## Institutionen för systemteknik Department of Electrical Engineering

Examensarbete

## Modeling of Fuel Dynamics in a Small Two-Stroke Engine Crankcase

Examensarbete utfört i Fordonssystem vid Tekniska högskolan vid Linköpings universitet av

Johan Andersson och Oscar Wyckman

LiTH-ISY-EX–15/4833–SE

Linköping 2015



Department of Electrical Engineering Linköpings tekniska högskola Linköpings universitet Linköpings universitet SE-581 83 Linköping, Sweden 581 83 Linköping

## Modeling of Fuel Dynamics in a Small Two-Stroke Engine Crankcase

### Examensarbete utfört i Fordonssystem vid Tekniska högskolan vid Linköpings universitet av

Johan Andersson och Oscar Wyckman

LiTH-ISY-EX–15/4833–SE





Linköping, 2 juni 2015



## **Abstract**

For any crankcase scavenged two-stroke engine, the fuel dynamics is not easily predicted. This is due to the fact that the fuel has to pass the crankcase volume before it enters the combustion chamber. This thesis is about the development of a model for fuel dynamics in the crankcase of a small crankcase scavenged two-stroke engine that gives realistic dynamic behavior.

The crankcase model developed in this thesis has two parts. One part is a model for wall wetting and the other part is a model for concentration of evaporated fuel in the crankcase. Wall wetting is a phenomenon where fuel is accumulated in fuel films on the crankcase walls. The wall wetting model has two parameters that have to be tuned. One is for the fraction of fuel from the carburetor that is not directly evaporated and one parameter is for the evaporation time of the fuel film.

The thesis treats tuning of these parameters by running the model with input data from measurements. Since not all input data are possible to measure, models for these inputs are also needed. Hence, development of simple models for air flows, fuel flow, gas mixing in the exhaust and the behavior of the *λ*-probe used for measurements are also treated in this thesis.

The parameter estimation for the crankcase model made in this thesis results in parameters that corresponds to constant fraction of fuel from the carburetor that evaporates directly and a wall wetting evaporation rate that increases with increasing engine speed. The parameter estimation is made with measurements at normal operation and three specific engine speeds. The validity of the model is limited to these speeds and does not apply during engine heat-up.

The model is run and compared to validation data at some different operation conditions. The model predicts dynamic behavior well, but has a bias in terms of mean level of the output  $\lambda$ . Since this mean value depends on the relation between input air and fuel flow, this bias is probably an effect of inaccuracy in the simple models developed for these flows.

## **Sammanfattning**

För alla tvåtaktsmotorer med bränslematning genom vevhuset är bränsledynamiken svårpredikterad. Detta beror på att bränslet måste passera vevhusvolymen innan det når förbränningskammaren. Denna uppsats handlar om utveckling av en modell som ger realistisk dynamik för bränslet i tvåtaktsmotorers vevhus.

Vevhusmodellen i denna uppsats har två delar. Den ena delen är en modell för bränslefilm på motorväggar och den andra delen är en modell för koncentration av förångat bränsle i vevhusvolymen. Bränslefilmsmodellen har två parametrar som måste trimmas. Den ena är andelen bränsle från förgasaren som inte förångas direkt och den andra är tidsåtgången för förångning av bränslefilmen.

Uppsatsen behandlar trimning av dessa parametrar genom körning av modellen med indata från mätningar. Eftersom inte all indata kan mätas behövs även modeller för dessa. Därför behandlar uppsatsen även utveckling av enkla modeller för luftflöde, bränsleflöde, gasblandning i avgasvolymen och beteende hos den för mätningar använda *λ*-sonden.

Parameterestimeringen för vevhusmodellen som är gjord i denna uppsats resulterar i parametrar som svarar mot konstant andel av bränslet från förgasaren som förångas direkt och en förångningshastighet för bränslefilmen som ökar med ökande motorhastighet. Parameterestimeringen är gjord med mätdata från normal körning vid tre olika motorhastigheter. Giltigheten för modellen är begränsad till dessa hastigheter och kan inte appliceras på körning av motorn vid kallstart.

Modellen är körd och jämförd med valideringsdata från olika körfall. Modellen förutser dynamiska beteenden väl, men har ett systematiskt fel gällande medelvärdet på *λ*. Eftersom detta medelvärde beror på förhållandet mellan luftflöde och bränsleflöde in i vevhuset är sannolikt detta systematiska fel en effekt av osäkerhet i de enkla modeller som utvecklats för dessa flöden.

## **Acknowledgments**

We would like to thank our examiner Lars Eriksson and Husqvarna AB for giving us the opportunity to write this thesis. We are thankful to Henrik Eklund, our supervisor at Husqvarna AB, for constructive discussions and for being very helpful.

The help of Xavier Llamas Comellas, our supervisor at the university, has been invaluable to us. He has spent a lot of his time and effort in guiding us in the right direction. We are truly grateful for this.

We want to thank Fredrik Hellquist at Husqvarna AB for helping us with measurements and tests. We would also like to thank Conny Andersson and Junting Qui for interesting discussions and enjoyable company in the project room.

Finally, we want to thank our families and our beloved Olivia and Louise for their support throughout this endeavor.

> *Linköping, June 2015 Johan Andersson and Oscar Wyckman*

## **Contents**





## **Notation**

<span id="page-14-0"></span>





# **1 Introduction**

<span id="page-16-0"></span>In this chapter the thesis work, a background and an explanation of the problem are introduced. The overall goal of the thesis will be explained as well as the expected results. In the end of the chapter, the outline of the thesis report is presented.

## <span id="page-16-1"></span>**1.1 Background**

Today, most chainsaws and some other products are powered by small two-stroke engines. In these small engines it is common to have carburetors and crankcase scavenging, which means that the air/fuel path goes through the engine crankcase. To achieve competitive engine performance, manufacturers spend a lot of time and money on control systems development. When designing such control systems it is important to get a good understanding of fuel dynamics, since control of the amount of fuel into the engine is one of the tasks these systems have.

<span id="page-16-2"></span>Design of control systems is made more efficient by having accurate engine models. It is therefore essential to create models that behave like their real-life counterparts. Since the crankcase volume is big relative to flows of fuel and air, it acts like an averaging volume and therefore affects fuel dynamics between the carburetor and the combustion chamber. In the crankcase, the fuel dynamics is affected by wall-wetting and evaporation processes. For small two-stroke engines, fuel dynamics in the crankcase is not fast enough to be considered instantaneous and thus a crankcase model that captures the physics is a key part for a good model of two-stroke crankcase scavenged engines.

## **1.2 Problem formulation**

Husqvarna has made a Matlab/Simulink two-stroke engine model, whose purpose is to be a tool for development of the engine control system. The crankcase component of the model does not handle air/fuel dynamics sufficiently well, and thus a correction factor has been introduced to improve the model accuracy. This factor has no physical meaning and therefore has to be experimentally determined, which is a time-consuming activity and it requires an actual product or prototype for testing.

There are ways to model air/fuel dynamics in the crankcase with good accuracy, for example high-resolution modeling such as computational fluid dynamics. However, these models are computationally demanding and cannot be utilized in Husqvarna's Matlab/Simulink model, since it has to be able to run in real-time. For these reasons, it would be desirable to have a crankcase model which is related to physical principles and does not require too much computational power. Husqvarna's model is an event-based model and a crankcase model has to be in that domain in order to fit their existing model. This poses new challenges since most physical phenomena are time domain based.

## <span id="page-17-0"></span>**1.3 Purpose and goals**

The purpose of the thesis is to investigate the air/fuel dynamics in the crankcase. This should lead to a greater knowledge about the underlying physical phenomena.

The goal of this thesis is to develop a model for the air and fuel dynamics in the crankcase. This new crankcase model must be possible to integrate in the full engine model used at Husqvarna and therefore it needs to be event-based.

## <span id="page-17-1"></span>**1.4 Expected results**

The final result expected by Husqvarna is an event-based model of the air/fuel dynamics in the crankcase. The model must be able to run in real time, or faster, and the trigger event will be one engine revolution.

The model is expected to be validated for different operating conditions. This should be done because it is important to get an understanding of the model.

## <span id="page-17-2"></span>**1.5 Outline**

In this first chapter, an introduction to the thesis and its objective have been presented. In chapter 2, fundamental concepts needed for understanding of the thesis contents are briefly explained. An introduction to research related to the thesis is then presented in the third chapter. Modeling of systems that are interfacing the crankcase, and results thereof, are presented in chapter 4. In the fifth chapter, the actual crankcase model and its implementation in Matlab/Simulink are described. In chapter 6, the method and results of parameter estimation for the crankcase model are presented. In chapter 7, a validation of the model is shown and its results are discussed. Finally, conclusions and suggestions on how the thesis work can be continued are presented in chapter 8.

# <span id="page-20-3"></span>**2 Basic theory**

<span id="page-20-0"></span>There are some concepts that are essential to be aware of in order to understand this thesis. These concepts are explained in this chapter. First, the two-stroke cycle operation is explained, followed by an explanation of important gas exchange parameters. The path of air and fuel through the engine is then discussed and in the end of the chapter the gas mixture variable, called air/fuel ratio, is expressed and explained.

## <span id="page-20-1"></span>**2.1 The two-stroke engine**

The two-stroke engine is the most common internal combustion engine type for small and very large engines. The small ones are used in applications such as mopeds, chainsaws and other forestry and gardening equipment. The engine type is relatively cheap, simple, robust, compact and light and therefore fits the needs for these applications. Large two-stroke engines are used for ships and for power-generation. The appeal for these applications is that the two-stroke engine is cost effective and has high efficiency. Unlike small two-stroke engines, these run on diesel and are not crankcase scavenged. [\[Heywood and Sher, 1999\]](#page-67-0)

## <span id="page-20-2"></span>**2.1.1 The two-stroke cycle for small engines**

The two-stroke engine cycle completes a full cycle during one engine revolution. Starting at the Top Dead Center (TDC) the compressed air/fuel mixture that is ignited causes a rise in pressure and temperature that drives the piston downwards in the power stroke.

When the piston is at the top, the inlet port is open so that the air/fuel mixture enters the crankcase. During the downward stroke the exhaust port opens, <span id="page-21-2"></span>which allows the exhaust gas to exit the combustion chamber. At the same time the air/fuel mixture in the crankcase is compressed by the piston moving downwards.

