ESSAYS ON KNOWLEDGE MANAGEMENT

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Wenli Xiao

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ESSAYS ON KNOWLEDGE MANAGEMENT

Approved by:

Dr. Cheryl Gaimon, Advisor
College of Management
Georgia Institute of Technology

Dr. Stylianos Kavadias
College of Management
Georgia Institute of Technology

Dr. Janice Carrillo
Warrington College of Business Administration
University of Florida

Dr. Beril Toktay
College of Management
Georgia Institute of Technology

Dr. Ravi Subramanian
College of Management
Georgia Institute of Technology

Date Approved:

To Dajun,
for supporting me the whole time,
and to Eric,
for bringing so much joy!

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SUMMARY

For many firms, particularly those operating in high technology and competitive markets, knowledge is cited as the most important strategic asset to the firm, which significantly drives its survival and success (Grant 1996, Webber 1993). Knowledge management (KM) impacts the firm's ability to develop process features that reduce manufacturing costs, product designs with the features and functionality to match consumer demand, and time to market. Unfortunately, many firms lack an understanding of how to develop and exploit knowledge capabilities for success. In this thesis I develop a rich and multifaceted understanding of how KM strategies lead to successful outcomes for a firm. The thesis comprises three essays, described below.

The first essay (Chapter 2) examines how volume-based learning influences the relationship between a buyer and supplier in a two-period Stackelberg game. Three types of knowledge management practices are considered. First, in contrast to the literature, I recognize that knowledge accumulated from current in-house production contributes to the buyer's future product and process development efforts. Second, I allow the supplier to invest in integration process improvement (a form of KD) to reduce the buyer's integration cost. Therefore, the supplier has two mechanisms to impact the buyer's demand: price and process improvement. Lastly, both the buyer and supplier benefit from volume-based learning that reduces their respective production costs. I provide conditions under which the buyer partially outsources component demand as opposed to fully outsourcing or fully producing in-house. In addition, I identify conditions for which the

supplier's price and investment in integration process improvement can serve either as substitutes or complements.

In the second essay (Chapter 3), I consider knowledge development (KD) strategies in a new product development (NPD) project with three stages of activities conducted concurrently: prototyping, pilot line testing, and production ramp-up. I capture the link between successive stages of engineering activities by recognizing that knowledge accumulated in one stage and transferred to another stage improves the efficiency of KD in the recipient stage. A Base Model and two extensions are introduced that differ in the manner in which knowledge transfer (KT) occurs. I find that the NPD manager pursues different dynamic strategies for KD in each stage of the project. In addition, I explore how the effectiveness of KD and the returns to KT impact the optimal strategies adopted in each stage.

In the third essay (Chapter 4), I introduce a dynamic model to explore the impact of KT on a manager's pursuit of an existing product improvement project and an NPD project. These two projects consume costly KD resources. A key feature of the model is the characterization of the KT process from the NPD project to the existing product improvement project. As a result of KT, the ability of the existing product improvement project to generate new knowledge is enhanced. However, the ability of the new product to generate expected net revenue when it is released to the marketplace is reduced due to the loss of proprietary knowledge. I obtain dynamic optimal strategies of KD in both projects and the optimal strategy of KT from the NPD project to the existing product improvement project.

CHAPTER 1

INTRODUCTION

To succeed in the dynamic competitive marketplace, companies need to develop, retain and deploy the knowledge of their workforce (Groff and Jones 2003). According to the knowledge-based view of the firm, knowledge is considered the most strategically important resource (Grant 1996). Knowledge management comprises strategies and practices to identify, create, distribute, transfer and enable the adoption of insights and experiences either embodied in individuals or embedded in technical systems and organizational practices (Gaimon 2008). In this dissertation, I present three essays that examine knowledge management strategies in two domains: manufacturing and outsourcing, and new product development (NPD). In the first essay, I study how learning influences the relationship between a buyer and supplier. Three types of knowledge management practices are considered. For example, the supplier can invest in integration process improvement (a form of KD) to reduce the buyer's integration cost. In the second essay, I consider how to manage the knowledge of the workforce involved in an NPD Project. More specifically, I examine the KD and KT activities in three successive stages of a single NPD project. In the third essay, I explore the impact of KT on the allocation of resources between a NPD project and an existing product improvement project. A key feature of the model is the characterization of the knowledge transfer (KT) process from the NPD project to the existing product improvement project. Due to KT, the ability of the existing product improvement project to improve the features and functionality of the existing product is enhanced.

In the first essay (Chapter 2), I consider the interaction between the outsourcing decision of a buyer and the pricing and integration process improvement decisions of a supplier. In a two-period Stackelberg game, the buyer determines the portion of demand to meet from in-house production versus outsourcing. In an important departure from the literature, the buyer recognizes that current production enhances the development of future products. In addition, I introduce the possibility that the supplier invests in integration process improvement to reduce the buyer's integration cost. Therefore, the supplier manipulates the buyer's outsourcing decision through price and its investment in integration process improvement. Both the buyer and supplier benefit from volume-based learning that reduces subsequent production costs. I provide conditions whereby the buyer partially outsources component demand as opposed to fully outsourcing or fully producing in-house. For example, when the future value is sufficiently large, the buyer optimally pursues a partial outsourcing strategy for component demand to the supplier. I provide important insights on the relationships among the supplier's price and investment in integration process improvement along with the ultimate impact on the buyer's outsourcing decision. In particular, I explore when the supplier's price and investment in integration process improvement are substitute strategies versus complementary.

In the second essay (Chapter 3), I consider KD in an NPD project with three stages of engineering activities: prototyping, pilot line testing, and production ramp-up. The rate that each activity is pursued over time drives the levels of prototyping knowledge, pilot line knowledge, and ramp-up knowledge, respectively. I capture the link between engineering activities by recognizing that, as a result of knowledge transfer (KT), the ability of the recipient stage to generate new knowledge is enhanced. The objective is to maximize the net revenue earned when the product is released to the marketplace less development costs. Net revenue is a function of the levels of knowledge realized at the predetermined product launch time. I introduce a base model and two model variations, as described below.

In the Base Model, KT is fluid, flowing continuously during the NPD project. This situation occurs when development teams are co-located and highly interactive. Moreover, KT occurs only in the forward direction, i.e., from prototyping to pilot line testing and from pilot line testing to production ramp-up. I find the optimal rate of each development activity follows an entirely different path over time. The prototyping team follows a front-loading strategy, the pilot line team follows a moderate delay strategy, and the ramp-up team follows an extreme delay strategy.

In the first extension of the Base Model, I recognize that knowledge accumulated in the production ramp-up stage may be continuously transferred back to the prototyping stage to enhance the ability of the prototyping stage to generate knowledge. Therefore, I consider both forward KT as well as feedback. I find that the returns to feedback not only influence the magnitude of the optimal rates of each development activity, but also significantly impact the optimal paths taken by each development activity during the NPD project. For example, when the returns to feedback are large, the prototyping team follows an extreme delay strategy, while both the pilot line and ramp-up teams follow front-loading strategies. Clearly, relative to the Base Model, entirely different strategies are adopted when the returns to feedback are considered.

In the second extension of the Base Model, knowledge is transferred from one stage to the next at discrete times during the NPD project. This situation is reasonable when product development teams work separately in different locations. The manager determines not only the optimal rates for each development activity over time, but also the optimal number and times for KT. I find the optimal rates of prototyping, pilot line testing and ramp-up activities follow strategies that are analogous to those in the Base Model. This insight is important since it demonstrates that the continuous model is an excellent approximation for the discrete model, which is more difficult to solve. Nevertheless, despite similarities in the structures of the optimal solutions, under certain conditions, I find that important differences arise between the optimal solutions to the continuous and discrete forward KT models.

In the third essay (Chapter 4), I introduce a dynamic model to explore a manager's pursuit of an existing product improvement project and a NPD project. A key feature of the model is the characterization of the KT process from the NPD project to the existing product improvement project. Due to the KT, the ability of the existing product improvement project to improve the features and functionality of the existing product is enhanced. While the existing product improvement project generates revenue throughout the development horizon with certainty, the NPD project generates expected revenue only when it is successfully released to the marketplace, which is uncertain. However, the expected revenue benefits obtained when the new product is successfully released to the marketplace suffers as a result of the KT since proprietary knowledge is lost. Naturally, the two projects consume costly KD resources. I obtain dynamic solutions characterizing how the NPD manager invests in KD activities for both the new and existing projects and the optimal KT strategy from the NPD project to the existing product improvement project. In addition, I explore factors that impact the optimal KD and KT strategies including the effectiveness of NPD and existing product improvement activities and the returns to KT.

CHAPTER 2

THE EFFECT OF LEARNING AND INTEGRATION INVESTMENT ON MANUFACTURING OUTSOURCING DECISIONS: A GAME THEORETIC APPROACH

2.1 Introduction

The problem of whether to manufacture a component in-house or outsource its production is well established. Typically, outsourcing is advocated as a cost reduction strategy. In the electronics industry, outsourcing grew 25% from 1989 to 1998 (Plambeck and Taylor 2005). In pharmaceuticals, outsourcing increased from 20% in 1988 to 50-60% in 1998 (van Arnum 2000). After decades of outsourcing assembly and manufacturing capabilities to Asia, serious concerns have emerged regarding the loss of knowledge gleaned from the actual production experience. This loss in knowledge is particularly threatening for high-tech firms whose product and process development capabilities are tightly linked (Ettlie and Reifeis 1987, Ettlie 1995). According to Pisano and Shih (2009), the outsourcing by U.S. firms of "critical components (such) as light-emitting diodes for the next generation of energy-efficient illumination; advanced displays for mobile phones…new consumer electronics products like amazon's Kindle e-reader…and many of carbon fiber components for Boeing's new 787 Dreamliner" limits the ability of these firms to develop products that will be competitive in the future marketplace.

Increasingly, firms are recognizing the value of manufacturing experience and undertake full or partial in-house production. Many major appliance makers pursue partial or full in-house production because their manufacturing processes embody design intellectual properties that drive product competitiveness (Linton 2011). Intel manufactures all of its state-of-the-art chips in-house since volume-based learning facilitates important enhancements in process technology that ultimately reduce the production cost and improve reliability (Zerega 1999). NEC has reduced its reliance on outsourcing because it recognizes that by exploiting the link between manufacturing and technology innovation it can realize volume-based learning benefits to improve process technology and reduce future production costs (Zerega 1999). To achieve successful development of copper interface and silicon-on-insulation innovations, IBM exploited "the close relationships among design, testing, and production" (Zerega 1999). Cypress Semiconductor only partially outsources production of logic chips so that it may benefit from learning derived from in-house production to develop future process technologies (Zerega 1999). Toyota designs all transmissions in-house and outsources 70% of the production. Toyota leverages the remaining 30% in-house production experience to improve the design of new transmissions and to enhance the development of future technology (Fine and Whitney 1996).

In response to the buyer's growing inclination toward more in-house production and to enhance the desirability of outsourcing, many suppliers make substantial investments in integration process improvement (written as IPI hereafter) to reduce the buyer's integration cost. A supplier may reduce the buyer's integration cost by developing alternative raw materials for components, creating specialized technology, re-designing the integration process, or co-locating manufacturing facilities (Seewer 2004, Dyer 1996). Auto-parts suppliers often locate plants near Toyota and Nissan or to invest in customized physical assets (Dyer and Ouchi 1993; Dyer 1994, 1996). In many cases the extent of a supplier's investment in IPI may be so substantial that it reflects a multi-year commitment between the buyer and supplier. In other words, to justify a substantial investment in IPI, the supplier may require the buyer to commit to do business for several years in the future (Dyer and Ouchi 1993). In fact, a supplier who invests in IPI to reduce a buyer's integration cost is typically considered a strategic partner and receives a long-term contract and commitment as well as future project awards from the buyer (Linton 2011).

In this paper, we introduce a two-period Stackelberg game of a supplier and buyer. Consistent with the literature, we assume both the buyer and supplier obtain reductions in their respective production costs in period 2 based on *volume-based learning* from period 1 production. In addition, motivated by the anecdotal literature cited above, we introduce another learning concept, the *future value*, to capture the benefits of transferring current manufacturing experience for the design and development of future products and technologies. Furthermore, we allow the supplier two mechanisms to impact the buyer's outsourcing decision: *price* and the investment in *integration process improvement* that reduces the buyer's unit cost of integration. A supplier's decision to invest in IPI considers the associated costs versus the value of the increase in outsourced demand generated by the buyer's lower integration cost. According to Johnson and Kaplan (1987), the buyer's integration cost significantly affects its outsourcing decision as well as the price charged by the supplier for outsourced components. Therefore, the supplier's investment in IPI may have strategic consequences (Linton 2011).

Our research considers key issues from the perspectives of both the buyer and supplier. We obtain analytic conditions whereby a buyer chooses a partial outsourcing strategy as opposed to producing all components in-house or outsourcing all components. Moreover, given the game theoretic framework, we develop a deep understanding of how the buyer's future value of manufacturing experience impacts the supplier's price and investment in IPI. In general, we develop analytic insights on the interplay between the supplier's two mechanisms to impact the buyer's outsourcing decision: the price and investment in IPI. In particular, we obtain conditions whereby the supplier's strategies serve as substitutes versus complements with respect to the impact on the buyer's level of outsourcing. As an example of the substitution effect, we identify conditions that drive the supplier to increase price while also increasing its investment in IPI. In addition, we explore the key role played by the buyer's integration cost to impact the supplier's investment in IPI as well as the price charged to the buyer. Lastly, we obtain insights on how the rates of volume-based learning for both the buyer and supplier impact the supplier's investment in IPI and price decisions as well as the buyer's outsourcing decision.

In Section 2.2, we review three streams of literature that relate to this work. The base model is presented in Section 2.3 and important analytic results are given in Sections 2.3 and 2.4. In Section 2.5, we relax assumptions used in the base model and present numerical insights. The conclusions appear in Section 2.6.

2.2 Literature Review

This section is devoted to a discussion of three research streams that relate to this work.

2.2.1 The Learning Curve Literature

Wright (1936) observed that the number of direct labor hours required to produce a unit of output decreases at a uniform rate as the quantity of units manufactured doubles. Most studies of learning curve theory, a fundamental part of management, focus on learning in terms of cost reduction or productivity improvement in relation to cumulative production (Yelle 1979). In our paper, we employ the standard power learning curve to capture reductions in the production costs of both the buyer and supplier in period 2 based on manufacturing experience in period 1.

Beyond this traditional notion of learning, we also recognize that manufacturing experience contributes to the future value of the firm through the link between manufacturing engineering and technology innovation (Ghemawat 1986, Dyer and Ouchi 1993). The concept of design for manufacturability demonstrates the importance of KT between product development and manufacturing engineering (Ettlie and Reifeis 1987). Bergen and McLaughlin (1988) and Ettlie (1995) find that superior product performance is realized when knowledge about manufacturing processes is leveraged in product design and development. In our paper, we consider the future value (beyond the two-period problem) of the buyer's production experience derived from in-house production.

2.2.2 Outsourcing

Few studies have considered partial outsourcing. In a single firm model, Anderson and Parker (2002) examine the outsourcing problem from the buyer's perspective, and discuss conditions that drive the buyer to undertake all in-house production, full outsourcing, or partial outsourcing. Parmigiani (2007) argues that a firm that concurrently insources and outsources may only need to manufacture a small portion of its demand in-house, while still reaping the benefits of learning. In a game theoretic model, Gray et al. (2009) study both the buyer and supplier's problems and provide conditions where the buyer optimally pursues all in-house production, full outsourcing or partial outsourcing. In addition, they discuss how the supplier's pricing decisions are affected by the buyer's outsourcing decisions and they consider volume-based learning in the buyer and supplier's production costs.

Our paper is most related to Gray et al. (2009) but with three important differences. First, in contrast to Gray et al. (2009) who consider piecewise linear learning in the production costs, we consider the traditional power learning. As a result, instead of the boundary solutions obtained by Gray et al. (2009), we are able to obtain interior solutions that facilitate a deeper understanding of the drivers of the buyer’s insourcing versus outsourcing decision. In addition, Gray et al. (2009) do not consider the supplier’s investment in IPI and its impact on the supplier’s price and the buyer’s outsourcing decision. Lastly, Gray et al. (2009) find that the supplier benefits from an increase in the buyer’s rate of learning that reduces the buyer’s production cost, whereas the buyer benefits from an increase in the supplier’s rate of learning only if the buyer’s rate of learning is positive. In contrast, we show that if the supplier’s rate of learning increases, both the buyer and supplier realize higher profit. Furthermore, we show that if the buyer has a higher rate of learning in its production cost, the buyer realizes higher profit and supplier’s profit declines.

2.2.3 Technology, Product and System Integration

Systems engineering is a well-established discipline (Kossiakoff et al. 2003). One element of systems engineering concerns the management of component integration (Fine and Whitney 1996). A firm with a strong systems engineering capability is better able to integrate in-sourced or outsourced components. Anderson and Parker (2002, 2008) consider integration as a driver of a buyer's outsourcing decision. In Anderson and Parker (2002) volume-based learning reduces the integration cost inside the firm, whereas in Anderson and Parker (2008) learning cost reductions are realized in relation to integration activities both inside and outside the firm's boundaries. However, these authors analyze the problem only from the buyer's perspective and do not allow the supplier to invest in IPI. In contrast, we allow the supplier to determine its investment in IPI. Specifically, we consider the important interplay between the supplier's pricing strategy, its investment in IPI and the buyer's outsourcing decision.

2.3 Base Model Formulation

In this section, we introduce the base model, a two period non-cooperative Stackelberg game of a buyer (b) and supplier (s). At the first stage of the first period, the supplier determines its investment in IPI, denoted by μ, that reduces the integration cost realized by the buyer in both periods 1 and 2. At the same time, the supplier determines the fixed price, denoted by P, to charge the buyer for outsourcing in both periods 1 and 2. The supplier selects its price and IPI strategies to maximize its two-period profit. In the second stage of the first period, the buyer reacts to the supplier's actions and determines the fixed amount of its component demand to outsource in both periods 1 and 2, denoted by w. The buyer and supplier benefit from volume-based learning derived from production experience in the first period that reduces their respective unit production costs in the second period. Despite fixing the decisions of the buyer and supplier in periods 1 and 2, a two-period model is needed to capture the volume-based learning cost reductions realized by both firms in period 2.

The two-period fixed price and outsourcing strategies simplify the model and enable us to obtain many important analytic results. Beyond that, however, the fixed price and outsourcing strategies reflect a relationship between the buyer and supplier that is consistent with practice (Dyer and Ouchi 1993, Linton 2011). The investment in IPI represents a major commitment by the supplier and benefits the buyer in both periods 1 and 2. It is therefore reasonable that the supplier expects a similar commitment from the buyer; i.e., the buyer must agree to a two-period outsourcing strategy. Naturally, the buyer would not enter into such an agreement without a two-period pricing strategy first announced by the supplier. Thus, the underlying modeling assumptions in the base model are reasonable. Nevertheless, in an extension (Section 5), we relax the fixed price and fixed outsourcing assumptions and allow the buyer and supplier to make different decisions in each period. In contrast to the analytic results for the base model given in Section 4, the managerial insights in Section 5 are derived from numerical experimentation.

In the next sections, we define the model mathematically. Table A.1 (Appendix) contains a summary of notation. Let $X'(y)$ and $X''(y)$ represent the first and second order derivatives of X with respect to y, respectively. Let X^* denote the optimal solution of X. To improve readability, all proofs appear in Appendix A.

2.3.1 Base Model Features

Buyer's Volume of In-house Production versus Outsourcing. Let $V > 0$ be the volume of component demand to be met by the buyer in both the first and second periods. The buyer produces $w \in [0, V]$ units in-house and outsources $V - w$ units in periods 1 and 2. Letting R denote the market price of the component, the buyer's revenue in periods 1 and 2 is given by $2RV$. In Section 5, we allow dynamic demand in the two-period problem.

Buyer and Supplier's Manufacturing Costs. Let C_b (C_s) be the unit manufacturing cost for the buyer (supplier) in the first period. Let $w^{-\alpha}$ and $(V - w)^{-\beta}$ represent the learning curves for the buyer and the supplier, respectively, where the rates of learning are denoted by α and $\beta \in (0,1)$. Let $C_b w^{-\alpha}$ be the 2nd period unit production cost for the buyer; let $C_s(V - w)^{-\beta}$ be the 2nd period unit production cost for the supplier. Naturally, each firm's period 2 unit manufacturing cost decreases at a decreasing rate in relation to its period 1 production volume (Yelle 1979).

Buyer's Integration and Outsourcing Costs. Integration costs are incurred for the assembly of internally produced components as well as components outsourced to a supplier (Iansiti 1995a, 1995b, Anderson and Parker 2002, 2008). We assume that the buyer's unit manufacturing cost in period 1, denoted by C_b, includes any costs associated with internal integration. (Therefore, volume-based learning from period 1 may reduce the buyer's manufacturing and internal integration costs in period 2.) In contrast, we explicitly define C_i as the unit integration cost incurred by the buyer for each unit outsourced to the supplier. Naturally, the buyer's unit outsourcing cost is the sum of the price charged for the outsourced component and the unit integration cost. Let $P > 0$

denote the price charged by the supplier per unit outsourced to the buyer. Therefore, the unit outsourcing cost incurred by the buyer is denoted as $P + C_i$.

Buyer's Future Value from Volume-based Learning. While we consider a two-period game, we recognize that the buyer may derive future benefits, beyond period 2, from volume-based learning. The benefits may be substantial and reflect the value of current manufacturing experience on the development of future products and technologies. We assume the future value is driven by the most recent production experience (i.e., production experience accumulated in the 2nd period). We introduce the future value, denoted by $f(w) = f_0 w^{f_1}$, with $f_0 > 0$ representing a scaling factor and $f_1 \in (0,1)$ representing the rate of diminishing returns. That is, the future value the buyer gains from volume-based learning increases at a decreasing rate as the quantity of the buyer's period 2 in-house production increases.

Supplier's Integration Process Improvement. We consider a supplier that has the ability to reduce the integration cost incurred by the buyer by investing in IPI. Process improvement activities may include the investment in new manufacturing equipment, the re-design of the integration process, and the hiring of more skilled employees (Carrillo and Gaimon 2004). At the beginning of period 1, the supplier determines the level of IPI, denoted as $\mu \in [0,1]$. The supplier incurs the cost $U\mu^{\gamma}$ for its investment in IPI where $U > 0$ and $\gamma > 1$. Note that U is a scaling factor and γ represents the diseconomies of scale associated with larger investments in IPI. The assumption of diseconomies of scale is standard in the large body of literature on process improvement (Carrillo and Gaimon 2004, Chand et al. 1996).

The supplier's investment in IPI directly impacts the buyer's integration cost. Let $L > 0$ represent the buyer's base unit integration cost, i.e., without any investment in IPI by the supplier. Let the buyer's unit integration cost be written as $C_i = L(1 - \mu)$ so that as the supplier's pursuit of IPI increases, the buyer's unit integration cost decreases. Of course, since the supplier invests in IPI at the start of period 1, the unit integration cost incurred is the same in both periods 1 and 2.

The two-period non-cooperative game is solved using backward induction: first we analyze the buyer' production decision (w); subsequently we determine the supplier's optimal price and its investment in IPI (P and μ).

2.3.2 Buyer's Decision

In response to the supplier's price, P, and investment in IPI, μ, the buyer optimally determines the amount of component demand to be met from in-house production versus outsourcing in periods 1 and 2, w. The buyer's objective is to maximize profit ($\Pi_b(w)$) over both periods as shown in Equation (1) subject to the constraints in Equation (2). The buyer's two-period profit is comprised of (i) the revenue realized in periods 1 and 2; (ii) the in-house production costs in periods 1 and 2; (iii) the outsourcing costs in periods 1 and 2; and (iv) the future value of volume-based learning from period 2 in-house production. The cost of outsourcing includes the cost of purchasing components from the supplier as well as integrating those components into the buyer's production process. Since each period's demand must be met from in-house production or outsourcing in that period, total revenue is fixed. The Lagrangian to be maximized appears in Equation (3) where λ_1 is the Lagrange multiplier corresponding to the constraint $w \leq V$. (The other constraints in Equation (2) are considered implicitly, as described in Theorem 1.) The first order Kuhn-Tucker conditions are stated in Equations (4)-(5) (Chiang and Wainwright 2005). The expression for $\frac{d\mathcal{L}_b}{dw}$ appears in Equation (6). Lastly, the sufficiency condition involves the second order derivative in Equation (7). With $X(w)$ given in Equation (8), we know $\frac{d^2\mathcal{L}_b}{dw^2}$ has the same sign as $X(w)$. We use this insight in the statement of Theorem 1, below.

$$\Pi_b(w) = 2RV - C_b w - C_b w^{1-\alpha} - 2\big(P + L(1-\mu)\big)\big(V - w\big) + f_0 w^{f_1} \quad (1)$$

$$0 \le w \le V, \text{ and } \Pi_b \ge 0 \quad (2)$$

$$\mathcal{L}_b(w, \lambda_1) = 2RV - C_b w - C_b w^{1-\alpha} - 2\big(P + L(1-\mu)\big)(V - w) + f_0 w^{f_1}$$
$$+\lambda_1(V - w) \quad (3)$$

$$\frac{d\mathcal{L}_b}{dw} \le 0, w \ge 0, \text{ and } w\frac{d\mathcal{L}_b}{dw} = 0 \quad (4)$$

$$\frac{d\mathcal{L}_b}{d\lambda_1} \ge 0, \lambda_1 \ge 0, \text{ and } \lambda_1\frac{d\mathcal{L}_b}{d\lambda_1} = 0 \quad (5)$$

$$\frac{d\mathcal{L}_b}{dw} =- C_b(1 + (1-\alpha)w^{-\alpha}) + 2\big(P + L(1-\mu)\big) + f_0 f_1 w^{f_1 - 1} - \lambda_1 = 0 \quad (6)$$

$$\frac{d^2\mathcal{L}_b}{dw^2} = \alpha(1-\alpha)C_b w^{-\alpha-1} - f_0 f_1(1-f_1)w^{f_1-2} = w^{f_1-2}[\alpha(1-\alpha)C_b w^{1-\alpha-f_1} -$$
$$f_0 f_1(1-f_1)] \quad (7)$$

$$X(w) = \alpha(1-\alpha)C_b w^{1-\alpha-f_1} - f_0 f_1(1-f_1) \quad (8)$$

Theorem 1 gives the buyer's optimal solution. In Section 4, we analytically explore drivers of the buyer’s optimal solution.

Theorem 1. *Four cases characterize the buyer’s optimal solution denoted by* w^* *and* λ_1^**. To obtain non-trivial solutions in which the buyer participates in the game, we assume* $\Pi_b(w^*) \ge 0$ *so that* $\mathcal{L}_b(w^*, \lambda_1^*) \ge 0$*. In addition, for Cases 2 and 3, let* $\lambda_1^{pos} =- C_b(1 + (1-\alpha)V^{-\alpha}) + f_0 f_1 V^{f_1-1}$.

- ***Case 1**: If $1-\alpha-f_1=0$ and $C_b<f_0$ or if $1-\alpha-f_1>0$ and $X(V)\leq 0$, then $\mathcal{L}_b(w,\lambda_1)$ is concave for $w\in[0,V]$ (see Figure 2.1, Case 1). The optimal solution is $w^*=w^{int}$ which satisfies Equation (6) with ${\lambda_1}^*$ = 0 (i.e., w^{int} is the interior solution).*
- ***Case 2**: If $1-\alpha-f_1=0$ and $C_b\geq f_0$ or if $1-\alpha-f_1<0$ and $X(V)\geq 0$, then $\mathcal{L}_b(w,\lambda_1)$ is convex for $w\in[0,V]$ (see Figure 2.1, Case 2). There are two possible optimal solutions. (a) $w^*=0$ and ${\lambda_1}^*=0$ if $\mathcal{L}_b(0,0)>\mathcal{L}_b(V,\lambda_1^{pos})$; or (b) $w^*=V$ and ${\lambda_1}^*=\lambda_1^{pos}>0$, otherwise.*
- ***Case 3**: If $1-\alpha-f_1>0$ and $X(V)>0$, then $\mathcal{L}_b(w,\lambda_1)$ is initially concave then becomes convex for $w\in[0,V]$ (see Figure 2.1, Case 3). There are two possible optimal solutions. (a) $w^*=w_H^{int}$ and ${\lambda_1}^*=0$ if $\mathcal{L}_b(w_H^{int},0)>\mathcal{L}_b(V,\lambda_1^{pos})$, where w_H^{int} satisfies Equation (6) (w_H^{int} is the interior solution that maximizes the concave domain); or (b) $w^*=V$ and ${\lambda_1}^*=\lambda_1^{pos}>0$, otherwise.*
- ***Case 4**: If $1-\alpha-f_1<0$ and $X(V)<0$, then $\mathcal{L}_b(w,\lambda_1)$ is initially convex then becomes concave for $w\in[0,V]$ (see Figure 2.1, Case 4). There are two possible optimal solutions. (a) $w^*=w_H^{int}$ and ${\lambda_1}^*=0$ if $\mathcal{L}_b(w_H^{int},0)>\mathcal{L}_b(0,0)$, where w_H^{int} satisfies Equation (6) (w_H^{int} is the interior solution that maximizes the concave domain; or (b) $w^*=0$ and ${\lambda_1}^*=0$, otherwise.*

An important result emerges from Theorem 1. If the buyer were to ignore the future value of experience (dropping the last term in Equation (1)), then $\Pi_b(w)$ is always convex. Thus, the buyer optimally produces everything in-house to meet demand or outsources all demand to the supplier depending on which is smaller: the marginal in-house production cost or the marginal cost of outsourcing. Said differently, Theorem 1 shows that even if the marginal outsourcing cost is lower than the marginal in-house production cost, the buyer may optimally pursue partial in-house production if it recognizes a sufficient future value of experience.

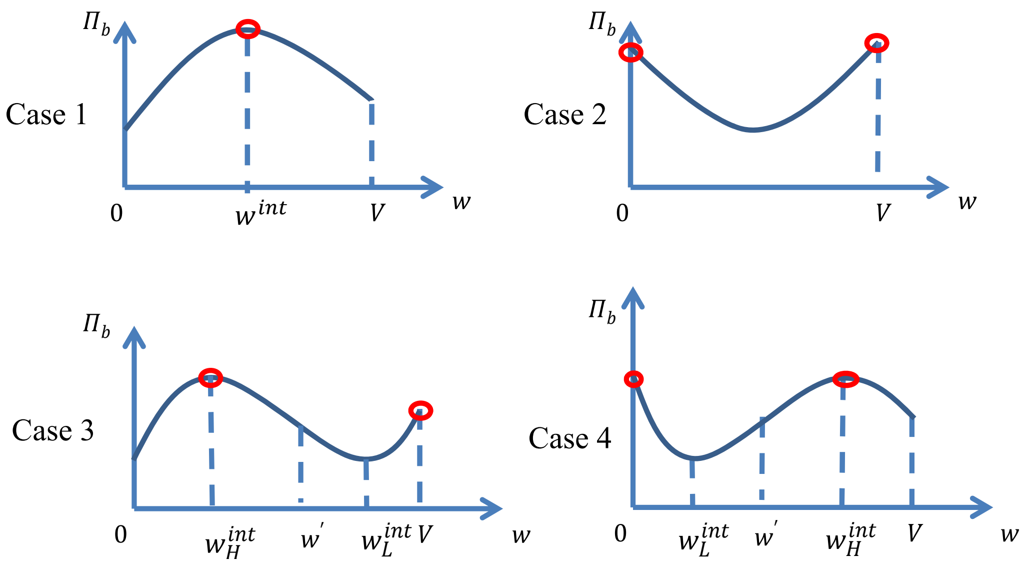

Figure 2.1: Four Cases for the Buyer's Problem

2.3.3 Supplier's Decisions

At the start of period 1, the supplier optimally determines the price, P, to charge the buyer in both periods 1 and 2, and the one-time investment in IPI, μ, to maximize its two-period profit as given in Equation (9) subject to the constraints in Equation (10). Naturally, the supplier's decisions reflect the subsequent response by the buyer (w^*). The first term in Equation (9) represents the revenue earned by the supplier from the buyer's outsourcing in periods 1 and 2. The second and third terms are the supplier's production costs to meet $V - w^*$ units of demand from the buyer in periods 1 and 2. Lastly, the fourth term is the one-time cost the supplier incurs for the investment in IPI. The Lagrangian to be maximized appears in Equation (11) where λ_2 and λ_3 are the Lagrange multipliers corresponding to the constraints $\Pi_b \geq 0$ and $\mu \leq 1$. The first order Kuhn-Tucker conditions for P, μ, λ_2, and λ_3 are stated in Equations (12)-(15) (Chiang and Wainwright 2005). Lastly, we define $H(P,\mu) = \begin{pmatrix} \partial^2\Pi_s/\partial P^2 & \partial^2\Pi_s/\partial \mathrm{P}\,\partial\mu \\ \partial^2\Pi_s/\partial\mu\,\partial \mathrm{P} & \partial^2\Pi_s/\partial\mu^2 \end{pmatrix}$ as the Hessian Matrix of the supplier's profit with respect to the solutions for P and μ. We assume the Hessian Matrix is negative definite (i.e., $H(P,\mu) < 0$) so that the supplier's profit is jointly concave with respect to P and μ and sufficiency is guaranteed. Note that the assumption $H(P,\mu) < 0$ may be violated under extreme conditions: L, C_b or C_s is extremely large; V or f_0 is extremely small; or γ is extremely close to 1. (Naturally, if the objective is not jointly concave in P and μ then the optimal solution for one or more of the decision variables lies on a boundary.)

