STUDENTS' USE OF METAPHOR AND GESTURE DURING COLLABORATIVE WORK ON TASKS DESIGNED TO FOSTER STUDENTS' COVARIATIONAL REASONING

by

PETER HORNBEIN

B.A., University of Colorado, Boulder, 1975

B.A., San Jose State University, 1978

A thesis submitted to the

Faculty of the Graduate School of the

University of Colorado in partial fulfillment

of the requirements for the degree of

Master of Science

Mathematics Education Program

2015

This thesis for the Master of Science degree by

Peter Hornbein

has been approved for the

Mathematics Education Program

By

Heather L. Johnson, Chair

Ron Tzur

Michael Ferrara

April 8, 2015

Hornbein, Peter (M.S.Ed, Mathematics Education)

Students' use of metaphor and gesture during collaborative work on tasks designed to foster students' covariational reasoning

Thesis directed by Assistant Professor Heather L. Johnson

ABSTRACT

Researchers have argued that gesture and speech, two elements of discourse, are neurologically related, and that language and mental imagery are intertwined. Because of this relationship between language, gesture and image, these discourse elements may allow a teacher to make inferences about the reasoning the student is using. In order for the teacher to make these inferences, students must engage in discourse, which I am initially defining here as written and spoken language and the accompanying gestures. This requires that students work on open ended, contextual problems that provide opportunities for discourse. An area that provides opportunities for discourse includes functions and the relationship between the covarying quantities that the function expresses.

By investigating discourse and covarying quantities, I will attempt to answer two, related research questions. *What is the nature of students' use of metaphor and gesture when working collaboratively on tasks designed to provide opportunities for covariational reasoning? What information might the students' use of metaphor and gesture provide about the student's covariational reasoning?* In order to answer these two questions, I analyzed data from four, ninth grade students during work on two task-based interviews in which the students completed a version of a widely-used bottle problem. The data analysis consisted of multiple passes coding for the quantitative operation, gesture and metaphor used by the students.

Gesture and metaphor helped make inferences about the quantitative operation the students were using and whether they were comparing or coordinating covarying quantities. The students' gesture allowed me to infer more about the underlying imagery they were using than did metaphor, however, the two were most powerful when considered together. Two of the four students were primarily comparing amounts of change in the two quantities and the other two students coordinated the two quantities. The results led me to a conjecture about the relationship of language, imagery and gesture, and how this relationship might be used in both educational and research settings.

I proposed a relationship between imagery, language and gesture that I referred to as the Language-Imagery-Gesture Triad with imagery and gesture forming the foundation supporting language. Linguistic structures such as metonymy and metaphor facilitate the relationship between imagery and language.

The form and content of this abstract are approved. I recommend its publication

Approved: Heather L. Johnson

DEDICATION

I dedicate this to my wife, Christina Mitchell. Her support of my return to school is cherished.

ACKNOWLEDGEMENT

I would like to thank Heather Johnson for all her support, advice and instruction. Her encouragement and faith in my abilities is testimony to her quality as a teacher and researcher.

I would also like to thank Ron Tzur for his encouragement in this pursuit. Without his support, I would have taken the easy road.

TABLE OF CONTENTS

CHAPTER I

INTRODUCTION

To make inferences about how a teacher's students are reasoning, that teacher could interpret his or her students' actions when they are working on open-ended, contextual problems that provide opportunities to engage in discourse with other students and the teacher (Herbel-Eisenmann & Otten, 2011; Herbel-Eisenmann, Cirillo & Skowronski, 2009). By discourse, I am referring to written and spoken language that includes the accompanying gestures that the speaker intends for one or more people in the context of conversation or collaborative work. I would argue that when engaging in discourse, the teacher might observe the students using academic or everyday language with varying types of gesture and metaphor when making sense of mathematical concepts. It follows, then, that if a teacher is intentional about the kind of reasoning that discourse might promote, he or she will be better able to advance students' reasoning. I would propose that any inferences a teacher makes during discourse can lead her to a better understanding of the imagery and reasoning process her students might be using when working on problems.

The mathematical concept of function is central to high school algebra (Common Core State Standards Initiative, 2010). One perspective of function is as a relationship between covarying quantities (Chazan, 2000; Confrey & Smith, 1995). To adopt a covariation perspective of function, a student would need to engage in covariational reasoning (Carlson, Jacobs, Coe, Larsen & Hsu, 2002; Clement, 1989; Johnson, 2012). Covariational reasoning refers to the dynamic mental activities involved when coordinating quantities that vary together and the ways in which their variation depends on each other (Carlson et al., 2002). Given the richness of covariational reasoning, I argue that students' discourse around covarying quantities and function are useful to investigate.

Discourse: Gesture and Metaphor and the Underlying Imagery

Stein, Engle, Smith & Hughes (2008) and Presmeg (1992) noted several important elements associated with classroom discourse, including the students' development of taken-as-shared knowledge and the evaluation of each other's constructed mathematical ideas. Presmeg (1992) noted that the development of concepts and the associated prototypical imagery (Sadoski and Paivio, 2009; Paivio, 2007) is idiosyncratic, and discourse is necessary to ensure that the created imagery is consistent with the taken-as-shared imagery. As I will more fully define in chapter two, I am using imagery here to refer to the visual images that one might associate with a word or concept. This imagery is what may lie at the heart of our metaphors and gestures (McNeill, 2005).

Four of the Gestures Comprising Discourse Are Studied in this Thesis

Gesture is a multidimensional element of discourse that is directly linked to an individual's underlying imagery and gesture may be a more direct representation of that imagery than the spoken word (McNeill, 2005; Goldin-Meadow, 1999); it is a part of language (McNeill, 2005). The four types of gesture of interest are metaphoric, iconic, beat and deictic (Edwards, 2009; McNeill, 2005). Metaphoric gestures are those that represent an abstraction, an example noted by Edwards (2009) is the palm-up, hand open gesture signifying that there is a change in the scene being talked about. This is the opposite of an iconic gesture that is concrete and acts out a shape or process—it reflects the actual object or action. For example, if something is shrinking, the thumb and pointer finger closing together could represent it. While metaphoric and iconic gestures are more or less opposites, deictic and beat gestures have no such relationship, but, instead, provide additional information or give an indication of the speaker's intent. A beat gesture provides emphasis, similar to foot tapping when listening to music, whereas deictic gesture identifies a space, either figuratively or literally. A figurative example of a deictic gesture could occur when one is talking about an increase in the temperature. If the individual's hand started very low, it could indicate that the temperature was very low in the beginning.

The Four Metaphors Comprising Discourse That Are Studied in this Thesis

Metaphor does not merely add flourish and color to one's writing or speech, it may also reflect how we are reasoning about something, or how we perceive either an object or concept. The use of metaphor helps to reduce or increase the level of abstraction, which, in the case of the former, may help an individual reason about an object or concept (Lakoff & Johnson, 1980). For example, …

Two commonly used types of metaphor are structural and ontological (Lakoff & Johnson, 1980). Structural and ontological metaphor are related in that they both map one concept onto another, although they map in different ways. A structural metaphor is used when one is thinking about one concept or thing in terms of another, possibly breaking down an abstraction or building one up. For example, in English, one will often equate *time and money*: "One must *spend one's time wisely",* or "*Give me more time because I haven't used it well."* (Lakoff & Johnson, 1980). Ontological metaphor is used when one is viewing something abstract as an entity in its own right; that is, as an entity that is something concrete that can be quantified, labeled or there is an aspect to the metaphor that we can concretely identify. One could use the metaphor, the *brutality* *of terrorism* to convey their feelings about terrorism's horror and the fear that it invokes (Lakoff & Johnson, 1980).

Two additional types of metaphor are orientational and container metaphors (Lakoff & Johnson, 1980). Orientational metaphor, in English, is based on the underlying constructions that *up is good, down is bad*; the *future is ahead of us, the past, behind* (Lakoff & Johnson, 1980). Thus, when one is entering a freeway, one needs to *speed up*; at graduation, one hears the phrase, "*You must look ahead to the future.*" The container metaphor breaks down an abstraction by mapping the abstraction onto something that may contain something else (Lakoff & Johnson, 1980). For example, even though one has paid off 90% of his or her outstanding debt, that individual is not *out of the woods.* The trees that make up the woods or forest form a container signifying trouble, and these woods contain the indebted individual. Thus, only when that person is free of debt will they be *out of the woods.*

A Task Fostering Students' Covariational Reasoning: Sketching Graphs Relating Volume and Height

Tasks fostering students' covariational reasoning address how students reason and work with quantities that covary and provide opportunities for discourse. By quantity, I mean an attribute that something has and this attribute can be measured, although its actual, numerical measurement is not necessary, only that the individual can conceive of the act of measuring (Thompson, 1994). It is important to note that researchers have found that middle and secondary students can engage in covariational reasoning prior to having a formal course in calculus (Johnson, 2012, 2015; Saldanha & Thompson, 1998; Thompson, 1994).

One of the original tasks investigating covariation was the Shell Centre's Bottle Problem, which had students sketch graphs representing a relationship between the volume of liquid in a bottle and time (Shell Centre for Mathematical Education,1985). Johnson adapted the Bottle Problem so that the students had to sketch the quantities of height and volume where the volume was on the vertical axis and the height on the horizontal axis (Johnson, 2012, 2013). Johnson's adaptation was intended to reduced the likelihood that students might operate on this task using time as the independent variable so the student must consider the relationship between less common quantities allowing greater insight into their reasoning; they cannot rely on previous experience. (Johnson, 2012).

Carlson et al. (2002) investigated how second semester, calculus students used covariational reasoning in a graph-sketching task and from this, derived five levels of covariational reasoning. Johnson (2015) noted that most of the students in Carlson et al.'s study were operating at Level-3 but not above, and this led Johnson to investigate why this might be happening. Johnson (2015) investigated the quantitative operations students use when they reason covariationally, and determined that Carlson et al.'s (2002) gradient may not be fine enough to explain why many students in Carlson et al.'s study did not engage in covariational reasoning above Level-3.

Johnson (2015) proposed two types of quantitative operations, comparison and coordination: an individual compares quantities by noting attributes like how one quantity changes more than another; an individual coordinates quantities by noting how the rate or intensity of one quantity's change depends on another quantity's continuous change. Johnson (2015) was able to divide each of these levels into 3 sub-levels. When a student is using the operation of comparison, they are considering the amount of change in one quantity with either the change or the amount of change in another quantity. At the upper-most level of comparison, the student may be considering an amount of change per unit of the second quantity. A student using the operation of coordination would be considering change in one quantity with continuous change in another, and at the higher two levels, the student will begin to consider whether the change in one quantity is happening faster than the change in the other.

To investigate students' covariational reasoning, researchers have implemented tasks that did not use numbers (Johnson, 2015; Thompson, 1994, 2011; Saldanha & Thompson, 1998). For example, a graphical representation of a function is presented, but the dependent and independent variables are only labeled by the quantity, in this case volume and height, with no numerical or metric presentation. Saldanha and Thompson (1998) used a graph that represented a car's path and the distance the car would be, relative to two cities. All the student has to work with is the qualitative behavior of the quantities and how they are covarying, and this requires her to keep simultaneous images of the quantities in mind, which may prevent the coordination of data points. For the purposes of this study, numerical and metric information was not provided in order to determine the students' level of covariational reasoning. The task used in this study had students consider two quantities qualitatively to provide them an opportunity to reason covariationally, which provided me an opportunity to study their discourse as they discussed their reasoning.

Research Questions

Through this investigation, I will attempt to answer two, related research questions:

- *What is the nature of students' use of metaphor and gesture when working collaboratively on tasks designed to provide opportunities for covariational reasoning?*
- *What information might the students' use of metaphor and gesture provide about the student's covariational reasoning?*

This thesis will attempt to answer these research questions by looking at the discourse of students as they work on an adaptation of the Bottle Problem. Possible pedagogical implications will be considered in later chapters, as well as a more thorough discussion of the literature and research methods.

CHAPTER II

LITERATURE REVIEW

Theoretical Framework and Fundamental Definitions Used

The Complementary Nature of Constructivist and Sociocultural Theories

In this chapter, I will review the literature related to covariational reasoning, differing levels of covaritational reasoning and the quantitative operations involved. I will set the stage by discussing how the constructivist and sociocultural theories are complementary. That is, how they are two sides of the same coin because they work together to ensure that what the student has constructed individually is consistent with the taken-as-shared construction (Cobb, 1994; Cobb & Bauersfeld, 1995). For my purposes, this is important because I am proposing that gesture and metaphor provide tools the researcher may use to make inferences about the underlying reasoning. I am using this complementary relationship between constructed and taken-as-shared information to make inferences about how gesture—representing internal imagery—and metaphor—the sociocultural representation of imagery and the contribution of the individual to the taken-as-shared imagery—come together to provide insight into how an individual is using her own mental imagery and make inferences about her reasoning. Gesture, by its neurological connectivity to language and ability to represent internal imagery through its non-linearity and spatial presentation (Goldin-Meadow, 1999; McNeill, 2005) may represent the internal process. Metaphor, because it is a part of spoken or written language and can facilitate the abstractness of concepts and imagery (Lakoff & Johnson, 1980) represents the sociocultural aspect of the complementary nature of constructivism and the sociocultural theories (Cobb, 1994; Cobb & Bauersfeld, 1995). Thus, this combination of metaphor and gesture allows me to draw inferences as to a student's mental imagery, quantitative operation used and, ultimately, her reasoning.

The Dynamic Mental Activity of Reasoning

Reasoning, as I am using it here, is a dynamic mental activity around some task that consists of and results in an internal, reflective process (Thompson, 1996, Simon, Tzur, Heinz & Kinzel, 2004) that is both augmented and supplanted by social interaction (Cobb & Bauersfeld, 1995). This same internal, reflective process helps the individual sort through existing personal and taken-as-shared information, restructuring and recombining that information and integrating new information into a network of concepts and procedures.

The process the individual employs when reasoning involves inner, internal and external speech (Vygotsky and Kozulin, 2012). Inner speech is not the internal monologue that we may think; Vygotsky and Kozulin (2012) used the term to refer to an entirely different entity that does not follow the normal syntax associated with either external speech or internal speech. Internal speech functions differently from inner speech in that inner speech is for the speaker, it is the formation of thought and the awareness of understanding (Vygotsky & Kozulin, 2012). In essence, the individual thinks words, rather than silently speaks them. I will use the term internal speech to refer to speech that is not spoken aloud but may be intended for others. In this sense, it is another form of external speech in that it is the physicality of thought and reasoning that the speaker may or may not intend for others (Vygotsky & Kozulin, 2012). It can consist of an internal monologue such as when composing a written piece or imagining a conversation with another; or it can be the things we are preparing to say in a conversation or discussion (Vygotsky & Kozulin, 2012) that comprises the spoken aspects of discourse.

Discourse as an Important Element of the Learning Process

Discourse, in addition to being the written and spoken language that includes accompanying gestures, is intended for one or more people. It is used in the context of conversation or collaborative work, and is the process of using language that depends on both social setting and the use of language; it is an essential part of the learning process (Stein et al., 2008; Herbel-Eisenmann et al., 2009). Thus, discourse may be the larger concept into which things like discussion, dialogue, and presentations fall. The social setting can determine whether the interchange is spoken or written, and if spoken, whether the spoken word is between two people forming a dialogue or in a group comprising a discussion. If the speaker uses this language face-to-face, it may be augmented by gesture and body language; and finally, the choice of word and language use in both spoken and written form is culturally dependent on whether the interchange is between peers in an informal setting or in a formal setting such as the classroom during a class discussion (Herbel-Eisenmann & Otten, 2011). During discourse, the individual is given additional opportunities to work with the material at hand by having to consider, for example, other students' solutions and compare these to their own solution (Yackel & Cobb, 1996). The role of discourse, whether between two people, in small-groups, or a classroom setting requires that the individual modify his or her network of concepts and procedures in such a way as to make sense of one's own knowledge base in concert with the cultural constructs, ultimately producing a personal understanding that is consistent with the taken-as-shared knowledge (Cobb & Bauersfeld, 1995). In other words, through discourse, that which we have constructed through an internal process and that which has arrived to us through a sociocultural process meld into a unified whole.

