum_{j=L_c}^{L}\binom{L}{j}\left(1-e^{-\frac{\gamma_T}{\bar{\gamma}}}\right)^{L-j}e^{-\frac{j\gamma_T}{\bar{\gamma}}}\right]\left[1-e^{-\frac{x-(L_c\gamma_T)}{\bar{\gamma}}}\left\{\sum_{k=0}^{L_c-1}\frac{1}{k!}\Big(\frac{x-L_c\gamma_T}{\bar{\gamma}}\Big)^{k}\right\}\right] \\
&\cdot U(x-L_c\gamma_T) \\
&+\sum_{i=1}^{L_c-1}\left[\binom{L}{i}(1-e^{-\frac{\gamma_T}{\bar{\gamma}}})^{L-i}e^{-\frac{i\gamma_T}{\bar{\gamma}}}\left(1-e^{-\frac{x-(i\gamma_T)}{\bar{\gamma}}}\left\{\sum_{K=0}^{i-1}\frac{1}{K!}\Big(\frac{x-i\gamma_T}{\bar{\gamma}}\Big)^{K}\right\}\right)\right] \\
&\cdot U(x-i\gamma_T) \\
&+(1-e^{-\frac{\gamma_T}{\bar{\gamma}}})^{L}
\end{aligned}
\tag{2.12}
$$

$U(\cdot)$ is the unit step function.

We perform a comparison between the above calculated analytical value and values obtained through simulation to validate the CDF analysis. The simulations are obtained using Monte Carlo methods. Figure 2.4 shows the analytical and simulation values obtained for $L_c = 2$ and $L_c = 3$ cases.

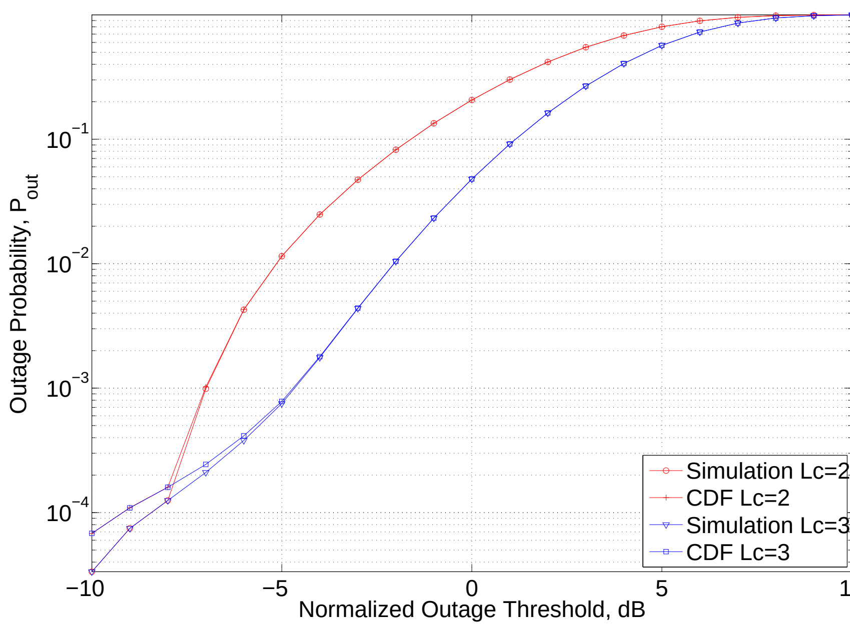

Figure 2.4: Comparison of analytical and simulation results in terms of the Outage probability for $L_c = 2, 3$ as a function of normalized outage threshold.

We observe that the simulation results matches the analytical result for the CDF hence validating the derived CDF in equation (2.12).

2.3.2 PDF of Combiner Output SNR

The PDF of the combiner output SNR, γ_c, $p_{\gamma_c}(x)$ is derived as

$$p_{\gamma_c}(x) = \frac{d}{dx} P_{\gamma_c}(x) \tag{2.13}$$

Substituting (2.12) in (2.13), carrying out the differentiation and appropriate simplifications, the analytical expression of the pdf is obtained as

$$\begin{aligned} p_{\gamma_c}(x) = \pi_{L_c} \cdot \sum_{k=0}^{L_c-1} \frac{1}{k!} e^{-\frac{x - L_c\gamma_T}{\bar{\gamma}}} \left(\frac{(x - L_c\gamma_T)^k}{(\bar{\gamma})^{k+1}} - \frac{k(x - L_c\gamma_T)^{k-1}}{(\bar{\gamma})^k} \right) \\ + \sum_{i=1}^{L_c-1} \left[T_i \sum_{k=0}^{i-1} \frac{1}{k!} e^{-\frac{x - i\gamma_T}{\bar{\gamma}}} \left(\frac{(x - i\gamma_T)^k}{(\bar{\gamma})^{k+1}} - \frac{k(x - i\gamma_T)^{k-1}}{(\bar{\gamma})^k} \right) \right] \end{aligned} \tag{2.14}$$

where π_{L_c} is defined in (2.7) and T_i is defined for conciseness as

$$T_i = \binom{L}{i} \left(1 - e^{-\frac{\gamma_T}{\bar{\gamma}}}\right)^{L-i} \left(e^{-\frac{i\gamma_T}{\bar{\gamma}}}\right) \tag{2.15}$$

2.3.3 MGF of Combiner Output SNR

The moment generating function of the combiner output SNR, γ_c, can be derived by observing the mode of operation of the L_C/L SEC and noticing that the number of r_j's in (2.1) remains between 0 to Lc. As they are exclusive and disjoint events, by applying the total probability theorem, the MGF of the combiner output γ_c can be expressed as [18] [Eq. 3]:

$$\mathcal{M}_{\gamma_c}(t) = \sum_{i=0}^{L_c-1} \pi_i \left(\mathcal{M}_{\gamma_j^+}(t) \right)^i \tag{2.16}$$

where π_i is the probability that there are exactly i_{r_j}'s in (2.1) that are greater than or equal to γ_T. Hence, π_i can be defined as:

- π_{L_c}: Probability that there are at least L_c paths with SNR greater than or equal to γ_T. and

- $Pr[L_a = i]$: Probability that there are exactly i, $i < L_c$, paths with SNR greater than or equal to γ_T.

Mathematically, using equations (2.7) and (2.9)

$$\pi_i = \begin{cases} \sum_{j=L_c}^{L} \binom{L}{j} \left(1 - e^{-\frac{\gamma_T}{\bar{\gamma}}}\right)^{L-j} e^{-\frac{j\gamma_T}{\bar{\gamma}}}, & i = 0, \cdots, Lc - 1 \\ \binom{L}{i} (1 - e^{-\frac{\gamma_T}{\bar{\gamma}}})^{L-i} e^{-\frac{i\gamma_T}{\bar{\gamma}}}, & i = Lc \end{cases} \tag{2.17}$$

$\mathcal{M}_{\gamma_j^+}(.)$ is the MGF of γ_j^+. Mathematically, it can be expressed as [18] [Eq. 6]:

$$\mathcal{M}_{\gamma_j^+}(t) = \int_{-\infty}^{+\infty} \gamma_j^+(x) e^{tx} dx \tag{2.18}$$

where $\gamma_j^+(.)$ is the pdf of γ_j^+ which is greater than or equal to γ_T, and can be expressed as

$$p_{\gamma_j^+}(x) = \frac{p_\gamma(x)}{1 - P_\gamma(\gamma_T)} \tag{2.19}$$

where $p_\gamma(.)$ is the pdf of the received SNR, which in case of Rayleigh fading channel is given in equation (2.4).

Substituting, (2.19) in (2.18),

$$\mathcal{M}_{\gamma_j^+}(t) = \frac{1}{1 - P_\gamma(\gamma_T)} \int_{\gamma_T}^{+\infty} p_\gamma(x) e^{tx} dx \tag{2.20}$$

Substituting (2.4) in (2.20) and solving the integral to get,

$$\mathcal{M}_{\gamma_j^+}(t) = \frac{e^{t\gamma_T}}{1 - t\bar{\gamma}} \tag{2.21}$$

Substituting (2.17) and (2.21) in (2.16), the expression for the MGF of the overall combiner output γ_c over Rayleigh fading channel is given as

$$\begin{aligned} \mathcal{M}_{\gamma_c}(t) = &\sum_{i=0}^{L_c-1} \binom{L}{i} (1 - e^{-\frac{\gamma_T}{\bar{\gamma}}})^{L-i} e^{-\frac{i\gamma_T}{\bar{\gamma}}} \left(\frac{e^{t\gamma_T}}{1 - t\bar{\gamma}}\right)^i \\ &+ \sum_{j=L_c}^{L} \binom{L}{j} \left(1 - e^{-\frac{\gamma_T}{\bar{\gamma}}}\right)^{L-j} e^{-\frac{j\gamma_T}{\bar{\gamma}}} \left(\frac{e^{t\gamma_T}}{1 - t\bar{\gamma}}\right)^{L_c} \end{aligned} \tag{2.22}$$

2.4 Performance Analysis of EE-GSEC

Based on the statistics of EE-GSEC output SNR derived in the earlier section, the performance of EE-GSEC over fading channels in terms of outage probability, average combined SNR, and average error probability is analyzed.

