

Essays on Macroeconomics in the Frequency

Domain

by

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Dissertation submitted in partial fulfillment of the requirements for the degree of
Doctor of Philosophy in the Department of Economics
in the Graduate School of Duke University
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ABSTRACT
(Economics)

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Abstract

This dissertation consists of three essays on macroeconomics in the frequency domain. In the first essay, I show that whereas the High-Frequency volatility of the majority of the macroeconomic series declined after the early 1980s, their Medium-Frequency volatility did not. Moreover, the Medium-Frequencies capture a large fraction of the volatility of these variables. In order to formally test whether a set of time-series is characterized by a break in their variance at any frequency, I construct a frequency domain structural break test. After deriving its asymptotic and small sample properties, I apply the test to the main U.S. real macroeconomic variables and conclude that the Great Moderation is just a High-Frequency phenomenon.

In the second essay I compute the welfare cost of the Great Moderation, using a consumption based asset pricing model. The Great Moderation is modeled according to the data properties of the stationary component of consumption, which displays a reduction of the volatility at high frequencies, and an unchanged volatility at medium frequencies. The theoretical model, calibrated to match the average asset pricing variables in the data, relies on the evolution of the habit stock, which depends on the lower frequencies of consumption. These two features generate a modest welfare gain of the Great Moderation (0.6 percent). I show that this result depends mainly on the medium frequency properties of consumption.

The third essay, which is joint work with Marija Vukotić, evaluates the effects of a change in monetary policy on the decline of the volatility of real macroeconomic

variables, and on its redistribution from high to medium frequencies during the post-1983 period. By using a dynamic stochastic general equilibrium (DSGE hereafter) model, we find that the monetary policy alone cannot account for the observed changes in the spectral density of output, investment, and consumption. However, when we also consider a change in the exogenous processes, a different monetary policy accounts for 40 percent of the decline in the high-frequency volatilities and partially accounts for the redistribution of the variance toward lower frequencies.

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Acknowledgements

I am truly grateful to Professor Craig Burnside for his guidance and constant support during my years at Duke. I am also grateful to the members of my committee Professors Pietro Peretto, Barbara Rossi, and Juan Rubio-Ramírez for all their help. I thank seminar participants of Duke's Macroeconomics Lunch Groups for their helpful comments and suggestions. I would also like to thank Professor Giorgio Calzolari for his advice throughout the years. I give special thanks to Marija Vukotić for her constant support and encouragement.

I am especially grateful to my mother, my father, my brother, and my grandparents for everything they have taught me.

Spectral Covariance Instability Test: An Application to the Great Moderation

1.1 Introduction

The investigation of the properties of economic cycles plays a central role in macroeconomics, since the movements and co-movements of the variables along their trend individuate booms and recessions. The literature on this topic is extensive, beginning early in the 20th century (see Schumpeter (1927, 1939), Kuznets (1940, 1961), Friedman and Schwartz (1963), Lucas (1977), and Hodrick and Prescott (1997)). As a consequence, the identification of the cyclical component of macroeconomic variables has drawn attention in the discipline, both from a theoretical and an empirical point of view. Burns and Mitchell (1946) are among the pioneers of business cycle analysis, defining the business cycle as those fluctuations that occur in the economy with periodicity up to ten years. Baxter and King (1999) formalized this definition and provided tools to isolate the business cycle component of a time series from its trend.

In the vast literature on the cyclical properties of the macroeconomic variable,

the above-mentioned definition of the business cycle provided by Burns and Mitchell (1946) is the conventionally adopted measure of the economic cycle. However, a recent paper by Comin and Gertler (2006) examines the features of a more broadly defined cycle, including fluctuations up to 50 years long. In this paper I follow a similar strategy, studying the cyclical behavior of the macroeconomic variables when the cycle includes fluctuations with larger periodicity than the conventional definitions of business cycle. In particular, I define the High-Frequency component, as the fluctuations included between 2 and 32 quarters, which corresponds to the common definition of the business cycle, and I define the Medium-Frequency component as those fluctuations between 32 and 80 quarters. The conventional business cycle literature includes only the High-Frequency component in the cyclical analysis, ignoring the Medium-Frequency component. In the empirical section of this paper I show that the Medium-Frequency component accounts for a larger fraction of the total fluctuations in output (37 percent) with respect to the High-Frequency component (25 percent). Therefore, ignoring the Medium-Frequencies fluctuations results in a relevant loss of information on the behavior of the economic cycle.

I also find that the behavior of the Medium-Frequency components of U.S. macroeconomic variables in the post-war period evolved differently from that of the High-Frequency component. Since the early 1980s the High-Frequency volatility of macroeconomic variables has sharply declined. The term Great Moderation was introduced to define this well-known and amply documented phenomenon. Kim and Nelson (1999), McConnell and Perez-Quiros (2000), Blanchard and Simon (2001), and Stock and Watson (2002), among others, have contributed to this literature. However, the Medium-Frequency component of the majority of the macroeconomic variables does not display a similar decline in volatility after the early 1980s. Therefore, in this paper I investigate whether the Great Moderation is a phenomenon robust to the adopted measure of cyclical behavior or whether it is just a High-Frequency phenomenon.

To formally test the presence of moderation of the macroeconomic variables at different frequencies, I introduce and define a tool, the Integrated Cospectrum, which computes the variance and the covariance of a set of variables at any given interval of frequencies. Since the goal of this paper is to study whether the variance and covariances of the variables at different cycles has significantly changed throughout the sample, I then define a structural break test in the frequency domain, the Spectral Covariance Instability test. Although the literature on structural break tests is large¹, their application to the frequency domain is one of the novelties introduced in this paper. I present three types of tests, the Spectral Average Wald test, the Spectral Exponential test, and the Spectral Nyblom test. After deriving the asymptotic proprieties of the tests, I analyze their small sample proprieties using Monte Carlo simulations. In addition, I compare the properties of the proposed frequency domain approach, with a time domain GMM-based approach, which is a natural alternative of calculating variances and covariances at any interval of frequencies. I show that the frequency domain approach I propose performs significantly better in small samples.

Finally, I apply the Spectral Covariance Instability test to three of the main U.S. macroeconomic variables, namely output, consumption, and investment. Consistent with the Great Moderation literature, the tests detect a break in the variance of the three variables when only the High-Frequencies are considered. However, when Medium-Frequencies are also included in the cyclical analysis, the tests suggest absence of any moderation. This finding, supported by the descriptive evidence mentioned above, reveals that the Great Moderation is just a High-Frequency phenomenon, which is at least mitigated, if not absent, when a broader measure of cycle is adopted.

The structure of the paper is as follows: in Section 1.2, I document the properties

¹ See Perron (2005) for a review

of the High-Frequency and Medium-Frequency components of U.S. macroeconomic variables. In Section 1.3, I define the Spectral Covariance Instability tests and assess their asymptotic properties. In Section 1.4, I analyze their small sample proprieties using Monte Carlo simulations, with particular emphasis on the advantage of this frequency domain approach with respect to the time domain GMM-based approach. In Section 1.5, I apply the Spectral Covariance Instability tests to consumption, output and investment. Section 1.6 concludes with some final remarks.

1.2 High Frequency and Medium Frequency Cycles

The cyclical behavior of economic variables has been one of the primary interests in macroeconomics since the early stages of the discipline. During most of the last century, research was devoted to the empirical characterization of the economic cycle. Burns and Mitchell (1946) defined the business cycles as follows:

A cycle consists of expansions occurring at about the same time in many economic activities, followed by similarly general recessions, contractions, and revivals which merge into the expansion phase of the next cycle; this sequence of changes is recurrent but not periodic; in duration business cycles vary from more than one year to ten or twelve years; they are not divisible into shorter cycles of similar character with amplitude approximating their own.

This definition was formalized by Baxter and King (1999), which identify the business cycle as those cycles with periodicity from 6 to 32 quarters. This definition is the commonly adopted measure in the cyclical analysis of macroeconomic variables. Thus, all fluctuations with a periodicity larger than 32 quarters are included in the trend, and therefore excluded from the investigation of the cyclical properties of the economic series. Figure 1.1 displays the log-level of real per-capita U.S. Gross

Domestic Product (solid line) and the trend identified by eliminating the fluctuations with periodicity up to 32 quarters (dashed line). The cyclical component is defined as the difference between the level and the trend. However, notice that the trend generated by this procedure displays evident waves that affect the medium-run behavior of the series. This source of fluctuations has been ignored in business cycle analyses that use the conventional definition of the cycle, as described above.

In this paper I study the properties of cycles with lower frequencies than the conventional definition of the business cycle, and their impact on the analysis of the cycle in the economy. The idea of a medium-run cycle was introduced at the beginning of the 20th century; Schumpeter (1954) decomposes a stationary series in four different waves, named after the economist that first introduced them, i.e.

- the Kitchin inventory cycle (2-5 years)
- the Juglar fixed investment cycle (7-11 years)
- the Kuznets infrastructural investment cycle (10-20 years)
- the Kondratieff wave or cycle (45-60 years) .

Therefore, the economic series is thought as a combination of these four components. The purpose of this paper is to apply an idea similar to Schumpeter's (1954). Whereas the conventional analysis of the cycle is based on the business cycle, which approximately includes the Kitchin inventory cycle and the Juglar fixed investment cycles, I also consider the role of the Kuznets infrastructural investment cycle, which corresponds to those fluctuations up to 20 years. Therefore, I ask whether a measure of the cycle defined more broadly, including the latter fluctuations with periodicity up to twenty years, provides additional and relevant information about the cyclical behavior of macroeconomic variables.

In order to clearly isolate the contribution of the different cyclical components, I will refer to the High-Frequency component as fluctuations included between 2 and 32 quarters, and to the Medium-Frequency component as the fluctuation included between 32 and 80 quarters. A formal definition of the cyclical components is provided as follows: Given a time series x_t , the High-Frequency component (HF), x_t^{HF} , corresponds to the cyclical component of x_t with periodicity between 2 and 32 quarters. In the frequency domain, these fluctuations belong to the interval $[\pi, \frac{\pi}{16}]$ for quarterly data. Given a time series x_t , the Medium-Frequency, x_t^{MF} , corresponds to the cyclical component of x_t with periodicity between 32 and 80 quarters. In the frequency domain, these fluctuations belong to the interval $[\frac{\pi}{16}, \frac{\pi}{40}]$ for quarterly data. Given a time series x_t , the High-to-Medium Frequency component (HM), x_t^{HM} , corresponds to the cyclical component of x_t with periodicity between 2 and 80 quarters. In the frequency domain, these fluctuations belong to the interval $[\pi, \frac{\pi}{40}]$ for quarterly data.

1.2.1 Cyclical Components of Output

To study the behavior of the different cycles defined above, in Figure 1.2 I plot the three cyclical components of output. Output is defined as the per-capita quarterly real GDP series, in the period 1947:1-2007:4². The solid line displays the High-Frequency component, the dashed line displays the Medium-Frequency component, and the star line displays the High-to-Medium-Frequency component. The series are computed using a bandpass filter, as presented by Christiano, Fitzgerald (2003). Comin and Gertler (2006) present a similar figure, plotting the *Medium Term Cycle* (whose maximum periodicity is 40 years) for the per-capita non-farm business output; the following findings are consistent with their analysis.

² Source: US Bureau of Economic Analysis (BEA)

First, note that the magnitude of the Medium-Frequency component is not negligible if compared to the High-Frequency component; therefore, the booms and the recessions identified by the High-to-Medium Frequency component are amplified with respect to those identified by the High-Frequency component. For example, the substantial upward movement of output during the 1960s caused a growth of roughly 10 percent in the High-to-Medium-Frequency cycle, versus a 4 percent growth in the High-Frequency cycle. Similarly, the decline of output at the end of the 1970s and at the beginning of the 1980s was almost 10 percent in the High-to-Medium-Frequency cycle and only 6 percent in the High-Frequency cycle.

In order to explore the contribution of different cycles to capturing the volatility of output, in Table 1.1 I report the standard deviation of the three different components and the total standard deviation of the linearly detrended output, both in levels and percentage: the volatility of the Medium-Frequency component is larger than that of the High-Frequency component. The former accounts for 38 percent of the total variability of output, whereas the latter accounts for only the 25 percent of the total variance of output.

Second, Figure 1.2 suggests that the correlation between the High-Frequency component and the High-to-Medium-Frequency component declined in the last part of the sample; whereas until the mid-1980s the two series have a similar pattern, in the last two decades the two cycles diverge. To show this fact, I divide the sample into two sub-samples: the first sub-sample includes observations in the period 1947:1-1983:4; the second sub-sample includes observation in the period 1984:1-2007:4. The sample correlation between the High-Frequency component and the High-to-Medium Frequency component is 0.70 in the first sub-sample, and 0.40 in the second sub-sample. This divergence can be attributed to the larger magnitude of the Medium Frequency component in the second sub-sample. To explain this point, in Table 1.2, I compute the standard deviations of the three components in both sub-samples.

Although the total standard deviation of the linearly detrended output has declined from 3.7 to 2.3 in the second sub-sample, this reduction might be located especially at high frequencies. In fact, whereas in the pre-1984 period, the High-Frequency component accounts for about 27 percent of the total volatility of output, this value declines to 18 percent in the post-1984 period. On the other hand, the relative contribution of the Medium-Frequency component doubled in the second sub-sample, from 31 percent to 62 percent. As a consequence, in the second sub-sample the behavior of the High-to-Medium Frequency component is mostly driven by the Medium Frequency component.

This descriptive evidence suggests that the Medium-Frequency component has become more relevant in the last part of the sample, thus implying a divergence between the High-Frequency cycle and the High-to-Medium-Frequency cycle. In other words, if a researcher who studies the fluctuations of the economy focuses just on the high frequencies, she would not take into account the larger amount of fluctuations now than in 1985. Obviously, this date is arbitrary, but the example above suggests the importance of exploring the contribution of the Medium Frequencies on capturing the fluctuations of the economy.

1.2.2 The High-Frequency and High-to-Medium Frequency Volatility Ratio

In the previous section, I showed that the correlation between the High-Frequency component and the High-to-Medium-Frequency component has declined in the last 20 years. This implies that in this period the Medium-Frequency component has increased its relative weight with respect to the High-Frequency component. Intuitively, if there were no fluctuations with periodicity between 32 and 80 quarters, the High-to-Medium-Frequency component would exactly coincide with the High-Frequency component; this obviously means that their correlation would be one. On the other hand, the more fluctuations belong to the Medium-Frequency cycle, the

more the High-Frequency component and the High-to-Medium-Frequency component diverge.

The intuition presented above is formally supported by Theorem 4, which provides a useful representation of this correlation in terms of the variances of the filtered process. Let $I_1 = [\omega_L^1, \omega_H^1]$ and $I_2 = [\omega_L^2, \omega_H^2]$ be two disjoint set of frequencies, and let $I_3 = I_1 \cup I_2$. Let x_t^1 , x_t^2 and x_t^3 be the filtered series obtained by the same process y_t , isolating respectively the frequencies in I_1 , I_2 , and I_3 . Then, the correlation between x_t^1 and x_t^3 is equal to the ratio of their standard deviation, i.e.

$$\rho(x_t^1, x_t^3) = \sqrt{\frac{Var(x_t^1)}{Var(x_t^3)}}. \quad (1.1)$$

It is worth noticing that this result holds when the frequencies in I_1 , I_2 , and I_3 are perfectly isolated.

For simplicity of notation, I denote $\rho = \rho(x_t^{HF}, x_t^{HM})$, $\sigma^{HF} = \sqrt{Var(x_t^{HF})}$, $\sigma^{HM} = \sqrt{Var(x_t^{HM})}$, and $\sigma^{MF} = \sqrt{Var(x_t^{MF})}$ henceforth. These four parameters are estimated using the sample correlation and the sample standard deviation of the series, filtered at the appropriate frequencies with a bandpass filter.

In order to visualize the evolution of $\hat{\rho}$ over time, I construct a rolling window statistic as follows:

$$\hat{\rho}_t = \hat{\rho} \left(\{x_J^{HF}\}_{j=t-k}^t, \{x_J^{HM}\}_{j=t-k}^t \right) \quad \text{for } t = k + 1, \dots, T,$$

where k indicates the length of the window, T is the length of the time series, and $\{x\}_{t_1}^{t_2}$ represent the subset of observations of the time series x included between the periods t_1 and t_2 .

Accordingly, $\hat{\rho}_t$ is the value of the correlation between the High-Frequency component and the High-to-Medium-Frequency component computed by considering the k observations of the series x prior to time t . Using a similar procedure, we can estimate $\hat{\sigma}_t^{HF}$, $\hat{\sigma}_t^{HM}$, and $\hat{\sigma}_t^{MF}$.

Figure 1.3 plots the rolling window statistics of output computed for a window-length of 20 years ($k = 80$). The first panel shows the evolution of $\hat{\rho}_t$, whereas the second panel shows the evolution of $\hat{\sigma}_t^{HF}$, $\hat{\sigma}_t^{HM}$, and $\hat{\sigma}_t^{MF}$. The correlation between the High-Frequency component and the High-to-Medium Frequency component has declined in the second part of the sample, from a maximum value of 0.8 to a minimum value around 0.2. As stated in the previous section, the decline of this correlation is explained by the increase in the relative importance of the Medium-Frequencies volatility with respect to the High-Frequencies volatility. In fact, the lower panel shows that the High-Frequency standard deviation and the Medium-Frequency standard deviation display a divergent pattern after the mid-eighties. In fact, whereas the standard deviations of output at high frequencies declines, in the same period the standard deviation of the medium frequencies increases. These facts have two implications: first, the correlation between the High-Frequency and High-to-Medium-Frequency components declines; second, the standard deviation of the High-to-Medium-Frequency cycle does not drop as sharply as the High-Frequency standard deviation does.

The decline of the volatility of the macroeconomic variables in the last twenty years has attracted scrutiny in the recent macroeconomic literature. The term Great Moderation was in fact created to refer to the evident stabilization of the macroeconomic variables³. However, the rolling window statistics presented above suggest that the reduction of the volatility of output is concentrated only at high frequencies, since the medium-frequency volatility actually increased. In the next sections of the paper, I will first show that this pattern is common to other macroeconomic variables, and then I will formally test the presence of moderation when the medium frequencies are considered.

³ See Stock and Watson (2002) for a survey of the literature.

In summary, the empirical evidence described in this section suggests that an analysis of the fluctuations of output conducted just at high frequencies might be misleading and incomplete on explaining the behavior of the volatility of the economy, since the medium cycle has captured an increasing share of the total volatility of output during the last twenty years.

1.2.3 Cyclical Components of Disaggregated Data

In the previous section I pointed out that the relative contribution of the Medium-Frequency component to the total volatility of output in the last 25 years increased. Similar descriptive evidence is found in the disaggregated components of GDP, as I show in this section. The data set is composed of the quarterly real per-capita NIPA series and covers the period 1947:1-2007:4. The list of the series and their identification number can be found in Appendix A.

As in the previous section, I plot the evolution of $\hat{\rho}_t$, $\hat{\sigma}_t^{HF}$, $\hat{\sigma}_t^{HM}$, and $\hat{\sigma}^{MF}$. From Figure 1.4 to Figure 1.24 I plot these rolling window statistics for all the NIPA series. Some evidence is common to the majority of the variables. The most important feature is that all the series, although to a different degree, display a decline in the variance of the High-Frequency component in the last part of the sample, which is not associated with a similar decline of the High-to-Medium-Frequency and Medium-Frequency variance. In fact, the Medium Frequency component generally has a larger variance in the last two decades in all the series. As a consequence, the correlation between the High-Frequency and the High-to-Medium-Frequency cycles of many series drops from values around 0.7 to values around 0.2 in the last part of the sample. These observations motivate the goal of this paper, which is to study whether the Great Moderation is a phenomenon robust to the different definitions of the cyclical components.

Consumption

Figure 1.4 shows that the correlation between the High-Frequency and High-to-Medium-Frequency components of the Personal Consumption Expenditure series has dropped from its maximum of 0.7 in the early-80s to 0.2 in the last years. This decline is due to the decrease of the High-Frequency volatility and to the increase of the High-to-Medium-Frequency volatility, mainly driven by the rising Medium Frequency volatility. The analysis of the component of Consumption (Durable, Non-Durable, and Services, respectively, in Figure 1.5, Figure 1.6, and Figure 1.7), shows that the standard deviation of the Medium-Frequency has risen especially for Durables and Services. In particular, in the last two decades the Durable series, the most volatile component of consumption, displays an evident divergence between the declining standard deviation of the High-Frequencies, and the rising standard deviation of the Medium-Frequencies. As a result, the correlation $\hat{\rho}_t$ constantly drops in the Durables series. The Service and Non-Durable series have similar features: the decline of the High-Frequency volatility in the last two decades (although less severe than in the Durables series), and the increase of the Medium-Frequency volatility.

Investment

The role of the Medium Frequency component is particularly evident in the Investment series, the most volatile component of GDP. As Figure 1.8 and Figure 1.9 show, the Gross Private Domestic Investment and the Fixed Investment series display a sharp decline of the correlation $\hat{\rho}_t$ after 1990, from 0.9 to almost 0.4. This drop is caused by both the decline of the High-Frequency volatility and the increase of the Medium-Frequency volatilities in the last twenty years. An additional implication of this fact is that in the last observations of the sample the standard deviation of the Medium-Frequency component is much larger than the standard deviation of the High-Frequency component.

I also analyze the behavior of the different components of investment, i.e. the Non-Residential (Figure 1.10) and Residential (Figure 1.13) investments. Several important implications can be inferred from comparing the two components. First, the High-Frequency volatility has declined in both Residential and Nonresidential investment in the last two decades. On the other hand, the standard deviation of the Medium-Frequency component has a different pattern in the two variables. In fact, in Nonresidential investment, the standard deviation $\hat{\sigma}_t^{MF}$ rises sharply after 1990 until the end of the sample, whereas it has increased in Residential Investment since the early 80s, and it slightly decreases at the end of the sample. As a consequence, in Residential Investment the volatility of the High-to-Medium-Frequency cycle follows the increasing Medium-Frequency volatility and in the Nonresidential it is driven by the decreasing High-Frequency volatility. For this reason, although Residential Investment is more volatile than Non-Residential in most of the sample, this spread has shrunk in the last decades.

Imports, Exports, and Government Spending

Since Imports, Exports and Government Spending together account for a smaller fraction of output than Consumption and Investment, their impact on the cyclical behavior of output is limited. However, Figure 1.14 and Figure 1.17 show that Exports and Imports, respectively, share the same features as most of the series analyzed above, i.e. the declining High Frequency volatility and the rising Medium Frequency volatility in the last part of the sample. In contrast, the Government Spending series, Figure 1.20, displays a large volatility only in the beginning of the sample, mainly because of Korean war military spending. Afterwards the rolling window standard deviations do not show relevant movements.

1.3 Spectral Covariance Instability (SCI) Tests

The empirical evidence shows that the Medium-Frequency component contains a large part of the information about the cyclical behavior of macroeconomic series, since it captures a large portion of their total volatility. Moreover, the Medium-Frequency components do not share similar properties with the High-Frequency component. For example, whereas there was a reduction of the High-Frequency volatility after the early 1980s, there was not a similar decline of the Medium-Frequency volatility in the same time period. Therefore, it is natural to ask whether the Great Moderation is robust to a different definition of the cyclical component.

For this purpose, I introduce the Spectral Covariance Instability (SCI) test, an useful tool to test whether a multivariate process has experienced a structural break in its variance or covariance at any interval of frequencies of interest. The basic concept at the base of this test is the Integrated Cospectrum, which computes the variance and covariance attributable to any interval of frequencies⁴.

1.3.1 The Integrated Cospectrum

Let \mathbf{y}_t be a $(N \times 1)$ multivariate linear stationary process whose infinite moving average, $MA(\infty)$, representation is:

$$\mathbf{y}_t = \boldsymbol{\mu} + \boldsymbol{\Psi}(L)\boldsymbol{\varepsilon}_t, \quad (1.2)$$

where L is the lag operator, $\boldsymbol{\mu}$ is the mean vector, and $\boldsymbol{\Psi}(L) = \sum_{k=0}^{\infty} \boldsymbol{\Psi}_k L^k$ with $\{\boldsymbol{\Psi}_k\}_{k=0}^{\infty}$ absolutely summable. The $(N \times 1)$ vector $\boldsymbol{\varepsilon}_t$ is a vector of white noise, i.e.:

$$E(\boldsymbol{\varepsilon}_t) = \mathbf{0}$$

$$E(\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}'_{\tau}) = \begin{cases} \boldsymbol{\Omega} & \text{for } t = \tau \\ \mathbf{0} & \text{otherwise} \end{cases} .$$

⁴ Ahmed, Levin, and Wilson (2004) used the Integrated Spectrum, the univariate version of the Integrated Cospectrum in their analysis.

The k -th autocovariance matrix of \mathbf{y} is given by:

$$E [(\mathbf{y}_t - \boldsymbol{\mu})(\mathbf{y}_{t-k} - \boldsymbol{\mu})'] = \Gamma^{(k)}. \quad (1.3)$$

and the autocovariance-generating function of \mathbf{y} is:

$$G_{\mathbf{Y}}(z) = \sum_{k=-\infty}^{\infty} \Gamma^{(k)} z^k, \quad (1.4)$$

with $\{\Gamma^{(k)}\}_{k=-\infty}^{\infty}$ absolutely summable and with z being a complex scalar. Then, the population spectrum of the vector \mathbf{y} is given by

$$\mathbf{s}_{\mathbf{Y}}(\omega) = (2\pi)^{-1} G_{\mathbf{Y}}(e^{-i\omega}) = (2\pi)^{-1} \sum_{k=-\infty}^{\infty} \Gamma^{(k)} e^{-i\omega k}, \quad (1.5)$$

where $i = \sqrt{-1}$ and ω is a real scalar. Thus, $\mathbf{s}_{\mathbf{Y}}(\omega)$, known as the cross-spectrum, is a $(N \times N)$ matrix in which the diagonal elements are real and the off-diagonal elements are complex. The cross-spectrum can be written in terms of its real and imaginary components, i.e.

$$\mathbf{s}(\omega) = \mathbf{c}(\omega) + i\mathbf{q}(\omega). \quad (1.6)$$

where $\mathbf{c}(\omega)$ and $\mathbf{q}(\omega)$ are known as the cospectrum and the quadrature of \mathbf{y}_t .

First, it is useful to recall some results of the frequency domain analysis in a univariate framework. Let x_t be a linear univariate stationary process with finite variance and with an $MA(\infty)$ representation,

$$x_t = \mu_x + \theta(L)\varepsilon_t, \quad (1.7)$$

with mean μ_x and whose spectrum is denoted by $s_x(\omega)$. Then, the integral between $-\pi$ and π of the spectrum of x_t is equal to its variance, that is:

$$\int_{-\pi}^{\pi} s_x(\omega) = E(x_t - \mu_x)^2.$$

For a given frequency range $[\omega_1, \omega_2]$, with $0 \leq \omega_1 < \omega_2$, the variance attributable at that interval of frequencies can be computed with the Integrated Spectrum, as in the following definition: Given a univariate process x_t as in (1.7), the Integrated Spectrum $H(\omega_1, \omega_2)$ for the interval of frequencies $[\omega_1, \omega_2]$, with $0 \leq \omega_1 \leq \omega_2$, is the integral of $s_x(\omega)$ between ω_1 and ω_2 , i.e.

$$H(\omega_1, \omega_2) = 2 \int_{\omega_1}^{\omega_2} s_x(\omega) d\omega.^5 \quad (1.8)$$

and it corresponds to the variance of the processes x_t due to cycles identified by the interval of frequencies $[\omega_1, \omega_2]$.

I now extend these results in the multivariate case; in fact, the integral of the cross-spectrum corresponds to the variance-covariance matrix of the multivariate process, i.e.:

$$\int_{-\pi}^{\pi} \mathbf{s}(\omega) d\omega = E [(\mathbf{y}_t - \mu)(\mathbf{y}_t - \mu)'] . \quad (1.9)$$

However, since $\mathbf{q}(\omega) = -\mathbf{q}(-\omega)$, the variance-covariance matrix can be computed from the area below the Cospectrum:

$$\int_{-\pi}^{\pi} \mathbf{c}(\omega) d\omega = E [(\mathbf{y}_t - \mu)(\mathbf{y}_t - \mu)'] , \quad (1.10)$$

It is then straightforward to extend the concept of the Integrated Spectrum to define the Integrated Cospectrum. Given a multivariate process \mathbf{y}_t as in (1.2), the Integrated Cospectrum for the interval of frequencies $[\omega_1, \omega_2]$, with $0 \leq \omega_1 \leq \omega_2$, is the integral of $\mathbf{c}(\omega)$ between ω_1 and ω_2 , i.e.

$$\mathbf{H}(\omega_1, \omega_2) = 2 \int_{\omega_1}^{\omega_2} \mathbf{c}(\omega) d\omega. \quad (1.11)$$

⁵ Note that $s_x(\omega) = s_x(-\omega)$.

and it corresponds to the variance-covariance matrix of the processes \mathbf{y}_t due to cycles identified by the interval of frequencies $[\omega_1, \omega_2]$.

To provide a consistent estimate for the Integrated Cospectrum, I generalize Priestley (1982)'s univariate approach to a multivariate process. Define the multivariate sample periodogram for a sample size T process \mathbf{y}_t as

$$\hat{\mathbf{I}}(\omega) = \frac{1}{2\pi} \sum_{j=1-T}^{T-1} \hat{\mathbf{\Gamma}}^{(j)} e^{-i\omega j} = \frac{1}{2\pi} \sum_{j=1-T}^{T-1} \hat{\mathbf{\Gamma}}^{(j)} \cos(\omega j), \quad (1.12)$$

where $\hat{\mathbf{\Gamma}}^{(j)}$ represents the sample autocovariance given by:

$$\hat{\mathbf{\Gamma}}^{(j)} = \frac{1}{T} \sum_{i=1}^{T-j} (\mathbf{y}_t - \bar{\mathbf{y}})' (\mathbf{y}_{t-j} - \bar{\mathbf{y}}). \quad (1.13)$$

A consistent estimate of the Integrated Cospectrum $\mathbf{H}(\omega_1, \omega_2)$ is given by

$$\begin{aligned} \hat{\mathbf{H}}(\omega_1, \omega_2) &= 2 \int_{\omega_1}^{\omega_2} \hat{\mathbf{I}}(\theta) d\theta & (1.14) \\ &= 2 \int_{\omega_1}^{\omega_2} \frac{1}{2\pi} \sum_{j=1-T}^{T-1} \hat{\mathbf{\Gamma}}^{(j)} \cos(\theta j) d\theta \\ &= \frac{1}{\pi} \sum_{j=1-T}^{T-1} \hat{\mathbf{\Gamma}}^{(j)} \int_{\omega_1}^{\omega_2} \cos(\theta j) d\theta \\ &= \frac{1}{\pi} \left[\hat{\mathbf{\Gamma}}^{(0)} + \sum_{j=1}^{T-1} \left(\hat{\mathbf{\Gamma}}^{(-j)} + \hat{\mathbf{\Gamma}}^{(j)} \right) \frac{\sin(\omega_2 j) - \sin(\omega_1 j)}{j} \right], & (1.15) \end{aligned}$$

where the second equality comes from (1.12), the third equality comes from switching summation and integral, and the fourth equality comes from $\int \cos(\theta j) d\theta = \frac{\sin(\theta j)}{j}$.

To prove the asymptotic proprieties of $\hat{\mathbf{H}}(\omega_1, \omega_2)$, it is convenient to recall some general results about weighted integrals of the periodogram, as in the following

Lemma⁶: Let $\phi_1(\omega)$ and $\phi_2(\omega)$, be two real valued functions defined in $-\pi \leq \omega \leq \pi$, each of which has at most a finite number of discontinuities and is both absolutely integrable and square integrable, i.e. for $i=1,2$,

$$\int_{-\pi}^{\pi} |\phi_i(\omega)| d\omega < \infty \quad \text{and} \quad \int_{-\pi}^{\pi} \phi_i^2(\omega) d\omega < \infty.$$

Let x_t be a general univariate linear process as in (1.7) with normal ε_t , and whose spectrum is $s_x(\omega)$. Let, for $i=1,2$,

$$\hat{\psi}_i = \int_{-\pi}^{\pi} \phi_i(\omega) \hat{I}(\omega) d\omega \tag{1.16}$$

$$\psi_i = \int_{-\pi}^{\pi} \phi_i(\omega) s_x(\omega) d\omega, \tag{1.17}$$

with $\hat{I}(\omega)$ being the sample periodogram of x_t . Then:

1.

$$\lim_{T \rightarrow \infty} E(\hat{\psi}_i) = \psi_i, \quad i = 1, 2.$$

2.

$$\lim_{T \rightarrow \infty} T \text{cov}(\hat{\psi}_1 \hat{\psi}_2) = 4\pi \int_{-\pi}^{\pi} \phi_1(\omega) \bar{\phi}_2(\omega) s_x^2(\omega) d\omega$$

$$\text{with } \bar{\phi}_2(\omega) = \frac{1}{2} [\phi_2(\omega) + \phi_2(-\omega)].$$

In particular, when $\phi_1(\omega) = \phi_2(\omega) = \phi(\omega)$, we have:

3.

$$\lim_{T \rightarrow \infty} T \text{var}(\hat{\psi}) = 4\pi \int_{-\pi}^{\pi} \phi(\omega) \bar{\phi}(\omega) s_x^2(\omega) d\omega$$

$$\text{with } \bar{\phi}(\omega) = \frac{1}{2} [\phi(\omega) + \phi(-\omega)]^7$$

⁶ For the proof, see Priestley (1982) pp. 427

The following Theorem shows the asymptotic proprieties of the element of the Integrated Cospectrum:

Let \mathbf{y}_t be a multivariate linear process as in (1.2), where ε_t is a multivariate normal. Then, the (m, n) -th element, $m = 1, \dots, N$, $n = 1, \dots, N$ of the sample Integrated Cospectrum in (1.15) has the following proprieties:

1. asymptotic unbiasedness:

$$\lim_{T \rightarrow \infty} E \left(\hat{\mathbf{H}}_{m,n}(\omega_1, \omega_2) \right) = \mathbf{H}_{m,n}(\omega_1, \omega_2)$$

2. consistency:

$$\hat{\mathbf{H}}_{m,n}(\omega_1, \omega_2) \xrightarrow{p} \mathbf{H}_{m,n}(\omega_1, \omega_2)$$

3. asymptotic normality:

$$\sqrt{T} \left[\hat{\mathbf{H}}_{m,n}(\omega_1, \omega_2) - \mathbf{H}_{m,n}(\omega_1, \omega_2) \right] \xrightarrow{d} N \left(\mathbf{0}, \mathbf{\Phi}_{m,n}(\omega_1, \omega_2) \right),$$

with

$$\mathbf{\Phi}_{m,n}(\omega_1, \omega_2) = 8\pi \int_{\omega_1}^{\omega_2} \mathbf{c}_{m,n}^2(\omega) d\omega. \quad (1.18)$$

In order to derive any inference result on the Integrated Cospectrum, we need an estimate of its asymptotic variance-covariance matrix in (1.18). An obvious procedure is to substitute the co-spectrum $\mathbf{c}(\omega)$ with the multivariate periodogram in (1.12). Moreover, I approximate the integral in (1.18) with a discrete sum, dividing the interval $[\omega_1, \omega_2]$ into q segments of length $\Delta\omega = \frac{\omega_2 - \omega_1}{q}$, with $q \rightarrow \infty$ as $T \rightarrow \infty$.