Discourse is important for three reasons: it is important in students' mathematical discussion (Yackel & Cobb, 1996), mathematical literacy (Cobb & Bauersfeld, 1995; Herbel-Eisenmann, Cirillo & Skowronski, 2009; Stein et al., 2008), and mathematics standards (National Governors Association Center for Best Practices, Council of Chief State School Officers, Common Core State Standards Initiative, 2010). Interaction with others around a problem or solution is an important element of mathematical discourse and contributes to the development of sociomathematical norms (Yackel & Cobb, 1996). Sociomathematical norms refer to the specific nature and format that is followed in mathematical discussion; for example, what counts as a different solution rather than a restatement of a previously-mentioned solution, what is mathematically sophisticated and elegant, and what is efficient and is part of the development of the taken-as-shared knowledge (Yackel & Cobb, 1996). The Common Core State Standards (Common Core State Standards Initiative, 2010) promote discourse and writing about mathematics in both their Standards of Mathematical Practice and within the standards, themselves. Thus, discourse is central to the newly established standards, as well as an important element in the representation and development of a student's reasoning.

Imagery and Its Role in Gesture and Metaphor

I will refer to the terms image and imagery frequently because these notions are central to this study and are central to the role that gesture and metaphor play in our observations of discourse and our use of language. When I use image or mental imagery, I am referring to those prototypical images associated with concrete terms; Paivio (2007) described a prototypical image as that which gives meaning to the associated word and the associated word provides a name for that prototypical image. This prototypical image expands as the individual constructs and negotiates a constructed class of objects, incorporating personal knowledge with the cultural, taken-as-shared knowledge ultimately forming a class of more abstract and a more inclusive set of images and words of which the original image and word represent and name. When one manipulates mental images, these types of images would be dynamic. Presmeg (1992) noted that imagery might be visual, auditory, tactile, gustatory and olfactory. She focused her attention on the visual aspect, and defined a visual image as any mental activity that involved either spatial or visual information. In this thesis when I use the terms, imagery, mental imagery or image, I will be specifically referring to visual imagery. Furthermore, because the information presented to the students in this thesis is visual, an underlying assumption is that the information will be processed through visual imagery. Thus, for the purposes of this thesis, I will assume that gesture represents the underlying imagery (McNeill, 2005; Edwards, 2009) present in the students' reasoning and quantitative operation.

Metaphor comprises a part of the spoken or written connection between the individual and others (Lakoff & Johnson, 1980; Presmeg, 1998). The four types of metaphor I will be studying—orientational, structural, container and ontological—give the speaker a way to convey concepts and mental imagery to others and, in that sense, can provide insight into the way that an individual is visualizing a situation or concept. While researchers have noted that gesture may go unnoticed by both speaker and listener (Goldin-Meadow, 1999; McNeill, 2005), our tendency to focus on the spoken language brings metaphor to the forefront and augments the gestural information. I say "augments the gestural information," because gesture is non-linear and spatial and provides a better vehicle to the underlying mental imagery, and it is for this reason that I am emphasizing gesture. However, the additional information provided by the students' use of metaphor helps paint the entire picture of the image the student has in mind.

Quantity and Covariation Defined

To effectively discuss covariation and the operations involved in analyzing the relationship between quantities, some fundamental terms need to be identified and defined. Fundamental to covariation is quantity because quantities can change together. Thompson (1994) described quantities as "conceptual entities" (p. 184), meaning that quantities are individuals' conceptions of attributes of some object that can be measured. Covariation, then is how quantities vary together, so covariational reasoning consists of the dynamic mental activities that a student will use when coordinating or comparing varying quantities and how these quantities change relative to each other (Carlson et al., 2002).

Gesture Provides Meaning Beyond Emphasis

Gesture can be rich in its meaning and use, and helps provide information about the quantitative operation and reasoning the student is using (McNeill, 2005; Pimm, 1988; Presmeg, 1992; Vygotsky & Kozulin, 2012). Gesture is a somatic component of the neurogestural system (McNeill, 2005; Goldin-Meadow, 1999) and is neurologically related to verbal language. McNeill (2005) referred to gesture as playing an active part in one's speaking and thinking, and forms a dialectic of image and language. McNeill (2005) noted that language cannot be separated from imagery and that gesture occurs universally, even in the blind. It is automatic with speech, thus, "…gestures are *part* of language" (location 95 of 5259). Because gesture is a part of language, I am using it as an important element in discourse analysis. If we consider gesture as a part of language and autonomic, as McNeill (2005) pointed out, when we think in order to speak we are utilizing a dynamic organization in a “dialectic of imagery and language” (Location 3434) that is expressed both verbally and gesturally.

The autonomic nature of gesture, which arises from its neurology (McNeill, 2005), is an important one because it eliminates or reduces the conscious control of gesture, thus gesture may provide an unmediated view of the student’s imagery. Vygotsky and Kozulin (2012) discussed the importance of gesture in conveying meaning of a child’s first words, noting that pointing and similar gestures are the precursors to human speech. Vygotsky considered psychological tools as mediators associated with higher functioning, and gesture was among these psychological tools (Vygotsky & Kozulin, 2012). McNeill (2005) outlines several areas of the brain that are responsible for both language and gesture, specifically, areas within Broca’s and Wernicke’s areas. Language and gesture are inextricably linked and together, and form the totality of language (McNeill, 2005, Goldin-Meadow, 1999). Unlike language that is both linear and non-spatial (Goldin-Meadow, 1999), gesture retains the underlying imagery, and its neurological connection to language suggests that it is not directly under conscious control. In that regard, and somewhat over-simplistically, I am proposing that gesture represents the underlying imagery that is feeding the more consciously controlled speech and so gesture may take precedence in the interpretation of the meaning of speech and the students’ use of metaphor.

When coupled with gesture, imagery is a part of language that McNeill (2005) and Vygotsky and Kozulin (2012) maintained is crucial to our development of language. The coupling of language and gesture, and I would submit, imagery, is an essential coupling in the development of our species’ ability to use language. With this approach in mind, it is reasonable to assume that gesture is not something separate from language. It is an integral part of language and cannot, therefore, be separated. We can choose our language and our use of metaphor, but gesture seems to be at the mercy of our cultural-linguistic background, arising naturally during speech, differing only across language and culture (Kendon, 1997). Thus, metaphor may provide insight into the sociocultural aspects of reasoning, and gesture may provide a lens to the internal processes and imagery of the speaker that have been mediated by the sociomathematical norms (Yackel & Cobb, 1996) of the classroom and the wider culture.

Despite the universal nature of gesture, many gestures go unnoticed by the listener (Goldin-Meadow, 1999; McNeill, 2005). They do, however, provide a subtext, or subconscious augmentation of the spoken words. These gestures, from the speaker's perspective, may reflect thoughts or images of which even the speaker is unaware (Goldin-Meadow, 1999). There are four general types of gesture, which McNeill defined as spontaneous movement of the arms and hands that is synchronized with speech. McNeill (2005) noted how close the synchronization is between language and gesture. As the speaker slows his or her speech, such as when groping for the correct word to use, the speaker's gesture slows to a halt, only to pick up again as the cadence of speech picks up again with normal fluency (McNeill, 2005). McNeill recognized that these gestures often do not occur singly, but are multidimensional and connected, flowing along with speech.

The Four General Types of Gesture Studied

The four general types of gesture outlined by McNeill (2005) are iconic, beat, deictic, and metaphoric. The iconic gestures essentially mimic the spoken word and refer to concrete objects, for example, if the speaker is referring to something moving upwards, this may be accompanied by a hand movement that is going up: a physical icon representing the event being spoken about.

For example, if I am discussing an element contained within an open interval, I may use a gesture that involves a cupping of both hands, representing a non-inclusive containment around the quantity in question, an iconic gesture in that I am referring to the delineation of an open interval by parenthesis. I may also repeat this gesture twice, quickly, adding emphasis. Thus, this single gesture could be considered as both an iconic and beat gesture, providing information about what I am referring to, as well as emphasizing my point; because beat gestures are akin to keeping the beat in music, they carry with them an element of accentuation, emphasizing the importance of the accompanying words. The deictic gesture involves locating something in space that the speaker carries out with the hands or any other body part. In an iconic gesture, going up represented by an upward motion with the hands and arms, may start with the speaker's hands quite low, a deictic gesture signifying that the speaker is starting at a very low point, for example, below zero when describing temperature. Finally, the metaphoric gesture is a physical metaphor, mapping an abstract concept onto a more concrete concept. The speaker will use the metaphoric gestures to represent some abstract object or concept (McNeill, 2005), much as she would use a spoken metaphor.

Edwards (2009) studied how students in a teacher education program used gesture when discussing fractions. One of Edwards (2009) results indicated that students tended to use fewer iconic gestures and more metaphoric gestures when discussing the mathematics in subsequent interviews, which I could interpret as meaning that the student teachers were using more abstract imagery. An interesting aspect to Edward's 2009 study was that it focused on fractions and rational numbers and their representation of parts of a whole, resulting in a large portion of gestures in Edward's (2009) study being iconic and consisting of a cutting motion, representing a separation of parts from the whole.

Imagery and the Use of Metaphor

Metaphor is a linguistic tool that goes beyond the artistic use of language to poetically convey meaning; metaphor is also involved in reasoning (Pimm, 1988) through the process of reifying the abstract, the creation of meaningful imagery and the extension of concept (Sadoski & Paivio, 2009). Sadoski and Paivio (2009) went on to note that the abstract is derived from the concrete metaphorically or through the construction of a class of indirect images and words stating that "[b]oth scientific and artistic language attempt to elegantly express the world as it is imagined to be…" (p. 8), connecting language and imagery in invention; metaphor's power "…lies in its use in making sense of new conceptions in terms of already existing conceptions…" (Presmeg, 1998, p. 29). Thus, through metaphor, we can talk about abstractions and imaginings that would otherwise be closed off to discourse because of an absence in linguistic structure to handle the abstraction. Furthermore, through metaphor we can understand new concepts in terms of concepts that already exist. Metaphor provides a foundation or skeleton for our thinking, allowing us access to the abstract (Zandieh & Knapp, 2006). From this, I can conclude that metaphor is a use of language that helps in the development and maintenance of sociomathematical norms in that it allows for the expression of abstract concepts and describes mental imagery through language (Zandieh & Knapp, 2006; Sadoski & Paivio, 2009). I am interpreting metaphor as also being associated with reflection and the construction of new concepts and the reorganization of existing concepts (Presmeg, 1988; Simon et al., 2004).

Metaphor in a Mathematical Context

Pimm (1988) noted that we use a "mathematical register" (p. 31) that consists of a set of everyday language, or natural language that has been redefined and repurposed to use as technical or lexical language, specific to mathematics. Much of this register comes from repurposing terms in everyday language, and as if this were not confusing enough for the student, mathematics will repurpose the same term multiple times giving us terms whose meaning is context dependent. Presmeg (1992) discussed this in the context of shared imagery that works in conjunction with an individual student's idiosyncratic imagery and therefore definition of a specific term. These terms, then, become defined through the use of metaphor, which then extends concepts and refines and redefines terms (Lakoff & Johnson, 1980), often by extending the metaphor through additional description.

When one adds descriptors, for example, a single adjective, we can extend or even alter the meaning of a mathematical concept. By way of example, consider the triangle that exists in Euclidean space: this triangle has three sides and angles that sum to 180 degrees. If we now add the descriptor, "spherical", we have a triangle whose sides are no longer lines, but portions of a great circle, the shortest distance between two points on a sphere, and whose angles sum to more than 180 degrees. This simple form of metaphor is the fundamental metaphoric structure in English (Pimm, 1980), created by the addition of an adjective. Presmeg (1992) added to this by introducing the concept of a visual-pictorial aspect to mathematics and to our reasoning process. This is especially important for the educator to keep in mind because not all students function verbally or in the realm of verbal-logical reasoning (Presmeg, 1992) and also points to the importance of gesture in the analysis of discourse. Imagery and vocabulary become linked to the symbols of mathematics with which the student must also be fluent. With increased representational fluency utilizing the symbols of mathematics, coupled with the imagery, gesture (Edwards, 2009) and vocabulary, comes full explanation of the student's reasoning; she may use metaphor, either spoken or in the form of gesture in her discourse as a way of either increasing or decreasing the level of abstraction, as appropriate.

Metaphor Construction and Metaphor as a Reflection of Concept

While the general perception of metaphor is that of a literary device, something to add a creative or poetic flourish to speech or writing, metaphor is far deeper and more complex, having many forms and the speaker's choice of form can provide some insight into the underlying imagery and concept (Lakoff & Johnson, 1980, Sadoski & Pavio, 2009). Lakoff and Johnson (1980) maintained that metaphor surrounds us and is essential in the way we think, and that metaphor reflects the structure of our concepts. Presmeg (1998) discussed two important aspects of a metaphor's structure that merit mention. A metaphor consists of two elements, the *ground* that consists of the similarities between the objects being compared and the *tension* constituting the dissimilarities. Presmeg (1998) noted that this simultaneity between the ground and the tension allows the metaphor to help structure new experiences based on the older ones; that is, the metaphor assists the student in her construction of knowledge based on previously learned information and through the mediation provided by discourse in a sociocultural context.

Unlike Presmeg (1998) who described the structure of metaphor using two elements, the ground and tension, Sadoski and Pavio (2009) discussed three components of the metaphor, the *topic*, the *ground* and the *vehicle*. The *topic* refers to what the metaphor is about, the subject that the student is learning or describing; the *vehicle* is what the topic is compared to, which may be similar or dissimilar, but has some meaning to the student. Finally, the *ground* is the concept that is common to both the vehicle and the topic. For example, we might describe a child with ADHD as a "bull in a China shop." The child with ADHD is the topic; it is the subject of our metaphor and the concept that we hope to learn. The China shop is the ground for the concept, and the vehicle is the bull; both the vehicle and the ground may be that additional information provided in the sociocultural aspect of one's construction of the concept. In this case, the bull and the child with ADHD are similar in that they are perceived as wild and ungraceful, running into things and bouncing off walls. What Sadoski and Pavio (2009) noted was that the individual must understand the essence of the topic, but once the vehicle is applied, it dominates the individual's perception of the topic.

This notion that metaphor reflects the structure of our concepts is not complete in the sense that it only partially structures the concept because the speaker can extend the conceptual structure underlying the metaphor (Lakoff & Johnson, 1980). Structural metaphors structure one concept in terms of another (Lakoff & Johnson, 1980), and allow us to think of one thing in terms of another, the breaking down, or possibly the building up, of an abstraction. Examples of structural metaphors might include *spending time,* or *give me more time, as I haven't spent my time well.* In this example, the concept of time, a somewhat abstract concept, is being mapped onto the very concrete concept of money, using the overarching generalization of *Time is Money* (Lakoff & Johnson, 1980). Related to the structural metaphor is the ontological metaphor. This metaphor involves viewing something abstract as an entity in its own right and breaks down into four types: 1) causality, *the pressure of the job lead to his drinking*; 2) quantification: *a lot of* *chutzpah*; 3) identification or personification: *brutality of terrorism*; and, finally 4) referential: *wearing boots because of a fear of insects* (Lakoff & Johnson, 1980).

