Essays on Time-Varying Volatility and Structural Breaks in Macroeconomics and Econometrics

by

Nyamekye Asare

A thesis submitted to the
Faculty of Graduate and Postdoctoral Studies
in partial fulfillment of the requirement for the degree of
Doctor of Philosophy in Economics

Department of Economics
Faculty of Social Sciences
University of Ottawa

Acknowledgements

I would like to thank my thesis co-supervisors Hashmat Khan and Kathleen Day for their comments, direction, suggestions, and advice for this thesis.

I would also like to thank Thesis Committee Members, Lilia Karnizova, for her comments from a macroeconomic perspective, and especially Lynda Khalaf, for her comments and advice from an econometric perspective. Professor Khalaf has provided me encouragement throughout my Ph.D. program and always made time for me whenever I needed her advice.

I am thankful to Thesis Committee Member Rafal Kulik from the University of Ottawa Mathematics Department and Mohamedou Ould Haye from Carleton University Mathematics and Statistics Department for their consultation and advice for my last two chapters. I thank the External Examiner, Dalibor Stevanovic from the Université du Québec à Montréal for his comments and advice for this thesis.

My gratitude goes to Aggey Semenov and Louis-Philippe Morin for their advice. I would also like to thank Catherine Deri-Armstrong, Jean-Francois Tremblay and their first-year seminar classes for providing insightful comments for my thesis. A special thanks to Susan Mowers for her assistance at the University of Ottawa Library locating data needed for the thesis.

Dedication

This is dedicated to my mother Perpetua Asare and my father Kweku Asare for their prayers, love, and support and to the memory of my grandfather, Dr.Atu Mensah Taylor, who graduated from Oxford University with a Ph.D. in Mathematics, a true inspiration to me.

Table of Contents

List of Tables

List of Figures

Chapter 1

Introduction

This thesis is comprised of three independent essays. One essay is in the field of macroeconomics and the other two are in time-series econometrics. The first essay, “Productivity and Business Investment over the Business Cycle”, is co-authored with my co-supervisor Hashmat Khan. This essay documents a new stylized fact: the correlation between labour productivity and real business investment in the U.S. data switching from 0.54 to -0.1 in 1990. With the assistance of a bivariate VAR, we find that the response of investment to identified technology shocks has changed signs from positive to negative across two subperiods: ranging from the time of the post-WWII era to the end of 1980s and from 1990 onwards, whereas the response to non-technology shocks has remained relatively unchanged. Also, the volatility of technology shocks declined less relative to the non-technology shocks. This raises the question of whether relatively more volatile technology shocks and the negative response of investment can together account for the decreased correlation. To answer this question, we consider a canonical DSGE model and simulate data under a variety of assumptions about the parameters representing structural features and volatility of shocks.

The second and third essays are in time series econometrics and solely authored by myself. The second essay, however, focuses on the impact of ignoring structural breaks in the conditional volatility parameters on time-varying volatility parameters. The focal point of the third essay is on empirical relevance of structural breaks in time-varying volatility models and the forecasting gains of accommodating structural breaks in the unconditional variance.

There are several ways in modeling time-varying volatility. One way is to use the autoregressive conditional heteroskedasticity (ARCH)/generalized ARCH (GARCH) class first introduced by Engle (1982) and Bollerslev (1986). One prominent model is Bollerslev's

(1986) GARCH model in which the conditional volatility is updated by its own residuals and its lags. This class of models is popular amongst practitioners in finance because they are able to capture stylized facts about asset returns such as fat tails and volatility clustering (Engle and Patton, 2001; Zivot, 2009) and require maximum likelihood methods for estimation. They also perform well in forecasting volatility. For example, Hansen and Lunde (2005) find that it is difficult to beat a simple GARCH(1,1) model in forecasting exchange rate volatility. Another way of modeling time-varying volatility is to use the class of stochastic volatility (SV) models including Taylor's (1986) autoregressive stochastic volatility (ARSV) model. With SV models, the conditional volatility is updated only by its own lags and increasingly used in macroeconomic modeling (i.e. Justiniano and Primiceri, 2010). Fernandez-Villaverde and Rubio-Ramirez (2010) claim that the stochastic volatility model fits better than the GARCH model and is easier to incorporate into DSGE models.

However, Creal et al. (2013) recently introduced a new class of models called the generalized autoregressive score (GAS) models. With the GAS volatility framework, the conditional variance is updated by the scaled score of the model's density function instead of the squared residuals. According to Creal et al. (2013), GAS models are advantageous to use because updating the conditional variance using the score of the log-density instead of the second moments can improve a model's fit to data. They are also found to be less sensitive to other forms of misspecification such as outliers. As mentioned by Maddala and Kim (1998), structural breaks are considered to be one form of outliers. This raises the question about whether GAS volatility models are less sensitive to parameter non-constancy.

This issue of ignoring structural breaks in the volatility parameters is important because neglecting breaks can cause the conditional variance to exhibit unit root behaviour in which the unconditional variance is undefined, implying that any shock to the variance will not gradually decline (Lamoureux and Lastrapes, 1990). The impact of ignoring parameter non-constancy is found in GARCH literature (see Lamoureux and Lastrapes, 1990; Hillebrand, 2005) and in SV literature (Psaradakis and Tzavalis, 1999; Krämer and Messow, 2012) in which the estimated persistence parameter overestimates its true value and approaches one. However, it has never been addressed in GAS literature until now. The second essay uses a simple Monte-Carlo simulation study to examine the impact of neglecting parameter non-constancy on the estimated persistence parameter of several GAS and non-GAS models of volatility. Five different volatility models are examined. Of these models, three –the GARCH(1,1), t-GAS(1,1), and Beta-t-EGARCH(1,1) models – are GAS models, while the other two – the t-GARCH(1,1) and EGARCH(1,1) models – are not. Following Hillebrand (2005) who studied only the GARCH model, this essay examines the extent of how biased the estimated persistence parameter are by assessing impact of ignoring breaks on the mean value of the estimated persistence parameter. The impact of neglecting parameter non-constancy on the empirical sampling distributions and coverage probabilities for the estimated persistence parameters are also studied in this essay. For the latter, studying the effect on the coverage probabilities is important because a decrease in coverage probabilities is associated with an increase in Type I error. This study has implications for forecasting. If the size of an ignored break in parameters is small, then there may not be any gains in using forecast methods that accommodate breaks.

Empirical evidence suggests that structural breaks are present in data on macro-financial variables such as oil prices and exchange rates. The potentially serious consequences of ignoring a break in GARCH parameters motivated Rapach and Strauss (2008) and Arouri et al. (2012) to study the empirical relevance of structural breaks in the context of GARCH models. However, the literature does not address the empirical relevance of structural breaks in the context of GAS models.

The third and final essay contributes to this literature by extending Rapach and Strauss (2008) to include the t-GAS model and by comparing its performance to that of two non-GAS models, the t-GARCH and SV models. The empirical relevance of structural breaks in the models of volatility is assessed using a formal test by Dufour and Torres (1998) to determine how much the estimated parameters change over sub-periods. The in-sample performance of all the models is analyzed using both the weekly USD trade-weighted index between January 1973 and October 2016 and spot oil prices based on West Texas Intermediate between January 1986 and October 2016. The full sample is split into smaller subsamples by break dates chosen based on historical events and policy changes rather than formal tests. This is because commonly-used tests such as CUSUM suffer from low power (Smith, 2008; Xu, 2013). For each sub-period, all models are estimated using either oil or USD returns. The confidence intervals are constructed for the constant of the conditional parameter and the score parameter (or ARCH parameter in GARCH and t-GARCH models). Then Dufour and Torres's (1998) union-intersection test is applied to these confidence intervals to determine how much the estimated parameter change over sub-periods. If there is a set of values that intersects the confidence intervals of all sub-periods, then one can conclude that the parameters do not change that much. The out-of-sample performance of all time-varying volatility models are also assessed in the ability to forecast the mean and variance of oil and USD returns. Through this analysis, this essay also addresses whether using models that accommodate structural breaks in the unconditional variance of both GAS and non-GAS models will improve forecasts.

Chapter 2

Productivity and Business Investment over the Business Cycle

Abstract

This paper documents a new stylized fact—the correlation between labour productivity and real business investment in the U.S. data since 1990 is -0.1. This correlation was 0.54 in the post-WWII data until the end of 1980s. The response of investment to identified technology shocks have switched signs across these two sub-periods from positive to negative, whereas the response to a non-technology shock has remained approximately the same. Since the volatility of technology shocks has decreased less relative to non-technology shock over the two sub-periods, we explore the hypothesis whether the relatively more volatile technology shocks and the negative response of investment can together account for the decreased correlation. We consider a canonical DSGE model and simulate data under a variety of assumptions about the parameters representing structural features and volatility of shocks. The results show that although the smaller decline in the volatility of technology shocks relative to non-technology shock has contributed to the decrease in the correlation, this channel alone is not sufficient. Structural features such as increased average duration of price contracts, unitary elasticity of substitution between labour and capital, and larger magnitudes of investment adjustment costs are needed for the model to produce a near-zero correlation between labour productivity and investment.

2.1 Introduction

We present a new stylized fact on the changing dynamics of U.S. business cycles.[1] The correlation between aggregate labour productivity and business investment in the post-WWII U.S. data was significantly positive (0.54) over the business cycle until the end of 1980s. Since then the correlation is statistically zero (-0.1) indicating that the two variables do not exhibit business cycle co-movement (see Table 2.1). Macroeconomic models that are widely used by academics and policy-makers deliver a strongly positive correlation. The intuition is that a positive unanticipated technology shock increases the marginal products of both labour and capital. Hence, labour productivity and investment moves together.[2] Understanding why this shift has occurred is, therefore, important from both macroeconomic model development and policy perspectives.

Examining the cyclicality of labour productivity and business investment *separately* reveals an important point about the stylized fact on their joint dynamics described above. Recently Gali and van Rens (2014) have pointed out that procylicality of labour productivity has declined in the U.S. economy.[3] We note that procyclicality of business investment, on the other hand, has increased since the 1990s. So a natural question is whether the decline in the cyclical correlation between labour productivity and business investment is simply due to the decline in cyclicality of the former. Two pieces of evidence suggest that this may not entirely be the case. First, the cyclicality of labour productivity sharply declined around 1984 (see Figure 2 in Gali and van Rens (2014)), whereas the correlation between labour productivity and business investment started to decline after 1990. Second, we present new evidence that the response of business investment to technology shocks changed from positive to negative after 1990. The the decline in the correlation between labour productivity and business investment after 1990, therefore, is likely to be related to shifts in the structural features of the U.S. economy. The question this paper seeks to answer is whether shifts in structural elements that are present in a standard Dynamic Stochastic General Equilibrium (DSGE) model, such as the real and nominal rigidities and volatility of shocks, can account for the changed co-movement between labour productivity and business investment.

Our paper is closest in spirit to Barnichon (2010) who studied the changed correla-

[1] This chapter is co-authored with my co-supervisor Hashmat Khan.

[2] More generally, other types of business cycle shocks such as investment-specific technology or marginal efficiency of investment also imply a positive co-movement.

[3] Related research on the changing cyclicality of labour productivity includes Stiroh (2009), Gordon (2010), and Barnichon (2010). Fernald and Wang (2015) provide a detailed discussion on the possible explanations for why the cyclicality of productivity has changed.

tion between productivity and unemployment before and after the mid-1980s. There are, however, two differences. First, our focus is on business investment whereas Barnichon (2010) abstracts from capital and investment. Second, Barnichon (2010) finds that the response of the variable of interest in his study— unemployment—to a technology shock did not change before and after the mid-1980s but the response to non-technology shock did. Interestingly, we find the opposite pattern. The response of the variable of interest in our study—business investment—to a technology shock changed after 1990 but not the response to a non-technology shock. This finding suggests that some structural features accounting for our stylized facts are likely to be different from the one proposed in Barnichon (2010) in the context of productivity and unemployment.

In our empirical analysis, we first establish using moving correlations of 10-year window that the decrease in correlation between labour productivity and business investment occurred since 1990. Although our focus is on total business investment, we document that the the post-1990 decrease in correlation between labour productivity and business investment occurs across all the sub-components of business investment, namely, residential, structures, machinery and equipment, and software.

Next, we estimate a bivariate VAR with labour productivity (measured as output per hour) and real business investment. We identify permanent shocks to labour productivity, labeled as technology shocks, using long-run restrictions as in Gali (1999). The impulse responses from this exercise reveal that the cyclical response of business investment to technology shocks has changed during the pre- and post-1990 periods. Since the volatility of technology shocks has decreased less relative to non-technology shock over the two sub-periods, we explore the hypothesis whether the relatively more volatile technology shocks and the negative response of investment can together account for the decreased correlation.

In our model-based analysis, we consider a canonical medium-scale DSGE model similar to Christiano et al. (2005) and Smets and Wouters (2007). Following Cantore et al. (2014), we allow a CES production function that nests the Cobb-Douglas specification. This allows us to examine the consequences of changes in the elasticity of substitution between labour and capital, in the context of our objective.

Using the DSGE model and simulate data under a variety of assumptions about the parameters representing structural features and volatility of shocks. The results show that although the smaller decline in the volatility of technology shocks relative to non-technology shock has contributed to the decrease in the correlation, this channel alone is not sufficient. Structural features such as increased average duration of price contracts, unitary elasticity of substitution between labour and capital, and larger magnitudes of investment adjustment costs are needed for the model to produce a near-zero correlation between labour productivity and investment.

The rest of the paper is organized as follows. Section 2 presents the empirical stylized facts (both unconditional and conditional). Section 3 presents the model. Section 4 presents the main results and section 5 concludes.

2.2 Empirical Analysis

In this section we provide new stylized facts on the changing co-movement between labour productivity and business investment.

2.2.1 Unconditional Correlations

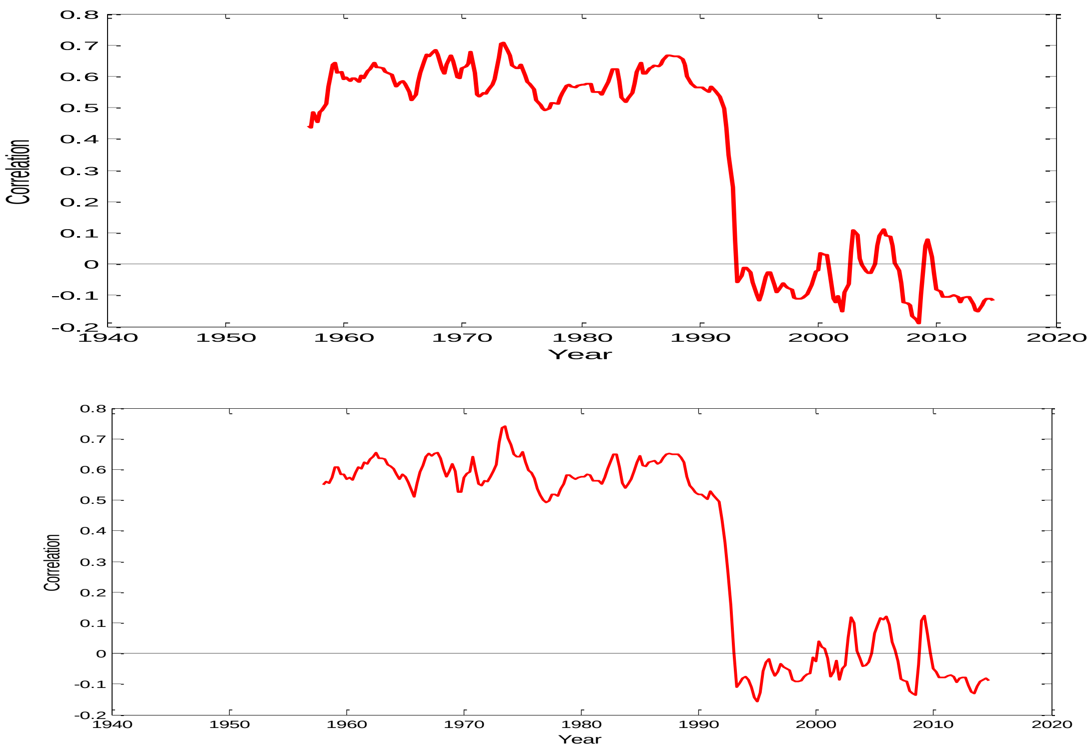
The cyclical correlation between labour productivity and real business investment is 0.38 over the 1947Q1-2014Q4 period. To study how this correlation has changed over time, we consider a moving correlation with a 10-year window. Figure 2.1 shows that the correlation between labour productivity and investment before 1990 was positive and strong. But after 1990 the correlation declines sharply and has fluctuated around -0.1 until the end of the sample period.[4] On the basis of this evidence we choose 1990 to be the break-point and divide the sample into pre- and post-1990 periods.

We split the sample into two periods 1947Q1-1989Q4 and 1990Q1-2014Q4. It is immediately evident from Table 2.1 that the moderate correlation of 0.38 for the entire period hides a change from a strong positive correlation before 1990 of 0.54 to a -0.1 correlation after 1990. The stylized facts are similar when using either the Hodrick-Prescott (HP) filter or the Baxter-King band-pass (BK) filter to extract the cyclical component.[5]

[4] An alternative definition of labour productivity, real output per person, also experienced a large change in correlation with real investment. It dropped from 0.6 before 1990 to around 0.3 afterwards. According to Santacreu (2015), real output per person captures other factors such as the composition of labour force, fertility and mortality rates. More information about the measurement can be found here - http://research.stlouisfed.org/publications/es/article/10328.

[5] Hamilton (2017) writes that the HP Filter should not be used because it produces spurious dynamics that is inconsistent with the underlying data generated process (DGP) and suggest his filter as a more robust alternative to the HP filter. So, we applied his filter to both labour productivity and investment and find that it also produces similar stylized facts about the drop in the correlation after 1990.

Figure 2.1: 10-year rolling correlation between labour productivity and real private business investment



Source: Bureau of Economic Analysis, Bureau of Labor Statistics, and FRED. The top panel represents the correlation between labour productivity and real investment. Bottom panel is the correlation between labour productivity and real investment per capita. Real private business investment by type and labour productivity measurements have been detrended using the HP filter. Both are indexes with 2009 = 100.

Table 2.1: Correlation between real investment and productivity

Filter	1947-2014	pre-1990	post-1990
	Labour productivity		
HP	0.38**	0.54**	-0.1
HP and per capita	0.18**	0.54**	-0.07
BK	0.39**	0.58**	-0.1

Labour productivity is measured as real output per hour. Significance is indicated by one asterisk (10-percent level) or two asterisks (5-percent level). Note that for real investment per capita, the period began in 1948 due to the availability of labour market data.

Did the components of business investment also experience a change in correlation with labour productivity? Table 2.2 reports the results.

Table 2.2: Correlations between real investment by type and labour productivity

Date	Residential	Structures	Machinery and Equipment	Software
1947-2014	0.50**	-0.30**	0.10*	-0.06
pre-1990	0.63**	-0.12	0.29**	-0.03
post-1990	0.09	-0.61**	-0.27**	-0.13

Data is from the Federal Reserve and Economic Data, Bureau of Economic Analysis and Bureau of Labor Statistics. Please note that all series were detrended using the HP filter. Significance is indicated by one asterisk (10-percent level) or two asterisks (5-percent level)

There are several notable stylized facts. Machinery and equipment spending experienced a sign switch in its correlation, moving from being positive before 1990 to being significantly negative after 1990. The correlation between residential investment and labour productivity dropped from being highly positive before 1990 (0.63) to becoming statistically zero after 1990 (0.09). For non-residential structures, the correlation drops from essentially zero for pre-1990 period to -0.61 in the post-1990 period. Finally, for software, the correlations are negative but not significant for both sub-periods. However, the magnitude does decrease in the post-1990 period. While there are differences in magnitudes and signs, the general pattern shows a decrease in the correlation across all four components, just as in the case of aggregate business investment shown in Table 2.1.

2.2.2 Response of business investment to technology and non-technology shocks

To help understand the source of the change in correlation, we examine the responses of labour productivity and investment to technology and non-technology shocks in the pre- and post-1990 period using the following system of equations.

$$\begin{bmatrix} \Delta x_t \\ \Delta i_t \end{bmatrix} = \Phi(L) \begin{bmatrix} \epsilon_t^x \\ \epsilon_t^i \end{bmatrix} \tag{2.1}$$

where x represents labour productivity and i denotes real investment, $\Phi(L)$ is the matrix of lagged polynomials, and ϵ_t^x and ϵ_t^i represent the technology and non-technology shocks respectively. Equation (2.1) can be written equivalently as

$$\begin{aligned} \Delta x_t &= \textstyle\sum_{j=1}^4 \beta_{xx,j} \Delta x_{t-j} + \sum_{j=1}^4 \beta_{xi,j} \Delta i_{t-j} + \epsilon_t^x \\ \Delta i_t &= \textstyle\sum_{j=1}^4 \beta_{ix,j} \Delta x_{t-j} + \sum_{j=1}^4 \beta_{ii,j} \Delta i_{t-j} + \kappa \epsilon_t^x + \epsilon_t^i \end{aligned} \tag{2.2}$$

We use the long-run identification assumption of Gali (1999) that only a technology shock has a permanent effect on the level of labour productivity.[6] This means that the long-run multiplier from investment to labour productivity is equal to zero. Using Shapiro and Watson's (1988) methodology, we impose the long-run restriction which is equivalent to allowing the second difference of investment to enter into the first equation. However, the innovations of labour productivity growth affect the contemporaneous values of investment growth, so we cannot estimate the model described in equation 2.2 using OLS.

Following Shapiro and Watson (1988) and Francis and Ramey (2004), the identification of technology shocks is achieved based on:

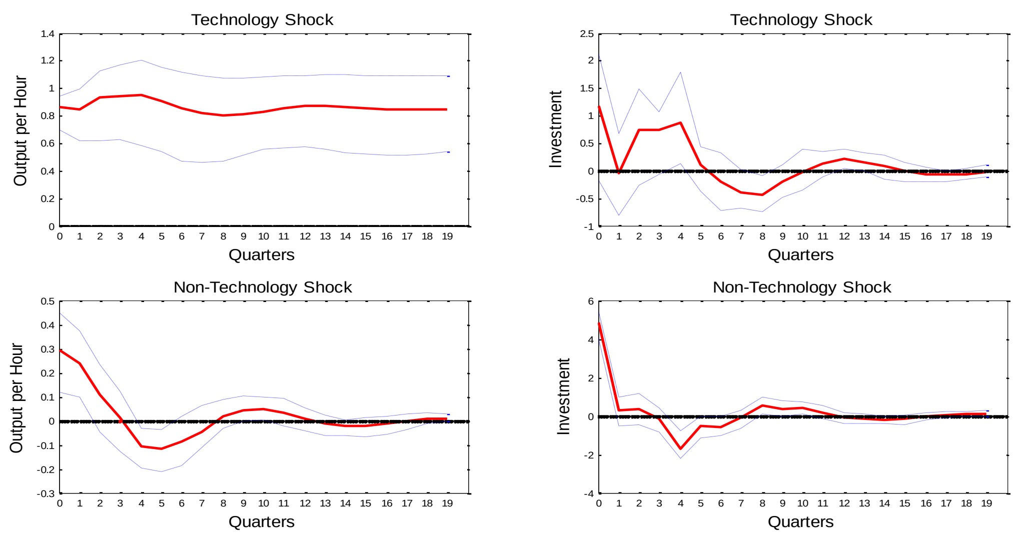
$$\begin{aligned} \Delta x_t &= \textstyle\sum_{j=1}^4 \beta_{xx,j} \Delta x_{t-j} + \sum_{j=0}^3 \beta_{xi,j} \Delta^2 i_{t-j} + \epsilon_t^x \\ \Delta i_t &= \textstyle\sum_{j=1}^4 \beta_{ix,j} \Delta x_{t-j} + \sum_{j=1}^4 \beta_{ii,j} \Delta i_{t-j} + \kappa \epsilon_t^x + \epsilon_t^i \end{aligned} \tag{2.3}$$

Since Δi_t is correlated with the error term ϵ_t^x, we use the Shapiro and Watson (1988) methodology by first taking 4 lags of Δx_t and Δi_t as instruments and estimating the first equation of (2) to obtain the residuals $\hat{\epsilon}_t^x$. Then, estimating the second equation of (2) through ordinary least squares (OLS) by replacing ϵ_t^x with $\hat{\epsilon}_t^x$, making it identifiable. Following Francis and Ramey (2004) and Barnichon (2010), the generalized method of moments (GMM) estimation is employed to jointly estimate both equations of (2).

These impulse responses are presented in Figures 2.2 and 2.3 for the pre- and post-1990 periods respectively.

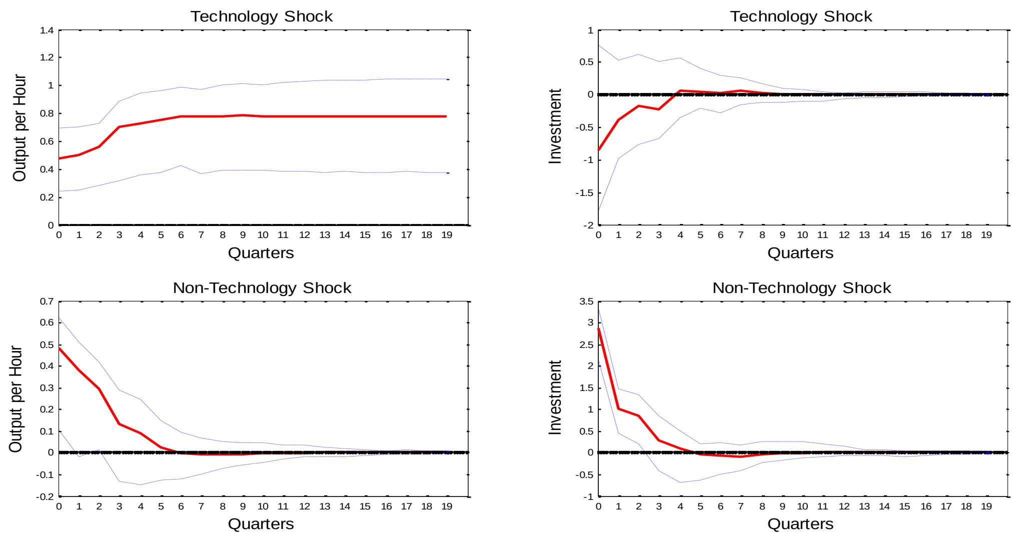
[6] We find using the Schwartz-Bayes Criterion and Akaike Information Criterion that using four lags provided the best fit for the model.

Figure 2.2: Pre-1990 empirical impulse response functions to technology and non-technology shocks:



Red line indicates the point estimate and the blue lines represent the 95% confidence interval

Figure 2.3: Post-1990 empirical impulse response functions to technology and non-technology shocks:



Red line indicates the point estimate and the blue lines represent the 95% confidence interval

The first row of Figures 2.2 and 2.3 reveals that the response of business investment to technology shock changed sharply across the two sub-periods. A 0.9% increase in labour productivity was associated with a 1 % *increase* in investment before 1990. By contrast, a 0.5 % increase in labour productivity is associated with a 1% *decrease* in investment in the post-1990 period. The second row of Figures 2.2 and 2.3 displays the effects of a non-technology shock. The response of productivity and investment to non-technology shocks after 1990 were relatively unchanged from the before 1990 period. The jump in productivity is slightly larger in the post-1990 period than in the pre-1990. However, for investment, the impact response is relatively small. Before 1990, a 0.3% increase in labour productivity is associated with a 5% increase in investment. After 1990, about a 0.5% increase in labour productivity is associated with close to 3% increase in investment.

Decline in the volatility of shocks

Barnichon (2010) documented although there has been a decline in the volatility of both technology and non-technology shocks, the latter volatility has fallen relatively more. We use a similar technique and compute the volatility of shocks measured as a 5-year rolling standard deviation. Figures 2.4 and 2.5 show the volatilities for technology and non-technology shocks, respectively, identified using the VAR in the previous section. The results show that the volatilities of technology and non-technology shocks have declined after the mid-1980s and early 1990s respectively. The technology shock volatility has decreased by 30%, and the non-technology shock has decreased by 38% after 1990. The technology shock results confirm the similar findings of Barnichon (2010). Since technology shocks generated a negative investment response, a relatively smaller decline in their volatility may be contribute to the sharp decrease in the correlation between labour productivity and business investment in the post-1990 period.

Figure 2.4: 10-year rolling volatility of technology shocks

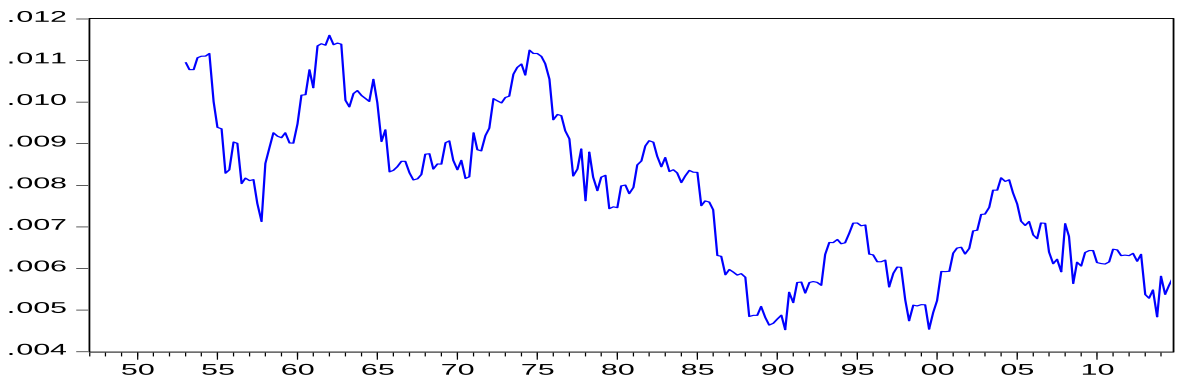
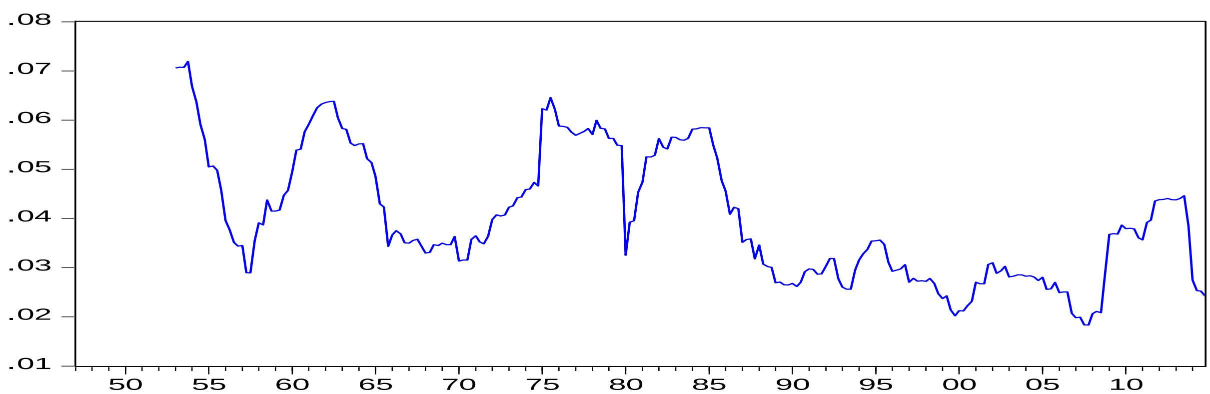


Figure 2.5: 10-year rolling volatility of non-technology shocks



2.3 Model

In this section we consider a DSGE model to study how shifts in certain structural features can deliver the near-zero correlation between cyclical labour productivity and business investment. The DSGE model is similar to the one developed in Christiano et al. (2005) and Smets and Wouters (2007) with a rich set of real and nominal frictions and shocks.

2.3.1 Households

Households are assumed to live infinitely and are numerous, represented by a unit interval, and supply heterogeneous labour (hours). Hours are then bundled together and sold to the labour packing firm. This firm assigns wages to households depending on the number of hours supplied. Furthermore, these households are also assumed to own firms; thus, they own capital stock and purchase investment. Hence, they choose consumption level C_t, investment level I_t, and bonds B_t to maximize their utility.

Labour Packers

Following Erceg et al. (2000), households are assumed to be heterogeneous in labour supply which is indexed by j $\in$ (0,1).

It is assumed that aggregate labour input is equal to:

$$N_t = (\int_0^1 N_t(j)^{\frac{\epsilon_w - 1}{\epsilon_w}} dj)^{\frac{\epsilon_w}{\epsilon_w - 1}} \tag{2.4}$$

where number of hours supplied by individual hours and wage elasticity of demand for labour are respectively represented by $N_t(j)$ $\epsilon_w > 1$.
The problem of the labour packer is to maximize hours:

$$\max_{N_t(j)} W_t^p(\int_0^1 N_t(j)^{\frac{\epsilon_w - 1}{\epsilon_w}} dj)^{\frac{\epsilon_w}{\epsilon_w - 1}} - \int_0^1 W_t(j) N_t(j) dj \tag{2.5}$$

After taking the first order condition of (3), we write individual hours in the following form:

$$N_t(j) = (\frac{W_t(j)}{W_t^p})^{-\epsilon_w} N_t \tag{2.6}$$

The labour packing firm rents out the heterogeneous labour supply to the intermediate firm, and it faces a downward-sloping demand because the households supply differentiated labour which is imperfectly substitutable. Therefore, they have some wage-setting power.

From the first order condition obtained in equation (4), the aggregate wage index, W_t, can be obtained as:

$$W_t^p = (\int_0^1 W_t(j)^{1-\epsilon_w} dj)^{\frac{1}{1-\epsilon_w}} \tag{2.7}$$

with $W_t(j)$ representing individual household j receiving wage based on hours supplied.

Household Problem

Household j chooses consumption $C_t(j)$, investment $I_t(j)$, capital goods $K_t(j)$, capital utilization $u_t(j)$, and bonds $B_t(j)$. With households receiving different wages based on different hours supplied, this implies that households would choose different consumption levels. In this model, the preferences are additively separable in consumption and hours. According to Erceg et al. (2000), households are identical in consumption, capital accumulation and utilization, and bonds but differ in wages they charge and labour supply, if preferences are separable in consumption and there is a state contingent that protects households from idiosyncratic wage risk. Hence, the subscript j can be dropped in this case. Based on C_t, u_t, B_t, I_t, K_t in each period, households maximize their utility represented by the following:

$$E_0 \sum_{t=0}^{\infty} \beta^t [log(C_t - bC_{t-1}) - \psi \tfrac{N_t(i)^{1+\eta}}{1+\eta}] \tag{2.8}$$

where $\beta \in (0,1)$ is the discount factor, η is the inverse Frisch elasticity of labour supply, and ψ represents the fixed cost of working. This paper follows Christiano et al. (2005) in allowing for habit formation in consumption.

The household gets utility from consuming goods but disutility from working and is subjected to the following constraints:

$$C_t + \tfrac{B_{t+1}}{P_t} \leq \tfrac{W_t(i)}{P_t} N_t(i) + R_t u_t K_t - (\chi_1(u_t - 1) + \tfrac{\chi_2}{2}(u_t - 1)^2) K_t + (1 + i_{t-1}) \tfrac{B_t}{P_t} + \tfrac{Div_t}{P_t} \tag{2.9}$$

$$K_{t+1} = (1 - \tfrac{\tau}{2}(\tfrac{I_t}{I_{t-1}} - 1)^2) I_t + (1-\delta) K_t \tag{2.10}$$

Equation (7) is the budget constraint of the household and equation (8) is the capital accumulation constraint. Households pay a price P_t for consumption goods, earn rental rates R_t, and hold nominal bonds B_{t-1} with nominal interest rate i_{t-1} and sell them (B_t) the next period. They do not pay any taxes. As owner of the firm, the representative household receives dividends from it. Furthermore, it owns capital stock, chooses the capital utilization level, and faces both quadratic investment and capital utilization costs.

The Lagrangian method is used to derive the following first order conditions for the household choices:

$$\frac{\partial \mathscr{L}}{\partial C_t} = 0 : \lambda_t = \frac{1}{C_t - bC_{t-1}} - \beta b E_t \frac{1}{C_{t+1} - bC_t} \tag{2.11}$$

$$\frac{\partial \mathscr{L}}{\partial u_t} = 0 : R_t = (\chi_1 + \chi_2(u_t - 1)) \tag{2.12}$$

$$\frac{\partial \mathscr{L}}{\partial B_{t+1}} = 0 : \lambda_t = \beta E_t \lambda_{t+1}(1 + i_t)\frac{P_t}{P_{t+1}} \tag{2.13}$$

$$\frac{\partial \mathscr{L}}{\partial I_t} = 0 : \lambda_t = \mu_t(1 - \frac{\tau}{2}(\frac{I_t}{I_{t-1}} - 1)^2 - \tau(\frac{I_t}{I_{t-1}} - 1)\frac{I_t}{I_{t-1}}) + \beta E_t \mu_{t+1}\tau(\frac{I_{t+1}}{I_t} - 1)(\frac{I_{t+1}}{I_t})^2 \tag{2.14}$$

$$\frac{\partial \mathscr{L}}{\partial K_{t+1}} = 0 : \mu_t = \beta E_t(\lambda_{t+1}(R_{t+1}u_{t+1} - \frac{1}{Z_{t+1}}(\chi_1(u_{t+1} - 1) + \frac{\chi_2}{2}(u_{t+1} - 1)^2)) + \mu_{t+1}(1 - \delta)) \tag{2.15}$$

In terms of setting wages, households cannot freely adjust their nominal wage each period. These wages are subject to Calvo (1983) pricing. This paper assumes that the households can change their nominal wages with probability $1 - \phi_w$. Partial indexation to inflation is allowed between wage changes in this paper and is represented by $\zeta_w \in (0, 1)$. In period t+k, a household with its last wage adjustment in period t has a real wage of:

$$w_{t+k}(j) = \frac{W_{t+k}(j)}{P_{t+k}} = \Pi_{s=1}^{k} \frac{(1+\pi_{t+s-1})^{\zeta_w}}{1+\pi_{t+s}} w_t(h) \tag{2.16}$$

The problem of households updating their wages is:

$$\begin{gathered} \max_{N_{t+k}(j), w_t(j)} E_t \sum_{k=0}^{\infty} (\beta\phi_w)^k (-\psi \frac{N_{t+k}^{1+\eta}}{1+\eta} + \lambda_{t+k} N_{t+k}^{1+\eta} \Pi_{s=1}^{k} \frac{(1+\pi_{t+j-1})^{\zeta_w}}{1+\pi_{t+s}} w_t(j)) \\ \text{s.t. } N_{t+k}(j) = (\frac{\Pi_{s=1}^{k} \frac{(1+\pi_{t+s-1})^{\zeta_w}}{1+\pi_{t+s}} w_t(j)}{w_{t+k}}) N_{t+k} \end{gathered} \tag{2.17}$$

When the first order condition is satisfied, then a reset wage is obtained which is common across all households.

2.3.2 Firms

The modeling for firms follows Christiano et al. (2005) and Smets and Wouters (2007) by incorporating the labour packing and price stickiness into the model. There are two types of firms in our model: intermediate firms and final goods firms. We use the Cantore et al. (2014) assumption that the intermediate firms produce intermediate goods using a CES technology.

Final goods firms

The representative firm is assumed to operate under perfect competition to produce a homogeneous good for consumption or investment. Prices for consumed and investment goods are assumed to be the same.

This representative firm buys $Y_t(m)$ units of each intermediate good at prices $P_t(m)$ in order to maximize profits. Hence, the final goods firm problem is as follows:

$$\begin{aligned} \max_{Y_t(j),Y_t} P_t Y_t - \int_0^1 P_t(m) Y_t(m) dm \\ \text{s.t. } (\int_0^1 Y_t(m)^{\frac{\epsilon_p - 1}{\epsilon_p}} di)^{\frac{\epsilon_p}{\epsilon_p - 1}} \geq Y_t \end{aligned} \tag{2.18}$$

We rearrange equation (16) to obtain the maximization problem with the firm needing only to choose $Y_t(j)$ and the respective first order condition:

$$\max_{Y_t(j)} P_t(\int_0^1 Y_t(j)^{\frac{\epsilon_p - 1}{\epsilon_p}} di)^{\frac{\epsilon_p}{\epsilon_p - 1}} - \int_0^1 P_t(j) Y_t(j) di \tag{2.19}$$

$$Y_t(j) = (\tfrac{P_t(j)}{P_t})^{-\epsilon_p} (\textstyle\int_0^1 Y_t(j)^{\frac{\epsilon_p - 1}{\epsilon_p}})^{\frac{\epsilon_p}{\epsilon_p - 1}} = (\tfrac{P_t(j)}{P_t})^{-\epsilon_p} Y_t \tag{2.20}$$

Intermediate goods firms

The intermediate firm m produces a homogeneous good at time t that depends on capital and labour under monopolistic competition. Each firm is assumed to face Calvo (1983) price adjustment costs.

$$Y_t(m) = A_t(\alpha(\tfrac{K_t(m) e^{z_t^K}}{K_0})^{\frac{\sigma - 1}{\sigma}} + (1 - \alpha)(\tfrac{N_t(m) e^{z_t^N}}{N_0})^{\frac{\sigma - 1}{\sigma}})^{\frac{\sigma}{1 - \sigma}} \tag{2.21}$$

These firms rent capital stock and household supply of hours. The intermediate firm's problem is to maximize profits. As in the case of households, firms are not free to change their prices every period. They are subject to Calvo (1983) pricing. Therefore, their problem is to minimize their inputs, subject to making sufficient demand for output.

$$\begin{aligned} &\min_{\tilde{K}_t(m),N_t(m)} \quad W_t^p N_t(m) + R_t^p \tilde{K}_t(m) \\ &\text{s.t. } A_t(\alpha(\frac{K_t\tilde{(m)}e^{z_t^K}}{K_0})^{\frac{\sigma-1}{\sigma}} + (1-\alpha)(\frac{N_t(m)e^{z_t^N}}{N_0})^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma}{1-\sigma}} \geq (\frac{P_t(j)}{P_t})^{-\epsilon_p} Y_t \end{aligned} \tag{2.22}$$

where ϵ_p represents the price elasticity of demand for intermediate goods.

Equation (18) represents demand for intermediate goods from the final goods firms. The Lagrangian equation is used in this case to obtain the following first-order conditions with respect to $\tilde{K}_t(m)$ and $N_t(m)$. The former denotes capital services, the product of capital utilization, u_t, and capital goods, K_t.

$$\mathcal{L} = -W_t^p N_t(m) - R_t^p \tilde{K}_t(m) + q_t(m)(A_t(\alpha(\frac{K_t\tilde{(m)}e^{z_t^K}}{K_0})^{\frac{\sigma-1}{\sigma}} + (1-\alpha)(\frac{N_t(m)e^{z_t^N}}{N_0})^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma}{1-\sigma}} - (\frac{P_t(j)}{P_t})^{-\epsilon_p} Y_t) \tag{2.23}$$

The first order conditions are taken with respect to $\tilde{K}_t(m)$ and $N_t(m)$ respectively:

$$\frac{\partial \mathcal{L}}{\partial \tilde{K}_t(m)} = 0 : R_t^p = q_t(m)\alpha A_t(\frac{Y_0}{K_0 e^{z_t^K}})^{\frac{\sigma-1}{\sigma}}(\frac{Y_t(m)}{\tilde{K}_t(m)})^{\frac{1}{\sigma}} \tag{2.24}$$

$$\frac{\partial \mathcal{L}}{\partial N_t(m)} = 0 : W_t^p = q_t(m)(1-\alpha)A_t(\frac{Y_0}{N_0 e^{z_t^N}})^{\frac{\sigma-1}{\sigma}}(\frac{Y_t(m)}{N_t(m)})^{\frac{1}{\sigma}} \tag{2.25}$$

Note that all the firms face the same prices, so the subscript m can be dropped. Furthermore, equations (22) and (23) can transformed into real terms if they are divided by price P_t.

$$R_t = mc_t \alpha A_t(\frac{Y_0}{K_0 e^{z_t^K}})^{\frac{\sigma-1}{\sigma}}(\frac{Y_t}{\tilde{K}_t})^{\frac{1}{\sigma}} \tag{2.26}$$

$$w_t = mc_t(1-\alpha)A_t(\frac{Y_0}{N_0 e^{z_t^N}})^{\frac{\sigma-1}{\sigma}}(\frac{Y_t}{N_t})^{\frac{1}{\sigma}} \tag{2.27}$$

where marginal cost $mc_t = \frac{q_t}{P_t}$.

The neutral technology shock A_t is assumed to follow the process:

$$lnA_t = \rho_a lnA_{t-1} + \epsilon_a; \tag{2.28}$$

and the labour and capital augmented technology shocks also follow an AR(1) process:

$$lnz_t^j = \rho_z lnz_{t-1}^j + \epsilon_z^j; forj = K, L \tag{2.29}$$

The real profits for intermediate firm j can be rewritten as:

$$\frac{\Pi_t^p(m)}{P_t} = \frac{P_t(m)}{P_t} Y_t(m) - mc_t Y_t(m) \tag{2.30}$$

As mentioned earlier, the intermediate firms cannot easily change their prices in every period. However, these firms can adjust their prices with probability $1-\phi_p$. In this paper, partial indexation to inflation is also allowed, but it is for changes in prices this time. This partial indexation is represented by $\zeta_p \in (0,1)$. The firm's problem is described as follows.

$$\begin{gathered}\max_{P_t(m),Y_t(m)} E_t \sum_{k=0}^{\infty} (\beta\phi_p)^k \frac{\lambda_{t+k}}{\lambda_t} (\frac{\Pi_{t-1,t+k-1}^{\zeta_p} P_t(m)}{P_{t+k}} Y_{t+k}(m) - mc_{t+k} Y_{t+k}(m)) \\ \text{s.t. } Y_{t+k}(m) = (\frac{\Pi_{t-1,t+k-1}^{\zeta_p} P_t(m)}{P_{t+k}}) Y_{t+k}\end{gathered} \tag{2.31}$$

The optimal price from equation (28) is called the reset price. All firms involved in updating will set their price.

2.3.3 Monetary Authority and Aggregation

The Central Bank is assumed to set its nominal interest rate according to the Taylor rule.

$$i_t = (1-\rho_i)i + \rho_i i_{t-1} + (1-\rho_i)(\phi_\pi(\pi_t - \pi) + \rho_y(lnY_t - lnY_{t-1})) + \epsilon_{i,t} \tag{2.32}$$

i_t is the interest rate determined by the central bank through the Taylor rule in period t. π_t measures the inflation of the consumed good. ρ_i is the interest rate smoothing parameter. Also, ρ_π and ρ_y are the monetary response to deviations from inflation and output respectively. The monetary policy shock, $\epsilon_{i,t}$, follows an AR(1) process:

$$\epsilon_{i,t} = \rho_{qu}\epsilon_{i,t-1} + \varepsilon_{i,t} \tag{2.33}$$

Under an aggregate economy, the following is obtained:

$$C_t + I_t = Y_t \tag{2.34}$$

2.4 Results

2.4.1 Log-linearization and calibration

We simulate the data for this model at a quarterly frequency. We use log-linearization of the first-order conditions for the households, firms, and monetary authority around their steady states. The calibrated values are similar those of Smets and Wouters (2007) posterior estimated parameters using Bayesian methods. We set the discount factor in the model to 0.98 and the capital share of output, α, to 0.33. The inverse of the Frisch elasticity of labour supply, ψ equals 2. The substitution between goods is represented by ϵ_p and the substitution between labour supply, ϵ_w, both equal 10, values consistent with the Kimball curvature (Smets and Wouters (2007)). The persistence of technology shock is 0.9. The standard deviation for technology shocks is decreased from 0.0088 to 0.0061 to examine how much it contributes to lowering the correlation between labour productivity and business investment the post-1990 period. We consider two separate values for the investment adjustment cost parameter (i) 2.5 which is the estimated value in Christiano et al. (2005) and (ii) 6 as per Smets and Wouters (2007). To capture the effects of different rates of substitution between capital and hours, we consider a baseline value of $\sigma = 0.35$ as in Cantore et al. (2014). Similarly for calibrating the volatility and persistence of capital-augmented technology shocks we use the Bayesian estimates of Cantore et al. (2017). The complete set of calibrated values is in the appendix. We use Dynare Adjemian et al. (2011) to solve the model and consider five thousand simulations of the model to generate correlations.

2.4.2 Shifts in technology shock volatility and structural features

Can the simulated model generate a low correlation between labour productivity and business investment? We pursue an approach similar to that in Barnichon (2010) and Gali and van Rens (2014) to address this question and explore shifts in the volatility of the technology shock volatility and structural features. First, we generate simulated data for two periods: pre-1990 and post-1990. The volatility of technology shock decreased from 0.0088 before 1990 to 0.0061 for after 1990. Table 2.3 shows the calibrated values of all the model parameters.

Table 2.3: Parameters and steady state calibration

Parameters	Definition	Pre-1990 value	Post-1990 value
ψ	Fixed cost of working	2	2
β	Discount factor	0.98	0.98
ϕ_p	Calvo price parameter	0.75	0.8
ϕ_w	Calvo wage parameter	0.75	0.8
α	Capital share of income	0.33	0.33
η	Inverse of Frisch elasticity	1.23	1.23
b	Habit formation parameter	0.64	0.64
χ_1	Capital utilization parameter 1	0.035	0.035
χ_2	Capital utilization parameter 2	0.01	0.01
τ	Investment adjustment cost magnitude	2.5	2.5
δ	Depreciation rate	0.025	0.025
ε_w	Wage elasticity of demand for labour	10	10
ε_p	Price elasticity of demand	10	10
ζ_p	Degree of indexation to past inflation	0.05	0.05
ζ_w	Degree of indexation to past wage inflation	0.44	0.44
ρ_i	Degree of interest rate smoothing	0.81	0.81
ϕ_π	Policy-controlled int. rate response to inflation	1.31	1.31
ϕ_y	Policy-controlled int. rate response to output gap	0.22	0.22
ρ_z^K	Capital-augmented shock persistence	0.8	0.8
ρ_a	Neutral technology shock persistence	0.95	0.95
ρ_z^H	Labour-augmented shock persistence	0.14	0.14
ρ_{qu}	Interest rate persistence	0.12	0.12
σ_{z^K}	Standard deviation of capital-augmented tech. shock	0.017	0.017
σ_{ea}	Standard deviation of neutral tech. shock	0.0088	0.0061
σ_{z^H}	Standard deviation of labour-augmented tech. shock	0.012	0.012
σ_{ei}	Standard deviation of interest rate persistence	0.0013	0.0008
$\pi*$	Trend inflation rate	0	0
Y/N	Steady state of Labor Productivity (in logs)	0.46	0.46
I	Steady state of Investment (in logs)	-2.34	-2.36

We simulate labour productivity and investment data using the model described in section 3.[7]

[7]We use the perturbation methods to solve the system of first order equations in log-linear form around the steady state. Next, we obtain each control variable (including labour productivity and investment) which is a function of the capital stock and also shocks. We then extract the cyclical components of the

Table 2.4 summarizes the U.S. data and model correlations. The results indicate that decreasing the technology shock volatility over the 1990 period would be insufficient to generate a low correlation between investment and labour productivity. In the case of the average duration of wage and price stickiness for 3 quarters ($\phi_w = \phi_p = 0.75$), the correlation decreased from 0.12 to 0.10 in the low elasticity of substitution case and from 0.20 to 0.19 in the constant elasticity of substitution case.

Additional assumptions about structural features such as price and wage rigidities are, therefore, needed for the model to produce lower correlation. One possible structural feature that may help generate a low correlation is an increase in the average duration of price contracts which is an indicator of price rigidity. The response of output and investment to technology shocks may be dampened in the presence of price rigidity. This is because when supply increases, demand is unable to rise enough to meet the increases in supply when prices are sticky. Thus, the response of output is smaller when prices are rigid than when prices are flexible.

As it turns out, the Cobb-Douglas specification of the production function with rigid prices and wages does generate a near zero correlation, suggesting that the model matches the post-1990 correlation between investment and productivity.

Table 2.4: U.S. data and model: Correlation before and after 1990 with standard investment adjustment cost parameter=2.5

	Pre-1990		Post-1990	
Data	0.54		-0.1	
Model and parameter assumptions	$\sigma = 0.35$	$\sigma = 1$	$\sigma = 0.35$	$\sigma = 1$
$\phi_p = \phi_w = 0.75$	0.12	0.20	0.10	0.19
$\phi_p = 0.75$, $\phi_w = 0.8$	0.07	0.15	0.06	0.14
$\phi_p = 0.8$, $\phi_w = 0.75$	0.08	0.16	0.07	0.14
$\phi_p = \phi_w = 0.8$	0.04	0.10	0.04	0.10

According to Table 2.4, the simulated model cannot generate a correlation close to the actual pre-1990 data in the presence of investment adjustment costs if it assumes that the average duration of wage contracts equals 0.75 (wages are rigid). However, the table shows that the model can generate a low investment and labour productivity correlation for the post-1990 period if the average durations of prices and wages increase.

simulated labour productivity and investment using the HP filter for quarterly data. We then calculate the unconditional correlation between cyclical components of labour productivity and investment.

Higher investment adjustment costs can also help generate a low correlation between labour productivity and investment is higher investment adjustment costs (IAC). This is because the IAC dampens the response of investment to technology shocks. Higher IAC means that it is more expensive to change the flow of investment, meaning that households would respond by consuming more and investing less than they would without the presence of IAC. Hence, the presence of IAC is important in helping to understand why there is a change in the investment and labour productivity correlation after 1990.

To further understand the effects of the IAC, we also generated the simulated correlations using the Smets and Wouters (2007) IAC parameter estimate equal to 6 instead of 2.5. Table 2.5 summarizes the results. The results show that the simulated model can generate an even lower correlation of 0.018 and 0.07 respectively for the $\sigma = 0.35$ and $\sigma = 1$ cases after 1990.

Following Barnichon (2010), monetary policy is used as a proxy for non-technology shock. When monetary policy shock volatility decreases, there is little change in the correlation. This is consistent with the empirical results in section 2, where we find that the change in the investment response to technology shocks after 1990 contributed to the low correlation between investment and labour productivity.

Table 2.5: US and model data correlation before and after 1990 with investment adjustment cost parameter=6

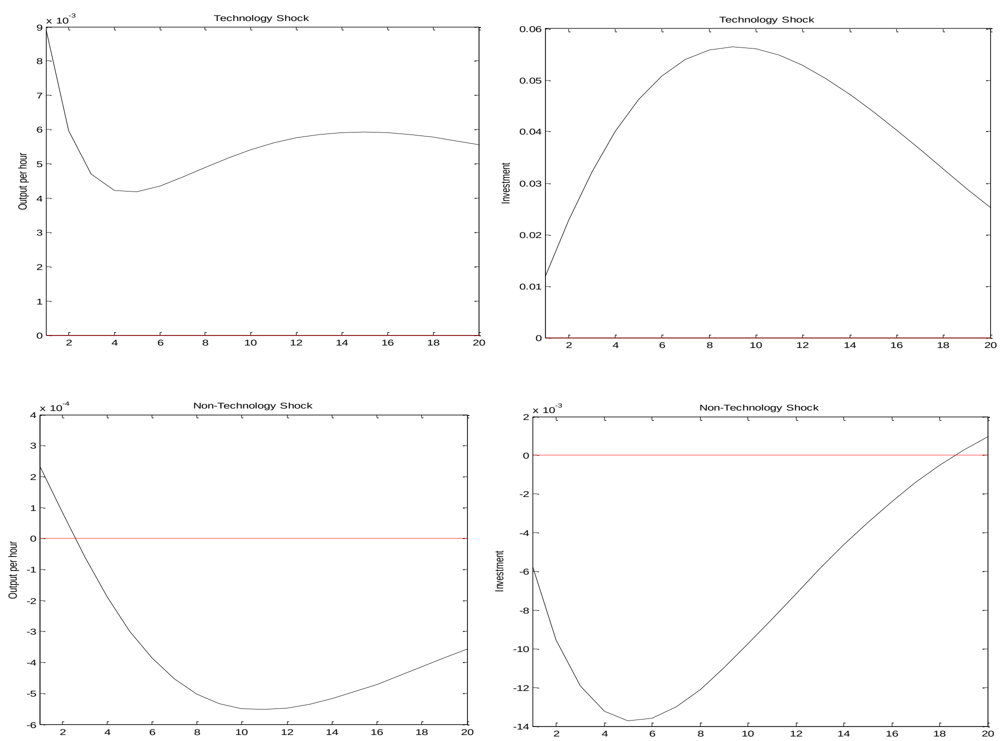
	Pre-1990		Post-1990	
Data	0.54		-0.1	
Model and parameter assumptions	$\sigma = 0.35$	$\sigma = 1$	$\sigma = 0.35$	$\sigma = 1$
$\phi_p = \phi_w = 0.75$	0.09	0.17	0.08	0.15
$\phi_p = 0.75$, $\phi_w = 0.8$	0.05	0.13	0.05	0.11
$\phi_p = 0.8$, $\phi_w = 0.75$	0.05	0.12	0.05	0.11
$\phi_p = \phi_w = 0.8$	0.02	0.08	0.02	0.08

In summary, the results from the simulated model indicate that a decline in the volatility of technology shocks, accompanied by an increase in both price and wage rigidities can help generate a low correlation between labour productivity and investment.

2.4.3 Impulse response analysis

Figures 2.6 and 2.7 display the simulated responses of productivity and investment to a technology and a monetary policy shock. The responses of labour productivity and investment to technology shocks slightly decrease after 1990. This is the only feature that matches the empirical model in section 2.2.

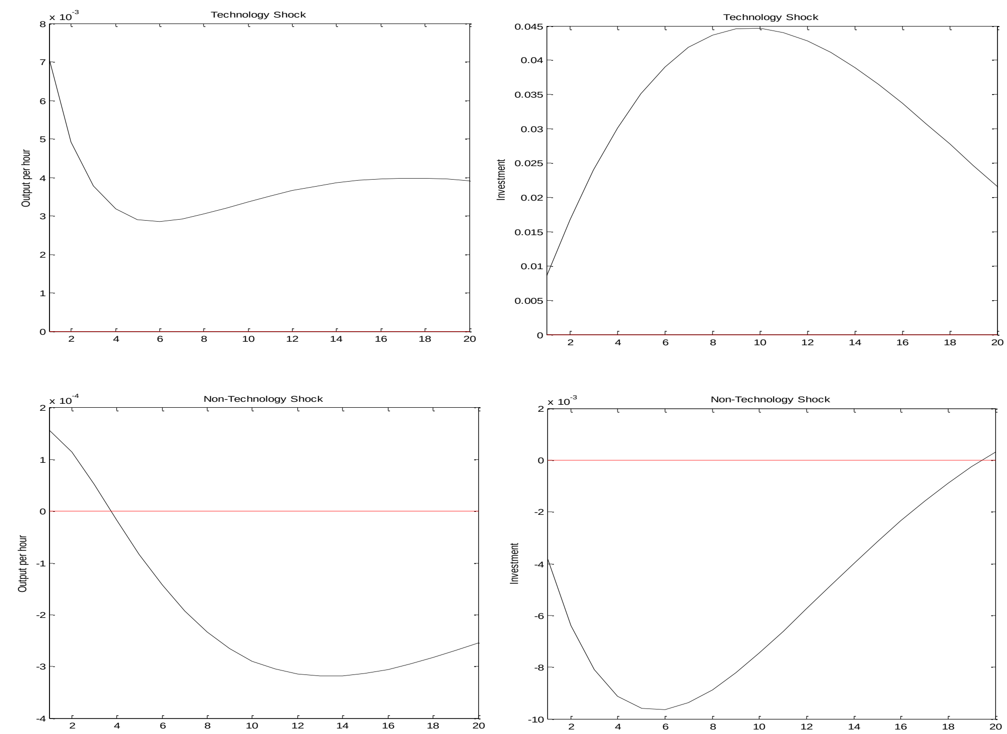
Figure 2.6: Pre-1990 simulated impulse response functions to technology and non-technology shocks:



Note: Impulse responses have been generated using $\sigma = 1$ (production function is Cobb-Douglas), $\phi_p = \phi_w = 0.75$, $\sigma_{ea} = 0.0088$, and $\sigma_{ei} = 0.0013$.

Generally, the model is not successful in matching the empirical responses found in section 2.2. With the exception of the investment response to technology shocks, the simulated responses to technology and non-technology shocks are remarkably different from

Figure 2.7: Post-1990 simulated impulse response functions to technology and non-technology shocks:



Note: Impulse responses have been generated using $\sigma = 1$ (production function is Cobb-Douglas), $\phi_p = \phi_w = 0.8$, $\sigma_{ea} = 0.0061$, and $\sigma_{ei} = 0.0013$.

the empirical responses. The magnitude of the model response of labour productivity to technology and non-technology shocks are smaller than the empirical responses. The sign of the investment response to non-technology shock for the simulated model is difference from the sign of the investment response for the empirical VAR model. This could be related to choosing monetary policy shock as a proxy to non-technology shocks for the investment and labour productivity analysis and other structural features not explored in this data mechanical exercise.

2.5 Conclusion

This paper documents a new stylized fact—the correlation between labour productivity and real business investment in the U.S. data since 1990 is essentially zero. This correlation was 0.54 in the post-WWII data until the end of 1980s. The response of investment to identified technology shocks have switched signs from positive to negative across these two sub-periods, whereas the response to a non-technology shock has remained approximately the same. Since the volatility of technology shocks has decreased less relative to non-technology shock over the two sub-periods, we explore the hypothesis whether the relatively more volatile technology shocks and the negative response of investment can together account for the decreased correlation. We consider a canonical DSGE model and simulate data under a variety of assumptions about the parameters representing structural features and volatility of shocks. The results show that although the smaller decline in the volatility of technology shocks relative to non-technology shock has contributed to the decrease in the correlation, this channel alone is not sufficient. Structural features such as increased average duration of price contracts, unitary elasticity of substitution between labour and capital, and larger magnitudes of investment adjustment costs are needed for the model to produce a near-zero correlation between labour productivity and investment.

Chapter 3

Effects of ignoring structural breaks in GAS volatility models

Abstract

Creal et al. (2013) recently introduced a new class of time-varying parameter models called generalized autoregressive score (GAS) models. Within the GAS volatility framework, the variance is updated by the scaled score of the model's density function instead of the squared residuals. Creal et al. (2013) claim that GAS models are advantageous to use because updating the conditional variance using the score of the log-density instead of the second moments can improve a model's fit to data. GAS models are also found to be robust to some forms of misspecification such as outliers, which raises the question of whether GAS volatility models are less sensitive to parameter non-constancy. This issue is important because ignoring structural breaks in the parameters can cause the conditional variance to exhibit unit root behaviour in which the unconditional variance is undefined. This implies that any shock to the variance will not gradually decline (Lamoureux and Lastrapes, 1990). This paper conducts a Monte-Carlo simulation study of the effects of ignoring parameter non-constancy on five different volatility models. Of these models, three – the GARCH(1,1), t-GAS(1,1), and Beta-t-EGARCH(1,1) – are GAS models, while the other two – t-GARCH(1,1) and EGARCH(1,1) – are not. The effects of the breaks are assessed by examining the behaviour of the mean value of the estimated persistence parameter and the coverage probabilities of the nominal 90% and 95% confidence intervals for the persistence parameter as the sample size approaches infinity. The first finding is that there are similarities between the responses of GAS and non-GAS models to a

failure to take structural breaks in the conditional volatility parameters into account in estimation. A second finding of this paper is that ignoring a break in any parameter that shifts the unconditional variance, such as the constant or the persistence parameter, causes the mean value of the estimated persistence parameter in all models to exceed its true value and approach one as the sample size approaches infinity and the break size increases. A third finding of this study is that a break in any parameter that does not shift the unconditional variance has little to no effect on the mean value of the estimated persistence parameter. Finally, the fourth finding of this study is that models based on the t-distribution tend to be less reactive to breaks than models based on the Gaussian distribution.

3.1 Introduction

There are several ways to model time-varying volatility. One way is to use the autoregressive conditional heteroskedasticity (ARCH)/ generalized ARCH (GARCH) class of models first introduced by Engle (1982) and then Bollerslev (1986). A prominent model that belongs to this class is the classical GARCH model of Bollerslev (1986), in which the conditional volatility is updated by its own squared residuals. ARCH/GARCH modeling is quite popular amongst econometric practitioners because these models can capture some stylized facts about asset returns such as volatility clustering and fat tails, and can be estimated using maximum likelihood methods (Engle and Patton, 2001; Zivot, 2009). Another way is to use Creal et al.'s (2013) generalized autoregressive score (GAS) volatility framework. Within the GAS volatility framework, the volatility is updated by the scaled score of the model's density function instead of the squared residuals. Different GAS volatility models emerge depending on the choice of distribution and scaling matrix. For example, if the density function is Student's t and the scaling matrix is Fisher's information matrix, then one obtains the Student's t-GAS volatility model (the t-GAS model hereafter). However, if the scaling matrix is again the inverse of Fisher's information matrix but the density is Gaussian, then the GAS framework reduces to Bollerslev's (1986) GARCH model. Harvey and Chakravarty's (2008) Beta-t-EGARCH model is obtained if the log density is Student's t and the scaling matrix is an identity matrix.

Creal et al. (2013) claim that GAS models are advantageous to use because updating the conditional variance using the score of the log-density instead of the second moments can improve a model's fit to data. GAS models also have the advantage of being easily estimated by maximum likelihood methods rather than the simulation-based estimation methods which are necessary for more complex models such as stochastic volatility models.

