Energy Efficient Cooperative Communication

by

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Abstract

This dissertation studies several problems centered around developing a better understanding of the energy efficiency of cooperative wireless communication systems. Cooperative communication is a technique where two or more nodes in a wireless network pool their antenna resources to form a "virtual antenna array". Over the last decade, researchers have shown that many of the benefits of real antenna arrays, e.g. spatial diversity, increased range, and/or decreased transmission energy, can be achieved by nodes using cooperative transmission. This dissertation extends the current body of knowledge by providing a comprehensive study of the energy efficiency of two-source cooperative transmission under differing assumptions about channel state knowledge, cooperative protocol, and node selfishness.

The first part of this dissertation analyzes the effect of channel state information on the optimum energy allocation and energy efficiency of a simple cooperative transmission protocol called "orthogonal amplify-and-forward" (OAF). The source nodes are required to achieve a quality-of service (QoS) constraint, e.g. signal to noise ratio or outage probability, at the destination. Since a QoS constraint does not specify a unique transmit energy allocation when the nodes use OAF cooperative transmission, minimum total energy strategies are provided for both short-term and long-term QoS constraints. For independent Rayleigh fading channels, full knowledge of the channel state at both of the sources and at the destination is shown to significantly improve the energy efficiency of OAF cooperative transmission as well as direct (non-cooperative) transmission. The results also demonstrate how channel state knowledge affects the minimum total energy allocation strategy. Under identical channel state knowledge

assumptions, the results demonstrate that OAF cooperative transmission tends to have better energy efficiency than direct transmission over a wide range of channel conditions.

The second part of this dissertation focuses on the development of an opportunistic hybrid cooperative transmission protocol that achieves increased energy efficiency by not only optimizing the resource allocation but also by selecting the most energy efficient cooperative transmission protocol from a set of available protocols according to the current channel state. The protocols considered in the development of the hybrid cooperative transmission protocol include compress-and-forward (CF), estimate-and-forward (EF), non-orthogonal amplify-and-forward (NAF), and decode-and-forward (DF). Instantaneous capacity results are analyzed under the assumption of full channel state knowledge at both of the sources and the destination node. Numerical results are presented showing that the delay limited capacity and outage probability of the hybrid cooperative transmission protocol are superior to that of any single protocol and are also close to the cut-set bound over a wide range of channel conditions.

The final part of this dissertation focuses on the issue of node selfishness in cooperative transmission. It is common to assume in networks with a central authority, e.g. military networks, that nodes will always be willing to offer help to other nodes when requested to do so. This assumption may not be valid in ad hoc networks operating without a central authority. This section of the dissertation considers the effect selfish behavior on the energy efficiency of cooperative communication systems. Using tools from non-cooperative game theory, a two-player relaying game is formulated and analyzed in non-fading and fading channel scenarios. In non-fading channels, it is shown that a cooperative equilibrium can exist between two self-interested sources given that the end of the cooperative interaction is uncertain, that the sources can achieve mutual benefit through cooperation, and that the sources are sufficiently patient in the sense that they value future payoffs. In fading channels, a cooperative conditional trigger strategy is proposed and shown to be an equilibrium of the two-player game. Sources following this strategy are shown to achieve an energy efficiency very close to that of a centrally-controlled system when they are sufficiently patient. The results in this section show that cooperation can often be established between two purely self-interested sources without the development of extrinsic incentive mechanisms like virtual currency.

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Chapter 1

Introduction

This chapter begins with a discussion of the motivation and context for the problems considered in this dissertation. An overview of the specific technical problems is provided in Section 1.2 and a summary of the main contributions of this dissertation is included in Section 1.3.

1.1 Motivation

Multipath fading [Stu96] is a significant impairment in a wide range of wireless mobile communication systems including radio, television, cellular telephones, and wireless internet. As shown in Figure 1.1, multipath fading results from the simultaneous arrival of radio frequency plane waves reflected from local scatterers. In a system with L paths between the transmitter and the recevier, the signal at the receiver can be modeled as

$$r(t) = \sum_{\ell=1}^{L} \alpha_\ell(t) s(t - \tau_\ell)$$

where $s(t)$ represents the waveform from the transmitter and α_ℓ and τ_ℓ denote the amplitude gain and delay of the ℓ^{th} path between the transmitter and the receiver, respectively. Multipath fading can result in intersymbol interference if the delay spread of the channel, i.e. $\Delta = \max \tau_\ell - \min \tau_\ell$, is greater than a small fraction of the symbol period T_s. In this case, the fading is called "frequency selective fading". When $\Delta \ll T_s$, multipath fading is called "flat fading". Although intersymbol interference is negligible in flat fading channels, the wireless transmissions over different paths arrive at the receiver at different phases and may combine destructively so that the aggregate signal is received weakly.

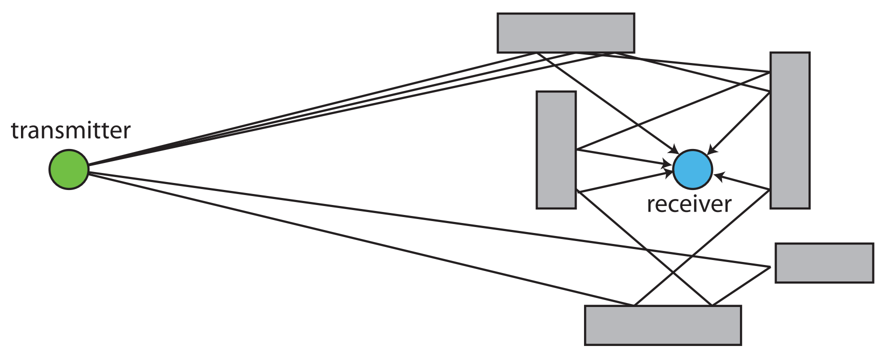

Figure 1.1: Multipath fading.

Figure 1.2 shows an example of a flat fading channel using a Jakes [Jak74] isotropic scattering fading channel simulator with mobile velocity 30 meters/second and carrier frequency 900MHz. The Matlab code used to generate this example is provided in Appendix A. This example clearly shows frequent fades in excess of 10dB with three deep fades of more than 30dB. It is clear that reliable communication is difficult to achieve in this type of channel. Moreover, energy efficient communication is difficult in systems with fading channels. If the transmitter does not know the channel state, significant excess transmission energy must be expended to provide enough link margin to overcome the frequent small fades. Even if the transmitter has knowledge of the channel state and can adjust its transmit power to ensure a target signal to noise ratio (SNR) at the receiver, the amount of dynamic range required to overcome the deep fades is often prohibitive in practical transmitters.

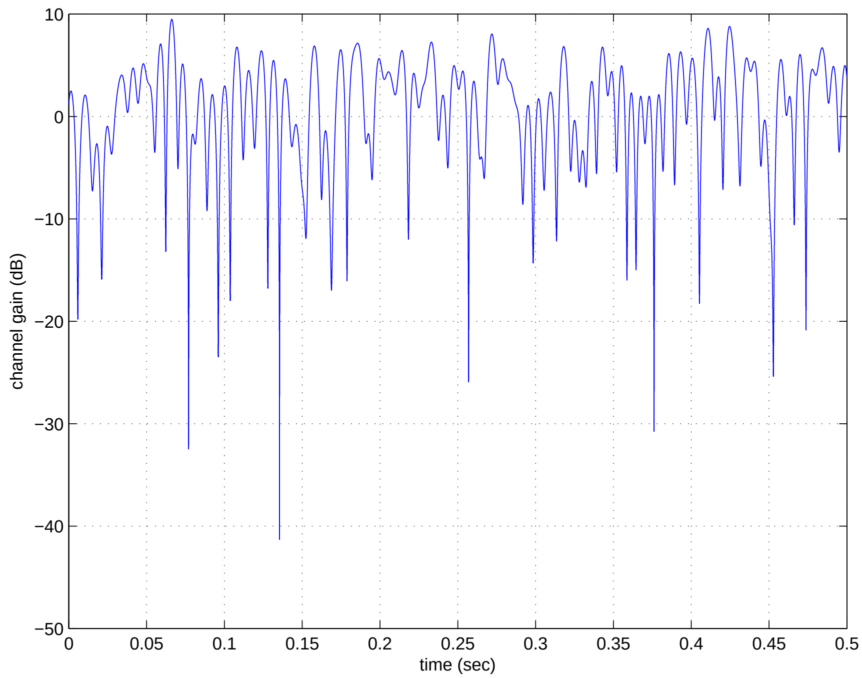

Figure 1.2: Flat fading example using a Jakes isotropic scattering fading channel simulator with mobile velocity 30 meters/second and carrier frequency 900MHz.

The key to reliable transmission over fading channels is *diversity* [Stu96]. Diversity techniques are based on the observation that the vast majority of reception errors occur when the channel is in a deep fade, e.g. at $t \approx 0.13$ seconds in Figure 1.2. If the transmitter can transmit its information over multiple channels, preferably ones with independent fading, then reliability is increased through the fact that it is less likely that multiple channels will experience simultaneous deep fades. Figure 1.3 shows an example of two independent flat fading channels with parameters identical to Figure 1.2. This example shows that, even though there are several individual channel fades in excess of 10dB, there are far fewer instances during which both channels are simultaneously in a fade greater than 10dB. If a receiver were able to simply select the better of the two channels, a technique called "selection diversity", the effective fading is reduced to that shown in the red dashed line in Figure 1.3 and reliability of the communication is significantly improved with respect to the channel shown in Figure 1.2.

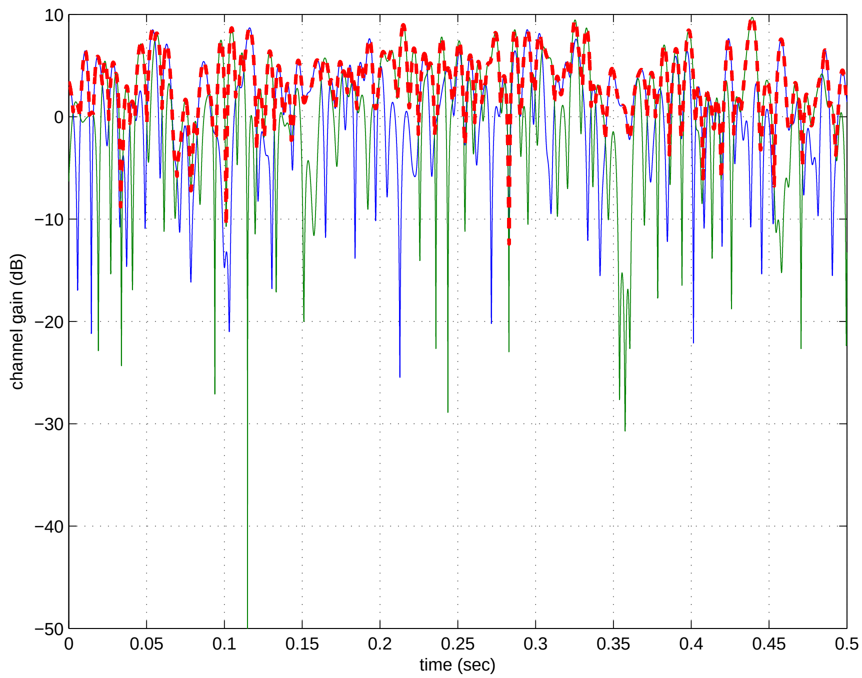

Figure 1.3: Independent flat fading example using a Jakes isotropic scattering fading channel simulator with mobile velocity 30 meters/second and carrier frequency 900MHz. The red dashed line follows the better of the two channels.

Depending on the application, diversity can be achieved through temporal (e.g., channel coding with interleaving), spectral (e.g., wideband signaling or frequency hopping), or spatial (e.g., multi antenna) techniques. Spatial diversity (see, e.g., [DADSC04]) is a particularly attractive technique in that it does not require additional bandwidth or a reduction of transmission rate. The downside of spatial diversity, however, is that it requires multiple antennas that must be separated by at least a few wavelengths in order to obtain signals that fade independently [Pro01]. At common transmission frequencies, the size of the required antenna array precludes its use in many applications.

Researchers recognized the fact that spatial diversity could be achieved in multiuser communication systems even if the nodes each have only one antenna. Sendonaris, Erkip, and Aazhang were the first to suggest the concept of *user cooperation diversity* where nearby users in a cellular system form cooperative "partnerships" by sharing their antennas to achieve increased rate or decreased outage probability in the uplink [SEA98]. Cooperation is motivated by the observation that uplink signals in a cellular system are omnidirectional and that these signals could be received and acted upon by other users in the system. Sendonaris, Erkip, and Aazhang describe specific CDMA cooperative protocols based on superposition block Markov encoding where the basic idea is that a cooperating user relays the information transmitted by its partner in the prior block. While the initial work in [SEA98] focused on transmit cooperation in cellular systems, the basic idea of cooperation extends easily to receive cooperation [Hos04] and ad-hoc networks [MYed]. Figure 1.4 shows the three canonical cooperative models.

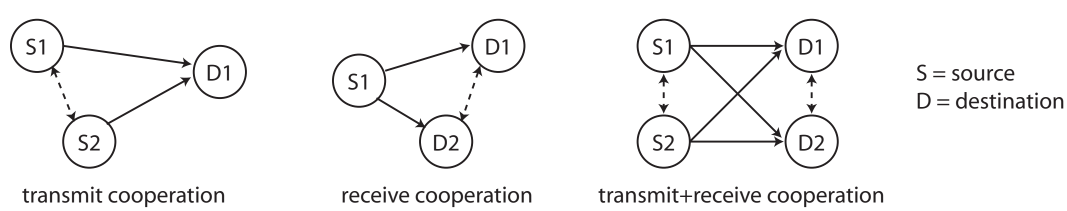

Figure 1.4: Canonical cooperative models. Cooperative links are shown as dashed lines. This dissertation is focused on two-source transmit cooperation.

Despite some apparent similarities, it is worth mentioning that wireless cooperative transmission has several fundamental differences with respect to conventional multi-hop transmission [Con07]. Multi-hop transmission is used extensively in wired networks and has also been applied to wireless mesh networks [Zha06]. The difference, however, between wireless cooperative transmission and wireless multi-hop transmission is that cooperative transmission explicitly leverages the undirected nature of single-antenna wireless transmission. When a source node transmits in a cooperative communication system, all of the nodes (including the destination) receive attenuated and noisy copies of the transmission. This is the key to obtaining diversity. As shown in Figure 1.5, multi-hop transmission typically does not take advantage of the source↔destination link. The source transmits to the relay and the relay forwards message to the destination. No diversity is obtained because the destination receives only one copy of the message. If the source↔relay link or the relay↔destination link is in a deep fade, then the message is likely to be received in error. The destination in a two-node cooperative transmission system, on the other hand, achieves diversity by receiving two (or more) copies of the message and optimally selecting or combining these copies to improve the resulting signal quality.

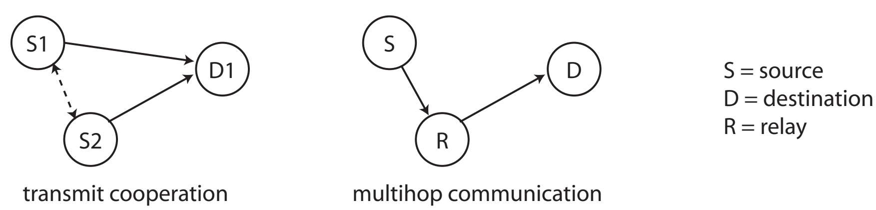

Figure 1.5: Transmit cooperation versus multi-hop transmission.

Another difference between cooperative transmission and classic multi-hop communications is that multi-hop transmission typically has clearly defined roles for the source and relay. In cooperative transmission systems, there may be no dedicated relays in the system and sources must behave as relays for other sources. The role of each source is usually dynamic and may be reconfigured on very short time scales.

The appeal of user cooperation diversity is clear: multiuser communication networks with simple, inexpensive, single-antenna transmitters can pool their antenna resources to form a virtual antenna array and achieve the increased reliability and energy efficiency promised by spatial diversity without the need for bulky, expensive "real" antenna arrays. Cooperative communication has the potential of increasing the energy efficiency of existing communication systems by an order of magnitude or more and can be applied to a wide range of multiuser communication systems including cellular systems and sensor networks. What differentiates user cooperation diversity from traditional antenna array techniques, including the large body of work on multi-input multi-output (MIMO) and space-time processing (see e.g., [GSsS^{+}03] and the other articles in this special issue), is (i) that cooperative links are not ideal, i.e., they may be fading and/or noisy, (ii) that the nodes in a cooperative partnership are at least *semi-autonomous* in the sense that they each typically have their own local resources (e.g., energy) and quality of service (QoS) requirements, and (iii) nodes may have limited and/or different knowledge about the channel state or channel statistics in the network. As will be shown in this dissertation, these differences are not trivial and require the development of new techniques that go beyond the classic antenna array processing literature to fully realize the potential gains of user cooperation diversity.

1.2 Dissertation Overview

This main body of this dissertation is organized into four chapters, followed by conclusions and a discussion of future research directions. An overview of these chapters is provided below.

Chapter 2 expands upon the material covered in Section 1.1 with a more rigorous review of fading channels and conventional diversity techniques. Receiver combining techniques for diversity reception are also discussed. A brief history of cooperative transmission is followed by a taxonomy of cooperative transmission protocols.

It was first suggested in [SEA98] that cooperative transmission could lead to an overall reduction in transmit energy and, consequently, increased battery life for battery-powered transmitters. It is not difficult, however, to construct examples where cooperative transmission is actually less energy efficient than direct (non-cooperative) transmission. These idea are investigated in **Chapter 3** where we consider the problem of how to optimally allocate transmission energy in a wireless communication system with two delay-constrained cooperative sources and one destination. To facilitate the development of analytical results, the focus in this chapter is on a simple cooperative transmission protocol called "Orthogonal Amplify and Forward" (OAF). Four different scenarios are studied: (i) both the sources and the destination have access to the full channel state information; (ii) the sources have access to only the channel statistics and the destination have access to the full channel state information; (iii) the sources have access to the full channel state information and the destination has no channel state information and (iv) the sources have access to only the channel statistics and the destination has no channel state information.

While the appeal of the OAF cooperative transmission protocol is good performance with minimum complexity, it is known that other cooperative protocols, e.g. "Decode and Forward", can offer better performance in some scenarios. **Chapter 4** extends some of the resource allocation results developed in Chapter 3 to more advanced cooperative transmission protocols and compares the performance of these protocols under optimal energy allocation. The problem of optimum energy allocation in cooperative networks depends not only on the assumptions about channel state knowledge but also on the choice of cooperative protocol and performance measure. This chapter also develops the idea of *dynamic protocol selection* where, given the instantaneous channel state, the cooperating sources not only optimally allocate their transmission energies but also optimally select the most energy efficient protocol to achieve the desired QoS at the destination.

A natural question to ask about cooperative transmission systems is how nodes establish and maintain cooperative partnerships. In some scenarios, e.g., military or emergency networks, all of the nodes in the network belong to a single authority and cooperation can be centrally controlled. In many civilian and commercial applications, however, the nodes in a network may be at least partially autonomous and cooperation can not be assumed. **Chapter 5** investigates the problem of whether cooperation can exist in networks without central control, incentive mechanisms, or altruistic nodes. Assuming rational and self-interested sources, a game theoretic framework is developed for the two-source OAF cooperative transmission protocol in both non-fading and fading scenarios.

1.3 Dissertation Contributions

This section summarizes the main contributions of this dissertation. Citations are given for the publications in which these results have appeared.

1.3.1 Optimal Resource Allocation for OAF (Chapter 3)

Under an outage probability constraint, we analytically derive the optimum (minimum energy) energy allocation strategy for the OAF cooperative transmission protocol for the cases

- the sources and the destination both have access to the full channel state information and
- the sources have access to the full channel state information but the destination has no channel state information

and explicitly describe the set of channel conditions under which cooperation is not energy efficient, i.e. cooperative transmission requires more energy than direct transmission. For the case when

- the sources have access to only the channel statistics (no instantaneous channel state information) and the destination has full channel state information
- the sources have access to only the channel statistics (no instantaneous channel state information) and the destination has no channel state information

we analyze the outage probability and derive explicit bounds for the case with independent Rayleigh fading channels. These bounds are then used to derive the optimum fixed energy allocation strategy for the case when the sources only know the channel statistics. In this case, we show that cooperative transmission is always more energy efficient than direct transmission. Numerical examples are presented for independent Rayleigh fading channels demonstrating that full knowledge of the channel state at both the sources and the destination significantly improves the energy efficiency of cooperative transmission. These results were published in [YB06] and [YB07a].

The main contribution of these results is a better understanding of how channel state knowledge affects the optimal energy allocation strategy as well as the expected gains of OAF cooperative transmission with respect to direct (non-cooperative) transmission. The results suggest that accurate channel state knowledge, possibly obtained via feedback from the the destination, enables significant gains in energy efficiency for both OAF cooperative transmission as well as direct transmission. Under identical channel state knowledge assumptions, OAF cooperative transmission tends to have better energy efficiency than direct transmission. The results in Chapter 3 show, however, that opportunistic direct transmission with full knowledge of the channel state by the source is often more energy efficient than cooperative transmission without source knowledge of the channel state.

1.3.2 Cooperative Protocol Comparison (Chapter 4)

In the case when the sources and the destination both have access to the full channel state information, we analytically compare the instantaneous capacity of the following cooperative transmission protocols[1] under optimal energy/time allocation:

- Non-orthogonal amplify and forward (NAF, a generalization of OAF)
- Opportunistic Decode and Forward (ODF)

[1] The details of these protocols are discussed in Chapter 2.

- Estimate and Forward (EF)
- Compress and Forward (CF)

Under our full channel state knowledge assumption, we show that the instantaneous capacity of EF is always no less than that of NAF. We also show that the instantaneous capacity of CF is always no less than that of EF. Numerical examples of minimum outage probability and maximum delay-limited capacity [HT98] are presented for a simple collinear network geometry with independent Rayleigh fading channels and the results are compared to the cut-set bound. The results demonstrate that, under optimal energy/time allocation, the CF and ODF cooperative transmission protocols provide the best outage probability and delay-limited capacity among all of the protocols considered. Nevertheless, the results also show that there is still a gap between the cut-set bound and the performance of the best protocol. Since NAF is the only protocol in which the optimal energy/time allocation can be computed analytically, the results also suggest that, the implementation complexity of NAF with optimal energy/time allocation is potentially lower than that of the other protocols. The delay-limited capacity results described in this chapter were published in [YGBE08].

The main contribution of these results is a better understanding of how the performance of cooperative protocols compares under optimal energy/time allocation. The literature has many comparisons between cooperative protocols that lead to conflicting conclusions. The source of this conflict can often be attributed to the fact that, given full channel state knowledge, each protocol has its own most favorable energy/time allocation; fixing this allocation and then comparing the performance of the protocols may lead to one protocol being compared in a more favorable energy/time allocation than another other protocol. The study performed in Chapter 4 compares each protocol in its own most favorable energy/time allocation and shows that, of all of the protocols considered, the best performance is achieved by either ODF or CF, depending on the channel statistics.

1.3.3 Dynamic Cooperative Protocol Selection (Chapter 4)

Inspired by the result that the CF and ODF cooperative transmission protocols provide the best outage probability and delay-limited capacity among all of the protocols considered, an opportunistic protocol selection strategy was developed to select between CF and ODF with optimal resource allocation. Since the hybrid protocol uses the least energy in each transmission interval, it also provides the best outage probability and delay-limited capacity performance of all protocols considered in this dissertation. Numerical results are provided and it is shown that the delay-limited capacity and outage probability of the hybrid CF/ODF protocol are close to the cut-set bound. The delay-limited capacity results for the hybrid CF/ODF protocol described in this chapter were published in [YGBE08].

The main contribution of these results is the development of the idea of opportunistic cooperative protocol selection and the numerical results showing performance close to the cut-set bound. The idea of opportunistic protocol selection is a natural extension of optimal resource allocation, yet it had not been described in the literature prior to [YGBE08].

1.3.4 Selfish Cooperation (Chapter 5)

In a two-source two-destination non-fading scenario, the results in Chapter 5 show that cooperation with the OAF cooperative transmission protocol and optimum energy allocation can exist between two self-interested sources given that the end of the cooperative "game" is uncertain and that the sources are sufficiently patient in the sense that sources do not excessively discount future payoffs with respect to current payoffs. In the fading scenario, we develop a conditional trigger cooperative strategy and show that this strategy is a Nash Equilibrium (NE) of the infinitely repeated game. An important feature of the conditional trigger strategy is that the sources cooperate using optimum resource allocation but with a ceiling placed on the optimized relay energy. If either source is asked to transmit with relay energy greater than their ceiling in a stage game, both sources use direct transmission in that stage game. We show that this ceiling goes to infinity as the sources become more patient. Our results also show that sources using the conditional trigger strategy can often achieve an overall system energy efficiency close to that of a centrally-controlled system, especially when the sources are patient. These results were published in [YB07b].

The main contribution of these results is that, unlike the prior literature that has considered central control, external incentive mechanisms, or altruistic nodes, we show that cooperation can often be established between two purely self-interested sources using only local utility functions (transmission energy). In systems with non-fading channels, we explicitly give conditions under which cooperation is mutually beneficial and describe a simple cooperative Nash Equilibrium strategy. When sources are infinitely patient, i.e. they do not discount future payments with respect to current payments, we explicitly describe a Nash Equilibrium strategy based on long-term mutual cooperation for systems with fading channels. When the sources discount future payments, we also explicitly describe a Nash Equilibrium strategy based on short-term mutual cooperation for systems with fading channels. Finally, we also show that the performance loss of cooperation between selfish sources with respect to centrally optimized energy allocation can be small in many cases, especially if the sources are patient.

1.4 Table of Abbreviations

CF	compress-and-forward
CSIT	channel state information at the transmitters (sources)
CSIR	channel state information at the receiver
DF	decode-and-forward
DLC	delay-limited capacity
EF	estimate-and-forward
EGC	equal gain combining
MIMO	multi-input multi-output
MRC	maximal ratio combing
NAF	non-orthogonal amplify-and-forward
NE	Nash equilibrium
OAF	orthogonal amplify-and-forward
ODF	opportunistic decode-and-forward
QoS	quality-of-service
SNR	signal-to-noise ratio

Chapter 2

Background

This chapter covers general background relevant to the entire dissertation. Specific background material relevant to only one chapter is provided in the introduction of the appropriate chapter.

2.1 Rayleigh Fading Channels

As discussed in Section 1.1, multipath fading is often a significant impairment in wireless communication channels. This section provides relevant background for the development of the *quasi-static Rayleigh fading* channel model used throughout this dissertation. We provide a very abbreviated overview in this section; the interested reader is referred to [Pro01] and [Stu96] for a more thorough treatment of this subject.

In a wireless communication system, the baseband signal at the transmitter is typically modulated onto a sinusoidal carrier for efficient transmission through the wireless medium as a bandpass signal. In typical systems, the bandpass signal generated by the transmitter can be represented as

$$s(t) \;=\; \mathrm{Re}\left\{m(t)e^{j\omega_c t}\right\} \tag{2.1}$$

for t on some transmission inverval $\mathcal{T}$ and where $m(t)$ represents the complex-valued baseband signal and ω_c denotes the carrier frequency (rad/sec). In a time-varying multipath environment with L paths between the transmitter and the receiver, the received signal can be written as

$$r(t) \;=\; \sum_{\ell=1}^{L} \alpha_\ell(t)s(t-\tau_\ell(t)) + w(t) \tag{2.2}$$

where $\alpha_\ell(t)$ and $\tau_\ell(t)$ represent the non-negative amplitude gain and delay of the ℓ^{th} path at time t, respectively, and $w(t)$ represents the real-valued additive noise in the channel. Substituting (2.1) into (2.2) and applying the standard receiver operations of downmixing (at ω_c) and low-pass filtering results in the complex-valued baseband received signal

$$x(t) \;=\; \sum_{\ell=1}^{L} \alpha_\ell(t)e^{-j\theta_\ell(t)}m(t-\tau_\ell(t)) + \tilde{w}(t) \tag{2.3}$$

where $\tilde{w}(t)$ is the receiver noise after downmixing and lowpass filtering and $\theta_\ell(t) := \omega_c\tau_\ell(t)$ is the resulting phase of the carrier waveform in the ℓ^{th} path.

It is evident from (2.2) that fading can result in intersymbol interference, i.e. the current symbol is received simultaneously with echoes from previous and/or future symbols, if the delay spread of the channel

$$\Delta \;:=\; \max \tau_\ell - \min \tau_\ell \tag{2.4}$$

is larger than a small fraction of the symbol duration T_s. To facilitate the development of analytical results, the focus in this dissertation is on systems with sufficiently low symbol rate such that $T_s \gg \Delta$. In these types of "flat fading" systems, the multipath channel affects all frequencies of the transmitted signal equally and intersymbol interference is negligible [Pro01]. Denoting $\tau(t) = \frac{1}{L}\sum_{\ell=1}^{L} \tau_\ell(t)$ and applying the approximation $m(t-\tau) \approx m(t-\tau_\ell(t))$ for all ℓ in $\{1,\ldots,L\}$, the baseband received signal in the flat fading case can be written as

$$x(t) \quad = \quad m(t-\tau(t))\sum_{\ell=1}^{L} \alpha_\ell(t) e^{-j\theta_\ell(t)} + \tilde{w}(t). \tag{2.5}$$

Intuitively, small movements by the transmitter and/or receiver will lead to small changes in the path amplitude gains $\alpha_\ell(t)$ and the path delays $\tau_\ell(t)$. Nevertheless, since ω_c is typically very large, small changes in path delay can lead to relatively large changes in received phase $\theta_\ell(t)$. For example, in system with a 900MHz carrier, a path length change of just 10cm results in a phase change at the receiver of 108°. It is the phase sensitivity to small path length changes that is the primary cause of flat fading, i.e. the sum of complex phasors

$$\sum_{\ell=1}^{L} \alpha_\ell(t) e^{-j\theta_\ell(t)} \quad = \quad \alpha(t) e^{-j\theta(t)} \tag{2.6}$$

can often be expected to be destructive and lead to a small envelope $\alpha(t)$. Even if the receiver is able to successfully recover the phase of the received signal, the probability of incorrect demodulation becomes large when $\alpha(t)$ is small.

A common assumption in richly scattered multipath channels is that the number of paths L is large and that the phase $\theta_\ell(t)$ of each path at the receiver is uniformly distributed on $[-\pi,\pi)$. Central limit theorem arguments can then be applied to show that (2.6) is well-modeled as a complex-valued Gaussian random process. In the absence of a strong line-of-sight path or a dominant reflector, the mean of the Gaussian random process at any time t can be expected to be close to zero and the phase of the process at any time t can be expected to be uniformly distributed on $[-\pi, \pi)$. It can be shown that the resulting envelope of the process, i.e. $\alpha(t)$, is Rayleigh distributed [Stu96] at any time t, i.e. it has the probability density function

$$f_{\alpha(t)}(x) = \frac{x \exp\left\{\frac{-x^2}{2\sigma^2}\right\}}{\sigma^2}.$$

The temporal characteristics of Rayleigh fading channels, e.g. various correlations, envelope level-crossing rates, average envelope fade durations, and much more, have been well-studied [Stu96]. Throughout this dissertation, unless otherwise stated, we assume that the wireless mulitpath channels are quasi-static Rayleigh fading in the sense that the sum in (2.6) is approximately constant over the duration of a transmission interval but is independent and identically distributed (i.i.d.) in different transmission intervals. To be explicit, in the k^{th} transmission interval denoted as $\mathcal{T}_k$, the expression in (2.5) can be written as

$$x(t) = \alpha_k m(t) + \tilde{w}(t) \qquad t \in \mathcal{T}_k$$

where α_k is the Rayleigh distributed channel envelope fading coefficient in transmission interval k and the mean delay τ has been omitted for notational simplicity. The quasi-static fading assumption implies that

$$\text{cov}\{\alpha_k, \alpha_m\} = \begin{cases} \frac{(4-\pi)\sigma^2}{2} & k = m \\ 0 & \text{otherwise} \end{cases}$$

for all k and m, and that

$$\text{E}[\alpha_k] = \sigma\sqrt{\frac{\pi}{2}}$$

for all k where σ is the single parameter of the Rayleigh distribution that is selected to match the expected path loss of a particular channel.

As a final note on fading channels, it is often more convenient to work with the squared envelope of the channel rather than the envelope. It is straightforward to show that when the envelope α_k is Rayleigh distributed with mean $\sigma\sqrt{\frac{\pi}{2}}$, the squared envelope $\gamma_k = \alpha_k^2$ is exponentially distributed, i.e.,

$$f_\gamma(x) \;=\; \lambda e^{-\lambda x}$$

with mean $\frac{1}{\lambda} = 2\sigma^2$.

2.2 A Short Introduction to Diversity Techniques

As discussed briefly in Section 1.1, the key to reliable and/or energy-efficient communication in fading channels is diversity. There are many types of diversity including temporal, frequency, spatial, multipath, polarization, and angle diversity. This section formalizes the concept of *diversity gain* and then provides a brief discussion of the first three techniques.

In rough terms, diversity can be thought of as receiving two or more copies of the same information through different (preferably independent) fading channels. If d denotes the number of copies of the message received through independent and identically distributed fading channels and p denotes the probability that the magnitude of the fading channel falls below a particular threshold, then the probability that all d channels fall below this threshold is p^d. This notion was recently formalized in [TVZ04] when the diversity gain of a particular diversity technique is defined asymptotically in terms of the rate of decay of the probability of error at large signal to noise ratios. Specifically, a diversity technique is said to have diversity gain of d if the error probability scales according to

$$P_e(\mathsf{SNR}) \sim \mathsf{SNR}^{-d}$$

for large SNR where SNR denotes the average signal to noise ratio of the system. It is clear from this definition that a system with diversity gain of $d = 2$ will significantly outperform a system with no diversity gain ($d = 1$) at large enough SNR since the diversity gain appears in the exponent of the SNR. Since it is often impossible to increase SNR due to transmitter power constraints and/or regulatory requirements, increasing the diversity gain of the system is often the only way of reducing the error probability in systems with fading channels.

A straightforward technique for increasing diversity gain is *temporal diversity*. In quasi-static flat fading channels, for example, the diversity gain could be increased by repeating the same message in different transmission intervals. Since the fading coefficients $\{\alpha_k\}$ are independent in different transmission intervals, repeated transmissions result in the desired effect: two or more copies of the same information are received through independent fading channels. While advanced temporal diversity techniques exist that are more efficient than simple message repetition, e.g. channel coding with interleaving, the main problem with temporal diversity is that the redundancy is conveyed temporally and leads to a reduction in the rate of the link.

A second approach called *frequency diversity* involves sending redundant information in different frequency bands, ideally with independent fading. Since the redundancy is conveyed simultaneously in different frequency bands, frequency diversity can be achieved without any loss of rate. The cost of this approach, however, is increased bandwidth requirements for the bandpass signals. This may be prohibitive in bandwidth-constrained applications.

A third approach called *spatial diversity* involves sending redundant information from different antennas, and/or receiving the redundant information on multiple antennas. Figure 2.1 shows an example of a multi-input multi-output (MIMO) communication system with three transmit antennas and two receive antennas. There are a total of six channels in this system and, if the antennas have sufficient separation, these channels can be assumed to be at least approximately independent. Space-time coding techniques for point-to-point MIMO channels have, somewhat surprisingly, been shown to achieve a maximum diversity gain equal to the product of the number of transmit and receive antennas [TVZ04]. In the example shown in in Figure 2.1, the maximum diversity gain of the system is equal to six.

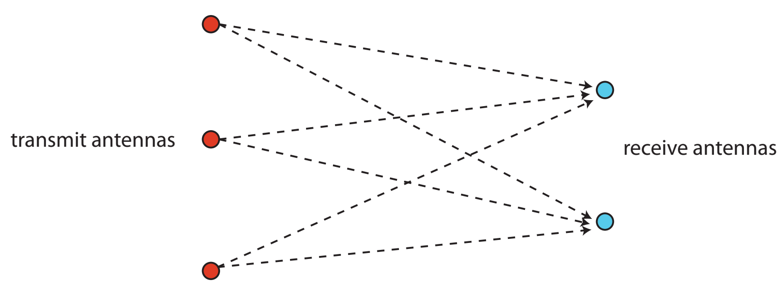

Figure 2.1: A MIMO system with three transmit antennas and two receive antennas.

The primary appeal of spatial diversity is that it does not require any additional bandwidth and that it does not reduce the rate[1] of the communication link. The problem with spatial diversity, however, is that antenna arrays with sufficient separation are too bulky for many applications. For example, in mobile phone systems, the base stations typically use antenna arrays but the mobile subscriber units do not. The notion of *user cooperation diversity* stems from the idea that, in networks with many single antenna nodes, it may be possible to pool the antenna resources of two or

[1] As discussed in [TVZ04], there is a fundamental tradeoff between diversity gain and multiplexing gain in MIMO systems. Nevertheless, the maximum diversity gain can be achieved in a MIMO system without any loss of rate with respect to the corresponding single-input single-output (SISO) system.

more nodes to achieve spatial diversity without real antenna arrays. A brief history of *user cooperation diversity* is provided in the next section.

2.3 A History of User Cooperation Diversity

The idea of user cooperation diversity is usually attributed to Sendonaris, Erkip, and Aazhang in [SEA98] but can also be traced back to the relay channel model first introduced in [Meu71] and shown in Figure 2.2. The relay channel generalizes the notion of a simple point-to-point channel with a single source and destination to include a relay whose sole purpose is to help transfer information from the source to the destination.

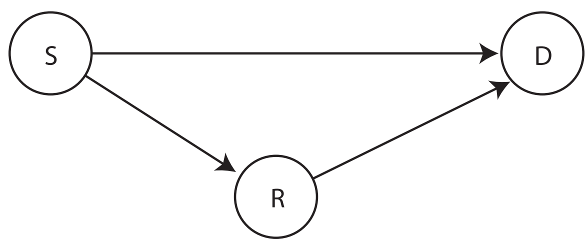

Figure 2.2: Relay channel.

