

Optimal Monetary and Fiscal Policy for Small
Open and Emerging Economies

by

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Dissertation submitted in partial fulfillment of the requirements for the degree of
Doctor of Philosophy in the Department of Economics
in the Graduate School of Duke University
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ABSTRACT
(Economics)

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Abstract

This dissertation computes the optimal monetary and fiscal policy for small open and emerging economies in an estimated medium-scale model. The model departs from the conventional approach as it encompasses all the major nominal and real rigidities normally found in the literature in a single framework. After estimating the model using Bayesian techniques for one small open economy and one emerging economy, the Ramsey solution for the optimal monetary and fiscal policy is computed. Results show that foreign shocks have a strong influence in the dynamics of emerging economies, when compared to the designed optimal policy for a developed small open economy. For both economies, inflation is low, but very volatile, while taxes follow the traditional results in the literature with high taxes over labor income and low taxes for capital income.

To Christiane, my wife

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List of Abbreviations and Symbols

Abbreviations

CE	Competitive Equilibrium.
CEE	Christiano, Eichenbaum and Evans.
CES	Constant Elasticity of Substitution.
DSGE	Dynamic Stochastic General Equilibrium.
EME	Emerging Economy.
EMEs	Emerging Economies.
ER	Exchange Rate.
ERR	Exchange Rate Regime.
GDP	Gross Domestic Product.
RBC	Real Business Cycle.
SGU	Schmitt-Grohé and Uribe.
SOE	Small Open Economy.
SOEs	Small Open Economies.
SW	Smets and Wouters.
UIP	Uncovered Interest Parity.
US	United States.

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Introduction

What is the optimal combination of monetary and fiscal policy instruments for a developed, small-open economy? Is it different from the policy prescribed for emerging economies? Are the allocations and prices under optimal policy for each these two types of economies comparable if the instrument set available for the benevolent planner changes? The literature has treated these questions as two separate topics. On the one hand, a lot of effort has been devoted to characterize optimal policy for developed small-open economies, without questioning if the welfare ranking of policy recommendations associated with the structural model is still the same for more volatile, less developed economies. On the other hand, recent developments of the literature solve the optimal policy problem for models with distinctive structural rigidities. These rigidities are usually designed to capture particular dynamics of emerging economies, assuming, in this case, that these are not interesting mechanisms to access business cycle moments of developed economies.

This dissertation departs from the approach of adding specific nominal or real rigidities to describe Emerging Markets Economies (EMEs, henceforth) in favor of a better understanding of the resulting allocations, transmission mechanisms and dynamics under optimal policy of a more conventional medium-scale model for open economies. In this dissertation, the main differences between EMEs and Small-Open Economies (SOEs, henceforth) will be restricted to the estimated parameter set. The exercise proposed here focus on the characterization of the Ramsey policy

steady state and dynamics, clarifying the trade-offs faced by the benevolent central planner.

The recent development of medium-scale dynamic stochastic general equilibrium (DSGE) models for closed economies¹, comprising a large set of nominal and real rigidities, has changed the research of optimal monetary and fiscal policies, not only because of the methodological departure from analytically solving Ramsey's (1927)[59] problem in small, tractable models, but also from a theoretical point of view. Models with a large number of nominal and real rigidities demand an equally large number of non-distortionary instruments in order to recover the first-best allocation as the equilibrium outcome of the optimal policy. Given the lack of such a large set of instruments, the literature now focus on the numerical characterization of the second-best outcome under optimal policy in models where the steady state of the economy is distorted as a consequence of the nominal and real rigidities. In these simulations, the benevolent government has access to a restricted set of instruments to maximize (minimize) an utility (loss) function.

Despite the recent adoption of medium-scale models for monetary policy analysis in some Central Banks, the research on optimal policy for SOEs in these models is still very incipient². The main focus of the literature is on the evaluation of optimal monetary policy in models with small departures from the basic sticky price framework proposed in Galí and Monacelli (2005)[36] and Monacelli (2005)[55]. The extensions try to deal with specific features of open economies: deviations from the Law of One Price (Kollmann (2002)[45], Ambler, Dib and Rebei (2004)[7]); incom-

¹ The models are usually some variation of the framework in Christiano, Eichenbaum and Evans (2005)[22]. See Schmitt-Grohé and Uribe (2005 and 2005b)[66][67] for optimal policy computation in those models.

² There is a significant literature on optimal monetary policy cooperation across countries with similar sizes, motivated by the creation of the European Monetary Union in the early 90's. Some authors, like Corsetti, Dedola and Leduc (2007)[27], still compute the optimal policy under the assumption of cooperation and commitment. This is not the focus of the analysis here.

plete foreign asset markets (Ambler, Dib and Rebei (2004)[7], Justiniano and Preston (2009)[44]); fiscal policy dimension of the open economy framework (Benigno and De Paoli (2009)[11]). However, none of these papers intended to compute optimal policy combine, at the same time, a large set of real and nominal frictions in a model where the main structural parameters are disciplined by some estimation procedure³. Furthermore, no distinctions are made between SOEs and EMEs, resulting in the same policy recommendations for both types of countries despite large differences documented in the literature between the structural parameters⁴.

The literature on EMEs is still concentrated on the structural description of these economies, adding features over the basic sticky-price model in order to capture distinctive aspects of data. The higher volatility in the aggregate time series, when compared with SOEs⁵, brings attention to topics like: foreign currency demand (Felices and Tuesta (2007)[35]); investment financed by foreign currency and “balance sheet effects” (Devereux, Lane and Xu (2006)[32] and Elekdag and Tchakarov (2007)[34], Batini, Levine and Pearlman (2009)[10]); a commodity sector, in order to highlight the importance of natural resources (Laxton and Pesenti (2003)[46], Batini, Levine and Pearlman (2009)[10]); households heterogeneity in credit market access (Batini, Levine and Pearlman (2009)[10]). Again, most of these papers focus on the computation of optimal monetary policy rules, with little focus on the estimation of

³ Ambler, Dib and Rebei (2004)[7] is a notable exception in terms of model’s scale. The authors compute optimal monetary policy for an estimated model with a large number of shocks. However, fiscal policy considerations are not explored. Also, the model is solved by a first order approximation around a steady state of price stability. This paper departs from these two assumptions.

⁴ Silveira (2006)[28] used Bayesian techniques to estimate the basic Galí and Monacelli (2005)[36] model using Brazilian data. The posterior estimates for the elasticity of the labor supply and the elasticity of substitution between imported and domestically produced consumption goods are outside the boundaries found in the literature. Another example is Elekdag, Justiniano and Tchakarov (2005)[73], with discrepancies in the values for the inverse of the intertemporal elasticity of substitution, when compared to the calibration used for closed economies, and on the elasticity of the labor supply.

⁵ See Aguiar and Gopinath (2007)[4] for the comparison of statistical moments between EMEs and SOEs.

the structural parameters⁶, or, sometimes, assuming a steady state that might be different from the Ramsey optimal solution. Among the papers in this non-exhaustive list, Batini, Levine and Pearlman (2009)[10] is the closest reference in terms of the theoretical framework adopted here, as they compute optimal monetary and fiscal policy rules in a model with several nominal and real rigidities. However, the flexible price allocation can be recovered as an optimal outcome of the policy due to the assumption of a lump sum taxation as the main fiscal policy instrument.

From a theoretical perspective, the model departs from the recent literature as it does not consider a full set of lump sum mechanisms in order to eliminate the distortions caused by nominal rigidities: the Ramsey planner has access to distortionary consumption, capital and labor income taxes, besides the control of the money supply and non-state contingent debt to balance the budget. As a consequence, the optimal policy allocations are not necessarily equivalent to those under flexible prices, just like the case for closed economies described in Schmitt-Grohé and Uribe (2005 and 2005b)[66][67]. The computation of optimal policy is based on the solution for the Ramsey problem, where monetary and fiscal authorities try to maximize the discounted expected utility of the representative household. This approach differs from several studies where the optimal policy is derived from the minimization of an arbitrary loss function as a measure of welfare⁷.

From an empirical point of view, the use of Bayesian procedures to estimate the structural parameters provides the necessary flexibility to deal with model comparisons and the impulse response analysis from the optimal stabilization policy.

The estimation of medium-scale models has been presented separately for SOEs

⁶ There is a recent effort to put these models in an estimated framework. As an example, Elekdag, Justiniano and Tchakarov (2005)[73] present an estimation of the working paper version of the model published later in Elekdag and Tchakarov (2007)[34].

⁷ Some examples of the loss function approach, not only for open economies, in Svensson (2000)[72], Levin and Williams (2003)[48], Laxton and Pesenti (2003)[46] and Justiniano and Preston (2009)[44]

and EMEs⁸. In this dissertation, optimal policy for SOEs and EMEs is characterized under the same theoretical model, but fully exploring the differences in parameter estimates, fiscal policy framework and nominal rigidities, as discrepancies in these characteristics might result in different prescriptions not only of short-run, dynamic policies, but also in terms of the steady state of the economy. In this sense, the main contributions of this dissertation are: 1) estimation of the main structural parameters for SOEs and EMEs in a medium-scale model containing a large set of nominal and real rigidities; 2) detailed description of the Ramsey steady-state solution; 3) computation of the Ramsey dynamics, characterizing optimal monetary and fiscal policies for both economies. The analysis of the Ramsey dynamics closes with a small criticism on the methodology for solving one point of major concern in DSGE modeling: the treatment of the zero lower bound for nominal interest rates. Adopting a slightly unconventional approach for dealing with this problem, the last section of the chapter describing the dynamics of the model performs a welfare analysis of the model assuming that the probability of violating the lower bound for interest rates is minimized.

In order to close this introduction, given the high volatility and the presence of structural changes characterizing EMEs, one might ask about the importance of characterizing optimal policy under commitment for these economies. As a matter of fact, most of the fiscal and monetary policy combinations observed in EMEs are characterized by some type of commitment. The adoption of controlled exchange rate regimes in the early 90's is an explicit commitment to keep exchange rate fluctuations under constraint. More recently, several EMEs adopted a combination of high fiscal surpluses and inflation targeting regimes, showing clear signs that little

⁸ Adolfson, Laseén, Lindé and Villani (2007)[2] have estimated open-economy versions of a model in line with Christiano, Eichenbaum and Evans (2005)[22]. Among others, Elekdag, Justiniano and Tchakarov (2005)[73], Castillo, Montoro and Tuesta (2006)[19], and Garcia-Cicco (2008)[37] have estimated models with specific characteristics highlighted in the EME literature.

(if none) interference will be made in the exchange rate markets – again, a new form of commitment. Even in abnormal periods, the government of a country suffering from a large shock usually sign “letters of intentions” to institutions like the IMF to call for additional funding, committing to a new macroeconomic arrangement in order to guarantee the emergency loans. Thus, the policy problem of EMEs can be viewed as setting the right commitment for these economies, instead of a proposition between “commitment versus discretionary policies”.

The dissertation is organized as follows. The next chapter presents the details of the structural model from the first principles of household and firms optimization. The chapter describes the main blocks of the model, with focus on the equilibrium equations and the connections between the structural frictions and the literature on DSGE models, finishing with the definitions of the competitive and the Ramsey equilibria. Chapter 2 presents a detailed analysis of the steady state of the model, assuming structural parameters normally observed in the literature. Thus, there is not inference regarding SOEs and EMEs. The main inference about these economies starts in chapter 3, with the results regarding the structural parameters and the dynamics under the competitive equilibrium for these economies. Chapter 4 presents the Ramsey dynamics under different assumptions regarding the fiscal policy framework. The chapter closes with a discussion based on the welfare analysis of imposing an additional constraint to deal with the zero lower bound problem in DSGE models.

The main results of this dissertation highlight the importance of the assumptions regarding the fiscal policy framework when dealing with the optimal policy problem, as the outcome of the problem generates significant variations in dynamics and in the steady state. For instance, optimal monetary policy might be characterized from the Friedman rule, where the nominal interest rates must be set at zero, to very high levels of inflation depending not only on the nominal and real rigidities of the model, but also to the number of instruments available to the Ramsey planner,

even if all the instruments distorts the prices and allocations from those verified at the Pareto optimal. Regarding the analysis of SOEs and EMEs, the structural parameters computed for Australia and Brazil show two economies with very different constraints, not only in terms of real and nominal rigidities, but also from the source of the main fluctuations of the economy. The Brazilian economy is largely influenced by shocks from the rest of the world, while the dynamics of the economy of Australia resemble those studied in close economies. Finally the Ramsey dynamics imply most of the times low but very volatile inflation for both countries. Even with a large set of instruments available to the Ramsey planner, the solution of the optimal policy implies large volatility of prices, irrespective of dealing with the structural parameters from Australia or Brazil.

It is quite obvious that the exercises performed here are not exhaustive in terms of the characterization of optimal policy for open economies in general. Several features regarding the dynamics of these economies still must be explored, both confronting the theoretical framework with the data and also computing the optimal policy. This is even more important when dealing with the nonlinearities and alternative propagation mechanisms studied in Emerging economies. Hopefully, this dissertation is a step in the right direction, dealing with a small piece of the enormous literature of optimal policy.

A Medium-Scale Model for a Small Open Economy

In this chapter the full model is described, with the characterization of the household and the firms' problem, the policy rules for the government in a competitive equilibrium, the foreign sector and aggregation¹. Beyond the description of the model, the definitions of a competitive and Ramsey equilibria are presented, as well as the procedures to deal with the problem of a zero lower bound for nominal interest rates and the welfare measure computation. The model is an extension for a small-open economy of the closed-economy model for monetary policy analysis proposed in Christiano, Eichenbaun and Evans (2005)[22] (CEE (2005), henceforth) and Altig, Christiano, Eichenbaun and Lindé (2005)[6]. Similar models are used in Adolfson, Laseén, Lindé and Villani (2007)[2] and, more recently, in Christiano, Trabandt and Walentin (2007)[23]. These models for small-open economies combine the basic sticky price framework proposed in Galí and Monacelli (2005)[36] and Monacelli (2005)[55] to add a set of nominal and real frictions based in the formulation of CEE (2005)[22].

In a brief overview, from the household perspective, the model presents external

¹ A full description of the model and the transformation for stationary form are available in a technical appendix upon request. Also, appendix A lists the final set of equilibrium conditions.

habit persistence in consumption, adjustment costs for investment, portfolio and changing the capacity utilization. Households own capital, demand money to buy consumption goods and set their wages after observing the demand for his specific type of labor. The objective of the household is to maximize the discounted value of expected utility. In order to achieve the objective, households in each period buy both domestically produced and imported goods for consumption, sell their labor to satisfy the demand by the firms after the acceptance of the proposed wage and set the rate of capital utilization. In order to transfer wealth across periods, households trade bonds domestically and in the international financial markets and accumulate capital built from both domestically produced and imported goods. Households are subject to a cash-in-advance constraint, requiring domestic currency to buy a share of total consumption goods.

Firms in the tradable and non-tradable sectors of the domestic economy rent capital and labor from the households to produce goods. They set prices in a Calvo style, with a probability α_i of not adjusting prices in period t . Firms from the tradable sector have to compete with imported goods retailers. These retail firms buy goods produced abroad and sell them domestically, also adjusting prices in a Calvo style in terms of domestic currency. On the other hand, firms from the tradable sector can sell goods for the exported goods retailers. These firms buy domestically produced goods and sell them abroad, setting price in a Calvo style in terms of foreign currency – thus, local currency pricing in both domestic and foreign markets justifies pricing-to-market discrimination and the deviations of the Law of One Price, as commonly seen in the literature². A demand for foreign currency is justified in the model by a working capital constraint for imported goods retailers, with those firms selling bonds to obtain foreign currency to finance the total acquisition of foreign inputs.

² Some of the references in models with at least partial local currency pricing are Kollmann (2002)[45], Ambler, Dib and Rebei (2004)[7], Devereux, Lane and Xu (2006)[32], Christiano, Trabandt and Walentin (2007)[23], Justiniano and Preston (2009)[44],

The government in a competitive equilibrium sets nominal interest rates according to a Taylor rule based on inflation, output gap and changes in the real exchange rate, in order to match an exogenous, time-varying inflation target. In terms of fiscal policy, the government has three instruments available to finance an exogenous stream of consumption: money, bonds sold domestically, and distortionary taxes. The government might tax in different rates consumption and the income from capital, labor and profits. In the competitive equilibrium, taxes on labor are set according to a simple policy rule based on total government liabilities. Taxes on consumption, capital and on profits are exogenous.

The foreign sector is described by a simple VAR including lags of the foreign money supply, output, inflation, interest rates and a measure of the risk premium. The VAR has all shocks identified by a Cholesky decomposition, following the traditional procedure in the literature. The model has a total of 16 shocks, with five of them being from the foreign sector (one for each variable of the VAR), plus the following: one on the price of imported goods in foreign currency; two stationary sectorial productivity shocks; a non-stationary aggregate productivity shock; a non-stationary, investment-specific shock; government spending; three tax shocks; monetary policy shock and a inflation target shock.

The chapter is organized as follows. Sections 1.1 and 1.2 present details of the optimization problem of households and firms, respectively, highlighting the role of the nominal and real frictions added with the literature. A characterization of the government and the foreign economy is provided in sections 1.3 and 1.4. These blocks of the model are very stylized, with shocks driving most of the dynamics of the government and a VAR describing the foreign economy. The aggregation problem and the macroeconomic identities are presented in section 1.5. Section 1.6 presents the steps to obtain the stationary representation of the model, the competitive equilibrium, the Ramsey equilibrium and the computation of welfare

measures.

1.1 Households

There is a continuum of infinitely-lived households i ($i \in [0, 1]$) populating the domestic economy, each one of them with an endowment of labor type i , $h_t(i)$. There is no population growth and labor can not be sold for firms in the rest of the world. In the intertemporal problem, households maximize discounted utility choosing current period's consumption capacity utilization and investment for each sector, wages, hours worked and the money demand, and next period's foreign and domestic bond holdings and physical capital stock. The general statement of the intertemporal household problem, given the non-Ponzi games constraints, is:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t [(1 - \gamma) \log(C_t(i) - \zeta C_{t-1}) + \gamma \log(1 - h_t(i))]$$

$$\begin{aligned} s.t. : \quad & P_t (1 + \tau_t^c) C_t(i) + \Upsilon_t^{-1} P_t (I_{x,t}^d(i) + I_{n,t}^d(i)) + P_t M_t(i) + R_{t-1} B_{h,t}(i) \\ & + S_t R_{t-1}^f I B_t(i) + W_t \frac{\phi_w}{2} \left(\frac{W_t(i)}{\pi_t^{xw} W_{t-1}(i)} - \mu^I \right)^2 + \frac{\psi_1}{2} Y_t \left(\frac{B_{t+1}(i)}{Y_t} - \frac{B}{Y} \right)^2 \\ & + \frac{\psi_2}{2} Y_t \left(\frac{S_t I B_{t+1}(i)}{Y_t} - \frac{rer I B}{Y} \right)^2 = P_{t-1} M_{t-1}(i) + (1 - \tau_t^h) W_t(i) h_t(i) \\ & + (1 - \tau_t^\phi) P_t \Phi_t(i) + (1 - \tau_t^k) P_t [(R_{n,t}^k \mu_{n,t} - \Upsilon_t^{-1} a(\mu_{n,t})) \bar{K}_{n,t}(i) \\ & + (R_{x,t}^k \mu_{x,t} - \Upsilon_t^{-1} a(\mu_{x,t})) \bar{K}_{x,t}(i)] + B_{h,t+1}(i) + S_t I B_{t+1}(i) \end{aligned}$$

$$\bar{K}_{j,t+1}(i) = (1 - \delta) \bar{K}_{j,t}(i) + I_{j,t}^d(i) \left(1 - \aleph \left(\frac{I_{j,t}^d(i)}{I_{j,t-1}^d(i)} \right) \right)$$

$$a(\mu_{j,t}) = \theta_1 (\mu_{j,t} - 1) + \frac{\theta_2}{2} (\mu_{j,t} - 1)^2$$

$$K_{j,t} = \mu_{j,t} \bar{K}_{j,t}$$

$$\aleph \left(\frac{I_{i,t}^d}{I_{i,t-1}^d} \right) = \frac{\phi_i}{2} \left(\frac{I_{i,t}^d}{I_{i,t-1}^d} - \mu^I \right)^2 \quad j = \{x, n\}$$

$$\frac{\Upsilon_{t+1}}{\Upsilon_t} = \mu_{t+1}^\Upsilon = (1 - \rho_\Upsilon) \mu^\Upsilon + \rho_\Upsilon \mu_t^\Upsilon + \epsilon_{t+1}^\Upsilon \quad \epsilon_t^\Upsilon \sim N(0, \sigma_\Upsilon)$$

$$h_t(i) = \left(\frac{W_t(i)}{W_t} \right)^{-\varpi} h_t$$

$$M_t(i) \geq \nu^m (1 + \tau_t^c) C_t(i)$$

In this problem, β is the intertemporal discount factor of the utility function. The utility function assumes a traditional, log-separable form in terms of consumption and labor, with consumption adjusted by external habit persistence³. The degree of habit persistence is defined by the parameter $\zeta \in [0, 1)$.

In the model, households accumulate physical capital, $\bar{K}_{j,t}$, for $j = \{x, n\}$ representing the sectors of the economy, buying from the firms investment goods that depreciate at a rate δ . Define Υ_t^{-1} as the non-stationary inverse of the relative price of investment in terms of consumption goods. The relative price of investment goods can also be interpreted as a technology shock affecting the linear production function available to households to transform consumption goods in investment goods⁴. Investment is subject to an adjustment cost $\aleph(\cdot)$, in the same fashion as in CEE (2005)[22] and Altig, Christiano, Eichenbaun and Linde (2005)[6] such that $\aleph(1) = 0, \aleph'(1) = 0, \aleph''(1) > 0$. The functional form adopted follows Schmitt-Grohé and Uribe (2005b)[67], with μ^I defining the steady state growth of investment. Households rent capital for the firms after setting the rate of capital utilization for each sector $(\mu_{j,t})$, paying a cost given by the function $a(\mu_{j,t})$ to change the utilization level in each period and in each sector. The after-tax private return of capital in each sector is defined, thus, as $(1 - \tau_t^k) P_t (R_{j,t}^k \mu_{j,t} - \Upsilon_t^{-1} a(\mu_{j,t})) \bar{K}_{j,t}(i)$.

³ In terms of notation, the general variable $x_t(i)$ represents the choice of household i on period t about x . The variable x_t is the aggregate value of $x_t(i)$ for the economy.

⁴ See Greenwood, Hercowitz and Krusell (2000)[40] and Schmitt-Grohé and Uribe (2005b)[67].

The supply of labor is decided by each household taking as given the aggregate wage of the economy, the aggregate demand for labor, h_t , and the quadratic adjustment cost function for wages. As a monopolist of a specific type of labor, the household chooses the nominal wage $W(i)$ and supplies all the demanded for labor $h_t(i)$ given the acceptance of $W(i)$. The elasticity of substitution across different types of labor $h_t(i)$ is given by $\varpi > 1$. The nominal wage adjustment cost function allows for partial indexation based on current inflation. The degree of indexation is determined by χ_w ($\chi_w \in [0, 1]$). The presence of sticky wages in the model results in an additional distortion, defined by mcw_t , which is equivalent to the markup households impose over real wages since they supply a specific type of labor to the firms. The use of a quadratic adjustment cost in the wage-setting process⁵ is consistent with the absence of lump sum instruments to correct for wealth dispersion across households. Wage setting processes based on the Calvo model create dispersion in the wage income across households, and the representative household is recovered through a lump sum subsidy scheme or an asset market structure that is capable to insure all households against this dispersion. Both instruments would be controversial with one of the main objectives of this paper, which is evaluating the optimal policy under the assumption that the government does not have access to any sort of lump sum scheme to support the agents. Another alternative to avoid the dispersion in wages is to assume the presence of a centralized union that coordinates the supply of labor among households, as proposed in Schmitt-Grohé and Uribe (2005b)[67]. The assumption of a labor union with such market power, however, does not seem reasonable for developed small-open economies outside Scandinavian countries⁶.

⁵ See, for instance, Chugh (2006)[24] and García-Cicco (2009)[37].

⁶ According to data from OECD (2004), only Denmark, Sweden, Finland, Iceland and Belgium presented a steady increase in the percentage of workers associated with an union (trade-union density) from 1960 to 2000. In Latin American economies like Mexico and Chile, recently associated with OECD, the trade-union density is significantly lower compared to Scandinavian countries, and declining since the 1990's (see Visser Martin Tergeist, 2008[74]).

Still in the budget constraint, households are able to allocate wealth over time buying one-period, non-state contingent nominal bonds from the government, $B_{h,t+1}(i)$, or from the rest of the world, $IB_{t+1}(i)$. In the later case, the bonds are priced in foreign currency, and S_t is the nominal exchange rate. In order to adjust its portfolio, and to induce stationarity in the model, the households incurs in adjustment costs, both domestically and in the international financial markets, based on the variance of the stock of bonds as a proportion of the GDP⁷. Households also receive (after-tax) dividends from the firms $\Phi_t(i)$.

Finally, following Schmitt-Grohé and Uribe (2007)[68], households demand money, $M_t(i)$, in a cash-in-advance constraint, in order to pay for a share $\nu^m \geq 0$ of the after-tax consumption. The constraint holds with equality as long as (gross) nominal interest rates, R_t , are larger than unity. The sequence of events in each period for the households is the same as in CEE (2005)[22], with the households first deciding consumption and capital allocation, then deciding, in sequence, the financial portfolio, wages and the labor supply, and the final composition of portfolio between bonds and money. Thus, domestic currency, in this model, is expressed as an end-of-period aggregate.

Define $\tilde{\lambda}_t/P_t$, $\tilde{\lambda}_t\tilde{q}_{j,t}$, $\lambda_t^m\tilde{\lambda}_t$ and $(\tilde{\lambda}_t(1-\tau_t^h)W_t)/(P_tmcw_t)$ the Lagrange multipliers on the budget constraint, on the capital accumulation equations, on the cash-in-advance constraint and on the labor demand function, respectively. After taking the first order conditions of the Lagrangian of the household's problem, and using the fact that the equilibrium is symmetric (note especially that, in equilibrium, $C_t(i) = C_t$ and $W_t(i) = W_t$), the final set of equilibrium conditions of the intertem-

⁷ See Schmitt-Grohé and Uribe (2003b)[65]. The functional form adopted is the same as the model proposed by those authors. However, the use of the ratio to GDP is adopted here to obtain the stationary form of the model.

poral problem of the household is given by:

$$\frac{(1 - \tau_t^h) \widetilde{W}_t}{(1 + \tau_t^c)(C_t - \zeta C_{t-1})} = \frac{\gamma}{(1 - \gamma)} \frac{mcw_t \left(1 + \nu^m \left(\frac{\widetilde{R}_t - 1}{\widetilde{R}_t}\right)\right)}{(1 - h_t)} \quad (1.1)$$

$$\frac{(1 - \gamma)}{C_t - \zeta C_{t-1}} = (1 + \tau_t^c) \widetilde{\lambda}_t \left(1 + \nu^m \left(\frac{R_t - 1}{R_t}\right)\right) \quad (1.2)$$

$$\widetilde{\lambda}_t \left[1 - \psi_1 \left(\frac{B_{h,t+1}}{Y_t} - \frac{B_h}{Y}\right)\right] = \beta R_t E_t \left(\frac{\widetilde{\lambda}_{t+1}}{\pi_{t+1}}\right) \quad (1.3)$$

$$\widetilde{\lambda}_t \left[1 - \psi_2 \left(\frac{S_t I B_{t+1}}{Y_t} - \frac{rer IB}{Y}\right)\right] = \beta R_t^f E_t \left(\frac{S_{t+1}}{S_t} \frac{P_t}{P_{t+1}} \widetilde{\lambda}_{t+1}\right) \quad (1.4)$$

$$\begin{aligned} \widetilde{\lambda}_t \widetilde{q}_{x,t} = \\ \beta E_t \left\{ \widetilde{\lambda}_{t+1} \left[(1 - \tau_{t+1}^k) (R_{x,t+1}^k \mu_{x,t+1} - \Upsilon_{t+1}^{-1} a(\mu_{x,t+1})) + \widetilde{q}_{x,t+1} (1 - \delta) \right] \right\} \end{aligned} \quad (1.5)$$

$$\begin{aligned} \widetilde{\lambda}_t \widetilde{q}_{n,t} = \\ \beta E_t \left\{ \left[\widetilde{\lambda}_{t+1} (1 - \tau_{t+1}^k) (R_{n,t+1}^k \mu_{n,t+1} - \Upsilon_{t+1}^{-1} a(\mu_{n,t+1})) + \widetilde{q}_{n,t+1} (1 - \delta) \right] \right\} \end{aligned} \quad (1.6)$$

$$K_{n,t} = \mu_{n,t} \overline{K}_{n,t} \quad (1.7)$$

$$K_{x,t} = \mu_{x,t} \overline{K}_{x,t} \quad (1.8)$$

$$\theta_1 + \theta_2 (\mu_{n,t} - 1) = \frac{R_{n,t}^k}{\Upsilon_t^{-1}} \quad (1.9)$$

$$\theta_1 + \theta_2 (\mu_{x,t} - 1) = \frac{R_{x,t}^k}{\Upsilon_t^{-1}} \quad (1.10)$$

$$R_t = \frac{1}{r_{t,t+1}} \quad (1.11)$$

$$\tilde{R}_t = R_t \left(1 - \psi_1 \left(\frac{B_{h,t+1}}{Y_t} - \frac{B_h}{Y} \right) \right)^{-1} \quad (1.12)$$

$$\begin{aligned} \tilde{\lambda}_t \Upsilon_t^{-1} &= \tilde{\lambda}_t \tilde{q}_{x,t} \left[1 - \Psi \left(\frac{I_{x,t}^d}{I_{x,t-1}^d} \right) - \left(\frac{I_{x,t}^d}{I_{x,t-1}^d} \right) \Psi' \left(\frac{I_{x,t}^d}{I_{x,t-1}^d} \right) \right] \\ &\quad + \beta E_t \left[\tilde{\lambda}_{t+1} \tilde{q}_{x,t+1} \left(\frac{I_{x,t+1}^d}{I_{x,t}^d} \right)^2 \Psi' \left(\frac{I_{x,t+1}^d}{I_{x,t}^d} \right) \right] \end{aligned} \quad (1.13)$$

$$\begin{aligned} \tilde{\lambda}_t \Upsilon_t^{-1} &= \tilde{\lambda}_t \tilde{q}_{n,t} \left[1 - \Psi \left(\frac{I_{n,t}^d}{I_{n,t-1}^d} \right) - \left(\frac{I_{n,t}^d}{I_{n,t-1}^d} \right) \Psi' \left(\frac{I_{n,t}^d}{I_{n,t-1}^d} \right) \right] \\ &\quad + \beta E_t \left[\tilde{\lambda}_{t+1} \tilde{q}_{n,t+1} \left(\frac{I_{n,t+1}^d}{I_{n,t}^d} \right)^2 \Psi' \left(\frac{I_{n,t+1}^d}{I_{n,t}^d} \right) \right] \end{aligned} \quad (1.14)$$

$$\bar{K}_{x,t+1}(i) = (1 - \delta) \bar{K}_{x,t}(i) + I_{x,t}^d(i) \left(1 - \aleph \left(\frac{I_{x,t}^d(i)}{I_{x,t-1}^d(i)} \right) \right) \quad (1.15)$$

$$\bar{K}_{n,t+1}(i) = (1 - \delta) \bar{K}_{n,t}(i) + I_{n,t}^d(i) \left(1 - \aleph \left(\frac{I_{n,t}^d(i)}{I_{n,t-1}^d(i)} \right) \right) \quad (1.16)$$

$$\begin{aligned} \left(\frac{1 - \varpi}{\varpi} + \frac{1}{mcw_t} \right) \varpi h_t (1 - \tau_t^h) &= - \frac{\phi_w}{\pi_t^{\chi_w - 1}} \left(\frac{\widetilde{W}_t}{\widetilde{W}_{t-1}} \right) \left(\frac{\widetilde{W}_t}{\pi_t^{\chi_w - 1} \widetilde{W}_{t-1}} - \mu^I \right) \\ &\quad + \beta E_t \left[\frac{\tilde{\lambda}_{t+1} \phi_w}{\tilde{\lambda}_t \pi_{t+1}^{\chi_w - 1}} \left(\frac{\widetilde{W}_{t+1}}{\widetilde{W}_t} \right)^2 \left(\frac{\widetilde{W}_{t+1}}{\pi_{t+1}^{\chi_w - 1} \widetilde{W}_t} - \mu^I \right) \right] \end{aligned} \quad (1.17)$$

From the first order equilibrium conditions, notice that the uncovered interest parity (UIP) condition between domestic interest rates and the interest rates in international financial markets can be recovered after the linearization of equations 1.3 and 1.4. The UIP condition holds in its strict sense only in the steady state, since the non-linear dynamics is also influenced by the presence of domestic and foreign

portfolio adjustment costs. This is a small departure from other studies for small-open economies, like Adolfson, Laseén, Lindé and Villani (2007)[2], where the only source of discrepancy between the domestic and foreign interest rates from the UIP condition is the debt-elastic foreign interest rate, like the one described in Schmitt-Grohé and Uribe (2003b)[65]. As the description of the foreign block of the model will make clear, the UIP condition adopted here is a combination of the debt-elastic foreign interest rate and the portfolio adjustment cost proposed in Schmitt-Grohé and Uribe (2003b)[65].

In the first stage of the decision in each period, the household also solves a sequence of minimization problems constrained by the CES aggregator function in order to choose the composition of the consumption and investment baskets. Expressing first the consumption problem, households decide the composition between imported and domestically produced goods in the tradable goods basket, and then chooses the optimal composition of tradable and non-tradable goods. For simplicity of exposition, assume also that the portfolio adjustment costs are paid with a share of the consumption goods acquired by the households. As a consequence, the cost minimization problem of the household is given by:

$$\min_{C_{n,t}, C_{i,t}, C_{m,t}, C_{x,t}} P_{n,t}C_{n,t} + P_{t,t}C_{t,t}$$

$$C_t + PAC_{b,t} + PAC_{ib,t} = \left[(1 - \omega)^{\frac{1}{\varepsilon}} C_{n,t}^{\frac{\varepsilon-1}{\varepsilon}} + \omega^{\frac{1}{\varepsilon}} C_{t,t}^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} \quad (1.18)$$

$$C_{t,t} = \left[(1 - \varkappa)^{\frac{1}{\varrho}} C_{x,t}^{\frac{\varrho-1}{\varrho}} + \varkappa^{\frac{1}{\varrho}} C_{m,t}^{\frac{\varrho-1}{\varrho}} \right]^{\frac{\varrho}{\varrho-1}} \quad (1.19)$$

$$PAC_{b,t} = \frac{\psi_1}{2} Y_t \left(\frac{B_{h,t+1}}{Y_t} - \frac{B_h}{Y} \right)^2$$

$$PAC_{ib,t} = \frac{\psi_2}{2} Y_t \left(\frac{S_t IB_{t+1}}{Y_t} - \frac{rer IB}{Y} \right)^2$$

Combine the first order conditions with the expenditure function to obtain the demand for each type of tradable good and the price index of tradable goods:

$$C_{m,t} = \varkappa \left(\frac{P_{m,t}}{P_{t,t}} \right)^{-\varrho} C_{t,t} \quad (1.20)$$

$$C_{x,t} = (1 - \varkappa) \left(\frac{P_{x,t}}{P_{t,t}} \right)^{-\varrho} C_{t,t} \quad (1.21)$$

$$P_{t,t} = \left[(1 - \varkappa) P_{x,t}^{1-\varrho} + \varkappa P_{m,t}^{1-\varrho} \right]^{\frac{1}{1-\varrho}}$$

By analogy, the optimal decision between tradable and non-tradable goods and the CPI index is given by:

$$C_{t,t} = \omega \left(\frac{P_{t,t}}{P_t} \right)^{-\varepsilon} (C_t + PAC_{b,t} + PAC_{ib,t}) \quad (1.22)$$

$$C_{n,t} = (1 - \omega) \left(\frac{P_{n,t}}{P_t} \right)^{-\varepsilon} (C_t + PAC_{b,t} + PAC_{ib,t}) \quad (1.23)$$

$$P_t = \left[(1 - \omega) P_{n,t}^{1-\varepsilon} + \omega P_{t,t}^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}$$

Households also solve an equivalent problem when setting the composition of the investment good for each sector. For simplicity, assume that the weights in the basket of goods and the elasticities of substitution among different types of investment goods is the same as the weights and the elasticities for consumption goods. Also, assume that the adjustment costs in capital utilization are paid in terms of aggregate investment. As a consequence, the demands for home produced and imported investment goods are given by:

$$\Upsilon_t^{-1} I_t = \Upsilon_t^{-1} (I_{n,t}^d + a(\mu_{n,t}) \bar{K}_{n,t} + I_{x,t}^d + a(\mu_{x,t}) \bar{K}_{x,t}) \quad (1.24)$$

$$I_{m,t} = \varkappa \left(\frac{P_{m,t}}{P_{t,t}} \right)^{-\varrho} I_{t,t} \quad (1.25)$$

$$I_{x,t} = (1 - \varkappa) \left(\frac{P_{x,t}}{P_{t,t}} \right)^{-\varrho} I_{t,t} \quad (1.26)$$

$$I_{t,t} = \omega \left(\frac{P_{t,t}}{P_t} \right)^{-\varepsilon} \Upsilon_t^{-1} I_t \quad (1.27)$$

$$I_{n,t} = (1 - \omega) \left(\frac{P_{n,t}}{P_t} \right)^{-\varepsilon} \Upsilon_t^{-1} I_t \quad (1.28)$$

$$\Upsilon_t^{-1} I_t = \left[(1 - \omega)^{\frac{1}{\varepsilon}} I_{n,t}^{\frac{\varepsilon-1}{\varepsilon}} + \omega^{\frac{1}{\varepsilon}} I_{t,t}^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

$$I_{t,t} = \left[(1 - \varkappa)^{\frac{1}{\varrho}} I_{x,t}^{\frac{\varrho-1}{\varrho}} + \varkappa^{\frac{1}{\varrho}} I_{m,t}^{\frac{\varrho-1}{\varrho}} \right]^{\frac{\varrho}{\varrho-1}}$$

1.2 Firms

There are four sectors in the economy, each sector composed by a continuum of firms operating in a monopolistic competitive framework. Firms in the non-tradable (n) and tradable (x) sectors demand labor and capital to produce. Firms in the imported goods (m) sector and in the exported goods (xp) sector buy the final good and sell it to consumers in the domestic economy (for the case of imported goods sector firms) or in the rest of the world (for the case of exported goods sector firms). Firms chooses the amount of production inputs to buy and set new prices according to a probability $\alpha_i, i = \{n, x, m, xp\}$, that is independent across sectors and across firms. If a firm is not allowed to optimize prices in period t , it changes prices according to an indexation rule based on past inflation. Imported goods' firms must finance the total amount of imported goods using only foreign currency. There is no firm entry into or exit out of sector i and also no change of firms across sectors.

The four sectors would result, in a log-linearized model around price stability, in four equations like the New Keynesian Phillips curve describing the dynamics of prices. However, since price stability might not be optimal policy for Ramsey planner, the recursive formulation for the first order condition of firms in terms of prices described in Schmitt-Grohé and Uribe (2005b)[67] is adopted. The recursive formulation is flexible enough to accommodate price stability as one special case,

and also allows, for estimation purposes, matching the steady state inflation with the average inflation in the sample.

1.2.1 Domestic non-tradable goods' producers problem:

Firms in the non-tradable sector use capital and labor to produce goods that are used for consumption, investment and spent by the government. The production technology is a simple Cobb-Douglas function with a sectorial stationary productivity shock and a non-stationary, labor-augmenting technological shock. Setting real profits for firm i_n as $\Phi_{n,t}(i_n)$, the problem of the domestic, non-tradable goods producers of type i_n product ($i_n \in [0, 1]$) is to maximize the expected discounted stream of profits, subject to the demand for good i_n , the production technology and the aggregate demand for non-tradable goods. In order to solve the problem, firms choose in each period the demand for labor, capital and, if allowed to do so with probability $1 - \alpha_n$, they optimize prices. The statement of the problem is given by:

$$\max E_0 \sum_{t=0}^{\infty} r_{0,t} P_{n,t} \left(\frac{P_{n,t}(i_n)}{P_{n,t}} D_{n,t}(i_n) - \frac{W_t}{P_{n,t}} h_{n,t}(i_n) - \frac{P_t}{P_{n,t}} R_{n,t}^k K_{n,t}(i_n) \right)$$

$$s.t. : \quad D_{n,t}(i_n) = \left(\frac{P_{n,t}(i_n)}{P_{n,t}} \right)^{-\eta_n} Y_{n,t}$$

$$Y_{n,t} = C_{n,t} + G_{n,t} + \Upsilon_t^{-1} \frac{P_t}{P_{n,t}} I_{n,t}$$

$$a_{n,t} K_{n,t}(i_n)^\theta (z_t h_{n,t}(i_n))^{1-\theta} - z_t^* \chi_n \geq D_{n,t}(i_n)$$

$$\Upsilon_t^{\frac{\theta}{1-\theta}} = \frac{z_t^*}{z_t}$$

$$\frac{z_{t+1}}{z_t} = \mu_{t+1}^z = (1 - \rho_z) \mu^z + \rho_z \mu_t^z + \epsilon_{t+1}^z; \quad \rho_z \in [0, 1); \quad \epsilon_t^z \sim N(0, \sigma_z)$$

$$\log a_{n,t+1} = \rho_n \log a_{n,t} + \epsilon_{t+1}^n; \quad \rho_n \in [0, 1); \quad \epsilon_t^n \sim N(0, \sigma_n)$$

In this problem, $a_{n,t}$ is a stationary, sector-specific technology shock, z_t is a labor-augmenting, non-stationary technology shock. The non-stationary shock z_t affects all the firms using labor in production. In order to guarantee zero profits in the

steady state, $z_t^* \chi_n$ introduces a fixed cost proportional to the evolution of the non-stationary shocks in production, following CEE (2005)[22], Schmitt-Grohé and Uribe (2005, 2005b)[66][67], among others. Describing the demand for good i_n , parameter η_n is the elasticity of substitution across varieties of non-tradable goods.