As the piston is in the area close to the Bottom Dead Center (BDC), both the inlet port from the crankcase and the exhaust port are open. The pressure in the crankcase is now higher than the pressure inside the cylinder, which forces the fresh charge to enter the cylinder and push out the burned gases in what is called the scavenge process.

The last step of the cycle is the upward stroke where the cylinder ports start closing. The point where the exhaust port closes is called the trapping point. At this point the scavenging process is over and the combustion chamber is now filled with both a fresh air/fuel mixture and exhaust gases that were trapped in the cylinder from previous combustion. After trapping of the gases, all ports are closed and the compression begins when the piston moves upwards for a new ignition. [\[Blair, 1996\]](#page-66-1)

<span id="page-21-1"></span>

Figure 2.1: Principle of 2-stroke cycle events.

The described events are graphically presented in Figure [2.1.](#page-21-1) The two strokes of the cycles are completed in one engine revolution. One stroke is completed when the piston moves upwards and the other stroke is completed when the piston moves downwards.

### <span id="page-21-0"></span>**2.1.2 Delivery ratio, trapping and scavenging efficiencies**

Since intake and exhaust of the combustion chamber are simultaneous in twostroke engines, loss of fresh fuel charge and mixing of fresh and burnt gases are two of the main drawbacks when compared to four-stroke engines. Engine operation is to a large extent depending on how well these drawbacks are minimized [\[Heywood and Sher, 1999\]](#page-67-0). To describe the characteristics of a two-stroke engine, parameters of the gas exchange process are essential. One of these parameters is the *delivery ratio*.

$$
\Lambda = \frac{\text{mass of fresh charge delivered}}{\text{mass of ideal fresh charge delivery}} \tag{2.1}
$$

<span id="page-22-2"></span>This ratio compares the actual amount of fresh charge delivered to the combustion chamber and the amount required in an ideal charging process.

Another important parameter is the *trapping efficiency*, which describes the delivered amount of fresh charge that is retained in the cylinder in the end of the gas exchange process.

$$
\eta_{tr} = \frac{\text{mass of fresh charge retained}}{\text{mass of fresh charge delivered}} \tag{2.2}
$$

The *scavenging efficiency* is used to express how much of the burnt exhaust gases are replaced with fresh charge.

$$
\eta_{sc} = \frac{\text{mass of fresh charge in the cylinder}}{\text{total mass of cylinder charge}} \tag{2.3}
$$

### <span id="page-22-0"></span>**2.2 Airhead scavenging**

One of the main challenges with two-stroke engines today is the high level of hydrocarbon emissions, due to fuel that passes the engine without being burnt. Many different techniques have been suggested to deal with this problem, among which sequentially stratified scavenging is one. Engines with sequentially stratified scavenging have a scavenging process that is divided into two parts, one with pure air and one with a mixture of fuel and air. [\[Bergman et al., 2003\]](#page-66-2)

The engine studied in this thesis utilizes an airhead for sequentially stratified scavenging. The airhead has separate air channels and a separate throttle. Through these, airhead air fills the scavenging channel at a certain point in the cycle. When the combustion chamber port opens, gases flow from the crankcase to the combustion chamber via the scavenging channel. Since the airhead air has filled the scavenging channel, it will be the first gas that enters the combustion chamber and thus being the gas that most probably flushes out exhaust gases. Since the airhead air does not contain any fuel, this limits fuel loss. [\[Martinsson et al.,](#page-67-1) [2008\]](#page-67-1)

### <span id="page-22-1"></span>**2.3 The air/fuel ratio**

A very common term for the gas mixture in internal combustion engines is the air/fuel ratio (AFR), which is defined as the ratio between air and fuel mass flow rates through an engine.

$$
AFR = \frac{\dot{m}_a}{\dot{m}_f} \tag{2.4}
$$

There is a specific ratio that equals the minimum amount of air needed for full oxidation of the fuel. This chemical equilibrium is defined as the stoichiometric air/fuel ratio, *AFRstoich*. Since the exhaust gas composition is very dependent on whether the actual ratio is larger or smaller than the stoichiometric ratio, it is common practice to use the normalized mixture parameter  $\lambda$  that is defined in

<span id="page-23-0"></span>the following way.

$$
\lambda = \frac{AFR_{actual}}{AFR_{stoich}}
$$
 (2.5)

A *λ*-value of 1 means that the engine operates under stoichiometric conditions. A *λ*-value larger than 1 means that there is more air than needed for complete fuel oxidation and the engine is thus running fuel-lean. A fuel-rich mixture where there is not enough air for full oxidation is represented by a *λ*-value smaller than 1. [\[Heywood and Sher, 1999\]](#page-67-0)

# <span id="page-24-2"></span><span id="page-24-0"></span>**3 Related research**

There is some research done about modeling of two-stroke engines, for example [Blair](#page-66-1) [\[1996\]](#page-66-1), [Hendricks](#page-67-2) [\[1986\]](#page-67-2), [Theotokatos](#page-67-3) [\[2009\]](#page-67-3). This research is based on large engines that run at low speeds. This thesis will mainly focus on fuel dynamics. The research on fuel dynamics is predominantly focused on four stroke engines, [Aquino](#page-66-3) [\[1981\]](#page-66-3), [Curtis et al.](#page-66-4) [\[1996\]](#page-66-4), [Simons et al.](#page-67-4) [\[2000\]](#page-67-4), [Locatelli et al.](#page-67-5) [\[2004\]](#page-67-5) to name a few. Much of this research can be applied to two-stroke engines as well. Most of the research regarding engine and fuel dynamics is time-based but there is also some research carried out in the event-based domain with either full cycles or crank angle as increment, for example [Amstutz et al.](#page-66-5) [\[1994\]](#page-66-5), [Chang](#page-66-6) [et al.](#page-66-6) [\[1993\]](#page-66-6). This chapter introduces research that is relevant to the subject of this thesis.

## <span id="page-24-1"></span>**3.1 Modeling of two-stroke engines**

There could be many different uses of engine models, for example simulation of engine performance and online prediction of dynamic response. The early model by [Hendricks](#page-67-2) [\[1986\]](#page-67-2) was developed for those reasons. By a combination of basic physics and simplifications of the more complex models at the time, Hendricks created a light-weight model that still was very accurate. The modeled engine was a large turbocharged two-stroke diesel engine.

In his paper, [Hendricks](#page-67-2) [\[1986\]](#page-67-2) presents a low order mean value model that takes fuel intake as its input variable, and engine shaft speed and scavenging pressure as state variables. The engine shaft speed is used as state variable since it is directly related to engine torque and power, and the scavenging pressure is used as state variable because it is simply related to air mass flow and thus also to fuel

<span id="page-25-1"></span>mass flow. The model is designed through equation simplification, coefficient identification and some variable dependency mapping. Through validation, it is concluded that the developed model, which is dynamic and non-linear, give high-accuracy results for a big operating range.

[Turesson](#page-67-6) [\[2009\]](#page-67-6) attempts to create a basic two-stroke engine mean value model, of a large-bore turbocharged engine of the type used for ship propulsion, in the software Simulink. He approaches the problem by splitting up the engine model componentwise into submodels. These submodels are the compressor, the intake manifold, the combustion chamber, the turbine, the turbo shaft and an external load.

Since the modeling is not made after a specific engine, some coefficients have to be estimated. Simulating the submodels gives a general idea about how different parameters affect each other and the component characteristics, but the lack of real engine data leaves the model without a clear validation.

[Theotokatos](#page-67-3) [\[2009\]](#page-67-3) presents two mean value models of different complexity for a marine diesel engine implemented in MATLAB/Simulink. These models are of the same type that [Turesson](#page-67-6) [\[2009\]](#page-67-6) attempted to create, but these models are parameterized and validated with experimental data. The two models presented by [Theotokatos](#page-67-3) [\[2009\]](#page-67-3) both show good correlation to the experimental data.

It is worth noting that the models mentioned above all are for large two-stroke engines. It is unclear how well the models can be generalized to be valid for small two-stroke engines as well, since they are different in many ways. Large two-stroke engines run on much lower angular velocities than small two-stroke engines, and often have direct fuel injection whereas the small ones have carburetors. There are many other design features that differ, such as crankcase scavenging which is only used in small two-stroke engines. [\[Heywood and Sher,](#page-67-0) [1999\]](#page-67-0)

One way to use engine models is to get a more detailed view of the engine dynamics and characteristics. [Blair](#page-66-1) [\[1996\]](#page-66-1) show how this kind of more advanced models could be used to see for example performance characteristics as well as single cycle pressure variations in the engine.

## <span id="page-25-0"></span>**3.2 Event-based modeling**

In event-based modeling the system dynamics is described in terms of events instead of time. According to [Chang et al.](#page-66-6) [\[1993\]](#page-66-6) the event-based dynamics are best described in the crank angle domain. Since physical models are often expressed in the time domain they need to be transferred to the event-based domain, in order to be used in this type of models.

<span id="page-26-4"></span>To switch between the time domain and the crank angle domain the following substitution can be used.

<span id="page-26-3"></span>
$$
dt = \frac{d\theta}{N} \tag{3.1}
$$

Here, *N* is revolutions per minute and  $\theta$  is crank angle in degrees. [Chin and Coats](#page-66-7) [\[1986\]](#page-66-7) shows a relation between the time domain and the crank angle domain using the Laplace transform.

<span id="page-26-1"></span>
$$
\theta = Nt \tag{3.2}
$$

Constant N in [\(3.2\)](#page-26-1) gives the following relation between engine variables in the two domains.

$$
Y_{\theta}(s_{\theta}) = N Y_t(N s_{\theta})|_{N=constant}
$$
\n(3.3)

This also gives a relation between transfer functions for the time domain and the crank angle domain.

$$
G_{\theta}(s_{\theta}) = G_t(Ns_{\theta})|_{N=constant}
$$
\n(3.4)

This gives an exact relation between the domains as long as the speed of revolution is held constant. In case it is not held constant, it is still said to be good enough for controlling purposes.

[Chin and Coats](#page-66-7) [\[1986\]](#page-66-7) also states that, with the exception of the fuel dynamics, most dynamics are less varying in the crank angle domain than in the time domain.

