Essays on Household Consumption and Labor Supply

by

Jutong Pan

B.A., Peking University, 2009
M.Phil., The Chinese University of Hong Kong, 2011

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF
THE REQUIREMENTS FOR THE DEGREE OF

DOCTOR OF PHILOSOPHY

in

The Faculty of Graduate and Postdoctoral Studies

(Economics)

THE UNIVERSITY OF BRITISH COLUMBIA

(Vancouver)

July 2017

Abstract

This dissertation studies how households adjust their consumption and labor supply in response to idiosyncratic shocks.

In the first chapter, I propose an empirical strategy for measuring consumption allocations within households over time. The strategy consists of imputing gender-specific consumption data from a cross-sectional dataset to a panel. I apply it on two publicly available datasets in the US: the Consumer Expenditure Survey and the Panel Study of Income Dynamics. The generated panel allows researchers to investigate questions such as how the sharing rule shifts in response to various shocks.

The second chapter studies how households insure themselves against idiosyncratic wage shocks and how this insurance interacts with intra-household bargaining. I set up an intertemporal household model and examine two channels of insurance, self-insurance and family labor supply adjustment. I consider two alternative specifications of this model: a unitary version in which I restrict sharing rules to be fixed within households, and a non-unitary one in which I allow sharing rules to change. I estimate the model using a panel that has information on consumption allocations within households. I find that intra-household allocations respond strongly to fluctuations in individual wages. Removing the restriction of fixed sharing rules does not reduce the extent of consumption smoothing within a household, but it significantly changes the relative importance of different channels. In particular, the relative contribution of family labor supply to household consumption smoothing decreases from roughly 60% in the unitary model to 30% in the non-unitary model. This is because the added worker effect – the increase in spousal labor supply following an adverse shock to a partner – is much milder in the non-unitary specification.

Non-stationary income processes are standard in quantitative life-cycle models, prompted by the observation that within-cohort income inequality increases with age. The last chapter generalizes Tauchen's (1986) and Rouwenhorst's (1995) discretization methods to non-stationary AR(1) processes. We evaluate the performance of both methods in the context of a canonical finite-horizon, income-fluctuation problem with a non-stationary income process. We find that the generalized Rouwenhorst's method performs extremely well even with a small number of states.

Lay Summary

What does a household do to insure against income shocks? What happens when the husband is shifted from a high-paying job to a low-paying job? The household may borrow money and use their savings, the husband may work more, and the wife may also choose to work for longer hours. This dissertation provides an economic analysis of such household behaviors in situations where the household members' income fluctuates. I start by proposing a new method for measuring consumption allocations in the first chapter. In the second chapter, I show that it is important to take bargaining between couples into the analysis. Accounting for strategic interactions suggests that the wife does not increase her working hours as much as previous studies find, which assume couples do not bargain. The last chapter generalizes two numerical methods for approximating income processes and evaluates their performance. We find that the generalized Rouwenhorst method is more accurate and robust than the generalized Tauchen method.

Preface

Chapter 3 *Markov-Chain Approximations for Life-Cycle Models* is a joint work with Professor Giulio Fella and Professor Giovanni Gallipoli. I was involved throughout each stage of the research: proposing the numerical algorithms, coding the simulations, organizing and presenting results, and writing several subsections of the manuscript.

Table of Contents

Abstract ii

Lay Summary iii

Preface iv

Table of Contents v

List of Tables vii

List of Figures viii

Acknowledgments ix

Introduction 1

1 Measuring Intra-Household Consumption Allocations over Time 4

1.1 Introduction 4
1.2 Methodology 7
1.3 Data 9
1.4 Results 10
1.5 Conclusion 12

2 Consumption Smoothing and Intra-Household Bargaining 15

2.1 Introduction 15
2.2 The Household Model 18
2.2.1 Wage Process 18
2.2.2 The Family Problem 20
2.2.3 The Solution to the Family Problem 21
2.2.4 Identification 23
2.3 Empirical Implementation 24
2.3.1 Data and Sample Selection 24
2.3.2 Consumption Imputation 25

2.3.3 Measurement Error . . . 26
2.3.4 Estimation Procedure . . . 26
2.4 Results . . . 27
2.4.1 Descriptive Statistics . . . 27
2.4.2 Wage Parameters . . . 29
2.4.3 Consumption, Labor Supply, and Bargaining Parameters . . . 29
2.4.4 Transmission of Wage Shocks to Consumption and Labor supply . . . 31
2.4.5 Insurance Accounting . . . 34
2.5 Conclusion . . . 36

3 Markov-Chain Approximations for Life-Cycle Models . . . 38
3.1 Introduction . . . 38
3.2 Discrete Approximations of AR(1) Processes . . . 39
3.2.1 Tauchen's (1986) Method . . . 40
3.2.2 Rouwenhorst's (1995) Method . . . 41
3.3 Evaluation . . . 43
3.3.1 Results . . . 45
3.4 Conclusion . . . 48

Conclusion . . . 50

Bibliography . . . 52

Appendices

A Appendix for Chapter 2 . . . 56
A.1 More on Wage Estimates . . . 56
A.2 Approximation of the First Order Conditions and Intertemporal Budget Constraint . . . 57
A.2.1 Linearization of the First Order Conditions . . . 57
A.2.2 Log-Linearization of the Lifetime Budget Constraint . . . 61
A.3 Moment Condtions in GMM estimation . . . 65
A.4 Moment Conditions with Measurement Errors . . . 69

B Appendix for Chapter 3 . . . 72
B.1 Normalized Problem with Unit-Root Labor Income . . . 72

List of Tables

1.1 Regression for Gender-Specific Clothing Consumption in the CEX 11

2.1 Descriptive Statistics: Sample Means 28
2.2 Estimates of Wage Parameters 29
2.3 Parameter Estimates 30
2.4 Average Estimates of Transmission Coefficients 32
2.5 Behavioral Responses to -10% Permanent Wage Shock to Husband 34
2.6 Behavioral Responses to -10% Permanent Wage Shock to Wife 34

3.1 Ratio of Model Moments Relative to Their Counterpart in the Quadrature Benchmark: (a) Markov Chain Simulation and (b) Continuous Random Walk Income Process 46

A.1 Estimates of Wage Parameters 56

List of Figures

1.1 Comparison of real versus imputed household-level clothing consumption . . . 13
1.2 Distribution for imputed husbands' share of consumption 14

Acknowledgments

First and foremost I want to thank my advisor, professor Giovanni Gallipoli. He has been very supportive throughout my Ph.D. studies, from the day I was only a second-year student taking his course to now. I would like to thank him for encouraging my research, for pushing me forward, and for caring about my future. This thesis would not have been possible without his guidance, and words are hardly adequate for expressing my gratitude.

I am also grateful for my committee members, professor David Green and professor Yaniv Yedid-Levi for their continuous guidance and attention. They also supported me intellectually and emotionally whenever I need them: when I was struggling in the middle of my research, when my confidence was getting low... You were always there, being tremendous mentors to me.

Special thanks to professor Giulio Fella at Queen Mary University of London, whom I worked with on the third chapter of this dissertation. Though we had to collaborate remotely, it has been a very rewarding and enjoyable experience. Thank you to professor Paul Beaudry, professor Matilde Bombardini, professor Patrick Francois, professor Hiro Kasahara, professor Paul Schrimpf, professor Michal Szkup and all other VSE professors who were ever at my presentations and offered me valuable suggestions and encouragement.

I would also like to give a heartfelt thank you to my parents and my sister for always believing in me and encouraging me to follow my dreams. Doing a Ph.D. is a huge investment and commitment, and my family backed me up through thick and thin. And finally to my loving fiancée Yawen Liang, whose faithful support during the final stages of my Ph.D. means the world to me. Thank you.

Introduction

Suppose that in a family, the husband's wage is cut down. How would the household respond to such a shock? There are two key channels for consumption smoothing available to the household. One channel is smoothing through financial markets, which is typically done by saving and borrowing. The other channel is family labor supply adjustments, that is, by increasing the husband's labor supply and/or increasing the wife's labor supply. How important is each channel? How should we measure the effectiveness of each channel? This dissertation attempts to address these questions.

I start by arguing that the relative importance of these two channels depends on how households make decisions. The theoretical and empirical literature on household intertemporal decisions has traditionally assumed that households behave as single agents. Under this assumption, the identity of the recipient of the shock cannot make a difference in terms of household behavior. Modeling based on this assumption is called the unitary approach. Recently, economists have developed models that address some of the limitations of the unitary approach as a framework used to answer policy questions. Those models explicitly recognize that household members have their own preferences. A popular approach to modeling the allocations that result from the intra-household decision making process is the collective model of the household, originally proposed in a static form by Chiappori (1988, 1992). There are now a substantial number of cross-sectional empirical studies (see, e.g. Browning et al., 1994; Chiappori et al., 2002; Blundell et al., 2007) demonstrating that allocations within households are related to the source of income and other factors such as the sex ratio and divorce legislation, providing supporting evidence to the non-unitary approach.

It is theoretically appealing to extend the collective model of the household to dynamic contexts for answering questions about intertemporal household decisions like ours, and there has been some progress (Mazzocco, 2007; Voena, 2015; Gallipoli et al., 2016. However, common household survey data typically does not provide information about the evolution of allocations within households, which is important for testing and characterizing intertemporal non-unitary models. To address this data limitation, the first chapter of the dissertation proposes a method for measuring intra-household consumption allocation over time when two types of data are available: one has information on intra-household consumption allocation but lacks time variation; the other is a traditional panel of household, in which information about intra-household allocation is missing. The strategy consists of imputing gender-specific consumption data from a cross-sectional dataset to a panel. I apply this method on two publicly available datasets in the US: the Consumer Expenditure Survey (CEX) and the Panel Study of Income Dynamics (PSID). The generated panel allows researchers to investigate questions such as how the sharing rule shifts in response to shocks over time.

Next, the second chapter systematically studies how households insure themselves against idiosyncratic wage shocks and how this insurance interacts with intra-household bargaining. I set up a life-cycle model of the household and consider two alternative specifications of the model: a unitary version in which I restrict sharing rules to be fixed within households, and a non-unitary one in which I allow sharing rules to change. Using the empirical strategy discussed in the first chapter, I estimate the model using a panel that has information on consumption allocation within households by the generalized method of moments. I find that intra-household allocations respond strongly to fluctuations in individual wages. Removing the restriction of fixed sharing rules does not reduce the extent of consumption smoothing within a household, but it significantly changes the relative importance of different channels. In particular, the relative contribution of family labor supply to household consumption smoothing decreases from roughly 60% in the unitary model to 30% in the non-unitary model. This is because the added worker effect – the increase in spousal labor supply following an adverse shock to a partner – is much milder in the non-unitary specification. Moreover, the estimated model generates rich heterogeneity in responses of allocations within households. Shocks of the same size have very different implications, depending on whose wage they change.

This study has several important policy implications. First, most families (i.e., poor or young families) do not have the assets that would allow them to smooth consumption effectively. Without the labor supply channel, one could conclude that they have little in the way of maintaining living standards when shocks hit. For a correct design of public and social insurance policies, it is important to know whether households can use labor supply as an alternative insurance mechanism and to what extent they do so. My research finds that it is important to incorporate intra-household bargaining into the analysis, otherwise, the effectiveness of labor supply mechanism would be over-estimated. Moreover, studying how well families smooth income shocks, how this changes over the life and over the business cycle in response to changes in the economic environment confronted, and how different household types differ in their smoothing opportunities, is an important complement to understanding the effect of redistributive policies and antipoverty strategies.

Finally, the last chapter focuses on the saving and borrowing channel for consumption smoothing. Quantitative studies on this channel (Huggett, 1993; Kaplan and Violante, 2010) often approximate continuous stochastic processes using discrete state-space representations;

e.g. Markov chains. The Tauchen's (1986) and Rouwenhorst's (1995) methods have been widely used in the context of stationary infinite horizon problems. However, covariance-stationary income processes are not consistent with the empirical fact that within-cohort income inequality increases with the age of a cohort. The last chapter extends both Tauchen (1986) and Rouwenhorst's (1995) methods to discretize non-stationary AR(1) processes and compare their respective performance within the context of a life-cycle, income fluctuation problem. We find that generalized Rouwenhorst's method performs extremely well even with a relatively small number of grid-points.

Chapter 1

Measuring Intra-Household Consumption Allocations over Time

1.1 Introduction

How would a multi-member household respond to a shock to one of its members? Does it matter if married individuals are taxed jointly or independently? Does a reform in the divorce legislation have an effect on married couples' behavior even if their marriage is stable? Apparently, a complete answer to any of these questions requires a systematic approach to modeling intertemporal household behavior. Until recently, the standard approach to modeling household decision making was based on versions of the so-called "unitary" approach, which assumes that a household can be represented by a single utility function. In a framework of this type, what exclusively matters, as far as household decisions are concerned, is the total amount of resources at the household's disposal. It ignores any policy effect on the spouses' bargaining positions and consequent decisions. Ever since the seminal work by Chiappori (1988, 1992), the non-unitary perspective, in particular the "collective approach", has been replacing the traditional approach. The collective approach explicitly recognizes that household members each have their own preferences, and thus factors such as the relative incomes of the household members may affect the final allocation decisions made by the household.

There are two types of collective models: a static one and an intertemporal one. While the static collective model has become influential in family economics and received much empirical support such as Browning et al. (1994), Chiappori et al. (2002), and Blundell et al. (2007), the dynamic counterpart is much less so. One crucial empirical challenge is that testing and characterizing an intertemporal collective model requires that intra-household allocation is observed (at least partially) over time. However, due to data limitations, in most cases consumption is often measured at the household level only; and in some cases where intra-household consumption allocation can be measured, the data does not follow the same households over time.[1]

[1] The only exception is the Japanese Panel Survey of Consumers, the one that is used in Lise and Yamada (2015). However, this dataset is not publicly available; and it would be still interesting if one could test and identify the intertemporal collective model using data in other countries such as the U.S. given the substantial

This paper proposes a method for measuring intra-household consumption allocation over time when two types of data are available: one has information on intra-household consumption allocation but lacks time variation; the other is a traditional panel of household, in which information about intra-household allocation is missing. In the US, the example of the former type is the Consumer Expenditure Survey (CEX), and that of the latter type is the Panel Study of Income Dynamics (PSID).

To be specific, I estimate the demand functions for men's clothing and women's clothing, respectively, within married households using the CEX data. The estimated demand functions are then used to predict the expenditure on men's clothing and women's clothing for the married households in the PSID. This measures the evolution of the sharing rule of clothing. Next, to adjust for the gender difference in preferences towards clothing, I estimate the functions for the shares of expenditure on clothing for men and women using the subsamples of single men and single women, respectively. Under the assumption that this share is independent of marital status, I use the estimated functions to predict the shares for husbands and wives in married households, which are then, combined with the imputed gender-specific clothing expenditure, to back out the intra-household consumption allocation.

This paper is related to several strands of literature. One obviously related literature is the research that tries to identify intra-household allocation in a static environment. Browning et al. (1994) derive conditions for identifying the intra-household allocation using assignable goods (which is also clothing in their empirical implementation). Blundell et al. (2007) use leisure as an assignable good. Both studies rely on some assignable good as this paper does. Some other researchers use equilibrium conditions in marriage market to identify the sharing rule based on distribution factors such as sex ratios and the nature of divorce laws (Chiappori et al., 2002). Most recent advance is Cherchye et al. (2012), who use Dutch data on the allocation of private and public consumption expenditures and individual time use to fully identify and estimate a household sharing rule. The data they use is from a single cross section, and as a result, they are identifying how allocations relate to differences in relative wages across households, and not necessarily how allocations would change within a household resulting from unanticipated income shocks. Overall, this literature typically finds strong evidence that rejects the unitary approach to modeling household behavior, but it is based on static models, which cannot be used to answer questions and evaluate policies that have any intertemporal dimension.

There is a small but growing literature that extends the collective approach to a dynamic context.[2] Without a panel of intra-household consumption allocation, researchers typically

cultural differences, and more useful when one could use publicly available data to do so.

[2]The key assumption in the collective approach is Pareto efficiency. Depending on how efficiency is defined within households, two types of intertemporal collective models exist, namely, a full-commitment model and a limited-commitment model.

assume that consumption and leisure enter the individual utility function in a separable way. Under this assumption, individual leisure as intra-household *time* allocation alone help in the identification of the evolution of intra-household decision powers, and thus one only needs a panel that has information on leisure (or labor supply), which is much more common than panels of (individual) consumption. While this separability assumption has been proven to be a profitable approach to estimating an intertemporal collective model (see, e.g. Voena, 2015; Gallipoli et al., 2016), it is not uncontroversial.[3] This paper suggests a method for overcoming the data limitations so that (i) researchers can estimate an intertemporal collective model without imposing the separability assumption; and (ii) even when one is willing to assume separability, the intra-household consumption allocation provides another source for the identification of the sharing rule.

This paper is also closely related to the consumption imputation literature. This literature stems from the lack of household consumption panel until very recently. For example, in the US, the CEX provides comprehensive data set on the spending habits of US households but it follows households for only four quarters at most. The PSID collects longitudinal annual data, but until 1999 it collects data only for a subset of consumption items, mainly food at home and food away from home. For a long time, researchers have to use food expenditures as a proxy for consumption, which is clearly a poor proxy given that food is largely a necessity and its consumption is much less elastic than other consumption items. Some economists thus propose methods to impute *total* consumption in the PSID. For this kind of imputation, the key original reference is Skinner (1987), who proposes to impute total consumption in the PSID using the estimated coefficients of a regression of total consumption on a series of consumption items (food, utilities, vehicles, etc.) that are present in both the PSID and the CEX. The regression is estimated with CEX data. From a statistical point of view, Skinner's approach can be formally justified by the idea of matching based on observed characteristics. This method is used in several articles including, among others, Palumbo (1999), Dynan (2000), and Bernheim, Skinner, and Weinberg (2001). The most notable practice, however, is Blundell et al. (2008). Their approach is slightly different from the original Skinner's: instead of fitting an equation for the total consumption, they start from fitting a standard demand function for food (a consumption item available in both surveys), and invert it (under monotonicity of food demands) to obtain the imputed total consumption in the PSID. My imputation strategy is a combination of Skinner (1987) and Blundell et al. (2008): similar to Skinner (1987), my dependent variable is not available in the PSID and the independent variables are common in both datasets; similar to Blundell et al. (2008), I fit a standard demand function for gender-specific clothing consumption. Finally, as far as I know, this is

[3] As Heckman (1974) first noted, the dynamic response of consumption to wage changes will depend on whether consumption and hours are complements or substitutes in utility. Separability between consumption and labor supply is rejected empirically in Browning and Meghir (1991) and Blundell et al. (2015).

the first imputation method that generates a panel of intra-household allocation.

The rest of the paper is organized as follows. Section 1.2 describes the empirical strategy in details. Section 1.3 describes the data and section 1.4 presents the results of applying the method on the CEX and the PSID. Section 1.5 concludes.

1.2 Methodology

For any married household i at any time t, the total expenditure can be decomposed into three components:

$$C_{it,m} = C_{i1t,m} + C_{i2t,m} + G_{it,m} \tag{1.1}$$

where the subscripts i,j,t are household, member, and time indexes, respectively, and m indexes 'married'. $C_{ijt,m}$ is member j's ($j = 1, 2$) private consumption, and $G_{it,m}$ is the household's public consumption. The distinction between private and public consumption follows Browning et al. (1994) and Lise and Seitz (2011).

In the micro-level data, each consumption category can be classified as either private or public consumption. However, among those private consumption categories, very few are exclusive to a specific member (or gender). We therefore only observe $C_{iPt} \equiv C_{i1t,m} + C_{i2t,m}$ (subscript P denotes "private"), but not $C_{i1t,m}$ or $C_{i2t,m}$, the split between the husband and the wife. There is one exception: the subcategory "men's clothing" can be considered exclusive to the husband; the same goes for "women's clothing" to the wife.[4]

The empirical strategy proceeds as the following:

First, using the sample of married households in the CEX, I regress the gender-specific clothing consumption, on the demographic characteristics and the total consumption:

$$cloth^{CEX}_{ijt,m} = \beta_t + X'^{CEX}_{it,m}\beta_{1j} + \beta_{2j} cloth^{CEX}_{it,m} + \beta_{3j} C^{CEX}_{it,m} + u_{ijt,m} \tag{1.2}$$

for gender $j = 1, 2$ separately. The dependent variable $cloth_{ijt}$ is the clothing consumption for gender j, whereas $cloth_{it}$ on the right-hand side refers to the household-level clothing consumption. I control for the year fixed-effect β_t. X are socioeconomic variables of the household and the household members, including age, age squared, education, and races of both partners, region of living[5], number of children, and family size. Finally, C_{it} is the total family-level nondurable consumption. The CEX covers more consumption categories

[4] Children's clothing is usually another subcategory that is separately counted.

[5] Although information on the state of residence is available in the CEX, it is suppressed for some observations to avoid some small-population areas being identified. On approximately 14% of the records on the CEX family files the state variable is blank, and approximately 4% of observations are recoded to states that are not where they actually reside. By contrast, the variable that identifies the region of living is never suppressed.

than the PSID. For the purpose of imputation, the total consumption is defined as the sum of the expenditure in all consumption categories in the PSID. These include the food consumption, household utilities, health-related expenditure, expenditure on home repair and furnish, clothing expenditure and expenditure on entertainment and trips. All consumption variables are in natural logarithms and are so throughout this paper unless explicitly stated in levels. In practice, I allow the elasticity β_{3j} to vary with time and with observable household characteristics by introducing interactions terms.[6] Equation (1.2) can be interpreted as an approximated demand function that relates the gender-specific clothing expenditure to the total expenditure.[7]

Then I predict the gender-specific clothing consumption for the married sample in the PSID using the estimated coefficients from the CEX regression, that is,

$$\widehat{cloth}_{ijt,m}^{PSID} = \hat{\beta}_t + X_{it,m}^{\prime PSID}\hat{\beta}_{1j} + \hat{\beta}_{2j} cloth_{it,m}^{PSID} + \hat{\beta}_{3j} C_{it,m}^{PSID} \tag{1.3}$$

where the $\hat{\beta}$s are the estimates of βs in equation (1.2).

The next step is to calculate the sharing rule for each household in the PSID using the imputed gender-specific clothing consumption. The simplest way is to assume that the husband's share of private consumption can be approximated by his share of clothing consumption, in which scenario the private consumption of partner j would be simply given by $\hat{C}_{ijt} = \frac{\widehat{cloth}_{ijt}}{\widehat{cloth}_{i1t}+\widehat{cloth}_{i2t}} C_{iPt}$, where C_{iPt} is the household-level private consumption, i.e., the sum of all private consumption categories for household i at t. The categories that belong to public consumption are enjoyed by the couple together and thus no splitting rule is required.

However, this assumption might be too restrictive. It is likely that the division for clothing does not represent the division for total private consumption, which will be true if, say, the wives on average spend more on clothing than the husbands do, relative to other private goods. In this case, one would under-estimate the husbands' shares. To correct for this bias, I exploit the information about how much men (women) on average spend on clothing out of total private consumption from the sample of single men (women), and adjust the division rule to take into account the preference difference between males and females.

Let me illustrate the steps using the women's case; the estimation for men is analogous. First, for each single female observation in the CEX, I calculate the proportion out of total consumption she spends on clothing: $\psi_{i2t,s} \equiv cloth_{i2t,s}/C_{i2t,s}$, where the subscript s indicates

[6] To be consistent with the imputation literature, which typically does not use the income information, I do not include income of household members as explanatory variables. While they are potentially important, I leave this for future research.

[7] It is worth pointing out that here I do not aim to establish any causal relationship between the dependent and independent variables. Rather, the goal of the regressions, as it is common in the consumption imputation literature, is to find the best fit that describes the matching between a husband's expense on clothing and all his family's observable characteristics and behaviors. Thus endogeneity problems arising from the fact the dependent variable is a subset of the independent variable is not an issue given this objective.

the single sample (note that the subscript 2 in this case only refers to the gender being female, not the second member in household i). This proportion $\psi_{i2t,s}$ is regressed on a set of individual observable characteristics that are commonly available in the CEX and in the PSID:

$$\psi_{i2t,s} = \gamma_{t,f} + X_{i2t,s}'^{CEX}\gamma_f + u_{i2t,f} \tag{1.4}$$

where the subscript f stands for *female*. Then the proportion that a wife spends on clothing out of her total private consumption in the PSID sample is predicted by:

$$\hat{\psi}_{i2t,m} = \hat{\gamma}_{t,f} + X_{i2t,m}'^{PSID}\hat{\gamma}_f \tag{1.5}$$

In fact, equations (1.4) and (1.5) are very similar to (1.2) and (1.3). The idea is to estimate the average proportion of expenditure on clothing for women (or for men), and I allow this proportion to vary across ages, races, etc. To justify this imputation, I assume that the preference for clothing relative to other private goods, once controlled for age, education, etc., does not depend on the marital status.

