ABSTRACT

AN OPTIMIZATION MODEL TO ALLOCATE BUDGET IN SCHOOL REHABILITATION PROJECTS

By

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Modernization and rehabilitation of existing school facilities is a key factor to increase the life expectancy of facilities and to benefit students by increasing educational standards. Since United States' last recession in 2009, many school districts have been confronted with difficulty in funding their projects. The existing process of decision making on distributing budget is subjective and depends on the judgment of decision makers. To avoid the errors involved in the existing approach, an optimization model is developed in this research study to optimize the project selection. This optimization system increases the horizontal and vertical equity in the educational system while it reflects the importance of different categories of facility projects and the various educational demands of students. Two powerful methods of Genetic Algorithm and Dynamic Programming are utilized to solve the optimization problem. Moreover, the proficiency of the developed model is shown in a case study.

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INTRODUCTION

Overview

Achieving high quality education standards and providing a safe and productive learning environment in modern facilities are fundamental goals of most education systems. One step to satisfy this goal is to maintain existing facilities in good condition through an ongoing maintenance program and continual upgrade via capital improvement projects (construction works) to increase the life expectancy of existing school buildings. In order for maintenance and/or capital improvement programs and projects to be successful, efficient use of resources and available funds is vital.

Research by Dana (2011) has shown that access to a quality education is a crucial component of quality of life standards and critical to the forward progress of society. Access to a high-quality education has a direct relationship with increased quantity and quality of employment opportunities, greater family stability, and greater productivity in society. Moreover, the rate of involvement in criminal activity and the demands on the public health system are inversely proportional to the quality of education. That is to say, that an increase in quality of education will directly reduce the chances of involvement in criminal activity. As an example, a case study by Dana (2011) shows there is a US $7.00 return for each dollar invested by taxpayers to increase the quality of pre-kindergarten education in Pennsylvania. This same study shows the state saves US $288 annually for lower crime rate when the school graduation rate increases about 5% in Pennsylvania.

According to the American Society of Civil Engineers (ASCE) Infrastructure Report Card (2013) one of the key elements in achieving high-quality education for students is having excellent educational facilities. ASCE report states that more than fifty percent (50%) of public school facilities were built before 1960 and most of these facilities are in need of immediate repair. Moreover, the report mentions that school facilities in the United States obtain a grade of D (on a standard American grading scale of A through F). This is a lower grade than the public parks and recreation centers infrastructure received. The current condition of the school buildings is a major concern for policymakers, school administrators and their constituents, and public communities. This same report states that student enrollment in the United States is anticipated to steadily increase through 2019 while state and local funding for school construction projects are in a continual state of decline. As a case in point, the amount of money spent on school construction projects in 2010 was about US $10 billion. This amount is approximately half of the money spent in a fiscal year before the US recession in 2009. Civil engineering professionals' evaluation for upgrading and renovating the existing school facilities in the United States extended to the year 2020 shows a US $270 billion shortfall in the collective budgets of the school districts. ASCE anticipates a need of US $390 billion to cover all costs while the funding is expected to reach only US $120 billion.

The quality of school facilities in the United States has been an increasingly important issue for many years. It is mentioned in a report published by ASCE (2009) that public school buildings in the United States have deteriorated over several decades and it was acknowledged that the existing conditions of Kindergarten through twelfth grade (K-12) school facilities may significantly affect classroom instruction or even be unsafe for the occupants. The ASCE report calls attention to a discrepancy between necessary and funding budgets. A budget of US $300 billion would be necessary to repair all of the deficiencies in existing facilities. At current Federal Government funding levels, school districts receive only US $40 billion for educational infrastructure maintenance and/or capital improvement projects / programs. Given that the available budget for rehabilitation and maintenance projects in school districts is usually insufficient to complete all the projects, funding sources must be properly allocated and must be based on a rational prioritization system to optimize the balance of available funds, the desires of the end users, and the criticality of the applicable rehabilitation projects (ASCE 2009).

In recent years, funding sources for school districts have shifted heavily from the State level to the local level. School Districts have lost funding from the state due to the recession and misunderstanding of the return on investment for educational dollars spent on the part of the legislature. School Districts have been forced to find local sources of funding. Although the shift in funding source was not deliberate on the part of the school districts, the end result was an increase in local control which better addresses the needs of the students and more efficiently allocates budgets. The increase of local control on funding system necessitates accountability and transparency for the responsible districts. . Each local school district knows best the local priorities and goals for its own area and follows those goals. Examples of priorities are pupil performance and achievements, school environment and climate, parent commitment and community involvement (Brown 2015).

The first stage of K-12 education funding and resource allocation is at the school level and it is frequently governed by school districts. As a result of an insufficient production of data (e.g., the lack of transparency), intra-district (school level) documentation of resource distribution patterns and processes have not been investigated thoroughly by researchers and obtained inadequate attention of constituent communities and policymakers. Rubenstein et al. (2007) mention that studies have shown education budgets are heavily dependent on the personal judgment of decision makers (Superintendents, School Boards, etc.) and the lack of state funding sources and its concomitant oversight has led to improper distribution of educational resources (funding for maintenance or capital improvement projects / programs) in school districts. One case study shows that in three relatively major metropolitan school districts in the cities of Columbus, Ohio, and, New York, student characteristics and school performance caused significant disparities in the resources made available to the schools within these school districts. Even though schools with higher rates of minorities and English Language Learner (ELL) students are provided with a greater number of teachers, these teachers are mostly less experienced as compared with the teaching staff makeup at schools with less number of these classes of students.

Available resources for school education are classified in fiscal, personnel, and facility resource categories. Fiscal fund services include, but are not limited to, the general education fund per pupil, instructional funds per pupil, and classroom material. Personnel resources serve the needs of teacher salaries, school administration expenses, staff and other human resource expenses. Facility resources include the funds for maintenance / operation of facilities and development / implementation of rehabilitation or capital improvement projects / programs (new school buildings, etc.) (Jimenez-Castellanos 2010).

Sources of funding for school districts include federal, state, and local tax revenue. Each of these revenue sources is classified in a specific category and comes with specific limitations. For example, the state source of capital project funds cannot be used for teacher salaries or everyday operating expenses (LAUSD Budget Services and Financial Planning Division 2013).

Jimenez-Castellanos (2010) introduces an Intra-district Multiple Resource Allocation (IMRA) framework and notes that studies on finding relationships between revenue and budget allocations within an education system should follow a methodology that considers when and how available resources make a difference in the outcome of a school. The author states that every school district needs to provide adequate resources for students of every socioeconomic class to proficiently meet the state requirements for standard level of education (See FIGURE 1).

The State of California budgets US $47.2 billion for K-12 education annually. This amounts to approximately 41.6% of the general fund expenditures for health, human services, natural resources, and higher education in California. Each year, California confronts budgetary deferrals and a portion of the new fiscal year budget is allotted to pay the districts back for the money spent on the rehabilitation projects. The 2015-2016 Budget for the State of California includes approximately US $250 million of the K-12 budget to cover budget overruns in the previous year. One of the reasons behind this lack of budget control is progressive increases in yearly expenditure per pupil after the recession in 2009. About 10.1% of the K-12 budget (US $4.72 billions) allocated to the maintenance and operations of existing facilities. Since 1998 to present, US $35 billion has been allocated to remodel and modernize school facilities for 6 million California students (Brown 2015).

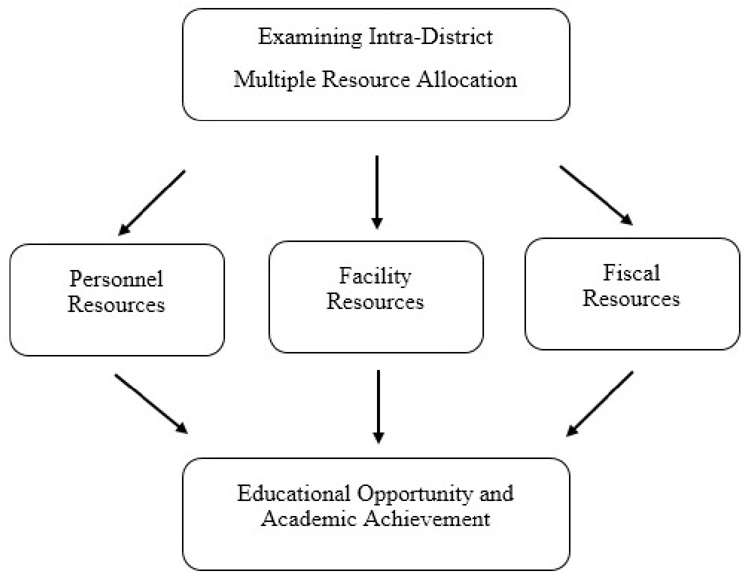


FIGURE 1. Intra-district multiple resource allocation framework (IMRA) (Jimenez-Castellanos 2010).

The competency and proficiency of the current funding system is debated in many aspects. Some of the deficiencies involved with the current system of money distribution between schools and districts are: inadequate local control of expenditures, inflexibility on allowing the local school districts to design school facility plans and first-come first-served basis of programs (Brown 2015).

Currently, all of the school districts in California are required to use the FIT (Facility Inspection Tools) as a standard format for evaluation of their existing facility conditions. Based on the FIT, evaluation of each school facility must be performed for different sub-areas with consideration for different parameters in that sub-area. A total assessment for all areas within a school is developed from the calculated partial assessments. The overall number of deficiencies for each school in the FIT system provides a school district with a system to prioritize rehabilitation and maintenance projects for all schools within that district.

The Board of Education (Board, School Board) for each applicable school district is in charge of decision-making with regard to disbursement of budgeted funds. The Board must consider information provided in FIT reports and consider additional factors such as current political priorities, the demographic arrangement of each school based on socioeconomic factors, availability of funds, and the time and cost effectiveness of each project. School Boards are typically composed of a limited number of people with varied backgrounds (not typically including construction) and their decision-making process is typically based only on the information obtained from FIT reports. Although the FIT reports are helpful in better prioritization of facility retrofit projects, they are not all-inclusive. As a result, Board decisions are susceptible to bias and varied degrees of understanding of the issues noted in the FIT reports.

Noureddine (2010) used the Analytical Hierarchy process (AHP) to improve the FIT prioritization system. His work shows that the FIT's disregard for the importance weight of each component in a school facility may lead to elevation of minor defects to major defects and consequently an inaccurate assessment. Moreover, he states that there are two limitations in the FIT prioritization system: 1) lack of importance weight of each decision element, 2) the risk of irrational decisions due to subjective judgment of each school district's Board of Education. Noureddine performed surveys and created an importance weight for each component of the FIT using AHP and developed a system to prioritize school rehabilitation projects considering those weights.

The goal of both prioritization systems offered by Noureddine (2010) and the FIT is to provide school districts with an overall rating of school infrastructure. However, with regard to the limited budget available to each school for facility maintenance, school-wide repairs and consideration of each school facility as one large project is usually impractical. Adding to that, both systems do not consider the importance of students' demographic data and demands in each school. Therefore, it comes to the attention that an optimization system needs to be developed to provide a selection of the rehabilitation projects that best meet budget limitations and maximize the average available resources for all students simultaneously.

Various researchers have investigated the relationship between the dollar amounts of available resources and student achievement. There is sufficient data that shows an increase in funding for schools directly increases performance and achievement. However, no practical method of measuring this relationship has previously been available (Hanushek 1996). The predominant theory in the 1990s was that there was a direct relationship between available funds for resources and student achievement. Provision of equal funds for every student ("equality"), regardless of any need factors was the common practice in funding schools. Later, studies disputed the reliability and practicality of equality, which led to a shift toward the concept of educational equity. If adequate opportunities were provided for each student based on his/her needs, this would balance and increase the overall performance of schools within a school district. (Jacques and Brorsen 2002).

Studies show that there is a great disparity in allocation of resources across schools within a school district and argue that resource allocation needs to follow a system which will lead to equity of available resources for all different groups of students (English Language Learners, students from socioeconomically disadvantaged families, minorities). More recently, discrepancies in resource allocation have shifted toward provision of more resources to the specific groups of students with greater resource demands to improve and balance the overall quality of education for all students in their district. Empirical evidence shows an overall trend in decision making in the New York City school district to disburse more money to the schools with a greater population of minority students and socioeconomically disadvantaged students (Iatarola and Stiefel 2003).

A study by Jacques and Brorsen (2002) assesses the effects of different types of district expenditures on student performance and test scores. They show that the rate of minority students, students with special educational needs, and students with Low-Income Family is influential in the overall performance of a school. They note the inverse relationship between the rate of these special categories of students and performance on standardized tests. Jacques and Brorsen analyze the direct relationship by increasing the amounts of different types of available resources to different schools in a large school district in Oklahoma and conclude that more available resources in schools with higher rates of special categories students will result in a significant improvement in average student performance on standardized tests.

It is demonstrated in the literature that there is a significant relationship between educational expenditures, student demographics, and student and school achievements. Researchers have examined the effects of different types of resources such as facility resources, personnel budgets, and fiscal resources on school performance while considering the influence of different categories of students and their population arrangements in each school within a large urban school district. Studies prior to 1965 show that the direction of available funds to schools with greater populations of minority, poor, and English Language Learner students was commonly lower than the funds directed toward schools with a lower population of students in these categories. To compensate for this defect in funding, an education law was enacted in 1965 to allocate more federal funding to schools with higher rates of Low-Income Family students. This was referred to as “Title One.” Recently, the States and local funding systems have taken account of this approach in their funding system by increasing the budgets for specific groups of students. The current approach is very subjective and depends on the judgment of decision makers (School Boards) rather than following a comprehensive method (Jimenez-Castellanos 2010). As a result of the high rate of socioeconomically disadvantaged students in California, the State allocated an additional 20% of the local education base fund to serve students from Low-Income Family and English Language Learners. These two categories of students consist of approximately 55% of the total students in California (Brown 2015).

Statistics shows that student enrollment in public schools is increasing and this will lead to requirements for more resources to provide sufficient facilities to accommodate incoming students. The allocation of budgets for the modernization of existing facilities and construction of new facilities is dependent on the provision of educational funding from the states based on per-pupil needs and eligibility of school districts. Funding can be utilized for purchasing new properties for schools, construction of new buildings and facilities on existing properties, or upgrading and/or retrofitting the existing facilities. The current school facilities data shows an unbalanced distribution of resources which resulted in some schools with overcrowded classrooms while some had vacant classrooms (Brown 2014). This may be a result of the first-come, first-served basis of distribution for grant funding instead of a system to choose the optimum selection of fund distribution.

Methodology

The goal of this research study is to develop a model to change the current subjective process of decision making to an objective function which will maximize the benefit of having a better conditioned facility for all students from different groups and classifications in a school district. This model attains the highest feasible selection of school facility projects on a limited available budget. The optimization model considers the importance of each parameter in a school facility found by Noureddine (2010). Also, this model considers the importance of maximizing overall benefit to all students (including minorities and socioeconomically disadvantaged students) within a school district on a limited budget. Two methods of Genetic Algorithm and Dynamic Programming are utilized to find a solution to this optimization problem. The Genetic Algorithm model is proposed with a combinatorial fitness model including per capita expenditure for all students, per capita expenditure for minority students, per capita expenditure for students from Low-Income Family, and facility criteria importance factors in quality of facility. The constraint of this optimization model is a limited fiscal budget available for school facilities in a school district. As an alternative method of solving the problem, the theory of Dynamic Programming in solving 0-1 Knapsack problems is applied to formulate the optimization model, the objective function, and the budget constraint. Finally, the properness of these two model is examined in a case study for a large urban school district. The Genetic Algorithm method is more flexible in a case that the input data or the constraints of this optimization method are changing by a user. The GA method answer is very sensitive to the definition of the first population of the data and to the number of iterations in finding new generation of a solution. On the other hand, Dynamic Programming method is very well suited to the current configuration of the problem and although the run time of the program in this method will be longer, it reaches the exact answer at the end. To reach the exact solution of the Dynamic Programming method with the use of GA, the number of the initial population and the number of generations needs to be selected large enough in the initial set up. This will significantly increase the run time of the program. The solution gained from both methods is reliable. However, the solution of the Dynamic Programming in the current definition of the problem in this research study is more accurate as of the limited number of generation selected for GA method to decrease the run time of the program. In contrast, for the future studies when the configuration of the problem changes, the GA method will be more flexible to incorporate additional variables and constraints with a different form of input data.

Thesis Organization

Chapter 2 is a literature review on the existing prioritization and funding system with a focus on the school facility projects and the concept of equity in education.

Chapter 3 discusses the development of the model, the software which has been used for formulating the problem, the objective function and constraints of the optimization model, and a detail description of the two algorithm utilized to find the optimal solution.

Chapter 4 demonstrates the applicability and proficiency of the program in a real world by conducting a case study of the large urban school district of Long Beach Unified School District (LBUSD)

Chapter 5 is a conclusion of the research study and discussion about the limitation of the model and the future studies that could be done to improve the optimization system.

CHAPTER 2

LITERATURE REVIEW

According to the paper published by Hiester (2008), the average life expectancy of public school buildings in the United States, is 42 years, and as a matter of fact, the lack of regular maintenance of these existing facilities has resulted in the fast deterioration of them after 40 years. Furthermore, Office of Environmental Health and Safety (2001) mentions that the condition of school facilities is playing a critical role in academic performance and achievement of students. The report points out that the well-serviced facilities have shown to improve student academic accomplishment and to decrease student's absenteeism. As a case in point, water leaks can result in unhealthy quality of air in the school environment and consequently, more chance of student illnesses and absences. Moreover, Schneider in 2002 mentions that the deficiency in maintenance will also affect the teachers' ability to accomplish their job. He also states that the performance of teachers and students directly relates to the air quality, climate control, noise and light in school facilities.

Office of Public School Construction State Allocation Board (2007) states that the relation between school facility quality and student achievement has been bolded since 2000, when Eliezer Williams together with more than 100 students from all over California filed a lawsuit against the state, accusing the education agencies of failure to maintain quality learning standards in public school facilities. The Williams lawsuit sheds more light on the mutual relationship of the quality between school facilities and students and teachers performance. The lawsuit brings to the attention that many schools are unable to provide safe and decent environment for learning. After four years of investigations, the court ordered the parties to come to an agreement. They finally reached a settlement that will lead to ensuring proper education and safe facilities will be accessible to all students. They estimated that Emergency Repair Program (ERP) needs $800 million which is necessary to provide resources for critical facility repairs. The ERP bases on the ranking and academic performance of the schools.

East and Liang (2006) discuss the difficulties in prioritizing the facility maintenance and rehabilitation projects in large organizations either private or public enterprises as often they own numerous facilities. These facilities, such as universities and colleges, highways, and schools, confront difficulties in prioritizing projects considering limited resources, time, budget, and manpower. Adding to that, the officials in Southern California school districts declare that school maintenance projects, usually fall behind due to the lack of funding especially nowadays since the cost of construction has increased. Funds are provided by local agencies and state and currently the process of "disbursement of these funds, including but not limited to the $800 million ERP bases on first-come, first-served basis" (Noureddine 2010).

The current method of granting schools does not guarantee that the facility which deserve money first get it first. Nowadays, school districts rely on visual inspection to determine the requirement for emergency repair in existing facilities. The district's management division sets up regular meetings, discusses the issues, and makes the decision based on personal judgment of the decision makers. Variable sources of recommendation and different levels of capabilities and experiences make personal judgments vulnerable to bias. Adding to this, neglecting the influence of priority weight of different components in each project leads to unwise judgment. The shortage of maintenance funds makes the process of decision-making vital in order to make the best decision (Noureddine 2010).

Current Prioritization Methods

Facility Inspection Tool

William lawsuit case lead to the development of the Facility Inspection Tool (FIT) which includes a standard format for an inspection report of existing facilities named "School Facility Condition Evaluation". These reports are filled out by surveyors, working for facility services division in school districts. If any significant deficiency is noted in the structure by the visual inspection, a consultant will be subcontracted from outside of the district to make a report and recommend a solution. The work orders are gathered and archived to be executed, when funds are available. This type of historical data collecting systems provides the school officials with the information of maintenance needs in the district. Subsequently, the regular meetings are held in the district to come up with a decision which is mostly rely on personal experience of the officials and the information provided by FIT (Rozzi 2008).

