Inventory and Procurement Management in the Presence of Spot Markets

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Abstract

This research develops mathematical models for inventory and procurement management in the presence of spot markets. More specifically, we consider those models by incorporating different types of supply contracts. Particular attention is paid to the quantity flexible contracts. This research is an attempt to understand how firms should adopt their operating policies in the presence of fluctuating commodity prices. In this thesis, we mainly consider the following three models.

In the first model, we study the optimal procurement strategy in a two-period framework when both the spot market and the forward contract are considered. The forward contract is agreed upon in the first period, and is then delivered in the second period, when the spot market is also available. This is followed by production and demand. The objective of the buyer is to minimize his expected cost. We study the problem for two scenarios: the buyer cannot and can sell to the spot market. Through our analysis, when the buyer can not sell to the spot market, there exists a threshold forward price, under which the buyer will enter into the forward contract. This threshold is lower than the expected spot price. Furthermore, we analytically show that the optimal order quantities via forward contract increase in the mean of the spot price, but decrease in the variability of the spot price. However, the buyer only speculates using the forward contract when he can sell to spot market.

In the second model, we consider a problem in which a buyer makes procurement decisions when he faces periodic random demand and two supply sources, one is a long-term contract supplier and the other is a spot market. When he procures from the contract supplier, a fixed unit price is charged and a predetermined minimum quantity for each period must be committed, and when he

procures from the spot market, a stochastic spot price plus a fixed setup cost is charged. The spot price is only realized at the beginning of each period. We show that the optimal policy consists three different (s, S) type policies. More important, we identify certain conditions under which there exist monotone properties between the policy parameters and the current spot price for a general Markov spot price process. Then, we can divide the price space into three regions, each of which corresponds to a specific policy, for each period. We also conduct numerical analysis to gain more insights into how the spot market impacts the buyer's performance. We find the buyers benefits from a more volatile market.

The last model extends the second model by incorporating an important feature that is widely seen; i.e., the procurement from the contract supplier should fulfill a total order quantity commitment (TOQC). The TOQC requires the buyer to procure no less than the predetermined commitment during the contract period, which we call the planning horizon. Thus, in each period, the buyer trades off between the possible lower cost now (by procuring from the spot market) and the reduced cost in the future (by reducing the remaining commitment). Two types of commitment contracts are considered: a minimal TOQC contract and a definite quantity contract. Our analysis characterizes an optimal procurement policy which depends on the spot price in each period and an optimal virtual remaining commitment level. Such a structured policy can be viewed as a combination of some policies of base-stock type, each of which can be computed through an equivalent system without any commitment. Moreover, some of these equivalent systems are of simple multiple-period newsvendor type. This greatly simplifies the computation of the optimal policies. We also numerically analyze how the TOQC and the spot market affects the buyer's performance.

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This work is dedicated to my family

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Chapter 1

Introduction

Procurement has gone through considerable changes since the 1970s. It used to be regarded as a clerical function with the objective of purchasing a good/service at the lowest price. However, the 1973-74 oil crisis and resulted raw material shortages drew significant attention to its importance (Farmer (1978)). Porter (1985), in his value chain framework model that helps to analyze specific activities through which firms can create value and competitive advantages, identified inventory control and procurement activity as two of the critical activities. Then, the strategic importance of the procurement and inventory management to the organization was beginning to receive recognition in the literature. This trend continues today and procurement becomes more and more important in the management of supply chains. Meanwhile, procurement cost always accounts for a large portion of the total operating costs in many industries. It is reported that material costs comprise a median half of U.S. plants' cost of goods sold (Butcher (2006)), and even reach 60-70% for some specific industries (HKTDC (2007)). Thus, an effective procurement strategy can increase firms' profits by reducing procurement cost. Therefore, procurement activity has been recognized as making a significant contribution to an organization's success.

Unfortunately, in today's dynamic business environment, significant uncertainties are present in customer demand, supply availability and supply cost, which make procurement activity very difficult to handle. Big loss has been reported in those companies with poor procurement strategies that can not well manage these uncertainties. For example, Dell's Computer announced the discouraging news of unexpected high memory prices in October 1999 and its stock price felt by 7% in just one single trading day (http://money.cnn.com/1999/10/18/technology/dell/). According to Peabody (2005), in the first half of 2005, St. Louis-based Peabody Energy Corp., one of the world's largest private-sector coal companies, reported US$ 34 million in loss due to contract breach by one of its suppliers. Another case came from Cisco, which reported a US$2.5 billion inventory write-down in April of 2001 due to a significant and unexpected drop in customer demand for its networking products (http://weissluie.com/case/cisco_system_inc). More news alike has been reported recently as the world economy goes to a downturn and the price of raw material is more volatile than ever before. As a result, more and more attention, from both academia and practitioners, has been paid to design a robust procurement and inventory strategy to deal with these uncertainties to enhance companies's overall performances.

A traditional procurement strategy to secure stable supply and insulate themselves from input price fluctuation is to enter into several long-term supply contracts, also called forward buy or fixed commitment contracts. These contracts specify a fixed quantity to be delivered at some point in the future, with fixed purchasing prices agreed by the supplier and manufacturer. However, such procurement mode always restricts the manufacturers' ability to respond quickly to changing conditions of the customer demand and the procurement cost. For example, if the demand happens to fall short of the expected amount, such a fixed supply quantity commitment results in a significant inventory build-up. Holding excess inventory not only incurs substantial holding cost, but also exposes the inventory to the risk of obsolescence-a problem that is particularly acute for those short life-cycle components. Alteratively, if the demand spikes higher than expected, such contracts limit the manufacturers' capability to satisfy customers. This results in the loss of potential profits and customers' goodwill. On the other hand, a drop of the raw materials price in the market makes the procurement with the price already fixed uneconomical. This can be further evidenced by the precious palladium punch of Ford Motor (http:johnhelmer.net?p=241). In 2002, by expecting a surge in the price of palladium due to the supply problem that occurred in Russia, the company decided to enter into forward contracts when the price spiked to over US$1,000 per ounce to secure the purchasing price and quantity. However, because of such high price, many users chose to switch to platinum as a substitute, reversing the price trend of palladium. The situation of Ford Motor was further aggravated as the Russian supply resumed normal and the price of palladium plummeted. A loss of US$1 billion because of the forward contracts was recorded according to the company's annual report of that year. Therefore, for those manufacturers who rely on such a rigid mode of supply, the optimality of its procurement planning decision depends solely on how accurately it could estimate the future demand and price, and on its ability to bargain a favorable price at the beginning of the planning horizon.

Due to the rigidity of the traditional procurement mode by fixed commitment contract, recent trends in industry begin to emphasize the analysis and use of flexible supply contracts in procurement. The purpose of such flexible contracts is to give manufacturers the desired flexibility which is helpful to partially mitigate various uncertainties that might occur in a supply chain. Among these approaches, the quantity flexible supply contracts are probably the most widely known and adopted. In these contracts, a fixed procurement quantity commitment is determined when the contract is signed, but the amount actually procured during the contract horizon may differ from the initial commitment within a given bound determined upon signing the contract. For example, the manufacturer can enter into a minimal total order quantity commitment (TOQC) contract, in which the manufacturer commits to purchase at least a minimum quantity during the entire horizon, to periodically review their procurement plans and to adjust the procurement quantities according to the most up-to-date market situations. If the customer demand happens to increase, such supply contract allows the manufacturer to increase the quantity to procure. On the other hand, if the demand happens to fall short of the initial forecast in some periods, this approach allows the manufacturer to procure less and wait for better market condition. Such minimal TOQC contracts are common practices in the electronic industry. Other forms of quantity flexible contracts can also be observed in practice, i.e., the periodic commitment contracts with flexibility, the supply contracts with buy/sell options, etc. (see Tsay (1999), Anupindi and Bassk (1998)) Approaches that help manufacturers to manage price fluctuation are always in a form of supply contracts with options giving buyer a lead-time flexibility, or incorporating price caps or floors (Li and Kouvelis (1999)). With these various approaches, manufacturers today can manage supply chain uncertainties better than ever before.

Although such a supply mode with flexible supply contracts has been widely used in practice, it changes quite little of the way in which supply is obtained. Manufacturers still rely on contract suppliers to secure supply. One main problem arisen in such supply mode is the information asymmetric between suppliers and manufacturers. This may cause additional cost for manufacturers to find suitable suppliers. Besides, the flexibility provided by suppliers is always limited, and there are some clauses in these contracts that restrict the manufacturers' behavior to the supplier's interest. Thus, the performance of procurement strategy depends largely on manufacturers' bargain power to solicit favorable contract terms.

The rapid growth of organized spot markets and the development of information technology present significant opportunities in the procurement of supplies. The organized exchanges, in which lots of traditional commodities are traded, have lasted for couple of centuries. London Commodity Exchange (LCE), Chicago Board of Trade (CBOT) are the prime markets for trading agricultural commodities such as coco, coffee, wheat, oat, corn and so on. Similarly, New York Mercantile Exchange (NYMEX) specializes in trading of crude oil, natural gas and related commodities. London Metal Exchange (LME) is one of the prominent marketplaces to trade metals such as copper, aluminum, zinc etc. Apart from these exchanges for traditional commodities, many new type of exchanges for industrial commodities have been found traded in recent years. For example, Converge and DRAM-Exchange are online spot markets that trade semiconductors, electronic components, computer products and networking equipment. Other similar markets for trading industrial commodities include E-steel for steel, Inter-continental paper exchange for paper related products, etc. On the other hand, the advance in information technology has reduced the cost of putting spot markets together. Today, suppliers and manufacturers can trade with a vast variety of goods all over the world in a freewheeling, frequently electronic, environment.

Price discovery is one main function of these spot markets. In general, the spot price of a commodity reflects the existing dynamics of supply and demand in the market. Moreover, the price dictated by the market contains more information on demand/supply dynamics than the information held by individual buyers and suppliers. As noted by Mendelson and Tunca (2007), "just as the stock market summarizes many pieces of information about the performance of a company, business-to-business spot markets can provide up-to-date information about the availability of raw materials, the cost of production, and consumer demand for the end product. Because all of this happens much closer to the time that the end product ships to the consumer, supply chain participants can update their plans to take into account real-time information ..."

Therefore, procurement managers can have a greater flexibility that cannot be obtained through the use of long-term supply contracts or flexible supply contracts by accessing to commodity spot markets. However, the proliferation of such commodities markets has also resulted in larger fluctuations of commodities prices, especially when the forces of international speculation are at work. Furthermore, the more you trade, the more you drive the price against yourself. For example, a manufacturer may want to buy 10,000 computer chips at $1 per chip. But trying to buy twice that amount may force the manufacturer to pay more per chip, as cheaper supplies will be exhausted. So buying only from the spot market raises the risk that you're going to spend more than you would have in a fixed-price contract. Thus, no matter how efficient spot markets can be, there will likely be a role for contract suppliers, and an efficient procurement strategy should consider a combination of contract suppliers and spot markets.

In practice, spot markets has already become an important procurement source in addition to supply contracts. For example, it is estimated that 5% of chemicals and steel sales, and up to 30% of memory chips sales are procured from the spot market, while the left volumes are procured through supply contracts. Similarly, crude oil has been procured via a combination of market-based and contract arrangements for decades. Similar trend is observed for a variety of commodities and industrial products ranging from natural gas (Hannon (2005)) to metals (Ann and Maguire (2005)). However, many companies still hesitate to participate in spot markets and they mainly use spot markets in an *ad hoc* manner as opposed to viewing them as an integral part of their procurement strategy. One main reason for such situation is the lack of quantitative tools for decision making. As Kaplan and Sawhney (2000) rightly note: "The matching mechanism works best when buyers and sellers are sophisticated enough to deal with dynamic pricing." Likewise, McKinsey Company and CAPS Research (2000) identified the need "to determine the appropriate contract-to-spot ratio for commodity purchases" in a broad survey on the impact of B2B e-marketplaces.

The purpose of this study is mainly to develop and analyze procurement strategies of a manufacturer in the presence of spot markets by incorporating supply contracts. Our main attention is paid to these supply contracts with quantity flexibility. Hewlett-Packard (HP), the largest technology company in the world, is one example related to our research. It is estimated that the company invests 50% of its procurement cost in long-term contracts, 35% in option contracts, and the rest to the spot market, see Carbone (2001). Some metal companies have also provided certain quantity flexible contracts to their customers. For example, Tally Metal Sales Inc., a Chicago based supplier for non-ferrous metals, provides a consignment stock program which bills the customers for only the quantity actually used with various pricing schemes, one of which is fixed price scheme. As we mentioned earlier, although supply contracts can keep the manufacturers from price fluctuation, the flexibility they provide is limited and the contract price is always higher than the current spot price. On the other hand, although the spot market can provide full flexibility to the manufacturers, the volatility of the spot price becomes the main problem. Thus, how to strategically trade off between these two procurement sources in procurement activities is a central problem these manufacturers have to face.

Our objective in this research is to understand how the internal operational decisions of the firm should be modified in the presence of spot markets. The results of our work provide quantitative tools for manufacturers to efficiently use the spot market as an integral part of their procurement strategy while maintaining a long-term contract supplier, to decide how to take the advantage of the spot market, and to study the effect of the key spot market characteristics on optimal contract quantities and cost. Specifically, we consider the following three models.

1. The first model is to study the optimal mix of the spot market and forward contract on procurement decisions under a two-period framework. A forward contract is a fixed commitment contract by which the buyer can buy a predetermined fixed amount of raw materials at a specified point of time in the future at a specified price. In our model, the forward contract is agreed upon in the first period, and is then delivered in the second period, when the spot market is also available. This is followed by production and demand. We study the procurement strategies for both scenarios under which

the manufacturer can and can not sell to the spot market, respectively.

2. The second model considers the optimal procurement problems in which the buyer faces periodic random demand and two supply sources: namely, the long-term contract supplier and the spot market. When he procures from the contract supplier, a fixed unit price is charged and a predetermined minimum quantity in each period must be committed; and when he procures from the spot market, a spot price plus a fixed setup cost is charged. The spot price is only realized at the beginning of each period. The objective of the buyer is to find an optimal procurement strategy to minimize its total expected cost.

3. The third model further extends the second model by incorporating an important feature that is widely seen in practice; i.e., the procurement from the contract supplier should fulfill a total order quantity commitment (TOQC). A TOQC contract is an agreement between the supplier and the buyer in which the buyer guarantees that his cumulative order for the whole (contract) duration will be at least the TOQC. Two types of commitment contracts are considered: a minimal TOQC contract and a definite quantity contract. Thus, in each period, the buyer should trade off between the possible lower cost now (by procuring from the spot market) and opportunity cost in the future (by reducing the commitment for future periods).

Through the analysis of the first model, we find when the buyer can not sell back to the spot market, there exists a forward price threshold under which the buyer will enter the forward contract. This threshold is lower than the expected spot price. Thus, in a risk-neutral framework in which the forward price is equal to the expected spot price, the cost minimizing buyer would not engage in a forward contract. However, when the buyer can sell back the spot market, he only speculate using the forward contract. Furthermore, we analytically show that the optimal order quantities via forward contract and the total cost increase in the mean of the spot price, but decrease in the variability of the spot price.

Then, we study the second model and show that the optimal policy consists three different policies of (s, S) type. More important, we identify certain conditions under which there exist monotone properties between the policy parameters and the current spot price for a general Markov spot price process. Then, we can divide the price space into three regions, each of which corresponds to a specified policy, for each period. To the best of our knowledge, a few studies investigate such monotone properties when price process follows a general Markov process. Furthermore, we numerically show that the buyer prefers a volatile market condition.

Our analysis on the third model shows that when the current spot price is lower than the commitment price, the buyer procures certain amount from the contract supplier first (while deferring some commitment to future periods), and then orders from the spot market. This differs from the traditional TOQC literature with deterministic environment, in which the buyer's optimal policy is always to fulfill the commitment first. This fact complicates our problem. Nevertheless, we prove the optimality of a structured procurement policy which can be viewed as a combination of some policies of base-stock types, which indicates that the optimal values can be calculated by some newsvendor-like models. This enhances the practicality of our model. Our numerical analysis implies that in the presence of spot market, the buyer benefits from the market volatility. But such benefits decrease as the TOQC increases.

The organization of this thesis is as follows. In Chapter 2, literature that related to procurement problems in the context of supply chain management is reviewed. We mainly review three streams: the traditional stochastic inventory models; the supply contract models and the procurement models in the presence of spot markets. In Chapter 3, the first model is studied and stochastic comparisons are conducted to analyze how the spot market affects the policy parameters. In Chapter 4, the second model is considered and conditions under which monotone properties hold are derived. Moreover, numerical analysis is conducted when the spot price follows a Geometric Brownian Motion process. In Chapter 5, the third model is considered and numerical analysis is performed to get more insights on how the incorporation of spot markets can improve the buyer's performance. Finally, the conclusion of this thesis is made in Chapter 6 by summarizing the major findings and scientific contributions of this study. Possible directions of further research are also discussed.

□ **End of chapter.**

Chapter 2

Literature Review

With the rapid development of spot markets for various commodities, manufacturers can now use spot purchasing as an alternative source of supply to the traditional long-term contracting. The purpose of this study is to investigate the procurement planning problem with flexible supply contracts in the presence of commodity spot markets. In this chapter, we first give an overview of traditional inventory models in supply chain management in Section 2.1. These models focus on either maximizing profit or minimizing cost. Particular attention is paid to multiple periods inventory problems, most of which assume the procurement cost to be known before the planning horizon and the only uncertainty is customer demand. Extensions to the multiple supply sources are also discussed. In Section 2:2, we focus on the multi-period inventory models with supply contracts. The original purpose of these contracts is to coordinate supply chain to achieve global optimal performance. But recently development begins to emphasize those flexible contracts which provides mechanisms to deal with supply chain uncertainties. Here, we mainly review those flexible quantity contracts with flexible commitment. In Section 2.3, we review the procurement planning problems in the presence of spot markets, which is a cash market where commodity prices are settled in cash on the spot at the current market prices. Special attention is paid to current developments combining spot market purchase with purchases made in advance from a specific long-term supplier. The combination of quantity flexible contracts and spot markets is discussed. At last, a brief comparison between our work and the existing literature is discussed.

2.1 An Overview of Inventory Models

Stochastic inventory theory is the application of quantitative methods into practical inventory management problems to efficiently match customer demand with on-hand inventory. The classical single-period newsvendor model is a building block of stochastic inventory theory. It was first proposed as early as 1940's, while the early publications of this model can date back to the early 1950's (Arrow et al. (1951); Dvoretzky et al. (1952)). This model considers how many newspapers a newsboy should order when facing a single period demand before demand is realized. To maximize his expected profit, the newsvendor must match the ordered quantity with the demand as closely as possible. Since newspapers are perishable products, if he ordered too many copies, unsold newspapers can only be sold at a low salvage value or even become worthless. On the other hand, if he ordered too few, the opportunity for gaining a higher profit is lost. Thus, the optimal ordering quantity should trade off the opportunity profits with the possible loss. To date, numerous works on the newsvendor problem have been reported. An excellent reviews of the classical newsvendor problem and its various extensions can be found in Khouja (1999).

A generalization of this newsvendor model is to consider a multi-period inventory control problem where the selling horizon is extended from one period to multiple periods, and a decision on order quantity in each period is made before the demand is realized. The key difference between the single-period model and the multi-period model is that the latter has replenish opportunities and stock leftovers, which makes the optimal choice of order quantities more complicated. It is generally beleivied that, Arrow et al. (1951) are the first publication which studies multi-period models with stochastic demand. In their model, there is a setup cost for each procurement. They assume the policy have the (s, S) form and find the best policy among this limited class (see Arrow (2002)). However, they do not show whether the optimal policy is of the (s, S) form. Two years later, Dvoretzky et al. (1953) show that one can find loss functions and distributions for which policies other than (s, S) are optimal. Scarf (1960) further analyzes this problem when the excess demand is backlogged in each period and shows for the first time that the (s, S) policy is optimal when the one period expected costs are convex. Veinott (1966) proves the optimality of (s, S) policy under another condition that the single period cost is quasi-convex and the its absolute minima of the one period costs is increasing (weakly) over time. In all these works, the ordering cost is simply linear with a setup cost. The case when the ordering cost is concave increasing with respect to the order quantity is first studied in Porteus (1971). In his work, a generalized (s, S) policy is shown to be optimal when the probability densities of demand is one-sided Polya densities (Karlin and Rubin (1956); Karlin (1958)). A generalization of the K-convex and quasi-convex functions to quasi-K-convex functions is introduced. Porteus (1972) establishes a similar result when the demand distribution is uniform. Schal (1976) finds new conditions for the optimality of an (s, S) policy which generalize those of

Scarf (1960) and Veinott (1966). Moreover, as a special case, he obtains a result very similar to that of Porteus (1971) without any assumption on the probability distribution of demand. His analysis is based on a general concept of convexity which includes convex functions in the usual sense, and monotone functions as special cases. The corresponding infinite horizon problem has also been studied (Iglehart (1963); Zheng and Federgruen (1991); Chen (2004)). When the setup cost equals to zero, the (s, S) policy is reduced to the base stock policy, and the mathematical complexity is much simpler in this case. These works stimulate a very large literature on dynamic inventory policies, i.e., Chen and Sethi (1999), Gallego and Hu (2004), Gallego and Toktay (2003), ect.

An extension of these traditional multi-period models is to include multiple supply sources. Here we only introduce the models with deterministic cost structure, and those with stochastic features will be reviewed in later sections. Such models have also been extensively researched in the operations management literature. Zhang (1996), Gallego and Zhang (2003) and Feng et al. (2005, 2006) concern with systems with more than two consecutive supply modes with different lead-times. For two supplier systems, we refer readers to Lawson and Porteus (2000), Moinzadeh and Nahmias (1988), Sethi et al. (2003), Fox et al. (2005), Scheller-Wolf et al. (2005) and papers therein. These papers differ in their consideration of expediting options, lead-time, cost structure (variable and set-up costs) associated with different modes of supply. We recommend readers to refer Minner (2003) for an excellent review on these models. Most research in this line considers different modes of supply with smaller lead-times in exchange for an additional known cost, except that in Fox et al. (2005). In Fox et al. (2005), the lead-times for the two supply modes are the same, but one charges a higher fixed per unit cost and the other one charges a lower per-unit cost with a setup cost. They find the optimal policy also has a form of generalized (s, S) type when the demand density is logconcave. The model considered in Porteus (1971) can also be viewed as a multi-supply problem, which is more general than Fox et al. (2005) but with stricter conditions.

2.2 Supply Contract Models

The academic literature on supply contracts is quite recent. For reviews, see Lariviere (1999) and Cachon (2004). The literature can be classified into two main categories. The first category focuses on designing the appropriate contracts so as to improve supply chain coordination. In a decentralized supply chain environment, only suboptimal performance of the whole supply chain is achieved because each party optimizes its own profit independently. Thus, the supply contracts in this category are to achieve a globally optimized supply chain while each party still optimizes its own profit. Various contracts and strategies have been proposed and extensively studied, i.e., buy-back contract (Pasternack (1985)), vertical integration (Alchian (1995); Handfield and Nichols (2002)), postponement (Bowersox et al. (2002); Iyer et al. (2003)) and revenue sharing contracts (Cachon (2004); Cachon and Lariviere (2005); Giannoccaro and Pontrandolfo (2004)) and option contracts (Barnes-Schuster et al. (2002)).

Another category focuses on replenishment policies and detailed contract parameters for a given type of contract. The study in this category focuses on optimizing the buyer's procurement strategy without considering the impact of the decision on the seller. Most of the contracts studied in this category are flexible supply contracts. The purpose of flexible supply contracts is to share the risk associate with uncertainty in demand, price and lead-time (Cachon and Lariviere (2001)) between the supplier and the buyer. For example, a quantity flexible supply contract can be used to enable the buyer to adjust its order quantities according to the most updated market forecast. The most obvious benefit of a quantity flexible supply contract to the buyer is that it provides an instrument for hedging uncertainties in both demand and procurement cost.

Bassok and Anupindi are probably the pioneers in the research of quantity flexible contracts (Anupindi (1993), Anupindi and Akella (1993); Bassok and Anupindi (1997);Bassok and Anupindi (2008)). Anupindi (1993) is the first to analyze quantity commitment contracts with flexibilities in which the decision maker makes a commitment for each period to purchase certain quantities during the horizon and can adjust the commitment within some pre-specified limits in that period. Bassok and Anupindi (1997) analyze a supply contract with a minimum total ordering commitment, under which the buyer agrees to purchase a total amount no smaller than agreed-upon quantity over a multi-period planning horizon, while orders are placed periodically. Bassok and Anupindi (2008) study a rolling-horizon flexibility (RHF) contracts, in which the buyer has a limited flexibility to purchase quantities different from the original commitments and is allowed to update the previously made commitments, within a given bound. Bassok et al. (1997) study a type of quantity flexible supply contract with periodic commitments and with a flexibility to update the required quantities. Their work is motivated by the practice in the IBM Printer System Company which was concerned with the sourcing of components for production over a multi-period time horizon. Chen and Krass (2001) further extend the model of Bassok and Anupindi (1997) to a more general situation with the non-stationary demand distribution and different cost structure. Anupindi and Bassok (1998) make a review on these quantity flexible models with commitment. For recent development of these models, we refer readers to Moinzadeh and Nahmias (2000), van Delft and Vial (2001), Li and Ryan (2004) and Tibben-Lembke (2004).

Flexible supply contracts with options have also an active research topic recently. Such contracts are designed by incorporating traditional long-term contracts with various options that prevailing in financial literature. Supply contracts with quantity options are studied by Agrawal et al. (1998), Tsay (1995), Tsay (1999), Tsay and Lovejoy (1999), Sethi et al. (2004), Milner and Rosenblatt (2002) and Wu (2005). Contracts with both quantity flexibility and time flexibility are studied in Li and Kouvelis (1999) with the assumptions of deterministic demand and uncertain procurement price. Capacity reservation options are investigated in Tan (2002). In Das and Abdel-Malek (2003), contracts with both quantity flexibility and lead-time flexibility are considered. The value of quantity flexibility and time flexibility is studied in Milner and Kouvelis (2005) under three different demand models.

2.3 Procurement from Spot Markets

Spot markets provide flexibility for market participants by allowing them to fine tune their stocking and production policies under uncertain demand and supply environment. As observed in Haksoz and Seshadri (2007), spot markets can be used more effectively and profitably for three main purposes: (1) procurement of products (raw materials, components, both customized and commodities); (2) trading in the market under different circumstances (such as low/high demand realizations, favorable price differentials, etc.); (3) design of supply chain contracts in which both buyers and sellers can conduct operations in the spot market. In this section, the literature related to optimal procurement planning in the presence of the spot market will be reviewed. There are two main streams of research that are relevant. The first stream is the work that considers inventory problems when buyers procure solely from the spot market with fluctuate spot price. The second one is the study of the supply chain procurement strategies combining spot market purchase with purchases made in advance from a specific long-term supplier.

Fabian et al. (1959) present a solution to the problem of determining inventories of raw material when the prices of this raw material fluctuate and are independent from period to period. They show the optimality of price-dependent base stock ordering policy. Kalymon (1971) extends Scarf's (1960) inventory model to include stochastic prices. He proves that the price-dependent (s, S) policy is optimal. Kingsman (1969) considers a finite horizon commodity-purchasing problem under stochastic prices so as to minimize its total costs over the horizon during which the buyer has many opportunities to make a purchase. He suggests the optimality of a price-dependent base stock policy, which was later proved by Golabi (1985) for the stationary and nonstationary price distribution cases. However, the demand for each period in their works is deterministic and known at the beginning of the planning horizon. Magirou (1982) considers a similar model with more general setting than Golabi (1985) where the price process is Markovian and the quantities can also be sold to spot market (see Magirou (1987)). Shastry (1993) studied the procurement problem of manufactured goods whose prices change in discrete jumps. Timing between two successive price jumps is a random variable with a known probability density function. Wang (2001) investigates inventory replenishment policy under the influence of decreasing prices, and stochastic demand in a multi-period model. He proves myopic policy to be optimal and characterizes the optimal procurement policies for the lost sales. Guzel (2004) proposes a new mode of analysis to inventory management systems when the unit-purchasing cost is allowed to be a general stochastic process, the demand is dynamic but deterministic and there is a constant lead-time. Arnold et al. (2006) present a deterministic optimal control approach optimizing the procurement and inventory policy for an enterprise that is processing raw material continuously when the purchasing price, holding cost, and the demand rate fluctuate in time.