Once we obtain $\hat{\psi}_{ijt,m}$ for $j = 1, 2$ for each married household in the PSID sample from (1.5), the private consumption for partner j is given by:

$$\hat{C}_{ijt} = \frac{\widehat{cloth}_{ijt}/\hat{\psi}_{ijt}}{\widehat{cloth}_{i1t}/\hat{\psi}_{i1t} + \widehat{cloth}_{i2t}/\hat{\psi}_{i2t}} C_{iPt} \tag{1.6}$$

1.3 Data

The main data I use in this paper is the Panel Study of Income Dynamics (PSID) for the years 2005-2013. The PSID data are collected biennially since 1999, so 5 waves are used: 2005, 2007, 2009, 2011, and 2013. Starting in 1999, in addition to income data and demographics, the PSID collects data about detailed assets holdings and consumption expenditures (at the household level), the latter of which cover many nondurable and services consumption categories, including utilities, health expenditure, transportation, education, and child care.[8] A few more consumption categories, including clothing, trips and vacation, and other recreation, were added in 2005. As the imputation procedure relies on matching the clothing consumption, I use the waves since 2005.

I focus on the core (non-SEO) sample of the PSID.[9] I select the households that are stably

[8] Before 1999, the PSID collected data on very few consumption items, mainly food at home and food away from home.

[9] SEO refers to the Census Bureau's Survey of Economic Opportunities. The SEO sample in the PSID consists of low-income families, which are over-sampled, and it distinguishes from the core sample, which is representative of the US households.

married over the sample period, with the male head aged between 25-60 and both partners are working. I drop the households with missing information on key demographic variables (age, race, and education) and households with zero consumption. I also drop the households in which the hourly wage of the husband or of the wife is lower than half of the minimum wage.[10] The remaining sample is an unbalanced panel of 9628 observations.

I also use the 1998-2013 Consumer Expenditure Survey (CEX) data. The CEX provides a comprehensive flow of data on the buying habits of American consumers. The data are collected by the Bureau of Labor Statistics and used primarily for revising the CPI. The definition of the head of the household in the CEX is the person or one of the persons who owns or rents the unit; this definition is slightly different from the one adopted in the PSID, where the head is always the husband in a couple. I make the two definitions compatible by restricting the CEX sample to be male-headed households.

The CEX is based on two components, the Diary survey and the Interview survey. The Diary survey is conducted over two consecutive one-week periods, designed to track detailed expenditures data on small and frequently purchased items, such as food, personal care, and household supplies. The Interview survey is conducted quarterly, and it covers about 95 percent of all household expenditure[11]. My analysis below uses only the Interview sample, because it is more comprehensive and it can be used to construct annual expenditure for households. I apply the same sampling restrictions on the CEX as on the PSID[12]: married households with the head being male and of age 25-60; both partners are working; and no missing information on key demographic characteristics.

1.4 Results

Table 1.1 reports the estimate of coefficients in the regressions of gender-specific clothing consumption in the CEX (equation 1.2). Column (1) is for the regression of men's clothing consumption and Column (2) for women's. Again, all consumption variables are in logarithms. The omitted group for education is the ones with less than high school, the base group for races is white, and for number of children I omit the household with no child.

[10] I use the highest minimum wage prescribed by federal law and state law. Before the Great Recession, the number of male workers that earn less than half of the minimum wage is very stable across years. In the post-crisis waves, 2011 and 2013, this number more than doubles. In fact, the whole wage distribution shifts to the left after the crisis. (The same thing happens to female workers.) To avoid excess truncation, I do not discard the observations whose wages are lower than half of the minimum wage after the crisis but not so before the crisis.

[11] With the exclusion of expenditures for housekeeping supplies, personal care products, and non-prescription drugs.

[12] Strictly speaking, these restrictions cannot be identical, because some sampling requirements imposed on the PSID rely on the fact that the households are observed for continuous years and thus cannot be applied on the CEX. For example, the households in the PSID sample here are stably married across years, but in the CEX I cannot observe whether a household is married in the adjacent years or not.

Table 1.1: Regression for Gender-Specific Clothing Consumption in the CEX

	(1)	(2)	
	Male's clothing	Female's clothing	Test $\beta_m = \beta_f$
Head's age	.0005	.0034	
	(.0088)	(.0074)	
Head's age squared	.0000	.0000	
	(.0001)	(.0001)	
Head's educ - high school	-1.1909	1.9926*	
	(1.2913)	(1.1293)	
Head's educ - some college or more	-.9032	2.4208**	
	(1.1987)	(1.0741)	
Spouse's educ - high school	-.9180	-1.7599	
	(1.6504)	(1.4272)	
Spouse's educ - some college or more	-1.2228	-1.9404	
	(1.6270)	(1.4254)	
Head's race - black	-.2549**	-.2771***	
	(.1164)	(.1009)	
Head's race - others	-.0390	-.0194	
	(.0714)	(.0688)	
Spouse's race - black	.3153**	.2458**	
	(.1248)	(.1069)	
Spouse's race - others	-.0033	-.0387	
	(.0664)	(.0633)	
Family size	-.3848***	-.2198***	
	(.0851)	(.0764)	
One child	-.9739	-1.8883***	
	(.6160)	(.5431)	
Two children	-1.5244**	-2.3802***	
	(.6206)	(.5688)	
Three children or more	-1.8814**	-2.2193***	
	(.9353)	(.8524)	
Household clothing consumption	.8034***	.9166***	p=0.000
	(.0147)	(.0129)	
Household total consumption	-.0915	.0618	p=0.030
	(.1315)	(.1143)	
Observations	6510	6715	
R-squared	0.4656	0.5937	

Notes: The last column reports the p-value for testing whether the coefficients for men's regression and for women's are the same. I also control for year fixed effects and region dummies. I include interactions between the household total consumption and observables to allow for flexible budget elasticity. The estimates for those coefficients are not shown for preserving space. Standard errors in parentheses are clustered at the household level. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Although not reported in Table 1.1 (for compactness), I also include the year fixed effects and dummies for region of living, and a set of interaction terms between the household total consumption and observables, which captures the idea that the budget elasticity may vary with those characteristics.

The last column in Table 1.1 is testing whether the men's coefficient is different from the women's. In particular, I'm interested in the differential responses between men's clothing consumption and women's when the household-level consumption changes, as they are important for identifying the sharing rule.[13] The last test (regarding total consumption) is a joint test of the total consumption alone and the interaction terms (not shown in the table) being no effect.

Figure 1.1 shows the comparison between the actual data on household-level clothing expenditure and the sum of imputed husband's clothing and wife's clothing consumption. The upper panel compares the histogram and the lower panel compares the kernel density estimates of the distributions. As it can be seen from the graphs, the imputed sum matches the actual data very well. In addition, the regression of actual household-level clothing expenditure on the imputed sum yields an R-squared of 0.8326.

The imputed sharing rule is shown in Figure 1.2. I draw both the distribution for the men's share of (imputed) clothing consumption and the distribution for men's share of private consumption adjusted for the gender preferences as shown in equation (1.6). The mean of men's share in consumption is 0.4067, and standard deviation 0.0815. This is in the range of what the literature found. Dunbar et al. (2013) estimate that men absorb 40-47% of household resources and a relatively small amount of variation using Malawi data. Using a Dutch cross-section dataset, Cherchye et al. (2012) report a roughly half-half division on average but the number is sensitive to the definition of private consumption. Lise and Seitz (2011) finds the mean of wife's share of consumption to be 0.33 in 1970 and 0.40 in 2000 using U.K. data, but this relatively lower wife's share on average is because they include all households and some of the wives do not participate in the labor market. My estimation, along with Dunbar et al. (2013)'s and Cherchye et al. (2012)'s, excludes families with non-participation members.

1.5 Conclusion

To analyze the intertemporal household behavior in a non-unitary setting, one needs to have information on the evolution of the sharing rule. The lack of panel data on intra-household consumption allocation presents an empirical challenge to estimating an intertemporal collective model. This paper proposes a method for measuring consumption allocation within a

[13] The differences in the coefficients on the observable characteristics in X, while might be interesting empirically, are not useful for the identification here, because in the GMM estimation all variables are "residuals" net of those observables.

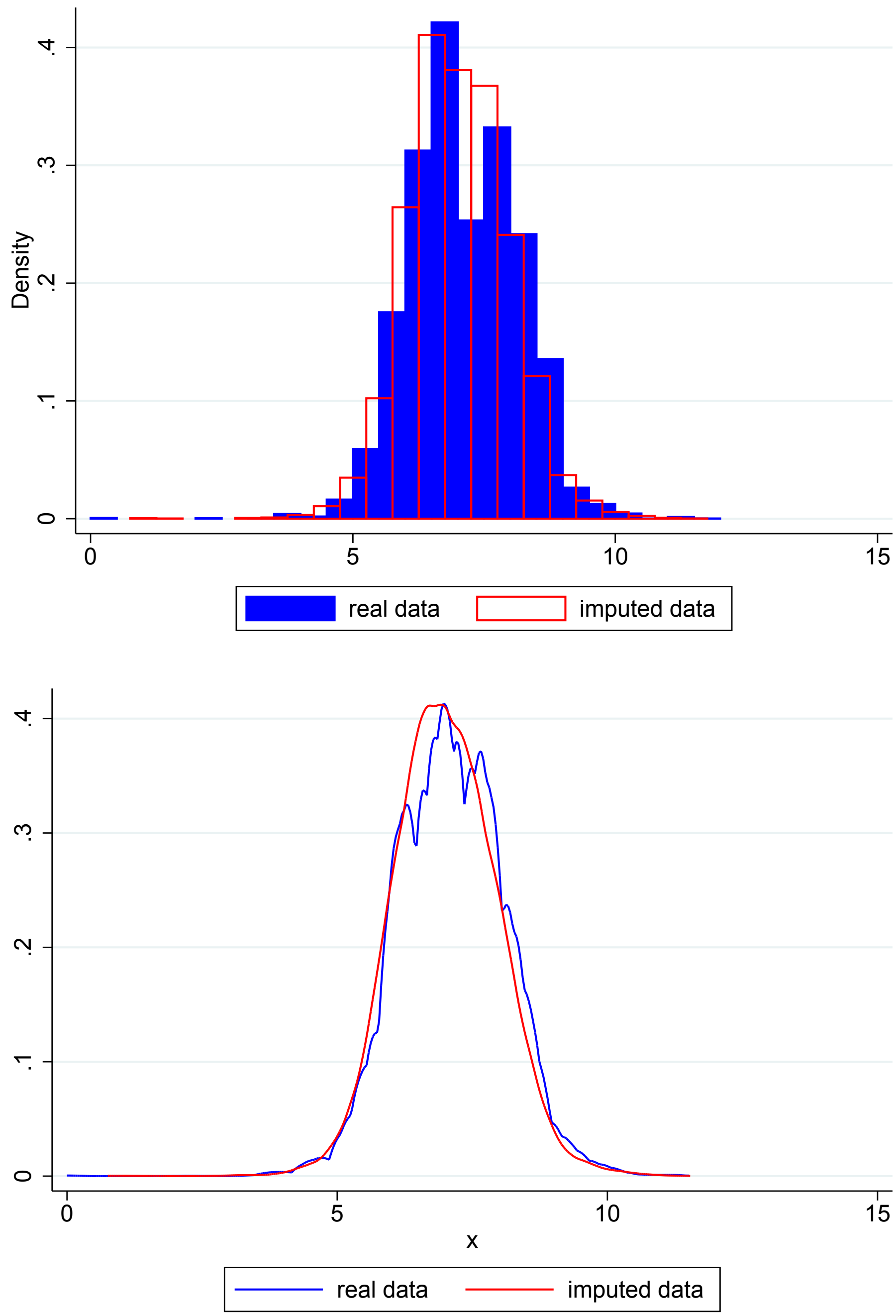

Figure 1.1: Comparison of real versus imputed household-level clothing consumption

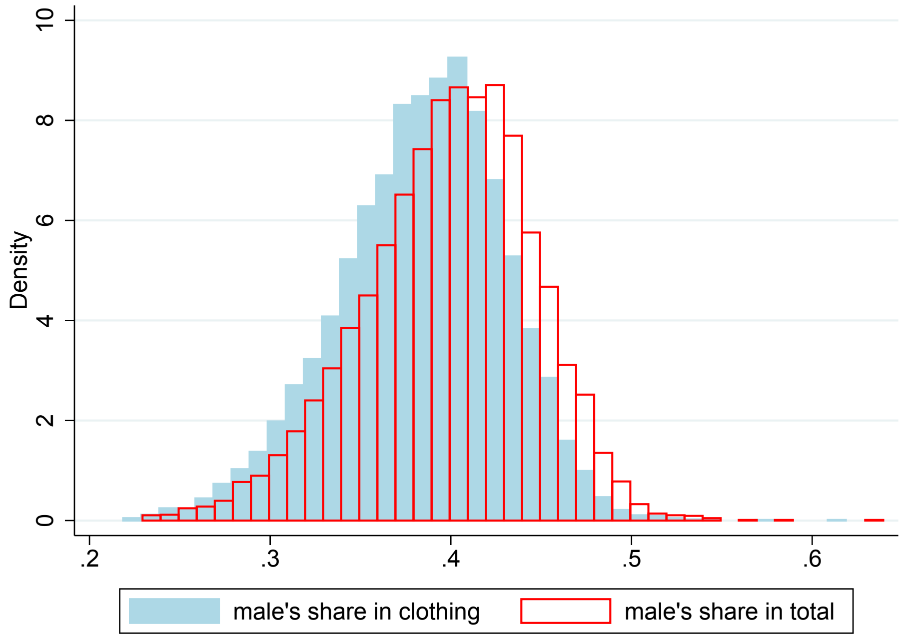

Figure 1.2: Distribution for imputed husbands' share of consumption

household over time. I apply it on two publicly available datasets in the US: the Consumer Expenditure Survey (CEX) and the Panel Study of Income Dynamics (PSID). Using the CEX data, I estimate the gender-specific consumption, clothing, as a function of socioeconomic variables that are commonly available in both the CEX and the PSID, and then use these estimated functions to predict the clothing consumption allocation within the PSID families. I also estimate the functions for the shares of expenditure on clothing for men and women using the subsamples of single men and single women, respectively. Under the assumption that this share is independent of marital status, I use the estimated functions to predict the shares for husbands and wives in married households, which are then, combined with the imputed gender-specific clothing expenditure, to back out the intra-household consumption allocation. Results are consistent with the empirical findings in the literature. The resulting panel provides rich *dynamic* information inside the "black box" by which a household is taken in a unitary model, and allows researchers to analyze the intertemporal effect of policies on household behavior based on the more micro-founded collective approach.

Chapter 2

Consumption Smoothing and Intra-Household Bargaining

2.1 Introduction

Much past and current literature has examined how households respond to income fluctuations. This line of research often attempts to quantify the degree of insurance achieved within households (see for example Blundell, Pistaferri, and Preston, 2008; Kaplan and Violante, 2010), but it also strives to understand the specific mechanisms used by households to smooth their consumption (e.g., Low, 2005; Heathcote, Storesletten, and Violante, 2014; Blundell, Pistaferri, and Saporta-Eksten, 2015). Both of these research objectives have crucial policy implications, which justifies the extensive attention they have received so far. Much of the existing studies treat the household as a single decision-maker, often ignoring the heterogeneity in intra-household allocations and, more importantly, how intra-household bargaining over such allocations affects the effectiveness of different insurance channels. These modeling choices are made in part because of data limitations that reduce the ability to reliably measure changes in intra-household allocations. Nonetheless, growing empirical evidence casts doubts on the innocuousness of modeling the household decision making as a unitary process (see, among others, Mazzocco, 2007; Lise and Seitz, 2011; Gallipoli, Pan, and Turner, 2016). In this paper, I show both theoretically and empirically that taking a non-unitary approach to household decision-making has non-trivial implications for the analysis of within-household labor supply patterns, consumption allocations, and insurance provision.

In the presence of wage uncertainty, there are several ways in which households can accommodate shocks and smooth consumption. Economists have mostly focused on two key smoothing channels.[14] The first one is self-insurance through borrowing and saving. This is probably the most studied source of insurance in the macro and labor literature. The other insurance channel encompasses all those adjustments to labor supply that members of the household make in response to individual wage fluctuations. Previous studies have traditionally looked at these two channels separately, until some more recent work has stressed the

[14] Other insurance channels include progressive taxation, social insurance programs, family transfers, informal networks, default or bankruptcy, et cetera.

importance of considering both mechanisms within the same framework.[15] In an influential paper, Blundell et al. (2015) estimate that following a 10% permanent decline in the husband's wage, roughly 3.9% of consumption is effectively insured. Of this 3.9%, 2.5 percentage points (63% of the total insurance effect) come from the family labor supply channel and 0.7 percentage point (17% of the total) comes from self-insurance.[16] This analysis is based on a unitary household framework, that is, a framework in which spouses pool consumption and make decisions as if they were one person.

In this paper I show that the relative importance of family labor supply as a consumption smoothing device decreases when couples can bargain over the intra-household allocation and that the bargaining power of each member of the household depends on his/her labor income relative to the spouse's. Intuitively, in the case of unitary decision making, when one earner in the family is hit by a negative permanent wage shock, there is an incentive for the other earner to work more in order to compensate for the family's income loss to the family. This is sometimes called the "added worker effect". This effect becomes less obvious in a non-unitary decision-making household. If an earner's bargaining power depends on his/her contribution to the total labor income of the family, the earner who experiences an adverse shock may not be willing to let own labor supply be substituted by the spouse's, because that would reduce their bargaining power and likely trigger a renegotiation leading to a new agreement (sharing rule) favoring the spouse.

I set up a life-cycle collective household model in which individual wages are subject to idiosyncratic (transitory and permanent) shocks. Individual members derive utility from individual consumption and labor supply. A household determines the intra-household allocation given Pareto weights on individual utilities. These weights are agreed upon by the couple. If the utility weights are fixed, the model assumes full-commitment and results in allocations consistent with a unitary specification.[17] For comparison with previous studies, I refer to

[15] The literature on the first insurance channel (self-insurance) dates back to at least Deaton (1991). Recent developments include Kaplan and Violante (2010) and Krueger and Perri (2006). This literature typically assumes exogenous labor supply. Studies on the responsiveness of individual labor supply to wage changes, as surveyed in Keane (2011), do not consider the joint consumption-labor supply choice and focus on the single earner case. There is a parallel, related literature in labor economics (e.g. Lundberg, 1985) asking to what extent a secondary earner's labor supply increases in response to negative wage shocks faced by the primary earner, known as the "added worker effect". However, this literature does not normally provide explicit measures of the consumption smoothing achieved by households.

[16] The remaining 0.7 percentage point is due to residuals capturing other formal and informal insurance channels.

[17] As Chiappori and Mazzocco (2015) noted, "There exists situations under which the unitary and collective models generate the same set of household decisions. This is the case, for instance, if the relative decision power is constant and therefore does not depend on prices, wages, income, and distribution factors." In addition, Mazzocco (2007) proves that the unitary model is a special case of the full-commitment collective model. This implies that if one rejects the full-commitment model, the unitary model is also rejected. Readers interested are referred to the technical proof in the appendix of Mazzocco (2007), but the proposition is basically a generalization of Gorman's aggregation theorem to an intertemporal framework with public consumption.

this specification of the model as "unitary". Otherwise, if the utility weights are subject to renegotiation, the model is a limited-commitment one, and I refer it to as "non-unitary". The non-unitary variant of the model encompasses the unitary one as a special case, when the response of the utility weights to wage shocks is restricted to be zero.

The literature on collective household models typically adopts a fully structural estimation approach. That is, one fully parameterizes the household model, making explicit assumptions about preferences, shocks, and the determination of the sharing rule. Examples of papers using this approach (albeit pursuing different research questions) include Lise and Seitz (2011) and Cherchye et al. (2012). The degree of insurance estimated by this kind of approach is, by construction, reliant on the chosen functional forms.[18] In this paper, I use a less restrictive approach: while keeping the preferences and bargaining process nonparametric, I approximate the household optimization conditions by log-linearization and derive a system of equations describing the transmission of the wage shocks to consumption and hours worked, which is then used for evaluating the importance of different channels of consumption smoothing.[19]

In the analytical section I show how the transmission coefficients, and the underlying Frisch elasticities and "bargaining parameter" (the elasticity of utility weights with respect to wages), can be identified using joint moments of individual wages, individual consumption, individual labor supply, and household assets. Importantly, the expression describing the shock transmission coefficients admits a "bargaining effect", in addition to the traditional substitution and wealth effects. The sign of this effect is unrestricted by the theory - in fact, I show that it is heterogeneous across households and has to be established empirically.

I use the 2005-2013 Panel Study of Income Dynamics (PSID) to estimate the model and I focus on continuously married households. To identify the bargaining effect, I need data on individual-specific consumption. However, while the PSID features a long time-series with rich information about individual labor market activities, it only provides consumption data at the household level. This is a common disadvantage of most existing consumption data sets - in fact, there is no panel of individual consumption in the US. This presents an empirical challenge for incorporating bargaining into the analysis of consumption smoothing.[20] To overcome this limitation, I use a method developed in Pan (2017), which approximates individual consumption expenditures by imputing the gender-specific private consumption

[18] For example, Lise and Seitz (2011) use a utility function that is separable between consumption and leisure. In contrast, Blundell et al. (2015) stress that non-separability may play an important role in the household responses to shocks.

[19] A drawback of this approach is that it is subject to approximation errors. See Blundell et al. (2013) for a discussion of the accuracy of the approximation approach in the context of unitary household models.

[20] The same problem is also present in data for Canada, the UK, and other countries. As far as I know, such a panel only exists in Japan; see Lise and Yamada (2015) for the details of the Japanese Panel Survey of Consumers.

based on estimates from the Consumer Expenditure Survey (CEX). In this way, I obtain a panel of households whose intra-household expenditures are continuously observable. Finally, I construct the data moments of individual consumption, as well as of labor supply and wages that are directly available in the PSID, and estimate the parameters by the generalized method of moments (GMM) using the moment conditions implied by the model structure.

I find that the utility weight strongly responds to individual wage shocks, lending support to the non-unitary specification of the model. The bargaining effect on the shock transmission is also economically important. In particular, in terms of consumption smoothing, allowing for intra-household bargaining lowers the relative contribution of family labor supply by roughly 16%-29%.[21] Consistent with the intuitive argument proposed above, this is mainly due to a weaker added worker effect in the non-unitary setting. Moreover, following a positive permanent shock to the husband's wage, his own consumption increases more and his spouse's consumption increases less than in the unitary case.

The remainder of this paper is organized as follows. Section 2.2 introduces the analytical framework, derives the model solutions, and discusses how the model can be identified and estimated using a panel of individual consumption and labor supply, as well as household assets. Section 2.3 describes the data and the empirical strategy. Section 2.4 presents and discusses the results. Section 2.5 concludes.

2.2 The Household Model

In this section, I develop a life-cycle model for a family consisting of two potential earners.

2.2.1 Wage Process

The primitive source of exogeneity and uncertainty to the family members are the hourly wages they earn. For each earner within the household, I posit a permanent-transitory wage process, assuming that the permanent component evolves as a unit root process. Formally, I assume that the log of individual j's real hourly wage in household i at t follows a permanent-transitory process and is given by

$$\ln W_{ijt} = X_{ijt}\zeta^j + w_{ijt} \tag{2.1}$$

$$w_{ijt} = w_{ijt}^P + u_{ijt} \tag{2.2}$$

$$w_{ijt}^P = w_{ijt-1}^P + v_{ijt}. \tag{2.3}$$

The vector X_{ijt} contains observed characteristics affecting wages and known to the household at time t. w_{ijt} is the residual wage, which can be decomposed into w_{ijt}^P, the permanent

[21]This figure varies depending on whether the shock is in the husband's wage or the wife's.

component, and u_{ijt}, the transitory shocks. The permanent component w^P_{ijt} follows a random-walk process and v_{ijt} is the permanent shock.

Deviations from the deterministic path for wages occur because permanent and transitory shocks, positive or negative, hit the individuals. A permanent shock shifts the value of one's skills in the market permanently (for example, an accident causing long-term disability, a sudden promotion); a transitory shock is mean reverting (for example, a short illness affecting productivity, a one-time bonus payment). When shocks hit, I assume the partners can perfectly observe and distinguish between them; moreover they hold no advance information about the shocks ($E_t[u_{i,j,t+1}] = 0, E_t[v_{i,j,t+1}] = 0$; E denotes subjective expectations).

I assume that earner j's permanent and transitory wage shocks are serially uncorrelated with variance $\sigma^2_{v_j}$ and $\sigma^2_{u_j}$, respectively. I also assume that permanent (transitory) shocks can be contemporaneously correlated within a family, with covariance $\sigma_{v_1 v_2}$ ($\sigma_{u_1 u_2}$). This correlation is theoretically ambiguous. For example, if spouses were to adopt sophisticated risk sharing mechanisms, they would select jobs where shocks are negatively correlated. Alternatively, assortative mating or other forms of sorting imply that spouses work in similar jobs, similar industries, and sometimes in the same firm - hence their shocks may be potentially highly positively correlated.[22] In the baseline specification, I impose stationarity for the variances and covariances of the shocks. Finally, I assume that transitory and permanent shocks are uncorrelated.