To obtain non-trivial solutions in which the buyer and supplier both participate in the game, we introduce bounds on P. Let $\underline{P}(\mu) > 0$ be the lower bound on P to ensure $\Pi_s \geq 0$ and let $\overline{P}(\mu) > \underline{P}(\mu)$ be the upper bound of P to ensure $\Pi_b(w^*) \geq 0$. It is easy to see that both bounds on P are impacted by μ, and the solutions of Equation (12)-(15) are functions of w^*. To focus on non-trivial solutions, we assume that $P \geq \underline{P}(\mu)$ such that $\Pi_s \geq 0$, i.e., the supplier participates in the game. The non-negativity of μ is implicitly guaranteed, as described in Theorem 2.

The supplier's optimal solution is given in Theorem 2. We analytically explore drivers of this solution in Section 4.

$$\Pi_s(P,\mu) = 2P(V - w^*) - C_s(V - w^*) - C_s(V - w^*)^{1-\beta} - U\mu^\gamma \quad (9)$$

$$0 \leq \mu \leq 1; \Pi_b \geq 0;\ P \geq 0;\ \Pi_s \geq 0 \quad (10)$$

$$\mathcal{L}_s(P,\mu,\lambda_2,\lambda_3) = 2P(V - w^*) - C_s(V - w^*) - C_s(V - w^*)^{1-\beta} - U\mu^\gamma$$

$$+\lambda_2\big(2RV - C_b w^* - C_b {w^*}^{1-\alpha} - 2\big(P + L(1-\mu)\big)\big(V - w^*\big) + f_0 {w^*}^{f_1}\big)$$

$$+\lambda_3(1-\mu) \tag{11}$$

$$\frac{d\mathcal{L}_s}{dP} \leq 0, P \geq 0, \text{and } P\frac{d\mathcal{L}_S}{dP} = 0 \tag{12}$$

$$\frac{d\mathcal{L}_s}{d\mu} \leq 0, \mu \geq 0, \text{and } \mu\frac{d\mathcal{L}_S}{d\mu} = 0 \tag{13}$$

$$\frac{d\mathcal{L}_s}{d\lambda_2} \geq 0, \lambda_2 \geq 0, \text{and } \lambda_2\frac{d\mathcal{L}_s}{d\lambda_2} = 0 \tag{14}$$

$$\frac{d\mathcal{L}_s}{d\lambda_3} \geq 0, \lambda_3 \geq 0, \text{and } \lambda_3\frac{d\mathcal{L}_s}{d\lambda_3} = 0 \tag{15}$$

Theorem 2. *The supplier's optimal solution consists of* $P^*, \mu^*, {\lambda_2}^*$ *and* ${\lambda_3}^*$. *Four solutions are obtained if the buyer's optimal solution is* $w^* = w^{int}$ *or* w_H^{int} *(Cases 1a-1d of Theorem 2); four solutions are obtained if the buyer's optimal solution is* $w^* = 0$ *(Cases 2a-2d of Theorem 2). To obtain non-trivial solutions in which the supplier participates in the game, we assume* $w^* \in [0, V)$ *and* $\Pi_s(P^*, \mu^*) \geq 0$ *so that* $\mathcal{L}_s(P^*, \mu^*, \lambda_2^*, \lambda_3^*) \geq 0$ *holds. (Therefore, we do not consider* $w^* = V$ *as in Theorem 1, Cases 2b and 3b.)*

- ***Case 1**:* $w^* = w^{int}$ *or* w_H^{int}.
  - ***Case 1a)**:* $(P^*, \mu^*, \lambda_2^*, \lambda_3^*) = (P^{1a}, \mu^{1a}, 0, 0)$ *when* $\mu^{1a} \in [0,1]$ *and* $P^{1a} \in [\underline{P}(\mu_{1a}), \overline{P}(\mu_{1a})]$. P^{1a} *and* μ^{1a} *are obtained by simultaneously solving* $\frac{d\mathcal{L}_s}{dP} = 0$ *and* $\frac{d\mathcal{L}_s}{d\mu} = 0$ *with* $\lambda_2 = \lambda_3 = 0$.
  - ***Case 1b)**:* $(P^*, \mu^*, \lambda_2^*, \lambda_3^*) = \big(P^{1b}, 1, 0, \lambda_3^{1b}\big)$ *with* $\lambda_3^{1b} = 2L(V - w^*) - U\gamma > 0$ *and* $P^{1b} \in [\underline{P}(1), \overline{P}(1)]$. P^{1b} *and* λ_3^{1b} *are obtained by simultaneously solving* $\frac{d\mathcal{L}_s}{dP} = 0$ *and* $\frac{d\mathcal{L}_s}{d\mu} = 0$ *with* $\lambda_2 = 0$ *and* $\mu = 1$.
  - ***Case 1c)**:* $(P^*, \mu^*, \lambda_2^*, \lambda_3^*) = (\overline{P}^{1c}, \mu^{1c}, \lambda_2^{1c}, 0)$ *where* $\overline{P}^{1c}$ *and* μ^{1c} *are obtained by maximizing* Π_s *given* $\Pi_b(w^*) = 0$. λ_2^{1c} *satisfies* $\frac{d\mathcal{L}_s}{dP} = 0$ *with* $\lambda_3 = 0$. *Note that* $\mu^{1c} \in [0,1]$.

- *Case 1d)*: $(P^*, \mu^*, \lambda_2^*, \lambda_3^*) = \left(\overline{P}^{1d}, 1, \lambda_2^{1d}, \lambda_3^{1d}\right)$ *where* $\overline{P}^{1d}$ *satisfies* $\Pi_b(w^*) = 0$, *and* λ_2^{1d} *and* λ_3^{1d} *satisfy* $\frac{d\mathcal{L}_s}{dP} = 0$ *and* $\frac{d\mathcal{L}_s}{d\mu} = 0$ *with* $\mu = 1$.

- ***Case 2***: $w^* = 0$.

→ *Suppose* $1 - \alpha - f_1 < 0$ *and* $X(V) > 0$ *or* $1 - \alpha - f_1 = 0$ *and* $C_b \geq f_0$.

- ***Case 2a):*** $(P^*, \mu^*, \lambda_2^*, \lambda_3^*) = (P^{2a}, \mu^{2a}, 0, 0)$, *where* P^{2a} *and* μ^{2a} *are obtained by maximizing* Π_s *given* $\Pi_b(0) = \Pi_b(V) \geq 0$. *Note that* $P^{2a} \in [\underline{P}(\mu^{2a}), \overline{P}(\mu^{2a})]$ *and* $\mu^{2a} \in [0,1]$.

- ***Case 2b):*** $(P^*, \mu^*, \lambda_2^*, \lambda_3^*) = (P^{2b}, 1, 0, \lambda_3^{2b})$ *where* P^{2b} *satisfies* $\Pi_b(0) = \Pi_b(V) \geq 0$ *and* λ_3^{2b} *satisfies* $\frac{d\mathcal{L}_s}{d\mu} = 0$ *with* $\mu = 1$. *Note that* $P^{2b} \in [\underline{P}(1), \overline{P}(1)]$.

→ *Suppose* $1 - \alpha - f_1 < 0$ *and* $X(V) < 0$.

- ***Case 2c):*** $(P^*, \mu^*, \lambda_2^*, \lambda_3^*) = (P^{2c}, \mu^{2c}, 0, 0)$, *where* P^{2c} *and* μ^{2c} *are obtained by maximizing* Π_s *given* $\Pi_b(0) = \Pi_b\left(w_H^{int}\right) \geq 0$. *Note that* $P^{2c} \in [\underline{P}(\mu^{2c}), \overline{P}(\mu^{2c})]$ *and* $\mu^{2c} \in [0,1]$.

- ***Case 2d):*** $(P^*, \mu^*, \lambda_2^*, \lambda_3^*) = (P^{2d}, 1, 0, \lambda_3^{2d})$, *where* P^{2d} *satisfies* $\Pi_b(0) = \Pi_b\left(w_H^{int}\right) \geq 0$ *and* λ_3^{2d} *satisfies* $\frac{d\mathcal{L}_s}{d\mu} = 0$ *with* $\mu = 1$. *Note that* $P^{2d} \in [\underline{P}(1), \overline{P}(1)]$.

2.4 Analysis of the Model

In this section, we provide results based on analytic sensitivity analysis. While we focus our attention on the interior solutions ($w^* = w^{int}$ or w_H^{int}, $P^* = P^{1a}$ and $\mu^* = \mu^{1a}$), we note that analysis of boundary solutions is analogous, and therefore omitted. Our key results are stated in corollaries and are derived using comparative statics (Chiang and Wainwright 2005).

2.4.1 Insights on Integration

U is the scaling factor of the supplier's cost for μ units of IPI, which reflects both the complexity and modularity of the component. A large (small) value of U indicates that each unit of IPI is more (less) costly to the supplier. Intuitively, if the component is more (less) complex or less (more) modular then efforts to improve the buyer's integration process are more (less) costly so that U is larger (smaller). As expected, we find that if U increases, the supplier reduces its level of IPI ($\mu^*\downarrow$) so that the buyer's unit integration cost increases. However, we also find that with larger U the supplier may charge a higher or lower price (P^*). Nevertheless, regardless of the change in price, the buyer always manufactures more components in-house ($w^*\uparrow$). The results on w^* and P^* are quite interesting and warrant further interpretation.

With the lower investment in IPI that occurs when U is larger, we expect the supplier to charge a lower price to entice outsourcing. We find, however, the supplier may charge a higher or lower price depending on the sensitivity of the buyer's profit to its level of in-house production. In particular, if the buyer's profit is highly sensitive to its amount of in-house production (or equivalently, insensitive to the supplier's decisions), then the supplier charges a higher price ($P^*\uparrow$) simply to increase its marginal revenue. Since $P^*\uparrow$ and $\mu^*\downarrow$ we know the buyer's unit cost of outsourcing is larger and it optimally undertakes more in-house production ($w^*\uparrow$). Alternatively, if the buyer's profit is insensitive to the portion of demand met from in-house production (or equivalently, highly sensitive to the supplier's decisions), then the supplier charges a lower price ($P^*\downarrow$) to attract more outsourcing from the buyer. However, the decrease in price only reduces the supplier's loss in total revenue and does not fully compensate for the increase in the unit integration cost. As a result, the buyer's unit outsourcing cost is larger and $w^*\uparrow$. Therefore, regardless of whether the supplier's price is higher or lower, if U increases, the buyer's unit outsourcing cost is larger, and the buyer pursues more in-house production.

Overall, the buyer's profit declines because the additional benefits derived from more in-house production are less than the additional outsourcing costs incurred. Similarly, driven by the loss in total revenue, the supplier's profit declines when U increases.

The above insights are summarized in Corollary 1. To state the results mathematically, we introduce the following notation. Let $Z \equiv \frac{d^2\Pi_b}{dw^2}|_{w=w^*} = \frac{d^2\mathcal{L}_b}{dw^2}|_{w=w^*}$ and note that Z < 0 holds. If Z is extremely small in relation to zero (i.e., $Z \ll 0$), then the buyer's profit is highly sensitive to the quantity of demand met by in-house production and relatively less sensitive to the supplier's decisions on price and IPI. Alternatively, if Z is close to zero (i.e., $Z \to 0^-$), then the buyer's profit is highly sensitive to the supplier's price and IPI decisions and relatively less sensitive to the quantity of in-house production. Also, let $\Phi_1 = \frac{1}{4}\frac{\partial^2\Pi_s}{\partial P^2}(\frac{d^2\mathcal{L}_b}{dw^2})^2 < 0$ and $\Phi_2 = \Phi_1 - Z$. Clearly, Φ_1 is more negative in relation to the decrease in the supplier's marginal profit with respect to price; and Φ_2 increases as Φ_1 increases or Z decreases.

Corollary 1. *Suppose the supplier's scaling factor for its investment in IPI (U) increases. The buyer's in-house production volume (w^*) is larger, and the supplier's level of IPI (μ^*) is smaller. If $\Phi_2 > 0$ ($\Phi_2 < 0$), then the supplier's price (P^*) is higher (lower). Lastly, both the buyer and supplier earn less profit.*

Another parameter related to the integration process is the buyer's base unit cost for integration (L). Intuitively, L is affected by the past collaborative relationship between the buyer and supplier, the modularity of the component, and the ability of the buyer's engineers to design an efficient component integration process. For instance, if the component is highly modular, integrating it with the production system is more straightforward and the base unit cost for integration is smaller. Similarly, if the buyer and supplier have collaborated well in the past, the buyer may benefit from a more efficient integration process. In an intriguing result, we find that the effect of an increase in L on the decisions of both the supplier and buyer depends on the magnitude of U, as described below.

Suppose the supplier's investment cost for IPI (U) is large and L increases. We find that the supplier's optimal investment in IPI may increase or decrease. There are two situations to consider. First, when L is large, we obtain μ^* is moderate. Due to diseconomies of scale and since U is large, the supplier incurs a substantial cost for even a small increase in μ^*. As a result, the increase in revenue for the supplier that would be obtained by reducing the buyer's integration cost does not compensate for the higher cost incurred for IPI. Therefore, when both L and U are large and L increases, the supplier optimally reduces its level of IPI ($\mu^*\downarrow$). Naturally, the reduction in μ^* and increase in L cause the buyer's unit cost of integration to increase ($L(1-\mu^*)\uparrow$). Second, when L is small, we obtain μ^* is small so that if L increases, the cost incurred by the supplier to increase its level of IPI is moderate even with large U. In fact, the increase in the supplier's revenue from more outsourcing fully compensates for the increase in the investment cost for IPI so that $\mu^*\uparrow$. Nevertheless, the increase in μ^* is not sufficient to compensate for the increase in L and the buyer's unit cost of integration is larger ($L(1-\mu^*)\uparrow$).

Again, we consider the situation where U is large and L increases, but shift our focus to the effect on the supplier's price and the buyer's level of outsourcing. Regardless of whether L is large or small, if L increases and the buyer's profit is highly sensitive to the portion of demand met from in-house production (i.e., insensitive to the supplier's price), then the supplier charges a higher price ($P^*\uparrow$) to increase marginal revenue. With the higher price and higher unit integration cost, the buyer incurs a higher unit cost for outsourcing and produces more in-house ($w^*\uparrow$). Alternatively, if the buyer's profit is highly sensitive to the supplier's decision and L increases, then the supplier charges a lower price ($P^*\downarrow$). However, the lower price is not sufficient to compensate for the larger base unit integration cost so that, again, the buyer's unit outsourcing cost is larger and $w^*\uparrow$. Clearly in this situation, the supplier charges a lower price simply to limit its loss in total revenue. Thus, when U is large and L increases, the supplier's seeks to limit the negative impact on profit due to the reduction in the buyer's outsourcing. Lastly, we find both the buyer and supplier earn lower profit as L increases.

The above insights are summarized in Corollary 2a. To state the results mathematically, we define $\Phi_3 = (V - w^*) - L\frac{dw^*}{dL}$. If $U > \frac{2L(V-w^*)\gamma^{\gamma-2}}{(\gamma-1)^{\gamma-1}}$, we refer to U as large; if $U = \frac{2L(V-w^*)\gamma^{\gamma-2}}{(\gamma-1)^{\gamma-1}}$, we refer to U as moderate; if $U < \frac{2L(V-w^*)\gamma^{\gamma-2}}{(\gamma-1)^{\gamma-1}}$, we refer to U as small. Note that Φ_3 increases in relation to the amount of outsourcing, decreases in relation to the base unit integration cost, and decreases in relation to the marginal increase in the buyer's amount of in-house production due to an increase in the base unit integration cost.

Corollary 2a. *Suppose the buyer's base unit cost for integration (L) increases and* U *is large. If* $\Phi_3 > 0$ ($\Phi_3 < 0$)*, then the supplier increases (reduces) its investment in IPI (*μ^**). If* $\Phi_2 > 0$ ($\Phi_2 < 0$)*, the supplier's price (*P^**) is higher (lower) and the buyer's quantity of in-house production (*w^**) increases. Both the buyer and supplier earn less profit.*

In Corollary 2b, suppose the cost of investing in IPI is small (i.e., U is small). If L increases, we find that the supplier increases its level of IPI to attract outsourcing from the buyer ($\mu^* \uparrow$). Since the cost of IPI is small, the increase in μ^* is sufficiently large so that it more than compensates for the increase in L and thereby drives a reduction in the unit cost of integration ($L(1-\mu^*)\downarrow$). As before, the supplier's price may increase or decrease due to the increase in L. First, if buyer's profit is highly sensitive to the amount of in-house production and relatively less sensitive to price, then the increase in L drives the supplier to lower its price ($P^* \downarrow$). With the lower unit cost for outsourcing, the buyer reduces its level of in-house production ($w^* \downarrow$). In this situation, we observe that the supplier's pursuit of μ^* and P^* are complements in terms of their impact on the buyer's decision, w^*. Second, if the buyer's profit is highly sensitive to the supplier's decisions and L increases, then the supplier charges a higher price ($P^* \uparrow$). Nevertheless, the decrease in the unit integration cost more than compensates for the higher price so that again the buyer's unit outsourcing cost is lower giving us $w^* \downarrow$. Thus, the supplier's pursuit of μ^* and P^* are substitutes in terms of their impact on the buyer's decision, w^*. Lastly, it is interesting to note that while the supplier earns less profit (the additional investment in IPI dominates the increase in total revenue), the buyer's profit increases (the lower unit cost for outsourcing compensates for the higher unit production cost and reduction in future value).

Lastly, when L increases and is U moderate, the supplier increases its level of IPI ($\mu^* \uparrow$) to precisely offset the increase in L so that the buyer's unit integration cost ($L(1-\mu^*)$) does not change. As a result, the buyer's in-house production and the supplier's pricing decisions do not change. Since revenue is constant but its investment in IPI increases, the supplier's profit is lower. Naturally, the buyer's profit does not change.

The above insights are summarized in Corollaries 2b and 2c.

Corollary 2b. *Suppose the buyer's base unit integration cost (L) increases and U is small. The buyer's optimal level of in-house production (w^*) decreases, and the supplier's optimal level of IPI (μ^*) increases. If $\Phi_2 > 0$ ($\Phi_2 < 0$) then the supplier's price (P^*) decreases (increases). The buyer's profit increases whereas the supplier's profit decreases.*

Corollary 2c. *Suppose the buyer's base unit cost for integration (L) increases and U is moderate. The buyer's optimal level of in-house production (w^*) and the supplier's price (P^*) do not change; the supplier's investment in IPI (μ^*) increases. The supplier's profit decreases; the buyer's profit does not change.*

2.4.2 Insights on Learning

Our model embodies two forms of learning. First, we analyze the effect of the rates of volume-based learning that reduce the period 2 manufacturing costs for both the supplier (Corollary 3) and buyer (Corollary 4). Second, we examine the effect of volume-based learning obtained by the buyer that enhances its development of future products and processes (Corollary 5).

Intuitively, if the supplier has a higher rate of volume-based learning (larger β), it has more incentive to attract outsourcing from the buyer to reduce its period 2 manufacturing costs. As such, the supplier increases its level of IPI ($\mu^*\uparrow$) to reduce the buyer's unit integration cost. However, the supplier's price can increase or decrease. If the buyer's integration cost is significantly reduced by the supplier's investment in IPI, the supplier is able to charge a higher price. Since the decrease in the buyer's unit integration cost more than offsets the increase in price, the buyer's unit outsourcing cost is smaller and the buyer's level of outsourcing increases ($w^* \downarrow$). Again, we see a substitution effect whereby the supplier relies more on μ^* and less on P^* to attract outsourcing. Alternatively, if the supplier's investment in IPI has a limited effect on reducing the buyer's integration cost, then to stimulate outsourcing from the buyer, the supplier charges a lower price ($P^* \downarrow$). Since both the price and unit integration cost are smaller, the buyer's unit outsourcing cost is smaller and the buyer pursues less in-house production ($w^* \downarrow$). In this situation, the supplier's price and investment in IPI are complementary in the sense that both seek to attract more outsourcing from the buyer. Therefore, regardless of the change in price, when the supplier's rate of volume-based learning is larger, the buyer outsources more. Finally, when the supplier's rate of learning increases, both the supplier and buyer earn higher profit.

The above discussion is summarized in Corollary 3. To state the results mathematically, let $\Phi_4 = 2L\mu^*(\gamma - 1)^{-1} + Z$. Clearly, Φ_4 increases in relation to the base unit cost of outsourcing, the optimal level of IPI, and the sensitivity of the buyer's profit to the supplier's price and IPI decisions. Additionally, Φ_4 decreases in relation to the diseconomies of scale associated with larger investments in IPI.

Corollary 3. *If the supplier's rate of learning (β) increases, then the supplier's optimal level of IPI (μ^*) is larger, and the buyer's optimal level of in-house production (w^*) is smaller. When $\Phi_4 > 0$ ($\Phi_4 < 0$), the supplier charges a higher (lower) price (P^*). Both the buyer and supplier realize higher profits.*

Continuing with our analysis of learning, we find that the buyer's optimal level of in-house production, w^*, is U-shaped with respect to the buyer's rate of volume-based learning, α. As such, we consider two cases as summarized in Corollaries 4a and 4b.

First, suppose the buyer's rate of volume-based learning is sufficiently small and increases. We expect the supplier needs to attract more outsourcing through its IPI and pricing decisions; and we expect the buyer undertakes more in-house production to reduce its period 2 unit production cost. Consistent with intuition, we do find that to further attract outsourcing, the supplier increases its level of IPI ($\mu^* \uparrow$). However, we also find that the supplier's optimal price may increase or decrease. First, when L is small or γ is large, increasing the level of IPI does not have a significant impact on the buyer's unit integration cost and thereby forces the supplier to focus on lowering its price to attract more outsourcing ($P^* \downarrow$). The decrease in the unit outsourcing cost more than compensates for the (small) increase in the buyer's period 2 in-house production cost, and the buyer optimally increases its level of outsourcing ($w^* \downarrow$). As such, the supplier uses complementary strategies (lower price and larger IPI) to attract outsourcing. Second, if L is large or γ is small, then the supplier charges a higher price ($P^* \uparrow$). In this situation, the supplier leverages the fact that increasing the level of IPI significantly reduces the buyer's unit integration cost. Despite the higher price, the buyer's unit outsourcing cost is lower so that the buyer outsources more to the supplier ($w^* \downarrow$). Thus, the supplier's higher investment in IPI more than compensates for the higher price. Although the supplier's earns more revenue from outsourced components, the high investment in IPI drives a reduction in the supplier's profit.

The above discussion is summarized in Corollary 4a. To state the results mathematically, we let $\Phi_5 = w^* - (V - w^*)(1 - 2\alpha) + (1 - \alpha)\big((1 - \alpha)w^* + V\alpha\big)log(w^*)$ and $\Phi_6 = \Phi_1 w^*\big(1 + log(w^*)(1 - \alpha)\big) + 2L(\gamma - 1)^{-1}\mu^* {w^*}^{4+2\alpha}\big(1 - 2\alpha - (1 - \alpha)\alpha log(w^*)\big)$. We see that Φ_5 increases in relation to the buyer's rate of volume-based learning and the quantity of in-house production; but Φ_5 decreases in relation to the quantity of outsourcing. Given our supposition that α is sufficiently small, we know that Φ_6 increases in relation to the base unit cost of integration and the optimal level of IPI; but Φ_6 decreases in relation to the buyer's rate of volume-based learning and the diseconomies of scale associated with investments in IPI. Note that if both L and γ are small or both are large, the impact on Φ_6 depends on the relative magnitude of other parameters.

Corollary 4a. *Suppose the buyer's rate of learning (α) is sufficiently small (i.e., $\Phi_5 < 0$) and increases. The buyer's optimal level of in-house production (w^*) is smaller, the supplier's optimal level of IPI (μ^*) is larger. If $\Phi_6 > 0$ ($\Phi_6 < 0$), then the supplier optimally charges a higher (lower) price (P^*). The supplier's optimal profit is smaller.*

Second, suppose α is sufficiently large and increases. Naturally, the buyer has incentive to pursue more in-house production because the high learning capability significantly reduces the in-house production cost. To attract more outsourcing, the supplier charges a lower price ($P^*\downarrow$). In contrast, since the buyer has a strong preference for in-house production, there is not enough incentive for the supplier to increase its level of IPI, as a result, $\mu^*\downarrow$. Therefore, the supplier's price and investment in IPI serve as substitutes when α is sufficiently large and increases. Ultimately, despite the supplier's lower price, the higher integration cost and the strong capability for volume-based learning in the production cost lead the buyer to produce more in-house ($w^* \uparrow$). As a result, the supplier's revenue from outsourcing is lower. Even though the supplier's investment in IPI is lower, the loss in revenue leads to a lower profit for the supplier.

Corollary 4b. *Suppose the buyer's rate of learning (α) is sufficiently large (i.e., $\Phi_5 \geq 0$) and increases. The buyer's optimal level of in-house production (w^*) is non-decreasing, the supplier's optimal level of IPI (μ^*) is non-increasing, and the supplier's optimal price (P^*) is non-increasing. The supplier's optimal profit is smaller.*

We conclude our discussion of learning by analyzing the value of the transfer of knowledge from period 2 in-house production that enhances the buyer's development of future products and processes. Suppose the future value obtained from volume-based learning in period 2 is larger (f_0). Given the buyer's greater incentive for in-house production, the supplier cannot justify the same investment in IPI so that $\mu^* \downarrow$. Interestingly, the supplier's optimal price may increase or decrease. If the buyer's profit is highly sensitive to the amount of demand met from in-house production (i.e., relatively insensitive to the supplier's decisions), then the supplier increases the price ($P^* \uparrow$) to raise marginal revenue despite the falling quantity of outsourcing. Given the larger unit cost for outsourcing and the larger value of future knowledge, the buyer pursues more in-house production ($w^* \uparrow$). As such, the supplier's solution is complementary in the sense that the changes in both IPI and price have the effect of reducing the buyer's level of outsourcing to limit the supplier's loss in profit. Alternatively, if the buyer's profit is highly sensitive to the supplier's decisions, then the supplier charges a lower price ($P^* \downarrow$) to attract outsourcing. Clearly, the lower price seeks to substitute for the supplier's lesser investment in IPI. However, given the buyer's strong incentive for in-house production driven by the higher future value, the decrease in price only limits the reduction in the buyer's level of outsourcing ($w^*\uparrow$). From this discussion, it is clear that when f_0 is larger, the buyer earns higher profit whereas the supplier's profit is smaller.

The above discussion is summarized in Corollary 5. To state the results numerically, we define $\Phi_7 = \Phi_2 w^* - (1 - f_1)(V - w^*)Z - 2L\mu^*(1 - f_1)(\gamma - 1)^{-1}$. Clearly, Φ_7 increases in relation to the optimal amount of outsourcing and the diseconomies of scale associated with investments in IPI; but Φ_7 decreases in relation to the sensitivity of the buyer's profit to the supplier's decisions, the optimal level of IPI, and the base unit integration cost.

Corollary 5. *Suppose the future value benefits realized by the buyer are larger (f_0 larger). The buyer increases its level of in-house production (w^*), and the supplier's level of IPI (μ^*) is smaller. If $\Phi_7 > 0$ ($\Phi_7 < 0$), then the supplier's price (P^*) is higher (lower). The buyer's profit is larger and the supplier's profit is smaller.*

2.5 An Extension: Dynamic Model

In this section, we relax several assumptions made in the base model (Section 3). We allow the supplier's price and the buyer's outsourcing decisions to differ in periods 1 and 2 (written as P_1, P_2, w_1, and w_2). However, we reasonably maintain that the supplier's investment in IPI occurs once at the beginning of period 1 and impacts the buyer's integration costs in both periods 1 and 2. Moreover, we allow dynamic demand for the buyer. Let component demand in period 1 be given by V and component demand in period 2 equal $V(1 + \rho)$, with $\rho \in [-1, \infty)$, where ρ indicates the percent demand change between periods. Naturally, if $\rho > 0$ ($\rho \leq 0$) demand increases (is non-increasing) from period 1 to 2. The buyer's revenue in periods 1 and 2 is written as RV and $RV(1 +$

ρ), respectively. The sequence of decision making in the model extension is described below.

In the first stage of the first period, the supplier determines its one time level of IPI (μ) and the price charged for an outsourced component in period 1 (P_1) to maximize the supplier's profit in two periods (Π_s). At the second stage of the first period, the buyer determines its level of in-house production in period 1 (w_1) in order to maximize its two period profit (Π_b). Again, the supplier and the buyer's profits are maximized over two periods simultaneously because the pricing and outsourcing decisions in period 1 impact the learning benefits in period 2 (Gray et al. 2009). In the first stage of the second period, the supplier determines the price to charge the buyer in period 2 (P_2) to maximize its period 2 profit (Π_s^2). In response, the buyer determines its level of in-house production in period 2 (w_2) to maximize its period 2 profit (Π_b^2).

While some analytic results are obtained, due to the complexity of the dynamic model, our interpretations rely entirely on insights obtained numerically. Moreover, to simplify the analysis we consider only non-boundary solutions, we assume the incentive compatibility constraints of the buyer and supplier are satisfied, and we assume $f_1 = \frac{1}{2}$. We use backward induction to solve the two-period Stackelberg game.

