A METHODOLOGY FOR QUANTITATIVE AND COOPERATIVE DECISION MAKING OF AIR MOBILITY OPERATIONAL SOLUTIONS

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John LaNay Salmon

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A METHODOLOGY FOR QUANTITATIVE AND COOPERATIVE DECISION MAKING OF AIR MOBILITY OPERATIONAL SOLUTIONS

Approved by:

Professor Dimitri N. Mavris, Advisor,
Committee Chair
School of Aerospace Engineering
Georgia Institute of Technology

Professor Daniel Schrage
School of Aerospace Engineering
Georgia Institute of Technology

Dr. Christopher Raczynski
Lead Systems Engineer
GE Power & Water

Professor Brian German
School of Aerospace Engineering
Georgia Institute of Technology

Mr. Philip Fahringer
Center for Innovation
Lockheed Martin Company

Date Approved: May 2013

For Them

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SUMMARY

Many complex and interdependent systems engineering challenges involve more than one stakeholder or decision maker. These challenges, such as the definition and acquisition of future air mobility systems, are often found in situations where resources are finite, objectives are conflicting, constraints are restricting, and uncertainty in future outcomes prevail. Air mobility operational models which simulate fleet wide behavior effects over time, in various mission scenarios, and potentially over the entire design life-cycle, are always multi-dimensional, cover a large decision space, and require significant time to generate sufficient solutions to adequately describe the design space. This challenge is coupled with the fact that, in these highly integrated solutions or acquisitions, multiple stakeholders or decision makers are required to cooperate and reach agreement in selecting or defining the requirements for the design or solution and in its costly and lengthy implementation. However, since values, attitudes and experiences are different for each decision maker, reaching consensus across the multiple criteria with different preferences and objectives is often a slow and highly convoluted process. In order to reach consensus on the solution requirements, their preferences must be in agreement.

Not only do the decision makers have different preferences or importance weightings for the multiple operational or logistical dimensions, they also have influence over each other in terms of persuading one to change their preferences. This consensus reaching process is traditionally performed through compromises, trades and other negotiation steps, all which are influenced by the relationships between decision makers. Each decision maker in the group must be willing to accept a set of

requirements all can agree upon in place of one that may benefit the decision maker directly but would not have sufficient support from all decision makers. Thus, an expensive, lengthy and ad hoc decision process is often employed where stakeholders slowly coalesce around a final solution after significant resistance and/or sacrifice of their individual preferences and self-interested objectives.

In response to these common deficiencies and provide quantitative data, this research investigates and proposes solutions to two challenges: 1) increase the speed at which operational solutions and associated requirements are generated and explored, and 2) systematize the group decision-making process, to both accelerate and improve decision making in these large operational problems requiring cooperation.

The development of the Air Mobility Operations Design (AirMOD) model is proposed to address the first challenge by implementing and leveraging surrogate models of airlift capability across a wide scenario space. In addressing the second major challenge, the proposed Multi-Agent Consensus Reaching on the Objective Space (MACRO) methodology introduces a process to reduce the feasible decision space, by identifying regions and requirements of high probability of reaching consensus. These consensus subregions of the entire design space are found by simulating multi-agent decision-making processes using game-theoretic techniques while applying the preferences of, and influence relationships between, the decision makers. The preferences of the decision makers are quantified through discrete choice experiments and inverse design filtering techniques to acquire distributions of the importance weightings for each of the objectives. Similarly, the relationships of influence between decision makers is computed with separate discrete choice experiments quantifying affinities to form coalitions with other decision makers.

The resultant decision space characterization can assist decision makers to take action more quickly and confidently from the responses to the two challenges by increasing the knowledge of the solution space through increased alternative generation

and identifying regions with high probability of reaching consensus within the group of stakeholders. The MACRO methodology is tested on a proof of concept involving the future acquisition decision of the fleet size and solution requirements for a heavy-cargo transport aircraft system, simulated by the AirMOD model.

AirMOD was found to perform remarkably well by being able to generate 1 million simulations in less than 50 minutes to cover the entire decision space. The visible trades and additional analysis only possible by this larger data set significantly increase the capability to explore air mobility solutions and potentially increase confidence and quality of decision making for requirements definition phases in system engineering problems. When the AirMOD data set was tested in a 5-agent group decision making scenario, the application of MACRO methodology was able to isolate a consensus region with less than 1% of the initial full set of operational solutions. Further analysis identified three solutions as most likely to be accepted in a consensus reaching exercise. These three solutions form the initial set of requirements that all stakeholders should initially consider to save time and resources, by removing unnecessary decision-making iterations. If one solution is found acceptable by all parties from this initial consensus region subset, then considerable resources will no longer be expended and time delays from lengthy negotiations will have been avoided. Simulation and experiments confirmed that influence relationships play a significant role in group decision-making activities and that discrete choice experiments offer an additional technique to extract not only preference distributions but power relationships. Cooperative game theoretic techniques also offer benefits when desiring to facilitate consensus reaching expeditiously by incentivizing cooperation through rewards of increased group influence and utility. Implementing the AirMOD model and MACRO methodology is found to potentially facilitate the requirements definition phase for future air mobility systems with increase speed and transparency.

CHAPTER I

INTRODUCTION AND MOTIVATION

1.1 Design and Decision Making

Engineering is about design and design is about making decisions.

Regardless of where one starts in the cycle of analyzing, decomposing or solving a problem, it invariably will end up at some point characterized as a decision which must be made. Even at the highest level of decomposition, answering the general question, "What is the problem?" requires a decision. Is the problem to meet the customer needs or stay employed? Is it to design something new or use an old design in a new way? Is to cut costs or improve performance? Is it both? Is it neither? Rittel and Webber take this notion once step further stating that "... designing systems [and therefore decision making] today is difficult because there is no consensus on what the problems are, let alone how to resolve them" [133].

Decision making is universal for almost every task and every step inside or outside of engineering activities. Although, traditionally, engineering might have been considered an activity *after* the decision is made and once a design is selected, the delineating line has become so blurred that now it has become almost a part of the decision making itself.

Today as decision making becomes an integral part of engineering, technical analyses, physics-based models, and statistics, to name a few, are brought earlier into the design or decision phases [103]. Systems engineers must now account for, understand, and design solutions with a larger set of dimensions, objectives and constraints. The additional dimensions, beyond the classical technical and performance based objectives, include financial constraints, operational logistics, and even political and societal implications.

This more recent requirement for engineers to design while maintaining a "bigger picture" of the world and the context in which a design will be implemented into a system, provided impetus to create a relatively new discipline designated *system engineering*, the development of which was accelerated through various organizations such as the International Council on Systems Engineering (INCOSE) founded in the early 1990's [78]. Further expansion of this notion, where independently functioning systems are applied or implemented across geopolitical boundaries, are physically separated, and exhibit emergent behavior providing capability that no one system can perform in isolation, requires *system-of-systems* (SoS) engineering. Examples of these problems for SoS engineering often include: power and energy production, trade and consumption, air, sea and ground transportation and the World Wide Web. The US World News has recently noted that universities themselves have moved to better educate the future problem solvers: "Engineering is at the core of so many complex global challenges—in healthcare, medicine, energy, food safety, manufacturing, communications, the environment—that grad programs have realized cross-disciplinary, even multi-disciplinary programs, are essential now to train new engineers" [65].

As these SoS engineering solutions become more complex, interdependent and sophisticated, the decision-making processes and algorithms themselves require concurrent development. Thus, the decision making that complement systems and SoS engineering must be equally multi-dimensional, adaptable and powerful. Borrowing the phraseology from a classical quote attributed to Einstein[1], one could state that "we cannot perform decision making in the same way we have for these more integrated and complicated problems."

Furthermore, these highly integrated and complicated problems are no longer

[1] "We can't solve problems by using the same kind of thinking we used when we created them." - Albert Einstein

solved by any one person or entity. They impact multiple people and they are solved by multiple people. Moreover, the funding for such solutions would likewise come from an even wider set of stakeholders: "In fact, it is much more likely that individual systems will be funded by a diverse group of organizations, and the goals of the funding sources for individual elements may not align with the goals of the SoS stakeholders" [72]. Likewise, the expertise in different dimensions of the solution is found in different individuals or entities and thus the decision making is similarly performed collectively as a group. Collaboration and cooperation become more than good ideas, they become essential. The final decisions may still slowly evolve over time such as standards for Internet protocols or be implemented relatively quickly such as in recent economic bailouts, but regardless, multiple stakeholders are involved and each can influence the final decision whatever it may be.

This multi-agent, or group decision making, in the context of systems engineering, and an approach to improving this often overlooked but highly crucial activity from an engineering perspective, is the general topic of this research.

1.2 Engineers as Decision Makers

Former Congressman Vern Ehlers once declared that "What the country desperately needs is more scientists and engineers in the public office at all levels" [147]. Current President Barack Obama has written "I wish the country had fewer lawyers and more engineers" [118]. Despite various backgrounds, educations and experiences, these public officers share the same perspective that what may be missing from public institutions and from the decision and policy making processes are not necessarily "engineers" in form but the engineering ideals and people possessing the desperately needed skills of analytical reasoning, objectivity and mathematical rigor that engineers commonly possess.

Many more have argued that government officials do not, but should, apply more of the scientific method in their leadership and decision-making activities, perhaps sharing the thought that "[s]ome quantitative researchers believe that systematic statistical analysis is the only road to truth in the social sciences" [89]. Some have suggested that policy makers do not take time to sift through data or use information pertinent to the current issues, resulting in significantly less effective decisions. Other individuals have wondered if engineering practices and scientific methods can and should be applied more often to social or economic challenges. At least one organization, Scientists and Engineers for America, have made it their mission "to promote evidence-based decision making at all levels of our government" [147].

But why are engineers, or more precisely the engineering principles, often turned to as a potential source for answers to difficult challenges? The responses may vary but a key element to the answer could be found in a statement by Hilberry: "All solutions in engineering are compromises" (quoted in [194]). Thus, any solution or engineering design comes as the result of testing and evaluating trades between options or variables, balancing the pros and cons, and comparing the candidate solutions across output metrics (e.g. time, cost, risk) in order to meet specified or implied requirements. These "compromises" are inherently a part of any engineering exercise where the resources are limited, arguably a necessary condition for any real challenge. The trade-offs and compromises a skillful engineer must grapple with in design are similar in kind to the challenges government leaders, business executives, and other professionals face continually. These challenges are always, to a certain extent, multidimensional, complex, interdependent, multidisciplinary and non-trivial.

Furthermore, they are always found in situations where resources are finite, objectives are conflicting, constraints are restricting, and uncertainty in future outcomes prevails. Blanchard and Fabrycky opined that "there is usually little assurance that predicted futures will coincide with actual futures. The physical and economic elements on which a course of action depends may vary from their estimated values because of chance causes...[T]his lack of certainty about the future makes decision making one of the most challenging tasks faced by individuals, industry, and government" [22]. However, decisions are and will be continually made at varying degrees of success for these important and multi-faceted problems.

Since the systems engineering field has experience in these types of problems and have developed tools to understand, decompose and analyze the relationships and characteristics of large complex problems, it should be of no surprise that government officials, including the President of the United States, are interested in applying these skills to decision making for current societal concerns as well. Although there may be areas and challenges where the scientific method and engineering skill sets are helpful, the over extension of these tools into so called "wicked" problems also deserves a warning from Rittel: "The kinds of problems that planners deal with-societal problems-are inherently different from the problems that scientists and perhaps some classes of engineers deal with" [133]. However, modeling and simulation, theory and experimentation, and computer and technological advancements have not only improved but accelerated in the four decades since this warning was given. True, there are problems which are ill-defined and ones that do not have a solution, or if they do, have a temporary solution, but limiting one's options for solving a problem, or at least, improving the current state would be unwise.

Therefore, a key caveat in this research is that the ideas presented herein are not a panacea for all problems. At the highest level, this research seeks to take a few steps to more fully involve and apply engineering principles into decision-making processes and push the state-of-the-art in group decision-making activities for multi-dimensional problems.

1.3 *The Era of Big Data*

One of the ways that computer technology has aided in quantifying or, at a minimum, understanding these highly complex problems is through both more data and better data. Improved statistical models and methods have allowed the analyst an ever increasing suite of tools with which to investigate the multi-dimensional solution spaces. Coupled with accelerated computational speeds and memory availability, the number of options, alternatives and candidate solutions to various problems have grown exponentially. Surrogate models and other advanced design methods have further enabled this explosion of multi-dimensional data [47]. The former challenge of "insufficient data" has become a challenge to find the "needle in the haystack" or piece of information among the gigabytes to terabytes of data stored and available. The relatively new multidisciplinary field of visual analytics has become increasingly important as the data to analyze and interpret becomes ever larger [104].

In terms of decision making, a larger set of alternatives can be offered and a literal uncountable set of solutions can be modeled and potentially implemented. The trend is likely to occur as decision makers seek greater levels of confidence that the best solution is available and can be found, tested and analyzed. The flip side to this "big data" revolution is the need for more sophisticated methods to explore and analyze, not one at a time, but in bulk or parametrically. Having a larger number of designs might be a good thing as it potentially increases the chances that the one ideal solution has been modeled, but it may require increased time to find the "needle in the haystack" and make the design selection. These two initially contradictory goals: 1) increase definition and resolution of the design space with more data points, and 2) accelerate decision times without sacrificing quality, are in fact possible, but require methodological advancements and improvements to modeling efforts such that better and faster decisions can be made.

In order to grapple with these goals and the ever increasing amounts of data, the analyst and/or decision maker need improved tools and decision making environments to play games and perform more in depth sensitivity analyses and exploration activities. Since the problems faced by these decision making groups are highly integrated and complex, the assumptions and objectives of all, or any one decision maker, may be initially unknown. Furthermore, their preferences can change, especially as each learns or gains more information about the problem from exploring the "big data" associated with better defined solution spaces. Similarly, no static analysis tool is sufficient and can adequately account for an expected evolving preference or group objective function over time. Tools must be quick, dynamic, and able to process large amounts of data to properly account for the multi-dimensional design spaces. The decision making in these types of problems will not be a simple, linear sequence of steps and thus the associated tools that facilitate group decision with big data must be made equal to the task.

1.4 Decision Making for Requirements Definition

In no place are decisions more significant than in the early phases of the acquisition process, before the pre-conceptual or requirement definition phases. The decisions made at these points in the process can drive a disproportionately larger percentage of the ultimate cost and designs of any future system. In essence, the desires and preferences of those individuals or organizations forming the requirements drive what eventually is generated as solution alternatives, and thus the design selection at later phases.

Efforts to better facilitate parts of this overall process has been recently revised from the Requirements Generation System (RGS) into the Joint Capabilities Integration and Development System (JCIDS). However, limitations still exists, and research addressing the various challenges of the JCIDS acquisition process has provided a number of potential improvements [72]. Two of those improvement areas involve accelerating the time at which parts of the process are implemented and increasing the traceability of decisions. Advancing the way in which groups of decision makers reach consensus for the requirements definition phase for major acquisition programs could potentially shorten the total time for system development and later assist with keeping system development programs from straying by resisting "requirements creep" from enhanced traceability. This, in turn, could save costs and increase the future success rate for acquisition programs.

1.4.1 Requirements Issues with the VH-71A U.S. Presidential Helicopter

A recent and very public example of one such failure, in part due to requirements issues, is the US presidential helicopter, the VH-71A. As winners of the 2002 VXX competition, the VH-71A was initially awarded to Lockheed Martin and AgustaWestland in 2005, to design and produce 23 helicopters which would replace the aging fleet of VH-3D and VH-60N helicopters. The initial contract was for $1.7 billion dollars [176] for the system development and demonstration phase.

However, very soon after the award, cost overruns and schedule slips were already projected due to changes or misinformation as understood between the various parties. A statement by Lockheed Martin encapsulated the major issue: "Immediately following contract award, Lockheed Martin and Navair [US Naval Air Systems Command] realized there was a difference in understanding about operational and technical requirements and how to develop and test the subsystems and the aircraft" and also that "additional required aircraft modifications, enhanced safety, additional testing and enhanced supportability" [184] were also part of the requirements creep that was observed. Much later these failures were described as "runaway requirements, program delays, and soaring costs" [90].

Some have suggested that the result of such requirements creep was that the "US

Figure 1: Conceptual Drawing of the VH-71A U.S. Presidential Helicopter which was canceled in 2009 (From [109])

Navy source-selection team was kept separate from the rest of Navair during the VXX competition" [184]. In other words, transparency between groups was not enforced and thus requirements came a surprise as the new group of decisions makers who held influence in later increments or stages had different preferences.

These changes of the decision makers and other unforeseen factors, which could have been mitigated, were not considered or accounted for in the overall program. The final outcome was a cancellation of the VH-71A program by Department of Defense (DOD) in 2009 after a doubling of the expected costs from 2005 to 2009. One of the identified reasons was again related to requirements: "Stringent performance requirements (some with no flexibility) were laid out for the system prior to the start of development and did not appear to involve significant consideration of trade-offs of cost, performance, and schedule negotiated between the customer and the developer" [179]. Analysis revealed the final cost was expected to total $13 billion, up from $6.5 billion estimated in 2005, for the 23 helicopters (or close to $600 million per vehicle). The U.S. Governmental Accounting Office reported total program expenditures up to the cancellation date summed to near $3 billion [179].

The requirements definition phase can be a severe stumbling block, if not performed adequately, to the success of a program. Furthermore, since so many decision makers have much at stake in the decision, the process can be held up much longer than desirable by the final user. After all, decisions makers are prone to change and may expect flexibility in others while ironically not recognizing it in themselves. Not only are the decision makers themselves replaced with others over time but their preferences can shift, reverse, or "move" based on experience, information and external pressures. This can have negative impacts on the overall process (e.g. cancellation) if these probable events are not considered. Therefore, a real need exists to explore ways to improve the group decision making process applied to this requirements definition phase of the acquisition process.

1.5 Motivating Problem: Requirements Definition for Future Air Mobility Systems

Since the fall of the Soviet Union in 1991, the US Department of Defense (DoD) has slowly moved away from traditional acquisition policies dominating the Cold War, into a policy where capability and affordability reign as the governing principles for making decisions about asset procurement. This paradigm shift most recently redefined in the 2012 Joint Capabilities Integration and Development System (JCIDS) [32], focuses on the process to obtain a more agile and flexible force to meet the needs of the modern war fighter.

Furthermore, under an environment of increased scrutiny and projected decreasing budgets into the next decade [44], the most recent DoD plan to cut costs "incorporates all areas from potential savings, to force structure enhancements, modifications, and adjustments" [120]. Some of these have already been initiated, most dramatically in the reduced procurements of F-35s (1591 to 365 from 2002 through 2017 [180]) and F-22s (750 to 195 [186]) in the last few years.

Other aircraft programs have been cut or reduced, and less expensive strategies such as service life extension programs (SLEP) have become more common to modify or upgrade aging, but existing, aircraft. One of these aircraft systems is the C-5 Galaxy.

1.5.1 The C-5 Galaxy

The C-5 Galaxy is the largest heavy-lift transport aircraft in the US military. It is the only cargo aircraft capable of transporting some of the heaviest combat equipment including two combat ready Abrams tanks, six Apache attack helicopters or the 74-ton mobile scissors bridge, up to a maximum payload of 270,000 pounds [175]. With cargo space for 36 standard pallets or 10 light armored vehicles coupled with both a rear and nose door with full width drive-on ramps allows for expedited loading of only 2 hours [46]. These and other features make the C-5's high carrying capacity a very attractive capability to maintain. This "astounding capability" of the C-5, as described by General John W. Handy, former commander of the US Transportation Command and Air Mobility Command, is why "[w]e certainly need to keep [it] at our fingertips for as far as I can see into the future." The C-5 was also described by Handy as an aircraft "whose value is dramatically underappreciated" [167].

However, the C-5 has suffered from low mission capable rates due to reliability and maintainability issues for years. For example, one estimate for the maintenance man hours per flight hour reached as high as 46 for the C-5A variant [71].

Strategic reviews found that C-5 platform met a key requirement for the US military and two programs were initiated to upgrade and modify the C-5 in response to the poor operational and sustainment metrics observed. The first of these two programs, the Avionics Modernization Program (AMP), seeks to "replace the existing flight and engine instrument system and the flight control system with integrated, state-of-the-art, cost-effective, highly reliable and capable systems " [45]. The second program, the Reliability Enhancement and Re-engining Program (RERP), will improve the logistics and sustainment metrics of the C-5 "by replacing the propulsion system and modifying the mechanical, hydraulic, avionics, fuel, and landing gear systems as well as other structural modifications" [177]. C-5 aircraft which have successfully completed both programs are designated C-5M Super Galaxy aircraft. As of December 2012, only 6 production C-5M's have been delivered out of 9 current C-5M's overall [101]. From the initial estimate of fully modernizing 112 C-5s, the number was reduced to only 52 in February 2008 [174] with the potential for even more reductions. As recent as October 2011, proposals to reduce the fleet by another 15 aircraft was offered as potential ways to save additional funds in an "Age of Austerity" [12].

Figure 2: The C5-M Super Galaxy During its First Flight in 2006 (From [175])

1.5.2 Strategic Airlift Comparison Tool (SACT)

In 2010, efforts to model the capability of the C-5M over the C-5A platform was conducted resulting in a tool called the Strategic Airlift Comparison Tool (SACT)

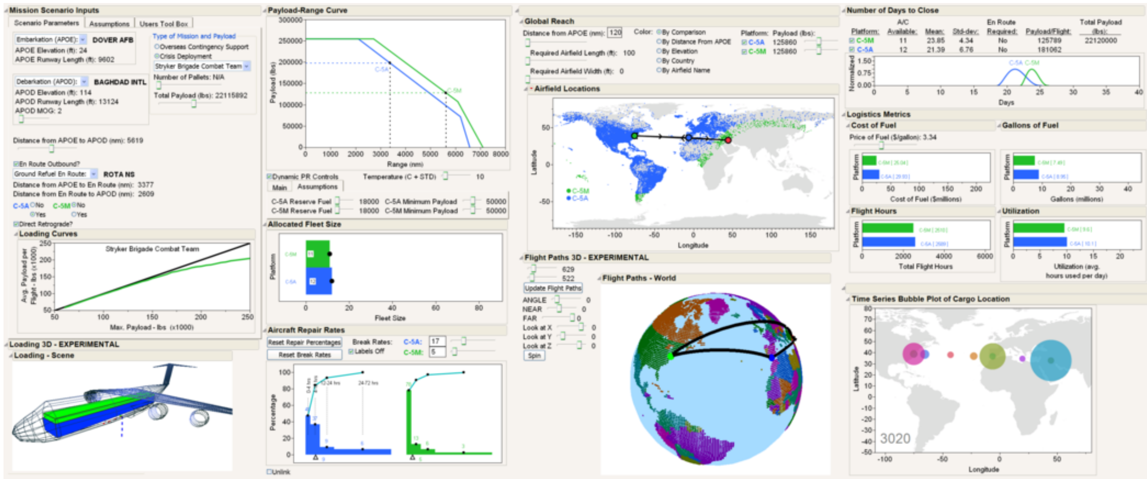


Figure 3: Screen Shot of the Strategic Airlift Comparison Tool (SACT) [142]

[142] to demonstrate and analyze the benefits from an operations and logistics perspective of the C-5M aircraft (see Figure 3). A user of the SACT tool was able to define various mission scenarios including mission payload, payload types, locations, maximum on ground (MOG) and other input scenario parameters. The logistic outputs included the mission time to close, fuel consumption flight hours, utilization, etc. Various trades with fleet sizes and refueling en route across platforms against the output metrics of interest were also available. SACT was developed to provide a modeling and simulations environment which could rapidly evaluate and compare scenarios with various enablers such as surrogate modeling techniques, to allow for increased and faster trade space analysis and improved decision making.

The tool has since been updated and enhanced internally at Lockheed Martin, with additional data integrated into the analysis framework.

1.5.3 Requirement for Future Military Transport Aircraft

Today the C-5 RERP program is still under careful scrutiny and the future uncertain with the continual looming possibility of further reductions. In fact, the most recent FY 2012 annual report from the Office of the Director, Operational Test & Evaluation reported a less than ideal observation that “C-5 Program Office continues to address deficiencies identified during the C-5M initial operational tests in 2010" [166].

Whether or not drastic reductions occur, the problem remains about how to maintain the carrying capacity of the current C-5 fleet into the future but at reduced cost. If upgrades and modifications become so expensive per unit, is it more reasonable to consider an all new design? Could the new design be a derivative of the C-5? Are schedule slips for modernization tolerable with an ever aging fleet? Could a smaller but more reliable fleet maintain the same capability as the current one? Would it be cheaper in the long term? Is it cheaper in the short term? Is the cost per flying hour still too high for the modified C-5? Can the military get by with a fleet with mostly C-17s?

These and other questions quickly cover the design or solution space for this multi-dimensional problem. Defining the requirements for a future fleet of heavy lift transport aircraft is a multi-agent, highly convoluted problem. The various stakeholders may have differing views on the future threats or locations in need for transport. Regarding the differences of these perspectives Ostrosi et al. conclude that "[d]ifferent disciplines, with the subjective nature of the opinions of actors, often reflect latent conflicts in actors' commitments" [124]. Thus, they may have conflicting opinions about funds available for such a fleet or diverging estimates on the actual needed carrying capacity. Lastly, they will likely differ in the importance they place on performance, cost, availability and maintainability described succinctly by Jemison and Sitkin: "Acquisitions are strategic, complex, occur sporadically (for most firms), and affect varied stakeholder groups and multiple actors whose involvement is temporally and functionally divided. These factors, in combination, result in an acquisitions process that is both discontinuous and fractionated" [82].

The two conflicting goals are again exemplified in this aforementioned challenge, namely, 1) generating alternatives or solutions, and 2) evaluating and selecting the design.

Firstly, the design space must be sufficiently defined and enough alternatives generated so as to adequately cover all possible combinations of solutions. This, in turn, requires a model fast enough to populate the entire design space with data such that all possibilities are considered and analyzed. A slow, non-parametric model will be ineffective in providing this capability. Surrogate models with Monte Carlo simulations in sufficiently wide ranges are needed to account for the various perspectives and preferences of all stakeholders and decision makers. If the model leaves spaces unexplored, the potential for one, many or all stakeholders to reject the data based on its biased or uneven exploration of the space is possible. Developing and generating the required data for these particular operational space decision-making activities is the first major objective of this research.

Secondly, the solution for such a problem will clearly involve more than one stakeholder or decision maker. The necessary decision-making process can be slow, inefficient and wasteful, or the process can facilitate clarity in determining preferences of all the stakeholders quickly and in identifying solutions that all can potentially agree upon as quickly as possible. Multiple decision makers will each seek to maximize their own objective or utility function. At the same time, the solution space requires the cooperation of all stakeholders and thus suggesting or guiding the group to the ultimate solution or at least to the region of the space at which cooperation will be most likely to occur is needed. This covers the second major objective of this research.

1.6 Other Group Decision-Making Challenges in Aerospace Systems

The following examples point out additional ramifications of group or multi-agent decision making for challenges involving aerospace systems. These examples should not be considered a complete discussion of all of the aspects of the challenge but a short summary to illustrate a particular point observed in multi-agent decision making. These examples, selected for their aerospace related elements, have been heavily publicized in both the domestic and global arenas.

1.6.1 The International Space Station

The International Space Station (ISS) is a multi-national endeavor to have a permanent human presence in space. The ISS, stationed in a low-earth orbit, is considered a research facility that provides an environment for space-related experimentation such as testing for future space exploration and other objectives.

Four countries, the United States, Russia, Japan, and Canada, as well as several European Union member countries, with their respective space agencies, are considered partners in the ISS program. Each has their own agenda and reasons for cooperating with the other countries to make the ISS successful. Each country has similarly contributed different parts to the station and have benefited in different ways. Currently, discussions are underway to decide the fate of the ISS in the next few decades. Although orbiting since 1998, criticisms of high cost in connection with little scientific contributions from the meager experiments conducted, has resulted in debates about funding continuation. Furthermore, other countries have expressed interest in joining the ISS program, including China, India and South Korea. Brazil was also part of the program before but has since departed from participating [88].

With the changing number of decision makers or countries with changing amounts of interest or ways to contribute, a constant decision-making process is in continual flux and with various support levels from different countries. How best can the partner nations meet the needs of the current and future stakeholders? Who has a say on the design, schedule and staffing of the ISS? How much will each country contribute and control in terms of life-cycle or utilization?

1.6.2 The Joint Strike Fighter

The Joint Strike Fighter (JSF) illustrates a similar agreement and partnership between nation states. However, the structure of partnerships establishes three levels within the JSF program. The United Kingdom is a level one partner, Italy and the Netherlands are level two, and Turkey, Australia, Norway, Denmark, and Canada are all level three [173]. Since partners at level two contributed more financially to the program, would they expect more influence over the design decisions than level three partners? How do non-financial contributions influence the design? Does the type of relationship with the United States (i.e. geographically, ideologically, or fiscally) give one nation more or less influence? Or do partner nations have almost no influence in the design and are simply "first customers" of the platform? How is Lockheed Martin, the prime contractor, influenced by partner nations or how do they influence others to increase or maintain current order levels?

1.6.3 USAF KC-X Competition

The USAF's KC-X air refueling tanker competition, which was rife with scandal, political maneuvering, and problems, required almost a decade to "decide" the winner. Multiple stakeholders including the USAF, Boeing, Northrop Grumman, EADS and others spent many years and millions of dollars to influence others and the key decision makers in various ways.

Many of the strategic maneuvers such as protesting evaluation methods or threatening to withdraw from the competition, were perhaps calculated, yet they still drove how the decision makers responded, underlying the fact that decisions perhaps are not entirely based on the engineering design and on, at least in part, subjective factors [150]. How can this type of group decision making be avoided? Or at least how can it be accelerated so as to decrease the time required?

1.6.4 Observations from Contemporary Examples

The above examples illustrate some common issues with multi-agent or group decision making. Often the set of decision makers is unspecified or at best changing. These same decision makers do not always recognize, or want to use the power or influence they have on other agents, especially if it does not further their own agenda or objectives. Similarly, they might not even know their agenda or at least the outcomes of any one decision since they, understandably, don't know how other agents will act. Lastly, the decision on how to act or what solution to implement can affect the relationship of the decision makers into the future, yet another dimension to consider when exploring the entire solution space.

For multi-agent decision making the attributes of, and relationships between, decision makers is as important or more important to the eventual outcome of an decision-making activity. Since engineering design and, in particular, systems engineering is so heavily composed of decision-making activities, in complex problems involving more than one decision maker, there exists a need to better understand and quantify how decisions are made in multi-agents systems. The interactions between decision makers and the changes to the individual preferences can potentially drive the decisions more than the actual objective data.

1.7 Focus of this Research

The main focus of this research is to 1) better define and characterize the design space by being enabled to generate greater number of alternative solutions quickly, and 2) develop a systematic methodology when dealing with problems where the engineering design or solution is dependent on the preferences of more than one decision maker and each decision maker must be in agreement or reach consensus on the final design point. Providing insight as to how one should act within a multi-agent decision-making environment can significantly improve how agents make decisions and reduce the time necessary for agents to take action. Furthermore, with an understanding of the general characteristics of the eventual agreed upon design point, decision makers can begin to hedge their design or contribution to the solution by investing early. In other words, if one knows with some level of confidence where all the decision makers will eventually "end up" in terms of an agreed upon decision point, they can take actions sooner, reducing costs and improve performance. For example, if an engine manufacturing company can ascertain with some degree of confidence that the eventually selected design point will have a certain thrust or other requirement, they can take steps to prepare in advance instead of waiting for the decision to be made. Thus, their actions can be robust against the possible decision outcomes of the multi-agent environment. Similarly, an agent can perform analysis to identify if entering into a negotiation or discussion with another agent is feasible by evaluating the likely output decision points based upon the preferences and political clout of the various agents.

The methodology developed throughout this research should not be considered a technique for ranking alternatives or selecting the best solution by combining agents' preferences, which have been shown to have significant limitations [5]. Instead, it should be viewed as a probabilistic or heuristic approach to identify areas of the design space where decision makers are more likely to reach a consensus. Potentially, this strategy will be independent of the type of decision-making technique ultimately used, since the focus will be on how the preferences of decision makers become aligned over the events (e.g. trading importance weightings or criteria preferences, forming alliances, etc.) usually held static within a decision-making technique.

Clearly, different preferences among the decision makers would make different points or solutions more attractive. In the general case, therefore, decision makers will prefer different solutions, at least initially. When the underlying assumption that decision makers are incentivized to cooperate and reach a consensus, or, that reaching a decision is more favorable for all decision makers than not reaching agreement, compromises are expected, trades will be performed and adjustments to one's preferences will occur.

How these compromises, trades and preference adjustments factor into the decision of a multi-agent system is not always analyzed from a systems engineering perspective. Applying engineering principles and techniques to the decision process directly can facilitate this consensus reaching thereby reducing the decision time and making preferences more transparent and tractable.

1.8 Dissertation Organization

This dissertation has been divided into six chapters. The next chapter, Chapter 2, reviews some of the observations about the motivating problems and other contemporary challenges summarized in this introductory chapter, followed by a formalization of the research objective, research questions and hypotheses. Chapter 3 describes some of the background information and technical foundation for the methodology. Chapter 4 presents an overview of the methodology with its constituent elements, AirMOD and MACRO. Chapter 5 describes the development and implementation of the first element, the AirMOD model. In Chapter 6, the second element, the MACRO methodology, is further tested and developed by stepping through a canonical problem while discussing the various assumptions with a variety of examples and experiments. Chapter 7 discusses the application of the MACRO methodology to the AirMOD data set and air mobility problem introduced in the current chapter and analyzes the results and findings. The final chapter concludes this research dissertation and underscores some of the key discoveries, contributions, benefits and limitations of the methodology.

CHAPTER II

RESEARCH OBJECTIVE, QUESTIONS AND HYPOTHESES

2.1 Review of Observations

The following list of observations is made regarding the air mobility challenge from the previous chapter and the other related aerospace examples with their attendant decision-making attributes.

Observation: Utility functions, such as an overall evaluation criterion, can be interdependent between decision makers. The various individuals in a group decision-making process such as selecting the fleet size and design of the future airlift capability are interdependent. Their preferences may be dependent on each other's perspectives and expertise, especially in highly integrated and complex problems.

Observation: Preferences or weightings on the dimensions or objectives are subject to change with different information or external influences. People are prone to change and can be persuaded to alter their opinion and preferences by others or by additional information.

Observation: Ultimate decisions are often influenced by external dimensions, constraints or factors which are not part of the classic decision-making process. Additional unaccounted dimensions to the problems, non-quantifiable constraints and other hidden objectives can often drive the underlying decision process. Group decision making with completely rational and objective stakeholders is at best rare and more likely non-existent.

Observation: The initial design point from which decision makers begin their deliberations has a strong impact or influence on the eventual final decision. The initially preferred design point for each individual, can and does affect how a group of decision makers negotiate, apply trade-offs and reach consensus, through anchoring and expected compromises.

Observation: For large scale, multi-agent multi-criteria challenges, the time lost to make a better or "best decision" can potentially be used for incremental improvements of an "early and good decision." The large amount of time necessary to make a decision for some acquisition involving large and highly integrated systems, could be displaced to improving the system once a faster but good enough decision has been make.

Observation: The number and attributes of decision makers can change over time. Decision makers can enter and leave a group decision-making process at increasing occurrences when the total time period to reach consensus is longer. Reducing the time at which decision makers need to reach consensus in group decisions, can avoid some of the negative problems of continual group restructuring.

Observation: Influence or power between decision makers is a real element to any group decision and can fluctuate based on various factors. Position, experience and expertise are just some of the intangible sources which play a role in group decision making. More transparent power is evident through group members' roles such as buyer/seller roles or investment willingness of the various agents, but many others are more private and elusive such as interpersonal abilities or negotiation skill-sets.

2.2 Research Objective

In a number of the aforementioned observations, the relationships between decision makers play a significant role in the final decision. As systems engineers seek to solve real challenges where:

- more than one agent has influence on the decision,
- the number of agents can vary over time,

- the agents are uncertain of their own preferences,
- the agents are influenced by subjective factors, and,
- the agents can change their preferences with more information,

the decision-making aspects of engineering design will need to account for these added complexities with greater skill in the future to minimize the bad decisions resulting in wasted time and/or money. Furthermore, a proper understanding of the dynamics between decision makers can allow designers, or decision makers themselves, to be more confident in taking robust action against the uncertainty in the multi-agent, decision-making process. Lastly, the decision space itself must be sufficiently defined and populated for confidence that enough solutions are on the table for selection. Within operational systems characterized by numerous entities with multiple dimensions and outputs metrics tracked, the models generating the alternative solutions is both slow and potentially burdensome.

Therefore, in connection with these observations and the need for more expeditious alternative generation, there exists a need to develop a systematic approach or methodology to address the concerns of multi-agent decision making concurrently with improved operational design space definition for ultimately enhanced requirement definition for future air mobility solutions. The objective of this proposed research is expressed as follows:

Research Objective: *Improve group decision making for requirements definition in cooperative air mobility operational solutions, by 1) generating greater numbers of operational design solutions, and 2) identifying the subset of feasible designs by accounting for preference uncertainty and the decision makers' influence relationships.*

2.3 Research Questions and Hypotheses

The research objective of this proposal leads to a number of research questions which will guide the formulation of the hypotheses and eventual development of the methodology.

2.3.1 Research Question #1 and Hypothesis #1

The first need as established in the objective is to increase the available designs which populate sufficiently the entire design space. With operational models often requiring large times to execute, most decisions are made with far fewer operation solutions than desired. The associated research question for the first half of the research objective is thus:

Research Question #1: *How can the number of operational design solutions originally considered increase to include a greater portion of the potential design space?*

At once, the answer to such a question hints at accelerating the speed at which the operational model is executed. Since most useful operational models are complicated discrete event simulation models, the possibility to regress a sufficient number of output simulations, and recast them as surrogate models can allow one to then quickly query or calculate any design point within the region for which the surrogate model is valid. Formalizing this hypothesis becomes:

Hypothesis #1: *Monte Carlo simulations of surrogate models developed around time-consuming operational models will provide capability to more rapidly define the design space by generating greater numbers of candidate solutions in the same time period.*

2.3.2 Research Question #2 and Hypothesis #2

One can assume that all agents seek individually to make better decisions and any additional information in assisting what the eventual outcome would be under various circumstances is desirable. If information was available providing some confidence about which designs or solutions would be ultimately selected as a group, then this would predictably help and guide the actions of the particular decision maker, either by causing them to act more quickly, or at least raising the confidence of future actions.

Even if the exact solution which the set of agents may agree upon cannot be predicted accurately, identifying the *region* of the decision space that may be most likely to include an area of agreement can benefit the decision makers, both individually and as a group. Therefore, the following research question formalizes this inquiry:

Research Question #2: *How can the feasible decision space be reduced to facilitate decision makers in reaching consensus?*

Individually, a decision maker who has confidence the dynamics of the group will reach agreement, for example, in region A of Figure 4, can investigate the characteristics of the nearby solutions and begin research, investment, etc. which will benefit them under these types of solutions. In this sense, they can hedge their efforts, time or resources to preparing for some design or solution in that area while the group is still reaching agreement, likely to be in that region. Although, the final decision may not be exactly in the "middle" of region A, the individual agent likely has had time to create a robust response and maximize the payoff of, or at a minimum begin development on some solution from within that region. Thus, one agent can strategize and take action sooner if information about common design characteristics, types or attributes of the solutions will be reached.

As a group, if decision makers can collectively accept that after days, weeks, months or some other time period, they are likely to be converging on region B (in Figure 4), for example, then their time should be devoted to analyzing solutions within or near that region; effectively shrinking down the total decision space from the black lined area to the blue lined area. With this method, the solution (or rather

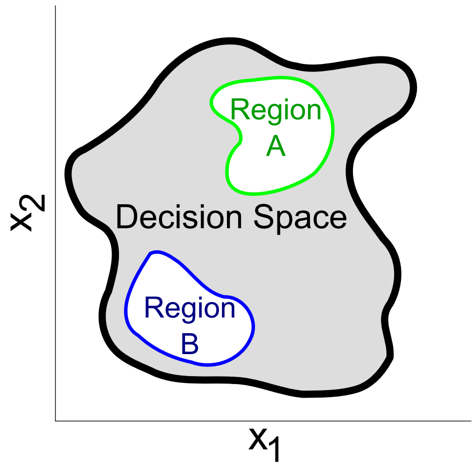


Figure 4: Potential Expected Regions Where Decision Makers may Reach Consensus

solution set or region) is one of "satisficing," in that the decision makers are willing to accept a solution which is sufficiently "better" for all agents at the cost of not exploring in detail the total decision space which may be prohibitively large. Since no real "optimal" solution exists without a known objective function, collectively accepting a satisficing solution earlier is also desirable as it can save both time and resources for many decision makers.

Modeling and simulating these actions with appropriate game theoretic techniques and data, in particular information about the individual preferences and power interrelationships, the related hypothesis can be stated as:

Hypothesis #2: *Simulating the multi-agent decision-making process with an iterated ultimatum game across all objectives, with the application of the preference distributions of, and power relationships between, agents, will significantly reduce the decision space and identify regions with high probabilities of reaching consensus.*

The metric to evaluate if Hypothesis #2 is valid or "not rejected" is the ratio between the number of designs at which the group decision could reach consensus and the full set of candidate points from the initial data. Ideally, this ratio is as small as possible (approaching one over the total number of initial designs at deterministic preferences) but can remain high with inputs such as the low numbers of discrete choices to which individual decisions makers chose to respond or a low number of iterations of other steps in the methodology. In this sense, Hypothesis #2 will be at least supported (marginally) if this ratio is less than 0.5, with a target value of less than 0.1 to show high utility.

2.3.3 Research Question #3 and Hypothesis #3

One of the essential pieces of information for testing Hypothesis #2 was the preference distribution information from each decision maker. Ideally preferences are rigid, non-changing and perhaps even deterministic, but the observations indicate a large propensity for decisions makers to potentially change their "minds" and thus their preferences, based on additional information or even in the company of others with influence. Acquiring these data more quickly without increasing the work load of any decision maker is a desirable goal. Formulating this into a research question, provides the following:

Research Question #3: *How can a decision maker's preference information, including the potential for changing preferences, across all objectives be acquired more quickly and accurately?*

This question addresses the need to quantify how a decision maker values each of the criteria or objectives. Any value system is based on the beliefs, experiences and information available to a decision maker. These three factors can result in an agent changing the importance of any particular dimension over time. For example, during times of economic prosperity, reducing negative environmental effects such as emissions from aircraft becomes more important. In times of slow economic growth, environmental concerns often take a secondary role to more short terms problems. But how quickly is one willing to change preferences between criteria and to what extent?

A weighting distribution can provide clues as to the range of weights a decision maker might be willing to apply under different circumstances. If a particular weighting distribution has zero probability of being less than 0.5, for example, it would mean that over many different circumstances that particular criterion is of most importance. Similarly, if the weight can cover a range from low importance (i.e. 0) to a high importance (i.e. 0.9), the decision maker may be either willing to trade this criteria for something else, or its importance is only valid in some situations. Extracting what reasonable combinations of weights could be acceptable by a decision maker shows insights into the value system and preferences for various dimensions; not only what is likely to be important, relative to each other, but how likely an agent is to change their mind about the importance of the criteria.

In fact, studies suggest that having an exact numerical value for importance weightings is not necessary, and that "elicitation procedures that are more natural for the user are likely to be more accurate" [121]. This "more natural" way could be using the design space more directly through pairwise or discrete choices comparisons to extract preferences.

This research hypothesizes that eliminating those combinations of weightings that do not satisfy a decision maker's preferences can be filtered down to a distribution (i.e. histogram of feasible weightings) that reflect the possible weightings that a decision maker may be willing to accept to reach consensus, or more formally:

Hypothesis #3: *Infeasible design or preference filtering on the range of possible weightings combinations, from a set of discrete choices employing candidate solutions, will identify a decision maker's preferences by providing feasible weighting distributions for each criterion or objective.*

Hypothesis #3's metric of interest is the difference between an assumed preference structure or truth model compared to the predicted weighting distributions by the methodology. If the implementation of the appropriate step cannot adequately predict this truth preference model within a reasonable level of error, then this difference metric is large and Hypothesis #3 can be rejected.

2.3.4 Research Question #4 and Hypothesis #4

The other information referred to in Hypothesis #2 was the power or influence relationships that exist between decision makers. These relationships can potentially drive the decisions and therefore the region of consensus by biasing those preferences of decision makers with greater influence over others. Any group decision-making technique requires a process to analyze these relationships and how one can extract or obtain those interactions is a relevant research question.

Research Question #4: *How can the influence relationships between decision makers be identified and quantified?*

Agents are susceptible to influence by other decision makers, and similarly decisions can be heavily impacted by the relationship between agents. For example, if one agent seeks to establish an agreeable relationship with another agent for a different or future reason (e.g. an upcoming opportunity to partner with them for a proposal), they may be willing to side with that other agent more quickly. In a particular case, one of their objectives would be the "satisfaction of the other agent" or pleasing the other agent. On the other hand, with cooperation required for many decisions, decision makers also want to have the others move "closer" or more aligned with their own importance weightings of the criteria. This may come as an expense by conceding the importance on one dimension (not as important to another agent) in order to acquire more of another preferred criteria. Lastly, two agents may be willing to "move nearer" each other in terms of preferences, but how much should each of them "move"; meeting half-way in the middle? The answer would be dependent on the power or influence exhibited on each other and the willingness of each agent to value cooperation versus sacrificing their own objective function.

Requiring each agent to select where (in terms of preferences) and with whom they would agree and form a coalition, can provide insight into how, if, and with whom they are likely to form coalitions. This hypothesis is structured below:

Hypothesis #4: *Discrete choice experiments between designs, and with whom an agent will form a coalition in the decision space, will identify relationships of influence, under the power constraints equations, between decision makers.*

Similar to Hypothesis #3, the metric to evaluate Hypothesis #4 is the difference between an assumed perceived influence truth model and the one predicted by the methodology. Close agreement would suggest that Hypothesis #4 should not be rejected.

2.4 Summary of Research Question and Hypotheses

For reference purposes, the following table summarizes the research questions and hypotheses introduced in the previous sections.

Table 1: Summary of Research Questions and Related Hypotheses

	Research Question	Hypothesis
1.0	How can the number of operational design solutions originally considered increase to include a greater portion of the potential design space?	Monte Carlo simulations of surrogate models developed around time-consuming operational models will provide capability to more rapidly define the design space by generating greater numbers of candidate solutions in the same time period.
2.0	How can the feasible decision space be reduced to facilitate decision makers in reaching consensus?	Simulating the multi-agent decision-making process with an iterated ultimatum game across all objectives, with the application of the preference distributions of, and power relationships between, agents, will significantly reduce the decision space and identify regions with high probabilities of reaching consensus.
3.0	How can a decision maker's preference information, including the potential for changing preferences, across all objectives be acquired more quickly and accurately?	Infeasible design or preference filtering on the range of possible weightings combinations, from a set of discrete choices employing candidate solutions, will identify a decision maker's preferences by providing feasible weighting distributions for each criterion or objective.
4.0	How can the influence relationships between decision makers be identified and quantified?	Discrete choice experiments between designs, and with whom an agent will form a coalition in the decision space, will identify relationships of influence, under the power constraints equations, between decision makers.

CHAPTER III

TECHNICAL BACKGROUND AND LITERATURE

The following sections will review and discuss some basic and foundational material contained in associated literature relevant to group decision making. Some of this information will serve as the building blocks upon which the methodology discussed in later chapters will be based. Other sections within this chapter serve as background information and will summarize the work previously performed by other researchers in related fields.

3.1 Decision Theory

Decision Theory is the broad and diverse field of study predominantly focused on investigating how decisions are made (descriptive), should be made (prescriptive), how they can be improved (or even optimized), and what factors drive the ultimate decisions in different environments and situations [14]. Like other major fields, the taxonomy for decision theory has differing perspectives but major categories would likely include:

- single-criteria vs. multi-criteria decision making
- single-agent vs. multi-agent (individual vs. group decision making)
- time-dependent vs. time-independent decisions
- deterministic vs. probabilistic (fuzzy or stochastic)
- certainty vs. uncertainty
- rational vs. irrational

These and other classifications and subcategories suggest an extensive discipline which is even further expanded when the various subfields are applied to different problems. Although numerous texts have been written defining large portions of this discipline such as [83], [63], and [128] to name a few, it continues to expand as evident from numerous contributions to new journals including the Journals of Cognitive Engineering and Decision Making, Decision Analysis, Decision Sciences and Decision Support Systems, which focus on human factors, theory and implementation, respectively, in terms of decision making.

With such a broad scope in decision theory, applications, and analysis, subfields have been introduced to further refine the areas and contributions of decision scientists. A few more specific subfields within the overarching umbrella of decision theory include:

- Expected Utility Theory (EUT)
- Game Theory
- Decision Field Theory (DFT)
- Prospect Theory

These subfields are further expanded with applications to various domains in economics, social behavior, cognitive psychology, and, of course, systems engineering. Many of these areas can be considered to have large overlap in fundamental principles and axioms, and some may suggest that they are more generalized theories of each other, under certain assumptions.

For example, Expected Utility Theory is often associated with the work of von Neumann and Morgenstern's [182] and pertains to how a rational or even ideal decision maker seeks to maximize the payout or utility for a decision situation. It is closely related to Game Theory, where similar goals exist to accept strategies or make decisions which maximize one's payout, but with additional factors involved such as the interdependence of players where payouts are dependent on the decisions of others for cooperative and competitive situations [66].

In Decision Field Theory (DFT), researchers are most interested in studying the dynamics of decisions, how they develop over time and how decision makers change preferences. Furthermore, DFT examines the mechanisms of how decisions makers analyze data, deliberate (sometimes at length), vacillate during conflict resolution or how the preference relation changes with time [29].

Lastly, Prospect Theory attempts to describe how decisions are made in reality, under uncertainty and risk, and with other human or cognitive limitations. In this respect, it is often considered at the other end of the spectrum away from the rational, ideal or optimal decision-making theories such as EUT [85].

Since the current research concerns group decision making with multiple agents, the principles of game theory apply and in particular cooperative games as discussed in later sections of this chapter. However, since another objective of the methodology discussed in this research seeks to facilitate and even accelerate decision making, ideas will also be drawn from DFT and Prospect theory in which it is acknowledged decision makers will change preferences and may be persuaded from non-rational factors. Lastly, essential evaluation functions, such as utility curves and formulations, will touch on some of the contributions of EUT.

Decision theory is also inextricably linked to optimization. In decision theory, one is concerned with identifying the "best" choice or alternative given a decision maker's preferences. Similarly, optimization seeks to find the optimal point which maximizes the utility function or minimize the objective function of the decision maker. In essence, all decisions require some form of optimization strategy or technique to find the best alternative (with or without mathematical tools and techniques). Likewise, all optimization processes require the application of a decision maker's preference and objective function with which to optimize.

3.2 History of Decision Theory

Although a detailed discussion or presentation about the history of the development of modern day decision making is beyond the scope of this research, a brief summary may be useful in introducing some of the topics discussed later and why they arose in the evolution of decision theory. In particular the ideas of bounded rationality and satisficing, and the need for improved methods for group decision making for the increasingly more common interdisciplinary problems and solutions, are placed in context of the overall research objectives.

Decision making is as early as self-awareness. Maximizing one's own chances of survival is manifestly good decision making required for intellectual evolutionary progression. However, as opined in the Harvard Business Review, up before the 17th century, analysis of risk were relegated to "priests and oracles" [28]. From the Renaissance onward, numbers became increasingly more useful and models, tools and theories were developed to support or refute the popular heuristics passed from generation to generation in the various trades and profession. In the 20th century, the mathematical foundation had reached sufficient sophistication to initiate advanced decision making methods, in particular as ideas about rationally such as that assumed in Expected Utility Theory were further developed in the works of Von Neumann and Morgenstern [182]. Empirical research quickly followed testing these rational theories, and differences were identified between the normative and descriptive theories of decision making. This check on the "unalloyed progress toward perfect rationalism" [28] resulted in various other principles and theories which matched more closely how humans made decisions in real-life. Generally described as "bounded rationality" [68], these principles are based on the limitations of the human mind, the return to heuristics allowing "fast and frugal" decision making, and even social or external pressures.

Today a compromised approach to decision making from both the rational choice and bounded rationality parts of the spectrum may be most promising and this research seeks to utilize both of these aspects from decision theory in the proposed methodology contained herein.

3.3 Aircraft Design and Decision Making in Practice

In terms of aircraft design, Raymer strongly suggests that one must "...let the numbers (not opinion, prejudice, or preconceived notions) make the final selection" within the conceptual design phase [132]. Realistically, however, there are a number of dimensions which are outside the control of the aircraft designer and these are not always governed by numbers, in the traditional sense. Just as propulsion engineers do not directly concern themselves with all variables of the other aerospace engineering disciplines, aircraft system engineers or designers do not always include trades across dimensions about which they do not have sufficient knowledge. For example, the aircraft designer can offer the best design to minimize cost, emissions and empty weight, but if agreements or contracts have already been made regarding material type, location of fabrication, or other constraints, a revisit to what is "best" under previously unknown dimensions is essential. The design itself will undergo an iterative process applying more information about more dimensions and added constraints, but the decision making itself will also undergo iteration.

It is well established that decision makers under different circumstances will weight different dimensions differently. For example, minimizing fuel consumption is more important if the price of fuel is likely to go up in the future. Similarly, reducing noise for a particular aircraft design may be a bigger driver than other dimensions if the aircraft will be used in a highly populous area. Clearly, changes to the weightings will occur as the number of dimensions used in the decision-making process increases or decreases. A dimension or factor previously considered not important can become increasingly important or even dominant in later stages of the decision making, as more information about the problem or design is established.

To complicate the decision-making process even more, decision makers will alter their preferences after familiarizing themselves with the process or are sometimes replaced with a different decision maker in the middle of the process. This is often the case in public office where the individuals filling many elected or appointed government positions change every two, four, or six years. If major acquisition programs take on average longer than six years, the likelihood that the solution, design or decision will be opposed, reevaluated or require buy-in from the next decision maker is almost guaranteed. Similarly, for the full aircraft research, design testing and evaluation RDT&E phases, which take many years to complete, the decision making can be dynamic and convoluted.

Finally, decisions are made in environments which are ever changing and in which rational decision theories are less likely to predict accurately. This makes "it hard to ignore the distinction between the objective environment in which the ... actor 'really' lives and the subjective environment that he perceives and to which he responds" [158].

3.4 Multi-Criteria Decision Making

Since all engineering problems are effectively trades within and across different dimensions, almost every decision is fundamentally multi-dimensional (i.e. multi-criteria or multi-objective). Practically speaking, one can argue that if only one dimension exists the decision-making process is trivial. Maximizing one's utility function is a straight forward process by identifying the minimum or maximum value along that dimension or at a certain value if a particular target is sought. Generally, only multi-objective decisions require some sort of external preference that will assist in weighing the importance of conflicting or negatively correlated dimensions. That is, the value in one dimension can be increased but perhaps at a reduction in another dimension, or vice versa. The decision making for these situations becomes much more challenging since the "right" solution depends heavily on an individual or group ascribing weightings or importance values to the various dimensions. This mapping of customer preferences to engineering characteristics is often done using qualitative techniques such as the Quality Function Deployment (QFD) [165] or HUDDLE [64].

Multi-Criteria Decision Making (MCDM) at its core is really just decision making, where the adjective "multi-criteria" can, in one sense, be considered redundant since almost every decision would be between conflicting choices or objectives. Even simple, mundane choices of what to eat or where to go has, to a certain degree, conflicting choices. A Dean from the Columbia Business School is recorded saying that "As for conflicting objectives - quality vs. lower cost, better product vs. cheaper raw materials, for example - just about any idiot can maximize a single function. Anybody can increase sales. After all, if nothing else matters, you can decrease the price to zero. In fact, you don't have to stop there. If they won't take it at zero, you pay them to take it" (quoted in [194]).

Thus, some could quickly argue that "Single Criterion Decision Making" does not exist. Or if it does, the answer is often trivial and the sole remaining challenge in this case would be to identify that single criterion. The decisions that face engineers, business leaders and politicians are always multi-criteria. No one dimension can be completely optimized without some cost or detriment to another criteria or dimensions.

As illustrated in Figure 5, each of the aerospace engineering disciplines depicted may seek to optimize their particular field. However, a structurally rigid aircraft may not be sufficiently aerodynamic, a noiseless aircraft may not be very fast, and a light

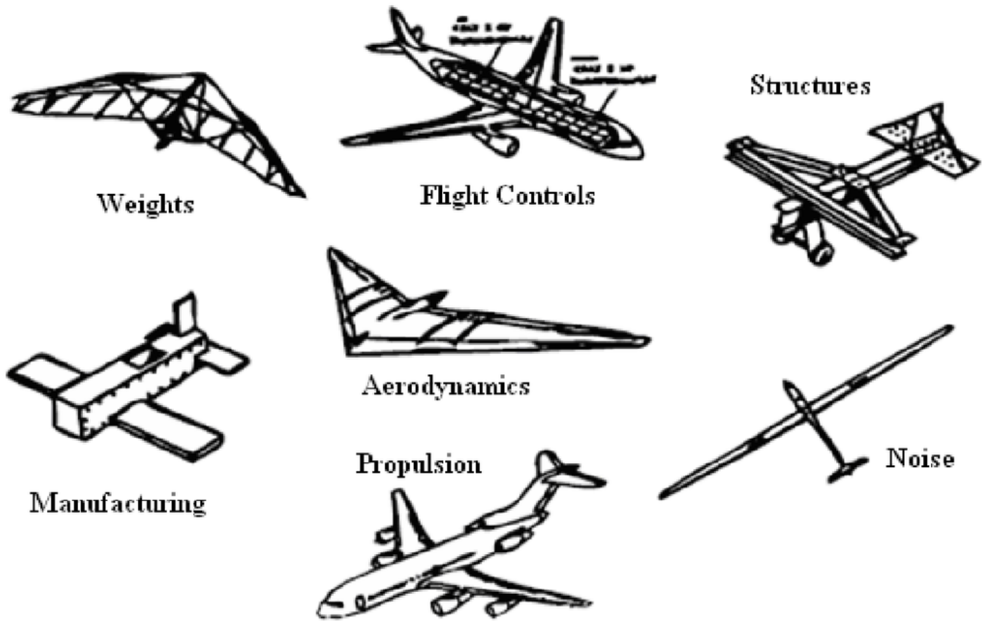


Figure 5: Aircraft Design from the Perspective of Each of the Disciplines (from [115])

aircraft may have little carrying capacity. Aircraft design is a multi-criteria or multi-objective decision-making activity where the various individuals or disciplines must reach agreement on many design variables to achieve success. Compromises must be made, iterations must be performed, and group decision making will be required continually throughout the design process.

3.5 Decision-Making Techniques

A relatively recent attempt [134] to enumerate the number of decision-making techniques in existence found more than 70 available multi-criteria decision-making techniques. Many of these techniques are, as could be expected, proposed to respond to the different types, domains, and scope of problems. However, they also each contain inherent limitations, different assumptions and shortcomings making them invalid if applied incorrectly to the wrong type of problem and even undesirable if incorrect results are obtained.

A comprehensive discussion of these techniques is beyond the scope of this research but some of the common features or characteristics of these techniques will be mentioned in various sections with three contrasted in the appendix of this dissertation. However, a summary and comparison of some of the popular techniques often applied to operations research is found in [169]. Furthermore, efforts to assist the decision maker in selecting the "right" or "best" technique based on the attributes of the decision problem itself have been made previously [96].

It is possible that any technique for a single decision maker, with appropriate characteristics, could be adapted for a group decision-making problem, which is the focus of this current research. However, the focus will be directed toward the steps in the overall methodology for multi-agent consensus reaching and not on the limitations or weaknesses of the candidate technique. In fact, throughout later discussions and examples in this research the application of a simple technique, a weighted additive utility function, will be preferable over more convoluted techniques to deemphasize the particular technique and underscore the properties of the group consensus reaching approach. Therefore, the attempt will be made to isolate many of the steps of the methodology to the valuation function of candidate designs.

Research has shown that formal processes or selection methods are not often employed in industry settings, with one survey suggesting that less than one in four companies use such techniques [143]. Therefore, a simple additive utility function could already improve decision making in a variety of organization without any need to apply any more sophisticated techniques. However, more sophisticated options are available if needed; for instance, 24 different types of additive utilities alone have been analyzed and compared in previous studies [59]. If these techniques are not used, human decision making can result in inconsistencies or sub-optimal choices, as a result of the human biases or irrationality often exhibited in multi-objective decision making discussed later [84].

3.5.1 A Common Preference Structure

A large portion of decision-making techniques make use of preference information. This is most readily described as importance values or weights w on each of the attributes, objectives or criteria on a set of alternatives or designs. Often these weights are normalized such that for n attributes or dimensions, the weight vector w is defined as:

$$w = \begin{bmatrix} w_1 & w_2 & \dots & w_n \end{bmatrix} \tag{1}$$

where:

n : Total number of objectives

subject to: $\sum_{k=1}^{n} w_k = 1, \text{ for all } k = 1, \dots, n$

which can satisfy the requirements of a generalized barycentric coordinate system [55].

This preference structure is the most commonly used, perhaps for its simplicity and transparency, but the suggested techniques, process, and methods to acquire such a structure involve a variety of principles such as Shannon's entropy [188] or Shapley values [187].

Evaluation of the alternatives themselves with different techniques take multiple forms (e.g. weighted product model, weighted sum model, The Technique for Order of Preference by Similarity to Ideal Solution - TOPSIS [77]) but many of them will make use of this simple preference structure from equation (1). (See Appendix B which compares three common decision-making techniques and their similarity to a simple additive weighting model).

There are other structures which can also be implemented with simple objective rank ordering which of course is possible if the objectives are also attached to weights directly. Thus, weightings can provide rank orders, but rank orders cannot provide weightings.

3.5.2 Preference or Weight Extraction

A variety of techniques have been proposed and implemented to obtain the actual preferences for each of the dimensions or objectives of a decision maker. Many make use of pairwise comparisons, with one of the most well known called the Analytical Hierarchy Process (AHP) [141]. In the traditional AHP implementation, the decision maker is given two objectives at a time and asked to provide the ratio that they prefer one to the other. For example, after being given the objectives of range and payload of a notional aircraft design, the decision maker may respond that payload is twice as important as range. Later, comparing the payload and cost objectives may result in cost being three times more important than payload. These ratios are then used to extract the weightings for the various objectives [49]. Assuming the decision maker is consistent one might conclude that cost would be six times more important than range. Consistency checks are of course possible but in decision problems with many objectives, the decision maker may not want to go through every pairwise comparison of two objectives. Recent efforts to account for these inconsistencies using optimization methods show some advantages [20]. However, the objectives and the important ratio between any two of them may depend on the values of other objectives. Just because the decision maker prefers a lower cost than increasing payload in the above example, their opinion may change if the range was excessively small. Pairwise comparisons usually assume that everything else is known or at least held constant within a particular context or situation. If that context is not known or fuzzy, AHP can provide considerably inaccurate preferences. Lastly, AHP suffers from rank reversals from the addition of irrelevant alternatives suggesting it can often be misused in decision-making activities [16].

The alternative is to compare the actual designs. Instead of pairwise comparison of objectives, the decision maker is given two or more designs and asked to rank them in some fashion. With two designs, they respond with which one is better or even how much more one is better than the other. With three or more designs, they can rank them ordinally or cardinally (if so desired). Although this may required additional effort and time to assess designs instead of objectives the results can be more reflective of reality since parts of the whole are not considered in isolation, which in the aggregate are not that significant. For example, two objectives may not be important overall, but comparing them side by side may unnecessarily exaggerate the one or the other.

This idea of ranking on the design space was recently shown to be effective in determining the preferences of experts regarding nanotechnology-enabled food products [60]. In the study, 26 hypothetical designs each with 10 objectives were analyzed and the experts were asked to select and rank the five best options and five worst options. The criteria weights were then calculated in various models and compared.

This shows a potential process in extracting the weights for other spaces with multiple dimensions or objectives. Requesting the decision makers to rank or designate a preferred design among a few designs, may be much more realistic (and reflects the actual decision-making process more accurately) than making decisions simply between objectives directly.

Furthermore, there is the potential to obtain uncertainty in the weights themselves as well. The equivalent in the AHP method might be to request the highest and lowest ratio between two objectives. However, the highest and lowest might be complete reversals of each other. That is, for some decision makers, objective 1 might be twice as important as objective 2, but objective 2 might be twice as important as objective 1 in some situations. Although initially this sounds less useful, it at least bounds the relationship the decision maker considers these two objectives can assume, but even this only provides the bounds on one objective pair. In reality, there is a range of values which all the objectives may possess in pairwise comparisons. Since preferences will be highly interdependent and context based, it is thus more useful to obtain them all together under the assumption that a decision maker can change opinions and will not hold their preferences so rigid. The results of such a process would allow a distribution of possible weights that the decision maker could potentially accept as reflective of their true preference structure.

3.6 Human Decision Making

Since the optimization or the decision-making process is so heavily dependent on the preferences of individuals (or a group) through the objective function, any change in the preferences of that individual or group will likely result in different solutions. In this sense, there is no global optimal solution. The optimum depends on a changing set of factors both internal and external to the decision makers, individually or as a group. "We all make decisions based on the information we have and the objectives we're pursuing, and these things vary from position to position" [127].

Only the single decision maker applying his or her current preferences can an optimum solution be selected, but this would be, at best, a temporary solution amid the dynamics of a large SoS engineering problem. Any change in the environment and the solution is less optimal. Similarly, any change in preferences, and the solution is likely to no longer be ideal. Flexibility and robustness can be designed into the system but often these design decisions assume the decision maker is true to their preferences. If the preferences and thus objective function itself changes, robustness is a much more difficult proposition.

This notion of preferences being "contrary to one's own real interests" due to "incorrect or on incomplete information" is considered in detail by Harsanyi [74]. He proposes that utility functions should be defined in terms of "hypothetical informed preferences rather than in terms of his actual preferences because the later will often contain some mistake preferences contrary to his real interests" [74]. The difference between these is accounted by how informed a decision maker may be. When increased information is available, one is likely to change their opinion according to new "informed preferences."

Since humans will "change their mind," and thus their preferences or importance weightings on the various criteria, based upon beliefs, knowledge, external conditions, etc., assuming a temporally static preference landscape for any decision should be questioned. In addition, for many decisions which are evolutionary and are made over time, the decision maker(s) will change. This is evident in many political positions where the decisions can take years to be implemented and require the support of many decision makers sequentially. If these political decision makers are replaced every few years, the preferences will likely change from person to person in addition to the change that any one individual will experiences over time after new information or experiences. This process is not only seen in government agencies. In business, CEOs or other management personnel can change just as quickly, making temporary decisions based on their own preferences, only to be replaced a few months or years later resulting in a new leader taking a new direction with new preferences.

It is with these types of ideas where Zeleny asks "What is so precious about assuming unchanging and continuous preferences, judgmental consistency, transitivity, inflexibility, utility maximization, and an inability to learn?" [194]. Although this rigidity in the normative approach of how human decision makers act is often described as rational, a much more fluid, uncertain and unpredictable algorithm, often referred to as *irrationality*, governs real-world decision making and warrants an equal, or perhaps greater amount, of attention.

This irrationality and its effect on decisions is most at play when the scenarios are evolving over time. Changing preferences is a challenging reality to manage in any decision environment. But with changing scenarios and changing preferences concurrently, the irrationality within human decision makers can be an even more substantial factor in the outcomes. When the scenarios change, the decision makers are likely to respond and alter the particular objective function or decision process implemented up to that point. Scenarios drive the objective function and any methodology must account for the specific situation, people and external environment in which both decisions and the solutions are made and implemented, respectively. This is another reason why tools and environments must be developed to account for scenario changes, which in turn cause modifications or corrections to the objective function and ultimately results in potential irrational behavior in humans, such as a change to one's preferences into something biased or even counter-intuitive.

3.6.1 Limitations of Human Decision Makers

The human mind is fraught with limitations. It is terrible at calculating probabilities of single events [39], its capacity to remember or transmit information is low [108], and falls victim to distraction [94], illusion and bias. Experimentalists have identified other more specific limitations, for example, the human mind can fall prey to a belief in a "hot hand" in events (e.g. "streak shooting" in basketball) [70], an expression of the gambler's fallacy [171]. Similarly, multiple dimensions become increasing difficult for humans to process and evaluate, summarized by a conclusion observed by Miller "people are less accurate if they must judge more than one attribute simultaneously" [108].

Some of these limitations are actually accounted for in some of the conclusions reached in the theory of evolution: "natural selection will favor strategies that make many incorrect causal associations in order to establish those that are essential for survival and reproduction."[62]. In other words, these so called limitations in human minds were (and potentially still are) advantageous in many environments where making the right decision, even a minority of the time, is crucial. Shermer provides a visual example of this phenomenon, where the cost associated with believing a rustle in the wind is actually one's predator is small, compared to the cost when believing a real predator is only the wind [153].

However, when applied to decision-making processes, recognizing and overcoming some of these "naturally selected" strategies could be in the decision maker's best interest especially when the decision environment is considerably different than the one for which the strategy was originally acquired.

The following sections discuss some of the more evident limitations of human decision makers which must be recognized for any attempt in improving the decision-making process in groups.

3.6.2 Manipulation of Decision Makers' Preferences

The fact that humans change their mind readily is universally accepted. Research in advertising is dedicated to studying how the human mind can be influenced to buy or support a particular product or issue, respectively. Many advertising techniques will take advantage of the innate limitations of the human mind to objectively evaluate candidates. One of the simplest techniques makes use of message repetition, working under the assumption that one's attitude will be, or can become, more positive when receiving stimuli that are more familiar [193]. Although some research suggests that too much repetition can incite attitudes of irrelevance or even negative responses [13, 30], the obvious goals remains the same to persuade, convince, and manipulate others or customers into making a decision that maximize their own objective function of increased profit, votes or prestige.

Preference reversal has been empirically shown in notable experiments by Tversky and Kahneman testing how framing decisions can have drastic impacts on the results.

In one experiment, respondents would reverse the preference between two hypothetical diseasing treating programs, depending on the focus of the question being on the expected number of lives saved or on the expected number of deaths. In other experiments, decision makers would choose the worse of two prospects and "exhibit patterns of preference which appear incompatible with expected utility theory" [172][1]. Thus, even the way in which data is presented to decision makers must be analyzed to avoid potential errors from not-objective judgments, qualitative comparisons, and biases.

Time pressure has also been shown to explain preference changes or reversal from a DFT theory perspective [48] and, more expectedly, identified as a crucial effect on decision accuracy and quality [125].

In a seminal study by Solomon Asch, the manipulation that was observed was more pronounced and caused directly by the "apparent" preferences or decisions of the group members. In the experiments, each member in a group was to state which line in a set of three was identical in length to a separate reference line a short distance away. When the group was composed of 11 confederates (responding to the task in the same wrong way) and one participant, the participant would conform to and agree with the majority's wrong decision one third of the time on average [6]. The results indicate that social pressures with a group setting can alter opinions. Since the true answer was obvious in the experiments, the willingness to join the group may be found to be even more exaggerated when uncertainty is greater or the differences in the data less clear or ambiguous. More recently, experiments testing conforming to the group or the majority was reproduced but without confederates [111].

Manipulation of decision makers has also been shown to exists under risk and uncertainty. Experiments illustrated how an "attraction factor" caused respondents

[1]Many years later, Daniel Kahneman would be awarded the Nobel prize in Economic Science "for having integrated insights from psychological research into economic science, especially concerning human judgment and decision-making under uncertainty" [116]

to reverse their opinions about which prospect was better even if the two options were identical in terms of utility [192]. The same study found the expected result that an additional method of manipulation (i.e. to change one's preference) is by allowing decision makers to gain information or consult with other agents. This has been shown for when information is gained from another decision maker [33] and when information is gained from independent experiences [92].

3.6.3 Biases in Decision Making

Biases can arise in a variety of ways in decisions making processes.

Decision makers will sometimes seek for data that confirms their preconceived notions about the problems and how it should be solved. At the same time, they may disregard data suggesting their decision made *a priori* is incorrect or less desirable. This *confirmation bias* can have negative effects as the final decision which may eventually be considered as poor, undesirable or even wrong [170].

In fact, some decision support tools can offer variable weightings on the metrics of interest (such as common slider bars for importance parameters). When the decision maker "plays" with those settings until their predetermined design is ranked or listed as the best, a confirmation bias has occurred if the decision maker uses the tool to support what they had already intended to select. In this case, the decision support system serves no purpose other than to support one's a priori decision (a not entirely useless endeavor) but there can exist a disconnect with what the decision maker wants as an outcome (an expression of their true preferences) and what they believe that answer should be.

A similarly related bias comes when decision makers are more likely to trust their own research, team or company over that of another when the question of credibility arises. Often one trusts their own model or results more than another's, even if no compelling evidence suggests this reflects reality. This *authority bias* can also exist when an individual or team claims an expertise either from formal education or experience and demands or at least persuades others to accept their conclusions as truth. In group decision-making activities, especially in cooperative multi-agent decisions, when a decision maker possesses a certain skill, position or knowledge necessary for the potential solution, others may unnecessarily or unconsciously believe their knowledge can be applied to other areas of the problem overextending the authority's influence in the group. In discussing process in group settings, Yalom describes that "... individuals high on the pyramid not only are more technically informed but also possess organizational information that permits them to influence and manipulate: that is, they not only have skills that have allowed them to obtain a position of power but, once there, have such a central place in the flow of information that they are able to reinforce their position" [189]. This bias thus comes to play a role in persuading others in group decision-making activities and should be accounted for in decision models.

An *overconfidence bias* can develop if historical trends have been favorable in some way, and the unfounded trend is expected to continue. This is most evident in quantification of risk where decision makers can become overconfident that a particular risky design can still meet the requirements because a design has performed well in the past. As a result, decisions can be selected which are riskier then what the data or models suggest is prudent. This is closely related to what Schwenk calls a *prior hypothesis bias* [146] where decision makers inappropriately maintain beliefs or attitudes despite additional evidence that strongly opposes their assumed views.

Relying to an inappropriate amount on one or a few pieces of information can result in an *anchoring bias.* In other words, the starting point can have a disproportionate amount of influence on the final decision. This is well known among professional salesman and negotiators and is exploited to further their own interests. Campell et al. [31] analyzed group consensus forecasts in finance and found "sizable predictable forecast errors" from this bias. However, the same phenomenon can exist in decision making regarding engineering solutions, which often make assumptions about future conditions, and thus attention should be given to avoiding unnecessarily emphasizing some designs over others prematurely or placing too much weight on previous performance.

These biases, and many other ones [171], can contribute significantly to some highly undesirable decisions. Good decision-making practices will always seek to remove these biases as much as possible or at least account for them in the decisions making process as various types of uncertainty.

3.6.4 Irrationality in Decision Makers

With so many limitations with which to grapple, the decision theorists have broadly classified the actions which go against one's sincere and objective beliefs and reasons as irrational. Similarly, when decisions are made with little or no reasoning and effort, or made under emotional stress, the term irrational is used to describe these types of decisions. For example, it might be considered irrational to forgo an operation now, if greater pain could be avoided in the future (assuming avoiding the most possible pain is desired) for the same cost.

Thus, although one would understandably seek to avoid making decisions irrationality, the time constraints, resources available, psychological biases present or even subjective and external factors can cause one to act irrational. This irrational behavior is shown in various experiments where individuals will punish others at the expense to themselves in order to establish fairness and encourage cooperation [56].

One of the best mathematical examples of irrationality can be illustrated in a game theoretic construct called the prisoner's dilemma (which is more formally defined in a later section). In the Prisoner's Dilemma, two prisoners are found to act (by testifying against the other) irrationally as a group because of the uncertainty in what the other prisoner will do. This uncertainty causes them to act in such a way that would be counter to their objective (i.e. freedom). If they had a chance to contemplate the entire situation and knew what the other prisoner was considering they might have both acted more rationally.

More formally, the 'rational man' is defined as "a man whose thought processes consist exclusively of logical propositions, or a man without prejudices, or a man whose emotions are inoperative" such that the term rational is efficient or, "maximizing the output for a given input, or minimizing input for a given output" [52].

Likewise, rationality in decision making typically refers to a decision maker acting in accordance with their beliefs or reasons. These reasons are usually a result of some combination of the information or data available to the decision maker. Often, a rational decision is one which maximizes the benefit and minimizes the cost. In other words, the action taken by a rational decision maker is one that maximizes their own utility so that "[r]ational individuals choose the alternative that is likely to give them the greatest satisfaction" [148]. Since the ability to choose the best option is required, a necessary condition is that the rational decision maker can evaluate or predict the outcome of different actions.

In many different fields, assumptions or knowledge about the information available to the decision maker and about their preferences for the various criteria are modeled to provide insight into how individuals will act. Under the assumption that decision makers act as rational agents, choice models can be developed to make predictions of how combinations of decision makers or groups will respond collectively. These are clearly simplifying assumptions in order to model, analyze and predict how individuals will or rather should make decisions but often humans are much less predictable and so modifications to these models have quickly emerged.

3.6.5 Bounded Rationality

When the predictive capability of the models of rational choice theory began to deviate from empirical data and experiments, the notion of bounded rationality was established and first proposed by Herbert Simon in 1957. He states that the "capacity of the human mind for formulating and solving complex problems is very small compared with the size of the problems whose solution is required for objectively rational behavior in the real world—or even for a reasonable approximation to such objective rationality" [157]. The basic tenet of bounded rationality is that the decision maker will not always choose or select the alternative that maximizing their own utility or satisfaction. Simon's description of bounded rationality is that it is "intendedly rational, but only limitedly so" [157]. Elsewhere, Radner describes it with at least three essential aspects, namely, 1) existence of goals, 2) searching for improvement, and 3) long-run success [129]. A number of ideas have been proposed as to why bounded rationality[2] may be a more accurate principle to guide decision-making analyses.

Firstly, humans, although perhaps considered rational in their ability to anticipate the outcomes of various situations, will likely not have the mental capacity to evaluate all outcomes or even be able to cognitively analyze varied and non-commensurate criteria. These "simple mental models" constructed by decision makers will be used even if there are inherent "ambiguities or contradictions" [160]. A non-quantitative and simplifying exercise may be employed and no attempt to quantify and evaluate every contingency or make every comparison between criteria is necessary or even possible.

[2]Even before the term had been coined, Benjamin Franklin had admitted to using a type of bounded rationality in a letter to Joseph Priestly. He writes: "...tho' the Weight of Reasons cannot be taken with the Precision of Algebraic Quantities... I have found great Advantage from this kind of Equation [in negating pros and cons of equal weight], in what may be called Moral or Prudential Algebra" [15]

Furthermore, time or other constraints will preclude any one from considering all possibilities which means the optimal solution may never have been actually considered. Gigerenzer and Goldstein share an example of bounded rationality: "...[A]n organism would choose the first object (a mate, perhaps) that satisfies its aspiration level–instead of the intractable sequence of taking the time to survey all possible alternatives, estimating probabilities and utilities for the possible outcomes associated with each alternative, calculating expected utilities, and choosing the alternative that scores highest" [69]. Not only would one fail in calculating all the scores or utilities of the numerous alternatives, but the fact that uncertainty as expressed as probabilities means that imperfect knowledge about some or all of the outcomes is likely, resulting in limitations to the rational processes described previously. Related to this issue is simply the capacity to process information, and keep sufficient information in working memory to make decisions. In [108], evidence suggests only around seven pieces of information can be properly kept in short-term memory for judgment and thus decision making. Computer aides and visual analytics can of course extend that capability but ultimately the decision maker can only process a small amount.

As an effect of Simon's and other's efforts in promoting a bounded rationality approach to decision making, a variety of subfields have been defined and adjustments to the traditional rational theories have been applied resulting in some new designations, including "behavioral economics", "behavioral game theory," and even "behavioral finance" [19].

3.6.6 Satisficing

In response to the inherent issues and limitations suggested by bounded rationality, the idea of "satisficing," a combination of the words or ideas of sufficing and satisfying, was proposed to account for the fact that very often decision makers will accept and choose an alternative that simply satisfies its own needs [156]. For example, an engineer can always run more experiments to increase the accuracy of a regression or for other analyses, but very often executing a sufficient number of cases is all that is needed for making certain decisions at the conceptual level. Obviously, considering all alternatives and evaluating their scores would be desirable but very often computational resources, time or both are limited and a subset of all the design points is enough. This idea is closely related to design of experiments and surrogate model creation, in that a small amount of evaluated points or designs can often provide insight into the other points or regions not directly sampled.

In this sense, satisficing does not seek the optimal solution as a decision-making process. Satisficing seeks to find a solution or solutions which are sufficient or adequate. The time, cost or effort required to gather all the information and perform an exhaustive evaluation on all the possibilities would be too large or impossible for one endorsing the principle of satisficing. Thus, one selecting a satisficing strategy for decision making could be considered one who applies as part of the decision process a preference or importance in saving time or money (the cost) in place of finding the optimal solution. This trade is evident between using up more time (or other resources) to find the optimal solution or accepting the first (or an early) design, solution or point that is "good enough" and still meets the requirements or needs as aspiration levels.

3.6.7 Evidence of Satisficing

One particularly evidential study of the feasibility of satisficing algorithms was performed by Gigerenzer and Goldstein [69]. They created a competition between a satisficing algorithm, called "Take the Best" (TTB), and other integration algorithms, such as Regression, Weighted and Unit-weighted linear models.

The TTB algorithm compares each factor or attribute (a "cue" in the experiment), one at a time, and will select the appropriate option when the two currently pertinent cue values sufficiently discriminate between the two candidates. If the two values do not discriminate between candidates, the algorithm continues to the next cue, ordered in terms of a ”ecological validity” or a measure of how often that cue would correctly predict the “best” decision. Therefore, in the TTB algorithm, the decision can be made after only one cue thus ignoring all the other information available for each particular comparison. This results in a satisficing strategy where limited knowledge and/or time leads to a decision deemed adequate.

On the other hand, the integration algorithms would make use of all information available with models inclusive of each of the factors, variables, or dimensions (i.e. “cue”) and even given additional information not explicitly used by the relatively simple TTB.

The competition in [69] evaluated how often a simulated decision maker correctly picked the larger, in terms of population, of two cities presented with different information (i.e. cues). Figure 6 summarizes the surprising result that the TTB algorithm performs just as good as some of the integration algorithms (e.g. tallying) and superior to some of them (e.g. Weighted Linear Model) for certain. Of note is that with no information about the candidates (or cities) the performance of all algorithms are equal at 50% accuracy (i.e. guessing) and at the other extreme with all information, the algorithms are also comparatively and equally good (≈74%). However, the most interesting is that with some but limited knowledge, the certain algorithms perform better than algorithms which implement features hailed as elements of classical rationality, such as “consider as much data as possible.”

In essence, in some situations decision makers can make just as good (or even better) decisions with limited knowledge. Furthermore, these better decisions can be made more quickly and with simpler heuristics if appropriate.

The conclusion drawn from this experiment is telling:

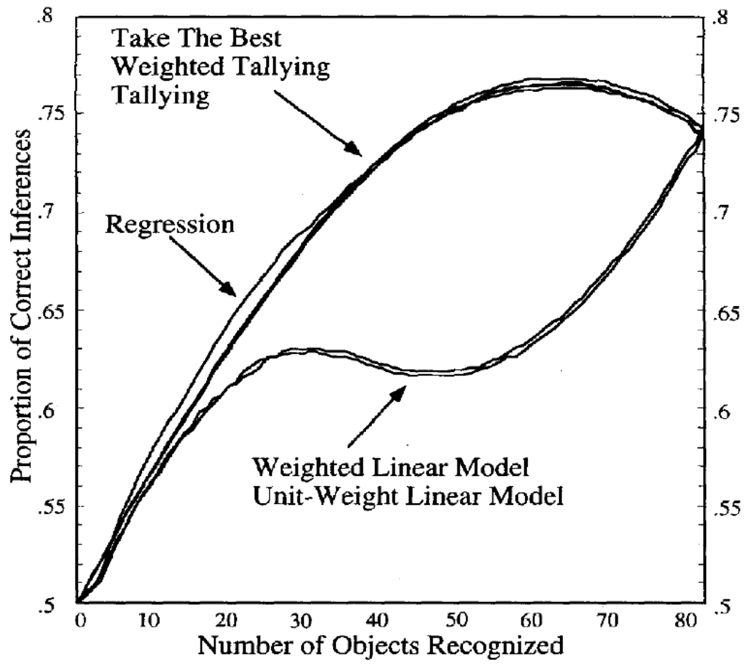


Figure 6: Comparing predictive accuracy for "Take the Best" against integration algorithms across different levels of tacit knowledge (i.e. Objects Recognized) (Reproduced from [69])

> The single most important result ... is that simple psychological mechanisms can yield about as many (or more) correct inferences in less time than standard statistical linear models that embody classical properties of rational inference. The demonstration that a fast and frugal satisficing algorithm won the competition defeats the widespread view that only "rational" algorithms can be accurate. Models of inference do not have to forsake accuracy for simplicity. [69]

This is understandably a useful finding, appropriate for the current research objective in trying to accelerate decision making without necessarily decreasing the quality of decisions. In the proposed methodology, the decision maker will be presented a relatively simple task of choosing the most preferred of two designs. The assumption that these decisions will be "good enough" and potentially faster without sacrificing accuracy is considered a key enabler.

3.7 Group Decision Making

Careful analysis reveals that we "often give our greatest responsibility to groups" including groups such as board of directors, board of regents, juries, the Supreme Court, Congress, and surgical teams [95]. Groups also dominate the workplace with a majority of large companies establishing work groups or work teams as the core unit. As problems grow in scope and breadth, increasingly collaboration between individuals with different skills or expertise will be needed. This collaboration is a key attribute of well-functioning groups even if co-location is not a defining feature. In engineering design, teams or groups, composed of experts from various disciplines, work together with the individuals taking on different roles. Decisions can impact many or all of the disciplines and thus decision making is an essential element to explore and facilitate within these groups or Integrated Product Teams (IPT) [144].

Group decision making is clearly when more than one individual or entity has some influence or "say" on the ultimate or final decision made. A dictatorship may be the best counter example of group decision making and yet even a dictator will likely have aides, counselors, etc. who influence their decisions. In the widest or all-inclusive definition of group decision making, one could argue that most of our seemingly personal or independent decisions are influenced by our backgrounds, culture or environment. So, although each individual is responsible for one's actions a realization that truly autonomous decisions are rarer than perhaps typically considered is warranted.

Forsyth [61] discusses four common approaches to making decisions in groups. The first is delegating or choosing an individual to make the decisions on behalf of the group. This can, of course, no longer be classified as "group" decision making, but if the appointed decision maker listens, is willing to try and discern the group's wishes and attempts to reflect the interests of the group (i.e. a kind of "benevolent dictator" [1]), the group still has a way to collectively express their preferences through their leader's decision. The second is averaging or combining the individual perspectives, decisions or preferences of the group members. Initially this can be done in private so as to not unnecessarily influence each other (i.e. Asch experiment) and then a method can be selected for averaging the individual decisions. The third is through voting, where the majority rules, and the minority are expected to accept the group's decision. (This and the previous approach can suffer from various limitations underscored by a discussion of Arrow's Impossibility Theorem in a later section.) The last approach is through group consensus where discussion, analysis, compromise, negotiation, etc. continue until all members are in agreement and a unanimous decision is supported by all. Understandably, this last approach is most desirable but not always the most efficient. Encouraging this approach by more quickly aligning preferences to each other is one of the main goals in satisfying the second objective of this research.

The approach taken, however, is only a part of the challenges to group decision making. Not only is group multi-objective decision making further complicated when more than one person has control over the weights of the objective function, but difficulties can arise even about what the criteria should be within the decision rule. Similarly, group decision making has to consider not only the conflicting nature of the various objectives but the conflict between individual preferences within the group. Each individual comes to the decision-making process with different values regarding the objectives or criteria. Some individuals may not consider some of the objectives established as even relevant and will base their preferences on a subset of the objectives, effectively weighting some criteria as zero or of no importance: "[D]ifferent importance may be assigned to [criteria] from design to design and from designer to designer" [194].

Furthermore, with engineering solutions to the large scale problems typically involving greater numbers of individuals, all with different expertise and preferences accounting for the various perspectives of the problem and solution, there exists a real need to study and understand influences between the players and, in general, the group dynamics in decision-making activities.

A number of different ideas or philosophies have arisen to try and reconcile the differences or at least forecast how a group will reach a decision. Some of these areas include Principal-Agent Theory [57], Strategic or Standard Groupthink [81], Evolutionary Theory [38] or Sociological Determinism [51]. These areas of research cover both rational and non-rational group action as well as different perspectives on society, including both individualistic and holistic views [3].

For engineering applications, group decision making is common in the design process such as in making trades between engineering groups, incorporating unquantifiable performance objectives and conflict resolution between disciplines [149]. In this sense, group decision making could be viewed as essentially an activity involving negotiation across preferences and objectives. Very often these negotiations can involve mean or additive weighting functions, voting methods or some combination of them, such as the Delphi method, to create knowledge-based systems to assist with decision making in a variety of situations [35].

In general, a group's objective function or utility function $u_g(x)$ will take the form:

$$u_g(x) = f(u_1(x), u_2(x), ...u_N(x)), \tag{2}$$

where $u_i(x)$ is the individual utility function of the ith decision maker $(i = 1, ..., N)$ of an alternative, described by x, where $x = [x_1, x_2, ...x_n]$, a set of attributes or objectives defining the specific alternative.

Combining individual utility functions in the literature is sometimes referred to as the "social welfare function", and is heavily influence by the work of Kenneth Arrow [5].

3.7.1 Arrow's Impossibility Theorem

In 1950, Kenneth Arrow published what is now known as Arrow's Impossibility Theorem [5]. This theorem proves that no aggregation method of ordinal or ranking preferences can be created or implemented such that it satisfies five generally acceptable criteria or conditions of fairness, namely:

- Unrestricted domain - A social function which rank orders the alternatives is complete and repeatable for any set of individual voters and their preferences.
- Positive association of social and individual values - Any individual in the group adjusting preferences for an alternative cannot allow an opposite change in the group's ranking. For example, an individual increasing the rank of one option cannot result in the group reducing its rank over all.
- Independence of Irrelevant Alternatives - The preferences of the group should not change for the winning alternative if an additional alternative which is inferior to all others is added into, or removed from, the pool of candidate alternatives.
- Citizen sovereignty - All possible rankings for the alternatives must be possible by some set of the individual preferences.
- Non-dictatorship - No one individual's preference can decide for the remainder of the group or population.

Since any attempt at combining ordinal preferences (i.e. rankings) in any particular decision-making technique could allow for the breaking of, at least, one of the above fairness criteria, dealing with paradoxes (e.g. in some voting methods), and how one will reconcile the proof of Arrow's Impossibility Theorem with any proposed solution, must be addressed.

3.7.2 Cardinal Utility Functions

In [87], Keeney responds to the apparent limitations of Arrow's Impossibility Theorem in using any aggregation method by proving that for some problems a group utility function holds to similar conditions when using cardinality utility functions. For situations when the alternatives are certain (i.e. deterministic), Keeny states that: "...given five assumptions analogous to Arrow's, using cardinal utilities rather than rankings, it is always possible to define consistent aggregation rules for a group cardinal utility function" [87].

Admittedly, this requires the individual decision makers to create their own von Neumann-Morgenstern utility function and thus not only define the rank or order of their preference but also the strength of preference between each alternative. However, this does provide a way to aggregate utilities while maintaining the acceptability conditions described previously.

More specifically, group cardinal utility functions over uncertain alternatives do not break the above assumptions, if

$$u_G = u(u_1, u_2, ...u_N) = \sum_{i=1}^{n} k_i u_i, \tag{3}$$

where, $k \geq 0$, $i = 1, ..., N$ and $k_i > 0$ for at least two k_i's [87].

3.7.3 Group Decision-Making Techniques and Consensus Reaching

In response to the added complications of group decision making, a plethora of decision-making techniques have been created to account for the different domains, various stakeholders and diverse types of problems.

Many of the individual techniques for decision making have been proposed and analyzed with the various adjustments to account for the multiple stakeholders. For example, TOPSIS has been extended for groups [154], for groups with preference aggregation within the procedure [155], and for groups in fuzzy environments [34].

