Robust and Resilient Control for Time Delayed Power Systems

by Mohammed K. Jamal Alden, Bachelor of Science

A Thesis Submitted in Partial
Fulfillment of the Requirements
for the Degree of
Master of Science
in the Field of Electrical Engineering

Advisory Committee:

Xin Wang, Chair

Scott Umbaugh

Ying Shang

Graduate School
Southern Illinois University Edwardsville
May, 2015

ABSTRACT

ROBUST AND RESILIENT CONTROL FOR TIME DELAYED POWER SYSTEMS

by

MOHAMMED K. JAMAL ALDEN

Chairperson: Professor Xin Wang

Power system is the backbone of modern society. Traditionally, over 90% of the electrical energy is produced by power generation systems driven by steam turbines. Recently, with the development of renewable energy resources, wind energy conversion systems are the proven solutions for the next generation sustainable energy resources. Stability and performance of these power systems are the primary concerns of power system engineers. To better characterize the dynamical behaviors of power systems in practical applications, time delays in the feedback state variables, systems modeling uncertainties, and external disturbances are included in the state space model of the power system in this work. Linear matrix inequality based robust and resilient controllers satisfying the H_∞ performance objective for time delayed power systems are proposed. Fixed time delays are assumed to exist within the system state and input signals. The system model is assumed to have unstructured bounded uncertainties and L_2 type of disturbances. Furthermore, controller gain perturbations are assumed to be of additive type. The proposed control techniques have been applied to variable speed permanent magnet synchronous generator based wind energy conversion systems, and electrical power generation systems driven by steam turbine. Computer simulations conducted in MATLAB show the effectiveness of the proposed control algorithms.

ACKNOWLEDGEMENTS

I would like to express my sincere appreciation and gratitude to my advisor Dr. Xin Wang for his unlimited support, guidance, motivation and advises, without which, this research will not be achieved. I would like to thank my committee members: Dr. Scott Umbaugh and Dr. Ying Shang for improving the quality of this work. I would also like to express my gratitude and thanks to my father Mr. Kais Jamal Alden and my mother Mrs. Thanaa Alkatib, who always supported me by their love and kindness since my first moments in this life. I would like to take the chance to express my thanks to my sisters, brother and my friends. I would like to express my special thanks to my friend Jacob Hostettler, for his help and support. Finally, I would like to thank the Graduate School, the International Student Office at Southern Illinois University Edwardsville for all of their help and support.

TABLE OF CONTENTS

ABSTRACT ii

ACKNOWLEDGEMENTS iii

NOMENCLATURE vi

LIST OF FIGURES viii

LIST OF TABLES ix

Chapter

1. INTRODUCTION AND BACKGROUND 1

1.1 Time Delay Systems 1
1.2 Motivations for Time Delay Analysis 3
1.3 Functional Differential Equations: A Brief Overview 3
1.4 Historical Review on the Stability of Time Delay System 4
1.5 Time Delay Systems Stability: Time Domain Methods 5
1.5.1 Lyapunov-Krasovskii Theorem 7
1.5.2 Razumikhin Stability Theorem 8
1.6 Robust Control 10
1.6.1 H_∞ norm 10
1.6.2 H_∞ control 13
1.7 Resilient Control 16

2. WIND ENERGY CONVERSION SYSTEMS 19

2.1 Introduction 19
2.2 Wind Energy Conversion System 20
2.3 WECS Control Schemes 21
2.4 Characterization of the Wind Energy Resource 22
2.5 Wind Turbine: Aerodynamics and Performance 24
2.5.1 Tip speed ratio 26
2.5.2 Power coefficient 26
2.5.3 Torque coefficient 27
2.5.4 Number of blades and safety procedures 28
2.6 WECS: Drive Train and Generator 28
2.6.1 Drive train 28
2.6.2 Generator 29
2.7 Wind Energy Conversion Systems Model 30
2.7.1 Nonlinear Model 31

2.7.2 Linearized Model . . . 32
2.8 Literature Survey . . . 34
2.9 Design of Robust and Resilient Controller for the Time Delayed WECS . . 35
2.10 Simulation Results . . . 41

3. POWER GENERATION SYSTEMS DRIVEN BY STEAM TURBINES . . . 45

3.1 Basic Components of Power Generation Systems . . . 45
3.1.1 Generation systems . . . 45
3.1.2 Transmission systems . . . 45
3.1.3 Distribution systems . . . 46
3.2 Literature Survey . . . 46
3.3 Model of Steam Turbine Based Power System . . . 46
3.3.1 Synchronous generator model . . . 47
3.3.2 Automatic voltage regulator and exciter circuit dynamics . . 50
3.4 Robust H_∞ Controller Design . . . 52
3.5 Simulation and Results . . . 59

4. CONCLUSION AND FUTURE WORK . . . 63

4.1 Conclusion . . . 63
4.2 Future Work . . . 63

BIBLIOGRAPHY . . . 64

APPENDICES . . . 69

A. PARK'S TRANSFORMATION . . . 69
A.0.1 Park's transformation . . . 69
A.0.2 Clarke's transformation . . . 71
A.0.3 Rotational transformation . . . 73

B. PERMANENT MAGNET SYNCHRONOUS GENERATOR MODEL IN THE $dq0$ COORDINATE FRAME . . . 74
B.0.4 Permanent magnet synchronous generator model in the *abc* coordinate frame . . . 74

NOMENCLATURE

Notation	Description
β	Blade pitch angle
δ	Rotor angle
λ	Tip-speed ratio
ν	Normalized frequency $\nu = \frac{\omega}{\omega_s}$
ω_e	Generator rotational frequency
ω_l	Low speed shaft rotational speed
ω_r	High speed shaft rotational speed
ω_B	Rated speed in electrical radians per second, $\omega_B = \omega_s$
ω_s	Synchronous speed
Ψ_m	Generator flux linkage
ρ	Air density
$\star$	Transposed value of the corresponding element
τ_e	Generator torque
τ_m	Mechanical Torque
$C_p(\cdot)$	Power coefficient
E'_{fd}	Transient excitation voltage
E'_q	Quadrature axis transient voltage
E_{fd}	Excitation voltage
E_q	Quadrature axis voltage
H	The shaft inertia constant is scaled by defining $H = \frac{\frac{1}{2}J(\omega_B\frac{2}{P})^2}{S_B}$
I_d	Direct axis current
I_q	Quadrature axis current
J	High speed shaft inertia
K_d	The damping factor

L_d	d-axis inductance
L_q	q-axis inductance
P	Total number of poles
R_t	Rotor radius
R_e	Transmission line resistance
R_s	Stator resistance
S_B	Rated three phase voltage ampere
T'_{do}	Equivalent transient rotor time constant
T_{mech}	Mechanical torque
u_d	d-axis voltage
u_q	q-axis voltage
U_{pss}	Power system stabilizer control signal
v	Wind speed
V_{ref}	Reference voltage
V_T	Terminal bus voltage
X'_d	Direct axis transient reactance
X_d	Direct axis reactance
X_e	Transmission linear reactance
X_q	Quadrature axis reactance
z_1, z_2, z_3	State variables of power system stabilizer

LIST OF FIGURES

Figure		Page
1.1	Standard H_∞ Control Problem Block Diagram	13
2.1	Two Blades HAWT WECS	20
2.2	The Actuator Disc of Energy Extraction	24
2.3	Wind Turbine Aerodynamic Performance Curve	27
2.4	Wind Turbine Output Power Versus Wind Speed	29
2.5	State Trajectory of d Axis Current	43
2.6	State Trajectory of q Axis Current	43
2.7	State Trajectory of Rotational Speed	44
2.8	Control Input Signals	44
3.1	Time Response of Rotor Angle	60
3.2	Time Response of Normalized Frequency	60
3.3	Time Response of Quadrature Axis Transient Voltage	61
3.4	Time Response of Excitation Voltage	61
3.5	Control Input	62
A.1	abc and $dq0$ Coordinate Frames	70
A.2	abc and $\alpha\beta0$ Coordinate Frames	72

LIST OF TABLES

Table | Page

2.1 Parameters of the Wind Energy Conversion System 42

CHAPTER 1

INTRODUCTION AND BACKGROUND

1.1 Time Delay Systems

It is widely assumed that the future evaluation of a system state variable is dependent on its current state only. However, this assumption does not hold for all systems, especially, the systems involved in material or information transfer. These systems have their future state variables dependent on both current and past states. Therefore, classical modeling and control techniques cannot provide satisfactory and reliable solutions for these systems, which are known as the time delay systems [9, 30, 37, 68]. The time delayed systems are also known as systems with after affect, dead time systems, hereditary systems and systems with deviating argument [9, 20, 36, 48, 61].

Time delays exist in a wide range of physical systems. The first observation of time delay as a phenomenon was in biological systems [61]. After that, researchers recognized that time delay also exist in engineering systems such as mechanical, fluid transmission systems, and wide area control systems [61]. Time delays are considered as a major source of instability and performance degradation. Therefore, tremendous volume of research work has been done in this field. Several factors lead a time delay to occur in a system. Time delay may appear due to the intrinsic property of the system. Moreover, measurements and feedback delays may also lead to time delays appearing in a system. Furthermore, time delay may be introduced on purpose to improve the stability. In this case, time delays are usually made to appear in the feedback path. Besides, time delay may also exist due to simplification of partial differential equations. Additionally, time delay occurs due to the system's limited capacity to process information and to disseminate data through the system channel. Finally, computational delays in digital processors may also lead to time delays [17, 36, 37].

Time delay can take various sizes and types. It can be long or short, fixed or varying, known or unknown, stochastic or deterministic. In some systems with long time delay intervals, the system behavior may not reflect the existence of a time delay. Analysis of such type of systems is more difficult compared to other cases, when a system behavior reflects the existence of a time delay in its dynamics [36, 68]. Based on the form of delay occurrence in a system, time delay systems can be classified into two broad categories:

- Lumped delay systems.
- Distributed delay systems.

For systems of the first category, time delays are encapsulated by a specific number of identified parameters. On the contrary, it is hard to characterize the delay by a specific number of states, for systems within second category [36].

Several techniques have been used to describe the dynamics of a time delay system. Each technique has its own advantages and disadvantageous. Time delay systems are mainly modeled by one of the following three techniques:

1. Infinite dimensional system theory may be used to describe the dynamical model of a time delay system. However, incomplete establishment of some system theories, such as stability, leads to difficulties in applying this method of characterization [9, 36].

2. Time delay system dynamics can be characterized using functional differential equation. This method provides flexibility in characterizing delays. Utilization of this approach allows the usage of finite dimensional space or the function space to perform system analysis and control design. Description of time delay systems as a finite dimensional systems provides the ability to use classical analysis and design techniques. Moreover, describing a time delay system by functional differential equations leads to simpler models and facilitate the usage of classical control and analysis tools. However, results obtained using functional differential equations may lead to conservative results [9, 15, 21, 22, 32, 36].

3. Over Ring Differential Equations can also be used to describe the dynamics of time delay systems. Moreover, this method provide powerful tools for system stability and observability, especially when the previous knowledge about delay is not a necessity [9, 15, 21, 22, 32, 36].

1.2 Motivations for Time Delay Analysis

Time delay studies are important in the following fields:

- Time delay exists widely in systems either as inherited phenomena or due model simplification. The design of high performance control systems require highly accurate mathematical description of the system. Therefore, including time delays in system models is necessary [17, 36, 48].

- Classical controllers are designed for finite dimensional systems. However, time delay systems are infinite dimensional systems. Therefore, classical control techniques are still not suitable for time delay system application, even when a time delay system is approximated by a functional differential equation[17, 36, 48].

- Time delays have unique characteristics. System stability can be improved by introducing time delay in the feedback paths of some systems [48].

- The subject of time delay systems appear to be a simple problem compared with other problems involving partial differential equations. Therefore, time delay systems are considered as simplified infinite dimensional systems [48].

1.3 Functional Differential Equations: A Brief Overview

Time delay systems are usually characterized by functional differential equations. The concept of functional differential equation can be introduced as follows. Assume that the set of continuous functions which maps the interval $[a,b]$ to $\mathbb{R}^n$ is represented by $C([a,b],\mathbb{R}^n)$.

The time delay system has a maximum delay defined by h. Based on this definition, the set of continuous functions which maps the interval $[-h, 0]$ to $\mathbb{R}^n$ can be referred to as $C = C([-h, 0], \mathbb{R}^n)$. Let A an arbitrary positive variable and let ψ be any continuous function such that $\psi \in C([t_0 - h, t_0 + A], \mathbb{R}^n)$, for which $t_0 \leq t \leq t_0 + A$. Moreover, denote $\psi_t \subset \psi \in C$ having the following definition $\psi_t(\theta) = \psi(t + \theta)$, $-h \leq \theta \leq 0$. The retarded functional differential equation is given as follows [17, 61]

$$\dot{x}(t) = f(t, x_t) \tag{1.1}$$

In Eqn. (1.1), $x(t) \in \mathbb{R}^n$ and $f : \mathbb{R} \times C \to \mathbb{R}^n$. Furthermore, Eqn. (1.1) shows that the time derivative of state x at time t depend on t and $x(\zeta)$, where $t - h \leq \zeta \leq t$. Therefore, the initial conditions of system states should be defined over a time interval of length h to determine the future states of the system

$$x_{t0} = \phi, \tag{1.2}$$

where $\phi \in C$ assumed to be given, i.e. $x(t_0 + \theta) = \phi(\theta)$, for $-h \leq \theta \leq 0$.

The main property of the retarded functional differential equation is that the highest order derivative does not have a delay. If a delay occurs in the highest order derivative, then the system under consideration is of the neutral type. The solution of a retarded functional differential equation x for a specific time interval should meet the following conditions [17, 61]:

- Continuous
- Satisfy the retarded functional equation over that interval
- (t, x_t) should be the domain of functional f satisfying the initial condition Eqn. (1.2)

1.4 Historical Review on the Stability of Time Delay System

Stability is one of the most important design considerations. Therefore, significant amount of research has been done in this area to study solutions for this problem [13, 42, 47, 61]. Initial research work on the stability of time delay systems dated back in the mid-twentieth century. Both of the frequency and time domain methods have been developed to study the stability of time delayed systems. Frequency domain approaches are used to determine the stability of a time delayed system which can be found in [17, 61, 68]. One of these schemes determines the stability by checking the distribution of the characteristic equation roots [18, 61]. The frequency domain approaches for time delay systems provide satisfactory results for systems with constant delays [61].

Time domain approaches provide powerful analysis alternatives. Lyapunov-Krasovskii functional and Razumikhin functions are the most common approaches for time domain stability analysis. Developing Lyapunov-Krasovskii functionals and Razumikhin functions was very difficult until the last century of the twentieth century. Therefore, the obtained results were only the existence conditions. Recently, advances in the solution of linear matrix inequalities and the introduction of Riccati equations led to the development of a general solution for Lyapunov-Krasovskii functional and Razumikhin functions [18, 61].

1.5 Time Delay Systems Stability: Time Domain Methods

A general form of a state delayed system is given by the following functional differential equation

$$\dot{x}(t) = Ax(t) + A_d x(t-h) \tag{1.3}$$

$$x(t) = \Phi(t), \quad t \in [-h, 0] \tag{1.4}$$

where, $x(t) \in \mathbb{R}^n$ represents the state vector. $h > 0$ is a scalar represents the state delay. A and $A_d \in \mathbb{R}^{n\times n}$ are the system and delay matrices respectively. $\Phi(t)$ is the initial condition.

As mentioned in section (1.4), Lyapunov-Krasovskii functional and the Razumikhin function are the most commonly used techniques to test the stability of time delayed system. The basic idea of Lyapunov-Krasovskii approach is to derive a suitable Lyapunov-Krasovskii functional to establish sufficient conditions for system stability. On the other hand, Razumikhin method uses a Lyapunov function to develop the sufficient conditions for stability.[61].

Stability analysis and control studies of time delay systems can be broadly categorized into delay independent and delay dependent cases. The delay independent case studies the stability of a time delay system without considering the size of the delay. This method leads to conservative results for systems with small delay size. A Lyapunov-Krasovskii functional of the following form is usually used

$$V_1(x_t) = x^T Px(t) + \int_{t-h}^{t} x^T(s)Qx(s)ds. \tag{1.5}$$

In Eqn. (1.5), P and Q are known as Lyapunov matrices, and they are positive definite. To obtain stability condition, an LMI problem is formulated by differentiating Eqn. (1.5) and substituting the solution of system (1.3) in the differential equation.

Delay dependent stability method studies the stability of a time delay system by first assuming that the system is stable without a delay. Then, an upper bound of delay for which the system remain stable is established. A Lyapunov-Krasovskii functional is usually used to determine stability conditions. This functional will be of the following form [61]

$$V(x_t) = V_1(x_t) + V_2(x_t) \tag{1.6}$$

where

$$V_2(x_t) = \int_{-h}^{0} \int_{t+\theta}^{t} X^T(s)Zx(s)dsd\theta$$

The derivative of $V_2(x_t)$ will be given by the following

$$\dot{V}_2(x_t) = hx^T Zx(t) - \int_{t-h}^{t} x^T(s)Zx(s)ds. \tag{1.7}$$

The integral term in the right hand side of Eqn. (1.7) imposes difficulties on the application of delay dependent stability cases. Several techniques has been developed to solve this issue. Among them, the most popular method is based on the discretization of the Lyapunov-Krasovskii functional. This method studies the stability of a time delay system by employing a discretization scheme to the Lyapunov-Krasovskii functional and present the results in the form of LMIs. This method give an approximate value of the maximum delay bound, which the system may have and remains stable. This method was introduced first by Gu in 1997 [61].

Fixed model transformation is another technique to treat the integral term in Eqn. (1.7). Several transformation techniques were proposed. Each technique has its merits and demerits. The usage of some transformation technique may lead to unrealistic results because these technique will add dynamics to the system under study. The parameterized model transformation can also be used to solve the integral issue of Eqn. (1.7). The stability problem can be divided into two problems, a delay independent problem and fixed model transformation. Each problem is treated separately [12, 19, 61].

1.5.1 Lyapunov-Krasovskii Theorem

Lyapunov-Krasovskii method is a powerful tool to study the stability of time delay systems. The main idea of this method is to develop a functional $V(t, x_t)$, which measures the deviation of the system state x_t from the system trivial solution. Moreover, the system state x_t is defined over the interval $[t-h, t]$.

To develop a more precise statement, $V(t, \phi) : \mathbb{R} \times C \rightarrow \mathbb{R}^n$ is assumed to be a differentiable functional. Moreover, $x_t(\tau, \phi)$ is assumed to be the solution of retarded functional differential equation (1.1) at time t with initial conditions given by $x_t = \phi$. Now, differentiating $V(t, x_t)$ and evaluating the solution at time $t = \tau$ yields the following equation

$$\begin{aligned}\dot{V}(\tau, \phi) &= \frac{d}{dt} V(t, x_t)|_{t=\tau,\ x_t=\phi} \\ &= \lim_{\Delta t \rightarrow 0} \sup \frac{V(\tau + \Delta t, x_{\tau+\Delta t}(\tau, \phi)) - V(\tau, \phi)}{\Delta t}\end{aligned}$$

If the derivative of functional $V(t, x_t)$ is negative, this means that x_t is not growing with the time. which also means that the time delay system defined by Eqn. (1.1) is stable [61, 17, 20].

Theorem 1.1. [17, 20, 61] *Lyapunov-Krasovskii Stability Theorem*
For the system described by Eqn. (1.1), it is assumed that $f : \mathbb{R} \times C \to \mathbb{R}^n$*. Moreover, it is assumed that there exist continuous nondecreasing functions* $u,\ v,\ w$*, which have the following properties*

- $u,\ v,\ w : \bar{\mathbb{R}}_+ \to \bar{\mathbb{R}}_+$
- $u(\tau)$ *and* $v(\tau) > 0$*, for* $\tau > 0$
- $u(0) = v(0) = 0$

Based on these assumptions, the following results hold

- *The system (1.1) is uniformly stable, if the existence of a continuously differentiable functional* $V : \mathbb{R} \times C$ *is satisfied, where* V *is identified by the following inequalities*

$$u(\|\phi(0)\|) \leq V(t, \phi) \leq v(\|\phi\|) \text{ and}$$
$$\dot{V}(t, \phi) \leq -w(\|\phi(0)\|)$$

- *Asymptotic stability of the trivial solution of system (1.1) is established, if the system is uniformly stable and* $w(t) > 0$ *when* $t = \tau$ *for* $\tau > 0$
- *Global, uniform and asymptotic stability of system (1.1) is guaranteed if uniform asymptotic stability conditions were satisfied and* $\lim_{\tau \to \infty} u(\tau) = \infty$.