The properties of the shocks can be summarized as follows:

$$E(v_{ij_1t}v_{ij_2t+s}) = \begin{cases} \sigma^2_{v_j} & \text{if } j_1 = j_2 = j, s = 0 \\ \sigma_{v_1,v_2} & \text{if } j_1 \neq j_2, s = 0 \\ 0 & \text{otherwise} \end{cases} \tag{2.4}$$

$$E(u_{ij_1t}u_{ij_2t+s}) = \begin{cases} \sigma^2_{u_j} & \text{if } j_1 = j_2 = j, s = 0 \\ \sigma_{u_1,u_2} & \text{if } j_1 \neq j_2, s = 0 \\ 0 & \text{otherwise} \end{cases} \tag{2.5}$$

$$E(u_{ij_1t}v_{ij_2t+s}) = 0 \text{ for all } j_1, j_2, s \tag{2.6}$$

Given the specification of the wage process (2.1)-(2.3) the growth in the residual log wages can be written as

$$\Delta w_{ijt} = \Delta u_{ijt} + v_{ijt} \tag{2.7}$$

[22] I do not attempt to model those potential sophisticated risk sharing mechanisms. Labor supply adjustment in my model only works at the intensive margin — by working longer hours. The sign of correlation between spousal wage shocks, which are taken as exogenous, is not restricted in the model and to be determined empirically, and there might be other possible explanations for non-zero correlation.

where Δ is a first difference operator and Δw_{ijt} is the log change in hourly wages net of observables.

2.2.2 The Family Problem

Given the exogenous wage process described above, a two-earner family maximizes a weighted sum of the husband's individual utility and the wife's:

$$\max \mathbb{E}_0 \sum_{t=0}^{T} \beta^t \left(\mu_{it} U_1(C_{i1t}, G_{it}, H_{i1t}; d_{i1t}) + (1-\mu_{it}) U_2(C_{i2t}, G_{it}, H_{i2t}; d_{i2t})\right) \tag{2.8}$$

subject to the intertemporal budget constraint

$$A_{i,t+1} = (1+r)(A_{it} + W_{i1t}H_{i1t} + W_{i2t}H_{i2t} - C_{i1t} - C_{i2t} - G_{it}) \tag{2.9}$$

where I denote the household's asset by A_{it}, partner j's individual private consumption by C_{ijt}, partner j's hours worked by H_{ijt}, and the family's public consumption by G_{it}. Note that the public consumption enters both partners' individual utilities. β is the time discount factor. I assume households have access to a risk-free asset that pays an exogenous interest rate of r, and all assets of a family are shared by the two earners. Finally, d_{ijt} are observable preference shifters, such as the number of children and the age of the earner; I account for these empirically by using the residual measures of consumption, wages and earnings.

I make a distinction between private and public consumption in the model. As public consumption is consumed by both members of the household, ignoring the presence of public consumption is likely to lead to overestimation of the degree of inequality within households. I follow the strategy of Browning et al. (1994) and partition all expenditures into either public or private expenditures. Intuitively, a good is deemed private if consumption of one unit of it by one partner implies that this given unit is no longer available to the other partner, whereas a good is deemed public if consumption by one family member does not reduce the available amount to the other family (i.e. it can be "shared"). More details about the classification are discussed in Section 2.3.1.

The last issue is how the intra-household allocation rule, i.e. the utility weight μ_{it}, is determined. Depending on whether the utility weight can change over time, there are two versions of the model.

In the first version, the utility weight is fixed so that

$$\mu_{it} = \mu_{it-1} = \cdots = \mu_{i0} = \mu^{\mathrm{u}}(z_{i0}). \tag{2.10}$$

In this case, all household decisions are efficient in the sense that they are always on the *ex-ante* Pareto frontier. In this case, the only thing that matters for determining the Pareto weight is the relative bargaining power at the time of marriage. In other words, the Pareto weight is only a function of information available at the time of marriage (including the forecastable components), $z_{i0} = \{E_0 z_{it}\}_{t=0}^{T}$. I refer to this version of the model as a unitary one.[23]

In the second version, I allow for intra-household bargaining and the utility weight is subject to change when there is news or shock to the household. In this case the Pareto weight depends both on the date-0 forecastable components z_{i0} and the realized deviations from this forecast $\varepsilon_{it} \equiv z_{it} - E_0 z_{it}$:

$$\mu_{it} = \mu^{\mathrm{n}}(z_{i0}, \varepsilon_{it}) \tag{2.11}$$

I call this version of the model a non-unitary one. Since the only exogenous shocks are in wages, ε_{it} consists of the accumulated wage shocks for the husband and for the wife. In fact, the non-unitary specification nests the unitary one as a special case: if μ_{it} is inelastic with respect to ε_{it}, then equation (2.11) would collapse to equation (2.10). In other words, if the bargaining power doesn't change in response to wage shocks, the non-unitary variant of the model would behave as the unitary one. This suggests a simple test of whether couples make decisions in a unitary way or not, which is equivalent to testing whether the elasticities of utility weight with respect to wages are zero.

2.2.3 The Solution to the Family Problem

The model outlined above does not have an exact analytical solution unless one imposes strong restrictions on the functional forms. Keeping the preferences non-parametric, I use an approximation approach to derive the solution. Similar approaches have been employed by Attanasio et al. (2002), Blundell et al. (2008), and Blundell et al. (2015), but only in the unitary framework. I extend the approach to apply in the collective household model.

The approximation can be summarized into two parts. First, I approximate the first order conditions to derive a system that links the endogenous variables (individual consumption, public consumption, etc.) to the exogenous shocks and the change in the marginal utility of wealth. Second, by log-linearizing the life-time budget constraint, I derive the change in the marginal utility of wealth as a function of the wage shocks. At the end, I derive the following expression of how the permanent and transitory shocks of individual wages affect the changes

[23] In fact, this is called the "full commitment" model in Mazzocco (2007). It is different from the purely unitary model in the sense that there are individual utilities in the full commitment model. However, the distinction between the pure unitary model and the full commitment model is only useful cross-sectionally. For a married household, whether it is purely unitary or full commitment, there is no time variation in the intra-household allocation.

in the consumption and earnings:

$$\begin{pmatrix} \Delta c_{i1t} \\ \Delta c_{i2t} \\ \Delta g_{it} \\ \Delta y_{i1t} \\ \Delta y_{i2t} \end{pmatrix} \approx \begin{pmatrix} \kappa_{c_1u_1} & \kappa_{c_1u_2} & \kappa_{c_1v_1} & \kappa_{c_1v_2} \\ \kappa_{c_2u_1} & \kappa_{c_2u_2} & \kappa_{c_2v_1} & \kappa_{c_2v_2} \\ \kappa_{gu_1} & \kappa_{gu_2} & \kappa_{gv_1} & \kappa_{gv_2} \\ \kappa_{y_1u_1} & \kappa_{y_1u_2} & \kappa_{y_1v_1} & \kappa_{y_1v_2} \\ \kappa_{y_2u_1} & \kappa_{y_2u_2} & \kappa_{y_2v_1} & \kappa_{y_2v_2} \end{pmatrix} \begin{pmatrix} \Delta u_{i1t} \\ \Delta u_{i2t} \\ v_{i1t} \\ v_{i2t} \end{pmatrix} \tag{2.12}$$

where κ_{lm} is the transmission coefficient of shock m into the choice variable l. Note that in general, κ's are heterogeneous across households and time (i.e., I should write $\kappa^{it}_{c_1u_1}$, etc.). To avoid cluttering, I leave this individual and age-dependence implicit.

These transmission coefficients are explicit (but very complicated) functions[24] of a set of Frisch elasticities, household assets, and the husband's and the wife's lifetime earnings:

$$\kappa_{lm} = \kappa^{lm}(\boldsymbol{\eta}, \pi_{it}, s_{it}) \tag{2.13}$$

where $\pi_{it} \approx \frac{A_{it}}{A_{it}+HumanWealth_{it}}$ is a "partial insurance" parameter (Blundell et al. (2008)). $HumanWealth_{it} = HumanWealth_{i1t} + HumanWealth_{i2t}$, and $HumanWealth_{ijt}$ is earner j's expected discounted lifetime labor income.[25] The higher π_{it} the lower the sensitivity of consumption to shocks because the household has more financial assets (relative to human wealth) to smooth the wage shocks. $s_{it} \approx \frac{HumanWealth_{i1t}}{HumanWealth_{it}}$ is the share of husband's human wealth over family human wealth. $\boldsymbol{\eta}$ is the vector of all elasticities, which includes gender j_1's Frisch elasticity of labor supply with respect to j_2's wage (denoted by $\eta_{h_{j_1},w_{j_2}}$)[26], gender j's consumption elasticity of intertemporal substitution ($\eta_{c_j,p}$)[27], and the "bargaining elasticity"(η_{μ,w_j}), the elasticity of utility weight with respect to changes in the wages.

[24] Full analytical expressions are presented in Appendix A.2.

[25] Formally, $HumanWealth_{ijt} = \sum_{s=t}^{T} E_{t-1}[\frac{Y_{ijs}}{(1+r)^{s-t}}]$, where T is the retirement age.

[26] Frisch elasticity of labor supply is, by definition, the elasticity of labor supply with respect to wage when the marginal utility of wealth is fixed (which is different from the Marshallian elasticity that captures both the substitution effect and wealth effect). It can be identified through labor supply response to transitory shocks, which by definition is mean-reverting and does not induce changes in lifetime earnings. However, I follow the common notation $\eta_{h,w}$ instead of $\eta_{h,u}$.

[27] p is the "price" of a unit of current consumption relative to future consumption.

2.2.4 Identification

The parameters of the wage process are identified independently of preferences. The following moments of the joint distribution of the individual wages deliver identification:

$$\begin{aligned}
\sigma^2_{u_j} &= -\mathbb{E}_t[\Delta w_{ijt}\Delta w_{ijt+1}], \quad j=1,2\\
\sigma^2_{u_1u_2} &= -\mathbb{E}_t[\Delta w_{i1t}\Delta w_{i2t+1}],\\
\sigma^2_{v_j} &= \mathbb{E}_t[\Delta w_{ijt}(\Delta w_{ijt-1}+\Delta w_{ijt}+\Delta w_{ijt+1})], \quad j=1,2\\
\sigma^2_{v_1v_2} &= \mathbb{E}_t[\Delta w_{i1t}(\Delta w_{i2t-1}+\Delta w_{i2t}+\Delta w_{i2t+1})]
\end{aligned}$$

where Δw_{ijt} is given by (2.7). Identification of the transitory variances rests on the idea that wage growth rates are autocorrelated due to mean reversion caused by the transitory component (the permanent component is subject to i.i.d. shocks). Identification of $\sigma^2_{u_1u_2}$ is an extension of this idea: between-period and between-earner wage growth correlation reflects the correlation of the mean-reverting components. Identification of the permanent variances rests on the idea that the variance of wage growth ($\mathbb{E}_t(\Delta w_{ijt})^2$), net of the mean reverting component ($\mathbb{E}_t[\Delta w_{ijt}(\Delta w_{ijt-1}+\Delta w_{ijt+1})]$), identifies the variance of innovations to the permanent component. Identification of $\sigma^2_{v_1v_2}$ follows a similar logic.

The transmission coefficients of wage shocks into consumption and earnings are identified by the covariances between these outcome variables and wages. Consider for example the transmission of wage shocks into the wife's private consumption. The coefficients $(\kappa_{c_2u_1}, \kappa_{c_2u_2}, \kappa_{c_2v_1}, \kappa_{c_2v_2})$ can be identified by the following moments:

$$\begin{aligned}
\mathbb{E}[\Delta c_{i2t}\Delta w_{i1t+1}] &= -\kappa_{c_2u_1}\sigma^2_{u_1} - \kappa_{c_2u_2}\sigma_{u_1u_2}\\
\mathbb{E}[\Delta c_{i2t}\Delta w_{i2t+1}] &= -\kappa_{c_2u_1}\sigma_{u_1u_2} - \kappa_{c_2u_2}\sigma^2_{u_2}\\
\mathbb{E}[\Delta c_{i2t}(\Delta w_{i1t-1}+\Delta w_{i1t}+\Delta w_{i1t+1})] &= \kappa_{c_2v_1}\sigma^2_{v_1} + \kappa_{c_2v_2}\sigma_{v_1v_2}\\
\mathbb{E}[\Delta c_{i2t}(\Delta w_{i2t-1}+\Delta w_{i2t}+\Delta w_{i2t+1})] &= \kappa_{c_2v_1}\sigma_{v_1v_2} + \kappa_{c_2v_2}\sigma^2_{v_2}
\end{aligned}$$

Suppose the wage parameters $(\sigma^2_{u_1}, \sigma^2_{u_2}, \sigma_{u_1u_2}, \sigma^2_{v_1}, \sigma^2_{v_2}, \sigma_{v_1v_2})$ have been identified. Then the first two moments identify the loading factors of transitory shocks $(\kappa_{c_2u_1}, \kappa_{c_2u_2})$, and the last two moments identify the loading factors of permanent shocks $(\kappa_{c_2v_1}, \kappa_{c_2v_2})$.

Identification of the remaining transmission coefficients follows the same logic. Note that the equations above serve to illustrate the idea of identification. More moment conditions are available and the model is over-identified.

Another thing worth mentioning is that, on the one hand, the PSID data are collected biennially since 1999; on the other hand, all variables (at least all the ones used in this paper) reported in the PSID are measured at the annual level. One has to be careful about this feature when deriving the empirical-relevant moment conditions. The full set of moment conditions in this context is presented in Appendix A.3.

2.3 Empirical Implementation

2.3.1 Data and Sample Selection

The main data I use in this paper is the Panel Study of Income Dynamics (PSID) for the years 2005-2013. The PSID data are collected biennially since 1999, so 5 waves are used: 2005, 2007, 2009, 2011, and 2013. Starting in 1999, in addition to income data and demographics, the PSID collects data about detailed assets holdings and consumption expenditures (at the household level), the latter of which cover many nondurable and services consumption categories, including utilities, health expenditure, transportation, education, and child care. A few more consumption categories, including clothing, trips and vacation, and other recreation, were added in 2005. As the imputation procedures relies on matching the clothing consumption, I use the waves since 2005.

I focus on the core (non-SEO) sample of the PSID. I select the households that are stably married over the sample period, with the male head aged between 25-60 and both partners are working.[28] I drop the households with missing information on key demographic variables (age, race, and education) and households with zero consumption. I also drop the households in which the hourly wage of the husband or of the wife is lower than half of the minimum wage. The remaining sample is an unbalanced panel of 9628 observations.[29]

I also use the 1998-2013 Consumer Expenditure Survey (CEX) data. The CEX provides a comprehensive flow of data on the buying habits of American consumers. The data are collected by the Bureau of Labor Statistics and used primarily for revising the CPI. The definition of the head of the household in the CEX is the person or one of the persons who owns or rents the unit; this definition is slightly different from the one adopted in the PSID, where the head is always the husband in a couple. I make the two definitions compatible by restricting the CEX sample to be male-head households.

[28]That is, I select the people who report positive annual working hours. People who are temporarily unemployed for less than a year remain in the sample. I focus on the couples who have at least some attachment to the labor market. While the extensive margin of labor supply adjustment is also important for family insurance, the non-parametric method for solving the household problem that I use in this paper assumes interior solutions only and is thus not suitable for analyzing the entering-and-exiting labor market choice. Incorporating the extensive margin requires one to impose more structure on the household behavior and is beyond the scope of the current paper.

[29]When calculating the relevant consumption, hourly wage and earnings moments, I do not use data displaying extreme "jumps" from one year to the next (most likely due to measurement error). A "jump" is defined as an extremely positive (negative) change from $t-2$ to t, followed by an extreme negative (positive) change from t to $t+2$. Formally, for each variable (say x), I construct the biennial log difference $\Delta^2 \log(x_t)$, and drop the relevant variables for observation in the bottom 0.1 percent of the product $\Delta^2 \log(x_t)\Delta^2 \log(x_{t-2})$.

The CEX is based on two components, the Diary survey and the Interview survey. The Diary survey is conducted over two consecutive one-week periods, designed to track detailed expenditures data on small and frequently purchased items, such as food, personal care, and household supplies. The Interview survey is conducted quarterly, and it covers about 95 percent of all household expenditure. My analysis below uses only the Interview sample, because it is more comprehensive and it can be used to construct annual expenditure for households. I apply the same sampling restrictions on the CEX as on the PSID: married households with the head being male and of age 25-60; both partners are working; and no missing information on key demographic characteristics.

To meet the data requirement of the model outlined in Section 2.2, I categorize and aggregate the consumption items in the PSID into private and public goods. The baseline categorization is that public consumption comprises food at home, rent, home insurance, health insurance, utilities, and child care, and all other items are considered private goods, such as clothing and apparel[30], telecommunications, food out, transportation, recreational goods, etc.

Finally, I also use the 1998-2013 series of Consumer Price Indexes, also collected by the U.S. Bureau of Labor Statistics, to deflate the nominal income, consumption, and assets.

2.3.2 Consumption Imputation

To identify the relative changes of bargaining power within families, I need information about how couples divide the resources, i.e. C_{ijt}, the private consumption of each partner across time. The PSID has rich dynamic information but only provides household-level consumption data. The CEX, on the other hand, has partial information on gender-specific consumption but is not a longitudinal data (follows the same family for only up to four quarters). I combine the strengths of these two datasets by imputing the gender-specific consumption in the CEX to the PSID.

Specifically, the CEX collects data on consumption of men's clothing and consumption of women's clothing.[31] The imputation utilizes the household and individual information that is commonly available in both the CEX and the PSID, and predicts the husband's and the wife's clothing expenditure separately for the married households in the PSID. Readers are referred to Pan (2017) for details about the imputation procedure.[32] I assume that the error

[30] Expenditure on children's clothing is categorized as public consumption. There might be concern that mothers care more (or less) about children's welfare than fathers do. In the current model, this difference can be (partially) accommodated by the different elasticities of public consumption between men and women. A more fundamental treatment of this issue is left for future study.

[31] Both are referred to adults clothing explicitly. There are additional categories for boys' (aged 2-15) clothing, for girls' (aged 2-15) clothing, and for clothing for children aged less than 2, separately.

[32] Browning et al., 1994 also makes use of data on clothing consumption to identify the sharing rule. A problem with using clothing is that some clothing expenditure is work-related and has nothing to do with

terms in the imputation regressions are not related to the intra-household allocations. That is, changes in intra-household allocations are mean-zero after controlling for the household and individual characteristics and household consumption.

2.3.3 Measurement Error

Earnings data and consumption data are subject to measurement errors. Ignoring the variance of measurement error in wages or earnings is problematic since it has a direct effect on the estimates of the structural parameters. Following Meghir and Pistaferri (2004), I use findings from validation studies to set *a priori* amount of wage or earning variability that can be attributed to error. I use the estimates of Bound et al. (1994), who estimate the share of variance associated with measurement error using a validation study for the PSID. Specifically, denoting the measurement error in variable x (in logs) by ξ_x, I set: $Var(\xi_w) = 0.13Var(w)$, $Var(\xi_y) = 0.04Var(y)$, $Var(\xi_h) = 0.23Var(h)$. These estimates can be used to correct all "own" moments (such as $E\left((\Delta y_{ijt})^2\right)$, $E\left(\Delta y_{ijt}\Delta y_{ijt+1}\right)$, etc.) with the only assumption (not entirely uncontroversial, see Bound and Krueger, 1991) that measurement error is not correlated over time. Cross moments (such as $E(\Delta w_{ijt}\Delta y_{ijt})$) involve the covariance between measurement errors in wages and in earnings. This covariance is non-zero by construction, since our wage measure is annual earnings divided by annual hours. This is the so-called "division bias". To correct for this, I write the relationship between measurement errors in log earnings, hours and wages as

$$Var(\xi_w) = Var(\xi_y) + Var(\xi_h) - 2Cov(\xi_y, \xi_h)$$

and given the variance of measurement errors in earnings, hours and wages, I can back out the covariance between measurement errors in wages and in earnings. Finally, for separable utility, log consumption is a martingale. Hence, the variance of the measurement error in consumption is directly identified from the moment $Var(\xi_c) = -E(\Delta c_{it}\Delta c_{it+1})$. I keep this exact identification also for the non-separable case.[33]

Using these estimates of measurement errors, I estimate the parameters using the moment conditions that are properly adjusted (See Appendix A.4).

2.3.4 Estimation Procedure

Here is a summary of my estimation procedure:

preference or bargaining. A man who gets promoted (a positive permanent shock) is likely to spend more money on suits, even if there is no bargaining. I make no attempt to address this issue in the current paper.

[33] In the case of non-separability between hours and consumption, $-E(\Delta c_{it}\Delta c_{it+1})$ gives the upper bound on the measurement error as long as the signs of κ_{cu_1} and κ_{cu_2} are the same, i.e. a transitory shock to the husband's wage affects consumption in the same direction as a transitory shock to the wife's wage does.

First, I impute the private consumption ($\hat{C}_{ijt}$) of each member in each household following Pan (2017).

Second, separately for the husbands and the wives, I regress the log of hourly wage, the log of annual labor income, and the log of imputed private consumption on observable characteristics and work with the residuals (the empirical counterparts of w_{ijt}, y_{ijt}, and c_{ijt}) in the following steps. The residual public consumption g_{ijt} is also obtained.

Third, I estimate the variances and covariances of the wage shocks ($\sigma^2_{u_j}$, $\sigma^2_{v_j}$, $\sigma_{u_1 u_2}$, and $\sigma_{v_1 v_2}$) by GMM using the wage moments.

Fourth, I estimate the smoothing parameters π_{it} and s_{it} using asset and (current and projected) earnings data.

Finally, given the estimates of the wage parameters and smoothing parameters, I estimate the transmission coefficients (κ's) and the underlying preference and bargaining parameters using the restrictions that the model imposes on the second order moments of Δc_{i1t}, Δc_{i2t}, Δg_{it}, Δy_{i1t} and Δy_{i2t}.

2.4 Results

2.4.1 Descriptive Statistics

Table 2.1 presents summary statistics for the PSID sample. The first panel reports the wages, hours worked, and earnings of males and females. Note that my sample is conditioning on working, so these are the conditional means of wages, hours, and earnings. Real wages for both males and females exhibit an inverse-U shape pattern across years, peaking at 2008 (the 2009 PSID actually reflects the situation in 2008 due to the retrospective nature of the survey) and then shrinking afterward. The average female labor supply changes very little (on the intensive margin); there is a slight drop in the male's hours worked during 2008-2010 but the change is minor. The earnings follow the same time pattern as the wages.

The second panel reports the average expenditure on different consumption categories. The public consumption items and the private consumption items are listed separately. Using this baseline categorization, the total public consumption accounts for roughly 59.2% of the household consumption on average. The time pattern of aggregate consumption is also interesting. Aggregate household consumption, whether private or public, starts to shrink from 2008, although wages and earnings have not decreased until 2010, suggesting that households have some forward-looking behaviors.

Table 2.1: Descriptive Statistics: Sample Means

	(1)	(2)	(3)	(4)	(5)	(6)
	2005-2013	2005	2007	2009	2011	2013
Panel A: Labor supply variables						
Male's hourly wage	30.13	29.4	29.47	31.63	30.58	29.54
Female's hourly wage	20.49	20.13	21.28	21.04	20.01	19.91
Male's hours worked	2245	2322	2317	2192	2174	2217
Female's hours worked	1671	1708	1650	1648	1665	1682
Male's earnings	66669	66206	67870	69118	64228	65760
Female's earnings	33859	33419	33320	34771	33890	33897
Panel B: Consumption variables						
Public consumption items						
Food at home expenditure	6044	6171	6119	5796	6112	6030
Rent or rent equivalent	13813	14963	15828	13295	12749	12105
Home insurance expenditure	734	723	750	712	749	738
Utility expenditure	2797	2655	2757	2860	2946	2766
Health insurance expenditure	2139	1899	1987	2033	2034	2770
Childcare expenditure	905	800	876	878	979	998
Total public consumption	26432	27211	28316	25574	25569	25407
Private consumption items						
Food out expenditure	2388	2581	2474	2203	2341	2342
Gasoline expenditure	2884	2609	3069	2327	3271	3169
Transportation (exc. gas)	3650	3891	3758	3474	3586	3538
Clothing expenditure	1808	2170	2053	1710	1619	1465
Education expenditure	2445	2525	2524	2289	2379	2511
Health care expenditure	1597	1445	1567	1614	1562	1806
Trips expenditure	2260	2297	2393	2218	2226	2160
Other recreation expenditure	1168	1257	1260	1172	1118	1022
Total private consumption	18200	18776	19097	17007	18103	18013
Observations	9628	1920	1983	1979	1895	1851

Notes: Data from the 2005-2013 PSID. Sample means of the variables are reported. All variables are annual (except for hourly wages). All wage, earnings, and consumption variables are expressed in dollars and deflated by the CPI index (base year is 2005).