Likewise, AHP has been extended to group decision making with goal programming in fuzzy environments [190].

Traditional multi-attribute or multi-objective group decision-making techniques are similarly enhanced or adjusted to account for the weights of different multiple decision makers, the aggregation processes, and the normalization operation. For example, one particular group decision-making techniques make use of Monte Carlo simulations for aggregating decision maker preferences [102]. It also incorporates incomplete information about the weights and the utility functions across the different decision makers which are then made more precise through a negotiation processes. The result is hopefully a consensus alternative for the entire group. Another study compared experimentally the effectiveness of three group decision-making techniques using multi-objective linear programming (MOLP), namely 1) the Group Naive Search (GNS) implementing a weighted-sums approach, 2) the Group Step Method, and 3) the Group Goal Programming Method [79]. GNS was found to take longer on average for the group to reach compromised solutions and resulted in lower quality solutions compared to the other two.

Furthermore, the VIKOR method [122], which is similar to TOPSIS but implements a slightly altered aggregation process as a function only of the distance from the ideal point, proposes a compromise solution, and uses linear normalization. TOPSIS, on the other hand uses both the distance from the ideal and detailed comparison between TOPSIS and VIKOR methods is presented in [123].

Yet another technique, extending the VIKOR method, has also been applied to multi-attribute group decision-making problems, where the attribute weights are given as generalized interval-valued trapezoidal fuzzy numbers [98]. This is similar to having a distribution on the weights themselves, although four points must be defined for each objective's "trapezoidal fuzzy number" which may become excessive for high dimensional spaces.

Of course, voting methods themselves can be applied to group decision making and often are [21]. Popular methods or techniques include the Borda Count [42], Plurality Voting [37], Condorcet Method [140], Instant Runoff [162] to name a few. Similarly, they also contain their own unique modifications to account for different problems or decision situations such as multi-stage voting or binary voting trees [126]. However, many of these suffer from limitations (described in an example presented in the appendix), and thus do not meet the requirements defined by Arrow's Impossibility Theorem summarized previously when there are three or more alternatives.

Regardless of the exact group decision-making technique implemented, at some level, the process must address the influence or power of each one of the stakeholders or decision makers within the group. Furthermore, sufficient distribution of power is expected among the decision makers or else one may approach breaking the 'Non-dictatorship' requirement for reasonable aggregation methods, resulting in a problem appropriate for a single decision maker.

3.7.4 The Delphi Technique

A similar technique to the overall methodology presented in this research is called the Delphi technique, originally proposed by Olaf Helmer-Hirschberg [75]. In general, the Delphi technique seeks to facilitate group decision making by iteratively collecting information from, and presenting results to, a set of experts which will reach consensus on a particular problem, typically focused on forecasting future states or planning. It has been successfully implemented in a variety of ways and in a variety of domains including medical care, government planning, and business [97].

Although this research and the Delphi method have generally similar goals in facilitating consensus reaching for multiple decisions making the domains of application are quite different as well as a number of other details. Still, some of the ideas and process involved in a Delphi-based decision making activity form the inspiration and foundation for parts of the steps in the methodology described herein. Some of the similarities and differences are discussed below.

First of all, the Delphi technique was designed initially to be used as a forecasting tool by aggregating expert knowledge, opinion, or, in particular, "informed intuitive judgment" about future states of technology and science. In some respects, the Delphi technique is designed to assist with prediction-making or planning. In other words, an agreement or consensus on the likely futures and the associated probabilities of these futures are desired outcomes of such an activity.

In contrast, the methodology presented in this dissertation is more focused on requirements definition (or concept selection from a different perspective). At these decision points, a future is assumed (perhaps as a result of some completed Delphi technique implementation) and the response to the this future is now under consideration. The future needs or scenario may already be created or exit (i.e. humanitarian needs after a natural disaster) but the solution to the future problem or needs requires cooperation amongst stakeholders. Simply worded, Delphi concerns itself more with consensus on the future scenario necessary for planning while the methodology herein focuses on the solution set or response to those futures.

Similarly, while the Delphi technique primarily attempts to use "experts," (however they may be defined), in isolation, the methodology presented here assumes no correlation with "expert" and "decision maker" and assumes that interaction is a key part of the process. The decision makers themselves are not necessarily experts in the fundamental sense but are evidently in positions of decisional power and exert influence over others and on the selected solution or requirements.

Furthermore, since many of the solutions which will meet the requirements for a particular problem demand cooperation between the various players, decisions in isolation are likely not acceptable as trades and compromises will be almost always required. This is one limitation of the Delphi technique which can end with two (or more) polarized groups quite confident about their respective opinions about the given situation. Since all stakeholders must be in agreement with one set of requirements or design, this is also unacceptable for the given motivating problem for air mobility requirements definition. For example, an RFP with two sets of design requirements because the decision makers could not agree would clearly be a disaster. The iterations of the Delphi technique, of course, can be repeated many more times but no guarantee on full group consensus is given.

On the other hand, the similarities between the proposed methodology and the Delphi technique also exist in a variety of ways. There is an initial information elicitation step in both processes which reveals the preferences of each of the individuals. In both methods, the opportunity to keep responses to the various questions can be made anonymous such that a truer opinion is extracted from the questionnaire (or discrete choices) due to increased confidence by the participants that no one else will see their answers. Yet, the reasons for such opinions are available to others for additional persuasion. Thus, both methods also account for changes in the decision makers preferences albeit in different ways. The Delphi technique requires updating each players opinion in each iteration while the proposed methodology accounts for this with a distribution of possible preferences. Lastly, the opportunity to weight the opinions of experts in the Delphi technique is matched by the power or influence relationships in the proposed methodology. The assumption that not all decision maker or expert is equal (in the sense of their ability to persuade others to view things their way) is an available option in both processes. Thus, in the Delphi technique, each expert may be assigned a weight, whereas in the methodology presented in this research, an influence relationships is obtained for each decision maker in the group.

3.8 Power and Influence in Groups

The research into power and influence, what they comprise, their difference and/or similarities, and their impacts on decision making is abundant. The studies, data and interpretations of various experiments span a wide range of fields.

A study by Raven, Schwarzwald and Koslowksy starts with the statement: "Social power can be conceived as the resources one person has available so that he or she can influence another person to do what that person would not have done otherwise" [131]. In a decision making context, this can equate to persuading someone else to change their preference to become more aligned with another decision maker. Although independently the first would not normally change their preference, expressions of power or influence would cause them to concede and compromise fully or at least partially in cooperative group decisions. In particular, one of 11 identified sources of power may stem from *legitimate reciprocity*, where one is obligated to respond to an agent's request if in the past (or in the future), positive action has been (or will be) granted to the target [131]. This is often the impetus of mutually beneficial contracts, the result of which often encourage partnering repetitively for other projects. The other power sources, such as power derived from expert knowledge, position, or ability to reward or punish, are also feasible ways that influence can be exerted over others in cooperative group decision making [131].

Although clearly inequitable from some perspectives, it may be desirable to have some differing levels of power or influence spread across the group if the problem is broad enough to cover a variety of perspectives, points of view or objectives. After all, these problems will likely require a variety of experts, each with experience, knowledge or competency in one a particular area of the problem.

This is almost guaranteed with large SoS engineering problems, which, by definition, incorporate multiple views, interfaces and functions, and employ remote, yet highly integrated, physical systems. Therefore, recognizing these perspectives and that some of them can be more accurate, pertinent or important than others is a crucial step in properly implementing useful acceptable group decision-making processes.

The weights attached to the decision maker's is sometimes referred to as a scaling constant, to distinguish the use of the term "weight" traditionally used to define the importance value for a particular dimension or attribute. However, the weight of a decision maker is still used in many references but is in essence an indication of that member's power or influence in the group, scaled or normalized appropriately.

3.8.1 Evaluating the Power or Influence of a Decision Makers

Just like the various attributes or objectives can be assigned a weight or importance value, the decision makers themselves can assume values indicative of the power or influence they have over the group. As the number of decision makers reach very large numbers (e.g. the population of a country) in some decision problems (e.g. political elections) the individual weight or power of just one decision maker is very small, but pools of similarly minded individuals, coalitions or subgroups can and do have significant clout in these situations (e.g. political parties, petitions, etc.).

According to [130] there are two basic ways of assigning the weight or influence to the decision makers: the *supra decision maker approach* and the *participatory approach*. If a supra decision maker is available to assign weights (i.e. scaling factors) to the others then this particular step precipitates to a single-agent decision-making problem, and the classical preference evaluation techniques can be used, not on the attributes or the objective, but on the importance of decision makers themselves. However, often in group decisions, there is no one decision maker which defines the power structure or importance of the others. In such situations, a participatory approach is required.

The following sections summarize a few of the ways that influence and power have been evaluated for group decision-making activities.

3.8.2 Shapley Values

Shapley values, named after Lloyd Shapley, provide a context to analyze coalitions in n-person decisions involving cooperation [152]. Each member in a group will have a Shapley value for a particular group decision. The value is conceptualized as the payoff to a particular member for joining the coalition or a group with similar preferences. The difference in payoffs between the coalition before and after some player joined is "claimed" by that same player as their compensation in joining the coalition. As each player calculates their marginal payoff of the coalition, a marginal vector is created, and a certain player's Shapley value will be the average of marginal vectors across all the possible orders of the players [10]. The summation of the Shapley values for all decision makers in a group must equal to 1.

3.8.3 Shapley-Shubik power index

A related value to the Shapley value called the Shapley-Shubik (SS) power index can also be used in situations where coalitions can be formed in some multi-agent decision-making processes [151]. This power index quantifies the power an individual decision maker may have in expressing and achieving their preferences, based on the full set of permutations of the votes for all players in the election or decision-making process.

For example, the SS index (ϕ_i) for player i is the value:

$$\phi_i = \frac{1}{n!} \sum_{S \subseteq N} (|S| - 1)!(n - |S|)!(v(S) - v(S \backslash \{i\})) \tag{4}$$

where, n is the number of players, and $v(S)$ represents the payoff of coalition S [10]. This value represents the contribution or payoff to the game of the difference with player i ($v(S)$) and without player i ($v(S \backslash i)$). When averaged over all the possible $n!$ permutations, the ith player has the contribution value or power index

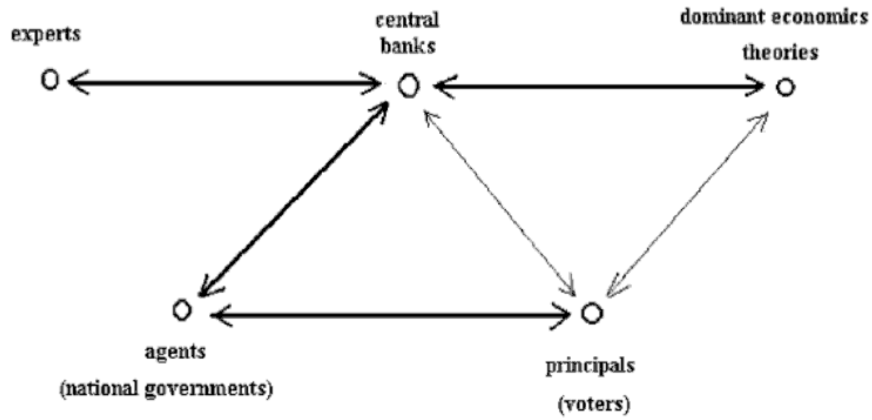


Figure 7: Example of Various Levels or Degrees (arrow thickness) of Influence Between Agents (Taken from [110])

of ϕ_i [10]. When each player is given a power index, a prediction on how any one agent will be influenced by another can be modeled, by assuming that one decision maker with more "power" will be able to persuade another agent to agree to their preferences more readily.

Figure 7 illustrates how multiple agents can have different levels of influence, if at all, on other entities. This example demonstrates potential relationships where voters have less influence on the central banks than financial experts or government agencies, considered a more likely scenario [110].

Assuming a dynamic number of decision makers at any one time, the value would require reevaluation since during negotiations or trades across different objectives, the power of any one player or agent is likely to change or shift.

3.8.4 Other Methods to Extract Decision Maker Weights

AHP, although used often to apply weights to objectives or attributes directly, has been used to extract the power between decision makers as well [130]. This approach has the decision makers define the ratio between their strength, weight or power directly with each other decision makers. An eigenvector method is then implemented to obtain the power or scaling factor for each stakeholder. Similarly, in [181], a discussion of calculating the decisional power of the members in a group environment is discussed using the REMBRANDT software suite implementing multiplicative AHP and SMART (Single Multi-Attribute Ranking Technique). In parallel with research implementing AHP, applications of TOPSIS have also been extended to determine the strength or power of each member in a group [191].

Likewise, game theory applications have been recast to also provide power indices of decision makers in groups [43]. Lastly, Banzhaf [11] and Coleman [36] power indices are two other options for calculating and describing the power or influence between members for collective groups.

3.9 Game Theory

Two individuals often considered the founders of game theory, von Neumann and Morgenstern [182] have provided the mathematical foundation upon which a considerable amount of research has been performed on how decisions are made amidst uncertainty. This uncertainty is typically found in how one's payoff, or some other metric of success, is dependent upon someone else's decisions. Since a decision-maker cannot know for certain what alternative or strategy others may choose, their expected payoff is uncertain. On the other hand, whichever alternative they choose can impact the payoff of other decision makers. Understanding, analyzing and evaluating this interdependency of decisions and payoffs among all the decision makers is at the heart of game theoretic research. Much of the effort focuses on identifying strategies considered "equilibria" from which a decision maker will select and apply continuously to the particular situation, since any other strategy will result in a less favorable outcome.

The analysis of these "games" has been useful for making decisions in many different fields from economics [53] to ecology [53] to engineering [26]. Although these games are essentially simplified models of real-world problems or challenges they provide a framework from which insight can be gained about how other decision makers will react or respond to one's own decisions; a useful tool in today's ever more networked and interdependent society.

3.9.1 Types of Games

As the number of areas grows for applying game theory, more precise and definitive characterizations of the types of games have resulted. These categories, described in the following sections, have allowed game theorists to understand and model the more complex assumptions that occur in real-life situations.

3.9.2 Symmetric and Asymmetric games

In symmetric games the payoff for a particular strategy regardless of the player is the same. If the payoffs are dependent on the player the game can be called asymmetric. For example, if in a particular game, the payoff is an apple or an orange for each player, and both players find both food items equally satisficing, the game is symmetric. Whereas if one player is allergic to oranges, the game payoffs would be asymmetric, with a negative payoff if receiving an orange and a positive payoff with an apple. The other player's payoffs remain the same as in the symmetric case.

In general, no two decision makers are identical. Each has his or her own beliefs, experiences and values. In most cases, asymmetric games are more reflective of reality as a result. The value of a good or service, for example, is often viewed differently between the buyer and seller. Still, the significant use and applicability of symmetric games is evident such as analyzing nuclear deterrence theory [80].

3.9.3 Perfect and Imperfect games

Perfect and Imperfect games refer to the information that each of the decision makers or players has regarding the history of the game. Since this type of categorization requires previous moves, a sequential game or one that has a number of choices over time is most commonly associated with a game of perfect information. Most one move games will be classified as games of imperfect information since knowledge of how the other player or players will act in the game is unknown.

3.9.4 Zero-sum and Non-zero-sum games

As the name suggests, zero-sum games refer to the inverse relationship of the players' payoffs. That is, for a two player game, if one player receives a negative payoff (i.e. -100) the other player will receive an equal in magnitude positive payoff (i.e. +100) and similarly for an n-player games, one player's payoff increase will be counted by an equivalent decrease for one or more of the other $n-1$ players' payoffs. On the other hand, the non-zero-sum game removes the restriction that one player's lost is another player's gain. For example, if one player can change their strategy and increase their own payoff without negatively impacting the payoff of the other players, the game is non-zero-sum.

3.9.5 Cooperative or Competitive Games

Another categorization that can be applied to game theory is cooperative or competitive games. Competitive, or non-cooperative, games are often used to model situations where coalitions cannot exist to increase the payoff of the group or coalition [73]. Very often non-zero-sum games are competitive games in the sense that one player's loss is another player's gain and thus both compete for the limited resources. Cooperative games are models of situations where the players can form alliances or coalitions to increase the groups', and potentially each individual's, payoff. Many games involving voting can be considered cooperative games in that a group of individuals can pool their votes together to reach a majority or some other threshold to guarantee their particular agenda across all the players both within and without the coalition. Since all players in a coalition seek a common result, before their "vote"

		Player 2	
		Cooperate	Defect
Player 1	Cooperate	3,3	0,5
	Defect	5,0	1,1

Figure 8: Example Payoff Matrix for the Classical Prisoner's Dilemma

is cast together, members within the coalition may have to make compromises, but these are considered acceptable concessions if the ultimate decision is more closely aligned with their own individual preference.

3.9.6 The Prisoner's Dilemma

The Prisoner's Dilemma is a 2-player game in which two prisoners, having been recently arrested for committing a crime, are each individually and separately offered a choice by the police: the prisoner can either provide evidence or testimony against the other prisoner or remain silent. If both prisoners remain silent (i.e. both cooperate with each other), there will not be enough evidence to convict either of the prisoners and they will both remain in prison on a smaller charge for a short sentence of one year. If one prisoner provides evidence (i.e. defects) and the other remains silent (i.e. cooperates), the first prisoner will go free and the second will go to prison for five years. If both prisoners testify against the other (i.e. both defect) they will both serve a sentence for three years.

A chart, called a payoff matrix, describing the dilemma with payoffs and strategies for both prisoners, is summarized in Figure 8.

The first prisoner, Prisoner 1, has their strategies down the rows, where "Cooperate" indicates cooperating or helping the other prisoner (i.e. staying silent) and

“Defect” indicates defecting or testifying against Prisoner 2. Prisoner 1’s payoffs are the first number in each quadrant. Prisoner 2 has the same two strategies and their payoffs are given as the second number in each quadrant. The payoffs are represented in Figure 8 by how many years in prison each will avoid for each combination of strategies. This is to keep all numbers positive such that each prisoner wants to maximize the number of years *avoided* for their individual sentence.

If Prisoner 1 cooperates and remains silent, Prisoner 2 would want to defect and go free as indicated in the top right quadrant. If Prisoner 1 defects and testifies, Prisoner 2 would want to defect as well, and have their sentence reduced since if they remained silent (bottom left quadrant) they would spend the maximum amount of time in prison. Regardless of what Prisoner 1 does, Prisoner 2’s best strategy is to defect. But the exact same process and logic can be applied to Prisoner 1 in that their best strategy is also to defect regardless of what Prisoner 2 does. As a result, both prisoners will defect and both will serve the maximum sentence (and avoid only 1 year). In this particular game, a Nash equilibrium is found in the bottom right quadrant (both defect), where neither prisoner can improve their payoff by switching to the cooperate strategy. However, there is a more Pareto-optimal point (both cooperate), where both prisoners can improve their payoffs. Interestingly, the selfish rationality of both prisoners seeking to minimize their sentence (maximize their payoff) resulted in an equilibrium which was sub-Pareto-optimal. Thus, the individual rationality resulted in group irrationality by both selecting a less optimal strategy, even though a strategy exists where both could improve their payoff.

The Prisoner’s Dilemma game can be generalize with the payoff matrix presented in Figure 9, where R is the reward for mutual cooperation, S is the Sucker’s payoff, T is the Temptation to defect and P is the Punishment for mutual defection, and $T > R > P > S$ with $R > (S + T)2$ for the game to be defined as the Prisoner’s Dilemma [8]. Comparing this generalized form to Figure 8, these equations hold true

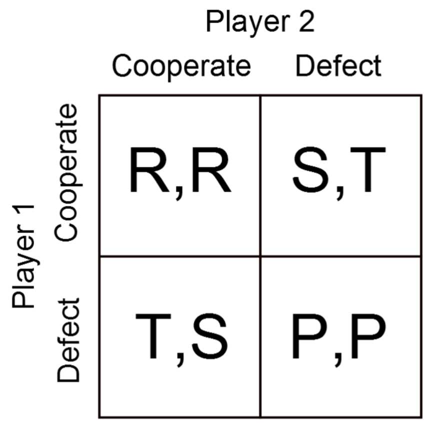


Figure 9: General Payoff Matrix for the Classical Prisoner's Dilemma

when considering that a higher payoff is less time in prison. For example, the inverse payoff matrix could have T=0, R=-1, P=-3, and S=-10, where the payoff represent years lost in prison and one would still want to maximize their payoffs (i.e. make less negative) as in Figure 9.

The game of Chicken, or sometimes known as Hawk-Dove, can take on the same general form but with different equations, namely, $S > P > T > R$ [105], and replacing "Cooperate" with "Continue Straight" (Hawk) and "Defect" with "Swerve" (Dove), under the assumption that the game is conceptually interpreted as two cars driving straight toward each other and it is a test of driver bravado. An equilibrium can be found where both players will swerve and receive payoffs equal to P. However, in this game, the dynamics are slightly different in that if one player changes their strategy to "Continue Straight", it does make sense for the other player to accept a lower payoff by also switching to "Continue Straight", although, they will have to accept a lower payoff by continuing with the "swerve" strategy. The concept of "brinkmanship" is one way to bring the other player back to the original "Swerve" strategy as discussed in more detail in [145].

In both these games, the actions of each decision maker are dependent on the actions of the other. If player 1 decides on a certain strategy and player 2 knew it, player 2 could maximize their own payoff, but if player 1 knew that player 2 knew what player 1 was going to do, player 1 could change their strategy also, and so on with this pattern continuing ad infinitum. To avoid this never ending circular logic, the equilibrium point(s) aforementioned were shown to be point(s) at which neither player is willing to change their position or strategy without external pressures or additional information. Thus, applying strategies to find the equilibrium, game theory can be effective in allowing one to identify what strategy they should implement which will best maximize their individual payoff, or meet their particular objectives.

3.9.7 Limitations to the Classic Prisoner's Dilemma

The classic Prisoner's Dilemma (PD) game has a number of useful attributes and is used in a variety of problems where more than one player has an influence on what all the players receive as payoffs. However, a number of limitations have been found and alternative games or different modifications to the classical prisoner's dilemma game have been investigated, with one candidate being the Snowdrift Game [50].

Some of the limitations with short descriptions of the PD are listed below:

- The classical PD involves only 2-players. Although 2-player models cover a large amount of real-world decisions, often more than two agents are involved in any decision. This is typical in any voting activity such as a company's board of directors or shareholders, and in elections.

- The payoffs are deterministic in a classical PD game. Although a number of different values for T,R, P, and S in Figure 9 satisfy the requirements for a PD game, those values are often probabilistic in nature, in that the actual payoff may be higher or lower than the expected payoff. In other words, the payoff is also uncertain regardless of the combination of strategies employed by the two agents. For example, the payoff if both players defect may be an actual time in prison above what was expected if the judge so decrees.

- The classical PD also assumes that there is no real knowledge about how the other player(s) will act. In reality, there is some history or available knowledge which will provide insight into the value system or expected choices of the opposite player. Most real-world multi-agent decisions are not made between two or more individuals with no knowledge of the other's background.

- Assuming only one PD game is played the players expected behavior is well predicted. However, very often players come in contact and reach multiple decision points with the same individual(s). This can be quickly accounted for with the iterated prisoner's dilemma, where a number of sequential PD games are played and behaviors are much more varied especially when the number of iterated PD games is unknown.

- Some forms of the PD game include no real investment or cost. That is, the players don't necessarily put any additional effort into their strategies. Real decisions and strategies required different amounts of investment or cost to the player. Furthermore, if one player has already invested heavily into one strategy they are more likely to select it.

As a result of these limitations, additional games are considered which are summarized in the next few sections, namely the snowdrift game and the ultimatum game.

3.9.8 The Snowdrift Game

The Snowdrift (SD) game is one answer to some of the limitations of the PD in describing real-world interactions between human decision makers. Researchers have explained that "the PD does not represent the frequent situation where individuals obtain immediate direct benefits from the cooperative acts they perform and costs of cooperation are shared between cooperators" [91].

The SD game is generally described as a snow drift blocking the road which two drivers (or players or actors) on either side of the snowdrift desire to travel through. They can let the other player clear the road and remove the snow while they wait or they can decide to help remove some or all of the snow [164].

Staying in the car and letting the other driver clear all the snow is the ideal strategy. However, it is also the best option for the other driver as well. The social dilemma, similar to the PD, is evident in that defecting or refusing to help remove the snow is best when the other driver is cooperating and is clearing the snow [41]. However, since the ultimate goal for both drivers is to pass to the other side, there is an incentive to not wait and begin to remove snow. Regardless of what the other driver does, eventually the one who removes the snow will gain passage. Thus, there is a cost or investment associated with removing the snow but a road clear of snow enabling passage is clearly a benefit and can be acquired regardless of what the other driver chose as their strategy.

The SD game has been shown to be a better model for high cooperation as shown in Figure 10 [91]. In Figure 10, the data show the results of how often cooperating acts were observed in the two games (PD and SD) between players who were both female (FF), both male (MM) or one of each gender (FM). The numbers below the bars represent the sample size in each of the six categories.

The SD game is a useful tool for analyzing decision making when there is an additional incentive to cooperate. In many business deals, the cooperation or consensus point is reached after each player or actor is willing to make a compromise (i.e. a cost or investment) to obtain the item. In such cases, it must be assumed that the cost expended by any one agent or player is less than the benefit. An agent will not trade or sell a product or position for one that is of lesser value.

In terms of decision making, reaching consensus is usually more favorable than

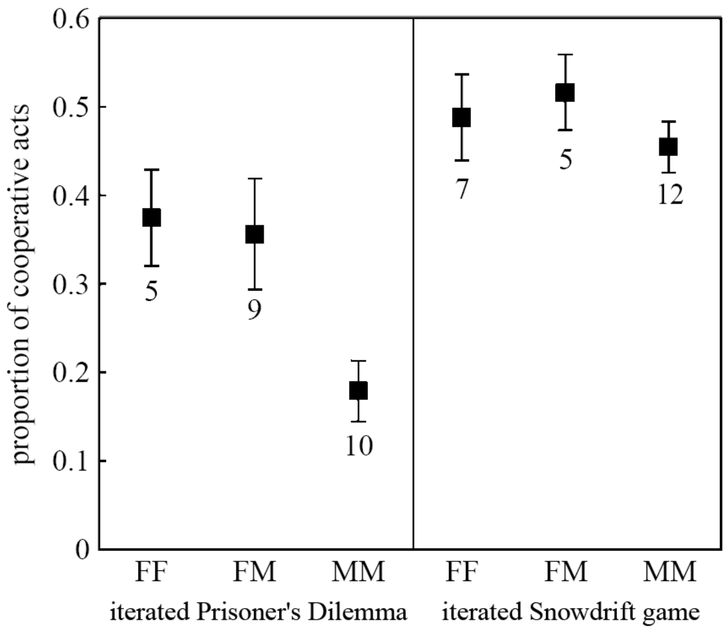


Figure 10: Comparing the Iterated PD and Iterated SD (from [91])

never reaching an agreement, since time can quickly become a cost if indecision negatively affects both or all parties. It also can be used to show that free-loaders can be modeled as a common occurrence in many group decision-making activities. Lastly, SD can potentially be used to account for the higher levels of cooperation observed in experimental tests compared to PD models and predictions [50].

3.9.9 The Ultimatum Game

Another game which improves upon PD in some areas is the Ultimatum game. Traditionally set up as a 2 player game, one player (the proposer) will propose how much (or a percentage) of some money, or other desirable resource, the proposer will receive and how much the other player (the responder) will receive. The responder can accept or reject the proposal. If he or she rejects the proposal, neither player receives any money [117]. A Nash equilibrium is found with a proposal that is just slightly greater than zero. This equilibrium is found since the responder, assuming he or she is rational, will take any proposal greater than zero [137]. However, experimental results indicate that this equilibrium does not predict actual human behavior very well, especially over time [119], [113].

Thus, in reality, experimental results have suggested that individuals are willing to penalize themselves in order to punish unfairness in others. The responder is willing to accept a loss to punish the proposer. As a result, over time if the ultimatum game is played multiple times, with sequential bargaining, proposals may tend to the 50-50 split, but other splits may be equally likely with asymmetric information or external influences [163]. For example, consider two companies where both have different requirements for return on investment or other economic metrics, and both desire to combine expertise to add value to a particular product line. The profit share of each of these companies will likely not be 50-50, based on their contribution to the design of the product. Thus, "fairness" can only be defined in the particular context when both parties agree to the split. Clearly, if one company invests significant more capital in a development program, they will understandably expect a higher return. On the other hand, if the smaller company does not contribute financially but possesses a skill or expertise, they may be also able to claim a larger percentage than what the capital investment input would suggest. Therefore, the "fair" value is dependent on each of the agents (i.e. companies) perceived deserved level of compensation. Still, as in previous examples, agreement is preferred over no agreement and thus attempts to propose and counter propose until a point is reached which is mutually acceptable will likely occur. However, the point at which that occurs, is again dependent on the relationship between the decision makers, the individual preferences or weightings for the particular criteria involved, and the knowledge about the design point.

3.10 Discrete Choice Modeling

Choice modeling is one name for the general principle of creating models representing (usually) preferences between attributes or about criteria, from data created from comparing two or more designs or concepts. Other names describing this research area include: “discrete choice”, “stated preference” or “choice experiments” [138]. Since the choice is often between two options, the term “binary choice” is also commonly used [158]. The resultant model can effectively predict how a human will make decisions and their individual behavior when given a choice between options.

Much of the recent theory on choice modeling is based on the work of Daniel McFadden who received the Nobel Prize in Economic Sciences in 2000 for his work on discrete choice [99]. Although McFadden’s work is expectedly founded in both empirical and theoretical economic models, choice modeling has been applied to other fields where choice and human behavior is an essential factor such as in occupational positions, recreational activities or transportation mode choices [107].

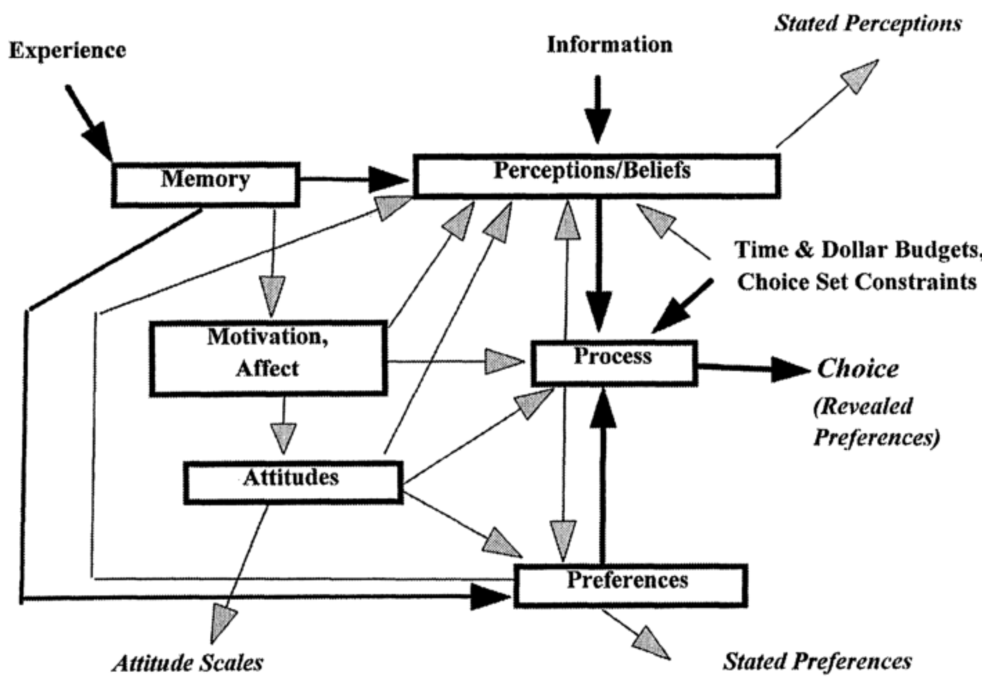


Figure 11: The Choice Process (from [107])

A recent model representing the choice process is shown in Figure 11. In this model, more inputs and outputs are evident than usually considered. Not surprisingly, data or information (shown entering the model at the top) is a requirement in any choice or decision-making process. Experience, on the other hand, is also a key input which is often couched in phrases such as "the art of design", "subject matter experts" or "tacit knowledge." Time, training or experience with previous decisions drive both preferences and perceptions through memory of previous choices. Whether this "memory" is actual historical trends, previous designs or established precedence, it influences how the data is perceived and what attributes or criteria are preferable. The process itself will result in a choice, where the data (and beliefs about the data) in combination with the preferences of the data or criteria are applied under the constraints of time, money or other limitations. A number of outputs in addition to the ultimate choice can be extracted from this process, namely attitude scales, stated perceptions, stated preferences and revealed preferences.

Attitudes (i.e. attitude scales) are "stable psychological tendencies" to view or consider an outcome approvingly or not, whereas perceptions are how one interprets a particular stimulus cognitively [18]. Preferences, similar to attitudes, are a perspective or measure of how one relates to a particular entity or criterion in terms of "like" or "dislike." Preferences can be rank ordered qualitatively or measured quantitatively usually with utility, a level of satisfaction as viewed by the decision maker often "correlative to Desire or Want" [100]. The preference for one attribute over another attribute can be revealed in the utility or score of each attribute. For example, if a decision maker receives more "utils" (i.e. general unit of pleasure or satisfaction) for a low risk design than a low cost design, their stated preference would be for lowering risk over lowering cost. However, since these two dimensions may not be independent (i.e. low risk designs do not necessarily mean low cost designs), and furthermore the utility could be a function of the magnitude of both of the measures, multi-dimensional utility functions are required to evaluate the utility of any particular design. For these multi-dimensional preferences, a utility function can be composed of the summation (or other mathematical combination) of the individual utilities of the various dimensions or objectives.

CHAPTER IV

METHODOLOGY DESCRIPTION AND DEVELOPMENT

In order to satisfy the research objectives and to perform experiments that can answer the various research questions and lend support to the related hypotheses, an overall methodology has been developed which allows one to test the pertinent variables in response to the motivating problem. The overall methodology with its two constituent parts is described in the following sections. A brief description of each of the two main parts is presented and then again in more detail of the specific steps.

4.1 Research Methodology Overview

The overall methodology discussed in this research is broken down into two major elements, 1) the Air Mobility Operations-based Design Model (AirMOD), and, 2) the Methodology for Multi-Agent Consensus Reaching on the Objective Space (MACRO). Together these two components offer a unique solution in addressing the motivating problem discussed in Chapter 1 to define the requirements of the fleet for a new heavy-lift cargo air transportation system.

Figure 12 illustrates where these two components fall within the classic systems engineering paradigm. After the need has been establish and the problem defined (i.e. requirements definition for air mobility systems) with the associated value, the set of possible solutions or alternatives, comprising a set of requirements, must be generated as potential candidates answering the problem. These solutions are design requirements or parameters which will drive future decisions within the acquisitions process.

The evaluation of these designs is then performed using utility scores across the operational metrics of interest. After a sufficiently large set of candidate solutions has

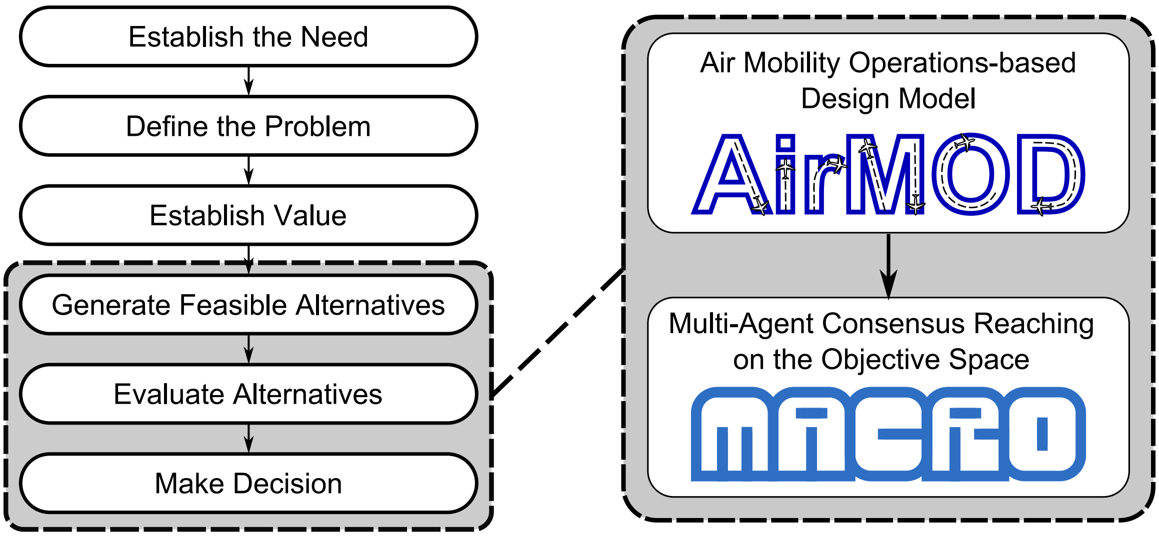


Figure 12: Methodology Application Areas to the Generalized Decision-Making Process

been generated and evaluated by AirMOD, the designs are passed onto the MACRO methodology in which the multiple stakeholders or decision makers reach a consensus on the design space in selecting a set of requirements for further investigation, testing and analysis.

The following two sections briefly describes these two elements followed by additional descriptions and details of the steps in the MACRO methodology.

4.2 AirMOD Model Overview

The Air Mobility Operations-based Design Model (AirMOD) is in direct response for the need of faster simulations to generate many more candidate solutions. With increase computational power and advanced modeling techniques, decision makers demand a better defined and characterized decision space. No longer are small data sets with just a few potential solutions adequate for the ever-changing and uncertain environments in which decision need to be made. Solutions which consider a variety of input scenarios, parameters, initial conditions and constraints all must be simulated and analyzed. Confidence in decision making can only come from a design or decision space which is well defined and explored. Only then can a decision maker make a decision.

AirMOD provides an operational and logistical perspective to system engineering solutions by calculating mission success in terms of the time to close or deliver certain total amounts of payload to various location throughout the world under different scenarios (discussed in more detail in later sections). For these operational models, with multiple objectives and potentially many more input variables, many simulations must be executed across the input space to increase the design space characterization. AirMOD leverages advanced design methods and techniques including surrogate models to execute each case much more rapidly compared to the full simulation model.

With the implementation of surrogate modeling techniques, AirMOD can generate a new solution every $\approx$3 ms, equivalent to about 1 million executions in less than 50 minutes. This in turn allows the identification of essential logistical and operational trades that are impossible with classical tools. Only with enough design points or solution can these trades and the associated Pareto fronts be explored and utilized in more advanced decision making processes.

Finally, these trades provide key levers on the decision space in which decision makers can utilize for making compromises and reaching consensus in group decision-making processes. Without the capabilities of AirMOD, group decision making on this air mobility operational design space would be severely limited with a higher probability of decisions made with greater uncertainty and with less confidence.

4.2.1 AirMOD Process Flow

The AirMOD model leverages the process flow, shown in Figure 13, from the Strategic Airlift Comparison Tool (described in more detail in the next chapter).

The discrete event simulation, written in SimPy [159], begins with all aircraft within the predefined fleet at the Aerial Port of Embarkation (APOE) and ends when

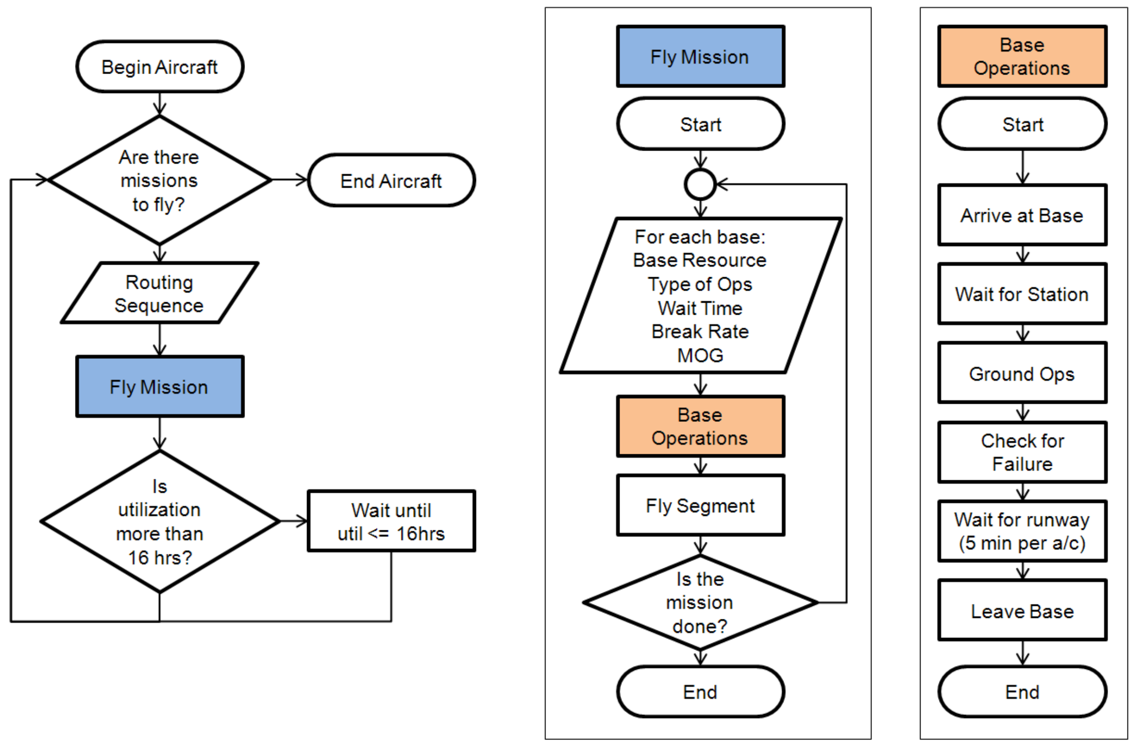


Figure 13: AirMOD Simulation Flow Block Diagram [From [142]

no more missions (i.e. no more cargo remains undelivered at the APOE) are required and the last delivery has been completed.

The aircraft in turn are assigned a route type by the user, which defines the flight path and the en route refueling locations if any. After the mission is flown, a delay is applied to account for scheduled maintenance such that the utilization never exceeds 16 hours. The unscheduled maintenance is randomly applied through the break rates parameter entered by the user and discussed in a later chapter.

During each mission (see the middle column of Figure 13), the sequence of events will occur, updating each aircraft status, such as loading, waiting, repair, or other operations, which are tracked and recorded for total up-time, down-time, flight hours, etc. on a per aircraft basis and then summed for the fleet wide statistics. Additional base operations occur within its subblock as indicated on the far right column of Figure 13. These events include additional wait times for limited resources (e.g.

depot or fueling stations, cleared runway), ground operations (e.g. time to refuel), and repair times if a failure is identified and requires maintenance.

The output metrics include the times associated for these various categories and ultimately rolled up to the metrics of interest including the total time to close the mission, fuel consumption, flight hours, and actual fleet wide utilization values.

Since the model is stochastic in nature, multiple runs for each scenario is repeated to obtain a distribution on the time to close, which then allows for appropriate statistics on these same distributions. A design of experiments with 50000 cases covered the design space was then executed with 1000 repetitions for each case. Neural networks were then employed to created the surrogate models which enable even faster computation of the output space. These surrogate models are then applied by the AirMOD model (with additional enhancements discussed later) to create candidate C-X designs very rapidly. Each design, in turn, represents a potential set of requirements that meets the stated problem of multiple decision makers agreeing upon the needs and requirements of a future air mobility system described in Chapter 1.

4.3 MACRO Methodology Overview

The Methodology for Multi-Agent Consensus Reaching on the Objective Space (MACRO) responds to the other half of the problem to improve decision making within an operational design space to increase transparency and quality of group decisions.

The MACRO methodology in the current research is broken down into three main steps, namely:

- Step 1: Calculating Weighting Distributions
- Step 2: Extracting Power Relationships
- Step 3: Reaching Preference Consensus

In Step 1, the focus is to obtain the preferences or importance weightings for all criteria or objectives from each decision maker. The uncertainty in the decision maker's preferences are expressed as weighting distributions and are calculated from the answers provided from discrete choices of pairwise comparisons of designs or solutions. This step is directly in response to exploring and testing the Research Question #3 and its complementary hypothesis, respectively.

Step 2 extracts the power or influence relationships between the decision makers. Discrete choice experiments are again invoked to quantify the willingness of each decision maker to form a coalition with others while potentially selecting a less desirable design such that a trade between utility and partnership is realized. Research Question #4 and Hypothesis #4 drive the creation of this step to obtain this data such that later consensus reaching processes can be possible.

Lastly, Step 3 evaluates, through simulation, the expected region, both in the preference space and later mapped to the design space, at which the set of decision makers are most likely to reach consensus. This model incentivizes cooperation between decision makers, under the assumption that coalitions can be formed and can exert greater power or influence over agents not part of a coalition. The second research question and hypothesis will be addressed with this step in the methodology, showcasing the overall feasibility of reducing the design space using weighting distributions, power relationships and game-theoretic applications.

This overall process is called the methodology for Multi-Agent Consensus Reaching on the Objective Space (MACRO). These three steps of the MACRO methodology are visualized in Figure 14 with the set of candidate designs as the major input and the subset or region of designs at which consensus is likely as the output, with the complementary preference space region as well.

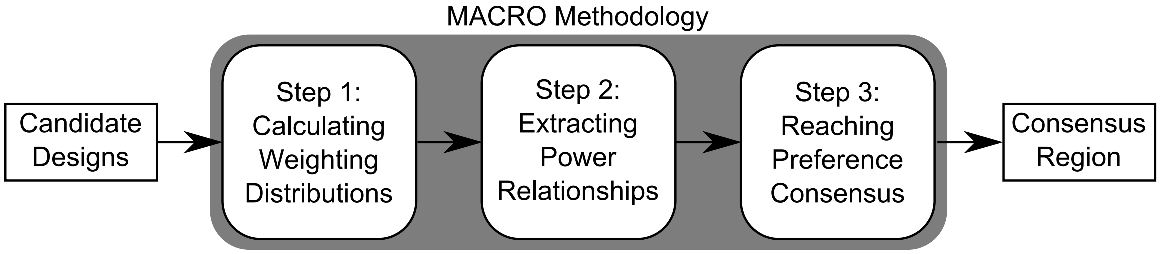


Figure 14: Overview of the MACRO Methodology

A preliminary step, sometimes referred to as "Step 0" in later chapters of this document, refer to the necessary processing steps of data or design points to create a feasible sets of candidates upon which the methodology acts. Although it is not a formal step in the methodology, it is addressed by the first half of the research objective and fulfills a crucial prerequisite assumed to have occurred before Step 1. Its importance and requirements are addressed in the canonical and case study problems in later chapters of this research.

The following three sections further expand upon and described in more detail the aforementioned methodology step summaries.

4.3.1 Step 1: Calculating Weighting Distributions

The first major step is to calculate the importance weightings or preferences of each decision maker on the various objectives as much as possible. Since there is uncertainty in the level of importance each decision maker places on the objectives, especially in group settings where changing opinions, or persuasion to differing viewpoints can exist, the preferences are expressed as "weighting distributions" reflecting the range of importance values or coefficients that the decision maker might apply under a variety of conditions.

In the previous chapter, a discussion of some of the alternatives for extracting this information (such as AHP or user-defined values) was discussed with some of the limitations inherent in those methods. AHP can suffer from rank reversal and

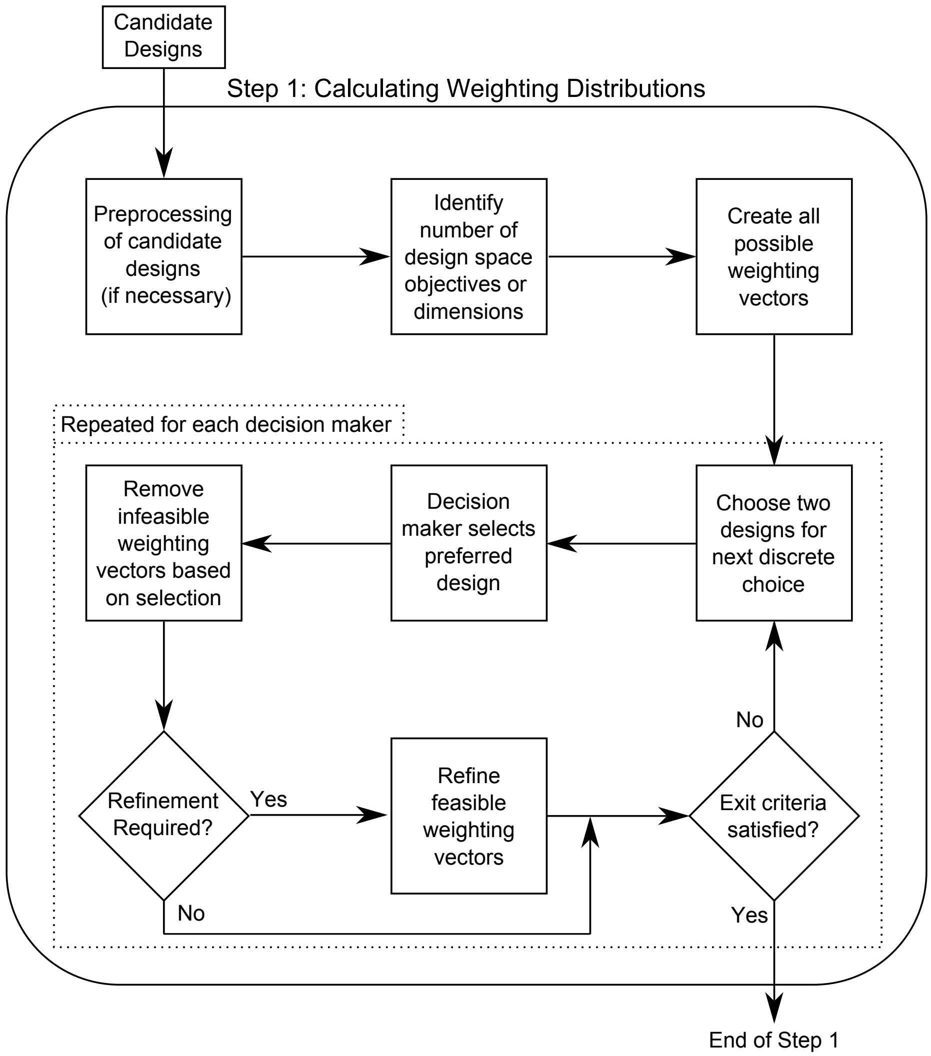


Figure 15: Flow Block Diagram of Step 1: Calculating Weighting Distributions

consistency issues. User-defined methods such as slider bars or other direct methods might not produce a distributions from which preference changes are accounted. Also, these methods are often scenario independent such that the decision maker is required to assume some general or average scenario and assign weightings to it. Furthermore, he or she will typically only look at small subsets of the multi-dimensional space and never consider the global perspective with all dimensions in mind. To remedy these concerns, a discrete choice experiment is implemented which does provide context and allows the decision maker to interact with "designs" and not "partial designs", in the sense of only a subset of the design parameters.

As shown in the block diagram of Figure 15, Step 1 begins by preprocessing the candidate designs among which decision makers seek to cooperate in choosing one design or set of requirements. This preprocessing, referred to above as Step 0, prepares the designs such that it meets the necessary requirements for later phases of the methodology. If necessary, quantifying the objectives that originally contain qualitative data would be performed within this sub-step. For example, a "low-medium-high" scale would necessarily need to be converted to numerical values (e.g. 1-3-5) if this dimension was to be included in some utility or valuation function for later steps. However, the discrete choice experiments themselves could still make use of the qualitative data, but numerically comparing various designs, a requirement for the methodology, is still needed.

Once the full design space is prepared and available, the number of objectives to include throughout the remainder of the methodology is defined. Ideally, the minimum amount of objectives will simplify and accelerate the analysis, but guaranteeing sufficient coverage in terms of the complete trade space of all essential objectives is required. Various guidelines and processes have been suggested to either expand or prune the list of necessary objectives [86].

With the dimensional size of the design space now defined, a set of all possible weights for each objective or the set of all weighting vectors in n-dimensions can be created at a particular resolution or discretization level. When n is large, the number of discretization levels should be lower to reduce data memory requirements, but higher resolved weighting vector can results in more precise weighting distributions.

Once the set of weighting vectors is defined, the one loop in Step 1 is entered (shown in Figure 15) by selecting two from the pool of candidate designs for the first discrete choice given to a decision maker. After the decision maker has responded, the infeasible weighting vectors are removed from the preference space, and the valid weighting distributions are updated.

This is followed by an optional refinement step where, if needed, the discretization level is increased for those areas of the preference space that are still valid and additional weighting vectors are concatenated to the set of feasible weighting vectors. This refinement step would likely only occur if very few designs or weighting vectors remain feasible and expanding the preference set is essential for additional discrete choice experiments.

The genesis or need for a refinement step was only developed after a recognition of the fact that higher dimensional design spaces required ever increasing weighting vectors to defined the objective space adequately. With sufficient computing power and memory available, this step could potentially be skipped but is likely still necessary for any problem of significant scope and with a large number of parameters or objectives.

Regardless of the refinement step, the stopping criteria will either continue the loop, where the next iteration of a discrete choice is provided to the decision maker with the reduced set of possible weighting vectors, or exit the loop, and thus Step 1, if the distribution of weighting vectors is sufficiently known or if time and resources will not allow for any additional discrete choice experiments of the particular decision maker.

Since each decision maker must respond individually to discrete choice experiments, the loop is executed for each agent or stakeholder in the group. Furthermore, each decision maker will have, potentially, a different stopping criterion. For example, one decision maker might have time to answer as many as time permits, while another may only desire to answer 5 or fewer. This is discussed further in later chapters with experiments to quantify the effects of differing stopping criteria.

Of course, with a different number of discrete choices for each decision maker, the certainty on the weighting distributions will be expectedly different. Although the methodology does not require equal amounts of certainty across decision makers, performance, measured by the relative "size" of the consensus region, is improved with less uncertainty. The following chapter will further explain and provide examples and analysis regarding these differences.

4.3.2 Step 2: Extracting Power Relationships

The second major step in the methodology is to obtain the power relationships from discrete choice experiments. This step makes use of the weighting distributions from Step 1 and is presented in an outline form in Figure 16.

Similar methods are available to extract this information as from Step 1 (e.g. AHP) but with the same limitations and issues. Since the information about the influence between decision maker can be highly sensitive, a method to extract these relationships while still allowing each decision maker to be comfortable with the provided information is requisite. Furthermore, a desire to use similar processes from before in Step 1, namely discrete choice experiments, such that the decision makers are not required to learn an additional technique for the current step. This not only encourages comfort with the process due to familiarity, but can also help in facilitating the speed at which responses are elicited.

Step 2 commences by defining the relationships which will be extracted from the set of discrete choices to be responded by the decision makers. Since a decision maker will possess a power relationship with all other decision makers, the set of power indices or relationships for all combinations can be defined with various power constraint equations, expressed in a matrix, A. The solution, x, to the system $Ax = b$ constitutes the relationships sought within this step.

Defining the right hand side b vector remains as the focus for the remainder of Step 2. Initially, it assumes large uncertainty, which is reduced after decision makers respond to various discrete choices about what designs and with whom they are more likely to form a coalition. Each response to a discrete choice will narrow the range of

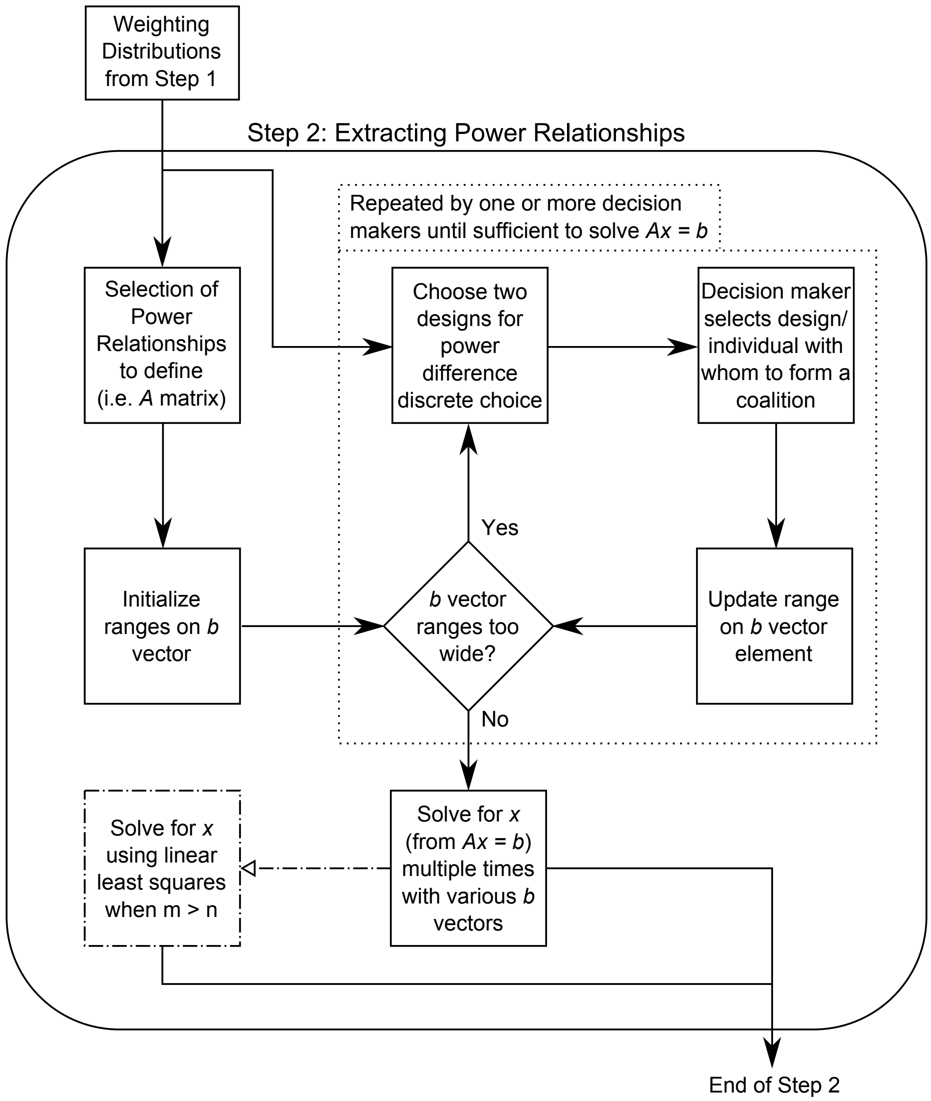


Figure 16: Flow Block Diagram of Step 2: Extracting Power Relationships

a particular element in the b vector and once the range is sufficiently narrow, Step 2 solves the system of equations multiple times to obtain a distribution or range of the power relationships as the output.

Depending on the maximum desirable b vector range, and on the number of decision makers, the number of discrete choice experiments necessary to satisfy the system will vary. However, a sufficient number of repetitions of the loop from one or more decision makers within Step 2 of Figure 16 is required to make $Ax = b$ solvable such that A is invertible. If more equations than unknowns are provided, a linear least squares approach to solving the system is available as indicated by the dashed line and block in the bottom left of Figure 16.

4.3.3 Step 3: Reaching Preference Consensus

The last major step of the methodology is obtaining the consensus region within the preference space and mapping those to the designs space to identify the set of designs which the group of decision makers are most likely to accept based upon their individual weighting distributions and power relationships and illustrated in Figure 17.

This step's origination comes from the requirement that to avoid many of the negative effects described by Arrow's Impossibility Theorem, if all decision makers are united in their preference structure then no conditions of fairness are broken. Furthermore, an assumption that consensus is reached intermediately before the final design or requirements are accepted by all parties. Coalitions are formed between decision makers and then these coalitions form super coalitions and so on until the full group consensus.

As explored in the previous chapter, voting techniques were first considered for consensus reaching but rejected due to a number of limitations, underscored by Arrow's Impossibility Theorem and described in more detail in the appendix. Analysis of other techniques including game theoretic approaches were more promising with the sequence of specific cooperative games considered including the Prisoner's dilemma, the Snow Drift game and the Ultimatum Game, the latter which offered the best attributes for the specific problem of bilateral and intermediate consensus reaching stages.

The two sets of data, individual weighting distributions and power relationships from steps 1 and 2 respectively, are inputs to a simulation of multiple executions of the ultimatum game which provides the region of consensus between only between

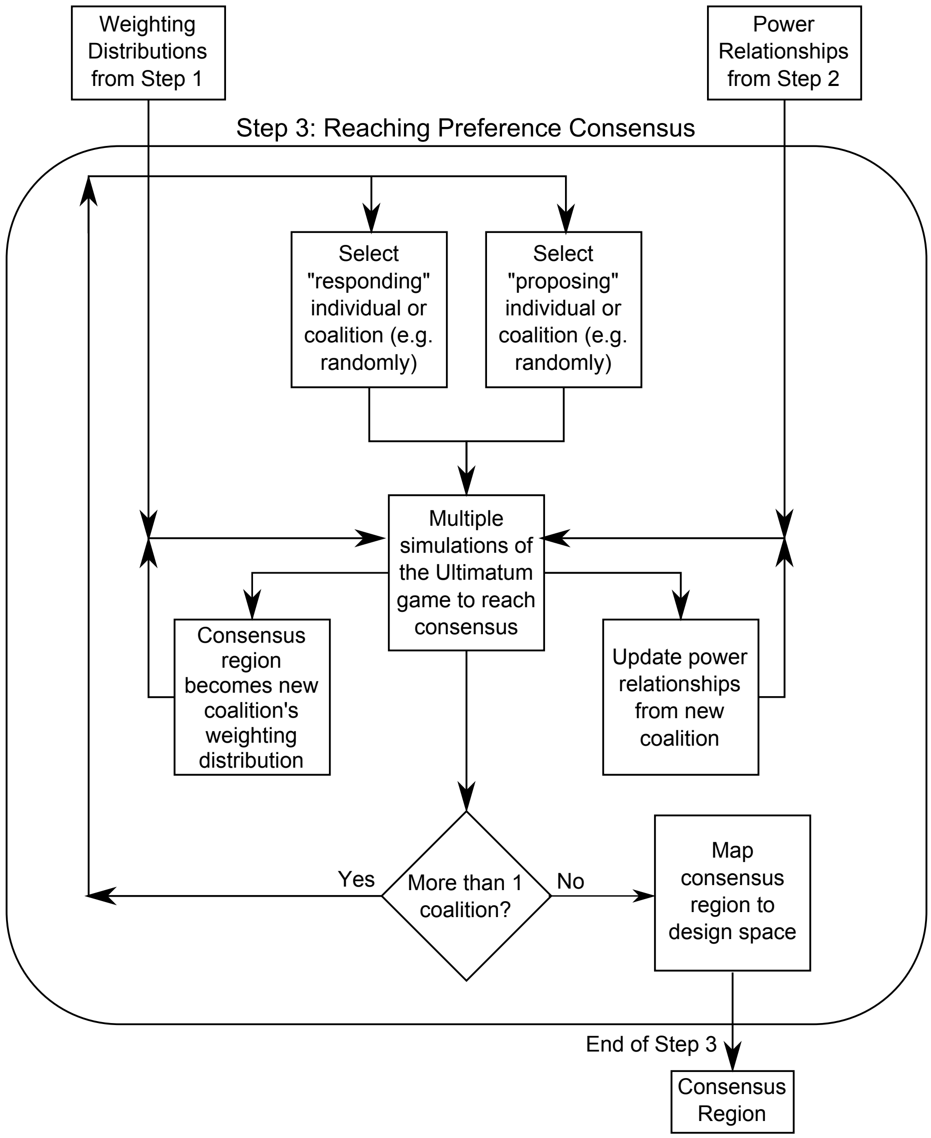


Figure 17: Flow Block Diagram of Step 3: Reaching Preference Consensus

those decision makers (or coalitions of decision makers).

This new consensus region becomes the preference or weighting distribution of the newly form coalition. Similarly, the power relationships are updated between decision makers and the coalitions.

If there is only one coalition (i.e. the group is now all in agreement), the group preferences are applied to the design space and Step 3 is complete, with the group consensus region defined.

If there remain two or more decision makers or coalitions, the process is repeated by selecting two of them, and repeating the ultimatum game simulations between the individuals or coalitions.

Since the order of how coalitions are formed can have some impact in the final consensus region, Step 3 can be repeated multiple times, if desired, to obtain a superset of designs that could be agreed upon, from a variety of coalition-forming sequences.

Lastly, variations on the simulation parameters are possible such as increasing one's propensity to accept proposals in later stages of the overall, simulation, coalition forming process. Likewise, a group themselves could have greater influence over a smaller group depending on the relative group sizes. The constituent group members would still drive the consensus process but a multiplier effect could be enabled to model other types of pressure such as going along with the majority (but not necessarily with a majority of the power).

4.3.4 Summary of the MACRO Methodology

The three major steps of the overall MACRO methodology are combined in Figure 18 to show some of the key interfaces between the steps.

These data and other methods will be described in more detail with examples in following chapters.

As with all model and methodologies, the extent of applicability is limited and implementing the MACRO methodology is no exception. However, at least three ways have been identified in which MACRO could be used:

- **By an outside analyst** attempting to forecast which design requirements a group of decision makers will select and study the potential outcomes from such predictions. In this use case, the analyst must make assumptions about how the various stakeholders will respond to discrete choice experiments on the design space. This is a possible source of uncertainty but can still reduce the set of potential designs if assumptions are founded on historical data or other

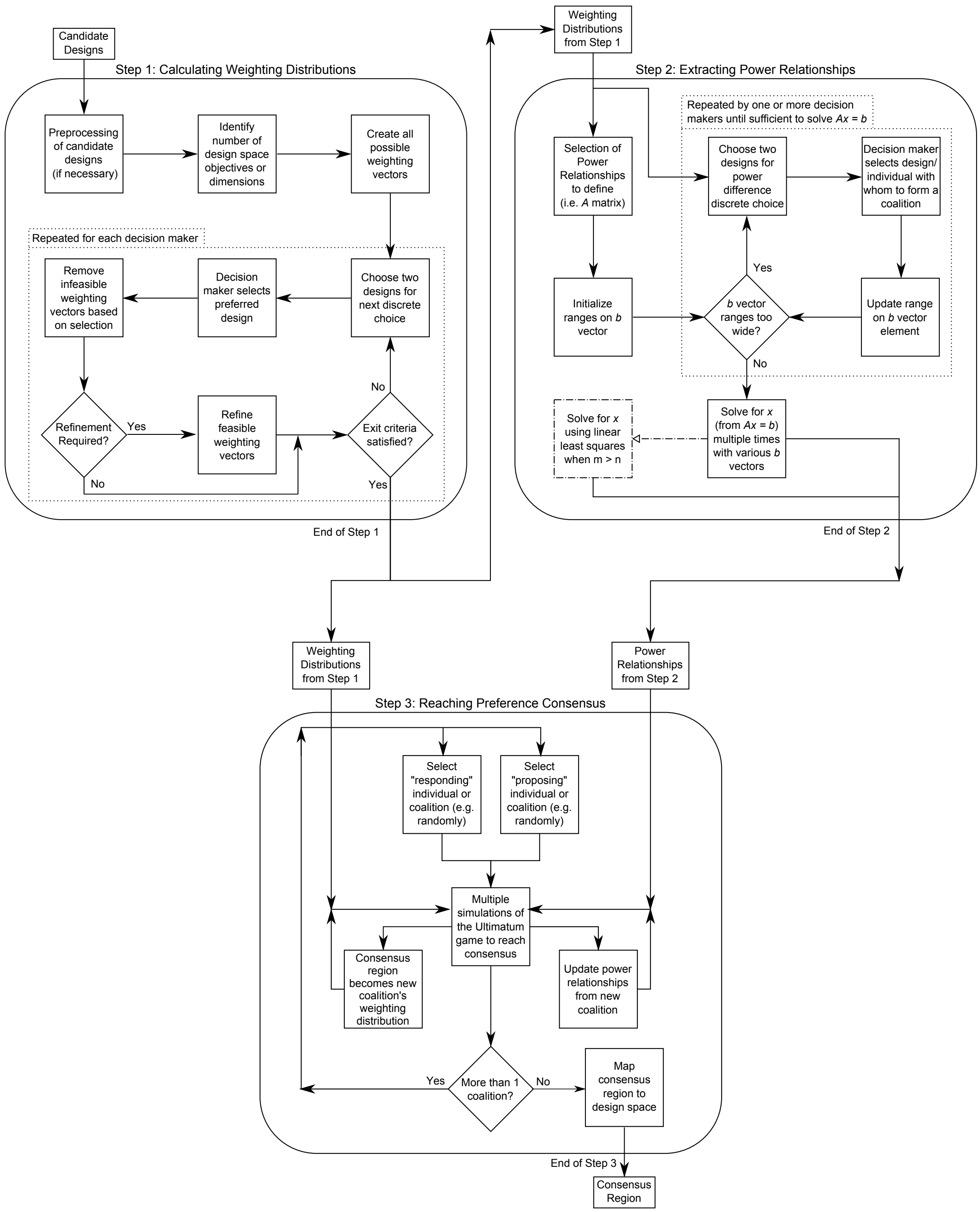


Figure 18: Flow Block Diagram of the MACRO Methodology

reasonable trends.

- **By one of the individuals in a group** attempting to identify the sensitivity of the consensus region with or without their cooperation. In this way, MACRO would be used as a type of meta-decision making tool, allowing an individual to prepare more for eventual discussion and negotiation which may take place between parties. Not only understanding the design space, but also exploring the group dynamics through MACRO can better prepare and individual for future interactions in decision meetings.

- **By the full group** willing to implement the methodology to accelerate the potentially lengthy consensus reaching process. In this use of MACRO, all decision makers are willing to respond to a sufficient number of discrete choices, such that preferences structures and influence relationships are defined, and are willing to work through the entire methodology to, at minimum, observe the set or region of solutions at which the model predicts high rates of consensus.

CHAPTER V

DEVELOPMENT AND IMPLEMENTATION OF AIRMOD

The current chapter summarizes and describes the first element of the overall methodology in response to the first half of the research objective. This element in the methodology generates the alternatives necessary for design down selection and, in turn, the requirements definition phase for future air mobility systems. The data created by this model enables the MACRO methodology, discussed in the following chapter, for consensus reaching in performing the group goal of defining the requirements expeditiously with increased transparency.

5.1 Air Mobility and Operations Design Model (AirMOD)

The Air Mobility and Operations Design Model (AirMOD) makes use of the underlying surrogate models previously developed in 2010 at the Georgia Institute of Technology as part of the Strategic Airlift Comparison Tool (SACT) project for Lockheed Martin's analysis on C-5 fleet logistical comparisons. However, no Lockheed Martin data from these efforts are included in the following sections and references to C-5 data, trends, and costs are publicly available and cited accordingly.

5.2 Strategic Airlift Comparison Tool (SACT) Description

The Strategic Airlift Comparison Tool (SACT) allows one to make comparisons between two platforms, namely the C-5A and C-5M, in a variety of scenarios across logistical and operational output metrics.

A screen shot of the tool at initial start-up, and before the user or decision maker has entered any scenario or comparative data is shown in Figure 19.

Inputs and assumptions such as Aerial Ports of Embarkation (APOE), Aerial

Ports of Debarkation (APOD), en route refueling locations and other mission scenarios parameters are defined on the left hand side of the tool. The right hand side is reserved for displaying the mapped scenario and feasible airfield locations and mission city-pair, in addition to the output metrics of interest such as the time to close the mission, fuel consumption, flight hours, etc.

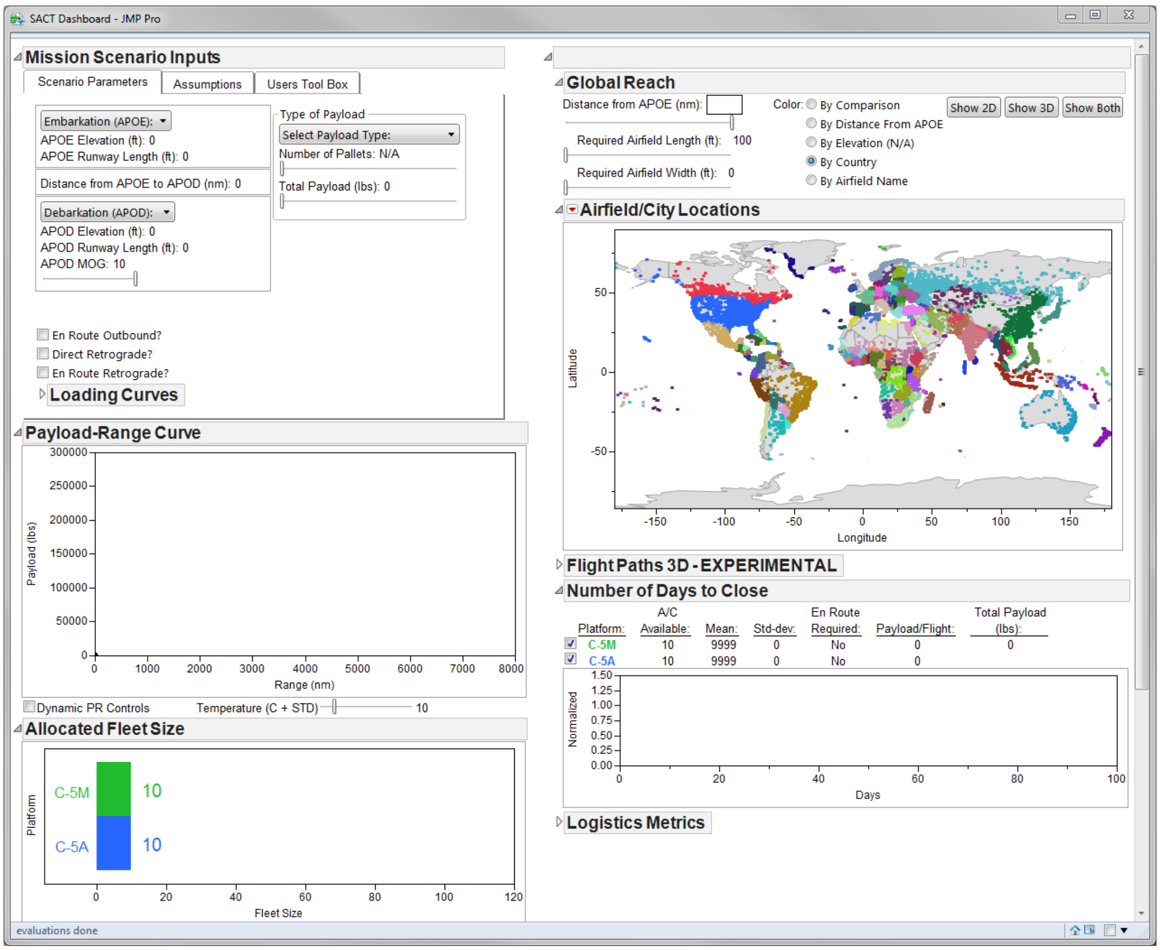


Figure 19: Screen Shot of Strategic Airlift Comparison Tool After Initial Start-up

In terms of layout, each input or output subgroup is designated by an outline box and associated blue triangle in the top left corner. Each outline box will contain particular controls to define the scenarios or other parameters, such as mission payload or fleet size. Using the JMP ®[1] standard toolbar, the SACT layout can be readjusted

[1] JMP is a statistical software package developed by the SAS Institute with advanced visual analytics capabilities

for different users or audiences by resizing, closing, and/or cloning various outline boxes. As a user or group interacts with SACT, different analyses can be executed or suppressed depending on preferences and specific perspectives such as performance, financial or political. Some of these boxes and analyses are further described in detail in the following sections.

5.2.1 Mission Scenario Inputs

The scenario and associated parameters are entered inside the Mission scenario outline box. An APOE and APOD is selected through a map or list box interface, followed by a total mission payload. Many parameters are defaulted at start-up such as the fleet size of 10 for both platforms. These parameters satisfy the requirements of surrogate models to calculate the logistic output metrics.

5.2.2 Aerial Ports

A variety of methods are available for entering the APOE or APOD desired locations. Selecting the airfield directly on the map, filtering based on country or airfield name, followed by accepting the selected airfield by clicking the button labeled “done,” are all valid ways to select an aerial port to define the scenario. Similar processes can be performed for selecting the en route or retro en route locations as well. Figure 20 shows one example of filtering to show only airfields in Germany.

5.2.3 Type of Payload and Loading Curves

An option to select the type of payload, such as Pallets or various Combat teams, is available within SACT to analyze the operational metrics that would be required for specific missions. A custom payload option can be adjusted for additional types of payload not listed in the drop down menu.

Each of the payload types will have an associated loading curves displaying the efficiency of the total carry capacity for the loads on average. For example, the C-5

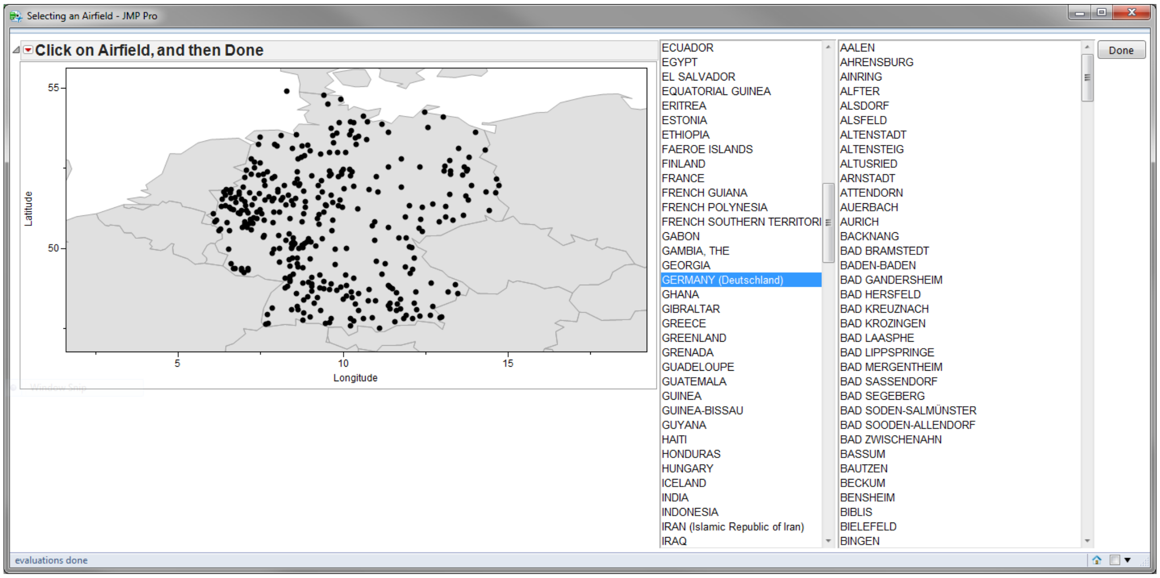


Figure 20: Selecting an APOE in SACT

only has a certain number of tie-down locations such that a limit of 36 pallets can be carried [175]. Thus, certain payloads will not allow the platform to use the total payload carrying capacity due to volume, called a "cubed-out" condition. Figure 21 shows the loading for such a payload where the average payload per flight will plateau even if larger payloads are possible.

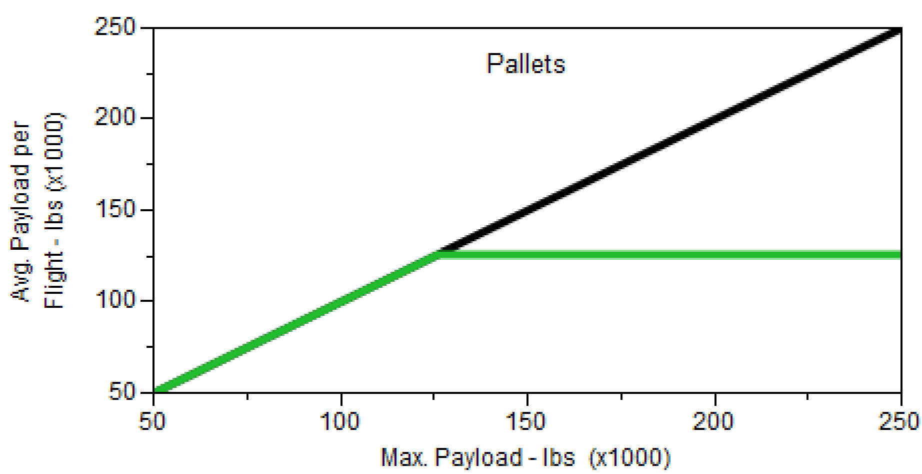


Figure 21: Loading Curves in SACT

5.2.4 En Route Location Selection

Since some scenarios include an APOE-APOD airfield pair with distances too far for C-5 platforms to reach directly, the option to include en route refueling can be applied. Also, some scenarios might be valid with very small payloads but at such small values to suggest infeasible operational modes. A threshold can be set to constrain the smallest payload possible within a scenario. Stopping over to refuel can open up the operational space at a cost of maintaining that base and the time lost for refueling. Thus, the main SACT objective to compare the two assets can include, not just "what" but "how" the platforms deliver cargo with potentially different flights paths used.

5.2.5 Payload-Range Curves

The Payload-Range (PR) Curve is the main performance differentiator between the two platforms, the C-5A and C-5M. Figure 22 illustrates the two notional PR curves for the two platforms. For the specified APOE-APOD distance around 3700 nmi, the C-5M has a carrying capacity of about 20000 lbs greater than that of the C-5A. This will equate to difference in output metrics discussed later. Similarly, at different scenarios the point at which both platforms will lie on the PR curve will update in real time when using the tool. Lastly, at different ambient APOE temperatures, the performance and thus PR curves will change accordingly, which can also be entered in by the user.

5.2.6 Allocated Fleet Size

Comparing the platforms within a fleet is also available within SACT. Adjusting dynamically the fleet size of one or both of the platforms will result in an appropriate increase or decrease in the time to close the mission and the other output logistics metrics. An interactive bar chart shown in Figure 23 shows the current fleet sizes used as input parameters to the operational surrogate models. Users can click and

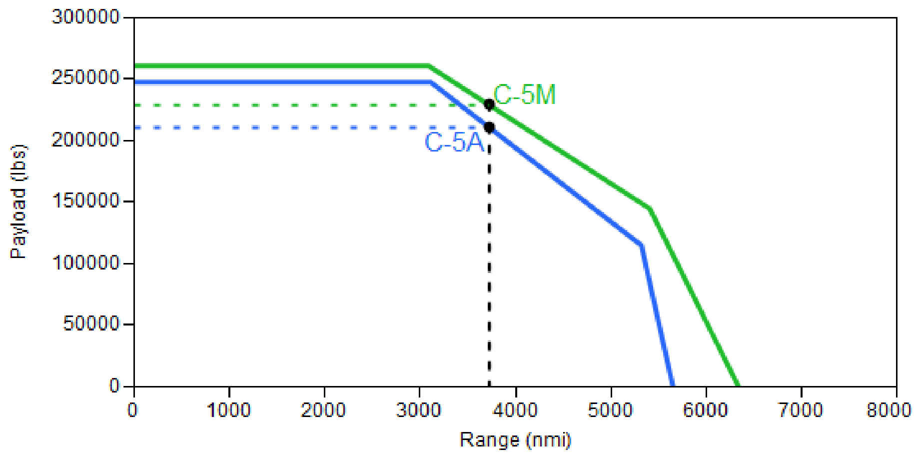


Figure 22: Notional Payload-Range Curve for the C-5A and C-5M

drag the bars based on potential and expected fleet sizes to analyze the impact of a larger (or a smaller) number of aircraft on the metrics of interest.

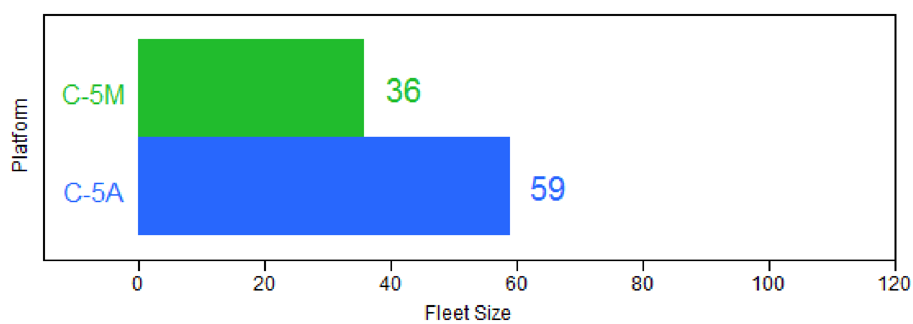


Figure 23: Fleet Size Bar Chart for the C-5A and C-5M

5.2.7 Aircraft Break and Repair Times

The break rate and repair times of the two platforms can likewise be updated in real-time and the output's sensitivity to these input parameters can be investigated. A user specifies the probability, in terms of a percentage, that each aircraft will be found to be non-mission capable due to a broken part, and the distribution of time required to repair those parts as shown in Figure 24.

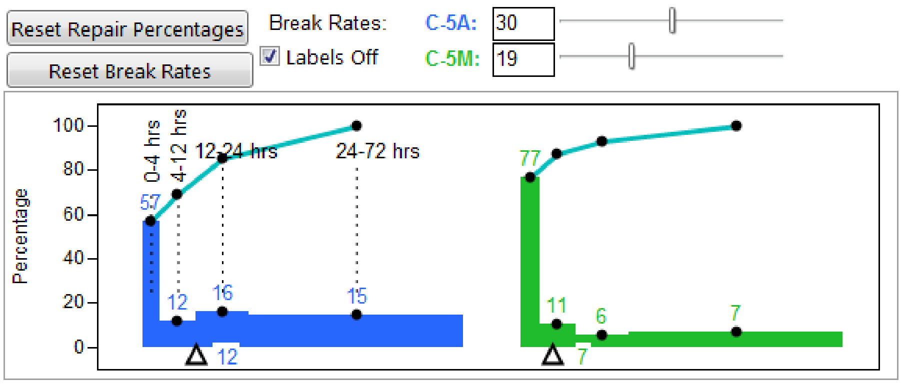


Figure 24: Break and Repair Rates for the C-5A and C-5M

5.2.8 Global Reach and Airfield Locations

The mission scenario is visualized in a Global Reach outline box (shown in Figure 25) which has a number of useful capabilities as well. Comparing or contrasting the number of airfields that each platform can reach flying direct with the same or different payloads can be applied. Furthermore, airfields only with sufficiently long and wide airfields can be shown by filtering out those runways that are too short and narrow for C-5 aircraft. Lastly, coloring the airfields with additional meta-data can be performed within this outline box such as elevation or other airfield attributes.

5.2.9 3D Flight Paths

Comparing different flights paths to identify the benefits of different en route refueling locations can be performed within the tool and projected on a 3D globe as shown in Figure 25. This capability allows the user to explore additional dimensions with the system such as political ramifications from requirements to enter various the airspaces of foreign nations. In Figure 25, three different ways are compared in delivering payload to a notional scenario in India from the continental US. The first employs a direct path while the other two have refueling stops in Europe. Some of the countries over which the flight paths extend may not allow US aircraft to enter their airspace, and thus the penalty in terms of the operational metrics can be identified

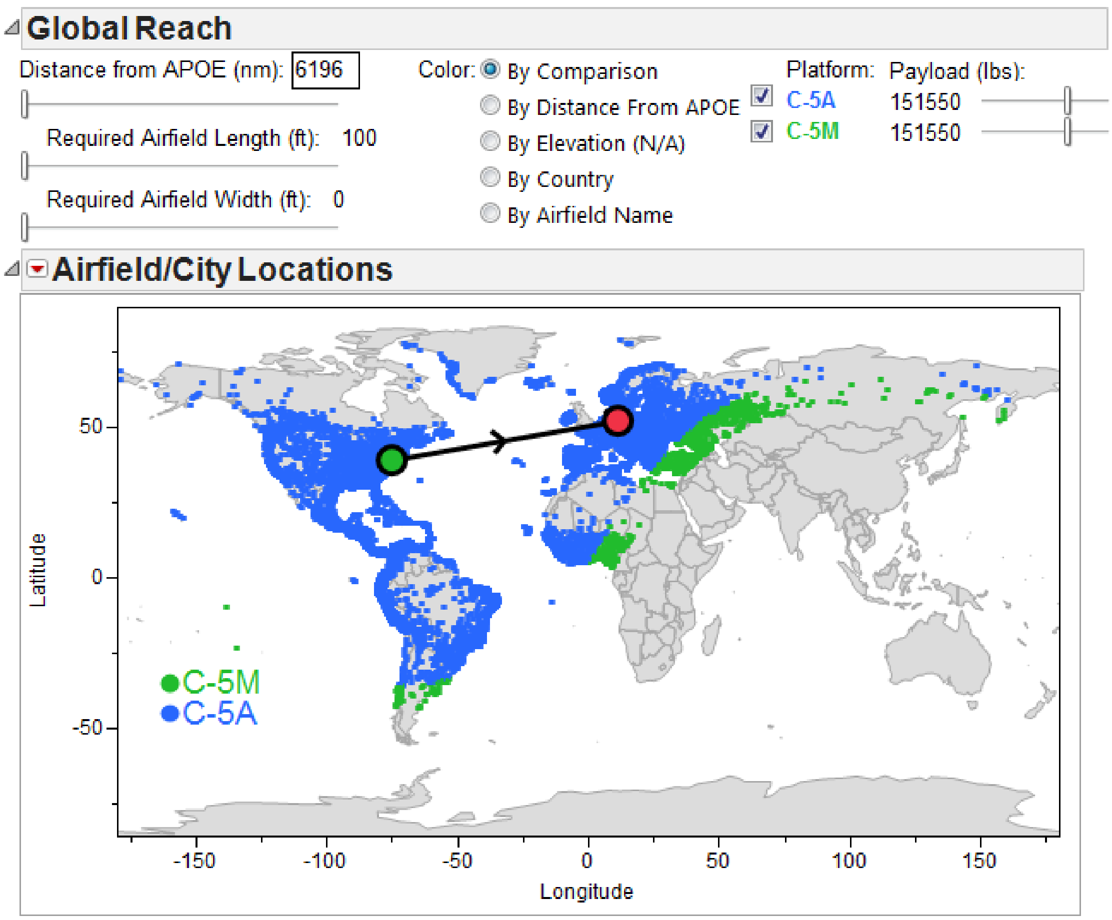


Figure 25: Map Display of Scenario and Airfields Reachable by Each Platform at Specified Payloads

when selecting those scenarios.

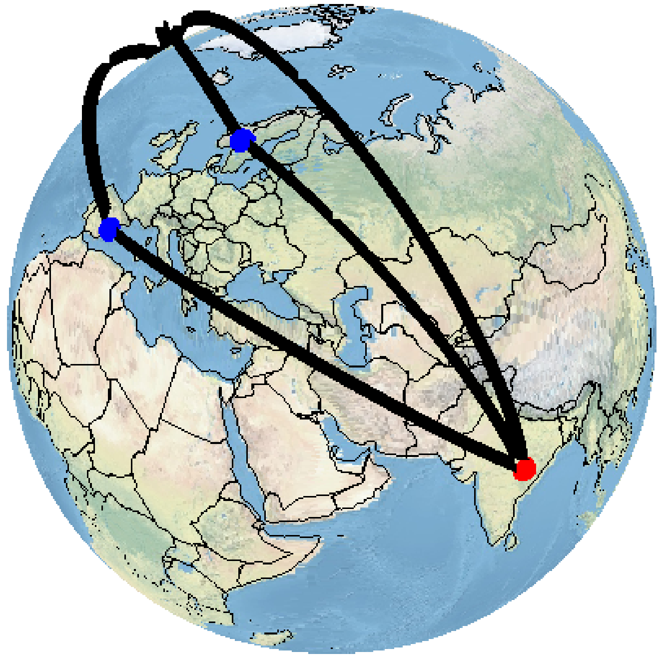


Figure 26: 3D Flight Paths of Three Scenarios

5.2.10 Number of Days to Close and Other Output Metrics

The number of days required to close a particular mission scenario and the uncertainty in that value (via the standard deviation) is presented as a distribution in the output metrics outline boxes. For a fleet size of 11 aircraft for both the C-5A and C-5M respectively, the output metrics are displayed on the right hand side of the tool shown with an example of notional output, in Figure 27.