Additionally, there is evidence suggesting that the heavy-tailed GAS volatility models are more successful empirically when compared to GARCH and stochastic volatility models. For example, Koopman et al. (2016) use simulated data to show that score-driven GAS(1,1) models have better forecast accuracy than GARCH models. Blazsek and Villatoro (2015) and Blazsek and Mendoza (2016) find that a Beta-t-EGARCH(1,1) model forecasts better than a simple GARCH(1,1) model during the post-financial crisis era, for nine global industry price indexes and 50 stocks from the S&P 500 index respectively.

GAS models are also found to be robust to some forms of misspecification. For example, GAS models perform well in forecasting even though the data are not generated from GAS models. Koopman et al. (2016) use simulated data to show that GAS models have similar forecast accuracy to that of state-space models, when the data are generated by state space models. Another example is that the Student's t-GAS model, in particular, is less sensitive to outliers than Bollerslev's (1987) t-GARCH model, which also assumes that the errors follow a Student's t-distribution. This is because in the t-GAS model, a large absolute realization of an observation does not necessarily lead to a large increase in the variance (Creal et al., 2013; Harvey, 2013). As mentioned by Maddala and Kim (1998), structural breaks can be considered to be one type of outlier. They can also be viewed as permanent outliers (see Harvey et al., 2001; Darné and Diebolt, 2005). Thus, this raises the question of whether GAS volatility models are less sensitive when parameter non-constancy is neglected in estimation than models not updated by the score.

The effect of ignoring parameter non-constancy on the estimated persistence of the conditional variance has yet to be fully addressed in the literature on GAS volatility models. This issue is important because if the persistence parameter equals one, then the conditional variance exhibits unit root behaviour and the unconditional variance is undefined, implying that any shock to the variance will not gradually decline (Lamoureux and Lastrapes, 1990). On the other hand, the impact of neglecting structural breaks in volatility has been raised only in the GARCH literature. Hillebrand (2005) shows that the persistence parameter of a standard GARCH(1,1) model is overestimated and approaches one if structural breaks in the parameters are neglected, while Lamoureux and Lastrapes (1990) find that ignoring structural breaks in unconditional volatility can give rise to IGARCH-like behaviour. Similarly, in their empirical study of oil price volatility, Ewing and Malik (2017) find that accounting for endogenous structural breaks in asymmetric GARCH models can decrease estimates of persistence. Ardia (2009) incorporates Markov-switching regimes into Glosten et al.'s (1993) asymmetric GARCH model and applies Bayesian methods. Using the Swiss Market Index log-return, he finds that incorporating Markov-switching regimes improves the fit and produces better out-of sample forecasts. Rohan and Ramanathan (2012) extend Ardia's (2009) study to general GARCH models and produce a formal test to identify break dates. They apply their model to both simulated data and empirical data such as the Dow Jones and S&P 500 stock price indexes and the USD/EURO exchange rate and are successful in identifying the first week of September 2008 as a break date coinciding with the 2008 Financial Crisis.

Hence, this paper contributes to the literature by extending the simple Monte Carlo simulations of Hillebrand (2005) to two GAS models – the t-GAS and Beta-t-EGARCH models – to examine what happens to the estimated volatility persistence parameter when structural breaks are ignored in estimation. This is done by generating data sets in which a break in one parameter occurs in the middle of the sample, and then estimating the two GAS models under the assumption that the parameters are constant. Leverage effects are included in the version of the Beta-t-EGARCH model examined.

For purposes of comparison, the paper also simulates the Gaussian GARCH model using two different generating processes, to see if the results are sensitive to whether the GAS formulation or standard GARCH formulation is used. Nelson's (1991) EGARCH model with leverage effects and Bollerslev's (1987) t-GARCH model are also simulated in this paper. Again, the simulated data sets include a break in one parameter in the middle of the sample, but the GARCH, EGARCH, and t-GARCH models are estimated under the assumption of constant parameters. As is the case for the t-GAS model, there do not appear to be any previous studies of the effects of breaks in volatility on estimates of the EGARCH and t-GARCH models.

For all the models considered, the effects of breaks on the empirical sampling distribution of the estimator of the persistence parameter are examined. Following Lumsdaine (1995), the coverage probabilities of the standard confidence interval for the persistence parameter are computed as well. Studying the impact of breaks on coverage probabilities is important because a decline in the coverage probability is associated with an increase in the probability of Type I error in hypothesis testing.

This paper is organized as follows. Section 3.2 briefly presents the time-varying volatility models considered. Section 3.3 describes the design of the simulation study, while section 3.4 presents the results. Finally, section 3.5 concludes and makes suggestions for the direction of future research.

3.2 GAS versus non-GAS time-varying volatility models

For all models considered in this paper, it is assumed that the conditional mean is specified as the following autoregressive (AR) process with one lag:

$$y_t = \mu + \phi y_{t-1} + u_t, \tag{3.1}$$

where

$$u_t = \sigma_t v_t \tag{3.2}$$

and y_t is the dependent variable of interest at time t. μ and ϕ are the conditional mean parameters. The error term u_t has a mean of 0 and variance σ_t^2. v_t is assumed to be generated by a standardized disturbance density $p(v_t)$. The mean of v_t is 0 and its variance is 1. How the conditional volatility is specified varies with each model.

The GAS volatility models considered in this simulation analysis are the t-GAS, GARCH, and Beta-t-EGARCH models. In Creal et al.'s (2013) formulation of GAS volatility models, the conditional variance is updated by the score of the log-density as follows:

$$\sigma_t^2 = \omega + \textstyle\sum_{i=1}^{p} A_i s_{t-i} + \sum_{j=1}^{q} B_j \sigma_{t-j}^2 \tag{3.3}$$

where $s_t = S_t \nabla_t$ is the scaled score function, $\nabla_t = \frac{\partial \ln p(u_t|u_1,u_2,...,u_{t-1},\sigma_1^2,\sigma_2^2,...,\sigma_{t-1}^2;\phi)}{\partial \sigma_t^2}$ is the score function of the log density of u_t with $\phi = \{\phi_1, \phi_2, ..., \phi_p\}$ representing the set of static parameters of the model, and $S_t = \mathcal{I}(\phi)_{t|t-1}^{-1} = (E_{t-1}[\nabla_t \nabla_t'])^{-1}$ is the scaling matrix function.[1] A_i represents the scaled score parameter and B_i denotes the persistence parameter of σ_t^2 for lag i. ω is the constant parameter in the conditional variance equation. Blasques et al. (2014) show that the condition $\sum_{j=1}^{l} B_j < 1$ must hold in order for the GAS(p,q) process to be stationary.

If v_t follows a standard Student's t distribution, then

$$p(u_t|u_1, u_2, ..., u_{t-1}, \sigma_1^2, \sigma_2^2, ..., \sigma_t^2; \phi, \eta) = \frac{\Gamma(\frac{\eta+1}{2})}{\sqrt{\pi(\eta-2)\sigma_t^2}\Gamma(\frac{\eta}{2})} \left(1 + \frac{u_t^2}{(\eta-2)\sigma_t^2}\right)^{-\frac{\eta+1}{2}}, \tag{3.4}$$

[1]Note that, as mentioned by Creal et al. (2013), the scaling matrix S_t can also take the form of an identity matrix I or the square root of $\mathcal{I}(\phi)_{t|t-1}^{-1}$, leading to different types of GAS models. For simplicity, this paper looks at only the Student's t and Gaussian volatility cases with $S_t = \mathcal{I}(\phi)_{t|t-1}^{-1}$.

where η represents the degrees of freedom of the distribution. Following Creal et al. (2013), the score of the density in this case is

$$\begin{aligned}\nabla_t &= \frac{\partial \ln p(u_t|u_1,u_2,...,u_{t-1},\sigma_1^2,\sigma_2^2,...,\sigma_{t-1}^2;\phi,\eta)}{\partial\sigma_t^2} \\ &= -\frac{0.5}{\sigma_t^2}+\frac{\eta+1}{2}\frac{\frac{u_t^2}{(\eta-2)\sigma_t^4}}{1+\frac{u_t^2}{(\eta-2)\sigma_t^2}}.\end{aligned} \tag{3.5}$$

Using equation (3.5), the scaling matrix for this model is

$$\begin{aligned}S &= \left(E_{t-1}\left[-\frac{0.5}{\sigma_t^2}+\frac{\eta+1}{2}\frac{\frac{u_t^2}{(\eta-2)\sigma_t^4}}{1+\frac{u_t^2}{(\eta-2)\sigma_t^2}}\right]^2\right)^{-1}. \\ &= \frac{\eta}{2\sigma_t^2(\eta+3)}.\end{aligned} \tag{3.6}$$

If equation (3.6) is multiplied by equation (3.5) and the product is substituted into equation (3.3), then one obtains the Student's t GAS(p,q) model, also known as the t-GAS(p,q) model of Creal et al. (2013):

$$\sigma_t^2 = \omega + \textstyle\sum_{i=1}^{p} A_i(1+3\eta^{-1}).\frac{(1+\eta^{-1})u_{t-i}^2}{(1-2\eta^{-1})\left[1+\frac{\eta^{-1}u_{t-i}^2}{(1-2\eta^{-1})\sigma_{t-i}^2}\right]} + \textstyle\sum_{j=1}^{q} B_j\sigma_{t-j}^2. \tag{3.7}$$

Assuming that the stationarity condition holds, the unconditional variance is

$$\sigma^2 = \frac{\omega}{1-\sum_{j=1}^{q} B_j} \tag{3.8}$$

for the t-GAS model. By setting $p = q = 1$, one obtains the t-GAS(1,1) model,

$$\sigma_t^2 = \omega + A(1+3\eta^{-1}).\frac{(1+\eta^{-1})u_{t-1}^2}{(1-2\eta^{-1})\left[1+\frac{\eta^{-1}u_{t-1}^2}{(1-2\eta^{-1})\sigma_{t-1}^2}\right]} + B\sigma_{t-1}^2 \tag{3.9}$$

and its unconditional variance

$$\sigma^2 = \frac{\omega}{1-B}. \tag{3.10}$$

Creal et al. (2013) demonstrate that Bollerslev's (1986) GARCH model is also a GAS model. If $v_t = \frac{u_t}{\sigma_t}$ has a standard Normal distribution, then

$$p(u_t|u_1,u_2,...,u_{t-1},\sigma_1^2,\sigma_2^2,...,\sigma_t^2;\phi) = \frac{1}{\sqrt{2\pi\sigma_t^2}}e^{-\frac{u_t^2}{2\sigma_t^2}}. \tag{3.11}$$

This implies that the score of the density is

$$\begin{aligned}\nabla_t &= \tfrac{\partial \ln p(u_t|u_1,u_2,\ldots,u_{t-1},\sigma_1^2,\sigma_2^2,\ldots,\sigma_{t-1}^2;\phi)}{\partial \sigma_t^2} \\ &= -\tfrac{0.5}{\sigma_t^2} + 0.5\tfrac{u_t^2}{\sigma_t^4}.\end{aligned} \tag{3.12}$$

Hence,

$$\begin{aligned}S &= \left(E_{t-1}\left[\nabla_t \nabla_t'\right]\right)^{-1} \\ &= \left(E_{t-1}\left[-\tfrac{0.5}{\sigma_t^2} + 0.5\tfrac{u_t^2}{\sigma_t^4}\right]^2\right)^{-1} \\ &= 2\sigma_t^4.\end{aligned} \tag{3.13}$$

Multiplying (3.13) by (3.12), setting $p = q = 1$, and substituting the result into equation (3.3) yields

$$\sigma_t^2 = \omega + A\left[u_{t-1}^2 - \sigma_{t-1}^2\right] + B\sigma_{t-1}^2. \tag{3.14}$$

As mentioned in Creal et al. (2013), equation (3.14) is more commonly written in the form

$$\sigma_t^2 = \omega + Au_{t-1}^2 + (B - A)\sigma_{t-1}^2, \tag{3.15}$$

$$\sigma_t^2 = \omega + \alpha u_{t-1}^2 + \beta\sigma_{t-1}^2, \tag{3.16}$$

where setting $\alpha = A$ and $\beta = B - A$ leads to the familiar GARCH(1,1) model, in which α is the ARCH parameter that captures the effect of past shocks and β is the GARCH parameter that captures the effect of the previous conditional variance. The unconditional variance is

$$\begin{aligned}\sigma^2 &= \tfrac{\omega}{1-\alpha-\beta} \\ &= \tfrac{\omega}{1-B}.\end{aligned} \tag{3.17}$$

Thus, if one chooses the scaling matrix to be the inverse of Fisher's information matrix and assumes that the errors are normally distributed, the GAS(1,1) model reduces to a GARCH(1,1) model under normality. The t-GAS(1,1) model reduces to the GARCH(1,1) model only if the degrees of freedom are infinite.

The Beta-t-EGARCH(1,1) model with leverage effects (referred to as, henceforth, the Beta-t-EGARCH(1,1) model) is another type of model encompassed by the GAS framework. Harvey and Chakravarty (2008) introduce this model to address the nonexistence of the unconditional moments of a Student's t-EGARCH(1,1) model with finite degrees of freedom. If the parameter of interest is instead $\log \sigma_t^2$, the scaling matrix is the identity matrix, v_t follows a Student's t-distribution, and a leverage effect is added to the model, then one obtains the Beta-t-EGARCH(1,1) model considered in this study. In this case, the scaled score of the log density with respect to the parameter of interest is

$$\begin{aligned} s_t &= -\tfrac{1}{2} + \tfrac{\eta+1}{2} \frac{\frac{u_t^2}{(\eta-2)\sigma_t^2}}{1+\frac{u_t^2}{(\eta-2)\sigma_t^2}} \\ &= \tfrac{1}{2}\left(\tfrac{(\eta+1)u_t^2}{(\eta-2)\sigma_t^2+u_t^2} - 1\right). \end{aligned} \tag{3.18}$$

Substituting (3.18) into (3.3), replacing σ_t^2 with $\log \sigma_t^2$, and adding a leverage effect gives the following equation:

$$\begin{aligned} \log \sigma_t^2 &= \omega + \textstyle\sum_{i=1}^{p} (A_i s_{t-i} + A_i^* 1(u_t < 0)(s_{t-1} + 1)) + \sum_{j=1}^{q} B_j \log \sigma_{t-j}^2 \\ &= \omega + \textstyle\sum_{i=1}^{p} \left[A_i \left(\tfrac{(\eta+1)u_{t-i}^2}{(\eta-2)\sigma_{t-i}^2+u_{t-i}^2} - 1\right) + A_i^* I(u_t < 0)(\tfrac{(\eta+1)u_{t-i}^2}{(\eta-2)\sigma_{t-i}^2+u_{t-i}^2})\right] + \sum_{j=1}^{q} B_j \log \sigma_{t-j}^2 \end{aligned} \tag{3.19}$$

With $p = q = 1$, the Beta-t-EGARCH(1,1) model and its unconditional variance are represented by

$$\log \sigma_t^2 = \omega + A\left(\tfrac{(\eta+1)u_{t-1}^2}{(\eta-2)\sigma_{t-1}^2+u_{t-1}^2} - 1\right) + A^* I(u_t < 0)\left(\tfrac{(\eta+1)u_{t-1}^2}{(\eta-2)\sigma_{t-1}^2+u_{t-1}^2}\right) + B \log \sigma_{t-1}^2 \tag{3.20}$$

and

$$\sigma^2 = exp\left(\tfrac{\omega}{1-B}\right), \tag{3.21}$$

respectively. A^* is the leverage effect parameter.

The two non-GAS models used for comparison are the t-GARCH(1,1) and EGARCH(1,1) models. Bollerslev's (1987) t-GARCH(1,1) model is identical to that in equation (3.15), except that the underlying distribution is a Student's t-distribution, not a Gaussian distribution. It is important to note that, although the normal GAS(1,1) model reduces to a GARCH(1,1) model, the t-GAS(1,1) model does not reduce to a t-GARCH(1,1) model. This is because the t-GAS(1,1) updating mechanism in the second term of (3.9) differs from the corresponding mechanism described by equation (3.15). The degrees of freedom appear in the second term of the t-GAS(1,1) model, while the second term of the t-GARCH(1,1) model depends only on the past squared error term. As Creal et al. (2013) and Harvey (2013) point out, the updating mechanism in the second term of the t-GAS(1,1) model provides a more modest increase than the second term of the t-GARCH(1,1) model; a large absolute realization of u_t would not necessarily lead to a large increase in the variance. Hence, the t-GAS(1,1) model is less sensitive to outliers than the t-GARCH(1,1) model.

Nelson's (1991) exponential GARCH (EGARCH) model also incorporates leverage effects and is not score-based. As in the Beta-t-EGARCH(1,1) model, the parameter of interest for the EGARCH model is also $\log \sigma_t^2$, which is given by

$$\log \sigma_t^2 = \omega + A(|v_{t-1}| - E|v_{t-1}|) + \lambda v_{t-1} + B \log \sigma_{t-1}^2. \tag{3.22}$$

If v_t is normally distributed, the conditional and unconditional variances are respectively

$$\log \sigma_t^2 = \omega + A(|v_{t-1}| - \sqrt{\tfrac{2}{\pi}}) + \lambda v_{t-1} + B \log \sigma_{t-1}^2, \tag{3.23}$$

and

$$\sigma^2 = exp\left(\tfrac{\omega}{1-B}\right), \tag{3.24}$$

where ω is the constant, λ captures the leverage effect, and B denotes the persistence parameter of the EGARCH(1,1) model.

Unlike the t-GAS(1,1) and GARCH(1,1) models, the Beta-t-EGARCH(1,1) and the EGARCH(1,1) models do not require restrictions to be imposed on their parameters to ensure that the conditional variance is positive. However, $B < 1$ is required for stationarity of the unconditional variance to hold. The next task is to examine, using a small-scale Monte-Carlo simulation, the impact of failing to take into account structural breaks in the parameters of the volatility models on the estimated persistence parameter $\hat{B}$. This task will include a comparison of the effects of neglected structural breaks on the GAS and non-GAS models.

3.3 Monte-Carlo simulation study design

The design of the simulation study follows that of Hillebrand (2005). Hillebrand conducts a study of the effects of neglecting parameter changes in α, β, and ω on the sum of the estimated parameters $\hat{\alpha}$ and $\hat{\beta}$ in the GARCH(1,1) model. He generates data for four different sample sizes – T = 200, 800, 4000, and 10000 – under this assumption. Also, he selects parameter values such that the generated data mimic the behaviour of financial time series data, with breaks taking the form of a change in parameters in the middle of each sample. Afterwards, he computes maximum likelihood estimates of the GARCH(1,1) model for each simulated data set using quasi-Newton methods. In doing so, he imposes constraints on the parameters ($\omega > 0$; $\alpha, \beta \geq 0$; $\alpha + \beta < 1$) to ensure that the unconditional variance is positive. Hillebrand (2005) finds that ignoring changes in the DGP of the conditional variance of GARCH(1,1) models can cause the parameters of the conditional variance to be substantially overestimated, leading the sum of $\hat{\alpha}$ and $\hat{\beta}$ to approach one.

This is a concern because in order for the GARCH(1,1) model to be covariance stationary, the sum of α and β must be strictly less than one.[2]

The simulation analysis carried out in this paper differs from that of Hillebrand (2005) in that the focus is on the effects of ignoring a structural break on the estimated persistence parameter, $\hat{B}$. The reason for focusing on the persistence parameter is that when GARCH models are applied to financial data, $\hat{B}$ tends to be high. High persistence may indicate that structural breaks are present (see Zivot, 2009; Hillebrand, 2005; Andreou and Ghysels, 2009). However, Hillebrand focuses only on the basic GARCH model. In this paper, the study of neglected breaks is expanded to the t-GAS, Beta-t-EGARCH, EGARCH, and t-GARCH models, with the standard GARCH model also included for purposes of comparison. Furthermore, this simulation study includes the impact of a break in η (models with Student's t-distribution only).

In the case of the GARCH model, the simulated data are generated using two alternative DGPs. The first is the standard GARCH DGP, given by equation (3.16). The second, called the GAS-GARCH model hereafter, is described by equation (3.14). The difference between the two formulations is that in the standard GARCH model, a change in the parameter A also shifts the parameter B (equal to $\alpha + \beta$), while in the GAS-GARCH model, changes in A do not change B. In both cases, it is the GAS-GARCH formulation of equation (3.14) that is estimated. Similarly, equation (3.16) is used for to generate data for the t-GARCH model, but the GAS parameterization of equation (3.14) was used in estimating it. This is done to ease the comparison between the effect of ignoring the breaks on the persistent parameter of the t-GARCH model with that of the other models. Note that changes in the parameter ω have no effect on the values of the other parameters of the DGP for the GARCH and t-GARCH models, regardless of the parameterization used.

For each simulation, the initial value of y at time $t = 1$ is a random number drawn from the standard normal distribution. The parameters of the conditional mean equation are fixed at $\mu = 0$ and $\phi = 0.7$. For the t-GAS(1,1), GARCH(1,1), GAS-GARCH(1,1), and t-GARCH(1,1) models, the initial parameter values of the conditional variance are $\omega = 0.2$, $\alpha = A = 0.1$, and $B = 0.8$. The corresponding unconditional variance is equal to one. The parameter values for the EGARCH(1,1) and Beta-t-EGARCH(1,1) models are $\omega = 0$ and $B = 0.8$, to ensure that the unconditional variance for these models is also equal to one. In the case of the three t-distribution models, the degrees of freedom are equal to five. For the EGARCH(1,1) model, the leverage effect parameter λ is equal to -0.09, and for the Beta-t-EGARCH(1,1) model, the leverage effect parameter A^* is equal to 0.08.[3]

[2]Recall that A and α are equivalent in this paper.

[3]In empirical studies, the EGARCH leverage effect parameter tends to be negative and the Beta-t-

Following Hillebrand (2005), multiple parameter switches are considered. Hillebrand considers ten, but this paper examines only five parameter switches for each model. For each model and sample size, the first simulation assumes no structural break. The unconditional standard deviation σ is set to $\sigma_1 = 1$. For all models, breaks in the parameter ω are chosen to correspond to the prespecified breaks in the conditional variance. After the break, $\sigma = \sigma_2$, where ω_2 ranges from 1.15 to 1.75 with increments of 0.15 in each subsequent simulation.[4] For the GARCH, GAS-GARCH, t-GARCH, and t-GAS models, changes in B are also chosen to correspond to changes in σ. Since the initial value of ω is zero in the EGARCH and Beta-t-EGARCH models, the same change in B is applied in these models. In the case of the GARCH and t-GARCH models, changes in A and β also reflect the changes in σ, with changes in A causing a simultaneous change in B. The parameter shifts in A that shift the unconditional variance of the GARCH and t-GARCH models are then applied to the GAS-GARCH, t-GAS, EGARCH, and Beta-t-EGARCH models, in which B remains constant when A changes. Although the parameter η does not affect the unconditional volatility of any of the t-distribution models, the effects of ignoring changes in it upon estimation are still assessed. Table 3.1 summarizes the changes in each parameter for the simulation study.

In each simulation, only one parameter of the volatility model is changed. For example, when ω changes as indicated in table 3.1, A, B, and η retain the same values as in the no structural break case. In each model, simulations are carried out for six different sample sizes: T = 250, 500, 1000, 2500, 5000, and 10000. These sample sizes reflect common sample sizes for macroeconomic and financial time series data. Although most macroeconomic time series tend to be of low frequency (i.e., quarterly or annual), high frequency data are available for some macroeconomic variables. For example, there are about 1000 monthly observations for U.S. inflation from 1947 to 2016, while the weekly federal funds rate series has over 3000 observations. In contrast, financial time series data tend to be of high frequency (i.e., hourly, daily, or weekly). A sample size of 10,000 observations corresponds to the size at which the asymptotic results become valid for high frequency data. For each simulation, two thousand datasets of each sample size are drawn.

For each model, the parameters μ, ϕ, A, B, and η (if applicable) are jointly estimated

EGARCH leverage effect parameter tends to be positive. See Zivot (2009), Harvey and Chakravarty (2008), and Harvey (2013).

[4] The 75% increase in volatility is motivated by the increase in US inflation volatility after March 1999. Note that the volatility of the inflation rate increased from approximately 0.0016 in the January 1984-March 1999 period to 0.0033 in the post-March 1999 period. However, volatilities have been normalized to equal 1 in order to have a clear understanding of the simulation results and to obtain simpler parameter values for the conditional variance constant, ω. Bataa et al. (2014) find that the US experienced a couple of breaks in (conditional) volatility in inflation, one in April 1992 and the other in March 1999.

using quasi-maximum likelihood estimation for each dataset.[5] During the estimation process, parameter restrictions are imposed on all models except the Beta-t-EGARCH(1,1) and EGARCH(1,1) models, to ensure that the conditional variance is positive. These restrictions are $\omega \geq 0$, $A > 0$, and $A \leq B < 1$ (for the GARCH(1,1), GAS-GARCH(1,1), and t-GARCH(1,1) models) or $0 < (1 + \frac{3}{\eta})A < B$ (for the t-GAS(1,1) model).[6]

3.4 Simulation results

This section summarizes the results of the Monte-Carlo simulations. For each simulation, the results are evaluated using two different methods. Firstly, the mean and standard deviation of the sampling distribution of the persistence parameter estimate, $\hat{B}$, are produced using the two thousand observations of $\hat{B}$. Secondly, following Lumsdaine (1995), coverage probabilities of the 90% and 95% confidence intervals for B are constructed in the following manner. First, confidence intervals for B are produced using the equation $\hat{B} \pm t_{1-\alpha/2}\sqrt{var(\hat{B})}$, where $t_{1-\alpha/2} = 1.645$ for the 90% case and $t_{1-\alpha/2} = 1.96$ for the 95% case. $var(\hat{B})$ is estimated using the element of the estimated variance-covariance matrix corresponding to $\hat{B}$. Afterwards, the number of times the true value falls within the confidence interval is divided by the number of datasets drawn (2,000). This measurement is known as the coverage probability.

The simulation results are presented in two tables for each parameter which experiences a structural break in the middle of the sample (see tables 3.2 to 3.7). In each table, the first row contains the base case of no structural breaks. The subsequent rows display the results when a structural break occurs in the middle of the sample, but is ignored in estimation. The first table summarizes the mean and standard deviation of the sampling distribution of $\hat{B}$ for each simulation. The second table contains the coverage probabilities of the 90% and 95% confidence intervals for B. In addition, it contains the standard error of the coverage probability. The standard error is constructed using the Bernouilli equation, as in Lumsdaine (1995): $\sqrt{\frac{\hat{x}(1-\hat{x})}{N}}$, where $\hat{x}$ is the coverage probability estimate and N (=2,000) is the number of draws in the simulations. These standard errors appear in parentheses below

[5] All analysis and results were generated using The Mathworks Inc. (2014) MATLAB programs. MATLAB packages for the estimation of GAS models were downloaded from the GAS Models website (http://www.gasmodel.com/code.htm), and Martin et al. (2013) provided examples on how to conduct Monte Carlo simulations.

[6] Conditions for ensuring positivity in the GAS model variances are from Creal et al. (2013) and Blasques et al. (2014).

the coverage probabilities. Lastly, relative frequency histograms are displayed for a sample size of $T = 10000$ for three cases: no structural break in the top panel, a medium-sized break in the middle panel, and a large break in the bottom panel.

3.4.1 Impact of ignoring a break in ω on $\hat{B}$

Table 3.2 summarizes the simulation results for $\hat{B}$ with and without the presence of a break in ω. The results show that when there is no break in ω and the sample size is small, the mean value of $\hat{B}$ lies below the true value of B, 0.8. The downward bias in $\hat{B}$ is most prevalent in the Beta-t-EGARCH(1,1) model, which has the lowest mean value of $\hat{B}$ for the sample sizes $T = 250$ and 500. On the other hand, the mean value of $\hat{B}$ for the t-GAS(1,1), GARCH(1,1), and GAS-GARCH(1,1) models is equal to 0.69, which is closer to the true value than the mean value for the other models. However, when the sample size is as large as 2500, the mean value of $\hat{B}$ for all the models equals 0.79, almost equal to the true value. Eventually, the mean value equals its true value when $T = 10000$ for all the models.

When both the sample and break sizes are small, the mean value of $\hat{B}$ is closer to its true value than when there is no break in ω. For example, for the t-GAS(1,1) and t-GARCH(1,1) models, the mean value of $\hat{B}$ is 0.79 when the unconditional volatility switches from 1 to 1.15 and $T = 1000$ as compared to 0.77 when there is no break. Similarly, at this sample size, the mean value of $\hat{B}$ for the GARCH(1,1), GAS-GARCH(1,1), and Beta-t-EGARCH(1,1) models is equal to the true value of B when the change in the unconditional volatility is equal to 0.15. Yet another example arises when T = 250 and the break in the unconditional volatility is as large as 0.45, in which case, the mean values of $\hat{B}$ for the EGARCH(1,1) and Beta-t-EGARCH(1,1) models are both equal to 0.79. This result can be explained by two biases offsetting each other. The first is that when the sample size is small, the mean value of $\hat{B}$ for all the models tends to underestimate the true value of B. The second is that the mean value of $\hat{B}$ tends to overestimate its true value when structural breaks are ignored in estimation.

For all the models, the results also show that the mean value of $\hat{B}$ eventually exceeds the true value of B as the size of the break increases, and rapidly converges to one as the sample size, T, approaches ∞. For instance, when the unconditional volatility switches from 1 to 1.45 at $T = 1000$, the mean value of $\hat{B}$ for the t-GAS(1,1) model is 0.91; similarly, for this model, the mean value of $\hat{B}$ equals 0.99 when the unconditional volatility changes from 1 to 1.75 at $T = 10000$. As for the t-GARCH(1,1) model, the mean value of $\hat{B}$ equals 0.99 when volatility increases by 0.75 and the sample size is equal to 2500. Given the same change in volatility, the mean value of $\hat{B}$ for the GARCH(1,1) model equals 1 when $T = 5000$, and the mean of $\hat{B}$ in the EGARCH(1,1) and Beta-t-EGARCH(1,1) models equals 1 and 0.98 respectively when $T = 10000$. Furthermore, a comparison of the GARCH and GAS-GARCH columns reveals that using the GAS parameterization of the GARCH model as the DGP does not affect the results. These results indicate that ignoring a break in ω affects GAS models and non-GAS models in a similar manner. In addition, the results for the GARCH(1,1) and t-GARCH(1,1) models are consistent with those of Hillebrand (2005).

Figures 3.1 to 3.6 display the relative frequencies of the persistence parameter of the five time-varying volatility models, for a sample size of 10000. The no structural break case is in the top panel, a medium-sized break of 0.45 is displayed in the middle panel, and in the bottom panel, there is a large break of 0.75. The figures show that when there is no break in ω, the sampling distributions of $\hat{B}$ in all the models center around the true value that equals 0.8. However, these figures also show that the sampling distributions of $\hat{B}$ become narrower and shift rightwards towards one as the size of the break increases. This result is consistent with those of table 3.2, where the mean value of $\hat{B}$ approaches one and the standard deviation of $\hat{B}$ decreases to zero as the size of the break in ω increases.

These same figures also indicate that Gaussian models are more sensitive to a break in ω than the Student's t models. If the size of the break is medium, then it is the EGARCH model that is most sensitive to the change in ω, because its distribution of $\hat{B}$ becomes a point mass at one. The figures also show that the GARCH model requires a smaller break in order for the sampling distribution of $\hat{B}$ to collapse to one than the t-GAS and t-GARCH models. The probability that $\hat{B} > 0.95$ for the GARCH(1,1) model is larger than the probability that $\hat{B}$ for the t-GAS(1,1), t-GARCH(1,1), and Beta-t-EGARCH(1,1) models. The performance of the GAS-GARCH model is similar to that of the GARCH model. The sensitivities of the t-GAS and t-GARCH models to a break in ω are similar as well. If the size of the break is large ($\Delta\sigma = 0.75$), the sampling distribution of $\hat{B}$ in all models collapses to one. With the large break size, the empirical sampling distributions of both the EGARCH and GARCH models are point masses at one. The GAS specification of GARCH model also has a distribution with a point mass at one, and the EGARCH model has the highest probability of $\hat{B}$ being equal to one (0.9). The GARCH model has the second-highest probability, followed by the t-GAS and t-GARCH models. The Beta-t-EGARCH(1,1) model has the smallest probability.

The fact that the sampling distributions of $\hat{B}$ for the Gaussian models collapses to one at smaller break sizes than the sampling distributions for the t-distribution models suggests that the presence of heavy tails dampens the effect of ignoring the breaks in ω. The presence of heavy tails may also explain why the t-GAS and t-GARCH models are less sensitive to breaks than the GARCH and EGARCH models. However, the Beta-t-EGARCH model also have Student's t errors and yet it has the smallest probability of $\hat{B}$ being equal to one. This phenomenon may be the result of the Beta-t-EGARCH model having a combination of features such as the log-variance being the time-varying parameter instead of the variance, volatility being updated by the score of log-density, and being a t-distribution model. The similarity of the GARCH and GAS-GARCH results implies that the choice of the data generating process does not alter the effect of ignoring a break in ω on $\hat{B}$.

Table 3.3 summarizes the coverage probabilities of the 90% and 95% confidence intervals for B. The results show that if there is a structural break in unconditional volatility and this break is ignored in estimation, then the 90% and 95% coverage probabilities for B decrease as the size of the break increases. In addition, as the sample size T approaches ∞, the coverage probabilities rapidly decrease to zero.

If there is no break, then the coverage probabilities for B in all models converge to 90% and 95% respectively as T approaches ∞. In this case, the t-GARCH model has the highest coverage probability for all sample sizes. In contrast, the EGARCH model has the lowest coverage probabilities for sample sizes $T = 250$, 500, and 1000; the t-GAS and t-GARCH models have the lowest coverage probabilities for $T = 2500$; and the Beta-t-EGARCH(1,1) model has the lowest for the larger sample sizes $T = 5000$ and 10000. In addition, for all sample sizes, the GAS-specified GARCH model has the same coverage probabilities as the GARCH model. Once again, this implies that using a different DGP for the GARCH model has no impact on the effect of a break in ω that is ignored in estimation.

Compared to the no break case, all models experience a decrease in the 90% and 95% coverage probabilities for B when a break occurs. In all cases, the GAS specification of the GARCH model has the same coverage probabilities as the standard specification of the GARCH model. For a small break, the GARCH model experiences the largest drop in the coverage probabilities. In contrast, the t-GAS model has the smallest decrease in coverage probabilities. Additionally, the t-GARCH model does not generally have the highest coverage probabilities for B when the break is small. For larger sample sizes, the t-GAS model has the highest coverage probabilities for the 90% confidence intervals when $T = 2500$ and 10000. The t-GAS model also has the highest 95% coverage probability for $T = 1000$, 2500, 5000, and 10000.

Table 3.3 shows that when the change in volatility is mid-sized ($\Delta\sigma = 0.3$ or 0.45), the Beta-t-EGARCH model has the highest coverage probabilities for all sample sizes except T = 250, in which case, the t-GAS model has the highest coverage probability, and T = 10000, where the EGARCH model has the highest coverage probability.

The results indicate that for all models, when a break in ω is ignored the coverage probability of the confidence intervals for B decreases to zero. The rate at which the coverage probabilities for $\hat{B}$ approach zero may differ for each model, but in the end the coverage probabilities are poor when the break is ignored. Also, the results indicate that re-parameterizing the GARCH model into a GAS model has no impact on the effect of ignoring a break in ω, since the coverage probability is the same. The results imply that problems in inference can arise when the break in ω is ignored, a decrease in coverage probability is associated with an increase in the probability of a Type I error in hypothesis testing.

3.4.2 Impact of ignoring a break in A on $\hat{B}$

Table 3.4 summarizes the Monte Carlo simulations when a break in A is ignored. The results show that the effect on the mean value of $\hat{B}$ varies by model. For the t-GAS(1,1), EGARCH(1,1), and Beta-t-EGARCH(1,1) models, not taking into account the parameter change in A has little to no effect on the mean value of $\hat{B}$. For example, on average, $\hat{B}$ for the t-GAS(1,1) model falls only slightly below its true value (equal to 0.8) if the parameter A increases by at least 0.08 when $T \to \infty$, while the mean value of $\hat{B}$ for the GAS-GARCH(1,1), EGARCH(1,1), and Beta-t-EGARCH(1,1) models converges to its true value when A increases by the same amount. The estimator of $\hat{B}$ remains consistent, as the standard deviation decreases and approaches zero. These results may be linked to the fact that changes in the parameter A in the t-GAS(1,1), GAS-GARCH(1,1), EGARCH(1,1), and Beta-t-EGARCH(1,1) models does not produce a shift in the unconditional variance.

In contrast, table 3.4 shows that the mean values of $\hat{B}$ for the GARCH(1,1) and t-GARCH(1,1) models are noticeably affected by a change in A that is neglected in estimation. The mean values converge towards one, but not as quickly as when ω changes. These results are consistent with Hillebrand's (2005). In contrast to the original GARCH specification, the neglected changes in A had relatively little effect on $\hat{B}$ for the GAS-GARCH model. Despite the GARCH and GAS-GARCH models being mathematically equivalent, the effect of the change in A is not the same for the two models. These conflicting results may be explained by the fact that in the standard GARCH parameterization, changes in A causes B to also shift in the GARCH model, but not in the GAS-GARCH parameterization. This implies that ignoring a break can affect the persistence parameter of the same model differently, when it is re-parameterized in different forms. It also implies that whether or not a neglected break occurs in a parameter that shifts the unconditional variance matters.

Figures 3.8 to 3.12 display the relative frequencies of $\hat{B}$ when the change in A is ignored upon estimation. For the GAS-GARCH(1,1), t-GAS(1,1), EGARCH(1,1), and Beta-t-EGARCH(1,1) models, these histograms show that the distribution of $\hat{B}$ remains virtually unchanged as the size of the break in A increases. In contrast, the distributions of $\hat{B}$ in the GARCH and t-GARCH models shift to the right as the size of a break in A increases. These results also indicate that if the break occurs in parameters that shift the unconditional variance and is ignored in estimation, then this break can shift the distribution of $\hat{B}$, collapsing to a mass point of one. These same results also show that neglected breaks in the parameters that do not shift the unconditional variance may not have any effect on the distribution of $\hat{B}$.

Table 3.5 displays the coverage probabilities of B when the break in A is ignored. For the t-GAS(1,1), GAS-GARCH(1,1), EGARCH(1,1), and Beta-t-EGARCH(1,1) models, the change in A has little to no effect on the mean value of $\hat{B}$, but the change did have some effect on the coverage probability for B. In sample sizes T = 250, 500, and 1000, the coverage probability for B is slightly higher when breaks in A are ignored than when there are no breaks in A under the t-GAS(1,1), EGARCH(1,1), and Beta-t-EGARCH(1,1) models. For example, for the t-GAS model, the 95% coverage probability for B increase by 2.3 and 2.9 percentage points when T = 500 and 1000 respectively if the break in A is as large as 0.13. Another example is that the 95% coverage probability for B for the GAS-GARCH, EGARCH, and Beta-t-EGARCH models increases by as much as 5 percentage points compared to the no break case when T = 500 and when the size of the break is 0.13. However, coverage probabilities decrease slightly in the presence of a break in A if T = 5000 or 10000 for the t-GAS(1,1) model. The decreases in the 90% and 95% coverage probabilities for the t-GAS model range from 0.3 to 3.5 percentage points and from 1 to 2 percentage points respectively. For the EGARCH(1,1) model, the coverage probability increases by only 1 percentage point when a break in A is ignored as compared to when there is no break if T = 2500 or 5000. With T = 10000, the EGARCH(1,1) coverage probabilities for B decrease slightly (1 to 2.5 pp) when the break in A is neglected for the 90% confidence interval and relatively unchanged for the 95% confidence interval. In the case of the Beta-t-EGARCH(1,1) model, when structural breaks are ignored the coverage probabilities for B is either slightly lower or almost the same as in the case where there are no structural breaks in A. In contrast, for the GARCH(1,1) and t-GARCH(1,1) models, the coverage probabilities for B decrease to zero as T approaches ∞ and the size of the break in A increases. The GARCH(1,1) and t-GARCH(1,1) coverage probabilities for B are also summarized in table 3.5.

The results imply that if a break occurs in a parameter that shifts the unconditional volatility, then the coverage probability for B converges to zero. Furthermore, these same results also indicate that if the break occurs in a parameter that does not shift the unconditional volatility, then the coverage probabilities for B also decrease, but only by a small amount. The decrease in coverage probability is associated with an increase in the probability of a Type I error in hypothesis testing. This implies that the probability of a Type I error is higher when the parameter shifts the unconditional volatility than when the parameter does not shift unconditional volatility.

3.4.3 Impact of ignoring a break in η on $\hat{B}$

Table 3.6 summarizes the Monte Carlo simulation results for the mean values of $\hat{B}$ of the t-GAS(1,1), Beta-t-EGARCH(1,1), and t-GARCH(1,1) models when the break in η is ignored. The results show that ignoring a break in η during estimation has no effect on the mean value of $\hat{B}$. This conclusion is confirmed in figures 3.13 to 3.15, since a break in η does not shift the distribution of $\hat{B}$ for any model. Note that η is another parameter that does not shift the unconditional variance. Once again, this suggests that a break in a parameter that does not change the unconditional variance has little to no effect on $\hat{B}$.

However, the change in η behaves in a similar manner to the change in A for the t-GAS, Beta-t-EGARCH, and EGARCH models. It does not affect $\hat{B}$, but it does slightly affect the coverage probability of B. According to table 3.7, the t-GAS model has slightly lower 90% and 95% coverage probabilities for B when a structural break in η is present than when no structural breaks are present. For the t-GAS model, the coverage probabilities for B decrease by about 1 to 3 percentage points. In the case of the Beta-t-EGARCH model, the coverage probability of B is relatively stable for $T = 500$, 1000, and 2500 as the size of the break increases. Additionally, the coverage probability of B for the Beta-t-EGARCH model generally decreases by 1 to 2 percentage points when $T = 5000$ and 10000 and the size of the break in η increases. With the exception of $T = 10000$, the coverage probability for B of the t-GARCH model is relatively unchanged for all sample sizes and as the break size increases. The t-GARCH coverage probabilities for B decrease slightly at $T = 10000$. Comparing the models, the results also indicate that the t-GAS model has the lowest coverage probability regardless of sample and break sizes. However, the t-GARCH and Beta-t-EGARCH models have similar coverage probabilities in the presence of a break in η.

As discussed in section 3.4.2, the results show that a break in parameter that does not shift the unconditional volatility has only a slight impact on the coverage probability. The unconditional variance for all three models does not depend on η. For the t-GARCH model, the effect of ignoring a break in A on the coverage probability for B differs from the effect of ignoring a break in η. This is because A shifts the unconditional variance of the t-GARCH model and η does not.

3.4.4 Impact of ignoring a break in B on $\hat{B}$

Table 3.8 displays the effect of ignoring a break in B on the sampling distribution of $\hat{B}$ for the t-GAS(1,1), EGARCH(1,1), and Beta-t-EGARCH(1,1) models.[7] A second simulation of the Beta-t-EGARCH model is carried out with the initial value of ω set equal to 0.2 rather than 0 because the original parameters for this model are such that changes in B would have no effect on the unconditional variance. The results of both simulations of the Beta-t-EGARCH model are included in the table.[8]

The results show that the impact of neglecting a break in B on $\hat{B}$ is similar to that of neglecting a break in ω. The mean value of $\hat{B}$ in all three models converges to one as T approaches ∞ and the size of the break in B increases. In addition, the rate at which the mean value of $\hat{B}$ approaches one varies. For example, for the t-GAS(1,1) model, the mean value of $\hat{B}$ never exceeds 0.97, even for the largest break size and $T = 10000$. However, for the other two models, it exceeds 0.97 at all sample sizes, in some cases for small and moderate breaks. As in the case of ω and A, when both the sample size and the size of the break in B are small, ignoring a structural break in B appears to bring the mean value of B closer to the true value.

Hence, the effects of breaks in ω and B on the mean value $\hat{B}$ are similar. This similarity may be due to the fact that both parameters have the ability to shift the unconditional variance in all models. The effects of ignoring a break in B on the coverage probabilities of the confidence intervals for parameter B is not studied here because B changes in mid-sample. Hence there is no true value of B, and thus no results for this particular case are discussed in this paper.

3.5 Other results

The focus of this paper is the impact of ignoring parameter non-constancy on the persistence parameter. However, the impact of ignoring structural breaks on the parameter estimates of A, ω, and η are in the Appendix. For instance, a change in ω causes $\hat{A}$ and $\hat{\eta}$ to fall below their true values on average and approach zero as T goes to infinity and the size of the break increases. The coverage probabilities for parameters A and η decrease to

[7] Some of the models were not analyzed because of time constraints. In addition, what the preliminary results for the other models are consistent with the finding that parameters that shift the unconditional variance affect $\hat{B}$ in all models.

[8] A similar approach would have been taken applied to EGARCH model, but due to time constraints the alternative simulation was not carried out.

zero, but at a slower rate than the coverage probability for B. Among the t-distribution models, the t-GARCH model is the most sensitive to breaks in ω because it has the lowest coverage probability for η when breaks are ignored.

A break in the parameter A has little effect on $\hat{\omega}$ for all the models. With the exception of the t-GARCH model, the mean value of $\hat{\omega}$ for all models either converges to or is close to the true value, irrespective of the size of the break. The coverage probabilities for ω for the t-GAS model slightly increase for T = 250, 500, and 1000 and slightly decrease for T = 2500, 5000, and 10000.

Also, a break in A has a small effect on $\hat{\eta}$. In some cases, the presence of a break in A helps bring the mean value closer to its true value. Similar to the impact on $\hat{B}$, the coverage probability for η either slightly decreases or remains relatively unchanged across all models and break sizes. This suggests that a break in parameters that do not change the unconditional volatility also have little to no effect on the estimated degrees of freedom parameter.

3.6 Conclusion

This paper conducts a Monte-Carlo simulation study of the effects of ignoring parameter non-constancy on five different volatility models. Of these models, three – the GARCH(1,1), t-GAS(1,1), and Beta-t-EGARCH(1,1) – are GAS models, while the other two – t-GARCH(1,1) and EGARCH(1,1) – are not. The effects of the breaks are assessed by examining the behaviour of the mean value of the estimated persistence parameter and the coverage probabilities of the nominal 90% and 95% confidence intervals for the persistence parameter as the sample size approaches infinity. This study produced several findings.

The first finding is that there are similarities between GAS and non-GAS models when structural breaks in the conditional volatility parameters are neglected. In all the models, when some parameters shift in the middle of the sample, the estimated persistence parameter converges to one as the size the break increases. Additionally, the coverage probabilities of the 90% and 95% confidence intervals collapses to zero depending on the size of the break in certain parameters and as the sample size approaches infinity. While this study was motivated by a desire to compare the responses of GAS and non-GAS models of volatility to breaks in their parameters, this study produces other interesting findings.

The second finding is that ignoring a break in any parameter that shifts the unconditional variance, such as the constant or the persistence parameter, causes the mean value of the estimated persistence parameter in all models to exceed its true value and approach one as the sample size approaches infinity and the break size increases. Additionally, the same parameter shifts cause the coverage probability to approach zero as the size of the break in the parameters increases.

The third finding of this study is that any parameter that does not shift the unconditional variance has little to no effect on the mean value of the estimated persistence parameter. Examples of these parameters are the degrees of freedom parameter and the score parameter. The same shift in the parameter can lower the coverage probability, which is associated with an increase in the probability of making a Type I error. However, the effect on the coverage probability is not as severe as the effect in the case of a parameter that shifts the unconditional volatility.

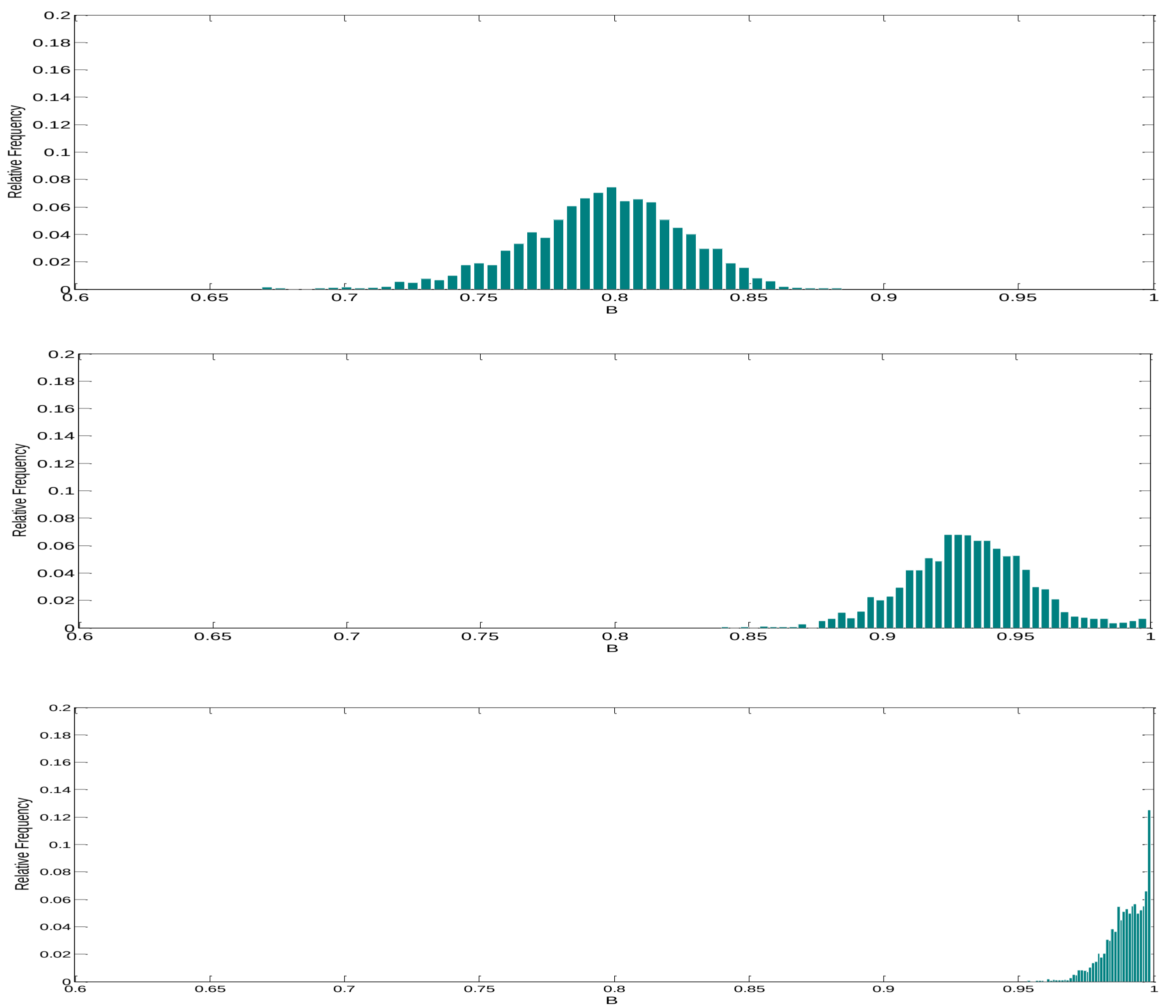
Finally, the fourth finding of this study is that models based on the t-distribution tend to be less reactive to breaks than models based on the Gaussian distribution. The results suggest that a larger break is required for the sampling distribution of the estimated persistence parameter to collapse to one for the t-distribution models than for the Gaussian models. Similarly, the probability of the estimated persistence parameter being equal to one is higher for the Gaussian models than for the t-distribution models.

Thus, this simulation study has several implications for practitioners. Firstly, the study suggests that ignoring parameter non-constancy does not always lead to spurious persistence. For instance, the results shows that a break in the score parameter can cause spurious persistence in the GARCH model, but if this same GARCH model is re-parameterized to become a GAS-type model, then a break in this same parameter has little or no effect on persistence. For practitioners, this suggests that it would be worthwhile to use the GAS specification of the GARCH model instead of the standard GARCH model for analysis. Secondly, the results from this study also suggest that if the break is small enough then there will be little effect on persistence. A small break in parameters that shift the unconditional variance has only a small effect on persistence in all the models. This implies that the size of the break also matters in estimating persistence in time-varying volatility models. Hence, practitioners should check for parameter non-constancy in both GAS and non-GAS models, especially for parameters that shift the unconditional variance. When checking for breaks with small sample sizes, practitioners can ignore the break in the estimated persistence parameter if they find that the percentage change in volatility is between 30% and 45%. Where the sample size is large, practitioners can ignore the break in the estimated persistence parameter if the size of the break is as large as 15%. Thirdly, the study has implications for forecasting. If the size of the break in parameters is small, then there may not be any gain in using forecast methods that accommodate breaks.

The findings of this study also suggest directions for future research. Since GAS models that allow for integrated processes do not exist, one direction would be to develop GAS models that allow for the persistence parameter to equal one. This type of model only exists in the ARCH/GARCH class of models, called the integrated GARCH (IGARCH) model. Another direction for future research would be to develop tests for parameter non-constancy in GAS models. These tests can be developed using a similar approach to that used by Smith (2008) for GARCH models. The results in this paper also raise questions such as when structural breaks are ignored, what is the impact on GAS models with non-normal distributions other than a Student's t? Future research could expand this simulation study to include GAS models with errors based on distributions other than Gaussian and Student's t. Future research could also extend Rapach and Strauss's (2008) study of the empirical relevance of structural breaks and their impact on forecasting to GAS models, using data on variables such as oil prices, stock market prices, and exchange rates.

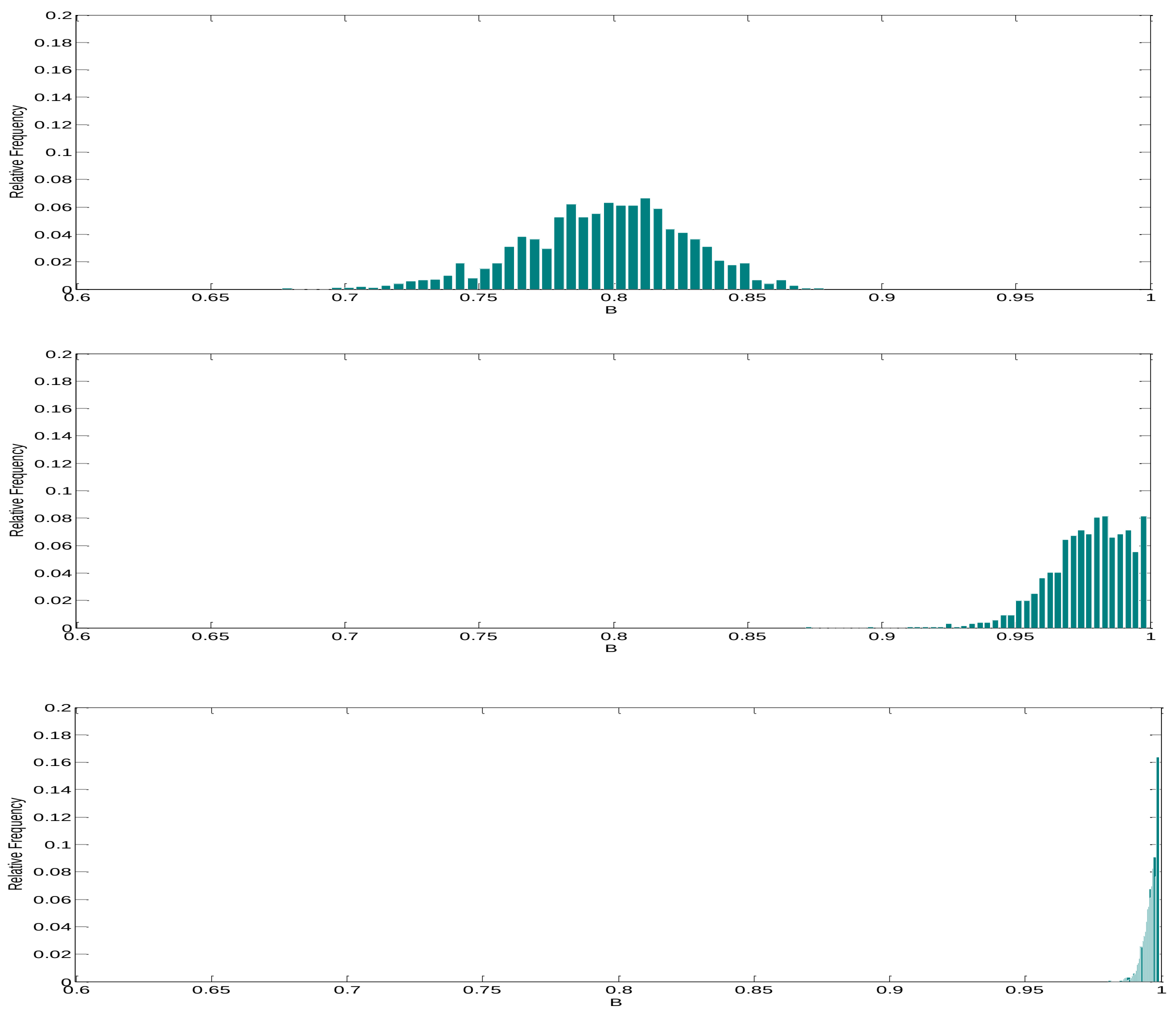
3.7 Empirical Distributions

Figure 3.1: The effects of ignored breaks in ω on the sampling distribution of $\hat{B}$ for the t-GAS model



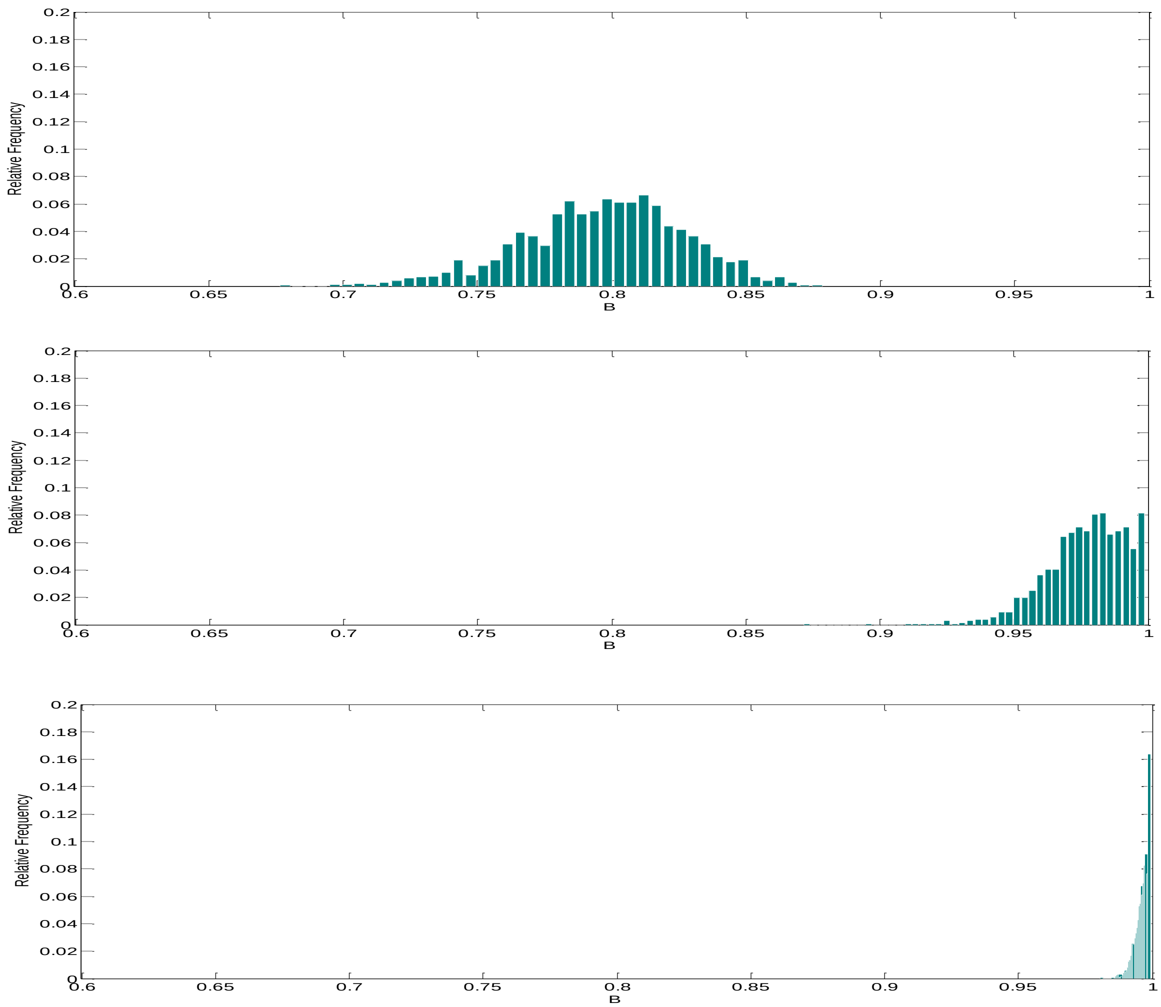
Notes: In all figures, $T = 10{,}000$ and the true value of B is 0.8. The top panel represents the empirical distribution of $\hat{B}$ in the no structural break case. The middle panel is the empirical distribution in the case of a mid-size structural break case with $\sigma_2 = 1.45$ ($\Delta\omega = 0.22$). The bottom panel presents the empirical distribution of $\hat{B}$ when the structural break in the middle of the sample is large, with $\sigma_2 = 1.75$ ($\Delta\omega = 0.41$).

Figure 3.2: The effects of ignored breaks in ω on the sampling distribution of $\hat{B}$ for the GARCH model



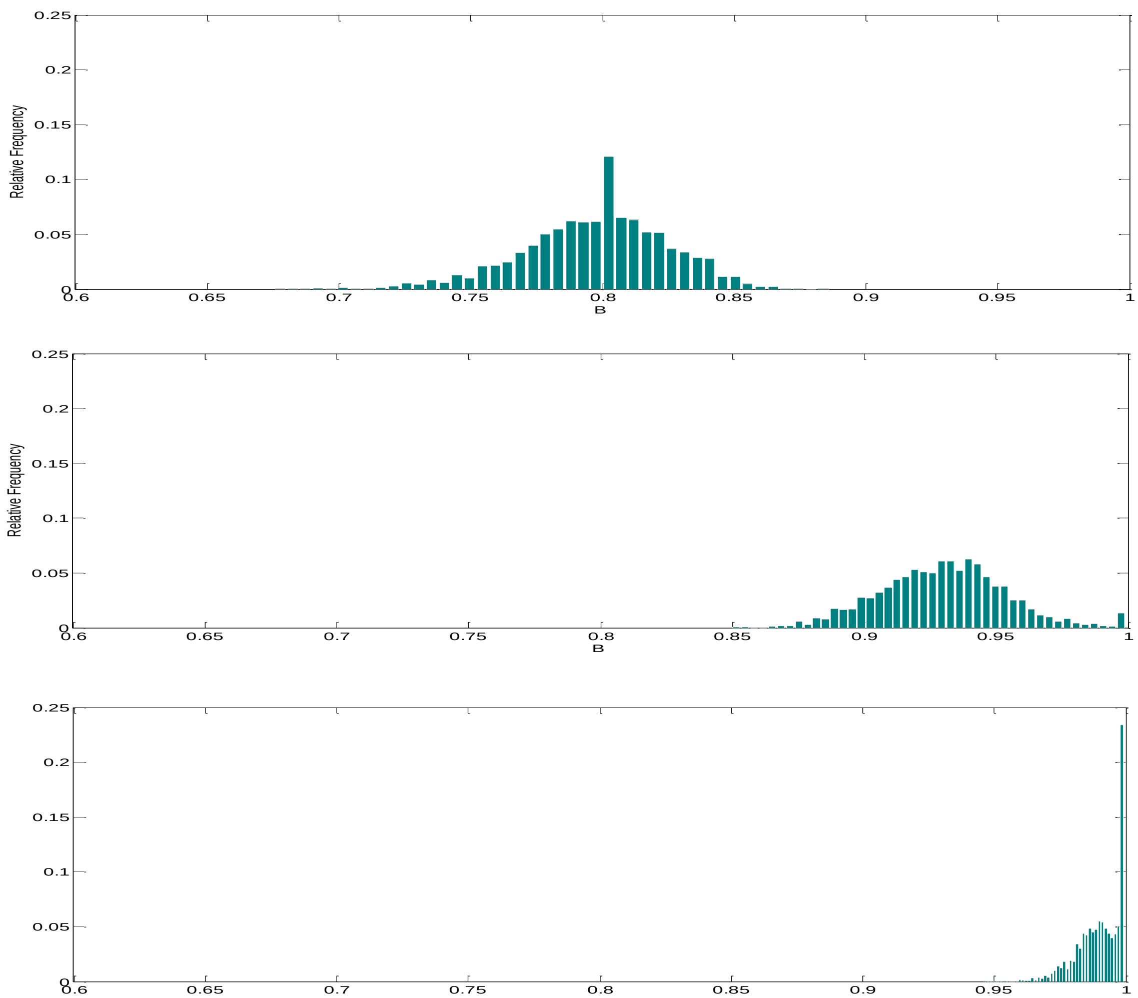
Notes: In all figures, $T = 10{,}000$ and the true value of B is 0.8. The top panel represents the empirical distribution of $\hat{B}$ in the no structural break case. The middle panel is the empirical distribution in the case of a mid-size structural break case with $\sigma_2 = 1.45$ ($\Delta\omega = 0.22$). The bottom panel presents the empirical distribution of $\hat{B}$ when the structural break in the middle of the sample is large, with $\sigma_2 = 1.75$ ($\Delta\omega = 0.41$).

