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Effects of Explicit, Strategic Teacher Directed Instruction with iPad Application Practice on the Multiplication Fact Performance of 5^{th} Grade Students with Learning Disabilities

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Effects of Explicit, Strategic Teacher Directed Instruction with iPad Application Practice on the Multiplication Fact Performance of 5th Grade Students with Learning Disabilities

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Dedication

This dissertation is dedicated to my Lord, who has begun, led, and ended this journey.

But the upright man will be living by his faith; and if he goes back,
my soul will have no pleasure in him. (Hebrews 10:38)

This dissertation is also dedicated to my parents and husband who have provided me endless love and support all the time.

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Effects of Explicit, Strategic Teacher Directed Instruction with iPad Application Practice on the Multiplication Fact Performance of 5^{th} Grade Students with Learning Disabilities

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It is critical that students develop computational skills with basic facts to attain more advanced mathematical skills (e.g., algebra and fractions). A limited ability in accuracy and fluency with basic facts by students with learning disabilities (LD) who have Individualized Education Program (IEP) goals in mathematics can hinder their performance with more advanced mathematical skills. Thus, it is imperative to provide effective instruction to help students with LD to improve their basic fact skills. Explicit, strategic instruction has been highly recommended as an effective method for helping students with LD to improve basic fact skills. In addition, recent studies reported tablet computers such as iPads have potential for teaching basic fact skills.

Thus, the purpose of this study was to investigate the effects of explicit, strategic teacher-directed instruction with iPad application practice on the multiplication fact performance of 5^{th} grade students with LD. A single-case, multiple probe design across participants was applied for this study. Four 5^{th} grade students with LD who had IEP goals in mathematics received fifteen 1:1 intervention sessions in multiplication facts (×4s and ×8s). Digits correct per minute in daily probes, use of a doubling strategy in strategy usage tests, and perspectives of students toward the intervention were measured. Results showed that all students improved their performance with multiplication fact proficiency; one student achieved the mastery level while the three other students approached mastery. All students also maintained the intervention gains, two weeks following the intervention. Additional findings showed that students increased their use of the doubling strategy to solve facts and were able to answer facts automatically following the intervention. Social validity interviews revealed that the intervention was viewed favorably by all students by their expression of positive perspectives toward using the doubling strategy and an iPad application to practice.

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Chapter 1: Introduction

According to the Common Core State Standards (CCSS) for Mathematics (National Governors Association Center for Best Practices, Council of Chief State School Officers [NGA/CCSSO], 2010) and the National Mathematics Advisory Panel (NMAP, 2008), computational fluency is one of the essential skills to be taught in mathematics. It is well known that the mastery of basic facts and computational skills are critical to develop advanced mathematical skills (e.g., algebra and fractions; Ball et al., 2005; Miller & Mercer, 1993; NGA/CCSSO, 2010; NMAP, 2008; Woodward, 2006).

Students who lack fluency in basic facts tend to struggle when performing complex and sophisticated mathematical problem solving tasks due to cognitive demands (Woodward, 2006). It has also been reported that students who have strong basic fact skills tend to get higher scores on achievement tests measuring advanced mathematics skills (Skiba, Magnusson, Marston, & Erickson, 1986), maintain mathematics skills learned for a longer period of time (Singer-Dudek & Greer, 2005), feel less anxiety about mathematics (Cates & Rhymer, 2003) and are willing to engage in mathematics activities (Skinner, Pappas, & Davis, 2005). The limited fact skills of students with learning disabilities (LD) often hinders their development in higher mathematical skills (Hasselbring, Goin, & Bransford, 1988; Vaughn, Bos, & Schumm, 2007), and limited fact fluency ultimately could impact their entrance into higher education as well as getting jobs in the future. According to Adelman (2006) and the National Center for Education Statistics [NCES] (2003), regardless of family income levels, students who enrolled in advanced mathematics classes had a higher likelihood of attending and graduating from 4-year colleges than students who did not. Today, many colleges and universities require students to be prepared with more advanced high school mathematics to gain admission.

Mathematics competency is also important in the job market. People with superior mathematics skills have a higher likelihood of landing a job and making more money than students with less advanced mathematics skills (U.S. Department of Education, 1997). According to a recent study (Ritchie & Bates, 2013), early mathematics success is one of the impact factors that predict socioeconomic status in adulthood; participants who had better mathematics skills as children ended up having better jobs, higher salaries and better housing. Finally, computational skills are frequently used in and necessary for everyday life (e.g., shopping, recreation) and for independent living (Nordness, Haverkost, & Volberding, 2011). Thus, it is apparent that proficiency in basic mathematics facts is important for success not only in school but also beyond academics.

MATHEMATICAL COMPETENCE OF STUDENTS WITH LD

Unfortunately, students with LD experience a variety of difficulties in mathematics (Bryant, Bryant, & Hammill, 2000; Fuchs & Fuchs, 2001; Geary, 2003; Mancl et al., 2012). According to Geary (2004), approximately 5% to 8% of school-aged students have LD in mathematics. In addition, more than half of students with LD have mathematics goals on their Individualized Education Programs (IEPs; Lerner, 2003). It is a defining characteristic of LD that their mathematics skills levels are significantly below those of their peers without disabilities (Mabbott & Bisanz, 2008; Wagner et al., 2003).

Low performance in mathematics is evident early and persists throughout students' academic career and continues after they leave school (Mercer & Miller, 1992). Students with LD tend to experience slower improvement and are less likely to pass mathematics tests than their peers without disabilities (Cawley & Miller, 1989). As proof, the results of The National Assessment of Educational Progress [NAEP] (e.g., 2011a, 2011b, 2013) consistently report a significant gap in mathematics achievement between students with disabilities and their peers without disabilities, not only in the primary grades but also the secondary grades. Moreover, the mathematics performance of students with LD is likely to plateau at the fourth grade level and remains at approximately a fifth or sixth grade level when students leave high school (Cawley & Miller, 1989; Deshler et al., 2001; McLeod & Armstrong, 1982; Warner, Schumaker, Alley, & Deshler, 1980). Some researchers paint an even more dismal picture. For example, Calhoon and colleagues (2007) reported high school students with LD tend to perform at approximately the third grade level of mathematics. According to Bottge et al. (2010), students with LD tend to have difficulties in a wide range of mathematics areas; they have challenges to obtain both basic skills (e.g., computation) and higher order skills (e.g., problem solving). Especially, deficiencies in basic mathematics facts and computation fluency are known as distinct characteristics of students with LD (Geary, 2004, 2005).

MATHEMATICAL COMPETENCE OF STUDENTS WITH LD ON MULTIPLICATION FACTS

According to NCTM standards (2006) and the CCSS for Mathematics (NGA/CCSSO, 2010), computational fluency is emphasized in mathematics curriculum for K-8th grades. NCTM (2000) defined computational fluency as "having efficient and accurate methods for computing” (p. 152). Students should develop fluent computation skills with both whole and rational numbers by the end of the eighth grade (NCTM, 2006; NMAP, 2008). However, it has been reported consistently that students with LD are likely to have difficulties with basic mathematics facts and computational fluency (Cawley & Miller, 1989; Calhoon et al., 2007; Geary, 2004; Gersten, Jordan, & Flojo, 2005). Students with LD often have difficulties with proficient fact retrieval, speed of processing, and use of effective strategies (Geary & Hoard, 2005; Rivera, 1997). Also, limited working memory abilities have been reported among for students with LD (Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007; Swanson & Sachse-Lee, 2001). Due to possible working memory deficits, students with LD may have difficulties storing facts in their memory and recalling them when solving mathematics problems (Gersten et al., 2009; Hallahan et al., 2005). In addition, students with LD also often have problems in effective, mature strategy acquisition and appropriate use of the strategy (Geary, 1994, 2004) because of their deficits in memory and cognitive and metacognitive abilities (Geary, 1990; Kroesbergen & Van Luit, 2002b, 2003). The deficits also often impede the ability of students with LD to generalize the skills and knowledge they learned and apply concepts they know to new situations (Kroesbergen & Van Luit, 2003).

Multiplication is one of the essential topics in the number and operation domain for upper elementary grades (Mabbott & Bisanz, 2008; NCTM, 2006; NGA/CCSSO, 2010; NMAP, 2008). Multiplication is typically introduced in the 3rd grade, and students are expected to possess fluent multiplication computation skills with whole numbers by the end of the 5th grade (NGA/CCSSO, 2010; NMAP, 2008). According to Stein et al.

(2006), competence in basic multiplication facts is one of the critical skills for success in school. However, many students with LD often struggle with multiplication; they have difficulties mastering multiplication facts and developing computational fluency when compared to the performance of their peers without disabilities (Mabbott & Bisanz, 2008; Mazzocco, Devlin, & McKenny, 2008; Rotem & Henik, 2013). Students with LD often exhibit slow speeds of fact retrieval (Swanson & Jerman, 2006; Zentall, 1990) and inaccurately solve multiplication problems (Mazzocco et al., 2008). According to a recent study (Rotem & Henik, 2013), the performance of 6th and 8th grade students with LD on multiplication is similar to that of 2nd grade students without disabilities. In addition, Koresbergen and Van Luit (2002b) reported that some students with LD have difficulty understanding what the multiplication sign symbol (×) means. For example, they may think the "×" symbol has the same meaning as the addition symbol (+) (Kroesbergen & Van Luit, 2002b). In addition, according to Sherin and Fuson (2005), children typically use more mature strategies as they get older for multiplication, but it was found that students with LD tend to use immature strategies over time (Geary, Hamson, & Hoard, 2000); for example, students with LD tend to continue using counting strategies (e.g., finger counting) for addition, which are less effective for multiplication (Koscinski & Gast, 1993b). Thus, it has been recommended to teach students with LD mature, efficient strategies for long-term retention and direct retrieval of multiplication facts (Wood & Frank, 2000). The poor multiplication skills of students with LD often blocks successful development of higher order mathematics skills (e.g., fractions, algebra; Lombardo &

Drabman, 1985). Therefore, it is imperative to teach multiplication fact skills to students with LD.

EXPLICIT, STRATEGIC TEACHER-DIRECTED INSTRUCTION

Given the importance of teaching basic mathematics skills to students with LD, it is important to provide effective mathematics instruction based on empirically based strategies and techniques (Geary, 1994). Over the past decades, there has been an effort to investigate effective instructional approaches for teaching mathematics for students with LD. Two instructional approaches, explicit instruction and strategy instruction, have been consistently recommended as effective instructional methods for teaching mathematics for students with LD (Mancl et al., 2012).

Explicit and systematic instruction paired with various scaffolds and supports has been highly recommended as one of the most effective methods for teaching academic skills (e.g., reading, writing, mathematics) for students with LD (Gersten et al., 2009; Kroesbergen & Van Luit, 2003; NMAP, 2008; Swanson, Hoskyn, & Lee 1999; Vaughn, Gertsen, & Chard, 2000). Research consistently has reported that explicit instruction is effective to help students with LD improve their performance on basic facts, computation and problem solving skills; the instruction can also help them to transfer skills and knowledge they learned to new situations (Kroesbergen & Van Luit, 2003; NMAP, 2008). Explicit instruction includes instructional components such as clearly stating goals and expectations, providing a cumulative review of previously learned information, activating prerequisite skills/knowledge, teaching logically sequenced skills, following a carefully planned sequence for instruction (e.g., modeling, guided practice and independent practice), using multiple examples, providing students multiple opportunities to respond, providing immediate corrective feedback to students, teaching at an appropriate pace, and monitoring student progress (Archer & Hughes, 2010; Jayanthi, Gersten, & Baker, 2008; Swanson, 2001).

Research also has reported that strategy instruction can be an appropriate and effective instructional approach to teach students with LD who struggle with acquiring and using effective strategies (Swanson, 2001). Many students with LD have difficulties with developing mature, effective strategies naturally or often select immature strategies (Geary, 1993; Goldman, Pellegrino, & Mertz, 1988). Strategy instruction can help them understand and acquire mature, effective strategies that good learners use (Luke, 2006; Swanson et al., 1999). Learning to use strategies could reduce students' burden of memorizing difficult basic facts (Erenberg, 1995). Mathematics professionals also have suggested that strategies can be helpful for long-term retention and direct retrieval of facts (Isaac & Carroll, 1999) and could be effective for solving extended facts (e.g., 4×3, extended fact 40×3; Woodward, 2006). Through strategy instruction, students learn about strategy selection, strategy implementation, and rationale for using the strategy; students then must practice the strategy systematically and evaluate their use of the strategy (Lovett et al., 1994; Swanson, 1999). Strategy instruction often includes using think-aloud models and verbalizing the steps of the strategies (Luke, 2006).

Explicit instruction and strategy instruction have both common and distinct features. Both instructional approaches have numerous similar components (e.g., multiple opportunities to ask and answer, multiple examples, sufficient practice, small group format; Bryant, Hartman, & Kim, 2003; Swanson, 1999). However, explicit instruction primarily emphasizes teaching specific skills while strategy instruction primarily emphasizes processes or usage of general rules (Montague, 2008; Swanson, 1999). Interestingly, in the results of three meta-analyses, Swanson and colleagues (1998a, 1999, 2001) found that when explicit instruction (or direct instruction) and strategy instruction were combined, their effect was higher than other instructional approaches for teaching students with LD. Thus, it is recommended that teachers use a combination of explicit and strategic instruction when teaching students with LD (Swanson, 2001). Moreover, not only explicit, strategic instruction, but also the use of technology is recommended as a possible effective method for teaching mathematics to students with LD (Bouck & Flanagan, 2009).

USE OF TABLET COMPUTERS IN MATHEMATICS INSTRUCTION

Technology has been recommended as a viable instructional method for teaching mathematics to students with LD (Bouck & Flanagan, 2009; Seo & Bryant, 2009). According to Bouck and Flanagan (2009), technology can successfully support these students by providing more opportunities to access mathematics ideas and experience more success. NCTM (2000) and NMAP (2008) have advocated the use of technology (e.g., computer software) in mathematics instruction. The use of technology can support students with LD to enhance their mathematics performance and also increase their motivation for learning (Bender, 2001; Okolo, 1992). Moreover, the use of technology in teaching can enhance students' other technology skills, which is essential for their future lives (NCTM, 2000). Computer-based instruction (CBI) is often recommended for mathematics instruction for students with LD (Bouck & Flanagan, 2009; NMAP, 2008; Vaughn & Bos, 2009).

CBI is defined as an instructional method, using computer software that provides instructional content for teaching knowledge and skills to foster academic achievement; CBI can be used separately or with teacher-directed instruction (TDI) as a supplementary tool (Okolo, Bahr, & Rieth, 1993; Ulman, 2005). Previous research has reported that CBI can be a useful supplementary instructional method for students with disabilities (Hughes & Maccini, 1996; Mastropieri, Scruggs, & Shiah, 1991); thus, in recent decades, CBI has been adapted and utilized for teaching students with disabilities (Fitzgerald, Koury, & Mitchem, 2008; Okolo et al., 1993). It has been found that CBI can be used effectively for supporting students with LD to compensate for their challenges and enhance their learning in a variety of academic areas such as reading, writing, and mathematics (e.g., Higgins & Raskind, 2005; MacArthur, 1998; McDermott & Watkins, 1983). CBI can also be useful to increase students' motivation (Okolo, 1992) and time and attention on tasks (Fisher, 1983; Okolo et al., 1993). CBI allows for providing adapted and individualized instruction for students with disabilities based on their individual needs. For example, CBI allows teachers to set a variety of goals for individual students, adjust difficulty levels and learning pace, provide more drill and practice opportunities, provide immediate feedback, and record student's progress consistently, which is often difficult for teachers to accomplish during whole class instruction (Grimes, 1981; Poplin, 1995; Hughes & Maccini, 1996; Xin, 1999). Due to these beneficial features, it has been recommended to use CBI for teaching mathematics to students with LD, especially, to support them to improve their basic mathematics skills including addition, subtraction, multiplication and division (Seo & Bryant, 2009).

As technology has evolved, a new type of computer, a tablet computer (e.g., iPad), has recently gained popularity in the special education field. Tablet computers are typically portable tablet-sized devices with touch-screen displays and Internet access features. Moreover, tablet computers could be useful learning tools for students with disabilities due to their various beneficial features (e.g., the availability of downloadable inexpensive apps, touch-screen features that allow students with disabilities to use a device without having to operate a mouse or a touchpad) (Nirvi, 2011). In particular, the use of iPads has increased in a surprisingly short time, even when there has been no empirical research on the effect of tablet computers (Nirvi, 2011). In a review of CBI in mathematics instruction for students with LD, it was found that in recent technology-based studies (e.g., investigating the effects of using tablet computers) for teaching mathematics to students with LD promising effects of integrating tablet computers into mathematics instruction (Ok, 2012). Thus, tablet computers and applications have potential for teaching basic facts and computational skills (Banister, 2010).