The distributions of the time to close indicate the expected, best and worst times that the surrogate models calculate a scenario mission can be complete. The standard deviation of the distributions will be functions of the break and repair times as entered by user. The logistics metrics include the cost and amount of fuel consumed for the entire mission as well as the total flight hours and the utilization. In the particular example presented in Figure 27, the C-5M fleet close the mission in about 10 days faster than the C-5A fleet. Furthermore, employing C-5M's instead of C-5A would

Figure 27: Time to Close and Other Output Metrics from SACT

save about 5$ million in fuel cost and over 300 flight hours. Lastly, the aircraft themselves would be used more at a higher utilization value due to a more reliable aircraft platform in terms of both break rates and repair times.

5.3 AirMOD Enhancements to SACT

In developing experiments for testing and investigating Hypothesis #1, the AirMOD platform builds upon the basic functionality of the SACT tool by allowing for batch mode capability enabling fast Monte Carlo (MC) simulations across scenarios, fundamental design parameters (through the payload-range curve), and reliability conditions. The output metrics, including the mean and standard deviation of the time to close the mission, total fuel consumption, utilization, and total flight hours, have been maintained from the original surrogate models with necessary changes to the underlying assumptions such as the threshold for the minimum payloads and fuel reserve requirements, which are both variable in AirMOD.

Within this updated platform, the comparative analyses are no longer performed between the C-5A and C-5M models but between different designs (designated as C-X) from the combinations of inputs and output metrics. Furthermore, large numbers of MC simulations can be quickly performed to cover the potential design and operation space for Pareto frontier extraction and eventual decision-making activities examined by the other research questions and hypotheses.

5.3.1 AirMOD Scenario Definition

A subset of the thousands of SACT airfields possible was down-selected to provide a manageable set of city-pair combinations for application within the AirMOD model. These 25 bases or cities are listed in Table 2.

These airfields were primarily selected based on their relative locations such that most regions of the earth are considered in the possible city-pair scenarios, with an accompanying wide range of distances. Many of the locations coincide with an

Table 2: Subset of Locations Used in AirMOD

1 Al Dhafra Air Base	14 Lima
2 Bombay	15 Manila
3 Cairo	16 Mexico
4 Cape Town	17 Moscow
5 Dover Air Force Base	18 Ramstein Air Base
6 Eielson Air Force Base	19 Rota Naval Base
7 Incirlik Air Base	20 Santiago
8 Jakarta	21 So Paulo
9 Joint Base Pearl Harbor-Hickam	22 Sydney
10 Joint Base San Antonio	23 Transit Center at Manas
11 Kandahar Airfield	24 Travis Air Force Base
12 Kunsan Air Base	25 Yokota Air Base
13 Lagos	

existing air base currently shared or used by U.S. military forces. Lastly, some city centers with large populations were included which often contain a relatively large airfield (longer than 11000 feet), such that most designs with a TOGW larger than the maximum TOGW for C-5 aircraft are still feasible. These cities would likely serve as the main service ports for major humanitarian efforts or as a supply base for regional conflicts.

The 25 locations are illustrated geographically in Figure 28, overlaid onto a world map shaded in gray.

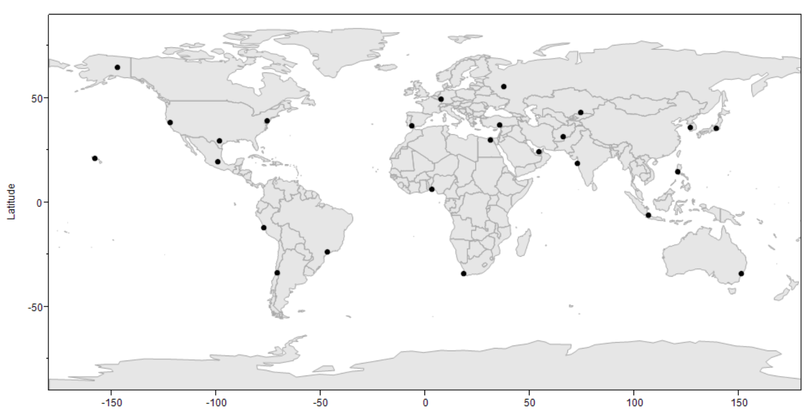


Figure 28: 25 Locations Available for Scenario Definitions

With 25 locations, the 600 (i.e. 25x24) city-pair or APOE-APOD combinations

(Aerial Port of Embarkation -Aerial Port of Debarkation combinations) cover a variety of distances and latitudes/longitudes for each flight path. The great circle distance between each city pair is projected onto the same world map in Figure 29.

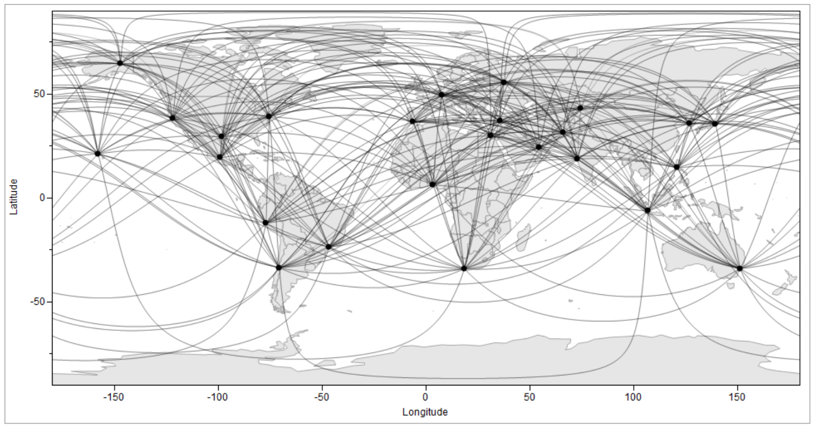


Figure 29: Great Circle Distance and Flight Paths from all 600 APOE-APOD Combinations Projected onto a World Map

With the projected great circle distances, airspace analysis for non-ally countries is also available within AirMOD with an example presented in Figure 30 where the countries whose airspace is entered during the great circle flight path are highlighted in red.

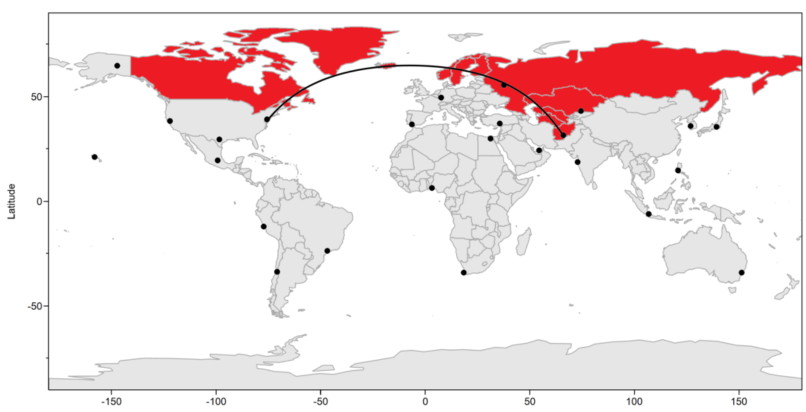


Figure 30: Great Circle Flight Path from Dover Air Force Base to Kandahar Airfield with highlighted countries with airspaces entered

Lastly, a histogram of the distances between the 600 APOE-APOD combinations is shown in Figure 31, indicating that for a large percentage (30%) of APOE-APOD combinations the range exceeds 7000 nmi (the maximum range for C-5 without cargo [175]). These combinations, of course, would necessitate stopping over en route for refueling, and similarly for all shorter distances when sufficiently large payloads are transported.

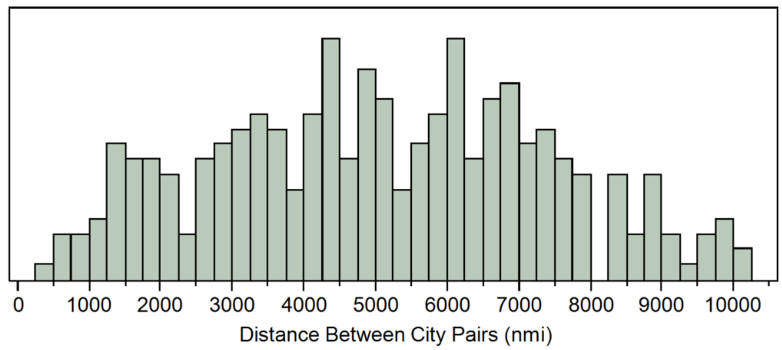


Figure 31: Great Circle Flight Path Distances for all 600 APOE-APOD combinations

For these more constraining cases, such as with higher payloads, initial filtering can be applied to the set of APOE-APOD combinations to investigate possible scenarios with other cargo weights (or even with other aircraft platforms) as shown in Figure 32 where only distances less than 3250 nmi are indicated.

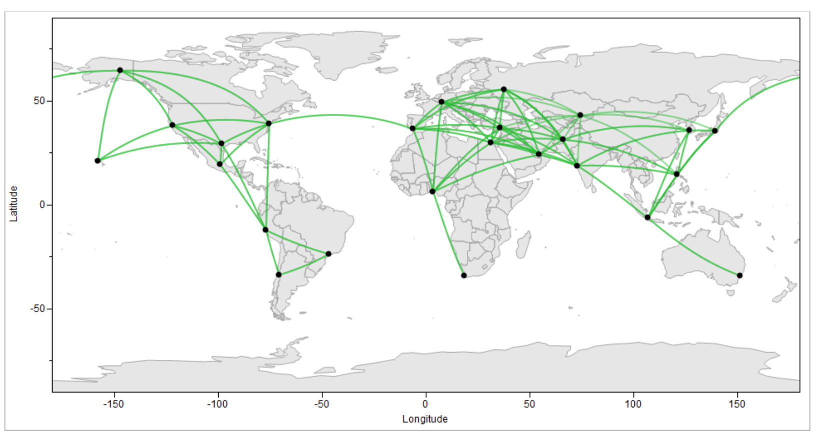


Figure 32: Great Circle Flight Path Distances for all APOE-APOD combinations with distances less than 3250 nmi

To account for refueling en route (or "retro" en route) AirMOD has 4 distinct types of scenarios with which to perform simulations:

- Type 1: Direct flights, no en route or retro en route refueling
- Type 2: En route and retro en route refueling occurs at the same location
- Type 3: En route refueling occurs with direct return flight from APOD to APOE
- Type 4: Refueling at en route and retro en route locations occur at different locations

The four modeling types are visualized in Figure 33. Although numerous additional types of combinations are possible (e.g. multiple en route refueling locations), these four types are sufficient in exploring the design space while requiring the given C-X platform to fly at the extremes of the payload range performance capabilities.

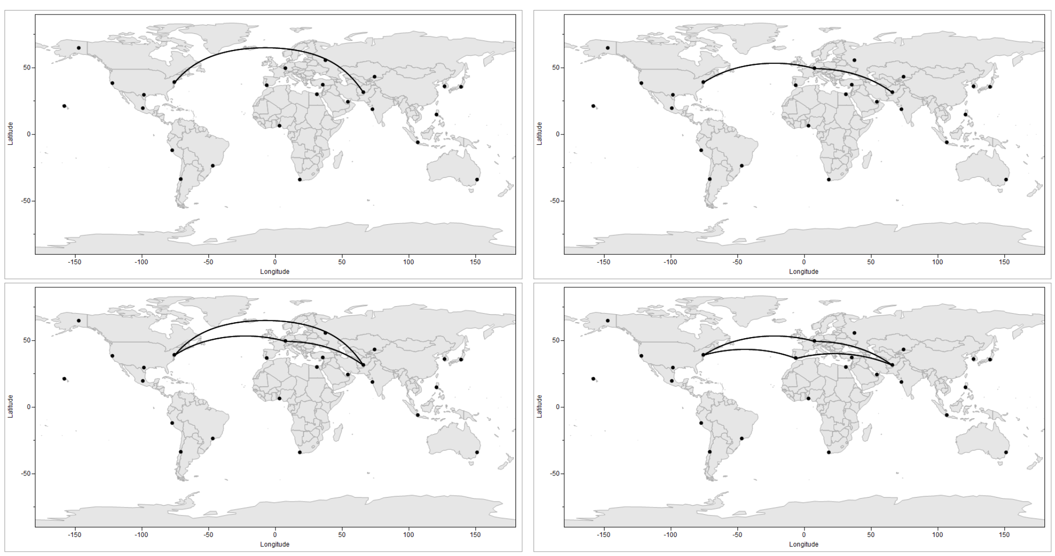


Figure 33: AirMOD Modeling Types: Type 1 (top left), Type 2 (top right), Type 3 (bottom left) and Type 4 (bottom right)

Another key parameter considered part of the scenario definition is the total mission payload. This is expectedly dependent on the mission type (e.g. humanitarian, regional conflict) but to test the high carrying capacity of the heavy airlift cargo aircraft, large total mission payloads are entered. A uniform distribution from 10 to

30 million pounds of cargo is used in randomly creating the scenarios' total mission payload in MC simulations.

Finally, to complete the scenario definition required for model simulation, the total number of aircraft (i.e. fleet size) can vary in a similar fashion with a random uniform distribution anywhere from 1 to 60. This maximum value of the range was selected based on the maximum total mission payload of 30 million, and the expected maximum C-X cargo per flight (i.e. $\approx$350000 lbs, larger than the C-5's 270000 lbs [175]), for 86 total flights. With more than 60 aircraft, this approaches an impractical level, where most of the aircraft would fly only one or two trips and be underused (or unused) a majority of the time.

5.3.2 Payload-Range Curve

A payload-range (PR) curve is used to define the model's multiple C-X designs from a capability stand point. AirMOD will then use this design (via a PR curve) to calculate the expected payload per flight, based on the scenario type, for further use as inputs to the associated discrete event surrogate models.

Four parameters can be entered (or randomly selected in MC simulation mode) to characterize the PR curve, namely, 1) the maximum take-off gross weight (MTOGW), 2) the maximum range at MTOGW, 3) the ambient temperature, and 4) the reserve fuel. A fifth parameter, the minimum or threshold payload at which the model will not execute or fly the missions, can be likewise set. This input represents the payload at which it is no longer practical to fly from APOE to APOD due to an extremely long distance. For example, a C-X may be able to transport a small 10000 lbs payload per flight a distance of 7000 nmi without refueling, but for any substantial amount of total mission payload (20 million lbs) the cost of transporting the total amount in such small "chunks" far outweighs the benefit of not stopping to refuel.

For the scope of this current research, and to make use of the surrogate models, the range of possible C-X transport capability is set to generally match that of the C-5 with slightly larger payloads. The range of the MTOGW is simulated between a range of 250000 and 350000 lbs (i.e. up to 30% more payload than the C-5). The distance for the MTOGW condition is similarly sampled but in a range from 2500 to 3500 nmi. Beyond this distance, the payload begins to be traded for longer ranges. The third parameter, temperature, can negatively impact the capability by reducing the MTOGW and therefore the payload for a given field length and elevation. The data and related surrogate model calculating this trend was less accurate and thus this parameter was defaulted to 10°C so that comparisons were consistent for all model runs. The fourth and fifth parameters, reserve fuel and minimum payload, were similarly defaulted to 10000 lbs and 25000 lbs respectively, since these were considered policy based decisions (i.e. safety policies and operations practicality) and need to be consistent across all designs.

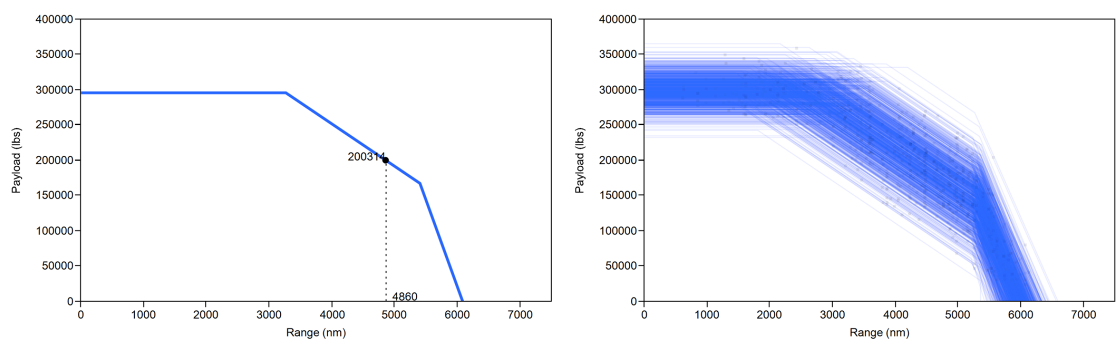


Figure 34: One Randomly Generated PR Curve with distance and payload at specified design point (left), Multiple PR curves from MC simulations (right)

The right hand side of Figure 34 shows 1000 randomly generated PR curves from the associated MC simulations of the aforementioned parameter distributions. Each PR curve represents a different design which the operational model will employ to calculate the logistical metrics within the particular scenario. (One such design example is shown in the left hand side of Figure 34).

5.3.3 Reliability and Repairability Inputs

The current model incorporates five different reliability and maintainability parameters which impact the availability of the aircraft.

The first, called the break rate (expressed as a percentage), defines the probability that any one flight will be delayed due to part failure, and will thus require maintenance crews to investigate and perform repair or part replacement operations. The break rate is effectively the inverse of reliability in that an aircraft with high reliability will possess a low break rate. The range from which the MC simulations generate the break rate values is between 0 and 60%, which covers a wide range of reliabilities, including a current estimate of the C-5's mission capability of 55% to 60% [174] (or converted to break rates near 40-45%).

The next four parameters define the likelihood or probability of time required to repair/replace parts once an aircraft is identified as non-mission capable and together these four parameters will sum to 100%. Each of these four percentages (p) specifies the probability that repairs will require one of four time periods (t_i, $i = 1...4$): 0-4 hours when i=1, 4-12 hours when i=2, 12-24 hours when i=3, and 24-72 hours when i=4. For example, if these four percentages (p_i, $i = 1...4$) are 35%, 25%, 30% and 10%, respectively, on average 35% of the flights that are non-mission capable will require an additional 0-4 hours of time delay to diagnose the problem, repair/replace parts, etc. Within the model, a uniform distribution within that time period (e.g. [0,4] hours) will be sampled for the actual simulated repair time.

These four parameters are further combined into an overall expected time to repair by calculating the weighted sum of the four categories of time periods. In the example listed above the expected time to repair would be: $t_e = \sum_{i=1}^{4} p_i \bar{t_i} = 0.35(2)+0.25(8)+0.30(18)+0.10(48) = 12.9$ hrs. On average, with all else being held constant, scenarios with identical t_e can be expected to produced similar responses, therefore only this expected time will be used to reduce 5 parameters down to 2 for design display and discrete choice experiments.

5.3.4 Model Inputs and Outputs Summary

A summary of the AirMOD model inputs and outputs metrics is presented in Table 3. Although the scenario is defined with two (or more) locations, these inputs are converted to great circle distances which is ultimately used in the model calculations. In a similar fashion, the AirMOD model will use the PR curve to extract the particular operations point (based on the most constraining leg and distance of the scenario) to calculate the intermediate payload per flight variable used in the surrogate models. The output metrics are also used as intermediate values and processed in various ways as discussed in more detail in the next sections.

Table 3: Summary of AirMOD Input and Output Variables

Variable Name/Category	Input or Output?	Range/Values	Units
APOE (Scenario)	Input	(Lat, Long)	degrees
APOD (Scenario)	Input	(Lat, Long)	degrees
En Route (Scenario)	Input	(Lat, Long)	degrees
Retro En Route (Scenario)	Input	(Lat, Long)	degrees
Type (Scenario)	Input	1, 2, 3, 4	-
Total Mission Payload (Scenario)	Input	[10,30]	lbs (millions)
MTOGW (PR curve)	Input	[250000,350000]	lbs
Range at MTOGW (PR curve)	Input	[2500,3500]	nmi
Break Rate	Input	[0,60]	%
Expected Repair Time	Input	[2,48]	hrs
Number of Aircraft	Input	[1,60]	-
Mean Time to Close	Output	-	days
Std. Dev. Time to Close	Output	-	days
Fuel Consumption	Output	-	lbs
Utilization	Output	0-24	hrs/day
Flight Hours	Output	-	hrs

5.4 Tradeoff Analysis

With the input ranges defined in the previous section, investigations into the available trades are necessary to identify the dimensions or factors across which decision makers will have different preferences.

Initially, only 40000 simulations were executed within the AirMOD model, 10000 of each type, to obtain a high level, or cursory summary, of the relationships between the various inputs and outputs.

Figures 35 and 36 show some of the most important input parameters versus the output metrics of interest, namely, Mean Time to Close (MTTC), Standard Deviation of the Time to Close (SDTTC), Total Fuel Consumption, Total Flight Hours, and Utilization. In Figure 35 all four types (i.e. Type 1, 2, 3, and 4) are included with Type 1 colored in red, Type 2 in green, Type 3 in blue, and Type 4 in orange. Since some of the points are occluded by each other in this figure, a filtered data set (presented in Figure 36) shows the same data but only for Type 1. Many of the same relationships were observed in each of the four types and thus observations about Type 1 trends and relationships can often be applied to the other three types unless otherwise specified. The individual subplots from Figure 36 are discussed in the succeeding figures in more detail.

Firstly, in Figure 36, the most visible trade is found between the payload per flight and the two outputs of fuel consumption and total flight hours. This is expected since flying with smaller payloads (perhaps due to "cubing-out" conditions) still requires the same flight hours (under the assumption of equal block speeds) and thus operational flight hours are closely correlated to the fuel consumption. In fact, this particular relationship is tied to many of the trades discussed in this section. That is, larger aircraft which can carry larger payloads can potentially decrease the total amount of fuel consumed for some conditions. On the other hand, many smaller vehicles may consume more resources (in addition to fuel) since more flight hours, crew members, hangar locations, etc. are necessary, albeit they are all individually smaller or less fuel consuming compared to a larger vehicle. This, of course, is the financial and operational motive behind mass public transportation.

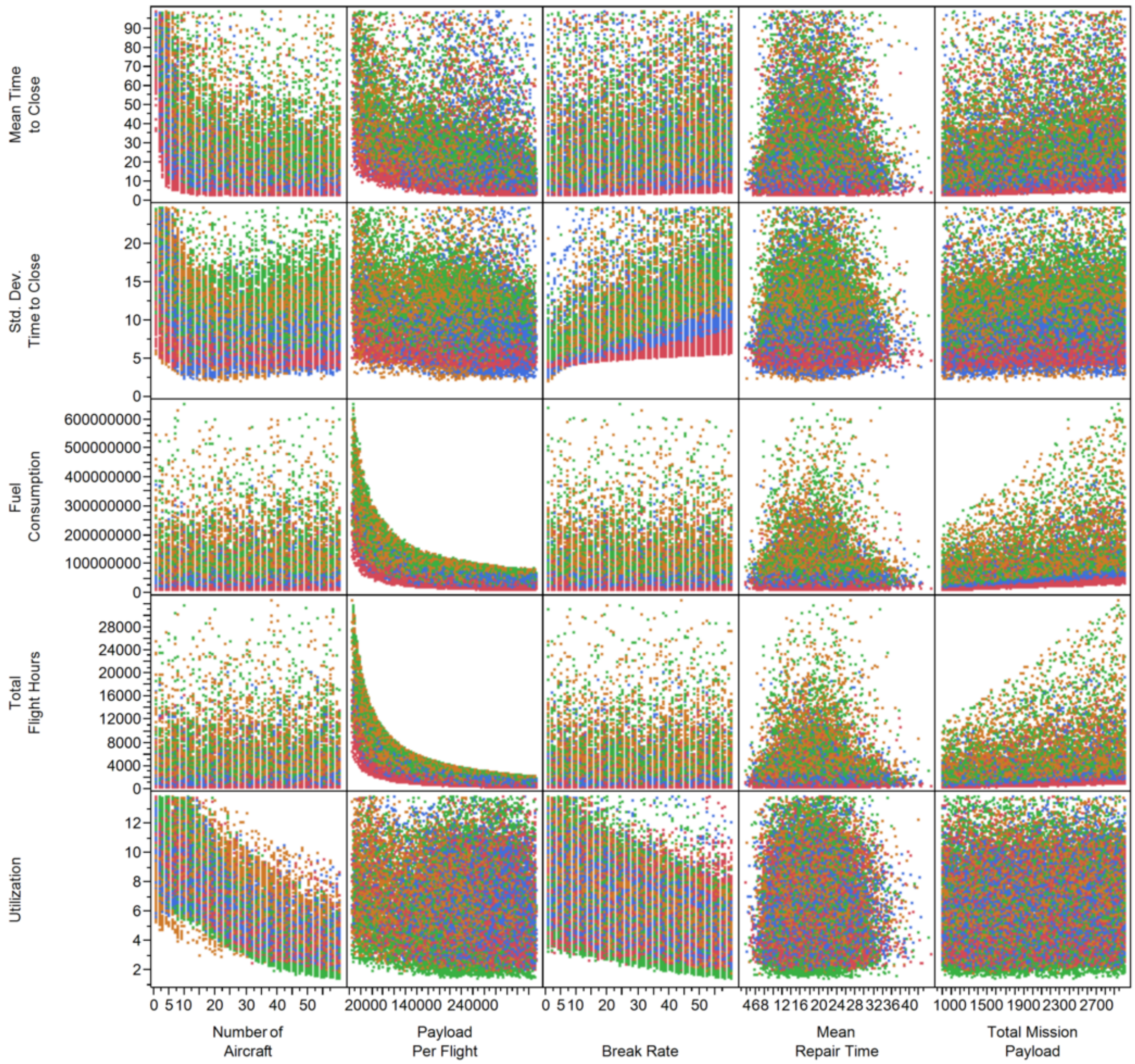


Figure 35: Scatterplot Matrix of various Input Parameters to Operational Output Metrics for All Types

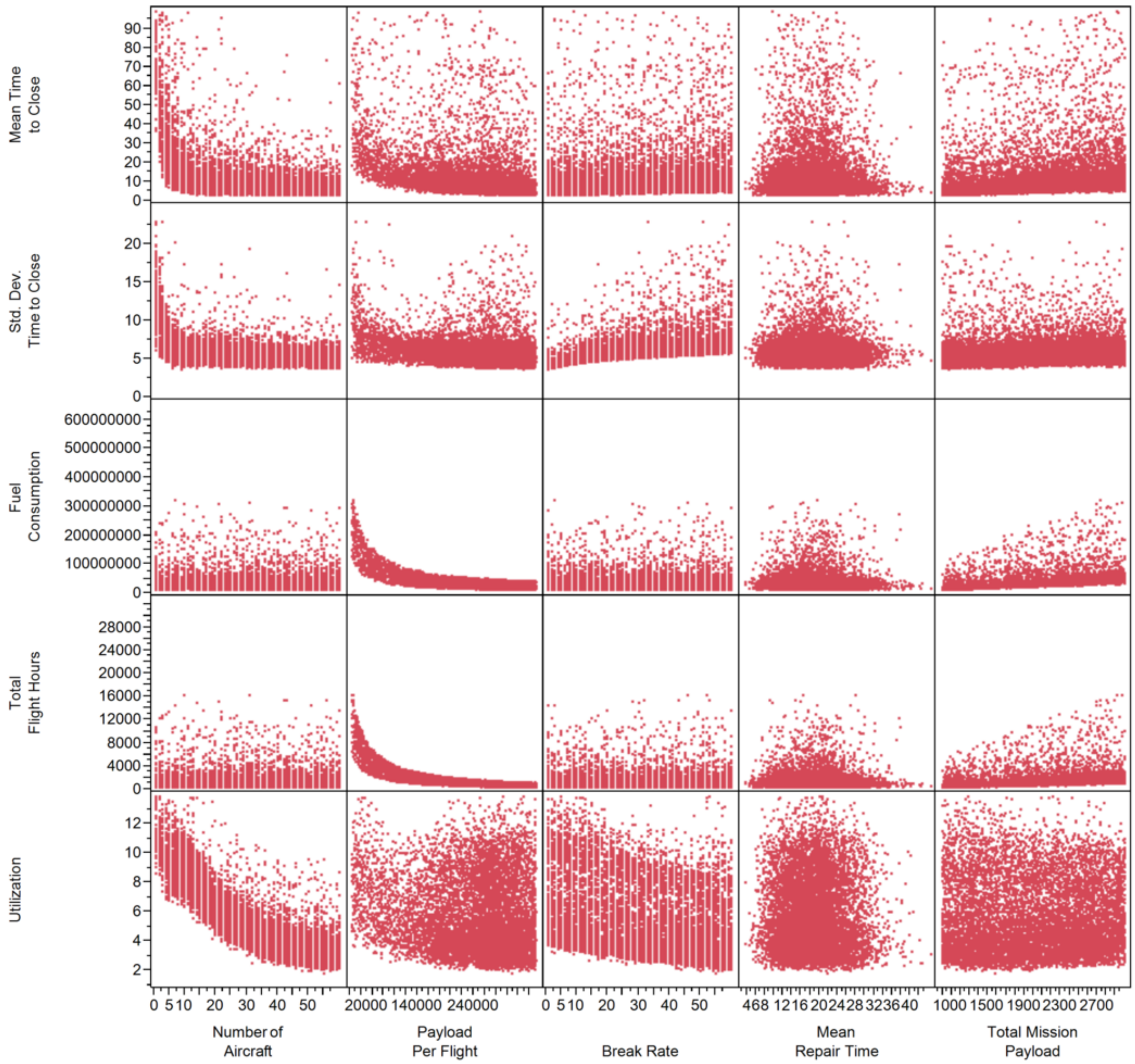


Figure 36: Scatterplot Matrix of various Input Parameters to Operational Output Metrics for Type 1. Individual subplots are discussed in Figures 37 through 45

5.4.1 Trades Involving Mean Time to Close

From Figure 36 another trade readily visible after zooming in on the range, is the Number of Aircraft versus the MTTC. Expectedly, more resources available (i.e. the number of aircraft) will help complete the mission more rapidly. Figure 37 shows a MTTC range from 0 to 50 with the full range of Number of Aircraft, 1 to 60, broken out by the four types, as compared to the equivalent top left subplot in Figure 35.

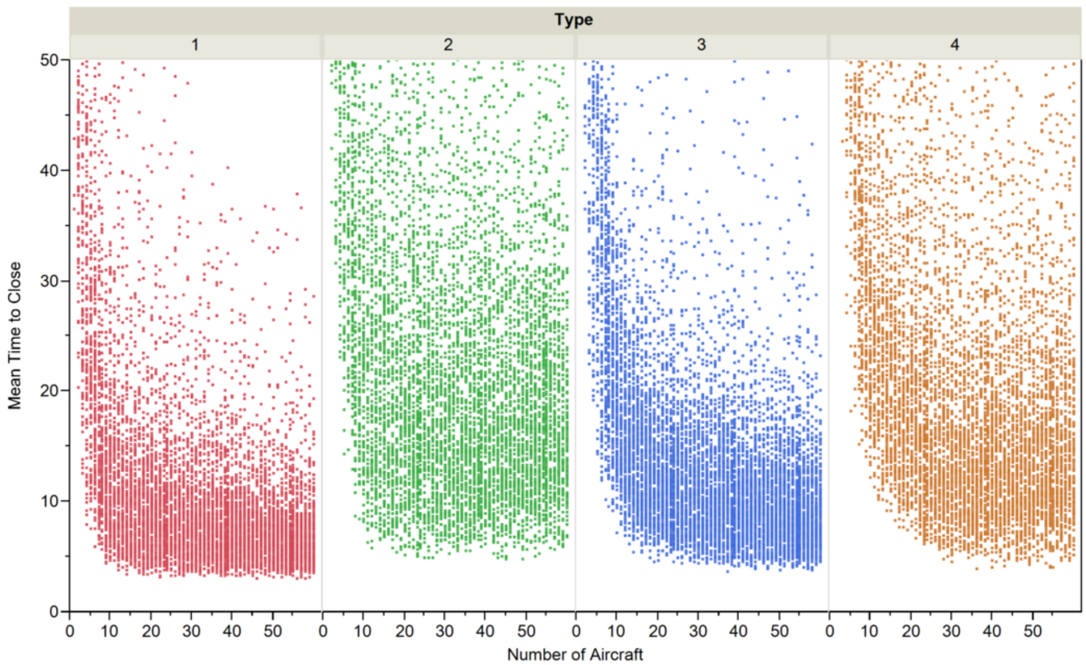


Figure 37: Number of Aircraft versus the MTTC Subdivided by Type

Operationally, the optimal strategy for this sub-trade is to minimize both the MTTC and number of aircraft. The minimization of these two objectives is inversely related. For example, increasing the number of available aircraft would decrease the MTTC, with the opposite result from decreasing the Number of Aircraft. The set of non-dominated points, which form the Pareto frontier, are found along the bottom left side edge in each of the four scatterplots of Figure 37. No points, designs or solutions exist to the left or below this frontier.

However, since only two of the multiple dimensions are shown in Figure 37, this edge is only a local Pareto frontier, and does not consider the influences from additional design or scenario variables. For example, focusing only on the Type 1 simulations, with an additional 1 million model runs, the simulations result in the two graphs of Figure 38 after processing.

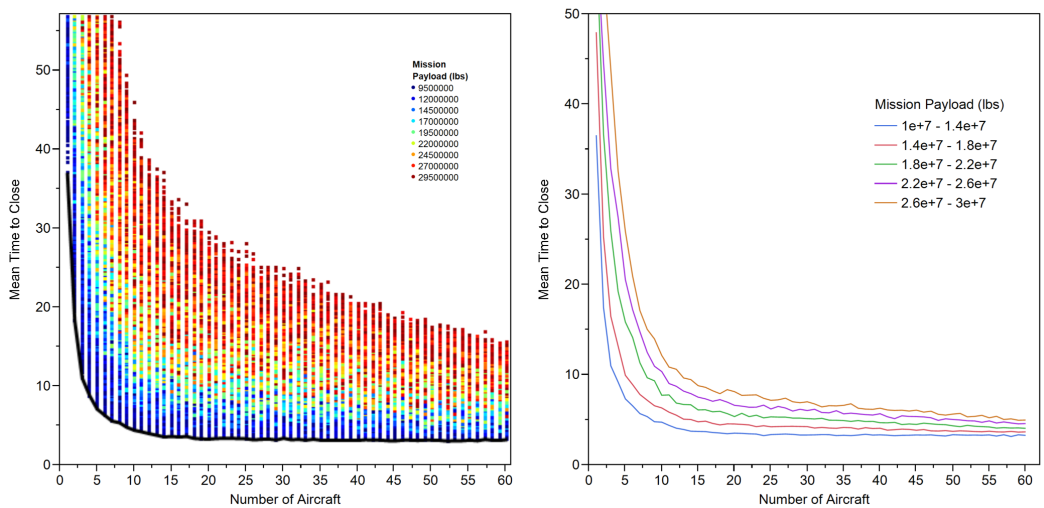


Figure 38: Number of Aircraft versus the MTTC for Type 1 with highlight Pareto frontier (left), Pareto frontiers of the same design space for various Mission Payloads (right)

The exact same relationship is found between MTTC and Number of Aircraft with the Pareto frontier designated with the black line as found on the left hand side of Figure 38. The points have been colored according to the total mission payload for the particular scenario.

For each integer value for the Number of Aircraft, the time to close increases for larger mission payloads, with the dark red points representing the largest payloads. This intuitively satisfying result visually reveals a similarly shaped edge separating the color bands of mission payloads. These edges represent the "set" of Pareto frontiers across this dimension and are part of the higher dimensional Pareto frontier which also includes the mission payload scenario parameter.

These are shown more explicitly in the right hand side of Figure 38, where the minimum MTTC or minimum Number of Aircraft (when the other is held constant) for a given mission payload range is indicated.

Of course, Figure 38 shows only three of the multiple dimensions. The dimensional space of the true Pareto frontier is much higher. For example, breaking out the points into the same five categories of Total Mission Payload results in Figure 39. This shows the thick band of impacts that other variables may have above the Pareto frontier (black lines) for each mission payload category.

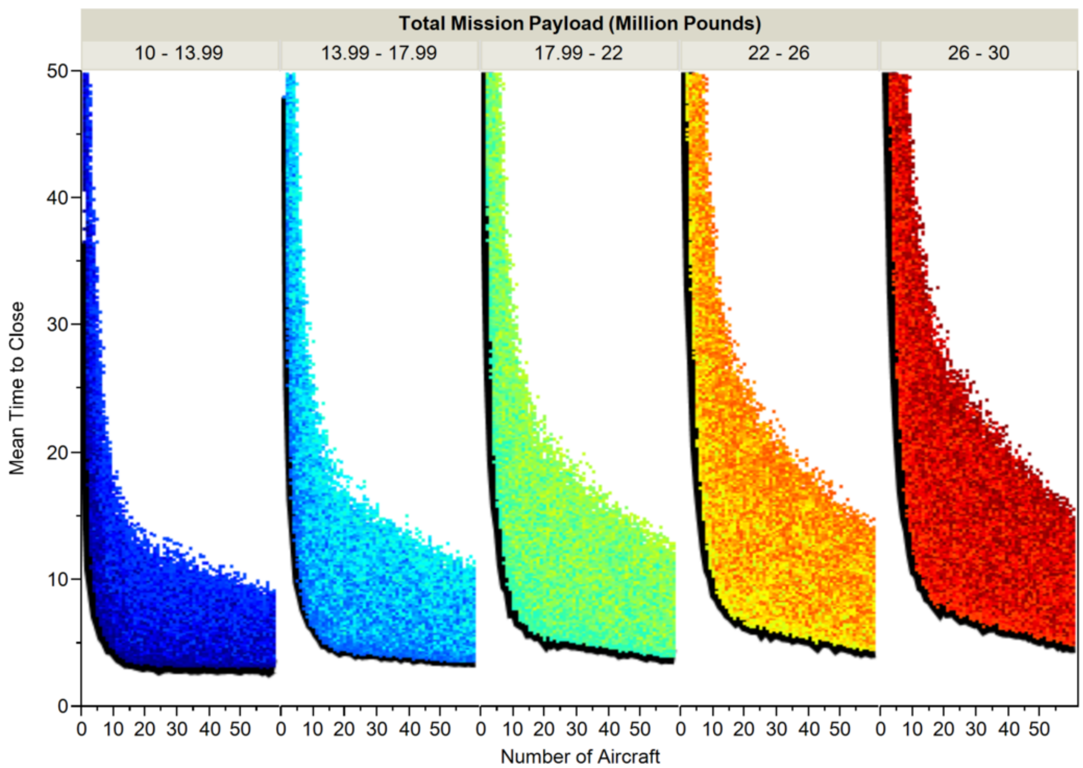


Figure 39: Number of Aircraft versus the MTTC for Type 1 with highlighted Pareto frontiers for each Mission Payload Category

Coloring the same points, not by Mission Payload, but by the size of the aircraft (i.e. by Empty Weight or Payload per Flight) as shown in Figure 40 more clearly shows that not only does the total mission payload affect the time to close but the carrying capacity of each of the aircraft. Not surprisingly, the MTTC is smaller with larger fleets of large aircraft.

An interesting feature is also observed in the lowest Mission Payload category

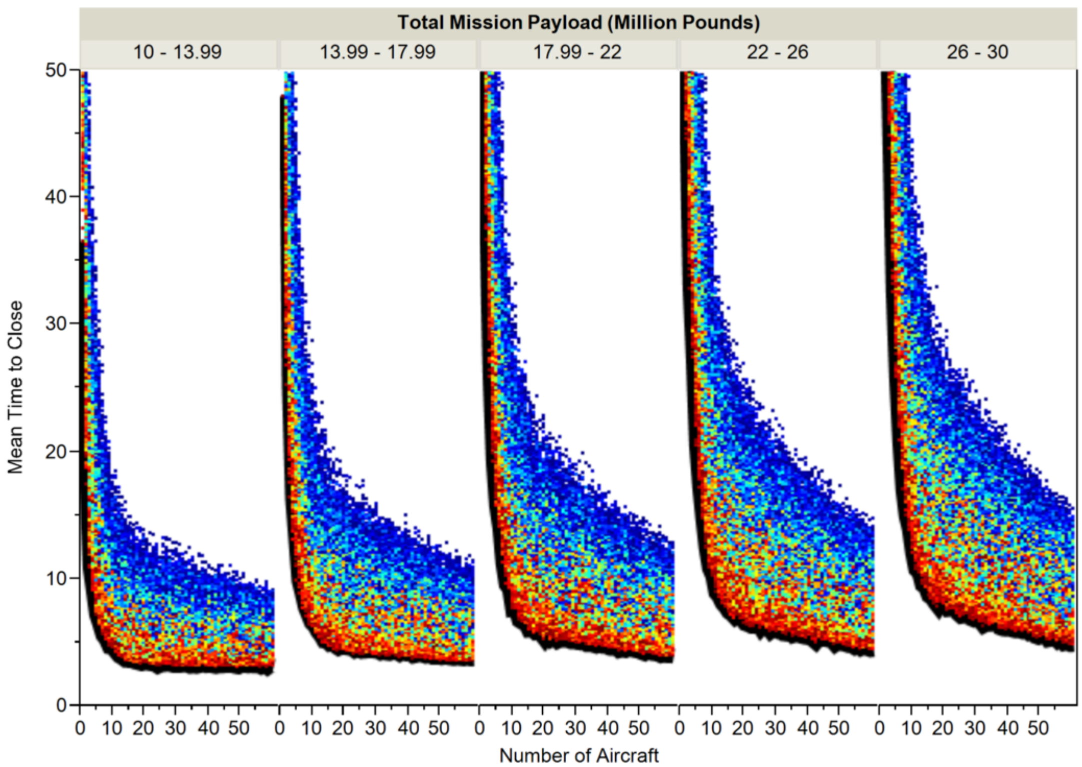


Figure 40: Number of Aircraft versus the MTTC for Type 1 with highlighted Pareto frontiers for each Mission Payload Categories and colored by Empty Weight

of Figure 40. The Mean Time to Close does not continually decrease after about 25 aircraft are employed to transport the cargo. For these cases, an excess number of aircraft are available. For example, if a fleet consists of 50 aircraft, and each can transport a payload of 250000 lbs, a 10 million pound total mission payload is transported with just 40 aircraft with one flight each while 10 aircraft are not used at all. This effect is, of course, more dramatic with larger aircraft in a large fleet with a scenario involving a small mission payload, but the shape of the Pareto frontier is consistent with this expected behavior. The minimum mean time to close is also readily shown by the lowest point in this same category. Regardless of how many additional aircraft are used, the time to close a mission cannot be faster than the actual trip time (with associated loading delays, airfield constraints, etc.) for one flight for every aircraft required to deliver the full mission payload.

Returning to Figure 37, comparing the local Pareto frontier for each of the four types shows another observation that could be predicted from the "Type" definitions above. Across all the combinations of high-level design parameters and scenarios, the MTTC is lowest for Type 1 followed by Type 3, Type 2 and Type 4, in that order.

Stopping to refuel twice, en route and retro en route, (i.e. for Types 2 and 4) will clearly take more time than Type 1 or 3 which has no refueling stops on the return flight and only one stop en route for Type 3. For small distances, Type 1 will dominate Type 3 in terms of time to close especially if distances are below the maximum range at maximum payload. However, at certain distances, the benefit to stopping over en route for refueling allows each aircraft to fly at a larger payload per flight, which may "make up" for the time lost due to refueling and other delays associated with the en route stop (e.g. delays associated with descending, taxing, taking-off, ascending, etc.). Furthermore, not every en route location will be directly along the great circle path between the APOE-APOD combination and therefore time and fuel are lost in traveling to those "off-direct-path" locations. Still, flying direct over a long distance with small payloads may be less efficient, more time consuming, or more costly depending on the scenario.

This trade is partially shown in Figure 41. The points are colored based on the distance between the APOE and APOD. For long distances (dark red), only Type 2 and Type 4 are able to achieve the missions since the aircraft would be unable to fly directly there (for Type 1) or back (for Types 1 and 3).

The MTTC is much better (lower) for Type 1 and 3 but for any specific APOE-APOD combination with a large great circle distance, these types are not feasible. However, the general trend, even across all types, that a smaller payload per flight results in a longer time required to close the mission is evident. Lastly, no points are seen below the constraint imposed upon the model with a practical limit of a minimum payload per flight of 25000 lbs.

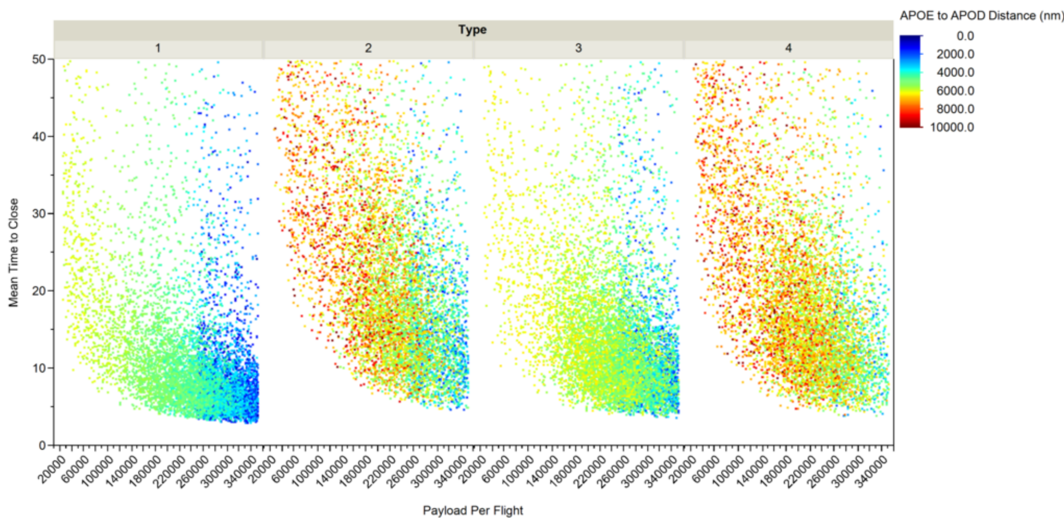


Figure 41: Payload Per Flight versus MTTC subdivided by Type, colored by Distance between APOE and APOD

5.4.2 Trades involving the Break Rate

Another interesting trend visible in the scatterplot matrix of Figures 35 of 36 is that between the break rates and the standard deviation for the time to close (SDTTC).

The same scatterplot, broken down by type, is shown in Figure 42. The overall behavior is consistent with expectations that when the break rate is higher, the SDTTC is likewise higher. An increased chance that an aircraft requires repairs for every flight, and the associated delay for the time to repair, will result in a wider spread in the distribution of the time to close output metric.

Type 1 performs the best among the 4 types, but with the aforementioned qualifier that many of these points are for shorter distances. The explanation for Types 2 and 4 performing the worst is found in the large number of take-off and landings for en route and retro en route refueling stops. This equates to additional chances for a break to occur, based on the parameter assumptions of the model, and will result in more delays for the time needed to repair or replace parts.

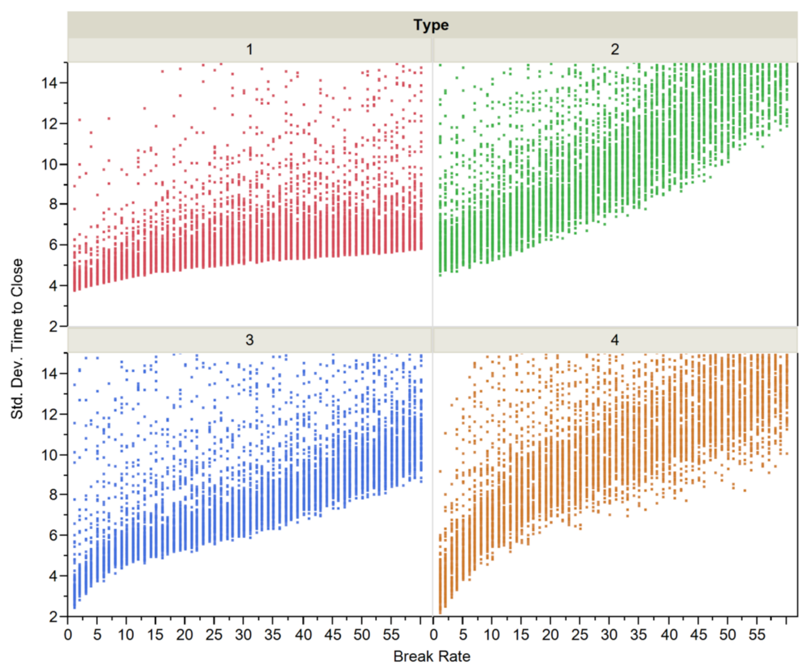


Figure 42: Break Rate versus SDTTC subdivided by Type

Investigating the impact of break rates and mission payload on the MTTC, is also interesting. In Figure 43, the points are subdivided by Type and also by Mission Payload. Each subplot shows an increase in the MTTC as the break rate increases to different degrees. A similar trend of Break Rate with SDTTC is seen with the MTTC metric, in that the larger the payload, the larger the number of flights are needed, and thus a larger number of chances to break down, resulting in a longer time to close the mission. As before, Types 1 and 3 are superior to Types 2 and 4 for similar reasons discussed previously and shown most explicitly in the lower graph of Figure 43 which presents the minimum MTTC for each Type and for each Mission Payload category.

5.4.3 Trades Involving Fuel Consumption and Flight Hours

The third and fourth rows of Figures 35, fuel consumption and total flight hours, are quite similar since a key intermediate variable used in the calculation for fuel is the flight hours for each leg of the mission.

The graph on the left hand side of Figure 44 shows the payload per flight versus the total flight hours with the points colored by the total mission payload, similar to the payload per flight versus the MTTC figure earlier (Figure 41). Being able to visualize all of these relationships for the four types on the same graph without occlusion (as shown on the right hand side of Figure 44) can be accomplished by normalizing the total flight hours by the total mission payload. The new output, "Flight hours per Million Pounds", is more useful in comparing scenarios that differ in terms of the mission payload. Since larger mission payloads would clearly require more flights and thus more flight hours, the ratio between these two metrics allows one to compare across the types in some situations. For example, the right hand side illustrates that, in terms of flight hours and payload per flight (i.e. a surrogate for the size of the aircraft), a Pareto frontier exists upon which decision makers can trade between these two objectives, independent of the mission payload. It also shows the similarity between Types 2 and 4 in at least these two dimensions. Of course, fuel, which is, again, highly correlated with flight hours, can be normalized with respect

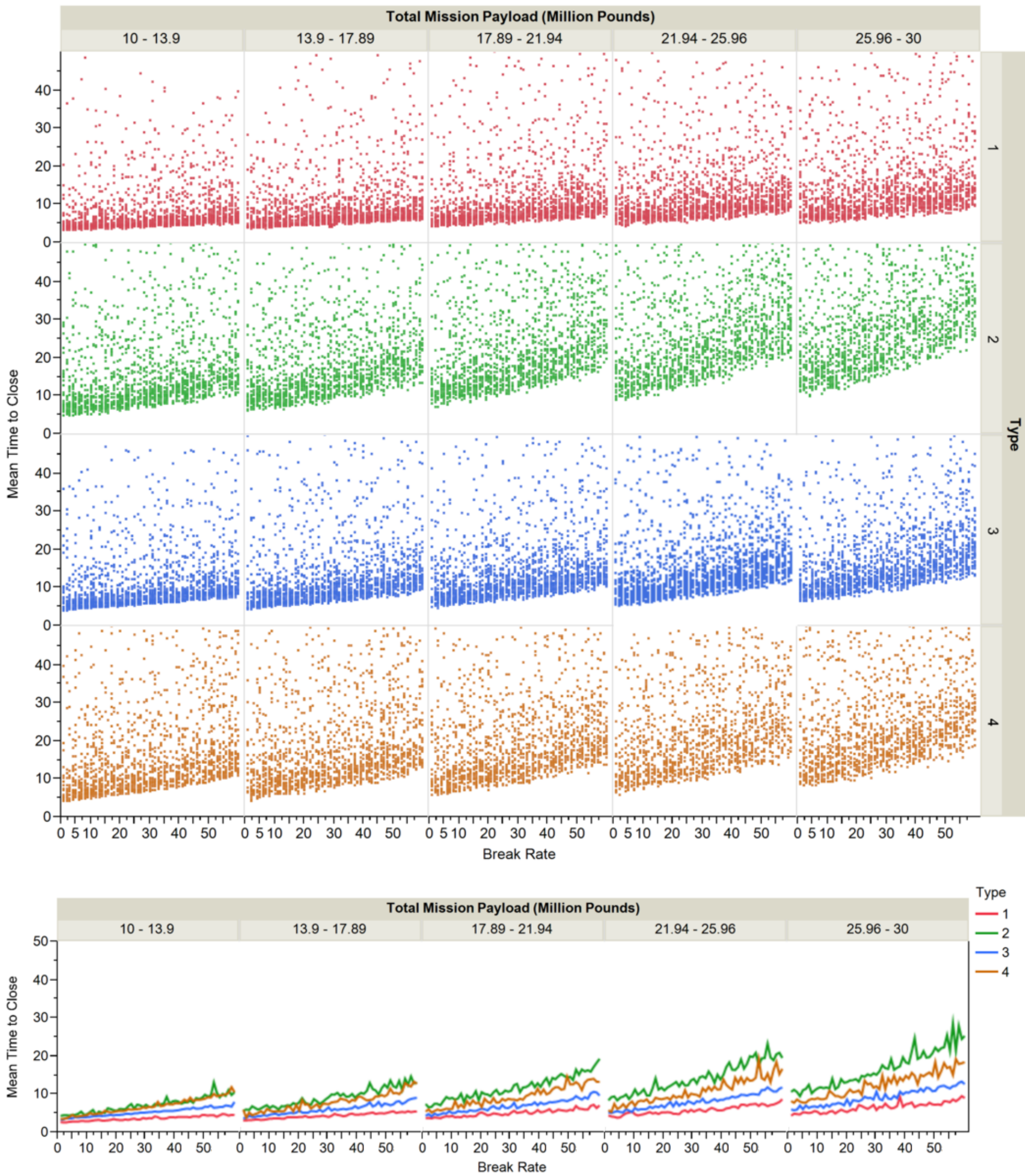


Figure 43: Break Rate versus MTTC subdivided by Type and Mission Payload Category (top), Break Rate versus Minimum MTTC for each Mission Payload Category (bottom)

to the mission payload to acquire the fuel consumed per pound (or a million pounds) in like manner.

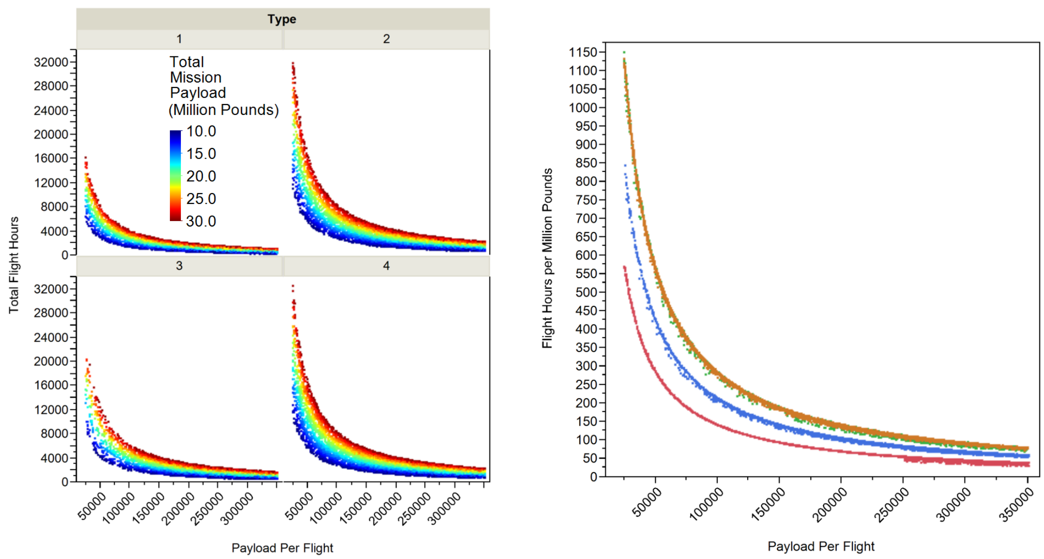


Figure 44: Payload Per Flight versus Total Flight Hours subdivided by Type (left), Payload Per Flight versus Total Flight Hours per Million Pounds (right),

5.4.4 Trades Involving Utilization

Utilization, found in the bottom row of the scatterplot matrix of Figure 35 has an interesting relationship with the Number of Aircraft. The left side of Figure 45 presents this same data with the four Types broken out into subplots.

The trade is again evident by noticing that the smaller the fleet size, the more an aircraft it needed and used assuming everything else being held constant (to meet a target date for mission completion, for example).

With this particular metric, Utilization, opinions on the correct amount of usage can vary significantly. Using the aircraft too much increases degradation rates, lowering the life of the aircraft, and potentially reducing reliability. However, a small number of aircraft may have its advantages by reducing the initial fleet acquisition cost and other life cycle costs such as maintenance, repair part inventories, and labor.

On the other hand, some may argue that a low utilization value is desirable because

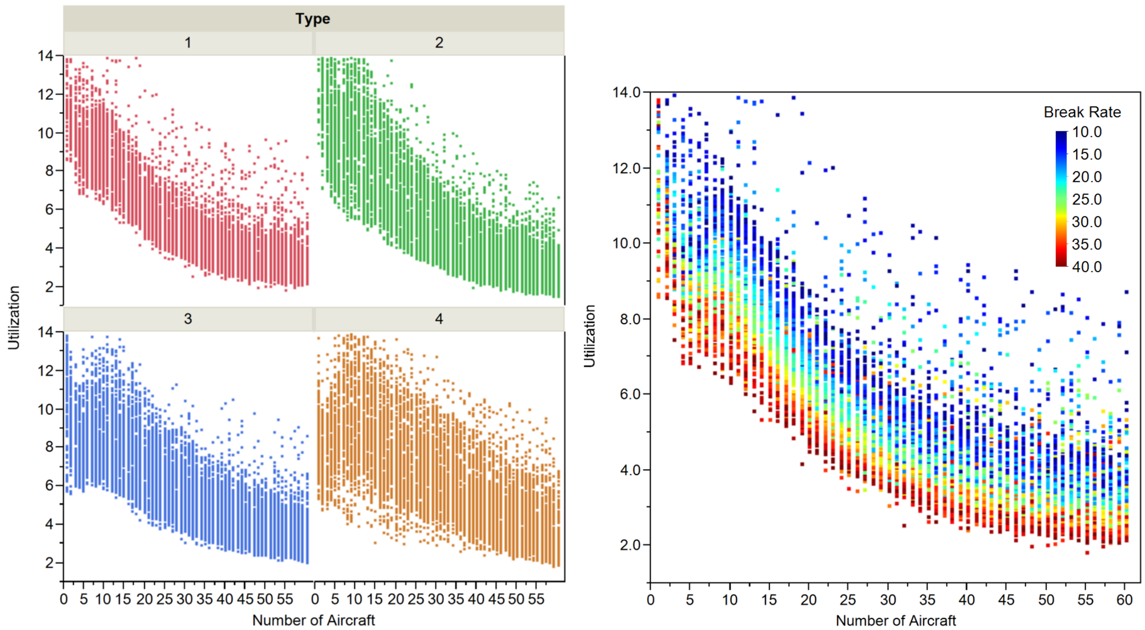


Figure 45: Number of Aircraft versus Utilization subdivided by Type (left), Number of Aircraft versus Utilization (only for Type 1), colored by Break Rate (right)

the aircraft are not degrading as quickly, costs for supporting the crew are lower (less flights equals less operating costs), inherent redundancy allows for a larger absolute number of aircraft to be down due to maintenance (resulting in higher fleet-wide reliability), and there is the potential to meet more demanding requirements (such as supporting two concurrent missions). Of course, these advantages come at a cost since acquiring, storing, and maintaining a larger fleet is clearly more expensive.

The right hand side of Figure 45 shows the same plot for Type 1 with individual designs colored by the break rate. In this graph, the dark red points on the bottom edge indicate simulations with a very large break rate. Since break rate would effectively increase the down time for each aircraft the utilization is correspondingly lower for these cases. This confirms that the model, at least in this situation, behaves correctly.

Another trade involving utilization is that which exists with respect to SDTTC. Figure 46 illustrates this relationship only for Type 1 with the designs colored by break rate (dark red equals higher break rates). In order to reduce the uncertainty in the time to close (i.e. SDTTC), one would necessarily see a higher utilization rate likely due to a lower break rate. This is also seen directly from the subplot between utilization and break rate in Figure 35 much earlier.

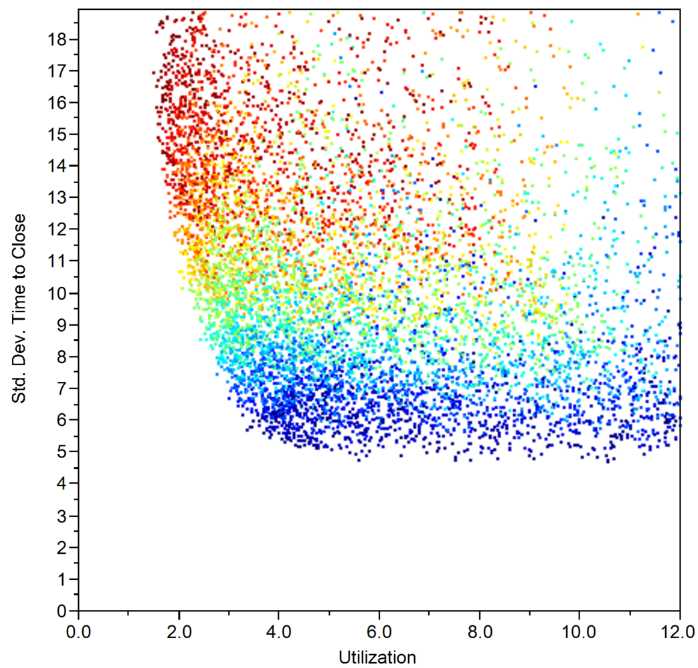


Figure 46: Utilization versus SDTTC, colored by Break Rate

5.4.5 Other Operational Trades

Plotting the output metrics with respect to each other underscores additional interesting relationships. In Figure 47, the normalized MTTC is plotted by type against the SDTTC. Coloring by total mission payload (dark red equals higher mission payloads), reveals that, in general, the larger the mission payload the larger the spread or uncertainty on the time to close (i.e. higher SDTTC). With more flights required for a higher mission payload come more chances for maintenance events and the corresponding repair time delays, which, in turn, cause wider variances in the time to close distribution.

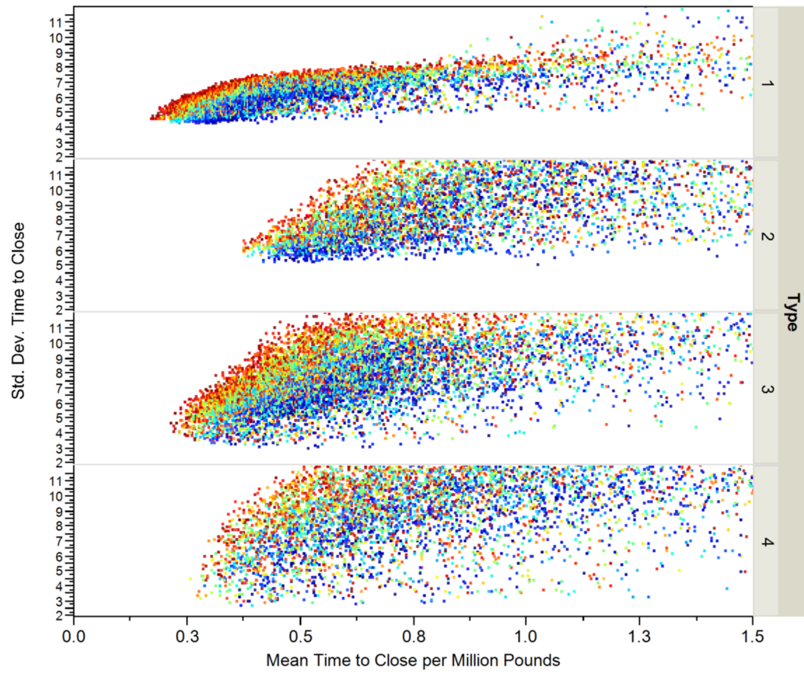


Figure 47: MTTC by Million Pounds versus SDTTC, colored by Mission Payload

Other interesting observations involve the minimum normalized MTTC. As described before, the Type 1 performs best for MTTC and SDTTC, but the associated scenarios and APOE-APOD combinations are nearer to each other with no en route refueling stops. Type 2's SDTTC is comparable to Type 3's, but Type 3 performs much better in terms of time to close, likely due to the direct return flight. Lastly, Type 4 has the largest SDTTC of all types but sometimes performs better than Type 2 in terms of MTTC. This can occur when the return flight selects a retro en route location more optimally situated along the great circle path than the en route refueling location.

These relationships often come back to one of the fundamental trades in the currently discussed operational space. Figure 48 displays this basic trade between the number of flights (which is highly correlated to the number of flight hours and therefore the fuel consumed) and the payload per flight (which is in turn highly correlated with the Maximum Payload, Empty Weight, and MTOGW).

For each one of the lines representing the same Mission Payload in Figure 48, the product of the size of the aircraft (expressed through the payload per flight) and the number of flights required to reach that payload is similar. Thus, many flights can be performed with smaller aircraft or fewer flights with larger aircraft. This relationship is independent of the number of available aircraft, but as shown in previous trades, the larger the fleet size the shorter the time to close. Figure 48 is created from 1 million simulations of Type 1 where the scenario is held constant, varying all other parameters as discussed previously.

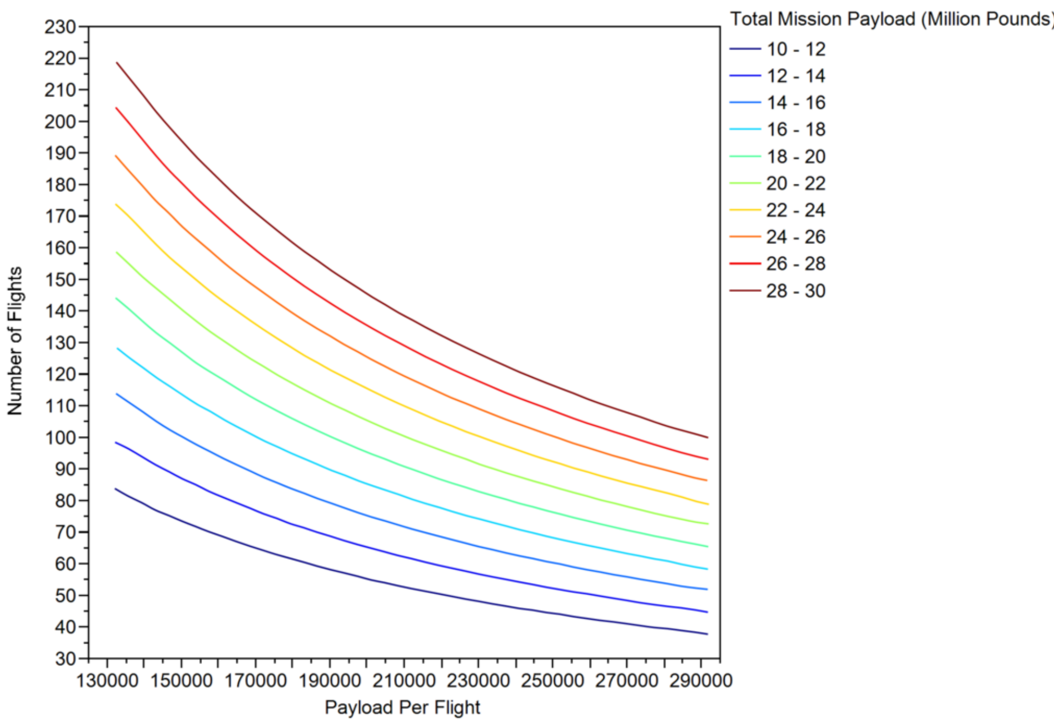


Figure 48: Payload Per Flight versus the Number of Flights, subdivided by Total Mission Payload Categories

This trade is made more obvious by investigating two points at the extreme ends of the 30 million pounds Mission Payload "isoline"[2]. At one point on this isoline, the payload per flight was set at 161300 lbs, delivering 30 million pounds in 186 flights. This selected data point had a MTTC of just over 19 days with a fleet size of 29.

[2]This "isoline" is in fact a mean line for the particular Mission Payload category indicated

However, a second point, also situated on the 30 million pounds Mission Payload isoline, only required 109 flights (41% less than the first), but with a much larger payload per flight of 277100 lbs. This second simulation had a fleet size of 23 and closed the mission with a MTTC of 14.3 days. Break rates were 42% and 51% respectively for these two example points. Other combinations of aircraft size, fleet sizes and break rates will provide other measures of MTTC and SDTTC that can satisfy a decision maker's preference on how quickly and with how many aircraft one can complete a given mission scenario to deliver a certain amount of payload.

For most of the preceding discussion, few comments have been given describing the costs associated with some of the data points. A decision maker would ideally seek to concurrently minimize flight hours (and at the same time total fuel consumption) while minimizing the time to close, the size (i.e. weight) of the aircraft, the break rate, etc. However, each of these has an associated cost, which typically increases as the metric's value decreases. These additional dimensions must be included in the list of objectives upon which any decision maker can perform trades.

For example, to decrease the break rate, significant R&D funds would be required to improve the reliability or life of the individual subsystems or other parts of the aircraft. To decrease the total flight hours, larger payloads per flight are possible but at a cost of acquiring generally heavier and costlier aircraft. Minimizing the time to close is always possible with larger fleets but that comes with increased acquisition and maintenance costs. Lastly, even minimizing the required stops for en route refueling (e.g. changing from Type 3 to Type 1 for a given scenario) necessitates a more capable, but simultaneously more expensive, aircraft.

For purposes of the creation of a Pareto optimal set of points on which a group of decision makers will reach consensus, three kinds of costs have been identified: 1) the cost to decrease the Break Rate, 2) the Operating Cost (per flight hour), and 3) the Acquisition Cost. Many other kinds of costs (e.g. manufacturing, maintenance, etc.)

are available, but the three listed here are sufficient for the scope of this research and can capture a large portion of the costs from a high-level operational and logistics view point.

The cost associated with improving the Break Rate is modeled after the behavior discussed in [2]. Figure 49 shows the general relationship of the relative cost to improve the reliability (i.e. lower the break rate) in a system. Often the reliability can be improved drastically for a relatively small cost increase, for example, by moving from point A to point B. Improving the marginal reliability after point B significantly increases the cost as a percentage of the total cost. Therefore, a much larger percentage of cost increases are required to go from point B to C than from point A to B.

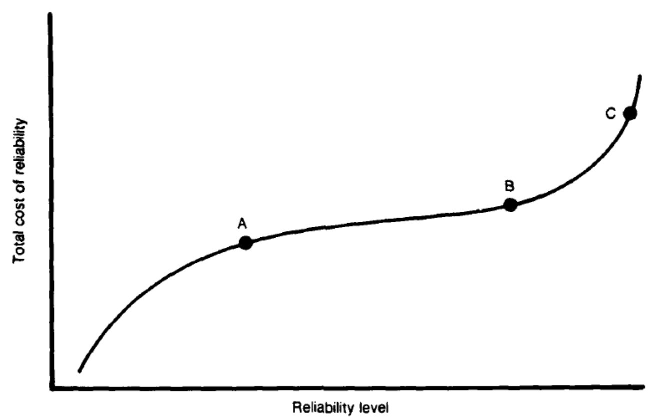


Figure 49: The General Shape of the Reliability-Cost Curve (From [2])

Since the DoD was initially willing to invest $12 billion into upgrading and improving the reliability of the C-5 fleet through the RERP and AMP Programs from a mission capable value near 60% to 75% [174] (i.e. improving the break rate from 40% to 25%), the assumption is made that this represents the "low hanging fruit" of the reliability-cost curve where the cost per reliability increase is flattest. The break rate, by investing nothing, would potentially remain at 40% (i.e. mission capable value of 60%) which gives the starting point for the reliability-cost curve used in this model.

Below break rates of 25% the cost would increases exponentially as the time, effort, and difficulty increases to cause the break rate to approach lower and lower values. Lastly, as seen in Figure 50, even small decreases in the break rate would require some associated RDT&E costs and therefore the cost is concave down in the region of break rates from 40% to 25%.

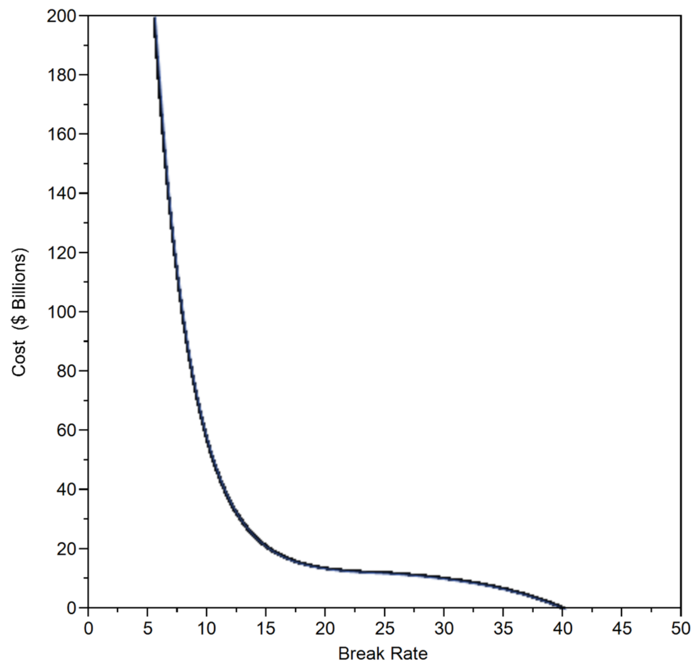


Figure 50: Reliability-cost Curve Used in the Current Analysis

The second kind of cost is the acquisition cost, or the unit price. Although a variety of cost estimating relationships exists from a manufacturing, engineering, tooling, or flight-test perspective [76], for purposes of this research a very basic relationship between the empty weight and unit cost will be sufficient.

Nicolai and Carichner [114] estimate that the price for the C-5B falls just above the $400/lb trend line in FY 1993 dollars for a total near $160 million. An official military specifications sheet estimates the unit cost of the C-5B at $179 million in FY 1998 dollars [175], which would convert to $252 million in FY 2012 dollars, or around $630/lb after applying an economic escalation factor (i.e. CPI = 1.59, from 1993 to

2012). For military aircraft, these values also concur with basic trend lines and ranges ($341/lb - $485/lb) estimated by Roskam [135]. This simple ratio will be applied for a first order approximation for calculating the cost of the larger aircraft designs whose empty weight (W_e) to MTOGW ratio is maintained from the C-5 platform estimate.

Lastly, the operating cost will typically include a variety of different sources such as fuel, oil, crew, day-to-day maintenance, and training, to name a few [114]. Often the fuel is considered one of the largest contributors to the operating cost which is highly correlated with the total number of flight hours. Furthermore, the maintenance costs can also be expressed in maintenance-man hours per flying hour (MMH/FH) [24]. Hence, total operating costs are often calculated on a per flight hour basis. The operating cost per flight hour (CPFH) of the C-5 aircraft has been estimated by one data analyst as high as $47,819 (or even higher by some models) [185] and as low as $23,100 [174] by the Governmental Accountability Office, who lowered their estimate to $20,947 [178] in 2009. This most recent value (based on DoD data) will be used for calculating the expected operating cost for any new aircraft platform.

With the equations or estimating relationships above, the operational metrics can be converted to a cost to achieve the particular level of performance. Therefore, the payload per flight versus the number of flights relationship, such as that shown in Figure 48 earlier, can be recreated in terms of acquisition cost and operating cost respectively, as shown in Figure 51.

The payload per flight is associated with a particular aircraft design and a certain MTOGW. Assuming the fuel consumption rates are consistent with current engine technology, the fuel and empty weight ratios can be extracted and the acquisition price can be calculated based on W_e. The number of flights is similarly translated to a summation of the total flight hours and then multiplied by the CPFH to obtain the total operating cost for the particular mission scenario.

In Figure 51, each of the five lines represent a particular mission payload category.

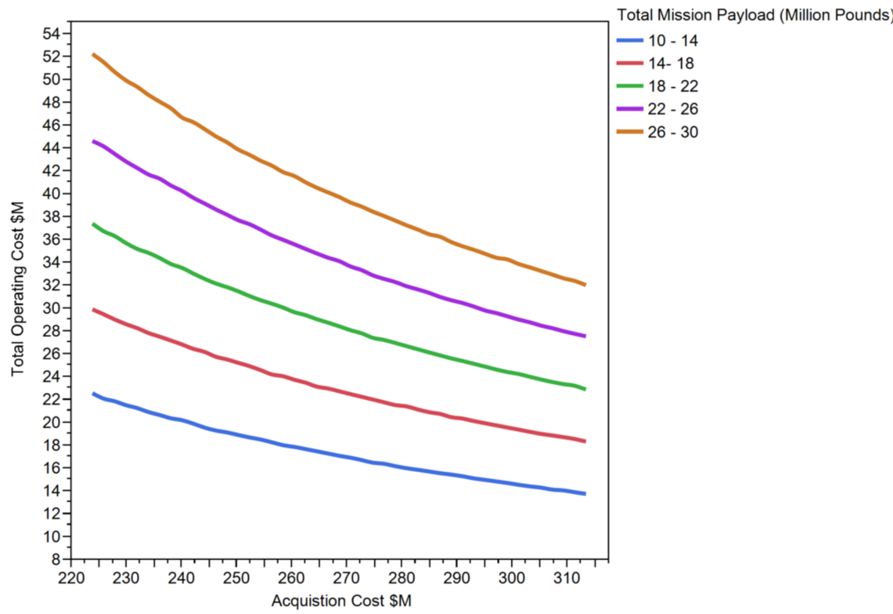


Figure 51: Acquisition Cost versus Total Operating Cost with Categories for Total Mission Payload

Thus, any point along a particular line can satisfy the mission requirements in terms of the cost for the two objectives. For example, more expensive (and thus larger aircraft) would be cheaper in terms of the total operating cost. Smaller and cheaper aircraft require more flight hours and thus, a corresponding higher operating cost, but with a lower unit cost.

Of course, the fleet size or number of aircraft would significantly impact the effective total mission cost (i.e. the sum of operating and acquisition costs), but too few aircraft in the fleet results in a much higher mean time to close. Thus, both of these costs can also be traded with the MTTC in this multi-objective decision space.

Furthermore, the cost to improve (lower) the break rate can be traded against the MTTC and the SDTTC. If a time constraint to deliver a certain amount of payload is imposed, one can either purchase more aircraft or make the current ones more reliable by reducing the break rate. Both of these options come at a cost, but either one may be less expensive in a particular mission scenario. Finally, the cost associated with maintaining an overseas base such as an en route location for refueling could also be included in these trades but no useful cost estimation was found for base maintenance, and was eventually deemed unnecessary for the scope of the current research, allowing it to focus on the group decision-making aspects more than the numerous potential and specific costs associated with the problem.

5.4.6 Summary of Trades

Some of the obvious and fundamental operational trades available in this data set, described in the previous sections, are listed below with short summaries. The arrows indicate the common direction (or target) viewed by decision makers on each of the objectives.

- Mean Time to Close($\downarrow$) versus Number of Aircraft($\updownarrow$): Shortening the time to close can always be accomplished (to a point) by having a larger fleet, but a larger fleet comes at a larger cost.

- Mean Time to Close($\downarrow$) versus Utilization(target): The MTTC can be reduced by using the aircraft more frequently over a period of time. However, increased use accelerates degradation and may reduce the life of the aircraft.

- SDTTC($\downarrow$) versus Number of Aircraft($\updownarrow$): Having extra aircraft can compensate for aircraft down for maintenance and would decrease the uncertainty on completion times. Again, more aircraft equal higher maintenance and acquisition costs.

- SDTTC($\downarrow$) versus Break Rate($\downarrow$): Making aircraft more reliable reduces the uncertainty in the time to close. Decreasing the break rate, though, will still require costs originated from RDT&E.

- Total Flight hours($\downarrow$) versus Payload per Flight($\updownarrow$): As one increases the other decreases assuming all other factors are held constant.

- Payload per flight(↕) versus Number of Flights(↓): This is similar to trading acquisition cost for operating cost, or vice versa.

- MTTC(↓) versus Payload per Flight(↕): Having larger aircraft can decrease the time, but larger aircraft are more expensive.

- Number of Aircraft(↕) versus Cost(↓): Increasing the fleet size always comes at an increased cost.

- Number of Aircraft(↕) versus Utilization(target): Having an excess of aircraft means every individual aircraft is used less, but that increases costs for maintenance, storage, and even the initial acquisition cost.

- En Route Refueling(↓) versus MTTC(↓): This trade becomes more interesting based on the distances and locations for the specific mission scenario. Stopping over to refuel might be faster but it could also be slower for locations significantly off the direct path.

Additional trades are clearly possible between these and other various inputs, such as engine technologies acting on the fuel consumption rate, or different constraints on the safety assumptions such as the required reserve fuel. These, and other variables like them, were either defaulted or assumed constant throughout this research to a tractable, but still sufficiently large, decision space.

CHAPTER VI

DEVELOPMENT AND APPLICATION OF MACRO TO CANONICAL PROBLEM

This chapter describes and tests the second major element in the overall methodology on a canonical data set to facilitate experimentation and explanation of the results. The three steps in the MACRO methodology are analyzed through a variety of experiments examining the effects of such factors as objective space discretization, methods to create discrete choices, the number of decision makers, the number of objectives, sequences of coalitions forming, and influence differences.

6.1 Step 1: Calculating Weighting Distributions

The approach described in this research makes the assumption that a set of points, designs or solutions are available to the decision maker. This is commonly known as the decision matrix (D) where each row representing a different design will have a value in each of the columns representing the objectives or dimensions (e.g. speed, weight, cost) of the design space. Although objectives can take values qualitative in nature, a further assumption that a translation or mapping from qualitative to quantitative values in terms of a utility or value function has occurred. A form of this process will be implemented for the proof of concept in the next chapter. This chapter assumes the completion of such a task has been applied to the decision space. Similarly in the canonical problem developed in the proceeding sections, the use of normalization for the decision space between lower and upper values of 0 and 1 respectively will be assumed where, unless otherwise specified, the higher the value the better, or more preferred. Correspondingly, the preference direction would be to prefer designs with higher values (i.e. utility scores) in each dimension. Thus, the best or most preferred choice would be that which maximizes the value function (e.g. utility function or overall evaluation function).

6.1.1 Creation of a Canonical Decision Space

In order to clearly describe and visualize the process of various steps in the methodology tested in later sections a simplified set of candidate design points in three dimensions is created. A set of 2000 points were randomly selected on the unit sphere with the constraints for each of the three dimensions (x_1, x_2 and x_3 test) are between 0 and 1, inclusive, and defining $x_j = u(x_j)$, or the utility score for the jth dimension for increased readability. Although many steps of the algorithm presented do not depend on the number of initial points or designs, the arbitrarily selected 2000 was considered sufficient to illustrate various features of the algorithm without significantly complicating the visualizations associated with each step.

With these above constraints applied, each point is considered Pareto-optimal in that each point could be the optimal design based upon the weightings for each dimension (in this case three) and the associated utility function or overall evaluation criterion used to quantify the utility or value of each design. Research in identifying, classifying, and describing the Pareto frontier and Pareto-optimal points is also outside the intended scope of this work but the reader is referred to [136] for recent efforts in this domain. However, the current research will assume that this subset of Pareto-optimal points is known, and the respective dimension values are available to the decision makers for application of preferences and ultimate design selection.

The 2000 non-dominated (or Pareto-optimal) points are illustrated in Figure 52. In the 3D graph on the left, the points are relatively equally distributed across the design space showing the shape of the Pareto frontier, extending from 0 to 1 in each dimension, x_1, x_2, and x_3. On the right are the histogram plots of the frequencies

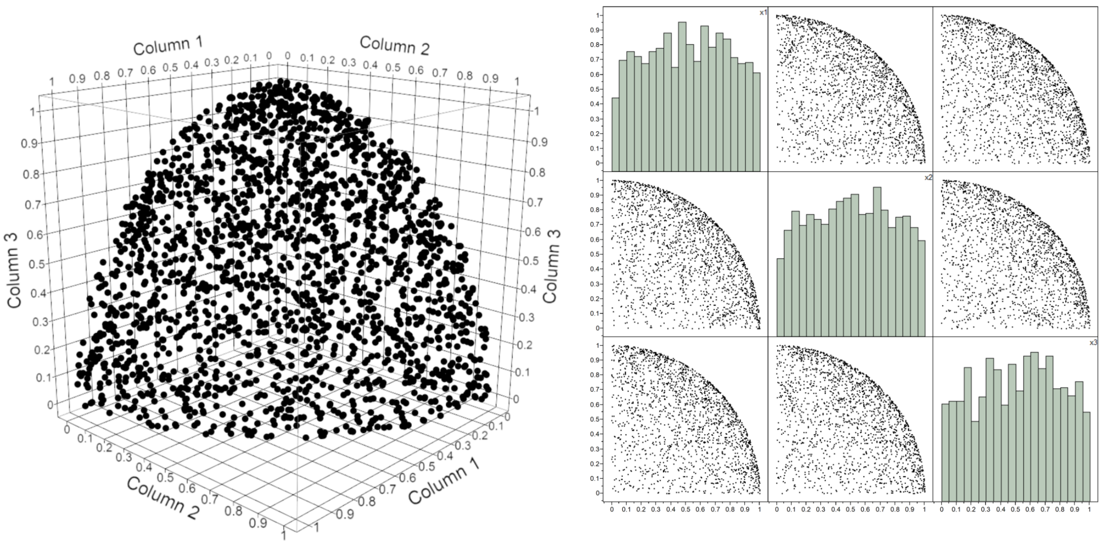


Figure 52: 2000 Points representing a Pareto set of possible decisions illustrated with scatterplot and multivariate Plots

of each value for each of the three dimensions. Furthermore, 2D scatterplots are positioned within the multivariate plot to illustrate the near equal and symmetric distributions of the points in each of the 2-dimensional combinations.

This data set or design space will be referred to numerous times throughout the analysis and explanations of future sections. In general, the terms design space, decision space or candidates will refer to this 2000 multi-dimensional set of points previously described.

6.1.2 Creation of a Set of All Possible Weights

Since the ultimate decision and selected design for a particular problem is so heavily dependent on the importance a decision maker gives to the various objectives or dimensions of a design, care must be taken to truly extract the real decision makers' preference or weights which will be eventually assigned to each objective. Therefore, the set of all possible weights as a starting point is required, and this set is further reduce as additional information is acquired or provided by the decision maker. In this way, the decision maker is not asked to arbitrarily assign values for the objective's importance but respond to comparisons of final designs that are either attractive and selected or repulsive and rejected.

The complete set of weights (W) can be defined by:

$$W = \begin{bmatrix} W_1 \\ W_2 \\ \vdots \\ W_m \end{bmatrix} \tag{5}$$

where:

m : Total number of possible weighting vectors

W_j : j^{th} possible weighting vector

and

$$W_j = \begin{bmatrix} w_{j1} & w_{j2} & \dots & w_{jn} \end{bmatrix} \tag{6}$$

where:

n : Total number of objectives

subject to: $\sum_{k=1}^{n} w_{jk} = 1, \;$ for all $j = 1, \dots, m$.

This results in the full W matrix of:

$$W = \begin{bmatrix} w_{11} & w_{12} & \dots & w_{1n} \\ w_{21} & w_{22} & \dots & w_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ w_{m1} & w_{m2} & \dots & w_{mn} \end{bmatrix} \tag{7}$$

The number of rows, m, of W is dependent on the discretization of the range available, [0, 1] for the weights, since there is an infinite number of combinations or weighting vectors that satisfy the constraint $\sum_{k=1}^{n} w_{jk} = 1$ and the number of objectives n. If, for example, it is assumed there are three objectives and the range is discretized into increments of $c = 0.5$, such that w_{mn} can take on any of three values 0, 0.5 or 1, all six possible weighting vectors are easily computed and are listed in Table 4.

Table 4: All possible weighting vectors with $n = 3$ and $c = \frac{1}{2}$

j	w_{j1}	w_{j2}	w_{j3}
1	1	0	0
2	0.5	0.5	0
3	0	1	0
4	0.5	0	0.5
5	0	0.5	0.5
6	0	0	1

Every row must satisfy the constraint condition above ($\sum_{k=1}^{n} w_{jk} = 1$). An equal number of each of the individual values (0, 0.5 or 1) will occur in each column of the table or matrix W.

The effect of halving the increment size and thus increasing the number of discrete weight levels from three (i.e. 0, 0.5, and 1) to five (i.e. 0, 0.25, 0.5, 0.75, and 1) increases the number of possible weighting vectors from six to 15, as shown in Figure 53, with the associated histogram for each dimension.

The number of occurrences or frequencies in each of the five bins (or discretization levels) is equal for each dimension but the number of counts of zero is the highest, since a large number of combinations of weighting values can exist when one of the n dimension is at zero. Likewise, at the highest bin or discretization level (i.e. 1) only one weighting vectors exists with that dimension taking on a weighting value of 1 (e.g. [1, 0, 0]).

By increasing the number of discretization levels from 3 to 20, (or similarly, by

j	w_{j1}	w_{j2}	w_{j3}
1	1	0	0
2	0.75	0.25	0
3	0.5	0.5	0
4	0.25	0.75	0
5	0	1	0
6	0.75	0	0.25
7	0.5	0.25	0.25
8	0.25	0.5	0.25
9	0	0.75	0.25
10	0.5	0	0.5
11	0.25	0.25	0.5
12	0	0.5	0.5
13	0.25	0	0.75
14	0	0.25	0.75
15	0	0	1

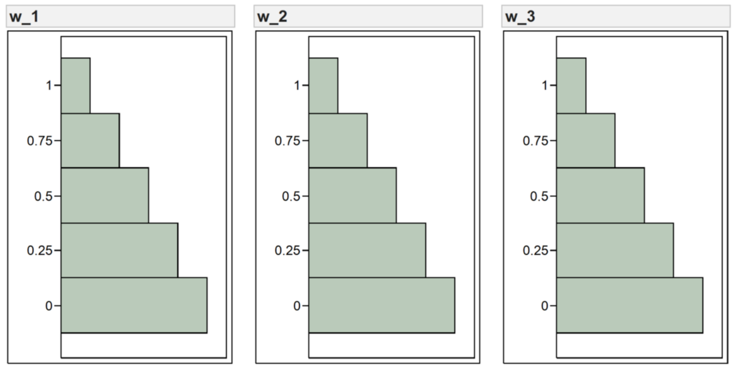


Figure 53: All possible weighting vectors with $n = 3$ and $c = 0.25$ with associated histogram

decreasing c from $\frac{1}{2}$ to $\frac{1}{3}$, and so on, to $\frac{1}{19}$) the relationship between the increment c and the number of weighting vectors m can be identified and is presented in the upper left graph of Figure 54 where n is held constant at 3. As the increment size decreases, m increases evidently faster than exponentially after converting the y-axis to a logarithmic scale in the upper right graph.

When holding the number of discretization levels constant at 3 (i.e. $c = 0.5$) while increasing the number of dimensions n from 3 to 20, the number of possible weighting vectors increases rapidly (bottom left), but slightly slower than exponentially, as shown in the logarithmic y-axis of the bottom right figure.

When varying both n and c over the same ranges (i.e. 3 to 10), the tabulated results of m in Table 5 shows that the number of possible weightings will be symmetric about the diagonal of the table or matrix.