Recently, researchers have explored the optimal mix of long-term and short-term contracts to reduce cost and enhance flexibility in supply chains. In this regards, Ritchken and Tapiero (1986) are perhaps the first to employ contingent claim analysis to mitigate price and demand risks in designing inventory management policies. Cohen and Agrawal (1999) evaluate the trade-off between long-term and short-term contracts by developing a bi-nominal model of price variation. They conclude that there is no single dominant strategy of procuring contracts. Gurnani and Tang (1999) obtain an optimal ordering policy for a retailer who orders a seasonal product from a manufacturer in two stages. In their model, a retailer faces the trade-off between a fixed price and uncertain demand in a stage versus a stochastic price and improved demand information in a second stage. Akella et al. (2002) model the optimal allocation of supply capacity before observing the realization of a random demand. In their model, if the realized demand is greater than the allocated capacity then the residual demand gets satisfied from the spot market at the market price. The buyer is risk-neutral and seeks to minimize its total expected cost. Their model has two periods. In period 1, before seeing the demand, the buyer has to decide the capacity reservation level from the contract supplier. And then, the demand is realized and has to be filled completely through both the contract supplier and spot markets. Kleindorfer and Wu (2003) perform a survey on the underlying theories and practices in using options on both capacity and output in the B2B markets. Seifert et al. (2004) explore the procurement and selling policies when purchasing both via forward contracts with fixed price and spot markets under fluctuating prices to minimize the mean-variance of the total cost. Goel and Gutierrez (2006) further extend this work to a multi-period setting in which prices evolve in continuous time. Martinez-de-Albeniz and Simchi-Levi (2005) present the optimal replenish policy in a multi-period environment with a portfolio contract combined by convex contracts and spot market. Similarly, Fu et al. (2006) design an optimal portfolio of optimal contracts for a single-period portfolio procurement problem when demand is uncertain and prices are stochastic. Mendelson and Tunca (2007) propose an endogenous model of interaction between a single supplier and multiple manufacturers where sourcing is done through a fixed price contract and spot procurement, and describe the strategic interaction amongst the players and its influence on the formulation of fixed-price contract and the supply chain efficiency. Secomandi (2008) characterize the optimal policy for storage of natural gas as a two price state variable dependent base stock levels. Yang and Xia (2008) study a continuous-review acquisition problem, in which the raw material price follows a discrete-state Markov process and demand is compound Poisson and show that an optimal policy is of the order-up-to type. Haksoz and Seshadri (2009) study a procurement problem of a commodity producer who sells via a long-term fixe price contract and spot market. They propose an optimal production policy and spot trading policy in the presence of production friction.

As an emerging topic in this stream, the optimal procurement policy under the combination of quantity flexible contracts and spot markets will discussed in detail here. Bonser and Wu (2001) explicitly incorporate spot procurement into a quantity flexible long-term supply contract in a fuel procurement problem. They study the fuel procurement planning problem for electricity companies facing uncertain demand and market price. Due to the terms set out in such a long-term supply contract, the decision maker has the quantity flexibility bounded by a minimum annual commitment and a maximum annual commitment. Spot procurement is used as an alternative source of supply to fulfil monthly demands. They first formulate the optimal fuel procurement problem as a multistage stochastic program. With the awareness of the difficulties in solving such a large-scale multistage stochastic program, they develop a two-phase dynamic heuristic to formulate plans for optimal procurement. Yi and Scheller-Wolf (2003) model procurement under two supply modes where one is a regular supplier, and the other is a spot market. The contract with the regular supplier specifies constrains on procurement volume for each period and a predetermined price, while the spot market has unlimited supply but a varying price. There is a setup cost when entering into the spot market. They show that the optimal policy is similar to the classic (s, S) policy. Seith et al. (2004) formulate a multi-period model to study procurement strategy under a quantity flexible contract with spot market purchasing opportunity. The quantity flexible contract is very similar to a call option. Feng and Sethi (2008) investigate a procurement problem with flexible contracts and spot markets. The flexible contract they consider contains a price-only contract for long-term order and an adjustment contract for short-term orders. There exists a capacity on the adjustment quantity from the supplier. They consider two types of capacity arrangement: dedicated capacity and overall capacity, and discuss the optimal procurement strategies and the criteria for capacity allocations when there is no setup cost for procuring from the spot market. Inderfurth and Kelle (2008) study a similar multi-period procurement problem with a capacity reservation contract in the presence of spot market. They consider a simpler (R, S) policy where the optimal parameters can be derived analytically.

The literature has not paid much attention to the time flexibility provided by spot markets in procurement decisions. In this context, Li and Kouvelis (1999) study supply contracts to satisfy a deterministic demand under procurement price uncertainty, and evaluate optimal procurement policies for time-flexible and time-inflexible contracts. They illustrate that adequately structured risk-sharing contracts provide opportunities for profit through sourcing from volatile price environments.

2.4 Comparison with Existing Literature

Our work in this thesis is to consider optimal procurement and inventory planning problems by incorporating long-term supply contracts. Here, we briefly compare our work with the existing literature. More detailed discussions will be deferred to corresponding chapters.

The first model considered in Chapter 3 is mainly based on Seifert et al. (2004). In their work, the spot procurement is used to satisfy excess demand after demand realizes, and the objective of the buyer is to minimize the mean-variance of his total cost. However, in our model, we consider a more realistic situation in which buyer procures the raw materials before the realization of demand to minimize its expected cost. Furthermore, we analytically study how the spot market affects the policy parameters.

The second model in Chapter 4 is an extension of Fox et al. (2006) to a fluctuate cost structure over time. This fluctuate cost structure is caused by incorporating the procurement via spot market. Moreover, the contract provided by the contract supplier specifies a pre-determined minimum quantity for each period. We identify certain monotone properties when prices between periods are dependent and follow a Markov process. This is different from the existing literature, which always assumes the prices between periods are independent when analyzing monotone properties.

The third model in Chapter 5 extends the models in Bassok and Anupindi (1997) and Chen and Krass (2001) to an environment where there is a spot market with fluctuate spot price. However, in contrast to Bassok and Anupindi (1997) and Chen and Krass (2001), in which the buyer always fulfills the commitment first, the incorporation of spot market allows the buyer to defer some commitment to future periods, and to procure via the spot market in current period. This makes the problem more interesting.

□ **End of chapter.**

Chapter 3

Procurement Decision with a Forward Contract and Spot Market

3.1 Introduction

The purpose of this chapter is to study the optimal procurement strategy in the presence of the spot market by incorporating a forward in a two-period setting. A forward contract is an agreement between two parties to buy or sell an asset at a specified point of time in the future. In reality, buyers always sign such contracts to fix their input prices before production begins. In this chapter, we model that the buyer is a price taker and is aim to choose an appropriate mix strategy of forward buy and spot procurement to minimize the expected cost. We consider two cases, in the first case, the buyer is not allowed to resell the raw materials back to the spot market, as in Martinez-de-Albeniz and Simchi-Levi (2005); and in the second, this restriction is relaxed.

Through our analysis, we find when the buyer can not sell back to the spot market, there exists a threshold forward price, under which the buyer enters the forward contract. This threshold is lower than the expected spot price. Furthermore, both the threshold and the optimal order quantities via forward contract increase in the mean of spot price, but decrease in the variability of the spot price. However, when the buyer can sell to the spot market, he speculates using the forward contract. That is, if the expected spot price is higher than the contract price, the buyer orders as much as possible via forward contract; otherwise, he only procures from the spot market. In both cases, the spot variability benefits the buyer in the sense of expected total cost. The results of our analysis can be used by decision makers to determine the effect of the key spot market characteristics on the optimal size of the forward contract and the corresponding cost, and to decide how to take the advantage of the spot market.

This chapter is organized as follows. **Section 3.2** reviews some related literature. **Section 3.3** formulates our mathematical model and derive the optimal procurement policy. **Section 3.4** conducts stochastic comparisons. **Section 3.5** extends the above model to allow the buyer to resell to the spot market. And **Section 3.6** concludes with discussion of some extensions.

3.2 Related Literature

Our current work is an extension of the classical newsvendor problem into a fluctuation environment. Akella et al. (2002) and Seifert et al. (2004) extend the newsvendor model by incorporating a long-term contract supplier and the spot market. In the model of Akella et al. (2002), the buyer is risk-neutral and seeks to minimize its total expected cost. Their model has two periods.

In period 1, before seeing the demand, the buyer has to decide the capacity reservation level from the contract supplier. And then, demand realizes and has to be filled completely through the contract supplier or the spot market. There is a maximum quantity that can be procured from the spot market. The authors show that a mixed procurement strategy comprising both the long-term contract and the spot market procurement is optimal. In Seifert et al. (2004), the buyer has two options of procurement. The first is a forward contract with known lead time and fixed unit price. The second option is to purchase from the spot market with negligible lead-time and at a stochastic spot price. The objective of the buyer is to minimize the mean-variance of the total cost. They conclude that significant profit improvements can be achieved if a certain percentage of commodities is procured via the spot market. However, both these two works assume the spot procurement is used to satisfy excess demand after demand is realized, and only numerical analysis is conducted to show how the spot market affects the optimal decision. Spinler et al. (2003) setup a framework, in which the buyer and the seller are to maximize their expected utility, for the valuation of options contracts with physical delivery. In their work, the spot market is also used to satisfy unmet demand after demand realizes. In this chapter, we model a more realistic situation in which the raw materials should be procured before the realization of the demand. Furthermore, we analytically show how the market parameters affect the parameters of optimal policy.

Our work is also related to the literature dealing with optimal hedge policies by futures (see Holthausen (1979), Lapan et al. (1991), Alghalith (2006), etc.). Most of these works focus on hedging the output price uncertainty by assuming all output can be sold to customers. There is no demand uncertainty. Our model differs from these works. First, we consider the hedge of input price by forward contract with physical delivery; and second, the demand is uncertain, and may not be fully satisfied.

3.3 Model Formulation

We consider the situation where a firm can enter into a forward contract with its suppliers at time 0 for the purchase of a certain amount of raw material for future production. This forward contract makes delivery at time 1. Then, at time 1, according to the current spot price c and his on-hand inventory level, the buyer needs to decide the procurement quantity from the spot market to adjust his raw material inventory. Then, the production begins and the demand realizes at time 2. Depending on the relative size of demand, a holding cost $h(u) = c_h u^+$, or a shortage cost $p(u) = c_s u^-$ occurs. The objective of the buyer is to choose an appropriate mix of long-term contracts and spot buying to minimize his expected cost. Without loss of generality, the excess inventory is assumed to be discarded without any salvage value and all backlogged demand is lost after time 2. The decision process is illustrated in **Figure 3.1**.

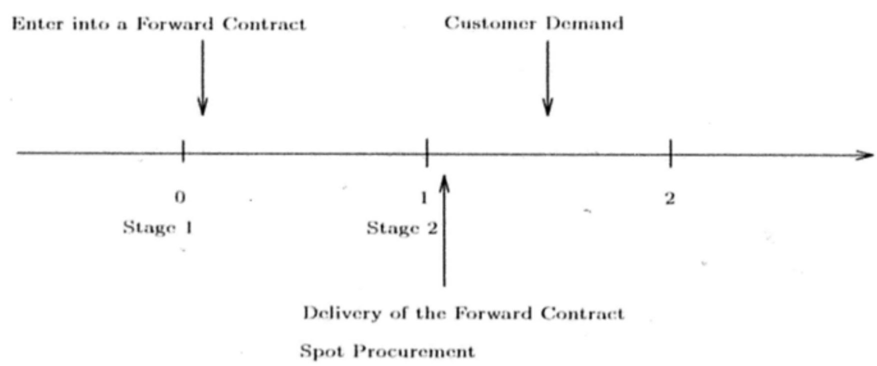


Figure 3.1: Two-stage decision process

Then, this problem can be separated into two stages. At the first stage, the buyer decides the size of the forward contract, Q, with contract price c_H, and then, at the beginning of the second stage, after the forward contract is delivered and the spot market price c reveals, the buyer should decide the optimal inventory position to maintain to satisfy future demand.

Other assumptions are as follows. The cumulative probability distribution function and probability density function for the spot price in time 2 are assumed to be $\Theta(\xi)$ and $\theta(\xi)$ on a positive support $[Z, M]$, respectively. Its mean is μ_c. We further let $c_s \geq Z$ and $c_s \geq c_H$, which avoid the optimality of no procurement. The demand distribution is assumed to be independent on the spot price. Let $\Phi(\eta)$ and $\phi(\eta)$ be the cumulative probability distribution function and probability density function for the demand distribution. The buyer is not allowed to sell to the spot market, that is, the presence of speculators is not allowed into our model, which is doubtless a good proxy for industrial markets (see Spinler et al. (2003)). The case when selling is allowed will be discussed later.

We first consider the problem for the second stage. The decision of the buyer is to find the optimal inventory level y, which is the solution for the following problem:

$$f(Q;c) = \min_{y \geq Q}[c_H Q + (y - Q)c + L(y)], \tag{3.1}$$

where $L(y) = \int_0^\infty p(y-\xi)\phi(\xi)d\xi + \int_0^\infty h(y-\xi)\phi(\xi)d\xi$. This is a classic newsvendor problem. Denote $S_s(c)$ as the optimal solution for the non-constraint variation of the above model when the spot price is c.

Then, the optimal order quantities are

$$(q_s, q_h) = \begin{cases} (0, Q), & Q \geq S_s; \\ (S_s - Q, Q), & S_s > Q, \end{cases} \tag{3.2}$$

where q_s and q_h are the order quantities from the forward contract and the spot market, respectively.

Some results about the properties of S_s are as follows.

Lemma 3.3.1 *$S_s(c)$ decreases in c.*

This is a direct application of the news-vendor solution, which is intuitive. Thus, we can find a c_0 such that $S_s(c_0) = Q$. ($c_0 = M$ or $c_0 = Z$ if c_0 is outside the support) Furthermore, we have the following result concerning the properties of the value function.

Lemma 3.3.2 *$f(Q, c)$ increases in c and convex in Q. Moreover, $f(Q, c)$ is concave in c for fixed Q.*

Proof. The optimal value function $f(Q, c)$ has the following form.

$$f(Q,c) = \begin{cases} c_H Q + L(Q), & c \geq c_0; \\ c_H Q + c(S_s(c) - Q) + L(S_s(c)), & c < c_0. \end{cases}$$

Thus, the convex property is straightforward, we neglect the detailed analysis and focus on the concavity of $f(Q, c)$ in c.

Note that when $c \geq c_0$, $\frac{\partial f(Q,c)}{\partial c} = 0$. When $c < c_0$, $\frac{\partial f(Q,c)}{\partial c} = S_s(c) - Q \geq 0$ and $\frac{\partial^2 f(Q,c)}{\partial c^2} = \frac{\partial S_s(c)}{\partial c} \leq 0$. Furthermore, $\frac{\partial f(Q,c)}{\partial c}|_{c \to c_0^-} = \frac{\partial f(Q,c)}{\partial c}|_{c \to c_0^+}$. The concavity is straightforward. ◇

This lemma means the marginal value of the spot price decreases as the spot price increases, since the buyer can hedge the spot market by reducing the order quantity from it.

Now we move backward to the first stage. The optimal decision is to choose a suitable Q to minimize the buyer's expected cost, that is,

$$\min_{Q \geq 0} \pi(Q) = \mathrm{E}_c f(Q, c).$$

We have the following result on this optimal problem.

Theorem 3.3.1 *$\pi(Q)$ is convex. The optimal size of forward contract, Q^*, is as follows:*

$$Q^* = \begin{cases} Q_2^*, & c_H \le Z; \\ 0, & c_H \ge \mu_c - \int_{Z+c_s}^{M}(c-c_s)\theta(c)dc; \\ Q_1^*, & otherwise, \end{cases}$$

where Q_1^* is the solution to the following equation

$$\int_{c_0(Q)}^{M}(c + L'(Q))\theta(c)dc + c_H - \mu_c = 0, \tag{3.3}$$

and $Q_2^* = \arg_Q\{c_H + L'(Q) = 0\}$. Furthermore, $L'(Q_1^*) \le 0$.

Proof. The formulation of $\pi(Q)$ is

$$\pi(Q) = \int_{c_0(Q)}^{M}(c_H Q + L(Q))\theta(c)dc + \int_{Z}^{c_0(Q)}(c_H Q + c(S_s(c) - Q) + L(S_s(c)))\theta(c)dc$$

and its first order and second order conditions are

$$\begin{aligned} \frac{\partial\pi(Q)}{\partial Q} &= \int_{c_0(Q)}^{M}(c_H + L'(Q))\theta(c)dc + \int_{Z}^{c_0(Q)}(c_H - c)\theta(c)dc = 0, \quad \text{and} \\ \frac{\partial^2\pi(Q)}{\partial Q^2} &= \int_{c_0(Q)}^{M}(L''(Q))\theta(c)dc \ge 0 \end{aligned}$$

Thus, the convexity of (3.3) is obtained. Let Q_1^* be the solution to the first order condition. Now, we discuss the optimal policy by the following three cases,

- Case 1: $c_H \ge \mu_c - \int_{c_s}^{M}(c - c_s)\theta(c)dc$. When $Q = 0$, $c_0(Q) = Z + c_s$ from the form of $S_s(c)$. Thus, substituting $Q = 0$ into the first order condition, we find $c_H = \mu_c - \int_{c_s}^{M}(c - c_s)\theta(c)dc$ if $Q_1^* = 0$. As Q_1^* decreases in c_H (proved in **Lemma** 3.4.2), which implies $Q_1^* < 0$ if $c_H > \mu_c - \int_{c_s}^{M}(c - c_s)\theta(c)dc$, thus, $Q^* = 0$ in this case.

- Case 2: $c_H \leq Z$. As in Case 1, substituting $Q = Q_2^*$ into the first order condition yields $c_H = Z$ if $Q_1^* = Q_2^*$. Thus, $Q^* = Q_2^*$ when $c_H < Z$.

- Case 3: $Z < c_H < \mu_c - \int_{c_s}^{M}(c - c_s)\theta(c)dc$. In this case, Q_1^* exists for3 (3.3), thus, $Q^* = Q_1^*$.

We next prove $E'(Q_1^*) \leq 0$. Define $\ddot{Q} = \arg_Q\{Z + L'(Q) = 0\}$. Then, since $c_H \geq Z$, we have $Q_1^* \leq \ddot{Q}$. And $Z \geq 0$, so we have $L'(Q_1^*) \leq 0$. ◇

This theorem characterizes the conditions when the forward contract and the spot market should or should not be used. It mainly associates with the comparison between the expected spot price and the forward price. It also shows that the threshold forward price below which the buyer enters the contract increases in the shortage penalty cost c_s. The optimal forward contract size is finite. Thus, if the commodity market is risk-neutral, i.e., the forward price is equal to the expected spot price, then the buyer does not enter a forward contract.

Remark 3.3.1 In this section, we assume all unmet demand is lost. This assumption can be further relaxed, i.e., the excess demand can be satisfied by the spot market. The analysis remains almost the same.

3.4 Stochastic Comparisons

We begin this section by reviewing some definitions from the theory of stochastic comparisons. To avoid triviality, in the following part, we always assume $Z \leq c_H \leq \mu_c - \int_{c_s}^{M}(c - c_s)\theta(c)dc$.

Define

$$I_F = \mu_c - \int_{c_s}^{M}(c - c_s)\theta(c)dc.$$

Then, I_F means the threshold forward price under which the buyer is willing to enter the forward contract.

Definition 1 If A and B are random variables and $\mathrm{E}f(A) \leq \mathrm{E}f(B)$ for all nondecreasing functions f for which the expectation exist, then A is stochastically smaller than B, i.e., $A \leq_{st} B$.

Definition 2 If A and B are random variables and $\mathrm{E}f(A) \leq \mathrm{E}f(B)$ for all convex functions f for which the expectation exist, then A is smaller than B according to the convex order, i.e., $A \leq_{cx} B$.

It can be shown that if $A \leq_{st} B$, then $\mathrm{E}(A) \leq \mathrm{E}(B)$; if $A \leq_{cx} B$, then $\mathrm{E}(A) = \mathrm{E}(B)$ and $\mathrm{Var}(A) \leq \mathrm{Var}(B)$.

As discussed previously, Θ is the distribution of the spot price c at time 1. In this section, we concern what happens when the spot price is altered to $\tilde{c}$ with distribution $\tilde{\Theta}$. Denote by $\pi^* = \max_Q \pi(Q)$, $\tilde{\pi}^* = \max_Q \tilde{\pi}(Q)$ the optimal forward sizes corresponding to Θ and $\tilde{\Theta}$, respectively. From **Lemma 3.3.2**, we know that when $c \leq_{st} \tilde{c}$, $\pi(Q) \leq \tilde{\pi}(Q)$, and when $c \leq_{cx} \tilde{c}$, $\pi(Q) \geq \tilde{\pi}(Q)$. Thus, we have the following result.

Lemma 3.4.1 *If $c \leq_{st} \tilde{c}$, then $\pi^* \leq \tilde{\pi}^*$. Furthermore, if $c \leq_{cx} \tilde{c}$, then $\pi^* \geq \tilde{\pi}^*$.*

This lemma shows that stochastically larger spot price increases the buyer's cost. This is intuitive. However, the second part implies that higher price variability always gives lower expected cost. For many common distributions including uniform, gamma, Weibull, lognomral and beta distributions, Muller and Stoyan (2002) provide examples to identify an ordering of their parameters that is equivalent to a convex order. Within each of these families, if two distributions have the same mean, then, the one with the smaller variance is smaller in the convex order. Thus, if the spot price is assumed to be Geometric Brownian motion, a large price volatility benefits the buyer. This is mainly because the buyer can hedge the price volatility by operationally controlling the inventory.

If c can be denoted in a form $c = \mu_c + \sigma_c\varepsilon$ where ε is a random variable with mean 0 and variance 1. Let ε's CDF and PDF be $\Psi(\cdot)$ and $\psi(\cdot)$, respectively. We have the following result.

Lemma 3.4.2 *Q^* decreases in c_H. Moreover, Q^* and I_F increase in μ_c.*

Proof. According to the first-order condition 3.3, we take its total derivative with respect to c_H and get

$$\frac{\partial Q}{\partial c_H}\int_{c_0(Q)}^{M} L''(Q)\theta(c)dc = -1.$$

Thus, Q^* decreases in c_H. For the second part, substituting $c = \mu_c + \sigma_c\varepsilon$ into 3.3 and taking derivative with respect to μ_c, we get

$$\frac{\partial Q}{\partial \mu_c}\int_{(c_0(Q)-\mu)/\sigma_c}^{(M-\mu_c)/\sigma_c} L''(Q)\psi(\varepsilon)d\varepsilon = 1 + \frac{(M + L'(Q))\psi(\frac{M-\mu_c}{\sigma_c})}{\sigma_c} - \int_{(c_0(Q)-\mu)/\sigma_c}^{(M-\mu_c)/\sigma_c}\psi(\varepsilon)d\varepsilon.$$

The right-hand side of the above equation is greater than 0 when paying attention to the fact that $M + L'(Q) \geq 0$. Thus, Q^* increases in μ_c. Analogous analysis shows that I_F increases in μ_c. ◇

This lemma tells us that the higher the expected future spot price compared to the contract price is, the more the buyer procures via forward contract, and the higher the threshold forward price I_F, which means the buyer is more willing to use the forward buy. Again, these results are intuitive.

Lemma 3.4.3 *If $c \leq_{cx} \tilde{c}$, then $Q^* \geq \tilde{Q}^*$, and $I_F \geq \tilde{I}_F$.*

Proof. Define a new function $g(c)$ as follows.

$$g(c,Q) = \begin{cases} c + L'(Q), & c \geq c_0(Q); \\ 0, & c < c_0(Q). \end{cases}$$

Then, $g(c, Q)$ is convex in c. Thus, when $c \leq_{cx} \tilde{c}$, for fixed Q, $\mathrm{E}_c g(c, Q) \leq \mathrm{E}_{\tilde{c}} g(\tilde{c}, Q)$. However, $\mathrm{E}(c) = \mathrm{E}(\tilde{c})$, thus, to insure $\mathrm{E}_c g(c, Q) = \mathrm{E}_{\tilde{c}} g(\tilde{c}, \tilde{Q})$, we have $Q^* \geq \tilde{Q}^*$ since $\mathrm{E}_c g(c, Q)$ increases in Q. The second part can be analyzed similarly. ◇

This lemma is interesting as it implies that a large variable spot price decreases both the order quantity via forward contract and the threshold forward price I_F. Thus, we can see that for a risk-neutral decision maker who can not sell to the spot market, the motivation to enter into the forward contract is to hedge the volatility of the spot price, but is to take advantages of lower forward price than the expected spot price.

3.5 Extension: When Reselling is Allowed

In previous analysis, we assume the buyer can not resell raw materials to the spot market. Here, we further extend the above analysis to the case when the buyer can sell to the spot market freely and other settings are the same.

In this case, the optimal problem is

$$f(Q; c) = \min_y [c_H Q + (y - Q)c + L(y)]. \tag{3.4}$$

And the optimal order quantities are

$$(q_s, q_h) = (S_s - Q, Q), \tag{3.5}$$

where a negative value of q_s means selling to the spot market. The optimal decision at the first stage is to choose an optimal Q to minimize the buyer's expected cost, that is,

$$\min_{Q \geq 0} \pi_1(Q) = \mathrm{E}_c f(Q, c).$$

We give the following result concerning $\pi_1(Q)$ without proof.

Theorem 3.5.1 $\pi_1(Q) = (c_H - \mu_c)Q + E_c[cS_s(c) + L(S_s(c))]$. *Furthermore, if* $c \leq_{cx} \tilde{c}$, *then* $\pi_1(Q) \geq \tilde{\pi}_1(Q)$.

This theorem implies that the optimal forward contract size only depends on $c_H - \mu_c$. That is, if the buyer's expectation on the spot price is higher than the contract price, the buyer orders as much as possible by forward contract in time 1; otherwise, he orders via the spot market only in time 2. Thus, when the buyer is risk neutral and can freely sell to the spot market, he speculates to use the forward contract. In this case, the variability of the spot price has no effect on the contract size.

We define

$$\Delta(Q) = \pi(Q) - \pi_1(Q)$$

to be the value of selling to the spot market when the buyer has Q forward contract on hand, and $\tilde{\Delta}(Q)$ be the counter-party for price distribution $\tilde{c}$. We characterize the property of $\Delta(Q)$ by the following result.

Lemma 3.5.1 $\Delta(Q)$ *increases and is convex. If* $c \leq_{cx} \tilde{c}$, *then* $\Delta(Q) \leq \tilde{\Delta}(Q)$.

Proof. $\Delta(Q)$ can be formulated as,

$$\Delta(Q) = \int_{c_0(Q)}^{M} [L(Q) + cQ - cS_s(c) - L(S_s(c))]\theta(c)dc.$$

Taking its first and the second derivative with respect to Q proves the first part. Now, define a new function $g(c)$ as follows.

$$g(c, Q) = \begin{cases} Q - S_s(c), & c \geq c_0(Q); \\ 0, & c < c_0(Q). \end{cases}$$

Since $Q - S_s(c) \geq 0$ when $c \geq c_0(Q)$, then, $g(c, Q)$ is convex in c. Thus, when $c \leq_{cx} \tilde{c}$, for fixed Q, $\mathrm{E}_c g(c, Q) \leq \mathrm{E}_{\tilde{c}} g(\tilde{c}, Q)$. This proves the second part. ◇

This lemma intuitively means that the value of selling to the spot market is higher when the buyer has a larger size of forward contract. Furthermore, the marginal value of reselling to the spot market is also increase in the contract size. We also note that the value of reselling to the spot market is higher when the spot price is more variable.

3.6 Summary

The chapter sets up a model to study the effect of the spot market and the forward contract on procurement decisions in a two-period framework. The objective of the buyer is to choose an appropriate mix strategy of the forward buy and the spot procurement to minimize his expected cost. We study the procurement strategies for both situations under which the buyer can and can not sell to the spot market, respectively. Furthermore, we show analytically how the spot market parameters affect the policy parameters and how much the value of selling to the spot market is from the buyer's perspective.

As an extension of our model, future research should take the risk measure into consideration. The expected value criterion used in this chapter is criticized for neglecting the extreme risk in the total cost, because in reality, a sudden big loss can bring the firm into bankrupt. Thus, procurement strategies under new objective functions incorporating certain risk criterions, such as mean-variance, CVaR, etc., should be more helpful for buyers.

□ **End of chapter.**

Chapter 4

Optimal Inventory Policy With a Long-term Supplier and Spot Market

4.1 Introduction

The objective of this study is to analyze a procurement model by incorporating traditional contract procurement with spot market operations. An example of our model is the procurement practice of metals or iron ores. Especially for iron ores, it is well-known that at the beginning of each year, the main ore sellers and iron producers enter a long-term contract that specifies the contract price and procurement quantities during that year. This contract price is negotiated by these two parities with reference to the current spot price and their forecasts about market trends (which is resulted by estimating future demand and supply). Besides the long-term contract, the buyers can also adjust their ores' inventory all year around by procuring from the spot market. However, the spot price is determined by market forces thus is volatile during that year. A snapshot of the Chinese imported iron ore spot price is illustrated in **Figure 4.1**, while the long-term contract price during this period is $91 per Tonne according to Metal Bulletin.