THEORETICAL FRAMEWORK

Operant Conditioning Theory

B. F. Skinner, a well-known behaviorist, created the operant conditioning theory, which is based on the idea that learning is observable changes in a learner's behaviors and these changes occur via stimuli in the environment (Bodnar & Lusk, 1979; Bos & Vaughn, 2001). The operant conditioning theory suggests that behaviors can be changed through the manipulation of not only antecedents but also consequences (Wolery, Bailey, & Sugain, 1988); Skinner found the use of consequences such as reinforcement and punishment could increase desirable behaviors or decrease undesirable behaviors. In addition, repetition and consistency are also important to gain behaviors desired as well as to increase the speed of learning (Gurganusm, 2007). The theory primarily advocates teacher-directed instruction (e.g., lectures, tutorials, drills, demonstration) because it is believed that students learn best when instruction is directly focused on the content to be taught (Gurganusm, 2007).

Therefore, direct or explicit instruction is based on operant conditioning theory (Magliaro, Lockee, & Burton, 2005; O'Shea, O'Shea, & Algozzine, 1998). Direct instruction is designed to help students develop knowledge and skills in the goal-focused and teacher-controlled environment (Ryder, Burton, & Silberg, 2006). In this type of instruction, teachers play a significant role in the effective learning of students by providing stimuli and reinforcements in a systematically structured manner in order to increase desirable behaviors (e.g., targeted skills) (Gurganusm, 2007; Silva, 2004). Teachers provide explicit modeling of skills or knowledge, guided practice, and independent practice opportunities to students until they successfully master and generalize the targeted skills or knowledge (Silva, 2004). Teachers teach pre-requisite skills, break down the skills or knowledge into small chunks, check students' understanding continuously, and provide immediate and correct feedback (Harasim, 2011). Moreover, the drill and practice type of CBI is based on the operant conditioning theoretical framework; CBI is designed and controlled by programmers, not learners, even though some individual customization is allowed (Harasim, 2011). In addition, drill and practice CBI for mathematics is primarily designed for the mastery and generalization of specific skills or knowledge and CBI provides immediate and corrective feedback to learners.

Information Processing Theory

Information processing theory (e.g., Simon, 1989; Sternberg, 2000), one of the primary cognitive theories, emphasizes how people receive, transform, reduce, elaborate, store, retrieve and use stimuli from the environment (Swanson, 1987). Cognitive theorists (e.g., Weiner, B., Sweller, J., Mayer, R., Piaget, J.) insisted that there are more factors, which can have an influence on the functional relationship between stimulus and response (Winn & Syner, 1996). Cognitive theorists place an emphasis on the understanding of the cognitive process of learners for effective learning (Harasim, 2011). According to cognitivists, learning is changes of the learner's mental structure and processes and the internal changes of learners can be observed in changes in their behaviors (Silva, 2004). They believe not only external stimuli learners received stimuli from the environment, but also their thoughts impact the changes in their behaviors (Pham, 2011).

The cognitivists described the following steps of storing and retrieving information to learn: (a) people receive information through stimuli from the environment; people can hold all of the information they perceived for approximately 1 second (b) the received information can be stored in short-term memory; without the use of strategies to remember the information, people will lose the information in 14 seconds,

(c) information in short-term memory can be transferred to or stored in long-term memory until it is needed, and (d) people retrieve information from their memory whenever the information is required; ease of recalling information stored depends on how the information is stored in their long-term memory (Bos & Vaughn, 2002; Silva, 2004).

From cognitivists' perspectives, learning strategies is important for effective learning; strategies can help learners to store information in their long-term memory meaningfully (Silva, 2004). Strategy instruction emphasizes instructing students how to learn rather than teaching specific skills or knowledge (Alley & Deshler, 1979). The ultimate goal of teaching strategies is to support students to become active and independent learners, so they can apply the skills learned to various situations (O'Shea et al., 1998). In addition, according to Montague and Dietz (2009), strategy instruction is rooted in both behavioral and cognitive theory. Therefore, operant condition theory and information processing theory are theoretical foundations of explicit, strategic TDI with added iPad application practice (See Figure 1.1).

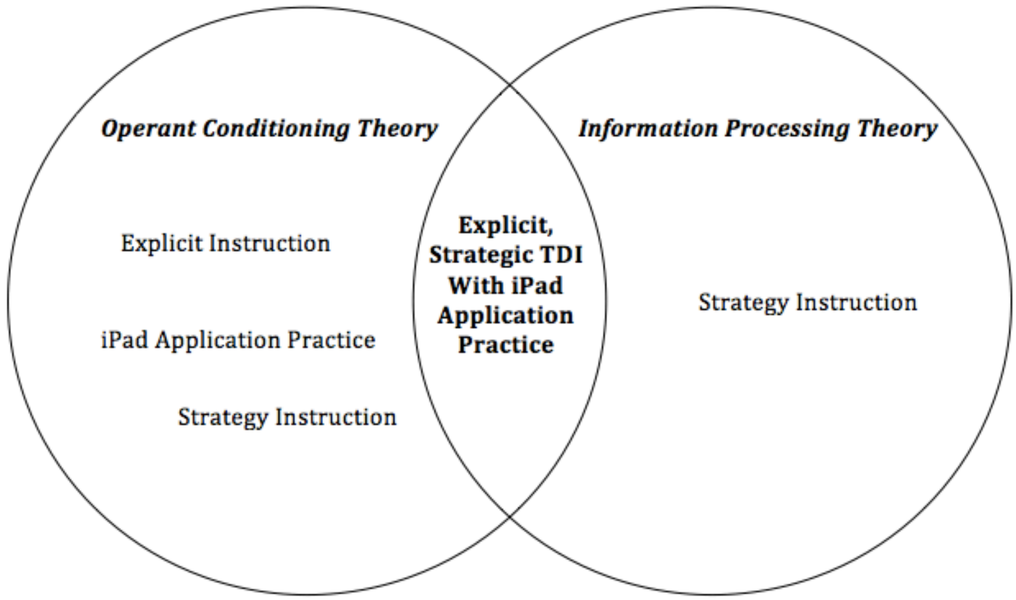


Figure 1. 1: Theoretical framework

STATEMENT OF PROBLEM

The Individuals with Disabilities Education Act (IDEA, 2004) determined that all students must have the opportunity to access the core curriculum. According to a report by the U.S. Department of Education (2012b), there are approximately 2.4 million students with LD in the United States. Students with LD receive over 60% of their instruction in general education classrooms (U.S. Department of Education, 2012a), and they sometimes receive “pull out” services for more intensive instruction. However, researchers have demonstrated that typical mathematics instruction in the general education classroom may be insufficient to help these students meet the instructional demands faced in the general education classroom (Cawley, Parmar, Yan, & Miller, 1998; Scruggs & Mastropieri, 1996). Many students with LD require empirically validated effective alternative instructional methods to help them learn (Gersten et al., 2009). Given the importance of teaching multiplication fact skills for students with LD, it is imperative to provide effective mathematics instruction based on empirical-based strategies and techniques to improve their multiplication fact skills. Without effective intervention, students with LD will continue to experience mathematical challenges and frustration (Mercer & Miller, 1992).

Howell and colleagues (1987) found that combined instruction that includes both TDI and CBI is more effective to teach mathematics for students with LD than CBI only. However, there has been a lack of research investigating the effect of the combined instruction for teaching mathematics to students with LD. Moreover, research is required to investigate the effects of newly emerging tablet computers and applications on mathematics instruction for students with LD. Therefore, it is important to examine the effect of explicit, strategic TDI with iPad application practice when teaching multiplication number facts for students with LD.

PURPOSE OF THE RESEARCH

Based on the need to conduct more research on combining TDI and CBI, this study was designed to investigate the effect of explicit, strategic TDI with iPad application practice on the performance in multiplication facts (i.e., ×4s and ×8s) performance of 5^{th} grade students with LD, who have mathematics IEP goals.

RESEARCH QUESTIONS

The following research questions guided this study:

1. What is the effect of explicit, strategic TDI with iPad application practice on the fluency of 5th grade students with LD, who have mathematics IEP goals, on single-digit multiplication number facts, ×4s and ×8s?
2. What is the effect of explicit, strategic TDI with iPad application practice on the use of a doubling strategy for solving single-digit multiplication number facts, ×4s and ×8s, among 5th grade students with LD, who have mathematics IEP goals?
3. How do 5th grade students with LD, who have mathematics IEP goals, maintain their fluency in single-digit multiplication number facts, ×4s and ×8s, 2 weeks following explicit, strategic TDI with iPad application practice?
4. What are the perspectives of 5th grade students with LD, who have mathematics IEP goals, toward explicit, strategic TDI with iPad application practice on learning multiplication facts, ×4s and ×8s?

Chapter 2: Review of Literature

RATIONALE

Mathematics competence is essential for all students, not only for academic success but also in daily their lives and future workplace opportunities (Bouck & Flangan, 2009; Nordness et al., 2011). Unfortunately, it is estimated that approximately 5-8% of school-age students have difficulties with mathematics (Geary, 2004). According to the National Assessment of Educational Progress (National Center for Education Statistics [NCES], 2013) in mathematics, 17% of 4th graders and 26% of 8th graders in the United States performed below the basic standard. The average scores in mathematics of 4th and 8th graders with disabilities were also significantly lower than their peers without disabilities. The gap was greater in 8th grade compared to 4th grade; their mathematics difficulties are likely to become greater as they get older (NCES, 2013).

It is well known that the majority of students with LD struggle with a variety of mathematics areas, in both basic skills (e.g., computation) and higher order skills (e.g., problem solving; Bottge et al., 2010). Research has consistently reported that students with LD experience greater difficulties compared to their peers without disabilities (Cawley, Parmer, Yan, & Miller, 1998; Mabbott & Bisanz, 2008; Wagner et al., 2003) and the gap in mathematics performance tends to be wider as they get older (Geary, 2010). The mathematics difficulties of students with LD tend to be persistent from primary through the secondary grades, and beyond (Judge & Watson, 2011; Strawser & Miller, 2001; Rotem & Henik, 2013). For example, a 6-year longitudinal study (Judge & Watson, 2011) reported that the low mathematics performance of students with LD was evident at the beginning of kindergarten and students gained less mathematical achievement growth than their peers without disabilities over a 6-year period, from kindergarten level through 5^{th} grade. Cawley and Miller (1989) noted that students with LD are likely to achieve only about 1 year of mathematics progress for every 2 years of schooling. Many 3^{rd} or 5^{th} grade students with LD often perform at a 1^{st} grade level of mathematics (Hallahan et al., 2005) and their mathematics performance tends to plateau at the 4^{th} grade level and remain at an approximately 5^{th} or 6^{th} grade level when leaving high school (Cawley & Miller, 1989). Similarly, a recent study by Rotem and Henik (2013) also found that the mathematics performance of 6^{th} and 8^{th} grade students with LD on multiplication is similar to that of 2^{nd} grade students without disabilities. As students get older, the achievement gap widens; between 14% and 27% of secondary students with LD remain more than two standard deviations below the average mathematics performance of their peers without disabilities (Wagner, Newman, Cameto, Levine, & Garza, 2006). The mathematics achievement level of students with LD in 12^{th} grade is similar to that of 5^{th} grade students without disabilities (Cawley & Miller, 1989).

Research has reported various potential causes of mathematical difficulties, such as deficits in the cognitive and meta-cognitive process, memory, acquisition, mastery, motivation, and vocabulary, and strategy acquisition (Geary, 2004). In addition, Carnine (1997) suggested that ineffective instruction that does not fit student needs and characteristics also could yield mathematics difficulties. Students with LD need special attention, extra help, and adapted mathematics interventions based on their needs (Kroesbergen & Van Luit, 2003; Maccini, Mulcahy, & Wilson, 2007). However, traditional instruction is often insufficient to support students with LD and to help them overcome the instructional challenges they face (Scheuermann, Deshler, & Schumaker, 2009; Scruggs & Mastropieri, 1996). Without effective instruction, students with LD will continue to experience mathematics difficulties and failure (Mercer & Miller, 1992). The challenges students with LD face with traditional instruction highlights the need for effective alternative, evidence-based instructional methods to help them develop mathematical skills and concepts (Gersten et al., 2009). No Child Left Behind (NCLB, 2001) requires that all students be assessed annually through a statewide assessment to prove adequate mathematics achievement progress (Bouck & Flanagan, 2009). For students with LD to be successful in these assessments, they require effective mathematics instruction throughout their schooling.

MATHEMATICS COMPETENCE OF STUDENTS WITH LD

Number Sense

According to NMAP (2008), number sense involves "an ability to immediately identify the numerical value associated with small quantities, a facility with basic counting skills, and a proficiency in approximating the magnitudes of small numbers of objects and simple numerical operations" (p. 27). Students typically develop number sense before kindergarten (Hallhan et al., 2005; Kibanoff, Levine, Huttenlocher, Vasilyeva, & Hedges, 2006; NCTM, 2008).

Number sense is a critical foundation for learning mathematics concepts and skills (Jordan, Kaplan, Locuniak & Ramineni, 2007) and it is a significantly reliable predictor of mathematics performance in later grades (Jordan, Glutting, & Ramineni, 2010; Jordan, Kaplan, Ramineni, & Locuniak, 2009). However, many young students who are at risk for LD have difficulties developing number sense; they often struggle with understanding these basic concepts and conceptualizing mathematics (Geary et al., 2009; Gersten et al., 2005; Mazzocco, Feigenson, & Halberda, 2011; Witzel, Ferguson, & Brown, 2007). According to Desoete and Grégoire (2006), average achieving students without disabilities tend to be already equipped with high number sense (e.g., number sequence, cardinality) in 1st grade, but students with LD tend to still lack number sense even in 3rd grade. Thus, students with LD often work on computation or problem-solving tasks with a lack of conceptual understanding of how and why mathematics works (Horowitz, n.d.). It is well known that number sense is essential for developing mathematics fact skills and computational fluency; a lack of number sense by students with LD often hinders developing adequate mathematics fact and computation skills (Locuniak & Jordan, 2008).

Computation Skills

After developing number sense, students typically learn basic mathematical operations including addition, subtraction, multiplication, and division (Kroesbergen & Van Luit, 2003). Mastery of these basic arithmetic skills plays an important role in developing a higher level of mathematics skills (Ball et al., 2005; NMAP, 2008; Woodward, 2006). Given the importance, CCSS for mathematics (NGA/CCSSO, 2010) and NMAP (2008) emphasized the need for students to possess proficient basic fact and computation fluency (e.g., addition, subtraction, multiplication and division). However, unfortunately, numerous research studies have consistently reported that many students with LD have great difficulties with developing basic fact automaticity and computation fluency (Bottge et al., 2010; Geary, 2011; Geary, Hoard, Nugent, & Bailey, 2012). Students with LD tend to struggle with understanding basic mathematics facts or developing fact automaticity (Geary, 2011; Geary et al., 2012; Mazzocco et al., 2008; Rotem & Henik, 2013). Students with LD are likely to make more errors in retrieving mathematics facts than their peers, due to their deficits in ability to store facts in long-term memory and recall the facts (Geary, 2011, Geary et al., 2000; Mazzocco et al., 2008). Basic mathematics fact skill deficits are common and a consistent characteristic among students with LD (Geary, 2011). In addition, Cawley and Miller (1989) found the performance of students with LD who are 8-9 years old on computation is similar to that of 1st grade students without disabilities. Cawley and colleagues (1998) also identified very minimal progress on computation skills of students with LD aged between 9 and 14 years old. According to Geary (2004), students with LD also often have more counting errors and use immature strategies in computation. Unfortunately, students with LD often leave elementary school without possessing fluent basic facts and computational skills (Kroesbergen & Van Luit, 2003). Their difficulties with computation fluency become more pronounced when they advance to the secondary grades (Bottge et al., 2010). For example, Rotem and Heink (2013) found secondary students with LD tend to be less accurate and respond slower on multiplication computation than their peers without disabilities. It was also noted that the mathematics performance of 6th and 8th grade students with LD was similar to that of 2nd grade students without disabilities (Rotem & Heink, 2013).

Problem Solving Skills

In addition to the basic computation skills, it is also important to develop sufficient problem solving skills (NCTM, 2006; NMAP, 2008). Mathematic problem solving requires complex and multiple cognitive processes; it requires both cognitive and metacognitive processes (Krawec, Huang, Montague, Kressler, & Alba, 2012; Montague, 1988). To solve problems proficiently, students must read and understand the problem, find important information, and use effective strategies to plan and solve the problems (Hudson & Miller, 2006; Montague, Warger, & Morgan, 2000). Students require sufficient basic mathematics skills and know how and when they need to apply the skills and concepts in new or various situations (Kroesbergen & Van Luit, 2003). NCTM (2006) emphasized the importance of problem solving skills; ultimately, mathematics education should support students to become proficient problem-solvers and logical thinkers. NMAP (2008) also highlighted the importance of problem-solving skills when noting, "for all content areas, conceptual understanding, computational fluency, and problem-solving skills are each essential and mutually reinforcing, influencing performance on such varied tasks as estimation, word problem, and computation" (p. 30). Unfortunately, students with LD are poor problem solvers; their difficulties may be caused by any number of factors, including deficits in cognitive and metacognitive processes (Gersten et al., 2005; Krawec et al., 2012; Montague, Enders, & Dietz, 2011; Rosenzweig, Krawec, & Montague, 2011).