In terms of angular crank velocity, [\(3.2\)](#page-26-1) can be expressed as the product of angular velocity and time.

$$
\theta = \omega t \tag{3.5}
$$

Differentiating this relation with the product rule, one gets the following.

<span id="page-26-2"></span>
$$
d\theta = d\omega \cdot t + \omega \cdot dt \tag{3.6}
$$

This is the basis for the domain switch method discussed. Both [Chang et al.](#page-66-6) [\[1993\]](#page-66-6) and [Chin and Coats](#page-66-7) [\[1986\]](#page-66-7) neglect the first term in [\(3.6\)](#page-26-2) when developing their methods. This implies that the angular acceleration of the crankshaft  $d\omega$  is so small it is neglectable.

## <span id="page-26-0"></span>**3.3 Wall wetting**

For the scope of this thesis, wall wetting in the crankcase of a crankcase scavenged two-stroke engine is to be considered. Current research on wall wetting is mainly focused on the intake manifold and port of larger engines, which are seldom crankcase scavenged. This wall wetting theory must be adapted for the relevant engine type, for the purpose of this thesis.

As fuel is injected in an intake manifold, only parts of it is directly mixed with air and inducted into the cylinder. The rest of the fuel is deposited on the manifold

<span id="page-27-4"></span>walls as fuel film or as fuel puddles and this phenomenon is called wall wetting. The dynamic delay between fuel injection and its response as a change in measured AFR can be partly explained by wall wetting. Wall wetting affects engine operation more during cold starts than when the engine has reached its working temperature and when it is running at high speeds. A simple model based on the fuel mass flows for wall wetting and into the cylinder can be derived from fuel injection flow, engine speed and intake manifold pressure, but there are also other more detailed models for wall wetting and fuel evaporation. [\[Eriksson and](#page-66-8) [Nielsen, 2014\]](#page-66-8)

#### <span id="page-27-0"></span>**3.3.1 Wall wetting modeling as a first order lag**

By studying the nature and causes of A/F transients in a five liter carbureted engine, [Aquino](#page-66-3) [\[1981\]](#page-66-3) developed a model that is given by a first order differential equation for the mass in the fuel film. It is based on the assumption that a percentage *X* of the injected fuel impacts the walls and there forms a film, and that the quantity of fuel evaporating from the film is proportional to the current amount of fuel in the film.

<span id="page-27-3"></span>
$$
\dot{m}_{ww} = X \dot{m}_{f,inj} - \frac{1}{\tau} m_{ww} \tag{3.7}
$$

This means that the change rate of evaporated fuel in the engine can be described by the following.

<span id="page-27-2"></span>
$$
\dot{m}_{EV} = (1 - X)\dot{m}_{f,inj} + \frac{1}{\tau}m_{ww}
$$
\n(3.8)

The time constant *τ* explains the rate at which the fuel film evaporates depending on its current mass. Equation [\(3.8\)](#page-27-2) is only valid when the effects of wall wetting are large compared to the effects of air charging. In other cases, this basic model has to be compensated for the air charging effects.

The parameters *X* and *τ* are model parameters that need to be fitted to experimental data. They could be functions of different engine variables, such as temperature, pressure, air velocity or fuel properties. As an example, [Aquino](#page-66-3) [\[1981\]](#page-66-3) modeled *X* as a function of the throttle angle and *τ* as a function of manifold temperature.

This kind of model treats a quite slow dynamic compared to the more physical models since only the main effect of the wall wetting and evaporation is accounted for and not the actual underlying physical principles. In the case that the fuel mass flow dynamics is slower than the chosen sample time, this model could be used as a discrete model.

### <span id="page-27-1"></span>**3.3.2 Physical modeling of wall wetting**

[Locatelli](#page-67-7) [\[2004\]](#page-67-7) describes a wall wetting model consisting of two parts, which are based on fundamental physical principles. The first part describes the evaporation of the injected fuel. The injected fuel is considered to be in droplet form and thus droplet evaporation is described. The amount of fuel that is not evaporated in this first model part becomes wall wetting. The second part of the model is the evaporation of the wall wetting. This two-part model describes an intake manifold injected four-stroke engine in continuous time.

The model output is the wall wetting mass flow, where the mass flow into the wall wetting is the injected fuel minus the evaporation of fuel droplets. The mass flow out of the wall wetting is the evaporation flow from the fuel film.

<span id="page-28-1"></span>
$$
\dot{m}_{ww} = \dot{m}_{ww,in} - \dot{m}_{ww,out} \tag{3.9a}
$$

$$
\dot{m}_{ww,in} = \dot{m}_{f,inj} - \dot{m}_{EV,D} \tag{3.9b}
$$

$$
\dot{m}_{ww,out} = \dot{m}_{EV,F} \tag{3.9c}
$$

The thesis is based on a fuel evaporation formula, which used for both droplet and fuel film evaporation.

<span id="page-28-0"></span>
$$
\dot{m}_{EV} = \frac{\rho_{lg} D_{AB} A_f}{d} Sh \cdot ln(1 + B) \tag{3.10}
$$

In this equation,  $\rho_{lq}$  is the surface density of an evaporating fluid and is assumed to be the average of its liquid and gas densities. The Spalding number, *B*, is dependent on fuel temperature and ambient temperature. The Sherwood number, *Sh*, is computed with experimentally estimated constants and the Reynolds, *Re*, and Schmidt, *Sc*, numbers.

$$
Re = \frac{u_{\infty}d}{\nu} \tag{3.11a}
$$

$$
Sc = \frac{v}{D_{AB}}\tag{3.11b}
$$

The variables in the evaporation mass flow [\(3.10\)](#page-28-0) are computed differently depending on whether fuel droplets or fuel film is considered. The evaporation is highly dependent on geometry and since droplet size changes rapidly, there is a separate differential equation for that included in the model. Also, the equation is true for one droplet only and thus has to be multiplied by the number of airborne droplets, which is a function of time and an experimentally evaluated time constant.

There are a number of issues with this model if it is to be implemented in a discrete model. Primarily, eq. [\(3.9a\)](#page-28-1) and thus its subequations have to be integrated over the sampling time. Since model dynamics is fast, the sampling time step size must be small. An alternative way would be to map variable behavior over sampling time with other variables as input. The mapping must be done through experimental data and therefore measurements need to be carried out. This model implementation manner prevents the model from being scalable, because a new mapping must be done for each new engine setup. Even if the modeling is carried out in continuous time, measurements need to be made. For example the mentioned time constant and droplet initial size have to be measured in some way.

<span id="page-29-1"></span>Also, the described model is built on a port fuel injected engine. For an engine with a carburetor, the fuel droplets are created through a different process. This would imply some of the droplet equations to be altered in order to use the model with this type of engine.

#### <span id="page-29-0"></span>**3.3.3 Statistical modeling of wall wetting**

A statistical approach for a detailed model of wall-wetting of a port fuel injected engine is the one developed by [Curtis et al.](#page-66-4) [\[1996\]](#page-66-4). Here the injected fuel is treated as particles using the Sauter Mean Diameter. The size of the fuel particles is said to be distributed according to the Rosin-Rammler distribution and every particle smaller than a certain size is treated as vapor. The particles that are larger than this threshold are being distributed as fuel films.

At certain points in the engine cycle, pressure distribution in the engine is such that the air flow direction is reversed. This is called the backflow process and can cause the fuel films to vaporize or break up into droplets. The droplets created in the backflow process are also distributed as a Rosin-Rammler distribution and are divided into vapor or fuel films. The vaporization of the fuel films follow the Reynolds analogy, see eq. [\(3.10\)](#page-28-0). The dynamic modeling of the fuel films also includes a dragging flow of liquid film by high velocity air and this is modeled by means of physical properties. In total, the model keeps track of the fuel puddle mass balances by integration of the mass flow rates of injection fuel film distribution, backflow vaporization and film flow.

The model output is highly dependent on the fuel puddle heights, which affect film vaporization, and fuel puddle areas, which affect film flow. Puddle growth and shrinkage is based on the ratio between the puddle and a reference puddle. An exponential constant *Z* is used to determine puddle height and area from this ratio. For model calibration, this constant has to be experimentally estimated for each puddle.

$$
\frac{h}{h_{ref}} = \left(\frac{V}{V_{ref}}\right)^2\tag{3.12a}
$$

$$
\frac{A}{A_{ref}} = \left(\frac{V}{V_{ref}}\right)^{1-Z}
$$
\n(3.12b)

[Arias](#page-66-9) [\[2005\]](#page-66-9) shows two different empirical correlations that could give an estimation of the Sauter Mean Diameter for the droplets created by the high air flow speed in the carburetor. The first is the Nukiyama-Tanasawa equation.

$$
D_{SMD} = \frac{585}{v} \sqrt{\frac{\sigma_l}{\rho_l}} + 597 \left(\frac{\mu_l}{\sqrt{\sigma_l \rho_l}}\right)^{0.45} \left(\frac{1000q_l}{q_a}\right)^{1.5}
$$
(3.13)

In this case, *v* is relative speed between the liquid and the air stream. This model is said to give a moderately good agreement with carburetors.

The second empirical correlation is the Hinze equation.

$$
We_{crit} = \frac{d_{max} \rho_a v_{init}^2}{\sigma_l}
$$
 (3.14a)

<span id="page-30-0"></span>
$$
D_{SMD} = 0.532 d_{max} \tag{3.14b}
$$

Here,  $v_{init}$  is the speed difference between air and fuel droplets at droplet creation. The critical Weber number  $We_{crit}$  is estimated to be 14.5. The relation between the Sauter Mean Diameter and the maximum droplet diameter is given by the Rosin-Rammler distribution with the assumption that its spread parameter  $n = 2$ . This model is said to predict the Sauter Mean Diameter for droplets created in the carburetor better than the Nukiyama-Tanasawa equation.

**4**

## <span id="page-32-3"></span><span id="page-32-0"></span>**Modeling of the crankcase interface**

Even though the crankcase model is meant to work with the full engine model developed by Husqvarna, the crankcase is isolated for modeling and validation purposes in this thesis. By isolating the crankcase model, its inputs and output can be taken from measurements rather than from modeled values. This way its internal parameters can be tuned in a way that makes the model give an approximation of the measured output from the measured inputs.