Plotting these values shows the increase in m when the number of dimensions increases and when the increment size decreases shown in Figure 55. The lowest line, where the number of discretization levels is three, is the same from Figure 54. The

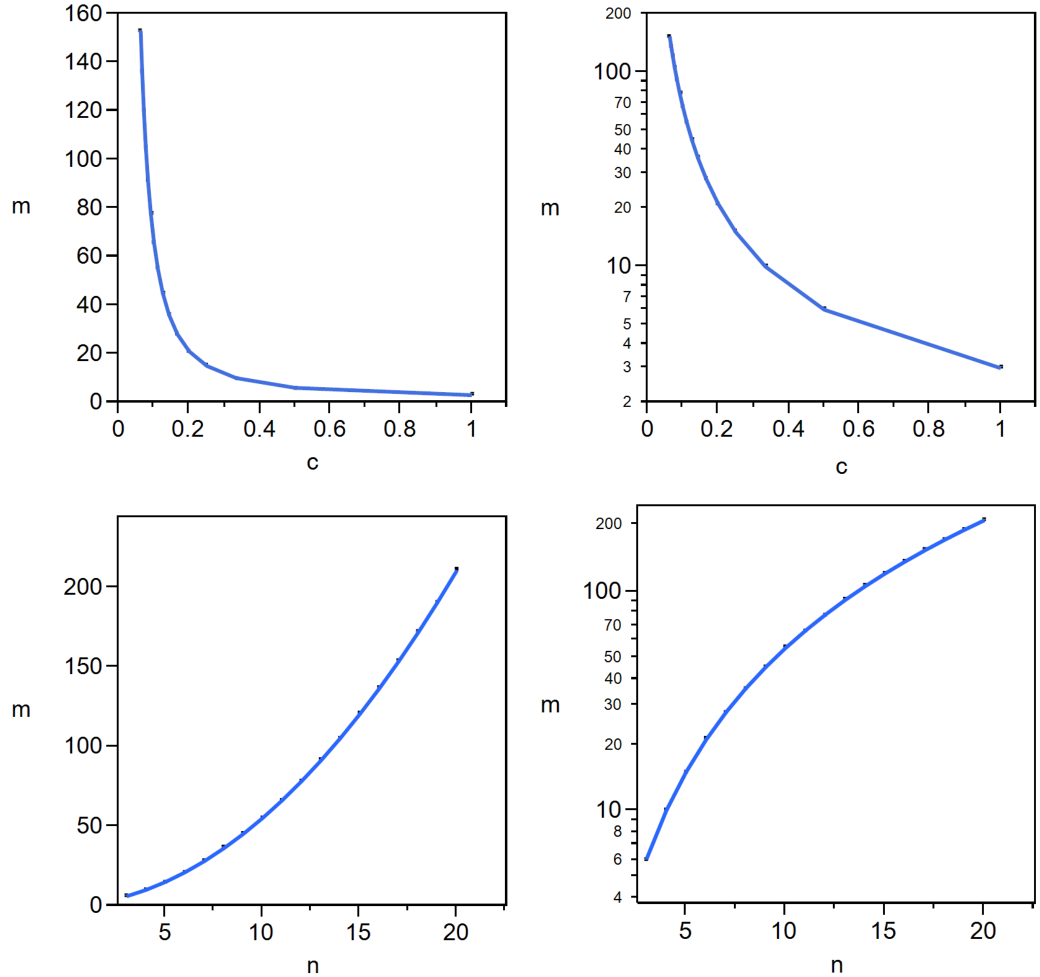


Figure 54: Number of weighting vectors over a range of c while $n = 3$ (top), and over a range of n while $c = 0.5$ (bottom)

Table 5: Number of Possible Weighting Vectors with $n = 3, ..., 9$ and $c = \frac{1}{2}, \frac{1}{3}, ..., \frac{1}{9}$

Number of Disc. Levels	c	n=3	n=4	n=5	n=6	n=7	n=8	n=9	n=10
3	0.5	6	10	15	21	28	36	45	55
4	0.333	10	20	35	56	84	120	165	220
5	0.25	15	35	70	126	210	330	495	715
6	0.2	21	56	126	252	462	792	1287	2002
7	0.167	28	84	210	462	924	1716	3003	5005
8	0.147	36	120	330	792	1716	3432	6435	11440
9	0.125	45	165	495	1287	3003	6435	12870	24310
10	0.111	55	220	715	2002	5005	11440	24310	48620

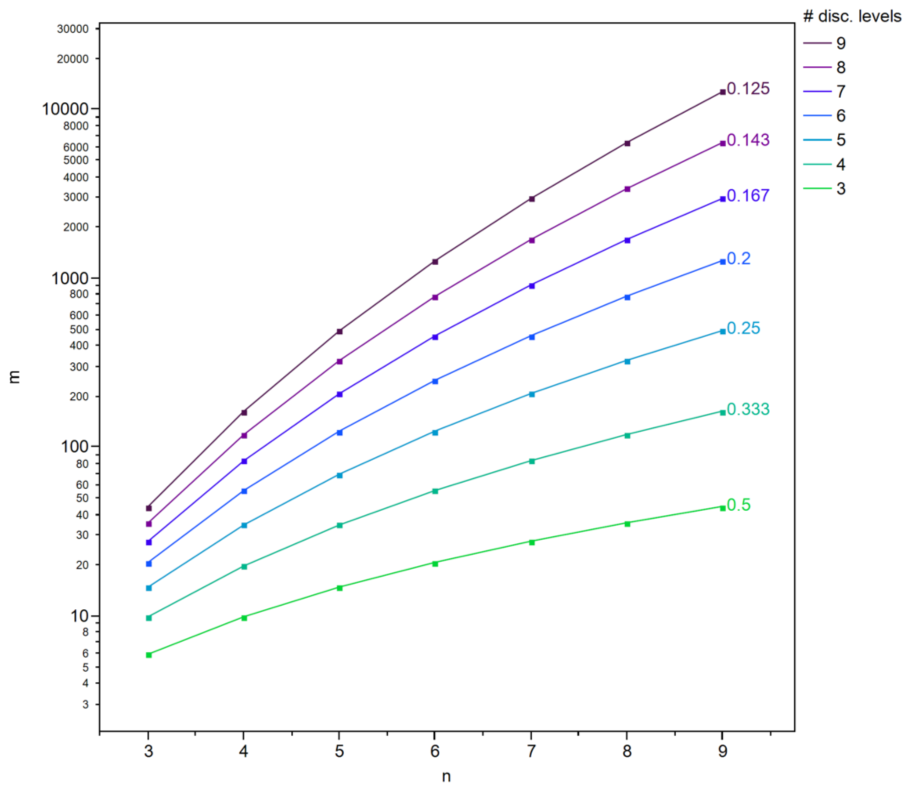


Figure 55: Number of possible weighting vectors for n and c (both varied from 3 to 9)

additional lines are for smaller values of c (or a larger number of discretization levels). The increment value is labeled on the graph for when n=9, and the legend indicates the color associated with each value for the number of levels.

Lastly, since Table 5 was symmetric about the diagonal for similar ranges, the current x-axis variable n can be traded with the c (or more specifically with the number of discretization levels) without changing the exact values in the graph, with the exception of the labels which would then be applied along the top most line associated with $n = 9$.

A recognition of this rapid increase in the size of W is crucial for further steps since its size has a large impact on the required computational expense. If the set of all possible weighting vectors is too large, the available memory for computer simulations can be insufficient and its computational speed can be prohibitively slow.

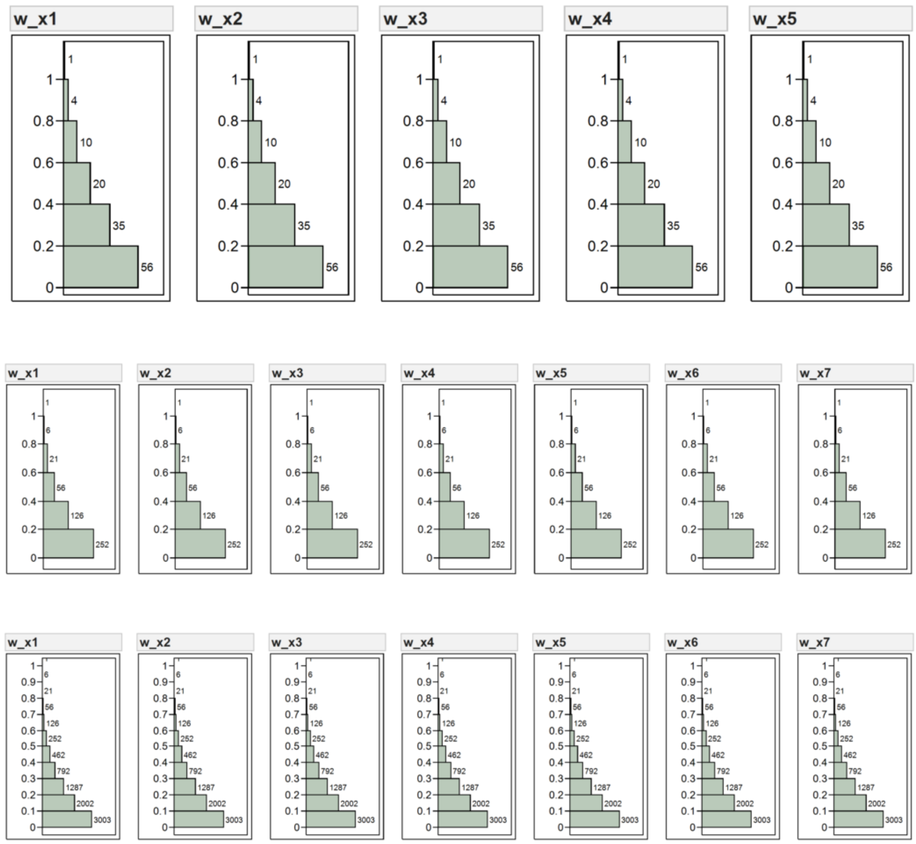


Figure 56: Histograms for the frequency of values of discretized levels in W for: $n = 5$ and $c = 0.2$ (top), $n = 7$ and $c = 0.2$ (middle), and $n = 7$ and $c = 0.1$ (bottom)

6.1.3 Visualizing the Set of Possible Weighting Vectors

The histogram introduced in Figure 53 will be one way to visualize W in explaining how discrete choices affect the decision space through the preference space. The particular shape of the histogram is likewise dependent on the values for c and n. A few different combinations are presented in Figure 56. Regardless of the number of dimensions, the most common value will be zero with only one occurrence of the value 1 contained within the respective dimension column (i.e. the standard basis vectors, e_i [25], for the axes in a generalized Cartesian coordinate system).

Similarly, a plot visualizing all the individual weighting vectors in n-space will facilitate the identification of regions or clusters of vectors that approach the true decision makers preference. Figure 57 created for n=3 at different increment values

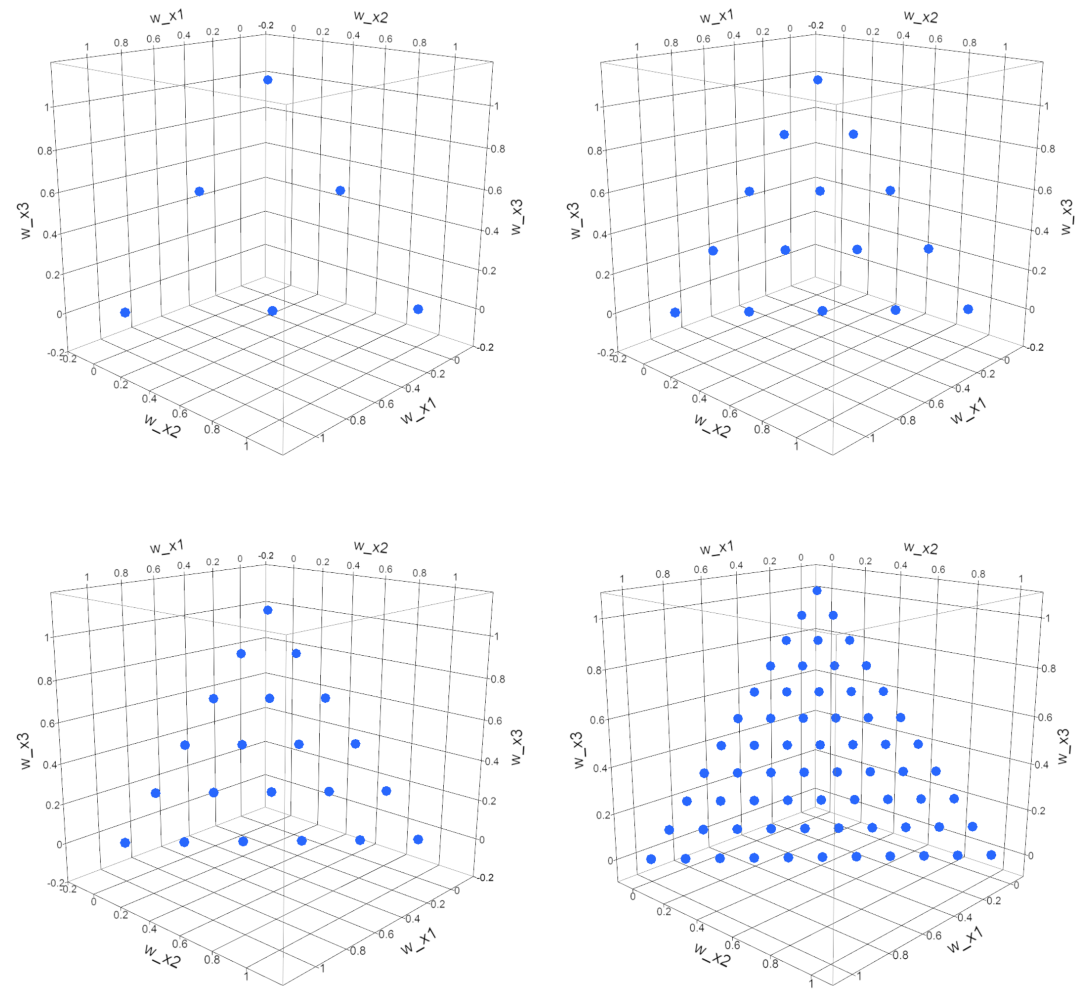


Figure 57: Weighting vectors (from W) where n=3: c=0.5 (top left), c=0.25 (top right), c=0.2 (bottom left), c=0.1 (bottom right)

illustrates the effect on the number of possible weighting vectors as m increases (or c decreases) and where they are located in a 3D graph. For n=2, the weighting vectors would be distributed equally along a line from [1, 0] to [0, 1] shown by the "lowest layer" in Figure 57 when $w_{x_3} = 0$. All but the bottom right plot in Figure 57 have an equivalent point in Figure 55.

For three dimensions (n=3), all of the weighting vectors will lie on a plane defined by the three points [1, 0, 0], [0, 1, 0], and [0, 0, 1]. In general, for n-dimensions, the weighting vectors from columns in the n-dimensional identity matrix (I) will define

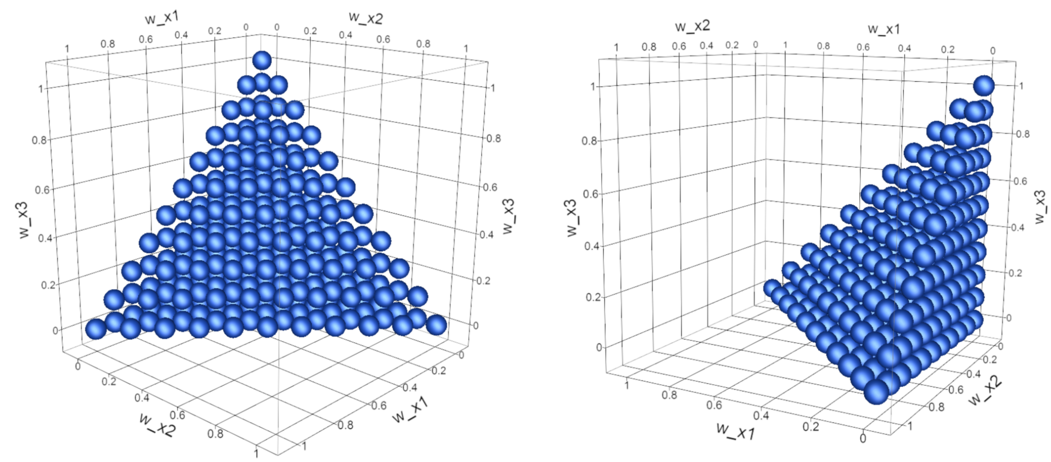


Figure 58: Two views of a discretized preference space of a 4-dimensional design space, $c = 0.1$

the hyper-volume within which all rows of W will be contained.

Four dimensions can be visualized since the preference space falls onto a n-1 dimensional space without overlap, as shown from two angles in Figure 58: one from the front and one from the side (turned 90° clockwise as viewed from the top). Also, the points have been given additional shading and size to help illustrate the depth or the extent of the volume when filled.

Making use of these visualizations can assist in understanding how a decision maker converges upon a preference for multi-dimensional design spaces without any explicit objective comparisons such as that required by other methods (e.g. AHP [141]).

6.1.4 Design Space Knowledge from a Single Discrete Choice

Under the assumption that a particular decision rule and associated utility function can be either explicitly or implicitly applied across the decision space, the first step in identifying a decision maker's preferences for the objectives themselves (i.e. the weighting distributions) and thus the more preferred designs, is by eliciting information through a discrete choice. This elicitation is described as a discrete choice

Which design do you prefer?

Design 1	Design 2
$x_1 = 0.571$	$x_1 = 0.266$
$x_2 = 0.765$	$x_2 = 0.832$
$x_3 = 0.297$	$x_3 = 0.486$

Figure 59: Example of a Discrete Choice Experiment of the Canonical Design Data Set

experiment summarized in a previous chapter 3.10.

By asking the decision maker to compare two from the 2000 candidate designs (such as that in Figure 59), and selecting the preferred one, information about the preferences or weightings can be extracted by eliminating weightings which are not consistent with the discrete choice. This, in turn, reduces the uncertainty about the preference structure or importance weighting on any one objective.

After only one discrete choice (between randomly selected designs #1569 and #946) the possible or feasible weights that correspond with this revealed preference are reduced by almost half as shown in Figure 60. In this figure (and in similar figures throughout the following discussion), the solid blue markers (i.e.$\bullet$) indicate possible weighting combinations, whereas the red open circle markers (i.e.$\circ$) indicate weights that are not possible under the assumed discrete choices made, or in other words, the preferences revealed through these preference decisions between two individual designs.

For example, the weighting vector $w_{52} = [0.4, 0, 0.6]$ which is located at the upper most solid circle in the current orientation of Figure 60, remains a potential weighting combination which could still represent the decision maker's true preferences. At this weighting, $w_{52} = [0.4, 0, 0.6]$, design #1569 has a utility values of 0.402 and design #946 has a utility values of 0.398. The response of the discrete choice is consistent with the possibility that the decision maker's true preference is [0.4, 0, 0.6]. After

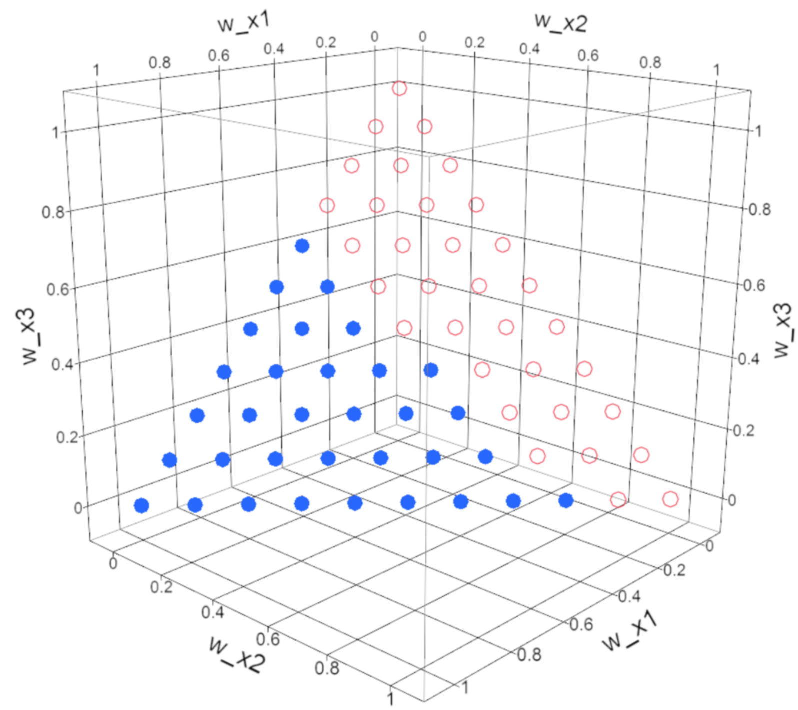


Figure 60: Feasible Weights (solid blue circle) After One Discrete Choice Comparing Design #1569 and #946

all, the vector does result in a higher utility value (i.e. 0.402) and the decision maker preferred it over design #946 suggesting this vector may yet represent their true preference. Thus, w_{52} is kept within the set of possible weighting vectors (and displayed with a blue solid circle in this and other later figures).

Table 6: Summary of Data from One Discrete Choice for Two Weighting Vectors

w_j	Weighting Vector	Design A	Design B	U(A)	U(B)	Design Preference?	Possible w?
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
w_{52}	[0.4, 0, 0.6]	1569	946	0.402	0.398	1569	✔
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
w_{57}	[0.3, 0, 0.7]	1569	946	0.375	0.42	1569	✗
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

On the other hand, the weighting vector of [0.3, 0, 0.7] (the red open circle marker just above and to the right of the previously discussed vector) is not possible in representing the decision maker's true preference. At this weighting, the utility values are 0.375 and 0.420 for design #1569 and design #946, respectively. Since the decision maker chose design #1569 as better, this weighting vector is inconsistent with the revealed preference and cannot be used as a potential or possible weighting vector. Hence, it is removed from the set of possible weightings.

These two particular weighting vectors from W and the related discrete choice data are shown in Table 6.

If the decision maker instead preferred design #946 over #1569 then the inverse is true. That is, the weighting vector [0.4, 0, 0.6] is not possible and the weighting vector [0.3, 0, 0.7] would be. This would of course flip the designations of each of these points in Table 6 and Figure 60 would be converted to the image on the right hand side of Figure 61.

Returning to original discrete choice (#1569 $\succ$ #946), an associated plot of the

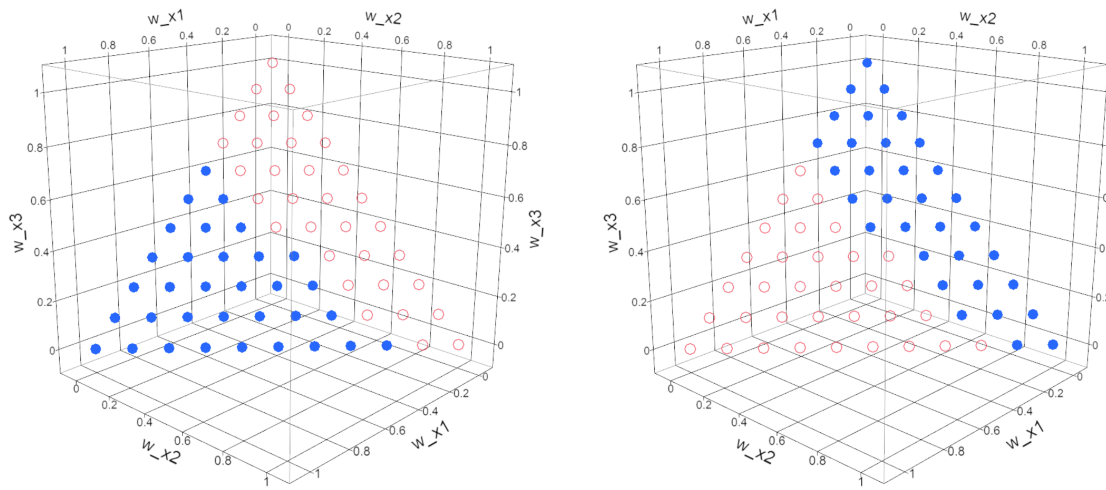


Figure 61: Feasible Weighting Vectors (filled in circles) After One Discrete Choice of Preferring Design #1569 Over #946 (left), and vice versa (right)

design space can now be updated to reflect the preference applied. (See Figure 62 in which design #1569 is indicated with a black square while design candidate #946 is a black triangle.)

The designs or points which are likely not to be desirable based on this discrete choice have been changed to unfilled circles colored in red, similar to the preference graph on the left of Figure 62. Since, from the preceding discussion, some of the weighting vectors cannot represent the decision maker's true preference they are eliminated. The corresponding designs which are optimal for those "now infeasible" weightings are similarly removed from the set of candidate designs. The apparent line of demarcation separating the feasible and infeasible designs represents the extent of information gleaned from one discrete choice.

Consider the previous example of investigating the weighting vectors $w_{52} = [0.4, 0, 0.6]$ and $w_{57} = [0.3, 0, 0.7]$, where the former w_{52} was kept as feasible. Two other points #296 [0.56, 0.047, 0.825] (feasible) and #1968 [0.520, 0.055, 0.853] (rejected) are relatively close to each other in terms of their objectives' values. Table 7 summarizes the utility values at the two weighting vectors for these two points.

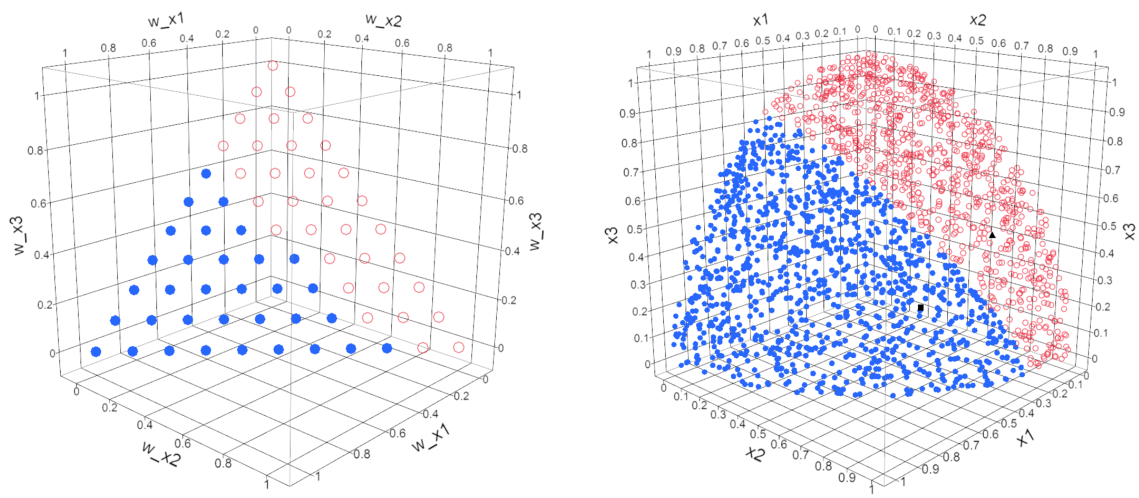


Figure 62: Feasible Weighting Vectors (filled in circles) After One Discrete Choice of Preferring Design #1569 Over #946 (left), and Preference Mapped to Design Space (right)

Table 7: Utility Values for Two Design Points for w_{52} and w_{57}

Design ID	Design Vector	U() with w_{52} =[0.4, 0, 0.6]	U() with w_{57} =[0.3, 0, 0.7]
#269	[0.56, 0.047, 0.825]	0.720	0.746
#1968	[0.520, 0.055, 0.853]	0.719	0.753

From the utility values from Table 7, if one's preference was w_{52} then #269 is preferred, but if w_{57} is the true preference then #1968 is preferred. However, since the discrete choice made previously already ruled out w_{57} as a possible weighting vector one can similarly rule out design #1968. The underlying assumption in this is that one's preferences must take on the values available to the weighting vectors at increment of c. In reality, there exists a weighting vector such that the utility values are identical for any two designs. In this instance, the weighting vector would clearly fall between the two points [0.4, 0, 0.6] and [0.3, 0, 0.7], and is solved with the system of equations:

$$U(d_{269}) - U(d_{1968}) = 0 \tag{8}$$

$$d_{269}^T(w) - d_{1968}^T(w) = 0$$

(for simple additive weight method)

subject to: $\sum_{k=1}^{n} w_k = 1,$

where:

w : A weighting vector

n : Total number of objectives.

In this simplified case discussed here, the weighting vector which satisfies the above equation is approximately [0.3879, 0, 0.6121], which is "between" the two previous weighting vectors above. If the increment value c was much smaller (as it is in Figure 63), then the preference space would have shown all weighting vectors in between [0.3879, 0, 0.6121] and [0.4, 0, 0.6] as feasible and those between [0.3879, 0, 0.6121] and [0.3, 0, 0.7] as infeasible. Not surprisingly, the highest accepted point and nearest the rejected weighting vector in Figure 63 are [0.4, 0, 0.6] and [0.39, 0, 0.61] respectively. The further refinement of the preference space will be performed on local regions about which additional discretization will be warranted.

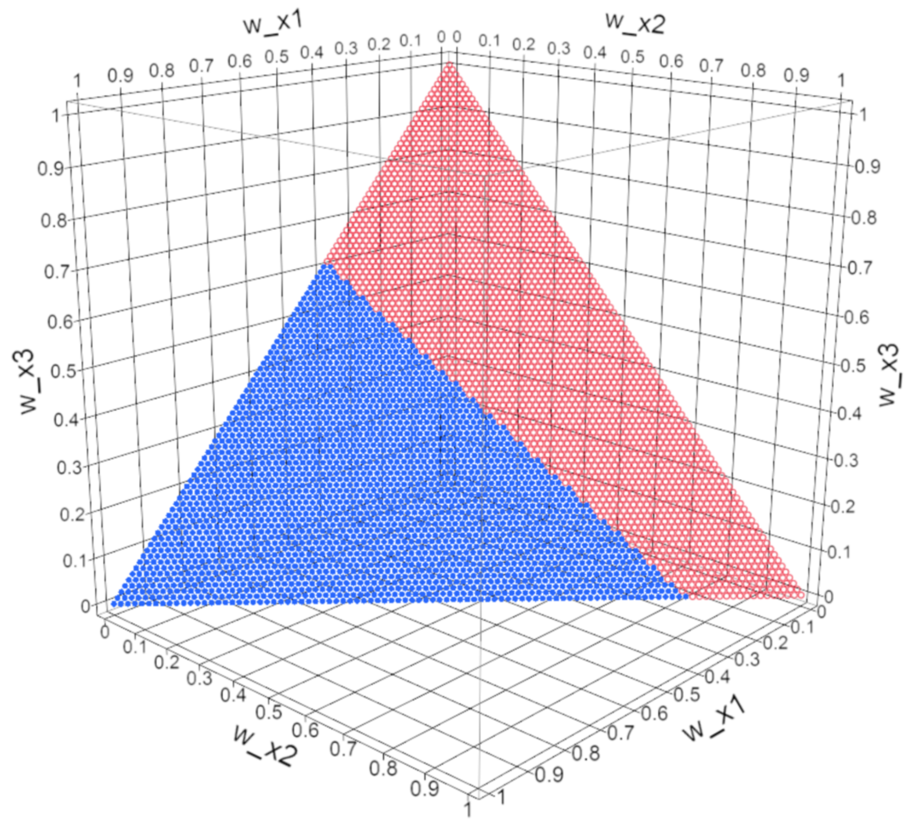


Figure 63: Feasible Weights (solid blue circles) After One Discrete Choice Comparing Design #1569 and #946 with c=0.01

To reduce the whole set of candidate designs, this process amounts conceptually to identifying the hyperplane normal to the vector between the two designs (from the discrete choice experiment) which also bisects the same vector. Any design point closer to the preferred option is still feasible, while any point closer to the rejected design is similarly rejected. This explains the apparent straight line along the Pareto frontier in Figure 62 separating blue and red points, or acceptable and rejected designs, respectively.

After one simple discrete choice, a large portion of the feasible design space has been removed. In fact, the number of feasible designs drops almost by 50% from the initial 2000 to 1098. Similarly, the number of possible weighting vectors, originally at 66 when c=0.1, is now reduced to 33, exactly half.

To complement Figure 62 a histogram of the weighting vectors can show this reduction where the highlighted or darker areas represent when w_j is feasible. Based upon the ranges of the highlighted bins, the data suggests that the first objective (i.e. x_1) *could* be more important than the others because most vectors with a w_{x_1} value less than 0.3 has been rejected. The word "could" is essential since there exists the possibility that the true preference of objective 1 is less than the others but never can w_{x_1} take on a value less than 0.2 based upon this one discrete choice. In fact, of the remaining 33 vectors possible, 24 of them have a w_{x_1} greater than the value of w_{x_2}, and 28 of them greater than w_{x_3}, suggesting that already some useful preference information is available after one simple discrete choice, in at least a simplified 3-dimensional design space.

6.1.5 Design Space Knowledge from Multiple Discrete Choices

In the previous sections, the canonical problem introduced has thus far only included one discrete choice - essentially a judgment about which of two designs is more preferred. All the preference information was inferred from this decision of a simple

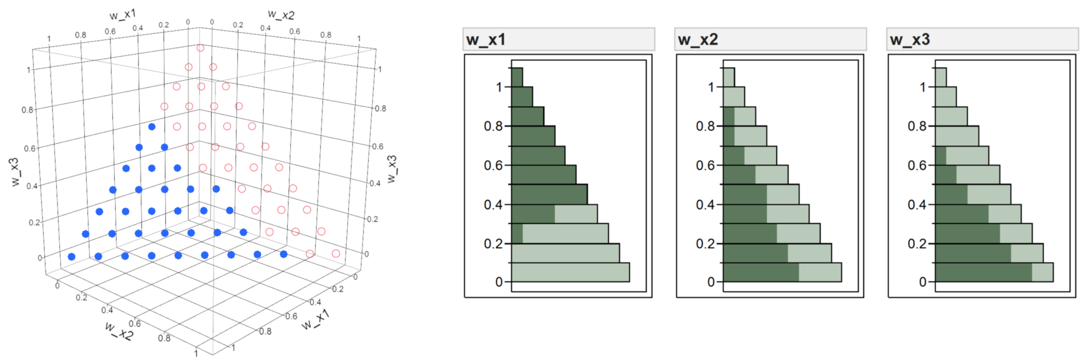


Figure 64: Histogram of Feasible Weighting Vectors (highlighted)

proposition, "which one is better?" resulting in some very rudimentary findings.

With additional discrete choice experiments, this greater knowledge is expressed as a reduction in uncertainty about the true preferences of a particular decision maker. Of course, this added information comes at a cost in terms of additional judgments or comparisons between designs by the design maker. Thus, the first occasion to implement a satisficing strategy has arisen. If sufficient certainty about the decision space has already been acquired, then there is no need for further discrete choices, and a "good enough" state has already been obtained. An example of this could be a decision-making technique that only requires the various objectives ordered in terms of most to least important. Although there remains large uncertainty about the order a rough estimate may be sufficient. Furthermore, if multiple decision makers are each asked one discrete choice, the distributions of ordered objectives for all decision makers might be insightful.

Still, a research question is needed to identify the relationship between how many discrete choices are needed and what level of accuracy is acceptable.

Research Question: *How many discrete choice experiments are needed to reach a particular level of certainty about the true preferences of a decision maker?*

The answer to this question is clearly dependent on some of the factors introduced in the previous sections including the number of objectives n, the size of the increment

c, the decision rules and potentially the number of candidate designs and shape of the Pareto frontier.

Consider for example four random discrete choices on the same design data set previously described, which are illustrated in Figure 65.

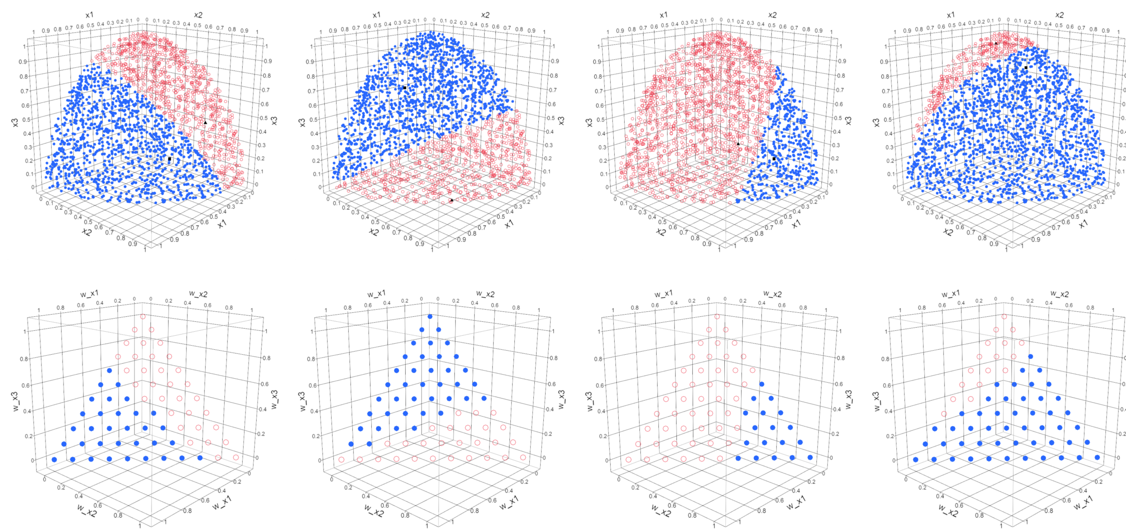


Figure 65: Feasible and Infeasible Designs and Weighting Vectors for four Different Randomly Selected Discrete Choices (one for each column)

The top row shows the designs which have been rejected (empty circles) and feasible (solid circles) based on the preferred design. The matching graph directly below shows the associated preference space for each discrete choice.

These examples all show a different portion of the design space excluded from further consideration. Furthermore, the number of feasible designs (from the original 2000) are different based upon the two randomly selected choice designs, with some including a large majority while others include a minority of the points. The choice represented in the second to last set of graphs has a subset of feasible points completely "inside" the last set of graphs. This means that no new useful information would have been acquired had the choices been asked in a left to right order. In other words, all the feasible vectors remain feasible after the last discrete choice. The assumption is that effort is wasted, in the form of useless experiments, if purely random discrete choices are made without accounting for their capacity (or lack thereof) to provide more information.

6.1.6 Experiment to Test Effects of Methods on Randomly Selected Discrete Choices

In order to test the above assumption, two methods to establish a sequence of discrete choices was performed drawing from the pool of 2000 candidate design points.

In the first method, two randomly selected designs are drawn from the full set of candidate designs in each iteration (i.e. discrete choices from any 2 of the 2000). The second method is identical to the first except that the pool of candidate designs decreases with each iteration after applying the filtering process of removing infeasible designs from previous discrete choices.

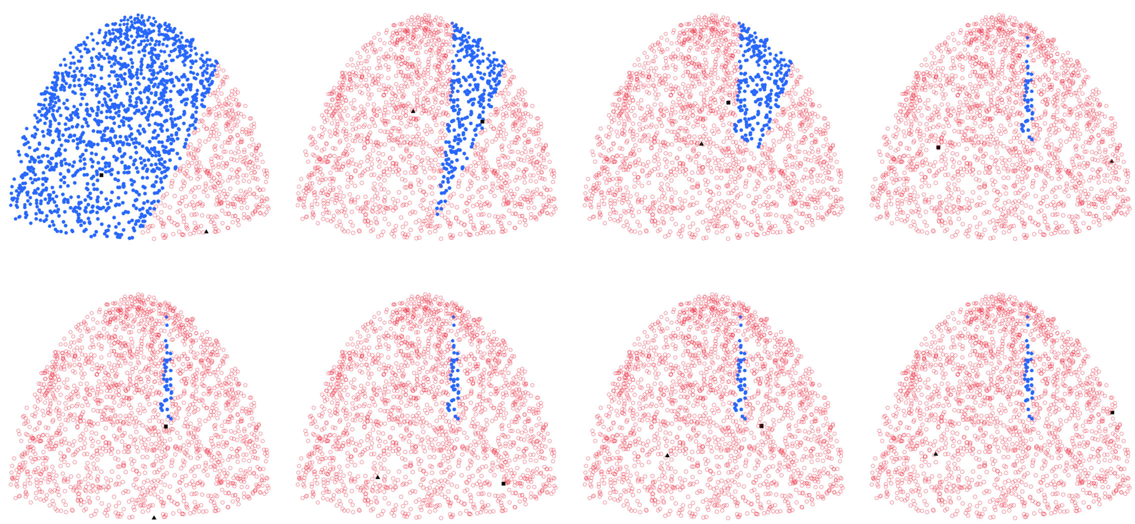


Figure 66: Method 1. Sequence of eight discrete choice experiments applied to the design space. Feasible Designs - filled in blue circles, Infeasible designs - open red circles. Axes removed for readability.

In Figure 66, method 1 is implemented with two designs randomly selected from the full set of candidate designs, both feasible and infeasible. Interestingly, after the first three discrete choices, no improvement in terms of reducing the set of feasible designs is observed until the 8th discrete choice. Although, the first three discrete choice experiments made large reductions in the feasible set, the first method fails to reduce the set for three iterations in a row, signifying wasted efforts in terms of unnecessary discrete choices or decisions. For method 1, after eight discrete choice experiments, the number of feasible designs has been reduced to 37.

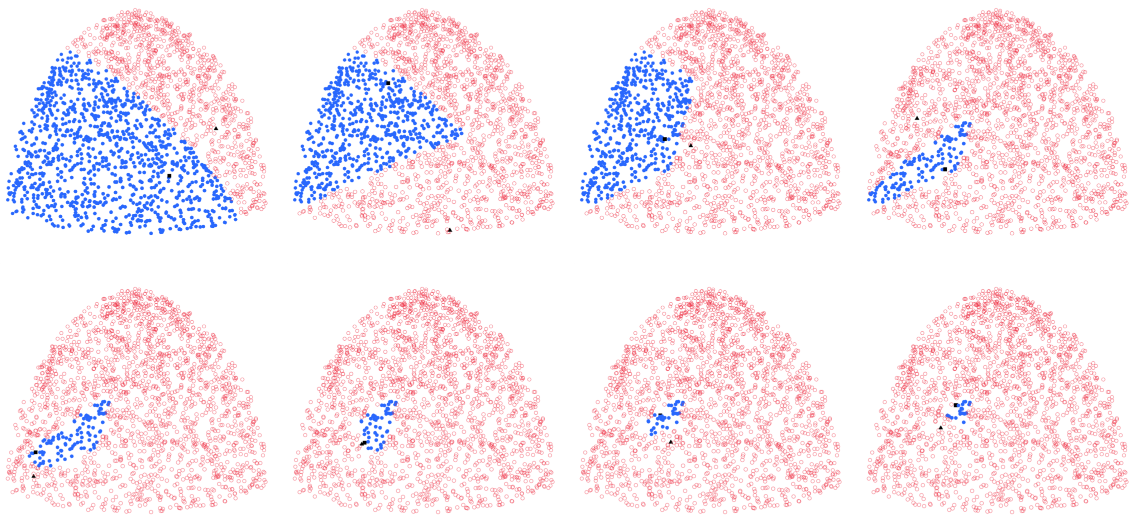


Figure 67: Method 2. Sequence of eight discrete choice experiments applied to the design space. Feasible Designs - filled in blue circles, Infeasible designs - open red circles. Axes removed for readability.

Figure 67 show a sequence of eight discrete choice experiments also mapped to the design space where the infeasible designs have been removed as they would not be preferred based on upon the responses from previous discrete choices. This figure is a representation of method 2 where randomly selecting designs within the feasible set is implemented as evident from the dark triangle and square (i.e. the two designs compared) both falling within the feasible region of the previous iteration. In other words, the '$i + 1$'th iteration will take feasible designs from the ith iteration to randomly select two designs. For method 2, after eight discrete choice experiments, the number of feasible designs has been reduced down to 19.

The difference between these two methods is most apparent when comparing the number of feasible designs for each iteration as shown in Figure 68.

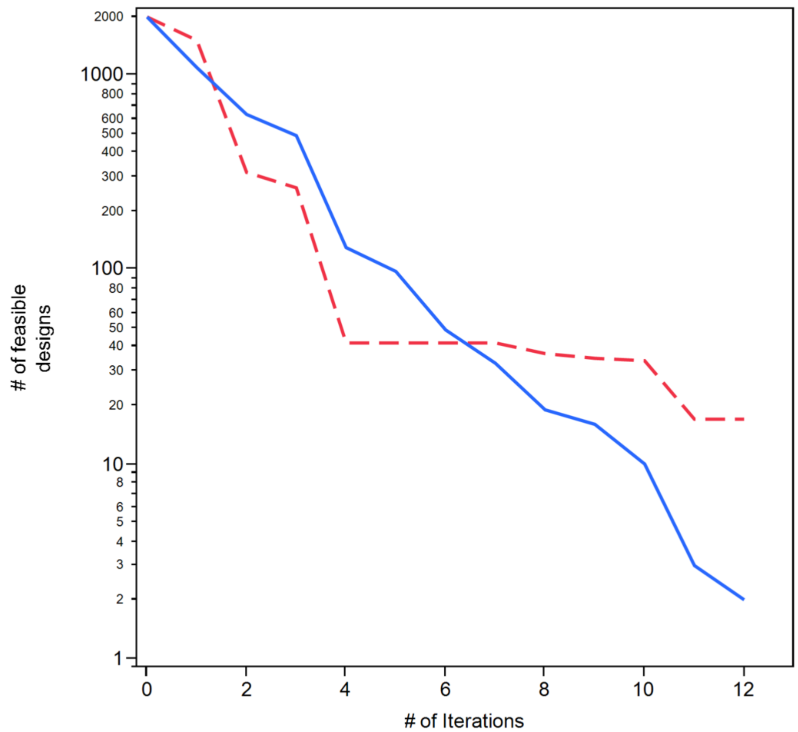


Figure 68: Reduction in feasible designs, Method 1 (red) vs. Method 2 (blue)

The lack of improvement from iterations 4 to 7 for method 1 is contrasted against the guaranteed reduction of method 2. With fewer than 6 iterations, method 1 appears to be better at reducing the feasible set, but since only one execution will not account for random effects, multiple repetitions are required to compare the difference more accurately when investigating this stochastic nature of the two methods.

A set of 1000 repetitions for the same two methods were executed and the results are shown in Figures 69 and 70. For method 1, after only two discrete choices, six of the total simulations (i.e. 0.6%) resulted in a feasible number of designs of just 1, but almost 15% of the simulations (146/1000) still had 100 or more feasible designs. Method 2 on the other hand required 4 iterations at a minimum to reach only 1 feasible design but with 10 iterations the greatest number of feasible designs remaining was 39.

The equivalent statistical figure on the right hand side of Figures 69 and 70 are more revealing, showing the minimum, maximum and mean number of feasible designs per iteration for each method. The approximately straight line in Figure 70 for iterations less than 8 suggest that on average the number of feasible designs will be halved, or, in other words decreases exponentially with approximately the decay constant of ln(2) or 0.693. Fitting the model with such an exponential, results in a calculated λ of 0.688. Method 1 apparently does not decay exponentially (i.e. sublinearly) and on average after 20 iterations will still have almost 15 feasible designs.

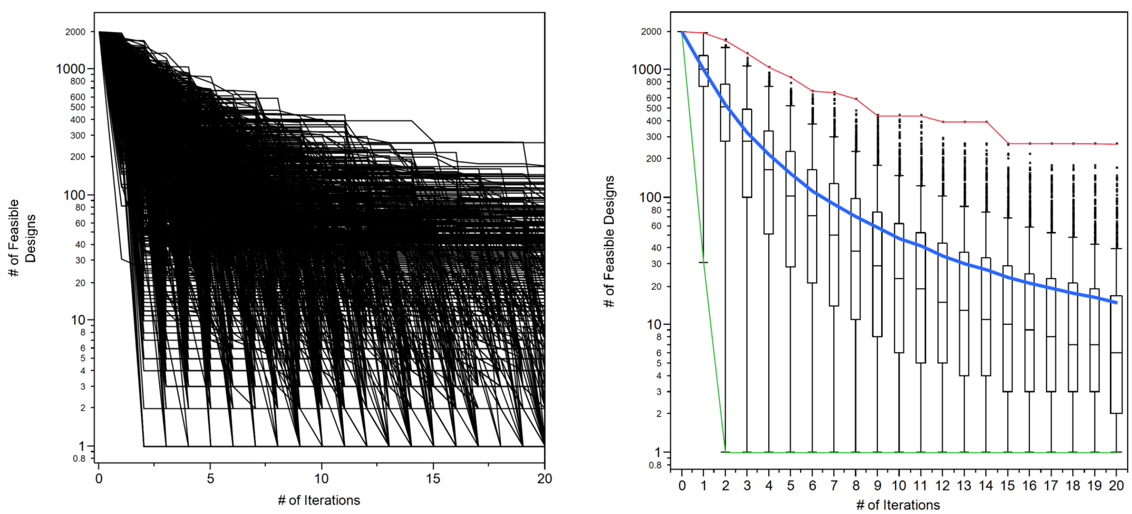


Figure 69: 1000 Executions of Method 1

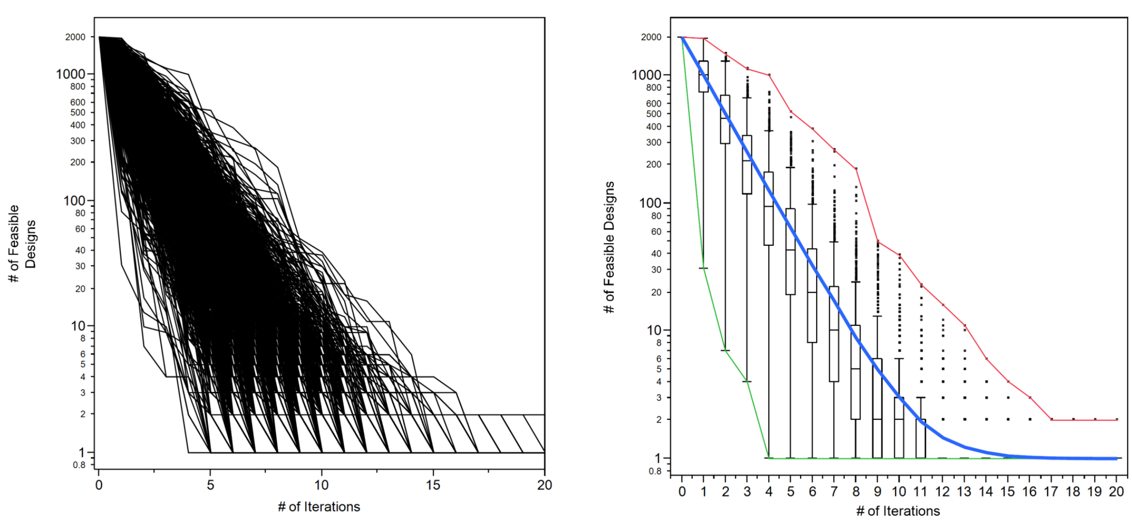


Figure 70: 1000 Executions of Method 2

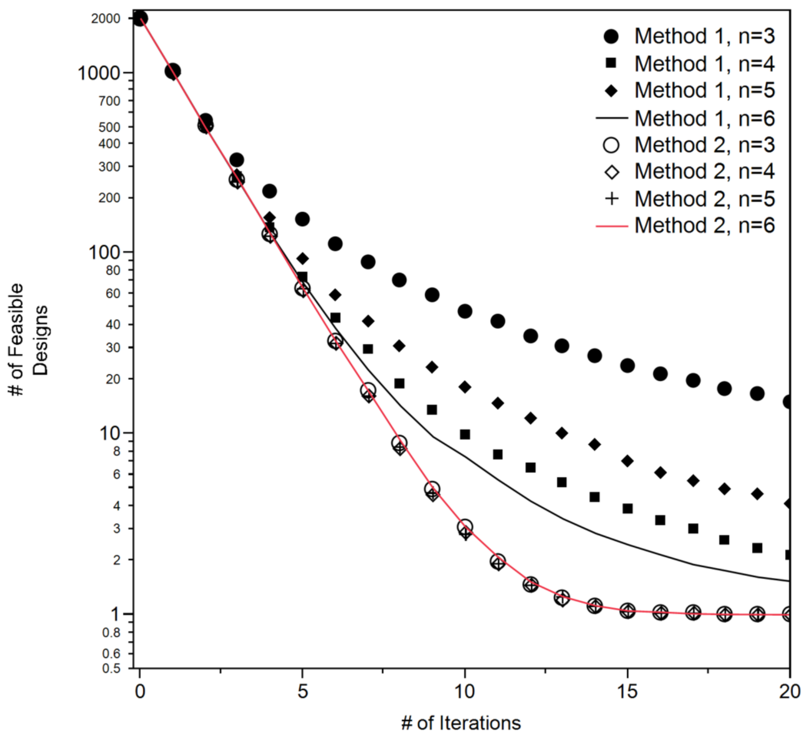


Figure 71: Comparing Method 1 and Method 2 for n = 3 to 6

An obvious question can be asked if these trends hold for more than three dimensions ($n > 3$).

Research Question: *What is the effect of increasing the number of dimensions on how discrete choices are selected?*

Repeating this same experiment for $n = 3$ to 6 results in Figure 71 where there is no increase in performance for method 2, where the three sets of points (i.e. $n = 3$, 4 and 5) directly overlap the red line (where $n = 6$). However, method 1 does show improvement with increased n, in that a fewer number of feasible designs are kept for a fewer number of discrete choices (or iterations). Still, method 2 is superior for all dimensions tested and likely only as n approaches very large numbers would method 1 asymptotically approach the behavior for method 2.

The results suggests that with more dimensions the likelihood that two randomly selected designs from the entire pool or set of designs will reject at least some designs is higher. That is, the chance that one of four numbers (a 4-dimensional design point) for example, is significantly different from the associated objective in a separate 4-dimensional design is higher than with 3 dimensions. But even this result could vary with a different Pareto frontier or design space, where grouping of designs and other concave/convex areas are possible.

6.1.7 Effects of Selection Method on Preference Space

Following the previous section's discussion on the reduction of feasible designs for every discrete choice, the equivalent preference space and possible weighting vectors are similarly reduced.

While considering method 2 as more efficient, as established from the analysis from the previous section, and applying the same set of discrete choices from before, results in Figure 72 shown across the preference space. For this set, only one possible weighting vector has not been rejected after 6 discrete choices. However, on the design side, 49 candidate designs remain as potentially the "best" design. In this example, the last remaining weighting vector, w_{25}, is [0.5, 0.3, 0.2]. Applying w_{25} to the design space results in the one optimal design point $d_{162} = [0.813\ 0.485\ 0.323]$, which, not surprisingly, has a fairly large x_1 value (i.e. the 83rd percentile) and decreasing in value for x_2 and x_3 in that order, for a utility score of 0.616.

To illustrate the same narrowing of the preference space for each of the dimensions, the histograms of only the feasible weight vectors are shown at the beginning and after each discrete choice in order from left to right in Figure 73. In this illustration the x_1 objective is set at the top followed by x_2 in the middle row and then x_3 across the bottom. For each objective the possible weighting vectors slowly coalesce around the weighting of $w_{25} = [0.5, 0.3, 0.2]$.

Of course, one could stop the algorithm at this stage and take d_{162} as the final solution but since there are 48 other designs which haven't been rejected and the increment value c was only set at 0.1, there remains uncertainty that w_{25} represents

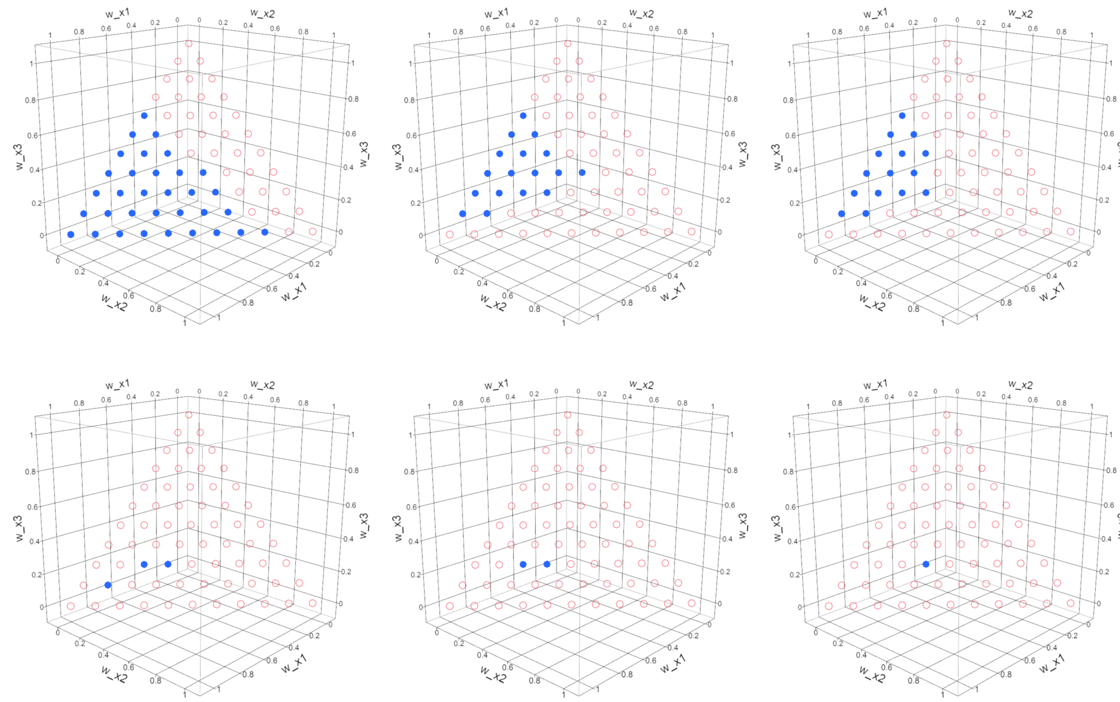


Figure 72: Possible weighting vectors after each discrete choice in the series

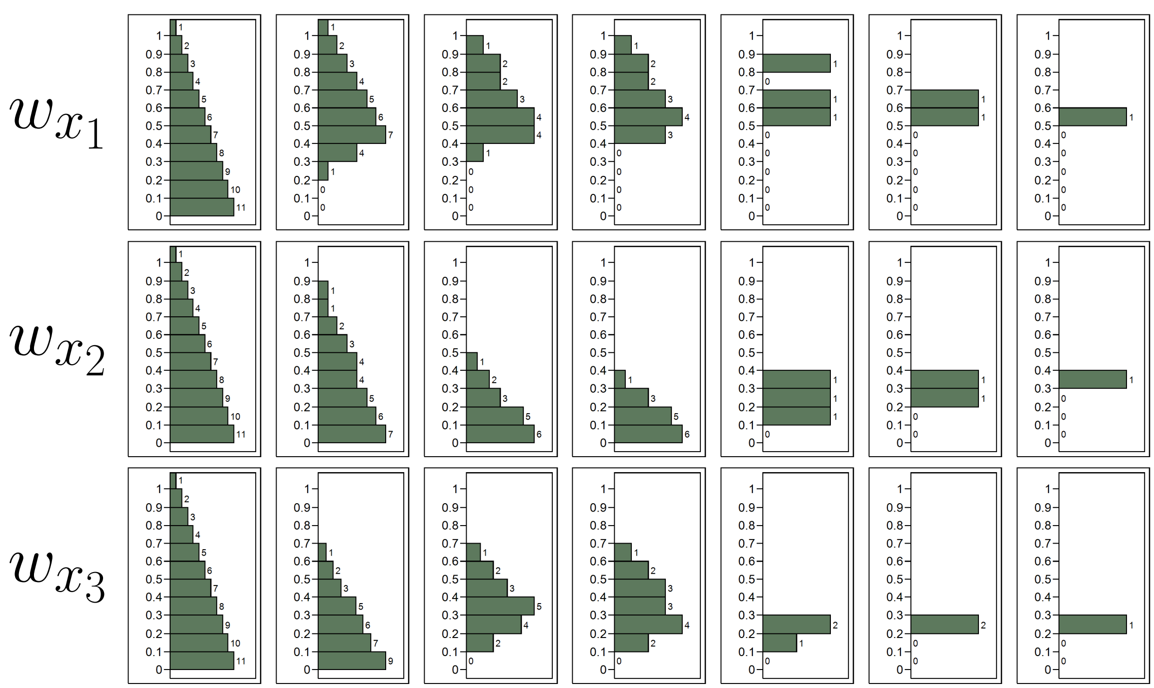


Figure 73: Histograms of possible weighting vectors after each discrete choice in order for x_1, x_2 and x_3

the true decision maker's preference and that d_{162} is their true optimum.

This leads to the next research question:

Research Question: *How can the preference space be further specified to guarantee that only one design choice is optimal based upon a set of discrete choices while minimizing the number of discrete choices?*

The initial answer to this question can of course be to decrease c from the very outset such as that in Figure 63. With $c = 0.01$ implementing the same of number discrete choices from the previous example (i.e. 6), m decreases to 49, but 76 possible weight vectors remain. An additional 7 more discrete choices are needed to reduce W to keep one valid w and one remaining design (i.e. d_{877}). This last w, with values [0.44, 0.28, 0.28], is expectedly close to the $w_{25}^{c=0.1} = [0.5, 0.3, 0.2]$, but clearly, greater precision about the preference space has been obtained (at the cost of more discrete choices). This results in the optimum design at point d_{877} which contains values [0.736, 0.475, 0.482]. Although, only one design remained for one possible w, a similar situation could arise where even an increment value of 0.01 was too large to completely specify on optimal design. However, this would unnecessarily increase the computational requirements for each step and, in addition, increase the number of discrete choices needed for low uncertainty about the true preference.

An experiment to test these opposing effects of reducing the number of discrete choices for minimizing the computational load while reaching as rapidly as possible a narrow distribution for w was performed. This experiment began with a "coarse" W (i.e. with $c = 0.25$) and then further refined W (i.e. local $c < 0.25$) after each discrete choice.

The series of discrete choices followed by a refining step of the feasible weighting vectors is show in Figure 74.

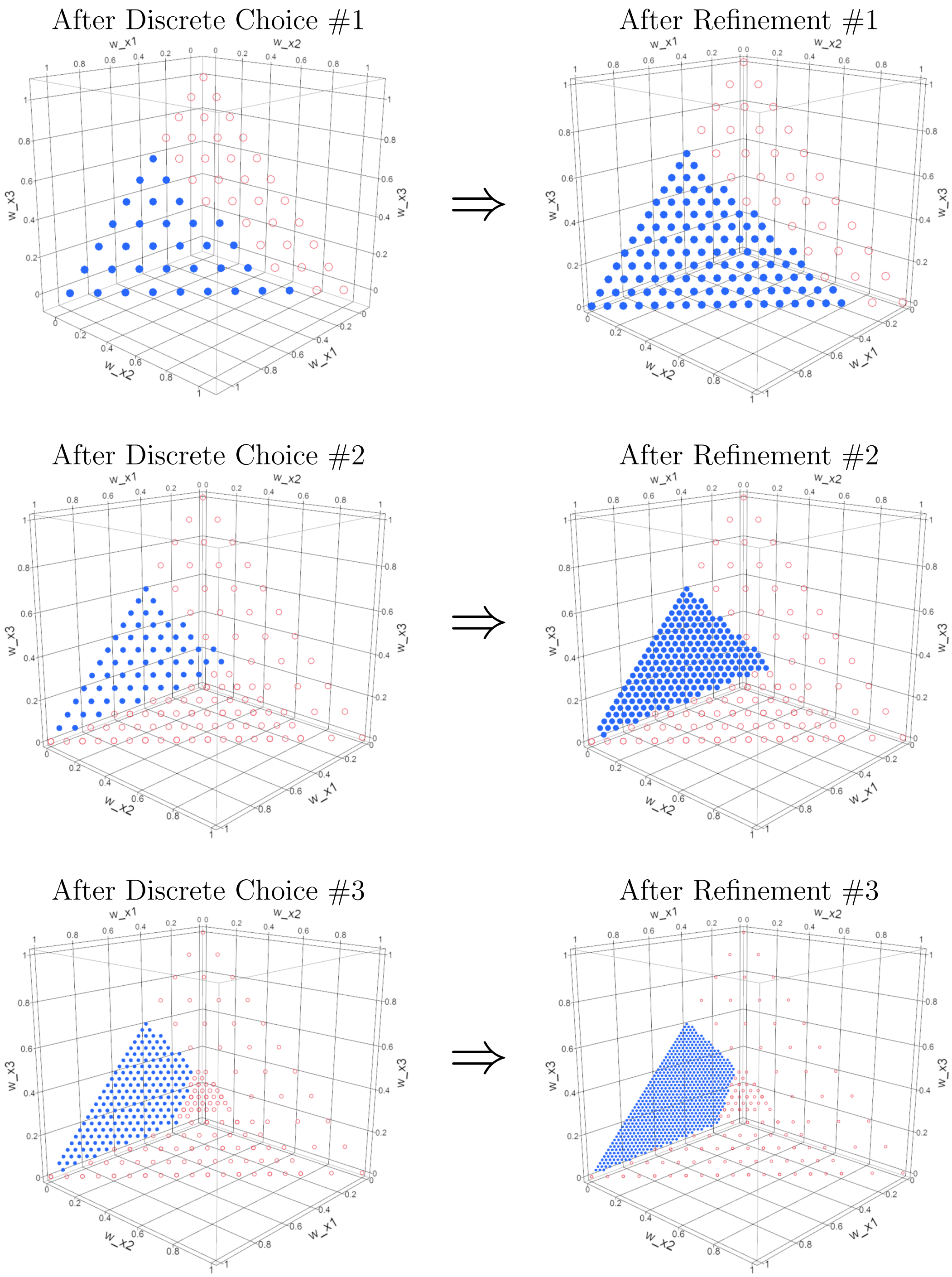


Figure 74: Three discrete choices each followed by a refinement step with $c = 0.1$, 0.05 and 0.025 respectively from top to bottom. (Marker size is reduced after each step to show detail.)

Expectedly, the total number of weighting vectors increase at each refinement step while the total *possible* weighting vectors can fluctuate based upon when, in a series of discrete choices, a refinement step is executed. This is shown in Figure 75 where, for the same set of discrete choices, the first five iterations are followed by a refinement step as indicated by the red line jumping up to a larger number of weighting vectors. The blue line represents the number of possible weighting vectors after the refinement process, which is in agreement with the previous discrete choices. The difference between these lines directly after the refinement step (i.e. the "vertical" portions) is equal to the number of weighting vectors before and after the refinement step which can appears differently on the logarithmic y-scale figure. For example, before refinement step #5: the number of possible w is 724 while the total number of w is 2052 (a difference of 1328). After refinement step #5: the number of possible w is 7195 and the total number of w is 8523 (the difference is again 1328). This confirms what the refinement algorithm is design to accomplish by only adding valid weighting vectors, which was also visible before in Figure 74.

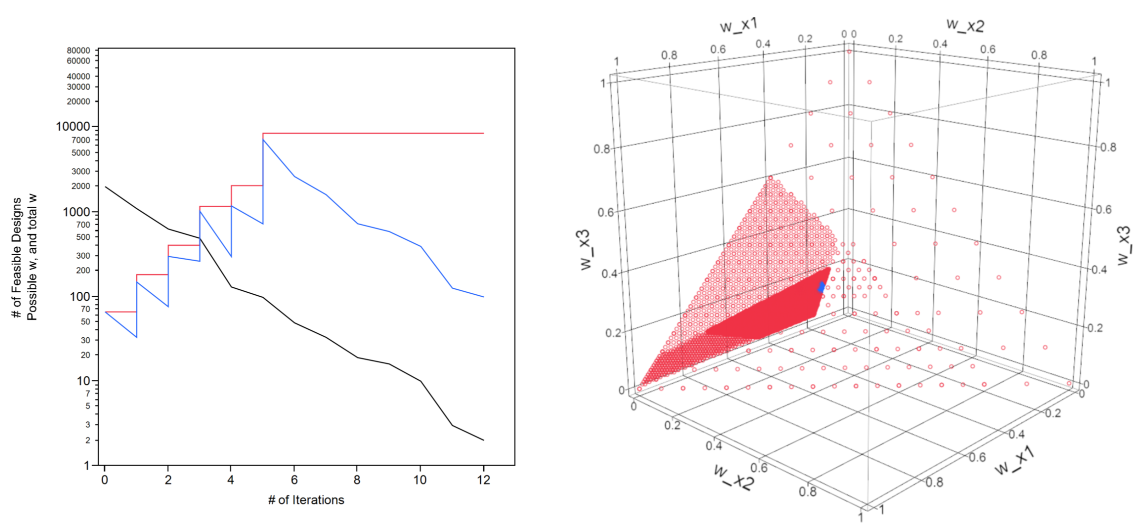


Figure 75: Number of iterations versus number of total weighting vectors, possible weighting vectors and remaining feasible designs with associated scatterplot representation

Also, in this example, after 5 refinement steps, the increment c has decreased initially from 0.1 to 0.003125 ($0.1(\frac{1}{2})^{i_r}$, where i_r is the number of refinement steps. Since the refinement algorithm takes into consideration the ranges of the acceptable weighting vectors in each dimension and with increasingly smaller values of c, the number of added possible weighting vectors could increase exponentially as well. This is also evident by the general positive slope of the "stair case" line of the total number of weighting vectors on a logarithmic y-axis.

Finally, with just over 8500 weighting vectors, only 99 remain possible or valid in a very small range for w_{x_1}, w_{x_2}, and w_{x_3} as shown by the very small blue region in the same figure. (Figure 75.)

The equivalent histogram for each iteration is shown in Figure 76 from top left to bottom right.

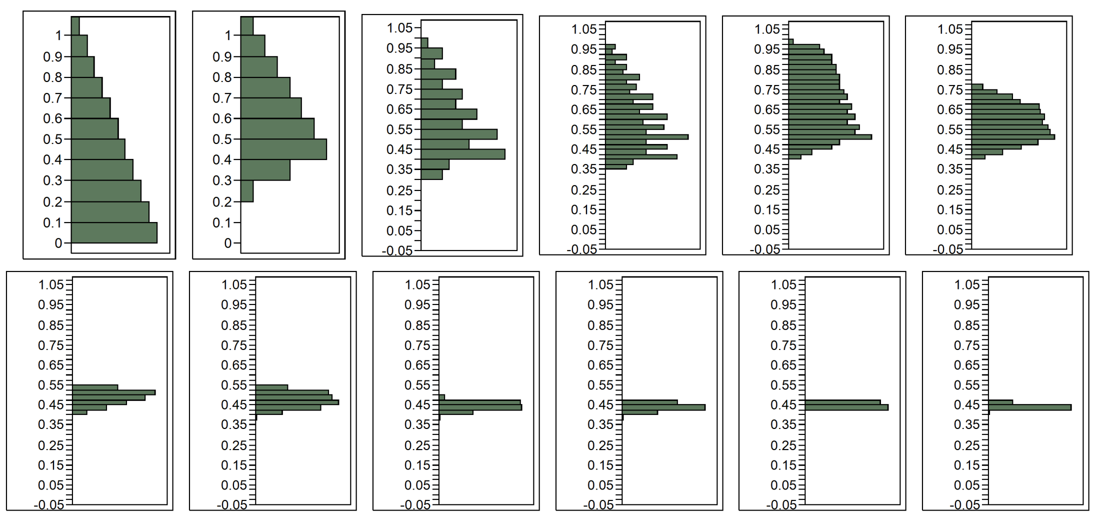


Figure 76: Histograms of possible weighting vectors (only for w_{x_1}) after each of 10 discrete choices with refinement steps between the first five in the series

Since multiple histograms are difficult to interpret for each iteration, the equivalent data is presented on the left side in Figure 77 but with all dimensions overlaid for the same iteration history.

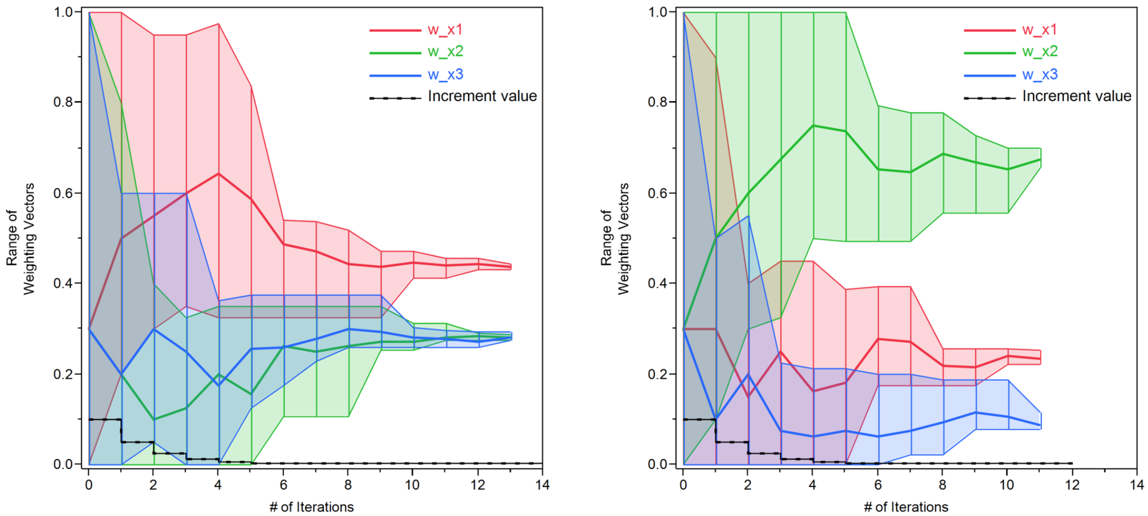


Figure 77: Number of iterations versus the overlapping range of possible weighting vectors

The thick colored solid lines in Figure 77 indicate the median value of the weighting vector for that particular dimension. In this example, each dimension started at a median value of 0.29 when $c = 0.1$, or, in other words, the centroid of the histogram from the full set of initial weighting vectors lies at a value of 0.29. This value will increase or decrease per iteration based upon the results of the previous discrete choice experiments and the weighting vectors that will be eventually removed from the feasible set.

Each dimension also shows the equivalent minimum and maximum value for each iteration. Therefore, the three points provide some statistical detail as to the shape of the histogram without recording it at each iteration for each dimension. The solid black line at the bottom indicates the increment size at each iteration. This values starts at $c = 0.1$ and is halved at each refinement step. On the right hand side of Figure 77, refinement steps are similarly applied after each of the first five discrete choices (with evidently different preferences responded to the discrete choices, such that the x_2 is most important followed by x_1 and then x_3). Interestingly, even after five discrete choices the potential for the x_2 objective to be the only objective remains possible (i.e. $w_{x_2} = 1$, when $i = 1...5$).

6.1.8 Analysis of Stopping Criteria on Preference Extraction

Another key difference between the two simulations in Figure 77 is the total number of iterations before the algorithm stopped based on the designs currently remaining and the random selection of those remaining designs. Referring back to Figure 70 (where on average 11 or more iterations are required to reduce the feasible design set to two or less), in the right hand side of Figure 77, the simulation required exactly 11 for the algorithm to conclude while the left hand side required 13. Since the number of feasible designs is not influenced by the increment value, this behavior is expected regardless of the number of refinement steps and at point in the series these steps are applied. The matrix of simulations in Figure 78 show the variability in the number of iterations needed for convergence of the weighting ranges and when, in terms of iterations, refinement steps are executed.

In the plots of Figure 78, the algorithm continued until no more feasible designs were available for discrete choices. However, the stopping criterion does not necessarily need to be the number of feasible designs remaining. In fact, since the decision maker's preference space is the real goal that this step seeks to define in the overall methodology, a more useful criterion could be a threshold on the ranges of the remaining weighting vectors (e.g. when each range per dimension for W is less than 0.1), or when a ranked order of the dimensions is evident (i.e. no overlap between ranges of dimensions of W), or, of course, simply setting the total number of discrete choices (i.e. total number of iterations) at a predetermined value which the decision maker is willing to consider. For example, if the decision maker has time to only answer 5 discrete choice experiments then this clearly would be the limiting factor in reducing the ranges of W.

A research question at this point is posed regarding the stopping criterion for this

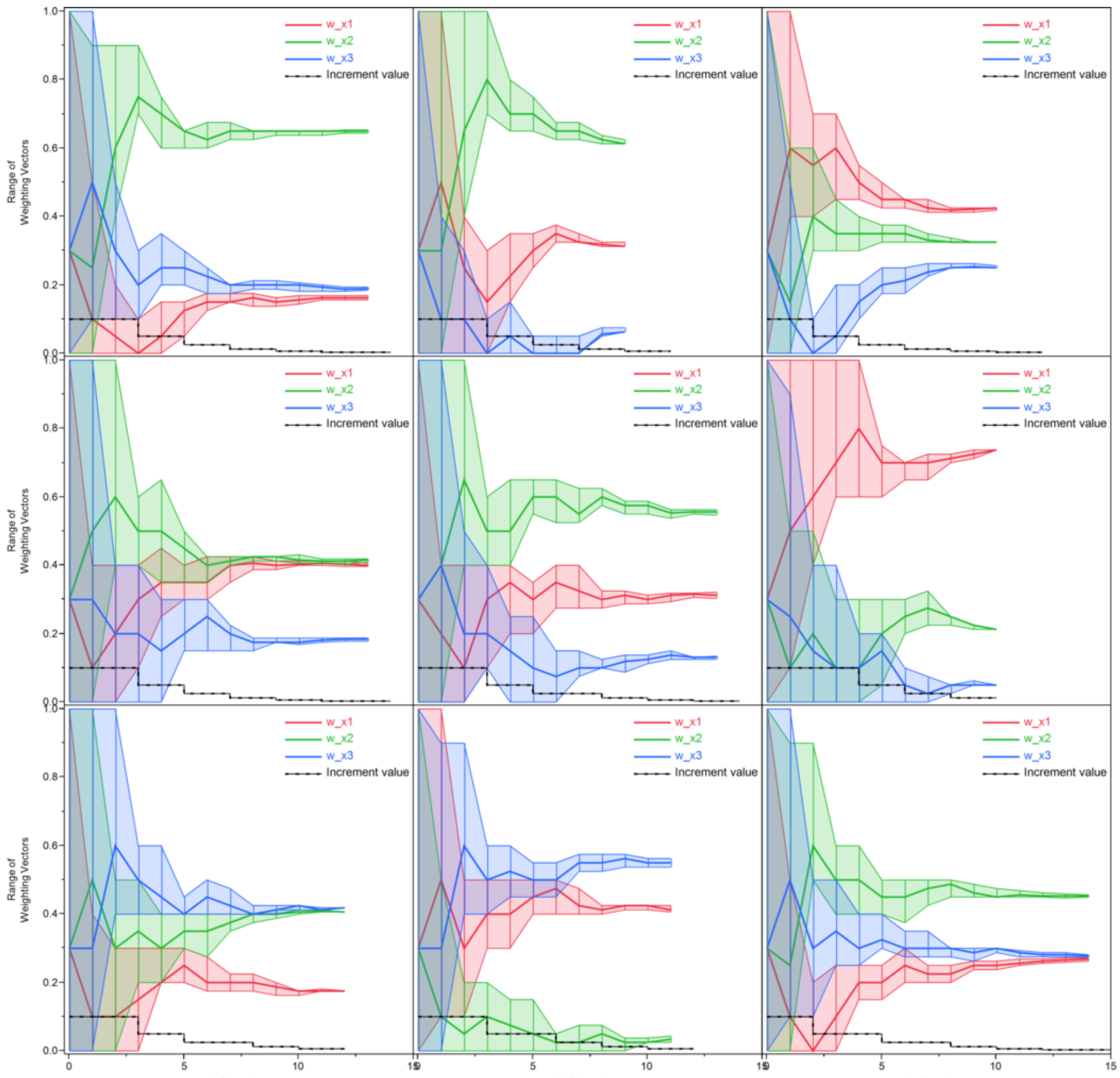


Figure 78: Matrix of 9 sample simulations of Step 1

step in the methodology.

Research Question: *What stopping criterion for discrete choice experiments should be used to extract the preferences of a decision maker?*

As mentioned, the answer for this is of course dependent on the number of discrete choices a decision maker is willing to consider. However, if a decision maker's "resources" are effectively unlimited, or in essence, the comparisons are relatively inexpensive and any number can be performed (within reason), the two other aforementioned possibilities can be subject to comparison (i.e. W ranges are less than x, or dimension order is identified) in experiments.

Simulating the above discrete choice experiments 1000 times, with the same 2000 candidate designs for each of these two stopping criteria, creates the outputs displayed in Figure 79. The minimum number of iterations to obtain a determined ordered preference is shown in a histogram on the left. However, slightly more than 10% of the simulations never reached sufficient separation in terms of the ranges for W suggesting that for some preferences, especially if two or more objectives have equal or near equal importance values, the ranges will be expectedly coincident and thus no true order can be ascertained. Still, since some decision-making techniques use preference order as opposed to preference weightings (or importance values for the objectives), this may be a more useful and efficient way to stop the algorithm.

On the right of Figure 79, the heat map illustrates when the stopping criteria is set for different range values, showing that the number of iterations required to reach increased levels of certainty in the ranges must likewise increase. The mean lines for the range of w_{x_1}, w_{x_2} and w_{x_3} are almost over top of one another indicating that each of the three dimensions follow similar behavior.

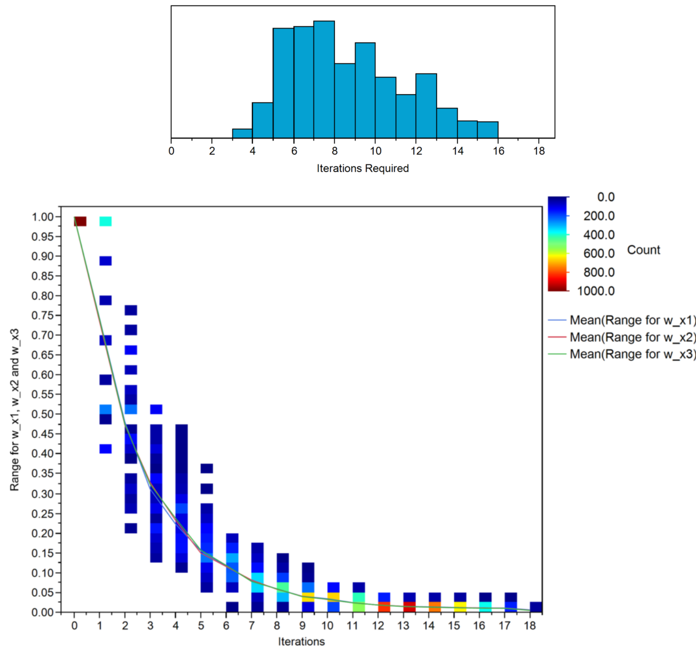


Figure 79: Number of required iterations for, 1) the preference order to be completely specified (top), and, 2) certain range values for w_{xj} (bottom)

A related topic is when and how the refinement steps should be applied. Refinement steps come at a cost computationally, but are essential to differentiate preferences when importance values for two or more objectives are equal or, at least, close in value (e.g. the left simulation example of Figure 77). In the previous simulations, a refinement step was executed when the number of possible weighting vectors was less than the arbitrarily set value of 100 weighting vectors (a number somewhat conservative for only 3 dimensions). This allows a trade between setting the increment c too small early in the simulation and the computation expense required for smaller values of c.

Further, in the above simulations, to guarantee that a weighting vector is always possible, the discrete choice experiments are selected based on halving the preference space in each iteration for the largest range of the n dimensions. Thus, if the range for w_{x_1} is currently from 0.3 to 0.6, a discrete choice will be given to make the range from 0.3 to 0.45 or from 0.45 to 0.6 after the decision maker's response. The range of the other dimensions will also likely be reduced but not to such an extent. If, after the discrete choice, one of the other ranges are now larger than 0.15, then the next discrete choice experiment would focus on that dimension to "halve."

6.1.9 Visualizing N-decision makers' preferences

Since the overall objective is to facilitate and accelerate group decision making, the desire is to have each decision maker, in parallel, perform a similar set of responses to a series of discrete choice experiments. This enables the rapid extraction of the individual preferences and allows for the execution of future steps (i.e. Steps 2 and 3) in the methodology which will use this information for power relationships and reaching consensus.

Since each decision maker will in general face a different series of discrete choices revealing their preferences, a useful graphic indicated which areas of the design space are more readily accepted or rejected is presented in Figure 80.

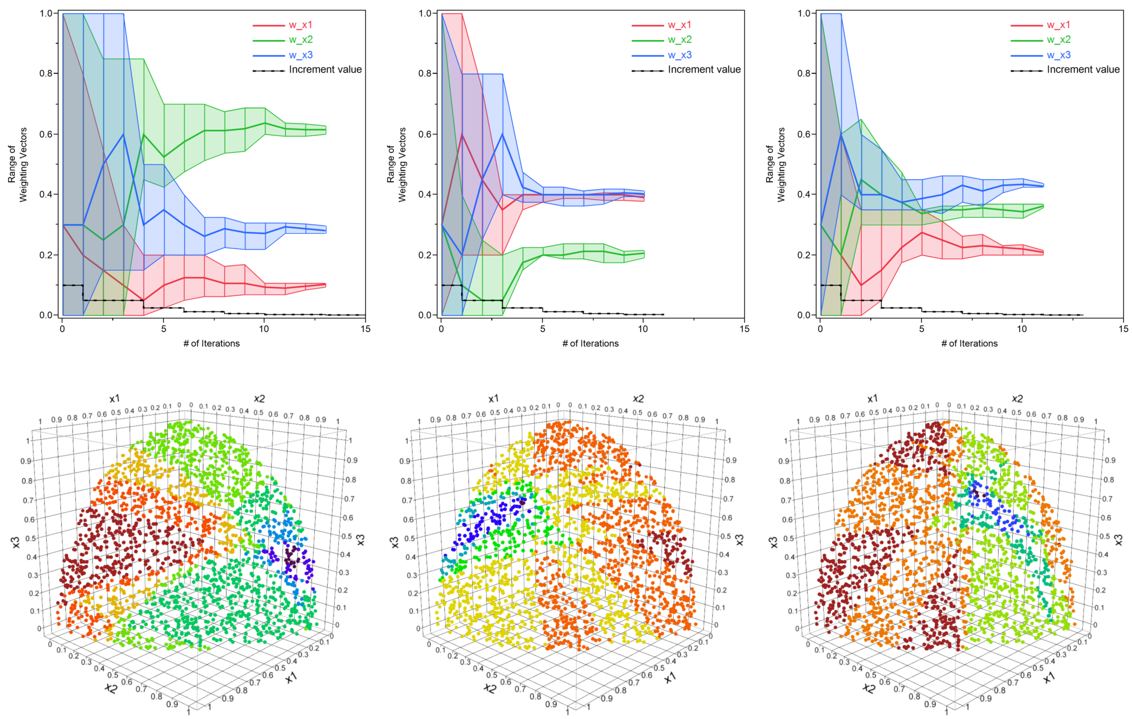


Figure 80: Three decision makers' revealed preferences and associated acceptable or rejected designs

The top row of plots in Figure 80 shows the histogram ranges and median weighting vector for each iteration. Decision Maker #2 responded to 10 while #3 responded to 13 discrete choices respectively. The bottom row illustrates the regions where designs would have been rejected more often based upon the responses of each decision maker. Therefore, the red regions or patches are designs rejected in a large majority of the discrete choices (i.e. 8 or more of the discrete choices), while the blue/purple patches suggest more acceptable or feasible designs according to the given preferences. (The dark purple markers indicate regions that have never been rejected in all discrete choices). The lines of demarcation are clearly visible for each additional discrete choice that removed a particular design from candidacy.

Of course, for consensus reaching in group decisions, investigating the design regions where the group has universally rejected designs can be useful. In Figure 81, the preference space applied to the design space is combined to identify design regions which together the group considered feasible or infeasible. These rejected design regions are again shown in red. In this figure, no decision maker was weighted more heavily than another. However, it is interesting to note that the random selection of feasible designs in the discrete choices result in a possible region of feasibility near the preferred regions of decision makers #1 and #3.

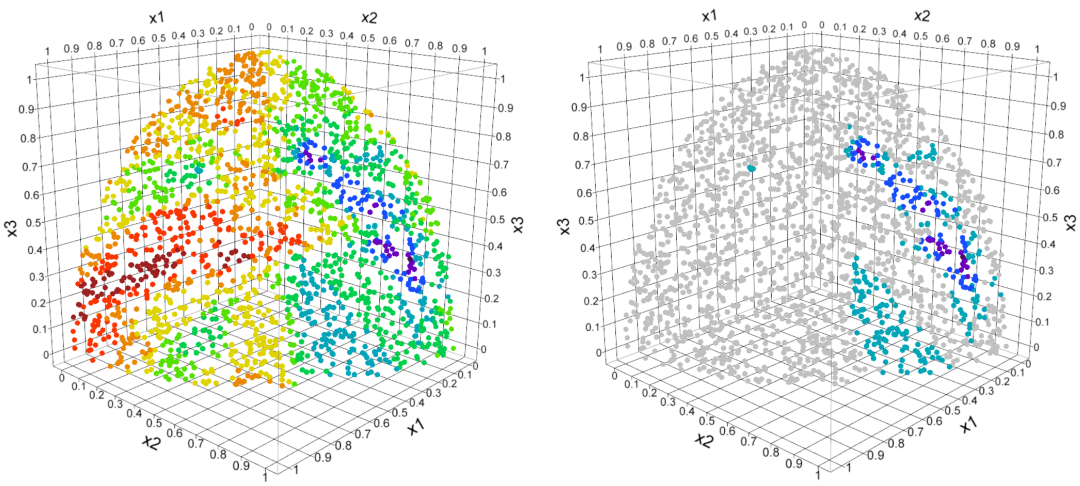


Figure 81: Combination (simple addition) of the three patch plots from Figure 80 and the removing (gray areas) of regions with greater than 10 "rejections" across all discrete choices for all three decision makers

Furthermore, the "path" each decision maker takes in revealing his or her preferences can be depicted using the median weighting vectors calculated at each iteration for each dimension. In Figure 82, this vector for each decision maker at each iteration is mapped to the design space and connected to the series of "optimal" designs for each median value of W.

The first decision maker is in blue, followed by the second in green and the third in red (contrast this to the previous two figures). The final "optimal" design, after the last discrete choice, is indicated with a large circular marker respectively. The intermediate designs are labeled as diamonds and the initial design in black is expectedly the “median” design near the point [0.577, 0.577, 0.577], which is initially the same for each decision maker.