2.4.1 Average Combined SNR

To derive the expression for average combined SNR, we utilize the fact that the first moment of a random variable is equal to its statistical average. It implies that the first derivative of the MGF of γ_c, $\mathcal{M}_{\gamma_c}(t)$, evaluated at zero yields the average combined SNR i.e.,

$$\bar{\gamma_c} = \left.\frac{d\mathcal{M}_{\gamma_c}(t)}{dt}\right|_{t=0} \tag{2.23}$$

Substituting (2.22) in (2.23), differentiating with respect to t, simplifying and setting $t = 0$ we obtain the closed form expression for the average combined SNR of EE-GSEC over i.i.d Rayleigh fading channel as

$$\begin{aligned}\bar{\gamma_c} &= (\gamma_T + \bar{\gamma}) \sum_{i=0}^{L_c-1} i \binom{L}{i} \left(1 - e^{-\frac{\gamma_T}{\bar{\gamma}}}\right)^{L-i} e^{-\frac{i\gamma_T}{\bar{\gamma}}} \\ &+ L_c(\gamma_T + \bar{\gamma}) \sum_{j=L_c}^{L} \binom{L}{j} \left(1 - e^{-\frac{\gamma_T}{\bar{\gamma}}}\right)^{L-j} e^{-\frac{j\gamma_T}{\bar{\gamma}}}\end{aligned} \tag{2.24}$$

From equation (2.24), we observe that $\bar{\gamma_c}$ is a continuous function of γ_T. We suspect that a certain value of γ_T will maximize $\bar{\gamma_c}$, which we refer to as the optimum switching threshold. The existence and behavior of the optimum switching threshold will be investigated in Chapter 3.

2.4.2 Outage Probability

Outage is typically defined based on a threshold, γ_{th}, that specifies the minimum SNR required for acceptable performance. In our case, outage probability is defined as the probability that the instantaneous combined SNR, γ_c, of the combiner falls below a certain specified threshold γ_{th}.

We can calculate the outage probability P_{out} as

$$P_{out} = Pr[\gamma_c < \gamma_{th}] = \int_0^{\gamma_{th}} p_{\gamma_c}(x)dx \tag{2.25}$$

Substituting (2.14) in (2.25) and solving to get a closed form expression for P_{out},

$$\begin{aligned} P_{out} = & \left[\sum_{j=L_c}^{L} \binom{L}{j} \left(1 - e^{-\frac{\gamma_T}{\bar{\gamma}}}\right)^{L-j} e^{-\frac{j\gamma_T}{\bar{\gamma}}}\right] \left[1 - e^{-\frac{\gamma_{th}-(L_c\gamma_T)}{\bar{\gamma}}} \left\{\sum_{k=0}^{L_c-1} \frac{1}{k!} \left(\frac{\gamma_{th} - L_c\gamma_T}{\bar{\gamma}}\right)^k\right\}\right] \\ & \cdot U(\gamma_{th} - L_c\gamma_T) \\ & + \sum_{i=1}^{L_c-1} \left[\binom{L}{i} (1 - e^{-\frac{\gamma_T}{\bar{\gamma}}})^{L-i} e^{-\frac{i\gamma_T}{\bar{\gamma}}} \left(1 - e^{-\frac{\gamma_{th} - (i\gamma_T)}{\bar{\gamma}}} \left\{\sum_{K=0}^{i-1} \frac{1}{K!} \left(\frac{\gamma_{th} - i\gamma_T}{\bar{\gamma}}\right)^K\right\}\right)\right] \\ & \cdot U(\gamma_{th} - i\gamma_T) \\ & + (1 - e^{-\frac{\gamma_T}{\bar{\gamma}}})^L \end{aligned} \tag{2.26}$$

In chapter 3 we perform a detail comparison between the outage performance of GSC, GSEC and EE-GSEC.

2.4.3 Average Error Probability

Another important metric for performance evaluation of a wireless system is the probability of error, defined relative to either symbol or bit errors. The average probability of error is calculated by integrating the instantaneous error probability over the fading distribution. In the case of EE-GSEC it can be calculated as

$$\bar{P}_R = \int_0^{\infty} P_R(x) p_{\gamma_c}(x) dx \tag{2.27}$$

where $P_R(x)$ is the instantaneous probability of symbol error for the combiner output and $p_{\gamma_c}(x)$ is PDF of the combiner SNR γ_c respectively.

However, the above integral may not lead to a closed-form solution and may also be difficult to compute numerically. [22] and [23] addresses the various numerical and approximation techniques for the computation of the above integrals over various fading channels. [22] provides a MGF-based approach to calculate the average error probability that eliminates the difficulties of integration of infinite integral. The MGF-based approach is able to evaluate the probability of error of wireless communication systems with and without diversity.

Based on the MGF approach as derived in [24–26] the average error probability can be expressed as

$$P(E) = \sum \int_{\tau_1}^{\tau_2} h(\theta)\mathcal{M}_{\gamma_c}(-g(\theta))d\theta \tag{2.28}$$

Note that the terms in the summation, the limits of the integral τ_1 and τ_2, and the functions $h(\cdot)$ and $g(\cdot)$ are the functions of the type of modulation in question i.e. they are independent of the combiner SNR γ_c. For example the average Bit Error Rate (BER) for binary signals can be expressed as [27]:

$$P_s(E) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \mathcal{M}_{\gamma_c}\Big(-\frac{g}{sin^2\theta}\Big)d\theta \tag{2.29}$$

where $g = 1$ BPSK, $g = 1/2$ orthogonal BFSK, and $g = 0.715$ for BFSK with minimum correlation.

Putting (2.22) in (2.29) we obtain the average bit error rate (BER) of BPSK with EE-GSEC as

$$\begin{aligned} P_s(E) = {} & \frac{1}{\pi}\sum_{i=0}^{L_c-1}\binom{L}{i}\big(1-e^{-\frac{\gamma_T}{\bar{\gamma}}}\big)^{L-i}e^{-\frac{i\gamma_T}{\bar{\gamma}}} \times \int_0^{\pi/2}\frac{exp\big(-\frac{i\gamma_T}{sin^2\theta}\big)}{\big(1+\frac{\bar{\gamma}}{sin^2\theta}\big)^i}d\theta \\ & + \frac{1}{\pi}\sum_{j=L_c}^{L}\binom{L}{j}\Big(1-e^{-\frac{\gamma_T}{\bar{\gamma}}}\Big)^{L-j}e^{-\frac{j\gamma_T}{\bar{\gamma}}} \times \int_0^{\pi/2}\frac{exp\big(-\frac{L_c\gamma_T}{sin^2\theta}\big)}{\big(1+\frac{\bar{\gamma}}{sin^2\theta}\big)^{L_c}}d\theta \end{aligned} \tag{2.30}$$

In chapter 3 we perform a detail comparison between the error performance of GSC, GSEC and EE-GSEC.

2.5 Complexity Analysis

To provide perspective to the complexity savings achieved by EE-GSEC it is important to quantify the average number of path estimations and the number of comparisons required during the path-update process. The complexity savings results here translate into design of less complex comparison circuitry and reduction of processing power consumption.

2.5.1 Average Number of Path Estimates

In order to obtain the expression for the average number of path estimates, N_E, required by EE-GSEC we need to examine the following two cases:

- There are at least L_c acceptable diversity paths: In this case the combiner will stop the path examination process when it has identified L_c acceptable paths out of the total L paths i.e. paths whose SNR is greater than a pre-determined threshold. The number of channel estimates in this case can be between L_c to L. Let $P_1^{(k)}$ denote the probability that the L_cth acceptable path is the kth path examined. This is equivalent to stating that $k - L_c$ ones of the first $k - 1$ examined paths are unacceptable. Hence, $P_1^{(k)}$ can be expressed as [18] [Eq. 26]:
$$P_1^{(k)} = \binom{k-1}{k-L_c}[1 - P_\gamma(\gamma_T)]^{L_c}[P_\gamma(\gamma_T)]^{k-L_c} \tag{2.31}$$

- There are less than L_c acceptable diversity paths: The probability, P_2, that there are less than L_c acceptable paths can be expressed as [18] [Eq. 25]:
$$P_2 = \sum_{k=L-L_c+1}^{L} \binom{L}{k}[1 - P_\gamma(\gamma_T)]^{L-k}[P_\gamma(\gamma_T)]^{k} \tag{2.32}$$

The expression for the overall average number of channel estimates,N_E, needed by EE-GSEC during the guard period is obtained by adding the two mutually exclusive cases

$$\begin{aligned} N_E &= \sum_{k=L_c}^{L} kP_1^{(k)} + LP_2 \\ &= \sum_{k=L_c}^{L} k\binom{k-1}{k-L_c}[1 - P_\gamma(\gamma_T)]^{L_c}[P_\gamma(\gamma_T)]^{k-L_c} \\ &\quad + L\sum_{k=L-L_c+1}^{L} \binom{L}{k}[1 - P_\gamma(\gamma_T)]^{L-k}[P_\gamma(\gamma_T)]^{k} \end{aligned} \tag{2.33}$$

For Rayleigh fading channels, the closed form expression for N_E is obtained after substituting (2.3) in the above expression

$$N_E = \sum_{k=L_c}^{L} k \binom{k-1}{k-L_c} e^{-\frac{\gamma_T L_c}{\bar{\gamma}}} [1 - e^{-\frac{\gamma_T}{\bar{\gamma}}}]^{k-L_c} + L \sum_{k=L-L_c+1}^{L} \binom{L}{k} e^{-\frac{\gamma_T (L-k)}{\bar{\gamma}}} [1 - e^{-\frac{\gamma_T}{\bar{\gamma}}}]^{k} \tag{2.34}$$

This is always smaller than L path estimates required by GSC and same as GSEC and GSECps as we will see in the next chapter.