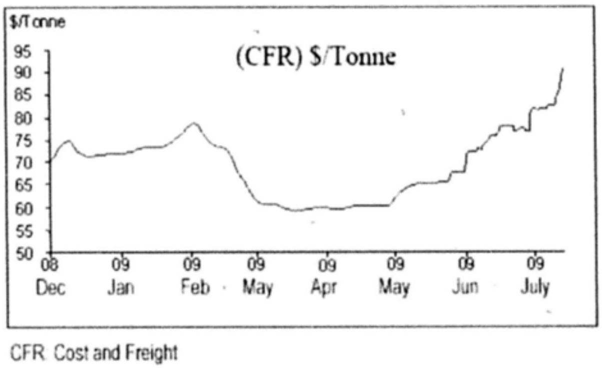


Figure 4.1: Spot price for Chinese imported iron ore

More specifically, we analyze a situation in which the buyer has two supply sources, namely, the contract supplier and the spot market. When the buyer procures from the contract supplier, he pays a fixed per unit cost but need to fulfill a minimum quantity commitment each period. However, when he procures from the spot market, he needs to pay a fixed setup cost besides the spot price, which is only known before each procurement. The buyer is not allowed to sell the excess inventory to the spot market, due to some constraints associated with real-world purchasing practices, such as either contractual agreements that restrict the buyer to resell to the spot market, or operational limitations, e.g., the final form of the product supplied is customized (such as micro-chips). This type of constraint implies that it is difficult or costly for the buyer to resell the materials back to the market.

Through our analysis, we learn the following about the buyer's procurement policy: 1) the optimal policy that minimizes the total discounted procurement inventory costs consists of three different policies of (s, S) type: a standard base-stock policy by procuring from the contract supplier; an (s, S) policy by procuring from the spot market; a generalized (s, S) policy in which there is a threshold inventory level to switch between these two supply sources; 2) the policy parameters (e.g., the reorder point, the order-up-to level) are monotonously changing with the spot price for a general Markov price process when it satisfies a plausible assumption. Such a property induces the existence of three spot price regions, each of which corresponds to a specific inventory policy. We also conduct numerical analysis to gain more insights into how the spot market impacts the buyer's performance.

The rest of the chapter is organized as follows. In **Section 4.2**, related literature is reviewed. In **Section 4.3**, the model is formulated, and the corresponding optimal inventory policies and monotone properties are explored. In **Section 4.4**, numerical experiments are conducted. The chapter is concluded with remarks in **Section 4.5**. All proofs are included in **Appendix**.

4.2 Literature Review

The literature incorporating spot market operation into long-term contract procurement is quite recent, as pointed out by Haksoz and Seshadri (2007). Sethi et al. (2004) formulate a multi-period model to study procurement strategy under a quantity flexible contract with spot market purchasing opportunity. The quantity flexible contract is very similar to a call option. Feng and Sethi (2008) investigate a procurement problem by incorporating the spot market to discuss the optimal procurement and capacity allocations strategies when the flexible contracts contain a price-only contract for long-term order and an adjustment contract for short-term orders. Our current work also considers the optimal inventory model by incorporating spot market operation into a long-term contract. One feature of our model is the inclusion of a periodic minimal order quantity contract.

Yi and Scheller-Wolf (2003) is the most relevant to our model. In their paper, they analyze a backlogged inventory problem in which the buyer can procure from both a contract supplier with a fixed order cost and the spot market with a given setup cost in addition to a random order cost. There is a capacity when procuring from the contract supplier. They show that the optimal policy is of an (s, S) structure, in which the order-up-to level from the contract supplier is inventory dependent. However, they do not specify what this state-dependent value should be, and their analysis can not be extended to the case of lost-sales. Our model may be regarded as a special case of their model – by relaxing the capacity constraint. However, our work contains two significant departures from theirs. First, we deal with the lost-sales case, while theirs does not allow lost-sales (i.e., an extension to such a case in their framework appears unlikely, because the value function is no longer K-convex in this case). Second, although we sacrifice some generality in the capacity constraint, we obtain a full characterization of the optimal policy. In the **Summary** (Section 5) and the **Appendix**, we highlight such a possible extension of our current model.

Within the literature considering inventory problems in the presence of the spot market, Fabian et al. (1959), to the best of our knowledge, are the first to present a solution to determine inventories of raw material when the price of this raw material fluctuates from period to period. Kalymon (1971) further studies an inventory model in which future purchasing prices are determined by a Markov process and there is a setup cost for each procurement. In these two papers, the authors conduct monotone analysis when prices between periods are independent. Kingsman (1969) and Golabi (1985) consider optimal inventory policies with stochastic price process, negligible setup cost, and deterministic but variable demand. Magirou (1982) considers a similar model with a more general setting where the price process is Markov and the quantities can also be sold to the spot market. The papers dealing with monotone properties when prices are general Markov process are few. Yang and Xia (2009) study a continuous-review acquisition problem, in which the demand distribution is compound Poisson, and prove the optimality of a policy of order-up-to type. They identify some conditions under which the order-up-to levels decrease in the spot price. Secomandi (2009) considers an optimal commodity trading problem with capacitated storage and shows that the optimal base-stock targets decrease in the spot price under certain assumptions. However, in these papers, the spot market is the only procurement source. The long-term contracts, especially quantity flexible contracts, are not considered.

Quantity commitment contracts with flexibilities are formally first analyzed by Anupindi (1993). In his work, the decision maker makes a commitment to purchase certain quantities in each period of the planning horizon. The commitment quantity in any given period is allowed to adjust within some pre-specified limits and the buyer is not allowed to update the commitments for future periods. Anupindi and Akella (1993) investigate a finite-horizon periodic commitment model with a response time to the adjustments in the order quantity. Anupindi and Bassok (1998) provide a general framework for the study of supply contracts and present a brief overview of supply contracts with quantity commitments.

However, these literature ignores the spot market.

The characterization of our model is related to the literature of periodic-review inventory problems with stochastic demand, which have been extensively studied (e.g., Scarf (1962), Veinott (1966), Porteus (1971), Wang (2001), Fox et al. (2006)). Here we discuss those most relevant to ours. Porteus (1971) analyzes inventory models with one supplier with concave order costs, and shows the optimality of a generalized (s, S) policy when the demand distribution is a one-sided Polya distribution. Porteus (1972) extents the results to the uniform demand distribution. Although both papers deal with the single supplier case, it can be easily extended to the multiple suppliers case in which suppliers have different linear and setup costs. Fox et al. (2006) consider a lost sale, but one that is extendable to a back-order, inventory problem in which the buyer faces two supply sources: one with a higher variable cost but no setup cost and one with both variable and setup costs. They prove that a simplified form of a generalized (s, S) policy is optimal when the density function of demand distribution is log-concave. Minner (2003) gives an extensive review on the multi-supplier procurement literature. However, variable order costs in all the above models are deterministic.

Our work extends the results in Fox et al. (2006) to a fluctuating cost environment. In particular, we allow the variable cost of the second supply source to be uncertain - to follow the spot market. Moreover, we go further and identify sufficient conditions (see **Assumption 4.3.2** in Section 3.2) on the Markov spot price process, under which the policy parameters are monotone with respect to the spot price. With these refinements, we are able to identify three spot price intervals, within which a specific policy applies. This is a significant enhancement to Fox et al. (2006) and Yi and Scheller-Wolf (2003).

4.3 Model Formulation

The setting of our model can be described as follows. A buyer manages his raw material inventory for a finite horizon to satisfy uncertain customer demand. There are N periods during the planning horizon. The demand occurs in each period and the buyer can only replenish his inventory level at the beginning of that period. We index the periods in a backward fashion, i.e., period n means that there are n periods left. There are two supply sources during the planning horizon: a contract supplier and a spot market. When the buyer procures from the contract supplier, a fixed unit price c_H is charged and a predetermined minimum quantity P_n for each period n must be committed. When he procures from the spot market, a spot price c_n plus a fixed setup cost K is charged. Such setup cost includes transportation, paperwork, inspection cost, etc. However, those costs are negligible when procuring from the contract suppliers because in practice, the suppliers provide certain consignment stock program to save on these costs (see http://www.tallymetal.com/tally3.html).

We assume the demands in different periods are independent and identically distributed with cumulative (density) distribution $\Phi(\xi)$ ($\phi(\xi)$), and the spot price c_n follows a Markov process. The Markov property of the spot price is a reasonable assumption, because the complete market hypothesis widely accepted in the financial literature guarantees that the current spot price contains all market information till now. Specifically, in our model, the distribution of the spot price, c_n, depends on the spot price c_{n+1} with probability distribution function $\Theta(c_n|c_{n+1})$ and density function $\theta(c_n|c_{n+1})$. The incorporation of the stochastic price process into inventory problems represents one major departure from Fox et al. (2006).

The sequence of events in each period is as follows. At the beginning of period n, the buyer first observes his on-hand inventory level x_n and the spot price c_n before ordering. Then, after fulfilling the committed quantity P_n, he decides which supply source to procure from and in what additional quantity (if any). We assume both orders arrive immediately, i.e., the lead-times for both supply sources are zero. Let y_n be the on-hand inventory after orders arrive. (The results in sequel nevertheless hold as long as the lead-times for these two types of replenishment are the same, and in this case, x_n should be the inventory position at the beginning of period n.) Then, demand realizes, and depending on its size, excess inventory u is taken over to the next period with a holding cost $h(u)$, or unsatisfied demand u is lost with a shortage cost $p(u)$. In this chapter, we assume $h(u)$ and $p(u)$ to take the linear form, i.e., $h(u) = c_h u^+$, $p(u) = c_s u^-$. Moreover, $c_s \geq c_H$, which rules out the trivial case in which the buyer does not buy from the contract supplier. The assumption of linear holding/shortage cost is common in the literature, the relaxation of which will also be discussed in **Appendix**. Without loss of generality, we assume the excess inventory is discarded without any salvage value at the end of the planning horizon.

Assumption 4.3.1 *The contract supplier has no capacity limit.*

This assumption is made for ease of exposition, the relaxation of which will result in additional complexity in both notation and optimal policy structure. This will be discussed in the **Summary**.

Define $f_n(x_n, c_n)$ to be the minimum expected discounted cost of managing inventory over periods $n, n-1, \cdots, 1$, starting with the inventory level x_n and the spot price c_n. As mentioned earlier, $f_0(x_0, c_0) = 0$.

Let

$$G_{n-1}(y_n, c_n) = L(y_n) + \alpha[\int_0^\infty \int_0^\infty f_{n-1}((y_n - \xi)^+, \epsilon)\phi(\xi)\theta(\epsilon|c_n)d\epsilon d\xi \tag{4.1}$$

be the cost-to-go function, which is a summation of the expected holding and shortage cost in the current period n and the optimal expected discounted cost over the next $n-1$ periods, where $L(y_n) = \int_0^\infty [p(y_n - \xi) + h(y_n - \xi)]\phi(\xi)d\xi$, and α is the discount factor $(0 < \alpha \leq 1)$. Then, the general dynamic equation governing the n-period problem is

$$f_n(x_n, c_n) = \min\{J_L(x_n, c_n), J_H(x_n, c_n)\}, \tag{4.2}$$

where

$$J_L(x_n, c_n) = \min_{y_n \geq x_n + P_n} [K\delta(y_n - x_n - P_n) + c_n(y_n - x_n - P_n) + c_H P_n + G_{n-1}(y_n, c_n)], \tag{4.3}$$

$$J_H(x_n, c_n) = \min_{y_n \geq x_n + P_n} [c_H(y_n - x_n) + G_{n-1}(y_n, c_n)], \tag{4.4}$$

and $\delta(z) = 1$ if $z > 0$, $\delta(z) = 0$ otherwise. Note that $J_L(x_n, c_n)$ corresponds to the case in which the buyer procures from the spot market after fulfilling the minimal commitment size, while $J_H(x_n, c_n)$ corresponds to the case in which the buyer procures only from the contract supplier.

Before ending this section, we introduce several definitions that will be used later.

Definition 1 A function $g(x)$ is *quasi-convex* if $g(\lambda x + (1-\lambda)y) \leq \max\{g(x), g(y)\}$ for all $x, y \in R$ and $\lambda \in [0, 1]$.

Definition 2 A function $g(x)$ is *K-convex* if for any $x \leq y$ and $\lambda \in [0, 1]$,

$$g(\lambda x + (1-\lambda)y) \leq \lambda g(x) + (1-\lambda)(g(y) + K).$$

Definition 3 A distribution $\mathbb{F}$ is *strongly unimodal* if $\mathbb{F}$ is continuous and its density $\mathbb{f}$ is log-concave. That is, for any $\lambda \in [0, 1]$,

$$\mathbb{f}^{\lambda}(x)\mathbb{f}^{1-\lambda}(y) \leq \mathbb{f}(\lambda x + (1-\lambda)y).$$

If a distribution function is strongly unimodal, then, its convolution with a quasi-convex function preserves the quasi-convex property (Dharmadhikari and Joag-dev (1988)). In our work, to ensure the tractability of our model, we assume the demand distribution to be strongly unimodal, as in Fox et al. (2006). Most of the commonly seen demand distributions belong to the class of strongly unimodal distributions, e.g., the normal distribution, exponential distribution, and uniform distribution.

In the following analysis, we first prove that the optimal inventory policy consists of three different polices of (s, S) type. Then, we identify certain conditions on the price process, under which there are monotone properties between the policy parameters and the spot price. To simplify the notation, we drop the subscript n of x_n and y_n when no confusion arises.

4.3.1 Optimal Inventory Policy

For ease of analysis, we first derive structured results by setting $P_n = 0$ for all n, and we later show that the incorporation of non-zero P_n's does not change the structure of the optimal policy by a transformation (see **Remark 4.3.1**).

Define

$$\begin{aligned}
S_{H,n}(c_n) &= \arg\min_{y \geq x}\{c_H(y-x) + G_{n-1}(y, c_n)\}, \\
S_{L,n}(c_n) &= \max \arg\min_{y \geq x}\{c_n(y-x) + G_{n-1}(y, c_n)\}, \quad \text{and} \\
s_{L,n}(c_n) &= \max\{\arg\min_{r}\{r | c_n r + G_{n-1}(r, c_n) = c_n S_{L,n}(c_n) + G_{n-1}(S_{L,n}(c_n), c_n) + K\}\}.
\end{aligned}$$

Note that $S_{H,n}(c_n)$ is a minimizer to the unconstraint problem of (4); $S_{L,n}(c_n)$ is a minimizer to the unconstrained problem of (3); and $s_{L,n}(c_n)$ is the reorder point to the unconstrained problem of (3). Furthermore, $S_{H,n}(c_n) > 0$. These are proved in **Appendix** (see **Lemma A3** and **Lemma A4**). Then, the optimal policy for $J_H(x, c_n)$ is a base-stock policy with an order-up-to level $S_{H,n}(c_n)$, and that for $J_L(x, c_n)$ is an $(s_{L,n}(c_n), S_{L,n}(c_n))$ policy.

Intuitively, we can image that when the spot price c_n is higher than the contract price c_H, the buyer should procure from the contract supplier by adopting a base-stock policy with an order-up-to level $S_{H,n}(c_n)$. When c_n is lower than c_H, the buyer should trade off between the cost saved by procuring at a lower spot price and the setup cost K saved by procuring from the contract supplier. Thus, to characterize the optimal policy, for $c_n < c_H$, we further define the following value:

$$s_{o,n}(c_n) = \frac{K + c_n S_{L,n}(c_n) + G_{n-1}(S_{L,n}(c_n), c_n) - c_H S_{H,n}(c_n) - G_{n-1}(S_{H,n}(c_n), c_n)}{c_n - c_H}. \tag{4.5}$$

Then, $s_{o,n}(c_n)$ is the threshold inventory level at which there is no difference on the costs between spot market procurement and contract supplier procurement. This is because, when initial inventory level is $s_{o,n}(c_n)$, the cost by procuring from the spot market at $c_n < c_H$, $K + c_n(S_{L,n}(c_n) - s_{o,n}(c_n)) + G_{n-1}(S_{L,n}(c_n), c_n)$, is equal to that from the contract supplier, $c_H(S_{H,n}(c_n) - s_{o,n}(c_n)) + G_{n-1}(S_{H,n}(c_n)$.

For simplicity, we drop the time subscript n out of $s_{o,n}(c_n)$, $s_{L,n}(c_n)$, $S_{L,n}(c_n)$ and $S_{H,n}(c_n)$ and suppress their dependence on c_n, i.e., we write them as s_o, s_L S_L and S_H, respectively. With the assistance of these values, we can obtain the following theorem concerning the optimal inventory policy for the whole planning horizon.

Theorem 4.3.1 *$f_n(x, c_n)$ is K-convex in x and $\frac{\partial f_n(x,c_n)}{\partial x} \geq -c_H$. Furthermore, the optimal inventory policy is one of the following three types, depending on the threshold value s_o.*

(a) If $c_n \geq c_H$ or $s_o \leq 0$, the buyer procures from the contract supplier with a base-stock policy of an order-up-to level S_H.

(b) If $c_n < c_H$ and $s_o \geq S_H$, the buyer procures from the spot market with an (s_L, S_L) policy.

(c) Otherwise ($c_n < c_H$ and $0 < s_o < S_H$), an (s_o, S_H, S_L) policy is optimal, i.e., the buyer orders up-to $S_L(\geq S_H)$ from the spot market if $x \leq s_o$, or up-to S_H from the contract supplier otherwise.

An illustration of the optimal inventory policy is depicted in **Figure 4.2**. Note that when $c_n < c_H$, the line with slope $-c_H$ is under the graph of $G_{n-1}(x, c_n)$ with a tangent point S_H, and the line with slope $-c_n$ starts from the inner of the graph of $G_{n-1}(x, c_n)$ with $S_L \geq S_H$, thus, we have $s_o \leq s_L$. Furthermore, s_o and s_L are on the same side of S_H. Then, for **Figure 4.2(a)**, i.e., $s_o \leq 0$ or $c_n \geq c_H$, the cost by procuring from the spot market is always higher than that by the contract supplier for any $x \geq 0$, thus, it is optimal to order up-to S_H from the contract supplier if $x \leq S_H$, or order nothing otherwise. For **Figure 4.2(b)**, i.e., $s_o \geq S_H$, the cost by procuring from the spot market is always lower than that by the contract supplier for any $x \geq 0$, thus, it is optimal to order up-to S_L from the spot market if $x \leq s_L$, or order nothing otherwise. For **Figure 4.2(c)**, i.e., $0 < s_o < S_H$, the cost by procuring from the spot market is higher than that from the contract supplier for $0 \leq x \leq s_o$, and lower than that from the contract supplier for $x > s_o$, thus, it is optimal to order up-to $S_L(\geq S_H)$ from the spot

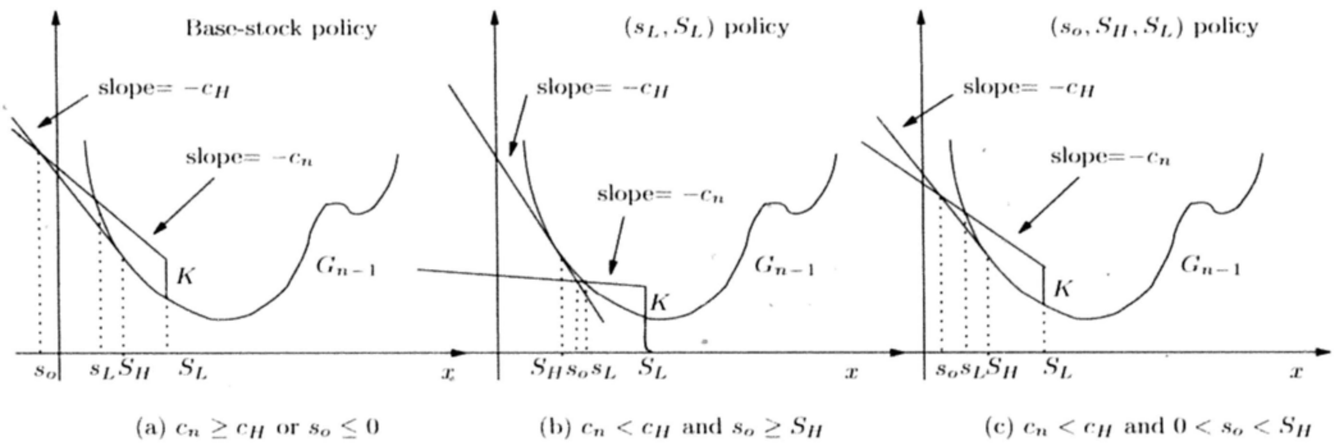


Figure 4.2: An illustration of the optimal inventory policy

market if $x \leq s_o$, or up-to S_H from the contract supplier if $s_o \leq x \leq S_H$, or order nothing otherwise.

The proof of this theorem is similar to the counterpart of Fox et al. (2006). However, we need to extend their analysis to handle the uncertain variable costs (see **Remark 4.5.1** in **Appendix**).

An intuitive result associated with procurement quantities when there is no difference in the costs by procuring from these two sources is given as follows.

Corollary 4.3.1 *In any period n, if $c_n \leq c_H$, then $S_H - s_o \leq \frac{K}{c_H - c_n} \leq S_L - s_o$.*

This lemma says that when $x = s_o$, i.e., when there is no difference in buying from these two sources, the quantity bought from the contract supplier is always less than that from the spot market.

Remark 4.3.1 The analysis above is conducted under the assumption that $P_n = 0$. When $P_n \neq 0$, we define a new state variable $z_n = x_n + P_n$ with state transition equation $z_{n-1} = (y_n - \xi)^+ + P_{n-1}$ where $y_n \geq z_n$. As the values of s_L, S_L, S_H and s_o depend only on the spot price c_n in period n, the whole procedure above can be replicated and essentially the same optimal inventory policy structure can be obtained.

4.3.2 Monotone Properties and Bounds

According to the above analysis, the optimal inventory policy is one of the three polices of (s, S) type. Here, we are interested in how the choice of these three policies depends on the spot price in each period after the spot price has been realized. For computational purposes, we would like to obtain the bounds for s_o, s_L, S_L and S_H as well.

This section is organized as follows. First, we explore the monotone properties of s_L, S_L, S_H and s_o with respect to spot price c_n if the price process follows a general Markov process that satisfies certain conditions. By applying the monotone properties, we identify three price intervals, within each of which a specific inventory policy applies. Finally, we develop the lower-and-upper bounds of S_H and S_L.

Monotone Properties

For a general Markov price process, we can write the relationship of prices between two excessive periods in the following form:

$$c_{n-1} = H(c_n) + V(c_n)\varepsilon.$$

Here, ε is a random variable defined on $[-A, B]$, with a mean of zero, variance of one, distribution function of $\Psi(\varepsilon)$, and probability density function of $\varphi(\varepsilon)$. $H(c_n)$ and $V(c_n)$ are two positive functions that stand for the mean and standard deviation of the spot price in period $n-1$, respectively, depending on the spot price in period n. Assume $c_{n-1} > 0$ for all ε, i.e., $A \leq \min_c \frac{H(c)}{V(c)}$. This form of price process is general, e.g., the Brownian Motion type processes belong to this form (see Gibson and Schwartz (1990), Schwartz (1997)).

Before further analyzing the monotone properties, we make the following assumption on $H(c)$ and $V(c)$. This assumption proposes a sufficient condition for deriving the monotone properties of policy parameters.

Assumption 4.3.2 *$H'(c) + V'(c)\varepsilon \geq 0$ for each ε, and $\alpha H'(c) \leq 1$.*

The first part of this assumption means that the spot price for the next period stochastic increases in the current spot price. That is, a higher current spot price is more likely to lead to higher prices for the next period. This condition is consistent with the continuity property in Yang and Xia (2009). (The stochastic increasing property can be verified as follows. Let c_n and $\widetilde{c}_n$ be two prices such that $c_n \geq \widetilde{c}_n$, and let $\mathbb{F}$ and $\widetilde{\mathbb{F}}$ be the distribution functions for the next period price c_{n-1}, corresponding to c_n and $\widetilde{c}_n$, respectively. For any $c > 0$, we find ξ and $\widetilde{\xi}$ which satisfy $c = H(c_n) + V(c_n)\xi$ and $c = H(\widetilde{c}_n) + V(\widetilde{c}_n)\widetilde{\xi}$. Thus, $H(\widetilde{c}_n) + V(\widetilde{c}_n)\widetilde{\xi} = c = H(c_n) + V(c_n)\xi \geq H(\widetilde{c}_n) + V(\widetilde{c}_n)\xi$, which follows from $H'(c) + V'(c)\varepsilon \geq 0$. Then, $\widetilde{\xi} \geq \xi$, which leads to $\mathbb{F}(c) = \Psi(\xi) \leq \Psi(\widetilde{\xi}) = \widetilde{\mathbb{F}}(c)$. This implies the stochastic increasing property.) The second part of this assumption indicates that the change of expected discounted price for the next period, induced by the current price change, is less than that of the current price. The serially independent price process and the Geometric Brownian Motion process with negative price drift satisfy this assumption. However, the mean-reverting price process and the Geometric Brownian Motions with positive price drifts do not satisfy the second part of this assumption (see Secomandi (2009)).

Now, we impose **Assumption 4.3.2** in the rest of this section, and let

$$g^{(1)}(x,y) = \frac{\partial g(x,y)}{\partial x}, \qquad g^{(2)}(x,y) = \frac{\partial g(x,y)}{\partial y} \qquad \text{and} \qquad g^{(12)}(x,y) = \frac{\partial^2 g(x,y)}{\partial x \partial y}, \tag{4.6}$$

for a general function $g(x,y)$ with two variables in our analysis.

The following lemma provides a result on how the marginal value of initial on-hand inventory changes with the spot price when applying the optimal inventory policy.

Lemma 4.3.1 *For $x \geq 0$, $f_n(x, c_n)$ increases in c_n and $f_n^{(1)}(x, c_n)$ is non-increasing in c_n; furthermore, $f_n^{(12)}(x, c_n) \geq -1$.*

This lemma has an immediate interpretation: the marginal value of initial on-hand inventory in a high spot price environment is greater than that in a low price environment, i.e., the marginal value of initial on-hand inventory increases in the spot price. However, the marginal value changes at a slower rate than the spot price.

With the aid of **Lemma 4.3.1**, we obtain the following theorem concerning the monotone properties.

Theorem 4.3.2 *Policy parameters S_L, s_L and s_o are non-increasing in c_n, S_H is non-decreasing in c_n.*

We further define two threshold prices as follows:

$$\begin{aligned} a_n^* &:= \max\{\arg_{c_n} s_o = S_H, 0\}, \\ b_n^* &:= \arg_{c_n}\{s_o = 0\}. \end{aligned} \tag{4.7}$$

These two values are well defined because of the non-increasing property of $s_o - S_H$ and s_o with respect to the spot price. The choice of the inventory policy, depending on the spot price, can be characterized by the following theorem.

Theorem 4.3.3 *It holds that $0 \le a_n^* \le b_n^* \le c_H$, and furthermore,*

(b) If $c_n > b_n^*$, a base-stock policy to procure from the contract supplier with an order-up-to level S_H is optimal.

(a) If $c_n < a_n^*$, an (s_L, S_L) policy to procure from the spot market is optimal.

(c) Otherwise, i.e., $a_n^* \le c_n \le b_n^*$, an (s_o, S_H, S_L) is optimal.

The result in this theorem can be illustrated by **Figure 4.3**. That is, in each period, we can divide the spot price state into three regions, each of which corresponds to a policy of (s, S) type we discussed previously.

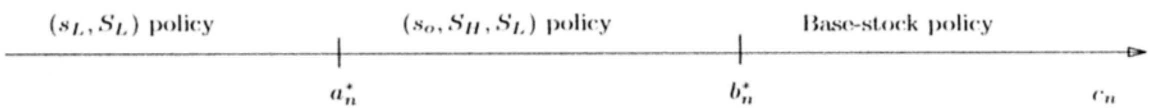


Figure 4.3: Choice of the optimal policy by the spot price

Remark 4.3.2 In Fox et al. (2006), c_n =constant and $c_n < c_H$. However, we can still define a_n^* and b_n^* that correspond to certain c_n values. **Theorem 3** further refines the structured result of Fox et al. (2006) by specifying the thresholds wherein one of the three polices applies.

Intuitively, there should exist a threshold price p_n, depending on the initial inventory level x, such that the buyer should switch between the contract supplier and the spot market according to the current spot price. This is stated in the following lemma.