1.5.2 Razumikhin Stability Theorem

Razumikhin methods is an alternative to Lyapunov-Krasovskii method, which uses functions instead of functionals. The main idea of Razumikhin methods is the development of two functions $V(x)$ and $\bar{V}(x_t)$. $V(x)$ represents the size of $x(t)$. while

$$\bar{V}(x_t) = \max_{\theta \in [-r, 0]} V(x(t + \theta)) \tag{1.8}$$

is used to measure the size of x_t. If $V(x(t)) < \bar{V}(x_t)$, this means that $\bar{V}(x_t)$ will not grow when $\dot{V}(x(t)) > 0$. To guarantee that $\bar{V}(x_t)$ will not grow, $\dot{V}(x_t)$ should always be negative when $V(x(t)) = \bar{V}(x_t)$ [17, 20, 61].

Theorem 1.2. [17, 20, 61] *Razumikhin Theorem*

For the system described by Eqn. (1.1), it is assumed that $f : \mathbb{R} \times C \to \mathbb{R}^n$. Moreover, it is assumed that there exist continuous nondecreasing functions $u,\ v,\ w$, which have the following properties

- $u,\ v,\ w : \bar{\mathbb{R}}_+ \to \bar{\mathbb{R}}_+$
- $u(\tau)$ *and* $v(\tau) > 0$, *for* $\tau > 0$
- $u(0) = v(0) = 0$

Based on these assumptions, the following results hold

- *The system (1.1), is uniformly stable if a continuously differentiable function $V : \mathbb{R} \times \mathbb{R}^n$ exists. Furthermore, V is given by the following inequalities*

$$u(\|x\|) \leq V(t, x) \leq v(\|x\|), \quad t \in \mathbb{R}, \quad x \in \mathbb{R}^n \tag{1.9}$$

$$\dot{V}(t, x(t)) \leq -w(\|x(t)\|) \quad \text{whenever } V(t+\theta, x(t+\theta)) \leq V(t, x(t)) \tag{1.10}$$

  for $\theta \in [-h, 0]$

- *Asymptotic stability of the trivial solution of system (1.1) is established, if the following conditions are satisfied*
  - $u(\|x\|) \leq V(t, x) \leq v(\|x\|), \quad t \in \mathbb{R}, \quad x \in \mathbb{R}^n$
  - $w(t) > 0$ *for* $t = \tau$ *and* $\tau > 0$
  - *Existence of a nondecreasing function* $p(t) > 0$ *for* $t = \tau$ *and* $\tau > 0$, *such that condition (1.10) can be rewritten as follows*

$$\dot{V}(t, x(t)) \leq -w(\|x(t)\|) \;\; if \;\; V(t+\theta, x(t+\theta)) \leq p(V(t, x(t)))$$

- *Global, uniform and asymptotic stability of system (1.1) is guaranteed, if the conditions of uniform asymptotic stability were satisfied and* $\lim_{\tau\to\infty} u(\tau) = \infty$.

1.6 Robust Control

Due to the existence of modeling uncertainties and the external disturbances, robust control is developed to provide the effective control solutions [39, 69, 70]. Significant amount of research work has been done. Major results in robust control theory were made during the last three decades of the twentieth century. Especially the advancement were made in[65] and [66] by introducing a systematic approach for designing a robust H_∞ controller for linear systems [31, 39].

Exact mathematical description of a system dynamics is impossible. Therefore, the robust control and stability techniques are intended to engage the control and stability problems of systems with uncertainties. Several factors make uncertainties appear in system dynamics

- Lack of full knowledge about some systems and process parameters.
- Simpler models are used to describe system dynamics due to limited capabilities of available mathematical tools. Therefore, some system dynamics are ignored
- The necessity for some control systems to operate over different operating conditions.

To encapsulate all these aspects and guarantee possible mathematical analysis, a bounded set is identified such that all uncertainties will be contained within it [17].

1.6.1 H_∞ norm

The H_∞ norm measures the size of a system represented by a transfer function

$$||G(s)||_\infty = \sup_{u(t)\neq 0} \frac{||y(t)||_2}{||u(t)||_2} \tag{1.11}$$

where $y(t)$ and $u(t)$ are the system output and input, respectively. For a stable system, the H_∞ norm can be computed as follows

$$\|G(s)\|_\infty = \sup_\omega |G(j\omega)| \tag{1.12}$$

It is obvious from Eqn. (1.12) that the H_∞ norm resembles the peak value of magnitude of a system frequency response.

According to the Bounded Real Lemma, the following condition,

$$\|G(s)\|_\infty \leq \gamma \tag{1.13}$$

can be satisfied, if and only if there exist matrix $P = P^t \geq 0$ which satisfies the following conditions [39, 16]

- Matrix P should be the solution of the following equation

$$PA + A^t P + C^t C + \gamma^{-2} PBB^t P = 0 \tag{1.14}$$

- The following matrix has stable roots

$$A + \gamma^{-2} BB^t P \tag{1.15}$$

The H_∞ norm can be interpreted as the gain in energy from input to output, i.e.

$$\|G(s)\|_\infty := \max_{u(t)\neq 0} \frac{\int_0^\infty y^t(t)y(t)dt}{\int_0^\infty u^t(t)u(t)dt} \tag{1.16}$$

The formulation of condition (1.13) can take two paths. For frequency response based system analysis, formulation of condition (1.13) is based on the H_∞ norm definition in Eqn. (1.11) and the application of the maximum singular value definition [39].

Defining the input and the output signals of the system as

$$u(t) = a_u e^{j\omega t} \qquad a_u = [a_{u1}\ a_{u2}\ ...\ a_{um}]^t$$

$$y(t) = a_y e^{j\omega t}, \qquad a_y = [a_{y1}\ a_{y2}\ ...\ a_{ym}]^t$$

where a_u and a_y are complex vectors. The vector a_y is given by

$$a_y = G(j\omega)a_u \tag{1.17}$$

Based on the maximum singular value definition, we can get the following relation, similar to Eqn. (1.11)

$$\frac{||a_y||_2}{||a_u||_2} \leq ||G(j\omega)||_\infty \Longrightarrow \sup_{a_u} \frac{||a_y||_2}{||a_u||_2} = \sigma_M(G(j\omega)) \leq ||G(j\omega)||_\infty \tag{1.18}$$

On the other hand, formulating condition (1.13) for time domain based system analysis involves the application of Parseval's theorem:

$$\begin{aligned} ||y(t)||_2^2 &= \frac{1}{2\pi}\int_{-\infty}^{\infty} y^*(j\omega)y(j\omega)d\omega \\ &= \frac{1}{2\pi}\int_{-\infty}^{\infty} ||y(j\omega)||_2^2 d\omega \end{aligned} \tag{1.19}$$

Since the relation between the input and output signals are given by

$$y(j\omega) = G(j\omega)u(j\omega)$$

where $G(j\omega)$ is the transfer matrix.

Assuming that the input and output have zero initial conditions, the following inequality for $||y(j\omega)||_2$ is defined

$$\begin{aligned} ||y(j\omega)||_2 &\leq \sigma_M(G(j\omega))||u(j\omega)||_2 \\ &\leq ||G(j\omega)||_\infty ||u(j\omega)||_2 \end{aligned} \tag{1.20}$$

Now, by substituting (1.20) into (1.19), we have

$$\begin{aligned} ||y(t)||_2^2 &\leq \frac{1}{2\pi}\int_{-\infty}^{\infty} ||G(j\omega)||_\infty ||u(j\omega)||_2^2 d\omega \\ &= ||G(j\omega)||_\infty \frac{1}{2\pi}\int_{-\infty}^{\infty} ||u(j\omega)||_2^2 d\omega \end{aligned} \tag{1.21}$$

By applying Parseval's theorem

$$||y(t)||_2^2 \leq ||G(j\omega)||_\infty ||u(t)||_2^2 \tag{1.22}$$

which implies that the following is true

$$\frac{||y(t)||_2^2}{||u(t)||_2^2} \leq \gamma \tag{1.23}$$

if the following condition holds

$$||G(j\omega)||_\infty \leq \gamma$$

Therefore, inequality (1.23) can be rewritten as

$$||y(t)||_2^2 - \gamma^2 ||u(t)||_2^2 < 0 \quad \forall u(t) \in \mathcal{L}_2 \tag{1.24}$$

1.6.2 H_∞ control

The standard H_∞ control problem block digram is shown in Fig. 1.1. $G(s)$

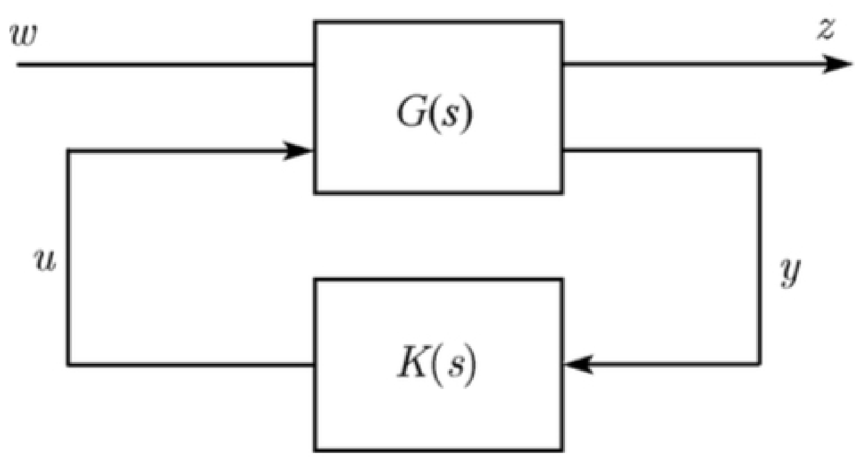

Fig. 1.1. Standard H_∞ Control Problem Block Diagram

is the system under consideration; $K(s)$ is the controller to be designed. It is assumed that the system and the controller belong to a class of finite dimensional linear time invariant systems. Furthermore, signals shown on the block diagram, $w,\ u,\ z,\ y$ are vectors representing external input, control input, controlled output, measurement output respectively. Assuming that $G(s)$ and $K(s)$ are rational, real and proper transfer function matrices. Based on Fig. 1.1, the following equation is developed

$$\begin{bmatrix} z \\ y \end{bmatrix} = G(s) \begin{bmatrix} w \\ u \end{bmatrix}$$

where $G(s)$ is given by the following

$$G(s) = \left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{array}\right] \tag{1.25}$$

Changing $G(s)$ into a decomposed form, we get

$$\begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} \tag{1.26}$$

Comparison between (1.25) and (1.26) yields

$$G_{ij}(s) = C_i(sI - A)^{-1}B_j + D_{ij}, \quad i, j = 1, 2 \tag{1.27}$$

Based on Eqn. (1.27), the closed loop transfer function from w to z is expressed by the following

$$T_{zw}(s) = G_{11}(s) + G_{12}(s)K(s)(I - G_{22}K(s))^{-1}G_{21}(s) := F_l(G, K) \tag{1.28}$$

where

$F_l(G, K)$: Lower linear fractional tranformation of $G(s)$ and $K(s)$.

Based on that, it can be seen that the H_∞ control problem involves searching for a controller $K(s)$ which stabilizes the system and minimize the $||T_{zw}(s)||_\infty$, i.e.

$$\min_K ||F_l(G, K)||_\infty$$

The optimal H_∞ controller is not unique for a MIMO system. Moreover, theoretical and computational steps to evaluate the optimal H_∞ norm can be complicated. Despite the theoritical advantage of the minimum value that H_∞ norm can achieve, the optimal H_∞ control solution is difficult to obtain. Moreover, it would be easier to find controllers with norm values in the vicinity of the optimal norm values, these controllers are called the suboptimal H_∞ controllers [39, 69, 70].

Suboptimal H_∞ control problem necessitate finding a controller $K(s)$ which stabilizes the closed loop system and guaranties $||T_{zw}(s)||_\infty < \gamma^2$ for $\gamma > 0$, i.e.

$$||F_l(G, K)||_\infty < \gamma^2$$

The H_∞ norm can be employed in the design of a state feedback controller of the following form

$$u(t) = Kx(t) \tag{1.29}$$

for a closed loop system described by

$$\begin{aligned} \dot{x}_(t) &= A_s x_s(t) + \Gamma w(t) \\ z(t) &= G_s x_s(t) + \Phi w(t) \end{aligned} \tag{1.30}$$

with

$$A_s = A + BK \qquad\qquad G_s = G + DK \tag{1.31}$$

In this case the H_∞ design problem involves finding the state feedback gain K, which guarantees both stability of the closed loop system and maintain $||z||_2 < \gamma^2 ||w||_2$ for a specific positive value of γ^2. A Lyapunov function is selected for the closed loop system to ensure stability. Based on the LaSalle's theorem, asymptotic stability is achieved, if the selected Lyapunov function V is a positive definite $V > 0$ and its derivative $\dot{V}$ is negative definite that is $\dot{V} < 0$. Therefore, the design of state feedback controller which guarantees asymptotic stability and the H_∞ norm condition, corresponds to finding a control K that makes the following inequality feasible [39, 69]

$$\dot{V}(x_s) + z^t z - \gamma^2 w^t w < 0 \tag{1.32}$$

1.7 Resilient Control

During the design phase of optimal and robust controllers, it is assumed that the controller will be exactly realized, so it can provide required level of performance and stability [38, 28]. However, this assumption doesn't hold in practice. Controller coefficients are exposed to various types of uncertainties from different sources, such as the limited length of digital words used in the system, accuracy issues in conversion between digital and analog systems, insufficient accuracy of measuring system, numerical computations issues, components aging, etc. This problem is known as controller fragility and it attracts the attention of researchers in the field of robust and optimal control, because the consequences of these perturbations may lead loss stability or performance deterioration. Controllers experiencing this issue are known as fragile controllers [38].

It is very important to have a comprehensive understanding about the effects of uncertainties in a robust controller feedback gain, because the existence of fragility in a controller may compromise controller performance for the accuracy in hardware realization. Therefore, it is important to include a model for controller uncertainties and develop design methods which provide the best performance in robustness and resiliency [38].

There are different approaches to address controller fragility problem such that the designed controller can be a non-fragile (resilient) robust controller. Most of these methods use the following procedure in the design a resilient controller [38].

- The choice of a controller of explicit formula and define necessary design parameters.

- The inclusion of multiple design performance requirement which guarantees appropriate judgment for the desired performance.
- Decide the components to be used in the control system before processing the of controller design.

The inclusion of uncertainties in controller parameters can be achieved by adding a matrix which consists uncertainties. As such, the formula of a resilient controller can be expressed as [38]

$$K(\Delta_c) = K + \Delta K(t) \tag{1.33}$$

where K is the original gain matrix, and ΔK is the gain uncertainty matrix.

The representation of the gain matrix shown in Eqn. (1.33) can take one of the following two models

- Additive Type : In this type, the gain uncertainty matrix will be given as

  $$\Delta K(t) = H_a \Delta_c(t) E_a \tag{1.34}$$

  where H_a and E_a are fixed constant matrices of appropriate dimensions. Moreover, Δ_c is an uncertain matrix which posses the following property

  $$\Delta_c \in \mathbf{\Delta} \stackrel{\Delta}{=} \{\Delta_c : \Delta_c^t \Delta_c \leq \gamma_c I\} \tag{1.35}$$

  Checking the robustness of designed controller involves tweaking the values of H_a and E_a matrices and observe the effects on performance. Controller uncertainty matrix ΔK can be a constant matrix to be found during controller synthesis by applying proper optimization techniques [38].

- Multiplicative Type: In this type, the gain uncertainty matrix will be given as

  $$\Delta K(t) = H_m \Delta_c E_m K \tag{1.36}$$

where H_m and E_m are the fixed constant matrices of appropriate dimensions. Moreover, Δ_c is an uncertain matrix. Based on Eqn. (1.36), we can rewrite Eqn. (1.33) as follows [11, 38]

$$K + \Delta K = \begin{bmatrix} k_{11}(1+\gamma_{11}) & \dots & k_{1n}(1+\gamma_{1n}) \\ \vdots & \ddots & \vdots \\ k_{m1}(1+\gamma_{m1}) & \cdots & k_{mn}(1+\gamma_{mn}) \end{bmatrix}$$

$$= \begin{bmatrix} k_{11} & \cdots & k_{1n} \\ \vdots & \ddots & \vdots \\ k_{m1} & \cdots & k_{mn} \end{bmatrix} + \begin{bmatrix} k_{11}\gamma_{11} & \cdots & k_{1n}\gamma_{1n} \\ \vdots & \ddots & \vdots \\ k_{m1}\gamma_{m1} & \cdots & k_{mn}\gamma_{mn} \end{bmatrix}$$

CHAPTER 2

WIND ENERGY CONVERSION SYSTEMS

Wind energy conversion systems are complicated systems subjected to unforeseeable, tumultuous operating conditions and inconsistent, demanding power grid. Furthermore, stability, reliability and efficiency issues are important challenges for a wind energy conversion system. Therefore, control systems play a dominant role in the advancement of wind energy industry [14].

2.1 Introduction

Limitations of existing fossil fuel resources, and the increasing demands to satisfy the energy requirements, make harvesting renewable energy resources one of the big challenges of the 21st century. Wind energy is the fastest developing renewable energy source. The development of seminal engineering techniques to improve wind energy harvesting is one of the major goals fof control and power engineers.

Wind energy conversion systems are the devices, which capture wind kinetic energy and convert it into electrical power. Significant developments have been achieved in upgrading the generating capacity of each single unit, some offshore units posses the ability to generate more than 7 MW of power. Out of the total globally generated energy, wind has a share of 2% and it is predicted that wind is going to take a bigger role in energy production during next decades [14].

Wind energy conversion systems are complicated systems subjected to unforeseeable, tumultuous operating conditions. Additionally, a wind energy conversion system has to overcome the challenges to integrate to the power grid. Furthermore, the output voltage of the wind energy conversion system should match the voltage, power sequence, and frequency. Furthermore, efficient, sustainable, and reliable energy production should be maintained. Therefore, control systems are playing critical role in the development of wind energy conversion systems [14].

2.2 Wind Energy Conversion System

A wind energy conversion system (WECS) is the system which captures the wind's kinetic energy and convert it into electrical power. The turbine rotor extract the energy from the wind by converting it to mechanical speed. An electrical generator converts the extracted mechanical energy into electrical energy [45].

There are two different arrangements of WECS, horizontal axis wind turbine (HAWT) and vertical axis (VAWT). The horizontal axis wind turbine is currently the dominating scheme in the wind energy industry. The horizontal axis wind turbine is shown in Fig. (2.1) [45].

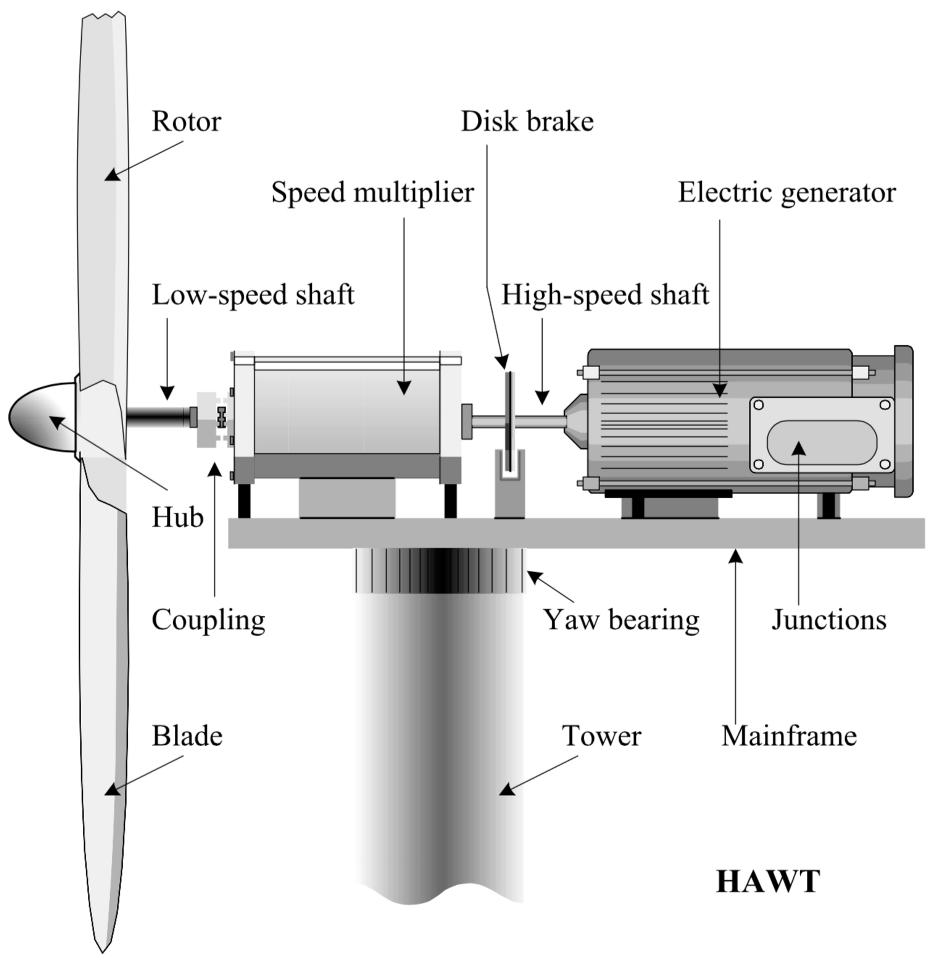

Fig. 2.1. Two Blades HAWT WECS

Wind energy conversion system consists of the following four sub-systems [45]

- The aerodynamic system: it basically contain parts of the WECS, which interact directly with the wind.
- Drive train system: this system is responsible of interfacing the wind turbine rotor shaft with the generator rotor shaft.
- Electromagnetic system: this part is basically the electrical power generation unit.
- Power electronics: this system is responsible to control the generated electrical energy and make it compatible with the grid.