The following Theorem assures the consistency of the estimate of $\mathbf{\Phi}_{m,n}(\omega_1, \omega_2)$.

Let \mathbf{y}_t be a multivariate linear process as in (1.2), where ε_t is a multivariate normal. Also, let $0 \leq \omega_1 \leq \omega_2$, and define $\Delta\omega = \frac{\omega_2 - \omega_1}{q}$, with $q \rightarrow \infty$ as $T \rightarrow \infty$.

Then, a consistency estimate of $\Phi_{m,n}(\omega_1, \omega_2)$, $m = 1, \dots, N$, $n = 1, \dots, N$ is given by

$$\hat{\Phi}_{m,n} = 4\pi \sum_{i=1}^q \left[\frac{1}{2\pi} \sum_{j=1-T}^{T-1} \hat{\Gamma}_{m,n}^{(j)} e^{-i\omega j} \right]^2 \Delta\omega. \quad (1.19)$$

Hence, the approximated distribution of $\hat{\mathbf{H}}_{m,n}(\omega_1, \omega_2)$ is given by:

$$\hat{\mathbf{H}}_{m,n}(\omega_1, \omega_2) \sim N \left(\mathbf{0}, \frac{\hat{\Phi}_{m,n}(\omega_1, \omega_2)}{T} \right). \quad (1.20)$$

1.3.2 Spectral Covariance Instability (SCI) Test

Once the asymptotic distribution of the Integrated Cospectrum is derived, I can use its estimate for testing purposes. In particular, in what follows I derive a structural break test for the element of the Integrated Cospectrum. In particular, let $y_{m,t}$ and $y_{n,t}$ be two components of the process \mathbf{y}_t . An estimate of the covariance between $y_{m,t}$ and $y_{n,t}$ at the frequencies in $[\omega_1, \omega_2]$ is given by $\hat{\mathbf{H}}_{m,n}(\omega_1, \omega_2)$. Even though in this section I focus on the covariance between two series ($m \neq n$), the same procedure can be applied when $m = n$, that is when the variance of $y_{m,t}$ is the object of the analysis

Suppose one wants to test whether the population covariance attributable to the frequencies in $[\omega_1, \omega_2]$, $\mathbf{H}_{m,n}(\omega_1, \omega_2)$, has experienced a one-time structural change in a known period τ . Let $\tau = [Tc]$, with $[\cdot]$ being the integer part operator and $c \in (0, 1)$. i.e. the null and the alternative hypothesis and that define the test are:

$$H_0 : \mathbf{H}_{m,n}^{(1,c)}(\omega_1, \omega_2) = \mathbf{H}_{m,n}^{(2,c)}(\omega_1, \omega_2) \quad \text{with } c \text{ known}$$

vs

$$H_1 : \mathbf{H}_{m,n}^{(1,c)}(\omega_1, \omega_2) \neq \mathbf{H}_{m,n}^{(2,c)}(\omega_1, \omega_2),$$

where $\mathbf{H}^{(1,c)}(\omega_1, \omega_2)$ denotes the value of the Integrated Cospectrum in the first sub-period $t = 1, \dots, \tau$, and $\mathbf{H}^{(2,c)}(\omega_1, \omega_2)$ denotes its value in the second sub-period

$t = \tau + 1, \dots, T$. Note that the value of these parameters depends on c , as indicated on the superscript. I define the Spectral Wald Statistic (S_T^W , henceforth) as:

$$S_T^W(\omega_1, \omega_2, c) = \frac{T \left[\mathbf{H}_{m,n}^{(1,c)}(\omega_1, \omega_2) - \mathbf{H}_{m,n}^{(2,c)}(\omega_1, \omega_2) \right]^2}{\left(\frac{\hat{\Phi}_{m,n}^{(1,c)}(\omega_1, \omega_2)}{c} + \frac{\hat{\Phi}_{m,n}^{(2,c)}(\omega_1, \omega_2)}{1-c} \right)}$$

where $\hat{\Phi}_{m,n}^{(1,c)}(\omega_1, \omega_2)$ is the estimate $\hat{\Phi}_{m,n}(\omega_1, \omega_2)$ computed in the first sub-sample $t = 1, \dots, \tau$, and $\hat{\Phi}_{m,n}^{(2,c)}(\omega_1, \omega_2)$ is the estimate computed in the second sub-sample $t = \tau + 1, \dots, T$. Under the null hypothesis $S_T^W(\omega_1, \omega_2, c)$ has a χ^2 distribution with 1 degree of freedom.

However, in this paper I test the presence of a structural change at an unknown date, i.e.

$$H_0 : \mathbf{H}_{m,n}^{(1,c)}(\omega_1, \omega_2) = \mathbf{H}_{m,n}^{(2,c)}(\omega_1, \omega_2) \quad \text{with } c \text{ unknown}$$

vs

$$H_1 : \mathbf{H}_{m,n}^{(1,c)}(\omega_1, \omega_2) \neq \mathbf{H}_{m,n}^{(2,c)}(\omega_1, \omega_2).$$

For this purpose, I introduce the following three types of Spectral Covariance Instability tests:

Let \mathbf{y}_t be a process as in (1.2), and $c \in \Pi \equiv (0, 1)$, for any interval of frequency $[\omega_1, \omega_2]$, with $0 \leq \omega_1 \leq \omega_2$, we define the following Spectral Covariance Instability tests:

- The Spectral Average Wald Test (SAW):

$$SAW = \int_{c \in \Pi} S_T^W(\omega_1, \omega_2, c) dc.$$

- The Spectral Exponential Wald Test (SEW):

$$SEW = \log \int_{c \in \Pi} \exp \left[\frac{1}{2} S_T^W(\omega_1, \omega_2, c) \right] dc.$$

- The Spectral Nyblom Test (SN):

$$SN = \int_{c \in \Pi} S_T^W(\omega_1, \omega_2, c) c(1 - c) dc.$$

An opportune choice of Π is $\Pi = [0.15, 0.85]$. The critical values for the three test presented above are equivalent to their counterpart in the time domain, and they are provided by Andrews, Lee, Ploberger (1996) for the SAW, and SEW test, and by Sowell (1996) for the SN test.

1.4 Monte Carlo Simulations

In this section, I analyze the small sample proprieties of the Spectral Covariance Instability tests by using Monte Carlo simulations. First, I compare the empirical rejection frequencies and the power of the three Spectral Covariance Instability tests, and then I examine the proprieties of an alternative approach to test a break in the variances and covariances at particular frequencies using a Generalized Method of Moments (GMM) based test.

1.4.1 Empirical Rejection Frequencies and Power

To study the small sample properties of the Spectral Covariance Instability tests, I conduct the following experiment. First, I consider a theoretical model shown to be able to generate macroeconomic variables with similar cyclical properties as their data counterpart. For this purpose I consider as a data-generating process a factor-hoarding real business cycle model introduced by Burnside and Eichenbaum (1995). This choice is motivated by the fact that this model has a well-functioning propagation mechanism that generates a relevant amount of low-frequency fluctuations. A brief description of the model and its calibration is reported in Appendix C. Then I use this model to simulate two series, output and investment. Given any interval

of frequencies, the parameters of interest are the variance of the two univariate processes, $H_{1,1}$ and $H_{2,2}$, respectively, for output and investment, and their covariance, $H_{1,2}$, attributable at those frequencies. Consistently with the definitions used in the empirical part of this paper, I consider two intervals of frequencies: the one that defines the High-Frequency component (2-32 quarters), and the one that defines the High-to-Medium Frequency component (2-80 quarters).

The first goal is to study the small sample properties of the Spectral Covariance Instability tests. Therefore, I compute the empirical rejection frequencies for the three tests when the nominal significance is 10 percent, 5 percent, and 1 percent. The reported rejection frequencies are based on 1000 Monte Carlo repetitions.

Table 1.3 reports the empirical rejection frequencies for the SAW, SEW, and SN tests when the sample size is $T = 200$. The coverage of the three tests is adequate, although the SEW tests appears to perform worse than the other two tests at the High-to-Medium frequencies. Table 1.4 reports the empirical rejection frequencies for the SCI tests when the sample size is $T = 500$. The performance of the test only slightly improves with respect to the previous case at High-Frequencies. However, the increase in the sample size implies significantly better results when the lower frequencies are taken into account. The same pattern is revealed when the sample size is $T = 1000$, as shown in Table 1.5. Overall the behavior of the three tests in the small sample is satisfactory, both at high and medium frequencies, and even when the sample size is similar to the one available for the majority of macroeconomic series.

The second goal is to compare the power of the three tests. For this purpose I impose that the model generates a break in the variance and covariance matrix of the vector composed by output, investment, and consumption at the middle of the sample. I assume that the break causes a given percentage decline in the variance and covariance between the first half of the sample and the second half of the sample.

I consider four different magnitudes of the break, i.e. a 10 percent, 25 percent, 50 percent, and 75 percent decline in the variance and covariance. In Table 1.6 I display the power of the tests at the 0.05 significance level, using a sample size of length $T = 500$. Note that when the break in the variance covariance matrix is small (a decline of 10 percent or 25 percent) the SEW test dominates the SAW test, which dominates the SN tests. However, since the break has a modest magnitude, the powers are overall low. However, as expected, when the magnitude of the break increases, the power of the test significantly improves: in particular, the SAW test is the most powerful when the decline of the covariance is 50 percent, although the SN test has a similar performance. Finally, with a 75 percent decline of the covariances, the three tests have a power greater than 0.95.

In conclusion, the Monte Carlo simulations suggest that the SCI tests have good small sample size performances, especially the SAW and SN test. Moreover, the SN and SAW tests have higher power than the SEW test when the magnitude of the break is large, in contrast to the small break case.

1.4.2 SCI Test Versus GMM: A Comparison

In this section I discuss an alternative approach for testing a break in the variance and covariance of a series at particular frequencies using a GMM approach. I show that the small sample proprieties of this approach are worse than for the SCI tests.

The GMM approach requires the following steps: first, the series of interest should be filtered at a particular interval of frequencies using a bandpass filter. Their variance, or covariance, and their standard errors is computed using a GMM estimator. Note that in order to calculate the optimal weighing matrix with the Newey and West (1994) procedure, a bandwidth and a smoothing window must be selected. Finally, the time domain equivalent of the SCI test, namely the Average LM test (ALM), the Exponential LM test, or the Nyblom test (NYB), can be directly applied to test

whether these parameters have experienced a structural break at an unknown date⁸.

The small sample proprieties, namely the empirical rejection frequencies, of these tests are presented in Table 1.7, Table 1.8, and Table 1.9. respectively for $T = 200$, $T = 500$, and $T = 1000$. I use a Bartlett window and its corresponding optimal bandwidth. In particular Newey and West (1994) shows that asymptotically the optimal bandwidth for this window is given by:

$$b = \left[4 \left(\frac{T}{100} \right)^{\frac{2}{9}} \right]$$

As the Tables show, the GMM approach performs considerably worse than the SCI test in the small sample, since its empirical rejection frequencies are far from their nominal values for the three sample sizes considered, both at High-Frequencies and at High-to-Medium-Frequencies, and for any for the sample sizes considered.

This result should not be surprising. den Haan and Levin (1996), and Kiefer, Vogelsang, and Bunzel (2000) have discussed the unsatisfactory small sample proprieties of GMM estimators, related in particular to the choice of bandwidth. In fact, whereas the Bartlett windows have been shown to have satisfactory properties, the choice of the bandwidth is a problematic issue. In fact, only asymptotic results related to the optimal rate of convergence of a bandwidth have been proposed in the literature, whereas there are no similar guidelines for the small sample problem. Second, the choice of the bandwidth implicitly implies a trade-off between the bias of the estimator and its variance. Therefore, the choice of the bandwidth in a small sample is not a trivial concern in practice, and with my calculation I show that although the choice of the bandwidth has been conducted considering an asymptotic optimal rule, the imprecision of the test statistics is evident. Similar results are obtained if the choice of the bandwidth is guided by the Andrew's (1991) procedure.

⁸ See Nyblom (1989), Andrew (1993), Andrew et al. (1996).

On the other hand, the Spectral Covariance Instability tests do not suffer from the same problem. In fact, as stated in Priestley (1982), the Integrated Cospectrum does not require any choice of a bandwidth. In fact, as shown in (1.14), the Integrated Cospectrum is estimated as the integral of the sample periodogram and the integration procedure along the frequencies works directly as a smoothing function. However, since the integration does not require the specification of any bandwidth parameter, the Integrated Cospectrum does not suffer from any trade-off between its bias and its variance of the estimation .

In conclusion, although a GMM approach can be followed to test for a break for the variances and covariances at particular frequencies, this procedure requires a not trivial choice of the bandwidth and has worse small sample properties than the frequency domain approach presented in this paper.

1.5 Application of the SCI Tests

The empirical evidence discussed in this paper indicates that the macroeconomic variables experienced a decline in the High-Frequency volatility, the Great Moderation phenomenon, but the High-to-Medium Frequency volatility did not drop in the same fashion. Using the Spectral Covariance Instability tests introduced in this paper, I formally test whether the macroeconomic variables display a break on their variance and covariance at these different frequency intervals.

I consider three important U.S. real per-capita macroeconomic variables, namely, output (measured as the gross domestic product), consumption (measured as consumption of nondurable goods and service goods), and investment (measured as private investment). I then apply the Spectral Covariance Instability tests presented in Section 3 to these series, to test the null hypothesis that there was no break in their variance or covariance at an unknown period, considering separately their High-Frequency and High-to-Medium Frequency components.

Table 1.10 shows the p-values of the SCI test applied to the variance-covariance matrix of consumption, investment and output. First consider the tests on the variance of these series, i.e. the diagonal element of the 3x3 matrices. When the High-Frequencies are considered, the three tests detect a break on the variance of output at a 5 percent level of significance, consistent with the theory on the Great Moderation. The same result is obtained when the High-Frequency variance of investment is analyzed. The SAW test rejects at the 5 percent level the hypothesis of stability of the High-Frequency variance of consumption, whereas the SAW and SN tests reject the hypothesis at the 10 percent level of significance. In summary, the Spectral Covariance Instability tests confirm the presence of a moderation on the High-Frequency variance of the main macroeconomic variables. However, when the High-to-Medium-Frequencies are taken into account these results change dramatically. In fact, all the tests fail to reject at 5 percent the hypothesis of a stable variance at these frequencies. Therefore, I conclude that the Great Moderation is just a High-Frequency phenomenon, that disappears when the Medium-Frequencies are taken into account.

Now consider the test on the covariance among the variables. At High-Frequencies the SEW test detects a break in the covariance between investment and output at 5 percent level of significance. The SN and the SAW tests find this break at the 10 percent level of significance. Although this is the only case in which the stability of a covariance is rejected, this result suggests that the high-frequency co-movement between output and investment has significantly changed over time. This stylized fact might be the starting point for a theoretical study on the dynamic relationship between these two variables. However, this result only holds for the high-frequencies, since the tests do not detect any breaks on any of the covariance when the High-to-Medium-Frequency components are considered.

1.6 Conclusions

The contribution of this paper is twofold. First, from an empirical point of view I investigate the behavior of the economic cycle, when it is defined more broadly than the conventional definition. In particular, I consider the role of the Medium-Frequency cycle, which includes fluctuations between 32 and 80 quarters, in addition to the role of the High-Frequency cycle, which includes fluctuations between 2 and 32 quarters. The conventional definition of the business cycle considers only the latter, totally ignoring the former. However, I show that the Medium-Frequency component captures the largest fraction of the fluctuation of many macroeconomic variables, in particular in the last two decades. Therefore, in order to fully characterize the properties of the economic cycle, a researcher should include the Medium-Frequencies in her analysis. In addition, the empirical investigation of the cyclical components of the macroeconomic variables provides another interesting stylized fact: whereas the volatility of the High-Frequency component has declined from the early 1980s, the so called Great Moderation, phenomenon, the volatility of the Medium-Frequency component does not have a similar pattern. This fact raises the question: is the Great Moderation just a High-Frequencies phenomenon?

The second contribution of this paper provides a tool to answer this question. In particular, I define the Spectral Covariance Instability tests, which are useful to test whether a set of variables experienced a break in its variance-covariance matrix at any given interval of frequencies. This battery of tests is based in the frequency domain and are built starting from the Integrated Cospectrum. After deriving the asymptotic properties of the Spectral Covariance Instability tests, I investigate their small sample properties, namely the empirical rejection frequency and the power, using Monte Carlo simulations. I show that the test has a good small sample behavior, in particular if compared with a time-domain GMM-based

alternative. In fact, the procedure proposed in this paper does not require the choice of any bandwidth parameter, a problematic choice in the GMM approach.

Finally, I apply the Spectral Covariance Instability tests to the some important U.S. macroeconomic variables, namely consumption, investment, and output in the postwar periods. The test formally detects a break in the variance of the three variables at High-Frequencies. However, the three tests are consistently unable to detect that break when the Medium-Frequencies are included in the analysis. This formal results shows that the Great Moderation is just a High-Frequency phenomenon.

1.7 Tables and Figures

Table 1.1: Standard Deviations of the Cyclical Components of Output and Relative Contribution in Two Sub-Samples

	Percent	
	Standard Deviation	Relative Contribution
High Frequency	1.63	25.4
Medium Frequency	1.99	63.8
High-to-Medium	2.59	37.7
Linearly Detrended	3.24	100

Note: Output is defined in real per-capita terms, measured as the Gross Domestic Product from NIPA. The cyclical components are isolated using a band-pass filter.

Table 1.2: Standard Deviations of the Cyclical Components of Output and Relative Contribution in Two Sub-Samples

	Percent			
	Standard Deviation		Relative Contribution	
	1947:1-1983:4	1984:1-2007:4	1947:1-1983:4	1984:1-2007:4
High Frequency	1.96	0.99	27.7	17.9
Medium Frequency	2.08	2.00	31.3	61.8
High-to-Medium	2.93	1.84	61.5	73.3
Linearly Detrended	3.73	2.34	100	100

Note: Output is defined in real per-capita terms, measured as the Gross Domestic Product from NIPA. The cyclical components are isolated using a band-pass filter.

Table 1.3: Empirical Rejection Frequencies for the SCI test. T=200

Nominal Significance	High-Frequencies			High-to-Medium Frequencies			
	0.100	0.050	0.010	0.100	0.050	0.010	
SAW Test	$H_{1,1}$	0.108	0.045	0.007	0.148	0.077	0.007
	$H_{2,2}$	0.109	0.045	0.008	0.147	0.080	0.008
	$H_{1,2}$	0.103	0.041	0.005	0.143	0.076	0.007
SEW Test	$H_{1,1}$	0.140	0.068	0.019	0.217	0.123	0.036
	$H_{2,2}$	0.145	0.070	0.018	0.218	0.124	0.037
	$H_{1,2}$	0.133	0.063	0.016	0.211	0.121	0.034
SN Test	$H_{1,1}$	0.090	0.038	0.004	0.121	0.058	0.004
	$H_{2,2}$	0.090	0.038	0.004	0.121	0.060	0.004
	$H_{1,2}$	0.082	0.029	0.004	0.118	0.059	0.004

Note: The Table displays the empirical rejection frequencies of the SCI test for testing the presence of a structural break of the variance and covariance of output and investment, generated with the factor-hoarding model described in the appendix. The series have a length of 200 periods. The table is based on 1000 Monte Carlo repetitions.

Table 1.4: Empirical Rejection Frequencies for the SCI test. T=500

Nominal Significance	High-Frequencies			High-to-Medium Frequencies		
	0.100	0.050	0.010	0.100	0.050	0.010
SAW Test	$H_{1,1}$	0.111	0.063	0.010	0.125	0.070
	$H_{2,2}$	0.115	0.064	0.010	0.126	0.071
	$H_{1,2}$	0.097	0.053	0.007	0.123	0.066
SEW Test	$H_{1,1}$	0.134	0.081	0.025	0.156	0.091
	$H_{2,2}$	0.136	0.083	0.027	0.158	0.094
	$H_{1,2}$	0.128	0.075	0.023	0.154	0.086
SN Test	$H_{1,1}$	0.096	0.052	0.004	0.104	0.049
	$H_{2,2}$	0.097	0.053	0.003	0.107	0.053
	$H_{1,2}$	0.092	0.048	0.002	0.104	0.047

Note: The Table displays the empirical rejection frequencies of the SCI test for testing the presence of a structural break of the variance and covariance of output and investment, generated with the factor-hoarding model described in the appendix. The series have a length of 500 periods. The table is based on 1000 Monte Carlo repetitions.

Table 1.5: Empirical Rejection Frequencies for the SCI test. T=1000

Nominal Significance	High-Frequencies			High-to-Medium Frequencies			
	0.100	0.050	0.010	0.100	0.050	0.010	
SAW Test	$H_{1,1}$	0.114	0.061	0.008	0.121	0.062	0.017
	$H_{2,2}$	0.114	0.062	0.008	0.124	0.063	0.017
	$H_{1,2}$	0.102	0.057	0.008	0.122	0.06	0.017
SEW Test	$H_{1,1}$	0.134	0.086	0.022	0.149	0.089	0.025
	$H_{2,2}$	0.135	0.086	0.023	0.148	0.090	0.027
	$H_{1,2}$	0.129	0.074	0.019	0.143	0.085	0.026
SN Test	$H_{1,1}$	0.109	0.055	0.007	0.111	0.057	0.014
	$H_{2,2}$	0.109	0.055	0.007	0.110	0.059	0.015
	$H_{1,2}$	0.096	0.051	0.006	0.109	0.055	0.014

Note: The Table displays the empirical rejection frequencies of the SCI test for testing the presence of a structural break of the variance and covariance of output and investment, generated with the factor-hoarding model described in the appendix. The series have a length of 1000 periods. The table is based on 1000 Monte Carlo repetitions.

Table 1.6: Power of the SCI test. T=500. The Nominal Significance is 0.05

Percentage Change in $\mathbf{H}_{m,n}$	High-Frequencies						High-to-Medium Frequencies						
	10	25	50	75	10	25	50	75	10	25	50	75	
SAW Test	$H_{1,1}$	0.068	0.136	0.570	0.988	0.055	0.145	0.602	0.994	0.055	0.145	0.602	0.994
	$H_{2,2}$	0.066	0.138	0.571	0.988	0.054	0.147	0.602	0.994	0.054	0.147	0.602	0.994
	$H_{1,2}$	0.059	0.132	0.554	0.986	0.053	0.143	0.597	0.994	0.053	0.143	0.597	0.994
SEW Test	$H_{1,1}$	0.084	0.140	0.522	0.988	0.089	0.148	0.570	0.993	0.089	0.148	0.570	0.993
	$H_{2,2}$	0.083	0.140	0.529	0.989	0.091	0.151	0.579	0.993	0.091	0.151	0.579	0.993
	$H_{1,2}$	0.073	0.136	0.505	0.987	0.086	0.145	0.565	0.993	0.086	0.145	0.565	0.993
SN Test	$H_{1,1}$	0.059	0.136	0.571	0.989	0.051	0.139	0.603	0.994	0.051	0.139	0.603	0.994
	$H_{2,2}$	0.060	0.136	0.570	0.989	0.050	0.143	0.605	0.996	0.050	0.143	0.605	0.996
	$H_{1,2}$	0.054	0.127	0.551	0.985	0.047	0.136	0.594	0.994	0.047	0.136	0.594	0.994

*Note:*The Table displays the power of the SCI test for testing the presence of a structural break of the variance and covariance of output and investment, generated with the factor-hoarding model described in the appendix. The break on the variance is modeled respectively as a 10%, 25%, 50%, and 75%, decline of the variance and covariance of the two series in the middle of the sample. The series have a length of 500 periods. The table is based on 1000 Monte Carlo repetitions.

Table 1.7: Empirical Rejection Frequencies for the GMM test. T=200

Nominal Significance	High-Frequencies			High-to-Medium Frequencies			
	0.100	0.050	0.010	0.100	0.050	0.010	
ALM Test	$H_{1,1}$	0.496	0.417	0.293	0.487	0.404	0.286
	$H_{2,2}$	0.496	0.418	0.295	0.487	0.403	0.286
	$H_{1,2}$	0.495	0.417	0.293	0.484	0.403	0.283
ELM Test	$H_{1,1}$	0.585	0.530	0.438	0.597	0.542	0.466
	$H_{2,2}$	0.588	0.530	0.442	0.594	0.540	0.470
	$H_{1,2}$	0.583	0.530	0.438	0.591	0.541	0.466
NYB Test	$H_{1,1}$	0.474	0.385	0.272	0.444	0.369	0.257
	$H_{2,2}$	0.474	0.383	0.273	0.449	0.371	0.259
	$H_{1,2}$	0.473	0.383	0.271	0.445	0.369	0.257

Note: The Table displays the empirical rejection frequencies of the GMM test for testing the presence of a structural break of the variance and covariance of output and investment, generated with the factor-hoarding model described in the appendix. The bandwidth is chosen using the Newey and West (1994) procedure. The series have a length of 200 periods. The table is based on 1000 Monte Carlo repetitions.

Table 1.8: Empirical Rejection Frequencies for the GMM test. T=500

Nominal Significance	High-Frequencies			High-to-Medium Frequencies			
	0.100	0.050	0.010	0.100	0.050	0.010	
ALM Test	$H_{1,1}$	0.348	0.258	0.141	0.315	0.235	0.137
	$H_{2,2}$	0.346	0.257	0.141	0.310	0.241	0.135
	$H_{1,2}$	0.347	0.257	0.141	0.312	0.235	0.135
ELM Test	$H_{1,1}$	0.424	0.350	0.26	0.384	0.319	0.244
	$H_{2,2}$	0.423	0.350	0.26	0.379	0.317	0.241
	$H_{1,2}$	0.425	0.350	0.26	0.382	0.318	0.241
NYB Test	$H_{1,1}$	0.321	0.238	0.123	0.303	0.218	0.121
	$H_{2,2}$	0.319	0.239	0.122	0.301	0.219	0.121
	$H_{1,2}$	0.320	0.238	0.123	0.300	0.218	0.119

Note: The Table displays the empirical rejection frequencies of the GMM test for testing the presence of a structural break of the variance and covariance of output and investment, generated with the factor-hoarding model described in the appendix. The bandwidth is chosen using the Newey and West (1994) procedure. The series have a length of 500 periods. The table is based on 100 Monte Carlo repetitions.

Table 1.9: Empirical Rejection Frequencies for the GMM test. T=1000

Nominal Significance	High-Frequencies			High-to-Medium Frequencies			
	0.100	0.050	0.010	0.100	0.050	0.010	
ALM Test	$H_{1,1}$	0.255	0.200	0.097	0.227	0.163	0.085
	$H_{2,2}$	0.255	0.202	0.095	0.223	0.160	0.083
	$H_{1,2}$	0.255	0.202	0.095	0.224	0.161	0.083
ELM Test	$H_{1,1}$	0.309	0.244	0.165	0.289	0.233	0.155
	$H_{2,2}$	0.310	0.241	0.164	0.285	0.234	0.158
	$H_{1,2}$	0.308	0.241	0.165	0.284	0.233	0.155
NYB Test	$H_{1,1}$	0.248	0.181	0.079	0.206	0.148	0.077
	$H_{2,2}$	0.248	0.183	0.080	0.206	0.149	0.076
	$H_{1,2}$	0.248	0.182	0.080	0.206	0.149	0.076

Note: The Table displays the empirical rejection frequencies of the GMM test for testing the presence of a structural break of the variance and covariance of output and investment, generated with the factor-hoarding model described in the appendix. The bandwidth is chosen using the Newey and West (1994) procedure. The series have a length of 1000 periods. The table is based on 1000 Monte Carlo repetitions.

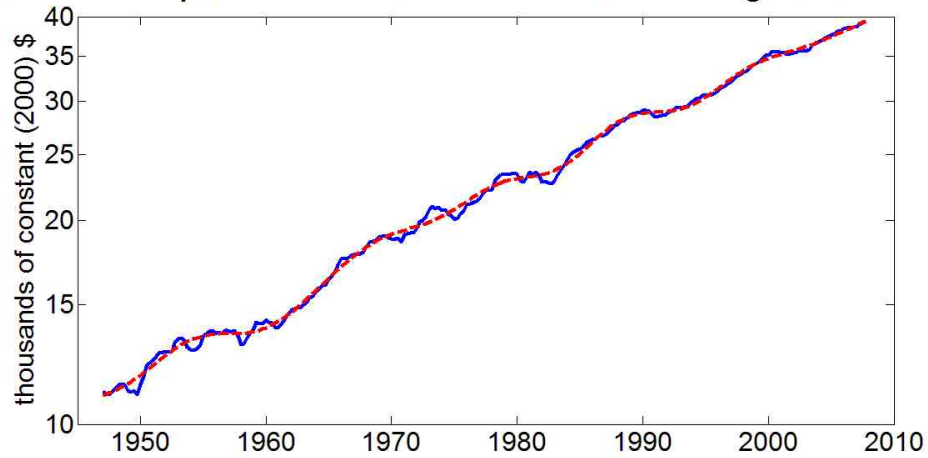
Table 1.10: P-values for the SCI Test

	High-Frequencies			High-to-Medium Frequencies		
	Consumption	Investment	Output	Consumption	Investment	Output
SAW Test	0.05	0.23	0.35	0.41	0.26	0.36
	Investment	0.05	0.06		0.50	0.35
	Output		0.01			0.10
SEW Test	0.07	0.14	0.34	0.29	0.09	0.29
	Investment	0.04	0.03		0.11	0.20
	Output		0.00			0.08
SN Test	0.06	0.23	0.34	0.47	0.29	0.43
	Investment	0.05	0.07		0.62	0.43
	Output		0.02			0.11

Note: The Table displays the p-values for the testing the presence of a structural break on the variance and covariance of consumption, investment, and output. The sample is 1947:1-2007:4. The variables are defined in real-per capita terms, and are obtained from NIPA.

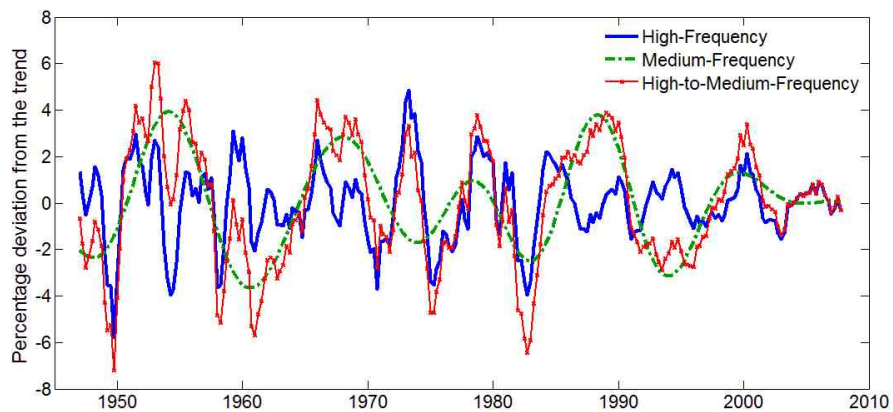
FIGURE 1.1: GDP: Level and Trend

Real Per-Capita Gross Domestic Product on a Logarithmic Scale



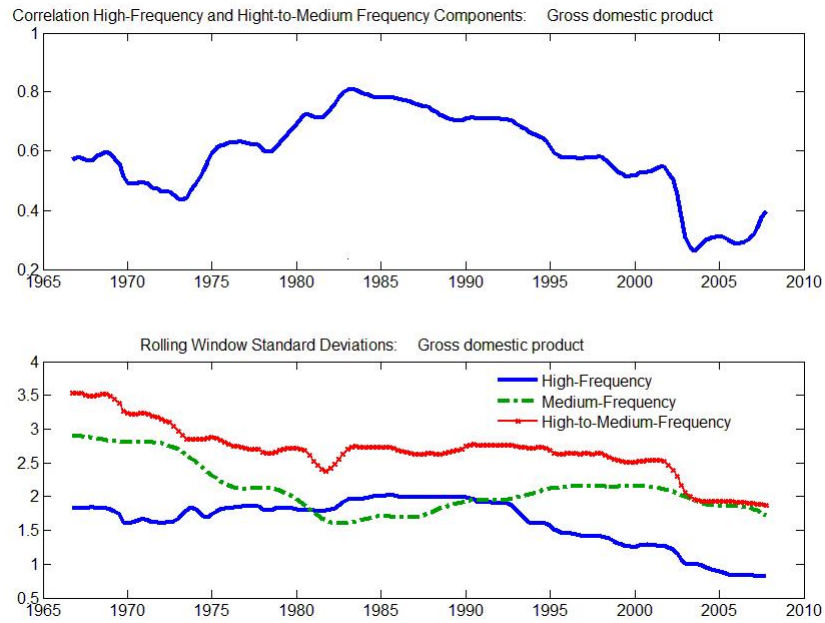
Note: Note: GDP is defined in real per-capita terms from NIPA. The sample includes quarterly observation from 1947:1 to 2007:4. The cyclical components, which are the High-Frequencies (HF, solid line), Medium-Frequencies (MF, dotted line), and High-to-Medium Frequencies (HM, dashed line) are isolated using a band-pass filter.