Figure 3.3: The effects of ignored breaks in ω on the sampling distribution of $\hat{B}$ for the GAS specification of the GARCH model



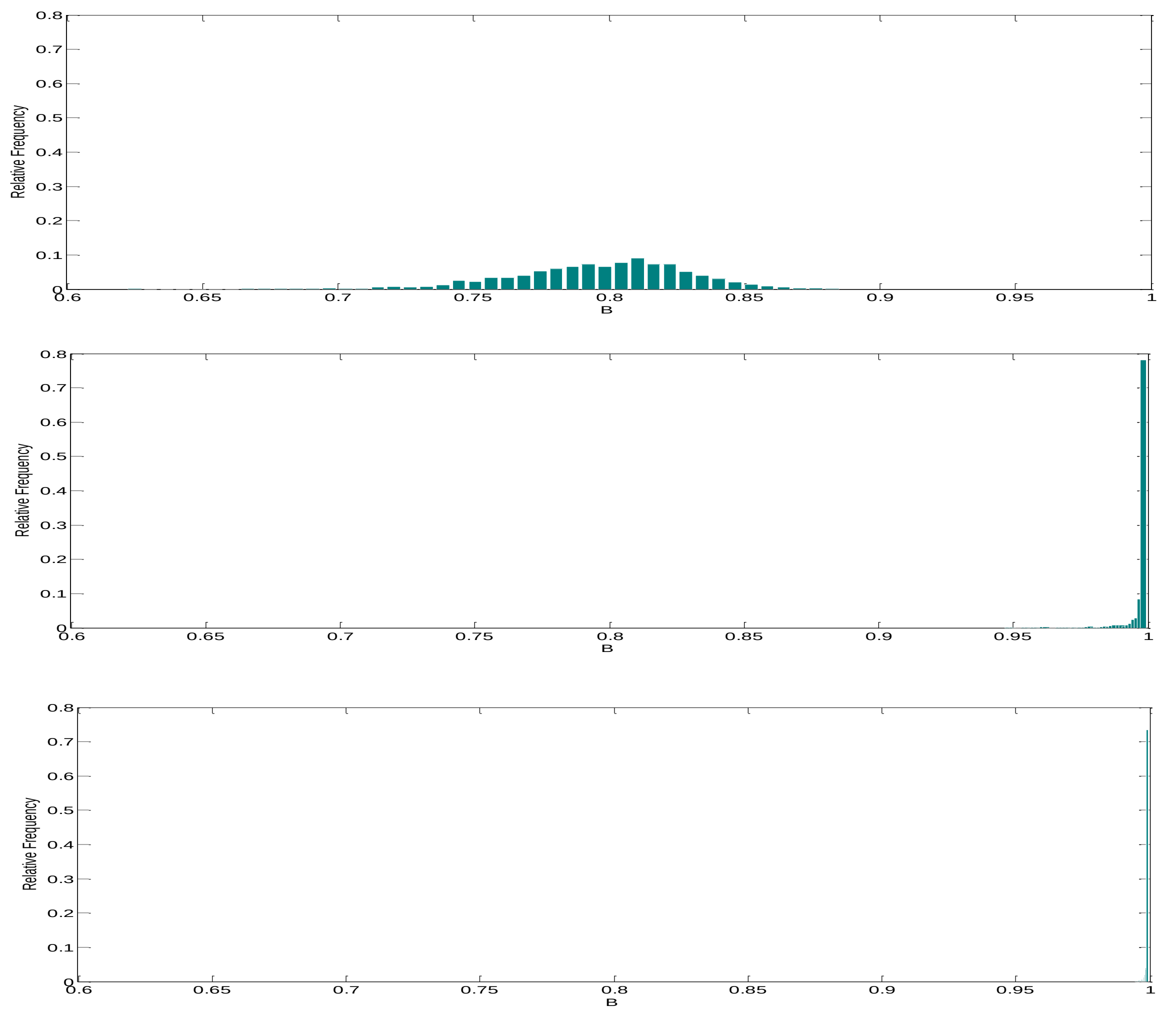
Notes: In all figures, $T = 10{,}000$ and the true value of B is 0.8. The top panel represents the empirical distribution of $\hat{B}$ in the no structural break case. The middle panel is the empirical distribution in the case of a mid-size structural break case with $\sigma_2 = 1.45$ ($\Delta\omega = 0.22$). The bottom panel presents the empirical distribution of $\hat{B}$ when the structural break in the middle of the sample is large, with $\sigma_2 = 1.75$ ($\Delta\omega = 0.41$).

Figure 3.4: The effects of ignored breaks in ω on the sampling distribution of $\hat{B}$ for the t-GARCH(1,1) model



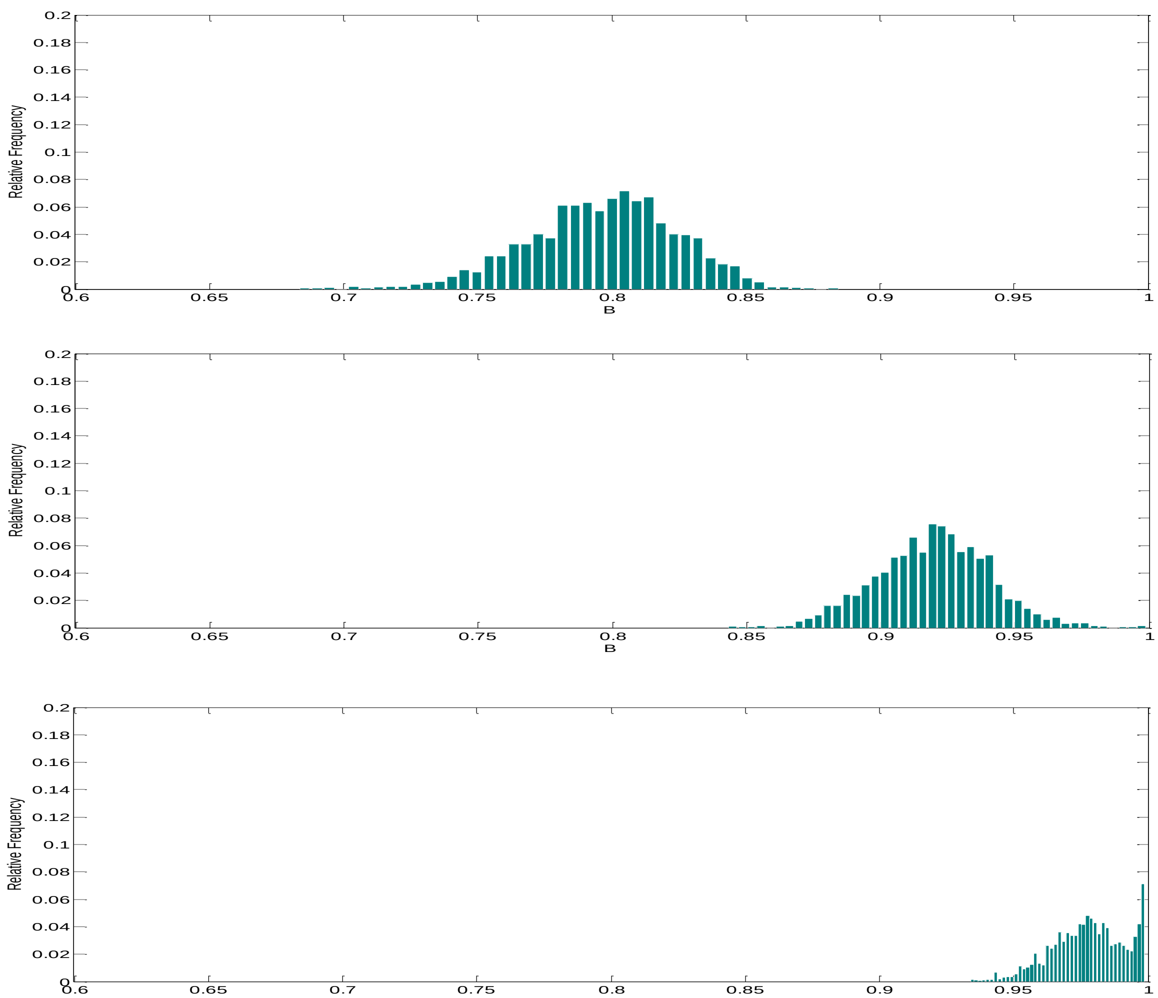
Notes: In all figures, $T = 10{,}000$ and the true value of B is 0.8. The top panel represents the empirical distribution of $\hat{B}$ in the no structural break case. The middle panel is the empirical distribution in the case of a mid-size structural break case with $\sigma_2 = 1.45$ ($\Delta\omega = 0.22$). The bottom panel presents the empirical distribution of $\hat{B}$ when the structural break in the middle of the sample is large, with $\sigma_2 = 1.75$ ($\Delta\omega = 0.41$).

Figure 3.5: The effects of ignored breaks in ω on the sampling distribution of $\hat{B}$ for the EGARCH model



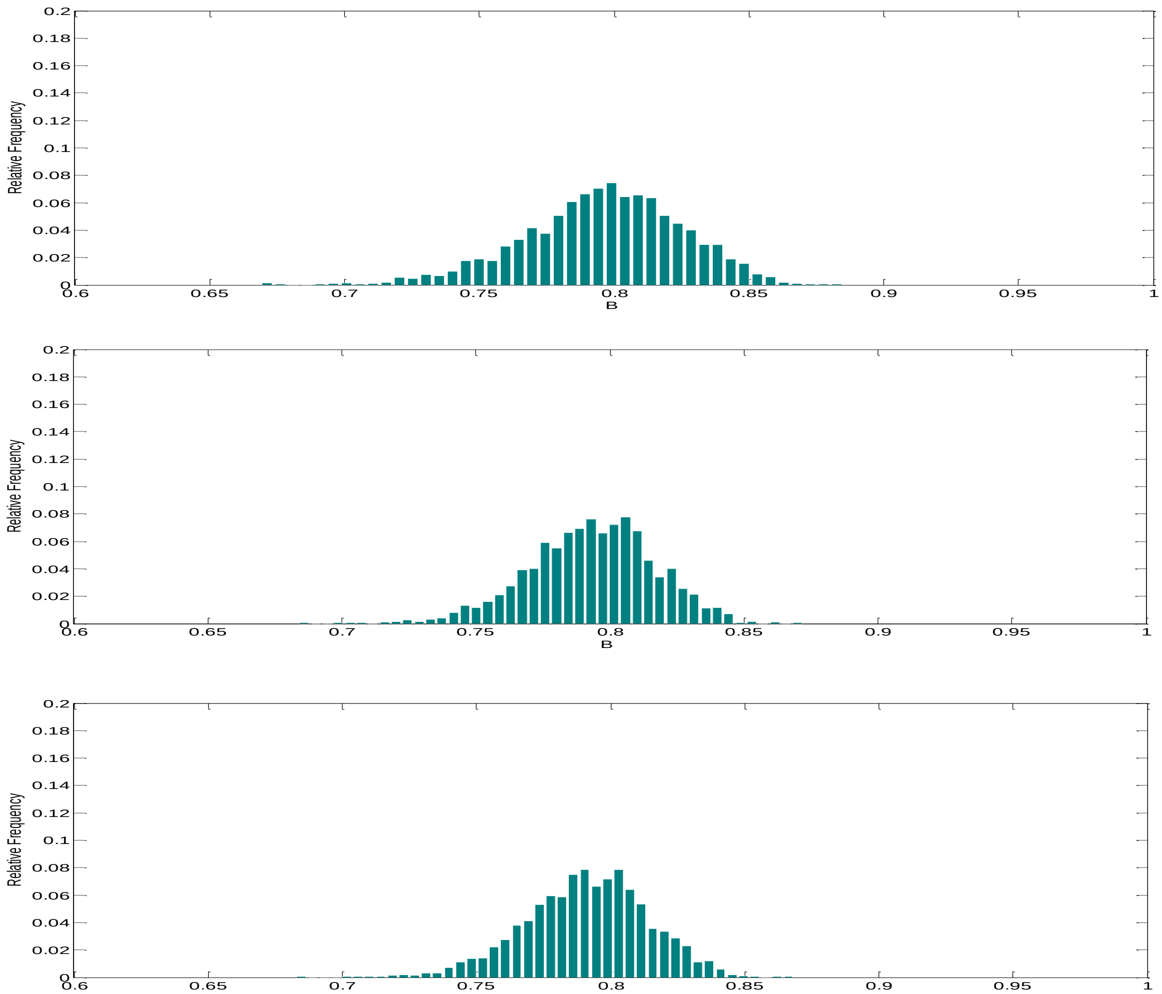
Notes: In all figures, $T = 10{,}000$ and the true value of B is 0.8. The top panel represents the empirical distribution of $\hat{B}$ in the no structural break case. The middle panel is the empirical distribution in the case of a mid-size structural break case with $\sigma_2 = 1.45$ ($\Delta\omega = 0.15$). The bottom panel presents the empirical distribution of $\hat{B}$ when the structural break in the middle of the sample is large, with $\sigma_2 = 1.75$ ($\Delta\omega = 0.22$).

Figure 3.6: The effects of ignored breaks in ω on the sampling distribution of $\hat{B}$ for the Beta-t-EGARCH(1,1) model



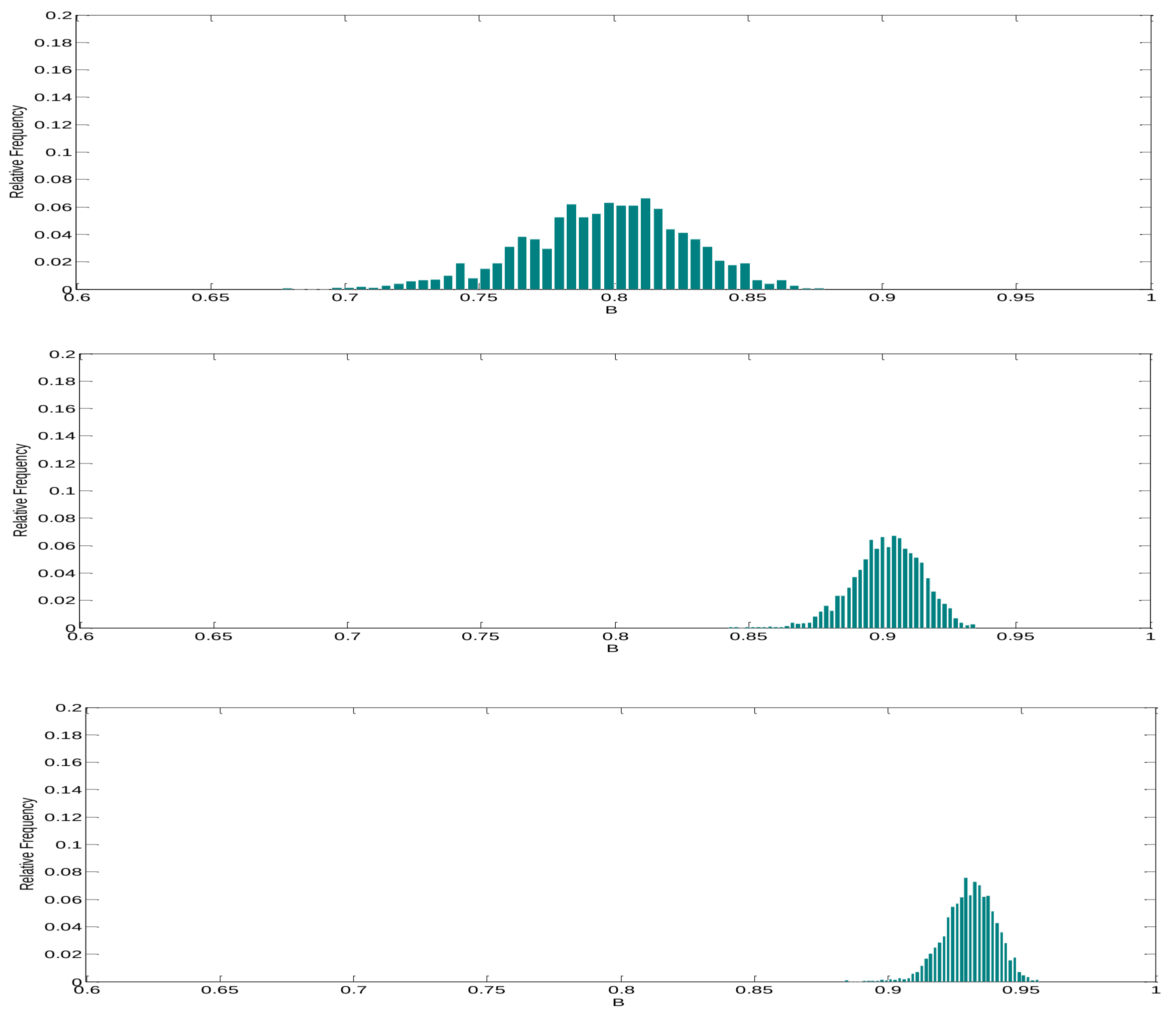
Notes: In all figures, $T = 10{,}000$ and the true value of B is 0.8. The top panel represents the empirical distribution of $\hat{B}$ in the no structural break case. The middle panel is the empirical distribution in the case of a mid-size structural break case with $\sigma_2 = 1.45$ ($\Delta\omega = 0.15$). The bottom panel presents the empirical distribution of $\hat{B}$ when the structural break in the middle of the sample is large, with $\sigma_2 = 1.75$ ($\Delta\omega = 0.22$).

Figure 3.7: The effects of ignored breaks in A on the sampling distribution of $\hat{B}$ for the t-GAS model



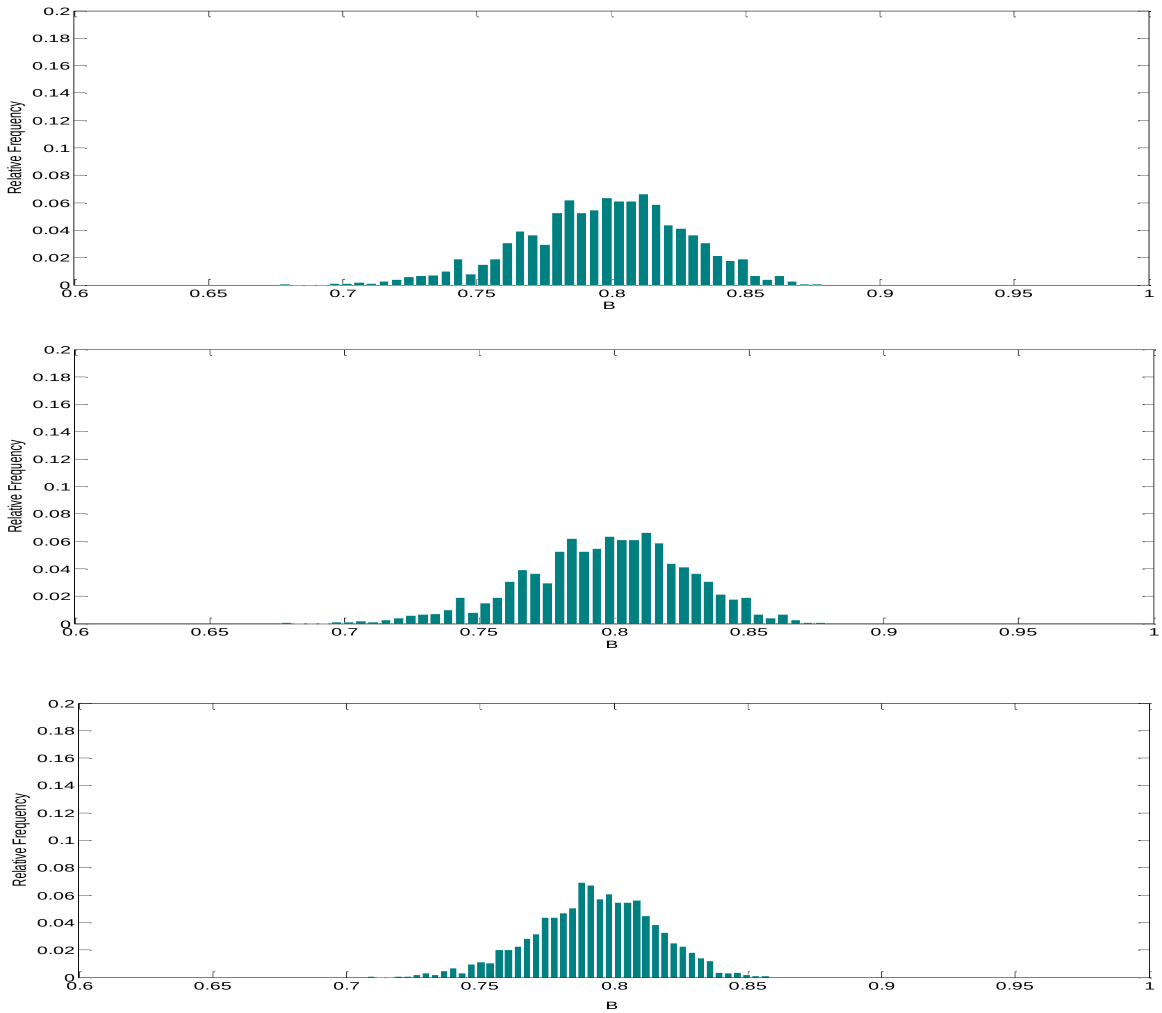
Notes: In all figures, T = 10,000 and the true value of B is 0.8. The top panel represents the empirical distribution of $\hat{B}$ in the no structural break case. The middle panel displays the curve of the empirical distribution of simulations in the case of a mid-size structural break with $\Delta A = 0.10$. The bottom panel presents the empirical distribution of $\hat{B}$ when the structural break in the middle of the sample is large, with $\Delta A = 0.13$.

Figure 3.8: The effects of ignored breaks in A on the sampling distribution of $\hat{B}$ for the GARCH(1,1) model



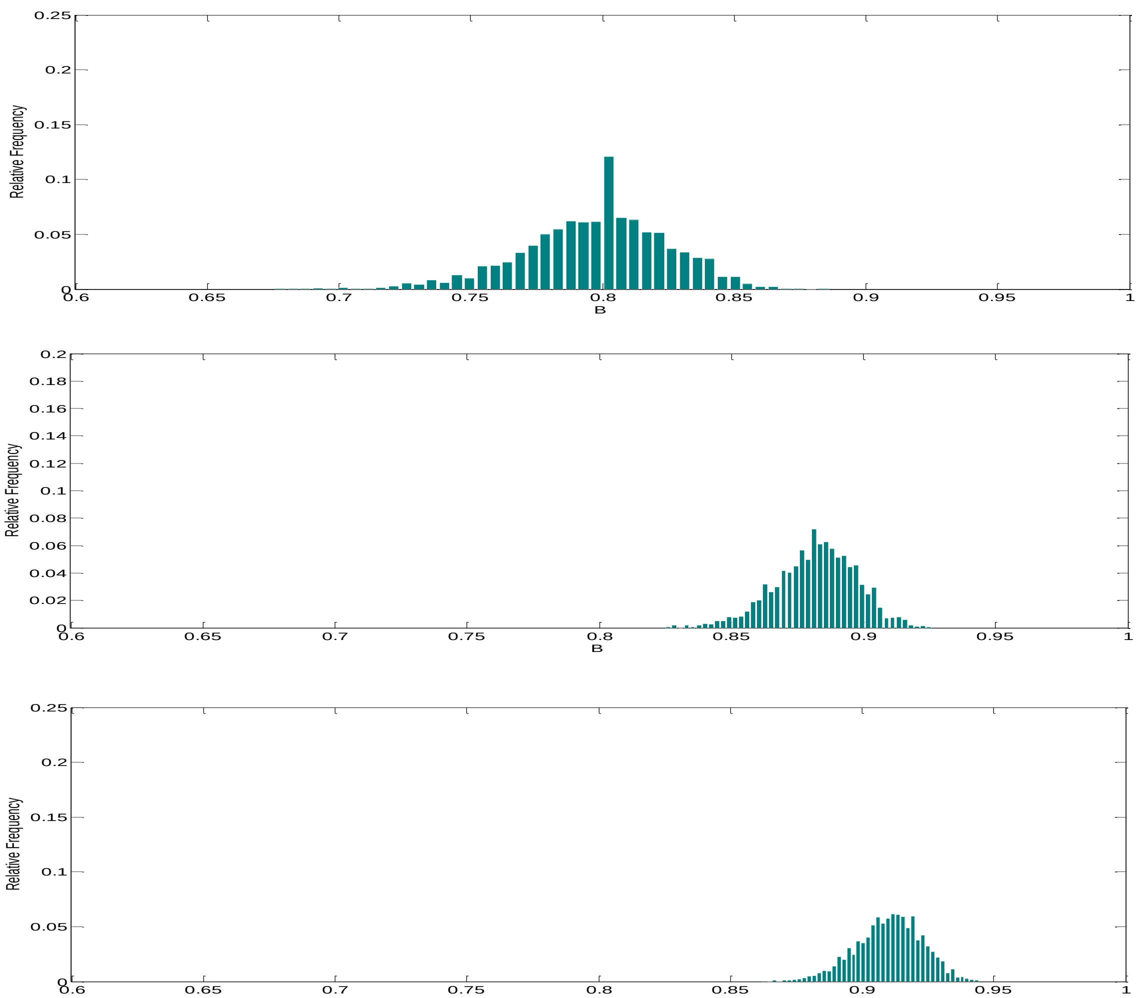
Notes: In all figures, $T = 10{,}000$ and the true value of B is 0.8. The top panel represents the empirical distribution of $\hat{B}$ in the no structural break case. The middle panel is the empirical distribution in the case of a mid-size structural break case with $\sigma_2 = 1.45$ ($\Delta A = 0.10$). The bottom panel presents the empirical distribution of $\hat{B}$ when the structural break in the middle of the sample is large, with $\sigma_2 = 1.75$ ($\Delta A = 0.13$).

Figure 3.9: The effects of ignored breaks in A on the sampling distribution of $\hat{B}$ for the GAS specification of the GARCH model



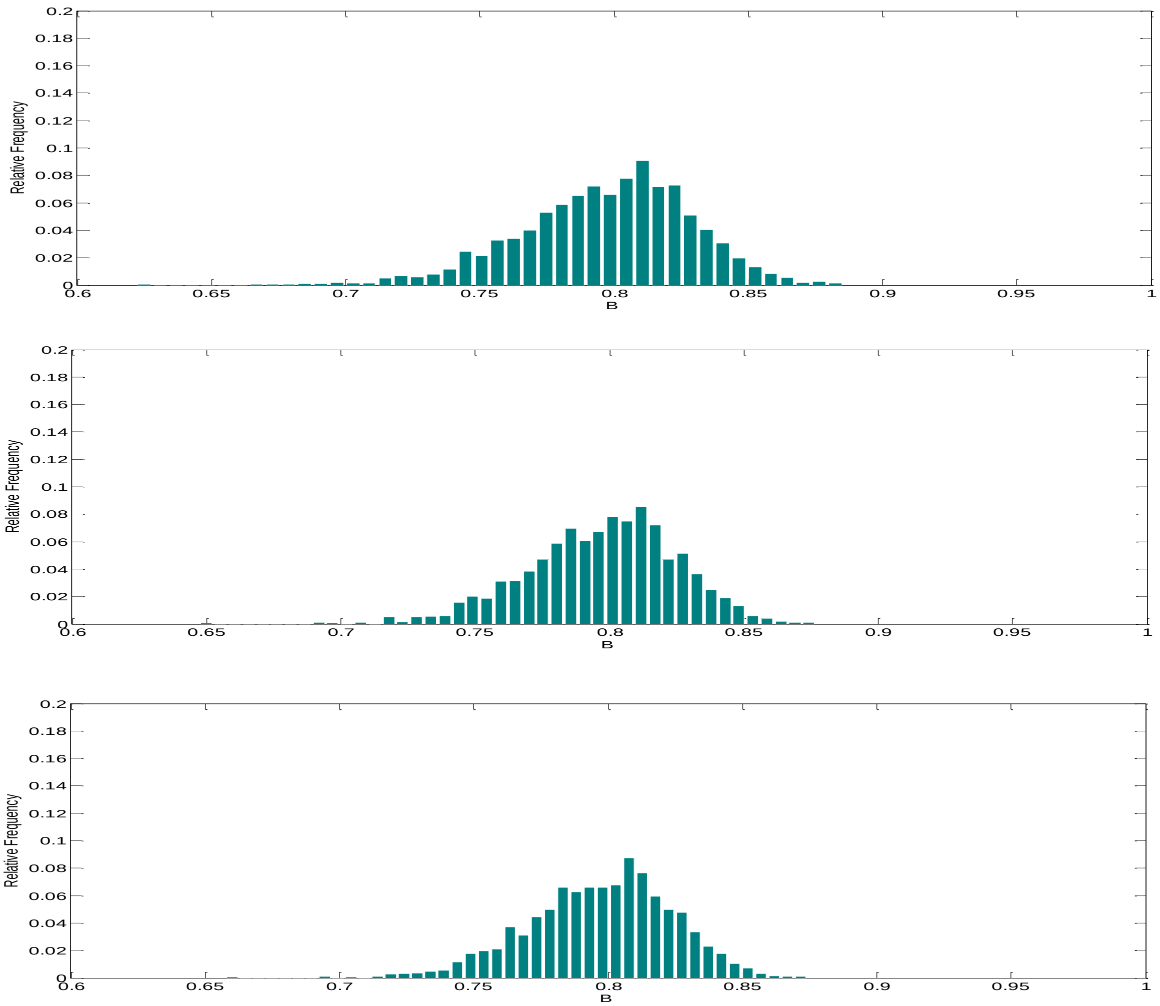
Notes: In all figures, T = 10,000 and the true value of B is 0.8. The top panel represents the empirical distribution of $\hat{B}$ in the no structural break case. The middle panel displays the curve of the empirical distribution of simulations in the case of a mid-size structural break with $\Delta A = 0.10$. The bottom panel presents the empirical distribution of $\hat{B}$ when the structural break in the middle of the sample is large, with $\Delta A = 0.13$

Figure 3.10: The effects of ignored breaks in A on the sampling distribution of $\hat{B}$ for the t-GARCH(1,1) model



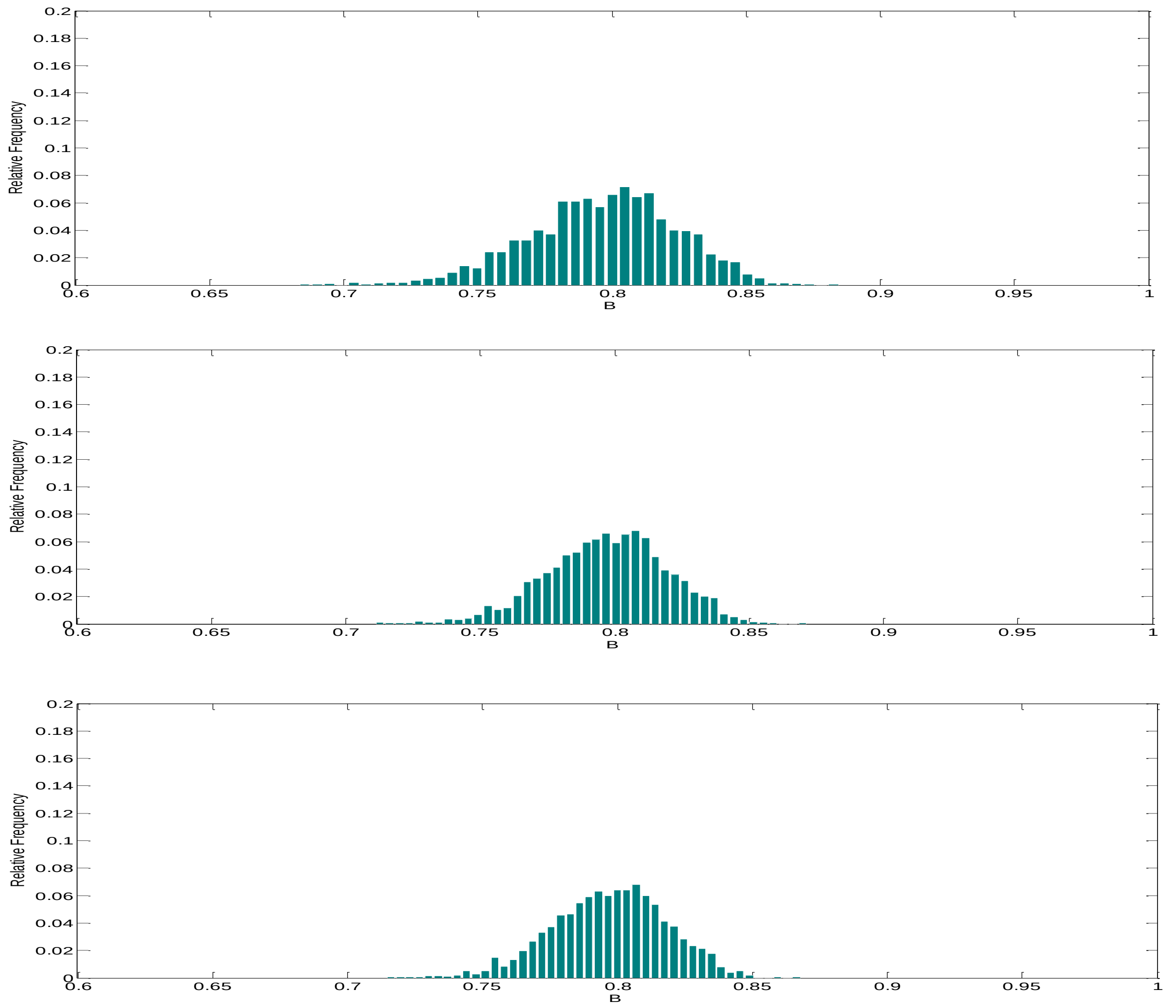
Notes: In all figures, $T = 10{,}000$ and the true value of B is 0.8. The top panel represents the empirical distribution of $\hat{B}$ in the no structural break case. The middle panel is the empirical distribution in the case of a mid-size structural break case with $\sigma_2 = 1.45$ ($\Delta A = 0.10$). The bottom panel presents the empirical distribution of $\hat{B}$ when the structural break in the middle of the sample is large, with $\sigma_2 = 1.75$ ($\Delta A = 0.13$).

Figure 3.11: The effects of ignored breaks in A on the sampling distribution of $\hat{B}$ for the EGARCH(1,1) model



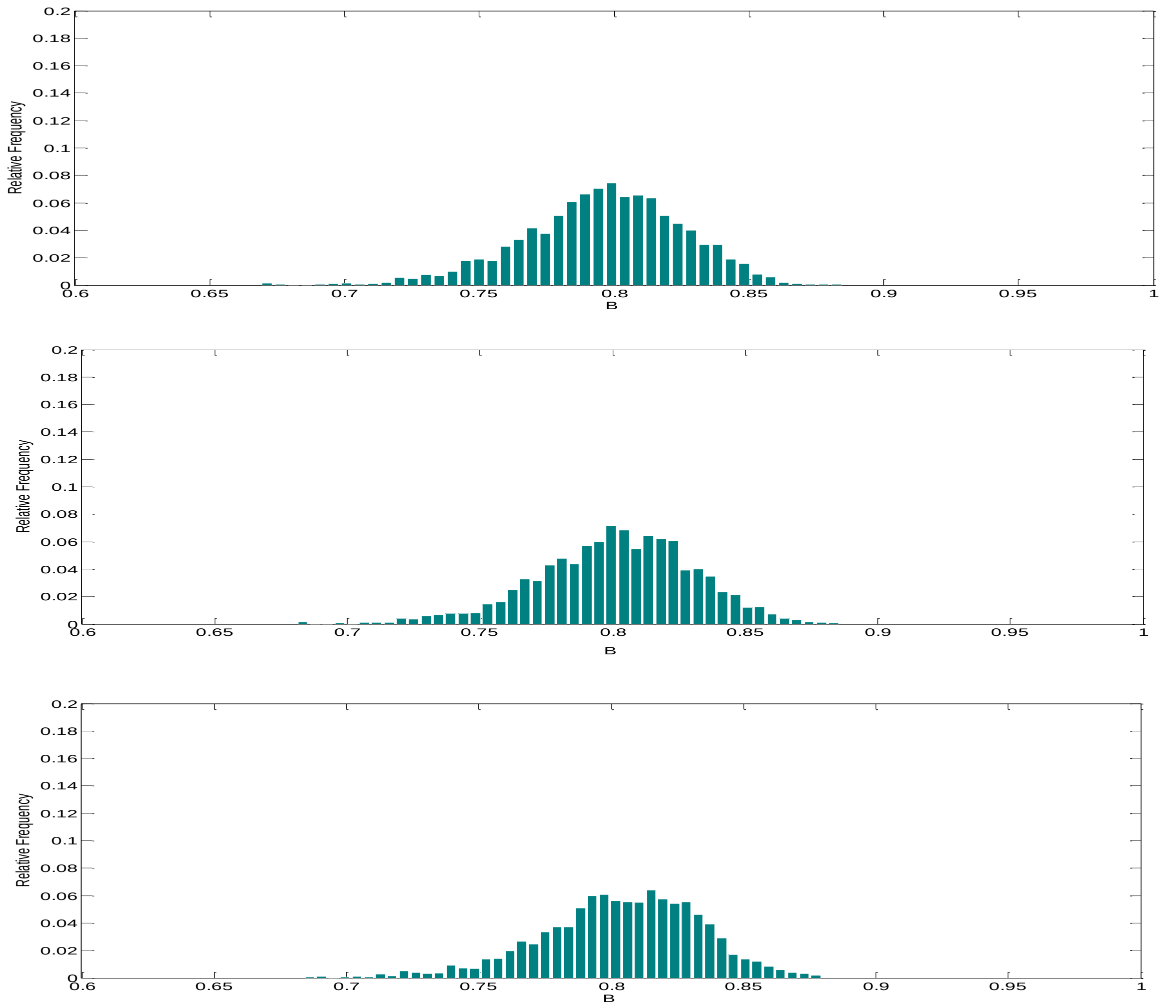
Notes: In all figures, T = 10,000 and the true value of B is 0.8. The top panel represents the empirical distribution of $\hat{B}$ in the no structural break case. The middle panel displays the curve of the empirical distribution of simulations in the case of a mid-size structural break with $\Delta A = 0.10$. The bottom panel presents the empirical distribution of $\hat{B}$ when the structural break in the middle of the sample is large, with $\Delta A = 0.13$.

Figure 3.12: The effects of ignored breaks in A on the sampling distribution of $\hat{B}$ for the Beta-t-EGARCH(1,1) model



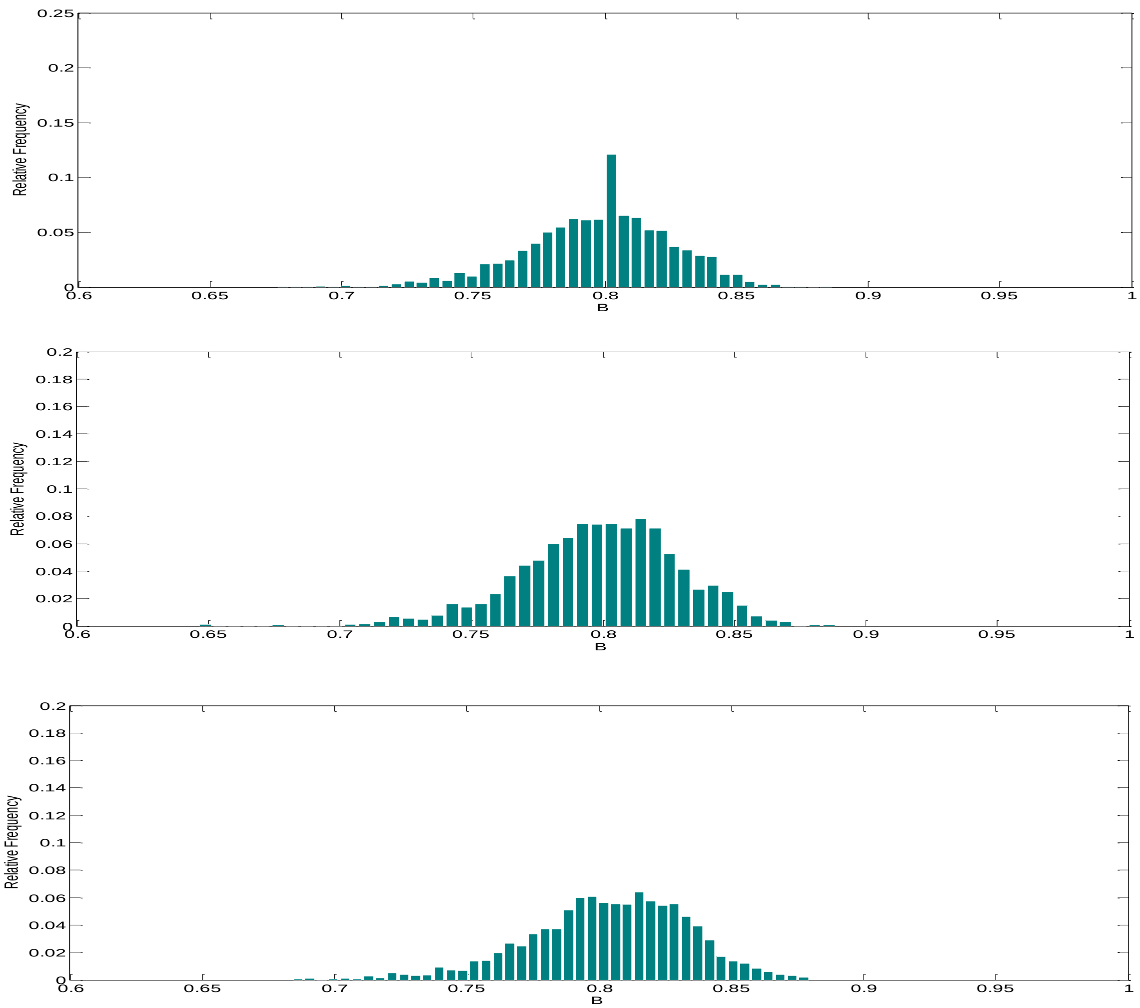
Notes: In all figures, T = 10,000 and the true value of B is 0.8. The top panel represents the empirical distribution of $\hat{B}$ in the no structural break case. The middle panel displays the curve of the empirical distribution of simulations in the case of a mid-size structural break with $\Delta A = 0.10$. The bottom panel presents the empirical distribution of $\hat{B}$ when the structural break in the middle of the sample is large, with $\Delta A = 0.13$.

Figure 3.13: The effects of ignored breaks in η on the sampling distribution of $\hat{B}$ for the t-GAS model



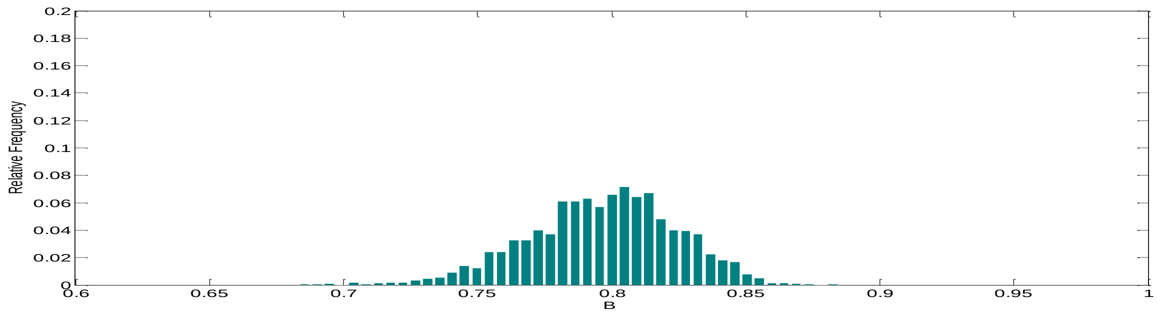
Notes: In all figures, T = 10,000 and the true value of B is 0.8. The top panel represents the empirical distribution of $\hat{B}$ in the no structural break case. The middle panel displays the curve of the empirical distribution of simulations in the case of a mid-size structural break with $\Delta\eta = 3$. The bottom panel presents the empirical distribution of $\hat{B}$ when the structural break in the middle of the sample is large, with $\Delta\eta = 5$.

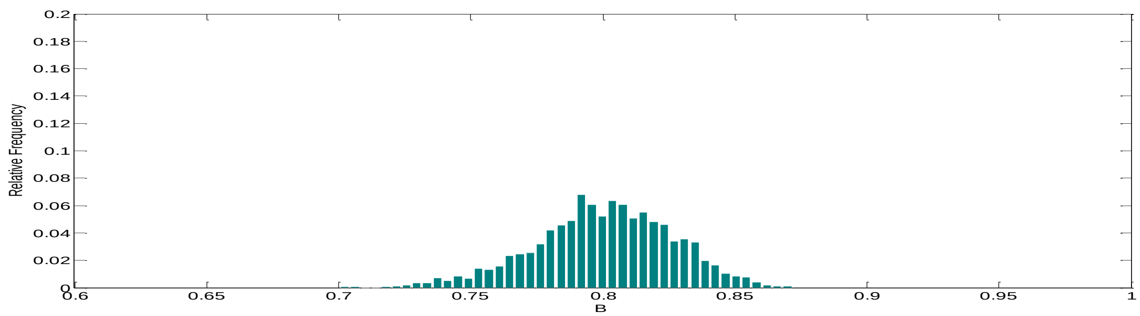
Figure 3.14: The effects of ignored breaks in η on the sampling distribution of $\hat{B}$ for the t-GARCH(1,1) model

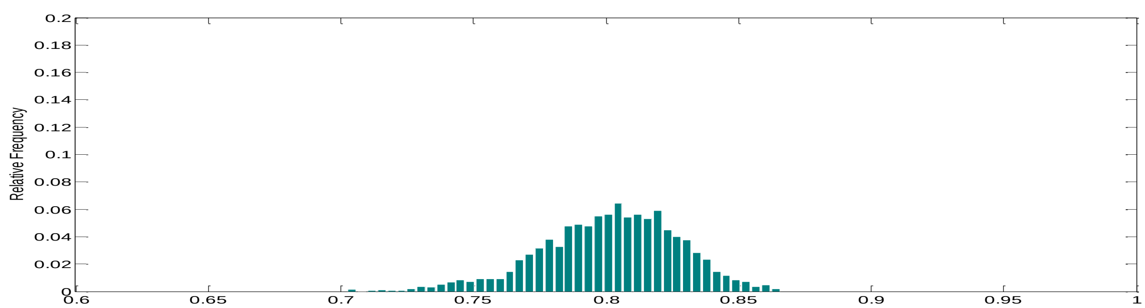


Notes: In all figures, T = 10,000 and the true value of B is 0.8. The top panel represents the empirical distribution of $\hat{B}$ in the no structural break case. The middle panel displays the curve of the empirical distribution of simulations in the case of a mid-size structural break with $\Delta\eta = 3$. The bottom panel presents the empirical distribution of $\hat{B}$ when the structural break in the middle of the sample is large, with $\Delta\eta = 5$.

Figure 3.15: The effects of ignored breaks in η on the sampling distribution of $\hat{B}$ for the Beta-t-EGARCH model







Notes: In all figures, T = 10,000 and the true value of B is 0.8. The top panel represents the empirical distribution of $\hat{B}$ in the no structural break case. The middle panel displays the curve of the empirical distribution of simulations in the case of a mid-size structural break with $\Delta\eta = 3$. The bottom panel presents the empirical distribution of $\hat{B}$ when the structural break in the middle of the sample is large, with $\Delta\eta = 5$.

3.8 Tables

Table 3.1: Table of initial parameter values and parameter switches

σ	ω^a	ω^b	B	A	η
1	0.2	0	0.8	0.1	5
1.15	0.26	0.06	0.85	0.15	6
1.3	0.34	0.1	0.88	0.18	7
1.45	0.42	0.15	0.9	0.2	8
1.6	0.51	0.19	0.92	0.22	9
1.75	0.61	0.22	0.93	0.23	10

Notes: ω^a is constant of the conditional variance for the t-GAS, GARCH, GAS-GARCH, and t-GARCH models. ω^b is constant for the Beta-t-EGARCH(1,1) and EGARCH models. ω^a and ω^b are chosen so as to generate the value of σ given in the left-most column of the table, holding the other parameters fixed. B is also chosen to generate the value of σ in the left-most column of the table, keeping the other parameters fixed. For the GARCH and t-GARCH models only, A is selected so as to generate the corresponding value of σ. The same values of A used for GARCH and t-GARCH models are also used for the t-GAS, GAS-GARCH, EGARCH, and Beta-t-EGARCH models. Additionally, for the t-distribution models only, changes to η are arbitrary.

Table 3.2: Effect of a change in ω on $\hat{B}$

T	Vol.	t-GAS		t-GARCH		GARCH		GAS-GARCH		EGARCH		Beta-t-EGARCH	
		Mean est.	Std. Dev.	Mean est.	Std. Dev.	Mean est.	Std. Dev.	Mean est.	Std. Dev.	Mean est.	Std. Dev.	Mean est.	Std. Dev.
	1	0.69	0.24	0.68	0.26	0.69	0.25	0.69	0.25	0.66	0.36	0.64	0.37
	1.15	0.71	0.24	0.72	0.26	0.72	0.25	0.72	0.25	0.67	0.36	0.67	0.35
250	1.3	0.77	0.22	0.78	0.24	0.80	0.22	0.80	0.22	0.72	0.35	0.72	0.33
	1.45	0.83	0.19	0.84	0.20	0.87	0.18	0.87	0.18	0.79	0.32	0.79	0.26
	1.6	0.88	0.16	0.89	0.17	0.91	0.14	0.91	0.14	0.85	0.30	0.85	0.21
	1.75	0.92	0.13	0.93	0.14	0.95	0.11	0.95	0.11	0.87	0.30	0.89	0.17
	1	0.73	0.19	0.74	0.20	0.73	0.20	0.73	0.20	0.72	0.27	0.71	0.26
	1.15	0.76	0.18	0.76	0.19	0.76	0.19	0.77	0.19	0.75	0.26	0.74	0.24
500	1.3	0.82	0.17	0.83	0.16	0.86	0.15	0.86	0.15	0.83	0.21	0.79	0.21
	1.45	0.88	0.13	0.89	0.13	0.92	0.10	0.92	0.10	0.92	0.16	0.86	0.15
	1.6	0.92	0.10	0.93	0.10	0.96	0.06	0.96	0.06	0.95	0.12	0.91	0.11
	1.75	0.96	0.07	0.96	0.07	0.98	0.04	0.98	0.04	0.97	0.10	0.94	0.08
	1	0.77	0.13	0.77	0.13	0.76	0.13	0.76	0.13	0.76	0.16	0.76	0.14
	1.15	0.79	0.12	0.79	0.12	0.80	0.12	0.80	0.12	0.81	0.14	0.80	0.12
1000	1.3	0.85	0.10	0.86	0.10	0.89	0.09	0.89	0.09	0.88	0.12	0.84	0.11
	1.45	0.91	0.08	0.91	0.08	0.95	0.06	0.95	0.06	0.96	0.07	0.90	0.08
	1.6	0.95	0.05	0.95	0.05	0.98	0.03	0.98	0.03	0.98	0.03	0.94	0.06
	1.75	0.97	0.03	0.98	0.03	0.99	0.01	0.99	0.01	0.99	0.01	0.96	0.04
	1	0.79	0.07	0.79	0.07	0.79	0.07	0.79	0.07	0.79	0.08	0.79	0.06
	1.15	0.81	0.07	0.81	0.07	0.82	0.06	0.82	0.06	0.83	0.07	0.82	0.06
2500	1.3	0.87	0.06	0.87	0.06	0.91	0.05	0.91	0.05	0.91	0.07	0.86	0.06
	1.45	0.93	0.05	0.93	0.05	0.97	0.03	0.97	0.03	0.99	0.02	0.92	0.04
	1.6	0.97	0.03	0.97	0.03	0.99	0.01	0.99	0.01	0.99	0.01	0.95	0.03
	1.75	0.98	0.01	0.99	0.02	0.99	0.01	0.99	0.01	1.00	0.00	0.97	0.02
	1	0.79	0.04	0.79	0.04	0.79	0.04	0.79	0.04	0.79	0.05	0.79	0.04
	1.15	0.82	0.04	0.82	0.04	0.83	0.04	0.83	0.04	0.84	0.05	0.82	0.04
5000	1.3	0.88	0.04	0.87	0.04	0.92	0.04	0.92	0.04	0.93	0.05	0.86	0.04
	1.45	0.93	0.03	0.93	0.03	0.97	0.02	0.97	0.02	0.99	0.01	0.92	0.03
	1.6	0.97	0.02	0.97	0.02	0.99	0.01	0.99	0.01	1.00	0.00	0.96	0.02
	1.75	0.99	0.01	0.99	0.01	1.00	0.00	1.00	0.00	1.00	0.00	0.98	0.02
	1	0.80	0.03	0.80	0.03	0.80	0.03	0.80	0.03	0.80	0.03	0.80	0.03
	1.15	0.82	0.03	0.82	0.03	0.83	0.03	0.83	0.03	0.84	0.03	0.82	0.03
10000	1.3	0.88	0.03	0.87	0.03	0.92	0.03	0.92	0.03	0.93	0.04	0.86	0.02
	1.45	0.93	0.02	0.93	0.02	0.98	0.01	0.98	0.01	1.00	0.01	0.92	0.02
	1.6	0.97	0.02	0.97	0.02	0.99	0.01	0.99	0.01	1.00	0.00	0.96	0.02
	1.75	0.99	0.01	0.99	0.01	1.00	0.00	1.00	0.00	1.00	0.00	0.98	0.01

Table 3.3: Effect of a change in ω on the coverage probability of B

T	Vol.	t-GAS		t-GARCH		GARCH		GAS-GARCH		EGARCH		Beta-t-EGARCH	
		90%	95%	90%	95%	90%	95%	90%	95%	90%	95%	90%	95%
	1	0.801	0.854	0.833	0.882	0.782	0.836	0.781	0.837	0.619	0.683	0.725	0.776
		(0.009)	(0.008)	(0.008)	(0.007)	(0.009)	(0.008)	(0.009)	(0.008)	(0.011)	(0.010)	(0.010)	(0.009)
	1.15	0.777	0.829	0.805	0.852	0.739	0.795	0.738	0.793	0.668	0.723	0.706	0.760
		(0.009)	(0.010)	(0.009)	(0.008)	(0.010)	(0.009)	(0.010)	(0.009)	(0.011)	(0.010)	(0.010)	(0.010)
	1.3	0.702	0.751	0.699	0.742	0.605	0.656	0.604	0.658	0.551	0.599	0.661	0.711
250		(0.010)	(0.010)	(0.010)	(0.010)	(0.011)	(0.011)	(0.011)	(0.011)	(0.011)	(0.011)	(0.011)	(0.010)
	1.45	0.599	0.648	0.571	0.621	0.467	0.511	0.467	0.511	0.432	0.479	0.591	0.644
		(0.011)	(0.011)	(0.011)	(0.011)	(0.011)	(0.011)	(0.011)	(0.011)	(0.011)	(0.011)	(0.011)	(0.011)
	1.6	0.482	0.536	0.441	0.487	0.316	0.357	0.316	0.357	0.370	0.352	0.498	0.549
		(0.011)	(0.011)	(0.011)	(0.011)	(0.010)	(0.011)	(0.010)	(0.011)	(0.010)	(0.011)	(0.011)	(0.011)
	1.75	0.388	0.438	0.322	0.363	0.228	0.260	0.228	0.260	0.234	0.260	0.420	0.477
		(0.011)	(0.011)	(0.010)	(0.011)	(0.009)	(0.010)	(0.009)	(0.010)	(0.009)	(0.010)	(0.011)	(0.011)
	1	0.799	0.855	0.827	0.874	0.751	0.820	0.752	0.821	0.690	0.759	0.774	0.823
		(0.009)	(0.008)	(0.008)	(0.007)	(0.010)	(0.009)	(0.010)	(0.009)	(0.010)	(0.010)	(0.009)	(0.009)
	1.15	0.766	0.825	0.775	0.835	0.715	0.778	0.716	0.777	0.668	0.723	0.751	0.810
		(0.009)	(0.008)	(0.009)	(0.008)	(0.010)	(0.009)	(0.010)	(0.009)	(0.011)	(0.010)	(0.010)	(0.009)
	1.3	0.638	0.690	0.625	0.671	0.531	0.580	0.530	0.579	0.542	0.597	0.691	0.747
500		(0.011)	(0.010)	(0.011)	(0.011)	(0.011)	(0.011)	(0.011)	(0.011)	(0.011)	(0.011)	(0.010)	(0.010)
	1.45	0.480	0.530	0.421	0.473	0.296	0.341	0.294	0.340	0.283	0.322	0.532	0.589
		(0.011)	(0.011)	(0.011)	(0.011)	(0.010)	(0.011)	(0.010)	(0.011)	(0.010)	(0.010)	(0.011)	(0.011)
	1.6	0.322	0.370	0.259	0.292	0.141	0.167	0.141	0.166	0.139	0.158	0.372	0.419
		(0.010)	(0.011)	(0.010)	(0.010)	(0.008)	(0.008)	(0.008)	(0.008)	(0.008)	(0.008)	(0.011)	(0.011)
	1.75	0.189	0.221	0.137	0.164	0.063	0.079	0.062	0.078	0.062	0.077	0.029	0.039
		(0.009)	(0.009)	(0.008)	(0.008)	(0.005)	(0.006)	(0.005)	(0.006)	(0.005)	(0.006)	(0.010)	(0.010)

Continued on next page

Table 3.3 – continued from previous page

T	Vol.	t-GAS		t-GARCH		GARCH		GAS-GARCH		EGARCH		Beta-t-EGARCH	
	1	0.823	0.876	0.841	0.892	0.824	0.878	0.824	0.877	0.773	0.832	0.832	0.884
		(0.009)	(0.007)	(0.008)	(0.007)	(0.009)	(0.009)	(0.009)	(0.009)	(0.009)	(0.008)	(0.008)	(0.007)
	1.15	0.786	0.844	0.799	0.840	0.766	0.816	0.766	0.816	0.727	0.782	0.782	0.838
		(0.009)	(0.008)	(0.009)	(0.008)	(0.009)	(0.009)	(0.009)	(0.009)	(0.010)	(0.009)	(0.009)	(0.008)
	1.3	0.614	0.676	0.572	0.630	0.447	0.505	0.448	0.505	0.501	0.552	0.667	0.723
1000		(0.011)	(0.010)	(0.011)	(0.011)	(0.011)	(0.011)	(0.011)	(0.011)	(0.011)	(0.011)	(0.011)	(0.010)
	1.45	0.377	0.430	0.301	0.354	0.169	0.200	0.169	0.200	0.149	0.181	0.419	0.477
		(0.011)	(0.011)	(0.010)	(0.011)	(0.008)	(0.009)	(0.008)	(0.009)	(0.008)	(0.009)	(0.011)	(0.011)
	1.6	0.160	0.205	0.117	0.142	0.042	0.053	0.042	0.052	0.040	0.044	0.207	0.255
		(0.008)	(0.009)	(0.007)	(0.008)	(0.004)	(0.005)	(0.004)	(0.005)	(0.004)	(0.005)	(0.009)	(0.010)
	1.75	0.068	0.082	0.033	0.042	0.015	0.019	0.015	0.019	0.015	0.018	0.094	0.126
		(0.006)	(0.006)	(0.004)	(0.004)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.007)	(0.001)
	1	0.865	0.910	0.862	0.916	0.871	0.917	0.873	0.919	0.850	0.914	0.871	0.923
		(0.008)	(0.006)	(0.008)	(0.006)	(0.007)	(0.006)	(0.007)	(0.006)	(0.008)	(0.006)	(0.008)	(0.006)
	1.15	0.803	0.873	0.794	0.859	0.789	0.848	0.789	0.849	0.747	0.815	0.803	0.861
		(0.009)	(0.007)	(0.009)	(0.008)	(0.009)	(0.008)	(0.009)	(0.008)	(0.010)	(0.009)	(0.009)	(0.008)
	1.3	0.521	0.600	0.438	0.513	0.295	0.354	0.295	0.354	0.394	0.452	0.575	0.658
2500		(0.011)	(0.011)	(0.011)	(0.011)	(0.010)	(0.011)	(0.010)	(0.011)	(0.011)	(0.011)	(0.011)	(0.011)
	1.45	0.175	0.223	0.102	0.141	0.038	0.053	0.038	0.053	0.033	0.046	0.188	0.257
		(0.008)	(0.009)	(0.007)	(0.008)	(0.004)	(0.005)	(0.004)	(0.005)	(0.004)	(0.005)	(0.009)	(0.010)
	1.6	0.034	0.044	0.012	0.015	0.007	0.008	0.006	0.007	0.010	0.012	0.035	0.053
		(0.004)	(0.005)	(0.002)	(0.003)	(0.002)	(0.002)	(0.001)	(0.002)	(0.002)	(0.002)	(0.004)	(0.005)
	1.75	0.007	0.011	0.002	0.002	0.004	0.004	0.004	0.004	0.008	0.009	0.009	0.016
		(0.002)	(0.002)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.002)	(0.002)	(0.002)	(0.003)

Continued on next page

Table 3.3 – continued from previous page

T	Vol.	t-GAS		t-GARCH		GARCH		GAS-GARCH		EGARCH		Beta-t-EGARCH	
	1	0.877	0.932	0.883	0.930	0.887	0.936	0.887	0.936	0.872	0.926	0.873	0.931
		(0.007)	(0.006)	(0.007)	(0.006)	(0.007)	(0.005)	(0.007)	(0.005)	(0.007)	(0.006)	(0.007)	(0.006)
	1.15	0.807	0.875	0.810	0.869	0.774	0.844	0.776	0.843	0.741	0.812	0.789	0.859
		(0.009)	(0.007)	(0.009)	(0.008)	(0.007)	(0.009)	(0.007)	(0.009)	(0.010)	(0.009)	(0.009)	(0.008)
	1.3	0.353	0.446	0.275	0.354	0.120	0.177	0.119	0.177	0.226	0.299	0.453	0.550
5000		(0.011)	(0.011)	(0.010)	(0.011)	(0.007)	(0.009)	(0.007)	(0.009)	(0.009)	(0.010)	(0.011)	(0.011)
	1.45	0.043	0.069	0.018	0.026	0.008	0.010	0.010	0.012	0.036	0.038	0.052	0.076
		(0.005)	(0.006)	(0.003)	(0.004)	(0.002)	(0.002)	(0.002)	(0.002)	(0.004)	(0.004)	(0.005)	(0.006)
	1.6	0.011	0.014	0.001	0.001	0.007	0.008	0.008	0.008	0.016	0.020	0.007	0.009
		(0.002)	(0.003)	(0.000)	(0.000)	(0.002)	(0.002)	(0.002)	(0.002)	(0.003)	(0.003)	(0.002)	(0.002)
	1.75	0.006	0.007	0.001	0.001	0.003	0.004	0.003	0.004	0.012	0.014	0.004	0.004
		(0.002)	(0.002)	(0.000)	(0.000)	(0.001	(0.001)	(0.001)	(0.001)	(0.002)	(0.003)	(0.001)	(0.001)
	1	0.881	0.938	0.896	0.950	0.887	0.941	0.887	0.941	0.890	0.942	0.879	0.938
		(0.007)	(0.005)	(0.007)	(0.005)	(0.007)	(0.005)	(0.007)	(0.005)	(0.007)	(0.005)	(0.007)	(0.005)
	1.15	0.781	0.854	0.777	0.848	0.653	0.775	0.655	0.776	0.633	0.744	0.749	0.827
		(0.009)	(0.008)	(0.009)	(0.008)	(0.011)	(0.009)	(0.011)	(0.009)	(0.011)	(0.010)	(0.010)	(0.008)
	1.3	0.129	0.207	0.104	0.155	0.014	0.025	0.013	0.025	0.081	0.134	0.242	0.329
10000		(0.007)	(0.009)	(0.007)	(0.008)	(0.003)	(0.003)	(0.003)	(0.003)	(0.006)	(0.008)	(0.010)	(0.011)
	1.45	0.001	0.004	0.000	0.000	0.008	0.009	0.006	0.007	0.051	0.058	0.003	0.004
		(0.001)	(0.001)	(0.000)	(0.000)	(0.002)	(0.002)	(0.002)	(0.002)	(0.005)	(0.005)	(0.001)	(0.001)
	1.6	0.004	0.005	0.001	0.001	0.008	0.009	0.007	0.009	0.026	0.029	0.002	0.003
		(0.001)	(0.001)	(0.000)	(0.000)	(0.002)	(0.002)	(0.002)	(0.002)	(0.004)	(0.004)	(0.001)	(0.001)
	1.75	0.005	0.006	0.001	0.001	0.002	0.003	0.002	0.003	0.022	0.026	0.004	0.005
		(0.002)	(0.002)	(0.000)	(0.000)	(0.001)	(0.001)	(0.001)	(0.001)	(0.003)	(0.004)	(0.001)	(0.002)