The final two types of metaphor that I will be addressing are related to each other in that they map one concept onto another spatial, concrete concept: the orientational and container metaphors. Orientational metaphor makes reference to directionality; in our culture, *up* is good, *down* is bad; the future is ahead of us; the past behind (Lakoff & Johnson, 1980). In the context of this study, the height of the water may *speed up* or *slow down*. The container metaphor utilizes the concept that something is contained in something else as a way of breaking down an abstraction (Lakoff & Johnson, 1980). We might talk about a clearing in the woods, where the trees form some fuzzy boundary, so that one may be *in the clearing*, or one may want to be *out of the woods*.

Covariational Reasoning and Quantitative Operations

In this section, I first define quantity and relate it to covariational reasoning, leading to a discussion of the quantitative operations Johnson (2015) proposed. Thompson (1994) distinguished between objects and qualities of those objects. To illustrate, I use Thompson's (1994) example: a child is aware that a passing car has the quality of motion, but is unable to conceive of the quality of speed or rate, that is, distance an object has moved during some amount of time. So when a student describes the motion of an object using the object's speed, this does not imply that the student understands the concept of speed or rate. The height of water in a bottle as the bottle is being filled can change, and it can change relative to the amount of change in the width or volume of the bottle. The student can observe that the height increases faster when the bottle is narrow, but this does not necessarily mean that the student is considering the speed or rate of change, only that the height seems to change differently when the bottle is narrower. Traditionally, teachers present speed algorithmically as the ratio of distance divided by time, but this begs the learning paradox (Steffe, 1991) because, for this to make sense, the student must already have the concept that motion involves two different quantities that vary in relation to one another. Even though the student has been taught the formulaic definition of speed ($s = d/t$), it does not mean the student has a conceptual understanding of what this means.

Covariational reasoning centers on the image of a constantly changing quantity that is connected to another changing quantity (Saldanha & Thompson, 1998), what Carlson, Jacobs, Coe, Larsen and Hsu (2002) described as reasoning by coordinating two varying quantities and noting how they change relative to one another. Students will often look at two quantities that can be linked together to form pairs of numbers. They can then locate these pairs of numbers on a coordinate axis and produce a plot or graph. Such a graph is static and the student may look at the graph as simply a picture, a representation of a situation. As the student gains experience working with quantitative operations, she may begin to consider the mental imagery, the physical nature of the situation and the dynamics of the relationship between the quantities, which allows her to hold the images of the changing height of the water in a bottle with the changes in the width of the bottle (Clement, 1989). This dynamic view of how the quantities are related may allow the student see the dynamic covariation in the relationship between the quantities (Clement, 1989).

Covariational reasoning can consist of holding a dynamic image in the mind's eye of two quantities interacting to produce a single, multiplicative value, or it can entail a static representation in which it is recognized that at certain instances of time, there are related values. Confrey and Smith (1995) demonstrated this static representation. Confrey and Smith provided an example of a student's work with a contextual problem that resulted in a data table exhibiting two sequences, one arithmetic and the other geometric. The student was able to link appropriate quantities to produce an exponential function relating the two quantities. The covariational approach used was one of static covariation in which the student considered *individual data points* from the domain and linked these to individual, corresponding points in the co-domain (Confrey & Smith, 1995); the student was comparing change in quantity to changes in the other quantity. The student using the static approach is reasoning with changes in data values. In contrast, a student may begin to reason by considering coordinated changes in each quantity, what Clement (1989) termed dynamic covariation. Saldanha and Thompson (1998) described continuous covariation as an outgrowth of static covariation and related it to one's image of time as a continuous quantity. Continuous covariation involves images of continuous change, rather than discrete imagery, which often corresponds to static covariation in that static reasoning considers change between individual, coordinated data points; continuous reasoning, then, is often dynamic in that it is operating on changes in each quantity.

Carlson et al. (2002) developed a progression of five mental actions, MA1 through MA5 that they equated to five levels of covariational reasoning. Coordination, or Level-1, involves simply coordinating the changes in one quantity with changes in another (Carlson et al., 2002). Behaviorally, the student is labeling axes and graphing the relationship as coordinated data points. In Level-2, termed Direction, the student is operating with the direction of change, that is, the output variable is increasing or decreasing as the input variable increases (Carlson et al., 2002); at this point, the student is considering directional change in quantities. When the student is describing the amount of change in the output, the student has attained Level-3, or Quantitative Coordination, and at Level-4, Average Rate, the student can verbalize a rate of change in the output with each unit of change of the input (Carlson et al., 2002), or continuous, dynamic covariation. Finally, in Level-5, Instantaneous Rate, the student can verbalize the concept of instantaneous rate of change over the entire domain, including concavities and inflection points, thus demonstrating a thorough understanding of a changing rate of change (Carlson et al., 2002). Although the students studied by Carlson et al. were successful in university mathematics courses, including second semester calculus, many did not reason consistently beyond Level-3, the level of quantitative coordination. The next section further elaborates on this issue.

Levels of Covariation: Comparison and Coordination of Quantity

Johnson (2015) argued that students' lack of progression beyond Level-3 might partly be due to how students think about rate as a relationship between varying quantities. Johnson argued that these students might be using two quantitative operations, which she differentiates: comparison and coordination. When a student is comparing quantities, she or he is associating amounts of change in quantities. For example, when water is filling a bottle at a constant rate, a student who is operating at the comparison level would be comparing a change in the height with a change in the width; that is, when the bottle is wide, the change in the height of the water is less than when the bottle is narrow. When the student is coordinating quantities, she is considering changes in one quantity that depend on simultaneous and continuous changes in the other quantity (Johnson, 2015). The coordinating student is noting, for example, changes in height that depend on the increases in volume, which involves images of continuously changing quantities.

Johnson (2015) proposed that Carlson et al.'s (2002) Level-3 could be split into two basic levels, comparison and coordination. Johnson proposed three sublevels at each. The key to Johnson's (2015) framework is that when the student is comparing quantities at the highest level, she is focusing on a single quantity resulting from a comparison between the changing quantities, which leads to coordination. When the student has moved to coordination, at Johnson's QO-1, the student is coordinating changes in one quantity with continual or simultaneous changes in another quantity. At QO-2, the student is coordinating how fast one quantity changes with continuous change in the other quantity, and using words like *faster* or *slower*; she is focusing on the concept of intensity of change. Finally, at QO-3, the student is dealing with a single quantity, and using phrases that address continuous change in speed like *decreasing faster* or *slower* when coordinating height and volume. This is summarized in Table 2.1:

Table 2.1: Quantitative Operations: Three Levels of Coordination and Comparison			
	QO-1	QO-2	QO-3
Quantitative Operation: Comparison	Comparing change in one quantity to change in another	Comparing amount of change in one quantity with amount of change in another	Determine single quantity indicating a comparison between change in quantities
Objects of Reasoning	Change in one quantity with change in a second	Amount of change in one with amount of change in another	Amount of change in quantity "per" amount of change in a second.
Quantitative Operation: Coordination	Coordinating change in one quantity with continuing change in another	Coordinating variation in intensity of change in one quantity with continuing change in another	Determine single quantity coordinating variation in intensity of change with continuing change in another
Objects of Reasoning	Change in one quantity with continuing change in a second	Variation in intensity of change happening in conjunction with continuing change in another	Variation in intensity of change in one quantity happening in conjunction with continuing change in another is a quantity itself.

Note. Adapted from Johnson (2015, p. 84).

Johnson's (2015) work helps the researcher and educator understand the operations a student is using as they reason through a covariational task, as long as the student is at the level where she is comparing quantities, that is, at Carlson et al.'s (2002) Level-3, or quantitative coordination. At this level, according to Carlson et al., the student is attending to the direction of change and the amount of change in broad terms. If the student is reasoning at Level-3, having moved beyond simply noting the direction of change, then Johnson's framework provides a valuable tool in this analysis.

CHAPTER III

RESEARCH METHODS

Methodology

This thesis is a qualitative, grounded theory study using a secondary analysis of data obtained from work carried out by Johnson in 2014 (Johnson, 2015), and is based on the inseparability of language, image and gesture. The use of language, in the form of metaphor and of gesture can give the researcher information about the underlying visual imagery the student is employing and thus lead to inferences about the quantitative operation being used. The analysis will focus on four students who presented with interesting, illustrative interchanges that demonstrated how their use of gesture and accompanying language and their use of metaphor reflected their use of quantitative operations and the associated mental images.

Subject Background and Description, Data Collection

Five students participated in this study, from which I selected four students for analysis because the four students analyzed presented with more data that could lead to greater insight in their underlying imagery and quantitative operation. I did not include the fifth student, Paola, following a very preliminary and rudimentary quantitative analysis that showed minimal changes in her use of metaphor and gesture. This led me to believe that greater insight could be had by focusing on the other four students, Ana, Lucia, Sofia and Elisa. Further, I did not include the work products from all the students because not all were meaningful or complete. The students selected for this activity were Mexican-American, ninth-grade females at a sixth – twelfth grade school. The five students took part in task-based, clinical interviews in two sessions. In the first session, there was a group of three students, Ana, Lucia and Sofia; and a pair, Elisa and Paola. In the second session, Elisa and Paola worked together, as did Sofia and Lucia; Ana worked alone. Audio and video recordings of each session were made and I analyzed these. I selected elements from the interviews of four students, Elisa, Ana, Sofia and Lucia, to discuss because these students demonstrated use of the quantitative operations described by Johnson (2015) and their use of metaphor or use of gesture provided insight. I analyzed all excerpts foregrounding gesture because gesture seems more nearly autonomic due to neurological associations and can represent the underlying mental images (McNeill, 2005).

Tasks Designed to Foster Students' Covariational Reasoning and Quantitative Operations

The tasks required that the students sketch a curve depicting the volume of a bottle as a function of height of the liquid, based on work originally carried out at Shell Centre for Mathematical Education, (University of Nottingham) (1985) and by, among others, Carlson et al. (2002) and Johnson (2012). The filling bottles I used were animations that appeared in a task published online (Meyer, 2014) and used by Johnson (2015) in both the pre and postinterview tasks; in the preinterview task, the bottle was triangular in the lower portion with a cylindrical

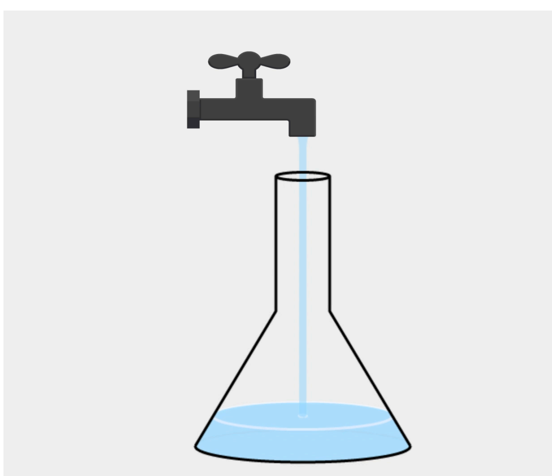

Figure 3.1. Triangular-shaped bottle used in the pre-interview video.

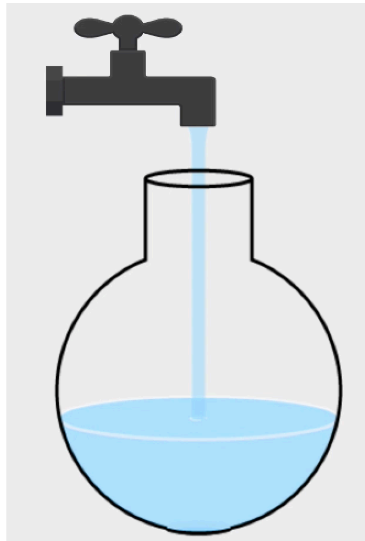

Figure 3.2. Spherical-shaped bottle used
in the post-interview video.

top portion (see Figure 3.1). In the postinterview task, the bottle was spherical, as shown in Figure 3.2.

Following some discussion about the task, the students repeated the task, however, in the second task, the students analyzed an animated, spherical bottle with a cylindrical top portion. The table below presents the order of questioning for both the preinterview and postinterview tasks:

Table 3.1: Schedule for Preinterview and Postinterview Questions	
Preinterview	Postinterview
1. Show video of filling bottle. – What changes and what stays the same.	1. Show video of filling bottle. – What changes and what stays the same?
2. How is the height of water changing as time is elapsing?	2. How is the height of water changing as time is elapsing?
3. How is the volume of water changing as time is elapsing?	3. How is the volume of water changing as time is elapsing?
4. How is the volume of water changing as the height of the water is increasing?	4. Are volume and height changing together, how?
5. Predict, then sketch a graph that relates the volume of water to the height of the water. If students are struggling prompt them to consider how the volume changes when the height goes up just a little bit	5. Sketch a graph that relates the volume of water to the height of the water. (labels with volume and height on the vertical and horizontal axes, respectively)
	6. Use volume and height to explain why the graph looks the way that it does

For both tasks, I transcribed the audio recordings and coded these transcriptions and the original videos for type of metaphor and gesture. From this analysis, I made inferences about the level of quantitative operation used.

Data Analysis: Coordination of Quantity and the Use of Metaphor and Gesture

I analyzed videos and transcripts of interviews, conducted by Johnson in 2014, from four students; the interviewer carried out the first interviews before discussion, the second, after discussion. I used the software MAXQDA 11, produced by VERBi GmbH, for data analysis and transcription. My initial pass consisted of watching the videos to get a broad overview and a sense of the students' gestures and speech patterns. I then reviewed the transcripts and coded for the general level of quantitative operation, either comparison or coordination (Johnson, 2015), see tables 3.2 and 3.3. I looked for terms that might indicate they were using coordination or comparison when referring to the quantities in question. Examples of these terms include, but are not limited to, the words or phrases: rate of change, slope, faster, slower, increase/decrease, decreasing increase/increasing decrease, not changing, and remaining constant.

I used the results of the first pass in the second pass, and began analyzing interchanges for use of metaphor. When metaphorical language was determined, I coded the term or phrase as *Structural, Orientational, Container* and *Ontological* as outlined in Table 3.2.

Table 3.2: Summary of Structural, Orientational, Container and Ontological Metaphors

	Structural	Orientational	Container	Ontological
Definition	Maps one concept onto another	Orientation in space	Something contained within something else	Viewing an abstract concept as an entity; concrete
Example	*Spending time*; maps time onto money "…and since it's like *a triangle*…"—Elisa	*Speeding up*; up implies more; down less *What lies ahead of us;* the future is in front "…it *goes down* because it's getting smaller…" —Elisa	*Getting out of the woods*; the trees form a boundary or container *In the clear*; in a clearing, perhaps surrounded by trees "*…out of nowhere…"*—Lucia	Causality: *Pressure led to drinking* Quantification: *a lot of Chutzpah* Personification: *brutality of terrorism* Referential: *avoids closets for fear of spiders* "…the height, *just keeps going…"* —Elisa

Where it was appropriate to capture the multi-dimensionality of the metaphor that both Lakoff and Johnson (1980) and McNeill (2005) discussed, I would code a second metaphor. An example of this multi-dimensionality might be the metaphor; "The test was a *walk in the park*." This metaphor falls in the category of structural, but could also fall into the second category of ontological. Structural, in that it represents the mapping of the concept of a test onto the concept of a park; ontological because it is viewing something abstract (the difficulty of the test) as a concrete entity (a park). The impression of a walk in the park is that it is pleasant, safe and not taxing. Thus, to refer to a test as being a *walk in the park* implies that it was easy, there was little stress, and that a high grade is expected.

The gestures I coded for in the third pass were those described by McNeill (2005) and used by Edwards (2009). The four gestures used were the Iconic, Metaphorical, Deictic and Beat. I summarized these gestures in the table below, Table 3.3.