Lemma 4.3.2 *In any period n, there is a threshold price $p_n(x) \in [a_n^*, b_n^*]$, which decreases in x, such that the buyer should procure from the contract supplier if $c_n \ge p_n(x)$, or the spot market otherwise.*

In addition, this lemma indicates that when the on-hand inventory is higher, the threshold price that the buyer procures from the spot market is lower. Then, in each period, when the spot price changes from low to high, the buyer switches from the spot market with order-up-to level S_L to the contract supplier with order-up-to level S_H.

Bounds on S_L and S_H

We define the values $\overline{S}$ and $\underline{S}$ as follows:

$$\begin{aligned}\overline{S}(c) &= \arg\min_y\{L(y) + [c - \alpha c_H]y\},\\ \underline{S} &= \arg\min_y\{L(y) + c_H y\}.\end{aligned}$$

The lower-and-upper bounds of S_L and S_H for each period can be characterized by the following result.

Lemma 4.3.3 *In any period n, if $c_n < c_H$, we have $\underline{S} \leq S_H \leq S_L \leq \overline{S}(c_n)$.*

Discussion of Assumption 4.3.2

The above analysis in this section bases on **Assumption 4.3.2**. If it is not met, the optimal policy may no longer exhibit monotonic properties. We demonstrate this point by a counter example, which will be detailed in **Figure 4.6** in the next section.

4.4 Numerical Illustration

In this section, we conduct an extensive numerical analysis to provide qualitative insights that may be useful in applications. To simplify the analysis, we assume the minimum commitment P_i to be same for all i, i.e., $P_i = P (i = 1, ..., N)$.

Nevertheless, different P_i's can also be incorporated. In particular, we first show the monotone properties we obtained under the condition (**Assumption 4.3.2**) we provided previously. Afterwards, we investigate the impacts of the spot price volatility, the spot price drift, and the contract price c_H on the buyer's performance.

The spot price used in our numerical analysis follows a Geometric Brownian Motion, which is formulated as

$$dc_t = c_t \mu dt + c_t \sigma dw_t,$$

where w_t is a standard Wiener process. Actually, more commodity price processes can be incorporated into our model to analyze varying market conditions. Such price processes are widely seen in financial literature (e.g., Hull (2000)), and their parameters (e.g., μ and σ) can be estimated from real data.

We first discretize the continuous GBM process by the (Cox-Ross-Rubinstein) binomial tree method. That is, we assume the process of $p_t e^{\mu_t}$ is a martingale under the risk-neutral probability, and the risk-neutral interest rate is zero (this is for simplicity as a non-zero interest rate has no impact on the following analysis), and we divide one period into k stages (thus, there are Nk stages during the planning horizon). The CRR model then tells us that there are two price states in the next stage, C_u and C_d, depending on the spot price, C, in the current stage. The relationships between these values are as follows:

$$C_u = e^{\frac{\mu}{Nk} + \sqrt{\frac{\sigma}{Nk}}} C \quad \text{and} \quad C_d = e^{\frac{\mu}{Nk} - \sqrt{\frac{\sigma}{Nk}}} C. \tag{4.8}$$

The probabilities associated with these two states, $\mathbb{p}_u$ and $\mathbb{p}_d$, are

$$\mathbb{p}_u = \frac{1 - e^{-\frac{\sigma}{Nk}}}{e^{\frac{\sigma}{Nk}} - e^{-\frac{\sigma}{Nk}}} \quad \text{and} \quad \mathbb{p}_d = 1 - \mathbb{p}_u.$$

Thus, such discreting approximation indicates that only the price volatility affects the upward and downward probability.

The parameters of our base model are listed in **Table 4.1**. Others are as follows: $k = 1$ (which is in fact without loss of generality), the initial inventory at the beginning of the planning horizon is 0 and the discount fact $\alpha = 0.8$.

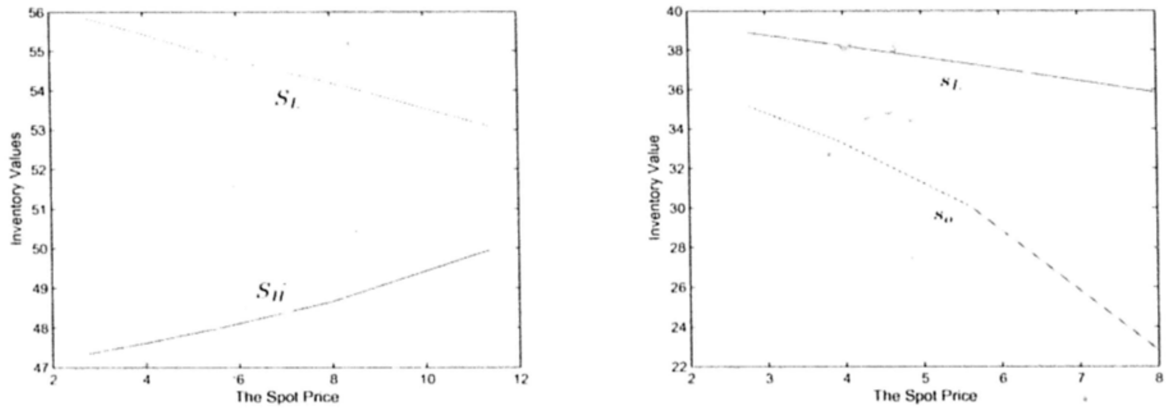
Table 4.1: Parameters settings

	Normal Dist.		Shortage Cost = 30
Demand Distribution	μ_d=50	Cost Paremeter	Setup Cost K = 200
	$\sigma_d = 10$		Holding Cost = 8
	Initial price $c_N = 10$	Min-Commitment	$P = 0$
Price Process (GBM)	$\mu = -0.3$	Contract Price	$c_H = 15$
	$\sigma = 0.5$	Number of Periods	$N = 8$

It is meaningful to discuss the settings of some of the parameters here. As we can see from **Figure 4.1**, the spot price for iron ore in December 2008 was about \$70/Tonne, while the long-term contract price was \$91/Tonne. Moreover, it is estimated that the average spot price for iron ore during January-July 2009 is \$58.64/Tonne (see http://www.metal-pages.com/news/story/36912/). To reflect this relative scale, we set $c_H = 15$, $c_N = 10$ and $\mu = -0.3$ in our numerical analysis. Nevertheless, the relative value of c_H and c_N will vary in subsequent experiments, which includes the case for $c_n < c_H$ as well.

The monotone properties between the policy parameters and the spot price of our base model are illustrated in **Figure 4.4**. They are consistent with our previous results.

In the following sections, we analyze the effect of the various parameters of our



(a) The value of S_H and S_L (b) The value of s_L and s_o

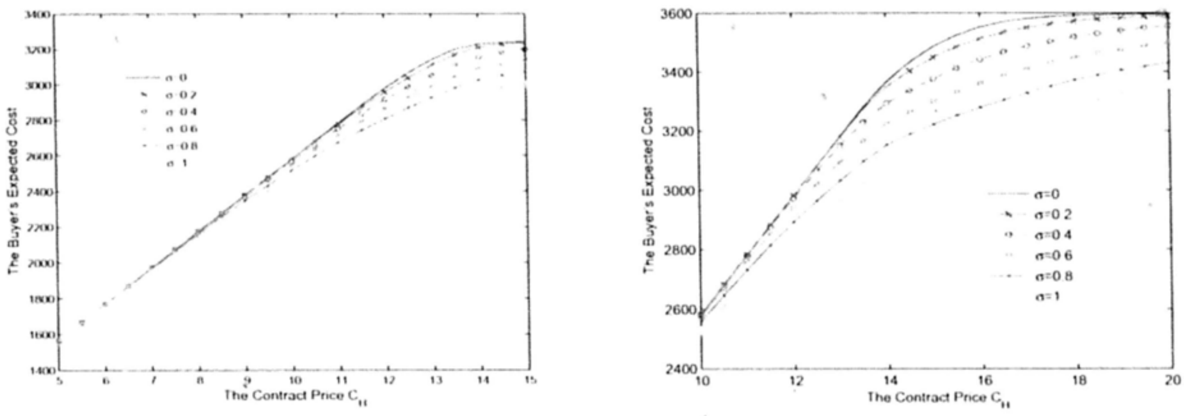
Figure 4.4: An example of monotone properties

model (i.e., contract price, market volatility, and price drift) on the performance of the buyer.

4.4.1 The effect of contract price, market volatility and drift

In this part, we analyze the effect of the contract price and the spot market parameters, i.e., the volatility and the price drift, on the buyer's performance, which is measured by his minimum expected cost. In doing so, we vary the value of σ in $[0, 1]$ and the value of c_H in $[5, 15]$ for the negative drift case (or $[10, 20]$ for the positive drift case) with the other parameters unchanged. The resulting performance is shown in **Figure 4.5**.

It is intuitive to see that the higher the contract price is, the higher the buyer's cost is. However, it is interesting to note that **Figure 4.5** indicates that the buyer's performance improves as the volatility of the spot market increases. This phenomenon can be explained as follows. First, when the volatility increases, the buyer can operationally adjust his inventory level to hedge the price move-



(a) $\mu = -0.3$ (b) $\mu = 0.3$

Figure 4.5: Effect of c_H on the buyer's discounted expected cost.

ments. Second, the contract supplier provides a hedging mechanism to the spot prices when they are higher than the contract price. Thus, the buyer prefers a more volatile spot market if he(she) has a minimum commitment contract with a predetermined contract price.

The effect of the price drift on the buyer's expected discounted cost is intuitive: a positive price drift always results in a higher cost for the buyer, and a price process with a higher drift increases the expected procurement quantity from the contract supplier.

Finally, we show an example in which **Assumption 4.3.2** is not met, and the monotone properties may no longer hold. In **Figure 4.6**, the price process follows a Geometric Brownian Motion with the initial price $c_N = 1$ (N=8) and $\mu = 2.5$, and we show the value of S_H in the fifth period (i.e., period 3). This is a Geometric Brownian Motion with a positive price drift that does not satisfy **Assumption 4.3.2**.

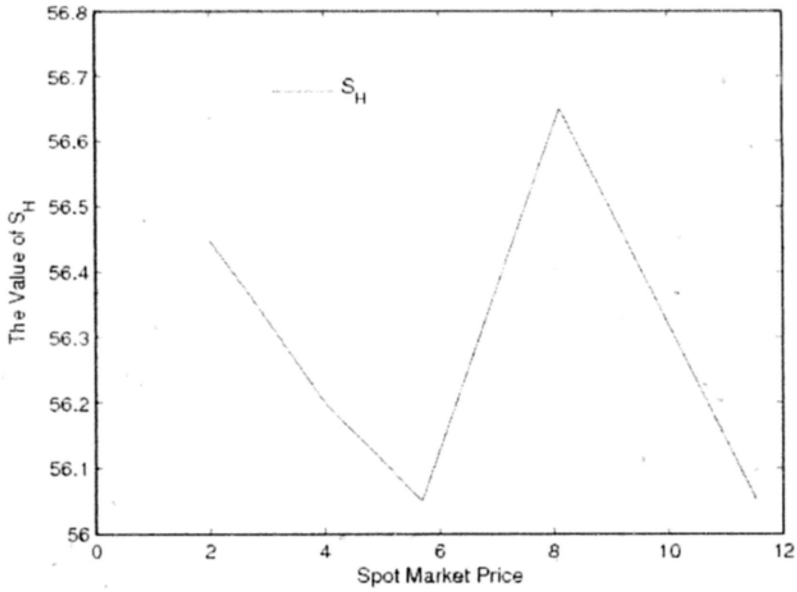


Figure 4.6: A counter example for order-up-to levels S_H in period 3

4.5 Summary

In this chapter, we consider the problem of procurement when the buyer faces a long-term contract supplier and a spot market. We prove that a reduced generalized (s, S) policy is optimal for the buyer. Furthermore, we identify certain conditions, under which the monotone properties of the various policy parameters exist when the price process follows a Markov process. Then, we analyze how the market environment and contract parameters impact the performance of the buyer through a numerical experiment. we find that the buyer prefers a more volatile environment as the contract supplier provides a natural hedge for the higher future spot price.

Our current research is just a preliminary work on the inventory policy in the presence of the spot market. More extensions can be made to this model. One extension can be generalizing our model to consider that the contract supplier provides a total minimum commitment contract for the planning horizon (see

Bassok and Anupindi (1997), Chen and Krass (2003)). In our current model, we only assume a predetermined minimum procurement quantity for each period during the planning horizon.

Another possible extension is to consider both a minimum commit quantity and a maximum capacity for the buyer to procure from the supplier in each period, i.e., we can relax **Assumption 4.3.1** by introducing a capacity, $P_{\max}$, which limits the maximum quantity the buying firm can order from the contract supplier in each period. This is important as knowing that the buyer will take advantage of the spot market and the long-term contract, the contract supplier may set lower-and-upper bounds so that the buyer's ability to "exploit" the contract supplier is restrained. We conjecture a modified (s, S) policy to be optimal. Specifically, the order-up-to level from the contract supplier should be the minimal value of two values. One is an inventory-independent level and the other is the order-up-to level by procuring all capacity from the contract supplier. However, to verify formally the optimality of such a policy entails a different approach. To illustrate, consider the case when $c_H < c_n$ in period n. Due to the capacity constraint, the firm may also buy from the spot market after the contract capacity has been exhausted yet the inventory level is too low. This leads to the failure of the condition $\frac{\partial \bar{f}_n(x,c_n)}{\partial x} \geq -c_H$ for the new value function $\bar{f}_n(x, c_n)$. This is a significant departure from our current model. Nevertheless, from the current analysis, we conjecture that a modified (s, S) policy is optimal and the order-up-to level from the contract supplier should be the minimal value of two values: an inventory-independent level and the order-up-to level by procuring all capacity from the contract supplier. These are all highlighted in the **Appendix** (see **Remark 4.5.3**). However, the failure of the condition just mentioned results in the failure of the K-convexity of the cost-to-go function for the lost-sales model. Such an extension is another direction for our future research.

Appendix: Proofs and Remarks

In this section, the following notation will be used: $\widetilde{f}_n(z, c_n) = f_n(z^+, c_n)$,

$$r_n(z) = \alpha \int_0^\infty \widetilde{f}_n(z, c_n)\theta(c_n|c_{n+1})dc_n + c_s z^- + c_h z^+$$

and $m_n(z) = r_n(z) + c_H z$.

We always assume the partial/corss partial derivatives exist for all functions in our model, e.g., $f_n(x, c_n)$, $G_n(x, c_n)$. This is made for simplicity and is based on the fact that there are at most a finite numbers of points without derivatives. At those points, the derivative is taken to be the maximum value within the right and left derivatives. And the partial/corss derivatives of these functions with two variables is denoted by the same way as (4.6). To reflect the dependence of policy parameters on the spot price, we restore the notation s_o, s_L, S_L and S_H to its original form, i.e., $s_{o,n}(c_n)$, $s_{L,n}(c_n)$, $S_{L,n}(c_n)$ and $S_{H,n}(c_n)$, respectively.

We need the next four lemmas to prepare the proof of **Theorem 1**. **Lemma A1** is a technical result to provide the condition under which a continuous left-linear expansion of a K-convex function is K-convex. **Lemma A3** and **Lemma A4** are about the optimal policies for the one period optimal inventory problems, $J_H(x, c_n)$ and $J_L(x, c_n)$, respectively.

Lemma A1 *Suppose $f(x)$ is K-convex on $[a, b]$ (b can be infinite). Given $d < a$ (d can be negative infinite), define $g(x)$ as*

$$g(x) = \begin{cases} c(x-a) + f(a), & x \in [d, a); \\ f(x), & x \in [a, b]., \end{cases}$$

where c is a constant. Then, $g(x)$ is K-convex if

$$c \le \frac{f(k(c)) + K - f(a)}{k(c) - a} \tag{4.9}$$

where $k(c)$ is the maximum value among the optimal points of $\min_{x\in[a,b]}[-cx + f(x)]$. As a special case, if $c \le f'(x)|_{x\to a^+}$, then, $g(x)$ is K-convex in $[d,b]$.

Proof: The existence of $k(c)$ for any c is guaranteed by the K-convexity of $f(x)$.

It is obvious that $g(x)$ is K-convex when it is restricted on the regions $[d,a)$ or $[a,b]$. Thus, we only show for any $d \le x \le a \le y \le b$, $g(\lambda x + (1-\lambda)y) \le \lambda g(x) + (1-\lambda)(g(y)+K)$.

If $\lambda x + (1-\lambda)y \in [d,a)$,

$$\begin{aligned} g(\lambda x + (1-\lambda)y) &= c(\lambda x + (1-\lambda)y - a) + f(a) \\ &\le \lambda g(x) + (1-\lambda)(g(y)+K) + (1-\lambda)(f(a) + c(y-a) - f(y) - K) \\ &\le \lambda g(x) + (1-\lambda)(g(y) - K(1-\lambda)(y-a)(\frac{f(y)+K-f(a)}{y-a} - c) \\ &\le \lambda g(x) + (1-\lambda)(g(y)+K). \end{aligned}$$

The last inequality holds since $\frac{f(y)+K-f(a)}{y-a} \ge \frac{f(k(c))+K-f(a)+c(y-k(c))}{y-k(c)+k(c)-a} \ge \frac{f(k(c))+K-f(a)}{k(c)-a} \ge c$.

If $\lambda x + (1-\lambda)y \in [a,b]$,

$$\begin{aligned} g(\lambda x + (1-\lambda)y) &= f(\lambda x + (1-\lambda)y) \le \lambda\frac{y-x}{y-a}f(a) + (1-\lambda\frac{y-x}{y-a})(f(y)+K) \\ &\le \lambda g(x) + (1-\lambda)(g(y)+K) + \lambda\frac{x-a}{y-a}[f(y)+K-f(a)-c(y-a)] \\ &\le \lambda g(x) + (1-\lambda)(g(y)+K). \end{aligned}$$

Thus, the first result obtained. When $c \le f'(x)|_{x\to a^+}$, $c \le \frac{f(y)+K-f(a)}{y-a}$ for all $y \in [0,\infty)$. Thus, $g(x)$ is K-convex. ◇

Lemma A2 *Suppose $f_n(x,c_n)$ is K-convex in x and $f_n^{(1)}(x,c_n) \ge -c_H$ on $[0,+\infty)$, then, $r_n(z)$ is K-convex in z on R. Furthermore, $G_n(y,c_{n+1})$ is K-convex in y on R.*

Proof: Since $f_n(x, c_n)$ is K-convex in x, $\widetilde{f}_n(z, c_n)$ is K-convex in z on $[0, +\infty)$. Furthermore, $U(z) = \alpha \int_0^{\infty} \widetilde{f}_n(z, c_n)\theta(c_n|c_{n+1})dc_n$, is K-convex in z and $U'(z) \geq -c_H$ on $[0, +\infty)$. Then,

$$r_n(z) = \begin{cases} U(z) + c_h z, & z \in [0, +\infty); \\ U(0) - c_s z, & \text{Otherwise.} \end{cases}$$

Thus, $r_n(z)$ is continuous on R, K-convex on $[0, \infty)$ and decreases in z on $(-\infty, 0)$. And $-c_s \leq -c_H \leq (U(z)+c_h z)'|_{z \to 0^+}$, we therefore see $r_n(z)$ satisfies the condition of **Lemma A1**. Hence, $r_n(z)$ is K-convex, and so is $G_n(y, c_{n+1}) = \mathrm{E}r(y - \xi)$. $\diamond$

Lemma A3 *Suppose $f_{n-1}(x, c_{n-1})$ satisfies $f_{n-1}^{(1)}(x, c_{n-1}) \geq -c_H$ on $[0, +\infty)$, then, $m_n(z)$ is quasi-convex and the following single period problem*

$$J_H(x, c_n) = \min_{y \geq x}\{c_H(y - x) + G_{n-1}(y, c_n)\}$$

has a base-stock policy with order-up-to level $S_{H,n}(c_n) > 0$. Furthermore, $J_H(x, c_n)$ is K-convex in x and $J_H^{(1)}(x, c_n) \geq -c_H$ on R.

Proof: We have

$$m_n(z) = \begin{cases} c_h z + c_H z + U(z), & z \geq 0; \\ -c_s z + c_H z + U(0), & z < 0. \end{cases}$$

where $U(x)$ is defined as that in **Lemma A2**. As $f_{n-1}^{(1)}(x, c_{n-1}) \geq -c_H$ when $x > 0$, we get $U'(z) \geq -c_H$ for $z \geq 0$. Then, $m_n(z)$ is continuous on R, decreases on $(-\infty, 0]$ and increases on $(0, +\infty)$. Thus, $m_n(z)$ is quasi-convex with minimum point 0.

Since $c_H y + G_{n-1}(y, c_n) = c_H \mu + \mathrm{E}m_n(y - \xi)$, where μ is the expected demand, and the demand distribution is strongly unimodal with a positive support, $c_H y + G_{n-1}(y, c_n)$ is quasi-convex in y. Furthermore, $[c_H y + G_{n-1}(y, c_n)]'|_{y=0} = \mathrm{E}m_n'(y-$

$\xi)|_{y=0} < 0$. Thus, the optimal y, denoted by $S_{H,n}(c_n)$, exists and $S_{H,n}(c_n) > 0$. That is, the optimal policy for J_H is a base stock policy with an order-up-to level $S_{H,n}(c_n)$, $G_{n-1}^{(1)}(y, c_n) \geq -c_H$ when $y \geq S_{H,n}(c_n)$ and $G_{n-1}^{(1)}(y, c_n) < -c_H$ when $y < S_{H,n}(c_n)$.

The minimum cost corresponding to the optimal policy is

$$J_H(x, c_n) = \begin{cases} G_{n-1}(x, c_n), & \text{if } x \geq S_{H,n}; \\ c_H(S_{H,n}(c_n) - x) + G_{n-1}(S_{H,n}(c_n), c_n), & \text{otherwise.} \end{cases}$$

Therefore, $J_H(x, c_n)$ is K-convex in x and $J_H^{(1)}(x, c_n) \geq -c_H$. ◇

Lemma A4 *Suppose $f_{n-1}(x, c_{n-1})$ is K-convex in x and $f_{n-1}^{(1)}(x, c_{n-1}) \geq -c_H$ on $[0, +\infty)$, then, the optimal policy for the following single period problem when $c_n \leq c_H$,*

$$J_L(x, c_n) = \min_{y \geq x}\{K\delta(y - x) + c_n(y - x) + G_{n-1}(y, c_n)\},$$

is an $(s_{L,n}(c_n), S_{L,n}(c_n))$ policy. Furthermore, $S_{H,n}(c_n) \leq S_{L,n}(c_n)$ for each c_n and $J_L(x, c_n)$ is K-convex in x on R.

Proof: As proved in **Lemma A2**, in our setting, $G_{n-1}(y, c_n)$ is K-convex in y, then, $c_n y + G_{n-1}(y, c_n)$ is K-convex in y. Thus, $S_{L,n}(c_n) = \max\{\arg\min_y c_n y + G_{n-1}(y, c_n)\}$ and $s_{L,n}(c_n) = \max\{\arg\min_r \{r | c_n r + G_{n-1}(r, c_n) = c_n S_{L,n} + G_{n-1}(S_{L,n}, c_n) + K\}\}$ exist and the optimal policy is an $(s_{L,n}(c_n), S_{L,n}(c_n))$ policy.

Because $G_{n-1}^{(1)}(y, c_n) \leq -c_H$ when $y \leq S_{H,n}(c_n)$ as in the proof of **Lemma A3**, $c_n y + G_{n-1}(y, c_n)$ strictly decreases in y whenever $y \leq S_{H,n}(c_n)$. Thus, $S_{H,n}(c_n) \leq S_{L,n}(c_n)$.

The optimal cost function corresponding to the optimal policy is

$$J_L(x, c_n) = \begin{cases} G_{n-1}(x, c_n), & \text{if } x \geq s_{L,n}(c_n); \\ K + c_n(S_{L,n}(c_n) - x) + G_{n-1}(S_{L,n}(c_n), c_n), & \text{otherwise.} \end{cases}$$

Hence, $J_L(x, c_n)$ is K-convex in x. ◇

Proof of Theorem 4.3.1: We show inductively that the following two properties are satisfied for all periods:

1) $f_n(x, c_n)$ is K-convex in x on $[0, +\infty)$;

2) $f_n(x, c_n)$ is continuous on $[0, +\infty)$ and $f_n^{(1)}(x, c_n) \geq -c_H$.

These two properties obviously hold when $n = 0$ since $f_0(x_0, c_0) = 0$. Then, for each other period n, we know

$$f_n(x, c_n) = \min\{J_L(x, c_n), J_H(x, c_n)\},$$

and suppose these 1) and 2) hold for $n = k$. We prove those also hold for $n = k+1$.

When $c_n \geq c_H$, it is obvious that the buyer procures from the contract supplier, thus, $f_n(x, c_n) = J_H(x, c_n)$ is K-convex in x, and $f_n^{(1)}(x, c_n) = J_H^{(1)}(x, c_n) \geq -c_H$ by **Lemma A3**.

When $c_n < c_H$, we discuss the buyer's optimal policy according to the following three scenarios.

(a) When $s_{o,n}(c_n) \geq S_{H,n}(c_n)$. In this case, for all inventory level $0 \leq x \leq s_{L,n}(c_n)$, $J_L(x, c_n) \leq J_H(x, c_n)$, and hence, the buyer should procure from the spot market and a pure $(s_{L,n}(c_n), S_{L,n}(c_n))$ policy is optimal. And $f_n(x, c_n) = J_L(x, c_n)$ is K-convex. Furthermore, because $s_{L,n}(c_n) \geq S_{H,n}(c_n)$, we have $G_{n-1}^{(1)}(x, c_n) \geq -c_H$ for $x \geq s_{L,n}(c_n)$ by **Lemma A3**. Hence, $f_n^{(1)}(x, c_n) = J_L^{(1)}(x, c_n) \geq -c_H$.

(b) When $s_{o,n}(c_n) \leq 0$. In this case, for all inventory level x satisfying $0 \leq x \leq s_{L,n}(c_n)$, $J_H(x, c_n) \leq J_L(x, c_n)$, thus, the buyer should procure from the contract supplier only, and a pure base-stock policy with order-up-to level $S_{H,n}(c_n)$ is optimal. Moreover, $f_n(x, c_n) = J_H(x, c_n)$ is K-convex and $f_n^{(1)}(x, c_n) = J_H^{(1)}(x, c_n) \geq -c_H$.

(c) When $0 \leq s_{o,n}(c_n) \leq S_{H,n}(c_n)$. In this case, for inventory level satisfying $0 \leq x \leq s_{o,n}(c_n)$, $J_L(x, c_n) \leq J_H(x, c_n)$, and for initial inventory level $x > s_{o,n}(c_n)$, $J_L(x, c_n) \geq J_H(x, c_n)$. Thus, an $(s_{o,n}(c_n), S_{L,n}(c_n), S_{H,n}(c_n))$ policy is optimal. Hence,

$$f_n(x, c_n) = \begin{cases} G_{n-1}(x, c_n), & \text{if } x \geq S_{H,n}(c_n); \\ c_H(S_{H,n}(c_n) - x) + G_{n-1}(S_{H,n}(c_n), c_n), & \text{if } s_{o,n}(c_n) \leq x < S_{H,n}(c_n); \\ c_n(S_{L,n}(c_n) - x) + G_{n-1}(S_{L,n}(c_n), c_n), & \text{otherwise.} \end{cases} \tag{4.10}$$

is K-convex in $[0, +\infty)$ by **Lemma A1**. And $f_n^{(1)}(x, c_n) \geq -c_H$ by **Lemma A3**.

To sum up, we have shown that 1) and 2) also hold for $n = k + 1$. The property that $f_n(x, c_n)$ is continuous is straight forward to verify. Therefore, the optimal policy is as the theorem indicates. ◇

Remark 4.5.1 The proof of this theorem is similar to that in Fox et al. (2006). However, we use the property that the linear combination of K-convex functions is also K-convex, to handle the uncertain variable price. Furthermore, we use **Lemma A1** to simplify the proof.

Remark 4.5.2 The analysis above can be applied to deal with the model when the excess demand is backlogged, by noting the fact that $f_n(x, c_n)$ decreases in x when $x \leq 0$. In this case, the policy when $c_H > c_n$ and $s_o \leq 0$ is the (s_o, S_H, S_L) policy discussed in **Theorem 4.3.1(c)**.

