

THE IMPACT OF THE INFINITE MATHEMATICS PROJECT ON TEACHERS'
KNOWLEDGE AND TEACHING PRACTICE: A CASE STUDY OF A TITLE IIB MSP
PROFESSIONAL DEVELOPMENT INITIATIVE

by

BARBARA J. SPONSEL

B.A., Wichita State University, 1987

M.S., Wichita State University, 1993

AN ABSTRACT OF A DISSERTATION

submitted in partial fulfillment of the requirements for the degree

DOCTOR OF PHILOSOPHY

Department of Curriculum and Instruction
College of Education

KANSAS STATE UNIVERSITY
Manhattan, Kansas

2010

Abstract

Ongoing, effective professional development is viewed as an essential mechanism for eliciting change in teachers' knowledge and practice in support of enacting the vision of NCTM's *Principles and Standards of School Mathematics*. This case study of the Infinite Mathematics Project, a Title IIB MSP professional development initiative, seeks to provide a qualitative examination of the characteristics and strategies used in the project and their impact on teacher learning and practice. The project embodied many features and strategies of effective professional development such as: mathematics content focus; sustained over time; reform activities (e.g., lesson study, teacher collaboration); active learning opportunities (e.g., implementing an action plan; developing differentiated instruction activities for a mathematics classroom); coherence with NCTM and state standards; and collective participation by IHE facilitators and participant K-12 teachers from partner districts. The findings reveal teachers gained both content knowledge (knowledge *about* mathematics, substantive knowledge of mathematics, pedagogical content knowledge, and curricular knowledge) and pedagogical knowledge (knowledge about strategies for differentiating instruction in a mathematics classroom, for supporting students' reading in the content area, for fostering the development of number sense, for implementing standards-based teaching, and for critically analyzing teaching). The study also provides some evidence that the project had an impact on teaching practice. In addition, an implication of the study suggests the positive impact of Title IIB MSP partnership requirements.

THE IMPACT OF THE INFINITE MATHEMATICS PROJECT ON TEACHERS'
KNOWLEDGE AND TEACHING PRACTICE: A CASE STUDY OF A TITLE IIB MSP
PROFESSIONAL DEVELOPMENT INITIATIVE

by

BARBARA J. SPONSEL

B.A., Wichita State University, 1987
M.S., Wichita State Univeristy, 1993

A DISSERTATION

submitted in partial fulfillment of the requirements for the degree

DOCTOR OF PHILOSOPHY

Department of Curriculum and Instruction
College of Education

KANSAS STATE UNIVERSITY
Manhattan, Kansas

2010

Approved by:

Co-Major Professor
Dr. David Allen

Approved by:

Co-Major Professor
Dr. Gail Shroyer

Copyright

BARBARA J. SPONSEL

2010

Abstract

Ongoing, effective professional development is viewed as an essential mechanism for eliciting change in teachers' knowledge and practice in support of enacting the vision of NCTM's *Principles and Standards of School Mathematics*. This case study of the Infinite Mathematics Project, a Title IIB MSP professional development initiative, seeks to provide a qualitative examination of the characteristics and strategies used in the project and their impact on teacher learning and practice. The project embodied many features and strategies of effective professional development such as: mathematics content focus; sustained over time; reform activities (e.g., lesson study, teacher collaboration); active learning opportunities (e.g., implementing an action plan; developing differentiated instruction activities for a mathematics classroom); coherence with NCTM and state standards; and collective participation by IHE facilitators and participant K-12 teachers from partner districts. The findings reveal teachers gained both content knowledge (knowledge *about* mathematics, substantive knowledge of mathematics, pedagogical content knowledge, and curricular knowledge) and pedagogical knowledge (knowledge about strategies for differentiating instruction in a mathematics classroom, for supporting students' reading in the content area, for fostering the development of number sense, for implementing standards-based teaching, and for critically analyzing teaching). The study also provides some evidence that the project had an impact on teaching practice. In addition, an implication of the study suggests the positive impact of Title IIB MSP partnership requirements.

Table of Contents

List of Figures	x
List of Tables	xi
Acknowledgements	xii
Dedication	xiii
CHAPTER 1 - Introduction	1
Preface	1
Overview of the Issues	2
Purpose of the Study	13
Research Questions	13
Research Design	14
Significance of the Study	15
Limitations of the Study	16
Definition of Terms	17
Summary	21
CHAPTER 2 - Literature Review	22
Effective Mathematics Teachers and Teaching	22
U.S. Mathematics Education Reform	27
Description and Challenges	27
Demands of Reform for Teachers	33
Elementary and Middle School Mathematics Content and Teachers	37
Elementary	37
Middle School	39
Teachers' Content Knowledge	40
Conceptualizing and Measuring Teachers' Content Knowledge	41
Development of Content Knowledge	48
Teaching Practice	54
Generalities about Teaching Practice	54
Traditional and Standards-based Teaching Practice	55

Obtaining Information about Teaching Practice	60
Professional Development	62
Increasing Awareness of the Importance of Professional Development	62
Elements of Effective Professional Development	65
Goals for the Professional Development of Teachers of Mathematics and Strategies Being Used to Address the Goals	72
Lesson Study	82
Differentiated Instruction	89
Reading in the Content Area and Using Math-Related Literature	99
Summary	102
CHAPTER 3 - Methodology	103
Overview	103
Setting	104
The University	104
Mathematics and Science Partnership (MSP) Program	105
The Project	105
Project Leadership	107
The Project Participants	107
Mini-Case Study Individuals	108
Researcher	110
Data Collection	114
Data Analysis	117
Validity and Reliability	120
Summary	123
CHAPTER 4 - Results	124
Introduction	124
The Effectiveness of the IMP Professional Development Model	124
Themes	133
Teachers Learned Mathematical Content	135
Theme: Teachers gained knowledge “about” mathematics (a component of subject matter knowledge)	137

Theme: Teachers gained substantive knowledge of mathematics (a component of subject matter knowledge)	153
Theme: Teachers gained mathematical curricular knowledge.....	165
Theme: Teachers gained pedagogical content knowledge.....	172
Teachers Gained Pedagogical Knowledge.....	176
Theme: Teachers learned about differentiating instruction in a mathematics classroom	177
Theme: Teachers learned about strategies for supporting students’ reading in the content area.....	191
Theme: Teachers learned pedagogical strategies for supporting the development of number sense.....	198
Theme: Teachers learned about lesson study and more general skills for analyzing practice.....	202
Theme: Teachers learned about standards-based instruction in mathematics	212
Impact of IMP Year 2 on Participants’ Teaching Practice	222
Theme: Short-term impact on teaching practice.....	223
Summary.....	227
CHAPTER 5 - Conclusions	228
Introduction.....	228
Summary of the Study Design	228
Summary of Results Related to Research Questions.....	229
Discussion.....	238
Discussion of results related to the impact of IMP on participants’ mathematical content knowledge.....	239
Discussion of results related to the impact of IMP on participants’ pedagogical knowledge.....	243
Discussion of results related to the impact of IMP on participants’ teaching practice...	248
Implications	251
Recommendations for Future Research.....	254
Summary.....	257
Appendix A - Informed Consent Form.....	260
Appendix B - Homework/Session Reflections	262

Appendix C - OEIE Summary of Data Analysis Pre-Post Content Test	263
Appendix D - OEIE Summary of Data Analysis Pre-Post Survey	269
Appendix E - Pre-Summer Institute Interview Protocol.....	275
Appendix F - Post-Summer Institute Interview Protocol	280
Appendix G - Pre-Classroom Observation Interview Protocol	284
Appendix H - Classroom Observation Protocol	286
Appendix I - Post-Classroom Observation Interview Protocol	292
References.....	295

List of Figures

Figure 4.1: Effective Professional Development	125
Figure 4.2: Themes	134

List of Tables

Table 3.1: Mini-Case Sampling Stratification	110
--	-----

Acknowledgements

I would like to thank the members of my doctoral committee: Dr. David Allen who provided guidance and encouragement throughout the whole process, who made deadlines seem attainable, and who gave me the opportunity to study his professional development project; Dr. Gail Shroyer whose wealth of experience provided sage guidance for the qualitative study; Dr. Jacqueline Spears whose editing contributions were invaluable; Dr. Thomas Vontz who provided valuable insights from a view outside of the project; and Dr. Andrew Bennett who agreed to serve on the committee at the last minute and who offered an important perspective.

I am grateful to the teachers and facilitators who participated in the Infinite Mathematics Project. Your commitment to education is inspiring. Thank you for generously allowing me to listen, observe, and be a part of your learning community. Furthermore, I would like to thank the mini-case participants for graciously sharing their time during interviews and observations.

I would also like to thank many colleagues and administrators at Newman University who provided advice and support along the way. Special thanks to Dr. Lori Steiner and Dr. Max Frazier who gave insight about what to expect throughout the process.

Finally, to the many members of my immediate and extended family who have provided encouragement. Your love, understanding, and assistance supported me throughout the process. Thank you!

Dedication

To my parents William and Patricia Bergman

You modeled the importance of faith, love, and perseverance.

Thank you for your love and support.

To my husband John and my children Kirk, Kelsey, Jessica, John Paul, and Christian

The many hours spent on the dissertation meant fewer hours spent with you.

Thank you for your love and generosity.

CHAPTER 1 - Introduction

Preface

The United States has always balanced precariously on the twin values of equity and excellence. As a people, we believe that birth in a log cabin should not be a barrier to the boardroom or the Oval Office and that all citizens should have access to opportunities that will help them realize their potential.

Similarly, we cling to a vision of the United States as representing the best. We stand for the fastest cars, the tallest buildings, the finest medical care, and the most innovative technology. We are committed to excellence....

To lose either equity or excellence as a guiding value would be to lose our identity. To maintain both, however, is a balancing act of the highest order. And the challenge is perhaps greatest in the schools that shape young people to be good stewards of these values. (Tomlinson, 2003, pp. 9-10)

Affirming the twin principles of excellence and equity, the National Council of Teachers of Mathematics (NCTM) described itself as an professional organization “committed to excellence in mathematics teaching and learning for all students” (2000, p. ix). In congruence with this commitment, NCTM has proposed Pre-K—12 education standards. However, standards alone will not improve mathematics education. Teachers are a critical factor related to efforts to improve education.

The escalation of the commitment to providing quality education to all students, at a time when the student population grows more diverse, has led to higher demands on teachers, who in turn need the support provided through appropriate professional development (Sowder, 2007, p. 159).

The U.S. Department of Education has made significant investments in professional development, particularly for mathematics and science teachers. The Infinite Mathematics Project (IMP) at Kansas State University was a federally-funded professional development program aimed at enhancing mathematics teachers’ content knowledge and teaching skills in order to improve student achievement. This study seeks to examine the impact of the IMP

initiative with respect to participant teachers' content knowledge, pedagogical knowledge, and teaching practice.

Overview of the Issues

Although NCTM currently expresses commitment to both excellence and equity (2000), a historical perspective reveals tension between balancing support for both values. The values are intertwined with the view that mathematics education has “a dual function: to prepare students to be mathematically functional as citizens of their societies—arguably provided equitably for all—and to prepare some students to be the future professionals in careers in which mathematics is fundamental” (Bishop & Forgasz, 2007, p. 1152).

During the twentieth century, the United States was internationally recognized as a leader in science and technology; furthermore, scientific and technological excellence undergirded the U.S.'s economic and military success (Haseltine, 2007). Despite being a recognized leader, the U.S. felt threatened by the Soviet Union in the middle of the twentieth century. “The military threat of Soviet space science and technology prompted a variety of political, business, and social groups to urge the critical examination of American mathematical, scientific, and technical education” (Fey & Graeber, 2003, p. 521). Although some reform activities had begun prior to 1957, the Soviet launching of Sputnik was a significant stimulus for U.S. actions focused on improving mathematics and science education. Prominent psychologists had been making recommendations for reformed teaching practices emphasizing student engagement in exploration and discovery learning. The professional mathematics community made recommendations for new mathematics topics. The National Science Foundation (NSF) funded several large-scale curriculum reforms in science and mathematics during the 50s and 60s (Elmore, 1996). The “new math” movement sought to modernize mathematics curriculum to reflect advancing technology and to move focus away from calculation to understanding of concepts (Walmsley, 2003). However, Becker and Perl (2003) noted that the “basic task of public education in the late 1950s and early 1960s shifted from providing education for *all* children to the creation of a technocratic elite to make the United States competitive with the Soviet Union” (p. 1093). Although a few initiatives addressed curriculum geared toward low-achieving students or minority populations, “the major curriculum reform triggered by *Sputnik* mainly addressed the education of the mathematical elite” (p. 1095).

In response to the challenging initiatives, “curriculum developers and teacher educators worked to transform the new content and pedagogical theories into working school mathematics programs and to enhance the understanding and skill of teachers at all levels” (Fey & Graeber, 2003, p. 522). Nonetheless, many teachers were expected to use the new math curricula with little training. Overall, despite significant funds being directed toward curriculum development and mathematics teacher education, relatively insignificant change occurred in teaching practice during this period (Elmore, 1996).

Much of U.S. mathematics education reform has been based on extreme positions of a swinging pendulum between constructivism and behaviorism (Walmsley, 2003). After the “new mathematics” movement, the pendulum swung toward drill and practice with the “back-to-basics” movement. In reaction to this movement, NCTM began to take a more active political role and made public recommendations for reform. In 1980, NCTM published *An Agenda for Action* recommending that “problem solving be the focus of school mathematics” (p. 1). Furthermore, Becker and Perl (2003) noted *An Agenda for Action* was the first of a string of reports from the last two decades of the twentieth century making equity a more visible goal in mathematics education. New concerns about international competitiveness were raised by publications such as *A Nation at Risk* (National Commission on Excellence in Education, 1983). Pessimism increased when results from the Second International Mathematics Study (SIMS) were released in 1987 indicating that U.S. eighth and twelfth graders had not scored significantly above the international level for any mathematics topic and had scored well below the international average for several topics (Senk & Thompson, 2003).

By 1989, NCTM provided leadership by outlining national mathematics curriculum standards in its landmark document *Curriculum and Evaluation Standards for School Mathematics* (1989). The initial document was supplemented with supporting documents including *Professional Standards for Teaching Mathematics* and *Assessment Standards for School Mathematics* (1991, 1995), and then refined and revised into the most recent NCTM standards document *Principles and Standards for School Mathematics* (2000). NCTM “has remained committed to the view that standards can play a leading role in guiding the improvement of mathematics education” (2000, p. 4). More recently, NCTM has continued to expand upon *Standards*. For example, NCTM has articulated important mathematical topics, “curriculum focal points,” for grade levels in order to enhance curricular coherence (2006, 2009).

Current international comparisons suggest the U.S. is struggling to maintain its leadership in science and technology in the 21st century. For example, although the U.S. is still internationally dominant in research and development expenditures, other countries are increasing their percentage of R&D expenditures (National Mathematics Advisory Panel, 2008; Ruvinsky, 2007). As another example, while the number of science and engineering undergraduates in China has doubled over the past ten years (Ruvinsky, 2007), the number of new U.S. university students choosing to pursue careers in engineering and science is barely holding steady (Haseltine, 2007).

Concerns about the U.S.'s ability to maintain leadership in science and technology are further fueled by reports on the status of U.S. mathematics and science education. International comparisons of mathematics and science achievement exist in reports from the Program for International Student Assessment (PISA) and the Trends in International Mathematics and Science Study (TIMSS). Findings from the 2003 TIMSS (Gonzales et al., 2004) indicate that although the average mathematics performance of U.S. fourth-grade students exceeded the international average and while U.S. fourth-graders outperformed their peers in 13 out of 24 countries, U.S. students performed at a lower level than their peers in eleven countries. For eighth-graders, the average score of U.S. students exceeded the international average, and U.S. students outperformed their peers in 25 of 44 participating countries. On the other hand, U.S. 8th-graders were outperformed by students in nine countries including five Asian countries and four European. With regard to science achievement, average performance for U.S. 8th-graders exceeded the international average; however, the average was boosted by stronger achievement in the life sciences than in chemistry or physics (Gonzales et al., 2004; Wenglinsky & Silverstein, 2007). TIMSS 2007 mathematics results indicate U.S. fourth-grade students scored higher on average than 23 out of 35 countries, lower than 8 countries (all in Asia and Europe), and not measurably different from 4 countries (Gonzales et al., 2008). For eighth-grade, the average U.S. mathematics score was higher than 37 out of 47 countries, lower than 5 countries (all in Asia), and not measurably different from 5 countries. Although rankings appear to have improved over the years, researchers (e.g., Kilpatrick, 2009; Schneider, 2008, December 9) have cautioned that improved standings in TIMSS 2007 may be ambiguous. Kilpatrick (2009) pointed out that some European countries (e.g., France, Spain) participated in TIMSS 1995, but no longer participate. An increasing variety of other countries, such as Algeria and Ghana, started

participating more recently. In addition, Schneider (2008, December 9) noted that the inclusion of many less-developed countries (e.g., South Africa) drives down the international average and thus enhances our relative performance.

In a different comparison, the Organization for Economic Cooperation and Development (OECD), composed of 30 member countries, sponsors PISA assessments to measure 15-year-olds' performance in reading, science, and mathematics (Baldi, Jin, Skemer, Green, & Herget, 2007; Schneider, 2008, December 9). Schneider (2008, December 9) argued that the 30 countries in the OECD represent our trading partners and competitors; thus, PISA provides a more suitable international comparison. Results from 2006 testing indicate the average U.S. mathematics literacy score and the average U.S. science literacy score were both lower than the OECD average score (Baldi et al., 2007).

A new report suggests that the U.S. will “relinquish its leadership in the 21st century” without “substantial and sustained changes to its educational system” (National Mathematics Advisory Panel, 2008, p. xi). However, substantial and sustained changes in U.S. education have not been evident in the history of U.S. mathematics education. Stigler and Hiebert (1999) said that U.S. teaching practices have changed very little despite years of reform. In contrast, Japan has experienced marked change in teaching practice over the past fifty years in large part due to a system of professional development supporting gradual, incremental improvements.

The United States' current mathematics reform movement emphasizes constructivism. Borasi and Fonzi (2002) described several constructivist assumptions about knowledge, learning, and teaching that characterize current mathematics reform efforts. First, knowledge is socially constructed and negotiated within a community of practice. Second, learning is a generative process whereby personal sense-making builds on prior knowledge and is influenced by context. Finally, teaching involves facilitating student learning by creating engaging situations conducive to inquiry and discussion and by supporting students as they attempt to solve problems and understand mathematical concepts.

Similarly, but described as broader than constructivism, the National Research Council (NRC) (2000) suggested theories about learning from a cognitive science perspective. First, students come to classrooms with pre-existing notions of how the world works. Teachers should draw out students' preconceptions in order to build on them, or challenge and possibly change them. In order to challenge students' preconceptions, teachers should “provide opportunities for

students to experience discrepant events that allow them to come to terms with the shortcomings in their everyday models” (NRC, 2005, p. 571). In addition, teachers should provide students with narrative accounts of the discovery or change of knowledge for a particular discipline. Second, students must have both factual knowledge and conceptual understanding in an organized framework (NRC, 2000). Rather than teaching a great number of topics in a cursory manner, teachers should teach fewer concepts in greater depth. Teachers should have both knowledge about the nature of the discipline (e.g., what it means to engage in doing mathematics) and knowledge of central concepts and relationships of a discipline (NRC, 2005). Finally, moving away from the earlier *tabula rasa* view of a passive mind as a blank slate on which to gradually record experience, a large body of research now suggests the active role of the learner (NRC, 2000, 2001). The brain does not just passively receive information; to effectively learn, the brain needs to actively engage in processing information (Silberman, 1996; Sousa, 2006). Students are less likely to retain information when the teaching method involves lecture or reading; students are more likely to retain information when instructional methods allow for discussion, practice by doing, and teaching others (Sousa, 2006). In order to process and store information, a person needs to reflect internally, reflect externally through discussion, and do something with the information (Silberman, 1996). With regard to external reflection, “if we discuss information with others and if we are invited to ask questions about it, our brains can do a better job of learning” (p. 3). The National Research Council (2000) recommended the teaching of metacognitive skills across disciplines in order that students might “learn to take control of their own learning by defining learning goals and monitoring their progress in achieving them” (p. 18). Teacher and student-guided classroom discussion (at times whole group and at times small group) is a pedagogical approach that supports students’ development of metacognitive skills (NRC, 2005). More specifically, teachers can cultivate metacognitive thinking by utilizing questions that encourage learners “to reflect on their learning, consider transfer possibilities, self-assess their performance, and set goals” (Tomlinson & McTighe, 2006, p. 79).

Based on constructivist assumptions, *Standards* documents describe classrooms in which *all* students are engaged in exploring, conjecturing, reasoning, communicating, making connections, interpreting representations, and problem-solving (NCTM, 1989, 2000). Current reform attempts radical changes in core dimensions of teaching and places high demands on

teachers' content knowledge (Floden, 1997). Teachers need to be able to select worthwhile tasks to engage students, to orchestrate classroom discourse where justification and sense-making are valued, to help students use technology to explore conjectures and examine multiple representations, and to help students seek connections to prior and developing knowledge (NCTM, 1991, 2000). Teachers need mathematical knowledge that is deep and flexible. Teachers need pedagogical knowledge in order to skillfully choose from a range of different teaching techniques and assessment strategies. Teachers need to understand the needs and strengths of students who come from diverse backgrounds, and teachers need to be able to effectively accommodate differences among students (NCTM, 2000).

Reformers have come to realize that new curriculum and high-stakes testing will not necessarily lead to reformed teaching practice (Wilson & Berne, 1999). Instead, many scholars, educators, and policymakers view ongoing professional development as an essential mechanism for eliciting change in teachers' knowledge, beliefs, and practice in support of school improvement (e.g., Desimone, Smith, & Ueno, 2006; Elmore, 2002). With mathematics education reform initiatives, teacher learning and professional development have received much attention. Consensus has been building about characteristics of effective professional development. For example, based on a literature review of professional development for a variety of subject areas, Elmore (2002) described a consensus view of characteristics of effective professional development including focuses on performance goals, building teachers' content knowledge and pedagogical knowledge, using theories of learning, using group settings, and moving learning closer to daily practice. Also based on a literature review of studies on in-service professional development, Richardson and Placier (2001) found "long-term, collaborative, and inquiry-oriented programs" (p. 921) appeared the most effective in eliciting change in teachers' beliefs and practices.

Of note, significant federal funding initiatives for professional development have been guided in part by consensus characteristics. For example, the Eisenhower Professional Development Program was established in 1984 and reauthorized in 1994 as part of Title II of the Elementary and Secondary Education Act of 1965, as amended by the Improving America's Schools Act of 1994 (Garet et al., 1999). The legislation intended to support sustained, high-quality professional development that reflected recent research about learning and teaching including focuses on content and pedagogical skills. The Eisenhower Program focused its

funding on professional development for mathematics and science teachers. In 1999, the U.S. Department of Education appropriated \$335 million for Part B of the Eisenhower Program through states to school districts and grantees.

Although consensus had been building about characteristics of effective professional development, little research had “explicitly compared the effects of different characteristics of professional development” (Garet, Porter, Desimone, Birman, & Yoon, 2001, p. 918). Based on their own review of the literature of features of professional development related to change in teacher knowledge and teaching practice, Garet et al. (1999) identified three structural features (characteristics of the organization of the activity) and three core features (characteristics of the substance of the activity) to study with regard to mathematics and science professional development associated with the Eisenhower Program. Structural features included **duration** of the activity (including total number of contact hours and span of the program), degree to which the activity emphasized **collective participation** of groups of teachers from the same school or district or grade level as opposed to individual teachers without any affiliation, and whether the activity was organized as a **reform type** (e.g., study group, teacher network, mentoring) versus a traditional type (e.g., workshops, conferences, courses for college credit). Core features included the degree of focus on deepening teachers’ **content** knowledge, the extent of opportunities for **active learning** (e.g., reviewing student work, analyzing teaching), and the degree of **coherence** of the project (e.g., alignment with state standards and assessments, consistency with teachers’ goals, and ability to nurture professional communication among teachers). Garet et al. (2001) surveyed a sample of teachers who had participated in Eisenhower-funded professional development activities. The researchers found the three core features all had significant, positive influence on teachers’ knowledge as self-reported by teachers. Furthermore, teachers who reported enhanced knowledge were more likely to report changes in teaching practice. The researchers also found that the three structural features appeared to influence the core features. For example, duration exerted considerable influence on the amount of time devoted to active learning and focused on subject matter content. In addition, professional development that was sustained over time supported more coherence. However, when reform and traditional activities were of the same duration, there was little difference on the impact of the professional development activities. Hence, the researchers concluded that their study provided empirical evidence confirming the importance of sustained professional development activities focused on

content, providing opportunities for active learning, promoting coherence with teachers' professional goals and students' learning goals, and providing opportunities for teachers to collectively support each other. Later, Desimone, Smith and Ueno (2006) reported that sustained and content-focused professional development "has emerged as perhaps the most important type of in-service teacher education" (p. 182).

Despite significant funding initiatives and some empirical evidence supporting consensus characteristics of effective professional development, Elmore (2002) suggested there was little evidence that consensus on characteristics of effective professional development had "had large-scale effect on the practices of schools and schools systems" (p. 10). Elmore described professional development for American schools as often organized around contractually specified days where sessions tend to be designed to serve a broad audience and topics are often disconnected. Blank and de las Alas (2009) suggested although strong research evidence "could contribute to improving teacher professional development methods and delivery, there still exists a significant gap in translating research into practice" (p. 4). However, other findings suggest that some professional development initiatives are embodying more of the characteristics associated with effective professional development. For example, Loucks-Horsley, Love, Stiles, Mundry and Hewson (2003) found that several positive developments were occurring in professional development including an expanded research base, more resources, more purposeful designs, more focus on deepening content knowledge and knowledge of student thinking, more strategies embedded in the daily work of teachers, and fewer short-term workshops.

The Eisenhower Program is no longer in existence, but new professional development funding has become available through the *No Child Left Behind Act of 2001* (NCLB). NCLB is a landmark federal intervention "designed to improve student achievement and change the culture of America's schools" (U.S. Department of Education, 2003, p. 1). The law mandates high-stakes testing in reading, mathematics, and science. Test results are used to gauge whether schools are making a state's standard of adequate yearly progress (AYP). If not, a school may be identified as needing improvement or restructuring. In addition, NCLB mandates that a "highly qualified" teacher be in every classroom. Darling-Hammond (2006b) noted that growing consensus about the importance of effective teachers and effective teaching have led to "reforms of teacher education, the development of professional teaching standards, and insistence under No Child Left Behind that schools hire 'highly qualified teachers'" (p. 18). In addition, the

teacher-quality provision draws “attention to the importance of ensuring equitable student access to high-quality teachers” (Darling-Hammond & Berry, 2006, p. 16).

As part of NCLB Title II, the U.S. Department of Education (2008a) has allocated almost three billion dollars annually for the Improving Teacher Quality State Grants program since 2002. Funding can be used to support a variety of activities, including teacher professional development, “so long as the activities are grounded in scientifically based research” (U.S. Department of Education, 2003, p. 20). The Mathematics and Science Partnership (MSP) program is a major funding initiative for the professional development of mathematics and science teachers and is administered by the Academic Improvement and Teacher Quality Program (AITQ) as a component of the No Child Left Behind Act of 2001, Title II, Part B (U.S. Department of Education, 2008c). Title IIB MSP funding began at about 100 million per year in 2003 (U.S. Department of Education, 2008a), and increased to about 180 million per year from 2005 to 2009 (U.S. Department of Education, 2009). The program’s goal is to improve academic performance of elementary and secondary students in mathematics and science by increasing content knowledge and instructional skills of teachers (U.S. Department of Education, 2008b). Partnerships are at the core of this improvement initiative. At minimum, partnerships are to include a high-need local education agency (LEA) and the science, technology, engineering or mathematics (STEM) department of an Institute of Higher Education (IHE). Each state is awarded funds based on student population and poverty rates; in addition, states are responsible for administering a competitive grant. Grant recipients must submit an evaluation plan and annual report on progress toward meeting objectives measuring the impact of the funded activities (U.S. Department of Education, 2004).

Title IIB MSP funding intends to support high-quality professional development. A variety of activities are eligible for funding so long as activities are grounded in scientifically based research and a partnership exists between a high-need LEA and a STEM department of an IHE (U.S. Department of Education, 2004). For example, funds can be used for recruiting mathematics and science majors to pursue teaching certification. However, the most prevalent professional development model funded by MSP grants reflects consensus recommendations for sustained and content-focused professional development (Blank, de las Alas, & Smith, 2008; Gummer & Stepanek, 2007). The model typically consists of mathematics and science teachers participating in two-week summer institutes and follow-up activities during the school year

supporting the improvement of teachers' content knowledge and/or the strengthening of teachers' skills in using research-based teaching methods.

A few studies have now reported on the nature of professional development projects being funded by Title II MSP. For example, Gummer and Stepanek (2007) from the Northwest Regional Educational Laboratory analyzed Title IIB MSP projects in their region. Analysis was based on proposal documents and evaluation reports produced by the projects and the states, and on interviews with project staff and evaluators. A portion of Gummer and Stepanek's descriptive analysis used the framework of structural and core features identified earlier by Garet et al. (1999) in their analysis of the Eisenhower Program: duration, activity type, collective participation, content focus, active learning, and coherence. With regard to duration, Garet et al. (1999) had examined known exemplary professional development programs and found that many consisted of 80 hours or more. Using 80 hours as a benchmark, Gummer and Stepanek found about 70 percent of the projects provided learning experiences of at least 80 hours. As a two-week summer institute with some follow-up during the school year is the prevalent model for Title IIB MSP projects in the Northwest Region, it is not surprising that most projects provided sustained activities. Although the projects used traditional activities such as summer institutes and workshops, more than half of the projects also included some reform activities such as study groups, lesson study, classroom coaching, and teacher teams. Almost two-thirds of the projects provided opportunities for collective participation by supporting collaboration among teachers from the same school or district. Most projects documented a focus on content. Most projects provided active learning opportunities for teachers. Some forms of active learning included observing modeled instruction, planning for classroom implementation, analyzing student work, and leading discussions. Finally, coherence was typically evidenced by project activities that were aligned with standards and that supported ongoing communication among teachers.

The Council of Chief State School Officers (CCSSO) has also recently studied mathematics and science professional development (Blank et al., 2008; Blank & de las Alas, 2009). For example, the CCSSO reviewed 25 professional development initiatives that were nominated as representing leading efforts to improve science and mathematics teaching (Blank et al., 2008). Although the majority of the nominated projects were funded through Title IIB MSP, the study was not limited to Title IIB MSP projects. Other sources of funding for projects

included Title IIA grants, NSF grants, other government funds and private foundations. Analysis was again generally organized around the framework identified by Garet et al. (1999). Most activities were rated as significantly focused on content knowledge. Most programs provided active learning opportunities for participant teachers. Coherence was commonly noted by alignment of activities with state content standards, with school curriculum, and with broader professional development goals. MSP funded projects commonly used summer institutes and follow-up workshops logging 100 or more hours of planned activities. Blank et al. (2008) identified positive trends in comparison with research findings on professional development activities in the mid-1990s: more sustained activities and the prevalence of active learning opportunities.

In a competitive MSP grant application for 3-year projects beginning in 2007, the Kansas Department of Education chose to target the area of K-8 mathematics (Kansas Department of Education, n.d.). Narrowed from federal guidelines, Kansas grant application guidelines stipulated that project partnerships were required, at minimum, to include high-need unified school districts (USD) and Kansas IHE faculty from both the mathematics teacher education department and from the mathematics and/or engineering department. In addition, each project was required to provide a content-focused two-week summer institute along with at least four days of follow-up training during the school year directly related to curriculum and academic areas in which the teacher provides instruction.

Dr. David Allen at Kansas State University initiated an application for a Kansas MSP grant in 2006 for 3 years of funding beginning in 2007 (Allen, 2007, January 4). Project partners included KSU's College of Education and Department of Mathematics and six Kansas public school partner districts. Funding was granted and the Infinite Mathematics Project (IMP) was studied in its second year. Several features and strategies implemented in the project have been identified as components of effective professional development. The project was a sustained, mathematics-focused initiative. The project format included a two-week summer institute with follow-up sessions during the school year. During the three years, content was focused on making some higher mathematics topics such as calculus, number theory, and algebra more accessible to elementary and middle-level teachers. The content focus for Year 2 was number systems, number patterns, and number theory. Mathematical content was also integrated with attention to pedagogical strategies including differentiating instruction, supporting reading in the

content area, fostering the development of number sense, implementing standards-based instruction, and implementing a lesson study. Active learning opportunities for teacher participants during the Year 2 summer institute included writing of action plans, creating differentiated instruction activities, and presenting a mathematics topic to the group. In addition, during the school year teachers participated in a lesson study cycle. Coherence was exhibited by alignment of project activities with national mathematics standards and by requirements for implementation of an action plan with lesson study addressing state standards, school goals, and school areas of weakness as identified by state assessment. Finally, teacher collaboration was supported by teacher involvement with lesson study groups at their schools.

Purpose of the Study

The purpose of this study was to examine the impact of a Kansas MSP-funded professional development project on K-12 mathematics teachers' knowledge and teaching practice. Although consensus has built up about strategies and characteristics of effective professional development (e.g., Elmore, 2002; Garet et al., 1999; Sowder, 2007), Borko (2004) suggested we know little about "what and how teachers learn from professional development" (p. 3). Results of Title IIB MSP professional development initiatives are only beginning to come in (e.g., Blank et al., 2008; Gummer & Stepanek, 2007). The Infinite Mathematics Project professional development model under study embodied several characteristics and utilized several strategies associated with "high quality" professional development. This case study of a Title IIB MSP project sought to provide a qualitative examination of the characteristics and strategies used in the project, and to understand their impact on teacher learning and practice.

Research Questions

The examination of the impact of a complex professional development project is challenging. In this study, the scope of the examination has been focused to address the following research questions:

1. What impact did the IMP program have on participants' content knowledge (e.g., subject matter content knowledge, pedagogical content knowledge, curricular knowledge)?

2. What impact did the IMP program have on participants' pedagogical knowledge (e.g., knowledge of differentiated instruction, knowledge of standards-based instruction)?
3. What impact did the IMP professional development program have on participants' teaching practices (e.g., differentiating instruction, implementing standards-based teaching, analyzing practice)?

Research Design

A case study methodology was used for this study in order to examine the impact of the IMP professional development program on teachers' knowledge and instructional practice. The macro case under study was the IMP project. Thirty-two teachers participated in the project. Sources of evidence as related to the entire group of participants included participant observation, documentary materials, participant homework and tests, participant action plans, participant reflections about summer institute homework and sessions, and existing project-collected survey data along with summary quantitative analyses reports.

In addition, four mini-case individuals were selected for closer examination of the impact of the IMP project on their knowledge and teaching practice. Analysis of evidence documenting the experiences of the four participants contributed to the overall analysis of the impact of the IMP project. Sources of evidence included pre- and post-summer institute semi-structured interviews, and two classroom lesson observations with pre- and post-observation interviews with each teacher.

A grounded theory approach was used during data collection and analysis. Data collection and organization began during the two-week summer institute. Participant observation was conducted throughout the institute. The content session leader administered pre- and post-summer institute content tests, and daily homework was assigned and collected. Copies of homework and tests were made for the researcher. Some homework was quickly reviewed by the researcher as it came in. Daily reflection about homework and sessions were requested by the researcher. The researcher initially read reflections as they came in as the data was sometimes used to direct the researcher's attention in later observations. Review of documentary materials was done as time allowed. In addition, pre- and post-summer institute interviews were conducted with the four mini-case individuals. During the school year, additional data was

collected through observations of lessons taught by the mini-case individuals and through participant observation of the final share fair follow-up activity.

After the summer institute, the researcher continued to organize and read the data. IMP program-collected data and associated analyses reports became available and were treated as additional data sources. Formal analysis of data started with preparation of textual data as necessary and entering text into a qualitative data analysis software program. The researcher searched for repetition and patterns of meaning related to the research questions while reading participant responses, and underlining or marking with color similar phrases. Memos were written and coding began with open coding as data units were compared with others for similarities and differences (Corbin & Strauss, 1990). The constant comparative method was used to identify new codes and to assign existing codes to quotations and data units. Concepts made their way into the theory by relevance to the research questions and “by *repeatedly* being present” (Corbin & Strauss, 1990, p. 7). Conceptually similar data units or codes were grouped together into themes. Decisions were made for presentation of the data and final report writing.

Significance of the Study

In their literature review, Brown and Borko (1992) emphasized the importance of viewing teacher development as a “life-long process” (p. 210) whereby teachers begin learning about teaching prior to formal teacher education and continue learning and changing throughout their teaching careers. Furthermore, Grouws and Schultz (1996) suggested that “teacher education must be viewed by researchers as a process rather than an event if the professional development of mathematics teachers is going to be positively influenced by research” (p. 453).

Current efforts in mathematics reform attempt radical change in core dimensions of teaching and places high demands on teachers’ content knowledge (Floden, 1997). Scholars, educators, and policymakers view ongoing professional development as an essential mechanism for eliciting change in teachers’ knowledge, beliefs, and practice in support of continual improvement (e.g., Desimone et al., 2006; Elmore, 2002).

Consensus has been building about strategies and characteristics of effective professional development (e.g., Elmore, 2002; Loucks-Horsley et al., 2003; Sowder, 2007). However, researchers have suggested there is still much to be learned about the impact of professional development on teachers’ knowledge and teaching practice. For example, Elmore bemoaned the

lack of empirical evidence concerning the impact of professional development on teachers' knowledge and teaching practice, and ultimately student learning (Elmore, 2002). In 2000, the National Research Council reported that "teacher learning is relatively new as a research topic, so there is not a great deal of data on it" (p. 190). In 2004, Borko suggested we are only beginning to understand "what and how teachers learn from professional development" (p. 3).

Title IIB MSP federal funding initiatives for mathematics and science professional development have promoted sustained, content-focused projects. Results of the impact of these funding initiatives are starting to come in. A few cross-project analyses have recently provided some insight into the quality and nature of professional development initiatives being funded by Title IIB MSP (e.g., Blank et al., 2008; Gummer & Stepanek, 2007).

This study can contribute to the knowledge base on teacher learning and teacher change as related to professional development. Descriptions and findings can inform teacher educators, education researchers, and education policy-makers in several ways.

- This study provides insights into professional development strategies and features that promote the development of content knowledge.
- This study provides insights into professional development strategies and features that promote the development of pedagogical knowledge.
- This study provides insights into what teachers may learn in a professional development Title IIB MSP project implementing research-based professional development strategies.
- This study provides insights into how teaching practice may change after participation in a professional development Title IIB MSP project implementing research-based professional development strategies.

Limitations of the Study

This study seeks to determine the impact of a single professional development initiative funded by Title IIB MSP. The IMP project under study was a professional development initiative launched by Kansas State University (KSU) in conjunction with partner public school districts in Kansas. Project participants primarily included elementary and middle school teachers who self-selected to participate in a sustained, mathematics-focused professional development experience. Although some data was collected for all project participants, only a small number of participants were interviewed and observed in their classrooms.

Borko (2004) suggested that professional development research can be differentiated into phases. Phase 1 research examines professional development at a single site to provide evidence that the program can help teachers increase their knowledge and change their instructional practices. Phase 2 research would involve research on a program enacted at multiple sites by different facilitators and commonly seeks to determine the resources needed to implement the program with integrity. Phase 3 research would provide comparative analysis of the effects of different programs and would be particularly useful for policy decisions on resource allocation.

This study is basically affiliated with Phase 1 research. However, as the study examines a Title IIB MSP project, results may provide further insight about the impact of professional development projects receiving this type of funding. In the strictest sense of generalization, results of this study do not generalize to other Title IIB MSP projects as there is considerable variation in contexts and professional development strategies amongst the projects. However, qualitative research often takes a revised view of generalization (e.g., Guba & Lincoln, 1981). Generalizability in naturalistic inquiry or naturalist evaluation may be viewed as applicability or transferability (Guba & Lincoln, 1981; Guba & Lincoln, 1989). This study used thick description of the context of the case and of the data collection methods and analysis. By doing so, “the case study facilitates the drawing of inferences by the reader which may apply to his or her own context or situation” (Guba & Lincoln, 1989, p. 224). Thus, findings of the study may serve as working hypotheses applicable to other Title IIB MSP projects with similar contexts (Guba & Lincoln, 1981).

Finally, qualitative methods are subject to researcher biases and interpretations. To mitigate subjectivity, the researcher has used triangulation of data sources and has followed interview and observation protocols.

Definition of Terms

Content knowledge: Described by Shulman (1986b) to include subject matter content knowledge, pedagogical content knowledge, and curricular knowledge.

Curricular knowledge: Described by Shulman (1986b) to include knowledge about available curricular alternatives, lateral knowledge about curriculum students might be studying in other subjects, and vertical knowledge about preceding and succeeding topics in the same subject area.

Differentiated instruction: Stems from the purpose of maximizing the capabilities of all students. Based on teachers' knowledge of students' learning preferences and students' readiness, critical elements of differentiated instruction include a focus on the big ideas (e.g., Rock, Gregg, Ellis, & Gable, 2008; Small, 2009), "choice, flexibility, on-going assessment, and creativity resulting in differentiating the content being taught, or how students are processing and developing understanding of concepts and skills, or the ways in which students demonstrate what they have learned and their level of knowledge through varied products" (Anderson & Algozzine, 2007, p. 50).

High-need unified school district: In the Kansas Mathematics and Science Partnership Program competitive grant application a high need USD was defined as having met any of the following criteria:

- Percent of free and reduced lunch is at or above the state average of 38.92 percent;
- High percent of teachers who teach mathematics are not endorsed in the content area;
- District did not make Adequate Yearly Progress (AYP) in mathematics;
- District is on improvement in mathematics (Kansas Department of Education, n.d., p.

20).

Inquiry: "An investigative process that involves posing questions, making conjectures, testing various alternatives, critically evaluating results and revising and retesting in light of the new information and insights" (C. Barnett, 1998, p. 84).

Inquiry into practice: Refers to teachers' dispositions towards analysis of teaching in order to learn from teaching and thus improve teaching (Ball & Cohen, 1999; Hiebert, Morris, Berk, & Jansen, 2007). Hiebert et al. (2007) outlined analyses skills to include: "(a) setting learning goals for students, (b) assessing whether the goals are being achieved during the lesson, (c) specifying hypotheses for why the lesson did or did not work well, and (d) using the hypotheses to revise the lesson" (p. 49).

Knowledge about mathematics: Ball (1990a, 1991a) expanded Schwab's (1978) general syntactic structures versus substantive structures of subject matter knowledge with specific regard to the domain of mathematics by recognizing two critical dimensions of subject matter knowledge: "substantive knowledge of mathematics" and "knowledge about mathematics." Knowledge about mathematics includes understandings about the nature of knowledge in the discipline: e.g., how truth is established in the field of mathematics, what reckons as a solution,

which ideas are based on convention and which are built on logic, and how mathematics has developed and changed over time.

Lesson study: A professional development approach credited for the steady improvement of teaching practice in elementary education in Japan. Lesson study cycles include: (a) collaborative planning of a lesson upon consideration of goals, (b) lesson implementation by one teacher with observation and data collection by other group members, (c) collective discussion and analysis of the lesson with attention to student learning and thinking, and (d) revision of the lesson with a new cycle of implementation (Lewis, Perry, & Murata, 2006).

NCTM Content Standards: Number & Operations, Algebra, Geometry, Measurement, Data Analysis & Probability (2000).

NCTM Process Standards: Problem Solving, Reasoning & Proof, Communication, Connections, Representation (2000).

Number Sense: The definition of number sense has lacked consistency (Van de Walle & Lovin, 2006b; Verschaffel, Greer, & De Corte, 2007). Van de Walle and Lovin (2006b) acknowledged Howden's (1989) description of numbers sense "as good intuition about numbers and their relationships. It develops gradually as a result of exploring numbers, visualizing them in a variety of contexts, and relating them in ways that are not limited by traditional algorithms" (p. 11).

Partnership: Refers to a "group of entities (organizations such as schools, colleges or universities, and for-profit or non-profit companies) that work together to accomplish a set of mutual goals" (Gummer & Stepanek, 2007, p. 9).

Pedagogical content knowledge: Refers to subject matter knowledge *for teaching* which includes knowledge for topics regularly taught in a subject area of useful representations, analogies, examples, explanations, and demonstrations. It also includes an understanding of what makes some topics easy or difficult to learn and what conceptions, preconceptions and misconceptions students might have at various ages (Shulman, 1987).

Pedagogical knowledge: Refers to knowledge of the art and science of teaching.

Professional development: Refers to activities aimed at developing the knowledge and skills of practicing teachers (Elmore, 2002).

Reading in the content area; content area literacy: Content area reading has a focus on reading to learn rather than learning to read. Reading in the content area is "characterized by the growing

importance of word meanings and of prior knowledge” (Chall, 1983, p. 21) as students learn about subject matter from reading subject matter textbooks, reference books, biographies, etc.

Standards: A term used for a series of documents published by NCTM “designed to set school mathematics goals for students and teachers” (Jacobs et al., 2006, p. 6) including: *Curriculum and Evaluation Standards for School Mathematics* (1989), *Professional Standards for Teaching Mathematics* (1991), *Assessment Standards for School Mathematics* (1995), and *Principles and Standards for School Mathematics* (2000).

Standards-based teaching; reform-based teaching; standards-based instruction; reform-based instruction; standards-based practices; reform-based practices: Teaching behavior that emphasizes addressing NCTM Content Standards via engaging students in NCTM Process Standards of reasoning, communicating, making connections, interpreting representations and problem-solving (2000).

Subject matter content knowledge: For Ball (1990a, 1991a), subject matter content knowledge includes two critical dimensions: substantive knowledge of mathematics (knowledge of topics, concepts, procedures, underlying principles and meanings, and relationships among the concepts) as well as “knowledge *about* mathematics” (an understanding of how truth is established in the field of mathematics, of what reckons as a solution, of which ideas are based on convention and which are built on logic, and of how mathematics is developed and changed over time).

Substantive structures of knowledge: Initially described by Schwab (1978) and referred to by Shulman (1986b) as how basic facts and concepts are organized within a domain.

Substantive knowledge of mathematics: Ball (1990a, 1991a) broadened Schwab’s (1978) substantive structures and provided more detail with specific regard to the domain of mathematics. For Ball, substantive knowledge of mathematics included knowledge of topics (e.g., trigonometry), concepts (e.g., infinity), procedures (e.g., factoring), underlying principles and meanings (e.g., what division with fractions means), and relationships among the concepts (e.g., how fractions are related to division).

Syntactic structures of knowledge: Initially described by Schwab (1978) and later explained by Shulman and colleagues as “the canons of evidence and proof that guide inquiry in the field” (Wilson, Shulman, & Richert, 1987, p. 114).

Traditional teaching; traditional instruction: Traditional U.S. teaching has been reported as characterized by “brief demonstrations of mathematics procedures followed by practice on many similar problems” (Jacobs et al., 2006, p. 29).

Summary

Professional development has been viewed as a critical component for school improvement and school reform. The U.S. Department of Education has made significant investments in professional development, particularly for mathematics and science teachers. Federal grants such as Title IIB MSP intend to fund high quality professional development initiatives that support increasing teachers’ knowledge and strengthening teachers’ instructional skills.

The Infinite Mathematics Project (IMP) professional development program combined many interesting features which are relevant to current recommendations and funding opportunities in professional development. This study seeks to examine the impact of the IMP initiative with respect to teachers’ content knowledge, pedagogical knowledge, and teaching practice.

CHAPTER 2 - Literature Review

Effective Mathematics Teachers and Teaching

As teachers occupy the largest share of the K-12 education budget (e.g., Ingersoll, 2002; Youngs, Odden, & Porter, 2003) and as educational researchers have long held the “core belief that *teachers make a difference*” (Wright, Horn, & Sanders, 1997, p. 57) in student achievement, it is not surprising that teachers have been a focus of much research. However, empirical evidence has not always corroborated the importance of teachers. For example, the Coleman Report of 1966 sparked debate about whether teachers and schools were important for student achievement. The report was interpreted as finding that families and peers were the primary determinants of student performance rather than teachers or schools (Harbison & Hanushek, 1992).

Research in the last fifteen years now empirically supports the notion that teacher effectiveness is a significant factor affecting student achievement (e.g., Hanushek, 1992; Harbison & Hanushek, 1992; Sanders & Horn, 1998). Economist Hanushek (1992) found “the estimated difference in annual achievement growth between having a good and having a bad teacher can be more than one grade-level equivalent in test performance” (p. 107). Upon developing and using the Tennessee Value-Added Assessment System, Sanders and colleagues found differences in teacher effectiveness “to be the dominant factor affecting student academic gain” (Wright et al., 1997, p. 66). In fact, Sanders and Rivers (1996, cited in Sanders & Horn, 1998) found teacher effects to be both additive and cumulative, with the residual effects of very effective or very ineffective teachers being measurable two years later.

Teacher effectiveness has been shown to be a major determinant of student academic progress; however, teacher qualities and classroom practices that characterize effective teaching have not been clearly identified. As Darling-Hammond (2000) noted, part of the problem is that identifying teacher qualities related to teacher effectiveness has been hampered by mixed results over the past 50 years. Teacher quality has been examined and proxied by a number of variables such as: measures of general academic ability, years of education, years of teaching experience, measures of subject matter knowledge and teaching knowledge, certification status, and teaching

behaviors. Upon review of many studies across a variety of content areas, Darling-Hammond (2000) and Hanushek (1986) found no one teacher quality that stood out as a major determinant of student achievement; furthermore, findings were inconsistent and subject to low statistical significance or statistical insignificance.

Although no one variable stands out across the studies, Darling-Hammond and Youngs (2002) reported that several aspects related to teachers' qualifications have some influence on student achievement including general academic/verbal ability, subject matter knowledge, knowledge about teaching and learning developed through education or professional development, teaching experience, and combined sets of qualifications measured by certification status. Several of these characteristics have been studied by using proxy variables, often referred to as teacher inputs, with a long history in production function studies (Wenglinsky, 2002). For example, Ferguson and Ladd (1996) found teacher performance on the ACT (a general academic ability test required in many states for application to college) and teacher education (whether or not the teacher had a master's degree) were positively linked with student achievement; whereas, teacher experience was not found to be significantly related to student achievement. On the other hand, other studies (Hanushek, 1992, 1986) have linked teacher experience with student achievement. In another study, Rowan, Chiang, and Miller (1997) found teachers' subject matter knowledge (crudely measured by a one-item math question and whether or not the teacher had majored in mathematics) had an influence on their students' mathematics achievement. Furthermore, the effects were larger for schools where students entered with lower levels of ability. In an oft-referred to study, Monk (1994) found that secondary teacher content preparation as measured by the number of courses taken in a subject area (a proxy measure for subject matter knowledge) was positively related to student achievement. Undergraduate course work in pedagogy also contributed to student achievement, and in fact had larger effects than additional undergraduate mathematics coursework. Furthermore, Monk found that "gross measures of teacher preparation (such as degree levels, undifferentiated credit counts, or years of teacher experience)" (p. 142) were not significantly related to student achievement.

Darling-Hammond (2000) suggested that certification status is a measure of teachers' qualifications that combines both aspects of subject matter knowledge and knowledge about teaching and learning. Upon a synthesis of results from many studies, Wayne and Youngs (2003) concluded that the effects of certification status are found only in the subject of

mathematics and appeared when teachers held standard certification in mathematics. They referenced a study by Goldhaber and Brewer (2000) who found students whose teachers were either not certified in mathematics or who held a private school certification had lower mathematics achievement than those students whose teachers had standard, probationary or emergency mathematics certification status. Upon examining data on public school teacher qualifications and other school inputs across a variety of states, Darling-Hammond (2000) found the teacher's certification status and holding a degree in the field to be taught were positively related to student achievement. Although teacher qualifications are only an indirect measure of teacher quality (Ingersoll, 2002), research studies using *combined* measures of teacher qualifications (e.g., experience, scores on a licensing examination, certification status, master's degree) indicate that teachers' qualifications are an important factor in student achievement (e.g., Ferguson, 1991; Greenwald, Hedges, & Laine, 1996).

A variety of generic teaching behaviors and practices also appear to be associated with student achievement. Darling-Hammond (2000) identified several recurring themes of teacher behavior associated with student achievement. For instance, a teacher's flexibility and adaptability are positively associated with student achievement, as typically the most successful teachers are able to use a wide-range of teaching strategies. Other variables linked to student achievement include the teacher's clarity, enthusiasm, task-oriented behavior, use of higher-order questions, and use of student ideas.

From a different perspective, Wenglinsky (2002) hypothesized that teacher quality has three main aspects that contribute to student achievement: the teacher's classroom practices, professional development that the teacher receives in support of teaching practice, and teacher inputs external to the classroom such as years of experience or education level. In order to investigate a broad spectrum of measures of teacher qualities, Wenglinsky used NAEP data of eighth grade students who took the 1996 mathematics assessment along with teacher information from a background questionnaire completed for NAEP. Three teacher input variables were studied including the teacher's education level, major in mathematics, and years of experience. Ten general professional development variables (with no variable addressing specific mathematics content) were identified. Finally, 21 classroom practice measures were used. The study revealed five aspects of teacher quality were positively related to student achievement: 1) the teacher's major, 2) professional development in higher-order thinking skills, 3) professional

development in learning how to teach different populations of students (collapsed from three measures including professional development in cultural diversity, in teaching limited-English-proficiency (LEP) students, and in teaching special-needs students), 4) teaching practices utilizing hands-on activities (collapsed from three measures of the relevant time students spent working with blocks, working with objects, and solving real-world problems), and 5) teaching practice incorporating higher-order thinking skills (from the single measure of the relative time students spent solving unique problems).

Although a variety of studies have examined the relationship between a limited number of variables representing teacher quality and student achievement, Darling-Hammond and Sykes (2003) suggested that few databases have enough data to allow for a large number of the variables to be examined at once; in addition, many variables representing teacher attributes are correlated. In fact, upon review of evidence, researchers (Brophy & Good, 1986; Darling-Hammond & Sykes, 2003) have reflected that effective teaching demands a *blend* of teacher characteristics and behaviors. An extensive list of teacher qualities and behaviors associated with student achievement might include verbal skills, subject matter knowledge, academic ability, professional knowledge, experience, enthusiasm, flexibility, perseverance, concern for children, and some specific teaching practices (Darling-Hammond & Sykes, 2003). In contrast, the National Mathematics Advisory Panel (2008) said that little is known from existing high-quality research about the skills and practices of effective teachers related to student achievement.

From a different perspective, some researchers (Hiebert & Grouws, 2007; Kennedy, 2006) have carefully distinguished between effectiveness in teaching and effective teachers. Kennedy (2006) suggested that both teacher quality and the conditions of teaching affect the quality of teaching. In addition, Hiebert and Grouws (2007) noted that while the characteristics of teachers can influence their teaching, teachers with different characteristics can teach in similar ways. They further suggested this may well be the reason that no clear connections can be found between teacher characteristics and student achievement.

Studies investigating effectiveness in teaching have faced many challenges due to the complex nature of teaching; as Brophy and Good noted in their 1986 review, what constitutes effective teaching in one setting may not be effective in another situation. For example, Hiebert and Grouws (2007) described three challenges to developing useful theories on classroom

teaching. First, different teaching methods may at times be more effective or less effective depending on the learning goals. Secondly, teaching is an interactive system whereby individual features cannot be isolated from the effects of other features in the system. Finally, methods of teaching effectiveness are mediated by students' thinking and contextual conditions.

Although there are challenges to developing useful theories of teaching effectiveness, researchers have expressed views on relationships between effective teaching and student learning in mathematics. For example, the National Research Council described effectiveness in mathematics teaching and learning as a “function of teachers’ knowledge and use of mathematical content, of teachers’ attention to work with students, and of students’ engagement in and use of mathematical tasks” (2001, p. 9). Some dimensions of effective teaching that can influence students’ opportunities and motivation for learning include having teachers who establish high expectations for students, who select demanding tasks, and who were able to interact with diverse student populations. However, isolated features need to be considered within the larger system of interactions between the teacher, student, and content.

Despite Hiebert and Grouws’ description of challenges (2007) to linking teaching effectiveness with student learning, they identified some patterns upon review of research. They noted that the opportunity to learn is a firmly established connection between teaching and learning. Hiebert and Grouws explained that “‘opportunity to learn’ is not the same as ‘being taught.’ Opportunity to learn includes considerations of students’ entry knowledge, the nature and purpose of the tasks and activities, the likelihood of engagement, and so on” (p. 379). Opportunity to learn can be influenced by forces such as individual students, schools, and the national education system. However, curriculum and teachers are potent forces on what students have the opportunity to learn (Hiebert & Grouws, 2007; National Research Council, 2001). Students in different curriculum tracks receive different opportunities to learn.

The emphasis teachers place on different learning goals and different topics, the expectations for learning that they set, the time they allocate for particular topics, the kinds of tasks they pose, the kinds of questions they ask and responses they accept, the nature of the discussions they lead—all are part of teaching and all influence the opportunities students have to learn. (Hiebert & Grouws, 2007, p. 379)

Hiebert and Grouws (2007) considered two major areas of opportunity to learn in mathematics: the opportunity to learn skill efficiency and the opportunity to develop conceptual

understanding. The researchers chose these areas because procedural fluency and conceptual understanding are two of the five strands of mathematical proficiency outlined by the National Research Council (2001), and because considerable research attention has been directed in these areas. Upon review of various studies, features of teaching that facilitate procedural efficiency include rapid-pacing, teacher modeling, and “smooth transitions from demonstration to substantial amounts of error free practice. Noteworthy in this set of features is the central role played by the teacher in organizing, pacing and presenting information to meet well-defined learning goals” (Hiebert & Grouws, 2007, p. 382). Research establishing links between teaching and students’ conceptual understanding is weaker. However, Hiebert and Grouws described two primary themes that have emerged. First, students can develop conceptual understanding when teaching explicitly attends to making connections between facts, procedures, and concepts. Secondly, students must engage in expending effort in making sense of mathematics in order to develop understanding. However, Hiebert and Grouws pointed out that the two features associated with providing students with opportunities to develop conceptual understanding have rarely been found in studies of teaching practices in the United States, including several fairly recent survey studies (e.g., Rowan, Harrison, & Hayes, 2004; Weiss, Pasley, Smith, Banilower, & Heck, 2003b).

Hence, although teacher effectiveness has been shown to be a major determinant of student academic progress, teacher qualities and classroom practice that characterize effective teaching have not been clearly identified. In addition, individual features can often not be isolated from the effects of other features in a complex teaching situation. Different features of teaching may contribute to different types of learning (e.g., procedural fluency vs. conceptual understanding). Nonetheless, the teacher is a key element in a complex system of interactions influencing student learning.

U.S. Mathematics Education Reform

Description and Challenges

In 1989, NCTM provided leadership and direction for our current reform movement with the publication of *Curriculum and Evaluation Standards. Professional Standards* (1991) and *Assessment Standards* (1995) soon followed. Initial visions from the original three standards documents were refined and synthesized, resulting in the single book, *Principles and Standards*

for School Mathematics (PSSM) (2000). Classrooms envisioned by NCTM would provide environments whereby *all* students were engaged in exploring, conjecturing, reasoning, communicating, making connections, interpreting representations, and problem-solving (1989, 2000). The image of effective teaching would require teachers who understood deeply the mathematics they would be teaching; who could select appropriate tasks to engage their students; who could foster understanding through the use of appropriate technology; who could orchestrate classroom discourse conducive to building students' mathematical understanding; and who could use a variety of assessment and pedagogical strategies (1991, 2000).

Another source of recommendations for reform came from the Committee on Mathematics Learning of the National Research Council in the publication *Adding it Up* (NRC, 2001). In order to attain *mathematical proficiency*, students would need to develop five interwoven strands: *conceptual understanding* (comprehension of connections and relationships between concepts and operations), *procedural fluency* (skill in carrying out procedures flexibly, efficiently, and accurately), *strategic competence* (ability to formulate, flexibly represent, and solve problems), *adaptive reasoning* (capacity to logically reason, explain, and justify), and *productive disposition* (tendency to see mathematics as useful, worthwhile, and sensible). "Helping children acquire mathematical proficiency calls for instructional programs that address all its strands" (p. 116). In addition, the strands should not be viewed as competing for attention; instead, many of the strands interact with each other. For example, understanding underlying mathematical concepts "makes learning skills easier, less susceptible to common errors, and less prone to forgetting" (p. 122). On the other hand, a certain level of procedural skill may be needed in order to learn mathematical concepts with understanding. As another example, adaptive reasoning particularly interacts with problem solving. Learners may draw on strategic competence to formulate and represent a problem, but learners must also reason to determine the effectiveness of a proposed strategy. Overall, the *Standards* documents, along with other publications such as *Adding it Up*, have provided a reformed vision for school mathematics education.

Describing the vision succinctly, in a way agreed upon by the majority, has been challenging and subject to interpretation. Floden (1997) described the current mathematics reform movement as an increased emphasis on teaching for understanding, learning with understanding, and application using real-world problems. Sherin (2002) believed that some key

phrases used to characterize the current movement include “‘teaching for understanding,’ ‘building a community of inquiry,’ or ‘mathematics for all’” (p. 121). Sowder (2007) chose to use Spillane’s term, “*principled knowledge*,” as a way of describing the goals of the current mathematics reform movement. Spillane (2000) wrote about the new movement:

Reformers want principled mathematical knowledge, as distinct from procedural knowledge, to receive more attention in school work. Whereas *procedural knowledge* centers on computational procedures and involves memorizing and following predetermined steps to compute answers, *principled knowledge* focuses on the mathematical ideas and concepts that undergird mathematical procedures. Procedural knowledge has dominated the K through 12 curriculum. Reformers also propose that students develop a more sophisticated appreciation for doing mathematics including framing and solving mathematical problems, articulating conjectures, and reasoning with others about mathematical ideas: Students need to appreciate mathematical activity as more than computation. (p. 144)

Enacting the vision has proven to be even more challenging and susceptible to debate than describing the vision.

Review of American educational reform attempts of the Progressive Period in the first half of the 20th century and of the “new math” movement of the 1950s and 60s suggest that classroom teaching was not significantly changed (e.g., Elmore, 1996). Upon review of the late-progressive period, Cuban (1984, cited in Elmore, 1996) concluded seldom had more than one-fourth of the classrooms in a district that tried to install progressive practices achieved substantial success. Similarly, although massive funds were directed toward mathematics teacher education during the “new math” movement of the 1950s and 1960s, relatively insignificant change occurred in teaching practices (Brown, Cooney, & Jones, 1990; Elmore, 1996). Although a few teachers changed their core teaching practice, large-scale reform failed. Elmore suggested some key flaws to the prior reform efforts included having placed the burden of reform on individuals and having had a lack of understanding on how to get reform successes to move from one setting to another. Scaling up had failed. Furthermore, the flaws in American reform efforts are rooted in a deep cultural teaching norm that posits “successful teaching is an individual trait rather than a set of learned professional competencies acquired over the course of a career” (Elmore, 1996, p. 16).

With regard to our current reform, Stigler and Hiebert (1999) found upon review of the TIMSS 1995 Video Study that typical American classroom teaching practice was still largely unchanged. However, Stigler and Hiebert noticed that some countries such as Japan were “continually improving their teaching approaches” (p. x). The teaching practices in Japan have changed considerably over the past fifty years through a system that supports gradual change. The Japanese educational system is characterized by collaboration among teachers in developing lessons, support from administration, and identification of clear learning goals for students. Japanese changes have been supported by expectations of on-going professional development as part of a teacher’s job. In contrast, U.S. reform movements have striven to make major changes in a short amount of time, and American teachers have often been assumed to be competent once they completed the teacher-training programs. In consequence, a teaching gap based on cross-cultural teaching differences is growing because the U.S. does not have effective mechanisms for sustaining reform that could result in improved teaching practice. A cross-cultural teaching gap results in a cross-cultural learning gap. Stigler and Hiebert suggested that improving American teaching quality must be put in the forefront for increasing student achievement.

Researchers have provided several recommendations for tackling the problem of scale in the United States that complement Stigler and Hiebert’s observations on reasons for Japanese success in changing teaching practice. For example, Elmore (1996) suggested that rather than rely on individuals, external normative structures should be constructed by professional bodies. Some external structures have already been constructed for our current reform efforts. For instance, NCTM has developed and communicated national standards over the past twenty years. However, providing a vision via written standards is not enough (e.g., Elmore, 1996; Spillane & Jennings, 1997; Spillane, 1999; Stigler & Hiebert, 1999). Studies (e.g., Hiebert & Stigler, 2000; Jacobs et al., 2006; Spillane, 1999) have revealed that among teachers who report having knowledge of reform and teaching practice aligned with reform, few of the teachers actually exhibit teaching practice reflecting reform recommendations. Thus, knowledge of reform standards does not directly translate into changes in core teaching practices. Weiss et al. (2003b) found that whereas standards were most frequently cited by teachers as influencing the lesson content, teachers reported their instructional practices were influenced by their own knowledge, beliefs, and prior experiences. Several researchers (e.g., Borasi & Fonzi, 2002; D. K. Cohen & Ball, 2001; Elmore, 1996; Spillane & Jennings, 1997; Weiss et al., 2003a) have exhorted that

more explicit and elaborated images of practice need to be communicated. However, providing detailed elaboration is challenging as inherent to teaching practice in our current reform is the notion that teachers and learners should interact, not that teachers should follow a specific script (Spillane & Jennings, 1997). Nonetheless, recommendations have been made. Researchers (Cohen & Ball, 2001; Spillane & Jennings, 1997) have emphasized that teachers need to be viewed as learners of reform practice and should thus be provided with substantial learning opportunities. Elmore (1996) suggested more elaborate images might be offered through communication; perhaps video tapes of teachers engaging in reform practices could be disseminated through professional organizations. In addition to video clips, Borasi and Fonzi (2002) suggested that vignettes of mathematics classrooms can provide an image of reform practice. Weiss et al. (2003a) suggested several possible interventions. For instance, they suggested that teachers need opportunities to analyze lessons with particular regard given to high quality instructional strategies related to teacher questioning and building conceptual understanding through a focus on sense-making. Lesson study might offer a context for analyzing lessons. The researchers also suggested teachers' instructional materials should be more explicit in identifying learning goals, describing research on student thinking for a content area, suggesting probes that teachers could use to check for student understanding, and outlining focal points teachers should emphasize to support student sense-making of the mathematical concepts. Furthermore, Weiss et al. (2003b) recommended that professional development opportunities should "themselves reflect the elements of high quality instruction, with clear, explicit learning goals; a supportive but challenging learning environment; and means to ensure that teachers are developing understanding" (p. 15). Spillane (1999) conjectured that changes in core instructional practice would require a social dimension beyond the teacher's individual classrooms. Teachers would need to have conversations with colleagues and experts about what reform means and about their efforts to enact reform ideas into practice. Teachers would also need a variety of curricular resources that were consistent with reform. Borasi and Fonzi explained that the vision of reform-oriented instruction is "grounded in views of knowledge, learning and teaching informed by a constructivist perspective" (2002, p. 14). As such, teachers would need to learn about constructivist assumptions. Borasi and Fonzi also suggested that research on how people learn complex tasks may reveal how teachers can learn new teaching practices. For example, Collins, Brown and Newman (1989) noted that in the traditional

apprenticeship model, an apprentice learns complex skills through observation, scaffolded practice, and increasingly independent practice. The corresponding teacher's responsibilities would include modeling, coaching, and fading. Hence, Borasi and Fonzi suggested that knowing that a new vision for teaching mathematics exists is not nearly sufficient for teachers to embrace new teaching practices. In order for teachers to be learners of new practice, they would first have to observe and examine new practice in a learning situation. They would also have to be supported as they engaged in new teaching practices.

In general, Elmore (1996) suggested that incentives, training, and time to observe and engage in new teaching strategies are elements that can support incremental growth in teaching practice. Whereas a small proportion of the teaching force may be intrinsically motivated to carry out reform, more explicit incentive and support structures need to be provided in order to bring reform to a large scale. Around the same time, Darling-Hammond and Sclan (1996) portrayed American schools as ones where a teacher's primary work was to teach large groups of students for most of working day; little time was left for tasks such as planning, preparation, or collaboration among teachers. In contrast, other countries like China, Japan, and Germany had structures whereby teachers taught groups of students about half of the week and the remaining half was used for activities such as preparation, joint planning, and tutoring.

NCTM exhorted that the "work and time of teachers must be structured to allow and support professional development" (2000, p. 19). To enact the vision described by *PSSM*, teachers would need opportunities for reflection on and refinement of instructional practice. In 2001, Ball, Lubienski and Mewborn lamented that there were few incentives for American teachers to take the time to work on more complex content or to organize richer experiences for their students. Instead, policymakers' focus on standardized testing often placed pressure on teachers to focus once again on "basic skills." Teachers continued to be isolated from one another and had little support or time for learning and trying out innovative practices. Furthermore, professional development was often observed as being "intellectually superficial, disconnected from deep issues of curriculum and learning, fragmented, and noncumulative" (p. 437).

Elmore (2002) noted that effective professional development would also require high organizational capacity. If a teacher gains new knowledge and skills from professional development, yet returns to the classroom where work conditions, students and content are

basically the same, the professional development experience will have little, or even a negative effect, on improving teaching practice. Thus, teachers need opportunities to learn new complex practices, and they need ongoing support during the process.

Demands of Reform for Teachers

Floden (1997) noted that the need for teacher learning due to reform initiatives is constantly present. However, some reform efforts require only weak modifications. For instance, the “back to basics” movement may have entailed some teachers needing a brief review or update on topics receiving renewed emphasis in the curriculum. At other times, reform proposals may be more radical in their attempt to change core dimensions of schooling (Elmore, 1996; Floden, 1997; Spillane, 1999). Elmore (1996) claimed the closer a reform attempts to change core dimensions of teaching, the less likely it will be successful on anything more than a small scale. Our current U.S. reform proposes dramatic changes in both content and core dimensions of teaching practice as it “combines its call for student understanding with advocacy for greater student engagement in active learning” (Floden, 1997, p. 16).

Spillane (1999) noted our current reform proposes changes to core dimensions of teaching which are arduous and complex as they involve the “knowledge represented in classroom tasks, classroom discourse patterns” (p. 143), and new roles and responsibilities in the classroom. Although teaching is generally viewed as involving complex interactions in unpredictable classroom settings, the vision being communicated by our current reform increases the uncertainties of teaching and involves even more complex interactions (e.g., Ball & Cohen, 1999; Floden, 1997). Schifter (1998) noted that when student thinking becomes the center of practice, classroom processes become less predictable and more difficult to manage. Heaton (2000) described her struggles as an experienced teacher trying to change her teaching practice in response to current mathematics reform recommendations. She said it was “not a matter of putting one’s current practice on hold, learning a new pedagogy, re-entering the classroom, and doing things differently” (p. 34). Instead, the journey was complex and uncertain; furthermore, it was embedded in the practice of teaching.

In addition to changes to teaching practice, Floden (1997) noted our current reform also involves teaching more challenging content. For example, teaching *why* an algorithm works was not a part of the curriculum for many of today’s teachers when they attended elementary school.

Teacher training may have involved models of teaching and learning that focused on memorization of facts instead of emphasizing understanding (Desimone et al., 2006). Hence, there exists a “sharp discrepancy between the content teachers have learned and the content they are now expected to teach” (p. 13). Borko and Putnam (1995) referenced two case studies (Heaton, 1992; Putnam, 1992) as providing evidence that a teacher’s limited subject matter knowledge may prove to be problematic in implementing curriculum involving more problem-solving and real-world contexts. For example, in a case study analysis, Heaton (1992) suggested that one teacher’s insufficient subject matter knowledge may have resulted in creating misunderstanding among students rather than understanding. The teacher had been focused on utilizing a cooperative learning activity to engage students in problem-solving related to a real-world context. The researcher observed the teacher leading the students to measure the area, rather than perimeter, to try to determine a length of fencing and resulting cost. As far as the teacher was concerned, the activity was a success because of the student engagement. However, the researcher suggested that errors in the mathematical content of the lesson may have resulted in student misunderstandings.

Researchers (Heaton, 2000; Schifter, Russell, & Bastable, 1999) have described that teaching for understanding requires a qualitatively different and richer understanding of mathematics than teachers might possess. Another researcher, Sherin (2002), outlined three important facets of reform practice that demand substantial teachers’ content knowledge and that are commonly agreed upon: (a) using new curricula that are designed to focus on conceptual understanding of concepts, multiple representation of concepts, and connections between concepts, (b) using a more adaptive teaching style by which teachers attend to students’ ideas raised during class, and (c) using classroom discourse to solicit and analyze students’ thinking and to insert mathematical explanations as appropriate. Unfortunately, many studies of both preservice and in-service teachers have illustrated weaknesses in the mathematics content knowledge of U.S. teachers (e.g., Ball, 1990b; Graeber, Tirosh, & Glover, 1989; Ma, 1999; Post, Harel, Behr, & Lesh, 1991). Overall, short-term workshops will not address the needs of teachers to enact our current reform; changes to content and teaching practice of this magnitude will likely require sustained learning over time on the part of teachers (e.g., Ball & Cohen, 1999; Borko & Putnam, 1995; Floden, 1997).

Floden (1997) suggested that there are two schools of thought about teacher understanding and standards-based teaching. One school of thought holds that “successful teaching for understanding is *impossible* for *topics* the teacher does not understand” (p. 15). This position would imply that teachers who lack understanding should not try to teach for understanding because they will be unsuccessful. A second school of thought asserts that teacher understanding is helpful for teaching for understanding, but not necessary. Floden suggested that teachers could teach more than they know by using additional resources including standards-based curricular materials (albeit teacher knowledge can remain a critical factor in directing student learning from the curriculum), information technology for additional access to content expertise, and students learning from each other through an emphasis on student discussions in the classroom. Floden argued that attention should be given to ways to help teachers learn while they are teaching concepts they may not have mastered. For instance, some priority should be assigned to help teachers develop habits of inquiry. Applications of habits of inquiry might include being able to guide classroom discussions that lead to justified conclusions, probe for student understanding, imagine alternatives, and reflect upon practice in light of its effect on students.

Other researchers have concurred that teachers need to develop a stance of inquiry into practice in order to be able to learn during the daily practice of teaching (e.g., Ball & Cohen, 1999; Darling-Hammond, 2006a; Hiebert et al., 2007; Schifter & Riddle, 2004). Ball and Cohen (1999) pointed out that knowledge of subject matter, learners, pedagogy, and learning were key components of the knowledge teachers would need in order to enact reform practice; however, teachers would also need to be able to learn how to anticipate, elicit, and interpret student thinking in the classroom setting. Teachers would need to learn how to use what they learned about student thinking to improve practice. Ball and Cohen proposed that this type of learning demands a stance of inquiry by which teachers can learn while teaching. Darling-Hammond (2006a) explained that increased expectations for teachers’ knowledge base and for being able to diagnose and assess diverse learners requires that teachers not only have access to more knowledge, but that they also develop classroom inquiry skills. The range of knowledge for teaching is so expansive that it cannot be mastered by a single teacher; teachers must have critical observation and analysis skills and teachers must be expert collaborators in order to learn from practice and to learn from each other. Similarly, Hiebert et al. (2007) suggested that

improvement in teaching will require not only an increased attention to subject matter knowledge, but also increased attention on helping teachers develop a collection of skills whereby they can test hypotheses about relationships between teaching and learning. Hiebert et al. (2007) considered hypotheses-testing skills as similar to components of Dewey's (1929) disciplined inquiry into teaching. Dewey had suggested that "systematic methods of inquiry" (p. 8) might serve to draw attention to an educational process of constructing, testing, and modifying hypotheses in order to understand and make better decisions. Similarly, Hiebert et al. (2007) claimed teachers need preparation that develops competencies including being able to set learning goals for students, to assess whether students are meeting lesson goals, to hypothesize whether a lesson did or did not work well, and to use hypotheses to revise lessons. Emphasis shifts from viewing teaching as an intuitive performance or an enactment of activities in the classroom to an analysis of teaching and learning through preparation and reflection *outside* the classroom; thus, teachers would be equipped to learn from the practice of teaching. Furthermore, Hiebert et al. (2007) identified Asian professional development activities (e.g., Lewis & Tsuchida, 1998; Ma, 1999; Stigler & Hiebert, 1999) as examples of evidence whereby intense focus on analysis of classroom lessons have been crucial in the improvement of teaching practice for several Asian countries.