Table 3.3: Summary of Structural, Orientational, Container and Ontological Gestures

	Iconic	Metaphorical	Beat	Deictic
Definition	Acts out a shape or process	Represents something abstract	Represents emphasis	Identifies space, size or location
Example	A triangular shape; *fingers pointing up, moves hands upward in straight lines, showing a triangular shape* —Elisa	Suddenly something happens; *Hand at shoulder height (elbow resting on table), makes a flicking motion with her fingers, flicking something away from her face* —Lucia	Emphasizing a point; *Fingers of both hands pointing up, but cupped, move in together and out, 5 times* —Elisa	Demonstrating height; *raises hand up from about 1 inch to head-high*—Elisa

I first viewed the video without sound so that I could capture all movement and as an attempt to lessen the influence of the students' utterances. After identifying these gestures, I classified them on a multi-dimensional scale (see Figure 3.3), based on McNeill's (2005) description of the multidimensionality of gesture. For example, a student might describe the domain of a function as an open interval and cup her hands around a space, accompanied by a couple of quick, vertical movements. This gesture would be an iconic gesture, but would have beat components. I coded this gesture primarily as an iconic gesture, but with a beat component signifying the student's confidence in her response, placing it in Quadrant IV.

Additionally, abbreviation was used for ease in recording and reporting results and this is presented in the following table, Table 3.4. Within the transcripts, gesture is italicized and set off with the use of italicized brackets. Metaphor is italicized and followed by a description of the type of metaphor set off by brackets that are not italicized.

Table 3.4: Summary of Abbreviations Used in Transcripts and Gesture Description

Gestural Notes	Po: Pointer or Index Finger R: Right L: Left	MF: Middle or Third Finger	RF: Ring or Fourth Finger	Pi: Pinky of Little Finger	CW/CCW: Clock or Counter Clockwise
Transcript Notes	Student	Date	Transcript Location	Time Stamp	Interview Number
	E: Elisa A: Ana L: Lucia S: Sofia	YYMMDD of interview	¶ ###: Paragraph number in transcript	HH:MM:SS	Int#
Gesture	For example: *[RPi was raised]*				
Metaphor	For example: the top part of the bottle *filled up* [container metaphor]				

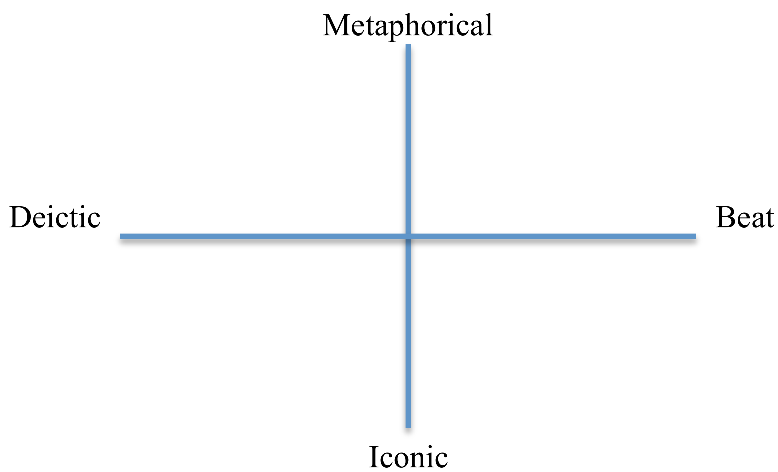

Figure 3.3: The four dimensions of gesture represented a coordinate axis.

Edwards (2009) reported that metaphoric gestures have iconic components. This naturally follows because the hands and arms are physical—concrete—and the metaphor represented is abstract; thus, the iconic aspects of the gesture provide some inferential support for the abstraction and imagery the students are referencing. In the coding scheme that I am using, that is, a coordinate axis, I have placed the metaphoric gesture opposite the iconic gesture. While there are elements of both in these types of gestures, I will be looking toward both the beat and deictic gesture as the elements uniting metaphor and icon. In Edward's study (2009), she was looking at how student teachers used gesture when discussing fractions. Edwards (2009) noted that many of the iconic gestures were not used to represent concrete items, but were used to represent mathematical symbolism; furthermore, in some instances, this metaphor-iconic gesture was used to represent not only the mathematical symbol, but also the operation it represents. Because I investigated students' use of gesture and metaphor that represent the underlying quantitative operation and imagery, and how the quantitative operation and imagery may change with exposure to tasks involving covariational reasoning and quantitative operations, I would expect some differences between my coding and Edwards' (2009).

I conducted all passes on two sets of interviews. The first set was from the preinterview period and the second set was from the postinterview period. My second pass and analysis consisted of rating the students' articulations based on Johnson's (2015) scale of quantitative operation. I was able to identify the students' quantitative operation by considering the students' descriptions of how the quantities of height of the liquid and the volume, or width, of the bottle changed relative to one another, as per Table 2.1. These descriptions were facilitated by the students' use of gesture and metaphor, which could alter my conclusion that was initially based on the students' verbal description.

Whereas gesture may be a more valuable analytic tool, it is also one that is difficult to use. In many instances, analysis of gesture required multiple passes of specific frames in the video because of the subtlety of the gesture and the multiple gestures that one can identify within one sentence. Furthermore, with experience, the interpretation and coding of gesture became more refined. For example, I first interpreted Elisa's use of a spiral gesture representing water flowing down a drain as metaphorical; however, with experience, it became evident that this was an iconic gesture. This came about as a result of my seeing more gesture that was clearly iconic, or that was metaphoric. Indeed, the researcher is a critical research tool. With the experience of seeing gesture that was definitely iconic, and considering it in the context of the task, for example, mimicking the shape of the bottle, I was able to more confidently identify certain gestures that could be ambiguous. Elisa's spiral motion is an excellent example of this progress.

CHAPTER IV

RESULTS

A Look at the Data: The Broad Overview

In this chapter, I will present my results by discussing the students' responses to interviewer questions and analyzing their gesture and metaphor. The students used a combination of metaphor, gesture, and verbal description in their discourse about how the height of water pouring into a bottle changed as the bottle's shape changed. Students rarely used the word volume, but appeared to think of volume as how much the bottle could hold. Instead of using volume, they appeared to refer to the bottle's volume by talking about the bottle's width. From the discourse of the four students I chose to analyze, all appeared to be comparing quantities at Johnson's (2015) QO-Comp-1 or 2 in most of their exchanges. In several instances, the graphs produced by the students were more representative of width as a function of height where height was plotted on the *x* axis and "width" on the *y* axis, and are, therefore, incorrect because the students were presented with coordinate axes marked with volume on the *y* axis and height on the *x* axis, or volume as a function of height.

The Students' Use of Metaphor and Gesture

Elisa: Pre-Interview Use of Metaphor and Gesture

<u>Elisa's Discussion of Volume.</u>

In the preinterview portion of the study, the students were referring to a triangular-shaped bottle, depicted in Figure 3.1. The preinterview video begins with some exchanges about the task. At the start of this portion of the video, the researcher is asking whether Elisa and Paola can graph how much water is in the bottle. The discussion immediately turned to how to graph when there are no numbers to identify locations on the grid. The exchanges about volume began with the researcher asking Elisa about the volume:

> R: (140421_Elisa&Paola_Int1_¶ 293) So, if I paused it here, and I say it went up this much in height, do you think I got a lot of volume or a little bit of volume?
>
> E: A lot of volume.
>
> R: So if you plotted a point…
>
> E: So it would be like, going down *[moves hand diagonally in a downward motion on the graph, from left to right in a straight line—deictic iconic gesture defining the space of the graph while representing the shape of the graph* (see Figure 4.1); *motion repeated once completely, and once, ending half way—gesture is iconic.]*
>
> R: What would be going down?
>
> E: The graph.
>
> R: How do you know?
>
> E: Because *it starts off big and it goes small, and right here it's the same, it would be like, constant* [structural metaphor] *[at "constant", moves hand—holding pen—across coordinate space, parallel to height axis—iconic]*.

Elisa considered the triangular-shaped portion of the bottle and discussed the changes in the shape of the bottle by noting that the bottle is large at the very bottom and then the width (volume) steadily decreases until the top, cylindrical portion. The iconic gesture that accompanies the structural metaphor defines the graph space on a coordinate axis with volume on the *y* axis and height on the *x* axis (Figure 4.1), so she is indicating that the change in the volume is decreasing as the height continues to increase. It is unclear if she was actually looking at the changes in both quantities, or if she was focusing only on the volume, that is, that the volume was decreasing without regard to the height. It is interesting to note that her gesture was a rapid sweeping motion from the general area of the upper left corner of the graph space, extending to the lower right corner and not referencing specific areas by pointing to one place on the axis then another, despite the researcher asking, "So if you plotted a point … " (see Fig. 4.2).

I am interpreting Elisa's response to mean that she was not thinking about any numerical

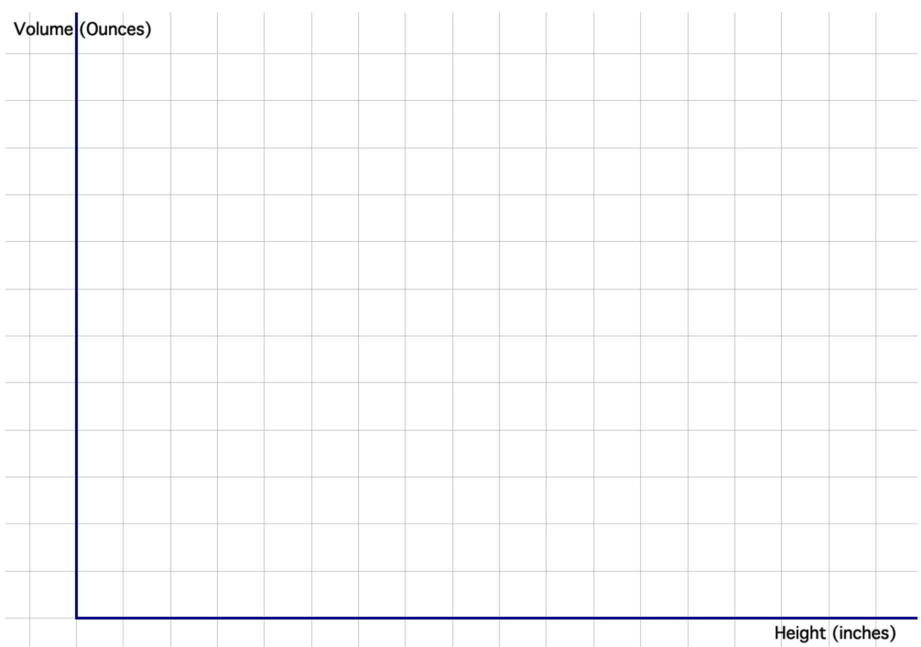

Figure 4.1. Coordinate axes that the students were asked to use in graphing portion of task.

representation of the situation because there was no scale associated with the graph; only the axes were labeled, so it seems reasonable to infer that she may also have thought that, since there was no scale associated with the graph, there was no identifiable starting point or frame of reference for her point and subsequent graph. Furthermore, as Johnson (2012) noted, if students do not have numbers available to use, they may be less likely to focus on algorithms. From this, I could speculate that she is reasoning covariationally, using QO-Coord-2, where she was considering the amount of change in the volume as the height changed continuously as indicated by the smooth sweeping gestures.

In this instance, the researcher set the question up by asking if Elisa could envision or physically plot a point, yet Elisa's response countered the question by not only not indicating a point, but by gesturally extending the graph. Her gesture was a quick, downward motion from the upper left corner of the graph space to the lower right with no hesitancy from start to finish. The speed of her hand and the surety of movement would seem to indicate that this also has a beat element to it that could demonstrate certainty. Verbally, she was simply describing how changes in the volume and height coordinate with one another by relating these to the bottle. Her initial gesture made a negatively sloped line across the graph space indicating that she may have been envisioning a constant decrease in volume, relative to height. The next iterations of the gesture, she repeated the motion twice, however, with a downward arc to these repetitions that seemed to add emphasis to her point. However, in these subsequent gestures Elisa made a definite arcing gesture as her hand moved across the graph space, which suggests that she may also have been considering that the volume was decreasing at a decreasing rate until it remained constant. Her graph, as seen below in Figure 4.2, indicates that she considered the decrease in volume as decreasing until a certain point at which the volume stopped decreasing, and held steady. The constant volume to which she refers in her graph is represented by her last gesture where she moved her hand across the graph, parallel to the *x*-axis as she said the word, "constant".

Elisa's Analysis of Volume.

Elisa's interpretation of volume in this task was unique, as demonstrated through her use of gesture and metaphor. Elisa's use of gesture when she responded to the researcher's question about whether there would be a lot or a little bit of volume, consisted of a linear, sweeping motion extending from the upper left to the lower right portion of the graph space (140421_Elisa&Paola_Int1_¶ 293). The graph indicated by her gesture suggests that the volume is decreasing as the height is increasing; from the perspective of only the volume, I could infer this to mean that the bottle is emptying. Since the height is also increasing, according to the gestured graph, this leads me to conclude that she was considering the decreasing volume of the bottle as water flows in, not the amount of water in it. Alternatively, she appears to be focusing on the width of the bottle, which does decrease as the height increases because the bottle is triangular-shaped. In fact, the bottle's volume is continually increasing but at a decreasing rate as the height from the bottom increases.

Might Elisa's gesture be a case of Alibali and Goldin-Meadow's (1993) discordant gesture? Alternatively, is she, in fact, considering the volume of the bottle and not the amount of liquid the bottle contains? I interpreted her gesture and metaphor as representing the volume of the bottle relative to the height of the liquid from the bottom, rather than her using a discordant gesture because her description of the volume of the bottle, her gesture and subsequent graph all agree. Her description of the graph and the subsequent drawing are both indicating that the graph "starts off big" and gets smaller until it reaches the cylindrical portion of the bottle when the graph becomes constant.

Elisa coordinated (Johnson, 2015) changes in the bottle's volume relative to continuous change in the liquid's height (QO-Coord-2) by her use of the structural metaphors, *big, small and constant* and the shape of her graph. The iconic sweeping gesture on the graph space that accompanied the structural metaphor of *big, small* and *constant* defines the graph space on a coordinate axis representing volume as a function of height, so the iconic aspect of the gesture, the downward motion that represents the graph, would indicate that she is coordinating the changes in the two quantities. She is indicating that the change in the volume is decreasing as the height continues to increase, but the decrease is linear as demonstrated by the linearity of her gesture. I can conclude from this and the fact that she did not plot a point, despite the researcher asking her to, that her reasoning about the covariation between the quantities was smooth and continuous. Her gesture was a quick, downward motion from the upper left corner of the graph space to the lower right with no hesitancy from start to finish. The speed of her hand and the surety of movement would seem to indicate that this also has a beat element to it demonstrating certainty. This gesture also foreshadows her interpretation in a subsequent interchange. Verbally, she is simply describing how changes in the volume and height are coordinated with one another by relating these to the bottle; yet, the continuity and smoothness of her gestures suggest more.

<u>Elisa's Discussion of the Graph.</u>

An exchange later, the researcher asked Elisa, "And tell me what's happening with that (referring to Elisa drew, as seen in Figure 4.2)."