FIGURE 1.2: GDP: Cyclical Components



Note: The cyclical components, which are the High-Frequencies (HF, solid line), Medium-Frequencies (MF, dotted line), and High-to-Medium Frequencies (HM, dashed line) are isolated using a band-pass filter.

FIGURE 1.3: GDP: Rolling Windows Statistics



Note: This figure, as well as Figures 1.4 to 1.24, displays the rolling window statistics of the the macroeconomic time series from NIPA. The upper panel shows the evolution of the correlation between the High-Frequency and Medium-Frequency components. The lower panel shows the evolution of the standard deviation, in percent, of the High-Frequency (solid line), Medium-Frequency (dotted line), and High-to-Medium Frequency (star line) components. The length of the window is 80quarters.

FIGURE 1.4: Personal Consumption Expenditure: Rolling Windows Statistics

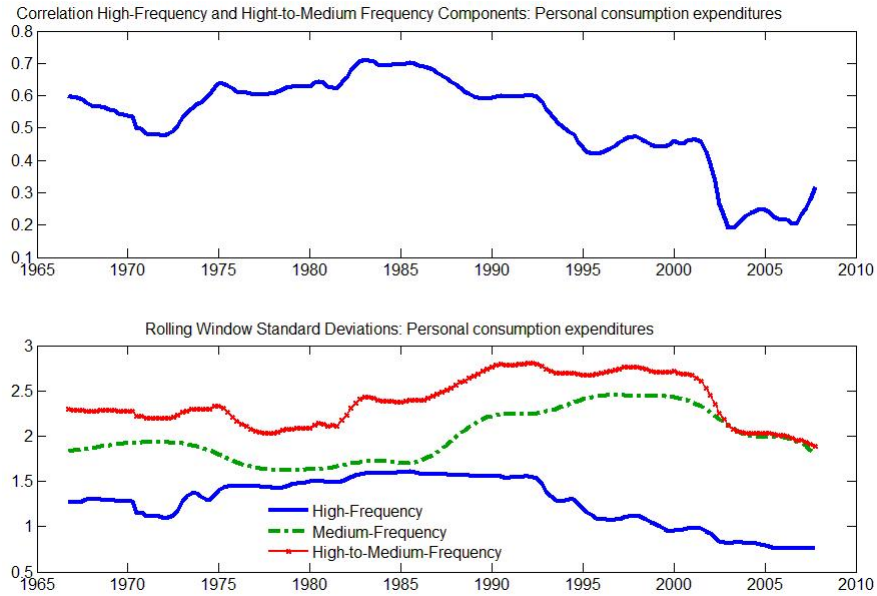


FIGURE 1.5: Consumption Durables: Rolling Windows Statistics

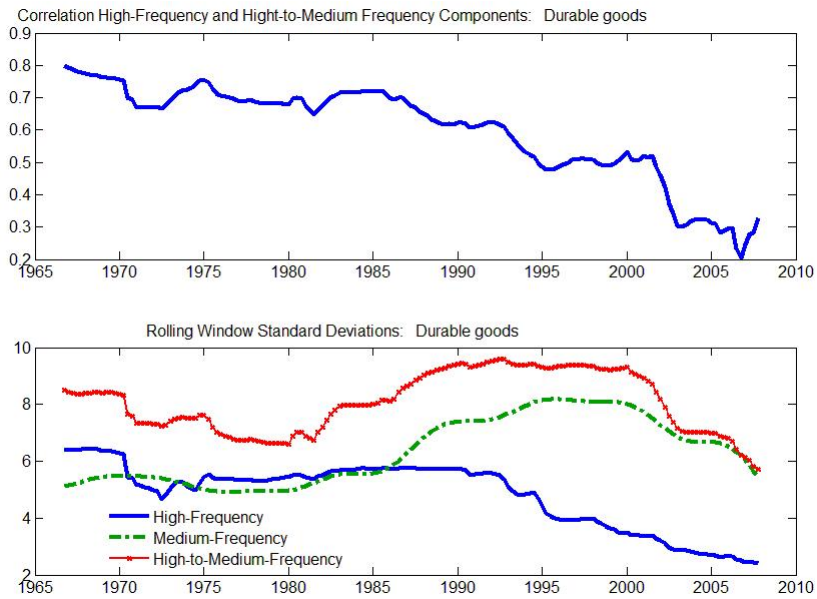


FIGURE 1.6: Consumption Nondurables: Rolling Windows Statistics

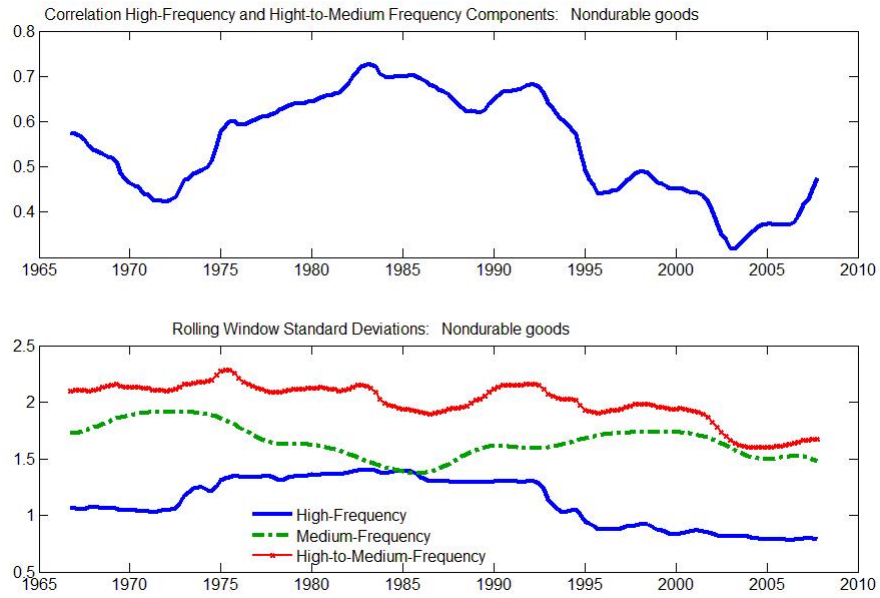


FIGURE 1.7: Consumption Services: Rolling Windows Statistics

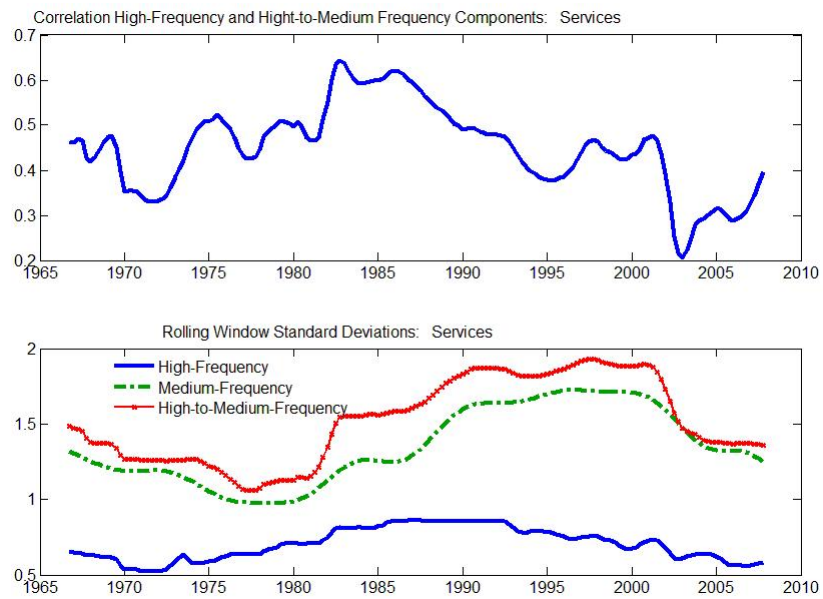


FIGURE 1.8: Gross Private Domestic Investment: Rolling Windows Statistics

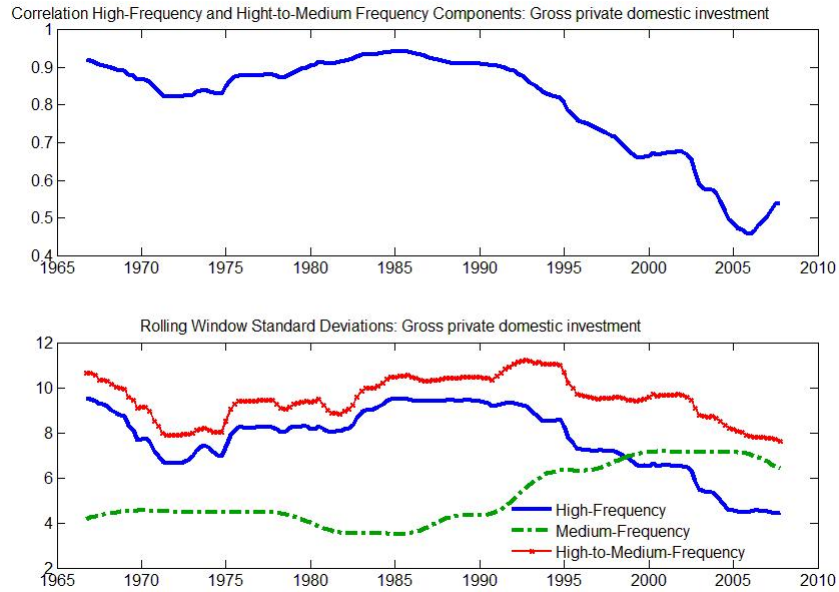


FIGURE 1.9: Fixed Investment: Rolling Windows Statistics

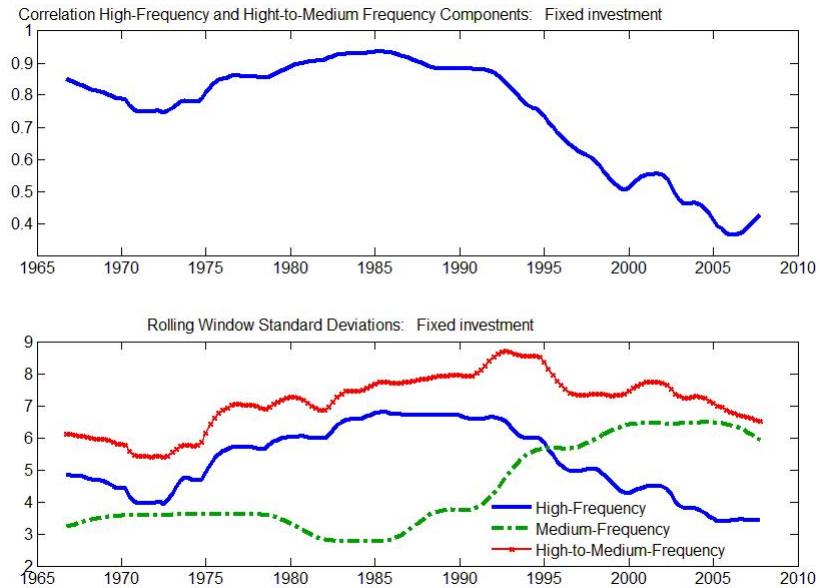


FIGURE 1.10: Nonresidential Fixed Investment: Rolling Windows Statistics

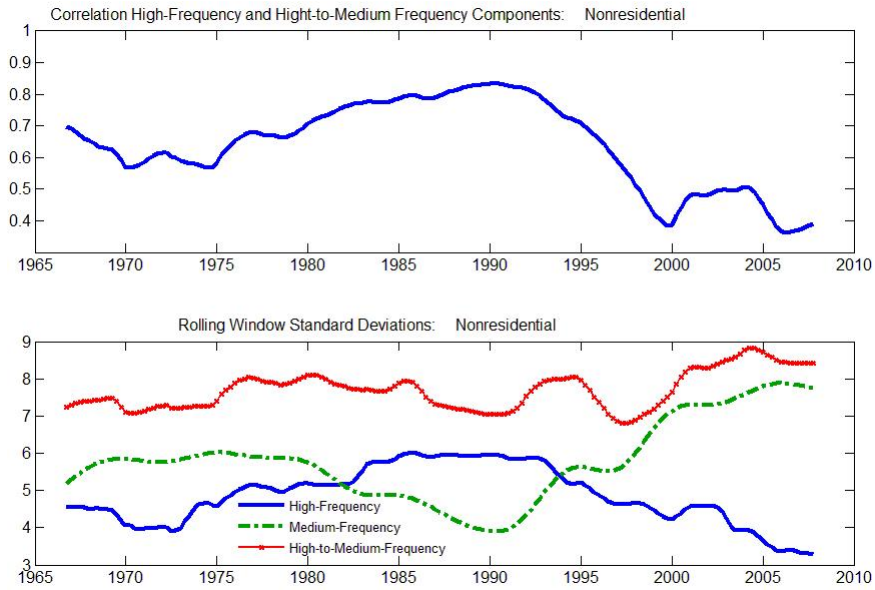


FIGURE 1.11: Structure Nonresidential Fixed Investment: Rolling Windows Statistics

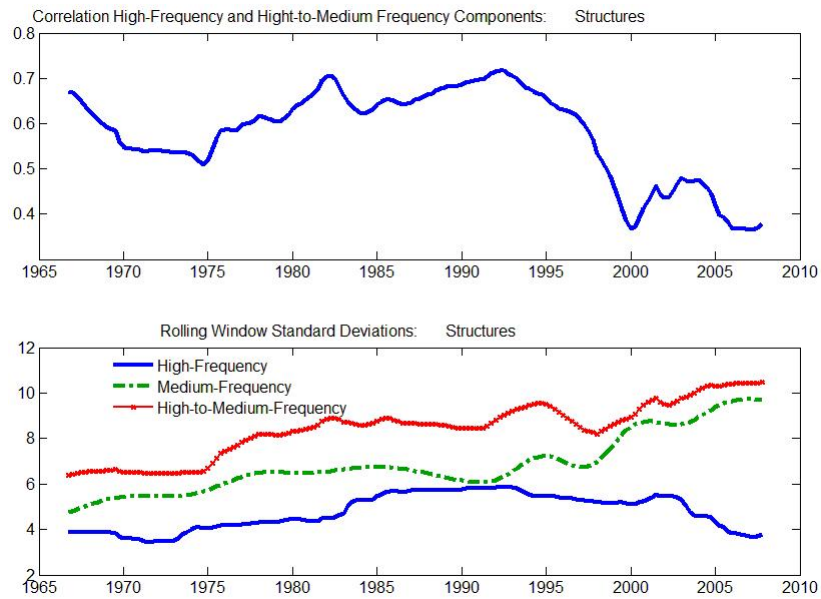


FIGURE 1.12: Equipment and Software Nonresidential Fixed Investment: Rolling Windows Statistics

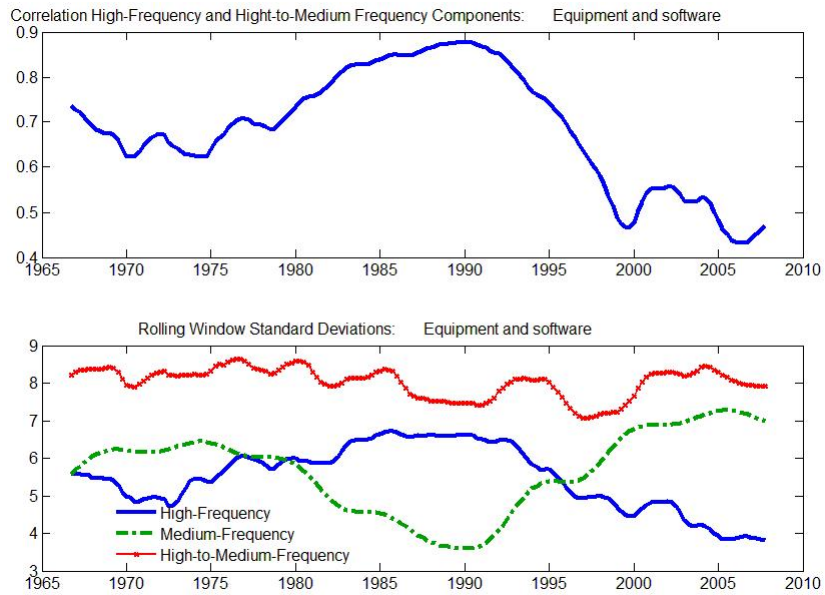


FIGURE 1.13: Residential Fixed Investment: Rolling Windows Statistics

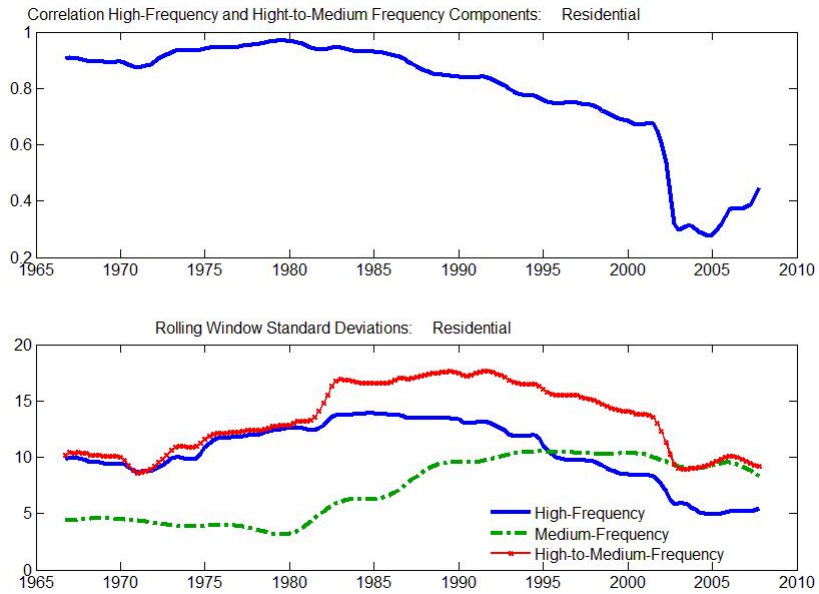


FIGURE 1.14: Exports: Rolling Windows Statistics

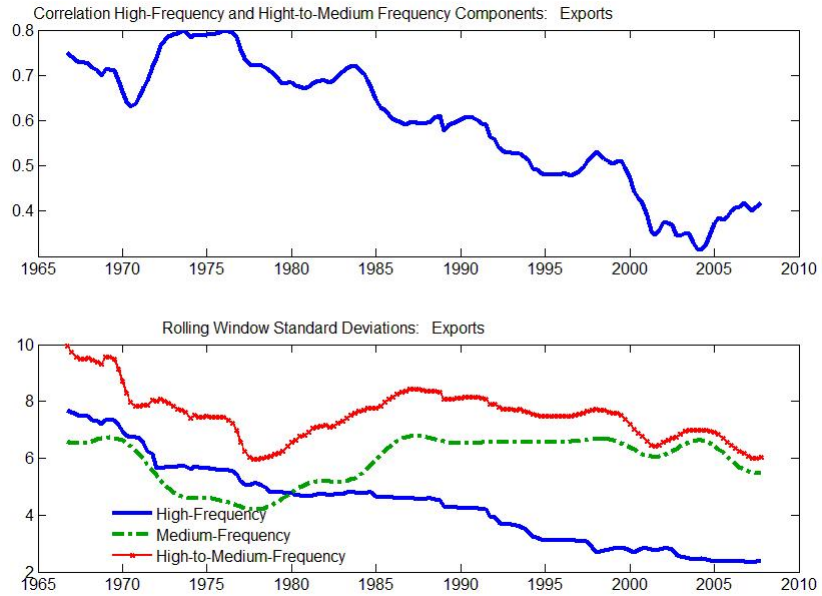


FIGURE 1.15: Exports, Goods: Rolling Windows Statistics

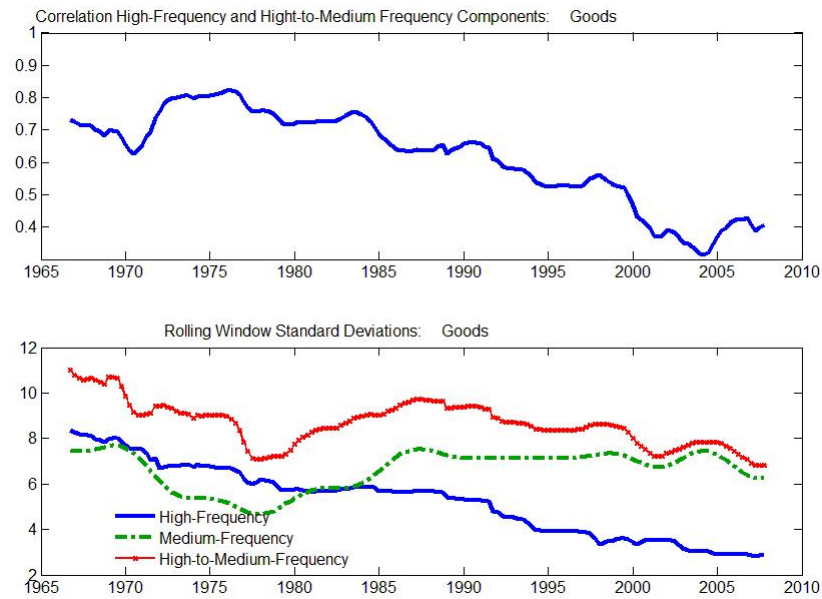


FIGURE 1.16: Exports, Services: Rolling Windows Statistics

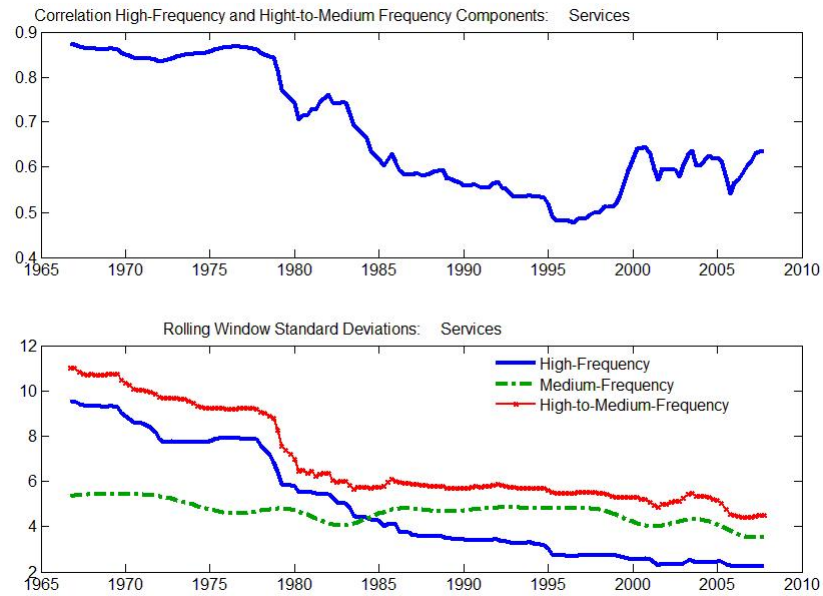


FIGURE 1.17: Imports: Rolling Windows Statistics

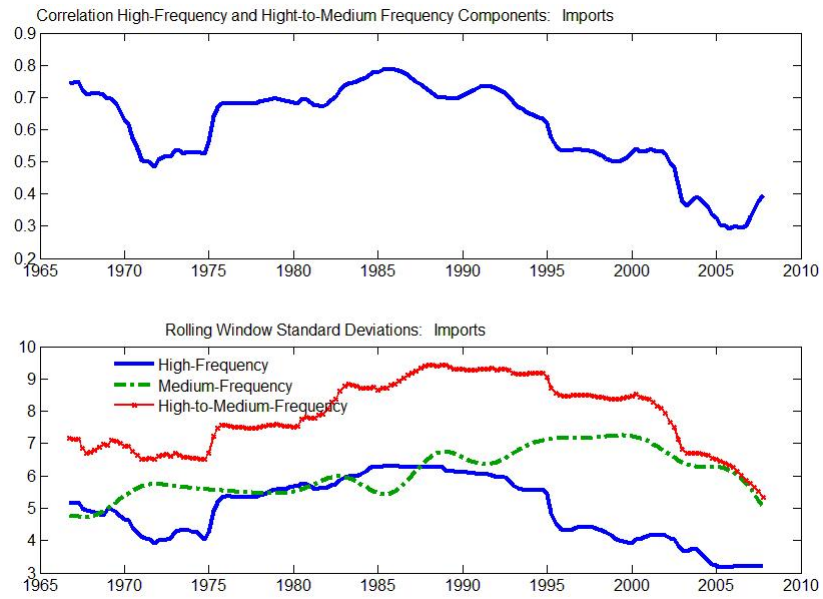


FIGURE 1.18: Imports, Goods: Rolling Windows Statistics

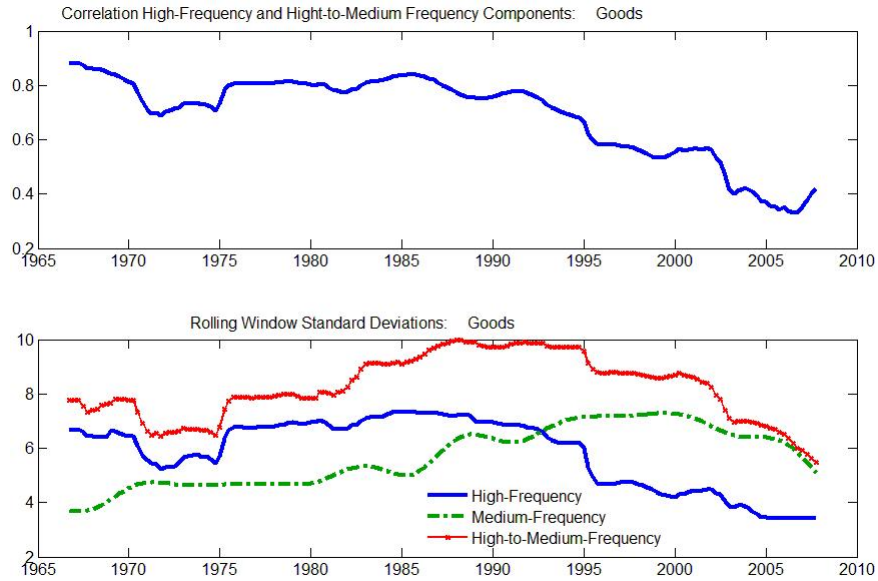


FIGURE 1.19: Imports, Services: Rolling Windows Statistics

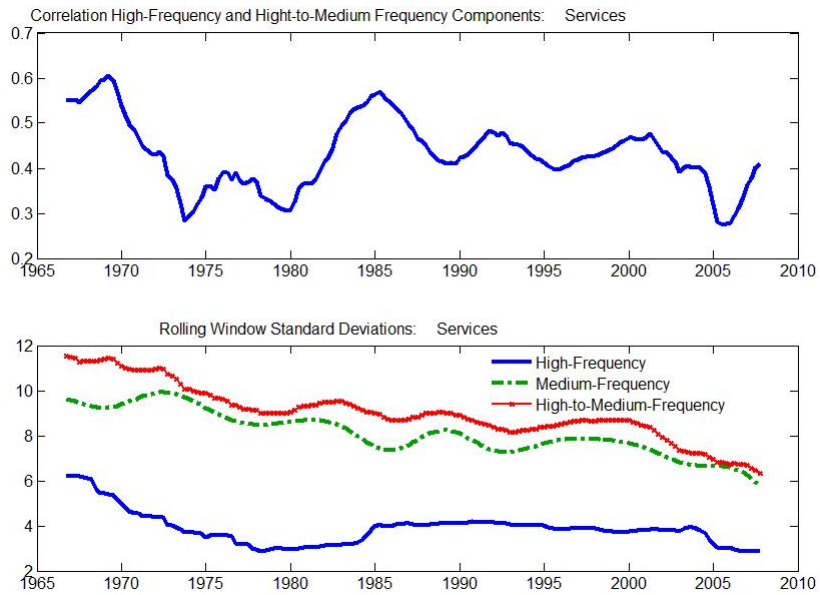


FIGURE 1.20: Government Consumption Expenditure and Gross Investment: Rolling Windows Statistics

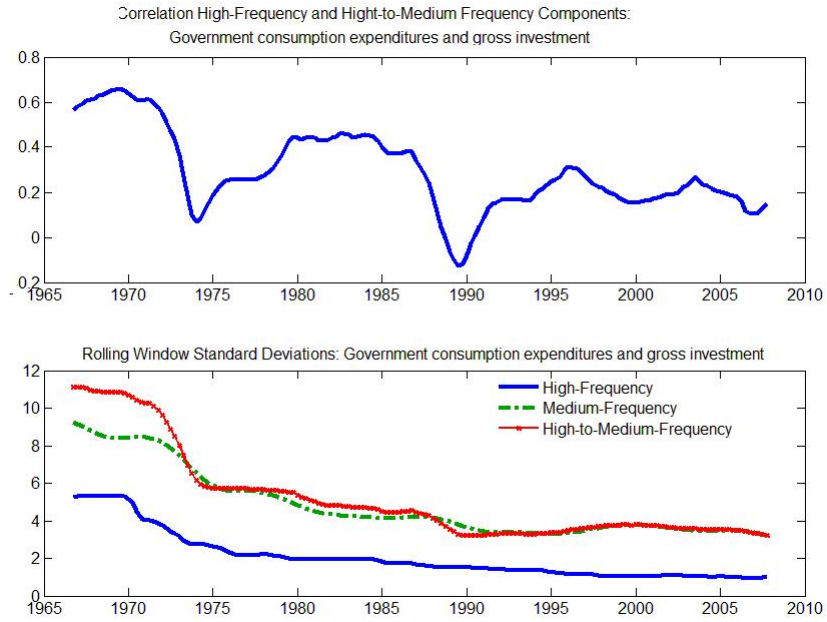


FIGURE 1.21: Federal Government Spending: Rolling Windows Statistics

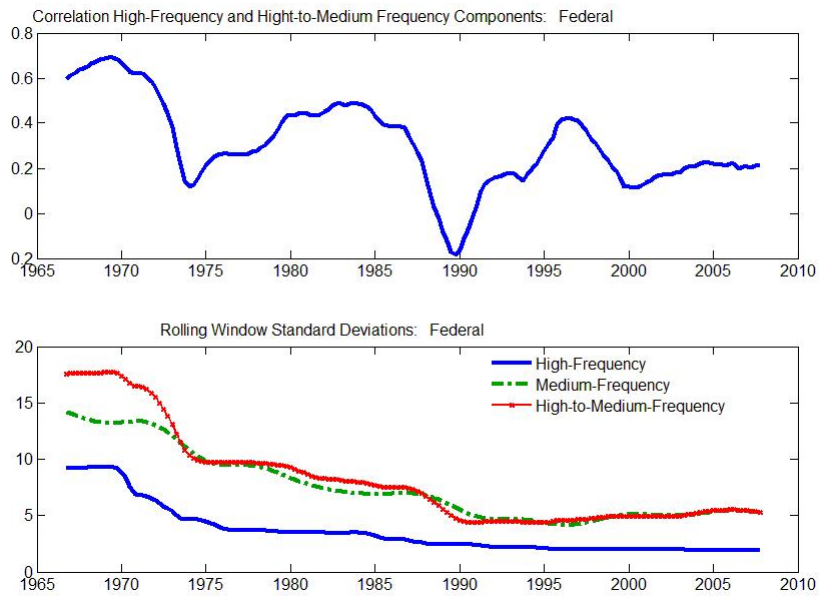


FIGURE 1.22: National Defense Government Spending: Rolling Windows Statistics

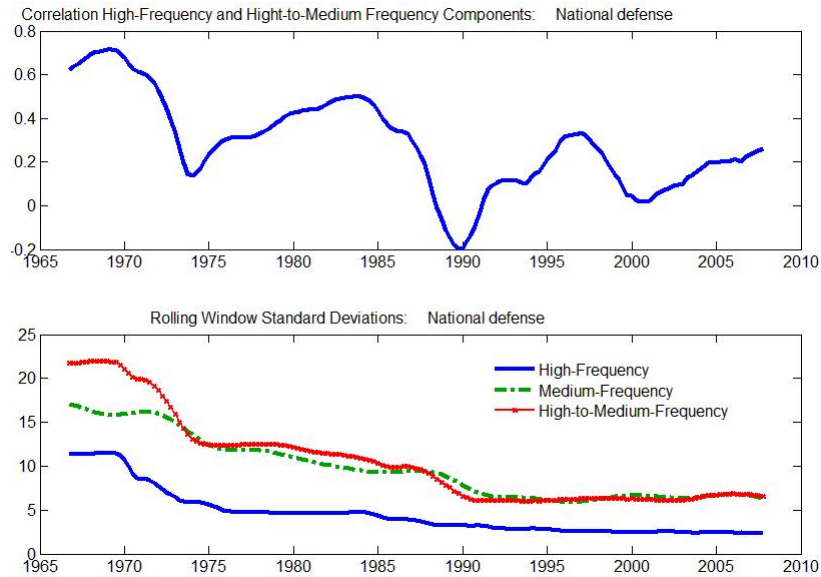


FIGURE 1.23: Nondefense Government Spending: Rolling Windows Statistics

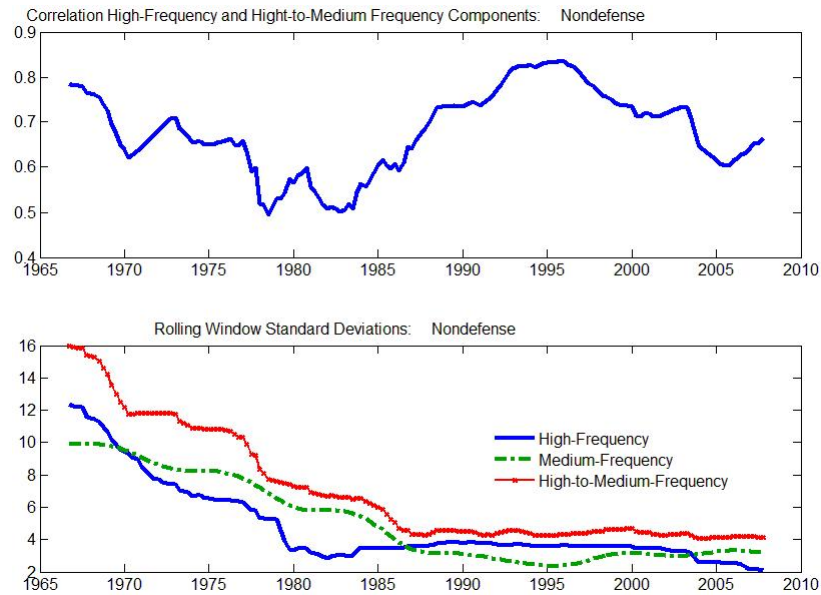
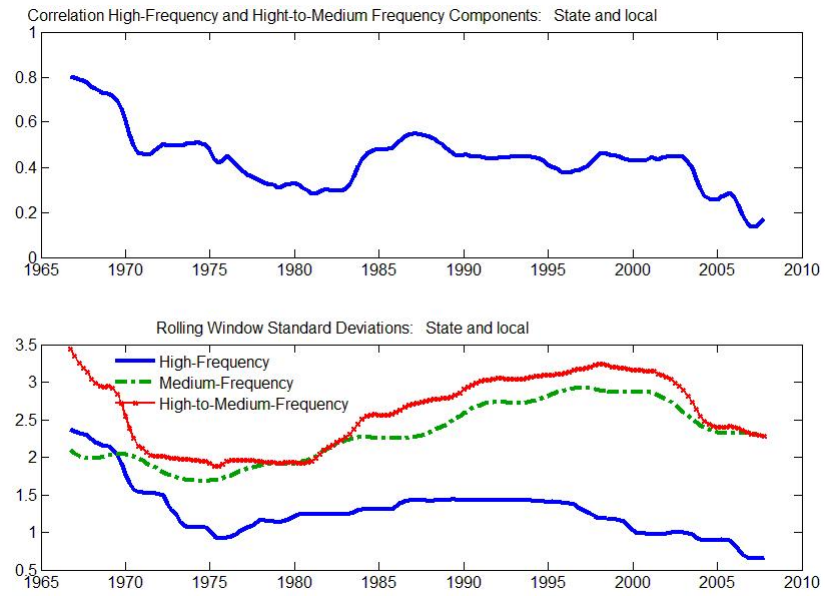


FIGURE 1.24: State and Local Government Spending: Rolling Windows Statistics



Who Cares About The Great Moderation?