The measurement values that are used as parameters in the model are carburetor valve level and air/fuel equivalence ratio,  $\lambda$ . Other inputs and outputs from the crankcase are not possible to measure and cannot simply be taken from the full engine model. This means that some modeling must be done for the parameters interfacing the crankcase model. The modeling methods and the results of the modeling are presented in this chapter.

## <span id="page-32-1"></span>**4.1 Discretization method**

The crankcase model, and the crankcase interface model, should be discrete, and the method for discretization is Euler's forward method. The main benefit of Euler's forward method in this case is that it is an explicit method [\[Glad and](#page-66-10) [Ljung, 2004\]](#page-66-10). This means that the next value is estimated using only data known in current step.

<span id="page-32-2"></span>
$$
x(k+1) = x(k) + \xi \cdot \frac{d}{dt}x(k)
$$
\n(4.1)

In this thesis the step size *ξ* is one since the model is sampled once each revolution. Concerning the transformation from time domain to event domain with one revolution as step size the following conversion of flows is suitable.

<span id="page-33-3"></span>
$$
\tilde{m} = \frac{\dot{m}}{N} \tag{4.2}
$$

### <span id="page-33-0"></span>**4.2 Test setup**

The measurements for the modeling of the crankcase interface have been collected in a test bench where the output shaft of the engine was connected to a dynamometer. This dynamometer controlled the speed, which made it possible to run the engine at constant speeds. There was equipment to measure the weight of the consumed fuel and the amount of air that flowed through the carburetor. The exhaust gases were led to an exhaust gas analysis system. A *λ*-probe, temperature sensors and pressure sensors were mounted on the engine. There was also a sensor measuring the carburetor valve opening.

The exhaust gas analysis system uses measured mass fractions of carbon monoxide and carbon dioxide in the exhaust gases to compute the *λ*-value. Using this value together with measured values for fuel consumption and air flow through the crankcase, other parameters, such as air flow through the airhead, are computed by the system.

Measurements were gathered during steady operation at three different engine speeds in the normal operating range of the engine. At each engine speed, measurements were made with varying carburetor valve opening to get different *λ*values.

### <span id="page-33-1"></span>**4.3 Fuel mass flow**

When air flows through the carburetor, fuel will flow with it. The amount of fuel is controlled by a sigma-delta (SD) signal, which opens or closes a carburetor valve. This signal is binary and the valve can for this reason not be controlled to be partly open. The valve is closed for pulses of a set number of engine revolutions and the SD signal controls the number of revolutions in between. There is a possibility to extend some of the closed pulses, which are then called lean outs since more closing means leaner combustion.

To get an estimate of the fuel flow into the crankcase a simple model is made. The amount of fuel is considered to be dependent on the air flow through the carburetor and the level of the carburetor valve.

<span id="page-33-2"></span>
$$
\dot{m}_{f,inj} = \begin{cases} c_1 \cdot \dot{m}_{a,cc} & \text{if the valve is open} \\ 0 & \text{if the valve is closed} \end{cases}
$$
 (4.3)

The fuel and air flows are measured as average flows over a measurement interval. In order to find the carburetor proportionality constant *c*1, one has to compensate

the ratio of the two flows with the amount of time the carburetor valve is open. This opening time is calculated via the SD level. In short,  $c_1$  is computed with the following formula.

<span id="page-34-0"></span>
$$
c_1 = \frac{\overline{m}_{f,inj}}{\overline{m}_{a,cc}} \cdot \left(\frac{255 - SD level}{255}\right)^{-1}
$$
 (4.4)

<span id="page-34-1"></span>By comparing fuel and air flow from 24 different measurements, the proportionality constant  $c_1$  in [\(4.3\)](#page-33-2) and [\(4.4\)](#page-34-0) is estimated. This value is computed as the average of the estimated proportionalities from the individual measurements.



Figure 4.1: Measured ratio between fuel and air mass flows for different engine speeds and throttle angles. The flows are measured as measurement averages and are thus not instantaneous values. Since air flow is continuous and fuel flow is varying with carburetor valve opening, the ratios are compensated with the percentage of the cycles the valve is open.

The values from the individual measurements are plotted in Figure [4.1.](#page-34-1) The standard deviation from the mean value is roughly 2.5%. This means that other operating parameters than SD level and air flow, such as *λ*-value or throttle angle, are assumed not to affect fuel flow into the engine.

### <span id="page-35-0"></span>**4.4 Air mass flow**

The air mass flow can be measured as a mean over a total measurement. Since the values are not instantaneous, the operation must be stationary for a mean value to represent every instant during a measurement.

A very simple air mass flow model is created for this thesis. The flow is assumed to only depend on engine speed and to vary linearly between engine speeds that have been measured. Extrapolation is done with the same linear relation as for the closest interval within the range of measured operating points.

By measuring air mass flow a number of times at some different engine speeds and using the linear least squares method, linear functions can be computed that connect the different engine speeds. Since the Simulink model is supposed to have a sampling size of one engine revolution, the mass flow unit has to be in mass flow per revolution. This is achieved by dividing the linear functions by engine speed, as in [\(4.2\)](#page-33-3). The type of function at each interval is thus the following.

<span id="page-35-1"></span>
$$
\tilde{m}_a(N) = \frac{c_1 + c_2 \cdot N}{N} \tag{4.5}
$$

The same type of function is made for both air flow through the carburetor and through the airhead. The carburetor air flows through the crankcase into the combustion chamber whereas the airhead air flows directly into the combustion chamber. Since these two air flows are not simply related to each other, it is important to separate the modeling of the air flows.

To find the coefficients  $c_1$  and  $c_2$ , measurements at three different engine speeds have been made. Since there are only three measurement engine speeds, each air flow model consists of only two linear functions.

<span id="page-36-0"></span>

Figure 4.2: Model of air flow through the carburetor and crankcase at different engine speeds. The left plot shows the air flow in terms of flow per second, whereas the right plot presents the same model in terms of flow per engine revolution. The blue line represents the model and the different colored crosses represent individual measurements.

<span id="page-36-1"></span>

Figure 4.3: Model of air flow through the airhead at different engine speeds. The left plot shows the air flow in terms of flow per second, whereas the right plot presents the same model in terms of flow per engine revolution. The blue line represents the model and the different colored crosses represent individual measurements.

The models are presented in both time domain and crank angle domain, in Figures [4.2](#page-36-0) and [4.3.](#page-36-1) The measurement data that has been used to model the flows are presented in the same figures.

## <span id="page-37-0"></span>**4.5 Trapping efficiencies**

The exhaust analysis system can calculate how much of the air is actually trapped in the combustion chamber at each revolution. In the same way as for the total air flows, these trapped flows are measured as mean values of measurements. This means that the measurements need to be run at stationary operation in order for the mean values to be possible to convert to instant values.

The trapping efficiency is modeled the same way as the air mass flows, which means with linear functions between the measured engine speeds. Since the efficiency is the same in both the time domain and the revolution domain, no conversion between these two is needed. The simple model can be written the following way.

<span id="page-37-1"></span>
$$
\eta_{tr}(N) = c_1 + c_2 \cdot N \tag{4.6}
$$

As with [\(4.5\)](#page-35-1), the coefficients of [\(4.6\)](#page-37-1),  $c_1$  and  $c_2$ , are different for different intervals of engine speeds. The coefficients are different for trapping of crankcase air and trapping of crankcase airhead air. Since trapping efficiency has been measured for three engine speeds, each efficiency model consists of two engine speed dependent linear functions. The trapping efficiency of fuel is assumed to be the same as for crankcase air since both flows follow the same path through the engine.

<span id="page-38-1"></span>

Figure 4.4: Model of trapping efficiencies at different engine speeds. The upper plot shows the trapping efficiency for fuel and air that has followed a path through the carburetor and crankcase. The lower plot presents the same but for air that has followed a path through the airhead. The blue lines represent the models and the different colored crosses represent individual measurements.

The models, along with measurement data, are graphically presented in Figure [4.4.](#page-38-1) Measurement data used for the modeling are also presented in the same figure.

## <span id="page-38-0"></span>**4.6 Gas mixing in the exhaust system**

Since the *λ*-probe is placed in the exhaust system, it is measuring total effects of both crankcase and exhaust dynamics. This means the crankcase dynamics is not separately measured. The exhaust system for the test engine was significantly larger than the engine volume, and it was hard to place the *λ*-probe in a way that would reduce the influence of the exhaust gas dynamics.

To model this mixing, the mass fraction of fuel is used. To keep track of the mass fractions of burned and unburned fuel, the same kind of model as for the concentration in the crankcase is used.

$$
C_{exh} = \frac{m_{f,exh} + m_{f,burned,exh}}{m_{tot,exh}}
$$
\n(4.7)

where

$$
\frac{d}{dt}(m_{f,exh} + m_{f,burned,exh}) = \dot{m}_{f,exh,in} + \dot{m}_{f,burned,exh,in} - C_{exh}\dot{m}_{tot,exh,out}
$$
 (4.8)

<span id="page-39-2"></span>The variable *mf ,exh* is the amount of fuel in the exhaust system that was not trapped during combustion. The burned fuel *mf ,burned,exh* is the mass of fuel in the exhaust system that was trapped during combustion. This burned fuel is not actual fuel since it has been combusted, but its mass is used for keeping track of the *λ*-value.

In the model this is made a bit easier since there is no need for keeping track of burned and unburned fuel separately and the two states are combined to one. If scavenging efficiency is assumed to be one, the concentration in the exhaust could be expressed by the following discrete model.

$$
C_{exh}(k) = \frac{m_{f,tot,exh}(k)}{m_{tot,exh}(k)}
$$
(4.9)

where

$$
m_{f, tot, exh}(k) = m_{f, tot, exh}(k-1) + \tilde{m}_{f, cc, out}(k-1) - C(k-1)\tilde{m}_{tot, exh, out}
$$
 (4.10)

Since the air flow models are modeled at close to steady state operation and there is no modeling of the pressure and temperature build-up, the total mass in the exhaust is kept constant. This means that  $\dot{m}_{tot,exh,out}$  is the same as the air from crankcase and airhead plus fuel entering the exhaust system. This is confirmed measurements of pressure and temperature in the exhaust, showing that there are very small cycle-to-cycle variations and variations between the operating points are small.

