Observational and Theoretical Issues in Early Universe Cosmology

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Abstract

In this thesis we evaluate and compare competing cosmological models for empirical and theoretical consistency and identify new ways of improving current paradigms of early universe cosmology. In the first part, we show that the most recent experimental data from the Planck2013 satellite measuring fluctuations in the cosmic microwave background favors a special class of "small-field plateau-like" models of inflation and disfavors the simplest inflationary potentials. We then identify a new kind of conceptual difficulty for the plateau models that we call the unlikeliness problem – namely, in an energy landscape that includes both plateau-like and simpler potential shapes, the plateau-like produces less inflation and, hence, is less likely to explain our observable universe. In addition, we show that the very same plateau-like models suffer from a new multiverse problem and a new initial conditions problem because they require that inflation starts at energy densities well below the Planck scale. Third, we comment on the impact of these results on the standard view of inflation and more recent versions of the theory invoking the multiverse and complex energy landscapes. In the second part of this thesis, imposing a single, simple, well-motivated constraint – scale-freeness – and using a general hydrodynamic analysis, we show that the unrestricted range of inflationary potentials reduces to a well-defined bundle of inflationary models. We classify and evaluate the scale-free inflationary models in light of Planck2013. We then repeat the construction to produce analogous scale-free bouncing cyclic models of the universe and compare with the inflationary results. In the third part, we introduce a new class of stable ekpyrotic/cyclic models that require less fine-tuning and generate negligible non-Gaussianity consistent with Planck2013 data.

Keywords: cosmology, inflation, cyclic universe, scalefreeness, non-Gaussianity

Zusammenfassung

In der vorliegenden Arbeit bewerten und vergleichen wir konkurrierende kosmologische Modelle im Hinblick auf theoretische Konsistenz und empirische Kohärenz. Ferner finden wir neue Wege, aktuelle kosmologische Paradigmen des frühen Universums weiter zu entwickeln. Im ersten Teil der Arbeit zeigen wir, dass die jüngsten empirischen Daten der Planck2013-Satellitenmission für eine spezielle Klasse inflationärer Modelle sprechen, nämlich sogenannte "plateauartige Modelle mit schmalem Feldbereich"; gleichsam werden die einfachsten inflationären Modelle von den Messdaten nicht gestärkt. Wir formulieren eine neuartige konzeptionelle Schwierigkeit, die für Plateau-Modelle entsteht und die wir 'unlikeliness problem' nennen. Das 'unlikeliness problem' besteht darin, dass in einer Energielandschaft, die sowohl plateauartige als auch einfachere Formen der inflationären Potenziale enthält, die plateauartigen weniger Inflation produzieren und es deshalb weniger wahrscheinlich ist, dass sie das observable Universum beschreiben. Wir zeigen ferner, dass dieselben Plateau-Modelle mit einem neuen Multiversumsproblem und einem neuen Anfangswertsproblem behaftet sind. Anschließend erläutern wir die Bedeutung dieser Probleme für das klassische inflationäre Modell sowie für jüngere Versionen der Theorie, die mit dem Multiversum und komplexen Energielandschaften operieren. Im zweiten Teil untersuchen wir die Implikationen einer einfachen und experimentell motivierten Zusatzbedingung, Skalenfreiheit. Wir zeigen, dass die uneingeschränkte Palette inflationärer Potenziale sich auf ein wohldefiniertes Bündel inflationärer Modelle reduziert. Dabei verwenden wir eine allgemeine hydrodynamische Beschreibung. Wir klassifizieren und bewerten diese skalenfreien inflationären Modelle im Licht von Planck2013. Anschließend wiederholen wir die Analyse, um ähnliche skalenfreie zyklische Modelle des Universums zu konstruieren. Diese Modelle vergleichen wir mit unseren Ergebnissen, die wir für die skalenfreien inflationären Theorie gewonnen haben. Im dritten Teil der Arbeit führen wir eine neue Klasse stabiler zyklischer Modelle ein. Wir zeigen, dass diese Modelle weniger Feinabstimmung der Anfangswerte benötigen. Gleichsam generieren sie vernachlässigbare Nicht-Gaussianität in Übereinstimmung mit den Planck2013-Messdaten.

Schlagwörter: Kosmologie, Inflation, Zyklisches Universum, Skalenfreiheit, Nicht-Gaussianität

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If you do what you always did,
you will get what you always got.
– Albert Einstein

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Chapter 1

Introduction

Thirty years of inflation have greatly changed modern cosmological thinking. Inflationary theory is based on the idea that for typical initial conditions emerging from the big bang, some regions of space have the properties required to undergo a period of accelerated expansion – inflation – that smoothes and flattens the universe, leaving only tiny quantum perturbations. Most importantly, by stretching quantum perturbations to cosmological distances, inflation provides a paradigm for the generation of primordial density fluctuations seeding the structure of our universe. Within this paradigm, we can easily design particular inflationary models that fit the observational data.

However, the physics governing the evolution of the very early universe before nucleosynthesis remains a challenge for modern theoretical cosmology. A main puzzle is – even after three decades – the *initial conditions problem*. Originally, inflation was supposed to smooth and flatten the universe beginning from *arbitrary* initial conditions after the big bang. However, the probability of a region of space having the right initial conditions to begin inflation is exponentially small [82, 34]. By standard classical statistical mechanical reasoning, even for simple inflationary potentials, there exist more homogeneous and flat cosmic solutions without a long period of inflation than with inflation [34].

A second open problem is *eternal inflation and the multiverse* [37]. A well-known property of almost all inflationary models is that, once inflation begins, it continues eternally [92, 98] – a direct consequence of quantum physics combined with accelerated expansion. Assuming smooth, classical evolution, inflation comes to an end in a finite time. However, classical evolution is sometimes punctuated by large quantum fluctuations, including ones that kick the inflaton field uphill, far from its expected classical course. These regions end up undergoing extra inflation that rapidly makes them dominant volumetrically. In this sense, inflation amplifies rare quantum fluctuations that keep space inflating, leading to eternal inflation. Continuing along this line of reasoning, there can be multiple quantum jumps of all sorts as the inflaton evolves with time leading to volumes of space (bubbles) with different inflaton trajectories and, consequently, different cosmological properties. For example, some are flat but some not; some have scale-invariant spectrum, some not; etc. This feature renders inflationary theory entirely unpredictive, insofar as no measure suggestion has proven successful in regulating infinities in the multiverse [54].

In principle, there are two ways to attack the cosmological problems. Either we look for solutions *within* the inflationary paradigm, assuming Einstein gravity, or we abandon the inflationary paradigm and look for alternatives, possibly including modifications of Einstein gravity. *A priori*, it is not obvious which of these two methods leads to success. Despite the conceptual problems, it is a great merit of inflation to have provided a semi-classical explanation for the generation of primordial density fluctuations such that abandoning the paradigm might be premature. For this reason, it seems most reasonable to start with revisiting the existing paradigm(s). In this thesis, we develop new methods to evaluate and compare competing cosmological models for observational and theoretical consistency and identify new ways of improving current paradigms of early universe cosmology.

Recent measurements from the Wilkinson Microwave Anisotropy Probe (WMAP), Atacama Cosmology Telescope (ACT) and Planck satellite (Planck2013) eliminate a wide spectrum of more complex inflationary models and favor a special class of models with a single scalar field, namely "small-field plateau-like models." In Chapter 2, we show that all the simplest single-field inflationary models are disfavored statistically relative to those with plateau-like potentials. Then, we argue that, in addition to having certain conceptual problems known for decades, the inflationary paradigm is, *for the first time*, disfavored by observations in the sense that the simplest models do not fit the data. We start with demonstrating a new kind of conceptual difficulty that we call *unlikeliness problem*: we argue that small-field plateau-like models that are currently favored by experimental data are, at the same time, disfavored by the inflationary paradigm. In addition, we find that the very same plateau-like models suffer from a new multiverse problem and a new initial conditions problem because inflation starts at energy densities well below the Planck scale. We show that this new initial conditions problem becomes even more serious if our current vacuum is metastable, as suggested, for example, by recent LHC results assuming a standard model Higgs. Chapter 2 is based on published work [40] done in collaboration with Abraham Loeb and Paul Steinhardt.

Guth, Kaiser and Nomura (GKN) and Linde have each published critiques, claiming that "cosmic inflation is on stronger footing than ever before." Their analysis rests upon the claim that there are two inflationary paradigms; they call the one "outdated" and do not name the alternative paradigm that revises the assumptions and goals of the former. We shall use the terms "classic" and "postmodern," which seem appropriate given the different cosmological outlooks. These two inflationary paradigms should be judged separately. In Chapter 3, we first review the situation for classic inflation – the theory described in textbooks and based on the idea that, beginning from typical initial conditions and assuming a simple inflaton potential with a minimum of fine-tuning, inflation can create exponentially large volumes of space that are generically homogeneous, isotropic and flat, with a nearly scale-invariant spectrum of density and gravitational wave fluctuations that is adiabatic, Gaussian and has generic predictable properties. Then, we will describe and briefly comment on postmodern inflation – a paradigm in which the physical laws and cosmological properties in our observable universe, although apparently uniform, may only be locally valid, with completely different laws and properties in regions outside our horizon and beyond any conceivable causal contact. This chapter is based on published work [41], a collaboration with Abraham Loeb and Paul Steinhardt and a response to [38] and [62].

Having studied the current observational status of inflationary cosmology, we turn to *theoretical issues*. In Chapter 4 and 5, we present new ways to evaluate and improve competing cosmological models.

It is well known that, besides the multiverse-unpredictability problem, inflation suffers from another unpredictability issue – *parameter unpredictability*. The problem is that the only constraint imposed on inflationary models is that they produce 60 e-folds (or more) of accelerated expansion consistent with the measured amplitude of primordial density perturbations. As a consequence, theorists can dream up (and have dreamed up) more baroque inflationary potentials with many parameters, dips and turns, and multiple stages of inflation such that literally any result for the spectral tilt, tensor-to-scalar ratio or other cosmological observables is possible, rendering inflation entirely unpredictive. In Chapter 4, to dramatically reduce degrees of freedom and improve predictability, we explore imposing an additional simple, physically well-motivated constraint – scale-freeness. Using a general hydrodynamic analysis, we find that the unrestricted range of more complex potentials collapses to a well-defined bundle of inflationary models. We also apply the same approach to bouncing cyclic models of the universe. Remarkably, in comparing the currently existing cosmological theories, we find there is a clear conceptual difference at background level: scale-free inflationary models produce a broad spectrum of outcomes that can be divided into three classes, requiring, for example, different initial conditions. We find that the observationally favored class is theoretically disfavored, *i.e.*, it suffers from an unlikeliness problem, and the theoretically favored class is strongly disfavored observationally. This is consistent with the results in Chapter 2, but more general since the conclusions are based on a hydrodynamic analysis and do not depend on the particular field or potential. Using the same type of analysis, we find there is only a single class of cyclic models such that the predictions for scale-free cyclic models are virtually parameter-independent at background level. Hence, cyclic theory does not suffer from an unlikeliness problem. At perturbative level, though, current versions of the cyclic theory require a certain conceptual restriction, namely a multi-component fluid for the generation of isocurvature fluctuations before the bang which are then converted into primordial density perturbations at some time during the transition from big crunch to big bang. This mechanism for generating density fluctuations is known as the *entropic mechanism*. We show that the entropic mechanism does not require any additional parameters or tuning in the scale-free hydrodynamic picture. More generally, our hydrodynamic analysis can be applied to evaluate and compare alternative cosmological theories. This chapter is based on published work [42] done in collaboration with Abraham Loeb and Paul Steinhardt.

As we have seen in Chapter 4, at background level and compared to inflationary solutions, cyclic/ekpyrotic models of the universe are remarkably simple – they do not suffer from an unlikeliness problem, neither do they produce a multiverse. However, it is well-known that standard ekyprotic solutions generating scale-invariant spectrum via the entropic mechanism are *unstable* and produce non-negligible non-Gaussianity *during the ekpyrotic phase*. In Chapter 5, we explore a new type of entropic mechanism in which there are two scalar fields, as before, but only one has a steep negative potential. This first field dominates the energy density and is the source of the ekpyrotic equation of state. The second field has a negligible potential, perhaps precisely zero potential, but its kinetic energy density is multiplied by a function of the first field with a a non-linear sigma-model type interaction. A specific example of this model was introduced by [60] and [83]. We show that scale-invariant adiabatic perturbations can be produced continuously as modes leave the horizon for any ekpyrotic equation of state. The corresponding background solutions are stable and the bispectrum of these perturbations vanishes, such that no non-Gaussianity is produced during the ekpyrotic phase. Hence, the only contribution to non-Gaussianity comes from the non-linearity of the conversion process during which entropic perturbations are turned into adiabatic ones. This chapter is based on yet unpublished work, a collaboration with Jean-Luc Lehners and Paul Steinhardt.

Chapter 2

Observational status of inflation after Planck2013

Summary. In this chapter we evaluate the observational status of inflationary theory in light of the most recent cosmic microwave background data gathered from WMAP and ACT and confirmed by Planck2013 and show that the inflationary paradigm is – for the first time – disfavored by experiment in the sense that the simplest models do not fit the data.

2.1 Introduction

The Planck satellite data reported in 2013 [3] shows with high precision that we live in a remarkably simple universe. The measured spatial curvature is small; the spectrum of fluctuations is nearly scale-invariant; there is a small spectral tilt, consistent with there having been a simple dynamical mechanism that caused the smoothing and flattening; and the fluctuations are nearly Gaussian, eliminating exotic and complicated dynamical possibilities, such as inflationary models with non-canonical kinetic energy and multiple fields. (Here, we will not discuss the marginal deviations from isotropy on large scales reported by the *Planck* Collaboration [4].) The results not only impose tight quantitative constraints on all cosmological parameters [2], but, qualitatively, they call for a cosmological paradigm whose simplicity and parsimony matches the nature of the observed universe.

The *Planck* Collaboration attempted to make this point by describing the data as supporting the *simplest* inflationary models [35, 64, 8]. However, the models most favored by their data (combined with earlier results from WMAP, ACT, SPT and other observations [87]) are simple by only one criterion: an inflaton potential with a single scalar field suffices to fit the data. By several other important criteria described in this chapter, the favored models are *anything but simple*: Namely, they suffer from exacerbated forms of initial conditions and multiverse problems, and they create a new difficulty that we call the inflationary "unlikeliness problem." That is, the favored inflaton potentials are exponentially unlikely according to the logic of the inflationary paradigm itself. The unlikeliness problem arises even if we assume ideal initial conditions for beginning inflation, ignore the lack of predictive power stemming from eternal inflation and the multiverse, and make no comparison with alternatives. Thus, the three problems are all independent, all emerge as a result of the data, and all point to the inflationary paradigm encountering troubles that it did not have before. We further speculate about how recent results from the Large Hadron Collider (LHC) suggesting a standard model Higgs could create yet another problem for inflation.

Our analysis is based on considering the "favored" models according to the current observations. (Here and throughout this thesis we use the ranking terminology of the *Planck* Collaboration). Although the simplest inflationary models are "disfavored" relative to these by 1.5 σ or more, it is too early in some cases to declare them "ruled out." We discuss in the conclusions how forthcoming searches for B-modes, non-Gaussianity and new particles could amplify, confirm, or resolve the problems for inflation.

2.2 Observationally favored inflationary models after Planck2013

Planck2013 has added impressively to previous results in three ways. First, it has shown that the non-Gaussianity is small. This eliminates a wide spectrum of more complex inflationary models and favors models with a single scalar field. This restriction to single-field models is what justifies focusing on the plot of r (the ratio of tensor to scalar fluctuations) versus n_s (the scalar spectral index), since it is optimally designed to discriminate among the single-field possibilities. In terms of the r-n_s plot, a second contribution of Planck2013 [3] has been to independently confirm the results obtained previously by combining WMAP with other observations. The data disfavors by 1.5σ or more all the simplest inflation models: power-law potential and chaotic inflation [66], exponential potential and power-law inflation [71], inverse power-law potential [10, 77]. Third, the r-n_s plot favors instead a special subclass of inflationary models with *plateau-like* inflaton potentials. These models – simple symmetry breaking [64, 8, 79], natural (axionic) [29], symmetry breaking with non-minimal (quadratic) coupling [86, 25, 16], R^2 [88], hilltop [93] – are simple in the sense that they all can be formulated (in some cases via changes of variable [73, 100, 26, 85]) as single-field, slow-roll models with a canonical kinetic term in the framework of Einstein gravity [3]. A distinctive feature of this subclass of models, following from the Planck2013 constraint on r ($r_{0.002} < 0.12$ at 95% CL), that will be important in our analysis is that the energy scale of the plateau ($M_{\rm I}^4$) is at least 12 orders of magnitude below the Planck scale $\sim M_{\rm Pl}^4$ [3],

$$M_{\rm I}^4 \lesssim \frac{3\pi^2 A_s}{2}\, r\, M_{\rm Pl}^4 \sim 10^{-12}\, M_{\rm Pl}^4\, \frac{r_*}{0.12} \tag{2.1}$$

at 95% CL, where A_s is the scalar amplitude and r_* the value of r evaluated at Hubble exit during inflation of mode with wave number k_*.

A classic example that we will consider first is the original *new inflation* model [64, 8] based on a Higgs-like inflaton, ϕ, and potential $V(\phi) = \lambda(\phi^2 - \phi_0^2)^2$, as illustrated in Fig. 2.1a. The plateau region is the range of small $\phi \ll \phi_0$. Other examples illustrated in Figs. 2.1b and 2.1c will then be considered.

An obvious difference between plateau-like models like this and the simplest inflationary models, like $V(\phi) = \lambda\phi^4$, is that the simplest models require only one parameter and absolutely no tuning of parameters to obtain 60 or more e-folds of

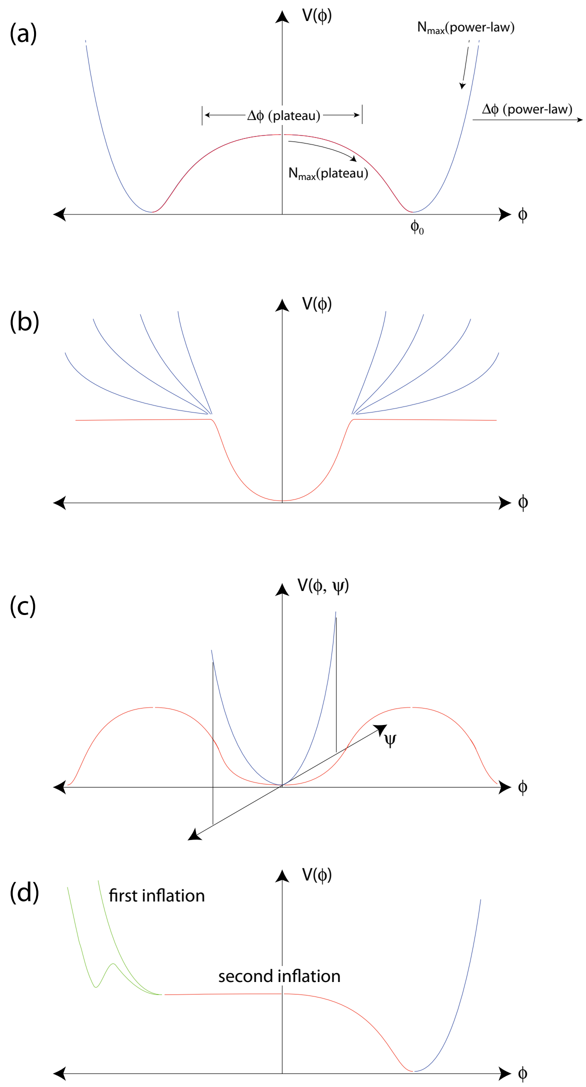

Figure 2.1: Plateau-like models favored by Planck2013 data: (a) Higgs-like potential V with standard Einstein gravity that has both plateau at $\phi \ll \phi_0$ (solid red) and power-law behavior at $\phi \gg \phi_0$ (dashed blue), where $N_{\rm max}$ is the maximum number of e-folds of inflation possible for the maximal range $\Delta\phi$; (b) unique plateau-like model (solid red) for semi-infinite range of ϕ if perfectly tuned compared to continuum of power-law inflation models (dashed blue) without tuning; (c) periodic (axion-like) plateau potential (solid red) for ϕ plus typical power-law inflation potential (dashed blue) for second field ψ; (d) designed inflationary potential with power-law inflation segment or false vacuum segment (dotted green) grafted onto a plateau model (solid red).

inflation while the plateau-like models require three or more parameters and must be fine-tuned to obtain even a minimal amount of inflation. For $V(\phi) = \lambda\phi^4$ all that is required is that $\phi \geq M_{\rm Pl}$, where $M_{\rm Pl}$ is the Planck mass. However, the fine-tuning of parameters is a minor issue within the context of the more serious problems described below that undercut the inflationary paradigm altogether.