2.5.2 Average Number of Comparisons

To calculate the average number of comparisons needed it is observed from the mode of operation of EE-GSEC that each path examination involves one path estimation and one comparison. When there are at least L_c acceptable paths out of available L paths the combiner stops the examination. In this case, the number of comparisons will be equal to number of path examination. The same holds true in the case when there are less than L_c acceptable paths where the combiner still estimates the remaining available paths. Therefore, the average number of comparisons, M, needed by EE-GSEC is same as the required average number of path estimates,N_E, given by (2.34):

$$M = \sum_{k=L_c}^{L} k \binom{k-1}{k-L_c} e^{-\frac{\gamma_T L_c}{\bar{\gamma}}} [1 - e^{-\frac{\gamma_T}{\bar{\gamma}}}]^{k-L_c} + L \sum_{k=L-L_c+1}^{L} \binom{L}{k} e^{-\frac{\gamma_T (L-k)}{\bar{\gamma}}} [1 - e^{-\frac{\gamma_T}{\bar{\gamma}}}]^{k} \tag{2.35}$$

2.5.3 Average Number of Active Paths

To illustrate the power saving advantages of EE-GSEC the average number of active paths,N_{active}, selected by EE-GSEC is calculated. This metric is important from the perspective of receiver design since less number of active paths during data reception results in considerable saving of receiver's processing power.

From the mode of operation of EE-GSEC it is observed that cases that contribute paths to N_{active} are

1. The probability that there are at least L_c acceptable paths and is given by equation (2.7)

2. The probability that there are $i, i < L_c$, acceptable paths and is given by equation (2.9)

Adding the above two mutually exclusive events, N_{active} For EE-GSEC in case of i.i.d Rayleigh fading channel is given as

$$\begin{aligned} N_{active} &= L_c \times \pi_{L_c} + \left[\sum_{i=1}^{L_c-1} L_a \times Pr[L_a = i] \right] \\ &= L_c \sum_{j=L_c}^{L} \binom{L}{j} \left(1 - e^{-\frac{\gamma_T}{\bar{\gamma}}}\right)^{L-j} e^{-\frac{j\gamma_T}{\bar{\gamma}}} \\ &+ \sum_{i=1}^{L_c-1} i \binom{L}{i} \left(1 - e^{-\frac{\gamma_T}{\bar{\gamma}}}\right)^{L-i} e^{-\frac{i\gamma_T}{\bar{\gamma}}} \end{aligned} \tag{2.36}$$

In chapter 3 we will observe though simulations the effect of choosing the optimum switching threshold on the average number of active paths. We will also perform analysis of average combined SNR per active paths and compare the same with the GSC scheme to illustrate the efficiency of EE-GSEC.

2.6 Conclusion

The proposed EE-GSEC scheme was introduced by describing the channel and system model. A detailed description of the mode of operation of the combiner was provided. Statistical analysis was performed based on the scheme's operation and the expressions for CDF, PDF and the MGF were derived. Based on the statistical results, we mathematically formulated the scheme's performance metrics in terms of average combined SNR, outage probability and the average BER. The complexity in terms of average number of path estimations and SNR comparisons required by EE-GSEC was also derived mathematically. The expression for average number of active paths during a given guard period was also derived.

In the next chapter we will evaluate EE-GSEC's performance and complexity by performing $MATLAB_{©}$ simulations. These will be compared against GSC and GSEC. We will also examine the energy efficiency of EE-GSEC over GSC in detail.

Chapter 3

Evaluation of Performance and Complexity of EE-GSEC Scheme

3.1 Introduction

We now evaluate, numerically, performance and complexity of the presented EE-GSEC scheme based on the mathematical deductions performed in Chapter 2. The performance and complexity trade-offs offered by EE-GSEC over other comparable diversity schemes namely GSC, GSEC is also examined.

The EE-GSEC's energy efficient claims are examined by evaluating the average combined SNR per active path at the MRC output for different channel conditions and comparing the same against GSC.

The numerical analysis and simulations have been performed using $MATLAB_{©}$.

3.2 Performance Evaluation and Comparisons

Based on the mathematical analysis performed in Chapter 2, Section 2.4 we now evaluate the performance of EE-GSEC numerically and compare it against other comparable schemes.

3.2.1 Average Combined SNR

Based on equation (2.24), Figure 3.1 shows the average combined SNR performance of EE-GSEC over four Rayleigh fading paths as function of the switching threshold, γ_T.

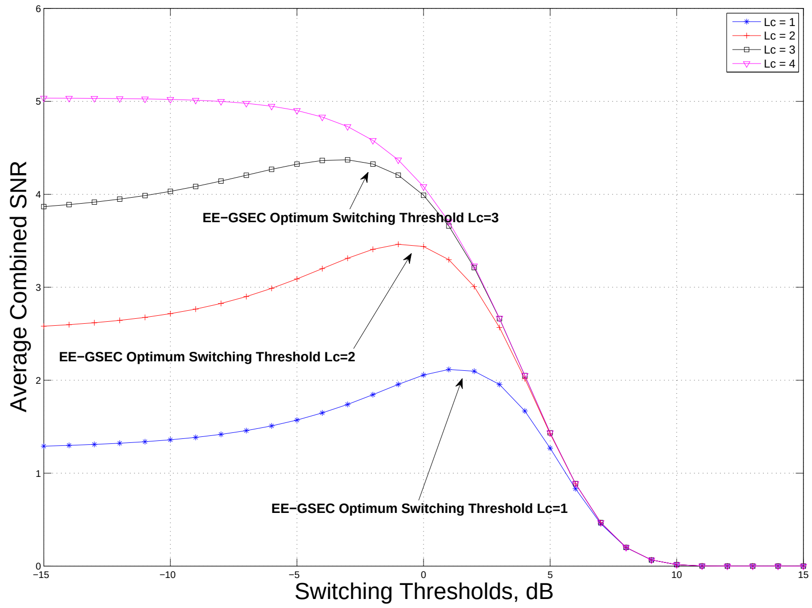

Figure 3.1: Average Combined SNR of EE-GSEC with L = 4 for different Lc as a function of switching threshold γ_T ($\bar{\gamma} = 0dB$)

We observe the following from figure 3.1:

- The average combined SNR of the combiner decreases as the value of the switching threshold increases. This is because with the increase in the switching threshold the $L_c/LSEC$ finds fewer and fewer acceptable paths i.e. paths with SNR greater or equal to γ_T. Fewer acceptable paths implies fewer paths for MRC resulting in decrease average combined SNR of the system.

- There exists an optimal choice of the threshold γ_T, indicated by the arrows, that maximizes the average combined SNR in case of $L_c < L$.

- The value of the optimal threshold decreases slightly with increase in L_c. From a system-performance perspective, having a lower threshold is advantageous since it will enable combining of more branches thus increasing the average combined SNR.

- The performance gain with the optimal switching threshold diminishes as the number of acceptable paths,L_c increases.

We also examine the average combined SNR of EE-GSEC as a function of the average SNR per path. Figure 3.2 shows the average combined SNR as a function of average SNR per path, $\bar{\gamma}$.

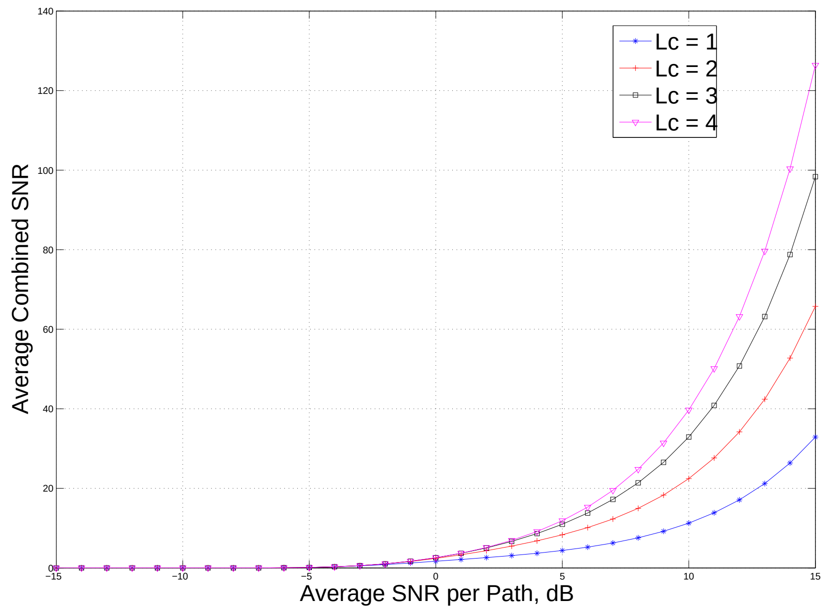

Figure 3.2: Average Combined SNR of EE-GSEC with L = 4 for different Lc as a function of average SNR per path, $\bar{\gamma}$, $\gamma_T = 0$dB

We observe that the average combined SNR increases with increase in the average SNR per path. Increase in the SNR per path results in paths with high SNR being combined and yielding an increase in the combiner's SNR. We further observe that as L_c increases so does the average combined SNR since combining SNR of more paths yields in increase of the SNR of the overall system. However, this gain diminishes with increasing L_c. To better observe the average combined SNR in case of low average SNR per path i.e. during poor channel conditions, Figure 3.3 shows the average combined SNR in the logarithmic scale. From figure 3.3 we observe that in case of low SNR per path there is no change in the average combined SNR for different cases of L_c. This is because as the SNR per path decreases, fewer paths are deemed acceptable and the paths that pass the threshold test have the SNR's in close range of each other.