Heaton (2000) described her experience in the "simultaneous acquisition and use of mathematical knowledge in the course of teaching" (p. 148). She took the risk of entering the classroom expecting to learn while teaching. But, rather than going into the classroom completely unprepared, Heaton considered teaching for understanding as preparing to improvise by viewing the textbook as a guide rather than script, by exploring the mathematical ideas herself, by predicting student thinking, and by then engaging in intellectual exchanges with students that may or may not result in what was predicted. Teachers would need support in order to learn how to recognize choices and make decisions about appropriate mathematical tasks, representations, and discourse that could foster students' understandings.

Standards-based teaching involves dramatic changes to both content and core dimensions of teaching. Teachers need to develop new knowledge and practice in order to enact the vision described by our current reform efforts. Teachers need incentives, training, and time in order to support these dramatic changes. In addition, more elaborate images of the vision need to be communicated to teachers.

Elementary and Middle School Mathematics Content and Teachers

Elementary

Principles and Standards for School Mathematics (PSSM) (NCTM, 2000) outlined five mathematics Content Standards to be addressed in all grade levels: Number and Operations, Algebra, Geometry, Measurement, and Data Analysis and Probability. The Algebra strand “emphasizes relationships among quantities, including functions, ways of representing mathematical relationships, and the analysis of change”(p. 37). In the lower grades, Algebra might involve describing patterns like 2, 4, 6, 8, ..., whereby skip counting by adding two to the previous number can be the beginning of recursive thinking. For Geometry, young children should identify shapes as well as focus on properties and relationships for classes of shapes. Measurement is important because of its propensity for use in daily life; children should become proficient in using measurement tools and applying formulas and techniques appropriately. For Data Analysis and Probability, teachers may scaffold learning experiences for younger children by framing the question and providing a tally sheet or chart for the children to record information in order to answer questions about data.

The remaining Content Standard, Number and Operations, is at the core for all mathematics in elementary and middle school (NCTM, 2000; National Research Council, 2001). In preschool and elementary school, children begin with the concept of whole number as it is the easiest number to understand and use (National Research Council, 2001). Many domains of the mathematics curriculum are intertwined with the concept of number. However, United States elementary education has typically focused on memorizing basic number facts to the neglect of developing other strands of mathematical proficiency including conceptual understanding, problem solving competence, reasoning processes, and mathematical power.

In *PSSM*, the development of number sense was described as central to the Number and Operations Standard. Number sense is a relatively new term that became popular in the late 1980s (Van de Walle & Lovin, 2006b). Verschaffel, Greer, and De Corte said “number sense is highlighted in current mathematics education reform documents as it typifies the theme of learning mathematics as a sense-making activity” (2007, pp. 580-581). However, number sense is difficult to characterize (e.g., Reys et al., 1999; Yang, Reys, & Reys, 2009), and therefore difficult to assess (Sowder, 1992). Even the definition of number sense has lacked consistency

(Van de Walle & Lovin, 2006b; Verschaffel et al., 2007). Some researchers (Yang, 2005; Yang et al., 2009) have used the definition: “Number sense refers to a person’s general understanding of numbers and operations and the ability to handle daily-life situations that include numbers. This ability is used to develop flexible and efficient strategies (including mental computation and estimation) to handle numerical problems” (Yang et al., 2009, p. 384). Van de Walle (2004) appreciated Howden’s (1989) description of numbers sense “as good intuition about numbers and their relationships. It develops gradually as a result of exploring numbers, visualizing them in a variety of contexts, and relating them in ways that are not limited by traditional algorithms” (p. 11). Despite the challenges in clearly defining number sense, some researchers have discussed various components of number sense (e.g., McIntosh, Reys, & Reys, 1997b; R. E. Reys et al., 1999; Sowder, 1992). For example, McIntosh, Reys and Reys (1997b) outlined major components of number sense as: (a) mental computation skills, (b) estimation skills, (c) understanding different but equivalent representations, (d) understanding relative size (including benchmarks), (e) understanding relationships and the effect of an operation between numbers, and (f) recognizing reasonableness.

Number sense is a complex process that develops over time (Reys, 1994; Yang & Reys, 2001). An over-emphasis on written calculation may result in children’s rote application of standard algorithms without regard to sense making (McIntosh, 2004). For example, if asked to perform a written calculation of $25 + 89$, most people would likely go through the standard algorithm for adding two-digit numbers. However, if a person was asked to perform the calculation mentally, he or she might be “creative, active, concentrating on number relationships and problem-solving” (p. 10) by breaking apart the addends in search of compatible pairs to group and add together. Teachers can play a primary role in helping children develop number sense through the tasks chosen and the questions asked (Yang & Reys, 2001). As “what is learned depends on what is taught” (National Research Council, 2001, p. 333), and as some mathematics curriculum place a heavy emphasis on computational procedures (Reys et al., 1999; Yang et al., 2009), the knowledge teachers have of number sense and the value they place on its importance may be critical factors for students’ opportunity to develop number sense (Yang et al., 2009).

Elementary teachers are trained as generalists and often teach all the subjects to a single class (Borasi & Fonzi, 2002). Many elementary teachers have taken only one college liberal arts

mathematics course. In addition, it is common for elementary teachers to express greater comfort and interest in teaching language arts than mathematics. Studies (e.g., Ma, 1999) have illustrated the weak conceptual understandings of U.S. elementary teachers for the mathematical content they are in charge of teaching. On the other hand, Schifter and Riddle (2004) asserted that elementary teachers should have a deep understanding “of the base-10 number system, the meaning of the basic operations, the logic of rational numbers, and the properties of geometric shapes” (p. 30). Mathematics teachers should also have mathematical knowledge that is connected and flexibly available in order to teach mathematics well.

Middle School

NCTM (2000) outlined ambitious expectations for middle school mathematics. NCTM suggested that all students in grades 6, 7 and 8 should have significant opportunities to learn algebra and geometry along with content on number, statistics, and measurement. Lest the curriculum become too fragmented, they suggested that proportionality could be used to support an integrated treatment of the content topics.

Middle school mathematics is of particular concern with regard to students’ future educational and career opportunities. Researchers and policy makers have voiced the importance for all students to take challenging mathematics courses in middle school as a gateway to advanced high school mathematics courses and subsequent college enrollment (Learning First Alliance, 1998; Riley, 1997; Silva & Moses, 1990). Repercussions for disadvantaged students are particularly relevant. Silva and Moses (1990) noted that enrollment rates in advanced mathematics courses are significantly lower for Blacks and Hispanics compared with Whites.

Many topics receiving emphasis in middle school mathematics can be considered gateways of their own accord. For instance, standard educational practice oftentimes identifies only “talented students,” who have mastered arithmetic and computational skills, for entry into algebra courses (Silva & Moses, 1990). The National Mathematics Advisory Panel claimed a “major goal for K-8 mathematics education should be proficiency with fractions (including decimals, percents, and negative fractions), for such proficiency is foundational for algebra and, at the present time, seems to be severely underdeveloped” (2008, p. xvii). In addition, recall *PSSM* (NCTM, 2000) expressed the centrality of number sense for the Number and Operations Standard. Researchers (Reys & Yang, 1998; Yang, 2005) have found that students highly skilled

in written computation may not have necessarily developed good number sense. Another aspect of Number and Operations, multiplicative reasoning, has been suggested to be the core foundation for students' conceptual understanding of many of the major arithmetic topics in middle school mathematics including rational number, ratio, rate, percent, and proportion (Sowder et al., 1998). However, multiplicative reasoning does not develop easily and requires schooling. Therefore, student understanding of middle school components of Number and Operation serves as a gatekeeper for enrollment in algebra.

Algebra has also been frequently associated with having a gatekeeper role (e.g., National Mathematics Advisory Panel, 2008; RAND Mathematics Study Panel, 2003; Silva & Moses, 1990). NCTM exhorted that "all students should learn algebra" (2000, p. 37). The RAND Mathematics Study Panel (2003) chose algebra as one of three areas of focus for long-term research because of the following: its foundation to other areas of mathematics, its gatekeeper role in educational and career options, and its growing prominence in high-stakes accountability testing and in proficiency testing for high school graduation.

Teacher training for middle school mathematics has a variety of paths (e.g., NCTM, 2000; Smith, Silver, & Stein, 2005). Although some teachers are specifically trained for middle mathematics, many others are initially prepared as secondary mathematics specialists or elementary generalists. Teacher preparation programs for elementary generalists often pay too little attention "to developing the specific proficiencies needed by mathematics teachers in the middle grades, where the mathematical ideas are more complex and difficult for students to learn" (Smith et al., 2005, p. xii). Secondary mathematics preparation programs often lack enough training about specific middle school issues including "adolescent development, pedagogical alternatives, and interdisciplinary approaches" (NCTM, 2000, p. 213). In addition, teacher preparation programs in general often lack coherence and relevance to real practice; mathematics is taught from a mathematics department and pedagogy from the education department (e.g., Smith et al., 2005). Hence, middle school mathematics teachers may well need additional support and development in order to increase their knowledge of mathematical content, pedagogy, and learners.

Teachers' Content Knowledge

Conceptualizing and Measuring Teachers' Content Knowledge

Philosophical arguments and admissions to common sense have long been used to assert that a teacher's own subject matter knowledge is an important determinant of his or her effectiveness in helping students learn subject matter (e.g., Ball & McDiarmid, 1990). However, early attempts to validate these arguments empirically were largely unsuccessful, as exemplified by Begle's (1979) widely cited study. Begle looked for relationships between teacher characteristics and beliefs with student mathematics achievement. Teacher characteristics included a substantial number of proxy measures for teachers' content knowledge (e.g., academic credits beyond a bachelor's degree, mathematics credits beginning with calculus, participation with in-service or extension courses, majoring or minoring in mathematics). Begle found the teacher characteristic which was the strongest indicator of teacher effectiveness, credits in mathematics methods, had only a positive significant relationship 24 percent of the time, whereas it had a negative relationship 6 percent of the time. Begle lamented in conclusion that "the effects of a teacher's subject matter knowledge and attitudes on student learning seem to be far less powerful than most of us realized" (p. 54).

With regard to Begle's work, Ball (1999) pointed out that "course-taking is not a good proxy for knowledge" (p. 21). Nonetheless, Floden and Meniketti (2005) reviewed and found some researchers are still investigating the relationship between teachers' subject matter knowledge and student achievement by using proxy measures of teachers' content knowledge such as whether the teacher majored or minored in the field or by the number of courses taken in the field. The most heavily studied subject is mathematics, particularly secondary mathematics. The majority of the studies meeting Floden and Meniketti's criteria for inclusion (e.g., Monk, 1994; Wenglinsky, 2002) reported a positive association between teachers' study of mathematics and student achievement; however, most of the studies also labeled the effect size as small.

Ball (1991a) noted that many researchers in the 1980s started to move away from using credits earned and course lists as proxy measures of teachers' subject matter knowledge to other conceptions for studying subject matter knowledge. Upon review, Ball found some researchers (e.g., Peterson, Fennema, Carpenter, & Loef, 1989; Thompson, 1984) were examining "teachers' *conceptions of or beliefs about mathematics*" (p. 6). For instance, Thompson (1984) used case studies of three teachers to examine the relationship between teachers' conceptions about mathematics and mathematics teaching with their instructional practice. Thompson found the

relationship between teachers' conceptions and practice to be complex, yet also found the conceptions played a "significant, albeit subtle, role in shaping the teachers' characteristic patterns of instructional behavior" (p. 125). Other researchers (e.g., Ball & McDiarmid, 1988; Leinhardt & Smith, 1985) were examining "teachers' *understanding of mathematical concepts and procedures*" (p. 6). For example, Leinhardt and Smith (1985) examined expert and novice teachers' subject matter knowledge using card-sorting tasks and interviews. Experts "exhibited a more refined hierarchical structure to their knowledge" (p. 252) while novices had a more horizontal structure of separate categories. Yet even upon classroom observation of a subset of the expert teachers, some exhibited well-developed conceptual understanding and connectedness between basic principles whereas others were tied to the efficient use of an algorithm.

In his 1985 Presidential Address to the American Educational Research Association (AERA) (published in 1986b), Shulman spoke extensively about forms of teacher knowledge. He noted researchers such as Bloom, Gagné, Schwab, and Peter had been describing various conceptions and organizations of content knowledge. For example, Schwab (1978) delineated substantive and syntactic structures of knowledge for a discipline. Shulman described Schwab's substantive structures as "the variety of ways in which the basic concepts and principles of the discipline are organized to incorporate its facts" (p. 9). On the other hand, Schwab defined syntactic structures as "different methods of verification and justification" (1978, p. 246) for a discipline.

Shulman (1986b, 1987), along with colleagues (Grossman, Wilson, & Shulman, 1989; Wilson et al., 1987), proposed an expanded conception of teacher knowledge. Shulman (1987) described categories of the knowledge base of teachers as including general pedagogical knowledge, knowledge of learners, knowledge of educational aims, knowledge of other content, subject matter content knowledge, pedagogical content knowledge, and curricular content knowledge. Wilson et al. (1987) suggested that teachers draw upon different types of knowledge when they are making decisions about the content they are teaching.

Most influential in his address to AERA, Shulman's (1986b) representation of content knowledge to include subject matter content knowledge (SMK), pedagogical content knowledge (PCK), and curricular knowledge highlighted the importance of content knowledge and brought it to the forefront. He suggested that research on teaching at the time was too focused on content free issues such as how teachers managed their classrooms, allocated time, and organized

activities; questions about teaching as related to a specific content area were missing. Shulman and colleagues (Shulman, 1986b; Wilson et al., 1987) described subject matter knowledge as knowledge of basic facts and concepts as well as an understanding of what Schwab had referred to as the structures of a discipline: how facts and principles are organized within a domain (substantive structures), and “canons of evidence and proof that guide inquiry in the field” (Wilson et al., 1987, p. 114) (syntactic structures). Later, Ball (1990a, 1991) expanded the concepts of substantive and syntactic structures with specific focus on mathematics. Per Ball, substantive knowledge *of* mathematics includes knowledge of topics (e.g., trigonometry), concepts (e.g., infinity), procedures (e.g., factoring), underlying principles and meanings (e.g., what division with fractions means), and relationships among the concepts (e.g., how fractions are related to division). Ball also broadened Schwab’s syntactic structure by describing another dimension of subject matter referred to as knowledge *about* mathematics. Knowledge *about* mathematics includes understandings such as: how truth is established in the field of mathematics; what reckons as a solution; which ideas are based on convention and which are built on logic; and how mathematics has developed and changed over time.

As another component of content knowledge, Shulman (1986b) suggested curricular knowledge includes knowledge about available curricular alternatives, lateral knowledge about curriculum students might be studying in other subjects, and vertical knowledge about preceding and succeeding topics in the same subject area. Finally, Shulman (1987) considered pedagogical content knowledge of special interest because of its blending of content and pedagogy. Pedagogical content knowledge was described as subject matter knowledge *for teaching* which includes “the most regularly taught topics in one’s subject area, the most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations” (p. 9). It also includes an understanding of what makes some topics easy or difficult to learn and what conceptions, preconceptions, and misconceptions students at various ages might have.

Based on a multitude of studies (as part of the Knowledge Growth in a Profession Project at Stanford University) from the mid to late 1980s investigating the role that subject matter knowledge played in beginning secondary teachers’ instruction, Grossman, Wilson, and Shulman (1989) summarized the findings by noting: “teachers’ subject matter knowledge affected both the content and processes of instruction, influencing both what teachers teach and

how they teach it.” For example, some teachers may avoid teaching topics they do not know well; or, they may choose lecture over soliciting questions from the students. Teachers’ knowledge of both substantive and syntactic structures of a discipline has implications for what they teach.

Ball and others (e.g., Ball, 1999; Ball et al., 2001) have often reflected about Shulman and colleagues’ proposed conception of teachers’ knowledge and its consequence for research in mathematics education. For example, attention placed on pedagogical content knowledge raised awareness that expert knowledge of mathematics did not necessarily translate to knowledge for teaching mathematics (Ball, 1999). In addition, Shulman’s conception of teachers’ knowledge was part of the impetus for new research whereby focus on teacher characteristics and mathematics course credit counting was transferred to a “qualitative focus on the nature of teachers’ knowledge” (Ball et al., 2001, p. 441). Furthermore, Shulman’s conception of teacher knowledge spurred two prominent lines of research: research on the interplay between teachers’ subject matter knowledge and student learning and on the interplay between “mathematics and pedagogy in teaching and teachers’ learning” (Ball, 1999, p. 22).

Other researchers (e.g., Ball, 1990a; Grossman, 1990; Leinhardt & Smith, 1985; Ma, 1999) proposed different organizations of teacher knowledge and made conjectures about what discipline-specific knowledge teachers need to have in order to teach (e.g., Hill, Schilling, & Ball, 2004; Hill, Sleep, Lewis, & Ball, 2007). Many descriptive studies have focused on teachers’ understandings of specific mathematics topics as related to teaching, such as generating a representation of division of fractions, rather than global understandings of mathematics (Ball et al., 2001). Unfortunately, many studies have revealed substantial weaknesses in U. S. teachers’ mathematical understandings (e.g., Ball et al., 2001; Ball, Hill, & Bass, 2005).

Ball (1990a, 1990b) investigated prospective teachers’ knowledge of division through tasks and interview probes. Tasks and probes had been written by Ball and others working with the Teacher Education and Learning to Teach project (TELT) at the National Center for Research on Teacher Education at Michigan State University (Ball, 1990a; Hill et al., 2007; Kennedy, Ball, & McDiarmid, 1993). The interview questions posed hypothetical teaching situations and asked teachers how they would explain or respond. For example, Ball (1990a, 1990b) asked prospective elementary and secondary teachers how they had been taught and how they would solve $1\frac{3}{4} \div \frac{1}{2}$. The prospective teachers were further asked to provide a real-world

situation or story problem that mathematically represented the problem in a way that could help a pupil understand the mathematical idea. Most of the prospective teachers were able to calculate the answer correctly. However, none of the elementary candidates and only about half of the secondary candidates could provide an appropriate representation for the problem. Ball surmised that the “prospective teachers’ knowledge of division seemed founded more on memorization than on conceptual understanding” (Ball, 1990b, p. 141). With Ball’s qualitative research on division, research on teachers’ knowledge turned “toward examining knowledge of mathematics that is specialized to teaching” (Hill et al., 2007, p. 129). Using fractions to solve a real-world problem is a reasonable expectation for a mathematically literate adult; however, providing an example of a real-life problem that illustrates the mathematical concept of division with fractions may be more likely limited to teachers’ knowledge.

A prominent cross-cultural comparative study was conducted by Liping Ma (1999). The mathematics educational community, including research mathematicians, was struck by Ma’s study of Chinese and American teachers, which revealed dramatic differences in subject matter knowledge and pedagogical content knowledge for the two groups (Ball et al., 2001). Ma (1999) used four mathematical task questions developed by Ball and others for the TELT project as interview questions. Once again division of fractions was used, but the other three scenarios included subtraction with regrouping, multidigit number multiplication, and the relationship between area and perimeter. Ma’s descriptions of interview data portrayed the fluency by which many Chinese teachers were able to provide conceptually correct models and explanations. In contrast, although the American teachers were able to show some algorithmic competence, their understandings and explanations were procedurally focused and lacked conceptual depth. Ma stated “the knowledge of the Chinese teachers seemed clearly coherent while that of the U.S. teachers was clearly fragmented” (p. 107). Ma observed that Chinese teachers had developed “knowledge packages” whereby “procedural topics and conceptual topics were interwoven” (p. 115). Furthermore, Ma found that about ninety percent of the Chinese teachers studied had developed partial or complete PUFM (profound understanding of fundamental mathematics) which encompasses more than sound conceptual understanding. Ma coined the concept of PUFM as mathematical understanding that “has connectedness, promotes multiple approaches to solving a given problem, revisits and reinforces basic ideas, and has longitudinal coherence” (p. 124). In contrast, U.S. teachers’ knowledge seemed fragmented and procedurally focused.

During the early 1990s, the nature of teachers' knowledge about division or rational numbers was also examined in other studies (Borko et al., 1992; Graeber et al., 1989; Post et al., 1991; Simon, 1993). Unfortunately, a dominant theme of this group of qualitative studies of both preservice and inservice U.S. teachers was the teachers' overall lack of deep understanding of mathematics they would be or were in charge teaching.

A few studies during the 1990s investigated teachers' knowledge of proof. Ball, Lubienski, and Mewborn (2001) surmised that the results of several studies (Ball & Wilson, 1990; Ma, 1999; W. G. Martin & Harel, 1989; Simon & Blume, 1996) suggested "that teachers are prone to accept inductive evidence, such as a series of empirical examples or a pattern, as being sufficient to establish the validity of a claim" (Ball et al., 2001, p. 447). For example, Martin and Harel (1989) found more than half the preservice elementary teachers studied accepted an inductive argument involving a single case of empirical evidence as a valid mathematical proof. In her cross-cultural study of U.S. and Chinese teachers, Ma (1999) found when teachers were posed a scenario of a student claim based on a single empirical example only three of the 23 (thirteen percent) U.S. teachers investigated the problem on their own and only one of those teachers successfully disproved the claim by producing a counterexample; on the other hand, sixty-nine percent of the Chinese teachers explored the problem and successfully disproved the claim. Ma also suggested that two factors may have hampered the U.S. teachers' investigations: for a few, their lack of computational proficiency, but more significantly, the U.S. teachers' attitude toward mathematics. "U.S. teachers behaved more like laypeople, while the Chinese teachers behaved more like mathematicians" (p. 104) as they demonstrated awareness that propositions needed to be proved.

Hill, Sleep, Lewis, and Ball (2007) noted another trend in the 1990s whereby some studies (Harbison & Hanushek, 1992; Mullens, Murnane, & Willet, 1996; Rowan, Chiang, & Miller, 1997) began to renew focus on how teachers' mathematical knowledge related to students' mathematics achievement. However, rather than use proxy measures such as mathematics coursework or degree attained, the researchers measured teachers' mathematical knowledge by administering problems to the teachers that had oftentimes been developed for students. Upon reflection on this group of studies, Hill et al. (2007) suggested that using assessment tools that had been originally developed for students with teachers might be theoretically limiting. Although they measured content that students should know themselves by

the end of schooling, they may be overlooking specialized knowledge for teaching. Furthermore, although much work had been done in the proposition of theoretical categories for teachers' content knowledge (e.g., Ball, 1990a; Leinhardt & Smith, 1985; Ma, 1999; Shulman, 1986b) and some elements of mathematical knowledge specialized to teaching had been uncovered (e.g., Ball, 1990a, 1991; Borko et al., 1992; Leinhardt & Smith, 1985), Hill, Schilling, and Ball (2004) asserted that there was "still much to be understood about the organization and structure of subject-matter knowledge in different disciplines and what these structures suggest for teaching" (p. 14). Much of the foundational work in ascertaining what mathematics teachers should know in order to teach had relied upon studies of prospective teachers, case studies of a single teacher, cross-cultural comparisons, and expert-novice comparisons. Scholars and evaluators recognized the need for more rigorous measurement tools in order to measure the relationship between a teachers' mathematical knowledge for teaching and student achievement and to track the development of teachers' mathematical knowledge over time (Hill et al., 2004; Hill et al., 2007). In the past decade, various research groups started to develop paper-and-pencil multiple-choice/short-answer assessments for which valid and reliable inferences could be made about the mathematical knowledge for teaching of individuals or groups (Hill et al., 2007). Hill et al. identified some instruments (e.g., Center for Research in Mathematics and Science Teacher Development, n.d.; Learning Mathematics for Teaching Project, n.d.; Schechtman et al., 2006) as having advanced the most in test development in terms of specifying domain maps, piloting and conducting psychometric analyses, and conducting some validity analyses. Setting the measures apart from other historical teacher tests, all of the measures contained some items representing specialized mathematics knowledge for teaching instead of only mathematical knowledge that is common to the general population. Hill et al. suggested that these types of efforts "have made an attempt to bring the measurement of teachers' mathematical knowledge closer to the actual practice of teaching" (2007, p. 138).

Many conceptions have been offered about teachers' knowledge (e.g., Ball, 1990a; Ma, 1999; Shulman, 1986b). Many approaches have been used to assess teachers' mathematical knowledge (see Hill et al., 2007, for a literature review). Different assessment approaches have revealed different things as assessments have different strengths and weaknesses. More recent conceptions of teachers' knowledge have been offered (e.g., Bransford, Darling-Hammond, & LePage, 2005), and promising assessment approaches are in development (Hill et al., 2007).

Development of Content Knowledge

Researchers (e.g., Ball & McDiarmid, 1990; Ball et al., 2001; Ma, 1999) have surmised that teacher subject matter knowledge can develop during various phases: elementary and secondary schooling, teacher preparation, and through the process of teaching and participating in professional development. For example, after Ma (1999) uncovered knowledge differences between U.S. and Chinese teachers, she delved further and examined two other groups of people in China: ninth-grade students and preservice teachers. Chinese participants from these two groups (as had the Chinese teachers group) displayed more conceptual understanding than the U.S. teachers. However, Ma also found differences between the understandings of the three Chinese groups. Chinese prospective teachers displayed more conceptual understanding and fewer misconceptions than Chinese ninth-grade students. In addition, Chinese teachers provided far more elaborate explanations than prospective teachers. Ma noted that in China, future teachers are attaining mathematical competence and understandings starting with their elementary schooling. Additional mathematical understandings for teaching develop throughout preservice education experiences and continue to develop through teachers' professional teaching experiences, potentially resulting in the development of PUFM. In contrast, Ma claimed U.S. teachers have often had low-quality elementary and secondary schooling and were "unlikely to have another opportunity to acquire it" (p. 145).

Ball and McDiarmid (1990) noted that a major portion of teachers' understandings are shaped prior to college and are likely to be procedurally focused rather than conceptually focused. Furthermore, U.S. teachers fail to gain subject matter understandings during teacher preparation. Researchers (Ball, 1990a; Grossman et al., 1989) have suggested that subject matter knowledge may have *not* figured prominently in teacher preparation due to time constraints in education courses. For example, based on a host of studies with novice secondary teachers, Grossman et al. (1989) said teachers were most likely, although not guaranteed, to attain substantive (explanatory frameworks) and syntactic (canons of evidence used in a discipline) structures of knowledge during their advanced undergraduate and graduate coursework in the disciplinary major. Furthermore, they noted considerable variation in beginning teachers' knowledge of syntactic structures. As knowledge of substantive and syntactic structures influenced teachers' instruction, Grossman et al. urged the integration of discussions about substantive and syntactic structures of disciplines in teacher education courses. In her study of

elementary and secondary teachers' understanding of division of fractions, Ball (1990a) found that although secondary education candidates had taken many more mathematics courses in college, the mathematics majors did not seem to convey considerably better understandings of underlying concepts and connections among mathematical ideas than the elementary candidates. When the secondary mathematics majors had to access mathematical concepts that they had been taught in elementary and high school, they often found their knowledge to be fragmented and lacking conceptual understanding. Ball recommended that more attention in teacher education courses needed to be given to the subject matter preparation of both elementary and secondary prospective teachers with regard to increasing their mathematical understandings for teaching. Similarly, Post et al. (1991) found only a minority of practicing intermediate level teachers knew enough about rational number concepts in order to be able to explain and discuss the topics in pedagogically acceptable ways. The researchers exhorted that preservice mathematics education courses needed a deep treatment of subject matter content rather than a superficial treatment that was more common at that time.

Some studies have described how preservice experiences affect the subject matter knowledge base of teachers. For instance, Floden and Meniketti (2005) noted although there was an overall lack of empirical evidence about what prospective teachers learn in preparation programs, the most heavily studied area was in mathematics. Floden and Meniketti reviewed 10 studies of prospective teachers' mathematics subject matter knowledge at or near the end of their teacher preparation education (e.g., Ball, 1990a, 1990b; Borko et al., 1992; Graeber et al., 1989; Simon, 1993). Unfortunately, Floden and Meniketti noted the dominant theme of the findings revealed that while prospective teachers may know basic rules and procedures, they "lacked a deeper understanding of the concepts they would later teach" (p. 270). The researchers cautioned that these analyses do not suggest that teacher preparation coursework had no effect, for prospective teachers' understandings might have been weaker before college. However, successful completion of college mathematics coursework does not imply that the prospective teachers have a deep understanding of mathematics concepts they would be in charge of teaching. "If the ability to explain basic concepts is important for teaching, then the subject matter courses teachers now typically take leave a large fraction of teachers without important subject matter knowledge" (p. 283).

Some studies (e.g., Frykholm, 2005; Geddis, 1993) have shown the impact of specific preservice experiences on the content knowledge of prospective teachers. For instance, based on case study analysis, Frykholm (2005) suggested that preservice elementary teachers' repeated experiences with reform-based middle school curriculum activities impacted their mathematical content knowledge, as well as their conceptions and beliefs about teaching and learning. Although the original intent for use of the curricular materials was to develop teachers' content knowledge, the use of the reform-based materials created a "rich context for learning simultaneously about mathematics, about children's thinking, and about pedagogy" (p. 32).

Illustrating a different impact, Simon and Blume (1996) investigated a program for prospective elementary teachers whereby the teacher educator strove to establish a classroom community committed to mathematical justification as opposed to holding the more traditional belief that the teacher or the textbook is the judge and authority. The teacher educator attempted to engage students in mathematical justification through questions based on students' contributions. For example, the teacher educator used questions such as "How do we know if they are right?", "Will it always work?", and "Why does this pattern exist?" It appeared that the prospective teachers progressed in developing an attitude of active participation in community justification.

Researchers (e.g., Ball & Cohen, 1999; Sherin, 2002) have suggested that implementing our current reform will require teachers to learn through the context of their practice. Practicing teachers may learn mathematics in both formal and informal settings. Some teacher learning may occur during a variety of informal day-to-day activities such as collaborating and sharing ideas with other teachers, reading, reflecting on their day's work, and examining student thinking (e.g., Darling-Hammond & McLaughlin, 1995; Wilson & Berne, 1999). Teachers may also learn in more formal settings such as professional development institutes.

Some studies have illustrated how teachers specifically gain content knowledge through experiences while teaching and participating in professional development. For instance, Ma (1999) interviewed three Chinese teachers who had developed what she called PUFM. Although the Chinese groups of ninth-graders and preservice teachers displayed more conceptual understanding than U. S. teachers, only some of the Chinese teachers had developed what Ma called PUFM (Profound Understanding of Fundamental Mathematics). She suggested that PUFM developed after they became teachers. Common themes emerged upon interviewing

three teachers with PUFM as to how they attained their mathematical knowledge. First, the teachers said they thoroughly studied teaching materials (curriculum materials). Second, the teachers expressed learning by collaborating with colleagues; they shared teaching ideas and collectively reflected on teaching. Third, the three teachers indicated that they learned from their students; that is, by appreciating and understanding a student's creative approach. Finally, the teachers suggested that they improved their mathematical knowledge by working mathematical problems beyond the content for which they were teaching; the teachers were enthusiastic about doing mathematics on their own.

Russell et al. (1995) suggested similar situations where teachers may learn mathematics through engagement in teaching practice based on qualitative data from their work with U.S. elementary teachers. First, they suggested that teachers may learn mathematics while exploring mathematics content in preparation of teaching. For example, learning may occur when a teacher examines a topic in several resource books, visits with colleagues about the content, and/or solves some mathematical problems. Second, learning may take place when a teacher assesses the reasonableness of a student strategy or representation that is different from his or her own. Finally, a teacher may learn while reflecting on student misconceptions and trying to understand why the misconception may have developed. Furthermore, the researchers cautioned that teacher learning of mathematics did not need to be viewed as remedial; they considered learning as a gradual process of deepening understanding and making new connections.

Remillard (2000) studied two fourth-grade teachers as they used a newly adopted "reform-oriented textbook" (p. 334) for one year. Remillard suggested the teachers learned mathematics in situations similar to those described by Russell et al. (1995). Some learning may occur when teachers read ideas in the text. However, Remillard found curricular materials were most likely to promote significant teacher learning when they engaged teachers in the processes of actively making adaptations to tasks or responding to unexpected student thinking in lieu of relying on familiar activities and ideas. Less learning occurred if the teacher used the curriculum examples verbatim.

Lesson study, a professional development model which originated in Japan and is now spreading in the U.S., provides a forum whereby teachers can increase their subject matter knowledge (Lewis, Perry, & Hurd, 2004). As teachers discuss essential concepts their students need to learn in the lesson, as they compare content development in different curricula, and as

they anticipate student thinking, the teachers “naturally generate many questions about the subject matter” (p. 19). The teachers can often answer their questions themselves; if not, the teachers can look for help from outside sources.

Simon (1997) described a model of teaching whereby students are encouraged to construct their own mathematical knowledge as an alternative to a teaching model focused on lecture and demonstration in order to impart knowledge. In Simon’s “Mathematics Teaching Cycle” (p. 76), the teacher tries to understand the students’ understanding of the mathematical ideas as opposed to trying to determine whether the student has gained some target knowledge. Through interactions with students such as problem posing, discourse facilitation, and listening to students’ ideas, a teacher may become aware of new mathematical representations or new connections between ideas. Thus, “a reflective teacher who is attentive to the mathematics of his students” (p. 80) is likely to gain knowledge during the teaching process.

As our current reform requires teachers to learn during the act of teaching, Sherin (2002) identified a type of teaching interaction whereby she suggested the most active teacher learning occurs. Upon analysis of videotaped lessons of two experienced teachers implementing a novel reform curriculum, she identified three types of interactions between teachers’ content knowledge and teaching practice. One type of interaction was labeled *transform*; in this interaction the teachers’ content knowledge did not change as the teacher transformed the new curriculum to reflect more traditional views. For the second type of interaction, *adapt*, some teacher learning occurred, but it was limited. Most commonly with *adapt*, a student’s comment or action triggered the teacher to provide a new explanation or make adjustments in his or her instruction. Finally, in the third type of interaction, *negotiate*, a student’s novel idea may have prompted the teacher to dramatically change the direction of the lesson. “Moreover, rethinking their knowledge of student understanding can prompt teachers to rethink other areas of their content knowledge as well” (p. 145). It is in the interaction of *negotiate* that Sherin believed the most active teacher learning occurs in connection with reform teaching. She also noted that the scarcity of *negotiate* interactions in her analysis reflects the complexity and difficulty in implementing reformed teaching practice.

Various viewpoints exist about the specific development of pedagogical content knowledge. In order for teachers to use appropriately metaphors and representations that illuminate substantive concepts, pedagogical content knowledge depends heavily on both

conceptual understanding and knowledge of students (Kennedy, 1997). A metaphor or explanation or representation that captures the essence of a big idea and works well with high school students may not work well with kindergartners. Grossman (1990) said past research suggested there were at least four possible sources for teachers to construct of pedagogical content knowledge: teachers' own experiences as students in classrooms of specific content, subject matter knowledge, teacher education (particularly subject-specific methods courses), and actual teaching experience. More succinctly, Wilson, Shulman and Richert (1987) proposed pedagogical content knowledge emerges as "teachers transform their content knowledge for the purposes of teaching" (p. 118). Ball and Bass (2000) claimed pedagogical content knowledge builds up "by teachers over time as they teach the same topics to children of certain ages, or by researchers as they investigate the teaching and learning of specific mathematical ideas" (p. 87). Barnett and Hodson (2001) provided specific occasions when they proposed pedagogical content knowledge most likely developed including "experience, discussion with more experienced colleagues, imitation, reflection on things seen and heard, attendance at professional conferences, and reading teacher journals" (p. 438). Sowder et al. (1998) found with their professional development work that practicing teachers tended to learn while thinking about how they would teach the concept themselves and how their students would think about the concept; however, preservice teachers were not able to anticipate student thinking. That is, they found practicing teachers made pedagogical decisions as they learned the content, and thus pedagogical content knowledge developed for the practicing teachers. On the other hand, Geddis (1993) suggested that a group of preservice science teachers uncovered pedagogical content knowledge while grappling with their own misconceptions or inability to support a correct answer, while considering why school children might develop misconceptions, and while discussing teaching strategies that could mitigate misconceptions.

In 1990, Ball & McDiarmid reviewed studies and reported that "although teachers' knowledge about learners, the curriculum, pedagogy, and the context seems to increase with their practice, whether they will learn enough about their subject matter from their teaching to shore up inadequate knowledge and understanding is unclear" (p. 446). With regard to this statement, discussion of professional development and studies revealing what teachers may learn through participation in professional development will be addressed later in the chapter.

Teaching Practice

Generalities about Teaching Practice

Teachers' beliefs and knowledge form an interrelated web with teaching practice. Upon review, some studies reported strong relationships between teachers' knowledge and teaching practice while other studies reported teachers' beliefs influenced teacher behavior (Pajares, 1992; A. G. Thompson, 1992). Other studies (e.g., Borko & Putnam, 1995; Putnam, 1992) have supported the influence of both knowledge and beliefs on teaching practice. Often, researchers assume there is a "one-way relationship between beliefs and practice, whereby teachers' beliefs change and changes in practice follow" (Philipp et al., 2007, p. 467). Although many professional development programs are based on the assumption that efforts to facilitate change in teacher beliefs should come before efforts to impact teaching practice, Guskey (1986) offered an alternative model. Based on evidence from several studies, Guskey posited that changes in teachers' beliefs would likely only occur after changes in student learning outcomes were evidenced. His model purports that professional development efforts should begin with efforts to change teaching practice, so that teachers would recognize change in student learning outcomes (as perhaps evidenced by test scores or student involvement during class), and subsequent changes in teachers' beliefs would result. Nonetheless, although the order of the process of teacher change might be argued, both knowledge and beliefs appear interrelated with teaching practice. In general, researchers agree that teacher "changes in beliefs, knowledge, and practice do not occur in isolation from one another" (Franke, Fennema, & Carpenter, 1997, p. 255). For example, Weiss et al. (2003b) found that while teachers most frequently cited state/district curriculum standards as influencing lesson content selection, teachers indicated their own knowledge, beliefs, and experiences influenced their instruction.

Researchers (e.g., Ball & Bass, 2000; Loucks-Horsley et al., 2003) commonly consider teaching practice as having both regularities and uncertainties. With regard to regularity, some topics are typically difficult for students. Ball and Bass suggested that teachers with pedagogical content knowledge can anticipate difficult topics and use approaches that "can help mediate the difficulties" (2000, p. 89), and thus exhibit preparedness for regularities. Loucks-Horsely et al. (2003) suggested past experience and an expert knowledge base can provide heuristics to guide decision making in practice. However, experience and knowledge do not provide a set of fixed

rules. Uncertainty and complexity in teaching practice make teaching more than passively enacting a plan. Novel situations in teaching require teachers be able to reason and flexibly call upon different kinds of knowledge such as knowledge of content, learners, and pedagogy (Ball & Bass, 2000).

No repertoire of pedagogical content knowledge, no matter how extensive, can adequately anticipate what it is that students may think, how some topic may evolve in a class, the need for a new representation or explanation for a familiar topic. (p. 88)

A mathematics teacher needs to be able to unpack his or her own compressed understandings into less polished forms that are more accessible to students; they need to know the content flexibly; they need to be able to listen to students.

Traditional and Standards-based Teaching Practice

Traditional U.S. teaching practice has been described as teacher presentation of definitions and procedures, followed by student practice of procedures (e.g., J. K. Jacobs et al., 2006; Stigler & Hiebert, 1999). Traditional teaching has emphasized algorithm and procedures for the purpose of student acquisition of skills (Franco, Sztajn, & Ortiagão, 2007). In their review of mathematics teaching practice, Franke, Kazemi, and Battey (2007) largely focused on classroom discourse as a primary feature of classroom practice. Communication in traditional mathematics classrooms has been dominated by the initiation-response-evaluation (IRE) (e.g., Cazden, 1986) discourse pattern, “where the lesson follows a teacher-dominated pattern of teacher-*initiated* questions, student *response*, and teacher *evaluation*” (Franke et al., 2007, p. 231).

In contrast, NCTM (2000) supports and encourages practices emphasizing student engagement in the Process Standards including reasoning and proof, communicating, making connections, interpreting representations, and problem solving. Thus, standards-based teaching emphasizes addressing NCTM Content Standards via engaging students in NCTM Process Standards.

Reasoning and Proof have been elevated to a prominent position as a NCTM Process Standard (Stylianou, Blanton, & Knuth, 2009). Today, “proof” is sometimes being interpreted subjectively as “what establishes truth for a person or community” and is “an activity that can permeate the whole mathematics curriculum” (Harel & Sowder, 2007, p. 806). Thus, proof has

received a “more prominent role throughout the *entire* school mathematics curriculum and is expected to be a part of the mathematics education of *all* students” (pp. 3-4). However, although all students should be exploring ideas, making conjectures, evaluating arguments, and justifying results, expectations should be consistent with students’ mathematical experience (NCTM, 2000). For example, students should reason inductively about patterns from specific cases at all levels. In later grades, students “should also learn to make effective deductive arguments” (p. 59).

The term “mathematical proof” (Harel & Sowder, 2007, p. 807) has been used to refer to proofs that “provide conclusive evidence for its truth by treating appropriately all cases covered by the generalization” (Stylianides & Stylianides, 2009, p. 315). Stylianides and Stylianides cautioned that although common student justification schemes such as empirical arguments (often relying on evidence from only a few cases) are valuable as methods of gaining insights, empirical arguments should not be treated “as equivalent to or as a substitute for” (p. 315) formal mathematical proof.

Formal mathematical proof “arose as response to a persistent concern for justification, a concern reaching back to Aristotle and Euclid” (Hanna, 1995, p. 46). The teaching of mathematical proof has long been a part of American school mathematics education, but historically has most been identified with the teaching of high school geometry (e.g., Stylianou et al., 2009). The status of proof in American mathematics education has been changing over the years. For example, amid “the demise of the ‘new math’, with its exaggerated emphasis on formal proof” (Hanna, 1995, p. 44), the stature of proof declined. Some interpretations of proof as authoritarian further undermined its status.

In response, some scholars made arguments about the importance and functions of mathematical proof in mathematics (e.g., Stylianou et al., 2009). Although historically the main function of proof was viewed in terms of verification (justification), de Villiers (1990) suggested more recent discussions expanded the notion of proof to include roles such as:

- *verification* (concerned with the **truth** of a statement)
- *explanation* (providing insight into **why** it is true)
- *systematisation* (the **organisation** of various results into a deductive system of axioms, major concepts and theorems)
- *discovery* (the discovery or invention of **new** results)
- *communication* (the **transmission** of mathematical knowledge). (p. 18)

Hanna argued that “proof conveys to students the message that they can reason for themselves, that they do *not* need to bow down to authority” (1995, p. 46). In fact, proof is an essential tool for encouraging understanding. Wu (1996) professed that although intuition and experimentation are important aspects of doing mathematics, there is no way of arriving at statements of truth for all cases without the use of formal mathematical proof. “Anyone who wants to know what mathematics is about must therefore learn how to write down a proof or at least understand what a proof is” (p. 222). Wu further suggested that the lack of proof outside of high school geometry distorted the field of mathematics. More recently, Stylianides and Stylianides suggested explanation and justification were functions of proof which “supported students’ engagement with mathematics as a sense-making activity” (2009, p. 318).

The lack of reasoning and proof in American mathematics has been evidenced in findings based on the TIMSS video studies of eighth-grade mathematics classrooms. Using data from the 1995 Video Study, Stigler and Hiebert (1999) described “there were *no* mathematical proofs in U.S. lessons” whereas “there were proofs in 53 percent of Japanese lessons and 10 percent of German lessons” (p. 59). The 1999 Video Study expanded participation to seven countries. Researchers (Hiebert et al., 2005) looking for special forms of mathematical reasoning (deductive reasoning, developing a mathematical justification, generalization from individual cases, use of counterexamples) found the U.S. had “low frequency or absence of deductive reasoning and use of counterexamples” (p. 118). Furthermore, the United States was the *only* country with *no* occurrences of development of a mathematical justification or generalization from individual cases. Although a majority of the U.S. eighth-grade teachers who participated in the 1995 & 1999 video studies indicated they were familiar with NCTM *Standards*, the findings from the Video Studies suggested “deductive reasoning and other special forms of mathematical reasoning were rarely evident” (J. K. Jacobs et al., 2006, p. 28) in classroom practices.

In *An Agenda for Action*, NCTM recommended problem solving “be the focus of school mathematics” (1980, p. 1). More recently, researchers have asserted the importance of problem solving in conjunction with procedural fluency and understanding of an organized set of concepts and facts (National Mathematics Advisory Panel, 2008; National Research Council, 2000; National Research Council, 2001). As an NCTM Process Standard, students should have opportunities to “build new mathematical knowledge through problem solving” (2000, p. 52) and students should be able to use a variety of strategies to “solve problems that arise in mathematics

and other contexts” (p. 53). Research (National Research Council, 2001) suggests that flexibility is a fundamental characteristic for proficiency in problem solving. Furthermore, “flexibility develops through the broadening of knowledge required for solving nonroutine problems rather than just routine problems” (p. 126). Not consistent with the NCTM Problem Solving Standard, analyses of the TIMSS 1995 and 1999 Video Studies revealed U.S. eighth-grade classroom teaching incorporated a “relatively strong emphasis on applying familiar procedures to a repetitive series of similar problems” (Jacobs et al., 2006, p. 29).

How mathematical ideas are represented is related to how they are understood. Representations that we may likely “take for granted—such as numbers expressed in base-ten or binary forms, fractions, algebraic expressions and equations, graphs, and spreadsheet displays—are the result of a process of cultural refinement that took place over many years” (NCTM, 2000, p. 67). In the last forty years, the increased availability of manipulative models, calculators, interactive software programs, and audiovisual materials has greatly influenced the way teachers can present mathematics (Seymour & Davidson, 2003). As students gain access to more representations, they are better able to think mathematically (NCTM, 2000). “In fact, mathematics can be said to be *about* levels of representation, which build on one another as the mathematical ideas become more abstract” (National Research Council, 2001, p. 19). As such, Representation is both a critical component of mathematics as well as an integral part of pedagogy for teaching of mathematics.

NCTM recommends that Connections be supported between students’ informal mathematical experiences and more-formal school mathematics, “between one mathematical concept and another, between different mathematics topics, between mathematics and other fields of knowledge, and between mathematics and everyday life” (2000, p. 132). Hiebert and Grouws (2007) identified a pattern across a range of studies (e.g., National Research Council, 2001) suggesting that one of the two key features linked with promoting students’ development of conceptual understanding of mathematics is for teaching to attend “explicitly to concepts—to connections among mathematical facts, procedures, and ideas” (2007, p. 383). Using TIMSS 1995 and 1999 Video Study data for U.S. eighth-grade classrooms, Jacobs et al. (2006) found problem solving emphasized correct implementation of algorithms and procedures rather than on making connections.

The Communication Process Standard interacts with the previous strands. *PSSM* asserts that when students are challenged to communicate their reasoning about mathematics, “they learn to be clear and convincing....Conversations in which mathematical ideas are explored from multiple perspectives help the participants sharpen their thinking and make connections” (2000, p. 60). Hence, Reasoning and Connections interact with Communication. Furthermore, as students try to communicate mathematical ideas, different Representations may be appropriate (e.g., some more formal, some less formal) for different grade levels.

Jacobs et al. (2006) suggested that external observable teaching practices which likely represent the intent of the *Standards* include actively “involving students in thinking critically about mathematical problems, and basing instruction on how students learn” (p. 13). On the other hand, observable practices such as incorporating technology, group work, and real-world problems may or may not represent the intent of the *Standards* depending on how they are implemented. So far, evidence (e.g., Jacobs et al., 2006; Spillane, 1999; Weiss, Pasley, Smith, Banilower, & Heck, 2003b) has not corroborated large-scale alignment of core teaching practice with recommendations of the *Standards*.

Researchers (e.g., Cohen & Ball, 2001; Elmore, 1996; Spillane & Jennings, 1997; Weiss et al., 2003a) have exhorted that more explicit and elaborate images of reformed practice need to be communicated. Furthermore, researchers (e.g., Borasi & Fonzi, 2002; Heaton, 2000) have suggested teachers need support during the process of changing teaching practice. Heaton (2000) described how changing her practice was not a mere matter of stopping an old pedagogy and starting a new one; rather, the learning process was complex and messy. “Framing the contrasts between old and new practice in terms of dichotomies of skills was not useful” (p. 147). Instead, she came to appreciate NCTM’s (1989) choice of heading titles whereby recommendations were made for instructional strategies needing “increased attention” (e.g., justification of thinking, problem-solving approach to instruction) and “decreased attention” (e.g., rote practice, teaching by telling). Furthermore, Heaton suggested:

emphasis of helping teachers change their practice should shift away from helping teachers learn new skills or strategies and away from supplying them with new math problems and manipulatives—specifics of what or how to teach—toward learning how to create and recognize choices and make decisions about appropriate math problems, representations, and responses to further students’ understandings. (p. 157)

Borasi and Fonzi (2002) suggested research on how people learn complex tasks may reveal what teachers need to experience in order to enact new teaching practices. For example, Collins et al. (1989) noted in the traditional apprenticeship model, apprentices learn through observation, scaffolded practice, and increasingly independent practice. The teacher's corresponding responsibilities in this model are modeling, coaching, and fading. Based on this apprenticeship model, Collins et al. theorized that a "cognitive apprenticeship" would require six basic teaching methods. Teachers would model a task, coach by observing and offering feedback as students carried out a task, and scaffold learning by helping the students if they could not complete the task on their own. Teachers would also try to promote students' articulation and reflection. Finally, teachers would push students to explore a task domain on their own. This apprenticeship model suggests that teachers learning new teaching practice should have opportunities to observe new practice and implement new practice in a supportive environment. Borasi and Fonzi (Borasi, Fonzi, Smith, & Rose, 1999; Borasi & Fonzi, 2002) described their professional development program whereby facilitators modeled inquiry-based teaching practices, teachers had experiences as learners of mathematics, teachers were supported by facilitators as they implemented a reform curriculum unit in their school, teachers had many opportunities to reflect on their experiences as teachers and as learners, and teachers were encouraged to participate in collaborative teams at their schools. Borasi et al. (1999) found that their program "*initiated* the process of rethinking beliefs and practices" (p. 63). Survey results also suggested that participants continued to use some of the reform strategies beyond the required curriculum unit implementation.

Obtaining Information about Teaching Practice

Describing the vision of mathematics reform has been subject to debate and interpretation. Analyzing classroom teaching is also subject to interpretation (e.g., Schoenfeld, 2008). To monitor the impact of efforts to reform education and teaching practices in the U.S., there has been a push for the routine collection of teaching practice data since the late 1980s (Mayer, 1999). Due to its cost-effectiveness, self-reported teacher survey data collection methods have often been employed (e.g., Weiss, Banilower, McMahon, & Smith, 2001) to obtain information about instructional practice for large scale studies (Mayer, 1999). However, findings of studies (Burstein et al., 1995; Hiebert & Stigler, 2000; Mayer, 1999) have revealed

inadequacies of using survey data. “Self-reports can be misleading” (J. K. Jacobs et al., 2006, p. 13). For example, Hiebert and Stigler (2000) found while the majority of teachers in the TIMSS video study reported they were implementing NCTM *Standards* in their classrooms, videotaped observation evidence did not corroborate the teachers’ self-reports. More often, teaching practice was aligned to the *Standards* in merely superficial ways.

Although less cost-effective and more time-consuming, classroom observations provide some advantages to survey data. Although survey data were inadequate, Mayer found classroom observation data were better able to “capture the *quality* of the interaction between teacher and student” (1999, p. 43). Burstein et al. (1995) also argued that some aspects of practice cannot be measured well without observing the interactions between teachers and students. For example, discourse patterns may reveal the extent of student participation and engagement during a lesson.

Lewis (2008) described the benefits of videotape as a powerful research tool. Although videotape does not contain the entire picture because we typically only see the view from one camera, “videotape is seen as one vehicle that suspends the action and holds it still long enough for it to be examined” (p. 7). The TIMSS video study of 1995 was the first of its kind as national samples of teachers from three different countries had been videotaped during classroom instruction (Stigler & Hiebert, 1999). Stigler and Hiebert described “figuring out how to analyze and summarize these videos was challenging” (p. x). In fact, the video data from both TIMSS 1995 and 1999 have been analyzed and reported on from various perspectives over the years (e.g., Hiebert & Stigler, 2000; J. K. Jacobs et al., 2006; Stigler & Hiebert, 1999). Similarly, an invitation by Alan Schoenfeld to several researchers to examine a six minute videotape of a classroom lesson resulted in a monograph illustrating “wide variations in interpretation” (J.M. Lewis, 2008, p. 5). Thus, besides the different methods of collecting data for analyzing classroom teaching, what researchers attend to in data collection and analysis also differs.

In conclusion, standards-based initiatives propose changes to core dimensions of teaching (Elmore, 1996; Floden, 1997; Spillane, 1999). So far, evidence (e.g., J. K. Jacobs et al., 2006; Spillane, 1999; Weiss et al., 2003b) has not corroborated large-scale alignment of core teaching practice with reform recommendations. Reformers (e.g., Borasi & Fonzi, 2002; D. K. Cohen & Ball, 2001; Elmore, 1996; Spillane & Jennings, 1997; Weiss, Pasley, Smith, Banilower, & Heck, 2003a) have suggested that more elaborate images of reform need to be communicated to teachers. In addition, changes to core dimensions of teaching will require opportunities for

teachers to observe new practice and implement new practice in a supportive environment (Borasi & Fonzi, 2002; Collins et al., 1989). Furthermore, ongoing professional development has been purported to be an essential mechanism for eliciting change in teachers' knowledge and teaching practice in support of school improvement (e.g., Desimone et al., 2006; Elmore, 2002; Hawley & Valli, 1999).

Professional Development

Increasing Awareness of the Importance of Professional Development

Sowder (2007) explained that the National Science Foundation (NSF) uses the term professional development to refer to both the preparation of prospective teachers and the continued development of practicing teachers. Rightfully, some research in the two areas intermingles and applies to both. For instance, research on beliefs about mathematics and learning mathematics, and changes in beliefs in response to professional development experiences, may relate to both prospective and practicing teachers. On the other hand, some issues, such as the effects of university methods courses, may apply to teacher preparation but not to professional development for practicing teachers. For this study, professional development refers to the narrower view of the continued development of practicing teachers. However, research on prospective teacher preparation may inform the study.

Teachers learn to teach through a variety of phases. Researchers (e.g., Nemser, 1983; Wilson & Berne, 1999) have described various phases of teacher learning occurring over time in various contexts; contexts prior to, during, and subsequent of teacher preparation. However, U.S. teachers have often been assumed to be competent once they have finished a teacher preparation program (e.g., Ball et al., 2001; Stigler & Hiebert, 1999).

Nemser (1983) characterized four phases of learning to teach. First, researchers have described informal influences on teacher learning occurring prior to and during formal teacher preparation that may powerfully shape teachers' beliefs about teaching. From a sociological perspective, Lortie (1975) described the influence of impressions from classroom experiences that teachers have collected during elementary, middle, high school, and college education. He suggested that "being a student is like serving an apprenticeship in teaching" (p. 61). The second phase of learning to teach involves preservice training whereby education courses provide the most formal part of exposing "future teachers to the knowledge base of the profession" (Nemser,

1983, p. 154). Researchers (e.g., Floden & Maniketti, 2005; Nemser, 1983) have claimed that we do not know very much about what is learned in this phase. However, Nesmer pointed out the limited research base suggests that formal teacher preparation is often not powerful enough to counteract the influence of early informal school experiences on teachers. Third, Nesmer distinguished an induction phase for learning to teach. Beginning teachers have described their first year of teaching as “intense and stressful periods of learning” (p. 158) during which some teachers come to feel their preservice education was inadequate. Teachers struggle to develop a system that allows them to cope with daily demands such as managing the class, preparing and teaching lessons, and grading papers. This becomes their style for teaching. Some teachers may develop a rigid, limited style of teaching that is resistant to future professional growth. Other teachers may feel motivated to continue to search for ways of improving their teaching. Finally, a fourth phase of learning to teach involves learning in the inservice phase. Nesmer suggested that some approaches to professional development, such as placing importance on teachers’ sharing their experiences with other teachers and viewing teachers as professionals, may stimulate ongoing professional growth during the final stage.

In 1999, Wilson and Berne noted that “calls for a commitment to teacher learning” (p. 173) had increased exponentially over ten years and surmised that several forces had likely influenced the increased attention on teacher learning. For instance, higher standards for students coincided with higher standards for teaching. In addition, reformers began to recognize that new curriculum and testing would not necessarily lead to reformed teaching practice. Putnam (1992) had provided a case study illustration of a teacher’s implementation of a newly adopted reform curriculum in California. Although being instructed to follow the new curriculum closely, the teacher enacted the curriculum through the lenses of her own knowledge and beliefs. She emphasized procedural aspects of the curriculum and deemphasized student sense-making and classroom discussion. With such images, reformers were recognizing the importance of professional development in supporting changes to teaching practice. A second force influencing attention to teacher learning involved increased efforts for recognition of the professionalism of the teaching profession. Addressing higher standards for professional teaching (e.g., NCTM, 1991) would require more professional development. Finally, calls were being made for more research on teacher learning. Teacher learning occurs over time across many contexts; however, what was known about teacher learning was scattered or perplexing

(Wilson & Berne, 1999). Informal learning experiences occurred in early school experiences described by Lortie's (1975) "apprenticeship of observation"; informal learning opportunities continued once teachers entered the profession, sometimes stemming from conversations with colleagues, observations while passing other teachers' classrooms, and through the daily practice of teaching (Wilson & Berne, 1999). More formal learning opportunities existed through teacher preparation and professional development for practicing teachers. However, while professional development opportunities existed, they typically consisted of inservice training workshops that were viewed by teachers as ineffective sources of learning (Smylie, 1989).

Also in 1999, Hawley and Valli outlined four converging developments that focused attention on needed changes in professional development in order that school improvement might be achieved. First, in congruence with Wilson and Berne (1999), higher standards for students necessitated greater demands on teaching. Professional development was recognized as a potential avenue for eliciting change in teachers' knowledge, beliefs, and practice in support of school improvement. Second, research was revealing a "symbiotic relation between professional development and school improvement efforts" (Hawley & Valli, 1999, p. 129). School improvement did not occur without a culture of professional development; conversely, professional development failed in the absence of a supportive school environment. Third, much of the research on learning was identifying learner-centered principles. For instance, one's past experiences serve as a foundation for future learning. Furthermore, learning is social, develops in common stages, is enhanced through motivation, and is supported by metacognitive skills. Teachers would need professional development to learn about the new research on learning and how it related to teaching practice. Finally, critics were describing professional development as "shallow and fragmented" (Hawley & Valli, 1999, p. 134). Research was confirming that the conventional strategies of professional development were often inadequate. Consensus was building about the need for more effective professional development.

The case study of Mrs. Oublier by D. K. Cohen (1990) has been offered as a classic illustration (National Research Council, 2000; Sowder, 2007) of how workshop professional development opportunities were inadequate for eliciting reformed teaching practice for a teacher. Mrs. Oublier was provided a written reform framework (which she read), reform curricula (which she used), and she was sent to a few summer workshops put on by the publisher. She embraced trying to enact change in her teaching practice to reflect reform initiatives.

Nonetheless, although she tried to use the curricular materials to teach for understanding, her practice often remained teacher-centered in discourse, rigid in classroom management, and knowledge was viewed starkly as correct or incorrect. Cohen described Mrs. Oublier's teaching as a remarkable mix of traditional and reform practices. Just as reform proponents argued that students should not be expected to understand math by simply being told, teachers should not be expected to learn to teach in new ways by merely being told. Teachers "would have to acquire a new way of thinking about mathematics, and a new approach to learning it" (p. 327). Cohen surmised that brief explanations and a few workshops had been inadequate support for Mrs. Oublier to fully enact reformed teaching.

Elements of Effective Professional Development

Despite claims (Hawley & Valli, 1999; Wilson & Berne, 1999) that professional development was commonly being touted as inadequate a decade ago, researchers (Borko & Putnam, 1995; Kennedy, 1999; Richardson & Placier, 2001; Wilson & Berne, 1999) were able to review research literature or examine exemplary in-service professional development programs to glean common characteristics of effective design principles. Richardson and Placier (2001) reviewed studies on in-service professional development and found "long-term, collaborative, and inquiry-oriented programs" (p. 921) often appeared to be successful in changing teachers' beliefs and practices. Borko and Putnam (1995) reviewed three professional development programs that resulted in successful changes in the teachers' knowledge base and teaching practices. Common characteristics included: (a) a focus on subject-specific pedagogy and knowledge, (b) professional development approaches that reflected teaching practice as advocated by reformers, and (c) ongoing support as teachers adapted new teaching strategies into their own practice. Wilson and Berne (1999) identified common themes across some exemplary programs. For instance, communities of learners had been established in the programs which had provided a platform whereby teaching practice could be reconceptualized. In addition, whereas traditional professional development focused on the packaging and delivering of knowledge, exemplary programs focused on activating teacher learning. Around the same time, Kennedy (1999) analyzed the few studies on in-service programs where the effectiveness of various teaching approaches in mathematics and science professional development were examined in conjunction with subsequent benefits to students. She found that programs with a

focus on subject matter knowledge, particularly as related to how students learn the particular subject, had stronger positive benefits for student learning than programs that focused mainly on generic teaching behaviors that could be applied to any subject area. Factors that appeared less predictive about benefits to students concerned the structure and form of the programs.

Over the past decade, professional development, teacher learning, and teacher change have continued to remain a primary focus for mathematics education and reform initiatives (e.g., Desimone et al., 2006). Elmore (2002) said that the “next stage of development in American education, propelled by the advent of performance-based accountability, requires the development of a practice of continuous school improvement” (p. 28). Furthermore, professional development is at the core of the process of improving teachers’ knowledge and practice in order to improve student learning. Our current mathematics reform initiatives involve dramatic changes to both core teaching practices and content (e.g., Floden, 1997). Research on both preservice and inservice U.S. teachers has revealed weaknesses in their mathematical understandings of the content they would be or were in charge of teaching (e.g., Ball, 1990a; Graeber et al., 1989; Ma, 1999; Simon, 1993). Research has also shown that even while teachers expressed that they were aware of current ideas about mathematics teaching and learning, and believed that they were implementing the reform ideas, evidence from their classroom practice suggested that many teachers retained the core of traditional practice (e.g., Hiebert & Stigler, 2000; Jacobs et al., 2006; Spillane, 1999).

Upon increased admonitions that teaching should be viewed as a career-long learning process, researchers and policymakers (e.g., National Research Council, Committee on Science and Mathematics Teacher Preparation, 2001) have earnestly called for high-quality ongoing professional development opportunities for teachers. To this end, there has been significant funding for professional development by the U. S. government and state and local programs. However, Borko described professional development opportunities for teachers as “woefully inadequate” (Borko, 2004, p. 3). Elmore (2002) explained that while consensus had been building about characteristics of effective professional development, including focus on performance goals, emphasis on building teachers’ content knowledge and developing pedagogical skills of effective instruction, using theories of learning, using group settings, and moving learning closer to daily practice, “there is little evidence that this consensus has had a large-scale effect on the practices of schools and school systems” (p. 10). American school

systems often organize professional development around days that are contractually specified. Sessions tend to be designed to serve broad audiences, and topics are often disconnected. Furthermore, Elmore identified several barriers in the American educational system for effectively using professional development as a mechanism for large-scale improvement. Large-scale improvements would require competence in several individual and organizational domains including the acquisition of new knowledge, use of incentives, and investments in resources and capacity. For example, an organization should only expect teachers to learn new knowledge and skills if the system has the capacity to support teachers as they develop their practice in the classroom. If teachers learn new skills, yet return to unchanged work conditions, improvement will not likely occur. The practice of school improvement requires change in three areas including people's values and beliefs, work structural conditions, and the ways in which people learn to do work. Elmore contended that learning would be difficult and uncertain, and thus should best be done in close proximity to practice. Furthermore, American school systems would need to reorganize themselves in order to support effective professional development and large-scale school improvement. On the other hand, Loucks-Horsley, Love, Stiles, Mundry and Hewson (2003) found several positive developments occurring in professional development including an expanded research base, more resources, more purposeful designs, more focus on deepening content knowledge and knowledge of student thinking, more strategies embedded in the daily work of teachers, and fewer short-term workshops.

Lists of elements of effective professional development have continued to be discussed and compared. Sowder (2007) compared lists of elements of effective professional development based on literature reviews (Elmore, 2002; Hawley & Valli, 1999) and practical experience (Clarke, 1994). Upon synthesis, she noted:

Primary commonalities include the role of determining the purpose of a professional development program, the role of teachers in deciding on foci (or, at least, a focus on issues relating to teachers' needs), the need to have support from other constituencies (e.g., administrators, peers, parents) to undertake changes in instruction, the important role of collaborative problem solving, the need for continuity over time, the necessity of modeling the type of instruction expected, and the need for assessment that provides teachers with feedback they need to grow. (p. 171)

Of interest, developing a strong content-knowledge base was missing from Hawley and Valli and Clarke's lists. Sowder considered that perhaps it had been assumed; or, content-knowledge has been considered more important for some areas, like mathematics, and less important for other areas. Lists by Friel and Bright (1997) and Ball and Cohen (1999) did include developing a strong content-knowledge base. For example, Friel and Bright's list referred specifically to mathematics professional development and included many of the above elements as synthesized by Sowder, but also recommended focus on subject matter knowledge, on understanding children's thinking, on using curriculum as a professional development tool, on developing teacher leadership, and on acknowledging and supporting teachers as they dealt with tensions of change. Borasi and Fonzi (2002) described several common features of successful professional development in support of school mathematics reform. Their list suggested that professional development opportunities should be sustained and intensive, informed by constructivist theories on how people learn, and focused on activities engaging teachers as learners and in close-proximity with practice. In addition, professional development should foster collaboration and offer a wide range of diverse experiences.

On a related note, the National Research Council (NRC) (2000) made recommendations for professional development learning environments based on research on how people learn. Learning is complex: new knowledge builds on prior knowledge; learning is active rather than passive; and deeply connected understanding as well as factual knowledge support transfer of knowledge to other contexts. The NRC suggested that principles of learning environments that should be cultivated in schools and classrooms also apply to adult learning in professional development programs. Effective professional development programs would be learner-centered, knowledge-centered, assessment-centered, and community-centered. The NRC also suggested that professional development should be evaluated according to these perspectives. Learner-centered environments would "attempt to build on the strengths, interests, and needs of the learners" (p. 192). Knowledge-centered environments include a content focus on developing understanding of mathematics and mathematics teaching and learning that a teacher would need for implementing the vision of reform. Assessment-centered environments of professional development would be designed to help teachers reflect on teaching practice. Teachers would have opportunities to test their understanding, and feedback would be provided. Finally, community-centered environments would "involve norms that encourage collaboration and

learning” (p. 197). Unfortunately, the researchers suggested many professional development programs violate the principles of environments that would support optimal learning.

Researchers (Borko, 2004; Garet et al., 2001) have noted that although the research base on teacher learning, professional development, and teacher change has grown, some areas have still received little attention. Borko (2004) claimed we are still only beginning to identify what and how teachers learn in professional development, and how teacher change might impact student outcomes. Garet et al. (2001) noted that very little research has been “conducted on the effects of alternative forms of professional development” (p. 917). Nonetheless, a few studies have provided some insight.

Garet et al. (2001) studied the effects of different characteristics of professional development on teachers’ learning. Results of their study indicated the degree to which three core features (degree of content focus, opportunities for active learning, and degree of coherence) were found in professional development all had significant, positive influence on teachers’ knowledge as self-reported by a national sample of teachers who had participated in Eisenhower-funded professional development activities. Furthermore, they found teachers who reported enhanced knowledge were more likely to report changes in teaching practice. The researchers also found the three structural features studied (whether or not it was reform type, duration, and degree of collective participation) appeared to influence core features. Hence, their results provided some empirical evidence that sustained professional development experiences were more likely to have an impact on teachers than short-term experiences. Later, Desimone, Porter, Garet, Yoon, and Birman (2002) used a longitudinal study over three points in time of a sample of 30 schools to build on the results of their national sample study. They found that professional development focused on specific teaching practices (technology use, use of higher order instructional methods, and use of alternative assessment practices) “had effects on the use of those practices in the classroom” (p. 99).

D. K. Cohen and Hill (2000, 2001) studied the influence of professional development on teacher practice by using survey data from 1994 of California elementary school teachers. They examined teachers’ opportunities to learn during professional development activities. Some teachers had participated in Marilyn Burns Institutes which often focused on teaching specific math topics or examining curricular replacement units consistent with reform. In contrast, some teachers participated in workshops focused on more generic topics (e.g., cooperative learning,

classroom management) or issues peripheral to subject matter (e.g., using math manipulatives). Teacher participation also varied on the amount of time spent in professional development activities. Cohen and Hill (2000) found that teachers who participated in more content specific workshops reported more reform-oriented practice and less traditional practice. In addition, the greater amount of time that teachers participated in content specific professional development corresponded to teacher reports of more frequent reform practice. Furthermore, when teachers had more opportunities to learn about new mathematics curricula or assessment methods, students scored higher on state mathematics assessments (Cohen & Hill, 2001).

Smith, Desimone, and Ueno (2005) studied teacher survey responses to the 2000 National Assessment of Education Progress (NAEP) Mathematics Assessment. They found that teacher “participation in content-related professional development, after control for experience, formal educational degrees, and self-reported content knowledge, is positively associated with increased use of reform teaching strategies” (p. 101). For the study, reform teaching was operationally defined as an emphasis on conceptual learning goals as opposed to procedural goals, and an increased use of conceptual learning strategies such as community discourse, cooperative learning, and using real-world situations.

Several studies about professional development and changes in teachers’ knowledge utilized measures developed as part of the Study of Instructional Improvement/Learning Mathematics for Teaching (SII/LMT) project (Learning Mathematics for Teaching Project, n.d.). Ball and researchers, as part of the SII project, began to write multiple choice items in 2001 intended to measure “mathematical knowledge used in teaching elementary mathematics” (Hill et al., 2004, p. 14). They started with items in three content areas: (1) number concepts, (2) operations, and (3) patterns, functions, and algebra. Within the content areas, they wrote items for different categories of knowledge as modified from Shulman, Wilson, Ball and others. Initial domain categories focused on knowledge of content and knowledge of students and content. But, knowledge of content was further differentiated between common knowledge and specialized knowledge. Common content knowledge items probed mathematical skills and knowledge in the public domain. Specialized content knowledge included items that engaged teachers in analyzing alternative algorithms, representing numbers with manipulatives, and providing explanations for mathematical rules. Knowledge of students and content corresponded to “Shulman’s ‘common student misconceptions’ portion of pedagogical content knowledge” (Hill

et al., 2007, p. 133). Items were assigned and balanced to various forms and piloted in California's Mathematics Professional Development Institutes (MPDIs) (Hill & Ball, 2004; Hill et al., 2004). The MPDIs were "designed to be content-focused, extended learning opportunities for teachers" (Hill & Ball, 2004, p. 331). Analyses based on the initial piloting tentatively suggested several things. For instance, results suggested that common content knowledge and specialized content knowledge were related, but not equivalent. That is, a person might "have well-developed common knowledge yet lack the specific kinds of knowledge needed to teach" (Hill et al., 2004, p. 24). On the other hand, a teacher might have some specialized knowledge for teaching, but exhibit weak common mathematical knowledge. If the result can be replicated, it supports the need for professional development activities that address teachers' specialized mathematical knowledge for teaching. Also, upon analyses of pre- and post-test performance for teachers participating in an MPDI, results suggested that teachers did learn mathematics for teaching in the context of a single professional development institute focused on mathematics content (Hill & Ball, 2004). Results also generally suggested the longer the institute, the greater the increase in teachers' knowledge; however, some counterexamples existed. Furthermore, a variable labeled as "opportunity to engage in mathematical analysis, reasoning, and communication" (p. 343) was positively and significantly related to teacher learning whereas other variables such as teachers' desire to learn and mathematical content covered were not significantly related. In another study, Hill, Rowen, and Ball (2005) reported they had found that teachers' mathematical knowledge for teaching (including common and specialized content knowledge) "positively predicted student gains in mathematics achievement during the first and third grades" (p. 399). Furthermore, the direct assessment of teachers' content knowledge for teaching was a better predictor of student gains than proxy measures such as courses taken and years of experience. Hill, Rowen, and Ball suggested the results support policy initiatives directed at improving teachers' mathematical knowledge through content-focused professional development and preservice education in order to improve student learning.

Desimone, Smith, and Ueno (2006) reflected that sustained, content-focused "professional development has emerged as perhaps the most important type of in-service teacher education" (2006, p. 182). In addition, emerging research suggests that professional development focused on both content and how children learn that content are important elements in changing instructional practice

Goals for the Professional Development of Teachers of Mathematics and Strategies Being Used to Address the Goals

As described earlier, current U.S. reform proposes dramatic changes in both content and core dimensions of teaching. Researchers (e.g., Ball & Cohen, 1999; Darling-Hammond, 2006a; Hiebert et al., 2007) have suggested that teachers need opportunities to develop subject matter knowledge and a stance of inquiry into practice in order to be able to learn from practice. Borasi and Fonzi claimed “the ultimate goal of any professional development program supporting school mathematics reform should be to develop among teachers the mindset that they are lifelong inquirers” (2002, p. 22). Researchers (e.g., Ball & Cohen, 1999; Elmore, 2002) have also suggested that professional development would best be done in close proximity to practice due to the complexity of the learning required. S. Cohen (2004) noted that broad outlines of features of professional development that support teacher learning and teaching practice have coalesced into a view for a “new genre of professional development” (p. 6) whereby the “focus of professional development is on both issues of subject matter and issues of teaching and learning as they come together in classroom practice, and as real students work at building new understandings of specific content” (p. 3). Researchers (e.g., Borasi & Fonzi, 2002; Sowder, 2007) have offered recommendations of goals for the professional development of mathematics teachers. Researchers (e.g., Borasi & Fonzi, 2002; S. Cohen, 2004; Sowder, 2007) have also provided illustrations of successful professional development strategies and successful professional development programs.

Sowder (2007) synthesized teacher needs for implementing reform as expressed by other researchers (Ball & Cohen, 1999; Borasi & Fonzi, 2002; Elmore, 2002; G. Sykes, 1999) into groupings identifying six goals of professional development programs serving mathematics teachers. The goals, sometimes intertwined, for supporting teachers’ needs include developing:

- (a) a shared vision for mathematics teaching and learning, (b) a sound understanding of mathematics for the level taught, (c) an understanding of how students learn mathematics, (d) deep pedagogical content knowledge, (e) an understanding of the role of equity in school mathematics, and (f) a sense of self as a mathematics teacher. (p. 161)

These six goals generally form a subset of nine goals identified by Borasi and Fonzi (2002). Borasi and Fonzi outlined several goals more strongly related to pedagogy including understanding pedagogical theories underlying mathematics reform, learning effective teaching

and assessment strategies, and developing “an attitude of inquiry toward one’s practice” (p. 10). In addition, they recommended that teachers needed the opportunity to become familiar with exemplary curricular resources and instructional materials.