Proof of Corollary 4.3.1: From (4.5), when $c_n < c_H$, we have

$$c_H(S_{H,n}(c_n)-s_{o,n}(c_n))+G_{n-1}(S_{H,n}(c_n),c_n) = K+c_n(S_{L,n}(c_n)-s_{o,n}(c_n))+G_{n-1}(S_{L,n}(c_n),c_n). \tag{4.11}$$

Because of the definition of $S_{L,n}(c_n)$, we have

$$\begin{aligned} G_{n-1}(S_{L,n}(c_n),c_n) + c_nS_{L,n}(c_n) &\leq G_{n-1}(S_{H,n}(c_n),c_n) + c_nS_{H,n}(c_n), \\ G_{n-1}(S_{H,n}(c_n),c_n) + c_HS_{H,n}(c_n) &\leq G_{n-1}(S_{L,n}(c_n),c_n) + c_HS_{L,n}(c_n). \end{aligned} \tag{4.12}$$

Combining (4.11) and (4.12) yields $S_{H,n}(c_n) - s_{o,n}(c_n) \leq \frac{K}{c_H-c_n} \leq S_{L,n}(c_n) - s_{o,n}(c_n)$. ◇

Lemma A5 *Suppose that $f_1(x)$ and $f_2(x)$ are both differentiable functions that have finite minimizers, and $f_1'(x) \leq f_2'(x)$. Let $S_i = \max\arg\min f_i(x)$, $i = 1,2$, then $S_1 \geq S_2$.*

Proof: Suppose to the contrary, $S_1 < S_2$. Then,

$$\begin{aligned} f_1(S_2) - f_1(S_1) &= \int_{S_1}^{S_2} f_1'(x)dx \\ &\leq \int_{S_1}^{S_2} f_2'(x)dx \leq f_2(S_2) - f_2(S_1), \end{aligned}$$

where the first formula is positive and the last formula is non-positive. Therefore, a contradiction arises, and the lemma holds true. ◇.

Proof of Lemma 4.3.1: We prove this lemma inductively. The result holds obviously for $n = 0$. We assume it also holds for $n-1$, we want to prove it is true for n. We assume the $(s_{o,n}(c_n), S_{L,n}(c_n), S_{H,n}(c_n))$ policy is used in period n. (The proof procedures for the other two are same.)

The value function, $f_n(x, c_n)$, is in a form as (4.10). Thus, the derivative of

$f_n(x, c_n)$ with respect to c_n, $f_n^{(2)}(x, c_n) =$

$$\begin{cases} \alpha \mathrm{E}[(H'(c_n) + V'(c_n)\varepsilon)W_n(x)], & \text{if } x \geq S_{H,n}(c_n); \\ \alpha \mathrm{E}[(H'(c_n) + V'(c_n)\varepsilon)W_n(S_{H,n}(c_n))], & \text{if } s_{o,n} \leq x < S_{H,n}(c_n); \\ (S_{L,n}(c_n) - x) + \alpha \mathrm{E}[(H'(c_n) + V'(c_n)\varepsilon)W_n(S_{L,n}(c_n))], & \text{otherwise,} \end{cases}$$

where $W_n(u) = f_{n-1}^{(2)}((u-\xi)^+, H(c_n) + V(c_n)\varepsilon)$ and the expectation is taken on both the demand uncertainty and price uncertainty. Because $f_{n-1}^{(2)}(x, c_{n-1}) \geq 0$ as we assume before, and $H'(c_n) + V'(c_n)\varepsilon \geq 0$ as in **Assumption 4.3.2**, we get $f_n^{(2)}(x, c_n) \geq 0$ for each x. That is, $f_n(x, c_n)$ also increases in c_n.

Now, taking derivative of $f_n^{(2)}(x, c_n)$ with respect to x, we get $f_n^{(12)}(x, c_n) =$

$$\begin{cases} \alpha \mathrm{E}[(H'(c_n) + V'(c_n)\varepsilon)f_{n-1}^{(12)}(x-\xi, H(c_n) + V(c_n)\varepsilon)|\xi \leq x], & \text{if } x \geq S_{H,n}(c_n); \\ 0, & \text{if } s_{o,n} \leq x < S_{H,n}(c_n); \\ -1, & \text{otherwise.} \end{cases}$$

Since $-1 \leq f_{n-1}^{(12)}(x, c_n) \leq 0$ as has been assumed for period $n-1$, we have $-1 \leq f_n^{(12)}(x, c_n) \leq 0$.

Thus, the results also hold for n. So, we can conclude the results hold for all n. ◇

Proof of Theorem 4.3.2: Let $S_{L,n}(c_n^1)$, $s_{L,n}(c_n^1)$, $S_{H,n}(c_n^1)$, $s_{o,n}(c_n^1)$ and $S_{L,n}(c_n^2)$, $s_{L,n}(c_n^2)$, $S_{H,n}(c_n^2)$, $s_{o,n}(c_n^2)$ be the policy parameters in period n when the spot market price are c_n^1 and c_n^2, respectively. Let $c_n^1 \geq c_n^2$. We prove this result when an $(s_{o,n}(c_n), S_{L,n}(c_n), S_{H,n}(c_n))$ policy is applied in period n, the other two polices can also be incorporated by the same procedure. The proof is divided into three parts: part (1) for the monotone property of $S_{L,n}(c_n)$; part (2) for that of $S_{H,n}(c_n)$; part (3) for that of $s_{L,n}(c_n)$ and $s_{o,n}(c_n)$.

(1) As we mentioned earlier, the value of $S_{L,n}(c_n^i)$ is the minimum point of

$g_i(y) = c_n^i y + G_{n-1}(y, c_n^i), i = 1, 2$. Furthermore, we have

$$g_1'(y) - g_2'(y) = c_n^1 - c_n^2 + \alpha \mathrm{E}[f_{n-1}^{(1)}((y-\xi)^+, H(c_n^1) + V(c_n^1)\varepsilon) - f_{n-1}^{(1)}((y-\xi)^+, H(c_n^2) + V(c_n^2)\varepsilon)]. \tag{4.13}$$

For each realized $\xi \leq y$ and ε, by the mean value theorem and **Lemma 4.3.1**, we get

$$\begin{aligned} & f_{n-1}^{(1)}((y-\xi)^+, H(c_n^1) + V(c_n^1)\varepsilon) - f_{n-1}^{(1)}((y-\xi)^+, H(c_n^2) + V(c_n^2)\varepsilon) \\ & \geq -[H(c_n^1) - H(c_n^2) + \varepsilon(V(c_n^1) - V(c_n^2))]. \end{aligned} \tag{4.14}$$

Combing (4.13) and (4.14) yields,

$$g_1'(y) - g_2'(y) \geq (c_n^1 - c_n^2) - \alpha[H(c_n^1) - H(c_n^2)] \geq 0,$$

where the last inequality is by **Assumption 4.3.2**. Then, $S_{L,n}(c_n^1) \leq S_{L,n}(c_n^2)$ by **Lemma A5** . Therefore, $S_{L,n}(c_n)$ is non-increasing with c_n.

(2) The value of $S_{H,n}(c_n^i)$ is the minimum point of $v_i(y) = c_H y + G_{n-1}(y, c_n^i), i = 1, 2$. Because we have

$$v_1'(y) - v_2'(y) = \alpha \mathrm{E}[f_{n-1}^{(1)}((y-\xi)^+, H(c_n^1) + V(c_n^1)\varepsilon) - f_{n-1}^{(1)}((y-\xi)^+, H(c_n^2) + V(c_n^2)\varepsilon)] \leq 0,$$

thus, $S_{H,n}(c_n^1) \geq S_{H,n}(c_n^2)$, that is, $S_{H,n}(c_n)$ is nondecreasing in c_n.

(3) We know that $s_{L,n}(c_n)$ is the root for the following equation,

$$H(x, c_n) := xc_n + G_{n-1}(x, c_n) - [c_n S_{L,n}(c_n) + G_{n-1}(S_{L,n}(c_n), c_n) + K] = 0.$$

To explore the monotone property of $s_{L,n}(c_n)$, we first prove $G_{n-1}^{(1)}(y, c_n)|_{y=s_{L,n}(c_n)} \leq -c_n$. If not, $xc_n + G_{n-1}(x, c_n)$ strictly increases in x at the point s_L, then, there exists $x_1 < s_{L,n}(c_n)$ such that

$$x_1 c_n + G_{n-1}(x_1, c_n) < s_{L,n}(c_n) c_n + G_{n-1}(s_{L,n}(c_n), c_n) = c_n S_{L,n} + G_{n-1}(S_{L,n}(c_n), c_n) + K.$$

This contradicts the fact that $xc_n+G_{n-1}(x,c_n) \geq c_n S_{L,n}(c_n)+G_{n-1}(S_{L,n}(c_n),c_n)+K$ whenever $x < s_{L,n}(c_n)$, thus, $G'^{(1)}_{n-1}(y,c_n)|_{y=s_{L,n}(c_n)} \leq -c_n$.

Now we turn to prove the monotone property of $s_{L,n}(c_n)$. First, deriving the total differentiation of $H(x,c_n)$ with respect to c_n and equating it to zero, we get

$$dH(x,c_n) = (c_n+G'^{(1)}_{n-1}(x,c_n))\frac{\partial x}{\partial c_n}+x+G'^{(2)}_{n-1}(x,c_n)-[S_{L,n}(c_n)+G'^{(2)}_{n-1}(S_{L,n}(c_n),c_n)] = 0. \tag{4.15}$$

In (4.15), $x + G'^{(2)}_{n-1}(x,c_n) - [S_{L,n}(c_n) + G'^{(2)}_{n-1}(S_{L,n}(c_n),c_n)] = x - S_{L,n}(c_n) + \alpha \mathrm{E}(H'(c_n)+V'(c)\varepsilon)[f'^{(2)}_{n-1}((x-\xi)^+, H(c_n)+V(c_n)\varepsilon)-f'^{(2)}_{n-1}((S_{L,n}(c_n)-\xi)^+, H(c_n)+V(c_n)\varepsilon)]$.

For each realization ξ and $0 \leq x \leq S_{L,n}(c_n)$, define

$$M(x) := f'^{(2)}_{n-1}((S_{L,n}(c_n)-\xi)^+, H(c_n)+V(c_n)\varepsilon) - f'^{(2)}_{n-1}(x-\xi)^+, H(c_n)+V(c_n)\varepsilon).$$

Then, $M(x)$ only takes one of the following values:

$$\begin{cases} f'^{(2)}_{n-1}(S_{L,n}(c_n)-\xi, H(c_n)+V(c_n)\varepsilon) - f'^{(2)}_{n-1}(x-\xi, H(c_n)+V(c_n)\varepsilon), & \text{if } \xi \leq x; \\ f'^{(2)}_{n-1}(S_{L,n}(c_n)-\xi, H(c_n)+V(c_n)\varepsilon) - f'^{(2)}_{n-1}(0, H(c_n)+V(c_n)\varepsilon), & \text{if } x \leq \xi \leq S_{L,n}(c_n); \\ 0, & \text{otherwise.} \end{cases}$$

Thus, $M(x) \geq -1(S_{L,n}(c_n)-x)$ by **Lemma 4.3.1**. Hence,

$$x+G'^{(2)}_{n-1}(x,c_n)-[S_{L,n}(c_n)+G'^{(2)}_{n-1}(S_{L,n}(c_n),c_n)] \leq (1-\alpha H'(c_n))(x-S_{L,n}(c_n)) \leq 0. \tag{4.16}$$

Combining (4.15) and (4.16), with the fact that $G'^{(1)}_{n-1}(y,c_n)|_{y=s_{L,n}(c_n)} \leq -c_n$, we have $\frac{\partial s_{L,n}(c_n)}{\partial c_n} \leq 0$. Thus, $s_{L,n}(c_n)$ is non-increasing in c_n.

The non-increasing property of $s_{o,n}(c_n)$ can be proved by similar procedure as that for $s_{L,n}(c_n)$ by substitute $G_{n-1}(x,c_n)$ with $J_H(x,c_n)$. We neglect the proof here. ◇

Proof of Theorem 4.3.3: If $c_n > b_n^*$, from the definition of b_n^* in (4.7), we know $s_{o,n}(c_n) \leq s_{o,n}(b_n^*) = 0$. Thus, a base-stock policy is optimal as in **Theorem 4.3.1**. The other two cases can also be proved accordingly. ◇

Proof of Lemma 4.3.2: We know that $s_{o,n}(c_n)$ is the threshold inventory level, according to which the buyer switch between these two supply sources. When the buyer's initial on-hand inventory level is $x \geq 0$, we can define $p_n(x)$ as follows:

$$p_n(x) = \max\{0, \arg_{c_n} s_{o,n}(c_n) = x\}.$$

$p_n(x)$ is well defined because of the monotonicity of $s_{o,n}(c_n)$. Furthermore, $p_n(x)$ decreases in x as $s_{o,n}(c_n)$ decreases in c_n. ◇

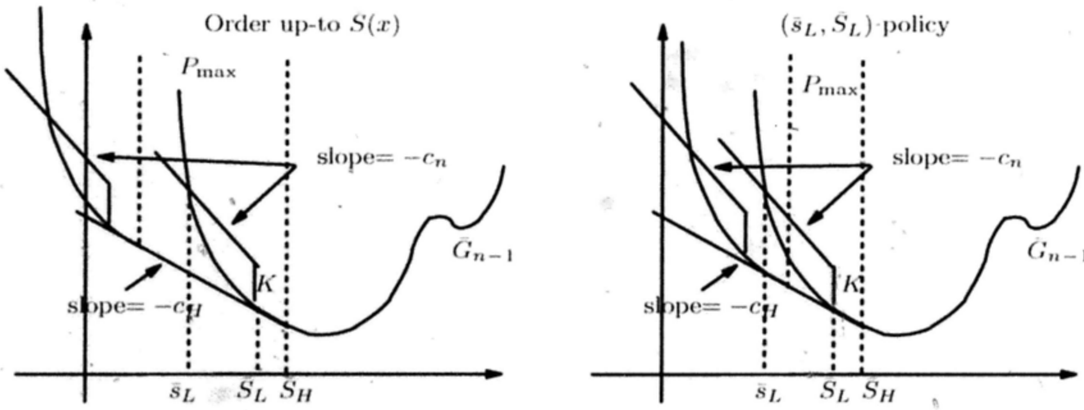
Proof of Lemma 4.3.3: Suppose $S_L > \overline{S}(c)$ when $c_k = c$ in period k, if the policy in this period is (s_L, S_L) (the case for other policies are same), we define a new policy that is $(s_L, \overline{S}(c))$ for this period while preserving the optimal policy for the following periods. Let $\widehat{f}_k(x)$ (we neglect the price variable for simplicity) be the optimal cost corresponding to this new policy when the inventory level is x. Then, when $x \leq s_L$,

$$\begin{aligned}\widehat{f}_k(x) - f_k(x) &= c(\overline{S}(c) - S_L) + L(\overline{S}(c)) - L(S_L) \\ &\quad +\alpha E[f_{k-1}((\overline{S}(c) - \xi)^+) - f_{k-1}((S_L - \xi)^+)] \\ &\leq c(\overline{S}(c) - S_L) + L(\overline{S}(c)) - L(S_L) + \alpha c_H(S_L - \overline{S}(c)) \leq 0,\end{aligned}$$

that is, the new policy achieves lower cost than the original one, that contradicts the optimality of (s_L, S_L). Therefore, $S_L \leq \overline{S}(c)$.

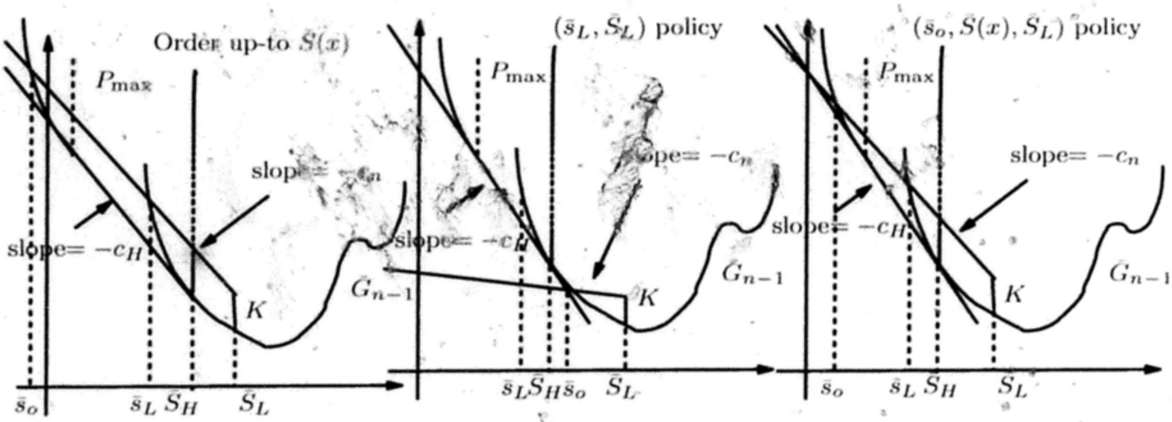
We prove $\underline{S} \leq S_H$ inductively. We first assume this holds for period k. Note that $f_k(x, c_k)$ is non-increasing in x when $x \leq \underline{S}$ by the form of $f_k(x, c)$ in (4.10) and the demand distribution has a positive support. For each realization ξ, $f_k((x - \xi)^+, c_k)$ is non-increasing in x when $x \leq \underline{S}$. Thus, $\mathrm{E}f_k((x - \xi)^+, c_k)$ is non-increasing when $x \leq \underline{S}$. And because $c_H y + L(y)$ is also non-increasing when $x \leq \underline{S}$. $c_H y + L(y) + \alpha \mathrm{E} f_k((x-\xi)^+, c_k)$ is non-increasing when $x \leq \underline{S}$. Thus, $\underline{S} \leq S_H$ in period $k+1$. This yields the fact that $\underline{S} \leq S_H$ for all periods. $\diamond$

Remark 4.5.3 Without loss of generality, we let $P_n = 0$, in this case, if we let $\bar{f}_n(x, c_n)$ and $\bar{G}_{n-1}(x, c_n)$ denote the value function and cost-to-go function for period n, then, according to our analysis in this chapter, we suggest the inventory policy would be one of those illustrated in **Figure 4.7** and **Figure 4.8**.



(a) $c_n \geq c_H$ and $\bar{s}_L \leq P_{\max}$ (b) $c_n \geq c_H$ and $\bar{s}_L > P_{\max}$

Figure 4.7: Inventory policy when $c_n \geq c_H$



(a) $c_n < c_H$ and $\bar{s}_o \leq 0$ (b) $c_n < c_H$ and $\bar{s}_o \geq S_H$ (c) $c_n < c_H$ and $0 < \bar{s}_o < S_H$

Figure 4.8: Inventory policy when $c_n < c_H$

That is, in each period, there exist $\bar{s}_L$, $\bar{S}_L$ and $\bar{S}_H$, which are independent with initial inventory level x. Let $\bar{S}(x) = \min\{\bar{S}_H, x + P_{\max}\}$. Then, when the contract price c_H is lower than the spot price c_n, the buyer orders from the contract supplier before using the spot market. In this case, **Figure 4.7(a)** shows that if $\bar{s}_L \leq P_{\max}$, the buyer should order from the contract supplier up-to $\bar{S}(x)$ if $x \leq \bar{S}_H$, or order nothing otherwise; **Figure 4.7(b)** shows that if $\bar{s}_L > P_{\max}$, the buyer should order $P_{\max}$ from the contract supplier, and then order from the spot market up-to $\bar{S}_L$ if $x + P_{\max} \leq \bar{s}_L$, or order nothing otherwise. When c_H is higher than c_n, the buyer only orders from one of these two sources. Thus, in this case, there exists a threshold inventory level $\bar{s}_o$, and **Figure 4.8(a)** shows that if $\bar{s}_o \leq 0$, the buyer should order from the contract supplier up-to $\bar{S}(x)$ if $x \leq \bar{S}_H$, or order nothing otherwise; **Figure 4.8(b)** shows that if $\bar{s}_o \geq \bar{S}_H$, the buyer should order from the spot market up-to $\bar{S}_L$ if $x \leq \bar{s}_L$, or order nothing otherwise; **Figure 4.8(c)** shows that if $0 < \bar{s}_o < \bar{S}_H$, the buyer should order from the spot market up-to $\bar{S}_L$ if $x \leq \bar{s}_o$, or order from the contract supplier up-to $\bar{S}(x)$ if $\bar{s}_o < x \leq \bar{S}_H$, or order nothing otherwise.

□ **End of chapter.**

Chapter 5

Optimal Procurement Policy with a Total Order Commitment Contract in the Presence of Spot Market

5.1 Introduction

The purpose of this chapter is to investigate a procurement model by combining a total order quantity commitment (TOQC) contract provided by a contract supplier with procurement opportunities through a spot market. A TOQC contract is an agreement between a contract supplier and a buyer in which the buyer guarantees that his cumulative order for the whole (contract) duration will be at least the TOQC. In return, the contract supplier offers a fixed price, typically at a certain discount of the prevalent spot price. In practice, the contract supplier provides a menu of (commitment price, total commitment) pairs from which the buyer chooses a commitment at the corresponding price. Two types of TOQC contracts, which differ in the price after the commitment quantity has been fulfilled, are considered. The first type is the so-called minimal TOQC, by which the buyer can order additional quantities at the commitment price from the contract supplier, after the committed quantity has been fulfilled. That is, the contract supplier is obligated to promise the same price for any quantities beyond the commitment, which is called to be the as-ordered quantities. In the second type of commitment contract, the TOQC is a definite amount that the buyer must fulfill and the contract supplier is obligated to provide at a pre-agreed upon price. However, as-ordered quantities are subject to the spot price. Furthermore, the buyer is not allowed to sell the materials to the spot market during the planning horizon. A justification of this assumption is given by some constraints associated with real world purchasing practices. These constraints are mostly contractual, i.e. the contract supplier does not allow the buyer to resell the product to the market. The buyer can also procure from the spot market to obtain full flexibility on procuring quantities. However, he should take the price risk as the spot price is volatile.

Both quantity commitment contracts differ from a forward contract that typical requires a definite schedule of deliveries (in both time and quantity), as with the former, the buyer can choose individual delivery sizes and their timings. In a risk-neutral framework, an expected cost minimizing buyer would not engage in a forward contract as it does not give the buyer any cost advantage (except non-economic factors, such as risk averse attitude, which are beyond the scope of this chapter). In our setting, the total quantity commitment contracts are similar to a forward contract, but it gives the buyer the flexibility in terms of order quantities and timings of deliveries within the contract duration. This flexibility is valuable because it allows the buyer to take advantage of the two supply sources.

The results of our analysis can be used by decision makers to quantitatively determine the optimal procurement and inventory planning over multiple periods by incorporating a quantity flexible contract, to decide how to take the advantage of the spot market while maintaining a relationship with a long-term contract supplier, and to study the effect of the key spot market characteristics on his total expected cost. Specifically, we find that when the spot price is lower than the comitment price, the buyer procures certain amount from the contract supplier first (while deferring some commitment to future periods), and then orders from the spot market. This differs from the traditional TOQC literature with deterministic costs, in which the buyer's optimal policy is always to fulfill the commitment first. Furthermore, the optimal policy can be viewed as a combination of some base-stock policies, each of which can be computed through an equivalent system without any commitment. Moreover, some of the equivalent systems are of simple multiple-period newsvendor type. This greatly simplifies the computation of the optimal policies. Our numerical analysis implies that the large volatility of the spot price benefits the buyer. This is because the TOQC contract provides a natural hedge to the higher spot price movements. However, such benefits become smaller when the buyer commit more quantities as it restricts the buyer's use of the spot market.

The rest of this chapter is organized as follows. In **Section 5.2**, related literature is reviewed. In **Section 5.3**, the mathematical model is formulated and the optimal inventory policy is derived. Then, the extension of the minimal TOQC contract to the definite quantity contract is presented in **Section 5.4**. In

Section 5.5, numerical experiment is conducted. In **Section 5.6**, conclusion is made and some extensions are discussed. All proofs are included in **Appendix**.

5.2 Related Literature

Our work is related to the literature that focus on understanding how the spot market can be used effectively and profitably for varying purposes. Haksoz and Seshadri (2007) make an extensive review and discussion on this research stream. Here, we briefly discuss two topics that consider the procurement strategies in the presence of the spot market. The first topic considers the inventory problems when procuring solely from the spot market with fluctuating spot prices (e.g., Fabian et al. (1959), Kalymon (1971), Kingsman (1969), Magirou (1982), Golabi (1985), Guzel (2004) and Arnold et al. (2006)). These works explore the optimal inventory policies with a single source and focus on analyzing how the spot price affects the buyer's optimal decisions.

Another topic is the study of the procurement strategies by combining the spot market purchase with purchases made in advance from a specific long-term contract supplier. Bonser and Wu (2001) study the fuel procurement problem for electric utilities in which the buyer can use a mix of long-term and spot purchases. The long-term contractual supply commitment are made at a preset price with the suppliers at the beginning of the planning horizon and the buyer can use fuel from these contracts or purchases fuel at the current spot price. The commitment contract contains a maximum quantity and minimum quantity the buyer can buy for each period. They propose a two-phase dynamic procedure to determine a procurement plan. Yi and Scheller-Wolf (2003) analyze the optimal policy in the presence of the spot market with a long-term contract, which specifies an order quantity capacity for each period in advance. They show that a generalized (s, S) type policy is optimal. Feng and Sethi (2008) investigate a flexible contract which contains a price-only contract for long-term order and an adjustment contract for short-term orders. The adjustment contract specifies two types of capacity arrangement: dedicated capacity and overall capacity. They discuss both the optimal procurement strategies and the criteria for capacity allocations. However, as appointed out by Haksoz and Seshadri (2007), "only a small beginning has been made with regard to optimal procurement and inventory planning over multiple periods by incorporating different types of supply contracts". Our current work also contributes to this topic. In this chapter, we explore the optimal policy by integrating a fixed-price TOQC contract with the spot market.

Our current work also contributes to the study of TOQC contract with stochastic demand. The TOQC contract was first explored by Bassok and Anupindi (1997). Specifically, they consider the model as follows: there is a TOQC over the contract duration for a single item, the per-unit price for as-ordered orders (those beyond the TOQC) is locked up with that for commitment purchases, and demand realizations follow an identical, independent distribution (iid). They show that for a given minimum quantity commitment, the optimal procurement policy for each period can be characterized by two order-up-to levels, one for each type of orders. The policy can be computed by solving two multi-period news-vendor problems without the commitment. Chen and Krass (2001) further extend the model in Bassok and Anupindi (1997) to incorporate non-stationary demand distributions and different per-unit prices for the as-ordered and the commitment purchase. They show that the optimal policy can still be characterized by two base-stock levels. In Bassok and Schuster (1995), the early termination of the TOQC is taken as an option with a fixed-plus-linear penalty. Sufficient conditions are obtained for decisions as to whether the termination option should be exercised. For more detailed reviews on supply contracts with quantity commitment, see Anupindi and Bassok (1998).

Our work is similar to Chen and Krass (2001) in that there are two price schemes (the commitment price and the as-ordered price). As showed in Chen and Krass (2001), it is optimal for the buyer to first fulfill the commitment before using the as-ordered price (even when the as-ordered price is lower than the commitment price). However, when we place such a TOQC contract in an environment where there is a spot market, we could imagine that the buyer may defer certain commitment to future periods, and procures via the spot market in the current period. That is, when the spot price is lower than the commitment price, the buyer should trade off the flexibility in future by fulfilling the commitment with the cost advantages by procuring from the spot market now. This fact makes the problem more complex.

5.3 Model Formulation

The setting of our model can be described as follows. A buyer manages his raw material inventory to satisfy uncertain demand for a finite horizon to minimize his total expected cost. There are N periods during the planning horizon and the buyer can only replenish his inventory level at the beginning of each period. We index the periods in a backward fashion, i.e., period n means that there are n periods left. There are two procurement sources in each period: a contract supplier and a spot market. The contract supplier provides a minimal TOQC contract with a fixed per-unit price c_H. The minimal TOQC contract specifies the buyer should fulfill a minimum total order quantity, $\tilde{Q}$, during the planning horizon. The per-unit price of the as-ordered quantities from the contract supplier is assumed to be the same as the commitment price, c_H, as in Bassok and Anupindi (1997). This assumption will be relaxed later. The spot market, on the other hand, provides the buyer opportunities of possible lower procurement price, c_n, in period n, although it is volatile over time. The buyer can not resell the materials to the spot market during the planning horizon as we discussed before. Thus, the problems faced by the buyers are two folds: which supply source he should choose and how the optimal procurement policy should be for each period.

The demand realizations in different periods are assumed to be an independent identical distributed random variables with cumulative distribution $\Phi_1(\xi)$ and probability density function $\phi_1(\xi)$. The spot price c_n follows a Markov process. The Markov property of the spot price is a reasonable assumption, because the complete market hypothesis widely accepted in the financial literature guarantees that the current spot price contains all market information till now. Specifically, in our model, the spot price of the next period, c_k, is a random variable with cumulative distribution function $\Phi_2(c_k|c_{k+1})$ and probability density function $\phi_2(c_k|c_{k+1})$ (we use $\Phi_2(c_k)$ and $\phi_2(c_{k+1})$ for simplicity).