Any school district in California is mandated to use Facility Inspection Tools (FIT) in order to report the present-day condition of each school buildings it administrates. To make a FIT report, fifteen different components which are: gas pipes; heat, ventilation and air conditioning (HVAC) and mechanical systems, windows, doors, gates and fences; interior surfaces such as walls, ceilings and floors; existence of hazardous materials; structural elements; fire safety precautions; electrical systems; pest and vermin infestation; drinking fountains; sewer systems; roofs; external toys such as slides on playgrounds; overall cleanliness of school and restroom fixtures have to be inspected (Rozzi 2008).

The FIT uses the CSI UniFormat for school building components, and for each of these components considers the same weight on a scale of 0 to 100 to determine the amount of deficiencies. Furthermore, the FIT uses the simple mean value of determined numbers for all components to come up with a percentage of overall rating of that school. The FIT takes a further step and proposes a prioritization method based on ranking the schools from the worst condition to the best condition. As a matter of fact, many school districts in their current rehabilitation projects consider these prioritization data to allocate their budget. There is some limitation with using FIT as a tool for decision-making system in school rehabilitation projects. As an example, it has been indicated that in many situations the age of a building or a component affects inspector's view of the percentage of deficiency and if there is a need of repair or not. Moreover, studies show that the percentage of the deficiencies and the cost of the projects is influential on the allocation of budget. The decision makers in school rehabilitation projects are less willing to allocate budget for the major repairs, which are mostly pricey such as the replacement of HVAC system, and usually tend to postpone these projects until the breakdown of the system due to limited budget (Noureddine 2010).

<u>Noureddine's (2010) AHP Based Prioritization System</u>

In 2010, Noureddine challenged the adequacy and accuracy of the Facility Inspection Tool, in giving a proper solution for prioritizing schools facility projects by stating that, FIT disregards the importance of each category in the overall improvement of the whole facility condition and quality of school building. Moreover, he proposed a hierarchy system for different parameters based on the CSI Autoformat categories which is shown in (FIGURE 2). In his hierarchy system, he considers 4 various levels of parameters for each school facility and assumes that each of these parameters has a different importance comparing to the other parameters in the same level. Furthermore, he made a survey utilizing Analytic Hierarchy Process (AHP) 1 to 10 scales to find the importance rates of each parameter. The surveys were distributed to the professionals in school rehabilitation projects and conducted an overall importance rate which is illustrated in the (TABLE 1).

Noureddine (2010) uses these priority coefficients to give an appropriate value to damage rates in each category based on the data available in the FIT. Finally, his model provides a list of schools, from more crucial to less critical in disbursing budget. In two case studies, he shows that his system offers a more rational and accurate solution than the FIT prioritization system. As of the limitations of his prioritization system, he mentions that there is no consideration of the limited budget in the system. Moreover, the ranking is on the school level, and there is no breakdown in the projects for funding purposes.

Definition of Quality in Educational System

Cheng and Tam (1997) mention that in policy and research discussion, education quality is a relatively inexplicit and controversial concept. There are various definitions of quality in education based on people's different point of view. For example, in some societies, uneven patterns of educational resource allocation to individuals may be

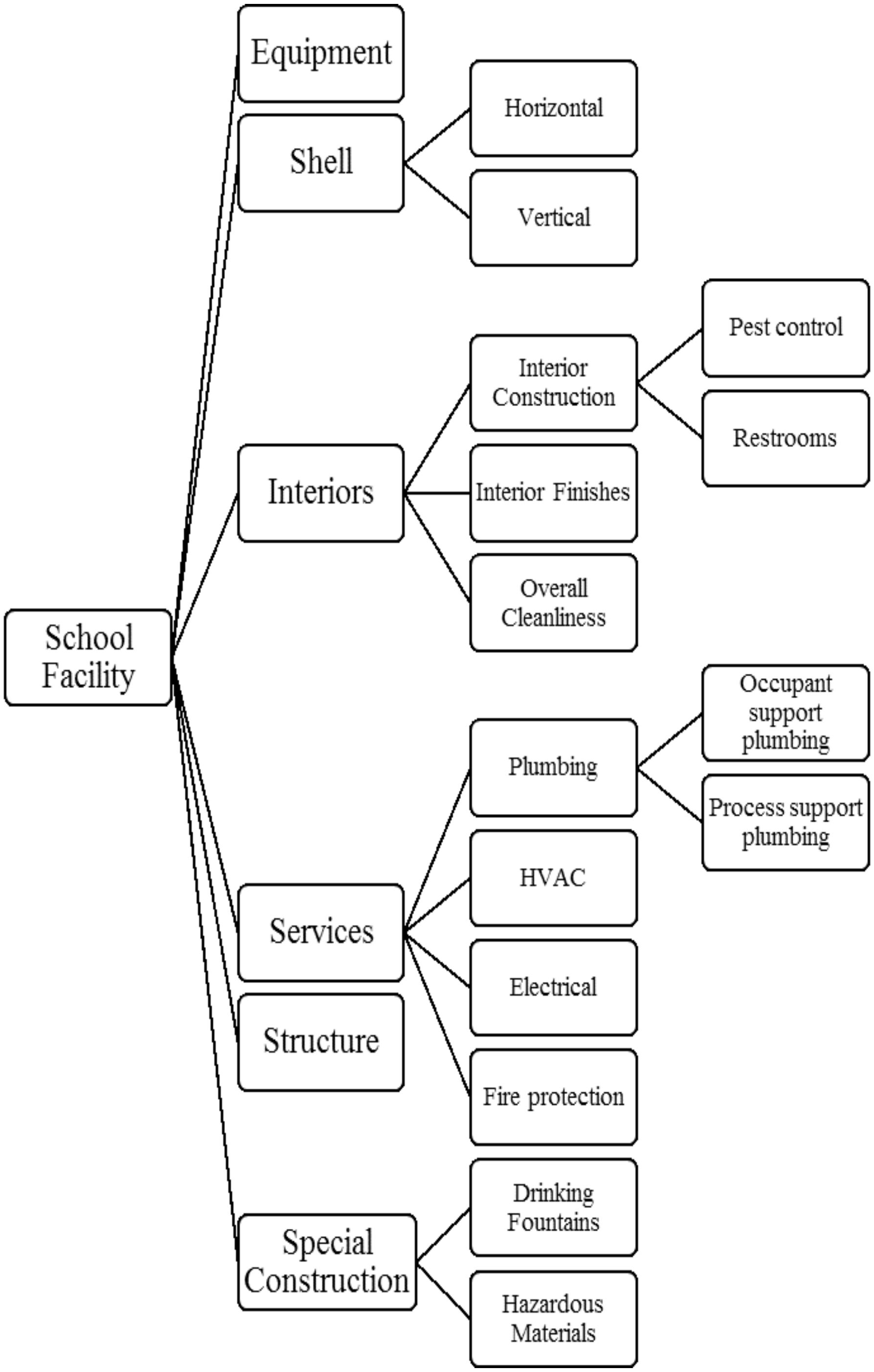


FIGURE 2. Hierarchy system for school facilities (Noureddine's 2010).

TABLE 1. Overall Priorities (Noureddine 2010)

Sub-factor	Overall Priority	Corresponding Attribute from FIT
Structural Damage	46.46%	Structural Damage
Vertical Enclosures	11.39%	Windows/Doors/Gates/Fences
Horizontal Enclosures	8.96%	Roofs
Restrooms	1.72%	Restrooms
Pest Control	4.53%	Pest/Vermin Infestation
Interior surfaces	1.49%	Interior Surfaces
Overall Cleanliness	2.02%	Overall Cleanliness
Occupant Support Plumbing	3.37%	Sewer
Process Support Plumbing	1.23%	Gas Leaks
HVAC	2.32%	Mech/HVAC
Fire Protection	4.30%	Fire Safety
Electrical	2.52%	Electrical
Equipment	3.60%	Playground/School Grounds
Drinking Fountains	1.09%	Drinking Fountains
Hazardous Material	5.02%	Hazardous Material

unacceptable while in others, it may be more of a concern to provide all children with at least a minimum base of resources. As another example, some districts may look for investment benefits' equally, while others may follow the pattern, in which they provide greater resources to students with low income families to compensate low family resources for these children.

Parkes and Stevens (2003) in their research study state that there is a potential that a school be recognized as low performance or low quality based on one characteristic of quality while it shows an extremely productive using another characteristic. This may sometimes lead to a school being subjected to sanctions, for showing little performance in progress and finally discouraging and disabling the school to increase its quality. Therefore, schools need to be funded to improve balance and to prevent the creation of a biased accountability which is also prohibited by the 14th amendment as of equal protection requirements.

Zvoch and Stevens (2008) mention that in order to provide equal chances of improvement in each school and student, the validity and reliability of different school quality indicators shall be confirmed in various characteristic indicators such as amount and type of available resources, student demographics, student outcomes, and internal processes and development.

Available Resources or Inputs

Cheong Cheng and Ming Tam (1997) mention that the quality of education follows a variety of concepts. The indicators used to describe education quality may be different in researcher's point of views. Some researchers put emphasis on the quality of inputs, whereas others emphasize on the quality of outcomes and processes. Aside from referring to input, output, process or any other aspects, most of the time definition of quality in education is linked with “fitness for use, the satisfaction of the needs of strategic constituencies (e.g. policy makers, parents, school management committee, teachers, students, etc.) or conformance to strategic constituencies’ requirements and expectations.”

Ladd and Loeb (2014) study the difficulties in measuring school qualities. They analyze the quality of education from various aspects and mention that to almost all appearances, the most appealing rubric for measuring quality of education is the amount of spending money per pupil. Besides the fact that price rates may be different in various places, most of the people have a sense of dollars’ scale on what they can or cannot purchase. The benefit of measuring school quality grounded on spending per pupil is that it does not base on any particular assumption or any preferred way to distribute the total budget between specific inputs. As a case in point, exactly the equal amount of money per pupil can be spend in two different schools with the characteristics of one with less experienced teachers and smaller classes and one with more qualified teachers and larger classes with in the other. One of the advantages of measuring the quality of education from inputs is, it enables very straightforward assessments between available resources and the outcomes in different schools or school districts. A very useful report could be generated with a form of comparing expenditure and various indicators of outcomes.

Darden and Cavendish (2012) in their paper mention that statistics shows that schools with more population of minority and poor students in urban and suburban school districts, usually have more overcrowded classes, less experienced instructors and overall, less available resources comparing to the schools with greater average socioeconomic rank of students. They state that the disparities among schools in resource allocation within a school district are typically higher than the disparities among the school districts in many regions.

Moreover, Unnever and et al. (2000) study the discrepancies in schools available resources and the student outcome. Their study shows that the amount of available resources is associated with socioeconomic status of school districts, and it is influential on student outcomes. They find that there is a collinear relation between the expenditure per pupil and socioeconomic status of the districts. Moreover, their study shows that increasing expenditure affects the aspiration and achievement of the students while they could not find any practical trend between expenditure per pupil and test scores.

Jimenez-Castellanos and Rodriguez (2009) discuss the necessity of the additional funding for English Language Learners and student from Low-Income Family to render the current inequalities in student achievements. They argue that, even though California has higher rate of English Language Learner and Low-Income Family students, the current approach of funding schools in California is less resolute to earmark sufficient financing for these categories of students than other states. They mention that there are evidence that adequate financing of minority and Low-Income Family students will improve the overall students' performance in public schools. They further, elaborate the demand of a proper resource allocation model which adequately meets the needs of all categories of students to promote educational achievements. In a case study in La Esperanza school district in California, the authors show that the higher amount of available resources allocated to the ELL and poor Latino students (both personnel and fiscal), significantly increases the outcome of education in schools. In their case study, the rate of Latino student in the district is 64.8 % while 32% of them are ELL students, and 40% of them are from Low-Income Family. They mention that, the number of ELL students in California has noticeably increased in the last decade, and this caused the escalation of the educational demands of schools. They demonstrate that the ELL and poor students are most of the times, in the need for more resources to be able to align successfully with their more advantaged peers.

Schwartz and Stiefel (2004) discuss that the dissimilarities in student needs has to be seen in the amount of spent money. They argue that a significant part of immigrants are from the categories of Low-Income Family and English Language Learners. Therefore, they confront with lower academic readiness at their grade level. Therefore, more allocated resources and inputs have to be considered to compensate the unbalanced situation of educational environments in schools and help these students to pursue their success in equal conditions.

The report published by Education Law Center (2013) mentions that equal educational opportunities need to be provided for students to achieve successful schools. The report argues that especial students with disabilities, English Language Learners, and students in poverty need to be supported additional resources, in order to have the same opportunity for high academic performance achievement in the schools. The report debates the appropriateness of the funding models which disregards differences of various socioeconomic class of students and their different needs. Moreover, it discusses the harmful consequences of the existing models in long-term application not only to the students but also to the communities and schools.

As a case in point, the report, references the $3000 gap of spending money per student in Pennsylvania on the comparison between wealthy schools and poor schools. The report argues that the local educational fund is significantly dependent on property taxes and, as a result, the inequalities will increase even more when the communities with high poverty are unable to raise sufficient funds to support schools. In fact, a reliable successful resource allocation system should reflect the existing inequalities and provide a system, which fairly funds the schools as the location of the school and the rate of the students with higher demand of resources do not affect the quality of education in a school (ELC 2013).

Equity vs. Equality in School Funding

Ramirez et al. (2011) contrast differences in definition of the equity and equality in education and bring to attention that, educational system has to follow equity in order to increase the output of the system and make students successful. The paper further defines the equality as an approach to treating every student the same way, while equity means to provide each student adequate resources, regarding the different needs of each. The authors study the efficiency of adequacy model in school funding in Colorado in providing sufficient resources for English Language Learner students to excel their academic achievements. Their study shows that the categorical funding system in Colorado has led to improvement of ELL students' academic performance and test scores. The authors bring to the attention that the state of Colorado considers different factors to make a formula for categorical funding. The total funding per pupil will increase by additional funding for students at-risk which in Colorado are the students eligible for reduced or free lunch program and English language learner student. The paper discusses about the pupil weight inserted in the formula to consider the extra cost associated with educating students in special categories and mentions that evidences in Colorado show that this extra funding will improve the implication of equity in educational system.

According to Banicki and Murphy (2014) school finance system and policies are revolutionized over the past two decades towards the point that “being fair” plays a significant role in addressing the students’ educational need. The authors mention that since early 1990s, researchers has been made a tremendous effort to change the general approach of funding privileged students, to the attitude which looks to provide sufficient funds for all students. Moreover, the authors state that understanding of researchers and school funding policy makers of the term “fair” has been shifted over the years and attained a more sophisticated definition. At the very beginning, the goal of policy makers was to provide a minimum amount of money for every student, this goal transformed to the concept which was established based on equalization of the available funds between students. The final formation of being fair nowadays is to provide sufficient resources, considering the student needs to provide a more balanced outcome of education in the whole system. It is mentioned in the paper that there are two different types of education equity. The first one is horizontal equity which focuses on all the students in the same situation, and similar needs have to receive an equal share of school funding. The second type is vertical equity which aims that student with unequal needs appropriately gets treated to make equal opportunity for success.

According to Satz (2007) on average, poor students in high schools, with high poverty rate, are less likely to complete the program and graduate from the school than their middle class peers in better funded schools. The paper also gives the information that the chance of poor students in poor schools attending colleges after graduation is significantly lower than students in wealthier schools. The paper argues the harmful result of disparities in educational funding between districts and as an example of these differences, the paper mentions that the wealthiest school district in New York City fund the schools about $25000 per student while at the same time a poor school district in Texas only allocates $1200 fund for each student. The author claims that a balance relation between equality and adequacy in education needs to be followed by policy makers in order to provide equal opportunity for every student.

Weighted Student Formula (WSF)

According to the Petko (2005) weighted student formula (WSF) is known by various names such as "student-based budgeting" and "student-weighted budgeting". This method of budgeting is relatively a new method of school funding and it is referred to a system of allocating resources between schools within a school district, on the basis of characteristics of students and their population in each school. The goal of WSF is to decentralize the allocation of budget from district level to the school levels. WSF considers the actual student's demographic data of schools in a district to distribute budget fairly. Although Britain education system has utilized WSF for many years, it has recently gained researchers' and policy makers' attention in America. The author counts four major advantage of this system as follows:

1. Increment of efficiency in funding schools as it provides a solid basis for perception of decision makers of budgeting.

2. Higher rate of adequacy and equity, as it will address the additional needs of educating various categories of students.

3. Provides competition element, as it motivates schools to retain their students by offering sufficient educational resources.

4. Develops a link between budgeting and schools effort on improving educational performance.

As a case in point, the report shows in a diagram the impact of WSF on funding schools in a hypothetical district by considering the percentage of students from Low-Income Family and how these disadvantaged students could get more money to prevent falling behind of their more advantaged peers (FIGURE 3).

Policy makers, in the last two decades and a gigantic size of legal activity, in an improvement of educational resource distribution methods, the approach of funding has been shifted to focus on resource allocation within school districts. The recent study demonstrates that there is a significant disparity in available resources across schools in many urban and suburban schools. To move towards performance of schools and students on the basis of accountability, various researches has been done to improve the funding distribution system. One of this very recent method which has attained much attentions by researchers is student-weighted budget allocation (Petko 2005).

Miles and Rosa (2006) study the applicability and efficiency of student-weighted allocating budget as a tool to attain a higher rate of equity in available resources. The article mentions that student-weighted system is a way to address student differences in demands of educational services by funding on the basis of student categories. The authors mention that this method of financing manifested as an alternative of the staff-based resource allocation and over the time obtained researchers attention as an efficient method of funding schools. The authors study the outcome of the application of this method in Houston Independent School District and Cincinnati Public Schools and illustrate that this method has led to a greater equity.

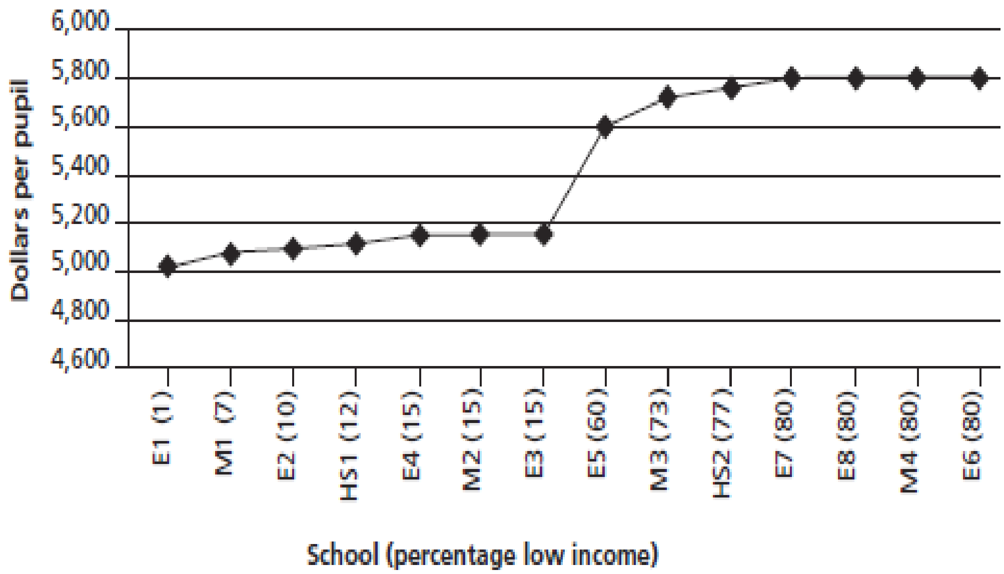


FIGURE 3. WSF impact on funding schools considering the low-income student percentage (Petko 2005).

The idea behind the weighted student funding is to favor a system on the basis of distributing dollars while it is incorporating a baseline for education and augment it based on the extra need of different classifications of students within a district. Student identified characteristics imply in a formula by means of coefficients to increase the amount of baseline. Most popular categories for weighting include but not limited to number of special education students, number of students eligible for free or reduced lunch program, number of students with limited English proficiency, the grade level of the school, and number of student study in the gifted education system. As a case in point, a school district wants to reduce the class size in its overcrowded K-3 level. The district may assign an additional 10% weight for all students study in K-3 level, and, therefore, the amount of resources for this category of students will increase about 10% of the resource baseline. All these percentages and numbers will be incorporated to allocate an additional dollar amount.