Researchers and expert practitioners (e.g., Borasi & Fonzi, 2002; Loucks-Horsley et al., 2003; Sowder, 2007) have also described different strategies being used in mathematics professional development. No single model of professional development works best for all; different strategies work better in different contexts in order to address different goals and needs (Borasi & Fonzi, 2002; Loucks-Horsley et al., 2003). Loucks-Horsley et al. (2003) organized eighteen strategies into six groupings: “(1) aligning and implementing curriculum, (2) collaborative structures, (3) examining teaching and learning, (4) immersion experiences, (5) practicing teaching, and (6) vehicles and mechanisms” (pp. 112-113). Designers of professional development may combine different strategies to address different goals or outcomes. Loucks-Horsley et al. suggested four common outcomes driving mathematics professional development designs include increasing content knowledge, increasing pedagogical content knowledge, building a professional learning community, and developing leadership. Loucks-Horsley et al. also provided some examples of appropriate combinations of strategies. For instance, “increasing teachers’ content knowledge is often best accomplished by immersing teachers in content as learners themselves” (p. 114). They suggested that this might well be accomplished through immersion strategies (such as focusing on problem solving or immersion into the world of mathematicians), through collaborative partnerships (perhaps between teachers and mathematics faculty at a university), and through the vehicle of a summer institute. “But learning content alone will not lead to changes in teaching, so designers must build in opportunities for teachers to put the content they learn into the context of teaching” (p. 114). This might be accomplished through aligning and implementing curriculum strategies or examining teaching and learning strategies via case discussions, examining student work, or lesson study. In addition, addressing the outcome of building a professional learning community might be accomplished through collegial arrangements including engagement in strategies such as study groups, lesson study, and demonstration lessons. Finally, developing leadership might be addressed through training specifically directed to developing facilitators.

Using a different categorization, Borasi and Fonzi identified “five main *types of professional development experiences*” (2002, p. 33) most frequently described in the literature

on professional development that supports mathematics reform. The types of experiences included: teachers engaging in mathematical experiences as learners; opportunities for analyzing student thinking; use of case examples of classroom practice such as video snippets or written narratives; teacher experimentation with innovative practice in a supportive setting; and traditional activities such as gathering and making sense of information in articles or presentations or conducting action research. Although the categories reveal the types of most frequently reported experiences, projects often used a wide variety of activities.

Sowder recently outlined an extensive review of examples of professional development contexts and strategies that have documented success in “providing teachers with the professional knowledge they need to teach mathematics well” (p. 173). As a qualifier, although she identified professional development projects as exemplifying a successful approach to developing teachers’ knowledge, she pointed out that many projects used more than one approach. The studies identified by Sowder often revealed content-focused professional development activities incorporating inquiry approaches to teaching and learning done in close proximity to practice. Sowder chose to organize a discussion about successful types of professional development around three relationships between knowledge and practice as expressed by Cochran-Smith and Lytle (1999). First, Cochran-Smith and Lytle described *knowledge-for-practice* as the commonly referred to “knowledge base” for teaching which is primarily determined and generated by scholars and researchers, and then shared with teachers. In this conception of knowledge and practice, “teachers are knowledge users, not generators” (p. 257). Sowder outlined four common approaches being used to provide settings for developing knowledge-for-practice: focus on student thinking, on curriculum, on classroom activities and artifacts by the use of case studies, or on mathematical knowledge for teaching through formal coursework. Cochran-Smith and Lytle’s second image, *knowledge-in-practice*, emphasizes knowledge in action. Practical knowledge is acquired by teachers through “reflection about and inquiry into experience” (p. 262). From this perspective, professional expertise comes largely from competent teachers themselves, rather than from scholars and researchers. Furthermore, knowledge is viewed as socially constructed. For example, experienced teachers may help induct novice teachers. Sowder described professional communities, professional development schools, and lesson study as settings for learning-in-practice. Finally, in the conception of *knowledge-of-practice*, teacher learning occurs as teachers do research in their classrooms in

order to become transformative agents for the larger educational community. Teachers learn “by challenging their own assumptions; identifying salient issues of practice; posing problems; studying their own students, classrooms, and schools; constructing and reconstructing curriculum; and taking on roles of leadership and activism in efforts to transform classrooms, schools and societies” (Cochran-Smith & Lytle, 1999, p. 278). Sowder suggested teachers engaged in action research by themselves, with inquiry teams or with university researchers might be developing knowledge-of-practice.

The following discussion generally follows Sowder’s (2007) organization of strategies aimed at developing teachers’ knowledge-for-practice through a focus on student thinking, on curriculum, on classroom activities and artifacts by the use of case studies, or on mathematical knowledge for teaching through formal coursework. Several projects are discussed at length in this section because of their extensive research base, unique features, and/or relationship to the study at hand. In addition, lesson study was identified by Sowder as a strategy that can be used to develop knowledge-in-practice and knowledge-of-practice. Lesson study will be fully reviewed in a separate section of the literature review because of its relevance to this study. Finally, professional development schools, another strategy for developing knowledge-in-practice, will be discussed in Chapter 3.

Both Sowder (2007) and Borasi and Fonzi (2002) highlighted the Cognitively Guided Instruction (CGI) project for its effectiveness in using a focus on student thinking as a professional development strategy. CGI researchers have generated an extensive body of research about their project. In one of their first publications, CGI researchers (Carpenter, Fennema, Peterson, Chiang, & Loef, 1989) described an experimental study of 40 first-grade teachers whereby half were randomly assigned to a treatment group and the other half to a control group. The teachers in the CGI treatment group participated in a four-week summer workshop focused on familiarizing the teachers with research-based knowledge about the development of children’s thinking and problem solving with regard to the content domains of addition and subtraction. Of note, Shulman (1986a) had specifically included teachers’ knowledge about both classifications of student conceptions on certain topics, and about stages of development that students will likely go through, as elements of pedagogical content knowledge. The teachers in the control group participated in four hours of professional development focused on solving non-routine problems. Pre- and posttests were administered to

teachers and students, and teachers were observed teaching mathematics over a four-month period. The teachers in the CGI group performed significantly better at predicting students' number facts strategies and problem-solving strategies. In addition, the CGI treatment affected teachers' beliefs and classroom practices. CGI teachers spent more time on problem solving and less time on number fact problems than did control teachers. CGI teachers also listened to students' processes for solving problems more often than control teachers. Furthermore, students in CGI classes outperformed students in control classes on some of the student achievement measures; on other achievement measures, there was no statistically significant difference. Finally, CGI students reported greater problem-solving confidence and understanding of mathematics than did control students.

After the experimental study, some CGI researchers conducted case studies of six of the teachers from the CGI treatment group and documented how the teachers used knowledge of student thinking in making instructional decisions. They found the "teachers listened to students and attempted to build on the students' knowledge" (Carpenter & Fennema, 1992, p. 468). The teachers were able to match problems and difficulty to their students' abilities. An additional report (Fennema, Carpenter, Franke, & Carey, 1992) was given on a four year study of one of the original six case study teachers. The teacher was observed making increased use of children's thinking in making instruction decisions over the four year period studied.

Evidence from a longitudinal study (Fennema et al., 1996; Franke et al., 1997) revealed long-term effects of teachers' participation with CGI. Changes in CGI teachers' beliefs, knowledge, and instruction occurred over time. By the end of a longitudinal study (Fennema et al., 1996) of 21 primary school teachers, 90 percent of the teachers were categorized as using instruction that epitomized the process standards of reform. About 80 percent of the teachers believed more strongly that their students could solve problems without being shown procedures for solving the problems. Furthermore, the researchers suggested the study provided "strong evidence that knowledge of children's thinking is a powerful tool that enables teachers to transform this knowledge and use it to change instruction" (p. 432). Finally, with regard to effects on student achievement, students in CGI classes showed improvement in problem solving and understanding concepts. Based on analyses, the researchers also hypothesized that gains were cumulative; that is, the longer the students were in CGI classes, the greater the gains.

A second successful professional development project, the Teaching to the Big Ideas (TBI)/Developing Mathematical Ideas (DMI) project, was highlighted by Sowder (2007) for its focus on student thinking and mentioned by Borasi and Fonzi (2002) for the DMI program's use of cases. Schifter and colleagues (Schifter et al., 1999) developed the TBI project based on previous professional development work and guiding principles derived from that work. Some of their guiding principles for professional development included recognizing: teachers' mathematical understandings were critical; instructional activities should facilitate teachers' construction of new practice; regular follow-up support was critical in promoting change in practice; reform-based curriculum materials can provide a platform "for teachers to continue to deepen their knowledge of mathematics content, of children's mathematical thinking, and of pedagogical approaches" (p. 33); and school wide collaborative sharing and support was essential for the success of reform.

The TBI project was supported with NSF funding and was structured as a four year professional development project. The project included four annual 2-week summer institutes and biweekly seminars during the 3 intervening school years. The institutes and seminars comprised of three basic strands: "participants engage in mathematics lessons, analyze students' mathematical thinking, and consider curricular issues" (p. 36). During the first two years, teachers become oriented to new practice, reach for new understandings of elementary mathematical topics, and develop skills in inquiring into students' thinking. Professional development facilitators model reform pedagogy and practice in mathematical lessons where teachers become learners. Participants explore mathematical content, identify their own conceptions and misconceptions, develop new understandings through group discussions and individual work, and reflect on the nature of mathematics. Inquiry into students' thinking begins through viewing and analyzing videotape of other educators working with children and through examining student work. Teachers also develop while writing scenarios about their own classroom experiences, including transcribing classroom discourse. Supporting teachers as their practice evolves, project staff regularly visit classrooms and establish one-on-one discussions with teacher participants about student thinking and mathematical ideas that students were grappling with. During year 3, participants transition to leadership roles. The teachers write about what they learned in the project and about classroom events illustrating the development of big ideas. During year 4, participants implement leadership plans including leading inservice

sessions, visiting other classrooms and initiating discussions between teachers, administrators, and parents. Although papers produced by the third year project participants were originally envisioned as materials for future preservice and inservice professional development, the project advisory board and staff recognized the broader potential of the participants' descriptions of student thinking from actual classroom situations. Hence, some of the participants' papers have become a central component of case-based professional development modules, Developing Mathematical Ideas (DMI) modules, which are now publicly available. Through the project's attention to teachers' engagement in both exploring mathematical ideas and in examining students' mathematical thinking, teachers developed habits of inquiry, and hence teaching practice was transformed (Schifter, 1998). In addition, scaling up was integrated into the project structure through developing new teacher leaders and creating nationally available professional development curriculum materials (Schifter et al., 1999).

S. Cohen (2004) examined and described teacher learning that occurred among DMI seminar participants. She started by providing her interpretation of three core aspects of DMI. First, topics in each module focus on fundamental mathematics that is complex for both children and adults who may not have had prior opportunities to explore the topics. Second, the modules were designed with a dual focus on both how children construct understanding of a topic and how teachers understand the topic. That is, seminar activities included opportunities for teachers to develop conceptual understandings of topics in the elementary curriculum. In addition, activities such as case reading, case writing, interview preparation, and analysis of student work offered teachers opportunities to engage in mathematical inquiry and analysis. Finally, Cohen considered another core feature of the DMI design was the congruence of the program's pedagogy with pedagogy envisioned by *Standards*. She described the program's commitment to fostering a learning community whereby students collaboratively considered ideas in a supportive environment. Cohen examined two groups of teachers as they participated in DMI seminars. Based on qualitative analyses, Cohen described how the teachers changed. She found that the teachers came to trust their own mathematical ability and their students' ability to understand mathematics. Teachers also came to more deeply understand the mathematics that they were in charge of teaching by learning mathematics during seminar meetings and during their own classroom teaching. Furthermore, Cohen found the teachers' changing classroom practices illuminated "the teachers' growing ability to support both student expression of

mathematical ideas and the mathematical investigation of those ideas” (p. 81). In addition, as teachers’ teaching practice changed, they reported that they saw increases in student engagement in mathematics. Students had increased enjoyment in mathematical work, confidence in their ability to do mathematics, willingness to delve more deeply into mathematical thinking, and opportunities to experience a mathematical community of learners. Hence, Cohen surmised that changes in teachers paralleled changes in students.

The Integrated Mathematics Assessment (IMA) program has also been described as a successful professional development program designed to include a focus on understanding student thinking (Borasi & Fonzi, 2002; Sowder, 2007). Gearhart, Saxe, and colleagues (Gearhart et al., 1999; Gearhart & Saxe, 2004; Saxe, Gearhart, & Nasir, 2001) have described their program and documented its positive impact on teaching practice and student learning. Program activities started with engaging “teachers as learners” while exploring complex fraction problems (Gearhart & Saxe, 2004). Next, videotape snippets of students working fraction problems and student work were used to engage “teachers as researchers” in analyzing student thinking. Finally, “teachers as professional educators” implemented activities in their classrooms with emphasis placed on ongoing assessment. During meetings teachers collaboratively reflected about challenges in using classroom strategies such as inquiry, classroom discourse, and assessment to understand student thinking. The researchers studied three groups of teachers and their students. Two groups were implementing reform curriculum in their classrooms and were participating in one of two year-long professional development programs. One of these groups participated in the IMA program. The second group participated in the Collegial Support (Support) program. In the Support program, teachers were provided with the opportunity to meet with a professional community of teachers engaged in similar efforts of implementing reform curriculum units. The third group of teachers (Traditional) had chosen to continue to use a traditional curriculum; these teachers were not involved in a professional development program. Findings on student performance indicated that IMA students had significantly greater gains in conceptual understanding of fractions than both the Support and Traditional groups (Saxe et al., 2001). Furthermore, there was no significant difference in student gain scores on fraction procedures for the IMA and Traditional groups (and the Support group had lower gain scores than the Traditional group). With respect to teaching practice, findings (Gearhart & Saxe, 2004) revealed that IMA and Support teachers were equally

likely, and more likely than Traditional teachers, to probe for student understanding and to engage students in conceptual thinking. In addition, IMA teachers were more likely than Support teachers to emphasize the relationship between graphic and numeric representations of fractions. The researchers concluded that IMA teachers were “engaging their students with a more complex treatment of fractions, one that provided children with a conceptual foundation for traditional fractions instruction” (2004, p. 310).

Thus, several professional development programs with emphasis on developing teachers’ understanding of students’ mathematical thinking have documented their positive effect on teaching practice and student learning. Through participating in program instruction and analyzing student thinking in video, cases, and their own classrooms, teachers developed skills in using student thinking to make decisions about teaching practice. Gains in students’ mathematical understandings were also documented in several of the studies.

A second category of professional development strategies being used to develop knowledge-for-practice focuses on using curriculum to foster teacher learning (Sowder, 2007). With the publication of *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989), the NSF provided funding for several mathematics curriculum projects (Sowder, 2007). Subsequently, many professional development activities focused on preparing teachers to use standards-based curricula.

Sowder (2007) highlighted several researchers who have described and studied professional development projects with a strong curricular component including Remillard and colleagues (Remillard, 2000; Remillard & Geist, 2002; Remillard & Bryans, 2004), Borasi, Fonzi, and colleagues (Borasi et al., 1999; Borasi & Fonzi, 2002), and Sowder and colleagues (e.g., Sowder, Philipp, Armstrong, & Schappelle, 1998). For example, Borasi, Fonzi, and colleagues’ (Borasi et al., 1999; Borasi & Fonzi, 2002) initial research component of their program “sought to develop a better understanding, in the context of middle school *inclusive* classrooms, of how an inquiry approach to mathematics instruction could respond to the recent call for school mathematics reform and to the needs of diverse learners” (Borasi et al., 1999, p. 52). To this end, three mathematics inquiry curriculum units were developed by a collaborative team aimed at addressing NCTM Standards in an environment capable of supporting better teaching for both regular students and students with learning disabilities. During an initial summer institute, facilitators modeled inquiry-based teaching while introducing curricular units.

Teachers participated as learners of mathematics. During the school year, teachers implemented one of the units developed by the program facilitators, and perhaps developed and implemented their own unit as well. During this phase teachers were supported by facilitators in their schools. Follow up meetings allowed teachers opportunities to share their experiences with others. Upon analyses from a qualitative perspective, the researchers found their program was “quite successful in accomplishing its main goal of *initiating* the process of rethinking beliefs and practices” (p. 63), although the extent of the change varied considerably among individuals. In addition, evidence revealed participants who had exhibited changes in beliefs and practices during the professional development program also exhibited sustained and increased changes in instructional practices over successive years. Furthermore, upon reflection, the professional developers identified several ways in which the more unique features of their program, “i.e., the role of the illustrative inquiry units, the participation of a diverse group of teachers, and the role of the school facilitator, contributed to the effectiveness of acknowledged professional development practices that had informed the design of the program” (Borasi et al., 1999, p. 69). Participants’ experiences as learners of mathematics designed around a reform curriculum unit were described by participants as a particularly influential feature of the summer institute. With respect to the feature of diversity amongst participants, participants included elementary, special education, and secondary mathematics teachers. Significant mathematical learning of all the participants created a meaningful illustration of the accessibility of implementing similar reform units to a group of diverse learners. With regard to unit implementation, several participants had positive experiences while implementing a reform unit. Seeing students’ success motivated teachers to continue implementing change in their practice. However, some teachers did not experience immediate positive outcomes in their students’ achievement in their initial attempts of a new instructional approach. Hence, the researchers concluded the scaffolded support offered to teachers in their first attempts at implementing reform practice were crucial to long-term success.

Sowder (2007) identified professional development programs focused on utilizing case studies (written, video and multimedia) as a third approach being used as an impetus for teacher learning in professional development programs. Cases have been used for developing knowledge in law, medical, and business professions for some time (Sowder, 2007; Stein, Smith, Henningsen, & Silver, 2000). However, the use of cases in teacher preparation is relatively recent. Researchers (e.g., Merseth, 1996; Sowder, 2007) have suggested that increased interest

in case study use for education stems from Shulman's 1985 Presidential Address to AERA (published in 1986b) where he described envisioning research-based teacher preparation programs using case literature. In the late 80s and early 90s, conferences and organizations examined case pedagogy (Merseth, 1996). By the early 90s, journals focused on case methods and books of cases became available. Cases were being used for both preservice and inservice education, but themes commonly addressed non-subject-specific domains such as classroom management (Barnett, 1991). More recent literature (e.g., Barnett, 1998; Lampert & Ball, 1998; Merseth, 2003; Smith, Silver, et al., 2005) reveals increased use of subject-specific case curriculum. "Cases are now frequently used in education to assist teachers in examining their practice and their students' reasoning and understanding" (Sowder, 2007, p. 180).

Barnett (1991) described one of the early attempts to use mathematics-focused written case curriculum in a professional development project for upper elementary and middle school teachers. Barnett and her colleagues at the Far West Laboratory for Educational Research and Development found that group discussions of cases resulted in elevated pedagogical thinking and reasoning. Barnett claimed that by prompting teachers to analyze situations and "argue the benefits and drawbacks of various alternatives, cases can play a critical role in expanding and deepening pedagogical-content knowledge" (p. 263). During discussions, one teacher would start with an idea related to a mathematical situation and then another teacher would build on the idea. Through a collaborative process, the group of teachers was able to construct understandings which may not have occurred to them on their own. In later work, Barnett (1998) described a professional development program whereby cases were used as a stimulus to "help teachers see teaching as shared inquiry" (p. 82).

A fourth approach identified by Sowder for addressing knowledge-for-practice makes use of formal coursework. Sowder briefly discussed continued coursework in master's degree programs and certificate programs as examples for this category. However, no professional development projects were highlighted for this category.

Lesson Study

Lesson study is a professional development approach that has been credited for the steady improvement of teaching practice in elementary education in Japan (e.g., Lewis & Tsuchida, 1998; Lewis, Perry, & Murata, 2006; Stigler & Hiebert, 1999). Although lesson study has been

practiced in Japan for a century (Lewis, Perry, & Murata, 2006), the American educational community took particular notice over the last twenty years due in part to the high achievement scores of the Japanese students on the Second and Third International Mathematics and Science Study (TIMSS) (e.g., Curcio, 2002). Interest was also sparked in 1999 with the publishing of “The Teaching Gap” by Stigler and Hiebert (C. Lewis, 2002). The report (Stigler & Hiebert, 1999) about the TIMSS video study of eighth-grade mathematics lessons in Japan, Germany, and the U.S. included a chapter on Japanese lesson study. Within four years lesson study became the focus of many conferences, reports, and research articles (Lewis, Perry, & Murata, 2006).

Lesson study has also been growing as a form professional development activity in the United States. In 2003, Loucks-Horsely et al. reflected that more examples of professional development strategies were incorporating learning through collaborative reflection and discussion that were embedded in daily practice; lesson study was highlighted as one such strategy. J. T. Sowder (2007) recognized lesson study as a form of professional development being used to develop teachers’ knowledge. In 2007, NCTM (2007) promoted lesson study by offering a Lesson Study Course for three hours of graduate credit developed as a three day face-to-face session followed by online activities throughout the school year.

Lesson study starts with the premise that the most effective place to improve teaching is in the context of a classroom lesson (Stigler & Hiebert, 1999). Researchers (e.g., Curcio, 2002; Lewis & Tsuchida, 1998; Stevenson & Stigler, 1992) have described several key features of Japanese lesson study. For instance, Lewis and Tsuchida (1998) investigated how “teaching as telling” in Japanese science elementary education had been replaced with “teaching for understanding”. The researchers found the Japanese teachers attributed improvement in teaching to the impact of “research lessons”. Research lessons are developed through collaborative planning with focus on a particular goal. After an initial planning phase, the lesson is taught by one teacher while other members of the group observe. Lessons are often recorded in a variety of ways (using videotape, audiotape, observation checklist, and/or copies of student work). Afterwards, the group meets to collectively discuss, analyze, and revise the lesson. Most often, a research lesson takes place in an ordinary elementary school. However, public research lessons are also conducted where large numbers of educators meet to observe lessons pioneering new developments in education. Hence, although lesson study starts in ordinary classrooms, public

research lessons provide a venue for examples of good practice to be disseminated throughout the country in order to enhance the improvement of Japanese education as a whole.

Lesson study embodies many features that have been identified by researchers as important to improvement in teaching practice. For example, Ball and Cohen (1999) suggested that knowledge of subject matter, learning, learners, and pedagogy is essential as teachers try to enact the vision of reform; however, teachers also need to develop an attitude of inquiry in order to use what they learn to improve instruction. Inquiry-oriented professional development would need to be “centered in practice”. Ball and Cohen suggested that records of practice including samples of student work, videotapes of classroom lessons, curriculum materials, and teachers’ notes could be used for grounding professional discussion in the analysis of practice. More recently, researchers (Hiebert et al., 2007; Lewis, Perry, & Murata, 2006) have described lesson study as a professional development activity that provides teachers with opportunities for developing habits of inquiry. Lewis, Perry, and Murata (2006) said that lesson study shares characteristics with other professional development records of practice that can be used to develop habits of inquiry including an analysis of student thinking by reviewing student work artifacts and an analysis of actual practice through videotaped case studies. However, lesson study with the “live classroom lesson as the centerpiece of study” (Lewis, Perry, & Murata, 2006, p. 3) distinguishes itself from the use of artifacts of practice. Lesson study provides a professional development experience focused directly on the daily practice of teaching whereby teachers reflect on practice “so as to enhance teacher-planning and decision-making processes” (Smith, 2001, p. 42). Recently, Hiebert et al. (2007) described competence in subject matter knowledge for teaching and competence in developing and testing hypotheses as two important attributes for contributing to teachers’ abilities to analytically study and improve teaching practice over time. One of their reasons for including knowledge and reasoning skills enabling effective development and testing of hypotheses stems from the similarity of the skills with Dewey’s (1929) components of disciplined inquiry. Another reason pertains to related evidence from cross-cultural research. Hiebert et al. credited Asian professional development activities, including a “relentless focus on analyzing classroom practice and testing hypothesized improvements” (p. 57), for the steady improvement of teaching practice in several Asian countries.

Lesson study also embodies many elements (i.e., collaborative lesson planning, observing, discussing, revising) that are similar to Little's (1982) school organizational characteristics identified as conducive to teachers' development of a perspective that teaching requires continuous learning. Little found continuous professional development was more apparent in schools characterized by four types of practices. First, continuous professional development was evident in schools where teachers engaged in frequent discussion of classroom practice; the teachers developed precise, shared language to describe teaching practice. Second, teachers observed and critiqued each other's teaching. Third, teachers collaborated in planning and designing teaching materials. Last, teachers considered sharing teaching and learning with each other as a collaborative way of improving teaching practice. Furthermore, Little summarized that norms of collegiality and continuous improvement within the school change the focus of professional improvement from an individual enterprise to a shared undertaking in the schools. In congruence, researchers have suggested that lesson study provides a practice-based environment whereby teachers can develop collegial networks (Lewis et al., 2004), and steady improvement can be fostered (e.g., Stigler & Hiebert, 1999).

Sowder (2007) described lesson study as a professional development approach that can develop both knowledge-in-practice and knowledge-of-practice. When teachers participate in "lesson study as observers and discussants, they are acquiring knowledge-in-practice by engaging in a community discussion of the lesson" (p. 194). On the other hand, when a teacher participates in lesson study by teaching and discussing the lesson, it is more likely that the teacher acquires knowledge-of-practice.

Whereas American teachers feel conflict about using other teachers' ideas, Asian teachers regularly "make use of examples that have been perfected by others and have become part of the lore of skilled teaching" (Stevenson & Stigler, 1992, p. 168). NCTM (2000) noted that reflection, analysis, and refinement of instructional practice are crucial for the enactment of the vision of reform described in *PSSM*. Furthermore, although reflection and analysis of instruction practice are often individual activities, collaboration with colleagues can greatly enhance the analysis process. NCTM highlighted lesson study as a "powerful, yet neglected, form of professional development in American schools" (2000, p. 19). Structures would need to be set in place for American teachers to have time to collaborate with colleagues in professional development activities.

Hiebert and Stigler (2000) argued that lesson study processes satisfy requirements for scaling up reform efforts in multiple ways. First, scaling up must provide for ways in which one can learn from their own experiences and be able to share knowledge with others. Hiebert and Stigler noted that the classroom lesson is the smallest unit that captures the system of teaching. Lesson study offers a forum for teachers to focus on interactions between student thinking, curriculum, and pedagogy; in this forum, teachers can improve their own knowledge. But, collaborative analyses of lessons also allow teachers to share knowledge and ideas with others. Secondly, scaling up requires processes that can start small and yet “accumulate to yield large-scale systems that work as well as small ones” (p. 16). Lesson study also satisfies the modularity requirement from a variety of perspectives. For instance, lessons can be accumulated with appropriate transitions to extend the effects from one lesson to units to the entire school year curriculum. Teachers can develop more general knowledge and pedagogy skills while working on a specific research lesson. Furthermore, lesson study can start in a school or district and spread outward. Hiebert and Stigler concluded that while lesson study could be a catalyst for improving teaching practice in the United States, it is unclear whether lesson study would have success here. Prior reform initiatives in the U.S. have come with expectations for quick, dramatic change. Lesson study offers a process whereby incremental change can occur over time; it cannot be rushed.

Lewis, Perry and Hurd (2004) cautioned that for lesson study to become a long-term agent of improvement rather than a short-term fad in American teaching practice, attention would need to be given to both the visible features of lesson study (such as collaborative planning, observing, and revising of lessons), but also underlying features that enable continual growth of teachers. Based on studies in Japan and the U.S., Lewis et al. identified seven critical pathways of lesson study that lead to improvement in instruction including: “increased knowledge of subject matter, increased knowledge of instruction, increased ability to observe students, stronger collegial networks, stronger connection of daily practice to long-term goals, stronger motivation and sense of efficacy, and improved quality of available lesson plans” (p. 19). First, increased knowledge of subject matter can result when teachers plan the lesson. Lesson study often begins with an examination and discussion of standards, curriculum, and essential concepts for students to learn in the lesson. These activities may lead to questions about the subject matter that are further examined. Second, through collaborative planning,

observing, and revising of lessons, teachers may learn more effective instructional strategies for engaging and motivating students. Third, lesson study provides teachers with opportunities to observe student thinking. While one member teaches, other team members observe and write down narrative about a single student or a small group of students. “After observing the research lesson, teachers can compare their predictions about student thinking with students’ actual thinking during the lesson, thereby gaining direct feedback on their own knowledge of how students think” (p. 20). Fourth, lesson study offers an environment for collaborative planning that is often lacking in the United States. In addition, professional relationships developed during lesson study may expand into collaborations beyond a single lesson. Fifth, lesson study in Japan addresses not only specific content goals for a lesson, but also broader long-term educational goals such as student motivation. Many U.S. educators believe there is a disconnection between daily educational practice and long-term educational goals. Lesson study provides an opportunity to strengthen the connection. Sixth, lesson study provides an environment that builds up a teacher’s belief that improvement can occur in teaching practice. Focusing on analyzing student thinking and teaching practice with a group goal of improvement emphasizes professionalism in teaching. Last, sharing lessons leads to sharing lessons learned, and future lessons can build on what teachers learned in prior lessons.

Lewis, Perry, Hurd and O’Connell (2006) reported on the evolution and ultimate success of lesson study conducted in a California school district over six years. Lesson study evolved from a narrow focus on surface features to a more enriching experience involving underlying principles of lesson study. First, teachers initially viewed lesson study as lesson polishing; later in the process teachers discussed what they were learning about mathematical content or about how students think. Second, in the beginning post-lesson discussions focused on outwardly visible student behavior such as whether the students followed directions or were on task. As time went on, post-lesson discussions focused on student solution strategies and misconceptions. Data collection also became more intentional; teams might plan to target observations on certain aspects of student thinking. Third, although initial lesson study teams were made up of members from the same school, subsequent groups enlisted help from educators outside the school. Also, during the first year the team relied exclusively upon the adopted curriculum and state standards. Later, other outside sources of knowledge were also consulted such as research articles and various examples of curricula. Last, whereas initially the “research lesson often felt like a final

performance rather than a catalyst for further study and improvement of practice” (p. 275), later lesson study cycles began with a review of student data and problems in learning that had been identified in the prior lesson study cycle. The researchers also noticed other qualities of the successful lesson study experience that emerged through multiple lesson study cycles in the California district. For instance, instructional coherence developed throughout classrooms and schools. Teachers developed a mutual sense of responsibility for enhancing students’ learning. Teachers pushed each other to make sense of subject matter. Overall, the process of lesson study fostered a culture whereby teachers came to expect ongoing learning to be a part of the job of teaching.

Fernandez, Cannon, and Chokshi (2003) suggested that successful implementation of lesson study in the United States would hinge upon how teachers viewed practice during the lesson study process. When Japanese teachers provided coaching for American teachers involved in a lesson study initiative, the researchers noticed the Japanese teachers applied three critical lenses to their analyses of lessons that enhance the power of lesson study: a researcher lens, a curriculum developer lens, and a student lens. First, Japanese teachers adopted a researcher lens as they developed hypotheses, collected data, tested hypotheses, and articulated their findings. Fernandez et al. found “American teachers had much difficulty adopting and maintaining this researcher lens while conducting lesson study” (p. 173). For instance, American teachers would select a lesson goal, but then the goal appeared absent in discussions. As another example, American teachers in the study took few notes during observations whereas Japanese teachers took extensively detailed notes. With regard to the second lens, Japanese teachers focused on understanding the curriculum in the text and its enactment in the classroom. They also displayed “remarkable insights into the development of content within a lesson” (p. 178). In contrast, American teachers did not examine reasons for lesson sequences. They also exhibited limited understanding of how to structure mathematical content for the students. Finally, Japanese teachers consistently focused on examining aspects of the lesson from the perspective of their students. They tried to understand students’ prior knowledge and they considered how they would help students reach new understandings. American teachers did not consistently examine student understanding. Fernandez et al. concluded that lesson study efforts in the U. S. should include coaching to enhance the building of researcher, curriculum developer, and student lenses.

In conclusion, lesson study is a professional development strategy that exhibits potential as an agent of improvement for U.S. education. Whether or not it is embraced and successfully implemented on a large scale is yet to be seen.

Differentiated Instruction

Researchers (e.g., Ball, 1990a; Ma, 1999; Shulman, 1986b) have offered various conceptions of teachers' knowledge. Recently, the National Academy of Education Committee on Teacher Education (Bransford et al., 2005) adopted a framework conceptualizing three important areas of knowledge for today's teachers: knowledge of learners and their development in social contexts; knowledge of subject matter and curriculum goals; and knowledge of teaching. Of note, within the area of knowledge of teaching was specific mention of knowledge of teaching diverse learners. Darling-Hammond (Darling-Hammond, 1998; Darling-Hammond, 2006a) has drawn attention to the importance of knowledge of teaching diverse learners. "Teachers need not only to be able to keep order and provide useful information to students but also to be increasingly effective in enabling a diverse group of students to learn ever more complex material" (Darling-Hammond, 2006a, p. 300). With specific reference to mathematics teaching, Ball and Bass suggested that high-quality teachers would be able to "reach all students, teach in multicultural settings, and work in environments that make teaching and learning difficult" (Ball & Bass, 2000, p. 94).

Although American classrooms are superficially homogenized by chronological age, remarkable diversity typically exists (Tomlinson et al., 2003). For example, classrooms are becoming more ethnically diverse. The proportion of minority students in public elementary and secondary schools increased from about thirty percent in 1986 to about forty-two percent in 2004 (Snyder, 2007). In addition, immigration is currently playing a crucial role in population growth for the U.S. (Lapkoff & Li, 2007). Of note, in 1970, about sixty percent of foreign-born people originated from Europe; in 2000, more than half of foreign-born people originated from Latin American and more than a fourth originated from Asia. As a result, greater demands have been placed on educational services such as including English as a second language (ESL) instruction.

Diversity in the range of academic abilities in general education classrooms has also increased (e.g., Banks et al., 2005; Rock et al., 2008). Some learners are identified as advanced learners whereas others may have learning difficulties (Tomlinson et al., 2003). Prior to 1975,

many students with disabilities were excluded from public schooling (Rock et al., 2008). During the decade after the enactment of the All Handicapped Children Act (EHCA) of 1975, children with disabilities were commonly included in public schooling, but were placed in special, self-contained classrooms. In 1976-77, eight percent of students in public schools were identified as children with disabilities (Snyder, 2007). By 2005-06, children with disabilities made up almost fourteen percent of public school enrollment. In addition, many students with disabilities were being educated for at least part of the day in a regular classroom (U.S. Department of Education, 2007). In 2003, almost half of all students with disabilities were educated in the regular classroom for most of the school day; more than three-quarters of all students with disabilities were in a regular classroom for at least forty percent of the school day. Moreover, 96 percent of students with disabilities were being educated in regular school buildings. The original EHCA has been reauthorized and amended to become the Individuals with Disabilities Education Improvement Act (IDEA) of 2004 which stresses the importance of educating students of disabilities alongside children who are not disabled to the maximum extent possible (IDEA, 2004; Rock et al., 2008). In fact, congress (IDEA, 2004) suggested that thirty years of research and experience had demonstrated the importance of access to the general education curriculum for students with disabilities.

In addition to diversity exhibited by the previous statistics, teachers encounter students with varying interests, preferred learning styles, and motivational levels (Tomlinson et al., 2003). Differentiation has been advocated as an approach whereby teachers “develop classroom routines that attend to, rather than ignore, learner variance in readiness, interest, and learning profile” (Tomlinson et al., 2003, p. 121). Tomlinson (2005) suggested elements of differentiation in U.S. education existed as far back as the one-room schoolhouse. Teachers had to be flexible in their instruction of students who varied in age, experience, and proficiency. More recently, researchers in the area of gifted education have advocated for differentiated education for at least four decades (e.g., Olenchak, 2001; Tieso, 2005). However, studies (Archambault et al., 1993; Westberg, Archambault, Dobyms, & Slavin, 1993) from the early 90s revealed little differentiation in instructional or curricular practices was being provided to gifted students in the regular classroom where gifted students typically spent most of their time.

Differentiated instruction has received increased attention over the past decade (Rock et al., 2008). Classrooms are becoming more diverse (e.g., Lapkoff & Li, 2007; Rock et al., 2008;

Snyder, 2007) and increasingly inclusive where students with disabilities (e.g., Haager & Klingner, 2005) and gifted students (e.g., Olenchak, 2001) are learning along with their peers in regular classrooms. Compound this with students who may struggle, but who do not qualify for support services under established criteria (Tobin & McInnes, 2008). Experts have been suggesting that differentiated instruction may be an effective solution for serving increasingly diverse student populations in contrast to the one-size-fits-all approach of traditional instruction (Rock et al., 2008). Westberg et al. (1993) suggested teachers need to be encouraged to try differentiated instructional strategies as given that increasing attention on equity and minimum competency testing may result in an inclination to “to teach the same thing, to all students, at the same time” (p. 142). Tomlinson (1999b) suggested differentiated instruction has the potential of addressing both U.S. values of excellence and equity by establishing communities of learning “built solidly on high-quality curriculum and instruction that strive to maximize the capacity of each learner” (p. 12).

Despite many years of theory, practice, and research, the interpretation and implementation of differentiation remains controversial (Olenchak, 2001). Differentiated instruction has been described as a philosophy (e.g., Tomlinson, 2000) or mindset (Wormeli, 2007) rather than an instructional strategy. By attending to learning preferences, student readiness, and individual interests, differentiated instruction implies schools should be maximizing the capabilities of each student (Anderson & Algozzine, 2007). Elements of differentiated instruction include “choice, flexibility, on-going assessment, and creativity resulting in differentiating the content being taught, or how students are processing and developing understanding of concepts and skills, or the ways in which students demonstrate what they have learned” (Anderson & Algozzine, 2007, p. 50). Winnowed down from eight key ideas of differentiation conveyed by Tomlinson (1999a), Rock et al. (2008) and Tieso (2003) concurred the current model for differentiated instruction has four guiding principles: “(a) a focus on essential ideas and skills in each content area, (b) responsiveness to individual student differences, (c) integration of assessment and instruction, and (d) an ongoing adjustment of content, process, and products” (Rock et al., 2008, p. 33). In slight variation, Small (2009) described general agreement about three common elements of differentiated instruction including: (a) a focus on the big ideas, (b) element(s) of choice for the student in content, process, or product, and (c) prior assessment in order to determine the needs of different

students. Furthermore, one of Tomlinson's (1999a) other key ideas for differentiated classrooms highlighted the importance of flexibility: flexibility in materials used, in pacing, in use of time, in instructional strategies, and in grouping.

Researchers (e.g., McTighe & Brown, 2005; Rock et al., 2008; Tomlinson, 2001; Tomlinson et al., 2003) have suggested that support for differentiation is grounded in theory and research. Key principles of differentiation are consistent with theories from the area of cognitive psychology on how people learn and are supported by educational research. First, research suggests that learning is enhanced when instruction and curriculum attend to focusing on the big ideas. With regard to brain research on how people learn, Sousa (2006) described transfer as a primary goal of learning. Rote learning of many facts with little opportunity to attach meaning likely hinders transfer. On the other hand, in-depth learning supports transfer. In addition, research on how experts differ from novices has revealed that experts often think in terms of core concepts whereas novices' knowledge is much less organized around big ideas (National Research Council, 2000). Furthermore, resources (Marzano, 2003; National Association of Secondary School Principals, 2004) outlining research-based practices associated with positive influences on student achievement recommend that instruction and curriculum be designed around a focus on essential learning.

Teaching that attends to individual student differences, including student variance in readiness, interests, and learning profiles, also has support from theory and research. For example, theories on cognitive development by Piaget and Vygotsky drew attention to the importance of considering children's readiness to learn (e.g., Brainerd, 1978; Tomlinson et al., 2003). Vygotsky (1962) suggested good instruction exists when a teacher leads development beyond what a student can currently do on their own. Vygotsky proposed a person learns in his or her "zone of proximal development" located somewhere between the person's present understanding and potential understanding; a point where a person may not be able to succeed on their own, but can succeed with support and scaffolding from a knowledgeable person (Steele, 1999; Tomlinson et al., 2003). Current research on learning suggests tasks should be at a "proper level of difficulty in order to be and to remain motivating: tasks that are too easy become boring; tasks that are too difficult cause frustration" (National Research Council, 2000, p. 61). Hence, theory and research support differentiated instruction as opposed to one-size-fits-

all instruction as using single tasks for all learners probably falls short of motivating many learners (Tomlinson et al., 2003).

Tomlinson et al. (2003) outlined several studies providing support for the notion that addressing learner interest can impact student motivation and learning. For example, upon review of the literature, Tobias (1994) concluded that “interest may have an energizing effect on learning and lead students to use deep comprehension processes” (p. 45). In addition, brain research suggests the probability of information being stored in long-term memory is best when “both sense and meaning are present” (Sousa, 2006, p. 49). Furthermore, meaning has a stronger impact. Information that has relevance to the learner is more likely to be stored.

Tomlinson (e.g., Tomlinson, 2001) referred to learning profile as the way in which an individual learns best, that can be influenced by factors including learning style, intelligence preference, gender, and culture. First, the Dunn and Dunn Learning-Style Model (e.g., Dunn & Dunn, 1992, 1993) is a model that has been clearly defined and extensively researched (Dunn & Dunn, 1992; Lovelace, 2005). Dunn and Dunn initially identified 12 variables, but by 1990 had proposed 21 learning style elements as affecting learners (Dunn & Dunn, 1992, 1993). Dunn and Dunn classified the elements into five categories of variables affecting learners:

- (1) *immediate environment* (sound, light, temperature, and furniture/seating designs);
- (2) *own emotionality* (motivation, persistence, responsibility [conformity versus nonconformity], and need for either externally imposed structure or the opportunity to do things in their own way);
- (3) *sociological preferences* (learning alone, in a pair, in a small group, as part of a team, or with either an authoritative or collegial adult; and wanting variety as opposed to patterns or routines);
- (4) *physiological characteristics* (perceptual strengths, time-of-day energy levels, and need for intake and/or mobility *while* learning);
- and (5) *processing inclinations* (global/analytic, right/left, and impulsive/reflective).

(Dunn & Dunn, 1992, pp. 3-4)

Dunn and Dunn suggested considerable research had established “the existence of individual differences among students—differences so extreme that the identical methods, resources, or grouping procedures can promote achievement for some and inhibit it for others” (1992, p. ix). Based on a meta-analysis of 76 original research studies, Lovelace (2005) reported that results “overwhelmingly supported the position that matching students’ learning-style preferences with

complementary instruction improved academic achievement and students' attitudes toward learning" (p. 180).

Two prominent contemporary theories about intelligence preferences suggest individuals have brain-based predispositions for learning (Tomlinson, 2001). Furthermore, research findings suggest that when students are allowed to approach learning in ways consistent with their intelligence preferences, learning is enhanced. First, Gardner (1983/1993) viewed human intelligence as an ability to solve problems or create effective products as valued in a culture. In contrast to a one-dimensional view of intelligence as measured by an IQ test, Gardner proposed a "pluralistic view of the mind" (1993, p. 6). Gardner originally proposed seven multiple intelligences (MI) domains: linguistic, musical, logical-mathematical, spatial, bodily-kinesthetic, interpersonal, and intrapersonal. Gardner suggested intelligence domains may be exploited as a means of learning in an educational encounter (1983/1993). Furthermore, material to be mastered may predominantly exist in a particular domain. Some descriptive studies (Emig, 1997; Greenhawk, 1997; Hoerr, 2004) on MI use in classrooms have revealed that students gained confidence when using their talents to learn. In addition, several studies (Campbell & Campbell, 1999; Greenhawk, 1997; Kornhaber, Fierros, & Veenema, 2004) have provided evidence that MI instruction is associated with improvements in academic achievement. In a study based on interviews with teachers and administrators at forty-one schools employing MI for three or more years, Kornhaber, Fierros, and Veenema (2004) found four-fifths of the schools reported improved standardized test scores and almost half of the schools associated the improvements with MI. Improvements were also commonly reported with regard to student discipline, parent participation, and for schooling of students with learning differences. Interview analysis suggested MI influences instruction and learning by supporting engagement of a wide-range of learners. In another example, Campbell and Campbell (1999) studied six public schools that had implemented MI programs for at least five years. All six schools documented achievement gains. Five of the six schools reported their students outperformed their peers locally and/or nationally. The researchers also discovered the achievement gap for white and minority student populations was substantially narrowed or eliminated. In addition to achievement gains, other positive student outcomes included enthusiasm for learning, increased school attendance, and enhanced self-perceptions.

From the teachers' perspective, Hoerr (2004) pointed out that "recognizing MI means realizing that children learn in different ways. Teachers who understand this try to provide opportunities for students to learn using a range of intelligences" (p. 43). Similarly, in a project guided by MI theory, Chen, Krechevsky, and Viens (1998) suggested nurturing children's strengths in different learning areas helped teachers "see *all* their students as talented individuals with the potential to learn and grow" (p. 68).

Soon after Gardner's work emerged, Sternberg proposed a theory about three patterns of intelligence (Sousa, 2006). Sternberg described "successful intelligence" as the ability to succeed in life by using one's skills in one or more of three areas: analytical, creative, and practical (Sousa, 2006; Sternberg & Grigorenko, 2002). Sternberg and colleagues (e.g., Sternberg, Ferrari, Clinkenbeard, & Grigorenko, 1996; Sternberg, Torff, & Grigorenko, 1998; Sternberg, Grigorenko, & Jarvin, 2001) have studied whether the model is empirically supported, and outcomes of applying the model in educational settings. For example, Sternberg, Ferrari, Clinkenbeard, and Grigorenko (1996) studied the triarchic model in the identification, instruction, and assessment of gifted high school students enrolled in a special summer psychology college course. During one component of instruction, some students received instruction emphasizing an area of strength, whereas others were mismatched. The researchers "found that students who were placed in a course whose instruction matched their pattern of abilities performed better than did students who were mismatched" (p. 136). In another study, Sternberg, Torff, and Grigorenko (1998) found elementary and middle school students assigned to an experimental group receiving triarchic instruction (analytical, creative, and practical) outperformed students assigned to either an analytic instruction control group or a conventional memory-based instruction control group. In fact, the students receiving triarchic instruction outperformed the other students on both a multiple-choice memory assessment and an analytical, creative, and practical performance assessment. Of note, Sternber and Grigorenko (2002) considered the theory of successful intelligence to be complimentary with Gardner's MI theory as "Gardner's theory specifies domains in which intellectual gifts may operate, whereas the theory of successful intelligence specifies kinds of processes" (p. 275).

Culture and gender also influence how we learn (Tomlinson, 2001). There are learning preferences that exist for a group of people; however, substantial variance exists within the group as well. Tomlinson said teachers should "come to understand the great range of learning

preferences that will exist in any group of people and to create a classroom flexible enough to invite individuals to work in ways they find most productive” (2001, p. 62). Heath (1983) wrote a compelling description of two communities and how their varied cultures influenced the learning of children. Between 1969 and 1978, Heath spent time with two communities that were in close proximity; one community was made up of white working-class families and the other community was made up of black working-class families. Heath worked with several teachers in the communities to enable them to participate in ethnographic work documenting their experiences during the beginning of desegregation in the South. Teachers noticed patterns of behaviors in groups of children. For example, the children from the two communities had different notions of story telling; furthermore, the notions for each group were at odds with school conventions for story telling. Many of the teachers changed some of their classroom routines and resources in order to accommodate group differences in culture and communication. In addition, some teachers tried to embed class materials with the life experiences of their students. The teachers believed their “altered ways of teaching allowed some children to succeed who might not otherwise have done so” (p. 354). Other researchers (e.g., Hatch & Gardner, 1993; Moll, Tapia, & Whitmore, 1993) have also illustrated the influence of culture in educational settings.

With regard to gender, Posner (2008) noted various studies have generally depicted females as “supportive collaborators in classroom interactions” and males as “dominant individuals, obtaining, directing, and holding the conversational floor for extended periods” (p. 133). In addition, numerous studies have illustrated how gender influences interactions in science and mathematics classrooms (see reviews by Posner, 2008; E. Seymour, 1995).

The differentiated instruction guiding principle promoting the “integration of assessment and instruction” (Rock et al., 2008, p. 33) is supported by current knowledge about how people learn and recommendations for effective teaching environments based on that knowledge. Based on contemporary knowledge of learning, the National Research Council (2000) recommended that effective learning environments would be learner centered, knowledge centered, assessment centered, and community centered. Effective learning environments that are assessment centered would provide many opportunities for assessments that elicit student understanding. Formative assessments would be used by teachers to improve both teaching and learning. Feedback would

be provided to students in order to support future progress. Furthermore, opportunities for feedback would “occur continuously, but not intrusively, as a part of instruction” (p. 140).

Learner centered and knowledge centered perspectives of effective learning environments are also in strong accord with the guiding principles of differentiated instruction. For example, teachers who are learner centered “recognize the importance of building on the conceptual and cultural knowledge that students bring with them to the classroom” (NRC, 2000, p. 134). Moreover, knowledge centered environments would focus on big ideas in order to support students’ development of an understanding of a discipline.

Tomlinson (1999a) also asserted flexibility (e.g., in grouping, materials) was another key component of differentiated classrooms. As part of a research team, Allington (2002) found a group of exemplary teachers were characterized by: (a) using multi-sourced and multi-leveled curriculum, (b) offering students “managed choice” as to what they learned and how they demonstrated the learning, and (c) using more personalized teaching and discussion and less whole-group lecture. In a meta-analysis, Lou et al. (1996) found within-class groupings, optimally of 3- to 4-member teams, “had a significantly positive effect on student learning when compared with traditional whole-class instruction and individual seatwork” (p. 448). Positive effects on student learning were maximized when within-class grouping was combined with adaptations of instructional methods and curriculum. In another study, Tieso (2005) found a differentiated mathematics curriculum along with flexible within-class grouping “can create significant student achievement gains” over whole-group instruction utilizing undifferentiated curriculum.

In addition to theoretical and educational research support for individual principles of differentiation, research also illustrates positive outcomes of differentiated instruction as a whole. In a study of four gifted student cases, Olenchak (2001) found a personally tailored development plan differentiated around a student’s unique interests and abilities resulted in positive improvements in school behavior. In another study, Tieso (2005) found students who were exposed to a differentiated mathematics curriculum unit combined with flexible within-class ability grouping experienced significantly higher mathematics achievement over students exposed to the regular textbook unit. DiMartino and Miles (2004) reported on the success of differentiated instruction at an alternative program in one school. Students in the Academic Improvement Magnet (AIM) program were ninth-grade repeaters who were “tough, hardened

kids” (p. 47). The teachers for the program used a variety of differentiated instruction strategies and approaches. The program was considered a success as many of the students achieved enough credits to catch up with their cohorts and go directly into the 11th grade after their year in the AIM program. Finally, Lewis and Batts (2005) described an elementary school where the majority of teachers had formerly used predominantly undifferentiated instruction approaches and students had an overall 79 percent proficiency rate on state-mandated end-of-grade tests. Five years after beginning the process of implementing differentiated instruction, about 95 percent of students scored at proficiency. In fact, improvement was achieved across all socioeconomic and racial groups. Furthermore, Lewis and Batts described a major benefit of the school’s adoption of the differentiated instruction approach was increased student “excitement and enthusiasm for learning” (p. 30).

Despite research identifying positive outcomes associated with differentiated instruction, some studies have revealed barriers to implementing differentiated instructional practices. For example, Schumm and Vaughn (1991) found that although teachers considered a variety of adaptations for special learners as desirable, teachers were most willing to provide support and encouragement and less willing to provide adaptations that required more individualization with respect to planning, instruction, and altering the environment. In a survey study, Schumm and Vaughn (1992) found teachers were more willing to make interactive planning adaptations (e.g., making an adjustment during a lesson in response to student progress) and less willing to make pre-planning and post-planning adjustments for mainstreamed students. Furthermore, frequently cited barriers to planning for mainstreamed students included class size, insufficient instructional time, and inadequate teacher preparation and training for working with mainstreamed exceptional students. Lastly, at a school where ongoing professional development was focused on building teachers’ professional knowledge of differentiated instruction, Lewis and Batts (2005) found that teachers considered time-consuming planning to be the biggest barrier for implementing differentiated instruction.

Small (2009) pointed out that although many teachers have been using some differentiated instruction in language arts (e.g., recognizing different students may need different reading material), “differentiated instruction in mathematics is a relatively new idea” (p. 1). Perhaps it is not as easy to differentiate instruction for mathematics. However, NCTM (2000) explained that achieving equity “does not mean that every student should receive identical

instruction” (p. 12). Instead, achieving equity requires accommodating differences among students. NCTM also asserted that achieving equity would require more professional development for teachers. Teachers need professional development in order to better understand and better accommodate the different needs of their students.

Congress (IDEA, 2004) suggested the education of children with disabilities can be enhanced by supporting high-quality professional education and preservice education for teachers who work with students with disabilities. Wenglinsky’s (2000, 2002) research has attested to the importance of professional development for learning to teach diverse learners. Wenglinsky (2000) used data of eighth graders who took the 1996 NAEP mathematics assessment and their teachers to study aspects of teacher quality in relationship to student achievement. He found students whose teachers had participated in professional development on learning to teach different groups of students (constructed from three measures including professional development involving cultural diversity, teaching LEP students, and teaching students with special needs) outperformed “their peers by more than a full grade level” (p. 7). Overall, there is a greater awareness about the importance of providing teachers with opportunities to develop knowledge about teaching diverse learners (e.g., Banks et al., 2005; Darling-Hammond, 2006a; Wenglinsky, 2000).

Reading in the Content Area and Using Math-Related Literature

Chall (1983) identified fourth grade as a critical juncture between students learning to read versus reading to learn. In the primary grades, children typically read narrative fiction in basal readers. Starting with the intermediate grades, reading is “characterized by the growing importance of word meanings and of prior knowledge” (p. 21) as students learn about subject matter from reading subject matter textbooks, reference books, biographies, etc. Hence, content area teachers and English/language arts teachers share the responsibility of literacy development for older students (e.g., Vacca, 2002). The Commission on Adolescent Literacy of the International Reading Association asserted adolescents deserve support in their continued development as readers, and thus outlined principles for supporting adolescent literacy. For example, “adolescents deserve instruction that builds both the skill and desire to read increasingly complex materials” (Moore, Bean, Birdyshaw, & Rycik, 1999, p. 102) including skills such as identifying and understanding key vocabulary, recognizing how a text is organized,

and interpreting diverse symbol systems. Adolescents also deserve teachers who “model and provide explicit instruction in reading comprehension and study strategies across the curriculum” (p. 104).

“At its most basic, teaching reading in the content areas is helping learners to make connections between what they already know and ‘new’ information presented in the text” (Billmeyer & Barton, 1998, p. 1). Consequently, teaching reading in the content area focuses on teaching students how to use reading as an effective tool for learning. A teacher who wants to help students improve their reading comprehension can focus their planning around three interactive elements of reading: (a) the reader including his/her prior knowledge and affective response toward reading, (b) the climate including the student’s feelings about acceptance, competence, and content relevance which can be influenced by teachers and peers, and (c) the text features including vocabulary, text structure, and reader aids. Billmeyer and Barton described narrative text and informational (expository) text as two major text structures. Narrative text is writing in which a story unfolds and the primary purpose is to entertain whereas the main purpose informational text is to inform or persuade.

Strategies have been identified whereby content area teachers can help students improve reading comprehension (e.g., Bean, Readence, & Baldwin, 2004; Beers & Howell, 2005; Billmeyer & Barton, 1998). For example, Beers and Howell (2005) outlined many general ideas about what teachers can do to help students become independent strategic readers. For example, to support students’ connection of new knowledge with existing knowledge, some strategies that can be employed include: using graphic organizers; encouraging students’ sharing of their perceptions about the reading; and providing reflective writing prompts. As another example, to help students plan for the reading task, teachers can provide advance questions to focus students’ reading or teachers can review vocabulary that will be important to the reading.

Some more specific reading strategies have also been provided and organized in different ways in resources. For instance, strategies have been organized according to pre-reading, during-reading, and post-reading activities (e.g., Beers & Howell, 2005; V. A. Jacobs, 2002). On the other hand, Billmeyer and Barton (1998) classified numerous strategies as pertaining to vocabulary development, narrative text, information text, and reflective strategies. For example, some vocabulary strategies included the Frayer Model, semantic mapping, and word sorts. Character maps, Venn diagrams, and story mapping were recommended as some strategies for

navigating the organizational structure of narrative text (e.g., setting, characters, plot, conflict). Other strategies such as anticipation guides, graphic organizers, partner reading, and group summarizing were suggested for addressing the common organizational patterns for information text (e.g., comparison/contrast, cause/effect). Finally, writing, discussion, and questioning strategies (e.g., learning logs, RAFT) were offered to support students' development of reflective or metacognitive skills.

Many content area reading strategies make use of graphic organizers (e.g., Beers & Howell, 2005; Billmeyer & Barton, 1998). "Graphic organizers provide a visual, holistic representation of facts and concepts and also the relationships that link them together" (Billmeyer & Barton, 1998, p. 109). In meta-analyses, use of graphic organizers (Moore & Readence, 1984) and concept maps (Nesbit & Adesope, 2006), a form of graphic organizer, have been found to benefit learners. At a school committed to adopting seven research-based reading and writing strategies, students consistently reported that the use of graphic organizers was the most helpful strategy employed (Fisher, Frey, & Williams, 2002). Appropriate use of graphic organizers can help focus students on the big picture, can help students make connections between new knowledge and existing knowledge, and can lead students to intended learning (Beers & Howell, 2005).

Another type of text intersects narrative and informational text: a trade book, defined as literature found in bookstores (Bean et al., 2004), may delve into a subject matter concept via a story, and thus may address dual purposes of entertaining and informing (Bean et al., 2004; Murphy, 2000). Whitin and Whitin (2004) wrote about math-related literature: "Good books can promote the Process Standards by incorporating a variety of representations (illustrations, charts, models, etc.), showing mathematics in interdisciplinary contexts, and, most important, inviting children to talk, reason, and solve problems in multiple ways" (p. 6). With specific regard to Problem Solving, educators have suggested the use of mathematical literature can provide students with nonroutine problem solving experiences (A. Jacobs & Rak, 1997) and problem solving activities that make mathematics more relevant (Melser & Leitze, 1999). Furthermore, the use of math-related literature provides natural links with Connections and Communication as children have opportunities to connect mathematical ideas with their own experiences (Altieri, 2009; Murphy, 2000) and communicate their knowledge with others (Altieri, 2009).

Summary

This literature review provided a description of the U.S. mathematics standards-based reform movement and the demands of the movement on teachers. Changes to core dimensions of teaching will require opportunities for teachers to observe new practice and implement new practice in a supportive environment (Borasi & Fonzi, 2002; Collins et al., 1989). Ongoing professional development has been purported to be an essential mechanism for eliciting change in teachers' knowledge and teaching practice in support of school improvement (e.g., Desimone et al., 2006; Elmore, 2002; Hawley & Valli, 1999).

Descriptions of features of professional development that support teacher learning and teaching practice have coalesced into a view of effective professional development whereby focus is given to both "subject matter and issues of teaching and learning as they come together in classroom practice" (S. Cohen, 2004, p. 3). The literature review provided illustrations of a vast array of effective professional development features (e.g., sustained, content-focused) and strategies (e.g., lesson study). Whereas the research base on characteristics of effective professional development has grown considerably (e.g., Loucks-Horsley et al., 2003; Sowder, 2007), Borko suggested the research base is thin as to "what and how teachers learn from professional development" (2004, p. 3).

CHAPTER 3 - Methodology

Overview

A case study methodology was used for this study in order to examine the impact of the IMP professional development program on teachers' knowledge and instructional practice. Experts have defined the case study strategy from different perspectives, thus illuminating different understandings of case study. For example, Yin (2003) defined case study in terms of its scope and process. First, "a case study is an empirical inquiry that investigates a contemporary phenomenon within its real-life context, especially when the boundaries between the phenomenon and context are not clearly evident" (p. 13). That is, a case study is appropriate when the researcher wants to consider contextual conditions. Second, Yin explained that case study is a comprehensive research strategy, not just a data collection tactic; the researcher copes with many variables of interest, "relies on multiple sources of evidence, with data needing to converge in a triangulated fashion", and "benefits from the prior development of theoretical propositions to guide data collection and analysis" (p. 14). On the other hand, Merriam (1998) considered the unit of study as the most defining characteristic of case study; a case should be a single entity or unit which is bounded. As such, Merriam described the case study as the product of an investigation whereby "case study is an intensive, holistic description and analysis of a single entity, phenomenon, or social unit" (p. 34). The IMP professional development intervention in 2008-2009 was intrinsically bounded by time, activities, and number of participants and coordinators. The program was situated in complex learning and teaching environments where variables could not be easily isolated. The end product was a holistic description of the impact of the professional development intervention on teachers' knowledge and practice. Hence, case study research was appropriate for this study.

Case study experts have provided various descriptions of case study designs. For instance, Yin (2003) outlined four typical designs including single-case, embedded units single-case, multi-case, and embedded units multi-case. Of interest for this study, an embedded design is informed by several units of analysis. Similarly, Patton (2002) suggested that a "single case study is likely to be made up of many smaller cases" (p. 297). For example, nested and layered

mini-case studies documenting individual experiences may contribute to an overall macro-case analysis. This study employed an embedded units single-case design. In this embedded case study, the main unit of analysis was the IMP professional development intervention, yet attention was also given to the impact of the professional development experience on specific teachers which served as smaller units of analysis.

Setting

The University

Kansas State University (KSU) is a large state university whose main campus is located in the college town of Manhattan, Kansas. KSU offers undergraduate and graduate degrees from a variety of disciplines. According to the Carnegie classification system, KSU is a doctoral-granting institution with very high research activity (The Carnegie Foundation for the Advancement of Teaching, 2007).

Since 1989, KSU's College of Education has entered into Professional Development School (PDS) partnerships with local school districts (Kansas State College of Education, n.d.). The idea of Professional Development Schools was promoted in 1986 by the Holmes Group (1986), now called the Holmes Partnership, a consortium of representatives from U.S. educational research institutions, public school districts, and teacher associations (Holmes Partnership, n.d.). Borrowing from medicine's teaching hospitals, educational partnerships between public school teachers, administrators, and university faculty were envisioned as collaborative sites where research and practice could intersect for the joint purpose of improving student learning (Holmes Group, 1986). KSU's Colleges of Education and Arts and Sciences entered into partnerships with three local school districts in 1989, and expanded to include six additional partner districts across Kansas in 2005 (Kansas State College of Education, n.d.). In the early 1990's, partnerships served as a major component of a NSF-supported mathematics, science, and technology (MST) professional development model which KSU's College of Education initiated to reform preservice and inservice elementary teacher education and to improve mathematics, science, and technology teaching (Shroyer, Wright, & Ramey-Gassert, 1996). In a 1995 publication, the Holmes Group highlighted KSU's PDS partnership model for its premise "that education should be viewed as a continuum from kindergarten through university, and that improvement in one part of the system is not possible without improvement

throughout” (Holmes Group, 1995, p. 8). KSU PDS partnerships continue to offer a collaborative environment supporting preservice and inservice education in efforts to enhance student learning (Kansas State College of Education, n.d.).

Mathematics and Science Partnership (MSP) Program

The Mathematics and Science Partnership (MSP) program is administered by the Academic Improvement and Teacher Quality Program (AITQ) as a component of the No Child Left Behind Act of 2001, Title II, Part B (U.S. Department of Education, 2008c). The program’s goal is to improve academic performance of elementary and secondary students in mathematics and science by increasing content knowledge and instructional skills of teachers (U.S. Department of Education, 2008b). Partnerships are at the core of the improvement initiative. At minimum, partnerships are to include a high-need local education agency (LEA) and the mathematics, science, or engineering department of an Institute of Higher Education (IHE). Each state is responsible for administering a competitive grant competition.

In a competitive MSP grant application for 3-year projects beginning in 2007, Kansas chose to target the area of K-8 mathematics (Kansas Department of Education, n.d.). Project partnerships were required, at minimum, to include high-need unified school districts (USD) and Kansas IHE faculty from both the mathematics teacher education department and from the mathematics and/or engineering department. In addition, each project would be required to provide a content-focused 2-week summer institute along with at least 4 days of follow-up training during the school year.

The Project

Kansas State University was well-prepared to apply for a Kansas MSP grant in 2007. First, partnerships were already established between several school districts and KSU’s Colleges of Education and Arts and Sciences. In addition, mathematicians and mathematics educators had collaboratively designed a professional development program for previously funded projects (MSP project, ACUMEN, NCLB 2003-2006; Department of Education (DOE) project, Equity and Access, DOE 2004-2009) (Allen, 2007, January 4). The program was named the C³ Academy for its foci on *C*ontent knowledge, *C*urriculum connections, and the *C*hild.

In the grant application, the Infinite Mathematics Project (IMP) was described as a 3-year professional development program aimed at improving “student achievement through a

comprehensive approach to improving teacher quality” (Allen, 2007, January 4, p. 1). Project partners included KSU’s College of Education and Department of Mathematics and six partner districts, five of which were high-need. Based on an assessment of student and teacher needs in the six districts, four goals were established for IMP:

Goal 1: Increase student achievement in mathematics in high needs schools, in particular to reduce the achievement gap of underrepresented populations.

Goal 2: Strengthen the content and pedagogical knowledge of K-8 teachers including increased number of teachers with middle school mathematics endorsement.

Goal 3: Increase the implementation of standards-based mathematics instruction and curriculum in K-8 classrooms.

Goal 4: Strengthen and expand existing partnerships to enhance collaboration to address the needs of K-8 schools with emphasis on underrepresented populations, while improving teacher quality in mathematics. (pp. 3-4)

The IMP grant proposal outlined specific activities to align with the C³ Academy professional development approach. First, each year had a content focus led by the IHE mathematics department during the morning summer institute sessions, seeking to make higher mathematics accessible to K-8 teachers. Year 1 (2007-2008) content tackled basic ideas of calculus with an emphasis on making connections to middle school mathematics in order to address the needs of several participants seeking a middle school mathematics endorsement. For IMP Year 2 (the year under study), the content focus engaged participants with math-related literature, *The Number Devil* by Enzensberger (2000), in order to examine patterns in basic number theory. Finally, content for the final year of IMP focused on patterns in algebra and real world applications using the topic of coding theory.

Second, curriculum connections were made through a variety of activities. For example, some standards-based curriculum activities were used during content sessions. In addition, project leaders emphasized “connections between the content and KSDE standards, high quality lessons from exemplary curriculum, research based teaching strategies and relevant mathematics manipulative tools” (Allen, 2007, January 4, p. 10). During afternoon summer institute sessions, teachers were guided to develop a “Content-specific Action Plan” (CAP). The action plan required groups of teachers to analyze data of their school’s performance on state mathematics

assessments. Based on areas of weakness, teachers identified goals and benchmarks to address in a lesson. As part of the summer institute, the Japanese lesson study protocol was modeled for the participants. During the school year, groups of teachers were supported as they developed lessons, carried them out, analyzed the lesson presentation and resulting student learning, and then revised and repeated the lesson in a different classroom.

Finally, a focus on the child was established by providing “teachers with strategies to meet the needs of **ALL** learners” (Allen, 2007, January 4, p. 10). Afternoon summer institute sessions identified and modeled differentiated instruction strategies. Guidance was provided on how to implement strategies in a mathematics classroom.

Of note, although content sessions were typically in the morning and pedagogy sessions were typically in the afternoon, a small modification occurred during Year 2. To compensate for losing two content morning sessions when a large number of participants attended required district pedagogy sessions during two full days of the IMP summer institute, content sessions took up two full days as recorded by the researcher as Days 2 and 3 of IMP Year 2. In addition, field notes of IMP specific days spanned Days 1 – 8.

Follow-up professional development activities took a variety of forms. First, teachers were supported as they developed their CAP plans and implemented a lesson study cycle during the fall. Project leadership dialogued with project participants throughout the year. Project leadership had face-to-face contact with participants during the school year by participating in the observation and discussion of research lessons. Finally, KSU hosted a final share fair for project participants to report their lesson activities.

Project Leadership

Two KSU faculty members from teacher education conducted the pedagogy sessions (Pedagogy Facilitator A, B). Four K-12 teacher leaders contributed as well. Two KSU faculty members and one KSU graduate research assistant from the mathematics department facilitated the content sessions (Content Facilitator A, B, C).

The Project Participants

Teachers from six KSU-PDS partner districts were invited to apply to participate in Year 2 (2008-2009) of the IMP program on a voluntary basis. Federal funding of the project allowed monetary incentives to be offered to participants; participants were paid a stipend for full

participation in the project. In addition, participants were given the option of applying the stipend for enrolling in up to five hours of KSU credit that could count towards a Middle Level Math Endorsement and/or a Masters in Curriculum & Instruction (Allen & Hancock, 2008).

Thirty teachers fully participated in Year 2 of the IMP program. In addition, two of the teacher leaders participated in the content activities while serving in leadership roles during the pedagogy activities. Seventeen teachers represented elementary schools; ten represented middle schools; and five represented high schools. Teachers' classroom experience as self-reported during the summer institute ranged from one to twenty-six years. One-quarter of the participants reported having 9 or fewer college math credits. On the other hand, five participants reported having 50 or more college math credits. Most of the participants taught in high need school districts. Half of the teachers had participated in the first year of IMP.

Mini-Case Study Individuals

Four project participants were selected for closer examination of the impact of the professional development program. Naturalistic inquiry commonly relies on purposeful sampling rather than random sampling more often associated with quantitative methods (e.g., Lincoln & Guba, 1985). Furthermore, Patton (2002) described how more than one qualitative sampling strategy may be used in sampling decisions. Several considerations were used in identifying potential candidates for the purposeful sample. First, teachers' grade level was considered for the sample. An analysis report of Year 1 IMP pre- and post-summer institute content test scores suggested that the program had a larger impact on teachers instructing at elementary and middle school grade levels (Kansas State University Office of Educational Innovation and Evaluation, 2007b). In addition, the grant focused on K-8 mathematics (Kansas Department of Education, n.d.). The researcher chose to further narrow the grade level criteria for the sample by focusing on teachers of grades 4-6 based on literature and consultation with the project director. In *PSSM*, NCTM highlighted the importance of ongoing professional development for teachers of the grade bands 3-5 and 6-8. Because of the "increasing mathematical sophistication of the curriculum in grades 3-5" (2000, p. 146), teachers having often received generalist training which provided "minimal attention to mathematics content knowledge" (p. 146) should seek opportunities to advance their own mathematical understandings through continuing professional development. For teachers of grades 6-8,

professional development is also especially important because few teacher preparation programs specifically train teachers for middle school mathematics. Middle school teachers often get their training in elementary programs or secondary programs. As such, they may need to develop their understandings of middle school mathematics and/or their understandings of adolescent development and pedagogical alternatives. Upon consultation with the program director, the grade level criterion was narrowed to grades 4-6.

A second consideration concerned whether or not participants had previously participated in Year 1 of the IMP program because of possible variation in the impact of the program based on years of participation. The researcher chose to select two participants who were participating in the IMP program for the first time in Year 2 and two participants who were in their second year of participation in the IMP program.

Another consideration for the sample involved the teachers' district locations. Upon discussion with advisors, homogeneity in location was considered in sampling. Participants from districts in a closer vicinity to the university were more likely to receive higher levels of follow-up with the project leadership. In addition, existing professional development school relationships with local districts would likely make it easier to establish consent for video-taping of classrooms lessons. Hence, teachers from districts in a close vicinity to the university were considered for selection.

Finally, a few participants were not considered for selection if they were a specialty area educator (e.g., special education teacher) or were involved with leadership. Of the remaining participants, ten teachers were identified by their application as teaching in grades four through six at districts in a close vicinity to the university. Of those ten, six were in their first year of participation in the IMP program. Wanting to capture some variation within the grade band and length of participation in the program, the researcher chose to select a participant representing each category as shown in Table 3.1.

Table 3.1: Mini-Case Sampling Stratification

1 st year of participation with IMP and Teaching in grade 4 or 5	2 nd year of participation with IMP and Teaching in grade 4 or 5
1 st year of participation with IMP and Teaching in grade 6	2 nd year of participation with IMP and Teaching in grade 6

A first contact teacher was identified for each category. A second contact teacher was also identified in case the first contact teacher declined to participate. First contact teachers were called or emailed to probe their willingness to participate as mini-cases involving summer institute interviews and two classroom observations during the school year (see consent form Appendix A). All four first contact teachers agreed to participate as mini-case study individuals.

Summer institute interviews with the four teachers revealed that experience was varied in the sample. One first year IMP participant and one 2nd year IMP participant were more experienced teachers. The other two teachers had only been teaching for one or two years. An analysis report of Year 1 IMP pre- and post-summer institute of teachers' self-reported perceptions of knowledge and comfort in teaching specific mathematical concepts suggested that the IMP project had a larger impact on teachers' ratings for less experienced teachers (Kansas State University Office of Educational Innovation and Evaluation, 2007a). Post-selection, the researcher noted that experience was conveniently varied in the mini-case sampling of Year 2 IMP participants.

Thus, mini-cases were purposely chosen from participants who were working in a school district with fairly close proximity to the university and who were teaching in grades four through six. The cases were purposely stratified across grades 4-6 and as to whether or not the teacher had participated in IMP during the previous year. Conveniently, the sample also varied by teachers' experience.

Researcher

Qualitative research challenges the inquirer to “be self-reflective, to acknowledge biases and limitations and to honor multiple perspectives” (Patton, 2002, p. 65). A researcher's model of teaching may determine what the researcher attends to, and hence define what counts as

teacher development for a study (Simon, 1997). Hence, I feel compelled to explain my own perspectives and conceptualizations of the nature of mathematics teaching and learning.

In December of 1987, I received a degree in secondary education in mathematics. In the spring, I taught as a six-week long-term substitute for mathematics at a middle school. However, I did not find my substitute teaching experience very fulfilling. As a novice teacher, I struggled to maintain discipline. I lacked support and mentoring. Hence, over the next year, I took a job outside of education. Later, I started teaching some mathematics classes at a university as a lecturer and had several positive teaching experiences. I went on to complete a master's degree in mathematics. Over the course of 16 years, I have taught mathematics, particularly algebra and calculus, at two universities. In 2002, I began to pursue a doctoral degree in curriculum and instruction at Kansas State University.

NCTM Standards had not yet been authored when I pursued my secondary education degree. Most of my education classes were not content-focused. As was typical for the time, most general pedagogy was taught from the education department and mathematics content was taught from the mathematics department. However, one instructional methods course stands out for its focus on teaching mathematics. The instructor taught at a high school and as an adjunct for the university. Looking back, I believe this instructor was reform-minded as she emphasized problem-solving. However, standards-based reform had not yet been formally communicated around 1986.

Over the years, my teaching practice has evolved and is still a work in progress. As I began to teach in college, I remember desiring to teach for understanding; however, I started out being rather teacher-centered. I focused on making sure I understood the material, on what I would say, and on what problems I would show the class. As with many others of my generation, my own mathematics education had been dominated by a focus on procedures. My conceptual understanding was weak for many topics. In addition, the curricula I used were often procedurally focused. Hence, my desire to teach for understanding was not very fulfilled.

In the second institution at which I taught, I entered a program where established curricula had some reform tendencies. Numerical, graphical, symbolic, and verbal representations were integrated. The curricula promoted using the graphing calculator for fostering conceptual understanding. Modeling was emphasized. A few years after I began to

work at the institution, I began work in a doctoral program. My world was opened up to constructivist theory, NCTM standards, and math-focused pedagogy.