Table 2.2: Estimates of Wage Parameters

		Estimate
Males	Trans. $\sigma^2_{u_1}$	.0266***
		(.0087)
	Perm. $\sigma^2_{v_1}$	.0391***
		(.0061)
Females	Trans. $\sigma^2_{u_2}$	.0180***
		(.0070)
	Perm. $\sigma^2_{v_2}$	.0503***
		(.0060)
Spousal Covariance	Trans. σ_{u_1,u_2}	.0052
		(.0038)
	Perm. σ_{v_1,v_2}	.0015
		(.0027)

Notes: Standard errors in parentheses are clustered at the household level. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

2.4.2 Wage Parameters

Table 2.2 reports the estimates of the wage variances and covariances. A few things are worth noting.

First, there is some evidence of "wage instability" (see Gottschalk and Moffitt, 2008) both for males and for females, as can be seen from the variances of the transitory components, and it is larger for males. Second, the variance of the more structural component (the variance of permanent shocks), in contrast, is larger for females, perhaps reflecting greater dispersion in the returns to unobserved skills, etc. Finally, neither the transitory components or the permanent components of the two spouses are significantly correlated.

In Appendix A.1, I also compare these estimates for 2005-2013 with those for 1999-2009 (which is the sample period used in BPS). On the one hand, for both men and women, the variances of the transitory components increase slightly in the later period, if any. On the other hand, the variances of the permanent shocks increase, from 0.032 to 0.039 for males and from 0.039 to 0.050 for females, perhaps reflecting the greater risks due to the Great Recession.

2.4.3 Consumption, Labor Supply, and Bargaining Parameters

Table 2.3 reports the estimates of gender-specific consumption and labor supply Frisch elasticities and bargaining parameters.

Some results are worth noting. First, I find an estimate of the consumption Frisch elasticity of $\eta_{c_1,p}$ around 0.41-0.46 for males, implying a relative risk aversion of around 2.3,

Table 2.3: Parameter Estimates

	(1)	(2)
	Unitary	Non-Unitary
Frisch elasticities		
$\eta_{c_1,p}$	.4139***	.4610***
	(.0722)	(.0519)
$\eta_{c_2,p}$	.3313***	.3622***
	(.0415)	(.0266)
η_{c_1,w_1}	-.0082	-.0233
	(.0209)	(.0340)
η_{c_2,w_2}	.0286	-.0126
	(.0174)	(.0084)
$\eta_{g,p}$	.1307***	.1622***
	(.0270)	(.0113)
η_{g,w_1}	-.0131	-.0189
	(.0481)	(.0261)
η_{g,w_2}	.0207	.0131*
	(.0131)	(.0060)
$\eta_{h_1,p}$	.0114**	.0120***
	(.0042)	(.0022)
$\eta_{h_2,p}$	.0342	.0421
	(.0384)	(.0271)
η_{h_1,w_1}	.9122***	.8630***
	(.0688)	(.0544)
η_{h_2,w_2}	1.1203***	1.0010***
	(.0743)	(.1025)
η_{h_1,w_2}	.2320***	.1072***
	(.0189)	(.0030)
η_{h_2,w_1}	.3912***	.1413***
	(.0455)	(.0202)
bargaining parameters		
η_{μ,w_1}	0	.2301***
	(n/a)	(.0067)
η_{μ,w_2}	0	-.3130***
	(n/a)	(.0101)

Notes: Standard errors in parentheses are clustered at the household level. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

which is in the plausible range of this parameter. The female consumption Frisch elasticity $\eta_{c_2,p}$ is lower, implying a higher relative risk aversion for females. Second, the Frisch labor supply elasticity of males is smaller than that for females, which is consistent with intuition and previous findings in the literature. In particular, I estimate η_{h_1,w_1} being around 0.86-0.91 for males. Keane (2011) surveys 12 influential studies and reports a range of 0.03-2.75 and an average estimate of 0.85.[34] For females, I estimate $\eta_{h_2,w_2} \approx 1$, which is similar to the estimate reported by Blundell et al. (2015) and Heckman and Macurdy (1980). Third, the elasticities of Pareto weights with respect to individual wages are significantly different from zero, which rejects the assumption of full commitment or unitary decision making. Empirically, *ceteris paribus*, if the husband's wage increase by 10%, the Pareto weight on the husband would increase by 2.3%; if the wife's wage increase by 10%, the Pareto weight on the husband would decrease by 3.1%. Finally, estimates of cross elasticities of labor supply are smaller in the non-unitary model, both for the husband and the wife. This suggests a weaker "added worker effect" once I allow for intra-household bargaining, which is consistent with the intuitive argument in the introduction of the paper.

2.4.4 Transmission of Wage Shocks to Consumption and Labor supply

Table 2.4 reports the estimates of transmission coefficients in (2.12). Note that in general, these transmission coefficients are heterogeneous across households and time. The values reported here are the sample averages.

A few results are worth noting. First, the signs of the transmission coefficients are mostly the same as in the unitary model and in the non-unitary model. Second, for the transmission to consumption, a positive permanent wage shock to the husband increases the husband's consumption (reflected by $\kappa_{c_1 v_1}$ in the table) in both models, but more so in the non-unitary case; the same shock leads to an increase in the wife's consumption ($\kappa_{c_2 v_1}$) as well, but less so in the non-unitary case. A symmetric pattern is found for a permanent shock to the wife's wage ($\kappa_{c_1 v_2}, \kappa_{c_2 v_2}$). This suggests that the permanent shocks to individual wages impact the intra-household allocation, consistent with the hypothesis that bargaining power changes as the spouses experience individual shocks. Third, for the labor supply response, I find some evidence for "added worker effects" in both cases ($\kappa_{h_1 v_2} < 0$ and $\kappa_{h_2 v_1} < 0$ in both columns), and it is stronger when the shocks come from the husband and the wife increases her labor supply ($|\kappa_{h_2 v_1}| > |\kappa_{h_2 v_1}|$). However, these effects become much weaker in the non-unitary case, that is, the spousal labor supply does not increase as much as in the unitary case when an adverse shock hits. Fourth, a negative shock to the wife's wage induces a decrease in her own labor supply ($\kappa_{h_2 v_2}$) in both models, but the decrease is smaller in the non-unitary case,

[34] Studies in the 1980s and 1990s typically find a Frisch elasticity close to zero. Studies after 2000 typically find larger estimates.

Table 2.4: Average Estimates of Transmission Coefficients

	(1)	(2)
	Unitary	Non-Unitary
$\kappa_{c_1u_1}$	-.0083	.0024
$\kappa_{c_1u_2}$	-.0026	-.0018
$\kappa_{c_1v_1}$	.2957	.3216
$\kappa_{c_1v_2}$	.1560	.1245
$\kappa_{c_2u_1}$	-.0103	-.0076
$\kappa_{c_2u_2}$	-.0091	-.0001
$\kappa_{c_2v_1}$	.3226	.2693
$\kappa_{c_2v_2}$	.1833	.2470
κ_{gu_1}	.0315	.0412
κ_{gu_2}	.0726	.0504
κ_{gv_1}	.3163	.3738
κ_{gv_2}	.1967	.2126
$\kappa_{h_1u_1}$	.9122	.8401
$\kappa_{h_1u_2}$	.2320	.2177
$\kappa_{h_1v_1}$	-.0043	.0323
$\kappa_{h_1v_2}$	-.2880	-.1792
$\kappa_{h_2u_1}$	.3912	.2500
$\kappa_{h_2u_2}$	1.1203	1.0329
$\kappa_{h_2v_1}$	-.7312	-.5715
$\kappa_{h_2v_2}$	.4205	.3683

Notes: κ's are heterogeneous across households. The numbers reported here are the average values in the sample.

suggesting that the bargaining power shifts to the husband following such a shock. Fifth, the response of labor supply to transitory shocks is larger than the response to permanent shocks (e.g. $\kappa_{h_1 u_1} > \kappa_{h_1 v_1}$), for both husband and wife, which is consistent with the finding in the literature and intuition. Finally, the transmissions of transitory shocks to consumption and to labor supply are different between the unitary case and the non-unitary case, but the discrepancies are not as large as those of permanent shocks.

We can use a numerical example to illustrate how these transmission coefficients reflect the behavioral responses of an *average* American household to various shocks. Consider a hypothetical family in which the couple earn the US average wages, work the average hours, and have the average consumption. (See the descriptive statistics in Table 2.1) In the PSID sample, the average hourly wage of married men is 30.13 dollars. Consider the scenario in which the husband suffers a permanent wage loss of 3 dollars per hour, which is a roughly 10% negative shock. According to the estimated transmission coefficients for the unitary model, this would induce 7.3% increase in the wife's hours work, whereas the change would be only 5.7% under the estimation for the non-unitary model. For the family considered, these translate into an increase of 122 hours per year if it is unitary and 94 if non-unitary. In both cases, however, the husband barely adjusts his labor supply. The couple adjusts their individual consumption differently between the two settings as well. The 10% permanent wage loss lowers the husband's consumption by 2.9%, or $215 per year, if we use the estimates obtained under the unitary specification and 3.2% ($234 per year) under the non-unitary one. Wife's consumption moves even more differently: she cuts down her own expenditure by $352 based on the unitary estimates, but only $293 based on the non-unitary estimates. These suggest that bargaining impact the intra-household allocation in the direction that the permanent wage loss to the husband hurts the husband's bargaining position, and thus *relatively* increase the wife's consumption and leisure. The behavioral responses in this scenario can be summarized in Table 2.5.

Next, I consider the same size of wage shock hits the wife in this hypothetical family: the wife's wage drops by 3 dollars per week permanently, which is roughly a 15% shock. (Married women's mean wage is $20.49 a week.) By similar calculations, the family adjusts the individual labor supply and consumption as in Table 2.6. Again, the added-worker effect is much weaker in the non-unitary case: the husband works 97 hours more a year in the unitary case but only 60 hours more in the non-unitary case. The difference is roughly one full-time week per year. When it comes to consumption, again the one who suffers the wage loss — in this case, the wife — suffers a greater loss of consumption in the non-unitary setting, and the spouse's welfare is hurt relatively less.

Table 2.5: Behavioral Responses to -10% Permanent Wage Shock to Husband

		Unitary	Non-Unitary
Labor Supply Response (in hours/year)	Husband	≈ 0	-7
	Wife	+122	+94
Consumption Response (in dollars/year)	Husband	-215	-234
	Wife	-352	-293

Table 2.6: Behavioral Responses to -10% Permanent Wage Shock to Wife

		Unitary	Non-Unitary
Labor Supply Response (in hours/year)	Husband	+97	+60
	Wife	-105	-92
Consumption Response (in dollars/year)	Husband	-170	-130
	Wife	-299	-405

Note: An numerical example of an "average" American household's response to wage shocks. Numbers are calculated based on the estimated transmission coefficients reported in Table 2.4.

2.4.5 Insurance Accounting

I now use the estimates of the transmission coefficients to understand the importance of various sources of insurance available to households.

First of all, to be consistent with the discussion in the literature, it is useful to calculate the shock transmission to household consumption, which in my framework can be approximated by

$$\Delta c \approx \frac{C_1}{C}\Delta c_1 + \frac{C_2}{C}\Delta c_2 + \frac{G}{C}\Delta g$$

Therefore, the transmission of, say, a permanent wage shock to the husband, to household-level consumption κ_{cv_1} can be approximated by $\frac{C_1}{C}\kappa_{c_1v_1} + \frac{C_2}{C}\kappa_{c_2v_1} + \frac{G}{C}\kappa_{gv_1}$, a weighted sum of the private consumption parameters and public consumption parameters. In the imputed data, the male's share in consumption is 16.8%, the female's share is 22.1%, and the public share 62.0%. Thus, in the non-unitary model, the transmission is $\kappa_{cv_1} \approx 0.3453$, which is slightly greater than the estimate in the unitary model (0.3171). For a permanent wage shock to the wife, the transmission to the household-level consumption $\kappa_{cv_2} \approx 0.2073$ in the non-unitary case and 0.1887 in the unitary one. In either case, the consumption is very smoothed even to permanent shocks, and even more so in the unitary case, i.e., the households achieve more risk sharing in the unitary case.

How about the relative contribution of different sources of insurance? Starting from the intertemporal budget constraint,

$$C = Y - S, \tag{2.14}$$

I decompose the response of household consumption growth to a permanent wage shock faced by the primary earner as: [35]

$$\frac{\partial \Delta c}{\partial v_1} \approx \frac{\partial \Delta y}{\partial v_1} - \frac{\partial \Delta (S/Y)}{\partial v_1}, \tag{2.15}$$

where S/Y is the average propensity to save out of family earnings. Thus, the first term on the right-hand side represents the extent of insurance achieved via family labor supply and the second term represents the insurance achieved through asset accumulation.

The first term, the response of household earnings to a permanent shock to the males hourly wage, can be decomposed as follows:

$$\frac{\partial \Delta y}{\partial v_1} \approx \omega \frac{\partial \Delta y_1}{\partial v_1} + (1-\omega) \frac{\partial \Delta y_2}{\partial v_1} \tag{2.16}$$

where $\omega = Y_1/(Y_1 + Y_2)$ is the male's share in household total earnings.

From the previous calculation based on the non-unitary estimates, a 10% permanent decrease in the husband's wage rate ($v_1 = -0.1$) induces a -3.5% change in household total consumption.

The response of consumption can be decomposed into several steps. Consider a case in which there is only one earner ($\omega = 1$), labor supply is fixed ($\frac{\partial \Delta h_1}{\partial v_1} = 0$), and there is no self-insurance through savings. Then $\frac{\partial \Delta c}{\partial v_1} = 1$ and consumption responds one-to-one to permanent shocks in hourly wages. Now if we bring in the second earner, the wife, but still assuming fixed labor supply and no savings, household earnings would fall by 7.0% ($\omega = 0.7$ in the data) and the fall in consumption is of the same magnitude given the absence of self-insurance through savings and labor supply behavioral responses.

The introduction of behavioral responses changes the picture slightly further. Assume, for example, that males can vary their labor supply (while keeping female labor supply exogenous). Since the husband's Marshallian elasticity is almost zero ($\hat{\kappa}_{h_1 v_1} = 0.0323$), $\frac{\partial \Delta c}{\partial v_1} = \frac{\partial \Delta y}{\partial v_1} = \omega \hat{\kappa}_{y_1 v_1} = 0.72$, almost the same as the case above. Allowing for added worker effects reduces the impact of a 10% decline in male permanent shock on consumption to only 6.3% ($\frac{\partial \Delta c}{\partial v_1} = \frac{\partial \Delta y}{\partial v_1} = \omega \hat{\kappa}_{y_1 v_1} + (1-\omega)\hat{\kappa}_{y_2 v_1} = 0.54$). Finally, with all insurance channels active, the fall in household earnings is still 5.4%, but the fall in consumption is greatly attenuated

[35] To derive this, first I take logs of both sides of (2.14), and then take first difference: $\Delta \log(C) = \Delta \log(Y - S) = \Delta \log(Y) + \Delta \log(1 - S/Y)$. And then use the approximation: $\log(1 - S/Y) \approx -S/Y$.

to 3.5%. In other words, I use (2.15) to calculate that, of the 35 percentage points (p.p.) of consumption "insured" against the shock to the males wage[36], 16 p.p. (45.7% of the total insurance effect) come from family labor supply adjustment and 19 p.p. (54.3%) come from self-insurance through borrowing and saving.

And how do the estimates from the unitary model and from the non-unitary model imply differently for insurance? Using my estimates of the unitary model, following the same decomposition approach as above, I calculate that 38% of consumption is insured when there is a permanent shock to the husband's wage. Out of 38 p.p of consumption insured against the shock to the male's wage, 23 p.p. (60.5% of the total insurance effect) come from family labor supply adjustment, and the remaining 15 p.p. (39.5%) come from self-insurance through credit markets. Therefore, although the total insurance achieved by the household estimated does not change much with and without intra-household bargaining, the *relative* importance of different channels of insurance does change. Allowing for intra-household bargaining lowers contribution of insurance from family labor supply (45.7% versus 60.5%) to consumption smoothing.

Finally, I discuss the consumption smoothing against a permanent wage shock to the wife. Based on the estimates for the unitary case, 12% of consumption is insured when there is a permanent shock to the wife's wage. Out of the 12 p.p., 7 p.p. (58.3% of the total insurance effect) come from family labor supply adjustment, and the remaining 5 p.p. (41.7%) come from self-insurance through credit markets. By contract, using the transmission coefficients estimated in non-unitary model, 10% of consumption in insured; And within this 10 p.p., only 3 p.p. (30% of the total insurance effect) come from family labor supply adjustment and the 7 p.p. (70%) come from self-insurance through credit markets. Again, the contribution from family labor supply channel decreases once intra-household bargaining is allowed.

2.5 Conclusion

This paper investigates how households insure themselves against idiosyncratic wage shocks and how this insurance interacts with intra-household bargaining. I merge information from the CEX and the PSID using an imputation procedure to obtain a panel data of households on individual consumption expenditures, income, and labor supply. Using a collective household model, I derive analytical equations describing how wage shocks are transmitted to consumption and labor supply, and how the transmission mechanism depends on preference parameters and bargaining parameters. The model is identified and estimated using the merged panel data. I find that intra-household allocations of expenditures and leisure

[36]The 36 p.p. figure is derived from the difference between the response of consumption with savings and family labor supply responses (a 3.5% decline) and without these (a 7.0% decline).

respond strongly to individual wage shocks, and the same shocks can have very different effects depending on whose income they perturb within a household.

The non-unitary approach has several interesting implications for the household members' behavioral responses. A permanent decline in the husband's wage induces an increase in the wife's labor supply in both the unitary and non-unitary specifications, but the increase is smaller in the non-unitary case. In particular, a 10% permanent decline in the husband's wage increases the wife's labor supply by 7.3% in the unitary model, but only 5.7% in the non-unitary model. Moreover, the husband's labor supply also increases following a negative permanent shock to the wife's wage but the increase is smaller when allowing for intra-household bargaining. A 10% permanent decline in the wife's wage increases the husband's labor supply by 2.9% in the unitary model versus 1.8% in the non-unitary model.

Individual consumption expenditures also respond to the individual wage shocks differently in the two model specifications. For example, a negative permanent shock to the husband's wage decreases his own consumption more and decreases his wife's consumption less in the non-unitary model relative to the unitary one. In terms of consumption smoothing, the overall insurance is not significantly altered, but the contribution of the family labor supply channel decreases from 60.5% in the unitary model to 45.7% in the non-unitary model when the shock hits the husband's wage; and from 58.3% to 30% when the shock hits the wife's wage.

These differences are fairly substantial and suggest that removing the restrictions implicit in the unitary model is of critical importance to make sense of observed changes in households' behavior, and to quantify the extent to which different channels contribute to consumption smoothing in the face of wage uncertainty.

Chapter 3

Markov-Chain Approximations for Life-Cycle Models

3.1 Introduction

In quantitative macroeconomic studies it is often necessary to approximate continuous stochastic processes using discrete state-space representations; e.g. Markov chains. Different methods are available to perform such approximations.[37] The properties of alternative discretization methods to approximate covariance-stationary AR(1) processes in the context of stationary infinite horizon problems have been studied in some detail by Kopecky and Suen (2010). They find that: (a) the choice of discretization method may have a significant impact on the model simulated moments; (b) the performance of Rouwenhorst's (1995) method is more robust, particularly for highly persistent processes.

While a covariance-stationary income process is convenient, it is not consistent with the fact, first highlighted by Deaton and Paxson (1994), that within-cohort income inequality increases with the age of a cohort. For this reason, most quantitative life-cycle analyses of consumption and income dynamics assume a non-stationary labor income process whose variance increases with age.[38] As a result, the difficulty of accurately approximating the income process with a small number of discrete states increases with age.

We show how to extend both Tauchen's (1986) and Rouwenhorst's (1995) methods to discretize non-stationary AR(1) processes and compare their respective performance within the context of a life-cycle, income-fluctuation problem. Both extensions keep the number of states in each time period constant, but they allow the state vector and transition matrix to change over time. In both cases, some property of the original stationary counterpart are

[37]The seminal contributions are Tauchen (1986), Tauchen and Hussey (1991) and Rouwenhorst (1995). Adda and Cooper (2003), Flodén (2008) and Kopecky and Suen (2010) introduce improvements for stationary, univariate, AR(1) processes. Markov-chain approximations for stationary, vector autoregressive processes have been proposed by Galindev and Lkhagvasuren (2010), Terry and Knotek (2011) and Gospodinov and Lkhagvasuren (2014). Farmer and Toda (2016) propose a method that can be applied to stationary, non-linear, multivariate processes.

[38]Non-stationarity in the income process can take the form of distributional assumptions on the initial conditions as in Huggett (1996), a unit root component as in Storesletten et al. (2004), or heteroskedasticity of the innovations as in Kaplan (2012).

preserved: Tauchen's method matches the transition probabilities implied by the normality assumption, while Rouwenhorst's method matches the conditional and unconditional first and second moments of the original process.

We evaluate the performance of both methods in the context of a finite-horizon income-fluctuation problem with a unit-root income process with normal innovations.[39] We find that Rouwenhorst's method performs extremely well even with a relatively small number of grid-points.

Our paper is related to several studies (see, among others, those listed in footnote 1). However, to the best of our knowledge, it is the first one to formally study the approximation of non-stationary AR(1) processes. Papers studying quantitative life-cycle problems with non-stationary stochastic processes have typically approximated those processes using a variety of intuitively appealing approaches. Storesletten et al. (2004) use a binomial tree, Huggett (1996) uses a variant of Tauchen discretization with a different conditional distribution at the initial age, Kaplan (2012) uses an age-varying, equally-spaced grid with range and transition probabilities chosen to match some moments of the original continuous process. In most cases these methods are only partially documented, hence we know very little about their performance. Our work is meant to provide a more systematic treatment of this approximation problem.

The remainder of this paper is structured as follows. Section 3.2 discusses how to extend Tauchen's (1986) and Rouwenhorst's (1995) methods to non-stationary AR(1) processes. Section 3.3 compares the accuracy of the two methods. Section 3.4 concludes.

3.2 Discrete Approximations of AR(1) Processes

Consider an AR(1) process of the following form,

$$y_t = \rho_t y_{t-1} + \varepsilon_t, \quad \varepsilon_t \overset{id}{\sim} N(0, \sigma_{\varepsilon t}) \tag{3.1}$$

with initial condition y_0, where y_0 can be deterministic or a random draw from some distribution. Let σ_t denote the unconditional standard deviation of y_t. It follows from equation (3.1) that

$$\sigma_t^2 = \rho_t^2 \sigma_{t-1}^2 + \sigma_{\varepsilon t}^2. \tag{3.2}$$

In general the above process is not covariance-stationary. Sufficient conditions for sta-

[39] As we discuss in the main text, the advantage of using such a process for our benchmark is that the associated optimization problem can be solved using extremely accurate numerical techniques.

tionarity are that the process in equation (3.1) is restricted to

$$y_t = \rho y_{t-1} + \varepsilon_t, \quad |\rho| < 1, \ \varepsilon_t \overset{iid}{\sim} N(0, \sigma_\varepsilon) \tag{3.3}$$

with constant persistence ρ, constant innovation variance σ_ε and y_0 randomly drawn from the asymptotic distribution of y_t; namely, $N(0, \sigma)$ where $\sigma = \sigma_\varepsilon / \sqrt{1 - \rho^2}$. We call this case the stationary case in what follows, to distinguish it from the general, unrestricted process[40] in equation (3.1).

The aim of these notes is to show how to adapt both Tauchen (1986) and Rouwenhorst (1995) methods to discretize a *non-stationary* AR(1) of the general form in equation (3.1).

3.2.1 Tauchen's (1986) Method

Stationary case

Tauchen (1986) proposes the following method to discretize a stationary AR(1) process. Construct a Markov chain with a time-independent, uniformly-spaced state space $Y^N = \{\bar{y}^1, \ldots, \bar{y}^N\}$ with

$$\bar{y}^N = -\bar{y}^1 = \Omega\sigma \tag{3.4}$$

where Ω is a positive constant.[41] If Φ denotes the cumulative distribution function for the standard normal distribution and $h = 2\Omega\sigma/(N-1)$ the step size between grid points, the elements of the transition matrix Π^N satisfy

$$\pi^{ij} = \begin{cases} \Phi\left(\frac{\bar{y}^j - \rho\bar{y}^i + h/2}{\sigma_\varepsilon}\right) & \text{if } j = 1, \\ \Phi\left(\frac{\bar{y}^j - \rho\bar{y}^i - h/2}{\sigma_\varepsilon}\right) & \text{if } j = N, \\ \Phi\left(\frac{\bar{y}^j - \rho\bar{y}^i + h/2}{\sigma_\varepsilon}\right) - \Phi\left(\frac{\bar{y}^j - \rho\bar{y}^i - h/2}{\sigma_\varepsilon}\right) & \text{otherwise.} \end{cases}$$

Basically, the method constructs the transition probabilities π_{ij} to equal the probability (truncated at the extremes) that y_t falls in the interval $(\bar{y}^j - h/2, \bar{y}^j + h/2)$ conditionally on $y_{t-1} = \bar{y}^i$.