2.3 WECS Control Schemes

During last decades, wind industry witnessed the emergence of several methods for energy extraction. The basic differences between these techniques are the applied control techniques and the innovations in power electronics. Therefore, different types of wind energy conversion system exist. Usually, wind energy conversion systems are classified based on whether a speed control or a power control framework is adopted [45].

The difference in the speed control technique leads to two different types of wind energy conversion systems, fixed speed WECS and variable speed WECS [45].

- Fixed speed wind energy conversion systems: These are the earliest discovered type of wind energy conversion systems. They offer a simple, and economical solution for harvesting wind energy. In these turbines, asynchronous generators are used, which are connected directly to the grid. Therefore, rotor speed are fixed to the grid frequency regardless of the wind speed [45].
- Variable speed wind energy conversion systems: They are the most commonly used type of wind energy conversion systems. Variable speed configuration has several advantageous

over fixed speed configuration. The generator is separated form the grid, which provide flexibility in applying various control techniques for optimal energy generation. The separation of generation unit from grid and the employment of advanced controllers requires improved power electronics. The increment in cost is compensated by both of the substantial controllability and hugely increased power generation levels. Wind energy conversion system can reach its maximum efficiency over a wide range of wind speeds by using this scheme. Moreover, variable speed wind turbines facilitate the application of multi objective control design techniques [45, 1].

Wind energy conversion systems can be categorized into three types based on the control design objective:

- Stall controlled WECS
- Pitch controlled WECS
- Active- stall controlled WECS

Major differences exist among these types of wind energy conversion systems. The stall controlled wind energy conversion systems have fixed rotor blade angle. Therefore, controlling the rotor blade is achieved without changing the rotor blade angle. On the other hand, pitch control scheme is based on changing blade's rotor angle of a wind turbine to reduce the impact of incoming high speed wind. The third technique uses a combination of the stall and pitch control. Moreover, each of theses techniques has its own merits and demerits. The stall technique is simple, cost effective, but it imposes stress on the turbine and reduces its efficiency. Pitch control provides flexibility for control. Furthermore, a turbine can achieve more efficient power generation with the higher cost and complexity [45].

2.4 Characterization of the Wind Energy Resource

Wind speed model mainly depends on the operation location of the turbine, and on the environmental conditions during the system operation. Therefore, the availability of useful wind speeds is time varying. The energy extracted from wind changes at a cubic rate of the wind speed. Therefore, the amount of produced energy is dependent on the wind speed. Therefore, prior to the establishment of a wind energy project, site survey is performed to asses the economical profit of the project, to specify the optimal location for each turbine, and to measure the maximum extreme wind speed for consideration in the turbine mechanical design.

Based on a time scale, variations in wind speed can be divided into three categories: large tine scales, medium time scale, and short time scale [45].

- Large time scale variations: for time scales of years or decades.
- Medium time scale variations (also called monthly variations): for time scales of months and up to year.
- Short time scale variations: for time scales of minutes or seconds.

Stochastic methods are usually used to describe monthly wind speed variations. The Weibull distribution is the most commonly used technique [45]. The Weibull distribution equation used for wind speed frequency is

$$P(v < v_i < v + dv) = P(v > 0)\left(\frac{k}{c}\right)\left(\frac{v_i}{c}\right)^{k-1} \exp\left[-\left(\frac{v_i}{c}\right)^k\right] \tag{2.1}$$

In Eqn. (2.1), c is the Weibull scale parameter, k stands for the Weibull shape parameter, v represent wind speed, v_i is a specific wind speed, dv is the increment in wind speed, $P(v < v_i < v + dv)$ is a probability for the wind speed to be within the range of $v + dv$. Moreover, $P(v > 0)$ is a probability for the wind speed to be greater that zero. The term $P(v < v_i)$ can be given as follows [45]

$$P(v < v_i) = P(v \geq 0)\left\{1 - \exp\left[-\left(\frac{v_i}{c}\right)^k\right]\right\} \tag{2.2}$$

Two methods may be used to compute Weibull distribution parameters $(c,\ k)$. The first one is based on using the natural logarithm and linear regression. The second method is based on the maximum likelihood, in which time series is used to represent wind speed data.

with only the knowledge about wind speed variations, one can not guarantee that maximum economical profit will be achieved. Therefore, a new parameter, known as the wind power density is introduced, representing the mixed effects of the wind speed frequency distribution, air density, and change in wind energy [45].

2.5 Wind Turbine: Aerodynamics and Performance

As the incoming wind hits the surface of wind turbine rotor, the aerodynamic behavior of the wind turbine is established. The aerodynamics are dependent on rotor blade position [45]. Fig. (2.2) shows the change in wind characteristics (speed and pressure) as it passes across

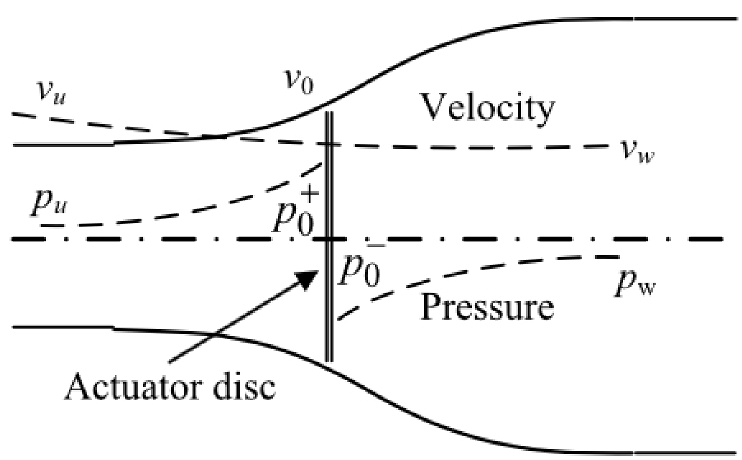

Fig. 2.2. The Actuator Disc of Energy Extraction

the actuator disc. In this figure, a subscript u will denote the wind pressure and speed before the wind hits the actuator disc. Moreover, the subscript w is used to refer to wind speed and pressure after the wind passes the actuator disc. Furthermore, wind speed and pressure having a subscript 0 represent the wind speed and pressure at the actuator disc.

Force created by the an air mass m passing through a cross sectional area A of the disc with a momentum $H = m(v_u - v_w)$, can be expressed by the following expression [45]

$$T = \frac{\Delta H}{\Delta t} = \frac{\Delta m(v_u - v_w)}{\Delta t} = \frac{\rho A v_0 \Delta t(v_u - v_w)}{\Delta t} = \rho A v_0(v_u - v_w) \tag{2.3}$$

Moreover, the force T can also be expressed as

$$T = A(p_0^+ - p_0^-) \tag{2.4}$$

$(p_0^+ - p_0^-)$ is the difference in pressure, and it can be expressed by the Bernoulli's equation as follows

$$p_0^+ - p_0^- = \frac{1}{2}\rho(v_u^2 - v_w^2) \tag{2.5}$$

Substituting (2.5) in (2.4), we have

$$T = \frac{1}{2}\rho A(v_u^2 - v_w^2) \tag{2.6}$$

Based on (2.6) and (2.3), v_0 can be expressed as

$$v_0 = \frac{1}{2}(v_u + v_w) \Longrightarrow v_u - v_w = 2(v_u - v_0) \tag{2.7}$$

Expression for the kinetic energy developed by the wind is given as

$$E_k = \frac{1}{2}mv^2 \tag{2.8}$$

Based on (2.8), the power developed of air mass passing through the disc in specific unit of time is characterized by

$$P = \frac{1}{2}\rho A v_0(v_u^2 - v_w^2) \tag{2.9}$$

An alternative expression for P is

$$P = \frac{1}{2}\rho A v^3 4a(1 - a)^2 \tag{2.10}$$

where $a = 1 - \frac{v_0}{v_u}$

Power coefficient can be expressed as

$$C_p = \frac{P}{P_t} = \frac{0.5.\rho A v^3.4a(1 - a)^2}{0.5.\rho A v^3} = 4a(1 - a)^2 \tag{2.11}$$

Wind turbine is responsible for converting from wind energy form to rotor mechanical energy. Efficiency of Energy conversion process dictates the amount of generated electrical energy. The performance of a wind energy conversion system is dependent on its output torque, thrust, and power. Therefore, dimensionless parameters representing these factors are developed to determine the performance of a wind energy conversion system. Among the most important performance indicators are the tip speed ratio, power coefficient, torque coefficient and number of blades.

2.5.1 Tip speed ratio

The tip speed ratio measures the ratio of turbine blade speed to actual wind speed. Tip speed ratio is characterized by λ, which shows how efficient a wind turbine in converting wind power to mechanical power. Furthermore, it can also be used to evaluate acoustic noise levels. Therefore, tip speed ratio plays an important role in development the control of a wind turbine. The tip speed ratio can be mathematically expressed as [45]

$$\lambda = \frac{R.\omega_r}{v} \tag{2.12}$$

where

R : Blade length

ω_r : Wind turbine rotor speed

v : Wind speed

2.5.2 Power coefficient

This performance index assess how efficient the wind turbine is in harvesting energy from the wind. Description of wind turbines aerodynamic performance is usually achieved using a curve which shows how power coefficient (C_p) value changes as tip speed ratio (λ) varies.

Fig. (2.3) shows the relation between C_p and λ for a wind turbine with two rotor blades. An expression for extracted power is developed based on (2.11) and it is given by [45].

$$P_{wt} = \frac{1}{2}\rho\pi R^2 v^3 C_p(\lambda) \tag{2.13}$$

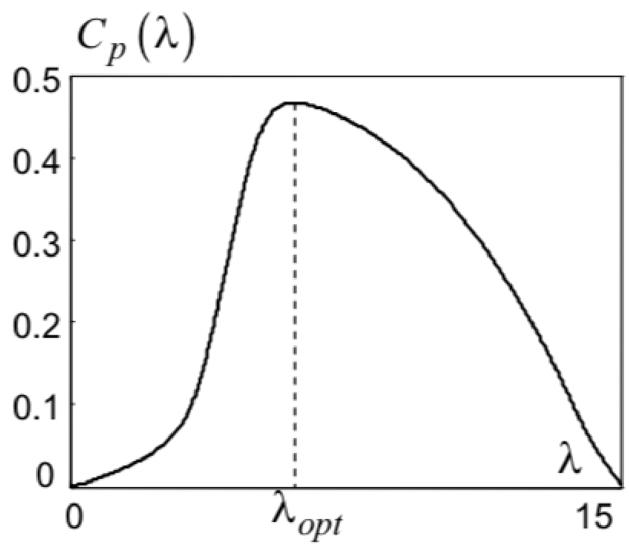

Fig. 2.3. Wind Turbine Aerodynamic Performance Curve

2.5.3 Torque coefficient

This coefficient is designated to characterize output torque from a wind turbine rotor. Thus, torque coefficient C_Γ provides a measure of turbine torque efficiency, which can be used for control design purposes. Mathematical expression is given by the dividing power coefficient by tip speed ratio [45]

$$C_\Gamma(\lambda) = \frac{C_p(\lambda)}{\lambda} \tag{2.14}$$

2.5.4 Number of blades and safety procedures

The number of blades in a wind turbine has several effects on torque, structure, rotor speed and cost. A wind turbine has either two or three blades. As the number of blades reduced, turbine weight will decrease. As such, less structural support is required, which lead to cost reduction. On the other hand, the higher the number of blades, the higher the torque that can be generated form the turbine [45].

Another aspect which affect the performance of a wind turbine is the level of maximum power to be extracted from wind. Wind turbines start to develop power as wind speed hits the blades at cut-in speed. Power output from a wind turbine continue to increase cubically as in (2.13) until wind speed hits a specific value, at which the turbine produces its rated power. Power at this wind speed is considered to be the maximum power to be generated form the turbine. Moreover, no further increase in generated power is allowed, even if wind speed continue to increase. This procedure is usually done for safety purposes. Restricting the turbine from developing torques is approached by reducing the power coefficient C_p. Several aerodynamic power control procedures are used to achieve this goal, which include passive and active control schemes. Figure (2.4) shows the developed power from a wind turbine as wind speed varies from cut-in to cut-out speed [45].

2.6 WECS: Drive Train and Generator

After discussing the aerodynamic part of the a WECS, a discussion about the drive train and the generator is presented in this section.

2.6.1 Drive train

The drive train is the part of WECS which is responsible for interfacing the wind turbine rotor with the generator rotor. Therefore, its primary role is to transmit power, torque and speed

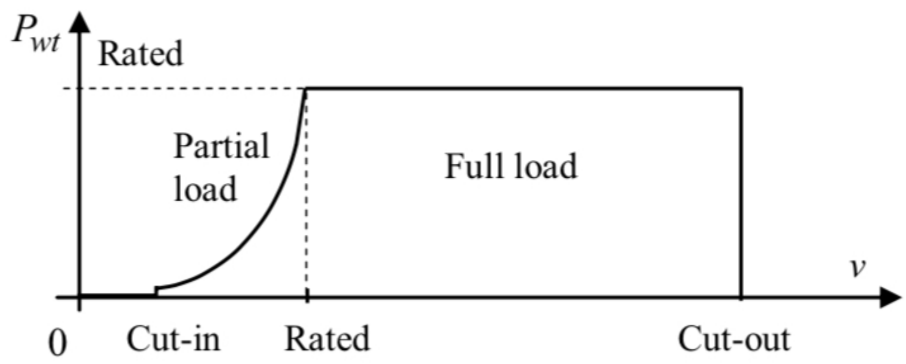

Fig. 2.4. Wind Turbine Output Power Versus Wind Speed

extracted from wind to generator. The design of a drive train is unique for each type of wind energy conversion system, specifically, it depends on the electrical generator equipped in the system. Wind energy conversion systems which use a synchronous machine as a generator, the turbine and the generator shafts are directly coupled on the same shaft. On the other hand, wind energy conversion systems utilizing an induction machine, use a different coupling arrangement. Adjustment of the wind turbine rotor speed and torque is required for this type of machines, which is usually accomplished using gearboxes that provide necessary multiplication ratio.

Using a gearbox with specific multiplication ratio in a wind energy conversion system will lead to differences between wind turbine rotor shaft speed and the generator rotor speed. Rigid or flexible coupling may be used to interface low and high speed shafts. Each of these coupling schemes has its own advantage and disadvantage. Flexible coupling adds cost and complexity, but it provides better damping [45].

2.6.2 Generator

Electrical generators are systems used to convert the mechanical power to electrical power. Usually, control of an electrical machine is accomplished using power electronics. From a modeling prospective, regardless the control approach to be used, stator and rotor voltages can be considered as inputs, while stator and rotor currents or fluxes may be considered as the states of the system.

Generally, generators consist of an electromechanical system, through which interface with mechanical power source is performed. As a generator connected to a load, an electromagnetic torque will be developed. The direction of the electromagnetic torque will be in the reverse direction of applied mechanical torque. The relation between electromechanical and electromagnetic torque can be expressed by [45]

$$J\dot{\omega}_m = \tau_{mec} - \tau_e \tag{2.15}$$

where

τ_{mec} : applied mechanical torque

τ_e : developed electrmagnetic torque

J : inertia of the high speed shaft

Currently, permanent magnet synchronous generator (PMSG) plays an important role in the wind energy industry. This trend of using the PMSG comes due the advantages and flexibility provided by PMSGs, which can be summarized as follows

- It operates with high efficiency and power factors due to the fact that a PMSG is a self excited machine.
- Excitation system has been waived due to the use of permanent magnet in the rotor. Therefore, maintenance is reduced.
- The usage of direct coupling which allows removing the gearbox. As a result, more reliable operation is achieved, due to high sensitivity of gearboxes.

2.7 Wind Energy Conversion Systems Model

In this section, a linearized state space model for the WECS will be developed.

2.7.1 Nonlinear Model

After applying the Prak's transformation to the (abc) coordinate frame permanent magnet synchronous generator (PMSG) model, The nonlinear model $d-q$ coordinate frame model is given as follows [24] [41] [55][40]

$$u_d = -R_s i_d + L_d \dot{i_d} + L_q i_q \omega_e \tag{2.16}$$

$$u_q = -R_s i_q - L_q \dot{i_q} - L_d i_d \omega_e + \Psi_m \omega_e \tag{2.17}$$

Consider the surface mounted permanent magnet synchronous generator (SPMSG) is used, then $L_d = L_q = L$. Eqn. (2.16) and Eqn. (2.17) can be further rewritten as:

$$\begin{bmatrix} \dot{i_d}(t) \\ \dot{i_q}(t) \end{bmatrix} = \begin{bmatrix} \frac{-R_s}{L} i_d(t) + \omega_e i_q \\ \frac{-R_s}{L} i_q - \omega_e i_d + \frac{\Psi_m \omega_e}{L} \end{bmatrix} + \begin{bmatrix} \frac{-1}{L} & 0 \\ 0 & \frac{-1}{L} \end{bmatrix} \begin{bmatrix} u_d \\ u_q \end{bmatrix} \tag{2.18}$$

where $\omega_e = \frac{P}{2}\omega_r$ is the electrical speed of rotor, and P is the number of the poles. On the other hand, based on Eqn. (2.15), the developed torque can be expressed as follows

$$\dot{\omega}_r = \frac{\tau_m}{J} - \frac{\tau_e}{J} \tag{2.19}$$

To achieve the control of the developed torque and rotor flux independently, the field oriented control approach is used, the electrical torque τ_e in Eqn. (2.19) has the following form

$$\tau_e = \frac{3}{2}\frac{P}{2}\Psi_m i_q \tag{2.20}$$

Combining Eqn. (2.20) and Eqn. (2.18) yields the nonlinear model of a variable speed surface mounted permanent magnet synchronous generator based wind energy conversion system as follows

$$\begin{bmatrix} \dot{i_d}(t) \\ \dot{i_q}(t) \\ \dot{\omega}_r(t) \end{bmatrix} = \begin{bmatrix} \frac{-R_s}{L} i_d + \frac{P}{2}\omega_r i_q \\ \frac{-R_s}{L} - \frac{P}{2}\omega_r i_d + \frac{\Psi_m \frac{P}{2}\omega_r}{L} \\ \frac{\tau_m}{J} - \frac{3P}{4J}\Psi_m i_q \end{bmatrix} + \begin{bmatrix} \frac{-1}{L} & 0 \\ 0 & \frac{-1}{L} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u_d \\ u_q \end{bmatrix} \tag{2.21}$$

2.7.2 Linearized Model

The system is linearized by taking the partial derivatives. Since the field oriented control framework is utilized, the operating conditions are obtained as follows [7, 6, 10].

$$i_d^* = 0$$
$$i_q^* = \frac{k^*}{k_t}(\omega_r^*)^2$$
$$\omega_r^* = \frac{\lambda_0 v}{R}$$

i_d^* is set to zero because it is desired to have the stator current to be aligned with the quadrature axis, which will lead to generate the maximum toque. Moreover, based on Fig. (2.3) the optimal value for the tip speed ratio ranges between 6 to 7 [6, 10]. Choosing the optimal $\lambda_0 = 7$, the optimal rotor shaft speed can be derived based on Eqn. (2.12). Furthermore, the mechanical torque produced by a wind turbine is given by

$$\tau_{mec} = \frac{P}{\omega_r} \tag{2.22}$$

Substituting Eqn. (2.13) in Eqn. (2.22) yields

$$\tau_{mec} = \frac{\rho\pi R^3 C_p(\lambda) v^2}{2\lambda} \tag{2.23}$$

Substituting the optimal rotor shaft speed and optimal tip sped ratio in Eqn. (2.23) yields

$$\tau_{max} = \frac{\rho\pi R^5 C_{max}(\omega_r^2)^*}{2\lambda_0^3} \tag{2.24}$$

Neglecting losses, $\tau_e = \tau_{mec}$. Therefore, the optimal value for i_q is given as

$$i_q^* = \frac{k^*}{k_t}$$

where k^* and k_t are given by

$$k^* = \frac{\rho\pi R^5 C_{pmax}}{2\lambda_0^3}$$
$$k_t = \frac{3\Psi_m P}{4}$$

Now taking the partial derivatives of the nonlinear model, the following linearized model for the WECS is obtained:

$$\begin{bmatrix} \Delta\dot{i}_d \\ \Delta\dot{i}_q \\ \Delta\dot{\omega}_r \end{bmatrix} = \begin{bmatrix} \frac{-R_s}{L} & \frac{P\omega_r^*}{2} & \frac{Pi_q^*}{2} \\ \frac{P\omega_r^*}{2} & \frac{-R_s}{L} & \frac{4P}{2L} - \frac{Pi_d^*}{2} \\ 0 & \frac{-P\Psi_m}{4J} & 0 \end{bmatrix} \begin{bmatrix} \Delta i_d \\ \Delta i_q \\ \Delta\omega_r \end{bmatrix} + \begin{bmatrix} \frac{-1}{L} & 0 \\ 0 & \frac{-1}{L} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta u_d \\ \Delta u_q \end{bmatrix} \tag{2.25}$$

For the sake of simplicity, let's drop the Δ terms. The delay appears in the system due to feedback delay in ω_r, we can rewrite the system in the following form

$$\dot{x} = A_0 x(t) + A_1 x(t-d) + B_0 u(t) \tag{2.26}$$

Considering model uncertainties and disturbances, we have

$$\dot{x} = (A_0 + \Delta A_0)x(t) + (A_1 + \Delta A_1)(t-d) + B_0 u(t) + Dw(t) \tag{2.27}$$

where ΔA_1 and ΔA_0 are the model uncertainties, $w(t)$ is the external disturbance.