E: (140421_Elisa&Paola_Int1_¶ 309) Oh my God, I think that, okay, so since the *bottom is like bigger or wider* [structural metaphor] *[both hands, tilting inward, move away from each other, sharply and crisply, and repeats—an iconic gesture with a beat component]* so there's more volume in there, and since it's *like a triangle* [structural metaphor] *[fingers point up, move hands upward in straight lines, showing a triangle shape—deictic iconic gesture]*, *it goes down because it's getting smaller* [structural metaphor] *[repeats triangular-shaped motion—beat iconic]*, but then when there's *[referring to the point on the bottle where the triangular portion merges with the cylindrical top—her hand makes a spiraling motion, pointing with pointer and middle fingers, moving her hand toward the computer—an iconic gesture]* the cylinder, top or whatever, it's constant, it's the same

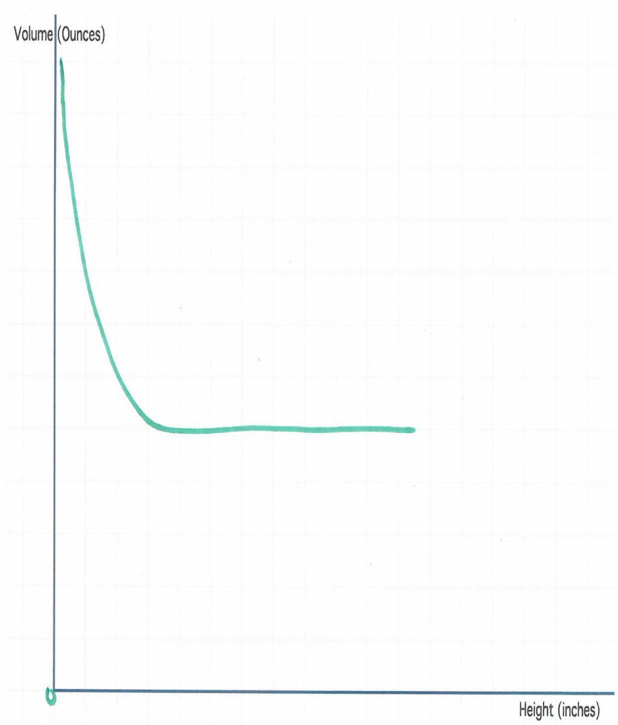

Figure 4.2. Elisa's graph depicting the height of water

[referring to the cylindrical portion—fingers pointing front, rigid, hands moving vertically and parallel, an iconic gesture], so it stays the same *[hand held rigidly, palm down, fingers outstretched and together, makes horizontal motion with hand twice—representing a beat iconic gesture]*.

In this series of gestures and description, Elisa was equating the width of the bottle to its volume and the fact that it is triangular-shaped. It has more width, therefore more volume, in the bottom. Elisa indicated that the bottle's width decreases as the bottle narrows toward the upper, cylindrical portion because it is triangular shaped. Referring to the point where the triangular portion of the bottle merges with the top, cylindrical portion, she made use of what I initially interpreted as a metaphorical gesture: a spiraling motion made with her pointer and middle fingers, moving away from her, toward the computer screen. I initially interpreted this as a metaphor for water accelerating down a drain; however, in subsequent passes, I decided that this gesture was iconic—still representing the same visual image of water spiraling down a drain, but that this was not a metaphorical gesture. I considered this gesture to be iconic because it did not seem to be representing an abstraction as metaphorical gesture does, but was mimicking the water's motion as it spirals down a drain much the same as a triangular motion is said to represent the triangular bottle—or any triangle, for that matter.

In the first part of the interchange, Elisa was discussing the volume as it relates to the bottle's shape; her graph represents an emptying bottle because the volume is decreasing, but, again, because the height is increasing, the graph appears to be referring to the remaining space in the bottle. She was consistent in her use of iconic gesture and structural metaphor. The spiraling motion she used iconically represents the motion of water as it speeds up and spirals down a drain. It is possible that her spiraling gesture represents her physically searching for the correct portion of the diagram on the computer screen; however, she had just pulled her hands from the screen, while continuing to look at the screen. She then refers to the top part with an iconic gesture representing a cylinder and talks about how the volume is constant. Elisa ends with a strong, iconic gesture (the beat component) indicating that the volume and height are changing at the same rate. From these uses of gesture, metaphor and verbal descriptions, I would infer that Elisa's quantitative operation is QO-Coord-2.

<u>Elisa's Analysis of the Graph.</u>

Elisa's iconic gesture (140421_Elisa&Paola_Int1_¶ 309) that I am interpreting as representing an image of water spiraling down a drain allows for some inferences about the images she may be holding in her mind. She begins this section with a structural metaphor, "…the *bottom is like bigger or wider…*" coupled with an iconic gesture *[both hands, tilting inward, move away from each other, sharply and crisply, and repeats]*. I interpreted these as representing an image in her mind of a slowing, or decreasing change in volume because the bottle is triangular and has less volume on the top while the height continues to change (QO-Coord-2). She then stated, "…but then when there's *[her hand makes a spiraling motion, pointing with pointer and middle fingers, moving her hand toward the computer—an iconic gesture]* the cylinder…" This progression of the verbal descriptions and their accompanying gestures seems to indicate that her quantitative operation shifted toward QO-Coord-3 because she seemed to be considering the velocity of the height with continuous changes in volume. Her spiraling gesture implies that she envisioned the velocity of the water's height increasing as the water spiraled "down a drain"; that is, when the water reaches the narrow portion of the cylindrical top, the increase in height speeds up as it enters the cylindrical portion of the bottle, where the change in volume remains constant. Finally, when the water is filling the upper, cylindrical portion, she motioned that the volume and height are both changing at a constant rate.

Thus, with the addition of her graph, subsequent gestures, metaphor and verbal descriptions, I can infer that she is considering an image of emptying, or of decreasing volume. This is consistent with her image of water accelerating as it goes down a drain—the water's velocity is increasing and the amount of liquid in the bottle is decreasing, although in this case, the remaining volume of the bottle is decreasing. I can conclude that she is coordinating the water's height with the remaining volume in the bottle based on the bottle's shape, something no other student did and operating at QO-Coord-3 because I am inferring that her gesture represents her use of the concept of speed.

Elisa: Post-Interview Use of Gesture and Metaphor

Elisa's Discussion of Height.

During the postinterview, Elisa was discussing the relationship between the height and the volume, using the bottle depicted in Figure 3.2. In this example, the researcher begins:

R: "…could you show me how the height would go?"

(140521_Elisa&Paola_front2_¶18). Elisa responds:

E: (140521_Elisa&Paola_front2_¶ 19) Like—could—the— height, would like a constant—well—not a constant—but like—a place where it goes slower, and this would go faster, and once it gets to the top, it's like, *whoosh* [structural metaphor] — increases by a lot— like goes up, like, let's say right here, —so it's— like right here—so it would just like, it would go fast— I don't know how to explain it.

When the water reaches the top, cylindrical portion, where she used an iconic, spiraling gesture in the preinterview, she uses no gesture but employed a common structural metaphor that is similar to other words used in other transcripts: the word *whoosh*. This seems consistent with her spiraling gesture in the preinterview in that the word *whoosh* is used to signify something passing by very quickly or moving quickly, akin to water speeding up as it goes down a drain and funnels into the narrow opening of the drain.

During the post-interview (140521_Elisa&Paola_front2_¶18), when she is discussing the height relative to the volume, she appeared to coordinate the speed of changes in the height of the water with a continual change in the bottle's width (QO-Coord-3). She used no gesture, however, she described how the speed of the height varies depending on where the water level is in the bottle, using terms like "slower" or "faster". Elisa finished her description of the change in the water's height by using a common structural metaphor *whoosh*. This metaphor and its use of onomatopoeia is often a mapping of motion onto the motion of a strong wind, or of a jet or fast-moving vehicle passing by and may have been used in place of the spiraling gesture from the pre-interview. Thus, it would appear from her use of *whoosh* that she is considering rapid motion consistent with the water's height at it enters the narrower portions of the bottle.

<u>Elisa's Discussion about Volume.</u>

In a subsequent interchange about volume, the concept of speed comes up. The researcher is asking her to explain the volume, "… And if you were to do the same thing for volume, what would it do?" (140521_Elisa&Paola_front2_¶ 20). Elisa responds (¶ 23):

> I think the volume would be like—um—it would only change speeds according to like how much—*[hands come off keyboard, F curved slightly, moves H apart, then brings*

together so fingers touch 2x—beat iconic], how big—like *right here it's less wide than right here* [container metaphor]—it would also change speeds here—again—so like, right there it would be like going kind of the same height, but like, I make it like right here, it would like *slowwww dowwwwn* [orientational metaphor]. I don't—*[shrug]*.

Her beat iconic gesture would imply that she is using the width of the bottle to identify where the speed of the volume will change and how it will change. This gesture was repeated, demonstrating the beat aspect of the gesture, but also demonstrates that she is considering the bottle's spherical shape as relating to the height of the water. She concluded this portion with an orientational metaphor, *slow down*.

In this interchange (140521_Elisa&Paola_front2_¶ 20), Elisa is discussing the rate of change of the volume, relative to the shape of the bottle. She appeared to be coordinating the change of the height, in her words, "the change in speed", with the shape or volume of the bottle (QO-Coord-2), coordinating the variation of change in height with the continuous changes in the bottle's volume. She noted that the change in the height would *slowwww dowwwwn* employing both orientational metaphor and onomatopoeia, at the widest portion of the bottle. I am interpreting her use of onomatopoeia as a way of adding emphasis to her speech, which is consistent with her use of beat gestures in both this interchange and in the previous.

Ana: Pre and Post Interview Examples of Metaphor and Gesture

<u>Ana's Discussion of Change in Both Height and Volume.</u>

In Ana's preinterview, the initial exchange, Ana, Sofia and Lucia are discussing the animation of the filling triangular shaped bottle. In response to the researcher's question, "So, tell me, what do you see when you are watching this?" (140421_AnaLuciaSofia_Int1_¶ 215), Ana begins:

> Ana: It goes with the shape…(140421_AnaLuciaSofia_Int1_¶ 218)
>
> L: (140421_AnaLuciaSofia_Int1_¶ 223) There's like more space over here so it's going slower. And it starts going fast, because there's not as much space, so it goes really fast.
>
> R: But it doesn't look like the stream is changing, like the—
>
> Ana: The amount of water that's *[Hand is up by her face, opens fingers as she says, "gushing"—metaphoric-deictic gesture]* gushing*?* (140421_AnaLuciaSofia_Int1_¶ 225)

Lucia, Sofia and Ana are discussing what the height of the water is doing as the bottle is being filled at a constant rate. Gesturally, at paragraph 225, Ana uses a metaphorical gesture that would seem to convey imagery related to a geyser, or a broken water pipe when she opens her fingers in front of her face. I coded this also as deictic because her hand was up by her face, from which I infer that she is envisioning a fountain of water.

Ana is using a metaphor that is very similar to Elisa's use of the word, *whoosh* (140521_Elisa&Paola_front2_¶18), to describe the behavior of the liquid at that same point of the bottle. The words, *whoosh* and *gush* are onomatopoeic metaphors, although the elicited imagery to me is different. In my mind, *whoosh* carries an image of wind, or something passing by very quickly; Webster defines it as, "to rush past, or to gush out". On the other hand, *gush,* to me, carries a liquid imagery and is something that broken water mains or geysers do; Webster defines it as, "to pour, issue, flow, or spout copiously or violently". In both cases, however, both girls, Ana and Elisa, appear to be considering similar imagery when the water gets to the cylindrical portion of the bottle. Elisa does not use a gesture accompanying her use of *whoosh*, so I can only interpret the imagery based on her word usage.

This first example in the postinterview focuses on Ana using the bottle depicted in figure 3.2. The researcher began the postinterview by stating, “So, just talk to me about what’s changing and what’s staying the same.” At paragraph 4 (140527_Ana_front1), Ana said, “It *fills up* [orientational metaphor (referring to volume of container)] fast from here and then it *slows down*. [orientational metaphor]” She continued:

> A: (140527_Ana_front1_ ¶ 6) And then as it gets like around here, it kinda starts going up fast, because it’s like *[brings hands up, bent slightly forward at fingers, and makes triangular shape--iconic gesture*] *coming in* [container metaphor]; and here, it’s like small, like this part is smaller than the others, so you can quickly think, or see that it *fills up* [orientational metaphor] faster.

Several things in this interchange merit note. At first glance, her gesture appeared somewhat discordant because her iconic gesture more closely resembled the triangular bottle from the pre-interview activity. Her description would indicate that she was referencing the upper portion of the spherical shape where the volume decreases as it moves in to the cylindrical top, thus giving the appearance of a triangular-shaped bottle.

In this first example interchange (140527_Ana_front1_ ¶ 6), Ana is coordinating quantities by comparing changes in the shape, or the perceived volume, to the continuous change in the height of the liquid flowing into the bottle (QO-Coord-1). She begins by stating, “…it *fills up faster*.” Ana’s use of the orientational metaphor, *fills up,* refers to the volume; that is, the amount of liquid in the bottle is increasing, and her use of the word “faster” implies that she is reasoning about how fast the volume is changing, which she is coordinating with the continuous change in height. Her change in phrasing before her observation about the rate of change of the volume, "… you can *quickly think*, or *see* …" seemed to represent a shift between a cognitive process (imagination) and a physical process (seeing). This could imply that she was not reasoning so much about what she saw in the animation, as she was imagining the process of the bottle filling up. Ana may be envisioning the process and reasoning about both quantities simultaneously.

<u>Ana's Use of Mental Imagery Instead of Technology.</u>

In the response to the above statement, the researcher asked in paragraph 7 (140527_Ana_front1):

> R: Could you know that it would fill up faster or slower without seeing the bottle filling? Like, could you predict that if we had not pressed play, would you have been able to tell me if the bottle was going to fill up slower or faster?
>
> A: (140527_Ana_front1_ ¶ 10) Yes.
>
> R: Ok.
>
> A: Because of the first one we did, where we got that the little *smallish space* [container metaphor] that was to *fill the quicker it fill up* [orientational metaphor], so this is kinda *small at the bottom* [container metaphor], but then as it goes up, it starts getting like *[Left Hand (LH) up from lap to above table, rests on table, thumb (T) and pointer (Po) spread, other F slightly spread, Palm Forward, Open Hand, raises up—iconic gesture]* bigger, so then take a smaller—longer time to *fill up* [orientational metaphor], but then as it like *[T and Po spread, F curing in and loosely together, then move together 2 times—beat*

iconic], curves in, it's like *there's less, like, there's still kind of less* [container metaphor] *[F round, forming spherical shape—deictic iconic]* so it *fills up* [orientational metaphor] faster. And then here it just like goes into, like a *[F curl, T and Po come together, H moves up and down—beat iconic]* little cylinder and … so that goes really fast.

Ana began by describing a portion of the bottle as a smallish space, using a container metaphor to refer to the upper portion of the triangular bottle from the preinterview. She compared the lower portion of the spherical bottle to the upper portion of the triangular bottle, both of which are less voluminous. She then began to discuss the middle section of the spherical bottle where it "…starts getting like bigger…" consistent with her iconic gesture of a spreading hand, and paired this with a statement referring to how much longer it will take to fill up, an orientational metaphor, and then moved into describing the spherical portion of the bottle using iconic gestures accompanied with a repeated, beat movement. She concluded with both an iconic gesture and a statement that the volume remaining in the bottle was less "curves in, it's like *there's less, like, there's still kind of less* [container metaphor]", referring to the upper part of the spherical portion of the bottle. Then, again with consistent gesture and language, she described the cylindrical portion, accompanied with the observation that it—the height—will go really fast, again seeming to compare the change in the height with the width.

At (140527_Ana_front1_ ¶ 10), Ana compared the lower portion of the spherical bottle to the upper portion of the triangular bottle, both of which are less voluminous, and therefore coordinated the change in the height with the continuous change in the volume (QO-Coord-1). She stated that the bottle fills up faster; that is, the height will increase faster because there is less volume. She then began to discuss the middle section of the spherical bottle where it "…starts getting like bigger …" consistent with her iconic gesture of a spreading hand. She then was coordinating the continually expanding width, or increasing volume with the change in the height by her use of the phrase, "…longer time to fill up …" She is coordinating the continuous change of the volume (use of word "fill") with smaller changes in height. The interchange end is marked by Ana's use of the term "goes really fast". Despite her reference to the quantity of speed, her iconic gesture indicates that she is referring to changes in the height while the volume remained constant thereby coordinating continuous changes between the height and volume (QO-Coord-1).