[40]Note that the general process does not restrict ρ_t to lie inside the unit circle.

[41]Tauchen (1986) sets $\Omega = 3$. Kopecky and Suen (2010) calibrate it so that the standard deviation of the Markov chain coincides with that of the original AR(1) process.

Non-stationary case

Our non-stationary extension of Tauchen (1986) constructs a state space $Y_t^N = \{\bar{y}_t^1, \ldots, \bar{y}_t^N\}$ with constant size N, but time-varying grid-points with

$$\bar{y}_t^N = -\bar{y}_t^1 = \Omega\sigma_t \tag{3.5}$$

and step size $h_t = 2\Omega\sigma_t/(N-1)$. The associated transition probabilities are

$$\pi_t^{ij} = \begin{cases} \Phi\left(\frac{\bar{y}_t^j - \rho\bar{y}_{t-1}^i + h_t/2}{\sigma_{\varepsilon t}}\right) & \text{if } j = 1, \\ \Phi\left(\frac{\bar{y}_t^j - \rho\bar{y}_{t-1}^i - h_t/2}{\sigma_{\varepsilon t}}\right) & \text{if } j = N, \\ \Phi\left(\frac{\bar{y}_t^j - \rho\bar{y}_{t-1}^i + h_t/2}{\sigma_{\varepsilon t}}\right) - \Phi\left(\frac{\bar{y}_t^j - \rho\bar{y}_{t-1}^i - h_t/2}{\sigma_{\varepsilon t}}\right) & \text{otherwise.} \end{cases}$$

The main difference between our extension and its stationary counterpart is that the range of the equidistant state space in equation (3.5) is time varying and, as a result, so are the transition probabilities.

3.2.2 Rouwenhorst's (1995) Method

The Rouwenhorst method is best understood as determining the parameters of a two-state Markov chain, with equally-spaced state space, in such a way that the conditional first and second moments of the Markov chain coincide with the same moments of the original AR(1) process.[42]

Stationary case

In the case of the stationary AR(1) process in equation (3.3), the state space for the two-state Markov chain is $\bar{y}^2 = -\bar{y}^1$ and the transition matrix is written as

$$\Pi^2 = \begin{bmatrix} \pi^{11} & 1-\pi^{11} \\ 1-\pi^{22} & \pi^{22} \end{bmatrix}. \tag{3.6}$$

The moment condition for the expectation conditional on $y_{t-1} = \bar{y}^2$ is

$$\mathrm{E}(y_t | y_{t-1} = \bar{y}^2) = -(1-\pi^{22})\bar{y}^2 + \pi^{22}\bar{y}^2 = \rho\bar{y}^2, \tag{3.7}$$

[42] In general, a Markov chain of order K is characterized by K^2 parameters (K states plus $(K^2 - K)$ linearly-independent transition probabilities) and can be uniquely identified by K^2 linearly-independent moment conditions. The Rouwenhorst method is, therefore, a special case of a general moment-matching procedure.

where the left hand side is the conditional expectation of the Markov chain and the right hand side its counterpart for the AR(1) process for y_{t-1} evaluated at the grid point $\bar{y}^2$. It follows that

$$\pi^{22} = \frac{1+\rho}{2} = \pi^{11}, \tag{3.8}$$

where the second equality follows from imposing the same condition for $y_{t-1} = \bar{y}^1 = -\bar{y}^2$.

The moment condition for the variance conditional on $y_{t-1} = \bar{y}^2$ is[43]

$$\text{Var}(y_t | y_{t-1} = \bar{y}^2) = (1-\pi^{22}) \left(-\bar{y}^2 - \rho\bar{y}^2\right)^2 + \pi^{22} \left(\bar{y}^2 - \rho\bar{y}^2\right)^2 = \sigma_\varepsilon^2, \tag{3.9}$$

which, after replacing for π^{22} from equation (3.8), implies

$$\bar{y}^2 = \sigma. \tag{3.10}$$

Having determined Π^2, the method scales to an arbitrary number of grid points N in the following way.[44] The state space $Y^N = \{\bar{y}^1, \ldots, \bar{y}^N\}$ is equally-spaced with

$$\bar{y}^N = -\bar{y}^1 = \sigma\sqrt{N-1}. \tag{3.11}$$

For $N \geq 3$, the transition matrix satisfies the recursion

$$\Pi^N = \pi \begin{bmatrix} \Pi^{N-1} & \mathbf{0} \\ \mathbf{0}' & 0 \end{bmatrix} + (1-\pi) \begin{bmatrix} \mathbf{0} & \Pi^{N-1} \\ 0 & \mathbf{0}' \end{bmatrix} + \pi \begin{bmatrix} 0 & \mathbf{0}' \\ \mathbf{0} & \Pi^{N-1} \end{bmatrix} + (1-\pi) \begin{bmatrix} \mathbf{0}' & 0 \\ \Pi^{N-1} & \mathbf{0} \end{bmatrix}, \tag{3.12}$$

where $\pi = \pi^{11} = \pi^{22}$ and $\mathbf{0}$ is an $(N-1)$ column vector of zeros.

The main difference between Rouwenhorst's and Tauchen's methods is that in the former the transition probabilities do not embody the normality assumption about the distribution of the shocks. Rather, Rouwenhorst matches exactly, by construction, the first and second conditional and, by the law of iterated expectations, unconditional moments of the continuous process independently from the shock distribution.

Non-stationary case

As for Tauchen, our non-stationary extension of Rouwenhorst (1995) constructs an equally-spaced, symmetric, state space $Y_t^N = \{\bar{y}_t^1, \ldots, \bar{y}_t^N\}$ with constant size N but time varying grid points and transition matrix Π_t^N. If $N = 2$, it follows that $\bar{y}_t^2 = -\bar{y}_t^1$ and the counterpart

[43] Symmetry implies that the second conditional-variance condition is linearly dependent with equation (3.9) and, therefore, satisfied.

[44] We refer the reader to Rouwenhorst (1995) and Kopecky and Suen (2010) for a rigorous derivation.

of the first-moment condition (3.7) becomes

$$\mathrm{E}(y_t|y_{t-1} = \bar{y}^2_{t-1}) = -(1-\pi^{22}_t)\bar{y}^2_t + \pi^{22}_t\bar{y}^2_t = \rho_t\bar{y}^2_{t-1},$$

with unique solution

$$\pi^{22}_t = \frac{1}{2}\left(1+\rho_t\frac{\bar{y}^2_{t-1}}{\bar{y}^2_t}\right) = \frac{1}{2}\left(1+\rho_t\frac{\sigma_{t-1}}{\sigma_t}\right) = \pi^{11}_t, \tag{3.13}$$

where the second equality follows from the counterpart of the second moment condition (3.9) which implies

$$\bar{y}^2_t = -\bar{y}^1_t = \sigma_t. \tag{3.14}$$

The third equality in equation (3.13) follows from the expression for the conditional first moment for $y_{t-1} = \bar{y}_{t-1}$.

As in the non-stationary version of Tauchen, the points of the state-space are a function of the time-dependent unconditional variance of y_t. Comparing equations (3.8) and (3.13) reveals that, relative to the stationary case, the probability of transiting from $\bar{y}^2_{t-1}$ to $\bar{y}^2_t$ depends on the rate of growth of the unconditional variance of y_t.

Equation (3.13) implies that the condition for the Markov chain to be well defined, and have no absorbing states, namely $0 < \pi^{11}_t = \pi^{22}_t < 1$, is equivalent to

$$\rho^2_t\frac{\sigma^2_{t-1}}{\sigma^2_t} < 1. \tag{3.15}$$

It follows from equation (3.2) that this condition always holds. Therefore Rouwenhorst's approximation can be applied to any process of the type defined in equation (3.1).[45]

As in the stationary case, the approach scales to an N-dimensional, evenly-spaced state space Y^N_t by setting

$$\bar{y}^N_t = -\bar{y}^1_t = \sigma_t\sqrt{N-1} \tag{3.16}$$

and Π^N_t to satisfy the recursion (3.12) with the transition matrices and the probability $\pi_t = \pi^{11}_t = \pi^{22}_t$ indexed by t.

3.3 Evaluation

This section assesses the performance of the two discretization methods above in solving a finite-horizon, income-fluctuation problem with a non-stationary labor income process.

[45]This is also trivially true for Tauchen's method.

Consider the following optimization problem in recursive form[46]

$$
\begin{aligned}
V_t(z_t, y_t) = \max_{c_t, a_t} & \log(c_t) + \beta \mathbb{E}_t V_{t+1}(z_{t+1}, y_{t+1}) \\
\text{s.t. } & z_t = (1+r)a_{t-1} + y_t \\
& a_t = z_t - c_t \\
& y_{t+1} = y_t \epsilon_t, \quad \log \epsilon_t \overset{i.i.d.}{\sim} N(0, \sigma_\varepsilon), \\
& a_t \geq 0, \quad a_t \text{ given.}
\end{aligned}
\tag{3.17}
$$

Individuals start life at age 1, with initial wealth $a_0 = 0$ and $y_0 = 1$, and live until age 40. Each model period is a year. In the computation we set the discount rate β to 0.96 and the interest rate r to .04 which are standard values. We set the variance of the labor income process $\sigma_\varepsilon^2 = .0161$, as in Storesletten et al. (2004). The parameterization implies an aggregate wealth-income ratio of about 0.6, in line with the baseline calibration in Carroll (2009) for a similar model with no retirement and deterministic lifetime.

Since the above problem does not have a closed-form solution, we evaluate the accuracy of the two discretization methods by comparing simulated moments under the two methods to those generated by a very accurate benchmark solution.

The advantage of problem (3.17) is that, as first shown in Carroll (2004), the combination of unit-root (in logs) income process and CRRA felicity function implies that the problem can be normalized using (permanent) labor income y_t, thereby reducing the effective state space to the single variable $\hat{z}_t = z_t/y_t$.[47]. It follows that, under the assumptions that income innovations are log-normally distributed, one can approximate the expectation in equation (3.17) using Gaussian-Hermite quadrature.

This allows one to solve the model using a very accurate procedure—the endogenous grid-point method—for the optimization step[48] and Gaussian-Hermite quadrature to approximate the expectation in (3.17). In particular, we compute the policy functions using an exponential grid G_z with 1,000 points for the normalized state variable $\hat{z}$ and 100 quadrature nodes for the shock $\log \varepsilon_t$. Given the well-known properties of quadrature,[49] the model solution using the endogenous gridpoint method and quadrature is extremely accurate.

We simulate the model by generating 2,000,000[50] individual histories for y_t using Monte

[46]The lower bound of zero for the choice of next period's assets is without loss of generality. It is always possible to rewrite the problem so that the lower bound on, the appropriately translated, asset space is zero.

[47]The Appendix reports the derivation

[48]See Barillas and Fernández-Villaverde (2007) for an assessment of the accuracy of the endogenous grid method.

[49]Given n quadrature nodes, Gaussian quadrature approximates exactly the integral of any polynomial function of degree up to $2n-1$.

[50]Increasing the number of individuals histories to 20,000,000 does not affect the results in any meaningful

Carlo simulation of the *continuous* $AR(1)$ process and linearly interpolating the policy functions for points off the discretized state space G_z. Since, by construction, the non-normalized policy function $a_t(z_t, y_t) = \hat{a}_t(\hat{z}_t)y_t$ is linear in labor income, our benchmark simulation does not require any approximation with respect to labor income. Therefore, the simulated moments generated by our benchmark method constitute a highly accurate approximation to the true model moments.

Next, we compute the same set of moments by applying the same optimization method as in the benchmark but using either Tauchen or Rouwenhorst's methods to discretize the labor income process. To be precise, in each case we solve the (non-normalized) decision problem (3.17) by replacing the continuous income process with the appropriate Markov chain with age-dependent grids Y_t^N and transition matrices Π_t^N and using a common exponential grid G_z with 1,000 points for z_t. We consider three different values for the income grid size N; namely 5, 10 and 25.

Given the policy functions thus obtained, we compute the model moments using a Monte Carlo simulation which again generates 2,000,000 income histories. This is done in two different ways. In the first case, we generate the income histories using the discrete Markov chain approximation. The simulation involves interpolating the policy functions linearly only with respect to z. In the second case, as in the benchmark quadrature case, we generate income histories using the *continuous* $AR(1)$ process. We then interpolate linearly over both z *and* labor income y.

The key difference between these two approaches has to do with the sources of the errors that they introduce. Both cases suffer from approximation errors for the policy function relative to quadrature due to: (a) the suboptimal approximation of the expectation in (3.17); (b) the fact that the policy functions solve the Euler equations exactly only at a relatively small number of grid points for labor income. Compared to the continuous $AR(1)$ simulation, the Markov chain simulation introduces an additional approximation error as the simulated policy functions are step, rather than piecewise-linear, functions along the income dimension.

3.3.1 Results

We evaluate the accuracy of Tauchen's and Rouwenhorst's discretization methods by comparing simulated moments obtained under the benchmark quadrature approach to those obtained under either of the two discretization methods. The moments are: (i) the unconditional mean; (ii) the unconditional standard deviation; and (iii) the Gini coefficient. Each set of moments is reported for the distributions of labor income, consumption, wealth and total income. Given the increasing interest in wealth concentration, at the bottom of each

way.

Table 3.1: Ratio of Model Moments Relative to Their Counterpart in the Quadrature Benchmark: (a) Markov Chain Simulation and (b) Continuous Random Walk Income Process

		$N=5$			$N=10$			$N=25$		
		R	T_{Ω^*}	$T_{\Omega=3}$	R	T_{Ω^*}	$T_{\Omega=3}$	R	T_{Ω^*}	$T_{\Omega=3}$
		(A) Markov chain simulation								
Labor income (y_t)	Mean	0.9960	0.9939	1.0880	0.9975	0.9952	1.0543	0.9983	0.9969	1.0070
	SD	0.9208	0.8798	1.3993	0.9618	0.9085	1.2329	0.9842	0.9490	1.0159
	Gini	0.9574	0.9608	1.1255	0.9815	0.9817	1.1069	0.9928	0.9942	1.0169
Consumption (c_t)	Mean	0.9966	0.9882	1.0755	0.9978	1.0006	1.0546	0.9984	0.9988	1.0093
	SD	0.9253	0.8606	1.3517	0.9640	0.9242	1.2140	0.9850	0.9558	1.0213
	Gini	0.9630	0.9634	1.1392	0.9851	0.9750	1.1045	0.9949	0.9926	1.0148
Assets (a_t)	Mean	1.0186	0.7385	0.5370	1.0083	1.2330	1.0706	1.0026	1.0795	1.1079
	SD	1.0611	0.7689	1.1376	1.0296	1.6243	0.9059	1.0110	1.2334	1.1987
	Gini	1.1088	1.3562	3.0492	1.0521	1.5599	0.6049	1.0198	1.2017	1.0961
Tot. inc. ($ra_{t-1}+y_t$)	Mean	0.9966	0.9882	1.0755	0.9978	1.0006	1.0546	0.9984	0.9988	1.0093
	SD	0.9232	0.8656	1.3723	0.9630	0.9190	1.2181	0.9846	0.9536	1.0190
	Gini	0.9607	0.9549	1.1071	0.9834	0.9885	1.0966	0.9938	0.9964	1.0174
Top 5% wealth share		1.0367	0.8449	1.6747	1.0217	1.3214	0.7299	1.0090	1.1451	1.0785
		(B) Random walk simulation								
Labor income (y_t)	Mean	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	SD	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	Gini	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Consumption (c_t)	Mean	1.0005	0.9941	0.9886	1.0002	1.0065	1.0807	1.0001	1.0023	1.0025
	SD	1.0027	0.9802	0.9694	1.0013	1.0339	0.8926	1.0005	1.0176	1.0094
	Gini	0.9994	1.0020	1.0200	0.9998	0.9939	0.5290	0.9999	0.9986	0.9974
Assets (a_t)	Mean	1.0232	0.7409	0.4950	1.0106	1.2889	1.0018	1.0039	1.1037	1.1111
	SD	1.0870	0.7862	0.9270	1.0418	2.2010	0.9888	1.0158	1.6298	1.3198
	Gini	1.1116	1.3533	2.8598	1.0527	1.7360	0.9890	1.0201	1.2945	1.1070
Tot. inc. ($ra_{t-1}+y_t$)	Mean	1.0005	0.9941	0.9886	1.0002	1.0065	1.0018	1.0001	1.0023	1.0025
	SD	1.0018	0.9861	0.9792	1.0009	1.0203	0.9877	1.0003	1.0104	1.0053
	Gini	1.0012	0.9944	0.9832	1.0006	1.0086	0.9942	1.0002	1.0030	1.0004
Top 5% wealth share		1.0670	0.8994	1.5462	1.0338	1.6214	0.7129	1.0130	1.2944	1.1086

Note: Parameter values: $\beta = 0.96$, $r = 0.04$, $\sigma_\varepsilon^2 = 0.0161$.
For columns T_{Ω^*}, $\Omega = 1.6919$ when $N = 5$, $\Omega = 2.0513$ when $N = 10$ and $\Omega = 2.5996$ when $N = 25$.

panel we also report the share of wealth held by the households in the top 5% of the wealth distribution.

Panel (A) and (B) in Table 3.1 report the ratio of the moments obtained from simulating the income process using Rouwenhorst and Tauchen's discretization methods to those computed for the Gaussian Hermite benchmark solution. In the table a value of one indicates that the approximation entails no error, relative to the benchmark solution. As shown in Flodén (2008) and Kopecky and Suen (2010), Tauchen's method is very sensitive to the choice of Ω. Tauchen (1986) originally sets $\Omega = 3$, while Kopecky and Suen (2010) calibrate Ω to match the variance of log income. The counterpart of the latter strategy for a non-stationary income process is not obvious. Hence we choose Ω to match the variance of log income over the whole population, and we report results both for this parametrization (columns T_{Ω^*}) and for the case in which $\Omega = 3$ (columns $\mathrm{T}_{\Omega=3}$).

Case 1: Markov chain simulation. Panel (A) shows results for the case in which the discretized income process is used both to compute the expectation in the decision problem and to simulate the model. In this case the Rouwenhorst method and the Tauchen method with "optimal" choice of Ω perform quite similarly in approximating the labor income moments and the first moment of the consumption distribution. As expected the Tauchen method with $\Omega = 3$ performs much worse. The Rouwenhorst method, though, is more accurate with respect to the standard deviation of consumption, and substantially more so with respect to the distribution of assets. In the latter case, the Rouwenhorst approximation has a maximum error (for any of the moments) of at most 11 per cent for $N = 5$ and of only 2 per cent for $N = 25$. In contrast, the Tauchen approximation is off by anywhere between 1/4 and 2 times relative to the benchmark quadrature method. Things are particularly worrying for the variance of assets, which is very poorly approximated under all Tauchen approximations. The top 5% shares of wealth are very badly approximated, even with a large number of points. Moreover, it is apparent that the approximation error does not necessarily shrink as the number of grid points increases. Intuitively, when comparing the range of the income grid for the Tauchen (equation (3.5)) and Rouwenhorst (equation (3.11)) methods, the range of the income grid increases faster with N for the latter method. This implies that, in the case of Tauchen, a larger number of simulated observations get piled onto the bounds relative to the benchmark method, reducing accuracy. This problem appears to be quite important when approximating the standard deviation of asset holdings. This conjecture is confirmed by the fact that the Tauchen method with $\Omega = 3$, hence with a larger labor income range, performs better than the one with the "optimal choice" of Ω in this respect.

Case 2: Random walk simulation. Panel (B) in Table 3.1 reports the approximation errors obtained through Monte Carlo simulation using the continuous income process. By construction, there is no approximation error for the income process in this case. As expected, the accuracy of both the Tauchen and Rouwenhorst methods generally improves relative to results for the Markov chain simulation. In fact, the accuracy of the Rouwenhorst method is extremely high even when $N = 5$.

Concerning the asset moments, the performance of the Rouwenhorst method is similar to that obtained for the Markov chain simulation. The performance of the Tauchen method is, if anything, worse suggesting that, given the narrower income grid relative to Rouwenhorst, extrapolation along the income dimension increases the overall error relative to the Markov chain simulation. In fact, for N larger than 5 the Tauchen method with $\Omega = 3$, hence with a larger labor income range, performs better than the one with "optimal choice" of Ω.

In sum, the Rouwenhorst method exhibits considerable accuracy even when using a small number of grid points, and its performance is substantially more robust across all moments considered and for all numbers of grid points. In addition, the accuracy of the Tauchen method, especially in terms of asset distributions, does not improve when adding more grid points.

3.4 Conclusion

Approximating non-stationary processes is commonplace in quantitative studies of life-cycle behavior and inequality. In such studies it is important to reliably model the fanning out over age of the cross-sectional distribution of consumption, income and wealth. Large approximation errors may result in misleading inference and the problem appears to be especially severe when approximating the distribution of wealth.

In this paper we provide the first systematic examination of the performance of alternative methods to approximate non-stationary (time-dependent) income processes within a life-cycle setting. We begin by explicitly deriving new generalizations of the Tauchen's and Rouwenhorst's approximation methods for the case of history-dependent state spaces, like the ones commonly employed in life-cycle economies. We then compare the relative performance of these approximation methods. For each method, we numerically solve a finite-lifetime, income-fluctuation problem, and compute a set of moments for the implied cross-sectional distributions of income, consumption and wealth. Next, we gauge the relative performance of the two methods by comparing these moments to the ones obtained from a quasi-exact solution of the same income-fluctuation problem.

The results of this comparison are quite clear and suggest that, in a life-cycle setting, Tauchen's method is generally much less precise than Rouwenhorst's. This discrepancy is most severe when considering the distribution of wealth. Perhaps more worrying is the fact that adding grid points to the income approximation does not seem to significantly improve the performance of Tauchen's method. In contrast, increasing the number of grid points does improve the accuracy of the Rouwenhorst approximation. However, we find that the latter method offers a very reliable approximation even with just 5 grid points.

Conclusion

This dissertation studies the households' intertemporal decision making, especially how they insure themselves facing idiosyncratic income risks. In order to analyze formally how different insurance mechanisms work and how bargaining impacts them, I develop a life-cycle collective model of the household. I show how the model can be identified using joint moments of individual wage changes, consumption changes, and hours changes. However, there is a lack of data that continuously measure the intra-household allocations within the same households, an empirical challenge faced by the researchers who want to test and estimate intertemporal non-unitary household models. The first chapter thus provides a new method for measuring the evolution of allocations over time using the existing data. Using the CEX data, I estimate the gender-specific consumption as a function of socioeconomic variables that are commonly available in both the CEX and the PSID, and then use these estimated functions to predict the consumption allocations within the PSID families. In the second chapter, I estimate the dynamic collective model using the imputed PSID panel. I find the Pareto weights change with wage shocks. This is a clear rejection of the unitary model. Second, the added worker effects are significantly weaker in the non-unitary model; roughly 25% weaker when compared to the estimates obtained for a unitary specification. Finally, this implies the contribution of the family labor supply channel to consumption smoothing is significantly lower in the non-unitary case. The last chapter presents a systematic treatment of numerical methods for approximating non-stationary income processes. We find that the generalized Rouwenhorst method is more efficient and accurate than the Tauchen method.

I believe that several contributions of this research are worth highlighting. First, this dissertation brings in the non-unitary approach, which has been proved fruitful in other contexts, into the analysis of the household response to wage shocks in a manageable way, without requiring strong functional form assumptions. Second, I suggest a new method for combining the information from the CEX and the PSID to get a continuous measure of consumption allocations within households. Third, when I bring the model and the data together, I do find evidence of a substantial bargaining effect, which cannot be captured in the unitary model and changes the inference we draw about the importance of alternative insurance channels. In particular, this implies that the traditional unitary approach leads to an upward bias in the estimates of the added worker effect and that the effect of family labor supply channel for consumption smoothing may be over-estimated when not accounting for renegotiation within the household. Fourth, we generalize the Tauchen's (1986) and Rouwenhorst's (1995) methods to approximating non-stationary income processes and compare their performance. A good approximation method for the process is important for obtaining accurate approximations for the statistics generated from the models and for understanding the household insurance mechanisms. Our findings suggest that in the context of a life-cycle, income fluctuation problem, the generalized Rouwenhorst method is highly recommended for its accuracy and robustness.

Bibliography

Adda, J. and R. W. Cooper (2003): *Dynamic Economics: Quantitative Methods and Applications*, MIT Press.

Attanasio, O., G. Berloffa, R. Blundell, and I. Preston (2002): "From earnings inequality to consumption inequality," *The Economic Journal*, 112, C52–C59.

Barillas, F. and J. Fernández-Villaverde (2007): "A Generalization of the Endogenous Grid Method," 31, 2698–2712.

Blundell, R., P.-A. Chiappori, T. Magnac, and C. Meghir (2007): "Collective Labour Supply: Heterogeneity and Non-Participation," *Review of Economic Studies*, 74, 417–445.

Blundell, R., H. Low, and I. Preston (2013): "Decomposing changes in income risk using consumption data," *Quantitative Economics*, 4, 1–37.

Blundell, R., L. Pistaferri, and I. Preston (2008): "Consumption Inequality and Partial Insurance," *American Economic Review*, 98, 1887–1921.