Baker et al. (2014) in a report assess the fairness of funding system for states and districts in United States. The authors discuss the factors which will affect the cost of education in states and local agencies. In a diagram, they show various parameters affecting state and districts funding (See FIGURE 4). They mention, to a greater picture, the states, and local agencies have to be financed based on two factors of student and location characteristics. The student factors such as poverty rate and percentage of students with disabilities and English Language Learners are utilized as a parameter in resource allocation across districts and within a district while the location factors are mostly used to distribute funds between states and districts.

Current Model and Formula of Funding

Toutkoushian and Michael (2008) mention that over 40 states in America use funding formulas or foundation aid program to distribute money to school districts. These formulas originate from legal actions on the claims of existing inequities in funding schools and disbursement of more money to the wealthier districts rather than poorer districts. The study mentions that these funding formulas are designed to utilize vertical and horizontal equity between districts and schools. Grounded on the local concerns, each state developed a model which may include various parameters such as the local revenues from taxpayers, the size of school districts, the demographic characteristics of the districts, the cost of living in the area and etc. It is shown in the paper that the formula frequently uses a base grant followed by defining various coefficients and rises the amount of budget for each of the parameters reflected in the state funding goal. The authors then, in a case study discuss the disadvantages and advantages of the state funding formula in State of Indiana. They claim that some changes in the coefficients and parameters will improve the efficiency of the formula.

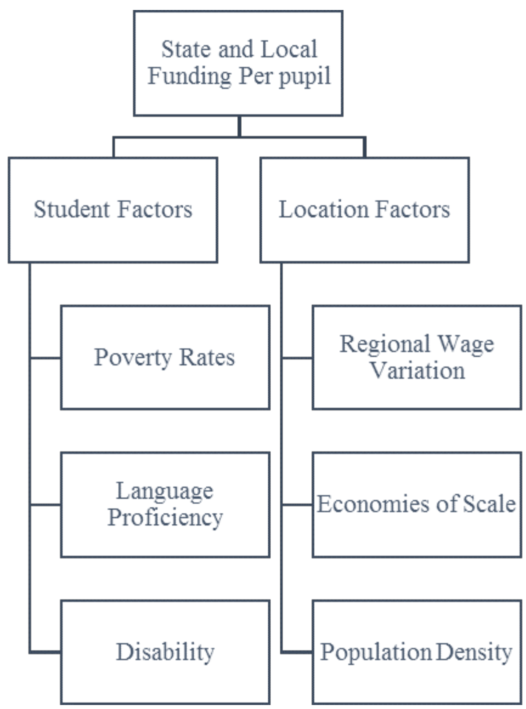


FIGURE 4. Cost effective factors in state and local education systems (Baker et al. 2014).

Many States in America developed a model for adequately funding schools based on the number of students enrolled in each school district, the poverty rank of the community and the amount of revenue collected from taxpayers in each area. This model considers the additional cost of educating students from lower socioeconomic class of society and the English Language Learners (ELL). To provide equal opportunities for all students, the total funding of school districts in a region will be divided based on the spending per student rather than providing an equal amount of money for each school district. This will allow the decision makers to consider the size of each district and also the demographic characteristics of the districts. Some states even consider district characteristics influential on the cost of education such as the higher cost of living in an area. The formula also implement more fund to the English learners and students with Low-Income Family by defining a coefficient grounded on the average rate of these students to the total number of enrolled students in a district (ELC 2013).

The funding formula hitherto pointed out consists of three components: base cost or the annual funding, formula factors which include students and district factors, and an adequacy goal.

Base Cost: Is the annual funding which does not consider any difference between districts, school, and students characteristics. It is calculated based on the school student's requirements to meet state standard for academic performance.

Student Factors: In order to provide additional resources for economically disadvantaged students or students with special needs such as English Language Learners or student with disability many states consider a factor to increase the level of funding for these categories of students. As a case in point, Pennsylvania uses a factor of 1.5 for students who are eligible for reduced or free lunch price and a factor of two for English Language Learners. 30 states use a student factor for socioeconomically disadvantaged students, and 27 states use a factor for ELL students. Overall, 37 states use at least one factor for these two categories of students.

School district factor: the size of each school district and the local educational revenue from property taxpayers are usually the parameters cogitated by states to find a coefficient for school districts.

Adequacy goal: Is the amount of budget needs to be provided in these models. Therefore, the adequacy goal will be on the right side of the equation as an indicator of the total required fund. To address the goal of equal opportunity for every student, the gap between available funds and required fund for standard academic achievement of students has to be investigated periodically and required resources have to be provided to reach adequacy.

As an example, the current funding formula in Pennsylvania is:

$$\begin{aligned} & Base\ Cost\ \times Total\ Student\ Enrollements \ldots \\ & \quad \times 1.1\ (for\ small\ school\ districts'\ higher\ costs\ of\ operating) \ldots \\ & \quad \times 1.1\ (for\ districts\ with\ a\ higher\ cost\ of\ living) \ldots \\ & \quad + Number\ of\ ELL\ students\ \times Base\ Cost\ \times 2.0 \ldots \\ & \quad + Number\ of\ students\ with\ disabilities\ \times Base\ Cost\ \times 1.5 \ldots \\ & \quad + Number\ of\ poor\ students\ \times Base\ Cost\ \times 0.5 = Adequacy\ Goal \end{aligned}$$

Pennsylvania State considers additional two times of resources for English learners and 1.5 times of the base fund for student with disabilities and about half of the base fund more for students from Low-Income Family.

The formula is schematically shown in the (FIGURE 5)**.**

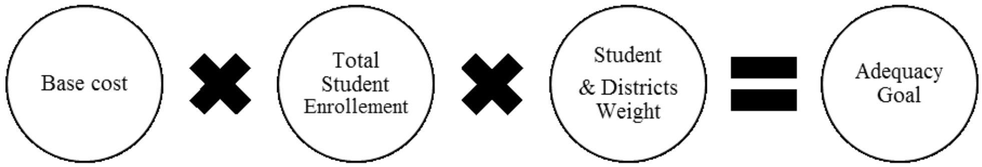


FIGURE 5. Adequacy model for school funding (ELC 2013).

According to the California Department of Education, local control funding formula (LCFF) review (2015) the State of California entitled educational policy makers to apply the funding formula from the fiscal year of 2014. The State does not consider weight factors for different categories of students in the funding formula as it was mentioned previously in this section. However, an additional concentration grant needs to be dedicated to each school based on the population of the English Language Learner and economically disadvantaged students in schools and respectively in districts. The review states that 20% more funding required by state for English Language Learners and student from Low-Income Family, when the average rate of these categories of students to the total number of enrolled students is less than 55% and in the case that this rate is more than 55% California State allocates 20% more for all student with additional 50% for the difference between the average percentage of the disadvantaged student and 55%. The coefficients of additional resources for disadvantaged student in California could be calculated from the following equations:

When the rate of ELL and Low-Income Family students is less than 55%:

$$Concentration\ Grant\ (Aditional\ Resources\ for\ disadvantaged\ students)$$

$$= Base\ Grant \ldots$$

$$\times \frac{Number\ of\ ELL\ and\ economically\ disadvantaged\ students}{Total\ enrolled}$$

$$\times 0.2$$

When the rate of ELL and Low-Income Family students is more than 55%:

$$Concentration\ Grant\ (Aditional\ Resources\ for\ disadvantaged\ students)$$

$$= Base\ Grant \ldots$$

$$\times \left[\left(\frac{Number\ of\ ELL\ and\ Low - economically\ disadvantaged\ students}{Total\ enrolled} \times 0.2 \right) \ldots \right.$$

$$+ \left(\left(\frac{(Number\ of\ ELL\ and\ economically\ disadvantaged\ students)}{Total\ enrolled} - (0.55) \right) \ldots \right.$$

$$\left. \left. \times 0.5 \right) \right]$$

As an example, if the base grant is $7000 and the average rate of ELL and Low-Income Family students are 70% in a school district then the additional grant will be calculated as follows:

$$Aditional\ grant = \$7000 \times (0.7 \times 0.2 + 0.15 \times 0.5) = \$1505$$

Algorithms for Optimization Model

Evolutionary Algorithms

Karaman, et al. (2012), in their research study, discuss about the application and practicality of evolutionary algorithms in optimization problems. They mention that, when the goal of an optimization model is to quickly achieve an optimal feasible solution, the use of stochastic search algorithm will allow the user to properly utilize randomness in the logic of the model. These types of algorithms choose a random set of input data and then direct the input data behavior, to achieve a better average performance in each step. Then after an anticipated runtime, the solution, converges to an appropriate value.

One of the most common systems to apply stochastic search methods to find an optimum solution for a combinatorial problem is evolutionary algorithms (EA). The idea of evolutionary algorithms rose from observation of the existing mutation process in selecting better generations in the nature. In these methods, a population or set of solutions, which is called chromosomes, will be repeatedly selected and manipulated to encode better candidates. In the process of improving each generation, different operations will be performed, such as selecting a set of chromosomes, crossover them and subsequently mutate them. A fitness model, which defines the goal of the optimization system, in each generation, will assess the conformity and properness of each candidate chromosome in the population (Karaman, et al. 2012).

The topmost three types of EAs are evolutionary programming, Genetic Algorithms (GAs), and evolutionary strategies. Between these three types of algorithms, genetic algorithm has been used more often by the researchers and is more popular than the other two. The first and the foremost phase of applying GA to an optimization system is that how to encode the chromosomes. The next important step, to develop a reliable GA model, is to define proper operators for crossovers and mutation in order to keep the validity of the solution in each step. In each specific problem the figuration of chromosomes and how to encode them and the definition of evolutionary operators, are different and should be specified exactly for that problem (Karaman, et al. 2012).

Xilin and Shiming (2013), use the Genetic Algorithm to develop an optimization model, to allocate resources in higher education system. In their study, they point out that, resource allocation projects, are typically nonlinear and multi-objective problems which will direct the researchers to use a type of heuristic algorithms such as Genetic Algorithm.

Campbell (1989) in his review states that, there are three difficulties in development of an optimization model for rehabilitation projects which need to be addressed. Firstly, it is not possible to use the ordinary optimization models, which have just one objective, as rehabilitation projects are multi-objective problems. Secondly, the variables included in objective functions such as resources and benefits, are discrete and nonlinear (i.e. the function value between successive input variables, is not continuous) so the use of linear optimization methods is not possible. Thirdly, this type of projects, are engaged with an analysis of an extremely massive search space, which is impossible to compute. He concludes that to overcome all of these difficulties, it is necessary to use evolutionary algorithms. Furthermore, he mentions that a method which has been proven to search nonlinear programming spaces effectively is Genetic Algorithm. The GA forte is considering solutions simultaneously while evaluating many subareas of the solution space. The GA processing, first identifies solutions which are better than average and then recombine them to find better solutions.

Orabi and El-Rayes (2012), use Genetic Algorithm for their optimization model in a transportation rehabilitation projects. They discuss that in optimization of rehabilitation projects, the decision maker needs to confront three main contests: 1-optimization problem: Is multi-objective in nature, 2-The planning objectives: Are nonlinear and non-continuous, and 3-The search space: Is large. Moreover, they mention that Genetic Algorithm is one of the most practical and powerful methods to solve this sort of problems.

Goel et al. (2010), use Genetic Algorithm for their resource allocation model. They discuss the ability of GA to solve the multi-objective problems, with a large number of data and how GA can reduce the run time of the computer processors, to reach the solution. They utilize GA to solve a combination of three different fitness models at the same time using different processor, to both cut the runtime and find the concurrent optimal solution.

Huang et al. (2010) use GA to develop an optimization model for Resource allocation in large-scale construction project management. They discuss that construction projects are most of the times engaged with constrained resources and it is important to use these limited number of resources efficiently, in order to optimize the benefit of the project. They compare the efficiency of different methods of programming; such as Dynamic Programming, deterministic algorithms such as linear programming and Genetic Algorithm. They state that, many research studies show the strong capacity of Genetic Algorithm, in finding the comprehensive, optimum solution for resource allocation projects.

Goldberg (1989) mentions that, the optimization models in rehabilitation projects are usually engages with discrete data and nonlinear multi-objective functions. As a result, the use of linear optimization methods, adds more complication and difficulty to the problem. Moreover, he mentions that in this type of projects, the model contains an extremely massive search space, which is problematic in computation. The author mentions that to overcome these difficulties evolutionary algorithms has been proven to be a convenient method. One method which has been proven to search nonlinear and discrete spaces effectively to find the optimal solution is Genetic Algorithm. It assumes that in large problems, the solution needs to be defined and after that categorized in “building blocks.” The GA forte is considering solutions simultaneously while evaluating many subareas of the solution space. The GA processing first identifies these building blocks which are better than average and then recombines them to find better solutions.

GA bases on Darwinian evolution theory. GA has been shown to be a very powerful method, which can solve almost every type of optimization problems. The process of solving the problem in GA, is to encode a set of possible solutions and improves them in certain iterations of running the program to optimize a defined fitness function. The fundamental operations and procedures of GA are to encode solutions or chromosomes, then calculate the fitness function value for that set of chromosomes, after that select a part of prior solution, crossover them and mutate them (Xilin and Shiming 2013).

Niazi and Leardi (2012) state that the idea of using Genetic Algorithm as an approach to solving optimization problems started in Holland in the early 1970s. They explain that the idea bases on using a simulation of evolutionary phenomena existed in living creatures to use random footsteps to achieve a particular optimal answer. The basic idea of GA is the finest, and fittest individuals have a higher chance of survival and more potential of spreading its characteristics to the next generation with its genome. As a result, reproduction of the two better individuals, will lead to a mix of better genomes and finally a better and finer individual. Moreover, in problems which there is no definite answer to the optimization model, Genetic Algorithm may be utilized to find the optimum feasible solution, after a certain evolution of the solutions.

In GA, a group of conceivable solutions form a set of chromosomes. The possible amount of these chromosomes is often gigantic. The encoded chromosomes through the process of mutation and crossover create a new generation of chromosomes. Finally, after a specific numbers of operations, attained chromosomes maximize the fitness function. This group of solutions are selected as the solution of optimization system. The chromosomes in a GA system are encoded into the binary strings in order to make a proper format for other operations (Xilin and Shiming 2013).

The process of finding the optimum solution using GA is shown in FIGURE 6.

The step by step method of solving the problem in GA may be formulated and classified as follows:

a) Generating a set of chromosomes in binary string format.

$$X_i = \{x_1, x_2, \dots, x_j\}, \qquad x_j \in \{0,1\}$$

b) Combine the generated chromosomes to form a population.

$$P = \{X_1, X_2, \dots, X_i, \dots\}$$

By this token the nth population will be indicated as:

$$P^n = \{X^n{}_1, X^n{}_2, \dots, X^n{}_i, \dots\}$$

c) Determine the value of the fitness function for each specific chromosomes.

d) Select m parent chromosomes from the chromosomes which has greater fitness value.

e) Choose the crossover chromosomes from the selected m parent chromosomes in a random process with probability of P_c^n.

f) With probability of P_m^n randomly select some chromosomes for mutation.

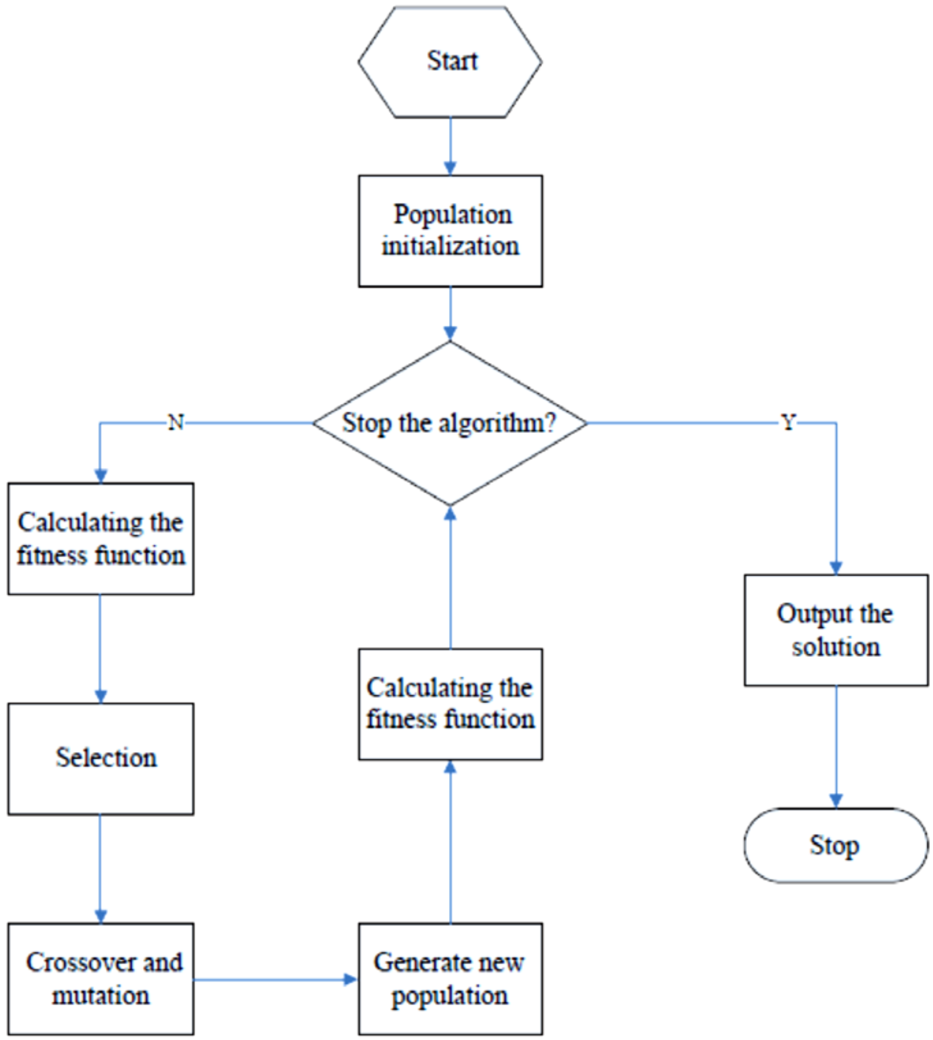


FIGURE 6. The flowchart of resource allocation optimization based on GA (Xilin and Shiming 2013).

Owing to the fact that the newly conducted generation needs to be more mature and the chromosomes' diversity needs to be reduced over the time, the probability of crossover has to be defined, reduce the time of running the program while the probability of mutation will be increased at the same time. In order to make this happen, the probability of crossover and mutation can be respectively defined as follows:

$$P_c^n = P_c(1 - {}^{n}/_{M})$$

$$P_m^n = P_m(1 - {}^{n}/_{M})$$

When M is defined as the total number of generations in the GA model and n is the current generation number.

g) Crossover: The act of swapping the genes in two parent's chromosomes to produce new chromosomes. Next, from the offspring chromosomes, a new population will be formed. Considering the crossover between first $(X^n{}_1)$ and second $(X^n{}_2)$ chromosomes in the nth population, will produce two offspring chromosomes of $(X^{n+1}{}_1)$ and $(X^{n+1}{}_2)$, each of these chromosomes and the crossover itself can be defined and formulated as the follows:

The nth chromosomes are:

$$X^n{}_1 = \{x^n{}_{11}, x^n{}_{12}, \dots, x^n{}_{1j}, \dots\}, \quad x^n{}_{1j} \in \{0,1\}$$

$$X^n{}_2 = \{x^n{}_{21}, x^n{}_{22}, \dots, x^n{}_{2j}, \dots\}, \quad x^n{}_{2j} \in \{0,1\}$$

The offspring chromosomes are:

$$X^{n+1}{}_1 = \{x^{n+1}{}_{11}, x^{n+1}{}_{12}, \dots, x^{n+1}{}_{1j}, \dots\}, \quad j = 1, 2, \dots, N$$

$$X^{n+1}{}_1 = \{x^{n+1}{}_{11}, x^{n+1}{}_{12}, \dots, x^{n+1}{}_{1j}, \dots\}, \quad j = 1, 2, \dots, N$$

The crossover can be defined as:

$$X^{n+1}{}_{1j} = rx^n{}_{1j} + (1 - r)x^n{}_{2j}$$

$$X^{n+1}{}_{2j} = rx^{n}{}_{2j} + (1-r)x^{n}{}_{1j}$$

$$r \in \{0,1\}$$

h) Mutation: As the chromosomes are encoded to the binary strings, in a random process with the previously defined probability (P_m^n), some of the bits in the string will change to make a new chromosome. The fittest chromosomes of the last generation will not change in this process, to get extremely fittest chromosomes in the process.

i) Stop point of the run: Two conditions may be defined to limit the runtime of the program. First, it can be defined that the number of generations in the program is M and the program will stop the algorithm, when the generation n=M and the chromosomes which have the fittest value will be selected as a result of the optimization system. Second, it can be defined if the maximum value of fitness function is not changing after a certain number of generations, as a result, the program will stop and the fittest population of chromosomes will be the answer to the system.