2.3 How do plateau-like inflationary models affect the initial conditions problem?

As originally imagined, inflation was supposed to smooth and flatten the universe beginning from arbitrary initial conditions after the big bang [35]. However, this view had to be abandoned as it was realized that large inflaton kinetic energy and gradients within a Hubble-sized patch prevent inflation from starting. While some used statistical mechanical reasoning to argue that the initial conditions required for inflation are exponentially rare [82, 34], the almost "universally accepted" [61] assumption for decades, originally due to Linde [66, 67, 65, 68, 49, 12, 96, 80, 50, 69], has been that the natural initial condition when the universe first emerged from the big bang and reached the Planck density is having all different energy forms of the same order. For the inflaton, this means $\frac{1}{2}\dot{\phi}^2 \sim \frac{1}{2}(\partial_i\phi)^2 \sim V(\phi) \sim M_{\rm Pl}^4$. Roughly speaking, the assumption is based on the notion that all these forms of energy density span the same range, from zero to $M_{\rm Pl}^4$, so it is plausible to have them of the same order at a time when the total energy density is $M_{\rm Pl}^4$. Evolving forward in time from these initial conditions, $V(\phi)$ almost immediately comes to dominate the energy density and triggers inflation before the kinetic and gradient energy can block it from starting.

After Planck2013, the very same argument used to defend inflation now becomes a strong argument against it. Because the potential energy density of the plateau $M_{\rm I}^4$ is bounded above and must be at least a trillion times smaller than the Planck density to obtain the observed density fluctuation amplitude, the only patches that exist have $\frac{1}{2}\dot{\phi}^2 \sim \frac{1}{2}(\partial_i\phi)^2 \gg V(\phi)$. In particular, beginning from these revised initial conditions and evolving forward in time, the kinetic energy decreases as $1/a^6$ and the gradient energy as $1/a^2$, where $a(t)$ is the Friedman-Robertson-Walker scale factor. Hence, beginning from roughly equal kinetic and gradient energy, gradients and inhomogeneities quickly dominate and the combination blocks inflation from occurring.

To quantify the problem, for inflation to initiate, there must be a seed region at the Planck density ($t = t_{\rm Pl}$) that remains roughly homogeneous until inflation begins ($t = t_{\rm I}$) and whose radius $r(t)$ has expanded to a size at least equal to a Hubble radius, $H^{-1}(t_{\rm I})$ at the time inflation initiates. After Planck2013, this requires, by simple comparison of the scales $M_{\rm Pl}/M_{\rm I} \sim 10^3 \cdot (10^{16}\,{\rm GeV}/M_{\rm I})$ as constrained by Planck2013, that there exist homogeneous initial volumes before inflation begins whose size is

$$
\begin{aligned}
r^3(t_{\rm Pl}) \;\; &\gtrsim \;\; \left[a(t_{\rm Pl})\int_{t_{\rm Pl}}^{t_{\rm I}} \frac{d\,t}{a}\right]^3 \sim \left[\frac{a(t_{\rm Pl})\,H(t_{\rm Pl})}{a(t_{\rm I})\,H(t_{\rm I})} H^{-1}(t_{\rm Pl})\right]^3 \\
&> \;\; 10^9 \left(\frac{10^{16}\,{\rm GeV}}{M_{\rm I}}\right)^3 H^{-3}(t_{\rm Pl})\,, \qquad (2.2)
\end{aligned}
$$

– initial smoothness on the scale of a billion or more Hubble volumes [61]!

In sum, by favoring only plateau-like models, the Planck2013 data creates a serious new challenge for the inflationary paradigm: the universally accepted assumption about initial conditions no longer leads to inflation; instead, inflation can only begin to smooth the universe if the universe is unexpectedly smooth to begin with!

2.4 Unlikeliness problem

All inflationary potentials are not created equal. The odd situation after Planck2013 is that inflation is only favored for a special class of models that is exponentially unlikely according to the inner logic of the inflationary paradigm itself. The situation is independent of the initial conditions problem described above; even assuming ideal conditions for initiating inflation, the fact that only plateau-like models are favored is paradoxical because inflation requires more tuning, occurs for a narrower range of parameters, and produces exponentially less plateau-like inflation than the now-disfavored models with power-law potentials. This is what we refer to as the inflationary "unlikeliness problem."

To illustrate the problem, we continue with the classic plateau-like model $V(\phi) = \lambda(\phi^2 - \phi_0^2)^2$. Like most plateau-like inflationary models, the plateau terminates at a local minimum, and then the potential grows as a power-law ($\sim \lambda\phi^4$ in this case) for large ϕ. The problem arises because *within this scenario* the same minimum can be reached in two different ways, either by slow-roll inflation along the plateau or by slow-roll inflation from the power-law side of the minimum. It is easy to see that inflation from the power-law side requires less tuning of parameters, occurs for a much wider range of ϕ, and produces exponentially more inflation: constraints on an inflationary model are determined by the amount of inflation ($N \sim 60$); the scale of density fluctuations ($\delta\rho/\rho \sim 10^{-5}$); and the condition called "graceful exit" (which ensures that inflation ends locally and marks the start of reheating). Using the well-known slow-roll approximation, $N \sim V/V''$, $d\rho/\rho \sim V^{3/2}/V'$, these constraints can be specified for both plateau-like $\sim \lambda\phi_0^4 - 2\lambda\phi_0^2\phi^2$ and power-law $\sim \lambda\phi^4$ inflation [74].

One immediately observes that the first constraint imposes no parameter tuning constraints on power-law models but does require fine-tuning for plateau-like models. For the plateau-like model, inflation occurs if ϕ lies in the range

$$\Delta\phi(\text{plateau}) \lesssim \phi_0 \sim M_{\text{Pl}}, \tag{2.3}$$

and the maximum number of e-folds is

$$\begin{aligned} N_{\max}(\text{plateau}) &= \int_{t_i}^{t_e} H\, dt \sim \frac{8\pi}{M_{\text{Pl}}^2} \int_{\phi_i}^{\phi_e} \frac{V}{V'}\, d\phi \\ &\sim 8\pi\phi_0^2/M_{\text{Pl}}^2\,. \end{aligned} \tag{2.4}$$

By comparison, coming from the power-law side of the same potential, inflation occurs for the range $\Delta\phi(\text{power-law}) \lesssim \lambda^{-1/4} M_{\rm Pl}$, so that

$$\Delta\phi(\text{power-law}) \gg \Delta\phi(\text{plateau}), \tag{2.5}$$

where we have followed convention in confining the power-law range to those values for ϕ for which $V(\phi)$ is less than the Planck density and used the fact that λ must be of order 10^{-15} to obtain the observed density perturbation amplitude on large scales. Also, the maximum integrated amount of inflation on the power-law side is

$$\begin{aligned} N_{\rm max}(\text{power-law}) &\sim \max\{8\pi(\phi_{\rm initial}^2 - \phi_{\rm end}^2)/M_{\rm Pl}^2\} \\ &\sim \lambda^{-1/2} N_{\rm max}(\text{plateau}) \\ &\gg N_{\rm max}(\text{plateau}). \end{aligned} \tag{2.6}$$

Obviously, given the much larger field-range for ϕ and larger amount of expansion, inflation from the power-law side is exponentially more likely according to the inflationary paradigm; yet Planck2013 forbids the power-law inflation and only allows the unlikely plateau-like inflation. This is what we call the inflationary unlikeliness problem.

Although we have demonstrated the principle so far for only a single potential, completion of most scalar field potentials, plateau-like or not, entails power-law or exponential behavior at large values of ϕ. There are notable examples that have no power-law completion, such as axion and moduli potentials. However, as discussed in Sec. 2.5, unless all scalar fields defining our vacuum are of this nature, inflation from a scalar field with power-law or exponential behavior is exponentially more likely; but this is disfavored by Planck2013.

Therefore, post-Planck2013 inflationary cosmology faces an odd dilemma. The usual test for a theory is whether experiment agrees with model predictions. Obviously, inflationary plateau-like models pass this test. However, this cannot be described as a success for the inflationary paradigm, since, according to inflationary reasoning, this particular class of models is highly unlikely to describe reality. The unlikeliness problem is an alarm warning us that a paradigm can fail even though observations favor a class of models if the paradigm predicts the class of models is unlikely.

2.5 Planck2013 data and the multiverse

A well-known property of almost all inflationary models is that, once inflation begins, it continues eternally producing a multiverse [92, 98] in which "anything that can happen will happen, and it will happen an infinite number of times" [37]. A result is that all cosmological possibilities (flat or curved, scale-invariant or not, Gaussian or not, *etc.*) and any combination thereof are equally possible, potentially rendering inflationary theory entirely unpredictive. Attempts to introduce a measure principle [33, 32, 6, 99, 104, 30] or anthropic principle [102, 103, 95] to restore predictive power have met with difficulty. For example, the most natural kind of measure, weighting by volume, does not predict our universe to be likely. Younger patches [70, 36] and Boltzmann brains/babies [7, 19] are exponentially favored.

Planck2013 results lead to a new twist on the multiverse problem that is independent of the initial conditions and unlikeliness problems described above. The plateau-like potentials selected by Planck2013 are in the class of eternally inflating models, so the multiverse and its effects on predictions must be considered. In a multiverse, each measured cosmological parameter represents an independent test of the multiverse in the sense one could expect large deviations from any one of the naive predictions. The more observables one tests, the greater the chance of many-σ deviations from the naive predictions. Hence, it is surprising that the Planck2013 data agrees so precisely with the naive predictions derived by totally ignoring the multiverse and assuming purely uniform slow-roll down the potential.

2.6 Is there any escape from these new problems?

In the previous sections we introduced three independent problems stemming from the Planck2013 observations: a new initial conditions problem, a worsening multiverse-unpredictability problem, and a novel kind of discrepancy between data and paradigm that we termed the unlikeliness problem. It is reasonable to ask: is there any easy way to escape these problems?

One approach that cannot work is the anthropic principle since the new problems that we discussed all derive from the fact that Planck2013 disfavors the simplest inflationary potentials while there is nothing anthropically disadvantageous about those models or their predictions.

The multiverse-unpredictability problem has been known for three decades before Planck2013 and, thus far, lacks a solution. For example, weighting by volume and bubble counting, the most natural measures by the inner logic of the inflationary paradigm, fail. For further discussion see below Sec. 3.3 of Chapter 3.

By contrast, one might imagine the unlikeliness problem first brought on by Planck2013 could be evaded by a different choice of potential. Above we used as an example the potential $V(\phi) = \lambda(\phi^2 - \phi_0^2)^2$, which has a plateau for $\phi \ll \phi_0$ and a power-law form for $\phi \gg \phi_0$. Here it was clear that inflation from the power-law side is exponentially more likely because inflation occurs for a wider range of ϕ and generates exponentially more accelerated expansion.

An alternative, in principle, is to have a plateau at large ϕ and no power-law behavior, as sketched in Fig. 2.1b. The problem with this is that the desired flat behavior, marked in red, is a unique form that only occurs for a precise cancellation order by order in ϕ (if one imagines V expanded in a power series in ϕ). Within the inflationary paradigm, this perfect cancellation is not only ultra-fine tuned, but also uncalled for since there are infinitely many power-law inflationary completions of the potentials (blue-dashed) in which V increases as a power of ϕ. The single plateau possibility is extremely unlikely compared to the continuum of blue-dashed possibilities. Yet now Planck2013 disfavors everything except for the unlikely plateau case. Examples of this type include the Higgs inflationary model with non-minimal coupling $f(\phi)R$ with $f(\phi) = M_{Pl}^2 + \xi\phi^2$ [16, 14, 13, 63, 31, 15] and the $f(R) = R + \xi R^2$ inflation model [88], where R is the Ricci scalar, once they are converted by changes of variable to a theory of a scalar field ϕ in the Einstein-frame. Note that a plateau only occurs if $f(\phi)$ or $f(R)$ are precisely cutoff at quadratic order, when there is no reason why there should not be higher order terms. Yet the addition of any one higher order term is enough to ruin plateau inflation.

A third possibility is periodic potentials of the type shown in Fig. 2.1c, as occurs for axion-like fields (*e.g.*, as in natural inflation [29] or in string theory moduli). This form is enforced by symmetry to be periodic and, unlike the previous cases, forbidden to have power-law behavior at large ϕ. This makes it the best-case scenario for evading the unlikeliness problem. The problem arises if there are any non-axion-like scalar fields that define the vacuum since they will generically have power-law behavior at large ϕ. The more ordinary scalar fields that exist in fundamental theory, the more avenues there are for power-law inflation, each of which is exponentially favored over plateau-like inflation from the periodic potential but disfavored by Planck2013.

Hence, none of these three cases evades the unlikeliness problem. At the same time, it is clear that none does anything to evade the new initial conditions problem caused by Planck2013. In each case, the plateau-like inflation begins well after the big bang, enabling kinetic and gradient energy to dominate right after the big bang.

A fourth possibility consists of models, like those sketched in Fig. 2.1d, in which complicated features are added for the purpose of turning an unlikely model into a likely one. For example, we have already shown that the plateau side (solid red) in Fig. 2.1a has exponentially less inflation than the power-law side and an initial conditions issue; so the fact that Planck2013 disfavors the power-law and favors the plateau is a problem. By grafting the sharp upward bend or false vacuum (dotted green) onto the plateau in Fig. 2.1d, the combination technically evades those problems, but at the expense of complicating the potential. So, in terms of the addressing our central issue – does Planck2013 really favor the simplest inflationary model? – this approach does not change the answer.

Furthermore, the only reason for grafting onto a plateau model rather than some other potential shape is because of the foreknowledge that the plateau model fits Planck2013 data. That means, effectively, what was supposed to be predicted output of the model has now been used as an input in its design. It does not make sense to apply the unlikeliness criterion to models in which the very same volume and initial conditions test criteria were already "wired in" as input. In fact, not only has the likeliness criterion been used as input, but all the Planck2013 data (tilt, tensor modes, spatial curvature, non-Gaussianity) have been used in selecting to graft onto a plateau potential rather than some other shape potential. If the only way the inflationary paradigm will work is by delicately designing all the test criteria and data into the potential, this is trouble for the paradigm.

2.7 More trouble for inflation from the LHC?

Thus far, we have only focused on recent results from Planck2013, but recent measurements of the top quark and Higgs mass at the LHC and the absence of evidence for physics beyond the standard model could be a new source of trouble for the inflationary paradigm and big bang cosmology generally [1, 51]. Namely, the current data suggests that the current symmetry-breaking vacuum is metastable with a modest-sized energy barrier ($(10^{12}$ GeV$)^4$) protecting us from decay to a true vacuum with large negative vacuum density [20]. This conclusion is speculative since it assumes no new physics for energies less than the Planck scale, which is unproven. Nevertheless, this is the simplest interpretation of the current data and its consequences are dramatic; hence, we consider the implications here.

The predicted lifetime of the metastable vacuum is large compared to the time since the big bang, so there is no sharp conflict with observations. The new problem is explaining how the universe managed to become trapped in this false vacuum whose barriers are tiny (by a factor of 10^{28}!) compared to the Planck density when it is obviously much more probable for the field to lie outside the barriers than within them. However, if the Higgs field lies outside the barrier, its negative potential energy density will tend to cancel the positive energy density of the inflaton and block inflation from occurring, unless one assumes large-field inflation and a certain kind of coupling between the inflaton and the Higgs [53, 48]. Even in the unlikely case that the Higgs started off trapped in its false vacuum and inflation began, the inflaton would induce de Sitter-like fluctuations in all degrees of freedom that are light compared to the Hubble scale during inflation. These tend to kick the Higgs field out of the false vacuum, unless the Hubble constant during inflation is smaller than the barrier height [24]. Curiously, a way to evade the kick-out is if all inflation (not just the last 60 e-folds) occurs at low energies where the de Sitter fluctuations are smaller than the barrier height. This would be possible if the only possible inflaton potentials are plateau-like with sufficiently low plateaus: the very same potentials that have the initial conditions and multiverse problems.

2.8 Discussion

In testing the validity of any scientific paradigm, the key criterion is whether measurements agree with what is expected given the paradigm. In the case of inflationary cosmology, this test can be divided into two questions: (A) *are the observations what is expected, given the inflaton potential X?*, here the analysis assumes classical slow-roll, no multiverse, and ideal initial conditions; and (B) *is the inflaton potential X that fits the data what is expected according to the internal logic of the paradigm?*. In order to pass, *both* questions must be answered in the affirmative.

The Planck2013 analysis, like many previous analyses of cosmic parameters, focused on Question A. Based on tighter constraints on flatness, the power spectrum and spectral index, and non-Gaussianity, the conclusion from Planck2013 was that single-field plateau-like models are the simplest that pass and they pass with high marks.

However, our focus has been Question B – are plateau-like models expected, given the inflationary paradigm? Based on the very same tightened constraints from Planck2013, we have identified three independent issues for plateau-like models: a dangerous new type of initial conditions problem, a twist on the multiverse problem, and, for the first time, an inflationary unlikeliness problem. The fact that a single data set like Planck2013 can expose three new problems is a tribute to the quality of the experiment and serious trouble for the paradigm.

Future data can amplify, confirm, or diffuse the three problems. Detecting tensor modes and constraining the non-Gaussianity to be closer to zero would ease the problems provided the r-n_s values are consistent with a simple power-law potential. Given the Planck2013 value for the tilt ($n_s = 0.9603 \pm 0.0073$), the only simple chaotic model that can be recovered is $m^2\,\phi^2$, predicting $0.13 \lesssim r \lesssim 0.16$ (depending on the value of N). Alternatively, if the observed r lies at 0.01 or below, power-law models are ruled out and all three current problems remain. Yet a third possibility is finding no tensor modes or detecting non-negligible non-Gaussianity (*e.g.,* $f_{\rm NL} \sim 8$ is well within Planck2013 limits but inconsistent with plateau models); measurements like these would create yet more problems for the inflationary paradigm and encourage consideration of alternatives.

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Chapter 3

Inflationary schism after Planck2013

Summary. Guth, Kaiser and Nomura and Linde have each published critiques, claiming that "cosmic inflation is on stronger footing than ever before." They do not dispute the problematic state of classic inflation – the theory described in textbooks. Instead, they describe an alternative inflationary paradigm that revises the assumptions and goals of inflation. In this chapter, we analyze this new paradigm and point out its implications for primordial cosmology.

3.1 Introduction

In Chapter 2, we have shown that in addition to having certain conceptual problems known for decades, the classic inflationary paradigm is for the first time also disfavored by data, specifically the most recent data from WMAP, ACT and Planck2013. In their response [38] to our analysis [40], Guth, Kaiser, and Nomura (GKN) countered that cosmic inflation is "on stronger footing than ever," [GKN1][1] and Linde [62] has expressed his support of that view. What is clear from GKN, though, is that two very different versions of inflation are being discussed.

One is the inflationary paradigm described in textbooks [74, 22], which we will call *classic inflation*. Classic inflation proposes that, beginning from typical initial conditions and assuming a simple inflaton potential with a minimum of fine-tuning,

[1]Throughout this chapter, [GKN#] refers to specific quotes from [38] that have been reproduced in the Appendix for convenience, though we suggest reading [38] in its entirety.

inflation can create exponentially large volumes of space that are generically homogeneous, isotropic and flat, with a nearly scale-invariant spectrum of density and gravitational wave fluctuations that is adiabatic, Gaussian and has generic predictable properties. Implicit in classic inflation is reliance on volume as being the natural measure: *e.g.,* even if the probability of obtaining a patch of space with the right initial conditions is small *a priori*, the inflated regions occupy an overwhelming volume *a posteriori* and so their properties constitute the predictions.

Until now, the problematic issues of classic inflation have been conceptual: the entropy problem [82], the Liouville problem [34], the multiverse unpredictability problem [92, 98, 36], etc. Our point in Chapter 2 was to show that, even if the conceptual problems are favorably resolved, classic inflation is now disfavored by observations. It is significant that neither GKN nor Linde dispute these points, as we will detail below [GKN2–6].

Instead, GKN label *classic inflation* as outdated and, over the course of their paper, they describe an alternative inflationary paradigm that has been developing in recent years and revises the assumptions and goals of inflation, and, as Linde suggests, perhaps of science generally. This makes clear that a schism has erupted between *classic inflation* and what might appropriately be called *postmodern inflation*. The two inflationary paradigms are substantially different and should be judged separately. We will first review the situation for classic inflation, where there is a consensus on its status. Then, we will describe postmodern inflation and briefly comment on its properties.

3.2 Classic inflation

Three independent inputs must be specified to determine predictions of any inflationary scenario, whether classic or postmodern: the initial conditions, the inflaton potential, and the measure. The initial conditions refer to the earliest time when classical general relativity begins to be a good approximation for describing cosmic evolution, typically the Planck time. (Here we are assuming for simplicity that inflation is driven by a scalar field slowly rolling down an inflaton potential, but our discussion can be easily generalized to other sources of inflationary energy.) Roughly, the *inflaton potential* determines a family of classical trajectories, some of which do and some of which do not include a long period of inflation; the initial conditions pick out a subset of trajectories; and the measure defines the relative "weight" among the subset of trajectories needed to compute the predictions.