Blundell, R., L. Pistaferri, and I. Saporta-Eksten (2015): "Consumption Inequality and Family Labor Supply," *American Economic Review (forthcoming)*.

Bound, J., C. Brown, G. J. Duncan, and W. L. Rodgers (1994): "Evidence on the Validity of Cross-sectional and Longitudinal Labor Market Data," *Journal of Labor Economics*, 345–368.

Browning, M., F. Bourguignon, P.-A. Chiappori, and V. Lechene (1994): "Income and Outcomes: A Structural Model of Intrahousehold Allocation," *Journal of Political Economy*, 102, 1067–96.

Browning, M. and C. Meghir (1991): "The Effects of Male and Female Labor Supply on Commodity Demands," *Econometrica*, 59, 925–951.

Carroll, C. (2004): "Theoretical Foundations of Buffer Stock Saving," Working Paper 10867, National Bureau of Economic Research.

Carroll, C. D. (2009): "Precautionary Saving and the Marginal Propensity to Consume out of Permanent Income," *Journal of Monetary Economics*, 56, 780–790.

Cherchye, L., B. De Rock, and F. Vermeulen (2012): "Married with Children: A Collective Labor Supply Model with Detailed Time Use and Intrahousehold Expenditure Information," *American Economic Review*, 102, 3377–3405.

Chiappori, P.-A. (1988): "Rational Household Labor Supply," *Econometrica*, 56, 63–90.

——— (1992): "Collective Labor Supply and Welfare," *Journal of Political Economy*, 100, 437–67.

Chiappori, P.-A., B. Fortin, and G. Lacroix (2002): "Marriage Market, Divorce Legislation, and Household Labor Supply," *Journal of Political Economy*, 110, 37–72.

Chiappori, P.-A. and M. Mazzocco (2015): "Static and Intertemporal Household Decisions," *Columbia University and UCLA, mimeo.*

Deaton, A. (1991): "Saving and Liquidity Constraints," *Econometrica*, 59, 1221–48.

Deaton, A. and C. Paxson (1994): "Intertemporal Choice and Inequality," 102, 437–67.

Dunbar, G. R., A. Lewbel, and K. Pendakur (2013): "Children's Resources in Collective Households: Identification, Estimation, and an Application to Child Poverty in Malawi," *American Economic Review*, 103, 438–71.

Farmer, L. E. and A. A. Toda (2016): "Discretizing Nonlinear, Non-Gaussian Markov Processes with Exact Conditional Moments," *Quantitative Economics*, forthcoming.

Flodén, M. (2008): "A Note on the Accuracy of Markov-chain Approximations to Highly Persistent AR(1) Processes," *Economics Letters*, 99, 516 – 520.

Galindev, R. and D. Lkhagvasuren (2010): "Discretization of Highly Persistent Correlated AR(1) Shocks," *Journal of Economic Dynamics and Control*, 34, 1260 – 1276.

Gallipoli, G., J. Pan, and L. Turner (2016): "Household Responses To Individual Shocks: Disability And Labor Supply," *University of British Columbia and University of Toronto, mimeo.*

Gospodinov, N. and D. Lkhagvasuren (2014): "A Moment-Matching Method for Approximating Vector Autoregressive Processes by Finite-state Markov Chains," *Journal of Applied Econometrics*, 29, 843–859.

GOTTSCHALK, P. AND R. MOFFITT (2008): "Trends in the Transitory Variance of Male Earnings in the U.S., 1970-2004," Boston College Working Papers in Economics 697, Boston College Department of Economics.

HEATHCOTE, J., K. STORESLETTEN, AND G. L. VIOLANTE (2014): "Consumption and Labor Supply with Partial Insurance: An Analytical Framework," *American Economic Review*, 104, 2075–2126.

HECKMAN, J. J. (1974): "Life Cycle Consumption and Labor Supply: An Explanation of the Relationship Between Income and Consumption Over the Life Cycle," *American Economic Review*, 64, 188–94.

HUGGETT, M. (1993): "The Risk Free Rate in Heterogeneous-Agents, Incomplete Insurance Economies," 17, 953–970.

——— (1996): "Wealth Distribution in Life-Cycle Economies," *Journal of Monetary Economics*, 38, 469–494.

KAPLAN, G. (2012): "Inequality and the Life Cycle," *Quantitative Economics*, 3, 471–525.

KAPLAN, G. AND G. L. VIOLANTE (2010): "How Much Consumption Insurance beyond Self-Insurance?" *American Economic Journal: Macroeconomics*, 2, 53–87.

KEANE, M. P. (2011): "Labor Supply and Taxes: A Survey," *Journal of Economic Literature*, 49, 961–1075.

KOPECKY, K. AND R. SUEN (2010): "Finite State Markov-chain Approximations to Highly Persistent Processes," *Review of Economic Dynamics*, 13, 701–714.

KRUEGER, D. AND F. PERRI (2006): "Does Income Inequality Lead To Consumption Inequality? Evidence And Theory," *The Review of Economic Studies*, 73, 163–193.

LISE, J. AND S. SEITZ (2011): "Consumption Inequality and Intra-household Allocations," *The Review of Economic Studies*, 78, 328–355.

LISE, J. AND K. YAMADA (2015): "Household Sharing and Commitment: Evidence from Panel Data on Individual Expenditures and Time Use," IFS Working Papers W14/05, Institute for Fiscal Studies.

LOW, H. (2005): "Self-Insurance in a Life-Cycle Model of Labor Supply and Savings," *Review of Economic Dynamics*, 8, 945–975.

LUNDBERG, S. (1985): "The Added Worker Effect," *Journal of Labor Economics*, 3, 11–37.

Mazzocco, M. (2007): "Household Intertemporal Behaviour: A Collective Characterization and a Test of Commitment," *Review of Economic Studies*, 74, 857–895.

Pan, J. (2017): "Measuring Intra-household Consumption Allocation over Time," Working paper.

Rouwenhorst, G. (1995): "Asset Pricing Implications of Equilibrium Business Cycle Models," in *Frontiers of business cycle research*, ed. by T. F. Cooley, Princeton University Press, chap. 10.

Skinner, J. (1987): "A Superior Measure of Consumption from the Panel Study of Income Dynamics," *Economics Letters*, 23, 213–216.

Storesletten, K., C. I. Telmer, and A. Yaron (2004): "Consumption and Risk Sharing over the Life Cycle," *Journal of Monetary Economics*, 51, 609–633.

Tauchen, G. (1986): "Finite State Markov-Chain Approximations to Univariate and Vector Autoregressions," *Economics Letters*, 20, 177–181.

Tauchen, G. and R. Hussey (1991): "Quadrature-Based Methods for Obtaining Approximate Solutions to Nonlinear Asset Pricing Models," *Econometrica*, 59, 371–96.

Terry, S. J. and E. Knotek (2011): "Markov-Chain approximations of vector Autoregressions: Application of General Multivariate-normal Integration Techniques," *Economics Letters*, 110, 4 – 6.

Voena, A. (2015): "Yours, Mine, and Ours: Do Divorce Laws Affect the Intertemporal Behavior of Married Couples?" *American Economic Review*, 105, 2295–2332.

Appendix A

Appendix for Chapter 2

A.1 More on Wage Estimates

Table A.1: Estimates of Wage Parameters

		(1)	(2)	(3)	(4)
		2005-2013	1999-2009	1999-2009	1999-2009 (BPS)
		sample selection I		sample selection II	
Males	Trans. $\sigma^2_{u_1}$	.0266***	.0226***	.0246***	.0275***
		(.0087)	(.0060)	(.0062)	(.0063)
	Perm. $\sigma^2_{v_1}$	.0391***	.0322***	.0299***	.0303***
		(.0061)	(.0040)	(.0049)	(.0049)
Females	Trans. $\sigma^2_{u_2}$	.0180***	.0169***	.0136**	.0125***
		(.0070)	(.0060)	(.0058)	(.0057)
	Perm. $\sigma^2_{v_2}$	.0503***	.0389***	.0383***	.0382***
		(.0060)	(.0046)	(.0045)	(.0044)
Spousal Covariance	Trans. σ_{u_1,u_2}	.0052	.0062**	.0052*	.0058**
		(.0038)	(.0029)	(.0028)	(.0027)
	Perm. σ_{v_1,v_2}	.0015	.0029	.0026	.0027
		(.0027)	(.0024)	(.0024)	(.0023)

Notes: Standard errors in parentheses are clustered at the household level. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Table A.1 presents a few sets of estimates of wage variances and covariances, for different periods and different sampling restrictions. Column (1) is the wage estimates used throughout in this paper, the same as reported in Table 2.2. Column (2) uses the same sampling restriction (labeled as "sample selection I" in the table; see Section 2.3.1 for details) and estimation procedure as (1), but uses the 1999-2009 PSID data instead.

Column (4) is the estimates reported by Blundell et al. (2015). The discrepancies between my main estimates (column 1) and Blundell et al. (2015)'s (column 4) can be attributed to two factors. One is that we use different waves of the PSID: I use 2005-2013 and they use 1999-2009. The other factor is that we apply slightly different sampling restrictions: first, I use the sample of male head aged 25-60 and they use 30-57; second, we both drop the observations with wages lower than half of the minimum wages, but they only consider the state-level minimum wages and I consider the combination of the federal level and the state levels (whichever is higher in each state); third, whenever a household has a head or wife change (due to divorce, death, or other reasons) in any year, I drop all year observations of this household, whereas they only drop the year of the change and treat the household unit as a new family starting with the observation following the change. In addition, the raw income data for 1999 in the PSID has been recalculated and updated (See the PSID documentation for the 1999 data). The data BPS uses is actually the old version, in which some income data do not match the data that are currently available from the PSID.

In Table A.1, I label their sampling criterion as "sample selection II". To show that I can replicate their wage estimates using my estimation procedure, I apply the same sample selection criterion as theirs and re-estimate the wage parameters using the 1999-2009 PSID, as reported in column (3). The replication is fairly close, as can be seen by comparing estimates in columns (3) and (4).

A.2 Approximation of the First Order Conditions and Intertemporal Budget Constraint

Household i maximizes:

$$U_i = \sum_{t=0}^{T} \beta^t \mathbb{E}_t U_{it},$$

$$U_{it} = \mu_{it} U^m(C_{i1t}, G_{it}, H_{i1t}) + (1 - \mu_{it}) U^f(C_{i2t}, G_{it}, H_{i2t}),$$

Household period budget constraint:

$$\sum_{j=1,2} C_{ijt} + G_{it} + A_{it+1} = \sum_{j=1,2} W_{ijt} H_{ijt} + (1 + r) A_{it}.$$

A.2.1 Linearization of the First Order Conditions

The first order condition for assets gives

$$\lambda_{it} = \beta(1 + r)\mathbb{E}[\lambda_{it+1}]$$

where λ is the Lagrangian multiplier on the budget constraint. Define $e^{\rho} = \frac{1}{\beta(1+r)}$, then we have

$$\mathbb{E}[\lambda_{it+1}] = e^{\rho} \lambda_{it}. \tag{A.1}$$

Write $\lambda_{it+1} = f(\ln \lambda_{it+1}) \equiv \exp(\ln \lambda_{it+1})$ and apply a second-order Taylor expansion of

$f(\ln \lambda_{it+1})$ around $\ln \lambda_{it} + \rho$:

$$\begin{aligned}\lambda_{it+1} &\approx f(\ln \lambda_{it} + \rho) + f'(\ln \lambda_{it} + \rho)(\ln \lambda_{it+1} - \ln \lambda_{it} - \rho) + \frac{f''(\ln \lambda_{it} + \rho)}{2}(\ln \lambda_{it+1} - \ln \lambda_{it} - \rho)^2 \\ &= \lambda_{it}e^{\rho} + \lambda_{it}e^{\rho}(\Delta \ln \lambda_{it+1} - \rho) + \frac{\lambda_{it}e^{\rho}}{2}(\Delta \ln \lambda_{it+1} - \rho)^2 \\ &= \lambda_{it}e^{\rho}\left(1 + (\Delta \ln \lambda_{it+1} - \rho) + \frac{1}{2}(\Delta \ln \lambda_{it+1} - \rho)^2\right)\end{aligned}$$

Taking expectation at time t yields

$$\mathbb{E}_t[\ln \lambda_{it+1}] = \lambda_{it}e^{\rho}(1 + \mathbb{E}_t(\Delta \ln \lambda_{it+1} - \rho) + \frac{1}{2}\mathbb{E}_t(\Delta \ln \lambda_{it+1} - \rho)^2)$$

Substituting for $\mathbb{E}_t[\ln \lambda_{it+1}]$ from (A.1) gives

$$\mathbb{E}_t(\Delta \ln \lambda_{it+1} - \rho) + \frac{1}{2}\mathbb{E}_t(\Delta \ln \lambda_{it+1} - \rho)^2 = 0$$

and thus

$$\mathbb{E}_t(\Delta \ln \lambda_{it+1}) = \rho - \frac{1}{2}\mathbb{E}_t(\Delta \ln \lambda_{it+1} - \rho)^2$$

which implies that we can write $\Delta \ln \lambda_{it+1}$ as

$$\Delta \ln \lambda_{it+1} = \varpi_t + \iota_{it+1} \tag{A.2}$$

where $\varpi_t \equiv \rho - \frac{1}{2}\mathbb{E}_t(\Delta \ln \lambda_{it+1} - \rho)^2$ is assumed to be fixed in the cross section and $\mathbb{E}_t(\iota_{it+1}) = 0$ by definition of a prediction error.

The first order conditions for C_{i1t}, C_{i2t}, G_{it}, H_{i1t}, and H_{i2t} are:

$$\begin{aligned}\mu_{it}U^m_{c_1}(it) &= \lambda_{it} \\ (1 - \mu_{it})U^f_{c_2}(it) &= \lambda_{it} \\ \mu_{it}U^m_g(it) + (1 - \mu_{it})U^f_g(it) &= \lambda_{it} \\ \mu_{it}U^m_{h_1}(it) &= \lambda_{it}W_{i1t} \\ (1 - \mu_{it})U^f_{h_2}(it) &= \lambda_{it}W_{i2t}\end{aligned}$$

where $U^m_{c_1}$ is the marginal utility of the husband with respect to his private consumption, etc. And I write $U^m_{c_1}(C_{i1t}, G_{it}, H_{i1t})$ as $U^m_{c_1}(it)$, etc., for compactness.

Taking log of both sides and then taking the time difference yields[51]

$$\Delta \ln U^m_{c_1}(it) = \Delta \ln \lambda_{it} - \Delta \ln \mu_{it} \tag{A.3}$$

$$\Delta \ln U^f_{c_2}(it) = \Delta \ln \lambda_{it} - \Delta \ln(1-\mu_{it}) \tag{A.4}$$

$$\psi_{it}\Delta \ln U^m_g(it) + (1-\psi_{it})\Delta \ln U^f_g(it) \approx \Delta \ln \lambda_{it} - \psi_{it}\Delta \ln \mu_{it} - (1-\psi_{it})\Delta \ln(1-\mu_{it}) \tag{A.5}$$

$$\Delta \ln U^m_{h_1}(it) = \Delta \ln \lambda_{it} + \Delta w_{i1t} - \Delta \ln \mu_{it} \tag{A.6}$$

$$\Delta \ln U^f_{h_2}(it) = \Delta \ln \lambda_{it} + \Delta w_{i2t} - \Delta \ln(1-\mu_{it}) \tag{A.7}$$

where $\psi_{it} \equiv \frac{\mu_{it}U^m_g(it)}{\mu_{it}U^m_g(it)+(1-\mu_{it})U^f_g(it)}$ is the husband's share of marginal utility of public goods.

Write $\ln U^m_{c_1}(C_{1t+1}, G_{t+1}, H_{1t+1}) = \ln U^m_{c_1}(e^{c_{1t+1}}, e^{g_{t+1}}, e^{h_{1t+1}})$ and apply a first order Taylor expansion around c_{1t}, g_t, h_{1t} (omitting the household index i for simplicity):

$$\Delta \ln U^m_{c_1}(t+1) \approx \frac{U^m_{c_1c_1}(t)}{U^m_{c_1}(t)}C_{1t}\Delta c_{1t+1} + \frac{U^m_{c_1g}(t)}{U^m_{c_1}(t)}G_t\Delta g_{t+1} + \frac{U^m_{c_1h_1}(t)}{U^m_{c_1}(t)}H_{1t}\Delta h_{1t+1}. \tag{A.8}$$

Similarly for U^m_g, $U^m_{c_2}$, $U^f_{c_2}$, U^f_g, and $U^f_{h_2}$ we have

$$\Delta \ln U^m_g(t+1) \approx \frac{U^m_{gc_1}(t)}{U^m_g(t)}C_{1t}\Delta c_{1t+1} + \frac{U^m_{gg}(t)}{U^m_g(t)}G_t\Delta g_{t+1} + \frac{U^m_{gh_1}(t)}{U^m_g(t)}H_{1t}\Delta h_{1t+1} \tag{A.9}$$

$$\Delta \ln U^m_{h_1}(t+1) \approx \frac{U^m_{h_1c_1}(t)}{U^m_{h_1}(t)}C_{1t}\Delta c_{1t+1} + \frac{U^m_{h_1g}(t)}{U^m_{h_1}(t)}G_t\Delta g_{t+1} + \frac{U^m_{h_1h_1}(t)}{U^m_{h_1}(t)}H_{1t}\Delta h_{1t+1} \tag{A.10}$$

$$\Delta \ln U^f_{c_2}(t+1) \approx \frac{U^f_{c_2c_2}(t)}{U^f_{c_2}(t)}C_{2t}\Delta c_{2t+1} + \frac{U^f_{c_2g}(t)}{U^f_{c_2}(t)}G_t\Delta g_{t+1} + \frac{U^f_{c_2h_2}(t)}{U^f_{c_2}(t)}H_{2t}\Delta h_{2t+1} \tag{A.11}$$

$$\Delta \ln U^f_g(t+1) \approx \frac{U^f_{gc_2}(t)}{U^f_g(t)}C_{2t}\Delta c_{2t+1} + \frac{U^f_{gg}(t)}{U^f_g(t)}G_t\Delta g_{t+1} + \frac{U^f_{gh_2}(t)}{U^f_g(t)}H_{2t}\Delta h_{2t+1} \tag{A.12}$$

$$\Delta \ln U^f_{h_2}(t+1) \approx \frac{U^f_{h_2c_2}(t)}{U^f_{h_2}(t)}C_{2t}\Delta c_{2t+1} + \frac{U^f_{h_2g}(t)}{U^f_{h_2}(t)}G_t\Delta g_{t+1} + \frac{U^f_{h_2h_2}(t)}{U^f_{h_2}(t)}H_{2t}\Delta h_{2t+1}. \tag{A.13}$$

[51] The first order condition for public goods G_{it} needs to be log-linearized as follows: $\Delta \ln(\mu_{it}U^m_g(it) + (1-\mu_{it})U^f_g(it)) \approx \frac{\Delta(\mu_{it}U^m_g(it)+(1-\mu_{it})U^f_g(it))}{\mu_{it}U^m_g(it)+(1-\mu_{it})U^f_g(it)} = \frac{\mu_{it}U^m_g(it)}{\mu_{it}U^m_g(it)+(1-\mu_{it})U^f_g(it)}\Delta \ln(\mu_{it}U^m_g(it)) + \frac{(1-\mu_{it})U^f_g(it)}{\mu_{it}U^m_g(it)+(1-\mu_{it})U^f_g(it)}\Delta \ln((1-\mu_{it})U^f_g(it))$.

Substituting (A.8)-(A.13) into (A.3)-(A.7) and rearranging them yields:

$$\begin{pmatrix} \Delta c_{1t+1} \\ \Delta c_{2t+1} \\ \Delta g_{t+1} \\ \Delta h_{1t+1} \\ \Delta h_{2t+1} \end{pmatrix} \approx \underbrace{\begin{pmatrix} \gamma_{c_1 w_1} & \gamma_{c_1 w_2} & \gamma_{c_1 \lambda} & \gamma_{c_1 \mu} \\ \gamma_{c_2 w_1} & \gamma_{c_2 w_2} & \gamma_{c_2 \lambda} & \gamma_{c_2 \mu} \\ \gamma_{g w_1} & \gamma_{g w_2} & \gamma_{g \lambda} & \gamma_{g \mu} \\ \gamma_{h_1 w_1} & \gamma_{h_1 w_2} & \gamma_{h_1 \lambda} & \gamma_{h_1 \mu} \\ \gamma_{h_2 w_1} & \gamma_{h_2 w_2} & \gamma_{h_2 \lambda} & \gamma_{h_2 \mu} \end{pmatrix}}_{\Gamma} \begin{pmatrix} \Delta w_{1t+1} \\ \Delta w_{2t+1} \\ \Delta \ln \lambda_{t+1} \\ \Delta \ln \mu_{t+1} \end{pmatrix} \tag{A.14}$$

where

$$\Gamma = A^{-1}B$$

$$A = \begin{pmatrix} \frac{U^m_{c_1c_1}(t)C_{1t}}{U^m_{c_1}(t)} & 0 & \frac{U^m_{c_1g}(t)G_t}{U^m_{c_1}(t)} & \frac{U^m_{c_1h_1}(t)H_{1t}}{U^m_{c_1}(t)} & 0 \\ 0 & \frac{U^f_{c_2c_2}(t)C_{2t}}{U^f_{c_2}(t)} & \frac{U^f_{c_2g}(t)G_t}{U^f_{c_2}(t)} & 0 & \frac{U^f_{c_2h_2}(t)H_{2t}}{U^f_{c_2}(t)} \\ \psi_t \frac{U^m_{gc_1}(t)C_{1t}}{U^m_g(t)} & (1-\psi_t)\frac{U^f_{gc_2}(t)C_{2t}}{U^f_g(t)} & \psi_t \frac{U^m_{gg}(t)G_t}{U^m_g(t)} + (1-\psi_t)\frac{U^f_{gg}(t)G_t}{U^f_g(t)} & \psi_t \frac{U^m_{gh_1}(t)H_{1t}}{U^m_g(t)} & (1-\psi_t)\frac{U^f_{gh_2}(t)H_{2t}}{U^f_g(t)} \\ \frac{U^m_{h_1c_1}(t)C_{1t}}{U^m_{h_1}(t)} & 0 & \frac{U^m_{h_1g}(t)G_t}{U^m_{h_1}(t)} & \frac{U^m_{h_1h_1}(t)H_{1t}}{U^m_{h_1}(t)} & 0 \\ 0 & \frac{U^f_{h_2c_2}(t)C_{2t}}{U^f_{h_2}(t)} & \frac{U^f_{h_2g}(t)G_t}{U^f_{h_2}(t)} & 0 & \frac{U^f_{h_2h_2}(t)H_{2t}}{U^f_{h_2}(t)} \end{pmatrix}$$

$$B = \begin{pmatrix} 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & \frac{\mu_t}{1-\mu_t} \\ 0 & 0 & 1 & -\psi_t + (1-\psi_t)\frac{\mu_t}{1-\mu_t} \\ 1 & 0 & 1 & -1 \\ 0 & 1 & 1 & \frac{\mu_t}{1-\mu_t} \end{pmatrix}.$$

Apply a first order Taylor expansion on $\mu_{it} = \mu(z_{i0}, \varepsilon_{it})$:

$$\Delta \ln \mu_{it} \approx \frac{\mu_2(z_{i0}, \varepsilon_{it})}{\mu_{it}} \varepsilon_{it} \Delta \ln \varepsilon_{it}$$

where I utilize the fact that $\Delta \ln z_{i0} = 0$ since z_{i0} is predetermined.

Recall that $\varepsilon_{it} = z_{it} - E_0 z_{it}$ is the deviation of z_{it} from its expected value and that the only exogenous shock is in individual wages; thus ε_{it} is a vector of accumulated individual wage shocks since time 0: $\ln \varepsilon_{it} = (\sum_{s=0}^{t}(\Delta u_{i1s} + v_{i1s}), \sum_{s=0}^{t}(\Delta u_{i2s} + v_{i2s}))$. This implies $\Delta \ln \varepsilon_{it} = (\Delta u_{i1t} + v_{i1t}, \Delta u_{i2t} + v_{i2t})$. Therefore, the equation above can be rewritten as

$$\Delta \ln \mu_{it} \approx \eta_{\mu,w_1}(\Delta u_{i1t} + v_{i1t}) + \eta_{\mu,w_2}(\Delta u_{i2t} + v_{i2t}) \tag{A.15}$$

where $\eta_{\mu,w_j} \equiv \frac{\mu_{w_j} w_j}{\mu}$ is the elasticity of μ with respect to changes in partner j's residual wage.

Now with (A.2), (A.14), and (A.15), I can write the changes in consumption and hours as functions of wage shocks and ι_{it}, the latter of which is the deviation of the marginal utility of wealth from its expectation. In the next section, I will derive ι_{it} as a function of wage shocks.