Based on Eqn. (2.27), the linearized model of a variable speed surface mounted permanent magnet synchronous generator based wind energy conversion system is given as follows:

$$\begin{bmatrix} \dot{i}_d(t) \\ \dot{i}_q(t) \\ \dot{\omega}_r(t) \end{bmatrix} = \begin{bmatrix} g_1 & g_2 & 0 \\ -g_2 & g_1 & 0 \\ 0 & g_3 & 0 \end{bmatrix} \begin{bmatrix} i_d(t) \\ i_q(t) \\ \omega_r(t) \end{bmatrix} + \begin{bmatrix} 0 & 0 & \frac{Pi_q^*}{2} \\ 0 & 0 & g_4 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_q(t-\tau) \\ i_q(t-\tau) \\ \omega_r(t-\tau) \end{bmatrix} +$$

$$\begin{bmatrix} \frac{-1}{L} & 0 \\ 0 & \frac{-1}{L} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta u_d \\ \Delta u_q \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} w \tag{2.28}$$

where:

$$g_1 = \frac{-R_s + \Delta R_s}{L + \Delta L}$$

$$g_2 = \frac{P\omega_r^*}{2}$$

$$g_3 = \frac{-3P\Psi_m}{4}$$

$$g_4 = \frac{4P}{2L + \Delta L} - \frac{pi_d^*}{2}$$

2.8 Literature Survey

Controlling a wind energy conversion system is one of the most challenging fields for control engineers. Control goals are set to achieve maximum power extraction, and to meet the grid requirements [23, 51]. Many different control algorithms have been developed in literature. In [33, 49, 50], robust controllers were developed to achieve robust performance, stability, and high efficiency of energy extraction under existence of uncertainties. The nonlinear model of the wind energy conversion system is linearized around multiple operating conditions corresponding for different wind speeds in [56]. To achieve maximum power extraction, A robust H_∞ controller is developed for each operating condition. Based of Lagrange interpolation, adaptive control techniques were used to switch to the corresponding controller for each wind speed. An adaptive gain scheduling based robust controller is presented in [54]. Based on wind speed, a model with variable parameters is established. Gain scheduling technique is used to switch around different models of wind energy conversion system. Taking into consideration the case of high fluctuations in wind speed, [23] discusses the implementation of a hardware based robust H_∞ controller for a grid connected wind energy conversion system. In [24], uncertainties in stator parameters were considered. Disturbance has been also taken into consideration. A robust H_∞ mixed sensitivity method is considered for disturbance rejection and to achieve the stability of the system. Linear matrix inequalities are used to formulate controller existence problem. [8] presents a control method for maximum point power tracking of a wind energy conversion system for wind speeds below rated levels. Perturb and observe control algorithm is adopted to extract the maximum energy form wind. [44] proposes a H_∞ controller to achieve two goals in the same time. First goal is optimize wind energy extraction, while the second target is the alleviation of cyclic load power train. [43] introduces a control scheme which uses pitch control along with H_∞. The H_∞ controller is used to regulate the power electronics in the wind energy conversion system . Results of [23, 24, 29] shows the effectiveness of a robust controller to achieve stability and performance requirements for a wind energy conversion system involving uncertainties and disturbances.

2.9 Design of Robust and Resilient Controller for the Time Delayed WECS

This section presents the main result of this thesis. The general time delay system model is given as

$$\begin{aligned}\dot{x}(t) &= (A_0 + \Delta A_0)x(t) + (A_1 + \Delta A_1)x(t-d) + (B_0 + \Delta B_0)u(t)+ \\ &\quad (B_1 + \Delta B_1)u(t-h) + Dw(t) \\ z(t) &= Ex(t)\end{aligned} \tag{2.29}$$

It is required to design a state feedback controller of the form

$$u(t) = (K + \Delta K)x(t) \tag{2.30}$$

Therefore, the closed loop system is given by the following

$$\dot{x}(t) = (A_c + \Delta A_c)x(t) + (A_1 + \Delta A_1)x(t-d) + [(B_1 + \Delta B_1)(K + \Delta K)]x(t-h) + Dw(t) \tag{2.31}$$

where

$$A_c = A_0 + B_0K \tag{2.32}$$

$$\Delta A_c = \Delta A_0 + B_0\Delta K + \Delta B_0K + \Delta B_0\Delta K \tag{2.33}$$

Before proceeding to state the main theorem, the following Lemma and Assumption is stated [25, 59].

Lemma 1.

$$AB^t + BA^t \leq \alpha AA^t + \alpha^{-1}BB^t$$

To prove this inequality, we can consider the following equivalent inequality which always holds, given arbitrary $\alpha > 0$:

$$(\alpha^{1/2}A - \alpha^{-1/2}B)(\alpha^{1/2}A - \alpha^{-1/2}B)^t \geq 0$$

Further more, if A and B are chosen to be $\begin{bmatrix} a^t \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ b^t \end{bmatrix}$ respectively, we get

$$\begin{bmatrix} 0 & a^t b \\ b^t a & 0 \end{bmatrix} \leq \begin{bmatrix} \zeta a^t a & 0 \\ 0 & \zeta^{-1} b^t b \end{bmatrix}$$

Assumption 1.

$$\Delta A_0^t \Delta A_0 \leq \gamma_{A_0} I$$

$$\Delta A_1^t \Delta A_1 \leq \gamma_{A_1} I$$

$$\Delta B_1^t \Delta B_1 \leq \gamma_{B_1} I$$

$$\Delta B_0^t \Delta B_0 \leq \gamma_{B_0} I$$

$$\Delta K^t \Delta K \leq \gamma_k I$$

Theorem 2.1. *Under the feedback control law (2.30), the system defined by (2.31) is asymptotically stable for all delays satisfying* $d, h \geq 0$, *and the* H_∞ *performance objective* $\|T_{zw}\|_\infty \leq \gamma^2$ *can be satisfied, if there exist symmetric positive definite matrix* X, Y, Q_t, Q_s *satisfies the following linear matrix inequality*

$$\begin{bmatrix} \Phi_1 & X & Y^t & A_1 X & B_1 Y & D \\ X & \Phi_2 & 0 & 0 & 0 & 0 \\ Y & 0 & -[\alpha_2 + \alpha_6] I & 0 & 0 & 0 \\ X A_1^t & 0 & 0 & \alpha_5^{-1} \gamma_{A_1} I - Q_t & 0 & 0 \\ Y^t B_1^t & 0 & 0 & 0 & \Phi_3 & 0 \\ D^t & 0 & 0 & 0 & 0 & -\gamma^2 I \end{bmatrix} < 0$$

Where

$$\Phi_1 = X A_0^t + Y^t B_0^t + A_0 X + B_0 Y + Q_t + Q_s + \alpha_1^{-1} \gamma_{A_0} I + (\alpha_2^{-1} + \alpha_4^{-1}) \gamma_{B_0} I + \alpha_3 B_0 B_0^t$$

$$\Phi_2 = -([(\alpha_1 + \alpha_5) + (\alpha_3^{-1} + \alpha_4^{-1} + \alpha_7 + \alpha_8) \gamma_k] I + E^t E)$$

$$\Phi_3 = (\alpha_6^{-1} + \alpha_8^{-1}) \gamma_{B_1} I + \alpha_7^{-1} B_1^t B_1 - Q_s$$

Proof. A Lyapunov-Krasovskii functional is chosen as follows:

$$V(x,t) = x^t(t)Px(t) + \int_{t-d}^{t} x^t(v)Q_1x(v)dv + \int_{t-h}^{t} x^t(v)Q_2x(v)dv \tag{2.34}$$

where $V(x,t)$ is a positive definite functional. P, Q_1, Q_2 are all positive definite.

taking derivative of Eqn. (2.34) yields

$$\begin{aligned}\dot{V}(x,t) = \dot{x}^t(t)Px(t) + x^t(t)P\dot{x}(t) + x^t(t)Q_1x(t) - x^t(t-d)Q_1x(t-d)+ \\ x^t(t)Q_2x(t) - x^t(t-h)Q_2x(t-h)\end{aligned} \tag{2.35}$$

Based on LaSalle's Theorem, asymptotic stability is achieved if $V > 0$ and $\dot{V} < 0$

In order to satisfy H_∞ performance objective, the following H_∞ performance criterion is adopted

$$J = \int_0^\infty (z^tz - \gamma^2 w^tw)dt < 0. \tag{2.36}$$

Sufficient condition to achieve both the asymptotic stability and H_∞ performance objective is given as

$$J = \int_0^\infty (z^tz - \gamma^2 w^tw + \dot{V})dt < 0. \tag{2.37}$$

Condition (2.37) implies

$$z^tz - \gamma^2 w^tw + \dot{V} < 0 \tag{2.38}$$

Substituting Eqn. (2.35) into Eqn. (2.38) leads to the following

$$\begin{aligned}\dot{x}^t(t)Px(t) + x^t(t)P\dot{x}(t) + x^t(t)Q_1x(t) - x^t(t-d)Q_1x(t-d)+ \\ x^t(t)Q_2x(t) - x^t(t-h)Q_2x(t-h) + z^tz - \gamma^2 w^tw < 0\end{aligned} \tag{2.39}$$

Substituting Eqn. (2.31) into Eqn. (2.39) results

$$[(A_c + \Delta A_c)x(t) + (A_1 + \Delta A_1)x(t-d) + [(B_1 + \Delta B_1)(K + \Delta K)]x(t-h)$$

$$+ Dw(t)]^t Px(t) + x^t P[(A_c + \Delta A_c)x(t) + (A_1 + \Delta A_1)x(t-d) + [(B_1 + \Delta B_1)$$

$$(K + \Delta K)]x(t-h) + Dw(t)] + x^t(t)Q_1 x(t) - x^t(t-d)Q_1 x(t-d) +$$

$$x^t(t)Q_2 x(t) - x^t(t-h)Q_2 x(t-h) + z^t z - \gamma^2 w^t w < 0 \tag{2.40}$$

Denote $\zeta = \begin{bmatrix} x & x(t-d) & x(t-h) & w \end{bmatrix}^t$ and rearranging the terms, inequality (2.40) can be written as $\zeta W \zeta^t < 0$. W is given by the following

$$W = \begin{bmatrix} W_1 & P(A_1 + \Delta A_1) & W_3 & PD \\ (A_1 + \Delta A_1)^t P & -Q_1 & 0 & 0 \\ W_2 & 0 & -Q_2 & 0 \\ D^t P & 0 & 0 & -\gamma^2 I \end{bmatrix} < 0 \tag{2.41}$$

where

$$W_1 = (A_c + \Delta A_c)^t P + P(A_c + \Delta A_c) + Q_1 + Q_2 + E^t E$$

$$W_2 = [(B_1 + \Delta B_1)(K + \Delta K)]^t P$$

$$W_3 = P(B_1 + \Delta B_1)(K + \Delta K)$$

Pre and post multiply W by $\Psi = diag[X\ I\ I\ I]$, with $X = P^{-1}$ and $Y = K.P^{-1} = K.X$, yields

$$\begin{bmatrix} F_1 & A_1 + \Delta A_1 & F_2 & D \\ (A_1 + \Delta A_1)^t & -Q_1 & 0 & 0 \\ F_3 & 0 & -Q_2 & 0 \\ D^t & 0 & 0 & -\gamma^2 I \end{bmatrix} < 0 \tag{2.42}$$

Where

$$F_1 = X(A_c + \Delta A_c)^t + (A_c + \Delta Ac)X + X(Q_1 + Q_2 + E^t E)X$$

$$F_2 = (B_1 + \Delta B_1)(K + \Delta K)$$

$$F_3 = (K + \Delta K)^t (B_1 + \Delta B_1)^t$$

Substitute for A_c and ΔA_c from Eqn. (2.32) and Eqn. (2.33) respectively, by Defining $Q_t = XQ_1X$ and $Q_s = XQ_2X$ and applying Shur complement, the following inequality is obtained

$$\begin{bmatrix} \theta_1 & (A_1 + \Delta A_1)X & \theta_2 & D \\ X(A_1 + \Delta A_1)^t & -Q_t & 0 & 0 \\ \theta_3 & 0 & -Q_s & 0 \\ D^t & 0 & 0 & -\gamma^2 I \end{bmatrix} < 0, \tag{2.43}$$

where

$$\theta_1 = X\,[A_0 + \Delta A_0 + (B_0 + \Delta B_0)(K + \Delta K)]^t + [A_0 + \Delta A_0 + (B_0 + \Delta B_0)(K + \Delta K)]\,X + Q_t + Q_s$$

$$\theta_2 = (B_1 + \Delta B_1)(K + \Delta K)X$$

$$\theta_3 = (K + \Delta K)^t(B_1 + \Delta B_1)^t$$

Inequality (2.43) can be partitioned into the sum of two matrices Ω_1 and Ω_2, where

$$\Omega_1 = \begin{bmatrix} XA_0 + Y^tB_0 + A_0X + B_0Y + E^tE & A_1X & B_1Y & D \\ XA_1^t & -Q_t & 0 & 0 \\ Y^tB_1^t & 0 & -Q_s & 0 \\ D^t & 0 & 0 & -\gamma^2 I \end{bmatrix} \tag{2.44}$$

And

$$\Omega_2 = \begin{bmatrix} \Omega_{21} & \Delta A_1X & \Omega_{22} & 0 \\ X\Delta A_1^t & 0 & 0 & 0 \\ \Omega_{23} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \tag{2.45}$$

with

$$\Omega_{21} = X\,[\Delta A_0 + \Delta B_0K + B_0K + \Delta B_0\Delta K]^t + [\Delta A_0 + \Delta B_0K + B_0K + \Delta B_0\Delta K]\,X$$

$$\Omega_{22} = [\Delta B_1K + B_1\Delta K + \Delta B_1\Delta K]\,X$$

$$\Omega_{23} = X\,[\Delta B_1K + B_1\Delta K + \Delta B_1\Delta K]^t$$

(2.44) is already in the form of linear matrix, but (2.45) is not. To convert (2.45) to linear matrix form, Lemma 1 and assumption need to be applied as follows

$$\Omega_{21} = X\Delta A_0^t + Y^t\Delta B_0^t + X\Delta K^t B_0^t + X\Delta K^t\Delta B_0^t + \Delta A_0 X + \Delta B_0 Y +$$

$$B_0\Delta KX + \Delta B_0\Delta kX \leq \alpha_1 XX + \alpha^{-1}\Delta A_0^t\Delta A_0 + \alpha_2 Y^t Y + \alpha_2^{-1}\Delta B_0^t\Delta B_0 +$$

$$\alpha_3 B_0 B_0^t + \alpha_3^{-1} X\Delta K^t\Delta KX + \alpha_4\Delta B_0^t\Delta B_0 + \alpha_4^{-1} X\Delta K^t\Delta KX = \alpha_1 XX +$$

$$\alpha^{-1}\gamma_{A_0} I + \alpha_2 Y^t Y + \alpha_2^{-1}\gamma_{B_0} I + \alpha_3 B_0 B_0^t + \alpha_3^{-1} X\gamma_K IX + \alpha_4\gamma_{B_0} I + \alpha_4^{-1} X\gamma_K IX$$

$$\Omega_2 \leq \begin{bmatrix} G_1 & 0 & 0 & 0 \\ 0 & \alpha_5^{-1}\gamma_{A_1} I & 0 & 0 \\ 0 & 0 & \alpha_6^{-1}\gamma_{B_1} I + \alpha_7^{-1} B_1^t B_1 + \alpha_8^{-1}\gamma_{B_1} I & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{2.46}$$

Where

$$G_1 = \alpha_1 XX + \alpha^{-1}\gamma_{A_0} I + \alpha_2 Y^t Y + \alpha_2^{-1}\gamma_{B_0} I + \alpha_3 B_0 B_0^t + \alpha_3^{-1} X\gamma_K IX +$$

$$\alpha_4\gamma_{B_0} I + \alpha_4^{-1} X\gamma_K IX + \alpha_5 XX + \alpha_6 Y^t Y + \alpha_7 X\gamma_K IX + \alpha_8 X\gamma_K I \tag{2.47}$$

Addition of inequality (2.44) and (2.46) yields the following

$$\begin{bmatrix} C_1 & A_1 X & B_1 Y & D \\ XA_1^t & \alpha_5^{-1}\gamma_{A_1} - Q_t & 0 & 0 \\ Y^t B_1^t & 0 & C_2 & 0 \\ D^t & 0 & 0 & -\gamma^2 I \end{bmatrix} < 0 \tag{2.48}$$

where

$$C_1 = XA_0^t + Y^t B_0^t + A_0 X + B_0 Y + E^t E + Q_t + Q_s + \alpha_1 XX + \alpha^{-1}\gamma_{A_0} I + \alpha_2 Y^t Y +$$

$$\alpha_2^{-1}\gamma_{B_0} I + \alpha_3 B_0 B_0^t + \alpha_3^{-1} X\gamma_K IX + \alpha_4\gamma_{B_0} I + \alpha_4^{-1} X\gamma_K IX + \alpha_5 XX$$

$$+ \alpha_6 Y^t Y + \alpha_7 X\gamma_K IX + \alpha_8 X\gamma_K I$$

$$C_2 = \alpha_6^{-1}\gamma_{B_1} + \alpha_7^{-1} B_1^t B_1 + \alpha_8^{-1}\gamma_{B_1} - Q_s$$

Applying Schur complement to inequality (2.48) we get the proposed inequality, which completes the proof

$$\begin{bmatrix} \Phi_1 & X & Y^t & A_1X & B_1Y & D \\ X & \Phi_2 & 0 & 0 & 0 & 0 \\ Y & 0 & -[\alpha_2+\alpha_6]I & 0 & 0 & 0 \\ XA_1^t & 0 & 0 & \alpha_5^{-1}\gamma_{A_1}I - Q_t & 0 & 0 \\ Y^tB_1^t & 0 & 0 & 0 & \Phi_3 & 0 \\ D^t & 0 & 0 & 0 & 0 & -\gamma^2 I \end{bmatrix} < 0 \qquad (2.49)$$

Where

$$\Phi_1 = XA_0^t + Y^tB_0^t + A_0X + B_0Y + Q_t + Q_s + \alpha_1^{-1}\gamma_{A_0}I + (\alpha_2^{-1} + \alpha_4^{-1})\gamma_{B_0}I + \alpha_3 B_0 B_0^t$$

$$\Phi_2 = -([(\alpha_1 + \alpha_5) + (\alpha_3^{-1} + \alpha_4^{-1} + \alpha_7 + \alpha_8)\gamma_k]I + E^tE)$$

$$\Phi_3 = (\alpha_6^{-1} + \alpha_8^{-1})\gamma_{B_1}I + \alpha_7^{-1}B_1^tB_1 - Q_s$$

□

2.10 Simulation Results

he proposed approach is applied to a variable speed fixed pitch permanent magnet synchronous generator based wind energy conversion system in by Eqn. (2.28). The following parameters are chosen for simulating the system

Based on model parameters, we obtain the state space model as follows.

$$A_0 = \begin{bmatrix} -0.0125 & 84 & 0 \\ -84 & -0.0125 & 0 \\ 0 & -0.4598 & 0 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} 0 & 0 & 284.2 \\ 0 & 0 & 288.74 \\ 0 & 0 & 0 \end{bmatrix}$$

Table 2.1

Parameters of the Wind Energy Conversion System

R	3.3
L	$41.56 * 10^{-3}$
P	6
Ψ	0.4832
ρ	1.25
R_t	2.5
$\lambda_{optimal}$	7
C_p	0.47
J	1
V	12

$$B = \begin{bmatrix} \frac{-1}{41.56*10^{-3}} & 0 \\ 0 & \frac{-1}{41.56*10^{-3}} \\ 0 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^t$$

$$E = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$$

Simulations are conducted with MATLAB robust control toolbox. Results show that the proposed controller effectively controlled the wind energy conversion system. Figures 1 to 4 show the time response of the wind energy conversion system state variables and control input.