As described in row 1 of Table 3.1, classic inflation is based on assuming simple initial conditions, simple potentials and a simple common-sense measure. The notion is that, for initial conditions emerging from the big bang, some regions of space have the properties required to undergo a period of accelerated expansion that smoothes and flattens the universe, leaving only tiny perturbations that act as sources of cosmic microwave background fluctuations and seeds for galaxy formation. Although most regions of space emerging from the big bang may not have the correct conditions to start inflation, this is compensated by the fact that inflation exponentially stretches the volume of the regions that do have the right conditions. Using volume-weighting as the measure, smooth and flat regions dominate the universe by the end of inflation provided the regions with the correct initial conditions are only modestly rare (though see discussion below). For potentials with a minimum of fields (one) and a minimum of fine-tuning of parameters, there are *generic* inflationary predictions: a spatially flat and homogeneous background universe with a nearly scale-invariant, red-tilted spectrum of primordial density fluctuations ($n_S \sim 0.94 - 0.97$), significant gravitational-wave signal ($r \sim 0.1 - 0.3$), and negligible non-Gaussianity ($f_{\rm NL} \sim 0$). Most but not all of these generic predictions are in accord with Planck2013, as emphasized by the Planck2013 collaboration, GKN, and Linde.

	Inflaton Potential +	Initial Conditions +	Measure $\Longrightarrow$	Predictions
Classic inflationary paradigm	**Simple** – Single, continuous stage of inflation governed by potentials with the fewest degrees of freedom, fewest parameters, least tuning.	**Insensitive** – Inflation transforms typical initial conditions emerging from the big bang into a flat, smooth universe with certain generic properties.	**Common-sense** – It is more likely to live in an inflated region because inflation exponentially increases volume $\Rightarrow$ measure = volume	**Generic** – Based on simplest potentials: - red tilt: $n_S \sim .94 - .97$, - large $r \sim .1 - .3$*, - negligible f_{NL}, - flatness & homogeneity
Conceptual problems known prior to Planck2013	**Not so simple** – Even simplest potentials require fine-tuning of parameters to obtain the right amplitude of density fluctuations.	**Sensitive** – The initial conditions required to begin inflation are entropically disfavored/exponentially unlikely. There generically exist more homogeneous and flat solutions without inflation than with.	**Catastrophic failure** – Inflation produces a multiverse in which most of the volume today is inflating and, among non-inflating volumes (bubbles), Inflation predicts our universe to be exponentially unlikely.	**Predictability problem** – no generic predictions; "anything can happen and will happen an infinite number of times." The probability by volume of our observable universe is less than $10^{-10^{55}}$.
Observational problems after Planck2013 [40]* **	**Unlikeliness problem** – *Simplest* inflaton potentials disfavored by Planck2013; favored (plateau) potentials require more parameters, more tuning, and produce less inflation.	**New initial conditions problem** – Favored plateau potentials require an initially homogeneous patch that is a billion times** larger than required for the simplest inflaton potentials.	**New measure problem** – All favored models predict a multiverse yet Planck2013 fits predictions assuming no multiverse.	**Predictability problem unresolved** – Potentials favored by Planck2013 do not avoid the multiverse or the predictability problems above. Hence, no generic predictions.

Table 3.1: Classic Inflation.

*The same arguments used to derive the "generic" predictions of tilt, flatness, etc., also predict the tensor-to-scalar ratio to be 10-30%, though GKN do not include this in their list. **GKN obtain a different value because they assume that inflation begins from a patch that is homogeneous and FRW at the start, without specifying any penalty for that selection. ***Future data can amplify, confirm, or diffuse the three problems introduced in [40]. Detecting tensor modes $r \gtrsim 0.13$ and constraining the non-Gaussianity, f_{NL}, to be closer to zero would ease the problems. Alternatively, if the observed r lies at 0.1 or below, power-law models are ruled out and all three current problems remain. Yet a third possibility is finding no tensor modes or detecting non-negligible non-Gaussianity, (e.g., $f_{NL} \sim 8$ is well within Planck2013 limits but inconsistent with plateau models); measurements like these would create more problems for the classic inflationary paradigm.

3.3 Known problems of classic inflation before Planck2013

Conceptual problems with classic inflation have been known for three decades; row 2 of Table 3.1. First, all inflationary *potentials* require orders of magnitude of parameter fine-tuning to yield the observed amplitude of the primordial density fluctuations ($\delta\rho/\rho \sim 10^{-5}$). Second, the probability of a region of space having the right initial conditions to begin inflation is exponentially small [82, 34]. By standard classical statistical mechanical reasoning, even for simple inflaton potentials, there exist more homogeneous and flat cosmic solutions without a long period of inflation than with inflation [34].

The most serious conceptual problem is the *multiverse problem* (sometimes called the *measure problem*) that results from eternal inflation [92, 98]. Assuming smooth, classical evolution of the inflaton, inflation comes to an end in a finite time according to when the inflaton reaches the bottom of the inflaton potential. However, generically, classical evolution is sometimes punctuated by large quantum fluctuations, including ones that kick the inflaton field uphill, far from its expected classical course. These regions end up undergoing extra inflation that rapidly makes them dominant volumetrically. In this sense, inflation amplifies rare quantum fluctuations that keep space inflating, leading to eternal inflation. Continuing along this line of reasoning, there can be multiple quantum jumps of all sorts as the inflaton evolves with time leading to volumes of space (bubbles) with different inflaton trajectories and, consequently, different cosmological properties. For example, some are flat but some not; some have scale-invariant spectrum, some not; etc.

Ultimately, the result is an eternal multiverse in which "anything can happen and will happen an infinite number of times" [GKN7]. What does inflation predict to be the most likely outcome in the multiverse? In the context of classical inflation, where volume is the natural measure, most volume today is inflating and most non-inflating volume (bubbles) is predicted to be exponentially younger than the observable universe [70, 36], [GKN8]. To be more specific, the volume-weighted prediction is that our observable universe is exponentially unlikely by a factor of $10^{-10^{55}}$ or more [GKN9]! Classic inflation is a catastrophic failure by this measure; numerically, it is one of the worst failures in the history of science.

How has a theory that fails catastrophically continued to survive in scientific discourse? For the most part, it is because, by ignoring the multiverse and assuming a continuous period of monotonic slow-roll, classic inflation seems to produce predictions that perfectly match observations. Our point in Chapter 2 was to show that this is no longer the case.

3.4 Problems of classic inflation after Planck2013

WMAP, ACT, and Planck2013 have passed an important milestone. Like previous experimental groups, they compare their results to an oversimplified version of classic inflation by ignoring the multiverse, as noted above. For the first time, observational data places pressure on this oversimplified classic inflation. The new pressure on classic inflation includes the "unlikeliness problem," a new initial conditions problem, and a new measure problem [40]; as summarized in row 3 of Table 3.1. We briefly describe the problems here.

The unlikeliness problem arises – as we explained in the preceding chapter – because Planck2013 disfavors the simplest (*e.g.*, power-law) inflaton potentials and favors small-field plateau-like potentials. Plateau-like potentials require more tuning, occur for a narrower range of parameters, and produce exponentially less inflation than would be produced by the disfavored power-law potentials[2], so it is surprising to find them favored. Furthermore, most energy landscapes with plateau-like inflation paths to the current vacuum also include simple power-law inflation paths to the same vacuum that generate more inflation, so it is exponentially unlikely that the current vacuum resulted from the plateau-like path. Yet this is what Planck2013 favors.

[2] In counting the maximal number of e-folds of inflationary smoothing for a given potential, one should only consider the final inflationary stage during which the density fluctuation $\delta\rho/\rho$ is much less than 1 and exclude inflaton field ranges where quantum fluctuations dominate classical evolution; see for further discussion Chapter 3 of this thesis and Sec. III.B of [42].

As described in Chapter 2, the new initial conditions problem arises because the energy density at the beginning of inflation M_b^4 is smaller by twelve orders of magnitude in the observationally favored models compared to the simplest inflaton potentials. In order for inflation to begin, a smooth patch of size M_b^{-3} Hubble volumes (as evaluated at the Planck time in Planck units) is required. Quantitatively, the observationally favored potentials require an initial smooth patch that is 10^9 Hubble volumes – a billion times larger than what is needed to begin inflation for the simplest inflaton potentials. Since larger smooth patches are exponentially rarer than smaller ones, the favored potentials require comparatively improbable initial conditions.

A third issue that arises due to observations is new challenges for resolving the multiverse measure problem. For classic inflation, volume-weighting was considered fine for making predictions until the discovery of the multiverse, when it was found that Hubble-sized patches of space like ours are highly improbable. The challenge for the last three decades has been to find an alternative weighting in the multiverse that will restore the naive volume-weighted predictions. That program has been unsuccessful to date, so there is no justification for expecting that a small-field plateau potential should produce values of n_s, r and $f_{\rm NL}$ that agree precisely with the naive volume-weighted predictions; yet these are the values that Planck2013 has found. This imposes a new tight constraint on any solution to the measure problem: one must seek a clever choice of weighting that can reproduce the naive volume-weighted predictions of classic inflation for plateau-potentials. However, then there is another twist. Using the same naive volume-weighting, we have shown in Chapter 2 that simple potentials are exponentially favored over the small-field plateau models. Hence, the solution to the measure problem must mimic naive volume-weighting for some predictions but not for others. These are new data-imposed restrictions for solving the measure problem.

	Inflaton Potential +	Initial Conditions +	Measure ⇒	Predictions
Postmodern Inflation	**Complex** – with many fields, parameters, dips, minima, and hence many metastable states, leading to multiple phases of inflation [GKN10-11] and making eternal inflation unavoidable [GKN12]	**Not important** – in considering the validity of inflation; any problems can be compensated by adjusting the measure [GKN19]	**To be determined** – from some combination of probability weighting and anthropic selection [GKN13,17,20]	**Generic** – predictions should generically agree with observations once the right complex potential and combination of measure and anthropic weighting is identified [GKN6,15]
Problems	**Unpredictability Part I** – A complex energy landscape allows virtually any outcome and provides no way to determine which inflaton potential form is most likely. [GKN17]	**Unpredictability Part II** – Without knowing initial conditions cannot make predictions even if energy landscape is known. [GKN14]	**Paradigm rests entirely on the measure** – yet, to date, no successful measure has been proposed and there is no obvious way to solve this problem. [GKN13]	**No predictions** – the simplest (volume) measure gives catastrophic results and different landscapes, initial conditions, and measures give different predictions [GKN6].

Table 3.2: Postmodern Inflation.

3.5 Postmodern inflation

From the three new problems we concluded after Planck2013 that classic inflation is observationally disfavored – a point which GKN are not disputing [GKN5]. Instead, they claim that the classic inflation must be replaced by a more recent paradigm; that we dub *postmodern inflation*[3]. The term seems to be appropriate to the new inflationary paradigm in which the physical laws and cosmological properties in our observable universe, although apparently uniform, may only be locally valid, with completely different laws and properties in regions outside our horizon and beyond any conceivable causal contact.

The postmodern approach makes different assumptions about the three inputs used to make inflationary predictions; row 1 of Table 3.2.

[3]Postmodern is a term used in literature, art, philosophy, architecture, and cultural or literary criticism for approaches that reject the idea of universal truths and, instead, deconstruct traditional viewpoints and focus on relative truths.

- Simple inflaton potentials should be replaced by highly complex potentials with many parameters, tunings, and fields because they are "very plausible according to recent ideas in high-energy physics" [GKN10–11]. The complex potentials inevitably lead to multiple stages of inflation and a multiverse in which anything can happen [GKN7].
- The validity of the postmodern inflationary paradigm should never be judged on whether it works for typical initial conditions since we do not know what those conditions are [GKN13]. Even if the initial conditions are determined some day they will not affect the validity of inflation; rather, the (yet unknown) measure will then be adjusted such that the observed properties of the universe are likely to emerge from those (yet unknown) initial conditions [GKN14].
- The volume measure is rejected in favor of complex measures that are to be (re-)adjusted (*a posteriori*) to ensure that the predicted outcome agrees with observations.

3.6 Problems of postmodern inflation

Postmodern inflation has its own issues. One problem arises from allowing highly complex potentials with more parameters than there are observables. Even if initial conditions were somehow fixed and the multiverse avoided, complex potentials introduce their own *parameter unpredictability* problem. For example, it has been shown [27] that a potential with a single field and only three parameters can be designed to fit any cosmological outcome for the standard cosmological observables. If so, then no observation can be said to test the theory. Introducing more degrees of freedom or a complex landscape further exacerbates the situation [GKN17].

A second issue relates to the claim that obtaining inflationary initial conditions following the big bang is unimportant to the validity of the paradigm. For some cosmologists, this revision will come as somewhat of a shock, since a common justification for introducing inflation is to explain how the current universe can naturally and robustly emerge from a wide range of possible big bang initial conditions. That is also why several groups have explored the dependence on initial conditions, with some ultimately concluding that the conditions required to have a long period of classic inflation after the universe emerges from the big bang are extremely rare [82, 34]. In postmodern inflation, it is conceded that the period of rapid accelerated expansion by itself does not explain how the universe emerged from typical initial conditions. Ignorance of initial conditions is claimed instead, and the resolution for how the current universe emerged from initial conditions is relegated to the measure, rather than inflation [GKN14].

Postmodern inflation rests entirely on the measure. It is the measure alone that is supposed to justify the choice of a particular highly complex potential among exceedingly many. At the same time, the measure is supposed to solve the initial conditions problem, and the very same measure is supposed to regulate infinities in the multiverse and restore predictiveness. Such a measure does not currently exist – "a persuasive theory of probabilities in the multiverse has not yet been found" [GKN6]. Common-sense volume-weighting of classic inflation is declared invalid, but not because there is a fundamental mathematical or logical or intuitive inconsistency with the volume measure. In fact, the volume measure may work well for some cosmologies [44]. Rather, volume-weighting is discarded because it produces an outcome for eternal inflation that is inconsistent with observations (see Table 3.1).

In postmodern inflation, volume-weighting is abandoned in favor of selecting a measure *a posteriori* to fit observations. In this approach, the notion of generic predictions is sacrificed. A paradigm that relies on a multiverse in which anything can happen, with initial conditions yet to be determined, with complex potentials consisting of multiple fields and parameters, and, then, with the freedom to select the measure *a posteriori* cannot have generic predictions. In fact, observations cannot falsify postmodern inflation – failure to match observations leads instead to a change of measure [GKN14]. This places postmodern inflationary cosmology squarely outside the domain of normal science. Linde concurs [62], quoting Steven Weinberg [103], "Now we may be at a new turning point, a radical change in what we accept as a legitimate foundation for a physical theory."

3.7 Discussion

The focus of [40] (as presented in Chapter 2) was what we call here the classic inflationary paradigm. We showed that Planck2013 data imposes new challenges by disfavoring the simplest inflaton potentials. As we emphasized in the conclusion of Chapter 2, the situation is subject to change depending on future data. For example, suppose that forthcoming analysis of the Planck polarization data will reverse the Planck2013 trend and find $r > 0.13$ with the value of n_S and $f_{\rm NL}$ unchanged. Then, the three observational challenges (row 3 in Table 3.1) posed in [40] disappear (though the conceptual problems in row 2 of Table 3.1 would remain). Other scenarios depending on future data are also discussed above.

GKN discount the classic inflationary paradigm as outdated and instead describe an alternative (postmodern) paradigm. Here, we have made it clear that these are two very different paradigms sharing the same name and being conflated. Henceforth, it is essential to distinguish the two paradigms; particularly when interpreting experiments.

Future data has no significance for the postmodern inflationary paradigm because the potential, initial conditions and measure are chosen *a posteriori* to match observations, whatever the results. For example, measuring $r > 0.13$ or $r < 0.13$ or not detecting any gravitational waves at all makes no difference.

The scientific question we may be facing in the near future is: If classic inflation is outdated and a failure, are we willing to accept postmodern inflation, a construct that lies outside of normal science? Or is it time to seek an alternative cosmological paradigm?

Chapter 4

Scale-free primordial cosmology

Summary. Having studied the current observational status of inflationary cosmology, we turn to theoretical issues. In this chapter, we present a new way of solving the so-called *parameter-unpredictability* problem of primordial cosmology. Based on a hydrodynamical approach and using scale-freeness as a guiding principle, we identify forms for the background equation-of-state for both inflationary and cyclic scenarios and use these forms to derive predictions for the spectral tilt and tensor-to-scalar ratio of primordial density perturbations. For the case of inflation, we show that the observationally favored class is theoretically disfavored because it suffers from an initial conditions problem and the hydrodynamical form of an unlikeliness problem similar to that we introduced in Chapter 2. We contrast these results with those for scale-free cyclic models.

4.1 Introduction

As we emphasized above, the recent *Planck* satellite measurements [2, 3, 5], together with earlier observations from WMAP, ACT, SPT, and other experiments [87], showed with high precision that the spectrum of primordial density fluctuations is nearly scale-invariant, Gaussian, and adiabatic. These results suggest that the universe is simple and the physics governing its early evolution on large scales is 'scale-free.' That is, the physics during that smoothing period in which the large-scale structure of the universe is determined is governed by dynamical equations that entail no dimensionful macroscopic scales and yield power-law solutions.

Scale-freeness was first conjectured as a guiding cosmological principle over four decades ago and was the historic motivation for both the Harrison-Zel'dovich-Peebles spectrum [39, 94, 81] and inflation [35, 64, 8]. In the intervening years, the principle seemed to lose favor as many baroque versions of inflationary (and other) models were proposed that explicitly introduce distinctive, scale-sensitive features on large scales. The problem is that, without a guiding principle such as scale-freeness, literally any result for the spectral tilt, tensor-to-scalar ratio or other cosmological observables is possible. Some have emphasized this as an 'attractive' feature of inflation on the grounds that the theory cannot be disproven (see for example [27]); but the other side of the coin is that this means the theory is entirely unpredictive.

Now that scale-freeness has substantial observational support, it is timely to examine how this guiding principle dramatically collapses the range of outcomes and makes cosmological theories like inflation meaningfully predictive. We use a hydrodynamical approach that is model-independent, *i.e.*, with no reference to scalar fields or potentials, to consider two well-known cosmological scenarios, the inflationary and cyclic (or ekpyrotic) theories of the universe. We identify forms for the background equation-of-state during the cosmological smoothing phase in each case consistent with *strict* scale-freeness. We also consider variations that "weakly" break scale-freeness. We then derive generic predictions for the spectral tilt and tensor-to-scalar ratio of primordial density perturbations resulting from the scale-free principle.

A hydrodynamical approach has been applied earlier to inflationary and cyclic theories [47, 75], without explicitly assuming scale-freeness. The hydrodynamical approach is attractive since it is powerful and simple at the same time; it enables us to derive generic results (given the assumptions) and leads us to an intuitive understanding of the underlying physical phenomena. It is also closer to observation, in the sense that it is easier to determine the equation-of-state from astrophysical data than to determine the microphysics (scalar fields and potentials) that caused it.

The goal of this chapter is to show how the combination of the hydrodynamical approach and the principle of scale-freeness impose restrictions on cosmological scenarios and their predictions. For inflation, the combination reveals the existence of three distinct classes of scale-free scenarios. We show that the class favored by current experiment suffers from an initial conditions problem and a series of other problems, including a hydrodynamic equivalent of the unlikeliness problem that we identified for certain inflaton potentials in Chapter 2. For the cyclic scenarios, where smoothing occurs during a period of ultra-slow (ekpyrotic) contraction, we find that there is only one class of scenarios and that none of the problems arise. In this analysis, we only consider a single contraction period without regard to whether the evolution repeats cyclically, so the same conclusions apply to bouncing cosmologies using ekpyrotic smoothing that have a single bounce or other variations.

For the cyclic (or other ekpyrotic) theories, most current versions use the entropic mechanism to generate curvature perturbations [55], which imposes the conceptual restriction that there be a two-component fluid to generate the perturbations. We find that handling two components rather than one in our approach is not a problem. We show that scale-freeness constrains the equations-of-state of both components, enabling us to derive generic predictions for the spectral tilt and tensor-to-scalar ratio analogous to the case of inflation.

We believe the approach adopted here based on scale-freeness and hydrodynamics provides what is arguably the predictions of the simplest, best-motivated, and observationally best-supported models of each given cosmological theory and sets a standard that can be applied to any scenario in which a smooth, *i.e.* scale-free background and nearly scale-invariant, adiabatic, and Gaussian perturbations are created at the same cosmological stage.