Cover and El Gamal [CG79, CT91] are credited with developing most of the information theoretic results on relay channel. They analyzed the capacity of the relay channel under the assumption that all nodes operate in the same band. Under this assumption, the system can be decomposed into a broadcast channel from the viewpoint of the source and a multiple access channel from the viewpoint of the destination. Since [SEA98] was presented at the 1998 International Symposium on Information Theory, the idea of user cooperation diversity first attracted the attention of the information theory community. Many of the ideas and results that appeared in the literature shortly after [SEA98] can be traced to [CG79].

While the body of literature on user cooperation diversity is now far too large to cover completely, we provide here some selected highlights. Sendonaris, Erkip, and Aazhang followed up on [SEA98] with a more detailed information theoretic study of two-source transmit cooperation in a mobile uplink scenario in [SEA03a, SEA03b]. This work was important in that it also exposed several practical implementation issues in cooperative transmission systems and attracted the interest of the communications and signal processing communities. Also noteworthy are the contributions of [Lan02, LW02, LTW04a] for studying the performance of several practical cooperative transmission protocols in fading environments. Yet another important set of contributions came in the form of novel information theoretic results and new insights into information theoretic coding in [KGG05]. New information theoretic results and results on power control were also discovered in [WZH05, HZ05a]. A variety of contributions to relaying including new bounds, cut-set theorems, power control strategies, LDPC relay code designs, and some of the earliest results on half-duplex relaying were proposed in [Kho04]. Researchers realized that relaying can mimic multiple-antenna systems even when the communicating entities were incapable of supporting multiple antennas. Prominent literature on the use of space-time codes with relays includes [LW02, NBK04, MOT05]. More theoretical characterizations and analysis for the physical layer of multihop wireless communications channels are presented in [HA03, BFY04, TG03, LV05].

For the most part, the user cooperation diversity literature has focused on developing fundamental results the case of cooperative transmission with two sources and one destination, where each source acts as a relay for the other source. This line of investigation is motivated by the hope that fundamental results developed for the two-source one-destination model will provide insight and suggest strategies for analysis of more general network architectures. As will be discussed in Chapter 4, however, some problems even in this simple case are analytically intractable and require numerical solutions. Extensions to more general network architectures have already appeared recently in the literature, e.g. [CSY08], but many of the fundamental results for the two-source one-destination model are still being actively pursued by the research community.

2.4 Cooperative Transmission Protocols

Since [SEA98], a focus area of the user cooperation diversity research community has been in the development of efficient protocols for the transmit cooperation scenario of Figure 1.4. All of these protocols operate in the general cooperative sense in that, once a source transmission has been received by a set of cooperating nodes, one or more of these nodes will transmit some amount of redundancy (e.g., repetition or parity bits) to the destination. There are three basic approaches in how this redundancy is conveyed to the destination:

1. Redundancy is conveyed by one or more cooperative nodes (and possibly the original source) transmitting in **orthogonal subchannels**. This is the simplest protocol category and includes the orthogonal amplify and forward (OAF), decode-and-forward (DF), and incremental schemes in [LW00a, LW00b, LWT01, LTW04b, GE07], the coded cooperation schemes in [SE02, SE03a, SE03b, HN02b, HN02a, HN03], and the "no phase knowledge at transmitters" scheme in [SEA03b].

2. Redundancy is conveyed by one or more cooperative nodes (and possibly the

original source) transmitting in **non-orthogonal subchannels**, e.g. non-cooperative and cooperative signals are transmitted in a single subchannel. These approaches have been studied in cite in [AES04, NBK04, KGG05, CdBSA06] and include the non-orthogonal amplify and forward (NAF), compress and forward (CF) and estimate and forward (EF) protocols.

3. Redundancy is conveyed by one or more cooperative nodes (and possibly the original source) transmitting in a single subchannel such that the bandpass transmissions **coherently combine** at the destination (i.e., distributed beamforming). This approach has been investigated in the context of repetition-based coherent cooperation in [SEA98, SEA03a, SEA03b], space-time coherent cooperation in [LW02, LW03], and space-time coded coherent cooperation in [JHHN04].

Each approach has distinct advantages and disadvantages. Protocols that fall into the orthogonal subchannel category tend to perform poorly in the high spectral efficiency regime [LW03, AES04]. While these protocols are attractive from the standpoint of simple implementation, the use of orthogonal subchannels demands a large price in the loss of achievable rate. This may be ameliorated somewhat through coded cooperation rather than repetition, but the diversity-multiplexing tradeoff [ZT03, TVZ04] of these protocols is not yet well understood. The non-orthogonal subchannel cooperative protocol in [AES04] was shown to achieve the optimal diversity-multiplexing tradeoff but requires the use of a more complicated receiver at the destination, i.e., sequence detection. Protocols that fall into the coherent combining category tend to have improved diversity-multiplexing tradeoff behavior [LW03] but suffer from the fact that strict transmitter synchronization is required to ensure that the bandpass signals coherently combine at the destination [BPM05, OB07, BIP08].

Because of the difficulty satisfying the strict synchronization requirements for the third category of coherent combining protocols, this dissertation focuses on protocols that fall into the first two catageries. We provide an overview of the most studied non-coherent cooperative transmission protocols within these two categories in the context of the relay channel model in Figure 2.2 in the subsections below.

2.4.1 Orthogonal Amplify and Forward (OAF)

The OAF cooperative transmission protocol was first described in [LWT01] and is generally considered the simplest of all of the cooperative transmission protocols since the relay (usually another source in the network) simply amplifies the signal received from the source and retransmits it to the destination. Since the relay does no processing to the received signal, it amplifies both the message and the noise in the source↔relay channel. As might be expected from its name, the OAF protocol is a member of the family of orthogonal subchannel protocols.

Figure 2.3 shows the schedule of the OAF cooperative transmission protocol. In the first half of the transmission interval, the source transmits while the relay and the destination receive faded and noisy copies of the source transmission. In the second half of the transmission interval, the source is silent while the relay retransmits the signal (and the noise) that it received in the first half of the transmission interval. Diversity is achieved because the destination receives two copies of the same message through independent[2] fading channels.

[2]It is commonly assumed that all of the nodes in the network are sufficiently separated so that the fading envelope of all of the channels can be assumed to be independent (but not necessarily identically distributed).

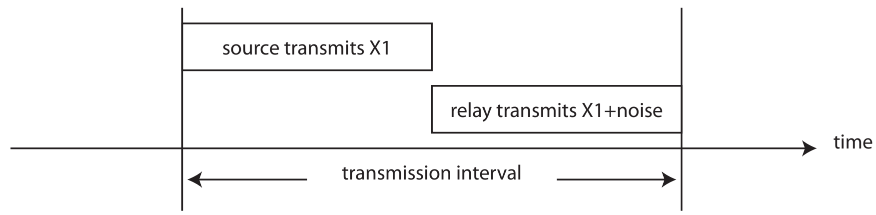

Figure 2.3: Orthogonal amplify and forward (OAF) protocol schedule.

2.4.2 Non-Orthogonal Amplify and Forward (NAF)

The simplicity of the OAF protocol comes at the cost of a loss of spectral efficiency since identical information is simply repeated the second half of the transmission interval. A generalization of the OAF protocol called "non-orthogonal amplify and forward" (NAF) was proposed in [NBK04] with the goal of improving the spectral efficiency of OAF. The basic idea is that the source is permitted to transmit additional information in the second half of the transmission interval. As might be expected from its name, the OAF protocol is a member of the family of non-orthogonal subchannel protocols. Figure 2.4 shows the schedule of the NAF cooperative transmission protocol.

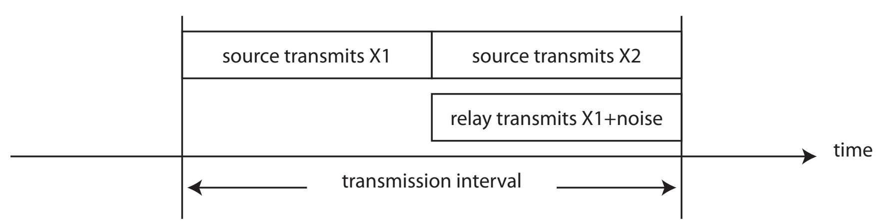

Figure 2.4: Non-orthogonal amplify and forward (NAF) protocol schedule.

In the first half of the transmission interval, the source transmits a signal to the relay and the destination while the relay is silent. In the second half of the transmission interval, the relay amplifies the received signal from the first half of transmission interval then retransmits this amplified signal while the source simultaneously transmits new information to the destination. Since the relay is assumed to be half-duplex, the relay does not receive the source's transmission in the second half of the transmission interval. With respect to OAF, additional processing is required at the destination for NAF in order to separate the signals received simultaneously at the destination in the second half of the transmission interval.

2.4.3 Decode and Forward (DF)

A fundamentally different approach to orthogonal-subchannel cooperative transmission called "decode and forward" was first described in [LW00a]. The basic idea is that, assuming a system in which the source transmits with forward error correction coding, rather than having the relay simply retransmit an amplified version of the signal (and noise) received in the first half of the transmission interval, the relay could instead attempt to decode the transmission, re-encode it, and then retransmit it to the destination. The advantage of this approach is that the relay transmits a "clean" copy of the original message, i.e. a noise-free copy, if it can successfully decode the message from the source in the first half of the transmission interval. Figure 2.4 shows the schedule of the DF cooperative transmission protocol.

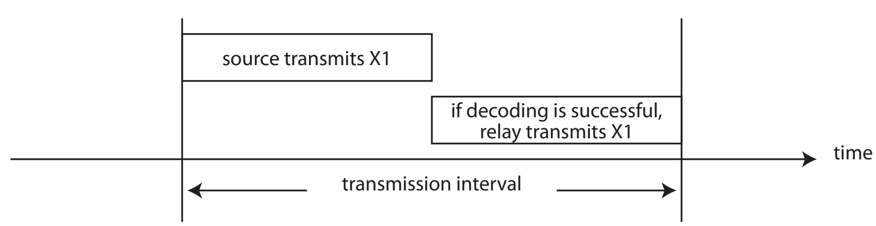

Figure 2.5: Decode and forward (DF) protocol schedule.

If the source has knowledge of the fading channel to the relay, the source can scale the rate of its transmission to ensure that the relay can successfully decode the message. In cases when this is not possible, there are various strategies that can be employed when the relay does not successfully decode the transmission from the source. The simplest approach is to have both the source and the relay remain silent in the second half of the transmission interval. Alternatively, if the source notices that the relay is not transmitting in the second half of the transmission interval, the source could retransmit its original message.

One big advantage of the DF protocol is that, unlike OAF and NAF, the time allocation between the source and relay is not required to be identical [GE07]. For example, if the source↔relay link is weak but the relay↔destination link is strong, it makes sense to increase the duration of the source transmission (since the capacity of the source↔relay link is relatively small) and decrease the duration of the relay transmission (since the capacity of the relay↔destination link is relatively large). Figure 2.6 shows the schedule of the DF cooperative transmission protocol with unequal time allocation between the source and relay.

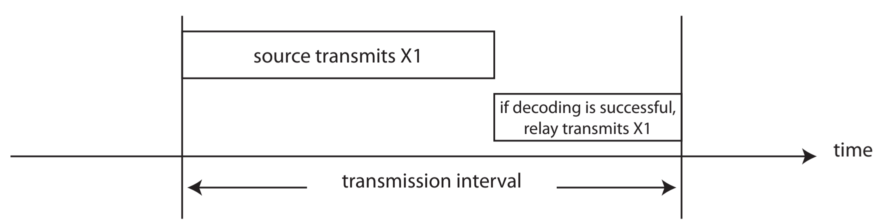

Figure 2.6: Decode and forward (DF) protocol schedule with unequal time allocation between the source and relay.

2.4.4 Opportunistic Decode and Forward (ODF)

An extension of the DF cooperative transmission protocol called "opportunistic decode and forward" (ODF) was proposed in [GE07]. The basic idea is that, when the source↔relay channel is in a deep fade, it is better for the source to attempt to transmit directly to the destination without any cooperation from the relay. The ODF protocol is *opportunistic* in the sense that, given knowledge of the channel states at the source, the source only uses the DF protocol when it offers a gain, e.g. lower transmission energy, with respect to direct transmission. Figure 2.4 shows the schedule of the ODF cooperative transmission protocol (allowing for unequal time allocation between the source and relay).

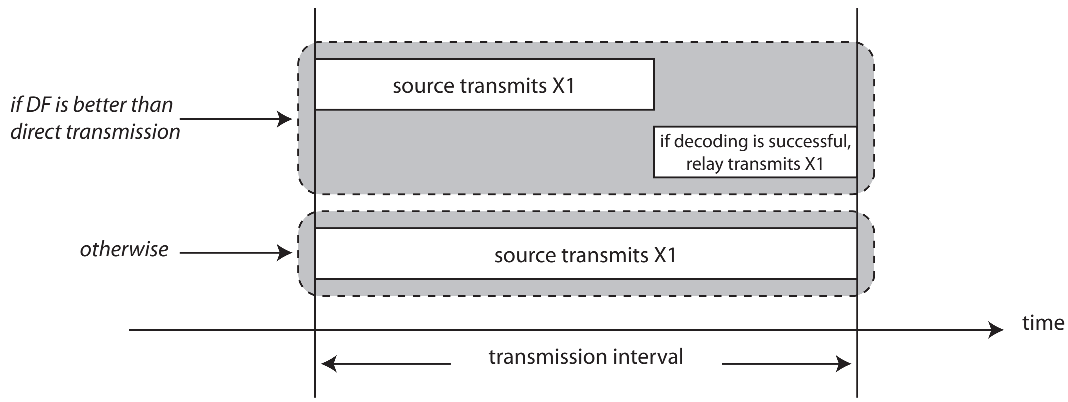

Figure 2.7: Opportunistic decode and forward (ODF) protocol schedule.

2.4.5 Compress and Forward (CF)

The compress and forward (CF) cooperative transmission protocol was first described in [CG79]. CF is a non-orthogonal cooperative transmission protocol in which the transmission interval is split into two parts which are not necessarily of equal duration. In the first part, the source transmits a signal to the relay and the destination while the relay is silent. The relay compresses the signal it received in the first part, and transmits the compressed version to the destination while the source simultaneously transmits independent information to the destination in the second part of the transmission interval. The compression is done in Wyner-Ziv [WZ76] sense by utilizing the destination's own correlated observation about the source signal of the first timeslot. Note that, since the time allocation between the first and second parts of the transmission interval is a design parameter of the CF protocol, CF with optimal time allocation includes direct transmission as a special case and is inherently an opportunistic protocol in the same sense of ODF as described in [GE07]. Figure 2.8 shows the schedule of the CF cooperative transmission protocol.

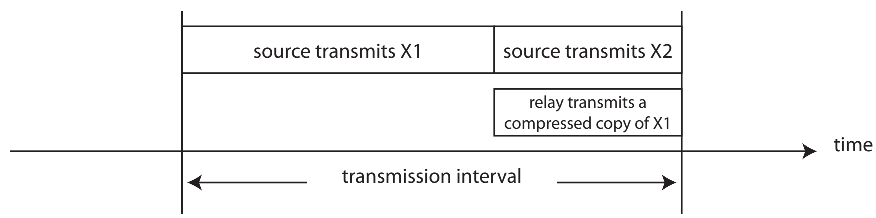

Figure 2.8: Compress and forward (CF) protocol schedule.

2.4.6 Estimate and Forward (EF)

The use of Wyner-Ziv compression in the CF cooperative transmission protocol may cause the relay encoder and the destination decoder to be prohibitively complicated in some cases. A simpler scheme in which the relay compresses its received signal ignoring the side information at the destination is called estimate-and-forward (EF) transmit cooperation [CdBSA06]. The capacity of EF is known to be slightly lower than that of CF [HZ05b], but the implementation of EF is simpler than that of CF. The schedule of the non-orthogonal EF cooperative transmission protocol is identical to the schedule shown in Figure 2.8.

2.5 Performance Metrics

Throughout this dissertation, several common performance metrics will be analyzed for cooperative transmission systems. These performance metrics are described below.

2.5.1 Signal to Noise Ratio (SNR)

Noise is unavoidable in all communication systems and can be especially prevalent in wireless communication systems. Signal to noise ratio (SNR) is simply defined as the ratio of signal power to noise power at the receiver. SNR is usually expressed in decibels, i.e. $\mathsf{SNR}(\mathrm{dB}) = 10\log_{10}(\mathsf{SNR})$. In systems with fading channels and constant transmit power, SNR is usually defined as an average, i.e.

$$\mathsf{SNR} = \mathrm{E}\left[\frac{\text{signal power}}{\text{noise power}}\right],$$

since the received signal power after propagation through the fading channel is stochastic.

2.5.2 Capacity

The capacity of a channel is the maximum rate (usually expressed in bits per second or bits per channel use) at which reliable communication[3] is possible between the transmitter and receiver. The concept of capacity was first described by Claude Shannon in [Sha48] and spawned the field of information theory that is still active today. The most famous result of information theory is the capacity formula for the

[3] Reliable communication at rate R means that there exists a forward error correcting code that conveys information through the channel at rate R and also achieves an error probability arbitrarily close to zero.

point-to-point nonfading channel with additive white Gaussian noise, given as

$$C = \log_2(1 + \mathsf{SNR}) \tag{2.7}$$

where C is the capacity in bits per second per Hertz of channel bandwidth. This result implies the intuitively satisfying notion that capacity is maximized when SNR is maximized.

While the capacity of the relay channel is unknown except in special cases, the maximum *achievable rate* of many of the most common cooperative transmission protocols including OAF, NAF, DF, CF, and EF is known [HZ05b]. It is customary in the cooperative transmission literature to refer to the maximum achievable rate of each protocol as its “capacity”. To be clear, this notion of capacity is for a particular choice of cooperative transmission protocol and does not refer to the general capacity of the relay channel. The capacities of the OAF, NAF, DF, CF, and EF cooperative transmission protocols are analyzed in Chapter 4.

2.5.3 Instantaneous and Delay-Limited Capacity (DLC)

In fading channels, the notion of capacity may be ambiguous when the SNR is stochastic. One approach in fading channels is to evaluate the *instantaneous capacity* of the channel by conditioning the capacity calculation on the current realization of the fading coefficient. In quasi-static fading scenarios, the instantaneous capacity of the channel is the maximum rate at which reliable transmission is supported in the transmission interval.

An alternative, and slightly more complex, capacity measure for systems with fading channels is *delay limited capacity* (DLC). Delay-limited capacity is an appropriate long-term performance metric for delay sensitive applications such as real-time voice and video communications and is defined as the maximum achievable rate that can be sustained independent of the channel state under a long-term average total transmit energy constraint [HT98]. Unlike instantaneous capacity, which is conditioned on a particular channel state realization, DLC is a "long-term" performance metric since it is computed over the joint channel state distribution. The DLC of direct (non-cooperative) transmission through a Rayleigh fading channel under any finite long-term average total transmit energy constraint is known to be zero [GE07] even if the transmitter knows the channel state. All of the cooperative protocols except DF, however, have non-zero DLC [YGBE08] when the source and relay know the channel state. This will be explored in more detail in Chapter 4.

2.5.4 Outage Probability

In scenarios where it is not necessary (or possible) to guarantee that a target rate is achieved for every channel state, outage probability is a more appropriate performance metric. Outage probability is another long-term performance metric that is computed over the joint channel state distribution and is typically defined as the probability that the instantaneous capacity falls below the current rate of transmission, i.e.

$$\text{outage probability} \;:=\; \text{Prob}[C < R]. \tag{2.8}$$

Since Shannon showed that reliable communication is impossible when $C < R$, the transmission is said to be in "outage" when this event occurs. A communication system transmitting at a rate no greater than its delay limited capacity can communicate with zero outage probability.

In scenarios where the capacity is monotonically increasing in SNR, e.g. the point-to-point channel with additive white Gaussian noise in (2.7), the outage condition

$C < R$ can equivalently be expressed as $\mathsf{SNR} < \rho$. Hence, an equivalent definition for outage probability in these scenarios can be written as

$$\text{outage probability} \;\; := \;\; \text{Prob}[\mathsf{SNR} < \rho]. \tag{2.9}$$

As will be discussed in Chapter 3, a cooperative transmission system using the OAF protocol is effectively a point-to-point channel with additive white Gaussian noise. Hence, (2.9) is used to analyze outage probability in Chapter 3. In Chapter 4, the more general definition of outage probability in (2.8) is used since the analysis focuses on the OAF, NAF, DF, CF, and EF cooperative transmission protocols.

Chapter 3

Optimum Energy Allocation for Orthogonal Amplify-and-Forward Protocol

This chapter considers the problem of how to efficiently allocate transmission energy in a wireless communication system with two delay-constrained cooperating sources and one destination. The sources in the system are assumed to cooperate via the orthogonal amplify-and-forward (OAF) protocol. The channels are assumed to be flat fading and the sources are each required to satisfy an outage probability constraint. The analysis focuses on optimum energy allocation and energy efficiency for different channel state information assumptions.

3.1 Background

Resource allocation[1] for general non-cooperative multiuser communication systems has been extensively studied (see, e.g., [Yat95, GS99, GS00]) but resource allocation in cooperative systems is a fundamentally different problem than non-cooperative systems since nodes can apply their resources to help other nodes. In most cooperative protocols, as a relay node increases its cooperative transmit power, the required transmit power of the other node is decreased. Assuming nodes are willing to cooperate, what is the most "efficient" allocation of their transmit energy? How does knowledge of the channel state affect the efficiency of the transmit energy allocation? Are there cases where it is better for the sources to not cooperate? These are some of the questions that we consider in this chapter.

Transmit energy allocation for decode-and-forward cooperation [LW03] was analyzed with the goal of maximizing rate in [HZ05b], minimizing outage probability in [GE05], and satisfying a fixed outage probability target in [CSY05]. Energy allocation for amplify-and-forward cooperation was analyzed with the goal of minimizing BER in [ARW05], minimizing total power subject to a rate constraint in [HHSL05], respectively. Minimum outage probability energy allocation has also been considered for a hybrid protocol in [AKA04]. While [GE05] considered the impact of partial channel state information (specifically, the instantaneous channel amplitudes) at the transmitters on the outage probability performance of the decode-and-forward protocol, the impact of channel state information on amplify-and-forward has not been studied. Zhao et al. consider optimum energy allocation for the amplify-and-forward protocol

[1]Resource allocation in this chapter refers specifically to the notion of transmit energy allocation. In Chapter 4, this notion of resource allocation is expanded to also include protocol timeslot allocation.

in [ZAL07] to minimize the outage probability, however, there is a mathematical error in their main result.

This chapter considers the problem of optimum energy allocation and energy efficiency of the amplify-and-forward protocol in four different scenarios: (i) both the sources and the destination have access to the full channel state information; (ii) the sources have access to only the channel statistics and the destination have access to the full channel state information; (iii) the sources have access to the full channel state information and the destination has no channel state information and (iv) the sources have access to only the channel statistics and the destination has no channel state information.

3.2 System Model

We consider the two-source one-destination cooperative transmission system model shown in Figure 3.1. Each source is assumed to transmit in an orthogonal subchannel (e.g. FDMA). Both sources wish to communicate distinct information to the destination and cooperate via the "orthogonal amplify-and-forward" protocol first described in [LWT01] as shown in Figure 3.2. We assume a half-duplex relay and normalize the time interval for each cooperative protocol (N symbols) to 1 unit. The two-source orthogonal amplify-and-forward cooperative transmission protocol divides the transmission interval into two time-slots of equal duration. Each source transmits its own information in the first timeslot (while receiving the transmission of the other source) and the second timeslot is used for cooperative retransmission of the signal received during the first timeslot. The channels are assumed to be flat and block-fading where their value is randomly generated but remains constant over the both timeslots in the cooperative frame. Note that each source transmits while receiving the transmission of the other source in the first timeslot. The sources operate in half-duplex mode, however, in the sense that transmission and reception does not occur simultaneously in any orthogonal subchannel. We denote $a := |h_{1D}|^2/\sigma^2$, $b := |h_{21}|^2/\sigma^2 = |h_{12}|^2/\sigma^2$, and $c := h_{2D}^2/\sigma^2$ as the normalized squared channel gains in the cooperative communication system shown in Figure 3.1, where $\sigma^2 > 0$ denotes the variance of the zero-mean Gaussian noise present in the channel. Denote channel state as $\mathbf{s} = (a,\ b,\ c)$, where $a,\ b,\ c$ are i.i.d. exponentially distributed with means $\mu_a,\ \mu_b,\ \mu_c$ respectively.

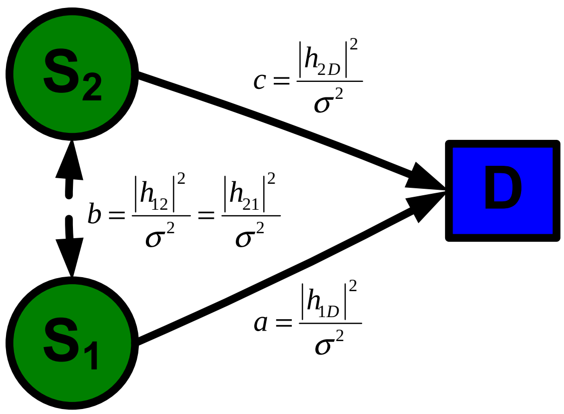

Figure 3.1: Two-source one-destination cooperative transmission system model.

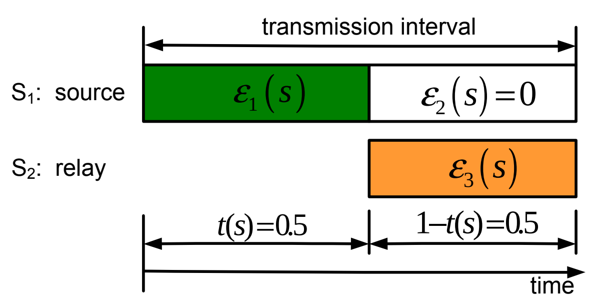

Figure 3.2: Orthogonal amplify-and-forward protocol.

From one source's point of view, the other source can be looked as the relay for that

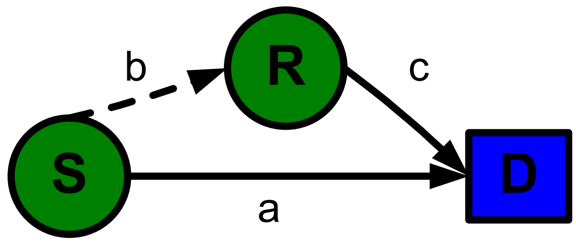

Figure 3.3: One-source, one-relay, one-destination model.

source. Due to the symmetry of the problem, we can represent our system model as a one-source and one-relay model as in Figure 3.3. Let $\mathcal{E}(\mathbf{s}) = (\mathcal{E}_1(\mathbf{s}), \mathcal{E}_2(\mathbf{s}), \mathcal{E}_3(\mathbf{s}), t(\mathbf{s}))$ be a resource allocation rule defined over the set of all possible network states $\mathbf{s} = (a, b, c)$, where $\mathcal{E}_1(\mathbf{s})$ is the source energy in the first timeslot of duration $t(\mathbf{s})$, and $\mathcal{E}_2(\mathbf{s})$, $\mathcal{E}_3(\mathbf{s})$ are the transmission energies of the source and the relay, respectively, in the second timeslot of duration $1 - t(\mathbf{s})$, $0 < t(\mathbf{s}) \leq 1$. $\mathcal{E}_{tot} := \mathcal{E}_1 + \mathcal{E}_2 + \mathcal{E}_3$ as the total transmission energy used in the transmission of the information from the i^{th} source to the destination. Note that in orthogonal amplify-and-forward protocol, we have $\mathcal{E}_2(\mathbf{s}) = 0$ and $t(\mathbf{s}) = 0.5$.

Define Ω as the set of all possible resource allocation functions. We have

$$\Omega = \{\mathcal{E}(\mathbf{s}) : \mathcal{E}_1(\mathbf{s}) \geq 0, \mathcal{E}_2(\mathbf{s}) \geq 0, \mathcal{E}_3(\mathbf{s}) \geq 0, 0 < t(\mathbf{s}) \leq 1\}.$$

Let $F(\mathbf{s})$ be the probability distribution function of the channel states. Then the long-term average total transmit energy constraint can be written as

$$\begin{aligned} \mathrm{E}[\mathcal{E}_{tot}(\mathbf{s})] &\triangleq \int_s [\mathcal{E}_1(\mathbf{s}) + \mathcal{E}_2(\mathbf{s}) + \mathcal{E}_3(\mathbf{s})] dF(\mathbf{s}) \\ &\leq \mathcal{E}_t. \end{aligned}$$

The long-term average total transmit energy constraint imposes a set of feasible resource allocation functions, $\bar{\Omega} \subseteq \Omega$, that is composed of energy allocation functions which satisfy the above inequality, i.e., $\bar{\Omega} = \{\mathcal{E}(\mathbf{s}) : \mathrm{E}[\mathcal{E}\mathbf{s}] \leq \mathcal{E}_t, \mathcal{E}(\mathbf{s}) \in \Omega\}$.

The problem we want to solve can be formulated as: minimize outage probability subject to a long-term average total energy constraint,

$$\min P_{\mathsf{out}},$$
$$\text{such that } \mathcal{E}(\mathbf{s}) \in \bar{\Omega}$$

In this chapter, we solve the dual problem of minimize long-term average total energy subject to an outage probability constraint.

$$\min \mathrm{E}[\mathcal{E}_{tot}(\mathbf{s})],$$
$$\text{such that } P_{\mathsf{out}} \leq p$$

3.3 Destination Processing and SNR Analysis

The performance measure that we consider in this chapter is outage probability, defined as the probability that the SNR of the source's information at the destination falls below a deterministic threshold ρ, i.e.,

$$P_{\mathsf{out}} := \mathrm{Prob}[\text{outage}] = \mathrm{Prob}[\mathsf{SNR} < \rho].$$

The SNR of the sources's information at the destination is determined not only by the channel states and the transmission energies but also by how the destination forms its decision statistic from the received source and relay transmissions. To better isolate the effect of channel state information at the source and relay, we first assume that the destination has full access to the channel states and transmit energies of both sources in both timeslots and uses maximal ratio combining (MRC) of the relevant source/relay observations in both timeslots to maximize the SNR of the decision statistic.

When the source and relay have access to full CSIT[2], they can dynamically allocate their transmission energies according to the instantaneous channel amplitudes in each transmission interval. The resulting instantaneous SNR at the destination, after MRC, can be expressed as

$$\mathsf{SNR}_1 = a\mathcal{E}_1 + \frac{b\mathcal{E}_1 c\mathcal{E}_3}{1 + b\mathcal{E}_s + c\mathcal{E}_3}. \tag{3.1}$$

When the source and relay do not have access to the channel state, they cannot dynamically allocate their transmission energies in each transmission interval. Instead, they must select a fixed transmission energy based only on knowledge of the channel statistics. The resulting instantaneous SNR at the destination, after MRC, can be expressed as

$$\mathsf{SNR}_2 = a\mathcal{E}_1 + \frac{b\mathcal{E}_1 c\mathcal{E}_3}{1 + \mu_b\mathcal{E}_1 + c\mathcal{E}_3}. \tag{3.2}$$

It may be somewhat surprising that (3.1) and (3.2) appear to be almost identical, the only difference being the expectation in the denominator of (3.2). In both cases, the instantaneous SNR at the destination is fully specified by the normalized channel amplitudes and transmit energies. The fundamental difference between (3.1) and (3.2), however, is in how the transmit energies $\mathcal{E}_1$ and $\mathcal{E}_3$ are selected. In (3.1), the transmit energies are functions of the current channel states a, b, and c whereas, in (3.2), these energies are based only on knowledge of the channel statistics, e.g., μ_a, μ_b, and μ_c. The following sections analyze the significance of this difference in terms of optimum energy allocation strategies and the energy efficiency of the two-source cooperative transmission system.

Now we assume CSI is not available at the destination, hence MRC can't be used. One approach in this scenario is to combine the observations with equal gain, i.e. EGC.

[2]Here, full CSIT means the source and the relay know the instantaneous channel amplitudes but not the phase, the same applies to Chapter 4 and 5.

When the source and relay have access to full CSIT, the resulting instantaneous SNR at the destination, after EGC, can be expressed as

$$\mathsf{SNR}_3 = \frac{a\mathcal{E}_1}{2} + \frac{c\mathcal{E}_1\mathcal{E}_3(b - \frac{c}{2}) + 2\mathcal{E}_1(abc\mathcal{E}_3\psi)^{\frac{1}{2}}}{2\psi + c\mathcal{E}_3} \tag{3.3}$$

where $\psi = b\mathcal{E}_1 + 1$.

While when the source and relay do not have access to the channel state, the resulting instantaneous SNR at the destination, after EGC, can be expressed as

$$\mathsf{SNR}_4 = \frac{\mathcal{E}_1(\sqrt{a} + \sqrt{\varphi bc})^2}{2 + \varphi c} \tag{3.4}$$

where $\varphi = \frac{\mathcal{E}_3}{\mu_b \mathcal{E}_1 + 1}$

The following sections analyze optimum energy allocation strategies and energy efficiency of the two-source cooperative transmission system based on (3.1)-(3.4).

3.4 Optimum Energy Allocation for OAF with Full CSIT/CSIR

3.4.1 Optimum Energy Allocation $p = 0$

To facilitate energy allocation analysis for general $p > 0$, we first consider the case when $p = 0$. The problem in this case is to select an energy allocation $\{\mathcal{E}_1, \mathcal{E}_3\}$ such that $\mathsf{SNR}_1 \geq \rho$ almost surely. Since the source and relay have access to the instantaneous channel amplitudes, they can dynamically allocate their transmission energies such that the randomness induced by the channel state in SNR_1 is removed and $\mathsf{SNR}_1 = \rho$. There are, however, an infinite number of energy allocations that satisfy $\mathsf{SNR}_1 = \rho$. The space of admissible energy allocations satisfying $\mathsf{SNR}_1 = \rho$ can be described as the region in $\mathbb{R}^2$ where $\mathcal{E}_3 \geq 0$ and $\frac{\rho}{a+b} < \mathcal{E}_1 \leq \frac{\rho}{a}$, where the upper limit to $\mathcal{E}_1$ corresponds to the case when $\mathcal{E}_3 = 0$ (direct transmission or, equivalently, no cooperation) and the lower limit corresponds to the case when $\mathcal{E}_3 \to \infty$ (infinite cooperation). In the case of direct transmission, the total energy required to meet the SNR target is $\mathcal{E} = \mathcal{E}_1 = \frac{\rho}{a}$.

Recall that we want to minimize long-term average total transmit energy subject to an outage probability constraint (here we let $p = 0$, i.e. $\mathsf{SNR}_1 = \rho$). When CSIT is available, the SNR threshold can be guaranteed. By minimizing instantaneous total transmit energy subject to the SNR threshold in each transmission interval, we actually minimize the long-term average total transmit energy.

Before deriving the optimum (minimum total energy $\mathcal{E}_1 + \mathcal{E}_3$) cooperative energy allocation strategy in this scenario, we first consider the question of when is it more efficient for the relay to *not* transmit. This is made formal in the following proposition.

Proposition 1. *There exists $\mathcal{E}_{tot} < \frac{\rho}{a}$ if and only if*

$$\frac{c}{a} > 1 + \frac{a}{b\rho}. \tag{3.5}$$

Proof. Using (3.1), the total energy required to satisfy the constraint $\mathsf{SNR}_1 = \rho$ can be written as

$$\mathcal{E}_{tot} := \mathcal{E}_1 + \mathcal{E}_3 = \mathcal{E}_1 + \frac{b\mathcal{E}_1^2 a + (a - b\rho)\mathcal{E}_1 - \rho}{c(\rho - (b + a)\mathcal{E}_1)}. \tag{3.6}$$

Define the interval $\mathcal{A} = \left(\frac{\rho}{a+b}, \frac{\rho}{a}\right]$. If

$$\arg\min_{\mathcal{E}_1 \in \mathcal{A}} \mathcal{E}_{tot} = \frac{\rho}{a}$$

then $\mathcal{E}_3 = 0$ and $\mathcal{E}_{tot}$ is minimized with direct transmission. Otherwise, $\mathcal{E}_3 > 0$ and cooperative transmission minimizes $\mathcal{E}_{tot}$.

In order to determine if the minimum of (3.6) on $\mathcal{A}$ occurs at the point $\mathcal{E}_1 = \frac{\rho}{a}$, we first establish that (3.6) can have only one minimum on $\mathcal{A}$ by proving that (3.6) is a strictly convex function of $\mathcal{E}_1$ on $\mathcal{A}$. The second derivative of (3.6) with respect to $\mathcal{E}_1$ can be written as

$$\mathcal{E}''_{tot} = \frac{2(1-\alpha)b\rho\left[(\rho+1)b+a\right]}{c((b+a)\mathcal{E}_1-\rho)^3}. \tag{3.7}$$

Note that the numerator of (3.7) is a negative quantity not dependent on $\mathcal{E}_1$. Since $\mathcal{E}_1(a+b) > \rho$ and $c > 0$, the denominator of (3.7) is also negative on the interval $\mathcal{E}_1 \in \mathcal{A}$, hence $\mathcal{E}''_{tot}$ is always positive on $\mathcal{A}$. This implies that $\mathcal{E}_{tot}$ is a strictly convex function of $\mathcal{E}_1$ on $\mathcal{A}$.