It is straightforward to show that Π_b^2 (Equation (16)) is concave with respect to w_2. Analytically, using first order condition we find $w_2^* = \left(\frac{f_0}{2(C_b w_1{}^{-\alpha} - P_2 - L(1-\mu))}\right)^2$ is the non-boundary solution for w_2^* that maximizes Π_b^2. Substituting w_2^* into the supplier's period 2 profit, we obtain Π_s^2 (Equation (17)). Again, it can be shown that Π_s^2 is concave with respect to P_2. We obtain an explicit non-boundary solution for P_2 that maximizes the supplier's period 2 profit and satisfies the first order condition $\frac{d\Pi_s^2}{dP_2} = 0$. In the next step, we substitute w_2^* and P_2^* into the buyer's two-period profit, Π_b (Equation (18)). Due to the mathematical complexity, we cannot show that Π_b is concave with respect to w_1, nor can we obtain an explicit solution for w_1. However, numerically, we find w_1^* such that $\frac{d\Pi_b}{dw_1} = 0$ and maximizes the buyer's two-period profit. Lastly, we substitute w_2^*, P_2^*, and w_1^* into the supplier's two-period profit and obtain Π_s (Equation (19)). Again, we cannot show that Π_s is jointly concave with respect to P_1 and μ, nor can we obtain explicit solutions for P_1^* and μ^*. Nevertheless, numerically we find optimal solutions of P_1 and μ that satisfy $\frac{d\Pi_s}{dP_1} = 0$ and $\frac{d\Pi_s}{d\mu} = 0$ and maximize the supplier's two-period profit.

$$\Pi_b^2 = RV(1+\rho) - C_b w_1^{-\alpha} w_2 - (P_2 + C_i)(V(1+\rho) - w_2) + f_0 w_2^{\frac{1}{2}} \tag{16}$$

$$\Pi_s^2 = P_2(V(1+\rho) - w_2^*) - C_s\left(V - w_1\right)^{-\beta}(V(1+\rho) - w_2^*) \tag{17}$$

$$\begin{aligned}\Pi_b = {} & RV + RV(1+\rho) - C_b w_1 - C_b w_1^{-\alpha} w_2^* - (P_1 + C_i)(V - w_1) \\ & -(P_2^* + C_i)(V(1+\rho) - w_2^*) + f_0 w_2^{*\frac{1}{2}}\end{aligned} \tag{18}$$

$$\begin{aligned}\Pi_s = {} & P_1(V - w_1^*) + P_2^*(V(1+\rho) - w_2^*) - C_s(V - w_1^*) - C_s(V - w_1)^{-\beta}(V(1+\rho) - w_2^*) \\ & -U\mu^{\gamma}\end{aligned} \tag{19}$$

We conducted extensive numerical experimentation to develop insights on how parameter values impact the optimal solutions. The full range of experimentation for all input parameters is given in Table A.2. In Section 2.5.1, we compare the base model analytic results with the numerical results obtained assuming $\rho = 0$. In this way, we can isolate the impact of allowing the supplier's price and the buyer's outsourcing decisions to vary in periods 1 and 2. In Section 2.5.2, we extend insights from the base model and analyze the case of dynamic demand ($\rho \neq 0$).

2.5.1 Comparison of Base and Dynamic Models

To perform a comparison with the base model, we assume that the buyer faces constant demand in periods 1 and 2 in the extension. We find that the impact of a parameter change on the decisions of the buyer and supplier in period 1 are consistent in both the base model and its extension, e.g., if w increases (decreases) in the base model then w_1 and w_2 increase (decrease) in the extension. Intuitively, w and w_1 move in the same direction since, in both models, the buyer and supplier optimize their two-period profit to obtain period 1 solutions. Similarly, w and w_2 move in the same direction due to volume-based learning (i.e., w is driven by volume-based learning in the production cost and the future value whereas w_2 is driven by volume-based learning in the future value). Furthermore, we find that the supplier's investment in IPI moves in the same direction in the base model and extension since, in both models, the supplier makes the investment decision only in period 1. Also, consistent with analytical results in the base model, we numerically find that the supplier's period 1 price, P_1, may increase or decrease depending on whether the buyer's profit is highly sensitive to the quantity of in-house production. In contrast, the direction of change in the supplier's period 2 price, P_2, differs in both models. In the base model, we analytically prove that an increase in a parameter may drive an increase or decrease in price depending on whether the buyer's profit is highly sensitive to the quantity of in-house production. In the extension, however, we find that an increase in a parameter changes P_2, in only one direction, independent of whether or not the buyer's profit is highly sensitive to the quantity of in-house production. Intuitively, this numerical result occurs since, in the extension, the supplier only has one mechanism to impact the buyer's outsourcing decision in period 2. Lastly, we find that in response to a parameter change the profit for the supplier and buyer in the base model and extension move in the same direction.

2.5.2 Effect of Dynamic Demand

In this section, we consider the effect of dynamic demand realized by the buyer. Given the focus on constant demand in the base model, these numerical results provide new insights that contribute to our understanding of the problem. We consider the situation where the buyer's demand is larger in period 2 than in period 1 ($\rho > 0$). Note that these numerical results hold over the ranges of the parameter values given in Table A.2.

With larger demand in period 2, the buyer has more incentive for in-house production in period 1 to reduce the period 2 in-house production cost. Observing the buyer's strong incentive for in-house production, the supplier invests in more IPI to attract outsourcing. Moreover, if the buyer's profit is less sensitive to the quantity of demand met from in-house production (i.e., more sensitive to the supplier's decisions), then the supplier charges a lower price in period 1 to attract more outsourcing. (This insight is consistent with analytic results in the base model.) As a result, the buyer's unit outsourcing cost is lower. Nevertheless, the buyer's incentive to accumulate more volume-based learning is sufficiently strong so that the quantity of period 1 in-house production increases. As such, the supplier's price and IPI strategies simply limit (but do not reverse) the loss in outsourcing in period 1. In period 2, the supplier observes the increase in the buyer's period 2 demand as well as the diminishing returns to the buyer's moderate future value of manufacturing experience and optimally charges a higher price. Naturally, given the buyer's smaller unit production cost and the supplier's higher price, the buyer undertakes more in-house production in period 2. However, although the quantity of period 2 in-house production (w_2^*) is larger, the portion of demand met from period 2 in-house production ($\frac{w_2^*}{V(1+\rho)}$) is smaller. Therefore, the supplier is able to charge a higher price in period 2 because it anticipates an increase in the buyer's portion of outsourcing. Lastly, in terms of profit in period 1, the buyer's profit is larger whereas the supplier's profit is smaller. Nevertheless, driven by the period 2 increase in demand realized by both firms, the total two-period profits of the buyer and supplier are larger.

2.6 Conclusion

We introduce a two-period Stackelberg game of a buyer and supplier from which we obtain results that contribute to the literature on two dimensions. First, we recognize that in many industries manufacturing experience may substantially impact a firm's ability to develop the next generation product and process technologies. For firms developing high tech products with short life cycles, the seamless KT from one product generation to the next is critical. We introduce a future value to the buyer's profit maximizing objective that reflects the benefits derived from current manufacturing experience to the successful development of future products and technologies. We show that if the future value is sufficient then the buyer optimally pursues a partial outsourcing strategy, even if the marginal cost of outsourcing is less than the marginal cost of in-house production. Analytic conditions are given providing insights on how the future value affects the buyer's level of outsourcing as well as the supplier's decisions. In contrast, we show that if the future value is ignored (zero) then the buyer does not optimally pursue a partial outsourcing strategy and instead meets all component demand either from in-house production or outsourcing; whichever is associated with the lower unit cost.

A second contribution of our research is that we permit the supplier to invest in IPI that enhances the buyer's efficiency at integrating the outsourced component into its manufacturing process. This is important since, along with the supplier's price, the buyer's integration cost significantly affects its outsourcing decision. Thus, the supplier has two mechanisms to influence the buyer's outsourcing decision: its price and its investment in IPI. Analytic results provide insights on situations where the supplier reduces its price and increases its investment in IPI (complementary strategy) versus situations where the supplier reduces both its price and investment in IPI (substitution strategy). Analytic results, described below, demonstrate that a buyer's partial outsourcing decision and the supplier's price and investment in IPI decisions are intertwined and impacted by volume-based learning realized by both firms, the cost of IPI incurred by the supplier, and the cost of integration incurred by the buyer.

2.6.1 Supplier's Two Mechanisms: Substitutes or Complements

Consider the situation where the buyer's manufacturing process becomes more complex so that the supplier's investment cost in IPI increases. We show analytically that if the buyer's profit is relatively less sensitive to the supplier's price and IPI decisions and more sensitive to the quantity of in-house production, then the supplier optimally reduces its investment in IPI and charges a higher price. Therefore, the supplier's two mechanisms to influence the buyer's behavior act as complements in sense that both reduce the buyer's pursuit of outsourcing in order to limit the supplier's loss in total profit. We also show that, beyond in-sourcing more component demand, the buyer realizes less profit. Alternatively, if the supplier's investment cost in IPI increases and the buyer's profit is relatively more sensitive to price and IPI decisions and less sensitive to the quantity of in-house production, then we prove that the supplier reduces its investment in IPI and reduces the price charged to the buyer. As such, we observe a substitution strategy whereby the supplier lowers its price in order to compensate for the buyer's higher unit cost of integration. Overall, however, our analytic results show that in both situations the unit cost of outsourcing increases and the buyer undertakes more in-house production. Again, both the buyer and supplier earn less profit.

Similar complementary versus substitution effects are obtained from analytical sensitivity analysis on the buyer's base unit integration cost. We find that as the base unit cost of integration increases, the supplier's actions only limit its reduction in profit, whereas the buyer's profit may increase, decrease, or remain unchanged depending on the magnitude of the supplier's cost for IPI (see Corollary 2).

2.6.2 The Rates of Learning on Cost Reduction for the Buyer and Supplier

Our analysis reveals that the rates of volume-based learning that reduce the production costs for the buyer and supplier impact decisions in different ways. When the supplier's rate of learning increases, we analytically show that the supplier increases its investment in IPI. Depending on the effectiveness of IPI on reducing the buyer's integration cost, the supplier's price may increase or decrease. In either situation, the buyer's unit outsourcing cost decreases and the buyer outsources more to the supplier. Moreover, both the buyer and supplier earn higher profit.

In contrast, consider the effect of volume-based learning for the buyer. When the buyer's rate of learning is small and increases, the supplier invests more in IPI to attract more outsourcing. Although the price may increase or decrease, the buyer's unit outsourcing cost declines due to the supplier's investment in IPI. The low unit outsourcing cost drives the buyer to increase its amount of outsourcing. Alternatively, if the buyer's rate of learning is large and increases the buyer undertakes more in-house production regardless of the supplier's decisions. The supplier limits the loss in outsourcing demand by lowering its price, and limits the reduction in profit by lowering costs (reducing IPI). Lastly, regardless of the magnitude of the buyer's rate of volume-based learning, if that rate increases, then we prove that the buyer's profit increases but the supplier's profit declines.

From the above we have shown that both the buyer and supplier realize higher profit when the supplier has a higher rate of volume-based learning in its production cost. Similarly, we show that the buyer realizes higher profit if it has a higher rate of volume-based learning in its production cost. Nevertheless, we find that the supplier does not benefit from an increase in the buyer's rate of volume-based learning despite the fact that it has two mechanisms to manipulate the buyer's outsourcing strategy. This finding is different from that of Gray et al. (2009).

2.6.3 An Extension

So far, we have presented analytic results derived for the base model, which assumes a fixed contractual agreement between the buyer and supplier in the two-period game. In Section 5, we present numerical results derived from an extension where we allow the supplier's price and the buyer's quantity of in-house production to vary over periods 1 and 2. The supplier's investment in IPI, however, remains a one-time decision made at the beginning of period 1. Furthermore, unlike the base model which assumes fixed demand, in the extension we allow the buyer's volume of component demand to vary between periods 1 and 2. Based on extensive numerical experimentation, we find that if the component demand is larger in period 2, the buyer has more incentive to reduce its unit production cost and pursues more in-house production in period 1. To attract more outsourcing, the supplier invests in more IPI and lowers the price in period 1. Nevertheless, the supplier's actions simply limit the increase in the buyer's period 1 in-house production. In period 2, the supplier charges a higher price because it recognizes that, while the buyer's component demand is larger, the buyer's future value benefits from manufacturing experience are moderate and exhibit diminishing returns. Ultimately, the buyer pursues a modest increase in the quantity of in-house production in period 2, which provides it with sufficient future value benefits. Moreover, the buyer outsources a larger portion of the increase in its component demand for period 2 to the supplier. Lastly, both firms earn higher profit over the two periods.

2.6.4 Future Research

In this paper, we do not consider the situation where the buyer realizes volume-based learning in the cost of its integration activities. According to Boone and Ganeshan (2001) and Anderson and Parker (2002, 2008), the learning phenomenon may exist in component integration. Future research may analyze how learning that reduces the buyer's component integration cost impacts the decisions of the buyer and supplier. Moreover, we limit our attention to the case of static learning curves. Since the learning rate may change over the product life cycle, relaxing the static assumption might provide interesting insights on outsourcing strategies.

CHAPTER 3

KNOWLEDGE CREATION AND KNOWLEDGE TRANSFER IN NEW PRODUCT DEVELOPMENT PROJECTS

3.1 Introduction

Due to time-based competition as well as short product life cycles a firm's ability to introduce new products to the marketplace has become increasingly important (Cohen et al. 1996, Ha and Porteus 1995, Loch and Terwiesch 1998). Thus, a firm must excel at new product development (NPD) to sustain or expand profitability (Dahan and Mendelson 2001, Terwiesch and Loch 2004). Managing NPD endeavors entails managing teams of highly skilled employees responsible for designing components of the product and process. Over time, the teams embed their knowledge into the development project. This leads us to the fundamental problem: how to manage the evolution of knowledge of the NPD teams.

In this paper, we introduce a Base Model and two model extensions that analyze how to manage knowledge throughout the NPD project. In all three models, progress in the NPD project is inferred by the growing levels of knowledge at three stages starting with prototyping, continuing to pilot line testing, and concluding with production ramp-up (Terwiesch and Loch 2004, Loch et al. 2001, Thomke 1998). The manager determines the rates and timing of development activities to be pursued to increase the levels of knowledge throughout the NPD project. Naturally, costs are incurred as these development activities are undertaken over time. Ultimately, the knowledge embedded into the NPD project by the product launch time determines the net revenue earned when the new product is released to the marketplace (Santiago and Vakili 2005, Chao et al. 2009, Gaimon et al. 2011, Ozkan et al. 2012). It is important to note that the three model formulations introduced are supported by the authors' interactions with NPD research analysts at a major U.S. consumer products firm, NPD managers from a U.S. firm in the energy industry, and a major electronic products manufacturer in Asia.

A key feature of our Base Model is the characterization of how knowledge at each stage increases over time. At the initial prototyping stage, the level of knowledge increases as the team pursues prototyping activities. At the second stage, the level of pilot line knowledge increases as the team pursues pilot line testing activities as well as through the transfer of knowledge from the prototyping stage. Prototyping knowledge improves the ability of engineers involved in pilot line testing to identify design features to be tested, to properly design the pilot line configuration, and to undertake pilot line testing (Thomke and Bell 2001). Similarly, the level of ramp-up knowledge increases over time as the team pursues more production ramp-up activities and through the transfer of knowledge from the pilot line stage. Knowledge from pilot line testing improves the ability of the production ramp-up team to help identify which experiments or engineering trials to perform as well as how to perform them (Terwiesch and Bohn 2001). In our Base Model, we assume KT is *continuous*. Therefore, the Base Model reflects an environment where the development teams are sufficiently small and highly interactive so that the flow of knowledge from one stage to the next is fluid and occurs continuously over time. Moreover, in the Base Model we assume that KT only occurs in the *forward direction* from an upstream to a downstream stage.

In an extension of the Base Model, referred to as the Feedback Model, we permit KT to occur continuously in both the *forward and backward directions* (Loch and Terwiesch 1998). Therefore, in addition to forward KT, we allow knowledge accumulated in a downstream stage to be continuously transferred backward to provide feedback to an upstream stage. For the three stage development process we consider, there are three feedback scenarios: (i) from the pilot line stage to the prototyping stage, (ii) from the ramp-up to the prototyping stage, and (iii) from the ramp-up to the pilot line stage. Feedback to the prototyping stage provides insights on design decisions including whether the materials chosen for the new product pass safety tests, and which design features ensure the manufacturability of the product. Feedback to the pilot line stage helps direct future testing efforts to ensure manufacturability.

In another extension of the Base Model, referred to as the Discrete Model, we consider the situation where the teams responsible for the NPD project are sufficiently large or work in diverse locations so that continuous KT is not practical. Instead, knowledge is transferred at discrete times during the development project. As such, knowledge is accumulated at the prototyping stage before being transferred to the pilot line testing stage, and knowledge is accumulated at the pilot line stage before being transferred to the ramp-up stage. Also, consistent with the Base Model, we assume that KT occurs in the forward direction. To summarize, in the Discrete Model, the NPD manager determines the optimal rates of development activities (e.g., prototyping, pilot line testing, and production ramp-up), and the number of forward KTs and the times they should occur.

We contribute to the literature by introducing a Base Model and two model variations to analyze how knowledge should evolve over time for three stages of an NPD project. First, by comparing the solutions of the Base Model and the Feedback Model, we analyze the effect of feedback on the manager's pursuit of prototyping, pilot line testing, and product ramp-up activities. Similarly, by comparing the solutions of the Base Model with the Discrete Model, we analyze the effect of different flows of KT (continuous versus discrete) on the manager's approach to knowledge management. In particular, we explore drivers of situations where the NPD manager prefers to transfer large amounts of knowledge less frequently versus small amounts of knowledge more frequently. In addition, for all models, we explore how the effectiveness of development activities at one stage impacts the evolution of knowledge in all stages of the NPD project. This is important since the manager has some control over the effectiveness of development activities, which may be enhanced with higher skilled team members or through superior technical support. Lastly, we explore how the returns to KT (either forward or backward) impacts the manager's pursuit of prototyping, pilot line testing, and production ramp-up activities throughout the NPD project. Again, the manager has some control over the returns to KT since she can formalize methods to document knowledge and can invest in advanced technical systems to facilitate the transfer.

This paper is organized as follows: In Section 2, a review of the literature is provided. In Section 3, we introduce and analyze the Base Model where forward KT occurs continuously between successive development teams. All of the results for the Base Model are analytic. In Section 4, we introduce and analyze the Feedback Model where, beyond the continuous forward flow of KT, feedback continuously occurs from the pilot line to the prototyping stage. Both analytic and numerical results are obtained. In addition, insights are given on the impact of feedback from the ramp-up stage. In Section 5, we introduce and analyze the Discrete Model where knowledge is batched and transferred in the forward direction at discrete times during the NPD project. While some results for the Discrete Model are analytic, other results are derived from extensive numerical experimentation. The concluding remarks are given in Section 6.

3.2 Literature Review

This research is related to the literature on KT and the literature on concurrent engineering in NPD projects.

3.2.1 The Knowledge Transfer Literature

Development knowledge that is product oriented includes knowledge about markets and consumer preferences as well as methods for market testing (Pisano 1997). In the context of process development, a firm's technical knowledge includes elements like "theories, principles, algorithms, conceptual models, specific analytical or experimental techniques, heuristics, and empirical regularities" (Pisano 1997, p. 205). Patents, documents, or computer models, etc. capture those elements of technical knowledge that can be codified (Pisano 1997). Naturally, some development knowledge remains tacit and is therefore more difficult to transfer. In the production literature, the level of knowledge of the manufacturing workforce is inferred by the number of units produced per unit time that meet quality standards (Argote et al. 1990, Argote 1999). Similarly, in the NPD domain, the team's level of knowledge may be inferred by both the number and quality of prototypes and pilot line experiments generated. Terwiesch and Bohn (2001) use the number of ramp-up experiments conducted to indicate the level of ramp-up knowledge.

Knowledge increases through learning activities, which may be categorized as either autonomous or induced (Dutton and Thomas 1984). Autonomous learning involves “automatic improvements that result from sustained production over long periods” (Dutton and Thomas 1984, p. 241). Induced learning requires managerial action and investment for the learning activities to occur (Terwiesch and Bohn 2001, Carrillo and Gaimon 2000, 2004). KT is a form of induced learning whereby one party passes knowledge to another either orally or with documentation (Ha and Porteus 1995, Argote 1999, Ozkan et al. 2012). Argote and Ingram (2000, p. 151) define KT as “the process through which one unit (e.g., group, department, or division) is affected by the experience of another”. The importance of KT to a firm’s performance has been shown empirically. Based on data from the construction of Liberty ships, Argote et al. (1990) find that a shipyard that begins operation late is more efficient than those starting early because of KT. Focusing on KT across shifts at a single plant, Epple et al. (1991) find that substantial but less than complete KT occurs. In a study of the global semiconductor industry, Salomon and Martin (2008) find that KT shortens the time a firm needs to ramp up to full production at a new manufacturing facility.

To understand factors that impact the effectiveness of KT, Lapre and Van Wassenhove (2001) examine factories in a steel manufacturing firm. They find that learning within the factory results in significant improvements in productivity. However, the knowledge transferred to other factories does not generate significant productivity improvements because of the lack of management buy-in and interdepartmental problem solving skills. Based on an eight year field investigation of Xerox Europe, Jensen and Szulanski (2007) examine whether the use of templates improves the effectiveness of KT.

A template refers to an existing business model that is observable, and is continuously used as a replication example (Winter and Szulanski 2001). Jensen and Szulanski (2007) find that using templates increases the likelihood of adopting a transferred routine, thereby enhancing the effectiveness of KT.

In this paper, we consider KT in the context of a three-stage NPD project. Consistent with the above-mentioned literature, we recognize that the knowledge transferred from a source team can improve the effectiveness of development activities of the recipient team. We examine KT in the forward and backward (feedback) directions, and we examine KT that occurs continuously versus at discrete times during the NPD project. We obtain insights on how these different models of KT impact the manager's pursuit of prototyping, pilot-line testing, and ramp-up activities throughout the NPD project.

3.2.2 The Concurrent Engineering Literature

We consider three distinct stages of engineering activities in an NPD project: prototyping, pilot line testing, and production ramp-up (Thomke 1998, Terwiesch and Bohn 2001, Loch et al. 2001, Terwiesch and Loch 2004). According to Thomke and Bell (2001), the transfer of prototyping knowledge improves the effectiveness of pilot line testing activities. Terwiesch and Xu (2004) analyze the situation where knowledge from pilot line testing improves the effectiveness of production ramp-up activities.

In NPD projects, time-to-market is a key source of competitive advantage (Loch and Terwiesch 1998, Ulrich et al. 1993). To shorten product development time, concurrent engineering is widely used (Wheelwright and Clark 1992, Terwiesch et al. 2002). Ha and Porteus (1995, p. 1431-1432) define concurrent engineering as a process “in which engineering activities are conducted concurrently rather than sequentially.” In concurrent engineering, the time that knowledge is transferred from one stage to the next is critical (Thomke and Bell 2001). If the KT occurs too early, the recipient benefits from a limited addition of knowledge. On the other hand, if the KT occurs too late, the recipient has limited time to leverage and utilize the knowledge (Loch and Terwiesch 2005). Thus, understanding how to manage the frequency and the timing of KT in concurrent engineering is vital (Loch and Terwiesch 1998).

Krishnan et al. (1997) provide a model-based framework to manage the concurrency of product development activities. They introduce the notion of the evolution of upstream information and the downstream sensitivity to the upstream information whereby development activities are overlapped. Loch and Terwiesch (1998) extend this concept by incorporating an information batching policy. They present a dynamic decision rule to determine the optimal time for KT, and provide the optimal level of concurrency between activities. They show that uncertainty in the rate of engineering changes and the dependence of upstream modification and downstream task decrease the optimal level of overlapping, and make concurrent engineering less attractive. When knowledge is transferred in batches (at discrete times), costs are incurred reflecting efforts expended to create reports, presentations, and attend meetings, as well as costs for transportation and telecommunication (Loch and Terwiesch 1998). Since KT is costly, the NPD manager needs to balance the benefits realized and the associated costs incurred (Ha and Porteus 1995, Terwiesch and Bohn 2001).

Our paper differs from the above NPD literature in three important aspects. First, to our knowledge, the NPD literature focuses exclusively on discrete KTs. However, KT may also occur continuously, either forward or backward, if engineering teams are small and highly interactive whereby no explicit cost is incurred (i.e., the cost of KT is subsumed in the cost of regular development activities). This notion of continuous KT is supported by the authors' interactions with NPD research analysts at a major U.S. consumer products firm, the case literature (Christensen 2006), as well as from the authors' observations of small entrepreneurial firms. By analyzing differences in the solutions for the Base Model and the Discrete Model, we explicitly observe how the characterization of KT (i.e., continuous or discrete) impacts the manager's pursuit of prototyping, pilot line testing, and ramp-up activities. By analyzing differences in the solutions for the Base Model and the Feedback Model, we explicitly observe how the direction of KT (i.e., forward or backward) impacts the manager's decisions.

Second, the literature focuses on how the impact of upstream knowledge to downstream activities (i.e., the returns to KT in our paper) affects the time of KT, but neglects to ascertain the impact on the rates of development activities (e.g., prototyping, pilot line testing and ramp-up production). In the discrete model, we extend the literature by considering the impact of the returns to KT, not only on the transfer times, but also on the rates of development activities in successive stages of an NPD project. In addition, we provide key insights on how the returns to KT impact the manager's optimal strategies differently in the discrete versus continuous model.

Lastly, most of the literature focuses on minimizing the product's time-to-market (Krishnan et al. 1997, Loch and Terwiesch 1998, Loch and Terwiesch 2005). In contrast, we recognize that for many products, the time-to-market is determined by seasonality conditions or external market forces. Firms in the automotive industry release products annually at the start of the year, whereas many computer electronics firms release new models in time for the annual holiday season. Therefore, in contrast to the literature, we focus on maximizing the net revenue derived from product development activities (i.e., the extent of functionality and features embedded in the NPD project) that is released to the market at a predetermined launch time. The features and functionality which drive net revenue are driven by the levels of knowledge from prototyping, pilot line, and ramp-up activities generated throughout the NPD project. We provide insights on how the marginal contribution to net revenue from each type of knowledge impacts the manager's decisions.

3.3 Continuous Forward KT Model (Base Model)

In this section, we present a Base Model of an NPD project where KT between stages occurs in real time, i.e., continuously, in the forward direction. This situation may arise if a small number of NPD teams work closely together in the same location such that KT is fluid and thereby occurs continuously over time from one stage to the next. Product development occurs over time $t \in [0,T]$, where 0 is the start of the NPD project and T is the given product launch time. The NPD manager determines the rates of prototyping, pilot line testing, and production ramp-up activities throughout the NPD project, as described below. In the remainder of the paper let $X_Z(Z)$ and $X_{ZZ}(Z)$ denote the first and second order derivatives of X(Z) with respect to Z.

3.3.1 The Levels of Knowledge

The manager determines the rate of prototyping activities to pursue over time, denoted by $y(t) \geq 0$ for $t \in [0,T]$ (control variable), which is measured in terms of hours of workforce effort. As prototyping activities occur, prototyping knowledge accumulates.

Let Y(t) denote the level of prototyping knowledge at time t for $t \in [0,T]$ with Y(0)>0 given (state variable). The level of prototyping knowledge at time t is comprised of the initial level (Y(0)) and learning benefits from the rate of prototyping activities undertaken through time t. The initial level of prototyping knowledge reflects the overall skill of the team at the project's inception and is based on the team's past experience, the amount of prior education and training, and peer reviews. The extent that prototyping activities at time t increase the level of prototyping knowledge at that time is driven by the rate of prototyping, the skill of the team, and the quality of the technical support available. With higher skill or superior technical support, the effectiveness of prototyping activities is higher as indicated by the parameter $\alpha_0>0$. This gives us Equation (1).

$$Y_t(t) = \alpha_0 y(t) \tag{1}$$

Similarly, the manager determines the rate of pilot line activities to pursue over time, denoted by $p(t)\geq 0$ for $t \in [0,T]$ (control variable), which can be measured in terms of hours of workforce effort. Let P(t) (state variable) denote the level of pilot line knowledge at time t, with P(0)>0 given. The level of pilot line knowledge at time t is comprised of the initial level (P(0)) and learning benefits from the rate of pilot line testing activities pursued through that time. The initial level of pilot line knowledge reflects the skill of the pilot line team and is based on previous experience, years of prior education and training, and peer reviews. The increase in the level of pilot line knowledge at a particular time is driven by the rate of pilot line testing activities at that time as well as the skill of the team and the quality of the technical support available. Moreover, the effectiveness of pilot line testing activities on increasing the level of pilot line knowledge is enhanced by knowledge transferred from prototyping (Mihm et al.

2003). Prototyping knowledge improves the ability of engineers involved in pilot line testing activities to identify design features to be tested, to properly design the pilot line configuration, and to undertake the actual pilot line testing. Workforce skill of the pilot line team and the quality of technical support available are inferred by the parameter $\beta_0>0$. The parameter $\beta_1\in(0,1)$ denotes the returns to KT and reflects the stickiness of the KT process. The value is close to 1 if the ability of the prototyping team to document and clearly communicate results is large (i.e., if knowledge is easily codified as opposed to tacit) (von Hippel 1994) and if the ability of the pilot line team to absorb the KT is substantial. This gives us Equation (2).

$$P_t(t) = \beta_0 p(t) Y(t)^{\beta_1} \tag{2}$$

Lastly, the manager determines the rate of production ramp-up activities to pursue over time, denoted by $n(t)\geq 0$ for $t\in[0,T]$ (control variable), which can be measured in terms of hours of workforce effort. Let $N(t)$ denote the level of ramp-up knowledge at time t, with $N(0)>0$ given (state variable). The level of ramp-up knowledge at time t reflects the initial level $N(0)$ as well as the learning benefits from the rate of production ramp-up activities and the transfer of pilot line knowledge through time t. The transfer of pilot line knowledge enhances the effectiveness of production ramp-up activities to increase the level of ramp-up knowledge by providing direction on the experiments or engineering trials needed (Terwiesch and Bohn 2001). In Equation (3), the workforce skill of the ramp-up team and the quality of technical support are indicated by the parameter $\gamma_0>0$, and the stickiness of the KT process is given by the parameter $\gamma_1\in(0,1)$.

$$N_t(t) = \gamma_0 n(t) P(t)^{\gamma_1} \tag{3}$$

3.2 The Objective

The profit-maximizing objective appears in Equation (4). The first terms (outside the integral) represent the net revenue earned when the product is released to the market at time T. The remaining terms consist of the costs incurred for development activities during the NPD project. Since KT is fluid and occurs continuously over time along with development activities, its costs are simply subsumed in those associated with the rates of prototyping, pilot line testing and production ramp-up activities. Below, we elaborate on each term.

$$\text{Maximize } r_1 Y(T) + r_2 P(T) + r_3 N(T) - \int_0^T [c_1 y(t)^{\sigma_1} + c_2 p(t)^{\sigma_2} + c_3 n(t)^{\sigma_3}] dt \quad (4)$$

The ability of the firm to earn net revenue is a function of the cumulative knowledge generated by NPD activities at the product launch time (Cohen et al. 1996, Kim 1998, Chao et al. 2009, Gaimon et al. 2011). We assume the levels of knowledge at the launch time T characterize the final product features, functionality and process efficiency for the new product. In addition, the levels of prototyping, pilot line and ramp-up knowledge may have value (i.e., contribute to net revenue) for future NPD projects. Let net revenue be denoted by $r_1 Y(T) + r_2 P(T) + r_3 N(T)$ with r_1, r_2, and $r_3 \geq 0$. (Carrillo and Franza 2006, Chao et al. 2009, Ozkan et al. 2012).

During the NPD project, costs are incurred for prototyping activities, pilot line testing and production ramp-up (Clark and Fujimoto 1991). Let $c_1 y(t)^{\sigma 1}$ denote the cost incurred for prototyping activities undertaken at time t, with $c_1>0$ and $\sigma_1>1$. The cost includes salaries for team members who conduct prototyping and the cost of the technical support systems, such as computer aided design workstations. Since $\sigma_1>1$ the cost is convex with respect to the rate of prototyping activities pursued at any instant of time

(Carrillo and Gaimon 2004, Chand et al. 1996, Terwiesch and Xu 2004). The cost increases at an increasing rate to reflect coordination costs and overtime or the use of less efficient methods as the finite development resources are increasingly strained. Similarly, we define $c_2p(t)^{\sigma 2}$ and $c_3n(t)^{\sigma 3}$, with c_2 and $c_3>0$, σ_2 and $\sigma_3>1$, as the costs for pilot line testing and production ramp-up activities at time t, respectively.