Chapter 4 is organized as follows. We begin in Sec. 4.2 by briefly reviewing the inflationary and cyclic (or ekpyrotic) scenarios and how they can create a scale-free background. To describe the background dynamics, in Sec. 4.3 we identify forms of the equation-of-state consistent with the principle of scale-freeness for the inflationary scenario. We demonstrate the existence of three distinct classes of scale-free solutions. Then, we use our background solutions to derive predictions for the spectral tilt and tensor-to-scalar ratio of primordial density perturbations. We also consider cases with deviations from scale-freeness on unobservably small scales. Our main aim is to make most generic statements from a minimal set of assumptions. In Sec. 4.4, we repeat the same type of analysis for the cyclic (ekpyrotic) model. We conclude in Sec. 4.5 by summarizing the constraints imposed by scale-freeness for both the inflationary and cyclic theories and comparing with constraints imposed by recent data.

4.2 Scale-freeness

Both inflation and the cyclic (or ekpyrotic) theory were introduced to explain how inhomogeneous and anisotropic initial conditions can be made smooth and (spatially) flat, resulting in a scale-free universe. Inflation [35, 64, 8] accomplishes the feat with a phase of accelerated expansion occurring very shortly after the big bang. Alternatively, flatness and homogeneity can be achieved by an ekpyrotic smoothing phase [46, 45], a period of ultraslow contraction before the big bang.

In both phases, the dynamics can be easily understood, using a hydrodynamical approach in which the background evolution is governed by a 'smoothing' fluid component (S) with equation-of-state parameter,

$$\epsilon \equiv \frac{3}{2}(1+w) \quad \text{with} \quad w \equiv \frac{\rho_S}{p_S}\,, \tag{4.1}$$

where w is the equation-of-state, ρ_S the energy density, and p_S the pressure of the smoothing component. Here and throughout this chapter we will restrict ourselves to the case that the speed of light is $c_s = 1$. (Although it is straightforward to extend the analysis to $c_s \neq 1$, current observations require $c_s > 1/3$ [5]; for this range of c_s, the difference from the $c_s = 1$ case is nominal.) To have accelerated expansion during the inflationary smoothing phase, the equation-of-state parameter must lie in the range $0 < \epsilon < 1$ since the scale factor increases with time as $a \propto t^{1/\epsilon}$. To have ultra-slow contraction in the ekpyrotic smoothing phase, the analogous condition is $\epsilon > 3$. In both cases, the condition on the equation-of-state guarantees that, in the Friedmann equation,

$$H^2 = \frac{1}{3\,M_{\rm Pl}^2}\left(-\frac{3k}{a^2} + \frac{\sigma_0^2}{a^6} + \frac{\rho_S}{a^{2\epsilon}} + [\text{matter, radiation, etc.}]\right), \tag{4.2}$$

the energy density in the smoothing component ($\rho_S \propto a^{-2\epsilon}$) can overtake all other forms of energy density, including matter ($\rho \propto a^{-3}$), radiation ($\rho \propto a^{-4}$), and gradient energy ($\rho \propto a^{-2}$), and can also overtake the anisotropy (σ_0^2/a^6) and spatial curvature (k/a^2). Generally, $\epsilon \equiv \epsilon(N)$ is a function of N, the number of e-folds before the end of the smoothing phase. (Here $M_{\rm Pl}^2 = (8\pi{\rm G})^{-1}$ is the reduced Planck mass and G is Newton's constant.)

In flattening the background with a single fluid of $\epsilon < 1$, inflation also generates a nearly scale-invariant, adiabatic, and Gaussian spectrum for the curvature perturbations on comoving hypersurfaces characterized by a spectral tilt $n_s(N) - 1$ [9, 76], which is also a function of N. The same is not true for ekpyrosis. If there is only a single fluid in the contracting phase, the growing-mode, adiabatic perturbations decay and cannot be the seed of structure in the post-bang universe [18]. Currently, the best understood way of creating primordial density perturbations is the *entropic* mechanism [17, 55]. Here, pre-bang isocurvature fluctuations are generated by adding a second fluid component; in the simplest case, one that does not affect the background evolution. These isocurvature modes are then converted into density perturbations which source structure in the post-bang universe. Another consequence of inflation is the generation of nearly scale-invariant tensor (gravitational wave) fluctuations. The ratio of the tensor-to-scalar amplitude as a function of N is labeled $r(N)$. For the ekpyrotic case, the tensor amplitudes are exponentially suppressed compared to inflation and can be considered negligible for the purposes of this discussion. Hence, the detection or non-detection of primordial gravitational waves is a key means of distinguishing the two scenarios.

Assuming only that there was a period of inflation, the point has been made by numerous authors (*e.g.*, see [27] for a recent example) that any observational outcome is possible, rendering the theory unpredictive. The purpose of this chapter is to use a hydrodynamical approach to determine how the predictions of inflationary and cyclic cosmologies are affected by the additional assumption that the underlying physics is scale-free. By a scale-free function we mean a power-law form up to a coordinate-shift, *i.e.*, $f : \mathbb{R} \to \mathbb{R}$ is a scale-free function iff there is a coordinate transformation $\pi : \mathbb{R} \to \mathbb{R}$, $x \mapsto x + C$, $C \in \mathbb{R}$, such that

$$(f \circ \pi)(x) = \beta x^{\alpha}, \quad \alpha, \beta \in \mathbb{R}. \tag{4.3}$$

Scale-invariant is the special case where $\alpha = 0$.

For our cosmological application, we describe a cosmology as *strictly scale-free* if both the background equation-of-state $\epsilon(N)$ and the perturbations, characterized by $n_S(N) - 1$ and $r(N)$, are scale-free. We shall show that this condition is highly constraining, leading to specific predictions for $n_S - 1$ and r. In particular, it is immediately apparent from the Friedmann equation, Eq. (4.2), which can be written as a sum of $a^{-2\epsilon_i}$, that for a scale-free background the equation-of-state parameter of all components relevant during the smoothing stage must be the same.

Since the case for scale-freeness is based on background evolution and observations on large scales, we also consider *background-only scale-freeness* in which ϵ is precisely scale-free but $n_S - 1$ can have deviations from scale-freeness on unobservably small length scales ($N = \mathcal{O}(\infty)$). In addition, we consider a class of models that *weakly break scale-freeness* where we analyze deviations in ϵ, $n_S - 1$, and r that only affect unobservably small scales.

4.3 Inflationary theory

In order to construct a model with N^* e-folds of inflation, the following two criteria must be satisfied:

I: (sufficient inflation) N^* e-folds inflation occur, *i.e.*, $\epsilon(N) < 1$ for $1 < N < N_*$, and

II: (graceful exit) inflation ends in the last e-fold, *i.e.* $\epsilon(N = 0) = 1$; plus $\epsilon(N > 0) < 1$ and $\epsilon(N < 0) \geqslant 1$.

where N is the number of e-folds of inflation remaining until its end $t_{\rm end}$, defined as

$$N = \int_t^{t_{\rm end}} H dt \,. \tag{4.4}$$

$N = 0$ marks the end of inflation. Here, without loss of generality we will assume a single continuous stage of inflation with N^* e-folds. If these are the only constraints imposed, then $\epsilon(N)$ can take many forms and the predictions can vary arbitrarily. To transform inflation into a predictive theory, an additional constraint is needed. We use scale-freeness as the added condition.

4.3.1 Scale-free inflationary theory

Scale-freeness, Eq. (4.3), combined with the two numbered criteria, determines the evolution of ϵ during inflation:

$$\epsilon(N) = \frac{1}{(N+1)^\alpha}, \quad \alpha > 0, \tag{4.5}$$

where α needs to be strictly positive to satisfy criterion I. That is, the equation-of-state $\epsilon(N)$ consistent with the scale-free principle is described by a simple power-law form with a *single* free parameter, α. The second free parameter in Eq. (4.3), β, is fixed by criterion II, the condition that $\epsilon(0) = 1$. Considering β as a second free parameter, as assumed in Ref. [75], violates criterion II. We will discuss the implications of this restriction below.

To analyze different inflationary solutions, we compute the evolution of the Hubble parameter in terms of $\epsilon(N)$. Note that we need to assume *both* criteria I and II for this type of analysis. Here we are being more precise than some previous hydrodynamical treatments. For example, Ref. [47] obtains Eq. (4.5), but through an inconsistent argument that first assumes ϵ = constant $\ll 1$ and, hence, violates criterion II. In Ref. [75], β is left as a free parameter, which is also inconsistent with criterion II.

For a homogeneous, isotropic, and spatially flat universe, the second Friedmann equation can be written as $\epsilon = -\dot{H}/H^2$. Since $dN = -d\ln a$, we can rewrite the relation as

$$\epsilon = \frac{d\ln H}{dN} \, . \tag{4.6}$$

Finally, integration of Eq. (4.6) together with our expression for ϵ in Eq. (4.5) yields a closed-form expression for H^2 (or, equivalently, the smoothing energy density ρ_S) as a function of N:

$$H^2/H^2_{\rm end} = \rho_S/\rho_{S,{\rm end}} = \exp\left[-2\int_N^0 \epsilon \, dN\right] \tag{4.7}$$

which reduces in the inflationary case to

$$H^2/H^2_{\rm end} \quad = \quad \begin{cases} (N+1)^2 \, , & \alpha = 1, \\ \exp\left[\frac{2\left(1-(N+1)^{1-\alpha}\right)}{\alpha-1}\right] \, , & \alpha \neq 1, \end{cases} \tag{4.8}$$

which is the relevant observable in inflationary dynamics. Note that the Hubble parameter at the end of inflation, $H_{\rm end}$, is arbitrary.

In Figure 4.1 we have plotted $H^2/H^2_{\rm end}$ during the inflationary phase as a function of N for different values of α. The dashed curve corresponds with the strictly scale-free case, $\alpha = 1$. The rest of the curves are background-only scale-free.

The curves divide into three classes: (i) the "plateau-like" class with $\alpha \gtrsim 1.5$ (bold curve) in which H^2 flattens out and is virtually independent of N over the range $N > 60$ (changing by less than 20%); (ii) the "power-law-like" class with $\alpha \lesssim 1$ in which H^2 is unbounded above; and (iii) an "intermediate class" with $1 < \alpha < 1.5$, that appears power-law-like during the last 60 e-folds (see Fig. 1) but which ultimately

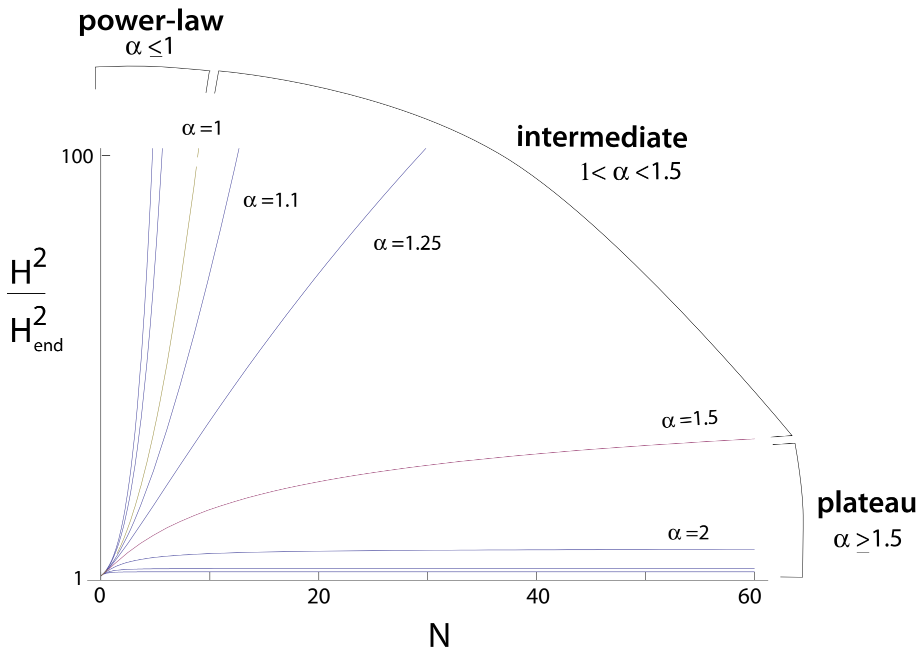

Figure 4.1: In the hydrodynamical picture, scale-free inflationary models can be divided into three classes characterized by α in Eq. (4.5): the *plateau-like* class (with $\alpha \geq 1.5$, where $\alpha = 1.5$ is the bold thick curve) in which H^2 flattens out rapidly (well before $N = 60$) as N increases; the *power law-like* class (with $\alpha \leq 1$, where $\alpha = 1$ is the dashed curve) in which H^2 is *unbounded* above and changes significantly as N increases; and the *intermediate class* (with $1 < \alpha < 1.5$), which rises like a power-law for $N < 60$ but which ultimately reaches a plateau at values of $N \gg 60$ that are irrelevant for cosmological predictions. The plateau-like class is most favored by current observations but encounters the problems described in this chapter. The power law-like models are strongly disfavored by current observations but do not suffer the same problems.

reaches a plateau at very large $N \gg 60$ (with H^2 increasing by more than 20% for $N > 60$).[1]

The expression for the equation-of-state parameter as defined in Eq. (4.5) enables us to derive predictions for the spectral tilt and the tensor-to-scalar ratio of primordial density perturbations. Since $\epsilon(N)$ does not change rapidly, *i.e.*,

$$\frac{d\ln\epsilon}{d\,N} = -\frac{\alpha}{N+1}, \; \frac{d^2\ln\epsilon}{d\,N^2} = \frac{\alpha}{(N+1)^2} \lesssim \mathcal{O}(1)\,, \tag{4.9}$$

[1]Note that "intermediate" here refers to the range of scale-free models that have a mix of characteristics between plateau and power-law scale-free behavior. This is distinct from Ref. [11], where "intermedidate" refers to cases where the scale-factor $a(t) \propto \exp(A\,t^f)$, which is not scale-free and so does not fit into our classification.

we can use the approximation [101]:

$$n_S - 1 \approx -2\epsilon + \frac{d\ln\epsilon}{d\,N}\,. \tag{4.10}$$

Substituting ϵ from Eq. (4.5) yields

$$n_S - 1 \approx -\frac{2}{(N+1)^\alpha} - \frac{\alpha}{N+1}\,. \tag{4.11}$$

It is instructive to note that $n_S - 1$ has a maximum value of

$$\begin{aligned} (n_S-1)(\alpha_0) &= -\frac{\ln\left[2(N+1)\ln(N+1)\right]+1}{(N+1)\ln(N+1)}, \\ \text{for}\quad \alpha_0 &= \frac{\ln\left[2(N+1)\ln(N+1)\right]}{\ln(N+1)}\,. \end{aligned} \tag{4.12}$$

For example, with $N=60$, we have $\alpha_0 \simeq 1.5$ and $(n_S-1)(\alpha_0) \simeq -.03$. This red tilt is the minimum deviation from Harrison-Zel'dovich-Peebles spectrum (HZP) for a scale-free inflationary model and is close to the observed value. (Without scale-freeness or criterion II, n_S can be arbitrarily close to HZP or yield a blue-tilt.) This extremum lies almost precisely at the borderline between the intermediate and plateau-like class. (The extremum is described as being at $\alpha \approx 2$ in [75], but, in our analysis, this crude approximation would give the wrong impression that it corresponds to the observationally favored models deep in the plateau range when it actually corresponds to a disfavored case.)

Finally, with the standard normalization, the tensor-to-scalar ratio is [47]

$$r \approx 16\epsilon = \frac{16}{(N+1)^\alpha}\,. \tag{4.13}$$

4.3.2 Cosmological problems

The plateau-like hydrodynamical class, especially near $\alpha = 2$, is the one favored by current observations [3], yet it suffers from a series of problems, some of which are analogous to those described in the analysis of scalar field potentials in [40] and some of which have not been discussed previously:

- *Extra parameters:* The plateau-like class has the property that H^2 is nearly flat except for the last e-fold or so when the expansion rate suddenly decreases; see the feature at small N in the plateau-like curves in Fig. 4.1. That means whatever microphysics accounts for $\epsilon(N)$ must have an extra parameter and/or field compared to the power-law-like models adjusted to rapidly cutoff the inflation after a long period of a nearly constant H^2. We will see this effect below when we translate our hydrodynamical results into models of scalar-fields and inflaton potentials.

- *Hydrodynamical initial conditions problem:* As originally imagined, inflation was supposed to smooth and flatten the universe beginning from arbitrary initial conditions after the big bang [35]. However, this view had to be abandoned as it was realized that large inflaton kinetic energy and gradients prevent inflation from starting. Consequently, inflation can only take hold if the entropy, kinetic energy, and gradients within a Hubble-sized patch is exceedingly small.

  We note that the later that inflation starts, the greater is the physical size of a Hubble patch and the more unlikely is the initial condition. A distinctive feature of the power law-like hydrodynamic class ($\alpha \leq 1$) is that H^2 is unbounded above. Hence, inflation can begin, in principle, at arbitrarily high H^2 or, equivalently, over a small patch where the initial conditions are less unlikely compared to cases where inflation starts later. This includes inflation beginning immediately after the big bang when the energy density is at the Planck scale.

  By contrast, inflation for models in which H^2 is bounded above, (*i.e.*, all $\alpha > 1$), can only begin after the universe expands enough for the energy density to drop to the level of the plateau, $M_{\rm I}^4$. The Planck2013 constraint on r ($r_{0.002} < 0.12$ at 95% CL) [3] yields

$$M_{\rm I}^4 \lesssim \frac{3\pi^2 A_s}{2}\, r\, M_{\rm Pl}^4 \sim 10^{-12}\, M_{\rm Pl}^4\, \frac{r_*}{0.12} \tag{4.14}$$

at 95% CL, where A_s is the scalar amplitude and r_* the value of r evaluated at Hubble exit during inflation of mode with wave number k_*. This is well below the Planck density at a time when the Hubble volume is, by simple comparison of the scales $M_{\rm Pl}/M_{\rm I} \sim 10^3 \cdot (10^{16}\,{\rm GeV}/M_{\rm I})$, a billion times (or more) greater [40]. In this case, some combination of gradient energy density, spatial curvature, and radiation must necessarily dominate immediately after the big bang and for a substantial period thereafter before inflation can ever take hold. A well-known problem, though, is that gradient energy and spatial curvature tend to block inflation by causing regions of space to collapse before inflation can start [40]. That is, inflation can only begin for the plateau-like models if there is the extraordinary additional assumption that the universe emerges from the big bang with a patch,

$$\begin{aligned} R^3(t_{\rm Pl}) \;&\gtrsim\; \left[a(t_{\rm Pl})\int_{t_{\rm Pl}}^{t_{\rm I}}\frac{d\,t}{a}\right]^3 \sim \left[\frac{a(t_{\rm Pl})\,H(t_{\rm Pl})}{a(t_{\rm I})\,H(t_{\rm I})}H^{-1}(t_{\rm Pl})\right]^3 \\ &>\; 10^9\left(\frac{10^{16}\,{\rm GeV}}{M_{\rm I}}\right)^3 H^{-3}(t_{\rm Pl}), \end{aligned} \tag{4.15}$$

that is smooth and flat on scales a billion times greater than required for the unbounded power-law-like case [61]. Our hydrodynamic analysis divides the inflationary models along the dashed line ($\alpha = 1$) in Fig. 4.1 between those that require this extraordinary assumption (plateau-like and intermediate with $\alpha > 1$) and those that do not ($\alpha \leq 1$).

- *Hydrodynamical unlikeliness problem:* Even assuming the rare initial conditions are satisfied, the observationally favored plateau-like models ($\alpha \approx 2$) produce exponentially less smooth and flat volume than the power-law-like or intermediate class models with $1 \lesssim \alpha < 1.5$. This leads to the hydrodynamic version of the "unlikeliness problem" similar to (but not identical to; see Sec. 4.5) the one discussed in Chapter 2: First, let's imagine a complex energy landscape in which there are many different kinds of paths corresponding to different a mix of power-law, intermediate and plateau-like classes that proceed to the same vacuum. The most most likely path is the one that produces the most number of e-folds of inflation.

For each α, we can compute the largest value of N for which the density fluctuation $\delta\rho/\rho(N)$ is less than 1. For larger N where $\delta\rho/\rho$ exceeds 1, quantum fluctuations totally spoil the homogeneity and curvature. Hence, $N_{\rm max}(\alpha)$, the maximum number of e-folds as a function of α, is determined by the condition

$$\delta\rho/\rho\,(N_{\rm max}) = 1. \tag{4.16}$$

The fluctuation amplitude is

$$\delta\rho/\rho\,(N) \simeq \frac{H(N)}{M_{\rm Pl}\sqrt{\epsilon(N)}} \tag{4.17}$$

(for the derivation use, for example, $\delta\rho/\rho = H/\dot{\phi}$ and $\dot{\phi}^2 = \rho + p$). Substituting the expressions we previously found for H^2 and ϵ, Eq. (4.16) and (4.17) together give

$$N_{\rm max}(\alpha) = -1 + \left(\frac{1}{2}\,\alpha\, W(z)\right)^{\frac{1}{1-\alpha}}\,, \tag{4.18}$$

where W is the Lambert W function, and its parameter

$$z = \frac{2}{\alpha}\left(10^5 \cdot 61^{\alpha/2} \cdot \exp\left(\frac{61^{1-\alpha}}{1-\alpha}\right)\right)^{\frac{2}{\alpha}(1-\alpha)}\,, \tag{4.19}$$

and $\delta\rho/\rho$ is normalized such that $\delta\rho/\rho(N = 60) = 10^{-5}$. For $\alpha = 1$, $N_{\rm max}(\alpha)$ is $61 \cdot 10^{10/3} \approx 10^5$.