2.1 Introduction

The term Great Moderation describes the reduction in macroeconomic volatility perceived by macroeconomists to have occurred after the early 1980s. The Great Moderation has received an enormous amount of attention in the literature, much of it devoted to assessing a range of possible causal factors.¹ Relatively little research, however, has addressed whether the Great Moderation is important in terms of improving household welfare. In this paper I calculate the welfare improvement caused by the Great Moderation, and conclude that it is more than likely modest, equivalent to roughly a 0.6 percent increase in household consumption.

The procedure I follow to measure the welfare gain from the moderation is characterized by two important features. First, I show that computed welfare gains depend crucially on the assumed laws of motion of consumption before and after the Great Moderation. Specifically, welfare calculations are sensitive to the spectral shape of consumption fluctuations. Therefore, a careful accounting of how macroeconomic

¹ Kim and Nelson (1999), McConnell and Perez-Quiros (2000), and Blanchard and Simon (2001) are among the pioneers of the literature on the Great Moderation. A survey of this literature can be found in Stock and Watson (2002).

volatility changed at different frequencies is required in order to assess changes in welfare. Additionally, because macroeconomic fluctuations are a source of risk for households, it seems natural that we should assess the gain from reducing this risk using a model which has empirically reasonable asset pricing implications². After all, these observed prices are our best measures of how actual agents value risk. A second feature of my analysis, therefore, is to pay close attention to the asset pricing implications of the models used in my analysis.

The vast literature on the Great Moderation focuses mainly on the significant reduction in the variance of either the growth rates of macroeconomic variables, or the business cycle components of these time series, isolated using a variety of filters. I show, however, that when a wider range of frequencies is considered, there was no apparent reduction in the variance after 1983. The decline in volatility at the relatively high frequencies studied in the literature actually coincided with a modest increase in volatility at medium to low frequencies. This finding is important, because depending on the preferences used to measure agents' welfare, we might actually expect to find no gain, or even a reduction in utility in the post-1983 period.

The following example serves to illustrate the importance of linking welfare calculations to asset prices. Lucas (1987) assessed the welfare cost of business cycle fluctuations using a simple representative-agent consumption-based asset pricing model with time separable constant relative risk aversion (CRRA) preferences. In his calibrated example the implied welfare gain of eliminating fluctuations is equivalent to a 0.01 percent increase in steady state consumption. Lucas' model, however, implies a negligible equity premium. If one calibrates the preference parameters, instead, so that the equity premium in the model is 6 percent (its average value in the post-war period), the welfare gain from eliminating fluctuations rises to 7.5 percent of steady

² Alvarez and Jermann (2004) also analyze the relation between welfare cost of cycles and asset pricing in a "model-free" environment.

state consumption.

With these considerations in mind, I proceed as follows. To model the Great Moderation, I estimate a fourth order autoregressive model of real U.S. per capita consumption for both the pre-1983 and post-1983 periods. This model is sufficiently rich that I am able to capture both the high and medium frequency behavior of the data in the two subsamples. To measure welfare gains, I use an endowment economy framework with habit formation preferences. The model parameters are calibrated such that the model is able to match key asset pricing moments (average risk-free return, average risky asset-return, average equity premium) across the two subsamples. The ability of the habit model to match the asset pricing moments is a consequence of the assumption that the agent's utility depends upon the consumption surplus, the distance between consumption and the habit stock. Since the latter is a smooth function of the past values of consumption, even a small degree of volatility in consumption results in significant volatility of the stochastic discount factor, and this generates a significant risk premium.

The increase in medium-frequency volatility experienced in the post-1983 period is due to increased persistence in the endowment process, while the decrease in high-frequency volatility is due to a decrease in the volatility of innovations to consumption. Consequently, in response to a negative shock to consumption, a representative agent expects his level of consumption to be close to the habit stock for several periods. Welfare losses stemming from this undesirable (from the perspective of the agent) feature of the post-1983 endowment process offset the welfare gain associated with the decline in high-frequency volatility. This explains my modest estimate of the welfare gain brought about by the Great Moderation: about 0.6 percent of steady-state consumption. To show that the medium-frequency behavior of consumption is indeed responsible for this small estimate of the welfare improvement, I consider a counterfactual scenario in which the variance of consumption is assumed

to decline at all frequencies. In this experiment the persistence parameters are held fixed at their pre-1983 values, while the variance of the innovation is set equal to its post-1983 value (which is 47 percent lower). In the counterfactual scenario, agents experience a bigger welfare gain, equal to about 2 percent of steady state consumption. Finally, using a bootstrap procedure I compute a 95 percent confidence band for my estimate of the welfare improvement: $(-2.8, 3.7)$ percent. The wide range of possibilities captured within the confidence set reflects the difficulty of precisely estimating the parameters governing the low-frequency properties of consumption. Consistent with my findings, Reis (2009) confirms that the persistence of the consumption process is a crucial determinant of the welfare cost and that the point estimate of this persistence is associated with large confidence bands.

My model of habit formation bears some similarity to the model proposed by Campbell and Cochrane (1999). Despite their model's attractive asset pricing qualities, I depart from it for important conceptual reasons. In their model, the habit stock depends non-linearly on past consumption. The particular form of the nonlinearity is problematic when computing the welfare change associated with a change in the law of motion of the endowment. In particular, the parameters of the law of motion of consumption implicitly affect the preference parameters that determine the sensitivity of the agent to consumption fluctuations. The lower the variance of consumption is, the more the habit stock responds to an endowment shock of a given magnitude. This mechanism plays an important role in the model's ability to match specific asset pricing facts, namely the first and second moments of the risk-free rate and the equity premium. However, unfortunately it obscures welfare calculations, because it is not possible to isolate the effects of the changes in the exogenous process while holding the preference parameters fixed³. On the other hand,

³ Ljungqvist and Uhlig (2009) also examines some related implications of the Campbell and Cochrane model for welfare calculations.

in my linear habit model preference parameters are independent of the law of motion consumption, which allows me to study the effects of the Great Moderation on welfare.

My model matches several asset pricing facts when calibrated to the full sample (1947–2007). However, the pre and post-1983 processes for consumption present a problem for my model, if I consider their separate asset pricing implications. In a time separable consumption-based model, the law of motion of consumption has similar implications for both the equity premium and welfare. A change in the law of motion leading to a big increase in welfare also significantly reduces the equity premium. Small changes in welfare are associated with small changes in the equity premium. However, in my habit model, even though there is only a small decline in welfare in the post-1983 period, there is a significant predicted decline in the average equity-premium. This is because in the habit model the equity premium displays more sensitivity to the behavior of the high frequency component of consumption than does welfare. This result is in line with Otrok (2001) and Otrok et al. (2002), which separately analyze the effects of habits on utility and on the equity premium. The predicted decline in the equity premium is at odds with the data, in which there is no significant change in the mean of the equity premium or other key asset pricing moments in the post-1983 period. While I could solve this problem by allowing for a change in the preference parameters, this solution would lead to the same criticism of my model that I gave of the Campbell and Cochrane model.

To assess whether other models predict a low gain from the Great Moderation, I consider two additional models in the recent macro-finance literature that have been shown to successfully match key asset pricing facts: the rare disaster model⁴ and the long-run risk model⁵. In these models only a small fraction of the equity premium

⁴ Rietz (1988), Barro (2005, 2009) among others.

⁵ Bansal and Yaron, (2006).

depends on the high-frequency properties of the consumption process, whereas it is in large part due to the probability and magnitude of rare disasters, and the importance of the very long-run component of consumption growth. If the Great Moderation is assumed to have left these features of the law of motion of consumption unchanged, there is little predicted change in the moments of financial variables, and only a very small welfare gain.

The paper is organized as follows. Section 2.2 presents the empirical analysis of the effect of the Great Moderation on macroeconomic variables and asset pricing. Section 2.3 illustrates the relationship between welfare cost, asset prices, and law of motion of consumption. Sections 2.4 and 2.5 present the asset prices model and its solution method. Section 2.6 illustrates the computed welfare costs of the Great Moderation. Section 2.7 discusses the effects of the medium frequency on the asset prices. Section 2.8 presents alternative models. Section 2.9 concludes with some remarks.

2.2 Great Moderation: Stylized Facts on Macroeconomic Variables and Asset Pricing

The extensive literature on the Great Moderation has mainly analyzed the stabilization of the high frequency volatility of macroeconomic variables by documenting the reduced variance of either the growth rate or the business cycle component of each series. In this section I extend the analysis of the Great Moderation in two directions. First, I analyze the behavior of some of the most relevant macroeconomic variables, namely consumption, output, and investment at the medium frequencies, in addition to the higher frequencies studied in the literature. I show that the large decline in volatility at high frequencies during the Great Moderation does not coincide with a reduction of the volatility at medium frequencies. Second, I investigate whether the Great Moderation affected any of the key moments of some asset prices, such as

the average values of the risk-free rate, the equity premium, and the price-dividend ratio.

2.2.1 Macroeconomic Variables

Consider the following three U.S. macroeconomic variables measured in real per capita terms: aggregate consumption, measured as non-durable goods plus services, output, and investment. The dataset includes observations from the period 1947Q1-2007Q4. I will refer to the subperiod 1947Q1-1982Q4 as Sample 1 (the period before the Great Moderation), and the subperiod 1983Q1-2007Q4 as Sample 2 (the period of the Great Moderation). The choice of 1983 as the break date for the beginning of the Great Moderation is in line with the large literature on this topic (see Stock and Watson, 2002).

Although many papers document the decline of the volatility of aggregate macroeconomic variables at the business cycle frequencies in the last twenty years, Pancrazi (2009) shows that the Great Moderation phenomenon disappears when medium frequencies are taken into account. In this section, I report some stylized facts that confirm the absence of moderation for the three macroeconomic variables when medium frequencies are considered.

For this purpose, consistent with Pancrazi (2009), I decompose a stochastic process as follows⁶:

Given a time series x_t , its High-Frequency component (HF), x_t^{HF} , corresponds to the cyclical fluctuations of x_t included in the period between 2 and 32 quarters. For quarterly data, in the frequency domain, these fluctuations belong to the interval $[\pi, \frac{\pi}{16}]$.

Given a time series x_t , its Medium-Frequency component (MF), x_t^{MF} , corresponds

⁶ Comin and Gertler (2006) also analyze the medium-cycle properties of some economic series. However their definition of medium-cycle includes fluctuations with periodicity up to 50 years.

to the cyclical fluctuations of x_t included in the period between 32 and 80 quarters. For quarterly data, in the frequency domain, these fluctuations belong to the interval $[\frac{\pi}{16}, \frac{\pi}{40}]$.

Given a time series x_t , its High-to-Medium frequency component (HM), x_t^{HM} , corresponds to the cyclical fluctuations of x_t included in the period between 2 and 80 quarters. For quarterly data, in the frequency domain, these fluctuations belong to the interval $[\pi, \frac{\pi}{40}]$.

Figures 2.1, 2.2, 2.3 plot the three components defined above for consumption, output, and investment, respectively. I use a band-pass filter (Christiano and Fitzgerald, 2003) to isolate the frequencies of interest in the data. The high-frequency components (solid line) display lower volatility in the post-1983 period compared to the pre-1983 period. This reduction of volatility at high-frequencies is a well-established fact in the literature about the Great Moderation. However, the analysis of the behavior of the three series at lower frequencies leads to some interesting and less familiar evidence. First, the magnitude of the fluctuations at medium-frequencies (dotted line) exceeds that of the high-frequencies fluctuations throughout the whole sample, as measured by the peak-to-trough distances. This suggests that a considerable part of the variability of consumption, output, and investment depends on fluctuations beyond the business cycle. Second, the Great Moderation period does not display any evident decline of the volatility of the medium-frequencies with respect to the previous subsample. These two facts explain why the fluctuations of the high-to-medium frequency component (dashed line) are mainly due to the medium-frequencies, especially in the Great Moderation period.

To quantify the stylized facts presented above, Tables 2.1, 2.2, and 2.3 report the standard deviations of the three macroeconomic variables at different intervals of frequencies in the two subsamples. The standard deviations of consumption, output, and investment at high-frequencies (2-32 quarters) declined by 44, 53, and

35 percent, respectively, during the Great Moderation. Similar results are obtained if the high-frequency component is defined using the first difference filter, rather than the bandpass filter.

However, once the medium-frequencies are taken into account, there is no apparent reduction in volatility during the Great Moderation. In fact, the standard deviation of the medium-frequency component (32-80 quarters) in the post-83 period actually increases by 25 percent for consumption, declines slightly, by 15 percent, for output, and more than doubles for investment, with respect to the pre-83 period. However, the standard errors suggest that these changes before and after the Great Moderation are not statistically significant. These results suggest that the impact of the Great Moderation on the spectrum of the macroeconomic variables is not homogenous throughout all the frequencies, since it has largely reduced their spectral mass only at high frequencies.

As the plots of the cycles show, the medium frequency component determines a substantial part of the total volatility of the three macroeconomics series. Therefore, it is not surprising that the behavior of the high-to-medium frequency component (2-80 quarters) during the Great Moderation is greatly affected by the medium frequency properties. The standard deviation of the high-to-medium frequency component of consumption decreased by only 6 percent during the Great Moderation, since the large reduction of high frequency volatility is offset by the increase of its medium frequency counterpart. The standard deviation of the high-to-medium frequency component of output declined by about 30 percent during the Great Moderation period, a value smaller than for the high frequency component. Finally, the standard deviation of the high-to-medium frequency component of investment increased by 15 percent during the Great Moderation.

These facts lead to another interesting consequence of the Great Moderation. Since the volatility at high frequencies for the macroeconomic variables was reduced

by a large amount and since the same reduction did not happen at lower frequencies, the percentage contribution of the medium frequency component to the total variance of the macroeconomic variables significantly increased in the second subsample. In fact, the medium frequency component accounts for 47 percent of the variance of the high-to-medium frequency component of consumption in the pre-1983 sample, 52 percent for output, and only 21 percent for investment. During the Great Moderation period, the medium frequency component accounts for more than 80 percent of the high-to-medium frequency variance for the three variables.

Given my analysis of the medium frequencies during the Great Moderation I draw two important conclusions: the stabilization of the fluctuations of macroeconomic variables is less evident at these frequencies, and the relative importance of medium frequency fluctuations rose considerably.

2.2.2 Asset Prices

In contrast to the vast literature on the stabilization of macroeconomic variables during the Great Moderation, relatively little attention has been paid to changes in the behavior of financial variables⁷. In this section I analyze some key moments of asset prices before and during the Great Moderation, to assess whether the reduction in the volatility of macroeconomic variables at high frequencies coincides with changes in the moments of financial variables. In particular, I first consider three time series: the real annualized return of a risk-free asset, measured as the return of 3-month Treasury bills, the annual real return of equity, measured using the value-weighted market return defined by Fama and French⁸, and the risk premium, measured as the difference between the risk-free return and the equity return.

Table 2.4 shows some moments of these variables in the two subsamples. The

⁷ Lettau, Ludvigson, and Wachter (2008) analyze the increase of the price-dividend ratio in the 1990s.

⁸ http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

average returns of the assets rose during the Great Moderation, by 2.3 percentage points for equities, and 1.6 percentage points for risk assets. As a result, the mean of the equity risk-premium grew slightly in the post-1983 period, by 0.7 percentage points. However, the standard errors and the Chow (1960) test statistic suggest that this small increase in the average risk premium is not statistically significant. Thus, I infer that the risk-premium did not change as a result of the Great Moderation. Although in this paper I focus mainly on the level of the risk premium, it is worth noting that its volatility was also approximately unchanged across the two subsamples.

Other asset pricing variables of interest are the price-dividend ratio and the price-consumption ratio⁹. Figure 2.4 displays these two series, where the first observation is normalized to unity. The averages of the two variables in the two subsamples are reported in Table 2.5. The price-dividend almost doubled during the Great Moderation period, whereas the price-consumption ratio increased by only 27 percent in the second subsample. The Chow tests suggest the presence of a structural break in the mean of the variables. It is interesting to compute the effect of the new-technology "bubble" in the late 1990s on the price-dividend and price-consumption ratios. To build a "bubble-free" scenario, I assume a linear pattern between 1995 and 2003 for the two variables. In this scenario, the average price-dividend ratio in the second subsample is 1.90, and the average price-consumption ratio is 1.89. When the "bubble" is eliminated, the Chow test cannot reject the null hypothesis of no changes in the mean of the price-consumption ratio, although the Chow test detects a break in the average price-dividend ratio.

⁹ These variable are extracted from Robert Shiller's online database. <http://www.econ.yale.edu/~shiller/data.htm>

2.3 Welfare and Asset Pricing

Lucas (1987) concludes that the welfare gain from eliminating business cycle fluctuations is negligible. In this section I demonstrate, however, that welfare calculations depend on two important features of a model: the specification of the exogenous consumption process and the asset pricing implications of the model structure.

2.3.1 Revisiting Lucas' Calculation

Lucas (1987) finds that the cost of business cycles is extremely low; an agent would agree to give up less than 0.04 percent of his consumption to avoid them entirely. However, to compute this cost Lucas (1987) uses two crucial assumptions: the logarithm of consumption is specified as an i.i.d. process around a linear trend, and CRRA utility is calibrated with a small coefficient of risk aversion. In this section I show that departures from these assumptions greatly affect the computed welfare cost. Specifically, I compute the welfare cost using the same CRRA utility specification as in Lucas, but I adopt the autoregressive process for consumption growth specified as in Mehra and Prescott (1985), which is fit to the 1889-1978 sample of U.S. data. I also calibrate the preference parameters so that the model matches the average equity premium and average risk-free rate in the Mehra and Prescott data set (1985).

To illustrate the sensitivity of welfare calculations to the model specification, in the first step, I assume that the preference parameters and the consumption process are specified as in Lucas (1987), i.e. the discount factor β is equal to 0.95, the coefficient of risk aversion is equal to 2, and the logarithm of consumption is i.i.d. around a linear trend, i.e.

$$\log(C_t) = gt + z_t \quad z_t \stackrel{iid}{\sim} N(0, \sigma_z^2).$$

Following Lucas'(1987) calibration, the mean growth rate of consumption, g , is set

equal to 0.03 and the standard deviation of the stationary component, σ , is set equal to 0.013. As the first column of Table 2.6 shows, the model predicts a negligible welfare cost from eliminating the fluctuations equal to 0.017 percent. Moreover, the model is not able to predict a significant equity premium, as Mehra and Prescott (1985) pointed out.

Similar results can be obtained maintaining the assumption of a CRRA utility function, but assuming that the growth rate of consumption is an i.i.d. normal random variable with mean μ and standard deviation σ , i.e.

$$\log(C_t) = \log(C_{t-1}) + \varepsilon_t \quad \varepsilon_t \stackrel{iid}{\sim} N(\mu, \sigma^2).$$

Calibrating $\mu = 0.03$ and $\sigma = 0.013$ as estimated in the post-war period, the second column of Table 2.6 shows that the model predicts an equity premium close to zero, and a low welfare cost from eliminating the fluctuations equal to 0.1 percent in consumption compensation. In both the trend stationary and difference stationary specifications of the consumption process, the model prediction of a low equity premium is associated with a low welfare cost of the fluctuations.

The link between asset pricing and welfare cost can be defined analytically in a basic consumption-based asset pricing model with time-separable CRRA utility, like the one considered for the previous calculations. In what follows, I show the tight relationship between the financial variables and welfare.

The representative agent maximizes the lifetime expected utility

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t),$$

where E_0 denotes the conditional expectations given the information at time 0, $U(\cdot)$ denotes the instantaneous utility function, C_t denotes consumption at time t , and β is the discount factor. There is a competitive market for trading assets (trees) which

pay dividends (fruits). Let P_t be the price of one unit of the asset and A_t be the agent's shareholding at time t , then the agent's budget constraint is

$$C_t + P_t A_{t+1} = (P_t + D_t) A_t,$$

where d_t denotes the exogenous stochastic flow of fruits at time t . Since there is no source of the consumption good other than the fruit, which is perishable, market clearing implies that $C_t = D_t$.

As in Lucas, the agent has CRRA preferences, i.e.

$$U(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma},$$

where $\gamma > 1$ is the coefficient of risk aversion. Since γ is positive, the agent in the economy is risk-averse.

The price of the asset is determined by the first order conditions as follows:

$$P_t = E_t \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} (P_{t+1} + C_{t+1}) \right]. \quad (2.1)$$

To link the asset pricing variables and the welfare cost of fluctuations, it is useful to rewrite (2.1) in terms of the price-dividend ratio $V_t = P_t/D_t$:

$$V_t = E_t \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{1-\gamma} (V_{t+1} + 1) \right]. \quad (2.2)$$

As in Lucas, I first assume that the logarithm of consumption, $c_t = \log(C_t)$, is an i.i.d. process around a linear trend, i.e.:

$$C_t = (1 + \mu)^t e^{z_t - \frac{1}{2}\sigma_z^2} \quad z_t \sim N(0, \sigma_z^2).$$

In this case, a first order approximation implies that the welfare gain from eliminating fluctuations, expressed in consumption compensation terms, is:

$$\lambda = \frac{1}{2}\gamma\sigma_z^2, \quad (2.3)$$

and the approximated expected value of the equity premium is

$$E(R^{EP}) = \{\beta^{-1}(1+\mu)^{-\alpha} - 1 + \gamma\}(1+\mu)\gamma\sigma_z^2. \quad (2.4)$$

where $\alpha = 1 - \gamma$. Given equation (2.3) and equation (2.4), it is evident that the model prediction about the equity premium is tightly related to the welfare cost. In fact, the two variables are proportional since:

$$\lambda = \frac{1}{2} \{\beta^{-1}(1+\mu)^{-\alpha} - 1 + \gamma\}^{-1} (1+\mu)^{-1} E(R^{EP}).$$

Moreover, notice that both the equity premium and the welfare cost are tied to the coefficient of risk aversion, γ .

Alternatively, assume that the growth rate of the consumption, $\Delta \log(C_t)$, is distributed as a normal with mean μ and standard deviation σ . Then the following expression for the price-dividend ratio holds¹⁰:

$$V = \frac{\beta \exp(\alpha\mu + \frac{1}{2}\alpha^2\sigma^2)}{1 - \beta \exp(\alpha\mu + \frac{1}{2}\alpha^2\sigma^2)}. \quad (2.5)$$

Under this assumption, the present value of the lifetime expected utility is a function of the expected price-dividend ratio:

$$U_0 = \frac{1 + E(V)}{\alpha}. \quad (2.6)$$

As stated in the previous section, the standard deviation of the growth rate of consumption has declined 46 percent in the Great Moderation period. What would the model predict about the effects of a significant decline of σ ?

Clearly equation (2.5) implies that a decline of the volatility of the growth rate of consumption, σ , would lead to a reduction of the price-dividend ratio. In fact, the

¹⁰ See Altug and Labadie (p.83, 1994)

derivative of V with respect to σ is given by

$$\frac{\partial V}{\partial \sigma} = \alpha^2 \sigma \beta \frac{e^{\frac{1}{2}\alpha^2 \sigma^2 + \mu \alpha}}{\left(\beta e^{\frac{1}{2}\alpha^2 \sigma^2 + \mu \alpha} - 1\right)^2} > 0,$$

and it is positive. Using equation (2.6) and the fact that $\frac{\partial V}{\partial \sigma} > 0$, I conclude that a decline in the variance of consumption growth leads to a welfare improvement in the economy, as long as $\gamma > 1$, ($\alpha < 0$).

2.3.2 Mehra and Prescott's Calibration

In the second step, I show that the specification of the consumption process affects the welfare computation. I follow Mehra and Prescott (1985), modeling the exogenous process as a first order autoregressive process for consumption growth:

$$\Delta \log(C_t) = \mu(1 - \rho) + \rho \Delta \log(C_{t-1}) + \sigma \varepsilon_t \quad \varepsilon_t \stackrel{iid}{\sim} N(0, 1),$$

where the mean μ is calibrated to be 0.0179, the autoregressive coefficient ρ is calibrated to -0.139 , and the standard deviation of the error term σ is calibrated to 0.0347. This model best fits the aggregate consumption data observed in Mehra and Prescott's (1985) sample period, 1889-1978. Using the same preference parameters as in the previous step ($\beta = 0.95$ $\gamma = 2$), the welfare cost of business cycles is now 0.65 percent, 30 times larger than Lucas' (1987) estimate. Obstfeld (1994) reaches similar conclusions: under the unit-root assumption, innovations in growth have cumulative effects, which greatly affect welfare. However, the cost of the business cycle is still modest and, as Table 2.6 displays, the equity premium predicted by the model is still small.

Finally, in the third step I maintain the assumption of the autoregressive process for consumption growth, but I calibrate the preference parameters such that the model predicts a risk-premium of 6 percent and a risk-free return of close to 1 percent,

the average values observed in Mehra and Prescott's (1985) sample. Table 2.6 shows that a coefficient of risk-aversion equal to 17 and a discount parameter greater than unity are able to generate asset returns whose first moments reasonably match the data¹¹. In this scenario, the welfare cost of the fluctuations is large, about 7.4 percent. This result suggests that the welfare cost of business cycle fluctuations implied by a model is tightly related to the ability of that model to generate a large price for risk.

However, as shown in the previous section, none of these calculations is appropriate for thinking about the effects of the Great Moderation because it did not lead to a decline in volatility at all frequencies. There was an increase in volatility at medium-frequencies coincident with the reduction in volatility at high frequencies. Therefore, a basic exercise in which consumption growth is the exogenous stochastic process, and the variance of its innovation declines, is completely silent about the effects of medium frequencies on welfare and asset pricing.

In this paper I propose a particular solution to this problem, introducing a model in which the consumption process is flexible enough to capture the behavior of the data at both high and medium frequencies, and in which preferences are such that the model can match the key moments of asset prices in the post-war period. With this model I compute the effect of the Great Moderation on welfare, and conclude that the gain implied by the Great Moderation is rather small, even though the model predicts a large equity premium and a small risk-free rate.

2.4 An Asset Pricing Framework

In this section I introduce a model which is able to match some of the basic asset pricing moments, that, given the analysis in Section 3, seem particularly relevant to welfare calculations: the risk-free rate, the equity return, and the price-dividend ratio. The model describes an endowment economy, in which the law of motion is

¹¹ Kocherlakota (1990) obtains similar results in an analogous exercise.

sufficiently rich that it captures both the high and medium frequency features of the data emphasized in Section 2.

2.4.1 The Model

The economy is similar to the one described above, but I assume that utility is time-nonseparable, by introducing external habits. The adoption of habits in asset price models was introduced by Abel (1990), and Constantinides (1990), and used by Campbell and Cochrane (1999) in their models. The representative agent maximizes his lifetime expected utility

$$E_0 \sum_{t=0}^{\infty} \beta U(C_t, X_t).$$

The agent's instantaneous utility depends on the consumption surplus, which is the difference between present consumption, C_t , and the habit stock, X_t :

$$U(C_t, X_t) = \frac{(C_t - X_t)^{1-\gamma} - 1}{1-\gamma}, \quad (2.7)$$

where C_t is the agent's consumption at time t , X_t is the external stock of habit, and γ governs the curvature of the utility function. In this setting the coefficient of relative risk aversion is time-dependent and is affected by the magnitude of the consumption surplus. The local coefficient of relative risk aversion is defined as

$$CRRA_t \equiv -\frac{C_t U_{cc}}{U_c} = \gamma \frac{C_t}{C_t - X_t}.$$

When consumption is close to the habit stock, the agent's utility declines and his aversion to risk increases, for any given γ .

All output in the economy is derived from an asset that produces a stochastic endowment of a single perishable good for each unit of the asset that the agent owns at the beginning of time t .

The budget constraint is

$$C_t + P_t A_{t+1} = (P_t + D_t) A_t,$$

where A_t is the quantity of asset owned at time t , P_t is the price of the asset, and D_t is the dividend generated by the asset.

The Euler equation governing the agent's optimal choice of consumption is

$$P_t (C_t - X_t)^{-\gamma} = \beta E_t (C_{t+1} - X_{t+1})^{-\gamma} (P_{t+1} + D_{t+1}),$$

which can be rewritten in terms of the price-dividend ratio, $V_t = P_t/D_t$, as:

$$V_t D_t (C_t - X_t)^{-\gamma} = \beta E_t (C_{t+1} - X_{t+1})^{-\gamma} (V_{t+1} + 1) D_{t+1}.$$

Since, in equilibrium, $D_t = C_t$, we have

$$V_t = E_t \left[M_{t+1} (V_{t+1} + 1) \frac{C_{t+1}}{C_t} \right], \quad (2.8)$$

where M_{t+1} is the stochastic discount factor, defined as:

$$M_{t+1} = \beta \left(\frac{C_{t+1} - X_{t+1}}{C_t - X_t} \right)^{-\gamma}. \quad (2.9)$$

I now specify the nature of the process of consumption C_t and of the external habit stock X_t . I assume that the level of consumption is the product of a deterministic time trend, e^{gt} , and a stationary component, \tilde{C}_t , that governs the fluctuations around the trend:

$$C_t = e^{gt} \tilde{C}_t.$$

Here g is the mean growth rate of consumption. Although one of the basic consumption-based models presented in the previous section and other habit-models, like Campbell and Cochrane (1999), parameterize consumption as a difference stationary process, my choice of a trend stationary process is motivated by the ability of the stationary

component \tilde{C}_t to capture the medium-frequency properties of the U.S. consumption time-series.

Denoting with lower-case the logarithm of a variable, i.e. $z_t = \log(Z_t)$, it follows that

$$c_t = gt + \tilde{c}_t. \quad (2.10)$$

I assume that the stock of habit is an infinite geometric average of the aggregate level of consumption, C_t^a i.e.

$$X_t = \left[\prod_{i=0}^{\infty} (C_{t-1-i}^a)^{\phi^i} \right]^{1-\phi}.$$

In logarithms we have:

$$x_t = (1 - \phi) \sum_{i=0}^{\infty} \phi^i c_{t-1-i}^a.$$

In equilibrium identical individuals choose the same level of consumption, therefore $C_t = C_t^a$. Thus, I drop the superscripts in what follows, since they are not essential.

Applying the decomposition in (2.10), we have:

$$x_t = (1 - \phi) \sum_{i=0}^{\infty} \phi^i [g(t - i - 1) + \tilde{c}_{t-1-i}] = gt - \omega + \tilde{x}_t,$$

where $\tilde{x}_t = (1 - \phi) \sum_{i=0}^{\infty} \phi^i \tilde{c}_{t-1-i}$, and $\omega = (1 - \phi) g \sum_{i=0}^{\infty} \phi^i (i + 1)$.