3.3.3 Continuous Forward KT Solution

In the remainder of the paper, the notation depicting time is suppressed whenever possible, all proofs appear in the Appendix, and "*" indicates an optimal solution. We solve the model using optimal control methods (Sethi and Thompson 2000). The Hamiltonian to be maximized is given in Equation (5). The adjoint variables $\lambda_1(t)$, $\lambda_2(t)$ and $\lambda_3(t)$ are introduced to represent the marginal values of the levels of prototyping, pilot line, and ramp-up knowledge at time t, respectively. Since the level of prototyping knowledge at time t is sustained from that time through the remainder of the development project, $\lambda_1(t)$ is interpreted as the marginal value of an additional unit of prototyping knowledge from time t to the product launch time, T. The optimality conditions for $\lambda_1^*(t)$ are given in Lemma 1. Similar interpretations hold for $\lambda_2(t)$ and $\lambda_3(t)$ whose optimality conditions are also given in Lemma 1.

$$\mathrm{H} = -c_1 y(t)^{\sigma_1} - c_2 p(t)^{\sigma_2} - c_3 n(t)^{\sigma_3} + \lambda_1(t)\alpha_0 y(t) + \lambda_2(t)\beta_0 p(t)Y(t)^{\beta_1}$$
$$+\lambda_3(t)\gamma_0 n(t)P(t)^{\gamma_1} \quad (5)$$

LEMMA 1: *(i)* $\lambda_{1t} = -\lambda_2(t)\beta_0 p(t)\beta_1 Y(t)^{\beta_1 - 1}, \lambda_1(T) = r_1;$

(ii) $\lambda_{2t} = -\lambda_3(t)\gamma_0 n(t)\gamma_1 P(t)^{\gamma_1 - 1}, \lambda_2(T) = r_2;$ *(iii)* $\lambda_3(t) = r_3 \; for \; t \in [0, T].$

In Lemma 1, we find that the marginal values of prototyping and pilot line knowledge are positive and decreasing over time and the marginal value of production ramp-up knowledge is constant over time. The marginal value of prototyping knowledge at time t is driven by the sum of the marginal contribution to net revenue from prototyping knowledge at T and the marginal benefit to pilot line testing from the transfer of prototyping knowledge at time t. It is important to see that an additional unit of prototyping knowledge transferred early enhances the effectiveness of the rate of pilot-line testing activities over the remainder of the development project. Thus, the marginal value of the level of prototyping knowledge decreases over time. Similarly, the marginal value of the level of pilot-line testing activities decreases over time. In contrast, the level of ramp-up knowledge derives value only in terms of the net revenue realized at the product launch. Thus, the marginal value of ramp-up knowledge is constant.

3.3.3.1 Optimal Rates of NPD Activities

The optimal rates of development activities are given in Theorem 1. From inspection, we see that the optimal rates of prototyping, pilot line testing, and production ramp-up activities at time t are functions of the marginal values and the marginal costs at that time. It is important to recognize that the marginal value of pilot line testing (production ramp-up) activities at time t is a function of the level of prototyping (pilot line testing) knowledge transferred at that time. In Corollary 1, we describe how the optimal rates of development activities change throughout the NPD project; the interpretations follow.

<u>THEOREM 1</u>: *The optimal rates the NPD manager pursues development activities are:*

(i) $y^*(t) = \left(\frac{\lambda_1{}^*(t)\alpha_0}{\sigma_1 c_1}\right)^{\frac{1}{\sigma_1 - 1}}$; *(ii)* $p^*(t) = \left(\frac{\lambda_2{}^*(t)\beta_0 Y(t)^{\beta_1}}{\sigma_2 c_2}\right)^{\frac{1}{\sigma_2 - 1}}$; *and* *(iii)* $n^*(t) = \left(\frac{\lambda_3{}^*(t)\gamma_0 P(t)^{\gamma_1}}{\sigma_3 c_3}\right)^{\frac{1}{\sigma_3 - 1}}$.

COROLLARY 1: *(a)* $y_t^* < 0$ *for t∈[0,T]; (b)* $n_t^* > 0$ *for t∈[0,T]; (c) (Case i)* $p_t^* > 0$ *for t∈[0,*t_s*),* $p_t^* = 0$ *at* t_s*, and* $p_t^* < 0$ *for t∈(*t_s*,T], where* t_s*∈[0,T]; (Case ii)* $p_t^* < 0$ *for t∈[0,T]; (Case iii)* $p_t^* > 0$ *for t∈[0,T].*

From Theorem 1 and Corollary 1 (a), we know that the optimal rate of prototyping is positive and decreasing over time until reaching $(r_1\alpha_0/\sigma_1 c_1)^{1/(\sigma_1-1)}$ at the end of the development project. (See Figure 3.1; for illustrative purposes only the solution is shown as convex in time.) We refer to this development strategy as *front-loading* (Blackburn et al. 1996, Thomke and Fujimoto 2000, Ozkan et al. 2012). Front loading optimally occurs since an additional unit of prototyping activity early in the development project increases the level of prototyping knowledge at that time and thereby enhances the effectiveness of pilot line testing from that time through the remainder of the development project. Said differently, front loading the rate of prototyping activities is advocated since, as the NPD project progresses, there is less opportunity to benefit from KT to pilot line testing.

At the other end of the spectrum, consider the result in Theorem 1 and Corollary 1 (b). We find that the optimal rate of production ramp-up activities is positive and increasing throughout the NPD project. (See Figure 3.1; for illustrative purposes only the solution is shown as convex in time.) This result is obtained since the rate of production ramp-up is more and more effective over time due to the transfer of more and more pilot-line knowledge. We refer to this solution as the *extreme delay* strategy since the maximum pursuit of production ramp-up occurs as the product launch time is reached.

Lastly, we consider the optimal rate of pilot line testing activities. Under reasonable parameter values we obtain the solution given in Theorem 1 and Corollary 1 I

(i) where the optimal rate of pilot line testing first increases, peaks, and then decreases over time. Since the peak rate of pilot line testing is delayed until later in the planning horizon we refer to this as the *moderate delay* strategy. (See Figure 3.1; for illustrative purposes only the solution is shown as concave in time.) (Ozkan et al. (2011) simply refer to this as the delay strategy since they do not obtain the extreme delay strategy.) Two forces drive this result. First, as time passes and the level of prototyping knowledge increases, pilot line testing activities are more effective as a result of KT. Thus, early in the development project the desirability of pilot line testing increases over time. However, as less time remains in the development project, lesser benefits accrue due to the transfer of knowledge from the pilot line to ramp-up stage. As a result, later in the project, the rate of pilot line testing activities decreases over time. Putting these two forces together we find that the maximum rate of pilot line testing is moderately delayed until later in the development project in order to take advantage of the higher level of prototyping knowledge while also providing sufficient pilot line knowledge to be utilized at the ramp-up stage.

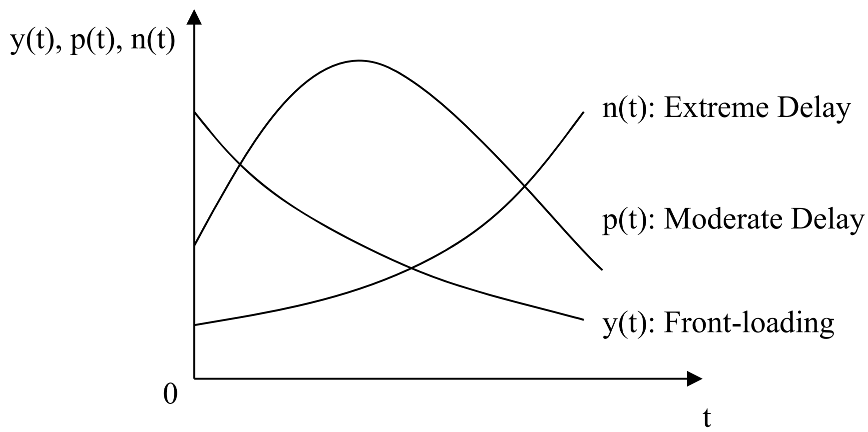

Figure 3.1: Rates of NPD Activities over Time

For completeness, note that two other solutions are possible (though highly unlikely) for the optimal rate of pilot line testing activities. First, if Y(0) is extremely large or r_2 is extremely small, we may obtain the solution in Corollary 1 I (ii). Starting at the initial time, the pilot line team leverages the extremely large level of prototyping knowledge so that the maximum rate of pilot line testing activities occurs at time t=0. Similarly, with extremely small r_2, if prototyping does not, in itself, contribute to net revenue then its only value is through the KT to the ramp-up stage. Since the marginal value of KT decreases over time, we see that the rate of pilot line testing activities has its maximum at the initial time and decreases thereafter. Alternatively, if Y(0) is extremely small or r_2 is extremely large, then the peak rate of pilot line testing occurs at t=T, as in Corollary 1 I (iii). The interpretation of case I (iii) is the reverse of case I (ii).

3.3.3.2 Analytic Sensitivity Analysis

Through analysis of the optimal solutions we find that if the effectiveness of the rate of any one development activity (α_0, β_0, γ_0) or the returns to any KT activity (β_1, γ_1) is larger, then the optimal rates of all development activities are larger and the optimal levels of knowledge are larger at all stages over all time. Similarly, if the cost of any development activity is larger, then the optimal rates of all development activities are smaller so that the levels of knowledge are smaller at all stages for all time. Hence we observe a synergistic relationship among the rates of prototyping, pilot line testing and production ramp-up activities. The source of the synergy is the mathematical structure that links each stage to the next through KT. To understand this result intuitively, consider the following.

Recall that β_0 represents the effectiveness of pilot line testing activities on increasing the level of pilot line knowledge. The manager has considerable control over the effectiveness parameter since she selects skilled team members and provides them with technical support resources. Intuitively, if β_0 is larger, the pilot line team is more capable of building pilot line knowledge so that the rate of pilot line testing activities is larger throughout the NPD project. Moreover, the rate of production ramp-up activities is larger for all time since the transfer of more pilot line knowledge enhances its effectiveness at increasing the level of production ramp-up knowledge. Similarly, for larger β_0 the marginal value of the level of prototyping knowledge is larger throughout the NPD project since prototyping knowledge provides more benefits when transferred to the pilot line stage. Therefore, larger β_0 is associated with larger levels of prototyping, pilot line and ramp-up knowledge throughout the development project. This insight is important because it shows how the manager's decisions regarding team skill and the resources provided for technical support at any one stage affect the pursuit of knowledge creation at other stages and ultimately impact net revenue at the product launch time. The above discussion is summarized in Corollary 2.

COROLLARY 2*: If workforce skill or technical support in pilot line testing (β_0) is larger, then the optimal rates of prototyping, pilot line testing, and production ramp-up activities are larger and the levels of knowledge in all stages of development are larger for $t \in (0,T]$. Analogous results are obtained for α_0, γ_0, β_1, and γ_1. The reverse results hold for c_1, c_2 and c_3.*

Next, we analyze the effect of the initial levels of knowledge at each stage of the NPD project. The existing knowledge provides a starting point for the NPD project and influences the knowledge development strategies pursued by the manager (Pisano 1997). For simplicity, we focus our discussion on the effect of the initial level of pilot line knowledge, P_0. Since knowledge transferred from the pilot line improves the effectiveness of production ramp-up activities, it would appear that larger P_0 leads to a larger rate of ramp-up activities. However, a larger rate of ramp-up activities is associated with a larger development cost. Therefore, if P_0 is larger, the NPD manager may need to control the amount of knowledge transferred to the ramp-up stage by reducing the rate of pilot line testing activities, which along with P_0 drives P(t). We find that the impact of P_0 on the optimal solution depends on the relationship between the returns of KT to the ramp-up stage (γ_1) and an expression indicating the extent of diseconomies of scale in the cost of ramp-up activities ($1-1/\sigma_3$), as described below.

Suppose $0<\gamma_1<1-1/\sigma_3$ holds and P_0 is larger. Since the extent of diseconomies of scale in the cost of ramp-up activities exceeds the returns of KT from the pilot line to the ramp-up stage, the manager has less incentive to develop pilot line knowledge and thereby reduces the rate of pilot line testing. Since the desirability of additional pilot line knowledge is small, the incentive to develop prototyping knowledge to improve the effectiveness of pilot line activities is small and the optimal rate of prototyping is reduced. Hence, we observe a substitution effect: the higher initial level of the pilot line knowledge is associated with smaller rates of pilot line testing and prototyping throughout the NPD project. Obviously, the levels of prototyping and pilot line knowledge are smaller.

Alternatively, suppose the returns to KT from the pilot line stage are larger than the extent of diseconomies of scale in the cost of ramp-up activities ($1-1/\sigma_3<\gamma_1<1$). If P_0

is larger, the manager optimally pursues a larger rate of pilot line testing activities throughout the NPD project in order to increase the effectiveness of ramp-up activities. Moreover, since prototyping knowledge improves the effectiveness of pilot line testing activities, the manager undertakes a larger rate of prototyping activities. As such, we observe a complementary relationship whereby a larger initial level of workforce knowledge drives the manager to pursue more pilot line testing and prototyping throughout the NPD project. Naturally, the levels of prototyping and pilot line knowledge are larger for all time. The larger level of pilot line knowledge increases the effectiveness of ramp-up activities, which are pursued at a larger rate, so that the level of ramp-up knowledge is larger throughout the NPD project. (As expected, if $\gamma_1=1-1/\sigma_3$, then the initial level of pilot line knowledge has no effect on the rates of NPD activities.)

To complete our analysis, we consider how P_0 impacts the level of ramp-up knowledge. It can be shown that if P_0 is larger, then the level of pilot line knowledge is larger throughout the NPD project. Therefore, even if $0<\gamma_1<1-1/\sigma_3$ holds so that larger P_0 leads to smaller rates of pilot line testing and prototyping activities for all time, the level of pilot line knowledge is always larger. As a result, the effectiveness of the rate of ramp-up activities is larger leading the manager to pursue a larger rate of ramp-up activities and driving a larger level of ramp-up knowledge throughout the NPD project.

The above results are summarized in Corollary 3. The impact of the initial level of prototyping knowledge Y_0 is analogous to the impact of P_0 and is omitted.

<u>COROLLARY 3.</u> *Suppose P_0 is larger. When $0<\gamma_1<1-1/\sigma_3$, the manager pursues smaller rates of prototyping and pilot line testing activities (y(t) and p(t)) for $t\in[0,T]$. When $1>\gamma_1>1-1/\sigma_3$, the manager pursues larger rates of prototyping and pilot line* *testing activities for $t\in[0,T]$. When $\gamma_1=1-1/\sigma_3$, the manager pursues the same rates of prototyping and pilot line testing for $t\in[0,T]$. For any $\gamma_1\in(0,1)$, the manager pursues a larger rate of ramp-up activities ($n(t)$) for $t\in[0,T]$.*

Lastly, the initial level of ramp-up knowledge (N_0) has no impact on development activities in any stage, so it has no effect to the levels of prototyping and pilot line knowledge. N_0 does, naturally, affect the level of knowledge in the ramp-up stage. An increase in N_0 results in a larger level of ramp-up knowledge throughout the development project.

3.3.3.3 Numerical Sensitivity Analysis

While considerable insights are obtained analytically, numerical sensitivity analysis allows us to explore how the predetermined product launch time, T, impacts the development activities in each stage. Holding all other input parameters fixed, we find that if the product is launched later (T larger), then the rates of prototyping, pilot line testing and ramp-up activities are all larger for $t\in[0,T]$. In addition, the knowledge levels of each stage of the NPD project are larger at the product launch time. Intuitively, since the recipient team has more time to leverage the KT from the source team, the marginal values of knowledge are larger for both teams. However, it is important to recognize that, due to competition, a later product launch may be associated with smaller marginal contributions to net revenue from the levels of knowledge. In that situation, the effect of a delayed product launch on the optimal rates of development activities throughout the NPD project is unclear.

3.4 Feedback Model

In Section 3.3, we consider the situation in which KT only occurs in the forward direction. However, in practice, feedback may occur whereby the knowledge accumulated in a downstream activity is transferred back upstream. In this section, we explore the situation in which KT occurs not only in the forward direction, but also in the backward direction in the form of feedback. More specifically, we allow the knowledge accumulated in the pilot line testing stage to be transferred back to the prototyping stage to improve the effectiveness of prototyping activities. For completeness, we also analyze the situation where feedback occurs from the ramp-up stage to the prototyping stage and the situation where feedback occurs from the ramp-up stage to the pilot line stage. We find that the general insights and implications of feedback are analogous in all situations.

3.4.1 The Levels of Knowledge with Feedback

The extent that prototyping activities at time t increase the level of prototyping knowledge at that time is driven not only by the rate of prototyping, the skill of the team, and the quality of the technical support available, but also by the level of pilot line knowledge transferred back, as given in Equation (6) (Loch and Terwiesch 1998). Feedback from the pilot line stage enables the prototyping team to better understand the impact of various elements of prototyping designs on pilot line configurations, (i.e., which product features work well versus fail during pilot line testing). Thus, feedback of pilot line knowledge enhances the effectiveness of prototyping activities on increasing prototyping knowledge. The parameter $\alpha_1 \in (0,1)$ denotes the returns to the feedback from the pilot line to the prototyping stage. A larger value of α_1 corresponds to the situation where the ability of the pilot line team to provide feedback is large and the ability of the prototyping team to absorb feedback is large. In contrast, a value of zero corresponds to the situation where feedback is not relevant as reflected in the Base Model. Lastly, note that as in the Base Model, Y(0) is given, $y(t) \geq 0$ for $t \in [0,T]$, and we have the same interpretation for α_0.

$$Y_t(t) = \alpha_0 y(t) P(t)^{\alpha_1} \quad (6)$$

By incorporating Equations (2), (3), and (4) from the Base Model, we complete our statement of the Feedback Model.

3.4.2 Optimal Solutions

The Hamiltonian for the Feedback Model appears in Equation (7), the optimal solutions for $\lambda_1(t)$, $\lambda_2(t)$ and $\lambda_3(t)$ (whose interpretations are the same as those in the Base Model) are given in Lemma 2. Theorem 2 states the optimal rates of KD activities in each stage of the NPD project. The interpretations follow.

$$H = -c_1 y(t)^{\sigma_1} - c_2 p(t)^{\sigma_2} - c_3 n(t)^{\sigma_3} + \lambda_1(t)\alpha_0 y(t) P(t)^{\alpha_1} + \lambda_2(t)\beta_0 p(t) Y(t)^{\beta_1}$$
$$+\lambda_3(t)\gamma_0 n(t) P(t)^{\gamma_1} \quad (7)$$

LEMMA 2: *The marginal values of prototyping, pilot line and ramp-up knowledge satisfy the following conditions:*

(i) $\lambda_{1t}(t) = -\lambda_2(t)\beta_0 p(t)\beta_1 Y(t)^{\beta_1 - 1} < 0, \lambda_1(T) = r_1;$

(ii) $\lambda_{2t}(t) = -\lambda_1(t)\alpha_0 y(t)\alpha_1 P(t)^{\alpha_1 - 1} - \lambda_3(t)\gamma_0 n(t)\gamma_1 P(t)^{\gamma_1 - 1} < 0, \lambda_2(T) = r_2;$

(iii) $\lambda_{3t}(t) = 0, \lambda_3(t) = \lambda_3(T) = r_3.$

From (i) and (ii) in Lemma 2, we know that the marginal values of prototyping and pilot line knowledge for the Feedback Model are positive and decreasing over time. Also, we observe that the marginal value of ramp-up knowledge is positive and constant over the development project, which is consistent with the Base Model. The interpretations of the marginal values of prototyping and ramp-up knowledge are analogous to the Base model and omitted. In contrast to the Base Model, however, given feedback we find that the marginal value of pilot line knowledge at time t is driven by the sum of the marginal contribution to net revenue from pilot line knowledge at T and the marginal benefit of feedback from the pilot line to the prototyping stage. The marginal value of pilot line knowledge decreases over time, since less time remains for the prototyping stage to benefit from feedback.

The optimal rates of prototyping, pilot line testing, and ramp-up activities are functions of the above-mentioned marginal value functions, the levels of knowledge, the effectiveness of each type of KT, and the costs of development, as shown in Theorem 2. A key difference between the results of the Base Model and the Feedback Model concerns the optimal rate of prototyping. In Theorem 2, beyond those drivers identified for the Base Model, we observe that the optimal rate of prototyping activities at time t is impacted by feedback from the pilot line stage at that time. In particular, the optimal rate of prototyping at time t is driven by the marginal value of prototyping knowledge, as well as the level of pilot line knowledge transferred to the prototyping stage that enhances the effectiveness of prototyping activities.

THEOREM 2: *The optimal rates the NPD manager pursues development activities are:*

(i) $y^*(t) = (\frac{\alpha_0 \lambda_1 P^{\alpha_1}}{c_1 \sigma_1})^{\frac{1}{\sigma_1 - 1}}$; *(ii)* $p^*(t) = (\frac{\beta_0 \lambda_2 Y^{\beta_1}}{c_2 \sigma_2})^{\frac{1}{\sigma_2 - 1}}$; *and (iii)* $n^*(t) = (\frac{\gamma_0 \lambda_3 P^{\gamma_1}}{c_3 \sigma_3})^{\frac{1}{\sigma_3 - 1}}$.

3.4.3 The Impact of Feedback

In this section, to investigate the differences between the Base Model and the Feedback Model, we examine how the returns to feedback from the pilot line to the prototyping stage (α_1) impacts the manager's optimal development strategies during the NPD project.

Of course, the manager has considerable control over the effectiveness parameter since she selects skilled team members and provides them with technical support resources. Moreover, understanding the effect of the returns parameter is critical since, if α_1=0, then the Feedback Model reverts to the Base Model.

<u>COROLLARY 4:</u> *If the returns to feedback from the pilot line to the prototyping stage (α_1) is larger, then $y^*(t)$, $p^*(t)$ and $n^*(t)$ are all larger for $t \in [0,T]$. It follows that $Y(t)$, $P(t)$, and $N(t)$ are all larger for $t \in [0,T]$.*

From Corollary 4, we see that if the feedback from the pilot line stage is more valuable in improving the capability of the prototyping team (α_1 is larger), the manager pursues a higher rate of prototyping activities throughout the NPD project and the level of prototyping knowledge is larger. Given the larger amount of prototyping knowledge transferred to the pilot line stage, the ability of the pilot line team to increase its level of knowledge is larger and the manager pursues a larger rate of pilot line testing throughout the NPD project. As a result, the level of pilot line knowledge is larger. Following the same logic, we know that the rate of production ramp-up activities and the level of ramp-up knowledge are larger throughout the development project.

<u>COROLLARY 5:</u> *If α_1 is sufficiently small then $y_t^*(t) < 0$ for $t \in [0,T]$ (front-loading strategy); if α_1 is sufficiently large then $y_t^*(t) > 0$ for $t \in [0,T]$ (extreme delay strategy). Note that the sign of $y_t^*(t)$ is determined by the sign of $-\beta_1\lambda_2(t)P(t) + \alpha_1\lambda_1(t)Y(t)$.*

<u>OBSERVATION 1:</u> *In the vast majority of numerical experiments, if α_1 is moderate, then $y^*(t)$ is inverse U-shaped (moderate delay strategy) over time, for $t \in [0,T]$. However it is possible, under extreme conditions, for $y^*(t)$ to be U-shaped (moderate front loading strategy).*

From Corollary 5 (analytic result) and Observation 1 (numerical result), we see that the optimal solution for the rate of prototyping activities is radically different due to feedback from the pilot line to the prototyping stage (Figure 3.2). The rate of change in $y^*(t)$ is determined by the sign of $-\beta_1\lambda_2(t)P(t) + \alpha_1\lambda_1(t)Y(t)$, where the first term indicates the value of the forward KT from the prototyping to the pilot line stage, and the second term is the value of the feedback from the pilot line to the prototyping stage. First, if the returns from feedback are relatively small (α_1 is small), the second term is dominated by the first term and prototyping activities are front-loaded. Essentially, the NPD manager undertakes a larger rate of prototyping activities *early* in the development project to accumulate and transfer prototyping knowledge forward and thereby improve the effectiveness of pilot line testing. This analytic result is consistent with the solution in the Base Model where $\alpha_1 = 0$. In contrast, suppose α_1 is large so that the value of feedback from the pilot line stage is instrumental to enhancing the effectiveness of prototyping activities. In this situation, we analytically find that the second term dominates the first term and prototyping activities follow the extreme delay strategy. The extreme delay strategy is advocated so that the maximum rates of prototyping activities are pursued late in the development project after most of the benefits from pilot line feedback are realized.

In contrast to the analytic results obtained when α_1 is either very small or large, given the complexity of the problem, we cannot obtain analytic results characterizing the rate of prototyping for intermediate values. However, based on extensive numerical investigation, we find that for intermediate values of α_1, in the vast number of experiments, the optimal rate of prototyping is inverse U-shaped. In other words, the manager pursues a moderate delay strategy. When the returns to feedback are moderate, the manager delays the maximum rate of prototyping until later in the development project to wait for more pilot line knowledge to be transferred. However, in extreme cases, when α_1 is moderate, we also find the manager may pursue a *moderate front-loading* strategy (i.e., $y^*(t)$ initially decreases then increases over time). For example, the moderate front-loading strategy is more likely to occur for very large γ_1. Early in the NPD project, the rate of prototyping is relatively large (but decreasing) and is driven by the desire to enhance the effectiveness of pilot line testing through forward KT (due to large γ_1). Later in the planning horizon, the rate of prototyping increases and is driven by the desire to leverage the enhanced effectiveness of prototyping activities due to the increasing amount of feedback from the pilot line stage (due to moderate α_1). Note that the moderate front-loading strategy did not occur in any solution of the Base Model and is unique to the Feedback Model.

<u>COROLLARY 6:</u> *As in the Base Model, for any* $\alpha_1 \in (0,1)$, *we have* $n_t^*(t) > 0$ *for* $t \in [0,T]$ *(extreme delay strategy).*

The interpretation of Corollary 6 is analogous to that of Corollary 1(b) thus omitted.

<u>OBSERVATION 3:</u> *In the vast majority of numerical experiments, if* α_1 *is small or moderate, then* $p^*(t)$ *is inverse U-shaped (moderate delay strategy) for* $t \in [0,T]$*. Also, if* α_1 *is large, then* $p_t^*(t) < 0$ *over time, for* $t \in [0,T]$ *(front-loading strategy). However under extreme conditions when* α_1 *is small or moderate, it is possible that* $p^*(t)$ *is U-shaped (moderate front-loading strategy).*

The rate of change in $p^*(t)$ is determined by the sign of $-\lambda_1\alpha_0\alpha_1 yYP^{\alpha_1-1} - \lambda_3\gamma_0\gamma_1 nYP^{\gamma_1-1} + \beta_1\lambda_2\alpha_0 yP^{\alpha_1}$, where the first term is the value of the forward KT from the prototyping to the pilot line stage, the second term is the value of the forward KT from the pilot line to the ramp-up stage, and the third term is the value of feedback from the pilot line stage to the prototyping stage. Analytically, we are not able to ascertain how the rate of pilot line testing changes over time in relation to α_1. However, through extensive numerical experimentation, we observe that if α_1 is small or moderate, the optimal rate of pilot line testing follows the moderate delay strategy; if α_1 is large then pilot line activities are front-loaded (Figure 3.3). The relationship between α_1 and $p^*(t)$ occurs because pilot line knowledge impacts the manager's pursuit of prototyping activities through feedback. When α_1 is small or moderate, pilot line knowledge has limited impact on the effectiveness of prototyping activities. In this situation, we obtain the Base Model solution in which the rate of pilot line testing follows the moderate delay strategy. However, as α_1 increases (while remaining small or moderate), the peak rate of pilot line testing occurs earlier reflecting the greater desirability of accumulating pilot line knowledge to serve as feedback for prototyping. In contrast, when α_1 is sufficiently large, the substantial desire to transfer large amounts of pilot line knowledge to the prototyping stage drives the manager to front-load pilot line testing.

3.4.4 Feedback from Ramp-up

In Section 3.4.3, we study the optimal KD strategies in a three stage NPD project when feedback occurs from the pilot line to the prototyping stage. In this section, we examine the optimal development strategies in the situation where feedback occurs from the ramp-up stage to either the prototyping or pilot line stage.

When the returns to feedback from the ramp-up to the prototyping stage or from the ramp-up to the pilot line stage are sufficiently small, we find that the shapes of the curves representing the optimal KD strategies are the same as those in the base model (though the actual values differ somewhat). In contrast, when the returns to feedback are sufficiently large, the shapes of the curves representing the optimal development strategies pursued by the NPD manager can be different from those in the base model, as described below.

With sufficiently large returns to feedback from the ramp-up to the prototyping stage, the NPD manager optimally pursues the extreme delay strategy for prototyping activities in order to wait for ramp-up feedback. Moreover, the manager front-loads pilot line activities since the accumulation of pilot line knowledge enhances KD in the ramp-up stage and thereby indirectly aids KD in the prototyping stage. Lastly, the manager front-loads ramp-up activities to rapidly build knowledge that is transferred as feedback to the prototyping stage.

With sufficiently large returns to feedback from the ramp-up to the pilot line stage, consistent with the base model, the manager optimally front-loads prototyping activities to enhance the effectiveness of pilot line activities early in the NPD project. In contrast, the manager pursues the extreme delay strategy for pilot line activities in order to wait for the accumulation of feedback from the ramp-up stage. Lastly, the manager front-loads ramp-up activities to provide rapid feedback to the pilot line stage.

Table 3.1 presents a summary of the optimal KD strategies when feedback does not occur or when the returns to feedback are sufficiently large and occur: (i) from the pilot line to the prototyping stage, (ii) from the ramp-up to the prototyping stage, or (iii) from the ramp-up to the pilot line stage.

Stages	No Feedback	Feedback from Pilot line to Prototyping	Feedback from Ramp-up to Prototyping	Feedback from Ramp-up to Pilot line
Prototyping Stage	Front-loading	Extreme Delay	Extreme Delay	Front-loading
Pilot line Stage	Moderate Delay	Front-loading	Front-loading	Extreme Delay
Ramp-up Stage	Extreme Delay	Extreme Delay	Front-loading	Front-loading

Table 3.1: KD Strategies When the Returns to Feedback Are Sufficiently Small (Analogous to No Feedback) and When the Returns to Feedback are Sufficiently Large

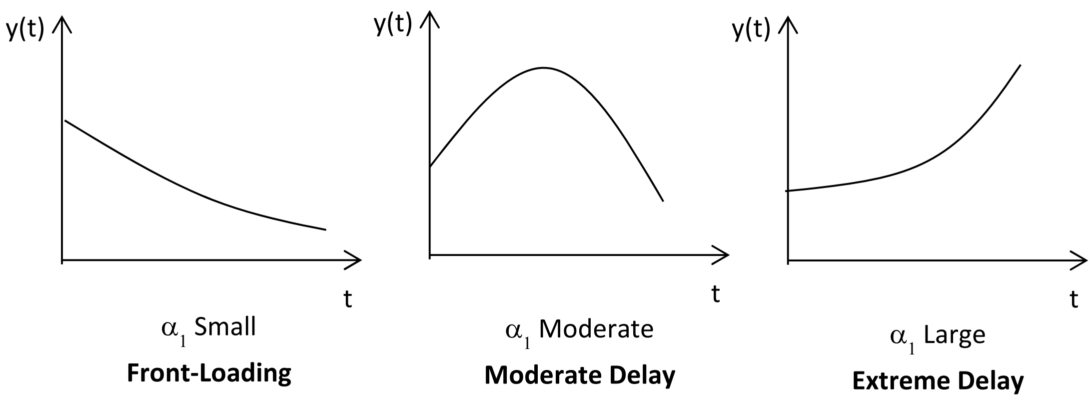

Figure 3.2: The Optimal Rate of Prototyping in Relation to α_1

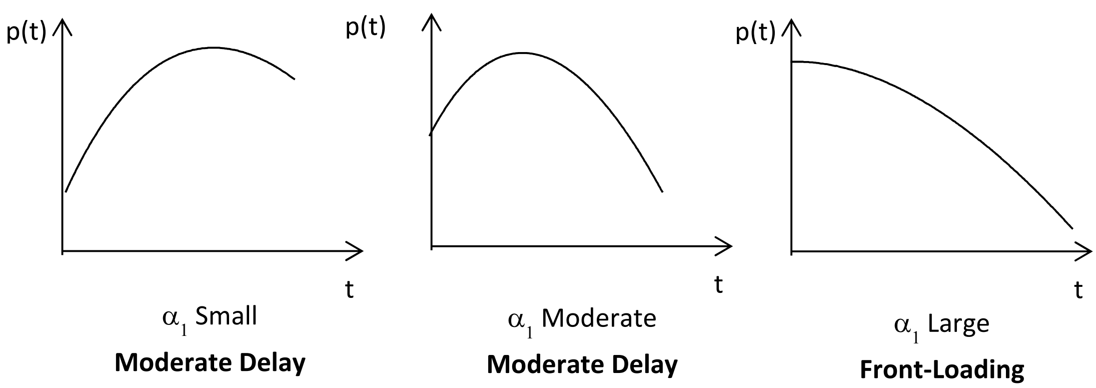

Figure 3.3: The Optimal Rate of Pilot Line Testing in Relation to α_1

3.5 Discrete Model

In this section, we present an extension of the Base Model, referred to as the Discrete Model, where the forward transfer of knowledge from one stage of product development to the next occurs at discrete times. The Discrete Model is appropriate when the development team responsible for the NPD project is sufficiently large or operates in different locations so that the continuous transfer of knowledge is not practical. To simplify the model presentation, we initially assume one KT occurs from the prototyping to the pilot line stage, and one KT occurs from the pilot line to the ramp-up stage.[1] In particular, we assume the level of prototyping knowledge accumulates from time 0 to t_1 when it is transferred to enhance the effectiveness of pilot line testing activities, and the level of pilot line knowledge accumulates from time 0 to t_2 when it is transferred to enhance the effectiveness of ramp-up activities, where $t_1<t_2$. Therefore, in the discrete model, in addition to obtaining the optimal rates of prototyping, pilot line testing and production ramp-up activities, we also determine the optimal times that KTs occur between successive stages of the NPD project.