Students with LD tend to solve problems slower than their peers without disabilities (Hanich, Jordan, Kaplan, & Dick, 2001). Students with LD also often have great difficulties with direct fact retrieval, number decomposition, and mathematical vocabularies that support effective problem solving (Bryant et al., 2000; Geary, 2011; Geary et al., 2012). Their difficulties with solving problems can increase when solving more complex problems (e.g., multistep problems; Fuchs & Fuchs, 2002). It is also well reported that students with LD tend to have difficulties with the problem solving process and use of appropriate strategies for solving problems; they often rely on a trial and error approach to solve problems rather than use effective or efficient strategies (Bryant et al., 2000; Montague & Applegate, 1993; Rosenzweig et al., 2011).

Use of Strategy in Mathematics

Students with LD often struggle with acquisition and use of effective cognitive and metacognitive strategies for mathematics (Montague & Applegate, 1993; Rosenzweig et al., 2011). According to Sherin and Fuson (2005), students typically develop and use more mature strategies as they get older, but students with LD tend to have difficulties with acquiring or selecting mature, efficient strategies and using strategies effectively (Geary et al., 2004; Geary, 2011; Swanson, 1990). Research has demonstrated that students with LD often rely on developmentally immature, inefficient strategies (e.g., finger-counting), rarely use strategies previously taught, and have difficulties with applying the strategies learned in various situations (Geary, 2004, 2011; Jordan, Hanich, & Kaplan, 2003a; Swanson, 1993). In addition, students with LD often struggle with using the strategies in an effective and efficient way. For example, when using counting strategies, they often select counting-all strategies rather than counting-on strategies (e.g., to solve 5 + 4, start from 5 and count 4 to add). In addition, when students with LD use counting strategies, they often start with a smaller number to count up; when solving 2 + 11, students with LD tend to start with 2 and count 11 to add up rather than start with 11 (Gersten et al., 2009; Woodward, 2006). Students with LD also tend to stick with using the same types of strategies (e.g., finger counting, verbal counting) while their peers without disabilities use a mix of strategies (Jordan et al., 2003a, 2003b). Students with LD also rarely use mature strategies such as the decomposition strategy (e.g., 28 + 7 = 28 + 5 + 2 = 30 + 5), while their peers without disabilities use the strategy for solving simple and complex addition problems (Geary et al., 2007). According to Geary and colleagues (2004), when solving mathematics fact problems, many students with LD also heavily rely on immature counting strategies over time, even when their peers without disabilities are likely to use direct fact retrieval strategies. Not only for computation, but also for problem solving, students with LD often struggle with using effective strategies. For example, Montague and Applegate (1993) found that when solving problems, students with LD have greater difficulties in knowledge, use and control strategies when compared to average-achieving and gifted students. Rosenzweig and colleagues (2011) also reported students with LD tend to have a lack of metacognitive skills and often fail to use metacognitive strategies successfully for solving problems when compared to their peers without disabilities. Even when solving more complex problems, students with LD tend to have more difficulties with using the strategies productively (Rosenzweig et al., 2011).

Summary

Students with LD experience greater difficulties in mathematics than their peers without disabilities, and the gap in mathematics achievement tends to grow wider over time (Geary, 2010). Students with LD tend to have difficulties with a wide range of mathematics areas including number sense, basic fact retrieval, computation fluency, and problem solving (Geary, 2011; Gersten et al., 2009; Jordan et al., 2007; Montague et al., 2011). In addition, students with LD often struggle with acquisition and use of mature and efficient strategies for mathematics (Montague & Applegate, 1993). Students with LD are likely to heavily rely on using immature strategies, and do so inefficiently (Geary et al., 2004). Thus, it is apparent that it is critical to provide effective mathematics intervention for students with LD to help them compensate for their deficits in mathematics and develop adequate mathematics skills.

REVIEWS OF LITERATURE ON MATHEMATICS INTERVENTIONS FOR STUDENTS WITH LD

Given the importance of effective mathematics instruction for students with LD, during the past decades, there have been great efforts to investigate and develop effective mathematics interventions for students with LD (Gersten et al., 2009, Miller, Butler, & Lee, 1998). In addition, several reviews and meta-analyses (e.g., Gersten et al., 2009; Kroesbergen & Van Luit, 2003; Miller et al., 1998) on this topic have attempted to summarize and analyze results of literature to determine effective instructional approaches.

Miller et al. (1998) reviewed 54 studies published between 1988 and 1998 that included 1,034 elementary and secondary students with LD. The studies were organized as (a) computation instruction (n = 26) and (b) problem-solving instruction (n = 28). For teaching computation, various interventions including use of constant time delay procedure (e.g., Koscinski & Gast, 1993a), manipulative devices and drawings (e.g., Harris, Miller, & Mercer, 1995), direct instruction (e.g., Rivera & Smith, 1988), strategy instruction (e.g., Van Houten, 1993), lecture-pause (Hawkins, Brady, Hamilton, Williams, & Taylor, 1994), goal structure (e.g., Fuchs, Bahr, & Rieth, 1989), and self-regulation instruction (e.g., Dunlap & Dunlap, 1989) were identified. In addition, for teaching problem solving skills, instructional methods including manipulative devices and drawings (e.g., Marsh & Cooke, 1996), strategy instruction (e.g., Montague, 1992), and direct instruction (e.g., Wilson & Sindelar, 1991) were identified. It was also identified that alternative delivery approaches including CBI (e.g., Bahr & Rieth, 1989), and videodisc instruction (Miller & Cooke, 1989), and peer-tutoring (e.g., Beirne-Smith, 1991) were used for both computation and problem solving instruction. As a result of the review, the authors noted that several interventions, including strategy instruction, self-instruction, direct instruction, and use of manipulative devices and drawings (e.g., Concrete-Representative-Abstract (CRA) technique) were effective for both teaching computation and problem-solving skills and CBI was promising for teaching both skills.

Kroesbergen and Van Luit (2003) analyzed 61 studies on mathematics instruction for kindergarten and elementary students with mathematics difficulties (e.g., students with LD, who are at risk or with mild disabilities) published between 1985 and 2000. Most of the studies (n = 35) focused on teaching basic skills (e.g., basic facts, computation), 13 studies focused on intervention for preparatory mathematics skills (e.g., classification, counting), and 17 studies taught problem solving skills. Various types of instructional approaches (e.g., CBI, direct instruction, self-instruction, and peer-tutoring) were identified. It was demonstrated that most of the studies (n=35) used direct instruction in their mathematics instruction; the effect size of direct instruction was large (.91, $p < .05$). The most effective instruction approach (n=16) was self-instruction with an effect size of 1.45 ($p < .05$). However, for teaching basic skills, direct instruction was more effective than self-instruction. CBI (n = 12) was also effective, but a smaller effect was observed than teacher-directed instruction. However, peer-tutoring (n = 10) was not statistically significantly effective.

Gersten et al., (2009) analyzed 42 studies that used randomized control trials or quasi-experimental research design on mathematics instruction for students with LD published between 1971 and 2007. The studies were organized based on the following five instruction components: (a) explicit instruction (n = 11), (b) use of heuristics (n = 4), (c) student verbalization (n = 8), (d) visual representations (n = 12), and (e) range and sequence of examples (n = 9). The studies also categorized the analysis according to types of feedback provided: (a) providing feedback to teachers on students' progress (n = 20), and (b) providing feedback to students on their performance (n = 24). Moreover, studies using peer tutoring were analyzed according to type of tutoring, cross-age tutoring (n = 2) and class-wide peer tutoring (n = 6). Results found that a multiple, heuristic strategy instruction was most effective with a very large effect size of 1.56 ($p = .00$). Explicit instruction (1.22, $p = 0.03$), cross-age tutoring (1.02, $p = 0.41$), student verbalization of mathematical reasoning (1.04, $p = .00$), and range and use of sequence of examples (.82, p = 0.01) reported large effect sizes. Except for providing feedback to students with goal setting and class-wide peer tutoring, all other instructional approaches also reported statistically significant effect sizes, with a range of .21 and .54.

Summary

All three studies (Gersten et al., 2009; Kroesbergen & Van Luit, 2003; Miller et al., 1998) indicated that both explicit instruction and strategy instruction could be a strongly effective instructional method for teaching mathematics for students with LD. In addition, CBI studies did not demonstrate large effect sizes overall, but it could be a viable instructional approach in mathematics instruction, especially for teaching basic mathematics skills. Above all, based on the results of the studies, it is apparent that explicit, strategic instruction could be the most effective instructional method for teaching mathematics for students with LD.

EXPLICIT, STRATEGIC INSTRUCTION

It has been well documented that both explicit instruction and strategy instruction are recommended as some of the most effective instructional approaches to teach mathematics for students with LD (Gersten et al., 2009; Kroesbergen & Van Luitt, 2003; NCTM, 2007; NMAP, 2008; Montague & Dietz, 2009; Swanson, 1999; Swanson & Hoskyn, 1998b). Explicit instruction is an instructional approach whereby teachers provide clear modeling for solving problems, provide multiple examples, provide sufficient practice opportunities of the skills newly learned, and provide extensive feedback (NMAP, 2008), rooted from direct instruction (Englemann & Carnine, 1991). A large body of research has not only examined the effect of explicit instruction but has also defined its principles. In 2010, Archer and Hughes (2010) summarized the principles that previous research suggested to provide 16 principles of explicit instruction. The principles include: (a) emphasize vital content in instruction, (b) teach logically sequenced skills, (c) break down complicated skills/strategies into smaller units, (d) design organized and structured lessons, (e) provide a clear statement of goals and expectations, (f) review prior knowledge and skills, (g) provide explicit modeling, (h) use clear and detailed instructions and explanations, (i) provide multiple examples, including appropriate examples and non-examples, (j) provide guided and independent practice, (k) provide multiple opportunities to respond, (l) monitor students' progress closely, (m) provide immediate and corrective feedback, (n) teach at an appropriate pace, (o) support students' organization of knowledge, and (p) provide a cumulative review.

Explicit instruction has been used as an effective instructional method for teaching a wide range of mathematics skills (e.g., predatory mathematics, computation, word problem-solving, fraction and algebra; Gersten et al., 2009; Swanson & Hoskyn, 1998). As results of their meta-analysis, Kroebergen and Van Luit (2003) reported direct instruction was more effective for teaching basic mathematics skills than other instructional methods. For example, Glover and colleagues (2010) used a direct instruction flashcard system for teaching division and multiplication facts for elementary students with LD and students improved the number of correct mathematics facts significantly (PND: 65%). Rivera and Smith (1988) investigated the effects of direct instruction on long division computation for middle school students with LD and found all students reached the mastery criterion (100%) in 2-9 days. A large effect size of 1.66

was reported. Hastings, Raymond and McLaughlin (1989) examined the effect of direct instruction on counting coins and bills for high school students with LD; all students decreased the amount of time required for counting money, and maintained and generalized the skills obtained (PND: 100%). Rogenberg (1989) compared effects of direct instruction alone versus direct instruction plus supplementary homework on the performance of mathematics facts for elementary students with LD. The study revealed direct instruction regardless of supplementary homework was strongly effective; a very large effect size of 1.31 was indicated. In addition, Van Houten and Ahmos (1990) examined the effect of direct instruction with a color mediation technique for teaching number identification and multiplication facts. All students demonstrated an immediate increase in targeted skills and maintained the skills after color mediation faded.

Strategy instruction, teaching strategies and activating cognitive and metacognitive processes, appears to be effective for teaching students with LD (Montague & Dietz, 2009). According to Swanson (1999), strategy instruction includes the following instructional variables: (a) explicit explanations or verbal descriptions of task performance, (b) modeling and questioning of the strategy procedures by teachers, (c) systematic cues and prompts to use strategies, (d) cognitive modeling using a think aloud technique. Compared to explicit instruction, strategy instruction focused more on the process of solving problems (Montague, 2008). Teaching strategies meet the needs of students with LD who have difficulties with selecting effective strategies or using the strategies effectively and efficiently (Swanson, 1990, 1993). Especially, cognitive strategy instruction (e.g., verbal rehearsal, visualization) designed for teaching strategies explicitly for learning of specific skills could be effective for students who lack strategies, while meta-cognitive strategy (e.g., self-instruction, self-monitoring) instruction could be effective for students who possess strategies, but who do not know how to use the strategies effectively and efficiently (Montague, 1997). The goal of strategy instruction is to support students to become proficient strategic users and problem-solvers (Montague & Dietz, 2009).

Similar to explicit instruction, strategy instruction has also been used for teaching a wide range of mathematics skills across grade levels. Previous studies have reported that strategy instruction can be an effective instructional approach for teaching basic mathematics skills such as facts and computations; especially, strategy instruction could support students with LD to maintain and generalize the skills obtained (e.g., Iseman & Naglieri, 2011; Naglieri & Gottling, 1995, 1997; Van Houten, 1993; Van Luit & Naglieri, 1999; Woodward, 2006). However, strategy instruction is, in particular, well suited in teaching higher-order skills such as word problem solving skills for teaching secondary students (Montague, 1997). Several reviews of literature also identified that cognitive strategy instruction has been commonly used for teaching word problem solving in mathematics education for students with LD (Gersten et al., 2009; Swanson & Hoskyn, 1998). There are various cognitive strategies developed and utilized; for example, a 7-step cognitive strategy (Read, Paraphrase, Visualize, Hypothesize, Estimate, Compute, Check) with metacognitive strategy (Say, Ask, Check; Montague, 1992; Montague et al., 1993), an 8-step cognitive strategy (read the problem aloud, paraphrase the problem aloud, state the problem, hypothesize, estimate, calculate, self-check; Montague & Bos, 1986), a 5-step cognitive strategy (read the problem out loud, look for important words and circle them, draw pictures to help tell what is happening, write down the mathematics sentence, write down the answer; Case, Harris, & Graham, 1992), and a 9-step cognitive instruction (FAST DRAW; Cassel & Ried, 1996). Many studies used cognitive strategies and metacognitive strategies (e.g., self-instruction, self-regulation) together.

Swanson and Hoskyn (1998) analyzed a vast number of intervention studies for teaching students with LD published between 1963 and 1997. Their meta-analysis did not focus on only mathematics instruction; it analyzed interventions teaching a wide range of skills including academic (e.g., reading, writing, and mathematics), social skills, and cognitive functioning. In their meta-analysis, Swanson and Hoskyn (1998) found larger effect sizes of direct instruction (or explicit instruction) and strategy instruction other than instructional methods for teaching students with LD. The authors also investigated which instructional approach (e.g., direct instruction, strategy instruction, or combined instruction) yielded a larger effect size. To answer the question, they first divided 173 intervention studies published between 1963 and 1997 into four groups (direct instruction alone, strategy instruction alone, combined instruction, and non-direct instruction/non-strategy instruction). However, it is often difficult to distinguish either direct instruction or strategy instruction because of their overlapped characteristics such as step-by-step sequence of instruction, use of multiple examples, visual cues, and reviews (Swanson, 1999). To answer the research question, Swanson and Hoskyn set up criteria to distinguish each instructional approach. Studies including at least four of the following instructional variables were identified as using direct instruction: “breaking down a task into small steps, administering probes, administering feedback repeatedly, providing a pictorial or diagram presentation, allowing for independent practice and individually paced instruction, breaking the instruction down into simpler phases, instructing in a small group, teacher modeling a skill, providing set materials at a rapid pace, providing individual child instruction teacher asking questions and teacher presenting the new materials" (Swanson, 1999, p. 130). To identify it as strategy instruction, studies included at least three of the following instructional variables: "elaborate explanations (systematic explanations, elaborations, and plan to direct task performance), modeling from teachers (verbal modeling, questioning and demonstration from teachers), reminders to use certain strategies or procedures (cues to use taught strategies, tactics, or procedures), step-by-step prompts or multiprocess instructions, dialogue (teacher and student talk back and forth), teacher asks questions, and teacher provides only necessary assistance" (Swanson, 1999, p. 130). Studies identified as combined instruction included minimum numbers of instructional variables, at least four for direct instruction and three for strategy instruction. In addition, studies that did not include the minimum number of instructional variables for both instructions were identified as non-direct instruction and non-strategy instruction (Swanson, 1999).