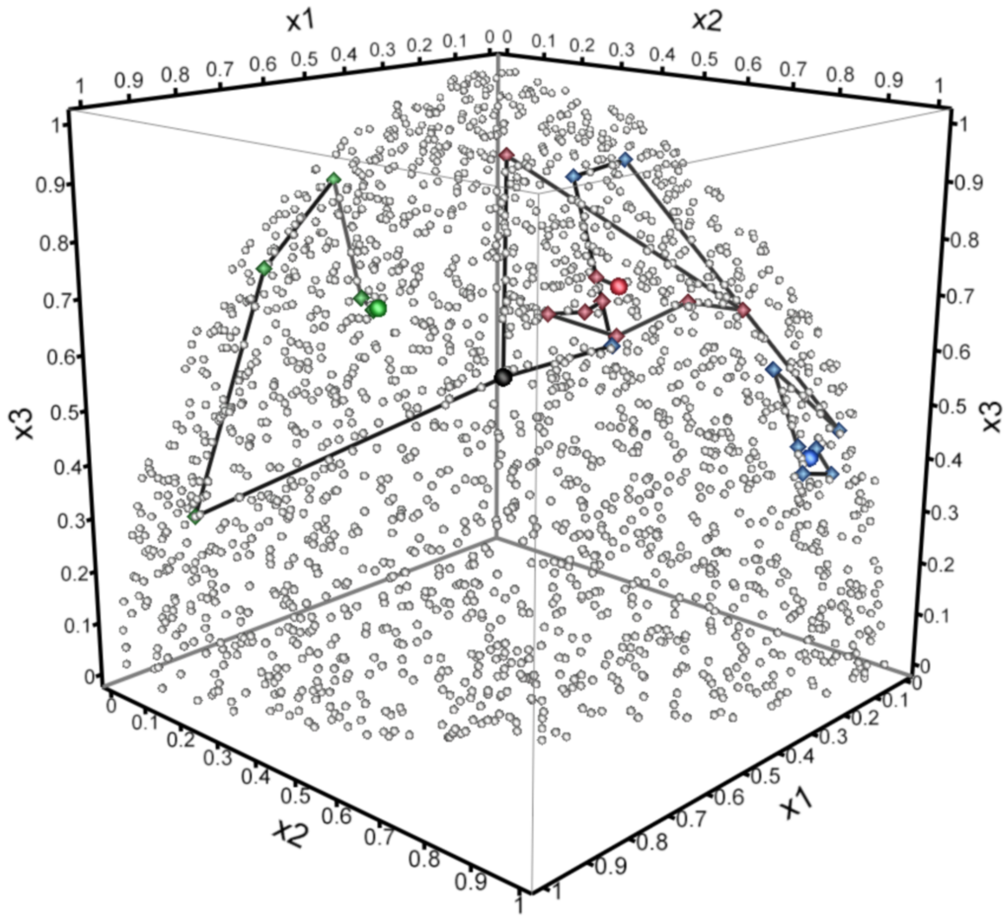
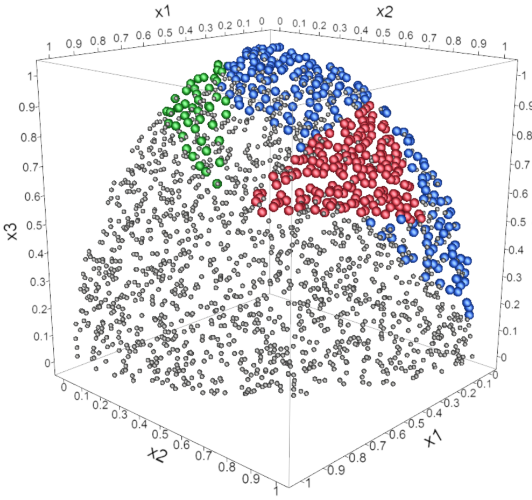
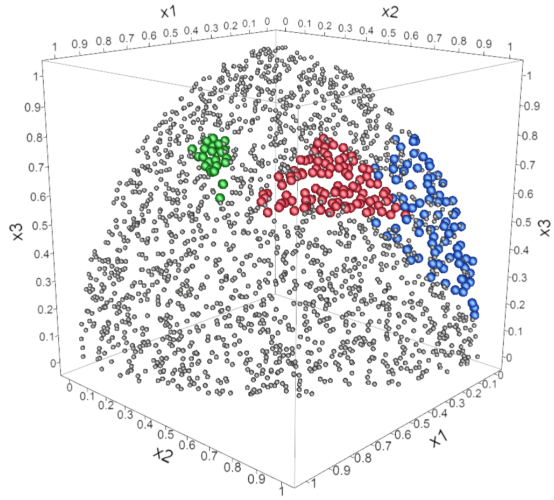


Figure 82: The optimal design for the median weighting vector for each decision maker after each iteration

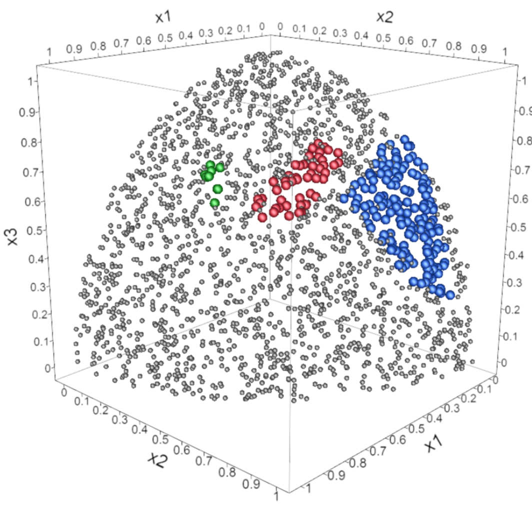
Finally, the set of “optimal” or best designs can be calculated from all the remaining possible weighting vectors at a particular iteration and projected onto a Pareto frontier to compare closeness and potential overlap of designs deemed acceptable by the revealed preferences for each decision maker.



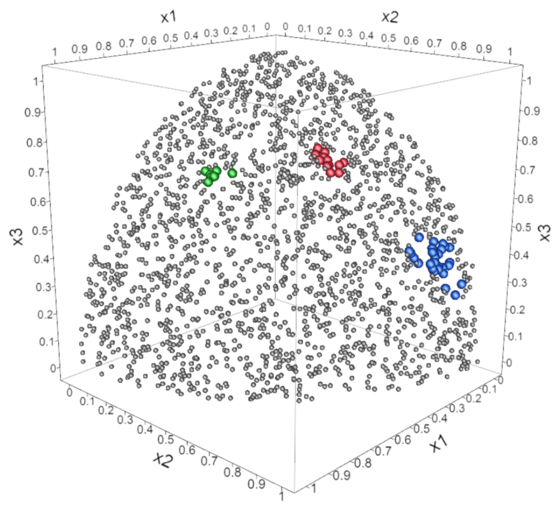
Acceptable designs after 4 iterations



Acceptable designs after 5 iterations



Acceptable designs after 6 iterations



Acceptable designs after 10 iterations

Figure 83: Three decision makers' revealed preferences and associated acceptable designs after i = 4, 5, 6, and 10.

In general, this process will hold for more than three dimensions. Figure 84 illustrates the overlap in each dimension for the three decision makers (red, blue and green) across the scatterplot showing a 4-dimensional design space. Each one of the points is Pareto optimal and colored points indicate a design which is potentially the global optimal design for that particular decision maker based on the responses of the discrete choice experiments.

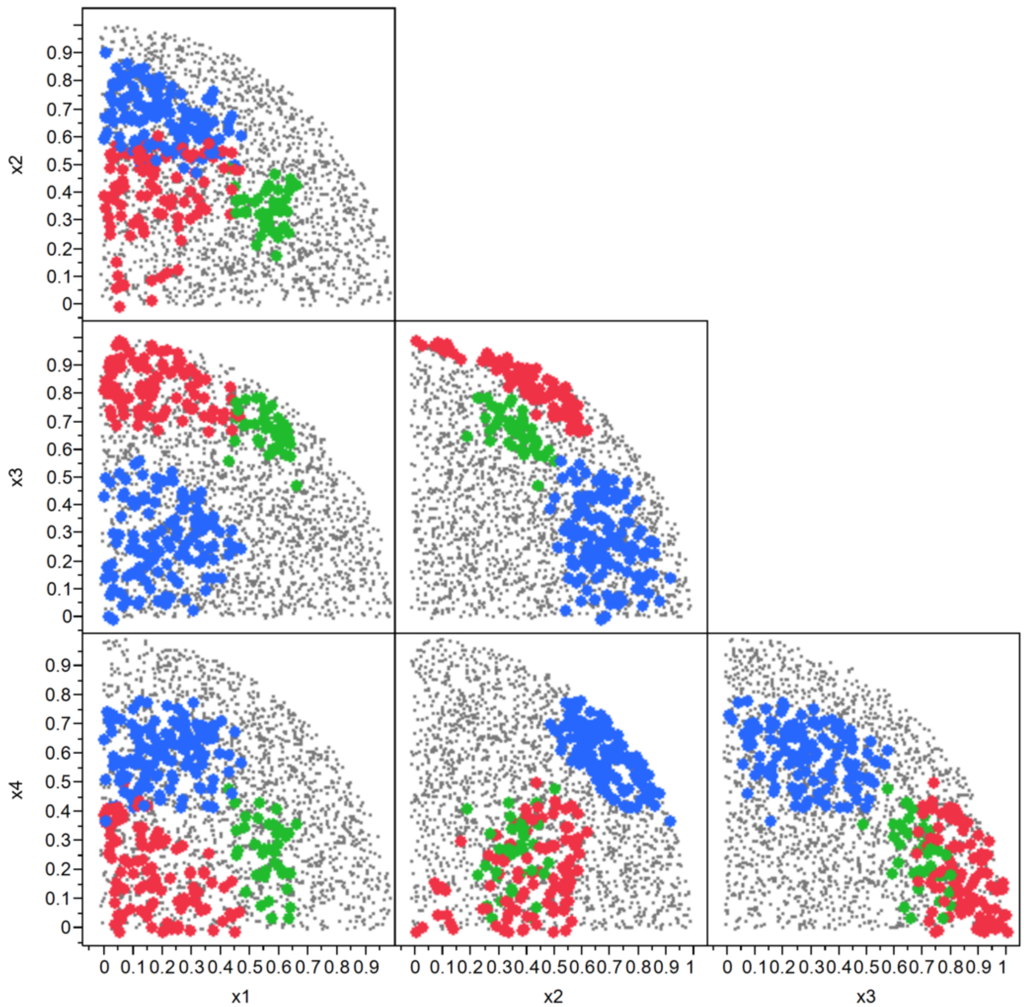


Figure 84: A four dimensional representation of possible candidate designs for three decision makers (red, green and blue) after seven discrete choice experiments.

6.2 Step 2: Extracting Power Relationships

Once the set of weighting vectors or weighting distributions are generally determined from the previous step for each decision maker, a process to extract the power relationships or amount of influence between decision makers is necessary.

The following discussion will make use of the identical set of candidate designs and the weighting vectors from Step 1 which represent the possible true preferences for each decision maker as illustrated in figures 80 and 83 in the previous section.

Also, to facilitate discussion, the first decision maker will be designated as DM_A (or A), and similarly, the second and third decision makers as DM_B and DM_C (or B and C) respectively. Table 8 summarizes the preferences of all three decision makers with the mean weighting value for each of the three dimensions after 10 discrete choices.

Table 8: Mean Values of Weighting Vectors for Each Decision Maker

Decision Maker	$\bar{w}_{j1}$	$\bar{w}_{j2}$	$\bar{w}_{j3}$
A	0.092	0.639	0.268
B	0.391	0.205	0.404
C	0.219	0.345	0.436

Decision Maker A has a much larger preference for x_2, while DM_B prefers x_1 and x_3 almost equally and DM_C prefers x_3 slightly more than x_2.

6.2.1 Power Assumptions and Constraints

For each of the three relationships among the three combinations (i.e. $A \leftrightarrow B$, $A \leftrightarrow C$, and $B \leftrightarrow C$), the total power in each relationship is shared between the two decision makers. If, for example, DM_A has full control or influence on DM_B's decisions, the power that A has over B is set at 1 or 100% (i.e. $P_{A \to B} = 1$). This, of course, would require that the power B has over A is zero, or $P_{B \to A} = 0$, If the total power is shared equally between A and B then $P_{A \to B} = P_{B \to A} = 0.5$, suggesting that A cannot impose their preferences onto B any more than B can onto A. The share of power between two decision makers can thus take on any split or division of 1 (or 100%).

This results in a set of three power constraints equations:

$$
\begin{aligned}
P_{A\to B} + P_{B\to A} &= 1 \\
P_{A\to C} + P_{C\to A} &= 1 \\
P_{B\to C} + P_{C\to B} &= 1
\end{aligned}
\tag{9}
$$

Knowledge about these 6 unknowns could then be used to calculate the power of any one decision maker over the entire group in aggregate.

Assuming that there is a finite amount of power for the entire group normalized to 1 and shared across all decision makers (potentially asymmetrically), results in the simple equation:

$$P_A + P_B + P_C = 1 \tag{10}$$

where P_A is the power held by DM_A, or more generally,

$$\sum_{i=1}^{k} P_i = 1 \tag{11}$$

where k is the total number of decision makers in the group.

The total power DM_A holds for the group can be defined as:

$$P_A = \frac{P_{A\to B} + P_{A\to C}}{3} \tag{12}$$

and more generally (for three decision makers),

$$P_i = \frac{\sum_{j=1}^{k} P_{i\to j}}{k}, i \neq j, k = 3 \tag{13}$$

which, if completely defined, would satisfy Step 2 in the overall methodology.

However, with only three equations available for the six unknowns, additional information must be acquired to solve this system of equations. Research Question

#4 and its associated hypothesis, repeated below, form the appropriate inquiry at this point in the analysis.

Research Question #4: *How can the influence relationships between decision makers be identified and quantified?*

Hypothesis #4: *Discrete choice experiments between designs, and with whom, an agent will form a coalition in the decision space will identify relationships of influence between decision makers, under the power constraints equations between decision makers.*

6.2.2 Power Information from one Discrete Choice Experiment

The hypothesis in response to Research Question #4 proposed that discrete choice experiments can recover the influence of one decision maker over another. These discrete choices must be sufficiently simple so as to not cognitively overload the decision maker and likewise take a small amount of time so that multiple discrete choices can be proposes to the same agent over a short time span.

The methodology presented in this research makes use of the preference information from the previous step, by establishing discrete choices between sub-optimal designs but with similar utility scores as established from the preference structures.

For example, starting with DM_A's preferences, which were summarized in Table 8, and calculating A's utility for each of the 2000 candidate designs using the mean weighting vector, $w_{mean} = [0.092, 0.639, 0.268]$, A's preferred region within the design space can be illustrated in the dark red region on the far right of Figure 85. This "mean weighting vector" was calculated after 10 discrete choices from Step 1 in the methodology.

The "best" design, after w_{mean} is applied, results in $d_{462} = [0.12, 0.920, 0.372]$ being selected with a total utility score of 0.6995 and indicated in black on Figure 85. Not surprisingly, since DM_A prefers x_2 to the other two dimensions a design with a

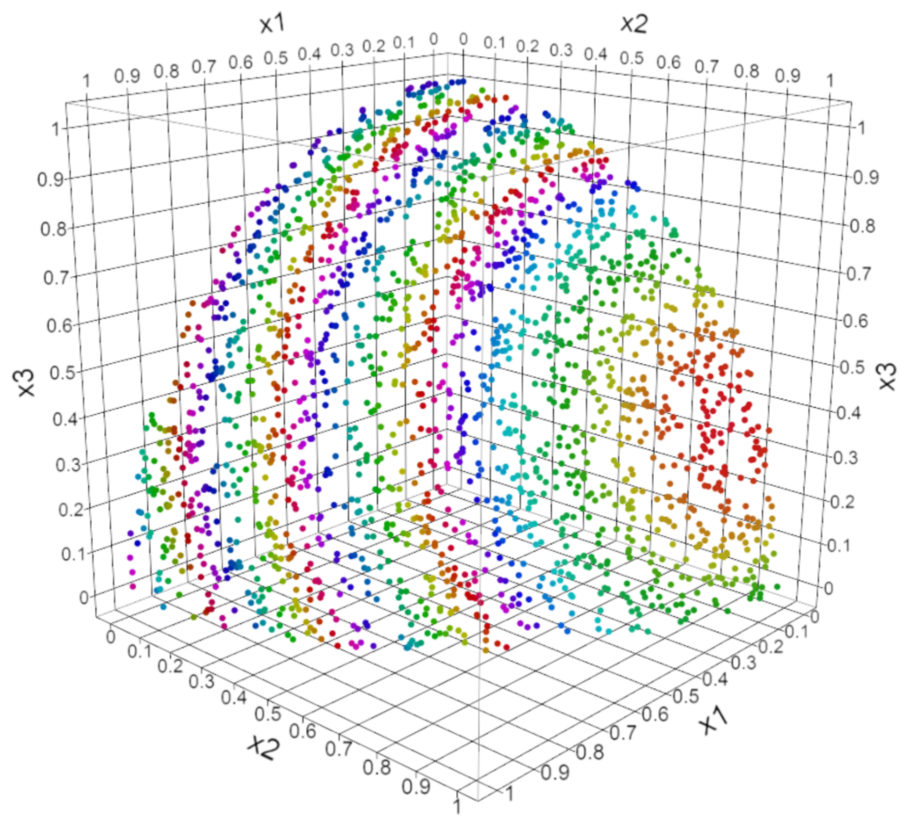


Figure 85: Candidate designs colored by utility value from DM_A's mean values of possible weighting vectors

very large score of x_2 is selected. Although there exists multiple weighting vectors which could represent DM_A's true preferences, the mean vector will serve currently as a "typical" weighting vector even though it may not even exists in the actual W matrix. Furthermore, the mean weighting vector would be clearly be different if $i \neq 10$, as it does in this example.

Each one of the apparent color bands in Figure 85 represents similar values or utility scores, informally described as "utility bands" here. The utility scores are in general continuous from the highest to the lowest utility score, and fall in the interval [0.1097, 0.6995]. A discrete choice between two designs within the same utility band for a particular decision maker (in this case DM_A) could be expectedly difficult to select the preferred design since the utility scores are so similar or close to the same value. That is, d_{954} (with a utility of 0.6107) and d_{1592} (with a utility of 0.6105)

If you had to form a coalition at one of the following two designs, whose design do you prefer?

Decision Maker B	Decision Maker C
$x_1 = 0.563$	$x_1 = 0.390$
$x_2 = 0.672$	$x_2 = 0.609$
$x_3 = 0.481$	$x_3 = 0.691$

Figure 86: Example #1 of a Discrete Choice Presented to Decision Maker A

could potentially achieve an indifference response from DM_A in a discrete choice experiment, since for these designs U(d_{954}) $\approx$ U(d_{1592}). A table of these two designs is found below:

Table 9: Scores for Two Designs in the Same Utility Band from DM_A's Perspective

Design ID	$U(d_?)$	x_1	x_2	x_3	w_{j3}	w_{j3}	w_{j3}
954	0.6107	0.5630	0.6719	0.4812	0.092	0.639	0.268
1592	0.6105	0.3896	0.6085	0.6913	0.092	0.639	0.268

Since there remains uncertainty in the true preference, (i.e. DM_A mathematically only slightly prefers d_{954} with the mean weighting vector as indicated over d_{1592}), the other design could easily be preferred with a different weighting vector other than the mean vector, which has not been invalidated from previous discrete choices in Step 1. Therefore, with the assumption that these two designs are approximately equal in utility from A's perspective, comparing them side by side should elicit an indifference preference response. This indifference can be leveraged to identify a hypothetical difference attached to each design. Thus, if DM_B is associated with d_{954} and DM_C is associated with d_{1592}, a discrete choice can be asked as to which design or, more precisely, with whom DM_A is more likely to join and form a coalition. One potential layout of the discrete choice presented to DM_A is shown in Figure 86.

Since the utility values are approximately equal in the discrete choice presented,

DM_A is "forced" to choose between the decision makers, which, from the utility based perspective, is the only real difference between these designs. In making this selection, a decision maker is likely to choose the design, that, in their perspective, will still somehow benefit them the most. For example, DM_A may chose to team up with DM_C as they may think DM_C has greater influence over others and will likely be able to maintain their position, increasing the probability that DM_A will also obtain at relatively high utility. Joining DM_B may be less certain in their ability to maintain the utility score of 0.6107. Also, there may be a historical precedent for working with DM_C based on past agreements, contracts or performance suggesting to DM_A that it is in their best interest to unite with the decision maker with the "better" record. Furthermore, a personal or private reason such as a desire to impress someone else, a need to avoid confrontation, or a penchant for risky actions can also exist. There are clearly a variety of reasons why one decision maker would prefer to work with, team up with, or form a coalition with someone else. The exact reason is not necessarily important, nor could it always be identified. Wagner even suggests that "[i]ndividuals may, in fact, be unaware of the exact values of these parameters [of power or influence]" [183]. However, recognizing the *existence* of such reasons, and considering the influence over another as an expression of those reasons, can be useful for improving group decision making.

One can make the assumption that if DM_C is preferred from the foregoing discussion then potentially:

$$P_{B \to A} \leq P_{C \to A}. \tag{14}$$

Of course, outside of the above causes or reasons, the decision maker is free to choose based on self-interest and their own preferences on just the objectives, effectively ignoring the associated decision makers. In this case, the two designs presented would essentially be an additional discrete choice of the type from Step 1.

To identify if this is the case, additional discrete choices are required to test out the strength or magnitude of DM_A desire to chose based on self-interest or based on the influence one of the other decision makers may have over DM_A.

For example, assume that DM_A chose DM_B's point but was too focused on the higher value of x_2 and made the decision based on the values of the design and not the decision maker. This could be tested by selecting two other designs with similar utility scores, but different values for the objectives, as shown in Figure 87 assuming DM_A did in fact choose DM_B in the first discrete choice.

Would you still form a coalition with Decision Maker B if only the following two designs were permitted?

Decision Maker B	Decision Maker C
$x_1 = 0.504$	$x_1 = 0.633$
$x_2 = 0.597$	$x_2 = 0.719$
$x_3 = 0.624$	$x_3 = 0.288$
$(U^B_{DM_A} = 0.5958)$	$(U^C_{DM_A} = 0.5953)$

Figure 87: Example #2 of a discrete choice presented to DM_A. Utility scores (inside parentheses for each design) would not be included or presented in an actual experiment.

In Figure 87 the utility scores are shown for each one of the designs in a smaller font and within parentheses, shown here for purposes of discussion but would not be shown in an actual discrete choice experiment.

If DM_A continues to select "DM_B's" design, then perhaps the evidence would suggest that DM_B has some influence over DM_A or at least some attractive or advantageous attribute with which to form a coalition. On the other hand, if DM_C is now chosen, the evidence could suggest that neither decision maker has relatively more influence or power over DM_A. Additional information from discrete choices would be needed.

6.2.3 Power Information from Multiple Discrete Choice Experiments

To further identify or quantify the potential existence of influence over DM_A, a third type of discrete choice is available where the utility scores are different.

Assuming you must choose to form a coalition at one of the two designs, with whom would you chose? Decision Maker A or Decision Maker B?

Decision Maker B	Decision Maker C
$x_1 = 0.703$	$x_1 = 0.605$
$x_2 = 0.635$	$x_2 = 0.756$
$x_3 = 0.320$	$x_3 = 0.248$
$(U^B_{DM_A} = 0.5568)$	$(U^C_{DM_A} = 0.6060)$

Figure 88: Example #3 of a discrete choice presented to DM_A. Utility scores (inside parentheses for each design) would not be included or presented in an actual experiment.

This type of discrete choice experiment, an example of which is shown in Figure 88, is useful in determining the strength of the influence that one of the decision makers may have over DM_A. If, in previous experiments, it is known that DM_A tends to form coalitions with DM_B and therefore likely has some sort of power or influence over DM_A, then a discrete choice can be presented where the utility value for "DM_B's design" is lower (0.5568) than that of "DM_C's design" (0.6060) some value (e.g. 0.05 or 0.6060-0.5568 = 0.0492 $\approx$ 0.05).

If DM_A still chooses to form a coalition with DM_B even with a lower utility value, then the assumption that the relationship between DM_B and DM_A, and more specifically that the benefit of agreeing with DM_B, must be at least equal to trading or giving up 0.05 of utility from DM_A's perspective. After another discrete choice, where DM_B had an even lower utility (e.g. 0.01) as compared to DM_C (i.e. $u^B_{DM_A} + 0.1 = u^C_{DM_A}$), but instead DM_A chooses DM_C's design, the amount of influence that B has over A (i.e. $P_{B \to A}$) is known to fall within 0.05 and 0.1 utils (units of utility). In equation form, this results in:

$$P_{B\to A} = P_{C\to A} + f(\Delta u_{DM_A}) \tag{15}$$

where,

$$f : \Delta U \to P \tag{16}$$

and,

$$0.05 \geq \Delta u_{DM_A} = u^B_{DM_A} - u^C_{DM_A} \leq 0.10, \tag{17}$$

for this example. The function f is a transformation of utility difference into an influence or power.

A valid function could be a proportional transformation across the entire utility space range. For example, the "best" point when the mean weighting vector is set at [0.092, 0.639, 0.268] is d_{462} with utility score, $u^{462}_{DM_A} = 0.6995$, while the "worst" is d_{889} with $u^{889}_{DM_A} = 0.1097$ (assuming that $w = w_{mean}$ discussed above). The entire utility range for DM_A would then be $u^{range}_{DM_A} = u^{462}_{DM_A} - u^{889}_{DM_A} = 0.6995 - 0.1097 = 0.5898$. If a discrete choice was presented to DM_A, with DM_B's design at the "worst" point, and DM_C at the "best" point, and DM_A still chose DM_B as the decision maker to form a coalition with, then B would clearly have 100% of the influence over A (i.e. $P_{B\to A} = 1$) and conversely A would have no power over B (i.e. $P_{A\to B} = 0$). Using this proportional ratio, results in a transformation of:

$$f(\Delta u_{DM_A}) = \frac{\Delta u_{DM_A}}{u^{range}_{DM_A}} = \Delta P^{B\sim C}_{DM_A}. \tag{18}$$

where $\Delta P^{B\sim C}_{DM_A}$ is the difference in power or influence and is positive since Δu_{DM_A} is always positive (or can be made positive by reversing DM_B and DM_C if needed). This ΔP is only the difference in power of DM_B and DM_C over A. For example, if

$\Delta P_{DM_A}^{B\sim C} = 0.2$ and $P_{B\to A} = 0.45$ then, $P_{C\to A} = 0.25$. Similarly, if $P_{B\to A} = 0.95$, then $P_{C\to A} = 0.75$, or in general,

$$P_{B\to A} - P_{C\to A} = \Delta P_{DM_A}^{B\sim C}. \tag{19}$$

Since the value of Δu_{DM_A} is actually only known to within a range, $0.05 \leq \Delta u_{DM_A} \leq 0.10$, the power difference can similarly only be known to within a range, $0.085 \leq \Delta P_{DM_A}^{B\sim C} \leq 0.17$, unless additional discrete choices are executed. These discrete choices can identify to a higher precision the values of Δu_{DM_A} and thus $\Delta P_{DM_A}^{B\sim C}$. The following table show a series of discrete choices and the bounded range of ΔP as a result of the difference in utility of the two designs and the selected decision maker with whom DM_A forms a coalition.

Table 10: Series of Discrete Choices Between DM_B and DM_C with Utility Values and Known Range of ΔP

Iter. #	Design # for DM_B	Design# for DM_C	$u_{DM_A}^B$	$u_{DM_A}^C$	Selected DM	Δu_{DM_A}	$\Delta P_{DM_A}^{B\sim C}$
1	1292	1481	0.625	0.625	B	0	$0 \leq \cdot \leq 1$
2	442	217	0.69	0.69	B	0	$0 \leq \cdot \leq 1$
3	892	672	0.468	0.568	B	0.1	$0.17 \leq \cdot \leq 1$
4	1055	1575	0.485	0.685	C	0.2	$0.17 \leq \cdot \leq 0.34$
5	10	1911	0.421	0.571	C	0.15	$0.17 \leq \cdot \leq 0.25$
6	1814	1267	0.527	0.652	B	0.125	$0.21 \leq \cdot \leq 0.25$
7	55	48	0.442	0.579	C	0.1375	$0.21 \leq \cdot \leq 0.23$

After the first two discrete choices (#1 and #2 in Table 10), the only information is that DM_A prefers DM_B when the utility scores are identical which may suggest that DM_B has influence over A's decisions, and that DM_B has some ΔP greater than zero relative to DM_C over A. To confirm this, the next discrete choice (#3) offers a considerably lower utility value for B's design. In this case, DM_A again chose B over C. This means that $\Delta P_{DM_A}^{B\sim C}$ is at least 0.17 and could be greater. The next discrete choice (#4 in Table 10) tests this possibility, but the difference is too extreme and A decides to select DM_C's design. This response places an upper bound on the influence of B onto A and thus the upper limit of $\Delta P_{DM_A}^{B\sim C}$ becomes 0.34.

Continuing this process by asking additional discrete choices with appropriately selected values of Δu_{DM_A}, each successive discrete choice will decrease the range of the possible difference in power or influence. For instance, after seven discrete choice, DM_A has revealed that B has more influence (over A) than C by about 0.21 to 0.23. The absolute power relationships are not known from this experiment alone but only the difference. However, this process provides an additional equation to the previous three, with all four reproduced below;

$$
\begin{aligned}
&P_{A\to B} + P_{B\to A} = 1 \\
&P_{A\to C} + P_{C\to A} = 1 \\
&P_{B\to C} + P_{C\to B} = 1 \\
&P_{B\to A} - P_{C\to A} = [0.21, 0.23]
\end{aligned} \tag{20}
$$

6.2.4 Identifying the Required Discrete Choices for Power Information

Since a number of discrete choices will be required to reduce the possible range on $P_{B\to A} - P_{C\to A}$, a related research question was posed to further explore this inquiry:

Research Question: *How many discrete choice experiments are needed to extract the power or influence difference between two decision makers?*

To answer this question, a number of experiments were conducted to explore potential answers.

For each series of discrete choices such as that presented in Table 10, a figure can be created from the range of possible Δu_{DM_A} over the number of iterations. Figure 89 illustrates the range of possible Δu_{DM_A} over 10 iterations of discrete choices on a logarithmic y-axis. The blue points and line indicate the upper bound on the maximum difference possible based on responses. This upper bound starts at 1 since in this formulation utility scores are normalized to a range of 0 to 1. This is, in fact, a wider range than necessary since the maximum utility range is only 0.5898, found by taking the difference between the largest and smallest utility score for all 2000 candidate designs. The green triangles and line indicate the lower bound on the utility difference after each successive discrete choice. The red circles indicate the utility score difference for the current discrete choice.

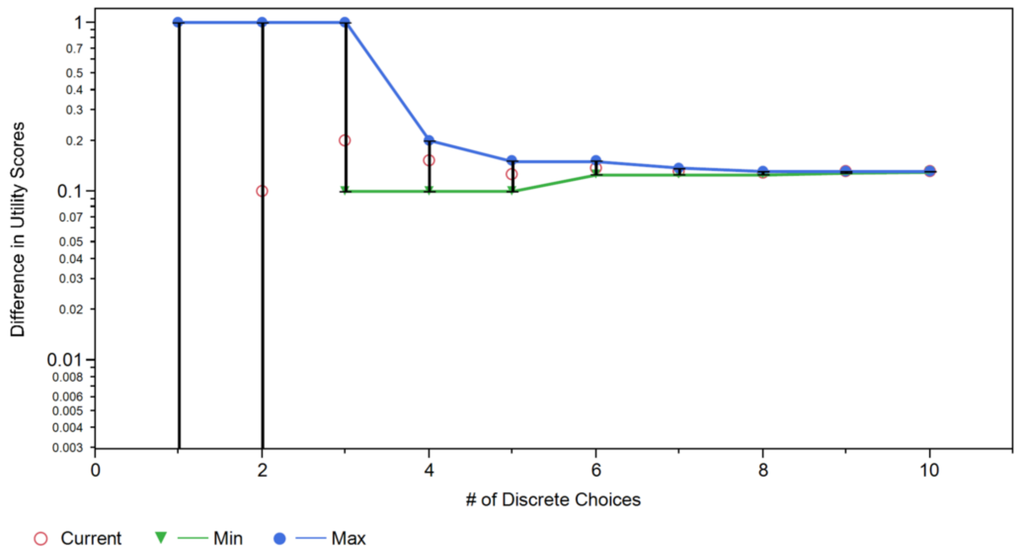


Figure 89: Range of utility difference versus number of discrete choices when Δu_{DM_A} = 0.13

The first discrete choice has a current difference in utility score of 0 (not shown on the logarithmically scaled y-axis) since the first discrete choice asks the decision maker to make a choice between two designs within the same utility band. However, for the second discrete choice, two designs with a utility difference of 0.1 are presented (shown by the first red circle at discrete choice #2 in Figure 89). The range at each successive discrete choice will contract based on the decision of each discrete choice. For example, after the decision maker chooses the design suggesting that the influence difference is larger than a utility difference of 0.1, the minimum value on the range takes on the previous current value (i.e. the green triangle for discrete choice #3). Similarly, the maximum value takes on the previous current value when the opposite design is selected (i.e. the blue line transition from discrete choice #3 to #4). Over multiple discrete choices the range will narrow in on the true influence difference (i.e.

utility difference) of the other two decision makers In this example and figure, Δu_{DM_A} is set to 0.13.

After 6 discrete choices the range in utility scores is quite small (i.e. $\max \Delta u_{DM_A} - \min \Delta u_{DM_A} \approx 0.01$). This range is likely sufficiently narrow, or even excessively narrow, especially in the view that the uncertainty in the mean weighting vector used for the above discrete choices in calculating the utility bands and representing the true weighting could be much larger than 0.01.

To further analyze this relationship, multiple simulations were conducted where the true utility difference was randomly selected between the values of 0 and 0.5898 (equivalent to an influence difference range of 0 to 1) and the starting difference for discrete choices was varied with discrete values 0.05, 0.1, 0.15 and 0.20. The simulations are shown in Figure 90 with the minimum and maximum for each first utility difference connected through the iterations of discrete choices, with a normal y-axis and logarithmic y-axis for the left and right figures respectively.

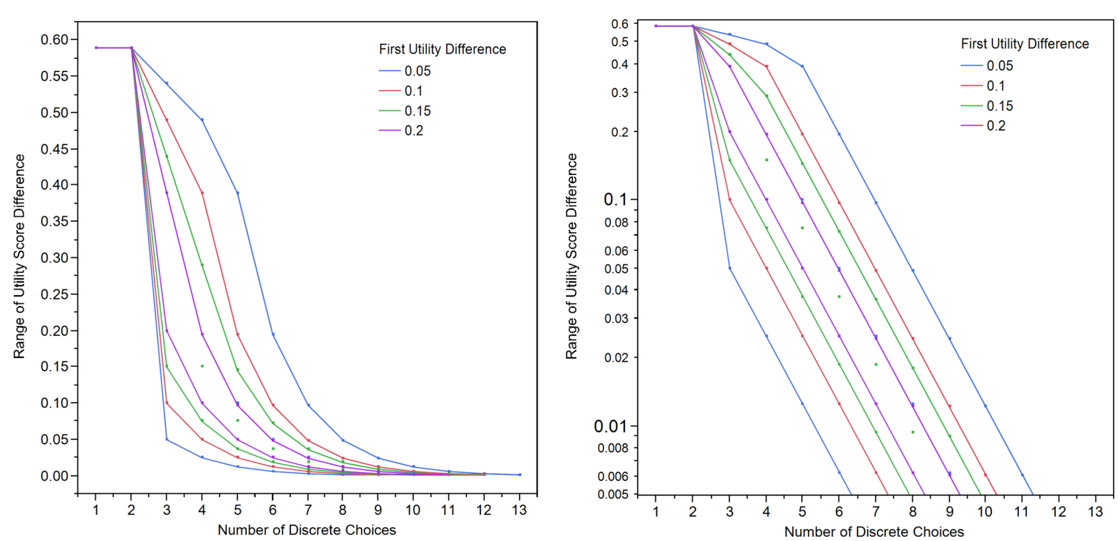


Figure 90: Multiple simulations of Step 2 showing the number of discrete choices to reach specified ranges on the utility score difference

When the first utility difference is set at 0.05, the range shrinks very quickly if the actual influence difference is very small (i.e. $\Delta u_{DM_A} \leq 0.05$). On the other hand, starting with an initial value of 0.05 when the actual influence difference is much larger results in many more discrete choices required to reach a certain threshold (i.e. 0.01). For the worst case, it requires 10 or more discrete choices, to reach a range of 0.01 and in the best case only 5. For the other values of the first utility difference, the best and worst cases are closer to each other (e.g. when $\Delta u_{DM_A} = 0.1$ best case is ≈ 6 and worst case is ≈ 9 discrete choices). In both these cases, the number of discrete choices approach the same value when the first utility difference is half the total range, and in this case, 0.5898/2, or ≈ 0.295. This means that each successive discrete choice can halve the previous range and thus a certain threshold on the range of Δu_{DM_A} can be defined by the number of discrete choices (when using this [particular strategy for defining the first utility difference) as:

$$\max \Delta u_{DM_A} - \min \Delta u_{DM_A} = u_{DM_A}^{range} (\frac{1}{2})^{n-n_e}. \tag{21}$$

where, n is the number of discrete choices, n_e is the number of discrete choices with identical utility scores, and $u_{DM_A}^{range}$ is the full range of utility values according to the mean weighting vector of DM_A. The "$n - n_e$" is to account for the first n_e discrete choices being used as to only identify which decision maker may have more influence, such that the first n_e discrete choices do not produce any knowledge on the range of Δu_{DM_A}. (In the above figures, n_e was set at 1 but was previously discussed with values of greater than 1 if needed.

This equation illustrates another potential use of a satisficing strategy. If time or resources are not abundantly available, and a relatively wide range about the influence of one decision maker over another is sufficient, then just a few discrete choices may allow some indication as to who is more likely to persuade or influence DM_A to change their preference. For example, if time only permits 5 discrete choices, and 2 of those are used as checks (i.e. $n_e = 2$) then the expected knowledge on the range of Δu_{DM_A} would be 0.074, and hence $\Delta P_{DM_A}^{B \sim C} = 0.125$, which could still be useful in determining influence relationships in solving equations (20).

6.2.5 Additional Steps to Solve for Power Relationships

Additional information necessary to solve the system in equations (9) or (20) can be obtained in at least two ways: 1) using discrete choices involving DM_A, or, 2) using power information from other decision makers.

The first strategy involves performing the similar types of discrete choice experiments as above but with the same decision maker making the choice (in this case DM_A) associated to one of the two options, such as in the following discrete choice:

If only these two designs were available, would you be more able to convince Decision Maker B to select yours, or would you select theirs?

Decision Maker A	Decision Maker B
$x_1 = 0.252$	$x_1 = 0.172$
$x_2 = 0.949$	$x_2 = 0.799$
$x_3 = 0.192$	$x_3 = 0.576$
$(U_{DM_A} = 0.6812)$	$(U_{DM_A} = 0.6814)$
$(U_{DM_B} = 0.3706)$	$(U_{DM_B} = 0.4642)$

Figure 91: Example #4 of a discrete choice presented to DM_A, where DM_A is associated with one of the designs. Utility Scores for each of the decision makers (inside parentheses for each design) are also shown.

In this example, the utility scores are shown for DM_A (who is also presented this particular choice) and DM_B using the mean weighting vector from Step 1 of the potential valid vectors from DM_B's responses.

Since the utility scores are effectively the same from A's perspective, the answer to this discrete choice provides insight as to how A views the power relationship between A and B. If DM_A chooses their own design, then a similar process from that described above can be initiated with successive discrete choices always involving Decision Makers A and B until the knowledge about the utility difference and therefore the difference in power or influence has reached a sufficiently tight range.

For example, DM_B would have to be convince to give up $\Delta u_{DM_A} = U^2_{DM_B} - U^1_{DM_B}$ = 0.4642 - 0.3706 = 0.0936, or almost 0.1 utils, if, in fact, DM_A was able to convince B to select A's design. This is of course viewed as A's influence or power over B (from A's perspective). Since, DM_A may not know, however, even an estimate on the weighting vector of DM_B, the opposite possibility can be tested, where B's utility score for the design associated with DM_A can be larger. Decision Maker A's choices for these experiments will suggest the initial view of who potentially maintains greater power over the other from DM_A's perspective.

This above process when after reaching the stopping criterion, provides the fifth equation below:

$$P_{A \to B} - P_{B \to A} = [min\Delta P^{A \sim B}_{DM_A}, max\Delta P^{A \sim B}_{DM_A}] \tag{22}$$

and also, a sixth equation, when a similar process is perform involving Decision Makers A and C:

$$P_{A \to C} - P_{C \to A} = [min\Delta P^{A \sim C}_{DM_A}, max\Delta P^{A \sim C}_{DM_A}] \tag{23}$$

Note that the two decision makers involved in the discrete choice are labeled in the power superscripts where the responding decision maker is located in the subscript.

This results in the full set of equations:

$$
\begin{aligned}
&P_{A\to B} + P_{B\to A} = 1 \\
&P_{A\to C} + P_{C\to A} = 1 \\
&P_{B\to C} + P_{C\to B} = 1 \\
&P_{B\to A} - P_{C\to A} = [min\Delta P^{B\sim C}_{DM_A}, max\Delta P^{B\sim C}_{DM_A}] \\
&P_{A\to B} - P_{B\to A} = [min\Delta P^{A\sim B}_{DM_A}, max\Delta P^{A\sim B}_{DM_A}] \\
&P_{A\to C} - P_{C\to A} = [min\Delta P^{A\sim C}_{DM_A}, max\Delta P^{A\sim C}_{DM_A}]
\end{aligned} \tag{24}
$$

and in matrix form:

$$
\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \end{bmatrix}
\begin{bmatrix} P_{A\to B} \\ P_{B\to A} \\ P_{A\to C} \\ P_{C\to A} \\ P_{B\to C} \\ P_{C\to B} \end{bmatrix}
=
\begin{bmatrix} 1 \\ 1 \\ 1 \\ [min\Delta P^{B\sim C}_{DM_A}, max\Delta P^{B\sim C}_{DM_A}] \\ [min\Delta P^{A\sim B}_{DM_A}, max\Delta P^{A\sim B}_{DM_A}] \\ [min\Delta P^{A\sim C}_{DM_A}, max\Delta P^{A\sim C}_{DM_A}] \end{bmatrix}. \tag{25}
$$

The 6x6 matrix above is not invertible as the 5th and 6th column vectors are not linearly independent, resulting in a matrix rank of 5 (which is less than the number of columns). However, the subset of equations involving $P_{A\to B}$ and $P_{B\to A}$ can be solved:

$$
\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}
\begin{bmatrix} P_{A\to B} \\ P_{B\to A} \end{bmatrix}
=
\begin{bmatrix} 1 \\ [min\Delta P^{A\sim B}_{DM_A}, max\Delta P^{A\sim B}_{DM_A}] \end{bmatrix}, \tag{26}
$$

and similarly, the values $P_{A\to C}$ and $P_{C\to A}$ are solved via:

$$
\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}
\begin{bmatrix} P_{A\to C} \\ P_{C\to A} \end{bmatrix}
=
\begin{bmatrix} 1 \\ [min\Delta P^{A\sim C}_{DM_A}, max\Delta P^{A\sim C}_{DM_A}] \end{bmatrix}. \tag{27}
$$

The fourth row or equation from (25) was never used in the above two solutions. It can be used as a check for consistency after the above systems have been solved or included in the system to solve the first two unknowns (i.e. $P_{B\rightarrow C}$ and $P_{C\rightarrow B}$) have been excluded resulting in the solvable system:

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & -1 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} P_{A\rightarrow B} \\ P_{B\rightarrow A} \\ P_{A\rightarrow C} \\ P_{C\rightarrow A} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0.2 \\ 0.3 \\ 0.4 \end{bmatrix}. \tag{28}$$

where the ranges have been replaced with an arbitrary single value (where the minimum and maximum values are the same) for purposes of illustration.

This system of equations is overdetermined and thus constitutes a general linear least squares problem [168]. Performing this operation using the unique solution:

$$x = (A^*A)^{-1}A^*b \tag{29}$$

where A is the 5x4 matrix above from equation (28), A^* is the Hermitian transpose of A, b is the vector of values on the right hand side and x represents the vector of unknowns (i.e. the power relationships).

The solution of this system becomes:

$$x = \begin{bmatrix} 0.65 \\ 0.3875 \\ 0.7 \\ 0.2625 \end{bmatrix}. \tag{30}$$

If one of the last three equations is removed from this same system, then the A matrix becomes 4x4 and x takes on respective values of:

$$x = \begin{bmatrix} 0.65 \\ 0.35 \\ 0.85 \\ 0.15 \end{bmatrix}, \begin{bmatrix} 0.5 \\ 0.5 \\ 0.7 \\ 0.3 \end{bmatrix} \text{ or } \begin{bmatrix} 0.65 \\ 0.35 \\ 0.7 \\ 0.3 \end{bmatrix}. \tag{31}$$

With three different answers (and four with the least squares result), the arbitrary values for the b vector, are inconsistent. For example, if b =[1, 1, 0.1, 0.2, 0.4] (a consistent answer) then x = [0.6, 0.4, 0.7, 0.3] for all four systems. Although this inconsistency is not desirable, it is quite likely since a decision maker may be potentially inconsistent through at least some of the discrete choices presented and across the various comparisons, such as the difference in influence between A and B, A and C, etc.

However, with only four unknowns, it is not required to ask at least one of the series of discrete choices that provided one of the last three equations. In terms of a satisficing strategy, two would have been sufficient. However, if time and resources permit additional testing than additional discrete choices can be performed, and consistency checks or least squares algorithms can be applied.

In fact, the last two unknowns, $P_{B \to C}$ and $P_{C \to B}$ could be estimated by DM_A by answering similar questions about whose design would win out in a tournament style decision between those two decision makers, similar to Figure 91 (but with DM_B and DM_C involved). This particular type of discrete choice is further removed from the knowledge of DM_A since they may known nothing or little about the relationship between two other individuals, but it at least can provide an estimate for $P_{B \to C}$ and $P_{C \to B}$ from the perspective of DM_A.

Similarly, one can ask DM_A about the responses that DM_B and DM_C would provide in other situations such as that in the initial discrete choice from Figures 87 and 88. These would obviously include a relatively high amount of uncertainty for the aforementioned reasons, but could still be useful in analyzing consistency in the responses of DM_A.

Therefore, the full set of possible equations in matrix form, solely from the perspective of Decision Maker A can be written as:

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} P_{A\to B} \\ P_{B\to A} \\ P_{A\to C} \\ P_{C\to A} \\ P_{B\to C} \\ P_{C\to B} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ [min\Delta P^{A\sim B}_{DM_A}, max\Delta P^{A\sim B}_{DM_A}] \\ [min\Delta P^{A\sim C}_{DM_A}, max\Delta P^{A\sim C}_{DM_A}] \\ [min\Delta P^{B\sim C}_{DM_\bullet}, max\Delta P^{B\sim C}_{DM_\bullet}] \\ [min\Delta P^{B\sim C}_{DM_A}, max\Delta P^{B\sim C}_{DM_A}] \\ [min\Delta P^{A\sim C}_{DM_\bullet}, max\Delta P^{A\sim C}_{DM_\bullet}] \\ [min\Delta P^{A\sim B}_{DM_\bullet}, max\Delta P^{A\sim B}_{DM_\bullet}] \end{bmatrix}, \qquad (32)$$

where the $DM_\bullet$ in the 6th, 8th and 9th elements in the b vector indicates those equations where DM_A is making an estimate on how the others will respond to discrete choices, that is, from *their* perspective. For example, $\Delta P^{B\sim C}_{DM_\bullet}$ is the estimate for how DM_B or DM_C will answer a discrete choice when they are given a particular discrete choice. In other words, DM_A acts as if they were DM_B or DM_C in responding to the discrete choice to give these equations.

The order in the above matrix has changed slightly from previous equations in that the first three rows are the constraint equations, the next three involve the tournament style discrete choices (where the decision maker given the discrete choice is associated with one of the designs) and the last three are those discrete choices where a particular decision maker identifies who of the other two decision makers has more power or influence over themselves. As mentioned above, only a subset of these nine equations is needed to solve the 6 unknowns, but additional ones could be helpful in applying checks or implementing a least squares approach to the solution.

In Figure 92, the power relationships are indicated with arrows and the circled numbers indicate the equation number (row number) in the above matrix that represents the power relationships with the related unknown power variables.

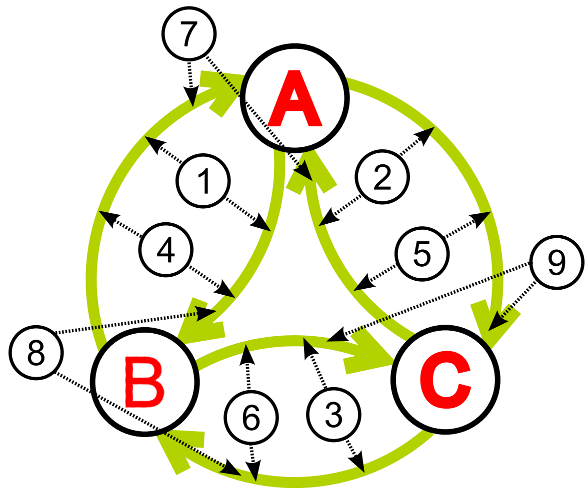


Figure 92: Diagram of potential power relationships equations

Since DM_A really only has some intimation of knowledge about equations ④, ⑤ and ⑦, the discrete choices that provide this information would be uncertain. Equations ①, ② and ③ are the known constraints. Next, equations ⑧ and ⑨ could be investigated through discrete choices presented to DM_A with even more uncertainty and lastly equation ⑥ could be estimated with the most uncertainty (which only involves the other two decision makers).

Clearly, 9 equations (and even 6 equations) with 6 or more series of discrete choices to extract the ranges for each of them is significant effort for one decision maker. Furthermore, the number of equations and effort only increases when more than 3 decision makers are involved. For example, four decision makers would result in 6 constraint equations and 12 other equations available for the 12 power relationship variables that need to be defined. This would indubitably require even more time and effort on the part of the decision maker.

To accelerate the process of extracting power relationships, the second method is more efficient by using the power information from multiple decision makers simultaneously. Up to this point, only DM_A's choices were used to define the 6 unknown power relationships. Understandably, each of the decision makers would seek to define their own influence (or perceived influence) over each other. This could result in additional inconsistencies which would likewise be interesting and useful to analyze but in order to simply satisfy the required number of equations (i.e. 6), only one equation provided by each of the three decision makers is necessary.

Since a decision maker is more likely to know the influence that others have on themselves, equations ⑦, ⑧, and ⑨ provided from discrete choices given to Decision Makers A, B and C respectively are all that are needed. Note that the above process will make use of the mean weighting vector for the other respective decision makers and not DM_A's mean weighting vector.

The system of equations, after these series of discrete choices are complete, now looks like:

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} P_{A\to B} \\ P_{B\to A} \\ P_{A\to C} \\ P_{C\to A} \\ P_{B\to C} \\ P_{C\to B} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ [min\Delta P^{B\sim C}_{DM_A}, max\Delta P^{B\sim C}_{DM_A}] \\ [min\Delta P^{A\sim C}_{DM_B}, max\Delta P^{A\sim C}_{DM_B}] \\ [min\Delta P^{A\sim B}_{DM_C}, max\Delta P^{A\sim B}_{DM_C}] \end{bmatrix}, \tag{33}$$

where the subscripts on the right hand side (e.g. $\bullet_{DM_B}$) indicate from whose perspective that particular vector element was provided (Decision Maker A, B or C). Assuming an arbitrary right hand side b vector of $[1, 1, 1, 0.3, 0.1, 0.2]$, the solution is calculated via $x = A^{-1}b$ (since A is invertible) and results in:

$$x = \begin{bmatrix} 0.3 \\ 0.7 \\ 0.6 \\ 0.4 \\ 0.8 \\ 0.2 \end{bmatrix}. \tag{34}$$

This is illustrated in Figure 93 where arrows are scaled and shaded by the relative values for the power associated for each relationship. Reading this figure is done by considering that the decision maker at the arrow's tail has power over the decision maker at the head of the arrow according to the value indicated. In this figure, Decision Maker B appears to have a significant high amount of power in the entire group with a power over A set at 0.7 and power over C at 0.8.

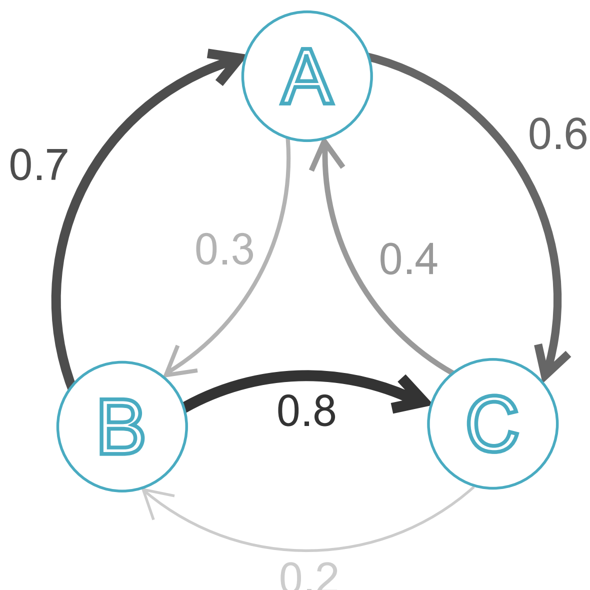


Figure 93: Illustration of power relationships between decision makers. Arrow color and thickness are scaled by power value.

Both the constraints, and similarly, the other equations are satisfied in the above solution and related figure, and no one decision maker was required to respond to more than one set of discrete choices for sufficient convergence. If more series of discrete choices are provided by one or more decision makers, making the matrix no longer square, then the strategy for solving the now overdetermined problem using a least squares method can be implemented.

6.2.6 Distributions of Power Relationships

The preceding discussion, however, ignored the fact that specific values are not given as elements in the b vector but instead as ranges (i.e. $[min\Delta P^{B\sim C}_{DM_A}, max\Delta P^{B\sim C}_{DM_A}]$). Since each decision maker reveals a range of influence differences for each of the equations, transforming these into the ranges for the actual power relationships is still needed.

This is done by taking the various combinations of minimum and maximum values (in this case, $2^3 = 8$) and solving the system of equations multiple times for each one. For example, if the b vector is:

$$b = \begin{bmatrix} 1 \\ 1 \\ 1 \\ [0.3, 0.4] \\ [0.05, 0.15] \\ [0.2, 0.35] \end{bmatrix}. \tag{35}$$

then the resultant solutions for x will be:

$$x = \begin{bmatrix} P_{A\to B} \\ P_{B\to A} \\ P_{A\to C} \\ P_{C\to A} \\ P_{B\to C} \\ P_{C\to B} \end{bmatrix} = \begin{bmatrix} [0.15, 0.325] \\ [0.675, 0.85] \\ [0.5, 0.675] \\ [0.325, 0.5] \\ [0.775, 0.95] \\ [0.05, 0.225] \end{bmatrix}. \tag{36}$$

Of the eight solutions these ranges contain, selecting all the minimum values would clearly not be a valid solution since each set of two rows (with the first row number of a set being odd) must sum to 1. Furthermore, assigning one to the minimum and one to the maximum values in each set would also not provide a valid answer since the values are highly interdependent. For example, if $P_{A \to B} = 0.15$ (minimum) and $P_{B \to A} = 0.85$ (maximum), the other four values cannot lie at any of the extremes (in at least this example). Thus, since the ranges displayed above are only the minimum and maximum values of x, a more precise and useful set of solutions to the system of equations is through distributions shown in Figure 94.

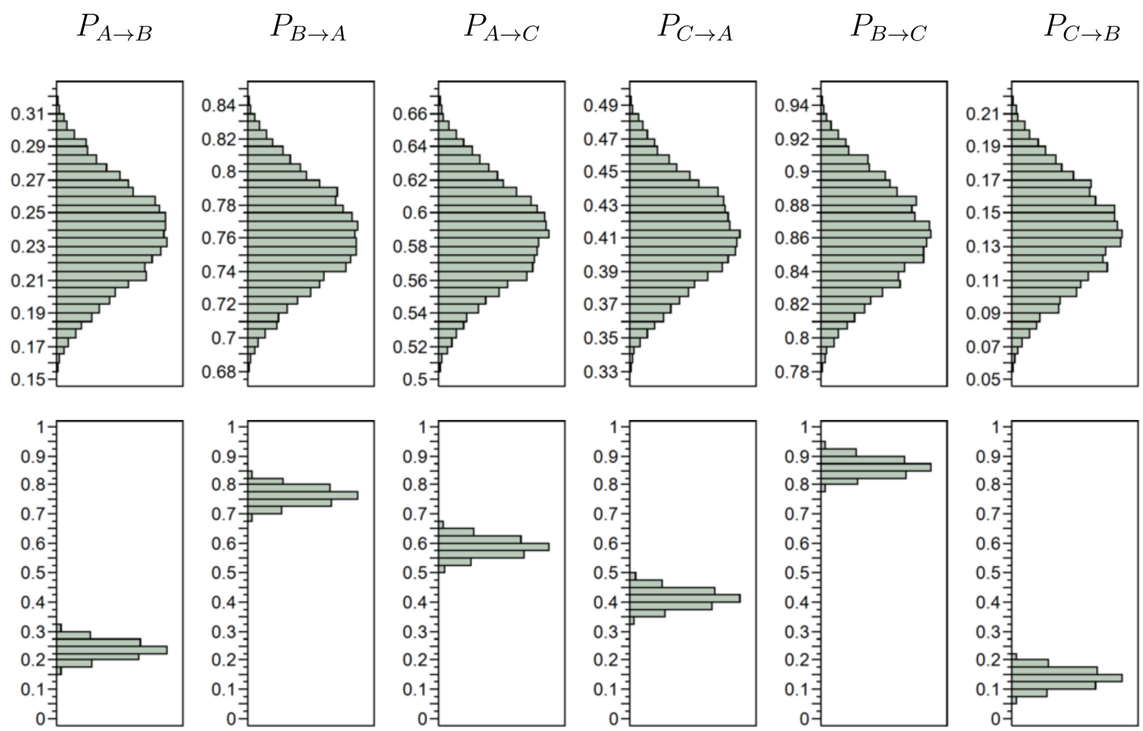


Figure 94: Distributions for decision maker power relationships generated by the execution of 10000 Monte Carlo simulations

These power distributions are created with a histogram of the solutions of x for 10000 randomly selected combinations of values within the appropriate ranges for the b vector in Equation (35). The distributions approach a normal distribution shape with a standard deviation close to 0.03, however, the tails are truncated at the maximum and minimum possible values listed above.

It is interesting to note that although the ranges are expectedly different (e.g. [0.15,0.325] vs. [0.675,0.85]), the difference between the maximum and minimum values for each of the ranges are the same (e.g. 0.325 - 0.15 = 0.175 = 0.85 - 0.675). This is tied to the size of the range for the values in b. The relative differences in the range for b in this example are 0.1, 0.1 and 0.15 respectively for the three last elements of this vector (see Equation (35)). In other words, the discrete choices presented to and answered by Decision Maker C have greater uncertainty than that of DM_A or DM_B.

Research Question: *What is the impact on the certainty of power relationships when decision makers respond to different numbers of discrete choices and provided different ranges?*

The experiment to answer this question involves simulating the above steps for multiple ranges for b and investigating the sensitivities of these ranges to the certainty in the power distributions.

The first range sweep will be by holding the range from DM_A and DM_B constant as above (i.e. $max(b[4]) - min(b[4]) = max(b[5]) - min(b[5]) = 0.1$) and sweeping the value of $max(b[6]) - min(b[6])$ from 0 to 0.8.

The one highlighted point in Figure 95 indicates the ranges in the b vector $b[4] = 0.1$, $b[5] = 0.1$ and $b[6] = 0.15$ which result in the power distribution range of 0.175. Expectedly, a higher uncertainty in b[6] results in higher uncertainty in the distribution ranges. The same point ($b[4] = 0.1$, $b[5] = 0.1$ and $b[6] = 0.15$) is highlighted in Figure 96 but with the values for $b[5]$ also varied between 0 and 0.95. Not surprisingly, the uncertainty increases when two decision makers are uncertain themselves about influence differences between decision makers. Finally, varying or sweeping all of the values from the b vector for each of the decision makers would produce a similar trend where each of the lines from Figure 96 is shifted even higher.

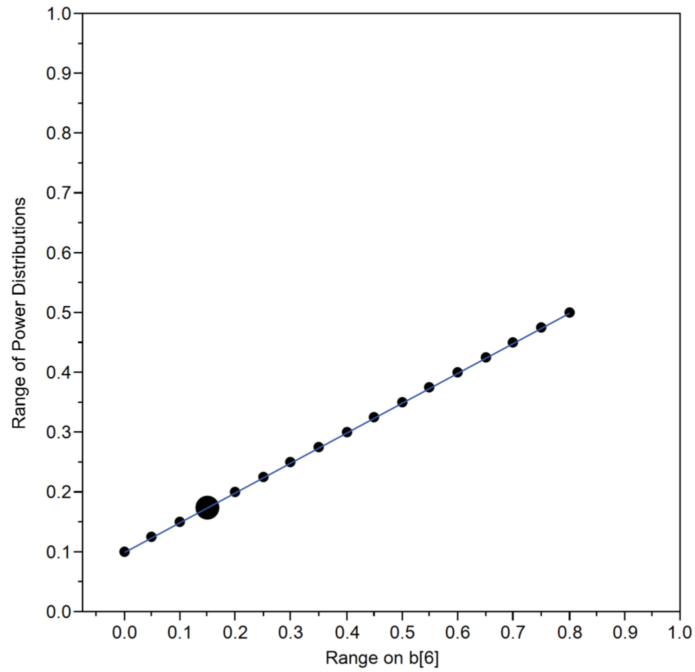


Figure 95: Range sweep of $max(b[6]) - min(b[6])$ from 0 to 0.8 while holding $max(b[4]) - min(b[4]) = max(b[5]) - min(b[5]) = 0.1$

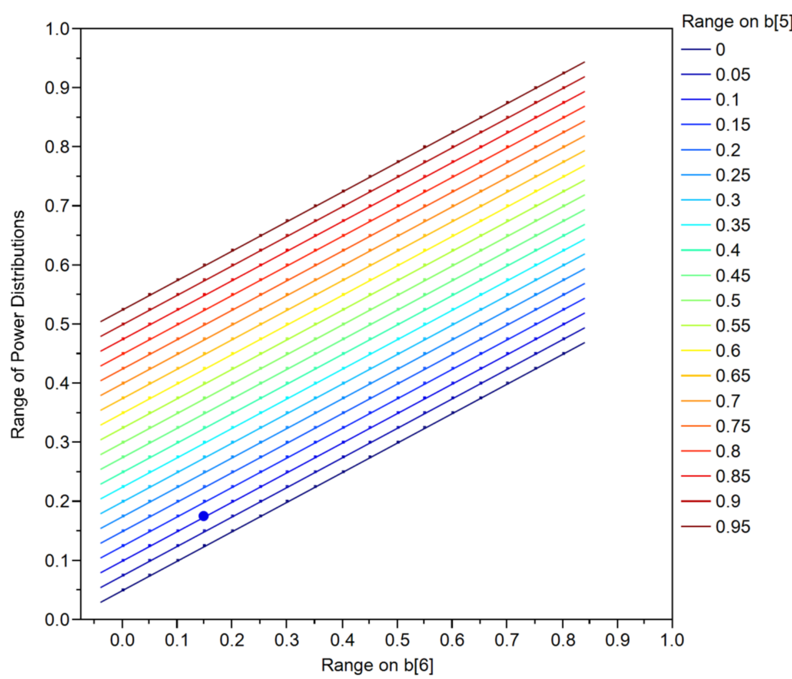


Figure 96: Range sweep for $b[5]$ and $b[6]$ while holding $b[4]$ at 0.1

An interesting result occurs if the sum of the uncertainties from the decision makers is greater than 2 (i.e. b[4]+b[5]+b[6] > 2). In this case, the ranges for the distributions of power relationships is always greater than 1, an impossible solution based upon the implicit constraints that $0 \leq P_{A \to B} \leq 1$, $0 \leq P_{B \to A} \leq 1$, etc. or that P_x is always non-negative and less than 1.

Moreover, when the sum is greater than 1 (but less than 2), solutions can likewise result with ranges greater than 1 but can also reach ranges as low as 0.5. For example, if $b[4] = 0.677$, $b[5] = 0.286$ and $b[6] = 0.038$, and therefore $b[4] + b[5] + b[6] = 1.001$, the ranges for the distributions are only 0.501 and the solutions for P_x are all between 0 and 1. However, if $b[4] = 0.869$, $b[5] = 0.002$ and $b[6] = 0.166$, and therefore $b[4] + b[5] + b[6] = 1.037$, the ranges span 0.519 but the values of P_x are greater than 1 or less than 0 such as $P_{C \to B} = 1.019$ and $P_{B \to C} = -0.019$ in this example. This would seem to indicate that in this particular relationship, DM_A holds all the power over DM_B. In these types of outcomes the power relationships are adjusted such that a value greater than 1 is set at 1 and values less than 0 are set at 0, such that $P_{C \to B} = 1$ and $P_{B \to C} = 0$ for this case. This same result occurs when the last three b vector elements are normalized such that when $b[4] = 0.869$, $b[5] = 0.002$ and $b[6] = 0.166$, they become $b[4] = 0.869/sum(b[4], b[5], b[6]) = 0.837$, $b[5] = 0.002/sum(b[4], b[5], b[6]) = 0.0019$ and $b[6] = 0.166/sum(b[4], b[5], b[6]) = 0.160$ which then provides the constrained values for the power relationships P_x to be non-negative as required.

With the normalization applied, the bottom half of Figure 95 will have a number of simulations which will be normalized due to $b[5] + b[6] > 1$. Those points will be normalized and recalculated with the updated values. The result appears in Figure 97 where the top portion with $b[5] + b[6] > 1$ is shifted or folded back onto the feasible region. The colors are again set to correspond with the range on b[5], such that the darkest red point corresponds to a $b[5]$ value of 0.9 and a $b[6]$ value of 0 ($b[4]$ is held constant at 0.1). As a consequence, any increase in the range of $b[6]$ will result in a normalization process with the portion of the triangle shape above 0.5 skewed to the left and resulting in ranges of the distributions as indicated on the y-axis for $P_{A \to B}$ for example.

Figure 98 shows the aforementioned normalization process when b[4] is swept across discrete values of 0.1, 0.35, 0.6 and 0.85. The top left section is identical to the entire top graph discussed above. The other three show increasing values of the distribution range of the power with increasing amounts of folding as normalization is required when the sum of the three b vector elements is greater than 1.

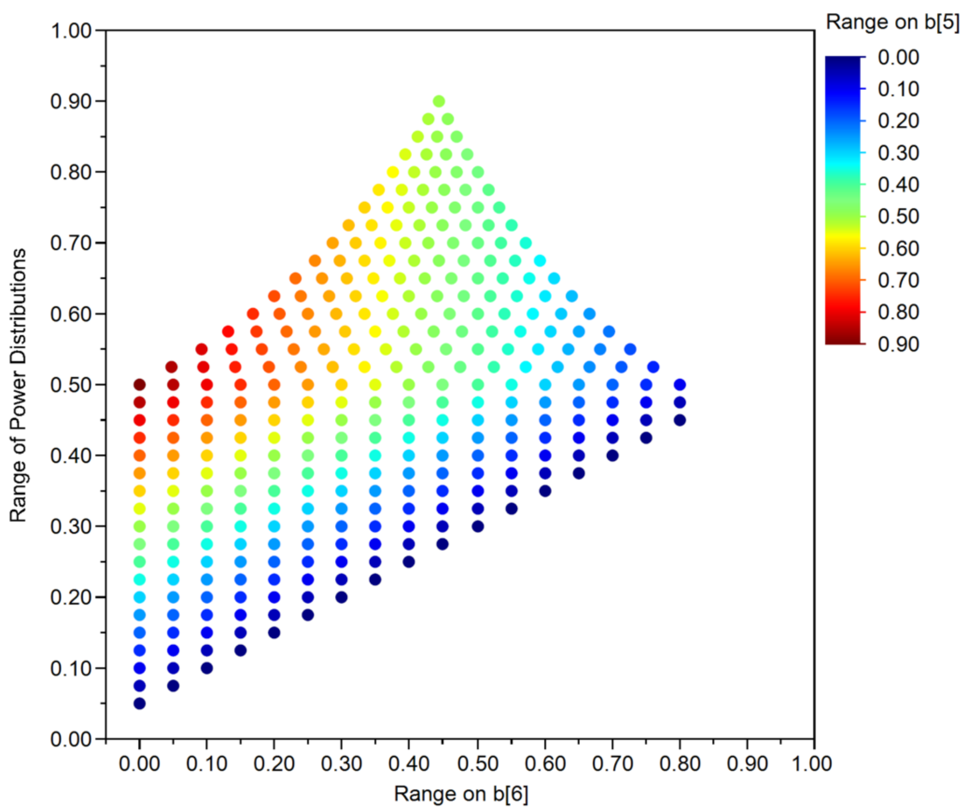


Figure 97: Normalized b vector values. Range sweep for $b[5]$ and $b[6]$ while holding $b[4]$ at 0.1

Under normalization, the power distributions can take on difference ranges. If the normalization is required, the following $b \Leftrightarrow x$ pair exist:

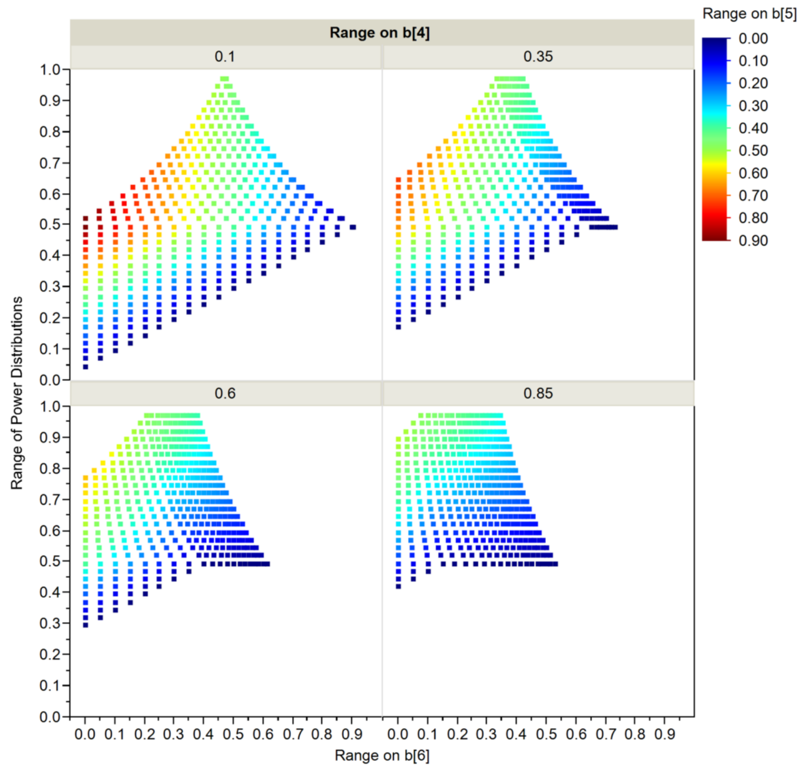


Figure 98: Normalized b vector values. Range sweep for $b[4]$, $b[5]$ and $b[6]$

$$b = \begin{bmatrix} 1 \\ 1 \\ 1 \\ [0.7, 0.8] \\ [0.05, 0.15] \\ [0.2, 0.35] \end{bmatrix} \qquad x = \begin{bmatrix} P_{A\to B} \\ P_{B\to A} \\ P_{A\to C} \\ P_{C\to A} \\ P_{B\to C} \\ P_{C\to B} \end{bmatrix} = \begin{bmatrix} [0.042, 0.143] \\ [0.857, 0.958] \\ [0.682, 0.826] \\ [0.174, 0.318] \\ [0.975, 1] \\ [0, 0.025] \end{bmatrix}$$

which has different ranges for the six elements of x namely 0.101 for the first two, 0.144 for the next two and 0.025 for the last two. The uncertainty is smallest for $P_{B\to C}$ and its complement due to the large difference in influence that DM_A reported between B and C each had over A (i.e. $P_{B\to A} > P_{C\to A}$). With such a large influence difference, in connection with the other values reported, there is strong evidence that $P_{B\to C}$ may be equal to 1. Of course this can be tempered significantly, if DM_C had responded with little or no difference between A and B within the $b[6]$ element or even a reversal of who had a majority of that relationship's power. For example, if, instead of [0.2, 0.35], $b[6]$ = [-0.35, -0.2], then $P_{B \to C}$ would have a min and max of [0.68, 0.82], with a range of 0.14 (much greater than 0.025). However, at the same time, this combination would potentially make $P_{A \to C} = 1$. Thus the uncertainty in P_x is ultimately highly dependent on the absolute values for the ranges but assuming all the values are relatively small such that the maximum extremes do not sum to greater than 1, the relationship between the uncertainty on the b vector and the uncertainty on the range x is linear through the summation of b. Therefore, halving the ranges of b will halve the range on x as shown in Figure 99 on the bottom (after halving). Since one more discrete choice will halve the uncertainty in the power difference, the power distribution uncertainty will halve with each successive discrete choice (assuming one more for each decision maker).

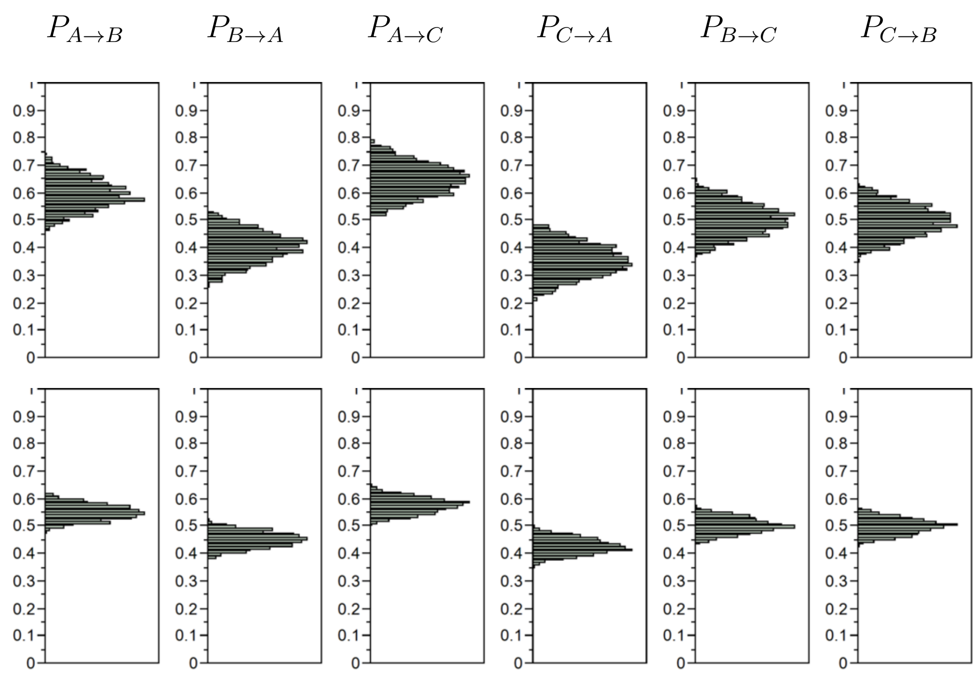


Figure 99: Before (top) and after (bottom) halving the uncertainty on the power distributions by halving the uncertainty on b

Of course, if only one decision maker responds to one more discrete choice, then the effect observed on the distribution may or may not be comparably significant. Ideally the decision maker with the largest uncertainty should be first to apply an additional discrete choice so as to maximize the benefits of one more experiment.

Lastly, although the range will contract (or expand) with b the overall shape of the histogram can also vary with the portion of uncertainty between the decision makers. For example, the left side of Figure 100 shows relatively equally uncertain ranges (r), $r_{b[4]} = 0.165$, $r_{b[5]} = 0.165$ and $r_{b[6]} = 0.17$, which is generally expected with a similar number of discrete choices, while the right hand side shows highly dissimilar ranges such that $r_{b[4]} = 0.45$, $r_{b[5]} = 0.025$ and $r_{b[6]} = 0.025$.

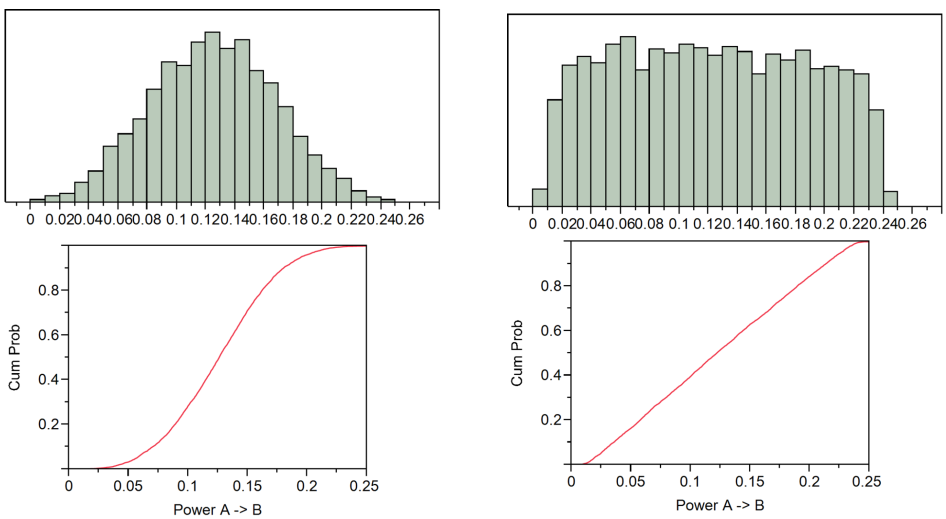


Figure 100: Examples of the effect of range differences on the distribution shape of power relationships

With this additional step of normalization required (i.e. the values given by the discrete choices when $b[4] + b[5] + b[6] > 1$), the full set of ranges for the b vector with 10000 Monte Carlo simulations can be executed and analyzed. Figure 101 shows the three power relationships and their complements in the upper 3x3 matrix. The correlation is exactly -1 for complementary power relationships as shown in the first three diagonal scatterplots. The last row and column illustrate the sum of the last three elements in b versus these same power relationships. All points are again colored by this same summation value.

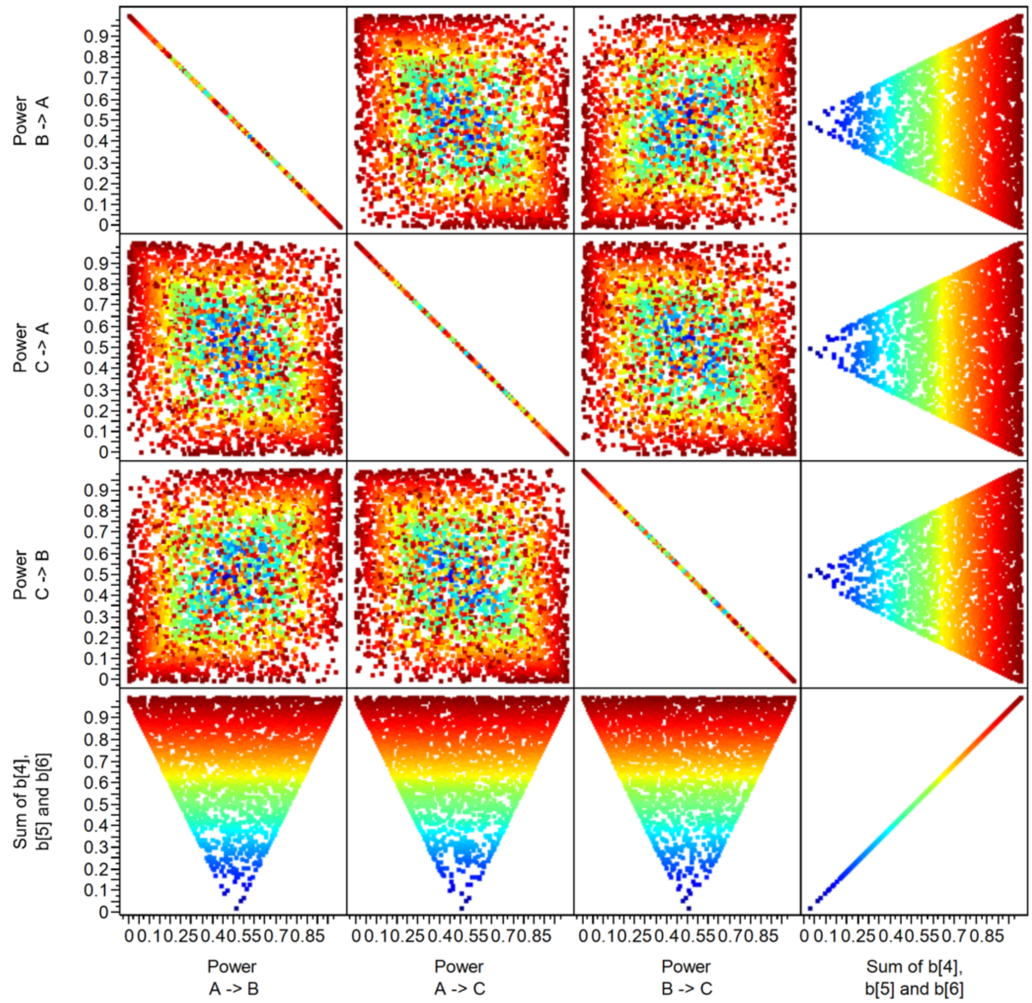


Figure 101: Monte Carlo Simulations for Power Relationships

Firstly, all combinations of power relationship are possible, which is to be expected with a simple MC covering the entire range of power or influence differences. Secondly, when the sum of the last three elements in b is small, the power relationships are more similar or at least some of them will be close to equal power (e.g. $P_{A \to B} = 0.49$ and $P_{B \to A} = 0.51$. Finally, when the sum of the last three elements in b approaches 1, possible combinations can exist across the entire "power space".

Technically, this "power space", is composed of all the possible combinations of power relationships that can exist. However, since the A matrix could be one of eight types below:

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & -1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & -1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & -1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & -1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 & 0 \end{bmatrix}$$

after the range for the b vector is made such that it is always non-negative, the power space is divided into these eight regions to which a particular A matrix can be mapped (through A^{-1}). Figure 102 shows the region or range (in a vector math sense) of the second type of the A matrix above.

In this figure, the power combinations suggest, for example, that:

$$P_{B\to C} + P_{C\to A} \leq 1 \tag{37}$$

and thus,

$$P_{B\to C} \leq 1 - P_{C\to A} \tag{38}$$

and finally:

$$P_{B\to C} \leq P_{A\to C}, \tag{39}$$

as suggested by the last row (sixth equation) from A when b is non-negative. The other inequalities established from the discrete choices with A would further define

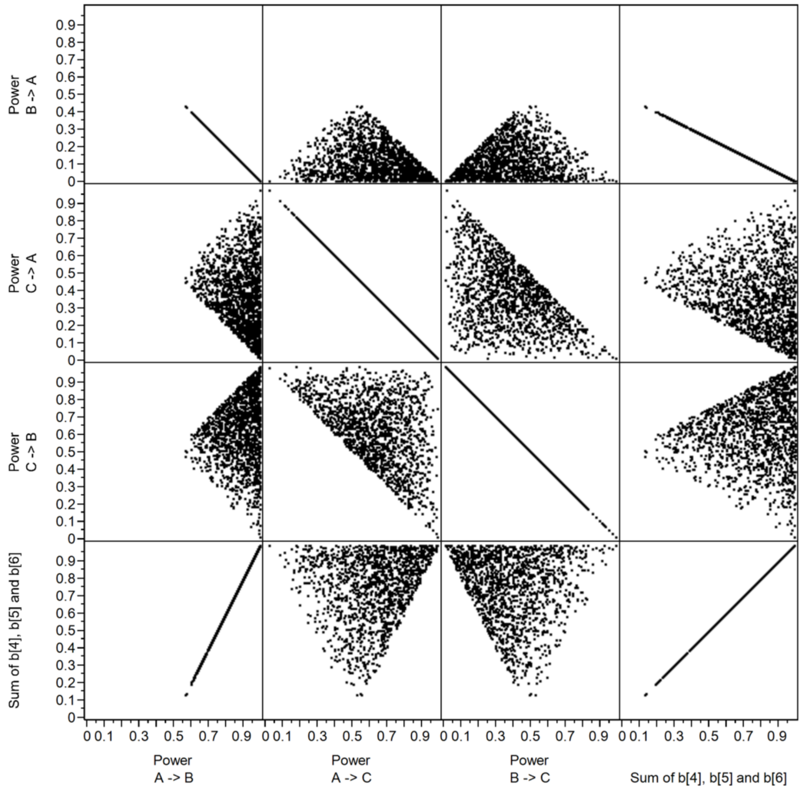


Figure 102: Type 2 of A matrix used for Monte Carlo Simulations and resultant power relationships

the other possible regions for power combinations. The total power space is the union of these spaces, so that the entire space as depicted in Figure 101 can capture completely all the possible power structures between decision makers.

6.2.7 Application of Vectors Other than the Mean Weighting Vector

Another factor initially overlooked in the above discussion was the use of the mean weighting vector for all of the comparisons. Extracting the power relationships assumes that this is in fact the weighting by which all comparisons can be referenced. In reality, it is known only to a certain level based on the set of all possible true weighting vectors as discussed in the previous section from the discrete choices in Step 1.

This suggests, therefore, that the ranges on the power relationships are likely less certain than originally presumed. For example, in Table 11, two designs which could be compared in a discrete choice, have a nearly equal utility value when using the mean weighting vector as above. This set could be used in a discrete choice to identify the initial power relationship between two decision makers. That is, if DM_B were attached to d_{35} and DM_C were assigned to d_{1481}, the respondent DM_A would likely not cognitively apply the mean weighting vector, but perhaps the w_{max} from d_{35}. When this different weighting vector is applied, the comparison is now between designs in different utility bands, namely between 0.6704 and 0.6197 (see the w_{max} for d_{35} and $w_{35_{max}}$ for d_{1481}), which is significantly different than the expected 0.6257 versus 0.6254.

Table 11: Mean Weighting Vector Contrasted Against Possible Minimum and Maximum Weighting Vectors

Design #35: $[0.3899, 0.9209, 0.0033]$	$U()$	w_{j1}	w_{j2}	w_{j3}
w_{mean}	0.6257	0.092	0.639	0.268
w_{min}	0.5925	0.1	0.6	0.3
w_{max}	0.6704	0.094	0.687	0.219
$w_{1481_{min}}$	0.6522	0.106	0.662	0.231
$w_{1481_{max}}$	0.6034	0.069	0.625	0.306
Design #1481: $[0.2297, 0.6356, 0.737]$	$U()$	w_{j1}	w_{j2}	w_{j3}
w_{mean}	0.6254	0.092	0.639	0.268
w_{min}	0.6388	0.106	0.662	0.231
w_{max}	0.616	0.069	0.625	0.306
$w_{35_{min}}$	0.6254	0.1	0.6	0.3
$w_{35_{max}}$	0.6197	0.094	0.687	0.219

To account for this source of uncertainty, the conservative or worst case for the upper and lower bounds can be propagated throughout the algorithm. In implementing this consideration, for each discrete choice, the range is now not necessarily halved but reduced by some amount less than half. For example, as in Figure 89, which is repeated on the top of Figure 103 but with a larger amount of iterations, the uncertainty in the utility difference slowly collapses around the particular power or influence difference (via utility). The bottom of Figure 103 shows the corrected minimum and maximum values for the difference in utility score implementing this conservative approach of only reducing the range down to the worst case.

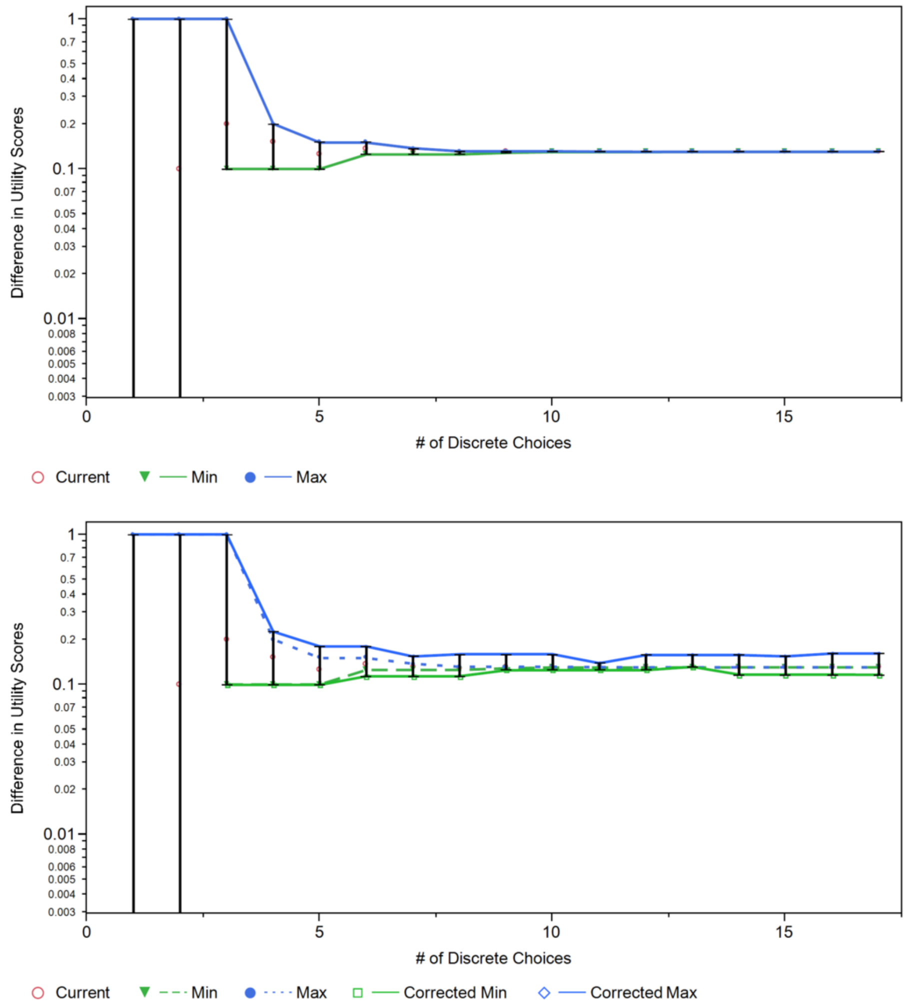


Figure 103: Accounting for uncertainty in using the mean weighting vector by propagating the worst case upper and lower bound throughout the set of discrete choices

As is visible on the far right of Figure 103, an increase in the number of discrete choices does not necessarily further reduce this range. This provides a potential stopping criterion for this step as no additional discrete choices are needed if the range is no longer reduced with another discrete choice. The reason for this originates in the random selection of the two designs in a specific utility band based on the mean weighting vector. If, in Step 1, a decision maker made fewer discrete choices, and thus the preference uncertainty is higher, the difference in the mean, maximum and minimum weighting vectors would also be much higher. An example of this is shown in Figure 104 when Step 1 only contained 5 discrete choices.

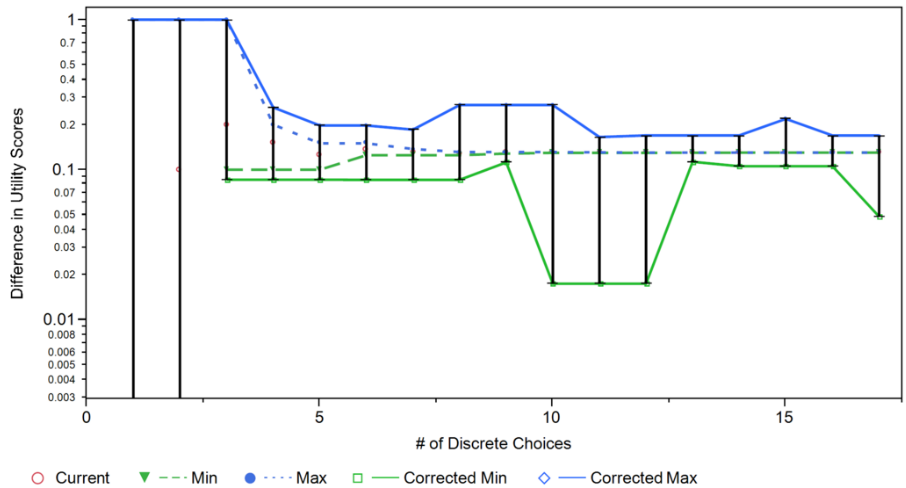


Figure 104: Example of erratic collapsing on the difference in utility score when Step 1 contains only 5 discrete choices

6.2.8 More than Three Decision Makers

To extract power relationships or distributions between more than three decision makers the above algorithm can be likewise implemented with additional discrete choice responses from each of the agents.

With four decision makers, 12 unknown power relationships can be identified but with six constraint equations already defined. The additional six equations can be acquired in a similar fashion as above by comparing two designs assigned to other decision makers and then requesting a third decision maker to choose with whom they are more likely to form a coalition. With four decision makers, two of them would be required to perform this process twice, while the others only once. However, a variety of combinations or series of discrete choices are available as discussed previously.

For example, valid equations could be obtained from DM_A choosing between 1) B and C, 2) B and D, or 3) C and D. Only a sufficient number (i.e. 6) are necessary to solve the system of equations, such as the example system below, where the A matrix is invertible:

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} P_{A\to B} \\ P_{B\to A} \\ P_{A\to C} \\ P_{C\to A} \\ P_{A\to D} \\ P_{D\to A} \\ P_{B\to C} \\ P_{C\to B} \\ P_{B\to D} \\ P_{D\to B} \\ P_{C\to D} \\ P_{D\to C} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ [min\Delta P^{B\sim D}_{DM_A}, max\Delta P^{B\sim D}_{DM_A}] \\ [min\Delta P^{C\sim D}_{DM_B}, max\Delta P^{C\sim D}_{DM_B}] \\ [min\Delta P^{A\sim B}_{DM_C}, max\Delta P^{A\sim B}_{DM_C}] \\ [min\Delta P^{B\sim C}_{DM_D}, max\Delta P^{B\sim C}_{DM_D}] \\ [min\Delta P^{B\sim C}_{DM_A}, max\Delta P^{B\sim C}_{DM_A}] \\ [min\Delta P^{A\sim D}_{DM_B}, max\Delta P^{A\sim D}_{DM_B}] \end{bmatrix}, \tag{40}$$

As with the case described previously with three decision makers, if more equations are available than unknowns, such that m > n (i.e. the number of rows (m) is greater than the number of columns (n)) a linear least squares solution can be implemented. This process may even be desirable to equalize the contributions of the four decisions makers since one or more agents may consider it unfair if two of them are fortunate to define more than one equation. On the other hand, minimizing the effort or time required to answer multiple discrete choice questions may be an equally valid reason to answer as little as possible.