The constant ω can be analytically computed as

$$\omega = (1 - \phi) g \sum_{i=0}^{\infty} \phi^i (i + 1) = \frac{g}{1 - \phi}.$$

The stochastic discount factor in (2.9) can be rewritten in terms of stationary variables:

$$M_{t+1} = \beta \left(\frac{C_{t+1} - X_{t+1}}{C_t - X_t} \right)^{-\gamma} = \beta e^{-g\gamma} \left(\frac{\tilde{C}_{t+1} - \Omega \tilde{X}_{t+1}}{\tilde{C}_t - \Omega \tilde{X}_t} \right)^{-\gamma},$$

where $\Omega = e^{-\omega}$.

Finally, the Euler equation in (2.8) becomes:

$$V_t = \beta e^{(1-\gamma)g} E_t \left[\left(\frac{\tilde{C}_{t+1} - \Omega \tilde{X}_{t+1}}{\tilde{C}_t - \Omega \tilde{X}_t} \right)^{-\gamma} (V_{t+1} + 1) \frac{\tilde{C}_{t+1}}{\tilde{C}_t} \right]. \quad (2.11)$$

Numerical methods, to be described below, allow me to solve this equation to obtain a pricing function that expresses the price-dividend ratio V_t as a function of the relevant state variables, which depend on the parametric specification of the exogenous process for the stationary component of consumption, \tilde{c}_t .

2.4.2 The Law of Motion of the Endowment

In order to close the model, I define a process for the stationary component of consumption, \tilde{c} . A simple first-order autoregressive $AR(1)$ process, which is commonly used in quantitative macroeconomic models, is not suitable for this purpose since its spectral shape, which is a function of only two parameters, is not flexible enough to match the reduction of the volatility at high-frequencies and the increase of the volatility at the medium frequencies observed in my sample period.

Therefore, I consider a higher-order autoregressive process, whose spectrum is more flexible since it is a function of a larger number of parameters. The Schwarz Information Criterion suggests that a fourth-order autoregressive, $AR(4)$, process of the form

$$\tilde{c}_t = \theta_1 \tilde{c}_{t-1} + \theta_2 \tilde{c}_{t-2} + \theta_3 \tilde{c}_{t-3} + \theta_4 \tilde{c}_{t-4} + \sigma_\varepsilon \varepsilon_t \quad \varepsilon_t \stackrel{iid}{\sim} N(0, 1),$$

is the best candidate among all the autoregressive processes, in the sense that it maximizes a penalized likelihood. Thus, I estimate the parameters of an $AR(4)$ process for consumption in each of the two subsamples. Define Θ^1 the set of parameters of the $AR(4)$ process estimated using data from Sample 1 (1947:1-1982:4) and define

Θ^2 the set of parameters of the $AR(4)$ process estimate in Sample 2 (1983:1-2007:4):

$$\Theta^1 = \begin{bmatrix} \theta_1 = 1.000 \\ \theta_2 = 0.030 \\ \theta_3 = -0.068 \\ \theta_4 = -0.097 \\ \sigma_\varepsilon = 0.0055 \end{bmatrix} \quad \Theta^2 = \begin{bmatrix} \theta_1 = 1.147 \\ \theta_2 = -0.127 \\ \theta_3 = 0.294 \\ \theta_4 = -0.360 \\ \sigma_\varepsilon = 0.0029 \end{bmatrix} .$$

As Table 2.7 shows, these estimated processes are able to match the pattern of the sample moments of consumption at different frequencies. The Sample 2 process is characterized by a large reduction in high frequency volatility and an increase in medium frequency volatility, relative to the Sample 1 process. As a result the standard deviation of the combined high and medium frequencies is similar for the two processes. Although the magnitude of the medium frequency volatility is smaller than in the data, the relative change of the implied standard deviations in the two subsamples is identical to their sample counterpart.

The ability of these processes to capture the changes in the shape of the spectrum of consumption relies on two factors, as Table 2.8 suggests. First, the decline of the standard deviation of the innovation, σ_ε , from Sample 1 to Sample 2 implies a proportional downward shift of the spectrum, resulting in a decline of the volatility at all frequencies. Second, the changes in the estimates of the autoregressive parameters imply an increasing persistence of the process from the first to the second subsample, as suggested by the higher largest root of the lag polynomial estimated in Sample 2. Such increased persistence generates a redistribution of the mass of the spectrum from higher to lower frequencies. Therefore, the effect of the decline of the variance of the error term is offset by the increase of the persistence at medium-frequencies, whereas the two effects go in the same direction at higher frequencies.

Finally, in some of my quantitative experiments I consider an additional counterfactual process for consumption in which the autoregressive parameters are held

constant at the estimates obtained in Sample 1, but the variance of the innovation is calibrated such that the resulting process has the same high frequency variance as in Sample 2. Let us define as Θ^C the parameters of the $AR(4)$ process that generates this counterfactual scenario:

$$\Theta^C = \begin{bmatrix} \theta_1 = 1.000 \\ \theta_2 = 0.030 \\ \theta_3 = -0.068 \\ \theta_4 = -0.097 \\ \sigma = 0.0034 \end{bmatrix}.$$

This counterfactual scenario is of interest because it allows me to assess what the welfare gain of the Great Moderation would have been, had there been an across-the-board decline in volatility at all frequencies. This helps me highlight the extent to which my welfare calculations depend on the spectral shape of consumption, not just the degree of volatility at high frequencies. Figure 2.5 plots the spectral density of the three different processes. In order to show more clearly the differences of the processes at medium frequencies, I truncate the x-axes to the frequency 0.6, since the three spectra have low power at higher frequencies. Note that the area below the spectrum in a particular interval of frequencies measures the variance of the process attributable to those frequencies. Figure 2.6 plots the log-spectra of the three processes in the support of frequencies $[0, \frac{\pi}{4}]$.

2.5 Solution Method: Parameterized Expectations

In order to solve the model presented in Section 4, I use the Parameterized Expectations approach¹². I assume that the price dividend ratio can be approximated by a parametric function of the 21 state variables defined by the complete set of

¹² See Judd (1998), Den Haan and Marcet (1990), Marcet and Marshall (1994), and Marcet and Lorenzoni (1998).

polynomials of total degree 2 in the five variables, $\tilde{C}_t, \tilde{X}_t, \tilde{C}_{t-1}, \tilde{C}_{t-2}, \tilde{C}_{t-3}$, i.e.

$$V_t \simeq \psi(s_t; \Phi),$$

where s_t is a vector containing the constant, the five variables listed above, their square values, and all the possible cross-product of degree 2, and Φ is a set of parameters. I assume that ψ is a linear function of Φ and s_t :

$$\psi(s_t; \Phi) = \Phi' s_t.$$

The Euler equation in (2.11) implies that

$$\psi(s_t; \Phi) \simeq \beta e^{(1-\gamma)g} E_t \left[\left(\frac{\tilde{C}_{t+1} - \Omega \tilde{X}_{t+1}}{\tilde{C}_t - \Omega \tilde{X}_t} \right)^{-\gamma} (V_{t+1} + 1) \frac{\tilde{C}_{t+1}}{\tilde{C}_t} \right].$$

Define

$$\vartheta(s_t, s_{t+1}, \Phi) = \beta e^{(1-\gamma)g} \left[\left(\frac{\tilde{C}_{t+1} - \Omega \tilde{X}_{t+1}}{\tilde{C}_t - \Omega \tilde{X}_t} \right)^{-\gamma} (\psi(s_{t+1}; \Phi) + 1) \frac{\tilde{C}_{t+1}}{\tilde{C}_t} \right].$$

The solution for Φ is the vector of parameters that minimizes the distance between $\psi(s_t; \Phi)$ and $E_t \{\vartheta(s_t, s_{t+1}, \Phi)\}$. In particular, given a vector of parameters $\Phi^{(n-1)}$, I can obtain successive $\Phi^{(n)}$ s using the recursion:

$$\Phi^{(n)} = \arg \min_{\Phi} M(\Phi, \Phi^{(n-1)}),$$

with

$$M(\Phi, \Phi^{(n-1)}) = E \left\{ \vartheta(s_t, s_{t+1}, \Phi^{(n-1)}) - \psi(s_t; \Phi) \right\}^2.$$

The first order conditions for this optimization problem imply

$$E \left(\left\{ \vartheta(s_t, s_{t+1}, \Phi^{(n-1)}) - \psi(s_t; \Phi) \right\} \frac{\partial \psi(s_t; \Phi)}{\partial \Phi_j} \right) = 0, \quad (2.12)$$

for $j = 1, \dots, J$, where J is the dimension of Φ .

Since $\psi (s_t; \Phi)$ is linear in Φ , we have that:

$$\frac{\partial \psi (s_t; \Phi)}{\partial \Phi_j} = s_{t,j},$$

where $s_{t,j}$ denotes the j -th element of s_t .

Therefore, the first order condition can be rewritten as

$$E \left(\left\{ \vartheta (s_t, s_{t+1}, \Phi^{(n-1)}) - \psi (s_t; \Phi) \right\} s_{t,j} \right) = 0 \quad \text{for } j = 1, \dots, J,$$

or, equivalently,

$$E \left\{ \vartheta (s_t, s_{t+1}, \Phi^{(n-1)}) s_{t,j} \right\} = E \left\{ \psi (s_t; \Phi) s_{t,j} \right\} ,$$

and since $\psi (s_t; \Phi) = \Phi' s_t$, we have

$$E \left\{ \vartheta (s_t, s_{t+1}, \Phi^{(n-1)}) s_{t,j} \right\} = E \left\{ [\Phi' s_t] s_{t,j} \right\} .$$

Defining a $J \times 1$ vector b whose j -th element is

$$b_j = E \left\{ \vartheta (s_t, s_{t+1}, \Phi^{(n-1)}) s_{t,j} \right\} ,$$

and a $J \times J$ matrix A whose ij -th element is

$$A_{ij} = E \left\{ s_{t,i} s_{t,j} \right\} ,$$

then the optimality condition is simply

$$b = A\Phi,$$

or

$$\Phi^{(n)} = A^{-1}b.$$

The procedure can be recursively iterated until $\Phi^{(n)} \approx \Phi^{(n-1)}$.

In order to compute the matrix A and the vector b we need to solve the expectations. For this purpose I use a simulation-based numerical approximation. Note that

$$b_j = E \left[\beta e^{(1-\gamma)g} \left[\left(\frac{\tilde{C}_{t+1} - \Omega \tilde{X}_{t+1}}{\tilde{C}_t - \Omega \tilde{X}_t} \right)^{-\gamma} (\psi(s_{t+1}; \Phi) + 1) \frac{\tilde{C}_{t+1}}{\tilde{C}_t} \right] s_{t,j} \right].$$

The expectation depends on the values of s_{t+1} and their probabilities, and the values of s_t and their probabilities. In order to calculate the unconditional expectations, I approximate the expectations with the mean of M Monte Carlo simulations for the process \tilde{c}_t .

Therefore, we have

$$b_j = \frac{1}{M} \sum_{m=1}^M \left\{ \beta e^{(1-\gamma)g} \left[\left(\frac{\tilde{C}_{t+1}^m - \Omega \tilde{X}_{t+1}^m}{\tilde{C}_t^m - \Omega \tilde{X}_t^m} \right)^{-\gamma} (\psi(s_{t+1}^m; \Phi) + 1) \frac{\tilde{C}_{t+1}^m}{\tilde{C}_t^m} \right] s_{t,j}^m \right\}.$$

Analogously, the elements of the matrix A are defined as

$$A_{ij} = \frac{1}{M} \sum_{m=1}^M (s_{t,i}^m s_{t,j}^m).$$

2.6 Welfare Gain from the Great Moderation

Throughout this paper I have emphasized that the welfare cost of the fluctuations computed from a model is related to its asset price implications. Thus, in order to compute a plausible welfare gain from the Great Moderation, the model needs to be calibrated to match key asset pricing moments. However, here I face a difficult challenge. Not only should the model do well at explaining asset prices, on average, in the post-war period, but it should also do well in the separate pre- and post-Great Moderation subsamples. Moreover, to isolate the effects of the different exogenous processes of consumption in the two periods, the preference parameters of the model must be assumed to be unchanged throughout the post-war period.

I first estimate an $AR(4)$ process for consumption in the entire sample 1947:1-2007:4. Let Θ^{ALL} denote my parameter estimates:

$$\Theta^{ALL} = \begin{bmatrix} \theta_1 = 1.072 \\ \theta_2 = 0.011 \\ \theta_3 = -0.020 \\ \theta_4 = -0.143 \\ \sigma = 0.0047 \end{bmatrix}.$$

Throughout the paper I set the value of the average growth rate of consumption, g , equal to its full-sample estimate 0.0052, and I assume the parameter governing the persistence of the habit stock is $\phi = 0.91$. Finally, I calibrate the value of the discount factor, β , and the parameter of the curvature of the utility function, γ , to match the average risk-free rate and average equity-premium in the post-war period. The resulting values of the preference parameters are $\beta = 0.996$, and $\gamma = 1.40$. Table 2.9 shows that the model is able to perfectly match these moments. However, most importantly, the sample average returns in the two subsamples belong to the 95 percent confidence band computed for the full sample calibration of the model by Monte Carlo simulations.

These results allow me to assume that the preference parameters, β and γ , did not change with the Great Moderation. In fact, the average returns from the data in the pre- and post-Great Moderation are plausibly generated by the model with a finite number of observations. This property is necessary to isolate the effects of a different law of motion of consumption during the Great Moderation on welfare.

Using the proposed model, which is able to match the average risk-premium before and during the Great Moderation, I calculate the percentage of consumption that the representative agent would be willing to give up, in order to have an alternative law of motion of consumption. This approach is equivalent to Lucas' calculation of the welfare cost of the business cycle. Let \mathbf{c} and $\tilde{\mathbf{c}}$ be two different processes for consumption. Let U be the present value of the lifetime expected utility under the

exogenous process \mathbf{c} , i.e.

$$U = E[U_0(\mathbf{c})].$$

I define the parameter λ such that the following equation holds

$$\bar{U} = E[U_0([1 - \lambda]\bar{\mathbf{c}})] = E[U_0(\mathbf{c})] = U,$$

which means that λ is the fraction of his consumption that an agent with income stream $\bar{\mathbf{c}}$ would be willing to give up to avoid the fluctuations associated with income stream \mathbf{c} .

The lifetime expected utility depends both on the set of preference parameters and on the law of motion of the exogenous process, Θ . To compute the cost or the gain brought about by the Great Moderation, I compute the cost in consumption terms, λ , when the agent's endowment evolves according to the law of motion estimated for the first subsample, Θ^1 , rather than the law of motion estimated for the second subsample, Θ^2 . As shown in the first row of Table 2.10, the Great Moderation causes a very modest gain in the agent's welfare, equal to about 0.6 percent in consumption-equivalent terms. This result might be surprising, since we observed a large reduction in the high-frequency volatility of macroeconomic variables during the Great Moderation. I attribute the small welfare gain to the role of the medium frequencies. In fact, as pointed out above, the exogenous process for consumption, when governed by Θ^2 , has more volatility at medium frequencies, as well as less volatility at high frequencies, compared to Θ^1 . Since in the proposed habit model the agent cares about lower frequency fluctuations because they drive the behavior of the habit-stock, the welfare loss due to the increase of the medium frequency volatility offsets the welfare gain due to the decline in high frequency volatility. This intuition is supported by the second row of Table 2.10, which shows the welfare gain implied by the counterfactual exogenous process, Θ^C . If the agent faced a consumption process characterized by the same decline of high-frequency variation as in the second

subsample as well as by a reduction of variance at all frequencies, then the welfare gain would be larger, equal to 2 percent in consumption-equivalent terms. This highlights the importance of medium frequency fluctuations in welfare calculations, if we adopt a model of habit formation. Indeed, we can also calculate the change in welfare if the agent faced a change in endowment process, from one governed by Θ^C to one governed by Θ^2 . Here the volatility of the endowment increases, but only at medium frequencies. The third row of Table 2.10 illustrates that this increase in medium frequency volatility has a large negative impact on welfare, equal to -1.45 percent in consumption-equivalent terms. Finally, the literature on the welfare cost of economic fluctuations usually considers the welfare gain that an agent would obtain from eliminating all the fluctuations in the economy. The fourth row of Table 2.10 computes the welfare benefit from completely eliminating the variance of the processes of consumption in the pre-Great Moderation period. It is equal to 3.10 percent in consumption-equivalent terms. This result indicates that the agent's welfare in the model is potentially largely affected by the volatility of the consumption process, since the welfare gain from having a deterministic path for consumption is considerable.

A natural question to ask is, how precise are these estimated welfare gains and losses? To address this question I use a bootstrapping procedure to compute confidence bands. The point estimate of 0.57 percent has a large 95 percent confidence band, whose bounds are -2.39 percent and 3.50 percent. The wide confidence interval is a consequence of the difficulty of obtaining precise estimates of the parameters that determine the importance of the medium frequency component of consumption in the two subsamples, and the fact that the welfare calculations are very sensitive to these parameters.

2.7 Medium Frequency Fluctuations and Asset Prices

I have shown that how we model the law of motion of consumption has a big impact on welfare calculations. In this section, I more thoroughly explore the role played by the medium frequency features of the law of motion of consumption in determining asset prices. The exercise runs as follows. Keeping the preference parameters β , γ , and ϕ as previously calibrated, I derive the expected value for the risk-free rate, the equity return, and the equity risk premium, when the three sets of parameters for the consumption process, Θ^1 , Θ^2 , and Θ^C are considered.

Table 2.11 reports the moments predicted by the model. The model cannot replicate the average returns in the two subsamples when the different processes for consumption are considered as exogenous processes, as Table 2.11 displays. In fact, the 7.70 percent equity premium in the pre-83 period is not included in the 95 percent confidence interval for its model counterpart, although the mean-equity return (8.68 percent in the data) and the mean risk-free rate (0.98 percent in the data) are close to the model predictions.

The performance of the model is more problematic when I consider the process estimated for the Great Moderation period, Θ^2 . In this case the model-predicted equity premium is very low (2.82 percent) compared to the data (8.11 percent). The small risk premium is a result of the large increase in the average risk-free rate and the contemporaneous decline of the equity return. Most importantly, the third row of Table 2.11 shows that the contribution of medium frequency fluctuations to asset prices is rather small, since the average returns implied by the counterfactual process, Θ^C , are very similar to the average returns implied by the process Θ^2 . Since the two processes are distinguishable only by medium frequency volatility, I conclude that the asset pricing variables in this model are mainly driven by the variance of the unpredictable component of consumption, rather than the variance

of the consumption process at medium frequencies.

This result is in contrast to the welfare implications of the model, where the medium frequencies play a big role. Therefore, the habit model does not share the close links between welfare calculations and asset prices that we saw for the simple consumption-based model. In addition, the asset prices moments implied by the model in the two sub-samples should be interpreted as steady-state values resulting from two different processes for consumption. On the other hand, the data moments do not have the same interpretation. In fact, it would not be realistic to think about the Great Moderation as of a sudden shift to a new steady-state. Moreover, since in the model the agent forms habit that depends on the past value of consumption, these calculations do not take into account the transitional dynamics of having a new process for consumption while forming habits driven by the old process of consumptions. The effects of this transition are not take into account in the moments of the asset prices presented in the table and are interesting material for future research.

2.8 Alternative Models

The inability of my model to match the behavior of the financial variables across the two subsamples might raise the question of whether my estimated small gain brought on by the Great Moderation is robust to alternative modeling choices. In this section I analyze the predictions of three alternative models that are successful in solving the equity premium puzzle: the habit model of Campbell and Cochrane (1999), the rare disaster model of Barro (2009) and the long-run risk model of Bansal and Yaron (2004).

2.8.1 Campbell and Cochrane Model

My model shares several features with the habit model introduced by Campbell and Cochrane (1999), which is successfully able to reconcile model predictions for many financial variables with their sample counterparts. However, one crucial difference distinguishes the two models, namely, the relationship between preference parameters and the parameters of the exogenous laws of motion. In conducting welfare calculations we generally want to hold preference parameters fixed, while experimenting with the law of motion of consumption. This is impossible with Campbell and Cochrane's model, and motivates the design of my model of habits.

There are three important features of the Campbell-Cochrane model. First, external habit formation, second, a slow response of habit to consumption, and third, a non-linear relationship between habit and consumption. In particular, the agent's instantaneous utility has the same form as in (2.7). Define the surplus consumption ratio, S_t as

$$S_t = \frac{C_t - X_t}{C_t}.$$

The law of motion of the habit stock is modeled specifying a heteroskedastic AR(1) process for the log surplus consumption ratio, s_t , i.e.

$$s_{t+1} = (1 - \phi) \bar{s} + \phi s_t + \lambda(s_t)(c_{t+1} - c_t - g),$$

where \bar{s} , g , and ϕ are parameters, and $c_{t+1} - c_t - g = v_{t+1}$ is an i.i.d. normal process with mean 0 and standard deviation σ . The function $\lambda(s_t)$ is the *sensitivity function* specified as follows

$$\lambda(s_t) = \begin{cases} \frac{1}{\bar{S}} \sqrt{1 - 2(s_t - \bar{s})} - 1 & \text{if } s_t \leq s_{\max} \\ 0 & \text{if } s_t > s_{\max}, \end{cases}$$

with $s_{\max} = \bar{s} + \frac{1}{2}(1 - \bar{S}^2)$. The parameter \bar{S} is the steady state surplus consumption ratio and is defined as follows, imposing some useful conditions on the sensitivity

function¹³:

$$\bar{S} = \sigma \sqrt{\frac{\gamma}{1 - \phi}}.$$

The sensitivity function measures the response of the surplus consumption ratio to innovations in consumption growth. Notice that since \bar{S} is proportional to σ , a less volatile consumption growth process, such as that experienced in the Great Moderation, implies a lower steady-state surplus consumption ratio. Moreover, the functional form of the sensitivity function indicates that a less volatile consumption growth process is associated with higher values of the sensitivity function, holding γ and ϕ constant. As a result, fixing the percentage deviation of the log-surplus consumption ratio from its steady state, $s_t - s$, the distribution of s_t associated with a less volatile consumption growth process shifts to the left and does not change its variance, since the lower volatility of the v_t process is amplified by a larger sensitivity function.

Although this mechanism helps to reconcile the model predictions with several otherwise puzzling asset pricing data moments, namely, the average risk-free return and the average equity premium as well as their volatilities, and the Sharpe ratio of equity returns, it also creates some counter-intuitive welfare implications. In fact, a reduction of the volatility of consumption growth leads to a decline of the surplus consumption ratio, which is the variable from which the agent gains utility. Thus, a less volatile growth rate of consumption has a negative effect on utility.

In the previous section I showed that the volatility of consumption growth has declined from 0.62 percent in the pre-1983 sample to 0.34 percent in the post-1983 sample. When these estimates are applied to the Campbell and Cochrane model, the welfare loss implied by the less volatile consumption process is 5 percent in consumption-equivalent terms. This result seems paradoxical since we usually expect

¹³ See Campbell and Cochrane (1999), p.213.

that a risk-averse agent would prefer a smoother consumption process. However, the left-ward shift on the distribution of the surplus consumption ratio is equivalent to a change in the preference parameters of the agent, or, in other words, to a re-scaling of the variable from which the agent gains utility. Therefore the decline in utility implied by this calculation is mainly due to the cardinal value of the utility function, which has no meaning in terms of welfare.

2.8.2 Rare Disaster Model

In the rare disaster model the equity premium is generated by two components; the first one is proportional to the variance of consumption growth, and the second one depends on the probability and magnitude of the rare disaster. In particular, the expression for the equity premium is

$$R^{RP} = \gamma\sigma^2 + pE(b[(1-b)^{-\gamma} - 1]),$$

where γ is the coefficient of relative risk aversion, σ^2 is the variance of consumption growth, p is the probability that a disaster occurs, and b is the magnitude of the disaster. Using Barro's calibration, the risk-premium implied by the model is 5.9 percent. However, only 0.16 percent is due to the first component. Therefore, the impact of the volatility of consumption growth is negligible if compared to the contribution of the rare disaster. This observation suggests that a reduction in the volatility of consumption growth will have a small impact on welfare calculations based on this model. In fact, assuming that the probability and the magnitude of a disaster did not change in the Great Moderation, a 50 percent decline of the standard deviation of consumption growth, as experienced in the post-1983 sample, implies a welfare gain of 0.84 percent. Since the agent in this model is mainly concerned about disaster risk, and this is what is being priced in the equity markets, a change in day-to-day "normal" volatility has only a limited effect on welfare.

2.8.3 Long-Run Risk Model

The Long-Run Risk model, introduced by Bansal and Yaron (2004), is an alternative model which is able to successfully predict several stylized facts about asset prices. The two main features of the model are the adoption of recursive preferences, and the presence of a small but very persistent component that drives the long-run behavior of consumption growth. Tallarini (2000) analyzes the welfare cost of fluctuations using recursive preferences and concludes, using an Epstein-Zin utility function, that the welfare cost of fluctuations is much higher than in Lucas' calculation.¹⁴

Croce (2006) separates the welfare effects of the short run component, which depends on the volatility of consumption growth, and the welfare effects of the long-run component. Assuming an intertemporal elasticity of substitution equal to unity to be consistent with Tallarini's calibration, Croce (2006) finds that the largest fraction of the welfare cost of fluctuations depends on the long-run component, since it increases the amount of risk and it increases the effective discount factor. In addition, the long-run risk component is the predominant source of the expected risk premium. In fact, the risk premium in this model is given by the sum of two components:

$$R^{RP} = c\sigma^2 + c_x\sigma_x^2,$$

where c and c_x are constants, σ^2 is the variance of the error term of consumption growth, and σ_x^2 is the variance of the long-run component of consumption growth. Using Croce's calibration to match a 6 percent annual risk premium, the first component accounts for 0.35 percentage points of the premium, while the second component accounts for the remaining 5.65 percentage points. Moreover, Croce (2006) finds that the total welfare cost of fluctuations is large, but its largest fraction (80 percent) depends on the long-run risk component.

¹⁴ See Epstein and Zin (1989, 1991), Kreps and Porteus (1978), and Weil (1990).

What would be the implied gain from the Great Moderation in this model? I assume that the variance of the error term of consumption growth decreases by 50 percent, as in the data, whereas the volatility of the long-run component does not change. Although changes in the long-run risk component are not directly observable in the data, my assumption is supported by two facts: first, the medium frequency component of consumption did not display a decline in its volatility, and second, the equity premium stayed approximately unchanged across the pre- and post-Great Moderation periods. Therefore, a Great Moderation characterized by a decline in the volatility of the growth rate of consumption and by an unchanged long-run risk component implies an unchanged risk-premium and a small effect on welfare, since it is the long-run risk component that mainly affects the two variables.

2.9 Conclusions

In this paper I estimated the welfare improvement brought about by the Great Moderation, the reduction in the high frequency volatility of macroeconomic variables after the early 1980s. Using simple consumption-based asset pricing models, I showed that the welfare estimates and the moments of asset prices are very sensitive to the time-series properties of the consumption processes that are fed into these calculations.

The contribution of this paper is to take very seriously the need for welfare calculations to be based on plausibly calibrated laws of motion of consumption, and on models which have reasonable predictions for asset prices. I document that the reduction in volatility in the Great Moderation period is a high frequency phenomenon, since medium frequency volatility did not change significantly after 1983. Therefore, I develop an asset pricing model with habit in which the law of motion of consumption captures the different behavior of consumption at high and medium frequencies. With a set of calibrated preference parameters, the proposed model delivers sensible

asset price behavior over the full sample. The implied welfare gain brought about by the Great Moderation is modest, and equal to 0.6 percent in consumption-equivalent terms. This result is not surprising, given that the welfare gain generated by the reduction in high frequency volatility is offset by the loss caused by the increasing persistence of the consumption process.

2.10 Tables and Figures

Table 2.1: Variability of Components of Consumption

		Standard Deviations (Percent)	
		Sample 1 1947Q1-1982Q4	Sample 2 1983Q1-2007Q4
First Differenced	$\Delta \log (C_t)$	0.62 [0.06]	0.34 [0.04]
High-Frequencies	C_t^{HF} (2-32Q)	0.90 [0.09]	0.51 [0.04]
Medium-Frequencies	C_t^{MF} (32-80Q)	0.91 [0.09]	1.14 [0.17]
High-to-Medium Frequencies	C_t^{HM} (2-80Q)	1.32 [0.06]	1.24 [0.15]

Note: Consumption is defined in real per-capita terms, measured as non durable goods plus services from NIPA. The cyclical components are isolated using a band-pass filter. Heteroskedasticity consistent standard errors computed with the Newey-West (1987) procedure in brackets.

Table 2.2: Variability of Components of Output

		Standard Deviations (Percent)	
		Sample 1 1947Q1-1982Q4	Sample 2 1983Q1-2007Q4
First Differenced	$\Delta \log (Y_t)$	1.18 [0.08]	0.56 [0.05]
High-Frequencies	Y_t^{HF} (2-32Q)	1.89 [0.18]	0.88 [0.07]
Medium-Frequencies	Y_t^{MF} (32-80Q)	2.05 [0.15]	1.73 [0.28]
High-to-Medium Frequencies	Y_t^{HM} (2-80Q)	2.84 [0.17]	1.92 [0.23]

Note: Output is defined in real per-capita terms, measured as Gross Domestic Product from NIPA. The cyclical components are isolated using a band-pass filter. Heteroskedasticity consistent standard errors computed with the Newey-West (1987) procedure in brackets.

Table 2.3: Variability of Components of Investment

		Standard Deviations (Percent)	
		Sample 1 1947Q1-1982Q4	Sample 2 1983Q1-2007Q4
First Differenced	$\Delta \log (I_t)$	3.02 [0.27]	1.79 [0.14]
High-Frequencies	I_t^{HF} (2-32Q)	5.53 [0.67]	3.55 [0.31]
Medium-Frequencies	I_t^{MF} (32-80Q)	3.04 [0.36]	6.83 [1.07]
High-to-Medium Frequencies	I_t^{HM} (2-80Q)	6.65 [0.83]	7.67 [0.93]

Note: Investment is defined in real per-capita terms, measured as private investment from NIPA. The cyclical components are isolated using a band-pass filter. Heteroskedasticity consistent standard errors computed with the Newey-West (1987) procedure in brackets.

Table 2.4: Moments of Asset Prices

Annualized Mean Return and Standard Deviations (Percent)	Whole Sample		Sample 1		Sample 2		Chow Statistic
	1947Q1-2007Q4	1947Q1-1982Q4	1947Q1-1982Q4	1983Q1-2007Q4			
Equity Return: Mean	8.68 [1.76]	7.74 [2.43]	7.74 [2.43]	10.03 [2.64]	0.30		
Risk Free Asset Return: Mean	0.98 [0.45]	0.33 [0.49]	0.33 [0.49]	1.92 [0.60]	24.41		
Risk Premium: Mean	7.70 [1.68]	7.41 [2.48]	7.41 [2.48]	8.11 [2.33]	0.03		
Risk Premium: Standard Deviation	15.91 [1.17]	15.85 [1.53]	15.85 [1.53]	15.99 [1.92]			

Note: The Chow Statistic in the fifth column tests the null hypothesis that the mean returns are equal in Sample 1 and Sample 2. The critical value of the Chow test at 5% is 3.84. Heteroskedasticity consistent standard errors computed with the Newey-West procedure in brackets.

Table 2.5: Mean Level of Price-Dividend and Price-Consumption Ratios

	Sample 1 1947Q1-1982Q4	Sample 2 1983Q1-2007Q4	Chow Stat. S1 vs S2	Bubble-Free Scenario	Chow Statistic S1 vs Bubble-Free
Price-Div. ratio	1.20 [0.10]	2.21 [0.36]	155.7	1.90 [0.23]	158
Price-Cons. ratio	1.76 [0.22]	2.24 [0.35]	23.6	1.89 [0.21]	2.75

Note: The Chow Statistic in the forth column tests the null hypothesis that the mean levels of the two ratios are equal in Sample 1 and Sample 2. The Chow Statistic in the sixth column tests the null hypothesis that the mean levels of the two ratios are equal in Sample 1 and in the "Bubble Free" scenario. The critical value of the Chow test at 5% is 3.84. Heteroskedasticity consistent standard errors computed with the Newey-West procedure in brackets.

Table 2.6: Asset Returns and Welfare Implications of a Time-Separable Model with Alternative Specifications of the Consumption Process

	Linear trend + iid	iid Cons. Growth	AR(1) Cons. Growth
Discount factor (β)	0.95	0.95	0.95
Risk aversion (γ)	2	2	2
	Annualized Mean Return (Percent)		
Equity Return	11.8	11.8	9.1
Risk Free Asset Return	11.7	11.8	8.8
Risk Premium	0.1	0.0	0.3
	Welfare change in terms of cons. compensation (Percent)		
Welfare gain from eliminating fluctuations	0.02	0.1	0.65
			7.4

Note: The model assumes a CRRA utility function. The “linear trend +iid” consumption process and the utility parameters in the second column are as in Lucas (1987). The “iid Consumption Growth” process in the third column is calibrated to match the post war data. The $AR(1)$ consumption growth process and utility parameters in the fourth column are as in Mehra and Prescott (1985). The $AR(1)$ consumption growth process in the fifth column has the same specification as in Mehra and Prescott (1985). However, the utility parameters are calibrated to match a 6 percent risk premium and a 1.4 percent risk free asset return.

Table 2.7: Model and Data Variability of Components of Consumption

Law of motion of consumption	Standard Deviations (Percent)					
	High-Frequencies		Medium-Frequencies		High-to-Medium Frequencies	
	Model	Data	Model	Data	Model	Data
AR(4) estimated in Sample 1 (Θ^1)	0.93	0.91	0.74	0.91	1.19	1.32
AR(4) estimated in Sample 2 (Θ^2)	0.57	0.52	0.94	1.14	1.10	1.24
Counterfactual AR(4) (Θ^C)	0.57		0.45		0.73	

*Note:*The model standard deviations are computed as the average standard deviations of the one thousand simulated series. The length of the each simulated series is two thousand quarters and its cyclical component is extracted using the appropriate band-pass filter.