I also began to have more interaction with K-12 education. As my own children have progressed through K-12, I have become more aware of K-12 curricula and learning. I have seen mile-wide and inch-deep high school curricula. I have come to appreciate the need for differentiated instruction by working with the different needs of my children. I have had many of my beliefs about learning challenged and altered by my involvement with the MATHCOUNTS program for middle school students. For example, while eighth graders may approach an application problem with algebra, a sixth grade student may come up with non-algebraic innovative reasoning for solving the same problem. I have observed how a cooperative learning environment for working with challenging mathematics problems can benefit students with a wide range of skills. I have also seen the results of K-12 education in my college classes. Students tend to reach me with very strong opinions about mathematics. Some students have had success and seem to enjoy mathematics; they tend to place in College Algebra or above and demonstrate a good understanding of mathematics. However, far too many students dislike mathematics. Often, mathematics has not made sense for a long time. Some students have had three or more years of high school mathematics, yet place in a beginning or intermediate algebra course. It is not unusual to have a student with some algebra skills, but very little ability in dealing with rational numbers. It is a challenge to build on students' prior knowledge in the setting of an algebra class when some of the students have a very weak understanding of rational numbers. Although I see the need for differentiated instruction and have been able to help my own children, I often find it a challenge to differentiate instruction for my college students.

I appreciate positions that recognize effective instructional practice may exist from a range of solutions rather than stark alternatives from the poles (e.g., National Mathematics Advisory Panel, 2008; RAND Mathematics Study Panel, 2003). The National Mathematics Advisory Panel (2008) noted that research does not support the exclusive use of either student-centered or teacher-directed instruction. I appreciate Franco, Sztajn and Ortigão's (2007) discussion of reform versus traditional teaching practice by comparing a one-dimensional model with a two-dimensional model. In a one-dimensional model, reform and traditional teaching are viewed in opposition to one another. To become more reform-oriented, one must let go of traditional practice. In a two-dimensional model, reform practice is represented on one axis and

traditional practice is represented on another axis. With this perspective, to become more reform-oriented, a teacher does not have to let go of certain practices that are considered more traditional.

Based on the two-dimensional model of teaching, I perceive my teaching practice as having taken on more features of reform practice over the years. For example, I have become more student-centered over the years. Whereas I have always sought to understand student thinking as expressed by work on open-ended homework and test problems, I have been expanding my attention to student thinking. Recently, I have implemented a variation of an activity recommended in an MAA publication (Prather, 2007). I am collecting student reflections (a couple of sentences or a more substantial paragraph) over homework assignments. The reflections have been quite revealing and I have sometimes altered my plans for the next class based on the reflections. I continue to learn more about the conceptual underpinnings of many topics and am better able to use that information in class. I use more activities that engage students, that build upon students' prior knowledge, and that connect physical representations and real-world contexts with symbolic representations. Recently, probably in connection with learning about differentiated instruction during my experiences with IMP, I have started to create some assignments with open questions (allowing for variety of responses or approaches) and choice of tasks (Small, 2009). Finally, I strive to create a classroom atmosphere whereby sense-making is valued.

Although I have incorporated more reform features into my practice over the years, I still retain some earlier habits. In most of the classes I teach in the university setting, I am still tied to covering certain content over the course of the semester. Although my current institution allows me some flexibility in content coverage as we do not have departmental finals driving faculty to cover "everything", I still feel compelled to get through critical material in order for the student to be prepared for potential future mathematics courses. As a consequence, I tend not to use lengthy exploration activities. However, I sometimes use exploration activities that do not take up a lot of time and I continue to add more exploration activities to my repertoire over time. I would consider many of my lessons to incorporate features of "interactive lectures" as opposed to "passive-student lectures." I question students and challenge them to make conjectures. We examine problems graphically and symbolically and look for connections. I may have the students try a problem during class. As the students attempt the problem, I walk around the

classroom to observe student work in order to informally assess student understanding and to provide additional support. Sometimes students bring forward questions for which the class brainstorms reasoning and conceptual underpinnings. However, although I try to promote discussion of mathematical ideas in the classroom, my tasks and questions are often not open-ended enough to generate a lengthy discussion among students.

Thus, I believe my practice has expanded to include more features of reform over the years. I am integrating more data modeling, physical demonstrations that connect abstraction with reality, and multiple representations. In addition, I try to create a classroom environment whereby reasoning, conjecturing, sense-making, and discussing alternative methods are valued.

Data Collection

The examination of the impact of a complex professional development project is challenging. In this study, the scope of the examination has been focused to address the following research questions:

1. What impact did the IMP program have on participants' content knowledge (e.g., subject matter content knowledge, pedagogical content knowledge, curricular knowledge)?
2. What impact did the IMP program have on participants' pedagogical knowledge (e.g., knowledge of differentiated instruction, knowledge of standards-based instruction)?
3. What impact did the IMP professional development program have on participants' teaching practices (e.g., differentiating instruction, implementing standards-based teaching, analyzing practice)?

This embedded units single-case study was informed by an examination of both qualitative and quantitative evidence. A variety of types of data collection procedures were used including: participant observation, interviews, documentary evidence, homework and test questions from content sessions, homework/session reflection questions, and direct observation of classroom lessons. In addition, existing project-collected data along with summary quantitative analyses reports were used to further inform the study.

For the macro-case study, sources of evidence included:

1. **Participant observation.** One year prior to the professional development project year under study, the researcher assumed a participant observer stance during the project's summer institute. The experience was invaluable as it allowed the researcher to become familiar with the general structure of the project. In addition, the researcher met members of the project leadership team and several participants who attended the project during both 2007-2008 and 2008-2009. Being already familiar with some people associated with the project and with the general structure of the project, the researcher was able to direct significant attention on participant and facilitator discourse and interactions during the 2008-2009 project activities. As a participant observer, the researcher took field notes about descriptions of the setting, people, activities, collective discussions, and conversations with small groups or individuals during the two-week summer institute and some follow-up activities. Some institute sessions were audio taped. Field notes were expanded and/or edited upon review of audio tapes.
2. **Documentary materials.** The IMP grant proposal and course materials were examined.
3. **Existing project-collected participant homework and test assessments.** The content sessions required participants to complete homework during the summer institute and pre- and post-summer institute content tests. Participant homework and test assessments were examined.
4. **Homework/session reflections.** The researcher asked participants to voluntarily consent to writing daily reflections about concerns/confidence with daily assignments and reflections about morning and afternoon session activities and discussions (see Appendix B). The researcher initially read the reflections during the summer institute and further reviewed them during later analysis.
5. **Existing project-collected survey data and summary quantitative analyses reports regarding scale survey data and content tests.** The existing project-collected survey data and summary quantitative analyses reports were examined so as to maximize their utility. The project administered pre- and post-summer institute surveys. The surveys posed some open-ended questions probing teachers' instructional strategies and knowledge of differentiated instruction. Responses were

available to the researcher for review. In addition, likert scale questions probed teachers' self-perceptions of knowledge of mathematical concepts and comfort in teaching mathematical concepts as related to project mathematical content. KSU Office of Educational Innovation and Evaluation (OEIE) provided data analysis reports of the pre- and post-summer institute content tests (see Appendix C) and the pre- and post-summer institute survey scale data (see Appendix D).

For the individual mini-cases, additional data sources included:

- 1. Pre- and post-summer institute semi-structured interviews.** Pre-summer institute interviews (see Appendix E) were conducted during the first and second day of the institute with four IMP teacher participants to probe typical lesson preparation habits, typical lesson characteristics, and mathematical knowledge for teaching. Post-summer institute interviews (see Appendix F) on the last day of the institute probed self-perceptions of gains in content knowledge, gains in pedagogical knowledge, and potential changes in teaching practice. Teachers were also asked to look at pre-summer institute mathematical knowledge for teaching questions to see if they wanted to change their answers in any way.
- 2. Classroom lesson observations.** Different data collection methods and analysis techniques have been utilized to study teaching practice. In congruence with an in-depth case study design, the researcher chose to do some classroom observations with videotaping. Two classroom lesson observations (see Appendix H) with pre-and post-observation interviews (see Appendix G and Appendix I) were conducted during the school year. During consideration of an observation protocol, the researcher reviewed research literature and several instruments. The researcher chose to create a protocol influenced by several resources and tied closely to the research questions of this study. Some observation protocols are very structured and are particularly useful for collecting data from a large number of classrooms by various observers. The instruments reviewed were the Classroom Observation Protocol (COP) developed by Horizon Research, Inc. (Horizon Research, 2005) and the Reformed Teaching Observation Protocol (RTOP) developed by Arizona State University-Arizona Collaborative for Excellence in the Preparation of Teachers (Piburn & Sawada,

2000). Both instruments utilize survey likert scale ratings for part of the protocol along with some quantitative analysis. Some other observation frameworks utilize data collection and analysis that differ from likert scale protocols. For instance, the researcher reviewed Jacobs et al.'s (J. K. Jacobs et al., 2006) discussion of features of teaching practice compared with NCTM *Standards* pedagogical process recommendations for middle school teachers. In their study, Jacobs et al. reviewed TIMSS video data, coded features of teaching practice including discourse and problem types, and recorded the frequencies of occurrence. Furthermore, the researcher reviewed NCTM's recommended Standards for the Observation, Supervision, and Improvement of Mathematics Teaching (T. S. Martin, 2007). The Standards provide a "framework from which to observe classroom activities" (p. 82) that focuses on viewing the interactions between the teacher and students. Reflection prompts provide direction on what observers should attend to. Finally, the researcher reviewed other resources to identify specific features to attend to related to differentiated instruction (Tomlinson, 2001) and skills for analyzing practice (Hiebert et al., 2007). Influenced by these sources, the protocol tightly attends to teaching strategies modeled during IMP Year 2. The protocol includes reflection prompts for instances of occurrences of features. Classroom observations were videotaped. Field notes from the observations were expanded and/or edited upon review of videotapes.

Before each lesson, the teacher was asked to answer interview questions (either via email or face-to-face interview with audiotaping) intending to probe content knowledge, skills for analyzing practice (Hiebert et al., 2007), and planned instructional practices. Then, the researcher observed, wrote field notes, and videotaped each lesson. Interview questions were also posed after the lesson to probe skills of inquiry into practice, typicality of instructional strategies, and use of differentiated instruction. Field notes from interviews were expanded and/or edited upon review of audiotapes.

Data Analysis

There is not a clear distinction between data collection and data analysis in qualitative research (e.g., Merriam, 1998; Patton, 2002). During qualitative data collection, ideas for

making sense of the data may emerge and constitute the beginning stages of data analysis. Patton described that when data collection ends, the researcher can organize further analysis based on two primary sources of information: “(1) the questions that were generated during the conceptual and design phases of the study, prior to fieldwork, and (2) analytic insights and interpretations that emerged during data collection” (2002, p. 437). Creswell (1998) described a qualitative data analysis spiral whereby the researcher engages in a process of moving in analytic circles rather than a fixed linear path. For a case study, the researcher begins by collecting and organizing data. The researcher gets a sense of the data by reading and writing memos. Later the researcher tries to describe, categorize and interpret the data. Finally, the researcher presents a narrative description. For this study, analysis involved repeated processes of circling up and down through stages.

Stage 1: Summer Institute Data Collection and Data Organization

An intense period of data collection along with initial data organization occurred during the two-week summer institute. Pre- and post-summer institute interviews were conducted with teachers participating in the mini-cases. In addition, participant observation was conducted throughout the summer institute. Participant homework/session reflections were collected and initially read during the institute. By reviewing reflections as they came in, analysis began and was used to direct the researcher’s attention during subsequent observations and interviews. Participant homework collected by the content session leader was photocopied by the researcher for later review.

Stage 2: Post-Summer Institute Data Organization, Modifications Made to the Literature Review and Methodology Chapters, Observation and Interview Data Collection for Mini-Cases Conducted during the School Year, Participant-Observation of Follow-Up Share Fair, and Transcription of Interviews

After the summer institute, researcher-collected data continued to be organized. Based on initial reactions and potential themes emerging from participant observation and interviews during the summer institute, the researcher made additions and modifications to the literature review. In addition, descriptive data (e.g., number of participants) from project-collected survey data was used to provide more details in the methodology chapter.

Data collection and organization continued during the school year. Mini-case teachers were interviewed pre- and post-lesson and observed for two mathematics lessons during the

school year. The researcher returned to a participant observer role for the project follow-up share fair activity. Data continued to be organized and aggregated. Interviews were transcribed. In order to help maintain a chain of evidence, the Atlas.ti qualitative data analysis software package was chosen to assist in the organization of textual data, memos, quotes, and coding.

Stage 3: Entering Data, Reading, Memoing, and Identifying Potential Themes through Coding

A grounded theory approach was used to identify themes emerging from analysis of the variety of data sources including: participant observation, documentary materials, summer institute homework and tests, summer institute reflections, interviews, classroom observations, and existing project-collected survey data and summary quantitative analyses reports data. Ryan and Bernard (2003) explained that themes come from both the data and the investigator's prior theoretical understandings. A priori themes stem from the research questions and the interview and observation protocols. But, many themes and subthemes are generated from reading, memoing, and coding the data.

Formal analysis of data started with preparation of textual data as necessary and entering text into a qualitative data analysis software program. The first set of data reviewed was the project-collected pre- and post-summer institute open-ended survey questions. Analysis commenced by searching for patterns of meaning related to the research questions while reading participant responses, and underlining or marking with color similar phrases. To identify patterns in the data, the researcher used two scrutiny techniques (Ryan & Bernard, 2003): looking for repetitions and employing a constant comparative method searching for similarities and differences across the units of data. Phrases representing similarities amongst responses were written and repetitions were counted. Representative phrases with corresponding counts and participant quote examples were typed in a document and entered into the qualitative software as a primary document for analysis. Researcher reactions and reflections about the open-ended survey data were written as memos for the document. In addition, codes were attached to some document quotes. Coding began with open coding as data units were compared with others for similarities and differences (Corbin & Strauss, 1990). Initial codes were created. Recurring patterns were coded from a list of existing codes. Subsequently, creating new codes and renaming old codes were done as needed.

The researcher continued analysis with data from the summer institute. Going in chronological order for each day, field notes were expanded based on audio recordings as rich discussions were transcribed from audio recordings. Daily homework was reviewed and oftentimes summaries of the content and general patterns were described in a document. Session/homework reflections which had already been transcribed were entered into the qualitative software as textual documents. Data units from the documents were coded and memos were written as described earlier.

Next, the researcher reviewed the transcribed mini-participant interviews and proceeded with coding and memoing for those textual documents. The researcher prepared the share fair data by expanding field notes based on audio recordings. Again, data coding and memoing transpired. Finally, classroom observation field notes and video tapes were reviewed, summarized, and coded.

Reading and memoing continued. The constant comparative method was used to identify new codes and to assign existing codes to quotations and data units. Concepts made their way into the theory by relevance to the research questions and “by *repeatedly* being present” (Corbin & Strauss, 1990, p. 7). Conceptually similar data units or codes were grouped together into themes.

Stage 4: Final Stages of Analysis and Presentation of the Data

In the final stages of analysis, the researcher considered whether description, discourse, exemplar quotes from reflections or survey data, homework or test performance, or other data sources best illustrated a theme. In the written report, attention was given to providing transitions and connections between data sources and units within a theme or between themes.

Validity and Reliability

Validity and reliability in qualitative research have been described from a variety of perspectives with a wide range of terms (e.g., Creswell & Miller, 2000; Patton, 2002). For instance, terms related to validity being used in qualitative literature include credibility, confirmability, and trustworthiness. Reliability has been described as dependability. Whereas a term like trustworthiness is an overarching, general term related to validity, some qualitative researchers differentiate between different types of validity such as face validity, internal

validity, theoretical validity, construct validity, and external validity (e.g., W. Sykes, 1990; Yin, 2003).

Researchers have outlined different strategies for promoting qualitative research validity and reliability (e.g., Creswell & Miller, 2000; Johnson, 1997; Patton, 2002; Yin, 2003). First, triangulation is a validity procedure that “strengthens a study by combining methods” (Patton, 2002, p. 247). Creswell and Miller (2000) described triangulation as a procedure whereby researchers search for convergence among multiple sources of information to form themes in a study. This study employed methods collection triangulation, qualitative data sources triangulation, and analyst triangulation (e.g., Patton, 2002). First with regard to methods collection triangulation, data was primarily collected through qualitative methods for the case study. However, the study was further informed by data collected through quantitative methods (i.e., IMP project collected pre- and post-institute survey data and analyses conducted by KSU OEIE). Second, data source triangulation was evidenced by the variety of qualitative data sources: participant observation; documentary materials; participants’ homework, tests, and reflections; and mini-case interviews and classroom observations. Per Patton, triangulating data sources means being able to compare the “the consistency of information derived at different times and by different means *within qualitative methods*” (p. 559). During analysis, information derived from various data sources (e.g., interviews, observations, content test performance, reflection comments) was compared. In addition, consistency and comparison over time could be checked through the collection of multiple reflection commentaries from each participant, pre- and post-institute interviews with mini-case teachers, lesson observations at two different times for mini-case teachers, pre- and post-institute project-collected survey and content test data, and project-collected daily homework. By triangulating with methods collection and data sources during data collection, data analysis attended to testing for consistency (Patton, 2002). Finally, Patton (2002) said the role of the doctoral committee as expert reviewers can increase the credibility of a study. Thus, a type of analyst triangulation was used in the study as the doctoral committee reviewed the study’s narrative and findings. Overall, a variety of triangulation strategies contributed to the credibility of the study.

Second, a search for disconfirming evidence was another strategy used for promoting validity. Although researchers have the natural inclination to search for confirming evidence, researchers should also strive to search for disconfirming evidence (Creswell & Miller, 2000;

Johnson, 1997). Final decisions about themes should reflect the majority of the evidence. After the researcher established some preliminary themes, data was searched for both confirming and disconfirming evidence. Thus, the search for disconfirming evidence provided further support of the credibility of the study.

Particularly in qualitative research, a potential threat to validity is researcher bias. A strategy for minimizing researcher bias is referred to as reflexivity by which “the researcher actively engages in critical self reflection about his or her potential biases and predispositions” (Johnson, 1997, p. 284). For this study, the researcher described her background and beliefs about mathematics learning and teaching. In addition, use of the constant comparison method during coding assisted the researcher in guarding against bias (Corbin & Strauss, 1990). Comparison of data with preconceived ideas challenged the appropriateness of concepts and themes.

Prolonged engagement in the field has also been described as a strategy for promoting validity (e.g., Creswell & Miller, 2000; Johnson, 1997). Prolonged engagement has no set duration; however, the longer a researcher stays, the better his or her understanding of the context of the participants’ views (Creswell & Miller, 2000). Participant observation of an intensive two-week summer institute and follow-up share fair day-long activity, along with limited classroom observations, do not necessarily constitute a long engagement in the field; however, the researcher also engaged in participant observation of the prior year’s summer institute and follow-up share fair. These earlier participant observation experiences supported establishing rapport with project leadership and with some of the participants who attended the professional development project for both years. In addition, the researcher gained a better understanding of the context of the professional development project by engaging in participant observation during both the year under study and the prior year. Furthermore, Bernard suggested participant observation in general gives the researcher a better understanding of the culture and thus “extends both the internal and external validity of what you learn from interviewing and watching people” (2006, p. 355).

The validity of a study can be further enhanced by establishing an “audit trail” (Creswell & Miller, 2000). Thus, the researcher strived to clearly document the inquiry process by establishing and maintaining organization of the raw data as well as organization of textually prepared data along with subsequent coding and memoing in the qualitative software file. These

activities also increased the reliability of the study (W. Sykes, 1990; Yin, 2003). By maintaining a chain of evidence from the research questions to the study conclusions, the researcher's documentation allows the reader to follow the evidentiary process (Yin, 2003). Furthermore, the use of protocols increases the reliability of the study. For this study, protocols for interviews and observations were created and followed. In addition, a case study protocol in the form of a dissertation proposal guided the data collection and analysis.

Finally, another validity strategy for qualitative research is the use of rich descriptions of the setting, participants, dialogue, and themes (Creswell & Miller, 2000). Generalizability in naturalistic inquiry or naturalist evaluation may be viewed as applicability or transferability (Guba & Lincoln, 1981; Guba & Lincoln, 1989). This study used thick description of the context of the case and of the data collection methods and analysis. By providing thick descriptions, the case study facilitated "the drawing of inferences by the reader which may apply to his or her own context or situation" (Guba & Lincoln, 1989, p. 224). In addition, as quotes are the lowest inference descriptor (Johnson, 1997), verbatim descriptive quotes were provided in the report when possible.

Summary

Professional development has been viewed as a critical component for U.S. school improvement and school reform. An embedded units single-case study design was utilized in order to examine a complex professional development program. A variety of data collection methods and data sources were used, thus adding to the credibility of the study. The researcher hoped to determine themes illuminating the impact of the IMP professional development program on teachers' knowledge and teaching practice.

CHAPTER 4 - Results

Introduction

A case study methodology was used in this study in order to examine the impact of the IMP professional development model on teachers' content knowledge, pedagogical knowledge, and instructional practice. A variety of types of data collection procedures were used, including participant observation, interviews, documentary evidence, homework and test questions from content sessions, homework/session reflection questions, and direct observation of classroom lessons. Furthermore, existing project-collected data along with summary quantitative analyses reports were used to inform the study. A grounded theory approach was used to identify themes emerging from analysis of the variety of data sources.

This chapter begins with a discussion of features and strategies associated with effective professional development and corresponding features and strategies employed by the IMP professional development model. The chapter will continue with themes outlining the findings of the study with regard to the impact of IMP on teachers' content knowledge, pedagogical knowledge, and teaching practice.

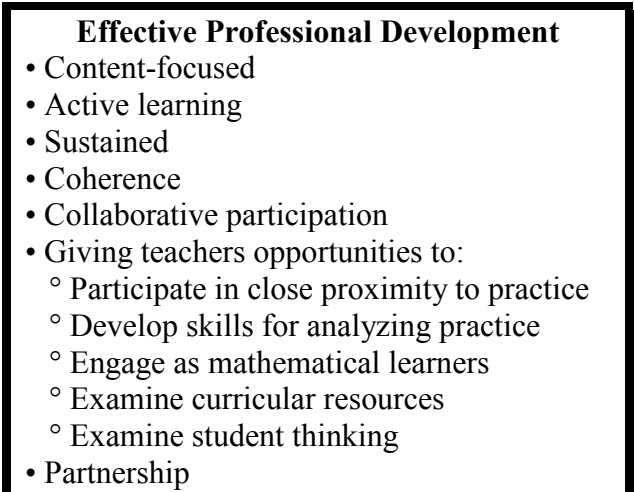
The Effectiveness of the IMP Professional Development Model

The Infinite Mathematics Project was grounded in a C³ model with foci on **C**ontent knowledge, **C**urriculum connections, and the **C**hild. The model was developed as part of a partnership between IHE faculty from a School of Education and a Department of Mathematics, and implemented with K-12 teachers from partner school districts. The program was intensive, sustained, and focused on mathematical content and research-based instructional strategies. Furthermore, IMP provided participants with many opportunities for active learning and collaboration.

Many characteristics and strategies associated with effective professional development have been reviewed, described, and recommended. For example, Loucks-Horsely et al. (2003) outlined seven principles of effective professional development and eighteen strategies for professional learning. Other researchers (e.g., Borasi & Fonzi, 2002; Sowder, 2007) have

synthesized and organized effective elements and strategies in different ways. Some characteristics and strategies with particular relevance to the study at hand (see Figure 4.1) are reviewed here.

Figure 4.1: Effective Professional Development

- 
- Effective Professional Development**
- Content-focused
 - Active learning
 - Sustained
 - Coherence
 - Collaborative participation
 - Giving teachers opportunities to:
 - Participate in close proximity to practice
 - Develop skills for analyzing practice
 - Engage as mathematical learners
 - Examine curricular resources
 - Examine student thinking
 - Partnership

First, six dimensions of effective professional development that were used as a framework for studying the Eisenhower Professional Development Program (Garet et al., 1999), and later used for research and program evaluation regarding Title IIB MSP programs (Blank et al., 2008; Gummer & Stepanek, 2007) include: (a) whether the activity is organized as a reform type, (b) the duration of the activity, (c) the degree to which the project emphasizes collective participation, (d) the degree to which the project has a content focus, (e) the degree to which the project offers opportunities for active learning, and (f) the degree to which the project promotes coherence. In addition to these six dimensions of effective professional development, other specific recommendations have been made as well. For instance, researchers and practitioners (Ball & Cohen, 1999; Borasi & Fonzi, 2002; Loucks-Horsley et al., 2003) have recommended that effective professional development should provide opportunities for teachers to participate in activities in close proximity to practice. In addition, teachers should have opportunities to develop skills and dispositions for analyzing practice. Furthermore, some specific strategies associated with effective professional development include engaging teachers as mathematical learners (e.g., Borasi & Fonzi, 2002; Loucks-Horsley et al., 2003), examining exemplary curricular resources, and examining student thinking (e.g., Borasi & Fonzi, 2002; Sowder, 2007). Finally, educational partnerships between public school teachers, administrators, and university

faculty have been envisioned as collaborative sites where research and practice can intersect for the joint purpose of improving student learning (Holmes Group, 1986).

The Infinite Mathematics Project embodied many elements and strategies associated with effective professional development. First, the IMP model was developed as part of a **partnership** between IHE faculty from a School of Education and a Department of Mathematics, and implemented with K-12 teachers from partner school districts. On a national level, a MSP Title IIB grant at minimum requires a partnership between a high-need local educational agency and IHE STEM faculty. The creation of Math and Science Partnerships in 2002 as part of No Child Left Behind Act was a push to involve university mathematics and science faculty with K-12 teacher professional development as opposed to previous attention on involving university education faculty (Brainard, 2005). However, education faculty can play a critical role in MSP projects by helping K-12 teachers recognize and understand how the mathematical knowledge they are learning can be used in teaching (Zhang et al., 2007). The IMP professional development program under study was funded under Kansas MSP Title IIB proposal guidelines which targeted K-8 mathematics and required not only a partnership with IHE STEM faculty and high-need school districts, but also with university mathematics teacher education faculty (Allen, 2007, January 4). As such, university faculty from both mathematics and mathematics education along with participant teachers having a variety of grade-level experience in mathematics education had opportunities to learn from each other with the goal of improving K-16 mathematics education. For example, K-12 teachers had opportunities to learn more about what “mathematicians do and how and why they do it” (Loucks-Horsley et al., 2003, p. 142). On the other hand, mathematicians had opportunities to learn more about the realities of K-12 mathematics education and to become more aware of their own teaching practices.

Second, IMP embodied all six dimensions of effective professional development being used as criteria for doing research and evaluation of Title IIB MSP programs (Blank et al., 2008; Gummer & Stepanek, 2007). By the very nature of the Kansas MSP Title IIB grant proposal guidelines requiring a two-week summer institute and four days of follow-up during the school year, IMP Year 2 professional development activities were **sustained over time**. Although the two-week summer institute has typically been categorized as a traditional type (Garet et al., 1999; Gummer & Stepanek, 2007), IMP Year 2 also incorporated **reform type activities** through its use of lesson study, opportunities for teacher collaboration, and mentoring. A **mathematics**

content focus was addressed in an integrated fashion by the following components: (a) project utilization of mathematical literature to examine Number and Algebra Content Standards as well as Process Standards, (b) project modeling and participant creation of differentiated instruction strategies/activities specifically related to mathematics, (c) project utilization of strategies for reading in the content area of mathematics, (d) pedagogy facilitator and lead teacher modeling of standards-based instruction supporting the development of number sense, (e) participant development of an action plan addressing the lowest mathematics indicator for his/her upcoming class, and (f) participant implementation of a mathematics lesson study during the school year.

Collective participation was evidenced by participation of teachers who were interested in mathematics professional development and who taught in KSU partner districts. During some aspects of the summer institute, teachers were grouped by grade level (e.g., when creating differentiated instruction process and/or product activities). In addition, some teachers came from the same school and/or district. **Coherence** was evidenced by project use of state standards and assessment results for addressing action plans, by encouragement of teacher collaboration and communication in flexible groupings, and by attention to IMP program goals and Title IIB MSP goals. Furthermore, IMP was designed to offer teachers the opportunity to apply their stipend for up to 5 hours of course credit that could count towards Middle Level Math Endorsement in Kansas and/or Masters in Curriculum and Instruction.

Finally, **active learning opportunities** for participants were abundant during IMP Year 2. Active learning experiences that were most related to Garet et al.'s dimensions (1999) included: (a) planning for instruction by collaborating with grade level teachers in creating differentiated instruction process activities and product assignments, (b) making a content mini-presentation to the group, (c) writing an action plan to meet identified student needs from data on lowest tested mathematics indicator, (d) participating in a lesson study during the school year (requiring planning for instruction, observing, being observed, and analyzing), and (e) presenting results of action plan initiatives and the lesson study process at the final share fair. In addition, specific pedagogical approaches were used that have been linked to strengthening learners' metacognitive skills and are therefore additional examples of actively engaging the participants in learning (National Research Council, 2000). For example, discussions were widely used throughout the summer institute. Discussions occurred in larger groups and smaller groups. Discussions were sometimes more facilitator led; at other times participants' questions and

reflections led the discussions. Research suggests as learners engage in discussing and questioning they become “better at monitoring and questioning their own thinking” (National Research Council, 2005, p. 577). Furthermore, the project facilitators used specific types of questions that encouraged learners “to reflect on their learning, consider transfer possibilities, self-assess their performance, and set goals” (Tomlinson & McTighe, 2006, p. 79), and thus cultivated metacognitive thinking. For instance, the content facilitators regularly posed “What questions do you have as you read?” as part of the reading assignment. As another example, for the follow-up Share Fair presentation about action plan results and lesson study experiences, participants were asked to describe what goals they had set, what were the strengths of the experience, what changes would they make, etc. Therefore, the project engaged participants in active learning on a variety of levels.

Features of IMP Year 2 also addressed other recommendations for effective professional development and/or other needs for teacher development. For example, researchers and practitioners have recommended that effective professional development provide teachers with opportunities to develop skills and dispositions for inquiry into practice (e.g., Ball & Cohen, 1999; Borasi & Fonzi, 2002). Teachers need to develop dispositions toward analysis of teaching in order to learn from teaching and thus improve teaching (Ball & Cohen, 1999; Hiebert et al., 2007). Hiebert et al. (2007) suggested analysis of teaching would require the following skills: “(a) setting learning goals for students, (b) assessing whether the goals are being achieved during the lesson, (c) specifying hypotheses for why the lesson did or did not work well, and (d) using the hypotheses to revise the lesson” (p. 49). Opportunities for supporting teachers in developing a mindset of inquiry into practice arose in several situations. For instance, teachers participated in **analyzing data and developing learning goals based on the data**. Teachers were given state assessment data for their district and were asked to determine their highest and lowest indicators of mathematics for their grade level. Also, teachers were to identify the cognitive category for the question: (a) memorize definition/formula, (b) perform procedure, (c) demonstrate understanding, (d) conjecture, generalize, prove, and (e) non-routine problems or make connections. Data was collected and displayed for the group as to content standard and cognitive category. A discussion ensued about patterns in the data. Next, participants were directed to address the low standard/indicator from the district data in the form of an action plan for the upcoming school year. The project required participant development of an action plan

using the *Understanding by Design* (UbD) template (Wiggins & McTighe, 2005). Wiggins and McTighe described weaknesses in traditional educational design stemming from activity-focused teaching (use of “engaging experiences that lead only accidentally, if at all, to insight or achievement” (p. 16)) and coverage-focused teaching (e.g., marching through a textbook). In contrast, UbD utilizes a “backward design” three stage approach to planning. Stages 1 and 2 complement the Hiebert et al.’s (2007) first two skills for analysis of teaching. In Stage 1, teachers establish learning goals, write essential questions associated with the goals, and communicate what students will understand, know, and be able to do. Stage 1 also complements differentiated instruction whereby focus is given to big ideas. IMP participants were directed to write their goals based on the acronym SMART (specific, measurable, attainable, results-oriented, and time-bound). In Stage 2, teachers predetermine assessment evidence that will be used to ascertain if students achieved the desired learning (related to Hiebert’s second skill of assessing whether goals are achieved). Finally, in Stage 3, teachers describe learning activities and instruction that will enable students to achieve the desired learning goals and results. Although the UbD acronym WHERETO (help students know **w**here the unit is going and **w**hat is expected, **h**ook the students, **e**quip students, allow students to **r**evise their understandings, allow students to **e**valuate their work, **t**ailor to different needs and interests of students, and **o**rganize to maximize engagement and learning) was available for Stage 3, participants were instructed that they did not have to use every letter of the acronym when writing their learning activities.

IMP participants also engaged in skills for analyzing teaching during the lesson study process. During the school year, each participant was to implement a lesson study either with fellow IMP participants or with other teachers at their school. During the process, teachers typically collaborated in dyads or triads to plan a lesson. One teacher from the group taught the lesson while other teacher(s) observed and collected data/information. After the lesson, teachers discussed, analyzed, hypothesized how the lesson could be improved, and revised the lesson. The lesson was taught at least a second time by another member of the group with discussion following.

Participant development of an action plan and implementation of a lesson study also satisfied recommendations that teachers have opportunities to **reflect upon student thinking**, and that professional development learning opportunities be in **close proximity to practice** (e.g., Ball & Cohen, 1999; Borasi & Fonzi, 2002; Elmore, 2002). Other project activities also

represented learning opportunities in close proximity to practice. In several pedagogy sessions, participants collaborated with grade level peers in making differentiated instruction process activities and product assessments for their classes. As another example, a videotape of a Japanese lesson study was viewed and discussed.

Another aspect of effective professional development involves **engaging teachers as mathematical learners with approaches they will use in the classroom** (Borasi & Fonzi, 2002; Loucks-Horsley et al., 2003). IMP Year 2 participants had many opportunities to experience pedagogical strategies while learning content themselves. In general, *The Number Devil* (Enzensberger, 2000) book was intended for use to introduce patterns in number theory (Allen, 2007, January 4; Allen & Hancock, 2008), thus addressing the NCTM Content Standards of Number & Operations and Algebra. More specifically, participants had opportunities to learn about many mathematical concepts including (but not limited to): numeral systems (particularly Roman numerals and base-ten); place value and the importance of zero; infinitely large and infinitely small (infinitesimal); number systems (e.g., natural numbers, rational numbers, irrational numbers, imaginary numbers); other types of numbers (e.g., prime, triangular); sequences (e.g., Fibonacci) and series; recursive and direct formulas; Pascal's triangle; Golden Ratio; Euler's formula; polyhedra; permutations and combinations; fractals; and cardinality. Thus, as some of the topics touched on Geometry (e.g., polyhedra) and Data Analysis & Probability (permutations and combinations), even more Content Standards were explored in the summer institute.

In addition to addressing the Content Standards, the use of *The Number Devil* served a springboard to accessing many other types of knowledge including: pedagogical knowledge about standards-based instruction via NCTM Process Standards (Whitin & Whitin, 2004); other areas of content knowledge (e.g., knowledge *about* mathematics); and pedagogical knowledge about reading in the content area. Whitin and Whitin (2004) included *The Number Devil* as a recommended book that can appeal to a wide range of ages and that has intriguing problem-posing potential. Specific topics of the book provided entry points to ideas that are important for K-8 teachers to know (National Research Council, 2001) as well as more advanced topics studied by mathematicians (e.g., Eves, 1990). For example, *The Number Devil* examines the Roman numerals in contrast with the modern base-10 place-value system. "Much mathematical insight can be gained by considering the genesis and development of" (National Research

Council, 2001, p. 96) numeral representation systems. Thus, this topic provided entry points to knowledge *about* mathematics (e.g., how mathematics developed and changed over time) as well as the Representation Process Standard and the Number & Operations Content Standard. As another example, the “handshake problem” was depicted in *The Number Devil*. In *Adding it Up*, the National Research Council (2001) highlighted the handshake problem as ideal for illustrating Connections between Number and Operations, Geometry, and Algebra. Representation (e.g., representing triangular numbers visually), Problem Solving (e.g., non-routine problems), and Communication (e.g., math-related literature) are also features of this problem. Furthermore, the Content Facilitators were able to engage participants in a constructivist learning environment (Borasi & Fonzi, 2002) as participants worked in small groups to act out and discuss the handshake problem (as well as other scenarios) revealing key aspects of combinations and permutations. The content facilitators also capitalized on book content and character banter (e.g., “You were right. It doesn’t work. How did you know?” (Enzensberger, 2000, p. 25); “Want me to prove it?” (p. 84)). That is, content facilitators led examinations of conceptual underpinnings for concepts, Reasoning and Proof, and knowledge *about* mathematics (e.g., how truth is established in the field of mathematics, which ideas are built on convention and which are built on logic). Furthermore, the content facilitators used constructivist methods such as discussion, open-ended questioning, and graphic representation (Wiggins & McTighe, 2005) by asking participants to examine and extend meaning by comparing, explaining, visualizing, etc. Finally, the content and pedagogy facilitators collaborated by integrating reading in the content area strategies (e.g., using graphic organizers and pair reading) with reading the trade book during the summer institute.

IMP Year 2 pedagogy sessions incorporated **examining curricular resources** as another effective professional development strategy (e.g., Friel & Bright, 1997; Sowder, 2007). For example, pedagogy facilitators and teacher leaders modeled curricular resources for fostering students’ number sense and differentiating instruction. More specifically, teacher leaders modeled curriculum and tasks that would be suitable for fostering number sense as appropriate for various grade levels (e.g., Creative Publications, 2001; McIntosh, Reys, & Reys, 1997a; McIntosh, Reys, & Reys, 1997b; V. Thompson & Mayfield-Ingram, 1998; Van de Walle, J. A. & Lovin, 2006b; Weinberg, Krulik, & Rudnick, 2002). As a result, Year 2 IMP participants had the opportunity to learn about number sense and to reflect on its importance while experiencing

curricula that could be used to foster students' number sense. Facilitators and teacher leaders also provided differentiated instruction curricular activities and templates for participant examination and use. Furthermore, content facilitators used good math-related literature as an entry to the Content and Process Standards.

Finally, the NCTM Teaching Principle outlines that effective mathematics teachers should know and understand: (a) mathematics, (b) students as learners, and (c) pedagogical strategies (NCTM, 2000). If considered as outcomes for improving participants' teaching practice, the IMP project's use of pedagogical strategies (exploring content via mathematical literature, exploring differentiated instructional strategies, exploring lesson study as a means of improving practice, and exploring reading in the content area) had the potential of promoting the outcomes. In fact, rather than one strategy impacting one outcome, a strategy often had the potential for impacting more than one outcome. For example, differentiated instruction was a project focus. Providing teachers with opportunities to learn about differentiated instruction afforded teachers opportunities to learn more about students as learners (as differentiated instruction attends to student readiness, learning preferences, etc.), about mathematics (as participants had the opportunity to learn about mathematics while collaboratively making differentiated instruction resources for their own use in mathematics lessons), and about pedagogical strategies (as specific instructional strategies associated with differentiated instruction were discussed and explored in pedagogy sessions and as some differentiated instructional strategies were modeled in content sessions). In addition, as participants had varying years of teaching experience in elementary, middle, or high schools from six different school districts, the summer institute afforded a meaningful backdrop for experiencing differentiated instruction by providing an educational experience amongst a group of diverse learners.

Overall, IMP Year 2 evidenced attention to elements of effective professional development design and use of multiple strategies to address project goals and participant needs associated with mathematics education. By doing so, IMP Year 2 provided participants with many opportunities to increase their understanding of mathematics, students as learners, curricular resources, and pedagogical strategies in order that participants might become more effective teachers.

Themes

Teachers' knowledge, (e.g., Bransford et al., 2005; Wilson et al., 1987), teaching practice (e.g., Brophy & Good, 1986), professional development for teachers (e.g., Loucks-Horsley et al., 2003), and teacher learning in professional development (e.g., Ball & Cohen, 1999; Elmore, 2002) are multifaceted. Thus, gauging the impact of IMP Year 2 on teachers' knowledge and teaching practice is also complex. However, results revealed that teachers constructed content knowledge and pedagogical knowledge. In addition, evidence illustrated the short-term impact of the professional development project on teaching practice. It is important to note that individual participants did not necessarily learn the same things. What each participant learned by his or her involvement with the IMP Year 2 project was influenced by his/her preexisting knowledge, by his/her choice in breakout sessions, by his/her involvement in small group conversations during the summer institute, by his/her lesson study experiences during the school year, etc.

The quotes and data selected for evidence supporting a particular theme or idea may also support other themes or ideas. The researcher is aware that it is not always clear cut as to which evidence best supports which theme; in fact, evidence may support several themes. In addition, teasing out different types of teachers' knowledge is open to interpretation as researchers (e.g., Bransford et al., 2005; Ma, 1999; Shulman, 1986b) have offered different conceptions and categories of teachers' knowledge. Nonetheless, evidence was chosen to support and illustrate the themes. A graphic organizer (see Figure 4.2) regarding the themes organized by the research questions will be included at the beginning of discussion for each research question and theme to support the reader.

Figure 4.2: Themes

- Themes Organized by Research Questions**
- Teachers learned mathematical content
 - Knowledge “about” mathematics
 - Substantive knowledge of mathematics
 - Curricular knowledge
 - Pedagogical content knowledge
 - Teachers gained pedagogical knowledge
 - Differentiating instruction
 - Supporting reading in the content area
 - Fostering number sense
 - Skills for analyzing practice
 - Implementing standards-based instruction
 - Impact on teaching practice
 - Short-term impact on teaching practice

The following conventions are used throughout the results: (a) extended written responses (from reflections, homework, tests, survey), extended verbal statements or dialogue based on audio recordings (from interview responses, summer institute session dialogue, share fair reporting), and extended written statements from session handouts are blocked off without quotation marks, (b) shorter written responses or verbal statements embedded in a paragraph are enclosed in quotation marks, (c) verbal statements will be reported as best as possible based on audio quality except for minor editing used to make them sound more seamless than the original (Bernard, 2006), (d) some verbal comments may be left out of the flow of dialogue (e.g., when two or more people started to comment at the same time, but one primary person took the floor, the secondary person’s starting comments were left out; when the overall flow of the dialogue could be followed without significant loss of meaning), (e) minor editing may be used for respondents’ written survey, test, homework, or reflection comments to fix grammar and/or spelling, and (f) researchers’ explanations and clarifications when added to blocked dialogue are enclosed in brackets.

In addition to a wide variety of data sources, some of the data was anonymously submitted while at other times data origination was known. Oftentimes, the researcher was able to link specific names with speakers during summer institute sessions. When known, the researcher identified facilitators and participants with pseudonyms so that the audience might be able to track repeated contributors to the dialogue. For example, participants involved in

dialogue during large group sessions or large breakout sessions were referred to as Participant A, Participant B ... in order of appearance in the results. When not known, the participant was identified as Participant. When dialogue occurred in small groups, the researcher may have identified the participants as Group Participant A, Group Participant B...in order of appearance in the group conversation if it contributed to ease in following the flow of the dialogue. As small group membership changed throughout the institute, Group Participant A in one dialogue may be different from Group Participant A in a different dialogue. Also, Participant G and Group Participant B could be the same person. For some other data sources, the data was submitted anonymously (e.g., project-collected survey data); therefore, no identifier could be attached with the data item. Furthermore, although some data sources could be identified by name, the researcher may have chosen not to identify the information in order to protect the participants (e.g., homework and tests). Finally, letters identifying Content and Pedagogy Facilitators remained consistent throughout the results.

Teachers Learned Mathematical Content

- Themes Organized by Research Questions**
- *Teachers learned mathematical content*
 - Knowledge “about” mathematics
 - Substantive knowledge of mathematics
 - Curricular knowledge
 - Pedagogical content knowledge
 - Teachers gained pedagogical knowledge
 - Differentiating instruction
 - Supporting reading in the content area
 - Fostering number sense
 - Skills for analyzing practice
 - Implementing standards-based instruction
 - Impact on teaching practice
 - Short-term impact on teaching practice

Participant performance on pre- and post-summer institute content tests broadly evidences increases in teachers’ mathematical content knowledge. Each test consisted of 15 problems (worth 2 points each) probing for participants’ understanding of concepts such as rational numbers, prime numbers, recursive formulas, place value, sequences, series, permutation, and

proof. Six of the pre- and post-questions were the same and four additional questions were the same conceptual question using different numbers. The five remaining paired questions were deemed similar, but not evenly matched by the researcher. Two of the remaining paired questions were very similar; however the post question was slightly easier (either because more explanation was provided in the post or the post question number was slightly easier to work

with). Two of the paired questions addressed different ideas related to Pascal's triangle; for one pair, the researcher viewed the post question as more difficult. The remaining pair generally probed understanding of the role of proof in mathematics, but the different questions required different explanations. Overall, questions posed on the pre- and post-summer institute content tests were fairly matched. However, the testing conditions were different as participants were allowed to make their own formula/information sheet for the post-test. This may have inflated participants' performance for a few questions on the post-test where knowledge of a formula or definition was helpful. All participants earned a higher score on the post-test than for the pre-test.

Quantitative analysis of pre- and post-summer institute content test scores was performed by the KSU Office of Educational Innovation and Evaluation (OEIE) (Office of Educational Innovation & Evaluation, 2008a). In the summary report, KSU OEIE reported the test scores were significantly different from pre- to post-summer institute, and that overall there was an increase in the mathematical content knowledge for teachers who participated in the summer institute [$F(1, 23) = 204.33, p < 0.001$] (see Appendix C). However, the researcher suggests that this conclusion must be viewed cautiously in light of the fact that participants were allowed a formula sheet for the post exam.

Participants also completed pre- and post-summer institute surveys. Survey questions included descriptive questions (e.g., years of teaching mathematics), likert questionnaire items for four scales (Knowledge of Math Topics, Knowledge of Teaching Strategies, Comfort with Math Topics, and Comfort with Teaching Strategies), and open-ended questions. Knowledge of Math Topics items asked participants to rate their knowledge level (no knowledge, a little knowledge, knowledgeable, very knowledgeable) for concepts related to number systems, number patterns, sequences, place value, etc. Participants anonymously filled out the survey; however, pre- and post-summer institute surveys were linked by participant use of an identification code.

KSU OEIE (2008b) performed an analysis for the Knowledge of Math Topics scale by using the pre- and post-summer institute mean scores of the 25 individual items composing the scale. KSU OEIE reported that pre- and post-summer institute mean scores were significantly different indicating participants reported improvement in Knowledge of Math Topics [$F(1, 22) = 36.41, p < 0.001$] (see Appendix D).

Therefore, reports based on quantitative analysis of project-collected pre- and post-summer institute content tests and survey data suggest teacher participants learned mathematical content over the course of the summer institute. However, this qualitative study will provide further insight as to what mathematical content the teachers learned. By examining the topics and concepts studied, the community discourse, participants' reflections, and the participants' responses on individual test and homework questions, more specifics emerge about the mathematical content that participants learned in IMP Year 2.

Theme: Teachers gained knowledge “about” mathematics (a component of subject matter knowledge)

- Themes Organized by Research Questions**
- Teachers learned mathematical content
 - ***Knowledge “about” mathematics***
 - Substantive knowledge of mathematics
 - Curricular knowledge
 - Pedagogical content knowledge
 - Teachers gained pedagogical knowledge
 - Differentiating instruction
 - Supporting reading in the content area
 - Fostering number sense
 - Skills for analyzing practice
 - Implementing standards-based instruction
 - Impact on teaching practice
 - Short-term impact on teaching practice

Ball (1990a, 1991) proposed knowledge *about* mathematics includes understandings about the nature of knowledge in the discipline which can include an understanding of: (a) which ideas are based on convention and which are built on logic; (b) how mathematics has developed and changed over time; (c) what reckons as a solution; and (d) how truth is established in the

field of mathematics. Scenarios described in *The Number Devil*, along with content facilitators' building upon the scenarios and direction of discussions in response to participants' questions, provided participants with opportunities to examine and learn knowledge *about* mathematics.

For example, participants engaged in thinking about whether certain mathematical ideas are based on convention or logic. One of the first significant discussions evolved from a participant's question arising during a facilitator led discussion about Night 2 of *The Number Devil*. In Night 2, Enzensberger used Roman numerals as a comparison with our modern base-10 place-value system. Stemming from Enzensberger's dialogue of the Number Devil with Robert, "I told you to *produce* the number, not scribble it down" (2000, p. 42), a content facilitator posed a question for which discussion follows.

Content Facilitator A: We go back to Robert's birth year, 1986...okay Robert, produce the number of the year you were born ...what do you think is the difference between writing the number and producing the number?

Participant A: Standard Notation.

Content Facilitator A: So he doesn't want him to just be writing out digits but he wants him to think about what that digit means...

Content Facilitator A: [Talking through and writing out what was described by the following sentence on the **session handout**. Note the information was similarly presented in *The Number Devil*.] Thus, he wants him to begin with $6 \times 1 = 6$, then continue with $8 \times 10 = 80$, and finish with $9 \times 100 = 900$ and $1 \times 1000 = 1000$, adding them all together to get 1986.

Participant: ...Does it matter if you start with the six or if you start with the thousand?... I know in my mind that it doesn't matter and I understand, but does it matter?

Participant A: I think about polynomials and when you do a polynomial in standard form you start with typically the highest power....my intention is for them to make that link...

Content Facilitator A: Well, what I would say is ...how do *you* know just by looking at it that that 1 means 1000?... you can look at it and immediately...maybe why for someone just learning it you would start on the right ...this is ones, then tens comes after ...if you're having troubles ...which is the 4th one over, then maybe you want to start right to left...

Content Facilitator B: Obviously, it's a convention. You could write them in the opposite order. There is an advantage to doing it this way [starting with 1×1000]...When we say a number like one thousand nine hundred and eighty-six, if you're doing approximation and number sense, if you said six and eighty and nine hundred and a thousand, then for the students to have a notion of how large that number is in doing an approximation, they have to wait for the end. Where if we write them one thousand nine hundred and eighty-six ...gives them the big number [first] ...it's about, it's in the thousands,..., somewhere between one and two thousand, and they get that information right away, whereas if I say six and something, they don't know right away how big a number that is. That is why we settled on writing numbers and speaking them with the biggest number first. It is just a convention that makes it easy to do number approximations, it would be perfectly valid to do it the other way; computers sometimes do internally.

Content Facilitator A: It seems like it depends on what you want to do.... If you are just at the level of what value is this number, how do you read this number off, what does each digit mean, you might want to be starting right to left, ... but if you really want to understand how big is this number, estimating then, ...[left to right]...

Another example of a discussion about mathematical ideas stemming from either convention or logic occurred during a content breakout session. Participants selected to go to a break out session either about mathematical content in the *Adventures of Penrose: The Mathematical Cat* (Pappas, 1997) or about some theorems and conjectures concerning prime numbers. The researcher joined the session about theorems and conjectures regarding primes.

The breakout session included some discussion about when primes were first identified and some theorems and conjectures concerning primes (e.g., Euler's proof by contradiction establishing there are infinitely many primes, Goldbach's conjecture). During about the last fifteen minutes of the session, Content Facilitator C provided a discussion prompt:

We've been talking about why you can't do things today, why you can't divide by zero, why 1 is not a prime number.... What would happen if $\frac{a}{0} = 0$, think about it in your groups... What are the craziest consequences that would come from this? Where a is any number...and then another one, you can choose which one you want to do... What would happen if 1 were a prime number?... How would that entirely change our system of mathematics.... Kind of an obvious one, if $\frac{a}{0} = 0$ that means $0 \cdot 0$ could be any number.

During our small group discussion, one participant pointed out that if 1 were prime, no other number would be prime. "It would be the only prime number." A different participant expressed confusion. "Why is 2 a prime number.... it has two factors 2 and 1..." Content Facilitator B had joined the discussion. The facilitator pointed out that a factorization tree that a participant was drawing was "never going to stop". More discussion occurred by participants trying to navigate group understanding. Content Facilitator B expressed:

We could have written a different definition [of prime]. But if we wrote down a definition that said 1 was prime....then this [prime factorization] becomes much harder to do...and there's no real advantage to us in calling 1 a prime...so I choose the definition this way because it makes my life much easier...

Group members brainstormed about similar mathematical situations along these lines (i.e., choosing a definition because it makes things easier to do) such as $0!$ needing to equal 1.

Content Facilitator B added order of operations.

We define an order of operations but we don't define that randomly. We chose to make the order of operations work in a particular fashion because it makes all the other rules work.... We could have written something different, but you find out all sorts of other things go wrong.

Small group discussions continued for several more minutes. Nearing the close of the session, Content Facilitator C solicited information from group discussions as to what would happen if 1 were prime and what would happen if $\frac{a}{0} = 0$. Finally, Content Facilitator B talked a little about order of operations and then assigned homework problems for participants attending the breakout session. “Why do we have an order of operations?” Secondly, “What would go wrong if we didn’t give addition and subtraction (or multiplication and division) the same precedence?”

Explanations for the order of operations homework problems varied. About 65 percent of the breakout session participants got 1 out of 2 points for the homework problems. The most common explanations suggested the order provided a way for people to be on the “same page”. The explanations could be construed as leaning towards associating the order with an arbitrary convention. The other 35 percent got 2 out of 2 points. Some of the participants getting full credit expressed the logical necessity for addition and subtraction to have the same precedence as addition and subtraction are inverse operations.

Although a variety of homework problems were assigned during the day, several participants attending the breakout session wrote specifically about the order of operations homework problems in their Homework/Session Reflection.

- I thought the order of operation questions ... were interesting.
- I had a hard time with the order of operations assignment.
- The questions on order of operations were thought provoking and either I answered too simply or totally wrong!
- The question regarding order of operations took longer to generate than anticipated.

Hence, participants in the break out sessions engaged in thinking about issues related to whether mathematical ideas were based on convention or logic.

The participants in the other breakout session had their own opportunities to engage in thinking about convention versus logic. The day after the breakout session discussion, Content Facilitator B described the order of operations problem to the whole group and provided the explanation that addition and subtraction must have the same precedence as addition and subtraction are inverse operations: the same operation, just alternate faces. Furthermore, participants in the other breakout session engaged in discussion about different base systems while exploring content in the *Adventures of Penrose: The mathematical cat* (Pappas, 1997). The predominant use of the base-10 numeration system in daily life is arbitrary (Ball, 1991b;

National Research Council, 2001). By grappling with some other base systems, participants in the second break out session may have identified the modern base-10 system as being constructed through human activity.

The examples used in the discussion of mathematical ideas based on convention or logic bridge with another dimension of knowledge *about* mathematics: an understanding of how mathematics has evolved and changed over time. In Night 2 of *The Number Devil*, Enzenberger's use of Roman numerals (which had no zero) as a comparison with the base-10 place-value system allowed for reflection about the significance of zero as being more than a number representing nothing. The use of zero as a place holder plays a significant role in our modern base-10 numeral system. In addition, Enzenberger used the image of "hopping" for exponentiation. After describing 10 "hopping" twice as being $10^2 = 100$, 10 "hopping" three times as being 1000, Enzenberger stated: "That's the beauty of the zero. It lets you hold a space and move on. You can always tell a number's value by its position: the farther to the left it is, the more it's worth" (p. 41).

Prior to discussion in a content session, participants were given a reading assignment over Nights 2 and 3. As they read Night 2, participants were asked to reflect upon "What is so special about powers of 10 and the role of the 0 in place value." Participants were given a "during-reading" graphic organizer as a means to express their understanding of the reading. The organizer was a Venn diagram. In one region of the graphic organizer, participants were asked to write about information they learned from the readings about Nights 2 and 3. In another region, participants were to write about information in the text that the participant already knew. Finally, in the overlapping region, participants were to express information for which they had some knowledge prior to the reading, but more understanding after the reading. Eleven participants classified as new information their consideration of the Roman numeral system as more cumbersome because of its lack of zero. For example, one participant wrote: "I never thought about Roman numerals being difficult to read because of the lack of zero". Eleven other participants considered as new information what one participant explained, "Each power of ten takes you to the next place value." Three from this set also wrote about the role of zero to hold a space. Hence, the mathematical literature and the reflection question spurred thinking about the significance of zero as a place holder in our modern base-10 system. As one participant wrote in his/her reflection about Day 1, "I always knew '0' was necessary, but on page 36, it tells me

why!” Another wrote, “I enjoy reading the book and the way the concepts are introduced. They made me think about ‘0’ in a different way.” A session discussion on the following day illustrates the concept is not tedious.

Participant B: The question I wrote on my paper is how do you explain to a kid... that the reason this [Roman numeral system] is difficult is because there aren’t zeros?

Content Facilitator A: ...It [Roman numeral system] is not a place value system. You need zero to have the kind of place value system we have because you need a place holder... If you write the number 705, ... we have 5 ones, we have 7 hundreds. If we didn’t have the zero...you would think 7 meant 7 tens.... [With Roman numerals] an I is always a 1. It never stands for a 10 by sitting someplace else.... In our place value system, the seven [in 700] doesn’t mean 7 anymore, it means 7 hundred because of where it stands...

Later in the session, a participant question prompted a content facilitator to provide more about the historical development of numeral systems.

Participant C: How do you write one million ... in Roman numbers?

Content Facilitator B: The Greeks and Romans topped out their numeral system at 10,000, which is a myriad... If you wanted to say one million you would say 100 myriads--you would have to write the word out myriads...that’s partly why you need a zero.... If you use a different symbol to keep track of what place it is...you run out....So they said okay 10,000 is a myriad. If you need more than 10,000 you have so many myriads.... Romans did not have decimals.... Both the Romans and the Greeks adopted the Babylonian place-value system, base-60, that had decimals but no zeroThey did that in their astronomy where they needed high precision calculations. We do still use that system today. Anyone know where?

Participant A: Angles.

Content Facilitator B: Angles.

Participant: Time.

Content Facilitator B: 60 minutes in an hour and 60 seconds in a minute. That actually originally begins with Ptolemy writing, I need to do fine work... I’m going to cut my angle into 60 minute [mi·nute] pieces ... we distinguish the word minute [mi·nute] for small and minute [min·ute] but they were originally the same word...And if you want it to be really fine, what do you do?

Participant A: Second.

Content Facilitator B: You cut it a second time, and that is the origin of the terms minute and second...the one ancient place value system, the base-60 system, but it has no zero, it

is an odd system to work in...They would not use Roman numerals for doing astronomy because they would realize these don't work...

Hence, mathematical literature and content session discussions provided a stage for participants to increase their understanding of the development of numeral systems as a human endeavor. The impact for one participant was further evidenced on her IMP Year 2 action plan by the inclusion of the importance of zero in place value as a desired understanding for students.

Other historical references were provided throughout the content sessions of IMP Year 2. For example, after leading participants in kinesthetically representing i , Content Facilitator B talked about Steinmetz, a GE electrical engineer, who discovered the abstract concept of complex numbers was useful in many engineering situations involving two-dimensions as complex numbers “spread into a number plane instead of a number line.” When reviewing the final chapter of *The Number Devil*, Content Facilitator A made many connections between names chosen by Enzensberger and historical names. For instance, the facilitator indicated if we knew German, we would get the connection between the names Dr. Happy Little and Felix Klein. The facilitator also brought a Klein bottle so that participants might have a concrete representation of what Enzensberger had described: “Imagine you wanted to paint the inside blue and the outside red.... There are no edges. You wouldn't know where to stop the blue or start the red” (p. 240). Furthermore, some historical proofs were examined during content sessions, thus bridging with another dimension of knowledge *about* mathematics: how truth is established in the field of mathematics.

Dialogue from *The Number Devil* and content session emphasis resulted in significant attention being placed on the role of proof in mathematics. Participants came to IMP Year 2 with a variety of background in proof. For example, participants with training in elementary education likely had very few prior experiences with formal mathematical proof, whereas secondary mathematics teachers likely had many experiences with proof in their college training and also in secondary mathematics teaching. On the content pre-test, participants were asked: “Why is proof important in mathematics?” Recall, de Villiers (1990) had suggested proof had multiple roles including verification (verifying the truth for all cases), explanation (providing insight into why something is true), systematization, discovery, and communication. Five participants (whose backgrounds were elementary or specialty area educators) received no credit

for the problem, of which four had left the question blank. Other participants received partial or full credit. Some examples of explanations that were less developed include:

- It shows the thinking behind the process.
- Proof is important in mathematics as it forms the basis for mathematics.
- You need to know how it works.

Some examples of more developed explanations of the role of proof in mathematics, particularly addressing the roles of verification and explanation, include:

- It gives the reasoning of why a theory is true and will always be true.
- It shows the logic behind the concepts, and it provides a statement of why the statement works! So the students can't argue.
- To understand a concept fully, you would need to know where it came from using proofs show a logical way of drawing conclusions in mathematics. So that you know a statement is true.

Thus, participants came to IMP Year 2 with different understandings of the role of proof in mathematics.

Material in Night 1 of *The Number Devil* and subsequent content session discussion initiated attention on the role of proof in mathematics. In Night 1, the Number Devil explained there are an infinite number of numbers. Robert questioned: "How can you be so sure Either I can count to the end, in which case there is no such thing as infinity, or there is no end and I can't count to it" (p. 16). Later, the Number Devil explained: "I don't really *need* to count them. All I need is a recipe to take care of anything that comes along" (p. 17). After participants read Night 1 during the first day of the summer institute, content facilitators presented the participants with questions for small group discussion about the chapter. In reference to Robert's concern, one of the questions on the handout asked: "How would you explain why Robert's idea is incorrect?" Our small group struggled at the beginning of the discussion regarding this question. A content facilitator joined the group discussion and helped the group get oriented.

Content Facilitator B: ...The devil is saying yes I can prove there are infinitely many even if I don't count them out...as long as I have the recipe to take care of anything that comes along...so it is possible to show that you will never stop without having to try it...

Group Participant A: Well good, I don't know if we would have gotten that. [Content Facilitator B left the group.]

Group Participant B: Does that mean that.... a big number you can count to that number and does that mean that you add one, add one, ... is that what the devil tried to explain?

Group Participant C: That's what I got, you can always add one more so that would be the formula...that's how I understood it.

Other discussion questions referred our attention to the final argument between Robert and the Number Devil in Night 1. The Number Devil had Robert examining $1 \times 1 = 1$, $11 \times 11 = 121$, $111 \times 111 = 12,321$, etc. The problems start with a very easy pattern, but the pattern breaks when you get to factors with ten ones $1,111,111,111 \times 1,111,111,111 = 1,234,567,900,987,654,321$.

Group Participant A: The book asks about what happens with 11 ones.

Group Participant C: I'm wondering even with 10 [ones]...

[The group uses a computer to check the answer.]

Group Participant B: Oh, so the 8 is missing... So there may be some recipe over here to find our value.

Group Participant C: ...Your 8 turned into a 9 because it's regrouping.

Group Participant A: Okay, so what happens with eleven?

While working on the problem by using a computer to generate answers, a content facilitator came and pushed us to generalize. "What do you think the next number in the pattern would be?... You have a computer, but don't let it do it for you." Our group kept working and two members came up with a new pattern for the larger factors.

Group Participant C: ...It breaks the pattern but it created a new one.

[Group Participant D asked for more clarification. More group discussion followed.]

Group Participant B: ...I don't know if I can do the proof of that...

Hence, participants began to think about the role of proof in mathematics.

Participants were called back to the full group setting. Content Facilitator A led a discussion to wrap up Night 1. In reference to looking for patterns for products whose factors had 1 in each digit, Participant D asked: "I'm thinking in terms of somebody who may not be interested in the subject of mathematics and they ask the question 'what's the purpose of doing this?'" Later in the discussion, Content Facilitator A offered: "The Number Devil is getting Robert to ask the questions himself; once he's asking the questions, he's got the motivation..."

Prior to Day 2 of the summer institute, participants were given a reading assignment over Nights 2 and 3. As part of the assignment, participants were prompted to describe questions they had over the readings. Responses indicated that several participants were asking questions about whether statements regarding prime numbers in Night 3 (e.g., between a number greater than 1 and its double, there exists at least one prime number) could be proven.

- Why can you add three prime numbers to equal any number?
- Is there a proof 2 primes add up to an even number? Is there a proof any odd number is the sum of 3 prime numbers?
- Why does any even number (larger than 2) always have two prime numbers that make up its sum...and any odd number (larger than 5) with three prime numbers? Why does this work every time?
- Why do we care about the trick of 2 prime numbers adding up to an even number? Do you know why it always works?
- Any even number larger than 2 has 2 prime numbers that add up to equal it. I wonder who figured this out and if nobody really knows why it works.

Some of the latter examples of responses about two prime numbers summing to every even number (larger than 2) were likely written in response to dialogue in *The Number Devil*. “Nearly every number devil of my acquaintance has tried to come up with an explanation. It always works, but no one knows why” (p. 63). During a Day 2 content session, a content facilitator (having already reviewed some of the participants’ reading assignment responses) drew attention to the topic.

Content Facilitator A: This is the one place in the book I think the Number Devil totally drops the ball because we do not know that this is true for every even number, we know it for many, many... this is actually a very famous unsolved problem known as Goldbach’s Conjecture...I think he should have said that this is not known...for those of you who asked...how can you show that’s always true...that’s a really good question.

Later in the day, Content Facilitator B extended the topic after having done a little research on the Internet. He described progress regarding the “weak” Goldbach conjecture that every odd number greater than 5 is the sum of 3 odd primes.

The “weak” Goldbach conjecture, up to 10^{18} , that’s numbers with 18 digits or fewer, is true. We have computers that have tested every odd number with 18 digits or fewer can be written as the sum of three prime numbers. There are also rules for finding for patterns for this that are based on things since there are so many small prime numbers, there are so many different ways to match them up, one of these must work. And, it has now been shown that it is also true for numbers with at least 1347 digits. What we don’t know is what’s in the middle. The techniques we have to prove that this has to happen only works for really large numbers, and the computers will check small numbers. But

these middle-sized numbers we don't know yet.... So if somebody gives you a number and it has between 18 and 1347 digits, we suspect it probably can be written as the sum of three odd primes, but I can't prove it...

For the "strong" Goldbach conjecture what we do know is that any even number bigger than 2 can be written as the sum of six prime numbers. In these ranges, four will work.... It has been shown, if you randomly pick an even number, the odds are better than 99% or 99.9%, big a percentage as you want, I can say the odds are better than that percentage that a randomly chosen even number can be written as the sum of two prime numbers, but that doesn't mean that there aren't a few out there that aren't...we just know that almost all of them can be...But we don't know all of them yet and we don't have a good sense of how....That's what we know today.

Hence, content facilitators capitalized on dialogue in *The Number Devil* and shared how mathematicians approach the discipline by emphasizing a primary role of proof in mathematics as verification of the truth of an assertion for all cases.

During the summer institute, not only was interest sparked as to what establishes truth in the field of mathematics, but some mathematical proofs were presented in breakout sessions. For example, during a morning breakout session on Day 2, Content Facilitator C talked more about some theorems related to primes (as prime numbers were the main topic of Night 3 of *The Number Devil*). The facilitator verified there are infinitely many primes by employing the method of proof by contradiction which the facilitator indicated had been similarly used by Euclid around 300 B.C. (e.g., Eves, 1990).

Later in the second day, participants read Night 4 and full group discussion followed about rational numbers (that can be written as either a finite or a repeating decimal) versus irrational numbers. A subsequent breakout session addressed a topic that was also hinted about in Night 4. After introducing the concept of irrational numbers, the Number Devil suggested "they're [irrational numbers] like sand on the beach, more common even than the other kind [rational numbers]" (p. 83). Based on the "teaser" per Content Facilitator A, the breakout session addressed the cardinality (size) of some number systems (e.g., natural, rational, irrational). Content Facilitator A pointed out that although intuition would suggest there are twice as many natural numbers (1, 2, 3, ...) as even (natural) numbers (2, 4, 6, 8, ...), they in fact have the same cardinality. The facilitator demonstrated a one-to-one correspondence between the two sets thus establishing the same cardinality for the two sets. Next, the facilitator proved the set of natural numbers and the set of rational numbers have the same cardinality by

demonstrating a proof employed by Cantor around 1874 (Eves, 1990) establishing a one-to-one correspondence between the sets of natural and rational numbers. Participants struggled with these abstract concepts. A participant asked for an example of something that was not the same size. The facilitator demonstrated a proof by contradiction, similar to the diagonal process used by Cantor (e.g., Eves, 1990), to establish that real numbers between zero and one could not be put in a one-to-one correspondence with the natural numbers.

Only about a third of the participants (overwhelmingly middle school and high school mathematics teachers) had self-selected to attend the breakout session. The participants had several questions about steps in the proof as they tried to better understand the proof by contradiction. The facilitator tried to reassure the audience that their struggle to understand the proof was understandable.

Content Facilitator A: Maybe this would be a good time to say the person who discovered this, Cantor, went insane... This was something that was highly controversial; this was in the 19th century that this was developed, the notion of different sizes of infinity. The idea is that the numbers between zero and one have a larger cardinality than the counting numbers; it's a bigger infinity, that's what the Number Devil was talking about... The irrationals are larger, a bigger collection, they have a larger cardinality... You can read about it in the packet I gave you from the textbook [sections from *The Heart of Mathematics: An Invitation to Effective Thinking* (Burger & Starbird, 2000, pp. 146-173)]... It talks about how this was really controversial...

Those attending the breakout session were able to demonstrate some understanding about the concept of cardinality through their performance on breakout session homework problems. About ninety percent of the breakout session participants were able to: (a) set up a 1-to-1 correspondence between whole numbers (i.e., 0, 1, 2, ...) and natural numbers, and (b) provide a new example of a set which was countable and could be put in a 1-to-1 correspondence with the natural numbers.

The concept of cardinality arose again in Night 9 of *The Number Devil*. After setting up a 1-to-1 correspondence between the natural numbers and the odd numbers, the Number Devil said to Robert: "Sorry to disappoint you, my boy, but as you see, there are exactly the same number of one as of the other" (p. 175). A breakout session on Day 4 similarly addressed cardinality as on Day 2. Participants were asked to attend the breakout session particularly if they had not attended the earlier breakout session on cardinality. Therefore, nearly all (or all)

IMP Year 2 participants had the opportunity to think about the counterintuitive idea of different “sizes” of infinity.

Although the researcher participated in the other breakout session on Day 4, the group walked in when the cardinality breakout session was wrapping up. A participant was asking Content Facilitator C specific questions about the proof by contradiction regarding the different cardinality of the sets of natural numbers versus real numbers between 0 and 1. A sequence of questions and answers regarding the proof exhibited some tension between the participant who was trying to understand the abstract concept and the facilitator who was trying to further explain the concept. Content Facilitator B stepped in and provided some more explanation of the historical context for the problem.

Content Facilitator B: It is true that Cantor ended up in an insane asylum... What doesn't always get noted, Cantor is actually working on a real world problem.... He's looking at an abstract mathematical concept, but in one particular context; suppose you have a radio signal and it has certain frequencies and you're missing bits of that, can you recover the signal.... What if you didn't know it at infinitely many points.... I can't just say there's one infinite set; there are some infinities that will foul up my signal and some infinities that I can recover from, so this is an actual real world situation that he's trying to analyze and is discovering sometimes with infinitely many things going wrong I can recover; sometimes with infinitely things going wrong I can't recover. I need a way to tell how big an infinity I've got here so that I can answer my question of when I can recover.... This is **very** abstract and difficult to catch...

To wrap up the discussion on cardinality, the content facilitators led the full group in lightheartedly singing the lyrics of “Hotel Infinity” (Lesser, 2000) to the tune of Hotel California. The facilitators also led a discussion about some of the mathematical ideas dealt with in the song. Although the song was likely intended to provide some closure for the topic, participants in a small group discussion on Day 6 started discussing cardinality on their own when Cantor's name came up again in reference to the last chapter (Night 12). The dialogue illustrates the participants in the group were still trying to understand the concept of cardinality.

Group Participant A: ...the proof for there are more real numbers than rational numbers [related to the proof that there are more real numbers between 0 and 1 than there are natural numbers]...we define real numbers as the rational and irrational numbers...why do you need a proof for that...you're adding something...

Group Participant B: But it doesn't follow with even versus natural...even though the natural include odd and even.

Group Participant C: ...that's the one that got me—how can both of those be equal sets; the natural numbers and the even numbers when the even numbers you're taking out half of the natural numbers so how can the two be the same?

Group Participant D: ...take out half of them...

Group Participant C: ...how can half of the set be the same of the whole set...

Overall, the participants had been challenged to engage in thinking about an abstract concept that had also been difficult for mathematicians to grasp during its development.

The content facilitators provided some other experiences with methods of proof during additional breakout sessions. For example, stemming from a discussion in Night 6 about Fibonacci numbers, Content Facilitator B (during a breakout session on Day 3) used mathematical induction to prove the sum of 1 with the first n Fibonacci numbers is equal to the $n + 2$ Fibonacci number. Some of the dialogue while the facilitator worked out the proof included:

Content Facilitator B: The real question is how do we know that's **always** going to be true...

Participant C: Proof by mathematical induction.

Content Facilitator B: ...Induction you always check at least the first one....Once I know this is true, I want to show it is true for all of them.... It is just like the Number Devil says in the beginning in Chapter 1, you don't have to actually count through all of these...you just have to have a rule that works for each of them and shows you how to handle anything that comes up...I just have to show I have a formula that works for all of them...

Thus, the content facilitator demonstrated a particular method of proof for the breakout session participants.

During a breakout session on Day 6, Content Facilitator C started with a discussion of propositions and logic and then talked about five specific methods of proof for which the first three stemmed from the proposition/logic framework. The proof techniques discussed included: direct proof, proof by contrapositive, proof by contradiction, proof by mathematical induction, and proof by well-ordering. The facilitator also led the group through some proofs demonstrating specific methods. The session concluded with an identification of proof methods that had been employed to prove specific assertions throughout the summer institute.

Overall, some participants attended several breakout sessions where content facilitators modeled different methods of proof. These experiences may have provided some participants with opportunities for strengthening their understanding of proof techniques. However, evidence is lacking as no homework or test problems required participants to write a formal proof. Furthermore, the researcher noticed high school and middle school mathematics teachers were the typical attendees of breakout sessions emphasizing employing methods of proof. Other participants had far fewer experiences with methods of proof as they may have only attended the one breakout session regarding cardinality that was offered at two different times in order to reach the entire group of participants.

Although it is unlikely that many participants significantly improved their ability in employing methods of proof, participants overwhelmingly learned about the role of proof in mathematics. As prior evidence demonstrated that the participants' interest was sparked as to the role of proof in mathematics during the first two days of the summer institute, additional evidence from the summer institute demonstrates that participants learned that proof was important in mathematics for verifying the truth of an assertion for all cases. For example, prior to Day 3, participants were prompted to write questions they had after reading Nights 5 and 6. Five participants expressed interest in knowing if a concept was **always** true. For example, "Is it really true that you can always find two or three triangular numbers to add up to any number (of choice)?" Another participant wrote: "Has the rule (any number can be written as the sum of 2 or 3 triangle numbers) ever been proven?" Content session discussions on Day 6 also revealed what participants learned about the role of proof in mathematics. After small group discussions, the full group was asked to reflect on questions regarding the final two chapters of the book. For example, participants were asked to describe their understanding of the role of proof in mathematics.

Participant E: Somebody in our group said it keeps math honest...

Participant F: It's an assurance that the way you're thinking...is correct or not correct.

Participant D: ...will hold true for all cases...