From the first order conditions in terms of $h_{n,t}(i_n)$ and $K_{n,t}(i_n)$, it is possible to prove that the capital-labor ratio is the same across firms in the non-tradable sector. As a consequence, the marginal cost across firms is also the same in this sector. Setting $mc_{n,t}$ as the Lagrange multiplier on the firm's demand constraint, the two equilibrium conditions are:

$$\widetilde{W}_t \frac{P_t}{P_{n,t}} = mc_{n,t} (1 - \theta) a_{n,t} z_t \left(\frac{K_{n,t}}{z_t h_{n,t}} \right)^\theta \quad (1.29)$$

$$R_{n,t}^k \frac{P_t}{P_{n,t}} = mc_{n,t} \theta a_{n,t} \left(\frac{K_{n,t}}{z_t h_{n,t}} \right)^{\theta-1} \quad (1.30)$$

Prices are formed in a Calvo style with indexation, where α_n is the probability that firm i_n is not allowed to optimally adjust its price in period t . In the case firms are not allowed to set up prices optimally, they follow the simple rule $P_{n,t}(i_n) = \pi_{n,t-1}^{\kappa_n} P_{n,t-1}(i_n)$, for $0 \leq \kappa_n \leq 1$ and $\pi_{n,t+1} = \frac{P_{n,t+1}}{P_{n,t}}$. Setting the Lagrangean of the problem, considering only the relevant terms for price determination:

$$\begin{aligned} n = E_t \sum_{s=0}^{\infty} \alpha_n^s r_{t,t+s} P_{n,t+s} & \left(\left(\frac{\widetilde{P}_{n,t}(i_n)}{P_{n,t+s}} \right)^{1-\eta_n} \prod_{k=1}^s \left(\frac{\pi_{n,t+k-1}^{\kappa_n}}{\pi_{n,t+k}} \right)^{1-\eta_n} Y_{n,t+s} \right. \\ & \left. - mc_{n,t+s} \left(\left(\frac{\widetilde{P}_{n,t}(i_n)}{P_{n,t+s}} \right)^{-\eta_n} \prod_{k=1}^s \left(\frac{\pi_{n,t+k-1}^{\kappa_n}}{\pi_{n,t+k}} \right)^{-\eta_n} Y_{n,t+s} \right) \right) \end{aligned}$$

In this problem, $r_{t,t+s}$ is the stochastic discount factor between periods t and $t+s$, and $\widetilde{P}_{n,t}(i_n)$ is the new price set by firms allowed to adjust prices in period t . The

first order condition for firms allowed to adjust prices is:

$$E_t \sum_{s=0}^{\infty} \alpha_n^s r_{t,t+s} Y_{n,t+s} P_{n,t+s} \left(\frac{\tilde{P}_{n,t}(i_n)}{P_{n,t+s}} \right)^{-\eta_n} \prod_{k=1}^s \left(\frac{\pi_{n,t+k-1}^{\kappa_n}}{\pi_{n,t+k}} \right)^{-\eta_n} \times \\ \left(\frac{(\eta_n - 1) \tilde{P}_{n,t}(i_n)}{\eta_n P_{n,t+s}} \prod_{k=1}^s \left(\frac{\pi_{n,t+k-1}^{\kappa_n}}{\pi_{n,t+k}} \right) - m c_{n,t+s} \right) = 0$$

As a consequence of the first order condition, given that mark-up over prices is the same across firms, the symmetric equilibrium is characterized by all firms in the non-tradable sector allowed to adjust prices in period t setting the same price: $\tilde{P}_{n,t}(i_n) = \tilde{P}_{n,t}$. Following Schmitt-Grohé and Uribe (2005b)[67], split the pricing function equation in two parts, X_t^1 and X_t^2 , and define $\tilde{p}_{n,t} = \frac{\tilde{P}_{n,t}}{P_{n,t}}$ in order to obtain the recursive solution for the problem of the non-tradable goods' producers:

$$X_t^1 = E_t \sum_{s=0}^{\infty} \alpha_n^s r_{t,t+s} Y_{n,t+s} P_{n,t+s} \left(\frac{\tilde{P}_{n,t}}{P_{n,t}} \right)^{-1-\eta_n} \prod_{k=1}^s \left(\frac{\pi_{n,t+k-1}^{\kappa_n}}{\pi_{n,t+k}} \right)^{-\eta_n} m c_{n,t+s} \\ X_t^2 = E_t \sum_{s=0}^{\infty} \alpha_n^s r_{t,t+s} Y_{n,t+s} P_{n,t+s} \left(\frac{\tilde{P}_{n,t}}{P_{n,t}} \right)^{-\eta_n} \prod_{k=1}^s \left(\frac{\pi_{n,t+k-1}^{\kappa_n}}{\pi_{n,t+k}} \right)^{-\eta_n} \left(\frac{(\eta_n - 1)}{\eta_n} \prod_{k=1}^s \left(\frac{\pi_{n,t+k-1}^{\kappa_n}}{\pi_{n,t+k}} \right) \right)$$

The system describing the evolution of non-tradable inflation is given by:

$$X_t^1 = Y_{n,t} \tilde{p}_{n,t}^{-1-\eta_n} m c_{n,t} + \alpha_n r_{t,t+1} E_t \left(\frac{\tilde{p}_{n,t}}{\tilde{p}_{n,t+1}} \right)^{-1-\eta_n} \left(\frac{\pi_{n,t}^{\kappa_n}}{\pi_{n,t+1}^{(1+\eta_n)/\eta_n}} \right)^{-\eta_n} X_{t+1}^1 \quad (1.31)$$

$$X_t^2 = Y_{n,t} \tilde{p}_{n,t}^{-\eta_n} \frac{(\eta_n - 1)}{\eta_n} + \alpha_n r_{t,t+1} E_t \left(\frac{\tilde{p}_{n,t}}{\tilde{p}_{n,t+1}} \right)^{-\eta_n} \left(\frac{\pi_{n,t}^{\kappa_n}}{\pi_{n,t+1}^{\eta_n/(\eta_n-1)}} \right)^{1-\eta_n} X_{t+1}^2 \quad (1.32)$$

$$X_t^1 = X_t^2 \quad (1.33)$$

1.2.2 Tradable goods' producers problem:

A tradable goods producer i_x ($i_x \in [0, 1]$) solves the same problem as the non-tradable producer, using labor and capital as production factors. The total production of the

tradable good is divided between domestic absorption (consumption, investment and government spending) and the demand of a continuum of i_{xp} exporting firms ($D_{xp,t}$).

The tradable goods' firm problem is given by:

$$\max E_0 \sum_{t=0}^{\infty} r_{0,t} P_{x,t} \left(\frac{P_{x,t}(i_x)}{P_{x,t}} D_{x,t}(i_x) - \frac{\widetilde{W}_t}{P_{x,t}} h_{x,t}(i_x) - \frac{P_t}{P_{x,t}} R_{x,t}^k K_{x,t}(i_x) \right)$$

$$s.t. : \quad D_{x,t}(i_x) = \left(\frac{P_{x,t}(i_x)}{P_{x,t}} \right)^{-\eta_x} Y_{x,t}$$

$$Y_{x,t} = C_{x,t} + G_{t,t} + \Upsilon_t^{-1} \frac{P_t}{P_{x,t}} I_{x,t} + D_{xp,t}$$

$$a_{x,t} K_{x,t}(i_x)^\theta (z_t h_{x,t}(i_x))^{1-\theta} - z_t^* \chi_x \geq D_{x,t}(i_{tr})$$

$$\Upsilon_t^{\frac{\theta}{1-\theta}} = \frac{z_t^*}{z_t}$$

$$\frac{z_{t+1}}{z_t} = \mu_{t+1}^z = (1 - \rho_z) \mu^z + \rho_z \mu_t^z + \epsilon_{t+1}^z; \quad \rho_z \in [0, 1); \quad \epsilon_t^z \sim N(0, \sigma_z)$$

$$\log a_{x,t+1} = \rho_x \log a_{x,t} + \epsilon_{t+1}^x; \quad \rho_x \in [0, 1); \quad \epsilon_t^x \sim N(0, \sigma_x)$$

χ_x is a fixed cost proportional to total output associated with the non-stationary shock in order to guarantee zero profits in steady state. Parameter η_x is the elasticity of substitution across varieties of tradable goods. Setting $mc_{x,t}$ as the Lagrange multiplier on the firm's demand constraint, the solution of the cost minimization problem of the firm in terms of $h_{x,t}(i_x)$ and $K_{x,t}(i_x)$, after using again the fact that the capital-labor ratio is the same across firms, become:

$$\widetilde{W}_t \frac{P_t}{P_{x,t}} = mc_{x,t} (1 - \theta) a_{x,t} z_t \left(\frac{K_{x,t}}{z_t h_{x,t}} \right)^\theta \quad (1.34)$$

$$R_{x,t}^k \frac{P_t}{P_{x,t}} = mc_{x,t} \theta a_{x,t} \left(\frac{K_{x,t}}{z_t h_{x,t}} \right)^{\theta-1} \quad (1.35)$$

Similar to the firms in the non-tradable sector, price adjustment is based on the Calvo mechanism with indexation to past inflation, with $0 \leq \kappa_x \leq 1$ defining the degree of indexation in the tradable sector. Taking the first order conditions in terms

of $\tilde{P}_{x,t}(i_x)$, and defining $\pi_{x,t+1} = \frac{P_{x,t+1}}{\tilde{P}_{x,t}}$, the optimal price set by each firm is:

$$E_t \sum_{s=0}^{\infty} \alpha_x^s r_{t,t+s} Y_{x,t+s} P_{x,t+s} \left(\frac{\tilde{P}_{x,t}(i_x)}{P_{x,t}} \right)^{-\eta_x} \prod_{k=1}^s \left(\frac{\pi_{x,t+k-1}^{\kappa_x}}{\pi_{x,t+k}} \right)^{-\eta_x} \times$$

$$\left(\frac{(\eta_x - 1)}{\eta_x} \frac{\tilde{P}_{x,t}(i_x)}{P_{x,t}} \prod_{k=1}^s \left(\frac{\pi_{x,t+k-1}^{\kappa_x}}{\pi_{x,t+k}} \right) - mc_{x,t+s} \right) = 0$$

As a consequence of the same mark-up over prices across firms, the symmetric equilibrium is characterized by all firms in the tradable sector setting the same price when allowed to optimize, $\tilde{P}_{x,t}(i_x) = \tilde{P}_{x,t}$. The recursive solution for the pricing problem of the importing firms is obtained after properly defining Z_t^1, Z_t^2 , such that $Z_t^1 = Z_t^2$, and $\tilde{p}_{x,t} = \frac{\tilde{P}_{x,t}}{P_{x,t}}$:

$$Z_t^1 = \tilde{p}_{x,t}^{-1-\eta_x} Y_{x,t} mc_{x,t} + \alpha_x r_{t,t+1} E_t \left(\frac{\tilde{P}_{x,t}}{\tilde{p}_{x,t+1}} \right)^{-1-\eta_x} \left(\frac{\pi_{x,t}^{\kappa_x}}{\pi_{x,t+1}^{(1+\eta_x)/\eta_x}} \right)^{-\eta_x} Z_{t+1}^1 \quad (1.36)$$

$$Z_t^2 = \tilde{p}_{x,t}^{-\eta_x} Y_{x,t} \frac{(\eta_x - 1)}{\eta_x} + \alpha_x r_{t,t+1} E_t \left(\frac{\tilde{P}_{x,t}}{\tilde{p}_{x,t+1}} \right)^{-\eta_x} \left(\frac{\pi_{x,t}^{\kappa_x}}{\pi_{x,t+1}^{\eta_x/(\eta_x-1)}} \right)^{1-\eta_x} Z_{t+1}^2 \quad (1.37)$$

$$Z_t^1 = Z_t^2 \quad (1.38)$$

1.2.3 Imported goods' firms problem:

Following Lubik and Schorfheide (2006)[50], deviations from the Law of One price in the model arises as a consequence of price stickiness in imported and exported goods. An imported goods' firm i_m ($i_m \in [0, 1]$) buys a bundle of the international homogeneous good⁸ and relabel it as an imported good type i_m . In order to buy the goods produced in the rest of the world, the firm needs to make payments using foreign currency. The firm sells intraperiod bonds in foreign markets in order

⁸ Note that, in the model, one country buys a combination of the goods from different countries. As a consequence, there is a gap between the world's CPI (P_t^*) and the price of the bundle imported by a given country ($P_{m,t}^*$).

to get foreign currency, but it does not transfer financial wealth over time. As a consequence, firms do not incur in exposure to risk in the international markets, just an increase in the marginal cost of production. As a timing convention, the bonds traded do not reflect in the end of period balance of payments. The same framework is adopted in Christiano, Trabandt and Walentin (2007)[23] and Mendoza and Yue (2008)[54]. The budget constraint of the exporting firm i_m , expressed in terms of domestic prices, is given by:

$$\frac{S_t P_t^*}{P_t} M_{m,t}^*(i_m) + \frac{S_t}{P_t} B_{m,t+1}^*(i_m) =$$

$$\frac{S_t}{P_t} P_{t-1}^* M_{m,t-1}^*(i_m) + \frac{S_t}{P_t} R_{t-1}^f B_{m,t}^*(i_m) + \left(\frac{P_{m,t}(i_m) - S_t P_{m,t}^*}{P_t} \right) D_{m,t}(i_m) - z_t^* \chi_m - \Phi_{m,t}(i_m)$$

where χ_m is a fixed cost associated with the non-stationary shock in order to guarantee zero profits in steady state. Following the assumption that firms do not keep any financial wealth across periods, and that all profits are distributed to the households, obtain the expression for real profits:

$$P_t^* M_{m,t}^*(i_m) + R_t^f B_{m,t+1}^*(i_m) = 0, \quad \forall t$$

$$\implies \frac{S_t P_t^*}{P_t} \left(M_{m,t}^*(i_m) - \frac{M_{m,t}^*(i_m)}{R_t^f} \right) = \left(\frac{P_{m,t}(i_m) - S_t P_{m,t}^*}{P_t} \right) D_{m,t}(i_m) - z_t^* \chi_m - \Phi_{m,t}(i_m)$$

$$\Phi_{m,t}(i_m) = \left(\frac{P_{m,t}(i_m) - S_t P_{m,t}^*}{P_t} \right) D_{m,t}(i_m) - z_t^* \chi_m - \frac{S_t P_t^*}{P_t} \left(\frac{R_t^f - 1}{R_t^f} \right) M_{m,t}^*(i_m)$$

where χ_m is a fixed cost associated with the non-stationary shock in order to guarantee zero profits in steady state. The imported goods' firm problem is given by:

$$\max_{\bar{P}_{m,t}(i_m)} E_0 \sum_{t=0}^{\infty} r_{0,t} \left[\left(\frac{P_{m,t}(i_m) - S_t P_{m,t}^*}{P_t} \right) D_{m,t}(i_m) - z_t^* \chi_m - \frac{S_t P_{m,t}^*}{P_t} \left(\frac{R_t^f - 1}{R_t^f} \right) \frac{P_t^*}{P_{m,t}^*} M_{m,t}^*(i_m) \right]$$

$$s.t. : D_{m,t}(i_m) = \left(\frac{P_{m,t}(i_m)}{P_{m,t}} \right)^{-\eta_m} \left(C_{m,t} + \Upsilon_t^{-1} \frac{P_t}{P_{m,t}} I_{m,t} \right)$$

$$M_{m,t}(i_m) \geq \frac{P_{m,t}^*}{P_t^*} D_{m,t}(i_m)$$

where $P_{m,t}^*$ is the price of the imported good bought by the domestic economy, quoted in foreign prices. Parameter η_m is the elasticity of substitution across varieties of imported goods. Taking the first order conditions in terms of $\tilde{P}_{m,t}(i_m)$, the price for those firms allowed to optimize prices in period t , and defining $\pi_{m,t+1} = \frac{P_{m,t+1}}{P_{m,t}}$ and $0 \leq \kappa_m \leq 1$ the degree of indexation in the imported goods' sector, the expression for the optimal price set by each firm becomes:

$$E_t \sum_{s=0}^{\infty} \alpha_m^s r_{t,t+s} P_{m,t+s} \left(C_{m,t+s} + \Upsilon_{t+s}^{-1} \frac{P_{t+s}}{P_{m,t+s}} I_{m,t+s} \right) \left(\frac{\tilde{P}_{m,t}(i_m)}{P_{m,t+s}} \right)^{-\eta_m} \prod_{k=1}^s \left(\frac{\pi_{m,t+k-1}^{\kappa_m}}{\pi_{m,t+k}} \right)^{-\eta_m} \times \\ \left(\frac{(\eta_m - 1) \tilde{P}_{m,t}(i_m)}{\eta_m P_{m,t+s}} \prod_{k=1}^s \left(\frac{\pi_{m,t+k-1}^{\kappa_m}}{\pi_{m,t+k}} \right) - \frac{S_{t+s} P_{m,t+s}^*}{P_{m,t+s}} \left(1 + \frac{R_{t+s}^f - 1}{R_{t+s}^f} \right) \right) = 0$$

In this problem, α_m is the probability that the importing firm i_m is not allowed to adjust its price in period t . As a consequence of the same mark-up over prices across firms (in this case, given by the real exchange rate deflated by the import price level), the symmetric equilibrium is characterized by $\tilde{P}_{m,t}(i_m) = \tilde{P}_{m,t}$. The recursive solution for the pricing problem of the importing firms is obtained after properly defining Y_t^1 and Y_t^2 such that $Y_t^1 = Y_t^2$, and $\tilde{p}_{m,t} = \frac{\tilde{P}_{m,t}}{P_{m,t}}$:

$$Y_t^1 = \tilde{p}_{m,t}^{-1-\eta_m} \left(C_{m,t} + \Upsilon_t^{-1} \frac{P_t}{P_{m,t}} I_{m,t} \right) \frac{S_t P_{m,t}^*}{P_{m,t}} \left(1 + \frac{R_t^f - 1}{R_t^f} \right) \\ + \alpha_m r_{t,t+1} E_t \left(\frac{\tilde{p}_{m,t}}{\tilde{p}_{m,t+1}} \right)^{-1-\eta_m} \left(\frac{\pi_{m,t}^{\kappa_m}}{\pi_{m,t+1}^{(1+\eta_m)/\eta_m}} \right)^{-\eta_m} Y_{t+1}^1 \quad (1.39)$$

$$\begin{aligned}
Y_t^2 = & \tilde{p}_{m,t}^{-\eta_m} \left(C_{m,t} + \Upsilon_t^{-1} \frac{P_t}{P_{m,t}} I_{m,t} \right) \frac{(\eta_m - 1)}{\eta_m} \\
& + \alpha_m r_{t,t+1} E_t \left(\frac{\tilde{p}_{m,t}}{\tilde{p}_{m,t+1}} \right)^{-\eta_m} \left(\frac{\pi_{m,t}^{\kappa_m}}{\pi_{m,t+1}^{\eta_m/(\eta_m-1)}} \right)^{1-\eta_m} Y_{t+1}^2 \quad (1.40)
\end{aligned}$$

$$Y_t^1 = Y_t^2 \quad (1.41)$$

1.2.4 Exported goods' firms problem:

On the exported goods' side, there is a specific sector consuming tradable goods and, in a Calvo style, setting prices in foreign currency. An exported goods' firm i_{xp} ($i_{xp} \in [0, 1]$) buys a share of the final tradable good in the domestic economy and sell it to the rest of the world. Prices are sticky in foreign currency. The exported goods' firm problem is given by:

$$\begin{aligned}
\max_{\tilde{P}_{x,t}^*(i_{xp})} E_0 \sum_{t=0}^{\infty} r_{0,t} \left[\left(\frac{S_t \tilde{P}_{x,t}^*(i_{xp}) - P_{x,t}}{P_t} \right) D_{xp,t}(i_{xp}) - \left(\frac{R_t - 1}{R_t} \right) M_{xp,t}(i_{xp}) - z_t^* \chi_{xp} \right] \\
s.t. : D_{xp,t}(i_{xp}) = \left(\frac{P_{x,t}^*(i_{xp})}{P_{x,t}^*} \right)^{-\eta_{xp}} X_t
\end{aligned}$$

where χ_{xp} is a fixed cost associated with the non-stationary shock in order to guarantee zero profits in steady state. Parameter η_{xp} is the foreign elasticity of substitution across varieties of domestic exported goods. Taking the first order conditions in terms of $\tilde{P}_{x,t}^*(i_{xp})$, and defining $\pi_{x,t+1}^* = \frac{P_{x,t+1}^*}{P_{x,t}^*}$ and $0 \leq \kappa_{xp} \leq 1$ the degree of indexation in the exported goods' sector, the expression for the optimal price set by each firm becomes:

$$\begin{aligned}
E_t \sum_{s=0}^{\infty} \alpha_{xp}^s r_{t,t+s} P_{x,t+s}^* X_s \left(\frac{\tilde{P}_{x,s}^*(i_{xp})}{P_{x,s}^*} \right)^{-\eta_{xp}-1} \prod_{k=1}^s \left(\frac{(\pi_{x,t+k-1}^*)^{\kappa_{xp}}}{\pi_{x,t+k}^*} \right)^{-\eta_{xp}} \times \\
\left(\frac{(\eta_{xp} - 1) \tilde{P}_{x,t}^*(i_{xp})}{\eta_{xp} P_{x,t}^*} \prod_{k=1}^s \left(\frac{(\pi_{x,t+k-1}^*)^{\kappa_{xp}}}{\pi_{x,t+k}^*} \right) - \frac{P_{x,s}}{S_s P_{x,s}^*} \right) = 0
\end{aligned}$$

In this problem, α_{xp} is the probability that an importing firm i_x is not allowed to adjust its price in period t , $P_{x,t}^*$ is the price of the tradable good from the domestic economy quoted in foreign prices. The symmetric equilibrium is again characterized by $\tilde{P}_{x,t}^*(i_x) = \tilde{P}_{x,t}^*$. The recursive solution for the pricing problem of the exporting firms is obtained after properly defining U_t^1 and U_t^2 , such that $U_t^1 = U_t^2$, and $\tilde{p}_{x,t}^* = \frac{\tilde{P}_{x,t}^*}{P_{x,t}^*}$:

$$U_t^1 = (\tilde{P}_{x,t}^*)^{-1-\eta_{xp}} X_t \frac{P_{x,t}}{S_t P_{x,t}^*} + \alpha_{xp} r_{t,t+1} E_t \left(\frac{\tilde{P}_{x,t}^*}{\tilde{P}_{x,t+1}^*} \right)^{-1-\eta_{xp}} \left(\frac{(\pi_{x,t}^*)^{\kappa_{xp}}}{(\pi_{x,t+1}^*)^{\frac{(1+\eta_{xp})}{\eta_{xp}}}} \right)^{-\eta_{xp}} U_{t+1}^1 \quad (1.42)$$

$$U_t^2 = (\tilde{P}_{x,t}^*)^{-\eta_{xp}} X_t \frac{(\eta_{xp} - 1)}{\eta_{xp}} + \alpha_{xp} r_{t,t+1} E_t \left(\frac{\tilde{P}_{x,t}^*}{\tilde{P}_{x,t+1}^*} \right)^{-\eta_{xp}} \left(\frac{(\pi_{x,t}^*)^{\kappa_{xp}}}{(\pi_{x,t+1}^*)^{\frac{\eta_{xp}}{(\eta_{xp}-1)}}} \right)^{1-\eta_{xp}} U_{t+1}^2 \quad (1.43)$$

$$U_t^1 = U_t^2 \quad (1.44)$$

1.3 Government

In the competitive equilibrium of the economy, the government follows basic rules to set monetary and fiscal policy. In terms of monetary policy, a standard Taylor rule includes an autoregressive component, plus the deviations of inflation from an exogenous, autocorrelated inflation target, deviations of output from its steady-state, and changes in the real exchange rate:

$$\log \left(\frac{R_{t+1}}{R} \right) = \rho_R \log \left(\frac{R_t}{R} \right) + (1 - \rho_R) \left[\alpha_\pi \log \left(\frac{\pi_{t+1}}{\pi_{t+1}^o} \right) + \alpha_y \log \left(\frac{y_{t+1}}{y} \right) + \alpha_{rer} \log \left(\frac{rer_{t+1}}{rer_t} \right) \right] + \epsilon_{t+1}^R \quad (1.45)$$

$$\pi_{t+1}^o = (1 - \rho_{\pi^o})\pi^o + \rho_{\pi^o}\pi_t^o + \epsilon_{t+1}^{\pi^o} \quad (1.46)$$

$$\epsilon_t^R \sim N(0, \sigma_R) \quad \epsilon_t^{\pi^o} \sim N(0, \sigma_{\pi^o})$$

The government, in order to finance its exogenous expenditures, G_t , collects distortionary taxes on consumption, labor, capital and profits income (τ_t^c , τ_t^h , τ_t^k and τ_t^ϕ), sells bonds domestically, $B_{g,t}$ and controls the money supply, M_t . The government budget constraint is given by:

$$\begin{aligned} P_t G_t + R_{t-1} B_{g,t} &= P_t T_t + P_t M_t + B_{g,t+1} - P_{t-1} M_{t-1} \\ G_t &= z_t^* g_t \\ g_t &= (1 - \rho_g) g + \rho_g g_{t-1} + \epsilon_t^g \quad \epsilon_t^g \sim N(0, \sigma_g) \end{aligned} \quad (1.47)$$

$$\begin{aligned} T_t &= \tau_t^c C_t + \tau_t^h \widetilde{W}_t h_t + \tau_t^\phi \Phi_t \\ &+ \tau_t^k [(R_{n,t}^k \mu_{n,t} - \Upsilon_t^{-1} a(\mu_{n,t})) \bar{K}_{n,t} + (R_{x,t}^k \mu_{x,t} - \Upsilon_t^{-1} a(\mu_{x,t})) \bar{K}_{x,t}] \end{aligned} \quad (1.48)$$

Following Schmitt-Grohé and Uribe (2005b)[67], after defining the total real government liabilities (L_t), the evolution of government debt is pinned down by a fiscal policy rule where the government sets income taxation as a function of the gap between the actual liabilities as a proportion of GDP and its steady state value, plus a term related with the output gap, in order to account for the stabilization of the business cycle. Use the definition of net government liabilities to rewrite the budget constraint:

$$L_{t-1} \equiv M_{t-1} + \frac{R_{t-1}}{P_{t-1}} B_{g,t} \quad (1.49)$$

$$\implies L_t = \frac{R_t}{\pi_t} L_{t-1} + R_t (G_t - T_t) - (R_t - 1) M_t \quad (1.50)$$

To close the dynamics of the fiscal block, assume that the government follows a fiscal policy rule to determine the labor income taxation, while taxes on capital

and profits are exogenous. The assumption of a fiscal policy rule for labor income taxation is an arbitrary choice, since the presence of portfolio adjustment costs in domestic financial markets ensures stationarity in the model. Also, for simplicity, assume that the taxation on profits is constant over time. Notice that taxes on profits are lump sum transfers from the households to the government. In this sense, it does not interfere with the dynamics under the competitive equilibrium, where profits are zero.

$$\tau_t^h - \tau^h = \psi_{li} \left(\frac{L_t}{Y_t} - \frac{l}{y} \right) + \psi_y (y_t - y) + \epsilon_t^\tau \quad (1.51)$$

$$\tau_t^k = (1 - \rho_{\tau k}) \tau^k + \rho_{\tau k} \tau_{t-1}^k + \epsilon_t^{\tau k} \quad (1.52)$$

$$\tau_t^\phi = \tau^\phi \quad (1.53)$$

$$\tau_t^c = (1 - \rho_c) \tau^c + \rho_c \tau_{t-1}^c + \epsilon_t^{\tau c} \quad (1.54)$$

$$\epsilon_t^{\tau h} \sim N(0, \sigma_{\tau h}) \quad \epsilon_t^{\tau k} \sim N(0, \sigma_{\tau k}) \quad \epsilon_t^{\tau \phi} \sim N(0, \sigma_{\tau \phi}) \quad \epsilon_t^{\tau c} \sim N(0, \sigma_c)$$

Additionally, the government solves an equivalent problem as the households to determine their optimal consumption of tradable and non-tradable goods. By assumption, the government does not consume imported goods⁹. The demand for each type of good is given by:

$$G_{n,t} = (1 - \omega) \left(\frac{P_{n,t}}{P_t} \right)^{-\varepsilon} G_t \quad (1.55)$$

$$G_{t,t} = \omega \left(\frac{P_{t,t}}{P_t} \right)^{-\varepsilon} G_t \quad (1.56)$$

$$G_t = \left[(1 - \omega)^{\frac{1}{\varepsilon}} G_{n,t}^{\frac{\varepsilon-1}{\varepsilon}} + \omega^{\frac{1}{\varepsilon}} G_{t,t}^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

1.4 International Financial Markets and World's Economy

The transmission of shocks from international financial markets assume the existence of an international bond market capable of evaluating country-specific risk on bonds

⁹ The same assumption is used in Lubik and Schorfheide (2006)[50].

issued outside the domestic economy. In this sense, a mechanism to induce stationarity in the style proposed in Schmitt-Grohé and Uribe (2003b)[65] can be used to determine (and estimate) the risk premium of the bonds issued in each country as a function of the net foreign position of the economy. The international interest rate is given by:

$$R_t^f = R_t^* (1 + \xi_t)^{\kappa_1} \left(\frac{S_t IB_{t+1}}{P_t Y_t} / \frac{IB}{Y} \right)^{\kappa_2} \quad (1.57)$$

In this equation, R_t^* is a baseline, risk-free nominal interest rate on bonds traded in international markets; ξ_t is an autonomous shock in the risk premium, associated with the general risk level of the world's economy, with expected value equal to the long run risk premium demanded from the domestic economy, ξ^* ; the last term is the gap between total external debt of the domestic economy and its long run level.

The world's economy is modeled by a VAR containing measures of output, y_t^* , inflation, π_t^* , interest rates, R_t^* , growth of money supply, ΔM_t^* , and the risk premium, ξ_t . The objective of the VAR with this specification is to be as close as possible to the empirical studies of identification of shocks in the line of CEE (2005)[22], without imposing a prior theoretical specification for the economy. When compared with García-Cicco (2009)[37], the system of equations here has a different identification assumption for the shocks. Also, the inclusion of the risk premium tries to capture financial shocks that are not only unrelated with country-specific events, but also not associated with changes in foreign monetary policy. International shocks are identified with the Cholesky decomposition of the variance-covariance matrix of residuals. The world's output is added in order to identify supply from demand shocks in changes in the international prices. Thus, the VAR for the rest of the world

will provide five shocks for the domestic economy.

$$\begin{bmatrix} \frac{\Delta M_t^*}{\Delta M^*} \\ \frac{\xi_t}{\xi^*} \\ \frac{R_t^*}{R^*} \\ \frac{\pi_t^*}{\pi^*} \\ \frac{y_t^*}{y^*} \end{bmatrix} = A \begin{bmatrix} \frac{\Delta M_{t-1}^*}{\Delta M^*} \\ \frac{\xi_{t-1}}{\xi^*} \\ \frac{R_{t-1}^*}{R^*} \\ \frac{\pi_{t-1}^*}{\pi^*} \\ \frac{y_{t-1}^*}{y^*} \end{bmatrix} + \begin{bmatrix} \epsilon_t^{m*} \\ \xi_t \\ \epsilon_t^{R^*} \\ \epsilon_t^{\pi^*} \\ \epsilon_t^{y^*} \end{bmatrix} \begin{bmatrix} \epsilon_t^{m*} \\ \xi_t \\ \epsilon_t^{R^*} \\ \epsilon_t^{\pi^*} \\ \epsilon_t^{y^*} \end{bmatrix} \stackrel{iid}{\sim} (0, \Sigma) \quad (1.58)$$

In the system, A is a 5 by 5 matrix of coefficients, Σ is a 5 by 5 upper triangular matrix of shocks estimated from the unrestricted model using the Cholesky decomposition. Variables are listed from the “more endogenous” to the “more exogenous” variable.

Two assumptions close the relation between prices and quantities of goods between the domestic country and the rest of world. First, assume that households in the rest of the world solve an expenditure minimization problem in order to set the optimal demand for home produced tradable goods. The solution of this problem is given by the demand equation:

$$X_t = \left(\frac{P_{x,t}^*}{P_t^*} \right)^{-\eta^*} z_t^* y_t^* \quad (1.59)$$

Finally, the terms of trade of the domestic economy are defined as the ratio between the exported goods and the imported goods price levels, both quoted in foreign currency. Also, the dynamics of the price of imported goods in foreign currency is given by an error-correction model that ensure the terms of trade becomes stationary, in the line of García-Cicco (2009)[37]. The dynamics of the terms of trade and imported goods’ prices are given by:

$$tot_t = \frac{\pi_{x,t}^*}{\pi_{m,t}^*} tot_{t-1} \quad (1.60)$$

$$\frac{\pi_t^{m*}}{\pi^{m*}} = v_1 \frac{\pi_{t-1}^{m*}}{\pi^{m*}} + v_2 \frac{tot_{t-1}}{tot} + \xi X_{t-1}^* + \epsilon_t^{\pi m} \quad \epsilon_t^{\pi m} \sim N(0, \sigma_{\pi m}) \quad (1.61)$$

with $X_t^* = \begin{bmatrix} \frac{\Delta M_t^*}{\Delta M^*} & \frac{\xi_t}{\xi^*} & \frac{R_t^f}{R^*} & \frac{\pi_t^*}{\pi^*} & \frac{y_t^*}{y^*} \end{bmatrix}$.

1.5 Aggregation and Relative Prices

In order to find an expression for the aggregate constraint of the economy, start from the demand faced by a non-tradable producer firm and integrate both sides over all the i_n firms, noting that $h_{n,t} = \int_0^1 h_{n,t}(i_n) di_n$, and that the capital-labor ratio is constant across all the firms:

$$a_{n,t} K_{n,t}^\theta (z_t h_{n,t})^{1-\theta} - z_t^* \chi_n = \int_0^1 \left(\frac{P_{n,t}(i_n)}{P_{n,t}} \right)^{-\eta_n} di_n \left(C_{n,t} + G_{n,t} + \Upsilon_t^{-1} \frac{P_t}{P_{n,t}} I_{n,t} \right)$$

Define $s_{n,t} = \int_0^1 \left(\frac{P_{n,t}(i_n)}{P_{n,t}} \right)^{-\eta_{n,t}} di_n$ to obtain:

$$a_{n,t} K_{n,t}^\theta (z_t h_{n,t})^{1-\theta} - z_t^* \chi_n = s_{n,t} \left(C_{n,t} + G_{n,t} + \Upsilon_t^{-1} \frac{P_t}{P_{n,t}} I_{n,t} \right) \quad (1.62)$$

Obtain the recursive form of $s_{n,t}$:

$$\begin{aligned} s_{n,t} &= \int_0^1 \left(\frac{P_{n,t}(i_n)}{P_{n,t}} \right)^{-\eta_n} di \\ \implies s_{n,t} &= (1 - \alpha_n) \tilde{p}_{n,t}^{-\eta_n} + \alpha_n \left(\frac{\pi_{n,t}}{\pi_{n,t-1}^{\kappa_n}} \right)^{\eta_n} s_{n,t-1} \end{aligned} \quad (1.63)$$

Also, from the definition of the non-tradable goods price index:

$$\begin{aligned} P_{n,t} &= \left[\int_0^1 P_{n,t}(i_n)^{1-\eta_n} di \right]^{\frac{1}{1-\eta_n}} \\ \implies 1 &= (1 - \alpha_n) \tilde{p}_{n,t}^{1-\eta_n} + \alpha_n \left(\frac{\pi_{n,t-1}^{\kappa_n}}{\pi_{n,t}} \right)^{1-\eta_n} \end{aligned} \quad (1.64)$$

Equivalent expressions can be written for the resource constraint, price dispersion and the price index of imported and domestically produced tradable goods and the

price index of exported goods in foreign currency respectively:

$$D_{m,t} - z_t^* \chi_m = s_{m,t} \left(C_{m,t} + \Upsilon_t^{-1} \frac{P_t}{P_{m,t}} I_{m,t} \right) \quad (1.65)$$

$$s_{m,t} = (1 - \alpha_m) \tilde{p}_{m,t}^{-\eta_m} + \alpha_m \left(\frac{\pi_{m,t}}{\pi_{m,t-1}^{\kappa_m}} \right)^{\eta_m} s_{m,t-1} \quad (1.66)$$

$$1 = (1 - \alpha_m) \tilde{p}_{m,t}^{1-\eta_m} + \alpha_m \left(\frac{\pi_{m,t-1}^{\kappa_m}}{\pi_{m,t}} \right)^{1-\eta_m} \quad (1.67)$$

$$a_{x,t} K_{x,t}^\theta (z_t h_{x,t})^{1-\theta} - z_t^* \chi_x = s_{x,t} \left(C_{x,t} + G_{t,t} + \Upsilon_t^{-1} \frac{P_t}{P_{x,t}} I_{x,t} + D_{xp,t} \right) \quad (1.68)$$

$$s_{x,t} = (1 - \alpha_x) \tilde{p}_{x,t}^{-\eta_x} + \alpha_x \left(\frac{\pi_{x,t}}{\pi_{x,t-1}^{\kappa_x}} \right)^{\eta_x} s_{x,t-1} \quad (1.69)$$

$$1 = (1 - \alpha_x) \tilde{p}_{x,t}^{1-\eta_x} + \alpha_x \left(\frac{\pi_{x,t-1}^{\kappa_x}}{\pi_{x,t}} \right)^{1-\eta_x} \quad (1.70)$$

$$D_{xp,t} - z_t^* \chi_{xp} = s_{xp,t} X_t \quad (1.71)$$

$$s_{xp,t} = (1 - \alpha_{xp}) (\tilde{p}_{x,t}^*)^{-\eta_{xp}} + \alpha_{xp} \left(\frac{\pi_{xp,t}^*}{(\pi_{xp,t-1}^*)^{\kappa_{xp}}} \right)^{\eta_{xp}} s_{xp,t-1} \quad (1.72)$$

$$1 = (1 - \alpha_{xp}) \tilde{p}_{xp,t}^{1-\eta_{xp}} + \alpha_{xp} \left(\frac{(\pi_{x,t-1}^*)^{\kappa_{xp}}}{\pi_{x,t}^*} \right)^{1-\eta_{xp}} \quad (1.73)$$

From the aggregation condition of the labor market, the total amount of work hours supplied by the domestic households is given by:

$$h_{x,t} + h_{n,t} = h_t \quad (1.74)$$

The external equilibrium assumes that the net foreign position of domestic households is proportional to the average trade balance result in steady state. Again, notice that the external equilibrium in the bond markets does not include the bonds issued by imported goods' firms, as they are negotiated and liquidated at the beginning and the end of each period. The description of the net foreign position in terms of

domestic currency is given by:

$$P_{x,t}X_t - P_{m,t}D_{m,t} \left[1 + \left(\frac{R_t^f - 1}{R_t^f} \right) \right] = S_t R_{t-1}^f P_t^* I B_t - S_t P_{t+1}^* I B_{t+1} \quad (1.75)$$

It's also necessary to determine the market clearing conditions for domestic bonds and money market. For simplicity, assume that foreign households and domestic firms do not demand home government bonds. As a consequence:

$$B_{g,t} + B_{h,t} = 0 \quad (1.76)$$

Finally, the gross domestic product is defined as:

$$Y_t = C_t + \frac{\psi_1}{2} Y_t \left(\frac{B_{t+1}}{Y_t} - \frac{B}{Y} \right)^2 + \frac{\psi_2}{2} Y_t \left(\frac{S_t I B_{t+1}}{Y_t} - \frac{rer I B}{Y} \right)^2 \\ + \Upsilon_t^{-1} I_t + G_t + \frac{P_{x,t}}{P_t} X_t - \frac{P_{m,t}}{P_t} D_{m,t} \left[1 + \left(\frac{R_t^f - 1}{R_t^f} \right) \right] \quad (1.77)$$

Aggregate profits are given by:

$$\Phi_t = Y_t - \widetilde{W}_t h_t - R_{n,t}^k \mu_{n,t} \overline{K}_{n,t} - R_{x,t}^k \mu_{x,t} \overline{K}_{x,t} \quad (1.78)$$

1.5.1 Relative prices

The model includes a set of relative prices that are strictly related to some observables of the economy. In terms of dynamics, the set of relative prices in the model is given by:

$$pt_t = \frac{P_{t,t}}{P_t} = \frac{\pi_{t,t}}{\pi_t} \frac{P_{t,t-1}}{P_{t-1}} \quad (1.79)$$

$$pn_t = \frac{P_{n,t}}{P_t} = \frac{\pi_{n,t}}{\pi_t} \frac{P_{n,t-1}}{P_{t-1}} \quad (1.80)$$

$$px_t = \frac{P_{x,t}}{P_{t,t}} = \frac{\pi_{x,t}}{\pi_{t,t}} \frac{P_{x,t-1}}{P_{t,t-1}} \quad (1.81)$$

$$pm_t = \frac{P_{m,t}}{P_{t,t}} = \frac{\pi_{m,t}}{\pi_{t,t}} \frac{P_{m,t-1}}{P_{t,t-1}} \quad (1.82)$$

$$pm_t^* = \frac{P_{m,t}^*}{P_t^*} = \frac{\pi_{m,t}^*}{\pi_t^*} \frac{P_{m,t-1}^*}{P_{t-1}^*} \quad (1.83)$$

$$rer_t = \frac{S_t P_t^*}{P_t} \quad (1.84)$$

1.6 Stationary Form and Equilibrium

The objective of this section is to describe the equilibrium conditions with the necessary adjustments to induce stationarity and characterize the competitive and Ramsey Equilibria. Define the stationary allocations with small letters, such that, for a generic variable X_t and the appropriate trend \check{Z}_t , the stationary variable is given by $x_t \equiv X_t/\check{Z}_t$. The model in stationary form is fully described by the stochastic processes for the following sets of variables:

- prices: $\pi_t, \pi_{n,t}, \pi_{x,t}, \pi_{t,t}, \pi_{m,t}, w_t, r_{x,t}^k, r_{n,t}^k, r_{t,t+1}, mcw_t, mc_{n,t}, mc_{x,t}, rer_t, \pi_t^*, \pi_{x,t}^*, \pi_t^{m*}, \tilde{p}_{n,t}, \tilde{p}_{x,t}, \tilde{p}_{m,t}, \tilde{p}_{x,t}^*, pt_t, pn_t, px_t, pm_t, pm_t^*, tot_t;$
- interest rates: $R_t, \tilde{R}_t, R_t^*, R_t^f;$
- allocations: $c_t, c_{t,t}, c_{n,t}, c_{m,t}, c_{x,t}, i_t, i_{t,t}, i_{n,t}, i_{m,t}, i_{x,t}, x_t, d_{m,t}, d_{xp,t}, \mu_{x,t}, \mu_{n,t}, i_{x,t}^d, i_{n,t}^d, y_t, \bar{k}_{x,t}, \bar{k}_{n,t}, k_{x,t}, k_{n,t}, h_t, h_{n,t}, h_{x,t}, x_t^1, x_t^2, z_t^1, z_t^2, y_t^1, y_t^2, u_t^1, u_t^2, ib_t, b_{h,t}, \xi_t, \Delta M_t^*, y_t^*, s_{n,t}, s_{m,t}, s_{x,t}, s_{xp,t}, \lambda_t, q_{x,t}, q_{n,t}, g_t, g_{n,t}, g_{x,t}, m_t, \phi_t;$
- government policies: $\tau_t^h, l_t, t_t, b_{g,t};$
- domestic shocks: $g_t, \tau_t^k, \tau_t^\phi, \tau_t^c, a_{x,t}, a_{n,t}, \mu_t^z, \mu_t^\Upsilon, \pi_t^o.$

The equations describing the law of motion of the variables are given by a set of equilibrium conditions for the household (equations 1.1-1.28), firms responsible for domestic production (equations 1.29-1.38), exporting and importing firms (equations

1.39-1.44), government (equations 1.45-1.56), foreign sector (equations 1.57-1.61), aggregation and price indexes (equations 1.62-1.78) and relative prices (equations 1.79-1.84). Additionally, there are 4 exogenous processes for sectorial productivity and aggregate productivity growth $(a_{x,t}, a_{n,t}, \mu_t^z, \mu_t^\Upsilon)$. As a consequence, there are 84 equations for endogenous variables¹⁰ and 9 domestic exogenous stochastic processes for a total of 93 variables in the model.