Given the convexity of $\mathcal{E}_{tot}$ on $\mathcal{A}$, we can determine whether the unique minimum of (3.6) on $\mathcal{A}$ occurs at the point $\mathcal{E}_1 = \frac{\rho}{a}$ by evaluating the first derivative of (3.6) at this point. If the first derivative is positive, then the minimum of (3.6) on $\mathcal{A}$ must occur at $\mathcal{E}_1 < \frac{\rho}{a}$ (corresponding to cooperative transmission), otherwise the minimum occurs at $\mathcal{E}_1 = \frac{\rho}{a}$ (corresponding to direct transmission). The first derivative of (3.6) evaluated at $\mathcal{E}_1 = \frac{\rho}{a}$ can be written as

$$\mathcal{E}'_{tot}\left(\frac{\rho}{a}\right) := \frac{\partial}{\partial \mathcal{E}_1}\mathcal{E}_{tot}\left(\frac{\rho}{a}\right) = 1 - \frac{a(b\rho+a)}{cb\rho}$$

This quantity is positive if and only if the conditions of (3.5) are satisfied, hence the unique minimum of (3.6) on $\mathcal{A}$ must occur at $\mathcal{E}_1 < \frac{\rho}{a}$ when the conditions of (3.5) are satisfied. Otherwise, the minimum of (3.6) on $\mathcal{A}$ must occur at $\mathcal{E}_1 = \frac{\rho}{a}$ and direct transmission is optimum. □

An intuitive explanation of the proof is given in Figure 3.4.

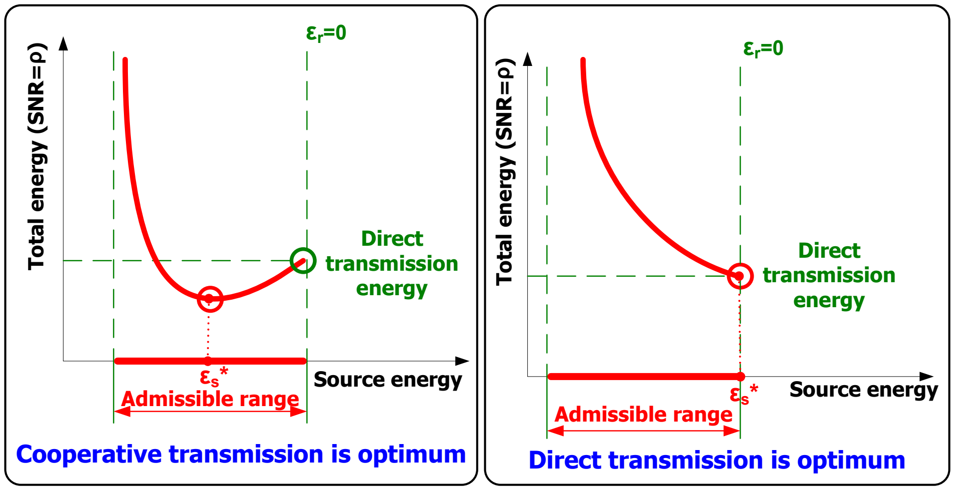

Figure 3.4: The intuitive proof of proposition 1.

Proposition 1 implies that it is possible through cooperative transmission to achieve a reduction in the transmit energy required to meet a fixed SNR target only if the ratio of the relay-destination to source-destination channel gains exceeds some threshold greater than one. If this condition is not satisfied, it is more efficient for the source to satisfy its SNR target through direct transmission and for no energy to be expended by the relay in the cooperative timeslot. Moreover, since c/a cannot be greater than one for both source 1 and source 2, Proposition 1 implies that at most only one source should cooperate in each transmission interval. In some cases, total energy is minimized if neither source cooperates and both sources satisfy their SNR targets via direct transmission.

When the conditions of Proposition 1 are satisfied, the energy allocation that minimizes $\mathcal{E}_{tot}$ while satisfying $\mathsf{SNR}_1 = \rho$ can be determined through standard calculus techniques. The value of $\mathcal{E}_1$ that minimizes $\mathcal{E}_{tot}$ subject to the fixed SNR constraint can be written as

$$\mathcal{E}_s^* = \frac{\rho}{b+a} + \frac{((1-\alpha)\,\rho b)^{1/2}(a+(1+\rho)b)^{1/2}}{(b+a)\,(\alpha c(b+a)-(1-\alpha)ba)^{1/2}} \tag{3.8}$$

and $\mathcal{E}_3^*$ is implied through (3.8) and (3.1) given $\mathsf{SNR}_1 = \rho$. The minimum total transmission energy is then $\mathcal{E}_{tot}^* = \mathcal{E}_1^* + \mathcal{E}_3^*$. Note that the random nature of the channel state implies that $\mathcal{E}_{tot}^*$ is random. We denote the cumulative distribution function of $\mathcal{E}_{tot}^*$ satisfying $\mathsf{SNR}_1 = \rho$ as $F_{\mathcal{E}_{tot}^*}(x) = \text{Prob}[\mathcal{E}_{tot}^* \leq x]$. It can be shown that when $\frac{c}{a} = 1 + \frac{a}{b\rho}$, (3.8) reduces to $\mathcal{E}_1^* = \frac{\rho}{a}$ and $\mathcal{E}_3^* = 0$, as implied by Proposition 1.

3.4.2 Optimum Energy Allocation $p > 0$

We now develop the optimum energy allocation strategy for the case when $p > 0$. Let t denote the value at which $F_{\mathcal{E}_{tot}^*}(t) = 1 - p$. Given the current channel state, if (3.5) is satisfied, solve for the optimum transmission energies $\mathcal{E}_1^*$ and $\mathcal{E}_3^*$ that satisfy $\mathsf{SNR}_1 = \rho$ via (3.8) and (3.1). If (3.5) is not satisfied, direct transmission is optimum and $\mathcal{E}_1^* = \frac{\rho}{a}$ and $\mathcal{E}_3^* = 0$. Note that the resulting minimum total energy $\mathcal{E}_{tot}^* = \mathcal{E}_1^* + \mathcal{E}_3^*$ will exceed the threshold x_t with probability p as shown in Figure 3.5. Since outage events are permitted with probability p, the strategy that minimizes average total transmission energy is to not transmit at all if $\mathcal{E}_{tot}^* > x_t$. If $\mathcal{E}_{tot}^* \leq x_t$, transmission occurs such that $\mathsf{SNR}_1 = \rho$ with the optimum energies $\mathcal{E}_1^*$ and $\mathcal{E}_3^*$. We note that this is essentially an opportunistic transmission strategy where the source and relay avoid transmission (and cause an outage) in cases when the channel state is unfavorable. The outage probability requirement is satisfied under this strategy since the SNR at the destination will be equal to ρ with probability $1 - p$ and equal to zero otherwise.

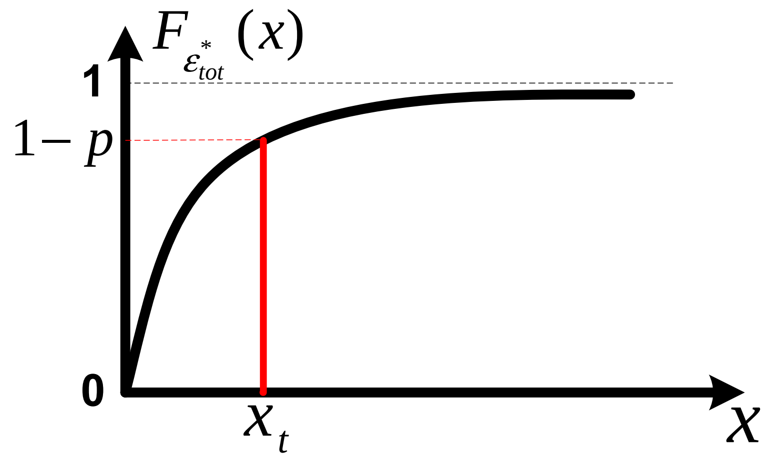

Figure 3.5: Cumulative distribution function of $\mathcal{E}^*_{tot}$.

3.5 Optimum Energy Allocation for OAF with Full CSIT and No CSIR

Recent work in resource allocation of cooperative wireless transmission systems has focused on the impact of the cooperative protocol and CSIT. In this section, we assume both the source and relay have access to full CSIT and derive the optimum energy allocation strategy for equal gain combining (EGC) to analyze the impact of receiver diversity combining on optimum energy allocation and overall energy efficiency.

Using the same approach as in section 3.4, we first consider the case $p = 0$. In this case, the relay node energy $\mathcal{E}_3$ can be written as a function of ρ and $\mathcal{E}_1$ by solving (3.3) for $\mathcal{E}_3$ when $\mathsf{SNR}_3 = \rho$. Note that (3.3) is quadratic in $\mathcal{E}_3$. The two roots for $\mathcal{E}_3$ can be written as

$$\mathcal{E}_{3_{1,2}} = \frac{(b\mathcal{E}_1 + 1)\left(ab\mathcal{E}_1^2 + 2\rho b\mathcal{E}_1 + a\mathcal{E}_1\rho - 2\rho^2\right)}{c(\rho - b\mathcal{E}_1)^2} \pm \frac{2\mathcal{E}_1\sqrt{ab\rho(2b\mathcal{E}_1 + a\mathcal{E}_1 - 2\rho)}}{c(\rho - b\mathcal{E}_1)^2}. \quad (3.9)$$

The admissible range for energy allocations that satisfy $\mathsf{SNR}_3 = \rho$ can be described as the region in $\mathbb{R}^2$ where $\mathcal{E}_3 \geq 0$ and $\frac{2\rho}{2b+a} \leq \mathcal{E}_1 \leq \frac{2\rho}{a}$. In the case when $\mathcal{E}_3 = 0$, (3.3) implies that $\mathcal{E}_1 = \frac{2\rho}{a}$ to satisfy the SNR constraint. This case corresponds to direct transmission and establishes the upper limit on the interval of admissible solutions for $\mathcal{E}_1$. The lower limit on the interval is established by the requirement for total energy to be a real-valued quantity. Inspection of the square root in the numerator of (3.9) reveals that $\mathcal{E}_3 \in \mathbb{R}$ only if $\mathcal{E}_1 \geq \frac{2\rho}{2b+a}$.

Denote the admissable range of $\mathcal{E}_1$ as $\mathcal{A}$. Since $\mathcal{E}_3$ must be a decreasing function of $\mathcal{E}_1$ on $\mathcal{A}$, the correct root of (3.9) must be $\mathcal{E}_{3_2}$. Hence

$$\mathcal{E}_3 = \frac{(b\mathcal{E}_1 + 1)\left(ab\mathcal{E}_1^2 + 2\rho b\mathcal{E}_1 + a\mathcal{E}_1\rho - 2\rho^2\right)}{c(\rho - b\mathcal{E}_1)^2} - \frac{2\mathcal{E}_1\sqrt{ab\rho(2b\mathcal{E}_1 + a\mathcal{E}_1 - 2\rho)}}{c(\rho - b\mathcal{E}_1)^2} \quad (3.10)$$

and the total energy required to satisfy the SNR constraint $\mathsf{SNR}_3 = \rho$ is $\mathcal{E}_{tot} = \mathcal{E}_1 + \mathcal{E}_3$.

The source energy minimization problem can now be stated as

$$\mathcal{E}_1^* = \arg\min_{\mathcal{E}_1 \in \mathcal{A}} \mathcal{E}_{tot}. \quad (3.11)$$

The solution to this problem is aided by the following result.

Proposition 2. *Total energy $\mathcal{E}_{tot}$ is a convex function of $\mathcal{E}_1$ on $\mathcal{A}$.*

Proof. To prove $\mathcal{E}_{tot}$ is convex, and hence has a unique minimum on $\mathcal{A}$, we will show that

$$\frac{\partial^2 \mathcal{E}_{tot}}{\partial \mathcal{E}_1^2} = \frac{b\rho\, \mathsf{f}(y)}{2c(b\mathcal{E}_1 - \rho)^4(ab\rho(2b\mathcal{E}_1 + a\mathcal{E}_1 - 2\rho))^{\frac{3}{2}}} \geq 0. \quad (3.12)$$

The function

$$\mathsf{f}(y) = \frac{(y - \rho a)^4\ \mathsf{r}(y)}{(2b + a)^2\rho^3 a^2}. \quad (3.13)$$

where $y := \sqrt{ab\rho(2b\mathcal{E}_1 + a\mathcal{E}_1 - 2\rho)}$. Note that $b\rho \geq 0$ and the denominator of (3.12) is nonnegative on $\mathcal{A}$. Hence, the condition $\frac{\partial^2 \mathcal{E}_{tot}}{\partial \mathcal{E}_1^2} \geq 0$ on $\mathcal{A} \Leftrightarrow \mathsf{f}(\mathsf{y}) \geq 0$ on $\mathcal{C}$ where

$\mathcal{C} = [0, 2\rho b]$. The function $\mathsf{r}(y)$ can be written as

$$\begin{aligned}\mathsf{r}(y) &= y^4 + 4a\rho y^3 - (12\rho^2 ab + 3\rho a^2 + 6\rho ab)y^2 \\ &\quad + (16\rho^3 b^2 a + 8\rho^2 ba^2 + 16\rho^2 b^2 a)y \\ &\quad + 4a^2 b 2\rho^4 + 2a^3 b\rho^3 + 4b^2 a^2 \rho^3.\end{aligned} \tag{3.14}$$

Note that $\frac{(y-\rho a)^4}{(2b+a)^2\rho^3 a^2} \geq 0$. Hence, the condition $\frac{\partial^2 \mathcal{E}_{tot}}{\partial \mathcal{E}_1^2} \geq 0$ on $\mathcal{A} \Leftrightarrow \mathsf{r}(y) \geq 0$ on $\mathcal{C}$. We consider the behavior of $\mathsf{r}(y)$ in two separate cases: $a \leq 4\rho b$ and $a > 4\rho b$.

Claim 1: $\mathsf{r}(y) \geq 0$ on $\mathcal{C}$ when $a \leq 4\rho b$.

Proof. Observe that $\mathsf{r}(0) = 4a^2 b^2 \rho^4 + 2a^3 b\rho^3 + 4b^2 a^2 \rho^3 \geq 0$. To prove that $\mathsf{r}(y) \geq 0$ on $\mathcal{C}$, it is only necessary to prove that $\mathsf{r}(y)$ is non-decreasing on $\mathcal{C}$. This will be shown by proving that the minimum of $\mathsf{s}(y) := \frac{\partial \mathsf{r}(y)}{\partial y} \geq 0$. We can write

$$\begin{aligned}\mathsf{s}(y) = 4y^3 + 12a\rho y^2 - 2(12\rho^2 ab + 3\rho a^2 + 6\rho ab)y \\ +16\rho^3 b^2 a + 8\rho^2 ba^2 + 16\rho^2 b^2 a.\end{aligned}$$

It can be shown that $\frac{\partial^2 \mathsf{s}(y)}{\partial y^2} \geq 0$ on $\mathcal{C}$. Hence, $\mathsf{s}(y)$ is convex and has a unique minimum on $\mathcal{C}$. The function $\mathsf{s}(y)$ achieves its unique minimum at the point $y^* := \arg\min_{y \in C} \mathsf{s}(y)$. This point can be written as

$$y^* = -a\rho + \sqrt{4a^2\rho^2 + 8\rho^2 ab + 4a\rho b + 2a^2\rho}.$$

It can be shown that $\mathsf{s}(y^*) \geq 0$ when $a \leq 4\rho b$. Hence, $\frac{\partial \mathsf{r}(y)}{\partial y} \geq 0$ for $y \in \mathcal{C}$ and $\mathsf{r}(y)$ is non-decreasing on $\mathcal{C}$. Since $\mathsf{r}(0) \geq 0$, this result implies that $\mathcal{E}_{tot}$ is convex on $\mathcal{A}$ when $a \leq 4\rho b$. □

Claim 2: $\mathsf{r}(y) \geq 0$ on $\mathcal{C}$ when $a > 4\rho b$.

Proof. Observe that $\mathsf{r}(0) \geq 0$ and $\mathsf{r}(2\rho b) \geq 0$. It is sufficient to show that $\mathsf{r}(y)$ is concave on $\mathcal{C}$ to imply that $\mathsf{r}(y) \geq 0$. It can be shown that $\mathsf{u}(y) := \frac{\partial^2 \mathsf{r}(y)}{\partial y^2}$ is convex. It can also be shown that $\mathsf{u}(0) \leq 0$ and $\mathsf{u}(2\rho b) \leq 0$ when $a > 4\rho H$. Hence, $\mathsf{u}(y) \leq 0$ on $\mathcal{C}$. This implies that $\mathsf{r}(y)$ is concave, which implies that $\mathsf{r}(y) \geq 0$ on $\mathcal{C}$ when $a > 4\rho b$. □

Claim 1 and Claim 2 imply that $\mathcal{E}_{tot}$ is convex on $\mathcal{C}$. □

Proposition 2 implies that there is a unique solution to (3.11). An explicit analytical solution to (3.11) requires computing the roots of a cubic polynomial. The convexity of $\mathcal{E}_{tot}$ on $\mathcal{A}$, however, implies that a variety of existing numerical methods can be used to solve (3.11). The optimum relay energy $\mathcal{E}_3^*$ is implied by (3.10) for $\mathcal{E}_1 = \mathcal{E}_1^*$.

Before we develop the energy allocation strategy for general $p > 0$, we still need to answer a question: when is it more efficient for the relay to *not* transmit. This is answered by the following proposition:

Proposition 3. *$\mathcal{E}_3^* > 0$ for all values of a, b, c, ρ when the destination uses EGC combining.*

Proof. Given the convexity of $\mathcal{E}_{tot}$ on $\mathcal{A}$, we can determine whether the unique minimum of $\mathcal{E}_{tot}$ on $\mathcal{A}$ occurs at the point $\mathcal{E}_1 = \frac{2\rho}{a}$ by evaluating $\frac{\partial \mathcal{E}_{tot}}{\partial \mathcal{E}_1}$ at this point. If $\frac{\partial \mathcal{E}_{tot}}{\partial \mathcal{E}_1} > 0$ at $\mathcal{E}_1 = \frac{2\rho}{a}$, then the minimum of $\mathcal{E}_{tot}$ on $\mathcal{A}$ must occur at $\mathcal{E}_1 < \frac{2\rho}{a}$ (corresponding to $\mathcal{E}_3^* > 0$), otherwise the minimum occurs at $\mathcal{E}_1 = \frac{2\rho}{a}$ (corresponding to $\mathcal{E}_3^* = 0$). It can be shown that

$$\left.\frac{\partial \mathcal{E}_{tot}}{\partial \mathcal{E}_1}\right|_{\mathcal{E}_1 = \frac{2\rho}{a}} = 1 > 0,$$

hence the unique minimum of $\mathcal{E}_{tot}$ on $\mathcal{A}$ must occur at $\mathcal{E}_1 < \frac{2\rho}{a}$. This implies that $\mathcal{E}_3^* > 0$ for all a, b, c, ρ. □

We now consider the optimum energy allocation strategy for the case when $p > 0$ using the same approach in section 3.4. Note that the random nature of the channel state implies that $\mathcal{E}_{tot}^*$ is random. We denote the cdf of $\mathcal{E}_{tot}^*$ satisfying $\mathsf{SNR}_3 = \rho$ as $F_{\mathcal{E}_{tot}^*}(x) := \text{Prob}[\mathcal{E}_{tot}^* \leq x]$. Let x_t denote the value at which $F_{\mathcal{E}_{tot}^*}(x_t) = 1 - p$. Given the current channel state, solve for the optimum transmission energies $\mathcal{E}_1^*$ and $\mathcal{E}_3^*$ that satisfy $\mathsf{SNR}_{tot} = \rho$ via (3.4), (3.10), and (3.11). Note that the resulting minimum instantaneous total energy $\mathcal{E}_{tot}^*$ will exceed the threshold x_t with probability p. Since outage events are permitted with probability p, the strategy that minimizes average total transmission energy is to *not* transmit if $\mathcal{E}_{tot}^* > x_t$. If $\mathcal{E}_{tot}^* \leq x_t$, transmission occurs such that $\mathsf{SNR}_3 = \rho$ with the optimum energies $\mathcal{E}_1^*$ and $\mathcal{E}_3^*$. We note that this is an opportunistic transmission strategy where the source and relay avoid transmission (and cause an outage) in cases when the channel state is unfavorable. The outage probability requirement is satisfied under this strategy since the SNR at the destination will be equal to ρ with probability $1 - p$ and equal to zero otherwise.

3.6 Optimum Energy Allocation for OAF with No CSIT and Full CSIR

When the source and relay do not have access to the channel state, they cannot dynamically allocate their transmit energy and must instead select fixed transmission energies based only on the channel statistics. In this section, we assume the destination has full access to the channel state information (CSIR). We begin our analysis of optimum energy allocation in this scenario by first deriving expressions for the outage probability of the two-source cooperative transmission system assuming fixed transmission energies.

3.6.1 Outage Probability Bounds Analysis

A general and exact expression for the outage probability in the fixed transmission energy scenario follows directly from the fact that the outage probability is the cumulative distribution function of the random variable SNR_2. Given the joint channel density $f_{a,b,c}(\boldsymbol{x})$, the outage probability can be expressed as

$$P_{\mathsf{out}} = \int_{R(\rho)} f_{a,b,c}(\boldsymbol{x})\, d\boldsymbol{x} \tag{3.15}$$

where the three-dimensional integration region $R(\rho)$ is derived from (3.2) and given as

$$R(\rho) = \Big\{ \boldsymbol{x} \in \mathbb{R}^3 \,:\, 0 \le x_1 \le \frac{\rho}{\mathcal{E}_1},\; 0 \le x_2 < \infty, \\ \text{and } 0 \le x_3 \le \frac{(\rho - \mathcal{E}_1 x_1)(1 + \mu_b \mathcal{E}_1 + \mathcal{E}_3 x_2)}{\mathcal{E}_1 \mathcal{E}_3 x_2} \Big\}.$$

While the explicit description of the integration region $R(\rho)$ is straightforward, explicit analytical solutions to (3.15) are difficult to obtain in many common cases. To facilitate analysis, we present a pair of bounds to (3.15) below.

Lower Bound: Perfect Source-Relay Channel

A lower bound on the outage probability can be obtained by assuming that the channel between the sources b is perfect, i.e. $b \to \infty$ and $\mu_b \to \infty$. In this case, (3.2) reduces to

$$\mathsf{SNR}_2 = a\mathcal{E}_1 + c\mathcal{E}_3.$$

Evaluation of the outage probability in this case involves only two-dimensional integration of the joint channel density $f_{a,c}(\boldsymbol{x})$ over the region

$$R_{lb}(\rho) = \left\{ \boldsymbol{x} \in \mathbb{R}^2 \,:\, 0 \leq x_1 \leq \frac{\rho}{\mathcal{E}_1} \text{ and } 0 \leq x_3 \leq \frac{\rho - \mathcal{E}_1 x_1}{\mathcal{E}_3} \right\}. \tag{3.16}$$

In the case of independent Rayleigh fading channels, this bound can be evaluated explicitly. The random variables a and c are exponentially distributed in this case, with means denoted as μ_a and μ_c, respectively. Evaluation of (3.15) with the integration region specified in (3.16) yields

$$P_{\textbf{out}} \geq \frac{\mu_a \mathcal{E}_1 \left(1 - \exp\left(\frac{-\rho}{\mu_a \mathcal{E}_1}\right)\right) - \mu_c \mathcal{E}_3 \left(1 - \exp\left(\frac{-\rho}{\mu_c \mathcal{E}_3}\right)\right)}{\mu_a \mathcal{E}_1 - \mu_c \mathcal{E}_3}. \tag{3.17}$$

Upper Bound: Rectangular Integration Region

An upper bound on the outage probability expression (3.15) can be obtained by noting that slices of the (exact) three-dimensional integration region $R(\rho)$ in the x_1, x_2 plane are triangular for all $x_3 > 0$. These triangular slices can be overbounded by enclosing rectangles with the identical intercepts to yield a slightly simplified integration region for (3.15) as

$$\begin{aligned} R_{ub}(\rho) = \Bigg\{ \boldsymbol{x} \in \mathbb{R}^3 \,:\, 0 \leq x_1 \leq \frac{\rho}{\mathcal{E}_1},\ 0 \leq x_2 < \infty, \\ \text{and } 0 \leq x_3 \leq \frac{\rho(1 + \mu_b \mathcal{E}_1 + \mathcal{E}_3 x_2)}{\mathcal{E}_1 \mathcal{E}_3 x_2} \Bigg\}. \end{aligned} \tag{3.18}$$

where the key advantage with respect to the exact integration region is that x_1 has been eliminated from the upper limit of the integral over x_3. In the case of independent Rayleigh fading channels, this bound can be evaluated explicitly. Evaluation of (3.15) with the integration region specified in (3.18) yields

$$P_{\textbf{out}} \leq \left(1 - \exp\left(\frac{-\rho}{\mu_a \mathcal{E}_1}\right)\right) \cdot \left(1 - \exp\left(\frac{-\rho}{\mu_b \mathcal{E}_1}\right) \psi K_1(\psi)\right) \tag{3.19}$$

where $K_1(\psi)$ is the modified Bessel function of the second kind and $\psi := 2\sqrt{\frac{\rho(1+\mu_b\mathcal{E}_1)}{\mathcal{E}_1\mathcal{E}_3\mu_c\mu_b}}$. Note that the relay transmission energy $\mathcal{E}_3$ appears only in the denominator of ψ in the upper bound (3.19).

As shown in Figure 3.6, these bounds can be fairly tight and show the trend of the exact outage probability.

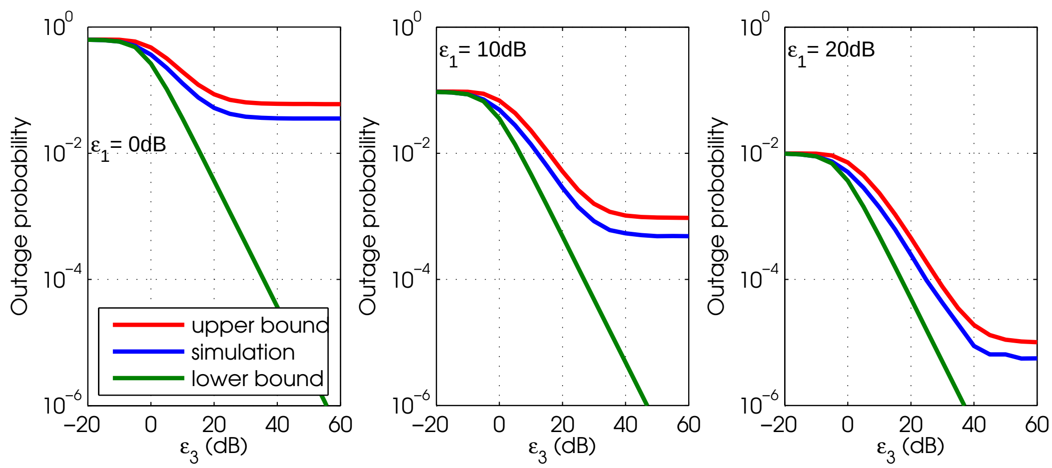

Figure 3.6: Outage probability bounds in independent Rayleigh fading.

Optimum Energy Allocation Analysis

While the outage probability bounds given in (3.17) and (3.19) are explicit and easily evaluated for any choice of transmit energies, analytical solutions to the optimum source/relay transmit energies based on these bounds are difficult to derive due to the fact that neither bound appears to yield an explicit solution for one transmit energy in terms of the other. Optimum energy allocations based on (3.17) and (3.19) can be obtained, however, using numerical optimization methods. Section 3.8 compares the numerical solutions to the optimum energy allocation and energy efficiency of cooperative transmission without CSIT to the analytical optimum energy allocation and energy efficiency of systems with full CSIT.

3.7 Optimum Energy Allocation for OAF with No CSIT and No CSIR

In this section, we consider the situation that neither the source and relay nor the destination knows the channel state information. From previous sections, we know that when the source and relay do not have access to the channel state, they cannot dynamically allocate their transmit energy and must instead select fixed transmission energies based only on the channel statistics. And when CSI is not available at the destination we simply choose to combine the observations with equal gain (EGC).

Recall that the outage probability is the cumulative distribution function of the random variable SNR. Given the joint channel density $f_{a,b,c}(\boldsymbol{x})$, the outage probability can be expressed as

$$P_{\mathbf{out}} = \mathbf{Prob}[\mathsf{SNR} < \rho] \tag{3.20}$$

$$= \mathbf{Prob}\left[\frac{(\sqrt{a}+\sqrt{\varphi bc})^2}{2+\varphi c} < \xi\right] \tag{3.21}$$

$$= \int_{R(\xi)} f_{a,b,c}(\boldsymbol{x})\, d\boldsymbol{x} \tag{3.22}$$

where $\xi = \frac{\rho}{\mathcal{E}_1}$ where the three-dimensional integration region $R(\xi)$ is derived from (3.4).

Following the approach in Section 3.6, we can derive a lower bound on (3.20) by assuming the cooperative channel is perfect. The expression (3.4) is reduced to

$$\mathsf{SNR} = \frac{1}{2}(\sqrt{a\mathcal{E}_1}+\sqrt{c\mathcal{E}_3})^2.$$

Evaluation of the outage probability in this case involves only two-dimensional integration of the joint channel density $f_{a,c}(\boldsymbol{x})$ over the region

$$R_{lb}(\rho) = \left\{ \boldsymbol{x} \in \mathbb{R}^2 \,:\, 0 \leq x_1 \leq \frac{2\rho}{\mathcal{E}_1} \text{ and } 0 \leq x_3 \leq \frac{(\sqrt{2\rho} - \sqrt{\mathcal{E}_1 x_1})^2}{\mathcal{E}_3} \right\}. \tag{3.23}$$

In the case of independent Rayleigh fading channels, this bound can be evaluated explicitly. Evaluation of (3.20) with the integration region specified in (3.23) yields

$$\begin{aligned}
P_{\mathbf{out}} \;\; \geq \;\; & -\sqrt{2\pi\rho}\, \mathbf{erf}\left(\sqrt{2}\sqrt{\rho}{\mu_a}^{-1}{\mathcal{E}_1}^{-1} \frac{1}{\sqrt{\frac{1}{\mu_a\mathcal{E}_1} + \frac{1}{\mu_c\mathcal{E}_3}}} \right) \\
\cdot \;\; & \exp\left(\frac{\rho\mu_a\mathcal{E}_1}{\mu_c\mathcal{E}_3\left(\mu_c\mathcal{E}_3 + \mu_a\mathcal{E}_1\right)} \right)^2 \\
\cdot \;\; & \exp\left(\frac{\rho\mu_c\mathcal{E}_3}{\mu_a\mathcal{E}_1\left(\mu_c\mathcal{E}_3 + \mu_a\mathcal{E}_1\right)} \right)^2 \exp\left(\frac{\rho}{\mu_c\mathcal{E}_3 + \mu_a\mathcal{E}_1} \right)^2 \\
\cdot \;\; & \exp\left(\frac{\rho}{\mu_a\mathcal{E}_1} \right)^{-2} \exp\left(\frac{\rho}{\mu_c\mathcal{E}_3} \right)^{-2} \\
\cdot \;\; & (\mu_c\mathcal{E}_3 + \mu_a\mathcal{E}_1)^{-1} \frac{1}{\sqrt{\frac{1}{\mu_a\mathcal{E}_1} + \frac{1}{\mu_c\mathcal{E}_3}}} - \mu_a\mathcal{E}_1 \\
\cdot \;\; & \exp\left(\frac{\rho}{\mu_a\mathcal{E}_1} \right)^{-2} (\mu_c\mathcal{E}_3 + \mu_a\mathcal{E}_1)^{-1} \\
- \;\; & \mu_c\mathcal{E}_3 \exp\left(\frac{\rho}{\mu_c\mathcal{E}_3} \right)^{-2} (\mu_c\mathcal{E}_3 + \mu_a\mathcal{E}_1)^{-1} \\
+ \;\; & \frac{\mu_c\mathcal{E}_3}{\mu_c\mathcal{E}_3 + \mu_a\mathcal{E}_1} - \sqrt{2\pi\rho} \exp\left(\frac{\rho\mu_a\mathcal{E}_1}{\mu_c\mathcal{E}_3\left(\mu_c\mathcal{E}_3 + \mu_a\mathcal{E}_1\right)} \right)^2 \\
\cdot \;\; & \mathbf{erf}\left(\sqrt{2}\sqrt{\rho}{\mu_c}^{-1}{\mathcal{E}_3}^{-1} \frac{1}{\sqrt{\frac{1}{\mu_a\mathcal{E}_1} + \frac{1}{\mu_c\mathcal{E}_3}}} \right) \exp\left(\frac{\rho}{\mu_c\mathcal{E}_3} \right)^{-2} \\
\cdot \;\; & (\mu_c\mathcal{E}_3 + \mu_a\mathcal{E}_1)^{-1} \frac{1}{\sqrt{\frac{1}{\mu_a\mathcal{E}_1} + \frac{1}{\mu_c\mathcal{E}_3}}} + \frac{\mu_a\mathcal{E}_1}{\mu_c\mathcal{E}_3 + \mu_a\mathcal{E}_1}
\end{aligned} \tag{3.24}$$

An upper bound on the outage probability expression (3.20) can be obtained by overbounding the x_1, x_2 plane by enclosing rectangles, i.e.

$$R_{ub}(\xi) = \left\{ \boldsymbol{x} \in \mathbb{R}^3 \,:\, 0 \leq x_1 \leq 2\xi + \varphi\xi x_3, 0 \leq x_2 \leq \frac{2\xi}{\varphi x_3} + \xi, \;\; \text{and } 0 \leq x_3 < \infty \right\}. \tag{3.25}$$

Given independent Rayleigh fading channels, the resulting upper bound can be computed to be

$$\begin{aligned} P_{\mathbf{out}} \;\leq\; & 2\sqrt{2}\left[\frac{\xi\mu_a}{\varphi\mu_b\mu_c(\mu_a+\varphi\xi\mu_c)}\right]^{\frac{1}{2}}\exp\left(-\frac{\xi}{\mu_b}-2\frac{\xi}{\mu_a}\right)K_1\left(2\sqrt{2}\left[\frac{\xi(\mu_a+\varphi\mu_c\xi)}{\varphi\mu_a\mu_b\mu_c}\right]^{\frac{1}{2}}\right) \\ - \;& \frac{\exp(-2\frac{\xi}{\mu_a})}{1+\frac{\varphi\xi\mu_c}{\mu_a}} - 2\sqrt{2}\left[\frac{\xi}{\varphi\mu_b\mu_c}\right]^{\frac{1}{2}}\exp\left(-\frac{\xi}{\mu_b}\right)K_1\left(2\sqrt{2}\left[\frac{\xi}{\varphi\mu_b\mu_c}\right]^{\frac{1}{2}}\right)+1. \quad (3.26) \end{aligned}$$

Figure 3.7 shows the efficacy of the bounds as a function of the transmit energy $\mathcal{E}_3$ for three values of $\mathcal{E}_1$. Note that the lower bound tends to be tight for small values of $\mathcal{E}_3$ but becomes loose as $\mathcal{E}_3$ becomes large. The upper bound is loose for small values of $\mathcal{E}_3$ but tends to become tight as $\mathcal{E}_3$ becomes large.

Note that, in contrast to the MRC case, the outage probability in the EGC case is not monotonically decreasing in the relay energy. As can be seen in Figure 3.7 for the cases $\mathcal{E}_1 = 10$dB and $\mathcal{E}_1 = 20$dB, increasing relay energy may lead to an increase in the outage probability. The reason for this behavior is that, when a relay using OAF increases its transmit energy, it also amplifies the noise. A destination using EGC combines the observation from the source and the relay with equal gain, which may lead to increased outage probabilities when the noisy relay transmission is at much greater transmit energy than the original source transmission. This implies that the relay energy must be carefully chosen when the destination uses EGC combining: too much relay power wastes energy and increases the outage probability. As seen in Figure 3.6, this behavior does not occur when the destination uses MRC.

3.8 Numerical Results

This section provides numerical results on the energy efficiency of OAF cooperative transmission for each of the four cases analyzed in this chapter. Comparisons between

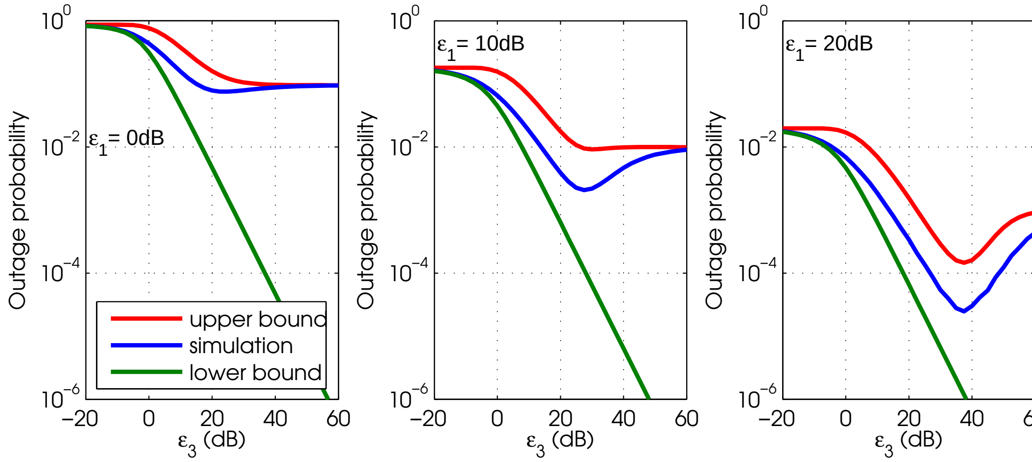

Figure 3.7: Outage probability bounds in independent Rayleigh fading for the case with no CSIT and no CSIR.

cases are also provided to quantify the value of channel state knowledge on the energy efficiency of OAF cooperative transmission.

3.8.1 Full CSIT/CSIR versus no CSIT/CSIR

This section presents numerical examples demonstrating the impact of CSIT on optimum cooperative energy allocation and energy efficiency for the case when the channels shown in Figure 3.3 are Rayleigh fading and independent. All of the results in this section assume $\mu_b = 100$, and $\rho = 10$dB. Figures 3.8 and 3.11 consider the case when the relay has a statistically *advantaged* channel to the destination, i.e. $\mu_c = 100$ and $\mu_a = 10$. Figures 3.9 and 3.12 consider the case when the source and relay face statistically *symmetric* independent Rayleigh fading channels to the destination, i.e. $\mu_c = \mu_a = 10$. Finally, Figures 3.10 and 3.13 consider the case when the relay has a statistically *disadvantaged* channel to the destination, i.e. $\mu_c = 10$ and $\mu_a = 100$.