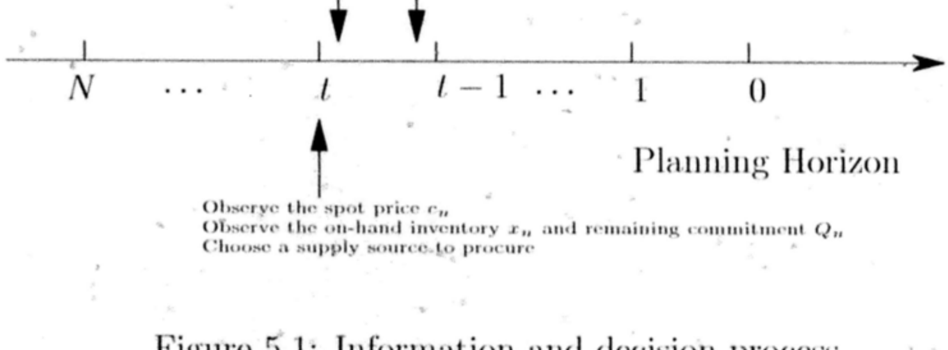


Figure 5.1: Information and decision process

The decision process of the buyer can be summerized by **Figure 1**. At the beginning of period n, the buyer observes the on-hand inventory level x_n, the spot price c_n and the remaining commitment size Q_n. Then, he decides which supply source he should procure from and the quantity he procures. We assume both orders arrive immediately, i.e., the lead-times for both supply sources are zero. (The results in sequel nevertheless hold as long as the lead-times for these two types of replenishment orders are the same.) We denote by q_n the order quantity from the contract supplier and by y_n the on-hand inventory after orders arrive. Then, demand realizes, and depending on the relative size of the demand, a holding cost $h(u) = c_h u$ if $u \geq 0$, or a shortage cost $h(u) = -c_s u$ occurs if $u < 0$ (where $c_s \geq c_H$ to avoid triviality). Without loss of generality, we assume the excess inventory is discarded without any salvage value and all backlogged demand is lost at the end of the planning horizon. For simplicity, we ignore discount effect.

Define $f_n(x_n, Q_n; c_n)$ to be the minimum expected cost of managing inventory over periods $n, n-1, \cdots, 1$, starting with the inventory level x_n, the spot price c_n and the remaining commitment size Q_n. As mentioned earlier, $f_0(x_0, c_0) = 0$.

Let

$$G_{n-1}(y_n, Q_n - q_n; c_n) = L(y_n) + \\ [\int_0^{\infty}\int_0^{\infty} f_{n-1}((y_n - \xi), (Q_n - q_n)^+; c_{n-1})\phi(\xi)\theta(c_{n-1}|c_n)dc_n d\xi,$$

where $L(y) = \int_0^{\infty} h(y-\xi)\phi(\xi)d\xi$, be the cost-to-go function, which is the sum of expected single period holding and shortage cost in the current period n and the minimal expected cost over the following $n-1$ periods. In the following analysis, we drop the subscript n of x_n, q_n and y_n when there are no confusions.

Then, in period 1, because all remaining commitment should be fulfilled, we have

$$f_1(x, Q_1; c_1) = \min_{y \geq x+Q_1} \{\min(c_1, c_H)(y - x - Q_1) + c_H Q_1 + L(y)\}, \quad (5.1)$$

and the general dynamic equation governing the n-period ($n \geq 2$) problem is

$$\begin{aligned} f_n(x, Q_n; c_n) &= \min\{\min_{y \geq x+q, q \geq 0}[c_H q + (y - x - q)c_n + G_{n-1}(y, (Q_n - q)^+; c_n)] \\ &\quad \min_{y \geq x}[(y - x)c_H + G_{n-1}(y, (Q_n - y + x)^+; c_n)]\}, \end{aligned} \quad (5.2)$$

with $Q_N = \widetilde{Q}$. That is, in each period, the optimal decision is the minimization of two optimization problems. The first problem is to procure certain amount from the contract supplier first, and then to procure from the spot market. This happens when the spot price is lower than the commitment price. The second problem is to procure solely from the contract supplier, and it occurs when the spot price is higher than the commitment price (recall that the buyer can buy any quantity from the contract supplier at the commitment price).

5.3.1 Single-Period Model

In the last period, the value function is defined as in (5.1). We consider the optimal policy according to the relative values of c_1 and c_H.

(i) If $c_1 \leq c_H$, the buyer only procures from the spot market for any quantity beyond the commitment. Thus, the buyer's optimal decision is to choose the optimal y for the following problem:

$$f_1(x, Q_1; c_1) = \min_{y \geq x+Q_1} [c_H Q_1 + (y - x - Q_1)c_1 + L(y)].$$

Denote by $S_{s,1}$ the optimal solution to the non-constrained variation of the above model. That is,

$$S_{s,1} = \arg\min\{c_1 y + L(y)\}.$$

Then, in the case of $c_1 \leq c_H$, the optimal inventory policy is of base-stock type with the order-up-to level $S_{s,1}$.

(ii) If $c_1 > c_H$, the buyer only procures from the contract supplier, and the buyer's optimal decision is to choose the optimal y for the following problem:

$$f_1(x, Q_1; c_1) = \min_{y \geq x+Q_1} [c_H(y-x) + L(y)].$$

Let

$$S_{H,1} = \arg\min\{c_H y + L(y)\}.$$

Then, in the case of $c_H < c_1$, the optimal inventory policy is the one with the order-up-to level $S_{H,1}$.

To summarize, the optimal procurement policy can be characterized by the following theorem. (In the following paragraph, we always use q_s and q_h to represent the optimal order quantities ordered from the spot market and the contract supplier, respectively.)

Theorem 5.3.1 *For the last period, suppose the spot price is c_1 and the remaining commitment size is Q_1.*

a) When $c_H \leq c_1$, the optimal order quantities are given by

$$(q_s, q_h) = \begin{cases} (0, Q_1), & x + Q_1 \geq S_{H,1}; \\ (S_{H,1} - Q_1 - x, Q_1), & x + Q_1 < S_{H,1}, \end{cases}$$

and $\frac{\partial f_1(x,Q_1;c_1)}{\partial Q_1} \geq 0$.

b) When $c_1 < c_H$, the optimal order quantities are given by

$$(q_s, q_h) = \begin{cases} (0, Q_1), & x + Q_1 \geq S_{s,1}; \\ (S_{s,1} - Q_1 - x, Q_1), & x + Q_1 < S_{s,1}, \end{cases}$$

and $\frac{\partial f_1(x,Q_1;c_1)}{\partial Q_1} \geq c_H - c_1$.

c) $f_1(x, Q_1; c_1)$ is jointly convex in (x, Q_1) for each c_1. Moreover, $\frac{\partial f_1(x,Q_1;c_1)}{\partial Q_1} \geq 0$, and $\frac{\partial f_1(x,Q_1;c_1)}{\partial x} - \frac{\partial f_1(x,Q_1;c_1)}{\partial Q_1} = -c_H$.

Proof. See the **Appendix**. ◇

Parts a) and b) are intuitive. It is also interesting to note that the difference between the marginal cost of on-hand inventory and that of commitment is constant and is equal to $-c_H$.

5.3.2 Two-Period Model

Now we turn to analyze the optimal problem in period 2. This two-period model allows the remaining commitment to be fulfilled partially in period 2. We examine the 2-period problem instead of a general N-period one because it is easier to gain insights, and in fact, with the 2-period problem being analyzed, the N-period problem becomes relatively easy to handle. As what we do earlier, we also consider the 2-period problem according to whether $c_2 \geq c_H$.

CASE $c_2 \geq c_H$

The buyer only procures from the contract supplier even if he has fulfilled the commitment (as he can still buy at the as-ordered price c_H). This is essentially equivalent to the model of Bassok and Anupindi (1997), because the spot market is not an option in period 2 when $c_2 \geq c_H$. Nevertheless, we need to provide a complete analysis here as 1) the cost-to-go function is different from Bassok and Anupindi (1997); and 2) it prepares the induction to the N-period problem.

By dynamic equation (5.2), the decision model can be formulated as:

$$f_2(x, Q_2; c_2) = \min_{y \geq x}[c_H(y - x) + L(y) + \mathrm{E}_{\xi, c_1} f_1(y - \xi, (Q_2 - y + x)^+; c_1)]. \quad (5.3)$$

Define $S_H^M = \Phi_1^{-1}(c_s/(c_h + c_s))$, the optimal "newsvendor" quantity. Then, we have the following theorem concerning the dynamic programming problem (5.3).

Theorem 5.3.2 *There exists a $S_{H,2}$, which solves*

$$\min_y [c_H y + L(y) + E_{\xi,c_2} f_1(y-\xi, 0; c_2)], \tag{5.4}$$

such that $S_{H,2} \le S_H^M$, and the optimal order quantities in period 2 are given by

$$(q_s, q_h) = \begin{cases} (0,0), & x \ge S_H^M; \\ (0, S_H^M - x), & x < S_H^M \text{ and } x + Q_2 \ge S_H^M; \\ (0, Q_2), & x + Q_2 < S_H^M \text{ and } x + Q_2 \ge S_{H,2}; \\ (S_{H,2} - Q_2 - x, Q_2), & x + Q_2 < S_{H,2}. \end{cases} \tag{5.5}$$

Proof. Define

$$\begin{aligned} H(x,y) &= c_H(y-x) + L(y) + \mathrm{E}_{\xi,c_1} f_1(y-\xi, Q_2 - y + x; c_1) \quad \text{and} \\ R(x,y) &= c_H(y-x) + L(y) + \mathrm{E}_{\xi,c_1} f_1(y-\xi, 0; c_1). \end{aligned}$$

Then, $H(x,y)$ and $R(x,y)$ are convex in y by **Theorem 5.3.1.c)**, and (5.3) can be further reformulated as follows:

$$f_2(x, Q_2; c_2) = \min\{\min_{Q_2+x \ge y \ge x} H(x,y), \min_{y \ge x+Q_2} R(x,y)\}. \tag{5.6}$$

To solve the above problem, taking the first derivative of $H(x,y)$ and $R(x,y)$ with respect to y yields

$$\begin{aligned} \frac{\partial H(x,y)}{\partial y} &= c_H + L'(y) + \mathrm{E}_{\xi,c_1}\left[\frac{\partial f_1(x_1,Q_1;c_1)}{\partial x_1} - \frac{\partial f_1(x_1,Q_1;c_1)}{\partial Q_1}\right]\Big|_{x_1=y-\xi, Q_1=Q2-y+x}, \\ \frac{\partial R(x,y)}{\partial y} &= c_H + L'(y) + \mathrm{E}_{\xi,c_1}\frac{\partial f_1(x_1,0;c_1)}{\partial x_1}\Big|_{x_1=y-\xi}. \end{aligned} \tag{5.7}$$

Note that the last block on the right-hand side of (5.7) is equal to $-c_H$ because of **Theorem 5.3.1.c)**.

Thus, the optimal solution to $\min_y H(x,y)$ is S_H^M. And the optimal solution to $\min_y R(x,y)$, denoted by $S_{H,2}$, solves

$$c_H + L'(y) + \int_0^{c_H}\int_{y-S_{s,1}}^{\infty} -c_1\phi_1(\xi)\phi_2(c_1)d\xi dc_1 + \int_0^{c_H}\int_0^{y-S_{s,1}} L'(y-\xi)\phi_1(\xi)\phi_2(c_1)d\xi dc_1$$
$$+ \int_{c_H}^{\infty}\int_{y-S_{H,1}}^{\infty} -c_H\phi_1(\xi)\phi_2(c_1)d\xi dc_1 + \int_{c_H}^{\infty}\int_0^{y-S_{H,1}} L'(y-\xi)\phi_1(\xi)\phi_2(c_1)d\xi dc_1 = 0. \tag{5.8}$$

This is the first order condition for the inner part of (5.4). The existence of $S_{H,2}$ is guaranteed by the fact that the left-hand side of (5.8) increases from negative to positive when y goes from $-\infty$ to ∞. Furthermore, because $\frac{\partial f_1(x_1,0;c_1)}{\partial x_1} \geq -c_H$, we have $L'(y)|_{y=S_{H,2}} \leq 0$. Thus, we see $S_{H,2} \leq S_H^M$. Now we analyze the optimal inventory policy by those critical values.

Suppose $x + Q_2 \leq S_{H,2}$, $R(x,y)$ can be minimized at $S_{H,2}$ in $[x+Q_2, +\infty)$. $H(x,y)$ decreases in y in $(-\infty, x+Q_2)$, and $R(x,y) = H(x,y)|_{y=x+Q_2}$. Thus, the optimal policy is to order up-to S_H^M. An analogous analysis can be conducted for $x + Q_2 \geq S_H^M$.

Suppose $S_{H,2} \leq x + Q_2 \leq S_H^M$, $R(x,y)$ increases in y in $[x+Q_2, +\infty)$, and $H(x,y)$ decreases in y in $(-\infty, x+Q_2)$ with $R(x,y) = H(x,y)|_{y=x+Q_2}$, then, the optimal policy is to order Q_2. ◇

The optimal policy for the case $c_H \leq c_2$ is illustrated in **Figure 5.2**. Depending on the on-hand inventory level and the remaining commitment size, we can divide the state space into four regions, i.e., I, II, III, IV. It is optimal for the buyer to order nothing in region I, to order up-to S_H^M from the contract supplier in region II, to order exactly Q_2 to complete the outstanding commitment in region III, and to act as in region III, but to order more from the contract supplier to bring y up-to $S_{H,2}$ in region IV.

We can obtain the following result concerning the properties of the value

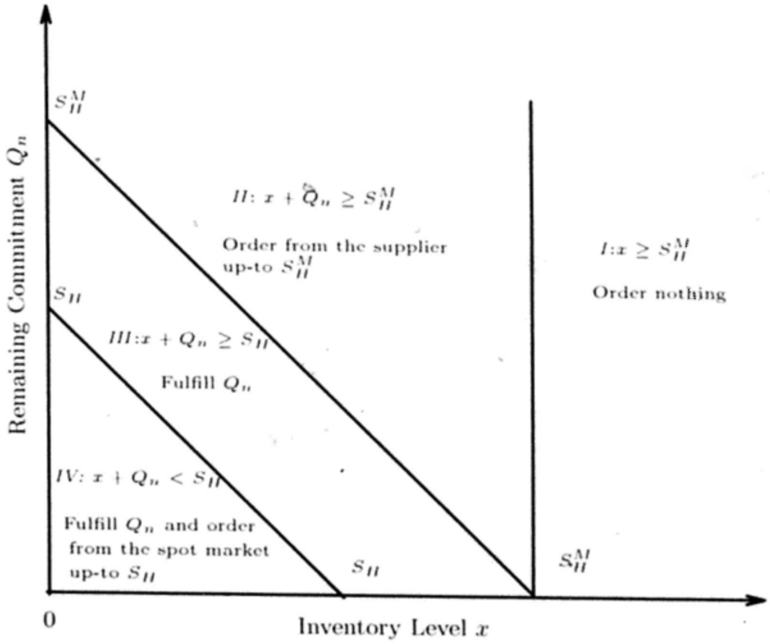


Figure 5.2: Optimal inventory policy when $c_H \leq c_2$

function as a consequence of the analysis above.

Lemma 5.3.1 *$f_2(x, Q_2; c_2)$ is jointly convex in (x, Q_2) for each c_2 when $c_2 \geq c_H$. Moreover, $f_2(x, Q_2; c_2)$ increases in Q_2 , and* $\frac{\partial f_2(x,Q_2;c_2)}{\partial x} - \frac{\partial f_2(x,Q_2;c_2)}{\partial Q_2} = -c_H$.

Proof. The value function with the optimal policy can be written as follows:

$$f_2(x, Q_2; c_2) = \begin{cases} G_1(x, Q_2, c_2), & x \geq S_H^M; \\ c_H(S_H^M - x) + G_1(S_H^M, Q_2 - S_H^M + x, c_2), & x < S_H^M \text{ and } x + Q_2 \geq S_H^M; \\ c_H Q_2 + G_1(x + Q_2, 0, c_2), & x + Q_2 < S_H^M \text{ and } x + Q_2 \geq S_{H,2}; \\ c_H Q_2 + c_H(S_{H,2} - x - Q_2) + G_1(S_{H,2}, 0, c_2), & x + Q_2 < S_{H,2}. \end{cases}$$

The convex and increasing properties in Q_2 can be verified case by case as in **Theorem 5.3.1.c)**.

To prove $\frac{\partial f_2(x,Q_2;c_2)}{\partial x} - \frac{\partial f_2(x,Q_2;c_2)}{\partial Q_2} = -c_H$, we just note that $\frac{\partial G_1(x,Q_2;c_2)}{\partial x} - \frac{\partial G_1(x,Q_2;c_2)}{\partial Q_2} = -c_H$, which is a direct application of **Theorem 5.3.1**. This completes the proof. ◇

CASE $c_2 < c_H$

The buyer needs to trade off between the two procurement sources. If he takes the advantage of the low spot price now, then the commitment may be pushed to the next period. Therefore, in this case, he may procure certain amount from the spot market at the current low spot price, and orders some quantities from the contract supplier at the same time to reduce the commitment to leave certain flexibility to the next period.

In this case, the dynamic programming problem can be written as

$$f_2(x, Q_2; c_2) = \min_{Q_2 \geq q \geq 0} V(q), \tag{5.9}$$

where

$$\begin{aligned} V(q) &= c_H q + \min_{y \geq x+q}[-c_2(x+q) + W(y,q)]; \\ W(y,q) &= c_2 y + L(y) + \mathrm{E}_{\xi,c_1} f_1(y-\xi,(Q_2-q);c_1). \end{aligned}$$

The analysis to follow next is to solve the relaxed version of (5.9), i.e.,

$$\min_{Q_2 \geq q} V(q). \tag{5.10}$$

It is called the relaxed problem because now q can be negative. This is for ease of mathematical derivation. One can regard the negative q as selling back to the contract supplier at the expense of increased commitment in period 1. Later, we restore the non-negative restriction on q to identify the optimal policy.

We first decide the optimal $y(q)$ for

$$\min_{y \geq x+q} W(y,q), \tag{5.11}$$

given $q \in (-\infty, Q_2]$. Then, $y(q)$ represents the optimal inventory level to maintain by ordering from the spot market after ordering q from the contract supplier.

By **Theorem 5.3.1.c)**, $W(y,q)$ is convex in y. Hence, $y(q)$ has the form as

$$y(q) = \begin{cases} S_2^*(q), & x \le S_2^*(q) - q; \\ x + q, & \text{otherwise}, \end{cases} \tag{5.12}$$

where $S_2^*(q)$ is the optimal value for the unconstrained version of (5.11). Furthermore, note that $S_2^*(q)$ is finite and satisfies the first order condition of $W(y,q)$.

We now obtain some properties concerning $S_2^*(q)$ as a function of q.

Lemma 5.3.2 *$S_2^*(q)$ increases and $S_2^*(q) - q$ decreases in q.*

Proof. See the **Appendix**. ◇

This lemma has a direct interpretation for the optimal order-up-to level and spot purchase. If the spot price is lower than the commitment price and the buyer has procured q from the contract supplier, then the optimal order-up-to level by ordering from the spot market increases in q. However, the amount procured from the spot market decreases in q.

In the following analysis, we determine the optimal q for (5.10). Before proceeding, we define three numbers: $S_{s,2}$, $S_{s,2}^Q$ and Q_2^*.

Let $S_{s,2}$ be the optimal solution to

$$\min_y [c_2 y + L(y) + \mathrm{E}_{\xi,c_1} f_1(y - \xi, 0; c_1)] (= \min_y W(y, Q_2)). \tag{5.13}$$

$S_{s,2}^Q$ be the optimal solution to

$$\min_y [c_2 y + L(y) + \mathrm{E}_{\xi,c_1} f_1(y - \xi, Q_2; c_1)] (= \min_y W(y, 0)). \tag{5.14}$$

and $Q_2^* (\ge 0)$ be the optimal solution to

$$\min_{Q \in [0,\infty)} [-(c_H - c_2)Q + \mathrm{E}_{\xi,c_1} f_1(S_H^M - \xi, Q; c_1)]. \tag{5.15}$$

Then, $S_{s,2} = S_2^*(Q_2)$ is the optimal order-up-to level when the buyer doesn't have any remaining commitment on hand; $S_{s,2}^Q = S_2^*(0)$ is the optimal order-up-to level via spot market when the buyer defers all commitment (Q_2) to period 1; Q_2^* is the optimal commitment size deferred to period 1 after the buyer trades off between the unit cost save in current period, $(c_H - c_2)$, by buying from the spot market and the opportunity cost saved for period 1, $(\frac{\partial E_{\xi,c_1} f_1(S_H^M - \xi, Q; c_1)}{\partial Q})$, by fulfilling the commitment.

Moreover, $S_{s,2}$ and Q_2^* are independent of Q_2, and there is a monotone property between $S_{s,2}^Q$ (or S_s^Q when no confusion arises) and Q_2 (or Q when no confusion arises) (**Figure 5.3**), which is showed by the following lemma.

Lemma 5.3.3 *$S_{s,2}^Q$ decreases in Q_2 and* $\frac{S_{s,2}^Q}{\partial Q_2} \geq -1$.

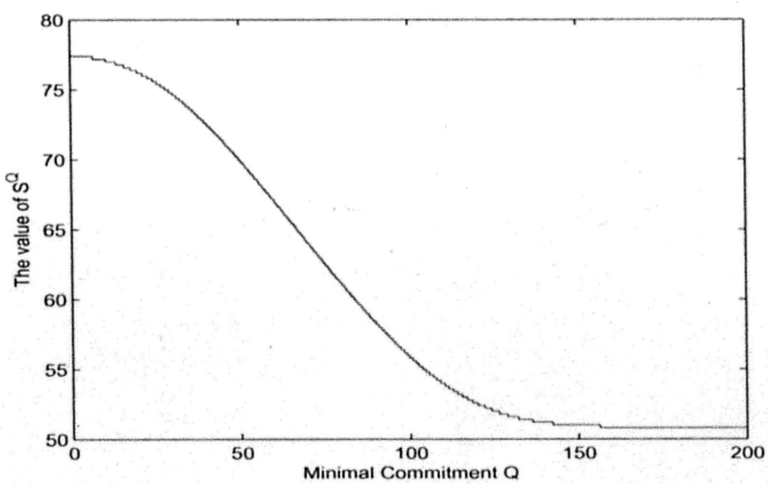


Figure 5.3: Relationship between S^Q and Q

Proof. See the **Appendix**. ◇

This lemma also implies that the more the commitment deferred to period 1, the lower the optimal order-up-to level (via the spot market) in period 2.

The next theorem states that it is optimal for the buyer to order from the contract supplier to bring the remaining commitment down-to Q_2^*, if the inventory level after ordering is lower than S_H^M, or just to order up-to S_H^M otherwise.

Theorem 5.3.3 *$V(q)$ is convex. Furthermore, let q^* be the optimal solution to (5.10), then the form of q^* is as follows.*

$$q^* = \begin{cases} Q_2 - Q_2^*, & x + Q_2 < S_H^M + Q_2^*; \\ S_H^M - x, & x + Q_2 \geq S_H^M + Q_2^*. \end{cases} \tag{5.16}$$

Proof. According to the above analysis (see (5.12)), $y(q)$ has different forms depending on whether $S_2^*(q) \geq x + q$ or not. We define q_x as follows: for those $x \geq S_2^*(Q_2) - Q_2$, define q_x as the solution to $S_2^*(q) - q = x$; for these $x < S_2^*(Q_2) - Q_2$, we just let $q_x = Q_2$. Then, as $S_2^*(q) - q$ decreases in q as **Lemma 5.3.2** shows, q_x exists and is unique. Furthermore, $y(q) = S_2^*(q)$ if $q \geq q_x$; $y(q) = x + q$ otherwise. Note that $q_x \leq Q_2$. Thus, our analysis is conducted for two cases: $q \leq q_x$ and $Q_2 \geq q > q_x$.

(1) Case $q \leq q_x$, $y(q) = S_2^*(q)$. Thus, the optimal problem defined by (5.10) can be reformulated as

$$\min_{q \leq q_x}[T(q) = (c_H - c_2)q + c_2(S_2^*(q) - x) + L(S_2^*(q)) + \mathrm{E}_{\xi,c_1} f_1(S_2^*(q) - \xi, Q_2 - q; c_1)]. \tag{5.17}$$

Denote by $q_2^{*(1)}$ the optimal solution to the unconstrained version of (5.17). Then, by **Lemma A6**, $T(q)$ is convex and $S_2^*(q_2^{*(1)}) = S_H^M$. Furthermore, if $Q_2^* > 0$, $q_2^{*(1)} = Q_2 - Q_2^*$ and if $Q_2^* = 0$, $q_2^{*(1)} \geq Q_2$.

(2) Case $Q_2 \geq q > q_x$, we have $y(q) = x + q$. Hence, the resulting formulation of (5.10) is as follows:

$$\min_{q \in (q_x, Q_2]} [\tilde{T}(q) = c_H q + L(x + q) + \mathrm{E}_{\xi,c_1} f_1(x + q - \xi, Q_2 - q; c_1)]. \tag{5.18}$$

Denote by $q_2^{*(2)}$ the optimal solution to the unconstrained version of (5.18). Then, by **Lemma A7**, $\tilde{T}(q)$ is convex and $q_2^{*(2)} = S_H^M - x$.

Combining cases (1) and (2) gives an equivalent formulation to the optimal problem (5.10) as follows.

$$\min\{\min_{q \le q_x} T(q), \min_{q \in (q_x, Q_2]} \tilde{T}(q)\}. \tag{5.19}$$

Furthermore, by **Lemma A8**, the optimal values of $q_2^{*(1)}$ and $q_2^{*(2)}$ are always in the same side of q_x. Since $\frac{\partial T(q)}{\partial q}|_{q \to q_x -} = \frac{\partial \tilde{T}(q)}{\partial q}|_{q \to q_x +}$, we see $V(q)$ is convex.

Now, we determine the optimal $q \in (-\infty, Q_2]$ for (5.10).

(i) If $Q_2^* > 0$, then $q_2^{*(1)} = Q_2 - Q_2^*$. If $q_2^{*(1)} < q_2^{*(2)}$, i.e., $Q_2 - Q_2^* < S_H^M - x$, we have $q_2^{*(1)} < q_2^{*(2)} \le q_x$, the optimal solution to (5.19) is the same as that to (5.17), i.e., $q^* = q_2^{*(1)} = Q_2 - Q_2^*$; if $q_2^{*(1)} \ge q_2^{*(2)}$, we have $q_2^{*(1)} \ge q_2^{*(2)} \ge q_x$, the optimal solution to (5.19) is the same as that to (5.18), thus, $q^* = q_2^{*(2)} = S_H^M - x$.

(ii) If $Q_2^* = 0$, then $q_2^{*(1)} \ge Q_2$. The optimal solution to (5.17) is q_x. If $q_2^{*(2)} = S_H^M - x \le Q_2$, then, $q_2^{*(2)} \ge q_x$ by **Lemma A8**, and thus, the optimal solution to (5.19) is the same as that to (5.18), i.e., $q^* = q_2^{*(2)} = S_H^M - x \le Q_2$; if $q_2^{*(2)} = S_H^M - x \ge Q_2$, then no matter what the value of q_x is, the optimal solution to (5.19) is $q^* = Q_2^*$.

Combining (i) and (ii) gives (5.16), which completes our proof. ◇

With the assistance of this theorem, we can go further to restrict q to be non-negative for formulation (5.10) to get the optimal policy for the dynamic programming problem (5.9), which is characterized by the following theorem.

Theorem 5.3.4 *In period 2, when the spot price $c_2 \le c_H$, the optimal policy can be classified by the value of Q_2^*. Specifically,* *if* $Q_2^* = 0$, *then* $S_H^M \geq S_{s,2}$, *and the the optimal order quantities are given by*

$$(q_s, q_h) = \begin{cases} (0,0), & x \geq S_H^M; \\ (0, S_H^M - x), & x < S_H^M \text{ and } x + Q_2 \geq S_H^M; \\ (0, Q_2), & x + Q_2 < S_H^M \text{ and } x + Q_2 \geq S_{s,2}; \\ (S_{s,2} - Q_2 - x, Q_2), & x + Q_2 < S_{s,2}. \end{cases} \quad (5.20)$$

If $0 < Q_2^* \leq Q_2$, *then the the optimal order quantities are given by*

$$(q_s, q_h) = \begin{cases} (0,0), & x \geq S_H^M; \\ (0, S_H^M - x), & x < S_H^M \text{ and } x + Q_2 - Q_2^* \geq S_H^M; \\ (S_H^M - (Q_2 - Q_2^*) - x, Q_2 - Q_2^*), & x + Q_2 - Q_2^* < S_H^M. \end{cases} \quad (5.21)$$

If $Q_2 < Q_2^*$, *then* $S_H^M < S_{s,2}^Q$, *and the the optimal order quantities are given by*

$$(q_s, q_h) = \begin{cases} (0,0), & x \geq S_{s,2}^Q; \\ (S_{s,2}^Q - x, 0), & x < S_{s,2}^Q. \end{cases} \quad (5.22)$$

Proof. The proof is divided into three cases in terms of the value of Q_2^*. The meanings of q_x, $T(q)$, $\tilde{T}(q)$, $q_2^{*(1)}$ and $q_2^{*(2)}$ here are the same as those in **Theorem 5.3.3**.