The prices and the shocks are stationary, but the allocations must be normalized in order to ensure stationarity. The set of variables given by $\{\bar{K}_{n,t+1}, K_{n,t+1}, \bar{K}_{x,t+1}, K_{x,t+1}, I_t, I_{t,t}, I_{n,t}, I_{m,t}, I_{x,t}, I_{x,t}^d, I_{n,t}^d\}$ must be normalized by $z_t^* \Upsilon_t$, while the variables $\{Y_t, C_t, C_{t,t}, C_{n,t}, C_{m,t}, C_{x,t}, W_t, X_t, D_{m,t}, D_{xp,t}, B_{h,t+1}, B_{g,t+1}, IB_{t+1}, M_t, X_t^1, X_t^2, Z_t^1, Z_t^2, Y_t^1, Y_t^2, U_t^1, U_t^2, G_t, G_{t,t}, G_{n,t}, G_{x,t}, L_t, T_t\}$ must be adjusted by z_t^* . Finally, the prices in each sector for renting capital from households $\{R_{x,t}^k, R_{n,t}^k\}$ and the shadow prices of investment $\{\tilde{q}_{x,t}, \tilde{q}_{n,t}\}$ are divided by Υ_t^{-1} , while the Lagrange multiplier of consumption, $\tilde{\lambda}_t$, is normalized by $(z_t^*)^{-1}$ to obtain λ_t .

1.6.1 Competitive Equilibrium

Given exogenous paths for shocks $\{g_t, \tau_t^k, \tau_t^\phi, \tau_t^c, a_{x,t}, a_{n,t}, \mu_t^z, \mu_t^\Upsilon, \pi_t^o\}$, foreign sector variables $\{\Delta M_t^*, \xi_t, R_t^*, \pi_t^*, y_t^*, \pi_{m,t}^*\}$, policy processes for interest rates $\{R_t, \tilde{R}_t, R_t^f\}$ and taxes τ_t^h , and initial values for prices $\{\pi_{-1}, \pi_{n,-1}, \pi_{x,-1}, \pi_{t,-1}, \pi_{m,-1}, w_{-1}, pt_{-1}, pn_{-1}, px_{-1}, pm_{-1}, pm_{-1}^*, tot_{-1}\}$ and allocations $\{c_{-1}, i_{x,-1}^d, i_{n,-1}^d, \bar{k}_{x,0}, \bar{k}_{n,0}, b_{h,-1}, b_{g,-1}, ib_{-1}, s_{n,-1}, s_{m,-1}, s_{x,-1}, s_{xp,-1}, l_{-1}\}$, a stationary competitive equilibrium is a set of processes for prices $\{\pi_t, \pi_{n,t}, \pi_{x,t}, \pi_{t,t}, \pi_{m,t}, w_t, r_{x,t}^k, r_{n,t}^k, r_{t,t+1}, mcw_t, mc_{n,t}, mc_{x,t}, rer_t, \pi_{x,t}^*, \tilde{p}_{n,t}, \tilde{p}_{x,t}, \tilde{p}_{m,t}, \tilde{p}_{x,t}^*, pt_t, pn_t, px_t, pm_t, pm_t^*, tot_t\}$ and allocations $\{c_t, c_{t,t}, c_{n,t}, c_{m,t}, c_{x,t}, i_t, i_{t,t}, i_{n,t}, i_{m,t}, i_{x,t}, x_t, d_{m,t}, d_{xp,t}, \mu_{x,t}, \mu_{n,t}, i_{x,t}^d, i_{n,t}^d, y_t, \bar{k}_{x,t}, \bar{k}_{n,t}, k_{x,t}, k_{n,t}, h_t, h_{n,t}, h_{x,t}, x_t^1, x_t^2, z_t^1, z_t^2, y_t^1, y_t^2, u_t^1, u_t^2, ib_t, b_{h,t}, b_{g,t}, s_{n,t}, s_{m,t}, s_{x,t}, s_{xp,t}, \lambda_t, m_t, q_{x,t}, q_{n,t}, g_{t,t}, g_{n,t}, g_{x,t}, t_t, l_t, \phi_t\}$

¹⁰ Note that equation 1.58 is a 5-variable VAR.

such that, after stationary transformations of the respective equations: a) households maximize utility; b) firms maximize profits; c) government balances its budget; d) markets clear.

1.6.2 Ramsey Equilibrium

The Ramsey equilibrium is evaluated by the “timeless perspective” described in Woodford (2003)[75], where the government is assumed to run the policy committed for a very long time. An alternative interpretation of this approach is that the government can not change its policy from the time when the Ramsey policy is implemented to the next periods. Given that capital is a predetermined variable in the model, the Ramsey planner, without this constraint, could maximize its revenues setting a very high value for τ_t^k at $t = 0$ and run an alternative policy for $t = 1, 2, 3...$ In this sense, this constraint eliminates any dynamics resulting from the initial state of the economy, and the economy fluctuates around its optimal policy steady state.

Given exogenous paths for shocks $\{g_t, \tau_t^k, \tau_t^\phi, \tau_t^c, a_{x,t}, a_{n,t}, \mu_t^z, \mu_t^\gamma, \pi_t^o\}$ and foreign sector variables $\{\Delta M_t^*, \xi_t, R_t^*, \pi_t^*, y_t^*, \pi_{m,t}^*\}$, previously defined, and a set of initial values for Lagrange multipliers, a Ramsey equilibrium is a set of processes for prices and allocations that maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t [(1 - \gamma) \log(C_t(i) - \zeta C_{t-1}) + \gamma \log(1 - h_t(i))]$$

subject to the equilibrium conditions of the competitive equilibrium and $R_t \geq 1$.

A couple of notes regarding the Ramsey equilibrium. First, the Ramsey equilibrium for a small-open economy must explicitly include an extra non-Ponzi game condition for the evolution of government liabilities. As explained in Schmitt-Grohé and Uribe (2003)[64], the absence of an explicit non-Ponzi game condition for liabilities allows the government to run explosive schemes against the rest of the world

as the optimal policy, using its own stock of liabilities to absorb all the shocks. In the model here, even with the government not trading international bonds, the optimal fiscal policy still could result in non-stationary behavior, as the government sets domestic interest rates low enough to induce households to use foreign bonds to allocate resources across time. The presence of portfolio adjustment costs, both in the domestic and in the international financial markets, combined with the risk premium function over foreign interest rate for borrowing in international markets, ensures that the Ramsey problem is stationary. To be more specific, the presence of portfolio adjustment costs in domestic financial markets imposes a discipline for the domestic household when setting its portfolio. The counterpart of the bonds traded in domestic markets is exactly the amount of debt issued by the government. Thus, the same constraint imposed on the household behavior is transferred for the government debt policy.

Second, the restriction that nominal interest rates must be at least larger than zero – the “zero lower bound problem” – presents an issue that must be carefully addressed. The model solution is obtained after a first-order log-linearization of the equilibrium conditions. As a consequence, for very large shocks, the lower bound for interest rates ($R_t \geq 1$) might be violated. In order to handle with this problem, Woodford (2003)[75], Adjemian, Pàriès and Moyen (2007)[56] and Batini, Levine and Pearlman (2009)[10], add one extra term to the welfare function of the households, penalizing for high deviations of the interest rates from its steady state level. Here, the penalty function is asymmetric, reducing the welfare only for very low values of the nominal interest rates:

$$E_0 \sum_{t=0}^{\infty} \beta^t \{ (1 - \gamma) \log (C_t(i) - \zeta C_{t-1}) + \gamma \log (1 - h_t(i)) + \exp [\omega_r (R_t/R)] \}$$

The asymmetric term allows the Ramsey planner to make a choice between

increasing the optimal level of inflation in steady state or reducing the variance of nominal interest rate changes in the dynamics of the optimal policy. Adjemian, Pàris and Moyen (2007)[56] document, in an estimated model for the Euro Area, a probability of 5% to violate the lower bound constraint for interest rates when the estimated model is centered around a steady state of 2% inflation per year under the competitive equilibrium. This probability increases to 13% if the steady state inflation is zero and to 37% under the Ramsey optimal monetary policy.

Parameter ω_r is calibrated to ensure that, in the ergodic distribution of the nominal interest rates, the probability of violating the lower bound of nominal interest rates is arbitrarily small.¹¹

1.6.3 Welfare Computation

In order to compute the welfare costs of an alternative policy relative to the time-invariant Ramsey optimal fiscal and monetary policy, denote \mathcal{U}_i the welfare associated with a given monetary and fiscal policy regime indexed by i , measured in terms of the period utility function of the households following the policy functions c_t^i and h_t^i for consumption and labor supply, respectively¹². The welfare *conditional on a initial state* at period zero of adopting policy i is defined as:

$$\mathcal{U}_i^c = E_0 \sum_{t=0}^{\infty} \beta^t U_t \left(\left(c_t^i - \frac{\zeta c_{t-1}^i}{\mu_t^z (\mu_t^r)^{\frac{\theta}{1-\theta}}} \right), h_t^i \right)$$

$$U_t(c_t, h_t) = (1 - \gamma) \log \left(c_t - \frac{\zeta c_{t-1}}{\mu_t^z (\mu_t^r)^{\frac{\theta}{1-\theta}}} \right) + \gamma \log(1 - h_t)$$

¹¹ The solution for the Ramsey problems is computed using the package Dynare for Matlab, combined with Andrew Levin's code to write the problem. For Levin's code, see Levin, Onatski, Williams, and Williams (2006)[47].

¹² We ignore here the term adjusting for the stationary process of consumption in the utility function, $(1 - \gamma) \log z_t^*$, since the policies compared here do not change the long run growth rate of the economy, z_t^* . As a consequence, the welfare cost of the alternative policies is not affected by this term.

Note that E_0 defines the expectations operator in terms of period zero. Using equivalent notation, the *unconditional welfare* of adopting policy regime i is defined as:

$$\mathcal{U}_i^u = E_t \sum_{t=0}^{\infty} \beta^t U_t(c_t^i, h_t^i)$$

Following Schmitt-Grohé and Uribe (2005 and 2005b)[66][67], the welfare cost λ_c of adopting the alternative policy regime i instead of the Ramsey monetary and fiscal policy r is measured in terms of the share of consumption the households give up in order to be indifferent between the two policy regimes:

$$\begin{aligned} \mathcal{U}_i^c &= E_0 \sum_{t=0}^{\infty} \beta^t U_t \left(\left(c_t^i - \frac{\zeta c_{t-1}^i}{\mu_t^z (\mu_t^r)^{\frac{\theta}{1-\theta}}} \right), h_t^i \right) \\ &= E_0 \sum_{t=0}^{\infty} \beta^t U_t \left((1 - \lambda_c) \left(c_t^r - \frac{\zeta c_{t-1}^r}{\mu_t^z (\mu_t^r)^{\frac{\theta}{1-\theta}}} \right), h_t^r \right) \quad (1.85) \end{aligned}$$

Using the period utility function of the households, the welfare cost λ_c can be expressed as:

$$\frac{\mathcal{U}_i^c - \mathcal{U}_r^c}{(1 - \gamma)} = \log \left((1 - \lambda_c) c_0^r - \frac{\zeta c_{-1}^r}{\mu_0^z (\mu_0^r)^{\frac{\theta}{1-\theta}}} \right) - \log \left(c_0^r - \frac{\zeta c_{-1}^r}{\mu_0^z (\mu_0^r)^{\frac{\theta}{1-\theta}}} \right) + \frac{\beta}{1 - \beta} \log(1 - \lambda_c)$$

Following again Schmitt-Grohé and Uribe (2005 and 2005b)[66][67], note that, by the expression above, λ_c is a function of the vector of states and shocks of the model, as they determine the welfare and the consumption in each period. In order to make the shocks relevant to welfare, λ_c is computed based on a second order approximation of the equilibrium conditions. Using the authors result, the final expression for the welfare costs of alternative policies, λ_c , for a vector of exogenous shocks with variance σ^2 , is given by:

$$\lambda_c = \frac{\mathcal{U}_{r,\sigma\sigma}^c - \mathcal{U}_{i,\sigma\sigma}^c}{(1 - \gamma) \left(\frac{\beta}{1 - \beta} + \frac{\mu^z (\mu^r)^{\frac{\theta}{1-\theta}}}{\mu^z (\mu^r)^{\frac{\theta}{1-\theta}} - \zeta} \right)} \times \frac{\sigma^2}{2}$$

where $\mathcal{U}_{r,\sigma\sigma}^c$ and $\mathcal{U}_{i,\sigma\sigma}^c$ are the second derivatives of the welfare function in terms of the vector of exogenous shocks σ . It is worth noting that this measure of welfare cost will also be used to compare the loss of the Ramsey policy under the constraint of the zero lower bound of interest rates.

Optimal Policy: The Ramsey Steady State

The main objective of this chapter is to evaluate the Ramsey planner's choices in terms of steady state policies and allocations with a calibration based on the literature of medium scale macroeconomic models. The simulations performed here do not target matching specific moments. Instead, the main goal is to understand the trade-offs presented in the planner's problem and the optimal responses given the restrictions imposed by parametric assumptions and by the number of instruments available for the planner. As a consequence, this section does not attempt to highlight differences between SOEs and EMEs, but, instead, tries to clearly state the priorities of the Ramsey planner when defining the optimal policy.

Few authors in the literature provide a comprehensive discussion about the properties of the steady state under the Ramsey policy. Woodford (2003)[75] provides a complete description of the steady state policy of the basic New Keynesian model for closed economies. The author explore the differences in the Ramsey setup when imposing additional restrictions like those included here, as the "timeless perspective" of the Ramsey formulation, discussed in the definition of the Ramsey equilibrium in chapter 1, and one possible solution for the zero lower bound constrain. Still in

the closed economy framework, but now dealing with variations of models similar in structure to CEE (2005)[22], Schmitt-Grohé and Uribe (2005, 2005b, 2007)[66] [67] [68] explore the properties of the steady state under Ramsey optimal monetary and fiscal policies. These medium-scale models do not have a closed form solution, like the basic formulations described by Woodford (2003)[75]. Therefore, the only way to understand and describe optimal policy is by means of numerical simulations. The results in terms of steady state of prices usually point out for price stability as the main outcome of the Ramsey planner, with small variations depending on the number of nominal and real rigidities included in the model.

The chapter is organized as follows. In the next section, the baseline calibration is described, with details about the big ratios and the range of the parameter simulations. Section 2.2 shows the optimal policy choices of taxes and interest rates for the general model, where the Ramsey planner can make use of all policy instruments available. The next two sections deal with two special cases: first, in section 2.3, the classical problem of choosing the optimal relative taxation between capital and labor is approached in a model where the government has no access to consumption taxes; next, the case where the government can not discriminate between production inputs using taxes is discussed, in a version of the model where the government sets optimal taxation using only income and consumption taxes. Introducing a limiting case, section 2.5 describes the optimal policy when the government has access only to an income tax. Finally, section 2.6 highlights the effects of the correction for the zero lower bound for nominal interest rates in the steady state of inflation and taxes and section 2.7 concludes.

In terms of results, price stability seems to be the main goal of the Ramsey planner around the parameters used in calibration. However, the number of taxes available for the government plays a key role in explaining how the government sets the optimal taxes and interest rates under different assumptions on nominal and real

rigidities. To be more specific, the inclusion of consumption taxes as one of the taxes available (but not the single tax instrument) in the model results in price stability as the optimal outcome, eliminating all the trade-offs related to the combination of nominal and real rigidities in the model only with income taxation. This result confirms, for a model designed for small open economies with a large number of real and nominal rigidities, the propositions in Correia, Nicolini and Teles (2008)[26] regarding the role of a tax over the final good of the economy, vis-a-vis a tax over intermediate inputs. However, it is worth noting that the classical result from Judd (2002)[43], of a high subsidy to capital relative to the returns of labor, is robust, irrespective to the set of instruments available to the benevolent government.

2.1 Calibration

The steady state of the model described in the previous chapter is fully characterized by the parameters listed in table 2.1 and the big ratios used to define the structural parameters under the competitive equilibrium. In a brief description of these ratios, assume that, in steady state, the domestic economy and the rest of the world stabilize the price level in all sectors. This assumption, which is common in models where the traditional New Keynesian Phillips curve is adopted¹, implies that there is no persistent loss due to price dispersion across firms in steady state. The growth rate of productivity is set at 2% per year, while, for simplicity, the growth in investment-specific technological shock is set at zero². The growth rate of productivity obviously implies that output per capita and other cointegrated variables grow at the same annual rate. It is assumed that households spend 20% of their time endowment at work ($h = 0.2$), following close the standard calibration proposed in Schmitt-Grohé

¹ It's worth remembering at this point that the New Keynesian Phillips curve is a log-linear approximation of the first order condition of the firms with respect to prices around a steady state of price stability.

² The same assumption is made during estimation in chapter 3.

and Uribe (2005b)[67]. When describing the government operations, it's important being able to compare the results in this chapter with other papers in the literature. In this sense, the calibration relies on standard numbers for the United States, with the ratios with respect to GDP of government spending, money supply and net public debt set at 17%, 16.95% and 42%, respectively, following Schmitt-Grohé and Uribe (2005b)[67]. The competitive equilibrium value for taxes in the US between 1990 and 2000 are taken from Carey and Rabesona (2003)[18], following an updated methodology derived from Mendoza, Razin and Tesar (1994)[53]. Taxes on capital (τ^k), labor (τ^h) and consumption (τ^c) are set at 39.5%, 23.4% and 6.4%. In the foreign sector, assume that the risk premium is set to zero and the trade balance is in equilibrium.

The choice of parameter values, presented in table 2.1, is based on the literature of medium scale models for economies with a similar set of rigidities as those presented here. Some values are standard, like the depreciation rate of capital at 10% per year ($\delta = 0.025$), the capital share representing 30% of output and the discount factor β targeting an annualized real interest rate of 4% in the balanced growth path. The final value of β , higher than the usual calibrations in RBC models, is comparable to other studies where there is a non-stationary component in productivity³. Due to the absence of empirical estimates for the share of tradable goods in the GDP, assume that these goods represent around 55% of the consumption basket. The share of imported goods in the aggregate consumption is set at 20%.

For didactic purposes, assume that the price elasticity of demand for each sector in the economy, $\eta_i, i = \{n, x, m, xp\}$, is the same. There is empirical evidence supporting the hypothesis that firms trading in foreign markets, especially exporting firms, present higher markups over prices when compared to firms trading only in the

³ It's worth noting that the value of β will change during the estimation procedure, since the growth rate of productivity will be calibrated to match the average growth of the economy in the dataset.

Table 2.1: Calibration for Steady State.

Parameter	Description	Value	Source
δ	Depreciation rate*	0.025	
θ	Capital share*	0.3	
β	Discount factor	0.9952	
ω	Share of tradable goods*	0.55	
\varkappa	Share of imports in tradable*	0.36364	
$\eta_n = \eta_x$	Price elasticity demand domestic goods	5	SGU (2007)[68]
η_m	Price elasticity demand imported goods	5	
η_{xp}	Price elasticity demand exported goods	5	
$\alpha_n = \alpha_x$	Calvo parameter domestic goods	0.6	CEE (2005)[22]
α_m	Calvo parameter imported goods	0.6	
α_{xp}	Calvo parameter exported goods	0.6	
ζ	Habit persistence	0.55	SW (2003)[70]
ϖ	Elast. subst. across labor types	21	CEE (2005)[22]
κ_1	Elast. R_t^f to exogenous risk premium	1	
κ_2	Elast. R_t^f to net foreign position	1	
η^*	Elast. subst. domestic exports to ROW	1	
θ_2/θ_1	Adjustment of capacity utilization	2.02	SGU (2005b)[67]

Note: (*) Same calibration used in estimation.

domestic goods market (see De Loecker and Warzynski, 2009[49]). When estimating the model, these differences will be noticed. Parameters η_i are calibrated such that the firm markup is set at 25% (see Basu and Fernald, 1997[9], and Schmitt-Grohé and Uribe, 2007[68]). The same reasoning is used to set, in the standard calibration, the Calvo parameter assigning the probability of a firm not adjusting its prices. Following the estimation of CEE (2005)[22], $\alpha_i, i = \{n, x, m, xp\}$, is set to 0.6.

Estimates of the habit persistence parameter in consumption are very unstable, with severe implication for the other parameters of the model. Justiniano and Preston (2009)[44] report, for different dataset and assumptions on the structural model, values in the range of [0.05, 0.82]. Garcia-Cicco (2008)[37] estimates a structural model with Mexican data and finds, in the baseline estimation, a value of 0.83, decreasing to 0.13 depending on the assumptions of the model structure. Given the

wide dispersion of estimates, assume for this chapter that, in the baseline scenario, the habit persistence parameter ζ is set at 0.55, following the estimate of Smets and Wouters (2003)[70] in a model for the Euro Area.

The elasticity of substitution across different labor types, ϖ , is usually calibrated in the literature. For emerging economies, Garcia-Cicco (2008)[37], based on the literature for emerging economies, sets a value where the markup of wages over the marginal rate of substitution between labor and leisure equals 100% ($\varpi = 2$). This is the same value calibrated in Smets and Wouters (2003)[70]. The estimation of CEE (2005)[22] is the baseline for the calibration used in Altig, Christiano, Eichenbaun and Lindé (2005)[6], Schmitt-Grohé and Uribe (2005, 2005b)[66][67] and Adolfson, Laseén, Lindé and Villani (2007)[2]. The estimation implies a markup of only 5% ($\varpi = 21$). For this chapter, the baseline scenario assumes the small markup value, leaving the analysis of different degrees of wage stickiness in the estimation.⁴

Given the functional form adopted for the cost of adjusting the capacity utilization, and the hypothesis that, in the steady state of the competitive equilibrium, the economy operates at full capacity ($\mu_n = \mu_n = 1$), the computation of the steady state demands a value for the ratio of parameters θ_2/θ_1 . The value used for the simulations follows Schmitt-Grohé and Uribe (2005b)[67], based on the estimation of Altig, Christiano, Eichenbaun and Lindé (2005)[6]. Despite the fact that this ratio is irrelevant for the steady state under the competitive equilibrium, the possibility of different levels of capacity utilization under the Ramsey policy forces an assumption regarding this value.

For the remaining parameters, the price elasticity of demand from the rest of the world for the domestic good, η^* , is set at unity, just like the elasticity of the domestic bonds traded in foreign markets to the world's risk premium and the domestic

⁴ Notice, however, that there is an equivalence in terms of results in the steady state depending on the combination of values between habit persistence and wage stickiness. This result is better explained in section 2.3.

economy's net foreign asset position (κ_1 and κ_2). These parameters are used only to determine the level of foreign variables, without any influence over the domestic economy's steady state.

2.2 An overview: The case with all taxes available

Consider the case where the government has access to all the fiscal policy instruments described in the model in chapter 1: taxes on consumption (τ^c), labor income (τ^h), capital income (τ^k), profits (τ^ϕ), the control of money supply (m) and debt (b_g). Despite providing large degrees of freedom for the Ramsey planner to optimize the objective function, this exercise provides a benchmark for the results in the following sections. The combination of income and consumption taxes has been explored in simple monetary models usually without capital. Examples can be found, in a flexible price environment for closed economies, in Chari, Christiano and Kehoe (1996)[20] and De Fiore and Teles (2003)[30] and for models with sticky prices in Correia, Nicolini and Teles (2008)[26]. The main focus of these papers is establishing the conditions under which the Friedman rule can be sustained as the optimal monetary policy framework. In the open economy framework, Adao, Correia and Teles (2009)[1] show, in a model without capital and with restrictions on labor mobility, that if each country in a single currency area can tax domestic consumption and labor income, the real exchange rate is completely irrelevant to characterize the optimal allocations and, as a consequence, welfare.

Table 2.2 describes the optimal choices of nominal interest rates and taxes on capital, labor and consumption under different assumptions regarding nominal rigidities, indexation, parameters characterizing the open economy and taxation on profits. The results in line 1 are based on the standard calibration described in the previous section. The baseline scenario assumes that the tax on profits is set at the same rate as the tax on capital ($\tau^\phi = \tau^k$) and that there is no indexation in prices.

Table 2.2: Optimal inflation and taxes.

	α_n	α_m	π	R	$\tau^k(\%)$	$\tau^h(\%)$	$\tau^c(\%)$	Obs.:
1	0.6	0.6	0.00	4.00	-15.35	100	-100	Baseline scenario
2	0	0	-3.85	0.00	-15.35	100	-100	
3	0.6	0	0.00	4.00	-15.35	100	-100	
4	0	0.6	0.00	4.00	-15.35	100	-100	
Indexation:								
5	0.6	0.6	-3.85	0.00	-15.35	100	-100	$\kappa_x = \kappa_n = \kappa_m = \kappa_{xp} = 1$
6	0.6	0.6	0.00	4.00	-15.35	100	-100	$\kappa_x = \kappa_n = 1$
7	0	0	-3.85	0.00	-15.35	100	-100	$\kappa_x = \kappa_n = 1$
8	0.6	0	-3.85	0.00	-15.35	100	-100	$\kappa_x = \kappa_n = 1$
9	0	0.6	0.00	4.00	-15.35	100	-100	$\kappa_x = \kappa_n = 1$
Open Economy:								
10	0.6	0.6	0.00	4.00	-15.35	100	-100	$\alpha_{xp} = 0$
11	0.6	0.6	-3.85	0.00	-15.35	100	-100	$\alpha_{xp} = 0, \kappa_x = \kappa_n = \kappa_m = \kappa_{xp} = 1$
12	0.6	0.6	0.00	4.00	-15.35	100	-100	$\alpha_{xp} = 0, \kappa_x = \kappa_n = 1$
13	0.6	0.6	0.00	4.00	-13.13	100	-100	$\varkappa = 0.01$
14	0.6	0.6	0.00	4.00	-13.11	100	-100	$\omega = 0.01$
15	0.6	0.6	0.10	4.10	-15.27	100	-100	$\pi = \pi^* = 1.0074$ (3%p.y.)
16	0.6	0.6	0.00	4.00	-15.35	100	-100	$\pi = 1.0074$ (3%p.y.)
Profit Taxation:								
17	0.6	0.6	0.00	4.00	-15.35	100	-100	$\tau^\phi = 0$
18	0.6	0.6	0.00	4.00	-15.35	100	-100	$\tau^\phi = 1$

Note: Baseline scenario: $\pi^* = \pi^o = 0$; $\varkappa = 0.363$; $\omega = 0.55$; $\alpha_{xp} = 0.6$; $\kappa_x = \kappa_n = \kappa_m = \kappa_{xp} = 0$; $tb/y = 0$; $\tau^\phi = \tau^k$.

There are two striking results in the first panel of table 2.2. First, the degree of nominal rigidity does not affect the optimal policy in terms of nominal interest rates, as there are only two possible outcomes regarding monetary policy, irrespective of the values for the Calvo parameters: price stability or the Friedman rule. The Friedman rule is the optimal policy under price flexibility or under conditions where the output loss due to sticky prices is removed, like some cases of indexation described below. For every other combination of parameters setting price rigidities, price stability is the optimal policy outcome. Second, also irrespective of the main parameters of the model, taxation on labor is set at 100%, while the tax on consumption is, actually,

a subsidy of 100%. As a matter of fact, the two results might be connected. De Fiore and Teles (2003)[30] and Correia, Nicolini and Teles (2008)[26] show that, if the conditions for uniform taxation on consumption goods are satisfied⁵ and the Friedman rule is the optimal policy, than consumption must be fully subsidized and labor income fully taxed. One of the reasons for this result is that the number of policy instruments is at least enough to eliminate the distortions generated from different frictions affecting the steady state of households and firms allocations. If this is the case, than money becomes nonessential, in the sense that any level of money holdings satisfies the households' equilibrium conditions, and the Ramsey planner eliminates the cost of shopping by fully subsidizing household consumption.

There are three reasons to believe that the result described in De Fiore and Teles (2003)[30] and Correia, Nicolini and Teles (2008)[26] also applies in this framework for small-open economies. First, the log-separable utility function in consumption and labor satisfies the implementation conditions for uniform taxation in consumption. Second, the robustness of results in terms of the taxes in consumption and labor even under different parameterization of the model. Third, as described in lines 17 and 18, is the fact that the Ramsey policy remained exactly the same as in the baseline calibration under different assumptions for taxation on profits. As discussed in Schmitt-Grohé and Uribe (2005b)[67], profits are a lump sum transfer from firms to the households. If allowed to set it optimally, the Ramsey planner chooses to confiscate all income from profits to finance its spending with minimum distortion of the households and firms allocations, setting $\tau^\phi = 1$. In the model proposed here, with the current number of policy instruments, the Ramsey planner is indifferent to the inclusion of a lump sum instrument.

The second panel of table 2.2 shows alternative scenarios regarding indexation.

⁵ These conditions are the separability between labor and consumption and homotheticity in consumption in the utility function.

Three possible combinations of scenarios can generate the Friedman rule as an outcome for monetary policy. In line 5, with the economy under full indexation, the output loss due to price dispersion across firms is eliminated, generating, in steady state, a similar framework to complete price flexibility. However, as line 6 shows, full indexation is necessary for both production and retail firms, since the policy with indexation present only on production is very similar to the baseline scenario in line 1. Lines 7, 8 and 9 show that the Friedman rule might return as a policy outcome with indexation in domestic production firms if prices for imported goods firms are flexible. Thus, the Friedman rule under the current tax system may still emerge as a solution under a restrict set of conditions: price flexibility; full indexation in prices of domestic firms and retailers; full indexation in prices of domestic firms with price flexibility of imported goods' retailers.

When structural parameters are changed, the tax instrument affected is the capital taxation, which is, under the baseline calibration, a subsidy. The intuition for the subsidy in capital was developed in Judd (2002)[43], where the presence of imperfect competition in product markets creates a distortion proportional to the price markup resulting from imperfect competition on the household's intertemporal substitution of consumption. This distortion is increasing over time. Schmitt-Grohé and Uribe (2005b)[67] explore the properties of the subsidy for the case with depreciation and time-varying capital utilization in a closed economy. Lines 13 and 14 in the table, however, show that parameters related to the open economy framework affect the size of the subsidy, as it declines with a reduction for the demand of imported goods. In order to understand the result, note that the steady state of capital taxes and the

return on capital are given by:

$$\tau^k = 1 - (r_i^k \mu_i - a(\mu_i))^{-1} \left(\frac{\mu^z (\mu^\Upsilon)^{\frac{1}{1-\theta}}}{\beta} - 1 + \delta \right)$$

$$r_i^k = mc_i \theta \left(\frac{k_i}{\mu^z (\mu^\Upsilon)^{\frac{1}{1-\theta}} h_i} \right)^{\theta-1}$$

As \varkappa or ω approach zero, the share of total investment based on domestic production increases, as the total demand of imported goods (c_m and i_m) decreases⁶. Without the imported good, the demand for domestic investment goods increases, also increasing the marginal return on capital (r_i^k) and reducing the subsidy necessary to reduce the distortions from the price markup. Figure 2.1 shows the size of the subsidy on capital, the capital-labor ratio, the rate of capital utilization and the marginal return of capital net of adjustment costs ($r_i^k \mu_i - a(\mu_i)$) as a function of the share of imported goods in the tradable goods basket \varkappa .

Finally, the only difference in allocations and policies in this setup of taxes was found when the inflation in steady state for the foreign economy was larger than zero. However, as the result in line 15 shows, the increase in domestic inflation is smaller than the change in foreign inflation: an inflation of 3% per year in the rest of the world results in an increase of less than 0.1% in the inflation under the optimal Ramsey policy. Also notice that this small deviation is a consequence only of foreign inflation, as a positive value in the underlying competitive equilibrium does not alter results from the baseline scenario.

Given the results on taxes and inflation, the next three sections gain importance, as the constraint in the number of policy instruments will describe the policy choices of the Ramsey planner. So far, it is already known, from results in this section, that

⁶ Remember, from the optimal choice of the households based on the CES aggregator, that \varkappa is the ratio of imported goods in the basket of tradable goods and ω is the share of tradable goods in the basket of total demand.

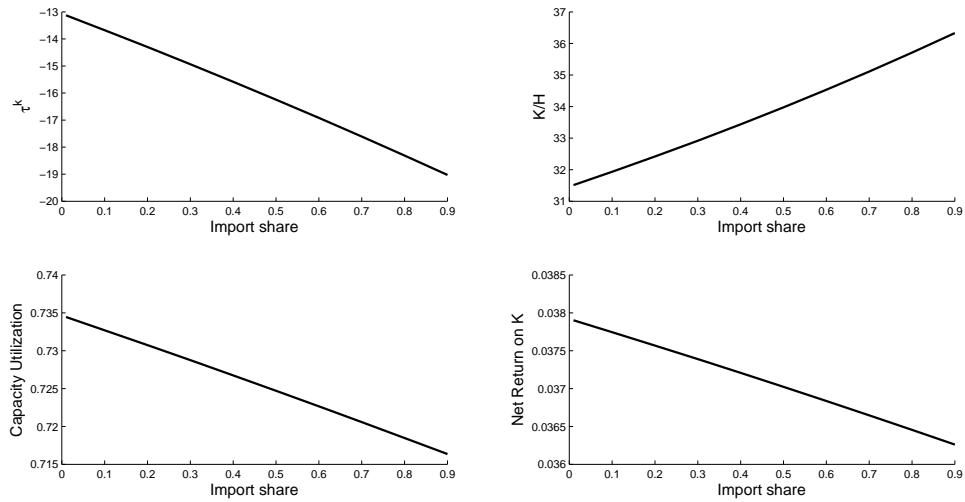


FIGURE 2.1: Optimal taxation on capital and openness

price rigidities restrict the policy choices between price stability and the Friedman rule. The presence of markups over prices also implies, just like in Judd (2002)[43], that the optimal tax on capital is actually a subsidy. The constraint in the number of instruments makes explicit a ranking of preferences in terms of which distortions in the model must be addressed with higher priority. It also provides a robustness test for the hypothesis of price stability as the major goal for monetary policy, as inflation, seen as a tax on money holdings, gains in importance as the alternative instruments become unavailable.

2.3 The case without consumption taxes

In this section, consider the case where the government can not directly tax consumption ($\tau^c = 0, \forall t$). Despite not being able to tax consumption, the Ramsey planner is still capable of perfectly discriminating the household's sources of income, as it is allowed to tax labor, capital and profits income at different rates. In order to understand the importance of the tax discrimination between labor and capital income,

consider first the stationary transformation of the intertemporal Euler equation of the households when consumption can not be taxed:

$$(1 - \tau^h) w (1 - h) = \frac{\gamma}{(1 - \gamma)} mcw \left(1 - \frac{\zeta}{\mu^z (\mu^r)^{\frac{\theta}{1-\theta}}} \right) c \left[1 + \nu^m \left(\frac{R - 1}{R} \right) \right]$$

The expression in the left-hand side is the (after-tax) value of leisure. On the right-hand side, the second term, $mcw = \frac{\varpi}{\varpi - 1}$ in steady state, is the wedge between wages and the marginal disutility of labor, generated by wage stickiness, while the third term, $\left(1 - \frac{\zeta}{\mu^z (\mu^r)^{\frac{\theta}{1-\theta}}} \right) c$, is the consumption adjusted for habit persistence. The last term, $\left[1 + \nu^m \left(\frac{R-1}{R} \right) \right]$, is a consequence of the money demand by the households. Thus, there are two policy instruments in this equation (nominal interest rates and labor income taxes) set by the Ramsey planner and three distortions: habit persistence, wage gap and the financial friction due to the cash-in-advance constraint. Note that, by construction, the distortions generated by habit persistence and wage gap operate with opposite signs: on the one hand, high elasticity of substitution across labor types, given by ϖ in the equation describing the demand for labor of household i , $h_t(i)$, implies low distortions from wage stickiness, with mcw having a lower bound at one; on the other hand, high values of ζ imply high degree of habit persistence on consumption, with the after-tax value of leisure approaching zero⁷. As a consequence, for an appropriate combination of ζ and ϖ , it is possible that the wage gap and the habit persistence offset each other, making the cash-in-advance constraint the only relevant distortion in the labor-leisure allocation.

The perfect discrimination between capital and labor income through taxes still allows the Ramsey planner to formulate fiscal and monetary policies resulting in prices and allocations close to the Friedman rule. In order to understand why, note

⁷ It's obvious that, for the model where the productivity growth is equal to zero, the expression $\left(1 - \frac{\zeta}{\mu^z (\mu^r)^{\frac{\theta}{1-\theta}}} \right)$ is exactly bounded between zero and one.

that with the taxation on capital designed to eliminate the wedge from the markup over prices, labor taxation can be used to minimize the gap between the efficient and the distorted intratemporal labor allocation. As seen above, for special cases of the parameters characterizing the rigidities from wage stickiness and habit persistence, the cost of holding money might emerge as the most relevant distortion affecting the labor allocation. Thus, in the limiting case of this special combination of parameters, the Friedman rule surges as the optimal policy outcome.

The role of real rigidities and the possibility of a monetary policy outcome close to the Friedman rule can be better visualized in figure 2.2. In the figure, the vertical axe shows the Ramsey inflation as a function of the Calvo probability of a firm in the domestic production sector (α_n) and the imported goods sector (α_m) changing prices. The figure has two surfaces, with the surface computed for the baseline scenario overlapping the surface of the scenario assuming a high degree of habit persistence on consumption ($\zeta = 0.90$). Note, in the figure, that inflation is decreasing as prices in both sectors become more flexible, with a policy close to the Friedman rule achieved when prices in both sectors are close to full flexibility. Also note that the economy with high habit persistence presents policies closer to the Friedman rule even in the presence of a larger degree of price rigidity. In the intratemporal Euler equation described above, the increase in ζ reduces the distortion generated by wage stickiness, allowing the Ramsey planner to reduce the cost of holding money even in the presence of price rigidity.

Table 2.3 shows the optimal combination of taxes and interest rates for different levels of nominal rigidities. Line number 1 presents the baseline scenario, with no indexation, taxation of profits in the same level as the taxation on capital and price stability in the underlying competitive equilibrium both domestically and in the rest of the world. The baseline scenario is characterized by a small deflation, and, as expected, a combination of subsidy on tax capital and high taxes on labor income.

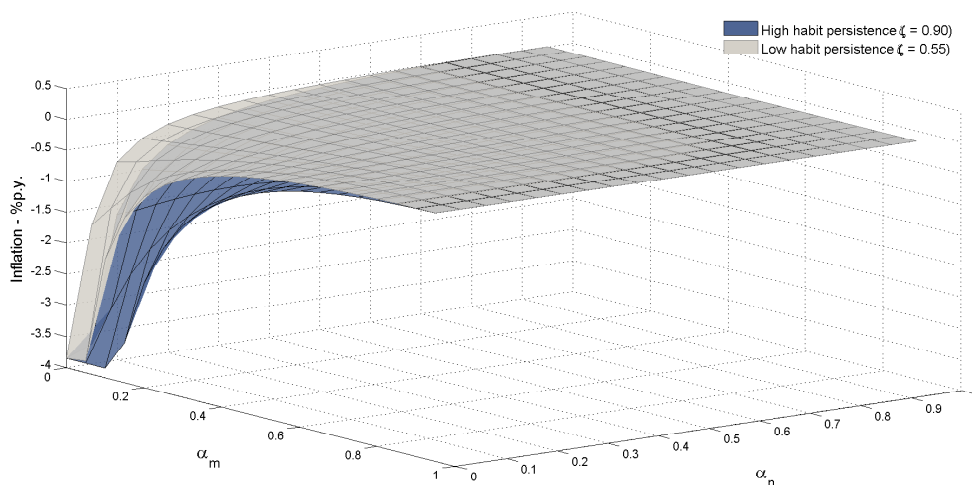


FIGURE 2.2: Real rigidities and optimal inflation: no consumption taxes

The second line of the table confirms the result that under flexible prices the Friedman rule is the optimal outcome of the Ramsey problem, as monetary policy tries to eliminate the cost of carrying money. However, as lines 3 and 4 show, the mild deflation returns as a result if there is heterogeneity in price rigidity across sectors. The second panel of table 2.3 confirms the scenarios presented in the previous section, regarding indexation, where the Friedman rule is optimal.

The third panel of table 2.3 details the optimal policy changing some parameters that are specific to the small-open economy described by the model. First, in lines 10-12, note that price flexibility for firms in the exported goods' sector do not alter the results in terms of indexation and price flexibility described in the first two panels. This is also the same result obtained in the previous section. In lines 13 and 14, reducing the share of tradable goods, ω , to 1% of the total domestic absorption, and the share of imported goods, \varkappa , to 1% of the domestic absorption of tradable goods, respectively, the “closed” economy features negative inflation in steady state, in a

Table 2.3: Optimal inflation and taxes – no consumption taxes.