As illustrated in Fig. 4.2, $N_{\rm max}$ is maximal overall for $\alpha \simeq 1.25$; among the power-law-like cases, $\alpha = 1$ is most favored; and among the plateau-like models $\alpha = 1.5$ is most favored. The differences in inflated volume in each case are exponentially large, of order $\exp(10^{5-8})$, so "favored" means "very strongly favored" [3]. Note that $\alpha = 2$ is strongly disfavored; yet, this is the inflationary type model that is currently most favored observationally.

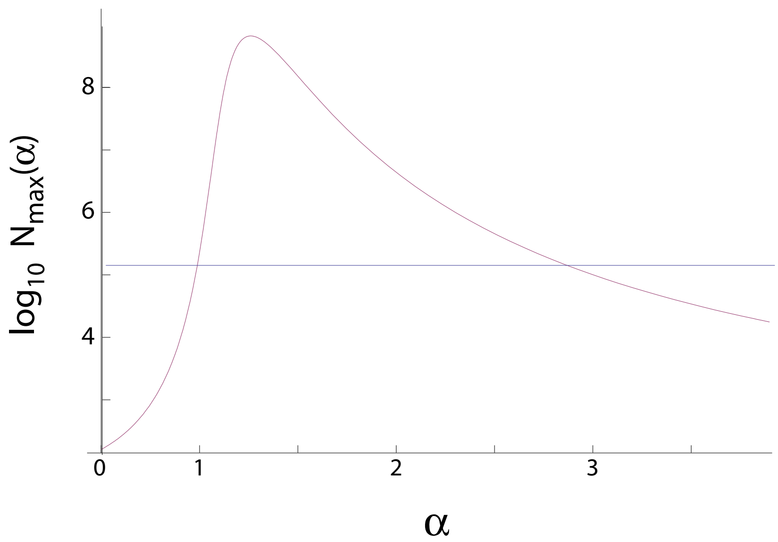

Figure 4.2: A logarithmic plot of the maximum number of e-folds $N_{\max}(\alpha)$ for scale-free models as a function of the hydrodynamic variable α. The plot assumes initial conditions can be set perfectly smoothly in the initial Hubble patch.

These estimates for $N_{\max}(\alpha)$ are, however, based on the idea of a complex energy landscape with many different types of paths to each minium, assuming that the initial conditions when the universe emerged from the big bang could be set with arbitrary accuracy so that the energy density in the smoothing component is the maximum possible, $3H^2(N_{\max}(\alpha))$ in Planck units. However, a more serious problem that applies for even simple energy landscapes is that most patches of space are likely to have large gradient energy that will spoil inflation altogether. Even if we eliminate those patches and consider only homogeneous patches, in each patch there remain different mixes of radiation, kinetic energy, potential energy, and other forms of energy such that, typically, we do not have patches at precisely the ideal potential energy to obtain $N_{\max}$. Hence we should imagine some flex of order x in the amount of the initial potential energy. A reduction of the average energy density in the patch by a factor x requires a revised estimate $N_{\max}(\alpha, x)$:

$$N_{\max}(\alpha, x) = \left(N_{\max}(\alpha, 0)^{1-\alpha} - \frac{\alpha - 1}{2} \ln(x) \right)^{\frac{1}{1-\alpha}} - 1, \tag{4.20}$$

which equals $61 \cdot 10^{10/3}\sqrt{x}$ for $\alpha = 1$. Because plateau-like models with $\alpha \geq 1.5$ are so flat for large N, a reduction in average H^2 by some factor x produces a much greater reduction in $N_{\max}(\alpha, x)$ relative to $N_{\max}(\alpha) \equiv N_{\max}(\alpha, 0)$ than is found for power-law-like models.

Fig. 4.3 shows $\log N_{\max}$ as a function of x for different values of α. The dashed line corresponds to the strictly scale-free, unbounded power-law-like case with $\alpha = 1$; the thin black curves to models with α-values of $1.25, 1.5, 2, 3$; the red horizontal line marks 60 e-folds. It is clear that the plateau-like models fail to reach $N = 60$ e-folds for even a small x, while the power law-like models and intermediate class models are comparatively insensitive to the initial distribution of energy in the patch.

In sum, there are three classes of scale-free inflationary scenarios. Power-law-like models ($\alpha \leq 1$) do not suffer from the initial conditions problem or unlikeliness problem. Models of the intermediate class have the initial conditions problem, but not the unlikeliness problem. However, these models are all observationally disfavored currently [3]. The observationally-favored plateau-like models with $\alpha = 2$ suffer from all the problems described above. Hence, the theoretically favored scale-free inflationay models are observationally disfavored and vice versa. The fact that the initial conditions and unlikeliness problems impose different constraints illustrates that they are logically distinct, a point that some have disputed in discussions of [40].

4.3.3 Deviations from scale-freeness

We have thus far considered $\epsilon(N)$ that have a scale-free form. The case $\alpha = 1$ is *strictly* scale-free in that the functions that describe the background, $\epsilon(N)$ and $H(N)$, as well as the functions that describe the perturbations

$$n_S(N) - 1 = -\frac{3}{N+1} \tag{4.21}$$

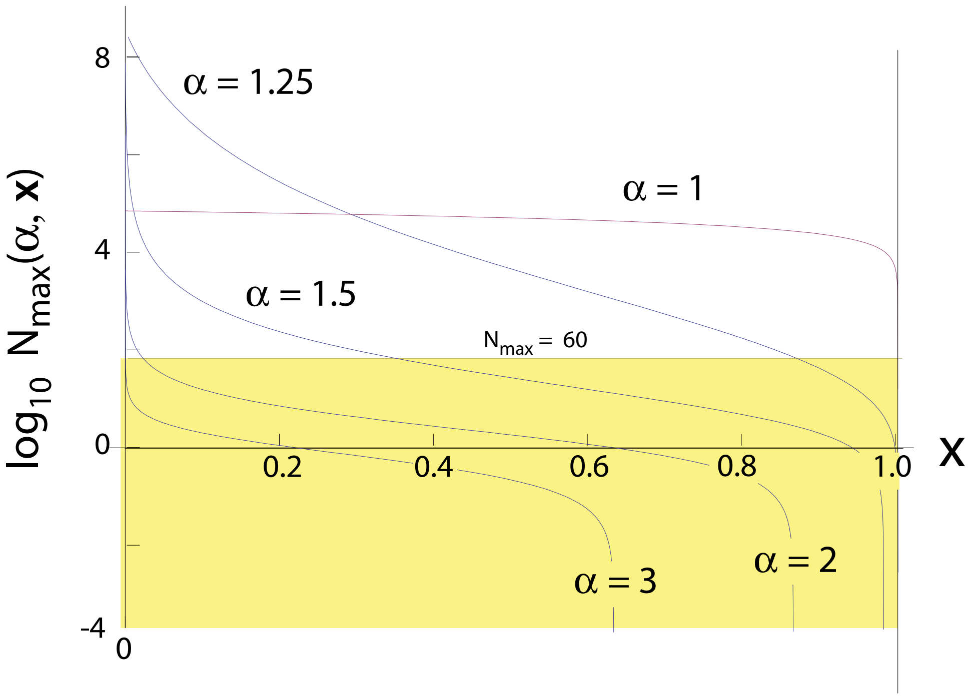

Figure 4.3: The sensitivity of $N_{\max}$ to the initial energy density in the smoothing component at the Planck time when the universe first emerges from the big bang. If the energy density in a patch could be set with perfect precision, the maximum number of e-folds of inflation would be $N_{\max}(\alpha) \equiv N_{\max}(\alpha, 0)$ plotted in Fig. 4.2. Due to contributions of other forms of energy (kinetic energy, radiation energy, *etc.*), we assume a variation of x percent from perfect precision and compute how this affects the maximum number of e-folds, $N_{\max}(\alpha, x)$, as shown in the logarithmic plot above. Note that the $N_{\max}(\alpha)$ in Fig. 4.2 is equal to $N_{\max}(\alpha, 0)$. The plot shows that $N_{\max}(\alpha, x)$ for $\alpha = 1$ (strictly scale-free power-law-like models) is rather insensitive to x. By contrast, plateau-like models ($\alpha \geq 1.5$) are so extremely sensitive to x that, unless the initial energy density of the smoothing component is set with extraordinary precision, the value of $N_{\max}(\alpha, x)$ is much less than that for the power-law-like class and less than the minimal 60 needed for inflation. The shade region corresponds to insufficient inflation.

are all simple power-laws (or power-laws with shifts).

For $\alpha \neq 1$, the background functions are still scale-free but the spectral index is not:

$$n_S(N) - 1 = -\frac{2}{(N+1)^\alpha} - \frac{\alpha}{N+1} \tag{4.22}$$

so there is only *background* scale-freeness.

For *weakly broken* scale-freeness, there can be no complete treatment since "weakly" is an imprecise term. Here we consider in this category deviations from scale-freeness at the background level but only on length scales that are unobservably small (corresponding to small N):

$$\epsilon = \frac{\beta}{(N+1)^\alpha} + \frac{1-\beta}{(N+1)^{\alpha+\gamma}}, \quad \text{with} \quad \beta, \gamma > 0,\ \beta \neq 1, \tag{4.23}$$

where this form is designed to still satisfy inflationary criteria I and II. For the deviation to be small, in addition, it is necessary that

$$|1 - 1/\beta| \ll (N+1)^\gamma \quad \text{and} \quad |\beta - 1| < 1 \tag{4.24}$$

for observable N. Then, with an additional free parameter, the predictions are modified:

$$\epsilon \approx \frac{\beta}{(N+1)^\alpha}, \quad n_S - 1 \approx \begin{cases} -\frac{2\beta}{(N+1)^\alpha}, & \alpha < 1, \\ -\frac{2\beta+1}{(N+1)}, & \alpha = 1, \\ -\frac{\alpha}{N+1}, & \alpha > 1, \end{cases} \quad r \approx \frac{16\,\beta}{(N+1)^\alpha}. \tag{4.25}$$

As we shall discuss below, the case $\alpha = 1$ is of particular interest as it corresponds to power-law inflaton (ϕ) potentials $V(\phi) \propto \phi^n$ with $n = 4\beta$. From Eq. (4.25), we note that the weakly scale-free breaking inflationary models ($\beta \neq 0$) entail two independent parameters while strictly scale-free inflationary theory involves exactly one free parameter.

4.4 Cyclic theory

In the following section, we carry out the same type of hydrodynamical analysis for the cyclic theory that we previously did for inflation. In order to construct a model with $\mathcal{N}^*$ e-folds of ultra-slow contraction (ekpyrosis) that flattens and smoothes the universe, the two criteria analogous to those used for inflation are as follows:

- I′: (sufficient ekpyrosis) $\mathcal{N}^*$ e-folds of ekpyrosis occur, *i.e.*, $\epsilon(\mathcal{N}) > 3$ for $1 < \mathcal{N} < \mathcal{N}^*$; and
- II′: (exit) ekpyrosis ends in the last e-fold, *i.e.*, $\epsilon(\mathcal{N} > 0) > 3$, and $\epsilon(0) = 3$.

We have introduced the dimensionless time variable $\mathcal{N}$, defined by

$$\mathcal{N} \equiv \ln\left(\frac{a_{\rm end}\, H_{\rm end}}{a\, H}\right) . \tag{4.26}$$

$\mathcal{N}$ measures the number of e-folds of modes that exit the horizon before the end of ekpyrosis. It is related to the time variable N used in the previous section by $d\mathcal{N} = (\epsilon - 1)\, dN$. For inflation $\mathcal{N} \approx N$, since $H \approx$ constant during accelerated expansion. For ekpyrosis, on the other hand, $\mathcal{N} \gg N$ because H grows significantly during ultra-slow contraction while a shrinks very slowly, so $\mathcal{N}$ is the correct time-variable to use. Here, in analogy with the treatment of inflation, we assume a single continuous stage of ekpyrosis with $\mathcal{N}^*$ e-folds.

4.4.1 Scale-free cyclic theory

Scale-freeness, combined with these two criteria, determines the evolution of ϵ during the ekpyrotic phase. From Eq. (4.3) together with criteria I′ and II′, we have

$$\epsilon(\mathcal{N}) = 3\,(\mathcal{N}+1)^{\alpha_1}, \quad \alpha_1 > 0. \tag{4.27}$$

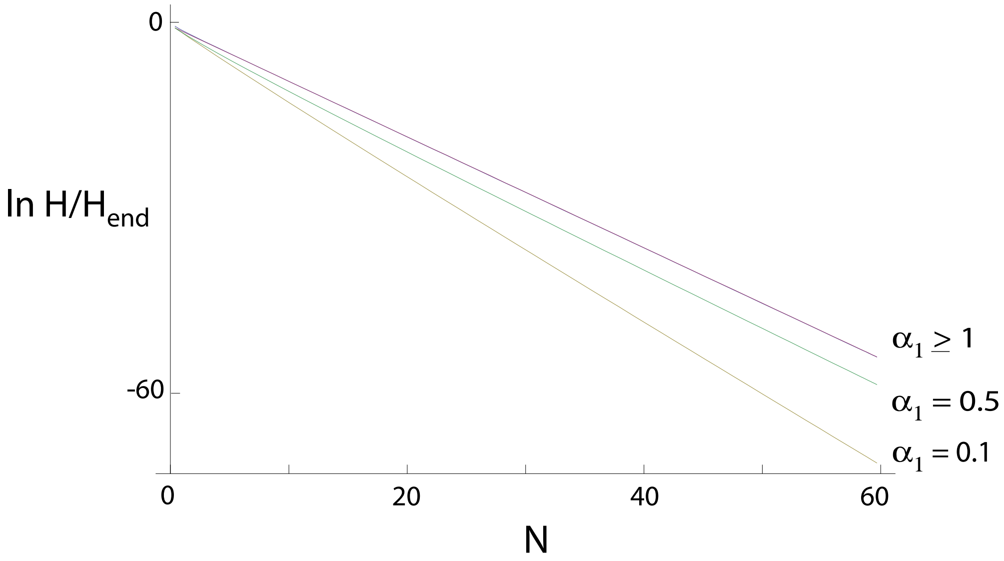

Figure 4.4: Plot of $\ln H^2/H^2_{\rm end}$ vs. $\mathcal{N}$ for the cyclic picture for a range of α_1.

That means, the shape of the equation-of-state parameter consistent with the scale-free principle is a simple power-law form with a single free parameter. The second free parameter, β_1, in Eq. (4.3) is fixed by criterion II′, which requires $\epsilon(0) = 3$.

To analyze different cyclic solutions, we study the evolution of the total energy density $H^2/H^2_{\rm end}$ during ekpyrosis. Substituting Eq. (4.27) into Eq. (4.7) yields

$$H^2/H^2_{\rm end} = \exp\left(-2\,\mathcal{N} + 2\int_{\mathcal{N}}^{0} \frac{d\mathcal{N}}{3(\mathcal{N}+1)^{\alpha_1} - 1}\right). \tag{4.28}$$

Note that this expression reflects a characteristic feature of an ekpyrotic phase that H^2 grows by many orders of magnitude during smoothing. Figure 4.4 shows a logarithmic plot of $H^2/H^2_{\rm end}$ for the ekpyrotic phase as a function of $\mathcal{N}$ for different values of α_1.

In contrast to inflation, cyclic models do not divide into different classes. In fact, for $\alpha_1 \gtrsim 1$ all of the H^2 curves lie virtually on top of one another such that the Hubble parameter proves effectively independent of α_1. Hence, the unlikeliness problem, based on comparing the probability of different classes, cannot arise for the cyclic theory. In addition, it follows from the α_1-independence that choosing a value of α_1 to fit observational data does not involve any special fine-tuning relative to the general class of models.

The initial conditions requirement is extremely mild. It suffices to have a volume of space on the scale of meters in diameter that is absent of black holes or non-linear structure at the beginning of the contraction phase [23]. The ekpyrotic mechanism will smooth and flatten this region and the bounce will transform this region during the expansion phase into a size of order the Hubble volume today. The initial condition can be reached in a number of ways, including by having an expanding phase precede the contraction phase. For example, in the cyclic scenario, the initial condition is easily achieved by having the ekpyrotic phase preceded by an expanding dark energy dominated phase just like the current phase of our universe. Consider that the present universe already contains exponentially many patches that satisfy the initial condition requirements and any further expansion only increases their number. Having an expanding dark energy phase turn into a contracting phase is known to be quite straightforward to achieve, *e.g.*, by having a scalar field roll or tunnel from a phase with positive potential density to a phase with a negative potential energy density [89, 91]. In order for ekpyrosis to occur, no further criteria need to be satisfied; expansion can turn into contraction at arbitrarily low energies for an α_1 since there is no (classical) limit in Fig. 4.4 on how low H can be when contraction begins for any α_1 (so the choice of α_1 does not require extra tuning). By contrast, for inflation, assuming an expanding phase after the bang is not sufficient since the natural conditions after the bang would have large gradient and kinetic energies that would block the initiation of inflation.

In sum, at background level, none of the problems pointed out above for inflation arise for the cyclic model. There is no fine-tuning or unlikeliness problem, and there is no initial conditions problem comparable to the inflationary case.

At the perturbative level there is a notable conceptual difference between inflation and the cyclic model, at least according to most current versions of cyclic theory. Namely, the generation of primordial density perturbations is assumed to be a two-stage process. First, entropy or isocurvature perturbations are created before the bounce. These perturbations are then converted into primordial density perturbations at some time during the transition from big crunch to big bang [55].

Modeling this scenario in a hydrodynamical approach requires a two-component fluid: one fluid component governs the background evolution and the other is responsible for the generation of isocurvature fluctuations. The background fluid component can be described by an equation-of-state parameter, $\epsilon_1(\mathcal{N})$, as defined in Eq. (4.27),

$$\epsilon_1(\mathcal{N}) = 3\,\beta_1(\mathcal{N}+1)^{\alpha_1}, \quad \alpha_1 > 0\,, \tag{4.29}$$

where $\beta_1 = 1$ according to criterion II. The equation-of-state parameter for the second fluid, $\epsilon_2(\mathcal{N})$, must also satisfy the requirement of scale- freeness. Hence, from Eq. (4.3), it is necessary (but not sufficient, as we point out below) for $\epsilon_2(\mathcal{N})$ to take the form

$$\epsilon_2(\mathcal{N}) = 3\,\beta_2\,(\mathcal{N}+1)^{\alpha_2}, \quad \alpha_2 \in \mathbb{R}\,. \tag{4.30}$$

If this component satisfies the null energy condition, β_2 must be greater than or equal to zero. Before imposing any further conditions, the general expression for the spectral tilt of density perturbations is

$$n_S(\mathcal{N}) - 1 = 3 - \sqrt{1+8\kappa}\left(1 + 3\cdot\frac{1-2\kappa}{1+8\kappa}\cdot\frac{2}{\epsilon_1} + \frac{8-5\kappa}{1+8\kappa}\cdot\frac{\epsilon_{1,\mathcal{N}}}{\epsilon_1}\right)\,, \tag{4.31}$$

where

$$\kappa(\mathcal{N}) = \epsilon_2/\epsilon_1 = (\beta_2/\beta_1)\,(\mathcal{N}+1)^{\alpha_2-\alpha_1} \tag{4.32}$$

(see Appendix B for the derivation). In the limit of constant $\kappa(N) \approx 1$, the expression reduces to

$$n_S - 1 \;=\; \frac{2}{\epsilon_1} - \frac{\epsilon_{1,\mathcal{N}}}{\epsilon_1} + \frac{4}{3}(1-\kappa)\,, \tag{4.33}$$

in agreement with [59, 17].