In contrast to the Base Model where the cost of KT is continuously subsumed in the development costs of prototyping and pilot line testing, in the Discrete Model the costs for each of the two KTs are explicit. Discrete KT may take many forms. Fundamentally, the source team prepares presentations or documentation to communicate knowledge to the recipient team. The transfer may take place remotely by sending reports or design drawings via telecommunications technology or in face-to-face meetings.

[1] Later, numerical solutions are given under other scenarios such as two stages with several KTs.

Naturally, when knowledge is "batched" and transferred at discrete times, costs are incurred reflecting meeting times, transportation, as well as efforts expended by the source team for documentation and by the recipient team to absorb and apply the new knowledge. If KT occurs earlier in the project, the cost is lower and the recipient has more time to benefit from the smaller amount of KT. Alternatively, if KT occurs later, the cost is higher and the recipient has less time to benefit from the larger amount of KT. Therefore, the manager needs to balance a complex set of tradeoffs when determining the times for KT. We formulate the model using impulsive control theory, described below.

3.5.1 Knowledge Transfer

We assume that forward KT occurs at the initial time from the prototyping to the pilot line stage and from the pilot line to the ramp-up stage.[2] The KTs at the initial time reflect the fact that when the pilot line (ramp-up) team begins its development activities it already benefits from the prototyping (pilot line) knowledge developed from prior NPD projects.

Let $\bar{Y}(t)$ denote the level of prototyping knowledge transferred at time t_1 to enhance pilot line testing activities, where $\bar{Y}(0)=Y(0)>0$. In other words, $\bar{Y}(t)$ is a step function whose value is $\bar{Y}(t)=Y(0)$ for $t\in[0,t_1]$ and $\bar{Y}(t)=Y(t_1)$ for $t\in(t_1,T]$, where $Y(t_1)$ is obtained from Equation (1). To formalize this mathematically, we introduce the impulsive control variable $\bar{y}(t_1)\in[0,1]$ which indicates whether a transfer of prototyping knowledge to the pilot line testing stage occurs at time t_1. It will be shown that $\bar{y}(t_1)$ appears linearly in the model so that its optimal solution lies on a boundary. As such,

[2] This assumption is easily relaxed and does not affect analysis of the model.

$\bar{y}(t_1)=1$ indicates a transfer optimally occurs at t_1; $\bar{y}(t_1)=0$ indicates a transfer does not optimally occur. The above discussion is summarized in Equations (8) and (9).

$$\bar{Y}_t(t) = 0, \quad t \neq t_1. \tag{8}$$

$$\bar{Y}(t_1^+)=\bar{Y}(t_1)+[Y(t_1)-\bar{Y}(t_1)]\bar{y}(t_1) = Y(0) + [Y(t_1)-Y(0)]\bar{y}(t_1), \quad t = t_1. \tag{9}$$

To determine the level of pilot line knowledge over the NPD project, we modify Equation (2) to reflect the fact that the transfer of prototyping knowledge occurs only at the initial time and at t_1. After time t_1, the effectiveness of the rate of pilot line testing activities is enhanced by the transfer of prototyping knowledge at t_1. The parameter β_1 indicates the returns to KT and reflects the ability of the prototyping team to transfer knowledge as well as the ability of the pilot line team to absorb it. These relationships are summarized in Equation (10).

$$P_t(t) = \beta_0\, p(t)\bar{Y}(t)^{\beta 1}, \quad t \in [0,T]. \tag{10}$$

Similarly, let $\overline{P}(t)$ denote the level of pilot line knowledge transferred at t_2 to enhance development activities at the production ramp-up stage, where $\overline{P}(0)=P(0)>0$ holds. Thus, $\overline{P}(t)$ is a step function whose value changes at t_2, giving us $\overline{P}(t)=P(0)$ for $t\in[0,t_2]$ and $\overline{P}(t)=P(t_2)$ for $t\in(t_2,T]$, where $P(t_2)$ is obtained from Equation (10). We introduce the impulsive control variable $\overline{p}(t_2)\in[0,1]$ that appears linearly in the model so that a boundary solution is obtained: $\overline{p}(t_2)=1$ if a transfer optimally occurs; $\overline{p}(t_2)=0$ if a transfer does not occur at t_2. This gives us the following.

$$\overline{P}_t(t)=0, \quad t \neq t_2 \tag{11}$$

$$\overline{P}(t_2^+)=\overline{P}(t_2)+[P(t_2)-\overline{P}(t_2)]\overline{p}(t_2)=P(0)+[P(t_2)-P(0)]\overline{p}(t_2), \quad t = t_2. \tag{12}$$

To determine the level of ramp-up knowledge over the NPD project, we modify Equation (3) to reflect the fact that the transfer of pilot line knowledge occurs only at the initial time and at t_2. After time t_2, the effectiveness of the rate of development activities at the ramp-up stage is enhanced by the transfer of pilot line knowledge. The parameter γ_1 indicates the returns to KT and reflects the ability of the pilot line team to transfer knowledge as well as the ability of the ramp-up team to absorb it. These relationships are summarized in Equation (13).

$$N_t(t)=\gamma_0 n(t)\bar{P}(t)^{\gamma_1}, \qquad t\in [0,T]. \tag{13}$$

3.5.2 The Objective

In contrast to the continuous model where the costs of KT are subsumed in development costs, in the discrete model explicit costs are incurred. The discrete transfer of prototyping knowledge to the pilot line stage occurs in meetings (either face to face or via telecommunications) or remotely by sharing computer generated drawings. Reflecting these transfer mechanisms, a cost is incurred for transportation, meeting facilities, telecommunications, as well as the time and effort by the prototyping team to document its results and the time and effort by the pilot line team to absorb those results. Also, KT disrupts ongoing development activities for both teams. Naturally, large amounts of KT are more disruptive and therefore more costly. Let the cost of transferring prototyping knowledge to the pilot line stage at time t_1 be denoted by $K_1+c_4(Y(t_1)-\bar{Y}(t_1))$. The parameter $K_1>0$ represents the fixed transfer cost whereas the parameter $c_4>0$ reflects the marginal cost in relation to the amount of the KT. Similarly, let $K_2+c_5(P(t_2)-\bar{P}(t_2))$ denote the cost of transferring pilot line knowledge to the ramp-up stage at t_2 with K_2 and $c_5>0$. If the KT process is difficult or the knowledge itself is sticky (von Hippel 1994), then the cost parameters are larger. From Equation (14), we see that the NPD manager maximizes the net revenue obtained at the predetermined product launch time T, while minimizing the costs associated with development activities incurred continuously over time and KT incurred at times t_1 and t_2.

$$\text{Maximize } r_1Y(T) + r_2P(T) + r_3N(T) - \int_0^T [c_1y(t)^{\sigma_1} + c_2p(t)^{\sigma_2} + c_3n(t)^{\sigma_3}]dt -$$

$$\bar{y}(t_1)\left(K_1 + c_4\left(Y(t_1) - \bar{Y}(t_1)\right)\right) - \bar{p}(t_2)\left(K_2 + c_5\left(P(t_2) - \bar{P}(t_2)\right)\right) \quad (14)$$

3.5.3 Discrete Knowledge Transfer Solution

The continuous and impulsive Hamiltonians are given in Equations (15) and (16), respectively. As in the Base Model, $\lambda_1(t)$, $\lambda_2(t)$ and $\lambda_3(t)$ denote the marginal values of the levels of prototyping, pilot line and ramp-up knowledge at time t. Let $\lambda_4(t)$ denote the marginal value of KT from the prototyping to the pilot line stage at time t, and $\lambda_5(t)$ denote the marginal value of KT from the pilot line to the ramp-up stage at time t. The optimal solutions for $\lambda_4(t)$ and $\lambda_5(t)$ are given in Lemma 3.

$$H = -c_1y(t)^{\sigma_1} - c_2p(t)^{\sigma_2} - c_3n(t)^{\sigma_3} + \lambda_1(t)\alpha_0y(t) + \lambda_2(t)\beta_0\beta_1p(t)\bar{Y}(t)$$

$$+\lambda_3(t)\gamma_0\gamma_1n(t)\bar{P}(t) \quad (15)$$

$$H^I = -\bar{y}(t_1)\left(K_1 + c_4\left(Y(t_1) - \bar{Y}(t_1)\right)\right) - \bar{p}(t_2)\left(K_2 + c_5\left(P(t_2) - \bar{P}(t_2)\right)\right)$$

$$+\lambda_4(t_1^+)\left(Y(t_1) - \bar{Y}(t_1)\right)\bar{y}(t_1) + \lambda_5(t_2^+)(P(t_2) - \bar{P}(t_2))\bar{p}(t_2) \quad (16)$$

From Lemma 3, we observe that the marginal values of the levels of prototyping and pilot line knowledge are step functions, and the marginal value of ramp-up knowledge is constant throughout the NPD project. Several intuitive results are reflected in Lemma 3. If the marginal cost of the KT from the prototyping to the pilot line stage is larger (smaller), the marginal value of the level of prototyping knowledge is smaller (larger) at time t_1. In addition, the marginal value of the level of prototyping knowledge is larger (smaller) if the effect on net revenue is larger (smaller) or if the KT to the pilot line stage is more (less) effective. Similar interpretations hold for $\lambda_2^*(t)$. Lastly, from Lemma

3 (iv) and (v), we see that the marginal value of KT from the prototyping (pilot line) to the pilot line (ramp-up) stage is non-negative throughout the NPD project and decreases continuously for $t \leq t_1$ and $t > t_1$ ($t \leq t_2$ and $t > t_2$). If the returns from KT to the pilot line stage are larger (smaller), the marginal value of KT to the pilot line stage is larger (smaller) since the KT from the prototyping stage is more (less) valuable. Additionally, the marginal value of KT to the pilot line stage is larger (smaller) if the effect of pilot line knowledge on net revenue is larger (smaller). Similar interpretations hold for $\lambda_5^*(t)$.

<u>LEMMA 3:</u> *The marginal values of prototyping, pilot line and ramp-up knowledge and the marginal values of the KT of prototyping and pilot line knowledge satisfy the following:*

(i) $\lambda_1^*(t) = r_1 + \lambda_4^*(t_1^+) - c_4 \; for \; t \in [0, t_1], \lambda_1^*(t) = r_1 \; for \; t \in (t_1, T];$

(ii) $\lambda_2^*(t) = r_2 + \lambda_5^*(t_2^+) - c_5 \; for \; t \in [0, t_2], \; \lambda_2^*(t) = r_2 \; for \; t \in (t_2, T];$

(iii) $\lambda_3^*(t) = r_3 \; for \; t \in [0, T];$

(iv) $\lambda_{4t}^* = -\lambda_2^*(t)\beta_0\beta_1 p^*(t)\bar{Y}(t)^{\beta_1 - 1} \; for \; t \neq t_1$, $\lambda_4^*(t_1) = \left(1 - \bar{y}(t_1)\right)\lambda_4^*(t_1^+) + c_4$, $\lambda_4^*(t_1^+) = \int_{t_1}^{T} r_2\beta_0\beta_1 p^*(t) Y(t_1)^{\beta_1 - 1} dt, \lambda_4^*(T) = 0;$

(v) $\lambda_{5t}^* = -r_3\gamma_0\gamma_1 n^*(t)\bar{P}(t)^{\gamma_1 - 1} \; for \; t \neq t_2$, $\lambda_5^*(t_2) = \left(1 - \bar{p}(t_2)\right)\lambda_5^*(t_2^+) + c_5$, $\lambda_5^*(t_2^+) = \int_{t_2}^{T} r_3 \, \gamma_0\gamma_1 n^*(t) P(t_2)^{\gamma_1 - 1} dt, \lambda_5^*(T) = 0.$

3.5.3.1 The Discrete KT Solutions

Theorem 3 characterizes the optimality condition indicating whether or not prototyping (pilot line) knowledge should be transferred to the pilot line (ramp-up) stage at time t_1 (t_2). We know $\lambda_4(t_1^+)$ is the marginal value of KT from the prototyping to the pilot line stage at time t_1, and $Y(t_1) - \bar{Y}(t_1)$ is the amount of KT. Therefore, $\lambda_4(t_1^+)\,[Y(t_1) - \bar{Y}(t_1)]$ is the value of transferring prototyping knowledge to the pilot line stage at time t_1, and

$c_4\big(Y(t_1) - \bar{Y}(t_1)\big) + K_1$ is the corresponding cost. If the benefit is larger than the cost incurred at time t_1, the manager pursues the KT and $\bar{y}^*(t_1) = 1$. Conversely, if the cost of transferring prototyping knowledge is large, KT is less likely to occur. Similar interpretations hold for the optimal decision regarding the transfer of knowledge from the pilot line to the ramp-up stage at time t_2. In the remainder of the paper, we assume $\bar{y}^*(t_1) = 1$ and $\bar{p}^*(t_2) = 1$ so that we focus on non-trivial solutions.

<u>THEOREM 3:</u> *$\bar{y}^*(t_1)$ and $\bar{p}^*(t_2)$ satisfy: (i) $\bar{y}^*(t_1) = 1$ if $(\lambda_4(t_1^+) - c_4)\big(Y(t_1) - \bar{Y}(t_1)\big) - K_1 \geq 0$ and $\bar{y}^*(t_1) = 0$ otherwise; (ii) $\bar{p}^*(t_2) = 1$ if $(\lambda_5(t_2^+) - c_5)\big(P(t_2) - \bar{P}(t_2)\big) - K_2 \geq 0$ and $\bar{p}^*(t_2) = 0$ otherwise.*

3.5.3.2 The Continuous Rates of Development Activities

The optimal solutions for the rates of prototyping, pilot line testing and production ramp-up are defined mathematically in Theorem 4 and illustrated in Figures 3.4. We find the optimal rate of prototyping follows a two-step function whose value changes at the time of KT, t_1. Initially, the rate of prototyping activities is driven by the (constant) marginal value of KT at t_1 and the (constant) marginal value of prototyping knowledge at T. With the constant marginal values and the convex cost, the manager optimally spreads out prototyping activities evenly over time so that from time 0 to t_1 the rate of prototyping is constant. After t_1, the rate of prototyping is only driven by the (constant) terminal value of the level of prototyping knowledge and the desire to smooth the convex cost so that, again, the rate of prototyping is constant after t_1. However, prior to t_1, prototyping is more desirable since it also reflects the value of increasing the effectiveness of pilot line testing activities. As a result, before the KT at t_1, the optimal rate of prototyping is larger (i.e., the steps go down). Therefore, the optimal rate of prototyping activities follows a discrete approximation of the *front-loading* strategy given in the Base Model.

At the other extreme, we also find that the optimal rate of production ramp-up follows a two-step function. Driven by the (constant) marginal value of the level of ramp-up knowledge at T, the (constant) pilot line knowledge transferred at t_2, and the convex cost, the optimal rate of ramp-up activities is constant after the KT at t_2. Similarly, driven by the terminal marginal value of the level of ramp-up knowledge and the convex cost, the optimal rate of ramp-up activities is constant before the KT. Moreover, following the transfer of pilot line knowledge, ramp-up activities are more effective at increasing the level of ramp-up knowledge. Thus, the rate of production ramp-up is larger after the transfer of pilot line knowledge at t_2 (i.e., the steps go up). As such, the optimal rate of ramp-up activities follows a discrete approximation of the *extreme delay* strategy given in the Base Model.

In contrast, under reasonable conditions the optimal rate of pilot line testing follows a three-step function whose value changes at the KT times, t_1 and t_2. Intuitively, given constant marginal values and the convex cost, the optimal rate of pilot line testing is constant from 0 to t_1, t_1 to t_2, and t_2 to T. After t_2, the optimal rate of pilot line testing activities is only driven by the terminal value of the level of pilot line knowledge and the convex cost. In contrast, from t_1 to t_2, the optimal rate of pilot line testing activities is higher because it is also driven by the marginal value of the transfer of pilot line knowledge to the ramp-up stage at t_2. As such, the step from t_1 to t_2 is higher than the step from t_2 to T. Similarly, the step from t_1 to t_2 is higher than the step from 0 to t_1 since, following the transfer of prototyping knowledge from stage 1 at t_1, the effectiveness (marginal value) of pilot line testing is larger. Therefore, the optimal rate of pilot line testing activities follows a discrete approximation of the *moderate delay* strategy given in the Base Model. Lastly, note that under extreme conditions, we obtain a two-step solution for the optimal rate of pilot line testing. If Y(0) is extremely large, the KT from the prototyping stage to the pilot line stage does not occur (t_1 does not exist) so that the first step is from 0 to t_2. This result is consistent with the continuous model. On the other hand, if r_2 is extremely large, the optimal rate of pilot line testing still follows a moderate delay strategy; this result is in contrast to the continuous model where the extreme delay case occurs. Thus, even with extremely large r_2, the step down in the pilot line activities optimally occurs since additional activities have less value after the KT to the ramp-up stage.

The above results are summarized in Theorem 4, below. The insights are important since they show that, for a three stage model with two KTs, the continuous model is a reasonable approximation of the discrete model. This observation is important in the sense that managers can apply the front loading, moderate delay, extreme delay strategies obtained in the continuous KT model to the discrete KT problem, which is much more difficult to solve.

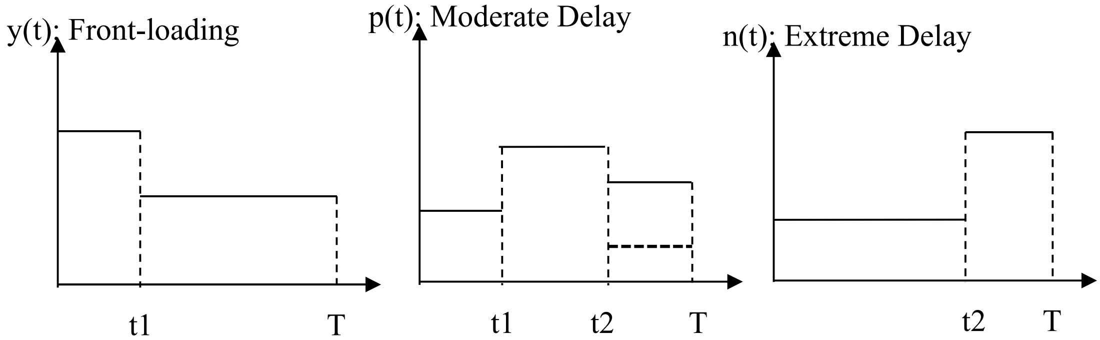

Figure 3.4: Discrete Forward KT Model: The Optimal Rates of NPD Activities

THEOREM 4: *(i) The optimal rate of prototyping activities follows a two-step function:* $y^*(t) = \left(\frac{\lambda_1{}^*(t)\alpha_0}{\sigma_1 c_1}\right)^{\frac{1}{\sigma_1 - 1}}$ *for* $t \in [0,t_1]$; $y^*(t) = \left(\frac{r_1\alpha_0}{\sigma_1 c_1}\right)^{\frac{1}{\sigma_1 - 1}}$ *for* $t \in (t_1,T]$. *(ii) The optimal rate of pilot line testing follows a three-step function:* $p^*(t) = \left(\frac{\lambda_2{}^*(t)\beta_0 Y(0)^{\beta_1}}{\sigma_2 c_2}\right)^{\frac{1}{\sigma_2 - 1}}$ *for* $t \in [0,t_1]$; $p^*(t) = \left(\frac{\lambda_2{}^*(t)\beta_0 Y(t_1)^{\beta_1}}{\sigma_2 c_2}\right)^{\frac{1}{\sigma_2 - 1}}$ *for* $t \in (t_1,t_2]$; $p^*(t) = \left(\frac{r_2\beta_0 Y(t_1)^{\beta_1}}{\sigma_2 c_2}\right)^{\frac{1}{\sigma_2 - 1}}$ *for* $t \in (t_2,T]$. *(iii) The optimal rate of production ramp-up activities follows a two-step function:* $n^*(t) = \left(\frac{r_3\gamma_0 P(0)^{\gamma_1}}{\sigma_3 c_3}\right)^{\frac{1}{\sigma_3 - 1}}$ *for* $t \in [0,t_2]$; $n^*(t) = \left(\frac{r_3\gamma_0 P(t_2)^{\gamma_1}}{\sigma_3 c_3}\right)^{\frac{1}{\sigma_3 - 1}}$ *for* $t \in (t_2,T]$.

3.5.3.3 The Optimal Times and Frequency of KT: Numerical Insights

We begin this section by continuing our analysis of a three-stage NPD project where one transfer of knowledge occurs between successive stages at times t_1 and t_2, with $t_1<t_2$. The times to pursue KT satisfy optimality conditions given in the Appendix. Analytically, the conditions are sufficiently complex to preclude interpretation. Therefore, we conduct extensive numerical experimentation to understand how certain parameters impact the optimal times for KT and, subsequently, the optimal rates of development activities during the NPD project. In Observation 4, we describe how the effectiveness of prototyping activities on increasing the level of prototyping knowledge (α_0) impacts the optimal solution. The interpretation follows.

OBSERVATION 4: *Suppose* α_0 *is larger.*

a) *The optimal rate of prototyping activities is larger for* $t \in [0,T]$.
b) *If the returns to KT from the prototyping to the pilot line stage is small (*β_1 *small), then* t_1 *occurs earlier, and the optimal rate of pilot line testing is larger for* $t \in [0,T]$.

However, if the returns to KT from the prototyping to the pilot line stage is large (β_1 large), then t_1 is delayed, and the optimal rate of pilot line testing is smaller for $t \in [0,t_1]$ and larger for $t \in (t_1,T]$.

c) *If the returns to both KTs are small (β_1 and γ_1 small), then t_2 occurs earlier, and the rate of production ramp-up activities is the same for $t \in [0,t_2]$ but smaller for $t \in (t_2,T]$. Otherwise, if one of the returns to KT is large (either β_1 or γ_1 large), then t_2 is delayed and the rate of production ramp-up activities is the same for $t \in [0,t_2]$ and larger for $t \in (t_2,T]$.*

It is particularly interesting to note that, while Observation (4a) is consistent with the Base Model, the insight in Observation (4b) and (4c) are unique to the discrete model. Recall that in the Base Model, if the effectiveness of prototyping (α_0) is large, the rates of prototyping, pilot line testing, and production ramp-up are large regardless of the returns to KT. In contrast, in the discrete model, the impact of the α_0 depends on the returns to KT: if α_0 is larger, the manager controls the timing of KTs as well as the rates of prototyping, pilot line testing and ramp-up production in relation to the returns to KT, as described below.

We find that if the returns to KT from the prototyping to the pilot line stage is small (small β_1), the manager is forced to increase the level of pilot line knowledge by undertaking a high rate of pilot line testing throughout the development project. In contrast, if the returns to KT is large (large β_1), the manager undertakes a low rate of pilot line testing before the transfer and a high rate after. Basically, the manager focuses more (less) efforts on pilot line testing after (before) the highly effective KT of prototyping knowledge occurs.

To determine the times for KT, the manager must balance the desire for transferring a large amount of knowledge (at a higher cost) later to drive a high level of returns at the recipient stage, versus transferring a small amount (at a lower cost) earlier so that the recipient has more time benefit from the KT. From Observation (4c), when the returns to both KTs are small (β_1 and γ_1 small), the optimal times for KT occur earlier. In essence, with small returns, KT has limited impact on improving the effectiveness of the recipient's development activities. Therefore, the manager optimally transfers smaller amounts of knowledge earlier so that the recipient has more time to derive benefits. With the smaller amount of KT from the pilot line stage and given the limited returns to KT, the rate of production ramp-up activities is smaller after the KT. However, the rate of production ramp-up prior to the KT does not change since it is not affected by the effectiveness of prototyping. In contrast, when at least one of the returns is large, KT from the pilot line to the ramp-up stage is delayed, and the rate of ramp-up activities is the same prior to the KT but larger after. As such, the manager leverages the high returns by transferring more knowledge later, and then exploits the KT by pursuing a higher rate of ramp-up activities.

In the remainder of this section, instead of focusing our numerical analysis on the times of KT, we focus on understanding drivers of the number of transfers that optimally occur. We now consider a two-stage NPD process where a series of KTs may occur from the prototyping to the pilot line testing stage. Therefore, we determine: the rate of prototyping activities, the rate of pilot line testing, and the sequence of times that KTs optimally occur from prototyping to the pilot line stage. The optimality conditions for the optimal times of KT are analogous to those for the three-stage two-transfer problem and are available upon request.

OBSERVATION 5: *(a) For a sufficiently small increase (decrease) in the returns to KT (β_1), the optimal number of KTs remains the same, but the times are optimally delayed (occur earlier)*[3]*. (b) For a sufficiently large increase (decrease) in β_1, the number of KTs increases (decreases).*

In Observation (5a), due to the delayed sequence of times, the manager transfers a larger amount of knowledge from the prototyping to the pilot line stage at each KT. The larger amounts of KT lead to a significant increase in the ability of pilot line testing activities to increase the level of pilot line knowledge. Thus, the benefits from utilizing a larger amount of knowledge in a shorter period of time dominate the benefits from utilizing a smaller amount of knowledge for a longer period of time. Observation (5b) is obtained since, if the returns to KT are sufficiently large, the benefits derived from more frequent KTs are large, as well. With a sufficient increase in the returns, the benefits from more KTs outweigh the associated costs and the manager optimally pursues a larger number of KTs.

In Observation 6, we explore the effect of the product launch time on the optimal solution. As reported in (i) (a) and (i) (b) and consistent with the Base Model, we find that if T is delayed, the optimal rates of all development activities increase. However, unique to the discrete model, we find that, when the product launch is delayed, the

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[3] We observe the same result in the 3-stage model: with a sufficiently small increase in β_1, KTs from the prototyping to the pilot line stage and from the pilot line to the ramp-up stage are optimally delayed.

optimal times of KT are delayed, as well, (i) (c). Despite the delayed KT, the pilot line team has sufficient time to leverage the larger amount of KT from the prototyping stage due to the later product launch. Lastly, consider the result in (ii). Suppose for a particular product launch time, the manger optimally pursues N KTs. If the product launch time is sufficiently delayed, the number of KTs increases to N+1, the first KT occurs earlier, and the last KT occurs later relative to the case when T is not delayed. Ultimately, the frequency of KT is driven by the need to balance the trade-off between the benefits (larger levels of knowledge and net revenue) and the associated costs.

OBSERVATION 6: *(i) With a sufficiently small increase (decrease) in the product launch time T, (a) the optimal rates of all development activities are larger (smaller) throughout the NPD project; (b) the optimal levels of knowledge at all stages are larger (smaller) throughout the NPD project; (c) the optimal times of KT are delayed (earlier). (ii) For a sufficiently large increase (decrease) in the product launch time T, the number of KTs increases (decreases).*

To complete our comparison between the Base Model and the Discrete Model, we present the following additional numerical insights. First, for a sufficiently small increase in α_0, the optimal number of KTs remains the same; and if the returns to KT are smaller (larger) (β_1), they occur earlier (later). This insight is analogous to Observation (2c). Moreover, for a sufficiently large increase (decrease) in α_0, the optimal number of KTs increases (decreases). This insight is analogous to Observation (3b). Lastly, if the effectiveness of pilot line testing (β_0) is larger, the marginal net revenue associated with the level of prototyping or pilot line knowledge (r_1 or r_2) is larger, or the costs of development activities (c_1 or c_2) or KT (c_4 or K) is smaller, the optimal number of KTs is larger. This final result, while interesting, is intuitive.

3.6 Conclusions and Future Research

In this paper, we examine KD strategies in an NPD project with three stages of activities conducted concurrently: prototyping, pilot line testing, and production ramp-up. The manager determines the optimal rates that development activities in each stage are pursued over the NPD project which drive the levels of prototyping knowledge, pilot line knowledge, and ramp-up knowledge, respectively. An important feature of our research is that we capture the link between successive stages of development activities. Specifically, we recognize that by transferring prototyping knowledge to the pilot line stage, the manager enhances the ability of pilot-line testing activities to increase the level of pilot line knowledge. Similarly, transferring pilot line knowledge enhances the ability of the production ramp-up activities to increase the level of ramp-up knowledge. Ultimately, the manager seeks to maximize the net revenue earned at the product launch time (which is a function of the levels of knowledge at that time) less the costs incurred for development activities.

A Base Model and two extensions are introduced that differ in the manner in which KT occurs. In the Base Model, KD activities are pursued by a relatively small number of persons who are co-located and who continuously transfer knowledge in the forward direction between successive stages. In the first extension of the Base Model, we allow the continuous feedback of knowledge from the ramp-up to the prototyping stage. In the second extension of the Base Model, we consider three large development teams who may reside in different locations. Since the continuous transfer of knowledge is not practical, knowledge is batched and transferred from one stage to the next at discrete times. In the remainder of this section, we describe the key results of each model.

3.6.1 Results of the Base Model

In the Base Model, we show analytically that the optimal rate of prototyping activities continuously decreases over time (follows a front-loading strategy); the optimal rate of pilot line testing continuously increases then decreases over time (follows a moderate delay strategy); and the optimal rate of production ramp-up activities continuously increases over time (follows an extreme delay strategy). Therefore, we provide a comprehensive perspective of how the manager should undertake knowledge management throughout the NPD project. Beyond results on the evolution of knowledge, we obtain insights based on analytic sensitivity analysis, as described below.

First, we show that an increase in the effectiveness of any development activity or the returns to forward KT leads to larger rates of all development activities throughout the NPD project. This result is particularly important since the manager has considerable control over the effectiveness of development activities as well as the returns to KT. The effectiveness of development activities is impacted by the manager's initial selection of team members (skills) as well as the nature of the technical support (for example, CAD technology) provided. The returns to KT embody more complex relationships. Higher returns are associated with a greater ability of the source team to articulate and document knowledge; as well as greater ability of the recipient team to understand and apply that knowledge. To some extent, higher returns reflect the capabilities of the technical support system that facilitates the KT as well as the ability of team members to utilize the system. Also, the returns to KT are impacted by the stickiness of the knowledge (knowledge is sticky if it is more tacit and more difficult to codify and transfer). Lastly, the ability of the source and recipient to drive the returns to KT also depends on the manager's ability to motivate the source team to document knowledge, and the manager's ability to motivate the recipient team to be receptive to utilize the knowledge.