Table 3.4: Effect of a change in A on $\hat{B}$

T	ΔA	t-GAS		t-GARCH		GARCH		GAS-GARCH		EGARCH		Beta-t-EGARCH	
		Mean est.	Std. Dev.	Mean est.	Std. Dev.	Mean est.	Std. Dev.	Mean est.	Std. Dev.	Mean est.	Std. Dev.	Mean est.	Std. Dev.
	0	0.69	0.24	0.68	0.27	0.69	0.25	0.69	0.25	0.66	0.36	0.64	0.37
	0.05	0.70	0.22	0.74	0.24	0.75	0.21	0.71	0.23	0.67	0.35	0.66	0.34
250	0.08	0.71	0.21	0.78	0.21	0.80	0.18	0.71	0.22	0.67	0.34	0.67	0.32
	0.10	0.72	0.20	0.81	0.20	0.83	0.16	0.72	0.21	0.67	0.34	0.67	0.31
	0.12	0.72	0.20	0.83	0.18	0.86	0.14	0.72	0.21	0.68	0.33	0.67	0.31
	0.13	0.72	0.19	0.85	0.18	0.88	0.13	0.72	0.20	0.68	0.33	0.68	0.30
	0	0.73	0.19	0.73	0.20	0.73	0.20	0.73	0.20	0.72	0.27	0.71	0.26
	0.05	0.75	0.16	0.79	0.16	0.79	0.15	0.74	0.17	0.72	0.26	0.73	0.22
500	0.08	0.75	0.15	0.83	0.13	0.84	0.12	0.74	0.16	0.72	0.25	0.74	0.20
	0.10	0.75	0.14	0.85	0.12	0.87	0.09	0.75	0.15	0.72	0.25	0.74	0.18
	0.12	0.76	0.14	0.87	0.10	0.90	0.07	0.75	0.15	0.73	0.23	0.75	0.17
	0.13	0.76	0.14	0.89	0.10	0.91	0.06	0.75	0.14	0.73	0.23	0.75	0.16
	0	0.77	0.13	0.77	0.13	0.77	0.13	0.76	0.13	0.76	0.16	0.76	0.14
	0.05	0.77	0.10	0.82	0.09	0.82	0.09	0.77	0.11	0.77	0.15	0.77	0.11
1000	0.08	0.78	0.09	0.85	0.07	0.87	0.06	0.78	0.10	0.77	0.14	0.78	0.10
	0.10	0.78	0.09	0.87	0.06	0.89	0.05	0.78	0.09	0.77	0.13	0.78	0.09
	0.12	0.78	0.09	0.89	0.05	0.91	0.04	0.78	0.09	0.77	0.13	0.78	0.09
	0.13	0.78	0.08	0.90	0.05	0.92	0.04	0.78	0.09	0.77	0.12	0.78	0.08
	0	0.79	0.07	0.79	0.07	0.79	0.07	0.79	0.07	0.79	0.08	0.79	0.06
	0.05	0.79	0.06	0.83	0.05	0.84	0.04	0.79	0.06	0.79	0.07	0.79	0.05
2500	0.08	0.79	0.05	0.86	0.04	0.88	0.03	0.79	0.05	0.79	0.06	0.79	0.05
	0.10	0.79	0.05	0.88	0.03	0.90	0.02	0.79	0.05	0.79	0.06	0.79	0.05
	0.12	0.79	0.05	0.90	0.03	0.92	0.02	0.79	0.05	0.79	0.06	0.79	0.05
	0.13	0.79	0.05	0.91	0.03	0.93	0.02	0.79	0.05	0.79	0.06	0.79	0.05
	0	0.79	0.04	0.79	0.04	0.79	0.04	0.79	0.04	0.79	0.05	0.79	0.04
	0.05	0.79	0.04	0.83	0.03	0.84	0.03	0.79	0.04	0.79	0.04	0.80	0.04
5000	0.08	0.79	0.03	0.86	0.03	0.88	0.02	0.79	0.03	0.79	0.04	0.80	0.03
	0.10	0.79	0.03	0.88	0.02	0.90	0.02	0.79	0.03	0.79	0.04	0.80	0.03
	0.12	0.79	0.03	0.90	0.02	0.92	0.01	0.79	0.03	0.79	0.04	0.80	0.03
	0.13	0.79	0.03	0.91	0.02	0.93	0.01	0.79	0.03	0.80	0.04	0.80	0.03
	0	0.80	0.03	0.80	0.03	0.80	0.03	0.80	0.03	0.80	0.03	0.80	0.03
	0.05	0.80	0.03	0.83	0.02	0.84	0.02	0.80	0.03	0.80	0.03	0.80	0.02
10000	0.08	0.79	0.02	0.86	0.02	0.88	0.02	0.80	0.02	0.80	0.03	0.80	0.02
	0.10	0.79	0.02	0.88	0.02	0.90	0.01	0.79	0.02	0.80	0.03	0.80	0.02
	0.12	0.79	0.02	0.90	0.01	0.92	0.01	0.79	0.02	0.80	0.03	0.80	0.02
	0.13	0.79	0.02	0.91	0.01	0.93	0.01	0.79	0.02	0.80	0.03	0.80	0.02

Table 3.5: Effect of a change in A on the coverage probability of B

T	ΔA	t-GAS		t-GARCH		GARCH		GAS-GARCH		EGARCH		Beta-t-EGARCH	
		90%	95%	90%	95%	90%	95%	90%	95%	90%	95%	90%	95%
	0	0.801	0.854	0.833	0.882	0.824	0.878	0.778	0.835	0.619	0.683	0.725	0.776
		(0.009)	(0.008)	(0.008)	(0.007)	(0.009)	(0.009)	(0.009)	(0.008)	(0.011)	(0.010)	(0.010)	(0.009)
	0.05	0.812	0.872	0.800	0.850	0.735	0.062	0.791	0.845	0.640	0.696	0.744	0.794
		(0.009)	(0.007)	(0.009)	(0.008)	(0.010)	(0.009)	(0.009)	(0.008)	(0.011)	(0.010)	(0.010)	(0.009)
	0.08	0.810	0.866	0.749	0.811	0.673	0.733	0.795	0.845	0.640	0.686	0.749	0.804
250		(0.009)	(0.008)	(0.010)	(0.009)	(0.010)	(0.010)	(0.009)	(0.008)	(0.011)	(0.010)	(0.010)	(0.009)
	0.10	0.807	0.864	0.705	0.769	0.606	0.659	0.799	0.850	0.654	0.706	0.746	0.809
		(0.009)	(0.008)	(0.010)	(0.009)	(0.011)	(0.011)	(0.009)	(0.008)	(0.011)	(0.010)	(0.010)	(0.009)
	0.12	0.817	0.870	0.640	0.708	0.516	0.579	0.800	0.856	0.647	0.706	0.763	0.820
		(0.009)	(0.008)	(0.011)	(0.010)	(0.011)	(0.011)	(0.009)	(0.008)	(0.011)	(0.010)	(0.010)	(0.009)
	0.13	0.799	0.863	0.614	0.684	0.473	0.531	0.801	0.856	0.644	0.705	0.771	0.827
		(0.009)	(0.008)	(0.011)	(0.010)	(0.011)	(0.011)	(0.009)	(0.008)	(0.011)	(0.010)	(0.009)	(0.008)
	0	0.799	0.855	0.827	0.874	0.838	0.878	0.751	0.820	0.690	0.759	0.774	0.823
		(0.009)	(0.008)	(0.008)	(0.007)	(0.008)	(0.007)	(0.010)	(0.009)	(0.010)	(0.010)	(0.009)	(0.009)
	0.05	0.812	0.871	0.780	0.831	0.731	0.782	0.775	0.841	0.722	0.778	0.794	0.846
		(0.009)	(0.008)	(0.009)	(0.008)	(0.010)	(0.009)	(0.009)	(0.008)	(0.010)	(0.009)	(0.009)	(0.008)
	0.08	0.813	0.879	0.681	0.741	0.623	0.686	0.786	0.854	0.723	0.784	0.811	0.865
500		(0.009)	(0.007)	(0.010)	(0.010)	(0.011)	(0.010)	(0.009)	(0.008)	(0.010)	(0.009)	(0.009)	(0.008)
	0.10	0.818	0.876	0.601	0.656	0.493	0.556	0.794	0.860	0.730	0.795	0.820	0.869
		(0.009)	(0.007)	(0.011)	(0.011)	(0.011)	(0.011)	(0.009)	(0.008)	(0.010)	(0.009)	(0.009)	(0.008)
	0.12	0.814	0.874	0.497	0.571	0.347	0.408	0.799	0.864	0.739	0.803	0.834	0.880
		(0.009)	(0.007)	(0.011)	(0.011)	(0.011)	(0.011)	(0.009)	(0.008)	(0.010)	(0.009)	(0.008)	(0.007)
	0.13	0.817	0.884	0.445	0.517	0.282	0.333	0.797	0.866	0.743	0.807	0.836	0.884
		(0.009)	(0.007)	(0.011)	(0.011)	(0.010)	(0.011)	(0.009)	(0.008)	(0.010)	(0.009)	(0.008)	(0.007)

Continued on next page

Table 3.5 – continued from previous page

T	ΔA	t-GAS		t-GARCH		GARCH		GAS-GARCH		EGARCH		Beta-t-EGARCH	
	0	0.823	0.876	0.841	0.892	0.867	0.909	0.823	0.878	0.773	0.832	0.832	0.884
		(0.009)	(0.007)	(0.008)	(0.007)	(0.008)	(0.006)	(0.009)	(0.007)	(0.009)	(0.008)	(0.008)	(0.007)
	0.05	0.844	0.902	0.778	0.826	0.746	0.806	0.844	0.893	0.790	0.851	0.848	0.897
		(0.008)	(0.007)	(0.009)	(0.008)	(0.010)	(0.009)	(0.008)	(0.007)	(0.009)	(0.008)	(0.008)	(0.007)
	0.08	0.842	0.900	0.620	0.691	0.514	0.590	0.847	0.900	0.804	0.857	0.845	0.899
1000		(0.008)	(0.007)	(0.011)	(0.010)	(0.011)	(0.011)	(0.008)	(0.007)	(0.009)	(0.008)	(0.008)	(0.007)
	0.10	0.846	0.905	0.475	0.560	0.311	0.385	0.846	0.903	0.805	0.868	0.849	0.907
		(0.008)	(0.007)	(0.011)	(0.011)	(0.011)	(0.011)	(0.008)	(0.007)	(0.009)	(0.008)	(0.008)	(0.007)
	0.12	0.840	0.901	0.332	0.402	0.147	0.191	0.848	0.903	0.809	0.872	0.851	0.907
		(0.008)	(0.007)	(0.011)	(0.011)	(0.008)	(0.009)	(0.008)	(0.007)	(0.009)	(0.007)	(0.008)	(0.007)
	0.13	0.840	0.905	0.267	0.322	0.093	0.121	0.846	0.908	0.814	0.877	0.847	0.911
		(0.008)	(0.007)	(0.010)	(0.010)	(0.006)	(0.007)	(0.008)	(0.006)	(0.009)	(0.007)	(0.008)	(0.006)
	0	0.865	0.910	0.862	0.916	0.889	0.931	0.874	0.920	0.850	0.914	0.871	0.923
		(0.008)	(0.006)	(0.008)	(0.006)	(0.007)	(0.006)	(0.007)	(0.006)	(0.008)	(0.006)	(0.008)	(0.006)
	0.05	0.872	0.919	0.702	0.774	0.674	0.752	0.875	0.926	0.859	0.916	0.871	0.925
		(0.007)	(0.006)	(0.010)	(0.009)	(0.010)	(0.010)	(0.007)	(0.006)	(0.008)	(0.006)	(0.007)	(0.006)
	0.08	0.860	0.920	0.411	0.495	0.280	0.369	0.874	0.927	0.859	0.919	0.876	0.924
2500		(0.008)	(0.006)	(0.011)	(0.011)	(0.010)	(0.011)	(0.007)	(0.006)	(0.008)	(0.006)	(0.007)	(0.006)
	0.10	0.868	0.922	0.206	0.277	0.091	0.124	0.877	0.931	0.857	0.921	0.875	0.928
		(0.008)	(0.006)	(0.009)	(0.010)	(0.006)	(0.007)	(0.008)	(0.006)	(0.008)	(0.006)	(0.007)	(0.006)
	0.12	0.862	0.922	0.085	0.118	0.013	0.023	0.876	0.924	0.857	0.925	0.872	0.925
		(0.008)	(0.006)	(0.006)	(0.007)	(0.002)	(0.003)	(0.007)	(0.006)	(0.008)	(0.006)	(0.007)	(0.006)
	0.13	0.862	0.919	0.047	0.068	0.005	0.006	0.872	0.925	0.860	0.923	0.869	0.925
		(0.008)	(0.006)	(0.005)	(0.006)	(0.002)	(0.001)	(0.007)	(0.006)	(0.008)	(0.006)	(0.008)	(0.006)

Continued on next page

Table 3.5 – continued from previous page

T	ΔA	t-GAS		t-GARCH		GARCH		GAS-GARCH		EGARCH		Beta-t-EGARCH	
	0	0.877	0.932	0.883	0.930	0.899	0.942	0.888	0.937	0.872	0.926	0.873	0.931
		(0.007)	(0.006)	(0.007)	(0.006)	(0.007)	(0.005)	(0.007)	(0.005)	(0.007)	(0.006)	(0.007)	(0.006)
	0.05	0.883	0.933	0.632	0.722	0.524	0.672	0.889	0.937	0.877	0.930	0.875	0.930
		(0.007)	(0.006)	(0.011)	(0.010)	(0.011)	(0.011)	(0.007)	(0.005)	(0.007)	(0.006)	(0.007)	(0.006)
	0.08	0.871	0.928	0.216	0.293	0.072	0.121	0.889	0.938	0.880	0.935	0.872	0.930
5000		(0.008)	(0.006)	(0.009)	(0.010)	(0.006)	(0.007)	(0.007)	(0.005)	(0.007)	(0.006)	(0.007)	(0.006)
	0.10	0.861	0.921	0.052	0.082	0.005	0.007	0.879	0.934	0.878	0.933	0.874	0.930
		(0.008)	(0.006)	(0.005)	(0.006)	(0.001)	(0.002)	(0.007)	(0.006)	(0.007)	(0.006)	(0.007)	(0.006)
	0.12	0.856	0.917	0.008	0.015	0.000	0.000	0.879	0.934	0.884	0.933	0.870	0.932
		(0.008)	(0.006)	(0.002)	(0.003)	(0.000)	(0.000)	(0.007)	(0.006)	(0.007)	(0.006)	(0.008)	(0.006)
	0.13	0.861	0.917	0.001	0.005	0.000	0.000	0.876	0.934	0.883	0.931	0.867	0.933
		(0.008)	(0.006)	(0.001)	(0.001)	(0.000)	(0.000)	(0.008)	(0.006)	(0.007)	(0.006)	(0.008)	(0.006)
	0	0.881	0.938	0.896	0.950	0.893	0.941	0.889	0.940	0.890	0.942	0.879	0.938
		(0.007)	(0.005)	(0.007)	(0.005)	(0.007)	(0.005)	(0.007)	(0.005)	(0.007)	(0.005)	(0.007)	(0.005)
	0.05	0.878	0.940	0.461	0.568	0.279	0.405	0.879	0.936	0.882	0.943	0.870	0.933
		(0.007)	(0.005)	(0.011)	(0.011)	(0.010)	(0.011)	(0.007)	(0.006)	(0.007)	(0.005)	(0.008)	(0.006)
	0.08	0.856	0.923	0.043	0.071	0.004	0.009	0.870	0.933	0.877	0.939	0.865	0.931
10000		(0.008)	(0.006)	(0.005)	(0.006)	(0.001)	(0.002)	(0.008)	(0.006)	(0.007)	(0.005)	(0.008)	(0.006)
	0.10	0.856	0.920	0.002	0.005	0.000	0.000	0.864	0.930	0.883	0.940	0.861	0.932
		(0.008)	(0.006)	(0.001)	(0.001)	(0.000)	(0.000)	(0.008)	(0.006)	(0.007)	(0.005)	(0.008)	(0.006)
	0.12	0.847	0.916	0.000	0.000	0.000	0.000	0.858	0.923	0.879	0.945	0.857	0.926
		(0.008)	(0.006)	(0.000)	(0.000)	(0.000)	(0.000)	(0.008)	(0.006)	(0.007)	(0.005)	(0.008)	(0.006)
	0.13	0.846	0.912	0.000	0.000	0.000	0.000	0.855	0.920	0.877	0.940	0.860	0.926
		(0.008)	(0.006)	(0.000)	(0.000)	(0.000)	(0.000)	(0.008)	(0.006)	(0.007)	(0.005)	(0.008)	(0.006)

Table 3.6: Effect of a change in η on $\hat{B}$

T	Size of break	t-GAS		t-GARCH		Beta-t-EGARCH	
		Mean est.	Std. Dev.	Mean est.	Std. Dev.	Mean est.	Std. Dev.
	0	0.69	0.24	0.68	0.27	0.64	0.37
	1	0.69	0.24	0.70	0.26	0.64	0.37
250	2	0.69	0.24	0.70	0.26	0.63	0.40
	3	0.70	0.24	0.70	0.26	0.63	0.40
	4	0.70	0.24	0.70	0.26	0.63	0.39
	5	0.69	0.24	0.69	0.27	0.63	0.40
	0	0.73	0.19	0.73	0.20	0.71	0.26
	1	0.74	0.18	0.73	0.20	0.72	0.24
500	2	0.74	0.19	0.73	0.21	0.71	0.27
	3	0.74	0.19	0.73	0.20	0.72	0.27
	4	0.73	0.19	0.73	0.21	0.72	0.26
	5	0.74	0.19	0.73	0.20	0.72	0.25
	0	0.77	0.13	0.77	0.13	0.76	0.14
	1	0.77	0.12	0.77	0.13	0.77	0.13
1000	2	0.77	0.12	0.77	0.13	0.77	0.14
	3	0.77	0.13	0.77	0.14	0.78	0.12
	4	0.77	0.13	0.76	0.15	0.77	0.13
	5	0.77	0.13	0.77	0.14	0.77	0.12
	0	0.79	0.07	0.79	0.07	0.79	0.06
	1	0.79	0.06	0.79	0.07	0.79	0.06
2500	2	0.79	0.06	0.79	0.07	0.79	0.06
	3	0.79	0.07	0.79	0.07	0.79	0.06
	4	0.79	0.07	0.79	0.07	0.79	0.06
	5	0.79	0.07	0.79	0.07	0.79	0.06
	0	0.79	0.04	0.79	0.04	0.79	0.04
	1	0.80	0.04	0.80	0.04	0.80	0.04
5000	2	0.80	0.04	0.79	0.04	0.80	0.04
	3	0.80	0.04	0.80	0.04	0.80	0.04
	4	0.80	0.04	0.80	0.04	0.80	0.04
	5	0.80	0.04	0.80	0.04	0.80	0.04
	0	0.80	0.03	0.80	0.03	0.80	0.03
	1	0.80	0.03	0.80	0.03	0.80	0.03
10000	2	0.80	0.03	0.80	0.03	0.80	0.03
	3	0.80	0.03	0.80	0.03	0.80	0.03
	4	0.80	0.03	0.80	0.03	0.80	0.03
	5	0.80	0.03	0.80	0.03	0.80	0.03

Table 3.7: Effect of a change in η on the on the coverage probability of B

T	Vol.	t-GAS		t-GARCH		Beta-t-EGARCH	
		90%	95%	90%	95%	90%	95%
	0	0.801	0.854	0.833	0.882	0.725	0.776
		(0.009)	(0.008)	(0.008)	(0.007)	(0.010)	(0.009)
	1	0.806	0.845	0.836	0.880	0.720	0.779
		(0.009)	(0.008)	(0.008)	(0.007)	(0.010)	(0.009)
	2	0.778	0.839	0.842	0.888	0.708	0.763
250		(0.009)	(0.008)	(0.008)	(0.007)	(0.010)	(0.010)
	3	0.778	0.831	0.845	0.881	0.702	0.759
		(0.009)	(0.008)	(0.008)	(0.007)	(0.010)	(0.010)
	4	0.781	0.837	0.838	0.881	0.723	0.773
		(0.009)	(0.008)	(0.008)	(0.007)	(0.010)	(0.009)
	5	0.783	0.839	0.840	0.881	0.721	0.784
		(0.009)	(0.008)	(0.008)	(0.007)	(0.010)	(0.009)
	0	0.799	0.855	0.827	0.874	0.774	0.823
		(0.009)	(0.008)	(0.008)	(0.007)	(0.009)	(0.009)
	1	0.783	0.843	0.813	0.876	0.780	0.837
		(0.009)	(0.008)	(0.009)	(0.007)	(0.009)	(0.008)
	2	0.776	0.833	0.815	0.868	0.774	0.835
500		(0.009)	(0.008)	(0.009)	(0.008)	(0.009)	(0.008)
	3	0.776	0.834	0.812	0.860	0.777	0.832
		(0.009)	(0.008)	(0.009)	(0.008)	(0.009)	(0.008)
	4	0.790	0.841	0.820	0.861	0.787	0.837
		(0.009)	(0.008)	(0.009)	(0.008)	(0.009)	(0.008)
	5	0.778	0.837	0.824	0.869	0.796	0.844
		(0.009)	(0.008)	(0.009)	(0.008)	(0.009)	(0.008)
	0	0.823	0.876	0.841	0.892	0.832	0.884
		(0.009)	(0.007)	(0.008)	(0.007)	(0.008)	(0.007)
	1	0.803	0.862	0.853	0.897	0.826	0.878
		(0.009)	(0.008)	(0.008)	(0.007)	(0.008)	(0.007)
	2	0.818	0.865	0.835	0.882	0.839	0.885
1000		(0.009)	(0.008)	(0.008)	(0.007)	(0.008)	(0.007)
	3	0.814	0.865	0.838	0.891	0.838	0.893
		(0.009)	(0.008)	(0.008)	(0.007)	(0.008)	(0.007)
	4	0.818	0.871	0.835	0.893	0.852	0.900
		(0.009)	(0.008)	(0.008)	(0.007)	(0.008)	(0.007)
	5	0.805	0.860	0.827	0.881	0.845	0.895
		(0.009)	(0.008)	(0.008)	(0.007)	(0.008)	(0.007)

Continued on next page

Table 3.7 – continued from previous page

T	Δη	t-GAS		t-GARCH		Beta-t-EGARCH	
2500	0	0.865	0.910	0.862	0.916	0.871	0.923
		(0.008)	(0.006)	(0.008)	(0.006)	(0.008)	(0.006)
	1	0.850	0.910	0.870	0.917	0.866	0.921
		(0.008)	(0.006)	(0.008)	(0.006)	(0.008)	(0.006)
	2	0.860	0.907	0.873	0.916	0.869	0.915
		(0.008)	(0.006)	(0.007)	(0.006)	(0.008)	(0.006)
	3	0.850	0.911	0.869	0.914	0.869	0.914
		(0.008)	(0.006)	(0.008)	(0.006)	(0.008)	(0.006)
	4	0.842	0.900	0.866	0.912	0.872	0.924
		(0.008)	(0.007)	(0.008)	(0.006)	(0.007)	(0.006)
	5	0.848	0.899	0.869	0.915	0.870	0.924
		(0.008)	(0.007)	(0.008)	(0.006)	(0.008)	(0.006)
5000	0	0.877	0.932	0.883	0.930	0.873	0.931
		(0.007)	(0.006)	(0.007)	(0.006)	(0.007)	(0.006)
	1	0.876	0.925	0.886	0.941	0.875	0.930
		(0.007)	(0.006)	(0.007)	(0.005)	(0.007)	(0.006)
	2	0.875	0.935	0.895	0.943	0.893	0.944
		(0.007)	(0.006)	(0.007)	(0.005)	(0.007)	(0.005)
	3	0.868	0.917	0.883	0.928	0.878	0.927
		(0.008)	(0.006)	(0.007)	(0.006)	(0.007)	(0.006)
	4	0.860	0.918	0.881	0.929	0.879	0.928
		(0.008)	(0.006)	(0.007)	(0.006)	(0.007)	(0.006)
	5	0.848	0.901	0.883	0.929	0.864	0.925
		(0.008)	(0.007)	(0.007)	(0.006)	(0.008)	(0.006)
10000	0	0.881	0.938	0.896	0.950	0.879	0.938
		(0.007)	(0.005)	(0.007)	(0.005)	(0.007)	(0.005)
	1	0.884	0.943	0.908	0.949	0.901	0.944
		(0.007)	(0.005)	(0.006)	(0.005)	(0.007)	(0.005)
	2	0.873	0.929	0.884	0.939	0.878	0.931
		(0.007)	(0.006)	(0.007)	(0.005)	(0.007)	(0.006)
	3	0.861	0.917	0.885	0.934	0.873	0.936
		(0.008)	(0.006)	(0.007)	(0.006)	(0.007)	(0.005)
	4	0.858	0.918	0.892	0.930	0.865	0.925
		(0.008)	(0.006)	(0.007)	(0.006)	(0.008)	(0.006)
	5	0.845	0.910	0.874	0.926	0.864	0.923
		(0.008)	(0.006)	(0.007)	(0.006)	(0.008)	(0.006)

Table 3.8: Effect of a change in B on $\hat{B}$

T	Size of break	t-GAS		EGARCH		Beta-t-EGARCH ($\omega = 0$)		Beta-t-EGARCH ($\omega = 0.2$)	
		Mean est.	Std. Dev.	Mean est.	Std. Dev	Mean est.	Std. Dev	Mean est.	Std. Dev.
250	0	0.69	0.24	0.66	0.36	0.63	0.40	0.63	0.40
	0.05	0.74	0.23	0.71	0.34	0.70	0.36	0.71	0.36
	0.08	0.79	0.21	0.81	0.30	0.81	0.27	0.81	0.27
	0.1	0.83	0.19	0.90	0.21	0.90	0.17	0.90	0.17
	0.12	0.88	0.16	0.96	0.13	0.96	0.07	0.96	0.07
	0.13	0.90	0.15	0.98	0.11	0.98	0.03	0.98	0.03
500	0	0.73	0.19	0.71	0.29	0.69	0.30	0.69	0.30
	0.05	0.79	0.17	0.79	0.23	0.78	0.24	0.78	0.24
	0.08	0.84	0.14	0.91	0.15	0.89	0.14	0.89	0.14
	0.1	0.88	0.12	0.97	0.06	0.95	0.08	0.95	0.08
	0.12	0.93	0.09	0.99	0.02	0.98	0.02	0.98	0.02
	0.13	0.94	0.07	0.99	0.01	0.99	0.01	0.99	0.01
1000	0	0.77	0.13	0.75	0.18	0.75	0.20	0.75	0.20
	0.05	0.82	0.11	0.84	0.13	0.83	0.14	0.83	0.14
	0.08	0.87	0.08	0.95	0.06	0.93	0.07	0.93	0.07
	0.1	0.91	0.06	0.99	0.01	0.97	0.03	0.97	0.03
	0.12	0.95	0.04	0.99	0.00	0.99	0.01	0.99	0.01
	0.13	0.96	0.03	0.99	0.00	0.99	0.01	0.99	0.01
2500	0	0.79	0.07	0.78	0.09	0.79	0.08	0.79	0.08
	0.05	0.84	0.06	0.87	0.06	0.86	0.06	0.86	0.06
	0.08	0.89	0.04	0.97	0.03	0.94	0.03	0.94	0.03
	0.1	0.93	0.03	0.99	0.01	0.98	0.01	0.98	0.01
	0.12	0.95	0.02	1.00	0.00	0.99	0.00	0.99	0.00
	0.13	0.97	0.02	1.00	0.00	0.99	0.00	0.99	0.00
5000	0	0.79	0.04	0.79	0.05	0.79	0.05	0.79	0.05
	0.05	0.84	0.03	0.88	0.04	0.86	0.04	0.86	0.04
	0.08	0.89	0.03	0.98	0.02	0.95	0.02	0.95	0.02
	0.1	0.93	0.02	0.99	0.00	0.98	0.01	0.98	0.01
	0.12	0.96	0.01	1.00	0.00	0.99	0.00	0.99	0.00
	0.13	0.97	0.01	1.00	0.00	0.99	0.00	0.99	0.00
10000	0	0.80	0.03	0.80	0.03	0.80	0.03	0.80	0.03
	0.05	0.84	0.02	0.88	0.03	0.87	0.03	0.87	0.03
	0.08	0.89	0.02	0.98	0.01	0.95	0.02	0.95	0.02
	0.1	0.93	0.01	1.00	0.00	0.98	0.01	0.98	0.01
	0.12	0.96	0.01	1.00	0.00	0.99	0.00	0.99	0.00
	0.13	0.97	0.01	1.00	0.00	1.00	0.00	1.00	0.00

Chapter 4

Structural breaks and GAS models of oil and exchange rate volatilities

Abstract

Empirical evidence suggests that structural breaks are present in data on macro-financial variables such as oil prices and exchange rates. The potentially serious consequences of ignoring a break in GARCH parameters motivated Rapach and Strauss (2008) and Babikir et al. (2012) to study the empirical relevance of structural breaks in the context of GARCH models. However, the literature does not address the empirical relevance of structural breaks in the context of GAS models. This paper contributes to this literature by extending Rapach and Strauss (2008) to include the t-GAS model and by comparing its performance to that of the GARCH model and two non-GAS models, the t-GARCH and SV models. The empirical relevance of structural breaks in the models of volatility are assessed using the formal test of Dufour and Torres (1998) to determine whether the estimated parameters change across subperiods. The performance of all the models is analyzed using both the weekly USD trade-weighted index between January 1973 and October 2016 and spot oil prices based on West Texas Intermediate between January 1986 and October 2016. Through this analysis, this paper also addresses whether accommodating structural breaks in the unconditional variance of both GAS and non-GAS models will improve forecasts. The results show that structural breaks are empirically relevant in GAS models of USD volatility but not for oil volatility in terms of modeling. They also indicate that using models that accommodate breaks can improve forecasts only in the short run.

4.1 Introduction

Time-varying volatility models used in forecasting the variance include Bollerslev's (1986) generalized autoregressive conditional heteroskedasticity (GARCH) model, Taylor's (1986) stochastic volatility (SV) model, and, more recently, Creal et al.'s (2013) generalized autoregressive score (GAS) volatility model in which volatility is updated by the score of the log-density.[1] Studies show that these models have empirical success in forecasting the variance. For example, Hansen and Lunde (2005) find after estimating 330 ARCH-type models that it is difficult for any other model to outperform a simple GARCH(1,1) model for exchange rate data. Clark and Ravazzolo (2015) use Bayesian estimation to find that autoregressive (AR) and vector autoregressive (VAR) models with SV improve the point and density forecast accuracies of real GDP growth, inflation, unemployment rate, and the 3-month Treasury bill rate. Koopman et al. (2016) use simulated data to show that score-driven GAS(1,1) models outperform various forms of the GARCH model in both in-sample and out-of-sample performance.

Empirical evidence suggests that structural breaks are also present in data on macro-financial variables such as oil prices and exchange rates. For example, Rapach and Strauss (2008) identify several breaks in the unconditional variance of seven US dollar exchange rate return series between 1980 and 2005. Arouri et al. (2012) identify structural breaks in the volatility of daily spot and future prices of gasoline and heating oil, but not in West Texas Intermediate crude oil prices. Salisu and Fasanya (2013) identify two breaks in West Texas Intermediate crude oil prices – in 1990 and 2008 – using Narayan and Popp's (2010) unit root test that allows for two structural breaks.

As for GAS models, the cited examples of their empirical success assume that the GAS process is stable and thus the unconditional volatility is constant. However, as shown in Chapter 3, if the assumption of a stable GAS process is violated due to ignored parameter non-constancy, then a break in a parameter that shifts the unconditional variance causes the estimated persistence parameter of the GAS model to be severely biased. Lamoureux and Lastrapes (1990) and Hillebrand (2005) produce similar results for the case of a break in any parameter of a GARCH model. Another consequence is that ignoring breaks in the variance can lead to biased forecasts of the variance, as the forecasts may converge to the wrong unconditional variance.

The potentially serious consequences of ignoring a break in GARCH parameters motivated Rapach and Strauss (2008) and Arouri et al. (2012) to study the empirical relevance

[1]The scaling matrix of these GAS models was the square root of the inverse of Fisher's identity matrix which makes the models different from the classical GARCH model.

of structural breaks in the context of GARCH models for exchange rates and oil respectively, and whether accommodating breaks can improve forecasts. Rapach and Strauss (2008) find that structural breaks are empirically relevant for GARCH models and accommodating breaks can improve volatility forecasts. Arouri et al. (2012) find that using rolling windows can improve forecasts for crude oil volatility. However, the issue of the relevance of structural breaks and how to best accommodate breaks in volatility when forecasting is not addressed in the GAS literature. This paper contributes to this literature by extending Rapach and Strauss (2008) to include the t-GAS model and comparing its performance to that of the GARCH model and two non-GAS models, the t-GARCH and SV models. The variables used are the weekly trade-weighted US dollar index and West Texas Intermediate crude oil prices are used for this analysis. The USD trade-weighted index was chosen simply for the fact that it is an aggregate measure of the US dollar against the currency of its major trading partners including Canada, the Euro Area, and Mexico.[2] West Texas Intermediate crude oil was chosen because it is produced in the US and considered to be a benchmark for crude oil prices in North and South America (US Department of Energy, 2014).

Although the focus of much of the paper is on forecasting the variance, the ability of the models to forecast the conditional mean of oil returns and exchange rate returns is also examined. This is because correctly modeling and forecasting the prices/returns and volatilities of oil and exchange rates are important for the following reasons. First, it can help policymakers and firms adopt a correct risk management approach. Second, firms and consumers depend on oil forecasts to make decisions regarding investments in reserve exploration, development and production (Pindyck, 1999), and on exchange rate forecasts for decisions regarding international trade, domestic wages, and employment.

In the case of the prices and returns of commodities such as oil, incorporating features such as mean-reversion and time-varying trends (Pindyck, 1999) and jumps (see Postali and Picchetti, 2006; Wilmot and Mason, 2013) have been found to improve model fit and forecasts. Beck (2001) finds that volatility directly affects the prices of storable commodities, and oil is considered to be a storable commodity. In determining whether adding jumps, mean-reversion, and/or adding volatility improves the forecasts of commodity prices, Bernard et al. (2008) find that Schwartz and Smith's (2000) mean-reversion model with a stochastic convenience yield produces the best forecasts. According to Alquist et al. (2013), AR and VAR models of the global oil market outperform a random walk forecast of the real price of oil only at short horizons, and no-change forecasts are difficult to beat.

[2]The definition of the USD trade-weighted index comes from Loretin (2005) and Board of Governors of the Federal Reserve System (US) (2014).

The papers cited above mainly focus on the mean forecasts of oil returns. Mohammadi and Su (2010) assess the modeling performance of both the mean and volatility of oil prices using various ARIMA-GARCH models. Since this paper assesses the forecasts of both the mean and the variance, it follows Mohammadi and Su's (2010) approach by specifying the conditional mean of oil returns as an ARIMA model. In their case, they specify the conditional mean as an MA(1) model, while this paper specifies the conditional mean as an AR(1) model. Also, the main focus of their paper is on conditional volatility models that incorporate asymmetry. Asymmetric models are not considered in this paper.

In the case of exchange rates, Meese and Rogoff (1983a,b) find that it is difficult to outperform a random walk model in forecasting exchange rates. This conclusion is known as the *Meese-Rogoff puzzle*. Meese and Rogoff (1983a) find that the inability of macroeconomic models for exchange rates to forecast better than a random walk model is not attributable to inconsistent or inefficient parameter estimation. In light of the Meese-Rogoff puzzle, the conditional mean of exchange rate return is assumed to be a constant (i.e., the exchange rate is a random-walk).

The in-sample performance of the models is assessed using the AIC and BIC. Also, in assessing the empirical relevance of structural breaks, structural breaks are based on historical events rather than hypothesis tests, because commonly-used structural break tests such as the CUSUM test suffer from low power (Smith, 2008; Xu, 2013).[3] The empirical relevance of structural breaks in the models of volatility is assessed using a formal test by Dufour and Torres (1998) to determine whether the estimated parameters change over sub-periods, in contrast to Rapach and Strauss (2008) and Arouri et al. (2012) who did not conduct any formal tests of the significance of changes in parameters. The in-sample performance of all the models is analyzed using the weekly data for both variables.

The out-of-sample performance of the models is assessed using one-step-ahead forecasts. Two forecasting strategies are applied – recursive and rolling window. The first strategy, the recursive method, is used frequently with models generated by a stable process. The second strategy, the rolling window method, is believed to better accommodate structural breaks. Two sizes of rolling windows are employed. The first is half the length of the initial estimation window of the recursive method. The second is one quarter the length of the initial estimation window. The out-of-sample forecasts begin in November 2010 and end in October 2016. This choice of period restricts the focus solely to the post-2008 Financial Crisis era. Two-year, four-year, and six-year horizons are analyzed in this paper. The

[3] Mavroeidis (2010) used a similar approach in his analysis of inflation by splitting the data between the pre- and post-Volcker years (1979). His focus is on the indeterminacy of US monetary policy using limited-identification robust methods.

out-of-sample performance is assessed for the mean, interval, and variance forecasts for both oil and USD returns.

This paper is organized as follows. Section 2 briefly discusses the competing volatility models considered in the paper. Section 3 describes the data used and the choice of structural breaks. Section 4 discusses the in-sample performance of all the models for oil and USD returns when structural breaks are ignored, and the empirical relevance of breaks in the two series. Section 5 assesses the out-of-sample performance of all the models when recursive or rolling window methods are employed. Finally, section 6 concludes and provides suggestions for future research.

4.2 Volatility Models

Four univariate models of the variance of a single random variable y_t are examined in this paper. They are Bollerslev's (1986) GARCH(1,1) model, Bollerslev's (1987) t-GARCH(1,1) model, Creal et al.'s (2013) t-GAS(1,1) model, and Taylor's (1986) ARSV(1) model. All these variance models are conditionally heteroskedastic. For each model, the conditional mean, $E_{t-1}[y_t]$, is assumed to contain no exogenous terms. $E_{t-1}[y_t]$ is specified either as a constant or as an AR(1) process, depending on the variable analyzed.[4]

The GARCH(1,1) model is the most well-known of the models used in this analysis. The conditional variance in this model is updated by the squared error term and its own lag:

$$\begin{aligned} y_t &= E_{t-1}[y_t] + u_t \\ u_t &= \sigma_t v_t \qquad v_t \sim \mathcal{N}(0,1) \\ \sigma_t^2 &= \omega + \alpha u_{t-1}^2 + \beta \sigma_{t-1}^2 \end{aligned} \tag{4.1}$$

where u_t is an error term with mean 0 and variance σ_t^2, and v_t is a sequence of independent and identically distributed (iid) standard normal random variables. ω is the constant of the conditional volatility model, α is the ARCH parameter, and β is the GARCH persistence parameter. The unconditional variance of the GARCH(1,1) model is

$$\sigma^2 = \frac{\omega}{1-\alpha-\beta}. \tag{4.2}$$

[4] The most common specification for modeling the conditional mean of a variable with a time-varying conditional variance is either with a constant or an AR(1) model. It is important to properly specify the mean because misspecification of the conditional mean can induce serially correlated residuals, which can in turn cause the squared residuals to be serially correlated, thereby leading the Engle (1982) ARCH-LM test to falsely reject the null hypothesis that the disturbances are not conditionally heteroskedastic (Lumsdaine and Ng, 1999).

The second model used in this analysis is the t-GARCH(1,1) model. The t-GARCH(1,1) model is a GARCH(1,1) model in which the error term u_t is assumed to follow a Student's t distribution instead of a Gaussian distribution; in other words, v_t is a sequence of iid standard Student's t random variables. The t-GARCH model should be considered as an alternative to the GARCH model because weekly data tend to be non-normally distributed. This model and its unconditional variance are described by the following equations:

$$\begin{aligned} y_t &= E_{t-1}[y_t] + u_t \\ u_t &= \sigma_t v_t \qquad v_t \sim St(0,1,\nu) \\ \sigma_t^2 &= \omega + \alpha u_{t-1}^2 + \beta \sigma_{t-1}^2 \end{aligned} \tag{4.3}$$

and

$$\sigma^2 = \tfrac{\omega}{1-\alpha-\beta}, \tag{4.4}$$

where ν is the degrees of freedom parameter.

The third model of volatility considered in this paper is the t-GAS(1,1) model:

$$\begin{aligned} y_t &= E_{t-1}[y_t] + u_t \\ u_t &= \sigma_t v_t \qquad v_t \sim St(0,1,\nu) \\ \sigma_t^2 &= \omega + A s_{t-1} + B \sigma_{t-1}^2, \end{aligned} \tag{4.5}$$

where u_t is the error term with mean 0 and variance σ_t^2, ν is the degrees of freedom parameter, A represents the scaled score parameter and B denotes the persistence parameter of σ^2 for lag 1. ω is the constant parameter in the conditional variance equation. The scaled score is defined as $s_t = S_t \nabla_t$, where

$$\nabla_t = \frac{\partial \ln p(u_t|u_1,u_2,...,u_{t-1},\sigma_1^2,\sigma_2^2,...,\sigma_{t-1}^2;\phi)}{\partial \sigma_t^2},$$

$\phi = \{\phi_1, \phi_2, ..., \phi_p\}$ represent the set of static parameters of the log density function, and $S_t = \mathcal{I}(\phi)_{t|t-1}^{-1} = (E_{t-1}[\nabla_t \nabla_t'])^{-1}$ is the scaling matrix function.[5] Following Creal et al. (2013), the scaled score of the density in this case is

$$\begin{aligned} s_t &= \tfrac{2\sigma_t^2(\nu+3)}{\nu} \left(-\tfrac{0.5}{\sigma_t^2} + \tfrac{\nu+1}{2} \frac{\frac{u_t^2}{(\nu-2)\sigma_t^4}}{1+\frac{u_t^2}{(\nu-2)\sigma_t^2}} \right) \\ &= (1+3\nu^{-1}) \frac{(1+\nu^{-1})u_t^2}{(1-2\nu^{-1})\left[1+\frac{\nu^{-1}u_t^2}{(1-2\nu^{-1})\sigma_t^2}\right]}. \end{aligned} \tag{4.6}$$

[5]Note that, as mentioned by Creal et al. (2013), the scaling matrix S_t can also take the form of an identity matrix I or the square root of $\mathcal{I}(\theta)_{t|t-1}^{-1}$, leading to different types of GAS models. For simplicity, this paper looks at only the Student's t and Gaussian volatility cases.

The t-GAS(1,1) unconditional variance is

$$\sigma^2 = \tfrac{\omega}{1-B}. \tag{4.7}$$

The t-GAS(1,1) model described in equation (4.5) is not equivalent to a t-GARCH(1,1) model (Creal et al., 2013). Both the t-GAS(1,1) and t-GARCH(1,1) models are equivalent if the degrees of freedom are infinite.[6]

The final model of volatility included in this analysis is the autoregressive stochastic volatility (ARSV) model. The ARSV model is popular in both macroeconomic and financial applications. Unlike the t-GAS, GARCH, and t-GARCH models, the ARSV model is not observation-driven, but parameter-driven. In this model, the conditional variance is dependent only on past conditional variances. A standard ARSV model with 1 lag, or alternatively the ARSV(1) model, is described by the following equations:

$$\begin{aligned} y_t \quad &= E_{t-1}[y_t] + u_t \\ u_t \quad &= \sigma_t v_t \qquad v_t \sim \mathcal{N}(0,1) \\ \log \sigma_t^2 &= \omega + \gamma \log \sigma_{t-1}^2 + \eta_t, \end{aligned} \tag{4.8}$$

where ω is a constant, γ is the coefficient of the log of the conditional variance in period t-1, and η_t is normally distributed with a mean of 0 and variance σ_η^2. η_t is also assumed to be uncorrelated with u_t. The unconditional variance of the ARSV(1) model is

$$\sigma^2 = \exp\left(\tfrac{\omega}{1-\gamma} + \tfrac{\sigma_\eta^2}{2(1-\gamma^2)}\right). \tag{4.9}$$

Three out of the four chosen models – the t-GAS(1,1), GARCH(1,1), and t-GARCH(1,1) models – are estimated using one-step quasi-maximum likelihood estimation.[7] In all three models, the conditional mean and the conditional variance are jointly estimated. Restrictions are imposed to ensure that the conditional and unconditional variances are positive and that stationarity holds. For the t-GAS model, the restrictions are that $\omega > 0$, $0 < (1 + \frac{3}{\nu})A < B$ and $B < 1$. For the GARCH and t-GARCH models, the restrictions are $\omega > 0$, $\alpha > 0$, $\beta > 0$ and $\alpha + \beta < 1$.

In the ARSV(1) model, all the parameters of the conditional variances are unobservable in all periods and the likelihood is not in closed form, so maximum likelihood estimation is

[6] See Creal et al. (2013); Harvey (2013) and Chapter 3 of this dissertation for further discussion.

[7] All estimates and results were generated using The Mathworks Inc. (2014) MATLAB programs. The t-GAS model codes are from the GAS models website, http://www.gasmodel.com/code.htm, and Martin et al. (2013).

not feasible. Hence, the ARSV(1) model is estimated using a two-step method which was proposed by Nelson (1998) and Harvey and Shephard (1996).[8] The first step is to estimate the conditional mean using ordinary least squares estimation and save the residuals. The second step uses the residuals to estimate the ARSV(1) model using quasi-maximum likelihood estimation based on the Kalman filter. According to Ruiz (1994), this quasi-maximum likelihood estimator is more efficient than a Generalized Method of Moments estimator. Restrictions have also been imposed on the parameters of this model: $\omega > 0$ and $0 < \gamma < 1$.[9]

4.3 Data

Two important indicators of the economic and financial health of an economy are oil prices and exchange rates. Alquist et al. (2013) write that if there are large unanticipated and persistent changes in oil prices, then the welfare of both oil-exporting and oil-importing countries are negatively impacted. Changes in the cost of crude oil can affect the decisions of households and firms when purchasing oil for heating, transportation, and manufacturing. In the case of exchange rates, their fluctuations can affect global trade. For example, an appreciation of the US dollar can make US goods relatively more expensive for other countries to purchase.

The volatilities of oil prices and exchange rates also matter. Oil price volatility can affect the price, production, and inventories of oil. An increase in oil volatility can increase the value of operating options held by firms and opportunity costs of holding these operation options, which in turn can cause oil production to decrease. Higher volatility of oil prices can also lead to greater demand for oil inventories which are necessary for smoothening production and deliveries and lower marketing costs (Pindyck, 2004a,b). Volatility of exchange rates is important because it reflects uncertainty about the prices of imports and exports, the value of international reserves, open positions in foreign currency, the domestic currency value of debt payments, and workers' remittance which in turn affects domestic wages, prices, output, and employment (Diebold and Nerlove, 1989).

Weekly data for the dollars per barrel price of West Texas Intermediate crude oil are

[8] Stochastic volatility models are frequently estimated using Bayesian methods in two steps. This type of estimation tends to be complicated. Another way of estimating a stochastic volatility model is using a closed-form method of moments estimator. See Dufour and Valery (2006).

[9] The stochastic volatility estimates and results were generated using the MATLAB code from the Pellegrini and Rodriguez (2007) course webpage (http://halweb.uc3m.es/esp/Personal/personas/spellegr/esp/Curso_Cordoba/index.html).

obtained from the Federal Reserve Economic Database (FRED) maintained by the Federal Reserve Bank of St. Louis. In this paper, the sample period for oil prices is January 10, 1986 to October 28, 2016 (1609 observations). Weekly data for the US dollar trade weighted index (USDTWI) from January 3, 1973 to October 26, 2016 (2287 observations) are also obtained from the FRED database.

Both series are then converted into returns by taking the first difference of their natural logs.[10] The resulting variables are oil returns and USD returns. Both are multiplied by 100 before carrying out the empirical analysis.

Figures 4.1 and 4.2 display the levels and the returns (or first differences) of both series. In figure 4.1, it can be seen that oil prices were relatively stable until the early 2000s. Then, oil prices steadily rose from \$20 to almost \$150 in 2008. Oil prices dropped by about \$100 in 2008, partially recovered in 2010, and then dropped once again in 2014. At the same time, oil returns show spikes in volatility in the early 1990s, late 1990s, 2008, and 2014. These time periods coincide with the First Iraq War, the Asian Financial Crisis of 1997, the 2008 Global Financial Crisis, and the decline in global demand for oil respectively. However, it is unclear from looking at figure 4.1 alone whether there are structural breaks in volatility.

Figure 4.1: Oil price and returns, 1986 to 2016

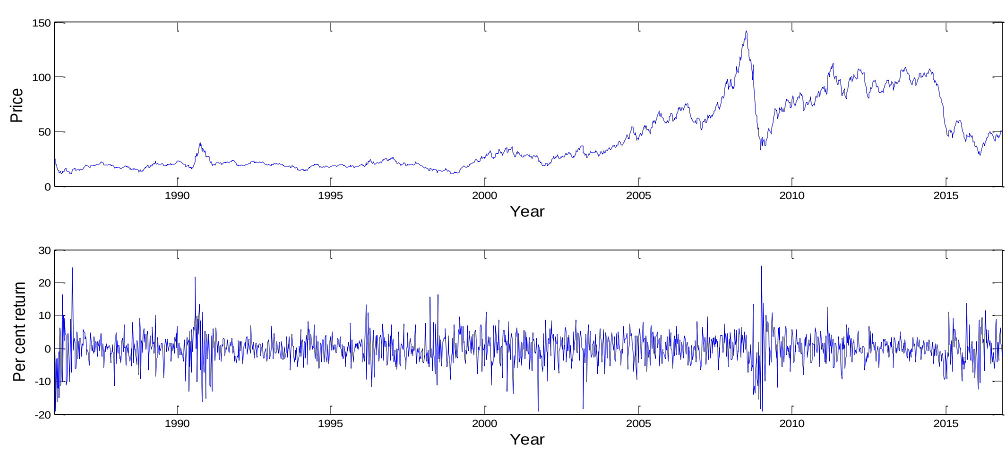
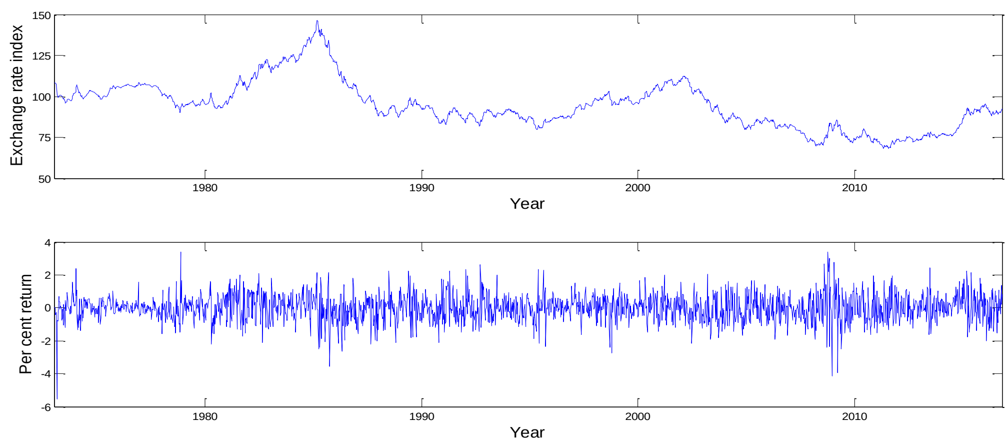


Figure 4.2 shows that the USDTWI reached a peak of 150 in the mid-1990s. Since

[10]The autocorrelations, partial autocorrelations, and unit root tests for the logs of oil prices and USDTWI are provided in Appendix A1. They show that the logs of both data series are not stationary in levels, but are stationary in first differences. Thus, the levels are transformed into first differences.

Figure 4.2: US Dollar trade-weighted exchange rate index and returns, 1973 to 2016



then, the index has fluctuated between 70 and 125. However, the second panel of figure 4.2 shows that there was a spike in volatility in 2008, indicating a possible break. This time period coincides with the 2008 Global Financial Crisis.

Once again, it is not easy to determine the properties of the data simply by looking at the figures. Hence, summary statistics are calculated as a first step towards understanding the properties of the data. These summary statistics are found in Table 4.1. The results indicate that the standard deviations of oil and USD returns are large relative to the mean. Furthermore, the data exhibit excess kurtosis. This means that models incorporating fat tails will likely fit the data better. Given that the frequency of the data is weekly, it is not surprising that the data exhibit non-normal behaviour. In addition, oil and USD returns are weakly and negatively skewed, suggesting that neither allowing for leverage effects nor assuming that the errors are from an asymmetric distribution may be necessary for this analysis.

The second step in the examination of the data's properties is to test for conditional heteroskedasticity. Table 4.1 also presents Ljung-Box Q statistics and Engle's (1982) ARCH-LM tests for the two variables. The Ljung-Box Q test shows that the null hypothesis of no serial correlation in the squared residuals is rejected at the 5% level in favour of the alternative hypothesis that the squared residuals exhibit autocorrelation. The ARCH-LM test statistics show that at both 5 and 10 lags, the null hypothesis that the error term is homoskedastic can be rejected in favour of the alternative hypothesis that the error term is conditionally heteroskedastic, at the 5% level of significance.

Table 4.1: Summary Statistics of the Series

Summary Statistics	Oil Returns	USD Returns
Mean	0.04	-0.01
Std. dev.	4.39	0.80
Skewness	-0.14	-0.20
Kurtosis	6.25	5.23
Minimum	-19.23	-5.52
Maximum	25.12	3.42
Ljung-Box Q(20)**	757.51 (0.000)	249.67 (0.000)
ARCH Lagrange Multiplier (5)	188.01 (0.000)	67.40 (0.000)
ARCH Lagrange Multiplier (10)	205.57 (0.000)	142.61 (0.000)
Number of observations	1608	2286

The number of lags used in the test is in brackets next to the names of the test. The p-values are contained in the brackets next to the test statistics. **Ljung-Box Q test is for the squared residuals based on AR(1) model. Time period for oil returns is January 10, 1986 to October 28, 2016. The period for USD returns is from January 3, 1976 to October 26, 2016.

Table 4.2: Summary statistics by sub-period

Variable	Subperiod	Mean	Std. Dev.	Skewness	Kurtosis
Oil Returns	January 1986 - July 1990	-0.11	4.92	-0.14	6.89
	August 1990 - June 1997	0.07	4.09	0.18	7.04
	July 1997 - May 2008	0.28	4.15	-0.38	4.64
	June 2008 - October 2016	-0.23	4.61	0.00	6.62
USD Returns	January 1973 - January 1994	-0.02	0.79	-0.32	6.09
	February 1994 - June 2008	-0.04	0.72	-0.12	3.47
	July 2008 - October 2016	0.06	0.92	-0.14	4.68

To see if there are differences in the sample moments due to structural breaks, table 4.2 provides summary statistics for sub-periods determined by historical events and policy changes. The structural breaks are defined by historical events rather than statistical testing for several reasons. Firstly, as pointed out by Hyndman (2014), structural breaks arise from historical events such as wars and major policy changes. Hence, there is no need to test for a break date such as August 1990, since it is known that the First Gulf

War began in that month. Secondly, the CUSUM- or LM-based tests commonly used for detecting structural breaks in the variance (e.g., Inclan and Tiao, 1994; Sanso et al., 2004) are problematic. Xu (2013) uses Monte-Carlo simulations to demonstrate that when the data have heavy tails, the CUSUM and LM tests have downward size distortion and suffer from a loss in power. He also finds that the power of these tests is sensitive to the break location.

For oil returns, the choice of break dates is based on the discussion in Baumeister and Kilian (2016).[11] The first break is assumed to be in August 1990, when the First Gulf War began. Another likely break date is July 1997, coinciding with the Asian Financial Crisis. June 2008, which marks the start of the global financial crisis that occurred during the summer of 2008, is the last break date selected for oil returns.[12] The summary statistics show that the returns do indeed exhibit changes in the mean, standard deviation, skewness, and kurtosis from one period to the next.

Thus, the results for oil returns suggest that one should allow for parameter changes in the conditional mean and variance. Additionally, the summary statistics show that it may not be necessary to allow the conditional distribution of oil returns to be skewed, since skewness is small across all sub-periods.[13] The kurtosis of oil returns for all three sub-periods exceeds 3, the level of kurtosis for a Gaussian distribution, by at least 1.64. This suggests that a t-distribution model would fit the oil returns data in all sub-periods better than a normally distributed model.

In the case of USD returns, two break dates are selected. The first break date is January 1994, when the Federal Reserve introduced a major policy change in the form of an increase in the Federal Funds Rate. Around that same date, Mexico experienced a currency crisis, Canada was experiencing fiscal deficit problems, and Europe experienced a collapse of the European Exchange Rate Commission, making the USD more attractive to investors. The next break date selected coincides with the global financial crisis in the summer of 2008.

Table 4.2 shows that both the mean and variance of USD returns experienced a change after June 2008, implying that it may be inappropriate to assume that the parameters of time-varying volatility models are constant. Although the skewness of USD returns changes

[11] Baumeister and Kilian (2016) discuss in detail trends in oil prices during the last forty years. Their article includes discussions of the impacts of events such as the First Gulf War and the Asian Financial Crisis on oil prices.

[12] Another potential break date is June 2014, but splitting the sample at that date would result in imprecise estimates of the time-varying volatility parameters because the sample size would be small, as shown in Lumsdaine (1995).

[13] Bulmer (1979) suggests that if skewness is between -0.5 and 0.5, then the sample is approximately symmetric.

after February 1994, it is still close to zero, implying that estimating a symmetrically distributed model (i.e., Gaussian, Student's t) for USD returns may be appropriate across all sub-periods. Also, the first and third sub-periods show signs of excess kurtosis, because the level of kurtosis exceeds 3. This implies that a t-distribution model would likely provide the best model fit to the data in the pre-February 1994 and post-July 2008 periods. The second sub-period also exhibits some excess kurtosis (0.47), but the value of excess kurtosis is less than 0.5 and much smaller than that for the first and third sub-periods (> 1.68). This suggests there may not be any gains in model fit using a t-distribution model in the February 1994 and June 2008 period.

4.4 In-sample performance

The next step of the analysis is to evaluate the in-sample performance of each model. All four models of volatility described in section 4.2 are applied to each variable. The conditional mean for oil returns is an AR(1) model, because this specification yielded the lowest AIC and BIC values for all the time-varying volatility models used in this paper.[14] Also, in specifying the conditional mean as an ARIMA-type model, this paper follows Mohammadi and Su (2010), who assess the modeling and forecasting performance of various ARIMA-GARCH models of oil returns and volatilities. On the other hand, the conditional mean for USD returns is assumed to be a constant (or equivalently, the conditional mean of the USDTWI is a random walk with drift), following Rapach and Strauss (2008). As discussed in section 4.3, both the mean and volatility of oil and USD returns change when structural breaks occur. Therefore, the models are first estimated using the full sample without taking structural breaks into account, and then re-estimated for each sub-period in order to allow for structural breaks.

After estimation, the in-sample performance of each model is assessed using two common model selection criteria, the AIC and BIC.[15] The model with the smallest AIC and/or BIC is said to have the best in-sample performance. Both criteria are considered in this analysis because although the AIC performs better than the BIC in finite samples, it is

[14]For oil returns, for each time-varying volatility model, the AIC and BIC values were lowest for the AR(1) conditional mean specification. The model selection was carried out using the full sample. The AR(1) conditional mean is used for oil returns for all subsamples as well.

[15]AIC $= -2LogL + 2K$ and BIC $= -2LogL + Klog(T)$ where K is the number of parameters and T is the sample size. To produce AIC and BIC values for the ARSV(1) model, I first estimate the ARSV model using a two step quasi-maximum likelihood estimation method, then take the estimated parameters and put them into the Gaussian log-likelihood function to obtain the log-likelihood value, $LogL$. Then, this log-likelihood value is substituted into the AIC and BIC equations to obtain the values.

biased towards selecting models that are over-parameterized. On the other hand, the BIC tends to select models that are more parsimonious but performs better than the AIC in large samples (Enders, 2010).

The empirical relevance of structural breaks in the parameters of the time-varying volatility models is assessed in two ways. First, the magnitude of each estimated parameter is examined to see there are changes in the estimates across sub-periods. Second, Dufour and Torres's (1998) union-intersection methods are used to determine whether changes in parameters over the sub-periods are statistically significant. This method is used because there is no formal way of testing for structural breaks in GAS models.

Dufour and Torres's (1998) methods are applied to the parameters ω and A (α in the GARCH and t-GARCH models) separately in each time-varying volatility model in the following manner.[16] The null hypothesis is that the parameters are not significantly different from each other or $H_0 : \theta_1 = \theta_2 = ... = \theta_m$ where $m = 4$ for oil returns, $m = 3$ for USD returns, and θ_i represents a parameter from a time-varying volatility model in sub-period i. The level of significance for this analysis is selected to be 10%. The confidence interval for θ for each subsample i is $C_i(y_t, \alpha_i) = [\hat{\theta}_i - t_{\alpha_i} * se(\hat{\theta}_i), \hat{\theta}_i + t_{\alpha_i} se(\hat{\theta}_i)]$, where α_i is significant level for sub-period i, t_{α_i} is the critical value for the test for subsample i based on the t-distribution, and $se(\hat{\theta}_i)$ is the standard error of $\hat{\theta}_i$ for sub-period i. All the individual significant levels are chosen such that their sum is equal to the overall significant level of 10%. For example, in the case of oil returns, 97.5% confidence intervals for θ_i are constructed for each sub-period separately, since 10%/4 = 2.5%. As for the USD returns case, 96.6% confidence intervals are constructed because 10%/3 = 3.33%.

As shown in Proposition 1 of Dufour and Torres (1998), the test of empty intersection is based on the rejection region $W(\alpha, m) = \{y \varepsilon Y : C_1(y_t, \alpha_1) \cap C_2(y_t, \alpha_2) \cap ... \cap C_m(y_t, \alpha_m) = \emptyset\}$. Y is the set of variables used in this analysis, oil and USD returns. y in this paper represents either oil returns or USD returns. If there is no intersection of the confidence intervals for all subsamples, then the null hypothesis that the parameter estimates are not significantly different from each other is rejected at the 10% level in favour of the alternative.[17]

[16] There is no parameter A in the ARSV model, so I looked at only at the parameter ω.

[17] Alternatively, Dufour and Torres (1998) state the null hypothesis can be rejected if one or more of the distances between two of the parameters is larger than the sum of their corresponding critical points.

4.4.1 Oil returns

Focusing on oil returns, table 4.3 contains the estimates of all four models using the full sample when structural breaks are ignored. The results indicate that the t-distribution models fit oil returns better than the Normal distribution models. This is confirmed by the fact that the estimated degrees of freedom parameter, $\hat{\nu}$, in the t-GAS(1,1) and t-GARCH(1,1) models is between 7 and 8. The AIC and BIC values indicate that the t-GARCH(1,1) model has the best fit, followed closely by the t-GAS(1,1) model. The fit of the ARSV(1) model is considerably worse than that of the other three models.

Table 4.3: Oil returns full sample estimates of time-varying volatility

Parameters	GARCH	t-GARCH	t-GAS	ARSV
μ	0.079	0.085	0.099	0.035**
	(0.080)	(0.084)	(0.093)	(0.000)
ϕ	0.149**	0.158**	0.150**	0.113**
	(0.030)	(0.025)	(0.029)	(0.000)
ω	0.595**	0.528**	0.542**	0.045**
	(0.167)	(0.147)	(0.155)	(0.000)
A	0.103**	0.107**	0.101**	
	(0.015)	(0.015)	(0.014)	
β	0.865**	0.866**		
	(0.014)	(0.016)		
B			0.970**	
			(0.012)	
γ				0.965**
				(0.000)
ν		7.345**	8.012**	
		(1.261)	(1.408)	
Unconditional Volatility	4.312	4.422	4.250	2.057
No. of obs	1607	1607	1607	1607
AIC	8973.6	8909.9	8912.0	12975.0
BIC	9000.5	8942.2	8944.3	12997.0

Note: The standard errors are in parenthesis. ** and * represent significance at the 5% and 10% levels respectively. For all models, the conditional mean is $E(y_t) = \mu + \phi y_{t-1}$. The GARCH, t-GARCH, t-GAS, and ARSV volatility models are given in equations (4.1), (4.3), (4.5), and (4.8) respectively. Note that α in equations (4.1) and (4.3) is equal to A in equation (4.5).

In terms of the parameter estimates, the estimated constant of the conditional mean varies from 0.03 to 0.1 amongst the four models for the full sample period. With the exception of the ARSV(1) model, the estimated constant is not statistically different from zero. In contrast, $\hat{\phi}$ is statistically significant. $\hat{\phi}$ ranges from 0.11 to 0.16, implying that persistence in the mean is low. Hence, past oil returns are not very useful for predicting and forecasting current oil returns.

The conditional variance parameter estimates of the GARCH(1,1), t-GARCH(1,1), and t-GAS(1,1) models are similar. The estimated constant parameter, $\hat{\omega}$, is greater than 0.5 for the GARCH(1,1), t-GARCH(1,1), and t-GAS(1,1) models and statistically significant. $\hat{\omega}$ equals 0.045 for the ARSV(1) model and is also statistically different from zero.[18]

All four models indicate the presence of high persistence in the variance, because $\hat{B}$ in the t-GAS(1,1) model, $\hat{\alpha} + \hat{\beta}$ in the GARCH and t-GARCH models, and $\hat{\gamma}$ in the ARSV(1) model exceed 0.96. A high persistence parameter value frequently indicates the presence of structural breaks not taken into account in these models, as discussed in Lamoureux and Lastrapes (1990), Hillebrand (2005), and Chapter 3. In addition, the estimated unconditional volatility of the t-GARCH(1,1) model is closer to the sample standard deviation than that of any other model. On the other hand, the estimated unconditional volatility of the ARSV(1) model is the furthest away from the sample standard deviation. This is not surprising since the ARSV(1) model did not do as well as the other models in terms of in-sample performance.