4.4.2 Deviations from scale-freeness

For the *strictly* scale-free case, both the background and the perturbations must be simple power-laws. For the background Friedmann equations, we mean that the dominant contribution to H^2 in Eq. (4.2) should be a simple power-law in a. As noted above in Eq. (4.3), this requires $\epsilon_1(\mathcal{N}) = \epsilon_2(\mathcal{N})$ with $\alpha_1 = \alpha_2 = 1$ and $\beta_1 = \beta_2$. Then, the prediction for the spectral tilt is

$$\begin{aligned} n_S - 1 &= \frac{2}{\epsilon_1} - \frac{\epsilon_{1,\mathcal{N}}}{\epsilon_1} \\ &= -\frac{1}{3\,(\mathcal{N}+1)}\,. \end{aligned} \tag{4.34}$$

For the *background-only* scale-free case, we still require $\beta_2 = \beta_1 = 1$ and $\alpha_1 = \alpha_2$, but the αs need not be 1. Then, the spectral tilt has a small deviation from scale-freeness;

$$\begin{aligned} n_S - 1 &= \frac{2}{\epsilon_1} - \frac{\epsilon_{1,\mathcal{N}}}{\epsilon_1} \\ &= \frac{2}{3\,(\mathcal{N}+1)^{\alpha_1}} - \frac{\alpha_1}{\mathcal{N}+1}\,, \end{aligned} \tag{4.35}$$

in agreement with [17, 55]. Note that, even though there are two fluid components, the expression for n_s has only one free parameter, as in the case of inflation.

Finally, we consider the *weakly broken* scale-free case in which deviations from scale-freeness occur only on unobservable scales. As with inflation, there is no absolute definition of weakly broken scale-free, but we consider two types of deviations that arise in microphysical models of scalar fields.

First, a very weakly broken scale-free background occurs if β_2 is close to but not equal to $\beta_1 = 1$, or, equivalently, $0 < |\kappa - 1| \ll 1$. In this case, the expression for the tilt reduces to the simpler form

$$
\begin{aligned}
n_S - 1 &= \frac{4}{3}(1-\kappa) + \frac{2}{\epsilon_1} - \frac{\epsilon_{1,\mathcal{N}}}{\epsilon_1} \\
&= \frac{4}{3}(1-\kappa) + \frac{2}{3\,(\mathcal{N}+1)^{\alpha_1}} - \frac{\alpha_1}{\mathcal{N}+1}\,,
\end{aligned}
\tag{4.36}
$$

in agreement with [17, 59]. A second type of deviation from background scale-freeness is to choose $\beta_1 \neq 1$, which generates additional contributions to n_S analogous to the inflationary case; see Eqs. (4.23) and (4.25). As with the background case, the weakly broken scale-free case for the two fluid-component cyclic scenario has the same number of free parameters as for inflation, so neither theory is advantageous by this measure.

4.5 Scale-free scalar fields and potentials

The problems we identified for inflationary theory are similar to but not identical to the issues identified previously in Chapter 2, using a model dependent analysis based on assuming that inflation is driven by scalar fields with specific potential forms. In order to compare the two approaches, we translate our hydrodynamical scale-free models into the field picture, first for inflation and subsequently for cyclic cosmology.

4.5.1 Scale-free inflationary potentials

The construction of scale-free inflationary potentials corresponding to the hydrodynamical models described in previous sections is based on assuming single-field, slow-roll inflation with canonical kinetic energy density and $\rho_S \simeq V(\phi)$, where $V(\phi)$ is the potential energy density for the inflaton scalar field ϕ. Following the method presented in [75], the Friedmann equations together with the identity $\dot{\phi}^2 = \rho_S + p_S$ yield

$$
\frac{\phi - \phi_{\text{end}}}{M_{\text{Pl}}} = \pm \int_N^0 \sqrt{2\epsilon}\, d\,N = \pm\sqrt{2} \cdot \begin{cases} -\ln(N+1), & \alpha = 2 \\ \frac{2}{2-\alpha}\left(1-(N+1)^{\frac{2-\alpha}{2}}\right), & \alpha \neq 2. \end{cases}
\tag{4.37}
$$

Then, with Eq. (4.7) we find the expression for the inflationary potential

$$V(\phi) = \begin{cases} \lambda\, \phi^4, & \alpha = 1, \\ V_{\rm end}\, \exp\left[2 - 2\, \exp\left(-\frac{\phi - \phi_{\rm end}}{\sqrt{2}\, M_{\rm Pl}}\right)\right], & \alpha = 2, \\ V_{\rm end}\, \left(3 - (N(\phi)+1)^{-\alpha}\right) \exp\left[\frac{2}{1-\alpha}\left((N(\phi)+1)^{1-\alpha} - 1\right)\right], & \alpha \neq 1, 2, \end{cases} \tag{4.38}$$

where $N(\phi)$ is given by Eq. (4.37).

In the hydrodynamical analysis, we found that the scale-free inflationary models divide into three classes, power-law-like ($\alpha \leq 1$), intermediate ($1 < \alpha < 1.5$) and plateau-like ($1.5 \leq \alpha$). In the scalar-field potential analysis, the first class, the power-law-like models, divides into two cases: the strictly scale-free $\alpha = 1$ case, corresponding to $V(\phi) = \lambda\phi^4$ with only a single *dimensionless* parameter; and $\alpha < 1$, for which the potential is exponential with a power-law pre-factor and a dimensionful parameter. Both cases are free of the hydrodynamical initial conditions and unlikeliness problems described here and the corresponding problems described for potentials in Chapter 2. However, in the latter case ($\alpha < 1$), graceful exit occurs since the power-law pre-factor becomes significant in the last e-fold, adding a feature to the exponential potential. The added feature breaks the appealing scale-free character. Hence, the scalar field potential analysis picks out the $\alpha = 1$ strictly scale-free case as being simplest among the power-law-like class.

The intermediate class of hydrodynamical models ($1 < \alpha < 1.5$) translates into plateau-potentials with large-field inflation. Unlike the $\alpha = 1$ case, these models require tuning one or more *dimensionful* parameters to satisfy cosmological constraints on the number of e-folds and the density fluctuation amplitude, $\delta\rho/\rho \sim 10^{-5}$. As in the hydrodynamical analysis, the predictions for $n_s - 1$ and r during the last 60 e-folds depend on the shape of the potential beyond the very flat part of the plateau as the potential dips sharply towards zero. Consequently, the predictions are very similar to expectations for monomial potentials, such as $V(\phi) \sim m^2\phi^2$. However, because the potentials are plateau-like at large ϕ, these models exhibit the initial conditions problem described here and in Chapter 2.

Finally, the plateau-like class of hydrodynamic models are split into two cases when translated into scalar-fields and potentials. For $1.5 \leq \alpha \leq 2$, they correspond to large-field models and include Higgs [16] (with action expressed in Einstein frame). They exhibit the initial conditions and unlikeliness problems and require tuning one or more dimensionful parameters to satisfy cosmological constraints. For $\alpha > 2$, the potentials correspond to small-field plateau-potentials such as new inflation [8, 64] which exhibit the initial conditions and unlikeliness problems and require two or more *dimensionful* parameters V_{end} and ϕ_{end} to yield the correct spectrum of primordial density fluctuations and sufficient e-folds of inflation.

In sum, the model dependent analysis based on inflaton fields and potentials gives a somewhat different view of the landscape of scale-free inflationary models and their problems, but on the whole confirms and sharpens the results of the hydrodynamic analysis. From either point of view, the strictly scale-free $\alpha = 1$ case is the least problematic among all the models and all classes. The analysis based on scalar fields with scale-free potentials splits two of the hydrodynamic classes into two distinct subgroups through the conversion from N to ϕ as the independent variable. It further suggests a hierarchy from least to most problematic, where the least problematic and requiring the least dimensionful parameters is the strictly scale-free $\alpha = 1$ model followed by the the power-law like models with $\alpha < 1$ and intermediate class models. Unfortunately, the inflationary models favored by present data does not belong to either of these groups. The results also show that, in the plateau-like class, large-field models with $\alpha < 2$ require fewer dimensionful parameters than small-field models ($\alpha > 2$).

We note that the hydrodynamic unlikeliness problem decribed in this chapter is more general than the version identified in Chapter 2. In Chapter 2 we have shown specifically for small-field plateau-like models that inflation is exponentially less likely in a generic energy landscape than monomial potentials $V \sim \phi^n$. The results in the present chapter based on scale-freeness show that the *entire* plateau-like class is theoretically disfavored compared to the entire power-law-like class, whether small-field or large field inflation.

Among monomial inflationary potentials $V \sim \phi^n$, the only strictly or background-only scale-free example is the conformally invariant case, $n = 4$, corresponding to $\alpha = 1$, which we have shown is the least problematic.[2] Recovering other power-law potentials requires explicitly breaking scale-freeness while still respecting the inflationary conditions, criteria I and II. For example, by introducing two additional non-zero parameters β and γ as defined in Eq. (4.23), the equation-of-state parameter can be made to follow closely the equation-of state that can be obtained for $n = 4\beta$. Note that ϕ^2 requires non-negligible scale-free breaking in the sense that β is significantly less than one. Power law models with yet smaller powers, such as $\phi^{2/3}$, require even greater deviations from scale-freeness.

However, introducing this extra scale-freeness breaking degree of freedom could be a dangerous course. There already exists a spectrum of inflationary cases parameterized by α in the background scale-free limit. Having a spectrum of cases reduces the predictive power of the paradigm. Applying the same scale-free breaking degree of freedom, β, for all α further broadens the range of possibilities and increases the number of parameters. This reduces the predictability to the point where there can be more parameters than observational constraints. Furthermore, the breaking of scale-freeness only complicates the model without resolving any of the problems identified for the scale-free cases. Given that the universe seems so simple based on observations, it is problematic to consider cases with more parameters than the inflationary paradigm requires or the data can constrain.

Not everyone would agree with this assessment. In order to address the initial conditions problem described earlier in Chapter 2 and in this chapter, authors have introduced potentials with double-inflation, first a power-law-like phase and then a

[2]Here we correct the crude approximation made in [75] which led to the incorrect conclusion that ϕ^6 is the strictly scale-free solution.

plateau-like like phase [21, 78, 27]; or they have introduced an energy landscape with false vacuum inflation tunneling to a plateau [97]. In these cases, the deviation from scale-freeness is intentionally designed to occur for modes outside the Hubble horizon beyond the range of observational tests. From a theoretical perspective, the logic is odd: if the physics underlying inflation is not truly scale-free, why should the deviation from scale- freeness only show up on unobservably large scales? The only purpose is to evade the initial conditions problem while remaining consistent with observations. But the cost is too precious. As evidenced by the example of Ferrara et al. [27], this approach introduces enough new parameters and enough tuning that any outcome for $n_S - 1$ and r becomes possible, such that inflationary cosmology loses all predictive power.

4.5.2 Scale-free cyclic potentials

As explained in Appendix B, a generic form for the scalar-field potential energy density in the cyclic model can be cast in the form:

$$V(\sigma, s) = V(\sigma, 0)\left(1 + \frac{1}{2}\,\kappa\,\frac{V_{,\sigma\sigma}}{V(\sigma, 0)}s^2 + \mathcal{O}(s^3)\right), \tag{4.39}$$

where σ corresponds with the fluid component governing the background evolution described by ϵ_1 and s is the field representation of the fluid with equation-of-state parameter ϵ_2 that generates the isocurvature fluctuations before the bounce (that are converted to the nearly scale-invariant curvature perturbations during the bounce). The background evolution is along the σ direction with $s = 0$. The parameter κ is the ratio ϵ_2/ϵ_1 defined in Eq. (4.32), which relates the curvature of the potential energy density along the s direction to the curvature along the σ direction. The strictly scale-free case corresponds to $\kappa = 1$ such that $V_{,ss}(\sigma, s) = V_{,\sigma\sigma}(\sigma, 0)$ [17].

The Friedmann equations together with Eq. (4.28) and (4.29) can be used to construct the potential given the background equation of state $\epsilon_1(\mathcal{N})$:

$$\begin{aligned} V(\sigma,0) &= -M_{\rm Pl}^2\left(\epsilon_1(\mathcal{N}-1)\right)H^2(\mathcal{N}) \\ &= -3\,M_{\rm Pl}^2\,H_{\rm end}^2\left((\mathcal{N}+1)^{\alpha_1}-1\right) \\ &\quad\times\exp\left(-2\mathcal{N}+2\int_{\mathcal{N}}^{0}\frac{d\mathcal{N}}{3(\mathcal{N}+1)^{\alpha}-1}\right), \end{aligned} \tag{4.40}$$

where $\mathcal{N}$ can be replaced by the background scalar field σ using the relation

$$\begin{aligned} \frac{\sigma-\sigma_{\rm end}}{M_{\rm Pl}} &= \pm\int_{\mathcal{N}}^{0}\sqrt{2\epsilon_1}\,(\epsilon_1-1)^{-1}d\mathcal{N} \\ &= \pm\sqrt{6}\int_{\mathcal{N}}^{0}\frac{(\mathcal{N}+1)^{\alpha_1/2}}{3(\mathcal{N}+1)^{\alpha_1}-1}d\mathcal{N}. \end{aligned} \tag{4.41}$$

For example, for $\alpha_1=1$ we have

$$V(\sigma,0)\simeq -3\,M_{\rm Pl}^2\,H_{\rm end}^2\left(\sigma^2/M_{\rm Pl}^2-1\right)\exp\left(-2\sigma^2/M_{\rm Pl}^2\right). \tag{4.42}$$

Here we set without loss of generality $\sigma_{\rm end}=1$ and assumed $\sigma-\sigma_{\rm end}>0$ during the smoothing phase. For all $\alpha>0$, the potential $V(\sigma,0)$ takes the same generic form: a steep negative potential that reaches a minimum before σ approaches $\sigma_{\rm end}$, the standard shape potential proposed for ekpyrotic and cyclic scenarios. (This can be checked by computing the derivative of Eq. (4.40), $dV/d\mathcal{N}$ for different α and by observing from Eq. (4.41) that the transformation from $\mathcal{N}$ to σ, $\mathcal{N}(\sigma)$, is strictly monotonic.)

This means that the potential picture gives the same simple result as the model independent hydrodynamic analysis, namely that the scale-free cyclic theory has only a single class of models all requiring a single dimensionful parameter, $H_{\rm end}^2$, to yield the correct spectrum of primordial density fluctuations, $\delta\rho/\rho\sim10^{-5}$. Hence, both pictures lead to the conclusion that there is no unlikeliness problem and no extra parameters or fine-tuning problem can arise.

4.6 Discussion

In this chapter, our aim has been to study different cosmological scenarios in a model independent way that does not refer directly to fields or potentials. Using a hydrodynamic approach, we derived algebraic forms for the equation-of-state parameter consistent with the scale-free principle for both inflationary and cyclic theory. In this section we discuss both theoretical and observational implications of this work.

Let us first consider inflationary cosmology alone. We found that, based on our hydrodynamical analysis, inflationary scale-free models divide into three distinct classes and identified a range of related problems: an initial conditions problem for the plateau-like and intermediate class, and an unlikeliness problem and a fine-tuning problem for the plateau-like class. The spectrum becomes even more divided when we translate the three cases into scalar-field potentials. Hence, even limiting ourselves to scale-freeness, there is a diversity of inflationary models and predictions.

In applying the same hydrodynamic analysis to cyclic scenarios, we found cyclic theory allows only a single scale-free class of models and does not suffer from the initial conditions or unlikeliness-type problems identified for inflation. At the perturbative level, current versions of cyclic theory require a two-component fluid for the generation of primordial isocurvature fluctuations, which are then converted into density fluctuations. This added condition compared to inflation appears to have no disadvantage in a hydrodynamical treatment assuming scale-freeness: there were no more parameters, fine-tuning, or other kinds of constraints compared to the inflationary one-fluid mechanism. Remarkably, translating this single cyclic class into scalar-field potentials, we found the same simple result.

One might ask if the problems found for inflation that were not found for cyclic may be related to the fact that a single fluid was assumed in the first case but not the second. The answer is no. As we discussed above in Sec. 2, in scale-free scenarios the background is always described by a single fluid component and the presence of multiple components becomes relevant only at perturbative level. However, the inflationary problems arise at background level such that adding multiple fluid components makes (at best) no difference whatsoever. In fact, the situation for inflation is typically made worse. For example, there is a well-known two-component fluid version of inflationary theory, known as the curvaton model [72]. As in the cyclic model, the background evolution is governed by one fluid component, the inflaton, and the perturbations are controlled by another, the curvaton. Since the inflaton must satisfy the same conditions on the equation-of-state as in the single-fluid case, there is no change whatsoever in the problems encountered by introducing the curvaton. Since both fluids are capable of generating density perturbations, extra fine-tuning is required to regulate the interplay of the inflaton and curvaton in order that only the curvaton affects the evolution of perturbations. That is, a curvaton is not automatically the leading order contributor to the perturbations; the model must be adjusted to make it so. In particular the curvaton construction requires setting $\epsilon_1(N)$ for the inflaton different from $\epsilon_2(N)$ for the curvaton, which explicitly breaks background scale-freeness. This is qualitatively different from the cyclic case where two fluids are required to generate the leading order contribution to the density perturbations and $\epsilon_1(\mathcal{N})$ can be set equal to $\epsilon_2(\mathcal{N})$, preserving scale-freeness, as was done in Sec. 4.

Finally, we relate our theoretical findings to current observations, in particular to recent Planck satellite measurements [3]. We see that strictly scale-free versions of both cosmological scenarios are observationally disfavored. The strictly scale-free ϕ^4-chaotic inflation potential is observationally disfavored by more than 4σ as a result of constraints on n_S and r. The strictly scale-free cyclic model is consistent with current bounds on r but predicts $n_S - 1 \simeq -.01$, which is disfavored by 3σ. That means, consistency with current observational data requires some deviation from strict scale-freeness in both scenarios.

In the cyclic theory the observational value of $n_S - 1$ can be obtained simply by introducing a very weak breaking of scale-freeness at the perturbative level (β_2 slightly different from 1 or, equivalently, $|\kappa - 1| \ll 1$), while leaving the dominant fluid and the background strictly scale-free ($\beta_1 = 1$). In inflation, by contrast, the current

<table>
<tr><th>r</th><th>$n_S - 1$</th><th>unlikeliness problem</th><th>favored model</th></tr>
<tr><td rowspan="3">$\gtrsim 10^{-4}$</td><td rowspan="2">scale-free satisfying Eq. (4.25) with $|\beta - 1| \ll 1$</td><td>no, if $r \gtrsim 0.1$</td><td rowspan="2">scale-free inflation</td></tr>
<tr><td>yes, if $0.1 \gtrsim r \gtrsim 10^{-4}$ *</td></tr>
<tr><td>violating Eq. (4.25)</td><td colspan="2">?</td></tr>
<tr><td rowspan="2">$\lesssim 10^{-4}$</td><td rowspan="2">scale-free satisfying Eq. (4.36) with $|\kappa - 1| \ll 1$</td><td>no</td><td>scale-free cyclic theory</td></tr>
<tr><td colspan="2">?</td></tr>
</table>

Table 4.1: Testing scale-free primordial cosmology with measurements of the tensor-to-scalar ratio r and the tilt $n_s - 1$. See discussion in text. *Note that the results from our model-dependent analysis in Sec. 5 based on scalar fields and potentials further divide plateau-like models into two groups: $\alpha \leqslant 2$, which requires $r \gtrsim 0.004$; and, $2 < \alpha$, which requires $10^{-4} \lesssim r \lesssim 0.004$, where this latter group requires more dimensionful parameters and has a more severe unlikeliness problem.

observations favor scale-freeness only for plateau-like models, which suffer from the initial conditions and unlikeliness problems described above. The only power-law-like models that are not strongly disfavored require significant breaking of scale-freeness ($|\beta - 1| \sim \mathcal{O}(1)$).

What will future observations tell us about scale-free primordial cosmology? Scale-free inflation is already in serious jeopardy given what we know: there are the historic entropy [82, 34] and multiverse [92, 98] problems that apply to *all* inflationary models [90]. Hence, at best, we have these problems to overcome. However, future observations could make matters worse for scale-free inflation. We summarize all possible scenarios in Table 4.1.

An important *prediction* for scale-free inflation that stems from this work is that the tensor-to-scalar ratio r should exceed 0.0001, which is within conceivable experimental sensitivity. (Here, as throughout this chapter, we assume $c_s > 1/3$, as implied by current observations [5].) This bound arises because smaller r requires $\alpha > 3$, which, in turn, requires $n_S < 0.95$ in disagreement with current measurements of the spectral tilt. Note that the tensor-to-scalar ratio, r, does not depend on the energy scale of inflation since it precisely cancels from the ratio. Models with r far below 10^{-4}

either violate existing observational constraints (such as the limit on $n_S - 1$) and/or introduce extra parameters that strongly break scale-freeness. If none of the scale-free combinations of $(r, n_S - 1)$ is found observationally, scale-free inflation is ruled out. If one of these combinations is observed with $10^{-4} < r \lesssim 0.1$, then scale-free inflation is possible, but it is necessary to resolve the initial conditions and unlikeliness problems discussed here. If a combination is found with $r > 0.1$, scale-free inflation without either of these problems is possible (though there would remain the entropy and multiverse problems common to all inflationary models).

The current situation is that observations indicate $r < 0.1$. Hence, unless future B-mode measurements bring a surprise that overrules this result, the only possible scale-free inflationary models remaining encounter the initial conditions and unlikeliness problems discussed here.