Knapsack Problems

Corman et al. (2009) mention that the knapsack problem got its name as of finding a solution to the problem of filling a knapsack with a certain size with most valuable items. Knapsack problems are classified as combinatorial optimization problems. The basic idea behind a knapsack problem is to find the best set of given items in which each of them has a defined value and mass to utilize the maximum possible value. The Knapsack problem has obtained significant attentions in programming not only for its numerous applications in solving optimization problems but also for its ability and flexibility to be utilized as a tool for various optimization systems.

There are two versions of Knapsack problems, one is when the items are continuous and divisible which is called Unbounded knapsack problem and one is when the items are discrete and indivisible which is called 0-1 Knapsack problem or exact K-item problems. The greedy algorithm will be applied to solve an unbounded knapsack problem and Dynamic Programming will be utilized as a proper algorithm to solve a 0-1 knapsack problem. Since the optimization problem in this research study is classified as a 0-1 Knapsack problem, and to avoid from giving unnecessary extra information, only this type of problems and the algorithm compatible to these problems will be discussed in this section (Corman et al. 2009).

Exact K-Item or 0-1 Knapsack Problems

Martello et al. (1999) state that a 0-1 Knapsack problem could be formulated as follows:

Considering that a subset of total number of n items has to be picked up to fill a knapsack with capacity of W. Each item i has a profit of p_i and a weight of w_i and the problem is how to maximize the profit with choosing the items which has the total weight of equal or less than the capacity.

Maximize the function:

$$z = \sum_{i=1}^{n} p_i x_i$$

Which is subject to:

$$z = \sum_{i=1}^{n} w_i x_i \leq W$$

While:

$$x_i \in \{0,1\}, i \in \{1, \dots, n\}$$

All of the coefficients in the equations are assumed to be positive integers and x_i will take the value of 1 if it is in the solution and it gets the value of 0 if it is not a part of solution (Silvano et al. 1999).

Example of a Knapsack problem

A thief wants to rob a store which has n items. The i th item has a value of v_i dollar and has a weight of w_i pounds, where both v_i and w_i are integers. The thief could fill and carry the maximum amount of W pounds with his knapsack. Therefore, he is looking for the best set of the items to fill the knapsack while he can take to maximize his profits. This problem is called 0-1 knapsack since each item has only the chance of being picked or not. There is no possibility that the thief can take a fraction of an item (Corman et al. 2009).

Dynamic Programming

Hillier and Lieberman (2001) mention that, Dynamic Programming is a technique, usually used to design and analyze algorithms to solve optimization problems in which a set of choices need to be examined to find an optimal solution. Dynamic Programming breaks a gigantic problem down to small subproblems and by linking the solutions of these subproblems, finds the solution for the original problem. In contrast to linear programming, Dynamic Programming does not follow a regular mathematic formulation to solve the problem. It has a general approach to solve problems, and a certain equation needs to be defined to fit a specific problem.

Dynamic programming is utilized when the subproblems have overlap with each other and each of the subproblems, could reappear in different stages of the solving process. Any of these subproblems could have a set of sub-subproblems which are engaged to each other. The efficiency and effectiveness of Dynamic Programming have been proven when there is a possibility that each of the subproblems appears in different set of choices of the solution of the problem. Dynamic Programming solves each of the subproblems one time and then saves and keeps the solution in order to prevent computing that sub problem again. To develop an algorithm based on Dynamic Programming four different steps has to be followed:

1. Characterize the structure of an optimum solution.
2. Repeatedly find the value of an optimum solution.
3. Conclude the optimum value in a backward fashion.
4. Develop the solution from calculated information. (Corman et al. 2009)

Kruger and Hattingh (2014) present that Dynamic Programming could be utilized to solve an optimization problem, both when the set of data is continuous or non-continuous. Finding a solution to the problem, within non-continuous media adds more complication to the algorithm. However, Dynamic Programming has the ability to solve discrete optimization problems such as 0-1 Knapsack problem as a very popular problem when the optimal solution needs to be found from integer numbers or in other word, when there are exact options which cannot be partially chosen as a part of solution.

Dynamic Programming could be applied to solve complicated knapsack problems. The running time of the Dynamic Programming in knapsack problems is pseudo-polynomial, as the running time over the value of the input is polynomial while the running time over the length of the inputs is exponential. Dynamic Programming by subtracting the subset of the solutions, saving them, and use them as a base solution of the further step, gives the option to the users to cut off a huge running time of the search over the searching space in order to find the optimal solution (Kruger and Hattingh 2014).

Rong et al. (2012) discuss, the application of Dynamic Programming algorithm for solving optimization problems. The authors mention that the best way to solve a complicated knapsack problem is to utilize Dynamic Programming. Since a large knapsack problem has a gigantic set of possible solutions, dynamic programing enables the possibility of solving a small domain of problem as a core solution and searching the new solution based on expanding the core solution. This will considerably shorten the duration of repetition and the effort of computation to find the optimum solution.

Dynamic Programming algorithm for a 0-1 knapsack problem, can be formulated as follows:

Considering that, $w_1, w_2, \dots, w_n, W$ are all positive integers, the maximum value that can be attained from the items with a weight of equal or less than w, can be defined as:

$$z[i, w]$$

This function recursively can be determined as:

When the weight of an item i is more than the weight limit:

$$z[i, w] = z[i-1, w] \; if \; w_i > w$$

When the weight of an item I is less than the weight limit:

$$z[i, w] = Max\{z[i-1, w], z[i-1, w-w_i] + z_i\} \; if \; w_i \leq w$$

The solution finally can be extracted from:

$$z[n, W]$$

Note that the value of items could be defined with any formulation either linear or non-linear to allow the users define and solve an optimization problem with various types of objective functions (Corman et al. 2009).

<u>A Simple Example of Using Dynamic Programming to Solve a Simple 0-1 Knapsack Problem:</u>

Assume that there are four items with different value and weights as shown in TABLE 2.

Considering that a selection of items with the most value has to fill a knapsack with the total weight of 10 ($W = 10$). The step by step solution of Dynamic Programming method to this problem could be shown as following:

First Step is to decompose the problem and characterize the structure with forming an array of $V[0 \dots n, 0 \dots W]$ for $1 \leq i \leq n$ and$1 \leq w \leq W$.

Second step is recursively to define the optimal value of subproblems solution. In each step there are two options that could be chosen; one is leaving the item i which means the best solution we can have, will be the set of items $\{1,2, \dots, i-1\}$ and the storage limit w is, $V[i-1, w]$; the second option is to take the item i, which is only possible, when the weight of item i is less than the total weight of the Knapsack or in mathematic formation of: $If\ w_i \leq w$. The total value could be obtained from this option will be a sum of the best value of the set of items $\{1,2, \dots, i-1\}$ with the storage of $(w - w_i)$ shown as $V[i-1, w-w_i]$ and the value of item i (v_i).

The third step is to calculate the optimum solution from the bottom to top using the following equation:

$$V[i, w] = Max\{V[i-1, w], V[i-1, w-w_i] + v_i\}$$

For this particular example, the array will be as shown in TABLE 3.

TABLE 2. Weight and Value in a Knapsack Example

Item i	1	2	3	4
Value (w_i)	10	40	30	50
Weight (w_i)	5	4	6	3

TABLE 3. Dynamic Programming Array

$V[i, w]$	$w = 0$	$w = 1$	$w = 2$	$w = 3$	…	W
$i = 0$	0	0	0	0	…	0 bottom
$i = 1$						
$i = 2$						
$i = n$						Top

Since W is equal to 10 in this example, the matrix will be formed as shown in TABLE 4.

The final output value is $V[4,10] = 90$.

And finally in the last step, the subset which gives the optimum solution of the problem will be items 2 and 4 (Hillier and Lieberman 2001).

TABLE 4. Dynamic Programming Matrix

$V[i,w]$	0	1	2	3	4	5	6	7	8	9	10
$i = 0$	0	0	0	0	0	0	0	0	0	0	0
$i = 1$	0	0	0	0	0	10	10	10	10	10	10
$i = 2$	0	0	0	0	40	40	40	40	40	50	50
$i = 3$	0	0	0	0	40	40	40	40	40	50	70
$i = 4$	0	0	0	50	50	50	50	90	90	90	90

CHAPTER 3

MODEL DEVELOPMENT

The development of an optimization model for school facility projects are necessary since the current subjective methods of distribution of funds is on the first-come-first-serve basis which involves lots of error in the results. As a matter of fact, significant steps have been taken to the current date to improve the applicability and efficiency of the decision making process but very few researchers have tried to develop an optimization system involving influential factors such as student demographic data, categorical demands of students, and the importance rate of each element in school facilities.

In this study, an optimization model is developed and introduced to allocate budget to school rehabilitation projects within a school district. This optimization system aims to achieve a better selection of the projects when the available resources are insufficient to accomplish all rehabilitation projects in a school district. An objective function has been formulated based on the increase in the overall equity of a limited accessible fund while addressing the differences in demands of various categories of students and incorporating the importance factor of each category of the rehabilitation projects in the process of decision making.

To find the optimal solution, two methods of Genetic Algorithm and Dynamic Programming have been utilized. Both methods are described in the chapter two and the optimal solutions achieved by each method is compared to other one in a case study. With these two models, the allocation of a limited budget in intra-district school facility projects is referenced to maximize the equity of budgeting in conjunction with the student and project characteristics.

Projects' Importance Coefficients in Optimization Model

Each school facility project can further be sub-divided into smaller projects which have different importance rates in overall performance of a facility. These importance factors have been investigated and formulated by Noureddine (2010), which had been addressed in the literature review section. In order to take a further step from the prioritization model that Noureddine developed in 2010 and to develop a more practical model which will consider the restraint of available resources, the level of the hierarchy to which the parameters of the system need to be subdivided should be decided. The first level of hierarchy system introduced by Noureddine (2010) defines six parameters in each school facility to break down a large size project into smaller subprojects. Each of these six subprojects will further be broken down to smaller sub-subprojects, and this division process will continue to the fourth level. The importance rates for each of the elements have been calculated by Noureddine (2010) where the sum of all elements in each level is equal to one. An optimization model could be developed on any of these four levels based on the expected level of detail in the output. Making decision to build the structure of the model on a certain level of hierarchy depends on how far the system needs to go into details and how much the decision maker needs to divide the projects into smaller projects. Formulation of the model and the optimization algorithm will not change significantly by changing the level of the hierarchy. As a result, only the size of input data will increase, and the optimal solution will contain more sub-categories of the projects. Regarding the fact that the categories introduced by Noureddine (2010) is slightly different from the districts definition of the facility subprojects, the use of 2nd, 3rd, and 4th level of the hierarchy will add more complexity to reclassification of the input data. Furthermore, the reliability of the system and efficiency of the model is almost independent of the level of hierarchy, thus choosing the level of the hierarchy will not affect the optimization algorithm of the model. Therefore, the first level of Noureddine's (2010) the hierarchy is utilized in the current optimization process, and the model runs on the following six categories defined in the first level of hierarchy:

1. Structure
2. Shell
3. Interiors
4. Services
5. Equipment/Furnishings
6. Special Construction

The importance factor for each of these categories corresponding to their numbers is as shown in TABLE 5.

TABLE 5. Projects' Importance Coefficients (Noureddine 2010)

No.	1	2	3	4	5	6
N_{Imp}	0.4646	0.2035	0.0976	0.1373	0.0360	0.0611

Where in the table $N_{Imp}$is the notation which shows Noureddine's (2010) importance factors for each category in a facility.

Equity in Distribution of a Limited Budget

According to the definition of horizontal equity in education, when the resources are not limited, the decision makers need to provide students with similar conditions the same amount of resources. In contrast, the vertical equity implements the requirement of providing all students equal educational opportunities by supplementing more resources to students with more demands. When there is a constraint on budget and a choice is needed to be made on which one of the two groups of students has to get the money, more complication will be added to the equity subject. Initiation of the idea of following equity in education was based on increasing the benefits of education for a larger group of students instead of a certain group of more advantaged students.

Due to lack of information and research studies at school levels, each sub-project either needs to be conducted as a whole or postponed to the future. There is no option to do a certain percentage of a rehabilitation project to improve the school performance. This leads to making a decision on which model will be fittest the most and be beneficial to more group of people. In this study, the objective function of the optimization model is formed based on allocating budget to benefit more groups of students. A parameter is defined based on the number of students divided by the project expenses in dollars in a specific school. The goal of the defined terms in the objective function is to maximize the number of students who will receive the budget. The horizontal equity has been applied to the objective function by the following term:

$$Eq_h = \frac{Number\ of\ all\ Students\ in\ the\ school}{\$\ amount\ of\ the\ facility\ project}$$

The vertical equity with a focus on two categories of English Language Learners and Low-Income Family students has been considered by the following two terms:

$$Eq_{v-ELL} = \frac{Number\ of\ English\ Language\ Learner\ students\ in\ the\ school}{\$\ amount\ of\ the\ facility\ project}$$

$$Eq_{v-LI} = \frac{Number\ of\ Low-Income\ Family\ students\ in\ the\ school}{\$\ amount\ of\ the\ facility\ project}$$

Adequacy and Student Weighted Formula

The literature review on the models of funding and resource allocation shows that there is a significant concern in appropriately addressing the extra demands of more disadvantaged students in the funding process. This consideration has been investigated by the researchers in the past decade which has resulted in shifting towards weighted student resource allocation. The goal of this study is to address the key concerns of decision makers and researchers in considering the influence of student categories in resource allocation. Therefore, the importance factors of Low-Income Family students and English Language Learner students is incorporated into the following two coefficients in the objective function:

$$\alpha = Coefficient\ of\ importance\ for\ ELL\ students$$

$$\beta = Coefficient\ of\ importance\ for\ low-income\ family\ students$$

The coefficient of the total number of students in the objective function is considered to be one and α, and β are always greater than one.

The coefficients of students' categories as mentioned above are the same as the coefficients that each state uses in its funding formula and could be changed from one state to another. Also, these coefficients can be assigned by decision makers such as the board of education in a district or the policy makers in the state.

Objective Function of Optimization Model

The objective function of an optimization model expresses the goal of the model and usually will maximize or minimize a set of variables with certain constraints. The objective function of the optimization model of this study is formed from a combination of the partial objectives with the coefficients introduced previously in this chapter. To find the optimal solution for resource allocation, the following formula has been developed and used as the objective function of the model:

$$O = \{[N_{Imp}]Eq_h\} + \{\alpha \times [N_{Imp}][Eq_{v-ELL}]\} + \{\beta \times [N_{Imp}][Eq_{v-LI}]\}$$

Where,

N_{Imp}, is the importance factors for each of six categories in a school facility defined in TABLE 5.

Eq_h is the horizontal equity.

Eq_{v-ELL} is the Vertical equity for language proficiency

Eq_{v-LI} is the vertical equity for Low-Income Family students

α is the importance coefficient for ELL students $\alpha \geq 1$

β is the importance coefficient for Low-Income Family Students $\beta \geq 1$

The limited budget is the constraint of this objective function. The total amount of the available budget needs to be defined by a school district.

This objective function needs to be maximized to increase the equity for all students and meet the additional demands of specific groups of students while considering the importance of different categories in a facility rehabilitation project. The objective function is bounded by a budget constraint which will be defined by the user as an available budget.

The purpose of the optimization model is to find an optimal solution for resource allocation in large and extra-large urban and suburban districts serving a great number of schools with various types and sizes of the facility projects. In these types of districts, usually the model has to search within a large number of data to find the optimal solution. Moreover, the objective function of this model is formed by combining smaller objective functions which will complicate the solving process. Genetic Algorithm (GA) and Dynamic Programming, known as two powerful tools for solving this type of optimization problems, are utilized here to find the optimal solution.

The optimization model is developed in MATLAB 2010 and is solved with both Genetic Algorithm and Dynamic Programming approaches. The input data needs to be saved in text document, or Microsoft Excel spreadsheet. The number of columns is the same for every district, but the number of rows will vary with the number of schools within the district. The output budgets correspond to the initial order of the schools in the input file respectively. Therefore, it is important to keep the order unchanged for the correct interpretation of the output. A sample format of the input data is shown in Table 6.

A school district could determine the cost of each category of rehabilitation projects. The total cost of the projects need to be defined for a fiscal year, and the scope of projects could be reclassified to meet the proposed categories. Data for six categories of rehabilitation projects could be reclassified as the proposed structure which is shown in FIGURE 7. Therefore, the cost of each scope of facility projects in a school district will be added to the correspondent category and the total amount of budget needed for that category will be calculated.

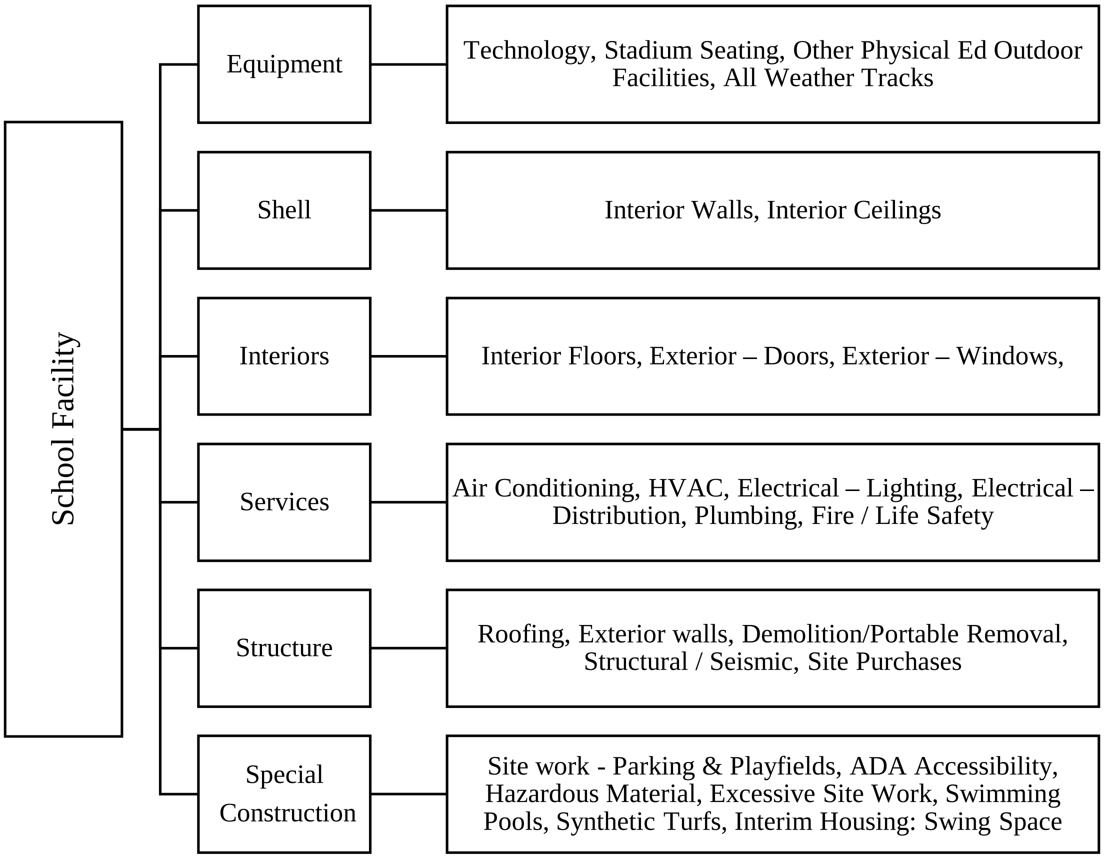


FIGURE 7. Proposed structure for reclassification of the facility projects.

The output of the program will be a table containing the costs correspond to the projects. There will be some projects which are funded based on their defined cost in the input, and there will be some projects which are not funded since the resources are limited. The correspondent fund of these projects in the output will be 0. A sample format of the output data is shown in TABLE 7.