Second, we find that the initial level of knowledge at each stage in the NPD project impacts the optimal solution. The manager has considerable control over the initial level of knowledge of the development teams since the manager selects members of the teams based on their past experience and ability as indicated in performance reviews. In a particularly interesting result, we show that the impact on the optimal solution due to the initial level of pilot line knowledge depends on the relationship between the returns from KT to the ramp-up stage and the extent of diseconomies of scale in the cost of ramp-up activities. If diseconomies of scale dominate the relationship, then corresponding to a larger initial level of pilot line knowledge, the incentive to develop additional pilot line knowledge is smaller and the manager reduces the rate of pilot line testing activities throughout the NPD project. Furthermore, given the reduced incentive to develop pilot line knowledge, the manager pursues less prototyping activities throughout the NPD project. Therefore, we observe a substitution effect: with a higher initial level of prototyping knowledge, the manager undertakes less prototyping and pilot line activities. Nevertheless, due to the larger initial level of pilot line knowledge and despite the smaller rates of prototyping and pilot line activities, we show that the level of pilot line knowledge is larger throughout the NPD project. As a consequence, ramp-up activities are more effective and are pursued at a larger rate throughout the NPD project. In contrast, suppose the returns to KT from the pilot line to the ramp-up stage dominate the effect of diseconomies of scale in the cost of ramp-up activities. We show that with a larger initial level of knowledge at the pilot line stage, the manager pursues more prototyping, pilot line testing and ramp-up activities throughout the NPD project. Terwiesch et al. (2002) also examine how preliminary information impacts KT strategies. However, they focus their analysis on features of the preliminary information: precision and stability.

Lastly, numerically we explore how the product launch time impacts the manager's knowledge management strategy. If the product launch time is delayed, holding other parameters constant, we find the manager optimally pursues more development activities in all stages over all time. This result reflects the longer period of time for which KT benefits are realized. It follows that, if the product launch time occurs earlier, then the manager optimally pursues less development activities in all stages over all time, unless the net revenue earned in relation to the levels of knowledge is larger.

3.6.2 Continuous KT with Feedback

In the Feedback Model, we consider not only the continuous forward KT between stages, but also the continuous backward KT from the pilot line to the prototyping stage, from the ramp-up to the prototyping stage, and from the ramp-up to the pilot line stage. A comparison of the Base Model and the Feedback Model shows that the optimal pursuits of prototyping, pilot line testing and ramp-up activities are significantly influenced by the returns to feedback. As expected, when the returns to feedback from the pilot line to the prototyping stage are sufficiently small, the optimal strategies for three development stages are the same as in the Base Model. However, when the returns to feedback from the pilot line to the prototyping stage are sufficiently large, the optimal strategies for KD differ dramatically from the Base Model. In particular, if the returns to feedback are sufficiently large, the optimal rate of prototyping follows the extreme delay strategy; the optimal rate of pilot line testing follows a front-loading strategy; and the optimal rate of production ramp-up activities follows an extreme strategy (as in the base model). The maximum rate of pilot line activities is pursued at the start of the planning horizon in order to quickly accumulate pilot line knowledge to transfer to the prototyping stage. Similarly, the maximum rate of prototyping activities is delayed until the end of the NPD project in order to wait and thereby benefit from the accumulation of more feedback from the pilot line stage. Analogous insights are found when feedback occurs from the ramp-up stage to either the prototyping or pilot line stage.

In another key result, we find that, corresponding to a larger rate of returns to feedback, the manager pursues more prototyping, pilot line testing, and ramp-up activities over all time. This latter result reflects the synergy among the three stages of the NPD project as driven by both forward and feedback KT.

3.6.3 Discrete KT

In the model where forward KT occurs at discrete times, we analytically show that the optimal rates of development activities satisfy step functions that mimic the continuous time solutions obtained for the Base Model. In particular, the optimal rate of prototyping activities steps down when knowledge is transferred to the pilot line stage; the optimal rate of pilot line testing steps up when KT is received from the prototyping stage, and later steps down when knowledge is transferred to the ramp-up stage; the optimal rate of ramp-up activities steps up when KT is received from the pilot line stage. Despite the similarities in the structures of the optimal solutions, based on analytic and numerical results, we find important differences between the continuous and discrete forward KT models, as described below.

For the Discrete Model, we show analytically that if the effectiveness of prototyping is larger, the optimal rate of prototyping activities is larger throughout the NPD project. This insight is consistent with the Base Model. However, we also show that if the effectiveness of prototyping activities is larger, then the impact on the optimal times of KT and on the optimal rates of pilot line and ramp-up activities directly depends on the corresponding returns to KT. This result is entirely different from the Base Model. Essentially, when the returns to KT are relatively small, KT has limited impact on improving the effectiveness of the recipient's development activities. As a result, the manager optimally transfers smaller amounts of knowledge earlier so that the recipient has more time to derive benefits. In contrast, when the returns to KT are relatively large, the manager optimally transfers larger amounts of knowledge later since the recipient is able to understand and absorb the knowledge.

Interesting insights are also obtained for the Discrete Model based on extensive numerical experimentation. We show that as the returns to forward KT increases, if the number of transfers remains the same, the manager optimally delays the times of KT and the magnitude of each transfer is larger. This insight is particularly interesting if interpreted from the perspective of the stickiness of knowledge. When the knowledge to be transferred is tacit, the nature of the documentation process undertaken by the source is unclear. Similarly, it is difficult for the recipient to interpret, absorb and deploy the KT to enhance the effectiveness of its development activities. In other words, the returns to transferring tacit knowledge may be smaller. As a consequence, the NPD manager pursues KTs earlier, and the magnitude of each KT is smaller. This timing strategy provides the recipient with sufficient time to digest and deploy the KT. In contrast, when knowledge is easily codified, the returns to KT may be higher since it is easier for the source to document the knowledge and easier for the recipient to absorb and deploy the knowledge. With higher returns, the times of the KTs are delayed and the magnitude of each KT is larger. Therefore, the manager is able to batch more knowledge in each KT since the source and recipient are better able to document and apply the knowledge.

Lastly, numerically we explore how the product launch time impacts decisions in the Discrete Model. As in the Base Model, for a sufficiently small delay in the product launch time, the manager optimally pursues more development activities in all stages over all time. In addition, we observe that the manager delays all KTs. Nevertheless, the delayed product launch times allow the recipient stages sufficient time to leverage the larger KTs. Furthermore, if the product launch time is sufficiently delayed, the manager optimally pursues a larger number of KTs to balance the trade-off between the benefits (larger levels of knowledge and net revenue) and the associated costs.

3.6.4 Future Research

In this paper, we assume the effectiveness of development activities is constant over time. However, the effectiveness of development activities may increase over time as technical and market uncertainty is resolved. In particular, as time passes development activities may be more effective because there are fewer errors to be identified and corrected. Future research can examine the situation where the effectiveness of KD activities increases over time. Also in this paper, we assume a predetermined product launch time. While this assumption is entirely appropriate in some environments where external forces determine the launch time, in other domains managers have the autonomy to determine the product launch time. Under time-based competition, an early product launch may drive higher product sales, whereas a later product launch may lead to a loss in market share and long-term sales. In future research, the product launch time can be determined optimally to maximize net revenue less development costs.

CHAPTER 4

THE DEVELOPMENT OF A NEW PRODUCT VERSUS THE IMPROVEMENT OF AN EXISTING PRODUCT AND THE VALUE OF KNOWLEDGE TRANSFER

4.1 Introduction

While the product improvement of an existing product is necessary to sustain a firm in the short-term, new product development (NPD) leads a firm to long-term success (Wheelwright and Clark 1992, Chao et al. 2009). The link between product improvement of an existing product and NPD projects are twofold. First, firms often rely on the revenue derived from the improvement of existing products for the cash flow necessary fund NPD projects. Therefore, since firms usually have limited resources to support innovation projects, competition between product improvement projects and NPD projects is commonplace (Loch and Kavadias 2002, Taylor 2010). Second, knowledge from the NPD project may be transferred to enhance the improvement of the existing product. The benefit of the knowledge transfer (KT) is clear: it serves as a buffer against the considerable uncertainty regarding the successful release of the new product to the marketplace. In other words, the knowledge developed from a risky NPD project may ultimately benefit the firm only in terms of the KT that enhances the existing product improvement project. On the downside, however, when knowledge is transferred from the new development project to an existing product improvement project, the proprietary benefits that would be obtained by the release of the new product are diminished.

Interestingly, to increase the chance for survival of the new product, some authors recommend the separation of the development of new products and the improvement of existing products (Tushman and O'Reilly 1997, Christensen 1997, Hlavacek and Thompson 1973). When a NPD project competes for resources and executive attention with an existing product improvement project, the likelihood of success of the new product is significantly reduced by the internal competition. Therefore, this internal competition is often considered as a hindrance to the adaptation and development of the firm.

In contrast, based on a field study of several software development firms, Taylor (2010) finds that internal competition can serve as one mechanism for knowledge diffusion which improves the overall firm performance in the marketplace. Taylor describes how, in one firm, the development team of an existing software product (E-Static) integrated knowledge transferred from the development team of a new software product (N-Monitor) to improve the features and performance of the existing product. In fact, although the NPD project for N-Monitor was eventually terminated, the knowledge accumulated in the development process benefited the firm by enhancing the development efforts of the E-Static team. In conclusion, the internal competition provides the opportunity for KT from a new to an existing development project which might improve the overall firm performance.

In this paper, we introduce a dynamic model to analyze a manager's pursuit of knowledge development (KD) for an existing product improvement project and KD for an NPD project during a pre-determined development cycle, referred to as a planning horizon. Progress in both projects is inferred by the growing levels of knowledge accumulated by the manager's pursuit of development activities (Santiago and Vakili 2005, Chao et al. 2009, Gaimon et al. 2011, Xiao et al. 2012). The rates of KD for the existing product improvement project as well as the NPD project are continuously determined throughout the planning horizon. Development activities include prototyping, simulation, and pilot line testing. In addition, the existing product improvement project may increase its level of knowledge through the transfer of knowledge from the NPD project. The rate of KT from the NPD project to the existing product improvement project is determined throughout the planning horizon. While the existing product improvement project generates revenue continuously and with certainty throughout the development project, uncertainty exists regarding the ability of the NPD project to generate revenue when it is successfully released to the marketplace. By investing in the accumulation of knowledge, the NPD team increases the probability of success in the marketplace. Therefore, we consider the expected revenue realized by the NPD project over the planning horizon (Chao et al. 2009, Ozkan et al. 2012). Moreover, the expected revenue earned by the NPD project is reduced as a result of knowledge transferred to the existing product development project since some of the benefits from the deployment of proprietary knowledge are lost. Naturally, costs are incurred as development and KT activities are undertaken over time. Ultimately, the manager seeks to maximize expected profit obtained from the product improvement project and the NPD project.

Our results indicate that the manager optimally pursue a front-loading strategy (the rate of development activities continuously decreases over time) for the KD activities in both the existing product improvement project and the NPD project. The optimal rate of KT from the NPD project to the existing product improvement project may follow a front-loading or an extreme delay strategy (the rate of KD increases continuously over time) or a moderate delay strategy (the rate of KD first decreases and then increases over time) depending on which of the following dominates: the benefits to the existing product improvement project from KT and the penalty and costs for KT.

This paper is organized as follows: In Section 4.2, a review of the literature is provided. Section 4.3 introduces the dynamic model. Section 4.4 contains the optimal solutions and interpretations. The concluding remarks and directions for future research are given in Section 4.5.

4.2 Literature Review

KT is a form of induced learning which requires managerial action for the learning activities to occur (Terwiesch and Bohn 2001, Carrillo and Gaimon 2000, 2004). Argote and Ingram (2000, p. 151) define KT as "the process through which one unit (e.g., group, department, or division) is affected by the experience of another". It requires one party to pass knowledge to the other party through discussion, seminars, presentations or documentation (Ha and Porteus 1995, Argote 1999, Ozkan et al. 2012, Xiao et al. 2012).

The NPD literature has studied the KT that occurs between stages within a single development project. Moreover, the KT from one product development stage (or team) to another may occur sequentially, simultaneously, or be overlapped in time. KT can occur in the forward direction only from an upstream stage to a downstream stage or from a product design team to a process design team (Ha and Porteus 1995, Krishnan et al. 1997, Loch and Terwiesch 1998, Xiao et al. 2012) or in both forward and backward directions

(Ozkan et al. 2012, Xiao et al. 2012). The overlapping of development activities in an NPD project has been well studied (Ha and Porteus 1995, Krishnan et al. 1997, Loch and Terwiesch 1998, Ozkan et al. 2012, Xiao et al. 2012).

Ha and Porteus (1995) investigate a model where product and process design are carried out in parallel to improve quality and to enhance manufacturability. During project development, progress reviews (a form of KT) are conducted to exchange information and discover design flaws. The flaws discovered reduce development time and the resources required later for redesign. Progress reviews also serve as a quality control process to evaluate the manufacturability of the product design. Unfortunately, progress reviews are costly. Thus, the manager must determine when to conduct progress reviews to minimize the total expected project completion time and the cost of progress review.

Krishnan et al. (1997) provide a model-based framework to manage the concurrency of product development activities which involves the KT from an upstream stage to a downstream stage. They introduce the view of the evolution of upstream information and the downstream sensitivity to the evolution. If the upstream activities are finalized early, the flexibility to make future changes is lost, and design quality suffers. On the other hand, if upstream information is used by downstream while upstream activities are still evolving, then the downstream activities are forced to iterate thereby delaying product launch. The authors formulate a model whereby upstream and downstream activities are overlapped to minimize the product launch time. Results indicate when preliminary information should be used by the downstream activity, and which parts of upstream information should be utilized early in downstream activities.

Loch and Terwiesch (1998) develop a model of concurrent engineering that allows the overlapping of upstream and downstream activities and KT from the upstream to the downstream activities. They incorporate the uncertainty in the average rate of engineering change and the dependence of upstream modification and downstream task. They present a dynamic decision rule to determine the optimal time for communication, and provide the optimal level of concurrency between activities. They observe that uncertainty and dependence decrease the optimal level of overlapping, and make concurrent engineering less attractive. Terwiesch et al. (2002) analyze the exchange of preliminary information in a concurrent engineering environment. Based on a field study, they develop a time-dependent model which coordinates parallel development activities. They define two sets of coordination strategies: iterative and set-based, and discuss the trade-offs in choosing each strategy and how they change over the development project.

Ozkan et al. (2012) consider NPD in the context of knowledge management. They examine the simultaneous and bi-directional KT between product and process design teams. They identify two possible KD and KT strategies for the product and process design teams: front-loading strategy (the rate of development effort decreases over time) and moderate delay strategy (the rate of development effort increases and then decreases over time). In addition, they examine how errors in KD and KT impacts the optimal strategies used. Xiao et al. (2012) examine forward and backward KT between three stages of an NPD project. They observe that feedback can significantly change the optimal KD strategy to be pursued. In addition to the front-loading and moderate delay strategies, we also introduce the extreme delay strategy (the rate of development effort decreases over time) and the moderate front-loading strategy (the rate of development effort decreases and then increases over time).

Note that the above literature focuses on the impact of KT either from one stage/team to another stage/team within one NPD project. In contrast we examine the KT between two NPD projects within one firm and assess how that KT impacts the overall firm performance. Based on a field study, Taylor (2010) examines the internal competition between a new technology development project and an existing technology improvement project. He finds that, as a result of internal competition, KT occurs from the new technology project to the existing-technology project. As a result, elements of the new technology are integrated into the development of the existing-technology product. Our approach, therefore, is consistent with Taylor's field study. We optimally determine the rate of KD for the NPD project and the rate of improvement of the existing project, as well as the rate of KT from the new to the existing project, throughout the planning horizon.

The aggregate project management problem has been addressed in the literature whereby a manager determines the development strategy for multiple NPD projects, typically subject to limited resources (Bower 1986, Rousell et al. 1991, Wheelwright and Clark 1992, Cooper et al. 1998, Kavadias and Loch 2003, Kavadias and Chao 2007, Chao et al. 2009). In the corporate environment, effective investment of resources directly impacts a firm's competitive advantage (Lock and Kavadias 2002). Nobeoka and Cusumano (1997) describe the importance of managing multiple product lines on firm performance. Cooper et al. (1998) introduce methods used to allocate resources to multiple product development projects under a budget constraint and introduce the notion of strategic buckets for the spending splits.

Normative research exists modeling the investment of resources under a budget constraint in the context of NPD. Using a linear programing (LP), Beaujon et al. (2001) examine the resource investment problem for multiple R&D projects where partial funding of projects is allowed. The partial funding assumption differs from the traditional assumption which characterizes funding as a zero or one decision. Kavadias et al. (2005) also consider project funding as a continuous decision. They assume project funding can be adjusted up or down with upper and lower bounds. A heuristic method is provided in which the marginal benefits determine the optimal resource investment. Loch and Kavadias (2002) introduce a dynamic programming model of resource investment under a budget constraint. They assume that the project investments within the product line may build up gradually over time. They show that it is optimal for the manager to invest the next dollar to the project with the highest marginal benefit. Recently, using a dynamic model, Chao et al. (2009) study how the funding authority and incentives impact the competition for resources between an existing product improvement project and a NPD project. They show that the manager invests more resources on the existing product improvement project under variable funding when the manager has the authority to use revenue derived from existing product sales to fund NPD effort.

The focus of the above-mentioned literature is the problem of resource investment in NPD projects under limited resources (i.e., a budget constraint). In contrast, we do not consider a budget constraint for the investment in development resources. Instead, we maximize the expected profit which is the difference between expected revenue less KD and KT costs. In addition, in contrast to the traditional resource allocation in multiple NPD projects, we explicitly recognize the potential value (increase in revenue) and the penalty and cost implications of KT from a new NPD project to an existing product improvement project. We identify the rates that KD activities should be pursued in both projects as well as the rate of KT from the NPD project to the existing product improvement project over time.

4.3 The Model

In this section, we consider product development in the context of knowledge management. We study a business division within a firm that produces and sells an existing product and develops a new product over a finite development cycle, $t \in [0, T]$. The terminal time of the development cycle (T), referred to as the planning horizon, is exogenous. We examine how KD and KT impact the level of knowledge embedded in the existing product improvement project as well as how KD impacts the level of knowledge embedded in the NPD project. A senior manager oversees the development of both projects and determines the rates of KD and the rate of KT. We describe how the level of knowledge of the existing development project enhances the ability to continuously generate revenue over time; and how the level of knowledge for the NPD project enhances the ability of the firm to derive expected revenue when the new product is successfully released to the marketplace. The manager's objective is to maximize expected profit which consists of revenue less the costs for KD and KT.

4.3.1 The Levels of Knowledge and Knowledge Transfer

Throughout the planning horizon, the manager invests in developing a new product based on a new technology that is fundamentally different from the underlying technology used in the existing product. Let $y_n(t) > 0$ denote the rate of KD pursed in the NPD project at time t (control variable), which can be measured in terms of hours of workforce effort. Let $K_n(t)$ (state variable) denote the level of knowledge for the NPD project at time t, with $K_n(0) > 0$ given. The level of new product knowledge at time t consists of the initial knowledge level ($K_n(0)$) and the additional knowledge associated with the KD activities pursued through that time. The initial level of knowledge for the NPD project reflects the skill of the team and is based on past experience, years of education and training, and peer reviews. The increase in the level of new product knowledge at time t is driven by the rate of KD at that time as well as the skill of the team and the quality of the technical support available. The skill level of the team and the quality of the technical support are inferred by the parameter $\alpha_n > 0$. This gives us Equation (1).

$$\dot{K}_n(t) = \alpha_n y_n(t) \tag{1}$$

In addition to developing the new product, the manager invests in improving the existing product to enhance its revenue generating potential. Improvements of an existing product include new product features, product feature upgrades, and process improvements that reduce manufacturing and distribution costs. Let $y_e(t) \geq 0$ denote the rate of product improvement activities (KD) for the existing product development project at time t (control variable), which can be measured in terms of hours of workforce effort. As additional KD activities are pursued, such as prototyping, simulation, and pilot line testing, the level of knowledge of the existing product improvement project accumulates. Let $K_e(t)$ denote the level of knowledge associated with the existing product at time t for t$\in [0, T]$ with $K_e(0) > 0$ given (state variable). The level of existing product knowledge at time t is comprised of the initial level of knowledge ($K_e(0)$), the benefits associated with the rate of KD activities undertaken through time t, and the learning benefits derived from KT from the NPD project to the existing product improvement project (Taylor 2010, Ozkan et al. 2012, Xiao et al. 2012). The initial level of existing product knowledge reflects the overall skill of the development team involved in the existing product improvement project and is based on the team's previous experience, the level of prior education and training, and peer reviews. The extent that KD at time t increases the level of existing product knowledge at that time is driven by the rate of development activities, the skill of the team, and the quality of the technical support available. The skill level of the team and the quality of technical support are indicated by the parameter $\alpha_e > 0$. This gives us the first term in Equation (3). The manner in which KT increases the level of knowledge of the existing product is described below.

The transfer of new product knowledge benefits the existing product improvement project by providing it with novel ideas on product or process features. If knowledge can be codified the transfer may be facilitated through documentation of actual product prototypes or designs. In contrast, if knowledge is more tacit, it may be transferred through the redeployment of human resources (Taylor 2010). The senior manager determines the portion of new product knowledge to be transferred during the planning horizon. Let $X(t)$ represent the cumulative amount of new product knowledge that has been transferred to the existing product improvement project by time t (state variable). Therefore, the level of new product knowledge that has *not* been transferred to the existing product improvement project by time t is given by $K_n(t) - X(t)$. In other words, $K_n(t) - X(t)$ indicates the availability of new product knowledge that may be transferred to the existing improvement project at time t. Let $\theta(t) \in [0,1]$ denote the portion of the available new product knowledge that is transferred to the existing product improvement project at time t (control variable). More specifically, the manager transfers the level of new product knowledge $\theta(t)(K_n(t) - X(t))$ to the existing product improvement project at time t, as shown in Equation (2). The parameter $\delta > 0$ denotes the marginal benefit of new product knowledge to the level of knowledge of the existing product improvement project as shown in the second term in Equation (3).

$$\dot{X} = \theta(t)(K_n(t) - X(t)) \quad (2)$$

$$\dot{K}_e(t) = \alpha_e y_e(t) + \delta\theta(t)(K_n(t) - X(t)) \quad (3)$$

4.3.2 Revenue

The business division generates revenue from the existing product and the new product. We define the revenue continuously generated from the existing product at time t as $R_e(K_e) = r_e(t)K_e(t)$. The ability of a product to deliver revenue at time t is a function of the cumulative knowledge embedded in the product at that time (Cohen et al. 1996, Kim 1998, Chao et al. 2009, Gaimon et al. 2011, Ozkan et al. 2012, Xiao et al. 2012). As such, we assume the level of existing product knowledge accumulated through time t characterizes the product specifications and features as well as the process efficiency, and thereby indicates the revenue generating ability of the existing product. In addition, we assume $r_e(t)$ is positive and decreases over time. Therefore, unless the manager invests in its improvement, the ability of the existing product to drive revenue declines over time due to competition or changes in the marketplace.

While the development of the new product is critical to the long-term survival of the firm, it usually involves high risk of product failure due to technical and market uncertainties. Nevertheless, the manager may influence the probability of successful development of the new product by investing in knowledge generating activities that reduce the market or technical uncertainties. We define $f\big(K_n(t)\big)$ as the probability that the NPD project is successfully developed at time t, with $f(K_n(t)) \in [0,1]$. Let $f\big(K_n(t)\big) = gQ(K_n(t))$ where $g \in [0,1]$ and $Q(K_n(t)) \in [0,1]$. The parameter g is a scaling factor. We assume that $\frac{dQ}{dK_n} > 0$ and $\frac{d^2Q}{dK_n{}^2} < 0$ so that $\frac{df}{dQ} > 0$ and $\frac{d^2f}{dQ^2} < 0$. That is, the probability that the new product is successfully developed at time t increases at a decreasing rate in relation to $K_n(t)$. In other words, as more knowledge is accumulated in the NPD project, the probability of success is higher.

If the new product is successfully developed at time t, it generates revenue given by $R_n(K_n, \theta) = r_n(t)K_n(t) - pX(t)$. In the first term, the level of knowledge embedded in the new product through time t impacts the respective revenue generating potential at that time. In addition, we assume that $r_n(t)$ is positive and decreasing over time reflecting time-based competition, (i.e., if the new product is successfully developed late (early), the revenue generated is lower (higher)). However, from the second term, there is a loss in potential revenue when new product knowledge is transferred to the existing product improvement project since valuable new product features are no longer proprietary to the new product project. In a sense, the existing product cannibalizes the potential sales of the new product. We assume p is positive and exogenous. By combining the probability of success for the new product, $f\big(K_n(t)\big)$ with the revenue function we have the expected revenue for the new product as $\int_0^T R_n(K_n, \theta) f\big(K_n(t)\big)dt$.

The final sources of revenue reflect the values of knowledge at the terminal time. The levels of knowledge for the existing and new products may have value for future

NPD projects after the planning horizon. Let $V_e(K_e) = \nu_e K_e(T)$ denote the future of the existing product knowledge beyond time T, with $\nu_e > 0$. Similarly, we define $V_n = \nu_n K_n(T)$ as the future value of the new product knowledge after T, with $\nu_n > 0$.

4.3.3. The Costs

During the development cycle, costs are incurred for efforts expended on improving the existing product and developing the new product (Clark and Fujimoto 1991). Let $C_e(y_e) = c_e y_e^2$ denote the cost incurred for KD activities undertaken at time t in the existing-technology project, with $c_e > 0$. The cost includes salaries for engineers who conduct KD activities and the cost of the technical support systems. We assume the cost is quadratic with respect to the rate of activities pursued at any instant of time. The cost increases at an increasing rate to reflect the coordination costs, overtime costs, or capacity constraints on specialized resources that occur when disproportionally large amounts of activities are pursued at any instant of time (Carrillo and Gaimon 2004, Chand et al. 1996, Terwiesch and Xu 2004, Chao et al. 2009). Similarly, we define $C_n(y_n) = c_n y_n^2$, with $c_n > 0$, as the costs for KD undertaken at time t in the new-technology project. In addition to the costs of development activities, we also consider the cost of KT. When the new product knowledge is transferred to the existing product improvement project, a cost is incurred for the time and effort by the NPD project to document its results and the time and effort by the existing product improvement project to absorb those results (Xiao et al. 2012). Also, KT disrupts ongoing development activities for both teams. We define the cost of KT as $C_k = c_k \theta^2 (K_n - X)$. Therefore, large amounts of KT are more disruptive and more costly.

4.3.4 Objective Function and Hamiltonian

The objective is to maximize the total expected profit obtained by the firm which is the difference between total revenue and the costs, as shown in Equation (4). The first term is the sum of the continuous (deterministic) stream of revenue earned by the existing product and the expected revenue earned by the new product. The second and third terms are the future values (beyond the current planning horizon) of the knowledge associated with existing and new products. The fourth term consists of the costs incurred for KD and KT during the planning horizon.

$$\max_{y_1(t),y_2(t),\theta(t)} \int_0^T (R_e + R_n f)dt + V_e + V_n - \int_0^T (C_e + C_n + C_k)dt \quad (4)$$

4.4 Optimal Solutions

In the remainder of the paper, the notation depicting time is suppressed whenever possible, "x^*" indicates an optimal solution of x, and all proofs appear in the Appendix. We solve the model using optimal control methods (Sethi and Thompson 2000). The Hamiltonian to be maximized is given in Equation (5) where we introduce the adjoint variables $\lambda_1(t)$, $\lambda_2(t)$, and $\lambda_3(t)$. Since the level of incremental knowledge at time t is sustained from t through the remainder of the planning horizon, $\lambda_1(t)$ is interpreted as the marginal value of an additional unit of knowledge for the existing development project from time t to the end of planning horizon, T. A similar interpretation holds for $\lambda_2(t)$. Lastly, $\lambda_3(t)$ is the marginal value of an additional unit of KT from the NPD project to the existing product improvement project at time t. The optimality conditions for the adjoint variables appear in Lemma 1.

$$H = R_e + R_n f - C_e - C_n - C_k + \lambda_1(\alpha_e y_e + \delta\theta K_n) + \lambda_2 \alpha_n y_n + \lambda_3 \theta(K_n - X) \quad (5)$$

Lemma 1. *The marginal values of existing product knowledge, new product knowledge, and the cumulative level of new product knowledge transferred to the existing product improvement project satisfy the following conditions:*

(i) $\dot{\lambda}_1(t) = -r_e < 0, \lambda_1(T) = v_e$*;*

(ii) $\dot{\lambda}_2(t) = -\theta(-c_k\theta + \delta\lambda_1 + \lambda_3) - g(Q(K_n)r_n + (-pX + K_n r_n)Q_{K_n}) < 0$, $\lambda_2(T) = v_n$*;*

(iii)$\dot{\lambda}_3(t) = pgQ(K_n) - \theta(c_k\theta - \delta\lambda_1 - \lambda_3) > 0, \lambda_3(T) = 0$*;*

In Lemma 1, we find that the marginal values of existing and new product knowledge are positive and decrease over time, and the marginal value of the cumulative amount of new product knowledge transferred is negative and increases over time. The marginal value of existing product knowledge at time t is driven by the marginal contribution to revenue from existing product knowledge over time. Since the existing product knowledge continuously generates revenue, an additional unit of existing product knowledge generated early creates revenue over the remainder of the development project. Thus, the marginal value of the level of existing product knowledge decreases over time. The marginal value of the new product knowledge at time t is driven by the sum of the marginal contribution to expected revenue from new product knowledge over time and the marginal benefit to the existing product improvement project from the transfer of new product knowledge at time t. Note that an additional unit of new product knowledge transferred early increases the level of existing product knowledge over the remainder of the planning horizon. Therefore, the marginal value of new product knowledge decreases over time as less time remains in the planning horizon to derive benefits. In contrast, the marginal value of the cumulative level of new product knowledge transferred at time t is negative since it is driven by the cost for KT and the penalty for losing the proprietary knowledge. As time passes and less time remains to incur the cost and penalty, the negative marginal value increases, reaching the value of zero at T.

4.4.1 Optimal Rates of NPD Activities

The optimal rates of KD in both projects and the optimal rate of KT from the NPD project to the existing product improvement project are given in Theorem 1. We observe that the optimal rates of KD and KT at time t are functions of the marginal values and the marginal costs at that time. In Corollary 1, we describe how the optimal rates of development activities and KT change throughout the planning horizon. The interpretations follow.

***THEOREM 1**: The optimal rates of existing product improvement and new product development activities, and the optimal rate of new product knowledge to be transferred to the existing product improvement project are*: (i) $y_e^*(t) = \frac{\alpha_e \lambda_1(t)}{2c_e}$; (ii) $y_n^*(t) = \frac{\alpha_n \lambda_2(t)}{2c_n}$; (iii) $\theta^*(t) = \max\left\{\min\left\{\frac{\delta\lambda_1(t)+\lambda_3(t)}{2c_k}, 1\right\}, 0\right\}$.

***COROLLARY 1:** For $t \in [0,T]$, (i) $\dot{y}_e^*(t) = \frac{\alpha_e \dot{\lambda}_1(t)}{2c_e} < 0$; (ii) $\dot{y}_n^*(t) = \frac{\alpha_n \dot{\lambda}_2(t)}{2c_n} < 0$; (iii) $\dot{\theta}^*(t) = \frac{\delta\dot{\lambda}_1(t)+\dot{\lambda}_3(t)}{2c_k}$.*

From Theorem 1 and Corollary 1 (i), we see that the rate of existing product improvement activities is positive and decreasing over time until reaching $\frac{\alpha_e v_e}{2c_e}$ at the end of the planning horizon (see Figure 4.1). This development strategy is referred to in the literature as *front-loading* (Blackburn et al. 1996, Thomke and Fujimoto 2000, Ozkan et al. 2012, Xiao et al. 2012). An additional unit of existing product improvement effort at time t increases the level of existing product knowledge and thereby the revenue generated at that time and throughout the remainder of the planning horizon. However, over time, there is less opportunity to benefit from the existing product knowledge so that KD declines.

The results in Theorem 1 and Corollary 1 (ii) show that the rate of NPD activities is positive and decreasing over time until reaching $\frac{\alpha_n v_n}{2c_n}$ at time T (see Figure 4.1). The manager pursues a front-loading strategy for the KD of the NPD project, as well. However, the driver of the front-loading strategy is different from that observed in the existing product improvement project. The front-loading strategy for KD of the NPD project optimally occurs since an additional unit of NPD effort at time t increases the amount of new product knowledge that can be exploited in the form of KT to the existing product from that time and throughout the remainder of the planning horizon.