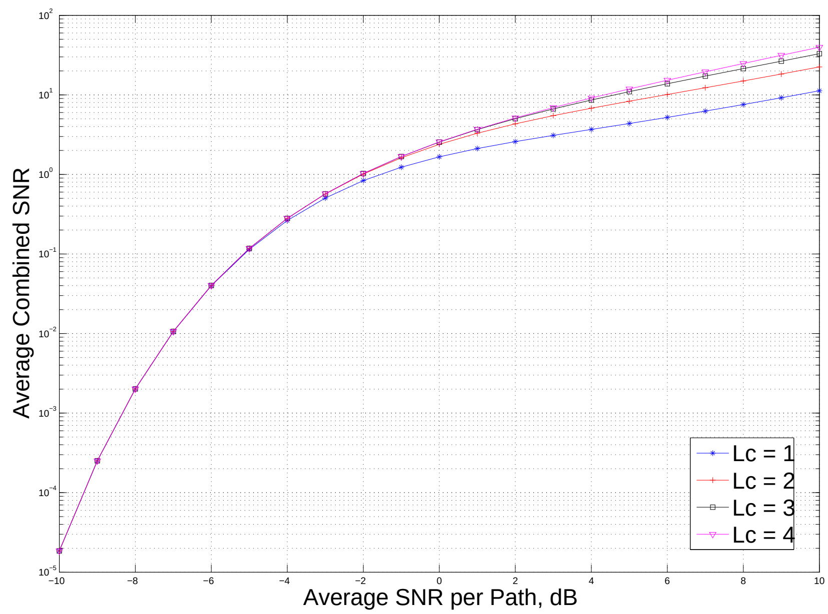

Figure 3.3: Average Combined SNR of EE-GSEC in dB with L = 4 for different Lc as a function of average SNR per path, $\bar{\gamma}$, $\gamma_T = 0$dB

Comparison with GSEC scheme

Figure 3.4 compares the performance of average combined SNR of EE-GSEC and GSEC.

We observe the following from Figure 3.4:

- For low switching thresholds, EE-GSEC provides the same average combined SNR as GSEC.
- The performance of EE-GSEC starts to degrade at approximately the optimum switching threshold level. This is due to the fact that for higher switching thresholds the number of acceptable paths available for EE-GSEC scheme starts decreasing.
- EE-GSEC's optimum switching threshold is slightly lower than GSEC's.

In summary, selecting the optimum switching threshold is vital to achieving a proportional average combined SNR performance for EE-GSEC when compared with GSEC.

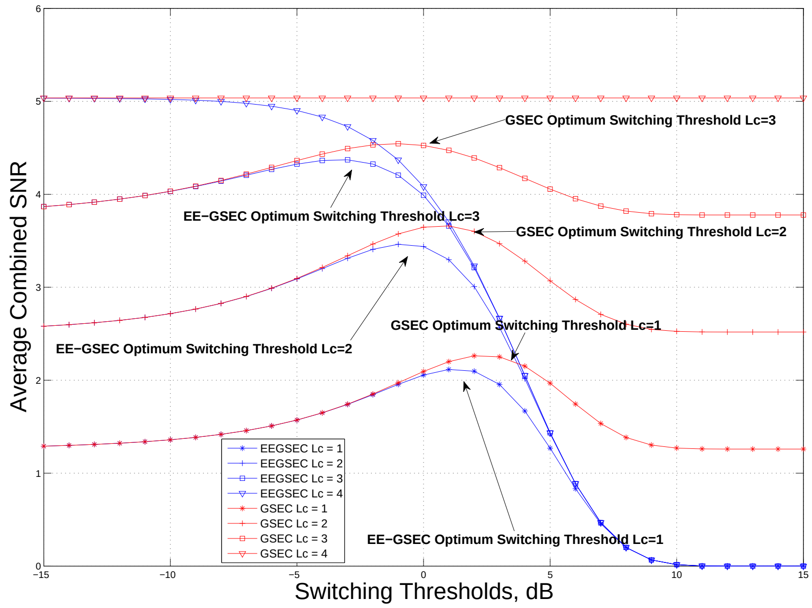

Figure 3.4: Comparison of the Average Combined SNR of EE-GSEC and GSEC with L = 4 for different Lc as a function of switching threshold, γ_T ($\bar{\gamma} = 0dB$)

3.2.2 Outage Probability

Using equation (2.26), the outage probability performance of EE-GSEC for different L_c's against the normalized outage threshold, $\gamma_t h/\bar{\gamma}$ is shown in Figure 3.5.

We observe that the outage performance improves with increasing L_c but with diminishing gains.

Comparison with GSC and GSEC schemes

Figure 3.6 compares the outage performance of EE-GSEC against GSEC and GSC schemes.It is observed that the outage performance of EE-GSEC aligns with GSEC. GSC, which combines paths based on the highest SNR, out-performs EE-GSEC. GSC's better performance, however, comes at a relatively high complexity cost. EE-GSEC is also able to provide better energy efficiency as compared to GSC as wee shall see in section 3.4.

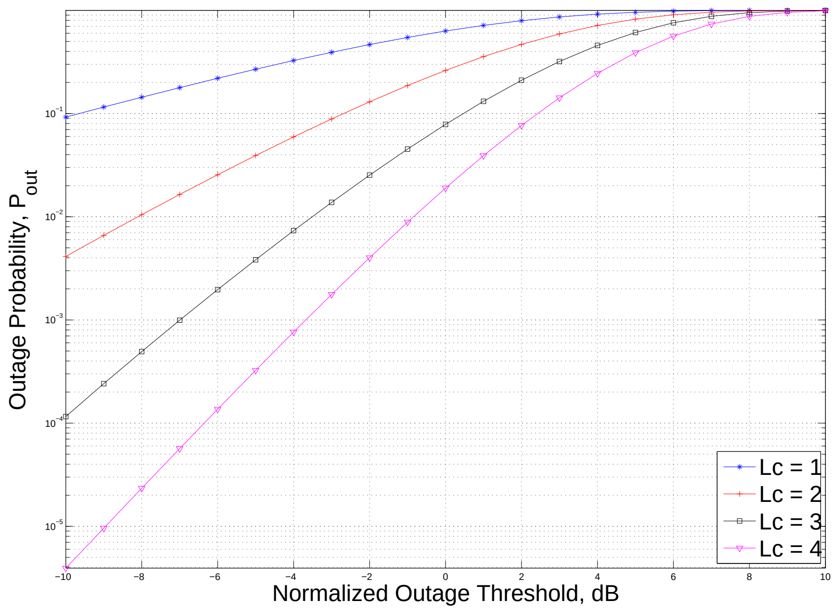

Figure 3.5: Outage Probability of EE-GSEC with $L = 4$ for different L_c as a function of normalized outage threshold, γ_{th}, $(\gamma_T/\bar{\gamma} = 0dB)$

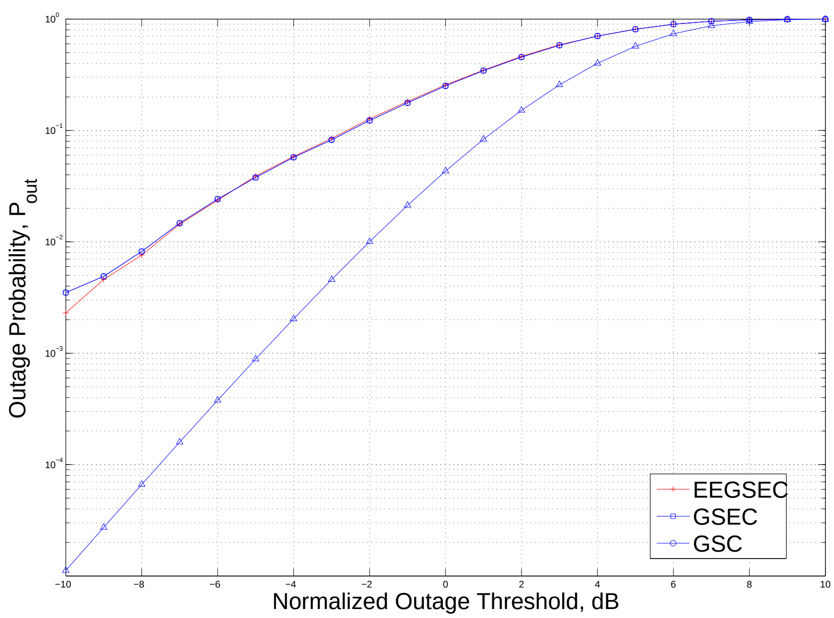

Figure 3.6: Outage Performance of EE-GSEC against GSEC and GSC EE-GSEC with $Lc = 2$ as a function of normalized outage threshold, γ_{th}, $(\gamma_T/\bar{\gamma} = 0dB)$

3.2.3 Average Error Probability

Using the expression derived for average bit error rate (BER) in equation (2.30), we evaluate the average error performance of EE-GSEC.