Alternatively, future observations could find that the measured values of r and $n_S - 1$ yield no scale-free combination consistent with Eq. (4.25), or $r < 0.0001$. Either case would eliminate all scale-free inflationary models and force extra degrees of freedom that allow virtually any outcome for $n_S - 1$ and r, as exemplified by the scale-freeness violating model of Ferrara et al. [27]. In this case, inflationary cosmology loses all predictive power.

As for scale-free cyclic models, the situation is somewhat different. There is no multiverse problem and the initial conditions and unlikeliness problems found for inflation are evaded. Observationally, the strictly scale-free cyclic case ($\alpha = 1$) is disfavored because of the current constraints on the spectral tilt. A best fit to the tilt requires a small deviation from scale-freeness at the perturbative level, by setting β_2 (or, equivalently κ) slightly greater than 1 instead of equal to 1 precisely. The forthcoming measurements of r are crucial to scale-free cyclic models because all predict no observable tensor modes. Detection of primordial gravitational waves would eliminate the entire spectrum of models. On the other hand, if there is no detection and r is proven to be less than 0.0001 – the conditions that eliminates scale-free inflation – scale-free cyclic would fit perfectly.

In the cyclic models considered here, we have assumed an entropic mechanism with two fluids for generating curvature perturbations. At least in currently known examples in which this is achieved with two scalar fields, the models generate non-negligible $f_{\rm NL}$ or $g_{\rm NL}$ or both. Current observational limits are consistent with predictions without requiring any additional tuning of parameters [59], but future measurements could result in detection or tighter constraints. Although non-Gaussianity is not directly predicted by hydrodynamical analysis and is more model-dependent in cyclic models, future measurements could be useful in distinguishing inflation versus cyclic scenarios and the testing the hypothesis of scale-free primordial cosmology.

In sum, introducing the scale-free principle makes cosmological theories – both inflationary and cyclic – meaningfully predictive and allows for observational test. Both for scale-free inflationary and cyclic cosmology, we could identify all combinations of parameters $(r, n_S - 1)$ consistent with the theory. If such a combination is not measured, the theory is falsified. Most interestingly, forthcoming measurements are capable of testing and eliminating scale-free inflationary models, scale- free cyclic models, or both, as indicated by the "?" in Table 4.1. Eliminating both means relinquishing scale-freeness and having to settle for unpredictive theory, like [27], or seeking another type of cosmological theory that retains scale-freeness and predictive power.

Chapter 5

Scale-invariant ekpyrotic perturbations with negligible non-Gaussianity

Summary. In Chapter 5, we explore a new type of entropic mechanism for generating density perturbations in a contracting phase in which there are two scalar fields, but only one has a steep negative potential. This first field dominates the energy density and is the source of the ekpyrotic equation of state. The second field has a negligible potential, but its kinetic energy density is multiplied by a function of the first field with a a non-linear sigma-model type interaction. We show that exactly scale-invariant adiabatic perturbations can be produced continuously as modes leave the horizon for any equation of state. Alternatively, the spectral tilt can be adjusted to be slightly red, in accordance with current data. The corresponding background solutions are stable and the bispectrum of these perturbations vanishes, such that no non-Gaussianity is produced during the ekpyrotic phase. Hence, the only contribution to non-Gaussianity comes from the non-linearity of the conversion process during which entropic perturbations are turned into adiabatic ones, resulting in a non-Gaussianity parameter $f_{\rm NL} \sim \mathcal{O}(5)$, in accordance with recent data from Planck2013.

5.1 Introduction

As we have seen in Chapter 4, at background level and compared to inflationary solutions, cyclic/ekpyrotic models of the universe are remarkably simple – they do not suffer from an unlikeliness problem, neither do they produce a multiverse. Using the field picture, in these models, the ekpyrotic phase is generated by a scalar field, ϕ, rolling down a steep negative potential

$$V = -V_0 e^{-\sqrt{2\epsilon}\phi}, \quad \epsilon > 3, \tag{5.1}$$

where V_0 is a constant and ϵ is the equation-of-state parameter, as introduced in Chapter 4. We have also discussed in Chapter 4 that, if there is only a single field in the contracting phase, the growing-mode, adiabatic perturbations decay and cannot be the seed of structure in the post-bang universe [18]. The currently best-understood way around this problem is the entropic mechanism, where pre-bang isocurvature fluctuations are generated by adding a second ekpyrotic field, ϕ_2 [17, 55]. These isocurvature modes are then converted into density perturbations which source structure in the post-bang universe.

The simplest action describing the standard ekpyrotic mechanism reads

$$S = \int d^4x\sqrt{-g}\left(\frac{R}{2} - \partial_\mu\phi_1\partial^\mu\phi_1 - \frac{1}{2}\partial_\mu\phi_2\partial^\mu\phi_2 + V_1 e^{-c_1\phi_1} + V_2 e^{-c_2\phi_2}\right), \tag{5.2}$$

where V_1, V_2, c_1, c_2 are constants and the two fields have separate ekpyrotic potentials. Throughout this chapter we choose units with $8\pi G = 1$. The background evolution is determined by the linear combination of these potentials, or equivalently, after performing a rotation in field space, by the adiabatic field, σ, (defined to point tangentially along the background trajectory, with $\dot\sigma = (\dot\phi_1^{\,2} + \dot\phi_2^{\,2})^{1/2}$) while the evolution of perturbations is governed by the entropy field, s (which is, by definition, perpendicular to the σ-field). At the end of the ekpyrotic phase and before the bounce, the background trajectory bends and the isocurvature perturbations are converted into adiabatic ones.

However, it is well-known that these ekyprotic solutions for ϕ_1 and ϕ_2 are *unstable*, in that the σ direction runs along a ridge in the potential that is unstable to variations in the s direction. Also, to obtain nearly scale-invariant spectra requires a steep negative potential which results in non-negligible non-Gaussianity *during the ekpyrotic phase* that dominates the non-Gaussianity arising from the conversion process. The steepness of the potential and instability involve additional tuning of parameters and initial conditions such that, from a theoretical point of view, it would desirable to find an alternative approach that avoids them.

In Chapter 5, we explore a new type of entropic mechanism in which there are two scalar fields, as before, but only one has a steep negative potential, $V(\phi)$. This first field, ϕ, dominates the energy density and is the source of the ekpyrotic equation of state. The second field, χ, has a negligible potential, perhaps precisely zero potential, but its kinetic energy density is multiplied by a function of the first field, $\Omega^2(\phi)$, with a a non-linear sigma-model type interaction. A specific example of this model was introduced by [60] and [83] where both the potential and the non-trivial kinetic coupling are identical and of the form $e^{-\lambda\phi}$, where λ is a positive constant. This model admits stable scaling solutions with scale-invariant spectrum and, as shown by [28], the bispectrum of this model vanishes such that no non-Gaussianity is produced during the ekpyrotic phase. Since it involves less tuning of parameters, such a model is theoretically attractive. Furthermore, it fits well the Planck2013 bounds on non-Gaussianity. Hence, it is worthwhile studying how general these results are.

In this chapter, we show that these results are not only valid for this special case, but can be extended to an entire class of ekpyrotic models. We show that scale-invariant adiabatic perturbations can be produced continuously as modes leave the horizon for any ekpyrotic background equation of state. This has the additional advantage of reducing fine-tuning constraints. The corresponding background solutions are stable and the bispectrum of these perturbations vanishes, such that no non-

Gaussianity is produced during the ekpyrotic phase. Hence, the only contribution to non-Gaussianity comes from the non-linearity of the conversion process during which entropic perturbations are turned into adiabatic ones.

Chapter 5 is organized as follows. In Sec. 5.2 we introduce a generic action involving two fields, derive the background equations of motions and briefly discuss their properties. In Sec.5.3 we derive the equations of motion at first order in perturbation theory and show that for each background potential, $V(\phi)$, we can define a non-trivial field-space metric such that the spectrum of entropy perturbations, produced by the χ-field, is scale-invariant. We illustrate our finding on a simple, generic class of ekpyrotic models with equation-of-state parameter $\epsilon = \bar{\epsilon}\tau^p$, where $p > 0$. In Sec.5.4 we compute the bispectrum of the perturbations and, using our example from above, we show that, for models with constant spectral tilt, no non-Gaussianity is generated during the ekpyrotic phase, whether the spectrum is scale-invariant, independent of the value of the spectral tilt. We conclude in Sec.5.5 by summarizing our results and discussing directions for future research.

5.2 Setup

We shall consider the following action involving two scalar fields and a non-trivial phase-space metric,

$$S = \int d^4x\sqrt{-g}\left(\frac{R}{2} - \frac{1}{2}\Omega^2(\phi)\partial_\mu\chi\partial^\mu\chi - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right). \tag{5.3}$$

With a steep negative potential, $V(\phi)$, the first field, ϕ, dominates the energy density and is the source of the ekpyrotic equation of state. The second field, χ, has a negligible potential, perhaps precisely zero potential, but its kinetic energy density is multiplied by a function of the first field, $\Omega^2(\phi)$, with a a non-linear sigma-model type interaction. Varying the action with respect to the metric and the fields, the scalar-field and Friedmann equations read

$$
\begin{aligned}
H^2 = \frac{1}{3}\Big(\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\Omega^2(\phi)\dot{\chi}^2 \quad &+ \quad V(\phi)\Big), && (5.4)\\
\ddot{\phi} + 3H\dot{\phi} - \Omega\,\Omega_{,\phi}\,\dot{\chi}^2 + V_{,\phi} \quad &= \quad 0, && (5.5)\\
\ddot{\chi} + \left(3H + 2\frac{\dot{\Omega}}{\Omega}\right)\dot{\chi} \quad &= \quad 0, && (5.6)
\end{aligned}
$$

where $H = \dot{a}/a$ is the Hubble parameter, a the scale factor, and dot denotes differentiation with respect to physical time t.

The crucial ingredient of our model is the non-trivial field-space metric combined with negligible mass of the χ-field: It is immediately apparent that $\dot{\chi} = 0$ is a solution for Eq. (5.6) – the non-canonical kinetic coupling acts as an additional friction, "freezing" χ, if $3H+2\dot{\Omega}/\Omega > 0$. Having no or negligible potential, in field-space, the χ direction is automatically perpendicular to the ϕ direction. Hence, the $\dot{\chi} = 0$ solution naturally defines χ as the entropy field generating first-order isocurvature fluctuations while ϕ remains the adiabatic field controlling the background evolution. Moreover, by standard stability analysis, it can be easily shown that scale-invariant (Ω^2, V) solutions for ϕ and H (that we shall discuss below) together with the constant-χ solution are stable [43].

5.3 Scale-invariance

Next, we shall show that for any arbitrary ekpyrotic potential $V(\phi)$ there is a non-canonical kinetic coupling $\Omega^2(\phi)$ such that the corresponding spectrum of entropy perturbations is scale-invariant.

5.3.1 The general case

In order to derive the equations of motion at first order in perturbation theory, we vary the second-order action

$$S = \int d^4\sqrt{-g}\,\Omega^2(\phi)\partial^\mu\delta\chi\partial_\mu\delta\chi \tag{5.7}$$

with respect to the entropy-field perturbation $\delta\chi$. With the canonically normalized variable $v_s \equiv a\delta s$, where $\delta s \equiv \Omega\delta\chi$ is the gauge invariant entropy perturbation and a the scale factor, in Fourier-space and using conformal time, τ, the linearized equation of motion reads

$$v_s'' + \left(k^2 - \frac{\Omega''}{\Omega} - 2\frac{a'}{a}\frac{\Omega'}{\Omega} - \frac{a''}{a^2}\right) v_s = 0. \tag{5.8}$$

Here, k denotes the wavenumber of the fluctuation mode and we use the convention that τ is running from large negative to small negative values during contraction with τ_{end} marking the end of ekpyrosis and prime denotes derivative with respect to conformal time.

Assuming standard Bunch-Davies initial conditions, *i.e.*, $v_s \to e^{-ik\tau}/\sqrt{2k}$ for $k\tau \to -\infty$, the solution of Eq. (5.8) is

$$v_s = \sqrt{\frac{\pi}{4}(-\tau)}H_\nu^{(1)}(-k\tau), \tag{5.9}$$

where $H_\nu^{(1)}$ is a Hankel function of the first kind and ν is given by

$$\nu^2 = \frac{1}{4} + \tau^2\left(\frac{\Omega''}{\Omega} - 2\frac{a'}{a}\frac{\Omega'}{\Omega} - \frac{a''}{a^2}\right). \tag{5.10}$$

In the late-time/large-scale approximation v_s reduces to

$$v_s \propto k^{-\nu}(-\tau)^{1/2-\nu}. \tag{5.11}$$

From here, we see that the spectral index is given by

$$n_S - 1 = 3 - 2\nu \tag{5.12}$$

and using Eq. (5.10) the condition for scale-invariance becomes

$$\Omega''(\tau) + 2\,\mathcal{H}\,\Omega'(\tau) + \left(\mathcal{H}^2 + \mathcal{H}' - \frac{2}{\tau^2}\right)\Omega(\tau) = 0, \tag{5.13}$$

where we introduced the conformal Hubble parameter

$$\mathcal{H} = a'(\tau)/a(\tau). \tag{5.14}$$

Eq. (5.13) is a homogeneous second-order linear differential equation. Hence, for all continuous $\mathcal{H}$ and all $\tau < \tau_{\mathrm{end}}$ there exists a scale-invariant $\Omega(\tau)$. That means, to any $V(\phi)$ we can find, at least locally, a $\Omega(\phi)$ such that the associated spectrum of entropy perturbations is scale-invariant. A global solution exists if the solution $\phi(\tau)$ is a $\mathcal{C}^1$-diffeomorphism, *i.e.*, continuously differentiable and invertible.

5.3.2 An example

To illustrate the above analysis we consider ekpyrotic models with equation-of-state parameter

$$\epsilon \equiv \bar{\epsilon}(-\tau)^p, \quad 0 < p < 1, \tag{5.15}$$

where $\epsilon > 3, \bar{\epsilon} =$ constant. In [28], where the $p = 0$ case was considered, $\epsilon = \bar{\epsilon} =$ constant and it was assumed that the potentials have some bend or cut-off at τ_{end} to reduce ϵ below 3 and end the ekpyrotic phase. Here, for ease of comparison with the constant ϵ case, we will do the same, taking $\tau_{\mathrm{end}} = -1$ so that $\epsilon \to \bar{\epsilon} =$ constant at the end of the ekpyrotic phase (and the potential has a bend or cut-off, as before).

From the second Friedmann equation, $\epsilon = 1 - \mathcal{H}'/\mathcal{H}^2$, we first get

$$\mathcal{H}^{-1} = -\int_{\tau}^{\tau_{\rm end}} \epsilon - 1 \, {\rm d}\tau = \tau \left(\frac{\bar{\epsilon}}{p+1}(-\tau)^p - 1 \right), \tag{5.16}$$

$|\mathcal{H}^{-1}(\tau_{\rm end})| \ll |\mathcal{H}(\tau)^{-1}|$. Substituting the expression for $\mathcal{H}$ into Eq. (5.13) yields

$$\Omega(\tau) = (\bar{\epsilon} - 1)^{1/p} \left(\frac{\bar{\epsilon}}{p+1}(-\tau)^p - 1 \right)^{-1/p} \exp\left(-\frac{\bar{\epsilon}}{\bar{\epsilon} - 1} \right), \tag{5.17}$$

where we defined the constants of integration such that $\Omega(\tau)$ corresponds to the constant ϵ solution for $p \to 0$.

The expression for the potential is given by the first Friedmann equation,

$$\begin{aligned} V(\tau) &= -\frac{\mathcal{H}^2}{a^2}(\epsilon - 3) \\ &= -(p+1)^2(\bar{\epsilon} - p - 1)^{2/p} \frac{\bar{\epsilon}(-\tau)^p - 3}{(\bar{\epsilon}(-\tau)^p - p - 1)^{2+2/p}}, \end{aligned} \tag{5.18}$$

with

$$a(\tau) = a(\tau_{\rm end}) \exp\left(\int_{\tau}^{\tau_{\rm end}} \mathcal{H} \, {\rm d}\tau \right) = \frac{1}{(-\tau)} \left(\frac{\bar{\epsilon}(-\tau)^p - p - 1}{\bar{\epsilon} - p - 1} \right)^{1/p}, \tag{5.19}$$

from Eq. (5.16), and $a(\tau_{\rm end}) = 1$.

Next, we want to find an expression for V and Ω as a function of ϕ. Again, we use the second Friedmann equation and find

$$\begin{aligned} \phi(\tau) &= \int_{\tau}^{\tau_{\rm end}} {\rm d}\tau \sqrt{2\epsilon}\, \mathcal{H} \\ &= \sqrt{2}(p+1) \int_{\tau}^{\tau_{\rm end}} {\rm d}\tau \sqrt{\bar{\epsilon}(-\tau)^p}\, \tau^{-1} \left(\bar{\epsilon}(-\tau)^p - p - 1 \right)^{-1} \\ &= \frac{\sqrt{2}\sqrt{p+1}}{p} \left(\ln\left(\frac{\sqrt{\bar{\epsilon}} - \sqrt{p+1}}{\sqrt{\bar{\epsilon}} + \sqrt{p+1}} \right) - \ln\left(\frac{\sqrt{\bar{\epsilon}(-\tau)^p} - \sqrt{p+1}}{\sqrt{\bar{\epsilon}(-\tau)^p} + \sqrt{p+1}} \right) \right). \end{aligned} \tag{5.20}$$

Note that $\phi(\tau) \to \sqrt{2/\bar{\epsilon}}\ln(-\tau)$ for $p \to 0$, in agreement with the $\epsilon \equiv \bar{\epsilon}$ solution. Inverting Eq. (5.20),

$$\tau(\phi) = \left(\frac{p+1}{\bar{\epsilon}}\right)^{1/p} \left(\frac{\frac{\sqrt{\bar{\epsilon}}+\sqrt{p+1}}{\sqrt{\bar{\epsilon}}-\sqrt{p+1}} \exp\left(\frac{p\phi}{\sqrt{2(p+1)}}\right) + 1}{\frac{\sqrt{\bar{\epsilon}}+\sqrt{p+1}}{\sqrt{\bar{\epsilon}}-\sqrt{p+1}} \exp\left(\frac{p\phi}{\sqrt{2(p+1)}}\right) - 1} \right)^{2/p} . \tag{5.21}$$

Substituting into Eq. (5.17),

$$\Omega(\phi) = \exp\left(\frac{-\bar{\epsilon}}{\bar{\epsilon}-1}\right) (\bar{\epsilon}-1)^{1/p} \left(\frac{\epsilon(\phi)}{p+1} - 1\right)^{1/p}, \tag{5.22}$$

where

$$\epsilon(\phi) = (p+1) \left(\frac{(\sqrt{\bar{\epsilon}}+\sqrt{p+1}) \exp\left(\frac{p\phi}{\sqrt{2(p+1)}}\right) + \sqrt{\bar{\epsilon}} - \sqrt{p+1}}{(\sqrt{\bar{\epsilon}}+\sqrt{p+1}) \exp\left(\frac{p\phi}{\sqrt{2(p+1)}}\right) - \sqrt{\bar{\epsilon}} + \sqrt{p+1}} \right)^2 , \tag{5.23}$$

and

$$\Omega(\phi) \to \exp(-\sqrt{\bar{\epsilon}/2}\,\phi) \quad \text{for} \quad p \to 0. \tag{5.24}$$

Finally, we express V as a function of ϕ. Eq. (5.18) and (5.21) yield

$$V(\phi) = -(p+1)^2(\bar{\epsilon}-p-1)^{2/p} \frac{\epsilon(\phi)-3}{(\epsilon(\phi)-p-1)^{2+2/p}}, \tag{5.25}$$

and

$$V(\phi) \to -\frac{\bar{\epsilon}-3}{(\bar{\epsilon}-1)^2} \exp(-\sqrt{2\bar{\epsilon}}\,\phi) \quad \text{for} \quad p \to 0. \tag{5.26}$$

In particular, we see that for constant equation-of-state, Ω^2 and V need to be identical (up to a constant coefficient) to yield a scale-invariant spectrum.

5.4 Non-Gaussianity from the ekpyrotic phase

In the following we show that with scale-invariant (Ω, V) pairs, as introduced in the previous section, no non-Gaussianity is produced during the ekpyrotic phase in the sense that the bispectrum of the perturbations vanishes. Hence, the only contribution to non-Gaussianity comes from the conversion process which is the subdominant contribution in standard ekpyrotic/cyclic theory [57, 59]. We extend this result to (Ω, V) pairs with constant spectral tilt.