TABLE 6. Input Data Format

School identification number	1	2	⋮	n
Number of Low-Income Family students	No.	No.	⋮	No.
Number of English Language Learner students	No.	No.	⋮	No.
Total number of enrolled students in the school	No.	No.	⋮	No.
cost of the project related to Special construction	$	$	⋮	$
cost of the project related to Equipment/Furnishings	$	$	⋮	$
cost of the project related to Services	$	$	⋮	$
cost of the project related to Interiors	$	$	⋮	$
cost of the project related to Shell	$	$	⋮	$
cost of the project related to Structure	$	$	⋮	$

TABLE 7. Output of Optimization Model

School identification number	1	2	⋮	n
cost of the project related to Special construction	0	0	⋮	$
cost of the project related to Equipment/Furnishings	0	$	⋮	$
cost of the project related to Services	0	0	⋮	$
cost of the project related to Interiors	$	$	⋮	$
cost of the project related to Shell	$	$	⋮	0
cost of the project related to Structure	$	$	⋮	0

The program is designed to give the two options of Genetic Algorithm and Dynamic Programming as a method of solving the problem. In the first screen of the program, a short description about GA and DP is given to the user and the user, needs to select one of the methods (See FIGURE 8)

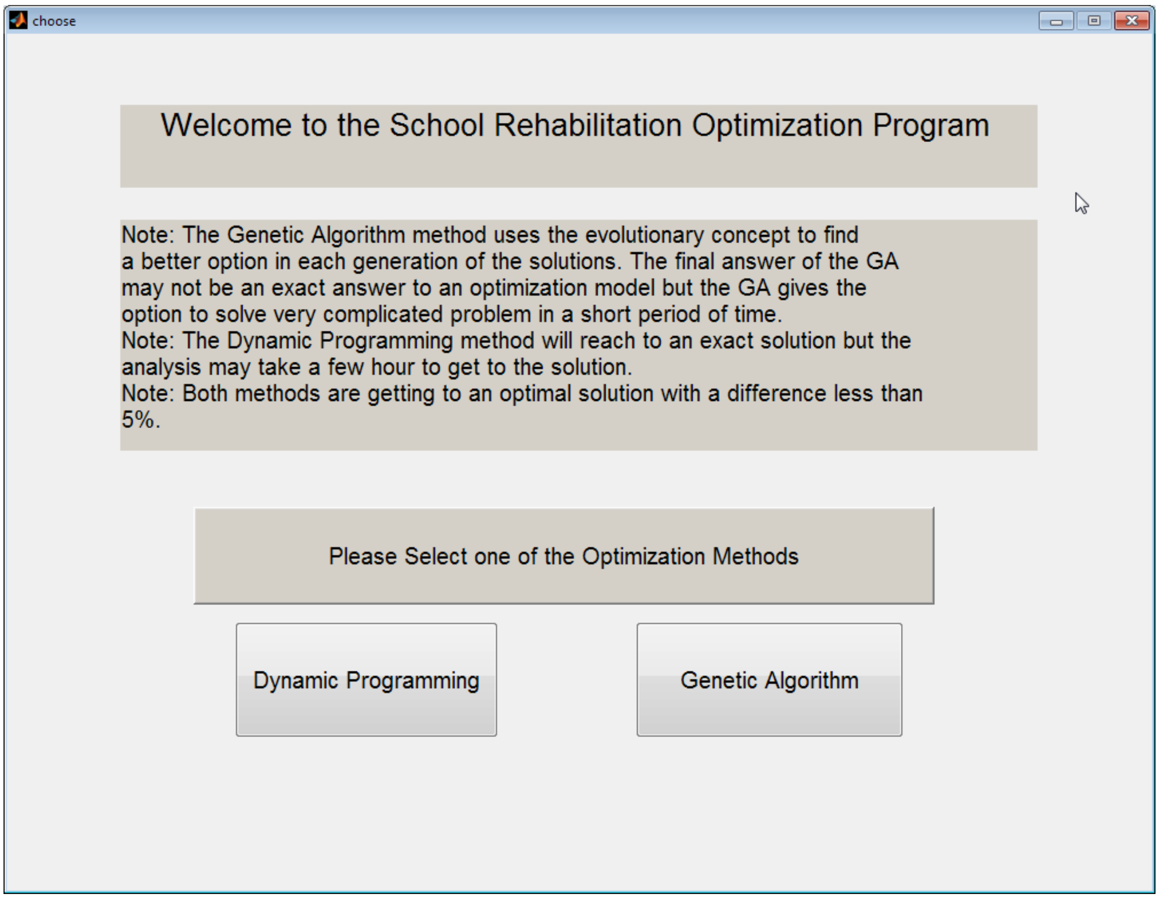


FIGURE 8. Select the optimization method.

When the user selects the optimization method another screen will open automatically and asks for the importance coefficients of English Language Learner students, Low-Income Family students, the total number of schools in the school district, and the amount of available budget in the school district for funding the projects (See FIGURE 9).

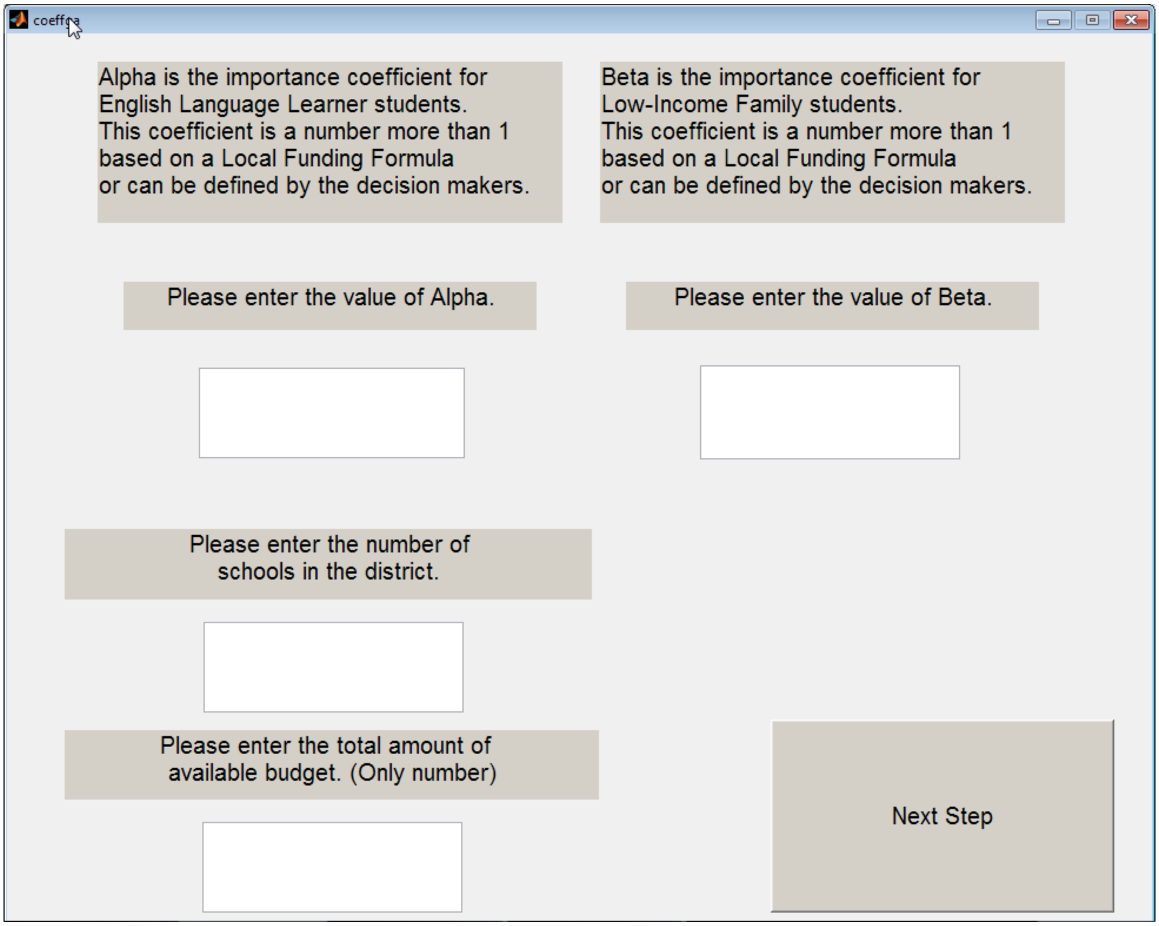


FIGURE 9. Students' importance coefficients, number of schools, available budget.

After the user enters the values and pushes the next step button, another screen will pop up and with a short description of input format, prompt the user to upload a text file with 9 columns (See FIGURE 10).

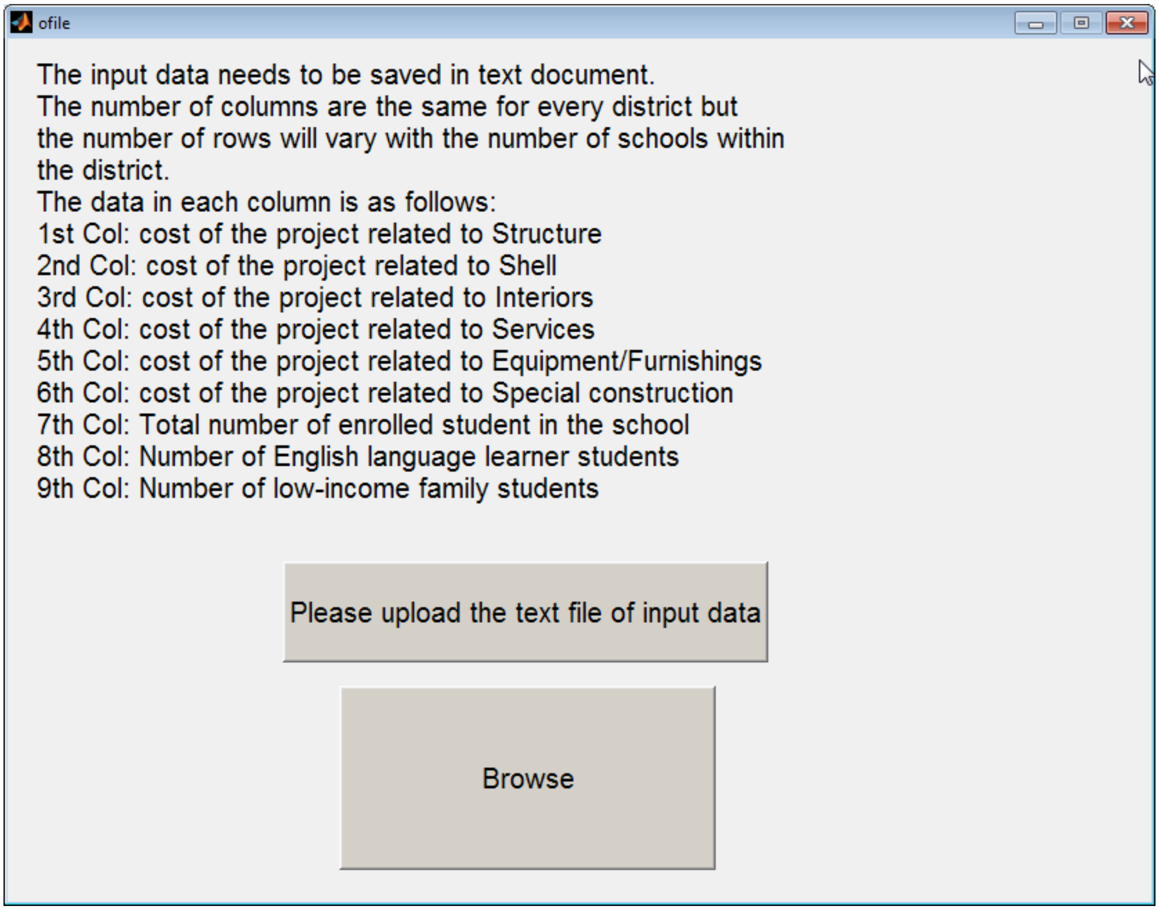


FIGURE 10. Upload the input data.

At the final step, in a new screen the user needs to hit the run button to start the optimization analysis (See FIGURE 11). The optimal solution of the program will be shown in the MATLAB workspace and can be exported to any spread sheet file.

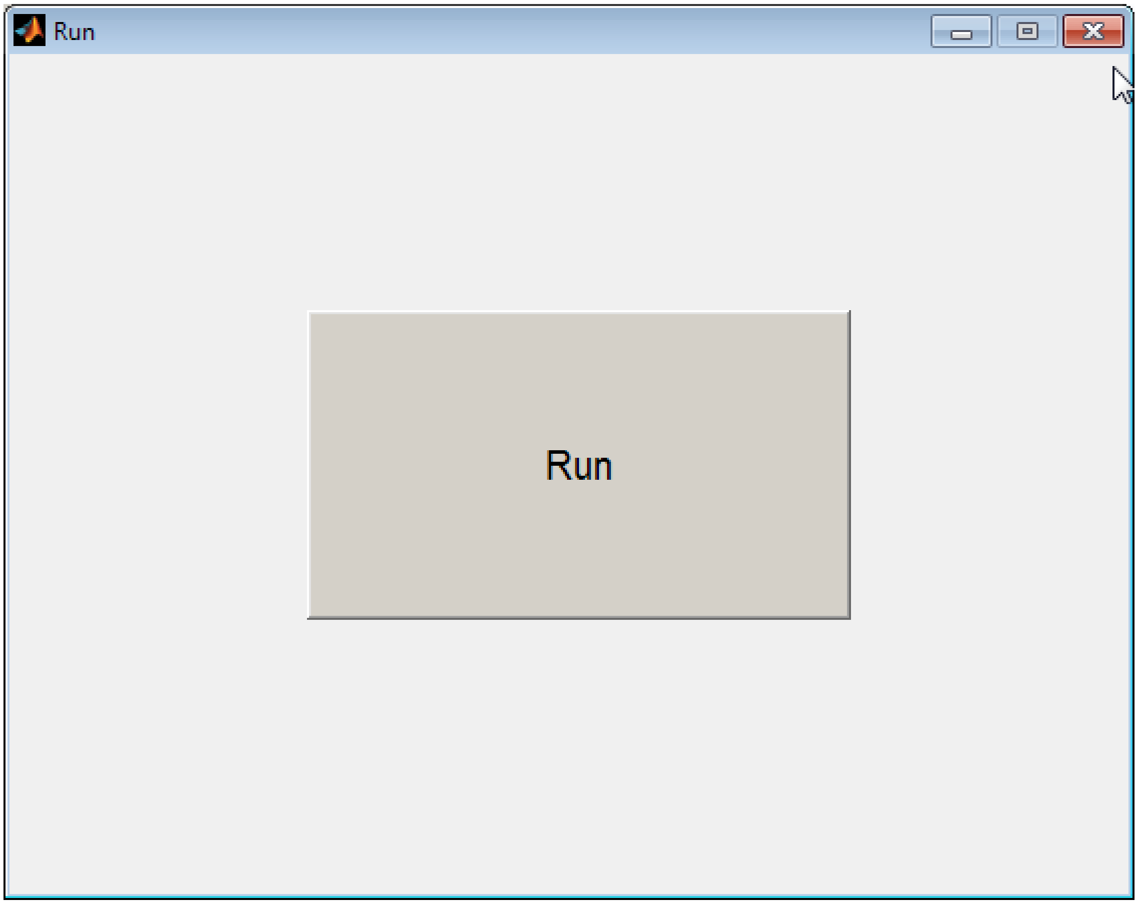


FIGURE 11. Run the program.

CHAPTER 4

CASE STUDY

In order to illustrate the efficiency and applicability of the developed model in the real world cases, a case study is conducted to optimize schools' facility rehabilitation projects in Long Beach Unified School District (LBUSD). Since the effectiveness of the model is better shown in larger school districts with a greater number of schools and facility projects, Long Beach Unified School District, which rules over a total number of 87 schools is selected. It is thought-provoking that even in 2007, LBUSD held community dialog and performed a survey to find out the community opinion on important criteria in prioritizing school facility projects. The criteria were used in the questionnaire includes but not limited to the age of the buildings, number of students, condition of the buildings, and new schools to address overcrowding. The result of the survey shows that 46% of the voters chose the condition of the building as the most important criteria in prioritization and 21% of the voters vote for number of students study in the school (Long Beach Unified School District 2008).

The definition and classification of scope of work in modernization and rehabilitation projects in LBUSD varies to some extent from the categories have been defined in this study. Therefore, minor modification is applied on the gathered data from the school district to reclassify the tasks and their associated costs.

In LBUSD facility master plan (2008), projects are classified in five different categories as follows:

1. New Building
2. Major Renovation
3. Moderate Renovation
4. Minor renovation
5. General maintenance (Long Beach Unified School District 2008)

Since the concern of this study is to make an optimization model for rehabilitation projects, construction of new facilities is not considered in the case study. Minor renovations include upgrading of some portion of the building with either repair or replace of flooring, ceiling, lighting, electrical upgrades, and painting. Moderate and Major renovation projects aim to upgrade the facility to meet the code requirements and future educational program demands. Although the extent of repairs in moderate renovation is narrower than major renovation, they both include the tasks such as handicapped accessibility, heating/ventilation/ air conditioning, roof, electrical, windows, flooring, ceiling, lighting, technology, infrastructure, and signal system.

The Long Beach Unified School District (LBUSD) facility master plan (2008) shows that about 74% of the schools within the district are subjected to a major or moderate renovation because of the condition of the existing buildings. The percentage of each class of facility master plan in LBUSD is shown in FIGURE 12.

The items which are typically included in LBUSD facility renovation and maintenance program and the items which are not included are listed as it is shown in TABLE 8.

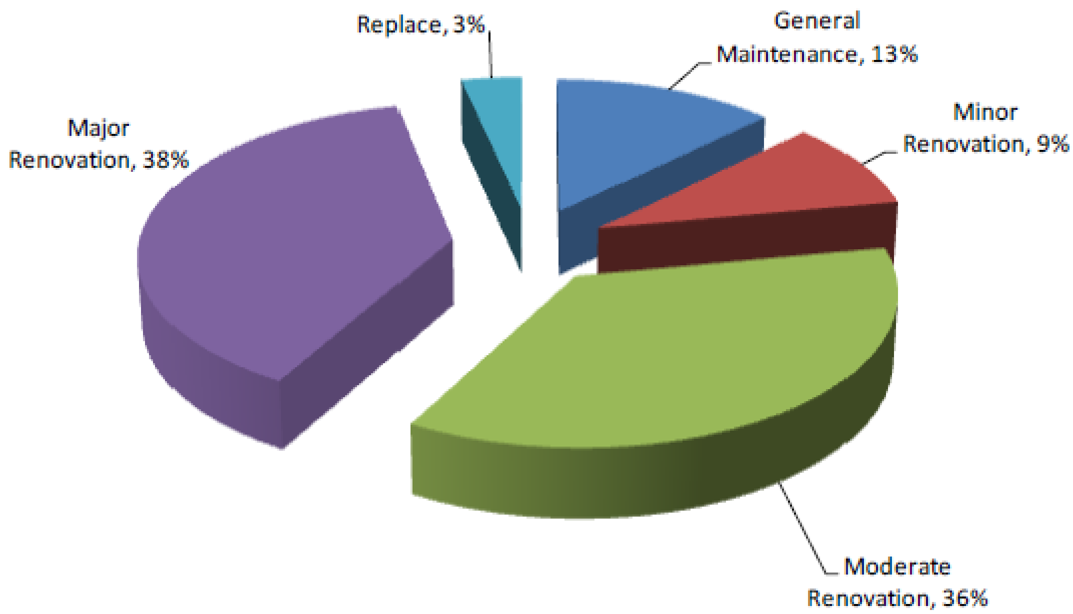


FIGURE 12. LBUSD facility projects break down (LBUSD 2008).

The data for facility modernization and maintenance projects in LBUSD is gathered from LBUSD Department of Facilities Development & Planning, the scope of work for each project is modified to conform to the 6 defined categories in this study.

From the data gathered from LBUSD on the current, complete and incomplete renovation and modernization projects the LBUSD renovation items are reclassified to the six categories as it is shown in TABLE 9.