6.2.9 Total Power Indices

With the power relationships defined (for three decision makers) in terms of distributions, the overall total power index for each decision maker is readily available.

For three decision makers the simple equation for the power or influence DM_A has in the entire group can be defined as:

$$P_A = \frac{P_{A\to B} + P_{A\to C}}{P_{A\to B} + P_{B\to A} + P_{A\to C} + P_{C\to A} + P_{B\to C} + P_{C\to B}} \tag{41}$$

with similar equations for DM_B and DM_C. More generally this is defined as:

$$P_i = \frac{\sum_{j=1}^{k} P_{i\to j}}{\frac{k(k-1)}{2}}, i \neq j \tag{42}$$

where k is the number of decision makers and i (or j) is the ith (or jth) decision maker, and P_i is the total power for DM_i within the group.

Assuming the ranges from equation (36), in a previous example, the total power for DM_A,DM_B, and DM_C is:

$$\begin{aligned} P_A &= \frac{[0.15, 0.325] + [0.5, 0.675]}{3} = [0.217, 0.333] \\ P_B &= \frac{[0.675, 0.85] + [0.775, 0.95]}{3} = [0.483, 0.6] \\ P_C &= \frac{[0.325, 0.5] + [0.05, 0.225]}{3} = [0.125, 0.242]. \end{aligned} \tag{43}$$

Since $P_A + P_B + P_C = 1$, only certain combinations of total power indices exist similar to the previous constraints placed upon the distributions of power for each relationship. The three maximum values cannot all be simultaneously retained nor can the three minimum values. The usefulness of these ranges allows for the testing of various scenarios involving power such as "best and worst outcome with power pessimism" or "coalition and majority forming" in Step 3 of the methodology described next when the decision makers must reach agreement on the preferences and, ultimately, on the design.

6.3 Step 3: Reaching Preference Consensus

Steps 1 and 2 described in the previous two sections respectively, and applied to the canonical problem, provided the two essential sets of data required to perform the proposed algorithm to calculated the regions where consensus is more likely to be reached.

In Step 1, the possible weighting vectors that could represent each decision maker's true preference was obtained. Each decision maker involved in a particular decision was required to respond to discrete choices between two designs. After a number of discrete choices the preference space representing the entire set of preferences was slowly reduced to a set of possible weighting vectors for each decision maker.

The output of this step, shown in Figure 105, summarizes the responses described in the previous sections from the canonical problem. Decision Maker A placed a high weighting on the second dimension (i.e. x_2), followed by x_3 and lastly by x_1. Decision Maker B's preference are in the exact opposite, preferring first x_1 and x_3 (at about the same weight of 0.4) and then x_2 (with w_{x_2} in a distribution grouped closely around 0.2). Lastly, Decision Maker C's preferences are also different with preferences for objective x_3 followed by x_2 and then x_1.

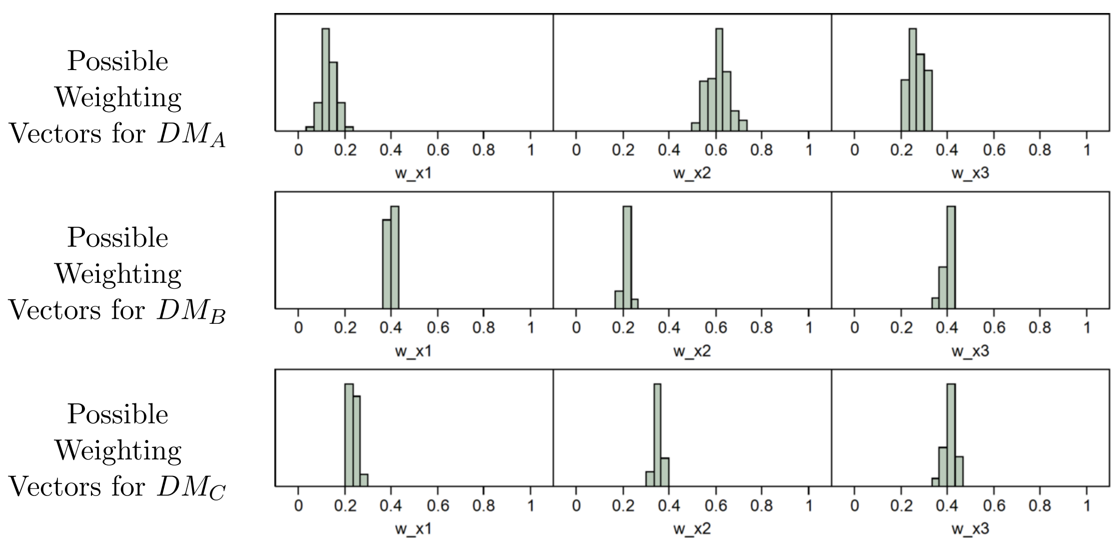


Figure 105: Output from Step 1: Possible Weighting Vectors for each Decision Maker

In Step 2, the power or influence relationships were extracted from between the decision makers. These relationships were expressed as the difference in influence that one of two other decision makers could potentially impose upon the responding decision maker. After a number of discrete choices, each which would slowly reduce the range of the power difference until the stopping criterion is reached, the equation produced from the difference in influence could then be combined with other constraint equations to solve for the power of each decision maker over the others.

The results of this step are shown in Figure 106, which illustrates, on the top half, that for each pair of decision makers, the total power sums to 1. Furthermore, on the bottom, the total power for each individual decision maker or the distribution of power shared for the entire group, is similarly represented, where DM_A likely holds the most influence with a mean power near 0.45, followed by DM_B ($\bar{P_B} \approx 0.4$) and then lastly by DM_C ($\bar{P_C} \approx 0.15$).

Since A and B have greater influence than C, the expected set of designs where consensus is more likely to be reached will be close to the preferred designs of A or B. However, to what degree this is the case can be determined by the process of Step 3 after reaching consensus through multiple ultimatum games between decision makers.

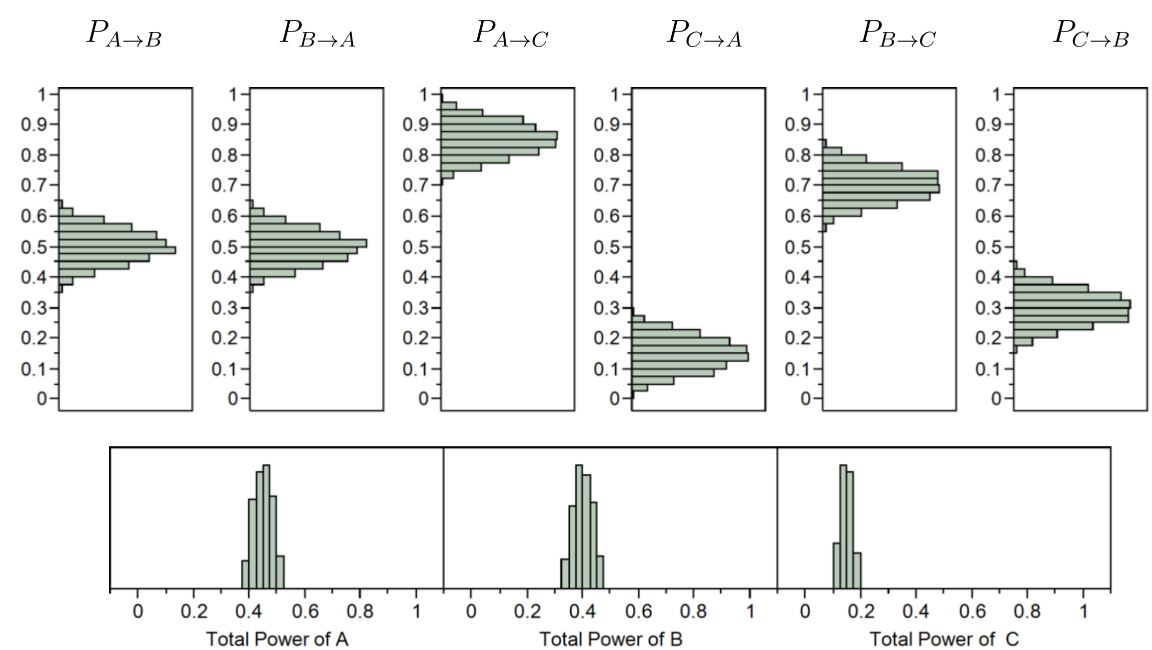


Figure 106: Output from Step 2: Distributions for Decision Makers' Power Relationships

6.3.1 The Ultimatum Game Between Decision Makers

Conceptually, the ultimatum game is played between two decision makers (or groups of decision makers) to identify the designs or rather the preferences at which the decision makers will reach consensus. The utility score or value of a proposed design will be lower than the "ideal" design for the preferences of both decision makers. Since both decision makers seek to maximize utility through their own preferences but recognize the requirement for reaching a compromise, they will trade utility value for cooperation or consensus, but only up to the amount that they cannot persuade the other to accept a preference closer to their own. This "willingness" to accept someone else's preferences (or a portion thereof) can be expressed in terms of the power relationships discussed in previous sections.

For example, in a particular encounter or game between A and B, decision maker A may assume any power from the distributions of $P'_{A \to B}$ from Figure 106 (where the prime on P' indicates a "perceived" power, which may or may not reflect reality). If they are optimistic in their ability to persuade, apply their reputation, or any other reason for exercising greater power, they may assume a value higher than the mean of 0.55, such as 0.65 for example. On the other hand, DM_B can be equally optimistic about their influence over A and assume a power index of 0.55 for $P'_{B \to A}$

Assuming DM_A proposes to reach agreement at a design (or preference) 15% closer (0.65-0.5=0.15) to A's design (or preference), B will reject the offer because they would only accept offers at values closer to their preferences by 5% (i.e. 0.55-0.5 = 0.05). After all, they assume that they can negotiate, persuade, hold out for, etc. a design that at least required A to come slightly more than half way to their own perspective and preferences. This will be called the *over-constrained* power condition.

If DM_B's perceived power over A was only 0.3 (again, see Figure 106), then

B would consider the offer advantageous to themselves, as they were offered something better than what they normally would accept on average. This is the *under-constrained* power condition.

Lastly, when $P'_{B\to A}$ is exactly 0.35, as perceived by DM_B (while the power index for A over B remains at 0.65), the offer is likewise accepted since B could not on average expect a more favorable offer. This is the *constrained* power condition.

These three situations or conditions are illustrated in Figure 107. In the top or over-constrained condition, the total perceived power for both follows the inequality $P'_{A\to B} + P'_{B\to A} > 1$. They both think that the other will concede more "ground" than what will occur in reality. In other words, the point proposed by A (small green circle) is "too far away" from an acceptable location (small blue circle) from B's perspective. This is equally true if the proposer and responder are reversed.

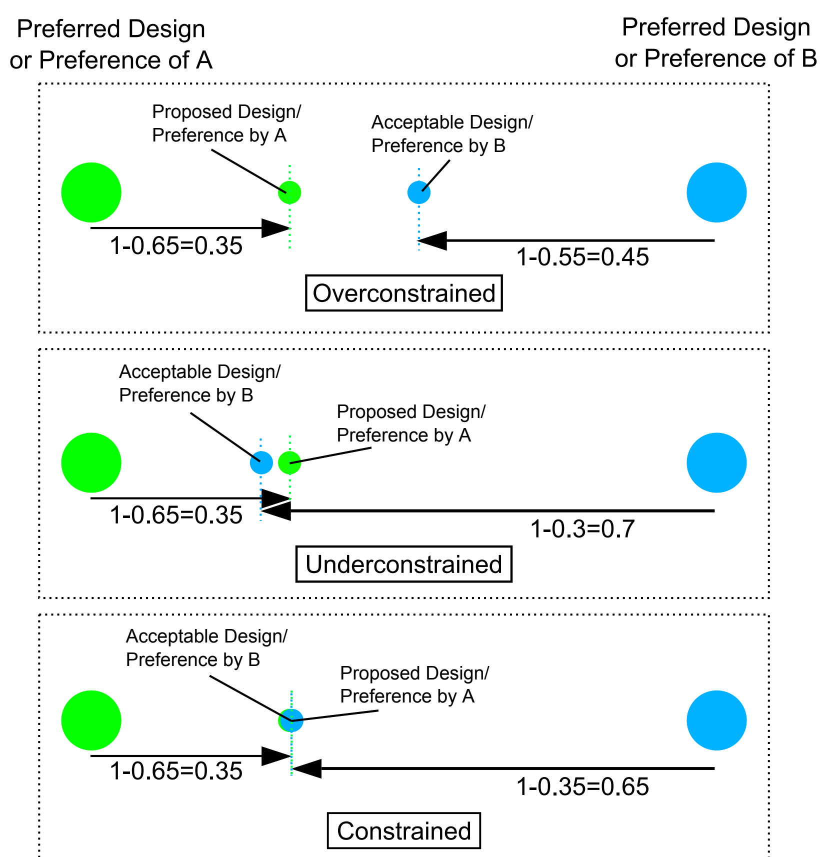


Figure 107: Examples of the Three Power Constraint Conditions

Table 12: Examples of Ultimatum Game Outcomes Between Decision Makers with Different Perceptions of Power Relationships

Description	Design	$[w_{x_1}, w_{x_2}, w_{x_3}]$	u_A	u_B	$P'_{A\to B}$	$P'_{B\to A}$	$P'_{A\to B} + P'_{B\to A}$	accept or reject
Pref. by A	705	[0.125, 0.6, 0.275]	0.672	0.416	n/a	n/a	n/a	n/a
Pref. by B	1038	[0.388, 0.2, 0.412]	0.466	0.600	n/a	n/a	n/a	n/a
Prop. by A	293	[0.217, 0.46, 0.323]	0.649	0.516	0.65	0.35	1	accept
Prop. by A	293	[0.217, 0.46, 0.323]	0.611	0.560	0.65	0.55	1.2	reject
Prop. by A	293	[0.217, 0.46, 0.323]	0.654	0.507	0.65	0.3	0.95	accept

In the middle or under-constrained condition, the inequality describing the total perceived power is $P'_{A\to B}+P'_{B\to A} < 1$. They are both willing to give up more "ground" than what is necessary. Agreement in these situations will be quick or highly probable. The proposed design or preference is closer to the responder's preferred design than what would be generally accepted. The outcome of proposed designs in these situations often results in agreement.

Finally, for the constrained condition, when $P'_{A\to B} + P'_{B\to A} = 1$, the proposed design lies exactly on the point which would satisfy the associated constraint equation. The responding decision maker could accept the design, since at this power relationship, they cannot expected to obtain a better design more than half the time, under the situation's perceived power assumptions.

A more concrete example is show in Table 12, where DM_A proposes design #293 to the responder (DM_B) who has assumed different influences onto A (i.e. $P'_{B\to A}$). The first two rows indicate the preferred design and utility scores before any consensus. The utility score for A's design is much larger than B's utility score of the same design, and vice versa for the other design.

The third row in the table shows the constrained condition where the sum of $P'_{A\to B}$ and $P'_{B\to A}$ equals 1. In this case, DM_B accepts this design since they will not expect to convince DM_A to approach or come closer to their preferred design (i.e. d_{1038}) any more than the design or preferences listed in row #3. The utility columns show that for this design, both decision makers concede some utility value. DM_A's utility dropped from 0.672 to 0.649 while DM_B's utility dropped from 0.6 to 0.516. Although both experienced a reduction in utility, the benefits of cooperation might compensate for the lower utility. In other words, before consensus was reached, both of their utility values were effectively zero if cooperation is essential (an assumption throughout this research). That is, without each other and a final agreement, no design can be selected, and thus no benefit or utility can be achieved for either one. Although, neither decision maker was able to achieve their own individual best value, both are able to gain from the relationship by finding at least one consensus point and by so doing potentially "pooling" their power over other decision makers, discussed later.

The fourth row in Table 12 shows when the over-constrained condition occurs and the sum of the powers over each other is greater than 1. DM_B expects that disagreeing will result in DM_A moving to a position more closely aligned with DM_B's preferred point. From the opposite perspective, DM_A might take a similar view that eventually DM_B will concede more as well. Regardless of the proposer or responder, the responder will reject the offer and wait for a new or counter-offer which is more acceptable.

Lastly, the fifth row shows the under-constrained case where both decision makers underestimate their influence over the other such that $P'_{A \to B} + P'_{B \to A} < 1$. In this case the responder will readily accept offers, since they view the proposed design as much better than they could have expected under the power assumptions made about relationship.

In all cases, if the utility value is equal or greater compared to the constrained case's utility value, the assumption is made that the responder will accept the proposed design or preference. (i.e. $0.56 > 0.516$ from the u_B column in Table 12).

6.3.2 Multiple Ultimatum Games Between Decision Makers

The limitation in the one game presented in Table 12 is that the actual preferred design is unknown, since only a range on the possible valid weighting vectors is known for each decision maker. Therefore, the above ultimatum game must be played multiple times from multiple initial weighting vectors.

Furthermore, only three combinations of power relationships were tabulated in Table 12. In fact, the power indices are also not precisely known with only a distribution of influence defining the relationships between the various decision makers. For this reason, the ultimatum game must be played multiple times across the various weighting vectors and concurrently with all the potential perceived power relationship combinations.

Consider two decision makers (A and B) both seeking to reach agreement on a design, with the set of feasible weighting vectors expressed in the design space as illustrated in Figure 108. Each design labeled with an A or B on the left and right of Figure 108, respectively, represents a "best" design associated with one or more of the feasible weighting vectors. These designs represent the mapped set of feasible weighting vectors onto the design space as the true preference from each of the decision makers from previous discrete choices in Step 1.

From before, DM_A prefers x_2 much more than x_1 with x_3 in between. DM_B has a near opposite perspective from that of DM_A. However, under the assumption that both prefer an agreement over no agreement both are willing, to different degrees, to reach consensus on some design likely in between their preferred regions.

To analyze at which designs reaching consensus is most likely, the initial preference vectors are randomly selected from among the set of weighting vectors whose corresponding designs are pictured in Figure 108. Concurrently, random values from the two related distributions of power relationships, namely from $P_{A\rightarrow B}$ and $P_{B\rightarrow A}$ in Figure 106, are selected providing the perceived power indices $P'_{A\rightarrow B}$ and $P'_{B\rightarrow A}$.

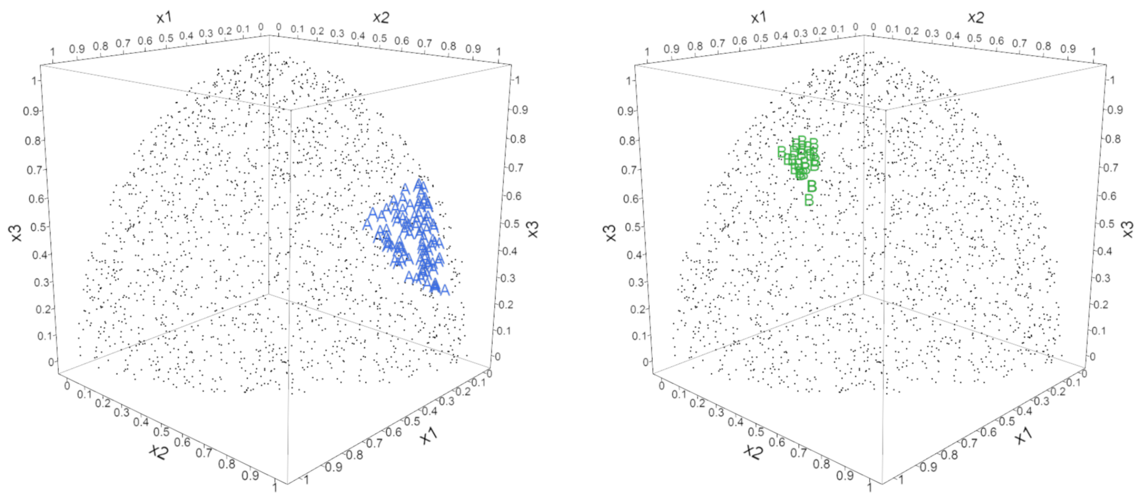


Figure 108: Feasible weighting vectors mapped to the design space for Decision Makers A and B

With the perceived power indices, one of the decision makers will propose to the other a mapped weighting vector (or a design) at which they would be in agreement. By assumption, the proposing decision maker will only propose a design which they would accept had the other proposed it. The other decision maker then evaluates the utility value (or the reduction in utility value) from their initial weighting vector and corresponding perceived power relationship. If the utility value is greater than what they can reasonably expect from the relationship (i.e. the offer exactly associated with the constraining power condition), they will accept the proposition and the consensus is reached.

This above process is repeated multiple times for randomly selected initial weighting vectors and power relationship indices. The count of how often each design becomes the point at which consensus was reached is recorded. Visualize these designs colored by the occurrences of consensus reached is illustrated in Figure 109. Designs which were never selected in the consensus reaching process are shown as small dark points. The designs with the highest number of occurrences where the two decision makers reached consensus ("consensus occurrence") are colored in red, with decreasing occurrences through a gradient of red, green and then blue.

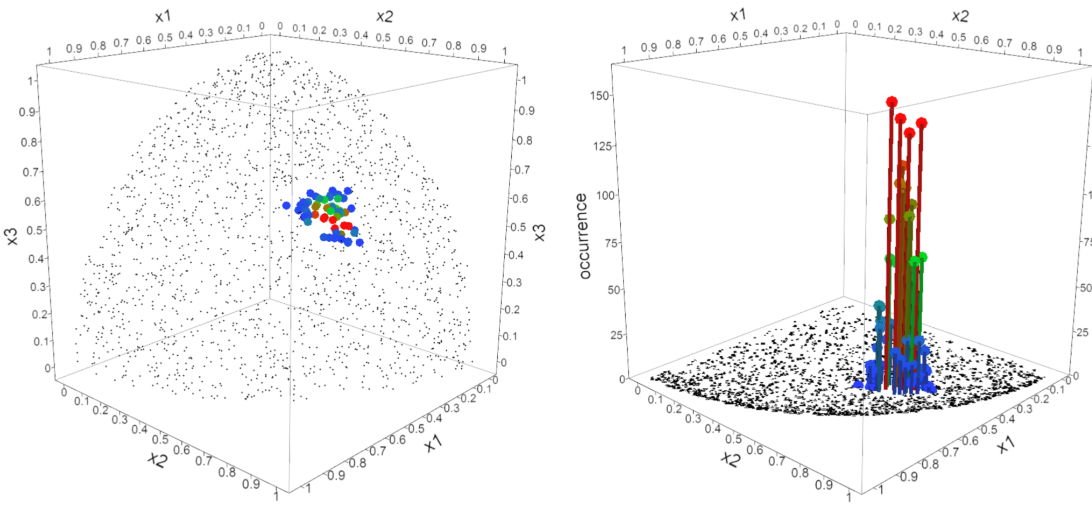


Figure 109: Left: Region of Design Space with consensus reaching. Right: Design Space Projected onto the x_1-x_2 plane vs. consensus occurrence number

The set of designs with consensus occurrence greater than one will generally fall between the designs representing the possible best weighting vector of each decision maker (compare Figure 108 to the left side of Figure 109). By projecting all the designs onto the x_1-x_2 plane, the consensus occurrence can be visualized in a type of 3-dimensional histogram using the third axis as the occurrence number (right side of Figure 109). The taller the vertical bars (or "redder"), the higher the probability that consensus is reached at that design (and thus at the complementary preference or weighting vector).

The one dimensional bar chart of the same data is shown in Figure 110, ordered by consensus occurrence. For these two decision makers, 60 designs were reached during the ultimatum game simulations, but only 30 labels (every other design) are listed on the x-axis. However, nearly 30% of all agreements are reached on only 5 of the designs. These are expectedly near the middle of the cloud of designs as shown in Figure 109. Statistics on the associated preferences relating to these designs

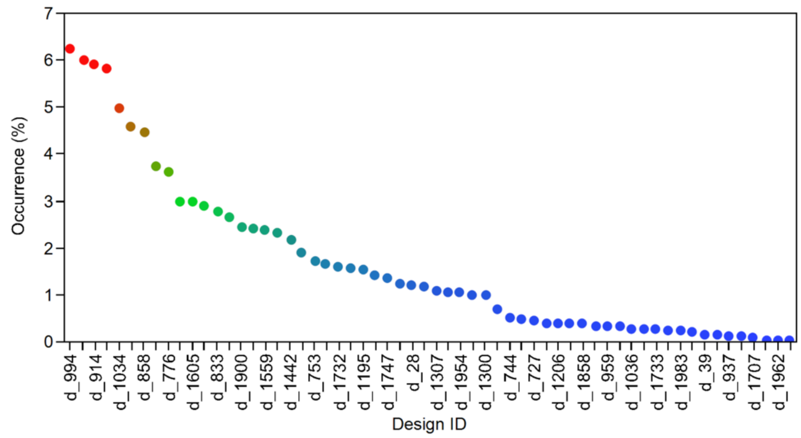


Figure 110: Designs sorted by the percent of all consensus occurrences between DM_A and DM_B after 5000 MC simulations

can suggest where preferences are most likely to coincide between these two decision makers.

As mentioned, these points fall between the initial set of designs preferred by DM_A and DM_B, but slightly closer to A's preferred designs. This is, of course, a consequence of the difference in power or influence these two decision makers have over each other. DM_A is able to convince or persuade DM_B to reach consensus on designs slightly closer to A's initial preferences. Although both must "give up" some utility in order to reach agreement, DM_A evidently will sacrifice less utility in terms of percentage compared to B.

If the ultimatum game was played between DM_A and DM_C, the shift would be even more skewed towards A's designs, since the influence of A over C is greater than that over B. The graph on the bottom of Figure 111 shows the output between these two decision makers. The top side of Figure 111 shows the respective regions that DM_A (top left) and DM_C (bottom left) preferred before reaching consensus is attempted.

The previous few figures only show the consensus region in terms of the design space. In the preference space, similar and additional observations can be made about

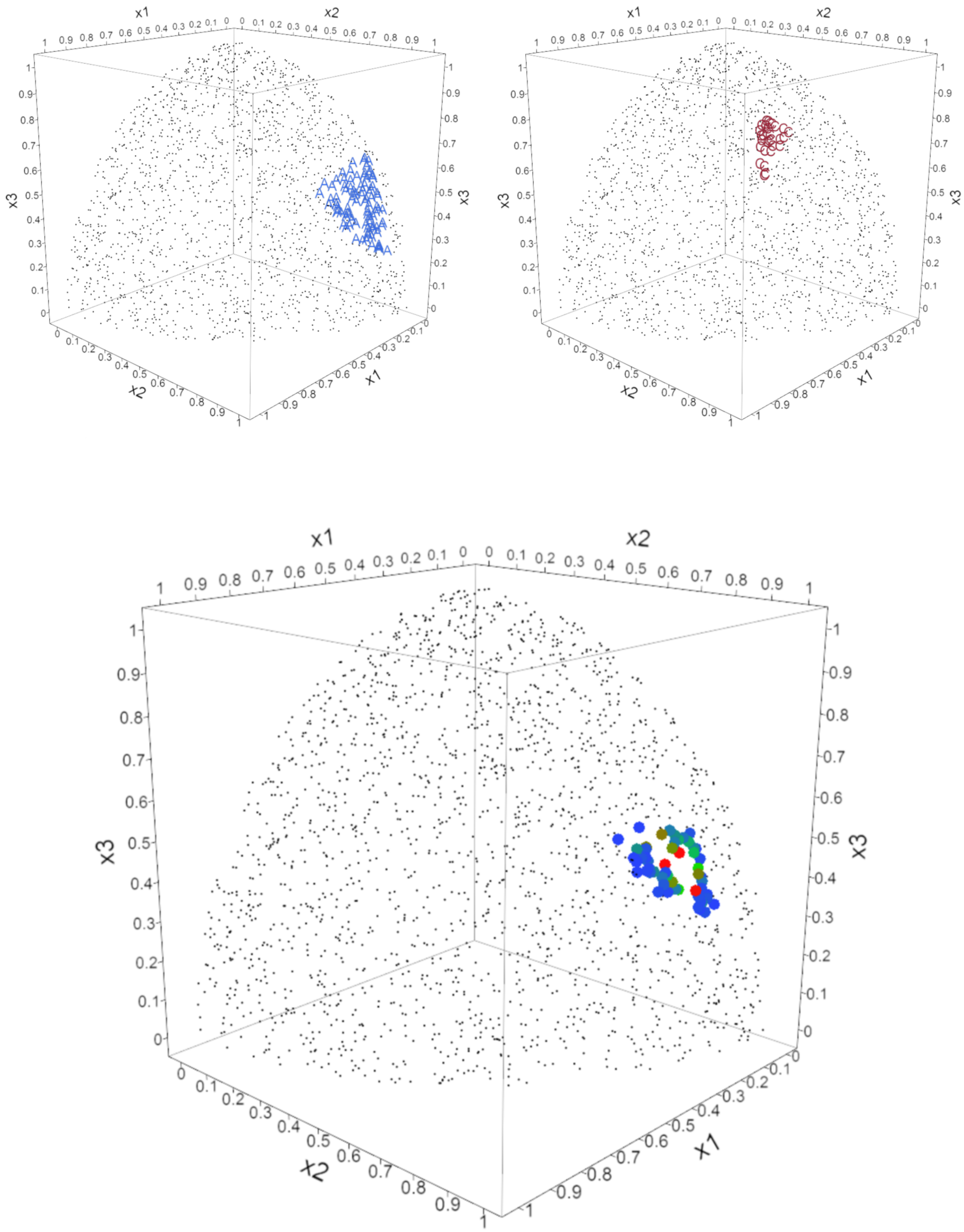


Figure 111: Region of design space with consensus reached between DM_A and DM_C

regions where consensus is most likely to occur. Figure 112 illustrates an example of the consensus region in the preference space. The original weighting vectors used for discrete choices from Step 1 in the methodology are shown as a reference plane. The two red regions on this plane indicate the set of weighting vectors that could represent the true preference of DM_A (right) and DM_B (left). The black cloud in the middle represent the set of weighting vectors at which decision makers reached agreement for at least one of the ultimatum games. As expected this cloud of points is close to the middle but slightly shifted towards A's preference weighting vectors. A magnified view of the same three regions with the reference plane removed is shown on the right of Figure 112. The discrete levels in the weighting vectors are visible along with the more random consensus region between them showing the lower density of points on the outer edges of the region. As mentioned, this set of 5000 weighting vectors converts to only 60 designs in the design space above. This is a consequence of multiple weighting vectors resulting in the same design point, since the resolution on the preference space can be much higher than the "resolution" ore density of the design space. If more than 2000 points existed in the design space or on the Pareto-front, then likely more than 60 designs would share in the points at which consensus occurred.

For the current example with three decision makers, the potential to form a coalition between any two decision makers is possible. Thus, the ultimatum game can be played between A and C or between B and C with different resultant regions of consensus. In Figure 113, these two other combinations are illustrated. Since both DM_A and DM_B have significantly more influence than DM_C the consensus region is much closer to A's (left hand side) or B's (right hand side) preferred region.

Comparing Figure 113 to the left hand side of figure 112, the three possible regions of consensus between any two of the decision makers take on different shapes and locations. As discussed previously, the power relationships will heavily influence

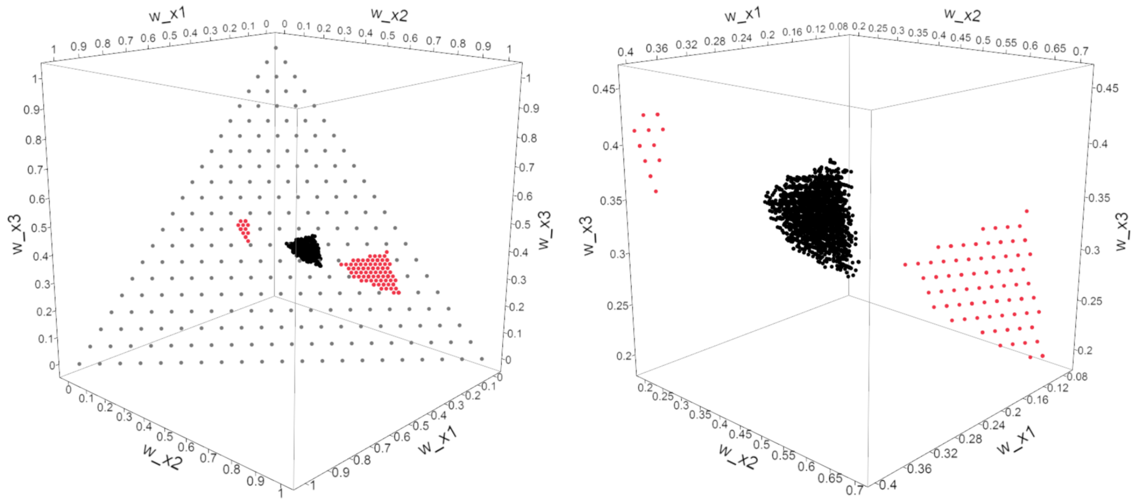


Figure 112: Consensus region in the preference space between decision makers A and B, with magnified view on the right.

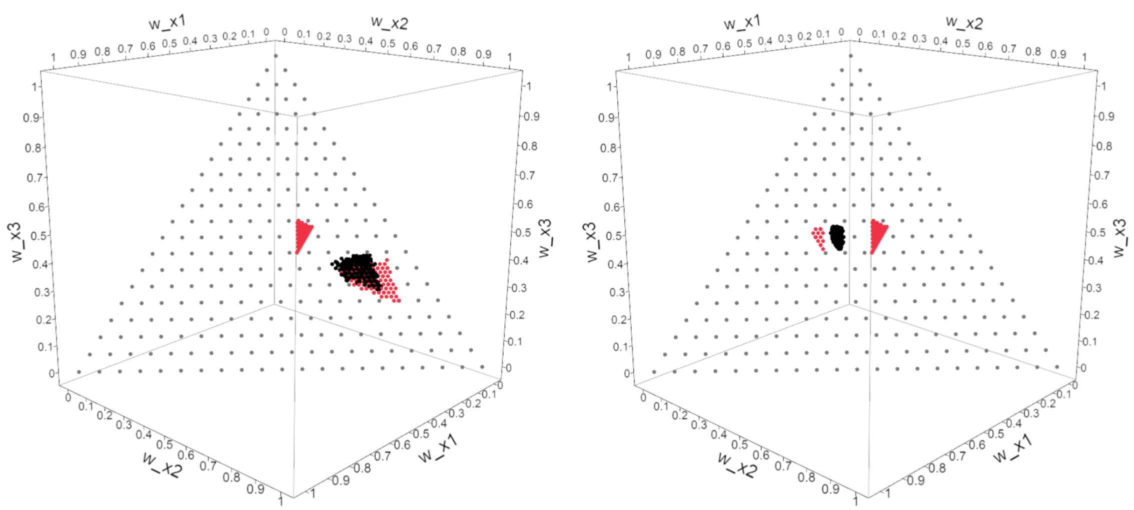


Figure 113: Region of consensus between DM_A and DM_C (left) and between DM_B and DM_C (right)

the relative "distance" between the initially preferred regions, but the shape and extent (or "size") of the consensus region is dependent on the range or uncertainty of the weighting vectors, which is in turn dependent on the number of discrete choices that each decision maker responded to within Step 1. If the randomly selected initial weighting vector comes from a smaller range, the consensus region will be expectedly smaller as well. In the current example the number of discrete choices presented to DM_A, DM_B and DM_C were 7, 5 and 8 respectively, which in part explains the differences in the "size" of the preferred regions but also the discretization or resolution of the set of weighting vector. However, the 7 randomly generated discrete choices for DM_A seemed to be, by chance, less effective in reducing the range of the weighting vectors compared to the 5 for DM_B when comparing the number of potential weighting vectors.

6.3.3 Sequence of N-player Consensus Reaching

The previous section discussed the points or rather the weighting vectors at which two decision makers will reach consensus. Although, only one weighting vector will be ultimately assumed and applied to a particular decision-making problem, identifying the range of these possible vectors allows one to analyze which vectors and thus which designs are more likely to be selected. This would not only potentially facilitate and accelerate decision making but also allow one to develop strategies for negotiating, cooperate more effectively or more precisely manage expectations.

After two decision makers have reached consensus, they have effectively traded utility for cooperation. In essence, they have become a unified decision maker with an updated set of vectors which represent their (now) new preferences. In other words, the consensus region between DM_A and DM_B becomes the preferences of the coalition DM_{AB} which now must reach consensus with the last decision maker DM_C.

Figure 114 shows this sequence pictorially where the consensus region between A and B is first evaluated and then that region (designated as AB) is used to reach consensus with C.

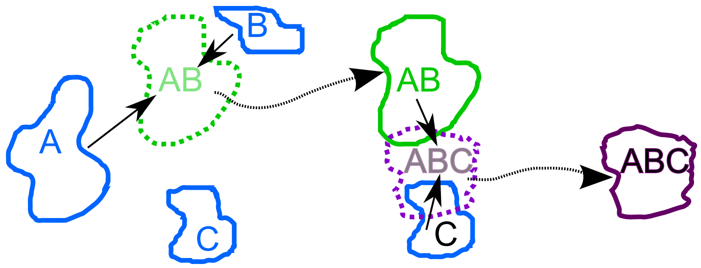


Figure 114: Pictorial example of sequence of regions through a consensus reaching process

With only three decision makers, there are only three sequences (with two consensus reaching stages) possible, namely: 1) A and B first reach consensus followed by AB with C, 2) A and C reach consensus followed by AC with B, and 3) B and C first reach consensus followed by BC with A.

Table 13: All Possible Sequences with Four Decision Makers

Sequence #	Stage 1	Stage 2	Stage 3
1	A—B	AB—C	ABC—D
2	A—B	AB—D	ABD—C
3	A—B	C—D	AB—CD
4	A—C	AC—B	ACB—D
5	A—C	AC—D	ACD—B
6	A—C	B—D	AC—BD
7	A—D	AD—C	ADC—B
8	A—D	AD—B	ADB—C
9	A—D	B—C	AD—BC
10	B—C	BC—A	BCA—D
11	B—C	BC—D	BCD—A
~~12~~	~~B—C~~	~~A—D~~	~~BC—AD~~
13	B—D	BD—A	BDA—C
14	B—D	BD—C	BDC—A
~~15~~	~~B—D~~	~~A—C~~	~~BD—AC~~
16	C—D	CD—A	CDA—B
17	C—D	CD—B	CDB—A
~~18~~	~~C—D~~	~~A—B~~	~~CD—AB~~

With four decision makers involved the number of sequences expands to 15 as many more combinations of coalitions are possible with additional sequences from the various orders. Table 13 shows the 15 sequences, each with three stages of consensus reaching. The symbol "—" separates the two decision makers or coalitions in each stage. Sequences 12, 15 and 18 are crossed off as they are repeated sequences of respective rows 9, 6 and 3. Therefore the 18-3 possible sequences accounts for the 15 unique sequences. Regardless of the number of decision makers, the number of stages is always $k - 1$, where k is the number of decision makers. The number of sequences increases much more quickly as a result of the various ways that 2, 3, ... $k-1$ decision makers can form coalitions in various orders.

Returning to the case with three decision makers, Figure 115 illustrates on the top one sequence (with stages B—C and then BC—A) within the preference space.

The black consensus region of the upper left graph (representing the stage B—C) becomes the preference region (in red) in the upper right graph (representing the stage BC—A).

On the bottom of Figure 115, the final set of design points upon which the entire group has reached agreement, is indicated on the bottom left, and the consensus occurrence is shown in the projected design space on the bottom right.

Assuming this "B—C, BC—A" sequence was in fact executed by the decision makers, the resultant design points of the final consensus region provide a set of designs from which these decision makers will "eventually" select. As mentioned above, this can potentially reduce the time needed to reach a decision as individuals realize that after multiple iterations, discussions, or negotiations, they still may "end up" in this region and so efforts may be more fruitful by only considering this smaller subset of designs, beginning at the designs most often reached during the consensus reaching simulation process.

Overlaying this consensus region with the mapped initial weighting vectors onto the design space of all of the decision makers results in Figure 116. Similar to previous

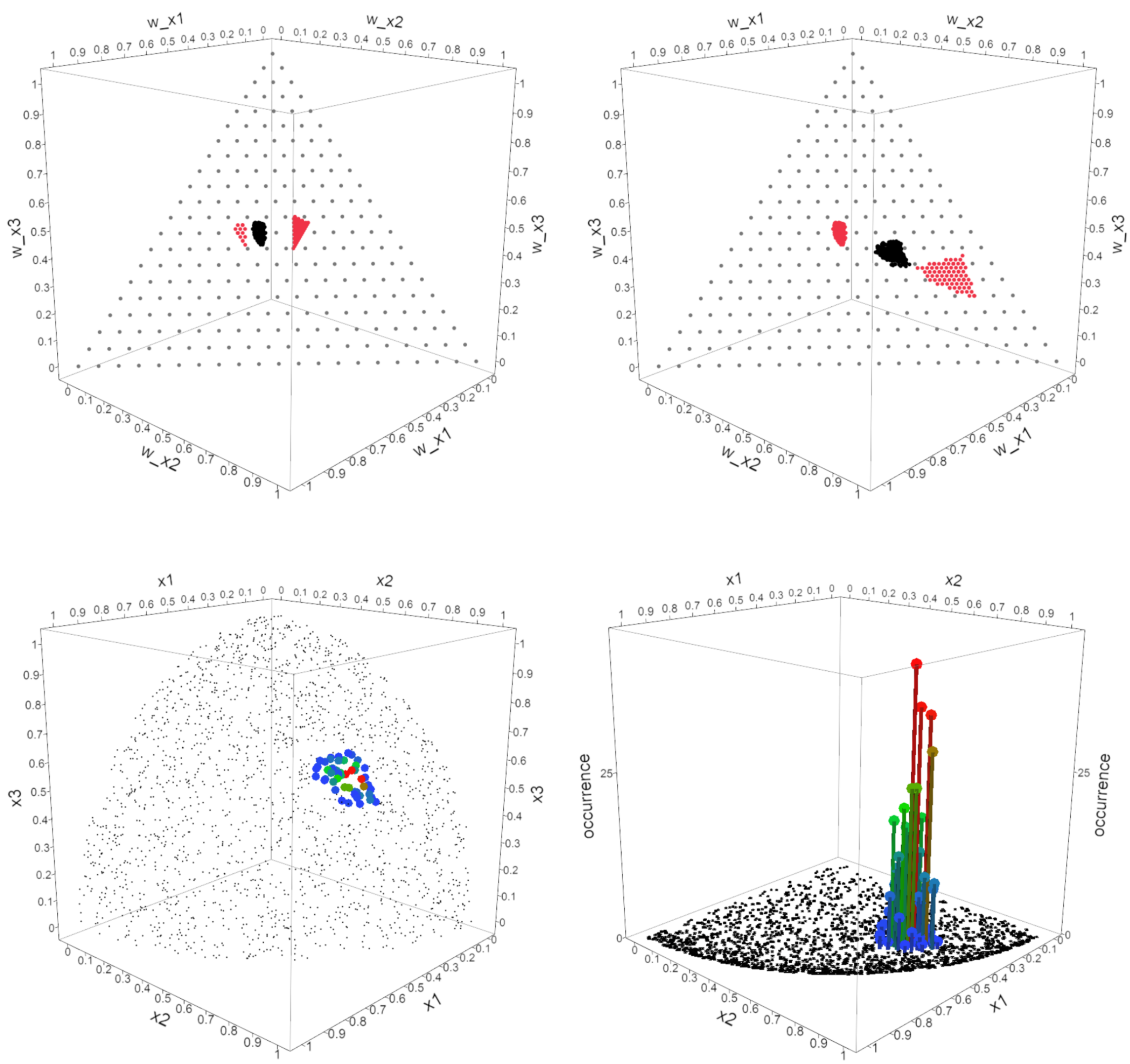


Figure 115: Consensus reaching sequence for "B—C, BC—A". Preference space of initial weighting vectors and consensus regions (top). Design space consensus region and number of occurrences onto projected design space (bottom).

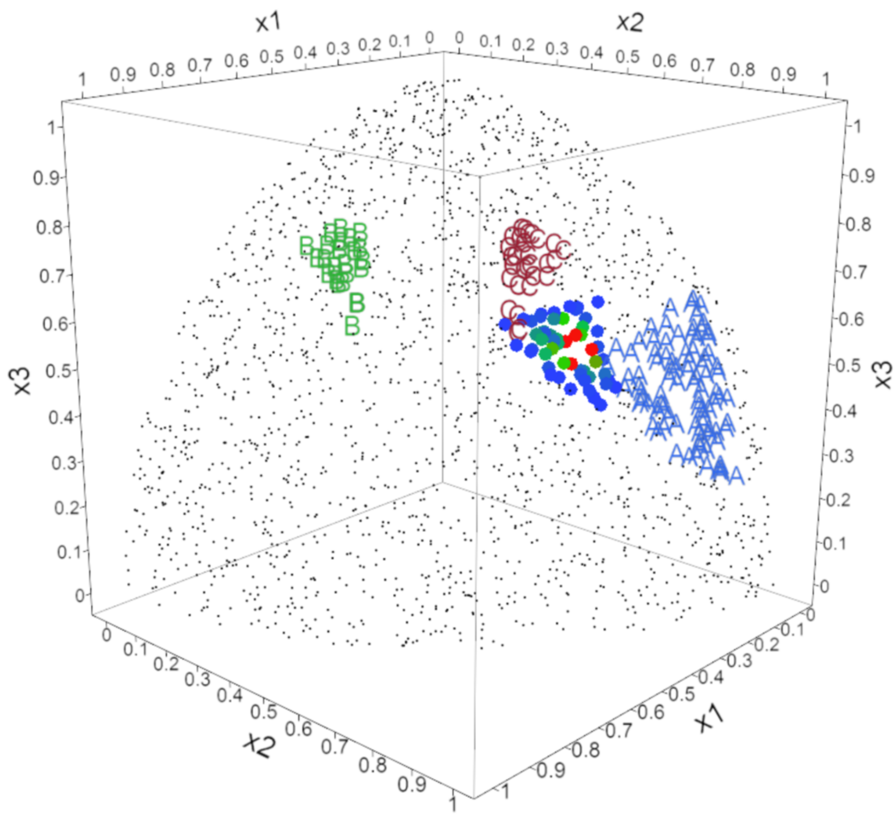


Figure 116: Consensus Region of sequence “B—C, BC—A”, overlaid with initial weighting vectors of all three decision makers

figures, the points that are most red are those designs at which consensus was reached most often.

This figure is contrasted against the other two consensus sequences in Figure 117 (i.e. "A—C, AC—B" and "A—B, AB—C").

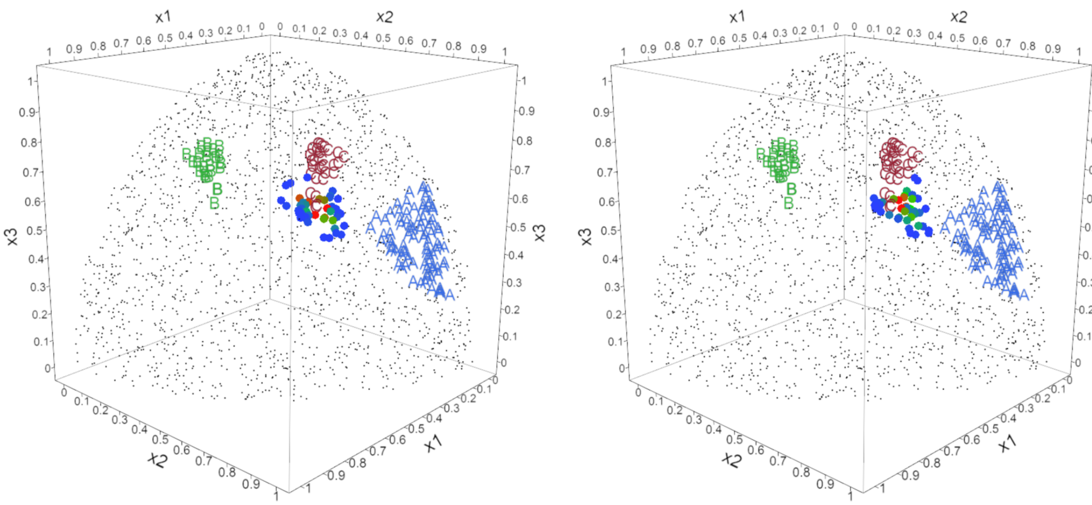


Figure 117: Consensus Region of sequences "A—C, AC—B" (left) and "A—B, AB—C" (right), overlaid with initial weighting vectors of all three decision makers

There are slight differences between the three sequences on account of the pooling and/or loss of power or influence over other decision makers through the appropriate consensus stages. Up until this point, the benefit of forming a coalition has been discussed only in terms of a requirement to eventually cooperate. However, combining a group's collective power to influence another decision maker is an even more motivating reason to cooperate and form coalitions.

In the first sequence (i.e. "B—C, BC—A"), DM_B and DM_C decide to form a coalition before either one of them does so with DM_A. If this is not in either of their best interests they would likely wait or seek some other combination (perhaps with A) resulting in a benefit to themselves individually. Since the desire for the overall methodology is to encourage cooperation and accelerate the decision-making processes through facilitating consensus reaching, the requirement that forming coalitions is desirable is needed. The appropriate research question follows:

Research Question: *How should power or influence be combined during the consensus reaching stage such that forming coalitions are advantageous to the individuals?*

The reversed question would be how to avoid penalizing a group for forming a coalition? Ideally, a reduction in influence of the group (or coalition) over the individual should not be possible. Similarly, some benefit of an increase in power over others should come as a result of, or an incentive for, forming a coalition.

In this current example, when DM_B and DM_C are unified and form a coalition, the power or influence relationships between them becomes irrelevant and are effectively removed from any further analysis. Assume that the power relationships for this example take on the values listed in Table 14.

Table 14: Power Relationships Between Three Decision Makers

Relationship	Value
$P_{A \to B}$	0.472
$P_{B \to A}$	0.528
$P_{A \to C}$	0.785
$P_{C \to A}$	0.215
$P_{B \to C}$	0.679
$P_{C \to B}$	0.324

If DM_B joined DM_C, together they would look to DM_B to influence DM_A since the influence that DM_C has over DM_A is minimal (0.215). The power relationship of B over A is $P_{B \to A} = 0.528$ which is much greater than 0.215. Therefore, after the stage one consensus, the relationships becomes $P_{BC \to A} = 0.528$ and $P_{A \to BC} = 0.472$.

DM_A loses their opportunity to influence DM_C, by remaining outside the coalition. DM_C reaps the benefits of joining DM_A which is having more influence over DM_A (through DM_B). Lastly, DM_B is able to express their entire influence onto DM_C ($P_{B \to C} = 0.679$) while removing the influence that DM_A would have on DM_B. If DM_B failed to form a coalition with DM_C, and instead DM_A and DM_C united,

DM_A would have been able to persuade DM_C to a region closer to DM_A than DM_B. Thus the consensus reaching in stage 2 would favor DM_A. The last sequence pools the influence of DM_A and DM_B, which together they would apply DM_A's influence over C more since $P_{A\to C} > P_{B\to C}$. Figure 118 summaries these three sequences.

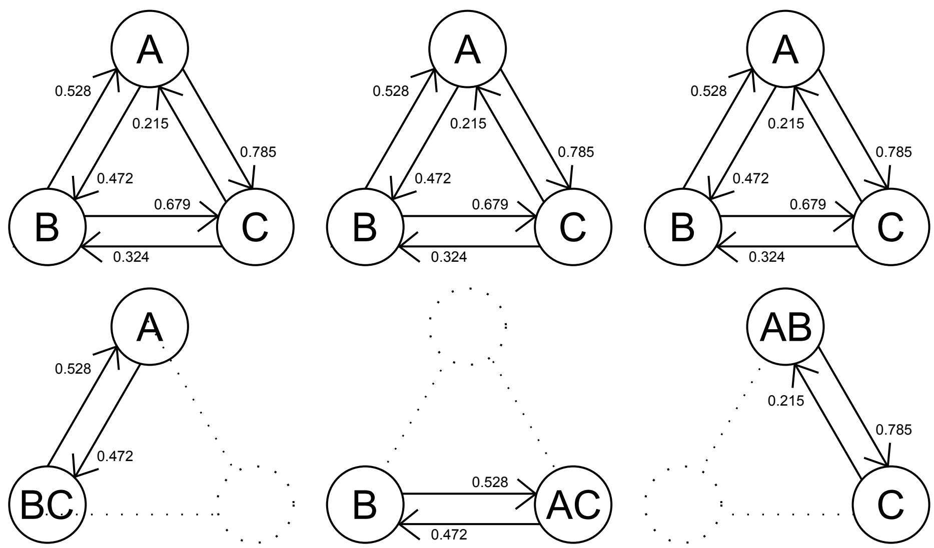


Figure 118: Three sequences with the power relationships indicated after stage 1. Left: "B—C, BC—A", Middle: "A—C, AC—B", Right: "A—B, AB—C"

6.3.4 Sequences with More than Three Decision Makers

More interesting combinations exist with more than three decision makers, since many more orders and coalitions can be formed. An experiment testing the impact of the order or sequence of forming coalitions with more than three decisions makers was performed.

While keeping the power relationships in Table 15 constant for more than one sequence, the consensus region was evaluated and visualized.

Even with the same power structure, based on the sequence the decision makers executed, the consensus region can be in significantly different areas. For example, if the sequences follow row 3 from Table 13 (i.e. "A—B , C—D , AB—CD") then the final consensus region for the group falls near DM_D's initially preferred region as shown in the left side of Figure 119. If the sequence executed is that from row 1

Table 15: Power Relationships Between Four Decision Makers

Relationship	Value	Relationship	Value
$P_{A\to B}$	0.890	$P_{B\to C}$	0.321
$P_{B\to A}$	0.110	$P_{C\to B}$	0.679
$P_{A\to C}$	0.380	$P_{B\to D}$	0.230
$P_{C\to A}$	0.620	$P_{D\to B}$	0.770
$P_{A\to D}$	0.871	$P_{C\to D}$	0.109
$P_{D\to A}$	0.129	$P_{D\to C}$	0.891

(i.e. "A—B , AB—C , ABC—D") of Table 13, then the consensus region touches and even overlaps some of the designs that DM_C initially preferred, as shown on the right hand side of Figure 119. As before the decision makers initially preferred regions or designs are labeled with the respective letter and colored accordingly.

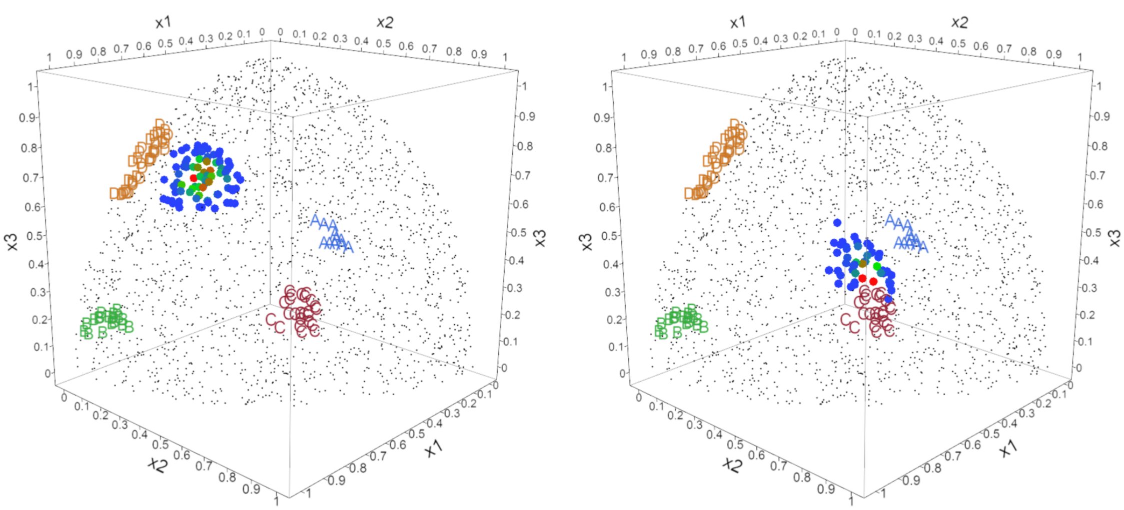


Figure 119: Consensus region of sequences "A—B , C—D , AB—CD" (left) and "A—B , AB—C , ABC—D" (right), overlaid with initial weighting vectors of all four decision makers

This experiment illustrates how the order of forming coalitions can result in different final consensus regions. Furthermore, analyzing the intermediate interactions is useful in understanding the dynamics within the group.

In the first sequence, DM_A and DM_B formed a coalition with DM_A having significant influence over B ($P_{A\to B} = 0.890$). The other two decision makers, DM_C and

DM_D formed a coalition with DM_D dominating the relationship ($P_{D \to C} = 0.891$). The benefit to DM_B in forming the coalition is the removal of both DM_C and DM_D influences over them. DM_B is relatively weak over all, but uniting with another decision maker early may help the final decision fall closer to their preferences and give them a little more "say" within their coalition. DM_C is the next weakest and formed a coalition with DM_D even though in this example DM_D's preferred region was the "furthest" away. The advantage in this relationship is more towards DM_D since DM_C holds a majority of the power between A and C. Since DM_D seeks this benefit, they quickly form a coalition (perhaps as a result of the influence D has over C) and then assume a power together over A of 0.620 (i.e. since, $P_{C \to A} = 0.620 => P_{CD \to A} = 0.620$). The final consensus stage has DM_{AB} with only 38% of the power over DM_{CD} and therefore the final consensus region is near DM_D's initially preferred region.

However, DM_C could have refused to form a coalition with D and instead unite with A, whose preference region is much "closer" to its own. This would have severe implications for the relationship between DM_{AC} and DM_D since $P_{A \to D} = 0.871$. No longer would DM_D have power over DM_C and therefore they would be required to concede significantly in the final consensus stage. This is in fact seen in the second sequence (right side of 119. As mentioned, DM_B is effectively a non-player in that their power over each of the others is less than 0.33 and thus they would likely join either A during stage 1 or AC during stage 2 without significantly altering the final result (which was shown to be true upon further experimentation). Since, the final consensus region lies very close to the initial preferred region of DM_C with the second sequence, and DM_C is aware of this, they will more readily form a coalition with DM_A before DM_D. Still, there are clearly reasons, events or additional factors for incentivizing DM_C to join D, but they would need to be significantly stronger than the benefits from joining A.

In general, however, coalitions are more likely to be formed with decision makers closer in terms of preferences (but not always) and with deceasing probabilities as the differences in the mean weighting vectors, and thus the difference in utility between decision makers, increases. The consequence of this assumption is that the above sequences will have different probabilities of being executed. For example, DM_C may be more likely to form a coalition with A than with D.

In the Step 3 algorithm for this methodology, the probability to initiate a coalition, or in other words to propose (in an ultimatum game) a design or preference to one of the other decision makers, is dependent on the utility of the designs of the others' mean weighting vectors, such that the probability:

$$P(DM_i(\text{proposer}) \text{ proposes to } DM_j(\text{responder})) = \frac{U(\bar{w_j})}{\sum_{m=1}^{k} U(\bar{w_m})}, i \neq m \tag{44}$$

where k is the number of decision makers, and i and j are the ith and jth decision maker respectively. In the above example, with four decision makers, the probability that C would propose to, and potentially form a coalition with D, evaluates to:

$$\begin{aligned} P(DM_C \text{ proposes to } DM_D) &= \frac{U(\bar{w_D})}{U(\bar{w_A}) + U(\bar{w_B}) + U(\bar{w_D})} \\ &= \frac{0.416}{0.585 + 0.454 + 0.416} \\ &= 0.286. \end{aligned} \tag{45}$$

A variety of other formulations are possible when defining a probability of selecting another decision maker such as a probability dependent on 1) the distance between weighting vectors, 2) the distance between the nearest design points of two preferred regions, or 3) the total group power at each consensus stage.

6.3.5 Effects of Power Relationship Changes on Consensus Reaching

Holding the power distributions constant and changing the sequence from above can also be reversed by investigating the effect when power distributions themselves change. This can occur when a decision maker is replaced by someone at some point during the decision-making process, who may have a different reputation, skill set, resource, etc. which changes the balance of power between decision makers.

For example, decision maker B was considered weak in terms of power over the other three decision makers in the previous section. If DM_B is replaced with DM'_B whose individual influence (for some reason) is greater than DM_B over the others, the results will be expectantly different even if the sequence executed is kept the same.

The two graphs in Figure 120 show the same sequence but with different power relationships for the four decision makers. On the left, DM_B is considered very weak and does not influence any decision maker more than 0.33 as described above. On the right, DM_B has significantly more influence only over DM_A but little on DM_C and DM_D. However, since the sequence under consideration has DM_A and DM_B forming a coalition in stage 1, the remaining stages are dominated by this partnership and therefore influence that DM_A has on the other two.

A recent example of changing decision makers and therefore changing power networks or relationships, in the middle of a decision process is the delay of the "sequestration" until March 2013 as part of the American Taxpayer Relief Act of 2012 passed on January 1, 2013 to avoid the so called "fiscal cliff" [161]. Since 33% of the US Senate and all 435 seats from the US House of Representatives were up for reelection in November 2012 many new individuals will became a part of the 113th United States Congress. Since the 112th United States Congress passed the Taxpayer Relief Act of 2012 two days before it ended, a new set of decision makers were put in place to grapple with major parts of the act currently used as negotiation tactics. A high chance exists that the influence or power relationships between individuals and

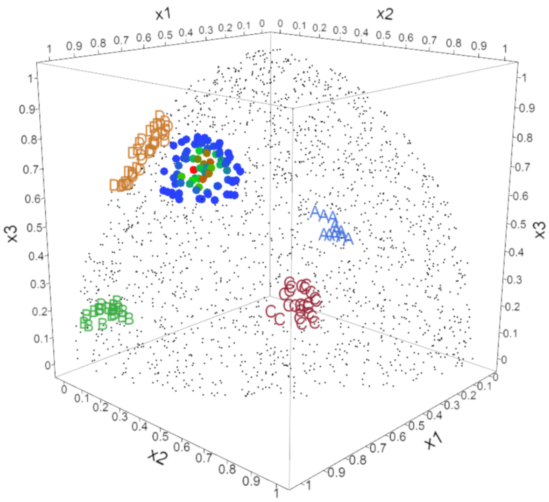


Figure 120: Consensus region of sequences “A—B , C—D , AB—CD” with DM_B “weak” (left) and “strong” (right)

across parties collectively had been altered significantly from 2012 to 2013.

Similar changes in power occur often on companies’ board of directors or on engineering project teams. Therefore accounting for changes in group decision making methodologies is requisite.

6.3.6 Multi-Dimensional Decisions with N-Players

Of course, in reality, problems will sometimes much larger than N-players or decision makers and the potential for many more dimensions than three. Both of these problem characteristics have the potential to change the accuracy and/or suitability of the methodology.

Research Question: *What is the impact of higher dimensional data sets with larger numbers of decision makers?*

In response to this question, and to support the overall hypothesis that this methodology is a valid process to facilitate decision making for multi-agents multi-objective decision making, the final set of experiments involves testing a similar canonical problem with 1) more than four decision makers and 2) more than a three dimensions.

Results from the first of these experiments are shown in the two figures on the next page. Figure 121 shows two simulation results with 10 different decision makers. Although the sequence executed by the collective decision makers is identical (i.e. the simple sequence "A—B , AB—C , ABC—D ...") the power relationships between the two simulations have been shuffled. The distribution means of the power relationships vary over a range from 0.5 to 0.95 in both simulations, but the assignments of these distributions to decision makers have been shuffled relative to each other.

In the first simulation (top of Figure 121), DM_I maintains a relatively high amount of influence despite the large group of 8 decision makers (A through H) with whom DM_I participates in a consensus stage close to the end of the sequence. In the second simulation (bottom of Figure 121), DM_I has significantly lower influence on the group DM_{A-H} at the second to last consensus stage and impacts very little the final consensus region. A similar situation is found for DM_J who has little influence at the end of the same sequence.

The last experiments explore higher dimensional data sets or design spaces. Most likely a real decision problem will include more than three objectives and thus testing if the methodology scales with dimensions or objectives is required.

Initially, the set of designs on the Pareto frontier assume points on a four dimensional hypersphere, where $x_1 ... x_4 > 0$. For the 4D case, the initial preferred regions along with the consensus region can still be partially visible in some visualizations of the design space. In Figure 122, three decision makers (A,B and C) reach consensus as indicated in the central region highlighted in dark blue. The other three initially preferred regions are similarly highlighted, with points as indicated from before. The graph in the top left shows the 4-dimensional design space in the $x_1 - x_2 - x_3$ coordinate system. The other three graphs show the same data and regions from three additional perspectives, namely, the top (along the x_3 axis), the left (along the x_1

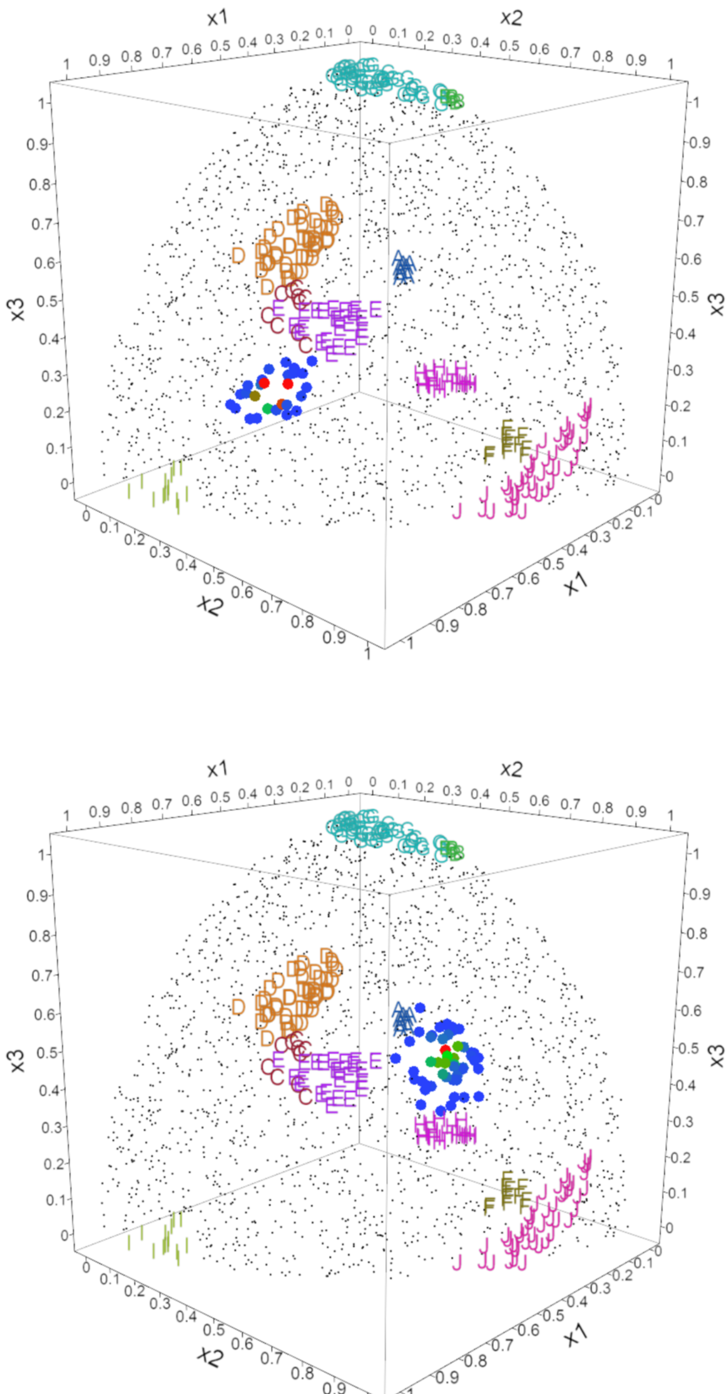


Figure 121: Consensus Reaching Examples with 10 Decision Makers (A-J). Power relationships for the two above simulations are the same relative values between the range of 0.5 and 0.95 but are shuffled with respect to each other.

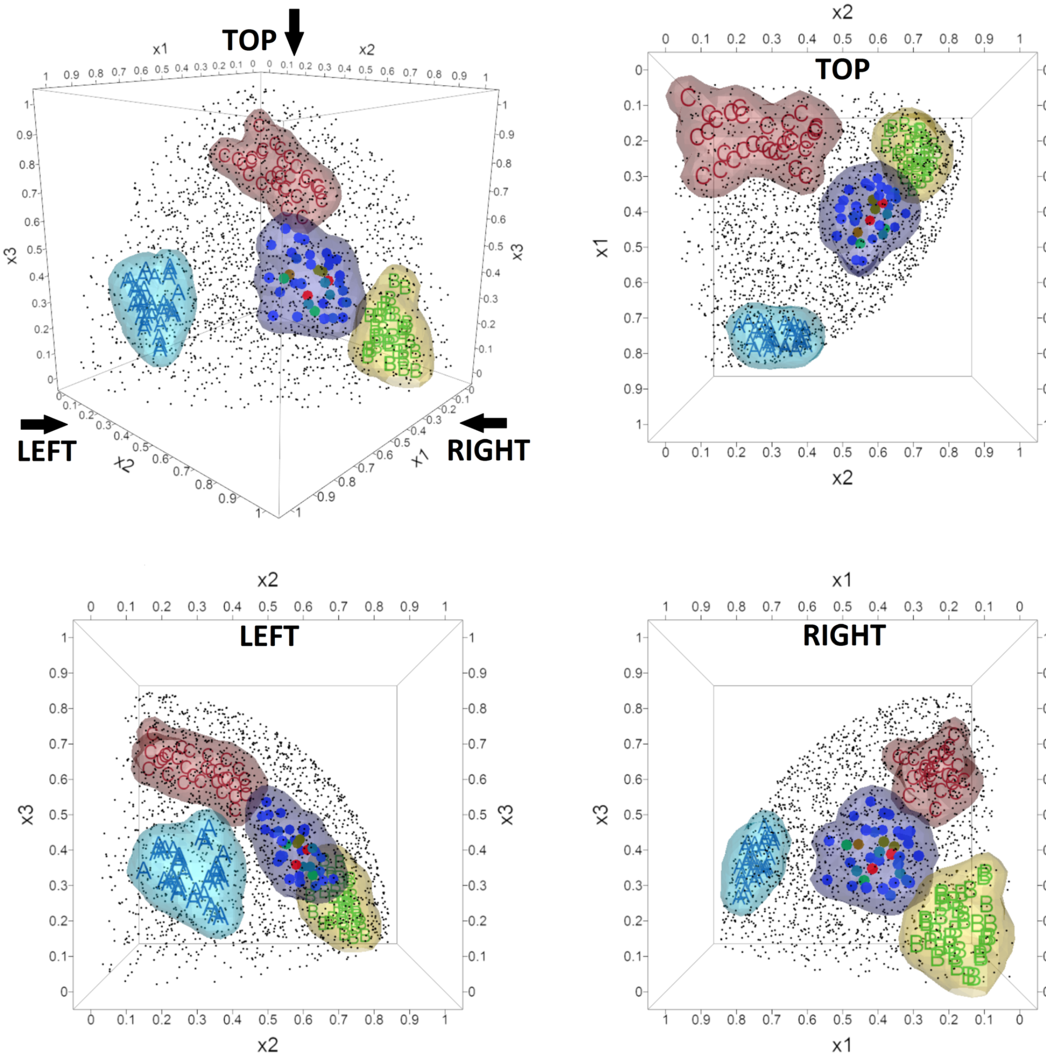


Figure 122: Consensus reaching examples with four dimensions in a 3-view visualization.

axis and the right (along the x_2 axis) as indicated. Since all points lie on the 4D Pareto frontier, the consensus region will not lie on the 3D Pareto frontier of only x_1, x_2 and x_3 as illustrated by a difference or offset from the "apparent 3D frontier" to the consensus region.

In general, more than 4 objectives could be included in the decision problems and therefore the application of a scatterplot matrix illustrating the same features but from a full set of 2D scatterplots as shown in Figure 123 is often more insightful. (Decision maker labels but not colors have been removed for clarity).

Similar to the apparent offset in Figure 122, in every individual scatterplot, the consensus region lies away from the local 2D Pareto frontier. If the consensus region falls onto a local lower level Pareto frontier the region would suggest that potentially one of the objectives is sufficiently insignificant and can be removed from the analysis with an effective weight of 0.

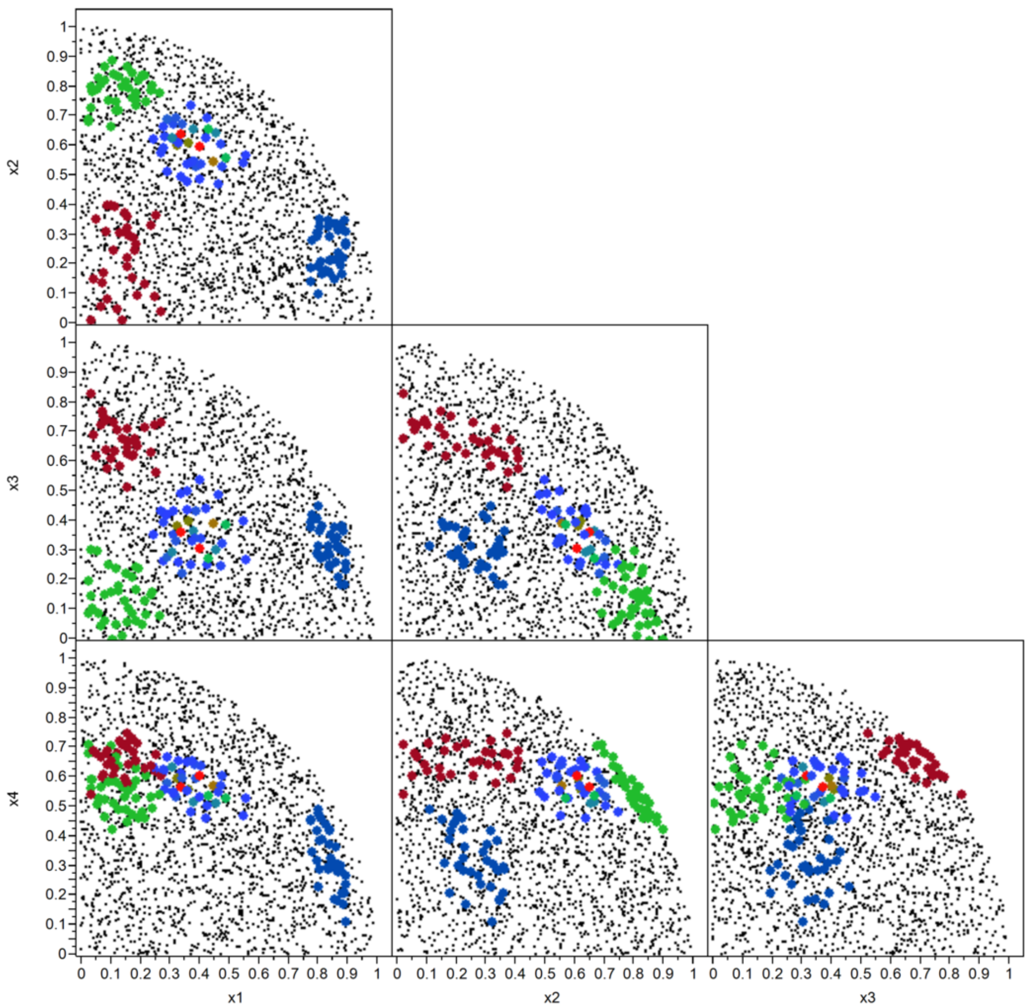


Figure 123: Consensus Reaching Examples with Four dimensions in a Scatterplot Matrix Visualization.

CHAPTER VII

CASE STUDY: IMPLEMENTATION OF MACRO TO AIR MOBILITY FUTURE SYSTEMS

This chapter implements and applies the MACRO methodology to a data set of future air mobility solutions generated by AirMOD. A case study is used to demonstrate and test the methodology on a notional example with five decision makers seeking to reach consensus on the requirements for a large future air mobility system. Each decision maker possesses differing views and preferences on the objective space and each hold varying levels of power or influence over the others.

7.1 Converting AirMOD Metrics to Utilities

In preparation for the application of the MACRO methodology, an essential step of processing the AirMOD simulation data involves converting the various metrics to a common or commensurate unit such that they can be combined for an overall evaluation or utility function.

The simplest approach involves non-dimensionalizing each metric of interest after which a simple additive function can be formulated. Equation (47) below shows one such method for non-dimensionalizing the variables and combining them into an OEC.

$$\begin{aligned} OEC_i = \quad & \alpha[\tfrac{MTTC_i}{\max(MTTC)-\min(MTTC)}] + \\ & \beta[\tfrac{SDTTC_i}{\max(SDTTC)-\min(SDTTC)}] + \\ & \gamma[\tfrac{TFH_i}{\max(TFH)-\min(TFH)}] + ... \end{aligned} \tag{46}$$

where i is the ith design or solution point, the coefficients $\alpha, \beta, \gamma ...$ are the weightings for each dimensions and sum to 1, $MTTC_i, SDTTC_i, ...$ etc. are the ith values for that dimension, and the denominators in each term represent the range from the lowest to the highest value of that dimension. Dividing by the same units as the numerator will non-dimensionalize each term such that the addition becomes a commensurate operation.

If a decision maker prefers reducing the time to close more than reducing the total flight hours, they would set $\alpha > \gamma$ in the above OEC. Since, for the three dimensions shown above, a decision maker would likely seek to minimize all of them, the operation design with the lowest OEC would be the best. If the decision maker sought to maximize one of the objectives, say payload per flight, then the ratio would be inverted such that another term in the equation would be $... + \delta\frac{\max(PPF)-\min(PPF)}{PPF_i} + ...$ and the OEC could continue to function as a minimization problem.

The full OEC minimization problem becomes:

$$
\begin{aligned}
min\{OEC_i = \quad & w_{MTTC}[\tfrac{MTTC_i}{\max(MTTC)-\min(MTTC)}] + \\
& w_{SDTTC}[\tfrac{SDTTC_i}{\max(SDTTC)-\min(SDTTC)}] + \\
& w_{TFH}[\tfrac{TFH_i}{\max(TFH)-\min(TFH)}] + \\
& w_F[\tfrac{F_i}{\max(F)-\min(F)}] + \\
& w_{BR}[\tfrac{BR_i}{\max(BR)-\min(BR)}] + \\
& w_{PPF}[\tfrac{\max(PPF)-\min(PPF)}{PPF_i}] + \\
& w_{NA}[\tfrac{NA_i}{\max(NA)-\min(NA)}] + \\
& w_{MP}[\tfrac{MP_i}{\max(MP)-\min(MP)}] + \\
& w_{OC}[\tfrac{OC_i}{\max(OC)-\min(OC)}] + \\
& w_{AC}[\tfrac{AC_i}{\max(AC)-\min(AC)}] + \\
& w_{BRC}[\tfrac{BRC_i}{\max(BRC)-\min(BRC)}] + \\
& w_{UT}[\tfrac{|(UT_{target}-UT_i)|}{\max(UT)-\min(UT)}] + \\
& w_{ER}[F(ER_i)]\},
\end{aligned} \tag{47}
$$

where, MTTC is the Mean Time to Close, SDTTC is the Standard Deviation on the Time to Close, TFH is the Total Flight Hours F is the Fuel Consumption, BR is the Break Rate, PPF is the Payload Per Flight, NA is the Number of Aircraft, MP is the Maximum Payload, OC is the Operating Cost, AC is the Acquisition Cost, BRC is the Cost to Reduce the Break Rate, UT is the Utilization, and ER is the En Route Type Selection.

Of the 13 objectives, only one, PPF, is inverted such that the larger the PPF the better. This may not necessarily be true since one may prefer smaller aircraft (i.e. perhaps because of the smaller unit cost). This objective illustrates that some of the potential objectives are not completely independent since PPF is clearly correlated with AC.

In fact, many of the objectives are dependent on another dimension. Although, an OEC can arbitrarily keep correlated objectives at the same time it is desirable to minimize the set of objectives to the least number while still capturing the most significant trades in the decision space. This can keep the problems manageable, especially when human decision makers involved who struggle to process multiple dimensions at the same time [108].

A correlation matrix was analyzed and PPF was found to be highly correlated to MP with $\rho = 0.85$. Furthermore, AC was correlated exactly with both PPF and MP since this CER uses the weight ratios directly. TFH was also highly correlated to OC and fuel consumption ($\rho = 0.94$) so only one of these three is really essential for a comprehensive equation. This reduces the equation by 4 terms to a set of 8 which include all costs, namely, MTTC, SDTTC, BR, OC, AC, BRC, UT, and ER. Although, these dimensions can be considered from one perspective unessential in calculating the OEC, the display of this information will be significantly more important within discrete choice experiments given to decision makers discussed later.

The term which includes Utilization (second to last) illustrates a slight difference in comparison to the other terms. A target utilization is established (i.e. UT = 8) and any solution which varies significantly away from the target value is considered a worse design (within that dimension).

Another problem from Equation (48) is found in the last term which can be understood as some mapping function that converts the selection of En Route Type used (i.e. no refueling, en route refueling, retro en route refueling) to some value based on the preferences of the decision maker. For example, if a decision maker prefers longer hauls, (i.e. a preference for Type 1) then the function output for a Type 2 or Type 4 design would be higher:

$$F(ER_i) = \begin{cases} A, & \text{if } ER_i = 1, \\ B, & \text{if } ER_i = 2, \\ C, & \text{if } ER_i = 3, \\ D, & \text{if } ER_i = 4, \end{cases} \tag{48}$$

where, $A < C < B < D$ and $0 \geq A, B, C, D \geq 1$.

Lastly, the normalization itself is somewhat unsatisfying. For example, completing the mission in a time to close of 20 days may be good whereas a time to close of 40 is unacceptable. On the other hand, reducing the time to close by 5 (i.e. reducing MTTC to 15) is much better but reducing it another 5 days may not be as useful compared to the first "5 day reduction." In the equation discussed above, a "5 day" reduction will contribute to the OEC in equal amounts regardless of the initial MTTC. Whether it is from 65 to 60 days or from 10 to 5 days, the OEC will decrease by the same absolute amount. However, often those two situations are drastically different, the former may be unacceptable before and after the reduction, while the latter is unnecessary or perhaps even too optimistic.

Many of these problems can be solved by converting the various objective ranges into utility curves which can capture other effects such as the law of diminishing returns or utility along some of the objectives.

For a particular mission payload, a constraint can be assumed that any time to close greater than 30 days is unacceptable. Consider, for example, a situation where significant amounts of supplies are needed in the few weeks after a major natural disaster in a highly populated area and that supplies (after 30 days) will be too late to help. From this perspective the utility is maximized (i.e. 1) at the minimum amount of time to close. The utility will likewise be 0 for MTTC of 30 days and later. In between these extremes the utility curve for MTTC will likely take on a concave shape as shown in Figure 124, where a decrease in the same amount of MTTC maps to a decreasing rate of utility increases (e.g. for 30 days to 20 days the utility may jump from 0 to 0.7, but from 20 days to 10 days, utility would increase from 0.7 to 0.8). The straight diagonal line would represent the utility of implementing the OEC equation described above with a constant slope between MTTC and the OEC value (utility).

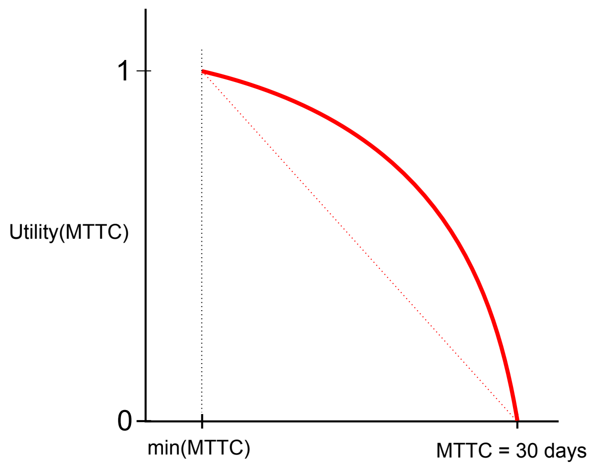


Figure 124: Notional Utility Function for the MTTC Objective

The details for this process to define utility functions are not included here but are documented in other sources such as [86]. However, a summary of the functions used to transform the above objectives into utility curves for this research is found in Table 16 and in the subplots of Figure 125.

Uncertainty in the time to close (SDTTC) would likely exhibit the same trend as the utility function for MTTC. That is, the marginal utility after the first improvements for the same change in SDTTC may not be as valuable. Furthermore, if the MTTC has already decreased by a significant amount, say by a full standard deviation lower, the decision maker may become less concerned about meeting the time constraint regardless of any improvements in SDTTC, resulting in a less amplified utility curve (in terms of concavity) for SDTTC.

For the cost objectives, utility (U()) will surely decrease as cost increases with the

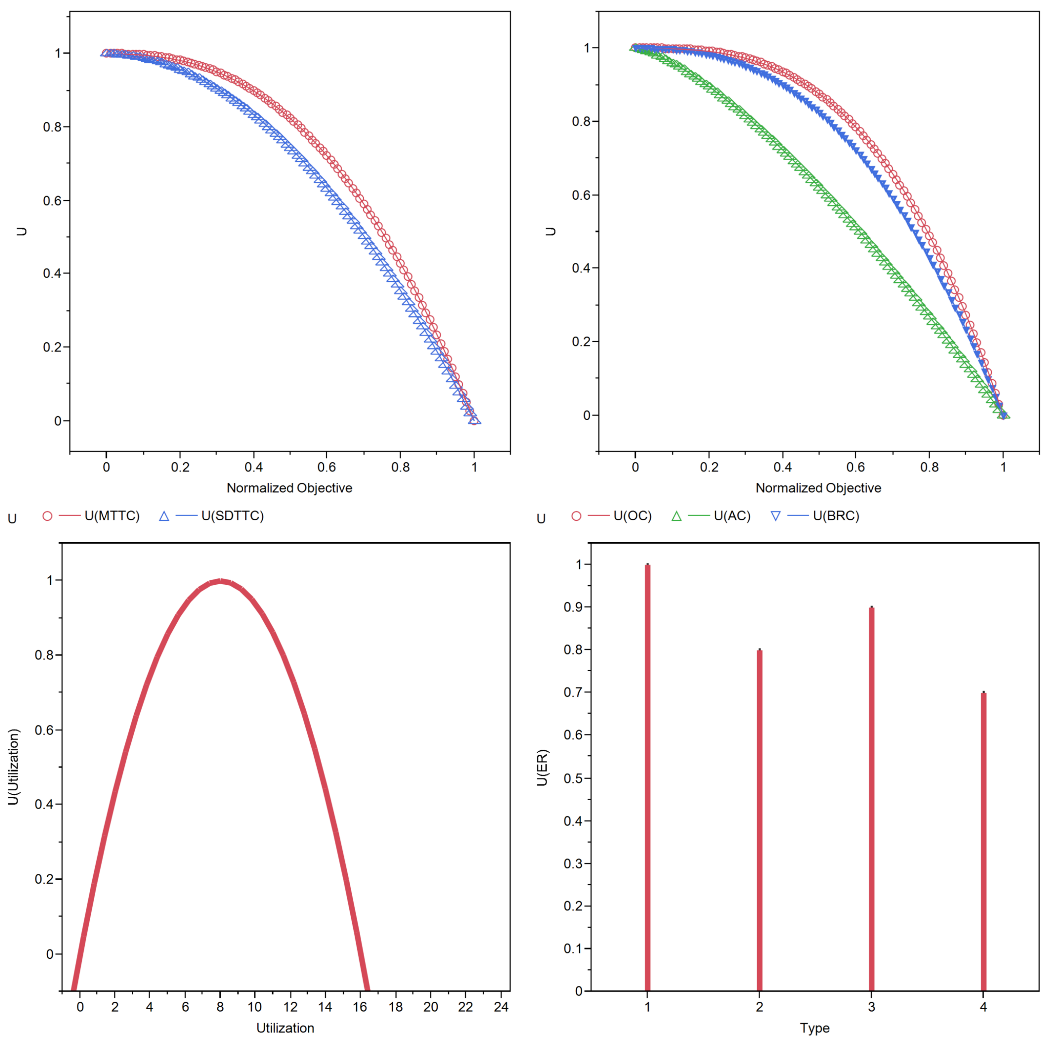


Figure 125: Utility Curve/Functions for the Operational Objectives

Table 16: Operational Objective Utility Functions

Variable	Variable Name	Utility Function
MTTC	Mean Time to Close	$1-(\frac{MTTC-\min(MTTC)}{\max(MTTC)-\min(MTTC)})^{2.5}$
SDTTC	Std. Dev. of Time to Close	$1-(\frac{SDTTC-\min(SDTTC)}{\max(SDTTC)-\min(SDTTC)})^{2}$
BR	Break Rate	$1-(\frac{BR-\min(BR)}{\max(BR)-\min(BR)})^{2.5}$
OC	Operational Cost	$1-(\frac{OC-\min(OC)}{\max(OC)-\min(OC)})^{3}$
AC	Acquisition Cost	$1-(\frac{AC-\min(AC)}{\max(AC)-\min(AC)})^{1.4}$
BRC	Cost to Reduce Break Rate	$1-(\frac{BRC-\min(BRC)}{\max(BRC)-\min(BRC)})^{2.5}$
UT	Utilization	$1-0.015625(UT-8)^2$
ER	En Route Selection Type	1 if Type = 1, 0.8 if Type = 2, 0.9 if Type = 3, 0.7 if Type = 4

similar concave shape again but to a different degree. Since AC (acquisition cost) is so large compared to OC (Operating Cost) in absolute terms, the concavity is likely to be less than that of U(OC). Regardless of the current price, reducing it by a million dollars translates to an almost equal marginal utility since the unit price is so large (e.g. $250 million). Operating Cost, on the other hand, has significant impacts throughout the life of the aircraft beyond the mission operating cost considered in this model. Therefore, a small decrease in OC, would have large benefits into the future over many years, resulting in a utility curve with high concavity. The cost to reduce the break rate also has long term benefits but likely not as extensive as the operating cost.

The break rate will follow this established pattern as well, but the utility for utilization will differ in that a target utilization may be preferred over simple minimization. In this case, the utility is maximized at the target utilization and falls off quickly at UT values greater than 16 or less than 1 (underused and overused conditions respectively). The utility function for the en route selection type is even more unique. With the assumption that less flights, more direct paths, and less stops are preferred over the other options, the general utility function will take on particular values for each of the 4 types.

These equations will be applied to calculate the utility for each design, by combining the individual objectives (transformed into utilities) in an additive weighted sum similar to the example problem from the previous chapter. The equation becomes:

$$\begin{aligned} U_i = \quad & w_{MTTC}U(MTTC_i) + \\ & w_{SDTTC}U(SDTTC_i) + \\ & w_{BR}U(BR_i) + \\ & w_{OC}U(OC_i) + \\ & w_{AC}U(AC_i) + \\ & w_{BRC}U(BRC_i) + \\ & w_{UT}U(UT_i) + \\ & w_{ER}U(ER_i) \end{aligned} \tag{49}$$

where, in this formulation, the decision maker seeks to maximize their utility (U_i).

7.2 Step 0: Decision Maker Definition

With the final utility maximization problem established in the previous section, the final step before implementation of the consensus reaching methodology is to define the decision makers themselves by designating the truth model of their individual preferences across the 8 objectives.

In selecting a program or solution to replace or enhance the current heavy airlift US military fleet of aircraft, a variety of key stakeholders would be heavily involved in the process to make decision worth billions of dollars over the life of the program. In creating this scenario, an arbitrary number of five individual players or agents have been identified that would need to agree upon the ultimate solution and reach consensus. Although more decision makers could be included within the analysis, the approach itself is not limited in this regard and these five represent a sufficiently diverse set of backgrounds, desires, expertise, and view points within the context of the problem to demonstrate the methodology.

The five agents or decision makers are as follows:

1. Chair of the Joint Requirements Oversight Council (JROC)
2. Representative of the Government (GOV)
3. Operations Director from USTRANCOM (OPS)
4. Industry Partner/Prime Contractor(IND)
5. Chief Technology Officer of the Prime Contractor(CTO)

Other stakeholders are clearly involved in the process, albeit in different roles, of selecting a design or defining requirements for such a program but some of these could be understandably grouped with one of the five "individuals" listed above, and would act in concert with them to a large degree.