Table 2.8: Properties of the Laws of Motion of Consumption

Law of motion of consumption	Largest Root	Standard deviation of innovations
AR(4) estimated in Sample 1 (Θ^1)	0.76	0.0055
AR(4) estimated in Sample 2 (Θ^2)	0.90	0.0029
Counterfactual AR(4) (Θ^C)	0.76	0.0034

Table 2.9: Model and Data Returns

	Mean Annualized Returns (Percent)		
	Model	Data	
	Whole Sample 1947Q1-2007Q4	Sample 1 1947Q1-1982Q4	Sample 2 1983Q1-2007Q4
Equity Return	8.68 [6.75 ; 11.38]	7.74	10.03
Risk Free Asset Return	0.98 [-1.17 ; 3.00]	0.33	1.92
Risk Premium	7.70 [5.76 ; 10.94]	7.41	8.11

Note: The 95 percent confidence bands of the model mean returns are computed with Monte Carlo simulations (10000 repetitions).

Table 2.10: Welfare Change

Consumption Compensation for the different laws of motion (Percent)		Welfare Change
From Sample 1 to Sample 2	$U(\Theta^1) \implies U(\Theta^2)$	0.57
From Sample 1 to Counterfactual	$U(\Theta^1) \implies U(\Theta^C)$	2.00
From Counterfactual to Sample 2	$U(\Theta^C) \implies U(\Theta^2)$	-1.45
From Sample 1 to Deterministic	$U(\Theta^1) \implies U(0)$	3.10

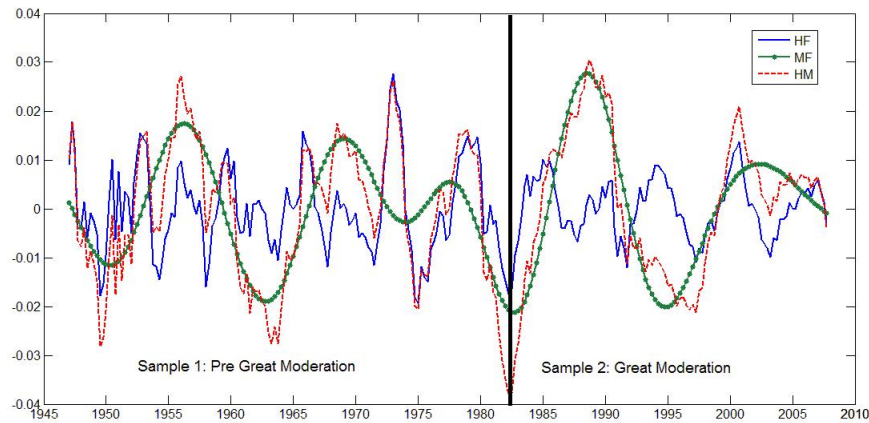
Note: The welfare change is computed with Monte Carlo simulation (100000 repetitions).

Table 2.11: Model Asset Pricing Returns

Law of motion of consumption	Annualized Returns (Percent)					
	Equity Return		Risk-Free Return		Risk-Premium	
	Model	Data	Model	Data	Model	Data
AR4 estimated in Sample 1 (Θ^1)	9.88 [8.08 ; 12.52]	7.74	-0.37 [-2.38 ; 1.68]	0.33	10.25 [7.90 ; 13.77]	7.41
AR4 estimated in Sample 2 (Θ^2)	6.20 [4.77 ; 8.13]	10.03	3.38 [1.84 ; 5.05]	1.92	2.82 [2.13 ; 4.00]	8.11

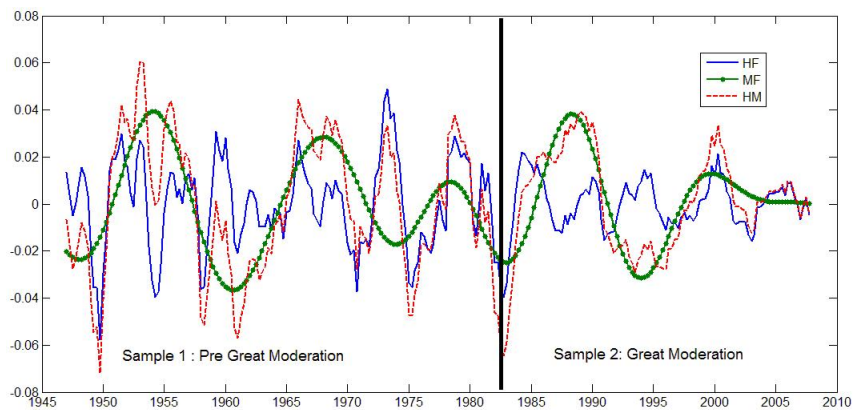
Note: The 95 percent confidence bands of the model mean returns are computed with a Bootstrap procedure (1000 repetitions).

FIGURE 2.1: Cyclical Components of Consumption



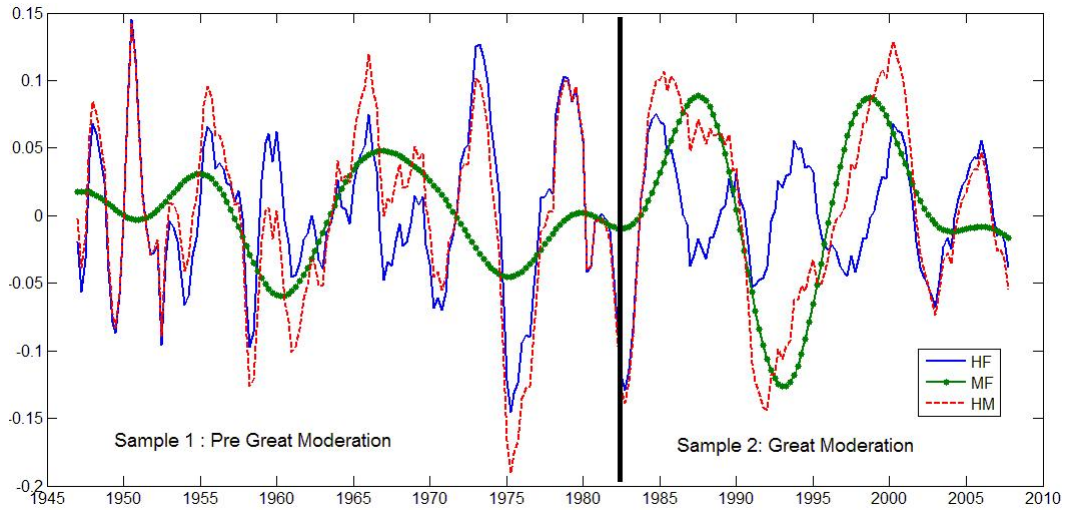
Note: Consumption is defined in real per-capita terms, measured as non durable goods plus services from NIPA. The sample includes quarterly observation from 1947:1 to 2007:4. The cyclical components, which are the High-Frequencies (HF, solid line), Medium-Frequencies (MF, dotted line), and High-to-Medium Frequencies (HM, dashed line) are isolated using a band-pass filter.

FIGURE 2.2: Cyclical Components of Output



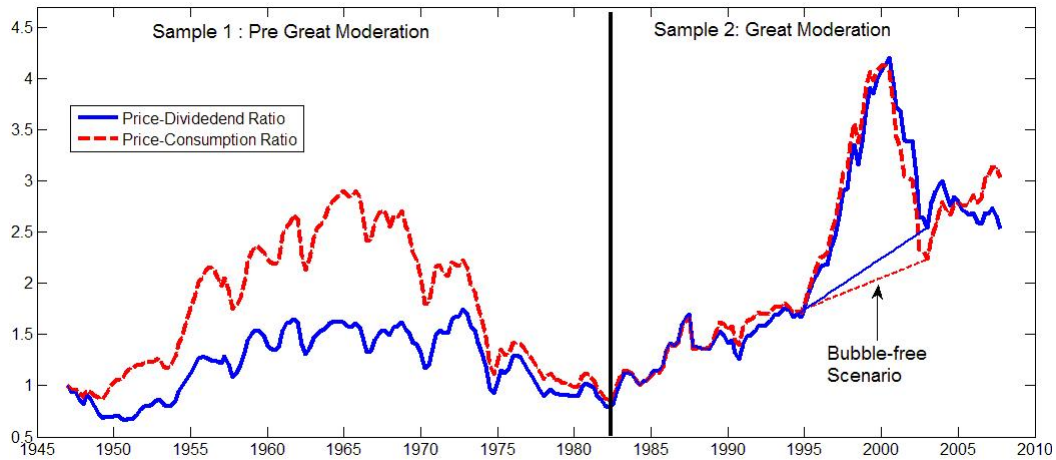
Note: Output is defined in real per-capita terms, measured as Gross Domestic Product from NIPA. The sample includes quarterly observation from 1947:1 to 2007:4. The cyclical components, which are the High-Frequencies (HF, solid line), Medium-Frequencies (MF, dotted line), and High-to-Medium Frequencies (HM, dashed line) are isolated using a band-pass filter.

FIGURE 2.3: Cyclical Components of Investment



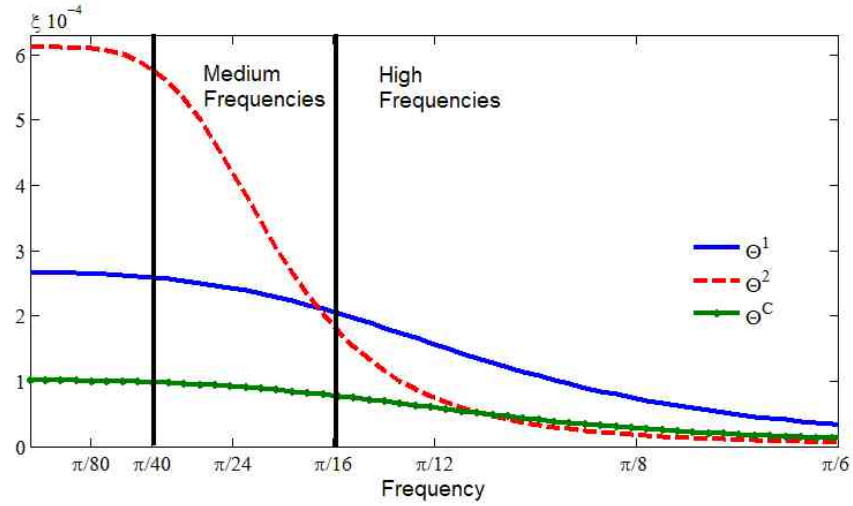
Note: Investment is defined in real per-capita terms, measured as private investment from NIPA. The sample includes quarterly observation from 1947:1 to 2007:4. The cyclical components, which are the High-Frequencies (HF, solid line), Medium-Frequencies (MF, dotted line), and High-to-Medium Frequencies (HM, dashed line) are isolated using a band-pass filter.

FIGURE 2.4: Price-Dividend and Price-Consumption Ratios



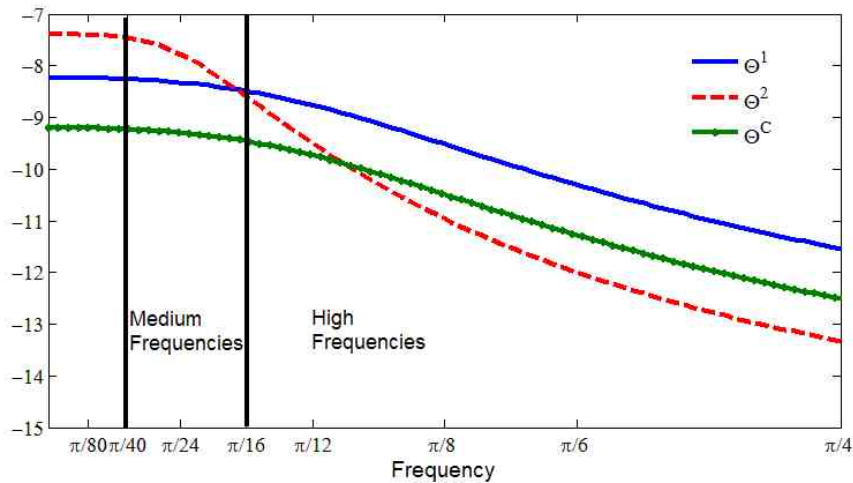
Note: The Price-Dividend ratio (solid blue line) and Price-Consumption ratio (dashed red line) are normalized to one in the first observation of the sample 1947:1-2007:4. The straight lines during the period 1995-2003 represent the bubble-free scenario for the evolution of the two variables.

FIGURE 2.5: Spectra of the AR4 Processes for Consumption



Note: The figure shows the spectral density of the three $AR(4)$ processes of consumptions within the frequencies 0 and $\pi/6$. The Medium Frequencies are defined in the interval $[\frac{\pi}{16}, \frac{\pi}{40}]$, the High-Frequencies are defined in the interval $[\pi, \frac{\pi}{16}]$.

FIGURE 2.6: Log-Spectra of the AR4 Processes for Consumption



Note: The figure shows the log-spectral density of the three $AR(4)$ processes of consumptions within the frequencies 0 and $\pi/4$. The Medium Frequencies are defined in the interval $[\frac{\pi}{16}, \frac{\pi}{40}]$, the High-Frequencies are defined in the interval $[\pi, \frac{\pi}{16}]$.

Medium-Run Effects of Monetary Policy During the Great Moderation

3.1 Introduction

The large reduction in macroeconomic volatility that occurred after the early 1980s has attracted an enormous amount of consideration in the last decade. Stock and Watson (2003) introduced the term “Great Moderation” to indicate this period of significant stabilization in economic fluctuations. Kim and Nelson (1999), McConnell and Perez-Quiros (2000), Blanchard and Simon (2001), Stock and Watson (2003) and many others, contributed formal tests of the presence of such moderation. Moreover, several authors investigated what are the sources of the reduction in macroeconomic volatility during the last three decades.¹ Whereas the majority of macroeconomists (see e.g. Stock and Watson (2002, 2003), Ahmed, Levin and Wilson (2004), Primiceri (2005), Galí and Gambetti (2009), Liu, Waggoner and Zha (2009)) attribute the decline in macroeconomic volatility to a reduction in the variance of exogenous shocks, others have focused on changes in the policy conducted by the monetary authority.

¹ Giannone, Lenza and Reichlin (2008) provide a detailed summary of the literature about the sources of the Great Moderation.

In fact, Clarida, Galí and Gertler (2000), Cogley and Sargent (2001,2005), Lubik and Schorfheide (2004), and Boivin and Giannoni (2006) have argued that monetary policy has become more aggressive since the early 1980s and that this change of attitude could have induced the observed changes in macroeconomic volatility.

The vast majority of the studies on the Great Moderation isolate the cyclical component of macroeconomic variables using high frequency filters, such as the first difference filter or the Hodrick-Prescott (1997) filter. These filters exclude a large portion of the total volatility of the variables from the analysis of cyclical behavior. More recently, Pancrazi (2009) documents that the Great Moderation is mainly a high-frequency phenomenon. By using a broader set of filters, Pancrazi (2009) shows that whereas the high-frequency variance of macroeconomic variables declined after the early 1980s, the medium-frequency variance did not. This implies that during the Great Moderation the spectral shape of real variables substantially changed, since the decline in volatility was not uniform at all frequencies. This result appears to be in contrast with Ahmed, Levin and Wilson (2004), who conclude that the spectral density of output growth before and during the Great Moderation period differs only by a proportional factor, and Stock and Watson (2002) who conclude that the coefficient of the univariate autoregressive model for GDP growth is time invariant. However, as pointed out, the first difference filter has high power mainly at high frequencies and therefore might be missing relevant information at medium frequencies.

The first contribution of this paper is, therefore, to explore the main sources of both observed properties of the Great Moderation: the reduction of high-frequency volatility of the real macroeconomic variables, and its observed spectral redistribution from high to medium frequencies. Specifically, we investigate whether these two facts are caused by the altered monetary policy or by different statistical properties of the shocks in the post-1983 period. Using a medium-scale DSGE model, we find that

the change in the variance and persistence of the exogenous shocks are the main contributors to the change in the level of the spectral density (i.e. on the variance) of consumption, output, and investment, as well as to the redistribution of their spectral density (i.e. on the shape of the spectrum). In fact, if the exogenous shocks had been the same as in the pre-1983 period, a more aggressive monetary policy would have had no effect on the volatility of the real variables. However, when we consider the statistical changes of the variance and persistence of the shocks as estimated in the post-1983 period, the different monetary policy contributes to approximately 40 percent of the total decline in the variance of the three macroeconomic variables. Moreover, whereas the changed monetary policy affects mainly the high-frequency volatility, it only slightly influences the medium-frequency volatility.

In order to provide some intuition about how a change in monetary policy could effect the spectral density and spectral distribution of the real variables, we first consider a simple New Keynesian model, as in Galí (2008). The model is characterized by two rigidities: imperfect competition in the goods market and price stickiness. The dynamics of the model are driven by two shocks: a technology shock and a monetary policy shock. Using this simple model, we show that a change of the monetary policy rule toward a more aggressive inflation targeting has a large effect on the shape of the spectrum of output, causing a decline in its level and a redistribution of its density from high to medium frequencies. In particular, we show that a change of the monetary policy affects the weights of the two shocks in the total variance of output; a larger response of monetary policy to inflation increases the relative weight of the technology shock with respect to the monetary shock. Since, in our calibrated, model the technology shock is more persistent than the monetary shock, its increased weight causes the redistribution of the volatility of output toward lower frequencies.

Even though this model is very useful for providing intuition, it is rather unre-

alistic, since it abstracts from the investment sector, it has few rigidities, and it is driven only by two shocks. For quantitative analysis of the effects of monetary policy and exogenous shocks, a rich medium-scale model is more appropriate. Therefore, as a theoretical framework, we use a fairly standard, DSGE model in the spirit of Christiano, Eichenbaum and Evans (2005) (CEE hereafter) and Smets and Wouters (2003). The model is augmented by a number of real and nominal rigidities. The nominal rigidities include price and wage stickiness, and indexation to past inflation. The real rigidities stem from habit formation in consumption, monopolistic competition in factor and product markets, and investment adjustment costs. The model is driven by four shocks: a neutral technology shock, an investment-specific shock, a fiscal policy shock, and a monetary policy shock.²

We consider two subsamples. The first subsample covers the period 1947:I-1978:IV, whereas the second subsample covers the period 1983:I-2007:IV. Following Ahmed, Levin and Wilson (2004), we eliminate the four years from 1978 to 1982 from the sample, since it is generally believed that the monetary policy rule followed in that period was rather different from the monetary policy rules used in all other sub-periods. We estimate the parameters governing the four exogenous processes separately in the two subsamples, using their data counterparts. Since the goal of this paper is to assess effects of different shocks on the change of variance of the real variables during the Great Moderation, we assume that the structural parameters of the model are constant throughout the whole sample. The structural parameters of the model are calibrated, using corresponding data statistics or conventional wisdom.

Our model, when driven by the estimated exogenous processes, generates realistic high-frequency dynamics of the macroeconomic variables, in both subsamples. The performance of the model at medium frequencies is less satisfying, as it largely under-

² For example, Giannone, Lenza and Reichlin (2008) show that a simple stylized model with few variables is subject to misspecification, which leads to an overestimate of the contribution of exogenous shocks to the overall behavior of macroeconomic variables.

estimates the standard deviations of real variables. Nevertheless, the model correctly predicts a large redistribution of the variance from high to lower frequencies during the Great Moderation, as observed in the data. Performing a counterfactual exercise in the spirit of Stock and Watson (2002, 2003), Ahmed, Levin and Wilson (2004), Primiceri (2005), Boivin and Giannoni (2006), we obtain several interesting results. First, if the statistical properties of the exogenous shocks are held fixed at their pre-1983 values, the change in monetary policy does not have any effect on the variance of the real variables. Second, when the persistence and the variance of exogenous shocks are as estimated in the post-1983 period, the change in monetary policy has a large effect on the reduction of high-frequency volatilities. In fact, approximately 40 percent of the overall decline of the high-frequency volatilities of consumption, output, and investment is due to the altered monetary policy rule. The rest of the high-frequency volatilities' decline is due to the changed parameters of the exogenous processes. Third, the main cause of the redistribution of the spectral density of the macroeconomic variables from high to medium frequencies is the increased persistence of the total factor productivity shock and investment-specific technology shock. However, the changed monetary policy partially contributes to the redistribution of the spectral density, since its effect on the volatilities is much smaller at medium frequencies than at high frequencies.

The rest of the paper is organized as follows. In Section 3.2 we provide an intuition about the role of a monetary policy change on the spectral density of the variables using a simple New Keynesian model. In Section 3.3, we describe a medium-scale DSGE model. In Section 3.4 we present the estimation and calibration procedures. In Section 3.5 we describe the main findings of the paper, and Section 3.6 concludes with several final remarks.

3.2 Monetary Policy and Spectral Density

In order to provide some intuition about the effects of the change in monetary policy on the level and on the redistribution of the spectral density of real variables, we first consider a basic New Keynesian Model, as in Galí (2008). This model is characterized by two rigidities. First, the perfect competition assumption is abandoned by assuming that each firm produces a differentiated good and sets its price. Second, firms set their prices a la Calvo (1983), i.e. in any given period, only a fraction of randomly picked firms is allowed to reset their prices.

The non-policy block of the model is composed of the New Keynesian Phillips Curve

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \tilde{y}_t,$$

and the dynamic IS equation, given by

$$\tilde{y}_t = -\frac{1}{\sigma} (i_t - E_t \pi_{t+1} - r_t^n) + E_t (\tilde{y}_{t+1}).$$

Here, E_t denotes expectation conditional on the information at time t , π_t denotes the inflation rate at time t , r_t^n is the natural real interest rate, \tilde{y}_t is the output gap defined as the deviation of output from its flexible price counterpart, β is the discount factor, $\kappa = \lambda \left(\sigma + \frac{\varphi + \alpha}{1 - \alpha} \right)$ with $\lambda = \frac{(1 - \theta)(1 - \beta\theta)(1 - \alpha)}{\theta(1 - \alpha + \alpha\varepsilon)}$, σ is the inverse of intertemporal elasticity of substitution, $1 - \alpha$ is the labor share in the production function, φ is the inverse of Frisch elasticity of labor supply, θ is the price stickiness parameter, and ε is the elasticity of substitution among the differentiated goods. The dynamics of the model are governed by two exogenous processes. First, the level of technology, which we denote as a_t , follows a first order autoregressive ($AR(1)$) process:

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a, \text{ where } \varepsilon_t^a \sim N(0, 1).$$

Second, the monetary policy shock, denoted as ν_t , follows a similar first order au-

autoregressive process:

$$v_t = \rho_v v_{t-1} + \varepsilon_t^v, \text{ where } \varepsilon_t^v \sim N(0, 1).$$

The monetary policy shock is considered to be an exogenous component of the nominal interest rate rule:

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \tilde{y}_t + v_t,$$

where i_t is the nominal interest rate at time t , and ρ is the household's discount rate, with $\rho = -\log(\beta)$. Up to a first order approximation, total output can be written as the following function of the two exogenous processes,

$$y_t = \Lambda_v(\phi_\pi, \phi_y, \rho_v, \Theta) v_t + \Lambda_a(\phi_\pi, \phi_y, \rho_a, \Theta) a_t, \quad (3.1)$$

where Λ_a and Λ_v are functions of the Taylor rule parameters (ϕ_π, ϕ_y) , the persistence parameters of the exogenous processes $(\rho_a$ or $\rho_v)$, and all the other structural parameters of the model gathered in the Θ . In particular, Galí (2008) shows that

$$\Lambda_v(\phi_\pi, \phi_y, \rho_v, \Theta) = -\frac{(1 - \beta\rho_v)}{(1 - \beta\rho_v)(\sigma(1 - \rho_v) + \phi_y) + \kappa(\phi_\pi - \rho_v)} \quad (3.2)$$

$$\Lambda_a(\phi_\pi, \phi_y, \rho_a, \Theta) = \psi \left(1 - \frac{\sigma(1 - \rho_a)(1 - \beta\rho_a)}{(1 - \beta\rho_a)(\sigma(1 - \rho_a) + \phi_y) + \kappa(\phi_\pi - \rho_a)} \right), \quad (3.3)$$

where $\psi = \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}$ and κ is defined as above. These expressions imply that the relationship between the persistence of the exogenous shocks and the level of output is non-linear in the monetary policy parameters. Assuming that ε_t^a and ε_t^v are independent, it is trivial to obtain the variance of output:

$$Var(y_t) = [\Lambda_v(\phi_\pi, \phi_y, \rho_v, \Theta)]^2 \frac{1}{1 - \rho_v^2} + [\Lambda_a(\phi_\pi, \phi_y, \rho_a, \Theta)]^2 \frac{1}{1 - \rho_a^2}, \quad (3.4)$$

and its spectrum:

$$S^y(\omega) = [\Lambda_v(\phi_\pi, \phi_y, \rho_v, \Theta)]^2 S^v(\omega) + [\Lambda_a(\phi_\pi, \phi_y, \rho_a, \Theta)]^2 S^a(\omega), \quad (3.5)$$

where $S^y(\omega)$ denotes the spectrum of output at frequency ω . Following Galí (2008), we can derive similar expressions defining the relationship between inflation and the two exogenous processes, as well as the variance and spectrum of inflation.³ Now assume that the parameters of the Taylor rule change from (ϕ_π, ϕ_y) to (ϕ'_π, ϕ'_y) , as a result of a change in monetary policy. Moreover, assume that all the parameters in Θ , and the persistence parameters of the exogenous process, ρ_a and ρ_v , remain unchanged. In this case, since Λ_a and Λ_v depend on the parameters of the Taylor rule, the unconditional variance of output changes. In addition, provided that $\rho_a \neq \rho_v$, the relative contributions of the two shocks to the variance of output also change, thus implying a different shape of the output spectrum.

To illustrate the magnitude of the effects of a change in monetary policy to the spectral density and spectral distribution of the economic variables, we perform a numerical exercise. First, we calibrate preference and technology parameters following Galí's baseline calibration: $\beta = 0.99$, $\sigma = 1$, $\alpha = 1/3$, $\varepsilon = 6$, and $\theta = 2/3$. The values of the autoregressive coefficients of the two shocks and the coefficients of the Taylor rule are the following: $\rho_a = 0.8$, $\rho_v = 0.5$, $\phi_\pi = 1.5$, and $\phi_y = 0.125$. Given this parameterization, using (3.4) and (3.5), we can compute several statistics of interest. In particular, we consider the standard deviation of output, the standard deviations of inflation, and the spectral distribution of the two variables. To obtain information about the shape of the spectrum of the two variables, following Pancrazi (2009) we consider two intervals of frequencies: the high frequencies, defined as the fluctuations with periodicity between 2 and 32 quarters, and the medium frequencies, defined as the fluctuations with periodicity between 32 and 80 quarters. The standard deviation

³ The expressions are: $\pi_t = \Lambda_v^\pi(\phi_\pi, \phi_y, \rho_v, \Theta)v_t + \Lambda_a^\pi(\phi_\pi, \phi_y, \rho_a, \Theta)a_t$
with $\Lambda_v^\pi(\phi_\pi, \phi_y, \rho_v, \Theta) = -\frac{\kappa}{(1-\beta\rho_v)(\sigma(1-\rho_v)+\phi_y)+\kappa(\phi_\pi-\rho_v)}$
 $\Lambda_a^\pi(\phi_\pi, \phi_y, \rho_a, \Theta) = -\psi\left(\frac{\sigma(1-\rho_a)\kappa}{(1-\beta\rho_a)(\sigma(1-\rho_a)+\phi_y)+\kappa(\phi_\pi-\rho_a)}\right)$

of output and inflation at these intervals of frequencies are reported in Table 3.1.

Now assume that the monetary authority decides to respond to inflation more aggressively, which implies a larger ϕ_π . Therefore, keeping all other parameters of the model constant, we set ϕ_π to be 6. The resulting standard deviations implied by the model are shown in Table 3.2. To illustrate the change from a different angle, in Table 3.3 we compute the percentage change of the variances driven by the new-monetary policy. These tables present some interesting findings. First, a change of the response of monetary authority to inflation has effects both on the stabilization of inflation itself and on the stabilization of output. In our exercise, the effect on inflation is larger: the variance of inflation declines by 87 percent, whereas the variance of output declines by 10 percent. Second, the decline of the volatility of output differs across the different frequencies. In fact, while at high frequencies the decline of the variance of output is 24 percent, the new monetary policy induces a slightly larger variance of output at medium frequencies. In contrast, the stabilization effect on inflation appears to be fairly uniform at all frequencies. This result can be visualized in Figures 3.1 and 3.2, where we plot the normalized spectrum of output and inflation under the two different monetary policies.⁴ As the figures show, the change in monetary policy largely affects the shape of the spectrum of output and inflation.

Why does the shape of the output spectrum change with a change of ϕ_π ? As equations (3.1), (3.2), and (3.3) suggest, a different ϕ_π leads to a change of the relative weight of the two shocks in output. In other words, Λ_v and Λ_a do not change proportionally, since they depend on the autocorrelations of the two different exogenous processes. To explore the effects of the change in ϕ_π we compute the variance decompositions of output and inflation, displayed in Table 3.4. The variance

⁴ The normalized spectrum is a useful tool for exploring the relative contribution of different frequencies to the total variance of a variable. Since output has different total variance in the two scenarios, we rescale the spectra so that those variables have variance equal to unity.

decomposition indicates the fraction of the variance attributable to the monetary shock and to the technology shock. Since in this experiment we assume that the exogenous processes do not change, the variance decomposition is affected only by the change in the Λ functions. In particular, the increase in ϕ_π implies that the dynamics of output are driven to a larger degree by the technology shocks, a_t , than by the monetary shocks, v_t with respect to the model with a lower ϕ_π . Since the technology shock is more persistent than the monetary shock, there is a redistribution of the volatility of output toward lower frequencies.

The purpose of this simple example was to illustrate that different monetary policy could have large effects on the level of the spectral density of the real macroeconomic variables and on its distribution. However, the model considered above is relatively unrealistic, since it abstracts from the investment sector, it features few rigidities, and it is driven only by two shocks. Therefore, in the following sections we consider a richer theoretical framework. This framework allows us to address the question whether a change in the monetary policy after the early 1980s affected the variances of the real variables and their temporal distributions.

3.3 Medium-Scale DSGE Model

We use a fairly standard DSGE model, in the spirit of CEE and Smets and Wouters (2003). The model is driven by four shocks: a neutral technology shock, an investment-specific shock, a fiscal policy shock, and a monetary policy shock. The model is augmented by a number of real and nominal rigidities. The nominal rigidities include price and wage stickiness, and indexation to the past inflation. The real rigidities evolve from the habit formation in consumption, monopolistic competition in factor and product markets, and investment adjustment costs.

3.3.1 Households

The economy is populated by a continuum of households indexed by $j \in [0, 1]$. Household's preferences are defined over consumption, c_{jt} , and labor, l_{jt} . Each household j maximizes lifetime utility that takes the following form:

$$U = E_t \sum_{t=0}^{\infty} \beta^t \left\{ \log(c_{jt} - bc_{jt-1}) - \psi \frac{l_{jt}^{1+\gamma}}{1+\gamma} \right\}, \quad (3.6)$$

where β denotes the subjective discount factor, b is the habit persistence parameter and γ is the inverse of Frisch labor supply elasticity.⁵

Households own physical capital. The capital stock, k_t , is assumed to evolve over time according to the following law of motion

$$k_{t+1} = (1 - \delta) k_t + \mu_t \left(1 - S \left(\frac{x_t}{x_{t-1}} \right) \right) x_t, \quad (3.7)$$

where δ is the depreciation rate of capital stock, x_t represents the gross investment, and μ_t is the investment-specific technology shock that follows an autoregressive process, given by

$$\log(\mu_t) = \rho_\mu \log(\mu_{t-1}) + \sigma_\mu \varepsilon_{\mu,t}, \text{ where } \varepsilon_{\mu,t} \sim N(0, 1).$$

The function $S(\cdot)$ is an investment adjustment cost function, as introduced by CEE. We assume that in the steady state $S = S' = 0$ and $S'' > 0$, which implies no adjustment costs in the vicinity of the steady state. We assume the following function form:

$$S \left(\frac{x_t}{x_{t-1}} \right) = \frac{\kappa}{2} \left(\frac{x_t}{x_{t-1}} - 1 \right)^2.$$

The first-order conditions with respect to consumption, capital, capacity utilization, and investment are fairly standard, whereas the first-order conditions with

⁵ We can omit the subscript j with consumption, because households are assumed to have access to a complete set of Arrow-Debreu securities, and can fully ensure against the idiosyncratic risks.

respect to labor and wages are more complex. We follow the set-up of Erceg, Henderson and Levin (2000) and assume that each household supplies differentiated labor services to the production sector. In order to avoid this heterogeneity spilling over into consumption heterogeneity, they assume that utility is separable in consumption and labor, and that, because of the existence of complete markets, households can fully ensure against the employment risks. In addition, we assume that a representative labor aggregator combines households' labor in the same proportion as firms would choose. This ensures that her demand for j -th household's labor is the same as the sum of the firms' demands for this type of labor.