	α_n	α_m	π	R	$\tau^k(\%)$	$\tau^h(\%)$	Obs.:
1	0.6	0.6	-0.11	3.88	-16.12	30.40	Baseline scenario
2	0	0	-3.85	0.00	-15.05	39.07	
3	0.6	0	-0.15	3.84	-16.12	30.41	
4	0	0.6	-0.44	3.54	-16.12	30.48	
Indexation:							
5	0.6	0.6	-3.85	0.00	-15.03	39.18	$\kappa_x = \kappa_n = \kappa_m = \kappa_{xp} = 1$
6	0.6	0.6	-0.44	3.54	-16.12	30.48	$\kappa_x = \kappa_n = 1$
7	0	0	-3.85	0.00	-15.07	38.91	$\kappa_x = \kappa_n = 1$
8	0.6	0	-3.85	0.00	-15.07	38.91	$\kappa_x = \kappa_n = 1$
9	0	0.6	-0.44	3.54	-16.12	30.48	$\kappa_x = \kappa_n = 1$
Open Economy:							
10	0.6	0.6	-0.11	3.88	-16.12	30.40	$\alpha_{xp} = 0$
11	0.6	0.6	-3.85	0.00	-15.05	39.07	$\alpha_{xp} = 0, \kappa_x = \kappa_n = \kappa_m = \kappa_{xp} = 1$
12	0.6	0.6	-0.44	3.54	-16.12	30.48	$\alpha_{xp} = 0, \kappa_x = \kappa_n = 1$
13	0.6	0.6	-0.19	3.80	-14.47	29.25	$\varkappa = 0.01$
14	0.6	0.6	-0.19	3.80	-14.46	29.24	$\omega = 0.01$
15	0.6	0.6	-0.01	3.99	-16.05	30.28	$\pi = \pi^* = 1.0074$ (3%p.y.)
16	0.6	0.6	-0.11	3.88	-16.12	30.40	$\pi = 1.0074$ (3%p.y.)
Profit Taxation:							
17	0.6	0.6	-0.02	3.98	-15.35	36.49	$\tau^\phi = 0$
18	0.6	0.6	-0.02	3.98	-15.34	36.49	$\tau^\phi = 1$

Note: Baseline scenario: $\pi^* = \pi^o = 0\%$; $\varkappa = 0.363$; $\omega = 0.55$; $\alpha_{xp} = 0.6$; $\kappa_x = \kappa_n = \kappa_m = \kappa_{xp} = 0$; $\tau^\phi = \tau^k$; $tb/y=0$.

result similar to Schmitt-Grohé and Uribe (2005b)[67]⁸. Note that the reduction of the capital tax subsidy as ω and \varkappa converges to zero also shows up in this framework. Lines 15 and 16 also confirm the results of a small effect of foreign inflation in the determination of domestic optimal level of inflation.

Finally, changes in the profit taxation indeed affect the allocations and the optimal policy in this framework for taxes. The setting of a fixed rate for taxes on profits, instead of a choice based also on the taxation of capital, as in the baseline

⁸ The baseline results in Schmitt-Grohé and Uribe (2005b)[67] of a small positive inflation in steady state are a consequence of the assumption of a lump sum transfer from the government to the households, with inflation acting as a tax on consumption. When this transfers are set to zero, the small deflation emerges as the optimal policy outcome.

scenario, allows the Ramsey planner to approximate the optimal policy to price stability. Most of the adjustment to the change in the tax structure is reflected in the optimal level of debt supported by the households.

In this exercise, one of the policy instruments was removed when compared to the setup in the previous section. Two results were robust to this formulation: the Friedman rule as the optimal outcome under flexible prices or indexation structure simulating the flexible price economy; and the subsidy to capital compared to the high taxes on labor income, in order to eliminate the distortions discussed in Judd (2002)[43]. Despite low, inflation under different levels of price rigidity was not zero, like in the previous section. The next section will show that the consumption tax play a critical role in this regard.

2.4 The case of income and consumption taxes

Assume, for this section, that the government is constrained on taxing all income at the same tax rate – thus, $\tau^k = \tau^h = \tau^\phi = \tau^y, \forall t$. However, differently from the previous section, the government can optimally set the taxation on consumption. Despite both taxes affect the intratemporal decision between labor and consumption of the household, the distinction and optimal mix between taxes on income and on consumption is relevant because of the additional dimensions of each tax in the model. Taxes on consumption will affect the transaction technology of the economy, since, for every unit of domestic or foreign good consumed, households must pay the tax. On the other hand, the taxation on income will distort the intratemporal decisions of capital accumulation, based on the net expected return of capital in the next period, and the use of the current stock of capital, since there is an allowance for changing the capacity utilization of the economy.

Table 2.4 shows the optimal combination of taxes and interest rates for different levels of nominal rigidities, with the first line again describing the baseline scenario.

The first block of results confirms the observations made in the previous section. The Ramsey planner still tries to tax labor more than capital, relying, in this case, in very high consumption taxes to pay for a subsidy on capital – in this case, the subsidy is implemented through negative income taxation. Regarding the optimal choice of interest rates, the model with income and consumption taxes replicates the results presented in section 2.2, where inflation departs from zero only in conditions of full price flexibility or in the case of indexation where the distortions from price stickiness are eliminated. In these cases, the optimal policy approaches the Friedman rule. In Correia, Nicolini and Teles (2008)[26], the authors prove that allocations in a closed economy without capital are the same under flexible and sticky prices as the optimal outcome of the Ramsey problem, irrespective of the degree of price stickiness. The important constraint necessary to prove the result is the minimum number of instruments (taxes) to operate fiscal policy, where consumption taxes is one of those taxes. Here, not only the model is designed to describe an open economy, but also the presence of capital adds one more source of revenues from income taxation, increasing the complexity of the model. In this sense, the results of Correia, Nicolini and Teles (2008)[26] seem to be robust to such expansion of the model.

The intuition for the result in Correia, Nicolini and Teles (2008)[26] is that, under the appropriate set of instruments, consumption tax operates as a state-contingent price used by the government to replicate the Pareto efficient allocations even under sticky prices. It is worth noting that it's only the consumption tax that is capable to operate like this: the same results can not be implemented if the tax system discriminates the income of capital and labor, as seen in the previous section. Thus, not only the number of instruments is relevant, but also that one of this instruments is the consumption tax. The optimal policy for prices, on the other hand, depends on the degree of price rigidity. Under flexible prices, taxes do not need to account for the distorting markups in production, and the optimal policy is the Friedman

Table 2.4: Optimal inflation and taxes – consumption and income taxes.

	α_n	α_m	π	R	$\tau^y(\%)$	$\tau^c(\%)$	Obs.:
1	0.6	0.6	0.00	4.00	-15.16	79.69	Baseline scenario
2	0.0	0.0	-3.71	0.15	-15.16	82.30	
3	0.6	0.0	0.00	4.00	-15.16	79.69	
4	0.0	0.6	0.00	4.00	-15.16	79.69	
Indexation:							
5	0.6	0.6	-3.79	0.06	-15.16	82.36	$\kappa_x = \kappa_n = \kappa_m = \kappa_{xp} = 1$
6	0.6	0.6	0.00	4.00	-15.16	79.69	$\kappa_x = \kappa_n = 1$
7	0	0	-3.79	0.06	-15.16	82.36	$\kappa_x = \kappa_n = 1$
8	0.6	0	-3.79	0.06	-15.16	82.36	$\kappa_x = \kappa_n = 1$
9	0	0.6	0.00	4.00	-15.16	79.69	$\kappa_x = \kappa_n = 1$
Open Economy:							
10	0.6	0.6	0.00	4.00	-15.16	79.69	$\alpha_{xp} = 0$
11	0.6	0.6	-3.79	0.06	-15.16	82.36	$\alpha_{xp} = 0, \kappa_x = \kappa_n = \kappa_m = \kappa_{xp} = 1$
12	0.6	0.6	0.00	4.00	-15.16	79.69	$\alpha_{xp} = 0, \kappa_x = \kappa_n = 1$
13	0.6	0.6	0.00	4.00	-13.12	83.84	$\varkappa = 0$
14	0.6	0.6	0.00	4.00	-13.12	83.84	$\omega = 0$
15	0.6	0.6	0.10	4.10	-15.10	79.85	$\pi = \pi^* = 1.0074$ (3%p.y.)
16	0.6	0.6	0.00	4.00	-15.14	79.60	$\pi = 1.0074$ (3%p.y.)

Note: Baseline scenario: $\pi^* = \pi^o = 0\%$; $\varkappa = 0.363$; $\omega = 0.55$; $\alpha_{xp} = 0.6$; $\kappa_x = \kappa_n = \kappa_m = \kappa_{xp} = 0$; $\tau^\phi = \tau^k$; $tb/y=0$.

rule. Under sticky prices, price stability over all periods and states is the optimal outcome, as the elimination of production markups dominate the objective function of the planner, just like in section 2.2. In this sense, consumption taxes assume the role of debt in models with complete markets, with very high volatility in order to insure households against all possible states of world.

In order to check the robustness of the Ramsey steady state with this combination of taxes, it becomes critical to evaluate prices and taxes under different parameterizations of the nominal and real rigidities affecting the intratemporal Euler equation describing the consumption-leisure trade-offs. The stationary transformation of the intratemporal Euler equation when the government has access to consumption taxes

is given by:

$$\frac{(1 - \tau^h)}{(1 + \tau^c)} w (1 - h) = \frac{\gamma}{(1 - \gamma)} mcw \left(1 - \frac{\zeta}{\mu^z (\mu^r)^{\frac{\theta}{1-\theta}}} \right) c \left[1 + \nu^m \left(\frac{R - 1}{R} \right) \right]$$

Again, as described in the previous section, there are three main rigidities affecting the intratemporal choice: habit persistence in consumption (given by parameter ζ), wage stickiness (setting a wedge given by mcw) and the cash-in-advance constraint (restricted by parameter ν^m). Figure 2.3 describe the taxes and nominal interest rate choices under all possible parameter combinations for these wedges. Each line of the figure shows how interest rate, taxes and the optimal labor supply responds when one of the parameters change along the interval of possible values. Notice that the Friedman rule is not a possible outcome, even eliminating most of the domestic rigidities present in the model. The optimal nominal interest rate does not diverge from 4% per year, implying that price stability is the optimal outcome. Income taxation also does not change significantly, always very close to the values found in the previous section as the subsidy for capital. The main adjustment, as expected, is on the consumption tax and on the labor supply, measured as the percentage deviation of the hours supplied under the Ramsey policy relative to the assumption of hours in the steady state of the competitive equilibrium.

This section characterized an extension of the results in Correia, Nicolini and Teles (2008)[26], showing the role of consumption taxes, even when the set of fiscal policy instruments is not as complete as in section 2.2. However, not only the number of instruments, but also the composition of the instrument set play a critical role in obtaining the result, as seen by the comparison of results with the previous section. In both cases, the Ramsey planner could set two taxes. However, in the previous section, a mild deflation was the optimal outcome in the Ramsey policy.

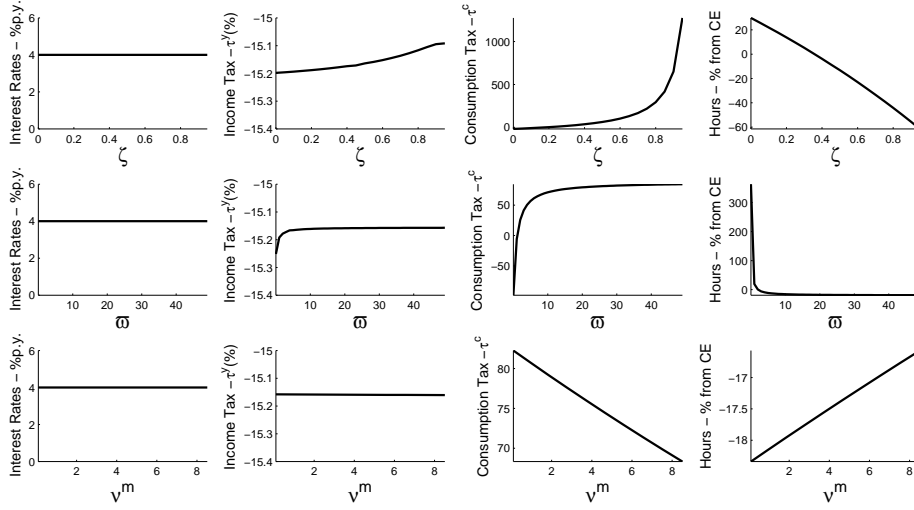


FIGURE 2.3: Optimal taxes and inflation: consumption and income taxes

2.5 The case of an income tax

In this section, consider the situation where the government is restricted to operate fiscal policy setting only a distortionary tax on total income ($\tau^h = \tau^k = \tau^\phi = \tau^y$, $\tau^c = 0$). The case of a single tax on income is extensively explored in the literature when considering a single distortionary fiscal policy instrument. For closed economies, Schmitt-Grohé and Uribe (2005b)[67] detail the steady state and the dynamics in a medium scale model with several rigidities. For an open economy, Benigno and De Paoli (2009)[11] explore the dynamics of a very simple model for small-open economies distorting the household intratemporal condition with a tax on total income. Ambler, Dib and Rebei (2004)[7], despite the focus on optimal monetary policy, also evaluates the dynamics of a small open economy with income taxes.

When the government has access to only one tax, it becomes impossible to set any sort of discrimination between production factors or between the demand of intermediate and final goods. With a single tax instrument and the money demand

Table 2.5: Optimal inflation and taxes – income taxes.

	α_n	α_m	π	R	$\tau^y(\%)$	Obs.:
1	0.6	0.6	1.89	5.97	8.63	Baseline scenario
2	0.01	0.01	174.41	185.38	-0.29	
3	0.6	0.01	2.45	6.55	8.60	
4	0.01	0.6	6.11	10.35	8.11	
Open Economy:						
5	0.6	0.6	1.89	5.97	8.63	$\alpha_{xp} = 0.001$
6	0.6	0.6	6.14	10.38	8.11	$\alpha_{xp} = 0, \kappa_x = \kappa_n = 1$
7	0.6	0.6	2.44	6.54	11.37	$\varkappa = 0.05$
8	0.6	0.6	2.47	6.57	11.49	$\omega = 0.05$
9	0.6	0.6	1.97	6.05	8.69	$\pi = \pi^* = 1.0074$ (3%p.y.)
10	0.6	0.6	1.89	5.97	8.65	$\pi = 1.0074$ (3%p.y.)

Note: Baseline scenario: $\pi^* = \pi^o = 0\%$; $\varkappa = 0.363$; $\omega = 0.55$; $\alpha_{xp} = 0.6$; $\kappa_x = \kappa_n = \kappa_m = \kappa_{xp} = 0$; $tb/y = 0$.

driven by the cash-in-advance constraint, the income tax is set in such a way that the government budget constraint is balanced. As a consequence of the lack of instruments, inflation is considered as an additional tax from the Ramsey planner’s perspective. This is the main result from the first panel of table 2.5, since, for low levels of price rigidity, inflation is significantly larger than zero. Note that this is the opposite relation presented, for instance, in section 2.3, where inflation was decreasing as a function of price rigidity. For a simple comparison with the previous case, figure 2.4 replicates the same simulation presented in figure 2.2, showing two surfaces relating the Calvo parameter for domestic and imported goods’ firms and the optimal of inflation for two different degrees of habit persistence in consumption.

There are two determinants of this result. First, notice, from the intratemporal Euler condition of the households setting the consumption-labor choice, that the money demand by the households sets up the nominal interest rate as an equivalent instrument as a tax in labor income. Thus, from the intratemporal allocation, a tax on labor income is observationally equivalent to an increase in nominal interest rates. Second, there is the objective of the Ramsey planner, present in all the simulations so

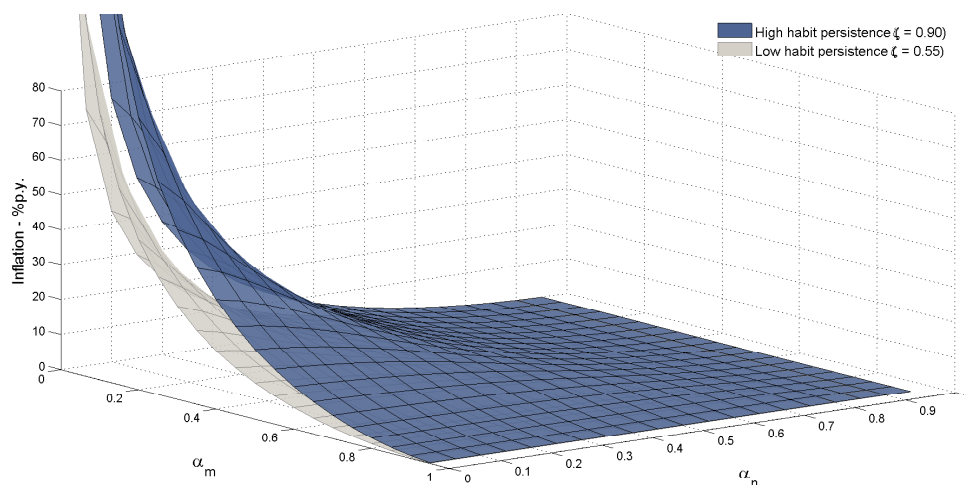


FIGURE 2.4: Real rigidities and optimal inflation: single income tax

far, in discriminating the returns from capital to the returns from labor, subsidizing the first at the expense of the later. Given that the nominal interest rate does not directly affect the capital allocation, raising inflation and the long run nominal interest rate is equivalent to imposing a large tax on labor. In order to confirm this result, simulations of the model without the cash-in-advance constraint result to price stability as the main outcome of the Ramsey policy with this configuration of taxes.⁹

Results in this section confirm, even under this extreme assumption regarding the number of taxes, the main priorities of the Ramsey planner, as discussed in the previous sections. The major difference is the instruments used in order to implement the optimal policy. Notice that, under this restricted set of instruments, the government still implements a relative subsidy on capital, even at the expense of an output loss due to the increase in inflation.

⁹ Results available upon request.

2.6 The zero lower bound and the steady state

The steady state of the Ramsey problem with the proposed correction for the zero lower bound problem for nominal interest rates highlights some of the trade-offs previously discussed in this chapter. The asymmetric term added in the objective function of the Ramsey planner shifts inflation to a higher level, reducing the probability that the zero lower bound for nominal interest rates is violated. Table 2.6 shows the optimal policy under different assumptions for domestic price rigidity and values for parameter ω_r , comparing the results with the baseline scenario presented in the previous sections. The first result detailed in table 2.6 is that the size of the adjustment is a function of the nominal rigidities in the model. The more flexible prices are, the smaller parameter ω_r must be in order to generate a significant deviation of the Ramsey policy for inflation from the baseline scenario. This is seen, for instance, in the first panel, comparing the results in lines 2 and 4, where the only difference between the two economies is the value of the Calvo parameter for domestic producers (α_n).

An important topic to be considered when implementing the adjustment to the zero lower bound problem is the new steady state values of the fiscal policy instruments. As the results in table 2.6 show, the number of instruments available plays a role again when a higher inflation is imposed as the Ramsey solution. In the first panel, when the government has all taxes available, the adjustment is made on the subsidy on capital, as the main result of the section 2.2 regarding taxes on consumption and on labor income remains the same. Without at least one of the taxes of the model, the final adjustment is distributed among the remaining instruments.

Notice, first, that the adjustment of the taxes related to capital is not linear as ω_r increases. In fact, there is an u-shaped behavior of capital taxes, mostly due to the cost of adjusting the capacity utilization in the model. Consider the

Table 2.6: Ramsey policy and the zero lower bound.

α_n	ω_r	π	R	$\tau^k(\%)$	$\tau^h(\%)$	$\tau^c(\%)$	Obs.:	
Full Set of Taxes								
1	0.6	0	0.00	4.00	-15.35	100.00	-100.00	Baseline
2	0.6	0.01	0.76	4.79	-15.36	100.00	-100.00	
3	0.6	0.1	6.09	10.33	-15.45	100.00	-100.00	
4	0	0.01	2.88	7.00	-15.36	100.00	-100.00	
5	0	0.1	15.67	20.30	-15.28	100.00	-100.00	
No Consumption Tax – $\tau^c = 0$								
6	0.6	0	-0.11	3.88	-16.12	30.40	0.00	Baseline
7	0.6	0.01	0.66	4.68	-16.16	29.90	0.00	
8	0.6	0.1	6.12	10.36	-14.97	37.62	0.00	
9	0	0.01	2.40	6.49	-16.16	29.50	0.00	
10	0	0.1	15.98	20.62	-14.79	36.23	0.00	
Consumption and Income Taxes – τ^c and $\tau^h = \tau^k = \tau^\phi = \tau^y$								
11	0.6	0	0.00	4.00	-15.16	-15.16	79.69	Baseline
12	0.6	0.01	0.77	4.80	-15.19	-15.19	79.68	
13	0.6	0.1	6.11	10.35	-15.47	-15.47	80.65	
14	0	0.01	2.88	6.99	-15.20	-15.20	78.37	
15	0	0.1	15.79	20.42	-15.68	-15.68	76.71	

Note: Baseline scenario: $\pi^* = \pi^o = 0\%$; $\varkappa = 0.363$; $\omega = 0.55$; $\alpha_{xp} = 0.6$; $\kappa_x = \kappa_n = \kappa_m = \kappa_{xp} = 0$; $\tau^\phi = \tau^k$; $tb/y=0$.

effects of an increase in ω_r when the government has access to all taxes. This is an interesting case to evaluate, as the capital tax is the only instrument adjusted with the increase in inflation. For low levels of ω_r , inflation raises the markup over prices, changing the returns on capital and on labor. In order to compensate for the distortion from inflation, the Ramsey planner initially increases the subsidy on capital and reduces the steady state of the capacity utilization. However, deviations from the full utilization of capital increases the quadratic cost of adjustment of capacity utilization, reducing the net return of capital. There is obviously a limit in the reduction of the net return of capital. This is exactly the point where the Ramsey planner stops increasing the capital subsidy together with ω_r , starting the movement in the opposite direction. Figure 2.5 shows the capital tax, inflation,

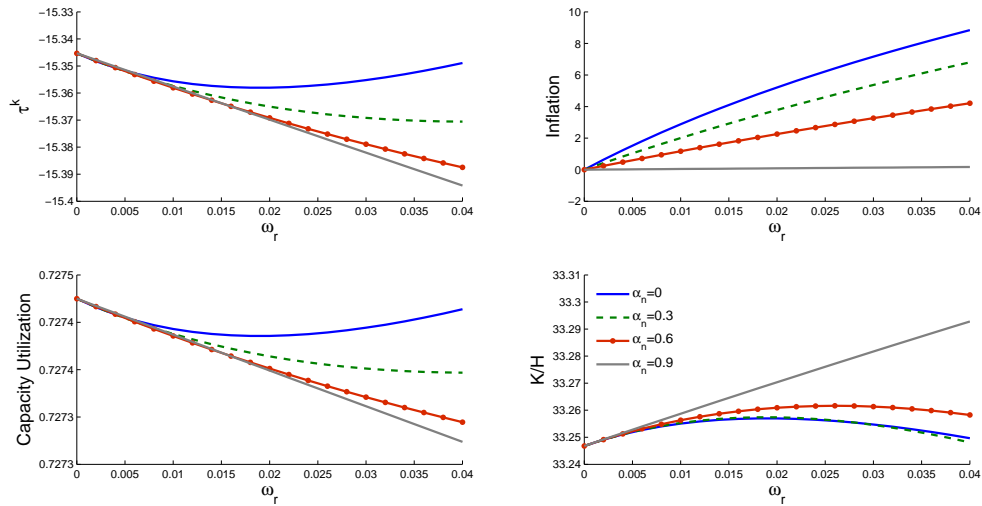


FIGURE 2.5: Optimal taxes and the zero lower bound for interest rates

marginal costs and the rate of capital utilization as a function of ω_r for different assumptions regarding nominal rigidity. Note that the u-shaped path for capital taxes follows a close path to the rate of capital utilization, just as described.

2.7 Conclusions

In this chapter, the main results confirm the propositions in Correia, Nicolini and Teles (2008)[26] about the relevance of consumption tax in the determination of the optimal policy in terms of inflation. For every scenario not associated with the conditions of flexible prices, price stability is the optimal outcome of the Ramsey problem. Also, the subsidy for capital is robust to every formulation in the model, confirming that the price markups distortions are the main target of the benevolent government when setting its policy. The small-open economy framework does not have a large influence in the determination of the steady state, mainly because the relevant distortions are still associated with the intertemporal and the intratemporal Euler equations of the household – structures that are irrelevant to the setup of an

open economy.

The next chapter presents the details of the estimation and the first comparison between emerging and developed, small-open economies. However, the characterization of the steady state of this economies will appear only in chapter 4, as the dynamics of Ramsey problem is described by a first order approximation of the non-linear problem.

3

Estimation

The main objective of this chapter is to estimate a complete set of parameters characterizing one EME, Brazil, and one SOE, Australia, in order to evaluate, in the next chapter, the differences in optimal policy steady state and dynamics due to parametric assumptions regarding these economies. The estimation uses Bayesian techniques, as it is now common in the literature, trying to fit a large number of time series in the model described in chapter 1, characterized not only by a large number of frictions, but also a large number of exogenous shocks. The estimation of medium-scale models, despite the recent advances in computational methods, is not a trivial task, as the high number of parameters to be estimated, combined with the non-linear structure of the model solution, might easily create problems to the econometrician, like, for instance, finding local maxima of the objective function of estimation instead of a global solution.

The choice of the countries was based on the policy framework across countries and data availability without notable structural breaks. It is well documented in the literature the high volatility in EMEs due not only to shocks, but also to policy shifts – if not both at the same time, with the policy regime change as a consequence of a

severe shock. Thus, finding a country operating under the same policy framework in a period sufficient to constitute a reasonable sample was the main task in selecting the countries for estimation. Also, Brazil has been operating monetary policy through an inflation targeting regime since 1999 and, despite some large shocks over the economy in the period, this framework has not been altered until the end of 2008. Australia also adopts a formal inflation targeting regime to conduct monetary policy, setting nominal interest rates to keep inflation inside a band of 2-3% annual inflation since early 1993. Thus, having countries conducting monetary policy under the same system allows a direct comparison of the estimates of the shocks derived from policy choices.

The literature has several examples of DSGE models estimated using Bayesian techniques. Most of the models are relatively small structures, when compared to the model presented in chapter 1, designed to highlight some specific characteristic of the data. Exception in the literature in terms of size are the models developed and estimated by several Central Banks¹. These models follow a tradition first established by Smets and Wouters (2003)[70] of fitting structures derived from first principles with a large number of shocks to an equally large number of observables. More recently, Adolfson, Laseén, Lindé and Villani (2007)[2] estimate for the Euro area a model very similar to the one presented here. They highlight the role of the large number of shocks and nominal rigidities in explaining data variability, as there was not a single shock dominating the dynamics of the aggregates used in estimation. For emerging economies, García-Cicco (2009)[37] estimates a model for Mexico, relying on data from input-output matrices to calibrate some parameters for intermediate production sectors.

This chapter is closely related to the work of Smets and Wouters (2005)[71]

¹ Among others, the models of the Riksbank (Sweden), RAMSES[3], Banco Central de Chile, MAS[52], the European Commission, QUEST III[60], and the model of the Norges Bank (Norway), NEMO[15], are estimated using Bayesian methods.

in setting a common methodology for the estimation of the model using different datasets. Smets and Wouters (2005)[71] use the same model, priors and time span to estimate a DSGE model for the Euro Area and the US, in order to compare their business cycle characteristics. In this chapter, the assumptions regarding priors will be the same across all countries, using very loose priors in order to add a minimum of country-specific information. The main difference from the procedure in Smets and Wouters (2005)[71] is the treatment of the sample, as the time series for each country considered here is restricted to periods when countries operated monetary policy under the same framework. As a consequence, datasets across countries here do not have the same size, as they start in different points in time.

In terms of models for small-open economies, this chapter also relates to the work in Justiniano and Preston (2009)[44], where the authors estimate a model using data from Canada, New Zealand and Australia using on the same priors. The objective is to find the set of policy parameters characterizing optimal monetary policy in an economy where the government minimizes an arbitrary loss function based on the variance of inflation, output and interest rates. The choice of the optimal parameter set is conditioned not only to the equations characterizing the equilibrium of the structural model, but also to the uncertainty related to the parameters describing the economy. The approach developed here is different, in the sense that the dynamics of the optimal policy is characterized by the solution of the Ramsey problem based on the maximization of households' welfare, and not an exogenous loss function. Second, instead of keeping the problem restricted to policies similar to the Taylor rule governing monetary policy, the main objective here is to obtain the structural parameters to compute, later in chapter 4, a solution to the non-linear policy described by the solution of the Ramsey problem. Third, the authors find that the optimal Taylor policy is muted in terms of interest rates response to changes in the exchange rates. It should be noted, however, that their sample is formed uniquely

by SOEs. The high volatility of exchange rates verified in EMEs can generate significant welfare effects that the non-linear Ramsey policy might consider relevant when setting the optimal policy. Finally, the model here also considers the optimal fiscal policy dimension of the Ramsey problem and the effects of distortionary taxes in the allocations.

As a general result, the model has a good fitting for both countries, despite some caveats regarding the foreign sector variables for Brazil. The estimation of the model using different datasets highlights the importance of the estimation of the same model for the two countries, as the propagation of shocks is significantly affected not only by the stochastic process of the shocks, but also by the nominal rigidities present in the model. Of particular importance is the system of equations describing sectorial prices, with the price rigidity associated with imported goods' retailers playing a crucial role in the dynamics of the model after foreign shocks.

This chapter is organized as follows. The next section describes the econometric procedure, with a very brief overview of the estimation of DSGE models using Bayesian methods, followed by a detailed description of the priors, dataset and the necessary conditions to solve for the steady state of the model for estimation. Section 3.2 show the results with focus on comparing the estimated values with the literature and establishing robustness of the procedure with respect to the priors used. The differences between SOEs and EMEs in terms of dynamics under the competitive equilibrium are shown in section 3.3. Despite not setting the log-linearization of the model around the same values of steady state, a variance decomposition provides a good benchmark for comparing the countries' dynamics. Section 3.4 concludes.

3.1 Econometric methodology: Bayesian estimation, priors and dataset

The objective of this section is to present the model's estimation procedure, followed by the description of the calibrated parameters necessary to derive the steady-state,

the set of parameters being estimated, its priors and the dataset. Given that the same model is estimated with different datasets, and one of the objectives of the estimation procedure is to check the parameter discrepancy across EMEs and SOEs, the estimation strategy is as agnostic as possible. In this sense, very little country-specific information is added in the estimation as a prior information, following the procedure of Smets and Wouters (2005)[71] to compare business cycles characteristics between the US and the Euro Area. Given the size of the model and the large number of parameters to be estimated, Bayesian techniques for DSGE models, described in details in Canova (2007)[16] and An and Schorfheide (2007)[8], constitute a very practical way to handle the estimation.

Due to the large literature available and the recent popularity of Bayesian techniques, the estimation procedure here is described without much detail. The general method follows these steps:

1. Set the priors for the set of parameters being estimated.
2. Solve the model after a log-linear approximation, obtaining a state-space representation of the dynamics and its likelihood by the prediction error decomposition.
3. Obtain 2,000,000 draws from the posterior distribution of the parameters, using the Metropolis-Hastings (MH) algorithm, and keep just the last 1,000,000 draws, in order to eliminate the effects of initial values in the simulation, and check for convergence of the chains.
4. Compute the marginal likelihood of the model and the statistics describing the relevant moments of the data.

The first step will be described in the next subsection, with a complete description

of the priors selection². For the second step, define Θ as the whole set of parameters of the model, θ as the vector of parameters being estimated, y_t as the vector of variables expressed as log-deviations from the steady-state level, ϵ_t as the vector of innovations and v_t a vector of expectation errors ($v_t = y_t - E_{t-1}(y_t)$). The log-linearized model can be expressed as a linear Rational Expectations system of the form:

$$\Pi_0(\Theta) y_t = \Pi_1(\Theta) y_{t-1} + \Pi_2(\Theta) \epsilon_t + \Pi_3(\Theta) v_t$$

The model is solved by perturbation methods up to first order and written in state-space form, where s_t is the vector of state variables and \hat{y}_t is a matrix observables in the form:

$$\begin{aligned} \hat{y}_t &= G(\Theta) + H s_t \\ s_{t+1} &= A(\Theta) s_t + B(\Theta) \epsilon_{t+1} \end{aligned}$$

In this case, note that the matrix $G(\Theta)$ is a function of the parameters, even if it is not directly associated with the state equations (e.g., the growth rate of output is a function of the technological shocks μ^r and μ^z), while matrix H selects the variables from the state equation. Once the model is set in the state-space representation, the (log-) likelihood can be obtained by the prediction error decomposition³.

The third step is the evaluation of the posterior distribution. For a given dataset $Y = \{\hat{y}_t\}_{t=0}^T$, priors set in the first step, $p(\Theta)$, and the likelihood function calculated in the second step, $L(\Theta|Y)$, the Bayes rule allows the derivation of an expression for the posterior distribution of the parameters:

$$p(\Theta|Y) \propto p(\Theta) L(\Theta|Y)$$

Given the final set of draws, the autocorrelogram of the chains and the Geweke

² The estimation is executed with the procedures written in the package Dynare for solving and estimating the model.

³ See Hamilton (1994)[41] page 385.

(1992)[38] statistic, comparing the mean of the first with the last one-third of the chain, are computed in order to check for normality and convergence of the chain.

3.1.1 Steady state conditions for estimation

Due to identification problems associated with the estimation, some parameters must be calibrated in order to solve for the steady state of the model.⁴ The set of calibrated parameters is exactly the same used in chapter 2 when the steady state of the Ramsey policy is computed. Thus, the capital share in the production function, given by θ , is set to 0.3, while the depreciation rate, δ , is set to 0.025, in order to obtain an annual value of approximately 10%. The tradable goods' share, ω , represents 55% of the consumption bundle. After normalizing the household's endowment to unity, assume they are working 20% of their time in steady state ($h = 0.2$).

Table 3.1 presents the data used to calibrate the estimations, with details also on the sample size and the monetary policy regime in each country. In terms of parameters and big ratios that are specific to each country in the sample, there are 12 values that must be assigned to complete the steady state computation for the estimation. First, assume that the long run economic growth is equal to its real GDP growth sample average, matching, in this case, the productivity growth of the economy, μ^z . Also, as a consequence, the drift μ^{Υ} in the investment-specific productivity shock is set to unity for both countries.

In order to derive the parameters associated with interest rates, assume, first, that the real interest rate is set at 4.5% per year. Setting a value for the real interest rate implies choosing a country-specific value for the discount factor, β , as the parameter becomes a function of the average growth of productivity. This choice also might imply a high value for β , real close to unity, but alternative setups used to reduce

⁴ The full derivation of the steady-state conditions of the competitive equilibrium is available in the appendix B.

Table 3.1: Steady state data by country.

Country	Australia	Brazil
Steady State Conditions		
π^o	2.75%	4.50%(*)
μ^z	3.8%	4.2%
R^*/R^f	1	1.06
G/Y	0.18	0.32
B/Y	0.10	0.36
M/Y	0.15	0.16
IMP/Y	0.20	0.12
τ^c	0.121	0.159
τ^k	0.494	0.345
Sample details		
Time Span	1993Q1 – 2008Q4	1999Q1 – 2008Q4
# Observations	64	36
Monetary Policy	I.T. since 1993:01Q	I.T. since 1999:03Q

Note: Data definitions and sources in appendix E. (*) Steady state based on inflation target set by monetary authority.

the discount factor would imply low productivity growth or, alternatively, very high real interest rates. The risk premium (R^*/R^f) is equal to the sample average of the Emerging Markets Bond Index Plus (EMBI+), calculated by J.P. Morgan, for those countries where data is available. In the estimation for Australia, as an SOE, it is assumed that the risk premium is negligible – thus, $R^f = R^* = R$. The nominal interest rate is set combining the real interest rate with the risk premium and a measure of the long run inflation target⁵.

It is assumed that the trade balance is in equilibrium in steady-state. As a consequence, external debt is also equal to zero. Also related with the foreign sector of the economy, the growth rates of the money supply and the measures of foreign inflation (ΔM^* , π^* , and π^{m*}) matches the average domestic inflation, in order to avoid changes in the nominal exchange rate in steady-state.

⁵ Since all the countries used in the estimation exercise are adopting inflation targeting regimes, some measure of the long run target for prices is available on the Central Bank websites.

Four big ratios are further used to characterize each economy: the government expenditure to output ratio, G/Y ; net debt to output, B/Y ; monetary base to output, M/Y ; and imports to output, IMP/Y . These ratios are the average computed from the National Accounts inside the sample period of each country. There is no visual evidence of non-stationary behavior of the ratios during the period considered.

Finally, the steady state of taxes is set based on studies derived from the methodology of Mendoza, Razin and Tesar (1994)[53], namely Carey and Rabesona (2003)[18] for Australia and Pereira and Ellery Jr (2009)[57] for Brazil. Carey and Rabesona (2003)[18] is the main reference for Australia for two reasons. First, the authors present the most updated tax ratio dataset known for the recent decade. Second, the authors reconsider issues regarding cross-border taxation, which are relevant for the computation of capital taxes in small-open economies. The authors also explore the OECD data on statutory tax arrangements to establish more realistic assumptions regarding the proportion between capital and labor taxes paid by the households.⁶

Another topic related to taxes and the big ratios of the economy is the government budget constraint. Given ratios with respect to the GDP for the money supply, debt and government spending, one of the taxes can not be used to set the calibration, as there will be one extra degree of freedom with respect to moments than variables to match. The choice here, robust to other alternatives, is to pick tax ratios for capital and consumption, using the value of the tax on labor, τ^h , to adjust the budget constraint.

The parameters of the foreign block of the economy were obtained estimating a VAR by MLE and using the Cholesky decomposition of the covariance matrix to

⁶ In Mendoza, Razin and Tesar (1994)[53], this proportion is, by assumption, equal to the capital and labor shares in total output. As Carey and Rabesona (2003)[18] point out, among other problems, this assumption ignores special tax arrangements in terms of social security, capital losses and dividends that are country-specific. Despite this adjustments, however, the correlation between the time series of tax ratios using the two methodologies, for the countries considered here, are not smaller than 0.91 (see table 3, page 143, in Carey and Rabesona, 2003[18]).

identify the shocks. The sample for the estimation of the VAR started in the first quarter of 1980, finishing in the last quarter of 2008. The selection of the sample respect other estimations for the US economy, based on the assumption that, after the 1970's there was a structural break in the equations describing the behavior of the monetary authority. The sample for this exercise starts in the same quarter as, for instance, the estimations in Canova (2005)[17] when investigating the transmission of shocks from the US to Latin American countries. The VAR contains only one lag, chosen by the SIC information criterion. All residuals seem to be well-behaved, without major outliers for concern.

3.1.2 Priors and dataset

The model was estimated for one SOE, Australia, and one EME, Brazil. The dataset for each economy has observables for six real variables in growth rates: output, consumption, investment, government expenditure, imports and exports; plus gross CPI inflation, nominal interest rates, imports and exports inflation (in levels and measured in foreign currency). The inclusion of imports and exports inflation in foreign currency provides information about sectorial prices, thus avoiding or, at least, restricting the identification problems related to the Calvo parameter for the imported goods sector in Lubik and Schorfheide (2006)[50]. All real variables are calculated by deflating the nominal value by CPI inflation, and the whole dataset is filtered by X-12 ARIMA to remove seasonal factors.

One feature of the dataset is that the time span selected for each economy is restricted to periods with a constant monetary policy framework. There are two main reasons for this choice, both related to the possibility of structural breaks in the time series. First, the high volatility of EMEs data is not only a consequence of bad shocks, but also a result of changes in economic policy. Drazen and Easterly (2001)[33] and Alesina, Ardagna and Trebbi (2006)[5] present empirical evidence that

Table 3.2: Economic crisis and monetary policy changes.

Country	Year	Old Policy Framework	New Policy Framework
Czech Republic*	1997	Fixed ERR	IT and floating ERR
Thailand**	1997	“Crawling peg” ER	Monetary targeting
Korea**	1998	Monetary targeting	IT and floating ERR
Poland*	1998	“Crawling peg” ER	IT and floating ERR
Brazil**	1999	“Crawling peg” ER	IT and floating ERR
Argentina**	2001	Currency board	Monetary targeting
Hungary*	2001	“Crawling peg” ER	IT and floating ERR

Note: (*) Information from Jonas and Mishkin (2004)[42]; (**) Information from the website of the Central Bank of each country. “Year” denotes the year of economic crisis

economic reforms, considered not only in the sense of stabilization policies but also structural changes, occur in countries after very negative economic outcomes. This is the basis of the so-called “reform follows crisis” hypothesis. In a small piece of evidence, table 3.2 shows recent cases of changes in the monetary policy framework that followed economic crisis in a group of seven countries.

The second reason for restricting the sample is that dealing with structural breaks through the addition of nonlinearities in the model might result in an increase in the number of state variables without necessarily a gain in terms of model fitting. In an extreme but relevant example, García-Cicco (2009)[37] notes that, conditioned on the solution method applied to solve the model, the inclusion of time-varying parameters in the Taylor rule might be useless to deal with changes in the response of monetary policy to the exchange rates. If the model is solved using a first order approximation of the equilibrium conditions, like here, the solution does not include the time-varying coefficient related to the changes in the exchange rate.

Table 3.3 presents the priors used for the estimation. The prior selection tried to cover most of the boundaries of the parameter space. As an example of this procedure, note that all the parameters related to the autoregressive components of shocks, price and wage indexation, Calvo pricing and the parameter associated with

households' habit persistence have priors based on the uniform distribution covering the unity interval. For the volatility of shocks, all priors are based in a inverse-gamma distribution with infinite variance. The priors for the standard deviation of the inflation target shock has a smaller mean relative to the remaining shocks because most of the countries adopting inflation targeting regimes set the target with some anticipation. Thus, changes in the target in consecutive periods constitutes a very rare event. On the other hand, priors for the volatility of the stationary sectorial shocks are assumed to be larger than the aggregate, non-stationary productivity growth. It is assumed that a prior with larger volatility for domestic shocks facilitated the data adjustments to the discrepancy in volatility between SOEs and EMEs without changing the assumption regarding aggregate shocks across countries.

Note also that, in order to ensure that capital has the same return in both sectors of the economy when the steady state of inflation is different than zero, the parameters related to price rigidity, price indexation and the sectorial price elasticity of substitution are equal across the two domestic production sectors.

In order to avoid problems in the MH algorithm with the acceptance rate target, parameters with simulated values close to the extreme of bounded priors were truncated. This was the case, as an example, with several estimates of shock's persistence parameters: the priors for all these parameters are based in an uniform distribution in the $[0, 1)$ interval. Since the simulations for some of those parameters were showing values very close to unity, the acceptance rates dropped to values outside the 30-40% target. In this case, the parameters were truncated at values very close to unity, and their confidence intervals not computed.

3.2 Estimation results

In this section, the main results from the estimation procedure are presented, highlighting the differences between the parametric properties of EMEs, based on the

Table 3.3: Prior selection.