Figures 3.8, 3.9, and 3.10 show the optimum source/relay energy allocations to achieve the outage probability target p for the cases when the relay faces a statistically

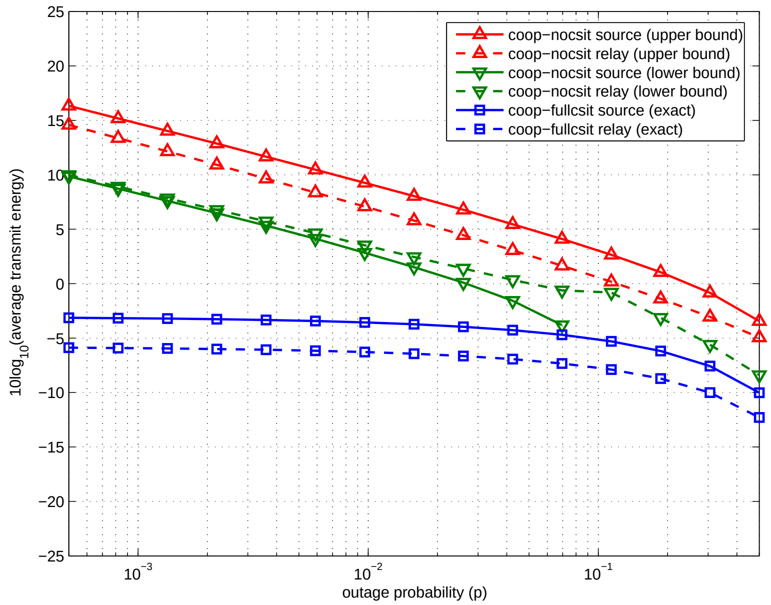

Figure 3.8: Optimum energy allocation when the relay has a statistically *advantaged* channel to the destination. Note that the source does not transmit for $p > 0.07$ when using the lower bound (3.17).

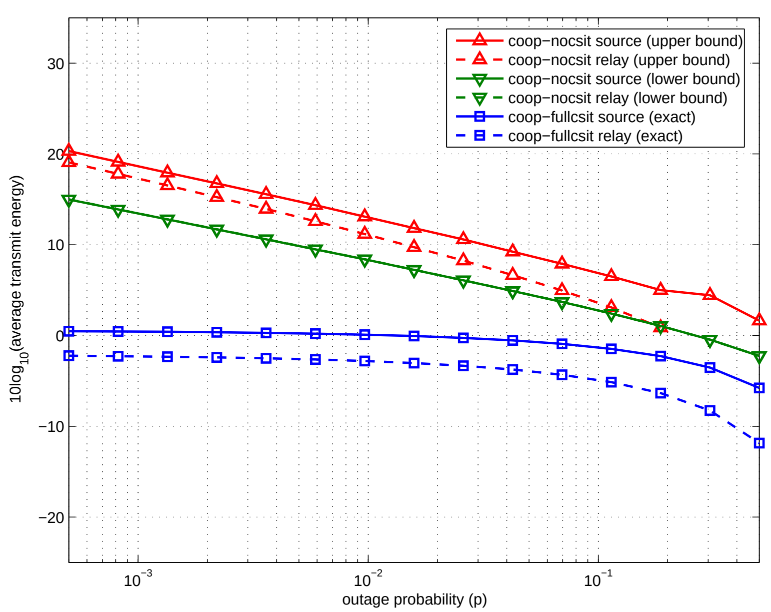

Figure 3.9: Optimum energy allocation when source/relay face statistically *symmetric* channels to the destination. Note that the source and relay transmit with identical energy when using the lower bound (3.17).

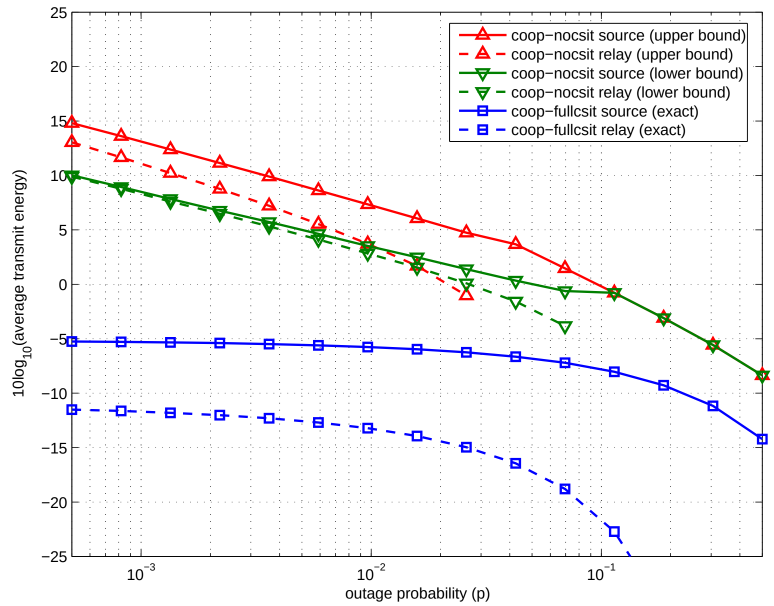

Figure 3.10: Optimum energy allocation when the relay has a statistically *disadvantaged* channel to the destination. Note that the relay does not transmit for $p > 0.07$ when using the lower bound (3.17).

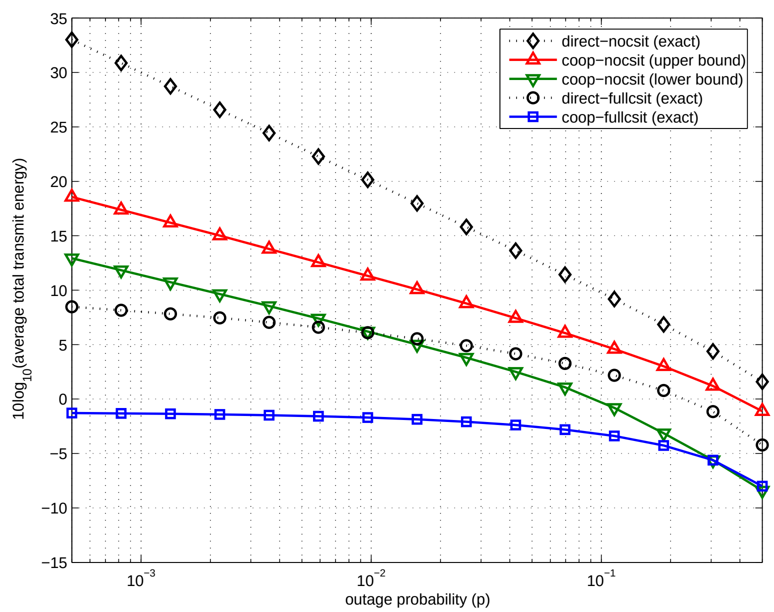

Figure 3.11: Average total transmission energy of optimum cooperative and direct transmission when the relay has a statistically *advantaged* independent Rayleigh fading channel to the destination.

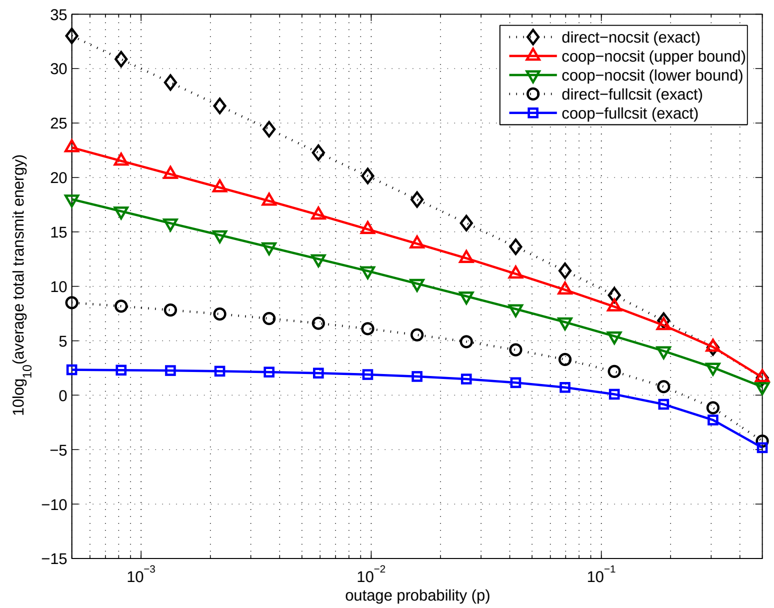

Figure 3.12: Average total transmission energy of optimum cooperative and direct transmission when source/relay face statistically *symmetric* independent Rayleigh fading channels to the destination.

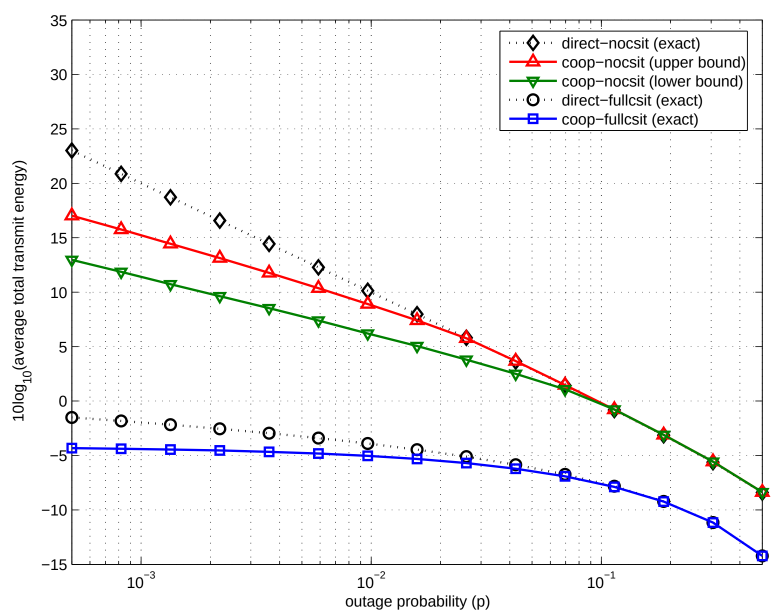

Figure 3.13: Average total transmission energy of optimum cooperative and direct transmission when the relay has a statistically *disadvantaged* independent Rayleigh fading channel to the destination.

advantaged, symmetric, and disadvantaged channel to the destination, respectively. With full CSIT, the source always transmits with more energy than the relay and the energy gap grows as the relay's channel becomes less advantaged. This can be attributed the fact that the source must always transmit, but the relay will not cooperate with high probability when it faces a statistically disadvantaged channel to the destination, as implied in Proposition 1. Without CSIT, the optimum source and relay energies were numerically determined using the lower and upper bounds in (3.17) and (3.19), respectively. The upper bound results show that, in all cases without CSIT, the source transmits with more energy than the relay. In fact, the upper bound results suggest that the relay should not cooperate when its channel is symmetric or disadvantaged with respect to the source and p is sufficiently large. The lower bound energy allocations are less accurate than the upper bound due to the ideal inter-source channel assumption. When the relay has an advantaged channel, for example, the lower bound results suggest that only the relay should transmit. The lower bound results do, however, tend to agree more closely with the upper bound results when the relay channel is disadvantaged.

Figures 3.11, 3.12, and 3.13 show the total transmit energies needed to satisfy the outage probability target p (assuming optimum energy allocation) and include results for direct transmission with and without CSIT for comparison. The results show, as expected, that cooperative transmission without CSIT achieves a fixed outage probability with less total energy than direct transmission without CSIT. Similarly, cooperative transmission with full CSIT achieves a fixed outage probability with less total energy direct transmission with full CSIT. In both cases, the energy gains tend to be large when the relay has a statistically advantaged channel to the destination and/or $p \to 0$. The potential energy gain of cooperative transmission diminishes as the relay's channel becomes less advantaged, and goes to zero at larger values of p due to the fact the minimum energy strategy is for the source to use direct transmission when the relay's channel is statistically symmetric or disadvantaged and p is large.

Figures 3.11, 3.12, and 3.13 also expose the impact of full CSIT on the overall energy efficiency the communication system shown in Figure 3.3. Both direct transmission and cooperative transmission are considerably more efficient when full CSIT is available. It is somewhat surprising to note, however, that direct transmission with full CSIT is more energy efficient than cooperative transmission without CSIT in almost all of the cases considered. In fact, when the relay has a statistically symmetric or disadvantaged channel to the destination, the energy required for direct transmission with full CSIT is less than even the lower bound results for cooperative transmission without CSIT for all p. In the case when the relay has an a statistically advantaged channel, the energy required for direct transmission with full CSIT is less than the upper bound results for cooperative transmission without CSIT for all p. These results demonstrate that a feedback channel providing full CSIT to a source may offer more benefit, at least in terms of transmission energy efficiency in fading channels, than cooperation without CSIT.

3.8.2 Full CSIT/CSIR versus full CSIT/no CSIR

This section presents numerical examples demonstrating the impact of receiver diversity combining on optimum cooperative energy allocation and energy efficiency for the case when the channels are Rayleigh fading and independent. Denote μ_a, μ_b, and μ_c as the mean of the exponential random variables a, b, and c. All of the results in this section assume $\mu_b = 100$, and $\rho = 10$dB. Figures 3.14 and 3.15 consider three separate cases: (i) when the relay has a statistically *advantaged* channel to the destination, i.e. $\mu_c = 100$ and $\mu_a = 10$; (ii) when the source and relay face statistically *symmetric* Rayleigh fading channels to the destination, i.e. $\mu_a = \mu_c = 10$; and (iii) when the relay has a statistically *disadvantaged* channel to the destination, i.e. $\mu_c = 1$ and $\mu_a = 10$.

Figure 3.14 shows the average minimum total transmit energies $\mathrm{E}[\mathcal{E}^*_{tot\_\mathsf{mrc}}]$ and $\mathrm{E}[\mathcal{E}^*_{tot\_\mathsf{egc}}]$ needed to satisfy the outage probability target p for the optimum energy allocation strategies developed in previous section. Direct transmission results (where $\mathcal{E}^*_1 = \frac{\rho}{a}$) are also included for comparison. These results show that the average total transmit energy decreases for both MRC and EGC as the relay channel becomes more advantaged and as $p \to 1$. As expected, MRC is more energy efficient than EGC. The energy gap between MRC and EGC grows as the relay channel becomes less advantaged, implying that knowledge of the channel state at the destination is more critical when the relay does not have a clearly advantaged channel. Cooperative transmission with MRC or EGC achieves a fixed outage probability with less average total energy than direct transmission in most cases, especially for $p \to 0$. Direct transmission outperforms cooperative transmission with EGC, however, when the relay's channel is statistically symmetric or disadvantaged and $p \to 1$.

Figure 3.15 shows the energy ratio $10 \log_{10} \frac{\mathrm{E}[\mathcal{E}^*_1]}{\mathrm{E}[\mathcal{E}^*_3]}$ as a function of p. These results show that the energy ratio tends to be smaller for EGC than MRC, implying that the relay assumes a larger role in cooperative communication systems where the destination does not have reliable CSI and uses EGC. As $p \to 1$, the relay tends to transmit with less relative energy for both MRC and EGC. For $p \to 0$, the relay tends to take on a larger fraction of the average total energy burden. The exception to both of these trends occurs when the relay has an advantaged channel to the destination.

In this case, the results in Figure 3.14 show that both the source and the relay benefit from the advantaged relay channel. The relay, however, tends to experience a greater reduction in optimum transmit energy than the source due to its advantaged channel to the destination.

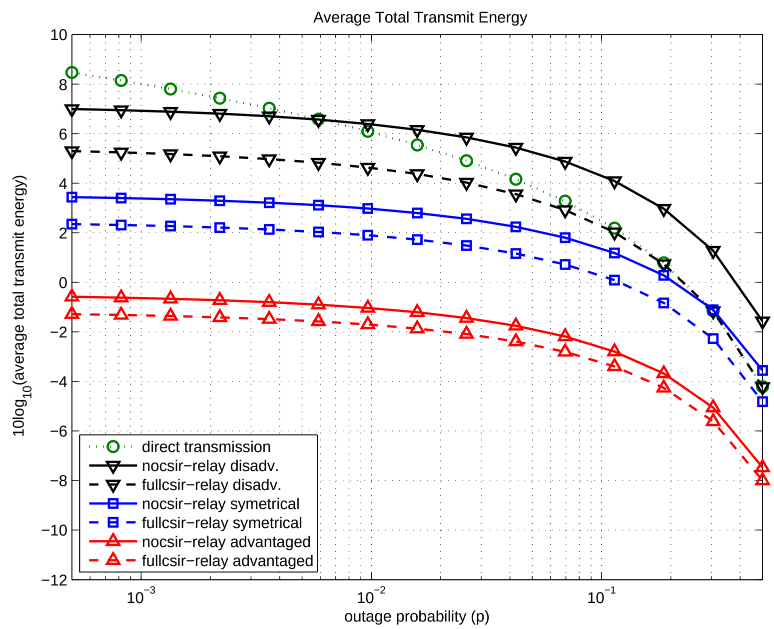

Figure 3.14: Average minimum total transmission energy vs. outage probability p. Direct transmission results are included for comparison.

3.8.3 Full CSIT/no CSIR versus no CSIT/no CSIR

This section presents numerical examples demonstrating the impact of CSIT on optimum cooperative energy allocation and energy efficiency when CSIR is not available at the destination. All of the results in this section assume $\mu_b = 100$, and $\rho = 10$dB. Figures 3.16 and 3.19 consider the case when the relay has a statistically *advantaged* channel to the destination, i.e. $\mu_c = 100$ and $\mu_a = 10$. Figures 3.17 and 3.20 consider the case when the source and relay face statistically *symmetric* indepen-

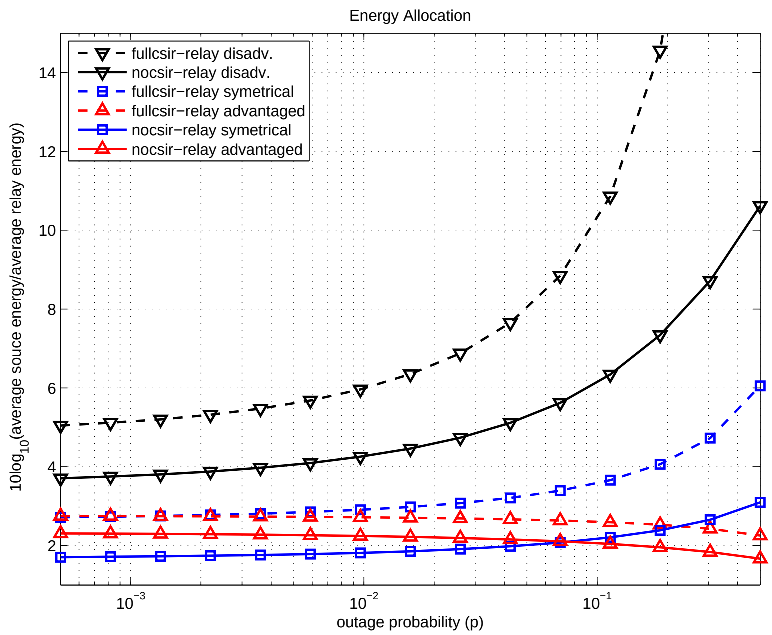

Figure 3.15: Optimum average source to relay energy allocation ratio vs. outage probability p.

dent Rayleigh fading channels to the destination, i.e. $\mu_c = \mu_a = 10$. Finally, Figures 3.18 and 3.21 consider the case when the relay has a statistically *disadvantaged* channel to the destination, i.e. $\mu_c = 10$ and $\mu_a = 100$. Matlab codes are included in Appendix B

Figures 3.16, 3.17, and 3.18 show the optimum source/relay energy allocations to achieve the outage probability target p for the cases when the relay faces a statistically advantaged, symmetric, and disadvantaged channel to the destination, respectively. Figures 3.11, 3.12, and 3.13 show the total transmit energies needed to satisfy the outage probability target p (assuming optimum energy allocation) and include results for direct transmission with and without CSIT for comparison. The results show, as expected, that the trend is pretty much the same as those in section 3.8.1, which shows that when CSIR is not available, a feedback channel providing full CSIT will offer more

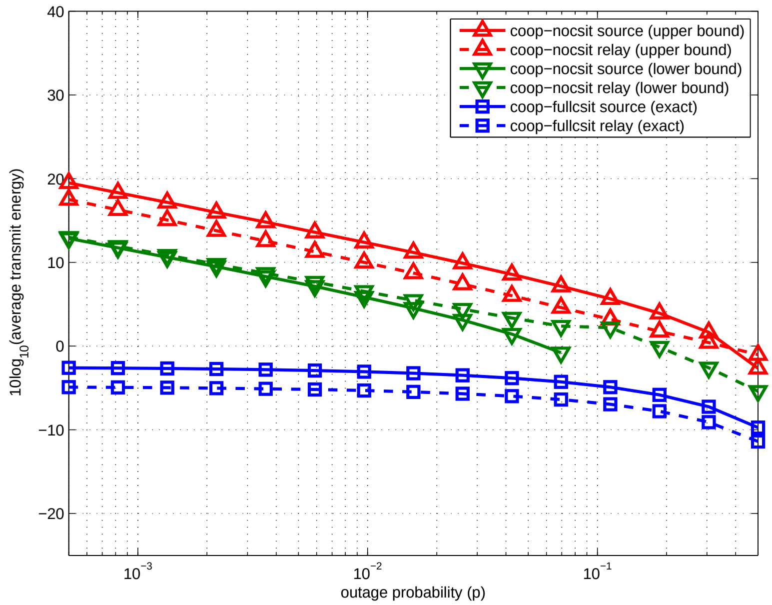

Figure 3.16: Optimum energy allocation when the relay has a statistically *advantaged* channel to the destination. Note that the source does not transmit for $p > 0.07$ when using the lower bound (3.17).

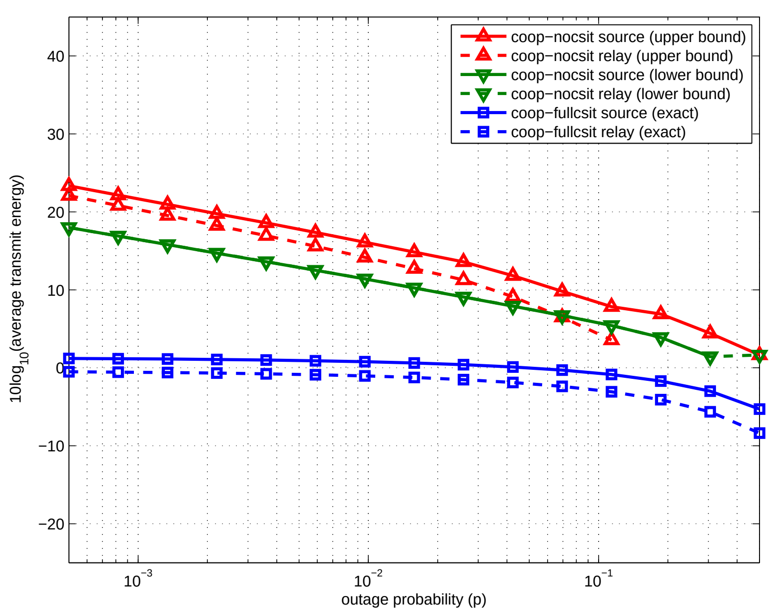

Figure 3.17: Optimum energy allocation when source/relay face statistically *symmetric* channels to the destination. Note that the source and relay transmit with identical energy when using the lower bound (3.17).

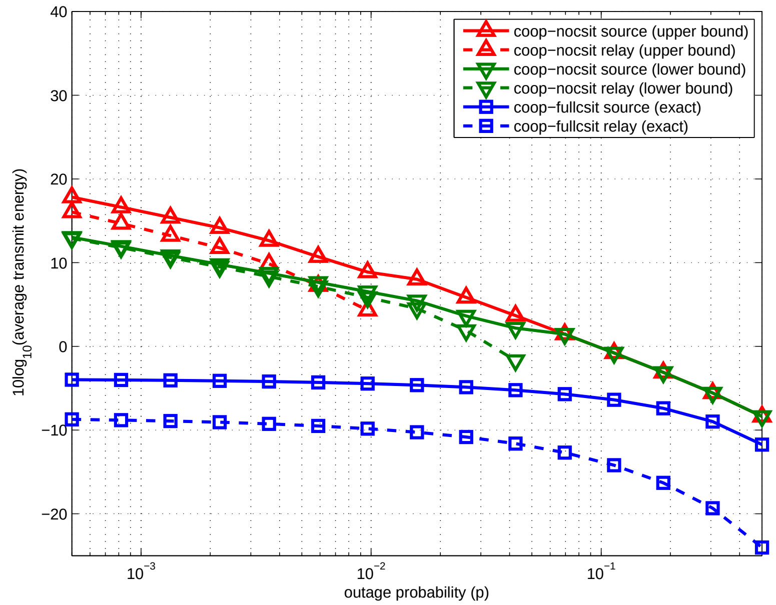

Figure 3.18: Optimum energy allocation when the relay has a statistically *disadvantaged* channel to the destination. Note that the relay does not transmit for $p > 0.07$ when using the lower bound (3.17).

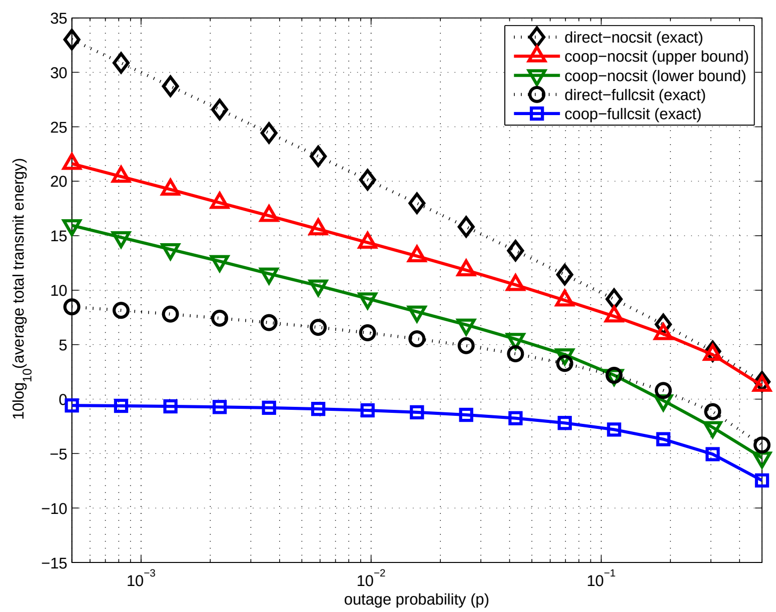

Figure 3.19: Average total transmission energy of optimum cooperative and direct transmission when the relay has a statistically *advantaged* independent Rayleigh fading channel to the destination.

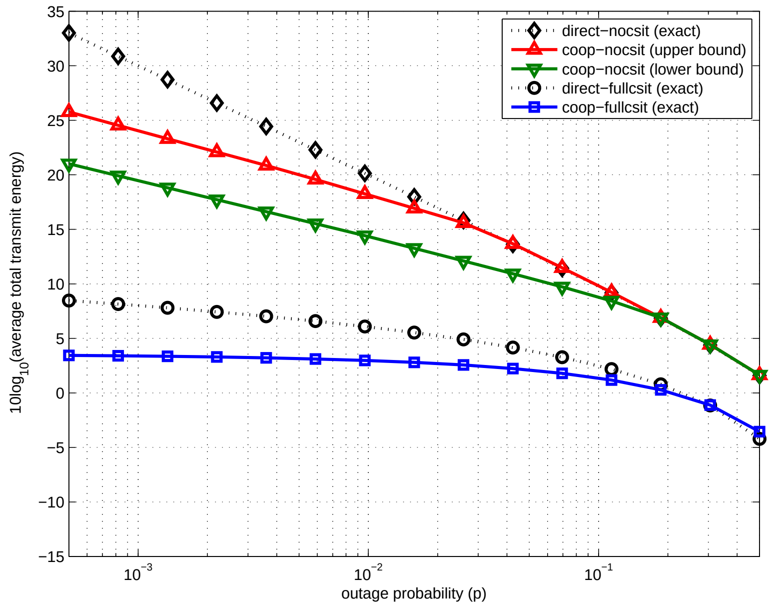

Figure 3.20: Average total transmission energy of optimum cooperative and direct transmission when source/relay face statistically *symmetric* independent Rayleigh fading channels to the destination.

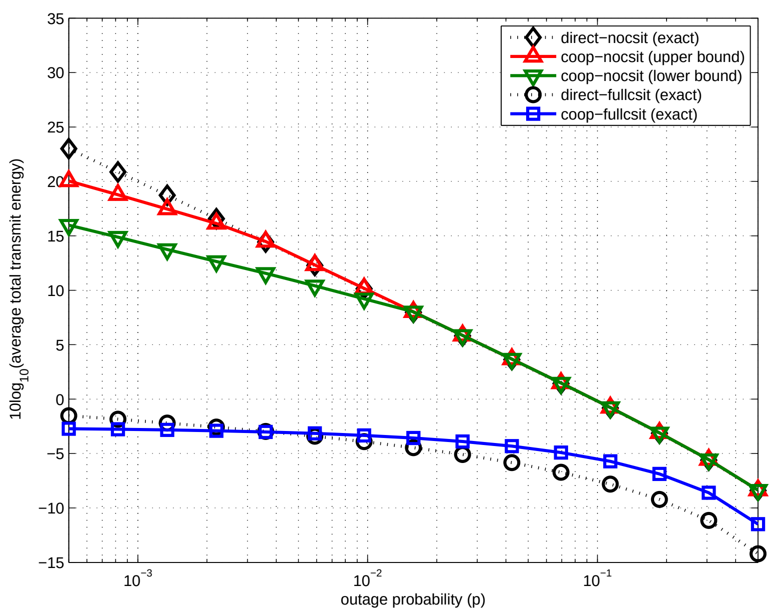

Figure 3.21: Average total transmission energy of optimum cooperative and direct transmission when the relay has a statistically *disadvantaged* independent Rayleigh fading channel to the destination.

benefit in terms of transmission energy efficiency in fading channels than cooperation without CSIT. However, we note that the energy gain by cooperation is decreased when CSIR is not available. When relay is disadvantaged, and the destination using EGC, direct transmission even can outperform cooperative transmission when p is large.

3.9 Conclusions

This chapter examines the impact of channel state information (CSIT and CSIR) on optimum energy allocation and energy efficiency of a wireless communication system with two delay-constrained cooperating sources and one destination using the orthogonal amplify-and-forward protocol. The sources are each required to satisfy an outage probability constraint. An explicit optimum (minimum total energy) source/relay energy allocation strategy is derived for the case when the sources have full CSIT (instantaneous channel amplitudes) and the destination has full CSIR/no CSIR. For the case without CSIT, outage probability bounds are derived.

Numerical examples with independent Rayleigh fading channels demonstrate that full CSIT can significantly improve the energy efficiency of both cooperative and direct transmission. The results also suggest that, while cooperative transmission tends to have better energy efficiency than direct transmission, cooperative transmission without CSIT is often less energy efficient than direct transmission with full CSIT. We also analyze how the receiver diversity combining technique affects both the optimum energy allocation and the overall energy efficiency of orthogonal amplify-and-forward cooperative transmission systems. Our results show that, unlike MRC, optimum cooperative transmission with EGC always requires transmission by the relaying node.

Numerical results for independent Rayleigh fading channels show that the ratio of optimum relay to source transmission energy tends to be greater for EGC than MRC. We also show that, while cooperative transmission with MRC is always the most efficient strategy, cooperative transmission with EGC tends to be more energy efficient than direct transmission when the outage probability constraint is small or when the relay-destination channel is advantaged.

Finally, we note that the results in this chapter can be extended to a one-source one-destination M-relay scenario by considering "best relay selection" [mK08]. This technique involves selecting one relay from a pool of $M > 1$ relays to forward the source's transmission to the destination. Depending on the availability of channel state information, the "best" relay selection criteria can be to optimize a short-term performance metric, e.g. minimize the total transmission energy in current transmission interval, or a long-term performance metric, e.g. minimize outage probability. The availability of more than one relay in the system can be expected to lead to improved performance with respect to the single-relay results presented in this chapter.

Chapter 4

Optimum Energy Allocation for Different Cooperative Protocols

This chapter extends the results of the previous chapter by considering the problem of optimum energy allocation for several cooperative transmission protocols. Like the previous chapter, the focus in this chapter is on a two-source one-destination network with the inter-terminal links in Figure 3.3 modeled as independent, frequency non-selective Rayleigh slow fading channels. We restrict our attention in this chapter to the case with full CSIT and full CSIR. The sources are assumed to satisfy a delay constraint on the transmission which requires that each channel codeword must be transmitted over one fading block of the network. Since the channel is not ergodic, we consider delay-limited capacity [HT98] and outage probability as our performance measures. Our goal is to maximize the delay-limited capacity of the system under a long-term average total transmit energy constraint. We also provide minimum outage probability analysis for the case when the available long-term average total transmit energy does not support the target delay-limited capacity [GE07].

To facilitate the development of opportunistic protocol selection with optimal resource allocation, we analyze optimal resource allocation for the compress-and-forward (CF), estimate-and-forward (EF) in which the received signal at the relay is compressed by ignoring the destination side information (Wyner-Ziv compression is not employed), and non-orthogonal amplify-and-forward (NAF) [NBM$^+$04] protocols. Note that, together with DF, these protocols have been extensively analyzed in terms of ergodic capacity as well as outage/error probability performance over static or fading channels. A comparative analysis of these protocols and the cut-set upper bound in the case of instantaneous CSI feedback has not been done, however. In this chapter, we compare the delay-limited capacity and outage probability of CF, EF and NAF under optimal energy and time allocation. We compare these results with the ODF performance obtained in [GE07].

It is worth emphasizing that all of the analysis in this chapter is based on cooperative transmission protocols *using optimum energy and time allocation* according to the current channel state under the full CSIT and full CSIR assumptions. This approach is consistent with the analysis of the ODF protocol in [GE07], but is different than the approach used in many other papers, e.g. [EGMZ06] and [NBM$^+$04], where a fixed resource allocation is assumed. As will be shown in this chapter, the performance of cooperative transmission protocols under an optimum energy and time allocation assumption can be quite different from the performance results obtained under a fixed resource allocation assumption.

In addition to a comparison of cooperative transmission protocols under optimum energy and time allocation, we also propose a hybrid opportunistic protocol that selects from all available protocols the protocol that achieves the rate target with the least total transmit energy. We show that, for protocols employing optimal resource allocation under a total energy constraint, the instantaneous rate of EF is at least that of NAF for any channel state. Since the instantaneous rate of CF is also at least that of EF for any channel state, the hybrid opportunistic protocol only needs to select between CF and ODF with optimal resource allocation in each channel state. Since the hybrid protocol uses the least energy in each transmission interval, it also provides the best delay-limited capacity performance of all protocols considered in this study. Our numerical results show that the hybrid protocol can offer delay-limited capacity close to the cut-set upper bound.

4.1 Problem Formulation

We consider a wireless communication system consisting of a single source (S), single destination (D), and an available relay (R) as shown in Figure 3.3. The links among the terminals are modeled as having independent, quasi-static Rayleigh fading as well as path loss. We assume zero-mean additive white Gaussian noise with unit variance at the receivers. The channel coefficients are assumed to be constant over a block of N symbols during which one codeword is transmitted, and are independent from one block to the other. We assume N is large enough to achieve instantaneous capacity. The squared channel amplitudes, denoted by $a = |h_1|^2$, $b = |h_2|^2$, and $c = |h_3|^2$ as in Figure 3.3, are exponentially distributed random variables with means λ_a, λ_b, and λ_c, respectively. The means capture the effect of path loss across the corresponding link. It is also assumed that the channel amplitude vector $\mathbf{s}$ is known at the source, the relay and the destination, while the phase information is only available at the corresponding receivers. The lack of channel phase information at the transmitters implies that the source and the relay can not beamform.

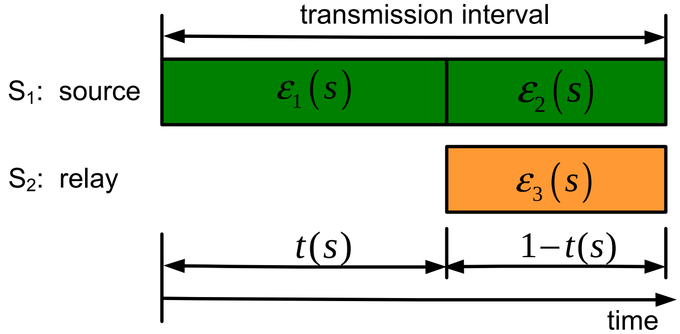

Figure 4.1: Half-duplex cooperative protocol.