Swanson and Hoskyn (1998) analyzed the studies and reported effect sizes .68 (SD = .51, N= 47) for direct instruction alone, .72 (SD = .47, N=28) for strategy instruction only, .84 (SD = .54, N = 55) for combined instruction, and .62 (SD = .53, N=43) for non-direct instruction/non-strategy instruction. Combined instruction reported a statistically significant, larger effect size (.84, $p < .05$) than the other instructional approaches. The results suggested that even though each instruction alone is promising to teach students with LD, when both approaches are combined, it could be more effective. Moreover, Swanson (1999) extended the investigation and identified which instructional components have an influence on the larger effect size of combined instruction. It was found that "sequencing (e.g., breaking down the task, step-by-step prompts), drill-repetition and practice-review (e.g., repeated practice, sequenced review), segmentation (e.g., breaking down targeted skill into smaller units then synthesizing the parts into a whole), directed questioning and responses (e.g., the teacher/students ask verbal questions), control difficulty or processing demands of a task (e.g., task sequenced from easy to difficult and only necessary hints and probes are provided), technology (e.g., use of computers), group instruction (e.g., small group instruction), a supplement to teacher and peer involvement (e.g., may include homework, parental assistance, or other assisted instruction), and strategy cue (e.g., use of think-aloud models)" (Swanson, 1999, p. 137).

Based on the findings, combined instruction, explicit, strategic instruction, is strongly recommended for teaching students with LD. Previous research has consistently reported that both explicit instruction and strategic instruction is effective for teaching students with LD, so it is not surprising a combination of both instructions could be more effective. Given its effectiveness, explicit, strategic instruction has been widely used in studies in mathematics instruction for students with LD for teaching a wide range of mathematics skills including both basic mathematics skills and higher-order skills.

Explicit, Strategic Instruction with Basic Mathematics Skills

Flores, Houchins and Shippen (2006) conducted a series of case studies to compare maintenance and generalization effects of constant time delay (CTD) and strategic instruction on the performance of multiplication for four middle school students with LD. The students that participated in this study were provided both types of instructions; constant time delay (CTD) and strategic instructional model (SIM) instruction. Students were first provided CTD instruction; they were asked to verbally respond to facts represented in flashcards, and then write the answer on the multiplication sheet. If students were not able to respond in a limited amount of time, the teacher provided the answer for the facts. During SIM instruction, students were explicitly taught how to use strategies (e.g., DRAW: Discover the sign, Read the problem, Answer, or draw tallies and/or circles and check your answer, Write the answer) for solving problems. Facts taught for each instruction were carefully selected so fact learning from CTD may be less likely to have an influence on learning through SIM. The study was designed to compare the maintenance and generalization effects of both instructional approaches. A one-minute timed probe was used for the maintenance effect (the number of correct digits per minute) after each 1 and 5 weeks after all interventions were terminated. Both 1-minute timed and untimed probes were used to measure the generalization effect (unknown facts). The results of this study indicated that strategic instruction was more effective for maintaining and generalizing skills obtained than CTD.

Joseph and Hunter (2001) investigated the effect of an explicit self-regulation strategy on the performance of fraction computations among three middle school students with LD. A multiple-baseline design across students was employed. During the baseline, students were asked to work on as many probes as they could, including 10 fraction computation problems. After the students entered the intervention phase, they received a cue card strategy instruction. Teachers first demonstrated explicitly how to use the cue card strategy to solve fraction problems including common and different denominators. Students were provided teacher-guided practice with examples, and corrective and immediate feedback was given. If students had difficulties with using the strategy, teachers continually providing prompts including self-reflection discussion (e.g., "Let's talk about how you did the worksheets"). During the maintenance period, students were asked to solve the probes without using the cue cards. Moreover, students were asked to chart their progress. Daily probes developed by researchers were administrated across the baseline, intervention and maintenance phases. The probes consisted of 10 addition and subtraction fraction computation problems (common and different denominators). It was indicated that explicit self-regulation strategy instruction was effective on the performance of fraction computation for students with LD. The PND calculated was 96%, considered to be a large effect.

Kelly, Gersten, and Carnine (1990) examined the effect of explicit, strategic instruction on teaching fractions for 34 high school low achieving students including 17 students with LD. All students were randomly assigned to two conditions: (a) explicit strategic instruction and (b) control. Control group instruction was adapted from a basal mathematics textbook (Abbott & Wells, 1985). Instructions for both groups were designed in direct instruction formats (teacher modeling-guided practice and independent practice), but crucial differences between two instructions were (a) explicit step-by-step strategy instruction for solving problems, (b) separation of confusing elements and vocabularies (e.g., introducing "numerator" in later lessons to prohibit confusing it with "denominator" which was introduced earlier), and (c) use of a wide range of examples (e.g., use of both proper and improper fraction examples). Especially, the explicit strategic instruction group was taught a step-by-step procedure (e.g., "First identifying the denominator, then the numerator in the answer") that they could follow to solve the problems. Both groups were given 10 30-minute sessions. A researcher-developed mathematics test was used to measure students' fraction skills (e.g., distinguishing numerator and denominator, writing fractions from pictures, and addition, subtraction, and multiplication of fractions). Both groups increased scores on the posttest compared to the pretest; however the explicit, strategic group (mean scores: 96.53) significantly outperformed the control group (mean score: 82.29). A large effect size of .88, which was statistically significant ($p < .01$) was reported.

Mancl and colleagues (2012) recently studied the effects of explicit, strategic instruction with a concrete-representational-abstract (CRA) sequence on addition and subtraction computation and word problem solving for elementary students with LD. A multiple-probe across participants design was used. During the intervention phase, five students with LD were provided 30-minute-explicit, strategic instruction involving the CRA technique. All 11 instructions were designed explicitly (teacher modeling, guided practice, independent practice). The first 5 lessons used concrete materials (e.g., base-ten blocks, place-value mats), the next 3 lessons used representational materials (e.g., drawings of base-ten blocks), on the 9th lesson, explicit strategy instruction (e.g., RENAME; Miller, Kaffar, & Mercer, 2011a, 2011b) was provided, and the last 2 lessons used abstract-level materials (e.g., problems with number symbols only). RENAME is a 6-step procedure for solving problems (e.g., Step 1, "Read the problem", Step 2, "Examine the ones column", Step 3, "Note ones in the ones column", Step 4, "Address the tens column", Step 5, "Mark tens in the tens column, and Step 6, "Examine and note hundreds; exit with a quick check"). Corrective and ongoing feedback was provided to students. The students and a teacher charted their progress at the end of each lesson. Researcher-developed probes consisting of 8 computation and 2 word problem solving problems were used to measure the effect of the intervention. The result of the study indicated explicit, strategic instruction using CRA was significantly effective for teaching both computation and word problem solving for students with LD. All students reached master criterion (higher than 80%) and they obtained significant gains on the posttest compared to the pretest as well as maintained the skills gained over time. The PND calculated was 95.73, which was a large effect.

McIntyre and colleagues (1991) examined the effect of using a count-by strategy on the performance of elementary students with LD on multiplication facts using a multiple-probe design. During the baseline phase, the students worked on a daily 1-minute multiplication fact probe. During the intervention phase, the student was taught multiplication facts using count-by strategies (e.g., counting by 3 three times for solving 3×3) for 10-15 minutes. The student practiced the technique with verbal as well as written approaches. The count-by strategy was taught using direct instruction, followed by teacher modeling, guided practice, and independent practice procedures. At the end of each session, the student worked on a daily 1-minute probe consisting of single-digit multiplication facts. The correctly written digits per minute (DC/M) on the daily probes were measured. Moreover, the generalization effect was measured with a test including all single-digit multiplication facts in random order and a test including a mixed probe randomly administered. The results reported the students increased their correct rate per minute on daily probes. The effect size was large, 1.14 (Swanson & Hoskyn, 1998). In addition, the students maintained and generalized the skills when the study was terminated.

Ross and Braden (1991) compared effects of explicit strategy instruction with verbalization, cognitive behavior modification (CBM; Meichenbaum, 1985), token reinforcement and direct instruction on the performance of addition and subtraction computation of elementary students with LD. Randomized control trial (RCT) was used for this study; 94 students were randomly assigned to the three instruction groups. Each group was provided with 19 1-hour sessions in 4 weeks. The direct instruction group was provided mathematics instruction in structured mathematics instruction, the token reinforcement group was provided typical mathematics instruction with token reinforcement system, and the CBM group was taught explicitly to use a CBM strategy (e.g., “What is my assignment for today?”, “What kind of problem is this, addition or subtraction?”) with verbalization. The performance of students on addition and subtraction computation was measured with a researcher-developed daily 2-minute mathematics probe and the Stanford Diagnostic Mathematics Test (SDMT; Beatty, Madden, Gardner, & Karlsen, 1986). All three groups significantly improved their scores on the 2-minute mathematics probes (F (2, 91) = 8.53, $p < .001$); no significant difference among groups was observed. However, CBM and token reinforcement groups significantly outperformed the direct instruction group (F (4, 182) = 4.03, $p < .01$) on the SDMT. Moreover, both groups maintained the skills obtained 2 weeks after the intervention was terminated. A small effect size of 0.26 was reported (Swanson & Hostkyn, 1998).

Tournaki (1993) investigated the effects of explicit minimum addition strategy instruction and drill and practice on teaching single-digit addition computation for elementary students with LD. Forty-two students were randomly assigned to three conditions: explicit minimum addend strategy instruction (ESI), drill and practice, (DP), and a control group. All groups received eight 15-minute instruction sessions. The ESI group was provided explicit strategy instruction; the strategy was demonstrated to students first, then they were asked to compute each problem given in the lesson as fast as they could. If students made any error, the strategy was reviewed again and students were asked to compute the problem again. The DP group also first asked to compute problems given in the lesson as fast as they could. Two alternate forms for each lesson were used and if criteria (90% accuracy in or less than 80 seconds) were met, the student moved to the next lesson. While students worked on the problems, no feedback was provided. After students completed solving the problems, errors were identified and students were asked to recompute the problems. The control group received supplementary instruction rather than typical classroom instruction. Researcher-developed mathematics probes (pre, post, and transfer test) were used to measure accuracy and latency of single-digit addition computation skills of the students. The results revealed that the explicit, strategic instruction group significantly outperformed the drill and practice or control groups on both the posttest (F (2,41) = 18.39, $p < .01$) and transfer test (F (2,77) = 24.33, $p < .01$). A very large effect size of 1.74 was reported (Gersten et al., 2009); explicit, strategic instruction was effective on teaching addition computation for students with LD.

Wood, Rosenberg and Carran (1993) investigated the effects of explicit, self-instruction with tape-recorded cues on the performance of elementary students with LD on addition and subtraction computation using a multiple-probe design. Nine students were assigned to three groups: (a) self-instruction group, (b) observation of self-instruction training group, and (c) control group. During baseline sessions, all students were asked to solve mathematics problems on worksheets. During intervention sessions, students in the self-instruction group were provided individualized thirty 40-minute self-instruction training. In the training, investigators taught students an 8-step self-instruction strategy (e.g., step 1. First, I have to point to the problem, step 2. Second, I have to read the problem) explicitly; then it was demonstrated how to solve problems with the strategy. While the investigators taught the students, the demonstration was tape-recorded and students in the self-instruction group were allowed to use the taped instruction as a prompt to complete other problems on the worksheet. Ongoing and corrective feedback was provided. Students in the observation group were paired with students in the self-instruction group; students in the observation group were not given self-instruction training, but they observed the training while students in the self-instruction group were provided the training. Students in the control group were not given any training or observed the training. Four dependent variables, (a) the number of problems attempted, (b) the number of problems completed correctly, (c) the time for task performance and (d) the number of self-instruction steps completed, on mathematics worksheets (10 addition and 10 subtraction problems) were measured. After the first self-instruction training, both the self-instruction group and the observation group were not able to use self-instruction successfully to solve problems. However, after the self-instruction group was trained on how to use taped-recorded cues, their scores on mathematics problems increased dramatically. Moreover, they maintained or improved after the use of tape-recorded cues faded. Even though both self-instruction and observation groups obtained higher scores than the control group, the observation group did not improve their scores comparably to students in the self-instruction group. In addition, a lack of generalization was observed for the observation group. A very large effect size (1.39) of using the self-instruction strategy was reported (Swanson & Hoskyn, 1998).

Explicit, Strategic Instruction with Higher-order Mathematics Skills

Explicit, strategic instruction has also been utilized for teaching higher-order skills such as word problem solving. Jitendra and colleagues (1998) compared the effects of explicit schema-based strategy instruction (SBI) versus traditional strategy instruction (TSI) on addition and subtraction word problem solving performance of elementary students with mild disabilities. Thirty-four students including 17 students with LD were randomly assigned to two conditions, SBI and TSI. In both groups, students were provided 19 43-minute intervention sessions. The TSI group was provided instruction adapted from the basal mathematics program, Addison-Wesley Mathematics (Eicholz, O'Daffer, & Fleenor, 1985). Students in the TSI group were first presented the Think Mathematics activities specified in the mathematics program. Then students were provided instruction on using a 5-step checklist procedure (e.g., Step 1, "Understanding the question by focusing on the question", Step 2, "Finding the needed data given in the problem", Step 3, "Planning what to do by guessing and checking") for solving word problems including all three types of problems (Change, Group, and Compare). Students were asked to work on mathematics worksheets and feedback on errors was provided. In contrast, the SBI group was instructed on how to use a schema-based strategy to solve all three types of word problems and instruction was provided using the explicit instruction approach (teacher-led modeling, guided practice and independent practice). Researcher-developed mathematics probes (pre, post, and delayed posttests) were measured to identify effects of interventions. It was indicated that both groups increased performance on word problem solving on the posttest compared to the pre-test, and maintained and generalized the skills they obtained. However, the SBI group obtained more gains significantly better than the TSI group on all measures. An explicit schema-based strategy was more effective than typical strategy instruction on teaching word problem solving for students with mild disabilities. A moderate effect size of .67 was reported.

Montague and colleagues (2011) recently examined the effects of explicit cognitive strategy instruction on word problem solving skills of middle school of students with LD. A cluster randomized design was used to assign a total of 779 students (78 with LD, 344 low achieving students (LA), and 357 average achieving students (AA)) from 24 schools into two groups; (a) explicit cognitive strategy instruction (n = 319) and (b) typical mathematics class instruction (n = 460). For 1 school year, the control group received their typical mathematics class instruction while the experimental group was provided a research-based cognitive strategy instructional program, Solve It! The program was designed to incorporate cognitive and metacognitive processes, which are integral for solving mathematics problems. The treatment group was provided explicit instruction on how to use and apply a 7–step cognitive strategy (read, paraphrase, visualize, hypothesize, estimate, compute, and check) and a 3-step self-regulation, metacognitive strategy (SAY, ASK, CHECK). Lessons were systematically organized and structured (modeling, guided practice, and instructional practice), appropriate scaffoldings (e.g., cues and prompts) and immediate and corrective feedback were provided. In contrast, the control group received typical mathematics class instruction. The performance of students on four basic operation word problem-solving skills was measured with seven curriculum-based measurements (CBM); data was collected 6 times across 1 school year. The results of the study revealed that the explicit, cognitive strategy instruction group showed significantly greater growth in problem solving than the control group. The control group did not demonstrate significant gains. Moreover, students with LD in the experimental group outperformed all students in the control group regardless of their ability (i.e., LD, LA, and AA).

Owen and Fuchs (2002) examined the effects of explicit strategy instruction on the word problem solving skills of elementary students with LD. Twenty-four students including 20 with LD were randomly assigned to four conditions: (a) acquisition, (b) low-dose acquisition plus transfer (LAT), (c) full-dose acquisition plus transfer (FAT), and (d) control. All three experimental groups received explicit instruction (6 30-minute sessions) on a 6-step strategy (e.g., 1. Read the problem, 2. Draw individual circles to show the number for which you will find half, 3. Draw a rectangle dividend in half, creating two boxes) for solving word problems which required detecting "half" of a number. As a large group, the strategy was broken down into component steps; the procedure was modeled with working examples. After the group instruction, students were asked to solve practice problems in pairs with high-achieving peers. All experimental groups had same instructional components except amount of time spent on transfer instruction (apply the skill of finding "half" to new problems). The acquisition group received only acquisition practice for 4 sessions. The LAT group was provided acquisition practice for 2 sessions and transfer instruction for 2 sessions while the FAT group was provided both acquisition and transfer instruction for 4 sessions. The control group was given typical mathematics instruction that included word problems involving "half". The performance was measured by researcher-developed probes (pretest and posttest); in particular, the number of problems corrected and amount of work (applying steps taught) were identified. The results of the study indicated that the FAT group obtained the greatest gains on both measures overall. On both measures, the two groups (FAT & LAT) that included transfer instruction had a significantly higher effect than the control group. A very large effect size of 1.39 was reported (Gersten et al., 2009). According to the results, explicit transfer instruction could be more effective for teaching word problem solving skills, but explicit, strategic instruction was effective whether or not transfer instruction was included.