Lucia: Pre and PostInterview Examples of Metaphor and Gesture

<u>Lucia's Preinterview Discussion.</u>

Lucia began by comparing the quantities of volume and height, and noted that the height increased "really fast" (140421_AnaLuciaSofia_Int1_¶ 216) and this was accompanied by a metaphoric gesture in which her hand moved up, to the right and then at the top, her fingers waved away as if the water went off into space, similar to Ana's gushing gesture. She continued comparing height and volume in paragraph 223, using structural metaphors: "There's like more *space over here* so it's going slower. And it starts going fast, because there's *not as much space*, so it goes really fast" (140421_AnaLuciaSofia_Int1_¶ 223). In this instance, she was using a structural metaphor to refer to the volume and was referring to it as *space*. She was comparing the change in the height, "…it goes really fast…" to the apparent volume, or space, employing a structural metaphor. This is demonstrated in paragraph 227, where she said, "It's just like the…, it's like *the area* [structural metaphor], I guess [Brings hands up, makes cupping/enclosed space with both hands—Iconic], it's smaller than at the bottom [then interlocks fingers at "smaller"—

deictic metaphor gesture]. That probably is—that makes it fill up faster" (140421_AnaLuciaSofia_Int1_¶ 227).

Lucia's Post-Interview Discussion.

In response to the researcher's initial question referring to the animation, "…when we press play, tell me what you're gonna see" (140521_Sofia&Lucia_front2_¶ 3).

> Lucia (140521_Sofia&Lucia_front2_¶ 5): Yeah, and then it goes like [F forward, P down, just above table, moves H up, then rotates hand, F still forward, Pi down and waves to right—iconic gesture] medium [H lowers about half-way, brief downward motion, then stiff hand up slightly and holds at the word, "bottom"—deictic iconic] like at the bottom, and then it goes like higher and higher [H goes up to face-level… —iconic] and it would like go […then F slowly waved away from face—metaphoric] faster.

In this passage, she was using predominantly iconic gesture, representing the actual height of the liquid; however, her final gesture was a metaphoric one that she has used previously: the waving of the fingers away conjuring up the image of something moving away, off into the distance.

In paragraph 9, she used an interesting container metaphor but coupled it with a beat gesture. In paragraph 9, she stated "Like if it goes up it's gonna go faster, because it has like the [F pointing up and curved away from the middle, indicating spherical shape—iconic] shape ["shape" is accented by beat, LH drops at the wrist, RH flips out—beat] it wouldn't *allow water to fit* [container metaphor], so it's gonna go faster" (140521_Sofia&Lucia_front2_¶ 11). She referred again to the cylindrical portion of the bottle later, using a structural metaphor coupled with a beat metaphoric gesture. At paragraph 35, she stated, "Well I'm guessing because that—cuz of that, it depends cuz if it's like a really [F tips are together, hands curving out and around—forming a spherical shape; then at the word, "thin" she brings the palms together—beat iconic] thin, *like a stick* [structural metaphor] or something, like probably gonna go really fast cuz it's like thin" (140521_Sofia&Lucia_front2_¶ 35). Her gesture iconically represented the shape of the bottle, and at the cylindrical portion, she brought her hands together, accenting the word "thin". She would appear to be comparing changes in the width, or volume, to the change in the height. She related, in paragraph 33, the width of the bottle and its shape, then in paragraph 35, where she thinks of the cylindrical top as something akin to a stick—thin, that, "…it's gonna go really fast…" (140521_Sofia&Lucia_front2_¶ 35). In this instance, then, she is describing how, when the bottle is very thin, the height will change very quickly. We do not know if she is focusing on the height of the water, or if she is thinking about some poorly represented "it" that could be an image of the water in the bottle, or of the height, specifically.

Lucia, in the preinterview, was comparing changes in the height to changes in the volume (QO-Comp-1). She was using structural metaphors referring to *space*, rather than actual volume (140421_AnaLuciaSofia_Int1_¶ 223), and was talking about how fast the height changes when compared to changes in the volume, and concluded that the confined space in the top, cylindrical portion of the bottle made it "fill up faster", referring to the increasing changes in the height. Lucia accompanied this phrase with a deictic metaphor gesture in which she interlocks her fingers when she says the word, "smaller". This gesture seems to suggest that she had a mental image of a small, closely confined space (140421_AnaLuciaSofia_Int1_¶ 227). In the postinterview, her increase in beat gesture indicates that she is emphasizing the changes between the height and volume. She describes the change in height as, "… goes like medium—like at the bottom, and then it goes like higher and higher, and it would, like, go—faster"

(140521_Sofia&Lucia_front2_¶ 5). Her gestures consisted of a series of iconic gestures representing the changes in the height; however, she moved her hand down slightly and held the gesture at the word, "bottom", adding emphasis to the word, "bottom". When she begins discussing how the height goes higher, she raised her hand, indicating that the water level was rising through the spherical portion until it reached the cylindrical portion when she waves her hand away from her face when she says the word, "faster". This series of gestures, when accompanied by her vocalization suggest that she is comparing the amount of change in the height of the water to the changes in amount of space, or volume (QO-Comp-2).

Lucia continued along this line, noting that the shape of the bottle at the top would not "*allow the water to fit"* (140521_Sofia&Lucia_front2_¶ 11) so the height must "go faster". This container metaphor carries the image that there is a tight space and something is not fitting into it; since there is not enough room for the water, the height of the water must speed up as it gets to the top part of the spherical shaped bottle, and then enters the smaller, cylindrical portion. Lucia used a structural metaphor several paragraphs later (paragraph 35), mapping the cylindrical portion of the bottle onto a stick—an image of something narrow and straight. At this point, she stated that the height of the water was going to go "really fast, 'cuz it's like thin" and accented the word "thin" by bringing her hands together with a beat iconic gesture. She was comparing the constant change in the height of the water, with the narrowness of the bottle's top, cylindrical portion (QO-Comp-2).

<u>Sofia and Lucia: Use of Metaphor when Graphing.</u>

Toward the end of the interview with Sofia and Lucia (140521_Sofia&Lucia_front2), their discourse turned toward graphing the situation, utilizing the graph in figure 4.1. Initially, Sofia and Lucia were confused because there were no numbers presented in the graph. Sofia was first to move beyond that, and used a container/structural metaphor, “It could be like a *small, little cupboard* [container/structural metaphor]” (140521_Sofia&Lucia_front2_¶ 45).

A few paragraphs later, Sofia was talking about the rate of change of the water’s height and was comparing the quantities of height and volume:

> R: (140521_Sofia&Lucia_front2_¶ 58) so … when my height…would the height of my water keep going up the same? Would there be times when the water would gets higher faster?
>
> S: Like the speed of the water doesn’t change, but the speed of it *filling up* [orientational metaphor] changes
>
> R: Yes; … when you say the speed of it filling up you’re looking at the water…
>
> S: *like from the drain* [structural metaphor] *[She is pointing at the faucet]*
>
> R: how does that speed change? What happens?
>
> S: Right here, it like, goes up faster *[She is pointing to the bottom of the bottle]*, and when it gets here it starts going slower and slower *[pointing to the middle section of the bottle, the widest portion of the bottle]* and then it goes fast again *[pointing to the top part where the bottle narrows into the cylindrical, top portion]*.

It is interesting here that she appeared to be using the verbal equivalent of Elisa’s spiral, metaphorical gesture referencing water spiraling down a drain in discourse about the rate of change of the water’s height at is enters the cylindrical, top portion. However, she was pointing to the spigot when referencing *drain*, so in this case, she is not alluding to water spiraling down a drain, but appears to be using the word *drain* instead of the word *spigot* or *faucet.* She then went on to compare the changes in the height with the changes in the width or volume of the bottle. At this point, the video shows that she has identified the water coming out of the faucet and that it is going into the bottle. When she is talking about "…it starts going slower and slower…", she was referring to the increase in height when the water is filling the middle section of the spherical portion of the bottle, where the bottle was at its widest. Again, as the spherical part of the bottle narrowed down into the top of the sphere and the cylindrical top, Sofia has recognized that the water again speeds up. I am not certain if she is focusing on the height of the water, or if she is simply considering that "the water", whatever it may be, is speeding up as the bottle gets narrower. It is evident to me that she is comparing the continuously changing width of the bottle to the speed of the height of the liquid and using QO-Comp-2.

Finally, at the end of the interview, Lucia sketched a graph representing the change in the water's height as a function of volume, and used a structural metaphor, based on a task the students had done in their math class. Figure 4.3 (below) is the sketch of the graph she drew at this point. She identified three sections of the bottle, and represented these three sections on her graph as linear. The task they had done in class involved a woman walking and jogging to her mailbox, and Lucia equated the first two actions, walking and jogging, to her graph. The first section she described was where the woman was jogging; the woman then slowed to a walk and then, in the third section, begins flying. At this point, Lucia is relating this newer experience of bottles filling with water to something she had done in the past that seemed relevant to this situation. The classroom activity involved graphing different speeds, relating the quantities of distance and time. She considered the section of the graph where the woman was jogging as being a steeper line and therefore faster, than the section where the woman was walking, the

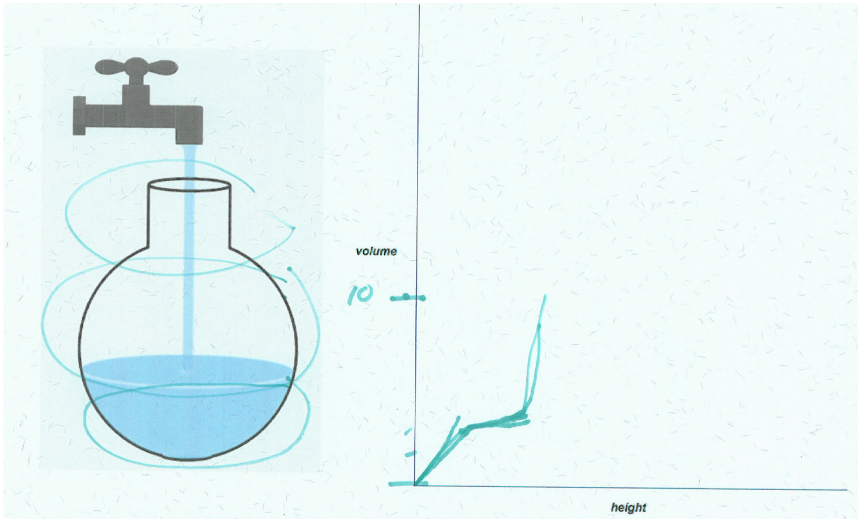

Figure 4.3. Lucia's graph representing the rate of change of the volume as a function of height.

most level part of the graph. Lucia equated this with the slower speed of walking and related it to the widest portion of the bottle. Finally, when the water arrived at the top cylinder, it "flew", represented by the steepest portion, and therefore fastest, portion of the graph.

Sofia and Lucia: Use of Metaphor to Reduce Level of Abstraction when Graphing.

At the end of the interview, the researcher asked Sofia and Lucia to graph the relationship of the volume as a function of the height of the water, and they were, at first, stymied, as there were no numbers associated with the coordinate axes. As noted by Johnson (2013), students use of numerical computation can block or limit their use of covariational reasoning; I have interpreted this brief impasse to the lack of numbers that forced their reliance on covariational reasoning to surmount. It was not until Sofia used a container/structural metaphor did they seem to be able to move on (140521_Sofia&Lucia_front2_¶ 45). Sofia's metaphor mapped the bottle's image onto the concept of a "small, little cupboard". At this point, the girls were able to continue.

Sofia compared the quantities of height and volume, noting that the water's speed does not change, but the speed of the bottle filling up does change. She pointed to specific areas of the bottle and stated that, "Right here, it like, goes up faster, and when it gets here, it starts going slower and slower…then it goes fast again", thereby comparing specific parts of the bottle that changed to changes in the height (QO-Comp-2). Lucia followed this up by relating this situation to a similar one that they had done in class. The classroom activity involved a woman walking and jogging to her mailbox, and Lucia applied that result to the bottle, apparently using the three regions of the bottle that Sofia had used. Lucia corresponded the lower portion of the bottle with a steeper section of the graph, and compared it to the point when the woman was jogging to her mailbox. She compared the widest part of the bottle to the region of the graph where the woman was walking, and then the final, steepest portion of the graph, corresponding to the top cylinder, she equated with the woman "flying". This indicated to me that she was comparing changes in the height of the water to specific regions of the bottle that produced different changes in the volume (QO-Comp-2).

The Use of Gesture and Metaphor in Covariational Reasoning and the Quantitative Operations

It was common for the students to reference the width of the bottle or the bottle's shape when talking about the volume. Their use of the term *volume* appeared very infrequently, and only in earlier parts of the first interviews did the students express what volume was. From their descriptions, I infer that they were visualizing it as the area of the two dimensional representation of the bottle, although there was some indication that they thought about the area as representing how much water there is in the bottle. Elisa, in the first part of the video when the bottle problem is first being discussed (140421_Elisa&Paola_Int1_¶281) equates how big the bottle is with its volume. The first instance of the word volume occurs in 140421_AnaLucia&Sofia_Int1_ ¶228, when the researcher used the term. Ana responded by noting that there was "…more volume down at the bottom. Because of how thick it is" (140421_AnaLucia&Sofia_Int1_ ¶229). Note, though, that she reverted to the structural metaphor of shape and width. In subsequent paragraphs, Sofia referred to the shape of the bottle as getting skinnier, rather than any reduction in volume. Finally, Elisa used the metaphor of a drain and how water spirals down and its speed increases in a gesture representative of what happens when water encounters the narrower opening of a drain. Thus, for the most part, they were considering the width of the two dimensional representation of the bottle as representing the bottle's volume.

For the most part, the students used iconic, beat iconic and deictic iconic gesture. I interpreted the beat gesture, when used in this context, as implying emphasis in what the student was saying, which could be interpreted as meaning confidence. The usual beat gesture, as described by McNeill (2005) is a repeated gesture; however, I interpreted other types of gesture as being beat. For example, I interpreted Elisa's gesture when describing a graph, as a beat gesture simply because the motion was fast, with no hesitation or pause. Their use of iconic gesture was in reference to the shape of the bottle and appeared in exchanges when they were discussing the shape of the container or the apparent volume of the container. There were few metaphorical gestures observed, although those proved to be the most satisfying from an analytical perspective. The metaphorical gestures all seemed to provide the greatest amount of information about their level of quantitative operation and the imagery used in their reasoning. There were very few deictic gestures used, but this seems reasonable to me because the students had the diagrams and animations available at all times, and this perhaps reduced the need to identify a space. When the students used deictic gesture, they used it to limit the space on a coordinate axis in conjunction with a graphing task or question.

CHAPTER V

DISCUSSION AND CONCLUSIONS

In this chapter, I will be considering the results, discussing conclusions about the mental imagery the students may have been using, their quantitative operations and the role both gesture and metaphor played in the analysis of the quantitative operations the students were using. I will be discussing this in the context of McNeill's (2005) and Lakoff and Johnson's (1980) demonstration of the role of gesture and metaphor, as well as a concept of mental imagery described by various authors (Sadoski & Paivio, 2009; Paivio, 2007; and Presmeg, 1992 & 1998). I will be enfolding these various concepts into what I am considering a melding of McNeill's (2005) description of Vygotsky's language process; that is, a merging of language and gesture that are so "…tight[ly coordinated] that they can be usefully regarded as two sides of a single thing/process" (Kindle Locations 1449-1450) and McNeill's (2005) imagery language dialectic into a functional triad of language-image-gesture. The language aspect represents the sociocultural elements of discourse, whereas the image-gesture aspect refers to the cognitive elements, thus resulting in a joining of the constructivist and sociocultural perspectives that Cobb (1994) and Bauersfeld and Cobb (1995) discuss.