Parameter	Distribution	95% C.I.	
Shocks: standard deviations and autocorrelations			
σ_i	Std.dev.shocks: $g, z, \Upsilon, \pi^m, R, \tau^i$	I-G(0.02, ∞)	[0.005, 0.111]
σ_x	Std.dev. trad. productivity shock	I-G(0.10, ∞)	[0.024, 0.557]
σ_n	Std.dev. non-trad. productivity shock	I-G(0.10, ∞)	[0.024, 0.557]
σ_{π^o}	Std.dev. inflation target shock	I-G(0.005, ∞)	[0.001, 0.028]
ρ_i	Shocks persistence	Uniform	[0.025, 0.975]
Prices, wages and markups			
$\alpha_n = \alpha_x$	Calvo price domestic production	Uniform	[0.025, 0.975]
α_m	Calvo price imported goods	Uniform	[0.025, 0.975]
α_{xp}	Calvo price exported goods	Uniform	[0.025, 0.975]
$\kappa_n = \kappa_x$	Indexation domestic production	Uniform	[0.025, 0.975]
κ_m	Indexation imported goods	Uniform	[0.025, 0.975]
κ_{xp}	Indexation exported goods	Uniform	[0.025, 0.975]
$\varpi - 1$	Elast. substitution labor types	G(10, 5)	[3.416, 19.38]
χ_w	Wage indexation	Uniform	[0.025, 0.975]
ϕ_w	Wage adj. cost	G(1.5,0.5)	[0.686, 2.627]
$\eta_i - 1$	Price elast. subst. goods	G(4,1)	[2.286, 6.185]
Policy parameters			
α_π	Inflation coeff. in Taylor rule	G(2,0.5)	[1.143, 3.093]
α_y	Output coeff. in Taylor rule	G(0.5,0.3)	[0.095, 1.236]
α_{rer}	Real exchange rate coeff. Taylor rule	G(0.5,0.3)	[0.095, 1.236]
ψ_{li}	Liabilities/GDP coeff. in fiscal rule	G(1,1)	[0.372, 0.939]
ψ_y	Output coeff. in fiscal rule	Normal(0,1)	[-1.96, 1.96]
Household's problem parameters			
ζ	Habit persistence	Uniform	[0.025, 0.975]
ε	Elast.subst.non-trad. and trad.	G(1.5,0.5)	[0.686, 2.627]
ϱ	Elast.subst.imported and domestic trad.	G(2,0.5)	[1.143, 3.093]
ϕ_i	Investment adj. cost	G(2.5,0.4)	[1.778, 3.343]
θ_2/θ_1	Capacity utilization adj, cost	G(1.5,0.5)	[0.686, 2.627]
Foreign sector and portfolio parameters			
η^*	Foreign elast. subst. of goods	G(2,0.5)	[1.143, 3.093]
κ_1	R^f elast. to risk	G(1,1)	[0.051, 2.996]
κ_2	R^f elast. to net foreign position	G(1,1)	[0.051, 2.996]
v_1	Persistence imported prices	Uniform	[0.025, 0.975]
v_2	Elast. π^m to terms of trade	Uniform	[0.025, 0.975]
ξ_i	Elast. π^m to foreign variables	Normal(0,1)	[-1.96, 1.96]
ψ_1	Portfolio adj. cost domestic bonds	I-G(0.02, ∞)	[0.005, 0.111]
ψ_2	Portfolio adj. cost international bonds	I-G(0.02, ∞)	[0.005, 0.111]

Note: Numbers in parenthesis are priors' mean and standard deviation. Distributions: "Uniform": Uniform between $[0, 1]$; "I-G": Inverse-gamma; "G": Gamma.

estimation for Brazil, and SOEs, based on results for Australia.

3.2.1 *Brazil*

The analysis of the estimated parameters for Brazil confirms that the main parameters of the model are quite in line with the literature on EMEs. The estimation presented a good convergence of the chains, and the statistic of Geweke (1992)[38] does not reject the hypothesis that the chains for each parameter is stable. However, in order to obtain these results regarding stability, several parameters had to be fixed in order to deal with the problems associated with the acceptance rate of the MH algorithm. The following set of parameters was fixed at the posterior mode: price indexation parameters $\{\kappa_n, \kappa_m, \kappa_{xp}\}$, wage indexation, χ_w , persistence parameters $\{\rho_\Upsilon, \rho_z, \rho_n, \rho_{\pi^o}, \rho_{\tau c}, \rho_g\}$, the portfolio adjustment cost on foreign bonds, ψ_2 , the elasticities of foreign interest rates with respect to the risk premium and the net foreign asset position, κ_1 and κ_2 , and the parameter relating the terms of trade to the imported goods' price in foreign currency, v_2 . Because of identification problems in estimation, the elasticity of substitution of imported goods, η_m , was truncated at the prior's mean. Tables 3.4 and 3.5 show the results, highlighting the priors' mean, the posteriors' mode and the 95% confidence interval for the posterior. The results are separated in two tables in order to facilitate the comparison of the volatility of shocks across countries.

Parameters describing the nominal rigidities associated with prices show significant heterogeneity across sectors, based on the estimates of price indexation and the Calvo parameters. On the one hand, the median value for the probability of a domestic producer firm not adjusting prices ($\alpha_n = 0.345$) implies a high degree of price flexibility, as the typical firm optimally adjust prices on average once every four to five months. On the other hand, the median of the same parameter estimate for imported goods ($\alpha_m = 0.911$) implies that firms are only allowed to optimally set

their prices on average every 2.7 years. The same discrepancy is verified in terms of indexation, as the estimates for domestic firms are not different than zero, while imported goods' firms are subject to almost full indexation on prices. The estimated values for the Calvo parameter and the indexation process for exporting firms ($\alpha_{xp} = 0.807$, $\kappa_{xp} = 0.017$) are in the range of other studies for the US economy. As an example, Cogley and Sbordone (2008)[25] find no substantial evidence of indexation for prices in the US, despite a lower value for the Calvo parameter when compared to the estimates presented here. Smets and Wouters (2005)[71] find similar coefficients in terms of the Calvo parameter for the US, with a median probability of 0.87, but a slightly larger degree of indexation, 0.17.

The empirical evidence from other estimated DSGE models offer mixed results to support the estimates presented here. For instance, Silveira (2006)[28] estimates a model equivalent to the baseline formulation of Galí and Monacelli (2005)[36], thus, without indexation or the presence of retail firms, and find a Calvo parameter of 0.91 using Brazilian data in a very similar time span as the sample used here. The author, however, observes that the estimation of models with other types of nominal rigidities, like price indexation and wage stickiness could possibly result in very different values for the posterior of the Calvo parameters, especially because of the high level and volatility of inflation in Brazil. When adding habit persistence in consumption and price indexation, Silveira (2008)[29] notices that the estimate of the Calvo parameter remains almost the same as in his previous study, while the price indexation parameter falls in the range of [0.13, 0.71]. Again, the model in Silveira (2008)[29] has only one sector in production and no deviations from the Law of One Price based on the existence of retail firms. The microevidence available for Brazilian prices, on the other hand, shows that the mean duration of a price spell is between 2.7 and 3.8 months⁷, in line with the price rigidity for domestic firms, but

⁷ See Gouvea (2007)[39].

significantly lower than the values obtained in the estimation for imported goods' firms. Notice, however, that studies based on disaggregated prices, like Bils and Klenow (2004)[13] are not capable of identifying the sources of price changes, as the main statistic computed is the frequency of price adjustment. In other words, the statistic does not capture if the change in price is due to an optimal decision of the firm or the result of an indexation process, as the stylized model proposes. In this sense, the microevidence seem to corroborate to the estimation results, given the high degree of indexation of imported goods in an environment of positive inflation in steady state, and the high flexibility of domestic prices, both factors pointing towards very low price spells at the most disaggregated level.

The estimation of the elasticity of substitution across different types of goods implies, under price stability in steady state, markups over prices of 20.6% for domestic producers and 25% for imported goods firms. Consistent with the microevidence for firms exporting goods presented in De Loecker and Warzynski (2009)[49], the estimation shows markup over prices for the exported goods firms of 25.3%.

The baseline estimates show a significant degree of habit persistence in consumption, $\zeta = 0.873$. This value is in the upper bound of the small model estimated in Silveira (2008)[29] for Brazilian data, but in line with the evidence presented for other EMEs, like in Castillo, Montoro and Tuesta (2006)[19] for Chile, with mode values between 0.75 and 0.94, depending on model and sample assumptions. Larger variations are found in the literature for this parameter, even for the same sample. García-Cicco (2008)[37] finds a posterior mean of 0.83 in his baseline model estimated with Mexican data. However, different assumptions regarding the model structure might reduce this value to 0.13.

The elasticity of substitution across labor types, ϖ , has a posterior mode located between the two extremes presented in the literature, as discussed in the previous chapter. Neither values of $\varpi = 2$, adopted in Smets and Wouters (2003)[71] and

García-Cicco (2009)[37], or $\varpi = 21$, as in CEE (2005), are contained in the posterior mode of $\varpi = 13.6$. This is a relevant result also because there were no identification problems during the estimation procedure: the mode is different from the prior's mean ($\varpi = 11$) and, despite the large confidence interval, the posterior is significantly more precise than the prior. The elasticity of substitution across labor types implies a markup over wages of 7.9%.

The parameter describing the wage adjustment cost is significantly lower than the value used in Chugh (2006)[24]. The author calibrated ϕ_w in order to match, in a Calvo-type of wage adjustment, a frequency of wage adjustment equal to three quarters, which is consistent with the estimation in CEE (2005)[22]. In the mapping derived by the author, this implies $\phi_w = 5.88$. The estimated value for Brazil implies that the wage spell lasts roughly a little bit more than two quarters. The extreme value found in the literature is in García-Cicco (2009)[37], whose estimate implies that approximately 75% of workers are able to adjust their wages every period, $\phi_w = 0.4$. Thus, the labor market in Brazil can be perceived as more flexible than the US labor market, but definitely more sluggish when compared with other EMEs, like Mexico. Also related to the dynamics of wages, the parameter describing wage indexation was very close to zero, and truncated at the posterior mode ($\chi_w = 0.026$) in order to avoid problems with the acceptance rate target of the MH algorithm.

The estimate of the ratio of the parameters governing the cost of adjusting capacity utilization (θ_2/θ_1) is in line with the benchmark estimated value in Altig, Christiano, Eichenbaum and Lindé (2005)[6], also used in Schmitt-Grohé and Uribe (2005b)[67]. On the other hand, the median of the parameter describing the capital adjustment cost function (ϕ_i) is slightly higher when related to the values calibrated in these two papers. The median is equal to 2.68, while the literature, based on the estimation with US data, uses a value of 2.49. However, it is worth noting that this value is contained in the confidence interval of the estimation performed here.

The policy parameters are roughly in line with standard estimations for the Taylor rule. Nominal interest rates are very persistent, based on the estimated mode for the autoregressive term of the Taylor rule, $\rho_R = 0.925$. Combined with the strong persistence, interest rates respond very aggressively to deviations of inflation from its target. Curiously, the response to changes in real exchange rate are small. Still related to the policy parameters, taxes on labor income suggest a strong pro-cyclical fiscal policy, as the median of the posterior of the parameter associated with changes in output is significantly different than zero ($\psi_y = -4.056$).

On the households' elasticity of substitution across goods, the model presents a very high elasticity of substitution between tradable and non-tradable goods, $\varepsilon = 4.82$, specially when compared to the values found in García-Cicco (2009)[37] for Mexico (between 2.6 and 3.4), Castillo, Montoro and Tuesta (2006)[19] for Chile (between 1.5 and 2.4) and Silveira (2008)[29] for Brazil (median of 0.06). It is worth noting, however, that only the first author considers the existence of non-tradable goods in the model, as the elasticity computed for Chile and Brazil is between domestic and imported goods⁸. In this sense, the value obtained for the elasticity between the domestic tradable and the imported good is more in line with the literature, with the posterior median equal to 1.25. Finally, the elasticity of demand for domestic tradable goods from the rest of the world, η^* , is close to unity and well defined in terms of the posterior density.

The estimation of the stochastic processes for the shocks shows overall the expected results, with very persistent stationary sectorial shocks, just like in the traditional RBC literature. The persistence of the government spending shocks are also very high, with ρ_g set at 0.956. It is also not surprising the value for the inflation target shock, with ρ_{π^o} set at 0.981. The major discrepancy with the literature is on

⁸ Furthermore, in the case of Silveira (2008)[29], the author assumes that the elasticity of substitution is the same for households in the domestic and in the foreign economy, the later estimated using a dataset for the US.

Table 3.4: Structural parameters: Brazil.

Parameters	Baseline Priors			Alternative Priors	
	Prior	Posterior	95%CI	Posterior	95%CI
Prices, wages and markups					
α_n	0.5	0.345	[0.271 0.424]	0.378	[0.317 0.442]
α_m	0.5	0.911	[0.889 0.933]	0.842	[0.796 0.890]
α_{xp}	0.5	0.807	[0.775 0.839]	0.753	[0.705 0.803]
κ_n	0.5	0.000	—	0.292	[0.089 0.483]
κ_m	0.5	0.999	—	0.871	[0.766 0.980]
κ_{xp}	0.5	0.017	—	0.250	[0.075 0.437]
$\varpi - 1$	10	12.579	[6.666 18.268]	11.234	[6.244 16.198]
χ_w	0.5	0.026	—	0.381	[0.159 0.588]
ϕ_w	1.5	1.303	[0.718 1.897]	1.018	[0.605 1.393]
$\eta_n - 1$	4.0	4.858	[3.321 6.533]	5.176	[4.012 6.284]
$\eta_m - 1$	4.0	4.0	—	3.099	[2.114 4.134]
$\eta_{xp} - 1$	4.0	3.943	[2.346 5.399]	4.107	[3.180 5.068]
Policy parameters					
α_π	2.0	2.856	[2.065 3.612]	2.492	[1.857 3.104]
α_y	0.5	0.163	[0.097 0.225]	0.123	[0.067 0.178]
α_{rer}	0.5	0.062	[0.022 0.100]	0.097	[0.043 0.148]
ψ_{li}	1.0	0.017	[0.005 0.028]	0.028	[0.014 0.043]
ψ_y	0.0	-4.056	[-4.556 -3.533]	-4.838	[-5.307 -4.344]
Household's problem parameters					
ζ	0.5	0.873	[0.834 0.913]	0.825	[0.778 0.873]
ε	1.5	4.819	[4.088 5.521]	4.497	[3.870 5.159]
ϱ	2.0	1.250	[0.936 1.571]	1.216	[0.905 1.505]
ϕ_i	2.5	2.677	[2.104 3.261]	2.554	[2.170 2.935]
θ_2/θ_1	1.5	2.131	[1.282 2.963]	2.234	[1.657 2.831]
Foreign sector and portfolio parameters					
η^*	2.0	1.122	[0.839 1.406]	0.929	[0.670 1.184]
κ_1	1.0	0.013	—	0.877	[0.000 1.830]
κ_2	1.0	0.416	—	1.294	[0.140 2.364]
ν_1	0.5	0.121	—	0.299	[0.155 0.447]
ν_2	0.5	0.03	—	0.03	[0.01 0.060]
ψ_1	0.020	0.0068	[0.004 0.009]	0.006	[0.004 0.007]
ψ_2	0.020	0.0087	—	0.015	[0.005 0.028]
ξ_1	0.00	0.194	[-1.139 1.497]	-0.476	[-1.818 0.777]
ξ_2	0.00	0.162	[-0.569 0.900]	0.388	[-0.426 1.197]
ξ_3	0.00	-0.230	[-0.462 -0.005]	-0.355	[-0.657 -0.055]
ξ_4	0.00	-0.375	[-1.945 1.169]	0.064	[-1.238 1.442]
ξ_5	0.00	0.184	[-0.169 0.533]	0.102	[-0.285 0.469]

Note: “Prior” refers to the prior’s mean; “Posterior” refers to the posterior’s mode.

Table 3.5: Structural shocks: Brazil.

Param.	Prior	Baseline Priors			Alternative Priors		
		Post.	95%CI	LR(σ)	Post.	95%CI	LR(σ)
ρ_g	0.5	0.956	—	0.041	0.913	[0.858 0.969]	0.029
σ_g	0.02	0.012	[0.010 0.015]		0.012	[0.010 0.014]	
ρ_z	0.5	0.0002	—	0.116	0.179	[0.082 0.277]	0.116
σ_z	0.02	0.116	[0.094 0.138]		0.114	[0.091 0.135]	
ρ_{Υ}	0.5	0.0005	—	0.049	0.281	[0.084 0.471]	0.054
σ_{Υ}	0.02	0.049	[0.040 0.058]		0.052	[0.041 0.061]	
ρ_x	0.5	0.977	[0.965 0.988]	0.183	0.949	[0.921 0.978]	0.298
σ_x	0.10	0.039	[0.026 0.051]		0.094	[0.070 0.120]	
ρ_n	0.5	0.993	—	0.694	0.744	[0.659 0.820]	0.082
σ_n	0.10	0.082	[0.060 0.104]		0.055	[0.039 0.070]	
ρ_R	0.5	0.925	[0.895 0.957]	0.011	0.900	[0.860 0.940]	0.011
σ_R	0.02	0.004	[0.003 0.005]		0.005	[0.004 0.005]	
ρ_{π^o}	0.5	0.981	—	0.015	0.921	[0.860 0.984]	0.012
σ_{π^o}	0.005	0.003	[0.002 0.004]		0.004	[0.002 0.005]	
ρ_{τ^k}	0.5	0.820	[0.764 0.881]	0.678	0.655	[0.567 0.746]	1.040
σ_{τ^k}	0.02	0.388	[0.200 0.557]		0.786	[0.418 1.145]	
ρ_{τ^c}	0.5	0.000	—	0.135	0.212	[0.045 0.355]	0.131
σ_{τ^c}	0.02	0.135	[0.088 0.179]		0.128	[0.086 0.170]	
σ_{τ^h}	0.02	0.012	[0.005 0.019]	0.012	0.013	[0.005 0.021]	0.013
$\sigma_{\pi m}$	0.02	0.029	[0.023 0.035]	0.029	0.028	[0.023 0.033]	0.028

Note: “Prior” refers to the prior’s mean; “Post.” refers to the posterior’s mode; “LR(σ)” is the long run variance evaluated at the posterior’s mode.

the non-stationary shocks. Both shocks, in this estimation, had their autoregressive coefficient calibrated to values very close to zero. In Adolfson, Laseén, Lindé and Villani (2007)[2], the persistence of both shocks for the Euro Area is estimated around 0.75. Altig, Christiano, Eichenbaum and Lindé (2005)[6] finds a lower value for the investment specific shock, $\rho_{\Upsilon} = 0.2$, but the parameter for the labor-augmenting technological shock is still very high, $\rho_z = 0.90$. Similar values for these parameters show when the estimation is compared with results from other EMEs. In García-Cicco (2009)[37], the persistence parameter of the non-stationary technological shock is estimated at 0.07 for Mexico.

Table 3.6 show the business cycle moments of the data compared with the filtered

moments generated by the model, while figure 3.1 show the time series path of empirical and simulated data. The table lists the results in terms of volatility of the time series, correlation with output, the relative volatility to output and the first-order autocorrelation. In terms of model fitting, the model does a good job in matching business cycle moments of the data, particularly of output, consumption, investment and inflation. The nominal interest rate also seems to be well described by the Taylor rule. The model has problems to characterize the government spending and the foreign sector, with a poor fitting of exports, imports and import prices. For exports, it is possible that the poor adjustment is explained by the description of the foreign demand for domestic goods, as the world's output is measured by the growth rate of the US economy. The empirical evidence shows that the share of the exports from Brazil to the US has significantly decreased over time⁹, making business cycle fluctuations in the US less relevant to determine Brazilian exports. For imports, note that the estimation matches pretty well the ratio of the standard deviation of this variable to total consumption, instead of output. By construction of the CES aggregator, the model assigns that imports are a direct function of aggregate consumption and investment. As the share of consumption is significantly higher than the share of investment, this is the expected result. Import prices, measured in foreign currency, are described by the same variables describing the VAR for the foreign sector. Thus, it is possible that the same factors explaining the bad performance of the model for exports also influence the forecast of imported goods' prices.¹⁰

To close the analysis for Brazil, the last two columns of tables 3.4 and 3.5 show a

⁹ In the sample used, the share of Brazilian exports to the US declined from 22% in the first quarter of 1999 to 13% in 2008.

¹⁰ Some preliminary evaluation of the model using information on the trade balance, instead of exports and imports, and the real exchange rate as observables, instead of export and import prices, did not improve the model's fitting.

Table 3.6: Business cycle moments: Brazil – Model and Data

	Volatility (σ)		$Corr(x_t, Y_t)$		$\sigma_{x_t}/\sigma_{Y_t}$		Autocorr.	
	Data	Model	Data	Model	Data	Model	Data	Model
Output	0.014	0.016	1.00	1.00	1.00	1.00	0.312	0.105
Consumption	0.018	0.014	0.608	0.817	1.270	0.878	-0.283	0.104
Government	0.041	0.009	-0.017	0.099	2.841	0.584	-0.257	0.889
Investment	0.070	0.058	0.729	0.898	4.838	3.615	0.187	0.108
Exports	0.098	0.028	-0.117	-0.415	6.811	1.725	0.159	0.265
Imports	0.068	0.046	-0.023	0.343	4.699	2.850	0.412	0.238
Inflation	0.011	0.017	-0.416	-0.519	0.789	1.084	0.527	0.517
Interest Rates	0.008	0.009	-0.175	-0.238	0.565	0.585	0.908	0.909
Export Prices	0.036	0.024	0.469	0.404	2.492	1.528	0.284	0.284
Import Prices	0.031	0.006	0.259	0.145	2.119	0.389	0.368	0.498

robustness analysis with all the priors based on the uniform distribution for parameters bounded in the unity interval replaced by priors based on the Beta distribution with mean 0.5 and standard deviation equal to 0.2. The new set of priors reduces the probability that the posterior mode of these parameters reaches extreme values. Under the new set of priors, only the posterior mode of three of the structural parameters estimated in the baseline procedure are outside of the 95% confidence interval of the parameters under the new priors: α_m , α_{xp} and ψ_y . This result shows some good properties of the estimation of these parameters in terms of stability, as most of them remained close to the original estimated values even after changing the priors. For the shocks, the major discrepancy occurred in the estimation of the stationary productivity shock for the non-tradable sector, with a significant reduction of the persistence and the long run standard deviation of the shock. For all the other shocks, the main properties previously discussed remained unaltered.

3.2.2 Australia

The estimation with data from Australia was clearly favored by the smaller variance and the longer time span of the dataset available. When compared to the estimation

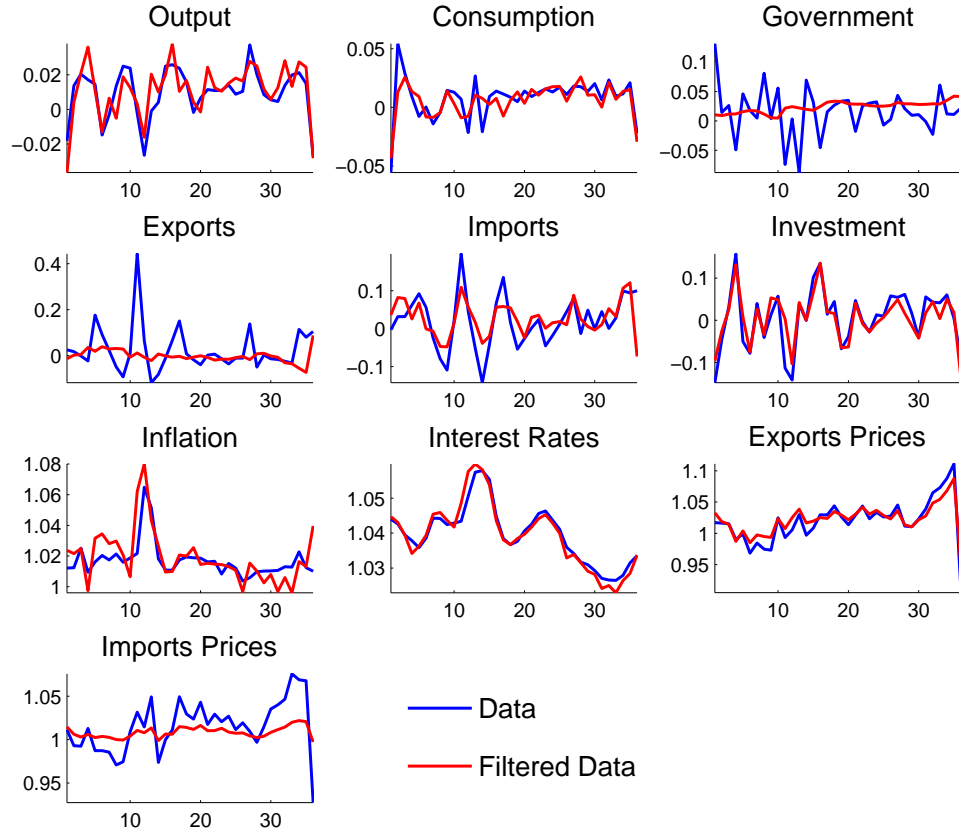


FIGURE 3.1: Data and model fitting: Brazil

for Brazil, a smaller number of parameters needed to be truncated in order to avoid problems with the MH algorithm. Also, as it will be shown later, the parameter estimates allowed a better fitting of the model in those time series that presented problems in the estimation for Brazil. The following set of parameters was fixed at the posterior mode: price indexation parameters $\{\kappa_n, \kappa_{xp}\}$, wage indexation, χ_w , persistence parameters $\{\rho_z, \rho_{\pi^o}, \rho_{\tau c}\}$, the elasticity of interest rates to the real exchange rate in the Taylor rule, α_{rer} , the foreign bonds portfolio adjustment cost, ψ_2 , the price elasticity of demand of the foreign economy with respect to domestic exports, η^* , the elasticities of foreign interest rates with respect to the risk premium

and the net foreign asset position, κ_1 and κ_2 , and the parameter relating the terms of trade to the imported goods' price in foreign currency, v_2 . There were no clear problems associated with parameter identification. Related to the subsection with results for Brazil, tables 3.7 and 3.8 show the priors' mean, the posteriors' mode and the 95% confidence interval for the posterior for the structural parameters and the shocks, respectively. Again, just like the exercise for Brazil, the last columns of the tables show the estimation exercise using priors for the parameters truncated in the unit interval based on the Beta distribution.

Overall, the parameter values, measured by the posterior's mode, significantly approximates to those verified in the literature for closed or developed small open economies. In terms of the parameters describing price dynamics, the estimate of the Calvo parameters for the domestic firms ($\alpha_n = 0.724$) approximates the results for the same country in Justiniano and Preston (2009)[44], implying a probability of firms optimally adjusting prices once every 11 months. Also very close to Justiniano and Preston (2009)[44], the indexation parameter, κ_n , is set at zero for this sector. On the other hand, there is a large discrepancy with the Calvo and indexation parameters for the imported goods' sector, with results in table 3.7 showing significant values for those parameters ($\alpha_m = 0.986$ and $\kappa_m = 0.776$). It is possible that this result is a consequence of the absence of an exported goods' sector in their model, as deviations from the Law of One Price are generated exclusively due to fluctuations in the imported goods when compared to the domestic production goods. With two sectors generating deviations from the Law of One Price, as it is the case here, price rigidity can be higher and still generate the same volatility of exchange rate. In fact, in Adolfson, Laseén, Lindé and Villani (2007)[2], in a model with both sectors estimated for the Euro Area, the indexation parameters increase significantly when one of the two sectors has its Calvo parameter generating higher price flexibility. Thus, a higher degree of price flexibility is compensated by an increase in price

indexation, highlighting the importance of price rigidity in driving the deviation of the Law of One Price. From this perspective, the estimation of the model using data from Australia presents very similar results, in terms of price rigidity, to the results obtained for Brazil, with a large degree of price rigidity for the exported goods and imported goods' sectors and high indexation for the imported goods's retailers. The results are also robust to the use of alternative priors to estimate the posterior of these parameters, as the last columns of table 3.7 show.

Domestic markups over prices are slightly lower when compared to the results obtained for Brazil. Again under the assumption of price stability in the steady state, domestic firms impose markups of 18.7% over prices ($\eta_n/(\eta_n - 1) = 1.187$), while markups from imported goods' retailers are estimated at 24.8%. The results confirm the estimated results for Brazil regarding the markups for exported goods' firms, with a markup over prices of 31.9%.

The estimation of the parameter for habit persistence in consumption is significantly higher than the value found in Justiniano and Preston (2009)[44], with the posterior mode located at the top of confidence interval of some estimates for emerging economies, like Castillo, Montoro and Tuesta (2006)[19]. In Justiniano and Preston (2009)[44], the authors use an informative Beta distribution as a prior, while here the uniform distribution is the standard for parameters bounded in the unit interval. It is worth noting that, despite the use of a non-informative prior, the estimation is very precise, with a very tight confidence interval. In Lubik and Schorfheide (2006)[50], the change in the priors for a model estimated with data from the US and the Euro Area altered the habit persistence parameter from 0.4 to 0.84. However, as the results assuming Beta priors for ζ show, a tighter prior is not enough to move the point estimate away from the original confidence interval.

Results regarding the elasticity of substitution across labor types, ϖ , the posterior mode for Australia approximates the results adopted in Smets and Wouters

(2003)[71] and García-Cicco (2009)[37]. The markup over wages is estimated at 19.7% at the posterior mode. However, it is worth noting that both standard value in the literature ($\varpi = 2$ or $\varpi = 21$) are outside the boundaries of the confidence intervals for Australia and Brazil. Despite a higher value of the posterior mode, the estimate of ϕ_w for Australia implies, using the same mapping as in Chugh (2006)[24], that wages are adjusted almost at the same frequency as wages in Brazil, around once every two quarters. Thus, considering also that indexation does not play a major role in the wage dynamics of both countries, the major difference across the two countries labor markets is the average markup.

It is worth noting that the robustness check shows that the wage indexation parameter, χ_w , is significantly higher when using the new priors, both for Brazil and Australia. This might be a consequence of a weak identification problem, with the priors having a strong influence over the shape of the posterior. This is not a new problem with the estimation of this type of models. Rabanal and Rubio-Ramírez (2005)[58], using data for the US in a standard New Keynesian model, find significant changes in the wage and price indexation parameters when the sample is reduced for robustness analysis. The estimated duration of a wage contract is less than three quarters in the model with wage indexation. Smets and Wouters (2003)[70] estimated a larger model for the Euro area, and the 95% confidence bands for the wage and price indexation parameters cover almost the entire parameter region.

Driving the dynamics of capital, the estimates for the parameters describing the capacity utilization adjustment cost and investment adjustment cost are significantly higher when compared to those estimated for Brazil. The use of these type of adjustment costs is justified by the lack of fitting of standard RBC models to the volatility of investment with respect to output verified in the data, as the model usually overestimates this ratio. Comparing table 3.9 with data in table 3.6, the volatility of investment in Brazil is higher than the volatility in Australia, not only based on the

Table 3.7: Structural Parameters: Australia.

Parameters	Baseline Priors			Alternative Priors	
	Prior	Posterior	95%CI	Posterior	95%CI
Prices, wages and markups					
α_n	0.5	0.724	[0.676 0.771]	0.722	[0.670 0.767]
α_m	0.5	0.986	[0.982 0.989]	0.977	[0.969 0.985]
α_{xp}	0.5	0.885	[0.871 0.900]	0.870	[0.847 0.891]
κ_n	0.5	0.000	—	0.044	[0.008 0.134]
κ_m	0.5	0.776	[0.709 0.847]	0.611	[0.451 0.732]
κ_{xp}	0.5	0.000	—	0.090	[0.022 0.190]
$\varpi - 1$	10	5.078	[1.409 8.642]	2.928	[1.382 5.256]
χ_w	0.5	0.046	—	0.424	[0.118 0.747]
ϕ_w	1.5	1.772	[0.966 2.524]	1.501	[0.872 2.300]
$\eta_n - 1$	4.0	5.363	[3.834 6.918]	4.711	[3.314 6.265]
$\eta_m - 1$	4.0	4.036	[2.355 5.531]	4.030	[2.694 5.364]
$\eta_{xp} - 1$	4.0	3.137	[1.895 4.260]	3.054	[1.983 4.223]
Policy parameters					
α_π	2.0	3.033	[2.146 3.897]	2.248	[1.910 2.719]
α_y	0.5	0.145	[0.095 0.193]	0.111	[0.079 0.148]
α_{rer}	0.5	0.004	—	0.006	[0.001 0.014]
ψ_{li}	1.0	1.089	[0.791 1.395]	1.088	[0.776 1.616]
ψ_y	0.0	1.200	[0.131 2.298]	0.711	[-0.251 1.676]
Household's problem parameters					
ζ	0.5	0.969	[0.958 0.981]	0.964	[0.945 0.977]
ε	1.5	0.497	[0.253 0.742]	0.611	[0.358 0.953]
ϱ	2.0	2.815	[2.486 3.124]	3.002	[2.619 3.480]
ϕ_i	2.5	3.658	[2.992 4.298]	3.656	[2.997 4.314]
θ_2/θ_1	1.5	2.633	[1.936 3.275]	2.544	[2.171 2.928]
Foreign sector and portfolio parameters					
η^*	2.0	0.257	—	0.244	[0.225 0.300]
κ_1	1.0	0.000	—	0.257	[0.011 0.982]
κ_2	1.0	0.622	—	0.428	[0.079 1.314]
ν_1	0.5	0.523	[0.447 0.601]	0.552	[0.450 0.639]
ν_2	0.5	0.030	—	0.027	[0.008 0.045]
ψ_1	0.020	0.035	[0.023 0.048]	0.039	[0.025 0.062]
ψ_2	0.020	0.009	—	0.012	[0.006 0.052]
ξ_1	0.00	-1.461	[-2.340 -0.581]	-1.275	[-1.940 -0.556]
ξ_2	0.00	0.142	[-0.301 0.576]	-0.024	[-0.424 0.431]
ξ_3	0.00	0.192	[-0.008 0.406]	0.191	[-0.052 0.420]
ξ_4	0.00	0.291	[-1.128 1.813]	0.407	[-0.951 1.721]
ξ_5	0.00	-0.056	[-0.260 0.160]	-0.013	[-0.256 0.228]

Note: "Prior" refers to the prior's mean; "Posterior" refers to the posterior's mode.

Table 3.8: Structural shocks: Australia.

Param.	Prior	Baseline Priors			Alternative Priors		
		Post.	95%CI	LR(σ)	Post.	95%CI	LR(σ)
ρ_g	0.5	0.838	[0.770 0.910]	0.005	0.823	[0.739 0.893]	0.006
σ_g	0.02	0.003	[0.003 0.004]		0.003	[0.003 0.004]	
ρ_z	0.5	0.100	—	0.044	0.283	[0.171 0.406]	0.044
σ_z	0.02	0.044	[0.038 0.050]		0.042	[0.036 0.050]	
ρ_Υ	0.5	0.742	[0.710 0.775]	0.049	0.718	[0.667 0.766]	0.046
σ_Υ	0.02	0.033	[0.028 0.037]		0.032	[0.027 0.039]	
ρ_x	0.5	0.941	[0.909 0.972]	0.157	0.944	[0.835 0.968]	0.149
σ_x	0.10	0.053	[0.039 0.067]		0.049	[0.036 0.073]	
ρ_n	0.5	0.965	[0.954 0.978]	0.221	0.962	[0.941 0.972]	0.227
σ_n	0.10	0.058	[0.040 0.075]		0.062	[0.046 0.084]	
ρ_R	0.5	0.918	[0.893 0.943]	0.008	0.902	[0.862 0.932]	0.006
σ_R	0.02	0.003	[0.002 0.003]		0.003	[0.002 0.003]	
ρ_{π^o}	0.5	0.998	—	0.016	0.995	[0.993 0.998]	0.011
σ_{π^o}	0.005	0.001	[0.001 0.001]		0.001	[0.001 0.002]	
ρ_{τ^k}	0.5	0.828	[0.805 0.850]	1.238	0.831	[0.808 0.856]	1.030
σ_{τ^k}	0.02	0.694	[0.460 0.939]		0.573	[0.348 0.914]	
ρ_{τ^c}	0.5	0.999	—	0.111	0.962	[0.903 0.990]	0.031
σ_{τ^c}	0.02	0.005	[0.004 0.007]		0.008	[0.005 0.014]	
σ_{τ^h}	0.02	0.011	[0.005 0.017]	0.011	0.012	[0.006 0.033]	0.012
$\sigma_{\pi m}$	0.02	0.029	[0.025 0.033]	0.029	0.029	[0.025 0.035]	0.029

Note: “Prior” refers to the prior’s mean; “Post.” refers to the posterior’s mode; “LR(σ)” is the long run variance evaluated at the posterior’s mode.

direct comparison, but also as a proportion of the standard deviation of GDP. Thus, in order to match such moments, a higher adjustment cost was expected.

The policy parameters bring two major differences when compared to the policy estimated for Brazil. First, in the Taylor rule, the elasticity of interest rates with respect of changes in the real exchange rate is even smaller than the value obtained for Brazil. Second, in terms of fiscal policy, taxes on labor characterize a countercyclical fiscal policy, as those taxes increase with a positive value for the output gap. In Brazil, on the other hand, fiscal policy takes a strong pro-cyclical stance with a negative coefficient for ψ_y .

The elasticity of substitution between tradable and non-tradable goods (ε) is sig-

nificantly lower than the value obtained for Brazil. On the other hand, the elasticity of substitution between domestic tradable and imported goods (ϱ) is pretty much in line with the values obtained in the literature, as discussed in the values obtained for Brazil. The estimated parameters are not directly comparable to Justiniano and Preston (2009)[44], as they assume that all goods are tradable in the model. In their baseline estimation, the elasticity of substitution between domestic and foreign goods is estimated at 0.58.

The structure of shocks can be easily compared across countries using the estimated long run standard deviation and the persistence parameter in order to understand the effects of the transmission to endogenous variables. The shocks originated from the government operations are very similar in Brazil and Australia. The major discrepancies are located in the government spending shock and the persistence of the consumption tax shock. In the first case, the estimated values for Australia results in smaller persistence when compared to Brazil. The autoregressive coefficient of consumption tax constitutes almost polar cases in terms of dynamics, with a near random walk in Australia and a truncated parameter ρ_{τ^c} at zero in Brazil. It is worth noting that changing the priors of the persistence parameters to the Beta distribution does not have any effect in terms of the structure of the shocks, as parameters are not really affected by the change.

In a small detour, from a historical perspective, it is curious that Brazil and Australia have a similar stochastic process for the path of the inflation target, especially for the long run standard deviation. The implementation of an inflation targeting regime in Australia followed a very smooth path, not being a consequence of a financial crisis or a change in policy regime¹¹. The inflation target, in Australia, was set from the beginning around a confidence band around the 2-3% interval for annual

¹¹ See, for instance, Bernanke, Laubach, Mishkin and Posen (2001)[12] about the implementation of the inflation targeting regime.

inflation. On the other hand, the decision about implementing a new monetary policy regime in Brazil was made in the middle of a crisis that forced the government to sharply raise nominal interest rates and devalue the currency¹². From that point on, the inflation target has systematically decreased from 8% per year in 1999 to the current annual target of 4.5%.

Back to the analysis, while shocks originated in fiscal and monetary policy decisions are almost equivalent both in size and persistence for both countries, productivity shocks are, in general, less volatile for Australia. The non-stationary shocks have more persistent effects in Australia, as both coefficients ρ_Y and ρ_z are truncated at values very close to zero for Brazil. The persistence of sectorial shocks is very similar in both countries.

Finally, table 3.9 show the fitting of the model in terms of business cycle moments, while figure 3.2 show the adjustment of the filtered series to the data. A quick inspection of the table shows the better adjustment of the model to the data, when compared to the results for Brazil. Despite the good adjustment, there are still problems in the forecast of exports. On the other hand, imports and foreign prices of exports and imports present a very good adjustment.

3.3 Dynamics under competitive equilibrium

This section presents simulations of the model under the competitive equilibrium using the parameters estimated for Australia and Brazil. The main objective here is to understand the relevance of each shock in the dynamics of the economy over different horizons, while also focusing on the contribution of foreign shocks to the dynamics of the domestic economy. These results will be useful in the next chapter, when the dynamics of Ramsey problem's solution is compared to the outcome of the competitive equilibrium.

¹² For details, see Bogdanski, Tombini and Werlang (2000)[14].

Table 3.9: Business cycle moments: Australia – Model and Data

	Volatility (σ)		$Corr(x_t, Y_t)$		$\sigma_{x_t}/\sigma_{Y_t}$		Autocorr.	
	Data	Model	Data	Model	Data	Model	Data	Model
Output	0.006	0.007	1.00	1.00	1.00	1.00	0.262	0.332
Consumption	0.006	0.005	0.717	0.672	0.862	0.755	0.182	0.141
Government	0.019	0.015	0.294	0.431	2.968	2.185	-0.189	0.902
Investment	0.020	0.024	0.589	0.631	3.185	3.437	0.731	0.559
Exports	0.031	0.013	0.137	-0.113	4.788	1.807	0.534	0.257
Imports	0.018	0.016	0.220	0.505	2.731	2.265	0.707	0.759
Inflation	0.006	0.006	-0.786	-0.353	0.889	0.889	0.205	0.339
Interest Rates	0.002	0.003	-0.165	-0.364	0.353	0.364	0.897	0.680
Export Prices	0.041	0.033	-0.019	0.006	6.373	4.659	0.355	0.288
Import Prices	0.031	0.019	-0.122	-0.274	4.753	2.705	0.276	0.298

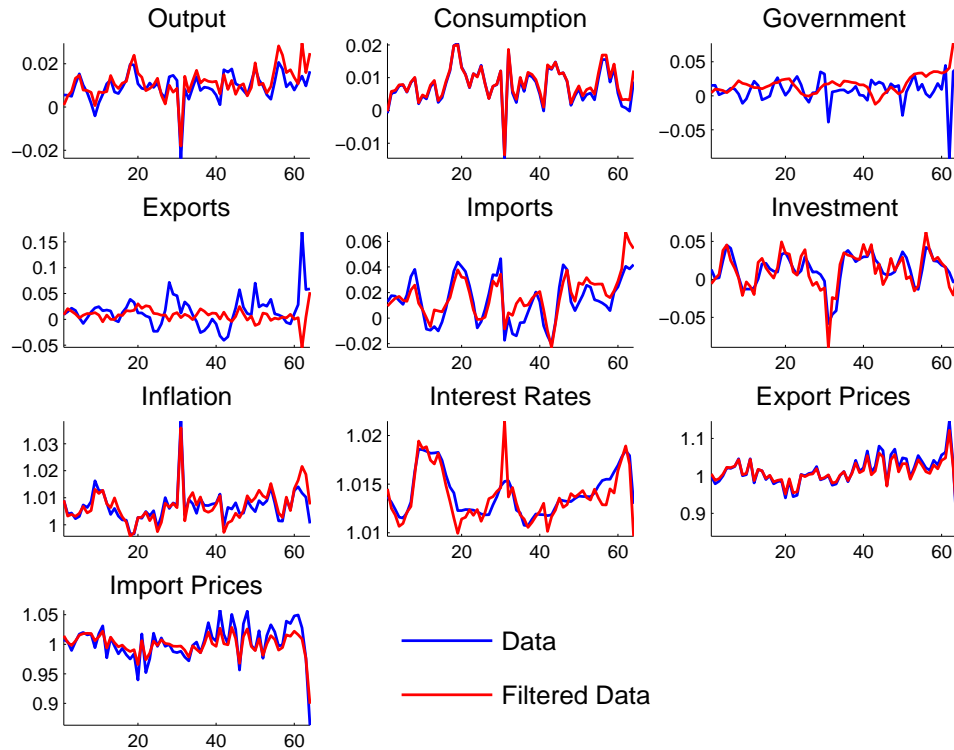


FIGURE 3.2: Data and model fitting: Australia

Tables 3.10, 3.11 and 3.12 show the unconditional variance decomposition and the conditional variance decomposition for the first quarter and for five years, respectively. The objective is to compute the share of each structural shock in explaining the fluctuations of the main aggregates of the model in different frequencies. Thus, the first table shows the effect of the shocks in the long run, while the last two highlight the immediate effects and the effects of shocks at the business cycle frequency.