Wilson and Sindelar (1991) compared the effects of strategy instruction with sequenced examples, strategy instruction and sequenced examples on the performance of addition and subtraction word problems of elementary students with LD. Sixty-two students were assigned to three different conditions: strategy plus sequence (SPS), strategy instruction only (SIO), and sequence only (SO). Students in the SPS and SIO groups were taught fact-family concepts during the first two days of intervention; they were instructed of the rules and when and how to apply the rules explicitly followed by boardwork (diagrammed applications were represented on the board) and seatwork practice (paper-based practice) activities. From the third day, students started learning how to apply the rules when solving word problems. Both groups were instructed with the same strategies, but problem examples were presented differently; the SPS group received all types of examples while the SIO group received sequenced examples (e.g., lessons 1-3: simple action problems, lessons 4-6: classification problems, lessons 7-9: comparison action problems, lesson 10-12: comparison problems). In contrast, the SO group was not taught any rules or strategies for solving problems, but provided a sequenced instruction (teacher instruction-boardwork practice-seatwork practice) with sequenced problems (simple action-classification-comparison action-comparison problems). Researcher-developed mathematics probes (pre, post, and follow-up tests) were used to measure their performance on addition and subtraction word problem solving. The results demonstrated that both the SPS (F (1, 57) = 7.49, $p < .05$) and SIO groups (F (1, 57) = 4.33, $p < .05$) significantly outperformed the SO group on the posttest and the follow-up test. However, the increase between SPS and SO was higher than that between SIO and SO. A large effect of .91 was reported (Gersten et al., 2009); strategy instruction combined with direct instruction was effective for teaching word-problems for students with LD regardless of whether sequenced examples were used or not.

Xin and colleagues (2005) compared the effects of explicit schema-based strategy instruction (SBS) versus general strategy instruction (GSI) on the word-problem solving performance of middle school students with LD. Twenty-two students with learning problems including 18 with LD were randomly assigned to two groups: (a) SBS and (b) GSI. Both groups were provided 12 1-hour instructions; in each instruction, a strategy was explicitly modeled with multiple examples first, followed by guided practice and independent practice. Students received corrective feedback and extra modeling if needed while practicing. The SBS group solved multiplicative compare problems (4 sessions), proportion problems (4 sessions), and mixed problems (4 sessions) while the GSI group solved both types of problems (multiplicative compare and proportion problems) across all sessions. The SBS group was instructed on how to recognize the two different problem types. A four-step procedure for solving problems (problem, planning, solving, and checking) was taught to both groups. However, step 2 and 3 were different according to how to plan and solve the problem. Step 2 for GSI was "Draw a picture to represent the problem" and step 3 was "Solve the problem" while step 2 for SBI was "identify the problem type, and use the schema diagram to represent the problem," and step 3 was "transform the diagram to a mathematics sentence". The SBI group was taught how to identify types of word problems and represent and solve the problems using schema-based diagrams while the GSI group was instructed to draw a picture to represent and solve the problem. Researcher-developed mathematics probes were used to measure the performance on multiplication and division word problems. Percentage of correct items across posts, maintenance and follow-up tests were measured. The results of this study revealed that the SBI group significantly outperformed the GSI group on all measures. Moreover, only the SBI group significantly improved performance on the generalization test. Schema-based instruction was more effective than general strategy instruction for teaching word problem solving for students with LD. A very large effect size of 2.15 was reported (Gersten et al., 2009).

Summary

The results of the studies indicated large effects of explicit, strategic instruction on teaching basic mathematics skills for students with LD. Various strategies (e.g., count-by, minimum addend, self-instruction, a cue card, SIM, CMB) were taught explicitly for enhancing basic skills (e.g., facts, computations and understanding concepts on four basic operations). It was also reported explicit, strategic instruction was more effective than other instructional approaches compared (e.g., direct instruction, token reinforcement, drill and practice, and typical mathematics class instruction). Moreover, explicit, strategic instruction yielded great effects on the maintenance and generalization of the skills obtained.

Similar results of the studies focusing on basic mathematics skills were found. A variety of strategies (e.g., schema-based, Solve It!) were used to teach word-problem solving on four basic number operations (addition, subtraction, multiplication, and division). Explicit, strategic instruction was effective for teaching word problem solving skills for students with LD; a large effect size overall was reported. Students were able to maintain and generalize the skills obtained. Interestingly, Montague et al. (2011) found that students with LD provided with an explicit, cognitive strategy instruction outperformed even low and average achieving students without disabilities in the control group. As a result of the literature review, explicit, strategic instruction is recommended as an effective instructional approach for teaching both basic mathematics skills and higher-order skills; it also supports students with LD to maintain and generalize skills obtained.

COMPUTER-BASED INSTRUCTION (CBI) IN MATHEMATICS INSTRUCTION

Throughout the past decades, the number of technology devices in use has dramatically increased in public school settings. The U. S. Department of Education (2010) reported that all public schools are equipped with at least one or more computers with Internet access used for instructional purposes and the ratio of students to computers was 3.1 to 1 as of 2008. Other than computers, it is also reported that a variety of technology devices, for example, projectors (97%), digital cameras (93%) and interactive whiteboards (73%), are available in public schools. Technology's development and increased availability in school settings have led to promising instructional methods to enhance learning mathematics of students with LD (Hughes & Maccini, 1996; Irish, 2002). According to Bouck and Flanagan (2009), technology could support these students by providing more opportunities to access mathematics ideas and experience more success. Moreover, NCTM (2000) and NMAP (2008) advocated the use of technology in mathematics instruction. Technology such as calculators and well-designed computer software are vital tools for high-quality mathematics instruction in the 21st century (NCTM, 2000). The use of technology tools can support students at various levels to enhance mathematics performance in mathematical reasoning and reflection, problem identification, problem solving and computation fluency. Technology also can increase students' motivation in learning especially for students with disabilities (Bender, 2001). Furthermore, the use of technology will enhance not only mathematics instruction but also student's technology skills, which is necessary for their future lives (NCTM, 2000). In particular, CBI is considered to be popular and a recommended method for using technology for mathematics instruction for students with LD (Bouck & Flanagan, 2009; Fitzgerald et al., 2008; Hughes & Maccini, 1996; Woodward & Rieth, 1997; Vaughn & Bos, 2009).

CBI is defined as an instructional method, using computer software programs that offer instructional content to enhance student's skills, knowledge or academic achievement. CBI can be used separately or as a supplementary tool when combined with traditional teacher-directed instruction (Okolo et al., 1993; Ulman, 2005). According to Watkins and Webb (1981), CBI is commonly delivered in the following six modes: (a) drill and practice, (b) tutorials, (c) games, (d) simulations, (e) problem solving, and (f) computer managed instruction. The instructional method was introduced in the 1960s and its usage and research has dramatically increased since the 1980s (Gleason, Carnine, & Boriero, 1990; Woodward & Rieth, 1997). Over the past decades, there have been efforts to adapt, utilize and evaluate CBI for the teaching and learning of students with disabilities (Fitzgerald et al., 2008; Okolo et al., 1993). Research has found that CBI can be a valuable supplementary teaching method for students with disabilities (Hughes & Maccini, 1996; Mastropieri et al., 1991).

In terms of students with LD, research has reported that CBI can be used effectively for helping these students compensate for their challenges and enhance their academic learning in, for example, reading (Higgins & Raskind, 2005; Higgins, Boone, & Lovitt, 1996; Raskind & Higgins, 1999), writing (Bahr, Nelson, & Van Meter, 1996; MacArthur, 1998; Zhang, 2000) and mathematics (Chiang, 1986; McDermott & Watkins, 1983). It has also been demonstrated that computers can be helpful to increase students' motivation (Okolo, 1992), attention in learning (Fisher, 1983), time on task (Okolo et al., 1993) and independence (Manset-Williamson, Dunn, Hinshaw, & Nelson, 2008). CBI also allows adapted and individualized instruction for students with disabilities to meet their individual needs. It also allows educators to adjust difficulty levels and learning pace, provide more practice opportunities or repetition, and set a variety of goals for individual students that is often difficult for a teacher to provide during whole class instruction (Grimes, 1981; Poplin, 1995; Xin, 1999). Moreover, CBI can provide immediate feedback and record student's individual progress (Hughes & Maccini, 1996).

Especially, to determine the effects of CBI on teaching mathematics for students with LD, a review of 14 studies published between 1980 and 2012 was conducted. Interestingly, it was detected that 10 out of 14 studies focused on teaching basic mathematics skills (i.e., facts or computations). In addition, seven out of 14 studies used drill and practice type of CBI. It was similar to the results determined in a meta-analysis by Seo and Bryant (2009); CBI is primarily used in teaching basic mathematics skills. It may be because CBI can be very helpful for motivating students to practice specific skills to be taught and providing ongoing feedback to students on their performance (Seo & Bryant, 2009). Among studies on basic mathematics skills, it was found that six studies (e.g., Howell et al., 1987; Irish, 2002; Koscinski & Gast, 1993a) focused on teaching multiplication skills and four studies (e.g., Christensen & Gerber, 1990; Nordness et al., 2011) focused on addition and subtraction. As results of the synthesis, it was revealed that even though effect size varied across the studies (zero to huge effect size), the majority of the studies (9 out of 14 studies) reported a medium effect size. Moreover, all of the studies that measured the maintenance effect reported a large effect size of CBI on teaching mathematics for students with LD.

CBI and Teaching Multiplication Skills

The six studies that used CBI for teaching multiplication skills revealed results similar to the CBI studies for teaching mathematics for students with LD. Four of the studies focused on teaching single-digit multiplication facts, but two studies did not report on this specifically. Various computer programs (e.g., Math Blaster, Drill, Memory Math, and Galaxy Math) were used; most of the programs were drill and practice programs (n = 4). It was also detected that three CBIs (Memory Math, Drill, and Multiplication facts Program) were designed with evidence-based instructional strategies (i.e., mnemonics, constant time delay, attribution feedback). In addition, most studies (n = 4) reported a medium effect size.

Howell et al. (1987) investigated the effect of Galaxy Math (Random House Inc., 1984), a drill and practice type of program, for teaching single digit multiplication facts to high school students with LD. The students were provided 13 20-minute instruction sessions. CBI had an initial positive effect on targeted skills (number of errors and average time to complete the multiplication problems), but errors and time increased when CBI was excluded. Negligible effect sizes (PND 12.5% for number of errors and 33% for response time) were detected.

In the following study, Howell et al. (1987) compared the effect of CBI only and combined instruction (CBI plus TDI) on single digit multiplication skills for a high school student with LD. Tutorial programs (MemorEase, Mind Nautilus Software, 1985) and game type drill and practice programs (Galaxy Math, Random House Inc., 1984) were used. A student was provided 30 20-minute sessions. In the combined instruction condition, the student was taught rules as well as using a CBI program for practice. The results showed both conditions had a positive effect on number of errors, but when CBI was used with TDI, the effect increased and was maintained after withdrawal. A larger effect size (Combined: PND of 85% vs. CBI only: PND of 50%) of combined instruction than CBI only was determined.

Irish (2002) investigated the effect of CBI designed with mnemonics (Memory Math, Irish, 2002) on single-digit multiplication fact skills for three students with LD. Memory Math is a combination type of CBI consisting of tutorial, game and drill and practice and is especially designed to teach facts using a mnemonics strategy to remember facts. A total of 20 24-29-minute sessions were provided to students and researcher-developed computer and paper-pencil based mathematics probes were used to measure the performance of students. As a result, all students improved accuracy and maintained the skills over time. A medium effect of intervention (PND 74%) and a large maintenance effect (91%) were observed.

Koscinski and Gast (1993a) examined the effect of a researcher-developed drill and practice type of CBI (Multiplication facts program, Authorware Professional Software, Inc. 1990) on multiplication skills of six elementary students with LD. The program was designed with a 5-second constant time delay technique. During the intervention, if students were not able to answer the problems within 5 seconds, a computer program provided the answer to the students. After 49 sessions, it was observed that all students improved achievement and learned all of the targeted facts. The gained skill was maintained among all students across probes. However, each student reported various generalizations of the targeted facts. A medium effect (PND 88%) of intervention and a large effect of maintenance (100%) were determined.

Okolo (1992a) examined a researcher developed drill and practice program (Drill, Davis, n.d.) on multiplication for 29 middle school students with LD. The program was allowed to control type of feedback (attribution retraining feedback vs. neutral feedback) provided to the students. Students were randomly assigned to two conditions: (a) attribution retraining feedback and (b) neutral feedback. After 8 30-minute intervention sessions, students worked on a researcher-developed multiplication test. The result of this study showed that the attribution retraining feedback group significantly improved targeted skills while the neutral feedback group did not report significant improvement. A large effect size (.9 and .87) of intervention was reported.

Wilson et al. (1996) investigated the effect of Math Blaster (Davidson & Eckert, 1987), a combination of tutorial, game and drill and practice, on single-digit multiplication facts for four elementary students with LD. An alternating treatment design was used to compare the effects of CBI and TDI. During TDI, students were involved in a direct instruction flashcard program. First, the teacher verbalized the facts and then students were asked to verbalize the facts together with the teacher. Finally, students were asked to verbalize the facts and answer alone. The teacher provided immediate and correct feedback. During CBI, students worked on the Math Blaster program to learn facts. Similar to TDI, the program was also designed to follow the sequence, demonstration, guided practice and game type practice. Students were provided 13-30 sessions; each session was 20 minutes consisting of 10 minutes of CBI and 10 minutes of TDI. A researcher-developed multiplication fact test was used to measure the performance of students on multiplication facts. As a result, all students mastered multiplication facts and increased automaticity in both conditions. However, the students in the TDI group mastered more facts than the CBI group. The TDI group reported a higher (2-4 times) response and success rate than the CBI group. A medium effect of intervention (PND 85%) and large maintenance effect (100%) were determined. The results of the CBI studies revealed that CBI could be effective for teaching mathematics, both basic skills and higher order skills, for students with LD.

Tablet Computing in Mathematics

As technology evolves, a new type of mobile devices (e.g., smart phones and tablet computers), small and handheld computing devices typically having a touch screen display and Internet access features, have emerged and have gained popularity since the late 2000s. Liu and colleagues (2013) found 63 studies on mobile learning in K-12 education settings between 2007 and 2013. It was identified that various mobile devices (e.g., smartphones, tablet computers, PDAs, Pocket PCs, and iPods) were being used in a variety of areas such as natural science (29%), mathematics (18%), language arts (16%), social studies (11%), and foreign language (6%). Liu et al. (2013) also found the number of studies published sharply increased in 2011; especially, studies published around 2007 used PDA or basic cellphones while studies published around 2012 focused on the use of new mobile devices such as smartphones or tablet computers. These studies illustrate the recent increases in the use of mobile devices such as tablet computers in K-12 settings.

Especially, one of the tablet computers, the iPad, developed by Apple, has gained surprisingly great attention and popularity in education settings as soon as it was released in 2010. In addition, even though there was a dearth of empirical research on the effect of iPads for teaching students with disabilities, iPads are widely used in special education settings (Nirvi, 2011). Nirvi (2011), one of the first researchers to demonstrate that iPads could be useful tools for students with disabilities, highlighted the devices' advantageous features: (a) the availability of inexpensive downloadable apps, (b) a touch-screen feature that allows students with disabilities to use the device without a mouse or a touchpad, (c) a user-friendly screen size, (d) portability due to its 10-hour battery life, and (e) multimodality (e.g., sound, touch).

In 2013, Kagohara and colleagues reviewed 15 studies on the use of iPods and iPads for individuals with developmental disabilities between 2008 and 2012. Eight of the studies (e.g., Achmadi et al., 2012; Kagohara et al., 2010) used the devices for enhancing communication skills using communication applications (e.g., Proloquo2Go™). The areas the other studies focused on were employment (e.g., Burke et al., 2010), leisure (e.g., Hammond, Whatley, Ayres, & Gast, 2010), academic learning (Kagohara, Sigafoos, Achmadi, O'Reilly, & Lancioni, 2012), and transitioning skills (Cihak, Fahrenkrog, Ayres, & Smith, 2010). It was also identified that all studies examined the use of iPods except for one study (Flores et al., 2012), which involved iPads with a speech generating application. This lack of research on iPads may be because the first generation of iPads emerged relatively recently. As a result of their review, Kagohara et al. (2013) concluded that iPods and iPads could be utilized successfully for supporting individuals with disabilities to teach a wide range of skills (e.g., academic learning, employment and leisure skills, and transitional skills).