We assume a half-duplex relay and normalize the time interval for each cooperative protocol (N symbols) to 1 unit as in Figure 4.1. Let $\mathcal{E}(\mathbf{s}) = (\mathcal{E}_1(\mathbf{s}), \mathcal{E}_2(\mathbf{s}), \mathcal{E}_3(\mathbf{s}), t(\mathbf{s}))$ be a resource allocation rule defined over the set of all possible network states $\mathbf{s} = (a, b, c)$, where $\mathcal{E}_1(\mathbf{s})$ is the source energy in the first timeslot of duration $t(\mathbf{s})$, and $\mathcal{E}_2(\mathbf{s})$, $\mathcal{E}_3(\mathbf{s})$ are the transmission energies of the source and the relay, respectively, in the second timeslot of duration $1 - t(\mathbf{s})$, $0 < t(\mathbf{s}) \leq 1$. We define Ω as the set of all possible resource allocation functions. We have

$$\Omega = \{\mathcal{E}(\mathbf{s}) : \mathcal{E}_1(\mathbf{s}) \geq 0, \mathcal{E}_2(\mathbf{s}) \geq 0, \mathcal{E}_3(\mathbf{s}) \geq 0, 0 < t(\mathbf{s}) \leq 1\}.$$

Let $F(\mathbf{s})$ be the probability distribution function of the channel states. Then the long-term average total transmit energy constraint can be written as

$$\begin{aligned} \mathrm{E}[\mathcal{E}(\mathbf{s})] &\triangleq \int_s [\mathcal{E}_1(\mathbf{s}) + \mathcal{E}_2(\mathbf{s}) + \mathcal{E}_3(\mathbf{s})] dF(\mathbf{s}) \\ &\leq \mathcal{E}_{avg}. \end{aligned}$$

The long-term average total transmit energy constraint imposes a set of feasible resource allocation functions, $\bar{\Omega} \subseteq \Omega$, that is composed of energy allocation functions which satisfy the above inequality, i.e., $\bar{\Omega} = \{\mathcal{E}(\mathbf{s}) : \mathrm{E}[\mathcal{E}(\mathbf{s})] \leq \mathcal{E}_{avg}, \mathcal{E}(\mathbf{s}) \in \Omega\}$. First, we try to maximize the delay-limited capacity of the system under a long-term average total transmit energy constraint, i.e.

$$\max_{\mathcal{E}(\mathbf{s})\in\bar{\Omega}} R, \tag{4.1}$$

$$\text{such that } C(\mathcal{E}(\mathbf{s}), \mathbf{s}) \geq R \text{ for all } \mathbf{s}.$$

where $C(\mathcal{E}(\mathbf{s}), \mathbf{s})$ is the instantaneous capacity with this resource allocation function $\mathcal{E}(\mathbf{s})$ at channel state $\mathbf{s}$. With the facilitation of 4.1, our goal of minimizing outage probability under a long-term average total transmit energy constraint, can be written as

$$\min_{\mathcal{E}(\mathbf{s})\in\bar{\Omega}} P_{\mathsf{out}} = \mathsf{Prob}[C(\mathcal{E}(\mathbf{s}), \mathbf{s}) < R], \tag{4.2}$$

.

4.2 Hybrid Optimum Energy Allocation with Full CSIT

4.2.1 Background

Without channel state information at the transmitters (CSIT) only a limited improvement can be achieved by statistical channel resource and energy allocation [LBC+07], [YB06]. However, in the case of instantaneous CSIT, it is possible to adapt to the channel state and achieve significant gains. Availability of CSIT is assumed in some recent literature on user cooperation as well. In [HZ05b], the ergodic capacity of a cooperative system is explored under both short-term and long-term average total transmit energy constraints. Host-Madsen and Zhang also explores outage capacity with short-term total transmit energy constraint for both synchronous and asynchronous relays. In [LVP07], resource allocation is considered to optimize the ergodic capacity under separate energy constraints at the source and the relay. In [AKA04], outage performance with long-term average total transmit energy constraint is investigated where full duplex relays cooperate irrespective of the channel state, thus no channel resource allocation needed. In [YB06], an opportunistic optimal energy allocation scheme for two-source amplify-and-forward protocol is proposed.

In addition to resource allocation, when the source and the relay have access to the instantaneous channel amplitudes they also have an opportunity to select a cooperative transmission protocol. In [GE07], the idea of opportunistic cooperation using decode-and-forward (DF) relaying is introduced, in which the terminals choose either DF (with optimal power and time allocation) or direct transmission (DT) depending on which protocol is more energy efficient in the current channel state. This hybrid protocol is called opportunistic decode-and-forward (ODF). The results in [GE07] show that the freedom of choosing among multiple transmission schemes improves both the delay-limited capacity and the minimum outage probability significantly. In particular, the ODF scheme is shown to achieve a nonzero delay-limited capacity, while both DF and DT individually have zero delay-limited capacities.

4.2.2 Delay-limited Capacity Analysis

Delay-limited capacity is defined as the highest achievable rate that can be sustained independent of the channel state [HT98]. This model is especially suitable for delay sensitive applications such as real-time voice and video communications. The availability of channel state information is essential to guarantee any non-zero transmission rate with zero outage probability.

In this section, we consider different cooperation protocols and dynamically allocate the relay transmit time and energy among the terminals, based on the channel states in order to maximize the delay-limited capacity. Let $\mathcal{E}(\mathbf{s})$ be the resource allocation function and $C(\mathcal{E}, \mathbf{s})$ be the instantaneous capacity of the underlying cooperation protocol with this resource allocation function at channel state $\mathbf{s}$. Then the delay-limited capacity maximization problem can be stated as follows[1].

$$\max_{\mathcal{E}(\mathbf{s}) \in \bar{\Omega}} R, \tag{4.3}$$

$$\text{such that } C(\mathcal{E}, \mathbf{s}) \geq R \text{ for all } \mathbf{s}.$$

In the following subsections, we introduce the specific cooperation protocols that will be analyzed in terms of delay-limited capacity.

4.2.3 Non-orthogonal Amplify and Forward

In the NAF protocol [NBM+04], the transmission slot is divided into two equal portions, that is $t(\mathbf{s}) = 1/2$ for all $\mathbf{s}$. During the first timeslot, the source transmits a signal to the relay and the destination while the relay is silent. In the second timeslot, the relay simply scales its received signal from the first timeslot and retransmits, and the source simultaneously transmits new symbols. For $i = 1, \ldots, N/2$, the input/output relationship for NAF can be characterized as

$$y_d[i] = h_1 \sqrt{\mathcal{E}_1} x_1[i] + n_1[i], \tag{4.4}$$

$$y_r[i] = h_2 \sqrt{\mathcal{E}_1} x_1[i] + n_2[i], \tag{4.5}$$

[1] In the following analysis, with abuse of notation, we sometimes omit the dependence on $\mathbf{s}$ and use $\mathcal{E}_1$, $\mathcal{E}_2$, $\mathcal{E}_3$ and t for the resource allocation functions.

and, for $i = N/2+1, \ldots, N$, the input/output relationship for NAF is

$$y_d[i] = h_1\sqrt{\mathcal{E}_2}x_1[i] + h_3\sqrt{\mathcal{E}_3}x_2[i] + n_1[i], \tag{4.6}$$

$$x_2[i] = \beta y_r[i - N/2]. \tag{4.7}$$

Here, $x_1[i]$, $x_2[i]$ are the source and the relay symbols at time i, $y_r[i]$ and $y_d[i]$ are the received symbols at time i at the relay and the destination, respectively, and β is the scaling factor at the relay that satisfies

$$\beta \leq \sqrt{\frac{\mathcal{E}_3}{|h_2|^2\mathcal{E}_1 + 1}}. \tag{4.8}$$

Define $\mathbf{x}_1 = [x_1[1], \ldots, x_1[N]]^T$, $\mathbf{x}_2 = [x_2[N/2+1], \ldots, x_2[N]]^T$ and $\mathbf{y}_d = [y_d[1], \ldots, y_d[N]]^T$. We have $E[\mathbf{x}_1^T\mathbf{x}_1 + \mathbf{x}_2^T\mathbf{x}_2] \leq 1$. The maximum instantaneous mutual information achieved by NAF with resource allocation $\mathcal{E}(\mathbf{s}) = (\mathcal{E}_1(\mathbf{s}), \mathcal{E}_2(\mathbf{s}), \mathcal{E}_3(\mathbf{s}), 1/2)$ at channel state $\mathbf{s}$ can be found as

$$\begin{aligned} &I(\mathbf{x}_1; \mathbf{y}_d \,|\, \mathbf{s}) = \\ &\frac{1}{2}\log\left(1 + a\mathcal{E}_1 + \frac{|\beta|^2 bc\mathcal{E}_1 + a\mathcal{E}_2}{1+|\beta|^2 c} + \frac{a^2\mathcal{E}_1\mathcal{E}_2}{1+|\beta|^2 c}\right). \end{aligned} \tag{4.9}$$

It can be shown that the maximum value of β also maximizes the mutual information. Then, substituting (4.8) in (4.9) we obtain

$$\begin{aligned} &C_{NAF}(\mathcal{E}(\mathbf{s}), \mathbf{s}) \triangleq I(\mathbf{x}_1; \mathbf{y}_d \,|\, \mathbf{s}) \\ &= \tfrac{1}{2}\log\left(1 + a\mathcal{E}_1 + \tfrac{bc\mathcal{E}_1\mathcal{E}_3 + a\mathcal{E}_2(1+a\mathcal{E}_1)(1+b\mathcal{E}_1)}{1+b\mathcal{E}_1+c\mathcal{E}_3}\right). \end{aligned} \tag{4.10}$$

The delay-limited capacity of NAF can be found by solving the optimization problem in (4.3), where we replace $C(\mathcal{E}, \mathbf{s})$ with $C_{NAF}(\mathcal{E}, \mathbf{s})$. Note that in (4.10) if we set $\mathcal{E}_1 = \mathcal{E}_2$ and $\mathcal{E}_3 = 0$, we get DT. If we set $\mathcal{E}_2 = 0$, we get OAF. Hence the optimization in computing the delay-limited capacity is opportunistic as in [GE07] and the relay is not used if DT is more energy efficient.

The following lemma shows that, for the NAF protocol with CSIT and optimal energy allocation, either the source transmits directly, or OAF is used in each channel state. Hence we can restrict our attention to optimal energy allocation for opportunistic OAF only.

Lemma 4.2.1. *Let $\mathcal{E}^*(\mathbf{s})$ be the optimal resource allocation function that maximize (4.10). Then at any channel state $\mathbf{s}$, we either have $\mathcal{E}^*(\mathbf{s}) = (\mathcal{E}_1^*, 0, \mathcal{E}_3^*, 1/2)$, i.e., we use OAF, or we have $\mathcal{E}^*(\mathbf{s}) = (\mathcal{E}_1^*, \mathcal{E}_2^*, 0, 1/2)$, i.e., we use DT.*

Proof. Let $\mathbf{s}$ be any channel state, and define $E(\mathbf{s}) \triangleq (\mathcal{E}_2^*(\mathbf{s}) + \mathcal{E}_3^*(\mathbf{s}))/2$ as the optimal energy allocated to the second timeslot by $\mathcal{E}^*(\mathbf{s})$. We consider the following maximization problem:

$$\begin{aligned} \max \quad & \tfrac{F\mathcal{E}_3 + B\mathcal{E}_2}{A + c\mathcal{E}_3} \\ \text{such that} \quad & \tfrac{\mathcal{E}_2 + \mathcal{E}_3}{2} \leq E, \end{aligned} \tag{4.11}$$

where $F \triangleq bc\mathcal{E}_1^*$, $B \triangleq a(1 + a\mathcal{E}_1^*)(1 + b\mathcal{E}_1^*)$, and $A \triangleq 1 + b\mathcal{E}_1^*$. It is easy to see that the optimal energy allocation for the above problem is

$$(\bar{\mathcal{E}}_2, \bar{\mathcal{E}}_3) = \begin{cases} (E, 0), & \text{if} \quad FA < B(A + cE) \\ (0, E), & \text{if} \quad \text{else} \end{cases} \tag{4.12}$$

Combining (4.12) with (4.10), we can argue that there exists an optimal energy allocation for which either the source or the relay is silent in the second timeslot. When $\mathcal{E}_3 = 0$, we let $\mathcal{E}_1 = \mathcal{E}_2 = (\mathcal{E}_1^* + \bar{\mathcal{E}}_2)/2$ without changing the achievable rate, which is equivalent to DT with constant energy over the whole timeslot. □

For both the OAF and DT protocols, the optimal energy allocation at each channel state can be found analytically [ZAL06]. Hence, an analytical solution for NAF can also be found by choosing between OAF and DT at each channel state.

4.2.4 Compress and Forward Relaying

The idea of compress and forward relaying stems from Theorem 6 in [CG79]. In CF the relay quantizes and compresses the signal it received in the first timeslot, and transmits the compressed version to the destination in the second timeslot, while the source continues sending independent information [KGG05]. The compression is done in Wyner-Ziv [WZ76] sense by utilizing the destination's own correlated observation about the source signal of the first slot (side information). Note that in the CF protocol it is not necessary to have $t(\mathbf{s}) = 1/2$, resulting in more flexibility compared to OAF and NAF.

The instantaneous capacity for CF using resource allocation function $\mathcal{E}(\mathbf{s})$ can be written as [HZ05b]:

$$C_{CF}(\mathcal{E}, \mathbf{s}) = t(\mathbf{s}) \log\left(1 + a\mathcal{E}_1 + \frac{b\mathcal{E}_1}{1+\sigma_w^2}\right) + (1 - t(\mathbf{s})) \log(1 + a\mathcal{E}_2), \tag{4.13}$$

where

$$\sigma_w^2 = \frac{1 + a\mathcal{E}_1 + b\mathcal{E}_1}{\left(\left(1 + \frac{c\mathcal{E}_3}{1+a\mathcal{E}_2}\right)^{\frac{1-t(\mathbf{s})}{t(\mathbf{s})}} - 1\right)(1 + a\mathcal{E}_1)}. \tag{4.14}$$

The delay-limited capacity of CF protocol is found by solving (4.3) where $C(\mathcal{E}, \mathbf{s})$ is replaced with $C_{CF}(\mathcal{E}, \mathbf{s})$.

While using Wyner-Ziv compression at the relay improves the performance, it also increases the complexity of the relay encoder and the destination decoder. We also consider a simpler scheme in which the relay compresses its received signal ignoring the side information at the destination. This scheme is called estimate-and-forward

(EF). The instantaneous capacity of EF with energy allocation $\mathcal{E}(\mathbf{s})$ at state $\mathbf{s}$ is

$$C_{EF}(\mathcal{E}(\mathbf{s}),\mathbf{s}) = t(\mathbf{s})\log\left(1+a\mathcal{E}_1+\frac{b\mathcal{E}_1}{1+\hat{\sigma}_w^2}\right) + (1-t(\mathbf{s}))\log(1+a\mathcal{E}_2), \tag{4.15}$$

where

$$\hat{\sigma}_w^2 = \frac{1+b\mathcal{E}_1}{\left(1+\frac{c\mathcal{E}_3}{1+a\mathcal{E}_2}\right)^{\frac{1-t(\mathbf{s})}{t(\mathbf{s})}}-1}. \tag{4.16}$$

As expected, EF has a larger quantization noise than CF, i.e., $\hat{\sigma}_w^2 \geq \sigma_w^2$. When we provide delay-limited capacity comparisons of different protocols, we will also consider simpler version of CF and EF with fixed and equal time allocation, that is, $\mathcal{E}(\mathbf{s}) = (\mathcal{E}_1, \mathcal{E}_2, \mathcal{E}_3, 1/2)$ for all $\mathbf{s}$. The instantaneous capacities for these schemes are denoted as $C_{CF}^{t=1/2}(\mathcal{E}(\mathbf{s}),\mathbf{s})$ and $C_{EF}^{t=1/2}(\mathcal{E}(\mathbf{s}),\mathbf{s})$. Their expressions can be found by setting $t = 1/2$ in equations (4.13)-(4.16). Note that both CF and EF protocols encompass DT as a special case, hence they are inherently opportunistic in the sense of [GE07].

Lemma 4.2.2. *For any given energy allocation and channel states, the instantaneous capacity of EF with fixed $t = 1/2$ is greater than or equal to the instantaneous capacity of NAF.*

Proof. The capacity of EF with fixed time allocation can be written as

$$C_{EF}^{t=1/2}(\mathcal{E}(\mathbf{s}),\mathbf{s}) = \frac{1}{2}\log\left\{1+a\mathcal{E}_1+a\mathcal{E}_2(1+a\mathcal{E}_1)+\frac{bc\mathcal{E}_1\mathcal{E}_3(1+a\mathcal{E}_2)}{(1+a\mathcal{E}_2)(1+b\mathcal{E}_1)+c\mathcal{E}_3}\right\}.$$

Using $\mathcal{E}_3 \geq 0$ and

$$\frac{1+a\mathcal{E}_2}{(1+a\mathcal{E}_2)(1+b\mathcal{E}_1)+c\mathcal{E}_3} \geq \frac{1}{1+b\mathcal{E}_1+c\mathcal{E}_3},$$

we get

$$\begin{aligned}
&C_{EF}^{t=1/2}(\mathcal{E}(\mathbf{s}), \mathbf{s}) \\
&\geq \frac{1}{2}\log\left\{1 + a\mathcal{E}_1 + \frac{a\mathcal{E}_2(1+a\mathcal{E}_1)(1+b\mathcal{E}_1)}{1+b\mathcal{E}_1+c\mathcal{E}_3} + \frac{bc\mathcal{E}_1\mathcal{E}_3}{1+b\mathcal{E}_1+c\mathcal{E}_3}\right\} \\
&= C_{NAF}.
\end{aligned}$$

□

4.2.5 Hybrid Opportunistic Optimum Energy Allocation

To maximize the delay-limited capacity for each protocols, we find the optimal resource allocation at each channel state so that the target rate is supported. However, there is no reason to be limited to a single cooperation protocol. Instead, at each channel realization, we can choose the optimal cooperation protocol along with its corresponding optimal resource allocation. This is similar to the ODF protocol in [GE07] where the choice is among DT and DF. Here, we include CF in the possible set of cooperation protocols. Note that, once we can choose among DF and CF we do not need to consider DT, NAF or EF, since DT is already a special case of CF, EF is inferior to CF, and NAF is inferior compared to EF by lemma 4.2.2.

The delay-limited capacity of the hybrid protocol can be found as

$$\max_{\mathcal{E}(\mathbf{s})\in\bar{\Omega}} R, \tag{4.17}$$

$$\text{such that } \max\{C_{CF}(\mathcal{E}, \mathbf{s}), C_{DF}(\mathcal{E}, \mathbf{s})\} \geq R, \text{ for all } \mathbf{s}.$$

4.2.6 Upper Bound to the Delay-Limited Capacity

Using the usual cut-set bounds for the half-duplex relay we find an upper bound (SCB) to the delay-limited capacity. For any energy and time allocation scheme, the instantaneous capacity can be upper bounded by

$$\begin{aligned}
&C_{CSB}(\mathcal{E}, \mathbf{s}) = \\
&\min \big\{ t \log(1 + (a+b)\mathcal{E}_1) + (1-t)\log(1 + a\mathcal{E}_2), \\
&t \log(1 + a\mathcal{E}_1) + (1-t)\log(1 + a\mathcal{E}_2 + c\mathcal{E}_3) \big\}.
\end{aligned}$$

Solving

$$\max_{\mathcal{E}(\mathbf{s}) \in \bar{\Omega}} R, \tag{4.18}$$

$$\text{such that } C_{CSB}(\mathcal{E}, \mathbf{s}) \geq R, \text{ for all } \mathbf{s}.$$

yields an upper bound to the delay-limited capacity since C_{CSB} is an upper bound to the instantaneous capacity at each channel realization.

4.3 Optimum Energy Allocation to Minimize Outage Probability

Recall that outage probability in this chapter is defined as

$$P_{\mathsf{out}} = \mathsf{Prob}[C(\mathcal{E}(\mathbf{s}), \mathbf{s}) < R]$$

We want to minimize outage probability subject to a long-term average total transmit energy constraint, i.e.

$$\min_{\mathcal{E}(\mathbf{s}) \in \bar{\Omega}} P_{\mathsf{out}},$$

Note that the optimum energy allocation $\mathcal{E}(\mathbf{s})$ that maximize delay-limited capacity is the same optimum energy allocation that can minimize the outage probability subject to the same long-term average total transmit energy constraint. Hence all the lemma in previous section can be applied to the outage probability case. As a result, the minimum outage probability with hybrid opportunistic optimum energy allocation can be found by opportunistically select from DF and CF with their respective optimum energy allocation.

The numerical results are demonstrated in section 4.4.

4.4 Numerical Results

4.4.1 Delay-limited Capacity Results

In this section, to consider the effect of the relay location on the performance of the network, we follow the model in Figure 4.2. We normalize the distance between the source and the destination, and assume that the relay is located between the source and the destination. For a fixed path loss exponent α, the effect of this normalization is scaling the long-term average total transmit energy. We denote the source-relay distance as d, where $0 < d < 1$, and the relay-destination distance as $1 - d$. Then the overall network channel state is denoted by $\mathbf{s} = (a, b, c)$, where a, b and c are independent exponential random variables with means $\lambda_a = 1$, $\lambda_b = \frac{1}{d^\alpha}$, and $\lambda_c = \frac{1}{(1-d)^\alpha}$, respectively. All of the results in this section assume $\alpha = 4$. Due to the reason that we do not have analytical results for the most of the protocols we considered, we use numerical methods to obtain the optimum energy allocation. The Matlab code used to generate the numerical results of compress-and-forward protocol are provided in Appendix C. For other protocols and CSB, to generate the results, simply replace the instantaneous capacity expression of CF with their corresponding expressions.

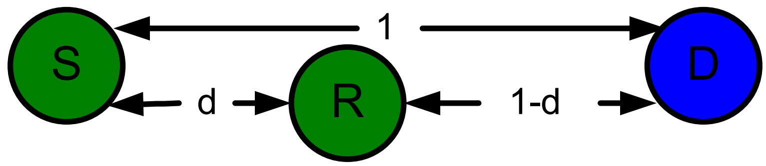

Figure 4.2: Simulation model.

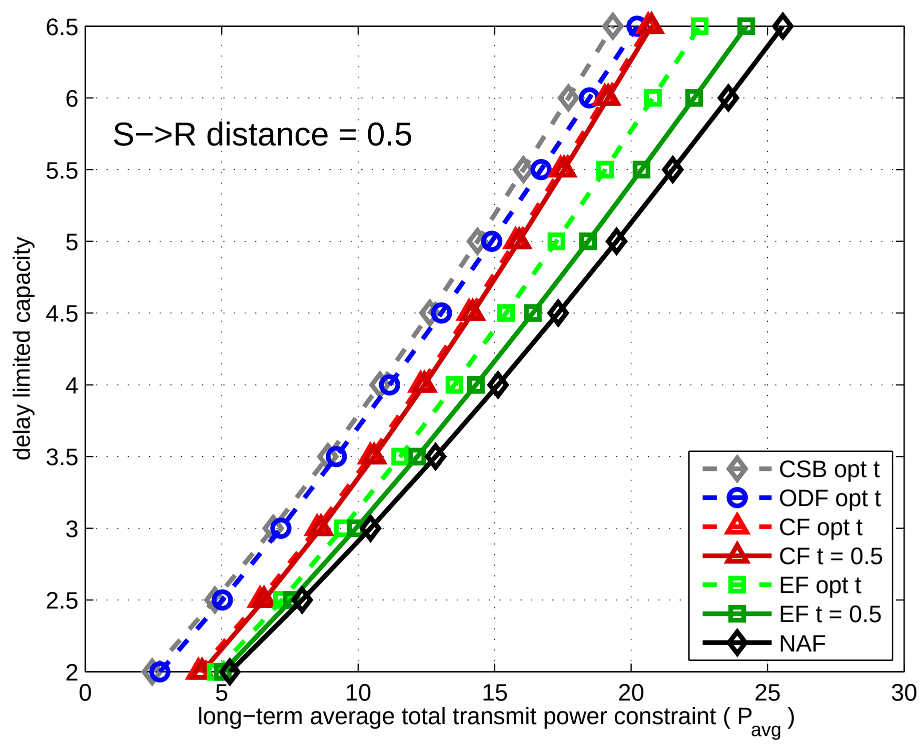

Figure 4.3: Delay-limited capacity versus the long-term average total transmit energy constraint $\mathcal{E}_{avg}$ when $d = 0.5$.

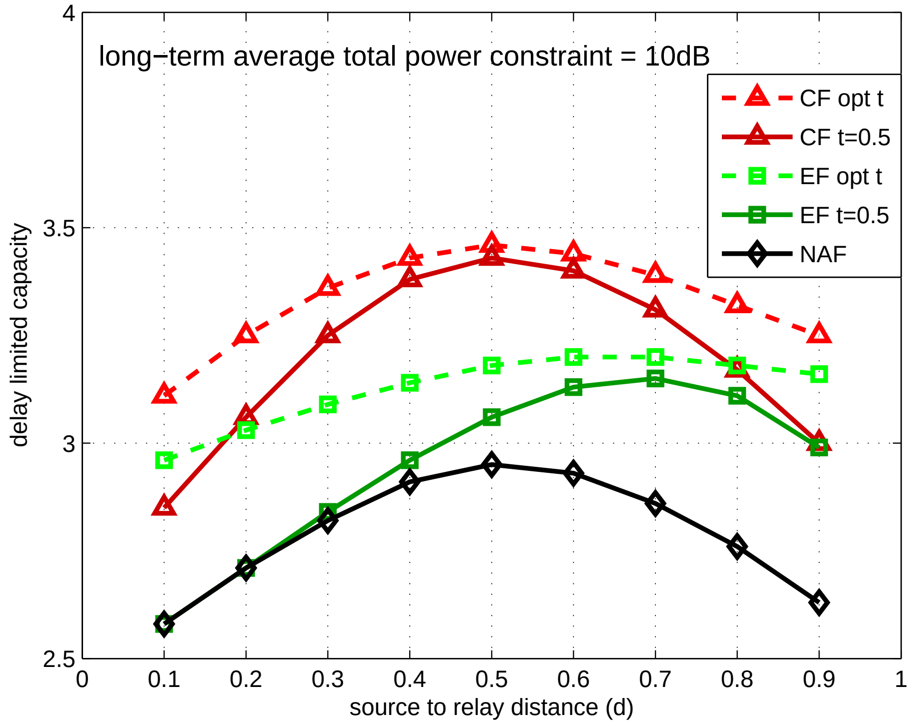

Figure 4.4: Delay-limited capacity of CF, EF and NAF protocol with and without optimal time allocation versus source to relay distance d.

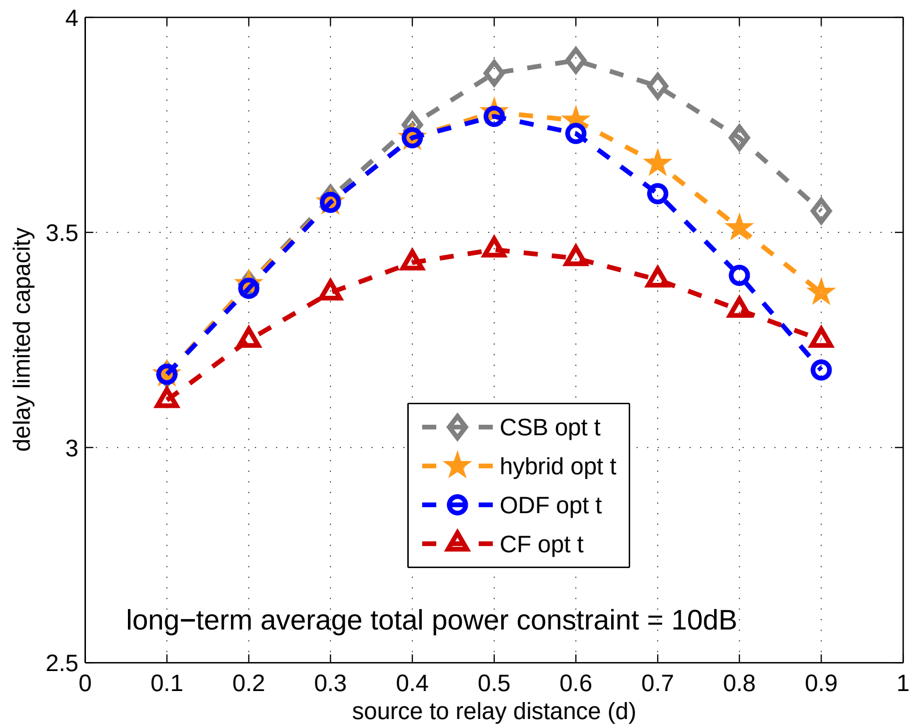

Figure 4.5: Delay-limited capacity of ODF, CF, the hybrid protocol, and the CSB with optimal time allocation versus source to relay distance d.

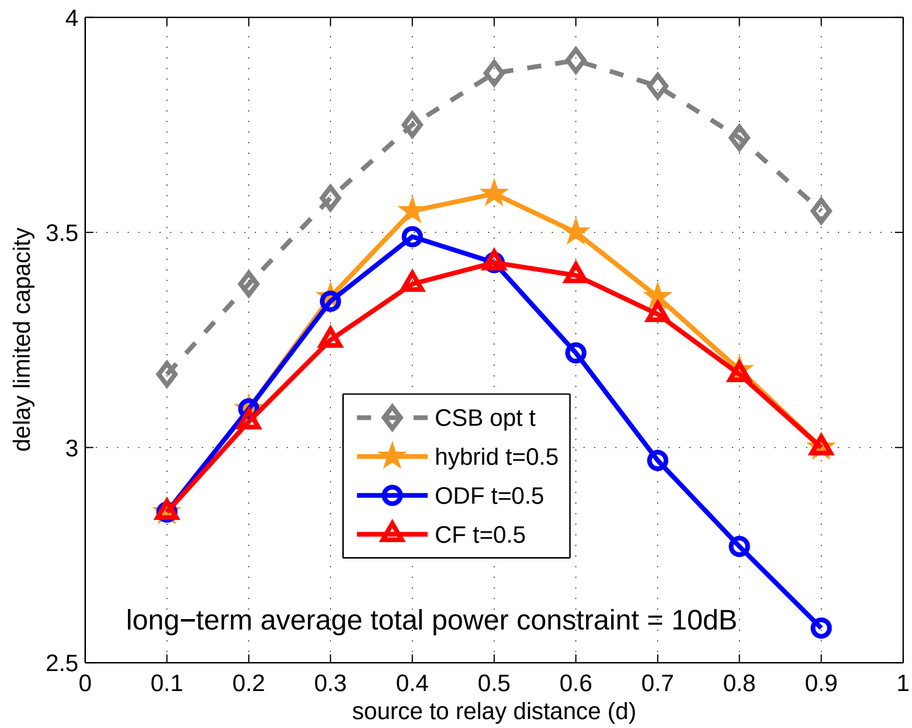

Figure 4.6: Delay-limited capacity of ODF, CF, and the hybrid protocol with $t = 0.5$, and the CSB with optimal t versus source to relay distance d.

Figure 4.3 demonstrates the delay-limited capacity as a function of the long-term average total transmit energy constraint for various relaying protocols for a relay location of $d = 0.5$. The cut-set bound (CSB) is also included for comparison. ODF with optimized time allocation performs closest to the CSB in this case. CF with fixed time allocation achieves almost the same performance as CF with optimized time allocation when $d = 0.5$. This observation is confirmed again in Figure 4.4, where the results show that time allocation is more important for CF when the relay is close to the source or the destination. The EF protocol, on the other hand, benefits more from optimal time allocation. The simplest protocol, NAF, although inferior to the other protocols still achieves a nonzero delay-limited capacity. This shows that even a simple cooperation strategy can improve the performance of delay-limited systems. Furthermore, in the low power regime, NAF still can be a viable alternative as the gains of higher complexity protocols become smaller.

Figures 4.4-4.6 show the variation of the delay-limited capacity with respect to relay location with the long-term average total power constraint of 10 dB. Figure 4.4 illustrates the delay-limited capacity of CF, EF and NAF with respect to different relay locations with and without optimal time allocation. The results show that EF with optimal time allocation can achieve higher delay-limited capacity than CF with fixed time allocation when the relay is very close to the source or to the destination. When the relay is close to the destination, EF benefits less from optimal time allocation. When the relay is close to the source, NAF performs almost as well as EF with fixed time allocation. Note also that the gap between NAF and CF with optimal time allocation is almost independent of the relay location.

Figures 4.5 and 4.6 show the delay-limited capacity of CF, ODF and the hybrid protocol with and without optimal time allocation, respectively. The CSB is also included for comparison. For the case with optimal time allocation, when the relay is close to the source, ODF and the hybrid protocol almost coincide with the CSB. As the relay moves towards the destination, the gap becomes larger. For CF, the gap between CSB becomes larger at first when relay moves towards the destination, then become smaller as the relay is very close to the destination. This is in accordance with the relative performances of these protocols in terms of their ergodic capacities [KGG05]. From Figure 4.5, we note that CF outperforms ODF when $d > 0.83$. Thus by adaptively choosing between CF and ODF, the hybrid protocol is superior to both CF and ODF. The trend is the same in Figure 4.6. We notice that without optimal time allocation, the performance gaps between the CSB and the other protocols become larger compared with the case with optimal time allocation. Among all the protocols, ODF is affected most by the absence of optimal time allocation. We note that CF begins to outperform ODF when $d = 0.5$ in Figure 4.6. In this case the advantage of the hybrid protocol is even more obvious.

4.4.2 Outage Probability Results

Outage probability results are extended from the delay-limited capacity results. Figure 4.7 and 4.8 demonstrates the outage probability as a function of the long-term average total transmit energy constraint for ODF, CF and the hybrid protocol with optimum time allocation for representative relay locations. The cut-set bound with optimum time allocation is included for comparison. When $d = 0.8$, ODF outperforms CF, while when $d = 0.9$, CF outperforms ODF. Figure 4.9 and 4.10 demonstrates the outage probability as a function of the long-term average total transmit energy constraint for ODF, CF and the hybrid protocol with fixed time allocation for representative relay locations. The cut-set bound with optimum time allocation is also included for comparison. The lines without markers show the long-term average transmit energy that is need to achieve zero outage probability, i.e. delay-limited capacity. When $d = 0.4$, ODF outperforms CF, while when $d = 0.6$, CF outperforms ODF. As expected, for all the cases the hybrid protocol performs closest to the CSB by adaptively choosing between CF and ODF. The above observations agrees with Figure 4.5 and Figure 4.6 in that when the relay is relatively close to the source, ODF performs better than CF, while CF outperforms ODF as relay moves towards the destination and the hybrid protocol performs closest to the CSB.

4.5 Conclusions

In this chapter, we analyze and compare the delay-limited capacity and outage probability of several cooperative protocols including CF, EF, ODF, and NAF under a long-term average total transmit power constraint and under the assumption that the instantaneous channel amplitudes are available at both the source and the relay prior to transmission. Given a particular cooperative protocol and an expression for its instantaneous mutual information in terms of the channel state and transmit powers, knowledge of the instantaneous channel amplitudes allows the source and the relay to minimize their instantaneous total power allocation while guaranteeing that the rate does not fall below a desired threshold in each channel realization. This knowledge also facilitates opportunistic transmission in the sense that the source and the relay can select a cooperative protocol from the family of available protocols that requires the minimum total transmit power in order to achieve the desired rate for the given channel state. This concept of opportunistic protocol selection has been explored on a

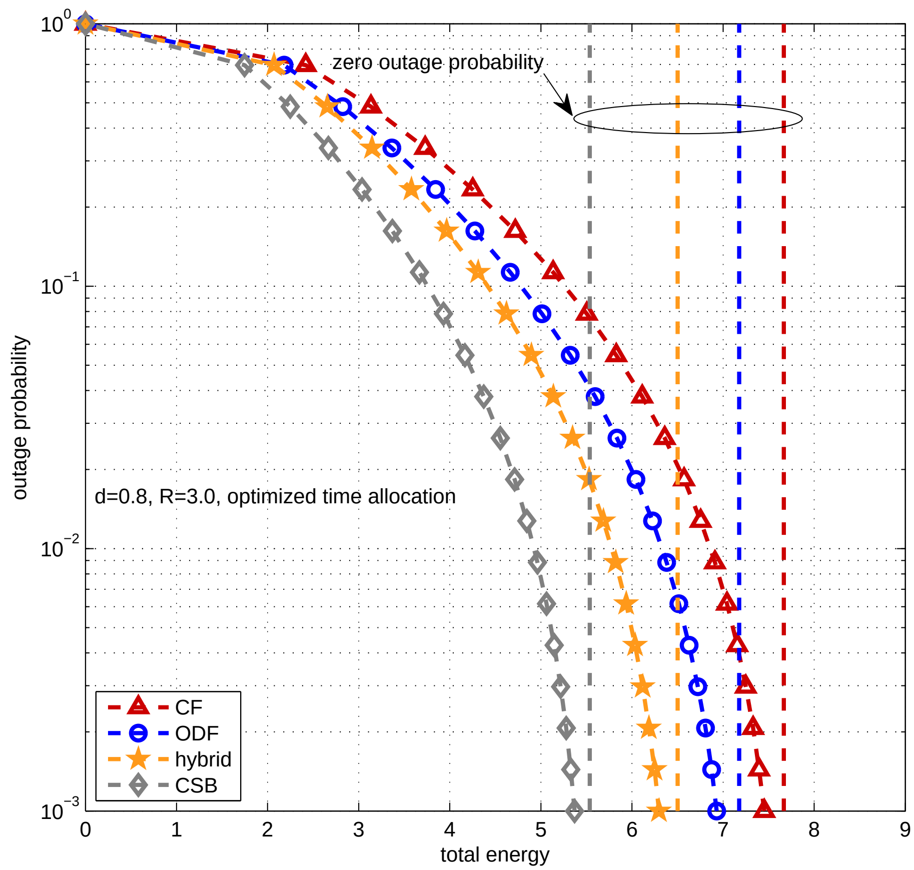

Figure 4.7: Outage probability of ODF, CF, the hybrid protocol, and the CSB with optimal time allocation versus the long-term average total transmit energy constraint $\mathcal{E}_{avg}$ when $d = 0.8$.