TABLE 8. Included and Not Included Items in LBUSD Facility Master Plan (2008)

Items Typically Included	Items are not included
Roofing	Demolition/Portable Removal
Exterior - Walls	Hazardous Material
Exterior - Windows	Excessive Site Work
Exterior - Doors	Structural / Seismic
Interior Floors	Site Purchases
Interior Walls	Interim Housing: Swing Space
Interior Ceilings	Swimming Pools
Air Conditioning	Synthetic Turfs
HVAC	All Weather Tracks
Electrical - Lighting	Stadium Seating
Electrical - Distribution	Other Physical Ed Outdoor Facilities
Plumbing	Inflation
Fire / Life Safety	
Technology	
ADA: Accessibility	
Site work - Parking & Playfields	

TABLE 9. Projects' Scope Reclassification for LBUSD

Optimization Model Categories	LBUSD Renovation Projects' Categories
Structure	Roofing, Exterior – Walls
Shell	Interior Walls, Interior Ceilings
Interiors	Interior Floors, Exterior – Doors, Exterior – Windows
Services	Air Conditioning, HVAC, Electrical – Lighting, Electrical – Distribution, Plumbing, Fire / Life Safety
Equipment and Furnishes	Technology
Special Construction	Site work - Parking & Playfields, ADA Accessibility

Based on the student demographic data from LBUSD, the percentage of English Language Learners and students from Low-Income Family in this district is 69.54 %. Therefore, the importance coefficients for adequacy will be calculated for this district as follows:

$$\alpha = \beta = 1 + 0.2 \times 0.6954 + 0.5 \times (0.6954 - 0.55) = 1.21185$$

The parameters in the objective function of the model will be:

$$[N_{Imp}] = [0.4646, 0.2035, 0.0976, 0.1373, 0.0360, 0.0611]$$

$$[Eq_h] = \frac{[Number\ of\ Students]}{[\$Str., \$Shell,\ \$Inter., \$Serv, \$Equ/Furn, \$Special\ Const.]}$$

$$[Eq_{v-ELL}] = \frac{[Number\ of\ English\ language\ learner\ students]}{[\$Str, \$Shell,\ \$Inter., \$Serv., \$Equ/Furn, \$Special\ Const.]}$$

$$[Eq_{v-LI}] = \frac{[Number\ of\ low-income\ family\ students]}{[\$Str, \$Shell,\ \$Inter., \$Serv., \$Equ/Furn, \$Special\ Const.]}$$

Therefore, the objective function of the model is:

$$O = \{[N_{Imp}]Eq_h\} + \{1.21185 \times [N_{Imp}][Eq_{v-ELL}]\} + \{1.21185 \times [N_{Imp}][Eq_{v-LI}]\}$$

The optimization problem is solved with consideration of a limited amount of budget defined by user, both with Genetic Algorithm and Dynamic Programming, and a comparison between the results of these two algorithms is conducted at the end.

The modified data for modernization and maintenance projects in Long Beach Unified School District are illustrated in TABLE 10. The cost of the modernization project in each six categories has been calculated using the district raw data for fiscal year of 2013-2014 and dollar amount of each category is rounded up to the nearest thousand. Although there are some completed projects from the last fiscal year, many modernization projects are incomplete and the actual cost of the project has not been determined yet. Student demographic information for each school is shown in the last three columns of the TABLE 10.

TABLE 10. LBUSD Facility Projects Costs and Student Demographic Data

Long Beach Unified School District										
No.	School	Structure	Shell	Interiors	Services	Equipment / Furnishings	Special Construction	Total Number of Students	Total number of ELL Students	Total Number of Socioeconomic Disadvantaged Students
1	Addams Elementary	$140,000	$120,000	$20,000	$190,000	$110,000	$20,000	995	487	945
2	Alvarado Elementary	$360,000	$70,000	$70,000	$310,000	$70,000	$140,000	412	128	356
3	Avalon K-12	$130,000	$80,000	$150,000	$100,000	$130,000	$130,000	636	266	464
4	Bancroft Middle	$100,000	$150,000	$70,000	$290,000	$70,000	$120,000	1,036	52	478
5	Barton Elementary	$30,000	$30,000	$20,000	$230,000	$110,000	$150,000	627	128	554
6	Beach High-Intensive Learning Program	$350,000	$180,000	$30,000	$220,000	$110,000	$170,000	343	120	281
7	Birney Elementary	$90,000	$170,000	$170,000	$210,000	$90,000	$20,000	683	164	519

TABLE 10. Continued

No.	School	Structure	Shell	Interiors	Services	Equipment / Furnishings	Special Construction	Total Number of Students	Total number of ELL Students	Total Number of Socioeconomic Disadvantaged Students
8	Bixby Elementary	$140,000	$180,000	$20,000	$220,000	$110,000	$130,000	544	57	292
9	Bryant Elementary	$360,000	$30,000	$180,000	$240,000	$50,000	$180,000	366	71	309
10	Burbank Elementary	$210,000	$25,000	$60,000	$110,000	$100,000	$180,000	820	418	761
11	Burcham K-8	$180,000	$20,000	$25,000	$100,000	$100,000	$150,000	616	146	400
12	Burnett Elementary	$350,000	$90,000	$170,000	$110,000	$130,000	$20,000	732	369	693
13	Cabrillo High	$250,000	$140,000	$130,000	$110,000	$70,000	$180,000	2,959	730	2,453
14	California Academy Of Mathematics And Science	$100,000	$20,000	$110,000	$300,000	$90,000	$180,000	677	20	337
15	Carver Elementary	$110,000	$110,000	$20,000	$290,000	$90,000	$180,000	458	24	113

TABLE 10. Continued

No.	School	Structure	Shell	Interiors	Services	Equipment / Furnishings	Special Construction	Total Number of Students	Total number of ELL Students	Total Number of Socioeconomic Disadvantaged Students
16	Chavez Elementary	$260,000	$20,000	$160,000	$250,000	$90,000	$170,000	494	242	465
17	Cleveland Elementary	$70,000	$140,000	$180,000	$110,000	$100,000	$130,000	598	65	238
18	Cubberley K-8	$90,000	$60,000	$40,000	$260,000	$90,000	$180,000	1,014	68	282
19	District Office	$190,000	$30,000	$120,000	$110,000	$90,000	$30,000	45	12	21
20	Dooley Elementary	$290,000	$110,000	$130,000	$275,000	$50,000	$90,000	1,126	426	1,051
21	Edison Elementary	$220,000	$40,000	$20,000	$320,000	$60,000	$90,000	725	444	690
22	Educational Partnership High	$380,000	$25,000	$25,000	$110,000	$100,000	$140,000	1,022	276	618
23	Emerson Parkside Academy	$240,000	$100,000	$190,000	$190,000	$70,000	$20,000	683	47	199
24	Ernest S. Mcbride, Sr. High	$330,000	$30,000	$170,000	$320,000	$70,000	$130,000	204	8	101

TABLE 10. Continued

No.	School	Structure	Shell	Interiors	Services	Equipment / Furnishings	Special Construction	Total Number of Students	Total number of ELL Students	Total Number of Socioeconomic Disadvantaged Students
25	Franklin Classical Middle	$340,000	$80,000	$20,000	$110,000	$100,000	$170,000	1,122	394	1,063
26	Fremont Elementary	$340,000	$25,000	$90,000	$100,000	$130,000	$60,000	512	30	140
27	Gant Elementary	$170,000	$20,000	$120,000	$110,000	$100,000	$160,000	640	23	151
28	Garfield Elementary	$380,000	$190,000	$70,000	$70,000	$90,000	$30,000	821	392	774
29	Gompers K-8	$330,000	$30,000	$190,000	$190,000	$90,000	$180,000	773	61	422
30	Grant Elementary	$110,000	$40,000	$20,000	$300,000	$80,000	$25,000	1,074	508	936
31	Hamilton Middle	$370,000	$70,000	$40,000	$100,000	$110,000	$20,000	927	267	839
32	Harte Elementary	$350,000	$160,000	$50,000	$240,000	$130,000	$190,000	959	355	858
33	Henry K-8	$360,000	$130,000	$170,000	$100,000	$90,000	$140,000	798	106	267

TABLE 10. Continued

No.	School	Structure	Shell	Interiors	Services	Equipment / Furnishings	Special Construction	Total Number of Students	Total number of ELL Students	Total Number of Socioeconomic Disadvantaged Students
34	Hill Classical Middle	$110,000	$25,000	$170,000	$230,000	$80,000	$120,000	783	145	652
35	Holmes Elementary	$370,000	$130,000	$80,000	$70,000	$80,000	$70,000	462	81	331
36	Hoover Middle	$300,000	$110,000	$130,000	$140,000	$10,000	$110,000	860	71	569
37	Hudson K-8	$110,000	$30,000	$110,000	$100,000	$100,000	$30,000	794	195	652
38	Hughes Middle	$380,000	$140,000	$100,000	$180,000	$90,000	$20,000	1,560	125	913
39	Intellectual Virtues Academy Of Long Beach	$60,000	$10,000	$100,000	$110,000	$130,000	$170,000	56	0	9
40	International Elementary	$380,000	$180,000	$190,000	$110,000	$100,000	$140,000	817	448	783
41	Jefferson Leadership Academies	$110,000	$190,000	$120,000	$260,000	$130,000	$110,000	973	273	885

TABLE 10. Continued

No.	School	Structure	Shell	Interiors	Services	Equipment / Furnishings	Special Construction	Total Number of Students	Total number of ELL Students	Total Number of Socioeconomic Disadvantaged Students
41	Jefferson Leadership Academies	$110,000	$190,000	$120,000	$260,000	$130,000	$110,000	973	273	885
42	Jessie Nelson Academy	$100,000	$180,000	$50,000	$240,000	$80,000	$190,000	837	207	768
43	Jordan High	$400,000	$190,000	$150,000	$360,000	$80,000	$130,000	3,481	781	2,866
44	Kettering Elementary	$410,000	$180,000	$130,000	$360,000	$90,000	$190,000	356	30	125
45	King Elementary	$170,000	$210,000	$60,000	$260,000	$80,000	$210,000	837	426	789
46	Lafayette Elementary	$390,000	$80,000	$30,000	$160,000	$10,000	$210,000	961	422	767
47	Lakewood High	$420,000	$60,000	$210,000	$270,000	$90,000	$190,000	3,693	197	1,918
48	Lee Elementary	$370,000	$170,000	$90,000	$360,000	$80,000	$190,000	972	540	934
49	Lincoln Elementary	$420,000	$210,000	$180,000	$170,000	$90,000	$210,000	1,146	711	1,074

TABLE 10. Continued

No.	School	Structure	Shell	Interiors	Services	Equipment / Furnishings	Special Construction	Total Number of Students	Total number of ELL Students	Total Number of Socioeconomic Disadvantaged Students
50	Lindbergh Middle	$270,000	$90,000	$140,000	$160,000	$20,000	$190,000	635	139	588
51	Lindsey Academy	$350,000	$180,000	$90,000	$260,000	$80,000	$210,000	872	207	801
52	Longfellow Elementary	$300,000	$140,000	$200,000	$350,000	$90,000	$140,000	1,091	113	491
53	Los Cerritos Elementary	$430,000	$130,000	$60,000	$90,000	$90,000	$30,000	565	40	267
54	Lowell Elementary	$250,000	$210,000	$140,000	$360,000	$90,000	$190,000	749	27	140
55	Macarthur Elementary	$190,000	$20,000	$150,000	$360,000	$80,000	$210,000	397	41	209
56	Madison Elementary	$380,000	$200,000	$40,000	$100,000	$70,000	$25,000	511	32	255
57	Mann Elementary	$170,000	$30,000	$25,000	$360,000	$80,000	$210,000	328	73	283

TABLE 10. Continued

No.	School	Structure	Shell	Interiors	Services	Equipment / Furnishings	Special Construction	Total Number of Students	Total number of ELL Students	Total Number of Socioeconomic Disadvantaged Students
58	Marshall Academy Of The Arts	$400,000	$80,000	$100,000	$130,000	$100,000	$30,000	916	125	540
59	Mckinley Elementary	$110,000	$200,000	$20,000	$160,000	$80,000	$170,000	710	330	668
60	Millikan High	$340,000	$210,000	$150,000	$80,000	$90,000	$140,000	3,954	427	2,078
61	Muir K-8	$540,000	$140,000	$90,000	$100,000	$60,000	$210,000	1,097	359	925
62	Naples Elementary	$70,000	$190,000	$170,000	$100,000	$60,000	$250,000	365	14	31
63	New City	$200,000	$180,000	$160,000	$190,000	$70,000	$260,000	428	153	362
64	Newcomb Academy	$150,000	$140,000	$250,000	$460,000	$110,000	$180,000	980	46	288
65	District Non-Public Non-Sectarian Schools	$100,000	$30,000	$140,000	$40,000	$70,000	$280,000	199	20	63

TABLE 10. Continued

No.	School	Structure	Shell	Interiors	Services	Equipment / Furnishings	Special Construction	Total Number of Students	Total number of ELL Students	Total Number of Socioeconomic Disadvantaged Students
66	Polytechnic High	$370,000	$60,000	$90,000	$460,000	$70,000	$280,000	4,497	602	2,678
67	Powell Academy For Success	$90,000	$290,000	$25,000	$410,000	$60,000	$290,000	1,328	549	1,216
68	Prisk Elementary	$560,000	$90,000	$110,000	$90,000	$60,000	$20,000	636	42	218
69	Reid High	$70,000	$260,000	$190,000	$420,000	$80,000	$180,000	297	93	236
70	Renaissance High School For The Arts	$550,000	$270,000	$260,000	$450,000	$30,000	$150,000	500	10	282
71	Riley Elementary	$100,000	$80,000	$50,000	$300,000	$110,000	$270,000	439	70	282
72	Robinson Academy	$70,000	$100,000	$30,000	$230,000	$110,000	$280,000	971	397	906
73	Rogers Middle	$250,000	$220,000	$270,000	$90,000	$80,000	$170,000	893	25	229
74	Roosevelt Elementary	$240,000	$120,000	$290,000	$460,000	$70,000	$190,000	1,034	711	945

TABLE 10. Continued

No.	School	Structure	Shell	Interiors	Services	Equipment / Furnishings	Special Construction	Total Number of Students	Total number of ELL Students	Total Number of Socioeconomic Disadvantaged Students
75	Select Community Day (Secondary)	$570,000	$30,000	$90,000	$210,000	$110,000	$290,000	8	2	6
76	Signal Hill Elementary	$540,000	$230,000	$250,000	$240,000	$120,000	$20,000	737	272	677
77	Stanford Middle	$250,000	$270,000	$250,000	$410,000	$100,000	$220,000	1,291	64	464
78	Stephens Middle	$100,000	$280,000	$180,000	$430,000	$90,000	$270,000	805	218	751
79	Stevenson Elementary	$80,000	$30,000	$200,000	$110,000	$60,000	$70,000	794	373	753
80	Tincher Preparatory	$410,000	$20,000	$30,000	$70,000	$90,000	$30,000	930	48	451
81	Twain Elementary	$110,000	$30,000	$290,000	$450,000	$70,000	$90,000	618	41	281
82	Two Harbors Elementary	$570,000	$50,000	$110,000	$320,000	$90,000	$180,000	4	0	0

TABLE 10. Continued

No.	School	Structure	Shell	Interiors	Services	Equipment / Furnishings	Special Construction	Total Number of Students	Total number of ELL Students	Total Number of Socioeconomic Disadvantaged Students
83	Washington Middle	$80,000	$280,000	$200,000	$70,000	$70,000	$30,000	1,107	380	1,068
84	Webster Elementary	$140,000	$270,000	$20,000	$100,000	$110,000	$20,000	643	221	594
85	Whittier Elementary	$440,000	$30,000	$25,000	$110,000	$80,000	$240,000	914	518	780
86	Willard Elementary	$380,000	$280,000	$270,000	$310,000	$110,000	$70,000	816	420	763
87	Wilson High	$130,000	$250,000	$20,000	$460,000	$110,000	$180,000	4,067	519	2,317

The total amount of budget which is needed to complete all the projects is about US $81 million. The assumption for available budget in this case study is US $40 million. The optimal selection of the Genetic Algorithm model is interpreted to the cost of each category in each school, and it is shown in TABLE 11**.**

TABLE 11. Genetic Algorithm Solution

Long Beach Unified School District (GA Solution)							
No.	School	Structure	Shell	Interiors	Services	Equipment / Furnishings	Special Construction
1	Addams Elementary	$140,000	$120,000	$20,000	$190,000	$110,000	$20,000
2	Alvarado Elementary	$0	$70,000	$70,000	$0	$0	$0
3	Avalon K-12	$130,000	$80,000	$150,000	$100,000	$0	$130,000
4	Bancroft Middle	$100,000	$150,000	$70,000	$0	$70,000	$120,000
5	Barton Elementary	$30,000	$30,000	$20,000	$0	$0	$0
6	Beach High-Intensive Learning Program	$0	$0	$30,000	$0	$0	$0
7	Birney Elementary	$90,000	$170,000	$170,000	$0	$0	$20,000
8	Bixby Elementary	$140,000	$0	$20,000	$0	$0	$0

TABLE 11. Continued

No.	School	Structure	Shell	Interiors	Services	Equipment / Furnishings	Special Construction
9	Bryant Elementary	$0	$30,000	$0	$0	$50,000	$0
10	Burbank Elementary	$210,000	$25,000	$60,000	$110,000	$100,000	$0
11	Burcham K-8	$180,000	$20,000	$25,000	$100,000	$0	$0
12	Burnett Elementary	$350,000	$90,000	$170,000	$110,000	$0	$20,000
13	Cabrillo High	$250,000	$140,000	$130,000	$110,000	$70,000	$180,000
14	California Academy Of Mathematics And Science	$100,000	$20,000	$0	$0	$90,000	$0
15	Carver Elementary	$110,000	$110,000	$20,000	$0	$0	$0
16	Chavez Elementary	$260,000	$20,000	$160,000	$0	$0	$0
17	Cleveland Elementary	$70,000	$140,000	$0	$110,000	$0	$0
18	Cubberley K-8	$90,000	$60,000	$40,000	$0	$0	$0
19	District Office	$0	$30,000	$0	$0	$0	$0
20	Dooley Elementary	$290,000	$110,000	$130,000	$0	$50,000	$90,000
21	Edison Elementary	$220,000	$40,000	$20,000	$0	$60,000	$90,000

TABLE 11. Continued

No.	School	Structure	Shell	Interiors	Services	Equipment / Furnishings	Special Construction
22	Educational Partnership High	$0	$25,000	$25,000	$110,000	$0	$0
23	Emerson Parkside Academy	$240,000	$100,000	$0	$0	$0	$20,000
24	Ernest S. Mcbride, Sr. High	$0	$30,000	$0	$0	$0	$0
25	Franklin Classical Middle	$340,000	$80,000	$20,000	$110,000	$100,000	$170,000
26	Fremont Elementary	$0	$25,000	$90,000	$100,000	$0	$60,000
27	Gant Elementary	$170,000	$20,000	$120,000	$110,000	$0	$0
28	Garfield Elementary	$380,000	$190,000	$70,000	$70,000	$90,000	$30,000
29	Gompers K-8	$330,000	$30,000	$0	$0	$0	$0
30	Grant Elementary	$110,000	$40,000	$20,000	$0	$80,000	$25,000
31	Hamilton Middle	$370,000	$70,000	$40,000	$100,000	$110,000	$20,000
32	Harte Elementary	$350,000	$160,000	$50,000	$240,000	$0	$0
33	Henry K-8	$0	$130,000	$170,000	$100,000	$0	$140,000

TABLE 11. Continued

No.	School	Structure	Shell	Interiors	Services	Equipment / Furnishings	Special Construction
34	Hill Classical Middle	$110,000	$25,000	$170,000	$0	$80,000	$120,000
35	Holmes Elementary	$0	$130,000	$80,000	$70,000	$0	$70,000
36	Hoover Middle	$300,000	$110,000	$130,000	$140,000	$10,000	$110,000
37	Hudson K-8	$110,000	$30,000	$110,000	$100,000	$100,000	$30,000
38	Hughes Middle	$380,000	$140,000	$100,000	$180,000	$90,000	$20,000
39	Intellectual Virtues Academy Of Long Beach	$60,000	$10,000	$0	$0	$0	$0
40	International Elementary	$380,000	$180,000	$190,000	$110,000	$100,000	$0
41	Jefferson Leadership Academies	$110,000	$190,000	$120,000	$0	$0	$110,000
42	Jessie Nelson Academy	$100,000	$180,000	$50,000	$240,000	$80,000	$0
43	Jordan High	$400,000	$190,000	$150,000	$360,000	$80,000	$130,000
44	Kettering Elementary	$0	$0	$0	$0	$0	$0
45	King Elementary	$170,000	$210,000	$60,000	$0	$80,000	$0