Even though a trend of increased research on the effect of using iPads in special education settings has been observed, there is still a lack of research on the effect of using iPads for teaching students with LD. Especially, in mathematics instruction, only two studies were noted (Cihak & Bowlin, 2009; Nordness et al., 2011) that investigated the effects of the new mobile computing devices (i.e., iPod Touch and Toshiba Pocket PC) sharing many common characteristics with iPads in a synthesis of literature on CBI in mathematics instruction for students with LD between 1980-2012 (Ok, 2012). Cihak and Bowlin (2009) investigated the effects of using a tablet computer (Toshiba e405 Pocket PC) on geometry problem solving for three high school students with LD. Teachers created a 10-minute video tutorial program for using the tablet computer to teach targeted skills. In the video, teachers provided explicit modeling of step-by-step procedures to solve problems. Students were asked to work on the video-based tutorial program and completed homework problems independently. Students were allowed to watch the tutorial as many times as they needed. The teacher provided corrective feedback on their work. All students improved their geometry solving skills (e.g., the perimeter of squares/rectangles, triangles/trapezoids, and missing sliders of various polygons). A large effect size (PND 100%) was reported and students were able to maintain the skills in 6 weeks (PND 100%). In addition, Nordness et al. (2011) examined the effects of the use of an iPod Touch application on teaching 2-digit subtraction computations for elementary students with LD. Students practiced subtraction computations with an iPod Touch application (Math Magic). All students improved the targeted skills. Both researcher-developed daily probes and a standardized test (The Nebraska Abilities Test, N-ABLES) were used to measure the effect of the instruction. A moderate effect (PND 70%) on N-ABLES and a large effect (PND 100%) on researcher-developed probes were reported. Moreover, a large maintenance effect (PND 100%) was observed. Even though the studies did not use iPads, based on the results of the studies, it seems like iPads would be effective for teaching mathematics for students with LD as well because the devices share many common characteristics (e.g., downloadable applications, touch screen, and mobility).

As is well documented, CBI could be an effective instructional approach for teaching mathematics, especially basic mathematics skills, for students with LD. Computer programs provide students with LD multiple opportunities for practice and drills as well as ongoing and immediate feedback on their performance. Recently, there have been various mathematics applications designed for iPads that have similar features of the computer programs. It is hypothesized that the use of iPad applications for teaching basic mathematics skills could be effective. Moreover, the new features of iPads (e.g., touch-screen and mobility) might yield even more positive effects compared to CBI. It is apparent that more evidence-based research on the newly emerging iPads and applications in mathematics instruction for students with LD is required. In addition, it is also hypothesized that when iPads are used with TDI, it would be more effective than iPads only based on the results of previous studies. For example, Howell et al. (1987) reported when CBI was combined with TDI (learning of rules), it could be more effective for teaching multiplication facts for students with LD compared to CBI only. Moreover, a recent study by Powell and colleagues (2009) also reported the effects of using CBI combined with TDI on enhancing mathematics fact retrieval for elementary students with LD. Thus, it is also required to use iPads with TDI for providing more effective instruction for students with LD.

Summary

It is apparent that CBI is one of the effective instructional methods for teaching mathematics for students with LD. Overall, previous studies over the past decades reported a moderate effect size using CBI in mathematics instruction for students with LD. Recently, the use of tablet computers (e.g., the iPad) has been highlighted in the special education field. Even though there has been a lack of research investigating the effect of using tablet computers for teaching mathematics for students with LD, recent studies reported promising results of using tablet computers in mathematics instruction for students with LD.

OTHER INSTRUCTIONAL COMPONENTS

In a review of previous literature on mathematics intervention for students with LD, Gersten et al., (2009) also reported the following instructional components, (a) using the concrete-representational-abstract (CRA) technique, (b) teaching logically sequenced and ranged mathematics skills, and (c) providing achievement data to students and feedback on their performance, are effective for teaching mathematics for students with LD.

Using Visual Representations: Concrete-Representational-Abstract (CRA)

Learning mathematics is often not easy for students with disabilities; it is required to understand abstract concepts of mathematics. Visual representations such as manipulatives, number lines, drawings, graphs, and pictures can be beneficial for understanding abstract or complex mathematics concepts (Steedly, Dragoo, Arafeh, & Luke, 2008). The NCTM (2007) summarized the results of several meta-analyses (Baker et al., 2992; Gersten et al., 2006; Kroesbergen & Van Luitt, 2003) and reported moderate effect sizes (.5) of using visual representations for students with mathematics disabilities. Especially, CRA is the most common approach in using visual representations (Steedly et al., 2008). According to Steedly et al. (2008), CRA is defined as "a three-part instructional strategy where the teacher uses concrete materials (manipulative) to model a concept to be learned, then uses representational terms (picture) and finally uses abstract, symbolic terms (numbers, math symbols)" (Steedly et al., 2008, p. 10). Over the past decades, there has been a large body of research that supports the effective use of CRA in mathematics instruction for students with LD (Manl et al., 2012). Research studies have proved the positive effects of CRA in teaching a wide range of mathematics skills, basic facts including addition (e.g., Mercer & Miller, 1992; Miller & Mercer, 1993), subtraction (e.g., Flores, 2009; Sealander, Johnson, Lockwood, & Medina, 2012), multiplication (e.g., Harris et al., 1995; Morin & Miller, 1998) and division (e.g., Mercer & Miller, 1992; Miller & Mercer, 1993), fractions (e.g., Butler, Miller, Crehan, Babbit, & Pierce, 2003), decimal place value (e.g., Peterson, Mercer, & O'Shea, 1988), and algebra (e.g., Maccini & Ruhl, 2000; Witzel, Mercer, & Miller, 2003) for students with mathematics disabilities. For example, Peterson et al. (1998) compared the effect of CRA versus abstract only on teaching initial place values for students with LD and the CRA group significantly outperformed the abstract only group. Butler et al. (2003) compared the use of CRA to the use of representational-only (RA) on fraction equivalency skills for students with high-incidence disabilities (most of them were students with LD) and it was reported that both instructions were effective, but CRA was more effective than RA for teaching students with LD. The effect of CRA has continuously been reported; a recent study (Sealander et al., 2012) also indicated CRA with direct instruction was effective for teaching subtraction skills to elementary students with LD.

Teaching Logically Sequenced or Ranged Mathematics Skills

According to Archer and Hughes (2010), teaching skills in logical sequence is one of the principles of explicit instruction. It is important to select and teach content in logical sequence (e.g., easier skills to harder skills, higher-frequency skills-less frequency skills, separate similar skills or strategies, review skills previously taught before moving to new skills). Gersten and colleagues (2009) analyzed 9 studies (Beirne-Smith, 1991; Butler et al., 2003; Kelly et al., 1990; Owen & Fuchs, 2002; Wilson & Sindelar, 1991; Witzel et al., 2003; Woodward, 2006; Xin et al., 2005) that examined content taught in a specific sequence or range. For example, Woodward (2006) organized multiplication facts into easy facts (×0s, ×1s, ×2s, ×5s, ×9s, and ×10s) and hard facts (×3s, ×4s, ×6s, ×7s, and x8s). The facts were taught sequentially from easy to hard. The facts taught were systematically reviewed before moving to learning a new fact. Wilson and Sindelar (1991) also divided word problems taught from simple to complex problems (simple action problems, classification problems, complex action problems, and comparison problems). As a result of the meta-analysis, Gertsen et al. (2009) reported a large effect size (.82) of teaching content thoughtfully sequenced or ranged on mathematics instruction for students with LD.

Providing Data and Feedback on Student Performance

It is well known that feedback is an integral instructional component of effective instruction (Rieth & Everston, 1988). Research has consistently reported feedback is strongly associated with the positive learning of students (Gersten, Carnine, & Woodward, 1987; Hattie & Timperly, 2007). According to NCTM (2007), providing assessment data to students (e.g., graph, charts, and scores) yielded a small to moderate effect size of .33 when combining results of three meta-analyses (Baker et al., 2002; Gersten et al., 2006; Krosebergen & Van Luit, 2003). Moreover, in a recent meta-analysis, Gersten and colleagues (2009) also reported a mean effect size of .23, which was statistically significant (p = .01) on providing assessment data to students. However, interestingly, their study found that providing feedback to students with goals (e.g., goal setting) was not effective for students with LD (effect size of .13, p = .29). Ongoing feedback on their performance or progress provided by teachers or classmates was effective for learning mathematics for students with LD, but it was not proved that participating students with LD in goal setting would be beneficial for their learning (Gersten et al., 2009).

Summary

According to the results of the previous studies, it is apparent that using CRA techniques, teaching logically sequenced and ranged mathematics skills, and providing data and feedback on performance to students are essential instructional components for effective mathematics instruction. Various effect sizes were reported; a large effect size for teaching logically sequenced and ranged mathematics skills, a moderate effect size for using CRA, and a small to moderate effect size for providing data and feedback to students.

SUMMARY OF CHAPTER

This chapter was designed to investigate and understand previous studies in effective mathematics instruction for students with LD. Especially the following areas were investigated: (a) mathematics competence of students with LD, (b) explicit, strategic instruction, (c) CBI, and (d) other effective instruction components. According to review of the literature, students with LD have great difficulties with a wide range of mathematics domains such as number sense, basic fact retrieval, computation fluency, and strategy usage. Thus, it is required to provide effective mathematics intervention to students with LD to compensate for their deficits in mathematics and support them to develop adequate mathematics skills. Especially, explicit, strategic instruction has been consistently reported as the most effective teaching method for students with LD for both basic and higher-order mathematics skills. CBI also has been recommended as one of the effective instructional methods for students with LD in mathematics, especially teaching basic mathematics skills. Recent research also reported that a new type of computer, tablet computers, could be promising as effective tools for teaching mathematics for students with LD. Moreover, previous studies determined using CRA techniques, instructing logically sequenced and ranged mathematics skills, and providing data and feedback on performance to students are effective instruction components for teaching mathematics for students with LD.

Chapter 3: Method

Computational fluency is essential to successfully develop higher-level mathematical skills such as geometry, algebra and fractions (Bana & Korbosky, 1995; Mazzocco et al., 2008; NGA/CCSSO, 2010; NMAP, 2008). According to NCTM (2000), computational fluency is defined as “having efficient and accurate methods for computing” (p. 152). Fluency with basic facts is a critical foundation not only for academic learning but also in many aspects of everyday life as well as in vocational settings (Bouck & Flangan, 2009; Miller & Heward, 1992; Nordness et al., 2011). Given the significance, the Curriculum Focal Points (NCTM, 2006) and the Common Core State Standards (CCSS) for mathematics (NGA/CCSSO, 2010) highlighted the importance of developing basic fact and computational fluency in mathematics curriculum. However, it is well documented that students with LD have difficulties with building basic fact fluency (Bottge et al., 2010; Geary et al., 2012). Many students with LD leave elementary school without mastering computational fluency (Koroesbergen & Van Luit, 2003). Students with LD often struggle with multiplication, which is an integral component in upper elementary mathematics curriculum (Cawley et al., 1996; Mabbott & Bisanz, 2008; Ozaki, Williams, & McLaughlan, 1996). Students with LD often struggle with recalling facts quickly (Mabbott & Bisanz, 2008; Rotem & Henik, 2013) and use immature strategies (e.g., finger-counting) for solving multiplication problems (Koscinski & Gast, 1993b). Their lack of multiplication skills often blocks their mathematics progress (Lombardo & Drabman, 1985).

Traditional mathematics instruction seems insufficient to develop adequate arithmetic skills for students with LD (Cawley et al., 1998) and many teachers are faced with the challenges of teaching mathematics to students with LD (Cihak & Bowlin, 2009). Thus, effective alternative instructional approaches based on empirical validation are required to support learning of students with LD in mathematics (Gersten et al., 2009; Koroesbergen & Van Luit, 2003). Without effective instruction, students with LD will continue struggling with mathematics and experience failure (Mercer & Miller, 1992).

Previous research has consistently found that explicit, strategic instruction is effective when teaching mathematics to students with LD (Gersten et al., 2009; Kroesbergen & Van Luitt, 2003; NMAP, 2008; Swanson, Hoskyn, & Lee, 1999). In addition, computer-based instruction (CBI) has been recommended as an effective approach to teach mathematics for students with LD (Vaughn & Bos, 2009) because CBI can be a valuable tool to help them build fact fluency by providing additional drill and practice opportunities (Christensen & Gerber, 1990; NMAP, 2008). As technology evolves, a new type of computer, the tablet computer (e.g., iPad), has emerged. Recent studies also found that educational applications available for tablet computers can be effective to teach mathematics for students with LD (e.g., Cihak & Bowlin, 2009). Previous research has found that when CBI is used with explicit teacher-directed instruction (TDI), its effectiveness is much greater than CBI only (Billingsley, Scheuermann, & Webber, 2009; Howell et al., 1987). Thus, research is required to investigate the effects of TDI with tablet computers to teach mathematics for students with LD.

Given the significant need of teaching basic math fact skills for students with LD and identifying effective mathematics instruction for them, this study is designed to investigate the effects of explicit, strategic TDI with iPad application practice on the performance in multiplication facts (i.e., ×4s and ×8s) of 5th grade students with LD who have IEP goals in mathematics The following research questions will guide this study:

1. What is the effect of explicit, strategic TDI with iPad application practice on the fluency of 5th grade students with LD, who have mathematics IEP goals, on single-digit multiplication number facts, ×4s and ×8s?
2. What is the effect of explicit, strategic TDI with iPad application practice on the use of a doubling strategy for solving single-digit multiplication number facts, ×4s and ×8s, among 5th grade students with LD, who have mathematics IEP goals?
3. How do 5th grade students with LD, who have mathematics IEP goals, maintain their fluency in single-digit multiplication number facts, ×4s and ×8s, 2 weeks following explicit, strategic TDI with iPad application practice?
4. What are the perspectives of 5th grade students with LD, who have mathematics IEP goals, toward explicit, strategic TDI with iPad application practice on learning multiplication facts, ×4s and ×8s?

This chapter describes the method that applied for this study including (a) a pilot study, (b) participants and setting, (c) research design, (d) measures, (e) materials, (f) procedures, and (g) data analysis.

A PILOT STUDY

A pilot study was conducted in the spring semester of 2012. Six 4th grade students with LD attending a university charter school in central Texas participated. All of the students had IEP goals in mathematics and had low performance on target multiplication number facts (×4s and ×8s). There were 4 boys and 2 girls and 4 of them were Hispanic and 2 were mixed race. The purpose of the study was to compare the effects of three instructional methods, teacher-directed instruction (TDI), applications only instruction (AI) or combined instruction (CI) in teaching multiplication number facts to students with LD. The following questions guided the study: (a) Which instructional approach, TDI, AI, or CI is more effective in teaching multiplication facts to students with mathematics learning disabilities?, and (b) What are the perspectives of students with mathematics learning disabilities and their mathematics interventionists towards TDI, AI, or CI after the intervention? A single-case design, alternating treatment design was used to compare the effects of the three instructional approaches on learning of multiplication facts for students with LD.

The participants were assigned to three groups based on their pre-test performance (e.g., ×2s test). Three instructional approaches were conducted five times each within a randomly determined sequence for a total of 15 sessions. A 30-minute intervention was conducted at the participant's special education classroom, 5 days for a week. Three trained interventionists provided instruction. In sessions 1 - 7, ×4s were taught, in sessions 8 - 10, ×8s were taught, and in sessions 11 - 15, a mix of ×4s and ×8s were taught. The dependent variable for this study was the performance on the target multiplication number facts; digits correct per minute (DC/M) on researcher-designed 2-minute daily probes containing 70 multiplication facts ($4 \times 0 - 4 \times 9$ and $8 \times 0 - 8 \times 9$) were recorded. Five alternative forms (A-E) were administered to each student in a counterbalanced order. Reliability of the alternative probes was between .84 and .94 (median = .93) and two researchers double-checked all scoring.

In TDI, explicit, strategic instruction was provided; a doubling strategy was taught for solving the target facts explicitly. The instructors reviewed pre-requisite skills and knowledge (preview: 3 minutes), provided explicit modeling on using a doubling strategy for solving target multiplication facts (modeled practice: 9 minutes), provided guided practice (9 minutes), students practiced the facts with paper-based worksheets (independent practice: 4 minutes) and worked on a daily probe (2 minutes). In AI, students used iPad applications to learn target multiplication facts. First of all, Math Drills, a flash-card type drill and practice application, was used for 8 minutes in the review mode providing a variety of scaffoldings and assistance (e.g., blocks, number lines, hints) that helped the students solve problems and for 8 minutes in the practice mode (solving problems without assistance or scaffoldings). Then students used Math Evolve, a game-type drill and practice application to solve problems for 8 minutes. In CI, TDI and AI were combined; for modeled practice and guided practice, TDI was provided, and for preview and independent practice, iPad applications were used. The interventionists strictly followed the scripted lessons during TDI and CI, and for AI, no instruction, assistance, and feedback was provided to students unless they experienced technical problems with their iPads or applications. Fidelity of implementation of intervention also was measured; the average fidelity reported was 96% for both TDI and CI, and 94% for AI. In addition, a social validity interview was conducted to examine students' and interventionists' thoughts toward instructional approaches.