The following descriptions summarize some of the backgrounds and perspectives these individuals may possess and thus what might be preferred by these five decision makers. These descriptions are purely for illustrative purposes only and should be considered notional, but have been established to provide a sense of reality to the problem. Similarly, the preference model they provide, (i.e. weighting vectors representing their true preferences), will be assumed, by mapping the subjective goals and ideals of each decision maker to the weights in each of the 8 objectives required by the analysis performed in the succeeding sections.

1. Council Members of JROC, (Vice Chiefs, Generals, etc.) from the various military services would likely take a mission-focused view in comparing various solutions. Completing the mission proficiently and following commands from

the commander-in-chief would dominate their preferences. As a result, time to close (mapped to MTTC importance) and the certainty that the mission could be achieved (SDTTC becomes important) within a particular time constraint would be of the utmost importance from their perspective. Clearly, doing so efficiently and inexpensively (i.e. cost metrics are important) would be accompanying goals to the capability priorities. Safety of the service men and women would also be high on the list of objectives which would translate to secure flights paths of various missions (i.e. ER metric) to avoid non-allied airspaces.

2. A representative from the government (GOV) , such as a congressman sitting on the U.S. Senate Appropriations Subcommittee on Defense or even the Secretary of the State, would potentially have a much wider set of objectives as compared to the JROC or other military officers. Since they represent the people of the United States, a more intense cost to benefit analysis would be required. Careful accounting of how tax dollar brought in are spent on defense spending has ramifications on other social programs and even on individual political careers, and therefore the costs (i.e. Acquisition Cost - AC and Operating Cost - OC become important) would likely be the most important objectives from a governmental point of view. Sensitivity to this bottom line will therefore be high, but the details about how the mission is accomplished may be less important to the public sector (lower important on Utilization, Break Rate, Repair Times). Lastly, supporting the troops during wartime and responding quickly (importance mapped to MTTC and SDTTC metrics) to natural disasters with strategic airlift capabilities is not only philanthropic but also politically wise.

3. An Operations Director (OPS) at USTRACOM or AMC would be in charge of the logistics surrounding the completion of a mission - where the aircraft are stationed, how quickly they can respond and how many are needed for each

mission. Fulfilling these goals requires properly using aircraft which have a high availability and reliability (importance mapped to UT and BR). In performing their duties, having extra aircraft can act as a safety net with low reliability aircraft. Not only can inherent redundancy improve with larger fleet sizes, but under extreme shortages cannibalization of parts can be less detrimental. Furthermore, larger aircraft might also be preferred. Reducing the flights and the flight hours, the associated crew costs, maintenance costs and in general, the operating cost without sacrificing throughput capacity would certainly be one goal for an operations planner or analyst. These ideals translate to setting a high importance on objectives related to completing the mission (importance mapped to MTTD and ER) and keeping the operating costs down (importance of OC).

4. In industry (IND), where shareholders ultimately defining the one governing objective of a company, profit, maximizing the objectives aligned with that one goal are likely to be heavily weighted, namely, the acquisition cost (AC) and the number of aircraft (This equates to a high cost/profit, which is mapped to a high capacity and throughput, and thus low importance on the price). Since the company will design, build, upgrade, manufacture, maintain, etc. the new C-X platform, the desire to maximize both revenue and profits can be expected. This type of preference structure would, of course, favors large purchase orders (i.e. high number of aircraft) with an ideally accompanying as high a unit price as possible (mapped to AC). Furthermore, training, maintenance and other life cycle contracts (mapped to OC) could be sought which would also help the bottom line if a desire existed on the side of the government to outsource some or many of these operating costs. Finally, since carrying capacity and performance is the ultimate product offered by the company, the weighting on the activities of research and development mapped through improving the reliability would

be high, such that the importance on the BR metric is also high.

5. Finally, the CTO, Chief Scientist or some other technologist from within the company or another subcontractor may prefer the challenge of designing, building and flying a new, larger and more reliable aircraft. Likely the design would require research and technological breakthroughs, and provide security to a plethora of other engineering, scientific and research jobs for the near future. From this technologist standpoint, the RDT&E funds which invest in reliability research and subsystem improvements would be most attractive (i.e. importance mapped to Break Rate - BR and the cost to improve reliability - BRC). Furthermore, implementing these new technologies without significantly increasing the unit price would also be of importance (importance mapped to AC).

Taking the above descriptions into consideration, and translating goals and desires into weights of the 8 objectives, the preference structure for each of the decision makers can be defined as shown in Table 17.

Table 17: Decision Makers' Preference Truth Model

DM	MTTC	SDTTC	BR	OC	AC	BRC	UT	ER
JROC	0.3	0.15	0.05	0.1	0.1	0.05	0.05	0.2
GOV	0.15	0.05	0.1	0.2	0.25	0.05	0.05	0.15
OPS	0.15	0.1	0.1	0.3	0	0	0.15	0.2
IND	0.4	0.2	0.2	0.05	0.05	0.1	0	0
CTO	0.2	0.2	0.35	0.05	0.1	0.1	0	0

The above table will be assumed to represent the preference truth model required from a simulation perspective for Step 1. The responses to discrete choices will be in accordance with these weightings even though the agents would not necessarily be able to reproduce these themselves without considerable difficulties or uncertainty. If Step 1 were to be performed in reality, the preferences would, of course, be derived from the discrete choices themselves and there would be no need for a simulation truth model.

The MC simulations for the 8 dimensional decision space are shown in Figure 126, with the solutions colored by the total utility using the function coefficients with the truth weights of DM_{GOV}. The point with the highest utility is selected and colored in black with decreasing utility values colored across a gradient from dark red to blue.

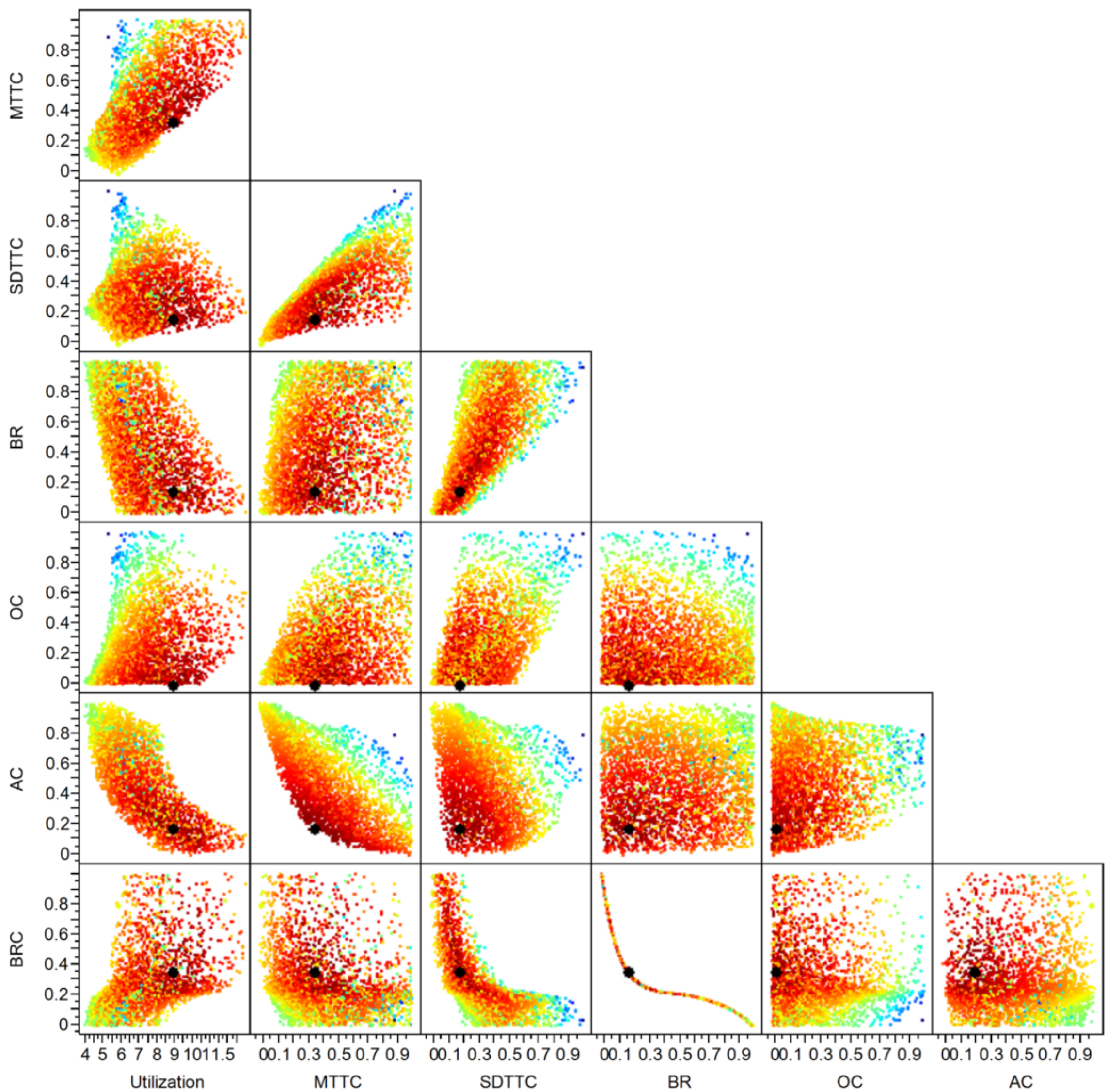


Figure 126: AirMOD Decision Space colored by utility from truth weights of DM_{GOV}

Figure 126 actually only shows 7 dimensions since the 8th (i.e. Type) was defaulted or filtered to only Type 1 mission scenarios. Also, the individual metric ranges have been normalized to a range of 0 to 1 within the particular APOE-APOD Type 1 mission scenario which includes post-processing and removal of all MTTC designs with values greater than 40 days to close. Since DM_{GOV} has a relatively uniform preference structure (i.e. [0.15 0.05 0.1 0.2 0.25 0.05 0.05 0.15]) compared to the other decision makers, the solutions or designs with the highest utility are not generally found on the edges of the 8D space.

An example of this occurs when the preference structure or weighting vector favors one, or just a few, of the objectives. For example, if "money was no object" and the decision maker only cared about completing the mission as quickly as possible (low MTTC) and with as little uncertainty as possible (low SDTTC), the weighting vector would be [0.5 0.5 0 0 0 0 0 0]. This would result in Figure 127 where the 8D utility space is shown and the U(MTTC) is almost 1 and the U(SDTTC) = 1. Interestingly, this also results in a very low break rate (i.e. high U(BR)) but an associated very expensive cost to improve this Break Rate. (In fact, the point selected has a BRC of $46 billion dollars!) Of course, spending a lot of money on increasing reliability has a direct impact on reducing MTTC and SDTTC, so those output metrics are consistent. In terms of the other costs, they take on extreme values as well. Having a larger and more expensive aircraft (made even more expensive with a larger fleet size) also helps with completing the mission more quickly. Again, costs are high for exceptional performance. This is acceptable if one was not to weight AC or OC very important, but those, in reality, drive or often constrain the performance metrics.

Turning now to the requirements of Step 2, additional inputs are needed only within the simulation mode of this methodology. Or, in other words, computer simulation requires these inputs whereas actual human responses to the Step 2 discrete choice experiments would provide this data.

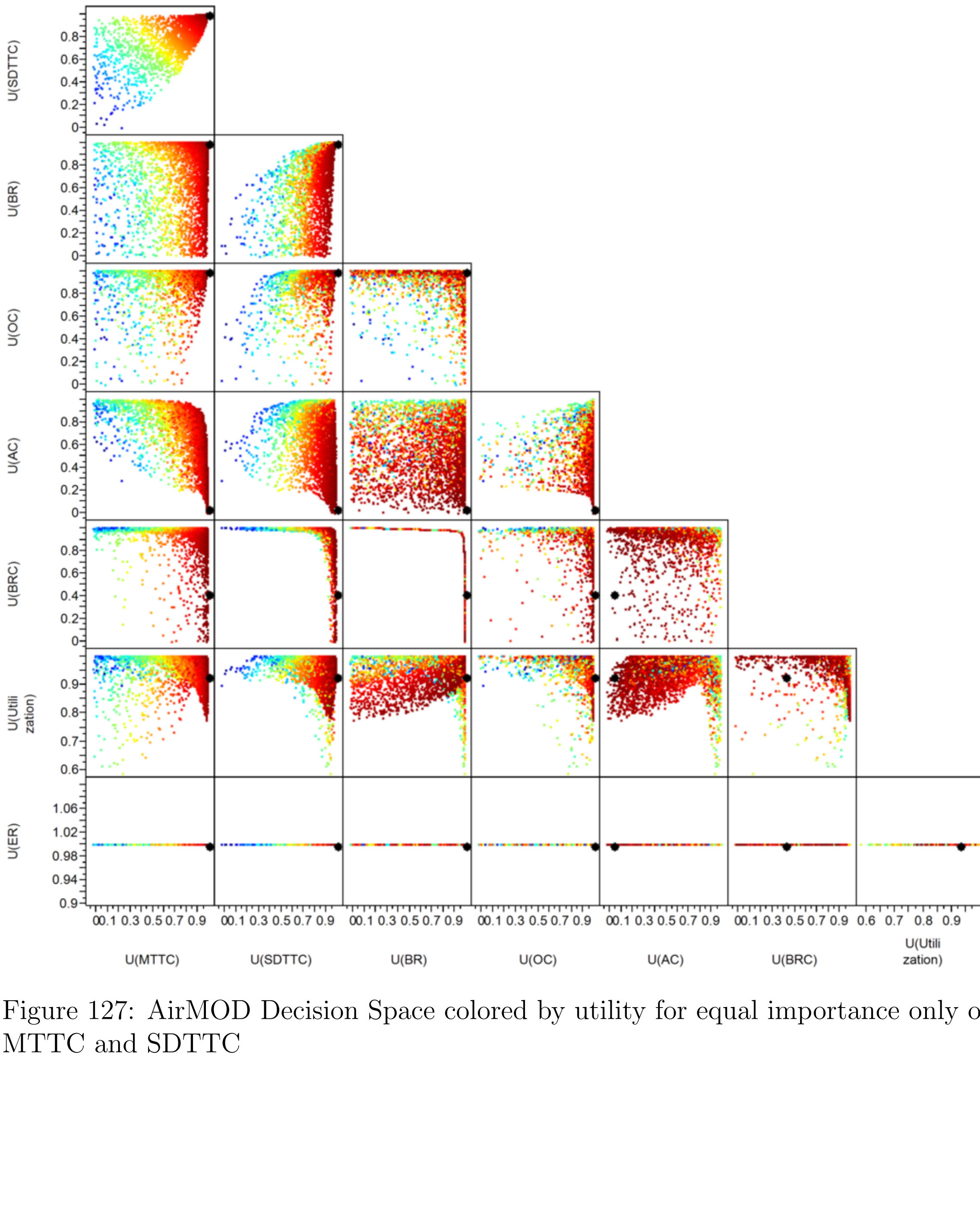


Figure 127: AirMOD Decision Space colored by utility for equal importance only on MTTC and SDTTC

Since universal cooperation is a key assumption for any solution in the current scope of this research, if any one of the players disagree with the final decision the program would not be considered "selected". That is, agreement must be reached by all parties to adequately define the region or points of consensus. For example, it is unlikely the CEO of the prime contractor will agree to a requirement if internal experts (e.g. CTO) do not think a particular requirement is possible at the negotiated price. Similarly, a congressman is unlikely to allocate funds if a military general or other JROC official does not approve of, or will not use, a specific design.

These relationships between the decision makers are an essential component to quantifying the power or influence that one may have on another in Step 2 of the approach. Since the decision makers and thus their perceived influence on each other must be modeled as inputs to the simulation, the following table. (Table 18 will serve as the perceived influence truth model in the methodology.)

Table 18: Decision Makers' Perceived Influence Truth Model

Perceived Influence of...	... on JROC	... on GOV	... on OPS	... on IND	... on CTO
JROC	-	0.6	0.7	0.7	0.5
GOV	0.6	-	0.6	0.8	0.5
OPS	0.6	0.4	-	0.4	0.3
IND	0.5	0.6	0.4	-	0.8
CTO	0.4	0.4	0.4	0.7	-

The table is read by selecting a decision maker down the rows, and then identifying the perceived influence they think they have over each of the other four in the table columns. Therefore, DM_{JROC} (military decision makers) has a perceived influence over the government official. DM_{JROC} may rationalize that since they with the services are the actual users of the asset and may know more about what level of capacity is truly needed, DM_{GOV} would be more willing to listen to their opinions. On the other hand, DM_{GOV} considers themselves as in control of the budget and may expect DM_{JROC} to "fall in line" as the defense arm (i.e. an appendage) of the government.

The other indices of power or influence could be defended with other notional explanations: Considering themselves as the customer, DM_{GOV} would perceive a high level of influence over DM_{IND}, but perhaps not so much over the contractor technical employees DM_{CTO}. DM_{CTO} might view themselves as able to persuade their CEO or DM_{IND} to consider things their way, but probably none of the other three. Lastly, DM_{OPS} would likely be able to persuade, remind, or convince the DM_{JROC} regarding their own military logistics and operations preferences but would not necessarily have as much influence over the others.

Table 18 contains examples for all three of the "constrained" (e.g. $P_{OPS \to GOV} + P_{GOV \to OPS} = 1$, "over-constrained" (e.g. $P_{JROC \to GOV} + P_{GOV \to JROC} > 1$) and "under-constrained" (e.g. $P_{IND \to OPS} + P_{OPS \to IND} < 1$) power conditions established in the previous chapter.

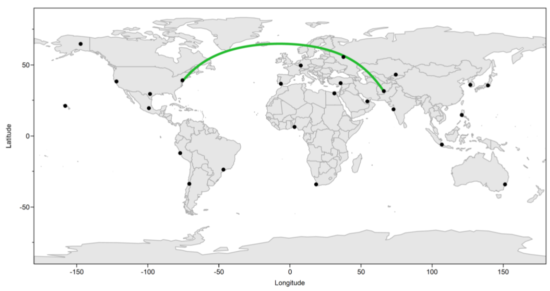
7.3 Step 1 Results: Calculating Weighting Distributions

The notional decision makers defined in the previous section are now given a set of discrete choices and required to designate which one they prefer to extract the weighting distributions. An example discrete choice experiment for this test problem is found in Figure 128.

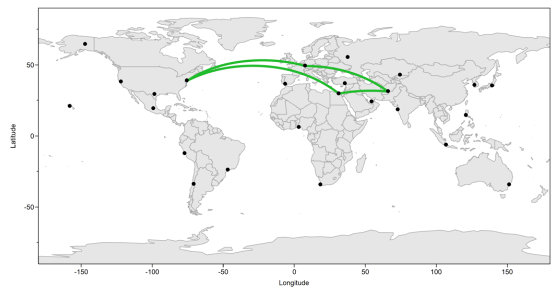
In general, some of the same scenario parameters will be similar for a valid discrete choice such that a decision maker can more easily make a comparison and select a preferred design. Therefore, in the discrete choice of Figure 128, the Mission payload is set to 20 million pounds for each and the APOE-APOD combination is the same as well. However, the types between two discrete choices need not be identical. In order to compare one's preferences for types across mission and the example in Figure 128 shows a discrete choice comparing a Type 1 solution to a Type 4 solution.

Which design do you prefer? A or B?

Solution/Design A



Solution/Design B



Metric	Design A	Design B	Units
Mission Payload	20	20	million lbs
Mean Time to Close (MTTC)	35.75	8.95	days
Std. Dev. TTC	7.8	8.1	days
Utilization	11.2	5.6	hours/day
Break Rate	21.3	20.7	%
Cost to Reduce Break Rate	13.2	13.5	$billion
Flight Hours	6027.3	2066	hours
Total Operating Cost	126.3	43.3	$million
Payload per Flight	47602	271628	lbs
Empty Weight	444758	386803	lbs
Fleet Size	15	41	
Acquisition Cost (per unit)	282.9	246	$million
Total Acquisition Cost	4.24	10.09	$billion
	Design A?	Design B?	
Preferred Design? ☑	☐	☐	

Figure 128: Air Mobility Example of a Discrete Choice from Step 1

If the mission payload is not similar, additional adjustments to the metrics themselves are required. For example, if the mission payload was 30 million pounds for one and 20 million pounds for the other, some of the metrics would have to normalized based on these different mission payloads. Thus, the time to close would be expressed in a "time to close per million pounds" or the flights hours would need to be expressed in "flight hours per million pounds," etc. This could cause unnecessary cognitive burden on decision makers trying to balance and make trades across these normalized or ratio-based metrics.

Thus, for any one discrete choice the scenario will be identical for the APOE-APOD combination and the mission payload. However, it is still valid to vary those parameters from discrete choice to discrete choice since extraction of weighting distributions is the intermediate objective for Step 1, and not necessarily to identify the precise solution.

The design parameters at the bottom illustrate some of the trades discussed in previous sections. The fleet size is 15 to 41 for Design A and B respectively and so total acquisition price is almost $6 billion greater for design B.

In fact, for Design A the aircraft individually are much larger and more expensive by about $37 million. They also carry much less payload per flight (almost one sixth that of Design B) as a result of an extremely long mission leg with no refueling stops. Since the flight hours are almost triple, the operating costs are correspondingly about three times as much. The break rate is approximately the same with the same level of investment for higher reliability. The SDTTC is also about the same on account of the comparable break rate, but the MTTC is much longer for Design A (36 days vs. 9 days). If time to close does not matter, then saving money with a smaller fleet size is attractive. But if a time constraint does exist, and funds are available to reduce meet this time target, then a larger fleet is an valid option. Of course, one major down side to Design B is the multiple stops in two other countries. Although Design

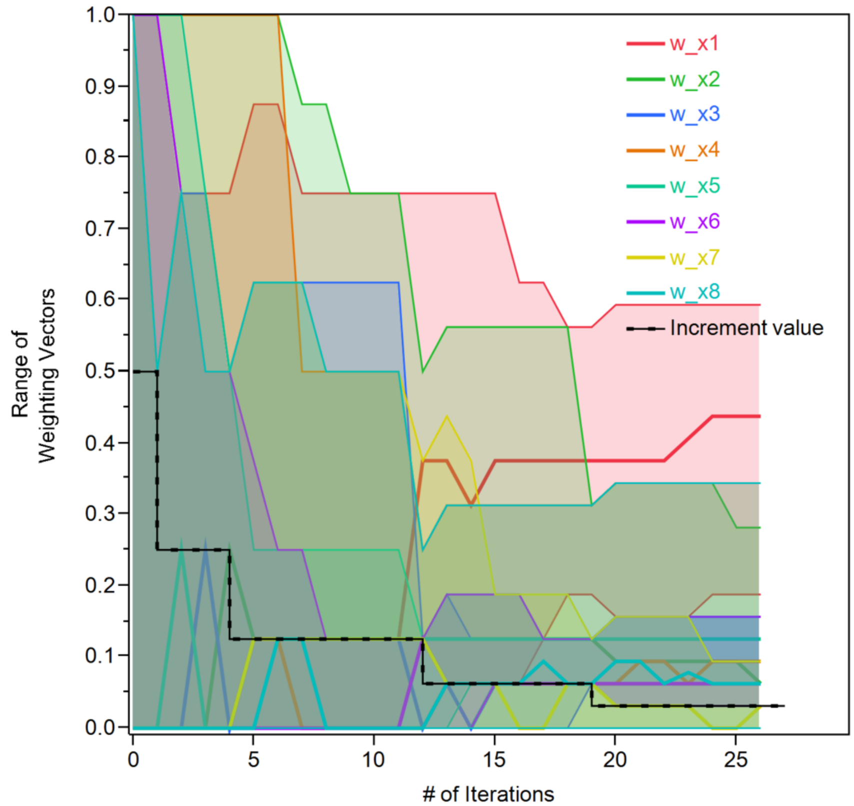


Figure 129: Reduction in Weighting Vector Ranges for all Objectives through a Series of Discrete Choices in Step 1

B closes the mission much more quickly, the cost associated with maintaining bases overseas may further add to the already more expensive design of the two solutions. The general trade is relatively clear: increased performance is more expensive. By responding to a set of these discrete choices, however, the preference of the decision maker between performance and cost can be obtained.

The outputs from Step 1 when the decision makers have responded to a set of discrete choices are the distributions of the weightings for each of the 8 objectives.

When simulating Step 1 with a set of discrete choices with DM_{JROC} the weighting distributions are summarized in Figure 129.

The range for each of the 8 objectives starts from 0 to 1 as in the canonical problem from the previous chapter. The increment value starts very large (0.5) but immediately after the first iteration a refinement step is executed with four more throughout the full set of 25 discrete choice experiments. Although w_{x_1} is clearly the most important objective as discovered by the simulation with a median weighting value near 0.45 after 25 discrete choices, the other 7 objectives occlude each other, and in particular the ones which end with lower weightings. To remedy this problem, these range histories are translated along the y-axis for easier comparison in Figure 130 on the following page. The objectives from x_1 to x_8 follow the same order as above such that x_1 = MTTC, x_2 = SDTTC, x_3 = BR, x_4 = OC, x_5 = AC, x_6 = BRC, x_7 = UT, and x_8 = ER.

Step 1 in the methodology appears to accurately predict the weighting distributions of JROC decision maker, DM_{JROC}, where the median of w_{x_1} (w_{MTTC}) is the largest of the 8 objectives (≈0.45), followed by a relatively wide range for w_{x_2} (w_{SDTTC}), w_{x_4} (w_{OC}) and w_{x_8} (w_{ER}). This generally coincides with the largest values for DM_{JROC}'s preference truth model. However, the median value for w_{x_1} (≈0.45) appears larger than that expected (0.3) while the other three are lower.

To aid in analyzing these findings, for any particular simulation, the maximum, median and minimum values for each objective are contrasted with the preference truth model used to model the decision maker. Figure 131 illustrates how the weighting distribution range spans the truth model for all objectives. The predicted (P) range on the left hand side of each objective in Figure 131 contains the three values, maximum, median and minimum in a boxplot with the mean for only those three points designated with the horizontal line. On the truth (T) side for each objective, the boxplot is compressed down to the one data point with the mean value designated as well. Comparing the median values to the truth data point with a correlation value provides a $\rho = 0.8$, suggesting that for at least this decision maker, Step 1 of the methodology functions reasonably well in predicting their preference structure.

To confirm this, Step 1 was executed multiple times for DM_{JROC}. A similar

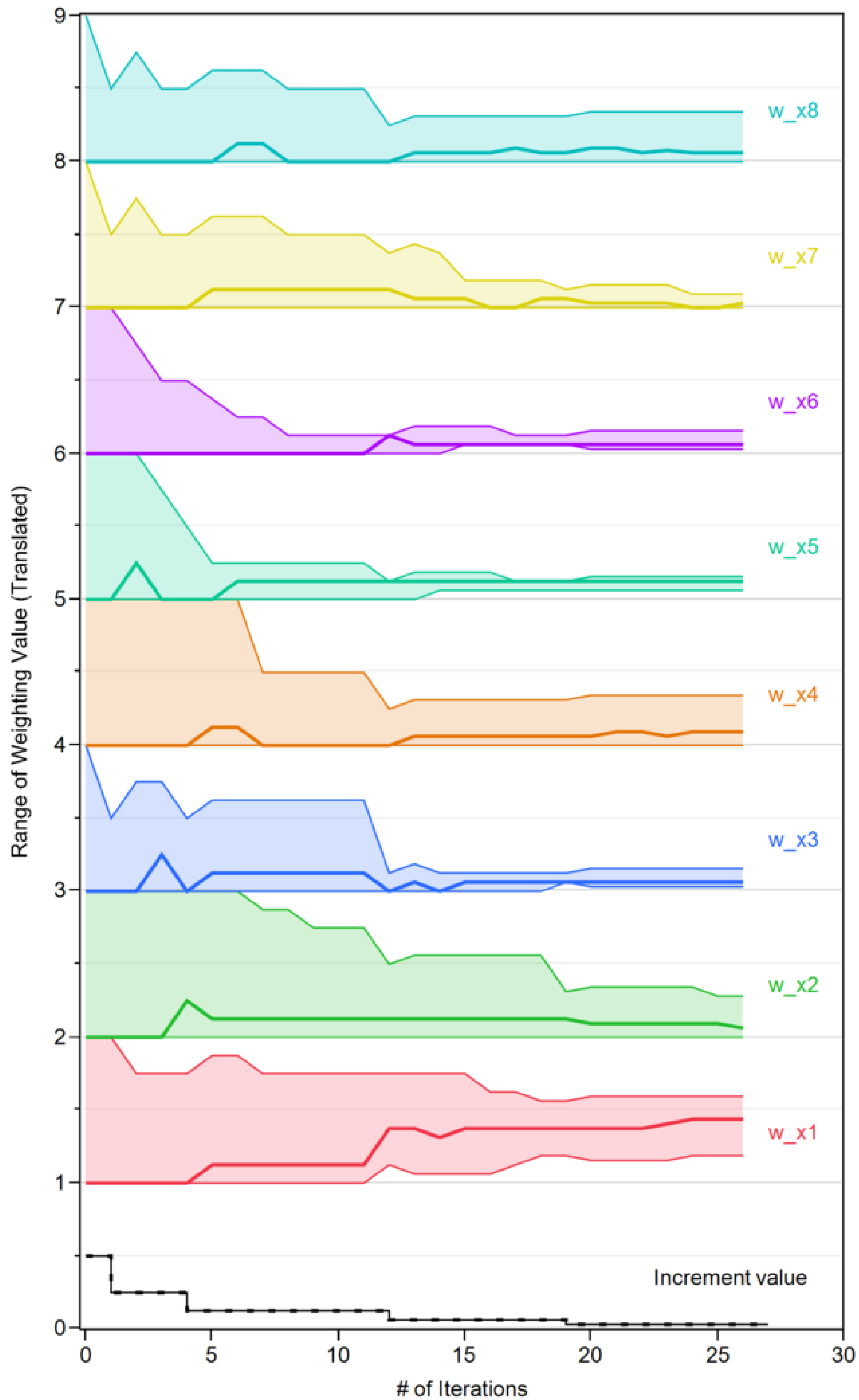


Figure 130: Reduction in Weighting Vector Ranges for all Objectives through a Series of Discrete Choices in Step 1 (separated by objective)

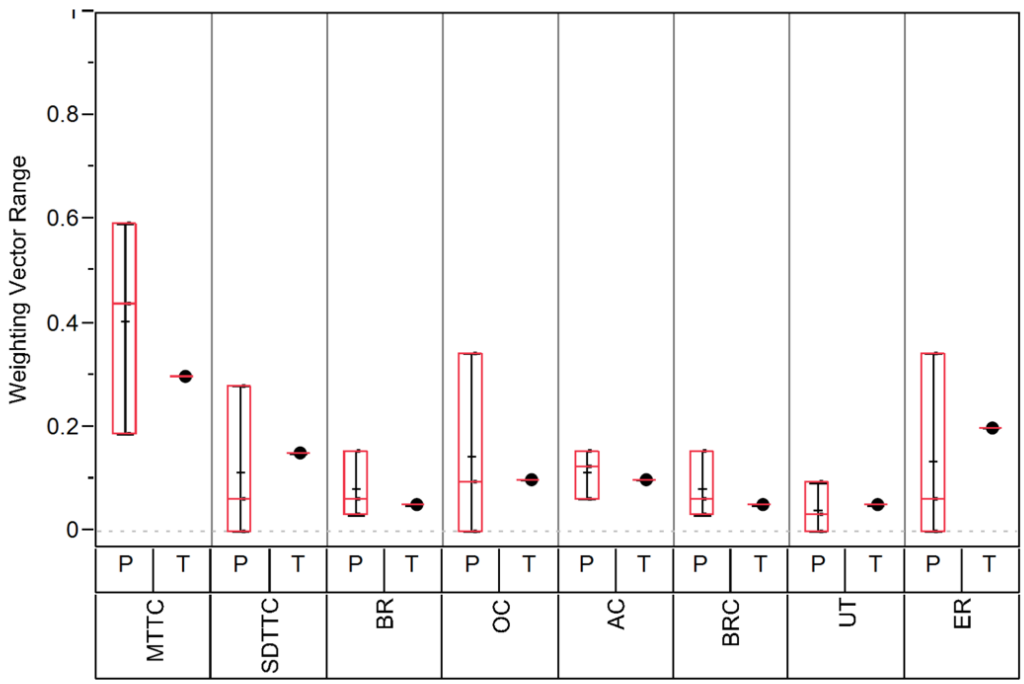


Figure 131: Variability Chart of Predictive accuracy of Step 1 from simulation using DM_{JROC} model with minimum, maximum and median values for each objective, compared to truth model

variability chart was created by showing the distribution of the median values from the 25 simulations as shown in Figure 132. The boxplots again show the range of the median values while the diamond plots span the 95% confidence interval of the mean.

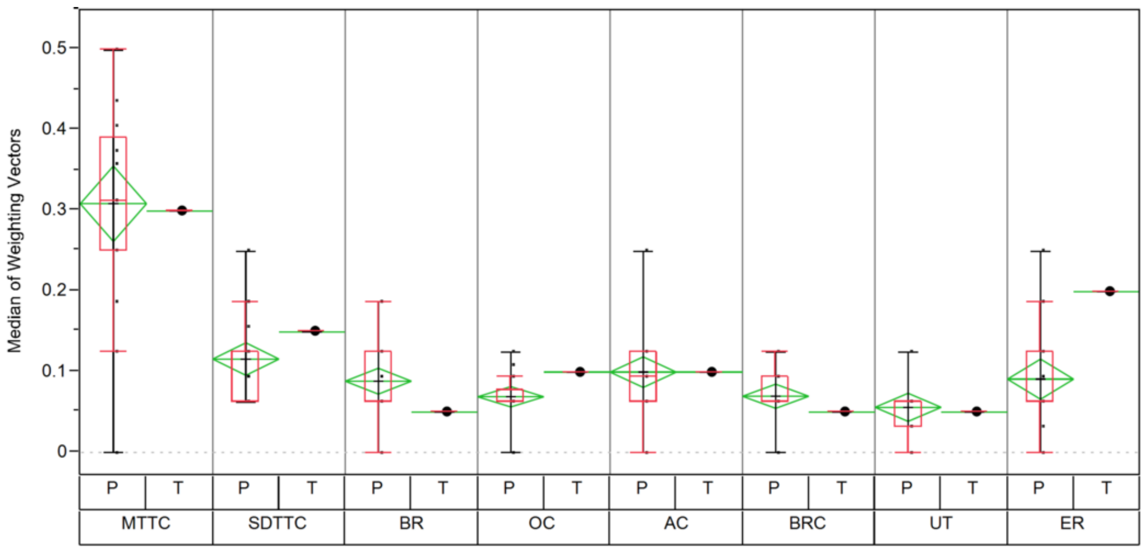


Figure 132: Variability Chart of 25 simulations of Step 1 for DM_{JROC} showing the median values for each weighting distribution

Plotting the possible weighting vectors for all the decision makers is shown in Figure 133 rotated 90 degrees counterclockwise. The 8-dimensional preference space is shown along the x-axis for each of the 5 decision makers DM_{JROC}, DM_{GOV}, etc. along the y-axis.

The top row (i.e. DM_{JROC}) shows a preference for good performance where the mean weighting vectors are higher for U(MTTC), U(SDTTC), and to a lesser extent U(ER) which is consistent with the preference truth model above for DM_{JROC}. The 5th objective for DM_{GOV} in the second row also appears shifted to higher importance values than the others and this too is consistent with DM_{GOV}'s preference for a lower cost followed by a preference for good performance for the weights of U(MTTC) and U(SDTTC). DM_{OPS}, according to the simulation, values U(ER) and U(UT) but does not set the weight for U(OC) as high as expected as defined in the truth model.

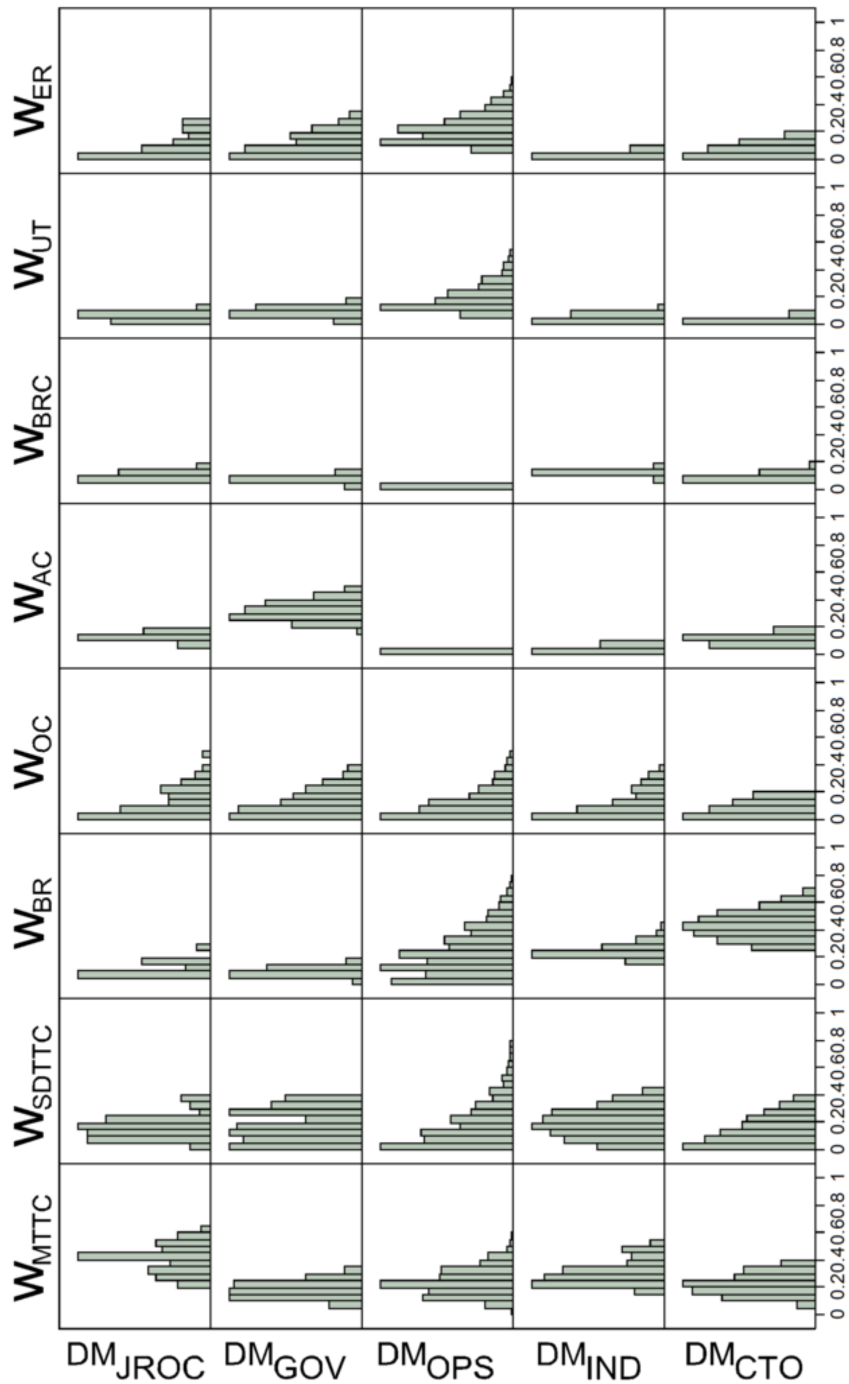


Figure 133: Possible Weighting Vectors (8-dimensional space along columns) for each of the 5 decisions maker (along rows)

DM_{IND} and DM_{CTO} show relative consistency in weighting heavily a more expensive but more capable design, with DM_{CTO} also valuing a more reliable design, which closely resembles the truth preference model for these last two decision makers.

The three tables (Tables 19, 20 and 21) assist in comparing the results of the simulation to the preference truth model. Table 19 is repeated from above with the preferences from the truth model used for the simulation of the discrete choice experiments. Table 20 is the predictive mean value of the weighting vectors for all 8 dimensions for each of the five decision makers. Comparing Tables 19 and 19 visually shows fairly good predictive capabilities for Step 1 in the methodology. To analyze this difference more fully, Table 21 shows the difference between the truth model and the predicted values. The various cells for each decision maker's preference on each of the 8 objectives are colored based on the absolute difference between the truth and simulation preference models. For a majority of the weighting values, the mean weighting vector is within 0.1 of the true preference. However, there are four weighting values where the absolute difference are larger than 0.1, namely DM_{JROC}'s mean value for U(ER), DM_{GOV}'s mean value for U(SDTTC), DM_{IND}'s mean value for U(MTTC), and DM_{OPS}'s mean value for U(OC).

Table 19: Decision Makers' Preference Truth Model (repeated)

DM	MTTC	SDTTC	BR	OC	AC	BRC	UT	ER
JROC	0.3	0.15	0.05	0.1	0.1	0.05	0.05	0.2
GOV	0.15	0.05	0.1	0.2	0.25	0.05	0.05	0.15
OPS	0.15	0.1	0.1	0.3	0	0	0.15	0.2
IND	0.4	0.2	0.2	0.05	0.05	0.1	0	0
CTO	0.2	0.2	0.35	0.05	0.1	0.1	0	0

This last value in particular is the largest error with a difference of more than 0.2 between the truth and simulation preference models. A possible cause of this large error, is supported by the correlation matrix between the 8 objectives of the design space presented in Table 22. The U(OC) objective is correlated with U(MTTC). This means that by placing a high weighting on the mean time to close (which DM_{OPS}

Table 20: Simulation Preference Model - Mean Weighting Value

DM	MTTC	SDTTC	BR	OC	AC	BRC	UT	ER
JROC	0.381	0.138	0.094	0.101	0.108	0.075	0.032	0.071
GOV	0.159	0.158	0.073	0.104	0.295	0.053	0.069	0.09
OPS	0.209	0.15	0.196	0.093	0	0	0.16	0.192
IND	0.291	0.184	0.232	0.094	0.033	0.108	0.039	0.019
CTO	0.196	0.12	0.411	0.06	0.088	0.067	0.008	0.051

Table 21: Difference Between Preference Truth Model and Simulation Preference Model

DM	MTTC	SDTTC	BR	OC	AC	BRC	UT	ER
JROC	0.081	-0.012	0.044	0.001	0.008	0.025	-0.018	-0.129
GOV	0.009	0.108	-0.027	-0.096	0.045	0.003	0.019	-0.06
OPS	0.059	0.05	0.096	-0.207	0	0	0.01	-0.008
IND	-0.109	-0.016	0.032	0.044	-0.017	0.008	0.039	0.019
CTO	-0.004	-0.08	0.061	0.01	-0.012	-0.033	0.008	0.051

does with a value of 0.15), one also values a lower operating cost, and vice versa with a correlation value of almost 0.4. Even more interestingly is the correlation between U(SDTTC) and both U(MTTC) and U(BR). As mentioned before, a more reliable aircraft (i.e. with low BR) directly relates to faster close times and less time uncertainty. Since these three are also correlated with each other, placing importance on one of them, also places importance on the others. With DM_{OPS} weighing all three "performance" objectives U(MTTC), U(SDTTC), and U(BR) with a total importance of 0.35 and U(OC) at 0.3, the model seems to result in favor of the performance objectives, and this is further suggested by the over-estimation of weights for U(MTTC), U(SDTTC), and U(BR) of 0.059, 0.05 and 0.096 respectively for DM_{OPS}.

Table 22: Correlation Matrix Between Objectives

	U(MTTC)	U(SDTTC)	U(BR)	U(OC)	U(AC)	U(BRC)	U(UT)	U(ER)
U(MTTC)	1	0.55	0.104	0.397	-0.335	-0.058	-0.175	0.005
U(SDTTC)	0.55	1	0.524	0.126	-0.006	-0.308	0.224	0.344
U(BR)	0.104	0.524	1	-0.046	0.015	-0.399	0.237	0.001
U(OC)	0.397	0.126	-0.046	1	0.081	0.033	-0.094	0.011
U(AC)	-0.335	-0.006	0.015	0.081	1	-0.008	0.732	0.002
U(BRC)	-0.058	-0.308	-0.399	0.033	-0.008	1	-0.104	0
U(UT)	-0.175	0.224	0.237	-0.094	0.732	-0.104	1	-0.096
U(ER)	0.005	0.344	0.001	0.011	0.002	0	-0.096	1

Furthermore, Table 22 also suggests a high correlation between U(UT) and U(AC). The higher the total cost of the fleet, the better one can match a desirable utilization value. With a fleet size which is too low, the utilization would be undesirably high since over taxing one's aircraft would be the only way to meet a target time to close. Lastly, this correlation matrix provides evidence that the 8 dimensions or objectives are not all completely orthogonal or independent one from another. Thus, the mean time to close, MTTC, or the variance on the time to close, SDTTC, might be beneficial to reduce the dimensionality without sacrificing too much fidelity in terms of design space coverage.

7.4 Step 2 Results: Extracting Power Relationships

In Step 2 of the methodology the discrete choice experiments are provided to allow the responding decision maker to quantify the difference in power than two other decision makers may have over the former.

A set of two designs are presented and the responding decision maker is required to make a choice with whom they are more willing to form a coalition. For example, Figure 134 illustrates one possible format for the discrete choice given to DM_{JROC} with scenario parameters (i.e. mapped flight paths) removed as compared to Figure 128. With most of the various parameters or objectives close to one another and assuming an almost comparable total utility score for each design, DM_{JROC} would potentially consider with which decision maker they would want to form a coalition. This in turn would provide information about the difference in influence between DM_{GOV} and DM_{OPS} have over DM_{JROC}.

A similar set of output graphs showing the converging process of the perceived influence difference of $P_{GOV \to JROC} - P_{IND \to JROC}$ is presented in Figure 135.

With whose design would you chose to form a coalition? DM_{GOV}'s Design or DM_{IND}'s Design ?

Metric	Design of DM_{GOV}	Design of DM_{IND}	Units
Mission Payload	20	20	million lbs
Time to Close (TTC)	27.69	24.19	days
Std. Dev. TTC	11.8	11.4	days
Utilization	9.8	9.9	hours/day
Break Rate	28	27	%
Cost to Reduce Break Rate	11.4	11.8	$billion
Flight Hours	3251.4	2876.5	hours
Total Operating Cost	68.1	60.3	$million
Payload per Flight	173897	194210	lbs
Empty Weight	352975	397452	lbs
Fleet Size	12	12	
Acquisition Cost (per unit)	224.5	252.8	$million
Total Acquisition Cost	2.69	3.03	$billion
	DM_{GOV}?	DM_{IND}?	
Form a Coalition With? ☑	☐	☐	

Figure 134: Air Mobility Example of a Discrete Choice from Step 2

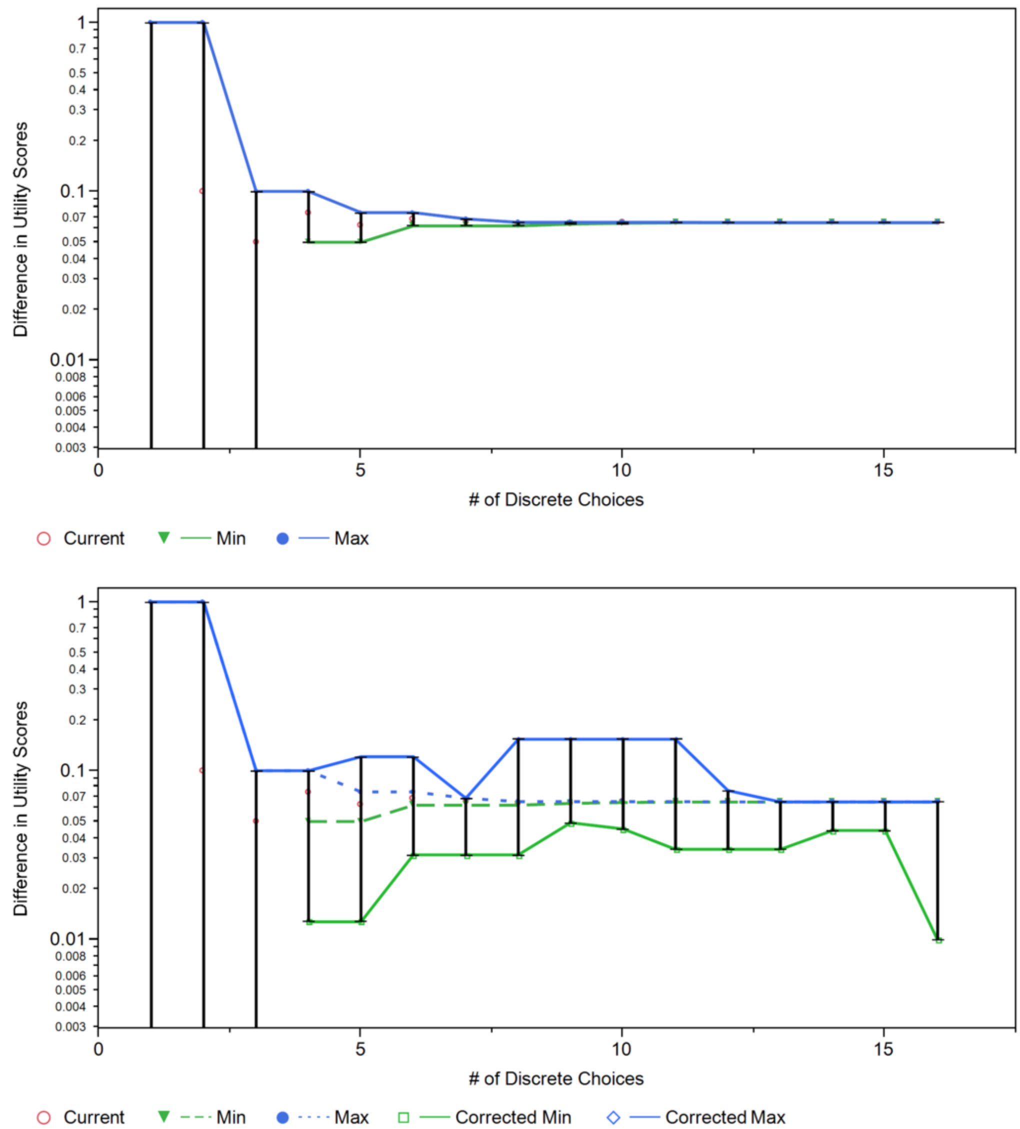


Figure 135: Converging Process of DM_{JROC}'s Perceived Influence difference between two other Decision Makers (DM_{GOV} and DM_{IND})

The expected perceived influence difference of $P_{GOV \to JROC} - P_{IND \to JROC}$ is 0.1, (i.e. 0.6 - 0.5 = 0.1, taken from the "...on JROC" column from Table 18). This is converted to the utility difference of 0.065 in Figure 135 since the range of u is $\approx$ 0.65. In other words, DM_{JROC} is willing to form a coalition with DM_{GOV} with utility scores 0.065 less than a design potentially offered by DM_{IND}. In reality, DM_{IND} might never position themselves at a design that much better than DM_{GOV}

from DM_{JROC} perspective, but the influence or power difference between these two decision makers is the sole goal of this step.

As described in previous sections, since a range of possible weighting vectors exists, one of which will represent the true DM_{JROC} weighting vector, the additional bounds are added above and below the range for the perceive influence difference as depicted on the bottom of Figure 135. This, of course, is dependent on the certainty in the ranges of weighting vectors from Step 1. If little or no certainty exists in predicting the weighting distributions, the corrections to the perceived influence differences will be minimized.

To quantify the methodology's ability to predict the power or influence differences, comparisons between the perceived influence truth model and the simulations outputs can be performed. Figure 136 shows the range of the perceived influence difference as calculated by the algorithm in step 2 of the methodology and the discrete choices (labeled as a P for predicted) and the true value (labeled with a T) taken from the perceived influence truth model from Table 18. In all of the discrete choices (i.e. two per decision maker), the model is able to bound the true perceived influence difference. However, there is significant uncertainty in some of the discrete choices.

The next substep is creating the A matrix and b vector to solve the system of equations, $Ax = b$, for $P_{i \to j}$, where $i \neq j$ and $i, j = DM_{JROC}$, DM_{GOV}, DM_{OPS}, DM_{IND}, or DM_{CTO}. The upper half of the b vector is defined by the power constraint equations for the 10 relationships, which each taking on the value of 1. The bottom half of the b vector assumes the ranges from the outputs of the 10 perceived influence discrete choices, two from each decision maker. These ranges are shown in the variability chart from Figure 136. For example, DM_{JROC} provides a response of the relative influence difference that DM_{GOV} and DM_{CTO} has on DM_{JROC} in the first discrete choice resulting in a range of influence difference between 0.049 and 0.105. This range is used in the '10+1'th element in the b vector and similarly for

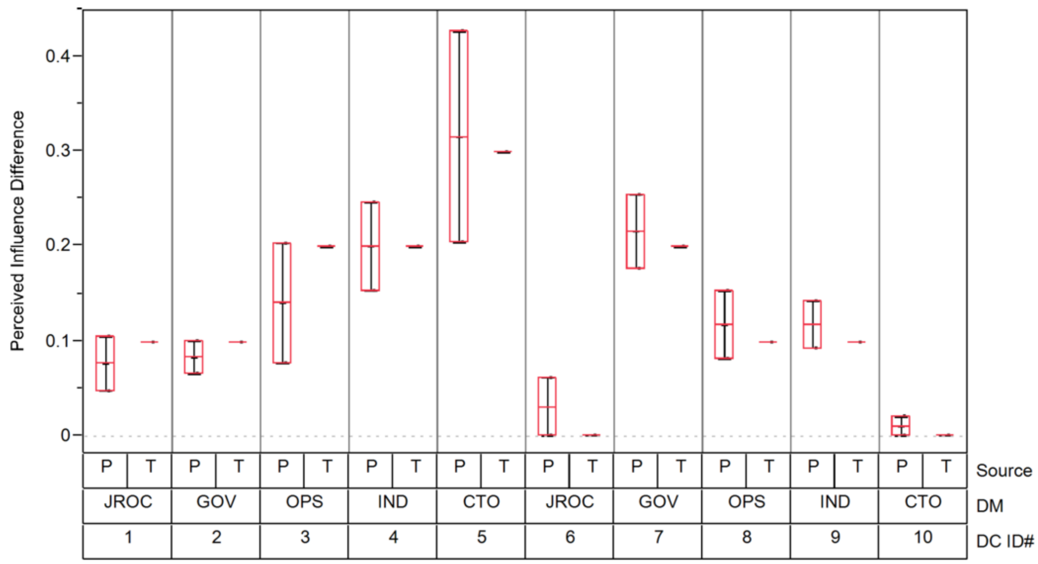


Figure 136: Variability Chart of Methodology's ability to Predict Perceived Influence Differences

the next 9 discrete choice output ranges. The set of discrete choices also define the A matrix with a 1 for the more influential decision maker and a -1 for the less influential decision maker for each row depending on who responded to the discrete choice experiments. Although two sets of discrete choices are given to each decision maker, unequal numbers of discrete choice per decision maker are possible but the A Matrix must be full rank or invertible in order to solve for x the power relationships. (Of course, more equations than unknowns are permissible, and in such cases, a linear least squares approximation is used, as described in the previous chapter.) For this particular example problem, the system of equations $Ax = b$ is defined as:

$$
\begin{bmatrix}
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\
0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\
0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
P_{JROC \to GOV} \\
P_{GOV \to JROC} \\
P_{JROC \to OPS} \\
P_{OPS \to JROC} \\
P_{JROC \to IND} \\
P_{IND \to JROC} \\
P_{JROC \to CTO} \\
P_{CTO \to JROC} \\
P_{GOV \to OPS} \\
P_{OPS \to GOV} \\
P_{GOV \to IND} \\
P_{IND \to GOV} \\
P_{GOV \to CTO} \\
P_{CTO \to GOV} \\
P_{OPS \to IND} \\
P_{IND \to OPS} \\
P_{OPS \to CTO} \\
P_{CTO \to OPS} \\
P_{IND \to CTO} \\
P_{CTO \to IND}
\end{bmatrix}
=
\begin{bmatrix}
1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\
[0.049, 0.105] \\
[0.066, 0.101] \\
[0.077, 0.204] \\
[0.154, 0.247] \\
[0.205, 0.427] \\
[0.001, 0.062] \\
[0.178, 0.255] \\
[0.082, 0.153] \\
[0.094, 0.143] \\
[0.001, 0.021]
\end{bmatrix}
\tag{50}
$$

and the solution with 5000 evaluations takes on the following ranges:

$$x = \begin{bmatrix} P_{JROC \to GOV} \\ P_{GOV \to JROC} \\ P_{JROC \to OPS} \\ P_{OPS \to JROC} \\ P_{JROC \to IND} \\ P_{IND \to JROC} \\ P_{JROC \to CTO} \\ P_{CTO \to JROC} \\ P_{GOV \to OPS} \\ P_{OPS \to GOV} \\ P_{GOV \to IND} \\ P_{IND \to GOV} \\ P_{GOV \to CTO} \\ P_{CTO \to GOV} \\ P_{OPS \to IND} \\ P_{IND \to OPS} \\ P_{OPS \to CTO} \\ P_{CTO \to OPS} \\ P_{IND \to CTO} \\ P_{CTO \to IND} \end{bmatrix} = \begin{bmatrix} [0.471, 0.534] \\ [0.466, 0.529] \\ [0.338, 0.659] \\ [0.341, 0.662] \\ [0.362, 0.67] \\ [0.33, 0.638] \\ [0.391, 0.467] \\ [0.533, 0.609] \\ [0.53, 0.779] \\ [0.221, 0.47] \\ [0.726, 0.891] \\ [0.109, 0.274] \\ [0.36, 0.448] \\ [0.552, 0.64] \\ [0.429, 0.746] \\ [0.254, 0.571] \\ [0.338, 0.655] \\ [0.345, 0.662] \\ [0.204, 0.446] \\ [0.554, 0.796] \end{bmatrix}. \tag{51}$$

A more useful graphic depicting additional information is found in the histograms of Figure 137 with the possible power relationships for each decision maker pair and its inverse directly beside it. The power that DM_{JROC} has over DM_{GOV} is near 0.50 and thus similarly for DM_{GOV} over DM_{JROC}. In fact, this first histogram suggests that neither DM_{JROC} or DM_{GOV} has significant influence over the other. The variance of these to mirrored distributions is also quite small suggesting that this state of nearly equal power between these two decision makers seems to be quite certain. On the other hand, the influence or power between DM_{JROC} or DM_{OPS} is less certain (top right histograms of Figure 137). Although the mean values are near 0.5, the potential for one (or the other) to be persuaded or influenced by the other decision maker is clearly possible.

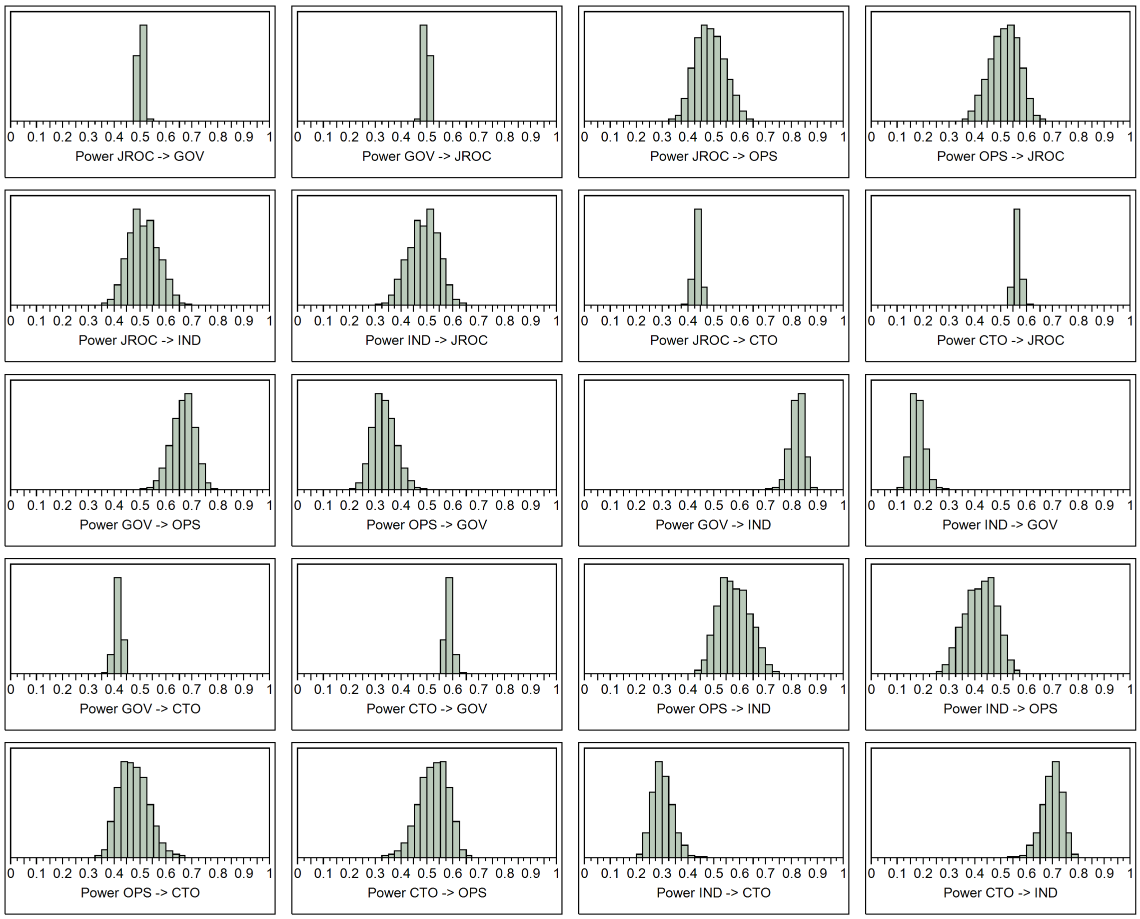


Figure 137: Step 2 Output Summarizing All 10 Decision Maker Power Relationships

Placing the mean influence values for each of the 10 relationships into a matrix, as in Table 23, illustrates additional results. The "row" decision maker, has an influence on the "column" decision maker at the value specified. For example, DM_{JROC} has a slight influence over DM_{GOV} of 0.502, and slightly more than that over DM_{IND} of 0.511. The cells are shaded based on the relative value from 0.5, such that 0 is red, 0.5 is white and 1 is green.

The most prominent relationship is that between DM_{GOV} and DM_{IND}. DM_{GOV}

has significant influence over the preferences of DM_{IND}. This is expected since "the customer" often drives the requirements or product's design and the ultimate choice whereas the "producer" is willing to concede some objectives to maximize profits or revenue. Interestingly, the DM_{CTO} has some influence over all other decision makers. Of course, the largest influence is over the DM_{IND} who would clearly seek approval or agreement on technical feasibility with their own technical experts. The other three might also be persuade by the assumed expertise about aircraft design or on other technical objectives, albeit at a significantly lower value compared to DM_{IND}. Lastly, DM_{IND} has power or influence of less than 0.5 over all the others. This again shows how the demand in any market (in this case the preferences of DM_{GOV} or DM_{JROC}) more or less "defining the agenda" or product requirements and designs.

Table 23: Actual Influence Relationships from Step 2

	JROC	GOV	OPS	IND	CTO
JROC	-	0.502	0.485	0.511	0.435
GOV	0.498	-	0.667	0.822	0.414
OPS	0.515	0.333	-	0.577	0.473
IND	0.489	0.178	0.423	-	0.301
CTO	0.565	0.586	0.527	0.699	-

Comparing this to the perceived influence model results in Table 24.

Table 24: Difference Between Evaluated and Perceived Influence Relationships

	JROC	GOV	OPS	IND	CTO
JROC	-	-0.098	-0.215	-0.189	-0.065
GOV	-0.102	-	0.067	0.022	-0.086
OPS	-0.085	-0.067	-	0.177	0.173
IND	-0.011	-0.422	0.023	-	-0.499
CTO	0.165	0.186	0.127	-0.001	-

The color scheme in Table 24 is similar in that a lower value compared to the perceived influence is in red while the opposite is in green. In other words, over estimating one's influence is highlighted in red and underestimate one's influence is in green. DM_{IND} appears to over-estimate their influence while DM_{CTO} was much more humble in estimating their influence over others. Likewise, DM_{JROC} slightly overestimated and DM_{OPS} underestimated their own power over others.

7.5 Step 3 Results: Reaching Preference Consensus

The final step in the overall methodology is identifying the region of consensus and evaluating where the group is likely to reach agreement in the preference space, and then mapped this to the consensus region or designs in the design space.

Using the weighting vector distributions from Step 1 as initial preferences, and then the power relationship distributions from Step 2, the ultimatum game can be played multiple times to identify the regions of consensus between two decision makers, after which that region becomes the preference structure of the coalition for later agreement with the other decision makers.

Although the sequence of coalitions formed can take on a variety of possible paths, the particular consensus sequence applied to the current simulations follows DM_{IND} with DM_{CTO}, then DM_{JROC} with DM_{GOV}, then $DM_{JROC-GOV}$ with DM_{OPS}, then finally, $DM_{JROC-GOV-OPS}$ with $DM_{IND-CTO}$.

This is depicted in Figure 138 where for each objective the sequence is repeated in the aforementioned order. The consensus at each stage is represent when two decision makers reach agreement about their preference structure as a coalition, and the lines will coincide for the remainder of the process. Only the mean weighting vector for the set of all potential consensus weighting vectors is shown. The degree to which one influences another in forming a coalition is represent by how "far away" a coalition weight vector is from one's initial preferred weighting vector. For example, the output from Step 2 revealed that DM_{CTO} had more than half the influence over DM_{IND} and as a result, the red line (the path for IND) reaches consensus with the blue line (the path for CTO) much closer to CTO's preferred weights. This is seen most strongly in the BR objective sequence at the top right of Figure 138. Different power relationships would result in a different final consensus mean weight vector as would a different sequence of coalitions.

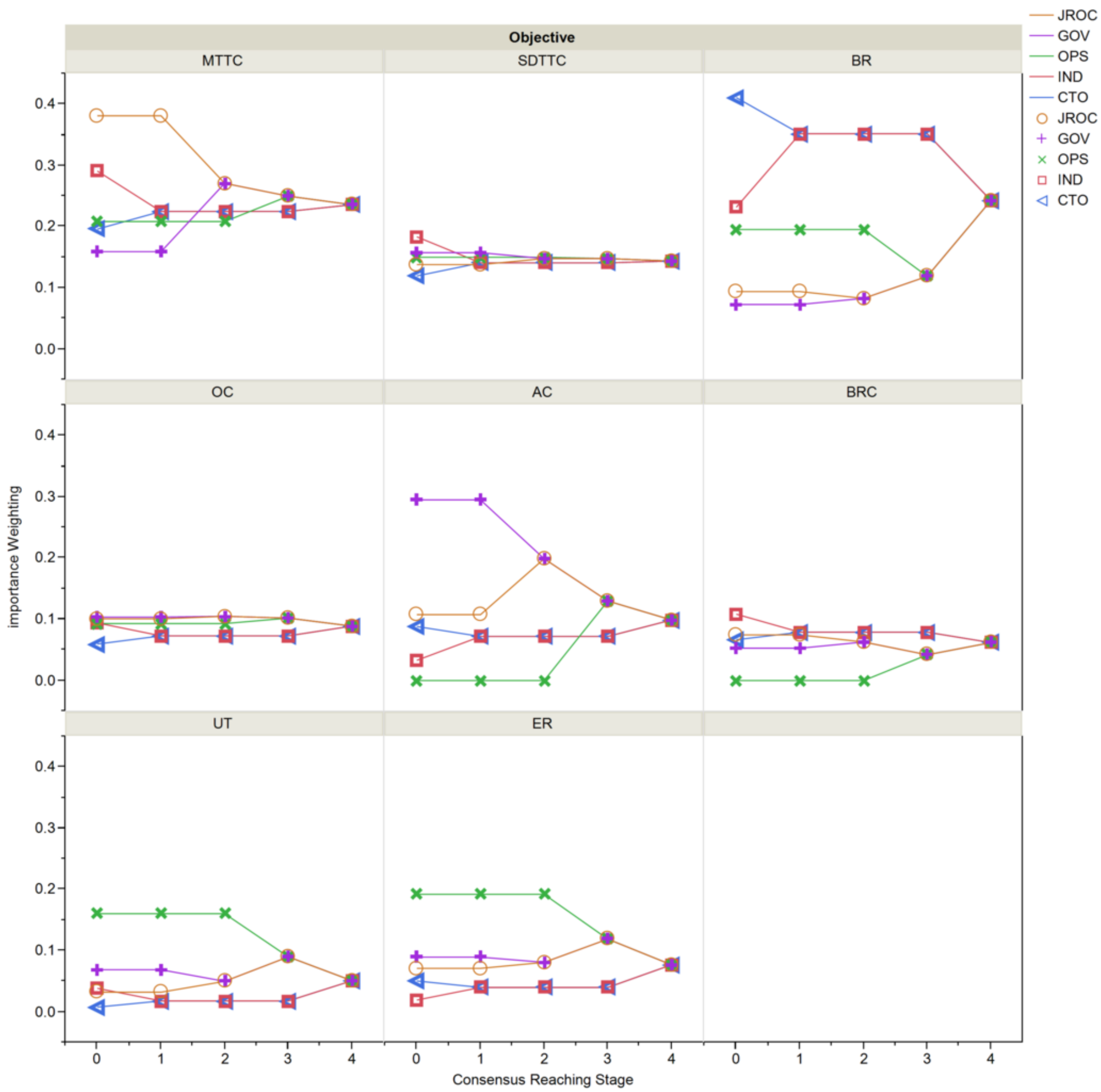


Figure 138: Mean weighting vector for each consensus sequence broken out by objective

The final consensus preference region after agreement of $DM_{JROC-GOV-OPS}$ and $DM_{IND-CTO}$ is of course in 8 dimensional space but illustrating a subset of those dimensions (i.e. 3 or less) can assist in visualizing this region. In Figure 139 shows the final group consensus region for the three dimensions w_{MTTC}, w_{SDTTC}, and w_{BR}. This is consistent with the previous figure where the dimension requiring the most compromise is w_{BR}. The weighting vectors for w_{MTTC} and w_{SDTTC} were already relatively in agreement as indicated by both Figures 138 and 139 by the last step in the coalition forming sequence. The small gray points show the original discretized preference space used for the first discrete choice in Step 1 of the methodology and is shown only for reference purposes.

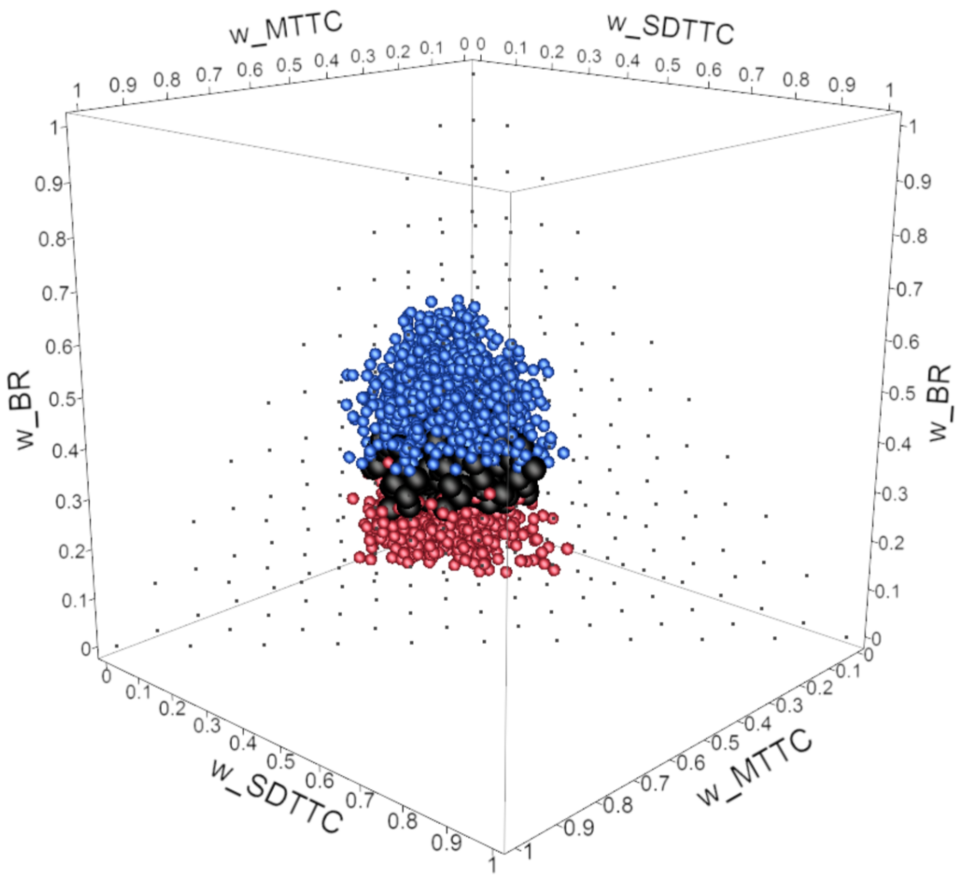


Figure 139: Final Group Consensus Region (black) with initial preference regions for $DM_{JROC-GOV-OPS}$ (blue) and $DM_{IND-CTO}$ (red) for only 3 dimensions w_{MTTC}, w_{SDTTC}, and w_{BR}

To visualize all the dimensions, a scatterplot such as that found in Figure 140 shows the 2D projections for all objective pair combinations. As in the previous figures, the objective with the greatest difference between the two coalitions for the fourth consensus step was w_{BR}. This is also shown by the relatively large separation of the blue and red preference clouds in the second row and third column of Figure 140.

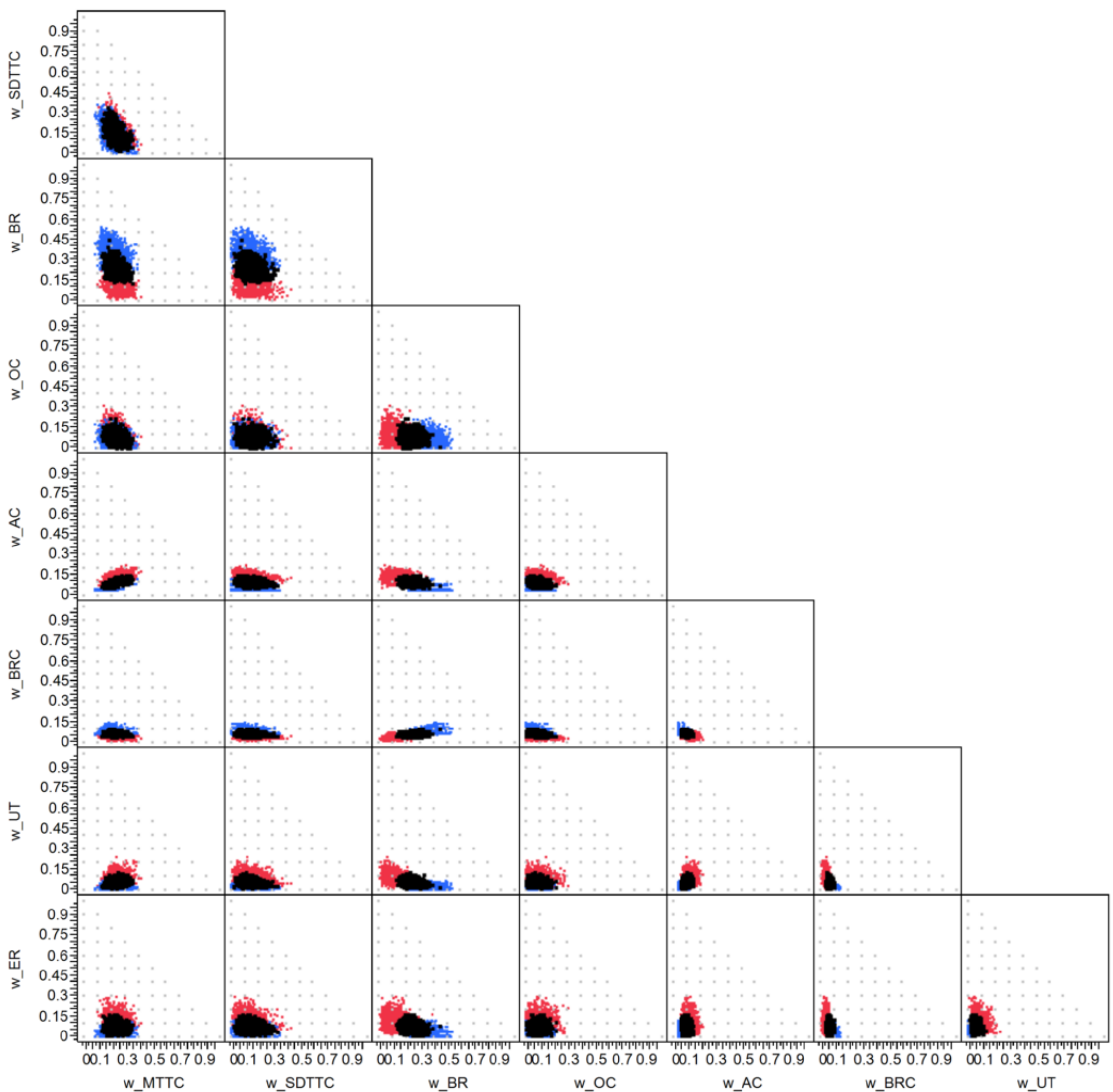


Figure 140: Final Group Consensus Region (black) with initial preference regions for $DM_{JROC-GOV-OPS}$ (blue) and $DM_{IND-CTO}$ (red) for all 8 objectives

The statistics on the final consensus region (the black points) are shown in Table 25. Interestingly, w_{BR} takes on the largest mean value of 0.245. Since, performance w_{MTTC} was heavily weighed by DM_{JROC} and DM_{IND}, and since w_{MTTC} is further correlated with a low break rate (i.e. high w_{BR}), these two objectives were likely

Table 25: Statistics on Objective Weighting Values for Final Group Consensus Region

Objective	Mean Weighting	Std. Dev.
w_{MTTC}	0.236	0.04
w_{SDTTC}	0.149	0.054
w_{BR}	0.245	0.048
w_{OC}	0.084	0.036
w_{AC}	0.099	0.018
w_{BRC}	0.063	0.012
w_{UT}	0.05	0.018
w_{ER}	0.075	0.028

amplified in the final result. Furthermore, this high value in w_{BR} is also likely due to the influence DM_{CTO} has over DM_{IND} at the first consensus reaching stage but also over the others at the last stage (see Figure 138).

The final action in the overall methodology is applying or mapping the group preference consensus region to the design space to identify the design parameters most likely to be preferred by the group. Analyzing the "best" design for each one of the weighting vectors in the final consensus preference space, results in only 19 designs that would potentially be selected from the full feasible set of 518000 designs. These are summarized in Table 26. Interestingly, all 19 designs were of Type 3 which made use of the fact that the Dover to Kandahar APOE-APOD combination was less than 6100 nmi and thus was in the range for the direct return flights for large aircraft with no payload. These designs dominated the Type 2 and Type 4 simulations since stopping over retro en route, to refuel penalized the time to close and other output metrics. Type 1 simulations are feasible, but for such a far distance, a low payload per weight would be required and thus many more flights and therefore flight hours would be needed. Furthermore, a large fleet size would be required to make up for a lower throughput capacity for long distances. The en route location, Ramstein Air Base, was also identical for all of the 19 "best" designs. Of the 23 candidate en route locations, Ramstein Air Base was closest to the midpoint on the great circle path from Dover to Kandahar Airfield, resulting in shorter flight times while maximizing

Table 26: Summary of the 19 "Best" Designs After Mapping the Group Preference Consensus Region to the Design Space

#	ID	W_e	MTTC	SDTTC	BR	OC	AC	BRC	UT
	-	lbs	days	days	%	\$M	\$M	\$B	/24hrs
1	137046	433139	10.94	5.64	15.4	29.16	3856.67	20.09	9.09
2	171131	424680	10.47	5.59	15.23	29.6	4051.44	20.55	8.99
3	154610	407377	11.41	5.71	15.59	30.93	3627.28	19.61	9.24
4	88967	490360	10.66	5.6	16.02	25.63	4054.3	18.63	8.83
5	145856	490131	11.25	5.57	14.43	25.63	3740.68	23.05	9.06
6	92207	432824	11.53	5.64	14.83	29.16	3578.59	21.72	9.29
7	166303	474406	10.18	5.5	15.05	26.51	4224.11	21.06	8.88
8	102926	380144	11.38	5.72	15.23	33.14	3626.57	20.54	9.27
9	130590	418850	11.81	5.7	15.07	30.04	3463.05	21	9.34
10	151019	457009	11.2	5.63	15.14	27.83	3778.55	20.79	9.13
11	99299	401989	10.85	5.61	14.64	31.37	3834.98	22.32	9.2
12	97768	431580	10.4	5.6	15.65	29.16	4117.27	19.47	8.92
13	91327	356491	10.38	5.72	15.23	35.35	4307.83	20.55	8.56
14	114164	361664	11.29	5.72	14.95	34.9	3680.3	21.34	9.23
15	93417	463154	11.84	5.69	15.02	27.39	3534.79	21.15	9.21
16	116317	420073	10.61	5.63	15.48	30.04	4007.5	19.89	9.01
17	126094	415202	11.19	5.62	14.6	30.49	3696.96	22.46	9.29
18	133637	356873	11.86	5.74	14.7	35.35	3404.57	22.14	9.48
19	145961	415343	11.15	5.59	14.38	30.49	3698.21	23.23	9.32
μ:	n/a	422699	11.07	5.64	15.1	30.11	3804.40	21.03	9.12

the payload capacity for each leg of the flight, which in turn translates to higher utility scores for those designs.

The mean aircraft empty weight was slightly larger than the C-5 (just over 422000 pounds vs. 380000), but the reliability was much higher compared to current mission capable rates of the C-5 (15% vs. 40% in terms of break rates). Although obtaining the lower break rates were significant, averaging just over 21 \$billion, the benefits in a low time to close were nevertheless considered substantial. In fact, relatively small fleet sizes were common in the 19 solutions, with the mean number of aircraft employed of 14.26 with the minimum and maximum of 12 and 19 respectively. With a worse break rate, the number of aircraft required to meet the same MTTC would be expectedly larger resulting in a higher acquisition cost, but a compensating lower cost for the higher break rate. The utilization values (mean of 9.12) are all close to the target of 8 hours per day established in this example. The acquisition cost per aircraft for the 19 designs had a mean value of just under \$269 million. Since the average number of aircraft used in the 19 simulations was slightly greater than 14, the product of these two results in the average total acquisitions cost of slightly over \$3.8 Billion.

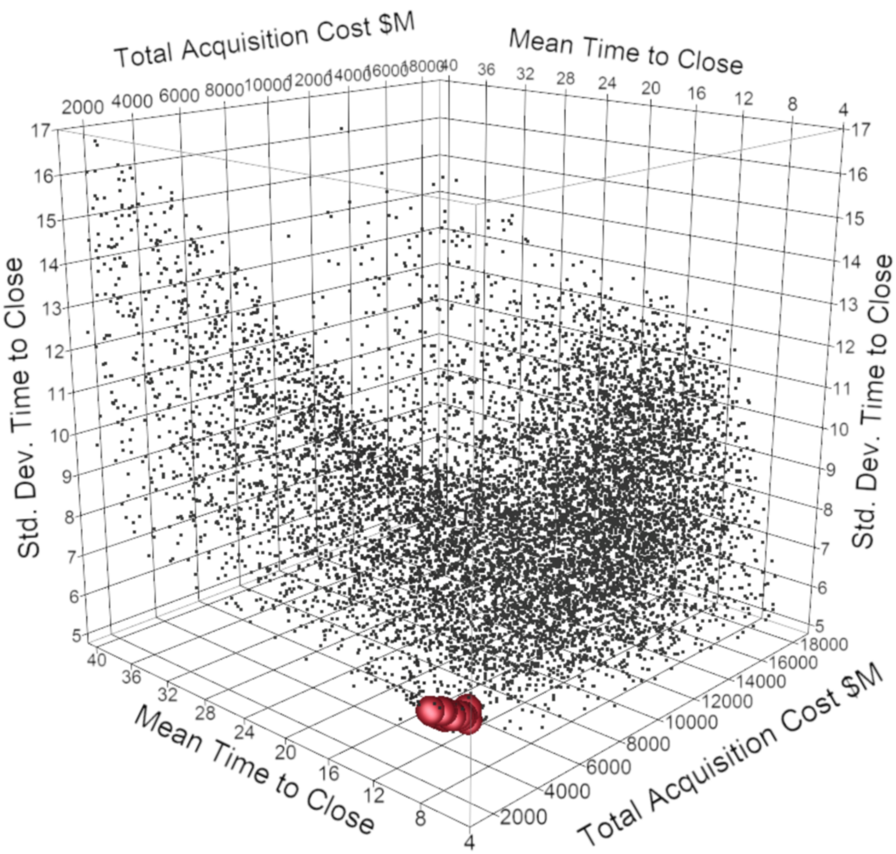


Figure 141: Design Space of Model Simulations with the 19 Final Group Consensus Designs shown in red (MTTC vs. AC vs. SDTTC)

These same 19 designs can be visualized in a similar manner as the preference space weighting vectors. In 3D graphs as in Figure 141, the designs (colored in red) can be shown to lie on the Pareto frontier of a cloud of points of MTTC, SDTTC and AC. As discussed previously, some solutions may close more quickly but would cost more (i.e. larger fleet size and AC).

In Figure 142, the designs are against displayed in the 3D graph but with axes replaced by the mean time to close, the utilization and the total operating cost. The cloud of designs representing the "best" designs as evaluated from the consensus preference region are located near the utilization values of 9 (i.e. 8 was the target value) and lower operating costs as expected. Lower MTTC values are possible (as shown in the previous figure as well), and might even lower the operating costs due to fewer flight hours, but the real trade is not visible in this graphic, since MTTC can be traded with AC or BRC more directly.

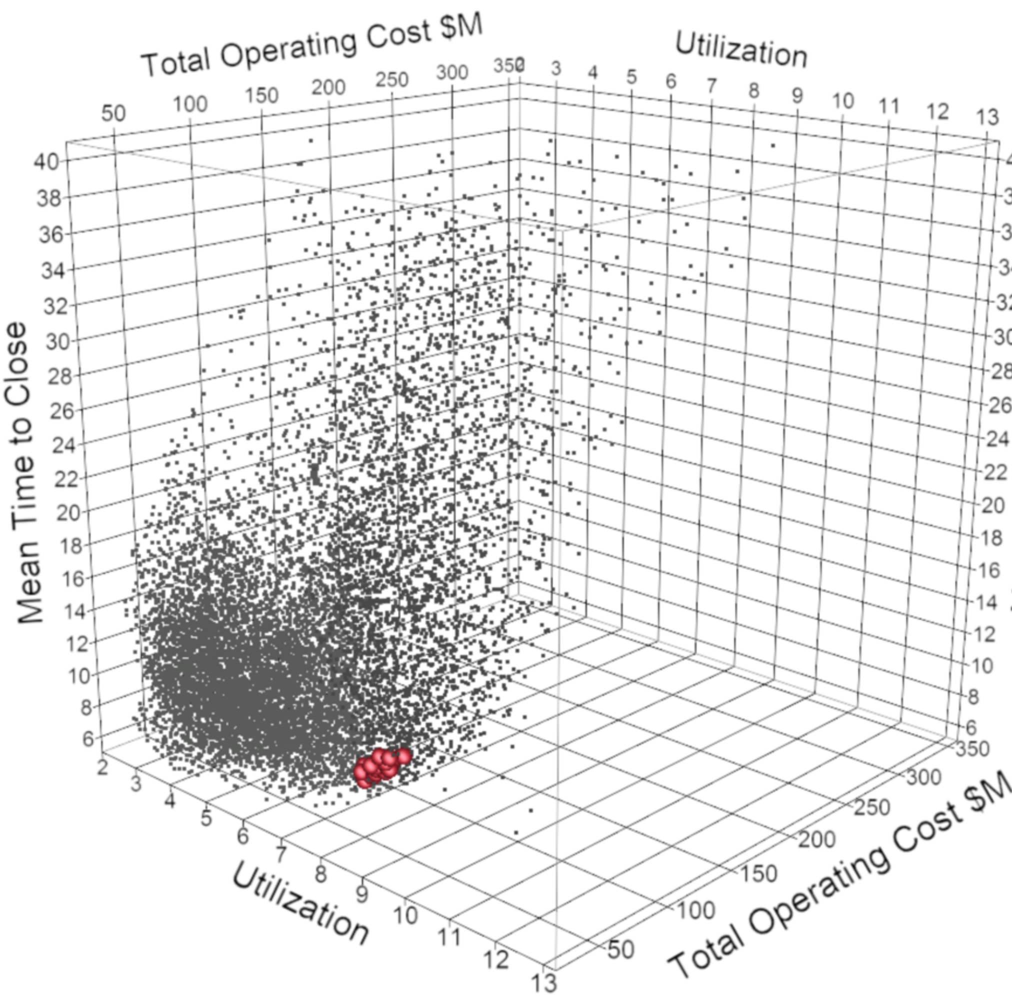


Figure 142: Design Space of Model Simulations with the 19 Final Group Consensus Designs shown in red (UT vs. OC vs. MTTC)

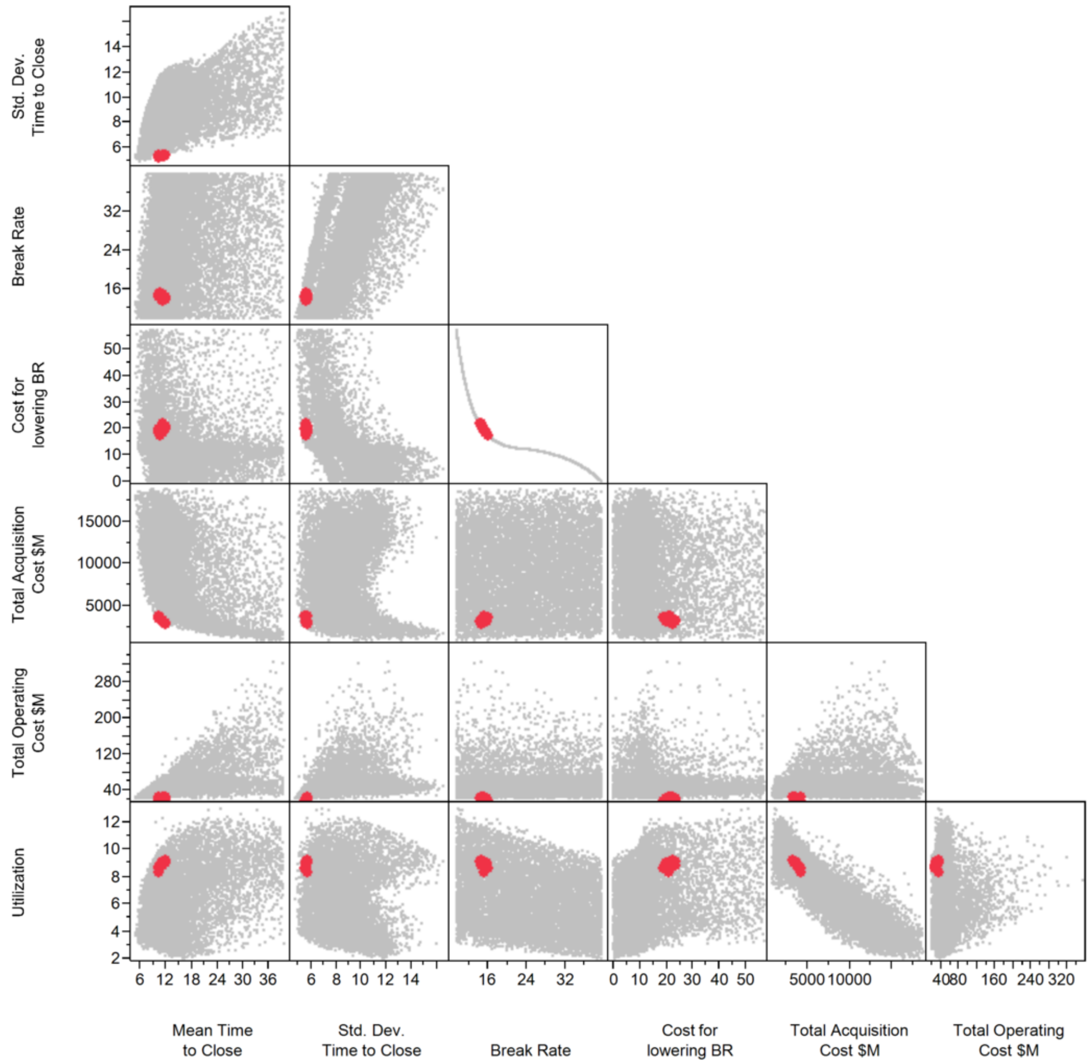


Figure 143: Design Space of Model Simulations with the 19 Final Group Consensus Designs shown in red. (All objectives shown except En Route Type Selection))

To help with these additional trades and to investigate more easily the designs in the multi-dimensional solution space, employing a scatterplot matrix as shown in Figure 143 is useful. The en route Type selection or U(ER) objective is not shown, resulting in only 7 of 8 objectives represented. The trade between MTTC and AC are again shown in the fourth row from the top and the explicitly defined trade between cost to lower BR and the break rate itself are also clearly visible. The cost to lowering the break rate is particularly interesting because the importance weighting on U(BR) was relatively high. Even though the cost increases exponentially below BR values of 25% the sequence taken by the decision makers and the influence relationships established by the step 2 discrete choices results in a higher break rate cost than the flat part of the curve. At some point the marginal improvement in break rate reduction is no longer attractive past this point in the curve for this group of decision makers. The other subplots show values in the middle of their ranges, suggesting and coinciding with the Step 3 results that these objectives have lower relative importance compared to the most important three objectives from Table 25, w_{MTTC}, w_{SDTTC}, and w_{BR}.