TABLE 11. Continued

No.	School	Structure	Shell	Interiors	Services	Equipment / Furnishings	Special Construction
46	Lafayette Elementary	$390,000	$80,000	$30,000	$160,000	$10,000	$0
47	Lakewood High	$420,000	$60,000	$210,000	$270,000	$90,000	$190,000
48	Lee Elementary	$370,000	$170,000	$90,000	$0	$80,000	$0
49	Lincoln Elementary	$420,000	$210,000	$180,000	$170,000	$90,000	$0
50	Lindbergh Middle	$270,000	$90,000	$140,000	$160,000	$20,000	$0
51	Lindsey Academy	$350,000	$180,000	$90,000	$0	$80,000	$0
52	Longfellow Elementary	$300,000	$140,000	$0	$0	$0	$0
53	Los Cerritos Elementary	$0	$130,000	$60,000	$90,000	$0	$30,000
54	Lowell Elementary	$250,000	$0	$140,000	$0	$0	$0
55	Macarthur Elementary	$190,000	$20,000	$0	$0	$0	$0
56	Madison Elementary	$0	$0	$40,000	$100,000	$0	$25,000
57	Mann Elementary	$170,000	$30,000	$25,000	$0	$0	$0

TABLE 11. Continued

No.	School	Structure	Shell	Interiors	Services	Equipment / Furnishings	Special Construction
58	Marshall Academy Of The Arts	$0	$80,000	$100,000	$130,000	$0	$30,000
59	Mckinley Elementary	$110,000	$200,000	$20,000	$160,000	$80,000	$0
60	Millikan High	$340,000	$210,000	$150,000	$80,000	$90,000	$140,000
61	Muir K-8	$0	$140,000	$90,000	$100,000	$60,000	$0
62	Naples Elementary	$70,000	$0	$0	$0	$0	$0
63	New City	$200,000	$0	$0	$0	$70,000	$0
64	Newcomb Academy	$150,000	$140,000	$0	$0	$0	$0
65	District Non-Public Non-Sectarian Schools	$100,000	$30,000	$0	$40,000	$0	$0
66	Polytechnic High	$370,000	$60,000	$90,000	$460,000	$70,000	$280,000
67	Powell Academy For Success	$90,000	$290,000	$25,000	$0	$60,000	$0
68	Prisk Elementary	$0	$90,000	$110,000	$90,000	$60,000	$20,000
69	Reid High	$70,000	$0	$0	$0	$0	$0

TABLE 11. Continued

No.	School	Structure	Shell	Interiors	Services	Equipment / Furnishings	Special Construction
70	Renaissance High School For The Arts	$0	$0	$0	$0	$30,000	$0
71	Riley Elementary	$100,000	$80,000	$50,000	$0	$0	$0
72	Robinson Academy	$70,000	$100,000	$30,000	$0	$110,000	$0
73	Rogers Middle	$250,000	$0	$0	$90,000	$0	$0
74	Roosevelt Elementary	$240,000	$120,000	$0	$0	$70,000	$190,000
75	Select Community Day (Secondary)	$0	$0	$0	$0	$0	$0
76	Signal Hill Elementary	$0	$230,000	$0	$0	$0	$20,000
77	Stanford Middle	$250,000	$270,000	$0	$0	$100,000	$0
78	Stephens Middle	$100,000	$280,000	$0	$0	$90,000	$0
79	Stevenson Elementary	$80,000	$30,000	$0	$110,000	$60,000	$70,000
80	Tincher Preparatory	$0	$20,000	$30,000	$70,000	$0	$30,000
81	Twain Elementary	$110,000	$30,000	$0	$0	$0	$90,000

TABLE 11. Continued

No.	School	Structure	Shell	Interiors	Services	Equipment / Furnishings	Special Construction
82	Two Harbors Elementary	$0	$0	$0	$0	$0	$0
83	Washington Middle	$80,000	$280,000	$200,000	$70,000	$70,000	$30,000
84	Webster Elementary	$140,000	$270,000	$20,000	$100,000	$0	$20,000
85	Whittier Elementary	$440,000	$30,000	$25,000	$110,000	$80,000	$0
86	Willard Elementary	$380,000	$280,000	$0	$0	$110,000	$70,000
87	Wilson High	$130,000	$250,000	$20,000	$460,000	$110,000	$180,000

After 140 generations, Genetic Algorithm model for this optimization problem reaches the optimum solution. Although the defined number of generation is 1000, changes in the solutions after 140 generations is inconsiderable, and this generation could be picked as the optimal solution of the program (See FIGURE 13).

FIGURE 14 shows the histogram of the parents in GA solving process. This diagram illustrates the number of children reproduced based on the number of parents.

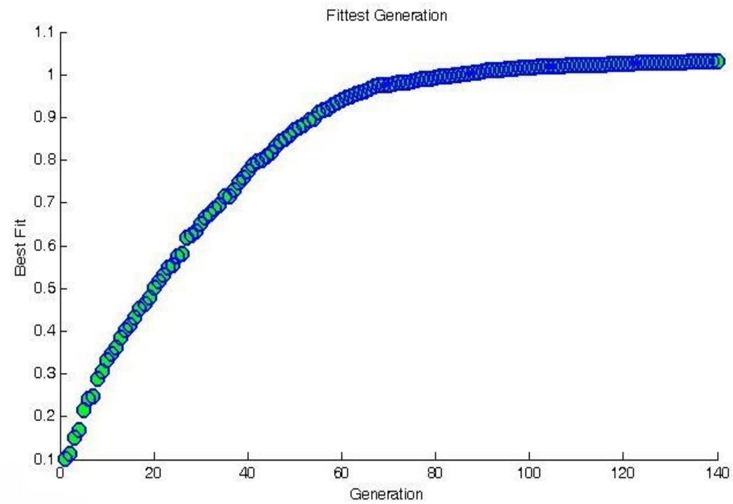


FIGURE 13. Best fit in generations.

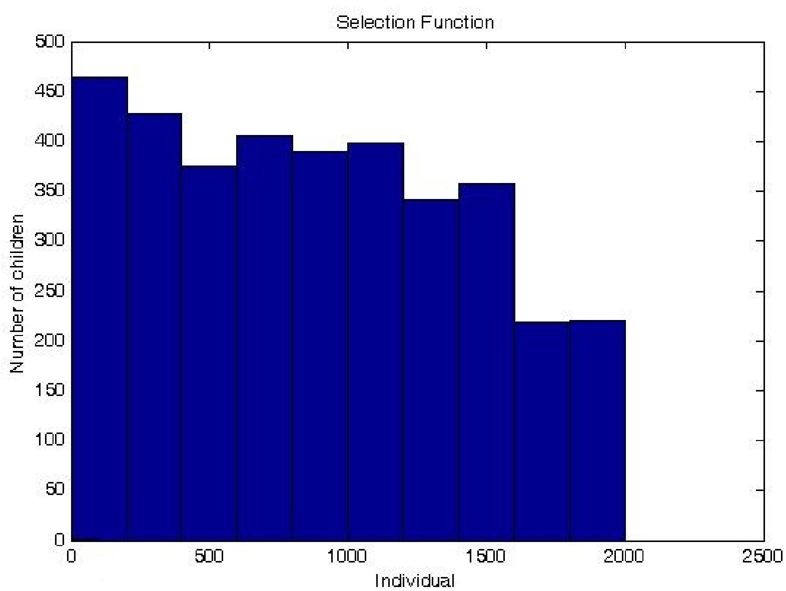


FIGURE 14. Histogram of the parents.

The average expenditure per all students, the average expenditure per ELL students, and the average expenditure per Low-Income Family students are shown in one graph in the FIGURE 15. The average expenditure is calculated by the sum of selected Projects cost in each school and then it is divided by the total number of the students and the number of ELL students and Low-Income Family students in the correspond school. The average expenditure per ELL students is significantly high in some schools as the low rate of these students in that school.

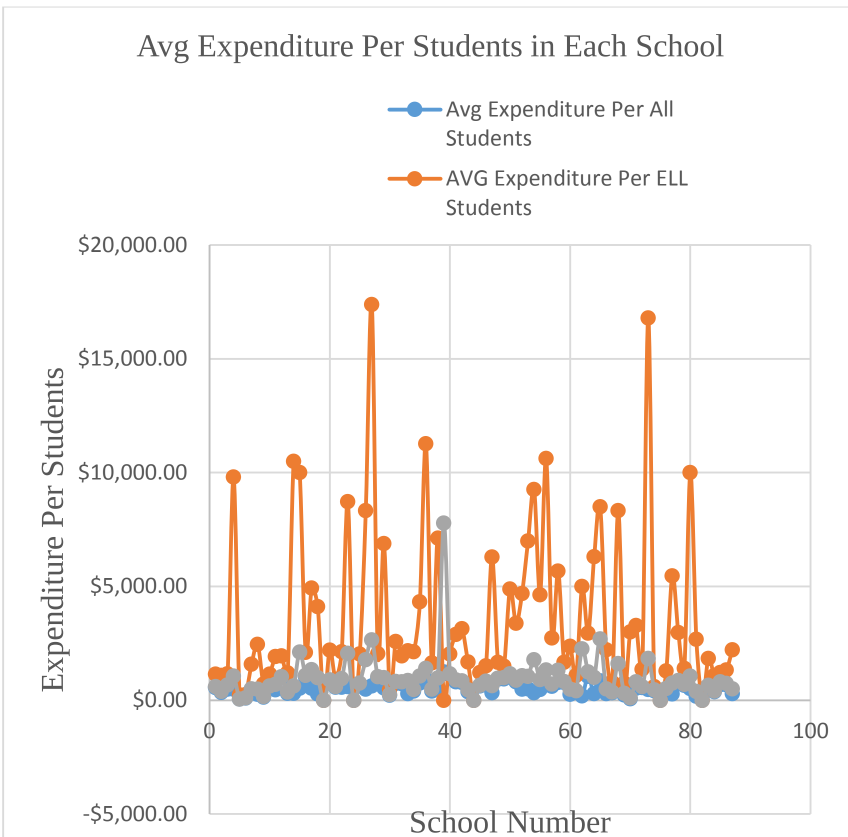


FIGURE 15. Average expenditure per students in each school in GA solution.

The optimal selection of the Dynamic Programming model is interpreted to the cost of each category in each school, and it is shown in the TABLE 12**.**

TABLE 12. Dynamic Programming Solution

Long Beach Unified School District (Dynamic Programming Solution)							
No.	School	Structure	Shell	Interiors	Services	Equipment / Furnishings	Special Construction
1	Addams Elementary	$140,000	$120,000	$20,000	$190,000	$0	$20,000
2	Alvarado Elementary	$360,000	$70,000	$70,000	$0	$0	$0
3	Avalon K-12	$130,000	$80,000	$0	$100,000	$0	$0
4	Bancroft Middle	$100,000	$150,000	$70,000	$0	$0	$0
5	Barton Elementary	$30,000	$30,000	$20,000	$0	$0	$0
6	Beach High-Intensive Learning Program	$0	$0	$30,000	$0	$0	$0
7	Birney Elementary	$90,000	$170,000	$0	$0	$0	$20,000
8	Bixby Elementary	$140,000	$0	$20,000	$0	$0	$0
9	Bryant Elementary	$0	$30,000	$0	$0	$0	$0
10	Burbank Elementary	$210,000	$25,000	$60,000	$110,000	$0	$0
11	Burcham K-8	$180,000	$20,000	$25,000	$100,000	$0	$0
12	Burnett Elementary	$350,000	$90,000	$170,000	$110,000	$0	$20,000
13	Cabrillo High	$250,000	$140,000	$130,000	$110,000	$70,000	$180,000
14	California Academy Of Mathematics And Science	$100,000	$20,000	$0	$0	$0	$0

TABLE 12. Continued

No.	School	Structure	Shell	Interiors	Services	Equipment / Furnishings	Special Construction
15	Carver Elementary	$110,000	$110,000	$20,000	$0	$0	$0
16	Chavez Elementary	$260,000	$20,000	$0	$0	$0	$0
17	Cleveland Elementary	$70,000	$140,000	$0	$110,000	$0	$0
18	Cubberley K-8	$90,000	$60,000	$40,000	$0	$0	$0
19	District Office	$0	$0	$0	$0	$0	$0
20	Dooley Elementary	$290,000	$110,000	$130,000	$275,000	$50,000	$90,000
21	Edison Elementary	$220,000	$40,000	$20,000	$0	$60,000	$90,000
22	Educational Partnership High	$380,000	$25,000	$25,000	$110,000	$0	$0
23	Emerson Parkside Academy	$240,000	$100,000	$0	$0	$0	$20,000
24	Ernest S. Mcbride, Sr. High	$0	$30,000	$0	$0	$0	$0
25	Franklin Classical Middle	$340,000	$80,000	$20,000	$110,000	$0	$0
26	Fremont Elementary	$0	$25,000	$0	$0	$0	$0
27	Gant Elementary	$170,000	$20,000	$0	$0	$0	$0
28	Garfield Elementary	$380,000	$190,000	$70,000	$70,000	$0	$30,000
29	Gompers K-8	$330,000	$30,000	$0	$0	$0	$0
30	Grant Elementary	$110,000	$40,000	$20,000	$300,000	$80,000	$25,000
31	Hamilton Middle	$370,000	$70,000	$40,000	$100,000	$0	$20,000
32	Harte Elementary	$350,000	$160,000	$50,000	$240,000	$0	$0
33	Henry K-8	$360,000	$130,000	$0	$100,000	$0	$0

TABLE 12. Continued

No.	School	Structure	Shell	Interiors	Services	Equipment / Furnishings	Special Construction
34	Hill Classical Middle	$110,000	$25,000	$0	$0	$0	$0
35	Holmes Elementary	$370,000	$130,000	$80,000	$70,000	$0	$0
36	Hoover Middle	$300,000	$110,000	$130,000	$140,000	$10,000	$0
37	Hudson K-8	$110,000	$30,000	$110,000	$100,000	$0	$30,000
38	Hughes Middle	$380,000	$140,000	$100,000	$180,000	$0	$20,000
39	Intellectual Virtues Academy Of Long Beach	$0	$0	$0	$0	$0	$0
40	International Elementary	$380,000	$180,000	$190,000	$110,000	$0	$0
41	Jefferson Leadership Academies	$110,000	$190,000	$120,000	$260,000	$0	$110,000
42	Jessie Nelson Academy	$100,000	$180,000	$50,000	$0	$0	$0
43	Jordan High	$400,000	$190,000	$150,000	$360,000	$80,000	$130,000
44	Kettering Elementary	$0	$0	$0	$0	$0	$0
45	King Elementary	$170,000	$210,000	$60,000	$260,000	$0	$0
46	Lafayette Elementary	$390,000	$80,000	$30,000	$160,000	$10,000	$0
47	Lakewood High	$420,000	$60,000	$210,000	$270,000	$90,000	$190,000
48	Lee Elementary	$370,000	$170,000	$90,000	$0	$80,000	$0
49	Lincoln Elementary	$420,000	$210,000	$180,000	$170,000	$90,000	$0
50	Lindbergh Middle	$270,000	$90,000	$0	$160,000	$20,000	$0
51	Lindsey Academy	$350,000	$180,000	$90,000	$0	$0	$0
52	Longfellow Elementary	$300,000	$140,000	$0	$0	$0	$0

TABLE 12. Continued

No.	School	Structure	Shell	Interiors	Services	Equipment / Furnishings	Special Construction
53	Los Cerritos Elementary	$0	$130,000	$60,000	$90,000	$0	$30,000
54	Lowell Elementary	$250,000	$0	$0	$0	$0	$0
55	Macarthur Elementary	$190,000	$20,000	$0	$0	$0	$0
56	Madison Elementary	$0	$0	$40,000	$100,000	$0	$25,000
57	Mann Elementary	$170,000	$30,000	$25,000	$0	$0	$0
58	Marshall Academy Of The Arts	$400,000	$80,000	$100,000	$130,000	$0	$30,000
59	Mckinley Elementary	$110,000	$200,000	$20,000	$160,000	$0	$0
60	Millikan High	$340,000	$210,000	$150,000	$80,000	$90,000	$140,000
61	Muir K-8	$540,000	$140,000	$90,000	$100,000	$60,000	$0
62	Naples Elementary	$70,000	$0	$0	$0	$0	$0
63	New City	$200,000	$180,000	$0	$0	$0	$0
64	Newcomb Academy	$150,000	$140,000	$0	$0	$0	$0
65	District Non-Public Non-Sectarian Schools	$100,000	$30,000	$0	$0	$0	$0
66	Polytechnic High	$370,000	$60,000	$90,000	$460,000	$70,000	$280,000
67	Powell Academy For Success	$90,000	$290,000	$25,000	$410,000	$60,000	$0
68	Prisk Elementary	$0	$90,000	$0	$90,000	$0	$20,000
69	Reid High	$70,000	$0	$0	$0	$0	$0
70	Renaissance High School For The Arts	$0	$0	$0	$0	$0	$0
71	Riley Elementary	$100,000	$80,000	$50,000	$0	$0	$0

TABLE 12. Continued

No.	School	Structure	Shell	Interiors	Services	Equipment / Furnishings	Special Construction
72	Robinson Academy	$70,000	$100,000	$30,000	$230,000	$0	$0
73	Rogers Middle	$250,000	$0	$0	$90,000	$0	$0
74	Roosevelt Elementary	$240,000	$120,000	$0	$0	$70,000	$0
75	Select Community Day (Secondary)	$0	$0	$0	$0	$0	$0
76	Signal Hill Elementary	$540,000	$230,000	$0	$0	$0	$20,000
77	Stanford Middle	$250,000	$270,000	$0	$0	$0	$0
78	Stephens Middle	$100,000	$280,000	$0	$0	$0	$0
79	Stevenson Elementary	$80,000	$30,000	$0	$110,000	$60,000	$70,000
80	Tincher Preparatory	$410,000	$20,000	$30,000	$70,000	$0	$30,000
81	Twain Elementary	$110,000	$30,000	$0	$0	$0	$0
82	Two Harbors Elementary	$0	$0	$0	$0	$0	$0
83	Washington Middle	$80,000	$280,000	$200,000	$70,000	$70,000	$30,000
84	Webster Elementary	$140,000	$270,000	$20,000	$100,000	$0	$20,000
85	Whittier Elementary	$440,000	$30,000	$25,000	$110,000	$0	$0
86	Willard Elementary	$380,000	$280,000	$0	$0	$0	$70,000
87	Wilson High	$130,000	$250,000	$20,000	$460,000	$110,000	$180,000

The average expenditure per all students, the average expenditure per ELL students, and the average expenditure per Low-Income Family students from the Dynamic Programming solution are shown in one graph in FIGURE 16.