A.2.2 Log-Linearization of the Lifetime Budget Constraint

The lifetime budget constraint is

$$\sum_{s=t}^{T} \frac{C_{i1s} + C_{i2s} + G_{is}}{(1+r)^{s-t}} = A_{it} + \sum_{s=t}^{T} \frac{W_{i1s}H_{i1s} + W_{i2s}H_{i2,s}}{(1+r)^{s-t}}. \tag{A.16}$$

First, write $\ln \sum_{s=t}^{T} \frac{C_{is}}{(1+r)^{s-t}} = \ln \sum_{s=t}^{T} \exp(c_{is} - (s-t)\ln(1+r))$ and apply a first order Taylor expansion around $\{E_{t-1}c_{is} - (s-t)\ln(1+r)\}_{s=t}^{T}$:

$$\begin{aligned} \ln \sum_{s=t}^{T} \frac{C_{is}}{(1+r)^{s-t}} &= \ln \sum_{s=t}^{T} \exp(c_{is} - (s-t)\ln(1+r)) \\ &\approx \ln \sum_{s=t}^{T} \exp(E_{t-1}c_{is} - (s-t)\ln(1+r)) + \sum_{s=t}^{T} \theta_s (c_{is} - E_{t-1}c_{is}) \end{aligned} \tag{A.17}$$

where $\theta_s \equiv \frac{\exp(E_{t-1}c_{is} - (s-t)\ln(1+r))}{\sum_{s=t}^{T}\exp(E_{t-1}c_{is} - (s-t)\ln(1+r))}$. Taking expectation of (A.17) with respect to time $t-1$ and t, respectively, and noting that θ_s is known (with no uncertainty) at time $t-1$ or at time t:

$$\begin{aligned} E_{t-1} \ln \sum_{s=t}^{T} \frac{C_{is}}{(1+r)^{s-t}} &\approx \ln \sum_{s=t}^{T} \exp(E_{t-1}c_{is} - (s-t)\ln(1+r)) + \sum_{s=t}^{T} \theta_s (E_{t-1}c_{is} - E_{t-1}c_{is}) \\ E_{t} \ln \sum_{s=t}^{T} \frac{C_{is}}{(1+r)^{s-t}} &\approx \ln \sum_{s=t}^{T} \exp(E_{t-1}c_{is} - (s-t)\ln(1+r)) + \sum_{s=t}^{T} \theta_s (E_{t}c_{is} - E_{t-1}c_{is}) \end{aligned}$$

Subtracting the first equation from the other:

$$E_t \ln \sum_{s=t}^{T} \frac{C_{is}}{(1+r)^{s-t}} - E_{t-1} \ln \sum_{s=t}^{T} \frac{C_{is}}{(1+r)^{s-t}} \approx \sum_{s=t}^{T} \theta_s (E_t c_{is} - E_{t-1}c_{is}). \tag{A.18}$$

By (A.2), (A.14), and (A.15), I have

$$\begin{aligned} \Delta c_{i1t} &\approx (\gamma_{c_1 w_1} + \gamma_{c_1 \mu}\eta_{\mu,w_1})(\Delta u_{i1t} + v_{i1t}) + (\gamma_{c_1 w_2} + \gamma_{c_1 \mu}\eta_{\mu,w_2})(\Delta u_{i2t} + v_{i2t}) + \gamma_{c_1 \lambda}(\varpi_t + \iota_{it}) \\ \Delta c_{i2t} &\approx (\gamma_{c_2 w_1} + \gamma_{c_2 \mu}\eta_{\mu,w_1})(\Delta u_{i1t} + v_{i1t}) + (\gamma_{c_2 w_2} + \gamma_{c_2 \mu}\eta_{\mu,w_2})(\Delta u_{i2t} + v_{i2t}) + \gamma_{c_2 \lambda}(\varpi_t + \iota_{it}) \\ \Delta c_{gt} &\approx (\gamma_{g w_1} + \gamma_{g \mu}\eta_{\mu,w_1})(\Delta u_{i1t} + v_{i1t}) + (\gamma_{g w_2} + \gamma_{g \mu}\eta_{\mu,w_2})(\Delta u_{i2t} + v_{i2t}) + \gamma_{g \lambda}(\varpi_t + \iota_{it}) \end{aligned}$$

Next, using the approximation that $\Delta c_{it} \approx \psi_{i1t-1}\Delta c_{i1t} + \psi_{i2t-1}\Delta c_{i2t} + \psi_{igt-1}\Delta g_{it}$, where $\psi_{i1t-1} = C_{i1t-1}/C_{it-1}$, $\psi_{i2t-1} = C_{i2t-1}/C_{it-1}$, and $\psi_{i1t-1} = G_{it-1}/C_{it-1}$, I write

$$c_{it} \approx c_{it-1} + (\gamma_{cw_1} + \gamma_{c\mu}\eta_{\mu,w_1})(\Delta u_{i1t} + v_{i1t}) + (\gamma_{cw_2} + \gamma_{c\mu}\eta_{\mu,w_2})(\Delta u_{i2t} + v_{i2t}) + \gamma_{c\lambda}(\varpi_t + \iota_{it})$$

where $\gamma_{cw_1} \equiv \psi_{i1t-1}\gamma_{c_1w_1} + \psi_{i2t-1}\gamma_{c_2w_1} + \psi_{igt-1}\gamma_{gw_1}$, etc. This implies

$$\begin{aligned}
E_t c_{it} - E_{t-1}c_{it} &\approx (\gamma_{cw_1} + \gamma_{c\mu}\eta_{\mu,w_1})(u_{i1t} + v_{i1t}) + (\gamma_{cw_2} + \gamma_{c\mu}\eta_{\mu,w_2})(u_{i2t} + v_{i2t}) + \gamma_{c\lambda}(\iota_{it}) \\
E_t c_{it+1} - E_{t-1}c_{it+1} &\approx E_t c_{it} - E_{t-1}c_{it} - (\gamma_{cw_1} + \gamma_{c\mu}\eta_{\mu,w_1})u_{i1t} - (\gamma_{cw_2} + \gamma_{c\mu}\eta_{\mu,w_2})u_{i2t} \\
E_t c_{it+2} - E_{t-1}c_{it+2} &\approx E_t c_{it+1} - E_{t-1}c_{it+1} \\
&\approx E_t c_{it} - E_{t-1}c_{it} - (\gamma_{cw_1} + \gamma_{c\mu}\eta_{\mu,w_1})u_{i1t} - (\gamma_{cw_2} + \gamma_{c\mu}\eta_{\mu,w_2})u_{i2t} \\
&\dots \\
E_t c_{iT} - E_{t-1}c_{iT} &\approx E_t c_{it} - E_{t-1}c_{it} - (\gamma_{cw_1} + \gamma_{c\mu}\eta_{\mu,w_1})u_{i1t} - (\gamma_{cw_2} + \gamma_{c\mu}\eta_{\mu,w_2})u_{i2t}
\end{aligned}$$

Substituting these into (A.18) yields

$$\begin{aligned}
&E_t \ln \sum_{s=t}^{T} \frac{C_{is}}{(1+r)^{s-t}} - E_{t-1} \ln \sum_{s=t}^{T} \frac{C_{is}}{(1+r)^{s-t}} \\
\approx& \sum_{s=t}^{T} \theta_s (E_t c_{it} - E_{t-1}c_{it}) - \sum_{s=t+1}^{T} \theta_s ((\gamma_{cw_1} + \gamma_{c\mu}\eta_{\mu,w_1})u_{i1t} + (\gamma_{cw_2} + \gamma_{c\mu}\eta_{\mu,w_2})u_{i2t}) \\
=& (E_t c_{it} - E_{t-1}c_{it}) \sum_{s=t}^{T} \theta_s - ((\gamma_{cw_1} + \gamma_{c\mu}\eta_{\mu,w_1})u_{i1t} + (\gamma_{cw_2} + \gamma_{c\mu}\eta_{\mu,w_2})u_{i2t}) \sum_{s=t+1}^{T} \theta_s \\
=& (E_t c_{it} - E_{t-1}c_{it}) - ((\gamma_{cw_1} + \gamma_{c\mu}\eta_{\mu,w_1})u_{i1t} + (\gamma_{cw_2} + \gamma_{c\mu}\eta_{\mu,w_2})u_{i2t})(1 - \theta_t) \\
=& (\gamma_{cw_1} + \gamma_{c\mu}\eta_{\mu,w_1})v_{i1t} + (\gamma_{cw_2} + \gamma_{c\mu}\eta_{\mu,w_2})v_{i2t} + \gamma_{c\lambda}\iota_{it} \\
&+ \theta_t((\gamma_{cw_1} + \gamma_{c\mu}\eta_{\mu,w_1})u_{i1t} + (\gamma_{cw_2} + \gamma_{c\mu}\eta_{\mu,w_2})u_{i2t})
\end{aligned}$$

where the last equality comes from the identity $\sum_{s=t}^{T} \theta_s = 1$. Now assume that θ_t (consumption today as a share of remaining lifetime consumption) is small and can be neglected. Then the result of the log-linearization of the left hand side of the lifetime budget constraint (A.16) is

$$E_t \ln \sum_{s=t}^{T} \frac{C_{is}}{(1+r)^{s-t}} - E_{t-1} \ln \sum_{s=t}^{T} \frac{C_{is}}{(1+r)^{s-t}} = (\gamma_{cw_1} + \gamma_{c\mu}\eta_{\mu,w_1})v_{i1t} + (\gamma_{cw_2} + \gamma_{c\mu}\eta_{\mu,w_2})v_{i2t} + \gamma_{c\lambda}\iota_{it} \tag{A.19}$$

Second, the log of the RHS of (A.16) is

$$\ln\left(A_{it}+\sum_{s=t}^{T}\frac{W_{i1s}H_{i1s}+W_{i2s}H_{i2s}}{(1+r)^{s-t}}\right)$$

$$=\ln(\exp(a_{it})+\sum_{s=t}^{T}\exp(w_{i1s}+h_{i1s}-(s-t)\ln(1+r))+\sum_{s=t}^{T}\exp(w_{i2s}+h_{i2s}-(s-t)\ln(1+r)))$$

(applying a first-order Taylor expansion around $E_{t-1}a_{it}$, $E_{t-1}w_{ijs}$, and $E_{t-1}h_{ijs}$)

$$\approx\ln\left(D_0+D_1+D_2\right)$$
$$+\frac{D_0}{D_0+D_1+D_2}(a_{it}-E_{t-1}a_{it})$$
$$+\frac{D_1}{D_0+D_1+D_2}\sum_{s=t}^{T}\frac{D_{1s}}{D_1}(w_{i1s}+h_{i1s}-E_{t-1}w_{i1s}-E_{t-1}h_{i1s})$$
$$+\frac{D_2}{D_0+D_1+D_2}\sum_{s=t}^{T}\frac{D_{2s}}{D_2}(w_{i2s}+h_{i2s}-E_{t-1}w_{i2s}-E_{t-1}h_{i2s})$$

where

$$\begin{aligned}D_0&=\exp(E_{t-1}a_{it})\\D_{1s}&=\exp(E_{t-1}w_{i1s}+E_{t-1}h_{i1s}-(s-t)\ln(1+r))\\D_{2s}&=\exp(E_{t-1}w_{i2s}+E_{t-1}h_{i2s}-(s-t)\ln(1+r))\\D_1&=\sum_{s=t}^{T}D_{1s}\\D_2&=\sum_{s=t}^{T}D_{2s}.\end{aligned}$$

Then the time difference in expectation of the log of the RHS of the lifetime budget constraint is (noting that $E_ta_{it}-E_{t-1}a_{it}=0$ because a_{it} is determined at $t-1$)

$$\begin{aligned}E_t\ln RHS-E_{t-1}\ln RHS\approx&\frac{D_1}{D_0+D_1+D_2}\sum_{s=t}^{T}\frac{D_{1s}}{D_1}(E_ty_{i1s}-E_{t-1}y_{i1s})\\&+\frac{D_2}{D_0+D_1+D_2}\sum_{s=t}^{T}\frac{D_{2s}}{D_2}(E_ty_{i2s}-E_{t-1}y_{i2s})\end{aligned}\tag{A.20}$$

Note that $\Delta y_{ijt} = \Delta w_{ijt} + \Delta h_{ijt}$ $(j = 1, 2)$. By (A.2), (A.14), and (A.15), I have

$$\Delta y_{i1t} \approx (1 + \gamma_{h_1 w_1} + \gamma_{h_1 \mu}\eta_{\mu,w_1})(\Delta u_{i1t} + v_{i1t}) + (\gamma_{h_1 w_2} + \gamma_{h_1 \mu}\eta_{\mu,w_2})(\Delta u_{i2t} + v_{i2t}) + \gamma_{h_1 \lambda}(\varpi_t + \iota_{it})$$
$$\Delta y_{i2t} \approx (\gamma_{h_2 w_1} + \gamma_{h_2 \mu}\eta_{\mu,w_1})(\Delta u_{i1t} + v_{i1t}) + (1 + \gamma_{h_2 w_2} + \gamma_{h_2 \mu}\eta_{\mu,w_2})(\Delta u_{i2t} + v_{i2t}) + \gamma_{h_2 \lambda}(\varpi_t + \iota_{it})$$

The items being sum in the first term of (A.20) then are

$$\begin{aligned}
E_t y_{i1t} - E_{t-1} y_{i1t} &\approx (1 + \gamma_{h_1 w_1} + \gamma_{h_1 \mu}\eta_{\mu,w_1})(u_{i1t} + v_{i1t}) + (\gamma_{h_1 w_2} + \gamma_{h_1 \mu}\eta_{\mu,w_2})(u_{i2t} + v_{i2t}) + \gamma_{h_1 \lambda}\iota_{it} \\
E_t y_{i1t+1} - E_{t-1} y_{i1t+1} &\approx E_t y_{i1t} - E_{t-1} y_{i1t} - (1 + \gamma_{h_1 w_1} + \gamma_{h_1 \mu}\eta_{\mu,w_1})u_{i1t} - (\gamma_{h_1 w_2} + \gamma_{h_1 \mu}\eta_{\mu,w_2})u_{i2t} \\
E_t y_{i1t+2} - E_{t-1} y_{i1t+2} &\approx E_t y_{i1t+1} - E_{t-1} y_{i1t+1} \\
&\cdots \\
E_t y_{i1T} - E_{t-1} y_{i1T} &\approx E_t y_{i1t+1} - E_{t-1} y_{i1t+1}
\end{aligned}$$

so the first summation in (A.20) is equal to

$$\begin{aligned}
&\sum_{s=t}^{T} \frac{D_{1s}}{D_1}(E_t y_{i1s} - E_{t-1} y_{i1s}) \\
\approx &(E_t y_{i1t} - E_{t-1} y_{i1t}) \sum_{s=t}^{T} \frac{D_{1s}}{D_1} - ((1 + \gamma_{h_1 w_1} + \gamma_{h_1 \mu}\eta_{\mu,w_1})u_{i1t} - (\gamma_{h_1 w_2} + \gamma_{h_1 \mu}\eta_{\mu,w_2})u_{i2t}) \sum_{s=t+1}^{T} \frac{D_{1s}}{D_1} \\
= &(E_t y_{i1t} - E_{t-1} y_{i1t}) - ((1 + \gamma_{h_1 w_1} + \gamma_{h_1 \mu}\eta_{\mu,w_1})u_{i1t} - (\gamma_{h_1 w_2} + \gamma_{h_1 \mu}\eta_{\mu,w_2})u_{i2t})(1 - \frac{D_{1t}}{D_1}) \\
= &(1 + \gamma_{h_1 w_1} + \gamma_{h_1 \mu}\eta_{\mu,w_1})v_{i1t} + (\gamma_{h_1 w_2} + \gamma_{h_1 \mu}\eta_{\mu,w_2})v_{i2t} + \gamma_{h_1 \lambda}\iota_{it}
\end{aligned}$$

where I utilize the assumption that $\frac{D_{1t}}{D_1}$ (labor income today as a share of remaining lifetime labor income) is small and can be neglected .

Similarly, the second summation in (A.20) is

$$\sum_{s=t}^{T} \frac{D_{2s}}{D_2}(E_t y_{i2s} - E_{t-1} y_{i2s}) \approx (\gamma_{h_2 w_1} + \gamma_{h_2 \mu}\eta_{\mu,w_1})v_{i1t} + (1 + \gamma_{h_2 w_2} + \gamma_{h_2 \mu}\eta_{\mu,w_2})v_{i2t} + \gamma_{h_2 \lambda}\iota_{it}$$

Therefore,

$$\begin{aligned}
&E_t \ln RHS - E_{t-1} \ln RHS \\
\approx &\frac{D_1}{D_0 + D_1 + D_2}((1 + \gamma_{h_1 w_1} + \gamma_{h_1 \mu}\eta_{\mu,w_1})v_{i1t} + (\gamma_{h_1 w_2} + \gamma_{h_1 \mu}\eta_{\mu,w_2})v_{i2t} + \gamma_{h_1 \lambda}\iota_{it}) \\
&+ \frac{D_2}{D_0 + D_1 + D_2}((\gamma_{h_2 w_1} + \gamma_{h_2 \mu}\eta_{\mu,w_1})v_{i1t} + (1 + \gamma_{h_2 w_2} + \gamma_{h_2 \mu}\eta_{\mu,w_2})v_{i2t} + \gamma_{h_2 \lambda}\iota_{it})
\end{aligned}$$

Define $\pi_{it} \equiv \frac{D_0}{D_0+D_1+D_2}$, financial wealth as a share of total (financial and human) wealth, and $s_{it} \equiv \frac{D_1}{D_1+D_2}$ (not to be confused with the time index s), the husband's share of human wealth. By "human wealth" I mean the remaining discounted lifetime labor income.

Now I combine the results of log-linearization of the left hand side and the right hand side of the lifetime budget constraint:

$$\begin{aligned}
&(\gamma_{cw_1} + \gamma_{c\mu}\eta_{\mu,w_1})v_{i1t} + (\gamma_{cw_2} + \gamma_{c\mu}\eta_{\mu,w_2})v_{i2t} + \gamma_{c\lambda}\iota_{it} \\
=&(1-\pi_{it})s_{it}\left((1+\gamma_{h_1w_1} + \gamma_{h_1\mu}\eta_{\mu,w_1})v_{i1t} + (\gamma_{h_1w_2} + \gamma_{h_1\mu}\eta_{\mu,w_2})v_{i2t} + \gamma_{h_1\lambda}\iota_{it}\right) \\
&+ (1-\pi_{it})(1-s_{it})\left((\gamma_{h_2w_1} + \gamma_{h_2\mu}\eta_{\mu,w_1})v_{i1t} + (1+\gamma_{h_2w_2} + \gamma_{h_2\mu}\eta_{\mu,w_2})v_{i2t} + \gamma_{h_2\lambda}\iota_{it}\right)
\end{aligned}$$

which implies ι_{it} can be written as

$$\iota_{i,t} = \gamma_{\iota v_1}v_{i,1,t} + \gamma_{\iota v_2}v_{i,2,t} \tag{A.21}$$

where

$$\begin{aligned}
\gamma_{\iota v_1} &\equiv \frac{(1-\pi_{it})s_{it}(1+\gamma_{h_1w_1} + \gamma_{h_1\mu}\eta_{\mu,w_1}) + (1-\pi_{it})(1-s_{it})(\gamma_{h_2w_1} + \gamma_{h_2\mu}\eta_{\mu,w_1}) - (\gamma_{cw_1} + \gamma_{c\mu}\eta_{\mu,w_1})}{\gamma_{c\lambda} - (1-\pi_{it})s_{it}\gamma_{h_1\lambda} - (1-\pi_{it})(1-s_{it})\gamma_{h_2\lambda}} \\
\gamma_{\iota v_2} &\equiv \frac{(1-\pi_{it})s_{it}(\gamma_{h_2w_1} + \gamma_{h_2\mu}\eta_{\mu,w_1}) + (1-\pi_{it})(1-s_{it})(1+\gamma_{h_2w_2} + \gamma_{h_2\mu}\eta_{\mu,w_2}) - (\gamma_{cw_2} + \gamma_{c\mu}\eta_{\mu,w_2})}{\gamma_{c\lambda} - (1-\pi_{it})s_{it}\gamma_{h_1\lambda} - (1-\pi_{it})(1-s_{it})\gamma_{h_2\lambda}}
\end{aligned}$$

Finally, plugging (A.2), (A.15), and (A.21) into (A.14) I get the transmission system in the main text. For example, the transmission equation for Δc_{i1t} is: $\Delta c_{i1t} = \kappa_{c_1u_1}\Delta u_{i1t} + \kappa_{c_1v_1}v_{i1t} + \kappa_{c_1u_2}\Delta u_{i2t} + \kappa_{c_1v_2}v_{i2t}$, where

$$\begin{aligned}
\kappa_{c_1u_1} &= \gamma_{c_1w_1} + \gamma_{c_1\mu}\eta_{\mu,w_1} \\
\kappa_{c_1v_1} &= \gamma_{c_1w_1} + \gamma_{c_1\lambda}\gamma_{\iota v_1} + \gamma_{c_1\mu}\eta_{\mu,w_1} \\
\kappa_{c_1u_2} &= \gamma_{c_1w_2} + \gamma_{c_1\mu}\eta_{\mu,w_2} \\
\kappa_{c_1v_2} &= \gamma_{c_1w_2} + \gamma_{c_1\lambda}\gamma_{\iota v_2} + \gamma_{c_1\mu}\eta_{\mu,w_2}.
\end{aligned}$$

A.3 Moment Condtions in GMM estimation

For simplicity, this section abstracts away measurement errors. The moment conditions with measurement errors are derived in Appendix A.4.

The PSID is biennial. The difference between year t and $t-2$ is actually $\Delta^2 w_t$: $\Delta^2 w_t \equiv w_t - w_{t-2} = w_t - w_{t-1} - (w_{t-1} - w_{t-2}) = \Delta w_t - \Delta w_{t-1} = \Delta u_t + v_t - \Delta u_{t-1} - v_{t-1} = u_t - u_{t-2} + v_t - v_{t-1} = \Delta^2 u_t + \Delta v_t$. Keeping this in mind, the moment conditions will be slightly different from the case of annual data. For example,

$$\begin{aligned} E[(\Delta^2 w_t)^2] &= E[(\Delta^2 u_t + \Delta v_t)^2] \\ &= E[(\Delta^2 u_t)^2] + E[(\Delta v_t)^2] \\ &= 2\sigma_u^2 + 2\sigma_v^2. \end{aligned}$$

Note that for the first difference (annual data):

$$\begin{aligned} E[(\Delta w_t)^2] &= E[(\Delta u_t + v_t)^2] \\ &= E[(\Delta u_t)^2] + E[(v_t)^2] \\ &= 2\sigma_u^2 + \sigma_v^2. \end{aligned}$$

The following formulas are used repeatedly in deriving the moment conditions:

$$\begin{aligned} E[(\Delta^2 u_{jt})^2] &= 2\sigma_{u_j}^2 \\ E[(\Delta v_{jt})^2] &= 2\sigma_{v_j}^2 \\ E[\Delta^2 u_{1t} \Delta^2 u_{2t}] &= 2\sigma_{u_1 u_2} \\ E[\Delta v_{1t} \Delta v_{2t}] &= 2\sigma_{v_1 v_2} \\ E[\Delta^2 u_{jt} \Delta^2 u_{jt-2}] &= -\sigma_{u_j}^2 \\ E[\Delta v_{jt} \Delta v_{jt-2}] &= 0 \\ E[\Delta^2 u_{1t} \Delta^2 u_{2t-2}] &= -\sigma_{u_1,u_2} = E[\Delta^2 u_{2t} \Delta^2 u_{1t-2}] \\ E[\Delta v_{1t} \Delta v_{2t-2}] &= 0 = E[\Delta v_{2t} \Delta v_{1t-2}]. \end{aligned}$$

They are easy to prove. For example, the first equation:

$$E[(\Delta^2 u_{jt})^2] = E[(u_{jt} - u_{jt-2})(u_{jt} - u_{jt-2})] = E[(u_{jt})^2] + E[(u_{jt-2})^2] = 2\sigma_{u_j}^2$$

Other equations are proved similarly.