The final analysis of these solutions can be the percentage or number of occurrences for each of the 19 designs. Since two (similar) weighting vectors applied to the design space can be mapped to the same "best" design, the number of occurrences that the weighting vectors from the consensus reaching process is mapped to each design can be insightful. This metric is important because it can suggest that from a range of importance weighting vectors the same design will be ultimately considered the "best". It may also mean that the design space is not "saturated" enough to discriminate between two almost equal weighting vectors. However, this problem can be readily overcome by executing additional model simulations within the appropriate range of inputs, a process which may be computationally expensive without the implementation of surrogate model techniques.

The number of occurrences of the 19 designs is shown in the vertical axis of Figure 144. The other two dimensions are the mean time to close and the total acquisition cost similar to Figure 141 above. As shown, two designs, in particular, have a consensus occurrence number of more than 200 (i.e. d_{137046} has 214 and d_{171131} has 205) and together they make up 40% of the entire consensus occurrences. This means that to properly predict the expected design parameters such as that from Table 26, the mean values should be weighted by the number of occurrences.

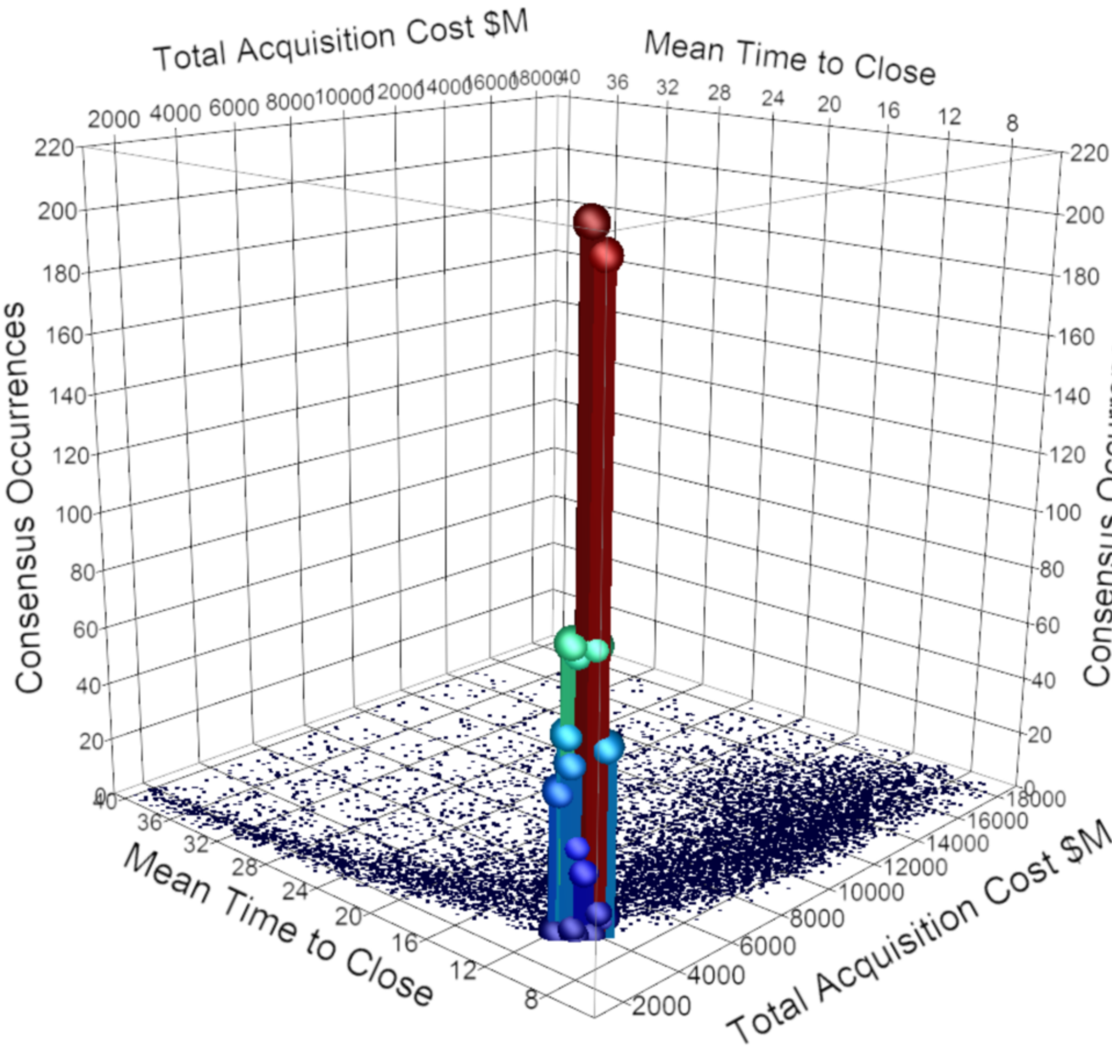


Figure 144: Design Space Projected onto MTTC-AC plane versus the consensus occurrence number. (Designs colored by number of occurrences)

Table 27 considers the effects of the occurrences of consensus on each the objectives for each of the 19 designs, which provide a more likely expected design across the consensus region. The difference between the "mean" and "weighted mean" values suggests a slightly larger (3.53% heavier) aircraft than originally predicted, but slightly less expensive in terms of operational cost (-3.89%). The cost associated with improving the break rate also goes down slightly because of a higher break rate accepted for the "weighted means" design.

Table 27: Weighted Mean Summary of the 19 "Best" Designs After Mapping the Group Preference Consensus Region to the Design Space

	W_e	MTTC	SDTTC	BR	OC	AC	BRC	UT
Units:	lbs	days	days	%	\$M	\$M	\$B	/24hrs
Mean:	422699	11.07	5.64	15.1	30.11	3804.4	21.03	9.12
Weighted Mean:	437615	10.95	5.62	15.24	28.94	3856.5	20.58	9.08
% Difference	3.53%	-1.08%	-0.35%	0.93%	-3.89%	1.37%	-2.14%	-0.44%

In predicting or forecasting how a group of decision makers will decide, the sequence of consensus stages can have a large impact on which design or solution the group will ultimately select. However, since the sequence cannot be determined a priori, all or at least many of the combinations of sequences can be executed with the distributions and power relationships from Step 1 and 2 respectively. The union of all the sets of designs at which consensus could be made (i.e. not just for one sequence but for all possible sequences) results in the superset of solutions at which the group could reach consensus. This combined set, much fewer than the original number of candidate designs, become the condensed list of candidates solutions from which the decision makers can collectively select.

In the above examples, the Step 3 sequence executed results in the 19 designs described previously. When focusing on only the Type 1, 2, and 3 solutions (177173 designs) all making use of the Ramstein Air Base (for Type 2 and 3), by repeating this process 100 times and allowing the sequence to be randomly assigned, the superset of designs will expand with the union of designs from each successive consensus sequence. For example, as shown in Figure 145, in a series of consensus sequences, the first sequence resulted in 15 different designs from the mapped grouped preference region. The second sequence resulted in 23 different designs but only 12 of them were unique as compared to the first sequence and thus the cumulative number of consensus designs is 27. The third sequence produced 20 designs but only two of those are unique. Thus, after three consensus designs, the cumulative number of consensus designs is 29. This process is repeated 100 times representing 100 randomly selected sequences for a total cumulative number of consensus designs of 71. The trend line approaches some absolute number of designs that could be possibly reached from an exhaustive combination of all weighting vectors, for all power relationships with all possible sequences. With additional computation time and resources, this upper limit could be calculated but reaching that point becomes too time-consuming as the additional consensus designs requires an increasing number of sequence simulations. An example of this is shown from sequences 69 to 90 in Figure 145 where no additional unique designs were reached. The 91st sequence reached one unique design and the 92nd reached three, but in general the trend where the number of sequences will exponentially increase per new unique design reached is evident.

Furthermore, discovering this upper limit is unnecessary from a practical standpoint. Since most of the later sequence are reaching the same designs, the cumulative number consensus at each of the designs will also increase, and the few designs which are reached in almost every sequence is of most interest.

Figure 146 projects the number of consensus occurrences for each of the 71 different designs from the 100 sequences. The design number is sorted by the total number of consensus occurrences after the 100 sequences. Therefore, design #157931 (in the current set of designs) was reached almost 7000 times across all sequences, more

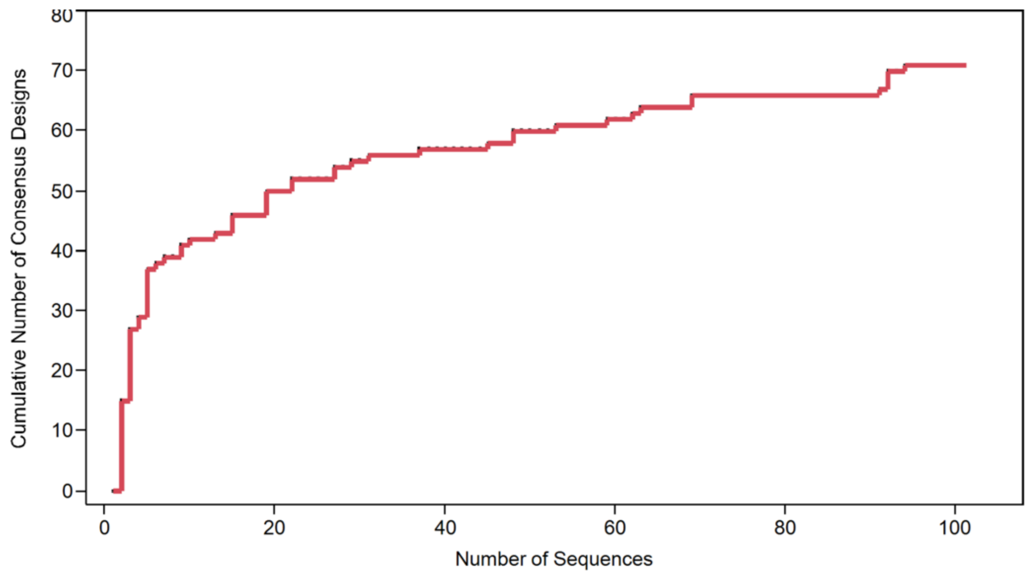


Figure 145: Cumulative number of unique designs at which consensus was reached for 100 randomly defined sequences

than any other design. The next highest was design #88967 with just under 5200 occurrences. Each successive design has a lower number of occurrences after the 100th sequence. The individual points and bars are colored by the cumulative number of occurrences. Interestingly, every sequence reached design #157931 at least once. For comparison, six different sequences never reached design #88967 in a consensus. The number of sequences that did not reach the various designs increases along the nominal axis of "Design Number" from left to right. Furthermore, 12 different unique designs are only reached once by one sequence (located at the far right of "Design Number"' axis, such as design #172980. These designs, of course, do not represent a high probability at which the group of design makers will reach agreement.

Lastly, Figure 146 shows that three designs #157931, #88967 and #154610 were most likely to be reached across all simulations. These same three account for over 60% of all simulations. In terms of predicting a design which is most likely to be collectively agreed upon, choosing one of these three, (and more specifically design #157931 which is reached by 27% of the simulations) would be the best strategy. Having decision makers discuss and contemplate this specific design (or similar ones) could then result in significant time savings from the reduction in less useful negotiations, deliberations, etc.

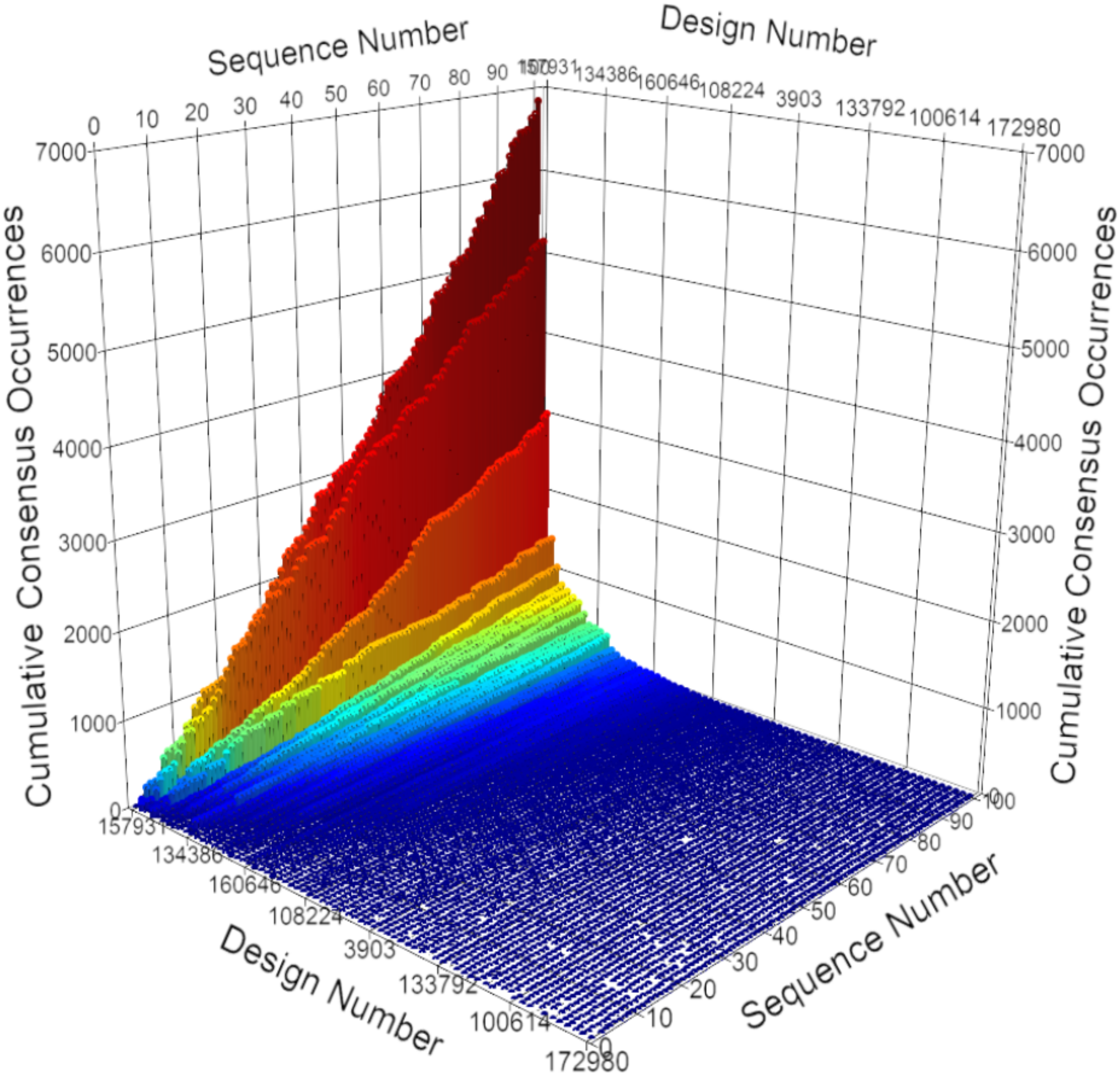


Figure 146: Cumulative number of occurrences at each of the 71 designs in Figure 145 for the 100 randomly defined sequences, sorted by total occurrence

These three particular designs are shown in Table 28 sorted by the cumulative occurrence number. As before, the decision makers as a group tend to prefer a slightly heavier and more capable aircraft, are willing to invest substantial amounts to make the fleet reliable so as to reduce the number or aircraft needed for a given level of performance with relatively low operation cost. However, with multiple sequences

Table 28: Summary of the 3 “Best” Designs After Mapping the Group Preference Consensus Region to the Design Space from Multiple Sequence Simulations

	ID	W_e	MTTC	SDTTC	BR	OC	AC	BRC	UT
Units:		lbs	days	days	%	\$M	\$M	\$B	/24hrs
1	157931	482152	12.55	5.87	16.02	26.07	3373.14	18.65	9.02
2	88967	490360	10.66	5.6	16.02	25.63	4054.3	18.63	8.83
3	154610	407376	11.41	5.71	15.59	30.93	3627.28	19.61	9.24

confirming similar results, the confidence one can have that one of these three designs could be ultimately selected is higher.

CHAPTER VIII

CONCLUSION

8.1 Summary of Findings

The MACRO methodology was found overall to function properly and was able to demonstrate in the proof of concept or air mobility problem from the previous chapter, that the original candidate design space was reduced from more than 500000 designs or solutions down to just 19. These 19 designs reflect the consensus region where the 5 stakeholders or decision makers would be expected to cooperate in reaching agreement for design selection. Since this consensus region depends on a specific sequence of coalition forming stages, multiple simulations were executed, covering a majority of the possible sequences. This analysis found that only 71 possible candidate designs exist in the superset of designs where consensus is possible. Furthermore, 3 specific designs were found to occur most often in the consensus regions from all simulations, suggesting an even smaller subset of designs at which the decision makers would likely reach consensus. (See Figure 143.)

The ratio of the number of designs in the consensus region to the initial design space (the metric for evaluating Hypothesis #2) is much less than 1% (in fact less than 0.1% in the proof of concept problem) suggesting that Hypothesis #2 should not be rejected. Since Step 3 in the methodology was directly related to testing Hypothesis 1, and Step 3 required the inputs from the other two steps, the successful results lends support that those proposed hypotheses (related to Steps 1 and 2) should also not be rejected.

More specifically, the difference between the truth model and predicted means of the weighting distributions (metric for evaluating Hypothesis #3) was found to be low, with the worse case difference calculated at 0.2 (the predicted weight for OC for DM_{OPS}). It was predicted that this discrepancy was due to a high correlation of OC with MTTC, which suggests, in general, that the methodology is likely more accurate for orthogonal and non-correlated objectives. (See Table 21.)

Comparing the perceived influence truth model to the predicted power relationships (metric for Hypothesis #4) also shows relatively good closeness. The few large differences were likely due to the original perceived influence truth model which contained over- and under- constrained power conditions. This resulted in the surprising results that the influence of DM_{CTO} was found to be larger than that expected, and that DM_{IND} significantly overestimated their own influence over DM_{GOV}. (See Table 24.)

The methodology, at least in part, and within the scope of the analyzed problems, fulfills the research objective in that it can significantly reduce the set of designs which a group of decisions makers should consider as most likely to ultimately be selected. This, of course, has the potential to reduce the time required to reach consensus by showing all decision makers where potential future actions, discussions, negotiations, and compromises will eventually bring the group to collectively agree within the decision space. Concurrently, the development of AirMOD was a successful response to the need of better defining the operational design space with more design solutions in shorter time frames. Almost 1 million points were generated across the design space in approximately 50 minutes. This not only facilitates additional analysis in bulk of the design space, but increases the decision makers' confidence that sufficient solutions have been modeled and are available for down selection and other decision-making activities.

The overall hypothesis that such a model and methodology will be able to assist a group of decision makers in reaching agreement about the requirements of a future air mobility system is promising. Compared to a "business as usual" approach, which often is lengthy and costly (in terms of opportunity costs, delayed implementation, etc.), together AirMOD and MACRO methodology offer a significantly better way of achieving cooperation between group members than simply waiting for the external pressure and forced "11th hour" decision making before finally beginning to concede in reaching a compromised solution. Furthermore, increased transparency is offered as a complementary benefit. Recording how the decision makers responded to discrete choices in revealing their preferences can be archived and used later on to contrast again current preferences, analyze requirements creep (and prepare action if observed), or provide guidance when new decision makers enter the group who may have their own agenda but are too late in being part of the initial decision making and requirements definition phases.

8.2 Research Questions and Hypotheses Revisited

Table 29 summarizes the research questions and how they were addressed throughout this dissertation. During the methodology development, additional research questions were posed and also addressed with various experimental tests and other investigations. These questions were posed during the development of the various methodology steps and are thus related and categorized accordingly in Table 29.

The related hypotheses at the highest level research questions are summarized in Table 30 with the key experimental observation supporting them.

8.3 Suitability and Limitations of the MACRO Methodology

As with all models, their sphere of applicability is always finite and the MACRO methodology from this research is no exception. A number of inherit limitations have been observed over the course of the research which need to be reviewed and underscored at this point in the dissertation. Not all of these limitations are insurmountable but some require actions performed before execution of the MACRO methodology as a whole.

First of all, the model assumption that cooperation by all agents is a necessary requirement is in fact relatively limiting. If cooperation is not a key aspect of the problem, or even desirable in the group decision-making process, determining the consensus region is clearly unsuitable. When competition is the driver, there is often no longer a state where all agents can benefit (i.e. zero-sum game). In such situations, both decision makers cannot simultaneously increase their levels of utility to the same amount (i.e. from 0 to $x > 0$) and one's reduction in utility could be the other increase. These competitive situations require a different set of game theoretic or simulation techniques, and are likely not facilitated by the MACRO methodology.

Second, in group decision making if an individual is replaceable in the sense that some other individual, company or entity could likely satisfy the particular expertise, skill, or resource, possessed by the first individual, the first individual could have little or no influence on decisions or over others. When this occurs in a group of two, and the replaceable individual is removed, the situation falls back to single-agent decision making and many of the techniques presented in this dissertation are no longer necessary. However, many supposed "single-agent decision making situations" have, in reality, more than one stakeholder or "decision maker." Consider a manager with a team of assistants and other coworkers trying to decide which strategy to implement in a company. Although, the decision-making power may reside in just

Table 29: Summary of How Research Questions were Addressed

	Research Question	Answer, Response or How Addressed
1.0	How can the number of operational design solutions originally considered increase to include a greater portion of the potential design space?	Surrogate models with Monte Carlo simulations enables fast operational design space characterization
2.0	How can the feasible decision space be reduced to facilitate decision makers in reaching consensus?	Step 3 in the MACRO Methodology
2.1	How should power or influence be combined during the consensus reaching stage such that forming coalitions are advantageous to the individuals?	A coalition assumes the highest power index value of all members over another decision maker (or coalition).
2.2	What is the impact of higher dimensional data sets with larger numbers of decision makers?	Methodology is slower and requires more work by individual decision makers but is still valid.
3.0	How can a decision maker's preference information, including the potential for changing preferences, across all objectives be acquired more quickly and accurately?	Step 1 in the MACRO Methodology
3.1	How many discrete choice experiments are needed to reach a particular level of certainty about the true preferences of a decision maker?	Depends on the number of dimensions or objectives.
3.2	What is the effect of increasing the number of dimensions on how discrete choices are selected?	Random selection (Selection method 1) approaches selection method 2 as number of dimensions increases.
3.3	How can the preference space be further specified to guarantee that only one design choice is optimal based upon a set of discrete choices while minimizing the number of discrete choices?	Refining the total preference space when necessary reduces the computational requirements and allows greater precision on weighting distributions.
3.4	What stopping criterion for discrete choice experiments should be used to extract the preferences of a decision maker?	Various, such as time available, variance of maximum distribution, established rank order, etc., depends on particular problem requirements.
4.0	How can the influence relationships between decision makers be identified and quantified?	Step 2 in the MACRO Methodology
4.1	How many discrete choice experiments are needed to extract the power or influence difference between two decision makers?	Depends on the uncertainty on weighting distributions, typically less than 10.
4.2	What is the impact on the certainty of power relationships when decision makers respond to different numbers of discrete choices and provide different ranges?	Range on the power relationships decrease with discrete choices up to a point related to the uncertainty in weighting distributions from Step 1.

Table 30: Summary of Experimental Results from Testing Hypotheses

	Hypothesis	Experimental Results
#1	Monte Carlo simulations of surrogate models developed around time-consuming operational models will provide capability to more rapidly define the design space by generating greater numbers of candidate solutions in the same time period.	The tested speed at which AirMOD can generate candidate solutions was measured at 1 million design solutions in 49.7 minutes or 2.98 ms/execution (on an Intel®Core^{TM} Duo CPU @2.20 Ghz). Individual SimPy model runs required from one to more than 30 seconds.
#2	Simulating the multi-agent decision-making process with an iterated ultimatum game across all objectives, with the application of the preference distributions of, and power relationships between, agents, will significantly reduce the decision space and identify regions with high probabilities of reaching consensus.	In the experimental case study, the methodology reduced the 500,000 designs down to 19 likely designs at which consensus is most probable. Further experimentation reduced those down to just 3 when the experiment included multiple sequences.
#3	Infeasible design or preference filtering on the range of possible weightings combinations, from a set of discrete choices employing candidate solutions, will identify a decision maker's preferences by providing feasible weighting distributions for each criterion or objective.	In the experimental case study, MACRO predicted 40 (5x8) weighting values within 0.041 on average (4% error) with two outliers of 0.21 and 0.12.
#4	Discrete choice experiments between designs, and with whom an agent will form a coalition in the decision space, will identify relationships of influence, under the power constraints equations, between decision makers.	In the experimental case study, MACRO was able to predict power relationships between all decision makers with a certainty, in terms of a maximum range, near 0.3.

one individual (the manager), power and influence reside in the others as well in the form of persuasion from expertise, experience, or information. The MACRO methodology is clearly applicable in these situations where power is more broadly defined.

Third, extremely high-dimensional design spaces would likely be impossible to use in such a methodology. Not only would the discrete choices be more challenging to respond to with so many trades to contemplate by decision makers, but the range on all preferences or the weighting distributions may all tightly hover near zero since so many weights sum to the constraint value of 1. However, most design spaces likely have a subset of objectives which capture a majority, if not all, of the key dimensions necessary for decision makers to evaluate the designs. Initial discrete choices could be used as tests to filter out objectives which have little or no impact on the utility score (i.e. objectives are removed that are found to be weighted with a value of zero).

Fourth, a high number of candidate designs require large computer memory and computational speed for storing and computing respectively the utility scores, intermediate regions, etc. If the number of designs becomes too large, implementing the methodology can be limited by external computer hardware or software constraints. For example, in the proof of concept problem discussed previously, 0.5 million designs were considered the maximum number of allowable designs that could be simultaneous processed in a reasonable amount of time without stalling or suspending the methodology on available computer resources.

Fifth, large numbers of decision makers in a group would also slow down the methodology and would require each to respond to a large number of discrete choices of pairwise comparisons, which is an unacceptable requirement. In the extreme example, every individual (i.e. citizen) could be modeled as agents (i.e. voter) in a group decision-making activity (i.e. federal election) to pick the design or solution (i.e. President), but many discrete choice experiments would be required to solve for the individual power relationships between each one. In such a case, the numerous agents would be "categorized" or "grouped together" and would be expected to decide (or vote) with similar preferences as a unit. This is essentially what the MACRO methodology does more formally, but with specific data from which to increase the confidence and accuracy of the output.

8.4 Summary of Contributions

The first major contribution of this research is the ability to more fully described and define the entire design space of an operational model. The AirMOD model enables rapid evaluation of the design space, extending the SACT tool for MC simulations, additional operational metrics including costs, and improved visualization of the scenario and logistical and operational metrics of interest. The analysis of such a design space was performed and observations and findings about the design space were only possible with a sufficient number of solutions compared and contrasted in a variety of visualizations.

The second major contribution is the MACRO methodology, a process to facilitate cooperative decision making within groups by identifying the candidate designs or solution and thus the region at which consensus is more likely to occur. The methodology itself was broken down into three major steps, each which offer unique contributions to the field of group decision making.

In Step 1, the application of discrete choices to extract preferences across the objective space was introduced, reflecting more realistic decisions as opposed to unrealistic objective comparisons out of context. This contribution allows one to identify the distributions of the preferences instead of deterministic weights of the objective space. It accounts for the fact that decision makers are likely to change preferences over time due to internal and external factors and are willing to concede their own preferences, to some degree, if cooperation is incentivized, either through time savings or benefits from forming coalitions with others.

The contributions from Step 2 provide a process to obtain the power indices, scaling factors or strength of the decision makers themselves using the same discrete choice technique as in Step 1 but with hypothetical design point associations to the other decision makers. The result from sufficient responses enables the evaluation of the power or influence relationships between decision makers. These indices can then be used to various consensus reaching algorithms to calculate the various willingness of decision makers to concede on the objective space with each other. As with Step 1, the power relationships are expressed as distributions between each pair of decision makers to account for the uncertainty and dynamic influence levels possible between them.

Step 3 contributions offer a unique consensus reaching algorithm implementing ultimatum game theoretic principles. Furthermore, the analysis of sequences in how decision makers form coalitions was conducted allowing one to compute the region in which the group is most likely to reach consensus across many sequences. Playing out the combination of sequences with the Step 1 distributions of weighting vectors and Step 2 influence relationships provides insight into candidate design points which should be initially considered as the subset of solutions for additional analysis and more formal negotiation.

Overall, the overall methodology with the two elements, AirMOD and MACRO, contribute to the conflicting requirements in air mobility operational problems of increasing the number of candidate designs more quickly and determining the set of design points at which consensus is probable for group decisions, thereby allowing a more expeditious and transparent requirements definition phase.

8.5 Future Research

A number of extensions to the research discussed in this dissertation have been identified as areas of potential future research.

Additional and alternative methods for the Design of Experiments, in particular for choice designs, could be explored and tested to evaluate their performance against the methods explored in this dissertation. This would be most applicable if the decision makers knew in advance how many discrete choices to which they are willing to respond or have time to complete. Knowing this constraint, the set of discrete choice experiments to maximize the information gained could then be designed.

A more detailed look into the sources of influence and in particular the influence differences among the objectives for different decision makers could be explored. For example, one decision maker may only have more influence over another in a subset of the objective space. In other words, their influence might extend to only 2 or 3 objectives and not across the whole space. Such an algorithm would potentially offer intermediate consensus regions on objective subsets with more complicated processes for reaching agreement. Furthermore, testing the methodology with real human subjects in group settings to evaluate the predictive capability of the methodology based on different influence relationships could also be investigated.

Reciprocity could also be explored in more detail, by analyzing the change in willingness of one decision maker to concede based on action or inaction of other decision makers. Similarly, metrics to track this willingness to concede over time as the "moment of decision" approaches, could reveal the impacts, both positive and negative, of constraining decision times. Penalty functions for not cooperating (or rewards for cooperating) could likewise be analyzed to identify strategies to accelerate decision making. These would be compared to studies into how the quality of the decision changes based on the level of reward or punishment.

The bilateral consensus reaching explored in this research could be expanded to account for trilateral, quadrilateral, etc. decision making or intermediate agreements throughout the sequence. In such a situation, the proposing decision maker might propose a design point or preference weighting vector somewhere between the 3 or more decision makers and any one of them can reject the proposal. As the intermediate coalition size approached the full group size the benefits would potentially decrease or at least take longer to reach consensus. However, coalitions with just a few more than two might be able to cooperate in more interesting ways than studied in this research.

Lastly, the effect of team or group structure could also be further investigated, especially on engineering project teams where managers may have more decisional power based on position but less expertise power. The ramifications between these and other factors for groups involved in decision making for system engineering applications would be similarly interesting.

8.6 Final Thoughts

It is important to note in the final analysis that this dissertation and the research herein never explicitly decides for the group of decision makers. No computer model ever "makes the decision." The best it can do is rank order, evaluate, suggest, or list the "best" designs or solutions according to specified assumptions and an objective function, provided by a human or other external source. However, simulating group decision-making with the appropriate data and assumptions, can make large advancements in assisting a group of decision makers in accelerating and facilitating the process saving both time and resources. Whether early in a system's life cycle, such as in the requirements definition phase, or in later phases, such as final design selection, this research is relevant by reducing the uncertainty about how a group of decision makers, seeking to cooperate, will act, by providing a design space or preference space region in which they are most likely to reach consensus.

APPENDIX A

VOTING METHODS

A common implementation of group decision making is voting, most often associated with political elections. Many different voting methods have been proposed for various situations such as plurality voting, Borda count, the Condorcet Method, instant runoff, etc. In describing and explaining the limitations of this partial list of voting methods, an example presented in [162] will be used.

Four cities are physically situated as shown in Figure 147 and will share one hospital. A vote is established to decide in which town the hospital should be located. Each town, and all individuals in each town, would prefer the hospital to be located as close as possible.

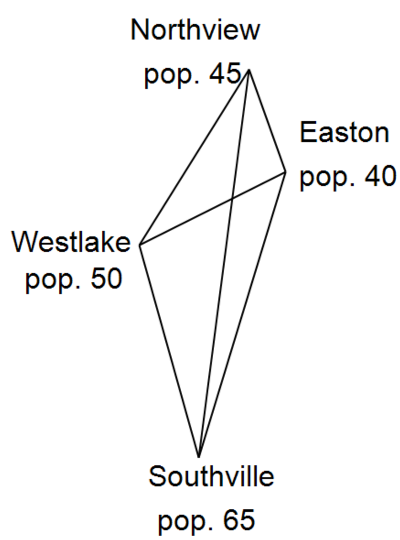


Figure 147: City Locations and Populations for a Vote (from [162])

A.1 Plurality Voting

With plurality voting, each individual in each town gets one vote and as a result Southville will win the vote since it has the largest population of 65 individuals. The other 135 people, however, would prefer a location other than Southville. Therefore, more than 2/3 of the towns combined population would receive their last choice.

This simple example quickly shows one weakness of plurality voting when there are more than two alternatives: a third alternative can change the ultimate winner. In terms of elections, suppose one candidate has 40% of the plural vote, a second has 35% and a third has 25%. If everyone who supports the third candidate also prefers the second candidate to the first, they would actually be indirectly supporting the candidate they least prefer, by taking "votes" away from a candidate (i.e. the second place winner) that would have won the election had the third candidate not run [106].

A.2 Borda Count

The Borda Count, named after Jean-Charles de Borda, is also vulnerable to manipulation [139]. Using a Borda mechanism, voters rank all the candidates or alternatives. The least desirable candidate receives 0 points, the next to last 1 point, the second to last 2 points, and so on. The scores for each candidate are added up and the winner is the one with the most points. The winner in the hospital example above would be Easton with 375 points followed by Westlake, Northview and Southville, with 365, 265, and 195 points, respectively, as summarized in Table 31.

Table 31: Borda Count for Each of the Four Candidate Hospital Locations

		Candidate Location and Borda rank			
City	Pop.	Northview	Easton	Southville	Westlake
Northview	45	3	2	0	1
Easton	40	2	3	0	1
Southville	65	0	1	3	2
Westlake	50	1	2	0	3
Total	-	265	375	195	365

Interestingly, if the exact Borda count process were repeated after removing Northview as one of the candidates (knowing a priori that it would not win anyway), Westlake wins with 250 points, followed by Easton at 220 and Southville with 130. The results are tabulated in Table 32.

Table 32: Borda Count for Each of the Three Remaining Candidate Hospital Locations After Northview Drops Out

		Candidate Location and Borda rank			
City	Pop.	Northview	Easton	Southville	Westlake
Northview	45	-	2	0	1
Easton	40	-	2	0	1
Southville	65	-	0	2	1
Westlake	50	-	1	0	2
Total	-	-	220	130	250

This illustrates a flaw in the Borda method. When a candidate considered an irrelevant alternative is removed, the actual winner can change. This is problematic since who wins is dependent on irrelevant alternatives that no one prefers. Fixes to this problem using "measure of dissimilarity" have been proposed but do not always work to remove the manipulation [54].

A.3 Condorcet Method

The Condorcet method applies the plurality voting method to each combination of candidates. Thus, Northview faces off head to head against all the other three candidates; Easton faces off against the remaining two as well, and so on. The candidate that wins head to head against all the other candidates is the winner.

The issue with this method is that there does not have to be a Condorcet winner. For example, applying the Condorcet method in a vote with three alternative, if the total population prefers candidate X to Y, Y to Z and Z to X, the preferences are cyclical resulting in the Condorcet Paradox [93]. In the example in Figure 147, Weston is the Condorcet winner, but with different populations, a cyclical population preference could be possible, especially with voters acting strategically. This Condorcet Paradox is more likely to occur with few voters as identified by [67].

A.4 Instant Runoff

In an instant runoff voting (IRV) method, the candidates with the fewest votes is eliminated and then the vote is recounted with $n - 1$ candidates. This process is repeated until a certain threshold (e.g. a majority) of the votes support one candidate, is reached. In the hospital example above, Northview will win the IRV method.

As with the other methods, IRV has interesting characteristics as well, such as a candidate losing the vote even though the popular opinion has moved closer to his or her position [162]. Similarly, paradoxes and other problems can arise in some situations of preferential voting discussed in [58].

A.5 Voting methods summary

From the foregoing sections, with Southville winning with plurality voting, Easton winning the Borda count method, Westlake winning the Condorcet method, and Northview taking the instant runoff election, voting methods clearly have serious limitations to unravel. Many attempts to avoid some of these issues have been proposed and currently two methods "approval voting" and "range voting" may be found to be better options to reduce gaming and manipulation of the voting systems [23].

APPENDIX B

COMPARING A SET OF THREE COMMON DECISION-MAKING TECHNIQUES

B.1 Decision Making Using the Overall Evaluation Criterion

Perhaps the most easy to understood and commonly used decision-making technique is when simple attributes or objectives are combined mathematically with weightings attached to each dimension as indicative of the importance or preferences of the various criteria [7]. This type of decision-making technique is called by various names including "parametric method", "simple additive weighting method (SAW)", "multi-attribute value" or "weighted sum model (WSM)" [17]. A related technique called the "weighted product model" (WPM) is very similar but the weighted objectives are multiplied instead of summed as in WSM [27]. Regardless of the name, the problem will seek to minimize (or maximize from the opposite perspective) the objectives based on the weightings of the objectives or criteria.

For the weighted sum approach, the score or value v_i for each alternative i takes the form of:

$$v_i = \textstyle\sum_{j=1}^{n} w_j x_{ij} \tag{52}$$

$$\text{subject to} \quad \textstyle\sum_{j=1}^{n} w_j = 1,$$

where w_j is the weight for criterion j and x_{ij} is the value or score of alternative i on criterion j, $j = 1, ..., n$.

Recasting the same approach into an optimization problem with the goal of maximization, results in:

$$\max_x \{\sum_{j=1}^{n} w_j f_j(x) = w^T F(x)\} \tag{53}$$

where, $f_j(x)$ is the jth objective (i.e. criteria) and $F(x) = [f_1(x) f_2(x) \dots f_n(x)]^T$. The optimization view is sometimes beneficial as the application of constraints, both equality and inequality, can be readily applied to identify feasibility of particular alternatives or designs [40].

The SAW formulation makes sense when the various criteria are of the same units. For example, if every criterion (i.e. speed, cost, performance) is transformed to constant units (i.e. "utils") through decision maker defined utility functions, then the above equation is valid.

With different dimensions, however, the WPM can often be more useful. It takes the form of:

$$\max_x \{v_i = \frac{\prod_{j=1}^{n} (x_{ij})^{w_j}}{\prod_{j=1}^{n} (x_j^*)^{w_j}}\} \tag{54}$$

where x_j^* is "best" value for the jth criteria, dimension or attribute. By employing these "best" values the upper-bound is established so that mapping all alternatives to the interval [0,1] can be performed [9].

A combination of the aforementioned ideas and approaches is developed through another weighted sum technique called the overall evaluation criterion (OEC), where the criteria are added (as in SAW) but are normalized against a baseline design (as in WPM), or other process, to established non-dimensional criteria. Equation 55 illustrates how the value in each criterion 1 through n for each alternative i is first divided by the baseline value of that particular criterion and then multiplied by the respective weighting factor w_j, $j = 1, ..., n$ before summation in the OEC. The best design is simply the alternative with the highest OEC.

$$\max_x\{OEC_i = w_1\frac{x_{i1}}{x_{baseline,1}} + w_2\frac{x_{i2}}{x_{baseline,2}} + \ldots + w_n\frac{x_{in}}{x_{baseline,n}}\} \tag{55}$$

Equation 55 assumes that it is more desirable to maximize all criteria 1 through n such that OEC scores of designs or alternatives with values greater than the baselines values will be proportionally larger. If minimizing certain criteria is better, the OEC formulation can be adjusted accordingly by inverting the fraction of the alternative's criterion value and baseline value as show in Equation 56.

$$\max_x\{OEC_i = w_1(\frac{x_{i,1}}{x_{baseline,1}})^{-1} + w_2(\frac{x_{i,2}}{x_{baseline,2}})^{-1} + \ldots + w_n(\frac{x_{i,n}}{x_{baseline,n}})^{-1}\} \tag{56}$$

Therefore, an example OEC with a combination of criteria from Equations 55 and 56 for one alternative would be:

$$\max_x\{OEC = w_{speed}\frac{x_{speed}}{x_{baseline,speed}} + w_{cost}(\frac{x_{baseline,cost}}{x_{cost}}\}, \tag{57}$$

assuming one desires to minimize cost and maximize speed.

One of the limitations of the weighted sums is described proficiently in [40]. For non-convex Pareto frontiers, certain points of the Pareto set cannot be solved for, and thus no combinations of weightings will ever identify those points as optimal based on a decision maker's preference.

Other issues of using decision matrices and OEC type analyses "...suffers from two major drawbacks; (i) some potentially optimal concepts may appear to be undesirable, because they never receive the highest total score, and (ii) the typical construction requires that the decision maker specify physically meaningless weights and ratings" [112]. These weights and ratings are the manifestations of human decision makers' preferences. In answer to these concerns, other methods have been developed to addressed these limitations.

The following sections will consider other techniques that are somewhat comparable to, and potentially improve upon, the simple OEC model. Although, this research is not intended to be a detailed analysis of different decision-making techniques, these sections are presented to help justify why the methodology presented is valid for an OEC model and could be applied to other decision-making techniques such as AHP and TOPSIS described next. In general, the same steps in reaching consensus could be technique independent under the assumption that preferences are an essential part of the technique and that the agents within the system are permitted to change preferences throughout the decision process.

B.2 AHP Compared to an OEC

In the AHP formulation [141], the weighting for each criterion (x_j) is established with a priority matrix which quantifies and tabulates pairwise comparisons of importance between the different criteria (x_1 to x_n) shown below. A decision maker will designate a value which represents the importance of one criterion with respect to another. For example, if criterion i is considered fives time more important than criterion j, then the a_{ij} element in the priority matrix will have a value of 5. Correspondingly, the a_{ji} element will assume the inverse of a_{ij}. Therefore, $a_{ji} = \frac{1}{5}$, and mathematically, $a_{ij} = a_{ji}^{-1}$. This reduces the number of importance pairwise comparisons in half. However, this can leave ambiguous actions if the values of importance ratios are not consistent. (If $a_{ij} = 5$ and $a_{jk} = 3$, does this necessarily mean that $a_{ik} = 15$ or can a pairwise comparison be valid for criteria i and k if it is different than 15? This is one of the short comings of AHP.) All diagonal elements of the priority matrix a_{ii} are intuitively given a value of one.

$$
\begin{array}{c|cccc}
 & x_1 & x_2 & \cdots & x_n \\
x_1 & 1 & a_{12} & \cdots & a_{1n} \\
x_2 & a_{21} & 1 & \cdots & a_{2n} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
x_n & a_{n1} & a_{n2} & \cdots & 1
\end{array} \tag{58}
$$

Once the above matrix has been filled, the priority vector for criteria x_1 through x_n can be calculated using the following equation in vector form:

$$
\begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix} = \begin{bmatrix} \frac{1}{n}\sum_{j=1}^{n}\left(\frac{a_{1,j}}{\sum_{i=1}^{n} a_{i,1}}\right) \\ \frac{1}{n}\sum_{j=1}^{n}\left(\frac{a_{2,j}}{\sum_{i=1}^{n} a_{i,2}}\right) \\ \vdots \\ \frac{1}{n}\sum_{j=1}^{n}\left(\frac{a_{n,j}}{\sum_{i=1}^{n} a_{i,n}}\right) \end{bmatrix} \tag{59}
$$

This equation is effectively an average of normalized pairwise comparisons between each criterion. The above vector only provides the importance weightings of the criteria (i.e. w_1). A similar process can now be applied to the actual attributes of each design, solution or choice for each criterion. Therefore, a matrix which contains the pairwise comparison for each of the m designs with respect to each criterion must be calculated.

$$
\begin{array}{c|cccc}
x_1 & d_1 & d_2 & \cdots & d_m \\
\hline
d_1 & a_{11}^{(x_1)} & a_{12}^{(x_1)} & \cdots & a_{1m}^{(x_1)} \\
d_2 & a_{21}^{(x_1)} & a_{22}^{(x_1)} & \cdots & a_{2m}^{(x_1)} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
d_m & a_{m1}^{(x_1)} & a_{m2}^{(x_1)} & \cdots & a_{mm}^{(x_1)},
\end{array} \tag{60}
$$

where, the superscript (x_1) for each element designates the criterion to which it corresponds. (Note, that there are 'm' designs, points or solutions that require pairwise comparison.)

Using the same format for the criteria importance vector above, the attribute importance vector is calculated by:

$$\vec{d}_{(x_1)} = \begin{bmatrix} d_{1_{(x_1)}} \\ d_{2_{(x_1)}} \\ \vdots \\ d_{m_{(x_1)}} \end{bmatrix} = \begin{bmatrix} \frac{1}{m}\sum_{j=1}^{m}\left(\frac{a_{1j}^{(x_1)}}{\sum_{i=1}^{m} a_{i1}^{(x_1)}}\right) \\ \frac{1}{m}\sum_{j=1}^{m}\left(\frac{a_{2j}^{(x_1)}}{\sum_{i=1}^{m} a_{i2}^{(x_1)}}\right) \\ \vdots \\ \frac{1}{m}\sum_{j=1}^{m}\left(\frac{a_{mj}^{(x_1)}}{\sum_{i=1}^{m} a_{im}^{(x_1)}}\right) \end{bmatrix} \tag{61}$$

After performing this calculation for each criterion, the number of attribute importance vectors can be calculated and combined into the attribute importance matrix shown below in element and vector forms:

$$\begin{bmatrix} \begin{bmatrix} d_{1_{(x_1)}} \\ d_{2_{(x_1)}} \\ \vdots \\ d_{m_{(x_1)}} \end{bmatrix} \begin{bmatrix} d_{1_{(x_2)}} \\ d_{2_{(x_2)}} \\ \vdots \\ d_{m_{(x_2)}} \end{bmatrix} \cdots \begin{bmatrix} d_{1_{(x_n)}} \\ d_{2_{(x_n)}} \\ \vdots \\ d_{m_{(x_n)}} \end{bmatrix} \end{bmatrix}_{m\times n} = \begin{bmatrix} \vec{d}_{x_1} & \vec{d}_{x_2} & \cdots & \vec{d}_{x_n} \end{bmatrix}_{m\times n}, \tag{62}$$

where m is the number of design points to compare and n is the number of criteria.

This process can be quite lengthy if the m and n are large and if the attributes are qualitative in nature, which requires a translation to a quantitative scale. (AHP also suffers from other limitations such as not being able to identify if some designs are infeasible due to constraints or other requirements [4].) Still, this process can be accelerated if all of the attributes have been quantified and simple equations to make these pairwise comparisons are implemented. (For example, if the range of a certain attribute for all the designs is from 40 to 90, and larger values are always preferred, linear comparisons are readily apparent (i.e. $90/40 = 2.25$). However, if 80 is twice as good as 40 but 90 is twice as good as 80, more sophisticated nonlinear comparisons are required.)

Once the attribute and criteria importance vectors are calculated, the evaluation process for all the designs takes the form of a simple matrix-vector product:

$$\begin{bmatrix} \vec{d}_{x_1} & \vec{d}_{x_2} & \cdots & \vec{d}_{x_n} \end{bmatrix}_{m\times n} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix}_{n\times 1} = w_1\vec{d}_{x_1} + w_2\vec{d}_{x_2} + \cdots + w_n\vec{d}_{x_n}, \tag{63}$$

which takes the familiar form of a simple OEC calculation,

$$\overrightarrow{OEC}_{m\times 1} = w_1\vec{d}_{x_1} + w_2\vec{d}_{x_2} + \cdots + w_n\vec{d}_{x_n} \tag{64}$$

or, separate OEC's for each m design points:

$$\begin{aligned} OEC_1 &= \quad w_1 d_{1_{x_1}} + w_2 d_{1_{x_2}} + \cdots + w_n d_{1_{x_n}} \\ OEC_2 &= \quad w_1 d_{2_{x_1}} + w_2 d_{2_{x_2}} + \cdots + w_n d_{2_{x_n}} \\ &\vdots \\ OEC_m &= \quad w_1 d_{m_{x_1}} + w_2 d_{m_{x_2}} + \cdots + w_n d_{m_{x_n}} \end{aligned} \tag{65}$$

B.3 TOPSIS Compared to an OEC

The Technique for Ordered Preference by Similarity to Ideal Solution (TOPSIS) is similar to AHP in that a process is used to quantify the score of a particular alternative by applying weights to the various criteria and comparing the alternatives to two idealize alternatives. Thus, the actual design points or alternatives are ordered in terms of preference with respect to each other, not by pairwise comparison, as in AHP, but by their relative distance to the absolute best (or worst) in each dimension or for each criterion [77]. These values are then multiplied by the weighting factors (just like in basic OEC or AHP) and ordered based on a cardinal scale.

Since TOPSIS requires the n-dimensional Euclidean distance(s) to a positive ideal and/or negative ideal solution, the first step in any TOPSIS algorithm is to define the ideal solution or design. This is accomplished by normalizing the best value for each criterion across all the solutions.

Starting with the decision matrix, D, defined as:

$$D = \begin{array}{c c} & \begin{array}{ccccc} x_1 & x_2 & x_3 & \cdots & x_n \end{array} \\ \begin{array}{c} A_1 \\ A_2 \\ A_3 \\ \vdots \\ A_m \end{array} & \begin{bmatrix} x_{11} & x_{12} & x_{13} & \dots & x_{1n} \\ x_{21} & x_{22} & x_{23} & \dots & x_{2n} \\ x_{31} & x_{32} & x_{33} & \dots & x_{3n} \\ \dots & \dots & \dots & \dots & \dots \\ x_{m1} & x_{m2} & x_{m3} & \dots & x_{mn} \end{bmatrix} \end{array} \tag{66}$$

where A_i is the alternative with x_{ij} as the values for alternative i in criteria j, $j = 1, ..., n$. For this general decision matrix D, there are n criteria and m alternatives.

The positive ideal solution A^+ is calculated by

$$A^+ = \begin{bmatrix} \frac{x_1^*}{\sqrt{\sum_{i=1}^m x_{i1}^2}} & \frac{x_2^*}{\sqrt{\sum_{i=1}^m x_{i2}^2}} & \cdots & \frac{x_n^*}{\sqrt{\sum_{i=1}^m x_{in}^2}} \end{bmatrix} = \begin{bmatrix} A_1^+ & A_2^+ & \dots & A_n^+ \end{bmatrix} \tag{67}$$

where, x_i^* is defined as the best value in column i of matrix D. A_j^+ is normalized by $\sqrt{\sum_{i=1}^m x_{j1}^2}$ in each column to non-dimensionalize the criteria. Since no indication of whether the decision maker seeks to minimize or maximize the particular criteria, the "best" value from each column will be the largest or smallest if he or she seeks to maximize or minimize the appropriate criterion, respectively. For example, the decision maker will take the lowest value for the cost criterion but the highest value for a performance criterion (such as fuel efficiency) to compose the positive ideal solution. Note that this positive ideal solution may not be itself a feasible solution within the design space.

The negative ideal A^- is similarly calculated, with the worst values in matrix D used after normalization:

$$A^- = \begin{bmatrix} \frac{x_1^-}{\sqrt{\sum_{i=1}^m x_{i1}^2}} & \frac{x_2^-}{\sqrt{\sum_{i=1}^m x_{i2}^2}} & \dots & \frac{x_n^-}{\sqrt{\sum_{i=1}^m x_{in}^2}} \end{bmatrix} = \begin{bmatrix} A_1^- & A_2^- & \dots & A_n^- \end{bmatrix} \tag{68}$$

The weighted Euclidean distance between each design alternative A_i and positive ideal A^+ is next calculated as:

$$S_i^+ = \sqrt{\sum_{j=1}^n (w_j x_{ij} - w_j A_j^+)^2} = \sqrt{\sum_{j=1}^n w_j^2 (x_{ij} - A_j^+)^2} \tag{69}$$

where w_j is the weight for criteria j, with $j = 1, ..., n$. The distance to the negative ideal is similarly evaluated:

$$S_i^- = \sqrt{\sum_{j=1}^n (w_j x_{ij} - w_j A_j^-)^2} = \sqrt{\sum_{j=1}^n w_j^2 (x_{ij} - A_j^-)^2} \tag{70}$$

TOPSIS will typically use both distances (i.e. the distance to both the positive and negative ideal solutions) to further differentiate between designs as shown in Figure 148. This is to maximize the distance to the negative ideal and, at the same time, minimize the distance to the positive ideal. Furthermore, in general, if two designs have the same distance to the positive ideal they will not have the same distance to the negative ideal, thus including both in a relative closeness parameter is beneficial.

This relative closeness parameter, R_i is calculated as the fraction of the distance to the negative ideal over the difference of the distances to the positive and negative ideals as shown in the following equation:

$$R_i = \frac{S_i^-}{S_i^+ - S_i^-} \tag{71}$$

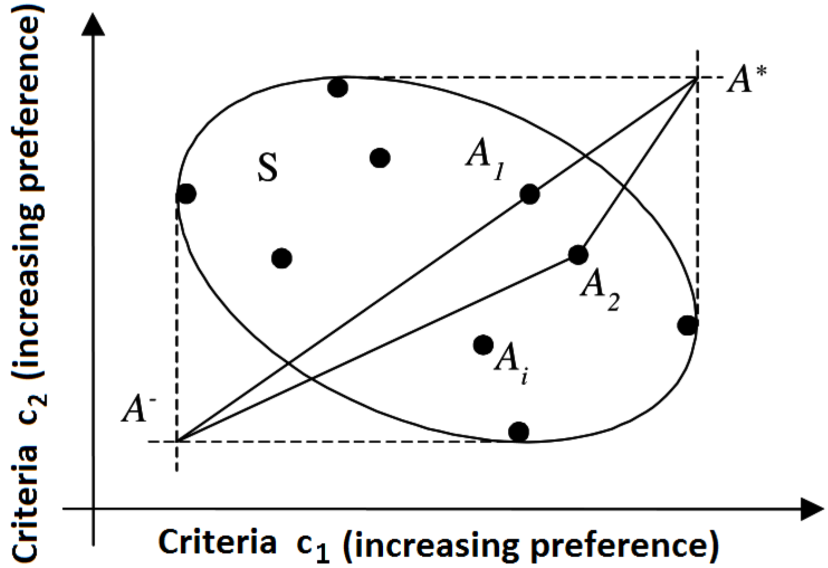


Figure 148: Euclidean Distances to Positive and Negative Ideals from Two Alternatives [77]

Expanding this equation, results in:

$$R_i = \frac{\sqrt{\sum_{j=1}^{n} w_j^2 (x_{ij} - \frac{x_j^-}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}})^2}}{\sqrt{\sum_{j=1}^{n} w_j^2 (x_{ij} - \frac{x_j^*}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}})^2} - \sqrt{\sum_{j=1}^{n} w_j^2 (x_{ij} - \frac{x_j^-}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}})^2}} \tag{72}$$

which is a function with the weights and elements from the original decision matrix x_{ij} and more simply:

$$R_i = f(w_j, x_{ij}), i = 1, ..., m, j = 1, ..., n \tag{73}$$

What the above equation illustrates is that the TOPSIS technique is just an additional combination of the same values and weights included in an OEC. Regardless of the decision-making technique selected as the process for evaluating the various alternatives, many of them can be compared to a process of weighting the various dimensions or criteria and then combining them in some simple (or complicated) form to extract the scores. The assumption that the OEC is representative of what other decision-making techniques use is thus made. Thus, although the group decision-making process, the MACRO methodology, proposed in this research can use potentially any preference-based decision-making technique, only a simple OEC in the proof of concept has been demonstrated.

REFERENCES

[1] Ackert, L. F., Gillette, A. B., Martinez-Vazquez, J., and Rider, M., "Are benevolent dictators altruistic in groups? A within-subject design," *EXPERIMENTAL ECONOMICS*, vol. 14, pp. 307–321, SEP 2011.

[2] Alexander, A. J., "The Cost and Benefits of Reliability in Military Equipment," *The RAND Corporation*, vol. AD-A207 344, 1988.

[3] Allan, P. and Dupont, C., "International relations theory and game theory: Baroque modeling choices and empirical robustness," *International Political Science Review*, vol. 20, pp. 23–47, Jan 1999.

[4] Antonsson, E. K. and Otto, K. N., "Imprecision in Engineering Design," *ASME Journal of Mechanical Design*, vol. 117, pp. 25–32, 1995.

[5] Arrow, K. J., "A Difficulty in the Concept of Social Welfare," *The Journal of Political Economy*, vol. 58, pp. 328–426, August 1950.

[6] Asch, S., *Effects of group pressure upon the modification and distortion of judgments*, pp. 177–190. Pittsburgh: Carnegie Press, 1951.

[7] Athan, T. W. and Papalambros, P. Y., "A Note of Weighted Criteria Methods for Compromise Solutions in Multi-Objective Optimization," *Engineering Optimization*, vol. 27, pp. 155–176, 1996.

[8] Axelrod, R. and Hamilton, W., "The Evolution of Cooperation," *Science*, vol. 211, no. 4489, pp. 1390–1396, 1981.

[9] Azar, F. S., "Multiattribute decision-making: Use of three scoring methods to compare the performance of imaging techniques for breast cancer detection," tech. rep., University of Pennsylvania, 2000.

[10] Bal, S., Rmila, E., and Solal, P., "Compensations in the shapley value and the compensation solutions for graph games," MPRA Paper 20955, University Library of Munich, Germany, Feb. 2010.

[11] Banzhaf III, J. F., "Weighted voting doesn't work: A mathematical analysis," *Rutgers L. Rev.*, vol. 19, p. 317, 1964.

[12] Barno, D. W. (Lieutenant General - Ret.), Bensahel, N., and Sharp, T., "Hard choices: Responsible defense in an age of austerity," 2011.

[13] Belch, G., "The Effects of Television Commercial Repetition on Cognitive Response and Message Acceptance," *Journal Of Consumer Research*, vol. 9, no. 1, pp. 56–65, 1982.

[14] Bell, D. and Raiffa, H., *Decision making: Descriptive, normative, and prescriptive interactions.* Cambridge University Press, 1988.

[15] Bell, W. J., ed., *Mr. Franklin: A Selection from His Personal Letters.* Yale University Press: New Haven, CT, 1956.

[16] Belton, V. and Gear, A., "On a Shortcoming of Saatys Method of Analytic Hierarchies in its Present Form," *Journal Of The Operational Research Society*, vol. 33, no. 12, p. A30, 1982.

[17] Belton, V., "A comparison of the analytic hierarchy process and a simple multi-attribute value function," *European Journal of Operational Research*, vol. 26, pp. 7–21, 1986.

[18] Ben-Akiva, M., McFadden, D., Grling, T., Gopinath, D., Walker, J., Bolduc, D., Brsch-Supan, A., Delqui, P., Larichev, O., Morikawa, T., Polydoropoulou, A., and Rao, V., "Extended Framework for Modeling Choice Behavior," *Marketing Letters*, vol. 10, pp. 187–203, 1999.

[19] Bendor, J., Diermeier, D., Siegel, D., and Ting, M., *A Behavioral Theory of Elections.* Princeton University Press, 2011.

[20] Benítez, J., Delgado-Galván, X., Gutiérrez, J., and Izquierdo, J., "Balancing consistency and expert judgment in AHP," *Mathematical and Computer Modelling*, vol. 54, no. 7, pp. 1785–1790, 2011.

[21] Black, D., "On the rationale of group decision-making," *Journal of Political Economy*, vol. 56, no. 1, pp. pp. 23–34, 1948.

[22] Blanchard, B. and Fabrycky, W., *Systems Engineering and Analysis.* Prentice-Hall, 1998.

[23] Brams, S. J. and Fishburn, P. C., "Approval Voting," *The American Political Science Review*, vol. 72, no. 3, pp. pp. 831–847, 1978.

[24] Brandt, S. A. and Stiles, R. J., *Introduction to Aeronautics: A Design Perspective (AIAA Education Series).* Amer Inst of Aeronautics &, 1997.

[25] Bretscher, O., *Linear Algebra With Applications.* Pearson College Division, 2012.

[26] Briceno, S., *A game-based decision support methodology for competitive systems design.* PhD thesis, Georgia Institute of Technology, November 2008.

[27] Bridgeman, P. W., *Dimensionless Analysis.* New Haven, CT: Yale University Press, 1922.

[28] Buchanan, L. and O'Connell, A., "A Brief History Of Decision Making," *Harvard Business Review*, vol. 84, no. 1, pp. 32 – 41, 2006.

[29] Busemeyer, J. R. and Townsend, J. T., "Decision field theory: A dynamic-cognitive approach to decision making in an uncertain environment.," *Psychological Review*, vol. 100, no. 3, pp. 432 – 459, 1993.

[30] Cacioppo, J. and Petty, R., "Effects Of Message Repetition And Position On Cognitive Response, Recall, And Persuasion," *Journal Of Personality And Social Psychology*, vol. 37, no. 1, pp. 97–109, 1979.

[31] Campbell, S. D. and Sharpe, S. A., "Anchoring Bias in Consensus Forecasts and Its Effect on Market Prices," *Journal of Financial and Quantitative Analysis*, vol. 44, pp. 369–390, Apr 2009.

[32] Chairman of the Joint Chiefs of Staff, *Chairman of the Joint Chiefs of Staff Instruction CJCSI 3170.01 H*, January 2012.

[33] Charness, G. and Rabin, M., "Expressed preferences and behavior in experimental games," *Games and Economic Behavior*, vol. 53, pp. 151–169, Nov 2005.

[34] Chen, C., "Extensions of the TOPSIS for group decision-making under fuzzy environment," *Fuzzy Sets And Systems*, vol. 114, pp. 1–9, Aug 16 2000.

[35] Chu, H.-C. and Hwang, G.-J., "A Delphi-based approach to developing expert systems with the cooperation of multiple experts," *Expert Systems with Applications*, vol. 34, pp. 2826–2840, May 4 2008.

[36] Coleman, J. S., "Control of collectivities and the power of a collectivity to act," *Social choice*, vol. 42, pp. 269–300, 1971.

[37] Cooper, D. and Zillante, A., "A comparison of cumulative voting and generalized plurality voting," *Public Choice*, vol. 150, pp. 363–383, Jan 2012.

[38] Cooper, W., "Decision-Theory as a Branch of Evolutionary-Theory - A Biological Derivation of the Savage Axioms," *Psychological Review*, vol. 94, pp. 395–411, Oct 1987.

[39] Cosmides, L. and Tooby, J., "Better Than Rational - Evolutionary Psychology And The Invisible Hand," *American Economic Review*, vol. 84, pp. 327–332, May 1994. 106th Annual Meeting of the American-Economic-Association, Boston, MA, Jan 03-05, 1994.

[40] Das, I. and Dennis, J., "A closer look at drawbacks of minimizing weighted sums of objectives for Pareto set generation in multicriteria optimization problems," *Structural and Multidisciplinary Optimization*, vol. 14, no. 1, pp. 63–69, 1997.

[41] Dawes, R., "Social Dilemmas," *Annual Review of Psychology*, vol. 31, pp. 169–193, 1980.

[42] De Borda, J., "Memoire sur les elections au scrutin, historie de lacademie royale des sciences," *Paris, France*, 1781.

[43] Deegan Jr, J. and Packel, E. W., "A new index of power for simplen-person games," *International Journal of Game Theory*, vol. 7, no. 2, pp. 113–123, 1978.

[44] Department of Defense, "Sustaining U.S. Global Leadership: Priorities for 21st Century Defense," January 2012.

[45] Department of Defense, Office of Inspector General, "Source Selection Procedures for the C-5 Avionics Modernization Program (D-2006-058)," tech. rep., Office of Inspector General, Arlington,VA, February 2006.

[46] Department of the Air Force, "Air mobility planning factors afpam10-1403," December 2011.

[47] Dickerson, C. and Mavris, D., *Architecture and Principles of Systems Engineering*. Boca Raton: Auderbach Publications, 2010.

[48] Diederich, A., "Dynamic stochastic models for decision making under time constraints," *Journal Of Mathematical Psychology*, vol. 41, pp. 260–274, Sep 1997.

[49] Dijkstra, T. K., "On the extraction of weights from pairwise comparison matrices," *Central European Journal of Operations Research*, vol. 21, no. 1, pp. 103–123, 2013.

[50] Doebeli, M. and Hauert, C., "Models of cooperation based on the Prisoner's Dilemma and the Snowdrift game," *Ecology Letters*, vol. 8, no. 7, pp. 748–766, 2005.

[51] Douglas, M., *How institutions think*. Syracuse, N.Y.: Syracuse University Press., 1986.

[52] Downs, A., *An economic theory of democracy*. Harper, 1957.

[53] Dugatkin, L. A., Mesterton-Gibbonsand, M., and Houston, A. I., "Beyond the prisoner's dilemma: Toward models to discriminate among mechanisms of cooperation in nature," *Trends in Ecology & Evolution*, vol. 7, no. 6, pp. 202 – 205, 1992.

[54] Dummett, M., "The Borda count and agenda manipulation," *Social Choice and Welfare*, vol. 15, pp. 289–296, Feb 1998.

[55] Ericson, C., *Real-Time Collision Detection*. No. v. 1 in Morgan Kaufmann Series in Interactive 3D Technology, Morgan Kaufmann, 2005.

[56] Fehr, E. and Gachter, S., "Altruistic punishment in humans," *Nature*, vol. 415, pp. 137–140, Jan 10 2002.

[57] Fernandez-Carro, R., "Principal-Agent Theory in Science and Technology Studies," *Arbor-Ciencia Pensamiento y Cultura*, vol. 185, pp. 809–824, Jul-Aug 2009.

[58] Fishburn, P. C. and Brams, S., "Paradoxes of Preferential Voting," *Mathematics Magazine*, vol. 56, no. 4, pp. 207–214, 1983.

[59] Fishburn, P. C., "Methods of estimating additive utilities," *Management science*, vol. 13, no. 7, pp. 435–453, 1967.

[60] Flari, V., Chaudhry, Q., Neslo, R., and Cooke, R., "Expert judgment based multi-criteria decision model to address uncertainties in risk assessment of nanotechnology-enabled food products," *Journal Of Nanoparticle Research*, vol. 13, pp. 1813–1831, May 2011.

[61] Forsyth, D., *Group Dynamics*. Cengage Learning, 2009.

[62] Foster, K. R. and Kokko, H., "The evolution of superstitious and superstition-like behaviour," *Proceedings Of The Royal Society B-Biological Sciences*, vol. 276, pp. 31–37, Jan 7 2009.

[63] French, S., *Decision Theory: An Introduction to the Mathematics of Rationality*. Ellis Horwood series in mathematics & its applications, Ellis Horwood, 1993.

[64] Fullmer, D. and Mavris, D. N., "Concurrent product data modeling for collaborative design environments," in *Proceedings of the 2010 Conference on Grand Challenges in Modeling & Simulation*, pp. 360–366, Society for Modeling & Simulation International, 2010.

[65] Gandel, C., "Revamped Engineering Programs Emphasize Real-World Problem Solving." `http://www.usnews.com/education/best-graduate-schools/articles/2013/03/14/revamped-engineering-programs-emphasize-real-world-problem-solving`, March 2013.

[66] Geckil, I. and Anderson, P., *Applied Game Theory and Strategic Behavior*. Taylor & Francis, 2010.

[67] Gehrlein, W., "Condorcet's paradox and the likelihood of its occurrence: Different perspectives on balanced preferences," *Theory and Decision*, vol. 52, pp. 171–199, Mar 2002.

[68] Gigerenzer, G. and Selten, R., *Bounded Rationality: The Adaptive Toolbox*. Dahlem Workshop Reports, Mit Press, 2002.

[69] Gigerenzer, G. and Goldstein, D. G., "Reasoning the fast and frugal way: Models of bounded rationality.," *Psychological Review*, vol. 103, no. 4, pp. 650 – 669, 1996.

[70] Gilovich, T., Vallone, R., and Tversky, A., "The Hot Hand in Basketball - On the Misperception of Random Sequences," *Cognitive Psychology*, vol. 17, no. 3, pp. 295–314, 1985.

[71] GlobalSecurity, "C-5 service life." `http://www.globalsecurity.org/military/systems/aircraft/c-5-serv.htm`.

[72] Griendling, K. A., *Architect: the architecture-based technology evaluation and capability tradeoff method*. PhD thesis, Georgia Institute of Technology, 2011.

[73] Harsanyi, J. C., "A Simplified Bargaining Model for the n-Person Cooperative Game," *International Economic Review,*, vol. 4, pp. 194–220, May 1963.

[74] Harsanyi, J. C., "Utilities, preferences, and substantive goods," *Social choice and welfare*, vol. 14, no. 1, pp. 129–145, 1996.

[75] Helmer-Hirschberg, O., "Analysis of the Future: The Delphi Method," *The RAND Corporation*, vol. P-3558, 1967.

[76] Hess, R. and Romanoff, H., "Aircraft Airframe Cost Estimating Relationships," *The RAND Corporation*, vol. R-3255-AF, December 1987.

[77] Hwang, C. L. and Yoon, K., *Multiple Attribute Decision Making: Methods and Applications A State-of-the-Art Survey*. Berlin: Springer-Verlag, 1981.

[78] INCOSE, "Genesis of INCOSE." `http://www.incose.org/about/genesis.aspx`.

[79] Iz, P. and Krajewski, L., "Comparative evaluation of three interactive multiobjective programming techniques as group decision support tools," *INFOR-OTTAWA-*, vol. 30, pp. 349–349, 1992.

[80] James, C. C., "Nuclear Arsenal Games: Coping with Proliferation in a World of Changing Rivalries," *Canadian Journal of Political Science / Revue canadienne de science politique*, vol. 33, no. 4, pp. pp. 723–746, 2000.

[81] Janis, I. L., *Victims of Groupthink*. Boston: Houghton Mifflin, 1972.

[82] Jemison, D. and Sitkin, S., "Corporate acquisitions: A process perspective," *Academy of Management Review*, vol. 11, pp. 145–163, 1986.

[83] Jones, J. M., *Introduction to decision theory / J. Morgan Jones.* Irwin series in quantitative analysis for business, Homewood, Ill. : R. D. Irwin, 1977., 1977.

[84] Kahneman, D., Slovic, P., and Tversky, A., *Judgment Under Uncertainty: Heuristics and Biases*. Cambridge University Press., 1982.

[85] Kahneman, D. and Tversky, A., "Prospect theory: An analysis of decision under risk," *Econometrica*, vol. 47, no. 2, pp. pp. 263–292, 1979.

[86] Keeney, R. L. and Raiffa, H., *Decisions with Multiple Objectives: Preferences and Value Tradeoffs (Probability & Mathematical Statistics)*. John Wiley & Sons Inc, 1976.

[87] Keeney, R., "Group Preference Axiomatization With Cardinal Utility," *Management Science*, vol. 23, no. 2, pp. 140–145, 1976.

[88] Kimura, E., "O Brasil na Estao Espacial Internacional." `http://www.gizmodo.com.br/conteudo/made-brazil-o-brasil-na-estacao-espacial-internacional/`, May 2009.

[89] King, G., Keohane, R. O., and Verba, S., *Designing social inquiry: scientific inference in qualitative research*. Princeton University Press, 1994.

[90] Kreisher, O., "The Saga of Marine One," *Air Force Magazine*, vol. 95, February 2012.

[91] Kuemmerli, R., Colliard, C., Fiechter, N., Petitpierre, B., Russier, F., and Keller, L., "Human cooperation in social dilemmas: comparing the Snowdrift game with the Prisoner's Dilemma," *Proceedings of the Royal Society B-Biological Sciences*, vol. 274, pp. 2965–2970, Dec 7 2007.

[92] Kuhberger, A., Komunska, D., and Perner, J., "The disjunction effect: Does it exist for two-step gambles?," *Organizational Behavior And Human Decision Processes*, vol. 85, pp. 250–264, Jul 2001.

[93] Kurrild-Klitgaard, P., "An empirical example of the Condorcet Paradox of voting in a large electorate," *Public Choice*, vol. 107, pp. 135–145, Apr 2001.

[94] Lavie, N., "Attention, Distraction, and Cognitive Control Under Load," *Current Directions in Psychological Science*, vol. 19, pp. 143–148, Jun 2010.

[95] Levy, P., *Industrial/Organizational Psychology*. Worth Publishers, 2009.

[96] Li, Y., *An Intelligent, Knowledge-based Multiple Criteria Decision Making Advisor for Systems Design*. PhD thesis, Georgia Institute of Technology, 2007.

[97] Linstone, H. and Turoff, M., *The Delphi method: techniques and applications*. Advanced book program: Addison-Wesley, Addison-Wesley Pub. Co., Advanced Book Program, 1975.

[98] Liu, P. and Wang, M., "An extended VIKOR method for multiple attribute group decision making based on generalized interval-valued trapezoidal fuzzy numbers," *Scientific Research and Essays*, vol. 6, no. 4, pp. 766–776, 2011.

[99] Manski, C. F. and Newey, W. K., "Economics to econometrics: In honor of Daniel L. McFadden," *International Economic Review*, vol. 48, pp. 1091–1092, November 2007.

[100] Marshall, A., *Principles of Economics: An introductory volume.* London: Macmillan, 8th ed., 1920.

[101] Martin, L., "Super Galaxy Delivered." http://www.lockheedmartin.com/us/news/press-releases/2012/december/super-galaxy-delivered.html, December 2012.

[102] Mateos, A., Jimenez, A., and Rios-Insua, S., "Monte Carlo simulation techniques for group decision making with incomplete information," *European Journal of Operational Research*, vol. 174, pp. 1842–1864, Nov 1 2006.

[103] Mavris, D. N. and DeLaurentis, D. A., "A probabilistic approach for examining aircraft concept feasibility and viability," *Aircraft Design*, vol. 3, no. 2, pp. 79 – 101, 2000.

[104] Mavris, D. N., Pinon, O. J., and Fullmer Jr, D., "Systems design and modeling: A visual analytics approach," *27th Congress of International Council of the Aeronautical Sciences ICAS*, pp. 1–28, 2010.

[105] Maynard Smith, John, *Evolution and the Theory of Games.* Cambridge University Press, 1982.

[106] McCannon, B. C., "Can the majority lose the election?," *Review of Economic Design*, vol. 13, pp. 305–317, Dec 2009.

[107] McFadden, D., "Economic Choices," *The American Economic Review*, vol. 91, pp. 351–378, June 2001.

[108] Miller, G. A., "The Magical Number Seven, Plus or Minus Two Some Limits on Our Capacity for Processing Information," *Psychological Review*, vol. 101, no. 2, pp. 343–352, 1955.

[109] Milliman, J., "Presidential helo is now 'VH-71A'." http://www.navair.navy.mil/index.cfm?fuseaction=home.NavairNewsStory&id=3273, July 2005.

[110] Mistri, M., "Multi-Objective Preferences and Conflicting Objectives: The Case of European Monetary Integration," in *Advances in Multiple Critieria Decision Making and Human Systems Management* (Yong Shi, Milan Zeleny, D. L., ed.), pp. 3–16, IOS Press, 2007.

[111] Mori, K. and Arai, M., "No need to fake it: Reproduction of the asch experiment without confederates.," *International Journal of Psychology*, vol. 45, no. 5, pp. 390 – 397, 2010.

[112] Mullur, A., Mattson, C. A., and Messac, A., "New Decision Matrix Based Approach for Concept Selection Using Linear Physical Programming," *AIAA paper*, no. Paper No. AIAA 2003-1446, 2003.

[113] Neelin, J., Sonnenschein, H., and Spiegel, M., "A further test of noncooperative bargaining theory - Comment," *American Economic Review*, vol. 78, pp. 824–836, Sep 1988.

[114] Nicolai, L. M. and Carichner, G., *Fundamentals of aircraft and airship design.* American Institute of Aeronautics and Astronautics Inc., 2010.

[115] Nicolai, L., *Fundamentals of Aircraft Design.* METS In, 1975.

[116] Nobelprize.org, "The Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel 2002." `http://www.nobelprize.org/nobel_prizes/economics/laureates/2002/`, March 2013.

[117] Nowak, M. A., Page, K. M., and Sigmund, K., "Fairness Versus Reason in the Ultimatum Game," *Science*, vol. 289, no. 5485, pp. 1773–1775, 2000. Read.

[118] Obama, B., *The Audacity of Hope: Thoughts on Reclaiming the American Dream.* Crown Publishers, 2006.

[119] Ochs, J. and Roth, A., "An Experimental-Study of Sequential Bargaining ," *American Economic Review*, vol. 79, pp. 355–384, Jun 1989.

[120] Office of the Under Secretary of Defense (Comptroller) / Chief Financial Officer, "Fiscal Year 2013 Budget Request," February 2012.

[121] Olson, D., Moshkovich, H., Schellenberger, R., and Mechitov, A., "Consistency and accuracy in decision aids: Experiments with four multiattribute systems," *Decision Sciences*, vol. 26, pp. 723–748, Nov-Dec 1995.

[122] Opricovic, S., "Multicriteria optimization of civil engineering systems." Belgrade, 1998.

[123] Opricovic, S. and Tzeng, G.-H., "Compromise solution by mcdm methods: A comparative analysis of vikor and topsis," *European Journal of Operational Research*, vol. 156, no. 2, pp. 445 – 455, 2004.

[124] Ostrosi, E., Haxhiaj, L., and Fukuda, S., "Fuzzy modelling of consensus during design conflict resolution," *Research In Engineering Design*, vol. 23, pp. 53–70, Jan 2012.

[125] Payne, J., Bettman, J., and Luce, M., "When time is money: Decision behavior under opportunity-cost time pressure," *Organizational Behavior and Human Decision Processes*, vol. 66, pp. 131–152, May 1996.

[126] Peress, M., "Selecting the Condorcet Winner: single-stage versus multi-stage voting rules," *Public Choice*, vol. 137, pp. 207–220, Oct 2008.

[127] Pfeffer, J., "Power Play," *Harvard Business Review*, vol. 88, pp. 84+, Jul-Aug 2010.

[128] Pratt, J., Raïffa, H., and Schlaifer, R., *Introduction to Statistical Decision Theory.* McGraw Hill, 1995.

[129] Radner, R., "Satisficing," in *Optimization Techniques IFIP Technical Conference Novosibirsk, July 1–7, 1974*, pp. 252–263, Springer, 1975.

[130] Ramanathan, R. and Ganesh, L., "Group-Preference Aggregation Methods Employed in AHP - An Evaluation And an Intrinsic Process for Deriving Members Weightages," *European Journal of Operational Research*, vol. 79, pp. 249–265, Dec 8 1994.

[131] Raven, B., Schwarzwald, J., and Koslowsky, M., "Conceptualizing and measuring a power/interaction model of interpersonal influence," *Journal of Applied Social Psychology*, vol. 28, pp. 307–332, Feb 15 1998.

[132] Raymer, D. P., *Aircraft design: A conceptual approach.* AIAA education series, Reston, Va.: American Institute of Aeronautics and Astronautics, 4. ed. ed., 2006.

[133] Rittel, H. W. J. and Webber, M. M., "Dilemmas in a General Theory of Planning," *Policy Sciences*, vol. 4, no. 2, pp. pp. 155–169, 1973.

[134] Roman, F., "Selection without reflection is a risky business...," in *10th AIAA/ISSMO Multidisciplinary Analysis and Optimization Conference*, vol. AIAA-2004-4429, 2004.

[135] Roskam, J., *Airplane Design.* No. pt. 8 in Airplane Design, DARcorporation, 2002.

[136] Rousis, D., *A pareto frontier intersection-based approach for efficient multi-objective optimization of competing concept alternatives.* PhD thesis, Georgia Institute of Technology, 2011.

[137] Rubinstein, A., "Perfect equilibrium in a bargaining model ," *Econometrica*, vol. 50, no. 1, pp. 97–109, 1982.

[138] Ryan, M. and Gerard, K., "Using discrete choice experiments to value health care programmes: current practice and future research reflections," *Applied Health Economics and Health Policy*, vol. 2, pp. 55–64, 2003.

[139] Saari, D., "Susceptibility to Manipulation," *Public Choice*, vol. 64, pp. 21–41, Jan 1990.

[140] Saari, D., "Symmetry Extensions of Neutrality .1. Advantage to the Condorcet Loser," *Social Choice And Welfare*, vol. 9, pp. 307–336, Oct 1992.

[141] Saaty, T., "How to Make Decisions: The Analytical Hierarchy Process," *European Journal of Operational Research*, vol. 48, no. 1, pp. 9–26, 1990.

[142] Salmon, J., Iwata, C., Mavris, D., Weston, N., and Fahringer, P., "Comparative Assessment and Decision Support System for Strategic Military Airlift Capability," in *Proceedings of the MODSIM World 2010 Conference & Expo*, (Virginia Beach, VA), MODSIM World 2010 Conference & Expo, 2010.

[143] Salonen, M. and Perttula, M., "Utilization of concept selection methods - A survey of Finnish industry," in *Proceedings of the ASME International Design Engineering Technical Conferences and Computers and Information in Engineering Conference, Vol 5*, (Three Park Avenue, NY, NY 10016-5990 USA), pp. 527–535, ASME, Design Engn Div; ASME, Comp & Informat Engn Div, American Society of Mechanical Engineers, 2005. 17th International Conference on Design Theory and Methodology/Power Transmission and Gearing Conference, Long Beach, CA, Sep 24-28, 2005.

[144] Schrage, D., DeLaurentis, D., and Taggart, K., "Ippd concept development process for future combat system," in *Atlanta, GA), Presented at the 9th AIAA/ISSMO Symposium on Multidisciplinary Analysis and Optimization, AIAA*, 2002.

[145] Schwarz, M. and Sonin, K., "A theory of brinkmanship, conflicts, and commitments," *Journal of Law Economics & Organization*, vol. 24, pp. 163–183, Mar 2008.

[146] Schwenk, C., "Cognitive Simplification Processes in Strategic Decision-Making," *Strategic Management Journal*, vol. 5, no. 2, pp. 111–128, 1984.

[147] Scientists and Engineers for America, "Why Run for Office?." `http://www.sefora.org/`.

[148] Scott, J., *Understanding Contemporary Society: Theories of the Presen*, ch. Rational Choice Theory, pp. 126–138. Sage Publications, 2000. editors are G.Browning, A. Halcli, and F. Webster.

[149] Scott, M. J. and Antonsson, E. K., "Formalisms for negotiation in engineering design," *in 8th International Conference on Design Theory and Methodology, ASME*, 1996.

[150] Shalal-Esa, A., "EADS: won't protest "lowball" Boeing tanker deal." Reuters, March 2011.

[151] Shapley, L. S. and Shubik, M., "A method for evaluating the distribution of power in a committee system," *American Political Science Review*, vol. 48, no. 3, pp. 787–792, 1954.

[152] Shapley, L. S., "A Value for n-Person Games." Santa Monica, CA: RAND Corporation, 1952.

[153] Shermer, M., "Patternicity," *Scientific American*, vol. 299, p. 48, Dec 2008.

[154] Shih, H., Lin, W., and Lee, E., "Group decision making for TOPSIS," in *Joint 9th IFSA World Congress and 20th NAFIPS International Conference, Proceedings, Vols. 1-5* (Smith, MH and Gruver, WA and Hall, LO, ed.), pp. 2712–2717, Int Fuzzy Syst Assoc; N Amer Fuzzy Informat Proc Soc; IEEE Syst, Man & Cybernet Soc; IEEE, Neural Networks Council, 2001. 9th International-Fuzzy-Systems-Association World Congress/20th North-American-Fuzzy-Information-Processing-Society, International Conference, Vancouver, Canada, Jul 25-28, 2001.

[155] Shih, H.-S., Shyur, H.-J., and Lee, E. S., "An extension of TOPSIS for group decision making," *Mathematical and Computer Modelling*, vol. 45, pp. 801–813, Apr 2007.

[156] Simon, H., "Rational Choice and the Structure of the Environment," *Psychological Review*, vol. 63, no. 2, pp. 129–138, 1956.

[157] Simon, H., *Models of man: social and rational; mathematical essays on rational human behavior in society setting.* Wiley, 1957.

[158] Simon, H., "Theories of Decision-Making in Economics and Behavioral-Science," *American Economic Review*, vol. 49, no. 3, pp. 253–283, 1959.

[159] SimPy, "SimPy Simulation Package." `http://simpy.sourceforge.net/`, June 2010.

[160] Srikantaiah, T., Srikantaiah, T., Koenig, M., and Al-Hawamdeh, S., *Convergence of Project Management and Knowledge Management.* G - Reference, Information and Interdisciplinary Subjects Series, Scarecrow Press, 2010.

[161] Steinhauer, J., "Divided House Passes Tax Deal in End to Latest Fiscal Standoff." `http://www.nytimes.com/2013/01/02/us/politics/house-takes-on-fiscal-cliff.html?pagewanted=all&_r=0`, January 2013.

[162] Stevens, S. P., "Games People Play: Game Theory in Life, Business, and Beyond." 24-part series on game theory for the Teaching Company, 2008.

[163] Straub, P. G. and Murnighan, J. K., "An experimental investigation of ultimatum games: information, fairness, expectations, and lowest acceptable offers," *Journal of Economic Behavior & Organization*, vol. 27, no. 3, pp. 345 – 364, 1995.

[164] Sugden, R., *The economics of rights, cooperation and welfare.* Oxford, UK; New York, NY: Blackwell, 1986.

[165] Sullivan, L., "Quality Function Deployment.," *Quality Progress*, vol. 19, pp. 39–50, June 1986.

[166] The Office of the Director, Operational Test & Evaluation, "FY2012 Annual Report," tech. rep., United States General Accounting Office, December 2012.

[167] Tirpak, J. A., "Saving the Galaxy." http://www.airforce-magazine.com/MagazineArchive/Pages/2004/January%202004/0104galaxy.aspx, January 2004.

[168] Trefethen, L. N. and Bau, D., *Numerical Linear Algebra*. SIAM: Society for Industrial and Applied Mathematics, June 1997.

[169] Triantaphyllou, E., Shu, B., Sanchez, S. N., and Ray, T., "Multi-criteria decision making: an operations research approach," *Encyclopedia of electrical and electronics engineering*, vol. 15, pp. 175–186, 1998.

[170] Tschan, F., Semmer, N. K., Gurtner, A., Bizzari, L., Spychiger, M., Breuer, M., and Marsch, S. U., "Explicit Reasoning, Confirmation Bias, and Illusory Transactive Memory A Simulation Study of Group Medical Decision Making," *Small Group Research*, vol. 40, pp. 271–300, Jun 2009. 3rd Annual Conference of the Interdisciplinary-Network-for-Group-Research, Kansas City, MO, Jul 17-19, 2008.

[171] Tversky, A. and Kahneman, D., "Judgment Under Uncertainty - Heuristics And Biases," *Science*, vol. 185, no. 4157, pp. 1124–1131, 1974.

[172] Tversky, A. and Kahneman, D., "The Framing of Decisions and the Psychology of Choice," *Science*, vol. 211, no. 4481, pp. 453–458, 1981.

[173] United States General Accounting Office, "Joint Strike Fighter Acquisition Observations on the Supplier Base," tech. rep., United States General Accounting Office, May 2004.

[174] United States General Accounting Office, "Timely and Accurate Estimates of Costs and Requirements Are Needed to Define Optimal Future Strategic Airlift Mix," Tech. Rep. GAO-09-50, United States General Accounting Office, November 2008.

[175] U.S. Air Force, "C-5 A/B/C Galaxy & C-5M Super Galaxy." http://www.af.mil/information/factsheets/factsheet.asp?id=84, August 2012.

[176] U.S. Department of Defense Office of the Assistant Secretary of Defense (Public Affairs), "Lockheed martin to build new presidential helicopter." http://www.defense.gov/releases/release.aspx?releaseid=8177, January 2005.

[177] U.S. Government Accountability Office, "Assessments of Selected Weapon Programs," tech. rep., U.S. Government Accountability Office, March 2009.

[178] U.S. Government Accountability Office, "Strategic Airlift Gap Has Been Addressed, but Tactical Airlift Plans Are Evolving as Key Issues Have Not Been Resolved." GAO-10-67, November 2009.

[179] U.S. Government Accountability Office, "Application of Lessons Learned and Best Practices in the Presidential Helicopter Program." GAO-12-437, June 2012.

[180] U.S. Government Accountability Office, "DOD Actions Needed to Further Enhance Restructuring and Address Affordability Risks." GAO-12-437, June 2012.

[181] Van den Honert, R., "Decisional power in group decision making: A note on the allocation of group members' weights in the multiplicative AHP and SMART," *Group Decision and Negotiation*, vol. 10, pp. 275–286, May 2001.

[182] Von Neumann, J. and Morgenstern, O., *Theory of Games and Economic Behavior*. Princeton: Princeton UP, 1944.

[183] Wagner, C., "Consensus through respect: a model of rational group decision-making," *Philosophical Studies*, vol. 34, no. 4, pp. 335–349, 1978.

[184] Warwick, G., "Presidential helicopter costs soar ." `http://www.flightglobal.com/news/articles/presidential-helicopter-costs-soar-213992/`, May 2007.

[185] Wheeler, W., "USAF Cost Over-Runs DoD Micro-Look." `http://www.phibetaiota.net/2011/09/winslow-wheeler-usaf-cost-over-runs-dod-micro-look/`, September 2011.

[186] Williams, M., *Superfighters : the next generation of combat aircraft*. Norwalk, Conn: AIRtime Pub, 2002.

[187] Wu, J., Liang, L., and Yang, F., "Determination of the weights for the ultimate cross efficiency using Shapley value in cooperative game," *Expert Systems With Applications*, vol. 36, pp. 872–876, Jan 2009.

[188] Wu, J., Sun, J., Liang, L., and Zha, Y., "Determination of weights for ultimate cross efficiency using Shannon entropy," *Expert Systems With Applications*, vol. 38, pp. 5162–5165, May 2011.

[189] Yalom, I. and Leszcz, M., *The Theory and Practice of Group Psychotherapy*. Basic Books, 2008.

[190] Yu, C., "A GP-AHP method for solving group decision-making fuzzy AHP problems," *Computers & Operations Research*, vol. 29, pp. 1969–2001, Dec 2002.

[191] Yue, Z., "Extension of TOPSIS to determine weight of decision maker for group decision making problems with uncertain information," *Expert Systems With Applications*, vol. 39, pp. 6343–6350, Jun 1 2012.

[192] Yukulov, V. I. and Sornette, D., "Manipulating decision making of typical agents," in *Proceedings of the FUR XV International Conference*, Foundations and Applications of Utility, Risk and Decision Theory, 2012.

[193] Zajonc, R., "Attitudinal Effects Of Mere Exposure," *Journal Of Personality and Social Psychology*, vol. 9, no. 2P2, pp. 1–&, 1968.

[194] Zeleny, M., *Multiple Criteria Decision Making.* McGraw-Hill Book Company, 1982.