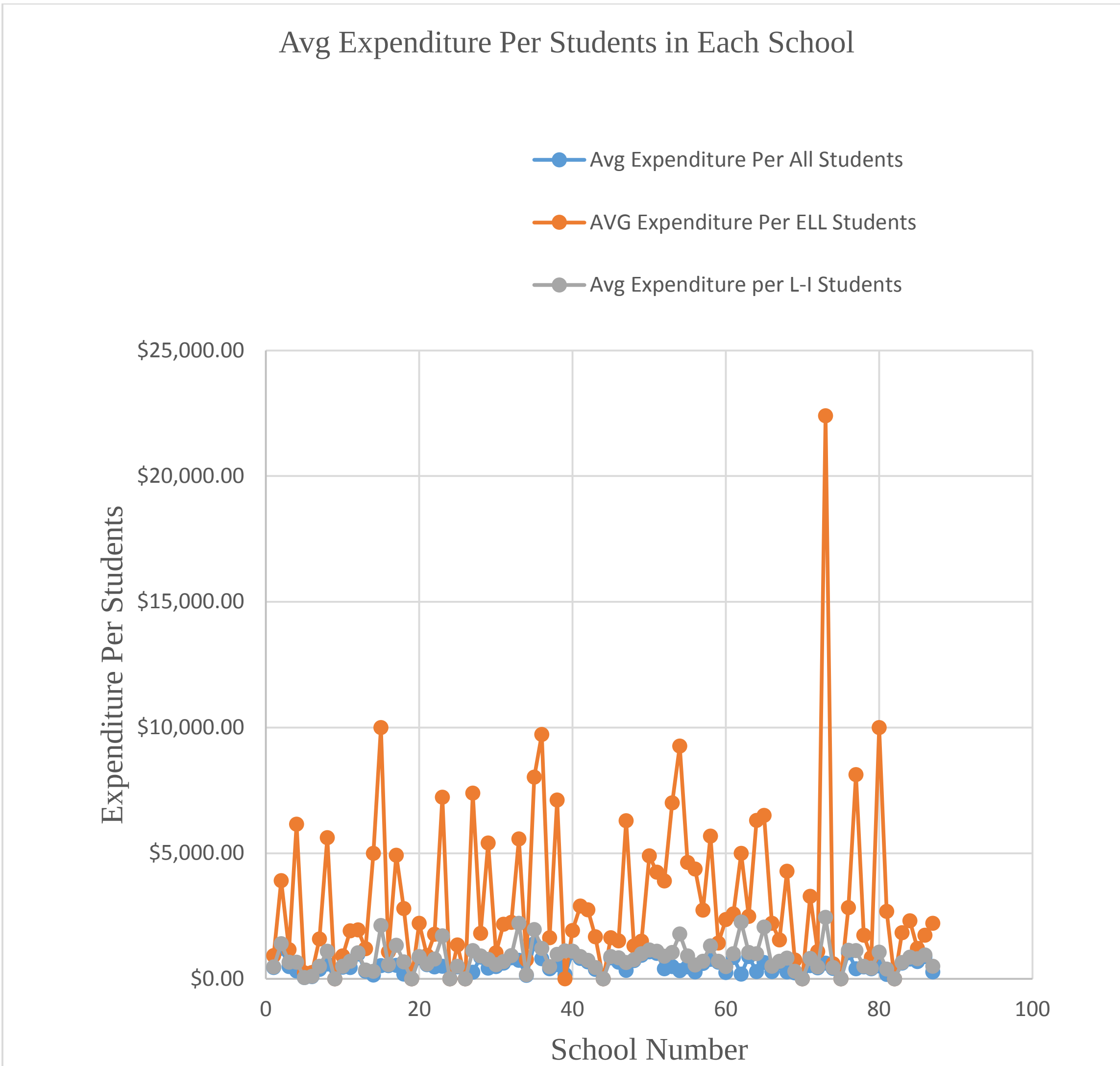


FIGURE 16. Average expenditure per students in each school in Dynamic Programming solution.

Comparison of the Two Methods

The sum of the selected projects cost from Genetic Algorithm solution is about US $39.9 million as GA will not provide an exact solution. The result of Dynamic Programming model uses the total available budget of US $40 by finding an exact solution to the problem. Both of the methods better maximize their fitness function on the output. The value of the fitness function on each method is calculated by the program and it is as follows:

1. Budget Used in Dynamic Programming: US $40000000
2. Budget Used in Genetic Algorithm: US $39995000
3. Genetic fitness function evaluated on Knapsack output 1.013477
4. Genetic fitness function evaluated on Genetic output 1.031773
5. Knapsack total value evaluated on Knapsack output 1.393728e+005
6. Knapsack total value evaluated on Genetic output 1.353071e+005

The result of both methods are reliable and valid as a solution to the optimization problem; however, the concept of reaching the answer is different for these two methods.

The result of this two method is different in the selection of the projects in schools. However, these differences are not significant. As a case in point, in school no. 1, Addams Elementary, the GA solution grants the project in Equipment and Furnishing category while this item is not selected by the Dynamic Programming solution.

Although the Dynamic Programming reaches the exact solution, the program run time is longer than the GA method. Moreover, Dynamic Programming is a suitable method of solving this specific problem. Therefore, if the objective function or the definition of inputs change, the applicability of this method needs to be investigated for the new problem. In contrast, GA method is more flexible to utilize any changes in the objective function and the inputs.

CHAPTER 5

CONCLUSION, SYSTEM LIMITATIONS, AND FUTURE WORK

The quality of school facilities in the United States has been always a key concern of the government, policy makers, and communities. The complexity of the maintenance and rehabilitation projects and the limited amount of available resources have provided decision makers with limited options for funding the projects. Even though the policy makers have developed Facility Inspection Tools (FIT) system to routinely control the condition of the existing facilities and to prioritize the facility repair projects, the suggested system still works on a first-come, first-served basis which does not necessarily guarantee that the project with the highest priority will receive the right funding. Further studies have been done by Noureddine (2010) to improve the results of the FIT system by focusing on the importance factor of various categories in facility projects. However, neither the conventional FIT nor Noureddine's (2010) system does not address the limitation of resources and the process of making optimal decision to meet the defined budget.

In this study, the above-mentioned problem has been addressed, and a solution has been offered by developing an optimization model based on the concept of equity in available resources. This model finds the optimal solution by taking the importance factor of each element of a school facility project into account. The developed model also addresses the limitation of the budget and deals with the decision makers concern about the distribution of resources to schools within a school district.

Further, the importance of resource allocation on school level is discussed, and the priority of specific categories of students such as English Language Learners and students from Low-Income Family are pointed out. Two aspects of horizontal and vertical equity in education are utilized to form the objective function of the optimization model. The model pursues the goal of increasing equity in both directions at the same time, because the budget is limited and not every school could be financed for its rehabilitation projects. The model will increase the overall equity of the system by selecting the projects in a more intelligent way. The importance factors of each facility element calculated by Noureddin (2010) are incorporated into the objective function, and the recent practices on weighted student funding are reflected in the objective function. Two solutions for the optimization problem are introduced based on Genetic Algorithm and Dynamic Programming. The two introduced algorithms are programmed to find the optimal solution. The developed program provides school districts with an option to input their coefficients for specific categories of students and to choose different amounts of available funds. The number of schools and students and the dollar amount of projects in schools needs to be inputted into the program, with either an Excel file or text file, to run the optimization model. The result of the optimization model is a table showing the projects that need to be funded and the projects that need to wait for further resources. The proficiency of the system in finding the optimal selection of projects is shown in a case study containing 87 schools within Long Beach Unified School District. The available budget for the case study was assumed to be half of the total budget required to complete all projects.

Although this optimization model introduces a more efficient process of decision making in school rehabilitation projects, it is restrained by some limitation. The first and foremost limitation of the system is that the projects have to be either selected or not. The complexity of the rehabilitation projects and the lack of information on what portion of a specific project could be acceptable and how much profit it would make, do not allow the system to further break down the projects. Furthermore, due to insufficient available data to break down the projects into more sub-projects, only the first level of Noureddine's (2010) hierarchy is considered in the optimization model.

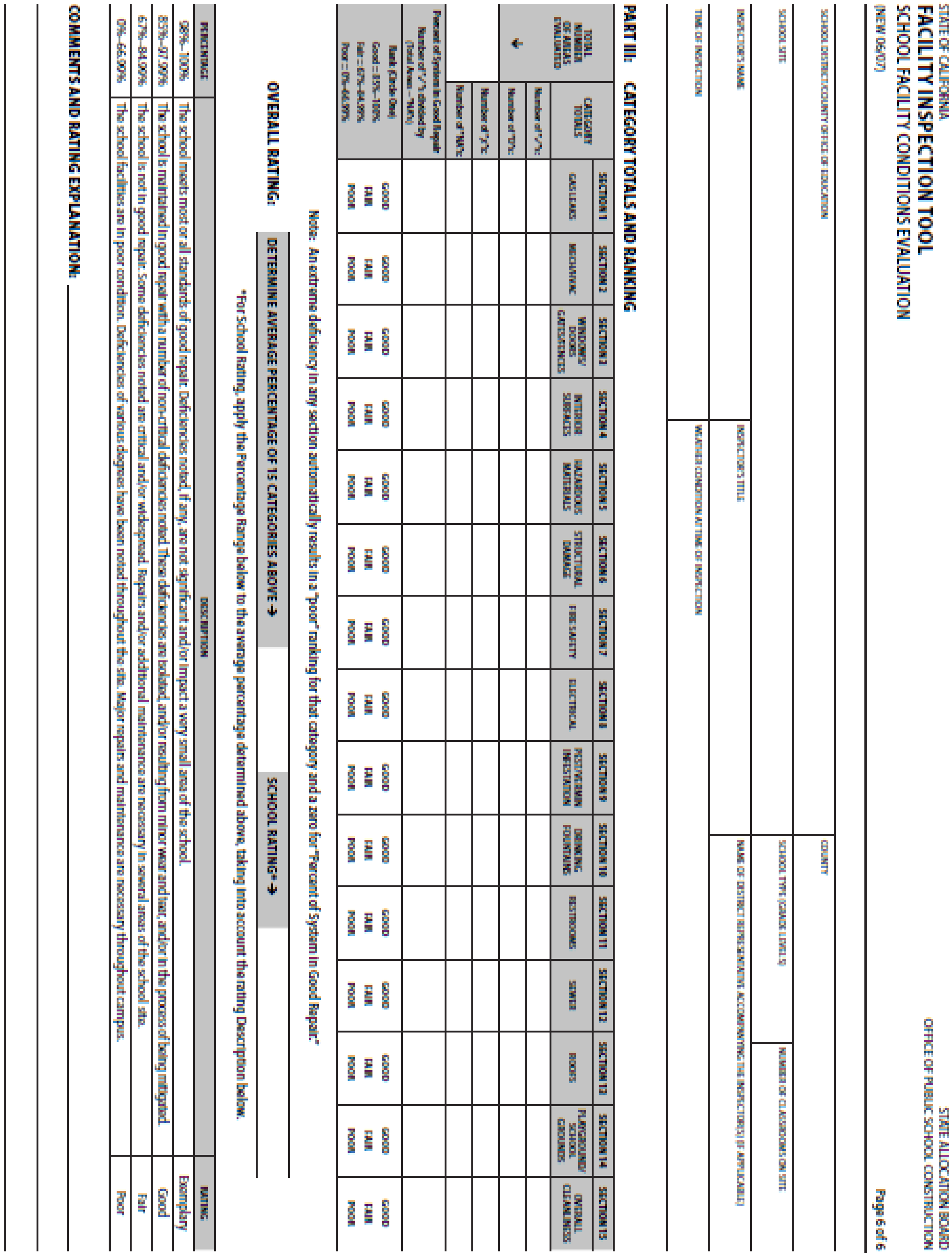
Since the relation between school facility and education outcomes has not been investigated in the literature and also owing to the fact that all the studies are mostly subjective, the importance of students' educational outcomes has been neglected in this research study. Further studies need to be done in the future to investigate the facility input and academic output relationship in schools education system in the form of equations.

Moreover, in optimization models for resource allocation, time is usually an influential parameter on decision making, however, the effect of time in this optimization model has only been considered in a fiscal year funding of school projects, while many projects take more than one fiscal year to be completed. Further research needs to be conducted to analyze and investigate the effect of time in the process of decision-making.

APPENDICES

APPENDIX A

SAMPLE OF FIT REPORT



STATE OF CALIFORNIA
FACILITY INSPECTION TOOL
SCHOOL FACILITY CONDITIONS EVALUATION
(NEW 06/07)

STATE ALLOCATION BOARD
OFFICE OF PUBLIC SCHOOL CONSTRUCTION
Page 6 of 6

SCHOOL DISTRICT/COUNTY OFFICE OF EDUCATION		COUNTY
SCHOOL SITE		SCHOOL TYPE (GRADE LEVELS) / NUMBER OF CLASSROOMS ON SITE
INSPECTOR'S NAME	INSPECTOR'S TITLE	NAME OF DISTRICT REPRESENTATIVE ACCOMPANYING THE INSPECTOR(S) (IF APPLICABLE)
TIME OF INSPECTION	WEATHER CONDITION AT TIME OF INSPECTION	

PART III: CATEGORY TOTALS AND RANKING

TOTAL NUMBER OF AREAS EVALUATED	CATEGORY TOTALS	SECTION 1	SECTION 2	SECTION 3	SECTION 4	SECTION 5	SECTION 6	SECTION 7	SECTION 8	SECTION 9	SECTION 10	SECTION 11	SECTION 12	SECTION 13	SECTION 14	SECTION 15
		GAS LEAKS	MECH/HVAC	WINDOWS/ DOORS GATES/FENCES	INTERIOR SURFACES	HAZARDOUS MATERIALS	STRUCTURAL DAMAGE	FIRE SAFETY	ELECTRICAL	PEST/VERMIN INFESTATION	DRINKING FOUNTAINS	RESTROOMS	SEWER	ROOFS	PLAYGROUND/ SCHOOL GROUNDS	OVERALL CLEANLINESS
	Number of "✓"s:															
↓	Number of "D"s:															
	Number of "X"s:															
	Number of "NA"s:															
Percent of System in Good Repair Number of "✓"s divided by (Total Areas – "NA"s)																
Rank (Circle One) Good = 85%–100% Fair = 67%–84.99% Poor = 0%–66.99%		GOOD FAIR POOR	GOOD FAIR POOR	GOOD FAIR POOR	GOOD FAIR POOR	GOOD FAIR POOR	GOOD FAIR POOR	GOOD FAIR POOR	GOOD FAIR POOR	GOOD FAIR POOR	GOOD FAIR POOR	GOOD FAIR POOR	GOOD FAIR POOR	GOOD FAIR POOR	GOOD FAIR POOR	GOOD FAIR POOR

Note: An extreme deficiency in any section automatically results in a "poor" ranking for that category and a zero for "Percent of System in Good Repair."

OVERALL RATING: DETERMINE AVERAGE PERCENTAGE OF 15 CATEGORIES ABOVE → SCHOOL RATING* →

*For School Rating, apply the Percentage Range below to the average percentage determined above, taking into account the rating Description below.

PERCENTAGE	DESCRIPTION	RATING
98%–100%	The school meets most or all standards of good repair. Deficiencies noted, if any, are not significant and/or impact a very small area of the school.	Exemplary
85%–97.99%	The school is maintained in good repair with a number of non-critical deficiencies noted. These deficiencies are isolated, and/or resulting from minor wear and tear, and/or in the process of being mitigated.	Good
67%–84.99%	The school is not in good repair. Some deficiencies noted are critical and/or widespread. Repairs and/or additional maintenance are necessary in several areas of the school site.	Fair
0%–66.99%	The school facilities are in poor condition. Deficiencies of various degrees have been noted throughout the site. Major repairs and maintenance are necessary throughout campus.	Poor

COMMENTS AND RATING EXPLANATION:

FIGURE 17. Sample FIT report (Rozzi 2008).

APPENDIX B

GENETIC ALGORITHM CODE

```
%% Genetic Algorithm Code

Data = importdata('input.txt');
coef = [.4646 .2035 .0976 .1373 .0360 .0611];
alpha = input('What is the value of alpha?'); %Alpha is the coefficient
for ElL students
beta = input('What is the value of alpha?'); %beta is the coefficient
for Low-Income Family students
Budget = input('What is the amount of available budget?');

Facility_Project_Costs = Data(:,1:6);
Facility_Project_Costs(Facility_Project_Costs == 0) = 10000000000;
Atmp = Facility_Project_Costs;
for i = 1 : 6
    Facility_Project_Costs(:,i) = Facility_Project_Costs(:,i)/coef(i);
end
Diversity = Data(:,7:9);

%output = sum over all the inverse capita
function o = fitness(V)
    if sum(V.*reshape(Atmp,1,length(V))) > Budget
        o = 1000;
    else
        B1 = zeros(size(Facility_Project_Costs));
        B2 = zeros(size(Facility_Project_Costs));
        B3 = zeros(size(Facility_Project_Costs));
        for i = 1 : 6
%total Students
            B1(:,i) = Diversity(:,1)./Facility_Project_Costs(:,i);
%ELL Students
            B2(:,i) = Diversity(:,2)./Facility_Project_Costs(:,i);
%Poor Students
            B3(:,i) = Diversity(:,3)./Facility_Project_Costs(:,i);
        end
        o = -sum(V.*reshape(B1,1,length(V)))...
        -alpha*sum(V.*reshape(B2,1,length(V)))...
        -beta*sum(V.*reshape(B3,1,length(V)));
    end
end

%creation function, initiates the population
%rate = probability 1
function Population = mycf(GenomeLength, FitnessFcn, options)
    rate = 0.01;
    Population = rand(10000,size(Facility_Project_Costs,1)*...
    size(Facility_Project_Costs,2)) > 1-rate;
end

function state = myplotfun(options,state,flag)
         plot(-state.Best,'.')
end


options = gaoptimset('PopulationType','bitstring','Generations',1000,
```

```
'PopulationSize',2000,'PlotFcns',{@myplotfun,@gaplotstopping,@gaplotgen
ealogy},'CreationFcn',@mycf,'CrossoverFcn',@crossoverscattered);

V = ga(@fitness,size(Facility_Project_Costs,1)...
*size(Facility_Project_Costs,2),options);

V2 = reshape(V,size(Facility_Project_Costs));
sum(V.*reshape(Atmp,1,length(V)))

fitness(V)
reshape(V,size(Facility_Project_Costs))
```

APPENDIX C

DYNAMIC PROGRAMMING CODE

```
%% Dynamic Programming Code

Data = importdata('input.txt');
coef = [.4646 .2035 .0976 .1373 .0360 .0611];
alpha = input('What is the value of alpha?'); %Alpha is the coefficient
for ElL students
beta = input('What is the value of alpha?'); %beta is the coefficient
for Low-Income Family students
Budget = input('What is the amount of available budget?');

Costs = Data(:,1:6);
N = size(Costs,1)*size(Costs,2);
Costs = round(Costs);
Diversity = Data(:,7:9);

Values = zeros(size(Costs));
for i = 1 : 6
    Values(:,i) =
(Diversity(:,1)+alpha*Diversity(:,2)+beta*Diversity(:,3))*coef(i);
%total
end
Values = reshape(Values,1,N);
Weights = reshape(Costs,1,N);
Values(Weights == 0) = 0;
Weights(Weights == 0) = 100;

function [best amount] = knapsack(weights, values, W)
    if ~all(is_positive_integer(weights)) || ...
       ~is_positive_integer(W)
        error('Weights must be positive integers');
    end
    %We work in one dimension
    [M N] = size(weights);
    weights = weights(:);
    values = values(:);
    if numel(weights) ~= numel(values)
        error('The size of weights must match the size of values');
    end
    if numel(W) > 1
        error('Only one constraint allowed');
    end

    % Solve the problem

    % Note that A would ideally be indexed from A(0..N,0..W) but MATLAB
    % does not allow this.
    A = zeros(length(weights)+1,W+1);
    % A(j+1,Y+1) means the value of the best knapsack with capacity Y
using
    % the first j items.
    for j = 1:length(weights)
        for Y = 1:W
            if weights(j) > Y
                A(j+1,Y+1) = A(j,Y+1);
            else
                A(j+1,Y+1) = ...
```

```
                    max( A(j,Y+1), values(j) + A(j,Y-weights(j)+1));
            end
        end
    end

   best = A(end,end);

   %Now backtrack
   amount = zeros(length(weights),1);
   a = best;
   j = length(weights);
   Y = W;
   while a > 0
       while A(j+1,Y+1) == a
           j = j - 1;
       end
       j = j + 1; %This item has to be in the knapsack
       amount(j) = 1;
       Y = Y - weights(j);
       j = j - 1;
       a = A(j+1,Y+1);
   end


    amount = reshape(amount,M,N);
end

function yn = is_positive_integer(X)
    yn = X>0 & floor(X)==X;
end


[BEST AMOUNT] = knapsack(Weights, Values, Budget);

%% Comparison Between Genetic Algorithm and Dynamic Programming

AAtmp = reshape(Atmp,1,length(AMOUNT));
AMOUNT(AAtmp > 1000000) = 0;
V(AAtmp > 1000000) = 0;

fprintf('Budget Used in Knapsack %d\n',sum(AMOUNT.*AAtmp));
fprintf('Budget Used in Genetic %d\n',sum(V.*AAtmp));

fprintf('Genetic fitness function evaluated on Knapsack output %d\n',-
fitness(AMOUNT));
fprintf('Genetic fitness function evaluated on Genetic output %d\n',-
fitness(V));

fprintf('Knapsack total value evaluated on Knapsack output
%d\n',sum(Values.*AMOUNT));
fprintf('Knapsack total value evaluated on Genetic output
%d\n',sum(Values.*V));

reshape(V,size(Costs))
reshape(AMOUNT,size(Costs)
```

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REFERENCES

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