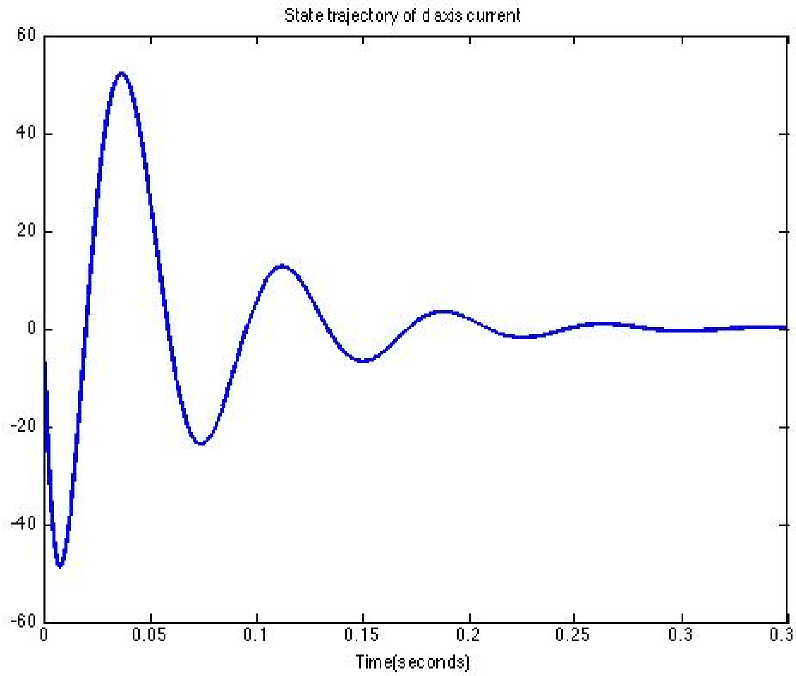

Fig. 2.5. State Trajectory of d Axis Current

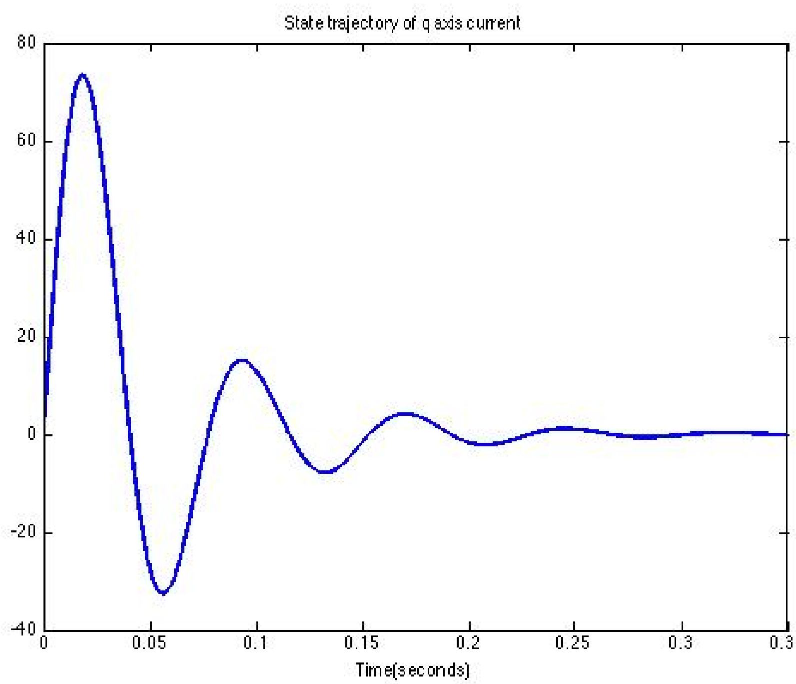

Fig. 2.6. State Trajectory of q Axis Current

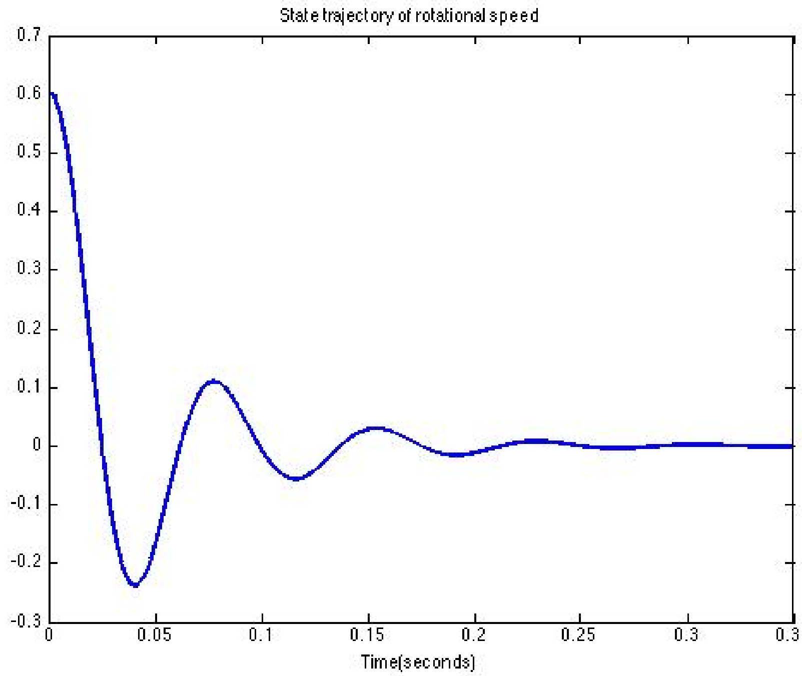

Fig. 2.7. State Trajectory of Rotational Speed

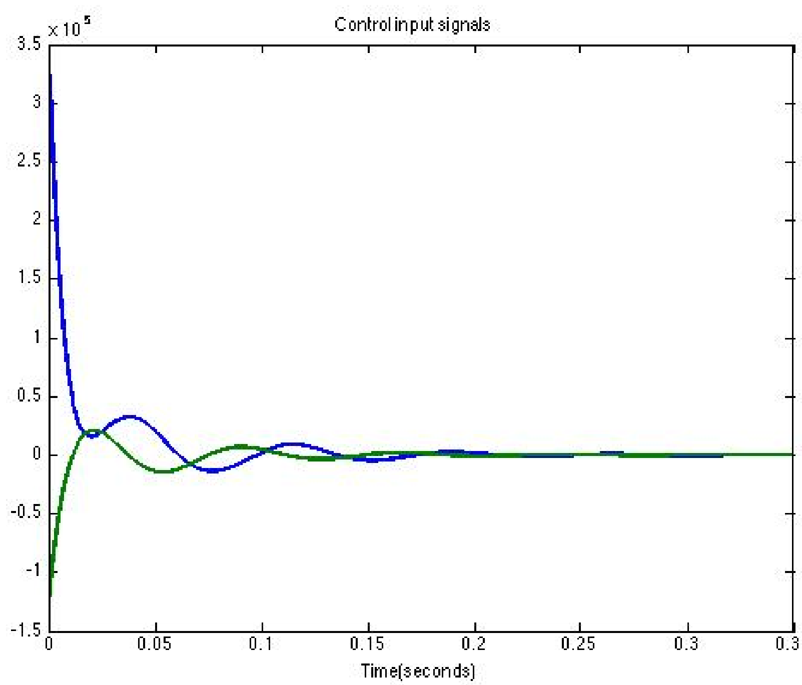

Fig. 2.8. Control Input Signals

CHAPTER 3

POWER GENERATION SYSTEMS DRIVEN BY STEAM TURBINES

Power systems are the basic infrastructure of modern civilization. Stability analysis and control system development of the smart power grid are becoming more and more important, due to the rapid deployment of the distributed energy resources (DER). In practical engineering applications, time delay plays a significant role in performance and stability of the overall power systems. Severe delays can even lead to catastrophic breakdown of the entire energy system due to instability [3, 4, 36, 46, 52, 60, 71]. For this reason, extensive research on the transient and the steady state stability analysis and controller design for power generation systems have been conducted during the past decade [27]. System design engineers should consider time delays in designing and implementing practical power systems due to their significance [3][60][62]. This chapter is mainly based on the discussion and results introduced in [25].

3.1 Basic Components of Power Generation Systems

A power generation system consists of: generation systems, transmission systems, distribution systems, and the loads [35].

3.1.1 Generation systems

The power generation systems are the devices by which energy conversion are taking place. This subsystem contains the prime mover, generator, generator control (excitation system and automatic voltage regulator) [35].

3.1.2 Transmission systems

Power systems are located near natural energy resources, and far away from load centers. Therefore, transmission lines are used to transmit electrical power from power plants to load centers [35].

3.1.3 Distribution systems

Distribution networks are intended to transfer electrical energy from transmission line to the consumers. Voltages of distribution networks are low to medium voltages [35].

3.2 Literature Survey

Many different control approaches have been studied in literature for effectively controlling the time delayed power systems. Based on an optimal control approach, the effect of time delays on the region of stability for small signals variation is studied in [26]. [67] proposes a robust control method to design a proportional-integral-derivative type load frequency control of power systems considering time delays. [53] introduces a robust controller for a wide-area power system involving input time delays. The controller is developed based on the model reduction and linear matrix inequalities. An adaptive wide area damping controller based on generalized predictive control and model identification for time delayed power system is proposed in [63]. [64] proposes a nonlinear robust control algorithm for power system considering signal delays and measurement incompleteness. [62] discusses the maximal allowable time delay margin for a stable power systems based on Lyapunov method, for a power system involving three generators and nine buses. [5] presents the nonlinear limit cycle effect of time delays on the local stability of the single machine infinite bus (SMIB) system. In [34], the Cluster treatment of eigenvalues is introduced to analyze the stability of a power system with time delays in the feedback loop.

3.3 Model of Steam Turbine Based Power System

In this section, the mathematical model of an infinite bus power system involving a synchronous generator is developed.

3.3.1 Synchronous generator model

A model for the synchronous generator is given as follows [58] :

$$\dot{E}_q' = -\frac{1}{T_{do}'}(E_q' - (X_d - X_d')I_d - E_{fd}) \tag{3.1}$$

$$\dot{\delta} = \omega - \omega_s \tag{3.2}$$

$$\dot{\omega} = \frac{\omega_s}{2H}[T_{mech} - (E_q' I_q + (X_q - X_d')I_d I_q + K_d(\omega - \omega_s))] \tag{3.3}$$

The stator algebraic equations are

$$V_T sin(\delta - \theta) + R_s I_d - X_q I_q = 0 \tag{3.4}$$

$$E_q' - V_T cos(\delta - \theta) - R_s I_q - X_d' I_d = 0 \tag{3.5}$$

Neglecting stator resistance by assuming $R_s = 0$, Eqns (3.4) and (3.5) can be rewritten as

$$V_T \sin(\delta - \theta) - X_q I_q = 0 \tag{3.6}$$

$$E_q' - V_T \cos(\delta - \theta) - X_d' I_d = 0 \tag{3.7}$$

Since

$$(V_d + jV_q)e^{j(\delta - \pi/2)} = V_T e^{j\theta} \tag{3.8}$$

therefore,

$$V_d = V_T \sin(\delta - \theta) \tag{3.9}$$

$$V_q = V_T \cos(\delta - \theta) \tag{3.10}$$

Substituting in Eqn (3.6) and Eqn. (3.7) yields

$$V_d - X_q I_q = 0 \tag{3.11}$$

$$E_q' - V_q - X_d' I_d = 0 \tag{3.12}$$

Assuming zero degree phase angle for the infinite bus voltage

$$(I_d + jI_q)e^{j(\delta - \pi/2)} = \frac{(V_d + jV_q)e^{j(\delta - \pi/2)} - V_\infty \angle 0^o}{R_e + jX_e} \tag{3.13}$$

Separating the imaginary and real parts of Eqn. (3.13)

$$R_d I_d - X_e I_q = V_d - V_\infty \sin(\delta)$$
$$X_e I_d - R_e I_q = V_q - V_\infty \cos(\delta) \tag{3.14}$$

Linearizing Eqn. (3.11) and Eqn. (3.12)

$$\begin{bmatrix} \Delta V_d \\ \Delta V_q \end{bmatrix} = \begin{bmatrix} 0 & X_q \\ -X'_d & 0 \end{bmatrix} \begin{bmatrix} \Delta I_q \\ \Delta I_q \end{bmatrix} + \begin{bmatrix} 0 \\ \Delta E'_q \end{bmatrix} \tag{3.15}$$

Moreover, linearizing Eqn. (3.14)

$$\begin{bmatrix} \Delta V_d \\ \Delta V_q \end{bmatrix} = \begin{bmatrix} R_e & -X_e \\ X_e & R_e \end{bmatrix} \begin{bmatrix} \Delta I_q \\ \Delta I_q \end{bmatrix} + \begin{bmatrix} V_\infty \sin(\delta) \\ -V_\infty \cos(\delta) \end{bmatrix} \Delta\delta \tag{3.16}$$

Equating the right hand sides of the Eqn. (3.15) and Eqn. (3.16) yields the following

$$\begin{bmatrix} R_e & -(X_e + X_q) \\ (X_e + X_d^{'}) & R_e \end{bmatrix} \begin{bmatrix} \Delta I_d \\ \Delta I_q \end{bmatrix} = \begin{bmatrix} 0 \\ \Delta E_q^{'} \end{bmatrix} + \begin{bmatrix} -V_\infty cos\delta \\ V_\infty sin\delta \end{bmatrix} \Delta\delta \tag{3.17}$$

$\Delta I_d, \Delta I_q$ can be obtained from Eqn. (3.15), and Eqn. (3.16) as

$$\begin{bmatrix} \Delta I_d \\ \Delta I_q \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} (X_e + X_q) & -R_e V_\infty cos\delta + V_\infty sin\delta (X_q + X_e) \\ R_e & R_e V_\infty sin\delta + V_\infty cos\delta (X_d^{'} + X_e) \end{bmatrix} \begin{bmatrix} \Delta E_q^{'} \\ \Delta\delta \end{bmatrix} \tag{3.18}$$

where

$$\Delta = R_e^2 + (X_e + X_q)(X_e + X_d^{'}) \tag{3.19}$$

Denote the normalized frequency $\nu = \frac{\omega}{\omega_s}$. The linearized synchronous generator model of Eqns. (3.1)-(3.3) is given as follows.

$$\begin{bmatrix} \Delta\dot{E}_q' \\ \Delta\dot{\delta} \\ \Delta\dot{\nu} \end{bmatrix} = \begin{bmatrix} -\frac{1}{T_{do}'} & 0 & 0 \\ 0 & 0 & \omega_s \\ -\frac{I_q^o}{2H} & 0 & -\frac{K_d\omega_s}{2H} \end{bmatrix} \begin{bmatrix} \Delta E_q' \\ \Delta\delta \\ \Delta\nu \end{bmatrix} + \begin{bmatrix} -\frac{1}{T_d'}(X_d - X_d') & 0 \\ 0 & 0 \\ \frac{1}{2H}(X_d' - X_q)I_q^o & \frac{1}{2H}(X_d' - X_q)I_d^o - \frac{1}{2H}E_q'^o \end{bmatrix} \begin{bmatrix} \Delta I_d \\ \Delta I_q \end{bmatrix}$$

$$+ \begin{bmatrix} \frac{1}{T_{do}'} & 0 \\ 0 & 0 \\ 0 & \frac{1}{2H} \end{bmatrix} \begin{bmatrix} \Delta E_{fd} \\ \Delta T_{mech} \end{bmatrix} \tag{3.20}$$

Substitute for $\Delta I_d, \Delta I_q$, results

$$\Delta\dot{E}_q' = -\frac{1}{K_3 T_{do}'}\Delta E_q' - \frac{K_4}{T_{do}'}\Delta\delta + \frac{1}{T_{do}'}\Delta E_{fd} \tag{3.21}$$

$$\Delta\dot{\delta} = \omega_s \Delta\nu \tag{3.22}$$

$$\Delta\dot{\nu} = -\frac{K_2}{2H}\Delta E_q' - \frac{K_1}{2H}\Delta\delta - \frac{K_d\omega_s}{2H}\Delta\nu + \frac{1}{2H}\Delta T_{mech} \tag{3.23}$$

where

$$\frac{1}{K_3} = 1 + \frac{(X_d - X_d')(X_q + X_e)}{\Delta} \tag{3.24}$$

$$K_4 = \frac{V_\infty(X_d - X_d')}{\Delta}[(X_q + X_e)sin\delta - R_e cos\delta] \tag{3.25}$$

$$K_2 = \frac{1}{\Delta}[I_q^o\Delta - I_q^o(X_d' - X_q)(X_q + X_e) - R_e(X_d' - X_q)I_d^o + R_e E_q'^o] \tag{3.26}$$

$$K_1 = -\frac{1}{\Delta}[I_q^o V_\infty(X_d' - X_q)\{(X_q + X_e)sin\delta - R_e cos\delta\}$$

$$+V_\infty\{(X_d' - X_q)I_d^o - E_q'^o\}\{(X_d' + X_e)cos\delta + R_e sin\delta\}] \tag{3.27}$$

Since

$$V_T^2 = V_d^2 + V_q^2$$

the differential terms is given as follows

$$\Delta V_T = \frac{V_d^o}{V_T}\Delta V_d + \frac{V_q^o}{V_T}\Delta V_q \tag{3.28}$$

Substituting Eqn. (3.18) into Eqn. (3.15), leads to the following

$$\begin{bmatrix} \Delta V_d \\ \Delta V_q \end{bmatrix} = \frac{1}{\Delta}\begin{bmatrix} X_q R_e & X_q(R_e V_\infty sin\delta + V_\infty cos\delta(X_d^{'} + X_e) \\ -X_d^{'}(X_q + X_e) & -X_d^{'}(-R_e V_\infty cos\delta + V_\infty (X_q + X_e) sin\delta) \end{bmatrix}\begin{bmatrix} \Delta E_q^{'} \\ \Delta\delta \end{bmatrix} + \begin{bmatrix} 0 \\ \Delta E_q^{'} \end{bmatrix} \tag{3.29}$$

Based on Eqn. (3.28), and Eqn. (3.29), the following result is obtained

$$\Delta V_T = K_5 \Delta\delta + K_6 \Delta E_q' \tag{3.30}$$

where

$$K_5 = \frac{1}{\Delta}\{\frac{V_d^o}{V_T} X_q [R_e V_\infty sin\delta + V_\infty cos\delta(X_d^{'} + X_e)] + \frac{V_q^o}{V_T}[X_d^{'}(R_e V_\infty cos\delta) - V_\infty (X_q + X_e) sin\delta]\} \tag{3.31}$$

$$K_6 = \frac{1}{\Delta}\{\frac{V_d^o}{V_T} X_q R_e - \frac{V_q^o}{V_T} X_d^{'}(X_q + X_e)\} + \frac{V_q^o}{V_T} \tag{3.32}$$

3.3.2 Automatic voltage regulator and exciter circuit dynamics

The following dynamical equations for Automatic Voltage Regulator (AVR) and excitation control system are adopted:

$$\dot{E}_{fd} = \frac{K_A}{T_A}(V_{ref} - V_T + U_{pss} - \frac{E_{fd}}{T_A}) \tag{3.33}$$

Linearizing (3.33) yields

$$\Delta\dot{E}_{fd} = \frac{K_A}{T_A}(\Delta V_{ref} - \Delta V_T + \Delta U_{pss}) - \frac{\Delta E_{fd}}{T_A} \tag{3.34}$$

Based on Eqn. (3.30), Eqn. (3.34) can be rewritten as

$$\Delta\dot{E}_{fd} = -\frac{\Delta E_{fd}}{T_A} - \frac{K_A K_5}{T_A}\Delta\delta - \frac{K_A K_6}{T_A}\Delta E_q' + \frac{K_A}{T_A}\Delta U_{pss} + \frac{K_A}{T_A}\Delta V_{ref} \tag{3.35}$$