For research question 1, mixed results were found. Figure 3.1 and 3.2 are graphs for students' progress data in daily probes (DC/M). It was detected that most of the students (n = 5) increased their DC/M across the study. However, because of the carry-over effect, it was not easy to examine which instruction was more effective for teaching students with LD with the graphs. Table 3.1 presents an analysis of levels (the mean scores for the intervention data; Morgan & Morgan, 2009). Four out of six students (students 1, 2, 3, 6) had the highest level and three students (students 1, 3, 6) had the lowest level. When analyzing the level of individual students, it was found that most of the students demonstrated the strongest performance in TDI. However, it should be interpreted cautiously because when establishing an average across six participants, CI was found to be slightly more effective than TDI, followed by AI. Table 3.2. presents an analysis of the trend (the slope of the best-fitting line describing data within a phase, Kratochwill et al., 2010). Microsoft Excel was used to find slopes for the best-fitting straight line for each instruction method (i.e., TDI, CI, AI). If an approach had the best slope, 3 points were assigned, 2 points for the second best and 1 point for the lowest slope. It was detected that TDI had the most points (14), CI had the second-highest number of points (13) followed by AI (10 points). Thus, it appears that TDI and CI were more effective than AI for most of the participants.

Student	TDI	CI	AI
1	42.00 (6.9)	40.20 (11.4)	31.20 (6.2)
2	47.75 (22.0)	32.75 (29.9)	41.60 (16.1)
3	10.60 (3.8)	9.40 (5.7)	8.40 (5.4)
4	11.60 (7.8)	14.80 (5.5)	18.20 (9.4)
5	20.00 (8.8)	27.20 (6.5)	23.60 (4.8)
6	50.75 (10.2)	55.20 (6.9)	49.50 (11.3)
Average	29.1 (19.2)	29.8 (19.3)	28.0 (16.0)

Table 3. 1: Means and Standard Deviations on Daily Probes for Three Intervention Approaches

Student	Beginning and Ending Score			Slope		
	TDI	CI	AI	TDI	CI	AI
1	37-42	34-48	25-38	2.5	5.2	3.2
2	15-60	6-55	15-60	14.1	20.1	7.2
3	7-7	4-8	4-7	-.01	1.8	-0.7
4	5-19	9-23	3-21	4.4	3.4	3.7
5	12-25	19-28	20-32	4.4	3.0	3.0
6	46-66	50-51	40-64	6.1	1.4	6.0

Table 3. 2: Data Depicting Performance for Each Student and Condition

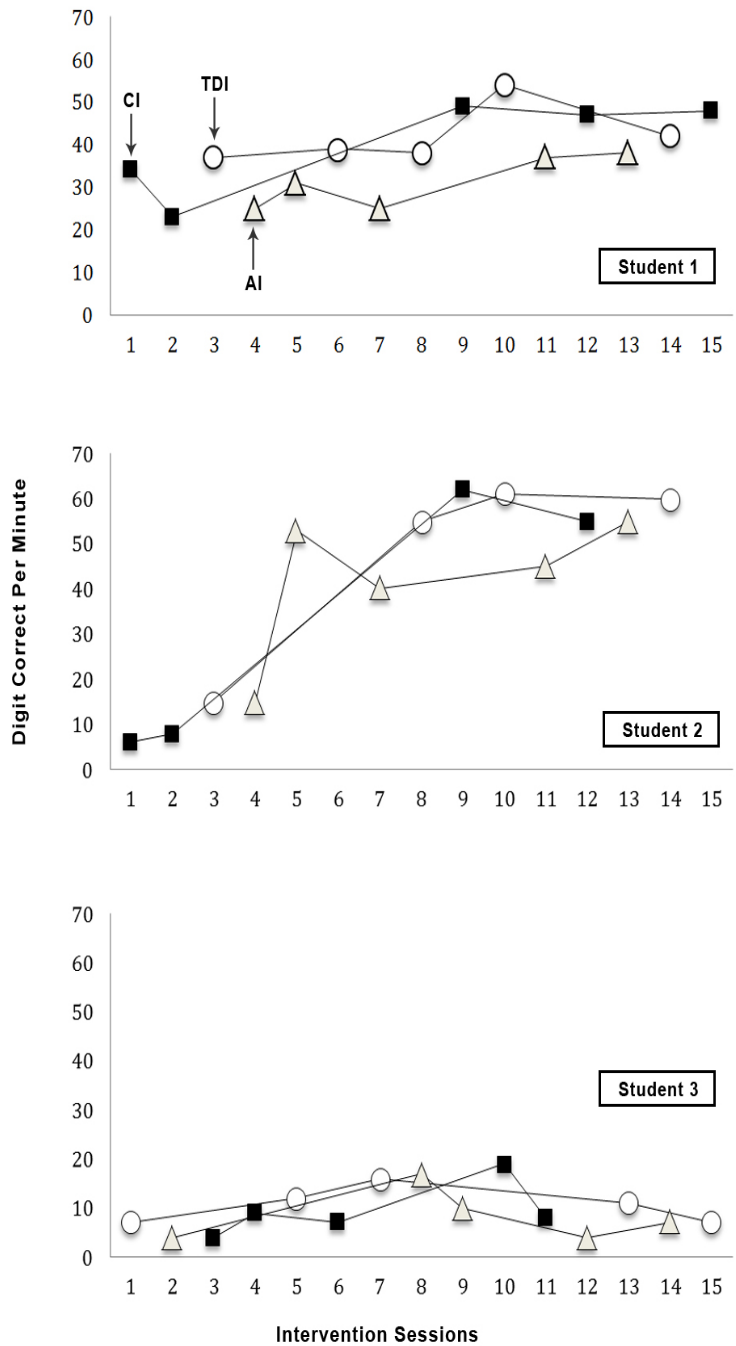


Figure 3. 1: Graphs for students 1, 2, and 3.

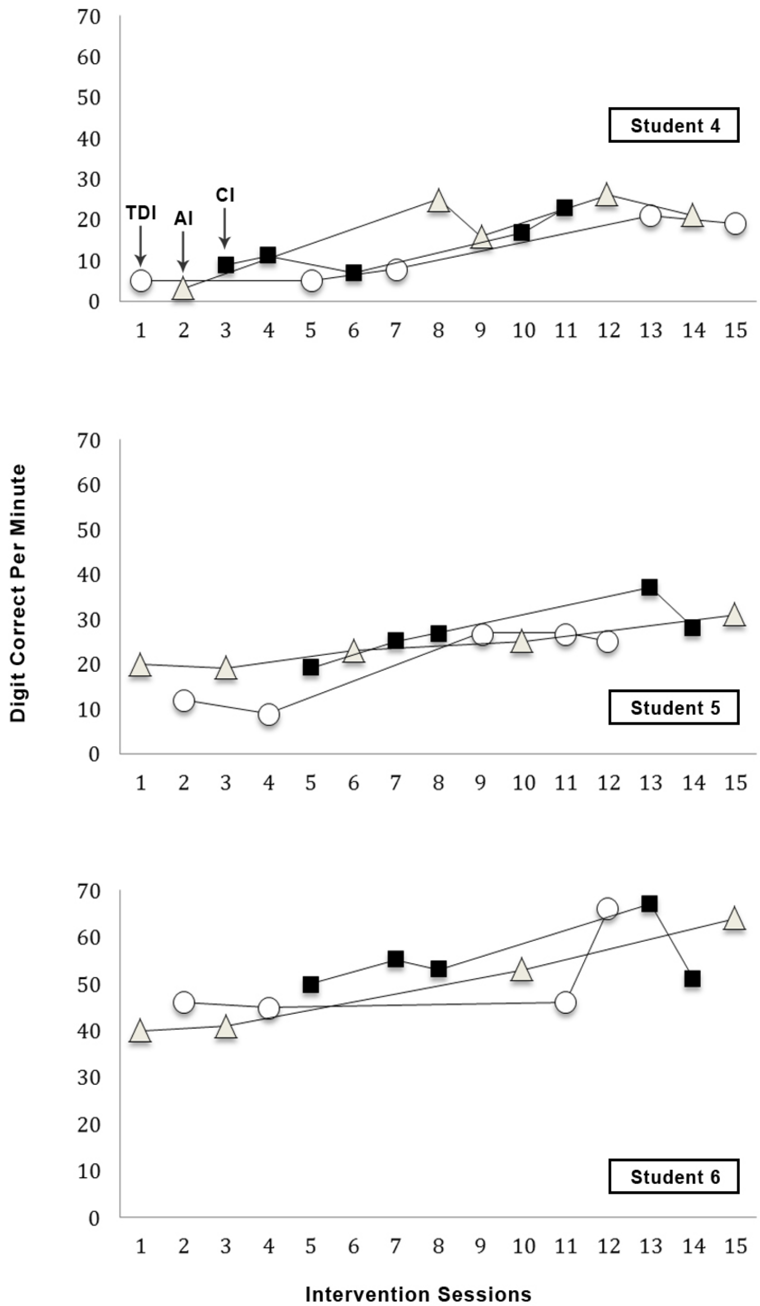


Figure 3. 2: Graphs for students 4, 5, and 6.

For research question 2, interviews were conducted with both students and interventionists. Mixed results were found; all three approaches were favorable for both students and interventionists. Students reported their favorite instructional approaches were TDI (n = 3) and AI (n = 3), the approaches that helped them be the most engaged was CI (n = 3), TDI (n = 2), and AI (n = 1), the approach helping them learn the most was CI (n = 5) and TDI (n = 1), and the approach helping them stay on-task was CI (n = 4) and TDI (n = 2). It was interesting to see that no students selected AI when they were asked which instructional approach helped them learn the most.

The results showed that all three instructional approaches (TDI, CI, AI) could be effective for enhancing the multiplication basic fact fluency of students with LD. However, CI was most effective, followed by TDI, and AI; it suggests CI or TDI could be more effective than AI for teaching students with LD basic fact skills. The findings align with what the previous literature reported (e.g., Howell et al., 1987); when technology is integrated in TDI, it could be more effective than technology-only instruction for teaching students with LD. It suggested that even though technology-based instruction could provide multiple opportunities and increase students' motivation to practice skills, teachers should be cautious before employing technology-based instruction to replace TDI to teach basic skills. Thus, educators should determine how to best use the tools available to them and effectively integrate technology-based instruction into their instruction. Based on the findings of the pilot study, it was decided that CI would be used for teaching students with LD for this study.

PARTICIPANTS AND SETTING

Participants

Four 5th grade students with LD attending two elementary schools (a public school and a state charter school) located in central Texas participated in this study. To recruit the participants, the investigator contacted the principals at the schools, informed them of the study and obtained permission for conducting the study in their schools. After giving their permission, the principals guided the investigator on how to obtain district or school board approval for the study. For the public school, the investigator filled out a district research proposal application and submitted it to a district administrator. The administrator reviewed it, had a phone conversation with the investigator to ask questions, and provided a site letter that indicated the district's permission to conduct the study in the school. For the charter school, the investigator was asked to submit a research proposal to the principal. She sent the proposal to the school management team for approval and provided a site letter after obtaining the management team's approval.

In addition, the investigator worked with special education teachers in each school to recruit participants in grades four or five who had mathematics IEP goals and who were interested in participating in the study. The investigator visited the teachers in person to provide specific information about the study (e.g., participant selection criteria, duration, intervention, measures). The investigator also provided parent consent and student assent forms to the teachers. The forms were written in English and included information about the study and participation (e.g., purpose of the study, procedures, possible benefits and risks, privacy and confidentiality protection). The forms were reviewed with the teachers and the investigator answered any questions that the teachers had. After the meeting, the teachers sent out the forms to the students' homes. The students' parents or legal guardian were asked to review the forms and submit the consent form with their signature if they allowed their child to participate in the study. Students were also asked to sign the assent form if they wanted to participate in the study.

Only students who submitted both the consent and assent forms were assessed with a pre-test to identify if they met the participant selection criteria (demonstrate low fluency on target multiplication number fact skills). To be eligible to participate in the study, students had to meet the following criteria: (a) be enrolled in the 4^{th} or 5^{th} grade, (b) be identified as having LD by their school district, (c) have mathematics IEP goals, (d) demonstrate low fluency on multiplication number facts (i.e., ×4s and ×8s), score at the frustration level (score lower than 20 digits correct per minute [DC/M]) on the pre-test, and (e) return parental consent and student assent forms. Only students who met all five criteria were selected to participate in this study.

Four students who met the criteria were invited to participate in this study. All students were in grade 5, identified as having LD by the school district, had mathematics IEP goals, and demonstrated lack of target multiplication fact skills (×4s and ×8s). Two students (James and Kate) were attending a public school and the other two students (Amy and Perry) were attending a charter school. These names (James, Kate, Amy and Perry) are pseudonyms for the participants' names. Table 3.3 provides participants' demographic and testing information including age, gender, race/ethnicity, disability, free or reduced lunch status, English Language Learner (ELL) status, standardized test scores (mathematics, reading, writing), and the pre-test scores.

James

James is a mixed-race male and at the start of the intervention, he was 10 years 5 months old. In addition to being identified as having a LD, he was also identified as having attention deficit hyperactivity disorder (ADHD) and he qualified for free/reduced lunch. According to the Woodcock-Johnson III Test of Achievement (WJ-III) results reported in his Full Individual Evaluation (FIE), which was conducted in May 2012, he scored a standard score (SS) of 84 in mathematics, a 98 SS in reading, and a 92 SS in writing. His Broad Math score, a comprehensive measure of mathematics achievement, was in the low average range (84) and in particular, he demonstrated deficits in computing mathematics problems accurately and quickly (a 81 SS in Math Calculation) as well as analyzing and solving practical problems (a 88 SS in Applied Problems). According to his IEP, in addition to his general education mathematics class, he was receiving one-hour mathematics intervention in his special education class for four days a week. He had mathematics IEP goals, which focused on place value, fractions and word problem solving skills. On the pre-test, he demonstrated fact fluency ability at the frustration level (13.5 DC/M); he was able to solve only two hard facts (4×3 and 4×4) and six easy facts (e.g., 4×0, 4×1, 4×2, 4×5, 8×0, 8×1). He solved 16 problems out of 60 in 2 minutes; his accuracy was 100%.

Kate

Kate is a Hispanic female. She was 11 years and 1 month at the start of the intervention. According to her FIE, in addition to being identified as having a LD, she was identified as an English Language Learner (ELL). She also qualified for a free/reduced lunch. According to the WJ-III results reported in her FIE, which was conducted in April 2012, she scored a SS of 87 in mathematics, a SS 78 in reading, and a SS 78 in writing. Her WJ-III scores indicated that in particular, she demonstrated deficits in the following areas: reading comprehension (a 75 SS in Reading Comprehension), reading fluency (a 80 SS in Reading Fluency), written expression (a 78 in Basic Writing Skills), and mathematics problem solving (a 84 SS in Math Reasoning and a 83 in Applied Problems). According to her IEP, in addition to her general education mathematics class, she was receiving one-hour mathematics intervention in her special education classroom for four days per week. Her mathematics IEP goals focused on developing problem solving skills. On the pre-test, she demonstrated fluency ability at the frustration level (13.5 DC/M); she was able to solve three hard facts (4×3, 4×4, 8×3) and seven easy facts (4×0, 4×1, 4×2, 4×5, 8×0, 8×1, 8×2). She solved 15 problems out of 60 in 2 minutes; her accuracy was 100%.

Amy

Amy is a Hispanic female. At the start of the intervention she was 10 years and 5 months old. According to her FIE, in addition to being identified as having a LD, she was also identified as an ELL; her FIE reported that her limited English proficiency might have impacted her educational achievement. The Kaufman Test of Educational

Achievement II (KTEA-II) reported in her FIE conducted in May 2011, she scored a 103 of SS in mathematics, a 98 SS in reading, and a 97 SS in writing. In FIE, her teachers reported her reading skills were below grade level; according to KTEA-II scores, she demonstrated significant educational and development deficits in the following areas, phonological processing (a 83 SS in Phonological Awareness), decoding skills (a 84 SS in Nonsense Word Decoding and a 85 SS in Decoding Composite), and reading fluency (a 81 SS in Sound-Symbol Composite). In addition, even though her mathematics skills were measured to be in the average range (a 100 SS in Math Computation and a 103 SS in Math Concepts and Applications), according to her FIE, her teacher estimated that her mathematics computation skills were below grade level because she had not mastered mathematics facts. In addition, her reasoning was also below grade level but stronger than her computational skills; she had difficulties with word problems. According to her IEP, in addition to her general education mathematics class, she was receiving one-hour mathematics interventions in her special education class for five days a week. Her mathematics IEP goals focused on developing fraction and word problem solving skills. On the pre-test, she demonstrated fact fluency ability at the frustration level (5.5 DC/M); she was able to solve only two hard facts (4 × 3, 4 × 4) and six easy facts (4 × 0, 4 × 1, 4 × 2, 4 × 5, 8 × 0, 8 × 1). She solved 8 problems out of 60 in 2 minutes; her accuracy was 100%.