Figure 3.7 shows the average error probability of Binary Phase Shift Keying (BPSK) modulation with EE-GSEC as a function of switching threshold for $L = 4$ and different values of L_c

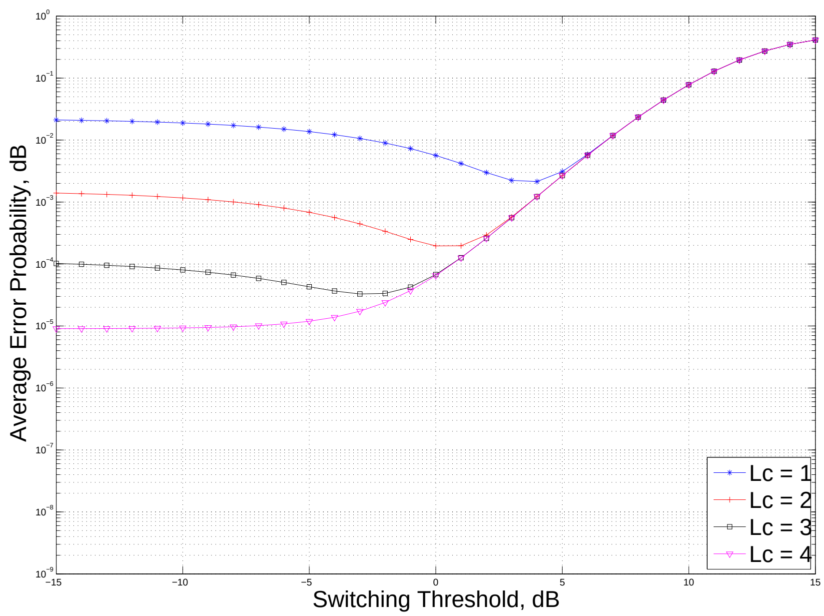

Figure 3.7: Average BER of BPSK with EE-GSEC with $L = 4$ for different L_c as a function of switching threshold, γ_T, ($\bar{\gamma} = 10dB$).

We observe the following from Figure 3.7

- Similar to the optimum switching threshold discussed for average combined SNR performance of EE-GSEC, we observe there exists a optimum switching threshold which minimizes the average error performance as well.

- As with the combined SNR, with increasing L_c the average error performance gains diminishes and the value of optimum threshold decrease.

- The optimum value of the switching threshold, γ_T, that minimizes the BER does not necessarily maximize the average combined SNR.

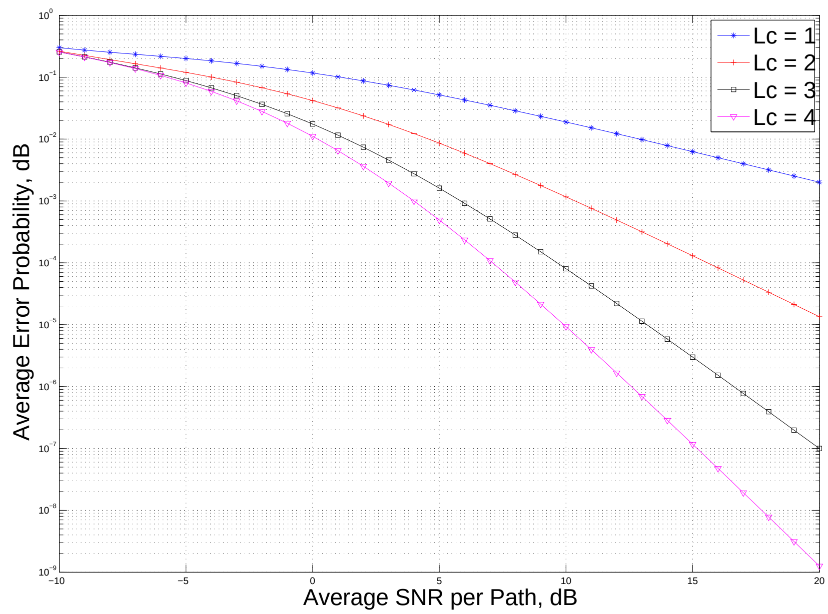

Figure 3.8: Average BER of BPSK with EE-GSEC $L = 4$ for different L_c as a function of average SNR per path, $\bar{\gamma}$, $\gamma_T = 0$dB

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The average error performance of BPSK with EE-GSEC as a function of the average SNR per path is shown in Figure 3.8. We observe the following:

- The average error rate decreases with increase in the SNR per path i.e. as the channel conditions improve the error rate of the system also improves. This is due to the increase in the number of paths considered acceptable.
- The BER decreases with diminishing gains as L_c increases and average SNR per path, $\bar{\gamma}$, increases.

Comparison with GSC and GSEC

Figure 3.9 compares the average error probability of EE-GSEC and GSEC schemes as a function of switching threshold, γ_T.

We make the following observation from Figure 3.9:

- The average error performance for both schemes matches for low switching thresholds.

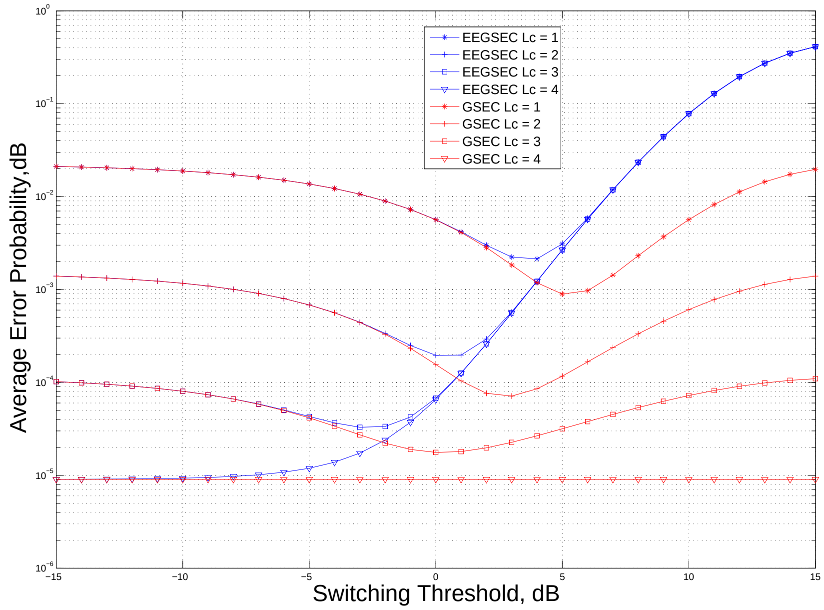

Figure 3.9: Comparison of Average BER of BPSK with EE-GSEC and GSEC $L = 4$ for different L_c as a function of switching threshold, γ_T, $\bar{\gamma} = 10dB$

- For thresholds greater than the optimum value, GSEC outperforms EE-GSEC specially when $L_c = L$. This is because when $L_c = L$ GSEC becomes equivalent to a L-branch MRC scheme.
- As with the case of average combined SNR, choosing the optimum switching threshold is important to achieve the same error performance as GSEC.

Figure 3.10 compares the average error performance of EE-GSEC with GSC and GSEC as a function of average SNR per path,$\bar{\gamma}$. Also, Table 3.1 compares the performance gap between the three schemes.

Table 3.1: Comparison of average SNR per path needed by GSC, GSEC and EE-GSEC to achieve a BER of 10^{-4} for different L_c.

Scheme	$\mathbf{L_c = 2}$	$\mathbf{L_c = 3}$
GSC	8.25 dB	7.5 dB
GSEC	10 dB	8 dB
EE-GSEC	11 dB	10.25 dB

We make the following observations from Figure 3.10:

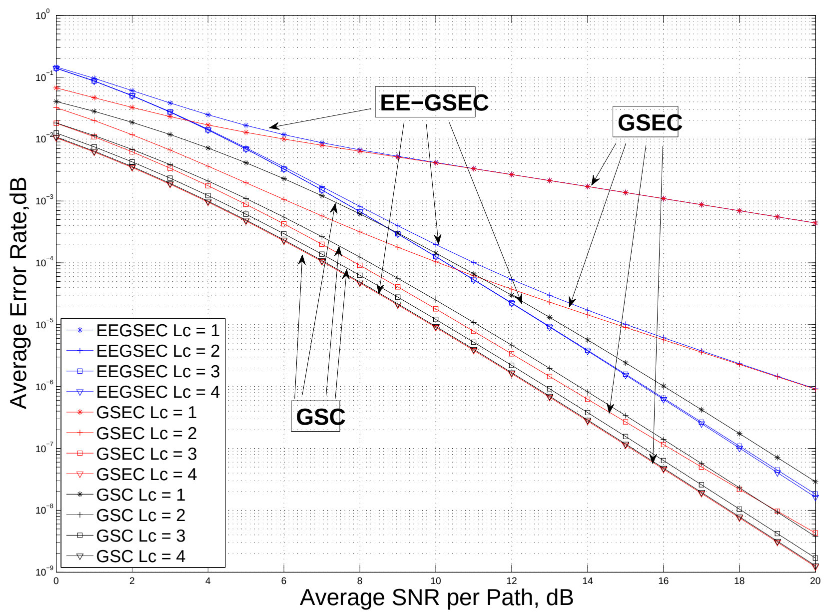

Figure 3.10: Comparison of Average BER of BPSK with EE-GSEC, GSC and GSEC $L = 4$ for different L_c as a function of average SNR per path, $\bar{\gamma}$, $\gamma_T = 0$dB

.

- Performance-wise GSC outperforms both EE-GSEC and GSEC.
- EE-GSEC's error performance is lower than GSEC but becomes equivalent with increase in SNR per path.
- The performance gap between the schemes reduces with increase in L_c. This suggests that the performance of EE-GSEC is comparable to GSC and GSEC in a diversity rich environment.

However, GSC's superior error performance comes at high complexity cost at the receiver as we will see in the next section.

3.3 Complexity Evaluation and Comparisons

Based on the mathematical analysis performed in Chapter 2, Section 2.5 we now analyze the complexity performance of EE-GSEC numerically and compare it against other schemes.

3.3.1 Average Number of Path Estimates

Based on equation (2.34) we plot, as Figure the average number of path estimates required by EE-GSEC scheme as a function of the switching threshold, γ_T.