The unconditional variance decomposition of shocks for Australia shows that foreign shocks does not have a major contribution to domestic fluctuations, as the major impact of foreign shocks is on the determination of real exchange rate, more specifically from the shocks in the price of imported goods. Among the domestic shocks, it is possible to make a clear distinction between the effects of policy shocks to the effects of productivity shocks, as the first group of shocks (given by shocks in taxes, the inflation target, interest rates and government spending) explains more than 80% of the movements in exports, imports, inflation, nominal interest rates and labor income taxes. The same set of shocks explains between 43 and 52% of the changes in output consumption, investment and wages. For these variables, the four productivity shocks play a larger role, with the investment-specific shock explaining 35% of the fluctuations in consumption and 26.5% of the fluctuations in real wages and the non-tradable goods stationary productivity shock responding for 25% of the changes in investment. Individually, the shocks on the inflation target, on capital income tax, on consumption tax and on the investment-specific technology are the most relevant, with the highest average participation in the fluctuations of the economy.

The conditional variance decomposition does not alter the analysis regarding the effect of foreign shocks in Australia. The major difference with respect to the unconditional decomposition is the larger effect of shocks in the imported goods prices and on the world's output in explaining the fluctuations of exports and the real exchange

Table 3.10: Variance decomposition: Australia

	y_t	c_t	i_t	x_t	imp_t	w_t	rer_t	π_t	R_t	τ_t^h
τ^h	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
τ^k	19.7	13.4	37.8	9.2	8.9	7.1	8.1	17.0	23.4	88.7
τ^c	1.3	0.9	1.2	43.4	20.0	3.2	21.5	1.7	40.8	2.6
π^o	30.0	28.6	5.9	29.2	56.0	35.3	14.7	61.7	18.5	2.3
R	0.2	0.1	0.7	0.3	0.4	0.4	1.8	0.7	0.9	0.0
g	0.3	0.2	1.4	0.1	0.1	0.1	0.4	0.1	0.1	0.2
a_x	4.2	2.9	10.3	1.4	2.9	5.5	1.2	3.4	3.3	0.6
a_n	12.9	9.4	25.0	4.1	3.7	19.8	5.4	6.4	5.5	1.2
μ^Y	23.0	35.0	14.3	5.9	5.4	26.5	14.6	6.5	5.3	3.6
μ^z	7.9	9.3	2.0	1.7	2.1	1.7	3.9	2.2	2.1	0.6
Domestic(σ)	99.5	99.8	98.7	95.3	99.3	99.6	71.5	99.5	99.8	99.9
π_m^*	0.5	0.2	1.0	3.9	0.5	0.3	23.1	0.3	0.2	0.01
ΔM^*	0.0	0.0	0.0	0.1	0.0	0.0	0.3	0.0	0.0	0.0
R^*	0.0	0.0	0.1	0.4	0.0	0.0	1.6	0.0	0.0	0.0
ξ	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
π^*	0.0	0.0	0.0	0.1	0.0	0.0	0.7	0.0	0.0	0.0
y^*	0.0	0.0	0.2	0.3	0.1	0.2	2.8	0.1	0.0	0.0
Foreign(σ)	0.5	0.2	1.3	4.7	0.7	0.5	28.5	0.4	0.2	0.01

rate. These two shocks combined explain 54% and 59% of exports fluctuations after one quarter and 5 years, respectively. Still related to foreign shocks, the nominal interest rate has some effect in explaining exports and the real exchange rate at the business cycle frequency, but its contribution is below 5% of total fluctuations otherwise.

At the highest frequency, the policy variables shocks lose importance, except for shocks in capital income tax, explaining more than 40% of the movements in inflation and 19% of changes in the nominal interest rates. In fact, at this frequency, the non-stationary technology shocks explain around 90% of the fluctuations in output, consumption and investment. The sectorial stationary shocks play a relevant role in explaining inflation, with the shocks in the non-tradable sector also explaining one-fifth of changes in real wages.

Table 3.11: Short-run variance decomposition: Unconditional decomposition – Australia – One quarter

	y_t	c_t	i_t	x_t	imp_t	w_t	rer_t	π_t	R_t	τ_t^h
τ^h	0.0	0.0	0.0	0.0	0.0	0.1	0.0	0.0	0.0	0.1
τ^k	3.4	3.4	3.3	6.0	1.0	8.9	4.9	40.6	19.0	99.2
τ^c	0.0	0.0	0.2	8.9	0.9	1.0	8.5	3.5	1.2	0.0
π^o	0.2	0.0	2.9	2.3	0.9	0.2	1.0	3.2	0.6	0.0
R	0.3	0.0	3.1	2.3	1.9	1.9	3.0	4.8	48.5	0.0
g	6.3	0.0	0.8	0.4	0.1	0.8	0.6	0.1	0.1	0.0
a_x	0.2	0.1	0.5	1.4	14.0	6.4	1.3	21.7	11.6	0.2
a_n	0.7	0.2	4.5	3.5	1.5	20.5	5.1	22.0	14.3	0.1
μ^Y	17.4	10.5	51.9	14.0	17.2	20.9	25.7	1.3	0.9	0.0
μ^z	71.6	85.8	32.7	2.5	62.3	38.9	9.2	1.6	3.9	0.4
Domestic(σ)	100.0	100.0	99.9	41.4	99.7	99.6	59.3	98.8	100.0	100.0
π_m^*	0.0	0.0	0.0	48.9	0.3	0.4	34.2	0.9	0.0	0.0
ΔM^*	0.0	0.0	0.0	0.6	0.0	0.0	0.4	0.0	0.0	0.0
R^*	0.0	0.0	0.0	3.3	0.0	0.0	2.7	0.1	0.0	0.0
ξ	0.0	0.0	0.0	0.1	0.0	0.0	0.0	0.0	0.0	0.0
π^*	0.0	0.0	0.0	0.2	0.0	0.0	0.8	0.0	0.0	0.0
y^*	0.0	0.0	0.1	5.5	0.0	0.0	2.5	0.2	0.0	0.0
Foreign(σ)	0.0	0.0	0.1	58.6	0.3	0.4	40.7	1.2	0.0	0.0

It's difficult to claim that one specific shock, or even a group of shocks, is responsible for fluctuations at the business cycle frequency. The foreign shocks still play a role in explaining only the movements of real exchange rate and imports, without major influences in other aggregates. Among the domestic variables, the shocks on capital income tax, investment-specific technology and in the non-tradable sector explain, on average, the largest share of fluctuations at this frequency.

Also, the results must be seen with caution when compared with the variance decomposition in Justiniano and Preston (2009)[44] for Australia. The authors not only have a different structure of exogenous shocks, but they also have a set of preference and markup shocks that are not present here. However, in common with their variance decomposition, the risk premium shock, ξ , does not have significant

Table 3.12: Business cycle variance decomposition: Unconditional decomposition – Australia – 20 quarters

	y_t	c_t	i_t	x_t	imp_t	w_t	rer_t	π_t	R_t	τ_t^h
τ^h	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
τ^k	39.5	26.7	45.3	6.6	20.8	12.8	5.3	48.4	58.9	96.5
τ^c	0.1	0.1	0.2	6.1	9.3	0.6	9.0	1.6	6.0	0.2
π^o	0.2	0.0	1.9	3.8	2.7	0.2	2.4	7.6	5.5	0.0
R	0.3	0.1	0.9	1.3	4.9	0.6	2.7	2.0	2.6	0.0
g	0.4	0.2	1.9	0.3	0.6	0.2	0.6	0.1	0.1	0.3
a_x	5.2	1.9	13.3	0.9	18.2	8.6	1.0	8.5	8.5	0.3
a_n	12.2	4.5	29.5	3.2	10.6	27.8	5.0	13.3	11.6	0.4
μ^Υ	26.7	44.9	5.6	8.4	17.4	45.4	19.3	12.0	3.5	1.8
μ^z	15.1	21.6	0.8	2.3	8.6	3.3	5.1	5.6	2.7	0.5
Domestic(σ)	99.7	99.9	99.3	32.8	93.1	99.4	50.4	99.0	99.6	100.00
π_m^*	0.3	0.1	0.6	56.3	5.1	0.5	40.3	0.7	0.3	0.0
ΔM^*	0.0	0.0	0.0	0.8	0.1	0.0	0.5	0.0	0.0	0.0
R^*	0.0	0.0	0.0	6.0	0.3	0.0	2.9	0.0	0.0	0.0
ξ	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.0	0.0	0.0
π^*	0.0	0.0	0.0	0.7	0.3	0.0	1.2	0.0	0.0	0.0
y^*	0.0	0.0	0.1	3.4	1.2	0.1	4.7	0.2	0.1	0.0
Foreign(σ)	0.3	0.1	0.7	67.2	6.9	0.6	49.6	1.0	0.4	0.0

impact in the economy. In their model, the foreign shocks explain around 10% of the movements in inflation, but the model does not have capital and the foreign economy, for that exercise, is treated as unobserved, which might increase the role of these shocks in the model as a whole. In an exercise to approximate the structure of shocks to the one in Justiniano and Preston (2009)[44], the variance of the shocks in the inflation target, taxes and the non-stationary productivity were set to zero. The foreign shocks, in this case, explained around 4% of the variability in inflation.

Tables 3.13, 3.14 and 3.15 replicate for Brazil the simulations previously performed for Australia. Three main features are evident from a first look at the data. First, foreign shocks explains a significant part of the dynamics in Brazil, especially the imported goods' inflation and the world's output shock. Just like Australia, the shocks in the nominal interest rates and risk premium are not relevant for Brazil,

which emphasizes the role of domestic policy in setting the interest rate charged in Brazilian bond in the international markets, R^f . Also, valid for all the decompositions, the shocks of world's output, despite low, are significant for all variables shown in the tables.

Second, the non-stationary sectorial productivity shocks are significantly more relevant in explaining the movements of the economy when compared to changes in the trend. This result goes against the proposition in Aguiar and Gopinath (2007)[4], since the authors identify changes in the non-stationary component as the main source of fluctuations in EMEs. One reason for this discrepancy is the sample selection procedure adopted here: Aguiar and Gopinath (2007)[4] claim that, among other reasons, fluctuations in the trend in EMEs are a consequence of “sudden stops” episodes or shifts in policy. The sample selected for Brazil does not present any significant policy shift, in terms of monetary policy regime, and the single episode that could be characterized as a “sudden stop” – the 2002 crisis, due to the uncertainty regarding the future economic policy, resulted in a devaluation of the domestic currency of 34.8% – did not alter the monetary policy framework of the economy.

The third feature of the simulations is the role of government spending. In the simulations for Australia, taxes on capital were the major exogenous component of the government budget constraint explaining the fluctuations of the economy. For Brazil, the effects of capital income taxation are significantly smaller, while government spending explains more than 12% of the fluctuations in investment, imports, real exchange rate, inflation and the nominal interest rate. One of the reasons for this result is the importance of government spending as a proportion of the GDP in the Brazilian economy. Additional simulations replacing the calibrated parameter G/Y with the value used for Australia (from 0.32 to 0.18, according to table 3.1) show that the government spending shock, under the new steady state, explains only 1.86%, 4.02%, 5.39% and 5.77% and 5.09% of the fluctuations in output,

Table 3.13: Variance decomposition: Brazil

	y_t	c_t	i_t	x_t	imp_t	w_t	rer_t	π_t	R_t	τ_t^h
τ^h	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
τ^k	0.5	0.4	0.8	2.2	3.1	0.7	3.6	2.4	3.0	0.8
τ^c	0.0	0.0	0.0	0.1	0.1	0.0	0.1	0.1	0.1	0.0
π^o	0.0	0.0	0.1	0.2	0.1	0.0	0.2	1.6	1.2	0.1
R	0.0	0.0	0.1	0.1	0.1	0.0	0.1	0.3	0.1	0.0
g	4.4	1.7	12.1	10.2	14.6	4.4	16.4	14.0	13.8	5.3
a_x	22.0	10.1	40.0	37.4	40.8	12.2	41.7	40.2	23.1	27.4
a_n	55.3	75.7	15.3	15.0	6.6	71.9	4.2	9.1	39.9	45.2
μ^Y	0.1	0.2	0.9	0.3	0.6	1.0	0.4	0.6	0.3	0.2
μ^z	1.7	1.4	8.0	4.3	8.9	2.3	7.5	7.8	3.2	2.2
Domestic(σ)	84.0	89.5	77.3	69.8	74.9	92.7	74.1	76.0	84.7	81.3
π_m^*	12.4	8.1	17.8	23.3	19.7	5.7	20.6	19.2	12.1	14.6
ΔM^*	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
R^*	0.4	0.2	0.5	0.6	0.6	0.2	0.6	0.6	0.4	0.4
ξ	0.0	0.0	0.0	0.1	0.0	0.0	0.0	0.0	0.0	0.0
π^*	0.5	0.4	0.7	1.0	0.8	0.2	0.7	0.7	0.4	0.6
y^*	2.7	1.8	3.6	5.1	4.0	1.2	3.9	3.5	2.3	3.1
Foreign(σ)	16.0	10.5	22.7	30.2	25.1	7.3	25.9	24.0	15.3	18.8

investment, imports, real exchange rate and inflation, respectively¹³.

The conditional variance decomposition for one quarter show almost the same results of the unconditional decomposition presented before. The foreign sector still has a major influence in explaining the fluctuations of the economy, while the government spending shock also keeps an elevated participation in the share of the decomposition. The major change with respect to the previous exercise is the high influence of the non-stationary productivity shock in output, consumption, inflation and the labor income taxes. The increase in the share of the labor-augmenting shock is compensated by the reduced role of the non-tradable productivity shock. This result also appeared in the simulations for Australia, thus showing that this effect is a characteristic of the model, instead of a consequence of the parameters used.

¹³ Results available upon request.

Table 3.14: Short-run variance decomposition: Unconditional decomposition – Brazil
– One quarter

	y_t	c_t	i_t	x_t	imp_t	w_t	rer_t	π_t	R_t	τ_t^h
τ^h	0.0	0.0	0.0	0.0	0.1	0.2	0.0	0.5	0.2	0.9
τ^k	1.3	0.0	4.3	7.9	0.1	1.4	3.9	0.1	4.4	0.9
τ^c	1.0	1.8	0.0	0.2	0.1	0.4	0.0	0.9	0.1	0.8
π^o	0.2	0.0	0.4	0.4	1.0	0.2	0.1	9.6	3.3	0.2
R	0.3	0.1	0.4	0.3	0.9	0.3	0.1	6.1	15.7	0.3
g	8.3	0.2	10.2	17.5	16.4	14.3	18.3	0.5	21.5	7.8
a_x	25.7	1.4	54.2	41.5	42.2	2.4	43.6	6.0	24.2	24.8
a_n	3.0	2.6	1.1	7.2	0.5	68.7	1.7	3.7	0.0	2.9
μ^Y	2.3	3.7	0.1	0.4	0.0	1.4	0.3	10.3	2.3	2.4
μ^z	41.9	88.6	0.0	9.1	2.0	0.3	4.8	61.5	5.2	43.5
Domestic(σ)	83.9	98.4	70.6	84.3	63.3	89.6	72.7	99.2	76.9	84.4
π_m^*	13.0	1.3	23.7	13.4	28.7	8.0	21.5	0.1	19.4	12.6
ΔM^*	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
R^*	0.4	0.0	0.7	0.3	0.8	0.2	0.6	0.0	0.6	0.4
ξ	0.0	0.0	0.0	0.1	0.1	0.0	0.1	0.0	0.0	0.0
π^*	0.5	0.1	0.8	0.1	1.2	0.4	0.8	0.1	0.4	0.4
y^*	2.3	0.3	4.1	1.8	6.0	1.8	4.4	0.5	2.6	2.3
Foreign(σ)	16.1	1.6	29.4	15.7	36.7	10.4	27.3	0.8	23.1	15.6

Another reason to view this result as a structural feature of the model is that, again, the importance of the non-stationary shock is reduced as the frequency of the decomposition moves from the one quarter to the 20 quarters period, as table 3.14 shows. At the business cycle frequency, in fact, the values of the variance decomposition are very similar to the unconditional exercise.

In order to finish the comparison, one major question that must be addressed is the amplifying mechanism that exacerbates the effect of foreign shocks in Brazil that is not present in Australia. Table 3.3 shows in the first column the unconditional variance decomposition of output for Australia. In the next columns, one structural parameter is changed for the value estimated for Brazil: the elasticity to the exogenous risk in the rest of the world, κ_1 ; the price elasticity of demand from world to domestic exports, η^* ; the indexation parameter for imported goods' retailers, κ_m ; and

Table 3.15: Business cycle variance decomposition: Unconditional decomposition – Brazil – 20 quarters

	y_t	c_t	i_t	x_t	imp_t	w_t	rer_t	π_t	R_t	τ_t^h
τ^h	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
τ^k	0.4	0.7	0.6	2.4	3.3	1.6	3.7	2.5	5.1	0.5
τ^c	0.0	0.1	0.0	0.1	0.1	0.1	0.1	0.1	0.1	0.0
π^o	0.0	0.0	0.0	0.2	0.1	0.1	0.2	1.3	1.7	0.0
R	0.0	0.0	0.1	0.2	0.1	0.1	0.1	0.3	0.2	0.1
g	6.0	4.1	12.5	11.7	15.2	7.1	16.7	14.8	16.8	6.4
a_x	43.4	25.9	46.1	35.0	41.3	28.7	42.0	42.3	41.6	43.4
a_n	16.8	38.6	3.7	2.6	3.7	40.8	3.2	4.9	3.6	16.4
μ^Y	0.2	0.6	1.1	0.4	0.6	3.5	0.4	0.6	0.5	0.2
μ^z	3.4	5.2	9.7	5.9	9.9	7.6	7.4	8.5	5.6	3.6
Domestic(σ)	70.4	75.1	73.8	58.4	74.4	89.5	73.7	75.3	75.3	70.7
π_m^*	23.1	19.3	20.6	32.1	20.0	8.4	20.9	19.8	19.9	22.9
ΔM^*	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
R^*	0.7	0.5	0.6	0.9	0.6	0.3	0.6	0.6	0.6	0.7
ξ	0.1	0.0	0.0	0.1	0.0	0.0	0.0	0.0	0.0	0.0
π^*	1.0	0.8	0.8	1.4	0.8	0.3	0.7	0.7	0.7	0.9
y^*	4.9	4.2	4.2	7.1	4.2	1.6	3.9	3.7	3.6	4.8
Foreign(σ)	29.6	24.9	26.2	41.6	25.6	10.5	26.3	24.8	24.8	29.3

the Calvo parameter for the imported goods' retailers, α_m . The last column shows the unconditional variance decomposition of output for Brazil. The table shows that nominal rigidities associated with the pricing mechanism for imported goods are, indeed, of great importance to the propagation of foreign shocks in the economy. The high degree of indexation, combined with lower price rigidity, when compared to the parameters estimated for Australia, significantly increases the effects of foreign shocks in the domestic economy. Notice that, if Australia had only the price elasticity for exports and the indexation parameter equivalent to Brazil, the effects of foreign shocks would be already very similar to those estimated for Brazil. The Calvo pricing for imported goods amplifies even more this effect.

Table 3.16: Variance decomposition and foreign shocks: from Australia to Brazil

	$y_t :$ Baseline	$y_t :$ $\kappa_1 = 0.01$	$y_t :$ $\eta^* = 1.12$	$y_t :$ $\kappa_m = 0.99$	$y_t :$ $\alpha_m = 0.91$	$y_t :$ Brazil
τ^h	0.0	0.0	0.0	0.0	0.0	0.0
τ^k	19.7	19.7	18.6	26.0	5.43	0.5
τ^c	1.3	1.3	1.6	0.1	0.0	0.0
π^o	30.0	30.0	34.0	0.8	0.0	0.0
R	0.2	0.2	0.2	0.2	0.0	0.0
g	0.3	0.3	0.2	0.3	0.1	4.4
a_x	4.2	4.2	4.0	7.3	2.7	22.0
a_n	12.9	12.9	11.8	14.4	1.2	55.3
μ^Υ	23.0	23.0	20.9	27.9	4.9	0.1
μ^z	7.9	7.9	7.15	9.2	1.2	1.7
Domestic(σ)	99.5	99.5	98.5	86.1	15.5	84.0
π_m^*	0.5	0.5	1.3	11.1	66.1	12.4
ΔM^*	0.0	0.0	0.0	0.1	0.8	0.0
R^*	0.0	0.0	0.1	0.7	4.1	0.4
ξ	0.0	0.0	0.0	0.0	0.2	0.0
π^*	0.0	0.0	0.0	0.4	2.6	0.5
y^*	0.0	0.0	0.1	1.6	10.7	2.7
Foreign(σ)	0.5	0.5	1.5	13.9	84.5	16.0

3.4 Conclusions

The estimation of the model showed the importance of nominal rigidities in the dynamics of the model and the differences between SOEs and EMEs not related to the volatility of the shocks that will be relevant in characterizing the optimal policy. The main differences are located in the parameters characterizing the nominal rigidities, with the pricing equations for retailers' firms playing a crucial role in the dynamics of the model. Recall from chapter 2 that nominal rigidities are also very important in the determination of the steady state of the optimal policy. The next chapter uses the parameters computed here to present the Ramsey dynamics and compare the theoretical moments with the competitive equilibrium discussed in the last section of this chapter.

Optimal Policy: The Ramsey Dynamics

In this chapter, the Ramsey problem of determining the optimal monetary and fiscal policy is solved for the two countries studied, Brazil and Australia. The next sections present the steady state values of the main aggregates, the ergodic moments derived from the model's solution and some impulse response functions to develop intuition about the dynamics of the economy under the Ramsey equilibrium. The results highlight some of the trade-offs existing in EMEs, when compared to SOEs, resulting exclusively from the structural parameters estimated in the previous chapter. Indeed, despite very similar behavior of the policy functions across countries after a shock, the parametric constraints imposed for the EME result in very different dynamics of the endogenous variables of the model.

The Ramsey equilibrium is solved after the log-linearization of the economy. There is evidence that the log-linearization of the Ramsey problem produces similar responses when compared to its exact solution¹. The use of a first-order approximation is also consistent with the adopted empirical procedure to estimate the structural parameters of the model, as it might be the case that estimations using high-order

¹ See, for instance, Schmitt-Grohé and Uribe (2005b)[67].

approximations of the model result in different parameter sets and moments for simulated data, as pointed out by Fernández-Villaverde and Rubio-Ramírez (2005)[63]. In this sense, this chapter leaves the optimal policy problem analysis of estimated models based on high-order approximations of the equilibrium conditions as a promising topic for a future research agenda².

The model analyzed here has a consumption tax and a single income tax as the instruments for the Ramsey planner. The other cases explored in the steady state description of chapter 2 are not presented due to the impossibility to solve the model under different fiscal policy frameworks. There are several sources of instabilities in these models. From a theoretical perspective, it is not possible to compute the dynamics of the model with all taxes available as instruments because the steady state is achieved only asymptotically³. From a practical perspective, the model with a single income tax and the model with capital income and labor income taxes do not have a stable solution for both countries with the estimated set of structural parameters from chapter 3⁴. However, being restricted to the analysis of only one combination of taxes does not harm the exercise proposed for two reasons. First, as the results will show, the generality of the conclusions regarding the steady state and the long run properties of the economy, discussed in chapter 2, still holds in the framework with two distortionary taxes. Second, the main objective of this work is to compare the dynamics of optimal policy for SOEs and EMEs, and the system with both taxes allows as much depth as any other version of the fiscal framework without requiring changes in the estimated parameters.

This chapter is organized as follows. The next section explore the optimal policy

² The feasibility of the estimation of a medium-scale model using high-order approximations, by itself, constitutes a topic for future research.

³ See Correia, Nicollini and Teles (2008)[26].

⁴ The model with only an income tax does not have a solution for Australia, while the model with capital income and labor income taxes was presenting, for both countries, a very persistent dynamics with very slow convergence to the steady state.

dynamics, with a detailed description of the moments generated by the simulation of the Ramsey problem. Section 4.2 discusses the problem of the zero lower bound of nominal interest rates and how the dynamics is affected when the probability of violating this constraint is reduced to acceptable levels. Section 4.3 concludes.

4.1 The optimal policy with income and consumption taxes

The ergodic moments of the solution to the Ramsey problem with consumption tax and a single income tax are presented in table 4.1. As discussed in chapter 2, the presence of consumption taxes in the model approximates the steady state solution to price stability, while keeping consumption taxes high when compared to taxes impacting capital income. The optimal inflation in this fiscal policy framework is estimated at 0.01% per year for Australia and 2.23% for Brazil, both values significantly below the calibration used in estimation (2.75% for Australia and 4.50% for Brazil, respectively). Despite low, inflation in both countries is very volatile, as the standard deviations assign a high probability of deflation episodes⁵. The high volatility of inflation confirms the results of Correia, Nicollini and Teles (2008)[26] about the presence of uniform consumption tax reducing the welfare costs of such fluctuations. The authors point out that the optimal policy in Schmitt-Grohé and Uribe (2005b)[67] results in low and stable inflation due to the absence of a fiscal policy instrument like the consumption taxes, capable of making government debt state-contingent.

The non-negative value for inflation in steady state is a consequence of the positive inflation in the world's economy, which was indeed the only case moving the optimal policy away from the choice between the Friedman rule and price stability in the results presented in table 2.4. As a robustness exercise, for both Australia and

⁵ Standard deviations reported on the second column of table 4.1 are presented on quarterly frequency.

Table 4.1: Ramsey moments: Income and consumption taxes.

	Mean		Volatility (σ)		$Corr(x_t, Y_t)$		Autocorrelation	
	Aus	Bra	Aus	Bra	Aus	Bra	Aus	Brazil
π_t	0.01	2.23	0.08	0.21	-0.02	0.19	0.86	0.98
R_t	4.01	6.62	0.77	0.29	-0.08	-0.07	0.59	0.94
τ_t^y	-3.02	-13.90	3.02	0.78	-0.56	-0.71	0.35	0.89
τ_t^c	25.41	90.48	3.19	2.11	0.74	0.54	0.79	0.97
y_t	-0.86	-0.87	1.06	0.86	1.00	1.00	0.97	0.98
c_t	-1.41	-1.74	0.68	1.00	0.50	0.32	1.00	0.99
i_t	-2.10	-1.94	4.35	3.22	0.95	0.85	0.97	0.98
w_t	0.39	0.39	0.97	0.61	0.58	0.76	0.87	0.99
x_t	-2.56	-2.74	0.27	0.80	0.61	-0.67	1.00	0.99
imp_t	-2.61	-2.75	2.82	2.85	0.91	0.49	0.99	0.98
rer_t	-0.24	-0.25	1.91	0.83	0.69	-0.50	0.90	0.98
li_t	0.53	0.47	1.47	0.48	0.80	0.89	0.82	0.96

Note: Inflation and nominal interest rates with mean in percent per year. Taxes are expressed in percent. All other moments are expressed in log-levels (for means) and in percentage deviation from the steady state. “Aus” = Australia and “Bra” = Brazil

Brazil, setting foreign inflation to zero would result in price stability as the optimal target for monetary policy while keeping the inflation volatility at the same level presented here⁶. Thus, foreign inflation provides a shift in level to domestic inflation without altering the main dynamics properties of the series under optimal policy.

As expected from the discussion in chapter 2, the Ramsey solution is characterized by high taxes on consumption and a subsidy on income tax. Notice that income receives a higher subsidy in Brazil. However, income taxes in Australia are more volatile and less persistent. This result is partly a consequence of the distortion generated by inflation, since when setting foreign inflation to zero, the gap between the tax subsidy in Australia to Brazil falls to seven percentage points, with the new values for the income subsidy set at 7.3% in Australia and 14.1% in Brazil. Evaluated by the contemporaneous correlation with output, the two taxes are used by the Ramsey planner in a very distinctive way, with the consumption tax smoothing

⁶ Results available upon request.

the business cycle and the income tax exacerbating fluctuations.

Figure 4.1 show the impulse response function of the effect of a normalized shock in government spending, assuming that the stochastic process for this shock is the same for both economies⁷. With the impulse response function normalized by the standard deviation of the shocks, the effects from the exogenous volatility of the economy are eliminated, thus helping the comparison across the two economies. Also, with the same stochastic process for both economies, the model does not change the allocations across time due to significant differences regarding the persistence of the shocks.

The difference in the estimates of nominal rigidities, especially the Calvo parameter for domestic prices and the indexation parameter for imported goods' retailers, show up in the high volatility of inflation for Brazil. After an increase in government spending, the immediate effect on prices is positive for both economies, but the order of magnitude is significantly different. The difference in nominal rigidities also implies a much more volatile nominal interest rate for Brazil. Consumption is smooth and persistent for both economies after the shock, with the effects not dying out even after 40 quarters. Consumption taxes and income taxes rates moves at opposite directions, with the first keeping close track to the changes in output. The difference in taxes' volatilities is also significant, with income taxes following a smoother path.

The exogenous shock in government spending reveals one of the dynamic choices of the optimal plan, as the Ramsey planner keeps the government intertemporal budget in equilibrium using on impact the tax that does not distort the households' choices over time. In other words, if the government increased income tax immediately after the increase in spending, not only current consumption would be affected,

⁷ In this case, the autoregressive coefficient for Brazil, $\rho_g = 0.956$, replaced the value estimated for Australia, $\rho_g = 0.838$.

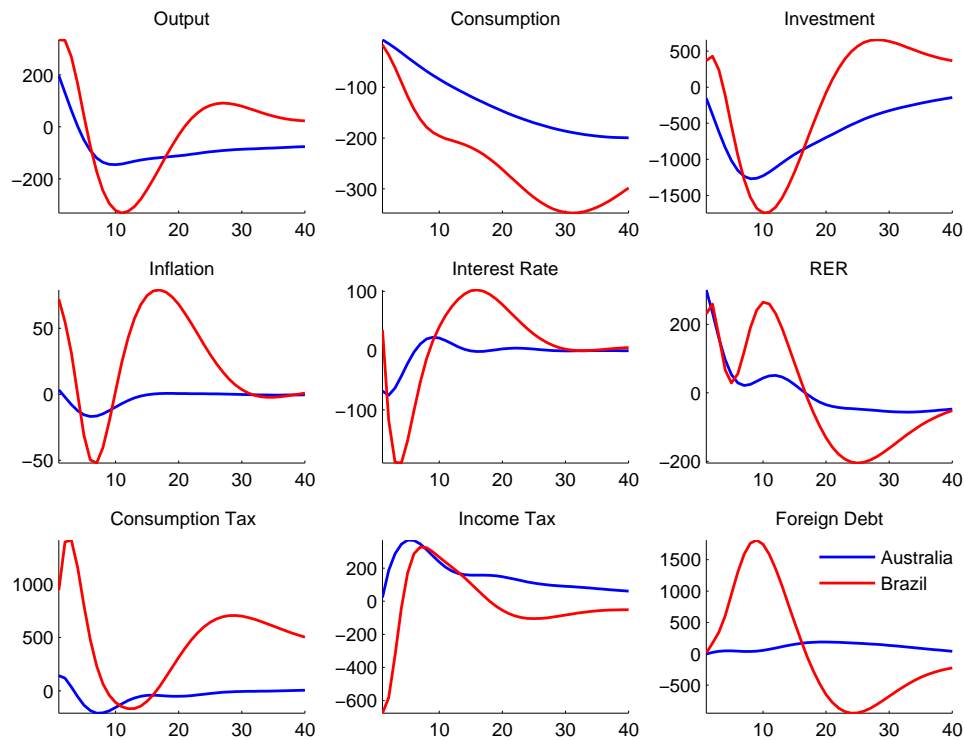


FIGURE 4.1: Impulse response function: government spending

but also investment and future output, as the real return of capital would decrease. In the optimal framework, the government reduces current aggregate consumption with an immediate increase in the consumption tax, but keeping the balance of the budget constraint over time adjusting the income tax.

Evaluating the optimal policy in response to foreign shocks, figure 4.2 show a normalized impulse-response function for a shock in foreign output. The plot shows two very different patterns after impact that are relevant to understand some of the policy constraints for EMEs. On impact, the economies of Australia and Brazil grow, based on the increase in exports to the rest of the world. However, the growth of domestic investment in Brazil generates an acceleration in imports, with an increase in foreign debt. This happens because the price elasticity of demand from the rest of

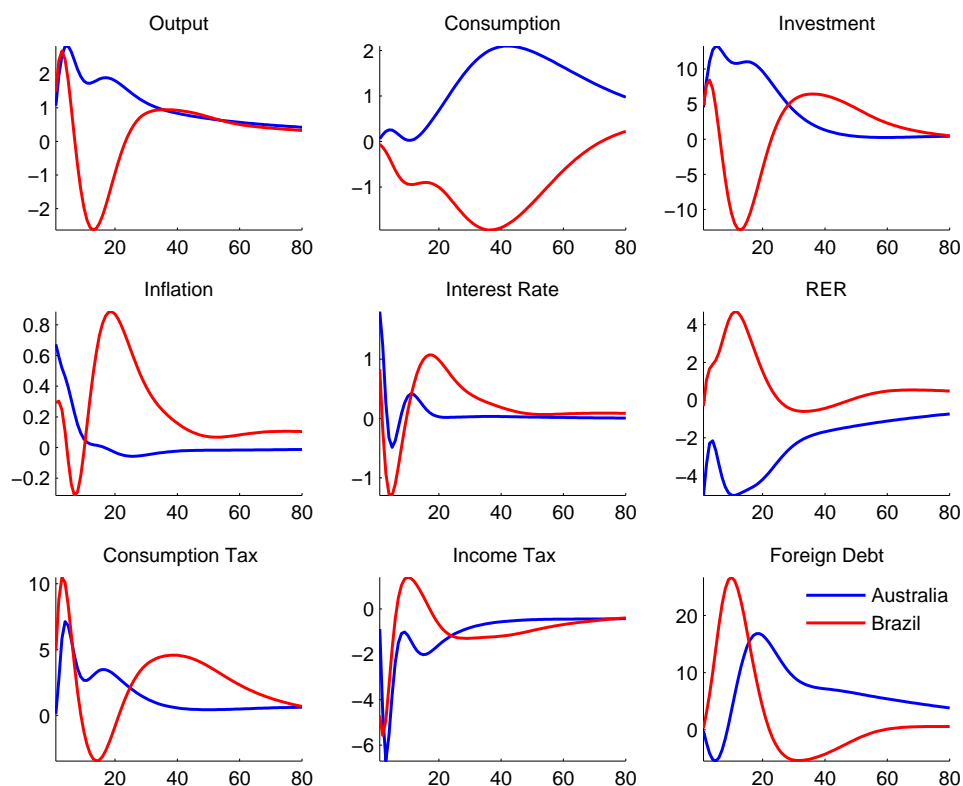


FIGURE 4.2: Impulse response function: world's output

the world for Brazilian goods is larger than the same coefficient for Australia. Thus, for Australia, the inelastic demand from the rest of the world allows larger increases in the total value of exports, avoiding trade deficits and the use of foreign debt to pay for the imports in that country. On the other hand, the increase in Brazil's foreign debt forces a real devaluation of the currency, inflation and an increase in nominal interest rates as a response. It is interesting to note that fiscal policy, in this case, does not seem to be effective, as the path of taxes both in Australia and in Brazil, despite the differences in volatility, are very similar.

The restrictions generated by the foreign demand for domestic goods, in a sense, also appeared in figure 4.1, where the increase in domestic demand generated by the

government spending shock also resulted in an increase in foreign debt, amplifying the effects over the real exchange rate and inflation in the medium term for the Brazilian economy. Again, this movement is much more smooth for Australia.

Another relevant point is the central role of the Euler equations describing the demand for domestic and foreign bonds by the households, representing the UIP condition in this model. More specifically, the transmission from the amount of debt issued by domestic households in international financial markets to the foreign interest rates in that system determines the real exchange rate fluctuations and the domestic response of prices and output. The closing device for small open economies described by Schmitt-Grohé and Uribe (2003b)[65], based on the transmission of movements in foreign debt to the interest rates in international markets, amplifies the exchange rate fluctuations and the impact of foreign shocks in the economies. The effect is more pronounced the larger the demand for imported goods, or the more elastic is the foreign demand for domestic goods.

In another way to see the effects of the UIP condition in the model, consider the effects of cost-push shocks generated from the foreign economy. Figure 4.3 show the impulse response function of a unit shock in foreign inflation, π_t^* . This shock affects the economies through several channels, as the estimated VAR for the foreign sector propagates the shock in prices through foreign output, the price of imported goods and the risk premium. There is also a common pattern in terms of output and investment in the two economies, with the optimal response of Australia and Brazil resulting in a large drop in output, mainly due to the fall in investment. The fall in output in Brazil is smoothed by an increase in exports. The shift of production from the domestic economy to the rest of the world slightly reduces domestic consumption and brings inflation in the medium term. For Australia, the increase in foreign inflation induces a strong real exchange rate depreciation and an increase in domestic interest rates to avoid stronger movements in capital flows. The

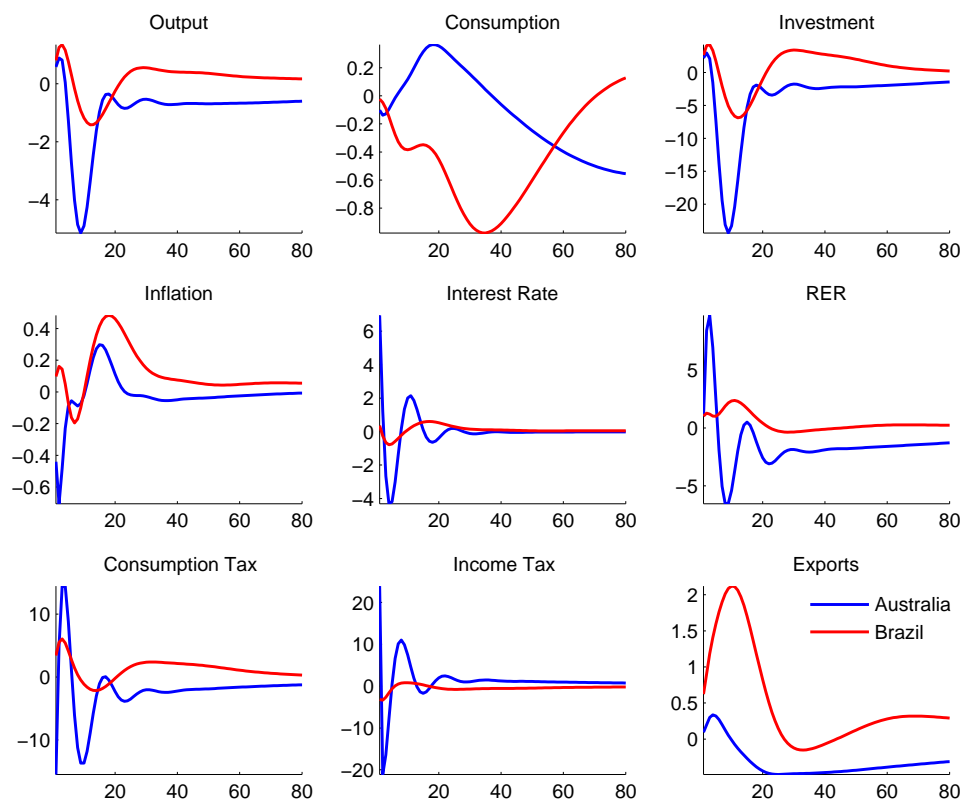


FIGURE 4.3: Impulse response function: world's inflation

shock, however, seems to have smaller persistence when compared to Brazil.

In terms of the UIP and the elasticities of demand for exports, notice that a shock in foreign prices results in a stronger response of Brazilian exports. This happens because, with the change in foreign prices, exporting firms in both countries can almost equally adjust their prices and obtain larger profits selling abroad, fluctuating just their markups. In this sense, when compared to the previous shocks, the price elasticity of demand for domestic exports is not relevant: firms don't face an increase in the quantity demanded, but, instead, explore the new prices in foreign markets for their goods. The larger fluctuation of the real exchange rate in Australia, in this case, is a consequence of the larger elasticity of the foreign interest rate to the

demand of foreign bonds by domestic households.

It is worth noting the different policy responses generated in the two economies. Taxes and interest rate moves very smoothly in Brazil, as the increase in exports guarantees the maintenance of capital flows to the economy. On the other hand, the foreign shock devaluates the Australian real exchange rate, resulting in strong movements of taxes and interest rates to adjust the economy. Again, the effects of shocks on the capital flows of the two economies determine the magnitude and the volatility of the optimal response of taxes and interest rates. Also, the effects of this shock justify the result presented in table , where the nominal interest rate, under the Ramsey policy, is much more volatile in Australia than in Brazil.

To sum up the results in this section, the key parameters to understand the Ramsey policy in this model are those describing the foreign demand for the domestic goods and the domestic demand for imported goods. These parameters characterize the final demand for foreign debt and, through the foreign interest rate channel, the real exchange rate fluctuations. The central role of the intertemporal Euler equation for foreign bonds and, for linear models, the UIP condition, also reinforces the need of a better effort in estimating the parameters associated with the elasticity of foreign interest rates with respect to debt in the “closing” device proposed in Schmitt-Grohé and Uribe (2003b)[65]⁸.