The Students' Use of Metaphor and Gesture

Comparison across Students: Similarities and Differences

Students' use of gesture and to a lesser degree metaphor link to the mental images the student has as she uses quantitative operations in a covariational problem (Alibali & Goldin-Meadow, 1993; Goldin-Meadow, 1999; Lakoff & Johnson, 1980; McNeill, 2005, Presmeg, 1992). In the study discussed in this thesis, students were considering a scenario that involved filling a bottle with water, and the students used gesture and metaphor in their discourse. One can use gesture and metaphor to make inferences about the quantitative operations students are employing; there are instances where the gesture provides enough additional information to allow a different interpretation of what the student is saying and how she is reasoning through a problem. For example, when Elisa is discussing her graph, her spoken language led me to conclude that her quantitative operation was QO-Coord-2. However, when I considered her spiraling gesture, I reevaluated her operation and concluded that she was operating at QO-Coord-3. There are indications, as well, that with exposure to covariational problems, students can consider the situation, changing the quantitative operation they use and this can be inferred through gesture and metaphor. Any inferred or observed changes may be the result of nothing more than exposure to and an awareness and familiarity with the topic and context, which, through reflection and discourse, helps the student to see and read more into the problem, thereby getting more information from the presented animations and problems.

The Use of Metaphor and the Quantitative Operations

The primary metaphors used by the students seemed to be the container and structural metaphors, and the least used was ontological. Given the context of the problem and the problems' presentation (the use of animation), I do not think this is unreasonable, because container metaphor maps the notion of a container onto differing aspects of containment (Lakoff & Johnson, 1980). This interpretation seems to be at play here because the students are discussing animated containers, and referring to how these bottles could contain a liquid. Orientational metaphor was frequently used because of the common expressions that are based on the archetypical orientation metaphor in English, namely that *up is good or more* (Lakoff &

Johnson, 1980); therefore, the use of *speeding up* or *filling up* is common. I also observed structural metaphor, although I did not see it as often in the post-interview discussion. Individuals commonly use structural metaphor to reduce the level of abstraction (Lakoff & Johnson, 1980), mapping an abstract construct onto something more concrete. Because of the ready availability of animations, it seems reasonable that there should not be many structural metaphors as there is little need for a reduction in abstraction either of the concepts or of the imagery.

Also noteworthy is the students' use of orientational metaphor. This may have happened because the students were often referring to the change of the height as speeding up, or the bottle filling up. As noted in chapter 4, I observed the ninth grade students in this study using more iconic gesture with fewer metaphoric gestures and this is somewhat counter to Edwards' (2009) finding that the graduate students in her study used gestures that are more metaphorical. This study differs from Edwards' (2009) study in that Edwards was looking at graduate students in a student teacher program who were discussing fractions and operations with fractions, and I was looking at ninth grade students. Perhaps more importantly, in my thesis, the students had ready access to not only static drawings, but also animations. The presence of these visual elements may be at the root of the students' use of iconic gestures.

The Use of Gesture in the Analysis of the Quantitative Operations

Attention to gesture as an analytical tool proved very useful, providing additional insight into the mental images and the quantitative operations the students were using. In several instances, the students' use of language indicated that they were using a lower level of quantitative operation than what their gesture indicated. Based on the examples I studied, it would appear that students' use of gesture provides more detail and insight into their use of quantitative operations than relying solely on their spoken word. In several instances, it was through the combination of gesture and metaphor that I was able to infer changes in the student's quantitative operation. One possible explanation for this observation lies in the ages of the students studied. In Edwards' (2009) study, the students were adults in a teacher education program. They have a more mature and developed vocabulary. The students I analyzed were in the ninth grade, so their vocabulary was, as one would expect, below that of the students in Edwards study. Perhaps the ninth graders were unable to find the words they needed to express their reasoning and had to rely on other communication methods.

I observed students using varying amounts of gesture. Even in those students who exhibited more gestures, for example, Elisa and Ana, they were not consistent in the amount of gesture used. Lucia and Sofia used few gestures, especially during the postinterview phase. Both Sofia and Lucia, who were working together in both the pre and postinterview, appeared to use fewer gestures in the postinterview than the others. An initial interpretation could be that this lack of gesture might fall under what Alibari and Goldin-Meadow (1993) referred to as discordant gesture where the student's use of gesture is not in agreement with his or her verbal discourse. In this instance, if the student was in a transitional stage and on the verge of mastery of a concept, as Alibari and Goldin-Meadow (1993) suggest, rather than presenting with discordant gesture, the student could present with little or no gesture. This conclusion seems reasonable because toward the end of the postinterview when the researcher asked Sofia and Lucia to graph the volume of water in the bottle, they appeared stymied because the *y* axis was only labeled "volume" and the *x* axis was only labeled "height". Yet, when Sofia used a container/structural metaphor, linking the bottle to a "small, little cupboard", she and Lucia were able to move beyond the confusion and demonstrate that they were able to graph the situation; Sofia demonstrating that she was operating at QO-Comp-2.

Metaphor and Gesture in the Classroom

Gesture in the Classroom

Within a classroom setting, unless one is very adept at recognizing gestures, only the least subtle, iconic, beat and deictic gesture are useful. As I stated, there were few metaphorical gestures, and they were the subtlest. However, one example of an iconic gesture merits mention. Reported by Moschkovich (2002), it involved an iconic gesture made by a bilingual student. She was unable to find the English or Spanish equivalent for rectangle, so she traced the shape of a rectangle. While this example from Moschkovich would not go unnoticed or not understood, there are other examples within this thesis that would go unnoticed and could play an important role in the classroom. Returning again to Elisa's spiraling gesture, or Ana's "gushing hand" gesture, these gestures could easily be overlooked in a classroom—or research setting, for that matter—and yet these gestures represented the students' underlying imagery and could lead to as powerful an insight into the student's reasoning or meaning as the less subtle gesture Moschkovich (2002) described.

In the context of collaborative discourse during class, the recognition of meaningful gesture is useful in assessing a group's progress toward the learning objective; however, its use demands changes in a teacher's monitoring practices. All too often, as teachers, we will approach a group to either answer a specific question, or to make sure all are on task and working toward the task's completion. We then move on. In order to take full advantage of an understanding of gesture, the teacher must use active monitoring. The teacher must approach a group with the intent of doing more than answering questions or ensuring on-task behavior. The teacher must engage the group in discussion focusing on the groups approach to the task and how they are reasoning about it.

During whole class discussion and collaborative group discussion, the teacher must do more than think about the next question or focus on the words the student is using; she must also consider the gesture. Two simple gestures, both discussed in McNeill (2005), can tell a teacher a great deal. However, the difference between them is very subtle. The palm open, hand up, fingers outstretched gesture signifies that someone or something is up next and the speaker is signaling that they are done, and that it is time for the next speaker or event. The same gesture, but with fingers curled and then opened straight signifies that the speaker has concluded, but the speaker is implying that they don't understand something, and are asking the next speaker to confirm, deny or clarify the first speaker's point. This illustrates why teachers should have an understanding of gesture. Too often, students will leave a class without fully understanding the material covered in class simply because they did not directly ask a question. Their gesture asked the question for them, but that was lost on either the teacher or classmates.

Metaphor in the Classroom

Students use metaphor constantly in the classroom, but because of the ubiquity of some metaphor, it often goes unnoticed by not only the teacher, but by other students. I would submit, that some metaphor is unnoticed by many in the general population. A common use, which I noted in this study, is its use as a way of reducing the level of abstraction, making the complexity and abstraction of the mathematics more accessible. In this study, the students used container metaphors frequently as a way of describing the effect the bottle's shape had on the changing quantities of volume and height. In this sense, they were mapping the concept of the bottle's volume onto the more familiar concepts of thick and thin, fat and wide. To this end, a teacher's awareness of how and what metaphor students are using can be useful in her ongoing formative assessments.

Metaphor and Gesture as Research Tools

Gesture is an integral part of speech in all cultures (Kendon, 1997; McNeill, 2005), whereas metaphor is not as highly integrated, although widely used (Lakoff & Johnson, 1980). Gesture has a neurological relationship with speech (McNeill, 2005), on the other hand, metaphor, while reflecting our individual language use, does not have the same neurological relationship with speech, and some metaphor has become so prevalent and incorporated into English, that we are unaware that we are using metaphor except when purposefully employing it as a linguistic or rhetorical device. Because of the ubiquitous nature of both of these elements of discourse, we are often unaware of their presence when we use them, and have near-total blindness at the conscious level to their use by others (Goldin-Meadow, 1999).

Research has established that gesture is a natural phenomenon, crossing culture and language, but the research also suggests that gesture is not fully under conscious control (McNeill, 2005, Goldin-Meadow, 1999). This reason warrants its use as a research tool in reasoning and makes it preferable to metaphor. This is not to say that the study and analysis of metaphor is without value, it is important; but I do not feel it represents the underlying reasoning well, limiting its use as a stand-alone tool. The reason for my belief lies in the fact that we can choose to use one metaphor over another; gesture, because of its neurological relationship with speech (Goldin-Meadow, 1998; McNeill, 2005), flows with our speech, arising from underlying mechanisms that are more autonomic than speech.

Metaphor in Educational Research

Metaphor poses other issues. While it is not difficult to understand the meaning behind some metaphors, even recognizing many others can be challenging. As I have stated before, many terms and phrases we use daily are metaphor. The majority of these are structural metaphors that map one concept onto another, often with the intention of reducing or increasing the level of abstraction (Lakoff & Johnson, 1980). In general, however, we have so ingrained many metaphors in our daily speech that their metaphorical quality is lost without an in-depth analysis. This, then, illustrates what I would consider a critical point concerning the use of metaphor in educational research. Does the use of certain metaphors add to our understanding of the role metaphor plays in the question the researcher is asking? I found that many added nothing of significance to my understanding of the students' reasoning or use of quantitative operation.

I noted previously that much of the language of mathematics has been repurposed from everyday language, and then, once in the mathematics domain, gets repurposed multiple times (Presmeg, 1992). These secondary, and even tertiary, repurposing events move toward a jargon that is context dependent and, ultimately, to the use of metonymy in addition to metaphor. As a very simple example, consider the term, slope: in everyday speech, this simply refers to how steep a hill is or to the side of the hill, itself, "We moved up the slope for a better view." When slope is transferred to mathematics, it refers to only how steep a line is—in its initial introduction. As the student advances, slope becomes a metonymy for rate of change, ultimately being absorbed into the metonomic structure of the derivative (Zandieh & Knapp, 2006), and how the word, derivative, becomes a metonymy for the many aspects of the concept of derivative, for example, the slope of a tangent line, or the velocity (Zandieh & Knapp, 2006). Like metaphor, individuals use the linguistic structure, metonymy, as a device for reducing the abstractness of a concept. Referring to Lakoff and Johnson (1980), Zandieh and Knapp (2006) demonstrated how students will use three types of metonymy to make 1) material easier to understand, 2) easier to remember, and 3) more easily used (Zandieh & Knapp, 2006). Thus, students use both metaphor and metonymy to mediate and help navigate the complexities and abstractions of mathematics.

Gesture in Educational Research

Gesture has been studied in education and has provided some insight into how students are learning and visualizing mathematics. At a fundamental level, researchers have commented on gesture that has aided in classroom discourse; for example, in a paper on bilingual education and discourse, Moschkovich (2002) described how one student did not know the word for "rectangle" in either Spanish or English, and resorted to an iconic gesture to make her point. This demonstrates the importance of attention to gesture during classroom discourse; however, as I noted, gesture is nearly invisible (Goldin-Meadow, 1999; McNeill, 2005), and this is the challenge for the observer or teacher. In the classroom Moschkovich (2002) referenced, the description of the student's iconic gesture suggested that the observed gesture was not subtle, unlike many gestures.

As suggested in this thesis, and described by Alibali and Goldin-Meadow (1999), and by Perry, Church and Goldin-Meadow (1988), gesture, or, what I would propose, the lack thereof, can indicate whether a student is fully understanding the topic or is in a transitory state, moving toward understanding. As Moschkovich (2002) reported, sometimes a student will use a gesture when he or she cannot find the correct word. A researcher or teacher can observe a circumstance that often occurs in the mathematics classroom when a teacher has just introduced a new concept; the students' gesture will be consistent, or concordant, with her speech. As she begins to work with the concept and is approaching, but not yet at, mastery, her gesture will become inconsistent with her speech, or discordant (Alibali & Goldin-Meadow, 1998). When the student reaches mastery, concordance returns to her gesture.

Underlying the discordant gesture during the transitory phase are competing and poorly integrated concepts (Alibali & Goldin-Meadow, 1998). The student will say one thing that differs from her imagery or conflicts with another concept, and her words will not be consistent with her gesture (Alibali & Goldin-Meadow, 1998), perhaps the more reliable indicator of the underlying imagery. This inconsistency can provide valuable formative information to the teacher and analytical information to the researcher.

Proposed Triad for Language, Gesture and Imagery

McNeill (2005) commented that one could not separate gesture and language and referred to an imagery-language dialectic. In studies with the blind, McNeill found that gesture continued unabated even in cases where one blind person is speaking with another or where the individual has been blind from birth. Additional work by Goldin-Meadow (1999) supports this idea that gesture represents mental imagery and language. From the work done by McNeill (2005) and Goldin-Meadow (1999), one may infer that gesture is, as McNeill (2005) put it, part of a "neurogestural model" (location 378) forming a "thought-language-hand link" (location 378). Similarly, when I consider the work done on image, gesture and language as a whole and taken as an integral part of discourse, links between them form what I consider an inseparable triad of language, gesture and image (McNeill, 2005; Goldin-Meadow, 1999; Paivio, 2007; Lakoff & Johnson, 1980), the Language-Imagery-Gesture Triad. The structure of this triad is pyramidal, with gesture and image forming the foundation, and language residing at the top. The body of work I have cited, coupled with examples I observed in the four students I studied, led to this structure, which was spurred by McNeill's (2005) neurological discussion. This corpus of work suggests that gesture and image are more primitive than language, but are necessary to language and not some vestigial artifact.

Linguistic structures, such as metaphor and metonymy contribute to this triad, as do the other structures such as onomatopoeia and simile, as studied and described by, among others, Lakoff and Johnson (1980), Pimm (1988), Presmeg (1992) and Zandieh and Knapp (2006).

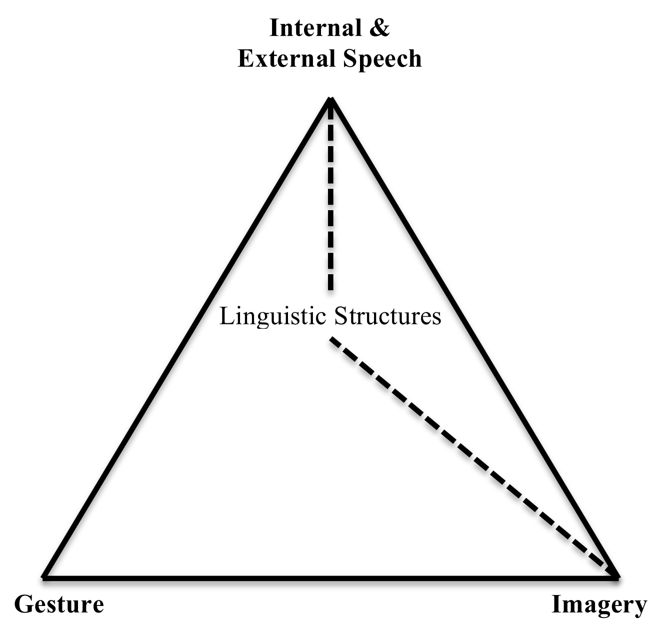

Figure 5.1. Structure of the proposed Language-Imagery-Gesture Triad, showing the facilitation provided by linguistic structures.