Case 1: $Q_2^* = 0$, or $q_2^{*(1)} \geq Q_2$.

1) If $x + Q_2 < S_H^M$, then, by **Theorem 5.3.3**, $q^* = Q_2 \geq 0$. By setting $q = Q_2$, the optimal y of problem (5.9) is equivalent to the solution to

$$\min_{y \geq x + Q_2} [c_2(y - x - Q_2) + c_H Q_2 + L(y) + \mathrm{E}_{\xi, c_1} f_1(y - \xi, 0; c_1)].$$

The optimal policy for above problem is of base-stock type with order-up-to level $S_{s,2}$, which is defined in (5.13). Note that $S_{s,2} = S_2^*(Q_2) \leq S_2^*(q_2^{*(1)}) = S_H^M$. Then, the optimal policy for (5.9) is, after ordering Q_2 from the contract supplier, to order $(S_{s,2} - Q_2 - x)$ from the spot market if $S_{s,2} \geq Q_2 + x$, and to order nothing otherwise.

2) If $x + Q_2 \geq S_{II}^M$, then $q^* = S_{II}^M - x$ for (5.10). If $q^* > 0$, i.e., $x < S_{II}^M$, the optimal q for (5.9) is the same as that for (5.10), with $y(q) = S_{II}^M (= x + q^*)$ (since $q_x \leq S_{II}^M - x$ as in **Lemma A8**); if $q^* \leq 0$, i.e., $x \geq S_{II}^M$, the optimal q for (5.9) is 0 (as $V(q)$ is convex) and $y(q) = x$. Thus, the optimal policy for (5.9) is to order $S_{II}^M - x$ from the contract supplier if $S_{II}^M \geq x$, and to order nothing otherwise.

Combining above two subcases, we obtain the optimal policy for $Q_2^* \leq 0$ as (5.20) shows.

Case 2: $0 < Q_2^* \leq Q_2$, or $0 \leq q_2^{*(1)} < Q_2$.

1) If $x + Q_2 - Q_2^* < S_{II}^M$, then $q^* = Q_2 - Q_2^* \geq 0$, which is optimal for (5.10), is also optimal for (5.9). As $q_x > S_{II}^M - x$ by **Lemma A8**, the order-up-to level $y(q^*)$ is equal to $S_2(q_2^{*(1)}) (= S_{II}^M)$. Thus, the optimal policy is to first order $(Q_2 - Q_2^*)$ from the contract supplier, then to order $S_{II}^M - Q_2 + Q_2^* - x$ via the spot market if $S_{II}^M - Q_2 + Q_2^* - x \geq 0$, and to order nothing otherwise.

2)If $x + Q_2 \geq S_{II}^M$, then, $q^* = S_{II}^M - x$ for (5.10). If $q^* > 0$, i.e., $x < S_{II}^M$, the optimal q for (5.9) is the same as that for (5.10), with $y(q) = S_{II}^M (= x + q^*)$ (since $q_x \leq S_{II}^M - x$ as in **Lemma A8**); if $q^* \leq 0$, i.e., $x \geq S_{II}^M$, the optimal q for (5.9) is 0 (as $V(q)$ is convex) and $y(q) = x$. Thus, the optimal policy for (5.9) is to order $S_{II}^M - x$ from the contract supplier if $S_{II}^M \geq x$, and to order nothing otherwise.

Combining above two subcases gives (5.21).

Case 3: $Q_2^* > Q_2$, or $q_2^{*(1)} < 0$.

In this case, if $x + Q_2 - Q_2^* < S_{II}^M$, then, $q^* = Q_2 - Q_2^* \leq 0$ is the optimal solution to (5.10). Since $V(q)$ is convex, the optimal solution to (5.9) is $q = 0$. If

$x + Q_2 - Q_2^* \geq S_H^M$, then $0 \geq S_H^M - x$, thus, the optimal solution to (5.9) is $q = 0$ too.

By setting $q = 0$, the optimal y of problem (5.9) is equivalent to the solution to

$$\min_{y \geq x}[c_2(y - x) + L(y) + \mathrm{E}_{\xi, c_1} f_1(y - \xi, Q_2; c_1)].$$

The optimal solution to the inner part of the above problem is $S_{s,2}^Q$, which is defined in (5.14). Note that $S_{s,2}^Q = S_2^*(0) \geq S_2^*(q_2^{*(1)}) = S_H^M$. Thus, the optimal policy for (5.9) is to order $(S_{s,2}^Q - x)$ from the spot market if $S_{s,2}^Q \geq x$, and to order nothing otherwise. This gives the policy (5.22). At last, combining Cases 1, 2 and 3 completes our proof. ◇

An illustration of the optimal policy is showed in **Figure 5.4**.

We can interpret the optimal policy as follows. In period 2, when the spot price is lower than the commitment price, there exists an optimal commitment level Q_2^*. If $Q_2^* = 0$ (**Figure 5.4a**), we divide the state space of on-hand inventory and the total remaining commitment into four regions: I, II, III, IV. In region I, order nothing; in region II, order up-to S_H^M from the contract supplier; in region III, just fulfill the commitment, and in region IV, first fulfill the commitment, and then order up-to $S_{s,2}$ from the spot market. If $Q_2^* > 0$, the situation is more complex. However, we can still divide the state space into four regions: I, II, III, IV, in **Figure 5.4b**. In region I, order nothing; in region II, order up-to S_H^M from the contract supplier; in region III, first order from the contract supplier to bring the commitment size for period 1 down-to Q_2^*, and then procure from the spot market to bring the inventory level up-to S_H^M, and in region IV, order up-to $S_{s,2}^Q$ from the spot market and order nothing from the contract supplier.

We have the following lemma concerning the value function $f_2(x, Q_2; c_2)$ for

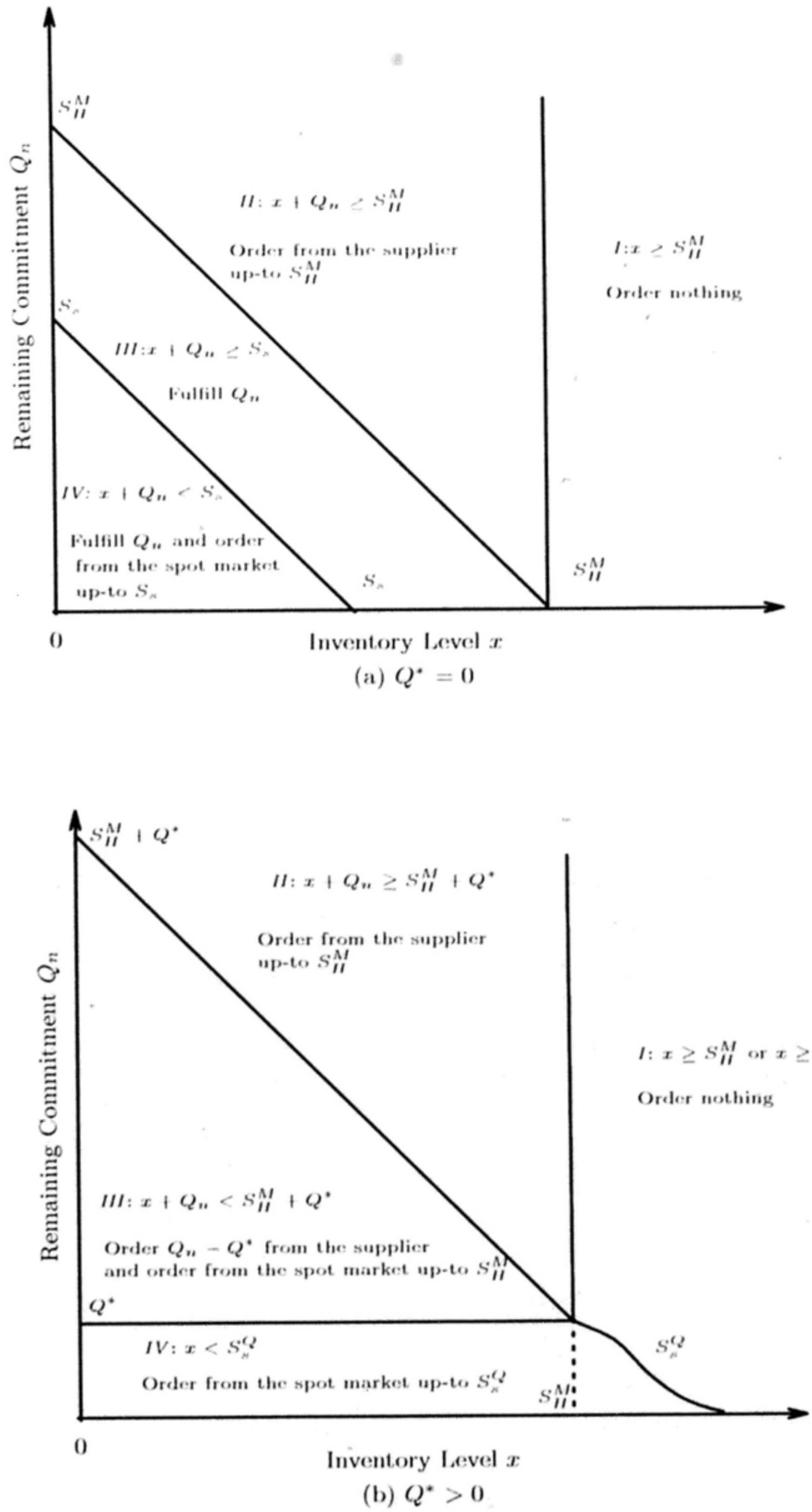


Figure 5.4: An illustration of optimal inventory policy

the case $c_2 < c_H$.

Lemma 5.3.4 *$f_2(x, Q_2; c_2)$ is jointly convex in (x, Q_2) when $c_2 < c_H$. Moreover, $\frac{\partial f_2(x,Q_2;c_2)}{\partial x} - \frac{\partial f_2(x,Q_2;c_2)}{\partial Q_2} = -c_H$, and $\frac{\partial f_2(x,Q_2;c_2)}{\partial Q_2} \geq 0$.*

Proof. Conditioning on the value of Q_2^*, the value function $f_2(x, Q_2; c_2)$ can be rewritten as follows,

if $Q_2^* = 0$

$$f_2(x, Q_2; c_2) = \begin{cases} G_1(x, Q_2, c_2), & x \geq S_H^M; \\ c_H(S_H^M - x) + G_1(S_H^M, Q_2 - S_H^M + x, c_2), & x < S_H^M \text{ and } x + Q_2 \geq S_H^M; \\ c_H Q_2 + G_1(x + Q_2, 0, c_2), & x + Q_2 < S_H^M \text{ and } x + Q_2 \geq S_{s,2}; \\ c_H Q_2 + c_2(S_{s,2} - x - Q_2) + G_1(S_{s,2}, 0, c_2), & x + Q_2 < S_{s,2}. \end{cases}$$

If $0 < Q_2^* \leq Q_2$

$$f_2(x, Q_2; c_2) = \begin{cases} G_1(x, Q_2, c_2), & x \geq S_H^M; \\ c_H(S_H^M - x) + G_1(S_H^M, Q_2 - S_H^M + x, c_2), & x < S_H^M \text{ and } x + Q_2 - Q_2^* \\ (c_H - c_2)(Q_2 - Q_2^*) + c_2(S_{H,2}^M - x) + G_1(S_H^M, Q_2^*, c_2), & x + Q_2 - Q_2^* < S_H^M. \end{cases}$$

If $Q_2 < Q_2^*$

$$f_2(x, Q_2; c_2) = \begin{cases} G_1(x, Q_2, c_2), & x \geq S_{s,2}^Q; \\ c_2(S_{s,2}^Q - x) + G_1(S_{s,2}^Q, Q_2, c_2), & x < S_{s,2}^Q. \end{cases}$$

As in **Theorem 5.3.1.c)**, for given c_2, the convex property can be verified case by case, and the equality $\frac{\partial f_2(x,Q_2;c_2)}{\partial x} - \frac{\partial f_2(x,Q_2;c_2)}{\partial Q_2} = -c_H$ is straightforward by noting that $\frac{\partial G_1(x,Q_2;c_2)}{\partial x} - \frac{\partial G_1(x,Q_2;c_2)}{\partial Q_2} = -c_H$. $\frac{\partial f_2(x,Q_2;c_2)}{\partial Q_2} \geq 0$ holds since $\frac{\partial G_1(x,Q_2;c_2)}{\partial Q_2} \geq 0$ and $c_2 < c_H$. ◇

To summarize, by combining cases $c_2 \geq c_H$ and $c_H > c_2$, we find that the properties of value function $f_1(x, Q_2; c_2)$ can be carried over to $f_2(x, Q_2; c_2)$. That is, $f_2(x, Q_2; c_2)$ is jointly convex in (x, Q_2); moreover, $\frac{\partial f_2(x,Q_2;c_2)}{\partial x} - \frac{\partial f_2(x,Q_2;c_2)}{\partial Q_2} = -c_H$ and $\frac{\partial f_2(x,Q_2;c_2)}{\partial Q_2} \geq 0$.

5.3.3 n-Period Model ($n \geq 2$)

Now, we consider the general n-period model for $n \geq 2$. The detailed proofs for the results in this section are given in **Appendix**.

Before we state the general theorem concerning the optimal procurement policy. We define the following several numbers for period $n \geq 2$: $S_{H,n}$, $S_{s,n}$, $S^Q_{s,n}$ and Q^*_n.

(1). Define the following multi-period dynamic problem:

$$I_{k+1}(x, c_{k+1}) = \min_{y \geq x}[\min(c_H, c_n)(y - x) + L(y) + \mathrm{E}_{\xi,c_k} I_k(y - \xi; c_k)], \qquad (5.23)$$

with $I_0 = 0$. Then, denote the optimal order-up-to level for this problem by $S_{H,n}$ if $c_H \geq c_n$ and by $S_{s,n}$ otherwise.

(2). $S^Q_{s,n}$ is defined as the order-up-to level for the following problem:

$$\min_{y}[c_n y + L(y) + \mathrm{E}_{\xi,c_{n-1}} f_{n-1}(y - \xi, Q_n; c_{n-1})]. \qquad (5.24)$$

(3). Q^*_n is defined as the optimal solution to

$$\min_{Q \geq 0}[-(c_H - c_n)Q + \mathrm{E}_{\xi,c_{n-1}} f_{n-1}(S^M_H - \xi, Q; c_{n-1})]. \qquad (5.25)$$

Then, $S_{s,n}$ is the optimal order-up-to level via the spot market when the buyer doesn't have any remaining commitment on hand in period n, and $S_{H,n}$ is that via the contract supplier, $S^Q_{s,n}$ is the optimal order-up-to level via the spot market when the buyer defers all remaining commitment size, (Q_n), to the following $n - 1$ periods, and Q^*_n is the optimal commitment level deferred to future periods after the buyer trades off between the cost saved by buying from the spot market and the opportunity cost saved by reducing commitment size for future periods. Moreover, $S_{s,n}$, $S_{H,n}$ and Q_n^* are independent of Q_n, and can be computed by equivalent systems of simple multiple-period news-vendor type (see dynamic problems (5.23) and (5.25)). $S_{s,n}^Q$ is an order-up-to level which decreases in Q_n and $\frac{\partial S_{s,n}^Q}{\partial Q_n} \geq -1$.

With the aid of these numbers, we have the following theorem which characterizes the optimal commitment and inventory policy.

Theorem 5.3.5 *In period n, suppose the spot price is c_n and the remaining commitment size is Q_n.*

When $c_H \leq c_n$, we have $S_{H,2} \leq S_H^M$ and the optimal order quantities are given by

$$(q_s, q_h) = \begin{cases} (0, 0), & x \geq S_H^M; \\ (0, S_H^M - x), & x < S_H^M \text{ and } x + Q_n \geq S_H^M; \\ (0, Q_n), & x + Q_n < S_H^M \text{ and } x + Q_n \geq S_{H,n}; \\ (S_{H,n} - Q_n - x, Q_n), & x + Q_n < S_{H,n}. \end{cases} \tag{5.26}$$

When $c_H > c_n$, the optimal policy is classified by the value of Q_n^*:

If $Q_n^* = 0$, then $S_H^M \geq S_{s,n}$, and the optimal order quantities are given by

$$(q_s, q_h) = \begin{cases} (0, 0), & x \geq S_H^M; \\ (0, S_H^M - x), & x < S_H^M \text{ and } x + Q_n \geq S_H^M; \\ (0, Q_n), & x + Q_n < S_H^M \text{ and } x + Q_n \geq S_{s,n}; \\ (S_{s,n} - Q_n - x, Q_n), & x + Q_n < S_{s,n}. \end{cases} \tag{5.27}$$

If $0 < Q_n^* \leq Q_n$, *then the optimal order quantities are given by*

$$(q_s, q_h) = \begin{cases} (0, 0), & x \geq S_H^M; \\ (0, S_H^M - x), & x < S_H^M \text{ and } x + Q_n - Q_n^* \geq S_H^M; \\ (S_H^M - (Q_n - Q_n^*) - x, Q_2 - Q_n^*), & x + Q_n - Q_n^* < S_H^M. \end{cases} \tag{5.28}$$

If $Q_n < Q_n^*$, *then* $S_H^M < S_{s,n}^Q$, *and the optimal quantities are given by*

$$(q_s, q_h) = \begin{cases} (0, 0), & x \geq S_{s,n}^Q; \\ (S_{s,n}^Q - x, 0), & x < S_{s,n}^Q. \end{cases} \tag{5.29}$$

$f_n(x, Q_n; c_n)$ *is jointly convex in* (x, Q_n). *Moreover,* $\frac{\partial f_n(x,Q_n;c_n)}{\partial Q_n} \geq 0$ *and* $\frac{\partial f_n(x,Q_n;c_n)}{\partial x} - \frac{\partial f_n(x,Q_n;c_n)}{\partial Q_n} = -c_H$.

We can see that the optimal policy for the general n-period model is similar to that for the two-period setting, thus it can also be illustrated by similar figures as **Figure 5.2** and **Figure 5.4**.

Therefore, we have generalized the minimal TOQC contract considered in Bassok and Anupindi (1997) and Chen and Krass (2001) to the case where there is a spot market for procurement. We can see that the optimal policy is characterized by three base-stock levels, S_H^M, $S_{H,n}$, and $S_{s,n}$, one optimal commitment level Q_n^*, and one commitment-dependent inventory level $S_{s,n}^Q$. Furthermore, the base-stock levels and the optimal commitment level can be computed through certain N-period problems of newsvendor type. When the spot price is deterministic and is equal to a constant during the planning horizon, the optimal policy is (5.26), which is the same as that in Bassok and Anupindi (1997).

Remark 5.3.1 Dynamic programming problem (5.23) is a multiple-period problem of newsvendor type, in which the buyer chooses the lower one between the commitment price and the spot price to procure in each period. $S_{s,n}$, $S_{H,n}$ are the corresponding base-stock levels for this dynamic problems. (5.25) is a problem to determine how much commitment size to enter into for future periods at the price $(c_H - c_2)$, when the on-hand inventory is S_H^M.

5.4 Extension to a Definite Quantity Contract

Till now, we have derived the buyer's optimal procurement policy by incorporating a minimal TOQC contract with spot market operations. The minimal TOQC contract assumes the buyer can buy the as-ordered quantity at the commitment price. However, when the spot market is rather volatile, the contract supplier may only provide fixed unit price for the quantities that the buyer commits at the beginning of the planning horizon, and charges the as-ordered quantities at the spot price. Tha is, the contract supplier only provides a definite quantity contract to the buyer. In this section, we extend the analysis in previous sections to incorporate such a situation.

Identical notation is adopted in this section. However, Q_n, although still the total remaining commitment size at the beginning of period n, is the exact quantity the buyer can and must buy at c_H for the following n periods. Then, the general dynamic equation governing the n-period ($n \geq 2$) problem is

$$f_n(x, Q_n; c_n) = \min_{y \geq x+q, Q_n \geq q \geq 0}[c_H q + (y - x - q)c_n + G_{n-1}(y, Q_n - q; c_n)], \tag{5.30}$$

with $Q_N = \tilde{Q}$.

In period 1, the buyer first fulfills all remaining commitment, and then orders at the spot price. The analysis is analogous to that in case (i) of the single period model (see **Section 5.3.1**). Here, we give the following lemma without proof.

Lemma 5.4.1 *For the single period problem, suppose the spot price is c_1 and the remaining commitment size is Q_1.*

a) The optimal order quantities are given by

$$(q_s, q_h) = \begin{cases} (0, Q_1), & x + Q_1 \geq S_{s,1}; \\ (S_{s,1} - Q_1 - x, Q_1), & x + Q_1 < S_{s,1}. \end{cases}$$

b) The value function $f_1(x, Q_1; c_1)$ is jointly convex in (x, Q_1) for given c_1. Moreover, $\frac{\partial f_1(x,Q_1;c_1)}{\partial x} - \frac{\partial f_1(x,Q_1;c_1)}{\partial Q_1} = -c_H$ and $\frac{\partial f_1(x,Q_1;c_1)}{\partial Q_1} \geq c_H - c_1$.

Then, we go back to period 2. In this period, the problem is

$$f_2(x, Q, c_2) = \min_{Q \geq q \geq 0; y \geq x+q} [c_H q + c_2(y - x - q) + L(y) + \mathrm{E}_{\xi, c_1} f_1(y - \xi, (Q - q); c_1)].$$

Thus, according to the analysis of case $c_2 \leq c_H$ for the two-period model in **Section 5.3.2**, we find that the optimal policy is similar to that in **Theorem 5.3.4**. $f_2(x, Q_2; c_2)$ is jointly convex in (x, Q_2) and $\frac{\partial f_2(x,Q_2;c_2)}{\partial x} - \frac{\partial f_2(x,Q_2;c_2)}{\partial Q_2} = -c_H$. But $\frac{\partial f_2(x,Q_2;c_2)}{\partial Q_2} \geq 0$ does not hold any more (since it is possible for the buyer to fulfill the remaining commitment first, and then to order from the spot market whose price is higher than the commitment price).

We keep using Q_n^*, $S_{s,n}^Q$ as in **Section 5.3.3**. Let $\bar{S}_{s,n}$ be the optimal order-up-to level for the following multi-period dynamic problem:

$$I_{k+1}(x, c_{k+1}) = \min_{y \geq x} [c_n(y - x) + L(y) + \mathrm{E}_{\xi, c_k} I_k(y - \xi; c_k)],$$

with $I_0 = 0$.

We state the next theorem concerning the optimal commitment and inventory policy for the definite quantity contract in a general n-period setting.

Theorem 5.4.1 *In period n, suppose the spot price is c_n and the remaining commitment size is Q_n.*

If $Q_n^* = 0$, then the optimal order quantities are given by

$$(q_s, q_h) = \begin{cases} (0, 0), & x \geq S_H^M; \\ (0, S_H^M - x), & x < S_H^M \text{ and } x + Q_n \geq S_H^M; \\ (0, Q_n), & x + Q_n < S_H^M \text{ and } x + Q_n \geq \bar{S}_{s,n}; \\ (\bar{S}_{s,n} - Q_n - x, Q_n), & x + Q_n < \bar{S}_{s,n}. \end{cases} \tag{5.31}$$

If $0 < Q_n^* \leq Q_n$, then the optimal order quantities are given by

$$(q_s, q_h) = \begin{cases} (0, 0), & x \geq S_H^M; \\ (0, S_H^M - x), & x < S_H^M \text{ and } x + Q_n - Q_n^* \geq S_H^M; \\ (S_H^M - (Q_n - Q_n^*) - x, Q_n - Q_n^*), & x + Q_n - Q_n^* < S_H^M. \end{cases} \tag{5.32}$$

If $Q_n < Q_n^*$, then $S_H^M < S_{s,n}^Q$, and the optimal order quantities are given by

$$(q_s, q_h) = \begin{cases} (0, 0), & x \geq S_{s,n}^Q; \\ (S_{s,n}^Q - x, 0), & x < S_{s,n}^Q. \end{cases} \tag{5.33}$$

Furthermore, $f_n(x, Q_n; c_n)$ is jointly convex in (x, Q_n) and $\frac{\partial f_n(x,Q_n;c_n)}{\partial x} - \frac{\partial f_2(x,Q_n;c_n)}{\partial Q_n} = -c_H$.

The proof of this theorem is similar to that for the case $c_2 \leq c_H$ in **Section 5.3.3**, and we neglect the proof here.

Thus, we can see that the procurement policy is similar to the case $c_n \leq c_H$ for minimal TOQC contract. Furthermore, if $c_n = c$ is deterministic for the planning horizon, then, we can inductively show that $\frac{\partial f_n(x,Q;c_n)}{\partial Q} \geq c_H - c$ for all n. By (5.25), $Q_n^* = 0$ for all n, thus, the inventory policy is degenerated to that in Chen and Krass (2001).

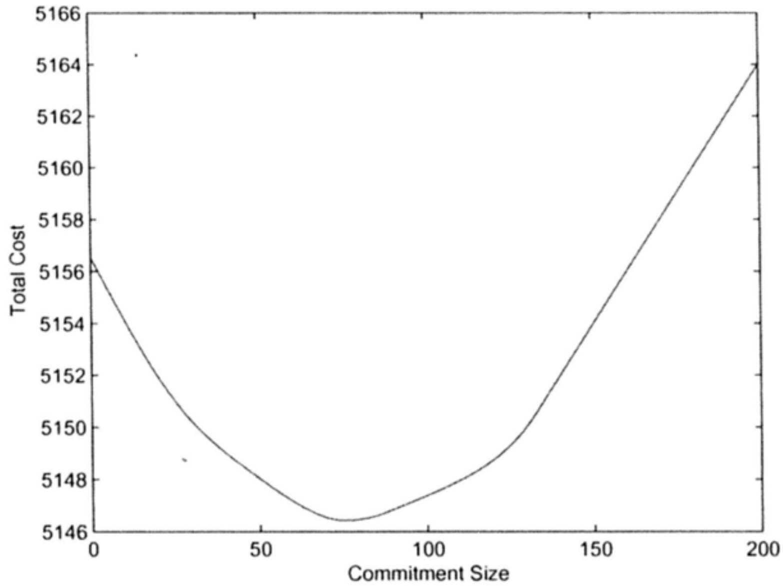


Figure 5.5: The effect of initial commitment

Furthermore, as $\frac{\partial f_n(x,Q;c_n)}{\partial Q} \geq 0$ does not hold any more for the definite quantity contract, the buyer should also determine the optimal size of total commitment at the beginning of the planning horizon to minimize his cost. This is because the definite quantity contract only allows the buyer to buy exactly the committed amount during the planning horizon, which is different from the minimal TOQC contract where the smaller the TOQC is, the more flexibility he obtains. This can also be seen in **Figure 5.5**, where we consider such a definite quantity contract.

5.5 Numerical Study

In this section, we conduct an extensive numerical analysis when the spot price during the planning horizon follows a Geometric Brownian Motion process. The results in this section may provide managerial insights. In particular, we investigate the impact of the spot price volatility on the the buyer's performance at different commitment levels.

In our numerical example, we assume that the spot price process follows a Geometric Brownian Motion, which is formulated as

$$dc_t = c_t \mu_t dt + c_t \sigma_t dw_t,$$

where w_t is a standard Wiener process. Furthermore, we assume the overall μ and σ are constants for the planning horizon. Actually, more general commodity price processes can be easily incorporated into our model to reflect varying market conditions.

We first discretize the continuous process to a discrete version by the (Cox-Ross-Rubinstein) binomial tree method. That is, we assume the process of $p_t e^{\mu_t}$ is a martingale under the risk-neutral probability, and the risk-neutral interest rate is zero (this is for simplicity as a non-zero interest rate has no impact on the following analysis), and we divide one period into k stages (thus, there are Nk stages during the planning horizon). The CRR model then tells us that there are two price states in the next stage, C_u and C_d, depending on the spot price, C, in the current stage. The relationships between these values are as follows:

$$C_u = e^{\frac{\mu}{Nk}+\sqrt{\frac{\sigma}{Nk}}}C \quad \text{and} \quad C_d = e^{\frac{\mu}{Nk}-\sqrt{\frac{\sigma}{Nk}}}C. \tag{5.34}$$

And the probabilities associated with these two states, $\mathbb{P}_u$ and $\mathbb{P}_d$, are

$$\mathbb{P}_u = \frac{1-e^{-\frac{\sigma}{Nk}}}{e^{\frac{\sigma}{Nk}} - e^{-\frac{\sigma}{Nk}}} \quad \text{and} \quad \mathbb{P}_d = 1 - \mathbb{P}_u.$$

The parameters of our base model are listed in **Table 5.1** as bellow. Other parameters are: $k = 1$ (which is in fact without loss of generality), the initial inventory $x = 0$.