Finally, explanations regarding a question on the post-summer institute content test provide further evidence that participants' learned about the role of proof in mathematics. Participants were asked: "What is the difference between 'showing' something and 'proving' something in

mathematics.” Over ninety percent of the participants earned 2 out of 2 points. Examples of responses (from teachers of different grade levels) include:

- When you show something in math it doesn’t necessarily mean that it will happen in every example. This is called a conjecture (when you have an idea something will work, but you haven’t been able to prove it yet). When you prove something in mathematics you have to be able to show that it will be correct in every possible example.
- Showing something works is only true for the number or set of numbers you use to show. A proof (proving) shows that it works logically for all situations (numbers).
- When showing something you are telling/explaining how. You are probably using examples. When proving something you are telling/explaining why. Here you are probably talking about a general case (not an example) and using some type of proof format.
- Showing is just stating a process and giving a pattern. Proving is finding a way to generalize and show that the statement is true for every case.
- Showing can be like teaching someone an example of how something can be done and how it works. Proving is showing why something works and leaves no doubt that the statement/theory is true in all cases. Proving has to work for all cases; showing can work for many and be fine.

Less than ten percent received partial credit for the question. For example, one participant wrote: “Showing something means working it out. It may have the wrong or right answer. Proving something means you have used the formulas and other means to make that answer correct.” Furthermore, all of the participants who had received no credit on the pre-summer institute proof question (“Why is proof important in mathematics?”) received full credit on the post-summer institute question (“What is the difference between ‘showing’ something and ‘proving’ something in mathematics?”).

Overall, participants learned that a primary function of proof in mathematics is to verify the truth of an assertion for all cases. Participants also learned that proof can provide insight into why an assertion is true. However, some participants struggled in trying to understand formal proof (see earlier discussions about cardinality) and a few participants reflected that the process of formal mathematical proof could be intimidating and/or might not be very applicable to their grade level. A few examples follow:

- The concepts were really complex, although I still found them interesting. I enjoy the logic of math and found some of [the content facilitators’] proofs very interesting.

Personally, I enjoyed the session, but it was not practical for my professional life (teaching 8th grade). [Reflection after Day 2]

- The 1-to-1 correspondence with countable and uncountable is still a mystery to me. I feel like the definitions of countable vs. uncountable were vague. It seems to me that there are many more countable than not, based on my level of experience. [Reflection after Day 2]
- Talking about proofs can be a little daunting. [Reflection after day 5]

In conclusion, participants of IMP Year 2 gained knowledge *about* mathematics.

Participants reflected on whether some mathematical ideas were based on logic or convention. Participants learned about how some mathematical concepts have developed over time. Finally, participants learned that formal mathematical proof is of primary importance for establishing truth in the field of mathematics.

Theme: Teachers gained substantive knowledge of mathematics (a component of subject matter knowledge)

<p>Themes Organized by Research Questions</p> <ul style="list-style-type: none">• Teachers learned mathematical content<ul style="list-style-type: none">◦ Knowledge “about” mathematics◦ <i>Substantive knowledge of mathematics</i>◦ Curricular knowledge◦ Pedagogical content knowledge• Teachers gained pedagogical knowledge<ul style="list-style-type: none">◦ Differentiating instruction◦ Supporting reading in the content area◦ Fostering number sense◦ Skills for analyzing practice◦ Implementing standards-based instruction• Impact on teaching practice<ul style="list-style-type: none">◦ Short-term impact on teaching practice

For Ball (1990a, 1991) substantive knowledge of mathematics included knowledge of topics (e.g., trigonometry), concepts (e.g., infinity), procedures (e.g., factoring), underlying principles and meanings (e.g., what division with fractions means), and relationships among the concepts (e.g., how fractions are related to division).

Many mathematical topics,

concepts, procedures, underlying principles and meanings, and relationships among concepts were studied during the two-week summer institute. More specifically, topics discussed in *The Number Devil* and expanded upon during content sessions included (but were not limited to): numeral systems (particularly Roman numerals and base-ten); place value and the importance of zero; infinitely large and infinitely small (infinitesimal); number systems (e.g., natural numbers, rational numbers, irrational numbers, imaginary numbers); other types of numbers (e.g., prime,

triangular); sequences (e.g., Fibonacci) and series; recursive and direct formulas; Pascal's triangle; Golden Ratio; Euler's formula; polyhedra; permutations and combinations; fractals; and cardinality. The variety of topics resulted in emphasis on the Number & Operations and Algebra Content Standards, but the other Content Standards were addressed as well (e.g., Geometry with polyhedra).

Recall, KSU OEIE (2008a) reported the content test scores were significantly different from pre- to post-summer institute, and that overall there was an increase in the mathematical content knowledge for teachers who participated in the summer institute (see Appendix C). As the majority of the pre- and post-summer institute content test questions addressed substantive knowledge about mathematics, the analysis of the pre- and post-content tests provides evidence that participants learned substantive knowledge about mathematics. However, this section will provide more detail about what participants learned. The researcher used several criteria to narrow the discussion. The researcher chose to focus on concepts for which evenly matched (exact same or conceptually same) pre- and post-summer institute questions indicated there was growth and for which the questions aligned with substantive knowledge. The researcher also looked for other supporting evidence.

Many topics in *The Number Devil* dealt with Number and Operations. For example, prime and composite numbers were the focus of Night 3 in *The Number Devil*. Participants came to the institute with varied levels of understanding of primes. On the pre-test, about forty percent of participants received only partial or no credit when asked to determine whether a specific number was prime or composite. The other sixty percent were able to make a basic decision about whether a given number was prime or composite. On the post-test, all but one participant received full credit. In addition, the researcher considered the post-question slightly more difficult because it dealt with a larger (odd) number.

In the book, the Number Devil defined prime numbers and emphasized that zero and one are neither prime nor composite. On the "during-reading" graphic organizer, nine participants (mostly elementary teachers) identified these concepts as new information that they gained from the reading. Of the nine, five participants wrote something about the definition of prime numbers. One wrote: "A prime number is a number that can only be divided by one and itself." The other four participants identified the concept that two is prime whereas one is not prime as new information. For example, "The number '2' is a prime number. Number '1' doesn't count."

Eight other participants included a definition of prime as knowledge that was both from the text and in their head; this can be construed as although the participant had some prior knowledge of the definition of a prime number, information in the book provided a refresher.

In addition to basic information about the definition of a prime number, other features regarding primes and composites were discussed. For example, in both a full-group content session and a breakout session, content facilitators pushed participants to think about why zero and one are neither prime nor composite as the book had not explained why. After Content Facilitator A tried to illustrate that zero and one behave differently and belong to a class of their own (neither prime nor composite), some participants struggled with the concept.

Participant C: I'm not too convinced that zero is not prime nor composite. Zero has more factors; why can't you include it as a composite number?

Content Facilitator A: It has any number as a factor that you could possibly want, so you can say it has factors but it means something different than other numbers having factors.

Participant G: Zero has an infinite number of factors and everything else has a finite [number of factors] because you can take zero times anything.

Participant C: So you are saying zero has [infinitely] many factors.

Participant G: Right.

An earlier vignette (in *Theme: Teachers gained knowledge "about" mathematics* regarding convention versus logic) also illustrated how some participants increased their understanding of the notion that one is neither prime nor composite through discussion.

Another topic stemming from *The Number Devil* and session discussion regarded using a sieve [Eratosthenes' sieve] to determine prime numbers. After writing out numbers between 2 and n , a sieve (crossing out multiples of 2, 3, 5, ...) can be used to determine prime numbers up to n . [Further discussion of this topic will be provided in a subsequent section pertaining to pedagogical content knowledge.] However, the Number Devil noted that this process is not especially effective for very large numbers. As part of their Night 3 reading assignment, participants were asked to write down any questions they had over the reading. Several participants similarly asked as one wrote: "Can you tell a number is prime just by looking at it?" On the next day, Content Facilitator A explained that "we don't have a formula for finding the next prime". The facilitator said that the largest known prime at that time was $2^{32,582,657} - 1$.

During the session, Content Facilitator A also drew attention to some “number curiosities” about primes that were discussed in Night 3 of *The Number Devil*. Many participants had written on their Venn diagram that the theorems (e.g., between a number and its double, there is always at least one prime) and conjectures (e.g., any even number larger than 2 can be written as the sum of two primes) about primes were new information. Rather than suggesting that the theorems and conjectures were important to know, the facilitator used the discussion to emphasize the difference between a conjecture (that may have been shown to be true for a finite number of cases) and a theorem (that has been proven to be true for all cases).

For homework after the content session discussions regarding Night 3, participants were asked to work the first problem and one of the remaining two of the following problems: (a) determine (and explain) [whether two given numbers] were prime or composite, (b) identify a prime number that had not been given in the book or in class, and (c) write a given number as the sum of three prime numbers. All participants except one correctly answered (a). Most of the participants worked both of the remaining two problems (instead of just one of the problems). All work was correct except for one (which was missing some explanation). Hence, after a refresher about some fundamental concepts regarding primes, homework and post-test performance indicates almost all of the participants could correctly answer basic questions about primes (whereas only sixty percent could on the pre-test). In addition, through discussions regarding aspects of primes, participants were challenged to think more deeply about why zero and one are neither prime nor composite and how truth is established in the field of mathematics.

Rational and irrational numbers were the focus of Night 4. Discussions addressed topics such as finite and repeating decimals and rational versus irrational numbers as well as procedures such as converting a fraction to a decimal and a repeating decimal to a fraction. On the pre-test, only about sixty-four percent of the participants could correctly convert a fraction to a repeating decimal. Later for a homework question, all participants were able to correctly write a fraction as a repeating decimal. On the post-test, all participants were able to write a fraction as a finite decimal.

Another type of question regarding irrational numbers was posed on a homework assignment and a quiz: participants were asked to give an example of an irrational number and to explain why a given number was irrational. On the pop-quiz (no notes), about eighty percent provided a correct irrational number (common examples were $\sqrt{2}$ and π) and an explanation for

why their number was irrational. Examples of accepted explanations were: “it cannot be written as a fraction” or its decimal form “is non-terminating and non-repeating.” While going over the quiz question, the facilitator checked for understanding and pressed participants to explain further how they might know that a number was irrational (particularly if it had a square root sign).

Content Facilitator A: Give an example of an irrational number and explain why it is irrational.

Participant: Irrational, I put π because if you write it as a decimal it's non-repeating and non-terminating.

Content Facilitator A: So it's a non-terminating, non-repeating decimal...

Participant: Does it have to be non-repeating?

Content Facilitator A: Yes.

Participant: There can be no pattern?

Content Facilitator A: There can be a pattern, but it can't be the same sequence of digits over and over....The example I gave yesterday, 0.101101110..., that has a pattern, but it's not a repeating decimal because it is not the same sequence over and over...

Participant H: And a rational number cannot be represented by a fraction.

Content Facilitator A: Right, it can't be represented by a fraction. Anybody else have an example they want to give?

Participant I: Square root of 10.

Content Facilitator A: Why is the square root of 10 irrational?

Participant I: Because it can't be expressed as a fraction.

Content Facilitator A: **Why** can't it be expressed as a fraction?

Participant: There's no even number or....number that multiplies by itself to equal ten.

Content Facilitator A: ...Ten is not a perfect square, it is not a square number, so there is no integer, no whole number that you can multiply by itself and get ten. Different from 9 because $9 = 3 * 3$...

Some additional discussion followed as to whether some specific numbers involving square roots and cube roots were irrational or rational. Later in the conversation, a participant wanted feedback as to whether her thinking was valid.

Participant B: Is it safe to say that any number you put under the square root symbol, as long as it's not a square number, it will be irrational?

Content Facilitator A: Right.

On the homework assignment, almost all of the participants were able to give an example of an irrational number not already given in the class or the book. However, a separate homework question asked: "Why is $\sqrt{7}$ irrational?" Less than twenty percent of the participants received full credit for their explanation of why $\sqrt{7}$ is irrational. Many participants received partial credit for describing the decimal representation for $\sqrt{7}$ was neither terminating nor repeating. However, participants were pushed to provide additional explanation for this specific case as full credit was only given for explanations that described $\sqrt{7}$ is irrational because 7 is not a perfect square.

On their reflections of Day 2, a few participants expressed their struggles in understanding rational and irrational numbers.

- I understood some but not all of the concepts discussed today. I am still a little confused about the difference between rational and irrational numbers.
- Content began in the realm that I am most comfortable in. I quickly realized that fractions and decimals expand into a much more wide area of mathematics (rational/irrational, countable/uncountable). I am interested in these concepts but find myself struggling with what this content will do for me as an educator.
- Why is it so important to know whether rational or irrational number? Is it simply that irrational numbers have no specific pattern to identify?

Overall, the evidence indicates that several participants grew in their understanding of rational and irrational numbers. Some participants likely already had good understanding of these topics as some participants performed well on the pre-test, post-test, quiz, and homework questions regarding irrational and rational numbers and as rational and irrational numbers were part of the curriculum that some participants taught. However, homework, quiz, and test performance as well as session discourse evidence an increase in understanding of rational and irrational numbers for several participants. Furthermore, project-collected survey data also supports participants' increase in rational and irrational number knowledge. Before the Summer

Institute, eleven participants indicated they had little knowledge of rational numbers. After the Summer Institute, only one participant reported that they had little knowledge of rational numbers; the other thirty-one participants reported they were knowledgeable or very knowledgeable. With regard to irrational numbers, nineteen participants indicated they had little or no knowledge prior to the Summer Institute. After the summer institute, only six reported they had little knowledge of irrational numbers; none reported having no knowledge. Therefore, evidence from multiple data sources reveals that many participants learned about rational and irrational numbers during the summer institute.

Some other topics in the book continued to deal with Number, but also significantly addressed Algebra by increased attention to patterns, relationships, algebraic symbols, and formulas. Many topics emphasizing both Number and Algebra were discussed on Day 3 of the summer institute. For example, in Night 5 the Number Devil introduced triangular numbers and square numbers and relationships between them. Content session discussions about triangular numbers allowed for opportunities to use multiple representations and represent relationships with both direct and recursive formulas. A question checking for understanding of relationships and representations of triangular and square numbers was posed on both the pre- and post-test: “Draw a picture illustrating an example of two consecutive triangular numbers whose sum is a square number.” On the pre-test, seventy-five percent of the participants received no credit, with most having left the question blank. Only one participant received full credit. Problem difficulty may have stemmed from a lack of knowledge about triangular numbers and/or lack of understanding of a pictorial relationship between triangular and square numbers. For the post-test, almost ninety percent of the participants improved their performance; almost seventy percent of the participants received full credit. Survey data also provides evidence of participants’ growth in knowledge of triangular numbers. Pre-summer institute, nineteen participants reported they had little or no knowledge of triangular numbers. On the post-summer institute survey, all participants reported they were knowledgeable or very knowledgeable about triangular numbers.

Fibonacci numbers were the focus of Night 6. Content session discussions provided participants with more experiences regarding Fibonacci numbers, sequences, and recursive formulas. Prior to the summer institute, twenty-six participants reported having little or no knowledge of Fibonacci numbers. After the summer institute, all but one participant reported

being knowledgeable or very knowledgeable about Fibonacci numbers. Participants also came to the institute with varied levels of understanding regarding recursive formulas. Less than thirty percent of the participants could determine the first 8 terms of a recursively defined sequence ($S_n = 2S_{n-1} + S_{n-2}, S_1 = 0, S_2 = 1$) on the pre-test whereas seventy-five percent of participants demonstrated understanding with a very similar post-test question.

Session discourse reveals how some of the participants strived to understand Fibonacci numbers, patterns, and recursive formulas. Content Facilitator A started a discussion by using the recursive formula for Fibonacci numbers to generate numbers in the sequence. Thus, for $F_n = F_{n-1} + F_{n-2}, F_1 = 1, F_2 = 1$, the Fibonacci sequence is $F_1 = 1, F_2 = 1, F_3 = 2, F_4 = 3, F_5 = 5, F_6 = 8, \dots$. After the introduction, some participants needed further clarification. The facilitator supported the participants as they tried to better understand recursive formulas by guiding the discussion as participants proposed and explored “what if” changes to the first two terms of the Fibonacci sequence.

Participant C: What happens if it starts with zero?

Content Facilitator A: Well, you have to start with two numbers. Do you want to start with zero and one?

Participant C: Zero and one.

Content Facilitator A: This is a little “what if” here. Why did we start with one and one? It’s kind of arbitrary....What if we start with something else... So what if we started with zero and one. Let’s try it. So we have zero and then one, so what is our next term going to be?

Participant(s): One. Two. Three.

Content Facilitator A: [0, 1, 1, 2, 3, ...] In this case, it doesn’t really change just everything gets shifted over one...

Content Facilitator A: [Participant D offered a suggestion for two numbers to start with.] You want to start with -1 and 0. [The facilitator guided the group to generate the resulting sequence -1, 0, -1, -1, -2, -3, -5, ...] So eventually, starting here [the facilitator pointed to the third term in the sequence], we get the negative version of the Fibonacci sequence.

Participant J: When we started to do this, both people chose to start with two different numbers, but the Fibonacci sequence you showed us, they start with the same number. Is there something that happens, something interesting, if you start with 2 and 2?

Content Facilitator A: Okay, let's start with 2 and 2. So we have 2 and 2, so what's going to be next?

Participant(s): [2, 2,] 4, 6, 10, 16, 26, 42, ...

Content Facilitator A: What do you notice?

Participant J: It's double the original one.

Content Facilitator A: ...It's double the original Fibonacci sequence.

Participant: Can you start with like 2 and 5...?

Content Facilitator A: Yes, let's get something so it looks totally different. So you want to start with 2 and 5. So we get 2, 5, 7, 12, 19, 31, So it looks different. It would be a good project topic for somebody if they were interested; look at Fibonacci-like sequences starting with something other than 1 and 1. We're going to talk about some properties of Fibonacci numbers. What properties still hold and which ones don't? Or what kinds of things go on? That would be a nice thing to investigate.

Thus, participants engaged in generating terms for Fibonacci-like sequences.

Through discussion, participants gained understanding of the Fibonacci formula. However, working with different recursive formulas still challenged some participants. On a quiz, participants were asked to determine the first ten terms of a recursively defined sequence: $S_n = 2S_{n-1}, S_1 = 2$. About sixty percent of the participants correctly worked the problem. However, almost thirty percent of the participants incorrectly generated a sequence by misconstruing the formula as direct: either $S_n = 2n$ or $S_n = 2(n-1)$. By the final exam, seventy-five percent of the participants correctly generated terms of the recursively defined sequence $S_n = S_{n-1} + 2S_{n-2}, S_1 = 0, S_2 = 1$ (which the researcher deems more challenging than the quiz question). Hence, although some participants likely had prior knowledge of recursive formulas (e.g., high school teachers as recursive formulas are often part of the curriculum), session discourse and performance on tests and quizzes evidences that some participants increased their understanding of recursive formulas.

Also on Day 3, participants were given time to read Night 7 and to use a during-reading strategy: Write-Pair-Share. Pascal's Triangle was the main topic of Night 7. As participants read in pairs, they were to consider the question: "How do sequences we've seen already arise in Pascal's triangle?" After pair reading and discussing and then sharing in small groups, the full

group was brought back together for further discussion about patterns arising in Pascal's Triangle.

Participants' reflection comments about Day 3 content suggest many participants were learning new number relationships as the participants found the patterns fascinating and were motivated to try to include the topics in the courses they taught. A few participants suggested they found the formulas and patterns confusing while other participants reflected that they liked working with the formulas.

- Wow! Even more patterns and connections to other concepts—and I get it! Good stuff.
- The triangular numbers are fascinating. I think my students would really enjoy them. The Fibonacci [numbers] are really fascinating—so many patterns.
- The patterns that we went over today were interesting. I could possibly use the Pascal's Pattern activity in class with my students. The Fibonacci sequence was interesting. The way many concepts overlap and twine together, and that is really interesting to me.
- I was able to see the various patterns discussed within Pascal's triangle.
- The numbers (tri-, quad-, Fibonacci) are very interesting to me. Patterns strengthen number sense. I want to understand how these mathematical concepts relate to number sense. Relationships between numbers are good to identify, but what is the value of knowing the patterns in isolation?
- I think the Fibonacci numbers are interesting, but when we break it down into a formula, it gets confusing.
- Night 5 exercise #7 [give a direct formula for the sequence of square numbers $S(n)$] still baffles me and I don't give up easily, so I keep trying. I hope we go over the exercise answers so I can know if I did them right, plus make sure my questions are answered. This content has definitely been a learning stretch for me, but one I needed!!
- Unsure what the bigger idea is for Bonacci [Fibonacci] numbers. Is it that there are many patterns or do mathematicians feel it is a link to a bigger picture not yet seen?
- I was unclear about the rabbit example with Fibonacci. If you weren't looking at the numbers, it would be a good representation of how quickly the numbers increase, but when thinking about the numbers it got tricky.
- It was hard to see the number groups in Pascal's triangle.
- I enjoyed reading and discovering about Fibonacci numbers. I didn't realize all of the patterns and relationships that go along with the rule.
- The content today was fun! I enjoy working with the formulas. [The facilitator] did a great job of giving concrete examples.
- Some of the assignments were hard because they required written explanation of patterns. I'm still a little unsure about how all the numbers on Pascal's triangle go together.
- The rabbit problem tied to Fibonacci was interesting but difficult to be conveyed.
- I was very interested and enjoyed the rabbit story of explaining "Bonacci" numbers. The pictures with the text really helped. I also liked that the book offered a chart as an additional way to show how the numbers worked.

Overall, the book and session content engaged participants in thinking and learning about number patterns and in describing number relationships with formulas.

Permutations and combinations were the main topic of Night 8 of *The Number Devil*. Only about thirty-five percent of the participants were able to successfully complete a basic arrangement (permutation) question on the pre-test. Participants had several learning experiences with permutation (arrangement) and combination (selection) during the summer institute. Prior to Day 4, participants were asked to read Night 8. In small groups on Day 4, participants acted out scenarios regarding arrangement (with order) and selection (without order) as had been similarly described in Night 8. Content facilitators checked on groups' progress and provided support as needed. Then, with the full group, Content Facilitator A built on participants' work by providing more formal mathematical language and notation for factorial, permutation, and combination. On subsequent homework, seventy-five percent of the participants received full credit regarding application problems using permutations and combinations. About sixteen percent of the participants only had an error on one of the three problems. Two participants struggled considerably with more than one problem. Nonetheless, on the post-test, one hundred percent of the participants successfully worked a permutation problem of comparable difficulty to the pre-test question. In addition, pre-summer institute about seventy-five percent of the participants reported having little or no knowledge of permutations and combinations. Post-summer institute, only about ten percent reported having little knowledge of permutations and combinations; none reported having no knowledge. Hence, multiple data sources provide evidence that participants grew in their understanding of permutations and combinations through summer institute learning experiences.

A final example of participants' growth in substantive knowledge of mathematics regards a topic from Night 10, Euler's formula for polyhedra. Prior to the summer institute, almost eighty-five percent of participants reported having little or no knowledge of Euler's formula. Post-summer institute, ninety percent reported being knowledgeable or very knowledgeable about Euler's formula. Performance on the pre- and post-summer institute content tests also supports dramatic growth in understanding Euler's formula. A problem on both the pre- and post-test was worded: "Show that Euler's formula $V - E + F = 2$ is true for the cube." Less than twenty percent of the participants successfully completed the problem on the pre-test. The rest of the participants received no credit with most having left the question blank. It is unclear as to

what caused so much difficulty for the problem. Brainstorming, the participants may have not been able to decipher what the letters represented (vertices, edges, faces). The participants may have not been able to determine the number of vertices, edges, and/or faces for a three dimensional solid. Or, perhaps the participants did not understand “show” meant they only had to demonstrate the formula worked for the single case of a cube. Nonetheless, although many participants struggled with the question on the pre-test, all of the participants received full credit on the post-test question. Hence, many participants learned about using Euler’s formula during the summer institute.

Self-reports about gains in substantive knowledge from mini-case participants during post-summer institute interviews also provide a picture of what learning stood out for participants who taught grades 4-6.

- I just think I learned more about the Fibonacci [numbers]. We had a lot of presentations about Fibonacci numbers. I just thought it might open up my eyes to looking for more patterns, in the real world. I thought the presentation on the Golden rectangle was interesting—how you see that all over the place. I think it piqued my interest more.
- Some as far as the patterns, and where they came from, and how they work together.
- A lot! I’ve heard a lot of things like Pascal’s triangle; I didn’t realize all the different functions of it. Same with Fibonacci numbers, it’s just a lot of things that I heard of but didn’t have a lot of experiences with, or how to exactly find them with the formulas to be able to prove them. It’s been [several years] since I’ve taken a college math class, so it was a good refresher, but most of what we learned this summer I didn’t know.
- Some of it ... was a refresher. [Some of it was new--] maybe relations, as far as Pascal’s triangle, and Fibonacci numbers, so the relationships that are there.

Overall, a variety of sources (i.e., improvement on matched pre- and post-summer institute content questions, performance on homework and quizzes, session dialogue, pre- and post-summer institute survey data, interviews with mini-case participants) provide evidence that participants gained substantive knowledge about mathematics by their participation in the summer institute.

Theme: Teachers gained mathematical curricular knowledge

Themes Organized by Research Questions

- Teachers learned mathematical content
 - Knowledge “about” mathematics
 - Substantive knowledge of mathematics
 - ***Curricular knowledge***
 - Pedagogical content knowledge
- Teachers gained pedagogical knowledge
 - Differentiating instruction
 - Supporting reading in the content area
 - Fostering number sense
 - Skills for analyzing practice
 - Implementing standards-based instruction
- Impact on teaching practice
 - Short-term impact on teaching practice

Participants had many opportunities to learn about curricular alternatives and about mathematical concepts that students might be learning at other grade levels. For example, content and pedagogy facilitators modeled using different types of curriculum. Pedagogy facilitators provided examples of how a concept is approached at different grade levels.

Furthermore, participants learned from each other about grade specific curricular standards and grade appropriate instructional approaches through small group discussions, content presentations, and share fair presentations.

First, content and pedagogy facilitators modeled using different types of curriculum. For example, by expanding upon concepts arising in *The Number Devil*, content facilitators modeled delving into mathematics content by using math-related literature. Content facilitators modeled other curricular options as well. For instance, other math-related literature options (i.e., *The Adventures of Penrose the Mathematical Cat* (Pappas, 1997), *The Further Adventures of Penrose the Mathematical Cat* (Pappas, 2004)) were explored during some breakout sessions.

Furthermore, content facilitators used sections or chapters from other curricula (i.e., *The Heart of Mathematics* (Burger & Starbird, 2000), *Elementary Mathematics for Teachers* (Parker & Baldrige, 2004)) as additional session resources.

Pedagogy facilitators modeled using a wide variety of curricular resources addressing number sense, reading in the content area, and differentiating instruction. During pedagogy sessions, teacher leaders (with experience at different grade levels) modeled lesson ideas for different grade bands using a variety of number sense curricular resources (e.g., *Teaching Student-Centered Mathematics: Grades K-3* (Van de Walle, J. A. & Lovin, 2006b), *Teaching Student-Centered Mathematics: Grades 3-5* (Van de Walle, J. A. & Lovin, 2006a), *Number Sense: Simple Effective Number Sense Experiences/ Grades 1-2* (McIntosh, Reys, & Reys,

1997a), *Number Sense: Simple Effective Number Sense Experiences/ Grade 6-8* (McIntosh, Reys, & Reys, 1997b), *Explain It! Answering Extended-Response Math Problems (Grades 7-8)* (Creative Publications, 2001), *Roads to Reasoning: Developing Thinking Skills Through Problem Solving (Grade 1)* (Weinberg et al., 2002), *Family Math: The Middle School Years* (V. Thompson & Mayfield-Ingram, 1998), *Mental Math in the Middle Grades* (Hope, Reys, & Reys, 1987)). In addition, as part of their participation with the IMP Year 2 project, participants received a grade-appropriate number sense resource (e.g., *Number Sense: Simple Effective Number Sense Experiences*). Pedagogy facilitators also provided information about teaching reading in the content area (e.g., *Teaching Reading in the Content Areas: If Not Me, Then Who?* (Billmeyer & Barton, 1998) and examples of reading strategy activities (e.g., *Reading Strategies for the Content Areas, Volume 2: An ASCD Action Tool* (Beers & Howell, 2005)). Some reading strategy activities were also used in conjunction with content session reading assignments for *The Number Devil*. Finally, pedagogy facilitators also used a variety of resources (e.g., *Differentiation: From Planning to Practice, Grades 6-12* (Wormeli, 2007), *How to Differentiate Instruction in Mixed-Ability Classrooms* (Tomlinson, 2001)) to provide information about key features and instructional strategies for differentiating instruction.

Feedback from participants evidences that participants valued and used or would use some of the curricular resources that had been modeled during the project. First, some participants named specific materials that had been modeled during IMP Year 2 that would be used as resources for their action plan. For example, one participant identified specifically *Roads to Reasoning* and *Teaching Student-Centered Mathematics (K-3)*, and more generally “Number Sense (I think we have them)”. During a share fair presentation, a participant said her lesson study group chose a “lesson from that Number Sense book that we all got.” In an interview, a mini-case participant expressed “there were some good resources that [a content facilitator] showed us--some books that we could use: the mental math and I guess where you can get questions and things like warm-up activities.” Some participants also expressed that they would strive to use more math-related literature; this topic will be addressed more fully as a theme in pedagogical knowledge. Overall, many participants expressed valuing or using curricular alternatives that had been modeled during IMP Year 2.

Participants also gained vertical curricular knowledge about mathematical concepts addressed, and instructional approaches employed, at other grade levels. Some learning

experiences were structured by project facilitators. For example, on Day 1 Pedagogy Facilitator B presented details about what nurturing number sense entails for different grade bands. In the early grades, children should have opportunities to identify several relationships: spatial, one and two more than or less than, benchmark numbers, and part-part-whole. In later grades, relationships get a little deeper. For instance, in the intermediate grades, attention should be given to relationships between operations (i.e., addition, subtraction, multiplication, division). Building upon the presentation, teacher leaders modeled activities appropriate for various grade bands that could be used to foster number sense throughout the summer institute. On the post survey, a participant reflected: “Also, the number sense activities were divided into levels (K-2), (3-5) and higher level. It was nice to see the difference between the concepts at the different grade levels”.

Pedagogy facilitators provided other opportunities and resources for participants to learn vertical curricular knowledge. For example, each participant received a set of four binders of Kansas Curricular Standards. Each binder addressed a single standard (Number, Data, Algebra, and Geometry) with a color coded progression through grade levels of benchmarks and indicators. Pedagogy Facilitator B described:

You need to know where your kids came from...and where they are going. So we wanted to give you the K-9 standards in a format...that if you're doing something on data ...you can look up and down, above and below your grade level.

As another example, on Day 1 Pedagogy Facilitator A led the participants in looking for patterns of student performance across grade levels. Each participant was asked to determine the highest and lowest tested indicators for his or her district and grade level. Data was coalesced as a visual display with green notes signifying the highest tested indicator and pink notes representing the lowest indicator. This allowed the participants to notice some patterns across grade levels. For example, the facilitator verbalized that there was a lot of pink (lowest indicator) for Geometry. The facilitator said that data analysis and probability and geometry are typically at the end of a traditional text and sometimes do not get covered. In the past we've had kids in Kansas struggle in geometry in high school. And, the whole idea behind state assessments is to drive instruction. The facilitator noted that the percent of state assessment questions for grade 3 addressing geometry (including measurement) had increased from sixteen to thirty. The state was trying to get teachers to teach more geometry at the lower grades. However, on the next day, Pedagogy

Facilitator B led a different discussion about tested indicators. The facilitator showed a slide representing 4th grade state standards and indicators. On the next slide, the tested indicators were highlighted. There was a huge difference between indicators versus tested indicators. The facilitator expressed concern if teachers were to focus on tested indicators (“teaching to the test”) for a particular grade level to the neglect of other indicators. What happens when the students go to the next grade and the tested areas change? “It’s not a 4th grade problem; it’s not a 5th grade problem; it’s a kindergarten through 12th grade problem.” Thus, session activities spurred discussions about the need for teachers to understand what kids were learning at various grade levels. In addition, teachers were encouraged to support their students in the development of mathematical understanding and power at their grade level in order that students might have strong foundations to build upon at the next level.

Grade level diversity amongst participants also provided many natural opportunities for participants to learn vertical curricular knowledge from each other. For example, participants (alone or in small groups) gave a content presentation over a topic of interest or related to their own teaching. The content facilitators encouraged the participants to pick a topic for which participants would like to increase their own understanding. Presentation topics were varied and were often related to the teacher’s grade level. For illustration, one group of intermediate teachers focused on divisibility rules; a group of sixth grade teachers presented on approaches for fostering understanding of operations with positive and negative numbers; a high school teacher presented on visually representing relationships amongst number systems. As expressed by a participant on the post-summer institute survey, “the project presentations allowed me to see how the concepts could be used at various grade levels”. Share fair presentations (regarding implemented action plans and lessons studies) also provided opportunities for participants to learn about curricular standards and instructional approaches affiliated with other grade levels. Furthermore, although participants combined efforts by working within grade bands to produce differentiated instruction activities during the summer institute, on the final day participants were invited to review all of the activities that were produced. Hence, participants could once again see examples of concepts covered and instructional approaches used at other grade levels.

Opportunities for learning vertical curricular knowledge also arose with communication between teachers of different grade levels. Discussions sometimes stemmed from content session activities. For instance, concepts addressed in *The Number Devil* initiated some

discussions about grade appropriate mathematical language or grade level curricular standards. As an illustration, exponentiation (e.g., $5^1 = 5$, $5^2 = 25$, $5^3 = 125$, ...) was referred to as “hopping” by the Number Devil. Several elementary teachers were concerned about the author’s word choice. As one participant described in a content session, “in kindergarten we say hop two spaces”.

Content Facilitator A: He’s using non-standard terminology here. You might be saying that I use hopping in my classroom to mean something different so you wouldn’t want to use the term hopping to talk about this because that would be confusing.... If you are thinking of hopping as making a jump on the number line of the same size each time, I guess that’s going to be misleading.... Is this an effective way to present this? Several of you mentioned this on your sheet—I’m not sure I like the word hopping; the word hopping is confusing.

[Several other comments were made by participants and the facilitator.]

Content Facilitator A: Every time you multiply by 10 it hops a place value...hopping in the sense of hopping from different place values. Again, we could get into a little philosophical argument about whether you like the term or not, but it makes more sense here.

Hence, the community of facilitators and participants was able to ponder how a non-standard word choice might result in confusion with regard to concepts or instructional practices associated with certain grade levels.

Discussions about prime and composite numbers provide another illustration of participants learning vertical curricular knowledge during the summer institute. For the reading assignment over Night 3 of *The Number Devil*, participants were to write about any questions they had over the reading. Questions from two participants follow.

- What is prime numbers importance? The only thing I can think of is to place a fraction in lowest possible terms...[6th grade teacher]
- How can I introduce the concept of prime numbers to first graders? When the Number Devil talks about large prime numbers would this be a good intro to divisibility rules—giving students a purpose for using/learning them?

Thus, content in *The Number Devil* sparked interest among some participants as to why primes were important. The question arose in other situations as well. For example, Content Facilitator C led a discussion about “Deep Math Questions”. In small groups, participants were to discuss math questions for which they wanted answers. Questions were often related to grounding procedures in conceptual understanding or contemplating how concepts were related to teaching.

One small group discussion amongst teachers of various grade levels considered prime and composite numbers.

Small Group Participant A: In 4th grade we don't get into prime and composite numbers, you might mention the word at the end of 4th grade. What is the benefit of knowing if something is prime or composite?

Small Group Participant B: Reducing fractions?

Small Group Participant C: A lot of times with a least common denominator you can find it by going down to primes, prime factorization of each, so being able to talk about primes is an approach to finding the least common denominator...

Small Group Participant A: Maybe since our kids struggle with finding the least common denominator in 5th grade, maybe that's a reason we need to start talking about prime numbers and factoring numbers in 4th grade. Our 5th graders bomb that every single year.

Small Group Participant C: That seems early. 5th graders, they're trying to get them to [find a least common denominator]...

Small Group Participant A: Yes, they have to find the least common denominator or greatest common multiple on the state test...

Small Group Participant C: They have to add and subtract fractions in 5th grade?

Small Group Participant A: Yes.

Small Group Participant C: Oh, I didn't know that.

Small Group Participant D: Don't they cover prime numbers and composite numbers in 2nd or 3rd grade?

Small Group Participant A: No, the vocabulary of prime and composite is not even in the standard for 4th grade. We introduce it at the end of 4th grade when we're done with the state test and we start looking at the vocabulary they need in 5th grade, so as teachers we introduce it...but some teachers don't...

Small Group Participant D: In which grade do you teach multiplication?

Small Group Participant B: 3rd grade.

Small Group Participant D: It can come immediately after multiplication...maybe it should be in 3rd grade.

Small Group Participant A: But now that I see why it's important for them to know it, now it makes sense to me because my kids always ask why do we have to know this, where are we going to use this...

Hence, a group of participants learned from each other about where and why prime numbers fit into school mathematics curriculum.

Evidence about participants learning vertical curricular knowledge also stems from interviews with mini-case participant at the end of the summer institute. Two participants brought up the content session focus on patterns gave them knowledge about more complicated patterns that their students might see in the future.

- We talked about patterns, which I teach that, but I guess my patterns...is you do the same thing each time, but these patterns, the Fibonacci numbers...it's different patterning, it's more advanced patterning.
- It gave me a window into why do we do patterns, why do we do sequences, and of course the ones at my level are much simpler, but that they are building to something they'll use later and then listening to the high school teachers and where they went with that...that was over my head, but I could see the bits and pieces of what we had learned during the week and how they were including it in their class.

Another mini-case participant reflected:

- We were sitting with a couple middle school teachers, they were like, "we do this, ...and this is what we do". I think it's good that I know now what they end up doing, because we kind of get in our mind set that this is fourth grade math, and everything revolves around fourth and fifth grade, you know not so much past the grade level above.

Hence, teachers learned vertical curricular knowledge through experiences with mathematical concepts examined during content sessions and through discussions between diverse grade level participants during the summer institute.

Vertical curricular knowledge learning opportunities also arose from communication between teachers of different grade levels as part of the participants' implementation of a lesson study. For example, in an interview with a mini-case participant during the school year, the participant described the impact of participating in a lesson study.

In our lesson study we had another 4th grade teacher, we had a 5th grade teacher, and then we had a K-State intern, and the intern actually gave us a lot of good ideas that they had just gotten when they were in their block classes.... The 5th grade teacher kind of made a big thing about—we need to do this with liquids, with different liquids and different sized containers because that's part of the test for 5th grade.... It was nice to be able to bounce

ideas off each other.... It was interesting to get the K-State perspective and it was interesting to see the 5th grade teachers' perspective...

As another example, during the share fair a participant described her group's modified lesson study experience.

We had a very interesting opportunity this year because we have a range of grade levels and we all did the same standard for our lesson study ...measurement.... We loved the opportunity to see how things intertwined with our mathematical standard from kindergarten to third grade...we all participated in everybody's action plan and lesson study but we all implemented them in our own grade level. It was really an awesome opportunity to see how measurement looks in kindergarten, and how it moved into first grade...and then we got to see it again in third. It really solidifies the need to collaborate above and below you with your grade level partners to make sure that you're hitting all those important concepts, skills, and ideas...

Therefore, some participants also gained vertical curricular knowledge through their lesson study experiences during the school year.

Overall, teachers gained curricular knowledge through their participation with IMP Year 2. Teachers gained knowledge about curricular alternatives: resources for fostering number sense, resources for differentiating instruction, activities for reading in the content area, and examples of math-related literature. Participants also gained vertical curricular knowledge through learning experiences structured by project facilitators, and more naturally, through discussions with teachers from a variety of grade levels.

Theme: Teachers gained pedagogical content knowledge

Themes Organized by Research Questions

- Teachers learned mathematical content
 - Knowledge “about” mathematics
 - Substantive knowledge of mathematics
 - Curricular knowledge
 - ***Pedagogical content knowledge***
- Teachers gained pedagogical knowledge
 - Differentiating instruction
 - Supporting reading in the content area
 - Fostering number sense
 - Skills for analyzing practice
 - Implementing standards-based instruction
- Impact on teaching practice
 - Short-term impact on teaching practice

IMP Year 2 participants had opportunities to increase their pedagogical content knowledge in a variety of situations. For example, content and pedagogy facilitators modeled some useful representations and explanations for K-12 mathematical concepts. In addition, IMP facilitators supported participants' development of pedagogical content knowledge by

focusing attention on conceptual understanding for mathematical ideas and on student thinking about ideas. Content presentations by teachers during the summer institute and share fair presentations about the lesson study process also provided evidence that teachers had gained pedagogical content knowledge.

First, performance by participants on a question of the pre- and post-summer institute content tests provides evidence that teachers gained pedagogical content knowledge. Participants were directed to use a sieve to find all the prime numbers between 2 and 30. Only twenty-five percent of the participants received full credit for the problem on the pre-test. Several participants received partial credit by correctly naming the primes; however, their work did not indicate that they used a sieve. Using a sieve to find [smaller] prime numbers was illustrated in Night 3 of *The Number Devil*. During a full group discussion over Night 3, one participant shared that she thought it was a very useful representation that kids would remember: “I love it; I try to do it every year”. On the post-test, all but one participant received full credit for using a sieve to find primes.

IMP pedagogy facilitators encouraged participants to attend to student thinking while implementing the lesson study, an action plan, standards-based instruction, and differentiated instruction. For example, when introducing the lesson study process, Pedagogy Facilitator B described student thinking regarding linear measurement. The facilitator suggested that teachers should try to identify ideas related to linear measurement for which students might be struggling (e.g., partitioning, unit iteration, transitivity, conservation, accumulation of distance). Teachers should also reflect upon appropriate instructional sequences. As another example, lead teachers discussed student understanding and effective instructional sequencing for some big ideas related to number sense as described in research-based resources by Van de Walle and Lovin (2006a, 2006b).

Content and pedagogy facilitators encouraged participants to understand more deeply the logic behind some mathematical procedures and ideas. For example, in conjunction with content in Night 4 of *The Number Devil*, the equivalence between 1 and $0.\overline{9}$ was explored. Content facilitator A demonstrated $1 = 0.\overline{9}$ with several explanations. For example, starting with the more common equivalence $\frac{1}{3} = 0.333\dots$, multiplying both sides by three results in

$3\left(\frac{1}{3}\right) = 3(0.333\dots)$ which means $1 = 0.999\dots$ or $1 = 0.\overline{9}$. Several participants seemed uncomfortable with the equivalence. The facilitator pointed out while the finite decimal is not equal to one (i.e., $0.999 \neq 1$), the repeating decimal is equal to one (i.e., $0.999\dots = 1$). Another explanation was offered to provide additional evidence for the equivalence. If we let $x = 0.999\dots$, and then if we multiply both sides by ten we would get $10x = 9.999\dots$. Next, if we subtract x from $10x$ and $0.999\dots$ from $9.999\dots$, we get the equation $9x = 9$ which implies $x = 1$. Hence, $x = 0.\overline{9} = 1$. Further discussion revealed some participants were still uncomfortable with the equivalence. Hence, the facilitator provided another explanation. “Suppose you were trying to put $0.999\dots$ on the number line and said it’s not equal to 1... what’s the difference between it and 1?” With repeated regrouping, one would reason that the difference is zero.

As another example, Pedagogy Facilitator B discussed the finger trick for multiplication with nine. The facilitator described that some teachers may show students the finger trick without taking the opportunity to talk about relationships underlying the trick. To determine 9×1 with your fingers, hold up your hands and tuck away the pinky on your left hand. You have nine fingers. To determine 9×2 by the trick, tuck away the ring finger on your left hand. On the left side of the tucked finger there is one finger representing one ten. On the right side, there are eight fingers representing eight ones. The total is eighteen. However, the facilitator recommended taking it a step further. Suppose you are at 9×2 , add a ten and subtract a one to determine 9×3 . That is, starting with 9×2 that has one ten and eight ones, add a ten by raising your ring finger and subtract a one by tucking the middle finger. The result is two tens and seven ones which is twenty-seven. As one participant wrote on a reflection: “Use of knuckles to do ‘9’s’. Explaining the concept behind this (why it works). I never thought about this...interesting!”

Also focusing attention on conceptual understanding, Content facilitator C led a group brainstorming activity for identifying “Deep Math Questions” for which participants wanted explanations. After small group discussion, questions were shared in the full group. About twenty questions were described. Later, the facilitator wrote the questions on a chart and identified which questions were explored in *The Number Devil* [N.D.]. Some examples of questions follow:

- How would you explain “i”? N.D.
- When you divide by a fraction, why do you multiply by the reciprocal?
- What can you use Pascal’s triangle to do? N.D.
- How many prime numbers are there? N.D.
- Why can’t you divide by zero? N.D.
- Why do we have an “order of operations”? N.D.
- Is zero positive, negative, both or neither?
- Why is a (negative number)(negative number) = positive number?
- Why does the “divisible by 3” rule work?

Several questions probed the conceptual basis underlying procedures. Over the course of the summer institute, some questions associated with concepts in *The Number Devil* were addressed by content facilitators (e.g., explanation for “i”, conceptual relationship between Pascal’s triangle and binomial coefficients, why division by zero is undefined, why we have an “order of operations”). Participants were also encouraged to consider the questions as potential ideas for content presentations. As such, some questions were investigated by participants and presented on to the group. For example, one group of sixth grade teachers presented on useful explanations and representations for adding, subtracting, and multiplying with positive and negative integers.

Finally, participants gained pedagogical content knowledge while reflecting upon student thinking and conceptual underpinnings for mathematical ideas as they implemented their lesson study and action plans during the school year. Some examples of comments by participants during the share fair include:

- Advise—we would definitely combine percents to the teaching of probability to help out with number sense because there are a lot of kids who we found didn’t understand that a fraction could be related to a percent... We also would do a fractions mini-unit because again, kids are scared of fractions and we definitely saw that a lot...
- Some [students] had really big misconceptions on how to measure.... They would get their yardstick and just walk across the room, “one, two, three”. It was crazy; we saw all these things that we never would have guessed that’s the way they would measure. It was really eye-opening to us.
- They would record the card they were given and then they would estimate fine as a group where it would go, but then as they converted, sometimes their conversions weren’t correct, and they didn’t get it that it wasn’t correct just by going off their estimations... For example $\frac{3}{5}$, they would know that that is larger than $\frac{1}{2}$... but then $\frac{3}{5}$ they would change to the decimal 0.35, so that would be their converting...and they didn’t realize 0.35 isn’t over $\frac{1}{2}$...

Hence, individual participants gained pedagogical content knowledge during the lesson study process, but also as the individual participants shared their reflections during the share fair, the larger group of participants may have gained learned pedagogical content knowledge as well.

Overall, participants gained pedagogical content knowledge. Some learning stemmed from facilitator-led examinations of useful representations, conceptual underpinnings for procedures, and student thinking. In addition, some participants gained pedagogical content knowledge while reflecting upon student thinking and conceptual underpinnings for mathematical ideas during their implementation of an action plan and lesson study during the school year.

Teachers Gained Pedagogical Knowledge

Themes Organized by Research Questions

- Teachers learned mathematical content
 - Knowledge “about” mathematics
 - Substantive knowledge of mathematics
 - Curricular knowledge
 - Pedagogical content knowledge
- ***Teachers gained pedagogical knowledge***
 - Differentiating instruction
 - Supporting reading in the content area
 - Fostering number sense
 - Skills for analyzing practice
 - Implementing standards-based instruction
- Impact on teaching practice
 - Short-term impact on teaching practice

Participants’ experiences in learning mathematical content and pedagogical strategies during the summer institute provided a backdrop for teachers to reflect on mathematics instruction pedagogy. Different pedagogical practices were used throughout the summer institute. For instance, participants had the opportunity to learn mathematics content via math-

related literature. Participants also had opportunities to experience and reflect upon elements and strategies for differentiating instruction, supporting content area reading, fostering number sense, and implementing standards-based instruction. For example, participants were engaged in the Process Standards while learning mathematical content introduced in *The Number Devil*. Sometimes content sessions revolved around whole group interactive lecture formats (lecture eliciting interactions with the participants) or large reading circle groups. At other times, participants discussed and acted out problems in small groups; participants engaged in pair reading math-related literature, followed by discussion regarding questions on a graphic organizer; participants tried activities from different curricula; etc. Participants were also actively

engaged while learning pedagogy. For examples, lead teachers modeled number sense activities and participants had opportunities for reflection and discussion about strategies for fostering number sense. As another example, participants made differentiated instruction activities with grade level peers. These varied experiences as learners provided a backdrop for teachers to reflect upon what were positive learning experiences for themselves and their peers.

In addition, pedagogy and content facilitators shared research-based strategies regarding differentiating instruction, reading in the content area, fostering number sense, and implementing standards-based instruction. Participants also learned about implementing a lesson study, an action plan, and thus gained skills for analyzing practice.

Theme: Teachers learned about differentiating instruction in a mathematics classroom

- Themes Organized by Research Questions**
- Teachers learned mathematical content
 - Knowledge “about” mathematics
 - Substantive knowledge of mathematics
 - Curricular knowledge
 - Pedagogical content knowledge
 - Teachers gained pedagogical knowledge
 - ***Differentiating instruction***
 - Supporting reading in the content area
 - Fostering number sense
 - Skills for analyzing practice
 - Implementing standards-based instruction
 - Impact on teaching practice
 - Short-term impact on teaching practice

An abundance of data evidences participants learned about differentiating instruction through their participation with IMP Year 2. Whereas some participants had very little prior knowledge or experience with differentiating instruction, other participants had some prior training. For example, about half of the participants of Year 2 had previously attended IMP Year 1. IMP Year 1

learning experiences tended to provide a general overview of differentiating instruction strategies; the experiences were less specific to differentiating instruction in a *mathematics* classroom. On the pre-summer institute survey, about half of the participants reported having very little training or minimal training (component of college classes and/or a few workshops). About twenty-five percent of participants identified having had differentiating instruction training in IMP Year 1 along with addition workshops or inservice training. However, even those participants having some prior knowledge learned more about differentiating instruction.

IMP Year 2 provided participants many learning opportunities regarding differentiating instruction. First, pedagogy facilitators shared information and spurred discussion about key elements for differentiating instruction as well as tips and strategies for implementing

differentiated instruction. Information came from resources by experts such as Rick Wormeli and Carol Ann Tomlinson. Some of the key elements that were discussed included: identifying the big idea that a teacher would want all students to understand; using pre-assessment to identify the needs of individual students; differentiating (and thus allowing elements of choice) in content, process, and product according to students' readiness, interests, and learning profile; and using flexible grouping. Some tips regarded: starting small but growing, combining efforts through collaboration, establishing routines, and managing differentiated instruction and assessment. Some specific strategies that were examined included anchor activities, tiered lessons, learning centers, learning contracts, multiple-entry journals, learning menus, cubing, ThinkDots, RAFTs, and Think-Tac-Toe.

Participants experienced some elements of differentiated instruction throughout the content and pedagogy sessions. For example, assessment was sometimes used to guide instruction. Participants started the summer institute with a pre-assessment content test. Throughout the summer institute, reading assignments regularly encouraged self-assessment by asking participants to write about what they did not understand from the reading. Sometimes the content facilitators followed up by discussing the questions or concerns with the full group. Furthermore, some quizzes were used to identify areas for which participants might need more explanation. As another example, content facilitators attended to participants' readiness, interest, and learning profile while differentiating content, process, and product. First, content facilitators attended to participants' readiness and interest by differentiating content in breakout sessions. Based on a brief description of the breakout session content, participants chose a session to attend. Sometimes addressing readiness, one session might explore additional representations and explanations of a concept while the other session might provide a more challenging extension of the concept. At other times, the sessions attended more to interest rather than readiness. Content sessions also provided some opportunities for flexible processing by using strategies such as partner reading and sharing, graphic organizers, small group discussions, and other varied learning experiences (e.g., acting out mathematical problems, multiple representations). Furthermore, participants were allowed some choice in product assignments. For example, oftentimes participants were allowed to choose a subset of homework problems from a larger set. In addition, participants were asked to choose a content presentation topic related to their own interest or teaching for which they wanted to increase their understanding.

Participants appreciated the differentiated instruction strategies that were applied in content sessions. On a post-summer institute survey question probing what differentiated instruction strategies were applied in content sessions and any recommendations for improvement, over seventy percent of the participants positively commented about the breakout sessions. Some examples include:

- In the morning sessions, the instructors provided separate sessions to appeal to different ability and interest levels that expanded on the topics of the day. I appreciated the opportunity to attend a session based on my personal ability or interest level.
- At some sessions, they allowed us to choose what break out we would go to depending on our comfort level with the content, so we could review or extend the content. Sometimes they would differentiate by the grade levels we taught to make the content more applicable in our teaching.
- They gave us choices based on our readiness level and interest. The breakout sessions that were available we got to choose by either readiness level or interest.
- The break-out sessions were helpful. Students (like me) who needed more help could go where we thought our needs would best be met.

Other strategies employed during content sessions for differentiating instruction were also identified (e.g., varying ways in which the participants worked, differentiated homework, flexible grouping):

- Instruction used visuals, kinesthetic, and auditory.
- [Content Facilitator B] incorporated kinesthetic movement into a lesson. [Content Facilitator A] created group question and answer sessions to explore the understanding of the group and observed for data to guide in further instruction of the concept.
- They allowed a choice in topic and presentation areas while presenting similar content. Visual aids were used. Media was used. We used some kinesthetic activity.
- Some mornings it was direct instruction and others it was group work. They tried to bring in different things to show us “real life” examples. They incorporated music, and movement also.
- Grouping, graphic organizers, content reading through partners...
- They differentiated through the breakout sessions and they used homework for differentiated instruction.

One strategy which was identified as an area of improvement regarded choice for the content presentation. Four participants suggested the presentation guidelines were too open-ended. For example,

- Project gave us choices, but it was almost too open.
- Open ended projects—however the assignment needed more guidelines and direction.

Therefore, participants experienced some differentiated instruction during the content sessions and were able to reflect upon their experiences.

Participants also experienced some elements and strategies of differentiated instruction during the pedagogy sessions. For example, a jigsaw reading activity (with participants being assigned to reading and discussing subsections of an article by a number found on their article) resulted in heterogeneous groupings of participants who may have not been grouped together before by readiness, interest, or grade level. In another learning experience, participants were able to combine efforts by working with grade level partners (homogeneous grouping) to create differentiated instruction activities for their classrooms. During the learning experience, participants moved around learning stations. At each station there was an explanation of the activity or instructional approach, example(s), and a blank template for the group to make their own. Groups were able to choose learning stations for which they had interest and to make decisions about which mathematical concepts (related to their grade level) to address with the activity. As another example, pedagogy sessions scheduled some time for facilitators and lead teachers to talk with individual participants (or small groups of participants) regarding their concerns or questions about the lesson study process, action plans, or differentiated instruction strategies. Hence, the pedagogy sessions providing learning experiences incorporating flexible grouping, choice, and flexible instructional strategies (e.g., at times addressing the individual needs of students in smaller groupings).

Next, lead teachers provided examples and modeled differentiated instruction strategies specific to mathematics, thus providing more learning opportunities for participants. During one afternoon session, participants were separated by grade level (K-5 and 6-12). In the 6-12 group, lead teachers (one middle and one high school) provided examples of differentiated strategies that they had created and used as well as some resources they had found. For example, one lead teacher described a tiered-lesson learning station activity regarding solving linear equations. She provided many details about how she managed the activity (e.g., pre-assessment, expectations at each station, teacher involvement). The other lead teacher shared a menu she had created pertaining to solving linear systems. Other resources were shared as well (e.g., examples of sponge/anchor activities).

Another significant learning opportunity was afforded by requiring participants to make their own differentiated instruction activities with grade level peers during an afternoon session.

Information about specific strategies was provided at a variety of learning stations (e.g., ThinkDots, Think-Tac-Toe, cubing, learning contracts). Groups chose and moved through several stations during the afternoon. The group the researcher participated with created high school activities (e.g., learning menu for circumference, area, and volume of a cylinder; tiered lesson about quadratic equations; RAFT about volume, surface area, etc.; anchor activity ideas; multiple-entry journal prompts).

After participating in IMP Year 2, participants demonstrated that they had learned differentiated instruction strategies in a variety of ways including (but not limited to): communicating key elements of differentiated instruction; identifying strategies that they would like to use during the school year; reporting on what they had learned during the summer institute that they would use in the coming school year; creating with grade level peers differentiated instruction activities and resources; and employing differentiated instruction strategies with content presentations (optional), lesson study (optional), and action plans (required).

A question on the pre- and post-summer institute survey probed participants' understanding of differentiated instruction: "How is differentiated instruction different than individualized instruction?" On the pre-survey, responses varied from little understanding to some understanding. Some responses indicated little understanding of differentiated instruction.

- I am not certain.
- My understanding is that they are the same.
- It is more effective.
- It is task divided by ability levels not individual students.

Some responses were weak or superficial or unclear. For example:

- Differentiated is designed the same for every student there are just many choices to choose from. Individualized instruction is differentiated, but more tailor-made to a specific student.
- Differentiated instruction is still looking at the whole group when you are trying to personalize every student's lesson.
- Different ways to approach a certain topic.
- DI you can group students by learning styles or by interests. It is more than one student.
- In differentiated instruction you are teaching the same concept keeping in mind the varying ability levels of the students.
- Differentiated instruction caters to the different activities a teacher does inside the classroom in terms of presenting the lessons to the students, whereas, individualized

instruction is preparing different activity for different students of different mathematical capabilities.

Some responses emphasized an element of choice for students in differentiated instruction.

- Differentiated instruction gives students options where they can pick ways of learning and communicating that learning in a way that works best for them. The teacher can give several options that will work for all individuals without actually individualizing the options.
- Differentiated instruction is different than individualized instruction because it is not a different plan for each child, but giving a few options to the class. There are other students in the room working on the same material during differentiated instruction.
- Differentiated instruction gives students choices on what they might do.

Some other responses focused on using different instructional strategies. For example,

- DI is different because it is strategies to meet the different learning styles of the kids. Individualized is more prescriptive instruction for specific learning deficiencies.
- I think differentiated instruction is presenting instruction in various ways to the entire class.
- I think differentiated instruction is using multiple strategies to achieve learning for all students.

A few responses reflected a more developed understanding of key elements for differentiated instruction.

- Differentiated instruction is a method of teaching to meet the learning needs and styles of all learners. Individualized instruction is teaching one to one instead of giving students the opportunity to learn with others. The skills content is not watered down for differentiated instruction.
- Differentiated instruction requires the teacher to create different types of activities that will match the readiness and multiple learning strategies for ALL students. Individualized instruction looks more at a single student and making adaptations for just that student.

Thus, prior to the summer institute, some participants had little or no understanding of differentiated instruction; other participants had some understanding of differentiated instruction.

For the same question on the post-summer institute survey, many participants provided a more developed description of differentiated instruction. Furthermore, the most common type of response (over one-third of responses) described differentiated instruction in a way that was fairly missing in pre-survey responses. The common description reflects what Pedagogy Facilitator A said about developing action plans, “everybody comes out with the same big idea; how they get there is differentiated”. Some examples follow:

- When instruction is differentiated it is organized so that all students' needs can be met. Differentiation can be done by modifying such things as the product. You must keep in mind that the essential content should be the same for all students. To me, individualized instruction is the type of "extra help" that you provide for a student when he/she is not being successful in your classroom.
- Differentiated instruction—you make sure that all students are getting the same concept but in different ways. Such as through music, writing, groups—individual, partner, small or large. Individualized instruction is working with the student one on one and using what they know to help develop the unknown.
- Differentiated instruction is ever evolving. Even though they may be in one group for one lesson they may be in a different group the next time. Students do not have to feel left out-or singled out-they are working on the same content, but maybe in a different way or at a different level.
- Differentiated instruction keeps the main concept the same for every student; it just gives them different ways of going about learning it. It also divides students up in groups by ability level or by interest. Individualized instruction focuses on one student and their needs.
- Differentiated instruction is gearing lessons towards students' learning styles. Every child learns the same skills but in a different way. Individualized instruction is more on each student's level.
- In differentiated instruction, all students are taught the standards necessary. Delivery of instruction, practice and assessment may be altered to meet students' needs according to readiness, ability, interest, and learning styles. All students are actively involved in the lessons. In individualized instruction, only a few are taught specific skills or lessons needed only by them.

Some other responses focused on different components of differentiated instruction and many tended to be less developed descriptions. About fifteen percent focused on choice when describing differentiated instruction. For example,

- I think that it is just providing opportunities for extension of topics and I think that it is geared toward the readiness of students. The teacher does not make each choice, but the students are given choices on what they are ready for.
- DI is different because it gives a student a choice to choose how they will learn or show their knowledge about a topic.
- DI allows some choice to students to keep them focused and interested in a topic. It also presents material at an instruction level for the child.

About another fifteen percent emphasized that differentiated instruction attempts to make learning accessible to all. For instance,

- Differentiated instruction is making activities understandable for all students. It could deal with learning styles of abilities. Individualized instruction is geared to creating something new for every student. Differentiated instruction might just be doing a lesson in another way; it can help everyone in the group not just one individual.

- Individualized instruction accommodates one student at a time. Differentiated instruction is set up to make learning accessible to all levels of learners.

A couple of responses suggested individualized instruction is a component of differentiated instruction. Also, a few responses did not seem to fit into a category. For example,

- Differentiated instruction is coming up with different activities that will facilitate student's learning based on the hierarchy of knowledge (that is, knowledge, comprehension, analysis, synthesis). Individualized instruction is preparing an activity that is suited for the ability of a particular student such that he or she could master a particular skill.

Finally, a few responses revealed a lack of understanding or a misconception about differentiated instruction. One participant wrote that she/he was not sure about the difference between differentiated instruction and individualized instruction. Three responses emphasized ability grouping (instead of flexible grouping).

- You can group students who are on the same level of understanding. Everyone can do the same type of activity with different levels of complexity.
- Individualized instruction is one on one; differentiated instruction is leveled for different groups of kids on about the same level.
- Differentiated instruction is used to cater for the interests of the whole class while individual instruction is focused on an individual. Differentiated instruction involves establishing stations based on ability but there is always a common goal that has to be achieved by the class.

Overall, prior to the summer institute participants' understanding typically ranged from little to some understanding of key elements for differentiated instruction. After the summer institute, many of the participants' responses revealed a more developed understanding of differentiated instruction. However, a few responses did not evidence an understanding of key elements for differentiated instruction.

Another way in which participants demonstrated learning about differentiated instruction was by their identification of differentiated instruction strategies they would like to use during the coming year in contrast to their reports on what they had previously used. On the pre-summer institute survey, participants were asked: "What differentiated instruction strategies have you implemented in your classroom?" Two participants could express no strategies.

- Honestly not sure.
- Probably in more ways that I know but as I am not sure what it is, I don't know!

A few responses primarily associated differentiated instruction with ability grouping.

- On Fridays we have “math academy” which groups students based on their math ability. Our reading program groups students by their reading level/ability, not their grade level.
- I have my students that have understood the concept quickly and completely help with explaining the concept to the students that might be struggling with that particular concept. As a team we group our math students up by ability level on Fridays and teach according to their levels (high end really try to challenge and maybe introduce the next concept; lower end go over concepts they struggle with).
- I am not certain. In reading groups, I have 5 different groups that I teach different levels of lessons to, in different ways.

About half of the response descriptions were rather generic, and some barely touched on aspects of differentiated instruction. For example,

- Creating assignments in which all students can contribute.
- Try some tactile exercises and a lot of visual. Usually auditory and visual.
- I use differentiated centers for both reading and math. I also differentiate by grouping students and the activities they work on. I try and differentiate my questioning during whole group to challenge all students.
- Hands on, peer tutoring, grouping students who are having trouble with the same topic.
- So far, what I have done in my room is to start with the knowledge they had already known, then going to the basic skills and then moving to complicated problems. Strategies include discovery, inductive, deductive and cooperative learning.
- Graphing calculators. Think pair share. Cooperative groups.
- Learning styles and ability.
- I have tried to give different levels of problems. For example, during white board practice I will have a regular problem and a challenge problem and the students can choose which problem they want to do.
- For math class, I have used student folders to give different work (or topics) to students that are all based on the same content.

A little over twenty percent of the participants provided one or two specific strategies that they were using in the classroom. Some also conveyed key elements of differentiated instruction.

For example,

- I have used a Tic/tac/toe activity. I have grouped students and adjusted activities based on readiness level. I have implemented group share options of round robin (kagan).
- Readiness groups, flexible groups, learning centers.
- I use pre and post tests to guide my planning for instruction. I used cooperative groups to complete a project in which all members of the group were responsible for the knowledge of their group’s members. Centers, small group instruction, music.
- I have used an exit card for understanding. I use some cooperative classroom groupings such as “Think, Pair, Share”, small groups, partners, varied how to create groups based on ability or random.

Finally, a little over ten percent of the participants provided three or more specific strategies being used in the classroom.

- Mini-lessons, contracts, vocabulary quilts (utilizing words, pictures and different languages), personalized projects (similar to recipes or menus).
- Cubing, RAFT, multiple journal options, compacting, open-ended project choice.
- Tic Tac Toe, learning stations, menus for choosing own assignment, open-ended projects.
- I have implemented corners, student contracts, tiered lessons and assignments, and student choice projects.

Overall, two potential reasons for the majority of the responses lacking specificity about differentiated instruction strategies being used would be: (a) some participants may have had little knowledge about specific strategies, and (b) some participants may have had knowledge, but were not using differentiated instruction strategies.

On the post-summer institute survey, participants were asked: “What differentiated instruction strategies will you implement in your classroom this next school year?” The greatly increased number of detailed responses (about ninety percent) likely indicates that participants had gained knowledge about differentiated instruction process and product strategies or that the participants planned on using strategies that they had not used before. Some examples of responses follow:

- Think tac toe. Tiering. Anchoring. Raft. Think dots.
- Cubing, RAFT, Think Tac Toe, Multiple journal entries, Menus, Think Dots, Tiered lessons and many more!! I can't wait!!
- I definitely plan to use the menu simply because I think it will be fun for the students. Although it will take more work and planning time, I think learning contracts would work very well for my students. I have some kids that do get bored and don't need to do all of the homework so contracting could be a good option.
- Tic Tac Toe, Choices, Learning Stations, Tiered Instruction
- I will implement menus and think dots.
- I will be using a tiered lesson, I want to try think dots, and I am going to try to have learning stations that serve as anchor activities.
- I use differentiated instruction in my classroom and will continue to refine my strategies. I will give students more choices and freedom in how they learn. I will develop lessons with the blank templates for Think Dots, Think-Tac-Toe, Menus, etc.
- I will implement the DOTS strategy, journal prompts, anchor strategy, and Tic Tac Toe.
- I love the menu ideas, think-tac-toe, etc. I think kids would see these more as fun than anything and be excited to do them.
- Learning profiles, think-tac-toes, cubing, tiered lesson, think dots, menus
- I would like to implement anchor activities, learning stations, and think-tac-toes in my classroom.

On the other hand, two responses did not offer any details about strategies that would be implemented and one response seemed focused on ability grouping (instead of flexible grouping).

- I will still be teaching special education next year in a class within a class model. I feel I need more time to study the strategies before I decide for sure.
- Not for sure yet. The reading will help.
- Grouping students based on own ability. Lesson plans and examples that are divided into catering for the ability of a group of students. Identifying students who need more attention and giving them work that is commensurate with their abilities and later expect all to achieve the same objective.

Overall the difference in the quality of the responses between the pre- (more generic) and the post-summer institute (more specific) regarding differentiated instruction strategies suggests some participants were learning new process and product strategies for differentiated instruction.

Pedagogy session reflection comments also provide some evidence that participants were learning differentiated instruction strategies. For example, on a day when two (elementary) lead teachers and two (one middle and one secondary) lead teachers modeled and described differentiated instruction strategies that they had used for their grade levels in mathematics, several participants reflected upon the learning experience.

- I liked the DI ideas we got. There were several useful examples. I just wish there was a whole book of Think-Tac-Toes or RAFTs or Think Dots. But I don't think such a thing exists!
- I am very confused about what tiering is. Differentiated instruction is becoming clearer. The terminology of the pedagogy boggles my mind at times—I know how to be a good teacher, but struggle putting it on paper. The work time was awesome.
- I liked getting lots of ideas for differentiated instruction. The menus were new to me, and I hope to use one in my classroom. It was nice to see different ways to do the same strategy.
- Loved the ideas again this afternoon! [The lead teachers] have done a great job giving us activities and time to discuss.
- I am looking forward to make and take on Wednesday. It will allow me to practice and make sure I understand correctly.
- I loved the ideas for DI—I think most are very doable.
- Differentiation with varying degrees of learning styles sounds wonderful. Just need to work on activities.
- The differentiated instruction strategies and activities were helpful. I'm looking forward to adapting them to my needs for my students.

Several participants reflected upon their learning after the next day's make and take session.

- Great hands-on activities for differentiation. Getting to do the activities makes it much easier to see how we can/should create them.
- Continued practice with activities. WOW, there are a lot!
- I appreciated the opportunity to create differentiated activities during the class. It's always hard to find time to do these things during the school year. I will use these activities in my teaching position.
- The DI activities were long, but I think beneficial.
- I really liked the RAFT activity. Good idea of setting up stations to learn a lot about each type. Better than just talking about them.
- It was great to have time to work with colleagues on developing DI activities.
- This was awesome! It was lots of fun to collaborate with others in my grade and to create exciting DI activities. I am looking forward to receiving copies of others' work.

Therefore, comments on reflections suggest some participants were learning new strategies for differentiated instruction.

The differentiated instruction activities and resources that the participants made with grade level peers also provided evidence of learning. On the final day of the summer institute, a room was set up with copies of activities that groups of participants had created; some types of products that were created include: learning menus, Think-Tac-Toes, contracts, ThinkDots, RAFTs, Tiered-Lessons, anchor activities, and multiple-entry journals. Furthermore, the two mini-case participants who had attended IMP Year 1, and had thus come to Year 2 with some knowledge about differentiated instruction, commented positively about the make and take learning experience.

- I love the fact that we could work with other people, because time is so crucial and so important and you have all these projects that you want to do, and I love that we were able to take a few hours and spend time with other teachers, my peers....and having gone through the motions with somebody else...
- We really fed off of each other, and one person would come up with an idea, "oh that was really good, but what if we did this..." I think the four of us together made better DI activities than if I had sat down by myself and tried to do it....here you're getting three other opinions...

Therefore, the session where participants created differentiated instruction activities with grade level peers provided a significant active learning experience for the participants.

Self-reports by participants about what they learned in the summer institute provides additional evidence that participants learned about differentiated instruction. On the post-summer institute survey, participants were asked: "What elements that you have learned in the past two weeks will you incorporate into a lesson introducing these topics (number systems,

whole number concept & operations, rational number concepts & operations, reasoning & proof) in the coming school year?”. The question was rather open-ended. Some participants responded about specific mathematical concepts that they learned and would incorporate in their lessons. Other participants focused on pedagogical strategies that they learned. Still others commented on both mathematical content and pedagogy that they learned and would use in the classroom. Over fifty percent of the participants reported that they had learned differentiated instruction strategies that they would like to incorporate in their lessons in the coming year. Some responses identified specific differentiated instruction strategies while other responses were general. Some examples of comments follow:

- I have learned a variety of differentiated instruction strategies that I hope to implement in my classroom. I like the idea of anchor activities to give the students something to work toward when they finish early in the classroom. I feel that these strategies combined with cooperative learning will enhance the students’ learning.
- I plan to differentiate more in my classroom and use rafts and tiered lessons and assignments more in my classroom so that my “high” kids aren’t bored and my “lower” kids are getting challenged at their level.
- I will use some of the DI strategies that we learned and I will also use some of the mathematical content that we learned. I will also try to use some of the number sense activities.
- I learned a lot of different ways to differentiate instruction. I will incorporate these concepts in my lesson.
- More of the DI. I really like the raft and think tac toe and the think dots.
- I have learned several usable differentiated instruction strategies that I will be able to incorporate. Both high prep strategies and some additional lower prep strategies that I do not already use.

Furthermore, when surveyed about their knowledge regarding differentiated instruction strategies, almost seventy percent of the participants reported having little or no knowledge on the pre-survey. On the post-survey, over ninety percent of the participants reported being knowledgeable or very knowledgeable about differentiated instruction strategies.

Finally, participants’ identification of differentiated instruction strategies that would be used (written action plan) or were used while implementing the action plan and lesson study (as presented during the share fair) provides additional evidence that participants learned about elements and strategies of differentiated instruction. Almost ninety percent of the participants described elements or strategies of differentiated instruction that would be or were implemented as part of their action plan or lesson study: some in writing on their action plan, some verbally at the share fair, and for many both.

First, an element of differentiated instruction is a focus on essential ideas. As part of their action plan, participants were asked to describe a SMART goal and desired key understanding(s). Most of the participants were able to articulate desired key understanding(s) for their action plan unit. A couple of examples of what participants wrote for desired understanding(s) follow:

- Students need to have an understanding of the vocabulary to compare measurements (greater, less than, equal to), how to estimate, be familiar with measurement tools (ruler), and be familiar with standard and nonstandard units of measure.
- Students need a deep understanding of the relationships that exist between linear, area, and volume measurements. In order to develop this sense they must have methods in place for calculating these measures and be able to make adjustments to find additional measures or describe changes in a figure.

Hence, due to the structure of the action plan template, many participants engaged in identifying essential ideas for student learning as a part of their action plan.

Next, many participants engaged in some level of integration of assessment and instruction during their implementation of the action plan. During share fair presentations, most of the participants provided data about pre- and post-assessments that were used to provide assessment evidence of student learning based on implementation of the action plan. Examples of descriptions of participant's pre- and post-assessment results are provided later in *Theme: Teachers learned about lesson study and more general skills for analyzing practice*. In addition, many participants identified several formative assessment(s) for monitoring student progress on their written action plan (e.g., warm-ups, daily observations, quizzes, exit slips, homework, journals, centers). Furthermore, a couple of participants described using assessment tool(s) in order to determine needs, and possibly adjust instruction, for different students.

- I did a lot of assessment and the data analysis to try to figure out where the students were at.
[Later in the presentation:] I actually looked at the data and actually paired up the students to do the assignment based on someone who understood the concept well with someone who was struggling with the concept ... Best way to learn was to teach someone else.
- Did data driven small group instruction.

Hence, by integrating assessment and instruction during implementation of the action plan, some participants evidenced employing another element of differentiated instruction.

Most of the participants described (either in their written action plans or their oral presentations) utilizing differentiated instruction strategies to offer choice and/or to attend to individual students' learning profile, readiness, and interests. Participants described using a variety of strategies such as: ThinkDots, RAFT, Think-Tac-Toe, anchor activities, tiered activities, flexible grouping, stations, and cooperative learning. Hence, by incorporating some elements of differentiated instruction in their action plan or lesson study, some participants may have actively learned about differentiating instruction in a mathematics classroom.

Overall, participants learned about key elements of differentiated instruction and specific process and product strategies. Participants came to IMP Year 2 with varied understanding of differentiated instruction. Nonetheless, whether they had little prior knowledge or substantial prior knowledge, most participants increased their knowledge of differentiated instruction. However, understanding of differentiated instruction may have been deficient for a small number of participants who primarily associated differentiated instruction with ability grouping.