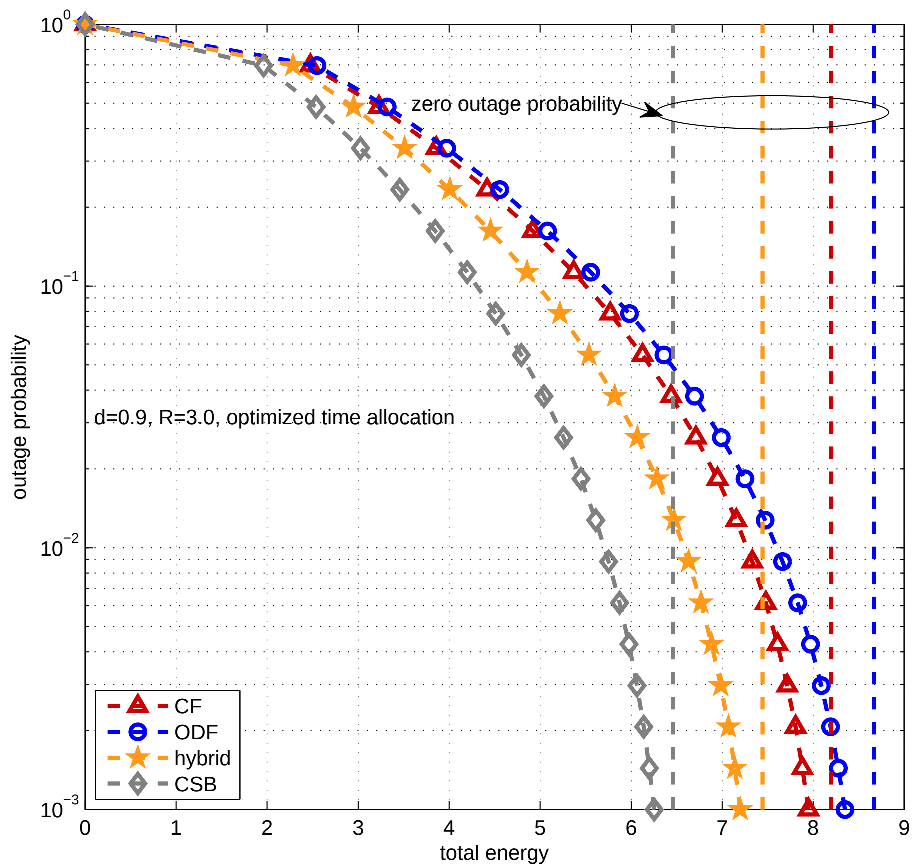

Figure 4.8: Outage probability of ODF, CF, and the hybrid protocol and the CSB with optimal time allocation versus the long-term average total transmit energy constraint $\mathcal{E}_{avg}$ when $d = 0.8$.

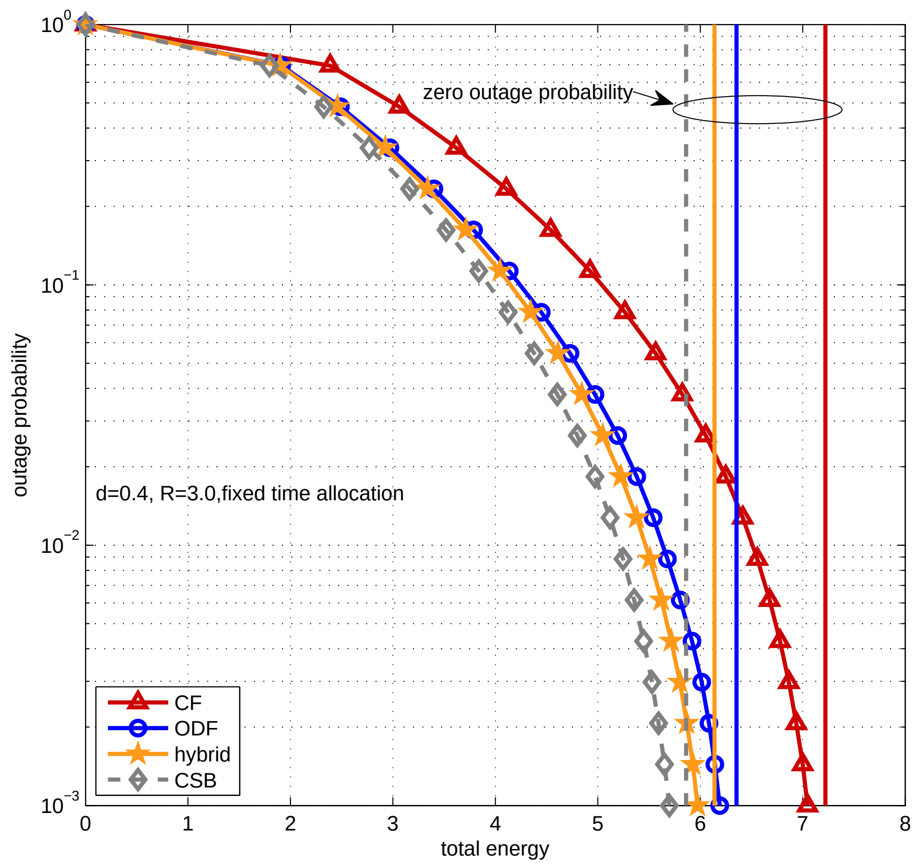

Figure 4.9: Outage probability of ODF, CF, the hybrid protocol, and the CSB versus the long-term average total transmit energy constraint $\mathcal{E}_{avg}$ when $d = 0.4$ and $t = 0.5$.

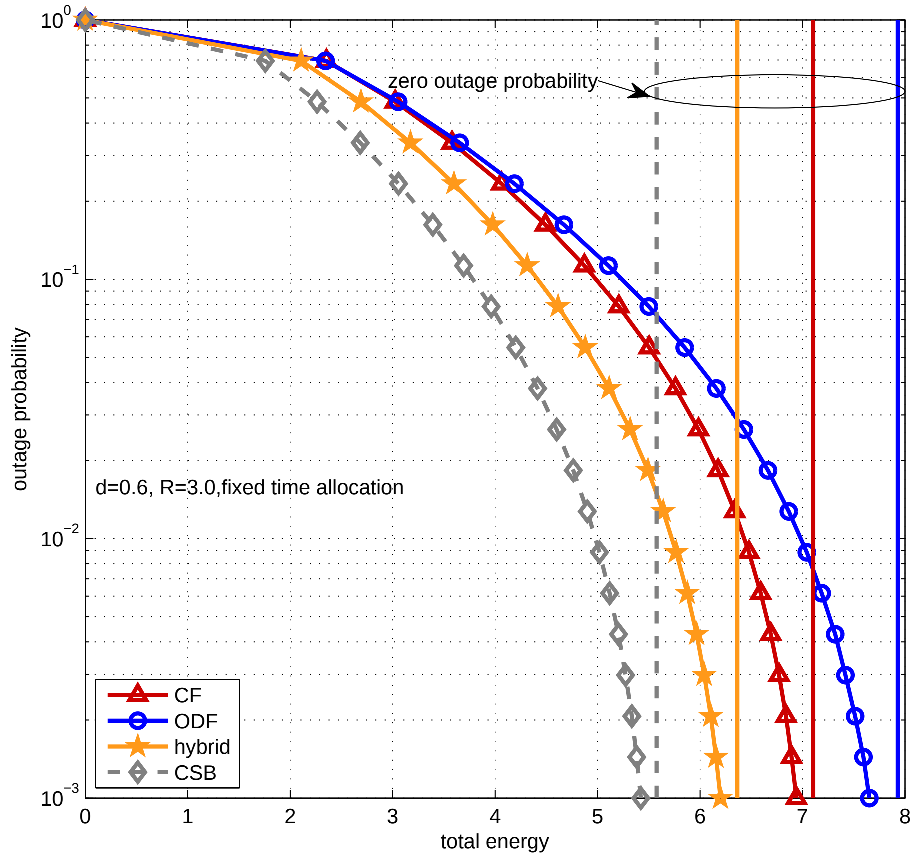

Figure 4.10: Outage probability of ODF, CF, the hybrid protocol, and the CSB versus the long-term average total transmit energy constraint $\mathcal{E}_{avg}$ when $d = 0.6$ and $t = 0.5$.

smaller scale in prior studies, e.g. opportunistic decode and forward where the choice is between DF and DT, but we are the first to consider opportunistic transmission over a large family of cooperative protocols with optimal resource allocation.

Our results show that, for protocols employing optimal resource allocation under a total energy constraint, the instantaneous rate of EF is at least as good as that of NAF for any channel state. Since the instantaneous rate of CF is also at least as good as that of EF for any channel state, we propose a hybrid opportunistic protocol in which the source and the relay choose between CF and ODF with optimal resource allocation in each channel state. The proposed hybrid opportunistic protocol offers the best delay-limited capacity performance, together with outage probability performance of all of the protocols considered since it always selects the protocol with the minimum total transmit power in each channel state. Our numerical results show that the hybrid opportunistic protocol tends to offer the most gain with respect to ODF when the mean of the relay-destination channel is better than that of the relay-source channel. The hybrid opportunistic protocol tends to offer the most gain with respect to CF when the mean of the relay-destination channel is similar to the mean of the relay-source channel.

While our results show that the delay-limited capacity of NAF is not as good as any of the other cooperative protocols considered in this study, it is the only protocol that we considered in which the optimal resource allocation can be computed analytically. Hence, NAF may still have a role in practical cooperative transmission systems since its complexity, both in terms of resource allocation and relay implementation, can be much lower than that of the other protocols considered in this chapter.

Chapter 5

Energy Efficiency of Selfish Cooperation

In wireless networks, intermediate nodes are often used as relays to reduce the transmission energy required to deliver a message to an intended destination. The selfishness of autonomous nodes, however, raises concerns about their willingness to expend energy to relay information for others. This chapter considers the effect of selfishness on energy efficiency using a non-cooperative game theoretic approach. A two-source relaying game is formulated for both non-fading and fading scenarios. We show that cooperative transmission with optimum energy allocation is a Nash Equilibrium in non-fading channels when the sources are sufficiently patient. In fading channels, cooperative transmission with optimum energy allocation is also a Nash Equilibrium when a ceiling is applied to the relay energy of each source. Simulation results show that sources acting in their own self-interest can achieve an energy efficiency close to that of centrally optimized energy allocation in many cases.

5.1 Background

In energy constrained wireless networks, multihop transmission can be used to deliver messages to a destination node outside the range of a source node. Cooperative transmission [SEA98, LTW04a] can also be used to reduce the amount of transmission energy required to deliver a message from a source node to a destination node in the network. If the nodes in the wireless network are all controlled by a central authority, e.g. a military network or an emergency network, then cooperative behavior among nodes in the network can be ensured. In many civilian applications, however, the nodes in a network may be partially or fully autonomous and cooperation can not be assumed. In order to save resources, e.g. battery energy, the nodes in these networks may behave selfishly by not forwarding messages and/or not cooperating with other transmitters. This in turn leads to inefficient use of the individual and overall network resources since messages have to be retransmitted and also possibly routed through suboptimal paths to the destination node [MGLB00].

Using tools from noncooperative game theory, four techniques have been proposed to stimulate cooperation between selfish nodes in wireless networks. These techniques, each of which modify the higher-level protocol layers, are summarized below:

1. **Reputation propagation:** The idea of reputation propagation involves having nodes monitor the transmissions of other nodes to see if they are forwarding other nodes' traffic. If a node is not forwarding traffic, its uncooperative reputation is propagated through the network and new routes are selected to avoid the uncooperative nodes [MGLB00, MM02, GxYh06, BB02a, BB02b, LY03]. Although reputation systems can provide incentive to induce cooperation, they still have some undesirable characteristics [ZCY03]. The additional network

traffic overhead generated by reputation propagation messages may be undesirable and it is not clear how misbehaving nodes are punished in reputation propagation systems [MGLB00].

2. **Virtual currency exchange:** The idea here is that nodes exchange units of virtual currency to pay for the cost of forwarding packets [HGLV01, BBC+01, BpH03]. Balances are kept locally and no centralized accounting is needed. Like reputation propagation, the virtual currency exchange approach requires some amount of additional network traffic overhead. Moreover, virtual currency exchange may be subject to fraud and may require a tamper-proof hardware at each node [BpH03, JHB03].

3. **Algorithmic mechanism design and pricing:** The idea here is that, instead of exchanging virtual currency directly between nodes, a central authority charges nodes for using network resources and reimburses nodes for cooperative behavior [ZCY03]. The pricing approach does not require tamper-proof hardware, however, it still requires a centralized accounting system to keep track of charges and reimbursements and to set market prices.

4. **Altruistic nodes:** It was shown in [LG06] that networks of selfish nodes could be induced to cooperate by introducing a vanishingly small fraction of altruistic nodes, i.e. nodes that always cooperate. The simplicity of this approach with respect to the prior approaches is appealing, but a centralized authority is needed to introduce the altruistic nodes into the network.

An overriding theme in the prior work in this area is that the short-term costs and benefits of cooperation are one-sided. Since nodes do not receive any immediate benefit for helping others, the assumption is that nodes that cooperate must be reimbursed with extrinsic payments that they can later use to buy cooperation from other nodes. Recent work in this area, however, has shown that cooperative behavior can be *mutually beneficial* even on a short-term basis. This implies that it may be possible to stimulate cooperation in wireless networks with selfish nodes in a fully-distributed manner without the introduction of extrinsic incentive mechanisms such as reputation propagation, virtual currency accounting, and/or altruistic nodes. This section focuses on the problems of when can cooperation between selfish sources be stimulated without central authority or a incentive mechanism and how does the energy efficiency of selfish cooperation compare to a system under central authority.

Especially, the problem of whether cooperation can exist without incentive mechanisms is considered in [UBG03, SPCR05, FHB06, GxYh06, CK07, LG06]. Most of the work in this area has focused on the network layer. It is claimed in [UBG03] that cooperation can be stimulated provided that no node has to forward more traffic than it generates. The nodes are classified into different energy classes and an energy efficient Nash Equilibrium strategy is proposed in [SPCR05]. Based on game theory and graph theory, the conditions when cooperation solely based on the nodes' self-interest can exist are proposed in [FHB06]. A joint analysis of cooperation stimulation and security is given in [GxYh06] and a set of reputation-based cheat-proof and attach-resistant cooperation stimulation strategies are derived. A few work investigate the effect of nodes' selfishness in the physical layer. It is shown in [CK07] that a mutually cooperative Nash Equilibrium can always be obtained when convex utility functions are used in Rayleigh fading channels for decode-and-forward protocol. Lai and Gamal [LG06] prove that full cooperation is possible by using a vanishingly small fraction of altruistic nodes . In this dissertation, we investigate the problem of whether cooperation can exist without incentive mechanisms or altruistic nodes of the two-source amplify-and-forward protocol in both non-fading and fading scenarios.

In both cases the sources are required to satisfy a instantaneous SNR constraint and are assumed to be rational and self-interested. The utility of each source is based on its own energy consumption. We model both scenarios as an infinitely repeated two-source relaying game. In the non-fading scenario, when channel state information is available, our results show that cooperation with optimum energy allocation can exist given sources are sufficiently patient. In the fading scenario, we propose a conditional trigger cooperative strategy and show that this strategy is a Nash Equilibrium of the infinitely repeated game. An important feature of the conditional trigger strategy is that the sources cooperate using optimum resource allocation but with a ceiling placed on the optimized relay energy. If either source is asked to transmit with relay energy greater than their ceiling in a stage game, both sources use direct transmission in that stage game. We show that this ceiling goes to infinity as the sources become more patient. Our results show that sources using the conditional trigger strategy can often achieve an overall system energy efficiency close to that of a centrally-optimized system, especially when the sources are patient. When channel state information is not available, both sources can only transmit with a fixed energy, hence the problem is essentially the same as the non-fading scenario.

5.2 System Model

We consider the two-source, two-destination system model shown in Figure 5.1. Source nodes S_1 and S_2 wish to communicate information to destination nodes D_1 and D_2, respectively, using the time and frequency slotted communication protocol described in Table 5.1. The unit duration *transmission interval* is divided into two *timeslots*, with transmission occuring on two different frequencies. In timeslot 1, both sources simultaneously transmit their local information to their intended destinations. We assume that the sources transmit on sufficiently separated frequencies to avoid interference and also to allow for reception of the other source's transmission during timeslot 1. In timeslot 2, each source can relay the transmission it overheard in timeslot 1 to the other source's destination. We note that the structure of the protocol specified in Table 5.1 is sufficiently general to include several common cooperative protocols, e.g. amplify-and-forward (AF), decode-and-forward (DF), and compress-and-forward (CF), as well as direct transmission when the duration of timeslot 1 is set equal to the full duration of the transmission interval.

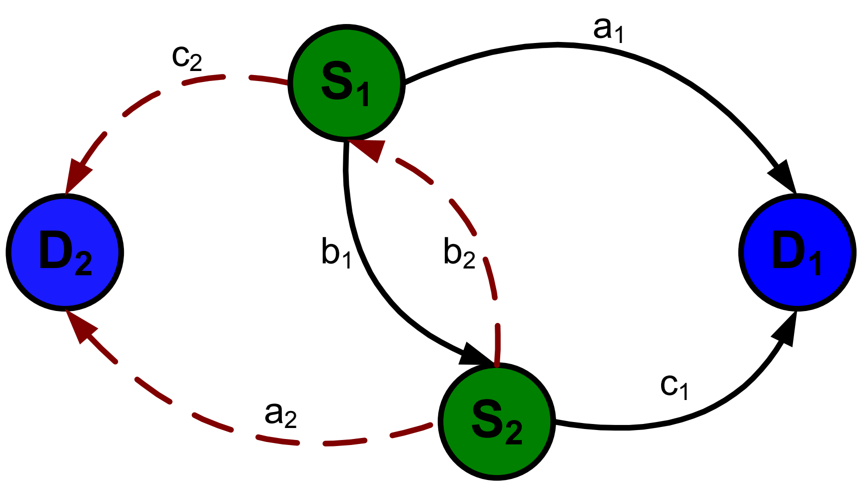

Figure 5.1: System model for two-user relaying game.

The channels shown in Figure 5.1 use the notation a_i to denote the squared magnitude of the direct link from S_i to D_i and c_i denotes the squared magnitude of the relay link from S_j to D_i, normalized by the energy of the additive white Gaussian noise at the receiver. The normalized squared magnitude of the channel from S_i to S_j is denoted as b_i. Since S_1 and S_2 transmit on different frequencies, we do not assume that

	transmission interval	
	timeslot 1	timeslot 2
	$\leftarrow$ $t(\boldsymbol{s})$ $\rightarrow$	$\leftarrow 1 - t(\boldsymbol{s}) \rightarrow$
frequency f_1	$\mathsf{S}_1 \rightarrow \mathsf{D}_1$ (overheard by S_2)	$\mathsf{S}_1 \overset{\text{relay}}{\rightarrow} \mathsf{D}_2$
frequency f_2	$\mathsf{S}_2 \rightarrow \mathsf{D}_2$ (overheard by S_1)	$\mathsf{S}_2 \overset{\text{relay}}{\rightarrow} \mathsf{D}_1$

Table 5.1: Two-source cooperative transmission interval.

$b_1 = b_2$. The normalized squared channel magnitude state $\boldsymbol{s} := \{a_1, a_2, b_1, b_2, c_1, c_2\}$ (which we henceforth will refer to as simply the "channel state" or the "state") is assumed to be quasi-static in the sense that it is constant over the duration of the transmission interval but is independent and identically distributed (i.i.d.) in different transmission intervals. We consider two scenarios: i) prior to the start of the transmission interval, both sources have perfect knowledge of $\boldsymbol{s}$, ii) both sources have only statistical knowledge of $\boldsymbol{s}$.

When the channel state $\boldsymbol{s}$ is known prior to the start of the transmission interval, a resource allocation rule can be specified as a function of the current channel state such that each source can achieve a particular performance target, e.g. signal-to-noise ratio (SNR) or rate, in each transmission interval. A resource allocation rule in this context specifies the timeslot duration $t(\boldsymbol{s})$ as well as the transmit energies $\mathcal{E}(\boldsymbol{s}) := \{\mathcal{E}_{1s}(\boldsymbol{s}), \mathcal{E}_{1r}(\boldsymbol{s}), \mathcal{E}_{2s}(\boldsymbol{s}), \mathcal{E}_{2r}(\boldsymbol{s})\}$, where $\mathcal{E}_{ij}(\boldsymbol{s})$ denotes the transmit energy of S_i in timeslot j, as a function of the current channel state. For example, if the sources do not cooperate (strictly use direct transmission) and are each required to achieve a SNR of 10 at their intended destination, a suitable resource allocation rule that satisfies the performance target in every transmission interval would be $t(\boldsymbol{s}) = 1$ and

$\mathcal{E}(\boldsymbol{s}) = \{10/a_1, 0, 10/a_2, 0\}$. We assume that each source is required to satisfy its performance target in every transmission interval and that the required performance target for each source does not change between transmission intervals.

When the channel state $\boldsymbol{s}$ is not known, a resource allocation rule can only be specified based on the statistics of the channel state. Instantaneous performance target such as SNR can not be guaranteed. Outage probability is a more appropriate performance measure.

When cooperation is allowed between the sources, the specification of a resource allocation rule guaranteeing that the performance target is satisfied in each transmission interval is not unique [Bro04]. In most cases, however, efficient resource allocation rules that achieve the performance target with less transmit energy in each transmission interval are preferred. The next section discusses how efficient resource allocation can be achieved in systems where resource allocation rule is centrally controlled and the sources do not act in their own self interest.

5.2.1 Two-user Relaying Game

In a centrally controlled system, sources are told to relay with a certain amount of energy to help each other, however, this cooperative behavior can not be taken for granted in a distributed system with selfish sources. In this section, we formulate a two-source relaying game to facilitate the game theoretical analysis in the following sections.

We consider the system in Figure 5.1 in the context of a two-source relaying game. Source 1 and source 2 formulate the player set $S = \{S_1, S_2\}$. They each have the strategy space Θ_i. Here we take the relay energy space as the strategy space, i.e.

$\Theta_i = \{\mathcal{E}_{ir} | \mathcal{E}_{ir} \in \Omega\}$, where Ω is the admissible range of the relay energy. Ω can be discrete or continuous. We denote the payoff function of S_i as $u_i(\theta_1,\ \theta_2)$, which is the payoff to source i if the sources choose the strategies $(\theta_1,\ \theta_2)$.

In the n-source game, the set of strategies $(\theta_1^*, ..., \theta_n^*)$ is a Nash Equilibrium (NE) if, for each source i, θ_i^* is source i's best response to θ_{-i}^*, where θ_{-i} denotes the strategies of all the sources except source i. It can be presented as [Gib92]

$$\pi_i(\theta_i^*, \theta_{-i}^*) \geq \pi_i(\theta_i, \theta_{-i}^*)$$

where π_i is the payoff function (utility) of S_i.

The concept of Nash Equilibrium is intuitively explained as follows: if all of the sources are following NE strategies, no source can increase their payoff by deviating from the NE strategy. Our goal is to find Nash Equilibria of the two-source relaying game in the case of non-fading and fading channels and to identify the conditions under which an equilibrium based on cooperation exists.

We consider the class of two-source relaying games whose payoff function can be presented as a linear combination of two functions. The first denotes the gain the source gets from the other. It is a function of the source energy of itself and the relay energy of the others. The other denotes the cost of the source paid to achieve the performance goal and to help the other. The payoff function can be written as

$$\psi_i = \alpha_i'(\mathcal{E}_{is}, \mathcal{E}_{jr}) - \beta_i'(\mathcal{E}_{is}, \mathcal{E}_{ir}) \tag{5.1}$$

$\beta_i'(\mathcal{E}_{is}, \mathcal{E}_{ir})$ can be further divided as

$$\beta_i'(\mathcal{E}_{is}, \mathcal{E}_{ir}) = \upsilon_i(\mathcal{E}_{is}) + \beta_i(\mathcal{E}_{ir}) \tag{5.2}$$

where, $\upsilon_i(\mathcal{E}_{is})$ denotes the cost that S_i spends to achieve its own performance gain and $\beta_i(\mathcal{E}_{ir})$ denotes the cost that S_i spends to help S_j. They are both increasing functions of $\mathcal{E}_{is}$ and $\mathcal{E}_{ir}$ respectively. Intuitively, the more energy S_j used to help S_i, the less energy S_i needs to achieve the performance goal. Hence it is reasonable to assume that $\mathcal{E}_{is}$ can be presented as a decreasing function of $\mathcal{E}_{jr}$. Consequently, $\upsilon_i(\mathcal{E}_{jr})$ is a decreasing function of $\mathcal{E}_{jr}$. Then equation (5.1) can be written as

$$\begin{aligned} \psi_i &= (\alpha_i'(\mathcal{E}_{jr}) - \upsilon_i(\mathcal{E}_{jr})) - \beta_i(\mathcal{E}_{ir}) \\ &= \alpha_i(\mathcal{E}_{jr}) - \beta_i(\mathcal{E}_{ir}) \end{aligned} \tag{5.3}$$

Note that since $\alpha_i'(\mathcal{E}_{jr})$ increases with $\mathcal{E}_{jr}$ while $\upsilon_i(\mathcal{E}_{jr})$ decreased with $\mathcal{E}_{jr}$, hence $\alpha_i(\mathcal{E}_{jr})$ is an increasing function of $\mathcal{E}_{jr}$. $\beta_i(\mathcal{E}_{ir})$ is also an increasing function of $\mathcal{E}_{ir}$, denoting the cost of S_i to help S_j. Assume $\beta_i(\mathcal{E}_{ir})$ and $\beta_j(\mathcal{E}_{jr})$ has the same increasing slope. It means that both sources' cost function increase with their relay energy at the same speed. For non-fading channels the function of $\alpha_i(\mathcal{E}_{jr})$ and $\beta_i(\mathcal{E}_{ir})$ remains the same for each stage game. Without loss of generality, we assume $\alpha_i(0) - \beta_i(0) = 0$.

It is known that selfish sources always aim at maximizing their own utility function shown in (5.3). From the systematic point of view, however, to achieve a certain performance goal for a particular source in one time interval, the system wants to find an optimum solution that can increase the gain for a certain source while keep the cost of both sources low. That is, the system tries to maximize the following function.

$$\Gamma_i = \alpha_i(\mathcal{E}_{jr}) - \beta_j(\mathcal{E}_{jr}) \tag{5.4}$$

Thus we have systematic optimum relay energy as

$$\mathcal{E}_{ir}^* = \arg\max_{\mathcal{E}_{ir} \in \Omega} \Gamma_j \tag{5.5}$$

In a lot of cases of cooperative applications, Γ_j is convex on the admissible range of $\mathcal{E}_{ir}$, which means Γ_j has a unique maximum [YB06] [YB07a].

In this chapter, we consider the case where the strategy space of each source contains only two actions, i.e. $\Theta_i = \{R, N\}$, where "R" denotes "relay" at the systematic optimum resource allocation relay energy and "N" denotes "do not relay". In this case, we only have four possible payoff outcomes for each source:

$$\begin{aligned}
&\text{temptation to defect: } T_i = \alpha_i(\mathcal{E}^*_{jr}) - \beta_i(0), \\
&\text{sucker's payoff: } F_i = \alpha_i(0) - \beta_i(\mathcal{E}^*_{ir}), \\
&\text{reward of mutual cooperation: } M_i = \alpha_i(\mathcal{E}^*_{jr}) - \beta_i(\mathcal{E}^*_{ir}), \\
&\text{punishment of mutual defection: } \alpha_i(0) - \beta_i(0) = 0
\end{aligned} \tag{5.6}$$

	θ_2=N	θ_2=R
θ_1=N	0, 0	T_1, F_2
θ_1=R	F_1, T_2	M_1, M_2

Table 5.2: Two-source relaying game payoff matrix

We present the payoffs using the payoff matrix shown in Table 5.2. In a general cooperative wireless transmission system, the temptation to defect, T_i, is usually larger than the reward for mutual cooperation, M_i. The sucker's payoff, F_i , is always the lowest. The relationship between mutual cooperation payoff, M_i and mutual defection payoff, 0, however, is tricker to decide. We know from [YB06] that under certain channel states, cooperation won't outperform direct transmission from a systematic view. It is also true from a individual source's point of view that the source might not get benefit by mutual cooperation, i.e. the energy it spends to help the other exceeds the energy it saves by getting help from the other. In the scenario where $M_i < 0$, mutual cooperation can not be an NE for both static game and finite/infinite repeated game. In the cases when cooperation is mutual beneficial, that is $M_1 > 0$ and $M_2 > 0$, noting that it is a Prisoners' Dilemma game if the value if the payoff matrix satisfy the following conditions.

$$\forall\, i \in \{1,2\} \quad T_i > M_i > 0 > F_i \;\text{ and }\; 2M_i > T_i + F_i$$

From (5.6), we know that given $M_i > 0$, $2M_i > T_i + F_i$ must be true. For Prisoners' Dilemma game, it is easy to check that for the one-shot or finite repeated two-source relaying game, the only NE is $\theta^* = (N, N)$ in each stage game. The situation changes, however, when the game is repeated infinitely. The goal of the sources now is to maximize the payoff that they accumulate over time, that is, they are willing to secure a high-payoff in the next stage by cooperating. From [YB06] we know that when cooperation is not systematic beneficial for S_i's transmission interval, the optimum relay energy $\mathcal{E}^*_{jr}$ will be zero (corresponds to direct transmission). In this case, the payoff matrix will be degenerated to $T_i = 0$ and $M_i = F_i \leq 0$. S_i will have a great motivation to defect.

5.3 Fixed channels Analysis

In the non-fading channels, when the two-source relaying game is repeated, the payoff matrix in Table 5.2 will remain the same in each stage game. Assume that for each time t, the outcomes of the $t-1$ preceding plays of the stage game are known before the present stage begins.

Denote the payoff of S_i of t^{th} stage game as π_{it}. It is reasonable to assume that when the sources begin the game, they do not know when the game will end. Thus we can model the finite repeated game with an unpredictable end as an infinite repeated game with discounted future payoffs. We define the *accumulated payoff* $\overline{\Pi}_i$ of S_i as

$$\overline{\Pi}_i = \sum_{t=1}^{\infty} \delta^{t-1} \pi_{it}, \tag{5.7}$$

where $0 \leq \delta \leq 1$ and t is the time index of the game. The discount factor δ represents the degree to which the payoff of each transmission session is discounted relative to the previous transmission session. In our case, δ can be looked at as the probability that the game doesn't end in this stage game. For example, δ can be considered a measure of mobility in the sense that it represents the probability of still having the other source in the neighborhood after the current transmission session. The sources do not have control over the value of δ. Nevertheless, the value of δ does affect the strategies that the sources will choose. In the analysis in this chapter, and as is customary in the analysis of repeated games, we assume both sources have the same value for δ. Our analysis can also be extended to the case when the sources have different values of δ.

Proposition 4. *There exists a Nash Equilibrium based on cooperation with optimum energy allocation for the infinitely repeated two-source relaying game in non-fading channels if and only if*

$$\max_{i \in \{1,2\}} \left(1 - \frac{M_i}{T_i}\right) \leq \delta \leq 1. \tag{5.8}$$

Proof. Suppose S_i adopts the *trigger strategy*, i.e. S_i cooperates in the first stage and keeps cooperating until S_j fails to cooperate, which triggers noncooperation forever. We want to show that if (5.8) is satisfied, the best response for S_j is to adopt trigger strategy. That is to say it is a Nash equilibrium of the infinitely repeated game for both sources to adopt the trigger strategy. Since S_i will punish source S_j by playing N forever if one stage's outcome differs from (R, R), S_j's best response is also to play N forever once one stage's outcome differs from (R, R).

Then what is S_j's best response in the first stage and the stages that all the preceding outcomes have been (R, R)? We notice that S_j can have a payoff of T_j by playing N at this stage but will trigger noncooperation forever after, hence the payoff in every future stage will be 0. The present value of the accumulated payoff is

$$\overline{\Pi_{jN}} = T_j + \delta \cdot 0 + \delta^2 \cdot 0 + ... = T_j \tag{5.9}$$

Alternatively playing R will yield a payoff of M_j and then lead to the exactly same choice in the next stage. Then the accumulated payoff is

$$\overline{\Pi_{jR}} = M_j(1 + \delta + \delta^2 + ...) = \frac{M_j}{1 - \delta} \tag{5.10}$$

Hence cooperation can be stimulated if and only if

$$\overline{\Pi_{jR}} \geq \overline{\Pi_{jN}} \tag{5.11}$$

From (5.9), (5.10) and (5.11), we have

$$\delta \geq 1 - \frac{M_j}{T_j} \tag{5.12}$$

The same analysis also applies to S_i and recall that both sources have the same discount factor δ, thus to stimulate cooperation, we have

$$\delta \geq \max_{i \in \{1,2\}} \left(1 - \frac{M_i}{T_i}\right) \tag{5.13}$$

We know that $\delta \in (0, 1)$, hence to make (5.13) meaningful, T_i must be no less than M_i, for all $i \in \{1, 2\}$. If the condition of (5.13) is satisfied then given S_i has adopted the trigger strategy, S_j's best response is playing R all the time. If both sources adopt the trigger strategy then (R, R) will be played in every stage of the infinitely repeated game. Thus cooperation can be naturally encouraged. $\square$

Proposition 4 has implied that $T_i \geq 1 - M_i$. From (5.6) we know it is equivalent to $\alpha_i(\mathcal{E}_{jr}^*) \geq \beta_i(\mathcal{E}_{ir}^*)$ for both sources. It denotes the *mutual beneficial* channel conditions that both sources can save energy by cooperation with optimum energy allocation. A source that can not have individual benefit by cooperation will always play N regardless of the interest of others. Thus cooperation can not be stimulated unless the mutual cooperation condition (5.8) is satisfied.

5.4 Fading channels Analysis

To analyze the two-source relaying game in the case of fading channels, we model our system as an infinitely repeated game. The payoff matrix of each stage game corresponds to one channel state realization of each transmission session. Each element in the payoff matrix of Table 5.2 is a random variable and will change with each new channel state realization in each stage game. Because of the "randomness" of the elements in the payoff matrix, for some time t, the stage game may satisfy the condition to be a Prisoners' Dilemma game, while for some other t, it may not be true. We model the payoffs in the payoff matrix as ergodic random processes. Denote the realizations of payoffs at time t as $T_i^{(t)}$, etc. Since the payoff matrix will change with each stage game, both sources are not able to know the exact value of the payoff matrix in the future.

5.4.1 Relaying Game with Full CSIT

To investigate the conditions of when cooperation can be stimulated between selfish nodes, we propose a *conditional trigger strategy*.

Suppose S_i adopts the conditional trigger strategy. In each stage game, S_i's strategy can be expressed as Figure 5.2. First, check the value of both sources' payoffs of $T_i^{(t)}$ and $M_i^{(t)}$. If either of their value of $T_i^{(t)} - M_i^{(t)}$ exceeds their corresponding ceiling value, C_i and C_j, S_i plays N. Otherwise, S_i checks the value of its optimum relay energy. If it is equal to 0, then S_i plays N (S_j uses direct transmission as specified by optimum energy allocation). If the optimum relay energy$\neq 0$ and neither source has *defected* in all the previous games, S_i plays R. Otherwise, S_i plays N.

Here we do not simply consider the behavior of "not relay" as "defect". S_i is considered to "defect" in stage game t if all of the conditions are satisfied for S_i to play R but, instead, S_i plays N. For the reason that only the "not relay" behavior under certain conditions will be considered "defect" and thus trigger non-cooperation forever after, we call our strategy "conditional trigger strategy". If S_i defects, it obtains an additional payoff in the current stage game since it does not expend any relaying energy. This short-term gain, however, is obtained at the cost of the loss of future payoffs since defection triggers non-cooperation for all future games.

Proposition 5. *Define*

$$I_{\{x\leq y\}} = \begin{cases} 1 & x \leq y \\ 0 & x > y. \end{cases}$$

If C_i satisfies

$$C_i = \frac{\delta}{1-\delta} E\left[M_i I_{\{T_1 - M_1 \leq C_1\}} I_{\{T_2 - M_2 \leq C_2\}}\right] \tag{5.14}$$

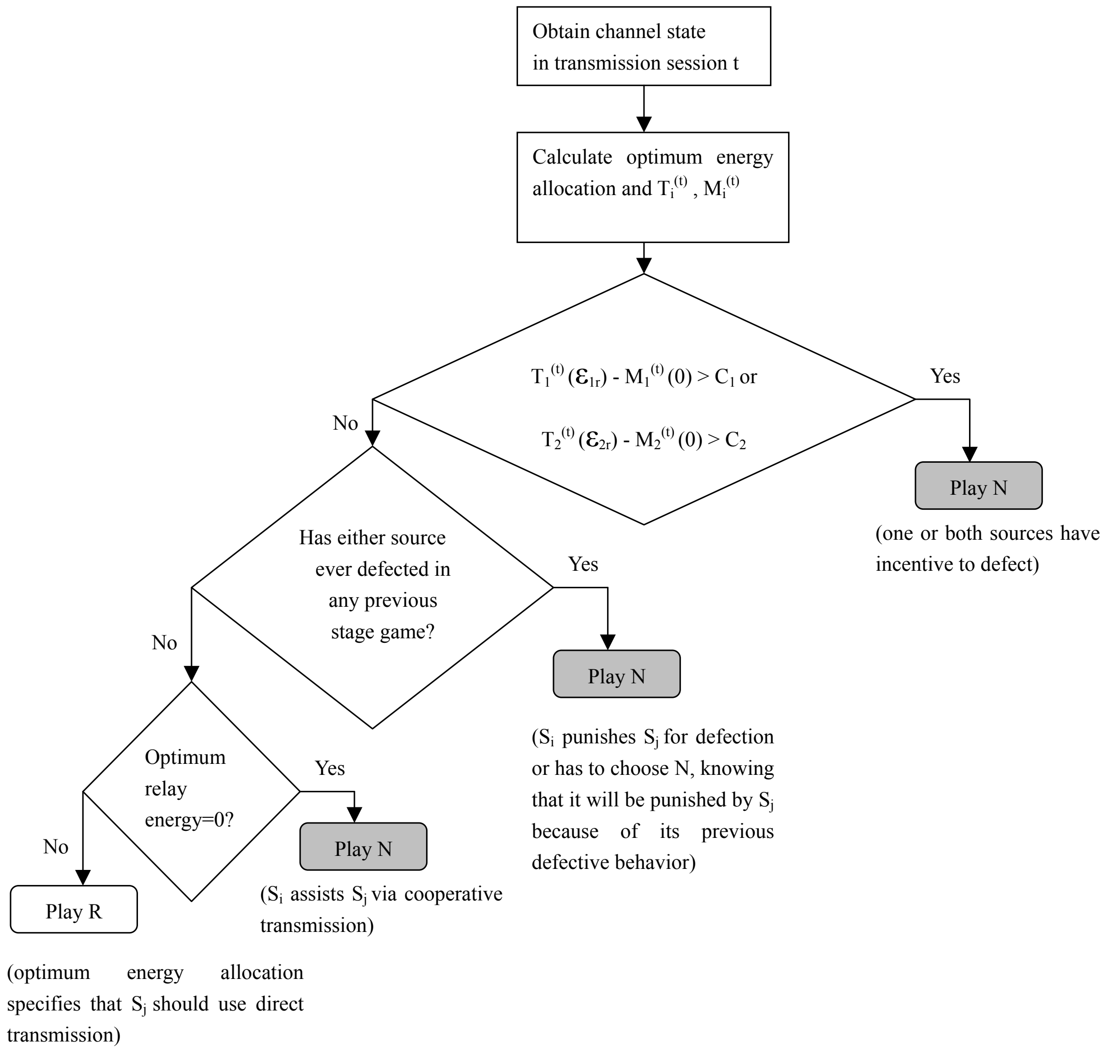

Figure 5.2: S_i's conditional trigger strategy in the t^{th} stage game.

for both $i = 1$ and $i = 2$, then the conditional trigger strategy *specified in Figure 5.2 is a Nash Equilibrium for the infinitely repeated two-source relaying game in fading channels when instantaneous channel state is available.*

Proof. For the stages that do not satisfy $T_i^{(t)} - M_i^{(t)} \leq C_i$, for both sources. since S_i plays N, S_j's best response is also to play N. When optimum relay energy $= 0$, it is trivial to show that when S_i should use direct transmission (specified by optimum energy allocation), S_j does not need to relay. For the reason that S_i plays N forever if either source has defected previously, S_j's best response is also to play N forever once S_i or S_j defect. The rest is to determine S_j's best response when the current stage game satisfies the condition of $T_i^{(t)} - M_i^{(t)} \leq C_i$ and neither source has defected in the previous games.