A typical Power System Stabilizer (PSS) control scheme include a washout filter and two lead-lag blocks. The retarded measure of ν propagates in the PSS equations. The linearized form of power system stabilizer can be modeled as:

$$\begin{aligned}
\Delta\dot{z}_1 &= -(K_w\Delta\nu(t-\tau)+\Delta z_1)/T_w \\
\Delta\dot{z}_2 &= [(1-\frac{T_1}{T_2})(K_w\Delta\nu(t-\tau)+\Delta z_1)-\Delta z_2]/T_2 \\
\Delta\dot{z}_3 &= \{(1-\frac{T_3}{T_4})[\Delta z_2+(\frac{T_1}{T_2}(K_w\Delta\nu(t-\tau)+\Delta z_1)]-\Delta z_3\}/T_4 \\
\Delta U_{pss} &= \Delta z_3+\frac{T_3}{T_4}[\Delta z_2+\frac{T_1}{T_2}(K_w\Delta\nu(t-\tau)+\Delta z_1)]
\end{aligned}$$

Hence, the overall linearized model of power generation system is given as follows

$$\begin{aligned}
\Delta\dot{\delta} &= \omega_s\Delta\nu && (3.36) \\
\Delta\dot{\nu} &= -\frac{K_2}{2H}\Delta E_q^{'}-\frac{K_1}{2H}\Delta\delta-\frac{K_d\omega_s}{2H}\Delta\nu+\frac{1}{2H}\Delta T_{mech} && (3.37) \\
\Delta\dot{E}_q^{'} &= -\frac{1}{K_3T_{do}^{'}}\Delta E_q^{'}-\frac{K_4}{T_{do}^{'}}\Delta\delta+\frac{1}{T_{do}^{'}}\Delta E_{fd} && (3.38) \\
\Delta\dot{E}_{fd} &= -\frac{\Delta E_{fd}}{T_A}-\frac{K_AK_5}{T_A}\Delta\delta-\frac{K_AK_6}{T_A}\Delta E_q^{'}+\frac{K_A}{T_A}\Delta V_{ref} \\
&\quad +\frac{K_A}{T_A}\{\Delta z_3+\frac{T_3}{T_4}[\Delta z_2+\frac{T_1}{T_2}(K_w\Delta\nu(t-\tau)+\Delta z_1)]\} && (3.39) \\
\Delta\dot{z}_1 &= -(K_w\Delta\nu(t-\tau)+\Delta z_1)/T_w && (3.40) \\
\Delta\dot{z}_2 &= [(1-\frac{T_1}{T_2})(K_w\Delta\nu(t-\tau)+\Delta z_1)-\Delta z_2]/T_2 && (3.41) \\
\Delta\dot{z}_3 &= \{(1-\frac{T_3}{T_4})[\Delta z_2+(\frac{T_1}{T_2}(K_w\Delta\nu(t-\tau)+\Delta z_1)]-\Delta z_3\}/T_4 && (3.42)
\end{aligned}$$

Denote $x=[\Delta\delta,\Delta\nu,\Delta E_q',\Delta E_{fd},\Delta z_1,\Delta z_2,\Delta z_3]^t$ and $u=[\Delta T_{mech},\Delta V_{ref}]^t$, the linearized model becomes

$$\dot{x}=A_0x(t)+A_1x(t-d)+B_0u(t)$$

where

$$A_0 = \begin{bmatrix} 0 & \omega_s & 0 & 0 & 0 & 0 & 0 \\ -\frac{K_1}{2H} & -\frac{K_d\omega_s}{2H} & -\frac{K_2}{2H} & 0 & 0 & 0 & 0 \\ -\frac{K_4}{T'_{do}} & 0 & -\frac{1}{K_3T'_{do}} & -\frac{1}{T'_{do}} & 0 & 0 & 0 \\ -\frac{K_AK_5}{T_A} & 0 & -\frac{K_AK_6}{T_A} & -\frac{1}{T_A} & \frac{K_AT_3T_1}{T_AT_4T_2} & \frac{K_AT_3}{T_AT_4} & \frac{K_A}{T_A} \\ 0 & 0 & 0 & 0 & -\frac{1}{T_w} & 0 & 0 \\ 0 & 0 & 0 & 0 & (1-\frac{T_1}{T_2})\frac{1}{T_2} & -\frac{1}{T_2} & 0 \\ 0 & 0 & 0 & 0 & (1-\frac{T_3}{T_4})\frac{T_1}{T_2}\frac{1}{T_4} & (1-\frac{T_3}{T_4})\frac{1}{T_4} & -\frac{1}{T_4} \end{bmatrix} \tag{3.43}$$

$$A_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{T_3T_1K_w}{T_4T_2} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{K_w}{T_w} & 0 & 0 & 0 & 0 & 0 \\ 0 & (1-\frac{T_1}{T_2})K_w\frac{1}{T_2} & 0 & 0 & 0 & 0 & 0 \\ 0 & (1-\frac{T_3}{T_4})\frac{T_1}{T_2}K_w\frac{1}{T_4} & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{3.44}$$

$$B_0 = \begin{bmatrix} 0 & 0 \\ \frac{1}{2H} & 0 \\ 0 & 0 \\ 0 & \frac{K_A}{T_A} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{3.45}$$

3.4 Robust H_∞ Controller Design

In this section, a novel design of the robust controller satisfying H_∞ performance objective is proposed. A system with state and input delays, uncertainties and disturbances is considered.

The system is of the form:

$$\dot{x}(t) = (A_0 + \Delta A_0)x(t) + (A_1 + \Delta A_1)x(t-d) + (B_0 + \Delta B_0)u(t) + (B_1 + \Delta B_1)u(t-h) + Dw(t) \quad (3.46)$$

the performance output is chosen as

$$z(t) = Ex(t)$$

and

$$x(t) = \phi(t) \quad for\ t \in [-d, 0]$$

It is required to design a state feedback controller of the following form

$$u(t) = Kx(t) \quad (3.47)$$

It is assumed that the state variables are available for feedback. Otherwise, estimators can be developed for state estimation purposes. Therefore, the closed loop system becomes:

$$\dot{x}(t) = (A_0 + \Delta A_0 + B_0K + \Delta B_0)x(t) + (A_1 + \Delta A_1)x(t-d) + (B_1 + \Delta B_1)Kx(t-h) + Dw(t) \quad (3.48)$$

Rearranging Eqn. (3.48), yields

$$\dot{x}(t) = A_c x(t) + \Delta A_c x(t) + (A_1 + \Delta A_1)x(t-h) + (B_1 + \Delta B_1)Kx(t-h) + Dw(t) \quad (3.49)$$

where

$$A_c = A_0 + B_0K$$

$$\Delta A_c = \Delta A_0 + \Delta B_0 K$$

Before proceeding to the theorem derivation, Assumption 1 and Lemma 1 are introduced [59].

Assumption 1. The general form of unstructured L_2 bounded uncertainties is used in this work:

$$\Delta A_0 \Delta A_0^t \leq \gamma_{A_0} I$$

$$\Delta A_1 \Delta A_1^t \leq \gamma_{A_1} I$$

$$\Delta B_0 \Delta B_0^t \leq \gamma_{B_0} I$$

$$\Delta B_1 \Delta B_1^t \leq \gamma_{B_1} I$$

Lemma 1.

$$AB^t + BA^t \leq \alpha AA^t + \alpha^{-1} BB^t$$

To prove this inequality, we can consider the following equivalent inequality which always holds, given arbitrary $\alpha > 0$:

$$(\alpha^{1/2} A - \alpha^{-1/2} B)(\alpha^{1/2} A - \alpha^{-1/2} B)^t \geq 0$$

Further more, if A and B are chosen to be $\begin{bmatrix} a^t \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ b^t \end{bmatrix}$ respectively, we get

$$\begin{bmatrix} 0 & a^t b \\ b^t a & 0 \end{bmatrix} \leq \begin{bmatrix} \zeta a^t a & 0 \\ 0 & \zeta^{-1} b^t b \end{bmatrix}$$

Based on Assumption 1 and Lemma 1, the main theorem of the paper is summarized as follows:

Theorem 1. Under the feedback control law (3.47), the system of (3.49) is asymptotically stable for all delays satisfying $d, h \geq 0$. And the H_∞ performance objective $\|T_{zw}\|_\infty \leq \gamma^2$ can be satisfied. if there exist symmetric positive definite matrix X, Y, Q_t, Q_s satisfies the following

LMI:

$$\begin{bmatrix} m_1 & A_1X & B_1Y & D & X & Y^t \\ XA_1^t & m_2 & 0 & 0 & 0 & 0 \\ Y^tB_1^t & 0 & m_3 & 0 & 0 & 0 \\ D^t & 0 & 0 & m_4 & 0 & 0 \\ X & 0 & 0 & 0 & m_5 & 0 \\ Y & 0 & 0 & 0 & 0 & m_6 \end{bmatrix} < 0 \tag{3.50}$$

where

$$m_1 = A_0X + B_0Y + XA_0^t + Y^tB_0^t + Q_t + Q_s + \alpha_1(\gamma_A + \gamma_B)I$$

$$m_2 = \alpha_2^{-1}I - Q_t$$

$$m_3 = \alpha_3^{-1}I - Q_s$$

$$m_4 = -\gamma^2 I$$

$$m_5 = -[\alpha_1^{-1}I + \alpha_2\gamma_{A1}I + E^tE]^{-1}$$

$$m_6 = -[\alpha_1^{-1}I + \alpha_3\gamma_{B1}I]^{-1}$$

Proof. A Lyapunov-Krasovskii function is chosen as follows:

$$V(x,t) = x^t(t)Px(t) + \int_{t-d}^{t} x^t(v)Q_1x(v)dv + \int_{t-h}^{t} x^t(v)Q_2x(v)dv \tag{3.51}$$

where $V(x,t)$ is a positive semi-definite functional and the matrices P, Q_1, Q_2 are all positive definite.

By taking derivative, we have

$$\dot{V}(x,t) = \dot{x}^t(t)Px(t) + x^t(t)P\dot{x}(t) + x^t(t)Q_1x(t) - x^t(t-d)Q_1x(t-d) + x^t(t)Q_2x(t) - x^t(t-h)Q_2x(t-h) \tag{3.52}$$

Based on LaSalle's Theorem, in order to achieve the asymptotic stability, the conditions $V > 0$

and $\dot{V} < 0$ need to be satisfied.

In order to satisfy H_∞ performance objective, the following H_∞ performance inequality needs be employed

$$J = \int_0^\infty (z^t z - \gamma^2 w^t w)dt < 0 \tag{3.53}$$

The sufficient condition to achieve both the asymptotic stability and H_∞ performance objective is

$$J = \int_0^\infty (z^t z - \gamma^2 w^t w + \dot{V})dt < 0 \tag{3.54}$$

Condition (3.54) implies

$$z^t z - \gamma^2 w^t w + \dot{V} < 0 \tag{3.55}$$

Substituting Eqn. (3.49) into Eqn. (3.52), results

$$\begin{aligned}\dot{V}(x,t) =& [A_c x(t) + \Delta A_c x(t) + (A_1 + \Delta A_1)x(t-d) + (B_1 + \Delta B_1)Kx(t-h) + Dw(t)]^t Px(t) + \\ & x^t(t)P[A_c x(t) + \Delta A_c x(t) + (A_1 + \Delta A_1)x(t-d) + (B_1 + \Delta B_1)Kx(t-h) + Dw(t)] + \\ & x^t(t)Q_1 x(t) - x^t(t-d)Q_1 x(t-d) + x^t(t)Q_2 x(t) - x^t(t-h)Q_2 x(t-h) < 0 \end{aligned} \tag{3.56}$$

Based on condition (3.54), we have

$$\begin{aligned}& [A_c x(t) + \Delta A_c x(t) + (A_1 + \Delta A_1)x(t-d) + (B_1 + \Delta B_1)Kx(t-h) + Dw(t)]^t Px(t) + \\ & x^t(t)P[A_c x(t) + \Delta A_c x(t) + (A_1 + \Delta A_1)x(t-d) + (B_1 + \Delta B_1)Kx(t-h) + Dw(t)] + \\ & x^t(t)Q_1 x(t) - x^t(t-d)Q_1 x(t-d) + x^t(t)Q_2 x(t) - x^t(t-h)Q_2 x(t-h) + z^t z - \gamma^2 w^t w < 0 \end{aligned} \tag{3.57}$$

Denote $\zeta(t) = \begin{bmatrix} x^t(t) & x^t(t-d) & x^t(t-h) & w^t(t) \end{bmatrix}^t$, then Eqn. (3.57) can be written as $\zeta^t(t) W_o \zeta(t) < 0$

where

$$W_o = \begin{bmatrix} c_1 & P(A_1 + \Delta A_1) & P(B_1 + \Delta B_1)K & PD \\ (A_1 + \Delta A_1)^t P & -Q_1 & 0 & 0 \\ K^t(B_1 + \Delta B_1)^t P & 0 & -Q_2 & 0 \\ D^t P & 0 & 0 & -\gamma^2 I \end{bmatrix} < 0 \qquad (3.58)$$

where

$$c_1 = P(A_c + \Delta A_c) + (A_c + \Delta A_c)^t P + Q_1 + Q_2 + E^t E$$

pre- and post-multiply Eqn. (3.58) with a diagonal matrix $diag(X, I, I, I)$, and denote

$$X = P^{-1}, \qquad Y = KX, \qquad Q_t = XQ_1X, \qquad Q_s = XQ_2X,$$

results in the following

$$\begin{bmatrix} \phi_{11} & \phi_{12} & \phi_{13} & D \\ \phi_{12}^t & \phi_{22} & 0 & 0 \\ \phi_{13}^t & 0 & \phi_{33} & 0 \\ D^t & 0 & 0 & \phi_{44} \end{bmatrix} < 0 \qquad (3.59)$$

where

$$\phi_{11} = (A_0 + B_0K + \Delta A_0 + \Delta B_0K)X + X(A_0 + B_0K + \Delta A_0 + \Delta B_0K)^t + Q_s + Q_t + XE^tEX$$

$$\phi_{12} = (A_1 + \Delta A_1)X$$

$$\phi_{13} = (B_1 + \Delta B_1)Y$$

$$\phi_{22} = -Q_t$$

$$\phi_{33} = -Q_s$$

$$\phi_{44} = -\gamma^2 I$$

Applying Lemma 1, yields

$$(\Delta A_0 + \Delta B_0 K)X + X(\Delta A_0 + \Delta B_0 K)^t = X\begin{bmatrix} I & K^t \end{bmatrix}^t \begin{bmatrix} \Delta A_0^t \\ \Delta B_0^t \end{bmatrix} + \begin{bmatrix} \Delta A_0 & \Delta B_0 \end{bmatrix}\begin{bmatrix} I \\ K \end{bmatrix} X$$

$$\leq \alpha_1 \begin{bmatrix} \Delta A_0 & \Delta B_0 \end{bmatrix}\begin{bmatrix} \Delta A_0^t \\ \Delta B_0^t \end{bmatrix} + \alpha_1^{-1} X \begin{bmatrix} I & K^t \end{bmatrix}\begin{bmatrix} I \\ K \end{bmatrix} X$$

Applying Assumption 1, results the following

$$(\Delta A_0 + \Delta B_0 K)X + X(\Delta A_0 + \Delta B_0 K)^t \leq \alpha_1(\gamma_{A_0} I + \gamma_{B_0} I) + \alpha_1^{-1} X \begin{bmatrix} I & K^t \end{bmatrix}\begin{bmatrix} I \\ K \end{bmatrix} X \quad (3.60)$$

Based on Lemma 1 and Assumption 1, the following matrix inequality is obtained:

$$\begin{bmatrix} 0 & \Delta A_1 X & \Delta B_1 Y & 0 \\ X^t \Delta A_1^t & 0 & 0 & 0 \\ Y^t \Delta B_1^t & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \leq \begin{bmatrix} \alpha_2 \gamma_{A_1} X^t X + \alpha_3 \gamma_{B_1} Y^t Y & 0 & 0 & 0 \\ 0 & \alpha_2^{-1} I & 0 & 0 \\ 0 & 0 & \alpha_3^{-1} I & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (3.61)$$

Substituting Eqn. (3.60) and Eqn. (3.61) into Eqn. (3.59) and applying Schur complement, results to the following linear matrix inequality

$$\begin{bmatrix} \zeta_{11} & \zeta_{12} & \zeta_{13} & D & X & Y^t \\ \zeta_{12}^t & \zeta_{22} & 0 & 0 & 0 & 0 \\ \zeta_{13}^t & 0 & \zeta_{33} & 0 & 0 & 0 \\ D^t & 0 & 0 & \zeta_{44} & 0 & 0 \\ X^t & 0 & 0 & 0 & \zeta_{55} & 0 \\ Y & 0 & 0 & 0 & 0 & \zeta_{66} \end{bmatrix} < 0 \quad (3.62)$$

where

$$
\begin{aligned}
\zeta_{11} =& A_0X + B_0Y + XA_0^t + Y^tB_0^t + Q_t + Q_s + \alpha_1(\gamma_{A_0} + \gamma_{B_0})I \\
\zeta_{12} =& A_1X \\
\zeta_{13} =& B_1Y \\
\zeta_{22} =& \alpha_2^{-1}I - Q_t \\
\zeta_{33} =& \alpha_3^{-1}I - Q_s \\
\zeta_{44} =& -\gamma^2 I \\
\zeta_{55} =& -[\alpha_1^{-1}I + \alpha_2\gamma_{A_1}I + E^tE]^{-1} \\
\zeta_{66} =& -[\alpha_1^{-1}I + \alpha_3\gamma_{B_1}I]^{-1}
\end{aligned}
$$

. □

3.5 Simulation and Results

The following parameters are used for simulations. Assuming that $R_e = 0$, $X_e = 0.5pu$, $V_T\angle\theta = 1\angle 15^o pu$, and $V_\infty\angle 0^o = 1.05\angle 0^o pu$. The generator, automatic voltage regulator and exciter parameters are $H = 3.2sec$, $T_{do}^{'} = 9.6sec$, $K_A = 400$, $T_A = 0.2sec$, $R_s = 0pu$, $X_q = 2.1pu$, $X_d = 2.5pu$, $X_d^{'} = 0.39pu$, $K_d = 0$, and $\omega_s = 377$. The power system stabilizer parameters are $K_w = 0.5, T_1 = 0.5, T_2 = 0.01, T_3 = 1, T_4 = 0.1, T_w = 10$. Based on (3.24), (3.25), (3.26), (3.27), (3.31) and (3.32), we can calculate the values: $K_1 = 0.9224, K_2 = 1.0739.K_3 = 0.296667, K_4 = 2.26555, K_5 = 0.005, K_6 = 0.3572$

The L_2 of disturbance is chosen as $w(t) = 5 \times 0.9^t$, notice that the disturbance energy is finite.

MATLAB robust control toolbox provide the capability to design the optimal control feedback with LMI. Computer simulation shows that our proposed controller effectively stabilizes the time response of rotor angle in Fig.1, normalized frequency in Fig.2, quadrature axis transient voltage in Fig.3 and excitation voltage in Fig.4. Simulation results have demonstrated the effectiveness and robustness of our proposed approach.