Theme: Teachers learned about strategies for supporting students' reading in the content area

- | |
|--|
| <p>Themes Organized by Research Questions</p> <ul style="list-style-type: none"> • Teachers learned mathematical content <ul style="list-style-type: none"> ◦ Knowledge “about” mathematics ◦ Substantive knowledge of mathematics ◦ Curricular knowledge ◦ Pedagogical content knowledge • Teachers gained pedagogical knowledge <ul style="list-style-type: none"> ◦ Differentiating instruction ◦ <i>Supporting reading in the content area</i> ◦ Fostering number sense ◦ Skills for analyzing practice ◦ Implementing standards-based instruction • Impact on teaching practice <ul style="list-style-type: none"> ◦ Short-term impact on teaching practice |
|--|

Supporting students' reading in the content area was one of the intended areas of focus for IMP Year 2. As reading *The Number Devil* was integral to the content sessions and as pedagogy facilitators sought to share research-based strategies for supporting content area reading, collaboration between content and pedagogy facilitators was critical. Learning opportunities about content

area reading arose as content in *The Number Devil* provided an organization for mathematics concepts studied in the content sessions, as pedagogy facilitators shared research-based strategies, as pedagogy and content facilitators modeled content area reading strategies during the summer institute, and as some participants implemented content area reading strategies during their content presentation, lesson study, or action plan.

Reading *The Number Devil* during the summer institute provided many opportunities for modeling content area reading strategies. Participants were encouraged to read strategically and reflect upon the mathematical content of the readings in a variety of ways. For example, participants were often provided with an advance question to focus their attention during the reading of a chapter. Participants were also encouraged to self-assess and to write about any questions they had over the reading. For several chapters, graphic organizers were provided for processing support so that participants (working alone or with partners) might have opportunities to: make connections between existing knowledge and new knowledge, focus on the big idea, and summarize what had been read and learned. Content facilitators often led discussions over readings. On one particular occasion after a homework reading assignment over Night 8, participants acted out and discussed similar scenarios in small groups to enhance understanding. Thus, various reading in the content area strategies were modeled as participants engaged in learning mathematics while reading math-related literature.

Pedagogy facilitators also played a significant role in encouraging understanding of reading in the content area. The facilitators pointed out that “reading” does not necessarily imply comprehension. The facilitators shared information about Billmeyer and Barton’s (1998) three interactive elements of reading and about choosing specific pre-reading, during-reading, and post-reading activities to support the development of strategic readers (e.g., Beers & Howell, 2005). A facilitator expressed the project would strive to expose participants to a variety of strategies. The pedagogy facilitators also modeled some reading strategies (preview of key vocabulary on wall, jigsaw with summarization, graphic organizers) as participants read articles concerning lesson study and number sense.

Discussions and reflections about the use of content area reading strategies demonstrated that participants were evaluating what they thought were more or less effective. A round robin reading activity during a content session on the first day was deemed less effective by many participants (and also spurred better coordination between pedagogy and content facilitators). Several participants provided positive feedback about later reading (individual and partner) and discussion activities during content sessions. Some examples of reflection comments follow:

- I enjoyed partner reading/discussing before the group discussion.
- Loved that I got to read with a partner instead of whole group.
- I loved how we broke into groups and read by ourselves and then discussed. Then we taught the other groups.

- Loved reading in small groups and meeting up with another group to discuss.
- I definitely need the class discussions we have to help me process the information we read.
- I feel lucky that I have people to talk to about the reading. I need that discussion time to process and think more deeply about the text.
- I enjoyed the reading and the graphic organizer... I am a “do it myself” person, and then “discuss as a class” person.

On another occasion, some participants reflected about a specific graphic organizer assigned for the first reading assignment. Participants were asked to reflect and fill out a Venn diagram identifying prior knowledge versus new knowledge gained from reading Nights 2 and 3 in *The Number Devil*. Participant reflections revealed mixed reviews of the strategy. Some examples of reflection comments follow:

- I had a little trouble using the Venn diagram. I had to keep going back and rereading the material to think of what to put in each section. That type of graphic organizer doesn't really fit my learning style. Outline format would be better for me.
- I really did not like the Venn diagram graphic organizer for the homework. I found it difficult to really put down any thoughts.
- I like the graphic organizer; it's making me think more deeply about my reading.
- The Venn diagram was a good exercise to complete to keep me focused on important concepts while reading.
- For homework, we read Chapters 2 and 3 and answered a couple of questions on a Venn diagram. I enjoy learning strategies I can use in my classroom, such as the Venn diagram. However, the explanation of what we should do on it was unclear and I was unable to decide how I could ever use it in my classroom.

Participants also gave mixed reviews about the jigsaw reading activity during a pedagogy session.

- Jigsawing was a good way for some people to get information. The review is important and also the think time.
- The Jigsaw activity helped generate some class discussion.
- As much as I am not a fan of Jigsaw-type grouping, I did force myself to keep my attention on the activity at hand. The articles are high interest, low workload, and the summaries are clear and well-presented. Now, that being said, what do I use at my level to support student success?
- I don't really get much out of Jigsaw. I would rather read it on my own, but that is how I learn.

In another instance, after a reading assignment over a chapter from *The Number Devil* regarding permutations and combinations, participants acted out mathematical situations that were similar to the book scenarios and answered mathematical questions. Many participants appreciated

being able to check their understanding of the concepts in the reading. In addition, some participants expressed gaining a better understanding of the concepts as they had struggled to make connections from the reading alone. Some examples follow:

- Enjoyed the format for Chapter 8 discussion (groups, acting out, drawing); our table worked well together and got all of the problems done.
- Opening session on Monday was very beneficial to explain and work with the information from Night 8.
- I liked having the chance to act out the situations in the handout from *The Number Devil* in order to find combinations and permutations. The concrete examples were helpful in finding patterns and understanding concepts.
- Acting out the problem helped to understand how and why the formulas work.

However, one participant described that the learning experience had not been as positive for her.

- This morning's session of small group exploration seemed relevant to groups around me, but when I attempted to speak or question the tasks, only one person in our group spoke; the rest just nodded in agreement now and then.

Thus, some participants used the varied experiences with content area reading strategies to reflect upon what strategies they considered the most effective and how the strategies might relate to their own learning preferences.

Discussions and homework also revealed that participants' experiences with math-related literature and non-standard language in *The Number Devil* served as a backdrop for reflection about student engagement with math-related literature and the importance of prerequisite knowledge and vocabulary knowledge for reading comprehension. Reflections from reading the first chapter on the first day of the summer institute revealed many participants enjoyed the reading and appreciated the way mathematical concepts were introduced while a few participants expressed concerns about the non-standard language. Some of the comments follow:

- The book is a very easy read. I enjoy reading the content and learning new and/or different ways to teach a concept.
- I enjoyed reading the book in the morning session. I think the book will be a great addition to this class. We had a great discussion as a group while reading.
- I enjoyed reading this book rather than a textbook. I like that we get to discuss the different interpretations and deciding if the Number Devil teaches in the creative and reasonable way.
- I have been looking forward to finding reading material I can use in my classroom. I love the way it introduces mathematical concepts in a story form. This book makes math more interesting and less cumbersome.
- So far the book is interesting and fast to read. I've picked up a few things I could use with my students.

- I really enjoyed the reading. I think the book is going to be really enjoyable and possibly something I could use in my classroom.
- I am enjoying reading the book. The characters and humor make it an easy read.
- The text was written as a story that is clear and understandable; some answers to Robert's questions posed to the devil were sometimes not clear on first reading.
- I enjoyed the reading; it is a very quick/easy read. I wish the author would have included the actual names alongside with his "funny" names. I never found prime numbers or composite numbers. I would like for kids to see that they relate—most kids will make connections, but some will not.
- The reading was hard for me. I was able to understand what the Number Devil was explaining because I know the mathematics. However, I am reading this book as if I were teaching it to my 6th graders and I am not able to decide if my students will be able to get it and for me to be able to have a discussion to make them believe.
- The reading was not something I was used to. I wanted to read it like a storybook, but the "math" got in the way. It may take a while for me to get used to it because I am not fluent with the math content.

As more of the book was examined, other opportunities arose for contemplation about non-standard language. For example, (as earlier discussed in the theme regarding curricular knowledge), some elementary teachers expressed concern about the use of the term "hopping" for exponentiation in Night 2 as "hopping" was often associated with skip counting in earlier grades. Later on, participants were challenged to pull meaning from reading Night 5 as a content facilitator posed a focusing question: "(Night 5) Compare 'quadrangle numbers' (as presented in this section) with 'square numbers' (presented in previous sections)." Many participants correctly surmised that quadrangle numbers and square numbers were the same. However, several participants expressed they were unsure or they did not explicitly say the names represented the same concept or they answered incorrectly.

- Square numbers and quadrangle numbers are multiplied by themselves. So in a sense if a number is a square number it is also a quadrangle number (?)
- Is a square number just any number "squared" and a quadrangle number the visualization of the squared number using actual squares? They seem to be the same.
- Quadrangle numbers and square numbers both "hop". The quadrangle numbers continue creating square numbers when you add the next odd number within the lines or outside edge of the square. [Later in a question about the reading the participant asked, "What exactly is the difference between square numbers and quadrangle numbers?"]
- Square numbers: 1, 4, 9, 16, 25, 36, 49, ...
Quadrangle numbers: 2, 6, 12, 20, 30, 42, ...
- Quadrangle numbers – 2 different triangular numbers added results in a quadrangle number. But square numbers – create the physical shape of a square with the number.

In another instance, a few teachers who lacked familiarity with the concept of factorial struggled when trying to pull meaning from Night 8 regarding the non-standard term “vroom” and the standard symbol “!”. Some of the questions the participants had about the reading illustrated they recognized they were missing a connection:

- How do you say 4!? In math what words stands for (!)?
- What’s the real name for vroom?
- What is the real number of !, $4!=24$? Why does the author consistently replace math vocabulary with other words?

During a content session, a participant asked why the author had used “vroom” for factorial. The content facilitator suggested that vroom was chosen to reflect that the sequence $1!=1$, $2!=2$, $3!=6$, $4!=24$, $5!=120$, $6!=720$, $7!=5040$, ... “gets really big really fast”. The facilitator further suggested that it was open to interpretation as to whether the choice of non-standard language was effective or not. As a final example, some participants provided mixed reactions about whether the rabbit story illustration about Fibonacci numbers in Night 6 was effective.

- I was very interested and enjoyed the rabbit story of explaining “Bonacci” numbers. The pictures with the text really helped. I also liked that the book offered a chart as an additional way to show how the numbers worked.
- I enjoyed reading and discovering about Fibonacci numbers.
- I did like the bunny problem with the “Bonacci numbers”. Great way to explain it to kids.
- The rabbit problem tied to Fibonacci was interesting but difficult to be conveyed.
- I was unclear about the rabbit example with Fibonacci.
- The Night 6 reading assignment on Bonacci numbers was confusing and I definitely will need the discussion to help me understand it.
- I enjoyed reading the book, but the rabbits comparison to the Fibonacci numbers was very confusing.
- I was a little confused on the Bonacci numbers—chapter was a little hard to understand.

Some participants benefitted from additional support provided by content facilitators for understanding the Fibonacci sequence. Overall, the participants experienced the use of math-related literature for introducing mathematics concepts and some participants reflected about their experiences with the literature and non-standard language. The experiences provided participants with a context to experience the engagement potential of math-related literature and to observe the importance of prerequisite knowledge and vocabulary knowledge for reading comprehension.

Further evidence reveals that many participants learned about content area reading strategies and valued the engagement potential of math-related literature. On the pre-summer institute survey, thirteen participants (about forty percent) reported having little knowledge about reading in the content area. On the post-survey, all but one participant reported being knowledgeable or very knowledgeable about reading in the content area. On an open-ended post-summer institute survey question participants were asked what elements they had **learned** about in the summer institute that they would be incorporating in their lessons to introduce number concepts or reasoning and proof. Three participants identified graphic organizers and three other participants recognized *The Number Devil* or math-related literature. One wrote: “Integrating interesting literature that introduces the concept in a fun and engaging way”. On another post-survey question, participants were asked what strategies they would use to teach number concepts and reasoning and proof (without expressly being limited to concepts they had learned during the institute). Nine participants identified that they would be using content area reading strategies or graphic organizers. Two comments further reveal IMP Year 2’s impact. One participant wrote: “I will incorporate graphic organizers as I see the support this provides for organizing understanding in the content area”. Another wrote that he or she would be “looking for different reading resources to focus more on reading in the content area”. Furthermore, of the twenty seven participants who reported on their action plan (and perhaps lesson study) at the share fair, almost seventy-five percent indicated they used math-related literature (e.g., *The Number Devil*, *I Know an Old Lady Who Swallowed a Fly*, *Inch by Inch*, *Grandfather Tang’s Story*) and/or content area reading strategies (e.g., graphic organizer, vocabulary builder) as part of their action plan. An observation of a mini-case participant revealed the participant had a metric conversion graphic organizer on the wall (developed with the help of a pedagogy facilitator). During the lesson, the researcher observed one student pointing to the graphic organizer as he tried to explain his reasoning about a metric conversion problem to another student. Hence, overall many participants learned about content area reading strategies and the engagement potential of math-related literature. In addition, evidence suggests teaching practice may have been impacted as well for some participants.

Theme: Teachers learned pedagogical strategies for supporting the development of number sense

Themes Organized by Research Questions

- Teachers learned mathematical content
 - Knowledge “about” mathematics
 - Substantive knowledge of mathematics
 - Curricular knowledge
 - Pedagogical content knowledge
- Teachers gained pedagogical knowledge
 - Differentiating instruction
 - Supporting reading in the content area
 - ***Fostering number sense***
 - Skills for analyzing practice
 - Implementing standards-based instruction
- Impact on teaching practice
 - Short-term impact on teaching practice

IMP Year 2 afforded participants with opportunities for learning pedagogical strategies that support the development of number sense. At the beginning of the summer institute pedagogy facilitators initiated interest in defining and recognizing number sense. For example, Pedagogy Facilitator A guided a Think-Pair-Write activity whereby participants

thought about and recorded their personal definition of number sense, shared and compared with their elbow partner, and revised definitions based on discussion. Later small groups formed to discuss definitions of number sense. Groups were asked to write definitions on small posters. Some examples of phrases from the group definitions are: “understanding of the characteristics of a number and its relationships to other numbers and how they are used” and “understanding of numbers and their values and how they work and how they relate to each other”. The facilitator pointed out “understanding” and “relationships” were recurring themes for the group definitions of number sense. In addition, the facilitator commented that although it is hard to find a good definition of number sense, several of the group definitions were very good. To follow up, the facilitators shared some definitions and characteristics of number sense as posited by researchers and educators. For example, a definition appreciated by Van de Walle (2004), describes number sense “as a good intuition about numbers and their relationships. It develops gradually as a result of exploring numbers, visualizing them in a variety of contexts, and relating them in ways that are not limited by traditional algorithms” (Howden, 1989, p. 11). With regard to characteristics, a facilitator noted that several activities throughout the summer institute would be coming from resources by McIntosh, Reys, and Reys (e.g., 1997a, 1997b) that outlined major components of number sense as: (a) mental computation, (b) estimation, (c) relative size

(including benchmarks), (d) multiple representation, (e) number relationships, and (f) reasonableness.

In addition to being able to define and recognize number sense, Pedagogy Facilitator A suggested it would be important for teachers to be able to choose activities that could help students develop number sense. Throughout the summer institute lead teachers modeled grade appropriate activities from a variety of curricular resources (e.g., *Teaching Student-Centered Mathematics: Grades K-3* (Van de Walle, J. A. & Lovin, 2006b), *Teaching Student-Centered Mathematics: Grades 3-5* (Van de Walle, J. A. & Lovin, 2006a), *Number Sense: Simple Effective Number Sense Experiences/ Grades 1-2* (McIntosh, Reys, & Reys, 1997a), *Number Sense: Simple Effective Number Sense Experiences/ Grade 6-8* (McIntosh, Reys, & Reys, 1997b), *Explain It! Answering Extended-Response Math Problems (Grades 7-8)* (Creative Publications, 2001), *Roads to Reasoning: Developing Thinking Skills Through Problem Solving (Grade 1)* (Weinberg et al., 2002), *Family Math: The Middle School Years* (V. Thompson & Mayfield-Ingram, 1998)). Also during one pedagogy session, Pedagogy Facilitator A shared some strategies for encouraging mental math (e.g., guiding students to share strategies; motivating students to mentally figure without using paper, pencil or calculator) and some curricular resources (e.g., Hope et al., 1987).

Mini-case interviews, survey comments, and share fair reflections reveal that some participants learned pedagogical strategies for supporting the development of number sense and some participants came to place a higher value on the importance of helping students develop number sense. For instance, when asked if they had gained any knowledge of pedagogy, three of the four mini-case participants said they learned some number sense strategies during the summer institute.

- Mental math, maybe teaching kids how to have a number sense, or showing them ways that they can gain it.
- The [number sense activity] packages were great.... I felt that I could take them and use them in my room.
- Just given more or different ways to look at the same things we already do, like taking a different spin on things, like the number sense things.

Post-summer institute survey comments from the full group of participants provide evidence as well. For example, one survey question asked: “What topics will you emphasize in number theory and algebra this school year?” Over forty percent of the participants named number sense

or a characteristic of number sense (e.g., mental math, estimation) as one of the topics they would be emphasizing the coming year. Another question referring to number and reasoning and proof concepts asked: “What strategies will you use to teach these concepts this next school year?” Almost thirty percent of the participants mentioned an aspect of number sense. Clips of some of the participants’ comments regarding number sense that provided more detail follow:

- I hope to incorporate more mental math. I have used it some, but have not given it as much time as I should.
- Teach the students to look at their answers and determine if they are reasonable. I will concentrate on estimation and rational numbers. The students will need to make personal benchmarks.
- I will use an interactive number line for kids to practice their number sense.
- Roads to Reasoning, and number sense activities; always checking for an answer’s reasonableness.

On another survey question, participants were asked: “What elements that you have learned in the past two weeks will you incorporate into a lesson introducing these topics in the coming school year?” About twenty-five percent of the participants noted they had learned about ideas related to number sense (e.g., mental math, estimation, number sense). A few examples follow:

- I will teach a strong understanding of number sense.
- I will also try to use some of the number sense activities.
- I will also try to incorporate more number sense activities and mental math strategies daily or at least weekly.
- I have learned many activities dealing with patterning and number sense.

Hence, survey responses reveal that some participants learned new strategies for fostering number sense and some participants came to recognize the importance of their role in helping students develop number sense.

Share fair comments and/or action plan statements also provide evidence that many participants used number sense strategies or addressed a component of number sense during their action plan and/or lesson study. Eleven of sixteen groups/individuals reporting at the share fair directly described an issue related to supporting number sense. Four other groups/individuals may have alluded to an issue related to number sense. One individual did not comment about number sense as she had focused on other pedagogical ideas. Five other individuals did not report at the share fair. Many of the action plans addressed measurement (geometry) as the weakest standard. Measurement lessons offered many natural opportunities for teachers to engage students in making estimations, using benchmarks, and checking for reasonableness.

Several other action plans addressed other standards (e.g., number) which also offered opportunities for comparing numbers, recognizing number relationships, using multiple representations of numbers, and using benchmarks. Comments taken from some of the group share fair presentations follow:

- And then we had a discussion of reasonable answers.
- We chose a lesson from that Number Sense book that we all got...“*How Long Is a Handful?*”
- I took a lesson out of number sense and then asked them how could you do this ...lesson at their grade level to develop number sense.
- They first had to decide ounces or pounds and then they had to estimate how many...
- ...have the kids choose an appropriate measurement and estimate.
- I said pick two numbers out of math and compare them...
- During our first lesson what we noticed was that some of the students were making guesses that were too large...suggestion that we talk about a “reasonable guess” and use that as a modification for our second lesson so we tried a comparison...
- ...after they recorded, they estimated how big it was...
- We would definitely combine percents to the teaching of probability to help out with number sense because there are a lot of kids who we found didn’t understand that a fraction could be related to a percent...

Hence, comments reveal that many participants attended to issues related to number sense in their action plan. Overall, while many participants already knew some strategies for fostering number sense, some participants learned more strategies for supporting the development of number sense during the summer institute. In addition, some participants came to recognize the importance of their role in helping students develop number sense and thus became more deliberate in using strategies to foster number sense.

Theme: Teachers learned about lesson study and more general skills for analyzing practice

Themes Organized by Research Questions

- Teachers learned mathematical content
 - Knowledge “about” mathematics
 - Substantive knowledge of mathematics
 - Curricular knowledge
 - Pedagogical content knowledge
- Teachers gained pedagogical knowledge
 - Differentiating instruction
 - Supporting reading in the content area
 - Fostering number sense
 - ***Skills for analyzing practice***
 - Implementing standards-based instruction
- Impact on teaching practice
 - Short-term impact on teaching practice

Hiebert et al. (2007)

suggested teachers need preparation that develops hypotheses-testing skills (setting goals for students, assessing whether goals are met, hypothesizing whether a lesson did or did not work well, and using hypotheses to revise lessons) in order that teachers would be equipped to analyze and learn from the practice of teaching. Several Infinite

Mathematics Project tasks supported teachers in developing a mindset of inquiry into practice.

During IMP Year 1, the Japanese lesson study cycle was modeled for participants. After two lead teachers developed a lesson, IMP participants observed the lesson being taught by one of the lead teachers (to a group of volunteer children). Participants filled out a personal reflection about the lesson for which they were encouraged to provide positive constructive feedback by focusing on the learners and how the learners responded to the lesson. Participants also observed the debriefing for the lesson where the lead teachers and outside observers shared their observations about what worked well and what could be improved. Subsequently, the lead teachers revised the lesson. Then, IMP participants observed the lesson being taught by the second lead teacher (with a new group of children) and the subsequent debriefing. Furthermore, during the school year, IMP Year 1 participants implemented a lesson study. IMP Year 1 participants also experienced data analysis and goal setting through their development of an action plan addressing their school’s weakest state assessment indicator for a particular grade level. Hence, IMP Year 2 participants who had attended IMP Year 1 already had some knowledge and experiences regarding lesson study, and more generally inquiry into practice. On the other hand, about half of the IMP Year 2 participants had not participated in Year 1. In addition, participants had other differences in prior learning experiences regarding inquiry into practice (e.g., years of teaching experience, other professional development experiences, undergraduate education). Hence, participants came to IMP Year 2 with various understandings

of lesson study and various experiences with inquiry into practice. During Year 2, pedagogy facilitators continued using lesson study and the development of an action plan as some of the pedagogical foci of the project; however, participants were guided through some different learning experiences than the previous year.

First, participants analyzed data and were assigned a task to develop an action plan to address an area of weakness. On the first day, Pedagogy Facilitator A led a data analysis activity. Participants were given district breakouts of KS mathematics assessment performance and were asked to identify the lowest and highest tested indicator along with the cognitive category (i.e., memorization definition/formula; perform procedures; demonstrate understanding; conjecture, generalize, prove; non-routine problems, make connections) for their grade level. The data was displayed as to weakness or strength, standard/indicator, and cognitive category. The group looked for patterns in the data. Next, the participants were charged to address the area of weakness in the form of an action plan (different template format from the previous year) for the upcoming school year. A brief overview was given of the action plan format. The participants were to follow the *Understanding by Design* (UbD) template (Wiggins & McTighe, 2005) which utilizes a “backward design” three stage approach to planning. Stages 1 and 2 complement Hiebert et al.’s (2007) first two skills for analysis of teaching. In Stage 1, teachers establish learning goals, write essential questions associated with the goals, and communicate what students will understand, know, and be able to do. IMP participants were directed to write their goals based on the acronym SMART (specific, measurable, attainable, results-oriented, and time-bound). In Stage 2, teachers predetermine assessment evidence that would be used to ascertain if students achieved the desired learning (related to Hiebert’s second skill of assessing whether goals are achieved). Finally, in Stage 3, teachers describe learning activities and instruction that would enable students to achieve the desired learning goals and results. Later, at various times during the summer institute, participants were allowed some time to work on the action plan. In addition, pedagogy facilitators and lead teachers made themselves available so that participants might ask specific questions regarding the action plan.

Participants also had opportunities to learn about lesson study during the summer institute. On the first day of the summer institute, Pedagogy Facilitator B shared (with a PowerPoint) a brief overview of lesson study and its impact on classroom teaching. The facilitator also expressed that participants from Year 1 had shared many great things about

pedagogy at the previous year's share fair. "Not that your content knowledge was missing, but we put so much of an emphasis on pedagogical improvement that we didn't hear a lot about content. So, I want to try to focus on that this year." After sharing some ideas from a Marilyn Burns talk about "scaffolding content", the facilitator challenged participants to think about what concepts are behind a process or idea and why concepts are taught in a particular sequence.

Later in the summer institute, pedagogy sessions provided additional opportunities for participants to learn about the lesson study process. For example, during a lunch session, participants had the opportunity to watch a video about lesson study. The video showed a Japanese lesson followed by a post lesson discussion. The video discussion included critical reflections by the teacher about the goals of the lesson and how the lesson went, and then constructive feedback by lesson observers. After the video, Pedagogy Facilitator B led IMP participants in a discussion about what was portrayed in the video.

During another session, participants learned about lesson study through a jigsaw activity. Small groups of participants reviewed and summarized a section of an article, "A Lesson is Like a Swiftly Flowing River" (C. Lewis & Tsuchida, 1998). Then, small groups presented their summaries to the full group.

The pre- and post-summer institute surveys probed participants' knowledge with respect to two aspects of lesson study. First, participants were asked to report their understanding of lesson study as a means of improving pedagogy. On the pre-survey, four participants indicated they had no knowledge and fourteen participants indicated they had little knowledge. On the post-survey, five participants indicated they had little knowledge and the other twenty-seven participants indicated they were knowledgeable or very knowledgeable about lesson study as a means of improving pedagogy. A second question probed participants' understanding of lesson study as a means of improving mathematical content knowledge. On the pre-survey, four participants reported they had no knowledge and seventeen participants reported they had little knowledge. On the post-survey, seven participants reported they had little knowledge while the remaining twenty-five participants reported they were knowledgeable or very knowledgeable about lesson study as a means of improving mathematical content knowledge.

Other significant learning opportunities regarding inquiry into practice occurred during the school year as IMP participants analyzed teaching practice by engaging in a lesson study and by carrying out an action plan. For the action plan, participants were asked to address an area of

weakness as identified by KS mathematics assessment data for their school/district. The lesson study could be a lesson within the action plan or it could be separate from the action plan.

About seventy percent (or greater) of the participants implemented a lesson study during the school year. A few participants did not implement a lesson study as they were carrying out other leadership or mentoring goals. Two participants carried out their action plan on their own and did not engage in a lesson study with other colleagues. Furthermore, a few participants did not report at the share fair; thus, others may or may not have carried out a lesson study.

Of those who carried out the lesson study, some expressed strengths and weaknesses of the experience. Some of the reflections about the strengths of lesson study include:

- What we thought were the strengths of our lesson study is that we got to collaborate, which we do very naturally anyway, but kind of a formal collaboration was different for us and we liked that. Getting positive feedback and ideas from other people....And I thought it was neat to be able to observe my own lesson....and it forced us to consider ways to deepen student content knowledge.
- The Japanese lesson study [with student teachers]....A lot of rich discussion in that. And I think you've all seen the lesson study is so non-threatening because they were correcting each other, "Well I don't know if I would write the formula up, I think I'd rather discover that..."
- The lesson study really is my favorite part...the whole team really does like the lesson study.
- What we loved about the lesson study process, just reflecting as a group: working with colleagues, it's a non-threatening environment, creating a lesson, getting input from other teachers and using each others ideas and seeing your ideas and their ideas in action; new ideas from other teachers that will help students get the concept better, getting that outside input...
- We really liked it, we liked the fact that we were able to work together...
- Because we had brand new teachers as our partners, we were mentors for them, so we encouraged them to participate in our lesson study. And I think that was really helpful to them...It was a really wonderful way to help them out.
- They did it [lesson study] with me last year and they really wanted to do it again.... After we did it last year...they said it's like the best form of collaboration ever—because you really have something to talk about and change...
- We're able to see and learn from the other teachers in the building. We're allowed the opportunity to collaborate and make a better lesson especially with the input from other colleagues outside the lesson study like [a Pedagogy Facilitator].
- Enjoyed going to other classrooms.... Debriefing—I enjoyed the time to discuss, "hey, what are you trying", "I don't understand this", "what can I say better, do better"?

Hence, many of the participants valued the lesson study process for its formal collaboration with the focus on jointly making a lesson better. On the other hand, participants suggested the time it takes to implement a lesson study and the trouble with getting substitutes are major hindrances.

- It's time consuming.
- Time to get together was hard.... nervous about presenting in front of other people...
- The weaknesses we thought were vague guidelines, not enough time to plan..., and getting subs in our district is not fun, so having to be out of our class too much...
- Our building does not have a vast supply of substitutes....struggling to find time to work together to get it planned and everything.

Overall, many participants found lesson study valuable for the collaborative effort to improve teaching practice. However, the considerable amount of time it takes to implement a lesson study was a serious drawback.

Participants had opportunities to experience, and possibly learn, skills associated with inquiry into practice. Many participants employed hypotheses-testing skills (setting learning goals for students; assessing whether students are meeting lesson goals; hypothesizing whether a lesson did or did not work well; and, using hypotheses to revise lessons) by their engagement in the action plan and/or lesson study. First, for the action plan, participants were to use KS mathematics assessment data for their district/school to identify the lowest indicator. Then, participants were to develop a plan to address the weakness. Thus, participants were to use data to establish learning goals and the goals were to be specific, measureable, attainable, results-oriented, and time-bound. Participants were to establish a plan for being able to assess whether goals were met. Action plan and share fair data reveal that many participants engaged in setting learning goals and critically assessing whether students met the learning goals. Some examples from written action plans include:

- **Standard, Benchmark & Indicator:** Geometry 3.2.A1

S.M.A.R.T. Goal: By April 2009 85% of the Kindergarten students will score proficient or above on a teacher-created rubric by demonstrating their understanding of the length of concrete objects using whole number approximations through the use of non-standard units of measure.

- **Standard, Benchmark & Indicator:** The student compares and orders: integers; fractions greater than zero, decimals greater than or equal to zero through thousandths place.

S.M.A.R.T. Goal: The student will be able to compare and order integers, fractions greater than or equal to zero through the thousandths place on post-test with 80% accuracy by 90% of the class.

• **Standard, Benchmark & Indicator:** 1.1.K2: Comparing and Ordering Fractions, Decimals, and Integers

S.M.A.R.T. Goal: By the end of the unit, 70% of students will achieve 70% or higher on the unit test. By the end of March, 70% of the students will achieve 70% or higher on this specific indicator of the state assessment.

• **Standard, Benchmark & Indicator:** 3.2.A2 Estimates to check whether or not measurement and calculations for length, width, height, volume, temperature, time and perimeter in real-world problems are reasonable.

S.M.A.R.T. Goal: Students will be able to estimate to check if measurements are reasonable to 80% accuracy by February...

Some examples follow from participants' (different participants than those linked to the previous written action plan goals) presentations at the share fair which includes both learning goals and results about whether the goals were met.

• Last year, we were looking through our data and this has been kind of a trend all along, the lowest state tested indicator is 4.1.K.3 which is compound probability which we have found is very difficult for 8th graders so we decided that needed to be our action plan goal for this year. Our goal was that 70% of our students would make the standard and at 8th grade the standard is 58%. We wanted to look at that standard and say we have 70% of our kids making 58% on that standard...

[Later in the talk the assessment results were given.] We just took questions from the Kansas Test Builder site and used those for our pre-assessment, and then our post-assessment was the Kansas State Math Assessment.... Here's data from my class and for the pre-assessment that we gave in August, 48% of all my students met the standard on this. For the state test [post-assessment], I was at 68% of my students. So I saw a big jump...

• Measurement and estimation as our lowest standard again [on action plan: 3rd Grade: 3.2.A1; 4th Grade: 3.2.A2]... These are our graphs from last year 2008; this is how we chose which ones. These are the 3rd graders from last year [61% lowest indicator on graph]...; and these are the 4th grade [45% lowest indicator on graph]... Clearly our lowest standard and they have been since I've been there.

[Later in the talk:] For pre- and post-test this year, we just used, it's required in our district, that we give the formative assessment using Test Builder... We have give that three times a year, so we thought we'd use that data. From the September formative, and then pulling out the data from those standards... For 3rd grade last year they started at around 12-13% on that standard ... and went up to 60%. This year they started out just a little bit higher, closer to 16 or 17% and went all the way up to 85-86%. 3rd graders really rocked... They only had one kid who did not make it for math... For 4th grade... last year: [32% to 42% on slide]. Again we started lower but made lots of gains this year

[25% to 52% on slide]. All of our lesson study and collaboration helped that. We made sure to focus every week on a center that was measurement...

- Our standard that we were working on was transformational geometry which sounds fancy. But what that means is if you had a rectangle and you were giving it scale factors, could you predict what a new rectangle's perimeter or area or volume... would be based on that scale factor.... Our original goal-that we wanted **all** of our students to meet standard for 3.3.A.1. We finally realized that **all** students meeting standard on something that was by far the lowest standard was a little lofty...adjust "to make significant progress"...

[Later in the talk each member of the group presented assessment results regarding their own students. One of the participants said:] On the KMA, we gave a practice assessment...they didn't do very well on that standard [3/22 scored at "meets standards" on slide] and that was before we had any sort of practice with it. Then actually on the KMA [Post 11/22 meets standard or higher on slide]... It's not advanced geometry so its half sophomores and half juniors so some are a little bit behind on some of their math skills. So relatively I guess that's something to be pretty happy about; 50% met standard when beforehand [only 3 met standard]...

- My lesson study was over converting within metric system... My goal is for students to score proficient or above on questions aligned with the 6th grade math standard 3.2.K.3b...

[Later in the talk:] My pre- and post-tests were exactly the same, an adaptation of ...; we wrote the questions exactly like they would have been written on the state assessment except for we did not give them multiple choice answers.... They learned a lot from pre- to post-test [for example, per the slide one class scored 12 on the pre and 80 on the post; another scored 24 on the pre and 87 on the post].... And my students were not only successful on my post-test but also on the KS Math State Assessment. I did not include the exact score because I had 27 students who were not here for this unit... We were somewhere around 80% on the KS State Assessment...

- I had chosen measurement and we specifically wanted to look at length...that's our low area... [Lower elementary grade]

[Later in the talk:] In my classroom ... 3 students chose the right way before and in the post test 6 students chose the correct way...

The above examples represent about sixty percent of the participants of IMP Year 2. The comments and explanations provide evidence of participants writing specific learning goals to address an area of weakness for their school and grade level. The participants also assessed and communicated whether the learning goals were achieved. Furthermore, the data tends to reflect a positive impact on student learning.

The remaining forty percent will be discussed. Five participants had action plans more focused on providing leadership or mentoring, and hence goals were structured differently. Although three of the five had clearly stated learning goals and provided some assessment data, the structure and explanations were different enough to not be grouped with the others. Two participants submitted an action plan with clearly defined learning goals; however the participants were not able to attend the share fair and hence no assessment data was reported. One participant who submitted an action plan (but did not attend the share fair) did not evidence clear goal setting. One participant from the share fair (for whom the researcher did not have access to a written action plan) did not have learning goals associated with a weak indicator. Three participants did not submit action plans (as far as the researcher was aware) nor attend the share fair.

A few participants reflected about the process of analyzing data to determine whether learning goals were met. Their share fair comments follow:

- We think because of doing IMP and coming here every summer that we really focus more on the math data. We have a half-time math coach this year which has helped, but otherwise we've been on our own for that. We have a huge focus on reading in our building so we have to look at that data; every week we're looking at reading data, but nobody was helping with the math. But because of coming here we kind of put that on ourselves...
- Japanese lesson study continues to be one of the most powerful pieces that we can do to implement—to improve our instruction and to improve student learning. Taking time in analyzing data...you can never assume anything, even when you are looking at data, you have to sit down with students and talk and get to know what are they thinking, but more importantly why are they thinking this...
- I also learned much more about data analysis and how to use it to drive instruction.

Hence, a few reflections reveal that some participants were engaging in more data analysis than previously typical as to whether learning goals were met.

Several participants also evidenced other hypotheses-testing skills during their share fair reflections about their lesson study. That is, some participants described being engaged in discussions about whether a lesson did or did not work well, and then using hypotheses to revise and teach the lesson in another classroom. Some examples follow:

- In the first lesson [teacher A] taught; she did a pretty good job but we ran out of time. There wasn't a lot of time at the end for what we were trying to do. They were a little bit confused... So in the next lesson, I did a lot of modeling ahead of time, but then there

wasn't enough time for them to do their exploration.... We made changes, we thought kindergarten is going to need even more modeling and talking...

- During our first lesson what we noticed was that some of our students were making guesses that were too large...[so a lesson study member] had a suggestion that we talk about “reasonable guess” and use that as a modification for our second lesson, so we tried a comparison, we added a little comparison section...
- The second lesson, we did make a few changes. Overall, our first lesson, it went pretty good...one of the biggest pieces of feedback came from [an intern] and they were like—when you do this put a basketball on the table, put a tennis ball there so they can physically compare it. We just didn't even think about doing that. We kind of thought they're going to play with that, that's not going to work. It really did work.
- That's also one thing that we changed on our prep activity... Before our first lesson we just had a conversion problem... So then we changed it. For the second and third lesson we had a number line where we just had the benchmarks 0, $\frac{1}{2}$, and 1; they were given a fraction, decimal, percent as they came in and they just put it about where it would go. The one thing I found out...they would record the card they were given and then they would estimate fine as a group where that would go, but then as they converted, sometimes their conversions weren't correct, and they didn't get it that it wasn't correct just by going off their estimation...
- Overall, we felt that our lesson went well and that the students enjoyed it... At the end, I felt like I wished I could try it [the lesson] again because I knew the lesson was better the second time and the third time. You think when you write the lesson, well how could you change it; you really spend a lot of time on it and you think it is really good to begin with...but then you find all of these little things like time, things like that that do make a difference. We did make some changes...

Hence, some participants engaged in hypothesizing whether a lesson did or did not work well, and then used hypotheses to revise and teach the lesson in a different classroom.

Pre-summer institute interviews with mini-case participants revealed that the teachers came to IMP Year 2 with habits of setting learning goals, often in conjunction with state standards. It was also evident that collaboration amongst colleagues was common for three of the mini-case participants. However, creating an action plan to address an area of weakness, analyzing data more formally, and engaging in more formal collaboration during implementation of a lesson study or action plan were likely outcomes of IMP. Reflection comments from a mini-case participant echo some of the previously identified strengths and weaknesses as well as further illuminate the impact of implementing a lesson study and an action plan. The participant

reflected on the lesson study experience from Year 1 (as at the time of the interview she had not yet conducted the lesson study for Year 2).

I know we took metric last year because we had to do the Japanese lesson study. We chose that we were going to spend more time on metric so I guess it forced us to spend longer teaching and I'm pretty sure that helped a great deal because metric was no longer our lowest, it was pretty much in the middle... we did very good last year in metric. So I think having to focus on our lowest indicator and we chose to stretch it out for a longer period of time—try to come up with more interesting lessons I suppose. It helped the kids' performance.

[Later in the conversation:] I wish that the lesson study was less time. I really do. I mean I think it's great! It's so much fun to see each other teach. I mean you learn from different teaching styles and even though it's the same lesson, it's good to see different kids, how they react. It's a great learning, it's just I wish there was more time for us teachers to get together.

Hence, the participant valued the formal collaboration, but considered the time requirement a considerable weakness. In addition, the participant thought the process of implementing the lesson study and an action plan to address an area of weakness had had a positive impact on student performance.

Due to insufficient pre-project and post-project teaching practice data regarding participants' disposition toward inquiry into practice, it is not clear whether participants acquired a disposition toward inquiry into practice based on their involvement with IMP. However, evidence suggests that at least some participants learned how to more critically analyze teaching through their engagement in more formal collaboration, data analysis, and hypotheses-testing during the lesson study and action plan development and implementation.

Theme: Teachers learned about standards-based instruction in mathematics

Themes Organized by Research Questions

- Teachers learned mathematical content
 - Knowledge “about” mathematics
 - Substantive knowledge of mathematics
 - Curricular knowledge
 - Pedagogical content knowledge
- Teachers gained pedagogical knowledge
 - Differentiating instruction
 - Supporting reading in the content area
 - Fostering number sense
 - Skills for analyzing practice
 - ***Implementing standards-based instruction***
- Impact on teaching practice
 - Short-term impact on teaching practice

IMP Year 2 explicitly taught strategies for differentiating instruction, supporting content area reading, fostering number sense, and participating in lesson study. However, a broader goal for the project was to increase the implementation of standards-based mathematics instruction. Standards-based teaching emphasizes addressing NCTM Content

Standards via engaging students in NCTM Process Standards: Problem Solving, Reasoning and Proof, Communication, Connections, and Representation (NCTM, 2000). However, orchestrating classrooms embodying the vision of reform is far from trivial or straight-forward. Studies (e.g., Hiebert & Stigler, 2000; J. K. Jacobs et al., 2006; Spillane, 1999) have revealed that among teachers who reported having knowledge of reform and teaching practice aligned with reform, few of the teachers actually exhibited teaching practice reflecting reform recommendations. Furthermore, Heaton (2000) suggested changing one’s teaching to reflect standards-based instruction involved a complex learning process. Hence, it is likely that participants came to IMP in many different places with regard to implementing standards-based instruction.

Some of the specific strategies for differentiating instruction, fostering number sense, supporting content area reading, and participating in a lesson study complement and support standards-based instruction. The researcher offers the following examples. Differentiating instruction by attending to students’ different learning styles and interests may necessitate using multiple Representations and making Connections. Activities for fostering number sense engage students in Reasoning, Problem Solving, making Connections, and employing multiple Representations. Supporting content area reading involves engaging students in Communication. And finally, lesson study often begins with setting learning goals by examining standards and curriculum. Theoretical connections exist as well. The *Standards* documents are based on

constructivist assumptions and best practices associated with theories of learning from a cognitive science perspective. Tomlinson (2001) identified connections between best practices and differentiation. Beers and Howell (2005) provided examples of how reading strategies can link with best practices and differentiation. In addition, number sense was described as central to Number and Operations in *PSSM*. Hence, some of the previous evidence suggesting that participants learned about strategies for differentiating instruction, fostering number sense, supporting content area reading, and participating in lesson study also provides evidence that teachers learned strategies for implementing standards-based instruction. The remaining portion of this section will be aimed at providing additional evidence that participants learned strategies for implementing standards-based instruction.

IMP Year 2 provided participants with opportunities to learn about standards-based instruction by engaging participants in the Process Standards while learning mathematical concepts considered in *The Number Devil*. Active engagement in the Process Standards was afforded by opportunities to experience learning mathematics via math-related literature along with facilitator guided activities involving reflection and discussion, critical thinking about mathematical problems, real world applications, and hands-on activities. Hence, the learning experiences provided a meaningful backdrop for participants to reflect upon the impact of standards-based instruction.

Opportunities for participants to engage in the Process Standards while learning mathematical content was afforded by experiences with good math-related literature along with IMP facilitator guidance. For example, dialogue from *The Number Devil* along with content facilitators' emphases resulted in significant attention being placed on the role of proof in mathematics. In addition, content facilitators shared some methods of proof and some examples of formal proofs with participants. (See *Theme: Teachers gained knowledge "about" mathematics*). Furthermore, the literature invited, and the content facilitators encouraged, participants to reflect critically and explain their reasoning regarding mathematical concepts and problems (see *Theme: Teachers gained substantive knowledge of mathematics*). Hence, participants engaged in Reasoning and considered the role of Proof in mathematics as the Process Standard was highlighted in *The Number Devil* and encouraged by content facilitators who challenged participants to reflect, discuss, and think critically.

Participants had opportunities to engage in the other four Process Standards as well through facilitator-led experiences with *The Number Devil*. First, participants engaged in the Representation Standard from multiple perspectives. For instance, participants had opportunities to reflect about different numeral representations (e.g., Roman numerals, base-10 system) and the advantages of modern base-10 system (see *Theme: Teachers gained knowledge “about” mathematics*). Participants also learned about mathematical concepts via multiple representations. In *The Number Devil*, Enzensberger explained mathematical concepts with number sequences, stories, drawings, and symbols. Content facilitators also supported learning by guiding participants in activities to better understand the concepts and representations, and by often sharing additional representations. For example, in Night 5 of *The Number Devil*, Enzensberger introduced triangular numbers and square numbers with a story, drawings (figurate numbers can be represented by a regular geometric pattern of equally spaced points), and a sequence. Content Facilitator A led a discussion reviewing the triangular number sequence and geometric form. In addition, the facilitator shared algebraic representations of the sequence as both recursive and direct formulas. The facilitator also guided group reflection about why the n th triangular number is useful as it represents the sum of the first n consecutive natural numbers. Furthermore, Content Facilitator B led a breakout session regarding other figurate numbers (e.g., pentagonal, oblong, cubical, tetrahedral) with multiple representations (number sequence, configuration, formula). As another example, in Night 6 Enzensberger described Fibonacci numbers with a sequence of numbers, a story about rabbits, and a picture chart. A few participants wrote in reflections that they liked the rabbit story and understood it. However, others were confused by it. A content facilitator led a variety of activities to support participants’ development of understanding about the Fibonacci sequence. The facilitator led discussions and reflections about the book representations and also shared an algebraic recursive formula representation for the Fibonacci sequence. In addition, the facilitator guided a discussion in critically thinking about other Fibonacci-like sequences. (See *Theme: Teachers gained substantive knowledge of mathematics*). As a final example, in Night 8 Enzensberger introduced the concepts of permutation and combination with story scenarios (seating, handshakes, broom-brigade), letter patterns, charts, and pictures. After reading the chapter on their own, several participants had questions regarding the reading (as illustrated by questions written with respect to the reading assignment prompt “What questions do you have as you

read?”). To support participants’ development of understanding, Content Facilitator A created questions for which participants met in small groups to act out and discuss similar scenarios as had been described in *The Number Devil*. The majority of the participants valued being able to act out and discuss the situations regarding combinations and permutations (see *Theme: Teachers learned about strategies for supporting students’ reading in the content area*). Hence, participants had many opportunities to actively engage in the Representation Standard as concepts were examined with multiple representations in *The Number Devil*, and as concepts were further developed in a supportive environment encouraging reflection, discussion, critical thinking, and application.

Next, participants engaged in Problem Solving as a variety of problems were posed throughout the summer institute as a part of *The Number Devil* readings, small group discussions, homework, quizzes, and tests. Although some problems were fairly routine applications of procedures, many other problems were less familiar. For instance, some problems were “open” in that a variety of responses were possible. Some examples follow:

- [From *The Number Devil*] Let me show you one last trick—if you haven’t dozed off, that is. It works with odd as well as even numbers. Think of a number, any number, as long as it’s bigger than five. Fifty-five, say, or twenty-seven. You can find prima-donna numbers that add up to them too, only instead of two you’ll need three. (p. 64)
- [Quiz] Give an example of an irrational number and explain why it is irrational.
- [Homework] Write down three examples of infinite sequences not given in the first part of Night 9 of *The Number Devil*.
- [Small group discussion question] *The Number Devil* avoids the question of explaining why any number to the 0 power is 1. How would you explain it?
- [Homework] Give another example of a set which is countable and give a one-to-one correspondence showing that it is countable.

As another example, a content facilitator challenged the participants with a hands-on, nonroutine problem that could result in more than one answer depending upon how things are defined. “Do the exercise given at the end of the Tenth Night, using the copy of the diagram to paste together attached. What do you get if you apply Euler’s formula?” For the 3-dimensional “donut” shape (non-simple polyhedron) in *The Number Devil*, $V - E + F \neq 2$. During a group discussion on the next day, participants were asked about their answers. Many answers were suggesting (e.g.,

0, 2, 15, 10, 1). Together the group examined the number of vertices, edges, and faces for the shape. The facilitator pointed out that two different values for edges could be justified depending on how an edge was interpreted. Therefore 0 and 10 were both valid answers. Some other problems may not have been open, but had some complexity. For example, the following homework problem regarding a recursive formula presented some challenge, particularly for participants who had not taught or taken algebra courses recently: “Write the first 10 terms of a recursive sequence given by $F_n = F_{n-1} + 2n$, $F_1 = 1$.” Furthermore, the content facilitators guided discussions where participants formulated their own questions. For example, a facilitator allowed participants to explore what would happen if the first two terms of the Fibonacci sequence were different values (see *Theme: Teachers gained substantive knowledge of mathematics*). Overall, participants engaged in Problem Solving as they were encouraged to solve nonroutine problems requiring more than repeating familiar procedures.

Participants also had many opportunities to reflect upon Connections between mathematical topics. First, Enzensberger’s book establishes many connections between mathematical topics and concepts. As just one of **many** examples, some number sequences established in earlier nights (powers of two: Night 2; even numbers, odd numbers, divisibility: Night 3; triangular numbers: Night 5; Fibonacci numbers: Night 6; natural numbers: Night 7) arise in Pascal’s triangle in Night 7 as diagonals, sums of rows, etc. Then, when combinations are examined in Night 8, it turns out each number in Pascal’s triangle represents a combination. Hence, *The Number Devil* book engaged participants in making connections. Furthermore, the content facilitators expanded upon the book and made many more connections. For example, in a breakout session, Content Facilitator A supported participants’ development in understanding the relationship between Pascal’s triangle and combinations with coefficients in binomial expansion. Also, historical connections were discussed in a previous theme (see *Theme: Teachers gained knowledge “about” mathematics*). Hence, the participants had many opportunities to reflect upon connections shared in *The Number Devil* and by content facilitators.

Finally, as participants engaged in reading math-related literature and as the Communication Standard interacts with the other Process Standards, participants were engaged in Communication. Participants engaged in Communication by reading about and making sense of mathematical concepts presented in *The Number Devil*. Furthermore, the other Process Standards interact with Communication. For instance, participants were challenged to

understand different representations and to communicate by using different representations. Participants were challenged to communicate their Reasoning during discussions and on homework and test questions (e.g., “explain”). Communication interacted with Connections via discussions about mathematical concepts from multiple perspectives. Finally, Communication and Problem Solving interacted via discussion and analysis of mathematical problems.

The Process Standard that received the most overt emphasis in *The Number Devil* and by content facilitators impacted participants as many participants expressed that they recognized the importance of Reasoning and Proof in mathematics instruction. In post-summer institute survey questions 3 & 4 probing “What strategies will you use to teach these concepts [Number, Reasoning & Proof] next school year?” and “What elements that you have learned in the past two weeks will you incorporate into a lesson introducing these topics [Number, Reasoning & Proof] in the coming school year?”, comment clips by some participants suggested they planned to incorporate more Reasoning and Proof in their teaching practice.

- I will let students work on proofs and conjectures. [Question 3]
- I think it will be important to show kids why you should PROVE your answers and look for a pattern. [Question 3]
- I plan to use some of the proof strategies that [Content Facilitator C] demonstrated during one of the breakouts. [Question 3]
- I am especially excited about talking about the proof issue. That just because you have an example it doesn't mean you have a proof. [Question 3]
- To teach mathematical reasoning, I will encourage my students to dig deeper than the surface of the concepts we are covering in order to truly understand the “why,” using prior knowledge to help lead the way. [Question 3]
- I have learned the importance of proof and reasoning. [Question 4]
- I will use proof to identify How and Why a formula works and compare it to ‘tricks’ used. [Question 4]
- I will encourage students to come to the “aha” moment on their own rather than just telling them what they need to know.” [Question 4]
- Types of proof [Question 4]

Thus, several participants expressed they wanted to include more Reasoning and Proof in their teaching practice. In addition, observations of mini-case participants may suggest that IMP Year 2 impacted some participants to emphasize Reasoning and Proof in teaching practice as the mini-case participants pushed their students (in grades 4 or 6) to explain their reasoning or to think about if a mathematical process would work every time. Some of the language is very much like discussions on the role of proof from IMP Year 2 summer institute. However, as there

is no pre-summer institute teaching observation data, it is not clear whether or not the participant used similar language emphasizing the role of proof prior to IMP.

- It worked that time. It worked on three examples, does it work every time?... if that is true for every case or not, but this is one way to prove your answer by drawing a picture. [Observation 1 mini-case participant A]
- If you and your partner get different answers you need to discuss [with your partner] and prove you answer. [Observation 1 mini-case participant B]

Hence, many participants came to a greater appreciation of the importance of engaging students in Reasoning and Proof. Furthermore, there is some evidence that valuing the importance of Reasoning and Proof may have had an impact on teaching practice.

Next, many participants expressed their appreciation of strategies used by content and pedagogy facilitators which engaged participants in learning (e.g., math-related literature, discussion and reflection, critical thinking, real world applications, multiple representations, hands-on activities). Furthermore, several participants expressed their desire to **engage** students in learning mathematics by using strategies employed by the content and pedagogy facilitators. First, the engagement potential of math-related literature was previously discussed in *Theme: Teachers learned about strategies for supporting students' reading in the content area*. Another strategy for engagement that the participants valued involved facilitator guided reflection and discussion in groups. Reflection comments from different days during the summer institute follow:

- I also enjoyed the fact that the instructors walked around and interacted and helped guide the group discussions. [Day 1]
- I did like talking about the content and concepts right away. Group members asked good questions that got us discussing. The discussion clarified concepts and made me think more deeply about the selection and what the author was wanting us to learn. [Day 1]
- The discussion and activities blended well with the book. I enjoyed working on the $11,111,111,111 \times 11,111,111,111$ problem (and extensions, patterns) with my group. [Day 1]
- I thought the questions were fitting to the content. I feel lucky that I have people to talk to about the reading. I need that discussion time to process and think more deeply about the test. [Day 2]
- I definitely need the class discussions we have to help me process the information we read. [Day 2]
- Enjoyed the format for Chapter 8 discussion (groups, acting out, drawing); our table worked well together and got all of the problems done. [Day 3]
- Jigsawing was a good way for some people to get information. The review is important and also think time. [Day 3]

- Loved reading in small groups and meeting up with another group to discuss. [Day 3]
- The most enlightening moment was when [a fellow participant] explained to me that S_{n-1} = one term back , S_{n-2} = two terms back, etc. This will help me tremendously, as I could not grasp what these terms were. [Day 4]
- Interesting discussion about size of infinity. I like the way we were shown multiple ways to understand this. [Day 4]
- Loved the ideas again this afternoon! [Pedagogy lead teachers] have done a great job giving us activities and time to discuss. [Day 4]
- The discussion about proof and the connection to why it's important for us to know about proof as math educators was a good way to end the book about Robert and the Number Devil. [Day 5]
- The collaboration time [to work on differentiated instruction activities] was great. We had good discussion and really came up with different levels of questions. It was wonderful to hear ideas that I had not thought of. Also, different ways to state the same question. [Day 5]

Some participants also expressed in reflections that they appreciated real world applications, multiple representations, and hands-on learning in content and pedagogy sessions.

- The content today was fun! I enjoy working with the formulas. [Content Facilitator A] did a great job of giving concrete examples. [Day 2]
- The content today was hands-on. I loved that we were active and encouraged to develop the sort of formula on our own. [Day 3]
- I liked having the chance to act out the situations in the handout from Number Devil in order to find combinations and permutations. The concrete examples were helpful in finding patterns and understanding concepts. [Day 3]
- Acting out the problem helped to understand how and why the formulas worked. [Day 3]
- Lots of new information. It was so much easier to understand when we acted it out. This also gave us real-life examples of how to apply this information. This makes it much easier to remember. [Day 3]
- I liked the Golden Ratio and Euler's applications—will be able to use them. [Day 4]
- The manipulatives were very helpful today for understanding. [Day 4]
- It was interesting figuring out about the Golden Ratio, and the ways it appears in nature. It was also interesting talking about how Euler's formula applies to planar graphs versus polyhedra. [Day 4]
- [Content Facilitator B's] presentation was very interesting and hands-on. [Day 4]
- Great hands-on activities for differentiation. Getting to do the activities makes it much easier to see how we can/should create them. [Day 4]
- The folding of the paper was fun. At first I was dreading it, but as I got into it, I found it to be fun! [Day 5]
- The folding the paper activity seemed tedious but I did enjoy figuring it out once I got started. [Day 5]
- I really enjoyed collaborating and working on projects that will be useful for our classrooms. Yah for projects! [Day 5]
- It was great to have time to work with colleagues on developing DI activities. [Day 5]

- This was awesome! It was lots of fun to collaborate with others in my grade and to create exciting DI activities. [Day 5]

Thus, participants experienced strategies that engaged them in learning mathematical content and pedagogy for a mathematics classroom.

Many participants described similar strategies for engaging students that they would use in their own teaching. The following comment clips come from post-summer institute survey questions 3 & 4 probing “What strategies will you use to teach these concepts [Number, Reasoning & Proof] next school year?” and “What elements that you have learned in the past two weeks will you incorporate into a lesson introducing these topics [Number, Reasoning & Proof] in the coming school year?” For some participants, this may provide evidence that they learned about standards-based instruction; however, it is unclear for question 3 whether or not the participants were already using these strategies prior to IMP. The following examples are clips that only address generic strategies of engagement (e.g., discussion and reflection, critical thinking, real world applications, multiple representations, hands-on activities); specific strategies regarding fostering number sense, supporting content area reading, engaging with math-related literature, differentiating instruction, and encouraging reasoning and proof have been previously discussed.

- real world application problems, moving from concrete hands-on to representative to abstract to solidify student understanding of math concepts. Also think-alouds, with students sharing their math strategies so others will grow and learn from each other. [Question 3]
- manipulatives, visual, modeling, hands-on [Question 3]
- As always, I will incorporate hands-on experiences when teaching new concepts to develop conceptual understanding. [Question 3]
- I will use lots of hands-on/kinesthetic learning centers as well as whole/small group activities. [Question 3]
- Hands-on activities, historical data, real life problems where the concepts can be used to find a solution [Question 3]
- I will use manipulatives and hands-on tools for beginning learning. [Question 3]
- I plan to use multiple strategies like manipulatives, modeling, visuals, discussions, and tiered lessons to help in the understanding of these concepts. [Question 3]
- visuals to make the numbers make sense (e.g., triangular numbers-coconuts), parts of NUMBER DEVIL, conceptual understanding of operations [Question 3]
- Will use scaffolding, triangular and square diagrams [Question 3]
- Number lines and concrete activities with manipulatives [Question 3]
- Real-world examples, use of manipulatives [Question 3]
- I learned some hands-on activities that I plan on taking back and using with my kids; the more concrete you can make it the better. [Question 4]

- Options, real-world examples, interactive (on-line) sites [Question 4]
- I learned about Pascal's triangle and I think that I can show my [students] how to make it and look for patterns they see. I will use the sieve with students to look for multiples of 2, 5, ... and see what is left. We will make a golden rectangle and Sierpinski triangle, (possibly Sierpinski tetrahedron). I am going to emphasize questioning, meaning I want students to learn how to ask them. [Question 4]

Participants' presentations at the share fair also suggest they valued using engagement strategies (e.g., discussion and reflection, critical thinking, real world applications, multiple representations, hands-on activities) during their action plans and/or lesson study. Some examples of comments follow:

- ...so we used a lot of those activities, a lot of hands-on things—rolling dice and drawing cards, picking marbles out of a bag, things like that, and we tried to apply it to real life. We did a lot of group work—playing games as a team to work together to try to figure out the probability of things, independent practice to make sure one kid is not doing all the work...
- ...every station had like three different things they had to do and you wanted to give them time to talk...time to really explore, but you had to keep moving them so our big change for the next time we did it was we decided that we'd at least take a couple things out... so it gave them a little more time to talk...
- We had many activities that kept kids engaged in both standard and metric units.... And the second one was "How far can you blow a pattern block versus a cotton ball". And the third one was body benchmarks. The coolest part was that all were actively engaged...
- We wanted to give them concrete examples of something that is one ounce and one pound. We started off having a really hard time finding things that were one ounce that the kids could relate to.... We got onto a [United Streaming Video clip] that talked about an ounce is about...half of a tennis ball... we found that an individual bag of chips is usually about right around an ounce...
- We also gave the kids more opportunities to work with measurement tools. After I did my measurement unit this year I had all the different measurement tools at different centers and I even let the kids play at the sink with capacity; that was a stretch but they did good.
- So we did an activity with Cheez-It crackers because that was one square unit... something about having that food in their hands, they were very engaged and very excited about doing it.
- In our planning stages we were trying to find ways to help them come to understanding using multiple representations.... We wanted to have three stations and related to some real world information that we dealt with.
- Most of the tweaks to our lesson were done in the questioning and the discussion part of the lesson. As they were going through we had them do a prediction if the area and perimeter would stay the same or not.... Then we wanted to use some higher level questioning....I had them go back and see "What relationships do you notice between the length, width, perimeter, and area?"

- We had so many great ideas and so many things we wanted to get done in that first day... we used gummy worms...we shot milliliters of water at them...
- Centers were from concepts I had taught the week before.... My goal for centers...was to have students actively engaged and learning ... using hands-on centers and then they were all differentiated and I did that by color coding...

Hence, IMP Year 2 may have influenced the teaching practice for some participants as several participants expressed valuing being able to engage their students.

Overall, participants experienced standards-based instruction through engagement in the Process Standards with a variety of strategies (e.g., discussion and reflection, critical thinking, real world applications, multiple representations, hands-on activities). Reflection comments evidenced that many participants valued learning mathematics while engaging in the Process Standards. In addition, many participants expressed that they wanted to employ strategies for engaging their students in learning mathematics. Finally, some evidence suggests that IMP may have impacted the teaching practice of some participants during the school year (particularly while implementing the action plan or lesson study) with regard to standards-based instruction.

Impact of IMP Year 2 on Participants' Teaching Practice

Themes Organized by Research Questions

- Teachers learned mathematical content
 - Knowledge “about” mathematics
 - Substantive knowledge of mathematics
 - Curricular knowledge
 - Pedagogical content knowledge
- Teachers gained pedagogical knowledge
 - Differentiating instruction
 - Supporting reading in the content area
 - Fostering number sense
 - Skills for analyzing practice
 - Implementing standards-based instruction
- ***Impact on teaching practice***
 - Short-term impact on teaching practice

In general, due to the data collected, it is difficult to determine the impact of IMP on participants' teaching practice. The researcher did not observe any of the participants' teaching practice prior to IMP Year 2. Although some pre-summer institute survey questions probed participants' teaching practice (e.g., “When teaching these concepts in your classroom, what

instructional strategies do you regularly use?”, “What differentiated instructional strategies have you implemented in your classroom?”), survey responses were anonymous and thus the teaching practice associated with individuals could not be identified. Hence, it could not be determined whether instructional strategies reported by groups or individuals at the share fair were new with

respect to anonymous pre-summer institute survey information. The researcher did observe two lessons each of four mini-case participants after the IMP Year 2 summer institute. Overall, the observations of the mini-case participants provided some evidence that the teachers were trying to engage students in the Process Standards. The researcher considered it unlikely that the teachers had developed dispositions toward implementing standards-based instruction solely based on their involvement in IMP. The researcher considered it more likely that the mini-case participants had some standards-based tendencies prior to their involvement in IMP. However, it appeared that some teaching strategies were likely influenced by participation with IMP. Furthermore, subtleties in reflections by participants during the share fair and by mini-case participants during the school year suggest that IMP made an impact on some of the participants' teaching practice. However, the evidence often only substantiates an impact on one lesson or unit as participants had reported on their lesson study and action plan.

Theme: Short-term impact on teaching practice

- Themes Organized by Research Questions**
- Teachers learned mathematical content
 - Knowledge “about” mathematics
 - Substantive knowledge of mathematics
 - Curricular knowledge
 - Pedagogical content knowledge
 - Teachers gained pedagogical knowledge
 - Differentiating instruction
 - Supporting reading in the content area
 - Fostering number sense
 - Skills for analyzing practice
 - Implementing standards-based instruction
 - Impact on teaching practice
 - ***Short-term impact on teaching practice***

Previous discussions about how IMP may have impacted teaching practice were included in some earlier themes regarding participant learning of pedagogy. First, one area was discussed in the previous theme: *Teachers learned about lesson study and more general skills for analyzing practice*. Project tasks requiring implementation of an action plan and lesson study resulted

in some participants engaging in a more critical analysis of their teaching practice. That is, some participants engaged in more formal collaboration, data analysis, and hypothesis-testing regarding a lesson or unit. Furthermore, the impact of IMP went beyond project participants as some participants (about seven) reported during the share fair that they had provided leadership in implementing a lesson study with other teachers (non-IMP participants) at their schools.

IMP may have also impacted teaching practice with regard to implementing strategies to support content area reading and the development of numbers sense. Share fair presentations

revealed that many participants used content area reading strategies (see *Theme: Teachers learned about strategies for supporting students' reading in the content area*) and/or strategies for fostering number sense (see *Theme: Teachers learned pedagogical strategies for supporting the development of number sense*) during their action plan or lesson study. Furthermore, IMP may have impacted participants' teaching practice with regard to standards-based instruction (see *Theme: Teachers learned about standards-based instruction in mathematics*).

Finally, the strongest evidence of IMP's impact on teaching practice regards differentiated instruction; the evidence has not been presented previously and will thus be discussed now. Some teachers came to IMP Year 2 with some understanding of differentiated instruction because of their involvement with IMP Year 1 or other professional development. Some teachers may have already been incorporating some elements of differentiated instruction in their classroom prior to Year 2. Determining the impact of IMP Year 2 (or perhaps IMP Year 1 & 2 for those who attended both) with regard to differentiating instruction is not clear cut. However, the researcher was able to look at subtleties in language in order to infer the impact of IMP on teaching practice with regard to differentiating instruction.

In share fair presentations, some of the participants used language (e.g., “we tried”, “I started”) when referring to a differentiated instruction strategy. Thus, participant comments provide some evidence that the participant(s) implemented a new differentiated strategy while teaching mathematics. In addition, several participants shared differentiated instruction strategies with teachers who had not participated in IMP, and thus the impact of IMP went beyond project participants.

- We tried to do some differentiated instruction—think dots, learning centers [while teaching probability].
- We did do a lot of Think-Tac-Toe—that was the one I liked—that was the one I thought was easiest to use. With four of us working together, it was easy for one of us to build one and for all of us to use it ... so really we built about one or two a week.
- Some of the things that we worked on this year is differentiated centers—I started centers for the first time last year and I really enjoy that time; I usually take small groups at that time...[participants had attended both Year 1 and Year 2 of IMP]
- We basically started throwing ideas together. It's good for me because... I'm the only geometry teacher [at my school] so it really does help to throw ideas off of other geometry teachers... We wanted to have three stations and related to some real world information that we dealt with...
- Another one that I did this year which I considered a DI activity was I wrote down all the state assessed indicators we had gone through...[and I told the students to] “pick your hardest one”.... Then, I made them create a Jingle, a RAFT, a chant, something to help

them remember and my hope was that they would understand it better plus they would [look] at somebody else's. I did have one student come up and say...“I remember this because so and so did this one”. I was so excited, so I'm going to do that again next year.

- What are my strong suits in reading and how can I transfer that over into math... I looked at things I could implement.... I differentiated by centers which were hands-on... I knew how to do it with reading; I had all the resources as far as data analysis; how could I do it with math.
- I was trying to encourage this person to differentiate their instruction and add rigor.... [Later in the presentation:] So I had already suggested to him about the ThinkDots. I ended up saying can I take this group and do a ThinkDots activity with them.
- They would then have choices when their initial investigation was completed, to further their investigation by choosing different questions out of the “Grow Pots” and solving for those. Not everybody got to do this on the day ...that we had the Japanese lesson study presented, but it proved to be a good anchor activity later on and in the weeks to follow.... They were out for the rest of the year for kids to come back to and use.
- So mainly I wanted to talk about how I've implemented [ideas from IMP] with the preservice teachers. First off, I went back to my syllabus and instead of saying you have these assignments, choose 3 out of 5 of these assignments.
- I got to present five differentiated instruction activity lessons to my staff and I chose specifically to teach those differentiated instruction strategies in content that was at the staff level... I used differentiated instruction strategies to work through the content of the meetings.
- My principal gives me a day about every six weeks to work with the teachers so I taught ThinkDots, Think-Tac-Toe, Menus, and RAFT. And, the one the teachers seemed to like to use were the ThinkDots so I have an example of that.
- And then the other thing that I tried to do as my role as an academic coach... I thought differentiated instruction activities would be something that I would work on this year. They take a lot of time to create. So I told the teachers at the beginning if they would give me a topic that I would create the first one for them; they could try it in the class and see if they felt comfortable with the activity or not... After they did one... they were happy to try some more; they still wanted help with them because they do take a lot of time to put together. [Some of the activities created were a perimeter and area Contract, Think-Tac-Toe with prime and composite numbers, Menu with division.]

Hence, teachers' language suggests that some IMP participants shared differentiated instruction strategies with non-IMP teachers and some participants tried out new differentiated instruction strategies while teaching mathematics. A few participants suggested that although they had already been using some differentiated instruction strategies in reading, they had now tried differentiating instruction in mathematics.

Interviews with mini-case participants during the school year provide further evidence that participants tried some new differentiated instruction strategies in the mathematics classroom. The mini-case participants were asked: “This year have you used any differentiated

instructional strategy or strategies in any mathematics lessons this year that you did not use in prior years?” and “Has the Summer Institute impacted your teaching in any way this year?”.

Responses follow:

- We’ve done RAFT this year, and menus for projects.

[Later in the conversation:] The make and take that we had a day where we could sit down, and if we couldn’t even come up with it, there were other ... teachers there from different parts of the state and they’re teaching the same standard as I do. So that kind of stuff I can just pull out of a folder and use, and anything that takes little to no planning and is good, is wonderful. To just be able to pull something out and use it and have it made from the summer. The RAFT was something that was made over the summer...

- [Comments from a two-year IMP participant:] Last year I did a tiered lesson and I’ve done that before. I’ve done stations, I did that last year. Tic-Tac-Toe was new for me. I always liked it and I never had used it so I used that for a homework assignment and then for my tutor group, they got to choose what activities they wanted to do to practice some of their Tic-Tac-Toe.

[Later in the conversation the participant talked about differentiating instruction by identifying a group of students with a pre-test as already having some basic knowledge of fractions, and then setting them up for some independent study.] You know, they went online and kind of did a web quest. So that was way new to me, and it was very successful. A lot of parents and kids emailed me and said how they loved it, and a lot of kids it boosted their idea that they were good in math. It was great. A lot of them who had not really struggled but had issues in class, and now I don’t have any problems with them.

- I think it’s just more awareness that there’s different ways that we can differentiate—that it’s not writing totally new lesson plans. It’s just changing little pieces of the lesson. That’s probably the biggest thing I took away from it. Try and focus on at least one little thing, make it manageable.

- I’ve done a lot of differentiated things with the Think-Tac-Toes, different work sheets that they’ve given us; we’ve done the menus a couple of times.

Therefore, comments by the mini-case participants during the school year provide additional evidence that some participants were trying differentiated instruction strategies that they had not used in previous school years.

Overall, evidence suggests that IMP impacted teaching practice at least with regard to implementation of an action plan and/or lesson study. Some participants engaged in a more critical analysis of their teaching. Many participants implemented strategies for supporting content area reading and/or for fostering number sense. More generally, some participants

implemented and valued engaging students in standards-based instruction. Finally, many participants implemented strategies for differentiating instruction.

Summary

The purpose of this case study was to examine the impact of a Kansas MSP-funded professional development project on K-12 mathematics teachers' knowledge and teaching practice. Chapter 4 began with a review of some elements and strategies associated with effective professional development and corresponding IMP elements and strategies. The chapter proceeded with the results of the qualitative study indicating that participants gained mathematical content knowledge and pedagogical knowledge. With regard to mathematical content knowledge, participants gained knowledge “about” mathematics, substantive knowledge of mathematics, curricular knowledge, and pedagogical content knowledge. Concerning pedagogical knowledge, participants learned about strategies for differentiating instruction, supporting students' reading in the content area, fostering the development of number sense, and implementing standards-based instruction. In addition, some participants also learned how to more critically analyze teaching. Finally, IMP Year 2 made at minimum a short-term impact on participants' teaching practice with regard to inquiry into practice and to implementing strategies in support of content area reading, development of number sense, standards-based instruction, and differentiated instruction.

CHAPTER 5 - Conclusions

Introduction

While consensus has been building about strategies and characteristics of effective professional development (e.g., Elmore, 2002; Garet et al., 1999; Sowder, 2007), Borko (2004) suggested we know little about “what and how teachers learn from professional development” (p. 3). Results of Title IIB MSP professional development initiatives are only beginning to come in (e.g., Blank et al., 2008; Gummer & Stepanek, 2007). The Infinite Mathematics Project professional development model under study embodied many characteristics and utilized several strategies associated with “high quality” professional development. This case study of a Title IIB MSP project sought to provide a qualitative examination of the characteristics and strategies used in the project, and to understand their impact on teacher learning and practice. This chapter includes: a summary of the study design; a summary of the results; a discussion about the connection between the results and the literature regarding teachers’ knowledge, teaching practice, and professional development; implications of the study; recommendations for the future; and a final summary.

Summary of the Study Design

An embedded units single-case study design was employed in order to examine the impact of a complex professional development program on teachers’ knowledge and teaching practice. Thirty-two teachers fully participated in IMP Year 2 along with four project facilitators, one graduate student facilitator, and two teacher leaders. The case study was informed by an examination of both qualitative and quantitative evidence. A variety of data sources were used including: participant observation; documentary evidence; participants’ content session homework; participants’ content session pre- and post-summer institute tests; participants’ homework/session reflections; existing project-collected survey data and summary quantitative analyses reports; interviews with mini-case participants; and, classroom lesson observations of mini-case participants. After data collection, data was organized and then later transcribed as needed, and entered as textual data into a qualitative data analysis software

program. A grounded theory approach was used to identify themes emerging from analysis of the variety of data sources. Coding began with open coding as data units were compared with others for similarities and differences (Corbin & Strauss, 1990). Recurring patterns were coded from a list of existing codes. A constant comparative method was used to identify new codes and to assign existing codes to quotations and data units. Concepts made their way into the theory by relevance to the research questions and by repetition. Conceptually similar data units and codes were grouped together into themes. In the final stages, the researcher considered whether project description, summer institute session discourse, exemplar quotes from participants' reflections or survey comments, homework or test performance, or other data sources best illustrated the theme. In the written report, attention was given to providing transitions and connections between data sources and units within a theme as well as between themes. Validity and reliability were attended to in a variety of ways. For example, validity was strengthened by employing a variety of strategies: methods collection triangulation, data sources triangulation, searching for disconfirming evidence, employing the constant comparison method, reflexivity, participant observation of both the year under study and the prior year, and use of thick description and many verbatim quotes. Reliability and validity were strengthened by establishing an audit trail through organization of the raw data and organization of textually prepared data along with subsequent coding and memoing in the qualitative software file. Reliability was also enhanced by the use of protocols for mini-case participant interviews and observations.

Summary of Results Related to Research Questions

This case study examined the impact of the Infinite Mathematics Project, a Title IIB MSP-funded professional development project, on teachers' knowledge and teaching practice. The study was guided by three research questions. The questions along with a summary of the results follow.

Effective Professional Development

- Content-focused
- Active learning
- Sustained
- Coherence
- Collaborative participation
- Giving teachers opportunities to:
 - Participate in close proximity to practice
 - Develop skills for analyzing practice
 - Engage as mathematical learners
 - Examine curricular resources
 - Examine student thinking
- Partnership

Throughout the summary of the results and discussion about the results, attention will be drawn to aspects of IMP related to characteristics and strategies of effective professional development that impacted learning and practice. The graphic organizer from Chapter 4 (Figure 4.1) is provided again as a reminder of characteristics and strategies of effective professional

development. Of note, IMP was a **sustained** professional development initiative that overwhelmingly **focused on mathematical content** and provided a multitude of **active learning opportunities** for its participants (see discussion throughout Chapter 4). As other strategies and characteristics appear as agents impacting learning and practice, attention will be drawn through bolding the phrase. Thus, discussion of the study results will serve to reaffirm research regarding characteristics and strategies for effective professional development. Details regarding strategies and their relationship with impacting certain areas of learning and practice may also contribute to research on effective professional development.