4.2 Evaluating constrained Ramsey policy: the zero lower bound

This section offers an alternative view of the methodology dealing with the additional non-linear constraint associated with the zero lower bound for nominal interest rates. The problem of the zero lower bound is relevant in the analysis of DSGE models due to the results commonly associated with the ergodic moments generated by

⁸ In the estimation exercises performed in chapter 3, parameter κ_2 had to be truncated for both countries.

these models after fitting the structural parameters to the data. Schmitt-Grohé and Uribe (2005, 2005b)[66][67], calibrating their medium-scale model to the US, claim that the zero lower bound is not relevant for the optimal policy analysis, as the probability of violating this constraint is very low. Their argument has been contested lately from two different perspectives. First, from a theoretical perspective, as discussed above, optimal monetary policy outcomes of low and stable inflation can be a result of the absence of a state-contingent instrument for the Ramsey planner. Once this instrument is added, as pointed out by Correia, Nicolini and Teles (2008)[26], inflation under the Ramsey policy becomes very volatile, increasing the probability of violating the constraint. Second, from an applied perspective, models calibrated or even estimated with a large number of shocks, like Adjemian, Pariès and Moyen (2007)[56] and Batini, Levine and Pearlman (2009)[10], dispute the validity of the original results, showing that the probability of hitting the lower bound in models with a higher number of structural shocks is significant for policy analysis, with important consequences also in terms of welfare.

Based on the results in the previous section and from the dynamics of the competitive equilibrium in chapter 3, the claim of Schmitt-Grohé and Uribe (2005, 2005b)[66][67] can also be contested in the current analysis. According to table 4.2, under the competitive equilibrium, using the ergodic moments of the model, the probability of hitting the lower bound for nominal interest rates is estimated at 49.5% and 43.5% for Australia and Brazil, respectively. Under the Ramsey policy, given that inflation in steady state is lower than the calibration used during estimation, these probabilities are estimated at 49.5% and 47.8% for the two countries. The table also shows the probability of negative inflation, and, indeed, it seems that the volatility of inflation is one of the main reasons to the violation of the zero lower bound constraint.

The traditional methodology, originally proposed in Woodford (2003)[75], im-

Table 4.2: Ergodic moments: Competitive and Ramsey Equilibria

Ergodic moments	Australia		Brazil	
	CE	RE	CE	RE
Mean				
Inflation	0.0068	0.0000	0.0110	0.0055
Interest rates	0.0166	0.0098	0.0215	0.0160
Standard deviation				
Inflation	1.7090	0.0834	0.1577	0.2132
Interest rates	1.3289	0.7732	0.1320	0.2876
Pr ($\pi \leq 0$)	49.8%	50.0%	47.3%	49.0%
Pr ($R \leq 0$)	49.5%	49.5%	43.5%	47.8%

Note: “CE” denotes the Competitive Equilibrium and “RE” the Ramsey Equilibrium

poses an exogenous term in the households’ utility function with a penalty to the variance of nominal interest rates. Both the level and the variance of nominal interest rates adjust to new values with this procedure, with the size of adjustment based on the weight given to the loss term. Woodford (2003)[75] computes the weight of the penalty function based on the structural parameters of the model after taking a second order approximation of the households’ welfare function. In Adjemian, Pariès and Moyén (2007)[56] and Batini, Levine and Pearlman (2009)[10], the weight for the penalty term is exogenously given, calibrating the weight in order to match the probability of violating the lower bound to an arbitrary very low level.

The exercise performed here changes the traditional methodology in two aspects. First, instead of controlling for the variance of interest rates, the extra term added in the households’ welfare function, described in chapter 1, explicitly penalizes only the level of nominal interest rates. As a consequence, given a value for the parameter ω_r , the adjustment of the volatility of the policy instrument is endogenous to the decision of the Ramsey planner. This is an important assumption, as the estimated high probabilities of hitting the lower bound imply a severe penalty for monetary authority if nominal interest rates volatility is exogenously adjusted. The second departure is

a consequence of the ability to solve the model with the extra term. Instead of calibrating the weight term ω_r , the procedure here performs a grid search of possible values up to the point where the probability of hitting the lower bound is minimized and the model has a stable and determinate solution. For the reasons discussed in the introduction of this chapter, it might be the case that the model, given the estimated parameters, violates the Blanchard-Kahn conditions regarding stability. Thus, the welfare analysis here shows the values of ω_r that can be implemented by the Ramsey planner.

Table 4.2 shows the value of ω_r , the ergodic moments of inflation, interest rates and other relevant aggregates under the constrained policy and the probability of hitting the zero lower bound of nominal interest rates. The last lines of the table show the welfare consequences of adopting the constrained policy, based on the discounted welfare and the consumption equivalent described in chapter 1. Despite a significant reduction in the probability of negative inflation, the chances of violating the zero lower bound in the model are still very high. Comparing with the results from table 4.1, the solution proposed here makes explicit that simply shifting the level of nominal interest rates in steady state might not be an appropriate solution, especially considering the results for Brazil. For that country, an increase of 18 percentage points in the steady state of interest rates reduced the probability of violating the zero lower bound only by seven points.

Still concerning the analysis for Brazil, besides the significant change in the level of nominal interest rates and inflation, the main consequences of the constrained Ramsey policy is the reduction in the level of consumption taxes, the persistence of inflation and interest rates, measured by the autocorrelation coefficient, and the slightly higher volatility of taxes and government debt. The decrease of consumption taxes was expected, as both inflation and the consumption taxes affect the money demand by household, distorting the intratemporal choice between labor and con-

Table 4.3: Business cycle moments and welfare under constrained policy.

	Mean		Volatility (σ)		$Corr(x_t, Y_t)$		Autocorrelation	
	Aus	Bra	Aus	Bra	Aus	Bra	Aus	Bra
π_t	2.66%	20.23%	0.038	0.119	-0.068	-0.550	0.565	0.940
R_t	6.76%	25.40%	0.066	0.240	0.334	-0.474	0.952	0.641
τ_t^y	-7.42%	-13.96%	1.073	0.885	-0.299	-0.727	0.182	0.874
τ_t^c	31.91%	81.85%	0.557	2.449	-0.030	0.636	0.976	0.953
y_t	-0.844	-0.879	0.709	0.922	1.000	1.000	0.995	0.978
c_t	-1.407	-1.760	0.683	1.016	0.750	0.252	0.997	0.993
i_t	-2.009	-1.951	2.054	3.460	0.812	0.871	0.929	0.979
w_t	0.417	0.377	0.498	0.633	0.608	0.718	0.826	0.992
x_t	-2.529	-2.756	1.100	0.832	0.693	-0.649	1.000	0.994
imp_t	-2.520	-2.768	1.967	2.724	0.818	0.490	0.996	0.982
rer_t	-0.288	-0.244	4.340	0.855	0.712	-0.429	1.000	0.959
li_t	-2.310	0.355	2.424	0.593	0.137	0.895	0.979	0.974
ω_r			1.2100	0.0352				
$\Pr(\pi \leq 0)$			43.0%	35.2%				
$\Pr(R \leq 0)$			40.1%	40.7%				
Welfare loss			142.12	34.49				
CE(%)			3.48%	0.23%				

Note: “Aus” = Australia and “Bra” = Brazil

sumption.

For Australia, the surprising result is the increase in consumption taxes and the larger subsidy for income. The constrained policy also delivers a significant loss in terms of welfare for that country, with the measure of consumption equivalence between the policies given by 3.48% of total consumption. The estimated loss for Brazil is not as large, 0.23% of total consumption. It seems that, for Brazil, the fact that the unconstrained Ramsey policy already imposes inflation different than zero results in the minimization of the losses from adopting the alternative policy. In other words, reducing consumption taxes to balance the budget constraint might be a mitigating factor in the welfare computation, as the unconstrained Ramsey policy for Brazil already assigns a very high level of consumption taxes. On the other hand, because the unconstrained policy assigns very low inflation and consumption taxes

for Australia, any increase in inflation can only be compensated, in terms of welfare, with the reduction of income taxes, reducing the burden of distortions both at the intratemporal and the intertemporal levels.

4.3 Conclusion

This chapter closes the discussion about the dynamics of optimal policy for SOEs and EMEs in this framework. The main results regarding the steady state, after having the estimated parameters available, confirms the analysis of chapter 2, with the Ramsey planner setting high taxes over labor income relative to the taxation over capital. Consistent with part of the literature that includes consumption taxes in the mix of fiscal policy instruments, inflation is low, but very volatile.

The optimal policy is characterized, in the model, by a large volatility of interest rates, a pro-cyclical response of income taxes and a countercyclical response of consumption taxes. Consumption taxes are more volatile than income taxes, as they simulate, affecting prices, the existence of a state-contingent bond in the economy.

In terms of the comparison of the Ramsey dynamics, the optimal policy for EMEs have to take into account the influence of foreign shocks, mostly generated by the price elasticities of demand related to exports and imports and the transmission of changes in foreign debt to the domestic economy. The UIP condition plays an important role in the model, but it is the structure of the demand for imported and exported goods that determines the difference in the policy for both economies.

Finally, the analysis of the zero lower bound for nominal interest rates show that the simple shift of the steady state level of inflation is not enough to solve the problem of excessive fluctuation of the monetary policy instrument in DSGE models with a large number of rigidities and frictions. The relationship between the number of shocks and the ergodic volatility of interest rates, as proposed in Adjemian, Pariès and Moyen (2007)[56], is still an open topic in these structural models and deserves

a more detailed study. These considerations, for obvious reasons, escape from the original topic proposed for this dissertation, serving as an idea for future research on the field.

Conclusion

This dissertation describe the optimal monetary and fiscal policy for SOEs and EMEs. In terms of steady state for the general model, the optimal policy prescribes in general price stability as outcome of the Ramsey problem. This result, however, is conditioned by the fiscal policy framework and, more specifically, by the number of instruments available to the planner. The subsidy for capital income, when compared to taxation over labor income, is robust to every formulation in the model.

The estimation highlighted the role of nominal rigidities in the dynamics of the model and some important differences between SOEs and EMEs that are relevant in the characterization of the optimal policy that goes beyond the simple difference in the volatility of shocks. The main differences between SOEs and EMEs are located in the parameters characterizing the nominal rigidities, with the pricing equations for retailers' firms playing a crucial role in the dynamics of the model. Also relevant for the design of the optimal policy are the estimates of the domestic price elasticity of demand for imported goods and the world's price elasticity for domestic exported goods.

Also related to the design of optimal policy, inflation is low, but very volatile, in-

come and consumption taxes follow opposite directions in terms of comovement with the business cycle and consumption taxes are very volatile, in order to replicate the effects of a state-contingent debt in the economy. Comparing the Ramsey dynamics between SOEs and EMEs, the optimal policy for EMEs have to take into account the strong influence of foreign shocks and the transmission of changes in foreign debt to the domestic economy. These results are not exactly the same for SOEs due to the smaller price elasticity of demand from the rest of the world for their exports. The structure of the demand, in this sense, is one of the most relevant factors to characterize the optimal policy between these economies.

This dissertation tried to address several relevant topics concerning the design of optimal policy in the general framework of small open economies. The characterization of the optimal policy between SOEs and EMEs can move even further with small steps, like the increment of the number of nominal and real rigidities or the use of alternative solution methods to solve the Ramsey policy or to estimate the model. However, without a first step, comparing the responses based on the most common structures used in the literature most of the recent advances in terms of modeling strategy would not make sense. From this perspective, the dissertation tried to close this small gap and suggest several paths where the research related to optimal policy might develop.

Appendix A

Stationary Equilibrium Conditions

In order to transform the model for the stationary form, first note that:

$$z_t^* = z_t \Upsilon_t^{\frac{\theta}{1-\theta}} \implies \frac{z_t^*}{z_{t-1}^*} = \frac{z_t}{z_{t-1}} \left(\frac{\Upsilon_t}{\Upsilon_{t-1}} \right)^{\frac{\theta}{1-\theta}} = \mu_t^z (\mu_t^\Upsilon)^{\frac{\theta}{1-\theta}}$$

Also:

$$\frac{z_t^* \Upsilon_t}{z_{t-1}^* \Upsilon_{t-1}} = \left(\frac{z_t^*}{z_{t-1}^*} \right) \mu_t^\Upsilon = \left(\frac{z_t \Upsilon_t^{\frac{\theta}{1-\theta}}}{z_{t-1} \Upsilon_{t-1}^{\frac{\theta}{1-\theta}}} \right) \mu_t^\Upsilon = \mu_t^z (\mu_t^\Upsilon)^{\frac{1}{1-\theta}}$$

The stationary equilibrium conditions of the model are:

$$\begin{aligned} \frac{(1 - \tau_t^h)}{(1 + \tau_t^c)} w_t &= \frac{\gamma}{(1 - \gamma)} \left(c_t - \zeta \frac{c_{t-1}}{\mu_t^z (\mu_t^\Upsilon)^{\frac{\theta}{1-\theta}}} \right) \frac{mcw_t \left(1 + \nu^m \left(\frac{\tilde{R}_t - 1}{R_t} \right) \right)}{(1 - h_t)} \\ \left(c_t - \zeta \frac{c_{t-1}}{\mu_t^z (\mu_t^\Upsilon)^{\frac{\theta}{1-\theta}}} \right)^{-1} (1 - \gamma) &= (1 + \tau_t^c) \lambda_t \left(1 + \nu^m \frac{R_t - 1}{R_t} \right) \\ \lambda_t \left[1 - \psi_1 \left(\frac{b_{h,t+1}}{y_t} - \frac{b_h}{y} \right) \right] &= \beta R_t E_t \left(\frac{\lambda_{t+1}}{\pi_{t+1} \left(\mu_{t+1}^z (\mu_{t+1}^\Upsilon)^{\frac{\theta}{1-\theta}} \right)} \right) \end{aligned}$$

$$\lambda_t \left[1 - \psi_2 \left(\frac{rer_t ib_{t+1}}{y_t} - \frac{rer ib}{y} \right) \right] = \beta R_t^f E_t \left(\frac{rer_{t+1}}{rer_t} \frac{\lambda_{t+1}}{\pi_{t+1}^* \left(\mu_{t+1}^z (\mu_{t+1}^\Upsilon)^{\frac{\theta}{1-\theta}} \right)} \right)$$

$$k_{x,t} = \mu_{x,t} \bar{k}_{x,t}$$

$$k_{n,t} = \mu_{n,t} \bar{k}_{n,t}$$

$$\theta_1 + \theta_2 (\mu_{x,t} - 1) = r_{x,t}^k$$

$$\theta_1 + \theta_2 (\mu_{n,t} - 1) = r_{n,t}^k$$

$$R_t = \frac{1}{r_{t,t+1}}$$

$$\tilde{R}_t = R_t \left(1 - \psi_1 \left(\frac{b_{n,t+1}}{y_t} - \frac{b_h}{y} \right) \right)^{-1}$$

$$\log a_{n,t+1} = \rho_n \log a_{n,t} + \epsilon_{t+1}^n$$

$$\log a_{x,t+1} = \rho_x \log a_{x,t} + \epsilon_{t+1}^x$$

$$\lambda_t q_{x,t} = \beta E_t \left\{ \frac{\left(\mu_{t+1}^z (\mu_{t+1}^\Upsilon)^{\frac{\theta}{1-\theta}} \right)^{-1}}{\mu_{t+1}^\Upsilon} \lambda_{t+1} \times \left[(1 - \tau_{t+1}^k) (r_{x,t+1}^k \mu_{x,t+1} - a(\mu_{x,t+1})) + q_{x,t+1} (1 - \delta) \right] \right\}$$

$$\lambda_t q_{n,t} = \beta E_t \left\{ \frac{\left(\mu_{t+1}^z (\mu_{t+1}^\Upsilon)^{\frac{\theta}{1-\theta}} \right)^{-1}}{\mu_{t+1}^\Upsilon} \lambda_{t+1} \times \left[(1 - \tau_{t+1}^k) (r_{n,t+1}^k \mu_{n,t+1} - a(\mu_{n,t+1})) + q_{n,t+1} (1 - \delta) \right] \right\}$$

$$\begin{aligned}
\lambda_t = \lambda_t q_{x,t} & \left[1 - \frac{\phi_i}{2} \left(\frac{i_{x,t}^d}{i_{x,t-1}^d} \mu_t^z (\mu_t^\Upsilon)^{\frac{1}{1-\theta}} - \mu^V \right)^2 \right. \\
& - \phi_i \left(\frac{i_{x,t}^d}{i_{x,t-1}^d} \mu_t^z (\mu_t^\Upsilon)^{\frac{1}{1-\theta}} \right) \left(\frac{i_{x,t}^d}{i_{x,t-1}^d} \mu_t^z (\mu_t^\Upsilon)^{\frac{1}{1-\theta}} - \mu^V \right) \Big] \\
& + \beta E_t \left[\lambda_{t+1} \left(\mu_{t+1}^z (\mu_{t+1}^\Upsilon)^{\frac{\theta}{1-\theta}} \right)^{-1} \frac{q_{x,t+1}}{\mu_{t+1}^\Upsilon} \times \right. \\
& \left. \phi_i \left(\frac{i_{x,t+1}^d}{i_{x,t}^d} \mu_{t+1}^z (\mu_{t+1}^\Upsilon)^{\frac{1}{1-\theta}} \right)^2 \left(\frac{i_{x,t+1}^d}{i_{x,t}^d} \mu_{t+1}^z (\mu_{t+1}^\Upsilon)^{\frac{1}{1-\theta}} - \mu^V \right) \right]
\end{aligned}$$

$$\begin{aligned}
\lambda_t = \lambda_t q_{n,t} & \left[1 - \frac{\phi_i}{2} \left(\frac{i_{n,t}^d}{i_{n,t-1}^d} \mu_t^z (\mu_t^\Upsilon)^{\frac{1}{1-\theta}} - \mu^V \right)^2 \right. \\
& - \phi_i \left(\frac{i_{n,t}^d}{i_{n,t-1}^d} \mu_t^z (\mu_t^\Upsilon)^{\frac{1}{1-\theta}} \right) \left(\frac{i_{n,t}^d}{i_{n,t-1}^d} \mu_t^z (\mu_t^\Upsilon)^{\frac{1}{1-\theta}} - \mu^V \right) \Big] \\
& + \beta E_t \left[\lambda_{t+1} \left(\mu_{t+1}^z (\mu_{t+1}^\Upsilon)^{\frac{\theta}{1-\theta}} \right)^{-1} \frac{q_{n,t+1}}{\mu_{t+1}^\Upsilon} \times \right. \\
& \left. \phi_i \left(\frac{i_{n,t+1}^d}{i_{n,t}^d} \mu_{t+1}^z (\mu_{t+1}^\Upsilon)^{\frac{1}{1-\theta}} \right)^2 \left(\frac{i_{n,t+1}^d}{i_{n,t}^d} \mu_{t+1}^z (\mu_{t+1}^\Upsilon)^{\frac{1}{1-\theta}} - \mu^V \right) \right]
\end{aligned}$$

$$\bar{k}_{x,t+1} = (1 - \delta) \frac{\bar{k}_{x,t}}{\mu_t^z (\mu_t^\Upsilon)^{\frac{1}{1-\theta}}} + i_{x,t}^d \left(1 - \frac{\phi_i}{2} \left(\frac{i_{x,t}^d}{i_{x,t-1}^d} \mu_t^z (\mu_t^\Upsilon)^{\frac{1}{1-\theta}} - \mu^V \right)^2 \right)$$

$$\bar{k}_{n,t+1} = (1 - \delta) \frac{\bar{k}_{n,t}}{\mu_t^z (\mu_t^\Upsilon)^{\frac{1}{1-\theta}}} + i_{n,t}^d \left(1 - \frac{\phi_i}{2} \left(\frac{i_{n,t}^d}{i_{n,t-1}^d} \mu_t^z (\mu_t^\Upsilon)^{\frac{1}{1-\theta}} - \mu^V \right)^2 \right)$$

$$\begin{aligned}
& \left(\frac{\varpi - 1}{\varpi} + \frac{1}{mcw_t} \right) \varpi h_t (1 - \tau_t^h) = \\
& - \frac{\phi_w}{\pi_t^{\chi_w - 1}} \left(\frac{w_t}{w_{t-1}} \mu_t^z (\mu_t^\Upsilon)^{\frac{\theta}{1-\theta}} \right) \left(\frac{w_t}{\pi_t^{\chi_w - 1} w_{t-1}} \mu_t^z (\mu_t^\Upsilon)^{\frac{\theta}{1-\theta}} - \mu^I \right) \\
& + \beta E_t \left[\frac{\lambda_{t+1} \phi_w}{\lambda_t \pi_{t+1}^{\chi_w - 1}} \left(\mu_{t+1}^z (\mu_{t+1}^\Upsilon)^{\frac{\theta}{1-\theta}} \right)^{-1} \times \right. \\
& \left. \left(\frac{w_{t+1}}{w_t} \mu_{t+1}^z (\mu_{t+1}^\Upsilon)^{\frac{\theta}{1-\theta}} \right)^2 \left(\frac{w_{t+1}}{\pi_{t+1}^{\chi_w - 1} w_t} \mu_{t+1}^z (\mu_{t+1}^\Upsilon)^{\frac{\theta}{1-\theta}} - \mu^I \right) \right]
\end{aligned}$$

$$c_t + \frac{\psi_1}{2} y_t \left(\frac{b_{h,t+1}}{y_t} - \frac{b_h}{y} \right)^2 + \frac{\psi_2}{2} y_t \left(\frac{rer_t ib_t}{y_t} - \frac{rer ib}{y} \right)^2 = \left[(1 - \omega)^{\frac{1}{\varepsilon}} c_{n,t}^{\frac{\varepsilon-1}{\varepsilon}} + \omega^{\frac{1}{\varepsilon}} c_{t,t}^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

$$c_{t,t} = \left[(1 - \varkappa)^{\frac{1}{\varrho}} c_{x,t}^{\frac{\varrho-1}{\varrho}} + \varkappa^{\frac{1}{\varrho}} c_{m,t}^{\frac{\varrho-1}{\varrho}} \right]^{\frac{\varrho}{\varrho-1}}$$

$$c_{m,t} = \varkappa (pm_t pt_t)^{-\varrho} c_{t,t}$$

$$c_{x,t} = (1 - \varkappa) (px_t pt_t)^{-\varrho} c_{t,t}$$

$$c_{t,t} = \omega (pt_t)^{-\varepsilon} \left(c_t + \frac{\psi_1}{2} y_t \left(\frac{b_{h,t+1}}{y_t} - \frac{B}{Y} \right)^2 + \frac{\psi_2}{2} y_t \left(\frac{rer_t ib_t}{y_t} - \frac{rer ib}{y} \right)^2 \right)$$

$$c_{n,t} = (1 - \omega) (pn_t)^{-\varepsilon} \left(c_t + \frac{\psi_1}{2} y_t \left(\frac{b_{h,t+1}}{y_t} - \frac{B}{Y} \right)^2 + \frac{\psi_2}{2} y_t \left(\frac{rer_t ib_t}{y_t} - \frac{rer ib}{y} \right)^2 \right)$$

$$i_t = i_{n,t}^d + a(\mu_{n,t}) \frac{\bar{k}_{n,t}}{\mu_t^z (\mu_t^\Upsilon)^{\frac{1}{1-\theta}}} + i_{x,t}^d + a(\mu_{x,t}) \frac{\bar{k}_{x,t}}{\mu_t^z (\mu_t^\Upsilon)^{\frac{1}{1-\theta}}}$$

$$i_{m,t} = \varkappa (pm_t pt_t)^{-\varrho} i_{t,t}$$

$$i_{x,t} = (1 - \varkappa) (px_t pt_t)^{-\varrho} i_{t,t}$$

$$i_{t,t} = \omega (pt_t)^{-\varepsilon} i_t$$

$$i_{n,t} = (1 - \omega) (pn_t)^{-\varepsilon} i_t$$

$$w_t = pn_t mc_{n,t} (1 - \theta) a_{n,t} \left(\frac{k_{n,t}}{\mu_t^z (\mu_t^\Upsilon)^{\frac{1}{1-\theta}} h_{n,t}} \right)^\theta$$

$$r_{n,t}^k = pn_t mc_{n,t} \theta a_{n,t} \left(\frac{k_{n,t}}{\mu_t^z (\mu_t^\Upsilon)^{\frac{1}{1-\theta}} h_{n,t}} \right)^{\theta-1}$$

$$w_t = px_t mc_{x,t} (1 - \theta) a_{x,t} \left(\frac{k_{x,t}}{\mu_t^z (\mu_t^\Upsilon)^{\frac{1}{1-\theta}} h_{x,t}} \right)^\theta$$

$$r_{x,t}^k = px_t mc_{x,t} \theta a_{x,t} \left(\frac{k_{x,t}}{\mu_t^z (\mu_t^\Upsilon)^{\frac{1}{1-\theta}} h_{x,t}} \right)^{\theta-1}$$

$$\begin{aligned} x_t^1 &= \tilde{p}_{n,t}^{-1-\eta_n} \left(c_{n,t} + g_{n,t} + \frac{i_{n,t}}{pn_t} \right) mc_{n,t} \\ &\quad + E_t \alpha_n r_{t,t+1} \left(\frac{\tilde{p}_{n,t}}{\tilde{p}_{n,t+1}} \right)^{-1-\eta_n} \left(\frac{\pi_{n,t}^{\kappa_n}}{\pi_{n,t+1}^{(1+\eta_n)/\eta_n}} \right)^{-\eta_n} \mu_{t+1}^z (\mu_{t+1}^\Upsilon)^{\frac{\theta}{1-\theta}} x_{t+1}^1 \end{aligned}$$

$$\begin{aligned} x_t^2 &= \tilde{p}_{n,t}^{-\eta_n} \left(c_{n,t} + g_{n,t} + \frac{i_{n,t}}{pn_t} \right) \frac{(\eta_n - 1)}{\eta_n} \\ &\quad + E_t \alpha_n r_{t,t+1} \left(\frac{\tilde{p}_{n,t}}{\tilde{p}_{n,t+1}} \right)^{-\eta_n} \left(\frac{\pi_{n,t}^{\kappa_n}}{\pi_{n,t+1}^{\eta_n/(\eta_n-1)}} \right)^{1-\eta_n} \mu_{t+1}^z (\mu_{t+1}^\Upsilon)^{\frac{\theta}{1-\theta}} x_{t+1}^2 \end{aligned}$$

$$\begin{aligned} z_t^1 &= \tilde{p}_{x,t}^{-1-\eta_x} \left(c_{x,t} + g_{t,t} + \frac{pt_t}{px_t} i_{x,t} + d_{xp,t} \right) mc_{x,t} \\ &\quad + E_t \alpha_x r_{t,t+1} \left(\frac{\tilde{p}_{x,t}}{\tilde{p}_{x,t+1}} \right)^{-1-\eta_x} \left(\frac{\pi_{x,t}^{\kappa_x}}{\pi_{x,t+1}^{(1+\eta_x)/\eta_x}} \right)^{-\eta_x} \mu_{t+1}^z (\mu_{t+1}^\Upsilon)^{\frac{\theta}{1-\theta}} z_{t+1}^1 \end{aligned}$$

$$\begin{aligned} z_t^2 &= \tilde{p}_{x,t}^{-\eta_x} \left(c_{x,t} + g_{t,t} + \frac{pt_t}{px_t} i_{x,t} + d_{xp,t} \right) \frac{(\eta_x - 1)}{\eta_x} \\ &\quad + E_t \alpha_x r_{t,t+1} \left(\frac{\tilde{p}_{x,t}}{\tilde{p}_{x,t+1}} \right)^{-1-\eta_x} \left(\frac{\pi_{x,t}^{\kappa_x}}{\pi_{x,t+1}^{\eta_x/(\eta_x-1)}} \right)^{1-\eta_x} \mu_{t+1}^z (\mu_{t+1}^\Upsilon)^{\frac{\theta}{1-\theta}} z_{t+1}^2 \end{aligned}$$

$$y_t^1 = \tilde{p}_{m,t}^{-1-\eta_m} \left(c_{m,t} + i_{m,t} \frac{pt_t}{pm_t} \right) rer_t \frac{pt_t pm_t^*}{pm_t} \left(1 + \frac{R_t^f - 1}{R_t^f} \right) \\ + E_t \alpha_m r_{t,t+1} \left(\frac{\tilde{p}_{m,t}}{\tilde{p}_{m,t+1}} \right)^{-1-\eta_m} \left(\frac{\pi_{m,t}^{\kappa_m}}{\pi_{m,t+1}^{(1+\eta_m)/\eta_m}} \right)^{-\eta_m} \mu_{t+1}^z (\mu_{t+1}^\Upsilon)^{\frac{\theta}{1-\theta}} y_{t+1}^1$$

$$y_t^2 = \tilde{p}_{m,t}^{-\eta_m} \left(c_{m,t} + i_{m,t} \frac{pt_t}{pm_t} \right) \frac{(\eta_m - 1)}{\eta_m} \\ + E_t \alpha_m r_{t,t+1} \left(\frac{\tilde{p}_{m,t}}{\tilde{p}_{m,t+1}} \right)^{-\eta_m} \left(\frac{\pi_{m,t}^{\kappa_m}}{\pi_{m,t+1}^{(1+\eta_m)/\eta_m}} \right)^{1-\eta_m} \mu_{t+1}^z (\mu_{t+1}^\Upsilon)^{\frac{\theta}{1-\theta}} y_{t+1}^2$$

$$u_t^1 = (\tilde{p}_{x,t}^*)^{-1-\eta_{xp}} x_t \frac{px_t pt_t}{rer_t pm_t^* tot_t} \\ + E_t \alpha_{xp} r_{t,t+1} \left(\frac{\tilde{p}_{x,t}^*}{\tilde{p}_{x,t+1}^*} \right)^{-1-\eta_{xp}} \left(\frac{(\pi_{x,t}^*)^{\kappa_{xp}}}{(\pi_{x,t+1}^*)^{\frac{(1+\eta_{xp})}{\eta_{xp}}}} \right)^{-\eta_{xp}} \mu_{t+1}^z (\mu_{t+1}^\Upsilon)^{\frac{\theta}{1-\theta}} u_{t+1}^1$$

$$u_t^2 = (\tilde{p}_{x,t}^*)^{-\eta_{xp}} x_t \frac{(\eta_{xp} - 1)}{\eta_{xp}} \\ + E_t \alpha_{xp} r_{t,t+1} \left(\frac{\tilde{p}_{x,t}^*}{\tilde{p}_{x,t+1}^*} \right)^{-\eta_{xp}} \left(\frac{(\pi_{x,t}^*)^{\kappa_{xp}}}{(\pi_{x,t+1}^*)^{\frac{(1+\eta_{xp})}{\eta_{xp}}}} \right)^{1-\eta_{xp}} \mu_{t+1}^z (\mu_{t+1}^\Upsilon)^{\frac{\theta}{1-\theta}} u_{t+1}^2$$

$$x_t^1 = x_t^2$$

$$z_t^1 = z_t^2$$

$$y_t^1 = y_t^2$$

$$u_t^1 = u_t^2$$

$$\log\left(\frac{R_{t+1}}{R}\right) = \rho_R \log\left(\frac{R_{t+1}}{R}\right) + (1 - \rho_R) \left[\alpha_\pi \log\left(\frac{\pi_{t+1}}{\pi_{t+1}^o}\right) + \alpha_y \log\left(\frac{y_{t+1}}{y}\right) + \alpha_{rer} \log\left(\frac{rer_{t+1}}{rer_t}\right) \right] + \epsilon_{t+1}^R$$

$$\pi_{t+1}^o = (1 - \rho_{\pi^o})\pi^o + \rho_{\pi^o}\pi_t^o + \epsilon_{t+1}^{\pi^o}$$

$$t_t = \tau_t^c c_t + \tau_t^h w_t h_t + \tau_t^k \left[(r_{n,t}^k \mu_{n,t} - a(\mu_{n,t})) \bar{k}_{n,t} + (r_{x,t}^k \mu_{x,t} - a(\mu_{x,t})) \bar{k}_{x,t} \right] + \tau_t^\phi \phi_t$$

$$g_t = (1 - \rho_g) g + \rho_g g_{t-1} + \epsilon_t^g$$

$$l_t = m_t + R_t b_{g,t+1}$$

$$l_t = \frac{R_t}{\pi_t} \frac{l_{t-1}}{\mu_t^z (\mu_t^\gamma)^{1-\theta}} + R_t (g_t - t_t) - (R_t - 1) m_t$$

$$\tau_t^h - \tau^h = \psi_{li} \left(\frac{l_t}{y_t} - \frac{l}{y} \right) + \psi_y (y_t - y) + \epsilon_t^\tau$$

$$\tau_t^k = (1 - \rho_{\tau^k}) \tau^k + \rho_{\tau^k} \tau_{t-1}^k + \epsilon_t^{\tau^k}$$

$$\tau_t^\phi = \tau^\phi$$

$$\tau_t^c = (1 - \rho_{\tau^c}) \tau^c + \rho_{\tau^c} \tau_{t-1}^c + \epsilon_t^{\tau^c}$$

$$g_{n,t} = (1 - \omega) (p n_t)^{-\varepsilon} g_t$$

$$g_{t,t} = \omega (p t_t)^{-\varepsilon} g_t$$

$$R_t^f = R_t^* (1 + \xi_t)^{\kappa_1} \left(\frac{rer_t i b_t}{y_t} / \frac{i b}{y} \right)^{\kappa_2}$$

$$\begin{bmatrix} \frac{\Delta M_t^*}{\Delta M^*} \\ \frac{\xi_t}{\xi^*} \\ \frac{R_t^*}{R^*} \\ \frac{\pi_t^*}{\pi^*} \\ \frac{y_t^*}{y^*} \end{bmatrix} = A \begin{bmatrix} \frac{\Delta M_{t-1}^*}{\Delta M^*} \\ \frac{\xi_{t-1}}{\xi^*} \\ \frac{R_{t-1}^*}{R^*} \\ \frac{\pi_{t-1}^*}{\pi^*} \\ \frac{y_{t-1}^*}{y^*} \end{bmatrix} + \begin{bmatrix} \varepsilon_t^{m^*} \\ \varepsilon_t^\xi \\ \varepsilon_t^{R^*} \\ \varepsilon_t^{\pi^*} \\ \varepsilon_t^{y^*} \end{bmatrix}$$

$$x_t = (p m_t^* t o t_t)^{-\eta^*} y_t^*$$

$$tot_t = \frac{\pi_{x,t}^*}{\pi_t^*} tot_{t-1}$$

$$\frac{\pi_t^{m*}}{\pi^{m*}} = v_1 \frac{\pi_{t-1}^{m*}}{\pi^{m*}} + v_2 \frac{tot_{t-1}}{tot} + \xi X_{t-1} + \varepsilon_t^{\pi m}$$

$$a_{n,t} \left(\frac{k_{n,t}}{\mu_t^z (\mu_t^r)^{\frac{1}{1-\theta}}} \right)^\theta h_{n,t}^{1-\theta} - \chi_n = s_{n,t} \left(c_{n,t} + g_{n,t} + \frac{i_{n,t}}{pm_t} \right)$$

$$s_{n,t} = (1 - \alpha_n) \widetilde{p}_{n,t}^{-\eta_n} + \alpha_n \left(\frac{\pi_{n,t}}{\pi_{n,t-1}^{\kappa_n}} \right)^{\eta_n} s_{n,t-1}$$

$$1 = (1 - \alpha_n) \widetilde{p}_{n,t}^{1-\eta_n} + \alpha_n \left(\frac{\pi_{n,t-1}^{\kappa_n}}{\pi_{n,t}} \right)^{1-\eta_n}$$

$$d_{m,t} - \chi_m = s_{m,t} \left(c_{m,t} + i_{m,t} \frac{pt_t}{pm_t} \right)$$

$$s_{m,t} = (1 - \alpha_m) \widetilde{p}_{m,t}^{-\eta_m} + \alpha_m \left(\frac{\pi_{m,t}}{\pi_{m,t-1}^{\kappa_m}} \right)^{\eta_m} s_{m,t-1}$$

$$1 = (1 - \alpha_m) \widetilde{p}_{m,t}^{1-\eta_m} + \alpha_m \left(\frac{\pi_{m,t-1}^{\kappa_m}}{\pi_{m,t}} \right)^{1-\eta_m}$$

$$a_{x,t} \left(\frac{k_{x,t}}{\mu_t^z (\mu_t^r)^{\frac{1}{1-\theta}}} \right)^\theta h_{x,t}^{1-\theta} - \chi_x = s_{x,t} \left(c_{x,t} + g_{t,t} + \frac{pt_t}{px_t} i_{x,t} + d_{xp,t} \right)$$

$$s_{x,t} = (1 - \alpha_x) \widetilde{p}_{x,t}^{-\eta_x} + \alpha_x \left(\frac{\pi_{x,t}}{\pi_{x,t-1}^{\kappa_x}} \right)^{\eta_x} s_{x,t-1}$$

$$1 = (1 - \alpha_x) \widetilde{p}_{x,t}^{1-\eta_x} + \alpha_x \left(\frac{\pi_{x,t-1}^{\kappa_x}}{\pi_{x,t}} \right)^{1-\eta_x}$$

$$d_{xp,t} - \chi_{xp} = s_{xp,t} x_t$$

$$s_{xp,t} = (1 - \alpha_{xp}) (\widetilde{p}_{x,t}^*)^{-\eta_{xp}} + \alpha_{xp} \left(\frac{\pi_{xp,t}^*}{(\pi_{xp,t-1}^*)^{\kappa_{xp}}} \right)^{\eta_{xp}} s_{xp,t-1}$$

$$1 = (1 - \alpha_{xp}) \widetilde{p}_{xp,t}^{1-\eta_{xp}} + \alpha_{xp} \left(\frac{(\pi_{x,t-1}^*)^{\kappa_{xp}}}{\pi_{x,t}^*} \right)^{1-\eta_{xp}}$$

$$h_{x,t} + h_{n,t} = h_t$$

$$y_t = c_t + g_t + i_t + px_t pt_t x_t - pm_t pt_t d_{m,t} \left[1 + \left(\frac{R_t^f - 1}{R_t^f} \right) \right] \\ + \frac{\psi_1}{2} y_t \left(\frac{b_{h,t+1}}{y_t} - \frac{b_h}{y} \right)^2 + \frac{\psi_2}{2} y_t \left(\frac{rer_t ib_t}{y_t} - \frac{rer ib}{y} \right)^2$$

$$b_{g,t} + b_{h,t} = 0$$

$$px_t pt_t x_t - pm_t pt_t d_{m,t} \left[1 + \left(\frac{R_t^f - 1}{R_t^f} \right) \right] = rer_t R_{t-1}^f \frac{ib_t}{\mu_t^z (\mu_t^\Upsilon)^{1-\theta}} - rer_t \pi_{t+1}^* ib_{t+1}$$

$$\phi_t = y_t - w_t h_t - r_{n,t}^k \mu_{n,t} \bar{k}_{n,t} - r_{x,t}^k \mu_{x,t} \bar{k}_{x,t}$$

$$m_t = \nu^m (1 + \tau_t^c) c_t$$

$$pt_t = \frac{\pi_{t,t}}{\pi_t} pt_{t-1}$$

$$pn_t = \frac{\pi_{n,t}}{\pi_t} pn_{t-1}$$

$$px_t = \frac{\pi_{x,t}}{\pi_{t,t}} px_{t-1}$$

$$pm_t = \frac{\pi_{m,t}}{\pi_{t,t}} pm_{t-1}$$

$$pm_t^* = \frac{\pi_{m,t}^*}{\pi_t^*} pm_{t-1}^*$$

$$\frac{\Upsilon_{t+1}}{\Upsilon_t} = \mu_{t+1}^\Upsilon = (1 - \rho_\Upsilon) \mu_t^\Upsilon + \rho_\Upsilon \mu_t^\Upsilon + \epsilon_{t+1}^\Upsilon$$

$$\frac{z_{t+1}}{z_t} = \mu_{t+1}^z = (1 - \rho_z) \mu_t^z + \rho_z \mu_t^z + \epsilon_{t+1}^z$$

Appendix B

Steady State Conditions: Competitive Equilibrium

This section describes the sequence of equations necessary to compute the steady state of the competitive equilibrium of the assuming that the values related to income taxation are known. The taxation on consumption is obtained using the government budget constraint, assuming that the steady state level of debt-output ratio is known. Given steady state values for taxes $\tau^h, \tau^k, \tau^\phi$, parameter values for $\beta, \theta, \delta, \omega, \varkappa, \mu^z, \mu^Y, \eta_x, \eta_n, \eta_m, \eta_{xp}, \varpi, \kappa_1, \alpha_x, \alpha_m, \alpha_{xp}, \alpha_n$, and steady state values for $h, R^*/R^f, tb/y, \tau/y, \pi^o, \pi^*, g/y, b/y, m/y, imp/y$ and the share of non-tradable goods in the output, there are 86 variables and 9 parameters to be computed in the steady state of the competitive equilibrium. The set of variables is given by:

$\left\{ \pi, \pi_n, \pi_m, \pi_t, \pi_x, \pi_m^*, \pi_x^*, a_n, a_x, q_n, q_x, \mu_x, \mu_n, mcw_t, \Delta M^*, pt, pn, px, pm, pm^*, R, r, \tilde{R}, R^f, \xi, R^*, \tilde{p}_x, s_x, \tilde{p}_m, s_m, \tilde{p}_x^*, s_{xp}, \tilde{p}_n, s_n, mc_n, mc_x, rer, r_n^k, r_x^k, h_x, h_n, \bar{k}_x, \bar{k}_n, k_x, k_n, i_x^d, i_n^d, i, g, g_t, g_n, c, c_t, c_n, c_x, c_m, i_n, i_t, i_x, i_m, ib, x, b_g, m, l, d_m, w, d_{xp}, \tau^c, tot, y^*, x^1, x^2, y^1, y^2, z^1, z^2, u^1, u^2, \lambda, y, \phi \right\}$. The set of parameters is given by: $\{ \theta_1, \theta_2, \nu^m, \chi_n, \chi_x, \chi_m, \chi_{xp}, \gamma \}$.