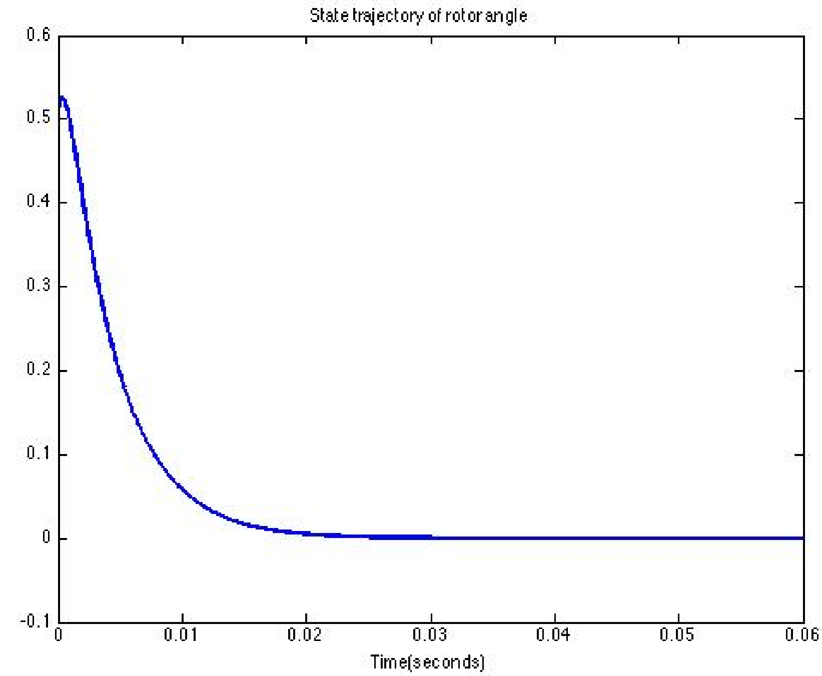

Fig. 3.1. Time Response of Rotor Angle

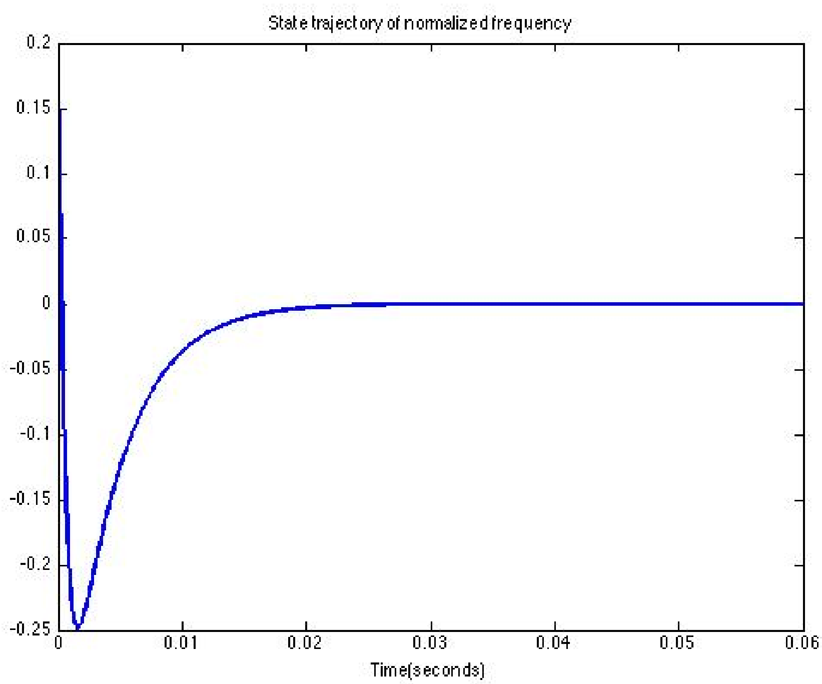

Fig. 3.2. Time Response of Normalized Frequency

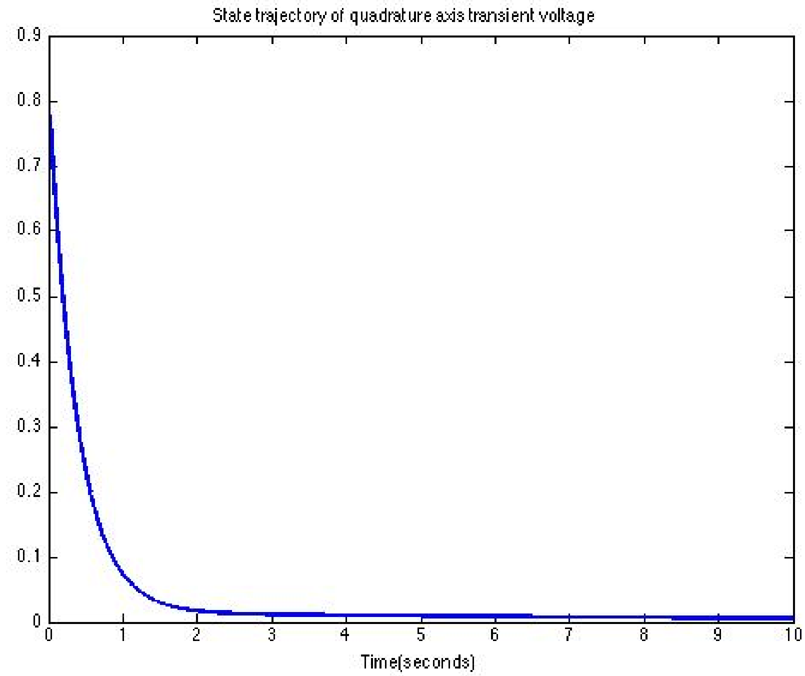

Fig. 3.3. Time Response of Quadrature Axis Transient Voltage

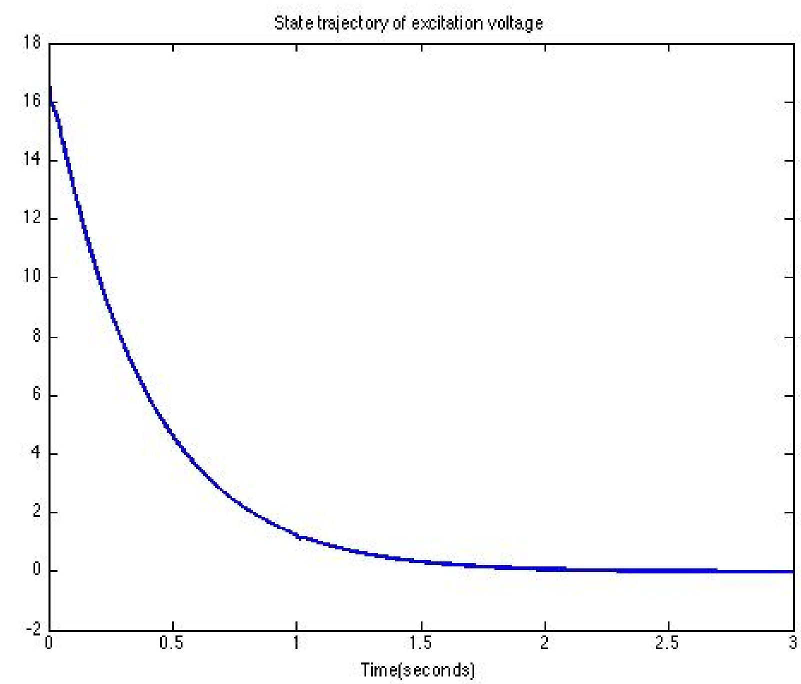

Fig. 3.4. Time Response of Excitation Voltage

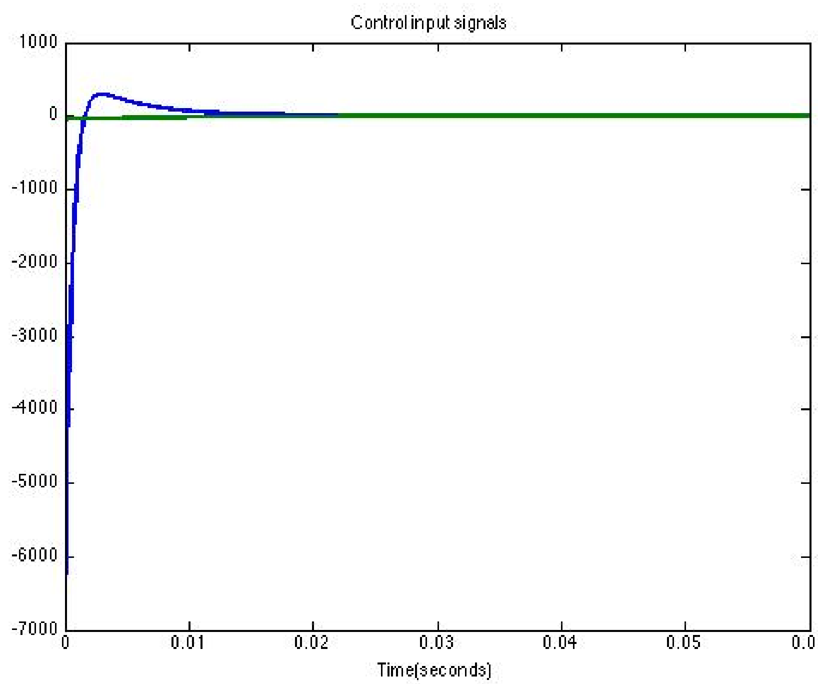

Fig. 3.5. Control Input

CHAPTER 4

CONCLUSION AND FUTURE WORK

4.1 Conclusion

A seventh order state space model for an electrical generator system driven by a steam turbine has been developed based on the nonlinear dynamics of a synchronous generator, automatic voltage regulator, power system stabilized, and exciter. Moreover, A linearized model for a variable speed surface mounted permanent magnet synchronous generator based wind energy conversion system has been developed. A linear matrix inequality based robust and resilient controller with H_∞ performance criterion for a time delayed power system has been developed to address the problems of time delays, model uncertainties, external disturbance, and controller feedback gain perturbations. Computer simulations conducted in MATLAB shows the efficacy of the proposed controller in power systems control applications.

4.2 Future Work

Further development for this are summarized as follows :

1. The discrete time system model will be studied and implemented by microcontrollers, Digital Signal Processors (DSP), and Field Programmable Gate Array (FPGA)

2. The controller in this thesis is developed using the delay independent cases. However, the delay dependent designs are based on the discretization of the Lyapunov-Krasovskii can be studied in the future. Therefore, the maximum allowable delay can be determined.

3. This work consider the case of fixed and known delays. However, further development of the work can consider the cases of varying and unknown time delays.

BIBLIOGRAPHY

[1] Ackermann, T. (2005). *Wind power in power systems*, volume 140. Wiley Online Library.

[2] Al Azze, Q. (2014). Field-oriented control of permanent magnet synchronous motors based on dsp controller. Master's thesis, Southern Illinois University Edwardsville.

[3] Alrifai, M. T., Zribi, M., Rayan, M., and Mahmoud, M. S. (2013). On the control of time delay power systems. *International Journal of Innovative Computing, Information and Control*, 9(2):769–792.

[4] Bayrak, A. and Tatlicioglu, E. (2013). Online time delay identification and control for general classes of nonlinear systems. *Transactions of the Institute of Measurement and Control*, 35(6):808–823.

[5] Bhanja Chowdhury, A., Kulhare, A., and Raina, G. (2011). A study of the smib power system model with delayed feedback. In *International Conference on Power and Energy Systems*, pages 1–6. IEEE.

[6] Bianchi, F. D., De Battista, H., and Mantz, R. J. (2006). *Wind turbine control systems: principles, modelling and gain scheduling design*. Springer Science & Business Media.

[7] Ciampichetti, S., Corradini, M., Ippoliti, G., and Orlando, G. (2011). Sliding mode control of permanent magnet synchronous generators for wind turbines. In *37th Annual Conference of the IEEE Industrial Electronics Society*, pages 740–745. IEEE.

[8] Dalala, Z. M., Zahid, Z. U., and Lai, J.-S. (2013). New overall control strategy for small-scale wecs in mppt and stall regions with mode transfer control. *IEEE Transactions on Energy Conversion*, 28(4):1082–1092.

[9] Dugard, L. and Verriest, E. I. (1998). *Stability and control of time-delay systems*. Springer London.

[10] El Mokadem, M., Courtecuisse, V., Saudemont, C., Robyns, B., and Deuse, J. (2009). Experimental study of variable speed wind generator contribution to primary frequency control. *Renewable Energy*, 34(3):833–844.

[11] Famularo, D., Abdallah, C., Jadbabale, A., Dorato, P., and Haddad, M. (1998). Robust non-fragile lq controllers: the static state feedback case. In *Proceedings of the American Control Conference*, volume 2, pages 1109–1113 vol.2.

[12] Fridman, E. (2001). New lyapunov–krasovskii functionals for stability of linear retarded and neutral type systems. *Systems & Control Letters*, 43(4):309–319.

[13] Fridman, E. and Shaked, U. (2003). Delay-dependent stability and H_∞ control: constant and time-varying delays. *International journal of control*, 76(1):48–60.

[14] Garcia-Sanz, M. and Houpis, C. H. (2012). *Wind energy systems: control engineering design*. CRC Press.

[15] Górecki, H. (1989). *Analysis and synthesis of time delay systems*. John Wiley & Sons Inc.

[16] Green, M. and Limebeer, D. J. (2012). *Linear robust control*. Courier Corporation.

[17] Gu, K., Chen, J., and Kharitonov, V. L. (2003). *Stability of time-delay systems*. Springer Science & Business Media.

[18] Gu, K. and Niculescu, S.-I. (2000a). Additional dynamics in transformed time-delay systems. *IEEE Transactions on Automatic Control*, 45(3):572–575.

[19] Gu, K. and Niculescu, S.-I. (2000b). Further remarks on additional dynamics in various model transformations of linear delay systems. In *Proceedings of the American Control Conference*, volume 6, pages 4368–4372. IEEE.

[20] Gu, K. and Niculescu, S.-I. (2003). Survey on recent results in the stability and control of time-delay systems*. *Journal of Dynamic Systems, Measurement, and Control*, 125(2):158–165.

[21] Hale, J. (1977). *Theory of functional differential equations*. Springer-Verlag, New York.

[22] Hale, J. K. (1993). *Introduction to functional differential equations*, volume 99. Springer Science & Business Media.

[23] Howlader, A. M., Urasaki, N., Yona, A., Senjyu, T., and Saber, A. Y. (2012). A new robust controller approach for a wind energy conversion system under high turbulence wind velocity. In *7th IEEE Conference on Industrial Electronics and Applications (ICIEA)*, pages 860–865. IEEE.

[24] Hui, Z., Guo-Bao, Z., Shu-min, F., Zi-Cong, W., and Hai-Rong, Z. (2011). H_∞ robust control of direct-drive permanent magnetic synchronous generator considering the uncertainty of stator parameters. In *Chinese Control Conference (CCC)*, pages 2368–2373.

[25] Jamal Alden, M. and Wang, X. (2015). Robust H_∞ control of time delayed power systems. *Systems Science & Control Engineering: An Open Access Journal*, 3(1):253–261.

[26] Jia, H., Yu, X., Yu, Y., and Wang, C. (2008). Power system small signal stability region with time delay. *International Journal of Electrical Power & Energy Systems*, 30(1):16–22.

[27] Jiang, Z. (2007). Design of power system stabilizers using synergetic control theory. In *IEEE Power Engineering Society General Meeting*, pages 1–8. IEEE.

[28] Keel, L. and Bhattacharyya, S. P. (1997). Robust, fragile, or optimal? *IEEE Transactions on Automatic Control*, 42(8):1098–1105.

[29] Kim, H.-W., Kim, S.-S., and Ko, H.-S. (2010). Modeling and control of pmsg-based variable-speed wind turbine. *Electric Power Systems Research*, 80(1):46–52.

[30] Kim, J. H. and Park, H. B. (1999). H_∞ state feedback control for generalized continuous/discrete time-delay system. *Automatica*, 35(8):1443–1451.

[31] Kokame, H., Kobayashi, H., and Mori, T. (1998). Robust H_∞ performance for linear delay-differential systems with time-varying uncertainties. *IEEE Transactions on Automatic Control*, 43(2):223–226.

[32] Kolmanovskii, V. and Myshkis, A. (1992). *Applied theory of functional differential equations*. Springer.

[33] Lima, M. L., Silvino, J. L., and de Resende, P. (1999). H_∞control for a variable-speed adjustable-pitch wind energy conversion system. In *Proceedings of the IEEE International Symposium on Industrial Electronics*, volume 2, pages 556–561. IEEE.

[34] Liu, Z., Zhu, C., and Jiang, Q. (2008). Stability analysis of time delayed power system based on cluster treatment of characteristic roots method. In *IEEE Power and Energy Society General Meeting-Conversion and Delivery of Electrical Energy in the 21st Century*, pages 1–6. IEEE.

[35] Machowski, J. and Bumby, J. R. (1997). *Power system dynamics and stability*. John Wiley & Sons.

[36] Mahmoud, M. S. (2000). *Robust control and filtering for time-delay systems*. CRC Press.

[37] Mahmoud, M. S. (2001). Robust control design of uncertain time-delay systems. *Systems Analysis Modeling Simulation*, 40(2):151–180.

[38] Mahmoud, M. S. (2004). *Resilient control of uncertain dynamical systems*, volume 303. Springer Science & Business Media.

[39] Mahmoud, M. S. and Xia, Y. (2012). *Applied control systems design*. Springer.

[40] Mesbahi, T., Ghennam, T., and Berkouk, E. (2011). Control of a wind energy conversion system with active filtering function. In *International Conference on Power Engineering, Energy and Electrical Drives (POWERENG)*, pages 1–6. IEEE.

[41] Mittal, R., Sandhu, K., and Jain, D. (2012). Ride-through capability of grid interfaced variable speed pmsg based wecs. In *IEEE Fifth Power India Conference*, pages 1–6. IEEE.

[42] Moon, Y. S., Park, P., Kwon, W. H., and Lee, Y. S. (2001). Delay-dependent robust stabilization of uncertain state-delayed systems. *International Journal of control*, 74(14):1447–1455.

[43] Muhando, B. E. and Wies, R. W. (2011). Nonlinear constrained feedback control for grid-interactive wecs under high stochasticity. *IEEE Transactions on Energy Conversion*, 26(4):1000–1009.

[44] Muhando, E. B., Senjyu, T., Uehara, A., and Funabashi, T. (2011). Gain-scheduled control for wecs via lmi techniques and parametrically dependent feedback part ii: Controller design and implementation. *IEEE Transactions on Industrial Electronics*, 58(1):57–65.

[45] Munteanu, I., Bratcu, A. I., Cutululis, N.-A., and Ceanga, E. (2008). *Optimal control of wind energy systems: towards a global approach*. Springer Science & Business Media.

[46] Okuno, H. and Nakabayashi, T. (2006). Basin of attraction and controlling chaos of five-synchronous-generator infinite-bus system. In *IEEE International Symposium on Industrial Electronics*, volume 3, pages 1818–1823. IEEE.

[47] Park, P. (1999). A delay-dependent stability criterion for systems with uncertain time-invariant delays. *IEEE Transactions on Automatic Control*, 44(4):876–877.

[48] Richard, J.-P. (2003). Time-delay systems: an overview of some recent advances and open problems. *automatica*, 39(10):1667–1694.

[49] Rocha, R. et al. (2003). A multivariable H_∞ control for wind energy conversion system. In *Proceedings of IEEE Conference on Control Applications*, volume 1, pages 206–211. IEEE.

[50] Rocha, R., Resende, P., Silvino, J. L., and Bortolus, M. V. (2001). Control of stall regulated wind turbine through h loop shaping method. In *Proceedings of the IEEE International Conference on Control Applications*, pages 925–929. IEEE.

[51] Sajedi, S., Rezabeyglo, H. J., Noruzi, A., Khalifeh, F., Karimi, T., Khalifeh, Z., and Branch, A. (2012). Modeling and application of pmsg based variable speed wind generation system. *Research Journal of Applied Sciences, Engineering and Technology*, 4(07):729–734.

[52] Scorletti, G. and Fromion, V. (1998). A unified approach to time-delay system control: robust and gain-scheduled. In *Proceedings of the American Control Conference*, volume 4, pages 2391–2395. IEEE.

[53] Snyder, A. F., Ivanescu, D., HadjSaid, N., Georges, D., and Margotin, T. (2000). Delayed-input wide-area stability control with synchronized phasor measurements and linear matrix inequalities. In *IEEE Power Engineering Society Summer Meeting*, volume 2, pages 1009–1014. IEEE.

[54] Tien, H. N., Scherer, C., and Scherpen, J. (2007). Robust performance of self-scheduled lpv control of doubly-fed induction generator in wind energy conversion systems. In *European Conference on Power Electronics and Applications*, pages 1–10. IEEE.

[55] Uehara, A., Pratap, A., Goya, T., Senjyu, T., Yona, A., Urasaki, N., and Funabashi, T. (2011). A coordinated control method to smooth wind power fluctuations of a pmsg-based wecs. *IEEE Transactions on Energy Conversion*, 26(2):550–558.

[56] Wang, W., Wu, D., Wang, Y., and Ji, Z. (2010). H_∞ gain scheduling control of pmsg-based wind power conversion system. In *The 5th IEEE Conference on industrial Electronics and Applications (ICIEA)*, pages 712–717. IEEE.

[57] Wang, X. (2015). Lecture notes in advance electric machines and drives. University lecture. Southern Illinois University Edwardsville.

[58] Wang, X. and Gu, K. (2014). Time-delay power systems control and stability with discretized lyapunov functional method. In *33rd Chinese Control Conference (CCC)*, pages 7004–7009. IEEE.

[59] Wang, X., Yaz, E. E., and Long, J. (2014). Robust and resilient state-dependent control of continuous-time nonlinear systems with general performance criteria. *Systems Science & Control Engineering: An Open Access Journal*, 2(1):34–40.

[60] Wu, H., Ni, H., and Heydt, G. (2002). The impact of time delay on robust control design in power systems. In *IEEE Power Engineering Society Winter Meeting*, volume 2, pages 1511–1516. IEEE.

[61] Wu, M., He, Y., and She, J.-H. (2010). *Stability analysis and robust control of time-delay systems*. Springer.