Tables 4.4 to 4.7 summarize the results for oil returns when structural breaks are controlled for by splitting the sample. These results suggest that the parameters of all the volatility models analyzed in this paper are indeed not constant. $\hat{\omega}$ and $\hat{A}$ in particular appear to be variable across models and sub-periods. With the exception of the ARSV(1) model, $\hat{\omega}$ ranges from 0.94 to 1.24 in the first sub-period, from 0.62 to 0.69 in the second sub-period, from 0.72 to 0.78 in the third sub-period, and from 0.28 to 0.33 in the final sub-period.

The statistical significance of $\hat{\omega}$ also varies across subsamples for the three models. For example, in the first sub-period, $\hat{\omega}$ is not statistically different from zero for the t-GAS, GARCH, and t-GARCH models. $\hat{\omega}$ for all models is statistically significant in the second sub-period. For the t-GAS and t-GARCH models, $\hat{\omega}$ is statistically significant in the third sub-period at the 5% and 10% levels respectively. However, $\hat{\omega}$ for the GARCH model is not significantly different from zero in the third sub-period. In the fourth and final sub-period, $\hat{\omega}$ for the t-GAS model is statistically significant at the 10% level. For the GARCH and t-GARCH models, $\hat{\omega}$ is not statistically different from zero in the final sub=period. On

[18] Note that the parameters of the ARSV(1) model relate to the log of the conditional variance and therefore are not directly comparable to those of the other three models.

Table 4.4: Oil returns estimates of time-varying volatility between January 1986 and July 1990

	GARCH	t-GARCH	t-GAS	ARSV
μ	-0.054	-0.156	-0.163	0.000
	(0.244)	(0.220)	(0.180)	(0.001)
ϕ	0.087	0.089	0.075	0.046**
	(0.078)	(0.074)	(0.061)	(0.000)
ω	1.236	0.940	0.974	0.071**
	(0.865)	(0.650)	(0.662)	(0.002)
A	0.121**	0.135**	0.141**	
	(0.058)	(0.064)	(0.057)	
β	0.814**	0.826**		
	(0.079)	(0.072)		
B			0.956**	
			(0.040)	
γ				0.947**
				(0.002)
ν		4.674**	4.895**	
		(1.113)	(1.162)	
Unconditional Volatility	4.361	4.910	4.700	2.044
No. of obs	238	238	238	238
AIC	1376.7	1356.9	1357.2	1833.6
BIC	1394.1	1377.7	1378	1847.5

Note: The standard errors are in parenthesis. ** and * represent significance at the 5% and 10% levels respectively. For all models, the conditional mean is $E(y_t) = \mu + \phi y_{t-1}$. The GARCH, t-GARCH, t-GAS, and ARSV volatility models are given in equations (4.1), (4.3), (4.5), and (4.8) respectively. Note that α in equations (4.1) and (4.3) is equal to A in equation (4.5).

the other hand, $\hat{\omega}$ for the ARSV model ranges between 0.03 and 0.11 and is statistically significant in all sub-periods.

Although there is some variability in $\hat{\omega}$ across sub-periods, the differences are not large for any of the models except for the ARSV model. Figure 4.3 displays the range of values that fall within the 90% confidence intervals of $\hat{\omega}$ for the GARCH, t-GARCH, t-GAS, and ARSV models. The null hypothesis that ω is not statistically different across subsamples can be rejected if there is no set of values that intersects the confidence intervals for all sub-periods. The results show that for the GARCH, t-GARCH, and t-GAS models, the confidence intervals of $\hat{\omega}$ all intersect somewhere between 0 and 0.5.[19] Hence, the null

[19] I imposed positivity constraints on the estimates for all models. However, the size of some standard

Table 4.5: Oil returns estimates of time-varying volatility between August 1990 and July 1997

	GARCH	t-GARCH	t-GAS	ARSV
μ	-0.007	-0.054	0.000	0.000
	(0.164)	(0.144)	(0.000)	(0.001)
ϕ	0.120*	0.096**	0.101**	0.037**
	(0.072)	(0.047)	(0.040)	(0.000)
ω	0.685**	0.632**	0.617**	0.111**
	(0.135)	(0.248)	(0.309)	(0.003)
A	0.125**	0.132**	0.125**	
	(0.038)	(0.032)	(0.031)	
β	0.812**	0.819**		
	(0.038)	(0.033)		
B			0.948**	
			(0.032)	
γ				0.923**
				(0.002)
ν		8.636**	9.560**	
		(3.821)	(3.821)	
Unconditional Volatility	3.297	3.591	3.445	2.214
No. of obs	361	361	361	361
AIC	1923.2	1911.9	1914	2307.4
BIC	1942.7	1935.2	1937.3	2323.0

Note: The standard errors are in parenthesis. ** and * represent significance at the 5% and 10% levels respectively. For all models, the conditional mean is $E(y_t) = \mu + \phi y_{t-1}$. The GARCH, t-GARCH, t-GAS, and ARSV volatility models are given in equations (4.1), (4.3), (4.5), and (4.8) respectively. Note that α in equations (4.1) and (4.3) is equal to A in equation (4.5).

hypothesis that the parameters are equal across the subsamples cannot be rejected. In contrast, for the ARSV model, this same null hypothesis for ω can be rejected in favour of the alternative hypothesis at the 10% level. This is because there is no intersection of the confidence intervals for all four subsamples. This suggests that only for the ARSV model is the $\hat{\omega}$ significantly different for the four subsamples.

Table 4.4 to 4.7 also indicate that the values of $\hat{A}$ are similar in the first, second, and

errors causes some parameter estimates to be statistically insignificant. When confidence intervals are constructed, the large standard errors cause some confidence intervals to have negative lower limits, and upper limits greater than one. Further testing for non-stationary behaviour in the variance may be required (i.e., Giraitis et al., 2006), but is left to future research.

Table 4.6: Oil returns estimates of time-varying volatility between August 1997 and June 2008

	GARCH	t-GARCH	t-GAS	ARSV
μ	0.314**	0.401**	0.403**	0.297**
	(0.169)	(0.171)	(0.166)	(0.000)
ϕ	0.146**	0.131**	0.138**	0.132**
	(0.039)	(0.038)	(0.038)	(0.000)
ω	0.762	0.723*	0.775**	0.030**
	(0.806)	(0.395)	(0.258)	(0.001)
A	0.025	0.029*	0.028*	
	(0.019)	(0.016)	(0.016)	
β	0.931**	0.929**		
	(0.058)	(0.032)		
B			0.955**	
			(0.014)	
γ				0.909**
				(0.002)
ν		9.377**	10.072**	
		(3.875)	(4.269)	
Unconditional Volatility	4.162	4.150	4.150	1.091
No. of obs	574	574	574	574
AIC	3260.2	3243.9	3244.4	8184.6
BIC	3281.9	3270.0	3270.5	8202.0

Note: The standard errors are in parenthesis. ** and * represent significance at the 5% and 10% levels respectively. For all models, the conditional mean is $E(y_t) = \mu + \phi y_{t-1}$. The GARCH, t-GARCH, t-GAS, and ARSV volatility models are given in equations (4.1), (4.3), (4.5), and (4.8) respectively. Note that α in equations (4.1) and (4.3) is equal to A in equation (4.5).

fourth sub-periods and across all models (there is no equivalent of $\hat{A}$ in the ARSV model). $\hat{A}$ is between 0.12 and 0.14 in the first sub-period, between 0.12 and 0.13 in the second sub-period, and between 0.10 and 0.12 in the fourth and final sub-period. In these three sub-periods, $\hat{A}$ is statistically different from zero in all models at the 5% level. On the other hand, in the third sub-period, $\hat{A}$ ranges from 0.025 to 0.029 across models. $\hat{A}$ is statistically different from zero at the 10% level for the t-GARCH and t-GAS models. For the GARCH model, $\hat{A}$ is statistically insignificant. Additionally, figure 4.4 shows that the differences in $\hat{A}$ are not significantly large for all the models. This is because the confidence intervals for $\hat{A}$ overlap, implying that the null hypothesis that the parameters are equivalent across sub-periods cannot be rejected in favour of the alternative.

Table 4.7: Oil returns estimates of time-varying volatility between June 2008 and October 2016

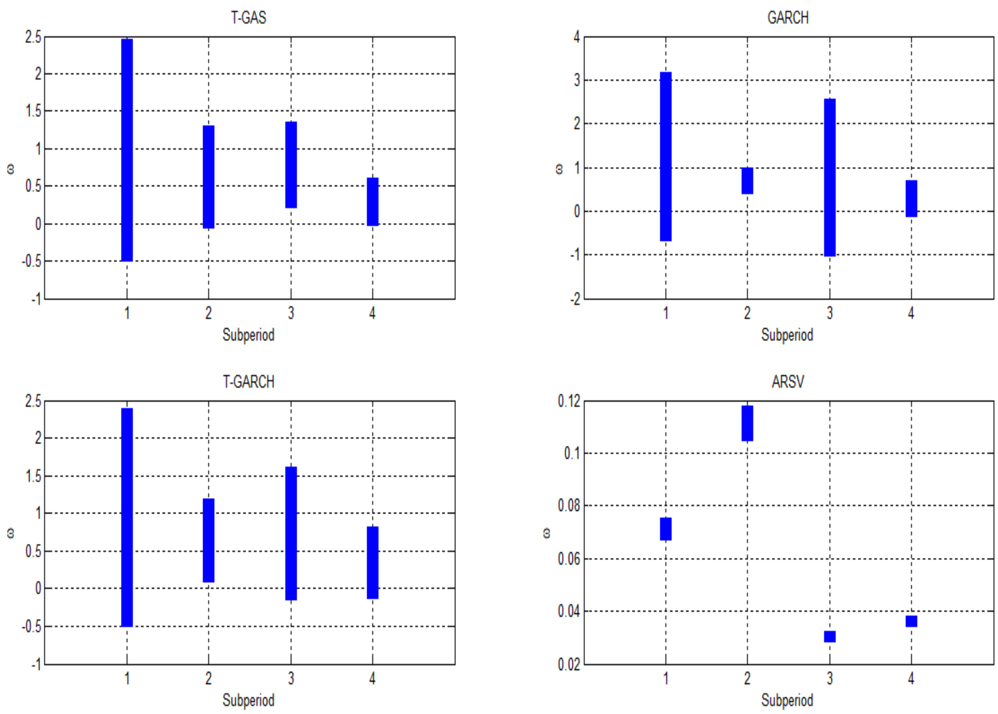
	GARCH	t-GARCH	t-GAS	ARSV
μ	-0.026	-0.06	-0.011	0.000
	(0.174)	(0.151)	(0.018)	(0.001)
ϕ	0.210**	0.251**	0.241**	0.170**
	(0.054)	(0.049)	(0.045)	(0.000)
ω	0.282	0.331	0.284*	0.036**
	(0.187)	(0.214)	(0.147)	(0.001)
A	0.117**	0.118**	0.103**	
	(0.039)	(0.043)	(0.029)	
β	0.872**	0.868**		
	(0.042)	(0.042)		
B			0.987**	
			(0.016)	
γ				0.978**
				(0.001)
ν		6.631**	8.222**	
		(1.718)	(2.714)	
Unconditional Volatility	5.063	4.862	4.673	2.612
No. of obs	435	435	435	435
AIC	2393.1	2387.1	2385.2	2831.5
BIC	2413.4	2411.5	2409.6	2847.8

Note: The standard errors are in parenthesis. ** and * represent significance at the 5% and 10% levels respectively. For all models, the conditional mean is $E(y_t) = \mu + \phi y_{t-1}$. The GARCH, t-GARCH, t-GAS, and ARSV volatility models are given in equations (4.1), (4.3), (4.5), and (4.8) respectively. Note that α in equations (4.1) and (4.3) is equal to A in equation (4.5).

The estimated persistence parameter – $\hat{B}$ in the t-GAS model, $\hat{\alpha} + \hat{\beta}$ in the GARCH and t-GARCH models, and γ in the ARSV model – remains high and statistically significant in all models and sub-periods. This implies that for oil returns, the presence of structural breaks is not the only factor causing persistence to be high in all models. Other factors that could cause persistence to be high in all models and all sub-periods include the presence of additional unknown breaks. Figure 4.5 shows the application of Dufour and Torres's (1998) test to the persistence parameter of the t-GAS, GARCH, and t-GARCH models. The results also indicate that the differences across sub-periods are also not statistically significant. Only γ for the ARSV model had there been some differences across sub-periods.

The results show that the estimated unconditional volatility changes slightly across the

Figure 4.3: 90% confidence intervals for ω by sub-period – oil returns

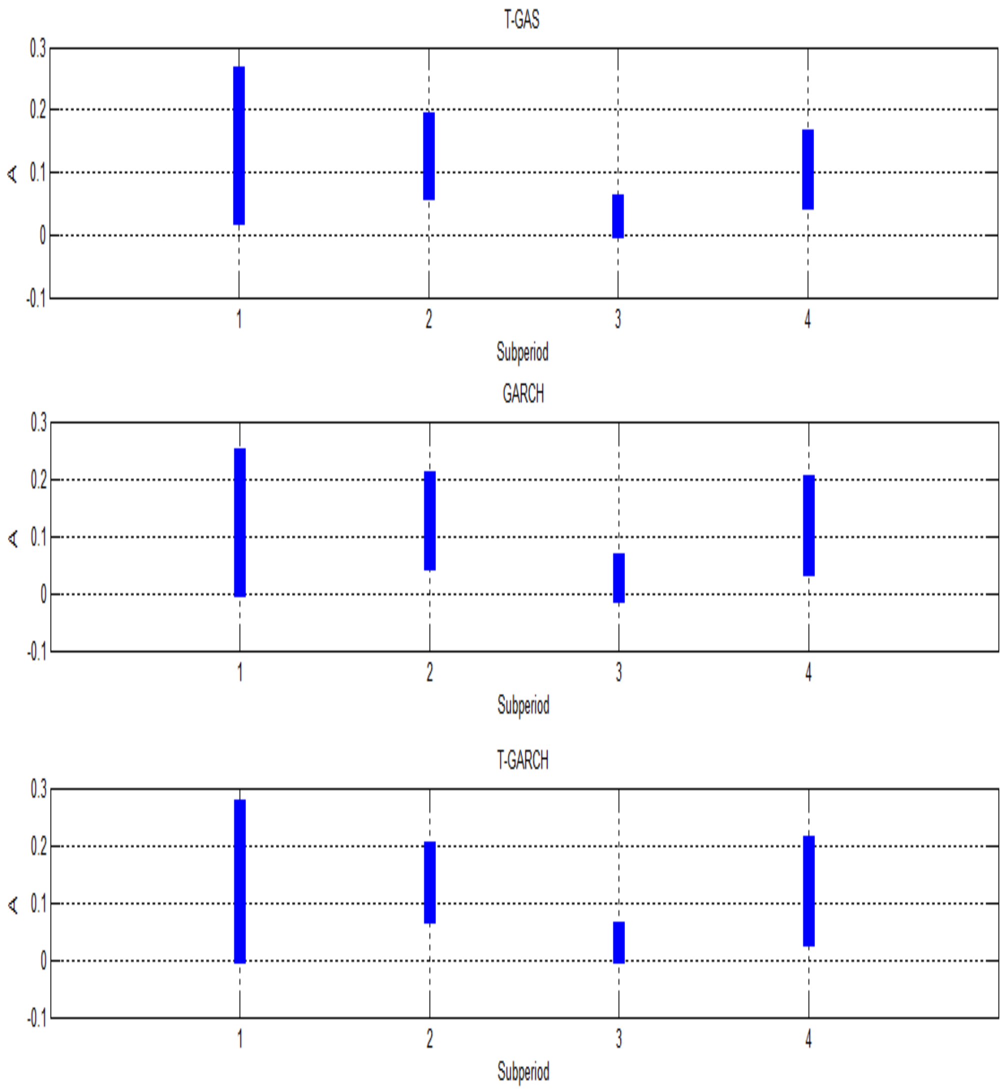


Note: The null hypothesis that ω is not statistically different from each other can be rejected if there is no intersection that falls between the ranges.

sub-periods for all models. This is because $\hat{\omega}$ in all models and $\hat{\alpha}$ for the GARCH(1,1) and t-GARCH(1,1) models are not significantly different across sub-periods. In tables 4.4 to 4.7, the results also show that either the t-GARCH(1,1) model or t-GAS(1,1) model have estimated unconditional volatilities closest to the sample standard deviation found in table 4.2. In all periods, the ARSV(1) model comes in last with respect to in-sample performance in terms of both its AIC and BIC values. Note that the estimated parameters of the conditional mean also vary across sub-periods in all models. Only in the third sub-period is $\hat{\mu}$ significantly different from zero for all models. In the last sub-period, the persistence of the conditional mean increases slightly for all models. Together, these changes imply that the unconditional mean also varies across subsamples.[20]

[20]However, when Dufour and Torres's (1998) test was applied to the parameter estimates of the con-

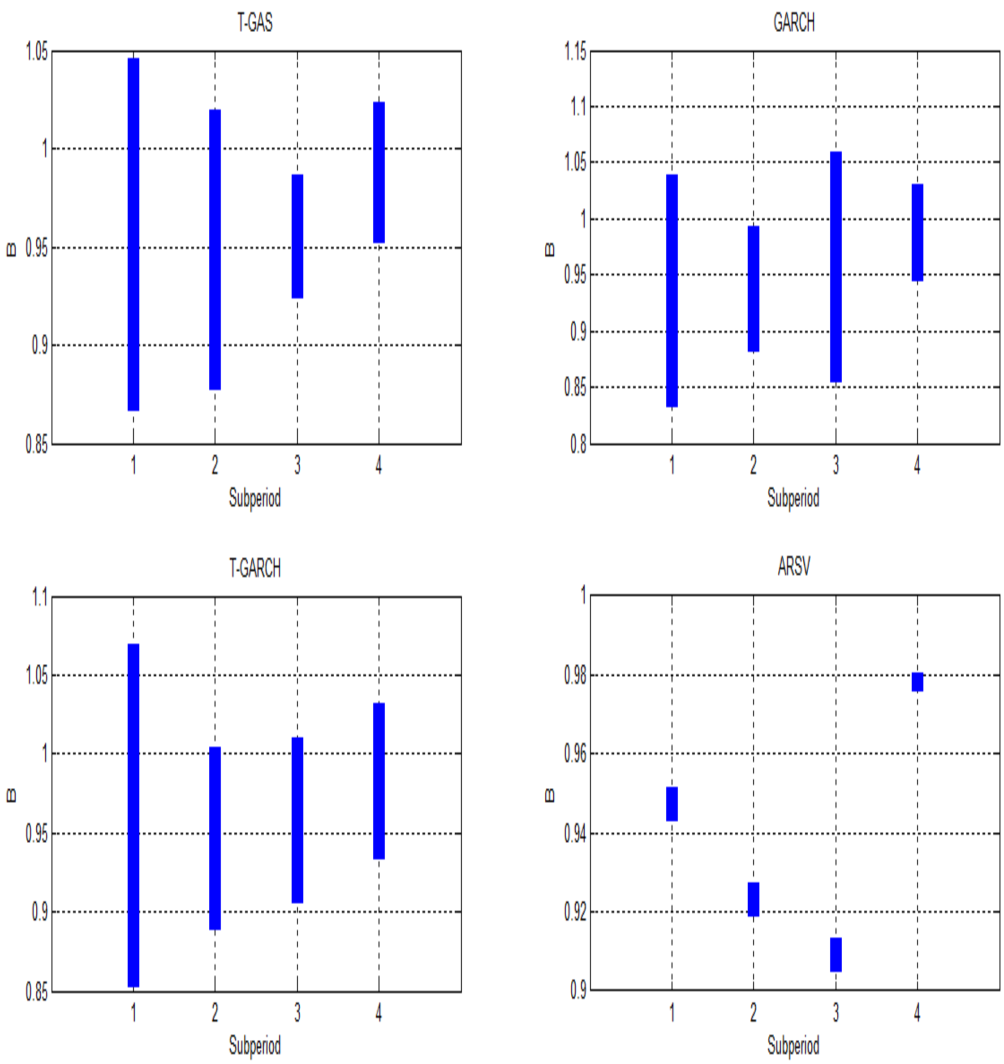
Figure 4.4: 90% confidence intervals for A by sub-period – oil returns



Note: The null hypothesis that A is not statistically different from each other can be rejected if there is no intersection that falls between the ranges.

ditional mean, the results show that the conditional mean estimates are statistically equivalent across

Figure 4.5: 90% confidence intervals for B by sub-period – oil returns



Note: The null hypothesis that B is not statistically different from each other can be rejected if there is no intersection that falls between the ranges.

subsamples for all models except for the ARSV model.

Although the parameter estimates of the conditional variance vary across subsamples, the differences are not large, implying that structural breaks are not empirically relevant for the volatility models of oil returns, except for the ARSV(1) model. For the GARCH(1,1) and t-GARCH(1,1) models, this finding is consistent with that of Arouri et al.'s (2012) for crude oil. Additionally, the poor performance of the ARSV(1) model compared to the other three models contradicts the findings of Chan and Grant (2016), in that they find that stochastic volatility models provide the best fit for oil returns.

4.4.2 USD returns

Table 4.8 summarizes the estimation results for USD returns for all four time-varying volatility models using the full sample. The results show that the t-GAS(1,1) model has better in-sample performance than the other three models when structural breaks are ignored, based on the AIC and BIC values. The next best model is the t-GARCH(1,1) model. Once again, the ARSV(1) model has the worst in-sample performance according to the AIC and BIC. According to table 4.8, the estimated degree of persistence in conditional volatility exceeds 0.98, indicating the possibility of structural breaks being present in USD returns. The estimated constant of the conditional volatility, $\hat{\omega}$, ranges from 0.008 to 0.015 in all the models. Only for the t-GARCH model is $\hat{\omega}$ statistically insignificant. Also, $\hat{A}$ is between 0.08 and 0.11 for the GARCH(1,1), t-GARCH(1,1), and t-GAS(1,1) models. The estimated degrees of freedom, $\hat{\nu}$, for the Student's t models are approximately 10. The finding that the Student's t models have a finite $\hat{\nu}$ as well as the lowest AIC and BIC values implies that USD returns are best fitted in-sample by a model based on the Student's t-distribution. As for the estimated mean $\hat{\mu}$, it is very close to zero in all models. With the exception of the ARSV(1) model, $\hat{\mu}$ is statistically insignificant for all the models.

The USD returns sample is then broken down into smaller subsamples based on the timing of the breaks, as discussed in section 4.3. Tables 4.9 to 4.11 summarize the estimates of the competing volatility models for three sub-periods.

One estimated parameter that shows variability across the models and sub-periods is the estimated constant of the conditional volatility, $\hat{\omega}$. The results show that in the first sub-period, $\hat{\omega}$ ranges from 0.006 to 0.085 and is statistically insignificant for all models, with the exception of the ARSV model. Additionally, table 4.10 shows that $\hat{\omega}$ ranges from 0.011 to 0.32 in the second sub-period. In contrast to $\hat{\omega}$ in the first sub-period, $\hat{\omega}$ in the second sub-period is statistically different from zero for all the models. As for the third sub-period, $\hat{\omega}$ is between 0.005 and 0.017 and is statistically significant at the 10% level only for the t-GAS model.

Table 4.8: USD returns full sample estimates of time-varying volatility

Parameters	GARCH	t-GARCH	t-GAS	ARSV
μ	-0.008	-0.006	-0.003	-0.007**
	(0.015)	(0.014)	(0.012)	(0.000)
ω	0.012*	0.010	0.008*	0.015**
	(0.007)	(0.007)	(0.005)	(0.000)
A	0.085**	0.108**	0.092**	
	(0.024)	(0.023)	(0.018)	
β	0.898**	0.884**		
	(0.029)	(0.027)		
B			0.992**	
			(0.010)	
γ				0.986**
				(0.000)
ν		10.084**	9.764**	
		(2.500)	(2.396)	
Unconditional Volatility	0.825	1.118	1.000	1.839
Number of Observations	2286	2286	2286	2286
AIC	5210.5	5140.8	5119.0	7096.4
BIC	5233.5	5169.5	5147.7	7113.6

Note: The standard errors are in parenthesis. ** and * represent significance at the 5% and 10% levels respectively. For all models, the conditional mean is $E(y_t) = \mu$. The GARCH, t-GARCH, t-GAS, and ARSV volatility models are given in equations (4.1), (4.3), (4.5), and (4.8) respectively. Note that α in equations (4.1) and (4.3) is equal to A in equation (4.5).

For all models, the results also indicate that the null hypothesis that ω is equal in all sub-periods can be rejected. This is because there is no set of values in ω that intersects with the confidence intervals for ω of all subsamples. This is illustrated in figure 4.6, which shows that the confidence intervals for ω for the three sub-periods have no points in common, for all four models.

The sub-periods in which $\hat{\omega}$ varies are different for each model. Although some confidence intervals reveal some pairwise overlaps, Dufour and Torres's (1998) test is looking for an overlap for all periods, and shows that the null hypothesis of the coefficient being equal across all subsamples can be rejected. For the t-GAS model, the confidence intervals for ω intersect only in the first and third sub-periods. As for the t-GARCH model, the confidence interval for ω in the second sub-period does not intersect with the confidence intervals for ω in the first and third sub-periods. In the case of the ARSV model, the first and second

Table 4.9: USD returns estimates between January 1973 and January 1994

Parameters	GARCH	t-GARCH	t-GAS	ARSV
μ	-0.007	0.000	0.000	-0.016**
	(0.022)	(0.000)	(0.000)	(0.000)
ω	0.006	0.009	0.060	0.085**
	(0.074)	(0.016)	(0.169)	(0.001)
A	0.070	0.161**	0.215**	
	(0.052)	(0.047)	(0.095)	
β	0.919**	0.920**		
	(0.060)	(0.054)		
B			0.912**	
			(0.266)	
γ				0.956**
				(0.001)
ν		7.006**	8.015**	
		(1.611)	(3.879)	
Unconditional Volatility	0.775	-	0.826	2.969
Number of Observations	1100	1100	1100	1100
AIC	2482.8	2400.8	2400.9	4232.1
BIC	2502.8	2425.8	2425.9	4247.1

Note: The standard errors are in parenthesis. ** and * represent significance at the 5% and 10% levels respectively. For all models, the conditional mean is $E(y_t) = \mu$. The GARCH, t-GARCH, t-GAS, and ARSV volatility models are given in equations (4.1), (4.3), (4.5), and (4.8) respectively. Note that α in equations (4.1) and (4.3) is equal to A in equation (4.5).

sub-periods do not intersect with each other, but they both intersect with the third sub-period. For the t-GARCH model, the second sub-period is significantly different from the first and third sub-period according to Dufour and Torres's (1998) union-intersection test, because there is no set of values of the confidence interval for the second sub-period that intersects with the confidence intervals for the first and third sub-periods. This implies that the differences in parameters across sub-periods are large for all four time-varying volatility models.

In the case of the estimated parameter $\hat{A}$, the parameter values vary across the models and across sub-periods. In the first sub-period, $\hat{A}$ ranges from 0.07 to 0.215, but it is statistically significant only for the t-GAS and t-GARCH models. As for the second sub-period, there is less variability between models, since $\hat{A}$ lies between 0.164 and 0.166. In the third subperiod, $\hat{A}$ lies between 0.102 and 0.107 and is statistically significant at the

Table 4.10: USD returns estimates between February 1994 and June 2008

Parameters	GARCH	t-GARCH	t-GAS	ARSV
μ	0.000	0.000	0.000	-0.034**
	(0.000)	(0.050)	(0.051)	(0.000)
ω	0.320**	0.317**	0.263**	0.011**
	(0.073)	(0.061)	(0.072)	(0.000)
A	0.164**	0.166**	0.164**	
	(0.036)	(0.031)	(0.028)	
β	0.237**	0.248**		
	(0.038)	(0.032)		
B			0.513**	
			(0.037)	
γ				0.966**
				(0.001)
ν		16.370**	16.289**	
		(0.000)	(0.000)	
Unconditional Volatility	0.731	0.735	0.735	1.181
Number of Observations	752	752	752	752
AIC	1636.5	1634.4	1634.4	1892.2
BIC	1655.0	1657.6	1657.6	1910.7

Note: The standard errors are in parenthesis. ** and * represent significance at the 5% and 10% levels respectively. For all models, the conditional mean is $E(y_t) = \mu$. The GARCH, t-GARCH, t-GAS, and ARSV volatility models are given in equations (4.1), (4.3), (4.5), and (4.8) respectively. Note that α in equations (4.1) and (4.3) is equal to A in equation (4.5).

5% level. However, the results in figure 4.7 show that although $\hat{A}$ appears to vary across subsamples, the differences are not significant. The figure shows that there exists a range of values of A such that the confidence intervals of all the subsamples intersect for all models. This implies that for the t-GARCH and GARCH models, in which A shifts the unconditional volatility, changes in A may not be large enough to cause changes in the unconditional volatility of the USD returns.

The results in tables 4.9 to 4.11 also show that the estimated persistence parameter – $\hat{B}$ of the t-GAS model, $\hat{\alpha} + \hat{\beta}$ of the GARCH and t-GARCH models, and $\hat{\gamma}$ of the ARSV model – varies across sub-periods in the t-GAS, GARCH, and t-GARCH models. For the GARCH(1,1), t-GARCH(1,1), and t-GAS(1,1) models, the estimated persistence parameter exceeds 0.9 in the first and last sub-periods, but is less than 0.43 in the second sub-period. In contrast, the estimated persistence parameter in the ARSV(1), $\hat{\gamma}$, process remains high, and in all periods exceeds 0.9. Dufour and Torres's (1998) test have also

Table 4.11: USD returns estimates between July 2008 and October 2016

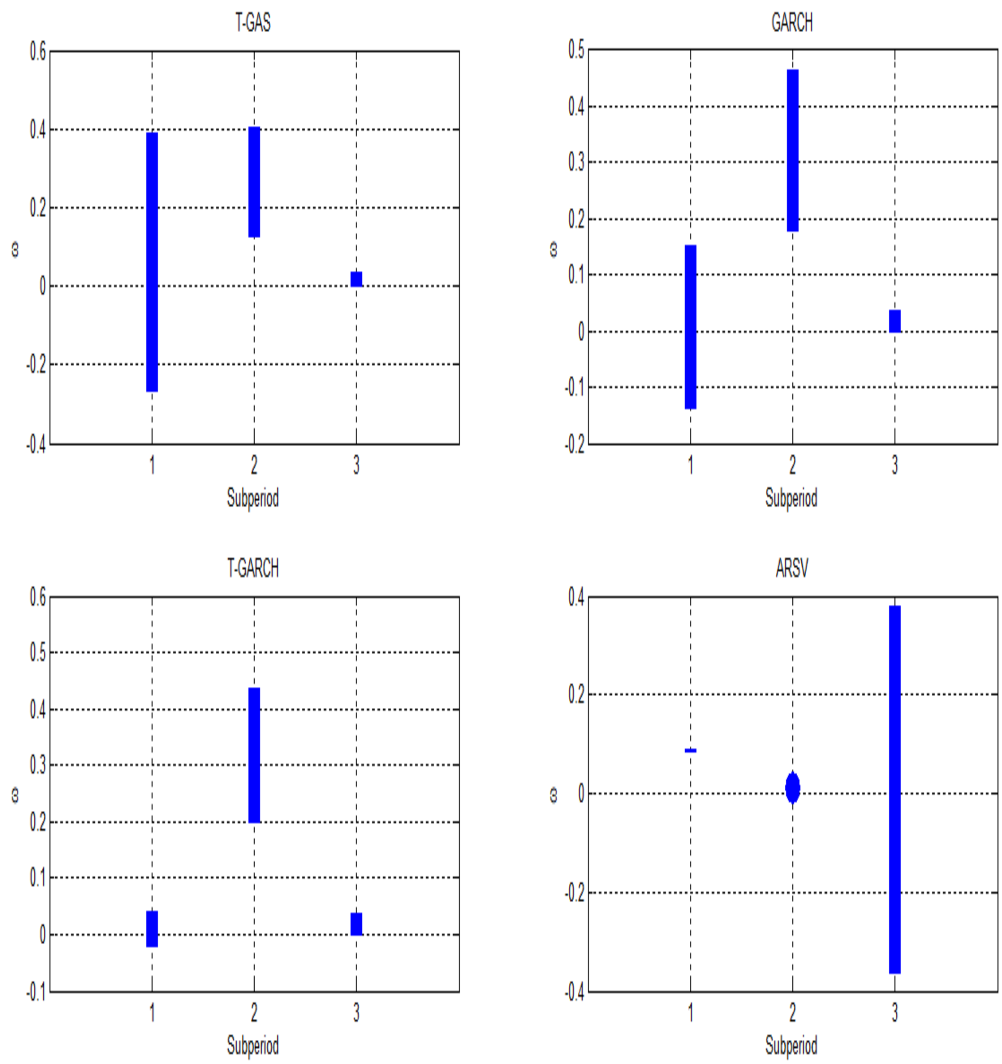
Parameters	GARCH	t-GARCH	t-GAS	ARSV
μ	0.063	0.064	0.063	0.062**
	(0.038)	(0.041)	(0.040)	(0.000)
ω	0.016	0.017	0.017*	0.005
	(0.010)	(0.011)	(0.010)	(0.190)
A	0.107**	0.107**	0.102**	
	(0.025)	(0.024)	(0.024)	
β	0.874**	0.874**		
	(0.022)	(0.022)		
B			0.980**	
			(0.018)	
γ				0.982**
				(0.501)
ν		29.861**	29.884**	
		(10.566)	(7.049)	
Unconditional Volatility	0.918	0.946	0.922	1.188
Number of Observations	434	434	434	434
AIC	1103.8	1106.3	1106.9	1227.0
BIC	1120.1	1126.6	1127.2	1239.3

Note: The standard errors are in parenthesis. ** and * represent significance at the 5% and 10% levels respectively. For all models, the conditional mean is $E(y_t) = \mu$. The GARCH, t-GARCH, t-GAS, and ARSV volatility models are given in equations (4.1), (4.3), (4.5), and (4.8) respectively. Note that α in equations (4.1) and (4.3) is equal to A in equation (4.5).

been applied to the estimated persistence parameter for the GARCH, t-GARCH, t-GAS, and ARSV models. As shown in figure 4.8, the results indicate that the changes in these parameters can be considered to be large only for the t-GAS model, since the null hypothesis that B is equivalent across sub-periods can be rejected in favour of the alternative. For the other three models, the null hypothesis cannot be rejected in favour.

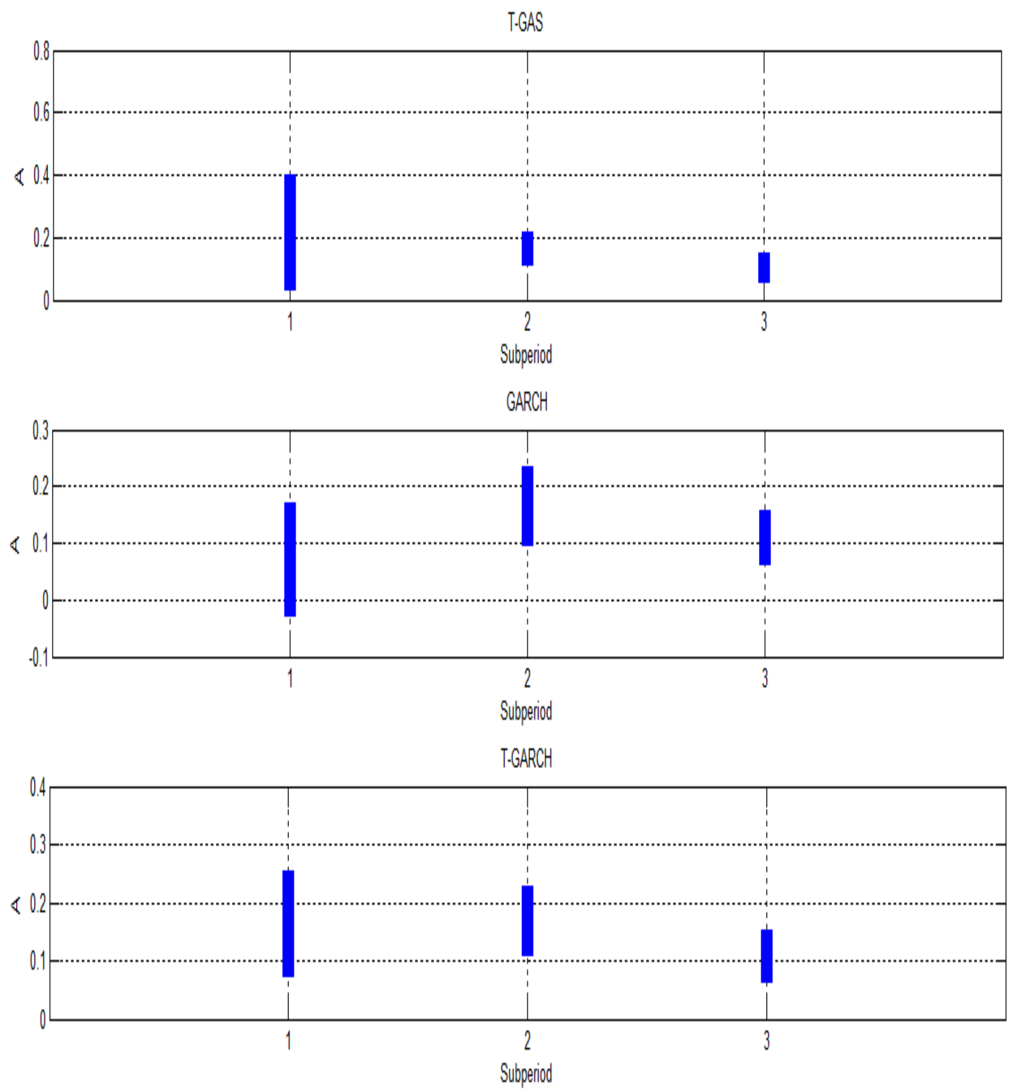
The estimate of the unconditional mean, $\hat{\mu}$ is close to zero in all periods and models. Finally, $\hat{\nu}$ in the t-GARCH(1,1) and t-GAS(1,1) models trends upwards, implying that over time, it becomes increasingly more favourable to estimate a Gaussian model of USD returns. The fact that the estimated degrees of freedom, $\hat{\nu}$, are large in the last sub-period confirms that it would be more appropriate to use a Gaussian GARCH model for that period. Also, figure 4.9 displays the confidence intervals of the ν across all three sub-periods. It shows that the differences in $\hat{\nu}$ across the sub-periods are also found to be

Figure 4.6: 90% confidence intervals for ω by sub-period - USD returns



Note: The null hypothesis that ω is not statistically different from each other can be rejected if there is no intersection that falls between the ranges. The dot in the figure represents a confidence interval that is actually the estimate since the standard error is zero.

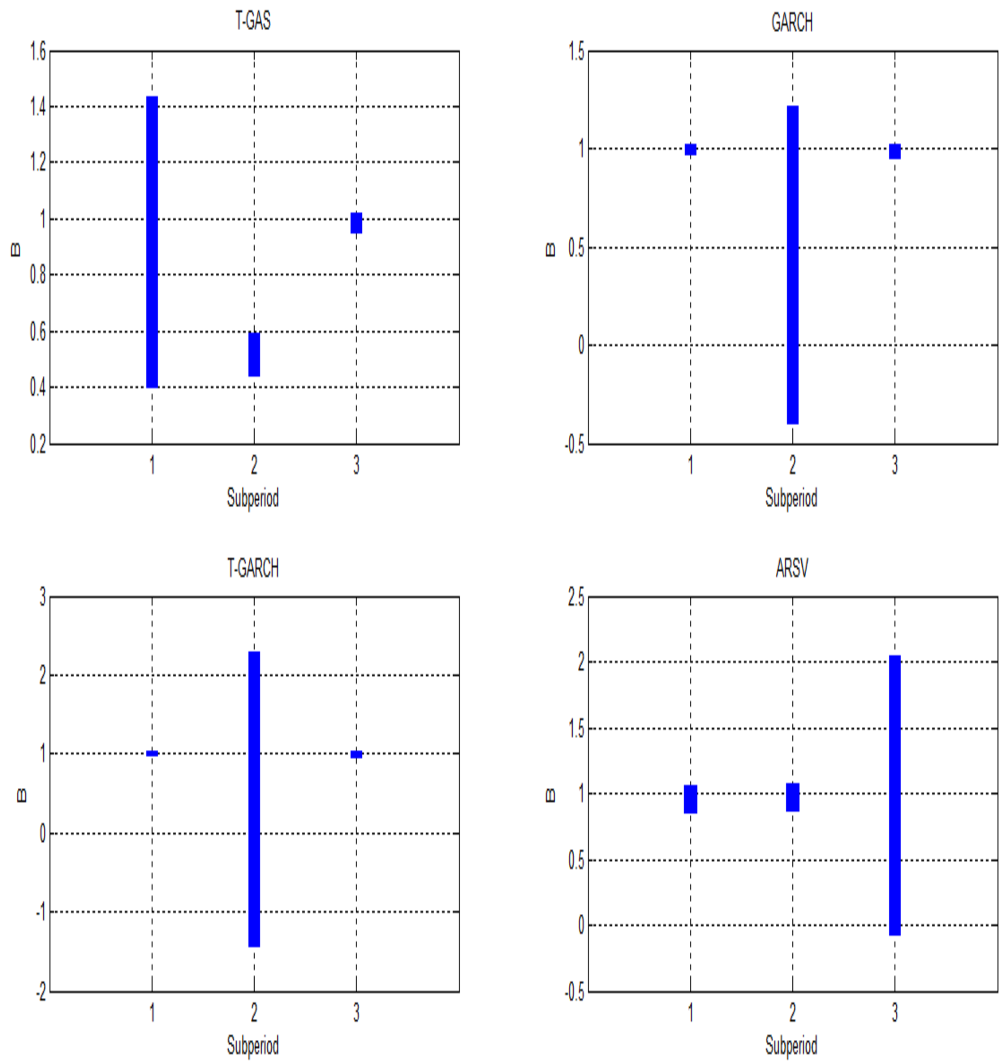
Figure 4.7: 90% confidence intervals for A by sub-period - USD returns



Note: The null hypothesis that A is not statistically different from each other can be rejected if there is no intersection that falls between the ranges.

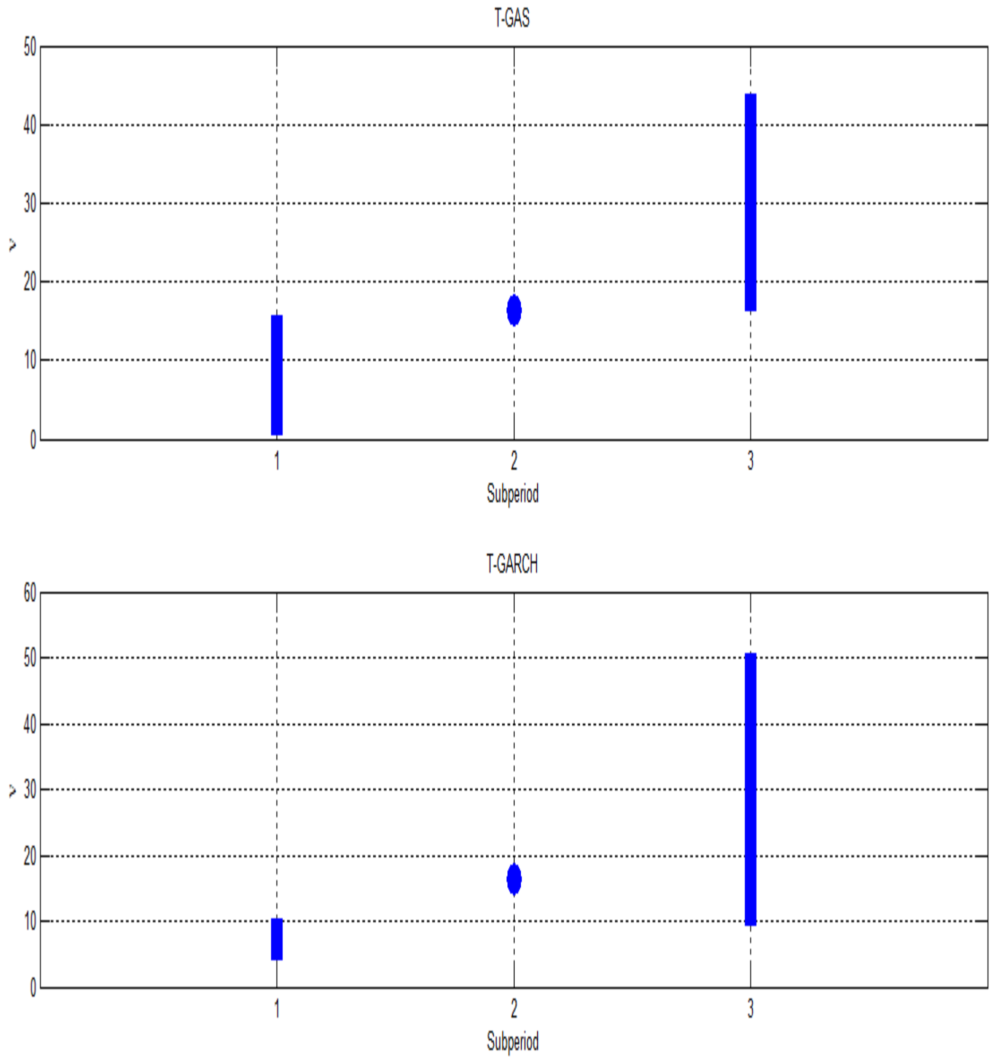
statistically significant at the 10% level.

Figure 4.8: 90% confidence intervals for B by sub-period - USD returns



Note: The null hypothesis that B is not statistically different from each other can be rejected if there is no intersection that falls between the ranges. The squares in the figure represents a confidence interval that is so small that it is difficult to see two confidence intervals that do not intersect.

Figure 4.9: 90% confidence intervals for ν by sub-period - USD returns



Note: The null hypothesis that ν is not statistically different from each other can be rejected if there is no intersection that falls between the ranges. The dot in the figure represents a confidence interval that is actually the estimate since the standard error is zero.

Tables 4.9 to 4.11 also show that the estimated unconditional volatilities of all models change over time. In addition, either the GARCH(1,1) model or the t-GAS(1,1) model has an estimated unconditional volatility closest to the sample standard deviation. Once again, the ARSV(1) model is farthest from the sample standard deviation, which is another indication that the ARSV(1) model does not perform well in-sample. Furthermore, the t-GARCH(1,1) model fare better than the other models in the first sub-period based on the AIC and BIC values, while the t-GARCH(1,1) and t-GAS(1,1) models fares better than the others in the second sub-period according only to the AIC value. Based on the same criterion, the GARCH(1,1) model slightly outperforms the t-GARCH(1,1) and t-GAS(1,1) models in the final sub-period.

4.4.3 In-sample performance: Summary and implications

For oil returns, it is unclear whether the t-GAS model or the t-GARCH model has the best performance. However, it is possible to say that a t-distribution model fits oil returns better than the Gaussian models, according to the AIC and BIC values. The AIC and BIC values for the t-GAS(1,1) and t-GARCH(1,1) models are close in all sub-periods. In the case of USD returns, either the t-GAS(1,1) model, t-GARCH(1,1) model, or the GARCH(1,1) model has the best performance when structural breaks are taken into account. An out-of-sample forecasting analysis is needed in order to determine which model has the best performance outside the sample period.

One thing that is clear is that for both oil and USD returns the ARSV(1) model has the worst in-sample performance for the entire period and each sub-period. This contradicts the findings of Chan and Grant (2016). Chan and Grant (2016) find that stochastic volatility models fit oil returns better than GARCH models. However, Chan and Grant (2016) use Bayesian methods to estimate their stochastic volatility models rather than quasi-maximum likelihood estimation with a Kalman Filter. The difference between the in-sample performance of the ARSV model in this paper and that of Chan and Grant (2016) suggests that alternative methods of estimation should be considered for the ARSV model.

The in-sample analysis shows that structural breaks are empirically relevant for all models of USD volatility, but not for the models of oil volatility. This is because, for the USD returns, the estimated parameters of the conditional volatility substantially change across the sub-periods. This is not the case for the models of oil volatility. In the case of the GARCH model, the USD result is consistent with that of Rapach and Strauss (2008) in which they find that the estimated parameters change substantially across sub-periods. The crude oil result is consistent with that of Arouri et al. (2012); they find no evidence of structural breaks in the volatility of the data after using the CUSUM test. The empirical relevance of structural breaks in both GAS models and non-GAS models indicates the need to assess which model has the best out-of-sample performance and whether accommodating structural breaks produces better forecasts, because in theory failing to accommodate breaks can give rise to biased forecasts.

4.5 Out of sample performance

This section analyzes the out-of-sample performance of all the time-varying volatility models described in section 4.2. Each of the four time-varying volatility models is combined with three types of forecasting method – the recursive window, which uses an expanding estimation window; a rolling window of one-half the size of the initial recursive estimation window, and a rolling window of one-quarter the size of the initial recursive estimation window – to generate out-of-sample forecasts of oil and USD returns and volatilities. Thus in total, 12 different forecasting models are compared in this analysis.

The recursive (expanding window) method is commonly used as a natural and appropriate benchmark for forecasting data generated from a stable process (Rapach and Strauss, 2008), while the rolling window method adapts to unknown structural breaks by removing pre-break data from the sample. As mentioned by Clark and McCracken (2009), the recursive method is the most accurate forecasting scheme with stable DGPs. On the other hand, rolling window methods are considered better at accommodating breaks than the recursive method. In principle, failing to accommodate structural breaks will induce a systematic bias in the forecasts. As a result, the forecast can converge to the wrong equilibrium values.

An alternative to the rolling window method would be to explicitly incorporate known breaks into each model by either adding dummy variables to the model or following Rapach and Strauss (2008) in using the last break date as the first observation of the estimation window. The problem with adding dummy variables to allow for a break in each parameter is that it leads to too many parameters to estimate. Generally, when the number of parameters is large, the estimates may not converge. A bigger problem is that one can only add dummy variables ex-post, after one knows when the breaks occurred. The last break date for oil and USD returns is the first week of June 2008, which is very close to the beginning of the forecast period (November 2010), which means that the sample would be too small to estimate each time-varying volatility model precisely. Therefore, neither option is pursued in this chapter.

One step-ahead forecasts of the mean and variance of oil and USD returns are generated for the following time horizons:

- two years ahead (November 2010 to October 2012);
- four years ahead (November 2010 to October 2014); and
- six years ahead (November 2010 to October 2016).

These forecast periods are of interest because the most recent recession and its aftermath gave academics and policy advisors the motivation to rethink and reassess existing models. It also prompted them to look for ways to improve forecasts of economic time series data and to provide a better understanding of their dynamics and behaviour.

For forecasts of both the mean and the variance, the first set of forecasting models involves estimation using a recursive window. The first step in conducting this forecasting exercise is to estimate the parameters of all four models using an estimation window. For the first forecast period, this estimation window starts at the beginning of the sample for each variable and ends at observation R. For oil returns, the initial estimation window begins in January 1986 and ends in October 2010. For USD returns, the initial estimation window is January 1973 to October 2010. The parameter estimates from the estimation window are used to compute the one-step-ahead forecast for the next period, $R+1$. The estimation window is then expanded by one period such that the estimation window runs from observation 1 to $R+1$, and a one-step-ahead forecast is generated for period $R+2$. This process continues until H forecasts have been produced. Using a recursive window to generate forecasts is appropriate if the mean and volatility processes are stable, in which case it should produce unbiased forecasts.

Following Rapach and Strauss (2008), the second set of forecasting models is estimated using rolling windows to better control for unanticipated breaks in the conditional mean and variance. When rolling windows are used for estimation, the estimation window is modified so that it does not include as many observations produced by a data generating process that is unrelated to current data (Clark and McCracken, 2009). This can reduce bias in the estimates and forecasts and reduce the mean squared forecast error. Unfortunately, reducing the size of the estimation sample also raises the variance of the estimates, causing the mean squared forecast error to increase. This is what Clark and McCracken (2009) and Pesaran and Timmermann (2007) refer to as the *bias-variance trade-off*. Pesaran and Timmermann (2007) find that it is in fact optimal to include some pre-break data in the estimation window, and derive conditions to determine how much of the pre-break data should be included in the estimation window. Unfortunately, Pesaran and Timmermann's

(2007) study is applicable only to a linear regression model for forecasting the mean. The conditions for selecting the appropriate number of pre-break observations to include in the estimation window do not yet exist for time-varying volatility models such as GAS models; hence, this paper follows Rapach and Strauss (2008) in selecting rolling windows that are one-half and one-quarter the size of the initial estimation window under the recursive method.[21] As mentioned earlier in this section, two types of rolling window are used in this analysis – a rolling window equal to one-half of the initial estimation window, and a rolling window equal to one-quarter of the estimation window. They are referred to as the 0.5 (or $0.5R$) and 0.25 (or $0.25R$) rolling windows respectively.

The rolling window forecasts are generated as follows. In the case of the 0.5 rolling window, the parameters of all four models are first estimated using an estimation window size of $0.5R$, consisting of observations $0.5R+1$ to R. Then, the first one-step-ahead forecast is produced. Afterwards, the first observation is dropped and another is added such that the estimation window now ranges from $0.5R+2$ to $R+1$. The estimation window keeps sliding by one observation and the process repeats until H forecasts have been produced. As for the 0.25 rolling window case, the forecasts are generated in the same fashion, except that the initial estimation window begins at $0.75R+1$ and ends at R for both oil and USD returns.

4.5.1 Forecast evaluation strategies

The approach used to evaluate forecasts of the mean is different from that used for the variance. For mean forecasts, two loss functions, the mean square prediction error (MSPE) and the mean absolute prediction error (MAPE), are employed in order to determine which model produces the best forecast. They are defined as follows:

$$MSPE = \frac{1}{H}\sum_{h=1}^{H}(y_{T+h} - \widehat{y}_{T+h|T+h-1})^2, \tag{4.10}$$

and

$$MAPE = \frac{1}{H}\sum_{h=1}^{H}|y_{T+h} - \widehat{y}_{T+h|T+h-1}|, \tag{4.11}$$

[21] Rapach and Strauss (2008) point out that conditions for optimal estimation windows for GARCH models are not yet in existence. I am not aware of any papers related to conditions for optimal estimation windows. The closest paper in relation to time-varying volatility models is Clark and McCracken (2009) who derive optimal choices of estimation window size and weights to combine recursive and rolling forecasts for linear regression models in the presence of conditional heteroskedasticity.

where y_{T+h} is the actual value of the variable in period $T+h$ and $\hat{y}_{T+h|T+h-1}$ is the forecasted value for period $T+h$ based on information available in the previous period.

However, the mean forecasts do not depend directly on the time-varying volatility part of the models. Therefore, this paper goes a step further in assessing the out-of-sample performance of the models by establishing prediction intervals around the mean forecasts. To construct these intervals, this paper follows the methodology of Granger et al. (1989). First, the forecast errors are obtained for each model. Then, the errors are standardized using their respective predicted standard deviations. The standardized errors are sorted and then regressed against a constant using quantile regression methods to obtain estimates at the 10% and 90% quantiles. These interval forecasts are then evaluated using Christoffersen's (1998) conditional coverage test.

Christoffersen's (1998) test is used to determine whether the interval forecasts have good unconditional and conditional coverage. The null hypothesis for both tests is that the nominal (unconditional or conditional) coverage equals 80%. He uses a likelihood ratio (LR) test to determine whether the null hypothesis for either unconditional coverage or conditional coverage can be rejected. If the null hypothesis is rejected, then one can say that the interval forecasts do not have good unconditional/conditional coverage.

The evaluation of variance forecasts is more difficult because the true variance is unobservable. Hansen and Lunde (2006), Patton and Sheppard (2009) and Patton (2011) point out that the use of squared error terms as a proxy for the variance σ_t^2 is noisy, and this noise can affect the power of tests of forecast accuracy and the ranking of volatility and variance forecasts. Patton and Sheppard (2009) and Patton (2011) further point out that traditional loss functions such as the MSPE and MAPE are not robust to this noise. An alternative approach to evaluating variance forecasts is to use the forecasting regression

$$\tilde{\sigma}^2_{T+h|T+h-1} = a_0 + b_0 \widehat{\sigma}^2_{T+h|T+h-1} + \varepsilon_{T+h}. \tag{4.12}$$

where H is the number of forecast horizons, $\tilde{\sigma}^2_{T+h|T+h-1}$ is the conditionally unbiased variance proxy at time $T+h$ and $\widehat{\sigma}^2_{T+h|T+h-1}$ is the variance forecast for period T+h given information at $T+h$-1. If a_0 and b_0 are equal to 0 and 1 respectively, then the forecasts are unbiased. Furthermore, if R^2 is high, then the forecasts are accurate.

However, equation (4.12) suffers from an "errors-in-variables" problem when GARCH parameter estimates are used to construct $\widehat{\sigma}^2_{T+h|T+h-1}$, which causes $\hat{b_0}$ to have a downward bias. This is also likely to happen with GAS and other volatility models. Hence, the focus when evaluating the variance forecasting performance is rather on R^2 (Andersen and Bollerslev, 1998; Zivot, 2009). As mentioned by Andersen and Bollerslev (1998), when the squared return is used as a proxy, then R^2 tends to be too low. They recommend the use of the realized variance (RV) as a proxy.[22]

RV is defined as

$$RV^m_{t+h} = \sum_{i=1}^{m} \tilde{y}^2_{t+h,i} \tag{4.13}$$

where, in this paper, $\tilde{y}$ represents the demeaned returns and m is the number of observations per week. Five daily values of oil and USD returns are used to construct the weekly realized variance for each series. In other words, $m = 5$ in this paper. Following Patton (2011), this constructed RV constitutes the conditional variance proxy $\tilde{\sigma}^2_{T+h|T+h-1}$. Then, RV is regressed against the conditional variance forecast $\widehat{\sigma}^2_{T+h|T+h-1}$ for each competing volatility model. R^2 is then obtained from each regression and used to assess the out-of-sample performance of each model. The model with highest R^2 is selected as the model with the best variance forecasts.

Alternatively, this paper also uses the Diebold-Mariano-West (DMW) test of Diebold and Mariano (1995) and West (1996) to evaluate the forecasts. Instead of using the MSPE and MAPE loss functions, this chapter uses the"QLIKE" and "MSE" loss functions which belong to Patton's (2011) family of loss functions that are robust to imperfect volatility proxies:

$$L(\widetilde{\sigma^2_t}, h_t; b) = \begin{cases} \frac{1}{(b+1)(b+2)}(\widetilde{\sigma}_t^{2(b+2)} - \hat{\sigma}^{2(b+2)}) - \frac{1}{b+1}\hat{\sigma}^{2(b+2)}(\widetilde{\sigma}^2_t - \hat{\sigma}^2_t) & b \neq -1, -2 \\ \hat{\sigma}^2_t - \widetilde{\sigma}^2_t + \widetilde{\sigma}^2_t \log \frac{\widetilde{\sigma}^2_t}{\hat{\sigma}^2_t} & b = -1 \\ \frac{\widetilde{\sigma}^2_t}{\hat{\sigma}^2_t} - \widetilde{\sigma}^2_t \log \frac{\widetilde{\sigma}^2_t}{\hat{\sigma}^2_t} & b = -2 \end{cases} \tag{4.14}$$

where $\hat{\sigma}^2_t$ is the conditional variance forecast and $\widetilde{\sigma^2_t}$ is the realized variance. Setting b = -2 gives rise to the "QLIKE" loss function, while setting b = 0 yields the "MSE" loss function. These robust loss functions are then used to calculate the loss. See Appendix C.3. for full details about Patton's (2011) approach to evaluating variance forecasts.

4.5.2 Oil return forecast results

Table 4.12 displays the MSPE and MAPE of the oil return point forecasts for all four time-varying volatility models. The mean forecasts for the ARSV model actually correspond to

[22]According to Andersen and Bollerslev (1998), using the RV, in particular with intra-day frequency, would result in a higher R^2. Due to limited data availability, daily oil and USD returns are used to construct weekly forecasts.

an AR(1) model with a constant variance because the ARSV model is estimated using a two-step method in which the conditional mean and the conditional variance are estimated separately. Only the predictive intervals, that are discussed later, take into account the conditional variance of the ARSV model. Hence, in table 4.12, the mean forecasts generated by the GARCH, t-GARCH, and t-GAS models are actually compared to forecasts generated by an AR(1) model with a drift and constant variance.

The results show that for the two-year horizon, the MSPE loss function selects the GARCH model with the recursive window and the MAPE selects the GARCH model with the 0.5 rolling window. However, for the four-year horizon, the MSPE selects the GARCH model with 0.5 rolling window and the MAPE favours the t-GARCH model with a recursive window. On the other hand, the MSPE and MAPE for the six-year horizon both select the GARCH model with the 0.5 rolling window. This implies that for the long-run, it is beneficial to use the GARCH model and to use a method that accommodates breaks when forecasting the conditional mean. On the other hand, in the short and medium run, there is an inconsistency between the two loss functions with respect to the usefulness of accommodating breaks using rolling window methods.

Table 4.12: Oil mean forecast performance

Model	2 year horizon		4 year horizon		6 year horizon	
	MSPE	MAPE	MSPE	MAPE	MSPE	MAPE
Recursive t-GAS	11.384	2.545	7.756	2.070	13.549	2.674
Recursive GARCH	**11.255**	2.515	7.713	2.057	13.353	2.636
Recursive t-GARCH	11.358	2.525	7.718	**2.053**	13.309	2.633
Recursive ARSV	11.321	2.530	7.743	2.066	13.491	2.654
t-GAS with 0.5 RW	11.416	2.545	7.735	2.069	13.278	2.638
GARCH with 0.5 RW	11.316	**2.505**	**7.692**	2.057	**13.227**	**2.626**
t-GARCH with 0.5 RW	11.406	2.538	7.757	2.068	13.316	2.642
ARSV with 0.5 RW	11.319	2.527	7.734	2.064	13.409	2.646
t-GAS with 0.25 RW	11.282	2.512	7.712	2.064	13.270	2.635
GARCH with 0.25 RW	12.636	2.639	8.686	2.190	14.493	2.761
t-GARCH with 0.25 RW	11.486	2.552	7.767	2.064	13.290	2.634
ARSV with 0.25 RW	11.309	2.525	7.702	2.058	13.354	2.636

Notes: 0.5 RW denotes 0.5 rolling window, and 0.25 RW denotes 0.25 rolling window. The ARSV model is in this case an AR(1) model with drift and with a constant variance. Bold values denotes the forecasting model with the smallest loss based on criteria and horizon.

Overall, all the models produce poor mean forecasts across all horizons. This is likely because of the assumption that the conditional mean follows an AR(1) process. Prices of commodities such as oil have been found to exhibit jumps, mean-reversion, and time-varying trends, none of which have been incorporated in the simple AR(1) model used here.[23]

The conditional mean forecasts do not depend on the variance parameters of any forecasting models used in this paper, but the prediction intervals do. The variance parameters are used to construct 80% confidence bands for each forecast. Figures 4.10 to 4.12 display oil returns, recursive forecasts of mean oil returns, and 80% confidence bands for the 2 year, 4 year, and 6 year horizons respectively for the GARCH, t-GARCH, t-GAS, and ARSV models. The figures show that the mean forecasts do not come close to the actual values of oil returns. This confirms the findings in table 4.12 that show that all the models yield poor mean forecasts.

Figures 4.10 to 4.12 show that it is difficult to distinguish with the naked eye which model's forecast interval has the best conditional coverage. With the exception of the six-year horizon, the GARCH, t-GARCH, and t-GAS models appear to have similar coverage. The ARSV model have appears to have straighter forecast intervals than the other three models.