$$\pi = \pi_n = \pi_m = \pi_t = \pi_x = \pi^o$$

$$\pi_m^* = \pi_x^* = \Delta M^* = \pi^*$$

$$a_n = a_x = 1$$

$$q_n = q_x = 1$$

$$\mu_x = \mu_n = 1$$

$$mcw_t = \frac{\varpi}{\varpi - 1}$$

$$pt = 1 \quad pn = 1 \quad px = 1 \quad pm = 1 \quad pm^* = 1$$

$$R = \frac{\pi}{\beta} \mu^z (\mu^Y)^{\frac{\theta}{1-\theta}} \quad r = \frac{1}{R} \quad \tilde{R} = R^f = R \quad \xi = \left(\frac{R^*}{R^f}\right)^{\frac{1}{\kappa_1}} - 1 \quad R^* = \left(\frac{R^*}{R^f}\right) R^f$$

$$\tilde{p}_x = \left(\frac{1 - \alpha_x \pi_x^{(\kappa_x - 1)(1 - \eta_x)}}{1 - \alpha_x} \right)^{\frac{1}{1 - \eta_x}} \quad S_x = \frac{(1 - \alpha_x) \tilde{p}_x^{-\eta_x}}{1 - \alpha_x \pi_x^{\eta_x (1 - \kappa_x)}}$$

$$\tilde{p}_m = \left(\frac{1 - \alpha_m \pi_m^{(\kappa_m - 1)(1 - \eta_m)}}{1 - \alpha_m} \right)^{\frac{1}{1 - \eta_m}} \quad S_m = \frac{(1 - \alpha_m) \tilde{p}_m^{-\eta_m}}{1 - \alpha_m \pi_m^{\eta_m (1 - \kappa_m)}}$$

$$\tilde{p}_x^* = \left(\frac{1 - \alpha_{xp} (\pi_x^*)^{(\kappa_{xp} - 1)(1 - \eta_{xp})}}{1 - \alpha_{xp}} \right)^{\frac{1}{1 - \eta_{xp}}} \quad S_{xp} = \frac{(1 - \alpha_{xp}) (\tilde{p}_x^*)^{-\eta_{xp}}}{1 - \alpha_{xp} (\pi_x^*)^{\eta_{xp} (1 - \kappa_{xp})}}$$

$$\tilde{p}_n = \left(\frac{1 - \alpha_n \pi_n^{(\kappa_n - 1)(1 - \eta_n)}}{1 - \alpha_n} \right)^{\frac{1}{1 - \eta_n}} \quad s_n = \frac{(1 - \alpha_n) \tilde{p}_n^{\eta_n}}{1 - \alpha_n \pi_n^{\eta_n(1 - \kappa_n)}}$$

$$mc_n = \tilde{p}_n \frac{1 - \alpha_n r \pi_n^{-\eta_n \left(\kappa_n - \frac{(1 + \eta_n)}{\eta_n} \right)} \mu^z (\mu^\Upsilon)^{\frac{\theta}{1 - \theta}} (\eta_n - 1)}{1 - \alpha_n r \pi_n^{(1 - \eta_n) \left(\kappa_n - \frac{\eta_n}{(\eta_n - 1)} \right)} \mu^z (\mu^\Upsilon)^{\frac{\theta}{1 - \theta}} \eta_n}$$

$$mc_x = \tilde{p}_x \frac{1 - \alpha_x r \pi_x^{-\eta_x \left(\kappa_x - \frac{(1 + \eta_x)}{\eta_x} \right)} \mu^z (\mu^\Upsilon)^{\frac{\theta}{1 - \theta}} (\eta_x - 1)}{1 - \alpha_x r \pi_x^{(1 - \eta_x) \left(\kappa_x - \frac{\eta_x}{(\eta_x - 1)} \right)} \mu^z (\mu^\Upsilon)^{\frac{\theta}{1 - \theta}} \eta_x}$$

$$rer = \tilde{p}_m \left(1 + \frac{R^f - 1}{R^f} \right)^{-1} \frac{pm}{pt pm^*} \frac{1 - \alpha_m r \pi_m^{(-\eta_m) \left(\kappa_m - \frac{(1 + \eta_m)}{\eta_m} \right)} \mu^z (\mu^\Upsilon)^{\frac{\theta}{1 - \theta}} (\eta_m - 1)}{1 - \alpha_m r \pi_m^{(1 - \eta_m) \left(\kappa_m - \frac{(1 + \eta_m)}{\eta_m} \right)} \mu^z (\mu^\Upsilon)^{\frac{\theta}{1 - \theta}} \eta_m}$$

$$r_n^k = (1 - \tau^k)^{-1} \left[\beta^{-1} \mu^\Upsilon \left(\mu^z (\mu^\Upsilon)^{\frac{\theta}{1 - \theta}} \right) - 1 + \delta \right]$$

$$r_x^k = (1 - \tau^k)^{-1} \left[\beta^{-1} \mu^\Upsilon \left(\mu^z (\mu^\Upsilon)^{\frac{\theta}{1 - \theta}} \right) - 1 + \delta \right]$$

$$\frac{k_x}{h_x} = \mu^z (\mu^\Upsilon)^{\frac{1}{1 - \theta}} \left(\frac{r_x^k}{mc_x \theta} \right)^{\frac{1}{\theta - 1}}$$

$$\frac{h_x}{h_n} = \frac{mc_x Y_x}{mc_n Y_n} \quad h = 0.2 \implies h_n = h \left(1 + \frac{mc_x Y_x}{mc_n Y_n} \right)^{-1} \quad h_x = \frac{Y_x}{Y_n} \frac{mc_x}{mc_n} h_n$$

$$k_x = \bar{k}_x = \frac{k_x}{h_x} h_x \quad k_n = \bar{k}_n = h_n \frac{k_x}{h_x} \left(\frac{mc_x}{mc_n} \right)^{\frac{1}{\theta}}$$

$$i_x^d = \left(1 - \frac{(1-\delta)}{\mu^z (\mu^\Upsilon)^{\frac{1}{1-\theta}}}\right) \bar{k}_x \quad i_n^d = \left(1 - \frac{(1-\delta)}{\mu^z (\mu^\Upsilon)^{\frac{1}{1-\theta}}}\right) \bar{k}_n$$

$$i = i_x^d + i_n^d \quad \theta_1 = r_x^k \quad \theta_2 = \theta_1 \frac{\theta_2}{\theta_1}$$

$$w = mc_n (1 - \theta) \left(\mu^z (\mu^\Upsilon)^{\frac{1}{\theta-1}} \frac{k_n \mu_n}{h_n} \right)^\theta$$

$$g = \frac{g}{y} \left(wh + r_x^k \frac{k_x}{\mu^z (\mu^\Upsilon)^{\frac{1}{1-\theta}}} + r_n^k \frac{k_n}{\mu^z (\mu^\Upsilon)^{\frac{1}{1-\theta}}} \right)$$

$$g_n = (1 - \omega) g \quad g_t = \omega g$$

$$c = \left(1 - \frac{tb}{y}\right) \left(wh + r_x^k \frac{k_x}{\mu^z (\mu^\Upsilon)^{\frac{1}{1-\theta}}} + r_n^k \frac{k_n}{\mu^z (\mu^\Upsilon)^{\frac{1}{1-\theta}}} \right) - g - i$$

$$c_n = (1 - \omega) c \quad c_t = \omega c \quad c_x = (1 - \varkappa) c_t \quad c_m = \varkappa c_t$$

$$i_n = (1 - \omega) i \quad i_t = \omega i \quad i_x = (1 - \varkappa) i_t \quad i_m = \varkappa i_t$$

$$\frac{ib}{y} = \frac{tb}{y} \left[\text{rer} \left(\frac{R^f}{\mu^z (\mu^\Upsilon)^{\frac{\theta}{1-\theta}}} - \pi^* \right) \right]^{-1}$$

$$ib = \frac{ib}{y} \left(wh + r_x^k \frac{k_x}{\mu^z (\mu^\Upsilon)^{\frac{1}{1-\theta}}} + r_n^k \frac{k_n}{\mu^z (\mu^\Upsilon)^{\frac{1}{1-\theta}}} \right)$$

$$x = \frac{tb}{y} \left(wh + r_x^k \frac{k_x}{\mu^z (\mu^\Upsilon)^{\frac{1}{1-\theta}}} + r_n^k \frac{k_n}{\mu^z (\mu^\Upsilon)^{\frac{1}{1-\theta}}} \right) + d_m \left(1 + \frac{R^f - 1}{R^f} \right)$$

$$m = \frac{m}{y} \left(wh + r_x^k \frac{k_x}{\mu^z (\mu^\Upsilon)^{\frac{1}{1-\theta}}} + r_n^k \frac{k_n}{\mu^z (\mu^\Upsilon)^{\frac{1}{1-\theta}}} \right)$$

$$b_g = \frac{b_g}{y} \left(wh + r_x^k \frac{k_x}{\mu^z (\mu^\Upsilon)^{\frac{1}{1-\theta}}} + r_n^k \frac{k_n}{\mu^z (\mu^\Upsilon)^{\frac{1}{1-\theta}}} \right)$$

$$\frac{l}{y} = \frac{m}{y} + R \frac{b_g}{y}$$

$$l = \frac{l}{y} \left(wh + r_x^k \frac{k_x}{\mu^z (\mu^\Upsilon)^{\frac{1}{1-\theta}}} + r_n^k \frac{k_n}{\mu^z (\mu^\Upsilon)^{\frac{1}{1-\theta}}} \right)$$

$$d_m = s_m \left(c_m + i_m \frac{pt}{pm} \right) \implies \chi_m = 0 \quad d_{xp} = s_{xp} x \implies \chi_{xp} = 0$$

$$d_{xp} = s_{xp} x$$

$$\tau^c = \left\{ R \left[g - \tau^h wh - \tau^k (r_n^k k_n + r_x^k k_x) \right] - (R - 1) m - l \left(1 - \frac{R}{\pi \mu^z (\mu^\Upsilon)^{\frac{\theta}{1-\theta}}} \right) \right\} (cR)^{-1}$$

$$\nu^m = \frac{m}{(1 + \tau^c) c}$$

$$tot = \frac{\eta_{xp}}{(\tilde{p}_x^*)(\eta_{xp} - 1)} \frac{1 - \alpha_{xp} r (\pi_x^*)^{(1-\eta_{xp})} (\kappa_{xp} - \frac{(1+\eta_{xp})}{\eta_{xp}})}{1 - \alpha_{xp} r (\pi_x^*)^{(-\eta_{xp})} (\kappa_{xp} - \frac{(1+\eta_{xp})}{\eta_{xp}})} \mu^z (\mu^\Upsilon)^{\frac{\theta}{1-\theta}} \left(\frac{px pt}{rer pm^*} \right)$$

$$y^* = x tot^{\eta^*}$$

$$\chi_n = \left(\frac{k_n}{\mu^z (\mu^\Upsilon)^{\frac{1}{1-\theta}}} \right)^\theta h_n^{1-\theta} - s_n \left(c_n + g_n + \frac{P}{P_n} i_n \right)$$

$$\chi_x = \left(\frac{k_x}{\mu^z (\mu^\Upsilon)^{\frac{1}{1-\theta}}} \right)^\theta h_x^{1-\theta} - s_x \left(c_x + g_t + \frac{P_t}{P_{x,t}} i_x + d_{xp} \right)$$

$$x^1 = \frac{\tilde{p}_n^{-1-\eta_n} \left(c_n + g_n + \frac{i_n}{pn} \right) m c_n}{1 - \alpha_n r \pi_n^{-\eta_n} (\kappa_n - \frac{(1+\eta_n)}{\eta_n}) \mu^z (\mu^\Upsilon)^{\frac{\theta}{1-\theta}}}$$

$$x^2 = \frac{\tilde{p}_n^{-\eta_n} \left(c_n + g_n + \frac{i_n}{pn} \right)}{1 - \alpha_n r \pi_n^{(1-\eta_n)} (\kappa_n - \frac{\eta_n}{(\eta_n-1)}) \mu^z (\mu^\Upsilon)^{\frac{\theta}{1-\theta}}} \frac{(\eta_n - 1)}{\eta_n}$$

$$y^1 = \frac{\tilde{p}_m^{-1-\eta_m} \left(c_m + i_m \frac{pt}{pm} \right) rer \frac{pt pm^*}{pm} \left(1 + \frac{R^f - 1}{R^f} \right)}{1 - \alpha_m r \pi_m^{(-\eta_m)} (\kappa_m - \frac{(1+\eta_m)}{\eta_m}) \mu^z (\mu^\Upsilon)^{\frac{\theta}{1-\theta}}}$$

$$y^2 = \frac{\tilde{p}_m^{-\eta_m} \left(c_m + i_m \frac{pt}{pm} \right)}{1 - \alpha_m r \pi_m^{(1-\eta_m)} (\kappa_m - \frac{(1+\eta_m)}{\eta_m}) \mu^z (\mu^\Upsilon)^{\frac{\theta}{1-\theta}}} \frac{(\eta_m - 1)}{\eta_m}$$

$$z^1 = \frac{\tilde{p}_x^{-1-\eta_x} \left(c_x + g_t + \frac{pt}{px} i_x + d_{xp} \right) mc_x}{1 - \alpha_x r \pi_x^{-\eta_x \left(\kappa_x - \frac{(1+\eta_x)}{\eta_x} \right)} \mu^z (\mu^\Upsilon)^{\frac{\theta}{1-\theta}}}$$

$$z^2 = \frac{\tilde{p}_x^{-\eta_x} \left(c_x + g_t + \frac{pt}{px} i_x + d_{xp} \right) (\eta_x - 1)}{1 - \alpha_x r \pi_x^{(1-\eta_x) \left(\kappa_x - \frac{\eta_x}{(\eta_x-1)} \right)} \mu^z (\mu^\Upsilon)^{\frac{\theta}{1-\theta}} \eta_x}$$

$$u^1 = \frac{x (\tilde{p}_x^*)^{-1-\eta_{xp}}}{1 - \alpha_{xp} r (\pi_x^*)^{(-\eta_{xp}) \left(\kappa_{xp} - \frac{(1+\eta_{xp})}{\eta_{xp}} \right)} \mu^z (\mu^\Upsilon)^{\frac{\theta}{1-\theta}}} \left(\frac{px \ pt}{rer \ pm^* \ tot} \right)$$

$$u^2 = \frac{x (\tilde{p}_x^*)^{-\eta_{xp}}}{1 - \alpha_{xp} r (\pi_x^*)^{(1-\eta_{xp}) \left(\kappa_{xp} - \frac{(1+\eta_{xp})}{\eta_{xp}} \right)} \mu^z (\mu^\Upsilon)^{\frac{\theta}{1-\theta}}} \frac{(\eta_{xp} - 1)}{\eta_{xp}}$$

$$\frac{\gamma}{(1-\gamma)} = \frac{(1-\tau^h) w (1-h)}{mcw (1+\tau^c) \left(1 + \nu^m \left(\frac{R-1}{R} \right) \right) c \left(1 - \frac{\zeta}{\mu^z (\mu^\Upsilon)^{\frac{\theta}{1-\theta}}} \right)}$$

$$\lambda = \left(c - \zeta \frac{c}{\mu^z (\mu^\Upsilon)^{\frac{\theta}{1-\theta}}} \right)^{-1} \frac{(1-\gamma)}{(1+\tau^c) \left(1 + \nu^m \frac{R-1}{R} \right)}$$

$$y = c + i + g + x - d_m \left(1 + \frac{R^f - 1}{R^f} \right)$$

$$\phi = y - wh - r_x^k \frac{k_x}{\mu^z (\mu^\Upsilon)^{\frac{1}{1-\theta}}} - r_n^k \frac{k_n}{\mu^z (\mu^\Upsilon)^{\frac{1}{1-\theta}}}$$

Appendix C

Ramsey Steady State

The Ramsey solution assumes the same parameters from the competitive equilibrium to compute allocations and prices, including those derived implicitly in the steady state computation. The Ramsey equilibrium is characterized by no inflation dispersion across sectors (thus, relative prices remain set at unity) and the Ramsey planner has the domestic nominal interest rates (R) and taxes (τ^h, τ^k, τ^c) as instruments to maximize the objective function, taking as given the values for domestic government expenditure, g , the taxation over profits, τ^ϕ , and the steady state values for the rest of the world.

$$\tau^h = \tau^h \quad \tau^k = \tau^k \quad \tau^c = \tau^c \quad \tau^\phi = \tau^\phi \quad R = R$$

$$R^* = R^* \quad g = g$$

$$\pi^* = \pi_x^* = \pi_m^* = \Delta M^*$$

$$\pi = \pi_n = \pi_m = \pi_t = \pi_x = \frac{\beta R}{\mu^z (\mu^\Upsilon)^{\frac{\theta}{1-\theta}}}$$

$$a_n = a_x = pm = px = pt = pn = pm^* = 1$$

$$mcw = \frac{\varpi}{\varpi-1} \quad r = \frac{1}{R} \quad R^f = \frac{\pi^*}{\pi} R \quad \xi = \left(\frac{R^*}{R^f}\right)^{\frac{1}{\kappa_1}} - 1 \quad \tilde{R} = R$$

$$\tilde{p}_x^* = \left(\frac{1 - \alpha_x (\pi_x^*)^{(\kappa_x p - 1)(1 - \eta_x p)}}{1 - \alpha_x p} \right)^{\frac{1}{1 - \eta_x p}} \quad S_{xp} = \frac{(1 - \alpha_x p) (\tilde{p}_x^*)^{-\eta_x p}}{1 - \alpha_x p (\pi_x^*)^{\eta_x p (1 - \kappa_x p)}}$$

$$\tilde{p}_n = \left(\frac{1 - \alpha_n \pi_n^{(\kappa_n - 1)(1 - \eta_n)}}{1 - \alpha_n} \right)^{\frac{1}{1 - \eta_n}} \quad S_n = \frac{(1 - \alpha_n) \tilde{p}_n^{-\eta_n}}{1 - \alpha_n \pi_n^{\eta_n (1 - \kappa_n)}}$$

$$\tilde{p}_x = \left(\frac{1 - \alpha_x \pi_x^{(\kappa_x - 1)(1 - \eta_x)}}{1 - \alpha_x} \right)^{\frac{1}{1 - \eta_x}} \quad S_x = \frac{(1 - \alpha_x) \tilde{p}_x^{-\eta_x}}{1 - \alpha_x \pi_x^{\eta_x (1 - \kappa_x)}}$$

$$\tilde{p}_m = \left(\frac{1 - \alpha_m \pi_m^{(\kappa_m - 1)(1 - \eta_m)}}{1 - \alpha_m} \right)^{\frac{1}{1 - \eta_m}} \quad S_m = \frac{(1 - \alpha_m) \tilde{p}_m^{-\eta_m}}{1 - \alpha_m \pi_m^{\eta_m (1 - \kappa_m)}}$$

$$rer = \tilde{p}_m \left(1 + \frac{R^f - 1}{R^f} \right)^{-1} \frac{pm}{pt pm^*} \frac{1 - \alpha_m r \pi_m^{(-\eta_m) \left(\kappa_m - \frac{(1 + \eta_m)}{\eta_m} \right)}}{1 - \alpha_m r \pi_m^{(1 - \eta_m) \left(\kappa_m - \frac{(1 + \eta_m)}{\eta_m} \right)}} \frac{\mu^z (\mu^\Upsilon)^{\frac{\theta}{1-\theta}} (\eta_m - 1)}{\mu^z (\mu^\Upsilon)^{\frac{\theta}{1-\theta}} \eta_m}$$

$$mc_x = \tilde{p}_x \frac{1 - \alpha_x r \pi_x^{-\eta_x \left(\kappa_x - \frac{(1 + \eta_x)}{\eta_x} \right)}}{1 - \alpha_x r \pi_x^{(1 - \eta_x) \left(\kappa_x - \frac{\eta_x}{(\eta_x - 1)} \right)}} \frac{\mu^z (\mu^\Upsilon)^{\frac{\theta}{1-\theta}} (\eta_x - 1)}{\mu^z (\mu^\Upsilon)^{\frac{\theta}{1-\theta}} \eta_x}$$

$$mc_n = \tilde{p}_n \frac{1 - \alpha_n r \pi_n^{-\eta_n} \left(\kappa_n - \frac{(1+\eta_n)}{\eta_n} \right) \mu^z (\mu^\Upsilon)^{\frac{\theta}{1-\theta}} (\eta_n - 1)}{1 - \alpha_n r \pi_n^{(1-\eta_n) \left(\kappa_n - \frac{\eta_n}{(\eta_n-1)} \right)} \mu^z (\mu^\Upsilon)^{\frac{\theta}{1-\theta}} \eta_n}$$

$$tot = \frac{\eta_{xp}}{(\tilde{p}_x^*) (\eta_{xp} - 1)} \frac{1 - \alpha_{xp} r (\pi_x^*)^{(1-\eta_{xp})} \left(\kappa_{xp} - \frac{(1+\eta_{xp})}{\eta_{xp}} \right) \mu^z (\mu^\Upsilon)^{\frac{\theta}{1-\theta}}}{1 - \alpha_{xp} r (\pi_x^*)^{(-\eta_{xp})} \left(\kappa_{xp} - \frac{(1+\eta_{xp})}{\eta_{xp}} \right) \mu^z (\mu^\Upsilon)^{\frac{\theta}{1-\theta}}} \left(\frac{px pt}{rer pm^*} \right)$$

$$q_x = 1 \quad q_n = 1$$

$$g_n = (1 - \omega) g \quad g_t = \omega g$$

$$\mu_n = \sqrt{\frac{2}{\theta_2} \left[(1 - \tau^k)^{-1} \left(\frac{\mu^\Upsilon \left(\mu^z (\mu^\Upsilon)^{\frac{\theta}{1-\theta}} \right)}{\beta} - 1 + \delta \right) - \theta_1 + \frac{\theta_2}{2} \right]}$$

$$\mu_x = \sqrt{\frac{2}{\theta_2} \left[(1 - \tau^k)^{-1} \left(\frac{\mu^\Upsilon \left(\mu^z (\mu^\Upsilon)^{\frac{\theta}{1-\theta}} \right)}{\beta} - 1 + \delta \right) - \theta_1 + \frac{\theta_2}{2} \right]}$$

$$r_x^k = \theta_2 (\mu_x - 1) + \theta_1 \quad r_n^k = \theta_2 (\mu_n - 1) + \theta_1$$

$$\frac{k_x}{h_x} = \mu_x \mu^z (\mu^\Upsilon)^{\frac{1}{1-\theta}} \left(\frac{r_x^k}{mc_x \theta} \right)^{\frac{1}{\theta-1}} \quad \frac{k_n}{h_n} = \mu_n \mu^z (\mu^\Upsilon)^{\frac{1}{1-\theta}} \left(\frac{r_n^k}{mc_n \theta} \right)^{\frac{1}{\theta-1}}$$

$$w = mc_n (1 - \theta) \left(\mu^z (\mu^\Upsilon)^{\frac{1}{\theta-1}} \frac{k_n \mu_n}{h_n} \right)^\theta \left(1 + \nu_f^m \frac{(R-1)}{R} \right)^{-1} \quad \frac{h_x}{h_n} = \frac{mc_x Y_x}{mc_n Y_n}$$

In order to calculate the amount of labor used in domestic production, use the non-tradable sector equilibrium condition:

$$s_n (c_n + g_n + i_n) + \chi_n = \left(\frac{1}{\mu^z (\mu^\Upsilon)^{\frac{1}{1-\theta}}} \frac{k_n}{h_n} \right)^\theta h_n$$

$$s_n (1 - \omega) (c + g + i_x^d + i_n^d + a(\mu_n) k_n + a(\mu_x) k_x) + \chi_n = \left(\frac{1}{\mu^z (\mu^\Upsilon)^{\frac{1}{1-\theta}}} \frac{k_n}{h_n} \right)^\theta h_n$$

$$s_n (1 - \omega) \left(c + g + \left(1 - \frac{1 - \delta}{\mu^z (\mu^\Upsilon)^{\frac{1}{1-\theta}}} + a(\mu_n) + a(\mu_x) \right) \left(\frac{k_n}{h_n} h_n + \frac{k_x}{h_x} h_x \right) \right) + \chi_n = \left(\frac{1}{\mu^z (\mu^\Upsilon)^{\frac{1}{1-\theta}}} \frac{k_n}{h_n} \right)^\theta h_n$$

$$s_n (1 - \omega) (c + g) + \chi_n = \left(\frac{1}{\mu^z (\mu^\Upsilon)^{\frac{1}{1-\theta}}} \frac{k_n}{h_n} \right)^\theta h_n - s_n (1 - \omega) \left(1 - \frac{1 - \delta}{\mu^z (\mu^\Upsilon)^{\frac{1}{1-\theta}}} + a(\mu_n) + a(\mu_x) \right) \left(\frac{k_n}{h_n} + \frac{k_x}{h_x} \frac{h_x}{h_n} \right) h_n$$

$$s_n (1 - \omega) \left(\frac{w (1 - \tau^h) R (1 - h)}{mcw (1 + \tau^c) (R + \nu^m (R - 1)) \gamma \left(1 - \frac{\zeta}{\mu^z (\mu^\Upsilon)^{\frac{1}{1-\theta}}} \right)} + g \right) + \chi_n =$$

$$\left(\frac{1}{\mu^z (\mu^\Upsilon)^{\frac{1}{1-\theta}}} \frac{k_n}{h_n} \right)^\theta h_n - s_n (1 - \omega) \left(1 - \frac{1 - \delta}{\mu^z (\mu^\Upsilon)^{\frac{1}{1-\theta}}} + a(\mu_n) + a(\mu_x) \right) \left(\frac{k_n}{h_n} + \frac{k_x}{h_x} \frac{h_x}{h_n} \right) h_n$$

$$\begin{aligned}
& s_n(1-\omega) \left(\frac{w(1-\tau^h)R}{mcw(1+\tau^c)(R+\nu^m(R-1))\gamma\left(1-\frac{\zeta}{\mu^z(\mu^\Upsilon)^{\frac{1}{1-\theta}}}\right)} + g \right) + \chi_n = \\
& \left(\frac{1}{\mu^z(\mu^\Upsilon)^{\frac{1}{1-\theta}}} \frac{k_n}{h_n} \right)^\theta h_n + \\
& s_n(1-\omega) \left[\left(\frac{w(1-\tau^h)R\left(1+\frac{h_x}{h_n}\right)h_n}{mcw(1+\tau^c)(R+\nu^m(R-1))\gamma\left(1-\frac{\zeta}{\mu^z(\mu^\Upsilon)^{\frac{1}{1-\theta}}}\right)} \right) \right. \\
& \quad \left. - \left(1 - \frac{1-\delta}{\mu^z(\mu^\Upsilon)^{\frac{1}{1-\theta}}} + a(\mu_n) + a(\mu_x) \right) \left(\frac{k_n}{h_n} + \frac{k_x h_x}{h_x h_n} \right) h_n \right]
\end{aligned}$$

Set:

$$HN_1 = s_n(1-\omega) \left(\frac{w(1-\tau^h)R}{mcw(1+\tau^c)(R+\nu^m(R-1))\gamma\left(1-\frac{\zeta}{\mu^z(\mu^\Upsilon)^{\frac{1}{1-\theta}}}\right)} + g \right) + \chi_n$$

$$HN_2 = \left(\frac{1}{\mu^z(\mu^\Upsilon)^{\frac{1}{1-\theta}}} \frac{k_n}{h_n} \right)^\theta$$

$$HN_3 = s_n(1-\omega) \left(\frac{w(1-\tau^h)R\left(1+\frac{h_x}{h_n}\right)}{mcw(1+\tau^c)(R+\nu^m(R-1))\gamma\left(1-\frac{\zeta}{\mu^z(\mu^\Upsilon)^{\frac{1}{1-\theta}}}\right)} \right)$$

$$HN_4 = s_n(1-\omega) \left(1 - \frac{1-\delta}{\mu^z(\mu^\Upsilon)^{\frac{1}{1-\theta}}} + a(\mu_n) + a(\mu_x) \right) \left(\frac{k_n}{h_n} + \frac{k_x h_x}{h_x h_n} \right)$$

Then:

$$h_n = \frac{HN_1}{HN_2 + HN_3 - HN_4}$$

$$h = \left(1 + \frac{h_x}{h_n}\right) h_n$$

$$h_x = h - h_n$$

Continuing with the steady state calculation:

$$k_x = \frac{k_x}{h_x} h_x \quad k_n = \frac{k_n}{h_n} h_n \quad \bar{k}_x = k_x / \mu_x \quad \bar{k}_n = k_n / \mu_n$$

$$i_x^d = \left(1 - \frac{1-\delta}{\mu^z (\mu^\Upsilon)^{\frac{1}{1-\theta}}}\right) \frac{k_x}{h_x} h_x \quad i_n^d = \left(1 - \frac{1-\delta}{\mu^z (\mu^\Upsilon)^{\frac{1}{1-\theta}}}\right) \frac{k_n}{h_n} h_n$$

$$i = i_x^d + i_n^d + a(\mu_n) \bar{k}_n + a(\mu_x) \bar{k}_x$$

$$i_n = (1 - \omega) i \quad i_r = \omega i \quad i_x = (1 - \varkappa) i_r \quad i_m = \varkappa i_r$$

$$c_n = \left(\left(\frac{1}{\mu^z (\mu^\Upsilon)^{\frac{1}{1-\theta}}} \frac{k_n}{h_n} \right)^\theta h_n - \chi_n \right) \frac{1}{s_n} - g_n - i_n$$

$$c = \frac{c_n}{(1-\omega)} \quad c_t = \omega c \quad c_x = (1 - \varkappa) c_t \quad c_m = \varkappa c_t$$

$$d_{xp} = \left(\left(\frac{k_x}{\mu^z (\mu^\Upsilon)^{\frac{1}{1-\theta}}} \right)^\theta h_x^{1-\theta} - \chi_x \right) \frac{1}{s_x} - c_x - g_t - i_x$$

$$x = (\chi_{xp} - d_{xp}) / s_{xp}$$

$$y^* = x \text{ tot}^{\eta^*} \quad d_m = \chi_m - s_m \left(c_m + \frac{pt}{pm} i_m \right)$$

$$ib = \frac{x - d_m \left(1 + \frac{(R^f - 1)}{R^f} \right)}{rer} \left(\frac{R^f}{\mu^z (\mu^\Upsilon)^{\frac{\theta}{1-\theta}}} - \pi^* \right)^{-1}$$

$$y = c + i + g + x - d_m \left(1 + \frac{R^f - 1}{R^f} \right)$$

$$m = \nu^m (1 + \tau^c) c$$

$$\phi = y - wh - r_x^k \frac{k_x}{\mu^z (\mu^\Upsilon)^{\frac{1}{1-\theta}}} - r_n^k \frac{k_n}{\mu^z (\mu^\Upsilon)^{\frac{1}{1-\theta}}}$$

$$b_g = (l - m) R^{-1}$$

$$\lambda = \left(c - \zeta \frac{c}{\mu^z (\mu^\Upsilon)^{\frac{\theta}{1-\theta}}} \right)^{-1} \frac{(1 - \gamma)}{(1 + \tau^c) \left(1 + \nu^m \frac{R-1}{R} \right)}$$

$$x^1 = \frac{\tilde{p}_n^{-1-\eta_n} \left(c_n + g_n + \frac{i_n}{pn} \right) m c_n}{1 - \alpha_n r \pi_n^{-\eta_n \left(\kappa_n - \frac{(1+\eta_n)}{\eta_n} \right)} \mu^z (\mu^\Upsilon)^{\frac{\theta}{1-\theta}}}$$

$$x^2 = \frac{\tilde{p}_n^{-\eta_n} \left(c_n + g_n + \frac{i_n}{pm} \right)}{1 - \alpha_n r \pi_n^{(1-\eta_n)\left(\kappa_n - \frac{\eta_n}{(\eta_n-1)}\right)} \mu^z (\mu^\Upsilon)^{\frac{\theta}{1-\theta}}} \frac{(\eta_n - 1)}{\eta_n}$$

$$y^1 = \frac{\tilde{p}_m^{-1-\eta_m} \left(c_m + i_m \frac{pt}{pm} \right) r e r \frac{pt pm^*}{pm} \left(1 + \frac{R^f - 1}{R^f} \right)}{1 - \alpha_m r \pi_m^{(-\eta_m)\left(\kappa_m - \frac{(1+\eta_m)}{\eta_m}\right)} \mu^z (\mu^\Upsilon)^{\frac{\theta}{1-\theta}}}$$

$$y_t^2 = \frac{\tilde{p}_m^{-\eta_m} \left(c_m + i_m \frac{pt}{pm} \right)}{1 - \alpha_m r \pi_m^{(1-\eta_m)\left(\kappa_m - \frac{(1+\eta_m)}{\eta_m}\right)} \mu^z (\mu^\Upsilon)^{\frac{\theta}{1-\theta}}} \frac{(\eta_m - 1)}{\eta_m}$$

$$z^1 = \frac{\tilde{p}_x^{-1-\eta_x} \left(c_x + g_t + \frac{pt}{px} i_x + d_{xp} \right) m c_x}{1 - \alpha_x r \pi_x^{-\eta_x \left(\kappa_x - \frac{(1+\eta_x)}{\eta_x} \right)} \mu^z (\mu^\Upsilon)^{\frac{\theta}{1-\theta}}}$$

$$z^2 = \frac{\tilde{p}_x^{-\eta_x} \left(c_x + g_t + \frac{pt}{px} i_x + d_{xp} \right)}{1 - \alpha_x r \pi_x^{(1-\eta_x)\left(\kappa_x - \frac{\eta_x}{(\eta_x-1)}\right)} \mu^z (\mu^\Upsilon)^{\frac{\theta}{1-\theta}}} \frac{(\eta_x - 1)}{\eta_x}$$

$$u^1 = \frac{x \left(\tilde{p}_x^* \right)^{-1-\eta_{xp}}}{1 - \alpha_{xp} r \left(\pi_x^* \right)^{(-\eta_{xp})\left(\kappa_{xp} - \frac{(1+\eta_{xp})}{\eta_{xp}}\right)} \mu^z (\mu^\Upsilon)^{\frac{\theta}{1-\theta}}} \left(\frac{px pt}{rer pm^* tot} \right)$$

$$u^2 = \frac{x \left(\tilde{p}_x^* \right)^{-\eta_{xp}}}{1 - \alpha_{xp} r \left(\pi_x^* \right)^{(1-\eta_{xp})\left(\kappa_{xp} - \frac{(1+\eta_{xp})}{\eta_{xp}}\right)} \mu^z (\mu^\Upsilon)^{\frac{\theta}{1-\theta}}} \frac{(\eta_{xp} - 1)}{\eta_{xp}}$$

$$l = \left\{ R \left[g - \tau^h wh - \tau^k \left((r_n^k - a(\mu_n)) k_n + (r_x^k - a(\mu_x)) k_x \right) - \tau^c c - \tau^\phi \phi \right] - (R - 1) m \right\} \left(1 - \frac{R}{\pi \mu^z (\mu^r)^{\frac{\theta}{1-\theta}}} \right)^{-1}$$

Appendix D

Welfare Cost Measurement

Following Schmitt-Grohé and Uribe (2006 and 2007)[?][68], the welfare cost λ_c of adopting the alternative policy regime i instead of the Ramsey monetary and fiscal policy r is measured in terms of the share of consumption the households give up in order to be indifferent between the two policy regimes:

$$\mathcal{U}_i^c = E_0 \sum_{t=0}^{\infty} \beta^t U_t \left(\left(c_t^i - \frac{\zeta c_{t-1}^i}{\mu_t^z (\mu_t^r)^{\frac{\theta}{1-\theta}}} \right), h_t^i \right) =$$
$$E_0 \sum_{t=0}^{\infty} \beta^t U_t \left((1 - \lambda_c) \left(c_t^r - \frac{\zeta c_{t-1}^r}{\mu_t^z (\mu_t^r)^{\frac{\theta}{1-\theta}}} \right), h_t^r \right)$$

Using the period utility function of the households, the welfare cost λ_c is given by:

$$\mathcal{U}_i^c = E_0 \sum_{t=0}^{\infty} \beta^t U_t \left((1 - \lambda_c) \left(c_t^r - \frac{\zeta c_{t-1}^r}{\mu_t^z (\mu_t^r)^{\frac{\theta}{1-\theta}}} \right), h_t^r \right)$$

Plug the period utility function for period zero and decompose the infinite sum:

$$\begin{aligned} \mathcal{U}_i^c = & (1 - \gamma) \log \left((1 - \lambda_c) c_0^r - \frac{\zeta c_{-1}^r}{\mu_0^z (\mu_0^r)^{\frac{\theta}{1-\theta}}} \right) \\ & + \gamma \log (1 - h_0^r) + E_0 \sum_{t=1}^{\infty} \beta^t U_t \left((1 - \lambda_c) \left(c_t^r - \frac{\zeta c_{t-1}^r}{\mu_t^z (\mu_t^r)^{\frac{\theta}{1-\theta}}} \right), h_t^r \right) \end{aligned}$$

Sum and subtract $(1 - \gamma) \log \left(c_0^r - \frac{\zeta c_{-1}^r}{\mu_0^z (\mu_0^r)^{\frac{\theta}{1-\theta}}} \right)$ in the right-side of the equation:

$$\begin{aligned} \mathcal{U}_i^c = & (1 - \gamma) \log \left((1 - \lambda_c) c_0^r - \frac{\zeta c_{-1}^r}{\mu_0^z (\mu_0^r)^{\frac{\theta}{1-\theta}}} \right) \\ & - (1 - \gamma) \log \left(c_0^r - \frac{\zeta c_{-1}^r}{\mu_0^z (\mu_0^r)^{\frac{\theta}{1-\theta}}} \right) + (1 - \gamma) \log \left(c_0^r - \frac{\zeta c_{-1}^r}{\mu_0^z (\mu_0^r)^{\frac{\theta}{1-\theta}}} \right) \\ & + \gamma \log (1 - h_0^r) + E_0 \sum_{t=1}^{\infty} \beta^t U_t \left((1 - \lambda_c) \left(c_t^r - \frac{\zeta c_{t-1}^r}{\mu_t^z (\mu_t^r)^{\frac{\theta}{1-\theta}}} \right), h_t^r \right) \end{aligned}$$

Decompose, from the infinite sum, the term of the welfare cost of the alternative policy, using the fact that the utility function is log-linear in consumption:

$$\begin{aligned} \mathcal{U}_i^c = & (1 - \gamma) \log \left((1 - \lambda_c) c_0^r - \frac{\zeta c_{-1}^r}{\mu_0^z (\mu_0^r)^{\frac{\theta}{1-\theta}}} \right) \\ & - (1 - \gamma) \log \left(c_0^r - \frac{\zeta c_{-1}^r}{\mu_0^z (\mu_0^r)^{\frac{\theta}{1-\theta}}} \right) + (1 - \gamma) \log \left(c_0^r - \frac{\zeta c_{-1}^r}{\mu_0^z (\mu_0^r)^{\frac{\theta}{1-\theta}}} \right) \\ & + \gamma \log (1 - h_0^r) + E_0 \sum_{t=1}^{\infty} \beta^t (1 - \gamma) \log (1 - \lambda_c) + E_0 \sum_{t=1}^{\infty} \beta^t U_t \left(\left(c_t^r - \frac{\zeta c_{t-1}^r}{\mu_t^z (\mu_t^r)^{\frac{\theta}{1-\theta}}} \right), h_t^r \right) \end{aligned}$$

Note that the third, fourth and last term of the right-hand side equal the welfare

of the Ramsey policy, \mathcal{U}_r^c :

$$\begin{aligned} \mathcal{U}_i^c &= (1 - \gamma) \log \left((1 - \lambda_c) c_0^r - \frac{\zeta c_{-1}^r}{\mu_0^z (\mu_0^r)^{\frac{\theta}{1-\theta}}} \right) \\ &\quad - (1 - \gamma) \log \left(c_0^r - \frac{\zeta c_{-1}^r}{\mu_0^z (\mu_0^r)^{\frac{\theta}{1-\theta}}} \right) + \mathcal{U}_r^c + E_0 \sum_{t=1}^{\infty} \beta^t (1 - \gamma) \log (1 - \lambda_c) \end{aligned}$$

Organizing the terms:

$$\frac{\mathcal{U}_i^c - \mathcal{U}_r^c}{(1 - \gamma)} = \log \left((1 - \lambda_c) c_0^r - \frac{\zeta c_{-1}^r}{\mu_0^z (\mu_0^r)^{\frac{\theta}{1-\theta}}} \right) - \log \left(c_0^r - \frac{\zeta c_{-1}^r}{\mu_0^z (\mu_0^r)^{\frac{\theta}{1-\theta}}} \right) + \frac{\beta}{1 - \beta} \log (1 - \lambda_c)$$

Now, approximate the welfare cost λ_c by a second-order Taylor expansion around the vector of disturbances σ to obtain:

$$\lambda_c \approx \bar{\lambda}_c + \lambda_{c,\sigma} \sigma + \lambda_{c,\sigma\sigma} \frac{\sigma^2}{2}$$

Following the results in Schmitt-Grohé and Uribe (2005b)[67][67], note that $\bar{\lambda}_c$ vanishes, because all the policies considered here do not alter the steady state of the economy, and $\lambda_{c,\sigma} = 0$. The second total derivative of the equation provides the welfare measure:

$$\begin{aligned} \frac{\mathcal{U}_{i,\sigma\sigma}^c - \mathcal{U}_{r,\sigma\sigma}^c}{(1 - \gamma)} &= - \left(\frac{\mu_0}{\mu_0 - \zeta} + \frac{\beta}{1 - \beta} \right) \lambda_{c,\sigma\sigma} \\ \implies \lambda_{c,\sigma\sigma} &= \frac{\mathcal{U}_{r,\sigma\sigma}^c - \mathcal{U}_{i,\sigma\sigma}^c}{(1 - \gamma) \left(\frac{\mu_0}{\mu_0 - \zeta} + \frac{\beta}{1 - \beta} \right)} \\ \implies \lambda_c &\approx \left(\frac{\mathcal{U}_{r,\sigma\sigma}^c - \mathcal{U}_{i,\sigma\sigma}^c}{(1 - \gamma) \left(\frac{\mu_0}{\mu_0 - \zeta} + \frac{\beta}{1 - \beta} \right)} \right) \frac{\sigma^2}{2} \end{aligned}$$

Appendix E

Data definitions and sources

Definitions and sources of data in table 1 for steady-state values:

- π^o : sample average of annualized CPI inflation or long run inflation target, depending on the information available for each country;
- μ^z : sample average of annualized real GDP growth;
- R^*/R^f : sample average of EMBI+ for countries where data is available
- G/Y : sample average of government expenditure-output ratio from National Accounts
- B/Y : 1991-2008 average of Net Financial Liabilities as a proportion of GDP according to OECD definition
- M/Y : sample average of monetary base-output ratio, with data from National Accounts and the country's Monetary Authority
- IMP/Y : sample average of imports-output ratio from National Accounts

Sources of data to compute steady-state values:

- Australia: Reserve Bank of Australia (<http://www.rba.gov.au>) and Australian Bureau of Statistics (<http://www.abs.gov.au>)
- Brazil: Central Bank of Brazil (<http://www.bcb.gov.br>) and Brazilian Institute of Geography and Statistics (<http://www.ibge.gov.br>)

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