Suppose the current stage is t_0 and S_j plays N, then S_j will get a payoff of $T_j^{(t_0)}$ but will trigger (N, N) forever after. Hence the present value of the accumulated payoff is

$$\overline{\Pi}_{jN} = T_j^{(t_0)} + \delta \cdot 0 + \delta^2 \cdot 0 + ... \tag{5.15}$$

On the other hand, playing R will yield a payoff of $M_j^{(t_0)}$ and then lead to the exactly same choice in the next stage that all the conditions are satisfied for S_j to play R (specified by the indicator function). We have the present value of the accumulated payoff as

$$\overline{\Pi}_{jR} = M_j^{(t_0)} + \delta \cdot M_j^{(t_1)} I_{\{T_j^{(t_1)} - M_j^{(t_1)} \leq C_j\}} I_{\{T_i^{(t_1)} - M_i^{(t_1)} \leq C_i\}} + ... \tag{5.16}$$

Although S_j does not know the exact value of payoffs in the payoff matrix in the future stage games, S_j knows what can *expect*. Hence equation (5.16) can be rewritten as

$$\overline{\Pi}_{jR} = M_j^{(t_0)} + \delta \cdot \mathrm{E}[(M_j) I_{\{T_j - M_j \leq C_j\}} I_{\{T_i - M_i \leq C_i\}}] + ...$$
$$= M_j^{(t_0)} + \tfrac{\delta}{1-\delta} \mathrm{E}[M_j I_{\{T_j - M_j \leq C_j\}} I_{\{T_i - M_i \leq C_i\}}] \tag{5.17}$$

Note that C_j satisfies the function (5.14), the above equation can be rewritten as

$$\overline{\Pi}_{jR} = M_j^{(t_0)} + C_j \tag{5.18}$$

Given the condition of $T_i^{(t_0)} - M_i^{(t_0)} \leq C_i$, for both sources, we know from (5.17) and (5.18) that $\overline{\Pi}_{jR}$ must be no less than $\overline{\Pi}_{jN}$. In this case, S_j has no strict incentive to defect in the stage game. Hence given that in the first game and in any stage game that all the preceding stage games' outcomes have no defective behavior, S_j's best response is to play R. The same analysis is applied to S_i. Hence it is a Nash Equilibrium for both sources to adopt the conditional trigger strategy of the infinitely repeated game in fading channels given both C_1 and C_2 satisfy (5.14). □

Proposition 5 implies that for the fading channel scenarios, sources decide their moves based both on the instantaneous value of the payoff matrix, $T_i - M_i$ and the statistics of the future payoffs, C_i. From (5.6), we have that $T_i - M_i$ is equivalent to $\beta_i(\mathcal{E}_{ir}^*) - \beta_i(0)$. From (5.3) we know that $\beta_i(\mathcal{E}_{ir}^*)$ is the cost of S_i to help the other and $\beta_i(0)$ is the cost that S_i does not help the other source, typically this value is 0. Intuitively, C_i can be looked as the ceiling value of the cost of helping the other. Its value can be decided by the function (5.14).

$$C_1 = \frac{\delta}{1-\delta} \mathrm{E}\left[(\alpha_1 - \beta_1) I_{\{\beta_1 \leq C_1\}} I_{\{\beta_2 \leq C_2\}}\right]$$
$$C_2 = \frac{\delta}{1-\delta} \mathrm{E}\left[(\alpha_2 - \beta_2) I_{\{\beta_1 \leq C_1\}} I_{\{\beta_2 \leq C_2\}}\right]$$

If S_i's cost of relaying exceeds the ceiling C_i, it means that the optimum energy allocation requires a large cost from S_i to help S_j. In this case, the cost in the current stage of playing R is so much that the expected future payoff is not tempting anymore. Thus a rational source will have the incentive to defect. However, the payoff matrix is also known by S_j. Knowing that S_i will not cooperate, S_j's best response is also not to cooperate. The no-cooperate behavior here is not considered "defection" and does not trigger non-cooperation forever. In the conditional trigger strategy, trigger will only happen if one or both sources do not cooperate with optimum energy allocation strategy when both sources has no incentive to defect in the stage game.

Now we consider the special case of $\delta = 1$. $\delta = 1$ implies that both sources are infinitely patient, i.e. the energy they spend in the future has the same importance as the energy they spend now or both sources are relative static (none of them have the idea of when they will run out of each other's neighborhood). When $\delta = 1$, the accumulated payoff defined in (5.7) goes to infinity. Hence we define the *average accumulated payoff* as

$$\overline{\Phi_i} = \lim_{T\to\infty} \frac{\sum_{t=1}^{T} \pi_i^t}{T} \tag{5.19}$$

Based on which, we have the following corollary

Corollary 1. *When both sources are infinitely patient, there exists a Nash Equilibrium based on full cooperation for the infinitely repeated two-source relaying game in fading channels if*

$$\min_{i\in\{1,2\}} E(\alpha_i - \beta_i) \geq 0. \tag{5.20}$$

The proof of Corollary 1 follows the steps in the proof of Proposition 5.

For Rayleigh fading, the expected value of α_i goes to infinity. This implies that even if δ is very small, the long term benefit of mutual cooperation with optimum energy allocation is still very large. The conditions in Proposition 5 will be satisfied. Rational sources in a Rayleigh fading environment will choose to cooperate with optimum energy allocation to achieve high payoff in the long run.

5.4.2 Relaying Game with No CSIT

When the channel state is not known, a resource allocation rule can only be specified based on the statistics of the channel state; an instantaneous performance target such as SNR can not be guaranteed to be satisfied. In this case, a performance metric such as outage probability is more appropriate. Without channel state knowledge, both sources can only choose a “fixed” strategy for all the transmission intervals based on the channel state statistics, which means the payoff matrix in Table 5.2 will remain the same in each stage game. As a result, the analysis for two-source relaying game with non-fading channels can be applied here.

Proposition 6. *When only channel state statistics is available, there exists a Nash Equilibrium based on cooperation with optimum energy allocation for the infinitely repeated two-source relaying game in fading channels if and only if*

$$\max_{i\in\{1,2\}} \left(1 - \frac{M_i}{T_i}\right) \leq \delta \leq 1.$$

Proof. The proof is similar to the proof of Proposition 4. □

5.5 Numerical Results

This section demonstrates the impact of sources’ selfish behavior on energy efficiency with respect to centrally optimized energy allocation under path loss channels and lognormal fading channels for orthogonal amplify-and-forward protocols. The distance between the two destinations in Figure 5.4 is normalized as 1. In the path loss model, the normalized channel gain is given as $b = d_b^{-\gamma}$, where d_b is the distance between source 1 and source 2 and γ is the channel attenuation exponent. a_i and c_i are defined in the same way. For lognormal fading channels, the channel states $\sqrt{a_i}$, $\sqrt{b_i}$ and $\sqrt{c_i}$ are lognormal distributed random variables with means $\mu_{c_i} = 1/d_{c_i}^{\gamma}$, and $\mu_b = 1/d_b^{\gamma}$, respectively. For both channel models, $\rho = 1$.

Figure 5.3 shows the region that cooperation is "naturally" encouraged in a wireless system under path-loss channels with different channel attenuation exponents. The results show that the region expands with γ, which indicates that with more sever channel attenuation, cooperation with optimum energy allocation is more preferable than direct transmission.

The model in Figure 5.4 is used to demonstrate the impact of sources' selfishness on energy efficiency. In this model, S_1 and S_2 is located on the line connecting the two destinations with $d_{c_1} = 0.25$. Energy efficiency is investigated while S_2 moves between S_1 and D_1. All of the following results assume $\gamma = 4$.

Figure 5.5 shows the discounted total saved energy of grim trigger strategy [Gib92] and centrally optimized energy allocation in the path loss channels. When d_b is small, d_{c_2} is large. S_2's relay energy required by optimum energy allocation exceeds the energy S_2 saved by help from S_1. In this case, S_2 refuses to relay. Thus no cooperation is stimulated by grim trigger strategy. When S_2 moving toward D_1, the required relay energy of S_2 becomes smaller and finally S_2 falls into the region where mutual benefit occurs. Cooperation with optimum energy allocation is stimulated and the discounted total saved energy of grim trigger strategy merges to the centrally optimized case. The matlab codes to generate this plot are in Appendix D.

Figure 5.6 shows the average discounted total saved energy of conditional trigger strategy in lognormal fading channels with $d_b = 0.4$. Centrally optimized energy allocation strategy is also included for comparison. Here "the average discounted total saved energy" denotes the discounted total saved energy averaged over channel distributions. The results show that the energy gap between the conditional trigger strategy and centrally optimized strategy becomes smaller as δ becomes larger. Both strategies merge as $\delta \to 1$.

Figure 5.7 shows the fraction of the stage games in which cooperation with optimum energy allocation (OEA) is stimulated with conditional trigger strategy. The fraction of the stage games using cooperation with optimum energy allocation increases with δ. As $\delta \to 1$, sources choose to cooperate with optimum energy allocation in almost all the stage games. In this case, the conditional trigger strategy is almost as energy efficient as the centrally optimized case.

Figure 5.8 shows the discounted total saved energy of grim trigger strategy [Gib92] and centrally optimized energy allocation in the fading channels. We used the upper bound expression of outage probability (equation 3.19) in section 3.6. We fix outage probability $P_{mathsfout} = 10^{-3}$. When d_b is small, d_{c_2} is large. S_2's relay energy required by optimum energy allocation exceeds the energy S_2 saved by help from S_1. In this case, S_2 refuses to relay. Thus no cooperation is stimulated by grim trigger strategy. When S_2 moving toward D_1, the required relay energy of S_2 becomes smaller and finally S_2 falls into the region where mutual benefit occurs. Cooperation with optimum energy allocation is stimulated and the discounted total saved energy of grim trigger strategy merges to the centrally optimized case.

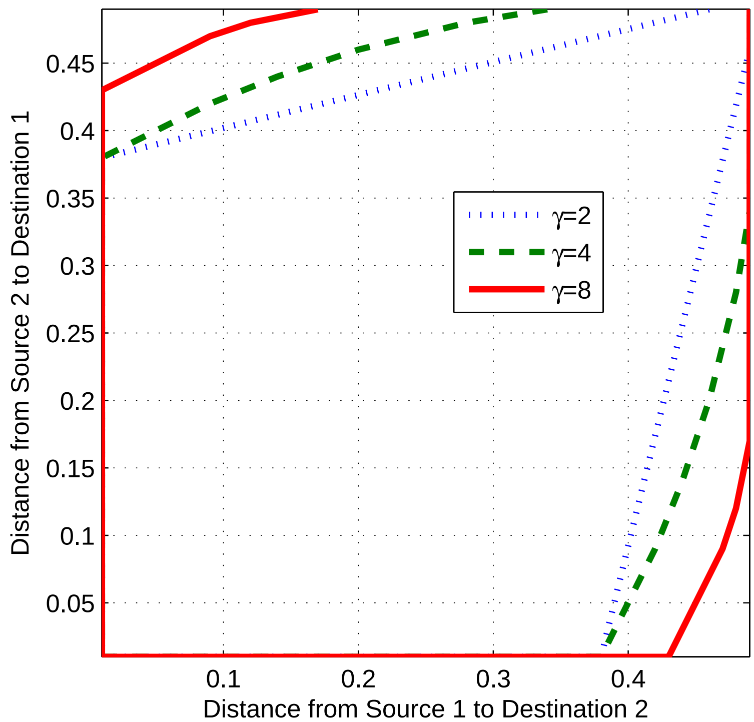

Figure 5.3: Region that cooperation with optimum energy allocation can emerge as a Nash Equilibrium between selfish sources for different channel attenuation exponent γ in path loss channels.

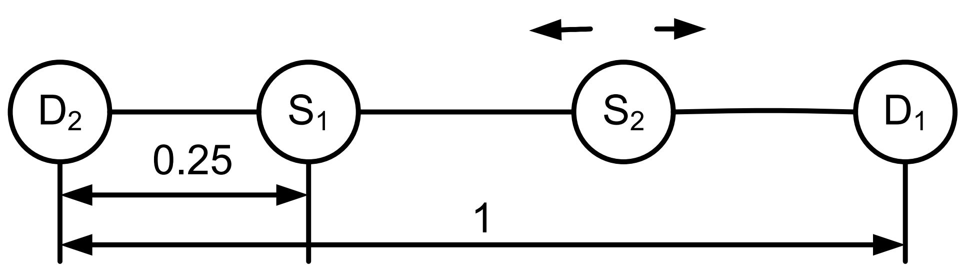

Figure 5.4: Simulation model.

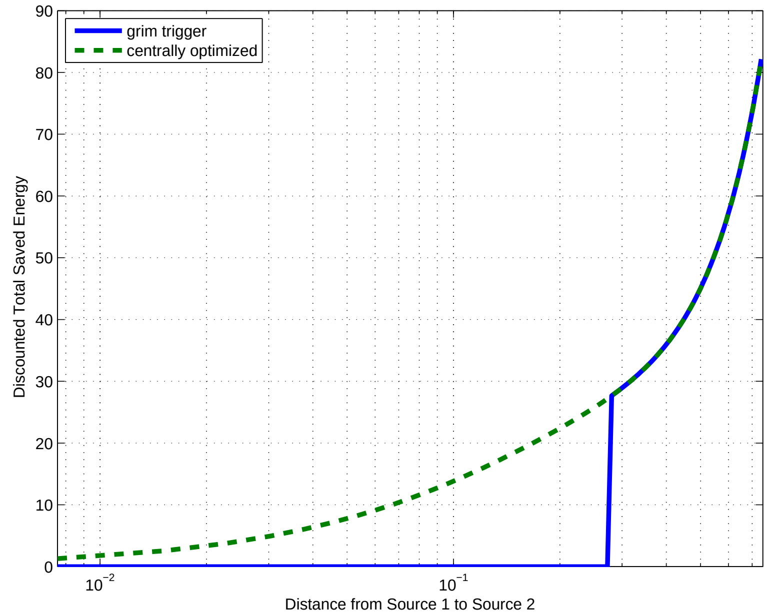

Figure 5.5: Discounted total saved energy versus d_H (distance between source 1 and source 2) in path loss channels with $\gamma = 4$.

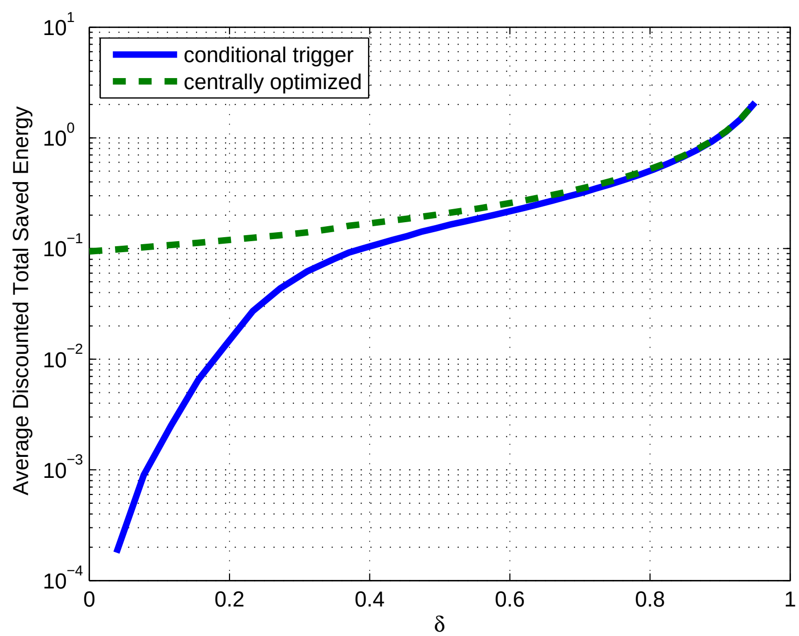

Figure 5.6: Average discounted total saved transmission energy versus δ in lognormal fading channels with $\gamma = 4$

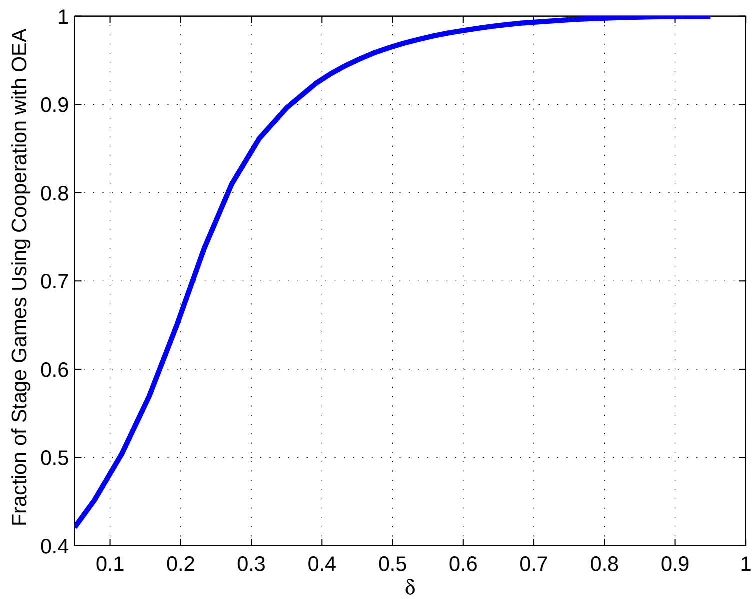

Figure 5.7: The fraction of stage games using cooperation with optimum energy allocation versus δ.

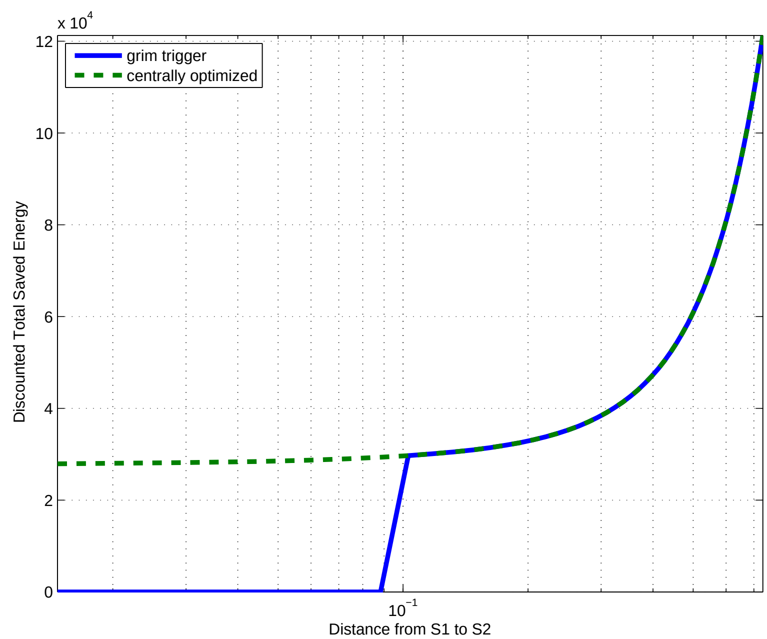

Figure 5.8: Discounted total saved energy versus d_b (distance between source 1 and source 2) in path loss channels with $\gamma = 4$.

5.6 Conclusions

This chapter considers the problem of under what conditions cooperation can be stimulated between rational and self-interest sources. A two-source relaying game is formulated for both non-fading and fading scenarios. We show that cooperative transmission with optimum resource allocation is a Nash Equilibrium in non-fading channels and fading channels without CSIT when the sources are sufficiently patient. In fading channels with CSIT, cooperative transmission with optimum resource allocation is also a Nash Equilibrium when a ceiling is applied to the relay energy of each source. Simulation results show that sources acting in their own self-interest can achieve an energy efficiency close to that of centrally optimized energy allocation in many cases. For networks with more than two nodes, we can pair a node with its adjacent node and our results can still be applied.

Chapter 6

Conclusions and Future Work

6.1 Summary

The essential idea of cooperative transmission in wireless systems is to exploit the broadcast nature and space diversity provided by the wireless medium. By having a relaying partner, transmit cooperation can offer many of the benefits of multi-antenna transmission, e.g. increased rate and/or reduced outage probability, reduced overall transmit energy. In this dissertation, we presented the analytical results of the optimum (minimum energy) energy allocation strategy for the OAF cooperative transmission protocol, analytically compared the instantaneous capacity of some of the most popular cooperative transmission protocols and quantized the condition when cooperation can be mutually beneficial so that autonomous nodes can cooperate without extrinsic incentive mechanisms. We summarize our results by chapter below.

- **Chapter 3** In this chapter, we presented an analysis of the optimum energy allocation strategy for the OAF cooperative transmission protocol for four different scenarios with different channel state assumptions. Numerical examples with independent Rayleigh fading channels demonstrated that full CSIT can significantly improve the energy efficiency of both cooperative and direct transmission. The main contribution of these results is a better understanding of how channel state knowledge affects the optimal energy allocation strategy as well as the expected gains of OAF cooperative transmission with respect to direct (non-cooperative) transmission. The results suggest that accurate channel state knowledge, possibly obtained via low rate feedback from the the destination, enables significant gains in energy efficiency for both OAF cooperative transmission as well as direct transmission. Under identical channel state knowledge assumptions, OAF cooperative transmission tends to have better energy efficiency than direct transmission. The results in Chapter 3 show, however, that opportunistic direct transmission with full knowledge of the channel state by the source is often more energy efficient than cooperative transmission without source knowledge of the channel state.

- **Chapter 4** In this chapter, we provided comparative analysis of the protocols of compress-and-forward (CF), estimate-and-forward (EF), non-orthogonal amplify-and-forward (NAF), decode-and-forward (DF) and the cut-set upper bound in the case of instantaneous CSI feedback. The analysis facilitates the development of opportunistic protocol selection with optimal resource allocation, which led to the proposal of a hybrid opportunistic protocol that selects from all available protocols the protocol that achieves the rate target with the least total transmit energy. We show that, for protocols employing optimal resource allocation under a total energy constraint, the hybrid opportunistic protocol only needs to select between CF and DF with optimal resource allocation in each channel state. Since the hybrid protocol uses the least energy in each transmission interval, it also provides the best delay-limited capacity performance of all protocols considered in this study. Our numerical results show that the hybrid protocol can offer delay-limited capacity close to the cut-set upper bound.

- **Chapter 5** In this chapter, we show that cooperation can often be established between two purely self-interested sources using only local utility functions (transmission energy). In non-fading channels and fading channels without access to the instantaneous channel state information, we show that cooperation with the OAF cooperative transmission protocol and optimum energy allocation can exist between two self-interested sources given that the end of the cooperative "game" is uncertain and that the sources are sufficiently patient in the sense that sources do not excessively discount future payoffs with respect to current payoffs. We explicitly give conditions under which cooperation is mutually beneficial and describe a simple cooperative Nash Equilibrium strategy. In fading channels with full channel state information, we develop a conditional trigger cooperative strategy and show that this strategy is a Nash Equilibrium (NE) of the infinitely repeated game. We show that this ceiling goes to infinity as the sources become more patient. Our results also show that sources using the conditional trigger strategy can often achieve an overall system energy efficiency close to that of a centrally-controlled system, especially when the sources are patient.

6.2 Future Research Directions

The results in Chapters 3-5 are all based on a small-scale network scenario (two sources and one destination or two sources and two destinations). These results can be applied to larger-scale networks in scenarios where nodes form cooperative partnerships but do not include the possibility *cooperative syndicates* with more than two nodes. The potential gains in these types of systems would be larger, but the analysis is more difficult and little research has been published in this direction. Chapters 3-5 also have a common assumption that the sources do not know the channel phase, hence beamforming can not be used. If the channel phase were known to both sources, then beamforming and/or space-time coding could be used. The expressions for the optimal resource allocations would also change for each protocol as well as the short-term and long-term performance measures. Given the recent progress on synchronizing sources for distributed beamforming [MBIMP09], this may be an promising direction for future research.

In Chapter 3, we presented explicit analytical results of optimum energy allocation for the ideal case of full CSIT and full CSIR and give Proposition 2 to help to solve the optimum energy allocation problem of full CSIT and no CSIR case. When channel state information is not available, lower and upper bounds are derived. Note that although we have an explicit expression for those bounds, the expressions are still very complicated (involving Bessel functions), hence explicit solution for optimum energy allocation still can not be obtained. More accurate and simpler bounds could be a valuable tool to make the solutions more practical. In Chapter 4, we give analytical comparison of the protocols of compress-and-forward (CF), estimate-and-forward (EF), non-orthogonal amplify-and-forward (NAF), decode-and-forward (DF) and the cut-set upper bound under the assumption of instantaneous CSI feedback. When there is no CSIT, which is a more practical scenario, the problem of how the delay-limited capacity will be affected is an interesting one to consider. When instantaneous channel state information is available, we can dynamically choose the "best" protocol with the optimum energy allocation. When channel state information is not available, however, the question of how to choose among the protocols for optimal performance is still open. In Chapter 5, we analyzed the condition under which selfish nodes can cooperate without extrinsic incentive mechanisms of the two-source two-destination non-fading/fading scenario. We used a generalized form of the payoff matrix and showed that the performance measure of saved transmit energy can be fit into the payoff function. Other performance measures, i.e. capacity or throughput, however, can not fit into this generalized form. Analysis based on a generalized payoff matrix may be an interesting area for future study. Also, we only considered repeated games of complete information (games in which the players' payoff functions are common knowledge) in Chapter 5. A more complicated yet reasonable assumption is that at least one player is uncertain about another player's payoff function. In this case, Bayesian Nash Equilibrium analysis comes into play and the results from such an analysis would lend insight into more practical scenarios.

Bibliography

[AES04] K. Azarian, H. El Gamal, and P. Schniter. On the achievable diversity-vs-multiplexing tradeoff in cooperative channels. In *Proceedings of the 38th Annual IEEE Conference on Information Sciences and Systems (CISS)*, pages 956–960, Princeton, NJ, March 17-19 2004.

[AKA04] N. Ahmed, M. A. Khojastepour, and B. Aazhang. Outage minimization and optimal power control for the fading relay channel. In *IEEE Information Theory Workshop*, pages 458–462, 2004.

[ARW05] J. Adeane, M. R. D. Rodrigues, and I. J. Wassell. Centralised and distributed power allocation algorithms in cooperative networks. In *2005 IEEE 6th Workshop on Signal Processing Advances in Wireless Communications*, pages 333–337, 2005.

[BB02a] S. Buchegger and J.Y.L. Boudec. Nodes bearing grudges: Towards routing security, fairness, and robustness in mobile ad hoc networks. In *Proceedings of the 10th Euromicro Workshop on Parallel, Distributed, and Network-based Processing*, 2002.

[BB02b] S. Buchegger and J.Y.L. Boudec. Performance analysis of CONFIDANT protocol: cooperation of nodes — fairness in dynamic ad-hoc networks.

In *Proceedings of IEEE/ACM Workshop on Mobile Ad Hoc Networking and Computing (MobiHOC)*, June 2002.

[BBC+01] L. Blazevic, L. Buttyan, S. Capkun, S. Giordano, J.-P. Hubaux, and J.-Y. Le Boudec. Self organization in mobile ad hoc networks: the approach of terminodes. *IEEE Communications Magazine*, 39(6):166–174, June 2001.

[BFY04] J. Boyer, D.D. Falconer, and H. Yanikomeroglu. Multihop diversity in wireless relaying channels. *Communications, IEEE Transactions on*, 52(10):1820–1830, Oct. 2004.

[BIP08] D. R. Brown III and H. V. Poor. Time-slotted round-trip carrier synchronization for distributed beamforming. *Signal Processing, IEEE Transactions on*, 56(11):5630–5643, Nov. 2008.

[BpH03] Levente Buttyn and Jean pierre Hubaux. Stimulating cooperation in self-organizing mobile ad hoc networks. *ACM/Kluwer Mobile Networks and Applications (MONET)*, 8:579–592, 2003.

[BPM05] D.R. Brown III, G. Prince, and J.A. McNeill. A method for carrier frequency and phase synchronization of two autonomous cooperative transmitters. In *Proceedings of the 5th IEEE Signal Processing Advances in Wireless Communications*, pages 278–282, New York, NY, June 5-8 2005.

[Bro04] D.R. Brown III. Resource allocation for cooperative transmission in wireless networks with orthogonal users. In *Conference Record of the Thirty-Eighth Asilomar Conference on Signals, Systems and Computers*, volume 2, pages 1473–1477 Vol.2, 2004.

[CdBSA06] Arnab Chakrabarti, Alexandre de Baynast, Ashutosh Sabharwal, and Behnaam Aazhang. Half-duplex estimate-and-forward relaying: Bounds and code design. *Information Theory, 2006 IEEE International Symposium on*, pages 1239–1243, July 2006.

[CG79] T. Cover and A.E. Gamal. Capacity theorems for the relay channel. *Information Theory, IEEE Transactions on*, 25(5):572–584, Sep 1979.

[CK07] Yingda Chen and Shalinee Kishore. A game-theoretic analysis of decode-and-forward cooperation in rayleigh fading channels. In *Information Sciences and Systems, 2007. CISS '07. 41st Annual Conference on*, pages 294–299, 2007.

[Con07] Marco Conti. *Mobile Multi-Hop Ad Hoc Networks: From Theory to Reality*. Hindawi Publishing Corporation, New York, NY, 2007.

[CSY05] Min Chen, S. Serbetli, and A. Yener. Distributed power allocation for parallel relay networks. In *IEEE Global Telecommunications Conference*, volume 3, pages 1177–1181, 2005.

[CSY08] Min Chen, S. Serbetli, and A. Yener. Distributed power allocation strategies for parallel relay networks. *Wireless Communications, IEEE Transactions on*, 7(2):552–561, February 2008.

[CT91] T.M. Cover and J.A. Thomas. *Elements of Information Theory*. Wiley, New York, 1991.

[DADSC04] S.N. Diggavi, N. Al-Dhahir, A. Stamoulis, and A.R. Calderbank. Great expectations: The value of spatial diversity in wireless networks. *Proceedings of the IEEE*, 92(2):219–270, February 2004.

[EGMZ06] A. El Gamal, M. Mohseni, and S. Zahedi. Bounds on capacity and minimum energy-per-bit for awgn relay channels. *Information Theory, IEEE Transactions on*, 52(4):1545–1561, April 2006.

[FHB06] M. Felegyhazi, J.P. Hubaux, and L. Buttyan. Nash euilibria of packet forwarding strategies in wireless ad hoc networks. In *IEEE Trans. Mobile Computing, Vol. 5, May. 2006*, pages 1044–1049, May 2006.

[GE05] D. Gunduz and E. Erkip. Outage minimization by opportunistic cooperation. In *2005 International Conference on Wireless Networks, Communications and Mobile Computing*, volume 2, pages 1436–1442, 2005.

[GE07] D. Gündüz and E. Erkip. Opportunistic cooperation by dynamic resource allocation. *Wireless Communications, IEEE Transactions on*, 6(4):1446–1454, April 2007.

[Gib92] Robert Gibbons. Game theory for applied economists. In *Princeton University Press*, 1992.

[GS99] D.L. Goeckel and W.E. Stark. Optimal diversity allocation in multiuser communication systems. I. system model. *IEEE tansactions on communications*, 47(12):1828–1836, December 1999.

[GS00] D.L. Goeckel and W.E. Stark. Optimal diversity allocation in multiuser communication systems. II. optimization. *IEEE tansactions on communications*, 48(1):45–52, January 2000.

[GSsS+03] D. Gesbert, M. Shafi, Da shan Shiu, P.J Smith, and A. Naguib. From theory to practice: an overview of MIMO space-time coded wireless systems. *Selected Areas in Communications, IEEE Journal on*, 21(3):281–302, April 2003.

[GxYh06] Wei Guang-xing and Qin Yan-hong. Incentive mechanism for team cooperation based on implicit side contract. In *Management Science and Engineering, 2006. ICMSE '06. 2006 International Conference on*, pages 1344–1349, 2006.

[HA03] M.O. Hasna and M.-S. Alouini. End-to-end performance of transmission systems with relays over rayleigh-fading channels. *Wireless Communications, IEEE Transactions on*, 2(6):1126–1131, Nov. 2003.

[HGLV01] J.-P. Hubaux, T. Gross, J.-Y. Le Boudec, and M. Vetterli. Towards self-organized mobile ad hoc networks: the terminodes project. *IEEE Communications Magazine*, 39(1):118–124, January 2001.

[HHSL05] Zhu Han, Thanongsak Himsoon, W. P. Siriwongpairat, and K. J. R. Liu. Energy-efficient cooperative transmission over multiuser ofdm networks: who helps whom and how to cooperate. In *2005 IEEE Wireless Communications and Networking Conference*, volume 2, pages 1030–1035 Vol. 2, 2005.

[HN02a] T.E. Hunter and A. Nosratinia. Coded cooperation under slow fading, fast fading, and power control. In *Proceedings of the Asilomar Conference on Signals, Systems, and Computers*, volume 1, pages 118–122, Pacific Grove, November 3-6 2002.

[HN02b] T.E. Hunter and A. Nostratinia. Cooperation diversity through coding. In *Proceedings of the 2002 IEEE International Symposium on Information Theory*, page 220, Lausanne, Switzerlan, June 30-July 5 2002.

[HN03] T.E. Hunter and A. Nosratinia. Performance analysis of coded cooperation diversity. In *Proceedings of the 2003 IEEE International Conference on Communications*, pages 2688–2692, 2003.

[Hos04] A. Host-Madsen. On the achievable rate for receiver cooperation in ad-hoc networks. In *Proceedings of the 2004 IEEE International Symposium on Information Theory*, page 272, Chicago, IL, June 27-July 2 2004.

[HT98] S.V. Hanly and D.N.C. Tse. Multiaccess fading channels. ii. delay-limited capacities. *Information Theory, IEEE Transactions on*, 44(7):2816–2831, Nov 1998.

[HZ05a] A. Host-Madsen and J. Zhang. Capacity bounds and power allocation for wireless relay channel. In *IEEE Trans. on Information Theory*, volume 51, pages 2020– 2040, June 2005.

[HZ05b] A. Host-Madsen and Junshan Zhang. Capacity bounds and power allocation for wireless relay channels. *IEEE Transactions on Information Theory*, 51(6):2020–2040, 2005.

[Jak74] W.C. Jakes. *Microwave Mobile Communications*. J. Wiley and Sons, New York, 1974.

[JHB03] Markus Jakobsson, J. P. Hubaux, and L. Buttyan. A micro-payment scheme encouraging collaboration in multi-hop cellular networks. In *Financial Crypto 2003*, 2003.

[JHHN04] M. Janani, A. Hedayat, T.E. Hunter, and A. Nosratinia. Coded cooperation in wireless communications: space-time transmission and iterative decoding. *Signal Processing, IEEE Transactions on*, 52(2):362–371, Feb. 2004.

[KGG05] G. Kramer, M. Gastpar, and P. Gupta. Cooperative strategies and capacity theorems for relay networks. *Information Theory, IEEE Transactions on*, 51(9):3037–3063, Sept. 2005.

[Kho04] Khojastepour, M. A. *Distributed Cooperative Communications in Wireless Networks*. PhD thesis, Rice University, May 2004.

[Lan02] Laneman, J.N. *Cooperative diversity in wireless networks: Algorithms and Architectures*. PhD thesis, Massachusetts Institute of Technology, May 2002.

[LBC+07] J. Luo, R.S. Blum, L.J. Cimini, L.J. Greenstein, and A.M. Haimovich. Decode-and-forward cooperative diversity with power allocation in wireless networks. *Wireless Communications, IEEE Transactions on*, 6(3):793–799, March 2007.

[LG06] L. Lai and H. El Gamal. On cooperation in energy efficient wireless networks: The role of altruistic nodes. In *submitted to the IEEE Transactions on Wireless Communications*, Aug 2006.

[LTW04a] J. N. Laneman, D. N. C. Tse, and G. W. Wornell. Cooperative diversity in wireless networks: Efficient protocols and outage behavior. *IEEE Transactions on Information Theory*, 50(12):3062–3080, 2004.

[LTW04b] J. N. Laneman, D. N. C. Tse, and G. W. Wornell. Cooperative diversity in wireless networks: Efficient protocols and outage behavior. *IEEE Transactions on Information Theory*, 50(12):3062–3080, 2004.