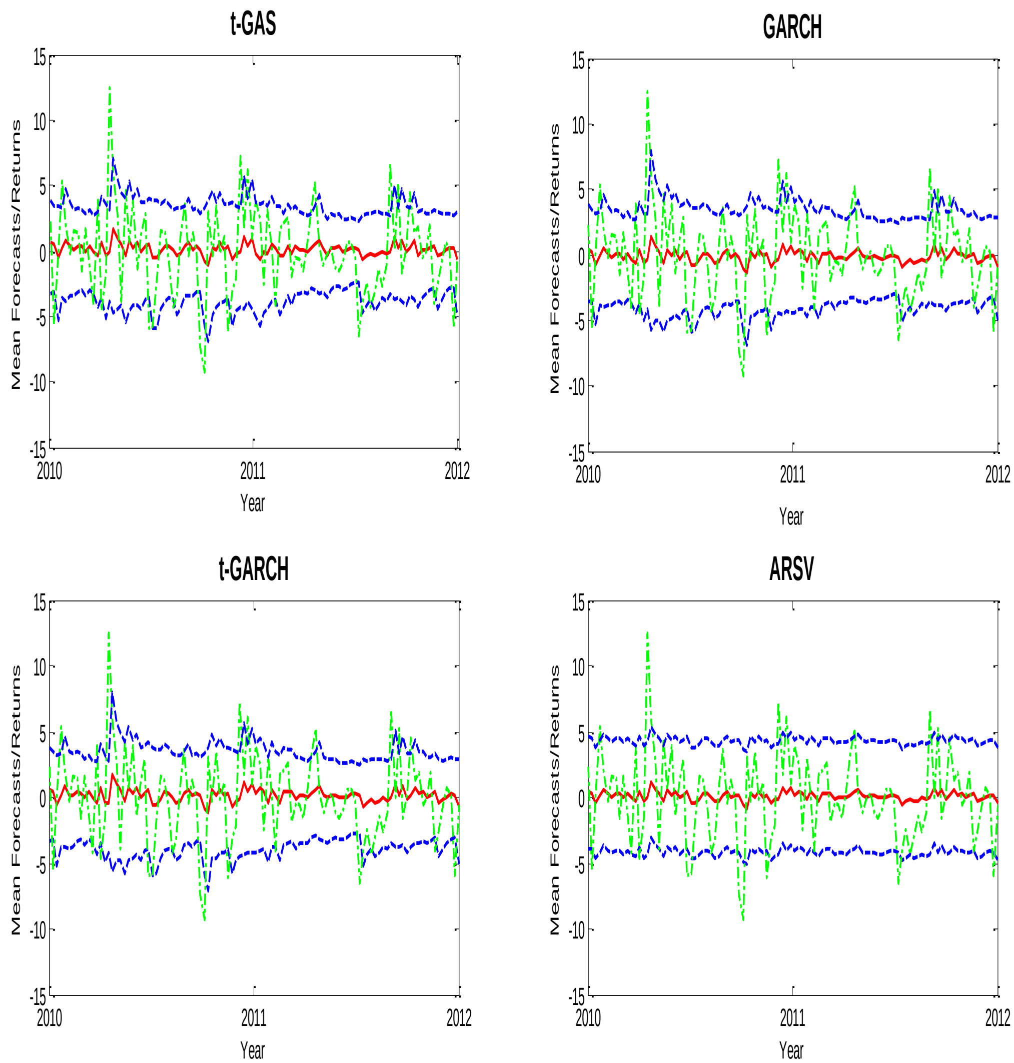
Figures 4.13 to 4.15 display the mean forecast for each model generated using the 0.25 rolling window.[24] The results indicate that once again the t-GAS(1,1), t-GARCH(1,1), and GARCH(1,1) models have slightly more coverage than the ARSV(1) model. For all the models, figures 4.13 and 4.14 also show that there is little difference between the coverage of the 0.25 rolling window forecasts and recursive forecasts of the conditional mean. It is only the ARSV(1) model for the six year horizon that displays some difference between the coverage for recursive forecasts and rolling forecasts.

Tests for conditional coverage are applied to each model to determine whether the actual coverage of interval forecasts is equal to the nominal coverage, which is 80%. Table 4.13 shows that all the models have good coverage, except for the t-GAS model with a recursive window at the six-year horizon and the GARCH model with a 0.25 rolling window at the two-year horizon. In the case of the t-GAS model with the recursive window at the six-year horizon, the null hypothesis that its intervals have the correct coverage can be rejected at the 10% level in favour of the alternative hypothesis. This implies that, in the

[23]See Schwartz and Smith (2000); Beck (2001); Bernard et al. (2008) for more information on modeling and forecasting commodity prices in the presence of jumps and mean-reversion.

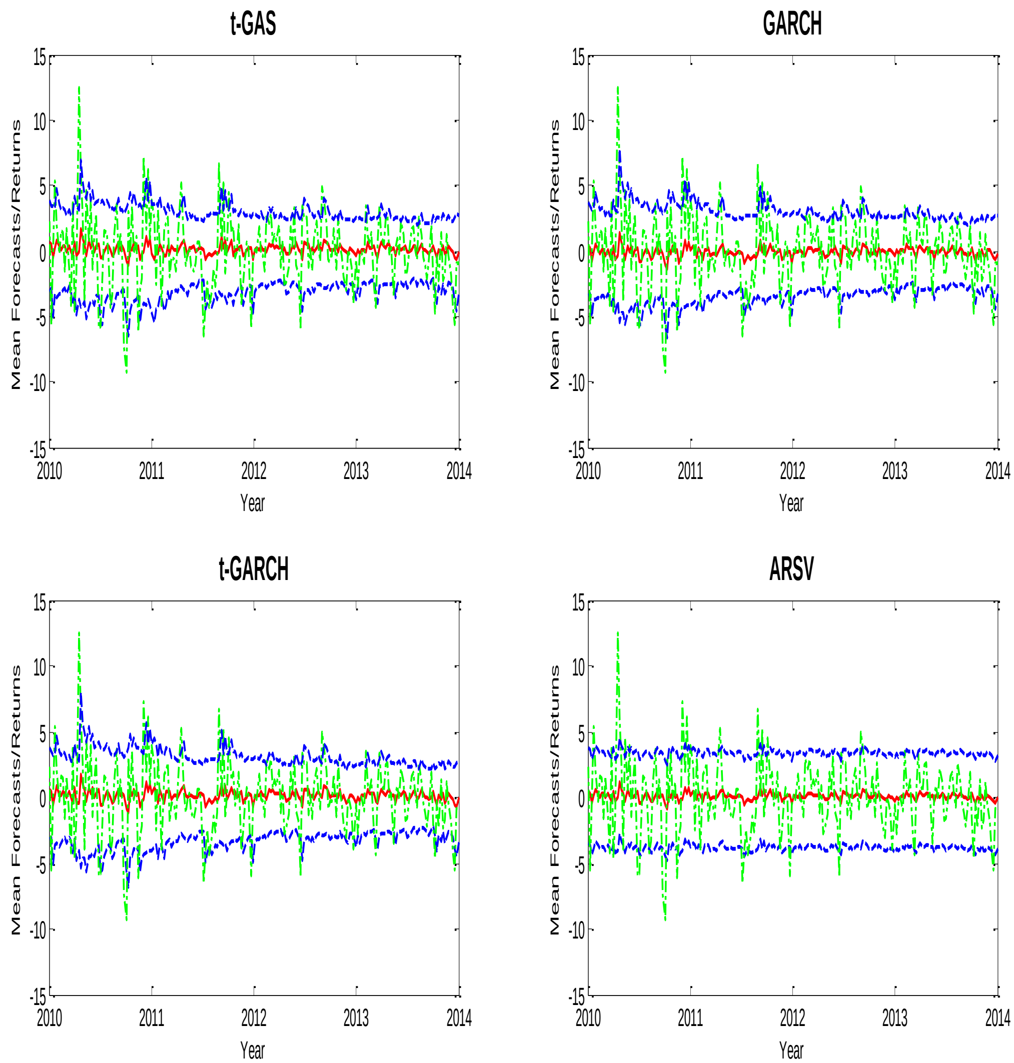
[24]I also created graphs for the 0.50 rolling window case, but they appeared to be identical to the recursive case.

Figure 4.10: Recursive oil returns mean forecast, 2 year horizon



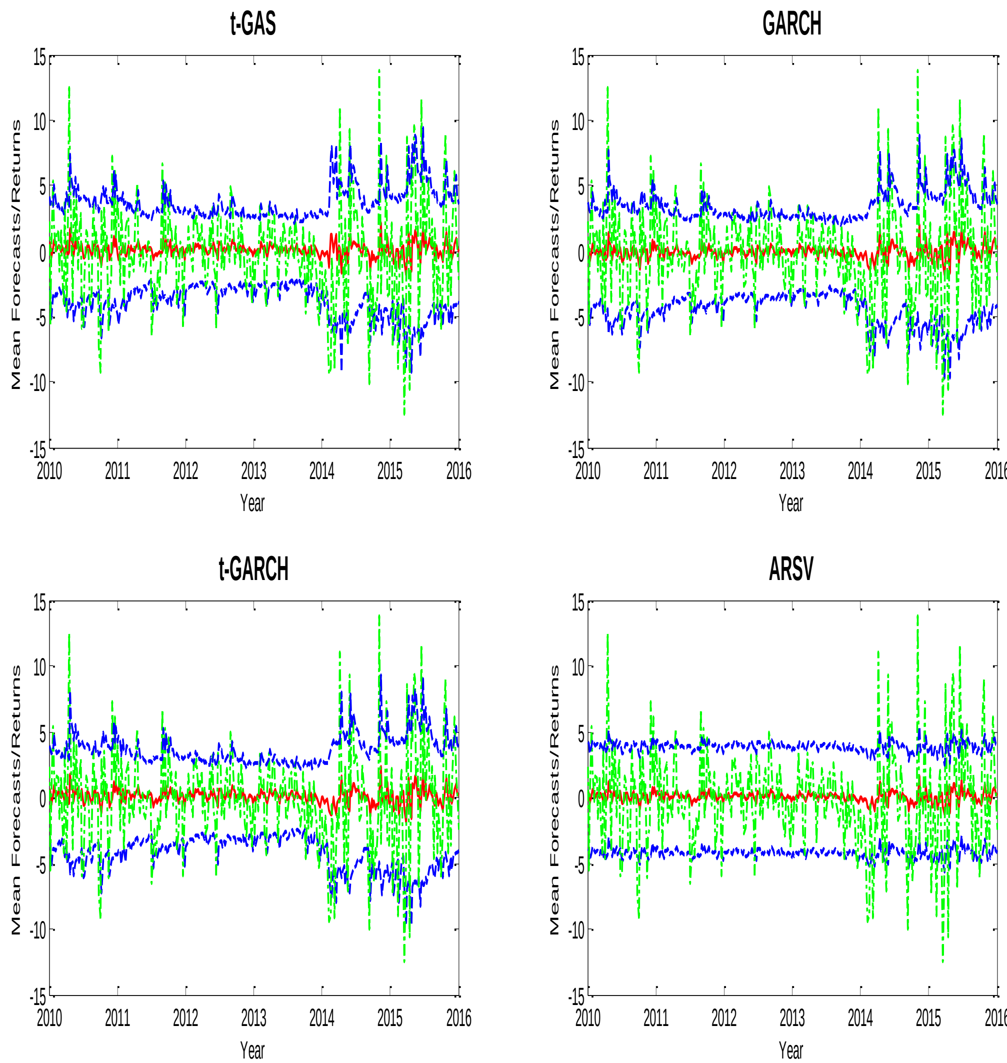
Note: The solid red line represents the mean forecasts, the green dash-dot line is the actual data and the blue dash lines represent the 80% forecast interval around the mean forecast.

Figure 4.11: Recursive oil returns mean forecast, 4 year horizon



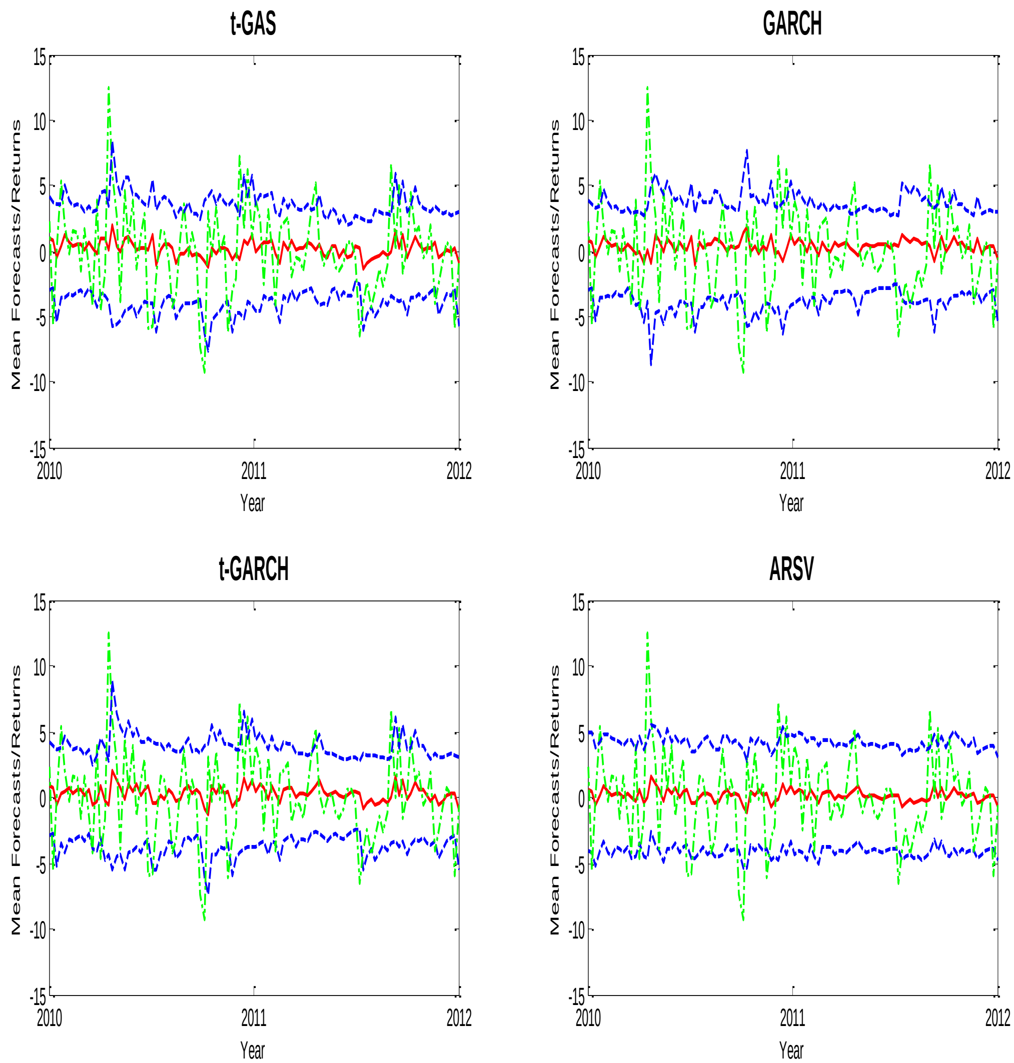
Note: The solid red line represents the mean forecasts, the green dash-dot line is the actual data and the blue dash lines represent the 80% forecast interval around the mean forecast.

Figure 4.12: Recursive oil returns mean forecast, 6 year horizon



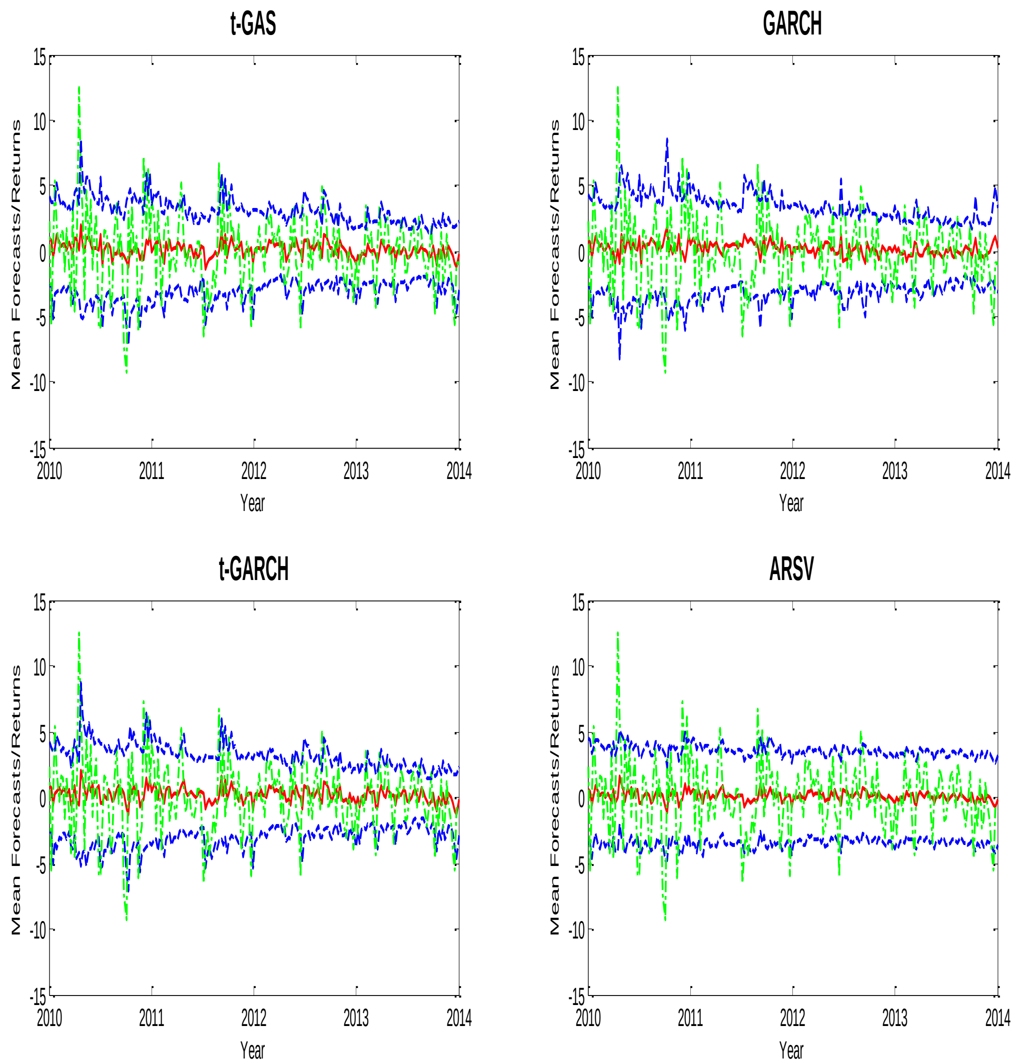
Note: The solid red line represents the mean forecasts, the green dash-dot line is the actual data and the blue dash lines represent the 80% forecast interval around the mean forecast.

Figure 4.13: 0.25 rolling window oil returns mean forecast, 2 year horizon



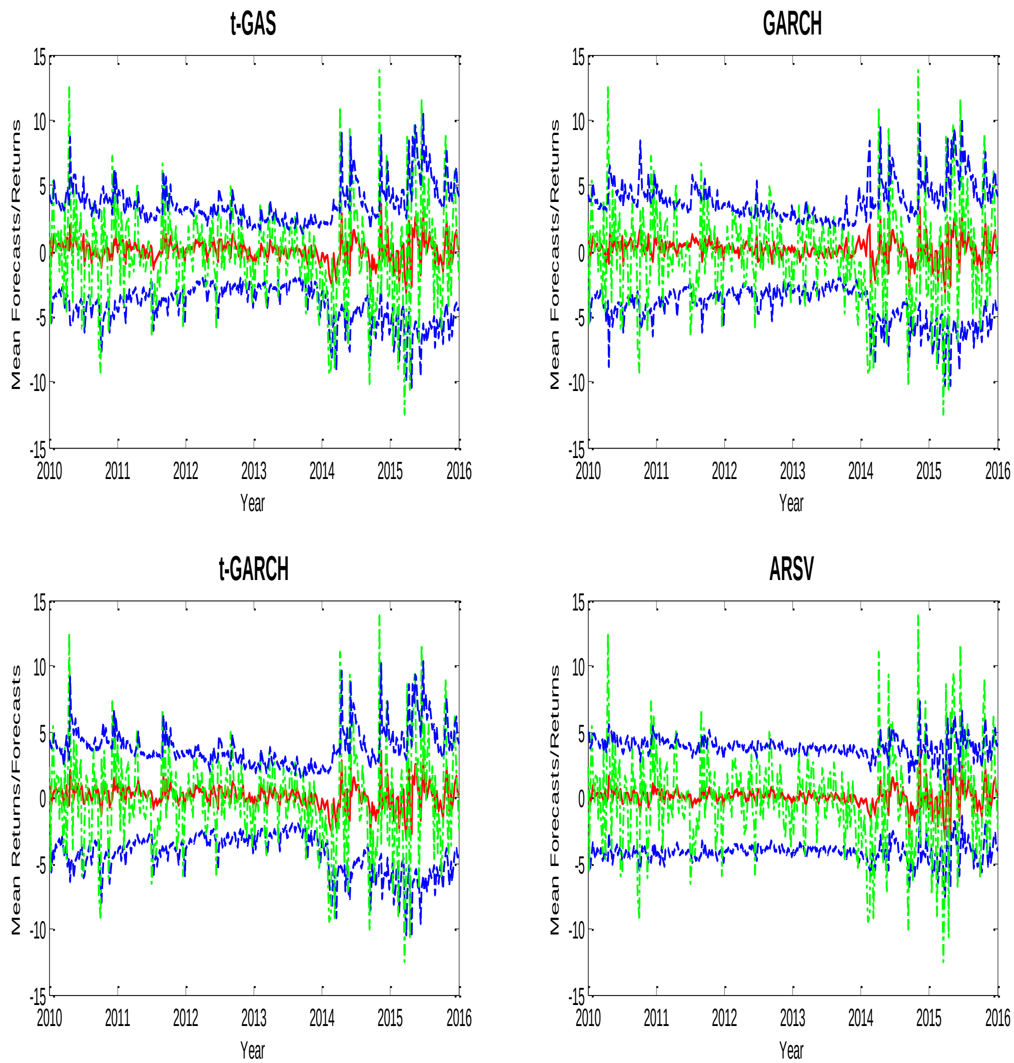
Note: The solid red line represents the mean forecasts, the green dash-dot line is the actual data and the blue dash lines represent the 80% forecast interval around the mean forecast.

Figure 4.14: 0.25 rolling window oil returns mean forecast, 4 year horizon



Note: The solid red line represents the mean forecasts, the green dash-dot line is the actual data and the blue dash lines represent the 80% forecast interval around the mean forecast.

Figure 4.15: 0.25 rolling window oil returns mean forecast, 6 year horizon



Note: The solid red line represents the mean forecasts, the green dash-dot line is the actual data and the blue dash lines represent the 80% forecast interval around the mean forecast.

long term, it is less favourable to ignore breaks in a t-GAS model in order to obtain good forecast coverage.

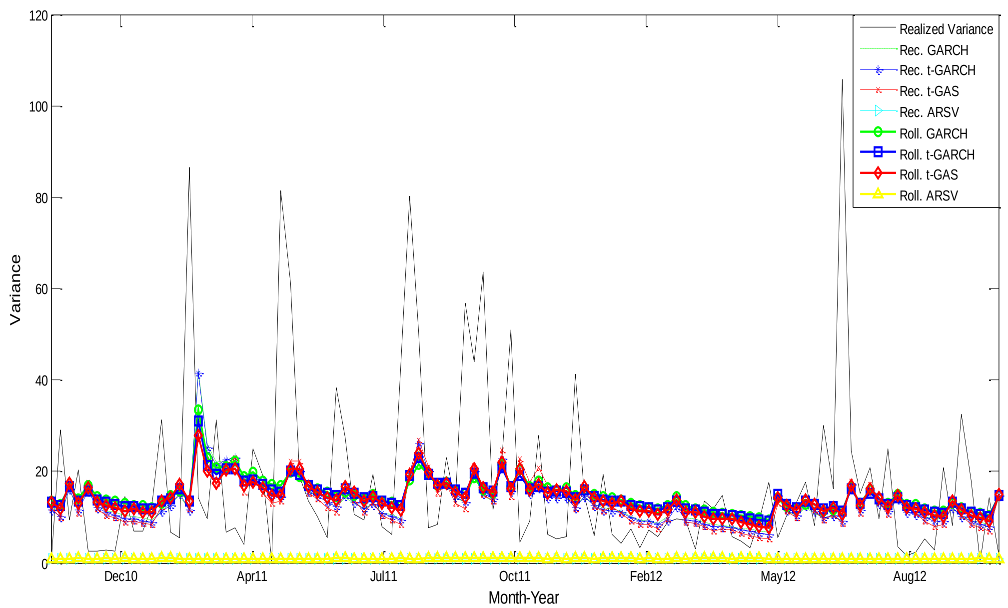
Table 4.13: P-values for Christoffersen's test of unconditional and conditional coverage for oil returns mean forecasts

		Recursive			0.5 Rolling Window			0.25 Rolling Window		
Model	Coverage	2 yr hor.	4 yr hor.	6 yr hor.	2 yr hor.	4 yr hor.	6 yr hor.	2 yr hor.	4 yr hor.	6 yr hor.
GARCH	Unconditional	0.29	0.65	0.22	0.56	0.43	0.27	0.08	0.92	0.42
	Conditional	0.30	0.89	0.46	0.68	0.48	0.47	0.21	0.97	0.72
t-GARCH	Unconditional	0.29	0.78	0.17	0.41	0.54	0.59	0.56	0.43	0.34
	Conditional	0.30	0.89	0.39	0.48	0.82	0.53	0.68	0.65	0.62
t-GAS	Unconditional	0.20	0.34	0.02	0.41	0.34	0.28	0.56	0.20	0.17
	Conditional	0.17	0.35	0.07	0.48	0.35	0.55	0.67	0.26	0.39
ARSV	Unconditional	0.92	0.94	0.98	0.92	0.92	0.91	0.92	0.67	0.86
	Conditional	0.72	0.72	0.45	0.98	0.82	0.20	0.91	0.70	0.36

Note: Null hypothesis is that actual coverage of confidence intervals equals nominal coverage of 80%.

In the case of the GARCH model with a 0.25 rolling window at the two-year horizon, the null hypothesis that the interval forecasts have good unconditional coverage rejected at the 10% level for a 0.25 rolling window forecast. This suggests that it is not beneficial to use a 0.25 rolling window to accommodate breaks in the GARCH model to produce good short-run interval forecasts for oil returns.

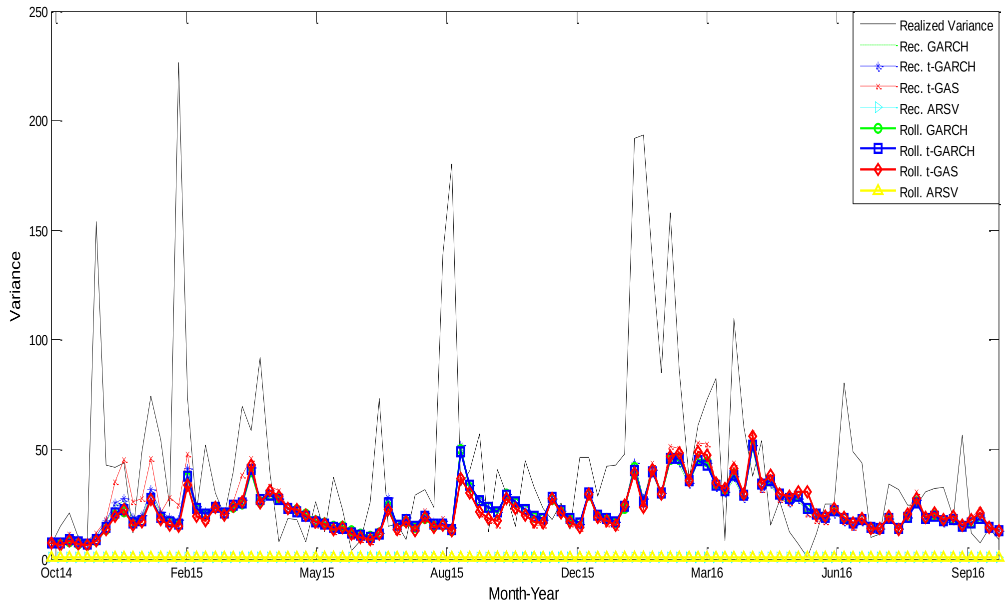
Figure 4.16: Oil returns variance forecasts, 2010 - 2012



Figures 4.16 and 4.17 display the first two years and last two years of the variance forecasts generated by all the models estimated using either the recursive or the 0.25 rolling window method. The results show that regardless of whether breaks are incorporated in the model or not, the performances of all the models except for the ARSV models, with forecasts appearing as straight lines near zero, appear identical. Also, for the ARSV model, similar variance forecasts are produced whether the recursive or rolling windows are used.

Table 4.14 summarizes the Mincer-Zarnowitz R^2 results for all twelve variance forecasting models and all three time horizons. The results show that in the short and medium run, it is beneficial to accommodate breaks using 0.5 rolling windows in order to produce the best forecasts. For the two-year horizon, the ARSV model with the 0.5 rolling window produces the best forecast, and t-GAS model with a 0.5 rolling window produces the best forecast for the four-year horizon. Meanwhile, the recursive t-GARCH model generates the

Figure 4.17: Oil returns variance forecasts, 2014 - 2016



best variance forecast for the six-year horizon. Since the t-GAS model produces the second best forecast, one can conclude that in the long-run, kurtosis matters for forecasting the variance.

Table 4.14: Mincer-Zarnowitz R^2 results for oil variance forecasts

	2 years			4 years			6 years		
	Rec.	0.5 RW	0.25 RW	Rec.	0.5 RW	0.25 RW	Rec.	0.5 RW	0.25 RW
t-GAS	0.023	0.035	0.023	0.097	**0.126**	0.100	0.272	0.235	0.208
GARCH	0.017	0.017	0.036	0.089	0.101	0.111	0.257	0.236	0.219
t-GARCH	0.024	0.033	0.019	0.100	0.123	0.097	**0.273**	0.242	0.214
ARSV	0.020	**0.131**	0.014	0.004	0.069	0.021	0.002	0.023	0.012

Note: This table contains the R^2 values for the Mincer-Zarnowitz regressions. Rec. denotes recursive forecast, 0.5 RW denotes 0.5 rolling window, and 0.25 RW denotes 0.25 rolling window. The bold value represents the forecasting model with the best forecast based on the Mincer-Zarnowitz R^2 values and horizon.

Tables 4.15 and 4.16 present the DMW statistics for pairwise comparisons of the oil returns variance forecasts, computed using Patton's (2011) "QLIKE" and "MSE" loss functions respectively. The results show that the t-GAS, GARCH, and t-GARCH models generally outperform the ARSV model in forecasting the variance. For the two-year horizon, the results for the ARSV model contradict the Mincer-Zarnowitz R^2 results in table 4.14. However, the results do not contradict the findings for the four-year and six-year horizons.

Table 4.15: DMW test statistics for oil variance forecasts based on "QLIKE" loss function

Model		2 years			4 years			6 years		
Model A	Model B	Rec.	0.5 Roll.	0.25 Roll.	Rec.	0.5 Roll.	0.25 Roll.	Rec.	0.5 Roll.	0.25 Roll.
t-GAS	GARCH	2.575**	0.139	1.505	1.613	-1.107	1.225	-0.541	0.863	2.470**
t-GAS	t-GARCH	2.553**	1.551	1.447	2.350**	0.380	1.169	0.546	2.000**	0.903
t-GAS	ARSV	-6.571**	-6.418**	-6.481**	-6.965**	-7.448**	-7.459**	-6.449**	-6.173**	-6.451**
GARCH	t-GARCH	0.192	1.794*	-1.418	0.657	-2.177**	-1.033	1.478	2.168**	-1.910*
GARCH	ARSV	-6.570**	-6.407**	-6.458**	-6.956**	-7.426**	-7.426**	-6.456**	-0.862	-6.450**
t-GARCH	ARSV	-6.569**	-6.411**	-6.476**	-6.961**	-7.433**	-7.454**	-6.452**	-6.172**	-6.450**

Note: This table contains the t-statistics from the Diebold-Mariano-West test of equal predictive accuracy for Models A and B. A positive (negative) value indicates Model A has a larger (smaller) loss function than Model B. * and ** indicate significance at the 10% and 5% levels. Rec. denotes recursive forecast, 0.5 Roll. denotes 0.5 rolling window forecast, and 0.25 Roll. denotes 0.25 rolling window forecast.

Table 4.16: DMW test statistics for oil variance forecasts based on "MSE" loss function

Model		2 years			4 years			6 years		
Model 1	Model 2	Rec.	0.5 Roll.	0.25 Roll.	Rec.	0.5 Roll.	0.25 Roll.	Rec.	0.5 Roll.	0.25 Roll.
t-GAS	GARCH	-0.108	-0.994	1.631	-0.124	-1.538	1.326	-1.857*	0.851	1.819*
t-GAS	t-GARCH	0.350	0.803	-0.029	0.482	0.327	0.028	-0.908	1.023	0.107
t-GAS	ARSV	-4.169**	-4.235**	-4.013**	-4.216***	-4.230**	-4.117**	-3.694**	-3.736**	-3.816**
GARCH	t-GARCH	1.423	1.891*	-1.754*	1.756*	1.891*	-1.392	2.120**	0.127	-2.373**
GARCH	ARSV	-4.145**	-4.163**	-3.962**	-4.193**	-4.138**	-4.011***	-3.733**	-0.851	-3.850***
t-GARCH	ARSV	-4.134**	-4.220**	-3.979**	-4.198**	-4.196**	-4.094**	-3.723**	-3.786**	-3.836**

Note: This table contains the t-statistics from the Diebold-Mariano-West test of equal predictive accuracy for Models A and B. A positive (negative) value indicates Model A has a larger(smaller) loss function than Model B. * and ** indicate significance at the 10% and 5% levels. Rec. denotes recursive forecast, 0.5 Roll. denotes 0.5 rolling window forecast, and 0.25 Roll. denotes 0.25 rolling window forecast.

Table 4.17: DMW test statistics for oil variance forecasts by forecasting method

	Model		GARCH		t-GARCH		t-GAS		ARSV	
	Model A	Model B	QLIKE	MSE	QLIKE	MSE	QLIKE	MSE	QLIKE	MSE
2 yr hor.	Rec.	0.5 Roll.	1.672*	1.550	1.983**	1.780*	2.555**	2.509**	1.199	1.499
	Rec.	0.25 Roll.	1.010	1.104	1.055	1.006	0.704	-0.109	-1.299	-0.693
	0.5 Roll.	0.25 Roll.	-0.589	-0.056	-1.050	-0.457	-2.425**	-1.516	-1.965**	-1.736*
4 yr hor.	Rec.	0.5 Roll.	0.062	1.011	0.424	1.347	1.031	1.980**	-1.806*	-1.003
	Rec.	0.25 Roll.	-0.098	0.824	-0.248	0.629	-0.868	-0.253	-4.663**	-3.595**
	0.5 Roll.	0.25 Roll.	-0.273	0.022	-1.095	-0.492	-2.253**	-1.429	-2.351**	-1.984**
6 yr hor.	Rec.	0.5 Roll.	-1.183	-0.820	-1.414	-1.678*	-1.215	-1.868*	-4.449**	-4.255**
	Rec.	0.25 Roll.	-0.417	-0.502	-1.048	-1.332	-1.958*	-2.007**	-5.738**	-5.423
	0.5 Roll.	0.25 Roll.	0.786	0.003	0.402	-0.047	-2.093**	-1.161	2.220**	1.812*

Note: This table contains the t-statistics from the Diebold-Mariano-West test of equal predictive accuracy for Models A and B. A positive (negative) value indicates Model A has a larger(smaller) loss function than Model B. * and ** indicate significance at the 10% and 5% levels. Rec. denotes recursive forecast, 0.5 Roll. denotes 0.5 rolling window forecast, and 0.25 Roll. denotes 0.25 rolling window forecast.

As part of the analysis of whether accommodating structural breaks improves forecasts, table 4.17 presents DMW test statistics that compare forecasts generated by the same volatility model, but using different forecasting methods. With the exception of the ARSV models, there are forecasting gains in using the 0.5 rolling window method to generate variance forecasts in the short run. For example, the test statistics for the two-year horizon indicate the t-GAS model generates better forecasts when a 0.5 rolling window is used than a recursive window. This result is significant at the 5% level regardless of the choice of loss function. For the same two-year horizon, the t-GARCH model also produces better forecasts using the 0.5 rolling window instead of a recursive window; this result is statistically significant at the 10% level. In the case of the GARCH model, only the QLIKE loss function implies that better forecasts are generated over the two-year horizon when a 0.5 rolling window is used instead of a recursive window. This QLIKE result is statistically significant at the 10% level. On the other hand, when the MSE loss function is used, there are no forecasting gains in applying the 0.5 rolling window method instead of the recursive window when using the GARCH model. In addition, there are no statistically significant forecast gains in using a 0.25 rolling window instead of the recursive window for any of the models. Thus, the results imply that for oil returns, using the 0.5 rolling window to accommodate structural breaks can improve variance forecasts in the short term, except for the ARSV model.

For longer horizons, the results in table 4.17 show that accommodating breaks generally does not produce gains in forecasting the variance. For example, the null hypothesis that the GARCH model with a recursive window and the GARCH model with a 0.5 rolling window have equal predictive ability cannot be rejected at the 10% level for four-year and six-year horizons, regardless of whether the QLIKE or MSE loss function is used. A similar result is produced when the GARCH model with a recursive window and the GARCH model with a 0.25 rolling window. In the case of the t-GARCH model, there are no forecasting gains in using either a 0.5 rolling window or a 0.25 rolling window to accommodate breaks at the four-year horizon and also at the six-year horizon for the QLIKE loss function case. However, if the MSE loss function is used, then the t-GARCH results show that there are no forecasting gains in using either a 0.5 rolling window or a 0.25 rolling window to accommodate breaks at the four-year horizon. Additionally, when the MSE loss function is used, a t-GARCH model with a recursive window produces better forecasts than a t-GARCH model with a 0.5 rolling window and forecasts just as good as the t-GARCH model with a 0.25 rolling window. The t-GAS results are mixed at the four-year horizon. On one hand, they show that a t-GAS model with a 0.5 rolling window produces better forecasts than a t-GAS model with a recursive window when the MSE loss function is used. On the other hand, a t-GAS model with a 0.5 rolling window produces forecasts just as well as a t-GAS model with a recursive forecast when the QLIKE function is used. For the six-year horizon, the t-GAS model with a recursive window produces better forecasts than the t-GAS model with a 0.5 rolling window based on the MSE loss function and performs equally well in forecasting based on the QLIKE loss function. Also, the t-GAS model with a recursive window produces better forecasts than the t-GAS model with a 0.25 rolling window regardless of the loss function used. The ARSV results show that with the exception of using the MSE loss function at the four-year horizon, using a recursive window produces better forecasts than using rolling windows. The results imply that in the long-run, there are no forecasting gains in using rolling windows to accommodate breaks in models of oil price volatility.

4.5.3 USD return forecasting results

As discussed in section 4.5.2, the ARSV model is estimated using a two-step method in which the conditional mean and conditional variance are estimated separately. In this case, the ARSV point forecasts are actually generated by a random walk with a drift and a constant variance. The random walk with a drift and constant variance serves as a benchmark in analyzing mean forecasts of USD returns. The mean forecasts of the GARCH, t-GARCH, and t-GAS models (whose conditional means are also in the form of a random walk) are compared with this benchmark model.

Table 4.18 contains the MSPE and MAPE of the point forecasts of USD returns. They generally show that there is no clear advantage for any model, which is consistent with the results of Meese and Rogoff (1983b). This is no surprise, since when the forecast of the mean is equal to a constant, it is unlikely that any model would have an advantage in forecasting the mean. This is the case regardless of the choice of forecasting method. The implication is that there are no forecast gains in using methods for accommodating for unknown breaks in the conditional mean, which is consistent with the finding of Burns and Moosa (2017) that structural breaks are not the cause of the Meese-Rogoff puzzle.

Figures 4.18 to 4.20 display actual USD returns, the recursive forecasts of the mean and 80% confidence bands for the two-year, four-year, and six-year horizons respectively. The figures show that the coverage of the mean forecast varies amongst the four time-varying volatility models. For the first 70 horizons, the t-GAS model generally has the widest and most volatile interval forecasts. This is followed by the t-GARCH and GARCH models. On the other hand, the interval forecasts are narrower for the t-GAS, GARCH, and t-GARCH models than that for the ARSV models between horizons 70 and 140 and from the 160th horizon onwards. Also, the ARSV has straight interval forecasts. This reflects the inability of the ARSV model to effectively capture changes in volatility.

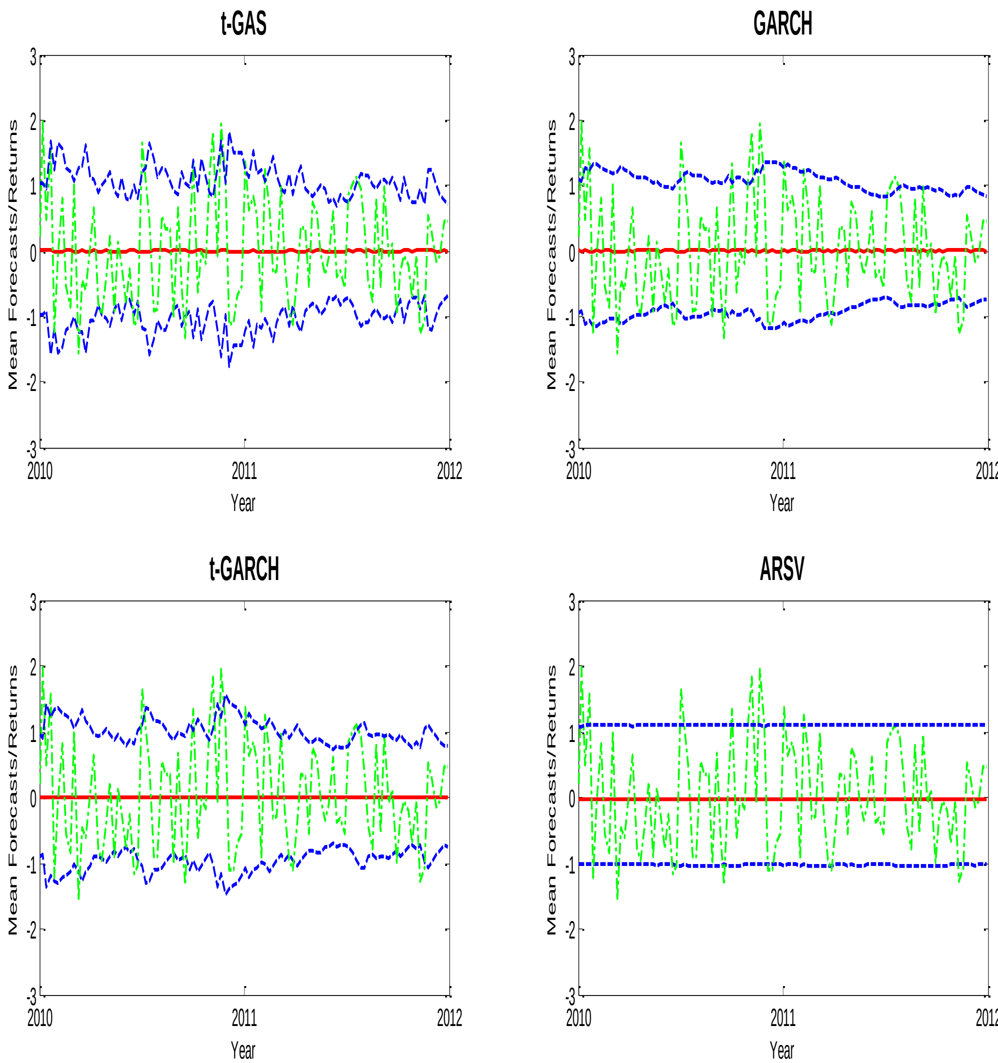
Table 4.18: USD mean forecast performance

Model	2 year horizon		4 year horizon		6 year horizon	
	MSPE	MAPE	MSPE	MAPE	MSPE	MAPE
Recursive t-GAS	0.663	0.682	0.528	0.589	0.615	0.631
Recursive GARCH	0.663	0.682	0.528	0.589	0.615	0.631
Recursive t-GARCH	0.663	0.682	0.528	0.589	0.615	0.631
Recursive ARSV	0.664	0.680	0.531	0.589	0.617	0.631
t-GAS with 0.5 RW	0.663	0.682	0.528	0.589	0.615	0.631
GARCH with 0.5 RW	0.663	0.682	0.528	0.589	0.615	0.631
t-GARCH with 0.5 RW	0.663	0.682	0.529	0.589	0.615	0.630
ARSV with 0.5 RW	0.664	0.680	0.531	0.589	0.616	0.631
t-GAS with 0.25 RW	0.663	0.682	0.528	0.589	0.614	0.630
GARCH with 0.25 RW	0.663	0.682	0.528	0.589	0.615	0.631
t-GARCH with 0.25 RW	0.663	0.682	0.528	0.589	0.615	0.631
ARSV with 0.25 RW	0.674	0.677	0.537	0.589	0.621	0.630

Note: 0.5 RW denotes 0.5 rolling window, and 0.25 RW denotes 0.25 rolling window. The ARSV model in this case is a random walk with drift, with constant variance. The bold value represents the forecasting model with the lowest forecast error based on the loss function and horizon.

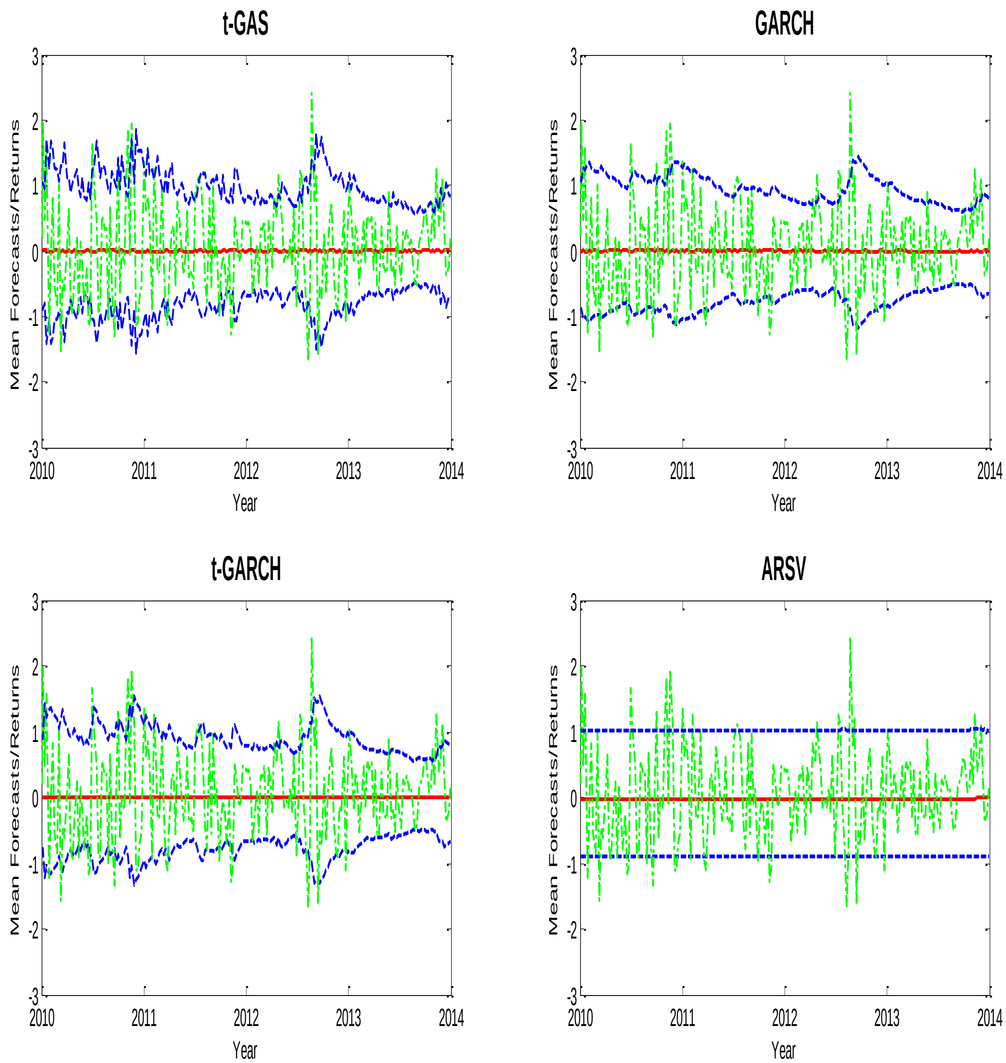
Figures 4.21 to 4.23 display the 0.25 rolling window mean forecasts and 80% forecast intervals for the two-year, four-year, and six-year horizons respectively. The figures indicate that the GARCH and t-GARCH models have similar coverage of the mean of USD returns forecasts. Additionally, the t-GAS model appear to have wider coverage of the mean forecasts than the GARCH, t-GARCH, and ARSV models for the first 50 horizons. After the 50th horizon, the t-GAS, GARCH, and t-GARCH appear to have similar coverage of the mean forecasts. After the 200th horizon, the coverage of the mean forecasts for the ARSV model is wider than that for the other three volatility models. In contrast to the oil returns case, the interval forecasts for all the models appear to be slightly different when the 0.25 rolling window forecasting method is used than when the recursive forecasting method is used. For example, the coverage of the mean forecasts for the ARSV model changes over the horizons when the 0.25 rolling method is used, as opposed to the coverage being flat when the recursive method is used. Another example, shown in figures 4.19 and 4.20, is that between the 150th and 250th horizons, the coverage of the mean forecasts is wider when the 0.25 rolling window is used than when the recursive window is used. The results suggest that accommodating unknown breaks can affect forecast uncertainty,

Figure 4.18: Recursive USD returns mean forecast, 2 year horizon



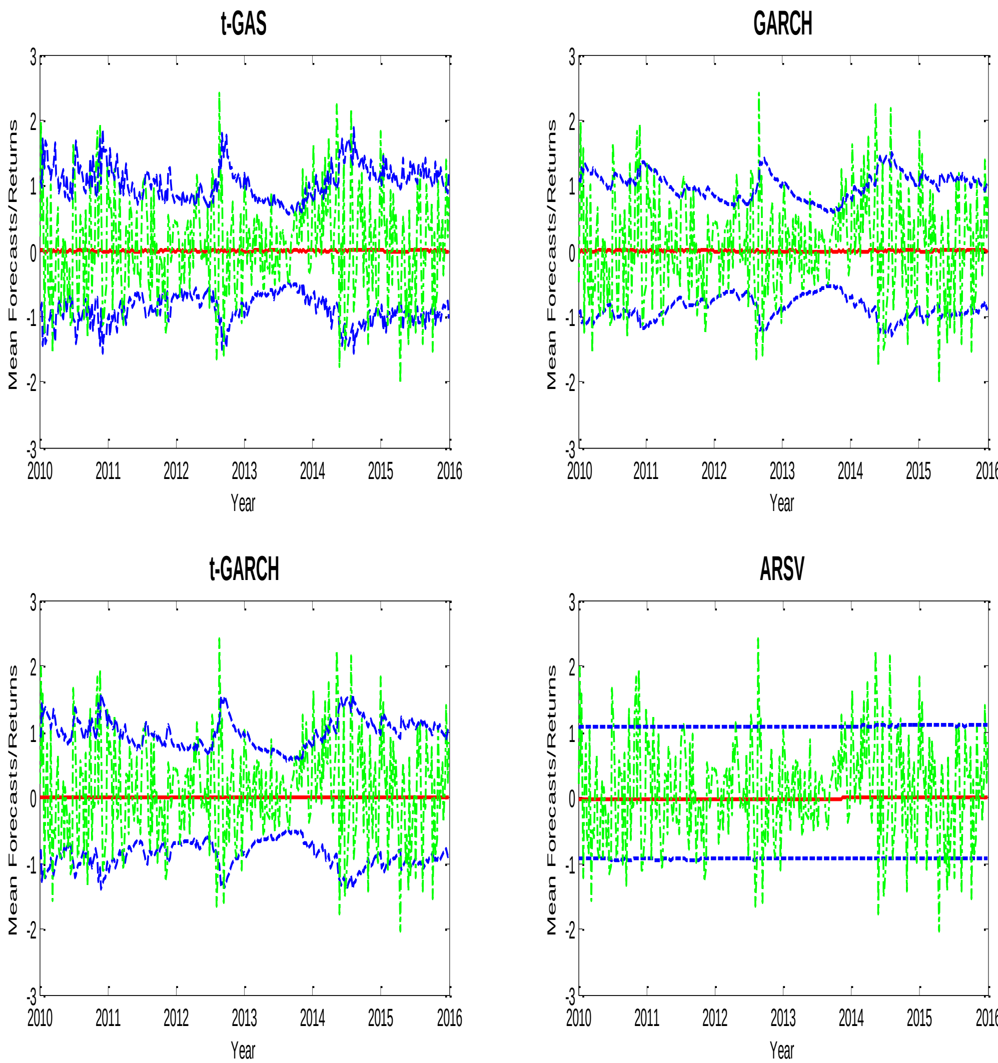
Note: The solid red line represents the mean forecasts, the green dash-dot line is the actual data and the blue dash lines represent the 80% forecast interval around the mean forecast.

Figure 4.19: Recursive USD returns mean forecast, 4 year horizon



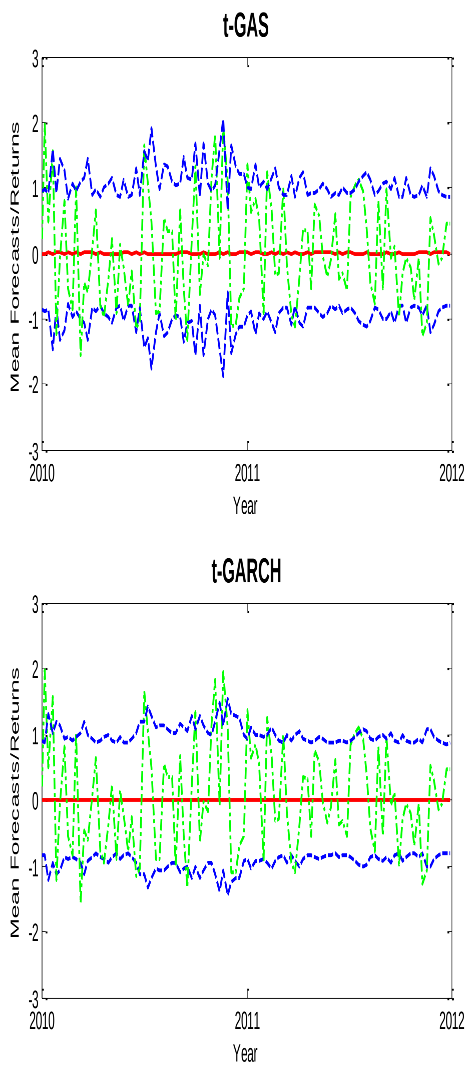
Note: The solid red line represents the mean forecasts, the green dash-dot line is the actual data and the blue dash lines represent the 80% forecast interval around the mean forecast.

Figure 4.20: Recursive USD returns mean forecast, 6 year horizon



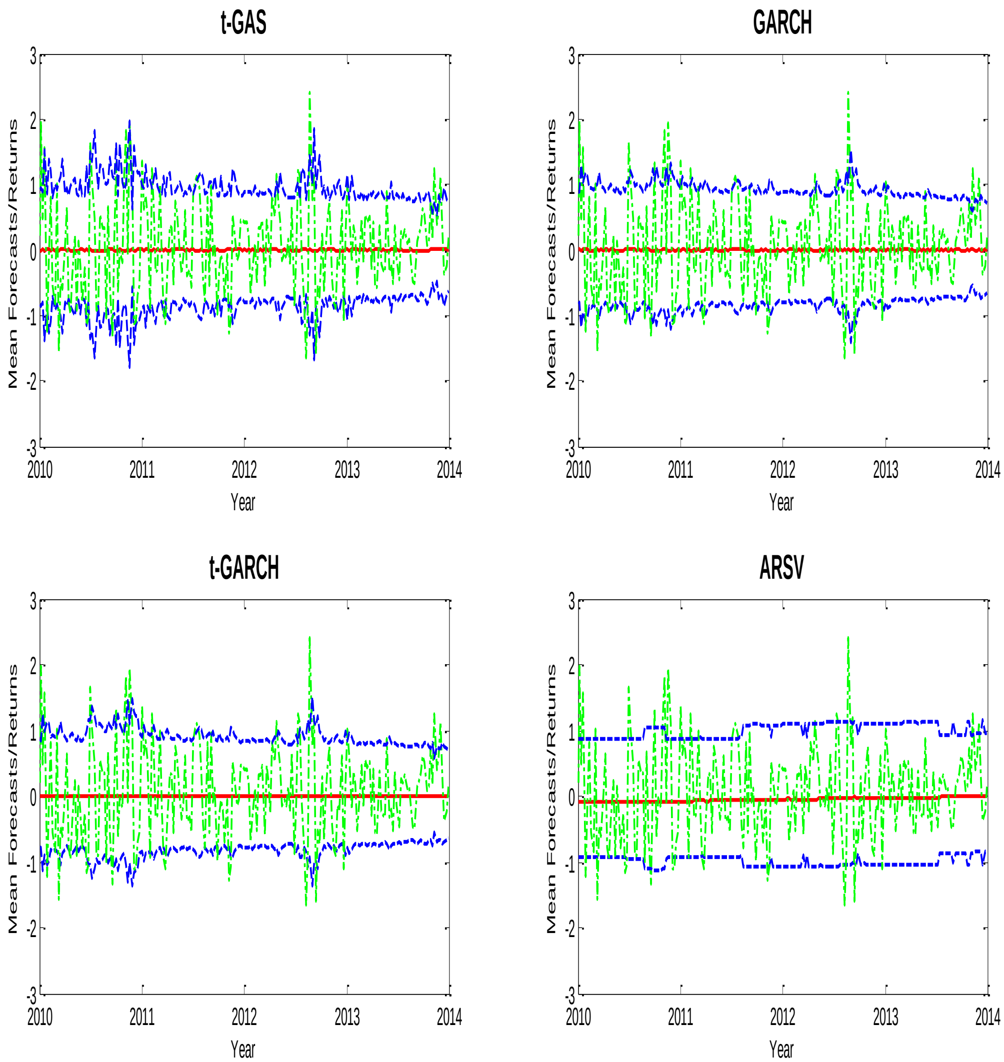
Note: The solid red line represents the mean forecasts, the green dash-dot line is the actual data and the blue dash lines represent the 80% forecast interval around the mean forecast.

Figure 4.21: 0.25 rolling window USD returns mean forecast, 2 year horizon



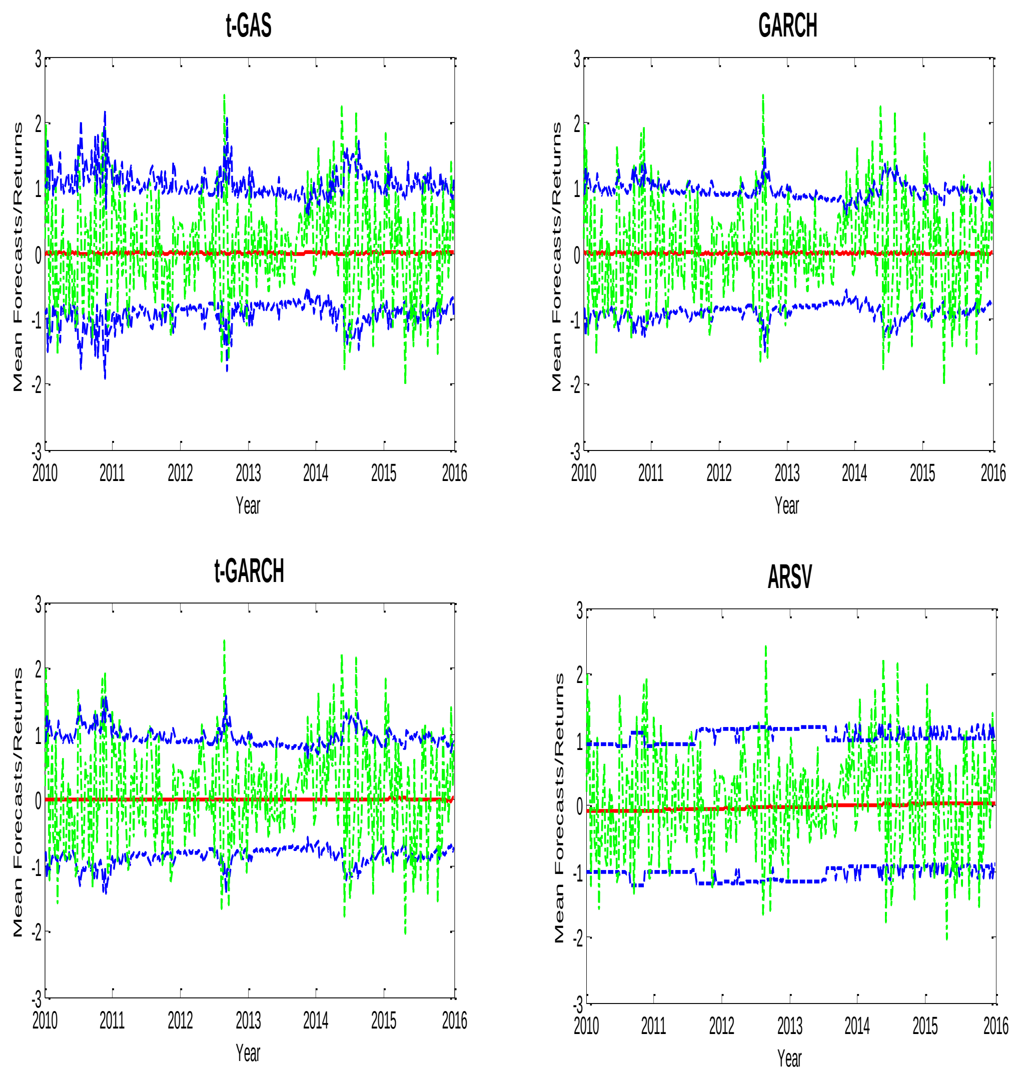
Note: The solid red line represents the mean forecasts, the green dash-dot line is the actual data and the blue dash lines represent the 80% forecast interval around the mean forecast.

Figure 4.22: 0.25 rolling window USD returns mean forecast, 4 year horizon



Note: The solid red line represents the mean forecasts, the green dash-dot line is the actual data and the blue dash lines represent the 80% forecast interval around the mean forecast.

Figure 4.23: 0.25 rolling window USD returns mean forecast, 6 year horizon



Note: The solid red line represents the mean forecasts, the green dash-dot line is the actual data and the blue dash lines represent the 80% forecast interval around the mean forecast.

implying that structural breaks may be relevant for interval forecasts of USD returns during the forecast period.

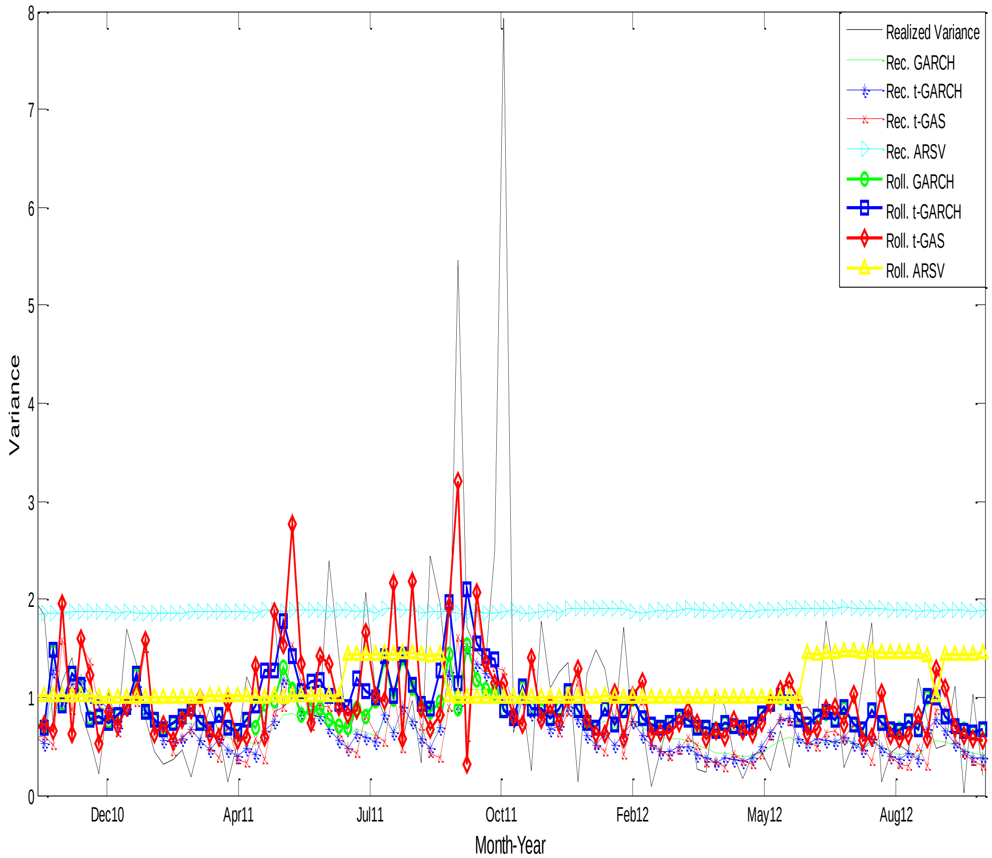
Table 4.19 displays test statistics for unconditional and conditional coverage. The results indicate that the null hypothesis that the interval forecasts have the correct coverage (80%) cannot be rejected in favour of the alternative for all the horizons and for all the t-GAS(1,1), GARCH(1,1), and t-GARCH(1,1) models. For the ARSV(1) model with a 0.5 rolling window, the null hypothesis that the model has the correct unconditional coverage cannot be rejected for four-year horizon. However, the null hypothesis that this same model has correct conditional coverage can be rejected in favour of the alternative hypothesis at the 5% level. This implies that using an ARSV model that accommodates breaks using a 0.5 rolling window does not provide good conditional coverage for USD mean forecasts.