Research Question 1. What impact did the IMP program have on participants' content knowledge (e.g., subject matter knowledge, pedagogical content knowledge, curricular knowledge)?

Themes Organized by Research Questions

- *Teachers learned mathematical content*
 - *Knowledge “about” mathematics*
 - *Substantive knowledge of mathematics*
 - *Curricular knowledge*
 - *Pedagogical content knowledge*
- Teachers gained pedagogical knowledge
 - Differentiating instruction
 - Supporting reading in the content area
 - Fostering number sense
 - Skills for analyzing practice
 - Implementing standards-based instruction
- Impact on teaching practice
 - Short-term impact on teaching practice

Researchers (e.g., Ma, 1999; Shulman, 1986b) have proposed different organizations of teachers’ content knowledge. The findings with regard to the first research question were framed around Shulman’s (1986b) conception of content knowledge as including subject matter knowledge, pedagogical content knowledge, and curricular knowledge. In addition,

using Ball’s representation (1990a, 1991), subject matter knowledge was further decomposed into knowledge “about” mathematics and substantive knowledge of mathematics. The findings revealed that participants gained subject matter knowledge, curricular knowledge, and pedagogical content knowledge. Furthermore, with regard to subject matter knowledge, participants gained both knowledge “about” mathematics and substantive knowledge of mathematics.

Knowledge *about* mathematics includes understandings such as: how truth is established in the field of mathematics; what reckons as a solution; which ideas are based on convention and which are built on logic; and how mathematics has developed and changed over time (Ball, 1990a, 1991). Scenarios described in *The Number Devil* (**curricular resource**), along with content facilitators’ (**mathematicians involved in partnership**) building upon the scenarios and skillful direction of discussions in response to participants’ questions, provided participants with opportunities to engage as **mathematical learners** with regard to knowledge *about* mathematics. That is, IMP participants reflected on whether some mathematical ideas were based on logic or convention. Participants learned about how some mathematical concepts have developed over time. Finally, participants learned that formal mathematical proof is of primary importance for establishing truth in the field of mathematics.

Substantive knowledge of mathematics includes knowledge of topics (e.g., trigonometry), concepts (e.g., infinity), procedures (e.g., factoring), underlying principles and meanings (e.g., what division with fractions means), and relationships among the concepts (e.g., how fractions

are related to division) (Ball, 1990a, 1991). Topics and concepts introduced in *The Number Devil*, and expanded upon during content sessions, provided participants with opportunities to learn about a variety of topics, concepts, procedures, underlying meanings, and relationships among concepts. The researcher chose to narrow the investigation by focusing on concepts for which evenly matched pre- and post-summer institute questions indicated there was substantial growth and for which there was additional supporting evidence. Thus, evidence from a variety of sources (i.e., improvement on matched pre- and post-summer institute content questions, performance on homework and quizzes, session dialogue, pre- and post-summer institute survey data, interviews with mini-case participants) revealed that participants **engaged as mathematical learners** of concepts related to substantive knowledge including (but not limited to): prime and composite numbers; rational and irrational numbers; triangular and Fibonacci numbers and sequences along with direct and recursive formulas; Pascal's triangle and related number patterns; permutations and combinations; and Euler's formula for polyhedra. Thus, participants increased their understanding of substantive knowledge of mathematics.

Per Shulman (1986b), curricular knowledge can include knowledge about curricular alternatives, lateral knowledge about curriculum students might be studying in other subjects, and vertical knowledge about preceding and succeeding topics in the same subject area. Participants had many opportunities to learn about **curricular resources** and to reflect upon **student thinking** as related to mathematical concepts that students might be learning at other grade levels. For example, content facilitators modeled delving into mathematics content by using math-related literature. In addition, pedagogy facilitators modeled a wide variety of curriculum resources addressing number sense, reading in the content area, and differentiating instruction. Some participants evidenced that they had learned about curricular alternatives through their expression of valuing the curricular resources that had been modeled or through their reports about using some of the curricular alternatives that had been modeled in IMP. Participants also had opportunities to gain vertical curricular knowledge. For instance, participants had opportunities to learn about vertical curricular knowledge during the summer institute as pedagogy facilitators provided examples of how a mathematical concept might be approached to support student thinking at different grade levels. Opportunities for learning vertical curricular knowledge also arose naturally as participants learned from each other about grade specific curricular standards and grade appropriate instructional approaches through group

discussions during content or pedagogy sessions, content presentations, lesson study and action plan implementation, and share fair presentations.

Finally, pedagogical content knowledge refers to subject matter knowledge *for teaching* which includes knowledge for topics regularly taught in a subject area of useful representations, analogies, examples, explanations, and demonstrations. It also includes an understanding of what makes some topics easy or difficult to learn and what conceptions, preconceptions, and misconceptions students might have at various ages (Shulman, 1987). IMP Year 2 participants had opportunities to increase their pedagogical content knowledge. For example, content and pedagogy facilitators modeled some useful representations and explanations regarding K-12 mathematical concepts. In addition, IMP facilitators (**partnership**) supported participants' development of pedagogical content knowledge by encouraging participants to **learn about the conceptual underpinnings of mathematical ideas** and to reflect upon **student mathematical thinking** during the summer institute. The facilitators also encouraged the participants to reflect upon student thinking and conceptual underpinnings for mathematical ideas as the participants carried out their action plans and lesson study during the school year. Participants' reflection comments during the summer institute, content presentations during the summer institute, and share fair presentations about the lesson study process, revealed that some participants had gained pedagogical content knowledge while reflecting upon student thinking and about conceptual underpinnings for mathematical ideas.

Research Question 2. What impact did the IMP program have on participants' pedagogical knowledge (e.g., knowledge of differentiated instruction, knowledge of standards-based instruction)?

Themes Organized by Research Questions

- Teachers learned mathematical content
 - Knowledge “about” mathematics
 - Substantive knowledge of mathematics
 - Curricular knowledge
 - Pedagogical content knowledge
- *Teachers gained pedagogical knowledge*
 - *Differentiating instruction*
 - *Supporting reading in the content area*
 - *Fostering number sense*
 - *Skills for analyzing practice*
 - *Implementing standards-based instruction*
- Impact on teaching practice
 - Short-term impact on teaching practice

Pedagogy facilitators shared research-based strategies for differentiating instruction, supporting reading in the content area, fostering the development of number sense, and implementing standards-based instruction. In addition, participants experienced different pedagogical strategies while learning mathematical content and pedagogical strategies. The

varied experiences provided a backdrop for teachers to reflect and learn about effective pedagogical strategies for mathematics instruction. Furthermore, some participants learned how to more critically analyze teaching during their implementation of a lesson study and action plan during the school year.

First, while participants came to IMP Year 2 with varied experiences and understanding of differentiated instruction, most participants increased their understanding of differentiated instruction. IMP Year 2 afforded participants with many learning opportunities regarding differentiating instruction. Pedagogy facilitators shared information and spurred discussion about key elements of differentiated instruction as well as tips and strategies for implementing differentiated instruction. Participants (as **mathematical and pedagogical learners**) experienced differentiated instruction as pedagogy and content facilitators employed elements and strategies for differentiating instruction during summer institute sessions. Lead teachers provided examples and modeled differentiated instruction specific to mathematics for grade level bands. Participants also had a significant **collaborative active learning** opportunity while making differentiated instruction activities for the mathematics classroom with grade level peers (**in close proximity to practice**) during a summer institute session. After participating in IMP Year 2, participants demonstrated they had learned differentiated instruction strategies in a variety of ways including (but not limited to): communicating key elements of differentiated instruction; identifying differentiated strategies that they had learned during the summer institute and that they would use in the coming school year; creating differentiated instruction activities

and resources with grade level peers; and employing differentiated instruction strategies during content session presentations (optional), lesson study (optional), and/or action plan implementation (required). The abundance of evidence from a variety of sources suggests that most participants increased their knowledge of differentiated instruction. However, understanding of differentiated instruction may have been deficient for a small number of participants who primarily associated differentiated instruction with ability grouping.

Second, many participants learned about strategies for supporting students' reading in the content area as well as the engagement potential of math-related literature. Learning opportunities about content area reading arose as *The Number Devil* (a **curricular resource** with potential for **engaging participants as mathematical learners**) provided an organization for mathematical concepts studied in the content sessions, as pedagogy facilitators shared research-based strategies, and as some participants implemented (**in close proximity to practice**) content area reading strategies during their content presentation, lesson study, or action plan. Furthermore, participants' summer institute experiences with math-related literature, non-standard language, and content area reading strategies served as a backdrop for reflection about the engagement potential of math-related literature, the importance of prerequisite knowledge and vocabulary knowledge for reading comprehension, and the benefits of strategies that support content area reading. Evidence in the form of summer institute discussions, content and pedagogy session reflections, reading assignment reflections, survey responses, and share fair reports revealed that many participants learned about content area reading strategies and valued the engagement potential of math-related literature.

Third, IMP Year 2 afforded participants with opportunities for learning pedagogical strategies in support of fostering the development of number sense. For example, pedagogy facilitators engaged participants in reflection about definitions of number sense and components of number sense. In addition, lead teachers modeled grade appropriate activities for fostering the development of number sense from a variety of **curricular resources**. Mini-case interviews, survey comments, and share fair reports provided evidence that some participants learned new or additional pedagogical strategies for fostering the development of number sense and some participants came to place a higher value on the importance of their role in helping students develop number sense.

Fourth, participants had opportunities to learn about standards-based instruction in mathematics. Of note, many strategies for differentiating instruction, supporting content area reading, and fostering number sense also support standards-based teaching. Hence, previously discussed learning regarding pedagogical strategies for differentiating instruction, supporting content area reading, and fostering number sense indirectly contribute to learning about standards-based instruction. In addition, participants learned about standards-based instruction through their **engagement in the Process Standards while learning mathematical concepts** considered in *The Number Devil*. **Active engagement** in the Process Standards was afforded by opportunities to experience **learning mathematics** via math-related literature (**curricular resource**) along with facilitator-guided (**partnership**) activities involving reflection and discussion, critical thinking about mathematical concepts, real world application, and hands-on activities. The learning experiences provided a setting for participants to reflect upon the impact of standards-based instruction. Reflections and survey comments revealed that participants valued learning mathematics while being engaged in the Process Standards. In addition, many participants expressed that they wanted to employ similar strategies for engaging their students in learning mathematics. Furthermore, some participants expressed valuing using engagement strategies (e.g., discussion and reflection, critical thinking, real world applications, multiple representations, hands-on activities) during the implementation of their action plan and/or lesson study during the school year (**in close proximity to practice**). Hence, participants experienced and learned about implementing standards-based instruction.

Finally, some participants learned how to **analyze teaching** more critically through their participation with more formal **collaboration**, data analysis, and hypotheses-testing (setting goals for students, assessing whether goals are met, hypothesizing whether a lesson did or did not work well, and using hypotheses to revise lessons) during the lesson study and action plan development and implementation (**in close proximity to practice**). In **coherence** with state and national standards, participants were guided in a summer institute activity requiring analysis of data to identify the lowest and highest tested indicator for their school and grade level. Furthermore, participants were charged by pedagogy facilitators (**partnership**) with creating an action plan to address the area of weakness identified by the data. In addition, participants had the opportunity to learn about the lesson study process as pedagogy facilitators shared information and facilitated discussions regarding lesson study. About seventy percent (or

greater) of the participants implemented a lesson study during the school year. At least sixty percent of the participants evidenced in their action plans and share fair reports being able to set clearly defined learning goals to address an area of weakness along with being able to assess and communicate whether the learning goals were achieved. During the share fair, some of the participants further described taking part in discussions about whether a lesson did or did not work well, and then using hypotheses to revise, teach, and observe the lesson in another classroom. Pre-summer institute interviews with mini-case participants generally revealed the four teachers came to IMP Year 2 with some habits of setting learning goals and collaborating with colleagues. However, creating an action plan to address an area of weakness, analyzing data more formally, and participating in more formal collaboration during implementation of a lesson study or action plan were outcomes of IMP for many participants.

Research Question 3. What impact did the IMP professional development program have on participants' teaching practices (e.g., differentiating instruction, implementing standards-based teaching, analyzing practice)?

<p>Themes Organized by Research Questions</p> <ul style="list-style-type: none"> • Teachers learned mathematical content <ul style="list-style-type: none"> ◦ Knowledge “about” mathematics ◦ Substantive knowledge of mathematics ◦ Curricular knowledge ◦ Pedagogical content knowledge • Teachers gained pedagogical knowledge <ul style="list-style-type: none"> ◦ Differentiating instruction ◦ Supporting reading in the content area ◦ Fostering number sense ◦ Skills for analyzing practice ◦ Implementing standards-based instruction • <i>Impact on teaching practice</i> <ul style="list-style-type: none"> ◦ <i>Short-term impact on teaching practice</i>
--

As the researcher did not observe the teaching practice of any participants prior to IMP Year 2, determining the impact of IMP on teaching practice was hindered. Nonetheless, subtleties in reflections by participants during the share fair and by mini-case participants during the school year suggested IMP made an impact on some participant's teaching practice. However, the

evidence often only substantiated IMP's impact on one lesson or unit as participants primarily reported on their lesson study or action plan.

Based on participant reports regarding implementation of the action plan and/or lesson study during the school year (**in close proximity to practice**), some participants engaged in more formal **collaboration** and **critical analysis of practice**. In addition, many participants used content area reading strategies and/or math related literature as part of their action plan.

Many participants used an activity or strategy for fostering the development of number sense. Some participants implemented and valued engaging students in standards-based instruction. Finally, the strongest evidence of IMP's impact on teaching practice pertained to differentiating instruction. Participants' use of language (e.g., "we tried", "I started", "I got to present") revealed that many participants tried new strategies for differentiating instruction or shared strategies with other teachers.

Discussion

IMP Year 2 embodied many features of effective professional development and utilized multiple strategies in order to provide an environment whereby teachers would have opportunities to gain knowledge and improve teaching practice. The results of this study provide evidence that participants of IMP Year 2 gained mathematical content knowledge and pedagogical knowledge. In addition, at minimum the project had a short-term impact on teaching practice.

IMP Year 2 was a prime example of a mixed-ability classroom. It was a classroom in that general goals for MSP Title IIB grants and specific goals for IMP included strengthening teachers' content and pedagogical knowledge and improving teaching practices. It was mixed-ability for a variety of reasons. Participants included elementary, middle, and high school teachers. Of the 32 participants, eight participants self-reported having 9 or fewer college math credits whereas five participants reported having 50 or more college math credits. Teaching experience ranged from one to twenty-six years. Furthermore, about half of the participants had participated in IMP Year 1, and hence had prior experience with the format of the project, lesson study, and differentiated instruction. Thus, participants came to IMP Year 2 with varied experiences and prior knowledge. As a result, participants did not necessarily learn the same things. For example, secondary mathematics teachers likely came to IMP with experience with recursively defined functions. On the other hand, elementary teachers with fewer prior experiences with recursive formulas were more likely to gain understanding about recursive functions during their participation with IMP. As another example, many participants had the opportunity to gain vertical curricular knowledge about grade bands that differed from their own. As a final example, participants came to IMP in many different places regarding their understanding of differentiated instruction; however, most participants increased their

understanding of strategies and elements for differentiating instruction. While some participants with fewer experiences were introduced to key elements and strategies for differentiated instruction, other participants with some prior knowledge learned additional strategies for differentiating instruction. Overall, the abundant and varied IMP Year 2 strategies and learning experiences provided opportunities for all participants to gain mathematical content knowledge and pedagogical knowledge. Furthermore, IMP impacted many participants' teaching practice, at least with regard to implementing a lesson study and/or action plan during the school year.

Discussion of results related to the impact of IMP on participants' mathematical content knowledge

Supporting teachers' development of content knowledge should be a major goal of professional development programs serving the needs of teachers engaging in mathematics reform (e.g., Borasi & Fonzi, 2002; Sowder, 2007). Professional development should provide teachers with opportunities to gain subject matter knowledge, pedagogical content knowledge, and curricular knowledge (Sowder, 2007).

Knowledge *about* mathematics, knowledge about the nature of the discipline of mathematics, and syntactic structures of knowledge for mathematics are related terms. Researchers and educators have suggested this component of subject matter knowledge is important for learning and teaching. For example, in order to support cognitive science principles of learning, the National Research Council (2005) suggested teachers should have both knowledge of the nature of the discipline (e.g., what it means to engage in doing mathematics) and substantive knowledge about core concepts fundamental for a discipline (e.g., functions). Kennedy (1997) suggested that through their pedagogy, teachers will represent the nature of the discipline. However, Ball (1991a) noted that mathematics students rarely have opportunities to learn "about the evolution of mathematical ideas or ways of thinking" (p. 7) mathematically; explicit curriculum in school or college rarely addresses knowledge *about* mathematics. Furthermore, Borko and Putnam (1995) suggested that mathematics teachers would particularly benefit from professional development focused on broadening understanding of syntactic structures of knowledge (how truth is established in mathematics). Upon review of several studies (e.g., Ma, 1999; W. G. Martin & Harel, 1989), Ball et al. (2001) suggested "teachers are prone to accept inductive evidence, such as a series of empirical examples or a pattern, as being sufficient to establish the validity of a claim" (p. 447). Sylianides and

Stylianides (2009) asserted that while not meaning to diminish the role of empirical exploration, empirical arguments relying on evidence from a few cases should not be considered as a substitute for formal mathematical proof. The researchers also pointed out that “there has been limited research about how instructors can help students develop their understanding of proof” (2009, p. 315). Stylianides and Stylianides hypothesized that in order to develop an understanding of proof, students would first have to see a need for learning validation methods for all cases covered under a generalization by realizing empirical arguments had limitations. In their research, “students” were prospective elementary teachers.

Scenarios described in *The Number Devil* (**curricular resource**) along with project involvement of mathematicians (**partnership**) who built upon the scenarios and skillfully directed discussions in response to participants’ questions provided participants with opportunities to **engage actively as mathematical learners** regarding both knowledge *about* mathematics and substantive knowledge of mathematics. First, the results revealed participants gained knowledge *about* mathematics (e.g., how truth is established in the field of mathematics, how mathematics has developed over time, which ideas are based on convention and which are built on logic). For example, participants engaged in thinking about whether certain mathematical ideas (e.g., order of operations) are based on convention or logic. In addition, the mathematical literature and content session discussions provided a stage for participants to increase their understanding of the development of numeral systems. Therefore, the project provided participants with the opportunity to learn that the development of mathematics is a “constructive process of continual invention and revision” (Ball, 1991b, p. 79). Furthermore, dialogue from *The Number Devil* and content session emphasis resulted in significant attention being placed on the role of proof in mathematics. Participants learned that a primary function of proof in mathematics is to verify the truth of an assertion for all cases; that empirical arguments relying on evidence from a few cases is not the same as formal mathematical proof. Participants also learned that proof can provide insight into why an assertion is true. Hence, the results of the study reveal strategies that can be employed in professional development to provide teachers with opportunities to gain knowledge *about* mathematics. The results also contribute to research about instructional strategies that can be implemented “to help students begin to realize the limitations of empirical arguments as methods for validating mathematical generalizations and

see an intellectual need to learn about secure methods for validation” (Stylianides & Stylianides, 2009, pp. 315-316).

IMP Year 2 also provided learning experiences for teachers to gain substantive knowledge of mathematics, another component of subject matter knowledge. Researchers have found that teachers’ knowledge about both substantive and syntactic structures of a discipline has implications for what they teach (Grossman et al., 1989). Studies have revealed that U.S. prospective teachers (e.g., Ball, 1990a, 1990b; Borko et al., 1992) and practicing teachers (e.g., Ma, 1999) lack a deep understanding of the concepts they teach. Participants had opportunities to learn about many mathematical concepts and topics. Topics discussed in *The Number Devil* and expanded upon during content sessions included (but were not limited to): numeral systems (particularly Roman numerals and base-ten); place value and the importance of zero; infinitely large and infinitely small (infinitesimal); number systems (e.g., natural numbers, rational numbers, irrational numbers, imaginary numbers); other types of numbers (e.g., prime, triangular); sequences (e.g., Fibonacci) and series; recursive and direct formulas; Pascal’s triangle; Golden Ratio; Euler’s formula; polyhedra; permutations and combinations; fractals; and cardinality. The variety of topics resulted in emphasis on the Number & Operations and Algebra Content Standards. A multitude of data sources (i.e., improvement on matched pre- and post-summer institute content questions, performance on homework and quizzes, session dialogue, pre- and post-summer institute survey responses, interviews with mini-case participants) provided evidence that IMP Year 2 participants gained substantive knowledge about mathematics. Some of these topics are related to an assertion by Schifter and Riddle (2004) that elementary teachers should have a deep understanding “of the base-10 number system, the meaning of the basic operations, the logic of rational numbers, and the properties of geometric shapes” (p. 30). In addition, some topics (e.g., infinitely large and infinitely small numbers, combinations and permutations) are related to content that middle and secondary mathematics teachers are in charge of teaching (NCTM, 2000).

Shulman (1986b) brought attention to another aspect of content knowledge: pedagogical content knowledge. Sowder (2007) recommended supporting the development of deep pedagogical content knowledge should be one of the goals for professional development of mathematics teachers. However, Ball and Bass (2000) claimed pedagogical content knowledge builds up in teachers over time as they teach the same topics. Thus, teasing out participants’

learning of pedagogical content knowledge was challenging due to the limited amount of time for the summer institute and the potential for interaction of other variables during the school year. There were only a few questions on the content pre- and post-summer institute tests that probed pedagogical content knowledge. Without having more knowledge about participants' pedagogical content knowledge prior to the summer institute, it was difficult to determine whether IMP Year 2 teachers learned about useful explanations, representations, or students' conceptions or misconceptions. However, prior research has suggested some possible sources for teachers' development of pedagogical content knowledge (e.g., J. Barnett & Hodson, 2001; Grossman, 1990; Kennedy, 1997; Sowder et al., 1998). For example, Sowder et al. (1998) found that practicing teachers made pedagogical decisions while learning content as they reflected on how their students would think about the concept. While learning mathematical content during the summer institute, many IMP Year 2 participants reflected out loud about how students might learn the content or about how the content might be related to their teaching. Also, Kennedy (1997) suggested that in order for teachers to appropriately use metaphors and representations that could illuminate substantive concepts, pedagogical content knowledge would depend heavily on both conceptual understanding and knowledge of students. IMP facilitators supported and encouraged participants to understand mathematical concepts more deeply and to understand student thinking. Hence, as the project provided participants with opportunities to engage as **mathematical learners** regarding the conceptual underpinnings of mathematical ideas and to reflect upon **student thinking**, so to the project supported the development of pedagogical content knowledge. Finally, Barnett and Hodson (2001) proposed pedagogical content knowledge might develop through discussions with more experienced colleagues. By their participation in IMP Year 2, teachers had many opportunities to talk with other teachers. Furthermore, because of the diversity of the group, different teachers were more experienced with different grade levels or pedagogical practices. Hence, there were many opportunities for participants to gain pedagogical content knowledge through discussions with other teachers. As described in the results, a few questions on the pre- and post-summer institute content tests, reflection comments, group discussions, and share fair reports provided evidence that participants gained pedagogical content knowledge.

Finally, the discussion turns to curricular knowledge. In his 1985 address to AERA, Shulman (1986b) claimed: "If we are regularly remiss in not teaching pedagogical knowledge to

our students in teacher education programs; we are even more delinquent with respect to the third category of content knowledge, *curricular knowledge*” (p. 10). After the publication of *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989), the NSF provided funding for several mathematics curriculum projects (Sowder, 2007). Subsequently, many professional development activities focused on preparing teachers to use standards-based curricula. In addition, using curriculum has been considered a strategy for fostering teacher learning. Borasi and Fonzi identified “becoming familiar with exemplary instructional materials and resources” (2002, p. 10) as one of nine main learning needs of teachers engaging in reform. Borasi and Fonzi suggested that exemplary instruction materials may be a comprehensive NSF-funded curriculum, or individual units designed to replace parts of a traditional curriculum in order to expand instructional goals and introduce some effective instructional practices.

The results of the study revealed that participants gained curricular knowledge. First, participants gained knowledge about **curricular resources** that could be supplements to a comprehensive curriculum. For instance, pedagogy facilitators shared resources about key elements of differentiated instruction along with some examples of differentiated instruction strategies and activities. Pedagogy facilitators and lead teachers also shared and modeled many activities and resources that could be used to foster the development of number sense. In addition, pedagogy facilitators shared graphic organizers that could be used to support content area reading and participants experienced using some graphic organizers as **learners**. Furthermore, as Whitin and Whitin (2004) suggested that good math-related literature could be a catalyst for engagement in the Process Standards, the use of *The Number Devil* served to **engage the participants in Process Standards while learning mathematical content**. Feedback from participants revealed many participants valued, and used or would use, some of the curricular alternatives that were modeled in the project. Finally, participants gained vertical curricular knowledge about mathematical concepts studied, and instructional approaches employed, at other grade levels. Some learning experiences were structured by pedagogy facilitators. Grade level diversity amongst participants also provided many natural opportunities for participants to gain vertical knowledge through conversations and discussions with each other.

Discussion of results related to the impact of IMP on participants’ pedagogical knowledge

The results of the study revealed that IMP Year 2 participants learned about strategies for differentiating instruction, supporting content area reading, fostering a development of

number sense, and implementing standards-based instruction. Many participants also learned how to engage in a more critical analysis of teaching during implementation of a lesson study and action plan.

First, the impact of IMP on participants' understanding of differentiated instruction will be discussed. Differentiating instruction is not a brand new idea; key elements existed as far back as the one-room classroom (Tomlinson, 2005) and researchers in the area of special education have advocated for differentiated education for at least four decades (e.g., Olenchak, 2001; Tieso, 2005). Furthermore, differentiated instruction has received increased attention over the past decade (Rock et al., 2008). However, while differentiating instruction appears more natural in language arts classroom, “differentiating instruction in mathematics is a relatively new idea” (Small, 2009, p. 1).

Researchers and educators have described key elements of differentiated instruction (e.g., Rock et al., 2008; Small, 2009; Tomlinson, 1999a) and strategies and activities for differentiating instruction (e.g., Tomlinson, 1999a; Wormeli, 2007). However, perhaps because differentiating instruction in mathematics is newer, there seems to be fewer resources specific to mathematics. Nonetheless, IMP Year 2 pedagogy facilitators placed attention on differentiating instruction in mathematics. Pedagogy facilitators shared resources about key elements and general strategies as a starting place; but then the facilitators and lead teachers (**partnership**) put the focus on differentiating instruction in mathematics. Lead teachers shared mathematics activities they had found from limited resources. The lead teachers also shared activities they had created themselves for differentiating in a mathematics classroom. Furthermore, the pedagogy facilitators created an **active learning** experience in **close proximity to practice** by setting up learning centers with differentiated instruction strategy templates in order that grade level peers might **collaboratively** create differentiated instruction mathematics activities that could be used in their own classrooms. Participant comments about the hands-on learning activity suggested they felt empowered by the experience—that as a teacher they would be able to create their own differentiated instruction activities for mathematics. For example, one participant reflected: “Getting to do the activities makes it much easier to see how we can/should create them”. Hence, not only did participants learn about key elements and generic strategies for differentiating instruction, participants learned about strategies and activities for differentiating instruction in a *mathematics* classroom.

Whereas most of the participants increased their understanding of differentiated instruction, a few participants may have developed a misconception. A few participant survey responses and reflections revealed an emphasis on associating differentiated instruction with ability grouping rather than flexible grouping. The notion was not anticipated by the researcher as flexible grouping was modeled fairly well during the project. Tomlinson (2001) provided guidelines for implementing flexible grouping in order to ensure all students have opportunities to work with other students similar and dissimilar from themselves in interest and readiness. For example, work groups assigned by teachers should at times match student readiness/interest and at other times ensure students work with a variety of classmates. When appropriate, some work groups can be created through student selection. A variety of group sizes were used throughout the IMP summer institute: pairs, small groups, large groups, and full group. At times, project leaders strategically assigned work groups to match teachers by certain criteria. For example, project leaders assigned participants to homogeneous grade level groups during the differentiated instruction make and take session. During another activity, participants were broken out by district in order to look at data from state assessments to determine the lowest indicator in their district for their grade level. These project activities supported **collective participation** amongst teachers with common interests or needs or backgrounds. In fact, many reflection and survey comments revealed that participants greatly appreciated the opportunity to participate collectively in making grade appropriate differentiated instruction content, process, and product activities for their mathematics classrooms. At other times, project leaders created groups to ensure teachers had opportunities to mix with teachers from different grade levels or schools. For example, during a jigsaw activity participants picked up articles with numbers written inside directing the participant to a reading section/group. Variety in readiness/interest/grade level/school was ensured as participants randomly walked up to pick up articles. As another example, participants were sometimes assigned to tables so that teachers from different grade levels and schools were grouped together. Small group discussions between teachers with varied experiences provided opportunities for participants to experience a key element of differentiated instruction and to broaden their knowledge. For instance, teachers had opportunities to share and learn about curriculum and instructional practices being used in different schools and at different grade levels. Thus, participants experienced working together with similar and dissimilar peers. The participants also learned how varied groupings can be constructed.

On the other hand, based on reflection about participant-observation during the summer institute, the researcher surmised that participant selection of content breakout session may have contributed to the misconception. The researcher reflected that she had tended to choose content breakout sessions quite similarly as high school and middle school mathematics teachers. Readiness and interest may have been strongly correlated for the breakout sessions and/or more breakout session material may have emphasized readiness. As content breakout sessions were prevalent during the summer institute, this may have been a reason why a few participants associated differentiated instruction more with ability grouping rather than flexible grouping. For example, on a post-summer institute survey question probing how differentiated instruction is different from individualized instruction, one participant wrote: “You can group students who are on the same level of understanding. Everyone can do the same type of activity with different levels of complexity”. However, the way the question was posed (differentiated versus individualized) could also be an explanation for the few responses emphasizing ability grouping as a component of differentiating instruction. Nonetheless, the preponderance of the data suggested that most participants learned about key elements and generic structures for activities to differentiate instruction as well as strategies and activities for differentiating instruction more specifically in a mathematics classroom.

Second, the use of math-related literature, *The Number Devil*, as an entry point to mathematical topics and concepts provided a natural connection for project focus on supporting content area reading. The Commission on Adolescent Literacy of the International Reading Association asserted that adolescents deserve teachers who will support their continued development as readers (Moore et al., 1999). The project facilitators modeled strategies aimed at supporting the participants as they **learned about mathematics** from **math-related literature** and as they learned about pedagogy from reading articles and materials. Of interest, strategies that support content area reading complement differentiated instruction and best practices from a cognitive science perspective (Beers & Howell, 2005). For instance, graphic organizers can help students make connections and focus students on the big picture; advance questions can also help students focus their reading. Thus, content area reading strategies can focus students on essential ideas; in turn, focusing on essential ideas is a key element of differentiating instruction. Furthermore, allowing students to discuss and write about their learning engages students in evaluating their understanding of the reading. The NRC (2005) suggested that classroom

discussion is a pedagogical approach that supports students' development of metacognitive skills. The project prompted much participant reflection and discussion in both small and large groups. Consequently, research-based strategies for supporting content area reading also complemented differentiated instruction and best practices.

Third, project focus on pedagogical strategies for fostering the development of number sense integrated nicely with standards-based teaching, examining **curricular resources**, and **learning mathematical content** in *The Number Devil*. For example, in *PSSM* (2000) the development of number sense was described as central to the Number and Operations Content Standard. In addition, as number sense typifies sense-making (Verschaffel et al., 2007), and as some components of number sense are number representations, number relationships, and reasonableness (McIntosh, Reys, & Reys, 1997b), number sense identifies with Process Standards such as Representation, Connections, and Reasoning. Pedagogy facilitators and lead teachers modeled using **curricular resources** to engage students in the Process Standards. Furthermore, topics arising in *The Number Devil* and explored in content sessions such as number systems also integrated with pedagogical focus on fostering number sense. Finally, Yang et al. (2009) suggested the knowledge teachers have of number sense and the value they place on its importance may be critical factors for students' opportunity to develop number sense. The results of the study revealed that some participants learned new strategies for fostering the development of number sense. In addition, some participants came to recognize the importance of their role in helping students develop number sense, and thus became more deliberate in using strategies to foster number sense.

Fourth, another project pedagogical focus involved implementing a lesson study and action plan during the school year, and thus provided participants with learning opportunities in **close proximity to practice** as recommended by researchers (e.g., Ball & Cohen, 1999; Borasi & Fonzi, 2002; Elmore, 2002). In addition, researchers (Ball & Cohen, 1999; Hiebert et al., 2007) have suggested that teachers need to develop dispositions toward analysis of teaching in order to learn, and thus improve, teaching. Researchers (e.g., Ball & Cohen, 1999; Borasi & Fonzi, 2002) have also recommended that professional development provide opportunities for teachers to develop skills and dispositions for inquiry into practice. Furthermore, Darling-Hammond (2006a) pointed out the knowledge base for teaching is too expansive for a single teacher to master; not only would teachers need critical observation and analysis skills to learn

from practice, teachers would also need to be expert collaborators in order to learn from each other. Results of the study revealed that participants learned how to **analyze their teaching** more critically by engaging in more **formal collaboration**, data analysis, and hypothesis testing during the lesson study and action plan implementation.

Finally, project foci on strategies for differentiating instruction, supporting content area reading, fostering the development of number sense, and critically analyzing teaching, along with mathematical content being framed around math-related literature, complemented and supported participants as they experienced and learned about standards-based instruction. For example, **exploring mathematical concepts** as introduced in the *The Number Devil* and extended by content facilitators provided participants with opportunities to **actively engage** in the Process Standards with good **math-related literature** (Whitin & Whitin, 2004). As number sense typifies sense-making (Verschaffel et al., 2007), and as some components of number sense are number representations, number relationships, and reasonableness (McIntosh, Reys, & Reys, 1997b), number sense identifies with Process Standards such as Representation, Connections, and Reasoning. Supporting **coherence**, lesson study and action plan development often started with examining state standards (for which national standards have served as a model) and curriculum. Finally, as strategies for differentiating instruction (Tomlinson, 2001) and supporting content area reading (Beers & Howell, 2005) have connections with best practices from a cognitive science perspective, the strategies also complement standards-based instruction. The results indicated that participants valued learning mathematics while engaging in the Process Standards. Furthermore, participants expressed that they wanted to employ strategies modeled during the summer institute (e.g., discussion and reflection, critical thinking, real world applications, multiple representations, hands-on activities) to engage their students in the Process Standards.

Discussion of results related to the impact of IMP on participants' teaching practice

In the results, the researcher noted it was difficult to determine the impact of IMP on participants' teaching practice due to the data collected. However, subtleties in reflections by participants during the share fair and by mini-case participants during the school year suggested that IMP made an impact on some of the participants' teaching practice at least with regard to implementation of an action plan and/or lesson study. Some participants engaged in a more **critical analysis of their teaching**. In addition, many participants implemented instructional

strategies in their **practice** related to foci of IMP Year 2: supporting content area reading, fostering number sense, implementing standards-based instruction, and differentiating instruction.

Several features and elements of IMP Year 2 complemented each other in providing participants with opportunities for strengthening teaching practice. For instance, assessment was considered a key component for differentiating instruction (e.g., Tomlinson, 2001) , for implementing an action plan (Wiggins & McTighe, 2005), and for participating in a lesson study (e.g., C. Lewis et al., 2004). Focusing on assessment in multiple contexts supported IMP participants in the development of both a mindset for differentiating instruction and a disposition toward inquiry into practice. First, with regard to differentiating instruction, the project highlighted the importance of pre-assessment to determine students' readiness and learning needs. Participants also **actively learned** how to create product assessments that allowed for student choice, and that could be used in their own **practice**. Second, assessment was also an integral component for the development and implementation of an action plan. Project directives for participant use of the *Understanding by Design* (Wiggins & McTighe, 2005) action plan template engaged participants in **close proximity with practice** with establishing learning goals, predetermining assessment evidence that would be used to determine whether goals were attained, and describing learning activities that would enable students to achieve the desired learning. Being able to establish learning goals and assess whether goals are achieved are some of the skills required for **analyzing teaching** (2007). Furthermore, during lesson study many participants strove to **interpret student thinking**, and to revise lessons based on their observations and assessments. Hence, project focus on assessment from multiple perspectives supported strengthening teaching practice.

A second example also reveals how harmonic features of IMP Year 2 provided participants with opportunities for strengthening teaching practice. *The Number Devil*, along with content facilitator direction, invited participants to reflect upon conceptual underpinnings for many number concepts. Project focus on number sense encouraged participants to consider number proficiency as far more than procedural aptitude; number sense is manifested by productive reasoning about numbers, mental computation, estimation, and understanding of number relationships, relative size, and multiple representations of numbers (e.g., McIntosh, Reys, & Reys, 1997a). Furthermore, many participants found measurement and estimation to be

low indicators for their grade level and school district. These coalescing components and features of IMP Year 2 impacted participants' teaching practice evidenced by many teachers employing strategies for developing number sense as part of their action plan or lesson study.

The strongest evidence of IMP's impact on teaching practice pertained to differentiated instruction. Levy suggested that "every teacher who has entered a classroom has differentiated instruction in one way or another" (Levy, 2008, p. 162). On the other hand, differentiated instruction has been described as a philosophy (e.g., Tomlinson, 2000) or mindset (Wormeli, 2007) rather than an instructional strategy. Differentiated instruction involves teacher responsiveness in meeting each student's needs through a set of strategies (Levy, 2008). What differentiated instruction looks like in one classroom may be different from what it looks like in another classroom (e.g., Tobin & McInnes, 2008). Wormeli (2007) recommended starting small by focusing on one element a week or a month. No one can accommodate every student in every class every single day.

The results of this study provide evidence that most participants learned about key elements of differentiating instruction and about strategies for differentiating instruction in a mathematics classroom. However, the results did not necessarily reveal that participants developed an overarching mindset of differentiation in their mathematics classrooms. On project-collected survey responses at the end of the summer institute, participants reported that they had learned specific strategies to differentiate instruction. However, many participants wondered how to best organize their time and classroom for differentiated instruction. Others suggested they needed to experience differentiated instruction in the classroom. Share fair presentations revealed most participants at minimum employed a strategy or element of differentiated instruction during the implementation of their action plan or lesson study in a mathematics classroom. However, the data did not reveal whether participants had developed a mindset for differentiation. Observations of mini-case participants revealed teachers were engaging diverse learners through cooperative learning, technology, and hands-on learning. In addition, students were encouraged to discuss strategies and justify their thinking. Most often, classroom observations revealed activities directed to the whole class albeit in a cooperative learning setting. There was at least one exception. During an observation interview, one teacher described how some students had been identified by pre-assessment for self-directed web-based learning during the unit. The identified students were to mostly work independently during a

fraction unit via web-based instruction. In addition, the students were assigned different homework and projects from the rest of the class. On my observation day, the students were brought back together with the whole group in order to learn a specific model associated with the textbook curriculum. Thus, the teacher demonstrated using compacting (allowing advanced learners to be exempt from whole-class instruction of material they already knew), and purposeful instructional groupings. But, the teacher also described this was the first time she was trying a breakout activity for a group of students to work individually through web-based instruction. Overall, evidence may not have revealed that participants developed a mindset toward differentiating instruction because of limitations of the study. Pre- and post-IMP observations, along with observations of many more teachers over longer periods of time would likely be needed in order to identify whether or not some participants had developed a mindset for differentiated instruction in a mathematics classroom.

Implications

Descriptions of features of professional development that support teacher learning and teaching practice have coalesced into a view of effective professional development whereby focus is given to both “subject matter and issues of teaching and learning as they come together in classroom practice” (S. Cohen, 2004, p. 3). However, whereas the research base on characteristics of effective professional development has grown considerably (e.g., Loucks-Horsley et al., 2003; Sowder, 2007), Borko suggested the research base is thin as to “what and how teachers learn from professional development” (2004, p. 3). Educational partnerships between public school teachers, administrators, and university faculty have been envisioned as collaborative sites where research and practice can intersect for the joint purpose of improving student learning (Holmes Group, 1986). Currently, partnerships are at the core for Title IIB MSP professional development initiatives intending to increase content knowledge and instructional skills of mathematics and science teachers in order to support student learning (U.S. Department of Education, 2008b). This case study on IMP Year 2 provides evidence of the impact of a Title IIB MSP professional development program on teacher learning and teaching practice.

Although programs of teacher education have traditionally separated knowledge of mathematics and knowledge of pedagogy by structuring separate course offerings from education departments and discipline departments, some recent programs have attempted to

bring content and pedagogy together (National Research Council, 2001). The findings of this study suggest the **partnership** requirements for MSP Title IIB grants have a positive impact for professional development opportunities. Albeit there was some tension between the partners at times, learning was sometimes an outgrowth of negotiating and talking through the conflict. Content facilitators took as their responsibility to provide focus on mathematics content and to share how mathematicians approach the study of mathematics. Pedagogy facilitators took as their primary responsibility an attention to best practices and research-based teaching methods. K-12 teachers brought current experiences with children, curriculum, and state assessments pertaining to their particular grade level. The partnership provided a check and balance system lest one area become too dominant to the neglect of other areas. Although each partner brought focus to their area of expertise, it was most fascinating how the partners learned from each other and integrated content, pedagogy, and practice together. As Darling-Hammond (2006a) pointed out, the knowledge base for teaching is too expansive for a single teacher to master; not only do teachers need critical observation and analysis skills to learn from practice, teachers also need to be expert collaborators in order to learn from each other. The partnership requirements for MSP Title IIB projects supported the **collaborative efforts** by facilitators and participants in IMP Year 2. For example, the content facilitators learned about differentiating instruction and reading in the content area, and then incorporated some of the strategies into their content sessions. As another example, as the content sessions had a major focus on Number and Operations (e.g., numeral systems, number sets), the pedagogy facilitators coordinated by drawing attention on how to develop students' number sense. Finally, K-12 teachers contributed by articulating and questioning how content and pedagogy might play out in their own classrooms. As with the Venn diagram depicting the National Academy of Education's framework (Bransford et al., 2005) of three general areas of knowledge, skills, and dispositions important to teachers (i.e., Knowledge of Teaching, Knowledge of Subject Matter & Curriculum Goals, Knowledge of Learners & their Development in Social Contexts), it is in the intersection where a "vision of professional practice" (p. 11) is realized. IMP Year 2 provided a supportive environment whereby participants and facilitators could learn and grow in combined efforts to support student learning.

As another implication, partnership requirements, along with IMP project goals and strategies, provided a vehicle whereby NCTM's "commitment to both excellence and equity"

(2000, p. 3) could be addressed in a **coherent** manner. For example, Title IIB MSP program goals included increasing teachers' content knowledge and improving teachers' instructional skills. IMP Year 2 content sessions challenged participants to understand some higher mathematical concepts (e.g., proof, conceptual underpinnings and mathematical relationships). In addition, the project intended to increase participants' implementation of standards-based instruction; and, standards-based instruction places high demands on teachers' content knowledge (e.g., Floden, 1997) and pedagogical knowledge (e.g., NCTM, 2000). Furthermore, project focus on differentiated instruction provided participants with opportunities to learn strategies for addressing NCTM Standards amongst a diverse student population. As such, the project provided participants with opportunities to develop skills for attending to both excellence and equity.

Finally, IMP Year 2 also provides an illustration of how professional development can address scaling up. In order for improvement in teaching practice to occur on a large scale in the U.S., Elmore (1996) suggested teachers would need incentives, training, and time. However, the structure in American schools requires teachers to spend most of their school day in teaching situations with students (Darling-Hammond & Sclan, 1996), and little time is left for preparation and joint planning which have been observed to be critical components for supporting improvement of teaching practice in Asian countries (e.g., Ma, 1999; Stigler & Hiebert, 1999). Scaling up was supported as Title IIB MSP funding provides teachers with incentives for **collaborating** and **learning**, much of which is during the summer. Furthermore, the impact of IMP Year 2 project went beyond participants. IMP Year 2 provided support for some participants who were interested in taking on leadership roles back in their schools and districts. Some lead teachers and participants created action plans to address leadership goals as opposed to low mathematics indicators. During the school year, lead teachers and some participants (pursuing leadership roles) shared strategies for differentiating instruction and participating in lesson studies. In addition, some participants (not pursuing leadership roles) participated in lesson studies with teachers who had not participated in IMP. Hence, the impact of IMP spread beyond project participants. If the structure in American schools does not change, funding for projects like Title IIB MSP will continue to be crucial for supporting improvement in U.S. education.

Recommendations for Future Research

Borko (2004) suggested the research base is thin with respect to what teachers learn during their professional development experiences. This case study of a Title IIB MSP project sought to provide a qualitative examination of the characteristics and strategies used in the project, and to understand their impact on teaching learning and practice. As such, the study contributed to a limited research base. However, further study regarding the impact of professional development on teacher's knowledge and practice is needed. Building upon the results of this study, future research could continue to provide insight as to the impact of professional development, particularly Title IIB MSP projects.

This study would fall under Borko's Phase 1 research of an "existence proof" that a single site, individual professional development program positively impacted teacher learning and practice. The results of the case study are tied to particular strategies employed in the project. Future study might seek to conduct Borko's Phase 2 research to determine whether the features and strategies of IMP Year 2 could be enacted with integrity "in different settings and by different professional development providers" (2004, p. 9).

In addition to expanding the level of research to Phase 2, further study could be made at Phase 1. For instance, although the qualitative study illuminated what participants learned during the project, additional research might employ more rigorous pre- and post-project assessment measures of teachers' mathematical knowledge for teaching for which validity analyses have been conducted (e.g., Center for Research in Mathematics and Science Teacher Development, n.d.; Learning Mathematics for Teaching Project, n.d.).

Furthermore, the data collected revealed the project's impact on teachers' knowledge far better than the impact on teaching practice. Additional study in Phase 1 might include more observations of participants' teaching practice over time. Observations of the four mini-case participants during the school year after the summer institute revealed the teachers were engaging diverse learners through cooperative learning, technology, discussion, and hands-on learning. In general, the teachers were enthusiastic, caring, and reflective practitioners. Thus, the observations were valuable in order to better understand the teachers who might be choosing to participate in the sustained, intensive professional development project. However, the observations revealed little about the impact of IMP Year 2 on teaching practice. Extended time in observations with more participants might better reveal the impact of IMP on teaching

practice. For example, as observations of four mini-case participants may have not been representative of the whole group, observations of more participants could provide further insight. In addition, no pre-project classroom observations were made for the current study. Pre-project observations could reveal a baseline about participants' practice with regard to supporting content area reading, fostering the development of number sense, differentiating instruction, and employing standards-based instruction. Then, the impact of the project on teaching practice could be better determined. Furthermore, additional observations per participant post-summer institute could be made as instructional practices for a participant might be more evident across multiple observations. For example, opportunities to observe differentiated instruction (e.g., flexible grouping, responsiveness to student readiness and interest, adjustments for products, integration of assessment and instruction, focus on big ideas) would likely be more possible across multiple observations. Finally, to better recognize the long-term impact of the project on teaching practice, observations could be made for several years after the project.

Future study might also seek to understand the impact of the project with regard to teachers who attended all three years of IMP. Although the project focused on lesson study, differentiated instruction, standards-based instruction, and mathematical content across all three years, the researcher noticed different emphases and different activities were used across the first two years of the project. For example, in the first year of IMP, an overview of differentiated instruction was presented by a speaker; however, not many examples of strategies were focused on mathematics. In the second year, differentiated instruction for a *mathematics* classroom was the focus. As another example, IMP Year 1 modeled the lesson study process. After two teacher leaders created a lesson together, participants observed as one lead teacher taught the lesson to a group of 4th and 5th grade summer volunteers. Participants also participated in a “debriefing” after the lesson. Afterwards, the lead teachers revised the lesson and the second teacher taught the lesson the next day to a new group of student volunteers followed by a group debriefing. The experience was very powerful. However, in Year 2 IMP, lesson study received less emphasis due to time constraints. As half of the participants had experienced Year 1 IMP, the lesson study component for Year 2 evolved. A video of a Japanese lesson with debriefing was viewed and discussed during the summer institute. In addition, participants took part in a jigsaw reading activity for the article “A Lesson is Like a Swiftly Flowing River” (C. Lewis & Tsuchida, 1998). As a final example, although the project sought to make higher mathematical concepts accessible

to K-8 teachers, the mathematical content addressed in each year was different. Year 1 content tackled basic ideas of calculus with an emphasis on making connections to middle school mathematics. For Year 2, the project used math-related literature to spark an examination of basic number theory. The final year of IMP would focus on patterns in algebra and real world applications using the topic of coding theory. A longitudinal study of teachers who participated in all three years might reveal whether long-term participation in the project had a more pronounced impact on teachers' practice. For example, further study might reveal whether some teachers developed habits of inquiry into practice, a mindset of differentiated instruction, and/or a disposition toward engaging students in the NCTM Process Standards.

Additional study might seek to better understand **how** the IMP project impacted participants' knowledge and teaching practice. During participant-observation, the researcher focused on gathering data revealing what the participants were learning and experiencing; thus, the study revealed what participants learned. To better understand **how** the project impacted participants' knowledge and teaching practice, data might also be collected from the views of the facilitators.

Finally, future research on IMP could include a closer examination of the impact of the project on student achievement. Borko (2004) suggested the research base regarding the impact of teacher change on student achievement is limited. Wenglinsky's study (2002) revealed five aspects of teacher quality were positively related to student achievement: 1) the teacher's major, 2) professional development in higher-order thinking skills, 3) professional development in learning how to teach different populations of students (collapsed from three measures including professional development in cultural diversity, in teaching limited-English-proficiency (LEP) students, and in teaching special-needs students), 4) teaching practices utilizing hands-on activities (collapsed from three measures of the relevant time students spent working with blocks, working with objects, and solving real-world problems), and 5) teaching practice incorporating higher-order thinking skills (from the single measure of the relative time students spent solving unique problems). IMP addressed several of these aspects through its focus on differentiating instruction in order to teach a diverse student population, and its engagement in the NCTM Process Standards (Reasoning, Problem Solving, Representation, Communication, Connections) which lends itself to higher-order thinking and hands-on activities. More generally, IMP Year 2 employed many strategies and features associated with high quality

professional development. Although participants reported about the impact of the action plan and/or lesson study on student achievement at the share fair, future research regarding the impact of IMP with respect to student achievement on a broader level could contribute to a limited research base.

Summary

Ongoing professional development has been purported to be an essential mechanism for eliciting change in teachers' knowledge and teaching practice in support of school improvement (e.g., Desimone et al., 2006; Elmore, 2002; Hawley & Valli, 1999). While consensus has built up about strategies and characteristics of effective professional development (e.g., Elmore, 2002; Garet et al., 1999; Sowder, 2007), Borko (2004) suggested we know little about "what and how teachers learn from professional development" (p. 3). In addition, results of Title IIB MSP professional development initiatives are only beginning to come in (e.g., Blank et al., 2008; Gummer & Stepanek, 2007). The Infinite Mathematics Project professional development model under study embodied several characteristics and utilized several strategies associated with "high quality" professional development. This case study of a Title IIB MSP project provides a qualitative examination of the characteristics and strategies used in the project, and of their impact on teacher learning and practice.

The Infinite Mathematics Project embodied many characteristics and strategies of effective professional development. This study provides details regarding strategies used in IMP as discussed in Chapter 4. In addition, attention was drawn to strategies that impacted specific domains of teacher learning and practice in Chapter 5. As such, the study reaffirms characteristics (content-focused, active learning, sustained, coherence, collaborative participation) and strategies (giving teachers opportunities to participate in close proximity to practice, to develop skills for analyzing practice, to engage as mathematical learners, to examine curricular resources, to examine student thinking) of effective professional development and their impact on teacher learning and practice.

This study provides detailed illustrations of what teachers learned through participation in IMP Year 2. Teachers gained both mathematical content knowledge and pedagogical knowledge. First with regard to content knowledge, teachers gained knowledge *about* mathematics, substantive knowledge of mathematics, pedagogical content knowledge, and

curricular knowledge (of both curricular resources and vertical curricular knowledge). In the discussion, it was noted that historically opportunities for learning knowledge *about* mathematics and curricular knowledge during teacher development have been fairly remiss. However, the study revealed that project participants gained knowledge *about* mathematics and curricular knowledge. In addition, although researchers (Ball & Bass, 2000) have suggested that pedagogical content knowledge builds up in teachers over time, IMP Year 2 provided a forum whereby teachers could gain pedagogical content knowledge. Furthermore, while past studies have revealed that prospective and practicing teachers lack a deep understanding of the concepts they teach (e.g., Ball, 1990aa, 1990b; Borko et al., 1992; Ma, 1999), IMP Year 2 provided learning experiences for teachers to gain substantive knowledge of mathematics.

Second with regard to pedagogical knowledge, IMP participants learned about effective pedagogical strategies for mathematics instruction. For example, participants learned about key elements and generic strategies for differentiating instruction. But more importantly, teachers learned about strategies and about creating their own activities for differentiating instruction in a mathematics classroom. Participants also learned about strategies for supporting students' reading in the content area and the engagement potential of math-related literature. Some participants learned new or additional strategies for fostering the development of number sense and some teachers came to place a higher value on the importance of their role in helping students develop number sense. Participants learned about implementing standards-based instruction as they experienced and learned about strategies (e.g., discussion and reflection, critical thinking, real world applications, multiple representations, hands-on activities) for engaging students in the NCTM Process Standards. In addition, participants learned how to more critically analyze teaching during their engagement in more formal collaboration, data analysis, and hypotheses-testing as part of their implementation of a lesson study and action plan during the school year.

Third, the study provides some evidence that IMP had an impact on teaching practice. However, due to the limited number of observations of which all were post-summer institute, determining the impact of IMP on teaching practice was hindered. Participant reports during the share fair regarding implementation of the lesson study or action plan revealed some participants used content area reading strategies or math-related literature, strategies for fostering the development of number sense, and strategies for implementing standards-based instruction. In

addition, some participants engaged in more formal collaboration, data analysis, and hypothesis-testing. Furthermore, the strongest evidence regarding teaching practice suggests many participants tried new strategies for differentiating instruction.

The results further indicate the partnership requirements for MSP Title IIB grants have a positive impact on professional development. IMP Year 2 provides an example of a professional development partnership between content facilitators (from an IHE Mathematics Department), pedagogy facilitators (from an IHE School of Education), and participants (K-12 teachers) that provided an environment whereby facilitators and participants learned about content, pedagogy, and practice with the common goal of supporting school improvement. In addition, the project provided participants with opportunities to develop skills for attending to both excellence and equity. Finally, the project illustrated how professional development can scale up and reach a larger audience. Overall, IMP Year 2 provides an illustration of a Title IIB MSP project that supports the improvement of U.S. mathematics education.

Appendix A - Informed Consent Form

Dear Infinite Mathematics Project (IMP) Participant:

I am a mathematics instructor at Newman University in Wichita, Kansas and I am currently pursuing a doctoral degree in Curriculum and Instruction from Kansas State University. As part of my dissertation research I am studying the impact of the IMP professional development program on teachers' knowledge and teaching practice. The results of this research will help inform how the IMP professional development program impacts teacher learning and teaching practice.

I will be observing the two-week 2008 IMP summer institute and collecting information as related to the program activities and discussions for the entire group of participants. I may also observe and collect information from IMP follow-up activities. I will be reviewing IMP summer institute homework. I will be reviewing survey data collected by the project. I will be reviewing homework and session reflections. In addition, I would like to gather more information from a smaller group of participants. Additional information would be gathered through interviews prior to and subsequent of the two-week summer institute, and through two classroom observations with interviews before and after the observations. Interviews may be audiotaped. Classroom observations will be videotaped.

If you are willing to participate in this study as described in the previous paragraph, please sign and date this consent form. Please be assured that care will be taken to remove individual and school identifiers in any publication of the results of this study. Data will be reported as grouped data or will be provided without being identified or will be identified by a pseudonym. Audio and video recordings will not be made available to the public.

If you have any questions or concerns regarding this research project, please contact me at Newman University, 316-942-4291 ext. 2247 or sponselb@newmanu.edu.

Your participation is strictly voluntary. Your decision whether or not to participate will not affect your IMP stipend and/or your course grade. If you do agree to participate, you are free to not answer any question or to withdraw from participating in the study at any time.

Thank you for reviewing this letter. If you do agree to participate, your cooperation is greatly appreciated. As a token of my appreciation, I will provide you with a total of \$60 in cash and/or gift cards, distributed at various times (at the end of the summer institute and at the two observation visits) throughout 2008-2009.

Sincerely,

Name _____
(Print)
Signature _____

Barbara Sponsel

Date _____

Appendix B - Homework/Session Reflections

1) Please reflect on the content (oftentimes morning) session activities and/or discussions.

2) Please reflect on the pedagogy (oftentimes afternoon) session activities and/or discussions.

3) Please reflect on the homework assignment.

Here are some examples of student homework reflections:

- “I didn’t think I was really going to catch on to this, but I did once I did a few problems on my own.”

- “I am apathetic towards this material so far. I don’t know if I like it or not yet. It seems fairly straightforward in class when we do examples, but it is a different story when I try to work homework problems in the evening.”

- “I must say that today’s material is my favorite section so far. I love it! I don’t know why, but it just works for me and I can figure out the problems really fast.”

- “When it comes to today’s material, I still need to look at the basic rules to solve the problem. But, the more problems I get to work out, the better I get since I start to remember more.”

- “I am having a difficult time with this, but I will keep working on it.”

**Appendix C - OEIE Summary of Data Analysis Pre-Post Content
Test**

**Summary of Data Analysis
Pre-Post Content Test
for
Infinite Math Project (IMP)**

Prepared for:
Dr. David Allen
249 Bluemont
Kansas State University

Summer, 2008

Submitted by:
**Office of Educational Innovation
and Evaluation (OEIE)**
2323 Anderson Avenue, Suite 220
Kansas State University
Manhattan, Kansas 66502
Phone: 785-532-5930
Fax: 785-532-7185



SUMMARY OF DATA ANALYSIS
PRE-POST CONTENT TEST for INFINITE MATH PROJECT
Summer 2008

Evaluation Questions

The IMP Principal Investigator (PI) contracted with the Office of Educational Innovation and Evaluation (OEIE) to assist in analyzing pre-collected data to answer three research questions:

1. Does teachers' mathematical content knowledge change after participating in the current intervention?
2. Is grade level taught, years of teaching mathematics, whether or not teachers are certified to teach mathematics, and whether or not teacher are classified as "highly qualified" by the Kansas State Department of Education associated with changes in teachers' mathematical knowledge?
3. Is there a correlation between teacher self-rated perceptions of knowledge and comfort in teaching mathematical concepts survey ratings and teacher mathematical content test scores?

Key Variables

Independent Variables

The four key independent variables were Grade Level Taught (K-5, 6-8, 9-12), Years Teaching Math (0-5, 6-15, over 15), Math Certification (Yes, No), and Highly Qualified Status (Yes, No/Don't know). The Highly Qualified Status categories of "No" and "Don't know" were collapsed for analysis.

Dependent Variables

One dependent variable was Mathematical Content Knowledge as measured by an exam. Each exam consisted of 15 questions worth 2 points a piece. The total score possible was 30 points. Participants had a pre-workshop score and a post-workshop score.

Four other dependent variables were Knowledge of Math Topics, Knowledge of Teaching Strategies, Comfort with Math Topics, and Comfort with Teaching Strategies. Participants had two scores on each dependent variable: a pre-workshop score and a post-workshop score. These scores were means of the individual items composing each particular scale. The Knowledge of Math Topics scale and the Comfort with Math Topics scale were composed of 25 items each, and the Knowledge of Teaching Strategies and the Comfort with Teaching Strategies scales were composed of five items each.

Data Sample Used in Analysis

There were 31 teachers who completed the pre-post survey and the pre-post content test. One participant indicated they taught an “other” grade, thus were not part of the population of interest and were excluded from the analysis yielding N=30.

Results

To address Evaluation Question 1 and 2, one mixed factors analysis of variance (ANOVA) was conducted on the key dependent variable. The SPSS output from the analysis is included in Appendix A. The repeated measures factor (i.e., dependent variable) was Mathematical Content Knowledge score. Participants had two scores on the dependent variable: a pre-workshop score and a post-workshop score. The four between subjects factors (i.e., independent variables) included in the analyses were Grade Level Taught (3 levels: K-5, 6-8, 9-12), Years Teaching Math (3 levels: 0-5, 6-15, over 15), Math Certification (2 levels: Yes, No) and Highly Qualified Status (2 levels: Yes, No/Don't know). To address Evaluation Question 3, two sets of simple bivariate correlations were performed on the dependent variables. One set was

performed on the pre-workshop scores, and another set was performed on the post-workshop scores. The SPSS output from these correlation analyses is included in Appendix B and C.

Evaluation Question 1

Results revealed that participants showed improvement between pre-workshop to post-workshop in their Mathematical Content Knowledge scores [$F(1, 23) = 204.33, p < .001$, partial eta squared = 0.90].

Evaluation Question 2

Inspection of the interaction effects revealed that none of the four key independent variables were associated with improvements related to Mathematical Content Knowledge scores. Participants were equally likely to experience improvement in content knowledge from the workshop regardless of their Grade Level Taught, Years Teaching Math, Math Certification or Highly Qualified Status. The statistics for the nonsignificant interactions are provided below. It is important to note that it was only feasible to test the effects of these independent variables using one-way interactions with the repeated measures factors (i.e., dependent variables). Low cell sizes made it impossible to investigate more intricate interactions.

Improvements between pre- and post-workshop related to content knowledge scores were not influenced by Highly Qualified Status [$F(1, 23) = 0.71, p = .41$, partial eta squared = 0.03], Grade Level Taught [$F(2, 23) = 1.42, p = .26$, partial eta squared = 0.11], Years Teaching Math [$F(2, 23) = 1.05, p = .37$, partial eta squared = 0.08], or Math Certification [$F(1, 23) = 3.94, p = .06$, partial eta squared = 0.15], although this final comparison did approach significance.

Evaluation Question 3

Results revealed that among pre-workshop scores, Mathematical Content Knowledge was significantly correlated with Knowledge of Math Topics ($r = .59, p < .001$) and Comfort with Math Topics ($r = .38, p = .040$). Generally speaking, the higher a participant's Mathematical

Content Knowledge prior to the workshop, the higher their Knowledge of Math Topics and Comfort with Math Topics at that time.

Among post-workshop scores, Mathematical Content Knowledge was not significantly correlated with any of the other measures. However, its correlations with Comfort with Teaching Strategies ($r = .37, p = .051$) and Comfort with Math Topics ($r = .36, p = .060$) approached significance. The lack of significant findings may have resulted from a ceiling effect experienced among the post-workshop Mathematical Content Knowledge scores. Post-workshop content knowledge exam scores were very high, thus reducing the variance ($M = 28.00, SD = 2.33$) compared to pre-workshop scores ($M = 17.10, SD = 4.33$).

Summary

Teacher Mathematical Content Knowledge scores were significantly different from pre- to post-survey indicating the treatment improved this knowledge of math content. There were no differences in improvements based on Grade Level Taught, Years Teaching Math, Math Certification or Highly Qualified status which indicates the treatment may be effective across all groups. This finding is very similar to the previous report which indicated that the workshop increased teachers self-rated knowledge and comfort in teaching math. Further, those analyses also indicated similar effectiveness of the workshop across groups.

Appendices

Appendix A: Analysis Pre-Post Mathematical Content Knowledge

Appendix B: Analysis Correlations for Pre-Workshop Mathematical Content Knowledge

Appendix C: Analysis Correlations for Post-Workshop Mathematical Content Knowledge

Appendix D - OEIE Summary of Data Analysis Pre-Post Survey

**Summary of Data Analysis
Pre-Post Survey
for
Infinite Math Project (IMP)**

Prepared for:
Dr. David Allen
249 Bluemont
Kansas State University
Summer, 2008

Submitted by:
Office of Educational Innovation
and Evaluation (OEIE)
2323 Anderson Avenue, Suite 220
Kansas State University
Manhattan, Kansas 66502
Phone: 785-532-5930
Fax: 785-532-7185



SUMMARY OF DATA ANALYSIS
PRE-POST SURVEY for INFINITE MATH PROJECT
Summer 2008

Evaluation Questions

The IMP Principal Investigator (PI) contracted with the Office of Educational Innovation and Evaluation (OEIE) to assist in analyzing pre-collected data to answer two research questions:

1) Does teachers' knowledge and comfort in teaching a variety of mathematical concepts change after participating in the current intervention?

2) Is grade level taught, years of teaching mathematics, whether or not teachers are certified to teach mathematics, and whether or not teachers are classified as "highly qualified" by the Kansas State Department of Education associated with changes in teachers' knowledge and comfort in teaching mathematical concepts?

Key Variables

Independent Variables

The four key independent variables were Grade Level Taught (K-5, 6-8, 9-12), Years Teaching Math (0-5, 6-15, over 15), Math Certification (Yes, No), and Highly Qualified Status (Yes, No/Don't know).

Dependent Variables

The four key dependent variables were Knowledge of Math Topics, Knowledge of Teaching Strategies, Comfort with Math Topics, and Comfort with Teaching Strategies.

Participants had two scores on each dependent variable: a pre-workshop score and a post-workshop score. These scores were means of the individual items composing each particular scale. The Knowledge of Math Topics scale and the Comfort with Math Topics scale were composed of 25 items each, and the Knowledge of Teaching Strategies and the Comfort with Teaching Strategies scales were composed of five items each.

Data Sample Used in Analysis

OEIE received 32 teacher responses to the pre-survey and 32 responses to the post survey. After matching teacher IDs there were 31 matched pre-post responses. One participant indicated they taught an “other” grade, thus were not part of the population of interest and were excluded from the analysis yielding N=30. There were 29 responses for the analyses of the two knowledge variables because one participant did not complete the post survey. There were 28 responses used in the analysis of the two comfort variables because one teacher did not complete the post survey (as previously mentioned) and one teacher completed the knowledge section of the post-survey but not the comfort section.

Results

A series of four mixed factors analyses of variance (ANOVAs) were conducted on four key dependent variables. The SPSS output from analysis is included in Appendices A - D. The repeated measures factors (i.e., dependent variables) were Knowledge of Math Topics, Knowledge of Teaching Strategies, Comfort with Math Topics, and Comfort with Teaching Strategies. Participants had two scores on each dependent variable: a pre-workshop score and a post-workshop score. The four between subjects factors (i.e., independent variables) included in the analyses were Grade Level Taught (3 levels: K-5, 6-8, 9-12), Years Teaching Math (3 levels: 0-5, 6-15, over 15), Math Certification (2 levels: Yes, No) and Highly Qualified Status (2 levels: Yes, No/Don't know).

Evaluation Question 1

Results revealed that participants reported improvements between pre-workshop to post-workshop on all four dependent measures. In other words, participants reported improvements in Knowledge of Math Topics [$F(1, 22) = 36.41, p < .001, \text{partial } \eta^2 = 0.62$], Knowledge of Teaching Strategies [$F(1, 22) = 29.53, p < .001, \text{partial } \eta^2 = 0.57$], Comfort with Math Topics [$F(1, 21) = 28.84, p < .001, \text{partial } \eta^2 = 0.58$], and Comfort with Teaching Strategies [$F(1, 21) = 30.78, p < .001, \text{partial } \eta^2 = 0.59$].

Evaluation Question 2

Inspection of the interaction effects revealed that none of the four key independent variables were associated with improvements related to knowledge or comfort scores. Participants were equally likely to experience improvement in any of the dependent variables from the workshop regardless of their Grade Level Taught, Years Teaching Math, Math Certification or Highly Qualified Status. The statistics for the nonsignificant interactions are provided below. It is important to note that it was only feasible to test the effects of these independent variables using one-way interactions with the repeated measures factors (i.e., dependent variables). Low cell sizes made it impossible to investigate more intricate interactions.

Math Certification did not influence participants' improvements between pre- and post-workshop related to Knowledge of Math Topics [$F(1, 22) = 0.07, p = .79, \text{partial } \eta^2 = 0.00$], Knowledge of Teaching Strategies [$F(1, 22) = 1.11, p = .30, \text{partial } \eta^2 = .05$], Comfort with Math Topics [$F(1, 21) = 0.10, p = 0.75, \text{partial } \eta^2 = 0.01$], or Comfort with Teaching Strategies [$F(1, 21) = 1.09, p = 0.31, \text{partial } \eta^2 = 0.05$].

Highly Qualified Status was not associated with participants' improvement between pre- and post-workshop related to Knowledge of Math Topics [$F(1, 22) = 0.56, p = .46, \text{partial } \eta^2 = 0.03$], Knowledge of Teaching Strategies [$F(1, 22) = 0.10, p = 0.75, \text{partial } \eta^2 = 0.01$], Comfort with Math Topics [$F(1, 21) = 0.51, p = 0.48, \text{partial } \eta^2 = 0.02$], or Comfort with Teaching Strategies [$F(1, 21) = 0.16, p = 0.69, \text{partial } \eta^2 = 0.01$].

Grade Level Taught was also not associated with participants' improvements between pre-and post-workshop related to Knowledge of Math Topics [$F(2, 22) = 0.38, p = .69$, partial eta squared = 0.03], Knowledge of Teaching Strategies [$F(2, 22) = 0.07, p = 0.94$, partial eta squared = 0.01], Comfort with Math Topics [$F(2, 21) = 0.37, p = 0.69$, partial eta squared = 0.03], or Comfort with Teaching Strategies [$F(2, 21) = 0.18, p = 0.84$, partial eta squared = 0.02].

Years Teaching Math was also not associated with participants' improvements between pre-and post-workshop related to Knowledge of Math Topics [$F(2, 22) = 1.20, p = .32$, partial eta squared = 0.10], Knowledge of Teaching Strategies [$F(2, 22) = 0.81, p = 0.46$, partial eta squared = 0.07], Comfort with Math Topics [$F(2, 21) = 0.58, p = 0.57$, partial eta squared = 0.05], or Comfort with Teaching Strategies [$F(2, 21) = 0.18, p = 0.84$, partial eta squared = 0.02].

Summary

Teacher ratings for Knowledge of Math Topics, Knowledge of Teaching Strategies, Comfort with Math Topics, and Comfort with Teaching Strategies were significantly different from pre- to post-survey indicating the treatment improved both knowledge and comfort in teaching math. There were no differences based on Grade Level Taught, Years Teaching Math, Math Certification or Highly Qualified status which indicates the treatment may be effective across all groups.

Appendices

Appendix A: Analysis Pre-Post Knowledge of Math Topics

Appendix B: Analysis of Pre-Post Knowledge of Teaching Strategies

Appendix C: Analysis Pre-Post Comfort of Math Topics

Appendix D: Analysis Pre-Post Comfort of Teaching Strategies

Appendix E - Pre-Summer Institute Interview Protocol

Teacher: _____

Interviewer: Barbara Sponsel

Date of interview: _____

Beginning time of interview: _____

Ending time of Interview: _____

Thank you for agreeing to let me interview you and observe your class at a later time. The purpose of the study is to identify the impact of the IMP professional development program on teachers' learning and teaching practice. Thus, after getting some background information, I will be asking you some questions about pedagogy, habits of teaching practice, and mathematics content. Please note that content questions are not being posed to check for mastery of any particular content. Instead, questions are being posed to probe for teacher understanding and ultimately changes in teachers' understanding based on experiences with the IMP program.

In advance, I apologize if some of the background questions seem redundant with questions you have answered for the IMP pre-institute survey.

INTERVIEW

Section I: Background information

- 1) How many years have you been teaching?
- 2) What grade level will you teach this coming year?
- 3) What grade levels have you ever taught and which have you taught most often?
- 4) What degrees do you hold?
- 5) What certification do you have (e.g., K-6, K-8, 9-12 mathematics, 7-12 mathematics)?

6) Do you have other endorsements/certificates (e.g., ESL endorsement, middle school math certificate)?

7) Did you participate in last year's IMP program?

If so, can you specify any learning or changes in teaching practice that were influenced by your participation in IMP last year?

8) What other professional development experiences have you had?

9) How comfortable do you feel in teaching mathematics at the grade level you will be teaching this coming year?

Section II: Probing typical lesson characteristics.

10) How do you typically prepare for teaching a mathematics lesson?

11) Do you have any typical resources that you consult while preparing a mathematics lesson?

12) a) For content you have taught over recent years, have you felt confident in your procedural fluency, conceptual understanding, and problem-solving skills?

12) b) Do you feel like you are regularly stronger in one or two of these areas compared with the other area(s)?

13) Has your procedural fluency, conceptual understanding or problem-solving skills for content topics affected how you teach the topic? If so, in what ways?

14) Describe some of the instructional strategies that you regularly used during the past year in mathematics lessons?

15) Do you feel like your instructional strategies for mathematics lessons have changed over the years? If so, in what ways?

16) How does a typical mathematics lesson unfold in your classroom for *introducing* a topic?

17) What training have you had in differentiated instruction?

18) Have you implemented any differentiated instructional strategies in mathematics lessons?

If yes, what types and how regularly?

Section III: Tasks probing teacher understanding of some specific math concepts.

19) Ms. Chambreaux's students are working on the following problem:

Is 371 a prime number?

As she walks around the room looking at their papers, she sees many different ways to solve this problem. Which solution method is correct? (Mark ONE answer.)

- a) Check to see whether 371 is divisible by 2, 3, 4, 5, 6, 7, 8, or 9.
- b) Break 371 into 3 and 71; they are both prime, so 371 must also be prime.
- c) Check to see whether 371 is divisible by any prime number less than 20.
- d) Break 371 into 37 and 1; they are both prime, so 371 must also be prime.

20) Ms. Dominguez was working with a new textbook and she noticed that it gave more attention to the number 0 than her old book. She came across a page that asked students to determine if a few statements about 0 were true or false. Intrigued, she showed them to her sister who is also a teacher, and asked her what she thought.

Which statement(s) should the sister select as being true? (Mark YES, NO, or I'M NOT SURE for each item below.)

	<u>Yes</u>	<u>No</u>	<u>I'm not sure</u>
a) 0 is an even number.	1	2	3
b) 0 is not really a number. It is a placeholder in writing big numbers.	1	2	3
c) The number 8 can be written	1	2	3

as 008.

Permission was granted by Heather Hill on March 30, 2008 to use questions # 19, 20 & 21 as interview probes. The items were developed by SII investigators (Ball, Hill, Rowan, & Schilling, 2002).

21) Did these problems seem easy/hard?

Did they seem related or unrelated to the mathematical content you might deal with in the classroom?

Appendix F - Post-Summer Institute Interview Protocol

Teacher: _____

Interviewer: Barbara Sponsel

Date of interview: _____

Beginning time of interview: _____

Ending time of Interview: _____

You've been through an intense two-week program. I realize that you may not have had a lot of time to think about how this professional development experience may influence your teaching. However, I'd like to ask a few questions to get your initial reaction to your participation with IMP this year. Thank you once again for being willing to share some of your time.

INTERVIEW

Section I: Reflection on teacher changes or potential changes due to IMP participation.

1) Do you believe you gained any “mathematical knowledge for teaching” over the past two weeks? That is, do you believe you have a better understanding of any math topics which relate to content you will be teaching in the coming year (or have taught in the past)? If so, what content knowledge?