We vary the price volatility, σ, from 0.2 to 1 with an increment of 0.2. And the corresponding performance is illustrated in **Figure 5.6**.

Table 5.1: Parameters settings

Demand Distribution	Normal Dist. $\mu_d = 60$ $\sigma_d = 20$	Cost Paremeter	Shortage Cost =20 Holding Cost =10
Price Process (GBM)	Initial $c_N = 12$	Min-Commitment	$\tilde{Q} = 160$
	$\mu = 0.2$	Commitment Price	$c_H = 15$
	$\sigma = 0.4$	Number of Periods	$N = 5$

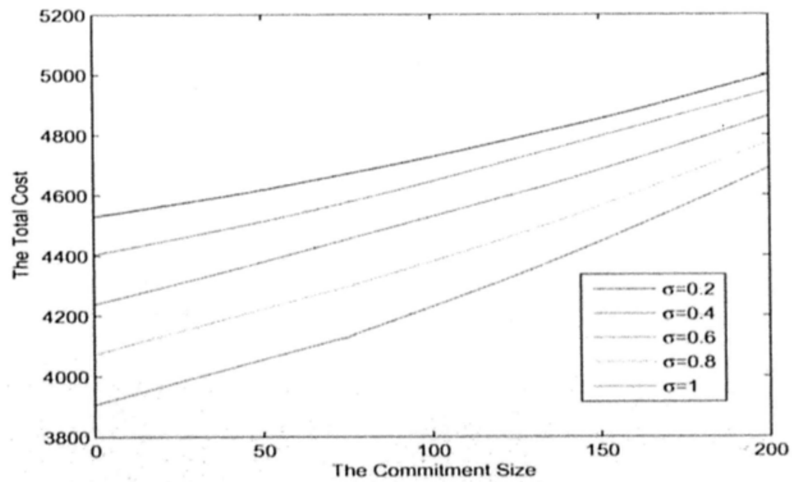


Figure 5.6: Relationship between total cost, price volatility and commitment size

From this figure, we can see that large volatility benefits the buyer. This is because the contract supplier provides a natural hedge to the higher price movements. However, the cost saved by high volatility decreases in the commitment size. This can be explained by the fact the the commitment size restricts the buyer's flexibility to use the spot market.

5.6 Summary

In this chapter, we consider an optimal procurement problem when the buyer can procure from a contract supplier with a minimal total order quantity commitment(TOQC) contract and the spot market. Our analysis characterizes an optimal procurement policy which depends on the spot price in each period. Specifically, when the spot price is higher than the commitment price, the buyer only procures from the contract supplier. Otherwise, there is an optimal commitment level for each period, and the buyer first procures from the contract supplier to bring the remaining commitment down-to this optimal level, and then procures from the spot market. Such a structured policy can be viewed as a combination of some base-stock policies, each of which can be computed through an equivalent system of simple multiple-period newsvendor type. We also numerically show how the existence of the spot market affects the buyer's performance with such a minimal TOQC contract.

One extension of our current model is to consider the case when there is a setup cost, K, each time when procuring from the spot market. Such a modification complicates the problem in a significant way. Another extension is to allow the buyer to trade through the spot market. But in this case, it is possible for the buyer to have an arbitrage opportunity through buying as more as possible from the contract supplier to sell to the spot market when the spot price is higher. To avoid such situation, a maximum total order quantity commitment (TOQC) during the planning horizon or a periodical maximum order quantity should be placed, in addition to the minimal TOQC contract. This should be our future research.

5.7 Appendix

Proof of Theorem 5.3.2 Part a), b) and c) are straightforward. We only prove part d) here. The formulation of $f_1(x, Q_1; c_1)$ can be denoted as follows:

$$f_1(x, Q_1; c_1) = \begin{cases} c_H Q_1 + (S_{s,1} - x - Q_1)c_1 + L(S_{s,1}), & x + Q_1 \leq S_{s,1} \text{ and } c_1 \leq c_H; \\ c_H(S_{H,1} - x) + L(S_{H,1}), & x + Q_1 \leq S_{H,1} \text{ and } c_1 > c_H; \\ c_H Q_1 + L(x + Q_1), & \text{otherwise.} \end{cases}$$

We only prove the case when $c_1 \leq c_H$, and that for $c_1 > c_H$ is similar to it.

It is straightforward that when $x + Q_1 \leq S_{s,1}$ or $x + Q_1 > S_{s,1}$, $f_1(x, Q_1; c_1)$ is jointly convex in (x, Q_1). What we need to prove is whether the combination of these two regions results in a convex function.

Denote (x', Q') and $(\bar{x}, \bar{Q})$ which satisfy $x' + Q' \leq S_{s,1}$ and $\bar{x} + \bar{Q} \geq S_{s,1}$ respectively. Let $\lambda \in [0, 1]$ and λ_0 satisfies

$$\lambda_0(x' + Q') + (1 - \lambda_0))(\bar{x} + \bar{Q}) = S_{s,1}.$$

It is apparent that $\lambda_0 \in [0, 1]$. Let $x_\lambda = \lambda x + (1 - \lambda))\bar{x}$, $Q_\lambda = \lambda Q' + (1 - \lambda))\bar{Q}$.

When $\lambda \leq \lambda_0$, $x_\lambda + Q_\lambda \geq S_{s,1}$, we have $(1 - \lambda)(\bar{x} + \bar{Q}) + \lambda S_{s,1} \geq x_\lambda + Q_\lambda$. Since $c_1 y + L(y)$ increases in y when $y \geq S_{s,1}$, we get

$$\begin{aligned} c_1(x_\lambda + Q_\lambda) + L(x_\lambda + Q_\lambda) &\leq c_1((1 - \lambda)(\bar{x} + \bar{Q}) + \lambda S_{s,1}) + L((1 - \lambda)(\bar{x} + \bar{Q}) + \lambda S_{s,1}) \\ &\leq (1 - \lambda)[c_1((\bar{x} + \bar{Q})) + L((\bar{x} + \bar{Q}))] + \lambda(S_{s,1} + L(S_{s,1})). \end{aligned}$$

Thus, we can derive the following result:

$$\begin{aligned} f_1(x_\lambda, Q_\lambda; c_1) &= c_H Q_\lambda + L(x_\lambda + Q_\lambda) \\ &\leq c_H Q_\lambda + (1 - \lambda)[c_1((\bar{x} + \bar{Q})) + L((\bar{x} + \bar{Q}))] + \lambda(S_{s,1} + L(S_{s,1})) - c_1(x_\lambda + Q_\lambda) \\ &= \lambda f_1(x', Q'; c_1) + (1 - \lambda) f_1(\bar{x}, \bar{Q}; c_1). \end{aligned}$$

The case when $\lambda > \lambda_0$ is almost the same as above analysis, we neglect the detailed analysis here. The property that $f_1(x, Q_1; c_1)$ increases in Q_1 is a combination of a) and b). And $\frac{\partial f_1(x,Q_1;c_1)}{\partial x} - \frac{\partial f_1(x,Q_1;c_1)}{\partial Q_1} = -c_H$ can be verified case by case. $\diamond$

Proof of Lemma 5.3.2 Taking derivative of $W(y, q)$ with respect to q yields

$$L'(S_2^*(q)) - \int_0^{c_H}\Big[\int_{A(q)-S_{s,1}}^{\infty} c_1\phi_1(\xi)\phi_2(c_1)d\xi - \int_0^{A(q)-S_{s,1}} L'(A(q)-\xi)\phi_1(\xi)\phi_2(c_1)d\xi\Big]dc_1$$
$$-\int_{c_H}^{\infty}\Big[\int_{A(q)-S_{H,1}}^{\infty} c_H\phi_1(\xi)\phi_2(c_1)d\xi - \int_0^{A(q)-S_{H,1}} L'(A(q)-\xi)\phi_1(\xi)\phi_2(c_1)d\xi\Big]dc_1 = -c_2, \tag{5.35}$$

where $A(q) = S_2^*(q) + Q_2 - q$. Let $M(q)$ represent the left-hand side of equation (5.35). Taking total derivative of $M(q)$ in q and letting $M'(q) = 0$, we have

$$L''(S_2^*(q))\frac{\partial S_2^*(q)}{\partial q} + \Big[\int_0^{c_H}\int_0^{A(q)-S_{s,1}} L''(A(q)-\xi)\phi_1(\xi)\phi_2(c_1)d\xi dc_1 +$$
$$\int_{c_H}^{\infty}\int_0^{A(q)-S_{H,1}} L''(A(q)-\xi)\phi_1(\xi)\phi_2(c_1)d\xi dc_1\Big]\Big(\frac{\partial S_2^*(q)}{\partial q} - 1\Big) = 0.$$

The convexity of $L(x)$ implies $\frac{\partial S_2^*(q)}{\partial q} \geq 0$. The first part of the lemma is thus proved. The above equation can also be rewritten as

$$L''(S_2^*(q))\frac{\partial S_2^*(q)-q}{\partial q} + L''(S_2^*(q)) + \Big[\int_0^{c_H}\int_0^{A(q)-S_{s,1}} L''(A(q)-\xi)\phi_1(\xi)\phi_2(c_1)d\xi dc_1 +$$
$$\int_{c_H}^{\infty}\int_0^{A(q)-S_{H,1}} L''(A(q)-\xi)\phi_1(\xi)\phi_2(c_1)d\xi dc_1\Big]\Big(\frac{\partial S_2^*(q)-q}{\partial q}\Big) = 0.$$

Thus, $\frac{\partial(S_2^*(q)-q)}{\partial q} \leq 0$, which completes the proof for the second part. $\diamond$

Proof of Lemma 5.3.3 Setting $q = 0$, we take total derivative with respect to Q_2 of (5.35), and obtain

$$L''(S_{s,2}^Q)\frac{\partial S_{s,2}^Q}{\partial Q_2} + \Big[\int_0^{c_H}\int_0^{S_{s,2}^Q+Q_2-S_{s,1}} L''(S_{s,2}^Q + Q_2 - \xi)\phi_1(\xi)\phi_2(c_1)d\xi dc_1$$
$$+\int_{c_H}^{\infty}\int_0^{S_{s,2}^Q-S_{H,1}} L''(S_{s,2}^Q + Q_2 - \xi)\phi_1(\xi)\phi_2(c_1)d\xi dc_1\Big]\Big(\frac{\partial S_{s,2}^Q}{\partial Q_2} + 1\Big) = 0.$$

By the convexity of $L(x)$, we conclude that $S_{s,2}^Q$ decreases in Q_2 and $\frac{S_{s,2}^Q}{\partial Q_2} \geq -1$. $\diamond$

Lemma A6 *$T(q)$ is convex. Let $q_2^{*(1)}$ be the optimal solution to $T(q)$, then, $S_2^*(q_2^{*(1)}) = S_H^M$. Furthermore, if $Q_2^* > 0$, $q_2^{*(1)} = Q_2 - Q_2^*$ and if $Q_2^* = 0$, $q_2^{*(1)} \geq Q_2$.*

Proof. Taking derivative of $T(q)$ with respect to q yields

$$T'(q) = -c_2 + \int_0^{c_H}\int_{A(q)-S_{s,1}}^{\infty} c_1\phi_1(\xi)\phi_2(c_1)d\xi dc_1 - \int_0^{c_H}\int_0^{A(q)-S_{s,1}} L'(A(q)-\xi)\phi_1(\xi)\phi_2(c_1)d\xi dc_1$$
$$+\int_{c_H}^{\infty}\int_{A(q)-S_{H,1}}^{\infty} c_H\phi_1(\xi)\phi_2(c_1)d\xi dc_1 - \int_{c_H}^{\infty}\int_0^{A(q)-S_{H,1}} L'(A(q)-\xi)\phi_1(\xi)\phi_2(c_1)d\xi dc_1,$$

where $A(q) = S_2^*(q) + Q_2 - q$. Substituting (5.35) into the above formulation, we get $T'(q) = L'(S^*(q))$.

Thus, $T''(q) = \frac{\partial L'(S^*(q))}{\partial q} \geq 0$, i.e., $T(q)$ is convex, and $S_2^*(q_2^{*(1)}) = S_H^M$.

Let $P = Q_2 - q_2^{*(1)}$. Then, as a consequence of $S_2^*(q_2^{*(1)}) = S_H^M$ and (5.35), we have

$$\int_{c_H}^{\infty}\int_{S_H^M-S_{H,1}+P}^{\infty} c_H\phi_1(\xi)\phi_2(c_1)d\xi dc_1 + \int_0^{c_H}\int_{S_H^M-S_{s,1}+P}^{\infty} c_1\phi_1(\xi)\phi_2(c_1)d\xi dc_1 +$$
$$\int_0^{c_H}\int_0^{S_H^M-S_{s,1}+P} -L'(S_H^M+P-\xi)\phi_1(\xi)\phi_2(c_1)d\xi dc_1$$
$$+\int_{c_H}^{\infty}\int_0^{S_H^M-S_{H,1}+P} -L'(S_H^M+P-\xi)\phi_1(\xi)\phi_2(c_1)d\xi dc_1 = c_2. \tag{5.36}$$

From the proof of **Theorem 5.3.1**, we know $\frac{\partial f_1(x,Q_1;c_1)}{\partial x} - \frac{\partial f_1(x,Q_1;c_1)}{\partial Q_1} = -c_H$ for all $Q_1 \in (-\infty,\infty)$, rearranging items finds that P satisfies the first order condition for the inner part of (5.15). Thus, if $Q^* > 0$, then, $P = Q^* > 0$; if $Q^* = 0$, then, $P \leq 0$. This completes the proof. ◇

Lemma A7 *$\tilde{T}(q)$ is convex. Let $q_2^{*(1)}$ be the optimal solution to $T(q)$, then, $q_2^{*(2)} = S_H^M - x$.*

Proof. The convexity follows from **Theorem 5.3.1.c)**. Then, the first order condition

$$\tilde{T}'(q) = L'(x+q) = 0$$

gives the desired result. ◇

Lemma A8 $q_2^{*(1)} \geq q_2^{*(2)} \geq q_x$ or $q_2^{*(1)} \leq q_2^{*(2)} \leq q_x$.

Proof. If $q_2^{*(2)} \geq q_x$, i.e., $S^*(q_2^{*(1)}) = S_H^M \geq x + q_x = S^*(q_x)$, then, $q_2^{*(1)} \geq q_x$. Furthermore, $q_2^{*(1)} \geq S^*(q_2^{*(1)}) - x = S_H^M - x = q_2^{*(2)}$.

If $q_2^{*(2)} \leq q_x$, i.e., $S^*(q_2^{*(1)}) = S_H^M \leq x + q_x = S^*(q_x)$, then, $q_2^{*(1)} \leq q_x$. Furthermore, $q_2^{*(1)} \leq S^*(q_2^{*(1)}) - x = S_H^M - x = q_2^{*(2)}$. ◇

Proof of Theorem 5.4.1 Without loss of generality, we assume the optimal policy for $n = k$ is as the theorem states. Furthermore, we assume $f_k(x, Q_k; c_k)$ is jointly convex in (x, Q_k), $\frac{\partial f_k(x,Q_k;c_k)}{\partial x} - \frac{\partial f_k(x,Q_k;c_k)}{\partial Q_k} = -c_H$ and $\frac{\partial f_k(x,Q_k;c_k)}{\partial x} \geq -c_H$. Then, we consider the case when $n = k + 1$.

(i)When $c_{k+1} \geq c_H$, the decision maker only procures from the contract supplier. The ordering cost can be formulated as follows.

$$f_{k+1}(x, Q_{k+1}; c_{k+1}) = \min_{y \geq x}[c_H(y - x) + L(y) + \mathrm{E}_{\xi,c_k} f_k(y - \xi, (Q_{k+1} - y + x)^+; c_k)]. \tag{5.37}$$

Define

$$\begin{aligned} H_{k+1}(x, y) &= c_H(y - x) + L(y) + \mathrm{E}_{\xi,c_k} f_k(y - \xi, Q_{k+1} - y + x; c_k); \\ R_{k+1}(x, y) &= c_H(y - x) + L(y) + \mathrm{E}_{\xi,c_k} f_k(y - \xi, 0; c_k). \end{aligned}$$

Then, $H(x, y)$ and $T(x, y)$ are convex in y, and we have

$$f_{k+1}(x, Q_{k+1}; c_{k+1}) = \min\{\min_{Q_{k+1}+x \geq y \geq x} H_{k+1}(x, y), \min_{y \geq x + Q_{k+1}} R_{k+1}(x, y)\}. \tag{5.38}$$

The optimal solution to $\min_y H_{k+1}(x, y)$ satisfies the following first order condition:

$$c_H + L'(y) + \mathrm{E}_{\xi,c_k}[\frac{\partial f_k(t, s; c_k)}{\partial t}|_{(y-\xi,Q_{k+1}-y+x)} - \frac{\partial f_k(t, s; c_k)}{\partial s}|_{(y-\xi,Q_{k+1}-y+x)}] = 0.$$

Thus, the optimal solution is S_H^M since $[\frac{\partial f_k(t,s;c_k)}{\partial t}|_{(y-\xi,Q_{k+1}-y+x)} - \frac{\partial f_k(t,s;c_k)}{\partial s}|_{(y-\xi,Q_{k+1}-y+x)}] = -c_H$.

Let $S_{H,n}$ be the optimal solution to $\min_y T(x,y)$. Then, $S_{H,n}$ satisfies the following first order condition:

$$c_H + L'(y) + \mathrm{E}_{\xi,c_k}\frac{\partial f_k(t,0;c_k)}{\partial t}|_{t=y-\xi} = 0.$$

Thus, as $\frac{\partial f_k(t,0;c_k)}{\partial t}|_{t=y-\xi} \geq -c_H$ $L'(y)|_{y=S_{H,n}} \leq 0$ and we get $S_{H,n} \leq S_H^M$. The following analysis on the optimal policy is almost the same as that in **Theorem 5.3.2**. Therefore, the optimal inventory policy in this case is as the theorems states .

(ii) When $c_{k+1} < c_H$, the dynamic equation can be formulated as follows.

$$\begin{aligned} f_{k+1}(x,Q_{k+1};c_{k+1}) &= \min\nolimits_{Q_{k+1}\geq q\geq 0}\{c_H q - c_{k+1}q + \inf\nolimits_{y\geq x+q}[c_{k+1}(y-x) + L(y) \\ &\quad + \mathrm{E}_{\xi,c_1} f(y-\xi,(Q_{k+1}-q);c_k)]\}. \end{aligned} \tag{5.39}$$

For a fixed q, we define

$$M_{k+1}(y) = c_{k+1}(y-x) + L(y) + \mathrm{E}_{\xi,c_k} f_k(y-\xi,(Q_{k+1}-q);c_k). \tag{5.40}$$

Then, $H(y)$ is convex on y. Denote the optimal minimizer of $H(y)$ by $S_{k+1}^*(q)$, then, for each fixed q, the optimal $y(q)$ for (5.39), as a function of q, is as follows.

$$y(q) = \begin{cases} S_{k+1}^*(q), & x \leq S_{k+1}^*(q) - q; \\ x+q, & \text{Otherwise}, \end{cases} \tag{5.41}$$

where $S_{k+1}^*(q)$ satisfies the first order condition

$$c_{k+1}y + L'(y) + \mathrm{E}_{\xi,c_k}\frac{\partial f_k(t,(Q_{k+1}-q);c_k)}{\partial t}|_{t=y-\xi} = 0. \tag{5.42}$$

Note $\frac{\partial^2 f_k(t,s;c_k)}{\partial t^2} = \frac{\partial^2 f_k(t,s;c_k)}{\partial t \partial s}$ because $\frac{\partial f_k}{\partial x} - \frac{\partial f_k}{\partial Q_k} = -c_H$, we take total derivative of the left-hand side of (5.42) with respect to q and have

$$L''(S^*_{k+1}(q))\frac{\partial S^*_{k+1}(q)}{\partial q} + [\mathrm{E}_{\xi,c_k}\frac{\partial^2 f_k(t,s;c_k)}{\partial t^2}|_{(t,s)=(S^*_{k+1}(q)-\xi,Q_{k+1}-q)}](\frac{\partial S^*_{k+1}(q)}{\partial q} - 1) = 0.$$

Thus, there is a similar result as **Lemma 5.3.2** concerning $S^*_{k+1}(q)$. Let $q^{*(1)}_{k+1}$ and $q^{*(1)}_{k+1}$ be the minimizer of $T(q)$ and $\tilde{T}(q)$, where

$$T_k(q) = (c_H - c_{k+1})q + c_{k+1}(S^*_{k+1}(q) - x) + L(S^*_{k+1}(q)) + \mathrm{E}_{\xi,c_k} f_k(S^*_{k+1}(q) - \xi, Q_{k+1} - q; c_k);$$

$$\tilde{T}_k(q) = c_H q + L(x+q) + \mathrm{E}_{\xi,c_k} f_k(x + q - \xi, (Q_{k+1} - q); c_k).$$

Then, if $q(x)$ satisfies $S^*_{k+1}(q_x) - q_x = x$, we have

$$f_{k+1}(x, Q_{k+1}; c_{k+1}) = \min\{\inf_{0 \le q \le q_x} T_k(q), \inf_{q_x \le q \le Q_{k+1}} \tilde{T}_k(q)\}.$$

The first order condition for $T(q)$ is

$$\begin{aligned} 0 = \frac{\partial T(q)}{\partial q} &= c_H - c_{k+1} - \mathrm{E}_{\xi,c_k}\frac{\partial f_k(S^*_{k+1}(q) - \xi, s; c_k)}{\partial s}|_{s=Q_{k+1}-q} \\ &= -c_{k+1} - \mathrm{E}_{\xi,c_k}\frac{\partial f_k(t, Q_{k+1} - q; c_k)}{\partial t}|_{t=S^*_{k+1}(q)-\xi} \qquad \text{(Assumption)} \\ &= L'(S^*_{k+1}(q)) \qquad \text{(Equation (5.42))}. \end{aligned}$$

Thus, $S^*_{k+1}(q^{*(1)}_{k+1}) = S^M_H$.

The first order condition for $\tilde{T}(q)$ is

$$\begin{aligned} 0 = \frac{\partial \tilde{T}(q)}{\partial q} &= c_H + L'(x+q) + \mathrm{E}_{\xi,c_k}[\frac{\partial f_k(t,s;c_k)}{\partial t}|_{(x+q-\xi,Q_{k+1}-q)} - \frac{\partial f_k(t,s;c_k)}{\partial s}|_{(x+q-\xi,Q_{k+1}-q)}] \\ &= L'(x+q). \end{aligned}$$

Thus, $q^{*(2)}_{k+1} = S^M_H - x$. Furthermore, similar results as **Lemma A6**, **Lemma A8** and **Lemma A8** can also be obtained for this general case.

Applying analogous analysis we used for proving **Theorem 5.3.4**, we can get the optimal policy for period $k+1$, which is just what **Theorem 5.4.1** states.

At last, we need to prove that the assumption at the beginning of this proof holds for $n = k+1$. Actually, the jointly convexity of $f_{k+1}(x, Q_{k+1}; c_{k+1})$, $\frac{\partial f_{k+1}(x,Q_{k+1};c_{k+1})}{\partial x} - \frac{\partial f_{k+1}(x,Q_{k+1};c_{k+1})}{\partial Q_k} = -c_H$ and $\frac{\partial f_{k+1}(x,Q_{k+1};c_{k+1})}{\partial x} \leq -c_H$ can be verified case by case when we explicitly writes out the form of $f_{k+1}(x, Q_{k+1}; c_{k+1})$. We neglect this process here.

Thus, the case for $n = k+1$ is proved. ◇

□ **End of chapter.**

Chapter 6

Concluding Remarks and Future Research

The rapid growth of organized spot markets and the development of information technology has significantly transformed the supply and sourcing environment. Instead of relying solely on the long-term supply contracts to secure input supply, manufacturers today prefer to maintain contractual relationship with their key suppliers, while using the spot market as an important supplementary supply source. However, as pointed in Haksoz and Seshadri (2007), although academic research has been extensively studied on the use of the spot market to manage procurement in supply chains, "only a small beginning has been made with regard to optimal procurement and inventory planning over multiple periods by incorporating different types of supply contracts".

In this thesis, we develop mathematical models to address the inventory and procurement decision problems when both supply contracts and spot markets are presented. Specifically, we consider the following three kinds of supply contracts: 1) a forward contract in which the buyer can buy a predetermined fixed amount of raw materials at a specified point of time in the future at a specified price; 2) a periodic minimal commitment contract in which a fixed unit price is charged and a predetermined minimum quantity in each period must be committed; 3) a minimal total order quantity commitment (TOQC) contract in which the buyer guarantees that his cumulative orders for the planning horizon will be at least a specified minimum quantity and the supplier provides fixed price for the quantity the buyer buys.

We first study the forward contract within a two-period framework to gain some preliminary results. We find that when the buyer can not sell to the spot market, there exists a threshold forward price, under which the buyer enters the forward contract. This threshold is lower than the expected spot price. Furthermore, such threshold and the optimal order quantities via forward contract, which are finite in this case, increase in the mean of the spot price, but decrease in the variability of the spot price. However, when the buyer can sell to the spot market, he only speculates to use the forward contract.

Then, we focus on the incorporation of the periodic minimal commitment contract and the TOQC contract, both of which provide quantity flexibilities to buyers, under a multi-period setting. We develop the optimal inventory and procurement policies for these two contracts by assuming the the buyer can not sell to the spot market.

For the periodic minimal order quantity commitment contract, we prove that the optimal policy for the buyer consists three different policies of (s, S) type. Moreover, we identify certain conditions under which monotone properties exist between the policy parameters and the spot price when the spot price during the planning horizon follows a general Markov process. Then, in each period, we can divide the spot price space into three regions, in each of which a specific policy applies. Furthermore, we numerically show that the buyer prefers a more volatile market condition when engaging a periodic minimal commitment contract.

For the minimal TOQC contract, we derive the optimal procurement policy which is much more complex than that in a deterministic environment. Such policy can be viewed as a combination of some base-stock levels, which can be calculated by some newsvendor-like models. This enhances the practicality of our model. Different from the traditional research on TOQC contract in a deterministic environment, in which the buyer always fulfills the commitment before using as-ordered price, we find the buyer may procures via the spot market in the current period before fulfilling the commitment in our model. We also extent the analysis on this minimal TOQC contract to incorporate a definite quantity contract, in which the buyer is only allowed to procure the exact size he committed during the planning horizon. We find that the optimal policy for this definite quantity contract is similar to that for the minimal TOQC contract. Our numerical analysis implies that the buyer benefits from the market volatility in the presence of spot market. But such benefits decrease as the TOQC increases.

As one direction of our future research, more types of flexible contracts can be taken into consideration in the presence of the spot market. The flexibility can be time flexibility, price flexibility and quantity flexibility (see Feng and Sethi (2008)). It is valuable to reconsider the optimal procurement policy with these flexible contracts, which are once considered in a deterministic environment, to see how firms should adopt their operating policies in the presence of fluctuating spot prices. Dynamic models that address the optimal decisions of both the buyer and the seller should be studied. More works need to be done to analyze what the optimal mix of the flexible contracts and the spot procurement is and how such mix strategy will effect the participants' performance.

There are other directions we think are fruitful for future research in this area. One of these is to integrate the dynamics of information revelation of the customer demand and spot prices into the procurement problems. As time passes, more information becomes available, therefore, demand and spot prices can be forecast better. It will be useful to establish the conditions under which using the updated demand/price forecasts and the flexible contracts reduces supply chain procurement costs.

Another direction is to introduce the commodity derivative markets into procurement problems. For example, futures prices reflect the anticipated demand and supply equilibrium in the future. And the information generated by the futures markets can be useful in negotiating forward contracts between buyers and suppliers of the commodity since the future price dictated by the market contains more information on demand/supply dynamics than the information held by individual buyers and suppliers. Therefore, the decision maker now needs to determine not only the inventory procurement quantities but also the futures markets holdings at the same time. The role of these derivative markets in hedging risk should be further considered when the buyer can use inventory control as a mean of operational hedge, which is mostly neglected in the literature of economics and finance.

In summary, we believe the incorporating of spot markets into traditional inventory problems will be a developing and interesting area ripe for new research, insights, and practical relevance.

□ **End of chapter.**

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