Specifically, the labor aggregator uses the following production technology:

$$l_t^d = \left(\int_0^1 l_{jt}^{\frac{\eta-1}{\eta}} dj \right)^{\frac{\eta}{\eta-1}}, \quad (3.8)$$

where $\eta \in [0, \infty)$ is the elasticity of substitution among different types of labor, and l_t^d is the aggregate labor demand. She maximizes profits subject to (3.8), taking as given all differentiated labor wages w_{jt} and the aggregate wage index w_t . Her demand for the labor of household j is given by

$$l_{jt} = \left(\frac{w_{jt}}{w_t} \right)^{-\eta} l_t^d \quad \forall j. \quad (3.9)$$

Households set their wages following Calvo setting, i.e. in any given period, a fraction $\theta_w \in [0, 1)$ of randomly picked households is not allowed to optimally set their wages. Instead, they partially index their wages to the past inflation, Π_{t-1} , which is controlled by the indexation parameter $\chi_w \in [0, 1]$. The remaining fraction of households who are allowed to reset their wages, choose the same optimal wage,

i.e. $w_t^* = w_{jt} \forall j$, that maximizes (3.6). The first-order condition to this problem is:

$$\begin{aligned} & \frac{\eta - 1}{\eta} w_t^* E_t \sum_{k=0}^{\infty} (\beta \theta_w)^k \lambda_{t+k} \left(\frac{\prod_{s=1}^k \Pi_{t+s-1}^{\chi_w}}{\Pi_{t+s}} \right)^{1-\eta} \left(\frac{w_t^*}{w_{t+k}} \right)^{-\eta} l_{t+k}^d \\ &= E_t \sum_{k=0}^{\infty} (\beta \theta_w)^k \left(\psi \left(\frac{\prod_{s=1}^k \Pi_{t+s-1}^{\chi_w}}{\Pi_{t+s}} \frac{w_t^*}{w_{t+k}} \right)^{-\eta(1+\gamma)} (l_{t+k}^d)^{1+\gamma} \right), \end{aligned}$$

where λ_t is the Lagrangian multiplier associated with the household's budget constraint.

3.3.2 The Final Good Producer

The final good producer aggregates intermediate goods, y_{it} , into the homogenous final good, y_t^d , using a Dixit and Stiglitz (1997) production function:

$$y_t^d = \left(\int_0^1 y_{it}^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}, \quad (3.10)$$

where ε is the elasticity of substitution among the intermediate goods. The final good producer chooses the bundle of goods that minimizes the cost of producing y_t^d , taking all intermediate goods prices p_{it} , final domestic good price p_t , and the quantity of intermediate goods y_{it} as given. The unit price of the output unit is equal to its unit cost p_t :

$$p_t = \left(\int_0^1 p_{it}^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}}.$$

The input demand function y_{it} for each intermediate good i is then given by:

$$y_{it} = \left(\frac{p_{it}}{p_t} \right)^{-\varepsilon} y_t^d \quad \forall i,$$

where y_t^d is the aggregate demand.

3.3.3 Intermediate Goods Producers

There is a continuum of intermediate goods producers indexed by i on the unit interval. Each differentiated good is produced by a single intermediate firm i that rents capital services k_{it} , and labor services l_{it}^d , using the production function:

$$y_{it} = A_t k_{it}^\alpha (l_{it}^d)^{1-\alpha},$$

where α is the capital share in the production function and A_t represents the neutral technology process, given by the following autoregressive process:

$$\log(A_t) = \rho_A \log(A_{t-1}) + \sigma_A \varepsilon_{A,t}, \text{ where } \varepsilon_{A,t} \sim N(0, 1).$$

Each intermediate goods firm chooses amount of k_{it} and l_{it}^d to rent, taking the input prices r_t and w_t as given. The standard static first-order conditions for cost minimization imply that real marginal cost is the same for all firms. Therefore it does not have a subscript i associated with it. The real marginal cost is given by

$$mc_t = \left(\frac{1}{1-\alpha} \right)^{1-\alpha} \left(\frac{1}{\alpha} \right)^\alpha \frac{w_t^{1-\alpha} r_t^\alpha}{A_t}.$$

We assume that the intermediate goods firms set their prices a la Calvo (1983) and Yun (1996). That is, in each period, a fraction $\theta_p \in [0, 1]$ of firms is not allowed to change their prices, and can only index them by the past inflation, which is controlled by the indexation parameter $\chi_p \in [0, 1]$. The remaining $1 - \theta_p$ firms that are allowed to reset their prices in period t , choose optimal price p_t^* , which is the solution to the following maximization problem:

$$\max_{p_{it}} E_t \sum_{k=0}^{\infty} (\beta \theta_p)^k \frac{\lambda_{t+k}}{\lambda_t} \left\{ \left(\prod_{s=1}^k \Pi_{t+s-1}^{\chi_p} \frac{p_{it}}{p_{t+k}} - mc_{t+k} \right) y_{it+k} \right\}$$

subject to:

$$y_{it+k} = \left(\prod_{s=1}^k \Pi_{t+s-1}^{\chi_p} \frac{p_{it}}{p_{t+k}} \right)^{-\varepsilon} y_{t+k}^d.$$

If we define recursively:

$$g_t^1 = \lambda_t m c_t y_t^d + \beta \theta_p E_t \left(\frac{\Pi_t^{\chi_p}}{\Pi_{t+1}} \right)^{-\varepsilon} g_{t+1}^1$$

$$g_t^2 = \lambda_t \Pi_t^* y_t^d + \beta \theta_p E_t \left(\frac{\Pi_t^{\chi_p}}{\Pi_{t+1}} \right)^{1-\varepsilon} \left(\frac{\Pi_t^*}{\Pi_{t+1}^*} \right) g_{t+1}^2,$$

where $\Pi_t^* = \frac{p_t^*}{p_t}$, the first-order condition to this problem can be written as $\varepsilon g_t^1 = (\varepsilon - 1) g_t^2$.

Finally, considering the price setting, the aggregate price index is:

$$p_t^{1-\varepsilon} = \theta_p (\Pi_{t-1}^{\chi_p})^{1-\varepsilon} p_{t-1}^{1-\varepsilon} + (1 - \theta_p) (p_t^*)^{1-\varepsilon}.$$

3.3.4 The Government Problem

The monetary authority follows the interest rate rule given by:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R} \right)^{\gamma_R} \left(\left(\frac{\Pi_t}{\Pi} \right)^{\gamma_\Pi} \left(\frac{y_t^d}{y^d} \right)^{\gamma_y} \right)^{1-\gamma_R} \exp(m_t), \quad (3.11)$$

where R_t is the nominal gross return on capital in period t , Π represents the target level of inflation which is equal to the inflation in the steady state, R is the steady state nominal gross return on capital, y^d is the steady-state level of output, and m_t represents the shock to monetary policy with the following law of motion:

$$m_t = \sigma_m \varepsilon_{m,t}, \text{ where } \varepsilon_{m,t} \sim N(0, 1).$$

Interest rate smoothing, i.e. the presence of R_{t-1} in the Taylor rule, is justified because we want to match the smooth profile of the interest rate, observed in the U.S. data.

The fiscal authority, or government, runs a balanced budget. Government spending, g_t , is modeled as an exogenous autoregressive process, given by

$$\log\left(\frac{g_t}{g}\right) = \rho_g \log\left(\frac{g_{t-1}}{g}\right) + \sigma_g \varepsilon_{g,t}, \text{ where } \varepsilon_{g,t} \sim N(0, 1).$$

Here g represents the steady-state level of government spending, defined as a constant portion, S_g , of the steady-state level of output.

3.3.5 Aggregation

The aggregate demand is given by

$$y_t^d = y_t + x_t + g_t + \mu_t^{-1} a(u_t) k_t, \quad (3.12)$$

where u_t is the variable capacity utilization and $\mu_t^{-1} a(u_t)$ is the physical cost of use of capital in resource terms. Following Altig, Christiano, Eichenbaum and Lindé (2005) and CEE, we assume that $u_t = 1$ in the steady state and $a(1) = 0$, and that the value of the curvature of a in the steady state, $a'(1)/a''(1) \geq 0$. We assume the functional form that satisfies these properties, given by

$$a(u_t) = \gamma_1 (u_t - 1) + \frac{\gamma_2}{2} (u_t - 1)^2.$$

After some manipulations, the goods market clearing condition is:

$$c_t + x_t + g_t + \mu_t^{-1} a(u_t) k_t = \frac{A_t (k_{t-1})^\alpha (l_t^d)^{1-\alpha}}{v_t^p},$$

where $v_t^p = \int_0^1 \left(\frac{p_{it}}{p_t}\right)^{-\varepsilon} di$ is the price dispersion term that is, considering the Calvo price setting, given by

$$v_t^p = \theta_p \left(\frac{\Pi_{t-1}^x}{\Pi_t}\right)^{-\varepsilon} v_{t-1}^p + (1 - \theta_p) (\Pi_t^*)^{-\varepsilon}.$$

Finally, the labor market clearing condition is obtained by integrating (3.9) over all households j :

$$l_t^d = \frac{1}{v_t^w} l_t,$$

where $v_t^w = \int_0^1 \left(\frac{w_{jt}}{w_t}\right)^{-\eta} dj$ is the wage dispersion term that is, considering the Calvo wage setting, given by

$$v_t^w = \theta_w \left(\frac{w_{t-1}}{w_t} \frac{\Pi_{t-1}^{Xw}}{\Pi_t} \right)^{-\eta} v_{t-1}^w + (1 - \theta_w) (\Pi_t^{w*})^{-\eta},$$

where $\Pi_t^{w*} = \frac{w_t^*}{w_t}$.

3.4 Estimation and Calibration

3.4.1 Estimation

The goal of this paper is to assess whether changes in the monetary policy after the early 1980s contributed to the decline of the variance of the real variables and to its temporal redistribution. To address this question, we first split the sample into two subsamples. The first subsample covers the period 1947:I-1978:IV, whereas the second subsample covers the period 1983:I-2007:IV. We eliminate the four years from 1978 to 1982 from the sample, since it is generally believed that the monetary policy rule being followed in that period was very different from the other sub-periods.⁶ We then estimate the processes for the investment-specific technology, the total factor productivity, the exogenous component of the monetary policy rule, and the exogenous process of government spending. We use their observable counterparts in the estimation process.

First, let us consider the monetary policy rule as in (3.11). We obtain ordinary least squares estimates of the Taylor rule parameters in the two subsamples and estimate the monetary policy shock, m_t , as the residual from this regression. This allows us to estimate the variance of the monetary policy shock. After taking logs,

⁶ Ahmed, Levin and Wilson (2006) use a similar approach.

equation (3.11) becomes:

$$\begin{aligned} \log\left(\frac{R_t}{R}\right) &= \gamma_R \log\left(\frac{R_{t-1}}{R}\right) + \\ &\quad (1 - \gamma_R) \gamma_\pi \log\left(\frac{\Pi_t}{\Pi}\right) + (1 - \gamma_R) \gamma_y \log\left(\frac{y_t^d}{y}\right) + m_t, \\ m_t &= \sigma_m \varepsilon_t^m, \end{aligned} \tag{3.13}$$

where Π is the average inflation in each subsample, and $R = \Pi/\beta$ is the steady state interest rate. Inflation, Π_t , is measured as the percentage change in the consumption deflator from NIPA. The real interest rate, R_t , is measured as three-month T-bills rate obtained from the International Financial Statistics, and y_t^d/y is the output gap, defined as the cyclical component of the real per capita gross domestic product. To take into account the role of the medium frequencies, we use a bandpass filter as implemented by Christiano and Fitzgerald (2003) to isolate the fluctuation between 2 and 80 quarters.

To obtain the parameters of the investment-specific technology process, we use the relative price of investment with respect to consumption as the observable. In fact, equation (3.7) and equation (3.12) imply that the relative price of the investment good with respect to the consumption good is $1/\mu_t$. Therefore, the level of the investment-specific technology, μ_t , can be estimated as the inverse of this relative price. National Income and Product Accounts (NIPA) provide data for both the investment deflator and the consumption deflator. The consumption deflator is computed as the real consumption price index of nondurables and services. The investment deflator is computed as the real price of private investment. The issue of the quality improvement of capital goods over time and its effects on the measured relative price of investment is well-explored in the macroeconomics literature. Gordon (1989) provides estimates of the quality adjusted price of several types of durable equipment. However, Gordon's time series covers only the postwar period

until 1983. Cummins and Violante (2002) and Pakko (2002) extended Gordon’s procedure using forward extrapolation to obtain updated quality-adjusted price series. However, Moulton (2001) revealed that NIPA currently takes into account the quality adjustment for electronic equipment, the component of investment intuitively more subject to quality changes. Since the two procedures deliver the same qualitative results, as illustrated by Pakko (2002), and since the forward extrapolation relies on some questionable assumption, e.g. that the quality bias in the price indexes has not changed since 1983, in this paper we use the price series from NIPA. We then estimate the parameters of an $AR(1)$ process on the relative price of investment to obtain point estimates of ρ_μ and σ_μ in each of the two subsamples.

We follow a similar approach to estimate the parameters of the TFP process. We account for the variable capacity utilization by constructing a measure of TFP as:

$$TFP_t = \left(\frac{Y_t}{L_t^{1-\alpha} (U_t K_t)^\alpha} \right).$$

We set the labor share, $1 - \alpha$, equal to 0.64, which is obtained as the average value of the labor share series recovered from the Bureau of Labor Statistics (BLS). From the same source we recover annual data on capital services, K_t . We interpolate the capital services series to obtain quarterly series, assuming constant growth within the quarters of the same year. Non-farm business measures of hours, H_t , and output, Y_t , are also retrieved from the BLS. Finally, the series of capacity utilization, U_t , is retrieved from the Federal Reserve Board. This measure is based on the manufacturing data.

Finally, we estimate the parameters governing the government spending process using data on government consumption expenditure from NIPA, and obtain the point estimates of ρ_g and σ_g in the two subsamples.

Table 3.5 shows the estimates of the parameters governing the four exogenous processes. First, the persistence of both the TFP and investment-specific technology

processes increased, with a larger increase of the TFP persistence. In contrast, the persistence of the government spending shock did not change. Second, the standard deviation of the innovations decreased for all the shocks, except for the monetary policy. The decrease of the standard deviation is more remarkable for the government spending innovation. This result is somewhat expected, since the first subsample includes the Korean War and since the government spending was more stable in the second subsample.

Since our goal is to explore the role of the different monetary policies and the different technology processes before and during the Great Moderation, we gather the estimated parameters of the Taylor rule in vectors Γ_i , and the parameters of the exogenous processes in Θ_i , with $i = 1, 2$, where i indicates the subsample used for the estimation.

3.4.2 Calibration

Since the goal of this paper is to assess the role of the different shocks in the change of variance of the real variables during the Great Moderation, we assume that the structural parameters of the model are constant during the entire post-war period. We calibrate the structural parameters of the model, using the corresponding data statistics or the conventional wisdom. We choose the subjective discount factor β to be $1.03^{-1/4}$, which corresponds to the annualized real interest rate of 3 percent. Following CEE, we set the habit persistence parameter, b , to 0.65. The preference parameter associated with labor, ψ , is chosen so that the agents allocate one-third of their time endowment to work. The depreciation rate parameter, δ , is set equal to 0.025, which implies an annual capital depreciation rate of 10 percent. We assign a value of 0.36 to the capital share in production function to match the steady state share of labor of 64 percent. Following Altig et al. (2005), we set the elasticity of substitution between different types of labor equal to 21, and the elasticity of sub-

stitution between differentiated intermediate goods equal to 6. The price stickiness parameter is set at 0.6, which implies price contracts lasting 2.5 quarters, whereas the wage stickiness parameter is set at 0.64, implying wage contracts lasting 2.8 quarters. Both values are taken from CEE. We assume no price indexation, following Cogley and Sbordone (2004) and Levin et al. (2005), who find that there is no indexation in product prices. Finally, the wage indexation is very close to unity, following Levin et al. (2005) who find a high degree of wage indexation. Values of the calibrated parameters are summarized in Table 3.6.

3.5 Results

3.5.1 Model Performance

After we estimate the exogenous processes of the model in both subsamples and feed them into the model, we assess if the model is able to generate reasonable predictions for the behavior of the macroeconomic variables. Table 3.7 displays the model predictions for the high-frequency standard deviations of output, consumption, and investment in two subsamples. In particular, we define as $M(\Gamma_i, \Theta_j)$, $i, j = 1, 2$, the model in which the Taylor rule parameters, Γ_i , are estimates of (3.13) using data from subsample i , and the parameters of the exogenous processes, Θ_j , are estimated using data from subsample j . We also report the estimates of the corresponding data moments for comparison.⁷

The model performs remarkably well in replicating the behavior of the standard deviations of the variables in the two subsamples. Although the model slightly underestimates the volatility of output and investment in the first subsample, it is

⁷ The data on consumption, output, and investment are retrieved from the NIPA. The consumption series is given by real per capita personal consumption expenditures on nondurables and services series. Output is measured by real per capita gross domestic product series, and investment by real per capita private investment. To obtain estimates of the high- and medium-frequency standard deviations as defined in Section 2, all data are filtered using the band pass filter implemented by Christiano and Fitzgerald (2003).

able to match the ratios of the standard deviations of output, consumption, and investment. Moreover, the model matches almost exactly the standard deviations of the variables in the second subsample, and also predicts the decline of the high-frequency volatility during the Great Moderation period. This result confirms that our model, driven by the estimated exogenous processes, generates realistic high-frequency dynamics of the macroeconomic variables.

However, since we want to explore the changes in the spectral shapes of the macroeconomic variables during the Great Moderation, we are also interested in the model predictions of the medium-frequency volatilities. Table 3.8 displays the model medium-frequency standard deviations of output, consumption, and investment, as well as their data counterparts. The model largely underestimates the standard deviations of the variables at medium frequencies. Therefore, the propagation mechanism governing the intertemporal dynamics of the model appears to be weak, since it cannot generate fluctuations at medium frequencies similar in magnitude to those observed in the data. Although the model fails to quantitatively capture the medium-frequency behavior of the macroeconomic variables, it produces rather interesting qualitative implications. The model correctly predicts the absence of moderation at medium frequencies, as observed in the data. Whereas the standard deviation at high frequencies largely declines from the first to the second subsample, the standard deviation at medium frequencies exhibits a different behavior. In fact, it largely increases for investment, slightly increases for output, and marginally declines for consumption. Hence, the model is able to qualitatively predict the spectral redistribution of the variance from high frequencies to low frequencies during the Great Moderation.

To assert this implication, in Table 3.9 we present the percentage contribution of the high-frequency variance to the total variance of the variables in the two subsamples, implied both by the data and by the model. Two important results emerge.

First, the data show a redistribution of the variance from high to medium frequencies during the Great Moderation. In fact, whereas the high frequencies account for approximately 40 percent of the total variance of the output and consumption and for 66 percent of the total variance of investment in the pre-1978 period, the contribution of the high frequencies for all the variables drops to about 20 percent during the Great Moderation. This result is a consequence of the specific nature of the Great Moderation, which is characterized by a sharp decline of the volatility only at high frequencies of the macroeconomic variables.⁸ Since the high-frequency volatilities declined remarkably, and the medium-frequency volatilities did not, the medium frequencies capture a larger fraction of the total variance during than before the Great Moderation. Second, as already pointed out, the model is not well suited to explain the medium-frequency fluctuations of the variables, since the largest fraction of the total variability of output, consumption, and investment is captured only by the high frequencies. Nevertheless, the model correctly predicts a large redistribution of the variance from high frequencies to low frequencies during the Great Moderation. Our primary goal in this paper is to explore what the main driving force is behind this redistribution of the variance from high to medium frequencies.

3.5.2 Counterfactuals: The Role of Monetary Policy

In Section 2, by using a simple model we showed that a change in the monetary policy parameters could potentially imply a redistribution of the spectral density of real variables. In this section, we evaluate the role of a change in the monetary policy during the Great Moderation period in explaining both the decline in the variance and its redistribution from high to medium frequencies, as observed in the data, performing two counterfactual exercises.

First, we compute the standard deviations implied by the model $M(\Gamma_2, \Theta_1)$,

⁸ See Pancrazi (2009).

where the exogenous processes are kept as estimated in the first subsample, Θ_1 , but we allow the monetary policy to adopt the rule estimated in the second subsample, Γ_2 . As Table 3.10 shows, the role of the monetary policy change is negligible. The model moments at both intervals of frequencies are essentially unaffected when we allow only the parameters of the Taylor rule to change. Therefore, we conclude that a different monetary policy during the post-1983 period alone could not have played a significant role in the decline of the high-frequency volatilities of the real macroeconomic variables, nor in the redistribution of their volatilities from high to medium frequencies.

Then, in the second counterfactual exercise, we consider the model $M(\Gamma_1, \Theta_2)$. We fix the coefficients of the Taylor rule as estimated in the first subsample, Γ_1 , but we now feed into the model the exogenous processes estimated in the second subsample, Θ_2 . Table 3.11 shows the implied model moments. In this scenario, the standard deviations of the real macroeconomic variables are strongly affected at both high and medium frequencies. However, the change in the estimates of the exogenous processes alone does not reproduce the same moments as in the case in which both the exogenous processes and the coefficients of the Taylor rule change, $M(\Gamma_2, \Theta_2)$. In particular, the decline of the high-frequency volatilities when the Taylor rule coefficients are fixed to their first subsample values, Γ_1 , is smaller than when the Taylor rule coefficients are estimated in the Great Moderation period, Γ_2 . The difference between these scenarios is much more pronounced at high frequencies than at medium frequencies. Therefore, once we assume that the exogenous processes have changed from the Pre-Great Moderation to the Great Moderation period, the role of monetary policy is not anymore negligible. In particular, when both exogenous processes and the Taylor rule coefficients change, the contribution of the monetary policy to the total reduction of the high-frequency volatilities of the macroeconomic variables is evident; 38 percent of the overall decline of the high-frequency variance

of output and investment, and 43 percent for consumption, is due to the different monetary policy.

The change in the exogenous processes and the change in the monetary policy have opposite effects at medium frequencies. In fact, the larger persistence of the total factor productivity and investment-specific technology causes an increase of the medium-frequency volatility of output and investment. In contrast, the change in monetary policy mitigates this increase, since it drives down the medium-frequency volatilities. However, the effect of the monetary policy on the medium-frequency volatilities is significantly smaller than on the high-frequency volatilities.

This finding is supported when we compute the percentage contribution of the high frequencies to the total variance of the macroeconomic variables in this counterfactual scenario, as reported in Table 3.12. The change in the exogenous processes alone implies a redistribution of the spectral density of output, investment, and consumption from high to medium frequencies, as suggested by the decline in the percentage contribution of high frequencies. This redistribution is mainly caused by the increase in persistence of the TFP, and of the investment-specific technology, since they are the major contributors to the overall variance of the real variables, as we show in the next section. However, the change in monetary policy amplifies this effect, since the different monetary policy causes a large decline in the high-frequency volatility and it has only a marginal effect on the medium-frequency volatility. Therefore, the monetary policy in the Great Moderation period contributed to the change of the spectral shape of the real macroeconomic variables, since it unequally altered the volatilities at different frequencies.

3.5.3 Variance Decomposition

Another interesting question we address is, what fraction of the high-frequency and medium-frequency variances is attributable to each of the four exogenous shocks that

drive the dynamics of the model? Tables 3.13 and 3.14 display the variance decompositions of output, consumption, and investment in the two subsamples, respectively. If we consider the model fitted to the pre-Great Moderation period, $M(\Gamma_1, \Theta_1)$, the largest fraction of the high-frequency variance of output depends on the TFP shock. Monetary shocks explain only about 2 percent of the total variance of the real variables, whereas the investment-specific and government spending shocks together are the source of less than 10 percent of the total variance for each of the three variables.

However, the percentage contributions of the shocks change significantly when we consider medium-frequency fluctuations. First, notice that although the TFP shock explains most of the medium-frequency variances of both output and investment, its share drops sharply with respect to the high-frequency contribution. The sharpest decline of the TFP shock share is for consumption; it drops from 83.6 percent at high frequencies to 30.8 percent at medium frequencies. Therefore, the other shocks become relatively more important when lower frequency fluctuations are considered. In fact, the government spending shock is the main source of medium-frequency movements in consumption. The importance of the investment-specific technology shock for all three macroeconomic variables increases notably when medium frequencies are considered. Finally, the effect of the monetary policy shock on the variances also remains negligible at medium frequencies.

Table 3.14 displays the variance decompositions of the three macroeconomic variables implied by the model driven by the exogenous processes estimated during the Great Moderation period, $M(\Gamma_2, \Theta_2)$. At high frequencies, the TFP shock drives the largest fraction of the fluctuations of all three real variables. The contribution of the monetary policy shock in the total variance drops essentially to zero for all variables. The investment-specific technology shock explains approximately 5 percent of the total variance, as in the model estimated in the pre-Great Moderation period, whereas the contribution of the government spending shock at high frequen-

cies significantly declines in the second subsample. At medium frequencies, the TFP remains the largest driving force for the fluctuations of the real variables. The role of the investment-specific shock increases, especially for consumption (from 4.8 percent to 24.2 percent) and investment (from 8 percent to 21.4 percent), which makes it the second most important shock. The government spending shock and the monetary policy shock play a minor role in explaining the medium-frequency fluctuations of the variables.

Using a simpler model, we showed that different monetary policy can potentially alter the variance decomposition of real variables. In order to explore whether that was the case during the Great Moderation period, we compute the variance decomposition of the counterfactual process $M(\Gamma_1, \Theta_2)$, shown in Table 3.15. Since in this model only the parameters of the exogenous processes are fixed to their second subsample values, the differences between the decompositions in Tables 3.14 and 3.15 are driven solely by the change in monetary policy. Therefore, by comparing the results, we infer the effects that the monetary policy adopted during the Great Moderation had on variance decompositions of output, consumption, and investment. The differences are negligible, not greater than 3 percent for any of the four shocks. Therefore, we conclude that the monetary policy did not alter the variance decompositions of the real macroeconomic variables.

3.6 Conclusions

In this paper, we focus on the two main characteristics of the Great Moderation: the significant reduction of the high-frequency volatility of real macroeconomic variables, which has been largely explored in the literature, and the absence of moderation of their medium-frequency volatility, as recently observed by Pancrazi (2009). In particular, using a medium scale DSGE model as in CEE, we explore whether the more aggressive monetary policy in the post-1983 period accounts for the reduction

of the variance and its different temporal distribution in the last three decades. We show that the “good luck” hypothesis, the notion that the nature of the exogenous processes has changed during the Great Moderation period, mainly accounts for both facts.

As a theoretical framework, we consider a model driven by four shocks: a neutral technology shock, an investment-specific technology shock, a government spending shock, and a monetary policy shock. The structural parameters of the model are calibrated and kept constant throughout the whole sample, whereas the parameters governing the exogenous processes are estimated in the two subsamples, pre-Great Moderation and the Great Moderation period, and fed into the model.

Using the predictions of this model when performing several counterfactual exercises, we conclude that a change in monetary policy during the post-1983 period alone did not play a significant role in accounting for the two facts characterizing the Great Moderation: the decline of the high-frequency volatilities of the real macroeconomic variables and the redistribution of their volatilities from high to medium frequencies. It is only with a simultaneous change in both the exogenous processes and monetary policy, that the contribution of the monetary policy to the total reduction of the high-frequency volatilities of the macroeconomic variables becomes evident; it accounts for 38 percent of the overall decline of the high-frequency variance of output and investment, and 43 percent of the decline in the case of consumption. Moreover, we document that the effects of the monetary policy are much larger at high frequencies than at medium frequencies.

The change in the exogenous processes alone largely accounts for the redistribution of the spectral density of output, investment, and consumption from high to medium frequencies. This redistribution is mainly caused by the increase in persistence of the TFP and the investment-specific technology processes, as two major contributors to the overall variance of the real variables. However, the change in

monetary policy amplifies this effect, since it has a much larger effect on the high-frequency volatility. This leads us to conclude that monetary policy in the Great Moderation period contributed to the transformation of the spectral shape of the real macroeconomic variables, even though much less than the change of the exogenous processes.

We also perform a variance decomposition exercise, and show that in both subsamples the TFP shock and the investment-specific technology shock are the two most important driving forces of the variances at both high and medium frequencies. In both subsamples, the role of the investment-specific shock increases at medium frequencies. The role of the monetary policy shock is negligible in both subsamples at all frequencies. The fiscal policy shock is relatively more important in the first subsample, and at medium frequencies. However, its role is only relevant in explaining medium-frequency movements of consumption. Finally, we show that these results are not affected by the change in monetary policy.

3.7 Tables and Figures

Table 3.1: Standard Deviations of Output and Inflation when $\phi_\pi = 1.5$

	Percentage		
	σ	σ^{HF}	σ^{MF}
Output	2.02	1.40	0.91
Inflation	0.43	0.32	0.19

Note: The table reports the standard deviation of output and inflation implied by the New Keynesian model when the inflation parameter in the Taylor rule is $\phi_\pi = 1.5$. The first column reports the total standard deviations, the second column reports the high-frequency standard deviations, defined as the fluctuations between 2 and 32 quarters, and the third column reports the medium-frequency standard deviations, defined as the fluctuations between 32 and 80 quarters.

Table 3.2: Standard Deviations of Output and Inflation when $\phi_\pi = 6$

	Percentage		
	σ	σ^{HF}	σ^{MF}
Output	1.92	1.22	0.97
Inflation	0.15	0.12	0.05

Note: The table reports the standard deviation of output and inflation implied by the New Keynesian model when the inflation parameter of the Taylor rule is $\phi_\pi = 6$. The first column reports the total standard deviations, the second column reports the high-frequency standard deviations, defined as the fluctuations between 2 and 32 quarters, and the third column reports the medium-frequency standard deviations, defined as the fluctuations between 32 and 80 quarters.

Table 3.3: Change of the Variances Driven by Different Monetary Policies

	Percentage		
	σ	σ^{HF}	σ^{MF}
Output	-10	-24	14
Inflation	-87	-86	-91

Note: The table reports the percentage change in the variance of output and inflation from an increase of the inflation parameter of the Taylor rule from 1.5 to 6. The first column reports the percentage change in the total variances, the second column reports the percentage change in the high-frequency variances, defined as the fluctuations between 2 and 32 quarters, and the third column reports percentage change in the medium-frequency variances, defined as the fluctuations between 32 and 80 quarters.

Table 3.4: Variance Decomposition with Two Different Monetary Rules

	Percentage			
	$\phi_\pi = 1.5$		$\phi_\pi = 6$	
	Technology Shock	Monetary Shock	Technology Shock	Monetary Shock
Output	58	42	91	9
Inflation	43	57	12	88

Note: The table reports the percentage contributions of the technology shock and monetary shock on the total variance of output and inflation, for the two cases when the inflation parameter of the Taylor rule is 1.5, and 6.

Table 3.5: Estimated Parameters of the Taylor Rule and Exogenous Processes

Parameter	Standard Deviation in Percent	
	Sample 1: 1947-1978	Sample 2: 1983-2007
Interest Rate Smoothing Coefficients	γ_R	Taylor Rule
Inflation Coefficients	γ_π	
Output Coefficients	γ_y	
Standard Deviation Innovation	σ_m	Monetary Policy Shock
Persistence	ρ_μ	Investment-Specific Technology Shock
Standard Deviation Innovation	σ_μ	
Persistence	ρ_A	Total Factor Productivity Shock
Standard Deviation Innovation	σ_A	
Persistence	ρ_g	Government Spending Shock
Standard Deviation Innovation	σ_g	

Note: The table displays the estimates of the parameters of the Taylor rule and of the exogenous shocks of the model. The first column reports the estimates using data in the first subsample (1947:1-1978:4). The second column reports the estimates using data in the second subsample (1983:1-2007:4). Standard deviations are reported in percent. Standard errors are displayed in brackets.

Table 3.6: Calibration of the Structural Parameters

Parameter	Calibrated value
β – Subjective discount factor	0.9926
b – Habit persistence in consumption	0.65
ψ – Preference parameter with labor	5
γ – Inverse of Frisch labor supply elasticity	1
δ – Depreciation rate of capital	0.025
α – Capital share in the production function	0.36
ε – EOS among differentiated intermediate goods	6
η – EOS among different types of labor	21
κ – Investment adjustment cost parameter	1.5
S_g – Share of Government spending in GDP	0.17
γ_2 – Coefficient of the capital utilization function	0.0655
θ_w – Wage stickiness	0.64
χ_w – Wage indexation	0.98
θ_p – Price stickiness	0.6
χ_p – Price indexation	0

Note: The table reports the values of the calibrated structural parameter of the DSGE model.

Table 3.7: Model and Data High-Frequency Standard Deviations

Percentage				
	$M(\Gamma_1, \Theta_1)$	Data: 1947-1978	$M(\Gamma_2, \Theta_2)$	Data: 1983:2007
Output	1.64	1.94 [0.20]	1.07	0.97 [0.09]
Consumption	0.81	0.88 [0.11]	0.51	0.60 [0.05]
Investment	4.59	5.44 [0.67]	2.97	2.96 [0.23]

Note: The table reports the high-frequency standard deviations of output, consumption, and investment implied by the model and estimated in the data. The high-frequencies correspond to fluctuations between 2 and 32 quarters. The first column reports the moments implied by the model $M(\Gamma_1, \Theta_1)$, where Γ_1 is the set of parameters of the Taylor rule estimated using data in the first subsample (1947:1-1978:4), and Θ_1 is the set of parameters of the exogenous processes estimated using also data in the first subsample. The second column reports the data moments in the first subsample. The third column reports the moments implied by the model $M(\Gamma_2, \Theta_2)$, where Γ_2 is the set of parameters of the Taylor rule estimated using data in the second subsample (1983:1-2007:4), and Θ_2 is the set of parameters of the exogenous processes estimated using also data in the second subsample. The fourth column reports the data moments in the second subsample.