[62] Xiaodan, Y., Hongjie, J., and Jinli, Z. (2008). A lmi based approach to power system stability analysis with time delay. In *IEEE Region 10 Conference TENCON*, pages 1–6. IEEE.

[63] Yao, W., Jiang, L., Wen, J., Cheng, S., and Wu, Q. (2009). An adaptive wide-area damping controller based on generalized predictive control and model identification. In *IEEE Power & Energy Society General Meeting*, pages 1–7. IEEE.

[64] Yu, G., Zhang, B., Xie, H., and Wang, C. (2007). Wide-area measurement-based nonlinear robust control of power system considering signals' delay and incompleteness. In *IEEE Power Engineering Society General Meeting*, pages 1–8. IEEE.

[65] Zames, G. (1981). Feedback and optimal sensitivity: Model reference transformations, multiplicative seminorms, and approximate inverses. *IEEE Transactions on Automatic Control*, 26(2):301–320.

[66] Zames, G. and Francis, B. A. (1983). Feedback, minimax sensitivity, and optimal robustness. *IEEE Transactions on Automatic Control*, 28(5):585–601.

[67] Zhang, C.-K., Jiang, L., Wu, Q., He, Y., and Wu, M. (2013). Delay-dependent robust load frequency control for time delay power systems. In *IEEE Power and Energy Society General Meeting (PES),*, pages 1–1. IEEE.

[68] Zhong, Q.-C. (2006). *Robust control of time-delay systems*. Springer Science & Business Media.

[69] Zhou, K. and Doyle, J. C. (1998). *Essentials of robust control*, volume 180. Prentice hall Upper Saddle River, NJ.

[70] Zhou, K., Doyle, J. C., Glover, K., et al. (1996). *Robust and optimal control*, volume 40. Prentice hall New Jersey.

[71] Zribi, M., Mahmoud, M., Karkoub, M., and Lie, T. (2000). H_∞-controllers for linearised time-delay power systems. *IEEE Proceedings Generation, Transmission & Distribution*, 147(6):401–408.

APPENDIX A

PARK'S TRANSFORMATION

Three phase machines are usually characterized by their voltage equations. The dynamical model of a three phase machine in the *abc* coordinate frame is time variant, which imposes difficulties in system analysis and controller design. Therefore, an alternative coordinate frame should be adopted to describe the machine dynamics, to result in a time-invariant model. To achieve this goal, Park's transformation is used to change systems description from the *abc* coordinate frame to the *dq*0 coordinate frame.

A.0.1 Park's transformation

Characterizing a three phase electrical machine using the stationary *abc* coordinates frame leads to time variant coefficients system. Therefore, Park's transformation is used to express the the model of a three phase electrical model in a rotating coordinate frame rather than the stationary one. This transformation is named after R. H. Park who introduces it in 1929 [2]. Park's transformation is used essentially to express the model in the *dq*0 coordinate frame, in which the machine inductances have fixed values. Moreover the Park's transformation simplifies the machine model because only two coordinates *dq* are used to model the system rather than three, since 0 coordinate frame is used only to facilitate the transformation and make matrices invertible. A schematic diagram of the *abc* and the *dq*0 coordinate frames are shown in Fig. (A.1). The transformation of a machine from the *abc* to *dq*0 is called the Park's transformation

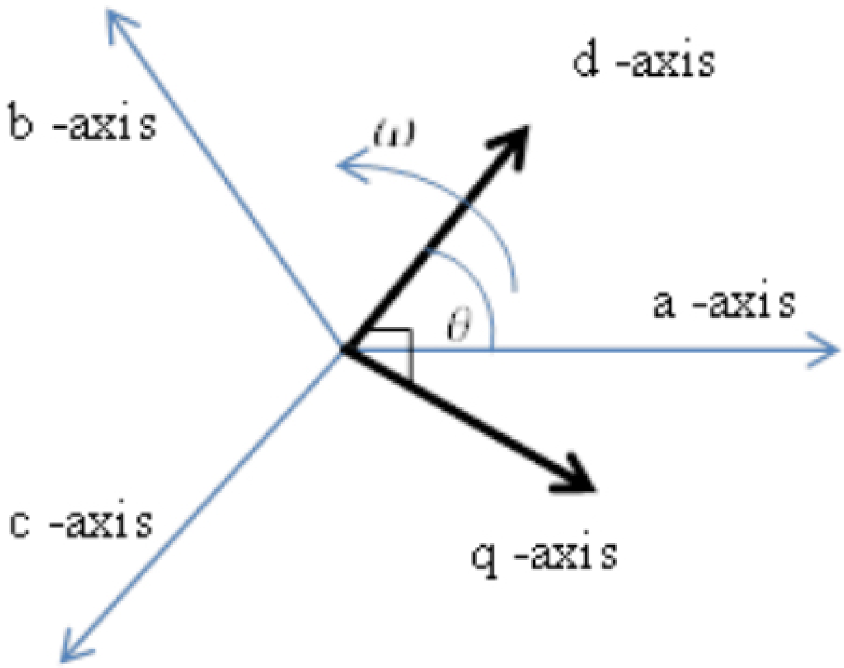

Fig. A.1. *abc* and *dq*0 Coordinate Frames

and it is given by

$$f_{dq0} = T_{dq0} f_{abc} \tag{A.1}$$

On the other hand, the transformation of the machine model from the *dq*0 to *abc* coordinate frame is called the inverse Park's transformation and it can be mathematically stated as

$$f_{abc} = T_{dq0}^{-1} d_{dq0} \tag{A.2}$$

In Eqn. (A.1) and Eqn. (A.2), f is a generic variable, which can represent the voltage, current, or flux linkage. T_{dq0} is the Park's transformation matrix and it is given by

$$T_{dq0} = \frac{2}{3} \begin{bmatrix} \cos(\theta) & \cos(\theta - \frac{2\pi}{3}) & \cos(\theta + \frac{2\pi}{3}) \\ \sin(\theta) & \sin(\theta - \frac{2\pi}{3}) & \sin(\theta + \frac{2\pi}{3}) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \quad \text{(A.3)}$$

$$T_{dq0}^{-1} = \begin{bmatrix} \cos(\theta) & \sin(\theta) & 1 \\ \cos(\theta - \frac{2\pi}{3}) & \sin(\theta - \frac{2\pi}{3}) & 1 \\ \cos(\theta + \frac{2\pi}{3}) & \sin(\theta + \frac{2\pi}{3}) & 1 \end{bmatrix} \quad \text{(A.4)}$$

θ in Eqn. (A.3) and Eqn. (A.4) is the angular position of the rotor.

The development of Park's transformation is based on the Calrke and rotational transformation.

A.0.2 Clarke's transformation

The concept of mapping the stationary *abc* coordinate frame into to the $\alpha\beta0$ rotating coordinate frame is first introduced by E. Clarke's, Fig. (A.2) shows the *abc* and the $\alpha\beta0$. The Clarke's transformation and its inverse are given mathematically by the following notations

$$f_{\alpha\beta0} = K f_{abc} \quad \text{(A.5)}$$

$$f_{abc} = K^{-1} f_{\alpha\beta0} \quad \text{(A.6)}$$

Where K and K^{-1} in Eqn. (A.5) and Eqn. (A.6) are given by [2]

$$K = \frac{2}{3}\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \tag{A.7}$$

$$K^{-1} = \begin{bmatrix} 1 & 0 & 1 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & 1 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & 1 \end{bmatrix} \tag{A.8}$$

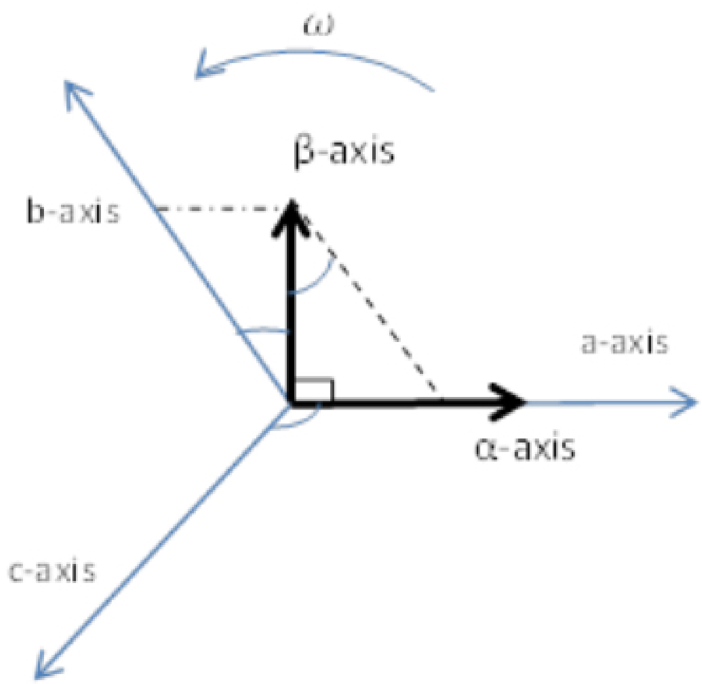

Fig. A.2. *abc* and *αβ*0 Coordinate Frames

A.0.3 Rotational transformation

The last step of the Park's transformation is the Rotational transformation. In this transformation, the $\alpha\beta 0$ coordinate frame is rotated to match the rotor $dq0$ coordinate frame. The Rotational transformation and its matrix inverse can be expressed as follows

$$f_{dq} = Q f_{\alpha\beta} \tag{A.9}$$

$$f_{\alpha\beta} = Q^{-1} f_{dq} \tag{A.10}$$

where Q and Q^{-1} in (A.9) and (A.10) are given by

$$Q = \begin{bmatrix} \cos(\sigma) & \sin(\sigma) \\ -\sin(\sigma) & \cos(\sigma) \end{bmatrix} \tag{A.11}$$

$$Q^{-1} = \begin{bmatrix} \cos(\sigma) & -\sin(\sigma) \\ \sin(\sigma) & \cos(\sigma) \end{bmatrix} \tag{A.12}$$

σ in Eqn. (A.11) and Eqn. (A.12) is the angle of the $dq0$ coordinate frame.

APPENDIX B

PERMANENT MAGNET SYNCHRONOUS GENERATOR MODEL IN THE *dq*0 COORDINATE FRAME

The development of the state space model for the permanent magnet synchronous generator in the (*dq*0) coordinate frame is presented in this chapter. Appendix B derivation is based on [57].

B.0.4 Permanent magnet synchronous generator model in the abc coordinate frame

The state space model of a permanent magnet synchronous generator in the (*abc*) coordinate frame is given as follows [57]

$$\begin{bmatrix} V_{abc} \\ 0 \end{bmatrix} = \begin{bmatrix} R_{ss} & 0 \\ 0 & R_{rr} \end{bmatrix} \begin{bmatrix} I_{abc} \\ I_{sdsq} \end{bmatrix} + \frac{d}{dt}\left\{ \begin{bmatrix} L_{ss} & L_{sr} \\ L_{rs} & L_{rr} \end{bmatrix} \begin{bmatrix} I_{abc} \\ I_{sdsq} \end{bmatrix} \right\} + \frac{d}{dt}\left\{ i_f \begin{bmatrix} L_{sf} \\ 0 \end{bmatrix} \right\} \tag{B.1}$$

Since $\Lambda = L \times I$, Eqn. (B.1) can be rewritten as

$$\begin{bmatrix} V_{abc} \\ 0 \end{bmatrix} = \begin{bmatrix} R_{ss} & 0 \\ 0 & R_{rr} \end{bmatrix} \begin{bmatrix} I_{abc} \\ I_{sdsq} \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} L_{ss} & L_{sr} \\ L_{rs} & L_{rr} \end{bmatrix} \begin{bmatrix} \Lambda_{abc} \\ \Lambda_{sdsq} \end{bmatrix} + \begin{bmatrix} E_{abc} \\ 0 \end{bmatrix} \tag{B.2}$$

Since $V_{abc} = T^{-1}V_{dq0}$, Eqn. (B.2) can be expressed as follows

$$\begin{bmatrix} V_{dq0} \\ 0 \end{bmatrix} = \begin{bmatrix} TR_{ss}T^{-1} & 0 \\ 0 & R_{rr} \end{bmatrix} \begin{bmatrix} I_{dq0} \\ I_{sdsq} \end{bmatrix} + \begin{bmatrix} T & 0 \\ 0 & U \end{bmatrix} \frac{d}{dt} \begin{bmatrix} T^{-1}\Lambda_{dq0} \\ \Lambda_{sdsq} \end{bmatrix} + \begin{bmatrix} E_{dq0} \\ 0 \end{bmatrix} \tag{B.3}$$

Expanding Eqn. (B.3) yields

$$V_{dq0} = \left(TR_{ss}T^{-1}\right) I_{dq0} + T\frac{d}{dt}\left(T^{-1}\Lambda_{dq0}\right) + E_{dq0} \tag{B.4}$$

$$0 = R_{rr}I_{sdsq} + \frac{d}{dt}\left(\Lambda_{sdsq}\right) \tag{B.5}$$

Since

$$T\frac{d}{dt}\left(T^{-1}\Lambda_{dq0}\right) = \dot{\Lambda}_{dq0} - \left(\dot{T}T^{-1}\Lambda_{dq0}\right) \tag{B.6}$$

and

$$\dot{T} = \omega G T \tag{B.7}$$

where

$$G = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{B.8}$$

$$T = \frac{2}{3}\begin{bmatrix} \cos(\sigma) & \cos(\sigma - \frac{2\pi}{3}) & \cos(\sigma - \frac{4\pi}{3}) \\ \sin(\sigma) & \sin(\sigma - \frac{2\pi}{3}) & \sin(\sigma - \frac{4\pi}{3}) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \tag{B.9}$$

$$T^{-1} = \begin{bmatrix} \cos(\sigma) & \sin(\sigma) & \frac{1}{2} \\ \cos(\sigma - \frac{2\pi}{3}) & \sin(\sigma - \frac{2\pi}{3}) & \frac{1}{2} \\ \cos(\sigma - \frac{4\pi}{3}) & \sin(\sigma - \frac{4\pi}{3}) & \frac{1}{2} \end{bmatrix} \tag{B.10}$$

Therefore, Eqn. (B.4) and Eqn. (B.5) can be expressed as follows

$$V_{dq0} = \left(TR_{ss}T^{-1}\right)I_{dq0} + \dot{\Lambda}_{dq0} - \omega G\Lambda_{dq0} + E_{dq0} \tag{B.11}$$

$$0 = R_{rr}I_{sdsq} + \dot{\Lambda}_{sdsq} \tag{B.12}$$

Since

$$\Lambda_{dq0} = \left(TL_{ss}T^{-1}\right)I_{dq0} + (TL_{sr})\,I_{sqsq} \tag{B.13}$$

$$\Lambda_{sdsq} = \left(L_{sr}^{t}T^{-1}\right)I_{dq0} + (L_{rr})\,I_{sdsq} \tag{B.14}$$

where

$$\left(TL_{ss}T^{-1}\right) = \begin{bmatrix} L_{dd} & 0 & 0 \\ 0 & L_{qq} & 0 \\ 0 & 0 & L_{00} \end{bmatrix} \tag{B.15}$$

$$(TL_{sr}) = \begin{bmatrix} L_{asdm} & 0 \\ 0 & L_{asqm} \\ 0 & 0 \end{bmatrix} \tag{B.16}$$

$$\left(L_{sr}^{t}T^{-1}\right) = \begin{bmatrix} \frac{3}{2}L_{asdm} & 0 & 0 \\ 0 & \frac{3}{2}L_{asqm} & 0 \end{bmatrix} \tag{B.17}$$

The flux linkage is expressed in the $dq0$ coordinate frame in Eqn. (B.13) and Eqn. (B.14). Therefore, all inductance matrices are time invariant. As such, the time derivative of the flux linkage can be expressed as follows

$$\dot{\Lambda}_{dq0} = \left(TL_{ss}T^{-1}\right)\dot{I}_{dq0} + (TL_{sr})\,\dot{I}_{sdsq} \tag{B.18}$$

$$\dot{\Lambda}_{sdsq} = \left(L_{sr}^{t}T^{-1}\right)\dot{I}_{dq0} + (L_{rr})\,\dot{I}_{sdsq} \tag{B.19}$$

Since

$$\left(TR_{ss}T^{-1}\right) = R_{ss} \tag{B.20}$$

Therefore, by substituting Eqns. (B.13), (B.14), (B.18), (B.19) and(B.20) into Eqn. (B.11) and Eqn. (B.12), we have

$$\begin{bmatrix} V_{dq0} \\ 0 \end{bmatrix} = \begin{bmatrix} R_{ss} & 0 \\ 0 & R_{rr} \end{bmatrix} \begin{bmatrix} I_{dq0} \\ I_{sdsq} \end{bmatrix} + \begin{bmatrix} -\omega G\left(TL_{ss}T^{-1}\right) & -\omega G\left(TL_{sr}\right) \\ 0 & 0 \end{bmatrix} \begin{bmatrix} I_{dq0} \\ I_{sdsq} \end{bmatrix} +$$

$$\begin{bmatrix} \left(TL_{ss}T^{-1}\right) & \left(TL_{sr}\right) \\ \left(L_{sr}^{t}T^{-1}\right) & \left(L_{rr}\right) \end{bmatrix} \begin{bmatrix} \dot{I}_{dqo} \\ \dot{I}_{sdsq} \end{bmatrix} + \begin{bmatrix} E_{dq0} \\ 0 \end{bmatrix} \tag{B.21}$$

Since

$$-\omega G\left(TL_{ss}T^{-1}\right) = \begin{bmatrix} 0 & -\omega L_{qq} & 0 \\ \omega L_{dd} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{B.22}$$

$$-\omega G\left(TL_{sr}\right) = \begin{bmatrix} 0 & -\omega L_{asqm} \\ \omega L_{asdm} & 0 \\ 0 & 0 \end{bmatrix} \tag{B.23}$$

$$R_{ss} = \begin{bmatrix} r_s & 0 & 0 \\ 0 & r_s & 0 \\ 0 & 0 & r_s \end{bmatrix} \tag{B.24}$$

$$R_{rr} = \begin{bmatrix} r_{sd} & 0 \\ 0 & r_{sq} \end{bmatrix} \tag{B.25}$$

Substituting Eqns. (B.15), (B.16), (B.17), (B.22), (B.23), (B.24), and (B.25) into Eqn. (B.21) yields

$$\begin{bmatrix} v_d \\ v_q \\ v_0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} r_s & 0 & 0 & 0 & 0 \\ 0 & r_s & 0 & 0 & 0 \\ 0 & 0 & r_s & 0 & 0 \\ 0 & 0 & 0 & r_{sd} & 0 \\ 0 & 0 & 0 & 0 & r_{sq} \end{bmatrix} \begin{bmatrix} i_d \\ i_q \\ i_0 \\ i_{sd} \\ i_{sq} \end{bmatrix} + \begin{bmatrix} 0 & -\omega L_{qq} & 0 & 0 & -\omega L_{asqm} \\ \omega L_{dd} & 0 & 0 & \omega L_{asdm} & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_d \\ i_q \\ i_0 \\ i_{sd} \\ i_{sq} \end{bmatrix} +$$

$$\begin{bmatrix} L_{dd} & 0 & 0 & L_{asdm} & 0 \\ 0 & L_{qq} & 0 & 0 & L_{asqm} \\ 0 & 0 & L_{00} & 0 & 0 \\ \frac{3}{2}L_{asdm} & 0 & 0 & L_{sdsd} & 0 \\ 0 & \frac{3}{2}L_{asqm} & 0 & 0 & L_{sqsq} \end{bmatrix} \begin{bmatrix} \dot{i}_d \\ \dot{i}_q \\ \dot{i}_0 \\ \dot{i}_{sd} \\ \dot{i}_{sq} \end{bmatrix} + \begin{bmatrix} e_d \\ e_q \\ e_0 \\ 0 \\ 0 \end{bmatrix} \quad \text{(B.26)}$$

Since

$$e_q = \omega \lambda_m \quad \text{(B.27)}$$

Neglecting damper winding effects and the ”0” coordinate, the state space model of a permanent magnet synchronous generator can be expressed as

$$\begin{bmatrix} \dot{i}_d \\ \dot{i}_q \end{bmatrix} = \begin{bmatrix} -\frac{r_s}{L_{dd}} & \omega \frac{L_{qq}}{L_{dd}} \\ -\omega \frac{L_{dd}}{L_{qq}} & -\frac{r_s}{L_{qq}} \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \begin{bmatrix} \frac{1}{L_{dd}} & 0 & 0 \\ 0 & \frac{1}{L_{qq}} & -\frac{\omega}{L_{qq}} \end{bmatrix} \begin{bmatrix} v_d \\ v_q \\ \lambda_m \end{bmatrix} \quad \text{(B.28)}$$