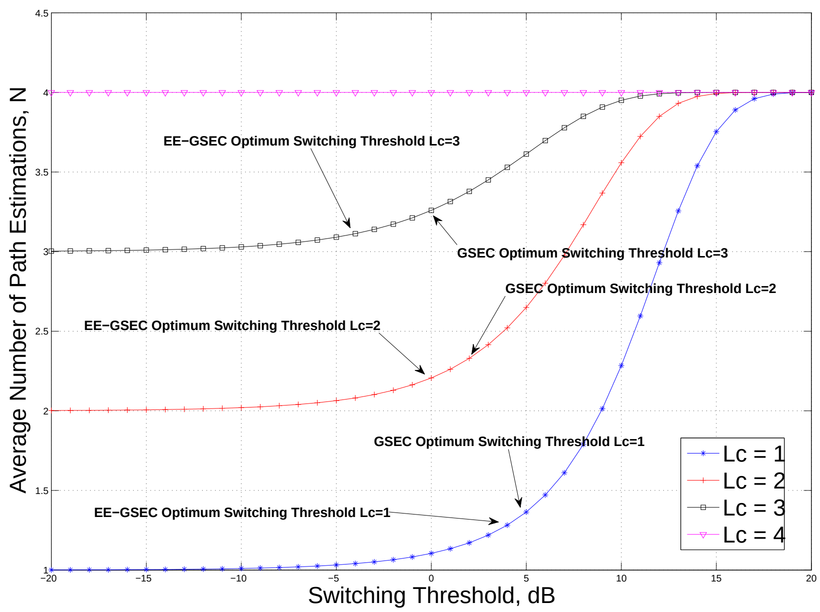

Figure 3.11: Average Number of channel estimates of EE-GSEC with $L = 4$ for different L_c as a function of switching threshold, $\gamma_T.(\bar{\gamma} = 10dB)$

.

The arrows point to the threshold level that minimizes the average error rate for EE-GSEC and GSEC as observed from Figure 3.9.

We make the following observations from Figure 3.11:

- The number of path estimates increases with increase in the switching threshold γ_T. This is due to the system need to examine and estimate more paths to find acceptable paths if the threshold is set high as less and less diversity paths pass the threshold test.

- For achieving the best error performance EE-GSEC does not require to estimate all paths. This is in contrast with GSC which has to estimate all L paths. Therefore, EE-GSEC is able to offer reduced processing power at the receiver.

Comparison with GSC and GSEC

Figure 3.12 compares the average number of path estimates required by EE-GSEC, GSEC and GSC against normalized switching threshold, $\gamma_T/\bar{\gamma}$.

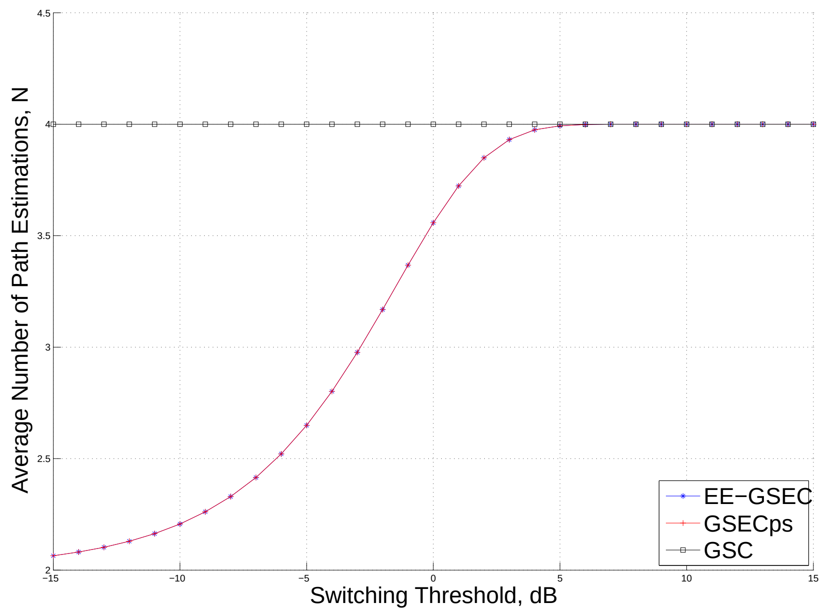

Figure 3.12: Comparison of average number of channel estimates of EE-GSEC, GSEC and GSC with $L = 4$, $L_c = 2$ as a function of normalized switching threshold, $\gamma_T/\bar{\gamma}dB$.

As stated earlier, GSC's superior outage and error performance, as compared to EE-GSEC and GSEC, comes at higher complexity cost. This is easily verified from Figure 3.12 by observing that GSC always needs to estimate all available paths, L whereas EE-GSEC and GSEC need to perform estimate all paths only when γ_T is greater than $\bar{\gamma}$ by more than 5 dB.

3.3.2 Average Number of Comparisons

As noted in section 2.5, the average number of SNR comparisons required by EE-GSEC'S is equal to the number of path estimates. This is due to the fact that the combiner needs to perform one estimation and one comparison to select an acceptable path. Therefore, equation (2.34) also provides the number of comparison needed to be performed by EE-GSEC.

Figure 3.13 compares the average number of comparison required to be executed by EE-GSEC, GSEC and GSC. We observe that both EE-GSEC and GSEC require the same number of SNR comparisons the value of which becomes equal to L with the increase in switching threshold when the scheme needs to estimate and compare all paths to find the acceptable one's. GSC on the other hand requires five comparisons to find two paths out of the available four paths. This again highlights the complexity cost carried by GSC.

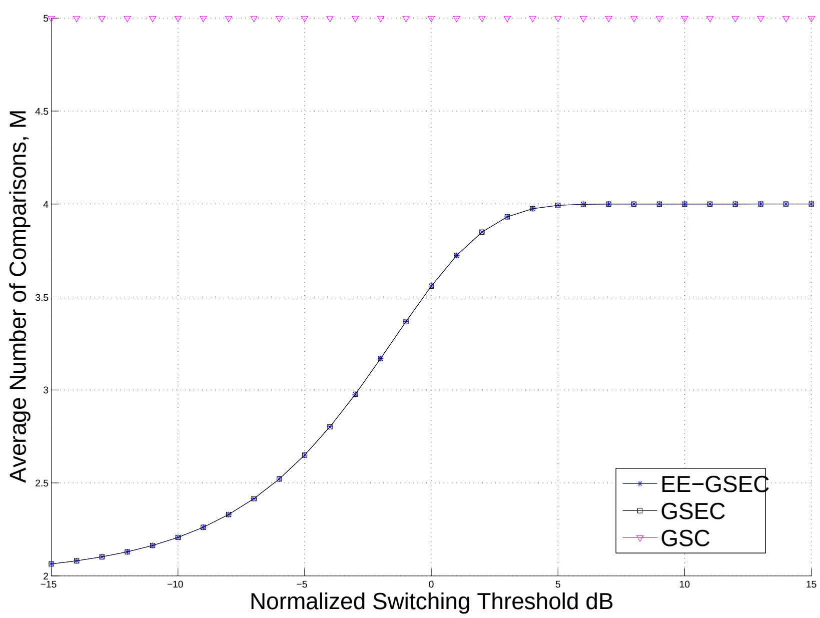

Figure 3.13: Comparison of average number of SNR comparisons needed of EE-GSEC, GSEC and GSC with $L = 4$, $L_c = 2$ as a function of normalized switching threshold, $\gamma_T/\bar{\gamma} dB$

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3.3.3 Average Number of Active Paths

We now look at the average number of active paths for MRC in EE-GSEC scheme. This is an important metric as the receiver can save significant processing power for every less active paths.

Figure 3.14 shows the average number of active paths in EE-GSEC as a function of switching threshold, γ_T. The arrows points to the threshold level that minimizes the average error rate for EE-GSEC Figure 3.9. We make the following observations:

- The number of active paths decreases with increase in the switching threshold γ_T. When the switching threshold is set too high the combiner is not able to find acceptable paths that matches the MRC input. Since acceptable paths when connected to the MRC input become the active paths for the system, at higher switching threshold this number decreases.

- For achieving the best error performance EE-GSEC requires, on an average, less than L_c active paths. This in contrast with GSC which requires to have all L_c paths active for MRC.

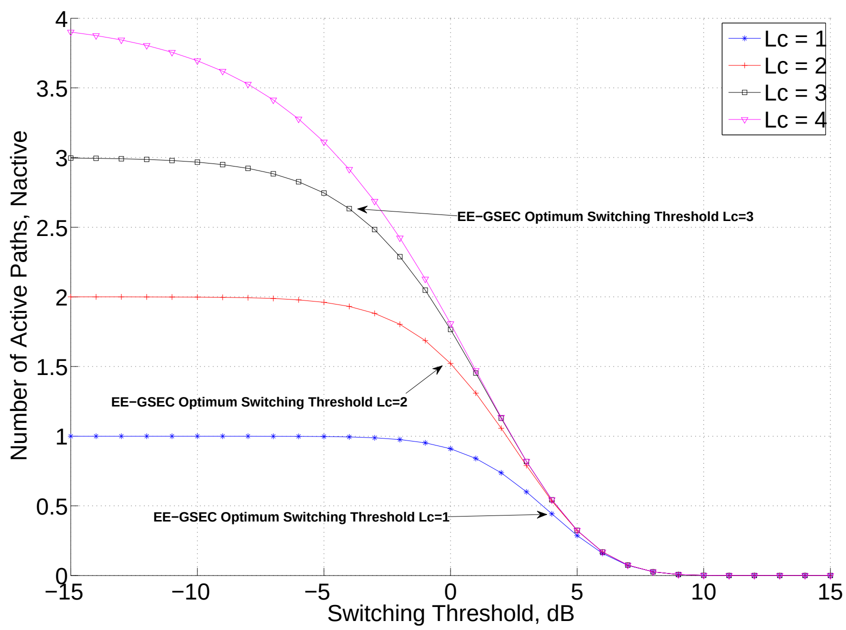

Figure 3.14: Average number of active paths $L = 4$ for different L_c as a function of switching threshold, γ_T

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Figure 3.15 shows the average number of active paths in EE-GSEC as a function of average SNR per path, $\bar{\gamma}$. We observe that number of active paths increases with the increase in average SNR per path as more paths are found to be acceptable.