### <span id="page-39-0"></span>**4.7 Dynamics of the** *λ***-probe**

The *λ*-probe does not instantly give corresponding voltages when the air/fuel mixture in its environment change. The sensor dynamics for a *λ*-probe is, according to [Eriksson and Nielsen](#page-66-8) [\[2014\]](#page-66-8), often modeled as a first order system.

<span id="page-39-1"></span>
$$
\frac{d}{dt}\lambda_{probe} = \frac{1}{\tau_{probe}}(\lambda_{exh} - \lambda_{probe})
$$
\n(4.11)

Converted into the discrete revolution based domain, [\(4.11\)](#page-39-1) will be the following.

$$
\lambda_{probe}(k) = \lambda_{probe}(k-1) + \frac{1}{N\tau_{probe}}(\lambda_{exh}(k-1) - \lambda_{probe}(k-1))
$$
\n(4.12)

The value for *τprobe* used is obtained from the probe specifications in its data sheet.

# <span id="page-40-4"></span><span id="page-40-0"></span>**5 Crankcase modeling**

The model used in this thesis is based on what has been suggested by research in the field of wall wetting dynamics. A model for the crankcase of a crankcase scavenged two-stroke engine is developed by complementing the wall wetting model with a gas mixing model.

In this chapter, this crankcase model is presented. It is also discussed how to ensure a mass conservative model and how the model is implemented in Matlab/Simulink.

## <span id="page-40-1"></span>**5.1 Choice of model**

The model of choice for this thesis is a three state model treating both the wall wetting phenomenon as well as the gas mixing in the crankcase. The states used in the model are mass of the fuel film, mass of the air/fuel mixture in the crankcase and concentration of evaporated fuel in the mixture.

### <span id="page-40-2"></span>**5.1.1 Model for fuel dynamics**

The model for the fuel dynamics is based on the model developed by [Aquino](#page-66-3) [\[1981\]](#page-66-3) and is presented in [\(3.7\)](#page-27-3) and [\(3.8\)](#page-27-2). The original model is given in the time domain, so it is for this thesis converted into the crank angle domain using the variable substitution presented in [\(3.1\)](#page-26-3). This is a one state model with the internal state  $m_{ww}$ , the input  $\tilde{m}_{f,ini}$  and the output  $\tilde{m}_{EV,in}$ .

<span id="page-40-3"></span>
$$
\frac{d}{d\theta}m_{ww} = X\frac{\dot{m}_{f,inj}}{2\pi} - \frac{1}{\omega\tau}m_{ww}
$$
\n(5.1)

<span id="page-41-1"></span>
$$
\frac{\tilde{m}_{EV,in}}{2\pi} = (1 - X)\frac{\tilde{m}_{f,inj}}{2\pi} + \frac{1}{\omega\tau}m_{ww}
$$
\n(5.2)

Considering that [\(5.1\)](#page-40-3) and [\(5.2\)](#page-41-1) should be implemented in a model with a sampling rate of one sample per revolution using [\(4.1\)](#page-32-2) it is discretized in the following way.

<span id="page-41-5"></span>
$$
m_{ww}(k) = m_{ww}(k-1) + X\tilde{m}_{f,inj}(k-1) - \frac{1}{N(k-1)\tau}m_{ww}(k-1)
$$
 (5.3)

<span id="page-41-6"></span>
$$
\tilde{m}_{EV,in}(k) = (1 - X)\tilde{m}_{f,inj}(k) + \frac{1}{N(k)\tau}m_{ww}(k)
$$
\n(5.4)

<span id="page-41-0"></span>where *N* is given in revolutions per second.

#### **5.1.2 Model for crankcase fuel concentration**

The crankcase concentration model has two states. These are the masses *mtot,cc* and  $m_{EV}$ . The inputs of this model are  $\dot{m}_{EV,in}$ ,  $\dot{m}_{a,cc,in}$  and  $\dot{m}_{a,cc,out}$ . The output is  $\dot{m}_{f,out}$ , but in this model other combinations of inputs and outputs could be used as long as the system can be solved.

<span id="page-41-2"></span>
$$
C = \frac{m_{EV}}{m_{tot,cc}}\tag{5.5a}
$$

<span id="page-41-3"></span>
$$
\frac{d}{dt}m_{tot,cc} = \dot{m}_{EV,in} + \dot{m}_{a,cc,in} - \dot{m}_{tot,cc,out}
$$
 (5.5b)

<span id="page-41-4"></span>
$$
\frac{d}{dt}m_{EV} = \dot{m}_{EV,in} - C\dot{m}_{tot,cc,out} \tag{5.5c}
$$

$$
\dot{m}_{EV,out} = C \dot{m}_{tot,cc,out} \tag{5.5d}
$$

The concentration model described in [\(5.5a\)](#page-41-2), [\(5.5b\)](#page-41-3) and [\(5.5c\)](#page-41-4) is also converted into the crank angle domain and discretized for the step size of one revolution.

$$
C(k) = \frac{m_{EV}(k)}{m_{tot}(k)}\tag{5.6a}
$$

$$
m_{EV}(k) = \tilde{m}_{EV,in}(k-1) + (m_{tot,cc}(k-1) - \tilde{m}_{tot,cc,out}(k-1)) C(k-1)
$$
 (5.6b)

$$
m_{tot,cc}(k) = \tilde{m}_{EV,in}(k-1) + \tilde{m}_{a,cc,in}(k-1) + m_{tot,cc}(k-1) - \tilde{m}_{tot,cc,out}(k-1)
$$
 (5.6c)

Since the model interface used for the parameter estimation process does not support difference between input and output flows, the state of total mass of gas in the crankcase is kept constant.

$$
\tilde{m}_{tot,cc,out}(k) = \tilde{m}_{EV,in}(k) + \tilde{m}_{a,cc,in}(k) \tag{5.7}
$$

### <span id="page-42-0"></span>**5.2 Mass balance of the model**

It is important to make sure the fuel dynamics model is mass conserving for both fuel and air. The mass of a system could be described using the following relation.

<span id="page-42-1"></span>
$$
\sum m_{in} - \sum m_{out} + m_{init} - m_{final} = 0
$$
 (5.8)

A simulation of a model in discrete time is always mass conserving if each individual step of the simulation is mass conserving. The expression in [\(5.8\)](#page-42-1) can therefore be replaced by the mass balance for one revolution.

<span id="page-42-2"></span>
$$
m_{in}(k) - m_{out}(k) + m(k-1) - m(k) = 0 \tag{5.9}
$$

If, for example, applying [\(5.9\)](#page-42-2) to fuel mass, it can be shown that the mass balance for the fuel always holds. The terms in this equation would in the case of fuel have to be replaced with the following. The mass of fuel in fuel film and fuel vapor, the fuel mass flow from the carburetor and the fuel mass out of the crankcase.

$$
m(k) = m_{EV}(k) + m_{ww}(k)
$$
 (5.10a)

$$
m_{in}(k) = \tilde{m}_{f,inj}(k) \tag{5.10b}
$$

$$
m_{out}(k) = \tilde{m}_{EV,out}(k) \tag{5.10c}
$$

## <span id="page-43-0"></span>**5.3 Model implementation**

When implementing the model in Matlab/Simulink, all the input and output flows are expressed in  $[mg/rev]$  instead of the SI derived unit  $[kg/s]$ . The reason for this is that it should be compatible with the already existing model explained in section [1.3.](#page-17-0)

### <span id="page-43-1"></span>**5.3.1 Model overview**

Even though the crankcase model in Simulink is isolated, some different model blocks are needed to support it. In the model created for this thesis, one supporting model block contains all manually set inputs and modeled parameters and vectors. There is also a block for the simple carburetor model presented in section [4.3](#page-33-1) that computes the fuel input to the crankcase. Another block contains models for the mixing of gases in the combustion chamber and the exhaust system. The output of this block is the concentration of burned and unburned fuel that is recorded by the *λ*-probe. This AFR is then compensated for sensor dynamics and converted to  $\lambda$  in another block. Measurement data in terms of valve level is used as input to the model.

<span id="page-43-2"></span>

Figure 5.1: All blocks used in the Simulink model. The blue block contains all input parameters. The purple block is the simple carburetor model, whereas the pink block is the actual crankcase model. The red block contains models for the mixing of gases and flows in the combustion chamber and the exhaust system. The gray block is used for tracking of the model mass balance and the green block is used to convert the concentration of fuel and air to *λ*-value. The blue and green bubbles contain the input and output for the model.

An overview of the crankcase model and its supporting model blocks is presented in Figure [5.1.](#page-43-2) Apart from the mentioned blocks, there is also a block that is used for keeping track of the mass balance [\(5.8\)](#page-42-1). By looking at the output of this block, one can double-check that the model is mass conserving.

The crankcase model contains the Aquino model for fuel dynamics presented in section [5.1.1](#page-40-2) and the fuel evaporation concentration model presented in section [5.1.2.](#page-41-0) This is the main content of this thesis and is presented more thoroughly in the two sections below.

### <span id="page-44-0"></span>**5.3.2 The Aquino model**

The discretized revolution based version of the Aquino model described in [\(5.3\)](#page-41-5) and [\(5.4\)](#page-41-6) is implemented in Matlab/Simulink. The needed inputs for the model are fuel flow and engine speed. The output from the model is the amount of evaporated fuel that enters the crankcase from the carburetor and the evaporation of fuel film. The tunable parameters in the model, *X* and *τ*, are expressed as constants for now, but they can be replaced with functions of engine operation parameters. This is discussed in a later section of this report. It is important to note that there is an initial value of the wall wetting mass to be set. If set incorrectly, the output of the model will initially be incorrect but the effects of the incorrect value will eventually disappear.

<span id="page-45-1"></span>

(a) Simulink model for the wall wetting mass.



(b) Simulink model for the fuel flow into the crankcase volume.

Figure 5.2: Simulink implementation of the wall wetting dynamics. The light blue ports represent inputs and parameters needed to run the model, whereas green ports are model outputs. The orange submodel block in (b) contains the model shown in (a).

The Simulink implementation of the wall wetting dynamics is presented in Figure [5.2.](#page-45-1) The inputs and outputs to the model are colorized for clarity.