Table 3.8: Model and Data Medium-Frequency Standard Deviations

Percentage				
	$M(\Gamma_1, \Theta_1)$	Data: 1947-1978	$M(\Gamma_2, \Theta_2)$	Data: 1983:2007
Output	0.80	2.44 [0.24]	0.95	1.94 [0.35]
Consumption	0.53	1.10 [0.09]	0.47	1.55 [0.23]
Investment	0.81	3.56 [0.37]	3.01	5.96 [1.10]

Note: The table reports the medium-frequency standard deviations of output, consumption, and investment implied by the model and estimated in the data. The medium-frequencies correspond to fluctuations between 32 and 80 quarters. The first column reports the moments implied by the model $M(\Gamma_1, \Theta_1)$, where Γ_1 is the set of parameters of the Taylor rule estimated using data in the first subsample (1947:1-1978:4), and Θ_1 is the set of parameters of the exogenous processes estimated using also data in the first subsample. The second column reports the data moments in the first subsample. The third column reports the moments implied by the model $M(\Gamma_2, \Theta_2)$, where Γ_2 is the set of parameters of the Taylor rule estimated using data in the second subsample (1983:1-2007:4), and Θ_2 is the set of parameters of the exogenous processes estimated using also data in the second subsample. The fourth column reports the data moments in the second subsample.

Table 3.9: Contributions of the High Frequencies to the Total Variance

Percentage				
	Data		Model	
	1947-1978	1983:2007	$M(\Gamma_1, \Theta_1)$	$M(\Gamma_2, \Theta_2)$
Output	38	20	80	55
Consumption	38	14	69	53
Investment	66	20	72	48

Note: The table reports the percentage contributions of the high frequency variance to the total variance for output, consumption, and investment. The first and second columns report these statistics estimated from the data in the two subsamples, respectively. The third and fourth columns report the statistics implied by the model.

Table 3.10: Model Standard Deviations: Counterfactual in Which Only Taylor rule Parameters Change

	High Frequencies			Medium Frequencies		
	$M(\Gamma_1, \Theta_1)$	$M(\Gamma_2, \Theta_2)$	$M(\Gamma_2, \Theta_1)$	$M(\Gamma_1, \Theta_1)$	$M(\Gamma_2, \Theta_2)$	$M(\Gamma_2, \Theta_1)$
Output	1.64	1.07	1.62	0.80	0.95	0.80
Consumption	0.81	0.51	0.80	0.53	0.47	0.53
Investment	4.59	2.97	4.52	2.82	3.01	2.80

Note: The table displays the high-frequency and medium-frequency standard deviations of output, consumption, and investment implied by the model. The model $M(\Gamma_2, \Theta_1)$ represents the counterfactual model in which the parameters of the Taylor rule, Γ_2 , are as estimated using data in the second subsample (1983:1-2007:4) and the parameters of the exogenous processes, Θ_1 , are as estimated using data in the first subsample (1947:1-1978:4).

Table 3.11: Model Standard Deviations: Counterfactual in Which Only Parameters of the Exogenous Processes Change

	High Frequencies			Medium Frequencies		
	$M(\Gamma_1, \Theta_1)$	$M(\Gamma_2, \Theta_2)$	$M(\Gamma_1, \Theta_2)$	$M(\Gamma_1, \Theta_1)$	$M(\Gamma_2, \Theta_2)$	$M(\Gamma_1, \Theta_2)$
Output	1.64	1.07	1.29	0.80	0.95	1.00
Consumption	0.81	0.51	0.60	0.53	0.47	0.49
Investment	4.59	2.97	3.59	2.82	3.01	3.18

Note: The table displays the high-frequency and medium-frequency standard deviations of output, consumption, and investment implied by the model. The model $M(\Gamma_1, \Theta_2)$ represents the counterfactual model in which the parameters of the Taylor rule, Γ_1 , are as estimated using data in the first subsample (1947:1-1978:4) and the parameters of the exogenous processes, Θ_2 , are as estimated using data in the first subsample (1983:1-2007:4).

Table 3.12: Contribution of High Frequencies to the Total Variance

	Percentage		
	Model		
	$M(\Gamma_1, \Theta_1)$	$M(\Gamma_2, \Theta_2)$	$M(\Gamma_1, \Theta_2)$
Output	80	55	62
Consumption	69	53	60
Investment	72	48	56

Note: The table reports the model implied percentage contributions of the high frequency variance to the total variance for output, consumption, and investment.

Table 3.13: Variance Decomposition in $M(\Gamma_1, \Theta_1)$

	Percentage											
	Monetary			Investment-specific			TFP			Government Spending		
	High Fr.	Medium Fr.	High Fr.	Medium Fr.	High Fr.	Medium Fr.	High Fr.	Medium Fr.	High Fr.	Medium Fr.	High Fr.	Medium Fr.
Output	2.2	0.7	3.0	20.0	93.8	74.3	1.1	5.0	9.8	40.3	3.1	9.6
Consumption	2.0	0.2	4.6	28.7	83.6	30.8						
Investment	2.0	0.6	7.6	37.7	87.3	52.1						

Note: The table represents the percentage contribution of the four shocks to the high-frequency, which corresponds to fluctuations between 2 and 32 quarters, and medium-frequency, which corresponds to fluctuations between 32 and 80 quarters, variance of output, consumption, and investment in the model $M(\Gamma_1, \Theta_1)$, where Γ_1 is the set of parameters of the Taylor rule estimated using data in the first subsample (1947:1–1978:4), and Θ_1 is the set of parameters of the exogenous processes estimated using also data in the first subsample.

Table 3.14: Variance Decomposition in $M(\Gamma_2, \Theta_2)$

	Percentage											
	Monetary			Investment-specific			TFP			Government Spending		
	High Fr.	Medium Fr.	High Fr.	Medium Fr.	High Fr.	Medium Fr.	High Fr.	Medium Fr.	High Fr.	Medium Fr.	High Fr.	Medium Fr.
Output	0.1	0.0	3.1	9.7	96.4	89.8	0.3	0.5				
Consumption	0.1	0.0	4.8	24.2	92.4	69.9	2.6	6.0				
Investment	0.1	0.0	8.0	21.4	91.2	77.5	0.7	1.0				

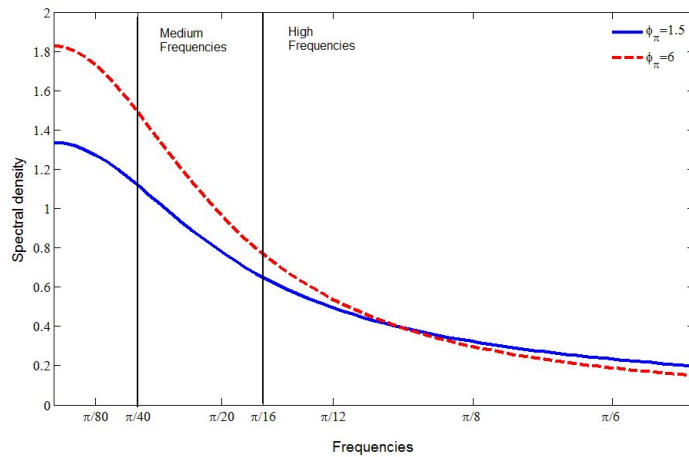
Note: The table represents the percentage contribution of the four shocks to the high-frequency, which corresponds to fluctuations between 2 and 32 quarters, and medium-frequency, which corresponds to fluctuations between 32 and 80 quarters, variance of output, consumption, and investment in the model $M(\Gamma_2, \Theta_2)$, where Γ_2 is the set of parameters of the Taylor rule estimated using data in the second subsample (1983:1-2007:4), and Θ_2 is the set of parameters of the exogenous processes estimated using also data in the second subsample.

Table 3.15: Variance Decomposition in $M(\Gamma_1, \Theta_2)$

	Percentage											
	Monetary			Investment-specific			TFP			Government Spending		
	High Fr.	Medium Fr.	High Fr.	Medium Fr.	High Fr.	Medium Fr.	High Fr.	Medium Fr.	High Fr.	Medium Fr.	High Fr.	Medium Fr.
Output	2.52	0.32	2.2	8.5	95.0	90.8	0.2	0.4				
Consumption	2.66	0.20	3.5	22.2	91.7	72.0	2.1	5.6				
Investment	2.33	0.35	5.5	19.0	91.5	79.7	0.6	1.0				

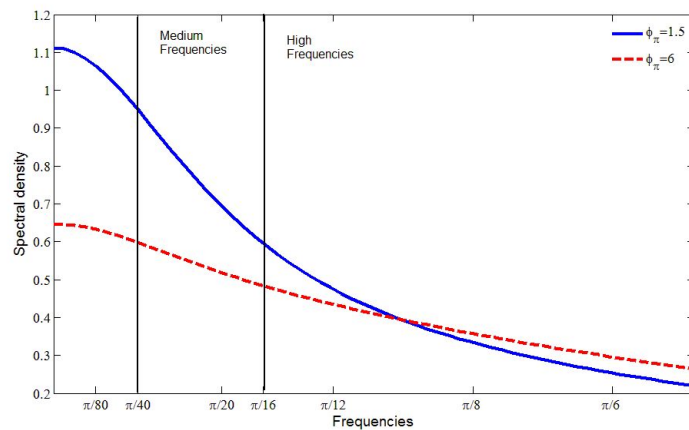
Note: The table represents the percentage contribution of the four shocks to the high-frequency, which corresponds to fluctuations between 2 and 32 quarters, and medium-frequency, which corresponds to fluctuations between 32 and 80 quarters, variance of output, consumption, and investment in the model $M(\Gamma_1, \Theta_2)$, where Γ_1 is the set of parameters of the Taylor rule estimated using data in the first subsample (1947:1-1978:4), and Θ_2 is the set of parameters of the exogenous processes estimated using data in the second subsample (1983:1-2007:4).

FIGURE 3.1: Spectrum of Output in the New Keynesian Model with Two Alternative Monetary Policies



Note: The figure plots the normalized spectrum of output implied by the New Keynesian model when the inflation parameter of the Taylor rule is 1.5, solid line, and 6, dashed line.

FIGURE 3.2: Spectrum of Inflation in the New Keynesian Model with Two Alternative Monetary Policies



Note: The figure plots the normalized spectrum of inflation implied by the New Keynesian model when the inflation parameter of the Taylor rule is 1.5, solid line, and 6, dashed line.

Appendix A

Appendix to Chapter 1

A.1 Data

The macroeconomic series analyzed in Section 2 are extracted by the NIPA dataset. The sample includes quarterly observation from 1947:1 to 2007:4. The series considered are the following:

- #1 Gross domestic product
- #2 Personal consumption expenditures
 - #3 Durable goods
 - #4 Nondurable goods
 - #5 Services
- #6 Gross private domestic investment
 - #7 Fixed investment
 - #8 Nonresidential
 - #9 Structures
 - #10 Equipment and software
 - #11 Residential

#12	Exports
#13	Goods Exports
#14	Service Exports
#15	Import
#16	Good Imports
#17	Services Imports
#18	Government consumption expenditures and gross investment
#19	Federal
#20	National defense
#21	Nondefense
#22	State and local

A.2 Proofs to the Results in Chapter 1

Theorem 5. Let $I_1 = [\omega_L^1, \omega_H^1]$ and $I_2 = [\omega_L^2, \omega_H^2]$ be two disjoint set of frequencies, and let $I_3 = I_1 \cup I_2$. Let x_t^1, x_t^2 and x_t^3 be the filtered series obtained by the same process y_t , isolating respectively the frequencies in I_1 , I_2 , and I_3 . Then, the correlation between x_t^1 and x_t^3 is equal to the ratio of their standard deviation, i.e.

$$\rho(x_t^1, x_t^3) = \sqrt{\frac{\text{Var}(x_t^1)}{\text{Var}(x_t^3)}} \quad (\text{A.1})$$

Proof. By definition

$$\rho(x_t^1, x_t^3) = \frac{\text{Cov}(x_t^1, x_t^3)}{\sqrt{\text{Var}(x_t^1) \text{Var}(x_t^3)}}.$$

Since $I_3 = I_1 \cup I_2$ and $I_1 \cap I_2 = \emptyset$, then $x_t^3 = x_t^1 + x_t^2$. Therefore,

$$\text{Cov}(x_t^1, x_t^3) = \text{Cov}(x_t^1, x_t^1 + x_t^2) = \text{Var}(x_t^1) + \text{Cov}(x_t^1, x_t^2) = \text{Var}(x_t^1)$$

where the last equality depends on the the fact that $I_1 \cap I_2 = \emptyset$. Thus,

$$\rho(x_t^1, x_t^3) = \frac{\text{Cov}(x_t^1, x_t^3)}{\sqrt{\text{Var}(x_t^1) \text{Var}(x_t^3)}} = \frac{\text{Var}(x_t^1)}{\sqrt{\text{Var}(x_t^1) \text{Var}(x_t^3)}} = \sqrt{\frac{\text{Var}(x_t^1)}{\text{Var}(x_t^3)}}.$$

□

Theorem 8. Let \mathbf{y}_t be a multivariate linear process as in (1.2), where ε_t is a multivariate normal. Then, the (m, n) -th element, $m = 1, \dots, N$, $n = 1, \dots, N$ of the sample Integrated Cospectrum in (1.15) has the following proprieties:

1. *asymptotic unbiasedness:*

$$\lim_{T \rightarrow \infty} E \left(\hat{\mathbf{H}}_{m,n}(\omega_1, \omega_2) \right) = \mathbf{H}_{m,n}(\omega_1, \omega_2)$$

2. *consistency:*

$$\hat{\mathbf{H}}_{m,n}(\omega_1, \omega_2) \xrightarrow{p} \mathbf{H}_{m,n}(\omega_1, \omega_2)$$

3. *asymptotic normality:*

$$\sqrt{T} \left[\hat{\mathbf{H}}_{m,n}(\omega_1, \omega_2) - \mathbf{H}_{m,n}(\omega_1, \omega_2) \right] \xrightarrow{d} N \left(\mathbf{0}, \mathbf{\Phi}_{m,n}(\omega_1, \omega_2) \right),$$

with

$$\mathbf{\Phi}_{m,n}(\omega_1, \omega_2) = 8\pi \int_{\omega_1}^{\omega_2} \mathbf{c}_{m,n}^2(\omega) d\omega \quad (\text{A.2})$$

Proof. Define the function $\phi(\omega)$ as

$$\phi(\omega) = \begin{cases} 2 & \text{for } \omega_1 \leq \omega \leq \omega_2 \\ 0 & \text{otherwise} \end{cases} \quad (\text{A.3})$$

Then, for any element (m, n) of the $N \times N$ matrices, we have from (1.16)

$$\hat{\psi}_{m,n} = 2 \int_{\omega_1}^{\omega_2} \hat{\mathbf{I}}_{m,n}(\omega) d\omega.$$

Note that this expression is the element (m, n) of $\hat{\mathbf{H}}(\omega_1, \omega_2)$ as defined in (1.14).

Therefore, the asymptotic unbiasedness comes directly from result (1) of Lemma 1.

Result 3 of Lemma 1 implies that

$$\lim_{T \rightarrow \infty} T \text{var} \left(\hat{\psi}_{m,n} \right) = 4\pi \int_{\omega_1}^{\omega_2} \phi(\omega) \bar{\phi}(\omega) \mathbf{c}_{m,n}^2(\omega) d\omega, \text{ with } \bar{\phi}(\omega) = \frac{1}{2} [\phi(\omega) + \phi(-\omega)],$$

which implies

$$\lim_{T \rightarrow \infty} T \text{var} \left[\hat{\mathbf{H}}_{m,n}(\omega_1, \omega_2) \right] = 4\pi \int_{\omega_1}^{\omega_2} \phi(\omega) \bar{\phi}(\omega) \mathbf{c}_{m,n}^2(\omega) d\omega. \quad (\text{A.4})$$

Since $\phi(\omega)$ as defined above is a fixed bounded function, it is clear that

$$\text{var} \left[\hat{\mathbf{H}}_{m,n}(\omega_1, \omega_2) \right] = O\left(\frac{1}{T}\right) \text{ as } T \rightarrow \infty,$$

thus implying the consistency of $\hat{\mathbf{H}}_{m,n}(\omega_1, \omega_2)$. Finally, the asymptotic variance

$$\sqrt{T} \left(\hat{\mathbf{H}}_{m,n}(\omega_1, \omega_2) - \mathbf{H}_{m,n}(\omega_1, \omega_2) \right)$$

can be derived substituting (A.3) into (A.4), i.e.:

$$\begin{aligned} \lim_{T \rightarrow \infty} T \text{var} \left[\hat{\mathbf{H}}_{m,n}(\omega_1, \omega_2) \right] &= 8\pi \int_{\omega_1}^{\omega_2} \mathbf{c}_{m,n}^2(\omega) d\omega \\ &= \Phi_{m,n}(\omega_1, \omega_2) \end{aligned}$$

□

Theorem 9. Let \mathbf{y}_t be a multivariate linear process as in (1.2), where ε_t is a multivariate normal. Also, let $0 \leq \omega_1 \leq \omega_2$, and define $\Delta\omega = \frac{\omega_2 - \omega_1}{q}$, with $q \rightarrow \infty$ as $T \rightarrow \infty$. Then, a consistency estimate of $\Phi_{m,n}(\omega_1, \omega_2)$, $m = 1, \dots, N$, $n = 1, \dots, N$ is given by

$$\hat{\Phi}_{m,n} = 4\pi \sum_{i=1}^q \left[\frac{1}{2\pi} \sum_{j=1-T}^{T-1} \hat{\mathbf{r}}_{m,n}^{(j)} e^{-i\omega j} \right]^2 \Delta\omega. \quad (\text{A.5})$$

Proof. The proof comes directly from the result showed by Lomnicki and Zaremba (1959) and Hannan (1960), which implies that

$$\begin{aligned} p \lim_{T \rightarrow \infty} \frac{1}{2} \int_{\omega_1}^{\omega_2} \hat{\mathbf{r}}_{m,n}^2(\omega) d\omega &= \int_{\omega_1}^{\omega_2} \mathbf{c}_{m,n}^2(\omega) d\omega \\ p \lim_{T \rightarrow \infty} \frac{1}{2} \int_{\omega_1}^{\omega_2} \left[\frac{1}{2\pi} \sum_{j=1-T}^{T-1} \hat{\mathbf{r}}_{m,n}^{(j)} e^{-i\omega j} \right]^2 d\omega &= \int_{\omega_1}^{\omega_2} \mathbf{c}_{m,n}^2(\omega) d\omega \end{aligned}$$

Moreover, since when $T \rightarrow \infty$, $\Delta\omega \rightarrow 0$, so that

$$p \lim_{T \rightarrow \infty} 4\pi \sum_{i=1}^q \left[\frac{1}{2\pi} \sum_{j=1-T}^{T-1} \hat{\Gamma}_{m,n}^{(j)} e^{-i\omega j} \right]^2 \Delta\omega = 8\pi \int_{\omega_1}^{\omega_2} \mathbf{c}_{m,n}^2(\omega) d\omega$$

□

A.3 The Model and its Parameterization

The Data Generating Process used for the Monte Carlo simulation is the factor hoarding model by Burnside and Eichenbaum (1996).

The economy is populated by a large number of infinitely-lived agents. The time t instantaneous utility of each agent is

$$\ln(C_t) + \theta \ln(T - \zeta - W_t f)$$

where T denotes the agent's time endowment, C_t denotes consumption, and W_t denotes labor effort. The time T output is produced according to

$$Y_t = (K_t U_t)^{1-\alpha} (N_t f W_t X_t)^\alpha$$

with $0 \leq \alpha \leq 1$, where K_t denotes the capital stock, U_t represent the capital utilization rate, N_t denotes the number of agents at work at time t , and X_t represent the level of technology. The stock of capital evolves according to:

$$K_{t+1} = (1 - \delta_t) K_t + I_t$$

where I_t denotes the gross investment, and the time dependent depreciation rate depends on the utilization rate, i.e.:

$$\delta_t = \delta U_t^\phi.$$

I assume that the only source of uncertainty is the level of technology, X_t , which is described by an autoregressive process:

$$\ln(X_t) = \rho \ln(X_{t-1}) + \varepsilon_t \quad \varepsilon_t \stackrel{iid}{\sim} N(0, \sigma_a).$$

The preference parameters of the models are calibrated as in Burnside and Eichenbaum (1996), i.e., $T = 1369$, $\beta = 1.03^{-\frac{1}{4}}$, $f = 324.8$, $\zeta = 60$, $\delta = 0.0195$, and $\alpha = 0.674$. The parameters for the evolution of technology are $\rho = 0.7$, $\sigma_a = 0.009$.

Bibliography

- [1] ABEL, A. B. Asset Prices under Habit Formation and Catching Up with the Joneses. *American Economic Review Papers and Proceedings 80* (1990), 38–42.
- [2] AHMED, S., LEVIN, A., AND WILSON, B. Recent U.S. Macroeconomic Stability: Good Policies, Good Practices, or Good Luck? *The Review of Economics and Statistics 86* (2004), 824–832.
- [3] ALTIG, D., CHRISTIANO, L., EICHENBAUM, M., AND LINDÉ, J. Firm-Specific Capital, Nominal Rigidities and the Business Cycle. *NBER Working Paper 11034* (2005).
- [4] ALTUG, S., AND LABADIE, P. *Dynamic Choice and Asset Markets*. Academic Press, San Diego, 1994.
- [5] ALVAREZ, F., AND JERMANN, U. J. Using Asset prices to Measure the Cost of Business Cycle. *Journal of Political Economy 112* (2004), 1223–1256.
- [6] ANDREWS, D. Heteroskedasticity and Autocorrelation Consistent Covariance Matrix Estimation. *Econometrica 59* (1991), 816–858.
- [7] ANDREWS, D. Tests for Parameter Instability and Structural Change with Unknown Change Point. *Econometrica 61* (1993), 821–856.
- [8] ANDREWS, D., LEE, I., AND PLOBERGER, W. Optimal Change Point Tests for Normal Linear Regression. *Journal of Econometrics 70* (1996), 9–38.
- [9] ANDREWS, D., AND PLOBERGER, W. Optimal Test When a Nuisance Parameter is Present Only Under the Alternative. *Econometrica 62* (1994), 1383–1414.
- [10] BANSAL, R., AND YARON, A. Risk for the Long-Run: a Potential Resolution of Asset Pricing Puzzles. *The Journal of Finance*.
- [11] BARRO, R. Rare Disasters and Asset Markets in the Twentieth Century. Mimeo, 2005.

- [12] BARRO, R. J. Rare Disasters, Asset Pricing, and Welfare Costs. *American Economic Review* 99 (2009), 243–64.
- [13] BAXTER, M., AND KING, R. Measuring Business Cycles: Approximate Band-Pass Filters for Economic Time Series. *The Review of Economics and Statistics* 81 (1999), 575–593.
- [14] BLANCHARD, O., AND SIMON, J. The Long and Large Decline in U.S. Output Volatility. *Brookings Papers on Economic Activity* 1 (2001), 135–164.
- [15] BOIVIN, J., AND GIANNONI, M. P. Has Monetary Policy Become More Effective. *The Review of Economics and Statistics* 88 (2006), 445–462.
- [16] BURNS, A., AND MITCHELL, C. M. Measuring Business Cycles. National Bureau of Economic Research, New York, 1946.
- [17] BURNSIDE, C., AND EICHENBAUM, M. Factor-Hoarding and the Propagation of Business-Cycle Shocks. *American Economic Review* 86 (1996), 1154–1174.
- [18] CALVO, G. A. Staggered Prices in a Utility-Maximizing Framework. *Journal of Monetary Economics* 12 (1983), 383–398.
- [19] CAMPBELL, J. Y., AND COCHRANE, J. H. By Force of Habit: a Consumption-Based Explanation of Aggregate Stock Market Behavior. *The Journal of Political Economy* 107 (1999), 205–251.
- [20] CHOW, G. Tests of Equality Between Sets of Coefficients in Two Linear Regression. *Econometrica* 28 (1960), 591–605.
- [21] CHRISTIANO, L. J., EICHENBAUM, M., AND EVANS, C. L. Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy. *Journal of Political Economy* 113 (2005), 1–45.
- [22] CLARIDA, R., GALÍ, J., AND GERTLER, M. Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory. *Quarterly Journal of Economics* 115 (2000), 147–180.
- [23] COGLEY, T., AND SARGENT, T. J. Evolving Post-World War II U.S. Inflation Dynamics. *NBER Macroeconomics Annual* 16 (2001), 332–373.
- [24] COGLEY, T., AND SARGENT, T. J. Drifts and Volatilities: Monetary Policies and Outcomes in the Post-WWII US. *Review of Economic Dynamics* 8 (2005), 262–302.

- [25] COGLEY, T., AND SBORDONE, A. M. A Search for a Structural Phillips Curve. *Computing in Economics and Finance* 291, Society for Computational Economics.
- [26] COMIN, D., AND GERTLER, M. Medium-term Business Cycle. *American Economic Review* 96 (2006), 523–551.
- [27] CONSTANTINIDES, G. Habit formation: a Resolution of the Equity Premium Puzzle. 519–543.
- [28] CROCE, M. Welfare Costs and Long-Run Consumption Risk in a Production Economy. Manuscript, 2006.
- [29] CUMMINS, J. G., AND VIOLANTE, G. L. Investment-Specific Technical Change in the US (1947-2000): Measurement and Macroeconomic Consequences. *Review of Economic Dynamics* 5 (2002), 243–284.
- [30] DEN HAAN, W., AND MARCET, A. Solving the Stochastic Growth Model by Parameterizing Expectations. *Journal of Business and Economic Statistics* 8 (1990), 31–34.
- [31] DIXIT, A. K., AND STIGLITZ, J. Monopolistic Competition and Optimum Product Diversity. *American Economic Review* 67 (1977), 297–308.
- [32] EPSTEIN, L., AND ZIN, S. Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: a Theoretical Framework. *Econometrica* 57 (1989), 937–969.
- [33] EPSTEIN, L., AND ZIN, S. Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: an Empirical Analysis. *Journal of Political Economy* 99 (1991), 263–286.
- [34] ERCEG, C. J., HENDERSON, D. W., AND LEVIN, A. T. Optimal Monetary Policy With Staggered Wage and Price Contracts. *Journal of Monetary Economics* 46 (2000), 281–313.
- [35] FRIEDMAN, M., AND SCHWARTZ, A. Money and Business Cycles. *Review of Economics and Statistics* 45 (1963), 32–64.
- [36] GALÍ, J. *Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework*. Princeton University Press, Princeton, 2008.

- [37] GALÍ, J., AND GAMBETTI, L. On the Sources of the Great Moderation. *American Economic Journal: Macroeconomics* 1 (2009), 26–57.
- [38] GIANNONE, D., LENZA, M., AND REICHLIN, L. Explaining the Great Moderation: it is not the Shocks. *Journal of the European Economic Association* 6 (2008), 621–633.
- [39] GORDON, R. J. *The Measurement of Durable Goods Prices*. University Chicago Press, Chicago, 1989.
- [40] HAAN, V. D., AND LEVIN, A. T. A Practitioner’s Guide to Robust Covariance Matrix Estimation. NBER Technical Working Paper 197.
- [41] HANNAN, E. *Time Series Analysis*. Methuen, London, 1960.
- [42] HODRICK, R., AND PRESCOTT, E. C. Postwar U.S. Business Cycles: An Empirical Investigation. *Journal of Money, Credit, and Banking* 29 (1997), 1–16.
- [43] JUDD, K. *Numerical Methods in Economics*. MIT Press, Cambridge, MA, 1998.
- [44] KIEFER, N., VOGELANG, T., AND BUNZEL, H. Simple Robust Testing of Regression Hypotheses. *Econometrica* 68 (2000), 695–714.
- [45] KIM, C., AND NELSON, C. R. Has the U.S. Economy Become More Stable? A Bayesian Approach Based on a Markov Switching Model of Business Cycle. *The Review of Economics and Statistics* 81 (1999), 608–616.
- [46] KIM, C.-J., AND NELSON, C. Has the U.S. Economy Become More Stable? A Bayesian Approach Based on a Markov Switching Model of Business Cycle. *The Review of Economics and Statistics* 81 (199), 608–616.
- [47] KOCHERLAKOTA, N. On tests of Representative Consumer Asset Pricing Models. *Journal of Monetary Economics*.
- [48] KREPS, D. M., AND PORTEUS, E. L. Temporal Resolution of Uncertainty and Dynamic Choice Theory. *Econometrica* 46 (1978), 185–200.
- [49] KUZNETS, S. Schumpeter’s Business Cycle. *American Economic Review* 30 (1940), 257–271.
- [50] KUZNETS, S. *Capital in the American Economy: Its Formation and Financing*. Princeton University Press, Princeton, N.J., 1961.

- [51] LETTAU, M., LUDVIGSON, S., AND WACHTER, J. The Declining Equity Premium: what Role Does Macroeconomic Risk Play? *Review of Financial Studies* 21 (2008), 1653–1687.
- [52] LEVIN, A. T., ONATSKI, A., WILLIAMS, J. C., AND WILLIAMS, N. Monetary Policy Under Uncertainty in Micro-Founded Macroeconometric Models. manuscript prepared for the NBER’s Twentieth Annual Conference on Macroeconomics, 2005.
- [53] LIU, Z., WAGGONER, D., AND ZHA, T. Asymmetric Expectation Effects of Regime Shifts in Monetary Policy. *Review of Economic Dynamics* 12 (2009), 284–303.
- [54] LJUNGQVIST, L., AND UHLIG, H. Optimal Endowment Destruction under Campbell-Cochrane Habit Formation. Mimeo, 2009.
- [55] LUBIK, T. A., AND SCHORFHEIDE, F. Testing for Indeterminacy: An Application to U.S. Monetary Policy. *American Economic Review* 94 (2004), 190–217.
- [56] LUCAS, R. E. Understanding Business Cycles. *Carnegie-Rochester Conference Series on Public Policy* 1 (1977), 19–46.
- [57] LUCAS, R. E. Asset Prices in an Exchange Economy. *Econometrica* 46 (1978), 1429–1445.
- [58] LUCAS, R. E. *Models of Business Cycle*. Basil Blackwell, Oxford, 1987.
- [59] MARCET, A., AND MARSHALL, D. Solving Nonlinear Rational Expectations Models by Parametering Expectations: Converge Results in the Stationary Case. Working Paper, Universitat Pompeu Fabra, 1994.
- [60] MCCONNELL, M., AND PEREZ-QUIROS, G. Output fluctuations in the United States: what has changed since the early 1980’s? *American Economic Review* 90 (2000), 1464–1476.
- [61] MOULTON, B. R. The Expanding Role of Hedonic Methods in the Official Statistics of the United States. Bureau of Economic Analysis Papers 0014, 2001.
- [62] NEWEY, W., AND WEST, K. A Simple, Positive Semi-definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix. *Econometrica* 55 (1987), 703–708.

- [63] NEWEY, W., AND WEST, K. Automatic Lag Selection in Covariance Matrix Estimation. *Review of Economic Studies* 61 (1994), 631–653.
- [64] NYBLOM, J. Testing for the Constancy of Parameters Over Time. *Journal of the American Statistical Association* 84 (1989), 223–230.
- [65] OBSTFELD, M. Evaluating Risky Consumption Paths: the Role of Intertemporal Substitutability. *European Economic Review* 38 (1994), 1471–1486.
- [66] OTROK, C. Spectral Welfare Cost Functions. *International Economic Review* 42 (2001), 345–367.
- [67] OTROK, C., RAVIKUMAR, B., AND WHITEMAN, C. Habit Formation: a Resolution of the Equity Premium Puzzle? *Journal of Monetary Economics* 49 (2002), 1261–1288.
- [68] PAKKO, M. R. What Happens When the Technology Growth Trend Changes? Transition Dynamics, Capital Growth, and the "New Economy". *Review of Economic Dynamics* 5 (2002), 376–407.
- [69] PANCRAZI, R. Spectral Covariance Instability Test: An Application to the Great Moderation. Manuscript, 2009.
- [70] PERRON, P. Dialing with Structural Breaks. *Palgrave Handbooks of Econometrics Vol. 1 Econometric Theory*.
- [71] PRIESTLEY, M. *Spectral Analysis and Time Series*. Academic Press, London, 1982.
- [72] PRIMICERI, G. Time Varying Structural Vector Autoregressions and Monetary Policy. *Review of Economic Studies* 72 (2005), 821–852.
- [73] RIETZ, T. The Equity Risk Premium: A Solution. *Journal of Monetary Economics* 22 (1988), 117–131.
- [74] SCHUMPETER, J. The Explanation of the Business Cycle. *Economica* 21 (1927), 286–311.
- [75] SCHUMPETER, J. *Business Cycles: A Theoretical, Historical, and Statistical Analysis of the Capitalist Process*. McGraw-Hill, New York, 1939.

- [76] SMETS, F., AND WOUTERS, R. An Estimated Stochastic Dynamic General Equilibrium Model for the Euro Area. *Journal of the European Economic Association* 1 (2003), 1123–1175.
- [77] SOWELL, F. Optimal Tests for Parameter Instability in the Generalized Method of Moments Framework. *Econometrica* 64 (1996), 1085–1107.
- [78] STOCK, J., AND WATSON, M. W. Has the Business Cycle Changed and Why? *NBER Macroeconomics Annual* 17 (2002), 159–218.
- [79] STOCK, J., AND WATSON, M. W. Has the Business Cycle Changed? Evidence and Explanations. *Monetary Policy and Uncertainty, Federal Reserve Bank of Kansas City* (2003), 9–56.
- [80] TALLARINI, T. Risk-Sensitive Real Business Cycle. *Journal of Monetary Economics* 45 (2000), 507–532.
- [81] WEIL, P. Nonexpected Utility in Macroeconomics. *Quarterly Journal of Economics* 105 (1990), 29–42.
- [82] YUN, T. Nominal Price Rigidity, Money Supply Endogeneity, and Business Cycles. *Journal of Monetary Economics* 37 (1996), 345–370.

Biography

My name is Roberto Panrazi, and I was born in Sansepolcro, Tuscany on January 26th, 1979. I earned my B.A. in Statistics and Economics, *cum laude*, from the Università di Firenze in 2003, under the supervision of Professor Giorgio Calzolari. Before joining the graduate program at Duke University in August 2004, I was a junior researcher at ENI S.p.A. in Rome, Italy. I was awarded M.A. in Economics from Duke University in 2005. I am planning on graduating from Duke University with doctorate degree in Economics in spring of 2010.