Table 4.19: P-values for Christoffersen's test of unconditional and conditional coverage for USD return mean forecasts

		Recursive			0.5 Rolling Window			0.25 Rolling Window		
Model	Coverage	2 yr hor.	4 yr hor.	6 yr hor.	2 yr hor.	4 yr hor.	6 yr hor.	2 yr hor.	4 yr hor.	6 yr hor.
GARCH	Unconditional	0.29	0.34	0.28	0.41	0.26	0.42	0.20	0.26	0.13
	Conditional	0.34	0.61	0.54	0.53	0.50	0.69	0.42	0.18	0.26
t-GARCH	Unconditional	0.13	0.26	0.13	0.29	0.34	0.17	0.13	0.43	0.08
	Conditional	0.31	0.53	0.26	0.57	0.63	0.39	0.31	0.57	0.17
t-GAS	Unconditional	0.41	0.65	0.28	0.41	0.81	0.98	0.41	0.81	0.65
	Conditional	0.53	0.89	0.54	0.53	0.60	0.97	0.70	0.83	0.90
ARSV	Unconditional	0.92	0.92	0.98	0.92	0.78	0.80	0.88	0.44	0.98
	Conditional	0.10	0.03	0.45	0.60	0.02	0.61	0.18	0.00	0.28

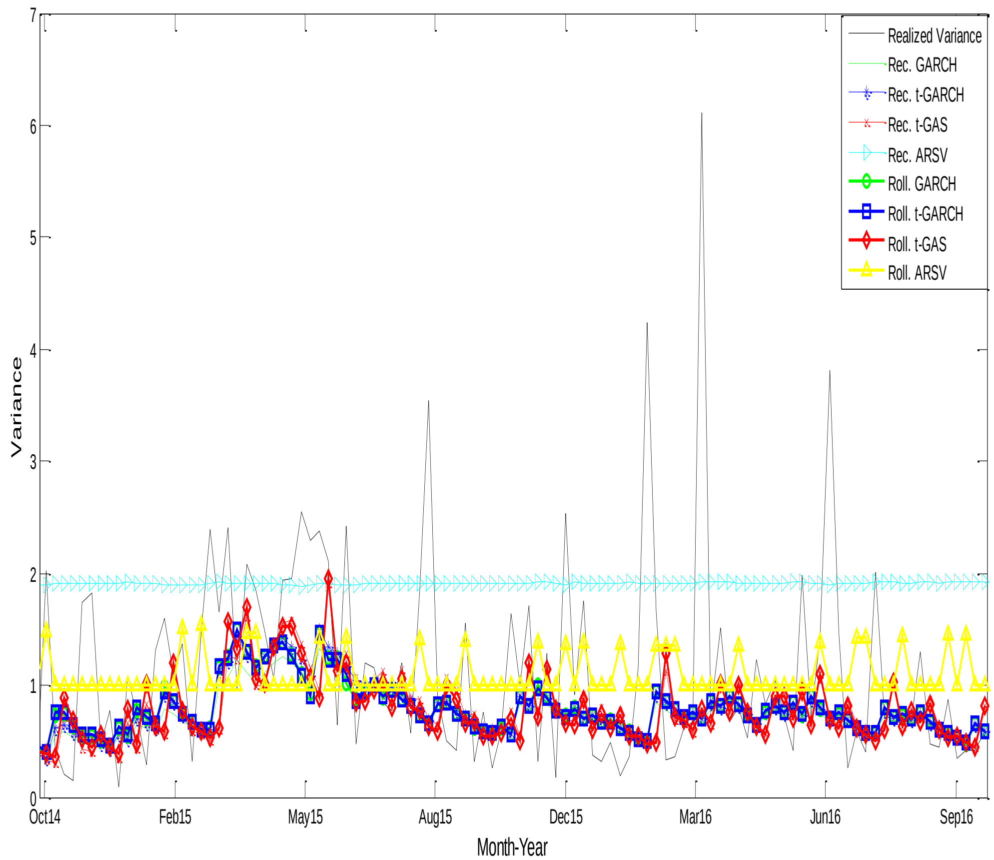
Note: Null hypothesis is that actual coverage of confidence intervals equals nominal coverage of 80%.

Figure 4.24: USD returns variance forecasts, 2010 - 2012



Figures 4.24 and 4.25 display the realized variance and the recursive and 0.25 rolling window variance forecasts generated by four models of volatility. As in the case of the variance of oil returns, no model appears to have a clear advantage in generating forecasts of the variance. Rather, the figures show that the ARSV models generate different forecasts for the variance of USD returns than the other models. Additionally, they show that the rolling window method produces more volatile forecasts than the recursive method, especially in the case of the t-GAS models. The figures also show that there are small differences between using the recursive and rolling forecast methods. However, the differences are most obvious for the ARSV model. These observations suggest that for the ARSV model, using forecast methods that accommodate unknown breaks may not improve

Figure 4.25: USD returns variance forecasts, 2014 - 2016



variance forecasts.

Table 4.20 presents the Mincer-Zarnowitz R^2s obtained after regressing the RV of USD returns on the variance forecasts for each forecasting model. The results indicate that for all forecast horizons, the t-GAS model combined with a 0.5 rolling window generates the best variance forecasts. Once again, the results indicate that the ARSV model generates the worst variance forecasts.

Table 4.20: Mincer-Zarnowitz R^2 results for USD variance forecast

	2 years			4 years			6 years		
	Rec.	0.5 RW	0.25 RW	Rec.	0.5 RW	0.25 RW	Rec.	0.5 RW	0.25 RW
t-GAS	0.106	**0.121**	0.112	0.172	**0.183**	0.175	0.147	**0.152**	0.112
GARCH	0.099	0.049	0.018	0.160	0.124	0.078	0.146	0.122	0.060
t-GARCH	0.068	0.045	0.042	0.135	0.118	0.107	0.131	0.120	0.070
ARSV	0.022	0.002	0.003	0.060	0.013	0.018	0.005	0.008	0.018

Note: This table contains the R^2 values for the Mincer-Zarnowitz regressions. Rec. denotes recursive forecast, 0.5 RW denotes 0.5 rolling window, and 0.25 RW denotes 0.25 rolling window. The bold value represents the forecasting model with the best variance forecast based on the Mincer-Zarnowitz R^2 values and horizon.

However, if forecast evaluation is based on Patton's (2011) loss functions and the DMW test, then the results in tables 4.21 and 4.22 show that the GARCH and t-GARCH models perform as well as the t-GAS model for all forecasting methods and forecast horizons. Also, the DMW statistics indicate that the ARSV model generally produces inferior variance forecasts of USD returns for the four-year and six-year horizons. The result that the t-GAS model with a 0.5 rolling window produces better forecasts than all the ARSV models in the medium and long run is consistent with the results in table 4.20. As for the comparison of the variance forecast performance of the t-GAS model with the 0.5 rolling window and the GARCH and t-GARCH models, it cannot be confirmed that the t-GAS model outperforms all the other models using the DMW test and Patton's (2011) loss functions.[25]

Table 4.23 presents DMW test statistics that compared the three different forecasting methods for each model. The results show that better forecasts can be generated for some volatility models if structural breaks are accommodated in the forecasting model in the short run. For example, the t-GARCH model with a 0.25 rolling window generates a better variance forecast than the t-GARCH model with either a 0.5 rolling window or a recursive window over the two-year horizon. The GARCH results in this paper are consistent with the findings of Rapach and Strauss (2008) whose forecast periods of 500 days which is almost one and a half years. They find that accommodating breaks in a GARCH model by using rolling windows is useful in forecasting exchange rate volatility. The ARSV model also generates better forecasts when structural breaks are accommodated in the forecasting model.

[25] However, the DMW test is a pairwise test and there are more general tests available such as Hansen's (2005) Superior Predictive Ability Test.

Table 4.21: DMW test statistics for USD variance forecasts based on "QLIKE" loss function

		2 years			4 years			6 years		
Model 1	Model 2	Rec.	0.5 Roll.	0.25 Roll.	Rec.	0.5 Roll.	0.25 Roll.	Rec.	0.5 Roll.	0.25 Roll.
t-GAS	GARCH	1.523	-0.629	0.157	0.691	-0.724	-0.591	0.789	-0.246	0.113
t-GAS	t-GARCH	0.611	-0.756	1.152	0.240	-0.888	0.179	0.803	-0.368	0.859
t-GAS	ARSV	-0.570	-0.520	-0.383	-4.207**	-4.021**	-4.530**	-3.220**	-2.477**	-2.011**
GARCH	t-GARCH	-2.252**	-1.504	1.767*	-1.055	-1.526	1.835*	-0.236	-1.026	1.872*
GARCH	ARSV	-1.287	-0.147	-0.609	-4.341**	-3.617**	-4.593**	-3.384**	-2.369**	-2.275**
t-GARCH	ARSV	-0.786	-0.068	-1.702*	-4.203**	-3.539**	-5.095**	-3.398**	-2.333**	-2.553**

Note: This table contains the t-statistics from the Diebold-Mariano-West test of equal predictive accuracy for Models A and B. A positive (negative) value indicates Model A has a larger (smaller) loss function than Model B. * and ** indicate significance at the 10% and 5% levels. Rec. denotes recursive forecast, 0.5 Roll. denotes 0.5 rolling window forecast, and 0.25 Roll. denotes 0.25 rolling window forecast.

Table 4.22: DMW test statistics for USD variance forecasts based on "MSE" loss function

		2 years			4 years			6 years		
Model 1	Model 2	Rec.	0.5 Roll.	0.25 Roll.	Rec.	0.5 Roll.	0.25 Roll.	Rec.	0.5 Roll.	0.25 Roll.
t-GAS	GARCH	-0.571	-1.253	-0.690	-0.692	-1.292	-0.815	-0.983	-1.173	-0.802
t-GAS	t-GARCH	-0.995	-1.295	-0.500	-1.068	-1.341	-0.659	-0.897	-1.189	-0.649
t-GAS	ARSV	-3.633**	-2.198**	-1.183	-7.799**	-6.096**	-4.685**	-6.927**	-4.454**	-3.394**
GARCH	t-GARCH	-1.124	-1.349	1.480	-0.815	-1.406	1.405	0.616	-1.000	1.378
GARCH	ARSV	-2.831**	-1.387	0.690	-6.934**	-5.155**	-4.591**	-6.258**	-3.931**	-3.179**
t-GARCH	ARSV	-2.751**	-1.387	-0.987	-6.907**	-5.081**	-4.922**	-6.378**	-3.897**	-3.393**

Note: This table contains the t-statistics from the Diebold-Mariano-West test of equal predictive accuracy for Models A and B. A positive (negative) value indicates Model A has a larger (smaller) loss function than Model B. * and ** indicate significance at the 10% and 5% levels. Rec. denotes recursive forecast, 0.5 Roll. denotes 0.5 rolling window forecast, and 0.25 Roll. denotes 0.25 rolling window forecast.

Table 4.23: DMW test statistics for USD variance forecasts by forecasting method

	Model		GARCH		t-GARCH		t-GAS		ARSV	
	Model A	Model B	QLIKE	MSE	QLIKE	MSE	QLIKE	MSE	QLIKE	MSE
2 year horizon	Rec.	0.5 Roll.	1.268	-1.201	-0.128	-0.809	2.296**	-0.376	5.639**	5.899**
	Rec.	0.25 Roll.	1.537	0.718	2.956**	2.135**	2.308**	0.783	2.375**	3.209**
	0.5 Roll.	0.25 Roll.	2.774**	2.304**	2.814**	2.200**	1.615	0.764	1.589	2.063**
4 year horizon	Rec.	0.5 Roll.	-0.571	-0.921	-0.157	-0.662	1.678*	-0.289	11.132**	11.362**
	Rec.	0.25 Roll.	-0.712	-0.621	-0.046	0.456	0.003	0.364	5.908**	7.571**
	0.5 Roll.	0.25 Roll.	-0.527	0.175	0.032	0.823	-0.664	0.388	4.130**	5.106**
6 year horizon	Rec.	0.5 Roll.	-0.654	-1.154	-0.647	-1.449	0.975	-1.116	7.859**	9.909**
	Rec.	0.25 Roll.	-0.475	-0.198	-0.056	0.206	-0.012	0.289	5.757**	7.487**
	0.5 Roll.	0.25 Roll.	-0.210	0.766	0.275	1.173	-0.449	0.615	3.495**	4.342**

Note: This table contains the t-statistics from the Diebold-Mariano-West test of equal predictive accuracy for Models A and B. A positive (negative) value indicates Model A has a larger(smaller) loss function than Model B. * and ** indicate significance at the 10% and 5% levels. Rec. denotes recursive forecast, 0.5 Roll. denotes 0.5 rolling window forecast, and 0.25 Roll. denotes 0.25 rolling window forecast.

In the case of the four-year horizon, the t-GAS results indicate that a t-GAS model with a 0.5 rolling window outperforms a t-GAS model with a recursive window in generating variance forecast, but only using the QLIKE loss function. The DMW statistics based on the MSE loss function indicate that there is no difference in out-of-sample performance between the recursive method and the rolling window methods. This suggests that in the medium run, accommodating breaks may or may not produce better USD variance forecasts. As for the GARCH model, the results indicate that there is also no difference in out-of-sample performance when either the rolling window or recursive method is used. They only indicate that the 0.25 rolling window outperforms the 0.5 rolling window. This implies that in the medium run, it is not always useful to accommodate breaks when forecasting the variance of USD returns.

Except for the ARSV model, accommodating breaks does not produce any forecast gains for longer horizons. The results in table 4.23 once again show that there is no significant difference in out-of-sample performance for the variance of USD returns if the rolling window or recursive window is used for the t-GAS, GARCH, and t-GARCH models. In contrast, the ARSV model produces better out-of-sample forecasts when the rolling window methods are used than when the recursive method is used. With that said it is worth noting that the ARSV models generally produce inferior forecasts than the GARCH, t-GARCH, and t-GAS models regardless of the forecasting method used. This implies that in the long run, it is generally not useful to accommodate for breaks when forecasting the variance.

4.6 Conclusion

This paper assesses the empirical relevance of structural breaks in GAS models – GARCH and t-GAS – of oil and USD returns relative to non-GAS models – t-GARCH and ARSV. First, in-sample performance is assessed by estimating the t-GAS, GARCH, t-GARCH, and ARSV models for the full sample and then breaking the sample into smaller subsamples to check for the existence of structural breaks. Dufour and Torres's (1998) union-intersection test is applied to all the estimated parameters in each model to determine whether the changes in the parameters are statistically significant. In all cases, the model selection criteria used are AIC and BIC. The estimated unconditional volatility is also compared to the sample standard deviation. Afterwards, the out-of-sample performance of the time-varying volatility models with respect to their ability to forecast the mean and the variance is compared. For the analysis, the four time-varying volatility models are combined with three forecasting methods – recursive window, 0.5 rolling window, and 0.25 rolling window

– to make 12 forecasting models. Recursive windows are frequently applied with models of stable processes. The 0.5 and 0.25 rolling windows are believed to better accommodate breaks than the recursive window.

This study produces several findings. First, structural breaks are empirically relevant in GAS and non-GAS models of USD volatilities, but not in those of oil volatilities. For the USD returns, the changes in the parameters of the conditional variance are statistically significant across sub-periods. The GARCH results are consistent with those of Rapach and Strauss (2008). As for the oil returns, the changes in parameters are not statistically significant across sub-periods. This result is consistent with the results of Arouri et al. (2012) since they found no structural breaks in GARCH models of crude oil prices.

Second, the ARSV model generally has the worst in-sample and out-of sample performances. The ARSV model consistently produced the highest AIC and BIC values, indicating that it has the worst model fit. Also, in terms of forecasting, both Mincer-Zarnowitz regressions and the Diebold-Mariano-West tests indicate that variance forecasts of the ARSV model are outperformed by the forecasts of the t-GAS, GARCH, and t-GARCH models. The ARSV result is inconsistent with the findings of Chan and Grant (2016) and Clark and Ravazzolo (2015), those who use Bayesian methods to estimate the ARSV model. Thus, it is possible that the problem may lie with the estimator. Hence, other estimation methods should be considered, such as that of Dufour and Valery (2006).

Third, with respect to oil returns forecasting, GARCH models that accommodate breaks perform the best in the long run. In the short and medium run, the GARCH model produces the best forecasts, but there is a disconnect between mean-squared error and mean absolute error values with respect to whether using rolling windows to accommodate breaks produces better forecasts than using recursive windows. As for forecasts of the variance of oil returns, incorporating structural breaks into the model produces better forecasts in the short and medium run. This result is consistent with those of Arouri et al. (2012) for GARCH models; they find that using a 0.5 rolling window and daily data produces better variance forecasts for crude oil returns. However, in the long run, this paper finds that kurtosis is an important feature, since the t-GARCH model produces better variance forecasts according to the Mincer-Zarnowitz regression and DMW tests for the six-year horizon case.

Fourth, all models are considered to be equal when it comes to forecasting the mean of USD returns. This is not surprising since it has been found by Meese and Rogoff (1983b) that it is difficult for to outperform the random walk model in terms of forecasting exchange rates. Accommodating breaks in the models does not improve the forecasts relative to the random walk model which is consistent with Burns and Moosa (2017). However, the t-GAS model estimated using a 0.5 rolling window produces the best variance forecast for all horizons, suggesting that it is useful to accommodate breaks in forecasting the USD variance. This finding is consistent with Patton's inferential approach which implies that t-GAS models with breaks produce better forecasts than t-GAS models that do not incorporate breaks in the short and medium run, but not in the long run.

Finally, accommodating structural breaks in models is helpful in forecasting the variance only in the short run. For the two-year horizon, models that accommodate breaks generally produce better variance forecasts than models that do not accommodate breaks. In the case of the medium-run, the gains are mixed but as for the long-run, there is no forecast gain for models that accommodate breaks. This implies that recursive methods should be used in forecasting oil and USD variances for very long horizons.

This study focused mainly on the empirical relevance of structural breaks in models of volatility. However, another important feature found in financial data is the presence of long memory. This feature has been incorporated in the study of Arouri et al. (2012) for oil volatilities, but not in this study. Future research could include the empirical relevance of structural breaks and long memory in GAS models. The role of asymmetry is not considered in this analysis; however, Mohammadi and Su (2010) find that using asymmetric models such as EGARCH can improve forecasts for crude oil returns and volatility. Extending the analysis of this study to include asymmetric GARCH and GAS models could be considered for future research.

The conditional mean specification in this study excludes explanatory variables. There are some studies suggesting that adding these variables improve the mean forecasts. Ferraro et al. (2015) find that oil prices have the ability to predict the Canadian-US Dollar nominal exchange rate only at daily frequencies, and Berg et al. (2016) find that energy and non-energy commodity prices and USD multilateral factor also have predictive ability for the Canadian-US Dollar nominal exchange rate. Hence, adding energy and non-energy commodity prices to the conditional mean equation of GAS models of exchange rate volatilities in assessing the empirical relevance of structural breaks could also be considered for future research.[26]

For the mean forecasts, alternative criteria such as directional accuracy, profitability, and the adjusted root mean square error are not considered in this chapter. According to Moosa and Burns (2014), models such as the time-varying parameter (TVP) models can outperform the random walk model in forecasting exchange rates based on these alternative criteria. A possible direction in future research could be to use a GAS model with the mean

[26]The Taylor rule and net foreign assets are also possible predictors according to a survey in exchange rate predictability by Rossi (2013).

as the time-varying parameter instead of the volatility for comparison with the TVP and random-walk models in forecasting exchange rates could also be a possible future research direction.

References

Adjemian, S., H. Bastani, M. Juillard, F. Karame, F. Mihoubi, G. Perendia, J. Pfeifer, M. Ratto, and S. Villemot (2011). Dynare: Reference Manual, Version 4. Working paper no.1, CEPREMAP, Paris, France.

Alquist, R., L. Kilian, and R. J. Vigfusson (2013). Forecasting the price of oil. In G. Elliott and A. Timmermann (Eds.), *Handbook of Economic Forecasting*, Volume 2, Chapter 8, pp. 427–507. Amsterdam: Elvesier.

Andersen, T. G. and T. Bollerslev (1998). Answering the skeptics: Yes, standard volatility models do provide accurate forecasts. *Intenational Economic Review 39*(4), 885–905.

Andreou, E. and E. Ghysels (2009). Structural breaks in financial time series. In T. G. Andersen, R. A. Davis, J.-P. Kreiß, and T. Mikosch (Eds.), *Handbook of Financial Time Series*, pp. 839–870. Berlin: Springer.

Ardia, D. (2009). Bayesian estimation of a Markov-switching threshold asymmetric GARCH model with Student-t innovations. *The Econometrics Journal 12*(1), 105–126.

Arouri, M. E. H., A. Lahiani, A. Lévy, and D. K. Nguyen (2012). Forecasting the conditional volatility of oil spot and futures prices with structural breaks and long memory models. *Energy Economics 34*(1), 283–293.

Babikir, A., R. Gupta, C. Mwabutwa, and E. Owusu-Sekyere (2012). Structural breaks and GARCH models of stock return volatility: The case of South Africa. *Economic Modelling 29*, 2435–2443.

Barnichon, R. (2010, November). Productivity and unemployment over the business cycle. *Journal of Monetary Economics 57*(8), 1013–1025.

Bataa, E., D. R. Osborn, M. Sensier, and D. van Dijk (2014). Identifying changes in mean, seasonality, persistence and volatility for G7 and Euro Area inflation. *Oxford Bulletin of Economics and Statistics 76*(3), 360–388.

Baumeister, C. and L. Kilian (2016). Forty years of oil price fluctuations: Why the price of oil may still surprise us. *Journal of Economic Perspectives 30*(1), 139–160.

Beck, S. (2001). Autoregressive conditional heteroskedasticity in commodity spot prices. *Journal of Applied Econometrics 16*, 115–132.

Berg, K., P. Guérin, and Y. Imura (2016). Predictive Ability of Commodity Prices for the Canadian Dollar. Staff Analytical Note 2016-2, Bank of Canada. Available at http://www.bankofcanada.ca/wp-content/uploads/2016/03/san2016-2.pdf.

Bernard, J.-T., L. Khalaf, M. Kichian, and S. Mcmahon (2008). Forecasting Commodity Prices: GARCH, Jumps, and Mean Reversion. *Journal of Forecasting 27*, 279–291.

Blasques, F., S. J. Koopman, and A. Lucas (2014). Maximum Likelihood Estimation for Score-Driven Models. Tinbergen Institute Discussion Paper 14-029/III. Available at SSRN:https://ssrn.com/abstract=2404276 or http://dx.doi.org/10.2139/ssrn.2404276.

Blasques, F., S. J. Koopmans, and A. Lucas (2014). Stationarity and ergodicity of univariate generalized autoregressive score processes. *Electronic Journal of Statistics 8*, 1088–1112.

Blazsek, S. and V. Mendoza (2016). QARMA-Beta-t-EGARCH versus ARMA-GARCH: an application to S and P 500. *Applied Economics 48*(12), 1119–1129.

Blazsek, S. and M. Villatoro (2015). Is Beta-t-EGARCH(1,1) superior to GARCH(1,1). *Applied Economics 47*(17), 1119–1129.

Board of Governors of the Federal Reserve System (US) (2014). Trade Weighted U.S. Dollar Index: Other Important Trading Partners [DTWEXO]. https://fred.stlouisfed.org/series/DTWEXO. Accessed: 2017-08-30.

Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics 31*(3), 307–327.

Bollerslev, T. (1987). A conditionally heteroskedastic time series model for speculative prices and rates of return. *Review of Economics and Statistics 69*(3), 542–547.

Bulmer, M. (1979). Concepts in the analysis of qualitative data. *Sociological Review 27*(4), 651–677.

Burns, K. and I. Moosa (2017). Demystifying the Meese–Rogoff puzzle: structural breaks or measures of forecasting accuracy? *Applied Economics 49*, 4897–4910.

Calvo, G. (1983). Staggered prices in a utility-maximizing framework. *Journal of Monetary Economics 12*, 383–398.

Cantore, C., F. Ferroni, and M. A. Leon-Ledesma (2017). The dynamics of hours worked and technology. *Journal of Economic Dynamics and Control 82*, 67–82.

Cantore, C., M. Leon-Ledesma, P. McAdam, and A. Willman (2014). Shocking stuff: Technology, hours, and factor substitution. *Journal of the European Economic Association 12*, 108–128.

Chan, J. C. and A. L. Grant (2016). Modeling energy price dynamics: GARCH versus stochastic volatility. *Energy Economics 54*, 182–189.

Christano, L. J. (1992). Searching for a break in GNP. *Journal of Business and Economic Statistics 10*, 237–249.

Christiano, L. J., M. Eichenbaum, and C. L. Evans (2005). Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of Political Economy 113*(1), 1–45.

Christoffersen, P. F. (1998). Evaluating interval forecasts. *International Economic Review 39*(4), 841–862.

Clark, T. E. and M. W. McCracken (2009). Improving forecast accuracy by combining recursive and rolling forecasts. *International Economic Review 50*(2), 363–395.

Clark, T. E. and F. Ravazzolo (2015). Macroeconomic forecasting performance under alternative specificiations of time-varying volatility. *Journal of Applied Econometrics 30*, 551–575.

Creal, D., S. J. Koopman, and A. Lucas (2013). Generalized autoregressive score models with applications. *Journal of Applied Econometrics 28*, 777–795.

Darné, O. and C. Diebolt (2005). Non-stationarity Tests in Macroeconomic Time Series. In C. Diebolt, C. Kyrtsou, and O. Darné (Eds.), *New Trends in Macroeconomics*, pp. 173–194. Springer.

Dickey, D. A. and W. A. Fuller (1981). Likelihood ratio statistics for autoregressive time series with a unit root. *Econometrica 49*(4), 1057–1072.

Diebold, F. X. and R. S. Mariano (1995). Comparing predictive accuracy. *Journal of Business and Economic Statistics 13*(3), 253–263.

Diebold, F. X. and M. Nerlove (1989). The dynamics of exchange rate volatility: a multivariate latent factor ARCH model. *Journal of Applied Econometrics 4*(1), 1–21.

Dufour, J.-M. and O. Torres (1998). Union-intersection and sample-split methods in econometrics with applications to MA and SURE models. In D. Giles and A. Ullah (Eds.), *Handbook of Applied Economic Statistics*, pp. 465–505. New York: Marcel Dekker.

Dufour, J.-M. and P. Valery (2006). On a simple two-stage closed-form estimator for a stochastic volatility in a general linear regression. In D. Terrell and T. B. Fomby (Eds.), *Advances in Econometrics: Econometric Analysis of Economic and Financial Time Series*, pp. 259–288. West Yorkshire, United Kingdom: Emerald Group Publishing Limited.

Enders, W. (2010). *Applied Econometric Time Series* (Third ed.). Hoboken, NJ: Wiley.

Engle, R. F. (1982). Autoregressive conditional heteroskedasticity with estimates of the variance of United Kingdom inflation. *Econometrica 50*, 987–1008.

Engle, R. F. and A. J. Patton (2001). What good is a volatility model? *Quantitative Finance 1*(2), 237–245.

Erceg, C. J., D. W. Henderson, and A. T. Levin (2000). Optimal monetary policy with staggered wages and price contracts. *Journal of Monetary Economics 46*, 281–313.

Ewing, B. T. and F. Malik (2017). Modelling asymmetric volatility in oil prices under structural breaks. *Energy Economics 63*, 227–223.

Fernald, J. and J. Wang (2015). Why has the cyclicality of productivity changed? What does it mean? Working Paper 15-6, Federal Reserve Bank of Boston.

Fernandez-Villaverde, J. and J. Rubio-Ramirez (2010). Macroeconomics and volatility: Data, models, and estimation. Working Paper 16618, National Bureau of Economic Research. http://www.nber.org/papers/w16618.pdf.

Ferraro, D., K. S. Rogoff, and B. Rossi (2015, June). Can oil prices forecast exchange rates? An empirical analysis of the relationship between commodity prices and exchange rates. *Journal of International Money and Finance 54*, 115–141.

Francis, N. and V. A. Ramey (2004, July). The source of historical economic fluctuations: An analysis using long-run restrictions. Working Paper 10631, National Bureau of Economic Research. http://www.nber.org/papers/w10631.

Fuller, W. A. (1976). *Introduction to Statistical Time Series*. New York: John Wiley and Sons.

Gali, J. (1999). Technology, employment, and the business cycle: Do technology shocks explain aggregate fluctuations? *American Economic Journal 89*, 249–271.

Gali, J. and T. van Rens (2014, March). The vanishing procyclicality of labor productivity. Working Paper 9853, Centre for Economic Policy Research.

Giraitis, L., R. Leipus, and A. Philippe (2006). A test for stationarity versus trends and unit roots for a wide class of dependent errors. *Econometric Theory 22*(6), 989–1029.

Glosten, L. R., R. Jagannathan, and D. E. Runkle (1993). On the relation between the expected value and the volatility of the nominal excess return on stocks. *The Journal of Finance 48*(5), 1779–1801.

Gordon, R. (2010). Okun's law, productivity innovations, and conundrums in business cycle dating. *American Economic Review: Papers and Proceedings 2*, 11–15.

Granger, C. W. J., H. White, and M. Kamstra (1989). Interval forecasting: an analysis based upon ARCH-quantile estimators. *Journal of Econometrics 40*(1), 87–96.

Hamilton, J. D. (2017). Why you should never use the Hodrick-Prescott filter. Working Paper 23429, National Bureau of Economic Research. http://www.nber.org/papers/w23429.

Hansen, P. R. (2005). A test for superior predictive ability. *Journal of Business and Economic Statistics 23*(4), 365–380.

Hansen, P. R. and A. Lunde (2005). A forecast comparison of volatility models: does anything beat a GARCH(1,1)? *Journal of Applied Econometrics 20*(7), 873–889.

Hansen, P. R. and A. Lunde (2006). Consistent ranking of volatility models. *Journal of Econometrics 131*(1), 97–121.

Harvey, A. C. (2013). *Dynamic Models for Volatility and Heavy Tails: With Applications to Financial and Economic Time Series*. Cambridge: Cambridge University Press.

Harvey, A. C. and T. Chakravarty (2008). Beta-t(E)GARCH. Working Paper 08340, University of Cambridge.

Harvey, A. C. and N. Shephard (1996). Estimation of an asymmetric stochastic volatility model for asset returns. *Journal of Business and Economic Statistics 14*, 429–434.

Harvey, D. I., S. J. Leybourne, and P. Newbold (2001). Innovational outlier unit root tests with an endogenously determined break in level. *Oxford Bulletin of Economics and Statistics 63*(5), 559–575.

Hillebrand, E. (2005). Neglecting parameter changes in GARCH models. *Journal of Econometrics 129*, 121–138.

Hyndman, R. J. (2014). Structural breaks. Hyndsight (blog), Last modified: May 23, 2014. http://robjhyndman.com/hyndsight/structural-breaks/.

Inclan, C. and G. C. Tiao (1994). Use of cumulative sums of squares for retrospective detection of changes of variance. *Journal of Time Series Analysis 89*, 913–923.

Justiniano, A. and G. E. Primiceri (2010). The time-varying volatility of macroeconomic fluctuations. *American Economic Review 98*(3), 604–641.

Koopman, S. J., A. Lucas, and M. Scharth (2016). Predicting time-varying parameters with parameter-driven and observation-driven models. *Review of Economics and Statistics 98*(1), 97–110.

Krämer, W. and P. Messow (2012). Structural change and spurious persistence in stochastic volatility. Ruhr Economic Paper 310, RWI - Leibniz-Institut fr Wirtschaftsforschung, Ruhr-University Bochum, TU Dortmund University, University of Duisburg-Essen.

Lamoureux, C. G. and W. D. Lastrapes (1990). Persistence in variance, structural change, and the GARCH model. *Journal of Business and Economic Statistics 8*(2), 225–234.

Loretin, M. (2005). Indexes of the foreign exchange value of the dollar. Federal Reserve Bulletin Winter 2005, Federal Reserve Board. Available at https://www.federalreserve.gov/pubs/bulletin/2005/winter05_index.pdf.

Lumsdaine, R. L. (1995). Finite-sample properties of the maximum likelihood estimator in GARCH(1,1) and IGARCH(1,1) models: A Monte Carlo investigation. *Journal of Business and Economic Statistics 13*, 1–10.

Lumsdaine, R. L. and S. Ng (1999). Testing for ARCH in the presence of a possibly misspecified conditional mean. *Journal of Econometrics 93*, 257–279.

Maddala, G. S. and I.-M. Kim (1998). *Unit Roots, Cointegration, and Structural Change.* Cambridge, UK: Cambridge University Press.

Martin, V., S. Hurn, and D. Harris (2013). *Econometric Modelling with Time Series.* New York: Cambridge University Press.

Mavroeidis, S. (2010). Monetary policy rules and macroeconomic stability: some new evidence. *American Economic Review 100*(1), 491–503.

Meese, R. and K. Rogoff (1983a). The out-of-sample failure of empirical exchange rate models: Sampling error or misspecification? In *Exchange rates and international macroeconomics*, pp. 67–112. University of Chicago Press.

Meese, R. A. and K. Rogoff (1983b). Empirical exchange rate models of the seventies: Do they fit out of sample? *Journal of International Economics 14*(1-2), 3–24.

Mohammadi, H. and L. Su (2010). International evidence on crude oil price dynamics: Applications of ARIMA-GARCH models. *Energy Economics 32*, 1001–1008.

Moosa, I. and K. Burns (2014). *Demystifying the Meese-Rogoff Puzzle.* London: Palgrave Macmillan UK.

Narayan, P. K. and S. Popp (2010). A new unit root test with two structural breaks in level and slope at unknown time. *Journal of Applied Statistics 37*(9), 1425–1438.

Nelson, D. B. (1991). Conditional heteroskedasticity in asset returns: A new approach. *Econometrica 59*(2), 347–370.

Nelson, D. B. (1998). *The time series behavior of stock market volatility and returns.* Ph. D. thesis, Massachusetts Information of Technology.

Patton, A. J. (2011). Volatility forecast comparison using imperfect volatility proxies. *Journal of Econometrics 160*, 246–256.

Patton, A. J. and K. Sheppard (2009). Evaluating volatility and correlation forecasts. In T. G. Anderson, R. A. Davis, J.-P. Krieß, and T. Mikosch (Eds.), *Handbook of Financial Time Series.* Berlin: Springer-Verlag.

Pellegrini, S. and A. Rodriguez (2007). Financial econometrics and SV models. Tutorial guide, Universidad Carlos III de Madrid. http://halweb.uc3m.es/esp/Personal/personas/spellegr/esp/Curso_Cordoba/Tutorial_Guide.pdf.

Perron, P. (1989). The great crash, the oil price shock, and the unit root hypothesis. *Econometrica 57*, 1361–1401.

Perron, P. (1997). Further evidence on breaking trend functions in macroeconomic variables. *Journal of Econometrics 80*, 355–385.

Perron, P. and T. J. Vogelsang (1993). The great crash, the oil price shock and the unit root hypothesis, erratum. *Econometrica 61*, 248–249.

Pesaran, M. H. and A. Timmermann (2007). Selection of estimation window in the presence of breaks. *Journal of Econometrics 137*(1), 134–161.

Phillips, P. C. and P. Perron (1988). Testing for a unit root in time series regression. *Biometrika 75*, 335–346.

Pindyck, R. S. (1999). The long-run evolution of energy prices. *Energy Journal*, 1–27.

Pindyck, R. S. (2004a). Volatility and commodity price dynamics. *Journal of Futures Markets 24*(11), 1029–1047.

Pindyck, R. S. (2004b). Volatility in natural gas and oil markets. *Journal of Energy and Development 30*(1), 1–19.

Postali, F. A. and P. Picchetti (2006). Geometric brownian motion and structural breaks in oil prices: a quantitative analysis. *Energy Economics 28*(4), 506–522.

Psaradakis, Z. and E. Tzavalis (1999). On regression-based tests for persistence in logarithmic volatility models. *Econometric Reviews 18*(4), 441–448.

Rapach, D. E. and J. K. Strauss (2008). Structural breaks and GARCH models of exchange rate volatility. *Journal of Applied Econometrics 23*, 65–90.

Rohan, N. and T. V. Ramanathan (2012). Asymmetric volatility models with structural breaks. *Communications in Statistics- Simulations and Computation 41*(9), 1519–1543.

Rossi, B. (2013). Exchange rate predictability. *Journal of Economic Literature 51*(4), 1063–1119.

Ruiz, E. (1994). Quasi-maximum likelihood estimation of stochastic volatility models. *Journal of Econometrics 63*(1), 289–306.

Salisu, A. A. and I. O. Fasanya (2013). Modelling oil price volatility with structural breaks. *Energy Policy 52*, 554–562.

Sanso, A., V. Arago, and J. L. Carrion-i-Silvestre (2004). Testing for changes in the unconditional variance of financial time series. *Revista de Economa Financiera 4*, 32–53.

Santacreu, A. M. (2015). Measuring Labor Productivity: Technology and the Labor Supply. Economic synposis, Federal Reserve Bank of St. Louis. Available at https://research.stlouisfed.org/publications/economic-synopses/2015/02/27/measuring-labor-productivity-technology-and-the-labor-supply/.

Schwartz, E. and J. E. Smith (2000). Short-term variations and long-term dynamics in commodity prices. *Management Science 46*(7), 893–911.

Shapiro, M. and M. Watson (1988). Sources of business cycle fluctuations. In S. Fischer (Ed.), *NBER Macroeconomics Annual*, Volume 3, pp. 111–156. MIT Press. http://www.nber.org/chapters/c10953.

Smets, F. and R. Wouters (2007). Shocks and frictions in US business cycles: A Bayesian approach. *American Economic Review 3*, 586–606.

Smith, D. R. (2008). Testing for structural breaks in GARCH models. *Applied Financial Economics 18*(10), 845–862.

Stiroh, K. (2009). Volatility accounting: A production perspective of increased economic stability. *Journal of the European Economic Association 7*(4), 671–696.

Taylor, S. J. (1986). *Modelling Financial Time Series*. Berlin: Wiley.

The Mathworks Inc. (2014). *MATLAB Student Version (R2014a)*. Natick, Massachusetts: The Mathworks Inc.

US Department of Energy (2014). Benchmarks play an important role in pricing crude oil. https://www.eia.gov/todayinenergy/detail.php?id=18571. Accessed: 2017-08-30.

West, K. D. (1996). Asymptotic inference about predictive ability. *Econometrica 64*(5), 1067–1084.

Wilmot, N. A. and C. F. Mason (2013). Jump processes in the market for crude oil. *Energy Journal 34*(1), 33–48.

Xu, K.-L. (2013). Powerful tests for structural changes in volatility. *Journal of Econometrics 173*(1), 126–142.

Zivot, E. (2009). Practical issues in the analysis of univariate GARCH models. In T. G. Anderson, R. A. Davis, J.-P. Krieß, and T. Mikosch (Eds.), *Handbook of Financial Time Series*, pp. 113–155. Berlin: Springer-Verlag.

Zivot, E. and D. W. Andrews (1992). Further evidence on the great crash, the oil price shock, and the unit root hypothesis. *Journal of Business and Economic Statistics 10*, 251–270.

APPENDICES

Appendix A

Chapter 2

A.1 Data

All data used in this section are quarterly indexes from 1947 to 2014. Each index is constructed by dividing each data point by the base year 2009. They were retrieved from the Federal Reserve of Economic Database (FRED) of the Federal Reserve Bank of St. Louis. The labour-related data source came from US Bureau of Labor Statistics and output-related data source came from the Bureau of Economic Accounts.

Table A.1: Data source for Chapter 2

Variable	FRED Variable Name	FRED ID Series
Labour Productivity	Nonfarm Business Sector: Real Output Per Hour of All Persons	OPHNFB
Investment	Real Gross Private Domestic Investment	B006RA3Q086SBEA
Residential Structures	Real Private Fixed Investment: Residential	B011RA3Q086SBEA
Non-Residential Structures	Real Private Fixed Investment: Nonresidential: Structures	B009RA3Q086SBEA
Equipment	Fixed Investment: Nonresidential: Equipment	Y033RA3Q086SBEA
Software	Nonresidential: Equipment: Information Processing Equipment	Y034RA3Q086SBEA

Note: All data in this analysis are transformed into logs

Labor productivity is defined as real output per hour for the US non-farm business section. There are other definitions of labour productivity such as output per person, however, the Federal Reserve Bank of St. Louis (see Santacreu, 2015) points out that the former is a better measurement than the latter.

Real gross private domestic investment represents investment. It is composed of non-residential structures, machinery and equipment, residential investment, and the change in private inventories. Non-residential structures includes new construction and renovating existing structures, and spending on mining exploration. Machinery and equipment consists of private businesses purchasing machines, furnitures, and dealers selling used machinery to private companies. Residential spending includes new construction of single and multi-housing family units, improving and maintenance of existing homes, equipment such as air conditioning and heating.

A.2 Unit Root testing in the Presence of Structural Breaks

Before estimating VAR models, we first use the Perron (1989) method to test for structural breaks in the presence of structural break in 1990.

This is important because in the presence of structural breaks, the various unit root tests can be biased towards failure to reject the hypothesis of a unit root. Given that the drop in correlation occurred around 1990, which indicates a sign of structural break.

The hypothesis is for no drift and no trend.

Null hypothesis is as follows:

$$H_0 : y_t = a_0 + y_{t-1} + \mu_1 D_P + \epsilon_t \tag{A.1}$$

Alternative hypothesis is:

$$H_a : y_t = a_0 + a_2 t + \mu_2 D_L + \epsilon_t \tag{A.2}$$

where D_P represent the pulse dummy variable and D_L represents the level dummy variable equal to 1 for t $>$ 1990Q1 and 0 otherwise.

The distribution for this test is the t-distribution and the critical values based on the proportion of observations prior to the break can be found in Perron and Vogelsang (1993).

For labour productivity, the t-statistic is -2.189 which is greater than the 5% critical value of 3.89 for the proportion of 0.6. This implies that we cannot reject the null hypothesis in favor of the alternative.

For investment, 2 additional lags are needed to remove autocorrelation. The t-statistic is -3.78 which is also larger than the critical value of -3.89 at the 5% level.

Residential and machinery and equipment t-statistics are -3.17 and -3.05 respectively. At the 5% level, their null hypotheses cannot be rejected.

Adding the drift and trend gives the following:

Null hypothesis:

$$H_0 : y_t = a_0 + y_{t-1} + \mu_1 D_P + \mu_2 D_L + \epsilon_t \tag{A.3}$$

Alternative hypothesis is:

$$H_a : y_t = a_0 + a_2 t + \mu_2 D_L + \mu_3 D_T + \epsilon_t \tag{A.4}$$

D_T represents the trend dummy where t - 1990Q1 for t > 1990Q1 and 0 otherwise.

The critical value, using the Perron and Vogelsang (1993) paper, is -4.18 at the 5% level. The t-statistics for labour productivity, investment, residential, and machinery and equipment spending are -2.24, -3.99, -3.61, and -3.03 respectively. All the t-statistics are larger than the critical value, implying that the null hypothesis of a unit root cannot be rejected for a break in 1990.

Due to criticism that the choice of the structural break is exogenous (see Christano (1992), Zivot and Andrews (1992)), The Perron (1997) test is used to test for unit roots in the presence of structural breaks. Once again, the null hypothesis of a unit root cannot be rejected whether there is an intercept or trend at the 5% level.

A.3 Procyclicality

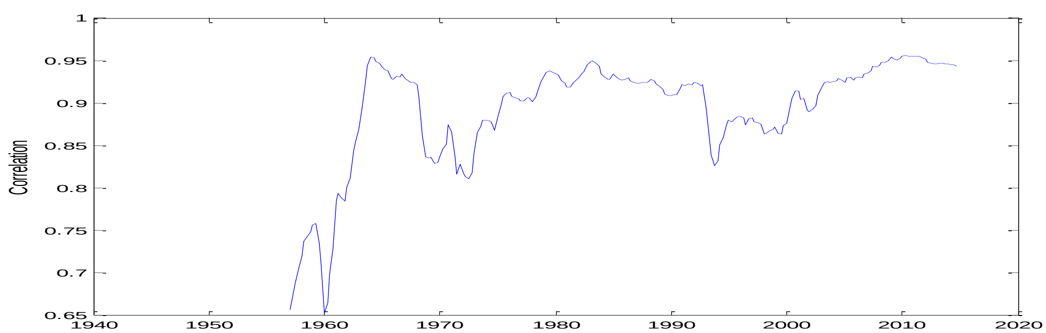
Procyclicality of labour productivity is a major issue in literature (Gali and van Rens (2014)). It was also discussed in Barnichon (2010). Table 5 below shows that procyclicality of labour productivity, investment and its components.

Table A.2 shows that for labour productivity, it became less procyclical after 1990, confirming the stylized facts by Gali and van Rens (2014). After 1990, the investment data appears to be procyclical. The rolling correlations are needed to see when the possible change in correlation took place.

Table A.2: Cyclicality of real investment by type

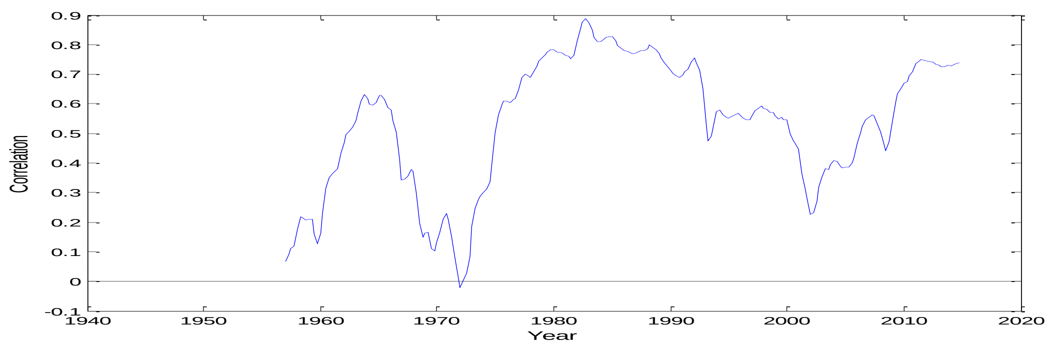
Correlation with real GDP	1947-2014	1947-1989	1990-2014
Labor Prod	0.4132 (1.22E-12)	0.5496 (5.82E-15)	-0.0909 (0.3684)
Investment	0.8404 (8.39E-74)	0.8211 (3.01E-43)	0.9246 (7.21E-43)
Residential	0.5319 (2.87E-21)	0.5067 (1.33E-12)	0.6339 (1.44E-12)
Non-res structures	0.4506 (5.28E-15)	0.4905 (8.47E-12)	0.5055 (8.13E-08)
Mach. and Equip.	0.8078 (6.36E-64)	0.8138 (6.30E-42)	0.8935 (7.50E-36)
Software	0.6439 (3.00E-33)	0.6063 (1.21E-18)	0.7932 (7.81E-23)

Figure A.1: 10-year rolling correlation between real private business investment and real GDP



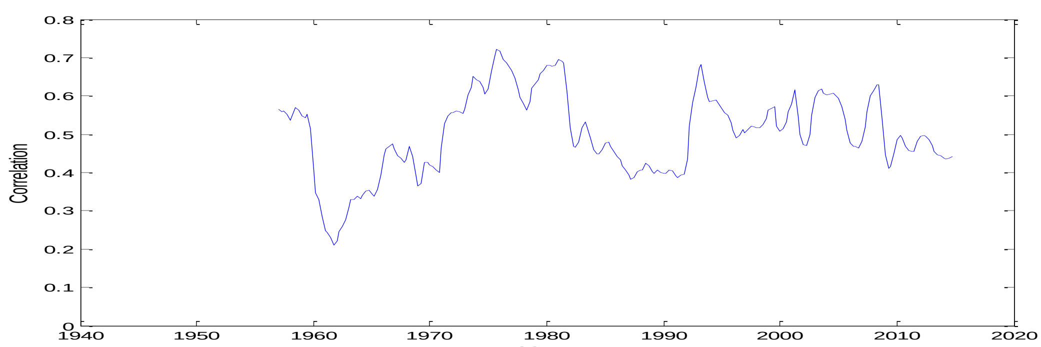
Source: Bureau of Economic Analysis and Bureau of Labor Statistics. Real private business investment and GDP have been detrended using the HP filter. Both are indexes with 2009 = 100.

Figure A.2: 10-year rolling correlation between real residential spending and real GDP



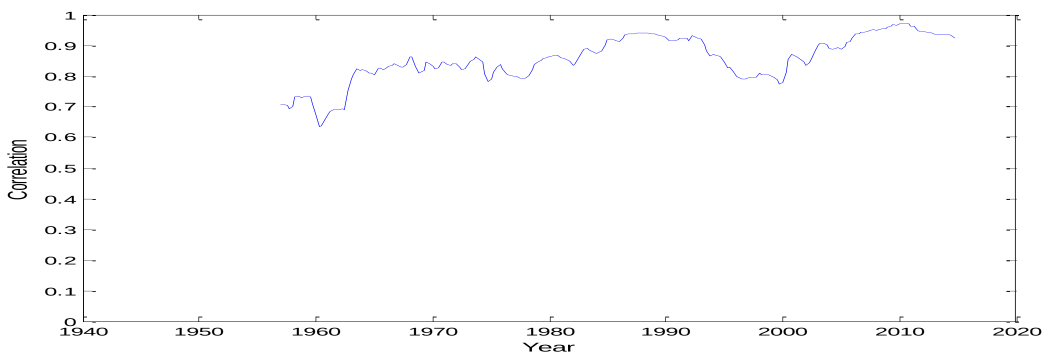
Source: Bureau of Economic Analysis and Bureau of Labor Statistics. Real private business investment and GDP have been detrended using the HP filter. Both are indexes with 2009 = 100.

Figure A.3: 10-year rolling correlation between real non-residential spending and real GDP



Source: Bureau of Economic Analysis and Bureau of Labor Statistics. Real non-residential investment and GDP have been detrended using the HP filter. Both are indexes with 2009 = 100.

Figure A.4: 10-year rolling correlation between real machinery and equipment spending and real GDP



Source: Bureau of Economic Analysis and Bureau of Labor Statistics. Real machinery and equipment spending and GDP have been detrended using the HP filter. Both are indexes with 2009 = 100.

Appendix B

Chapter 3

B.1 Effect of a break in ω on other parameters

Table B.1: Effect of a change in ω on A

T	Vol	t-GAS		t-GARCH		GARCH		EGARCH		Beta-t-EGARCH	
		Mean est.	Std. Dev.	Mean est.	Std. Dev.	Mean est.	Std. Dev	Mean est.	Std. Dev	Mean est.	Std. Dev
	1	0.11	0.07	0.11	0.09	0.11	0.07	0.05	0.16	0.13	0.09
	1.15	0.11	0.07	0.12	0.09	0.11	0.07	0.08	0.15	0.13	0.09
250	1.3	0.11	0.07	0.12	0.09	0.12	0.07	0.11	0.15	0.13	0.09
	1.45	0.11	0.07	0.13	0.09	0.12	0.07	0.15	0.14	0.14	0.09
	1.6	0.11	0.07	0.14	0.10	0.13	0.07	0.17	0.13	0.14	0.09
	1.75	0.12	0.07	0.14	0.10	0.13	0.07	0.18	0.12	0.14	0.09
	1	0.10	0.05	0.11	0.06	0.10	0.05	0.08	0.10	0.12	0.06
	1.15	0.11	0.05	0.11	0.06	0.11	0.05	0.10	0.09	0.12	0.06
500	1.3	0.11	0.05	0.11	0.06	0.10	0.05	0.12	0.09	0.12	0.06
	1.45	0.10	0.05	0.11	0.07	0.10	0.05	0.13	0.08	0.12	0.06
	1.6	0.10	0.05	0.11	0.07	0.10	0.05	0.13	0.07	0.12	0.06
	1.75	0.10	0.05	0.11	0.07	0.09	0.05	0.14	0.07	0.12	0.06
	1	0.10	0.03	0.10	0.04	0.10	0.04	0.09	0.06	0.11	0.04
	1.15	0.10	0.03	0.10	0.04	0.10	0.04	0.11	0.06	0.11	0.04
1000	1.3	0.10	0.04	0.11	0.05	0.10	0.04	0.12	0.06	0.11	0.04
	1.45	0.09	0.04	0.10	0.05	0.08	0.04	0.10	0.06	0.11	0.05
	1.6	0.09	0.04	0.09	0.05	0.08	0.04	0.10	0.05	0.11	0.05
	1.75	0.08	0.04	0.09	0.05	0.07	0.03	0.10	0.04	0.11	0.05
	1	0.10	0.02	0.10	0.02	0.10	0.02	0.10	0.03	0.11	0.02
	1.15	0.10	0.02	0.10	0.03	0.10	0.02	0.11	0.03	0.11	0.03
2500	1.3	0.10	0.02	0.10	0.03	0.09	0.03	0.11	0.05	0.11	0.03
	1.45	0.09	0.03	0.10	0.04	0.07	0.03	0.07	0.04	0.11	0.03
	1.6	0.08	0.03	0.08	0.04	0.06	0.03	0.07	0.03	0.10	0.04
	1.75	0.06	0.03	0.07	0.04	0.06	0.02	0.07	0.02	0.09	0.04
	1	0.10	0.01	0.10	0.02	0.10	0.02	0.10	0.02	0.10	0.02
	1.15	0.10	0.01	0.10	0.02	0.10	0.02	0.12	0.02	0.11	0.02
5000	1.3	0.10	0.02	0.10	0.02	0.09	0.02	0.11	0.04	0.11	0.02
	1.45	0.09	0.02	0.09	0.03	0.06	0.03	0.05	0.03	0.11	0.02
	1.6	0.07	0.03	0.07	0.04	0.05	0.02	0.05	0.02	0.09	0.03
	1.75	0.06	0.03	0.06	0.03	0.05	0.02	0.05	0.01	0.08	0.03
	1	0.10	0.01	0.10	0.01	0.10	0.01	0.10	0.02	0.10	0.01
	1.15	0.10	0.01	0.10	0.01	0.10	0.01	0.12	0.02	0.11	0.01
10000	1.3	0.10	0.01	0.10	0.01	0.09	0.02	0.11	0.04	0.11	0.01
	1.45	0.09	0.02	0.10	0.02	0.06	0.02	0.03	0.02	0.11	0.02
	1.6	0.07	0.02	0.07	0.03	0.05	0.02	0.03	0.01	0.10	0.02
	1.75	0.05	0.02	0.06	0.03	0.04	0.01	0.04	0.01	0.08	0.03

Table B.2: Effect of a change in ω on the coverage probability of A

T	Vol.	t-GAS		t-GARCH		GARCH		EGARCH		Beta-t-EGARCH	
		90%	95%	90%	95%	90%	95%	90%	95%	90%	95%
	1	0.887	0.934	0.875	0.913	0.886	0.926	0.746	0.803	0.904	0.950
		(0.007)	(0.006)	(0.007)	(0.006)	(0.007)	(0.006)	(0.010)	(0.009)	(0.007)	(0.005)
	1.15	0.865	0.916	0.877	0.915	0.867	0.909	0.788	0.838	0.896	0.945
		(0.008)	(0.006)	(0.007)	(0.006)	(0.008)	(0.006)	(0.009)	(0.008)	(0.007)	(0.005)
	1.3	0.853	0.908	0.853	0.901	0.827	0.873	0.792	0.849	0.879	0.938
250		(0.008)	(0.006)	(0.008)	(0.007)	(0.008)	(0.007)	(0.009)	(0.008)	(0.007)	(0.005)
	1.45	0.834	0.892	0.847	0.897	0.816	0.872	0.802	0.856	0.877	0.934
		(0.008)	(0.007)	(0.008)	(0.007)	(0.009)	(0.007)	(0.009)	(0.008)	(0.007)	(0.006)
	1.6	0.839	0.895	0.853	0.906	0.834	0.892	0.805	0.872	0.883	0.929
		(0.008)	(0.007)	(0.008)	(0.007)	(0.008)	(0.007)	(0.009)	(0.007)	(0.007)	(0.006)
	1.75	0.843	0.899	0.858	0.913	0.859	0.911	0.790	0.864	0.871	0.875
		(0.008)	(0.007)	(0.008)	(0.006)	(0.008)	(0.006)	(0.009)	(0.008)	(0.007)	(0.007)
	1	0.850	0.904	0.856	0.905	0.831	0.894	0.789	0.853	0.910	0.953
		(0.008)	(0.007)	(0.008)	(0.007)	(0.008)	(0.007)	(0.009)	(0.008)	(0.006)	(0.005)
	1.15	0.847	0.895	0.855	0.896	0.820	0.879	0.806	0.867	0.902	0.943
		(0.008)	(0.007)	(0.008)	(0.007)	(0.009)	(0.007)	(0.009)	(0.008)	(0.007)	(0.005)
	1.3	0.818	0.866	0.810	0.860	0.784	0.834	0.796	0.850	0.893	0.936
500		(0.009)	(0.008)	(0.009)	(0.008)	(0.009)	(0.008)	(0.009)	(0.008)	(0.007)	(0.005)
	1.45	0.783	0.851	0.775	0.827	0.753	0.808	0.803	0.858	0.871	0.925
		(0.009)	(0.008)	(0.009)	(0.008)	(0.010)	(0.009)	(0.009)	(0.008)	(0.007)	(0.006)
	1.6	0.777	0.830	0.742	0.803	0.767	0.819	0.855	0.905	0.861	0.921
		(0.009)	(0.008)	(0.010)	(0.009)	(0.009)	(0.009)	(0.008)	(0.007)	(0.008)	(0.006)
	1.75	0.775	0.821	0.726	0.801	0.802	0.850	0.901	0.938	0.869	0.921
		(0.009)	(0.009)	(0.010)	(0.009)	(0.009)	(0.008)	(0.007)	(0.005)	(0.008)	(0.006)
	1	0.872	0.917	0.873	0.915	0.852	0.907	0.825	0.890	0.916	0.960
		(0.007)	(0.006)	(0.007)	(0.006)	(0.008)	(0.007)	(0.009)	(0.007)	(0.006)	(0.004)
	1.15	0.882	0.938	0.861	0.908	0.837	0.895	0.836	0.898	0.908	0.947
		(0.007)	(0.005)	(0.008)	(0.006)	(0.008)	(0.007)	(0.008)	(0.007)	(0.006)	(0.005)
	1.3	0.845	0.888	0.810	0.865	0.797	0.843	0.770	0.834	0.897	0.939
1000		(0.008)	(0.007)	(0.009)	(0.008)	(0.009)	(0.008)	(0.009)	(0.008)	(0.007)	(0.005)
	1.45	0.796	0.844	0.723	0.777	0.735	0.785	0.734	0.802	0.861	0.907
		(0.009)	(0.008)	(0.010)	(0.009)	(0.010)	(0.009)	(0.010)	(0.009)	(0.008)	(0.006)
	1.6	0.736	0.786	0.633	0.699	0.711	0.760	0.827	0.878	0.834	0.881
		(0.010)	(0.009)	(0.011)	(0.010)	(0.010)	(0.010)	(0.008)	(0.007)	(0.008)	(0.007)
	1.75	0.697	0.750	0.579	0.650	0.717	0.779	0.900	0.934	0.821	0.874
		(0.010)	(0.010)	(0.011)	(0.011)	(0.010)	(0.009)	(0.007)	(0.006)	(0.009)	(0.007)

Continued on next page

Table B.2 – continued from previous page

T	Vol.	t-GAS		t-GARCH		GARCH		EGARCH		Beta-t-EGARCH	
	1	0.884	0.945	0.878	0.939	0.884	0.935	0.873	0.926	0.918	0.950
		(0.007)	(0.005)	(0.007)	(0.005)	(0.007)	(0.006)	(0.007)	(0.006)	(0.006)	(0.005)
	1.15	0.882	0.938	0.879	0.933	0.895	0.933	0.859	0.919	0.902	0.949
		(0.007)	(0.005)	(0.007)	(0.006)	(0.007)	(0.006)	(0.008)	(0.006)	(0.007)	(0.005)
	1.3	0.886	0.932	0.821	0.890	0.858	0.897	0.740	0.816	0.887	0.939
2500		(0.007)	(0.006)	(0.009)	(0.007)	(0.008)	(0.007)	(0.010)	(0.009)	(0.007)	(0.005)
	1.45	0.838	0.877	0.710	0.777	0.712	0.762	0.589	0.648	0.856	0.910
		(0.008)	(0.007)	(0.010)	(0.009)	(0.010)	(0.010)	(0.011)	(0.011)	(0.008)	(0.006)
	1.6	0.725	0.772	0.566	0.617	0.592	0.658	0.599	0.664	0.813	0.870
		(0.010)	(0.009)	(0.011)	(0.011)	(0.011)	(0.011)	(0.011)	(0.011)	(0.009)	(0.008)
	1.75	0.624	0.683	0.452	0.503	0.527	0.605	0.641	0.718	0.796	0.842
		(0.011)	(0.010)	(0.011)	(0.011)	(0.011)	(0.011)	(0.011)	(0.010)	(0.009)	(0.008)
	1	0.908	0.951	0.898	0.950	0.887	0.941	0.870	0.923	0.917	0.958
		(0.006)	(0.005)	(0.007)	(0.005)	(0.007)	(0.005)	(0.008)	(0.006)	(0.006)	(0.004)
	1.15	0.904	0.950	0.891	0.950	0.903	0.956	0.823	0.896	0.911	0.950
		(0.007)	(0.005)	(0.007)	(0.005)	(0.007)	(0.005)	(0.009)	(0.007)	(0.006)	(0.005)
	1.3	0.922	0.958	0.848	0.911	0.884	0.927	0.668	0.753	0.889	0.941
5000		(0.006)	(0.005)	(0.008)	(0.006)	(0.007)	(0.006)	(0.011)	(0.010)	(0.007)	(0.005)
	1.45	0.870	0.910	0.736	0.802	0.682	0.750	0.468	0.523	0.875	0.929
		(0.008)	(0.006)	(0.010)	(0.009)	(0.010)	(0.010)	(0.011)	(0.011)	(0.007)	(0.006)
	1.6	0.718	0.767	0.526	0.580	0.441	0.526	0.388	0.448	0.836	0.876
		(0.010)	(0.009)	(0.011)	(0.011)	(0.011)	(0.011)	(0.011)	(0.011)	(0.008)	(0.007)
	1.75	0.544	0.616	0.355	0.403	0.322	0.417	0.369	0.452	0.790	0.835
		(0.011)	(0.011)	(0.011)	(0.011)	(0.010)	(0.011)	(0.011)	(0.011)	(0.009)	(0.008)
	1	0.909	0.953	0.912	0.959	0.890	0.942	0.881	0.933	0.917	0.954
		(0.006)	(0.005)	(0.006)	(0.004)	(0.007)	(0.005)	(0.007)	(0.006)	(0.006)	(0.005)
	1.15	0.911	0.953	0.903	0.953	0.907	0.958	0.729	0.831	0.886	0.940
		(0.006)	(0.005)	(0.007)	(0.005)	(0.007)	(0.005)	(0.010)	(0.008)	(0.007)	(0.005)
	1.3	0.920	0.959	0.859	0.907	0.900	0.941	0.646	0.731	0.824	0.900
10000		(0.006)	(0.004)	(0.008)	(0.007)	(0.007)	(0.005)	(0.011)	(0.010)	(0.009)	(0.007)
	1.45	0.909	0.949	0.751	0.831	0.606	0.718	0.484	0.542	0.831	0.894
		(0.006)	(0.005)	(0.010)	(0.008)	(0.011)	(0.010)	(0.011)	(0.011)	(0.008)	(0.007)
	1.6	0.691	0.778	0.462	0.542	0.265	0.372	0.454	0.502	0.885	0.917
		(0.010)	(0.009)	(0.011)	(0.011)	(0.010)	(0.011)	(0.011)	(0.011)	(0.007)	(0.006)
	1.75	0.396	0.521	0.250	0.299	0.136	0.212	0.441	0.503	0.840	0.875
		(0.011)	(0.011)	(0.010)	(0.010)	(0.008)	(0.009)	(0.011)	(0.011)	(0.008)	(0.007)

Table B.3: Effect of a change in ω on η

Number of Periods	Vol.	t-GAS		t-GARCH		Beta-t-EGARCH	
		Mean est.	Std.Dev.	Mean est.	Std.Dev.	Mean est.	Std.Dev.
250	1	6.38	4.10	6.43	4.12	6.52	4.40
	1.15	6.24	3.96	6.26	4.02	6.35	4.26
	1.3	5.97	3.74	5.94	3.80	6.14	4.06
	1.45	5.72	3.56	5.63	3.54	5.82	3.80
	1.6	5.50	3.43	5.38	3.36	5.57	3.61
	1.75	5.28	3.20	5.16	3.12	5.37	3.46
500	1	5.56	1.89	5.57	1.91	5.58	1.99
	1.15	5.48	1.86	5.48	1.88	5.48	1.89
	1.3	5.30	1.72	5.26	1.76	5.34	1.72
	1.45	5.13	1.59	5.06	1.64	5.14	1.57
	1.6	4.98	1.50	4.89	1.54	4.98	1.50
	1.75	4.86	1.46	4.76	1.49	4.88	1.45
1000	1	5.27	0.99	5.27	1.00	5.27	0.99
	1.15	5.21	0.96	5.21	0.97	5.20	0.96
	1.3	5.08	0.91	5.04	0.91	5.10	0.92
	1.45	4.96	0.87	4.89	0.86	4.95	0.87
	1.6	4.85	0.83	4.77	0.83	4.85	0.84
	1.75	4.77	0.81	4.67	0.80	4.77	0.82
2500	1	5.08	0.52	5.09	0.53	5.08	0.52
	1.15	5.03	0.51	5.03	0.52	5.02	0.51
	1.3	4.91	0.49	4.88	0.49	4.93	0.49
	1.45	4.81	0.47	4.75	0.47	4.80	0.47
	1.6	4.73	0.46	4.66	0.46	4.72	0.45
	1.75	4.68	0.45	4.60	0.46	4.67	0.45
5000	1	5.05	0.36	5.05	0.37	5.05	0.36
	1.15	5.00	0.36	5.00	0.37	4.99	0.36
	1.3	4.88	0.34	4.85	0.35	4.90	0.34
	1.45	4.78	0.33	4.73	0.34	4.78	0.33
	1.6	4.71	0.32	4.65	0.33	4.70	0.32
	1.75	4.67	0.32	4.60	0.33	4.66	0.32
10000	1	5.02	0.25	5.02	0.25	5.02	0.25
	1.15	4.97	0.24	4.97	0.25	4.96	0.24
	1.3	4.86	0.23	4.83	0.23	4.87	0.23
	1.45	4.76	0.22	4.70	0.22	4.75	0.22
	1.6	4.69	0.22	4.63	0.22	4.68	0.22
	1.75	4.65	0.21	4.57	0.22	4.64	0.21