### <span id="page-45-0"></span>**5.3.3 The concentration model**

The fuel concentration model treats the mass fraction of evaporated fuel in the crankcase. In this model the initial value of air mass is very important. Since there are no available models for the mass flows in and out of the volume, the initial value adds a bias error if not set correctly. Besides the initial air mass there is also an initial value for the concentration of evaporated fuel in the crankcase. This value will eventually settle to the right level, after the initial transients fade out.

<span id="page-46-1"></span>

(a) Simulink model for the mass of evaporated fuel in the crankcase.



(b) Simulink model for the fuel mass fraction in the crankcase volume.

Figure 5.3: Simulink implementation of the fuel concentration in the crankcase. The light blue ports represent inputs and parameters needed to run the model and green ports represent model outputs. The grey submodel block in  $(b)$  contains the model in  $(a)$  and the orange block is the model shown in Figure [5.2.](#page-45-1)

<span id="page-46-0"></span>The Simulink implementation of the concentration model is presented in Figure [5.3.](#page-46-1) The inputs and outputs to the model are colorized for clarity.

### **5.4 Model stability**

Since the model used for the fuel film is a first order system with a static gain of 1, this model is considered stable in terms of input/output behavior in the time domain. However, in the discrete case there is a problem concerning stability.

All of the models in this thesis are discretized using Euler's forward method described in [\(4.1\)](#page-32-2). The discretization implies some stability problems in the model. The model for the fuel film mass in [\(5.3\)](#page-41-5) with the input set to zero it becomes the following.

$$
m_{ww}(k) = m_{ww}(k-1) - \frac{1}{N(k-1)\tau} m_{ww}(k-1)
$$
\n(5.11)

Expanding this expression for *k* steps with *N* kept constant it can be described with one equation.

$$
m_{ww}(k) = \left(1 - \frac{1}{N\tau}\right)^{k+1} m_{ww,init} \tag{5.12}
$$

This means that  $N\tau$  must be more than one to get a stable system. This kind of instability regarding the step size could be a problem in the concentration model as well. If  $\tilde{m}_{tot,cc,out}(k-1)C(k-1)$  is bigger than the amount of fuel in the volume it would result in a negative concentration in the next step. For the concentration model it is also important to keep in mind that the total mass flow in and out of the model must be set equal if no pressure or temperature build-up occurs. Otherwise, the mass state will keep increasing or decreasing, depending on the whether difference is positive or negative.

**6**

## <span id="page-48-0"></span>**Parameter estimation**

The aim of the model tuning is to find values for *X* and  $\tau$  that give realistic behavior of the crankcase dynamics. In short, this is done by setting measurement data as inputs and outputs of the created Simulink model and running the in-built tool for parameter estimation. This estimation method is run multiple times for different operating conditions. By correlating the found values to the changes in operating conditions, one can express *X* and  $\tau$  as functions of certain engine variables. The estimation method and the found results are presented in this chapter.

## <span id="page-48-1"></span>**6.1 Estimation data collection**

Measurements for the parameter estimation were collected for the same three speeds as the crankcase interface modeling measurements were made. Additionally, some data was collected for four other speeds as well. The measurements were run during lean out operation with different SD levels for each speed.

In the test bench that was used, the engine output shaft was connected to a dynamometer.  $\lambda$  in the exhaust muffler, to be used as output in the Simulink model, was measured with a *λ*-probe. The exhaust analysis system discussed in section [4.2](#page-33-0) was used to map measured values of *λ*-probe outputs. The level of the carburetor valve was measured to be used as input in the Simulink model.

## <span id="page-49-5"></span><span id="page-49-0"></span>**6.2 Comparing crankcase model output to** *λ***-measurements**

To correlate the *λ*-probe output to the real *λ*-values, a correction map is created. The map gives information about the relation between the mixture in the exhaust system and the *λ*-probe voltage. It is important to make sure that the map covers the range of operating conditions that it is intended to be used for. To achieve the map, a series of stationary measurements are done where the *λ*-probe voltage is coupled with the *λspindt*-value given by the exhaust gas analysis system. Information about *λspindt* can be found in [Spindt](#page-67-8) [\[1965\]](#page-67-8).

Given this map and the flow measurements, one can easily calculate other *λ*values in the system. Other values could be  $\lambda_{CO/CO}$ , which represents the AFR during combustion, and *λcc* which represents the AFR in the crankcase.

Even though the *λ*-values are calculated in a different way, the following relations show what they actually represent.

<span id="page-49-1"></span>
$$
\lambda_{CO/CO_2} = \frac{\dot{m}_{a,cc,tr} + \dot{m}_{a,ah,tr}}{\dot{m}_{f,tr} AFR_{stoich}} \tag{6.1}
$$

<span id="page-49-2"></span>
$$
\lambda_{spindt} = \frac{\dot{m}_{a,cc} + \dot{m}_{a,ah}}{\dot{m}_f AF R_{stoich}} \tag{6.2}
$$

<span id="page-49-3"></span>
$$
\lambda_{cc} = \frac{\dot{m}_{a,cc}}{\dot{m}_f AF R_{stoich}}\tag{6.3}
$$

The AFR of the gas leaving the crankcase could be converted to the AFR of the gas that enters either the combustion chamber or the exhaust, since the flows and trapping efficiencies are known. This requires the relations [\(6.1\)](#page-49-1), [\(6.2\)](#page-49-2) and [\(6.3\)](#page-49-3) and the assumption that no mass from the previous cycle remains in the combustion chamber, which means the scavenging efficiency is 1. This kind of conversion is necessary since the *λ*-probe is placed in the exhaust system and the model is to output AFR out from the crankcase. To go from the  $\lambda$  exiting the crankcase to the  $\lambda$  entering the exhaust, the following calculation is used.

<span id="page-49-4"></span>
$$
\lambda_{spindt} = \lambda_{cc} + \frac{\dot{m}_{a,ah}}{\dot{m}_f A F R_{stoich}}
$$
(6.4)

If [\(6.4\)](#page-49-4) is used together with the models for exhaust gas mixing and the *λ*-probe dynamics, described in [4.6](#page-38-0) and [4.7,](#page-39-0) the model output can be compared with the *λ*-measurements.

<span id="page-50-1"></span>

Figure 6.1: Linear approximation for the conversion from *λ*-probe output voltage to *λspindt*. The blue circles represent measurements at the lowest engine speed, whereas red represent the highest speed and black a speed in between these. The cross colors represent the same speeds, but these measurements are made with partly open carburetor throttle.

The *λ*-probe voltages are plotted against measured *λspindt*-values in Figure [6.1.](#page-50-1) The measurements are performed for three speeds and are also done with a partly open throttle. Since the relationship between the probe voltage and *λspindt* seems linear it is approximated with the following equation.

$$
\lambda_{spindt} = c_1 + c_2 u \tag{6.5}
$$

<span id="page-50-0"></span>The coefficients  $c_1$  and  $c_2$  are found with the linear least squares method.

### **6.3 Simulink estimation method**

The parameter estimation tool in Simulink uses a non-linear least squares method. This tool optimizes the parameters decided by the user. Parameter values that give local minima for the sum of the squares are found by running the model with this tool. The model inputs are the crankcase model interface discussed in

<span id="page-51-2"></span>chapter [4](#page-32-0) and measurement data in terms of valve level. Measurement data in terms of  $\lambda$  is used as model output.

The tool is used to find optimal values for the set parameters for measurements at different operating conditions. Plotting the estimated values against the operation variables, relations on how they vary together can be found.

### <span id="page-51-0"></span>**6.3.1 Treatment of initial values during parameter estimation**

Some of the submodels have states where initial values need to be set. Two of these states are the mass of gas in the crankcase and the exhaust system. The initial values for these states are set using the ideal gas law assumption with pressure and temperature measurements for each set of estimation data.

For the internal states of the fuel dynamic model it is more difficult to estimate the initial conditions. If an estimation data measurement has a somewhat stationary operating point in the beginning, the initial values for the fuel film mass and the evaporated fuel in the crankcase could be estimated together with the other model parameters. To avoid the initial value to be good for just one estimation, the parameter estimation is always run in pairs of measurements for equal operating conditions.

There is also a need for setting the initial values for all of the model inputs, such as flows of air and fuel and the engine speed. These flows could be set to zero but is preferably set to the same as the first value in the measurement, in other words the value at index  $k = 1$ .

## <span id="page-51-1"></span>**6.4 Estimation of the fuel impaction percentage parameter**

The size of the fuel droplets when they are created in the carburetor can be used to estimate the amount of fuel that is directly vaporized, according to the research by [Curtis et al.](#page-66-4) [\[1996\]](#page-66-4) presented in section [3.3.3.](#page-29-0) The Hinze equation [\(3.14\)](#page-30-0) shows how the fuel droplet size is dependent of air speed. For this reason, the model coefficient describing the amount of fuel that instantly vaporizes, *X*, could probably be modeled as a function of air speed through the carburetor.

Air speed cannot be directly measured, so a proxy variable needs to be used. The air mass flow can be measured and since it is varying with air speed it is used as the proxy variable. Primarily, estimation data has been has been gathered at engine speeds where mass flows measurements for [\(4.5\)](#page-35-1), [\(4.6\)](#page-37-1) and [\(4.3\)](#page-33-2) have been made. The reason for this is that these models, the crankcase interface models, are probably more accurate at these speeds. Estimation data has also been gathered at other speeds, but this data is not been used to determine *X* of the final model.

<span id="page-52-0"></span>

Figure 6.2: Estimated *X* values for each set of estimation data plotted against air mass flows from the crankcase interface. The red circles are for measurement at the highest engine speed, blue for the lowest speed and black for a speed in between these. The dotted line is the mean value of these circles and is the parameter value determined for the crankcase model. The magenta colored circles show the estimated *X* for measurements at engine speeds where mass flow measurements have not been made. These speeds are higher, lower and in between the other speeds. The magenta colored circles have not been used to determine the final model coefficient value.

Comparing the results of the parameter estimations and air mass flow through the crankcase, there does not seem to be any relation between the two (see Figure [6.2\)](#page-52-0). There can be a number of reasons for this. One reason could be that the number of measurements are so few that the variation of the parameter make it seem random. Another reason could be that one or more of the crankcase interface submodels are not accurate enough and therefore makes the parameter estimation somewhat inaccurate. The estimation tool itself can also be an error source. It can find a set of parameter values that are optimal but unrealistic. It can also find local optima that seem realistic but are not the most reasonable optima in terms of parameter values.