These linguistic structures facilitate between the more abstract and the concrete (Lakoff & Johnson, 1980; Presmeg, 1992), and between the primitive foundation formed by gesture and imagery and the spoken word. This triad (Figure 5.1) is a useful way to think about and visualize the relationship between language, gesture and imagery and may provide additional ways to analyze discourse. This suggests further investigation.

Limitations

To improve the generalizability of these implications, I would want to repeat this study using students from a different setting, for example, a suburban, ninth – twelfth high school. Furthermore, I would also want to investigate any gender differences. One could find that in classes where the students are more adept at using academic language, they use fewer metaphors of different types, that is, orientational, structural, container and ontological, and the gesture structure could change. I observed a decline of some students' use of gesture in this study, and could explain this either as a cultural issue or simply as subject fatigue. I could also explain this apparent decline in gesture use through Alibali and Goldin-Meadow's (1993) work on gesture-speech mismatch, and this reduction in participation a manifestation of discordant gesture.

Implications for future research

Results of this study suggest that students can use different quantitative operations when working on covariational tasks. The results of the gesture and metaphor analysis suggest that this approach to exploring the quantitative operation students use when working on mathematical tasks is viable, providing insight into the underlying processes and mental imagery students are using.

This study has implications for two areas of future research. The first centers on my proposed triad relating gesture, language and image. While this proposed triad appears reasonable on the surface, it requires additional research focusing on, among other areas, the neurology and psycholinguistics involved in gesture. Furthermore, considering Paivio's (2007) dual coding theory, does this structure provide additional insight into dual coding and its role in reasoning and quantitative operations? The second area of research lies in the reduced number of gestures observed during Sofia and Lucia's postinterview video and whether this is an aspect of discordant gesture that Alibali and Goldin-Meadow (1993) discussed.

On a more general level, of interest to me is a role reasoning might play in the formation of mathematical resources used in problem solving and the interplay between procedural fluency and conceptual understanding. Reasoning is a process that does not stand alone, but may be the link between procedural fluency and conceptual understanding. Future research may be able to use gesture and, to a lesser degree, metaphor found in discourse during collaborative work to determine the role reasoning has in this link between conceptual understanding and procedural fluency. Furthermore, one can use how students reason mathematically to guide curriculum and lesson development, so research in this area of reasoning could provide valuable information and direction for pedagogy.

Other areas of research lie in how different cultures and speakers of different languages use gesture and metaphor when carrying out mathematics. At the heart of this question is whether the language or culture dictates the gesture and metaphor, or if the subject that the individual is thinking about dictates the gesture and metaphor. For example, were the gestures seen and the metaphors used common to all ninth grade mathematics students, or are they truly specific only to those students participating in the study? If I were to carry out a similar study with other students at the same school, could I reasonably expect to see similarities? It seems to me that the native language and the individual's culture are the determining elements in the nature of the gesture or metaphor (Kendon, 1997). What are the effects of a different school and different classes at that school? I would be interested in exploring any differences between suburban and urban settings; this suggests other, related questions: I would be curious to see what differences there are in students' use of gesture and metaphor between special education and regular education students. Do these discourse elements vary by level of mathematics, or how well the students are doing in school over all? These questions, however, beg the underlying issue of the relationship between our reasoning and how gesture and metaphor reflect reasoning.

Reflection

Metaphor provided a sociocultural lens to view the students' mental images, but there are several issues inherent with the use of metaphor. First, there are many metaphors that can go unrecognized as metaphor to an untrained ear. The orientational metaphor is a perfect example. In English, the notion that *up is more* is so prevalent that we are unaware of its use, let alone the metaphorical aspect. There also seems to be a contextual element to students' use of metaphor; the central object the researcher asked the students to attend to was a bottle that was being filled with water; therefore one would expect an abundance of container and orientational metaphor, coupled with iconic gesture. This raises a question as to whether the phrase, *filled up*, is an orientational metaphor. It can also represent an actual description of the circumstance as the water's height is increasing. I was expecting more deictic gesture as well, however, as I progressed in the analysis, it became apparent that the set up of the study and follow up questions reduced the appearance of this type of gesture.

This posed some problems for me in the analysis of the data for this thesis. For example, a common phrase heard in this study in the context of a bottle filling was *fill up*. Technically, this is an orientational metaphor, but I questioned whether I should code it as such. On the one hand, it is an orientational metaphor in that it is founded on the concept that *up is good* and *up is more*. However, the students used it in the context of filling a bottle with water. In this case, there is only one direction for the water to go, and that is up. I coded this as an orientational metaphor where it could possibly add to my understanding of the students' quantitative operations and mental imagery; however, I could just as easily argue that the students are not using it metaphorically, but that the use of *up* is simply a descriptive redundancy that we use out of habit. As I reviewed my coding, I decided to code this as a metaphor only when it added to the objective of understanding the students' reasoning through their use of metaphor and gesture.

I found that gesture provided great insight into the underlying reasoning; however, this was only after a great deal of thought and practice coding and analyzing gesture. During my initial coding of one of Elisa's interchanges, I recognized only two iconic gestures referring to the bottle's shape; what I missed was potentially important. The gestures she used first, I initially coded as a single iconic beat gesture; however, with practice, I could see that there were actually two separate gestures, one I coded as deictic iconic and the other as beat iconic. Two other gestures I completely missed were a beat and iconic gesture that I initially did not interpret as gestures, but simply as her hands coming to rest. In fact, she was demonstrating the bottom portion of the bottle, and emphasizing that the bottom was the widest part, preparing to iconically show the changes in shape of the bottle. This emphasizes the need for practice at coding gesture, and demonstrates the difficulty one might face using gesture in the classroom. Furthermore, in another instance, I attributed much to Sofia's use of the term *drain*, associating it with Elisa's iconic gesture representing water running down a drain. Upon further study of her gestures, it was apparent that Sofia was using the word *drain* instead of the word faucet, as her gesture ultimately demonstrated.

A word of caution, however: in other instances, I found that the combination of both metaphor and gesture was powerful and that the metaphor added significantly to the analysis and understanding of the gesture and the underlying reasoning. In the discussion of Elisa's reasoning in the subsection entitled *Elisa's Analysis of the Graph*, I decoded her metaphorical gesture consisting of a spiral motion through the combined use of container and structural metaphor and her gesture. In my initial analysis, I thought the gesture to be simply her finding the right part of the animation to point to; however, when I coupled it with both her use of metaphor and the fact that she never looked away from the screen, in the end, I considered it an iconic gesture, instead. Metaphor has, therefore, a notable place in educational research, but I would suggest that it cannot be separated from any accompanying gesture.

As a researcher, it was not until I began an in-depth look into the data that I became aware of the extensive presence of these aspects of discourse. I will not use the metaphor *gained expertise* because my lack of experience precludes the use of the term *expertise*. However, as I delved deeper into the transcripts and videos, I became more adept at recognizing gesture and metaphor, and as I became more adept recognizing these discourse forms, I was better able to analyze the subtext and see the relationship to the underlying reasoning and imagery. From a research perspective, this element—experience at recognition and coding—is vital because of the subtleties of these forms of discourse. If one is going to study reasoning and other cognitive processes during collaborative tasks, one cannot successfully do so without a solid understanding of the importance of gesture and metaphor. From my work on this thesis, it became apparent that, while useful, most of the metaphors used by the students did not illustrate the underlying quantitative operations well; however, when taken with the accompanying gesture, their value increased. I found, however, the gesture proved to be the more useful.

Closing remarks

In closing, this experience provided me an opportunity to explore students' use of discourse in mathematics and how they use gesture and metaphor. It afforded me the opportunity to increase my understanding of covariation and the subtle differences in the varying levels of covariation and quantitative operations that students employ. I am convinced that covariation lies at the heart of the function because it feels to me to be more intuitive and natural. Furthermore, it appears to take little effort to improve students' abilities to reason covariationally resulting in a stronger understanding of the function. More to my fundamental interest, is the information I gained from exploring the role gesture and metaphor play in understanding how students reason. After completing this thesis, it seems that gesture and metaphor can provide a window into the imagery and reasoning students are using when they are carrying out mathematics in general and, specifically, covariational tasks. The connections between gesture and imagery appear to me to be quite remarkable, and could provide a means for further work in the area of reasoning.

REFERENCES

Alibali, M. W., & Goldin-Meadow, S. (1993). Gesture-speech mismatch and mechanisms of learning: what the hands reveal about a child's state of mind. *Cognitive Psychology, 25*, 468 – 523.

Chazan, D. (2000). *Beyond Formulas in Mathematics and Teaching: Dynamics of the High School Algebra Classroom.* New York: Teachers College Press.

Cobb, P. & Bauersfeld, H. (Eds.). (1995). *The Emergence of Mathematical Meaning: Interaction in Classroom Cultures*. [Kindle Edition]. Retrieved from Amazon.com.

Cobb, P. & Bowers, J. (1999). Cognitive and situated learning perspectives in theory and practice. *Educational Researcher, 28*(2), 4 – 15.

Carlson, M. P., Jacobs, S., Coe, E., Larsen, S., & Hsu, E. (2002). Applying covariational reasoning while modeling dynamic events: A framework and a study. Journal for Research in Mathematics Education, 33(5), 352–378.

Clement, J. (1989). The concept of variation and misconceptions in Cartesian graphing. *Focus on Learning Problems in Mathematics, 11*(1-2), 77 – 87.

Cobb, P. (1994). Where is the mind? Constructivist and sociocultural perspectives on mathematical development. *Educational Researcher, 23*(7), 13 – 20.

Confrey, J., & Smith, E. (1995). Splitting, Covariation, and Their Role in the Development of Exponential Functions. *Journal for Research in Mathematics Eduation, 26*(1), 66 – 86.

Edwards, L. D. (2009). Gestures and conceptual integration in mathematical talk. *Educational Studies in Mathematics, 70*(2), Gestures and Multimodality in the Construction of Mathematical Meaning, 127 – 141.

Goldin-Meadow, S. (1999). The role of gesture in communication and thinking. *Trends in Cognitive Sciences, 3*(11), 419 – 429.

Harel, G., Behr, M., Lesh, R. & Post, T. (1994). Invariance of ratio, the case of children's anticipatory scheme for constancy of taste. *Journal for Research in Mathematics Education, 25*(4), 324 – 345.

Herbel-Eisenmann, B. A., Cirillo, M. & Skowronski, K. (2009). Why discourse deserves our attention. In A. Florio (Ed.), *Mathematics for Every Student: Responding to Diversity, Grades 9 – 12*. Reston, VA: NCTM.

Herbel-Eisenmann, B. A., & Otten, S. (2011). Mapping mathematics in classroom discourse. *Journal for Research in Mathematics Education, 42*(5), 451 – 485.

Johnson, H. L. (2012). Reasoning about variation in the intensity of change in covarying quantities involved in rate of change. *Journal for Mathematical Behavior, 31*, 313 – 330.

Johnson, H. L. (2013). Designing covariation tasks to support students reasoning about quantities involved in rate of change. In C. Margolinas (Ed.), *Task design in Mathematics Education*. Proceedings of ICMI Study 22 (Vol. 1, pp. 213-222). Oxford.

Johnson, H. L. (2015). Secondary students' quantification of ratio and rate: A framework for reasoning about change in covarying quantities. *Mathematical Thinking and Learning, 17*(1), 64 - 90.

Kendon, A. (1997). Gesture. *Annual Review of Anthropology, 26*, 109 – 128.

Lakoff, G. & Johnson, M. (1980). *Metaphors We Live By.* [Kindle version]. Retrieved from Amazon.com.

McNeill, D. (2005). *Gesture and Thought*. [Kindle version]. Retrieved from Amazon.com.

Meyer, D. (2014). Waterline & Taking Textbooks Out of Airplane Mode. http://blog.mrmeyer.com/2014/waterline-taking-textbooks-out-of-airplane-mode/

Moschkovich, J. (2002). A situated and sociocultural perspective on bilingual mathematics learners. *Mathematical Thinking and Learning, 4*(2), 189 – 212.

National Governors Association Center for Best Practices, Council of Chief State School Officers. (2010). *Common Core State Standards—Mathematics.* Washington, D.C.: National Governors Association Center for Best Practices, Council of Chief State School Officers.

Paivio, A. (2007). *Mind and Its Evolution: A Dual Coding Theoretical Approach.* New York: Psychology Press.

Perry, M., Church, R. B., & Goldin-Meadow, S. (1988). Transitional knowledge in the acquisition of concepts. *Cognitive Development, 3*, 359 – 400.

Pimm, D. (1988). Mathematical metaphor. *For the Learning of Mathematics, 8*(1), 30 – 34.

Presmeg, N. (1992). Prototypes, metaphors, metonymies and imaginative rationality in high school mathematics. *Educational Studies in Mathematics, 23*(6), 595 – 610.

Sadoski, M. & Paivio, A. (2009). *Imagery and Text: A Dual Coding Theory of Reading and Writing.* New Jersey: L. Erlbaum Associates.

Saldanha, L., & Thompson, P. W. (1998). Re-thinking covariation from a quantitative perspective: Simultaneous continuous variation. In S. B. Berensen, K. R. Dawkins, M. Blanton, W. N. Coulombe, J. Kolb, K. Norwood, & L. Stiff (Eds.), *Proceedings of the 20th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education,* (Vol. 1, p. 298 – 303). Columbus, OH: Eric Clearinghouse for Science, Mathematics, and Environmental Education.

Shell Centre for Mathematical Education (University of Nottingham). (1985). *The language of functions and graphs: An examination module for secondary schools:* Shell Centre.

Steffe, L. P. (1991a). The learning paradox. In L. P. Steffe (Ed.) *Epistemological foundations of mathematical experience*. New York: Springer-Verlag.

Stein, M. K., Engle, R. A., Smith, M. S., & Hughes, E. K. (2008). Orchestrating productive mathematical discussion: five practices for helping teachers move beyond show and tell. *Mathematical Thinking and Learning, 10*, 313 – 340.

Thompson, P. W. (1994). The Development of the Concept of Speed and Its Relationship to Concepts of Rate. In *The Development of Multiplicative Reasoning in the Learning of Mathematics*, G. Harel & J. Confrey, (Eds.). New York: State University of New York Press.

Thompson, P. W. (1996). Imagery and the development of mathematical reasoning. In L. P. Steffe, P. Nesher, P. Cobb, G. A. Goldin & B. Greer (Eds.), Theories of mathematical learning (pp. 267-284). Mahwah, NJ: Lawrence Erlbaum Associates

Thompson, P. W. (2008). Conceptual analysis of mathematical ideas: Some spadework at the foundation of mathematics education. In O. Figueras, J. L. Cortina, S. Alatorre, T. Rojano & A. Sepulveda (Eds.), *Proceedings of the 32nd Annual Meeting of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 31-49). Morelia, Mexico

Vygotsky, L. S. & Kozulin, A. (2012). *Thought and Language*. [Kindle version]. Retrieved from Amazon.com.

Yackel, E. & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Eductation, 27*(4), 458 – 477.

Zandieh, M. J. & Knapp, J. (2006). Exploring the role of metonymy in mathematical understanding and reasoning: The concept of derivative as an example. *Journal of Mathematical Behavior, 25,* 1 – 17.