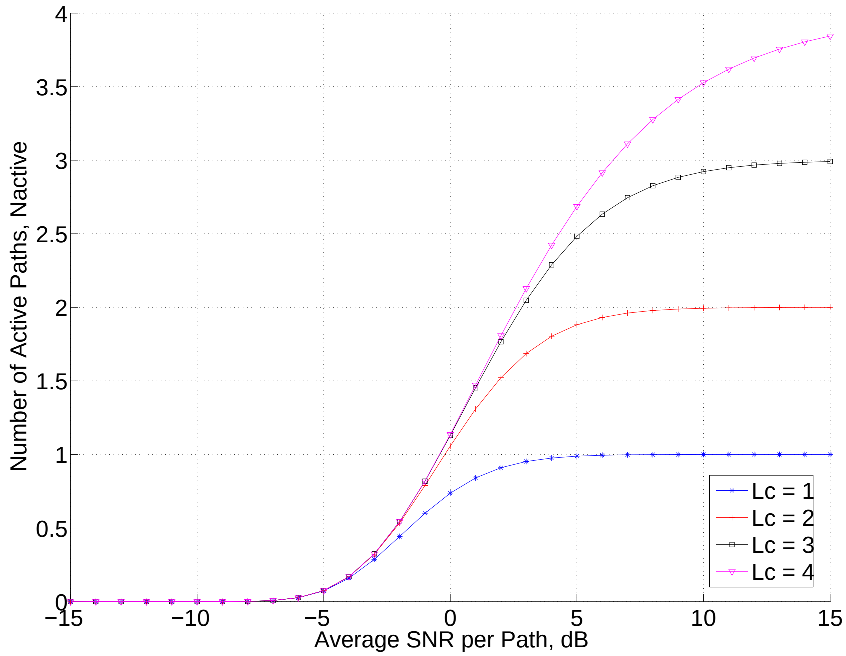

Figure 3.15: Average number of active paths over $L = 4$ for different L_c as a function of average SNR per path $\bar{\gamma}$, $\gamma_T = 0$dB

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3.4 Evaluation of Energy Efficiency of EE-GSEC

Based on the active paths analysis we will now evaluate the energy efficiency provided by EE-GSEC by comparing its Average Combined SNR,γ_c, per active path with that of GSC.

We start by observing Figure 3.16 that compares the Average Combined SNR per Active Path as a function of switching threshold for EE-GSEC and GSC. We observe that for EE-GSEC as the switching threshold increases, the average combined SNR per active path increases. This is because at higher switching thresholds paths that are deemed acceptable, and hence active, will have a high SNR. Since GSC does not perform a threshold test, its average combined SNR per active path remains constant. Hence, EE-GSEC provides better efficiency per active path as compared to GSC especially for higher switching threshold.

We now compare, though figure 3.17, the Average combined SNR per active path for EE-GSEC and GSC as a function of average SNR per path. We observe that for EE-GSEC the average combined SNR per active path increases with increase in

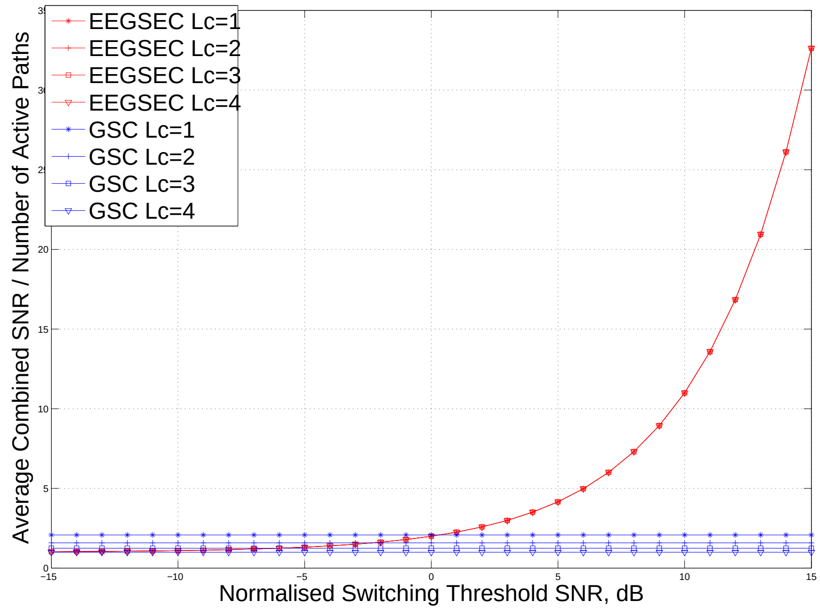

Figure 3.16: Comparison of Average combine SNR per active paths for EE-GSEC and GSC $L = 4$ as a function of switching threshold, γ_T

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the average SNR per path. This is due to the system combining paths with high SNR in good channel conditions. GSC, on the other hand, provides the same average combined SNR per active paths for higher L_c as the EE-GSEC for all paths. Hence, EE-GSEC provides better per path efficiency for average combined SNR. Also, EE-GSEC is able to provide higher average combined SNR per active path under poor channel conditions i.e. when the average SNR per path is low. This because when the channel conditions are poor, EE-GSEC only selects paths with high SNR for combining and ignores the weak paths.

The energy efficiency for EE-GSEC is viewed from the perspective of using less active paths to achieve comparable average combined SNR. The MRC system under EE-GSEC will combine, on an average, fewer paths than GSC that results in power savings at the receiver.

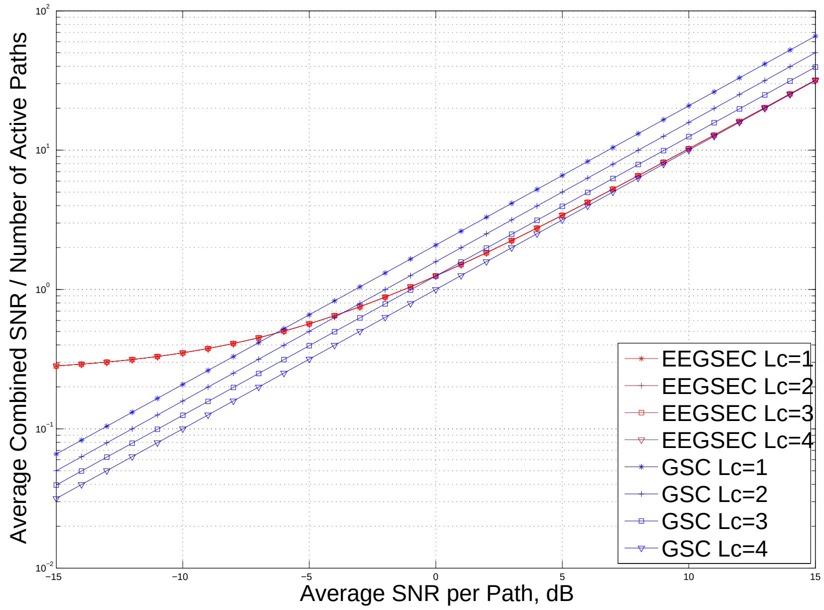

Figure 3.17: Comparison of Average combined SNR per active paths for EE-GSEC and GSC $L = 4$ as a function of average SNR per path, $\bar{\gamma}$, $\gamma_T = 0$dB

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3.5 Conclusion

In this chapter the performance and complexity evaluations of the presented EE-GSEC scheme were performed. These were compared and examined against two other competing schemes, GSC and GSEC. We observed the complexity-performance trade-off between these competing schemes. We also examined the energy efficiency of EE-GSEC over GSC in terms of average combined SNR per active path. We observed that EE-GSEC is able to provide better per active path efficiency over GSC.

Chapter 4

Summary and Future Research Path

4.1 Summary

An energy-efficient diversity scheme based on the well researched Generalized-Switch-and-Examine Combining (GSEC) is presented. The presented scheme is more efficient in terms of providing better average combined SNR per active path. This results in considerable processing power savings of the receiver especially compared to the GSC scheme. EE-GSEC performance in terms of the average combined SNR, outage probability and average bit error rate (BER) are comparable to GSEC under certain conditions. EE-GSEC's hardware complexity is lower than GSC and same as GSEC. This results in a considerable hardware cost savings at the receiver. However, the complexity savings come at the cost of performance when compared to GSC. This is a natural trade-off and needs to be considered when designing a wireless communication system.

In chapter 1 we examined the need for diversity in wireless communication systems by considering the phenomenon of fading and its effects on the transmitted wireless signal. A brief overview of the traditional and advanced diversity combining schemes was presented. The advanced diversity techniques offer a complexity-performance trade-off that have been well researched. We briefly introduced the mode of operation of the advanced diversity schemes namely GSC, GSEC and GSECps and observed the advantages/disadvantages offered by each of these schemes.

In chapter 2 the system and channel model for the presented EE-GSEC scheme is examined. Through an illustrative example the mode of operation of the EE-GSEC scheme was described. Based on the mode of operation of the $L_c/LSEC$ a detailed statistical analysis of the combiner output SNR was performed. We mathematically formulated the expression for CDF, pdf and the MGF of the combiner output SNR. Using these statistical analysis we obtained the mathematical expressions for the outage probability, average combined SNR and the average bit error rate. We also performed mathematical analysis of the complexity and also for the average number of active paths.