[LV05] Y. Liang and V.V. Veeravalli. Gaussian orthogonal relay channels: Optimal resource allocation and capacity. *Information Theory, IEEE Transactions on*, 51(9):3284–3289, Sept. 2005.

[LVP07] Yingbin Liang, V.V. Veeravalli, and H.V. Poor. Resource allocation for wireless fading relay channels: Max-min solution. *Information Theory, IEEE Transactions on*, 53(10):3432–3453, Oct. 2007.

[LW00a] J.N. Laneman and G.W. Wornell. Energy-efficient antenna sharing and relaying for wireless networks. In *Proc. of the IEEE Wireless Communications and Networking Conf.*, pages 7–12, Chicago, IL, Sep. 2000.

[LW00b] J.N. Laneman and G.W. Wornell. Exploiting distributed spatial diversity in wireless networks. In *Proceedings of the Allerton Conference on Communications, Control, and Computing*, Urbana-Champaign, IL, October 2000.

[LW02] J.N. Laneman and G.W. Wornell. Distributed space–time-coded protocols for exploiting cooperative diversity in wireless networks. In *Proceedings of the IEEE Global Communications Conference (GLOBECOMM)*, pages 77–81, Taipei, Taiwan, November 2002.

[LW03] J. N. Laneman and G. W. Wornell. Distributed space-time-coded protocols for exploiting cooperative diversity in wireless networks. *IEEE Transactions on Information Theory*, 49(10):2415–2425, 2003.

[LWT01] J. N. Laneman, G. W. Wornell, and D. N. C. Tse. An efficient protocol for realizing cooperative diversity in wireless networks. In *Proceedings of the IEEE International Symposium on Information Theory (ISIT)*, page 294, June 24-29 2001.

[LY03] Y. Liu and Y.R. Yang. Reputation propagation and agreement in mobile ad-hoc networks. In *Proceedings of the IEEE Wireless Communications and Networking Conference (WCNC)*, volume 2, 2003.

[MBIMP09] Raghuraman Mudumbai, D.R. Brown Iii, Upamanyu Madhow, and H.V. Poor. Distributed transmit beamforming: challenges and recent progress. *Communications Magazine, IEEE*, 47(2):102–110, February 2009.

[Meu71] Edward C. Van Der Meulen. Three-terminal communication channels. *Advances in Applied Probability*, 3(1):120–154, 1971.

[MGLB00] Sergio Marti, T. J. Giuli, Kevin Lai, and Mary Baker. Mitigating routing misbehavior in mobile ad hoc networks. In *in Proc. ACM/IEEE Annual International Conference on Mobile Computing and Networking (Mobicom)*, pages 255–265, 2000.

[mK08] Najmeh Forouzandeh mehr and Hossein Khoshbin. Performance analysis of relay selection methods for ieee802.16j. *Electrical and Computer*

Engineering, 2008. ICECE 2008. International Conference on, pages 484–487, Dec. 2008.

[MM02] Pietro Michiardi and Refik Molva. Core: A collaborative reputation mechanism to enforce node cooperation in mobile ad hoc networks. In *Communication and Multimedia Security*, pages 107–121, 2002.

[MOT05] P. Mitran, H. Ochiai, and V. Tarokh. Space-time diversity enhancements using collaborative communications. *Information Theory, IEEE Transactions on*, 51(6):2041–2057, June 2005.

[MYed] I. Maric and R.D. Yates. Maximum lifetime of cooperative broadcast in wireless networks. *Selected Areas in Communications, IEEE Journal on*, submitted.

[NBK04] R.U. Nabar, H. Bolcskei, and F.W. Kneubuhler. Fading relay channels: performance limits and space-time signal design. *Selected Areas in Communications, IEEE Journal on*, 22(6):1099–1109, Aug. 2004.

[NBM+04] Rohit U. Nabar, Helmut Bölcskei, Senior Member, Felix W. Kneubhler, and Student Member. Fading relay channels: Performance limits and space-time signal design. *IEEE J. Select. Areas Commun*, 22:1099–1109, 2004.

[OB07] Ipek Ozil and D.R. Brown III. Time-slotted round-trip carrier synchronization. In *Proceedings of the 41st Asilomar Conference on Signals, Systems, and Computers*, pages 1781 – 1785, Pacific Grove, CA, November 4-7 2007.

[Pro01] J.G. Proakis. *Digital Communications*. McGraw-Hill, New York, NY, fourth ed. edition, 2001.

[SE02] A. Stefanov and E. Erkip. Cooperative coding for wireless networks. In *Proceedings of the IEEE 4th International Workshop on Mobile and Wireless Communication Networks*, pages 273–277, September 9-11 2002.

[SE03a] A. Stefanov and E. Erkip. Cooperative space–time coding for wireless networks. In *Proceedings of the IEEE Information Theory Workshop (ITW)*, pages 50–53, Paris, France, March 31 – April 4 2003.

[SE03b] A. Stefanov and E. Erkip. On the performance analysis of cooperative space–time coded systems. In *Proceedings of the IEEE Wireless Communications and Networking Conference (WCNC)*, volume 2, pages 729–734, 2003.

[SEA98] A. Sendonaris, E. Erkip, and B. Aazhang. Increasing uplink capacity via user cooperation diversity. In *Proc. 1998 IEEE International Symp. on Information Theory*, page 156, August 16-21 1998.

[SEA03a] A. Sendonaris, E. Erkip, and B. Aazhang. User cooperation diversity — part i: System description. *IEEE tansactions on communications*, 51(11):1927–1938, Nov. 2003.

[SEA03b] A. Sendonaris, E. Erkip, and B. Aazhang. User cooperation diversity — part ii: Implementation aspects and performance analysis. *IEEE tansactions on communications*, 51(11):1939–1948, Nov. 2003.

[Sha48] Claude Shannon. A mathematical theory of communication. *Bell System Technical Journal*, 27:379–423 and 623–656, July and October 1948.

[SPCR05] V. Srinivasan, P.Nuggehalli, C.Chiasserini, and R.R.Rao. An analytical approach to the study of cooperation in wireless as hoc networks. In *IEEE Trans. Mobile Computing, Vol. 5, May. 2006*, pages 1044–1049, Mar 2005.

[Stu96] Gordon L. Stuber. *Principles of Mobile Communication*. Kluwer Academic Publishers, Boston, 1996.

[TG03] S. Toumpis and A.J. Goldsmith. Capacity regions for wireless ad hoc networks. *Wireless Communications, IEEE Transactions on*, 2(4):736–748, July 2003.

[TVZ04] D.N.C. Tse, P. Viswanath, and Lizhong Zheng. Diversity-multiplexing tradeoff in multiple-access channels. *Information Theory, IEEE Transactions on*, 50(9):1859–1874, Sept. 2004.

[UBG03] A. Urpi, M.A. Bonuccelli, and S. Giordano. Modelling cooperation in ad-hoc networks: a formal description of selfishness. In *WiOpt'03 Workshop: Modeling and Optimization in Mobile, Ad Hoc and Wireless Networks*, pages 303–312, Mar 2003.

[WZ76] A. Wyner and J. Ziv. The rate-distortion function for source coding with side information at the decoder. *Information Theory, IEEE Transactions on*, 22(1):1–10, Jan 1976.

[WZH05] B. Wang, J. Zhang, and A. Host-Madsen. On the capacity of mimo relay channels. *Information Theory, IEEE Transactions on*, 51(1):29–43, Jan. 2005.

[Yat95] R. Yates. A framework for uplink power control in cellular radio systems. *Selected Areas in Communications, IEEE Journal on*, 13(7):1341–1347, September 1995.

[YB06] J. Yang and D.R. Brown. The effect of channel state information on optimum energy allocation and energy efficiency of cooperative wireless transmission systems. *Information Sciences and Systems, 2006 40th Annual Conference on*, pages 1044–1049, March 2006.

[YB07a] J. Yang and D.R. Brown III. The effect of receiver diversity combining on optimum energy allocation and energy efficiency of cooperative wireless transmission systems. In *ICASSP 2007*, Honolulu, HI, April 2007.

[YB07b] Jie Yang and D.R. Brown. Energy efficient relaying games in cooperative wireless transmission systems. *Signals, Systems and Computers, 2007. ACSSC 2007. Conference Record of the Forty-First Asilomar Conference on*, pages 835–839, Nov. 2007.

[YGBE08] Jie Yang, Deniz Gündüz, D.R. Brown III, and Elza Erkip. Resource allocation for cooperative relaying. In *Proceedings of the Conference of Information Sciences and Systems (CISS 2008)*, Princeton, NJ, Mar 2008.

[ZAL06] Yi Zhao, Raviraj Adve, and Teng Joon Lim. Improving amplify-and-forward relay networks: Optimal power allocation versus selection. *In-*

formation Theory, 2006 IEEE International Symposium on, pages 1234–1238, July 2006.

[ZAL07] Y. Zhao, R. Adve, and T.J. Lim. Improving amplify-and-forward relay networks: optimal power allocation versus selection. *Wireless Communications, IEEE Transactions on*, 6(8):3114–3123, August 2007.

[ZCY03] S. Zhong, J. Chen, and Y.R. Yang. Sprite: a simple, cheat-proof, credit-based system for mobile ad-hoc networks. *INFOCOM 2003. Twenty-Second Annual Joint Conference of the IEEE Computer and Communications Societies. IEEE*, 3:1987–1997 vol.3, March-3 April 2003.

[Zha06] Yan Zhang. *Wireless Mesh Networking: Architectures, Protocols and Standards.* Auerbach Publications, Boca Raton, FL, 2006.

[ZT03] L. Zheng and D.N.C. Tse. Diversity and multiplexing: A fundamental tradeoff in multiple antenna channels. *IEEE Trans. on Information Theory*, 49(5):1073–1096, May 2003.

Appendix A

Jakes Flat Fading Simulator Matlab Code

```
% Rician fading channel simulator using improved Jake's method
% based on "Limitations of Sum-of-Sinusoids Fading Channel Simulators" by
% Pop and Beaulieu, TCOM April 2001

% ---------------------------------------------------------------------
% User Parameters
% ---------------------------------------------------------------------
c = 3e8;                    % speed of light (meters/sec)
fc = 900e6;                 % carrier frequency (Hz)
v = 30;                     % mobile velocity (meters/sec)
M = 32;                     % number of low frequency oscillators
t = 0:1e-6:0.5;             % time vector
K = 0;                      % noncentrality parameter for Rician

% ---------------------------------------------------------------------
% Preliminary calculations
% ---------------------------------------------------------------------

N = 2*(2*M+1);              % number of sinusoids in Jake's model
lambdac = c/fc;             % carrier wavelength (meters)
fm = v/lambdac;             % maximum doppler frequency

n = 1:M;
f = fm*cos(2*pi*n/N);

beta = pi*n/M;
alpha = 0;
```

```
%------------------------------------------------------------------
% Jake's sum-of-sinusoids from (2.169) in Stuber second edition
% Modified according to Pop and Beaulieu TCOM April 2001
%------------------------------------------------------------------

gi = sqrt(2)*cos(alpha)*cos(2*pi*fm*t+2*pi*rand);
gq = sqrt(2)*sin(alpha)*cos(2*pi*fm*t+2*pi*rand);
for n=1:M,
    gi = gi + 2*cos(beta(n))*cos(2*pi*f(n)*t+2*pi*rand);
    gq = gq + 2*sin(beta(n))*cos(2*pi*f(n)*t+2*pi*rand);
end
gi = (2/sqrt(N)*gi+sqrt(K))/sqrt(1+K);
gq = (2/sqrt(N)*gq+sqrt(K))/sqrt(1+K);

g = gi+j*gq;

envelope = abs(g);
ph = angle(g);

%------------------------------------------------------------------
% Plots
%------------------------------------------------------------------

plot(t,10*log10(envelope.^2));
grid on;
xlabel('time (sec)');   ylabel('channel gain (dB)');
```

Appendix B

Optimum Energy Allocation of Orthogonal amplify-and-forward Matlab Code

```
% ———————————————————————————————————————
% Generate energy allocation and enrgy efficiency plot
% for NoCSITNoCSIR case
%
% ———————————————————————————————————————
% User parameters below
% (relay has a statistically advantaged channel)
% ———————————————————————————————————————
clc
clear all
% exponential rv parameters
mu_s = 10;                    % source -> destination channel
mu_r = 100;                   % relay -> desintation channel
mu_H = 100;                   % source -> relay channel
rho_dB = 10;                  % outage level (dB)
p_test = 5*logspace(-4,-1,15);  % outage probability

N = 2E6;                      % number of channel realizations (for CSIT)

% ———————————————————————————————————————

format compact
rho = 10^(rho_dB/10);         % compute outage level

% generate channel realizations (N of each)
G_s = (randn(N,1).^2+randn(N,1).^2)/2*mu_s;
G_r = (randn(N,1).^2+randn(N,1).^2)/2*mu_r;
H = (randn(N,1).^2+randn(N,1).^2)/2*mu_H;

results = zeros(11,length(p_test));
```

```
checks = zeros(9,length(p_test));

% ///////////////////////////////////////////////////////////////
% Zero outage probability part first
% ///////////////////////////////////////////////////////////////

% energy needed for direct transmission (no cooperation)
% when p=0 and source has CSIT
e_nocoop_csit = rho./G_s;
% sort energies for later use
[junk,i_nc] = sort(e_nocoop_csit(:),1,'descend');
% output minimum Es and totoal energy
Es_min=zeros(N,1);
Et=zeros(N,1);

%find minimum total energy for EGC with CSIT
for i=1:N
    Gs=G_s(i);
    Gr=G_r(i);
    h=H(i);
    p=rho;
    b=[-2*p/(2*h+Gs);2*p/Gs];
    [Es_min(i),Et(i)]=fminbnd(@(x) f_et(x,Gs,Gr,h,p),2*p/(2*h+Gs),2*p/Gs);
    i=i+1;
end
% source and relay energies needed for cooperative transmisssion
% when p=0 and source/relay both have CSIT
e_s_coop_egc_csit = Es_min;
e_r_coop_egc_csit = (G_s.*H.*e_s_coop_egc_csit.^2+2*rho*H.*e_s_coop_egc_csit+...
    G_s.*e_s_coop_egc_csit*rho-2*rho^2-2*e_s_coop_egc_csit.*(-G_s.*H*rho.*...
    (-2*H.*e_s_coop_egc_csit-G_s.*e_s_coop_egc_csit+2*rho)).^(1/2)).*...
    (H.*e_s_coop_egc_csit+1)./(-H.*e_s_coop_egc_csit+rho).^2./G_r;
e_coop_egc_csit = e_s_coop_egc_csit+e_r_coop_egc_csit;

% sort energies for later use

[junk,i_egc_c] = sort(e_coop_egc_csit(:),1,'descend');

% ///////////////////////////////////////////////////////////////
% Now do positive outage probability part
% ///////////////////////////////////////////////////////////////

tic
i1 = 0;
for p = p_test,

    i1 = i1+1;

    % ///////////////////////////////////////////////////////////////
    % non-cooperative with CSIT p>0
```

```
    % ////////////////////////////////////////////////////////////////

    e_nocoop_csit_out = e_nocoop_csit;
    % zero out the N*p highest energies
    e_nocoop_csit_out(i_nc(1:round(N*p))) = 0;

    % ////////////////////////////////////////////////////////////////
    % non-cooperative without CSIT p>0
    % ////////////////////////////////////////////////////////////////

    e_nocoop_nocsit_out = -rho/mu_s/log(1-p);

    % ////////////////////////////////////////////////////////////////
    % cooperative with CSIT p>0
    % ////////////////////////////////////////////////////////////////

    e_s_coop_egc_csit_out = e_s_coop_egc_csit;
    e_r_coop_egc_csit_out = e_r_coop_egc_csit;
    e_s_coop_egc_csit_out(i_egc_c(1:round(N*p))) = 0;
    e_r_coop_egc_csit_out(i_egc_c(1:round(N*p))) = 0;
    e_coop_egc_csit_out = e_s_coop_egc_csit_out+e_r_coop_egc_csit_out;

    % ////////////////////////////////////////////////////////////////
    % cooperative without CSIT p>0
    % ////////////////////////////////////////////////////////////////

    % compute optimal energy allocation via lower bound

    E_coop_nocsit_out_lb = min_e_lb(e_nocoop_nocsit_out,mu_s,...
        mu_r,p,rho,1000,0.99);


    % compute optimal energy allocation via upper bound
    E_coop_nocsit_out_ub = min_e_ub(e_nocoop_nocsit_out,mu_s,...
        mu_r,mu_H,p,rho,1000,0.99);

    % ////////////////////////////////////////////////////////////////
    % store results
    % ////////////////////////////////////////////////////////////////

    results(1,i1) = mean(e_nocoop_csit_out);
    results(2,i1) = e_nocoop_nocsit_out;
    results(3,i1) = mean(e_s_coop_egc_csit_out);
    results(4,i1) = mean(e_r_coop_egc_csit_out);
    results(5,i1) = mean(e_coop_egc_csit_out);
    results(6:7,i1) = E_coop_nocsit_out_lb;
    results(8,i1) = sum(E_coop_nocsit_out_lb);
    results(9:10,i1) = E_coop_nocsit_out_ub;
    results(11,i1) = sum(E_coop_nocsit_out_ub);

```

```

    indx=find(results<10^(-5));
    results(indx)=0;

    [p toc]

end % for p = p_test

save 'nocsitnocsir_ad'

% energy allocations plot
figure(1)
set(0,'DefaultAxesColorOrder',[1 0 0; 1 0 0; 0 0.5 0 ;0 0.5 0 ; 0 0 1 ; 0 0 1])
semilogx(p_test,10*log10(results(9,:)),'^-',...
    p_test,10*log10(results(10,:)),'^--',...
    p_test,10*log10(results(6,:)),'v-',...
    p_test,10*log10(results(7,:)),'v--',...
    p_test,10*log10(results(3,:)),'s-',...
    p_test,10*log10(results(4,:)),'s--',...
    'LineWidth',2,'MarkerSize',9);
legend('coop-nocsit source (upper bound)',...
    'coop-nocsit relay (upper bound)',...
    'coop-nocsit source (lower bound)',...
    'coop-nocsit relay (lower bound)',...
    'coop-fullcsit source (exact)',...
    'coop-fullcsit relay (exact)')
axis([5e-4 0.5 -25 40]);
grid on
xlabel('outage probability (p)');
ylabel('10log_{10}(average transmit energy)')

r_adv = results([2 11 8 1 5],:);

% total energies plot
figure(2)
set(0,'DefaultAxesColorOrder',[0 0 0; 1 0 0; 0 0.5 0 ;0 0 0 ; 0 0 1])
semilogx(p_test,10*log10(results(2,:)),'d:',...
    p_test,10*log10(results(11,:)),'^-',...
p_test,10*log10(results(8,:)),'v-',...
p_test,10*log10(results(1,:)),'o:',...
p_test,10*log10(results(5,:)),'s-',...
'LineWidth',2,'MarkerSize',9);
legend('direct-nocsit (exact)',...
    'coop-nocsit (upper bound)',...
    'coop-nocsit (lower bound)',...
    'direct-fullcsit (exact)',...
    'coop-fullcsit (exact)');
axis([5e-4 0.5 -15 35]);
grid on
xlabel('outage probability (p)');
```

```
ylabel('10log_{10}(average total transmit energy)')

figure(1); print -depsc ea_ra_nocsitnocsir.eps
figure(2); print -depsc e_ra_nocsitnocsir.eps
```

```
% min_e_ub.m
% no csit
% outage probability p>0
% find the optimum energy allocation

function Y = min_e_ub(e,mus,mur,muh,p,rho,Npoints,rate)

e_s = linspace(0,e,Npoints);
e_s = e_s(2:end-1);                 % don't use zeros at either end
e_r = e-e_s;

outage_prob = op_ub(e_s,e_r,mus,mur,muh,rho);
[min_op,index] = min(outage_prob);
while min_op<p,
    e = e*rate;                     % decrease energy
    e_s = linspace(0,e,Npoints);
    e_s = e_s(2:end-1);             % don't use zeros at either end
    e_r = e-e_s;
    outage_prob = op_ub(e_s,e_r,mus,mur,muh,rho);
    [min_op,index] = min(outage_prob);
end
e = e/rate;                         % bump energy back up
e_s = linspace(0,e,Npoints);
e_s = e_s(2:end-1);                 % don't use zeros at either end
e_r = e-e_s;
outage_prob = op_ub(e_s,e_r,mus,mur,muh,rho);
[min_op,index] = min(outage_prob);

Y=[e_s(index);e_r(index)];

% handle special cases
if (index==1),                      % very low source energy
    e_s = linspace(0,e_s(1),Npoints);
    e_s = e_s(2:end);               % chop off e_s=0
    e_r = e-e_s;
    outage_prob = op_ub(e_s,e_r,mus,mur,muh,rho);
    [min_op,fineindex] = min(outage_prob);
    if (fineindex==1)&(op_ub(0,e,mus,mur,muh,rho)<min_op),
        Y = [0;e];
    else
        Y = [e_s(fineindex);e_r(fineindex)];
    end
elseif (index==(Npoints-2)),        % very low relay energy
    e_s = linspace(e_s(Npoints-2),e,Npoints);
    e_s = e_s(1:end-1);             % chop off e_r=0
    e_r = e-e_s;
    outage_prob = op_ub(e_s,e_r,mus,mur,muh,rho);
    [min_op,fineindex] = min(outage_prob);
    if (fineindex==(Npoints-1))&&(op_ub(e,0,mus,mur,muh,rho)<min_op),
        Y = [e;0];
```

```
    else
        Y = [e_s(fineindex);e_r(fineindex)];
    end
end
```

```
% min_e_lb.m
% no csit
% outage probability p>0
% find the optimum energy allocation

function Y = min_e_lb(e,mu_s,mu_r,p,rho,Npoints,rate)

e_s = linspace(0,e,Npoints);
e_s = e_s(2:end-1);                 % don't use zeros at either end
e_r = e-e_s;

outage_prob = op_lb(e_s,e_r,mu_s,mu_r,rho);
[min_op,index] = min(outage_prob);
while min_op<p,
    e = e*rate;                     % decrease energy
    e_s = linspace(0,e,Npoints);
    e_s = e_s(2:end-1);             % don't use zeros at either end
    e_r = e-e_s;
    outage_prob = op_lb(e_s,e_r,mu_s,mu_r,rho);
    [min_op,index] = min(outage_prob);
end
e = e/rate;                         % bump energy back up
e_s = linspace(0,e,Npoints);
e_s = e_s(2:end-1);                 % don't use zeros at either end
e_r = e-e_s;
outage_prob = op_lb(e_s,e_r,mu_s,mu_r,rho);
[min_op,index] = min(outage_prob);

Y=[e_s(index);e_r(index)];

% handle special cases
if (index==1),                      % very low source energy
    e_s = linspace(0,e_s(1),Npoints);
    e_s = e_s(2:end);               % chop off e_s=0
    e_r = e-e_s;
    outage_prob = op_lb(e_s,e_r,mu_s,mu_r,rho);
    [min_op,fineindex] = min(outage_prob);
    if (fineindex==1)&(op_lb(0,e,mu_s,mu_r,rho)<min_op),
        Y = [0;e];
    else
        Y = [e_s(fineindex);e_r(fineindex)];
    end
elseif (index==(Npoints-2)),                % very low relay energy
    e_s = linspace(e_s(Npoints-2),e,Npoints);
    e_s = e_s(1:end-1);             % chop off e_r=0
```

```
        e_r = e-e_s;
        outage_prob = op_lb(e_s,e_r,mu_s,mu_r,rho);
        [min_op,fineindex] = min(outage_prob);
        if (fineindex==(Npoints-1))&&(op_lb(e,0,mu_s,mu_r,rho)<min_op),
            Y = [e;0];
        else
            Y = [e_s(fineindex);e_r(fineindex)];
        end
end
```

```
% function to set up nonlinear constraints based on upper bound analysis
% NoCSITNoCSIR
% Notation
% e_s = source energy
% e_r = relay energy
% mu_s = mean value of normalized source-destination channel
% mu_r = mean value of normalized relay-destination channel
% p = outage probability = P(SNR<rho)
% rho = outage threshold

function outage_prob = op_ub(es,er,mus,mur,muh,z)

if length(es)>1,
    ep=z./es;
    a=er./(muh*es+1);
    outage_prob = 2*2^(1/2)*(ep*mus/mur/muh./a./(a.*ep*mur+mus)).^(1/2).*...
        exp(-2/mus*ep-1/muh*ep).*besselk(1,2*2^(1/2)*((a.*ep*mur+mus).*...
        ep/mur/mus/muh./a).^(1/2))-2*2^(1/2)*(1/mur/muh./a.*ep).^(1/2).*...
        exp(-1/muh*ep).*besselk(1,2*2^(1/2)*(1/mur/muh./a.*ep).^(1/2))-...
        exp(-2/mus*ep)./(1+1/mus*a.*ep*mur)+1;
else
    ep=z/es;
    a=er/(muh*es+1);
    outage_prob = 2*2^(1/2)*(ep*mus/mur/muh/a/(a*ep*mur+mus))^(1/2)*...
        exp(-2/mus*ep-1/muh*ep)*besselk(1,2*2^(1/2)*((a*ep*mur+mus)*...
        ep/mur/mus/muh/a)^(1/2))-2*2^(1/2)*(1/mur/muh/a*ep)^(1/2)*...
        exp(-1/muh*ep)*besselk(1,2*2^(1/2)*(1/mur/muh/a*ep)^(1/2))-...
        exp(-2/mus*ep)/(1+1/mus*a*ep*mur)+1;

end
```

```
% function to set up nonlinear constraints based on lower bound analysis
% NoCSITNoCSIR
% Notation
% e_s = source energy
% e_r = relay energy
% mu_s = mean value of normalized source-destination channel
% mu_r = mean value of normalized relay-destination channel
% p = outage probability = P(SNR<rho)
% rho = outage threshold

function outage_prob = op_lb(e_s,e_r,mu_s,mu_r,rho)

outage_prob = (mu_s*e_s.*(1-exp(-2*rho/mu_s./e_s))-mu_r*e_r.*...
    (1-exp(-2*rho/mu_r./e_r)))./(mu_s*e_s-mu_r*e_r);
```

Appendix C

Optimum Energy Allocation of Compress and Forward Matlab Code

```
% Matlab script to generate capacity versus total energy results for
% various cooperative protocols.
%

% This version of the code DOES NOT generate lots of random channel states.
% Rather, we generate deterministic channel states according to the
% exponential distributions. The channel states are picked according to
% an exponential variable substitition to avoid having to approximate an
% indefinite integral and also to improve the accuracy of the results
%

%
% ----------------------------------------------------------------------
% User parameters
% ----------------------------------------------------------------------
Cmin = 2.9;                  % minimum capacity of interest
Cmax = 3.5;                  % maximum capacity of interest
Nrough = 20;                 % number of energy points to test (rough)
Nfine = 50;                  % number of energy points to test (fine)
N = 25;                      % number of channel states to generate
                             %in each channel
Econstraint = 10;            % average total energy constraint
DLCresolution = 0.01;        % resolution of delay limited capacity results
dtest = 0.1:0.1:0.9;         % S->R distances to test
gamma = 4;                   % path loss exponent
NE = 50;                     % resolution of the energy allocation
NA = 20;                     % resolution of the time allocation
% ----------------------------------------------------------------------

tic
```

```
format compact

DLCtest = Cmin:DLCresolution:Cmax;  % delay limited capacities to compute

% Channel states after exponential substition
A = linspace(1/(2*N),1-1/(2*N),N);  % A = exp(-lama*a) (a: source->dest)
B = linspace(1/(2*N),1-1/(2*N),N);  % B = exp(-lamb*b) (b: source->relay)
C = linspace(1/(2*N),1-1/(2*N),N);  % C = exp(-lamc*c) (c: relay->dest)
dA = 1/N;
dB = 1/N;
dC = 1/N;
W = dA*dB*dC;

% allocate space for capacity and energy results
Copt = zeros(length(dtest),N^3,Nfine);
Eopt = zeros(length(dtest),N^3,Nfine);
maxCerr = zeros(length(dtest),length(DLCtest));
meanE = zeros(length(dtest),length(DLCtest));
DLC = zeros(1,length(dtest));
Eerr = zeros(1,length(dtest));

% Channel a = Gs (source to destination) is not a function of d
mua = 1;
lama = 1/mua;
a = -log(A)/lama;

dindex = 0;
for d = dtest,
    dindex = dindex+1;

    % Channel b = H (source to relay)
    mub = (1/d)^gamma;
    lamb = 1/mub;
    b = -log(B)/lamb;

    % Channel c = Gr (relay to destination)
    muc = (1/(1-d))^gamma;
    lamc = 1/muc;
    c = -log(C)/lamc;

    for ia = 1:N,

        % direct transmission energy required to reach Cmax
        Emax = (2^Cmax-1)/a(ia);
        Erough = logspace(log10(Emax)-3,log10(Emax),Nrough);

        for ib = 1:N,
            for ic = 1:N,

                stateindex = (ia-1)*N^2+(ib-1)*N+ic;
```

```

                    % first find the energy range the covers Cmin to Cmax
                    %Crough = Coaf(a(ia),b(ib),c(ic),Erough);
                    Crough = Ccf(a(ia),b(ib),c(ic),Erough,NE,NA);
                    if Crough(1)>1.00001*Cmin,
                        disp('oh no! - rough energy range is not low enough')
                        pause
                    end
                    if Crough(end)<0.99999*Cmax,
                        disp('oh no! - rough energy range is not high enough')
                        pause
                    end
                    [junk,index1] = min(abs(Crough-Cmin));
                    [junk,index2] = min(abs(Crough-Cmax));
                    if (Crough(index1)>Cmin)&&(index1>1),
                        index1=index1-1;
                    end
                    if (Crough(index2)<Cmax)&&(index2<Nrough),
                        index2=index2+1;
                    end

                    % now do fine optimized resource allocation
                    Efine = logspace(log10(Erough(index1)),log10(Erough(index2)),Nfine);
                    Cfine = Ccf(a(ia),b(ib),c(ic),Efine,NE,NA);

                    Copt(dindex,stateindex,:) = Cfine;
                    Eopt(dindex,stateindex,:) = Efine;

                end
            end
        end

        % Now that we have the results for each channel state and over various
        % total energies in the capacity range of interest, we can compute the
        % delay limited capacity...

        % extract relevant part of capacity results
        CC = squeeze(Copt(dindex,:,:))';
        % extract relevant part of total energy results
        EE = squeeze(Eopt(dindex,:,:))';

        DLCindex = 0;

         % compute average energy for each DLC of interest
        for CAP = DLCtest,
            DLCindex = DLCindex+1;

            % find closest capacity over energy index (not channel state)
            [Cerr,index] = min(abs(CC-CAP));
            % record maximum capacity error for later checking
```

```
            maxCerr(dindex,DLCindex) = max(Cerr);
            % generate linear index for binning energies
            linearindex = sub2ind(size(EE),index,1:N^3);
            % compute mean energy acheiving C
            meanE(dindex,DLCindex) = sum(EE(linearindex))*W;

        end

        % find mean energy closest to contraint
        [Eerr(dindex),index] = min(abs(meanE(dindex,:)-Econstraint));
        % the index of this energy yields the delay limited capacity
        DLC(dindex) = DLCtest(index);
        [d toc]
        save DLCcf_optalpha.mat

end
```

```
% function Cmax = Ccf(a,b,c,E,NE,NA)
%
% Inputs:
%   a, b, c = scalar channel states
%   NA = number of points to test for time allocation on (0,1)
%         note: set NA = 1 if you want to only compute alpha=0.5;
%   NE = number of points to test for energy allocation
%   E = vector of total energies
%
% Outputs:
%   Cmax = vector of achieved capacities for each of the total energies
%
% Notes:
%       The program selects the better capacity between compress and forward
%       and direct transmission (DT). Technically, CF includes DT as a
%       possibility, but we do not check the alpha = 1 case directly, hence
%       we must compare with DT.

function Cmax = Ccf(a,b,c,E,NE,NA)

    E = E(:)';                        % make sure E is a row vector
    Cmax = zeros(1,length(E));  % allocate space to store maximum capacities

    % capacity of direct transmission
    Cdt = log2(1+E*a);

    for Efrac1 = 1/NE:1/NE:1,              % percentage of energy in E1
        for Efrac2 = 0:1/NE:(1-Efrac1),      % percentage of energy in E2

            E1 = E*Efrac1;
            E2 = E*Efrac2;
            E3 = abs(E-E1-E2);
            % Note the use of abs() here for E3. This is because Matlab
            % sometimes generates very small negative values for E3, due to
            % precision limitations.

            for alpha = 1/(2*NA):1/NA:(1-1/(2*NA)),

                P1 = E1/alpha;
                P2 = E2/(1-alpha);
                P3 = E3/(1-alpha);

                % CF
                T1 = 1+(a+b)*P1;
                T2 = 1+(c*P3)./(1+a*P2);
                T3 = T2.^((1-alpha)/alpha)-1;
                T4 = 1+a*P1;
                sig2w = T1./(T3.*T4+eps); % +eps here to avoid divide by zero
                C_cf = alpha*log2(1+a*P1+b*P1./(1+sig2w))+(1-alpha)*log2(1+a*P2);

```

```
                C = max([C_cf ; Cdt]);

                % update maximum capacities
                i = find(C>Cmax);
                Cmax(i) = C(i);

            end
        end
    end
```

Appendix D

Fixed Channel Matlab Code

```
% Discounted total saved energy as a function of the distance
% between the two sources for Fixed channel

clc
clear all


N=.75;     % range of the distance between the two sources
z=1;
x_t=linspace(0,N,100);
x_t=x_t(1:end-1);
G1 = 1/.75^4;    % souce 1 to destination 1 channel
G12 = 1/.25^4;   % souce 1 to destination 2 channel
delta=0.99;      % discounted factor
discd=delta/(1-delta);
Etot_in=zeros(1,length(x_t));     % grim trigger results
Etot_full=zeros(1,length(x_t));   % centrally optimized results

i1=0;

for x=x_t

    i1=i1+1;

    if (x==0)
        G2=1/(.25+x)^4;
        E1=z/G1;
        E2=z/G2;

        Etot_full(i1)=E1+E2;
        Etot_in(i1)=E1+E2;


    else

```

```
        G2=1/(.25+x)^4;
        H = 1/x^4;
        G22=1/(.75-x)^4;

        % optimum energy allocation results obtained from Chapter 3

        E11opt=(G22*z*H-G1*H*z+G22*G1*z+((G1+H+z*H)*z*H*(G1*G22-...
            G1*H+G22*H))^(1/2))/(G1+H)/(G1*G22-G1*H+G22*H);
        E22opt=(-2*z*H^2*G1^2+G1^2*H*G22*z-G1^2*(-(G1+H+z*H)*z*...
            H*(-G22*G1+G1*H-G22*H))^(1/2)-2*G1*H^3*z^2-2*G1*z*...
            H^3+2*G1*G22*z*H^2+G22*z^2*H^2*G1-G1*H*z*(-(G1+H+z*H)*...
            z*H*(-G22*G1+G1*H-G22*H))^(1/2)-G1*(-(G1+H+z*H)*z*H*...
            (-G22*G1+G1*H-G22*H))^(1/2)*H+G22*z*H^3+H^3*G22*z^2+...
            H^2*z*(-(G1+H+z*H)*z*H*(-G22*G1+G1*H-G22*H))^(1/2))/...
            (G1+H)^2/(-(G1+H+z*H)*z*H*(-G22*G1+G1*H-G22*H))^(1/2)/G22;
        E21opt= (G12*G2*z+G12*z*H-G2*H*z+(z*H*(G12*G2+G12*H-G2*H)...
            *(H+z*H+G2))^(1/2))/(H+G2)/(G12*G2+G12*H-G2*H);
        E12opt=(-2*G2^2*z*H^2+G2^2*H*G12*z-G2^2*(-z*H*(-G12*G2-...
            G12*H+G2*H)*(H+z*H+G2))^(1/2)-2*G2*H^3*z^2-2*G2*z*...
            H^3+2*G2*G12*z*H^2+G12*z^2*H^2*G2-G2*(-z*H*...
            (-G12*G2-G12*H+G2*H)*(H+z*H+G2))^(1/2)*H-G2*H*z*...
            (-z*H*(-G12*G2-G12*H+G2*H)*(H+z*H+G2))^(1/2)+...
            H^3*G12*z^2+G12*z*H^3+H^2*z*(-z*H*(-G12*G2-G12*H+...
            G2*H)*(H+z*H+G2))^(1/2))/(H+G2)^2/(-z*H*(-G12*G2-...
            G12*H+G2*H)*(H+z*H+G2))^(1/2)/G12;
        E1=z/G1;
        E2=z/G2;

        % test the conditions in Propostion 4
        chk1=((E11opt+E22opt)<E1)*(sign(E11opt)~=-1)*(sign(E22opt)~=-1);
        chk2=((E21opt+E12opt)<E2)*(sign(E21opt)~=-1)*(sign(E12opt)~=-1);

        E11opt=chk1*E11opt+(1-chk1)*E1;
        E22opt=chk1*E22opt;
        E21opt=chk2*E21opt+(1-chk2)*E2;
        E12opt=chk2*E12opt;


        Etot_full(i1)=discd*(E1+E2-(E11opt+E22opt+E21opt+E12opt));

        %incetive cooperation
        if (E1>(E11opt+E12opt))&&(E2>(E21opt+E22opt))
            Etot_in(i1)=discd*(E1+E2-(E11opt+E22opt+E21opt+E12opt));
        else
            Etot_in(i1)=0;
        end


    end

```

```

end

semilogx(x_t, Etot_in, '-', x_t, Etot_full, '--', 'LineWidth', 3)
grid on
xlabel('Distance from Source 1 to Source 2');
ylabel('Discounted Total Saved Energy')
legend('grim trigger', 'centrally optimized', 'location', 'northwest')
axis([0 .75 0 90])
figure(1); print -depsc nonfading.eps
```