For a general variable x, where x can be c_1, c_2, y_1, y_2, h_1, h_2, or g, write the transmission equation from wage shocks $\{u_{1t}, u_{2t}, v_{1t}, v_{2t}\}$ to variable x as

$$\Delta x_t = \kappa_{x,u_1} \Delta u_{1t} + \kappa_{x,u_2} \Delta u_{2t} + \kappa_{x,v_1} v_{1t} + \kappa_{x,v_2} v_{2t}$$

Again, as the PSID is biennial, what I measure is actually $\Delta^2 x_t$, the transmission equation of which is

$$
\begin{aligned}
\Delta^2 x_t &= \Delta x_t - \Delta x_{t-1} \\
&= \kappa_{x,u_1}\Delta^2 u_{1t} + \kappa_{x,u_2}\Delta^2 u_{2t} + \kappa_{x,v_1}\Delta v_{1t} + \kappa_{x,v_2}\Delta v_{2t}
\end{aligned}
$$

First, I have the following moments of x itself:

$$
\begin{aligned}
E[(\Delta^2 x_t)^2] &= E[(\kappa_{x,u_1}\Delta^2 u_{1t} + \kappa_{x,u_2}\Delta^2 u_{2t} + \kappa_{x,v_1}\Delta v_{1t} + \kappa_{x,v_2}\Delta v_{2t})^2] \\
&= \kappa_{x,u_1}^2 E[(\Delta^2 u_{1t})^2] + \kappa_{x,u_2}^2 E[(\Delta^2 u_{2t})^2] + \kappa_{x,v_1}^2 E[(\Delta v_{1t})^2] + \kappa_{x,v_2}^2 E[(\Delta v_{2t})^2] \\
&\quad + 2\kappa_{x,u_1}\kappa_{x,u_2} E[\Delta^2 u_{1t}\Delta^2 u_{2t}] + 2\kappa_{x,v_1}\kappa_{x,v_2} E[\Delta v_{1t}\Delta v_{2t}] \\
&= 2\kappa_{x,u_1}^2\sigma_{u_1}^2 + 2\kappa_{x,u_2}^2\sigma_{u_2}^2 + 2\kappa_{x,v_1}^2\sigma_{v_1}^2 + 2\kappa_{x,v_2}^2\sigma_{v_2}^2 \\
&\qquad + 4\kappa_{x,u_1}\kappa_{x,u_2}\sigma_{u_1u_2} + 4\kappa_{x,v_1}\kappa_{x,v_2}\sigma_{v_1v_2} \\
E[(\Delta^2 x_t)(\Delta^2 x_{t-2})] &= E[(\kappa_{x,u_1}\Delta^2 u_{1t} + \kappa_{x,u_2}\Delta^2 u_{2t} + \kappa_{x,v_1}\Delta v_{1t} + \kappa_{x,v_2}\Delta v_{2t}) * \\
&\qquad (\kappa_{x,u_1}\Delta^2 u_{1t-2} + \kappa_{x,u_2}\Delta^2 u_{2t-2} + \kappa_{x,v_1}\Delta v_{1t-2} + \kappa_{x,v_2}\Delta v_{2t-2})] \\
&= \kappa_{x,u_1}^2 E[\Delta^2 u_{1t}\Delta^2 u_{1t-2}] + \kappa_{x,u_2}^2 E[\Delta^2 u_{2t}\Delta^2 u_{2t-2}] \\
&\qquad + \kappa_{x,u_1}\kappa_{x,u_2} E[\Delta^2 u_{1t}\Delta^2 u_{2t-2} + \Delta^2 u_{2t}\Delta^2 u_{1t-2}] \\
&= -\kappa_{x,u_1}^2\sigma_{u_1}^2 - \kappa_{x,u_2}^2\sigma_{u_2}^2 - 2\kappa_{x,u_1}\kappa_{x,u_2}\sigma_{u_1u_2},
\end{aligned}
$$

and the cross moments between the variable x and the wages:

$$
\begin{aligned}
E[(\Delta^2 w_{1t})(\Delta^2 x_t)] &= E[(\Delta^2 u_{1t} + \Delta v_{1t})(\kappa_{x,u_1}\Delta^2 u_{1t} + \kappa_{x,u_2}\Delta^2 u_{2t} + \kappa_{x,v_1}\Delta v_{1t} + \kappa_{x,v_2}\Delta v_{2t})] \\
&= \kappa_{x,u_1} E[(\Delta^2 u_{1t})^2] + \kappa_{x,u_2} E[\Delta^2 u_{1t}\Delta^2 u_{2t}] + \kappa_{x,v_1} E[(\Delta v_{1t})^2] + \kappa_{x,v_2} E[\Delta v_{1t}\Delta v_{2t}] \\
&= 2\kappa_{x,u_1}\sigma_{u_1}^2 + 2\kappa_{x,u_2}\sigma_{u_1u_2} + 2\kappa_{x,v_1}\sigma_{v_1}^2 + 2\kappa_{x,v_2}\sigma_{v_1v_2}
\end{aligned}
$$

$$
\begin{aligned}
E[(\Delta^2 w_{2t})(\Delta^2 x_t)] &= E[(\Delta^2 u_{2t} + v_{2t})(\kappa_{x,u_1}\Delta^2 u_{1t} + \kappa_{x,u_2}\Delta^2 u_{2t} + \kappa_{x,v_1}\Delta v_{1t} + \kappa_{x,v_2}\Delta v_{2t})] \\
&= \kappa_{x,u_1} E[\Delta^2 u_{1t}\Delta^2 u_{2t}] + \kappa_{x,u_2} E[(\Delta^2 u_{2t})^2] + \kappa_{x,v_1} E[\Delta v_{1t}\Delta v_{2t}] + \kappa_{x,v_2} E[(\Delta v_{2t})^2] \\
&= 2\kappa_{x,u_1}\sigma_{u_1u_2} + 2\kappa_{x,u_2}\sigma_{u_2}^2 + 2\kappa_{x,v_1}\sigma_{v_1v_2} + 2\kappa_{x,v_2}\sigma_{v_2}^2
\end{aligned}
$$

$$\begin{aligned}
E[(\Delta^2 x_t)(\Delta^2 w_{1t-2})] &= E[(\kappa_{x,u_1}\Delta^2 u_{1t} + \kappa_{x,u_2}\Delta^2 u_{2t} + \kappa_{x,v_1}\Delta v_{1t} + \kappa_{x,v_2}\Delta v_{2t})(\Delta^2 u_{1t-2} + \Delta v_{1t-2})] \\
&= \kappa_{x,u_1} E[\Delta^2 u_{1t}\Delta^2 u_{1t-2}] + \kappa_{x,u_2} E[\Delta^2 u_{2t}\Delta^2 u_{1t-2}] \\
&= -\kappa_{x,u_1}\sigma^2_{u_1} - \kappa_{x,u_2}\sigma_{u_1 u_2} \\
E[(\Delta^2 w_t)(\Delta^2 x_{1t-2})] &= -\kappa_{x,u_1}\sigma^2_{u_1} - \kappa_{x,u_2}\sigma_{u_1 u_2} \\
E[(\Delta^2 x_t)(\Delta^2 w_{2t-2})] &= -\kappa_{x,u_1}\sigma_{u_1 u_2} - \kappa_{x,u_2}\sigma^2_{u_2} \\
E[(\Delta^2 w_t)(\Delta^2 x_{2t-2})] &= -\kappa_{x,u_1}\sigma_{u_1 u_2} - \kappa_{x,u_2}\sigma^2_{u_2}.
\end{aligned}$$

Let z_t be another endogenous variable different from x_t. The covariance between x and z can also be used for identifying the transmission parameters of x and z:

$$\begin{aligned}
&E[(\Delta^2 x_t)(\Delta^2 z_t)] \\
&= E[(\kappa_{x,u_1}\Delta^2 u_{1t} + \kappa_{x,u_2}\Delta^2 u_{2t} + \kappa_{x,v_1}\Delta v_{1t} + \kappa_{x,v_2}\Delta v_{2t}) * \\
&\quad (\kappa_{z,u_1}\Delta^2 u_{1t} + \kappa_{z,u_2}\Delta^2 u_{2t} + \kappa_{z,v_1}\Delta v_{1t} + \kappa_{z,v_2}\Delta v_{2t})] \\
&= \kappa_{x,u_1}\kappa_{z,u_1} E[(\Delta^2 u_{1t})^2] + \kappa_{x,u_2}\kappa_{z,u_2} E[(\Delta^2 u_{2t})^2] + (\kappa_{x,u_1}\kappa_{z,u_2} + \kappa_{x,u_2}\kappa_{z,u_1}) E[\Delta^2 u_{1t}\Delta^2 u_{2t}] \\
&\quad + \kappa_{x,v_1}\kappa_{z,v_1} E[(\Delta v_{1t})^2] + \kappa_{x,v_2}\kappa_{z,v_2} E[(\Delta v_{2t})^2] + (\kappa_{x,v_1}\kappa_{z,v_2} + \kappa_{x,v_2}\kappa_{z,v_1}) E[\Delta v_{1t}\Delta v_{2t}] \\
&= 2\kappa_{x,u_1}\kappa_{z,u_1}\sigma^2_{u_1} + 2\kappa_{x,u_2}\kappa_{z,u_2}\sigma^2_{u_2} + 2(\kappa_{x,u_1}\kappa_{z,u_2} + \kappa_{x,u_2}\kappa_{z,u_1})\sigma_{u_1 u_2} \\
&\quad + 2\kappa_{x,v_1}\kappa_{z,v_1}\sigma^2_{v_1} + 2\kappa_{x,v_2}\kappa_{z,v_2}\sigma^2_{v_2} + 2(\kappa_{x,v_1}\kappa_{z,v_2} + \kappa_{x,v_2}\kappa_{z,v_1})\sigma_{v_1 v_2} \\
&E[(\Delta^2 x_t)(\Delta^2 z_{t-2})] \\
&= E\left[(\kappa_{x,u_1}\Delta^2 u_{1t} + \kappa_{x,u_2}\Delta^2 u_{2t} + \kappa_{x,v_1}\Delta v_{1t} + \kappa_{x,v_2}\Delta v_{2t}) * \right. \\
&\quad \left. (\kappa_{z,u_1}\Delta^2 u_{1t-2} + \kappa_{z,u_2}\Delta^2 u_{2t-2} + \kappa_{z,v_1}\Delta v_{1t-2} + \kappa_{z,v_2}\Delta v_{2t-2})\right] \\
&= \kappa_{x,u_1}\kappa_{z,u_1} E[\Delta^2 u_{1t}\Delta^2 u_{1t-2}] + \kappa_{x,u_2}\kappa_{z,u_2} E[\Delta^2 u_{2t}\Delta^2 u_{2t-2}] \\
&\quad + \kappa_{x,u_1}\kappa_{z,u_2} E[\Delta^2 u_{1t}\Delta^2 u_{2t-2}] + \kappa_{x,u_2}\kappa_{z,u_1} E[\Delta^2 u_{2t}\Delta^2 u_{1t-2}] \\
&\quad + \kappa_{x,v_1}\kappa_{z,v_1} E[\Delta v_{1t}\Delta v_{1t-2}] + \kappa_{x,v_2}\kappa_{z,v_2} E[\Delta v_{2t}\Delta v_{2t-2}] \\
&\quad + \kappa_{x,v_1}\kappa_{z,v_2} E[\Delta v_{1t}\Delta v_{2t-2}] + \kappa_{x,v_2}\kappa_{z,v_1} E[\Delta v_{1t-2}\Delta v_{2t}] \\
&= -\kappa_{x,u_1}\kappa_{z,u_1}\sigma^2_{u_1} - \kappa_{x,u_2}\kappa_{z,u_2}\sigma^2_{u_2} - (\kappa_{x,u_1}\kappa_{z,u_2} + \kappa_{x,u_2}\kappa_{z,u_1})\sigma_{u_1 u_2} \\
&E[(\Delta^2 z_t)(\Delta^2 x_{t-2})] \\
&= -\kappa_{x,u_1}\kappa_{z,u_1}\sigma^2_{u_1} - \kappa_{x,u_2}\kappa_{z,u_2}\sigma^2_{u_2} - (\kappa_{x,u_1}\kappa_{z,u_2} + \kappa_{x,u_2}\kappa_{z,u_1})\sigma_{u_1 u_2}
\end{aligned}$$

To summarize:

The own moments of x_t provide 2 moment conditions: $E[(\Delta^2 x_t)^2]$ and $E[(\Delta^2 x_t)(\Delta^2 x_{t-2})]$.

The cross moments of x_t and w_{jt} provide 6 moment conditions: $E[(\Delta^2 x_t)(\Delta^2 w_{1t})]$, $E[(\Delta^2 x_t)(\Delta^2 w_{2t})]$, $E[(\Delta^2 x_t)(\Delta^2 w_{1t-2})]$, $E[(\Delta^2 w_t)(\Delta^2 x_{1t-2})]$, $E[(\Delta^2 x_t)(\Delta^2 w_{2t-2})]$, and $E[(\Delta^2 w_t)(\Delta^2 x_{2t-2})]$.

These together provide 8 moment conditions, which already over-identify the 4 transmission parameters from wage shocks to x_t.

If the tranmission parameters of x_t and z_t are estimated jointly, in addition to those 8×2 moment conditions, we can also use 3 cross moments of x_t and z_t: $E[(\Delta^2 x_t)(\Delta^2 z_t)]$, $E[(\Delta^2 x_t)(\Delta^2 z_{t-2})]$, and $E[(\Delta^2 z_t)(\Delta^2 x_{t-2})]$. In total, we have 19 moment conditions to identify 8 parameters.

In general, suppose we have n endogenous variables and we estimate the transmission parameters of them jointly. The moment conditions are:

1. $2n$ own moments;
2. $6n$ cross moments with w_{jt};
3. $3 \times C_2^n$ cross moments between any two endogenous variables.

In total this gives $8n + 3 \times C_2^n$ moment conditions for identifying $4n$ transmission parameters, as illustrated in the following table.

Number of endogeous variables n	Number of parameters $4n$	Number of moments $8n + 3 \times C_2^n$
1	4	8
2	8	19
3	12	33
4	16	50
5	20	70

For my application, there are 5 endogenous variables $\{c_1, c_2, g, y_1, y_2\}$, and thus I use 70 moments in the GMM estimation.

A.4 Moment Conditions with Measurement Errors

With measurement errors taken into account, the moment conditions presented in Appendix A.3 are no longer precise. Denote ξ_x be the measurement error of variable x. Then for $j = 1, 2$,

$$\Delta^2 w_{jt} = \Delta^2 u_{jt} + \Delta v_{jt} + \Delta^2 \xi_{w_j t}$$

and

$$\Delta^2 x_t = \kappa_{x,u_1} \Delta^2 u_{1t} + \kappa_{x,u_2} \Delta^2 u_{2t} + \kappa_{x,v_1} \Delta v_{1t} + \kappa_{x,v_2} \Delta v_{2t} + \Delta^2 \xi_{xt}$$

Some moment conditions need to be adjusted for measurement errors. For example, the wage moments: for $j = 1, 2$,

$$\begin{aligned}
E[(\Delta^2 w_{jt})^2] &= E[(\Delta^2 u_{jt} + \Delta v_{jt} + \Delta^2 \xi_{w_j t})^2] \\
&= E[(\Delta^2 u_{jt})^2] + E[(\Delta v_{jt})^2] + E[(\Delta^2 \xi_{w_j t})^2] \\
&= 2\sigma^2_{u_j} + 2\sigma^2_{v_j} + 2\sigma^2_{\xi_{w_j}} \\
E[(\Delta^2 w_{jt})(\Delta^2 w_{jt-2})] &= E[(\Delta^2 u_{jt} + \Delta v_{jt} + \Delta^2 \xi_{w_j t})(\Delta^2 u_{jt-2} + \Delta v_{jt-2} + \Delta^2 \xi_{w_j t-2})] \\
&= -\sigma^2_{u_j} - \sigma^2_{\xi_{w_j}}.
\end{aligned}$$

Own moments of x:

$$\begin{aligned}
E[(\Delta^2 x_t)^2] &= E[(\kappa_{x,u_1}\Delta^2 u_{1t} + \kappa_{x,u_2}\Delta^2 u_{2t} + \kappa_{x,v_1}\Delta v_{1t} + \kappa_{x,v_2}\Delta v_{2t} + \Delta^2 \xi_{xt})^2] \\
&= \kappa^2_{x,u_1} E[(\Delta^2 u_{1t})^2] + \kappa^2_{x,u_2} E[(\Delta^2 u_{2t})^2] + \kappa^2_{x,v_1} E[(\Delta v_{1t})^2] + \kappa^2_{x,v_2} E[(\Delta v_{2t})^2] \\
&\quad + 2\kappa_{x,u_1}\kappa_{x,u_2} E[\Delta^2 u_{1t}\Delta^2 u_{2t}] + 2\kappa_{x,v_1}\kappa_{x,v_2} E[\Delta v_{1t}\Delta v_{2t}] + E[(\Delta^2 \xi_{xt})^2] \\
&= 2\kappa^2_{x,u_1}\sigma^2_{u_1} + 2\kappa^2_{x,u_2}\sigma^2_{u_2} + 2\kappa^2_{x,v_1}\sigma^2_{v_1} + 2\kappa^2_{x,v_2}\sigma^2_{v_2} \\
&\quad + 4\kappa_{x,u_1}\kappa_{x,u_2}\sigma_{u_1 u_2} + 4\kappa_{x,v_1}\kappa_{x,v_2}\sigma_{v_1 v_2} + 2\sigma^2_{\xi_x} \\
E[(\Delta^2 x_t)(\Delta^2 x_{t-2})] &= E[(\kappa_{x,u_1}\Delta^2 u_{1t} + \kappa_{x,u_2}\Delta^2 u_{2t} + \kappa_{x,v_1}\Delta v_{1t} + \kappa_{x,v_2}\Delta v_{2t} + \Delta^2 \xi_{xt}) * \\
&\qquad (\kappa_{x,u_1}\Delta^2 u_{1t-2} + \kappa_{x,u_2}\Delta^2 u_{2t-2} + \kappa_{x,v_1}\Delta v_{1t-2} + \kappa_{x,v_2}\Delta v_{2t-2} + \Delta^2 \xi_{xt-2})] \\
&= \kappa^2_{x,u_1} E[\Delta^2 u_{1t}\Delta^2 u_{1t-2}] + \kappa^2_{x,u_2} E[\Delta^2 u_{2t}\Delta^2 u_{2t-2}] \\
&\quad + \kappa_{x,u_1}\kappa_{x,u_2} E\left[\Delta^2 u_{1t}\Delta^2 u_{2t-2} + \Delta^2 u_{2t}\Delta^2 u_{1t-2}\right] + E[\Delta^2 \xi_{xt}\Delta^2 \xi_{xt-2}] \\
&= -\kappa^2_{x,u_1}\sigma^2_{u_1} - \kappa^2_{x,u_2}\sigma^2_{u_2} - 2\kappa_{x,u_1}\kappa_{x,u_2}\sigma_{u_1 u_2} - \sigma^2_{\xi_x}
\end{aligned}$$

I assume that the measurement errors of consumption are uncorrelated with the measurement errors of earnings or those of wages. This assumption implies that the cross moments between consumption and earnings, and the cross moments between consumption and wages, have the same expressions as in the case of no measurement errors. I also assume that the measurement errors of the husband's variable (wage, earnings, ...) and the measurement errors of the wife's variable are uncorrelated. Thus, the cross moments between the husband's variable and the wife's variable are also unchanged.

The cross moments between wages and earnings change. Due to the fact that $w = y - h$ and thus $\xi_w = \xi_y - \xi_h$, we have

$$\begin{aligned}
Var(\xi_w) &= Var(\xi_y) + Var(\xi_h) - 2Cov(\xi_y, \xi_h), \\
Var(\xi_h) &= Var(\xi_y) + Var(\xi_w) - 2Cov(\xi_y, \xi_w).
\end{aligned}$$

The cross moments between wages and earnings become:

$$\begin{aligned}
E[(\Delta^2 w_{jt})(\Delta^2 y_{jt})] =& 2\kappa_{y_j,u_1}\sigma_{u_1}^2 + 2\kappa_{y_j,u_2}\sigma_{u_1u_2} + 2\kappa_{y_j,v_1}\sigma_{v_1}^2 + 2\kappa_{y_j,v_2}\sigma_{v_1v_2} + E[\Delta^2\xi_{w_jt}\Delta^2\xi_{y_jt}] \\
=& 2\kappa_{y_j,u_1}\sigma_{u_1}^2 + 2\kappa_{y_j,u_2}\sigma_{u_1u_2} + 2\kappa_{y_j,v_1}\sigma_{v_1}^2 + 2\kappa_{y_j,v_2}\sigma_{v_1v_2} + 2Cov(\xi_{w_j}, \xi_{y_j}) \\
=& 2\kappa_{y_j,u_1}\sigma_{u_1}^2 + 2\kappa_{y_j,u_2}\sigma_{u_1u_2} + 2\kappa_{y_j,v_1}\sigma_{v_1}^2 + 2\kappa_{y_j,v_2}\sigma_{v_1v_2} \\
& + Var(\xi_{w_j}) + Var(\xi_{y_j}) - Var(\xi_{h_j}) \\
=& 2\kappa_{y_j,u_1}\sigma_{u_1}^2 + 2\kappa_{y_j,u_2}\sigma_{u_1u_2} + 2\kappa_{y_j,v_1}\sigma_{v_1}^2 + 2\kappa_{y_j,v_2}\sigma_{v_1v_2} \\
& + 2Var(\xi_{y_j}) - 2Cov(\xi_{y_j}, \xi_{h_j}) \\
E[(\Delta^2 w_{jt})(\Delta^2 y_{jt-2})] =& -\kappa_{y_j,u_j}\sigma_{u_j}^2 - \kappa_{y_j,u_{-j}}\sigma_{u_j,u_{-j}} + E[\Delta^2\xi_{w_jt}\Delta^2\xi_{y_jt-2}] \\
=& -\kappa_{y_j,u_j}\sigma_{u_j}^2 - \kappa_{y_j,u_{-j}}\sigma_{u_j,u_{-j}} - Cov(\xi_{w_j}, \xi_{y_j}) \\
=& -\kappa_{y_j,u_j}\sigma_{u_j}^2 - \kappa_{y_j,u_{-j}}\sigma_{u_j,u_{-j}} - Var(\xi_{y_j}) + Cov(\xi_{y_j}, \xi_{h_j})
\end{aligned}$$

where the variances of ξ_w, ξ_y, ξ_h and their covariances are obtained by the the method described in Section 2.3.3.

Appendix B

Appendix for Chapter 3

B.1 Normalized Problem with Unit-Root Labor Income

In the case in which the (log) income process has a unit root and the felicity function has the CRRA form $u(c) = c^{1-\gamma}/(1-\gamma)$, it is well known from Carroll (2004) that it is possible to normalize problem (3.17) by (permanent) labor income y_t, thereby reducing the effective state space to z_t.

To see this, replace for $c_t = z_t - a_t$ in (3.17) and consider the problem in the second-to-last period

$$\mathbb{V}_{T-1}(z_{T-1}, y_{T-1}) = \max_{a_{T-1}} u(z_{T-1} - a_{T-1}) + \beta \mathbb{E}_{T-1} u(z_T) \tag{B.1}$$

If one defines the state variables $\hat{z}_t = z_t/y_t$ and $\hat{a}_t = a_t/y_t$, equation (B.1) can be rewritten as

$$\begin{aligned} \mathbb{V}_{T-1}(z_{T-1}, y_{T-1}) &= \max_{\hat{a}_{T-1}} u(y_{T-1}(\hat{z}_{T-1} - \hat{a}_{T-1})) + \beta \mathbb{E}_{T-1} u(y_T \hat{z}_T) \\ &= y_{T-1}^{1-\gamma} \left\{ \max_{\hat{a}_{T-1}} u(\hat{z}_{T-1} - \hat{a}_{T-1}) + \beta \mathbb{E}_{T-1} \epsilon_T^{1-\gamma} u(\hat{z}_T) \right\} \end{aligned} \tag{B.2}$$

Note that by definition

$$\hat{z}_t = (1+r) \frac{a_{t-1}}{y_{t-1}\epsilon_t} + 1 = (1+r) \frac{\hat{a}_{t-1}}{\epsilon_t} + 1, \tag{B.3}$$

which implies that the curly bracket in (B.2) is equal to $V_{T-1}(\hat{z}_{T-1})$ where the latter satisfies the Bellman equation

$$V_{T-1}(\hat{z}_{T-1}) = \max_{\hat{a}_{T-1}} u(\hat{z}_{T-1} - \hat{a}_{T-1}) + \beta \mathbb{E}_{T-1} \epsilon_T^{1-\gamma} V_T(\hat{z}_T) \tag{B.4}$$

with $V_T(\hat{z}_T) = u(\hat{z}_T)$.

Equations (B.2) and (B.4) imply that $\mathbb{V}_{T-1}(z_{T-1}, y_{T-1}) = y_{T-1}^{1-\gamma} V(\hat{z}_{T-1})$. The same logic implies that this holds also for any $t < T-1$.

Therefore the Bellman equation for the problem in normalized form satisfies

$$V_t(\hat{z}_t) = \max_{\hat{a}_t} u(\hat{z}_t - \hat{a}_t) + \beta \mathbb{E}_t \epsilon_{t+1}^{1-\gamma} V_{t+1}(\hat{z}_{t+1}), \tag{B.5}$$

for all t. It follows from (B.3) and the envelope condition that the associated Euler equation is

$$u'(\hat{c}_t) = \beta R \mathbb{E}\left[\epsilon_{t+1}^{-\rho} u'(\hat{c}_{t+1})\right] \tag{B.6}$$

The advantage of the normalized problem (B.4) is that one can solve for the saving function $\hat{a}_t(\hat{z}_t)$ which is independent of the income realization y_t and use $a_t(z_t, y_t) = \hat{a}_t(\hat{z}_t) y_t$ to recover the policy function for a_t.

Under the assumption that ϵ_t is i.i.d. and log-normally distributed the expectation in equation (B.4) can be computed using Gaussian Hermite quadrature.