Lastly, in Theorem 1 and Corollary 1 (iii), we observe that the optimal rate of KT from the NPD project to the existing product improvement project is bounded by 0 and 1. The rate of change in $\theta^*(t)$ over time depends the values of $\delta\dot{\lambda}_1(t)$ and $\dot{\lambda}_3(t)$. The first term $\delta\dot{\lambda}_1(t)$ represents the benefits of KT to the existing product improvement project. The second term $\dot{\lambda}_3(t)$ indicates the penalty and cost for transferring the new product knowledge to the existing product improvement project. There are five cases that characterize how KT changes over time. At the extremes, we obtain $\theta^*(t) = 0$ or 1 for $t \in [0, T]$. In other words, if $\frac{\delta\lambda_1(t)+\lambda_3(t)}{2c_k} \geq 1$ $\left(\frac{\delta\lambda_1(t)+\lambda_3(t)}{2c_k} \leq 0\right)$over the entire planning horizon then KT is fixed at one (zero) for all time. The remaining three non-boundary solutions are described below.

In Case 3, we have $\theta^*(t) \in (0,1)$ for some non-zero interval of time and $\delta\dot{\lambda}_1(t) + \dot{\lambda}_3(t) > 0$ for $t \in [0, \mathrm{T}]$, i.e., KT is a non-decreasing function of time throughout the planning horizon. (The conditions for four possible scenarios are described in the Appendix, which also contains illustrations.) In Case 3, we find that the maximum rate of KT is delayed to the end of the planning horizon. In the literature, this is referred to as the *extreme delay* strategy (Xiao et al. 2012). This strategy is driven by the desire to delay KT until later in the planning horizon when the marginal penalty and cost of KT are driven to zero.

In Case 4, we have $\theta^*(t) \in (0,1)$ for some non-zero interval of time and $\delta\dot{\lambda}_1(\mathrm{t}) + \dot{\lambda}_3(\mathrm{t}) < 0$ for $\mathrm{t} \in [0, \mathrm{T}]$, i.e., KT is a non-increasing function of time throughout the planning horizon. (The conditions for four possible scenarios are described in the Appendix, which also contains illustrations.) In Case 4, we find that the maximum rate of KT occurs at the beginning of the development project. That is, the manager pursues a front-loading strategy for transferring the new product knowledge to the existing product improvement project. Front-loading occurs when the extent to which the benefits of KT dominate the corresponding penalty and cost is non-increasing over time.

In Case 5, we have $\theta^*(t) \in (0,1)$ for some non-zero interval of time $t \in [t_1, t_2]$ where $t_1 \geq 0$ and $t_1 < t_2 \leq T$; $\delta\dot{\lambda}_1(t) + \dot{\lambda}_3(t) \leq 0$ for $t \in [t_1, t_m]$; $\delta\dot{\lambda}_1(t) + \dot{\lambda}_3(t) > 0$ for $t \in (t_m, t_2]$; and $\theta^*(t_m)$ is the minimum value of $\theta^*(t)$ for $t \in [0, T]$. In other words, during a non-zero interval of time $t \in [t_1, t_2]$, KT is convex decreasing, reaches a minimum, and becomes convex increasing. Furthermore, for $t \in [0, t_1)$ and $t \in (t_2, \mathrm{T}]$ we have $\theta^*(t) = 1$ (i.e., KT is at its upper bound). Intuitively, this solution occurs when the marginal benefits of KT exceed the marginal penalty and cost throughout the planning horizon; but initially at a non-increasing rate then later at a non-decreasing rate. (The conditions for four possible scenarios are described in the Appendix, which also contains illustrations.) We refer to the solution of Case 5, as the *moderate front-loading* strategy (Xiao et al.2012). Managerially, throughout the planning horizon the marginal benefits of KT exceed the corresponding marginal penalty and cost. Early in the planning horizon the extent that the marginal benefits exceed the marginal penalty and cost are substantial (but decreasing). This situation occurs for two reasons. First, early in the planning horizon the potential to exploit KT by the existing product improvement project is large but decreasing. Second, early in the planning horizon when new product knowledge is relatively small so that the probability of success in the marketplace for the new product is small, then the marginal penalty and cost for KT is small but increasing. In contrast, later in the planning horizon, the level of new product knowledge grows sufficiently large such that, while the benefits still dominate the penalty and cost (i.e., KT optimally occurs), the rate of increase in the penalty and cost of KT dominates the rate of decrease in the benefits. Therefore, KT is delayed until later in the planning horizon when the marginal penalty and cost of KT increases (from a negative value) to zero.

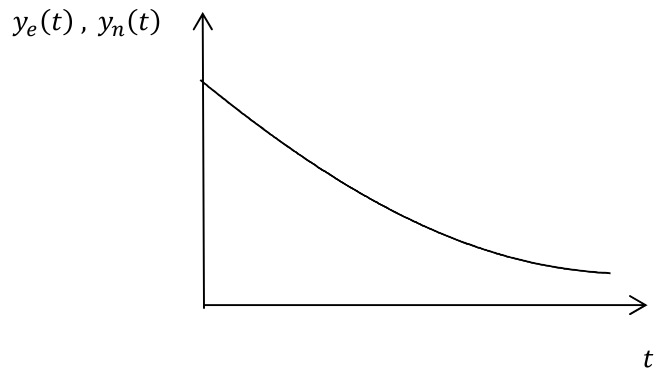

Figure 4.1: The Optimal Strategy for Existing Product Improvement and New Product Development Projects: ***Front-loading***

4.4.2 Sensitivity Analysis

In this section, we study the impact of parameter values on the optimal solutions and overall firm performance. In Corollaries 2 and 3, we explore how the effectiveness of development activities in the existing product improvement project and the NPD project (α_e and α_n) impact the optimal rates of KD and KT, respectively. Corollary 4 discusses the impact of the returns KT (δ) on the optimal solution. Interpretations follow.

COROLLARY 2: *If workforce skill or technical support in the NPD project (α_n) is larger, then the optimal rate of KD for the NPD project is larger and the optimal rate of KT from the new development project to the existing product improvement project is smaller for $t \in [0,T]$. The optimal rate of KD for the existing product improvement project remains the same. The levels of knowledge associated with the NPD project and the existing product improvement project are larger $t \in (0,T]$. The overall expected profit is larger. The reverse results hold for c_n.*

Intuitively, if the effectiveness of KD for the NPD project is larger, then it is more desirable so that the rate of KD is larger throughout the planning horizon. Naturally, the level of new product knowledge is larger. With a larger amount of new product knowledge, although the manager pursues a smaller rate of KT, the cumulative amount of new product knowledge transferred to the existing product improvement project is still larger. Therefore, the level of existing product knowledge is larger. The increase in expected revenue and revenue that occurs due to the higher levels of new product and existing product knowledge, respectively, plus the smaller penalty for KT dominate the increase in costs due to more KT and more KD activities for the NPD project. Thus, the firm's expected profit increases.

COROLLARY 3: *If workforce skill or technical support in the existing product improvement project (α_e) is larger, then the optimal rate of KD for the existing product improvement project is larger for $t \in [0,T]$, and the level of existing product knowledge is larger for $t \in (0,T]$. The rate of KD for the NPD project, the rate of KT from the NPD project to the existing product improvement project, and the level of new product knowledge remain the same. The expected profit is larger. The reverse results hold for c_e.*

To interpret Corollary 3, note that the effectiveness of workforce skill and technical support for the existing product improvement project (α_e) only impacts KD in the existing product improvement project. In particular, if α_e is larger, KD is more effective at increasing the level of knowledge for the existing product thus the rate of KD is larger and the level of existing knowledge is larger. Since the rate of KD for the NPD project and the rate of KT to the existing product improvement project are not impacted by the value of α_e, the level of new product knowledge remains the same. The increase in revenue due to higher level of existing product knowledge dominates the additional cost for more KD in the existing product improvement project, and the expected profit is larger.

COROLLARY 4: *If the returns to KT (δ) is larger, then the optimal rate of KD for the new product is larger and the optimal rate of KT to the existing product improvement project is larger for $t \in [0,T)$. The optimal rate of KD for the existing product improvement project remains the same. The levels of knowledge for the NPD project and the existing product improvement project are both larger for $t \in (0,T]$. The expected profit is larger.*

Lastly, we analyze the impact of the returns to KT. When the returns to KT are larger, the manager pursues a larger rate of NPD activities so that more new product knowledge is available for transfer to the existing product improvement project. Furthermore, the manager pursues a larger rate of KT throughout the planning horizon. As a result, the cumulative amount of new product knowledge transferred to the existing product improvement project is larger and the level of existing product knowledge is larger even though the rate of KD for the existing product improvement project remains the same. The expected profit is larger since the increase in revenue from the existing product plus the increase in expected revenue from the NPD project dominate the increase in penalties due to the loss of proprietary knowledge plus the increase in costs from more KD and KT.

4.5 Conclusion and Future Research

In this paper, we explore a manager's pursuit of KD for an existing product improvement project and KD for an NPD project, which increase the respective levels of knowledge embedded in the existing product and the new product over time. Revenue is obtained with certainty and throughout the development project in relation to the level of knowledge in the existing product; but the amount of revenue is relatively limited. In contrast, substantial revenue may be realized when the NPD project is successfully released to the marketplace. The probability of success in the marketplace is driven by the level of knowledge in the NPD project over time. An important feature of our research is that we capture the link between the NPD project and the existing product improvement project through KT. More specifically, the knowledge accumulated in the NPD project can be transferred to the existing product improvement project for the design of new features or capabilities that enhance the existing product and thereby increase its revenue generating potential. However, the loss in proprietary knowledge due to the KT reduces the expected revenue earned by the new product. The objective is to maximize the revenue earned from the existing product, plus the expected revenue from the new product, less the penalty for loss in proprietary knowledge and the costs of KD and KT.

We obtain dynamic solutions characterizing how the NPD manager pursues KD for both the new and existing projects and KT from the NPD project to the existing product improvement project. We analytically show that the optimal rates of KD for the new product and the existing product continuously decrease over time (follow a front-loading strategy); the optimal rate of KT may follow a front-loading, an extreme delay, or a moderate delay strategy depending on which of the two dominates: the benefits to revenue for the existing product improvement project from KT versus the penalty and cost associated with KT (see details in Section 4.4.1).

We obtain insights on the impact on the optimal solution due to the effectiveness of KD for both an existing product improvement project and an NPD project. In addition, results are given depicting how the returns to KT impact the optimal solution. Next, we plan to explore the impact of the following on the optimal solutions: the penalty for losing proprietary knowledge associated with the NPD project; the rate at which the probability of success in the marketplace increases in relation to the level of NPD knowledge; the rate that revenue increases in relation to the level of knowledge for the existing product improvement project; the initial levels of new product and existing product knowledge; and the unit cost of KT.

APPENDIX A

A1. Tables

Table A.1: Notation for the Base Model

w	Buyer's level of in-house production in periods 1 and 2 (decision variable)
P	Supplier's outsourcing price in periods 1 and 2 (decision variable)
μ	Supplier's level of integration process improvement (IPI) in period 1 (decision variable)
R	Unit market price earned by the buyer.
C_b	Buyer's unit in-house production cost in period 1
α	Buyer's rate of volume-based learning in its production cost in period 2
C_s	Supplier's unit production cost in period 1
β	Supplier's rate of volume-based learning in its production cost in period 2
U	Scaling factor of the supplier's investment cost in IPI
γ	Diseconomies of scale associated with the supplier's investments in IPI
L	Base unit integration cost for the buyer (i.e., cost without IPI)
C_i	Buyer's unit integration cost
$C_i + P$	Buyer's unit cost of outsourcing
V	Component demand to be met by the buyer in each period
f_0	Scaling factor of the buyer's future value benefits from volume-based learning
f_1	Rate of diminishing returns associated with the future value benefits
Π_b	Buyer's profit in periods 1 and 2
Π_s	Supplier's profit in periods 1 and 2

Table A.2: Range of Input Parameter Values for Numerical Experiments

$R\in[10,100]$; $\alpha, \beta\in(0,1)$; $f_1=½$; $\gamma\in(1,2]$; C_b, $C_s\in[1,20]$; $L\in[1,10]$; $U\in[1,400]$; $f_0\in[1,100]$; $V\in[1,200]$; $\rho\in[-0.9,5]$

A2. Proofs of Theorems and Corollaries

Proof of Theorem 1.

Proof of Case 1. $\mathcal{L}_b(w,\lambda_1)$ is concave in two situations. First, given $1-\alpha-f_1=0$ and $C_b<f_0$, we have $X(w)<0$ and $\frac{d^2\mathcal{L}_b}{dw^2}<0$. Second, given $1-\alpha-f_1>0$, we know that

$X(w)$ increases as w increases. From Equation (8), we have $X(V) \leq 0$. Since $X(w)$ increases as w increases and $X(V) \leq 0$, we have $X(w) \leq 0$ and $\frac{d^2\mathcal{L}_b}{dw^2} \leq 0$ for $w \in [0, V]$. With the concavity of the Lagrangian, we obtain an interior solution $w^{int} \in (0, V)$ where w^{int} satisfies Equation (6). If $\Pi_b(w^{int}) \geq 0$ then the buyer participates in the game and $w^* = w^{int}$. From Equation (5), we obtain $\lambda_1{}^* = 0$. (Note that if $\Pi_b(w^{int}) < 0$, the buyer does not participate in the game.)

Proof of Case 2. $\mathcal{L}_b(w, \lambda_1)$ is convex in two situations. First, re-writing $1 - \alpha - f_1 = 0$ as $f_1 = 1 - \alpha$ and given $C_b \geq f_{0,}$ we have $X(w) = \alpha(1 - \alpha)(C_b - f_0) \geq 0$ so that $\frac{d^2\mathcal{L}_b}{dw^2} \geq 0$. Second, given $1 - \alpha - f_1 < 0$, we know that $X(w)$ decreases as w increases. From Equation (8), we have $X(V) \geq 0$. Since $X(w)$ decreases as w increases and $X(V) \geq 0$, we know that $X(w) \geq 0$ and $\frac{d^2\mathcal{L}_b}{dw^2} \geq 0$ for $w \in [0, V]$. With the convexity of the Lagrangian, the Case 2 optimal solution occurs on a boundary: $w^* = 0$ or V. If $w^* = 0$ ($w^* = V$) then from Equation (5) we obtain $\lambda_1{}^* = 0$ ($\lambda_1{}^* = \lambda_1^{pos}$) and $\mathcal{L}_b(0,0) > \mathcal{L}_b(V, \lambda_1^{pos})$ $\left(\mathcal{L}_b(V, \lambda_1^{pos}) > \mathcal{L}_b(0,0)\right)$.

Proof of Case 3. Given $1 - \alpha - f_1 > 0$ and $X(V) > 0$, since $X(0) < 0$ there exists w' such that (i) for $w \in [0, w')$ we have $\frac{d^2\mathcal{L}_b}{dw^2} < 0$; (ii) for $w = w'$ we have $\frac{d^2\mathcal{L}_b}{dw^2} = 0$ (i.e., $X(w') = 0$); and (iii) for $w \in (w', V]$ we have $\frac{d^2\mathcal{L}_b}{dw^2} > 0$, where $\frac{d^2\mathcal{L}_b}{dw^2}$ is given in Equation (7). That is, the Lagrangian is initially a concave and then becomes a convex function of w. Let the maximum of the concave domain be given by w_H^{int} and the minimum of the convex domain be given by w_L^{int}, where $w_H^{int} < w' < w_L^{int}$. There are two possible optimal solutions: $w^* = w_H^{int}$ and $\lambda_1{}^* = 0$, or $w^* = V$ and $\lambda_1{}^* = \lambda_1^{pos} > 0$, depending on whether $\mathcal{L}_b\left(w_H^{int}, 0\right) > \mathcal{L}_b\left(V, \lambda_1^{pos}\right)$ or $\mathcal{L}_b\left(V, \lambda_1^{pos}\right) > \mathcal{L}_b\left(w_H^{int}, 0\right)$, respectively.

Proof of Case 4. Given $1 - \alpha - f_1 < 0$ and $X(V) < 0$, since $X(0) > 0$ there exists w'' such that (i) for $w \in [0, w'')$ we have $\frac{d^2\mathcal{L}_b}{dw^2} > 0$; (ii) for $w = w''$ we have $\frac{d^2\mathcal{L}_b}{dw^2} = 0$ (i.e., $X(w'') = 0$); and (iii) for $w \in (w'', V]$ we have $\frac{d^2\mathcal{L}_b}{dw^2} < 0$, where $\frac{d^2\mathcal{L}_b}{dw^2}$ is given in Equation (7). That is, the Lagrangian is initially a convex and then becomes a concave function of w. Let the minimum of the convex domain be given by w_L^{int} and the maximum of the concave domain be given by w_H^{int}, where $w_L^{int} < w'' < w_H^{int}$. Again, there are two possible optimal solutions: $w^* = w_H^{int}$ and $\lambda_1{}^* = 0$, or $w^* = 0$ and $\lambda_1{}^* = 0$, depending on whether $\mathcal{L}_b\left(w_H^{int}, 0\right) > \mathcal{L}_b(0,0)$ or $\mathcal{L}_b(0,0) > \mathcal{L}_b\left(w_H^{int}, 0\right)$, respectively. #

Q.E.D.

Proof of Theorem 2

Since we assume that $H(P, \mu)$ is negative definite, then the supplier's profit is jointly concave with respect to P and μ. As a result, the maximum profit is achieved at the interior solutions (P^{1a}, μ^{1a}), obtained by simultaneously solving $\frac{d\mathcal{L}_s}{dP} = 0$ and $\frac{d\mathcal{L}_s}{d\mu} = 0$ with the associated Lagrange multipliers equal to zero. Note that $\underline{P}(\mu)$ is defined as the lower bound on P to ensure $\Pi_s \geq 0$ and $\overline{P}(\mu)$ is the upper bound of P to ensure $\Pi_b(w^*) \geq 0$. Both bounds on P are impacted by μ. Recall that since we consider non-trivial solutions, we assume the supplier's profit is non-negative so that the supplier participates in the game.

Proof of Case 1.

Case (1a) When $\mu^{1a} \in [0,1]$ and $P^{1a} \in [\underline{P}(\mu_{1a}), \overline{P}(\mu_{1a})]$, we obtain $\lambda_2^* = \lambda_3^* = 0$ (Equations (14) and (15)) so that $(P^*, \mu^*, \lambda_2^*, \lambda_3^*) = (P^{1a}, \mu^{1a}, 0, 0)$. Moreover, we have $P^{1a} = \frac{1}{2}((V - w^*)(f_0 f_1(1 - f_1)w^{*f_1 - 2} - C_b\alpha(1-\alpha)w^{*-1-\alpha}) + C_s(V - w^*)^{-\beta}(1 + (V - w^*)^{\beta} - \beta))$ and $\mu^{1a} = \left(\frac{2L(V-w^*)}{U\gamma}\right)^{\frac{1}{\gamma - 1}}$.

Case (1b) When $\mu^{1a} > 1$, since the supplier's profit is concave in μ, we obtain $\mu^* = 1$. If $P^{1b} \in [\underline{P}(1), \overline{P}(1)]$ then $\lambda_2^* = 0$ (Equation (14)) and $\lambda_3^* = 2L(V - w^*) - U\gamma$ (Equation (15)).

Case (1c) If $P^{1a} > \overline{P}(\mu_{1a})$, to ensure the buyer participates in the game, the supplier determines P^* and μ^* such that $\Pi_b = 0$. Since the supplier's profit is concave in P, solving $\Pi_b = 0$, we obtain $\overline{P}^{1c} = \overline{P}(\mu) = \frac{1}{2(V-w^*)}\left(2RV - C_b w^* - C_b w^{*1-\alpha} + f_0 w^{*f_1}\right) - L(1-\mu)$. Substituting $\overline{P}^{1c}$ for P into $\Pi_s(P,\mu)$ and differentiating with respect to μ gives us μ^{1c} which maximizes the supplier's profit. Note that the substitution reduces the two-variable problem into a one-variable problem, where it is easy to show that $\Pi_s(\overline{P}^{1c}, \mu)$ is concave in μ. From $\frac{d\mathcal{L}_s}{dP} = 0$ (Equation (12)), we obtain $\lambda_2^* = \lambda_2^{1c}$. If $\mu^{1c} \in [0,1]$ then $\lambda_3^* = 0$ (Equation (15)) giving us $(P^*, \mu^*, \lambda_2^*, \lambda_3^*) = (\overline{P}^{1c}, \mu^{1c}, \lambda_2^{1c}, 0)$.

Case (1d) If $\mu^{1c} > 1$, then set $\mu^* = 1$ so that $\overline{P}^{1d} = \overline{P}(\mu = 1) = \frac{1}{2(V-w^*)}\left(2RV - C_b w^* - C_b w^{*1-\alpha} + f_0 w^{*f_1}\right)$. From $\frac{d\mathcal{L}_s}{dP} = 0$ and $\frac{d\mathcal{L}_s}{d\mu} = 0$ with $\mu^* = 1$, we obtain $\lambda_2^* = \lambda_2^{1d}$ and $\lambda_3^* = \lambda_3^{1d}$.

Proof of Case 2. Focusing on the non-trivial solutions, the supplier sets P^* and μ^* such that $\Pi_b(0) \geq \Pi_b(V)$ giving us $\lambda_2^* = 0$. Setting $\Pi_b(0) = \Pi_b(V)$, we obtain P as a function of μ expressed as $Y(\mu)$, where $P = Y(\mu) = \frac{1}{2}(C_b(1+V)^{-\alpha} - f_0 V^{f_1 - 1}) -$

$L(1-\mu)$. Substituting for P into $\Pi_s(P,\mu)$ and differentiating with respect to μ gives us $\mu^{2a} = \left(\frac{2LV}{U\gamma}\right)^{1/(\gamma-1)}$. Note that the substitution reduces the two-variable problem to a one-variable problem, where it is easy to show that $\Pi_s(Y(\mu),\mu)$ is concave in μ.

Case (2a) If $\mu^{2a} \in [0,1]$ then $\mu^* = \mu^{2a}$ and $\lambda_3^* = 0$. Substituting μ^{2a} into the expression for P gives us $P^* = P^{2a} = \frac{1}{2}(C_b(1+V)^{-\alpha} - f_0V^{f_1-1}) - L\left(1-\left(\frac{2LV}{U\gamma}\right)^{1/(\gamma-1)}\right)$. The supplier's optimal solution is $(P^*,\mu^*,\lambda_2^*,\lambda_3^*) = (P^{2a},\mu^{2a},0,0)$.

Case (2b) If $\mu^{2a} > 1$, then we obtain $\mu^* = 1$ and $\lambda_3^* = \lambda_3^{2b}$, where λ_3^{2b} satisfies $\frac{d\mathcal{L}_s}{d\mu} = 0$. From $P = Y(\mu)$, we have $P^* = P^{2b} = Y(1)$ so that $(P^*,\mu^*,\lambda_2^*,\lambda_3^*) = (P^{2b},1,0,\lambda_3^{2b})$.

The proofs of Cases (2c) - (2d) are analogous to those of Cases (2a) – (2b) and are omitted. Note that when $1-\alpha-f_1 > 0$ and $X(V) > 0$, then $w^* = w_H^{int}$ or V. Alternatively, when $1-\alpha-f_1 = 0$ *and* $C_b < f_0$ or $1-\alpha-f_1 > 0$ and $X(V) < 0$, then $w^* = w^{int}$. Lastly, given our focus on non-trivial solutions, we do not consider the case $w^* = V$. # *Q.E.D.*

Proof of Corollary 1

Define $F_1 = \frac{d\mathcal{L}_b}{dw}|_{w^*=w^{int}} = -C_b(1+(1-\alpha)w^{*-\alpha}) + 2(P+L(1-\mu)) + f_0f_1w^{*f_1-1} = 0$. Using the implicit-function theorem (Chiang and Wainwright, 2005), we obtain $\frac{dw^*}{dP} = -\frac{dF_1/dP}{dF_1/dw^*}$ and $\frac{dw^*}{d\mu} = -\frac{dF_1/d\mu}{dF_1/dw^*}$. Substituting (P^{1a},μ^{1a}) from Theorem 2 into F_1, we obtain F_2 given in Equation (A-1).

$$F_2 = -C_b\left(1+(1-\alpha)w^{*-\alpha-1}(V\alpha + w^*(1-\alpha))\right) + f_0f_1w^{*f_1-2}(V - f_1(V-w^*))$$
$$+C_s(1+(V-w^*)^{-\beta}(1-\beta) + 2L\left(1-\left(\frac{2L(V-w^*)}{U\gamma}\right)^{\frac{1}{\gamma-1}}\right) \quad \text{(A-1)}$$

Since the sign of dF_2/dw^* is the same as that of $-|H(P^*,\mu^*)|$ and given $|H(P^*,\mu^*)| > 0$, then $dF_2/dw^* < 0$. Using the implicit-function theorem, we obtain $\frac{dw^*}{dU} = -\frac{dF_2/dU}{dF_2/dw^*}$. The sign of $\frac{dw^*}{dU}$ is the same as that of $\frac{dF_2}{dU}$, with $\frac{dF_2}{dU} = \frac{2L}{U(\gamma-1)}\left(\frac{2L(V-w^*)}{U\gamma}\right)^{\frac{1}{\gamma-1}} > 0$. Therefore, we have $\frac{dw^*}{dU} > 0$ (i.e., w^* increases as U increases). Using the chain rule, we have $\frac{d\mu^*}{dU}$ and $\frac{dP^*}{dU}$, given in Equations (A-2) and (A-3), respectively. Note that $\frac{d\mu^*}{dw^*}$ is obtained by taking the derivative of μ^* with respect to w^*. Clearly, $\frac{d\mu^*}{dU} < 0$ holds. That is, μ^* decreases as U increases. In addition, the sign of $\frac{dP^*}{dU}$ depends on the sign of Φ_2. When $Z \ll 0$ ($Z \to 0^-$), we have $\Phi_2 > 0$ ($\Phi_2 < 0$) so that $\frac{dP^*}{dU} > 0$ ($\frac{dP^*}{dU} < 0$).

$$\frac{d\mu^*}{dU} = \frac{\partial\mu^*}{\partial U} + \frac{d\mu^*}{dw^*}\frac{dw^*}{dU} = -\frac{1}{U(\gamma-1)}\left(\frac{2L(V-w^*)}{U\gamma}\right)^{\frac{1}{\gamma-1}} - \frac{2L}{U\gamma(\gamma-1)}\left(\frac{2L(V-w^*)}{U\gamma}\right)^{\frac{2-\gamma}{\gamma-1}}\frac{dw^*}{dU} \quad \text{(A-2)}$$

$$\frac{dP^*}{dU} = \frac{\partial P^*}{\partial U} + \frac{dP^*}{dw^*}\frac{dw^*}{dU} = \frac{\Phi_2}{2}\frac{dw^*}{dU} \quad \text{(A-3)}$$

Substituting w^* and (P^*,μ^*) into Π_b and Π_s, we have Π_b^* and Π_s^* given in Equations (A-4) and (A-5). Taking derivative of Π_b^* with respect to U, we have $\frac{d\Pi_b^*}{dU}$ as shown in Equation (A-6). Using the Envelope Theorem, we have $\frac{d\Pi_b^*}{dw^*} = 0$ and $\frac{d\Pi_b^*}{dU} < 0$. That is, Π_b^* decreases as U increases. Taking derivative of Π_s^* with respect to U and applying the chain rule, we have $\frac{d\Pi_s^*}{dU}$ as shown in Equation (A-7). It follows that $\frac{d\Pi_s^*}{dU} < 0$ holds. That is, Π_s^* decreases as U increases.

$$\Pi_b^* = 2RV - C_b w^* - C_b w^{*1-\alpha} - (V-w^*)^2\left(f_0 f_1(1-f_1)w^{*f_1-2} - C_b\alpha(1-\alpha)w^{*-1-\alpha}\right)$$
$$-C_s(V-w^*)^{1-\beta}\left(1+(V-w^*)^{\beta}-\beta\right) - 2L(V-w^*)\left(1-\left(\frac{2L(V-w^*)}{U\gamma}\right)^{\frac{1}{\gamma-1}}\right)$$
$$+f_0 w^{*f_1} \quad \text{(A-4)}$$

$$\Pi_s^* = (V-w^*)^2(f_0 f_1(1-f_1)w^{*f_1-2} - C_b\alpha(1-\alpha)w^{*-1-\alpha}) - C_s\beta(V-w^*)^{1-\beta}$$

$$-U(\frac{2L(V-w^*)}{U\gamma})^{\frac{\gamma}{-1+\gamma}} \qquad \text{(A-5)}$$

$$\frac{d\Pi_b^*}{dU} = \frac{\partial\Pi_b^*}{\partial U} + \frac{d\Pi_b^*}{dw^*}\frac{dw^*}{dU} = \frac{\partial\Pi_b^*}{\partial U} = -\frac{2L(V-w^*)}{U(\gamma-1)}\left(\frac{2L(V-w^*)}{U\gamma}\right)^{\frac{1}{\gamma-1}} < 0 \qquad \text{(A-6)}$$

$$\frac{d\Pi_S^*}{dU} = \frac{\partial\Pi_S^*}{\partial U} + \frac{d\Pi_S^*}{dw^*}\frac{dw^*}{dU} = -\left(\frac{2L(V-w^*)}{U\gamma}\right)^{\frac{\gamma}{\gamma-1}} < 0 \qquad \text{(A-7)}$$

 # Q.E.D.

Proof of Corollary 2

Using the implicit-function theorem, we obtain $\frac{dw^*}{dL} = -\frac{dF_2/dL}{dF_2/dw^*}$. Since $dF_2/dw^* < 0$, the sign of $\frac{dw^*}{dL}$ is the same as that of $\frac{dF_2}{dL}$, with $\frac{dF_2}{dL} = 2\left(1 - \frac{\gamma^{\frac{\gamma-2}{\gamma-1}}}{\gamma-1}\left(\frac{2L(V-w^*)}{U}\right)^{\frac{1}{\gamma-1}}\right)$. When $U > \frac{2L(V-w^*)\gamma^{\gamma-2}}{(\gamma-1)^{\gamma-1}}$, we have $1 - \frac{\gamma^{\frac{\gamma-2}{\gamma-1}}}{\gamma-1}\left(\frac{2L(V-w^*)}{U}\right)^{\frac{1}{\gamma-1}} > 0$. Therefore $\frac{dF_2}{dL} > 0$ and $\frac{dw^*}{dL} > 0$ hold (i.e., w^* increases as L increases). Otherwise, when $U < \frac{2L(V-w^*)\gamma^{\gamma-2}}{(\gamma-1)^{\gamma-1}}$, we have $1 - \frac{\gamma^{\frac{\gamma-2}{\gamma-1}}}{\gamma-1}\left(\frac{2L(V-w^*)}{U}\right)^{\frac{1}{\gamma-1}} < 0$ so that $\frac{dw^*}{dL} < 0$ holds. Using the chain rule, we obtain $\frac{d\mu^*}{dL}$ and $\frac{dP^*}{dL}$ as given in Equations (A-8) and (A-9), respectively. Clearly, the sign of $\frac{d\mu^*}{dL}$ depends on the sign of $V - w^* - L\frac{dw^*}{dL}$. If $\frac{dw^*}{dL} > 0$ and L is sufficiently large (small), we have $\frac{d\mu^*}{dL} < 0$ ($\frac{d\mu^*}{dL} > 0$). However, if $\frac{dw^*}{dL} < 0$, we have $\frac{d\mu^*}{dL} > 0$. In addition, the sign of $\frac{dP^*}{dL}$ depends on the sign of Φ_2 and $\frac{dw^*}{dL}$. If $\frac{dw^*}{dL} > 0$, then $\frac{dP^*}{dU} > 0$ ($\frac{dP^*}{dU} < 0$) whenever $\Phi_2 > 0$ ($\Phi_2 < 0$). Otherwise, if $\frac{dw^*}{dL} < 0$, then $\frac{dP^*}{dU} < 0$ ($\frac{dP^*}{dU} < 0$) when $\Phi_2 > 0$ ($\Phi_2 < 0$).

$$\frac{d\mu^*}{dL} = \frac{\partial\mu^*}{\partial L} + \frac{d\mu^*}{dw^*}\frac{dw^*}{dL} = \frac{2}{U(\gamma-1)\gamma}\left(\frac{2LV-2Lw^*}{U\gamma}\right)^{\frac{2-\gamma}{\gamma-1}}\left(V - w^* - L\frac{dw^*}{dL}\right) \qquad \text{(A-8)}$$

$$\frac{dP^*}{dL} = \frac{\partial P^*}{\partial L} + \frac{dP^*}{dw^*}\frac{dw^*}{dL} = \frac{\Phi_2}{2}\frac{dw^*}{dL} \qquad \text{(A-9)}$$

Taking derivative of $\Pi_b{}^*$ (Equation (A-4)) with respect to L, we have $\frac{d\Pi_b^*}{dL}$ as shown in Equation (A-10). Using the Envelope Theorem, we have $\frac{d\Pi_b^*}{dw^*} = 0$ so that $\frac{d\Pi_b^*}{dL} = \frac{\partial\Pi_b^*}{\partial L}$. When $U > \frac{2L(V-w^*)\gamma^{\gamma-2}}{(\gamma-1)^{\gamma-1}}$ ($U \leq \frac{2L(V-w^*)\gamma^{\gamma-2}}{(\gamma-1)^{\gamma-1}}$), we have $\frac{d\Pi_b^*}{dL} < 0$ ($\frac{d\Pi_b^*}{dL} \geq 0$). That is, Π_b^* decreases (is non-decreasing) as L increases. Taking derivative of Π_s^* (Equation (A-5)) with respect to L and using the chain rule, we have $\frac{d\Pi_s^*}{dL}$ given in Equation (A-11). It follows that $\frac{d\Pi_s^*}{dL} < 0$. That is, the Π_s^* decreases as L increases.

$$\frac{d\Pi_b^*}{dL} = \frac{\partial\Pi_b^*}{\partial L} + \frac{d\Pi_b^*}{dw^*}\frac{dw^*}{dL} = \frac{\partial\Pi_b^*}{\partial L} = -2(V-w^*)\left(1 - \frac{\gamma^{\frac{\gamma-2}{\gamma-1}}}{\gamma-1}\left(\frac{2L(V-w^*)}{U}\right)^{\frac{1}{\gamma-1}}\right) \quad \text{(A-10)}$$

$$\frac{d\Pi_s^*}{dL} = \frac{\partial\Pi_s^*}{\partial L} + \frac{d\Pi_s^*}{dw^*}\frac{dw^*}{dL} = -2(V-w^*)\left(1 - \left(\frac{2L(V-w^*)}{U\gamma}\right)^{\frac{1}{\gamma-1}}\right) = -2(V-w^*)(1-\mu^*) < 0 \quad \text{(A-11)}$$

Q.E.D.