5.4.1 Non-Gaussianity from the ekpyrotic phase

The standard (phenomenological) parameterization of non-Gaussianities is by way of introducing a non- linear correction to a Gaussian perturbation, ζ_G,

$$\zeta(\mathbf{x}) = \zeta_G(\mathbf{x}) + \frac{3}{5} f_{\rm NL}^{\rm loc.} \left[\zeta_G^2(\mathbf{x}) - \langle \zeta_G^2(\mathbf{x})\rangle\right]. \tag{5.27}$$

This definition is local in real space and thus $f_{\rm NL}^{\rm loc.}$ is called non-Gaussianity of the *local* type.

More generally, the leading non-Gaussian correction is given by the 3-point correlation function, or its Fourier-equivalent, the bispectrum

$$\langle \zeta_{\mathbf{k}_1}\zeta_{\mathbf{k}_2}\zeta_{\mathbf{k}_3}\rangle = B_\zeta(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3). \tag{5.28}$$

For perturbations around an FRW background, the momentum dependence of the bispectrum simplifies considerably. Homogeneity, or translation invariance, means that the bispectrum must be proportional to a delta function of the sum of the momenta, $B_\zeta(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) \propto \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)$, *i.e.*, the sum of the momentum 3-vectors must form a closed triangle. Isotropy, or rotational invariance, dictates that the bispectrum only depends on the magnitudes of the momentum vectors, but not on their orientations,

$$B_\zeta(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = (2\pi)^3\delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)B_\zeta(k_1, k_2, k_3). \tag{5.29}$$

Different types of non-Gaussianities are described by different shapes of the closed triangle formed by their three momenta. For $f_{\rm NL}^{\rm loc.}$ the triangle is "squeezed," *i.e.*, $k_1 \ll k_2 \sim k_3$. Here we have ordered the momenta such that $k_1 \leq k_2 \leq k_3$. Higher-derivative interactions can lead to large non-Gaussianities. A key feature of such interactions is that they are suppressed when any individual mode is far outside the horizon. Hence, the bispectrum arising from higher-derivative interactions peaks when all three modes have wavelengths equal to the horizon size, *i.e.*, the triangle has a shape $k_1 = k_2 = k_3$, generating non-Gaussianity of the *equilateral* type, $f_{\rm NL}^{\rm equil.}$. A shape that is orthogonal to both the local and equilateral templates is called non-Gaussianity of the *orthogonal* type, $f_{\rm NL}^{\rm ortho.}$. This shape also arises in the presence of higher-derivative interactions.

In the absence of four-derivative or higher-order kinetic terms in the action as in Eq. (5.3), no non-Gaussianity of the equilateral or orthogonal type is produced [52]. Therefore, we will focus on the 3-point function of *local* shape.

During the ekpyrotic phase, non-Gaussianities of the local type can be generated in two ways, by second-order entropy perturbations, $\delta s^{(2)}$, (*intrinsic* non-Gaussianity) and by first-order entropy perturbations, $\delta s^{(1)}$, that source second-order curvature perturbations, $\zeta^{(2)}$, [56]. Here, we indicate the perturbative order by a superscript.

At second order and in co-moving gauge ($\delta\sigma^{(1)} = \delta\sigma^{(2)} = 0$), using the method from [84] for the perturbation in the fields ϕ and χ we find

$$\delta\phi^{(2)} = \frac{1}{2}\delta s^{(1)}\left(\frac{\Omega_{,\phi}}{\Omega}\delta s^{(1)} - \frac{\delta s^{(1)\prime}}{\phi'}\right), \tag{5.30}$$

$$\delta\chi^{(2)} = \Omega^{-1}\delta s^{(2)}. \tag{5.31}$$

Since the χ-field is massless and "frozen" at background level, there is no source term for the second-order entropy perturbation, $\delta s^{(2)}$, and, hence, no *intrinsic* non-Gaussianity is generated during the ekpyrotic phase; in agreement with [28].

In order to compute the non-Gaussianity from second-order curvature perturbations that are sourced by first-order entropy perturbations, we use the formula

$$\dot{\zeta} = \frac{2H\delta V}{\dot{\phi}^2 - 2\delta V}, \tag{5.32}$$

first derived in [58] and valid to all orders in perturbation theory. It was shown in [28] that this formula applies to actions with non-canonical kinetic term as in Eq. (5.3). With Eq. (5.30), we have

$$\zeta^{(2)\prime} = \frac{\mathcal{H}a^2 V_{,\phi}}{\phi'^2}\delta s^{(1)}\left(\frac{\Omega_{,\phi}}{\Omega}\delta s^{(1)} - \frac{\delta s^{(1)\prime}}{\phi'}\right). \tag{5.33}$$

In the late-time/large-scale approximation as in Eq. (5.11) the expression for $\zeta^{(2)\prime}$ reduces to

$$\zeta^{(2)\prime} = \frac{\mathcal{H}a^2 V_{,\phi}}{\phi'^3}\left(\frac{v_s}{a}\right)^2\left(\frac{\Omega_{,\tau}}{\Omega} - \frac{1-2\nu}{2\tau} + \mathcal{H}\right). \tag{5.34}$$

Approximating the ekpyrotic background equation of state by $\epsilon = \bar{\epsilon}(-\tau)^p$ and substituting our expressions for $\mathcal{H}$ from Eq. (5.16) and Ω from Eq. (5.17), we have

$$\zeta^{(2)\prime} = 0. \tag{5.35}$$

That means, scale-invariant $(\Omega, \mathcal{H})$ pairs generate no non-Gaussianity during the ekpyrotic phase. Furthermore, repeating the analysis with the same background equation of state but allowing for deviation from exact scale-invariance, $n_S - 1 \equiv -\delta$, from Eq. (5.10) we first get

$$\Omega \propto \tau^{-\delta/2}\left(\frac{\bar{\epsilon}(-\tau)^p}{p+1} - 1\right)^{-1/p}. \tag{5.36}$$

Then, substituting into Eq. (5.34) yields $\zeta^{(2)\prime} = 0$. That means, during the ekpyrotic phase no non-Gaussianity is generated even for this broader class of ekpyrotic models with non-zero tilt (*e.g.*, tilts in accord with cosmic microwave background measurements).

5.4.2 Non-Gaussianity from the conversion process

As we discussed in previous chapters, cosmic microwave background experiments always measure curvature perturbations, ζ, *i.e.*, local perturbations in the scale factor that is given by

$$ds^2 = -dt^2 + a^2(t)e^{2\zeta(t,x^i)}dx^i dx_i \tag{5.37}$$

Short of a complete theory of the bounce, it is assumed that the entropic perturbations generated during the ekpyrotic phase are converted into primordial curvature perturbations during the bounce. Depending on the concrete bounce model, the conversion process produces local non-Gaussianity, though, for an efficient conversion, the final bispectrum remains small [28]. Since our model describes the physics during the ekpyrotic phase, the contribution from non-Gaussianity from conversion process remains the same as in standard ekpyrotic theory. However, while in case of previously studied ekpyrotic models the contribution to non-Gaussianity from the conversion process is subdominant, in the theory presented here the only and dominant contribution comes from the conversion. Numerically, $f_{\text{NL}}^{\text{conversion}}$ is expected to be $\sim \mathcal{O}(5)$, as shown by [59].

5.5 Discussion

In this Chapter, we explored a new class of two-field ekpyrotic models with a massive ekpyrotic field governing the background evolution and a second field with no or negligible mass and non-canonical kinetic term.

The crucial ingredient of our model is the non-trivial coupling of the background field to the kinetic term of the second, massless field, which plays the role of the entropy field governing the perturbations. Remarkably, we have found that for each background equation of state there is a non-trivial kinetic coupling such that our model admits scale-invariant solutions at first-order in perturbation theory.

At second-order, we have found that the bispectrum of these perturbations vanishes, such that no non-Gaussianity is produced during the ekpyrotic phase in the sense that the bispectrum of the perturbations vanishes. Hence, the only contribution to non-Gaussinity comes from the non-linearity of the conversion process during which entropic perturbations are turned into adiabatic ones. This process is model-dependent, though, for an efficient conversion, the final bispectrum remains small, with $f_{\rm NL}^{\rm loc.} \sim \mathcal{O}(5)$, which is in accord with current cosmic microwave background measurements [5].

This analysis leaves many avenues for future work. A natural extension of our analysis is the calculation of the 4-point function and predictions for the trispectrum, particularly, since forthcoming data release from the Planck satellite will be able to tightly constrain the primordial trispectrum. Throughout our analysis, we worked with a minimal extension of the standard ekpyrotic theory, studying a two-field Lagrangian. It might be worthwhile to see if a multi-field generalization adds to our model in improving cyclic theories. Similarly, it would be interesting to explore the implications of a slowly-varying, time-dependent spectral index or to include a non-negligible mass for the entropy field.

Chapter 6

Conclusion and outlook

In this thesis we have studied inflationary and cyclic/ekpyrotic cosmologies for empirical and theoretical consistency and suggested new ways of improving them.

From the observational perspective, we have shown that, for inflation, in addition to the classic conceptual difficulties known for decades, new issues arise from WMAP, ACT, and Planck2013 data (Ch. 2). Most importantly, we have pointed out that the inflationary paradigm is, for the first time, disfavored by experiment in the sense that the simplest inflationary models are disfavored by data (Ch. 2.Sec. 4). By contrast, standard cyclic/ekpyrotic theory is in accord with current cosmic microwave background measurements.

From the theoretical perspective, we have discussed four different kinds of conceptual problems – parameter unpredictability (Ch. 4), initial conditions problem (Ch. 2 Sec. 3; Ch. 3 Sec. 3-4,6;), unlikeliness problem, and multiverse unpredictability problem. Here, we identified the *unlikeliness problem* as a new kind of conceptual difficulty, arising for the inflationary paradigm because the theoretically favored model class within the paradigm is disfavored by data while the theoretically disfavored class is at the same time favored by data (Ch. 2 Sec. 4). We found no unlikeliness problem for cyclic/ekpyrotic cosmologies (Ch. 4 Sec. 4).

Parameter unpredictability is the problem that can be eased rather straightforwardly. We have shown that this problem arises if a paradigm does not pose sufficiently many constraints for model building such that, as a result, any observation can be accommodated by varying parameters. Using a simple, observationally well-motivated guiding principle, scale-freeness, we could restore predictability for both inflationary and cyclic models in the sense that the number of degrees of freedom is smaller than the number of predictions (Ch. 4).

As a result of data, the inflationary *initial conditions problem* is worse than ever before. We have pointed out that, by lowering the energy-scale for the start of inflation to 10^{16} GeV, current microwave background data exacerbates the old initial conditions problem; huge (superhorizon) smooth and flat initial patches at the Planck density are required for inflation to begin, and such patches are exponentially improbable (Ch. 2 Sec. 3). For cyclic/ekpyrotic cosmologies, on the other hand, we could ease the initial instability issue by modifying the standard ekpyrotic mechanism, introducing a non-canonical kinetic term in the Lagrangian for a (massless) entropy field (Ch. 5 Sec. 1–2).

The most serious difficulty is the *inflationary multiverse-unpredictability* problem. In a multiverse, scanning over all bubbles, anything can happen and does happen an infinite amount of times (Ch. 2 Sec. 5; Ch. 3 Sec. 6). A measure – a highly non-trivial one, as we have learned from thirty years of an unsuccessful hunt for the right one – is required to set sensible predictions from the inflationary multiverse. Thus far, non has been found. Furthermore, the measure is defined over volumes of space that are forever causally disconnected from our observable universe, and, hence, the measure can never be properly tested observationally. Allowing the freedom to choose the measure, as suggested by [38], renders the underlying cosmology entirely unpredictive.

For cyclic/ekpyrotic cosmologies, no multiverse-unpredictability problem is known. However, the theory is to date incomplete as a full theory of the bounce is missing. It might also be desirable to find a simpler mechanism for the generation of primordial density perturbations than the current entropic mechanism (which entails creating first isocurvature perturbations that need be converted into density perturbations during the bounce).

We think that early universe cosmology has reached a critical stage. The old problems remain unresolved – we are in need of a meaningfully predictive mechanism that smooths and flattens the background while creating primordial density fluctuations seeding the structure of our universe with a nearly scale-invariant, adiabatic, and Gaussian spectrum. But today we are in an exceptionally privileged situation: observational data puts remarkably tight constraints on model-building and we have learned from inflation of the conceptual flaws that we need to avoid – and that's why it's a good time to search for an alternative paradigm.

Appendix A

List of quotes from [38]

[GKN 1] Recent experimental evidence, including the impressive measurements with the *Planck* satellite of the CMB temperature perturbation spectrum and the strong indication from the LHC that fundamental scalar fields such as the Higgs boson really exist, put inflationary cosmology on a stronger footing than ever. [p. 8]

[GKN 2] ISL further argue that the plateau shape of the low-energy part of the potential is not a consequence of inflation, but instead is chosen only to fit the *Planck* data, a situation which they describe as "trouble for the [inflationary] paradigm." It is of course true that inflation does not determine the shape of the potential, and indeed most inflationary theorists, including us, would consider a $m^2\phi^2$ or a $\lambda\phi^4$ potential to be a priori quite plausible for the low-energy part of the potential. [p. 4]

[GKN 3] We agree that if the observable inflation occurred on a plateau-like potential, eternal inflation seems very likely. It can occur either while the scalar field is at or near the top of the plateau, or in a metastable state that preceded the final stage of inflation. We also agree that this leads to the measure problem: in an infinite multiverse, we do not know how to define probabilities. [p. 5]

[GKN 4] ... since the measure problem is not fully solved, ISL are certainly justified in using their intuition to decide that eternal inflation seems unlikely to them. [p. 5]

[GKN 5] If the physical system consisted of a single scalar field ϕ which started with random initial conditions at the Planck scale, then ISL's argument would be persuasive. [p. 6]

[GKN 6] We agree with Ijjas, Steinhardt, and Loeb that important questions remain. A well-tested theory of physics at the Planck scale [*initial conditions*] remains elusive, as does a full understanding of the primordial singularity and of the conditions that preceded the final phase of inflation within our observable universe [*potential*]. Likewise, although significant progress has been made in recent years, a persuasive theory of probabilities in the multiverse has not yet been found [*measure problem*].[p. 8]

[GKN 7] anything can happen and will happen an infinite number of times [p. 5]

[GKN 8] While the proper-time cutoff measure seems intuitive, it has been found to lead to a gross inconsistency with experience, often called the "youngness problem." [p. 6]

[GKN 9] Pocket universes as old as $\Delta t = 14$ billion years, for example, are suppressed by a factor such as $e^{-3\Delta t/\tau_{\min}} \sim 10^{-10^{55}}$. [p. 6]

[GKN 10] ... the effective theory below the Planck scale may contain multiple – often separate – sectors), we find it very plausible that V (ϕ) is much more complicated than that, with multiple fields and many local minima. [p. 3]

[GKN 11] ... we wish to emphasize ... inflation with what we consider a realistic form of V (ϕ), containing many local minima and hence many metastable states, lead[ing] to multiple phases of inflation. [p. 4]

[GKN 12] But once we consider a potential energy function with more than one metastable local minimum – ... eternal inflation seems unavoidable. [p. 3]

[GKN 13] ... the measure problem: in an infinite multiverse, we do not know how to define probabilities ... We do not yet know what is the correct method of regularization, or even what physical principles might determine the correct answer. [p. 5]

[GKN 14] Unlike ISL, we would view the success or failure of such predictions [of conditions at the Planck scale] not as a test of the inflationary paradigm, but rather as part of our exploration of the measure problem. [p. 5–6]

[GKN 15] Anthropic selection effects can then make it plausible that we live in a pocket universe that evolved in this way. [p. 4]

[GKN 16] These generic predictions are consequences of simple inflationary models, ... confirmed to good precision, most recently with the *Planck* satellite. [p. 1]

[GKN 17] ... the relative probabilities of the two starting points for the last stage of inflation – plateau-like or outer wall – become the issue of complicated dynamics in the multiverse, and we are unable to compute which will dominate with our current knowledge and technology. [p. 7]

[GKN 18] the possibility that the final stage of inflation was preceded by a bubble nucleation event is at least one way that fine-tuning issues can be avoided. [p. 3]

[GKN 19] We also believe, as a matter of principle, that it is totally inappropriate to judge inflation on how well it fits with anybody's speculative ideas about Planck-scale physics – physics that is well beyond what is observationally tested. ... and we should similarly not even consider rejecting the inflationary paradigm because it is not yet part of a complete solution to the ultimate mystery of the origin of the universe. [p. 2–3]

[GKN 20] ... important advances have been made in recent years on topics such as eternal inflation, the multiverse and various proposals to define probabilities, and the possible role of anthropic selection effects. [p. 2]

Appendix B

Derivation of Eq. (4.31)

In order to derive the general hydrodynamic expression for the spectral tilt of primordial density fluctuations in cyclic theories, we follow the same procedure as for inflation [101]. Namely, we first solve for the perturbations, assuming the fluids can be represented as scalar fields with potentials, and then we convert the potential parameters in the expression derived for the tilt into hydrodynamic variables. To represent the two-component fluid we choose two fields, σ and s, where σ corresponds to the fluid component governing the background evolution described by equation of state ϵ_1 and s is the field representing the fluid that generates the isocurvature fluctuations before the bounce that are later converted to curvature perturbations during the bounce. The second fluid has equation-of-state parameter ϵ_2. The perturbation equation is given by

$$\delta\ddot{s} + 3H\delta\dot{s} + \left(\frac{k^2}{a^2} + V_{,ss}\right)\delta s = 0\,, \tag{B.1}$$

where dot denotes derivation with respect to physical time, k is the adiabatic mode.

For the cyclic potential we choose the form

$$V(\sigma, s) = V(\sigma, 0)\left(1 + \frac{1}{2}\kappa\frac{V_{,\sigma\sigma}}{V(\sigma, 0)}s^2 + \mathcal{O}(s^3)\right), \tag{B.2}$$

in agreement with [17, 59]. Here κ is the ratio of the equation-of-state parameters , $\kappa \equiv \epsilon_2/\epsilon_1$ as in Eq. (4.32). $V(\sigma, s)$ is constructed such that for $\kappa = 1$ it yields scale-free solutions; this corresponds to the case $V(\sigma, s)_{,ss} = V(\sigma, 0)_{,\sigma\sigma}$. Parameterizing the cyclic potential in this way is useful since the form naturally incorporates the entropic mechanism by dividing the potential into a first factor, that describes the background evolution along the σ direction and the second factor, which describes the direction of the isocurvature perturbations. Furthermore, this form encompasses all known simple cyclic potentials, such as models that can be written as sums of exponentials of independent fields.

After rescaling $\delta S \equiv a(\tau)\delta s$ and assuming standard Bunch-Davies initial conditions, $\delta s \to e^{-ik\tau}/(2k)^{3/2}$, the solution of Eq. (B.1) is the Hankel function

$$\delta s = \frac{\sqrt{-\pi\tau}}{2} H_\nu^{(1)}(-k\tau)\,, \tag{B.3}$$

with

$$\nu^2 = \frac{1}{4} + \eta^2 \left(\frac{a''}{a} - a^2 \kappa\, V_{,\sigma\sigma}\right)\,. \tag{B.4}$$

Here prime denotes derivative with respect to conformal time τ. On large scales, $k \ll aH$, $\delta s \sim k^{-\nu}$. This corresponds to a spectral tilt

$$n_S - 1 = 3 - 2\nu. \tag{B.5}$$

To express the tilt in hydrodynamical language, we follow [55] and rewrite H, a, and $V_{,\sigma\sigma}$ in terms of the background equation-of-state parameter $\epsilon_1(\mathcal{N})$,

$$\begin{aligned}
(a\,H)^{-1} &\simeq \epsilon_1 \tau \left(1 - \frac{1}{\epsilon_1} - \frac{\epsilon_{1,\mathcal{N}}}{\epsilon_1}\right), && \text{(B.6)}\\
\frac{a''}{a} &\simeq 2\,a^2 H^2 \left(1 - \frac{1}{2\,\epsilon_1}\right), && \text{(B.7)}\\
V_{,\sigma\sigma} &\simeq -H^2 \left(2\epsilon_1^2 - 6\epsilon_1 - \frac{5}{2}(\epsilon_1 - 1)\epsilon_{1,\mathcal{N}}\right). && \text{(B.8)}
\end{aligned}$$

After some algebra, we find

$$\nu^2 \simeq \frac{1}{4} + 2\left(\kappa + 3 \cdot \frac{1-2\kappa}{2\epsilon} + \frac{8-5\kappa}{4} \cdot \frac{\epsilon_{,\mathcal{N}}}{\epsilon}\right)\,, \tag{B.9}$$

where we neglected terms of order $1/\epsilon^2$. Finally, substituting into Eq. (B.5) yields the hydrodynamic expression for the spectral tilt as stated in Eq. (4.31).

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Selbständigkeitserklärung

Hiermit erkläre ich, die vorliegende Arbeit selbständig ohne fremde Hilfe verfasst und nur die angegebene Literatur und Hilfsmittel verwendet zu haben.

Princeton, 1. März 2014

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