Even though it is not presented in this report, the estimation results have also been compared to values for other operation variables. No apparent relation for *X* has been found with these. Since relating the *X* to any other parameters has

<span id="page-53-1"></span>been unsuccessful, it is chosen to be constant. This constant is computed as the average of the estimated values for each estimation data set.

## <span id="page-53-0"></span>**6.5 Estimation of the boiling time constant**

The boiling time constant describes how quickly the wall film evaporates. [Aquino](#page-66-3) [\[1981\]](#page-66-3) models this constant as a function of wall temperature whereas [Locatelli](#page-67-7) [\[2004\]](#page-67-7) finds even more dependencies, such as wall film thickness. Since low temperatures give low evaporation of the wall film, the wall film thickness will be high at low temperatures. A high wall film thickness in turn gives a low evap-oration rate. According to [Locatelli](#page-67-7) [\[2004\]](#page-67-7), the effect on  $\tau$  of low temperatures and high wall film thickness is substantial in comparison to opposite operating conditions.

For this thesis, some estimation data has been collected from measurements during engine heat-up. This has been done with the purpose of finding a relationship between temperature and the boiling time constant. With the parameter estimation method used for this thesis, the results from such estimations vary heftily and no clear trend can be seen. This is probably an effect of unusually high wall film thickness, which gives an engine behavior that is relatively difficult to predict. Since initial wall wetting mass is essential for model outcome, an unpredictable change of wall film thickness further increases the difficulty of estimating reasonable parameter values.

Because of the uncertainty and unpredictability of the states during heat-up measurements, these measurements are neglected for the parameter estimation. Instead, the data sets used for estimation of the boiling time constant is the data that has been gathered at normal operating conditions.

<span id="page-54-0"></span>

Figure 6.3: Estimated *τ* values for each set of estimation data plotted against engine speed. The blue circles are for measurement at the lowest engine speed, red for the highest speed and black for a speed in between these. The dotted line shows the estimated linear function of engine speed for *τ* that is determined for the crankcase model. The magenta colored circles show the estimated  $\tau$  for measurements at engine speeds where mass flow measurements have not been made. These speeds are higher, lower and in between the other speeds. The magenta colored circles have not been used to determine the final model coefficient function.

Comparing the outcome of the parameter estimations and corresponding engine speeds, a trend where *τ* decreases with increasing engine speed can be seen. The results are shown in Figure [6.3.](#page-54-0) Since the number of estimated values is small, it is reasonable to adapt a simple model for this relationship. By using the linear least squares method, a linear model is determined.

$$
\tau(N) = c_1 + c_2 \cdot N \tag{6.6}
$$

Since this relationship is observed from data at normal operating conditions, the validity range of the model cannot be extended to cold engine operation. However, it seems evident from related research that the boiling time constant would have significantly higher values at lower temperatures.

# **7 Validation**

<span id="page-56-0"></span>By testing the conceived model on data that has not been used for modeling, one can get an idea of its predictive capabilities. This chapter presents a validation of the model. The complete model is used for the validation, so the results are depending on both the crankcase model and the crankcase interface models.

## <span id="page-56-1"></span>**7.1 Validation data collection**

The measurements for validation data were made with the same test setup that was used for collection of estimation data. The validation data was also collected for the same seven engine speeds. The measurements were run during lean out operation and during operation with steps in SD level without lean outs. Measured values were  $\lambda$  in the exhaust muffler and carburetor valve level.

## <span id="page-56-2"></span>**7.2 Validation results**

When validating the model it is first simulated for an amount of time so that the effects of possibly inaccurate initial values disappears. The validation data has not been used for parameter estimation. When comparing the model output to this data, the model shows a good dynamic behavior for most of the measurement. The absolute level of  $\lambda$  is however wrong in some of the measurements. Bias errors of the *λ*-value can have many reasons. Most probably it is an effect of imprecise flow estimations in the simplified model interface, but it could also be due to measurement errors.

<span id="page-57-0"></span>

Figure 7.1: Validations for the lowest speed used for estimation. The left plot shows richer mixture than the plot to the right.

<span id="page-57-1"></span>

Figure 7.2: Validations for a speed between the highest and lowest used for estimation. The left plot shows richer mixture than the plot to the right.

It can be seen in Figure [7.1](#page-57-0) that for the lower speed the model gives a good estimation of the *λ*-value. Both the level and dynamics looks to be close to the measurement. It can be seen in richer case that the dynamic raises a bit too high compared to the measurements. In the leaner case the model output does not match lowest points of the measurements.

The validation experiments for the medium speed, presented in Figure [7.2,](#page-57-1) also show a reasonable dynamic behavior but in this experiment there is a bias error in the richer case. It can also be seen that there are some variations in the measurement signals that are not covered by the model, for instance the varying heights of the *λ*-peaks. As for the measurements in Figure [7.1](#page-57-0) the model output does not match lowest points of the measurements in the leaner case.

<span id="page-58-0"></span>

Figure 7.3: Validations for the highest speed used for estimation. The left plot shows richer mixture than the plot to the right.

The validation experiments for the highest speed are presented in Figure [7.3.](#page-58-0) In the experiment with the richer mixture the match between model and measurement is good. In the leaner case there is a bias between the measurement and model output, giving a too rich model output.

The validations show that the model gives good estimations of the fuel dynamics in the system for the kind of impulses that lean out experiments give. However, there are bias errors in some of the validation experiments. For lower speeds, the model output is too lean for rich operation. Contrarily, for the higher speed the model output is too rich for leaner operating points.

For further validation, some validation experiments were made to compare the model with measurements for input steps in SD level. There were also validation made for engine speeds between the parameter estimation points and slightly outside the speed range of the estimation measurements.

<span id="page-59-0"></span>

Figure 7.4: Validation experiment for speeds not used for parameter estimation. The plot to the left is slightly below the lowest estimation speed and the right plot is slightly above the lowest estimation speed.

For the lower speeds not used for estimation, the validation results are shown in Figure [7.4.](#page-59-0) The model performs well for speeds where it is not estimated, close to the lower of speeds used for parameter estimation. For the slowest speed the model is a little to lean compared to the measurement. The model rises at the same time as the measurement but it is a little bit slower before it turns down again. For the higher of the two speeds, the peaks are a bit too low but otherwise the model predicts the *λ*-value well.

<span id="page-60-0"></span>

Figure 7.5: Validation experiment for speeds not used for parameter estimation. The plot to the left is slightly below the highest estimation speed and the right plot is slightly above the highest estimation speed.

<span id="page-60-1"></span>

Figure 7.6: Validation experiment at the lowest speed used for estimation with a step in the fuel input signal. Both plots are for the same operating conditions, but with opposite step direction.

The results for validation of the higher speeds not used in estimation, shown in Figure [7.5,](#page-60-0) show a large bias for the higher of the two speeds, but not for the lower. The *λ*-peaks for the lower of these two speeds are too low compared measurements. Together with the previous Figure [7.4](#page-59-0) it can be seen that the model predicts dynamic behavior well for engine speeds that are just outside the range of which estimation data have been collected. The bias is different for these for engine speeds.

Figure [7.6](#page-60-1) shows that the model predicts the main dynamics well of a fuel step for lowest speed also used for estimation. The model output for this step is generally too lean. In this particular case, the choice of input signal and engine speed gives a damping behavior of the model that is not observed in the measurements. This behavior did not appear in any other validation experiment.

<span id="page-61-1"></span>

Figure 7.7: Validation experiment at the highest speed used for estimation with a step in the fuel input signal. Both plots are for the same operating conditions, but with opposite step direction.

<span id="page-61-0"></span>The step shown in Figure [7.7](#page-61-1) is for the highest speed also used for validation. It predicts the main dynamics during the step response well. The model does not capture the full amplitude of the oscillations around the static level. The model output of this validation experiment was a bit too rich, compared to the measurements.

## **7.3 Discussion of the results**

From the validation it seems that the model is able to follow the dynamics of the system quite well. However, the model is only validated for close to steady operation. Minor speed deviations and the dynamics of the carburetor and air flow are neglected during validation and parameter estimation. Since some small deviations are noticed in the measurements, this neglection possibly have some influence on the validation. For example, the small speed deviations probably give a somewhat uneven mass flow through the engine.

To be able to isolate the crankcase from the intake manifold and exhaust system in terms of airflow, the engine was operated at constant speeds and constant SD level for crankcase interface modeling. The air mass flows are assumed to be fuel independent, meaning that each engine speed is said to give the same flow, no matter of amount of input fuel. If the airflow is affected by the amount of fuel, this would give a different AFR in the crankcase. This simplification could be the reason the model is not matching the lowest points of the measurements in Figures [7.1](#page-57-0) and [7.2.](#page-57-1) During the presence of lean outs, the model does not give sufficiently lean outputs.

When creating the model for the carburetor in section [4.3,](#page-33-1) a very simple relation between airflow and fuel flow was used. The model has no leakage and instant opening and closing of the valve. If leakage and opening and closing dynamics were considered in the carburetor model, it would probably give a smoother fuel flow input to the model. An error in the input fuel to the model gives a direct error in the *λ*-level and could be the explanation for the bias errors that occur in some of the validation experiments. The observed trend is that the leaner the experiment goes, the model is more shifted to being too rich. This implies that the carburetor model should be more complex than the current constant proportionality. The fact that the carburetor model is not tuned during lean out operation could also be the reason for the mismatch with lowest points in Figures [7.1](#page-57-0) and [7.2.](#page-57-1)

The exhaust system is modeled as a perfect averaging volume with constant mass. This means that possible transport delays and zones of different gas concentrations are neglected. A too simple exhaust gas model, as in this case a perfect averaging volume, could give a damping behavior. The exhaust gas model could therefore be the reason for the model output not capturing all the highest peaks or the oscillations, in for example Figure [7.7.](#page-61-1) Another simplification is that the scavenging is assumed optimal, which means no exhaust gas remains in the combustion chamber between revolutions. This probably leads to a somewhat wrong value for  $\lambda$  in the exhaust. The magnitude of this error has not been investigated.