Proofs of Corollaries 3, 4, and 5

The proofs of Corollaries 3, 4 and 5 are analogous to Corollary 2 and are omitted. # *Q.E.D.*

APPENDIX B

Proofs of Theorems and Corollaries

Proofs of Lemma 1

Follows from standard optimality conditions, for example, $\lambda_{1t} = -H_Y$ (Sethi and Thompson (2000)). #Q.e.d.

Proof of Theorem 1

Follows from standard optimality conditions, for example, $H_y = 0$ (Sethi and Thompson (2000)). #Q.e.d.

Proof of Corollary 1

Taking the first order derivatives of y, p, and n with respect to t, we obtain:

$$y_t = \left(\frac{\alpha_0}{\sigma_1 c_1}\right)^{\frac{1}{\sigma_1-1}} \frac{\lambda_1^{\frac{2-\sigma_1}{\sigma_1-1}}}{\sigma_1-1}\lambda_{1t} \qquad \text{(B-1)}$$

$$p_t = \left(\frac{\beta_0}{\sigma_2 c_2}\right)^{\frac{1}{\sigma_2-1}} \frac{(\lambda_2 Y(t)^{\beta_1})^{\frac{2-\sigma_2}{\sigma_2-1}} Y(t)^{\beta_1-1}}{\sigma_2-1}(\lambda_{2t}Y(t)+\lambda_2\beta_1 Y_t(t)) \qquad \text{(B-2)}$$

$$n_t = \left(\frac{r_3\gamma_0}{\sigma_3 c_3}\right)^{\frac{1}{\sigma_3-1}} \frac{\gamma_1}{\sigma_3-1} P(t)^{\frac{1+\gamma_1-\sigma_3}{\sigma_3-1}} P_t \qquad \text{(B-3)}$$

In Equation (B-1), since $\lambda_{1t}<0$, we have $y_t<0$. In Equation (B-3), since $P_t>0$, we have $n_t>0$. The sign of p_t depends on the sign of the expression $\lambda_{2t}Y+\lambda_2\beta_1Y_t$, given in Equation (B-2). We know $\lambda_{2t}<0$ and Y, Y_t and $\lambda_2>0$ so that the first term is negative while the second term is positive. Case i: First, early in the planning horizon, since both λ_2 and Y_t are decreasing in time, $\lambda_2\beta_1Y_t$ has its maximum value at the initial time 0. Second, $\lambda_2\beta_1Y_t$ has its minimum value at time T. Thus for reasonable parameter values, $\lambda_{2t}Y+\lambda_2\beta_1Y_t$ is positive at the initial time, and negative at the terminal time. Therefore, p^* first increases and then decreases over time. Let t_s denote the peak time such that the expression equals zero: $\lambda_{2t}(t_s)Y(t_s)+\lambda_2\beta_1 Y_t(t_s)=0$ where $t_s \in [0,T]$.

Theoretically, two other cases are possible though they require unrealistic parameter values. For completeness, we state these two cases. Case ii: For parameter values such as very Y(0) or very small r_2, we have $\lambda_{2t}Y+\lambda_2\beta_1 Y_t$ is negative so that $p_t<0$ holds for $t \in [0,T]$. Case iii: For parameter values such as very small Y(0) or very large r_2, $\lambda_{2t}Y+\lambda_2\beta_1 Y_t$ is positive so that $p_t>0$ holds for $t \in [0,T]$. #Q.e.d.

Proof of Corollary 2

a) Taking derivative of y^* with respect to α_0, we obtain:

$$\frac{dy^*}{d\alpha_0} = \frac{\partial y}{\partial \alpha_0} + \frac{\partial y}{\partial \lambda_1}\left[\frac{\partial \lambda_1}{\partial \lambda_2}\frac{\partial \lambda_2}{\partial P}\left(\frac{\partial P}{\partial p}\frac{\partial p}{\partial \lambda_2}\frac{d\lambda_2}{d\alpha_0} + \frac{\partial P}{\partial Y}\frac{dY}{d\alpha_0}\right) + \frac{\partial \lambda_1}{\partial Y}\frac{dY}{d\alpha_0}\right]$$

From the above, we see that $\frac{dy^*}{d\alpha_0}$ includes first order, third order, fifth order and sixth order effects. We reasonably assume that the fifth and sixth order effects are negligible compared to the first and third order effects. (Similar assumptions are made in Carrillo and Gaimon 2004, Heiman et al. 2001, Carrillo and Franza 2006, Gaimon et al 2011, Ozkan et al. 2012. Also see Chiang and Wainwright 2005.) Therefore, we simplify $\frac{dy^*}{d\alpha_0}$ as:

$$\frac{dy^*}{d\alpha_0} \approx \frac{\partial y}{\partial \alpha_0} + \frac{\partial y}{\partial \lambda_1}\frac{\partial \lambda_1}{\partial Y}\frac{dY}{d\alpha_0}$$

$$= \left(\frac{1}{\sigma_1 c_1}\right)^{\frac{1}{\sigma_1-1}} \frac{1}{\sigma_1-1}\left(\lambda_1{}^*(t)\alpha_0\right)^{\frac{2-\sigma_1}{\sigma_1-1}} \left(r_1 \int_t^T \frac{\left(\lambda_{2(t)}\beta_0\right)^{\frac{\sigma_2}{\sigma_2-1}} \beta_1 Y(t)^{\frac{\sigma_2(\beta_1-1)+1}{\sigma_2-1}}}{\left(\sigma_2 c_2\right)^{\frac{1}{\sigma_2-1}}} \left(1 + \frac{\sigma_2(\beta_1-1)+1}{(\sigma_2-1)Y}\frac{dY}{d\alpha_0}\alpha_0\right) dt \right).$$

The sign of $\frac{dy^*}{d\alpha_0}$ is the same as the sign of $1 + \frac{\sigma_2(\beta_1-1)+1}{(\sigma_2-1)Y}\frac{dY}{d\alpha_0}\alpha_0$. Proof by contradiction: When $\sigma_2(\beta_1-1)+1<0$, we assume that $\frac{dy^*}{d\alpha_0}<0$. Clearly, $\alpha_0\frac{dY}{d\alpha_0} = \int_0^t \Big(\alpha_0 y(t) +$

${\alpha_0}^2 \frac{dy^*}{d\alpha_0}\Big) dt < \int_0^t \alpha_0 y(t) dt < Y(t)$. Therefore, we have $1 + \frac{\sigma_2(\beta_1 - 1) + 1}{(\sigma_2 - 1)Y} \frac{dY}{d\alpha_0} \alpha_0 > 1 + \frac{\sigma_2(\beta_1 - 1) + 1}{(\sigma_2 - 1)} = \frac{\sigma_2 \beta_1}{(\sigma_2 - 1)} > 0$, which means $\frac{dy^*}{d\alpha_0} > 0$. By contradiction, we obtain $\frac{dy^*}{d\alpha_0} > 0$. When $\sigma_2(\beta_1 - 1) + 1 > 0$, we need to determine the sign of $\frac{dY}{d\alpha_0}$. Taking derivative of Y with respect to α_0, we obtain $\frac{dY}{d\alpha_0} = \frac{\partial Y}{\partial \alpha_0} + \frac{\partial Y}{\partial y} \frac{dy}{d\alpha_0} = \frac{\partial Y}{\partial \alpha_0} + \frac{\partial Y}{\partial y} \left(\frac{\partial y}{\partial \alpha_0} + \frac{\partial y}{\partial \lambda_1} \frac{\partial \lambda_1}{\partial Y} \frac{dY}{d\alpha_0} \right)$. We reasonably assume that the first and second order effects dominate the fourth order effect. Thus, we have $\frac{dY}{d\alpha_0} = \int_0^t y dt + \int_0^t \alpha_0 \left(\frac{\lambda_1}{\sigma_1 c_1} \right)^{\frac{1}{\sigma_1 - 1}} \frac{\alpha_0^{\frac{2 - \sigma_1}{\sigma_1 - 1}}}{\sigma_1 - 1} > 0$ giving us $\frac{dy^*}{d\alpha_0} > 0$.

b) Taking derivative of p* with respect to α_0, we obtain $\frac{dp}{d\alpha_0} = \frac{\partial p}{\partial \lambda_2} \frac{\partial \lambda_2}{\partial P} \left(\frac{\partial P}{\partial p} \frac{\partial p}{\partial \lambda_2} \frac{d\lambda_2}{d\alpha_0} + \frac{\partial P}{\partial Y} \frac{dY}{d\alpha_0} \right) + \frac{\partial p}{\partial Y} \frac{dY}{d\alpha_0}$.

We reasonably assume that the fourth and fifth order effects are negligible, so that $\frac{dp}{d\alpha_0} \approx \frac{\partial p}{\partial Y} \frac{dY}{d\alpha_0} = \left(\frac{\lambda_2(t) \beta_0}{\sigma_2 c_2} \right)^{\frac{1}{\sigma_2 - 1}} \frac{\beta_1}{\sigma_2 - 1} Y^{\frac{\beta_1 - \sigma_2 + 1}{\sigma_2 - 1}} \frac{dY}{d\alpha_0} > 0$. Taking derivative of P with respect to α_0, we obtain $\frac{dP}{d\alpha_0} = \frac{\partial P}{\partial p} \frac{dp}{d\alpha_0} + \frac{\partial P}{\partial Y} \frac{dY}{d\alpha_0} > 0$.

c) Taking derivative of n* with respect to α_0, we obtain $\frac{dn}{d\alpha_0} = \frac{\partial n}{\partial P} \frac{dP}{d\alpha_0} > 0$ and $\frac{dN}{d\alpha_0} = \frac{\partial N}{\partial n} \frac{dn}{d\alpha_0} > 0$.

The proofs for β_0, and γ_0 are analogous and omitted. #Q.e.d.

Proof of Corollary 3

The proof of Corollary 3 is analogous to Corollary 2 and is omitted. # Q.e.d.

Proof of Lemma 2

Follows from standard optimality conditions such as $\lambda_{1t} = -H_Y$ (Sethi and Thompson (2000)). #Q.e.d.

Proof of Theorem 2

Follows from standard optimality conditions such as $H_y = 0$ (Sethi and Thompson (2000)). #Q.e.d.

Proof of Corollary 4

Taking derivative of y^* with respect to α_1, we obtain:

$$\frac{dy^*}{d\alpha_1} = \frac{\partial y^*}{\partial \alpha_1} + \frac{\partial y^*}{\partial \lambda_1}\frac{d\lambda_1}{d\alpha_1} + \frac{\partial y^*}{\partial N}\frac{dN}{d\alpha_1} = \frac{\mathrm{Log}(N)}{\sigma_1 - 1}\left(\frac{\alpha_0\lambda_1}{c_1\sigma_1}\right)^{\frac{1}{\sigma_1-1}} N^{\frac{\alpha_1}{\sigma_1-1}} + \frac{\partial y^*}{\partial \lambda_1}\frac{d\lambda_1}{d\alpha_1} + \frac{\partial y^*}{\partial N}\frac{dN}{d\alpha_1}.$$

The first term (first order effect) is positive, and it is easy to prove that $\frac{\partial y^*}{\partial \lambda_1} > 0$ and $\frac{\partial y^*}{\partial N} > 0$. Both $\frac{d\lambda_1}{d\alpha_1}$ and $\frac{dN}{d\alpha_1}$ contain third and higher order effects. We assume that the first and second order effects dominate the third or higher order effects. Therefore, we have $\frac{dy^*}{d\alpha_1} > 0$. Similarly, we can prove $\frac{dp^*}{d\alpha_1} > 0$ and $\frac{dn^*}{d\alpha_1} > 0$. #Q.e.d.

Proof of Corollary 5

Taking derivative of y^* with respect to t, we obtain:

$$y_t^* = \left(\frac{\alpha_0}{c_1\sigma_1}\right)^{\frac{1}{\sigma_1-1}} \frac{1}{\sigma_1 - 1}(\lambda_1 P^{\alpha_1})^{\frac{2-\sigma_1}{\sigma_1-1}} P^{\alpha_1-1}(\lambda_{1t}P + \lambda_1\alpha_1 P_t)$$

$$= \left(\frac{\alpha_0}{c_1\sigma_1}\right)^{\frac{1}{\sigma_1-1}} \frac{1}{\sigma_1 - 1}(\lambda_1 P^{\alpha_1})^{\frac{2-\sigma_1}{\sigma_1-1}} P^{\alpha_1-1}\left(-\beta_1\lambda_2\beta_0 pPY^{\beta_1-1} + \alpha_1\lambda_1\beta_0 pY^{\beta_1}\right)$$

$$= \left(\frac{\alpha_0}{c_1\sigma_1}\right)^{\frac{1}{\sigma_1-1}} \frac{1}{\sigma_1 - 1}(\lambda_1 P^{\alpha_1})^{\frac{2-\sigma_1}{\sigma_1-1}} P^{\alpha_1-1}\beta_0 pY^{\beta_1-1}(-\beta_1\lambda_2 P + \alpha_1\lambda_1 Y)$$

The sign of y_t^* is determined by the sign of $X_1 = -\beta_1\lambda_2 P + \alpha_1\lambda_1 Y$. Taking the derivative of X_1 with respect to α_1, we obtain $\frac{dX_1}{d\alpha_1} = \frac{\partial X_1}{\partial \alpha_1} + \frac{\partial X_1}{\partial \lambda_1}\frac{d\lambda_1}{d\alpha_1} + \frac{\partial X_1}{\partial \lambda_2}\frac{d\lambda_2}{d\alpha_1} + \frac{\partial X_1}{\partial Y}\frac{dY}{d\alpha_1} + \frac{\partial X_1}{\partial P}\frac{dP}{d\alpha_1}$. The first order effect is given by $\frac{\partial X_1}{\partial \alpha_1} = \lambda_1 Y > 0$. We assume that the first order effect dominates the third and higher order effects given by $\frac{d\lambda_1}{d\alpha_1}, \frac{d\lambda_2}{d\alpha_1}, \frac{dY}{d\alpha_1}$, and $\frac{dP}{d\alpha_1}$. Therefore, we have $\frac{dX_1}{d\alpha_1} > 0$. When α_1 is small, $-\beta_1\lambda_2 P$ dominates and $X_1 < 0$ so that y_t^*<0. If α_1 is sufficiently large, $\alpha_1\lambda_1 Y$ dominates giving us $X_1 > 0$ and y_t^*>0. #Q.e.d.

Proof of Corollary 6

The proof of Corollary 6 is analogous to Corollary 5 and omitted. # Q.e.d.

Proof of Lemma 3

(i)-(iii) follow from optimality conditions in standard control theory; (iv)-(v) follow from optimality conditions in impulsive control theory (Sethi and Thompson (2000)). #Q.e.d.

Proof of Theorem 3

Follows from optimality conditions in impulsive control theory (Sethi and Thompson (2000)). #Q.e.d.

Proof of Theorem 4

Follows from standard optimality conditions, for example, $H_y = 0$ (Sethi and Thompson (2000)). #Q.e.d.

The numerical results reported in Section 5.3.3, employ the optimality conditions for the times of KTs, t_1 and t_2. The optimality condition in impulsive control theory, gives us t_i such that $H(t_i^+) = H(t_i) + H_{t_i}^I$, for i = {1,2}. For our problem and for i = {1,2}, we have $H_{t_i}^I = 0$ as well as $H(t_i^+)$ and $H(t_i)$ given below.

$$H(t_i^+) = -c_1 y(t_i^+)^{\sigma_1} - c_2 p(t_i^+)^{\sigma_2} - c_3 n(t_i^+)^{\sigma_3} + \lambda_1(t_i^+)\alpha_0 y(t_i^+)$$
$$+\lambda_2(t_i^+)\beta_0\beta_1 p(t_i^+)\overline{Y}(t_i^+) + \lambda_3(t_i^+)\gamma_0\gamma_1 n(t_i^+)\overline{P}(t_i^+)$$

$$H(t_i) = -c_1 y(t_i)^{\sigma_1} - c_2 p(t_i)^{\sigma_2} - c_3 n(t_i)^{\sigma_3} + \lambda_1(t_i)\alpha_0 y(t_i) + \lambda_2(t_i)\beta_0\beta_1 p(t_i)\overline{Y}(t_i)$$
$$+\lambda_3(t_i)\gamma_0\gamma_1 n(t_i)\overline{P}(t_i)$$

APPENDIX C

C.1 Proofs of Theorems and Corollaries

Proofs of Lemma 1

Follows from standard optimality conditions, for example, $\dot{\lambda}_1 = -H_{K_e}$ (Sethi and Thompson (2000)). #Q.e.d.

Proof of Theorem 1

Follows from standard optimality conditions, for example, $H_{y_e} = 0$ (Sethi and Thompson (2000)). #Q.e.d.

Proof of Corollary 1

Taking the first order derivatives of y_e, y_n, and θ with respect to t, we obtain:

$$\dot{y}_e^*(t) = \frac{\alpha_e \dot{\lambda}_1(t)}{2c_e} = -\frac{\alpha_e r_e}{2c_e} < 0 \quad \text{(C-1)}$$

$$\dot{y}_n^*(t) = \frac{\alpha_n \dot{\lambda}_2(t)}{2c_n} < 0 \quad \text{(C-2)}$$

$$\dot{\theta}^*(t) = \frac{\delta \dot{\lambda}_1(t) + \dot{\lambda}_3(t)}{2c_k} \quad \text{(C-3)}$$

Obviously, from Equation (C-1), we know that $\dot{y}_e^*(t) < 0$ holds. Since $\dot{\lambda}_2(t) < 0$, from Equation (C-2), we have that $\dot{y}_n^*(t) < 0$. We know $\dot{\lambda}_1(t) = -r_e < 0$ and $\dot{\lambda}_3(t) = pgQ(K_n) - \theta(c_k\theta - \delta\lambda_1 - \lambda_3) > 0$. Next, we look at how $\theta^*(t)$ changes over time in Cases 3-5. In each case, we consider four scenarios.

Case 3 (see Figure C.1): First, if $\theta^*(t) \in (0,1)$, then the rate of KT increases over time from time 0 to T. Second, if $\theta^*(0) \in (0,1)$ and there exists $t_1 \in [0, \mathrm{T}]$ that $\theta^*(t) \in (0,1)$ for $t \in [0, t_1]$ and $\theta^*(t) = 1$ for $t \in (t_1, \mathrm{T}]$. In other words, $\theta^*(t)$ increases over time before time t_1, and then remains constant ($\theta^*(t) = 1$) after t_1. Third, if $\theta^*(0) = 0$ and $\theta^*(t) \in [0,1)$, then there exists $t_2 \in [0, \mathrm{T}]$ that $\theta^*(t) = 0$ for $t \in [0, t_2]$ and $\theta^*(t) \in$

(0,1) for $t \in (t_2, \mathrm{T}]$. That is, $\theta^*(t)$ remains constant before time t_2, and increases over time after t_2. Fourth, if $\theta^*(0) = 0$ and $\theta^*(T) = 1$, then there exists $t_3, t_4 \in [0, \mathrm{T}]$ ($t_3 < t_4$) that $\theta^*(t) = 0$ for $t \in [0, t_3]$, $\theta^*(t) \in (0,1)$ for $t \in (t_3, t_4]$, and $\theta^*(t) = 1$ for $t \in (t_4, \mathrm{T}]$. Note that $\theta^*(t)$ increases over time from time t_3 to t_4.

Case 4 (see Figure C.2): First, if $\theta^*(t) \in (0,1)$, then the rate of KT decreases over time from time 0 to T. Second, if $\theta^*(0) \in (0,1)$ and $\theta^*(T) = 0$, there exists $t_5 \in [0, \mathrm{T}]$ that $\theta^*(t) \in (0,1)$ for $t \in [0, t_5]$ and $\theta^*(t) = 0$ for $t \in (t_5, \mathrm{T}]$. In other words, $\theta^*(t)$ decreases over time before time t_5, and then remains constant ($\theta^*(t) = 0$) after t_5. Third, if $\theta^*(0) = 1$ and $\theta^*(t) \in (0,1]$, then there exists $t_6 \in [0, \mathrm{T}]$ that $\theta^*(t) = 1$ for $t \in [0, t_6]$ and $\theta^*(t) \in (0,1)$ for $t \in (t_6, \mathrm{T}]$. That is, $\theta^*(t)$ remains constant ($\theta^*(t) = 1$) before time t_6, and increases over time after t_6. Fourth, if $\theta^*(0) = 1$ and $\theta^*(T) = 0$, then there exists $t_7, t_8 \in [0, \mathrm{T}]$ ($t_7 < t_8$) that $\theta^*(t) = 1$ for $t \in [0, t_7]$, $\theta^*(t) \in (0,1)$ for $t \in (t_7, t_8]$, and $\theta^*(t) = 0$ for $t \in (t_8, \mathrm{T}]$. Note that $\theta^*(t)$ decreases over time from time t_7 to t_8.

Case 5(see Figure C.3): First, if $\theta^*(0)$ and $\theta^*(T) \in (0,1)$, then the rate of KT decreases over time from time 0 to t_9 and increases over time from time t_9 to T. Second, if $\theta^*(0) = 1$ and $\theta^*(T) \in (0,1)$, then there exists $t_{10} \in [0, \mathrm{T}]$ that $\theta^*(t) = 1$ for $t \in [0, t_{10}]$, and $\theta^*(t)$ decreases over time from time 0 to t_9 and increases over time from time t_9 to T. Third, if $\theta^*(0) \in (0,1)$ and $\theta^*(T) = 1$, then there exists $t_{11} \in [0, \mathrm{T}]$ that $\theta^*(t)$ decreases over time from time 0 to t_9 and increases over time from time t_9 to t_{11}, and then $\theta^*(t) = 1$ for $t \in [t_{11}, \mathrm{T}]$. Fourth, if $\theta^*(0) = \theta^*(T) = 1$, then there exists $t_{12}, t_{13} \in [0, \mathrm{T}]$ ($t_{12} < t_{13}$) that $\theta^*(t) = 1$ for $t \in [0, t_{12}]$ and (t_{13}, T], and $\theta^*(t)$ decreases over time from time t_{12} to t_9 and increases over time from time t_9 to t_{13}.

Q.e.d.

Proof of Corollary 2

Taking derivative of $y_n{}^*$ with respect to α_n, we obtain:

$$\frac{dy_n{}^*}{d\alpha_n} = \frac{\partial y_n{}^*}{\partial \alpha_n} + \frac{\partial y_n{}^*}{\partial \lambda_2}\left[\frac{\partial \lambda_2}{\partial K_n}\frac{dK_n}{d\alpha_n} + \frac{\partial \lambda_2}{\partial Q}\frac{\partial Q}{\partial K_n}\frac{dK_n}{d\alpha_n} + \frac{\partial \lambda_2}{\partial \theta}\frac{\partial \theta}{\partial \lambda_3}\frac{\partial \lambda_3}{\partial Q}\frac{\partial Q}{\partial K_n}\frac{dK_n}{d\alpha_n}\right] \quad \text{(C-4)}$$

From the above, we see that $\frac{dy_n{}^*}{d\alpha_n}$ includes first order, third order, fourth order and sixth order effects. We reasonably assume that the first and third order effects dominate (Similar assumptions are made in Carrillo and Gaimon 2004, Heiman et al. 2001, Carrillo and Franza 2006, Gaimon et al 2012, Ozkan et al. 2012, Xiao et al. 2012. Also see Chiang and Wainwright 2005.) We simplify Equation (C-4) as:

$$\frac{dy_n{}^*}{d\alpha_n} = \frac{\partial y_n{}^*}{\partial \alpha_n} + \frac{\partial y_n{}^*}{\partial \lambda_2}\frac{\partial \lambda_2}{\partial K_n}\frac{dK_n}{d\alpha_n} = \frac{\lambda_2}{2c_n} + \frac{\alpha_n}{2c_n}\int_t^T \left(gr_n Q_{K_n}\int_0^t y_n dt\right) dt > 0 \quad \text{(C-5)}$$

Since $Q_{K_n} > 0$, we have $\frac{dy_n{}^*}{d\alpha_n} > 0$ so that $y_n{}^*$ increases as α_n increases. Analogously, we can prove that $\frac{dy_e{}^*}{d\alpha_n} = 0$, $\frac{d\theta^*}{d\alpha_n} < 0$, $\frac{dK_n}{d\alpha_n} > 0$, $\frac{dK_e}{d\alpha_n} > 0$ and $\frac{dProfit}{d\alpha_n} > 0$. # Q.e.d.

Proof of Corollaries 3 and 4

The proofs of Corollaries 3 and 4 are analogous to Corollary 2 and are omitted. # Q.e.d.

C.2 Figures

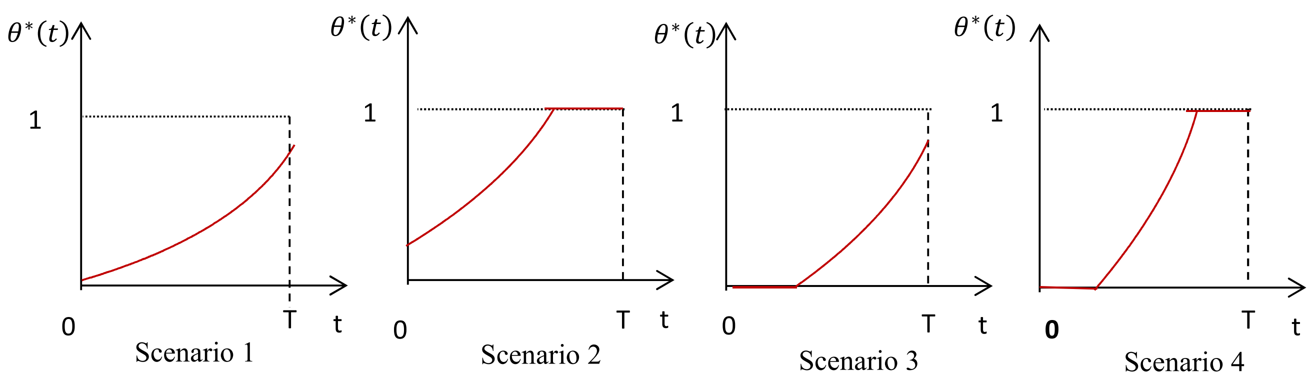

Figure C.1: Case 3-the Changes of θ^* with respect to t.: Extreme Delay

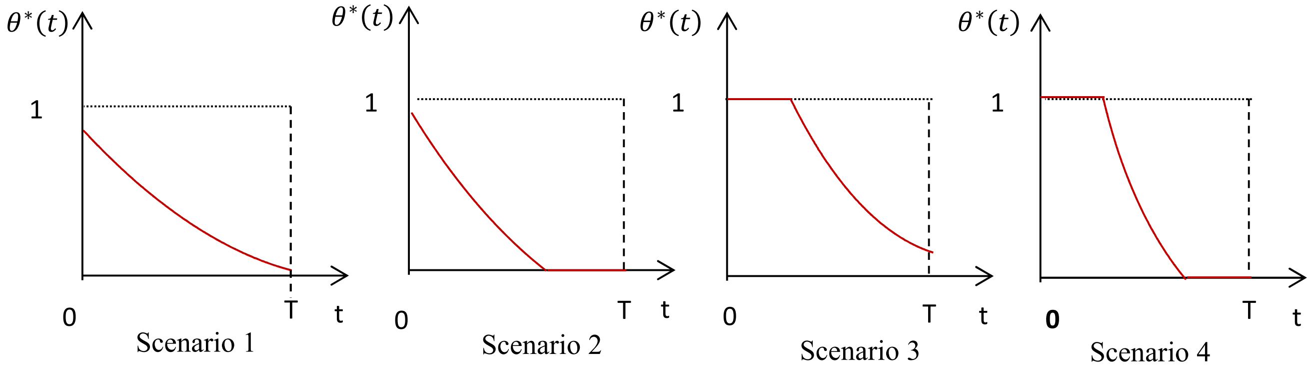

Figure C.2: Case 4-the Changes of θ^* with respect to t: Front-loading

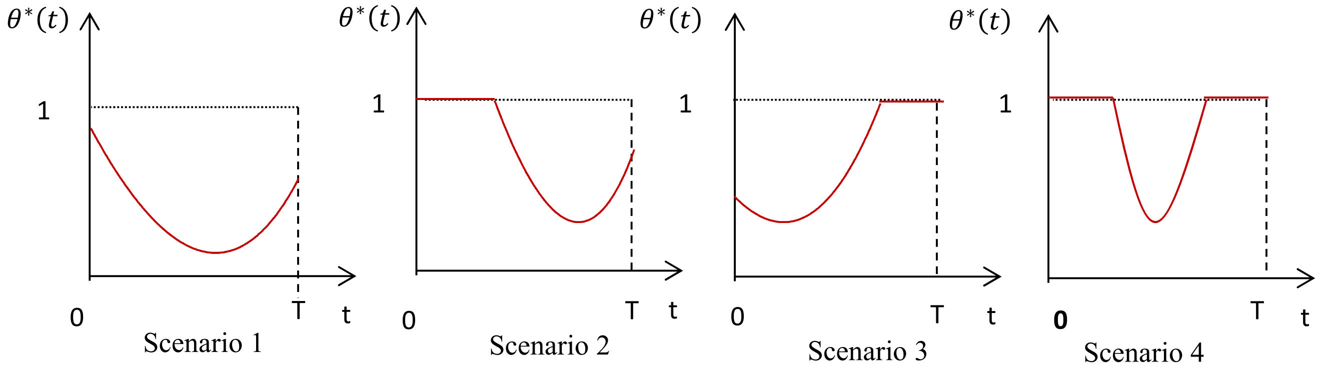

Figure C.3: Case 5-the Changes of θ^* with respect to t: Moderate Front-loading

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