Table B.4: Effect of a change in ω on the coverage probability of η

T	Vol.	t-GAS		t-GARCH		Beta-t-EGARCH	
		90%	95%	90%	95%	90%	95%
250	1	0.920	0.945	0.917	0.942	0.909	0.931
		(0.006)	(0.005)	(0.006)	(0.005)	(0.006)	(0.006)
	1.15	0.914	0.937	0.905	0.932	0.900	0.921
		(0.006)	(0.005)	(0.007)	(0.006)	(0.007)	(0.006)
	1.3	0.900	0.930	0.887	0.918	0.886	0.911
		(0.007)	(0.006)	(0.007)	(0.006)	(0.007)	(0.006)
	1.45	0.883	0.913	0.858	0.891	0.868	0.901
		(0.007)	(0.006)	(0.008)	(0.007)	(0.008)	(0.007)
	1.6	0.865	0.901	0.831	0.871	0.847	0.884
		(0.008)	(0.007)	(0.008)	(0.007)	(0.008)	(0.007)
	1.75	0.848	0.888	0.807	0.844	0.839	0.871
		(0.008)	(0.007)	(0.009)	(0.008)	(0.008)	(0.007)
500	1	0.923	0.953	0.928	0.950	0.923	0.950
		(0.006)	(0.005)	(0.006)	(0.005)	(0.006)	(0.005)
	1.15	0.922	0.949	0.920	0.945	0.913	0.947
		(0.006)	(0.005)	(0.006)	(0.005)	(0.006)	(0.005)
	1.3	0.900	0.935	0.896	0.927	0.901	0.935
		(0.007)	(0.006)	(0.007)	(0.006)	(0.007)	(0.006)
	1.45	0.882	0.920	0.865	0.903	0.876	0.917
		(0.007)	(0.006)	(0.008)	(0.007)	(0.007)	(0.006)
	1.6	0.866	0.906	0.824	0.875	0.859	0.898
		(0.008)	(0.007)	(0.009)	(0.007)	(0.008)	(0.007)
	1.75	0.845	0.893	0.788	0.851	0.847	0.885
		(0.008)	(0.007)	(0.009)	(0.008)	(0.008)	(0.007)
1000	1	0.895	0.942	0.911	0.952	0.883	0.939
		(0.007)	(0.005)	(0.006)	(0.005)	(0.007)	(0.005)
	1.15	0.890	0.937	0.904	0.943	0.882	0.939
		(0.007)	(0.005)	(0.007)	(0.005)	(0.007)	(0.005)
	1.3	0.883	0.927	0.892	0.927	0.875	0.922
		(0.007)	(0.006)	(0.007)	(0.006)	(0.007)	(0.006)
	1.45	0.873	0.915	0.859	0.902	0.863	0.911
		(0.007)	(0.006)	(0.008)	(0.007)	(0.008)	(0.006)
	1.6	0.846	0.894	0.820	0.878	0.846	0.895
		(0.008)	(0.007)	(0.009)	(0.007)	(0.008)	(0.007)
	1.75	0.826	0.879	0.783	0.849	0.822	0.875
		(0.008)	(0.007)	(0.009)	(0.008)	(0.009)	(0.007)

Continued on next page

Table B.4 – continued from previous page

T	Vol.	t-GAS		t-GARCH		Beta-t-EGARCH	
	1	0.892	0.947	0.896	0.954	0.872	0.932
		(0.007)	(0.005)	(0.007)	(0.005)	(0.007)	(0.006)
	1.15	0.884	0.942	0.887	0.945	0.874	0.931
		(0.007)	(0.005)	(0.007)	(0.005)	(0.007)	(0.006)
	1.3	0.856	0.919	0.852	0.913	0.851	0.918
2500		(0.008)	(0.006)	(0.008)	(0.006)	(0.008)	(0.006)
	1.45	0.810	0.884	0.783	0.857	0.803	0.871
		(0.009)	(0.007)	(0.009)	(0.008)	(0.009)	(0.008)
	1.6	0.764	0.844	0.725	0.800	0.749	0.835
		(0.010)	(0.008)	(0.010)	(0.009)	(0.010)	(0.008)
	1.75	0.726	0.813	0.682	0.756	0.715	0.801
		(0.010)	(0.009)	(0.010)	(0.010)	(0.010)	(0.009)
	1	0.885	0.938	0.889	0.941	0.866	0.926
		(0.007)	(0.005)	(0.007)	(0.005)	(0.008)	(0.006)
	1.15	0.874	0.930	0.885	0.930	0.859	0.921
		(0.007)	(0.006)	(0.007)	(0.006)	(0.008)	(0.006)
	1.3	0.828	0.889	0.819	0.881	0.836	0.897
5000		(0.008)	(0.007)	(0.009)	(0.007)	(0.008)	(0.007)
	1.45	0.741	0.829	0.725	0.804	0.737	0.818
		(0.010)	(0.008)	(0.010)	(0.009)	(0.010)	(0.009)
	1.6	0.683	0.761	0.638	0.728	0.664	0.751
		(0.010)	(0.010)	(0.011)	(0.010)	(0.011)	(0.010)
	1.75	0.642	0.725	0.578	0.681	0.623	0.717
		(0.011)	(0.010)	(0.011)	(0.010)	(0.011)	(0.010)
	1	0.883	0.942	0.899	0.947	0.861	0.925
		(0.007)	(0.005)	(0.007)	(0.005)	(0.008)	(0.006)
	1.15	0.881	0.929	0.893	0.937	0.857	0.917
		(0.007)	(0.006)	(0.007)	(0.005)	(0.008)	(0.006)
	1.3	0.788	0.866	0.770	0.853	0.802	0.875
10000		(0.009)	(0.008)	(0.009)	(0.008)	(0.009)	(0.007)
	1.45	0.638	0.733	0.585	0.684	0.633	0.725
		(0.011)	(0.010)	(0.011)	(0.010)	(0.011)	(0.010)
	1.6	0.539	0.637	0.457	0.553	0.521	0.617
		(0.011)	(0.011)	(0.011)	(0.011)	(0.011)	(0.011)
	1.75	0.492	0.583	0.350	0.455	0.454	0.562
		(0.011)	(0.011)	(0.011)	(0.011)	(0.011)	(0.011)

B.2 Effect of a break in A on other parameters

Table B.5: Effect of a change in A on ω

Number of Periods	Vol.	t-GAS		t-GARCH		GARCH	
		Mean est.	Std. Dev.	Mean est.	Std. Dev.	Mean est.	Std. Dev
250	1	0.31	0.24	0.30	0.25	0.30	0.24
	1.15	0.29	0.21	0.29	0.24	0.28	0.21
	1.3	0.28	0.20	0.28	0.23	0.28	0.20
	1.45	0.28	0.19	0.26	0.22	0.27	0.19
	1.6	0.28	0.18	0.25	0.21	0.27	0.19
	1.75	0.27	0.18	0.24	0.20	0.27	0.19
500	1	0.26	0.18	0.26	0.18	0.27	0.19
	1.15	0.25	0.15	0.24	0.17	0.26	0.16
	1.3	0.24	0.14	0.22	0.14	0.25	0.15
	1.45	0.24	0.13	0.21	0.13	0.25	0.14
	1.6	0.24	0.13	0.20	0.12	0.24	0.14
	1.75	0.24	0.12	0.19	0.12	0.24	0.14
1000	1	0.23	0.13	0.23	0.12	0.23	0.13
	1.15	0.22	0.10	0.21	0.10	0.23	0.10
	1.3	0.22	0.09	0.20	0.09	0.22	0.09
	1.45	0.22	0.08	0.19	0.08	0.22	0.09
	1.6	0.22	0.08	0.17	0.07	0.22	0.09
	1.75	0.22	0.08	0.17	0.07	0.22	0.08
2500	1	0.21	0.07	0.21	0.07	0.21	0.07
	1.15	0.21	0.06	0.20	0.05	0.21	0.05
	1.3	0.21	0.05	0.18	0.05	0.21	0.05
	1.45	0.21	0.05	0.17	0.04	0.21	0.05
	1.6	0.21	0.05	0.16	0.04	0.21	0.05
	1.75	0.21	0.05	0.16	0.04	0.21	0.05
5000	1	0.21	0.04	0.20	0.04	0.21	0.04
	1.15	0.21	0.04	0.19	0.03	0.21	0.04
	1.3	0.21	0.03	0.18	0.03	0.21	0.03
	1.45	0.21	0.03	0.17	0.03	0.21	0.03
	1.6	0.21	0.03	0.16	0.03	0.21	0.03
	1.75	0.21	0.03	0.16	0.02	0.21	0.03
10000	1	0.20	0.03	0.20	0.03	0.20	0.03
	1.15	0.20	0.03	0.19	0.02	0.20	0.03
	1.3	0.21	0.02	0.18	0.02	0.20	0.02
	1.45	0.21	0.02	0.17	0.02	0.21	0.02
	1.6	0.21	0.02	0.16	0.02	0.21	0.02
	1.75	0.21	0.02	0.15	0.02	0.21	0.02

Table B.6: Effect of a change in A on the conditional coverage of ω

T	Vol.	t-GAS		t-GARCH		GARCH	
		90%	95%	90%	95%	90%	95%
	1	0.803	0.854	0.833	0.874	0.773	0.835
		(0.009)	(0.008)	(0.008)	(0.007)	(0.009)	(0.008)
	1.15	0.819	0.864	0.835	0.870	0.784	0.833
		(0.009)	(0.008)	(0.008)	(0.008)	(0.009)	(0.008)
	1.3	0.802	0.860	0.821	0.866	0.780	0.843
250		(0.009)	(0.008)	(0.009)	(0.008)	(0.009)	(0.008)
	1.45	0.811	0.871	0.810	0.861	0.780	0.840
		(0.009)	(0.008)	(0.009)	(0.008)	(0.009)	(0.008)
	1.6	0.811	0.867	0.798	0.854	0.777	0.834
		(0.009)	(0.008)	(0.009)	(0.008)	(0.009)	(0.008)
	1.75	0.840	0.902	0.801	0.848	0.776	0.833
		(0.008)	(0.007)	(0.009)	(0.008)	(0.009)	(0.008)
	1	0.808	0.857	0.833	0.874	0.749	0.815
		(0.009)	(0.008)	(0.008)	(0.007)	(0.010)	(0.009)
	1.15	0.823	0.879	0.836	0.882	0.775	0.836
		(0.009)	(0.007)	(0.008)	(0.007)	(0.009)	(0.008)
	1.3	0.828	0.884	0.822	0.871	0.779	0.843
500		(0.008)	(0.007)	(0.009)	(0.008)	(0.009)	(0.008)
	1.45	0.826	0.881	0.805	0.849	0.786	0.854
		(0.008)	(0.007)	(0.009)	(0.008)	(0.009)	(0.008)
	1.6	0.821	0.884	0.778	0.830	0.778	0.849
		(0.009)	(0.007)	(0.009)	(0.008)	(0.009)	(0.008)
	1.75	0.828	0.886	0.768	0.818	0.782	0.852
		(0.008)	(0.007)	(0.009)	(0.009)	(0.009)	(0.008)
	1	0.832	0.882	0.845	0.895	0.824	0.871
		(0.008)	(0.007)	(0.008)	(0.007)	(0.009)	(0.007)
	1.15	0.850	0.901	0.839	0.884	0.835	0.888
		(0.008)	(0.007)	(0.008)	(0.007)	(0.008)	(0.007)
	1.3	0.844	0.898	0.814	0.868	0.842	0.901
1000		(0.008)	(0.007)	(0.009)	(0.008)	(0.008)	(0.007)
	1.45	0.851	0.906	0.784	0.840	0.845	0.896
		(0.008)	(0.007)	(0.009)	(0.008)	(0.008)	(0.007)
	1.6	0.851	0.904	0.744	0.809	0.841	0.896
		(0.008)	(0.007)	(0.010)	(0.009)	(0.008)	(0.007)
	1.75	0.846	0.897	0.720	0.782	0.842	0.894
		(0.008)	(0.007)	(0.010)	(0.009)	(0.008)	(0.007)

Continued on next page

Table B.6 – continued from previous page

T	Vol.	t-GAS		t-GARCH		GARCH	
	1	0.867	0.914	0.871	0.924	0.871	0.917
		(0.008)	(0.006)	(0.008)	(0.006)	(0.008)	(0.006)
	1.15	0.876	0.922	0.850	0.902	0.869	0.923
		(0.007)	(0.006)	(0.008)	(0.007)	(0.008)	(0.006)
	1.3	0.867	0.921	0.787	0.854	0.857	0.913
2500		(0.008)	(0.006)	(0.009)	(0.008)	(0.008)	(0.006)
	1.45	0.863	0.919	0.715	0.785	0.858	0.921
		(0.008)	(0.006)	(0.010)	(0.009)	(0.008)	(0.006)
	1.6	0.855	0.919	0.630	0.703	0.861	0.913
		(0.008)	(0.006)	(0.011)	(0.010)	(0.008)	0.006
	1.75	0.854	0.918	0.568	0.657	0.853	0.917
		(0.008)	(0.006)	(0.011)	(0.011)	(0.008)	(0.006)
	1	0.882	0.937	0.896	0.938	0.893	0.933
		(0.007)	(0.005)	(0.007)	(0.005)	(0.007)	(0.006)
	1.15	0.888	0.939	0.865	0.917	0.878	0.931
		(0.007)	(0.005)	(0.008)	(0.006)	(0.007)	(0.006)
	1.3	0.870	0.931	0.774	0.840	0.872	0.927
5000		(0.008)	(0.006)	(0.009)	(0.008)	(0.007)	(0.006)
	1.45	0.865	0.928	0.635	0.732	0.866	0.928
		(0.008)	(0.006)	(0.011)	(0.010)	(0.008)	(0.006)
	1.6	0.854	0.919	0.460	0.559	0.873	0.928
		(0.008)	(0.006)	(0.011)	(0.011)	(0.007)	(0.006)
	1.75	0.856	0.917	0.370	0.465	0.861	0.924
		(0.008)	(0.006)	(0.011)	(0.011)	(0.008)	(0.006)
	1	0.882	0.941	0.905	0.952	0.893	0.938
		(0.007)	(0.005)	(0.007)	(0.005)	(0.007)	(0.005)
	1.15	0.882	0.938	0.863	0.910	0.871	0.933
		(0.007)	(0.005)	(0.008)	(0.006)	(0.008)	(0.006)
	1.3	0.859	0.918	0.696	0.777	0.862	0.932
10000		(0.008)	(0.006)	(0.010)	(0.009)	(0.008)	(0.006)
	1.45	0.849	0.918	0.482	0.578	0.857	0.924
		(0.008)	(0.006)	(0.011)	(0.011)	(0.008)	(0.006)
	1.6	0.846	0.911	0.256	0.345	0.855	0.919
		(0.008)	(0.006)	(0.010)	(0.011)	(0.008)	(0.006)
	1.75	0.840	0.902	0.168	0.236	0.845	0.914
		(0.008)	(0.007)	(0.008)	(0.009)	(0.008)	(0.006)

Table B.7: Effect of a change in A on ω of models with leverage effects

Number of Periods	Volatility	EGARCH(1,1)		Beta-t-EGARCH(1,1)	
		Mean est.	Std. Dev.	Mean est.	Std. Dev.
	1	-0.01	0.06	0.04	0.12
	1.15	-0.01	0.06	0.03	0.12
250	1.3	-0.01	0.06	0.03	0.11
	1.45	-0.01	0.06	0.03	0.11
	1.6	-0.01	0.06	0.03	0.11
	1.75	-0.01	0.06	0.02	0.11
	1	0.00	0.03	0.02	0.08
	1.15	0.00	0.03	0.02	0.07
500	1.3	0.00	0.03	0.02	0.07
	1.45	0.00	0.03	0.01	0.06
	1.6	0.00	0.03	0.01	0.06
	1.75	0.00	0.03	0.01	0.06
	1	0.00	0.02	0.01	0.04
	1.15	0.00	0.02	0.01	0.04
1000	1.3	0.00	0.02	0.01	0.04
	1.45	0.00	0.02	0.01	0.04
	1.6	0.00	0.02	0.01	0.04
	1.75	0.00	0.02	0.01	0.04
	1	0.00	0.01	0.00	0.02
	1.15	0.00	0.01	0.00	0.02
2500	1.3	0.00	0.01	0.00	0.02
	1.45	0.00	0.01	0.00	0.02
	1.6	0.00	0.01	0.00	0.02
	1.75	0.00	0.01	0.00	0.02
	1	0.00	0.01	0.00	0.01
	1.15	0.00	0.01	0.00	0.01
5000	1.3	0.00	0.01	0.00	0.02
	1.45	0.00	0.01	0.00	0.02
	1.6	0.00	0.01	0.00	0.02
	1.75	0.00	0.01	0.00	0.02
	1	0.00	0.00	0.00	0.01
	1.15	0.00	0.00	0.00	0.01
10000	1.3	0.00	0.00	0.00	0.01
	1.45	0.00	0.00	0.00	0.01
	1.6	0.00	0.00	0.00	0.01
	1.75	0.00	0.00	0.00	0.01

Table B.8: Effect of a change in A on the conditional coverage of ω

T	Vol.	EGARCH		Beta-t-EGARCH	
		90%	95%	90%	95%
	1	0.897	0.930	0.932	0.963
		(0.007)	(0.006)	(0.006)	(0.004)
	1.15	0.907	0.938	0.924	0.958
		(0.007)	(0.005)	(0.006)	(0.004)
	1.3	0.898	0.941	0.925	0.963
250		(0.007)	(0.005)	(0.006)	(0.004)
	1.45	0.905	0.946	0.917	0.961
		(0.007)	(0.005)	(0.006)	(0.004)
	1.6	0.903	0.943	0.915	0.957
		(0.007)	(0.005)	(0.006)	(0.005)
	1.75	0.902	0.946	0.915	0.961
		(0.007)	(0.005)	(0.006)	(0.004)
	1	0.929	0.962	0.933	0.971
		(0.006)	(0.004)	(0.006)	(0.004)
	1.15	0.934	0.967	0.924	0.963
		(0.006)	(0.004)	(0.006)	(0.004)
	1.3	0.936	0.967	0.917	0.962
500		(0.005)	(0.004)	(0.006)	(0.004)
	1.45	0.936	0.969	0.912	0.957
		(0.005)	(0.004)	(0.006)	(0.005)
	1.6	0.938	0.966	0.914	0.957
		(0.005)	(0.004)	(0.006)	(0.005)
	1.75	0.936	0.967	0.908	0.953
		(0.005)	(0.004)	(0.006)	(0.005)
	1	0.928	0.969	0.927	0.966
		(0.006)	(0.004)	(0.006)	(0.004)
	1.15	0.925	0.967	0.916	0.959
		(0.006)	(0.004)	(0.006)	(0.004)
	1.3	0.923	0.964	0.909	0.952
1000		(0.006)	(0.004)	(0.006)	(0.005)
	1.45	0.919	0.967	0.900	0.948
		(0.006)	(0.004)	(0.007)	(0.005)
	1.6	0.923	0.964	0.900	0.944
		(0.006)	(0.004)	(0.007)	(0.005)
	1.75	0.917	0.961	0.895	0.939
		(0.006)	(0.004)	(0.007)	(0.005)

Continued on next page

Table B.8 – continued from previous page

T	Vol.	EGARCH		Beta-t-EGARCH	
	1	0.917	0.960	0.920	0.965
		(0.006)	(0.004)	(0.006)	(0.004)
	1.15	0.918	0.959	0.901	0.957
		(0.006)	(0.004)	(0.007)	(0.005)
	1.3	0.910	0.955	0.887	0.953
2500		(0.006)	(0.005)	(0.007)	(0.005)
	1.45	0.907	0.954	0.888	0.947
		(0.006)	(0.005)	(0.007)	(0.005)
	1.6	0.911	0.952	0.886	0.948
		(0.006)	(0.005)	(0.007)	(0.005)
	1.75	0.910	0.953	0.886	0.948
		(0.006)	(0.005)	(0.007)	(0.005)
	1	0.913	0.958	0.919	0.963
		(0.006)	(0.005)	(0.006)	(0).004
	1.15	0.901	0.955	0.905	0.951
		(0.007)	(0.005)	(0.007)	(0.005)
	1.3	0.899	0.948	0.892	0.946
5000		(0.007)	(0.005)	(0.007)	(0.005)
	1.45	0.897	0.949	0.890	0.944
		(0.007)	(0.005)	(0.007)	(0.005)
	1.6	0.890	0.948	0.883	0.939
		(0.007)	(0.005)	(0.007)	(0.005)
	1.75	0.884	0.949	0.882	0.936
		(0.007)	(0.005)	(0.007)	(0.005)
	1	0.896	0.947	0.914	0.967
		(0.007)	(0.005)	(0.006)	(0.004)
	1.15	0.884	0.941	0.891	0.949
		(0.007)	(0.005)	(0.007)	(0.005)
	1.3	0.876	0.939	0.883	0.939
10000		(0.007)	(0.005)	(0.007)	(0.005)
	1.45	0.872	0.935	0.884	0.935
		(0.007)	(0.006)	(0.007)	(0.006)
	1.6	0.865	0.931	0.884	0.936
		(0.008)	(0.006)	(0.007)	(0.005)
	1.75	0.862	0.929	0.880	0.933
		(0.008)	(0.006)	(0.007)	(0.006)

Table B.9: Effect of a change in A on η

T	Size of break	t-GAS		t-GARCH		Beta-t-EGARCH	
		Mean est.	Std.Dev.	Mean est.	Std.Dev.	Mean est.	Std.Dev.
250	0	6.38	4.10	6.43	4.12	6.52	4.40
	0.05	6.35	4.06	6.36	4.12	6.54	4.43
	0.08	6.28	3.92	6.29	4.09	6.53	4.42
	0.1	6.26	3.88	6.23	4.09	6.51	4.38
	0.12	6.20	3.77	6.17	4.05	6.48	4.35
	0.13	6.18	3.76	6.13	4.01	6.46	4.32
500	0	5.56	1.89	5.57	1.91	5.58	1.99
	0.05	5.54	1.88	5.52	1.91	5.58	1.99
	0.08	5.49	1.80	5.46	1.91	5.57	1.99
	0.1	5.46	1.76	5.41	1.90	5.55	1.97
	0.12	5.43	1.72	5.35	1.84	5.53	1.96
	0.13	5.41	1.70	5.32	1.81	5.52	1.95
1000	0	5.27	0.99	5.27	1.00	5.27	0.99
	0.05	5.25	0.97	5.24	0.99	5.27	0.98
	0.08	5.22	0.95	5.18	0.97	5.25	0.97
	0.1	5.20	0.94	5.14	0.95	5.24	0.97
	0.12	5.17	0.93	5.10	0.94	5.23	0.96
	0.13	5.15	0.92	5.07	0.93	5.22	0.96
2500	0	5.08	0.52	5.09	0.53	5.08	0.52
	0.05	5.07	0.52	5.05	0.52	5.08	0.52
	0.08	5.04	0.51	5.00	0.52	5.06	0.51
	0.1	5.02	0.50	4.97	0.51	5.05	0.51
	0.12	5.00	0.50	4.93	0.50	5.04	0.50
	0.13	4.98	0.49	4.91	0.50	5.03	0.50
5000	0	5.05	0.36	5.05	0.37	5.05	0.36
	0.05	5.03	0.36	5.02	0.37	5.04	0.36
	0.08	5.01	0.35	4.97	0.36	5.03	0.36
	0.1	4.99	0.35	4.94	0.36	5.02	0.35
	0.12	4.96	0.34	4.90	0.35	5.00	0.35
	0.13	4.94	0.34	4.88	0.35	5.00	0.35
10000	0	5.02	0.25	5.02	0.25	5.02	0.25
	0.05	5.00	0.25	4.99	0.25	5.01	0.25
	0.08	4.98	0.24	4.94	0.24	5.00	0.24
	0.1	4.96	0.24	4.91	0.24	4.99	0.24
	0.12	4.93	0.24	4.87	0.24	4.97	0.24
	0.13	4.92	0.24	4.85	0.24	4.97	0.24

Table B.10: Effect of a change in A on the conditional coverage of η

T	Vol.	t-GAS		t-GARCH		GARCH	
		90%	95%	90%	95%	90%	95%
	0	0.920	0.945	0.917	0.942	0.909	0.931
		(0.006)	(0.005)	(0.006)	(0.005)	(0.006)	(0.006)
	0.05	0.917	0.945	0.908	0.936	0.906	0.930
		(0.006)	(0.005)	(0.006)	(0.005)	(0.007)	(0.006)
	0.08	0.907	0.935	0.904	0.930	0.908	0.935
250		(0.007)	(0.006)	(0.007)	(0.006)	(0.006)	(0.006)
	0.1	0.907	0.938	0.899	0.923	0.906	0.931
		(0.007)	(0.005)	(0.007)	(0.006)	(0.007)	(0.006)
	0.12	0.902	0.931	0.893	0.921	0.908	0.931
		(0.007)	(0.006)	(0.007)	(0.006)	(0.006)	(0.006)
	0.13	0.901	0.932	0.887	0.918	0.909	0.932
		(0.007)	(0.006)	(0.007)	(0.006)	(0.006)	(0.006)
	0	0.923	0.953	0.928	0.950	0.923	0.950
		(0.006)	(0.005)	(0.006)	(0.005)	(0.006)	(0.005)
	0.05	0.918	0.952	0.926	0.948	0.921	0.950
		(0.006)	(0.005)	(0.006)	(0.005)	(0.006)	(0.005)
	0.08	0.899	0.952	0.915	0.942	0.920	0.954
500		(0.007)	(0.005)	(0.006)	(0.005)	(0.006)	(0.005)
	0.1	0.905	0.942	0.910	0.942	0.915	0.955
		(0.007)	(0.005)	(0.006)	(0.005)	(0.006)	(0.005)
	0.12	0.900	0.941	0.902	0.931	0.915	0.955
		(0.007)	(0.005)	(0.007)	(0.006)	(0.006)	(0.005)
	0.13	0.901	0.937	0.900	0.930	0.913	0.952
		(0.007)	(0.005)	(0.007)	(0.006)	(0.006)	(0.005)
	0	0.895	0.942	0.911	0.952	0.883	0.939
		(0.007)	(0.005)	(0.006)	(0.005)	(0.007)	(0.005)
	0.05	0.893	0.948	0.907	0.952	0.873	0.938
		(0.007)	(0.005)	(0.006)	(0.005)	(0.007)	(0.005)
	0.08	0.880	0.935	0.905	0.944	0.878	0.939
1000		(0.007)	(0.006)	(0.007)	(0.005)	(0.007)	(0.005)
	0.1	0.874	0.929	0.899	0.940	0.883	0.938
		(0.007)	(0.006)	(0.007)	(0.005)	(0.007)	(0.005)
	0.12	0.873	0.930	0.897	0.934	0.880	0.934
		(0.007)	(0.006)	(0.007)	(0.006)	(0.007)	(0.006)
	0.13	0.872	0.924	0.893	0.929	0.873	0.931
		(0.007)	(0.006)	(0.007)	(0.006)	(0.007)	(0.006)

Continued on next page

Table B.10 – continued from previous page

T	Vol.	t-GAS		t-GARCH		GARCH	
	0	0.892	0.947	0.896	0.954	0.872	0.932
		(0.007)	(0.005)	(0.007)	(0.005)	(0.007)	(0.006)
	0.05	0.891	0.948	0.897	0.949	0.869	0.934
		(0.007)	(0.005)	(0.007)	(0.005)	(0.008)	(0.006)
	0.08	0.868	0.933	0.883	0.943	0.873	0.936
2500		(0.008)	(0.006)	(0.007)	(0.005)	(0.007)	(0.005)
	0.1	0.864	0.932	0.881	0.936	0.865	0.933
		(0.008)	(0.006)	(0.007)	(0.005)	(0.008)	(0.006)
	0.12	0.866	0.932	0.865	0.925	0.866	0.933
		(0.008)	(0.006)	(0.008)	(0.006)	(0.008)	(0.006)
	0.13	0.859	0.925	0.861	0.920	0.864	0.931
		(0.008)	(0.006)	(0.008)	(0.006)	(0.008)	(0.006)
	0	0.885	0.938	0.889	0.941	0.866	0.926
		(0.007)	(0.005)	(0.007)	(0.005)	(0.008)	(0.006)
	0.05	0.887	0.943	0.891	0.936	0.856	0.921
		(0.007)	(0.005)	(0.007)	(0.005)	(0.008)	(0.006)
	0.08	0.858	0.928	0.870	0.931	0.848	0.915
5000		(0.008)	(0.006)	(0.008)	(0.006)	(0.008)	(0.006)
	0.1	0.856	0.924	0.857	0.919	0.844	0.914
		(0.008)	(0.006)	(0.008)	(0.006)	(0.008)	(0.006)
	0.12	0.846	0.911	0.848	0.905	0.846	0.909
		(0.008)	(0.006)	(0.008)	(0.007)	(0.008)	(0.006)
	0.13	0.846	0.905	0.836	0.889	0.842	0.904
		(0.008)	(0.007)	(0.008)	(0.007)	(0.008)	(0.007)
	0	0.883	0.942	0.899	0.947	0.861	0.925
		(0.007)	(0.005)	(0.007)	(0.005)	(0.008)	(0.006)
	0.05	0.882	0.941	0.894	0.943	0.846	0.916
		(0.007)	(0.005)	(0.007)	(0.005)	(0.008)	(0.006)
	0.08	0.860	0.920	0.874	0.923	0.840	0.910
10000		(0.008)	(0.006)	(0.007)	(0.006)	(0.008)	(0.006)
	0.1	0.839	0.918	0.847	0.906	0.829	0.905
		(0.008)	(0.006)	(0.008)	(0.007)	(0.008)	(0.007)
	0.12	0.828	0.894	0.817	0.882	0.821	0.898
		(0.008)	(0.007)	(0.009)	(0.007)	(0.009)	(0.007)
	0.13	0.817	0.896	0.795	0.869	0.816	0.894
		(0.009)	(0.007)	(0.009)	(0.008)	(0.009)	(0.007)

B.3 Impact of ignoring parameter changes on $\hat{B}$ for alternative models

Table B.11: Effect of a break in parameters on $\hat{B}$ of t-GAS model with B = 0.5

T	Break Size	ω		A	
		Mean est.	Std. Dev.	Mean est.	Std. Dev.
250	No break	0.47	0.29	0.47	0.29
	Medium	0.66	0.32	0.49	0.22
	Large	0.83	0.25	0.49	0.21
500	No break	0.46	0.26	0.46	0.26
	Medium	0.74	0.28	0.48	0.18
	Large	0.92	0.17	0.48	0.17
1000	No break	0.47	0.22	0.47	0.22
	Medium	0.82	0.21	0.48	0.15
	Large	0.98	0.08	0.48	0.14
2500	No break	0.48	0.15	0.48	0.15
	Medium	0.90	0.15	0.48	0.10
	Large	1.00	0.01	0.48	0.10
5000	No break	0.49	0.10	0.49	0.10
	Medium	0.95	0.10	0.48	0.07
	Large	1.00	0.00	0.48	0.07
10000	No break	0.50	0.07	0.50	0.07
	Medium	0.97	0.07	0.49	0.05
	Large	1.00	0.00	0.48	0.05

Table B.12: Effect of a break in parameters on $\hat{B}$ for the alternative EGARCH model

Number of Periods	Break Size	ω		A		B	
		Mean est.	Std. Dev.	Mean est.	Std. Dev.	Mean est.	Std. Dev.
250	No break	0.66	0.36	0.66	0.36	0.66	0.36
	Medium	0.81	0.30	0.72	0.31	0.76	0.31
	Large	0.91	0.20	0.75	0.29	0.85	0.26
500	No break	0.72	0.27	0.72	0.27	0.72	0.27
	Medium	0.92	0.15	0.79	0.20	0.85	0.17
	Large	0.98	0.06	0.83	0.17	0.94	0.11
1000	No break	0.76	0.16	0.76	0.16	0.76	0.16
	Medium	0.96	0.07	0.83	0.11	0.89	0.09
	Large	0.99	0.01	0.87	0.09	0.97	0.03
2500	No break	0.79	0.08	0.79	0.08	0.79	0.08
	Medium	0.99	0.02	0.85	0.06	0.91	0.04
	Large	1.00	0.00	0.88	0.05	0.97	0.02
5000	No break	0.79	0.05	0.79	0.05	0.79	0.05
	Medium	0.99	0.01	0.86	0.04	0.91	0.03
	Large	1.00	0.00	0.89	0.03	0.98	0.01
10000	No break	0.80	0.03	0.80	0.03	0.80	0.03
	Medium	1.00	0.01	0.86	0.03	0.91	0.02
	Large	1.00	0.00	0.89	0.02	0.98	0.01

Appendix C

Chapter 4

C.1 Checking for non-stationarity

C.1.1 Autocorrelation and Partial Autocorrelations

Figure C.1: Autocorrelation of the Oil Prices

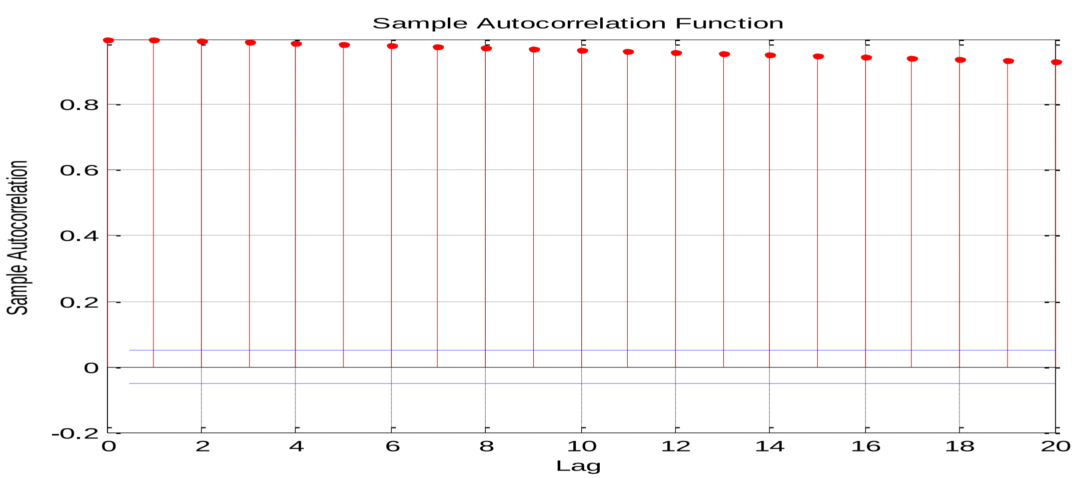


Figure C.2: Partial Autocorrelation of the Oil Prices

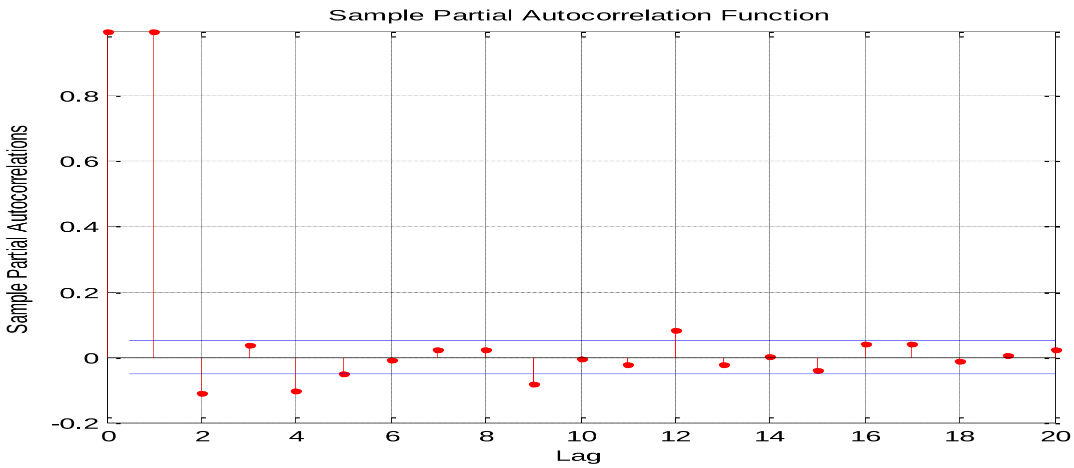


Figure C.3: Autocorrelation of the US Trade Weighted Index

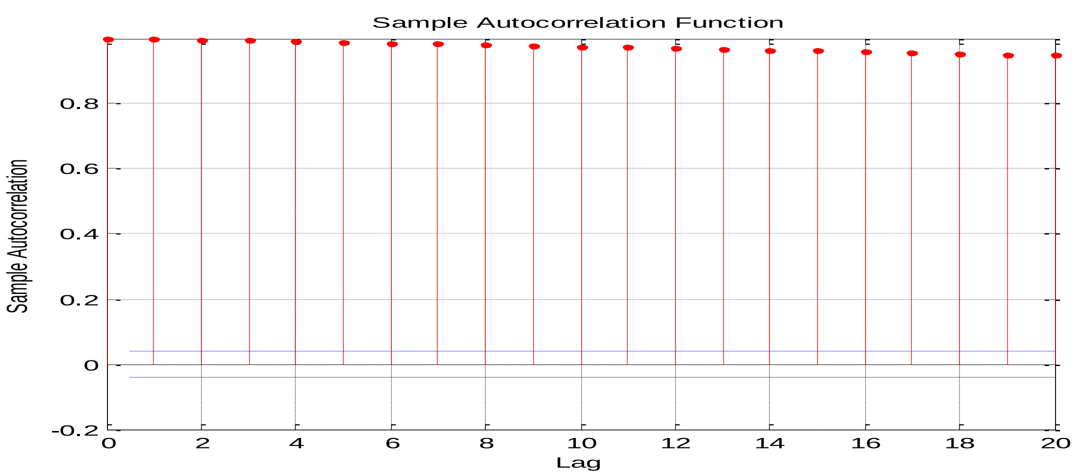
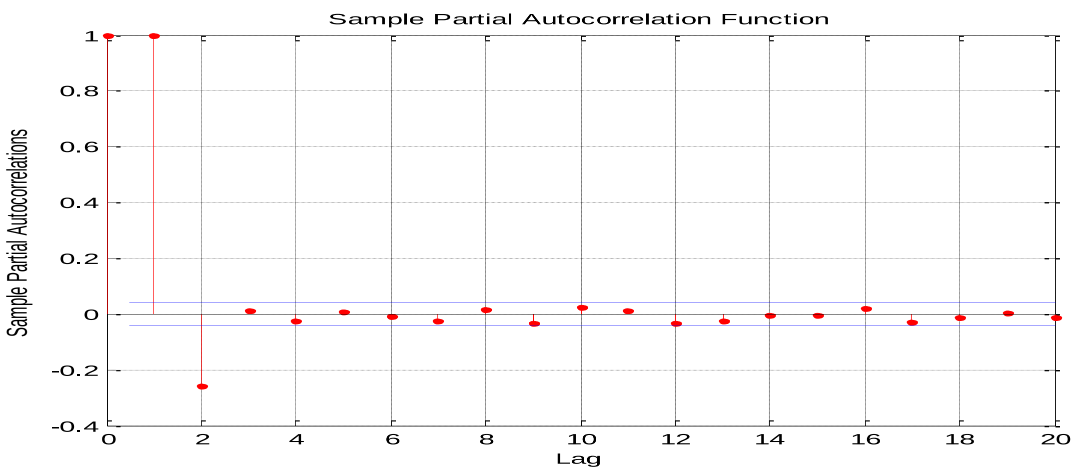


Figure C.4: Partial Autocorrelation of the US Trade Weighted Index



C.1.2 Unit Root Tests

This section shows that both oil prices and USD trade-weighted index contain a unit root. Three types of unit root tests used are the Augmented Dickey-Fuller (ADF) test by Dickey and Fuller (1981) and the Phillips-Perron (PP) test by Phillips and Perron (1988). In the case of the ADF and PP tests, the null hypothesis states that the time series of interest contains a unit root. The alternative is that the series does not contain a unit root. As for the KPSS test, the null hypothesis is that the series is I(0) or stationary. Table C.1 presents the results.

Table C.1: Unit root tests by series

Variables/Unit Root Test	ADF	PP	KPSS
Oil Prices	-1.61	-1.29	2.92
USD Trade-Weighted Index	-1.81	-1.47	2.28
Oil Returns	-9.10	-35.77	0.07
USD Returns	-17.25	-36.77	0.08

Notes: The drift but no trend case is applied to oil prices and USD Trade-Weighted Index and the no drift, no trend case is applied to oil and USD returns. For the ADF and PP tests, the critical values at the 5% level are -2.86 and -3.41 for the drift but no trend and no drift, no trend cases respectively. For the KPSS test, critical value at the 5% level for the no trend case is 0.446.

The distribution for the test is the t-distribution and the values for the ADF tests are found in Fuller (1976). For oil prices, the appropriate number of lags is 3 and the ADF test statistic is -1.61. In the case of the USD trade-weighted index, the number of appropriate lags is 1 and the ADF test statistic is -1.81. The ADF test statistics for both series are larger than the critical value of -3.41,at the 5% level. This implies that the null hypothesis that the series contain a unit root cannot be rejected in favour of the alternative, meaning that the oil price and USD trade-weighted index series are not stationary. These results are also consistent with the PP tests because the PP test statistics for both series are also larger than the critical value at the 5% level. In the case of the KPSS test, the null hypothesis of stationarity can rejected for both series.

The first difference of the log of oil prices and log of the USD trade-weighted index are constructed to make the data stationary. The former is called oil returns and the latter is called USD returns. Table C.1 also show the test statistics for both series. These results indicate that the series are stationary because the null hypothesis that the series contains a unit root is rejected in both the ADF and PP tests. Also, using the KPSS test, the null hypothesis that the series is stationary cannot be rejected for both oil and USD returns.

C.2 Supplementary Data Information

Figures C.5 and C.6 display the oil return squared residuals and autocorrelations of the AR(1) model of oil prices. Figure C.5 also shows some signs of volatility clustering. This same figure reveals an especially

Figure C.5: Squared residuals of oil returns

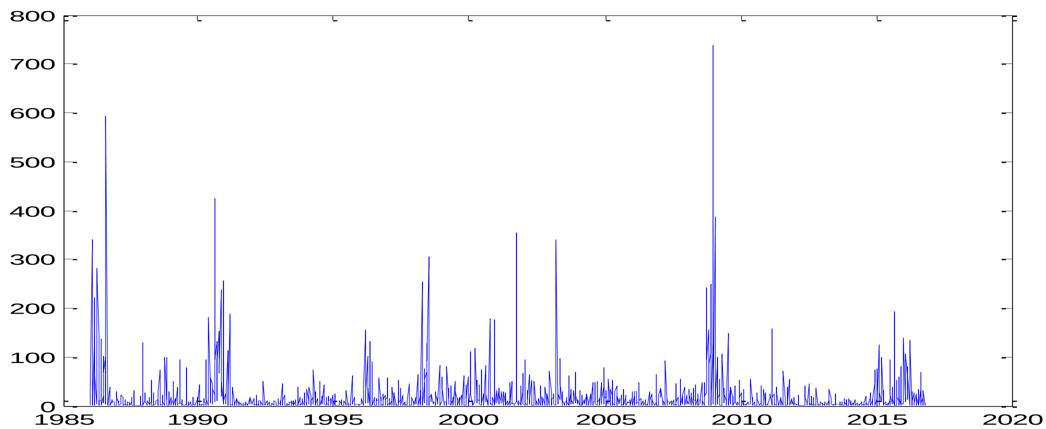
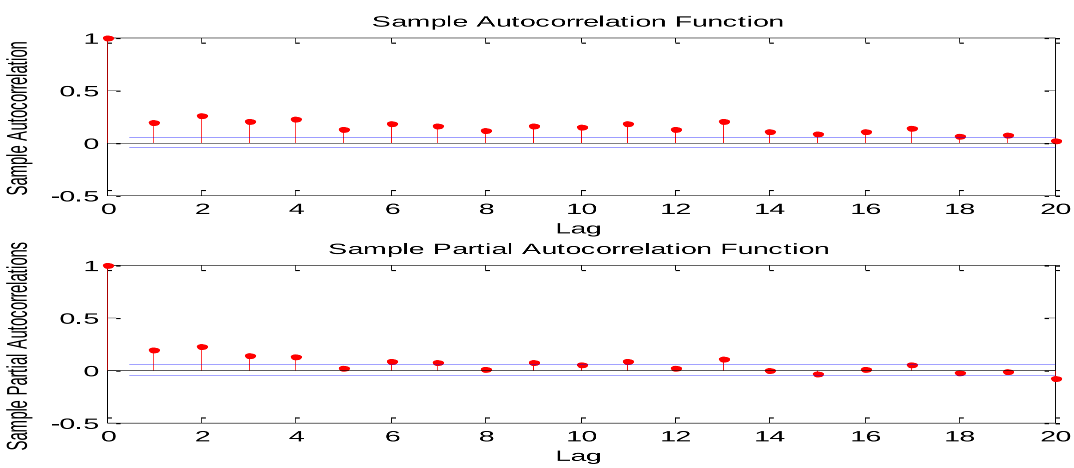


Figure C.6: Autocorrelation and partial autocorrelation of oil return squared residuals



large spike around 2008, which points to the effects of the 2008-2009 recession. Figure C.6 also displays statistically significant evidence of clustering, as there are significant autocorrelation for up to 7 lags and from lags 11 to 13. Turning to partial autocorrelations, the first 7 lags (except for lag 4) are significant as well as lags 11 and 13. Thus, there are also signs of conditional heteroskedasticity in the oil returns residuals.

Figures C.7 and C.8 display the USD return squared residuals and autocorrelation of the AR(1) model of USD returns. These results show that volatility clustering also occurs in USD returns. There are some large spikes that occur at the beginning of the sample as well as in the late 1970s and in 2008-2009 period.

Figure C.7: Squared residuals of USD returns

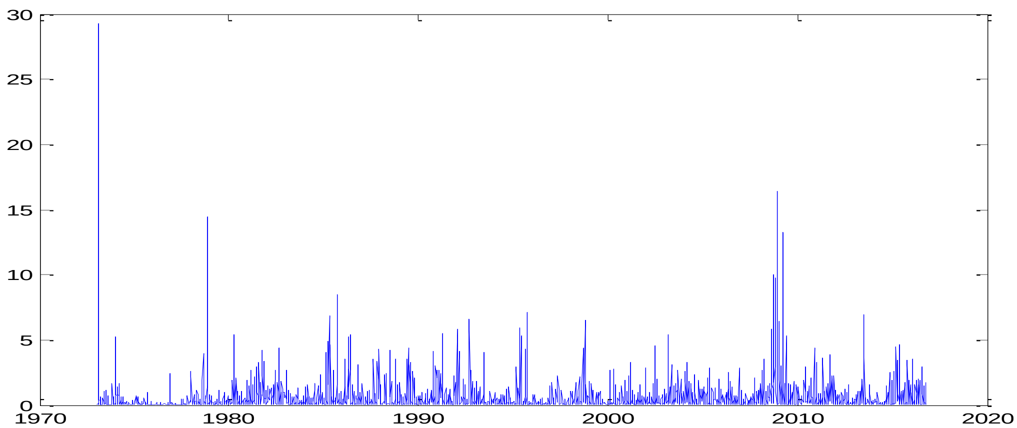
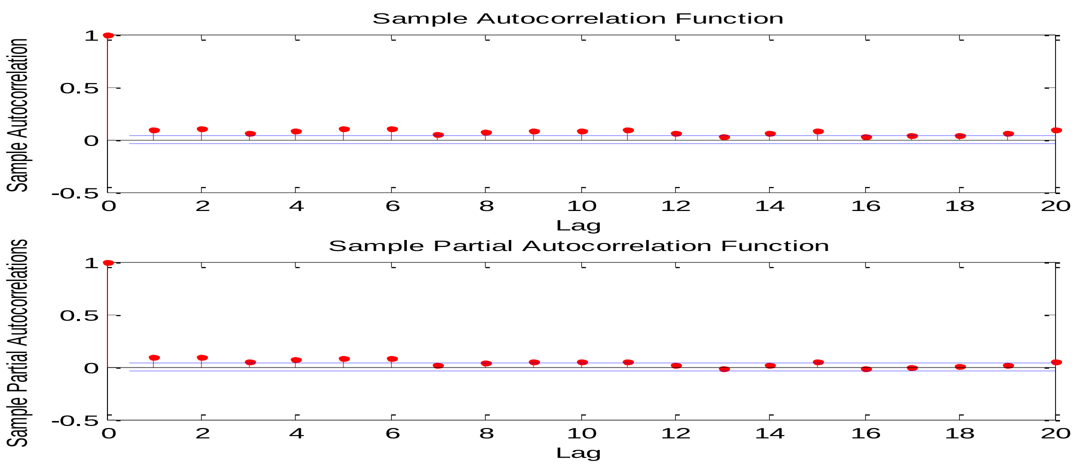


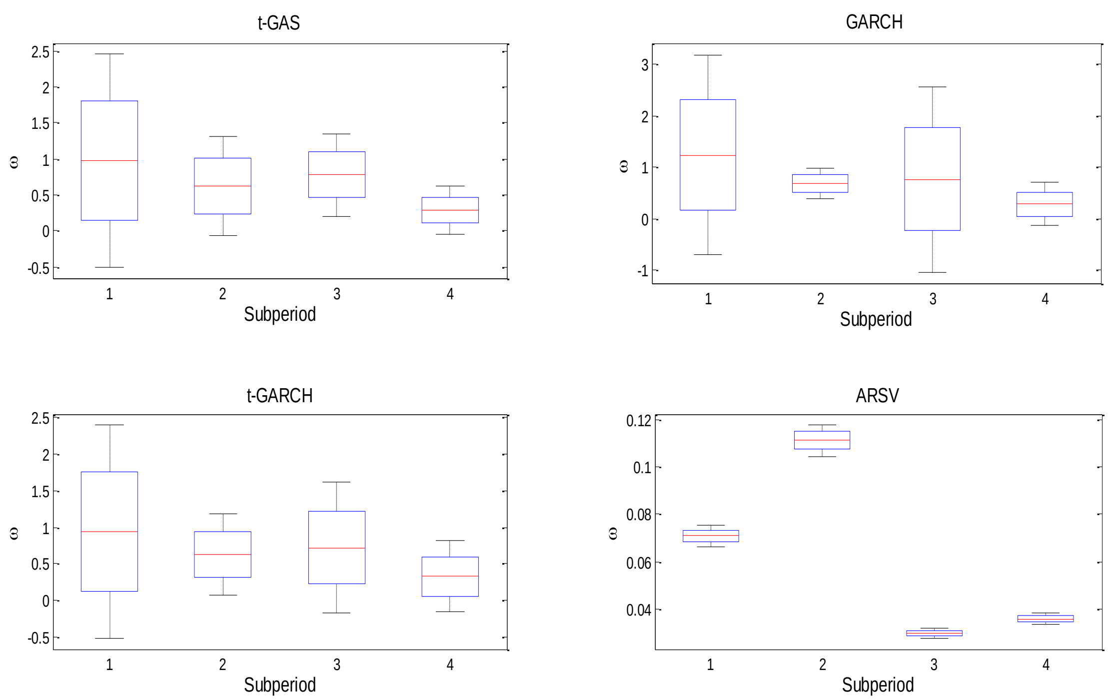
Figure C.8: Autocorrelation and partial autocorrelation of USD return squared residuals



According to figure C.8, significant autocorrelation occurs from lags 1 to 12 and at lags 14, 15, and 20. There is also significant partial autocorrelations at lags 1 to 6.

C.3 Alternative figures for the application of Dufour and Torres's (1998) test

Figure C.9: 90% confidence intervals for ω by sub-period – oil returns



Note: The null hypothesis that ω is not statistically different from each other can be rejected if there is no intersection that falls between the ranges.

Figure C.10: 90% confidence intervals for A by sub-period – oil returns

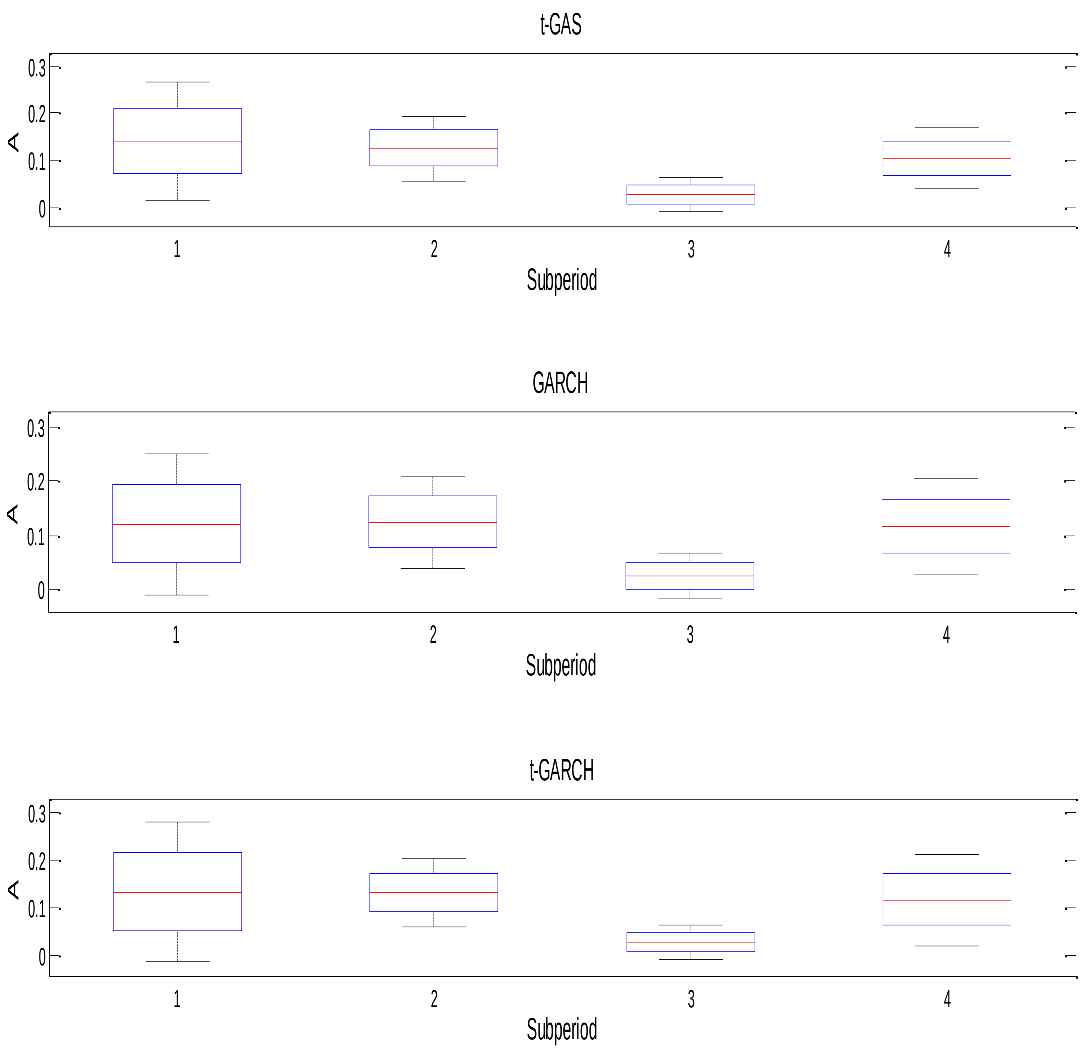


Figure C.11: 90% confidence intervals for ω by sub-period - USD returns

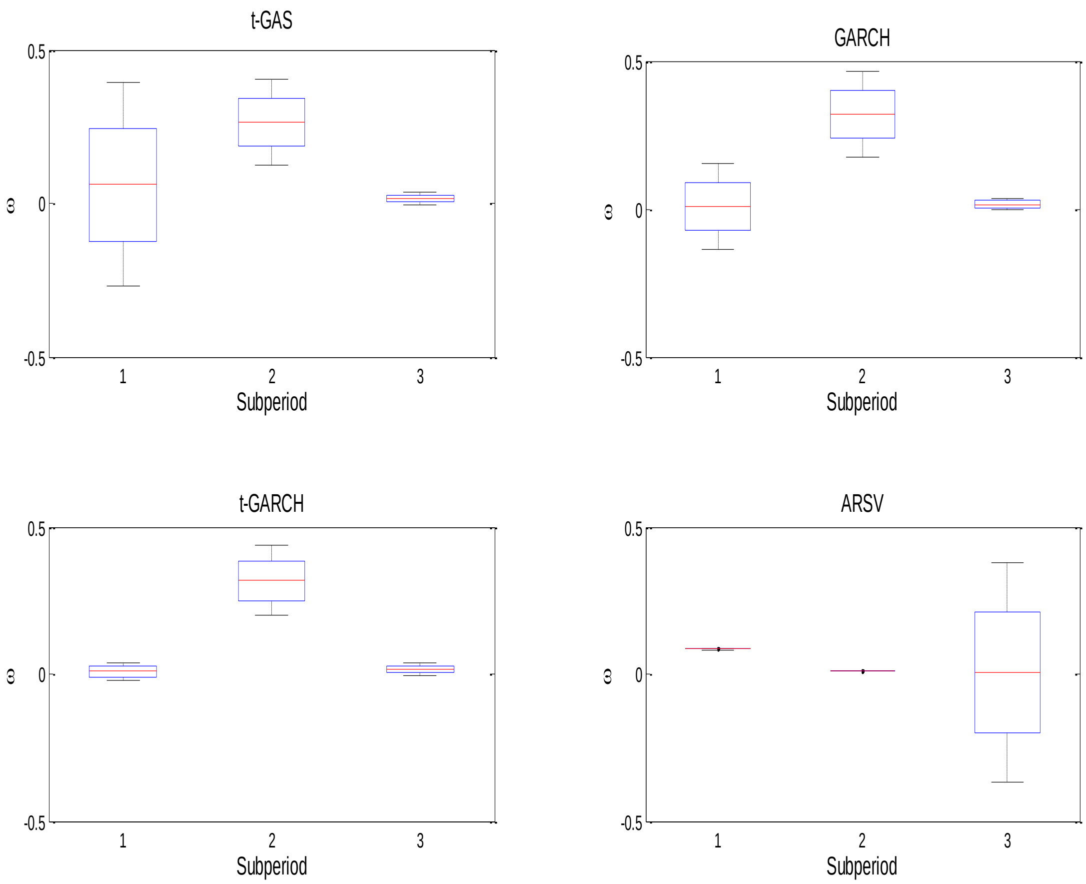
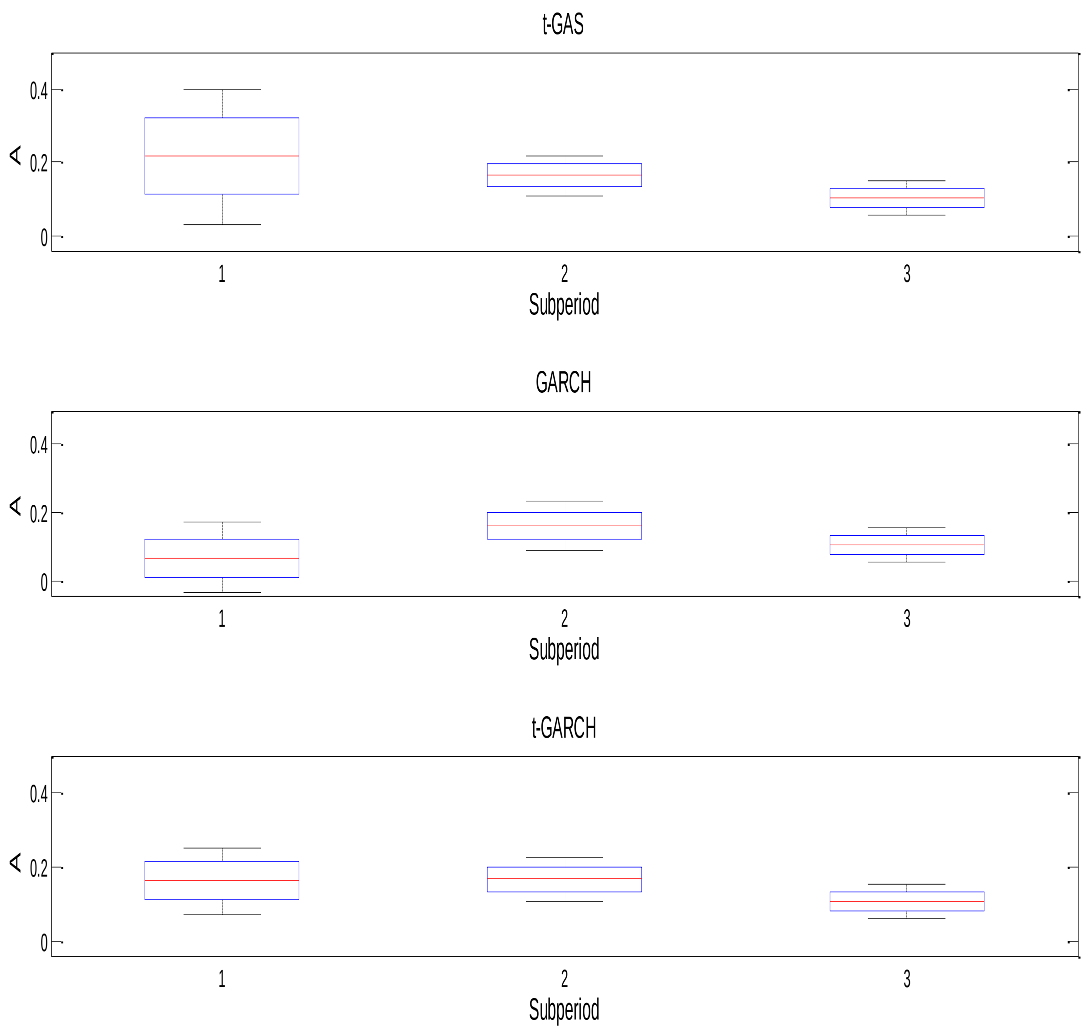


Figure C.12: 90% confidence intervals for A by sub-period - USD returns



C.4 Using Patton's (2011) loss functions for evaluating variance forecasts

Patton (2011) proposed the following family of robust loss functions:

$$
\begin{aligned}
L(\widetilde{\sigma_t^2}, h_t; b) &= \tfrac{1}{(b+1)(b+2)}(\widetilde{\sigma}_t^{2(b+2)} - \hat{\sigma}^{2(b+2)}) - \tfrac{1}{b+1}\hat{\sigma}^{2(b+2)}(\widetilde{\sigma}_t^2 - \hat{\sigma}_t^2), b \neq -1, -2 \\
&= \hat{\sigma}_t^2 - \widetilde{\sigma}_t^2 + \widetilde{\sigma}_t^2 \log \tfrac{\widetilde{\sigma}_t^2}{\hat{\sigma}_t^2}, b = -1 \\
&= \tfrac{\widetilde{\sigma}_t^2}{\hat{\sigma}_t^2} - \widetilde{\sigma}_t^2 \log \tfrac{\widetilde{\sigma}_t^2}{\hat{\sigma}_t^2}, b = -2
\end{aligned}
\tag{C.1}
$$

where $\hat{\sigma}_t^2$ is the conditional variance forecast in this paper. Setting b = -2 gives rise to the "QLIKE" loss function, while setting b = 0 yields the "MSE" loss function. These robust loss functions are then used to calculate the loss.

Once the loss has been calculated, the Diebold-Mariano-West (DMW) test is used to determine which forecast is best. The formula for the DMW statistic is:

$$DMW_t(b) = \frac{\sqrt{T}D(b)_T}{\sqrt{a\hat{v}ar[D(b)_T]}}, \tag{C.2}$$

where

$$D_t(b) = L(\widetilde{\sigma^2}_t, \hat{\sigma}^2_{A,t}; b) - L(\widetilde{\sigma^2}_t, \hat{\sigma}^2_{B,t}; b). \tag{C.3}$$

$\hat{\sigma}^2_{A,t}$ is the forecast generated by model A and $\hat{\sigma}^2_{B,t}$ is the forecast generated by model B. Equation (C.2) is the difference in the loss functions. If D_T is positive (negative), then model B (A) produces a better forecast. As mentioned in Patton and Sheppard (2009), the asymptotic variance of the average differences, $\widehat{avar}$, are computed using the Newey-West variance estimator.

Both the "QLIKE" and "MSE" are considered in this paper because Patton (2011) find that both functions are robust to imperfect proxies for the conditionally unbiased variance. Although, it is noted that Patton and Sheppard (2009) and Patton (2011) find that the "QLIKE" loss function produces the greatest power in conducting tests in comparing forecasts. For the DMW test, the null hypothesis, H_0 is that the forecasts have equal accuracy. The alternative hypothesis is that one forecast is more accurate than the other.