2) Now I'm going to ask about specific types of content knowledge as defined by some researchers (Shulman in particular). Do you believe you gained any knowledge in these specific areas? (Start with a definition, if something from question #1 seems to already fit in the area, restate it and ask if there is anything else):

- Subject matter knowledge (knowledge of key mathematical facts, central concepts and relationships among concepts)

- Pedagogical content knowledge (useful representations, examples, explanations, demonstrations, understandings of what makes some topics easy or difficult to learn, understandings of what conceptions, preconceptions and misconceptions students might have at various ages)

- Curricular knowledge (knowledge about available curricular alternatives, lateral knowledge about curriculum students might be studying in other subjects, and vertical knowledge about preceding and succeeding topics in the same area)

3) Do you believe you have gained any knowledge of pedagogy over the past two weeks? If so, what knowledge of pedagogy? (If not already addressed, use extra probes: Knowledge of standards-based instruction? Knowledge of differentiated instruction?)

4) Can you foresee any changes in your teaching practice based on experiences over the past two weeks? If so, what changes?

Section II: Revisiting mathematical content tasks. Intent is to probe for changes in teacher understanding of some specific mathematics concepts.

I will give the teacher the same two mathematical content questions from the pre-institute interview (#19 & 20).

5) Ms. Chambreaux's students are working on the following problem:

Is 371 a prime number?

As she walks around the room looking at their papers, she sees many different ways to solve this problem. Which solution method is correct? (Mark ONE answer.)

- a) Check to see whether 371 is divisible by 2, 3, 4, 5, 6, 7, 8, or 9.
- b) Break 371 into 3 and 71; they are both prime, so 371 must also be prime.
- c) Check to see whether 371 is divisible by any prime number less than 20.
- d) Break 371 into 37 and 1; they are both prime, so 371 must also be prime.

6) Ms. Dominguez was working with a new textbook and she noticed that it gave more attention to the number 0 than her old book. She came across a page that asked students to determine if a few statements about 0 were true or false. Intrigued, she showed them to her sister who is also a teacher, and asked her what she thought.

Which statement(s) should the sister select as being true? (Mark YES, NO, or I'M NOT SURE for each item below.)

	<u>Yes</u>	<u>No</u>	<u>I'm not sure</u>
a) 0 is an even number.	1	2	3
b) 0 is not really a number. It is a placeholder in writing big numbers.	1	2	3

- c) The number 8 can be written
as 008. 1 2 3

Permission was granted by Heather Hill on March 30, 2008 to use questions # 4, 5 & 6 as interview probes. The items were developed by SII investigators (Ball et al., 2002).

7) Do you think you got the same or different answers today than at the beginning of the institute? If you think you got different answers, what during the last two weeks might have affected your work on these problems?

8) Do you feel confident or unsure about how to work these problems? Has your confidence changed from the first time to the second time?

9) Please explain what you were thinking as you worked the problems.

Appendix G - Pre-Classroom Observation Interview Protocol

To be answered via email or a phone interview or a face-to-face conversation prior to the lesson observation.

- 1) What is the topic of the lesson?

- 2) Why are you teaching this topic? If there are several reasons, please describe them all.

- 3) How does the topic relate to other concepts your students have studied and/or will study?

- 4) Have you outlined any learning goals for this lesson? If so, what are they?

- 5) How did you prepare to teach this lesson? Is this typical for how you've been preparing lessons for this school year? If not, how is it different?

6) Will you execute an existing lesson essentially as it was organized in a resource or did you modify a resource lesson(s) or did you make up your own lesson?

7) Describe how comfortable you are in using the planned instructional strategies for this lesson.

8) Are there any students with special needs in this class? If so, describe some of their needs.

For example:

Are there students for whom English is not their first language?

Are there students with learning disabilities?

Are there heterogeneous ability levels?

Other considerations for students with special needs?

Appendix H - Classroom Observation Protocol

Section I: Background Information

Teacher: _____

Observer: Barbara Sponsel

Grade Level: _____

Observation Date: _____

Start time: _____

End time: _____

Section II: Contextual Background

Number of students:

Gender of students:

Ethnicity of students:

Teacher aide?:

Learning space/Classroom setting:

Section III: Lesson Description

Write details about the lesson.

Section IV: Reflection prompts for the observer

A. Teacher Content Knowledge (T. S. Martin, 2007):

- Evidence of teacher knowledge of mathematics:

- Teacher “demonstrates a sound knowledge of mathematical concepts and procedures” (T. S. Martin, 2007, p. 84)
- Teacher “represents mathematics as a network of interconnected concepts and procedures” (T. S. Martin, 2007, p. 84)
- Teacher “emphasizes connections between mathematics and other disciplines and connections with daily living” (T. S. Martin, 2007, p. 84)
- Teacher “models and emphasizes aspects of problem solving” (T. S. Martin, 2007, p. 84)
- Teacher “recognizes reasoning and proof as fundamental aspects of mathematics” (T. S. Martin, 2007, p. 85)
- Teacher “models and emphasizes mathematical communication to help students organize and consolidate their mathematical thinking” (T. S. Martin, 2007, p. 85)

- In contrast, evidence suggesting weak content knowledge:

- Teacher “making frequent mathematical mistakes, using limited or inappropriate representations, or presenting mathematics as a static subject from which meaning can be derived solely from symbolic representations” (T. S. Martin, 2007, p. 85)

B. Standards-based instruction (instances or approximate rate of occurrence):

- Evidence of the lesson supporting student engagement in reasoning (J. K. Jacobs et al., 2006, p. 20):

- Using deductive reasoning; deriving a conclusion(s) “from stated assumptions using a logical chain of inferences” (p. 20)
- Developing a rationale; “explaining or motivating, in broad mathematical terms, a mathematical assertion or procedure” (p. 20)
- Making a generalization; “recognizing that several examples share more general properties” (p. 20)

- Providing a counterexample; “finding one example that does not work to prove that a mathematical conjecture cannot be true” (p. 20)
- Other
- Evidence of students engaged in communication:
 - Students presenting or discussing alternative solutions (J. K. Jacobs et al., 2006, p. 21)
 - Students were involved in communicating their ideas to others “using a variety of means and media” (Piburn & Sawada, 2000, p. 31)
 - Other
- Evidence of the lesson supporting student engagement in making connections (J. K. Jacobs et al., 2006):
 - Problem type(s) (inference)
 1. A problem is related to a preceding problem in a mathematically significant way by “using the solution to a previous problem to solve this problem, extending a previous problem by requiring additional operations to solve this problem, highlighting some operations of a previous problem by considering this problem as a simpler example, or elaborating a previous problem by solving this similar problem in a different way” (p. 22)
 2. A rich mathematical task prompting students to apply a familiar concept to another context
 - Way the problem(s) played out publicly (direct measure)

“Explicit references were made to the mathematical relationships and/or mathematical reasoning involved while solving the problem.” (p. 25) (As opposed to giving results only, focusing on procedures rather than underlying concepts, and stating concepts without discussing relationships)
 - Other

- Evidence of the lesson supporting student engagement in interpreting representations
 - Physical materials (e.g., protractors, tiles), drawings or diagrams, tables, and/or graphs (mathematically relevant to the problem) being used by the teachers and/or student(s) when presenting or solving a problem (J. K. Jacobs et al., 2006)
 - Other
- Evidence of student engagement in problem-solving (J. K. Jacobs et al., 2006):
 - Students were doing something other than just repeating learned procedures (e.g., developing solution procedures that were new to students, modifying solution procedures)
 - Students were presented with a problem that could be solved in different ways and:
 - 1) Students were given a choice in how to solve a problem
 - 2) Multiple solution methods were publicly presented
 - 3) “Class critiqued, examined, or compared the methods” (J. K. Jacobs et al., 2006, p. 18)
 - Teacher challenged students with a problem(s) that had more than one correct solution
 - Procedural complexity of the problem(s) was moderate or high by requiring more than four decisions by the students and/or contains one or more subproblems
 - Problem(s) incorporated real-life contexts
 - Other

C. Activities unrelated to standards-based instruction (instances or approximate rate of occurrence) (Horizon Research, 2005):

- Students are passively listening to the teacher
- Students are passive recipients of information in the textbook
- Students are involved in activity that lacks relationship to a mathematical topic

D. Differentiated Instruction: Rock et al. (2008) and Tieso (2003) concurred the current model for differentiated instruction has four guiding principles, winnowed down from eight key ideas of differentiation conveyed by Tomlinson (1999a): “(a) a focus on essential ideas and skills in each content area, (b) responsiveness to individual student differences, (c) integration of assessment and instruction, and (d) an ongoing adjustment of content, process, and products” (Rock et al., 2008, p. 33) according to individual students’ readiness, interests, and learning profile (Tomlinson, 1999a). One of Tomlinson’s (1999a) other key ideas for differentiated classrooms highlighted the importance of flexibility: flexibility in materials used, flexibility in pacing, flexibility in use of time, flexibility in instructional strategies, and flexibility in grouping.

- Evidence of attention to big ideas (e.g., essential facts or terms, essential questions to engage students, key concepts that help students organize and relate information studied (Tomlinson, 1999b))
 - Evidence of integration of assessment and instruction
 - Evidence of content being differentiated (according to individual students’ readiness, interests, and learning profile)
 - Evidence of process being differentiated (according to individual students’ readiness, interests, and learning profile)
 - Evidence of product being differentiated (according to individual students’ readiness, interests, and learning profile)

E. Evidence of Inquiry into Practice during the lesson (Hiebert et al., 2007):

- Teacher focuses attention on observing and/or collecting evidence about student thinking

Section V: Additional Comments

Appendix I - Post-Classroom Observation Interview Protocol

To be answered via email or a phone interview or a face-to-face conversation after the lesson observation.

- 1) Describe how you feel the lesson played out.

- 2) (If you had learning goals) Do you believe the students (or some of the students) achieved the learning goals? Do you have any evidence or will you have any evidence?

- 3) As you reflect now, do you think you would change anything the next time you taught the lesson?

- 4) Have you taught this lesson in previous years?

If so, can you think of any differences in how you taught it today compared with previous years?

If yes:

- Please describe them.

- Are there any particular characteristics of this group of students that led you to plan the lesson in this way?

- Can you think of any other reasons why you modified the lesson from previous years?

5) Describe any way(s) in which you deviated from your original lesson plan (if any) and why you deviated.

6) Were there any unexpected student behaviors/comments today? If so, what were they?

7) Were the instructional strategies you used today typical for mathematical lessons you've been teaching this year? If not, how were they different?

8) Did you use any differentiated instructional strategy or strategies in your lesson today? If so, what were they? How often do you use the strategy? How long have you been using the strategy?

9) Do you have any additional comments about this lesson?

After Observation 2

10) Are you using any instructional strategies for mathematics this year that you have not used in prior years? If so, what are they and how often are you using them?

11) Have you used any differentiated instructional strategy or strategies in any mathematics lessons this year that you did not use in prior years? If so, what are they and how often have you used them this year?

12) Has the Summer Institute impacted your teaching in any way this year? If so, please describe.

13) Has your participation in Lesson Study impacted your teaching in any way this year? If so, please describe.

14) Do you have any additional comments about your teaching practice in mathematics as related to this school year?

References

- Allen, D. S. (2007, January 4). *2007-2008 application for the Kansas Mathematics and Science Partnerships Program grant*. Unpublished funded proposal, Kansas State University, Manhattan.
- Allen, D. S., & Hancock, M. J. (2008). *The Infinite Mathematics Project: C³ summer academy, year 2, June 16-27, 2008*. Unpublished brochure, Kansas State University, Manhattan.
- Allington, R. L. (2002). You can't learn much from books you can't read. *Educational Leadership, 60*(3), 16-19.
- Altieri, J. L. (2009). Strengthening connections between elementary classroom mathematics and literacy. *Teaching Children Mathematics, 15*(6), 346-351.
- Anderson, K. M., & Algozzine, B., (Ed.). (2007). Tips for teaching: Differentiating instruction to include all students. *Preventing School Failure, 51*(3), 49-54.
- Archambault, F. X., Westberg, K. L., Brown, S., Hallmark, B. W., Zhang, W., & Emmons, C. (1993). Classroom practices used with gifted third and fourth grade students. *Journal for the Education of the Gifted, 16*, 103-119.
- Baldi, S., Jin, Y., Skemer, M., Green, P. J., & Herget, D. (2007). *Highlights from PISA 2006: Performance of U.S. 15-year-old students in science and mathematics literacy in an international context* (NCES 2008-016). Washington, DC: National Center for Education

Statistics, Institute of Education Sciences, U.S. Department of Education. Retrieved from <http://nces.ed.gov/pubsearch/pubsinfo.asp?pubid=2008016>

Ball, D. L. (1990a). The mathematical understandings that prospective teachers bring to teacher education. *Elementary School Journal*, 90(4), 449-466.

Ball, D. L. (1990b). Prospective elementary and secondary teachers' understanding of division. *Journal for Research in Mathematics Education*, 21, 132-144.

Ball, D. L. (1991a). Research on teaching mathematics: Making subject matter part of the equation. In J. Brophy (Ed.), *Advances in research on teaching* (pp. 1-48). Greenwich, CT: JAI.

Ball, D. L. (1991b). Teaching mathematics for understanding: What do teachers need to know about subject matter? In M. M. Kennedy (Ed.), *Teaching academic subjects to diverse learners* (pp. 63-83). New York, NY: Teachers College Press.

Ball, D. L. (1999). Crossing boundaries to examine the mathematics entailed in elementary teaching. In T. Lam (Ed.), *Contemporary mathematics* (pp. 15-36). Providence, RI: American Mathematical Society.

Ball, D. L., & Bass, H. (2000). Interweaving content and pedagogy in teaching and learning to teach: Knowing and using mathematics. In J. Boaler (Ed.), *Multiple perspectives on the teaching and learning of mathematics* (pp. 83-104). Westport, CT: Ablex.

Ball, D. L., & Cohen, D. K. (1999). Developing practice, developing practitioners: Toward a practice-based theory of professional education. In L. Darling-Hammond, & G. Sykes

(Eds.), *Teaching as the learning profession: Handbook of policy and practice* (pp. 3-32).
San Francisco, CA: Jossey-Bass.

Ball, D. L., Hill, H. C., & Bass, H. (2005). Knowing mathematics for teaching: Who knows mathematics well enough to teach third grade, and how can we decide? *American Educator*, *Fall 2005*, 14-22.

Ball, D. L., Hill, H. C., Rowan, B., & Schilling, S. G. (2002). *Measuring teachers' content knowledge for teaching: Elementary mathematics release items, 2002*. Ann Arbor, MI: Study of Instructional Improvement. Retrieved from http://www.sii.soe.umich.edu/documents/released_items02.pdf

Ball, D. L., Lubienski, S. T., & Mewborn, D. S. (2001). Research on teaching mathematics: The unsolved problem of teachers' mathematical knowledge. In V. Richardson (Ed.), *Handbook of research on teaching* (4th ed., pp. 433-456). Washington, DC: American Educational Research Association.

Ball, D. L., & McDiarmid, G. W. (1988). Research on teacher learning: Studying how teachers' knowledge changes. *Action in Teacher Education*, *10*(2), 17-23.

Ball, D. L., & McDiarmid, G. W. (1990). The subject-matter preparation of teachers. In R. Houston (Ed.), *Handbook of research on teacher education* (pp. 437-447). New York, NY: MacMillan.

- Ball, D. L., & Wilson, S. M. (1990). *Knowing the subject and learning to teach it: Examining assumptions about becoming a mathematics teacher* (Research Report No. 90-7). East Lansing: Michigan State University, National Center for Research on Teacher Education.
- Banks, J., Cochran-Smith, M., Moll, L., Richert, A. E., Zeichner, K. M., LePage, P., . . . , & McDonald, M. (2005). Teaching diverse learners. In L. Darling-Hammond, & J. D. Bransford (Eds.), *Preparing teachers for a changing world: What teachers should learn and be able to do* (pp. 232-274). San Francisco, CA: Jossey-Bass.
- Barnett, C. (1991). Building a case-based curriculum to enhance the pedagogical content knowledge of mathematics teachers. *Journal of Teacher Education*, 42(4), 263-272.
- Barnett, C. (1998). Mathematics teaching cases as a catalyst for informed strategic inquiry. *Teaching and Teacher Education*, 14(1), 81.
- Barnett, J., & Hodson, D. (2001). Pedagogical context knowledge: Toward a fuller understanding of what good science teachers know. *Science Education*, 85(4), 426-453.
- Bean, T. W., Readence, J. E., & Baldwin, R. S. (2004). *Content area literacy: An integrated approach* (9th ed.). Dubuque, IA: Kendall/Hunt Publishing Company.
- Becker, J. R., & Perl, T. (2003). The mathematics education community's response to a diverse and changing student population. In G. Stanic, & J. Kilpatrick (Eds.), *A history of school mathematics* (pp. 1085-1141). Reston, VA: National Council of Teachers of Mathematics.
- Beers, S., & Howell, L. (2005). *Reading strategies for the content areas, volume 2: An ASCD action tool*. Alexandria, VA: ASCD.

- Begle, E. G. (1979). *Critical variables in mathematics education: Findings from a survey of the empirical literature*. Washington, DC: Mathematical Association of America and National Council of Teachers of Mathematics.
- Bernard, H. R. (2006). *Research methods in anthropology: Qualitative and quantitative approaches* (4th ed.). Lanham, MD: AltaMira Press.
- Billmeyer, R., & Barton, M. L. (1998). *Teaching reading in the content areas: If not me, then who?* (2nd ed.). Alexandria, VA: Association for Supervision and Curriculum Development.
- Bishop, A. J., & Forgasz, H. J. (2007). Issues in access and equity in mathematics education. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 1145-1167). Reston, VA: National Council of Teachers of Mathematics.
- Blank, R. K., & de las Alas, N. (2009). *Effects of teacher professional development on gains in student achievement: How meta analysis provides scientific evidence useful to education leaders*. Washington, DC: Council of Chief State School Officers. Retrieved from <http://www.ccsso.org/Resources/Publications.html>
- Blank, R. K., de las Alas, N., & Smith, C. (2008). *Does teacher professional development have effects on teaching and learning? Evaluation findings from programs in 14 states*. Washington, DC: Council of Chief State School Officers. Retrieved from <http://www.ccsso.org/Resources/Publications.html>
- Borasi, R., & Fonzi, J. (2002). *Professional development that supports school mathematics reform* (Foundations series of monographs for professionals in science, mathematics and

- technology education ed.). Arlington, VA: National Science Foundation. Retrieved from <http://www.nsf.gov/pubs/2002/nsf02084/>
- Borasi, R., Fonzi, J., Smith, C., & Rose, B. (1999). Beginning the process of rethinking mathematics instruction: A professional development program. *Journal of Mathematics Teacher Education*, 2, 49-78.
- Borko, H. (2004). Professional development and teacher learning: Mapping the terrain. *Educational Researcher*, 33(8), 3-15.
- Borko, H., Eisenhart, M., Brown, C. A., Underhill, R., Jones, D., & Agard, P. C. (1992). Learning to teach hard mathematics: Do novice teachers and their instructors give up too easily? *Journal for Research in Mathematics Education*, 23, 194-222.
- Borko, H., & Putnam, R. T. (1995). Expanding a teacher's knowledge base: A cognitive psychological perspective on professional development. In T. R. Guskey, & M. Huberman (Eds.), *Professional development in education: New paradigms and practices* (pp. 35-65). New York, NY: Teachers College Press.
- Brainard, J. (2005, May 27). Juggling the numbers. [electronic version]. *Chronicle of Higher Education*, Retrieved from <http://chronicle.com/weekly/v51/i38/38a02001.htm>
- Brainerd, C. J. (1978). *Piaget's theory of intelligence*. Englewood Cliffs, NJ: Prentice-Hall.
- Bransford, J. D., Darling-Hammond, L., & LePage, P. (2005). Introduction. In L. Darling-Hammond, & J. D. Bransford (Eds.), *Preparing teachers for a changing world: What teachers should learn and be able to do* (pp. 1-39). San Francisco, CA: Jossey-Bass.

- Brophy, J., & Good, T. L. (1986). Teacher behavior and student achievement. In M. C. Wittrock (Ed.), *Handbook of research on teaching* (3rd ed., pp. 328-375). New York, NY: Simon & Schuster.
- Brown, C. A., & Borko, H. (1992). Becoming a mathematics teacher. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 209-239). New York, NY: Macmillan.
- Brown, S., Cooney, T. J., & Jones, D. (1990). Mathematics teacher education. In W. R. Houston (Ed.), *Handbook of research on teacher education* (pp. 639-656). New York, NY: Macmillan.
- Burger, E. B., & Starbird, M. (2000). *The heart of mathematics: An invitation to effective thinking*. Emeryville, CA: Key College Publishing.
- Burstein, L., McDonnell, L. M., Winkle, J. V., Ormseth, T. H., Mirocha, J., & Guiton, G. (1995). *Validating national curriculum indicators*. Santa Monica, CA: RAND. Retrieved from http://www.rand.org/pubs/monograph_reports/2007/MR658.pdf
- Campbell, L., & Campbell, B. (1999). *Multiple intelligences and student achievement: Success stories from six schools*. Alexandria, VA: Association for Supervision and Curriculum Development.
- Carpenter, T. P., & Fennema, E. (1992). Cognitively guided instruction: Building on the knowledge of students and teachers. *International Journal of Educational Research*, 17, 457-470.

- Carpenter, T. P., Fennema, E., Peterson, P. L., Chiang, C., & Loef, M. (1989). Using knowledge of children's mathematics thinking in classroom teaching: An experimental study. *American Educational Research Journal*, 26(4), 499-531.
- Cazden, C. B. (1986). Classroom discourse. In M. C. Wittrock (Ed.), *Handbook of research on teaching* (3rd ed., pp. 432-463). New York, NY: Macmillan.
- Center for Research in Mathematics and Science Teacher Development. (n.d.). *Diagnostic mathematics assessments*. Louisville: University of Louisville. Retrieved November 1, 2010 from http://louisville.edu/education/research/centers/crmstd/diag_math_assess_elem_teachers.html
- Chall, J. S. (1983). *Stages of reading development*. New York, NY: McGraw-Hill.
- Chen, J., Krechevsky, M., & Viens, J. (1998). *Building on children's strengths: The experience of project spectrum*. New York, NY: Teachers College Press.
- Clarke, D. (1994). Ten key principles from research on the professional development of mathematics teachers. In D. B. Aichele, & A. F. Coxford (Eds.), *Professional development for teachers of mathematics, 1994 yearbook* (pp. 37-48). Reston, VA: National Council of Teachers of Mathematics.
- Cochran-Smith, M., & Lytle, S. (1999). Relationships of knowledge and practice: Teacher learning in communities. In A. Iran-Nejad, & P. D. Pearson (Eds.), *Review of research in*

- education* (Vol. 24 ed., pp. 249-305). Washington, DC: American Educational Research Association.
- Cohen, S. (2004). *Teachers' professional development and the elementary mathematics classroom: Bringing understandings to light*. Mahwah, NJ: Erlbaum.
- Cohen, D. K. (1990). A revolution in one classroom: The case of Mrs. Oublier. *Educational Evaluation and Policy Analysis*, 12(3), 327-345.
- Cohen, D. K., & Ball, D. L. (2001). Making change: Instruction and its improvement. *Phi Delta Kappan*, 83(1), 73-77.
- Cohen, D. K., & Hill, H. C. (2000). Instructional policy and classroom performance: The mathematics reform in California. *Teachers College Record*, 102(2), 294-343.
- Cohen, D. K., & Hill, H. C. (2001). *Learning policy: When state education reform works*. New Haven, CT: Yale University Press.
- Collins, A., Brown, J. S., & Newman, S. E. (1989). Cognitive apprenticeship: Teaching the craft of reading, writing and mathematics. In L. B. Resnick (Ed.), *Knowing, learning and instruction: Essays in honor of Robert Glaser* (pp. 453-494). Hillsdale, NJ: Erlbaum.
- Corbin, J., & Strauss, A. (1990). Grounded theory research: Procedures, canons, and evaluative criteria. *Qualitative Sociology*, 13(1), 3-21.
- Creative Publications. (2001). *Explain it! Answering extended-response math questions (grades 7-8)*. Chicago, IL: Wright Group/McGraw-Hill.

- Creswell, J. W. (1998). *Qualitative inquiry and research design: Choosing among five traditions*. Thousand Oaks, CA: Sage Publications.
- Creswell, J. W., & Miller, D. L. (2000). Determining validity in qualitative inquiry. *Theory into Practice, 39*(3), 124-130.
- Cuban, L. (1984). *How teachers taught: Constancy and change in American classrooms, 1890-1980*. New York, NY: Longman.
- Curcio, F. R. (2002). *A user's guide to Japanese lesson study: Ideas for improving mathematics teaching*. Reston, VA: National Council of Teachers of Mathematics.
- Darling-Hammond, L. (1998). Teacher learning that supports student learning. *Educational Leadership, 55*(5), 6-11.
- Darling-Hammond, L. (2000). Teacher quality and student achievement: A review of state policy evidence. *Education Policy Analysis Archives, 8*(1).
- Darling-Hammond, L. (2006a). Constructing 21st-century teacher education. *Journal of Teacher Education, 57*(3), 300-314.
- Darling-Hammond, L. (2006b). Securing the right to learn: Policy and practice for powerful teaching and learning. *Educational Researcher, 35*(7), 13-24.
- Darling-Hammond, L., & Berry, B. (2006). Highly qualified teachers for all. *Educational Leadership, 64*(3), 14-20.

- Darling-Hammond, L., & McLaughlin, M. W. (1995). Policies that support professional development in an era of reform. *Phi Delta Kappan*, 76(8), 597-604.
- Darling-Hammond, L., & Sclan, E. M. (1996). Who teaches and why: Dilemmas of building a profession for twenty-first century schools. In J. Sikula, T. Buttery & E. Guyton (Eds.), *Handbook for research on teacher education* (2nd ed., pp. 67-101). New York, NY: Macmillan.
- Darling-Hammond, L., & Sykes, G. (2003). Wanted: A national teacher supply policy for education: The right way to meet the "highly qualified teacher" challenge. *Education Policy Analysis Archives*, 11(33), 1-55. Retrieved from <http://epaa.asu.edu/ojs/article/view/261>
- Darling-Hammond, L., & Youngs, P. (2002). Defining "highly qualified teachers": What does "scientifically-based research" actually tell us? *Educational Researcher*, 31(9), 13-25.
- de Villiers, M. (1990). The role and function of proof in mathematics. *Pythagoras*, 24, 17-24.
- Desimone, L. M., Porter, A. C., Garet, M. S., Yoon, K. S., & Birman, B. F. (2002). Effects of professional development on teachers' instruction: Results from a three-year longitudinal study. *Educational Evaluation and Policy Analysis*, 24(2), 81-112.
- Desimone, L. M., Smith, T. M., & Ueno, K. (2006). Are teachers who need sustained, content-focused professional development getting it? An administrator's dilemma. *Educational Administration Quarterly*, 42(2), 179-215.
- Dewey, J. (1929). *The sources of a science of education*. New York, NY: Horace Liveright.

- DiMartino, J., & Miles, S. (2004). Equity in the classroom. *Principal Leadership*, 5(4), 44-48.
- Dunn, R., & Dunn, K. (1992). *Teaching elementary students through their individual learning styles: Practical approaches for grades 3-6*. Boston, MA: Allyn & Bacon.
- Dunn, R., & Dunn, K. (1993). *Teaching secondary students through their individual learning styles: Practical approaches for grades 7-12*. Boston, MA: Allyn & Bacon.
- Elmore, R. F. (1996). Getting to scale with good educational practice. *Harvard Educational Review*, 66(1), 1-26.
- Elmore, R. F. (2002). *Bridging the gap between standards and achievement: The imperative for professional development in education*. Washington, DC: Albert Shanker Institute.
- Emig, V. B. (1997). A multiple intelligences inventory. *Educational Leadership*, 55(1), 47-50.
- Enzensberger, H. M. (2000). *The Number Devil*. New York, NY: Owl Books.
- Eves, H. (1990). *An introduction to the history of mathematics* (6th ed.). New York, NY: Saunders.
- Fennema, E., Carpenter, T. P., Franke, M. L., & Carey, D. A. (1992). Learning to use children's mathematical thinking: A case study. In R. Davis, & C. Maher (Eds.), *Schools, mathematics, and the world of reality* (pp. 93-117). Needham Height, MA: Allyn & Bacon.
- Fennema, E., Carpenter, T. P., Franke, M. L., Levi, M., Jacobs, V. R., & Empson, S. (1996). A longitudinal study of learning to use children's thinking in mathematics in instruction. *Journal for Research in Mathematics Education*, 27, 403-434.

- Ferguson, R. F. (1991). Paying for public education: New evidence on how and why money matters. *Harvard Journal on Legislation*, 28, 465-498.
- Ferguson, R. F., & Ladd, H. F. (1996). How and why money matters: An analysis of Alabama schools. In H. F. Ladd (Ed.), *Holding schools accountable* (pp. 265-298). Washington, DC: Brookings Institution.
- Fernandez, C., Cannon, J., & Chokshi, S. (2003). A U.S.-Japan lesson study collaboration reveals critical lenses for examining practice. *Teaching and Teacher Education*, 19(2), 171-185.
- Fey, J. T., & Graeber, A. O. (2003). From the new math to the *Agenda for Action*. In G. Stanic, & J. Kilpatrick (Eds.), *A history of school mathematics* (pp. 521-558). Reston, VA: National Council of Teachers of Mathematics.
- Fisher, D., Frey, N., & Williams, D. (2002). Seven literacy strategies that work. *Educational Leadership*, 60(3), 70-73.
- Floden, R. (1997). Reforms that call for teaching more than you understand. In N. Burbules, & D. Hansen (Eds.), *Teaching and its predicaments* (pp. 11-28). Boulder, CO: Westview Press.
- Floden, R., & Maniketti, M. (2005). Research on the effects of coursework in the arts and sciences and in the foundations of education. In M. Cochran-Smith, & K. M. Zeichner (Eds.), *Studying teacher education: The report of the AERA panel on research and teacher education* (pp. 261-308). Mahwah, NJ: Lawrence Erlbaum.

- Franco, C., Sztajn, P., & Ortiagão, M. I. R. (2007). Mathematics teachers, reform, and equity: Results from the Brazilian national assessment. *Journal for Research in Mathematics Education, 38*, 393-419.
- Franke, M. L., Fennema, E., & Carpenter, T. P. (1997). Teachers creating change: Examining evolving beliefs and classroom practices. In E. Fennema, & B. S. Nelson (Eds.), *Mathematics teachers in transition* (pp. 255-282). Mahwah, NJ: Lawrence Erlbaum Associates, Inc.
- Franke, M. L., Kazemi, E., & Battey, D. (2007). Mathematics teaching and classroom practice. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 225-256). Charlotte, NC: Information Age Publishing.
- Friel, S. N., & Bright, G. W. (1997). Common components and guiding principles for teacher enhancement programs. In S. N. Friel, & G. W. Bright (Eds.), *Reflecting on our work: NSF teacher enhancement in K-6 mathematics* (pp. 7-21). New York, NY: University Press.
- Frykholm, J. (2005). Innovative curricula: Catalysts for reform in mathematics teacher education. *Action in Teacher Education, 26*(4), 20-36.
- Gardner, H. (1983/1993). *Frames of mind: The theory of multiple intelligences*. New York, NY: Basic Books.
- Gardner, H. (1993). *Multiple intelligences: The theory in practice*. New York, NY: BasicBooks.

Garet, M. S., Birman, B. F., Porter, A. C., Desimone, L. M., Herman, R., & Yoon, K. S. (1999). *Designing effective professional development: Lessons from the Eisenhower Program*.

Washington, DC: U.S. Department of Education.

Garet, M. S., Porter, A. C., Desimone, L. M., Birman, B. F., & Yoon, K. S. (2001). What makes professional development effective? Results from a national sample of teachers. *American Educational Research Journal*, 38(4), 915-945.

Gearhart, M., & Saxe, G. B. (2004). When teachers know what students know: Integrating mathematics assessment. *Theory into Practice*, 43(4), 304-313.

Gearhart, M., Saxe, G. B., Seltzer, M., Schlackman, J., Ching, C. C., Nasir, N., Fall, R., . . . , & Sloan, T. F. (1999). Opportunities to learn fractions in elementary mathematics classrooms. *Journal for Research in Mathematics Education*, 30(3), 286-315.

Geddis, A. (1993). Transforming subject-matter knowledge: The role of pedagogical content knowledge in learning to reflect on teaching. *International Journal of Science Education*, 15, 673-683.

Goldhaber, D. D., & Brewer, D. J. (2000). Does teacher certification matter? High school teacher certification status and student achievement. *Educational Evaluation and Policy Analysis*, 22(2), 129-145.

Gonzales, P., Guzman, J. C., Partelow, L., Pahlke, E., Jocelyn, L., Kastberg, D., & Williams, T. (2004). *Highlights from the trends in international mathematics and science study (TIMSS) 2003* (NCES 2005-005). Washington, DC: U.S. Government Printing Office.

Gonzales, P., Williams, T., Jocelyn, L., Roey, S., Kastberg, D., & Brenwald, S. (2008).

Highlights from TIMSS 2007: Mathematics and science achievement of U.S. fourth- and eighth-grade students in an international context (NCES 2009-001). Washington, DC: U.S. Department of Education, National Center for Education Statistics. Retrieved from <http://nces.ed.gov/pubs2009/2009001.pdf>

Graeber, A. O., Tirosh, D., & Glover, R. (1989). Preservice teachers' misconceptions in solving verbal problems in multiplication and division. *Journal for Research in Mathematics Education*, 20, 95-102.

Greenhawk, J. (1997). Multiple intelligences meet standards. *Educational Leadership*, 55(1), 62-64.

Greenwald, R., Hedges, L. V., & Laine, R. D. (1996). The effect of school resources on student achievement. *Review of Educational Research*, 66(3), 361-396.

Grossman, P. L. (1990). *The making of a teacher: Teacher knowledge and teacher education*. New York, NY: Teachers College Press.

Grossman, P. L., Wilson, S. M., & Shulman, L. S. (1989). Teachers of substance: Subject matter knowledge for teaching. In M. C. Reynolds (Ed.), *Knowledge base for the beginning teacher* (pp. 23-36). New York, NY: Pergamon.

Grouws, D. A., & Schultz, K. A. (1996). Mathematics teacher education. In J. Sikula (Ed.), *Handbook of research on teacher education* (pp. 442-458). New York, NY: Macmillan.

- Guba, E. G., & Lincoln, Y. S. (1981). *Effective evaluation: Improving the usefulness of evaluation results through responsive and naturalistic approaches*. San Francisco, CA: Jossey-Bass.
- Guba, E. G., & Lincoln, Y. S. (1989). *Fourth generation evaluation*. Newbury Park, CA: Sage.
- Gummer, E., & Stepanek, J. (2007). *Analysis of Title IIB Mathematics and Science Partnerships in the northwest region*. Portland, OR: Regional Educational Laboratory Northwest.
- Guskey, T. R. (1986). Staff development and the process of teacher change. *Educational Researcher*, 15(5), 5-12.
- Haager, D., & Klingner, J. K. (2005). *Differentiating instruction in inclusive classrooms*. Columbus, OH: Merrill.
- Hanna, G. (1995). Challenges to the importance of proof. *For the Learning of Mathematics*, 15, 42-49.
- Hanushek, E. A. (1986). The economics of schooling: Production and efficiency in public schools. *Journal of Economic Literature*, 24(3), 1141-1177.
- Hanushek, E. A. (1992). The trade-off between child quantity and quality. *Journal of Political Economy*, 100(1), 84-117.
- Harbison, R. W., & Hanushek, E. A. (1992). *Educational performance for the poor: Lessons from rural northeast Brazil*. Oxford, England: Oxford University Press.

- Harel, G., & Sowder, L. (2007). Toward comprehensive perspectives on the learning and teaching of proof. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 805-842). Reston, VA: National Council of Teachers of Mathematics.
- Haseltine, W. (2007, October). Losing the lead. *Discover*, 80-81.
- Hatch, T., & Gardner, H. (1993). Finding cognition in the classroom: An expanded view of human intelligence. In G. Salomon (Ed.), *Distributed cognitions: Psychological and educational considerations* (pp. 164-187). Cambridge, MA: Cambridge University Press.
- Hawley, W. D., & Valli, L. (1999). The essentials of effective professional development: A new consensus. In L. Darling-Hammond, & G. Sykes (Eds.), *Teaching as the learning profession: Handbook of policy and practice* (pp. 127-150). San Francisco, CA: Jossey-Bass.
- Heath, S. B. (1983). *Ways with words: Language, life, and work in communities and classrooms*. Cambridge, MA: Cambridge University Press.
- Heaton, R. M. (1992). Who is minding the mathematics content? A case study of a fifth-grade teacher. *Elementary School Journal*, 93(2), 153-162.
- Heaton, R. M. (2000). *Teaching mathematics to the new standards: Relearning the dance*. New York, NY: Teachers College Press.

- Hiebert, J., & Grouws, D. A. (2007). The effects of classroom mathematics teaching on students' learning. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 371-404). Reston, VA: National Council of Teachers of Mathematics.
- Hiebert, J., Morris, A. K., Berk, D., & Jansen, A. (2007). Preparing teachers to learn from teaching. *Journal of Teacher Education, 58*(1), 47-61.
- Hiebert, J., & Stigler, J. W. (2000). A proposal for improving classroom teaching: Lessons from the TIMSS video study. *Elementary School Journal, 101*(1), 3-20.
- Hiebert, J., Stigler, J. W., Jacobs, J. K., Givvin, K. B., Garnier, H., Smith, M. S., Hollingsworth, H., Manaster, A., Wearne, D., & Gallimore, R. (2005). Mathematics teaching in the United States today (and tomorrow): Results from the TIMSS 1999 video study. *Educational Evaluation and Policy Analysis, 27*(2), 111-132.
- Hill, H. C., & Ball, D. L. (2004). Learning mathematics for teaching: Results from California's mathematics professional development institutes. *Journal for Research in Mathematics Education, 35*, 330-351.
- Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal, 42*(2), 371-406.
- Hill, H. C., Schilling, S. G., & Ball, D. L. (2004). Developing measures of teachers' mathematics knowledge for teaching. *Elementary School Journal, 105*(1), 11.
- Hill, H. C., Sleep, L., Lewis, J. M., & Ball, D. L. (2007). Assessing teachers' mathematical knowledge: What knowledge matters and what evidence counts? In F. K. Lester (Ed.),

- Second handbook of research on mathematics teaching and learning* (pp. 111-155). Reston, VA: National Council of Teachers of Mathematics.
- Hoerr, T. (2004). How MI informs teaching at New City School. *Teachers College Record*, 106(1), 40-48.
- Holmes Group. (1986). *Tomorrow's teachers*. East Lansing, MI: Author.
- Holmes Group. (1995). *Tomorrow's schools of education*. East Lansing, MI: Author.
- Holmes Partnership. (n.d.). *The Holmes Partnership*. Retrieved November 1, 2010 from <http://www.holmespartnership.org/>
- Hope, J. A., Reys, B. J., & Reys, R. E. (1987). *Mental math in the middle grades*. Palo Alto, CA: Dale Seymour Publications.
- Horizon Research, I. (2005). *2005-06 Local Systemic Change classroom observation protocol*. Chapel Hill, NC: Author. Retrieved from <http://www.horizon-research.com/instruments/lsc/pdop.pdf>
- Howden, H. (1989). Teaching number sense. *Arithmetic Teacher*, 36(6), 6-11.
- Individuals with Disabilities Education Improvement Act (IDEA) of 2004, Public Law 108-446, 20 U.S.C. 1400 et seq. (2004). Retrieved from <http://idea.ed.gov/download/statute.html>
- Ingersoll, R. M. (2002). *Out of field teaching, educational inequality, and the organization of schools: An exploratory analysis*. University of Washington: Center for the Study of

Teaching and Policy. Retrieved from

<http://www.ecs.org/html/Document.asp?chouseid=4390>

Jacobs, J. K., Hiebert, J., Givvin, K. B., Hollingsworth, H., Garnier, H., & Wearne, D. (2006).

Does eighth-grade mathematics teaching in the United States align with the NCTM standards? Results from the TIMSS 1995 and 1999 video studies. *Journal for Research in Mathematics Education*, 37, 5-32.

Jacobs, A., & Rak, S. (1997). Mathematics and literature--A winning combination. *Teaching Children Mathematics*, 4(3), 156-157.

Jacobs, V. A. (2002). Reading, writing, and understanding. *Educational Leadership*, 60(3), 58-61.

Johnson, R. B. (1997). Examining the validity structure of qualitative research. *Education*, 118(3), 282-292.

Kansas Department of Education. (n.d.). *Kansas Mathematics and Science Partnership program (MSP): Competitive grant application*. Retrieved March 15, 2008 from <http://www.ksde.org/Default.aspx?tabid=1803>

Kansas State College of Education. (n.d.). *K-state professional development schools*. Retrieved November 2, 2010 from <http://coe.k-state.edu/about/pds/>

Kansas State University Office of Educational Innovation and Evaluation. (2007a). *Infinite Math Project (IMP): Pre- and post-survey responses on teachers' self-perceptions of knowledge*

and comfort in teaching mathematical concepts. Unpublished report, Kansas State University, Manhattan.

Kansas State University Office of Educational Innovation and Evaluation. (2007b). *Infinite Math Project: Pre- and post-content test scores in mathematical concepts and correlations between teacher knowledge and comfort in teaching mathematical concepts survey ratings and teacher mathematical content test scores*. Unpublished report, Kansas State University, Manhattan.

Kennedy, M. M. (1997). *Defining optimal knowledge for teaching science and mathematics* (Research Monograph 10). Madison, WI: National Institute for Science Education, University of Wisconsin.

Kennedy, M. M. (1999). *Form and substance in mathematics and science professional development*. *National Institute for Science Education Brief* (November 1999): pp. 1-7.

Kennedy, M. M. (2006). From teacher quality to quality teaching. *Educational Leadership*, 63(6), 14-19.

Kennedy, M. M., Ball, D. L., & McDiarmid, G. W. (1993). *A study package for examining and tracking changes in teachers' knowledge* (Technical Series 93-1). East Lansing, MI: The National Center for Research on Teacher Education.

Kilpatrick, J. (2009). TIMSS 2007 mathematics: Where are we? *MAA FOCUS*, 29(2), 4-7.

Kornhaber, M. L., Fierros, E., & Veenema, S. (2004). *Multiple intelligences: Best ideas from theory and practice*. Needham Heights, MA: Allyn & Bacon.

- Lampert, M., & Ball, D. L. (1998). *Teaching, multimedia, and mathematics: Investigations of real practice*. New York, NY: Teachers College Press.
- Lapkoff, S., & Li, R. M. (2007). Five trends for schools. *Educational Leadership*, 64(6), 8-15.
- Learning First Alliance. (1998). *Every child mathematically proficient: An action plan of the learning first alliance*. Washington, DC: Author.
- Learning Mathematics for Teaching Project. (n.d.). *Measures of teachers' knowledge for teaching mathematics*. Ann Arbor, MI: Author. Retrieved from <http://www.sitemaker.umich.edu/lmt/home>
- Leinhardt, G., & Smith, D. A. (1985). Expertise in mathematics instruction: Subject matter knowledge. *Journal of Educational Psychology*, 77(3), 247-271.
- Lesser, L. M. (2000). *Hotel Infinity*. Retrieved June 14, 2010 from <http://www.math.utep.edu/Faculty/lesser/hotelinfinity.html>
- Levy, H. M. (2008). Meeting the needs of all students through differentiated instruction: Helping every child reach and exceed standards. *Clearing House: A Journal of Educational Strategies, Issues and Ideas*, 81(4), 161-164.
- Lewis, J. M. (2008). Through the looking glass: A study of teaching. In A. H. Schoenfeld (Ed.), *A study of teaching: Multiple lenses, multiple views* (pp. 1-12). Reston, VA: National Council of Teachers of Mathematics.

- Lewis, C. (2002). Does lesson study have a future in the United States? *Nagoya Journal of Education and Human Development*, (1), 1-23.
- Lewis, C., Perry, R., & Hurd, J. (2004). A deeper look at lesson study. *Educational Leadership*, 61(5), 18-22.
- Lewis, C., Perry, R., Hurd, J., & O'Connell, M. P. (2006). Lesson study comes of age in North America. *Phi Delta Kappan*, 88(4), 273-281.
- Lewis, C., Perry, R., & Murata, A. (2006). How should research contribute to instructional improvement? The case of lesson study. *Educational Researcher*, 35(3), 3-14.
- Lewis, C., & Tsuchida, I. (1998). A lesson is like a swiftly flowing river: How research lessons improve Japanese education. *American Educator*, 22(4), 12-17, 50-52.
- Lewis, S. G., & Batts, K. (2005). How to implement differentiated instruction? Adjust, adjust, adjust. *Journal of Staff Development*, 26(4), 26-31.
- Lincoln, Y. S., & Guba, E. G. (1985). *Naturalistic inquiry*. Beverly Hills, CA: Sage.
- Little, J. W. (1982). Norms of collegiality and experimentation: Workplace conditions of school success. *American Educational Research Journal*, 19(3), 325-340.
- Lortie, D. C. (1975). *Schoolteacher: A sociological study of teaching*. Chicago, IL: University of Chicago Press.
- Lou, Y., Abrami, P. C., Spence, J. C., Poulsen, C., Chambers, B., & Apollonia, S. (1996). Within-class grouping: A meta-analysis. *Review of Educational Research*, 66(4), 423-458.

- Loucks-Horsley, S., Love, N., Stiles, K. E., Mundry, S., & Hewson, P. W. (2003). *Designing professional development for teachers of science and mathematics* (2nd ed.). Thousand Oaks, CA: Corwin Press.
- Lovelace, M. K. (2005). Meta-analysis of experimental research based on the Dunn and Dunn model. *Journal of Educational Research, 98*(3), 176.
- Ma, L. (1999). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States*. Mahwah, NJ: Erlbaum.
- Martin, T. S. (Ed.). (2007). *Mathematics teaching today: Improving practice, improving student learning* (2nd ed.). Reston, VA: National Council of Teachers of Mathematics.
- Martin, W. G., & Harel, G. (1989). Proof frames of preservice elementary teachers. *Journal for Research in Mathematics Education, 20*(1), 41-51.
- Marzano, R. J. (2003). *What works in schools: Translating research into action*. Alexandria, VA: Association for Supervision and Curriculum Development.
- Mayer, D. P. (1999). Measuring instructional practice: Can policymakers trust survey data? *Educational Evaluation and Policy Analysis, 21*(1), 29-45.
- McIntosh, A. (2004). Where we are today? In A. McIntosh, & L. Sparrow (Eds.), *Beyond written computation* (pp. 3-14). Perth, Western Australia: Mathematics, Science & Technology Education Centre, Edith Cowan University.

- McIntosh, A., Reys, B. J., & Reys, R. E. (1997a). *Number sense: Simple effective number sense experiences/ grades 1-2*. U.S.A.: Dale Seymour Publications.
- McIntosh, A., Reys, B. J., & Reys, R. E. (1997b). *Number sense: Simple effective number sense experiences/ grades 6-8*. U.S.A.: Dale Seymour Publications.
- McTighe, J., & Brown, J. L. (2005). Differentiated instruction and educational standards: Is detente possible? *Theory into Practice*, 44(3), 234-244.
- Melser, N. A., & Leitze, A. R. (1999). Connecting language arts and mathematical problem solving in the middle grades. *Middle School Journal*, 31(1), 48-54.
- Merriam, S. B. (1998). *Qualitative research and case study applications in education*. San Francisco, CA: Jossey-Bass.
- Merseth, K. (1996). Cases and case methods in teacher education. In J. Sikula (Ed.), *Handbook of research on teacher education* (2nd ed., pp. 722-744). New York, NY: Macmillan.
- Merseth, K. (2003). *Windows on teaching math: Cases of middle and secondary classrooms*. New York, NY: Macmillan.
- Moll, L., Tapia, J., & Whitmore, K. F. (1993). Living knowledge: The social distribution of cultural resources for thinking. In G. Salomon (Ed.), *Distributed cognitions: Psychological and educational considerations* (pp. 139-163). Cambridge, MA: Cambridge University Press.

- Monk, D. H. (1994). Subject area preparation of secondary mathematics and science teachers and student achievement. *Economics of Education Review*, 13(2), 125-145.
- Moore, D. W., Bean, T. W., Birdyshaw, D., & Rycik, J. A. (1999). Adolescent literacy: A position statement. *Journal of Adolescent & Adult Literacy*, 43(1), 97-112.
- Moore, D. W., & Readence, J. E. (1984). A quantitative and qualitative review of graphic organizer research. *Journal of Educational Research*, 78(1), 11-17.
- Mullens, J. E., Murnane, R. J., & Willet, J. B. (1996). The contribution of training and subject matter knowledge to teaching effectiveness: A multilevel analysis of longitudinal evidence from belize. *Comparative Education Review*, 40, 139-157.
- Murphy, S. J. (2000). Children's books about math: Trade books that teach. *New Advocate*, 13(4), 365-374.
- National Association of Secondary School Principals. (2004). *Breaking ranks II: Strategies for leading high school reform*. Reston, VA: Author.
- National Commission on Excellence in Education. (1983). *A nation at risk: The imperative for educational reform*. Washington, DC: Government Printing Office.
- National Council of Teachers of Mathematics. (1980). *An agenda for action: Recommendations for school mathematics of the 1980s*. Reston, VA: Author.
- National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: Author.

- National Council of Teachers of Mathematics. (1991). *Professional standards for teaching mathematics*. Reston, VA: Author.
- National Council of Teachers of Mathematics. (1995). *Assessment standards for school mathematics*. Reston, VA: Author.
- National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.
- National Council of Teachers of Mathematics. (2006). *Curriculum focal points for prekindergarten through grade 8 mathematics: A quest for coherence*. Reston, VA: Author.
- National Council of Teachers of Mathematics. (2007). *2007-08 NCTM lesson study course*. Retrieved September 13, 2007 from <http://www.nctm.org/lessonstudy/asp>
- National Council of Teachers of Mathematics. (2009). *Focus in high school mathematics: Reasoning and sense making*. Reston, VA: Author.
- National Mathematics Advisory Panel. (2008). *Foundations for success: The final report of the national mathematics advisory panel*. Washington, DC: U.S. Department of Education. Retrieved March 25, 2008 from <http://www.ed.gov/about/bdscomm/list/mathpanel/index.html>
- National Research Council. (2000). In Bransford J. D., Brown A. L. and Cocking R. R. (Eds.), *How people learn: Brain, mind, experience and school (expanded edition)*. Washington, DC: National Academy Press.

- National Research Council. (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: National Academy Press.
- National Research Council. (2005). In Donovan M. S., Bransford J. D. (Eds.), *How students learn: Mathematics in the classroom*. Washington, DC: The National Academies Press.
- National Research Council, Committee on Science and Mathematics Teacher Preparation. (2001). *Educating teachers of science, mathematics, and technology: New practices for the new millennium*. Washington, DC: National Academy Press.
- Nemser, S. F. (1983). Learning to teach. In L. S. Shulman, & G. Sykes (Eds.), *Handbook of teaching and policy* (pp. 150-170). New York, NY: Longman.
- Nesbit, J. C., & Adesope, O. O. (2006). Learning with concept and knowledge maps: A meta-analysis. *Review of Educational Research*, 76(3), 413-448.
- Office of Educational Innovation & Evaluation. (2008a). *Summary of data analysis: Pre-post content test for Infinite Math Project*. Unpublished report. Manhattan, KS: Kansas State University.
- Office of Educational Innovation & Evaluation. (2008b). *Summary of data analysis: Pre-post survey for Infinite Math Project*. Unpublished report. Manhattan, KS: Kansas State University.
- Olenchak, F. R. (2001). Lessons learned from gifted children about differentiation. *Teacher Educator*, 36(3), 185-198.

- Pajares, M. F. (1992). Teachers' beliefs and educational research: Cleaning up a messy construct. *Review of Educational Research, 62*(3), 307-332.
- Pappas, T. (1997). *The adventures of Penrose the mathematical cat*. San Carlos, CA: Wide World Publishing/Tetra.
- Pappas, T. (2004). *The further adventures of Penrose the mathematical cat*. San Carlos, CA: Wide World Publishing/Tetra.
- Parker, T. H., & Baldrige, S. J. (2004). *Elementary mathematics for teachers*. Okemos, MI: Sefton-Ash Publishing.
- Patton, M. Q. (2002). *Qualitative research & evaluation methods* (3rd ed.). Thousand Oaks, CA: Sage.
- Peterson, P. L., Fennema, E., Carpenter, T. P., & Loef, M. (1989). Teachers' pedagogical content beliefs in mathematics. *Cognition and Instruction, 6*, 1-40.
- Philipp, R. A., Ambrose, R., Lamb, L. L., Sowder, J. T., Schappelle, B. P., Sowder, L., ..., & Chauvot, J. (2007). Effects of early field experiences on the mathematical content knowledge and beliefs of prospective elementary school teachers: An experimental study. *Journal for Research in Mathematics Education, 38*(5), 438-476.
- Piburn, M., & Sawada, D. (2000). *Reformed teaching observation protocol (RTOP): Reference manual* (ACEPT technical report IN00-3). Phoenix, AZ: Arizona Collaborative for Excellence in the Preparation of Teachers. Retrieved from

<http://www.public.asu.edu/~anton1/AssessArticles/Assessments/Chemistry%20Assessments/RTOP%20Reference%20Manual.pdf>

Posner, T. (2008). Equity in a mathematics classroom: An exploration. In A. H. Schoenfeld (Ed.), *A study of teaching: Multiple lenses, multiple views. JRME monograph series* (pp. 127-168). Reston, VA: National Council of Teachers of Mathematics.

Post, T. R., Harel, G., Behr, M. J., & Lesh, R. (1991). Intermediate teachers' knowledge of rational number concepts. In E. Fennema, T. P. Carpenter & S. J. Lamon (Eds.), *Integrating research on teaching and learning mathematics* (pp. 177-198). New York, NY: State University of New York Press.

Prather, J. (2007). Teaching time savers: Homework without grading. *Focus*, 27(5), 19.

Putnam, R. T. (1992). Teaching the "hows" of mathematics for everyday life: A case study of a fifth-grade teacher. *Elementary School Journal*, 93(2), 163-177.

RAND Mathematics Study Panel. (2003). *Mathematical proficiency for all students: Toward a strategic research and development program in mathematics education*. Santa Monica, CA:

Author. Retrieved from

http://www.rand.org/pubs/monograph_reports/MR1643/MR1643.pref.pdf

Remillard, J. T. (2000). Can curriculum materials support teachers' learning? Two fourth-grade teachers' use of a new mathematics text. *Elementary School Journal*, 100(4), 331-350.

- Remillard, J. T., & Bryans, M. B. (2004). Teachers' orientations toward mathematics curriculum materials: Implications for teacher learning. *Journal for Research in Mathematics Education, 35*, 352-388.
- Remillard, J. T., & Geist, P. K. (2002). Supporting teachers' professional learning by navigating openings in the curriculum. *Journal of Mathematics Teacher Education, 5*(1), 7-34.
- Reys, B. J. (1994). Promoting number sense in the middle grades. *Mathematics Teaching in the Middle School, 1*(2), 114-120.
- Reys, R. E., Reys, B. J., McIntosh, A., Emanuelsson, G., Johansson, B., & Yang, D. C. (1999). Assessing number sense of students in Australia, Sweden, Taiwan, and the United States. *School Science and Mathematics, 99*(2), 61-70.
- Reys, R. E., & Yang, D. C. (1998). Relationship between computational performance and number sense among sixth- and eighth-grade students in Taiwan. *Journal for Research in Mathematics Education, 29*(2), 225-237.
- Richardson, V., & Placier, P. (2001). Teacher change. In V. Richardson (Ed.), *Handbook of research on teaching* (4th ed., pp. 905-947). Washington, DC: American Educational Research Association.
- Riley, R. W. (1997). In math and college-going, middle school makes all the difference. *Middle School Journal, 29*(2), 3-7.
- Rock, M. L., Gregg, M., Ellis, E., & Gable, R. A. (2008). REACH: A framework for differentiating classroom instruction. *Preventing School Failure, 52*(2), 31-47.

- Rowan, B., Chiang, F., & Miller, R. J. (1997). Using research on employees' performance to study the effects of teachers on students' achievement. *Sociology of Education*, 70(4), 256-284.
- Rowan, B., Harrison, D. M., & Hayes, A. (2004). Using instructional logs to study mathematics curriculum and teaching in the early grades. *Elementary School Journal*, 105(1), 103.
- Russell, S. J., Schifter, D., Bastable, V., Yaffee, L., Lester, J. B., & Cohen, S. (1995). Learning mathematics while teaching. In B. S. Nelson (Ed.), *Inquiry and the development of teaching: Issues in the transformation of mathematics teaching* (pp. 9-16). Newton, MA: Center for the Development of Teaching.
- Ruvinsky, J. (2007, December). Planet science. *Discover*, 42-43.
- Ryan, G. W., & Bernard, H. R. (2003). Techniques to identify themes. *Field Methods*, 15(1), 85-109.
- Sanders, W. L., & Horn, S. P. (1998). Research findings from the Tennessee Value-Added Assessment System (TVAAS) database: Implications for educational evaluation and research. *Journal of Personnel Evaluation in Education*, 12(3), 247-256.
- Sanders, W. L., & Rivers, J. C. (1996). *Cumulative and residual effects of teachers on future student academic achievement*. (Research Progress Report). Knoxville, TN: University of Tennessee Value-Added Research and Assessment Center.

- Saxe, G. B., Gearhart, M., & Nasir, N. (2001). Enhancing students' understanding of mathematics: A study of three contrasting approaches to professional support. *Journal of Mathematics Teacher Education*, 4(1), 55-79.
- Schifter, D. (1998). Learning mathematics for teaching: From a teachers' seminar to the classroom. *Journal of Mathematics Teacher Education*, 1(1), 55-87.
- Schifter, D., & Riddle, M. (2004). Teachers become investigators of students' ideas about math: Learning involves seeing how children think about numbers. *Journal of Staff Development*, 25(4), 28-32.
- Schifter, D., Russell, S. J., & Bastable, V. (1999). Teaching to the big ideas. In M. S. Solomon (Ed.), *The diagnostic teacher: Constructing new approaches to professional development* (pp. 22-47). New York, NY: Teachers College Press.
- Schneider, M. (2008, December 9). Reading the TIMSS results: Why the good news may not be so good. *Education Week*, Retrieved May 22, 2009 from <http://www.edweek.org/ew/articles/2008/12/04/15schneider.h28.html>
- Schoenfeld, A. H. (Ed.). (2008). *A study of teaching: Multiple lenses, multiple views*. Reston, VA: National Council of Teachers of Mathematics.
- Schumm, J. S., & Vaughn, S. (1991). Making adaptations for mainstreamed students: General classroom teachers' perspectives. *Remedial and Special Education*, 12(4), 18-27.
- Schumm, J. S., & Vaughn, S. (1992). Planning for mainstreamed special education students: Perceptions of general classroom teachers. *Exceptionality*, 3, 81-98.

- Schwab, J. J. (1978). Education and the structure of the disciplines. In I. Westbury, & N. J. Wilkof (Eds.), *Science, curriculum, and liberal education: Selected essays* (pp. 229-272). Chicago, IL: University of Chicago Press.
- Senk, S. L., & Thompson, D. R. (2003). School mathematics curricula: Recommendations and issues. In S. L. Senk, & D. R. Thompson (Eds.), *Standards-based school mathematics curricula: What are they? What do students learn?* (pp. 3-27). Mahwah, NJ: Erlbaum.
- Seymour, D., & Davidson, P. S. (2003). A history of nontextbook materials. In G. Stanic, & J. Kilpatrick (Eds.), *A history of school mathematics* (pp. 989-1035). Reston, VA: National Council of Teachers of Mathematics.
- Seymour, E. (1995). The loss of women from science, mathematics, and engineering undergraduate majors: An explanatory account. *Science Education*, 79, 437-473.
- Shechtman, N., Roschelle, J., Knudsen, J., Vahey, P., Rafanan, K., Haertel, G., & et al. (2006). *SRI teaching survey: Rate and proportionality*. Unpublished manuscript, SRI International, Menlo Park, CA:
- Sherin, M. G. (2002). When teaching becomes learning. *Cognition and Instruction*, 20(2), 119-150.
- Shroyer, M. G., Wright, E. L., & Ramey-Gassert, L. (1996). An innovative model for collaborative reform in elementary school science teaching. *Journal of Science Teacher Education*, 7(3), 151-168.

- Shulman, L. S. (1986a). Paradigms and research programs in the study of teaching: A contemporary perspective. In M. C. Wittrock (Ed.), *Handbook of research on teaching* (3rd ed., pp. 37-49). New York, NY: Macmillan.
- Shulman, L. S. (1986b). Those who understand: Knowledge growth in teaching. *Educational Researcher*, 15(2), 4-14.
- Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review*, 57(1), 1-22.
- Silberman, M. (1996). *Active learning: 101 strategies to teach any subject*. Needham Heights, MA: Allyn & Bacon.
- Silva, C. M., & Moses, R. P. (1990). The algebra project: Making middle school mathematics count. *Journal of Negro Education*, 59(3), 375-391.
- Simon, M. A. (1993). Prospective elementary teachers' knowledge of division. *Journal for Research in Mathematics Education*, 24, 233-254.
- Simon, M. A. (1997). Developing new models of mathematics teaching: An imperative for research on mathematics teacher development. In E. Fennema, & B. S. Nelson (Eds.), *Mathematics teachers in transition* (pp. 55-86). Mahwah, NJ: Lawrence Erlbaum Associates, Inc.
- Simon, M. A., & Blume, G. W. (1996). Justification in the mathematics classroom: A study of prospective elementary teachers. *Journal of Mathematical Behavior*, 15(1), 3-31.

- Small, M. (2009). *Good questions: Great ways to differentiate math instruction*. Reston, VA: National Council of Teachers of Mathematics.
- Smith, M. S. (2001). *Practice-based professional development for teachers of mathematics*. Reston, VA: National Council of Teachers of Mathematics.
- Smith, M. S., Silver, E. A., & Stein, M. K. (2005). *Improving instruction in rational numbers and proportionality: Using cases to transform mathematics teaching and learning, volume 1*. New York, NY: Teachers College Press.
- Smith, T. M., Desimone, L. M., & Ueno, K. (2005). "Highly qualified" to do what? The relationship between NCLB teacher quality mandates and the use of reform-oriented instruction in middle school mathematics. *Educational Evaluation and Policy Analysis*, 27(1), 75-109.
- Smylie, M. A. (1989). Teachers' views of the effectiveness of sources of learning to teach. *Elementary School Journal*, 89(5), 543-558.
- Snyder, T. D. (2007). *Mini-digest of education statistics 2006* (NCES 2007-067). Washington, DC: National Center for Education Statistics, Institute of Education Sciences, U.S. Department of Education. Retrieved July 7, 2008 from <http://nces.ed.gov/pubsearch/pubsinfo.asp?pubid=2007067>
- Sousa, D. A. (2006). *How the brain learns* (3rd ed.). Thousand Oaks, CA: Corwin Press.
- Sowder, J. T. (1992). Estimation and number sense. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 371-389). New York, NY: Macmillan.

- Sowder, J. T. (2007). The mathematical education and development of teachers. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 157-223). Reston, VA: National Council of Teachers of Mathematics.
- Sowder, J. T., Armstrong, B., Lamon, S., Simon, M. A., Sowder, L., & Thompson, A. G. (1998). Educating teachers to teach multiplicative structures in the middle grades. *Journal of Mathematics Teacher Education*, *1*(2), 127-155.
- Sowder, J. T., Philipp, R. A., Armstrong, B., & Schappelle, B. P. (1998). *Middle-grade teachers' mathematical knowledge and its relationship to instruction: A research monograph*. Albany, NY: SUNY Press.
- Spillane, J. P. (1999). External reform initiatives and teachers' efforts to reconstruct their practice: The mediating role of teachers' zones of enactment. *Journal of Curriculum Studies*, *31*(2), 143-175.
- Spillane, J. P. (2000). Cognition and policy implementation: District policymakers and the reform of mathematics education. *Cognition and Instruction*, *18*(2), 141-179.
- Spillane, J. P., & Jennings, N. E. (1997). Aligned instructional policy and ambitious pedagogy: Exploring instructional reform from the classroom perspective. *Teachers College Record*, *98*(3), 449-481.
- Steele, D. F. (1999). Learning mathematical language in the zone of proximal development. *Teaching Children Mathematics*, *6*, 38-42.

- Stein, M. K., Smith, M. S., Henningsen, M., & Silver, E. A. (2000). *Implementing standards-based mathematics instruction: A casebook for professional development*. New York, NY: Teachers College Press.
- Sternberg, R. J., Ferrari, M., Clinkenbeard, P., & Grigorenko, E. L. (1996). Identification, instruction, and assessment of gifted children: A construct validation of a triarchic model. *Gifted Child Quarterly, 40*(3), 129-137.
- Sternberg, R. J., & Grigorenko, E. L. (2002). The theory of successful intelligence as a basis for gifted education. *Gifted Child Quarterly, 46*(4), 265-277.
- Sternberg, R. J., Grigorenko, E. L., & Jarvin, L. (2001). Improving reading instruction: The triarchic model. *Educational Leadership, 58*(6), 48-52.
- Sternberg, R. J., Torff, B., & Grigorenko, E. L. (1998). Teaching for successful intelligence raises school achievement. *Phi Delta Kappan, 79*(9), 667-669.
- Stevenson, H. W., & Stigler, J. W. (1992). *The learning gap: Why our schools are failing and what we can learn from Japanese and Chinese education*. New York, NY: Simon & Schuster.
- Stigler, J. W., & Hiebert, J. (1999). *The teaching gap: Best ideas from the world's teachers for improving education in the classroom*. New York, NY: Free Press.
- Stylianides, G. J., & Stylianides, A. J. (2009). Facilitating the transition from empirical arguments to proof. *Journal for Research in Mathematics Education, 40*(3), 314-352.

- Stylianou, D. A., Blanton, M. L., & Knuth, E. J. (2009). Introduction. In D. A. Stylianou, M. L. Blanton & E. J. Knuth (Eds.), *Teaching and learning proof across the grades: A K-16 perspective* (pp. 1-12). New York, NY: Routledge.
- Sykes, G. (1999). Teacher and student learning: Strengthening the connection. In L. Darling-Hammond, & G. Sykes (Eds.), *Teaching as the learning profession: Handbook of policy and practice* (pp. 151-179). San Francisco, CA: Jossey-Bass.
- Sykes, W. (1990). Validity and reliability in qualitative market research: A review of the literature. *Journal of the Market Research Society*, 32, 289-328.
- The Carnegie Foundation for the Advancement of Teaching. (2007). *Classifications*. Retrieved March 12, 2008 from <http://www.carnegiefoundation.org/classifications/>
- Thompson, A. G. (1984). The relationship of teachers' conceptions of mathematics and mathematics teaching to instructional practice. *Educational Studies in Mathematics*, 15(2), 105-127.
- Thompson, A. G. (1992). Teachers' beliefs and conceptions: A synthesis of the research. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 127-143). New York, NY: Macmillan.
- Thompson, V., & Mayfield-Ingram, K. (1998). *Family math: The middle school years*. Berkeley, CA: The Regents of the University of California.
- Tieso, C. (2003). Ability grouping is not just tracking anymore. *Roeper Review*, 26(1), 29-36.

- Tieso, C. (2005). The effects of grouping practices and curricular adjustments on achievement. *Journal for the Education of the Gifted*, 29(1), 60-89.
- Tobias, S. (1994). Interest, prior knowledge, and learning. *Review of Educational Research*, 64(1), 37-54.
- Tobin, R., & McInnes, A. (2008). Accommodating differences: Variations in differentiated literacy instruction in grade 2/3 classrooms. *Literacy*, 42(1), 3-9.
- Tomlinson, C. A. (1999a). *The differentiated classroom: Responding to the needs of all learners*. Alexandria, VA: Association for Supervision and Curriculum Development.
- Tomlinson, C. A. (1999b). Mapping a route toward differentiated instruction. *Educational Leadership*, 57(1), 12-16.
- Tomlinson, C. A. (2000). Reconcilable differences? Standards-based teaching and differentiation. *Educational Leadership*, 58(1), 6-11.
- Tomlinson, C. A. (2001). *How to differentiate instruction in mixed-ability classrooms* (2nd ed.). Alexandria, VA: Association for Supervision and Curriculum Development.
- Tomlinson, C. A. (2003). Deciding to teach them all. *Educational Leadership*, 61(2), 6-11.
- Tomlinson, C. A. (2005). Traveling the road to differentiation in staff development. *National Staff Development Council*, 26(4), 8-12.
- Tomlinson, C. A., Brighton, C., Hertberg, H., Callahan, C. M., Moon, T. R., Brimijoin, K., ..., & Reynolds, T. (2003). Differentiating instruction in response to student readiness, interest,

and learning profile in academically diverse classrooms: A review of literature. *Journal for the Education of the Gifted*, 27(2-3), 119-145.

Tomlinson, C. A., & McTighe, J. (2006). *Integrating differentiated instruction and understanding by design: Connecting content and kids*. Alexandria, VA: ASCD.

U.S. Department of Education. (2003). *No child left behind: A parents guide*. Washington, DC: Author.

U.S. Department of Education. (2004). *Part B - Mathematics and Science Partnerships*. Retrieved April 10, 2008 from <http://www.ed.gov/policy/elsec/leg/esea02/pg26.html>

U.S. Department of Education. (2007). *27th annual (2005) report to congress on the implementation of the individuals with disabilities education act*. Retrieved July 7, 2008 from <http://www.ed.gov/about/reports/annual/osep/2005/parts-b-c/index.html>

U.S. Department of Education. (2008a). *Fiscal year 2001-2008 state tables for the U.S. department of education*. Retrieved February 2, 2008 from <http://www.ed.gov/about/overview/budget/statetables/index.html>

U.S. Department of Education. (2008b). *Mathematics and science partnerships*. Retrieved March 13, 2008 from <http://www.ed.gov/programs/mathsci/index.html>

U.S. Department of Education. (2008c). *Mathematics and science partnerships program welcome*. Retrieved March 13, 2008 from <http://www.ed-msp.net/>

- U.S. Department of Education. (2009). *Fiscal year 2008-FY 2010 state tables for the U.S. department of education*. Retrieved May 22, 2009 from <http://www.ed.gov/about/overview/budget/statetables/index.html#update>
- Vacca, R. T. (2002). From efficient decoders to strategic readers. *Educational Leadership*, 60(3), 6-11.
- Van de Walle, J. A. (2004). *Elementary and middle school mathematics: Teaching developmentally* (5th ed.). Boston, MA: Pearson Education.
- Van de Walle, J. A., & Lovin, L. H. (2006a). *Teaching student-centered mathematics: Grades 3-5*. Boston, MA: Pearson.
- Van de Walle, J. A., & Lovin, L. H. (2006b). *Teaching student-centered mathematics: Grades K-3*. Boston, MA: Pearson.
- Verschaffel, L., Greer, B., & De Corte, E. (2007). Whole number concepts and operations. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 557-628). Reston, VA: National Council of Teachers of Mathematics.
- Vygotsky, L. S. (1962). *Thought and language*. Cambridge, MA: MIT Press.
- Walmsley, A. L. E. (2003). *A history of the "new mathematics" movement and its relationship with current mathematical reform*. Lanham, MD: University Press of America.
- Wayne, A. J., & Youngs, P. (2003). Teacher characteristics and student achievement gains. *Review of Educational Research*, 73(1), 89-122.

- Weinberg, S. L., Krulik, S., & Rudnick, J. A. (2002). *Roads to reasoning: Developing thinking skills through problem solving (grade 1)*. Chicago, IL: Wright Group/McGraw-Hill.
- Weiss, I. R., Banilower, E. R., McMahon, K. C., & Smith, P. S. (2001). *Report of the 2000 national survey of science and mathematics education*. Chapel Hill, NC: Horizon Research, Inc. Retrieved from <http://2000survey.horizon-research.com/reports/status.php>
- Weiss, I. R., Pasley, J. D., Smith, P. S., Banilower, E. R., & Heck, D. J. (2003a). *Highlights report, looking inside the classroom: A study of K-12 mathematics and science education in the United States*. Retrieved from <http://www.horizon-research.com>
- Weiss, I. R., Pasley, J. D., Smith, P. S., Banilower, E. R., & Heck, D. J. (2003b). *Looking inside the classroom: A study of K-12 mathematics and science education in the United States*. Retrieved from <http://www.horizon-research.com>
- Wenglinsky, H. (2000). *How teaching matters: Bringing the classroom back into discussions of teacher quality*. Princeton, NJ: Educational Testing Service. Retrieved from <http://www.ets.org/Media/Research/pdf/PICTEAMAT.pdf>
- Wenglinsky, H. (2002). How schools matter: The link between teacher classroom practices and student academic performance. *Education Policy Analysis Archives*, 10(12). Retrieved from <http://epaa.asu.edu/epaa/v10n12/>
- Wenglinsky, H., & Silverstein, S. C. (2007). The science training teachers need. *Educational Leadership*, 64(4), 24-29.

- Westberg, K. L., Archambault, F. X., Dobyms, S. M., & Slavin, T. J. (1993). The classroom practices observation study. *Journal for the Education of the Gifted, 16*, 120-146.
- Whitin, D. J., & Whitin, P. (2004). *New visions for linking literature and mathematics*. Reston, VA: National Council of Teachers of Mathematics.
- Wiggins, G., & McTighe, J. (2005). *Understanding by design* (2nd ed.). Alexandria, VA: ASCD.
- Wilson, S. M., & Berne, J. (1999). Teacher learning and the acquisition of professional knowledge: An examination of research on contemporary professional development. *Review of Research in Education, 24*, 173-209.
- Wilson, S. M., Shulman, L. S., & Richert, A. E. (1987). "150 different ways" of knowing: Representations of knowledge in teaching. In J. Calderhead (Ed.), *Exploring teachers' thinking* (pp. 104-124). London: Cassell.
- Wormeli, R. (2007). *Differentiation: From planning to practice, grades 6-12*. Portland, ME: Stenhouse Publishers.
- Wright, S. P., Horn, S. P., & Sanders, W. L. (1997). Teacher and classroom context effects on student achievement: Implications for teacher evaluation. *Journal of Personnel Evaluation in Education, 11*(1), 57-67.
- Wu, H. (1996). The role of Euclidean geometry in high school. *Journal of Mathematical Behavior, 15*, 221-237.

- Yang, D. C. (2005). Number sense strategies used by 6th-grade students in Taiwan. *Educational Studies, 31*(3), 317-333.
- Yang, D. C., & Reys, R. E. (2001). One fraction problem: Many solution paths. *Mathematics Teaching in the Middle School, 7*(3), 164-166.
- Yang, D. C., Reys, R. E., & Reys, B. J. (2009). Number sense strategies used by pre-service teachers in Taiwan. *International Journal of Science and Mathematics Education, 7*(2), 383-403.
- Yin, R. K. (2003). *Case study research, design and methods* (3rd ed.). Thousand Oaks, CA: Sage Publications.
- Youngs, P., Odden, A., & Porter, A. C. (2003). State policy related to teacher licensure. *Educational Policy, 17*(2), 217-236.
- Zhang, X., McInerney, J., Frechtling, J., Nyre, G., Michie, J., Miyaoka, A., ..., J., & Hershey-Arista, M. (2007). *Effect of STEM faculty engagement in MSP--A longitudinal perspective*. Rockville, MD: Westat. Retrieved August 22, 2007 from <http://esfe.mspnet.org/index.cfm/14376>