Chapter 3 presented the mathematical results obtained in Chapter 2 through $MATLAB_{©}$ simulations that allowed comparison with GSC and GSEC. We observed that the average combined SNR of the combiner for EE-GSEC maximizes for the optimum switching threshold. EE-GSEC's outage performance is on par with GSEC but GSC outperforms the former. The average error performance of EE-GSEC can also be maximized with the selection of optimum switching threshold. On comparing with GSEC and GSC we noted that although they outperform EE-GSEC, in diversity rich environments the performance gap is considerably reduced and EE-GSEC is able to offer similar error performance as the other two schemes. The complexity analysis of EE-GSEC substantiated its claims of complexity savings cost over GSC. EE-GSEC, on an average, requires less path estimations and SNR comparisons than GSC. We performed a energy efficiency evaluation between EE-GSEC and GSC and found that the former is able to provide better average combined SNR per active path. This results in EE-GSEC's system utilizing its resources in terms of active paths more efficiently than GSC.

4.2 Recommended Future Research Path

4.2.1 Examining EE-GSEC under Rician and Nakagami-m Fading Models

This thesis evaluated the operation and performance of EE-GSEC by assuming that the diversity paths undergo Rayleigh fading model. However, we can easily analyze the same for Rician and Nakagami-m fading models. The CDF, pdf and MGF under equations (2.3), (2.4) and (2.5) respectively, of the received SNR γ per path for Rician and Nakagami-m fading is shown in Table 4.1. By substituting these in appropriately we can evaluate EE-GSEC's performance for these fading models as well.

Table 4.1: Statistics of Fading Signal SNR γ for Rician and Nakagami-m Fading Models.

Model	**Rice**	**Nakagami-m**
Parameter	$K \geq 0$	$m \geq \frac{1}{2}$
PDF, $p_\gamma(x)$	$\frac{(1+K)}{\bar{\gamma}} exp(-K - \frac{(1+K)x}{\bar{\gamma}}) I_0\left(2\sqrt{\frac{(1+K)Kx}{\bar{\gamma}}}\right)$	$\left(\frac{m}{\bar{\gamma}}\right)^m \frac{x^{m-1}}{\Gamma(m)} exp(-\frac{mx}{\bar{\gamma}})$
CDF, $P_\gamma(x)$	$1 - Q_1\left(\sqrt{2K}, \sqrt{\frac{2(1+K)x}{\bar{\gamma}}}\right)$	$1 - \frac{\Gamma(m, \frac{mx}{\bar{\gamma}})}{\Gamma(m)}$
MGF, $M_\gamma(t)$	$\frac{1+K}{1+K-t\bar{\gamma}} exp(\frac{t\bar{\gamma}K}{1+K-t\bar{\gamma}})$	$\left(1 - \frac{t\bar{\gamma}}{m}\right)^{-m}$

4.2.2 Analyzing EE-GSEC's performance trade-off with Adaptive Combining Techniques

To better utilize the receiver's processing resources under favorable conditions adaptive diversity combining has been proposed recently. The goal behind adaptive combining is to meet a certain quality requirement by adaptively utilizing the diversity combiner resources. The receiver performs just enough combining operations such that the quality of the combiner output signal becomes acceptable. Examples of adaptive diversity combining techniques include minimum selection GSC (MS-GSC), output threshold MRC (OT-MRC), output threshold GSC (OT-GSC) and minimum estimation and combining GSC (MEC-GSC). Refer to [6] for more details on adaptive transmission.

A recommended future research topic would be to evaluate EE-GSEC's performance against the above mentioned schemes. The results would define the performance trade-off between these schemes.

Bibliography

[1] M. Kavehrad and P. J. McLane, "Performance of low-complexity channel coding and diversity for spread-spectrum in indoor, wireless communication," *AT & T Tech. J.*, vol. 64, pp. 1927–1965, Oct 1985.

[2] T. Eng, N. Kong, and L. B. Milstein, "Comparison of diversity combining techniques for rayleigh-fading channels," *IEEE Trans. Commun.*, vol. 44, pp. 1117–1129, Sep 1996.

[3] K. J. Kim, S. Y. Kwon, E. K. Hong, and K. C. Whang, "Comments on comparison of diversity combining techniques for rayleigh-fading channels," *IEEE Trans. Commun.*, vol. 46, pp. 1109–1110, Sep 1998.

[4] M. J. Gans, "The effect of gaussian error in maximal ratio combiners," *IEEE Trans. Commun. Technol.*, vol. COM-19, pp. 492–500, Aug 1971.

[5] B. R. Tomiuk, N. C. Beaulieu, and A. A. Abu-Dayya, "General forms for maximal ratio diversity with weighting errors," *IEEE Trans. Commun.*, vol. 47, pp. 488–492, Apr 1999.

[6] H. C. Yang and M. S. Alouini, *Order Statistics in Wireless Communications.* Cambridge University Press, 2011.

[7] T. Eng, N. Kong, and L. Milstein, "A selection combining scheme for rake receivers," in *Proc. IEEE Int. Conf. Universal Personal Communications*, (Tokyo, Japan), pp. 308–310, Nov 1995.

[8] M. Z. Win and Z. A. Kostic, "Virtual path analysis of selective rake receiver in dense multipath channels," *IEEE Commun. Lett.*, vol. 3, pp. 308–310, Nov 1999.

[9] Y. Roy, J. Y. Chouinard, and S. A. Mahmoud, "Selection diversity combining with multiple antennas for mm-wave indoor wireless channels," *IEEE J. Select. Areas Commun.*, vol. 14, pp. 674–682, May 1998.

[10] N. Kong and L. Milstein, "Average snr of a generalized diversity selection combining scheme," *IEEE Commun. Lett.*, vol. 3, p. 5759, Mar 1999.

[11] N. Kong and L. B. Milstein, "A closed form expression for the average snr when combining an arbitrary number of diversity branches with nonidentical rayleigh fading statistics," in *Proc. IEEE Int. Conf. Communications*, (Vancouver, BC, Canada), pp. 1864–1868, Jun 1999.

[12] M. S. Alouini and M. K. Simon, "Performance of coherent receivers with hybrid sc/mrc over nakagami-m fading channels," *IEEE Trans. Veh. Technol.*, vol. 48, pp. 1155–1164, Jul 1999.

[13] M. Z. Win and Z. A. Kostic, "Impact of spreading bandwidth on rake reception in dense multipath channels," *IEEE J. Select. Areas Commun.*, vol. 17, pp. 1794–1806, Oct 1999.

[14] M. S. Alouini and M. K. Simon, "An mgf-based performance analysis of generalized selective combining over rayleigh fading channels," *IEEE Trans. Commun.*, vol. 48, pp. 401–415, Mar 2000.

[15] A. Annamalai and C. Tellambura, "Error rates of hybrid sc/mrc diversity systems on nakagami-m channels," in *Proc. IEEE Wireless Communication, Networking Conf.*, (Chicago, IL), pp. 211–215, Sep 2000.

[16] Y. Ma and C. Chai, "Unified error probability analysis for generalized selection combining in nakagami fading channels," *IEEE J. Select. Areas Commun.*, vol. 18, pp. 2198–2210, Nov 2000.

[17] A. Annamalai and C. Tellambura, "Analysis of hybrid selection/maximal-ratio diversity combiner with gaussian errors," *IEEE Trans. Wireless Commun.*, vol. 1, pp. 498–512, Jul 2002.

[18] H. C. Yang and M. S. Alouini, "Generalized switch and examine combining (gsec): A low-complexity combining scheme for diversity rich environments," *IEEE Trans. Commun.*, vol. 52, pp. 1711–1721, October 2004.

[19] H. C. Yang and M. S. Alouini, "Improving the performance of switched diversity with post-examining selection," *IEEE Trans. Wireless Commun.*, vol. 5, pp. 67–71, January 2006.

[20] A. F. Molisch, M. Z. Win, and J. H. Winters, "Capacity of mimo systems with antenna selection," in *Proc. IEEE Int. Conf. Communications*, (Helsinki, Finland), pp. 570 – 574, Jun 2001.

[21] H. C. Yang and M. S. Alouini, "Performance analysis of generalized selection combining with threshold test per branch (t-gsc)," *IEEE Trans. Commun.*, vol. 51, pp. 782–794, May 2003.

[22] M. K. Simon and M. S. Alouini, *Digital Communications over Generalized Fading Channels: A Unified Approach to Performance Analysis.* New York John Wiley & Sons, 2000.

[23] I. S. Gradshteyn and I. M. Ryzhik, "Table of integrals, series, and products," San Diego, CA, 1994. Academic.

[24] J. W. Craig, "A new, simple, and exact result for calculating the probability of error for two-dimensional signal constellations," in *Proc. IEEE Military Communications Conf. (MILCOM 91)*, (McLean, VA), pp. 571–575, Oct 1991.

[25] M. K. Simon, "A new twist on the marcum q-function and its applications," *IEEE Commun. Lett.*, vol. 2, pp. 39–41, Feb 1998.

[26] C. W. Helstrom, *Elements of Signal Detection and Estimation.* Prentice Hall, 1995.

[27] M. S. Alouini and A. J. Goldsmith, "A unified approach for calculating the error rates of linearly modulated signals over generalized fading channels," *IEEE Trans. Commun.*, vol. 47, pp. 1324–1334, September 1999.