It has not been discussed in the thesis, but the valve level input signal is considered very trustworthy compared to other measurements. The characteristics of the *λ*-probe on the other hand could give oscillations and noise that do not correspond to the actual *λ*-value in the exhaust. This would also be an explanation for the model not capturing all measurement oscillations.

# **8 Conclusion**

<span id="page-64-0"></span>The purpose of this thesis has been to create a model for the fuel dynamics in the crankcase of a small crankcase scavenged two-stroke engine. This has been done with a model that treats the mass of fuel vapor in the crankcase volume and the mass of fuel film on the crankcase walls. To get a model behavior for the fuel dynamics that predicts output AFR in the studied operating range, the fuel impact parameter *X* is set constant and the evaporation rate  $\tau$  is set to vary with engine speed.

The main problem with this kind of model is the tuning, where the two parameters *X* and *τ* affect each other and are therefore estimated together. The model only predicts dynamic behavior, which makes parameter estimation impossible during real steady state operation. This means that dynamic estimation is needed.

Since neither the inputs nor the outputs of the system are measurable, simplified models have been made for these. Together with this simplified interface the fuel dynamics model have shown a good dynamic behavior compared to measurement data. Sometimes the model gives a bias error which probably occurs because of simplifications in the crankcase interface models.

## <span id="page-64-1"></span>**8.1 Future work**

In this thesis, a model is created that estimates the dynamics of fuel in a limited range of operation. One objective to work with in the future is how the dynamics are affected during other conditions, for example part throttle, idle speeds, different weather conditions and cold start. Another issue that could be investigated is whether this kind of model also applies for other fuels and for other crankcase

<span id="page-65-4"></span>scavenged two-stroke engines.

### <span id="page-65-0"></span>**8.1.1 Speed variations**

The conversion between time domain and crank angle domain used in this thesis is done under the assumption of constant speed. Even though the model probably works reasonable well during varying speed, it would be interesting to investigate how much error the current gives since it is not adapted for speed deviations according to the differentiation shown in [3.6.](#page-26-2)

### <span id="page-65-1"></span>**8.1.2 Improved model interface and measurements**

The focus for this thesis has been the fuel dynamics in the crankcase and therefore there are too simple models in the crankcase interface. For example, a bias error in the validation could indicate that the model of either air or fuel flow is inaccurate and therefore more should be put into these. The flow models should also be extended for a greater operating range. It would also be beneficial with an interface treating dynamic changes in airflow, pressure and temperature in the volumes for crankcase and exhaust.

### <span id="page-65-2"></span>**8.1.3 Air flow effects**

The air flow is one of the dependencies mentioned in the related research by for example [Locatelli](#page-67-7) [\[2004\]](#page-67-7). For research done on larger port fuel injected engines it has been shown that the impact parameter *X* could vary with air flow given in mass per revolution. However, the operating range studied in this thesis had very similar air flow over the whole range. To get flow dependencies for the model parameters, the estimations could be made for part throttle measurements as well.

### <span id="page-65-3"></span>**8.1.4 Temperature effects**

When cold starting the engine the model probably looks a lot different than if cooled down during operation. [Locatelli](#page-67-7) [\[2004\]](#page-67-7) shows a theory that the evaporation rate of the fuel film could be a function of the thickness of the film. During cold start the amount of fuel in the crankcase is probably much larger due to the start-up process. This would result in a different dynamic behavior during cold start than at steady state operation.

To get a model that is valid during other temperatures the model should be compared with measurements where an active cooling or heating system is applied to the crankcase. In this way, measurements for temperature dependency could be done during conditions that are close to steady state.

## **Bibliography**

- <span id="page-66-5"></span><span id="page-66-0"></span>Alois Amstutz, Nicholas P Fekete, and J David Powell. Model-based air-fuel ratio control in si engines with a switch-type ego sensor. Technical report, SAE Technical Paper, 1994. Cited on page [9.](#page-24-2)
- <span id="page-66-3"></span>Charles F Aquino. Transient a/f control characteristics of the 5 liter central fuel injection engine. Technical report, SAE Technical Paper, 1981. Cited on pages [9,](#page-24-2) [12,](#page-27-4) [25,](#page-40-4) and [38.](#page-53-1)
- <span id="page-66-9"></span>Diego A Arias. Numerical and Experimental Study of Small Engine Carburetors. PhD thesis, University of Wisconsin-Madison, 2005. Cited on page [14.](#page-29-1)
- <span id="page-66-2"></span>Mikael Bergman, R U K Gustafsson, and B I R Jonsson. Emission and performance evaluation of a 25 cc stratified scavenging two-stroke engine. Technical report, SAE Technical Paper, 2003. Cited on page [7.](#page-22-2)
- <span id="page-66-1"></span>Gordon P Blair. Design and simulation of two-stroke engines. Society of Automotive Engineers Warrendale, PA, 1996. Cited on pages [6,](#page-21-2) [9,](#page-24-2) and [10.](#page-25-1)
- <span id="page-66-6"></span>Chen-Fang Chang, Nicholas P Fekete, and J David Powell. Engine air-fuel ratio control using an event-based observer. Technical report, SAE Technical Paper, 1993. Cited on pages [9,](#page-24-2) [10,](#page-25-1) and [11.](#page-26-4)
- <span id="page-66-7"></span>Yuen-Kwok Chin and Francis E Coats. Engine dynamics: Time-based versus crank-angle based. Technical report, SAE Technical Paper, 1986. Cited on page [11.](#page-26-4)
- <span id="page-66-4"></span>Eric W Curtis, Charles F Aquino, David K Trumpy, and George C Davis. A new port and cylinder wall wetting model to predict transient air/fuel excursions in a port fuel injected engine. Technical report, SAE Technical Paper, 1996. Cited on pages [9,](#page-24-2) [14,](#page-29-1) and [36.](#page-51-2)
- <span id="page-66-8"></span>Lars Eriksson and Lars Nielsen. Modeling and control of engines and drivelines. John Wiley & Sons, 2014. Cited on pages [12](#page-27-4) and [24.](#page-39-2)
- <span id="page-66-10"></span>Torkel Glad and Lennart Ljung. Modellbygge och simulering. Linköping University. Second edition. Studentlitteratur, 2004. Cited on page [17.](#page-32-3)
- <span id="page-67-2"></span>Elbert Hendricks. A compact, comprehensive model of large turbocharged, twostroke diesel engines. Technical report, SAE Technical Paper, 1986. Cited on page [9.](#page-24-2)
- <span id="page-67-0"></span>John B Heywood and Eran Sher. The two-stroke cycle engine: its development, operation, and design. Taylor & Francis, 1999. Cited on pages [5,](#page-20-3) [6,](#page-21-2) [8,](#page-23-0) and [10.](#page-25-1)
- <span id="page-67-5"></span>Marzio Locatelli, Christopher H Onder, and Hans P Geering. An easily tunable wall-wetting model for pfi engines. Technical report, SAE Technical Paper, 2004. Cited on page [9.](#page-24-2)
- <span id="page-67-7"></span>Marzio A Locatelli. Modeling and compensation of the fuel path dynamics of a spark ignited engine. IMRT Press, 2004. Cited on pages [12,](#page-27-4) [38,](#page-53-1) and [50.](#page-65-4)
- <span id="page-67-1"></span>Par Martinsson, Mikael Bergman, and Reine Gustafsson. Two-stroke engine, December 30 2008. US Patent 7,469,666. Cited on page [7.](#page-22-2)
- <span id="page-67-4"></span>Michael R Simons, Marzio Locatelli, Christopher H Onder, and Hans P Geering. A nonlinear wall-wetting model for the complete operating region of a sequential fuel injected si engine. Technical report, SAE Technical Paper, 2000. Cited on page [9.](#page-24-2)
- <span id="page-67-8"></span>Roderick S Spindt. Air-fuel ratios from exhaust gas analysis. Technical report, SAE Technical Paper, 1965. Cited on page [34.](#page-49-5)
- <span id="page-67-3"></span>Gerasimos Theotokatos. A comparative study on mean value modelling of twostroke marine diesel engine. 2009. Cited on pages [9](#page-24-2) and [10.](#page-25-1)
- <span id="page-67-6"></span>Martin Turesson. Modelling and simulation of a two-stroke engine. Master's thesis, Department of Signals and Systems, Chalmers University of Technology, Gothenburg, 2009. Cited on page [10.](#page-25-1)



## Upphovsrätt

Detta dokument hålls tillgängligt på Internet — eller dess framtida ersättare under 25 år från publiceringsdatum under förutsättning att inga extraordinära omständigheter uppstår.

Tillgång till dokumentet innebär tillstånd för var och en att läsa, ladda ner, skriva ut enstaka kopior för enskilt bruk och att använda det oförändrat för ickekommersiell forskning och för undervisning. Överföring av upphovsrätten vid en senare tidpunkt kan inte upphäva detta tillstånd. All annan användning av dokumentet kräver upphovsmannens medgivande. För att garantera äktheten, säkerheten och tillgängligheten finns det lösningar av teknisk och administrativ art.

Upphovsmannens ideella rätt innefattar rätt att bli nämnd som upphovsman i den omfattning som god sed kräver vid användning av dokumentet på ovan beskrivna sätt samt skydd mot att dokumentet ändras eller presenteras i sådan form eller i sådant sammanhang som är kränkande för upphovsmannens litterära eller konstnärliga anseende eller egenart.

För ytterligare information om Linköping University Electronic Press se förlagets hemsida <http://www.ep.liu.se/>

## Copyright

The publishers will keep this document online on the Internet — or its possible replacement — for a period of 25 years from the date of publication barring exceptional circumstances.

The online availability of the document implies a permanent permission for anyone to read, to download, to print out single copies for his/her own use and to use it unchanged for any non-commercial research and educational purpose. Subsequent transfers of copyright cannot revoke this permission. All other uses of the document are conditional on the consent of the copyright owner. The publisher has taken technical and administrative measures to assure authenticity, security and accessibility.

According to intellectual property law the author has the right to be mentioned when his/her work is accessed as described above and to be protected against infringement.

For additional information about the Linköping University Electronic Press and its procedures for publication and for assurance of document integrity, please refer to its www home page: <http://www.ep.liu.se/>

© Johan Andersson och Oscar Wyckman