Perry

Perry is a Caucasian male. He was 11 years and 1 month old at the beginning of the intervention. He was identified as having LD. According to the KTEA-III results reported in his FIE, which was conducted in November 2012, he scored a 93 SS of in mathematics, a SS 81 in reading, and a SS 72 in writing. In particular, he demonstrated significant developmental deficits in basic reading skills (a 82 SS in Reading Composite), reading fluency (a 71 SS in Reading Fluency), and written expression (a SS 72 in Written Language Composite). In addition, even though his overall mathematics skills (a 93 SS in Math Composite and a 103 SS in Math Concepts and Applications) were measured to be in the average range, in his FIE, his teachers reported his computation skills were below grade level (a 84 SS in Math Computation) and he was not automatic with mathematics facts. In addition to his general education mathematics class, he was receiving one-hour mathematics intervention in his special education classroom for five days a week. His mathematics IEP goals focused on developing fractions and word problem solving skills. He demonstrated fact fluency ability at the frustration level (16.5 DC/M) on the pre-test; he was able to solve six hard facts (4×3, 4×4, 4×6, 4×7, 4×8, 4×9) and six easy facts (4×0, 4×1, 4×2, 4×5, 8×0, 8×1). It seemed like Perry was knowledgeable at $\times 4$s, but he was very slow to compute the facts. He solved 19 out of 60 problems in 2 minutes; his accuracy was 100%.

	James	Kate	Amy	Perry
Age (Years. Months)	10.5	11.1	10.5	11.1
Grade	5	5	5	5
Gender	M	F	F	M
Race/Ethnicity	Mixed Race	Hispanic	Hispanic	Caucasian
Disability	LD	LD	LD	LD
Free/Reduced Lunch	Y	Y	N	N
ELL	N	Y	Y	N
Mathematics	84*	87*	103**	93**
Reading	98*	78*	98**	81**
Writing	92*	78*	97**	72**
Pre-test (DC/M)	13.5	13.5	5.5	16.5

Note. M = male; F = female; Y = yes; N = no; *Woodcock-Johnson III Test of Achievement; ** Kaufman Test of Educational Achievement II (KTEA-II).

Table 3. 3: Participants' Demographic and Testing Information

Setting

This study was conducted in two elementary schools located in Central Texas. One of the schools was a public elementary school in a school district serving about 8,000 students. The school served 605 students in pre-K through 5th grade. The students consisted of 74% White, 12% Asian, 10% Hispanic, 2% Multi-Race, and 1% African-American. In addition, 10% of the students were served in special education, 9% were gifted and talented students, 4% had limited English proficiency and 1% was economically disadvantaged. The school was considered high achieving; their 2011 Federal Accountability Rating was "Exemplary", the highest possible ranking in school performance. The second school was a state charter primary school serving 305 students in pre- K through 5^{th} grade. More than half (64%) of the students were Hispanic, 19% were White, 15% were African American, and 1% was Asian. Moreover, 10% of the students were served in special education, 7% were gifted and talented students, 8% had limited English proficiency and 61% were economically disadvantaged. This information was obtained from each school's website.

All intervention sessions occurred within the participants' elementary school. One-on-one instruction was delivered during tutoring time in small classrooms available, which were located near the participants' special education classroom. During most of intervention sessions, only the researcher and a participant occupied the classroom. However, other teachers sometimes stayed in the classroom; they worked with their computers quietly, so their existence was not disruptive to the tutoring. Each intervention session lasted 30 minutes and a total of 15 sessions occurred for 5 days per week (Monday through Friday). The intervention was delivered in a different time period for each participant (James: 12:10 – 12:40 pm, Kate: 3:15 – 3:45 pm, Amy: 9:30 – 10:00 am, Perry: 10:10 – 10:40 am).

RESEARCH DESIGN

A single-case, multiple probe design across participants, was employed to conduct this study (Kennedy, 2005). Single-case design is a rigorous, experimental design to investigate intervention effect on outcomes (Horner et al., 2005; Kenney, 2005). Outcomes are measured repeatedly across time to determine if a causal relation exists between changes in intervention and outcomes (Kenney, 2005). The single-case design is highly flexible and beneficial to understand individual participants, so it has been well suited and widely used in the special education field (Horner et al., 2005; Kratochwill et al., 2010), especially, when examining effects of interventions for basic fact math skills, multiple baseline design has been widely used (e.g., Codding, Archer, & Connell, 2010; Irish, 2002; Walker, McLaughlin, Derby, & Weber, 2012; Wood, Frank, & Wacker, 1998). Traditional multiple baseline designs establish two or more baselines simultaneously and introduce intervention conditions sequentially across participants, settings or behaviors after a stable baseline is observed (Horner et al., 2005; Kennedy, 2005; Kratochwill et al., 2010). To establish experimental control, single-case design studies determine whether a functional relation exists (Kratochwill et al., 2010). According to Kennedy (2005), a functional relation stands for "establishing a consistent effect on a dependent variable by systematically manipulating an independent variable" (p. 28). For example, in multiple baselines across participants design, experimental control is demonstrated when a change in the dependent variable is observed only after the intervention is introduced and when this trend is observed across participants (Horner et al., 2005; Kratochwill et al., 2010). This design is ethically desired because it does not require withdrawing potentially beneficial intervention for participants unlike other single-case designs such as ABAB design (Kennedy, 2005).

However, traditional multiple baseline designs have some drawbacks due to continuously repeated baseline measurements. For example, if there are six participants in the study, the last participant needs to be assessed for a long time before entering the intervention phase. According to Kazdin (2003), changes in outcomes could occur due to the repeated measurement. Also, according to Horner and Baer (1987), the continuous measurement is not necessary if sufficient baseline data representing a stability of levels, trends, and variability is obtained. In addition, the continuous measurement could be impractical, impossible, or too expensive (Cooper, Heron, & Heward, 2007). Thus, Horner and Baer (1987) introduced the multiple probe design, a variation of the multiple baseline design, as a more efficient design for researchers. In the multiple probe design, the researcher only collects baseline data intermittently but consistently, rather than continuously (Kennedy, 2005). Data collection occurs at a strategic time, before and after the treatment introduction for each intervention (Kennedy, 2005). The multiple probe baseline design can also help researchers decrease the amount of time and effort required for collecting and scoring data (Kennedy, 2005). This design has been widely used for basic fact skill intervention studies (Koscinski & Gast, 1993a, 1993b; McCallum, Skinner, Turner, & Saecker, 2006; Rao & Mallow, 2009; Williams & Collins, 1994). Thus, to investigate the effects of explicit, strategic TDI with iPad application practice on performance on single-digit multiplication number facts for this study, the multiple probe design across participants was used.

In multiple probe baseline design across participants, staggered intervention is introduced over time across the participants (Kratochwill et al., 2010). For example, if there are four participants, after the consistent pattern of fluency on daily probes for the first participant is observed, he or she enters the intervention phase while the other participants remain in the baseline phase. The second participant enters the intervention phase after the first participant establishes a consistent pattern of levels, trends, and variability representing an intervention effect (e.g., immediately increased level, nonoverlapped data with baseline data) while the third and the fourth participants stay in the baseline phase. This was repeated until the last participant enters the intervention phase. In addition, it is recommended that the participant's order for beginning the intervention phase is decided randomly (Kratochwill et al., 2010).

Experimental control is demonstrated when the outcome changes for each participant only after the intervention has been introduced and when this trend is observed across participants (Horner et al., 2005; Kennedy, 2005). When the functional relation is replicated for more participants, the experimental effect would be more convincing (Kennedy, 2005; Kratochwill & Levin, 2010). To meet the evidence standards, at least three replications are recommended (Kratochwill et al., 2010). Thus, for research question 1, the experimental effect of explicit, strategic TDI with iPad application practice was demonstrated if participants improved their fluency (DC/M) on single-digit multiplication number facts (i.e., ×4s and ×8s) on daily probes only after explicit, strategic TDI with iPad application practice was implemented without influencing the fluency of other participants remaining in the baseline phase, and this trend was replicated across participants (Kratochwill et al., 2010). For research question 2, the experimental effect of explicit, strategic TDI with iPad application practice was determined if participants improved using a doubling strategy for solving multiplication number facts only after the intervention was introduced without having an impact on the use of the strategy for other participants remaining in the baseline phase and this pattern was replicated across the participants.

Independent Variable

The independent variable of this study was explicit, strategic TDI with iPad application practice. The intervention consisted of three main components: (a) TDI using explicit, strategic instruction for teaching multiplication number facts (i.e., warm-up, modeling, and guided practice); (b) independent practice with iPad application practice; and (c) daily probes and graphing data.

Dependent Variable

Table 3.4 summarizes the dependent variables for each research question for this study. First of all, for research questions 1 and 3, the dependent variable was the fluency of participants in solving single-digit multiplication number fact equations, ×4s (i.e., facts with a factor of 4 and the other factor is 0-9) and ×8s (i.e., facts with a factor of 8 and the other factor is 0-9). Computational fluency is defined as “having efficient and accurate methods for computing” (NCTM, 2000, p. 152). Teaching multiplication fact fluency skills met the CCSS guidelines for mathematics grade 3 (3.0A, 3.NBT) and the State of Texas Assessments of Academic Readiness (STAAR) grade 3 (3.4, 3.6) and grade 4 (4.4, 4.6). Participant’s performance was measured with the digits correct per minute (DC/M) in 2-minute researcher-developed daily probes (Deno & Mirkin, 1977; Shapiro, 2010). The number of digits correct for 2 minutes was calculated and then divided by 2 to calculate DC/M (Shapiro, 2010). According to Deno and Mirkin (1977), DC/M is a more efficient way to understand and provide scores for partial math competencies of students than traditional scoring methods, which usually score the total number of correct items. Two multiplication facts (i.e., ×4s and ×8s) were selected to teach in this study because the facts are considered to be harder than other facts (e.g., ×0s, ×1s, ×2s, ×5s; Stein, Kinder, Silbert, & Carnine, 2006; Woodward, 2006); additionally, the same strategy (i.e., doubling strategy) can be used to solve both ×4s and ×8s multiplication number facts. Second, dependent variables for research question 2 were (a) accuracy of solving multiplication number fact equations (total number of items correct divided by number of items attempted, multiplied by 100) on the strategy usage test, and (b) percentage of the use of a doubling strategy (total number of uses of the doubling strategy divided by total number of items in the probe (10) and then multiplied by 100) observed to examine if participants improved over time regarding the use of the doubling strategy for solving the targeted multiplication number facts (i.e., ×4s and ×8s). Finally, the dependent variable for research question 4 was the participant's response to 20 open-ended and close-ended questions to understand participants' perspectives toward explicit, strategic TDI with iPad application practice. The mean scores of 5 scales in the close-ended questions were recorded and answers to the open-ended questions were analyzed.

Research Question	DV	Measures
1. What is the effect of explicit, strategic TDI with iPad application practice on the fluency of 5th grade students with LD on single-digit multiplication number facts, ×4s and ×8s?	✓ Fluency (Digits correct per minute (DC/M))	✓ 2-minute researcher-developed daily probes (60 items, A-E forms)
2. What is the effect of explicit, strategic TDI with iPad application practice on the use of a doubling strategy for solving multiplication number facts, ×4s and ×8s, among 5th grade students with LD?	✓ Accuracy: (total # of correct items/# of items attempted in the probe) x 100 ✓ Percentage of doubling strategy usage: (total # of the use of the doubling strategy /total # of items in the probe) x 100	✓ Researcher-developed strategy usage probe (10 items) ✓ Researcher-developed strategy usage observation form (8 strategy categories)
3. How do 5th grade students with LD maintain their fluency on single-digit multiplication number facts, ×4s and ×8s, 2 weeks following explicit, strategic TDI with iPad application practice?	✓ Fluency (DC/M)	✓ 2-minute researcher-developed maintenance test (Two daily probes were used)
4. What are the perspectives of 5th grade students with LD toward explicit, strategic TDI with iPad application practice on learning multiplication number facts, ×4s and ×8s?	✓ 16 Close-ended question answers (mean score of scales) ✓ 4 Open-ended question answers	✓ Researcher-developed social validity interview form (20 open/ended questions)

Table 3. 4: Research Questions, Dependent Variables, and Measures

MEASURES

Pre-Test

This study was designed to improve multiplication number fact fluency (i.e., ×4s and ×8s) of students who had low fluency in the target skill. Thus, students were assessed on their performance on the target skills. Form A, one of the five forms (A-E) developed as daily probes, was used as a pre-test. Students were asked to solve as many as problems as they could in 2 minutes (Shapiro, 2010). Only students who scored at the frustration level (i.e., 0-19 DC/M; based on the criteria levels for assessments [Deno & Mirkin , 1977]) were selected to participate in this study. The survey level of assessments can be used for determining the level of instructional materials for students (Shapiro, 2010). The following are the levels of assessment for students who are in grade 4 or above: (a) frustration level (0-19 DC/M): material is too challenging, (b) instructional level (20-39 DC/M): material is appropriately challenging, and (c) mastery level (40+ DC/M): material is mastered.

Daily Probes

At each baseline session and at the end of each intervention session, participants were asked to complete a 2-minute daily probe on targeted multiplication number facts (i.e., ×4s and ×8s). The researcher-developed paper-and-pencil based daily probes contained 60 multiplication facts of ×4s and ×8s (30 of each) and all problems were represented in vertical formats. The form consisted of 6 rows and each row had 10 problems. All possible facts including single-digit multiplication ×4s and ×8s facts (i.e., 4 × 0 - 4 × 9, 8 × 0 - 8 × 9, and the commutative property facts) were listed first then assigned in the probes. The first two rows contained ×4s, the next two rows contained ×8s and the last two rows contained a mix of ×4s and ×8s. This sequence (i.e., ×4s, ×8s, and a mix) was decided based on lesson sequence (i.e., teaching ×4s, ×8s, then a mix). Not only target facts (i.e., 4 × 3, 4 × 4, 4 × 6, 4 × 8, 4 × 9, 8 × 3, 8 × 6, 8 × 7, 8 × 8, 8 × 9 and the commutative property facts) but also easy, review facts (i.e., 4 × 0, 4 × 1, 4 × 2, 4 × 5, 8 × 0, 8 × 1, 8 × 2, 8 × 5 and the commutative property facts) were included in daily probes. The easy, review facts were located at the beginning of the each row that ×4s and ×8s problems were first introduced (i.e., the first row and the third row). According to Cybriwsky and Schuster (1990), the known facts in the probes improve student's opportunity for responding correctly and help students to stay on-task (i.e., solving the problems). After the easy, known facts were placed, facts randomly selected from all target facts including commutative property facts were placed in the form. Five alternate forms of the probe (A-E) were developed (Kratochwill et al., 2010). After assigning the facts in the probes, it was verified whether all targeted facts were assigned an equal number of times across the five forms (Pfannestiel, 2011) based on the recommendation for math CBM, "different but equivalent math sheets" (Hosp et al., 2007, p. 98). Participants were assessed with the daily probes in the order of A, B, C, D, and E in each session during the baseline phase and intervention phase. Because Form A was used as a pre-test, participants were assessed from Form B when starting the baseline phase. When finishing Form E, participants went back to Form A and the order was repeated by the end of the study. DC/M was calculated (i.e., total number of DC/M divided by 2) and recorded to measure participants' progress (Shapiro, 2010). An example of daily probes is located in Appendix A.

Maintenance Test

A maintenance test was administered to participants at 2 weeks after the intervention phase was terminated to determine whether the explicit, strategic TDI with iPad application practice influenced participants' performance on multiplication number facts (i.e., ×4s and ×8s) over time. The maintenance effect was assessed with two of the 2-minute daily probes. For example, if participants used Form C in the last intervention session, they were assessed with Form D and Form E for the maintenance test (in 2 weeks). The average DC/M of two tests was calculated and recorded.

All tests including the pre-test, daily probes and the maintenance tests were administrated based on recommendations for mathematics CBM (Hosp et al, 2007; Shapiro, 2010). The investigator followed the following directions for administrating the tests (Hosp et al., 2007; Shapiro, 2010): (a) "Eyes on me." (Check student's attention), (b) "All problems are single-digit multiplication facts, ×4s and ×8s. When I say 'Begin,' you will have 2 minutes to do as many multiplication problems as you can. Work from left to right in each row." (Model with a finger left to right on the probe), (c) "Keep going until I say 'Stop' or until you come to the end of the page. If you see a problem you do not know, skip it and go on and try the next one.", (d) "Ready?, Begin." (Start a timer), and (e) "Stop. Put your pencil down." (When the timer vibrates). Participants were also provided a blank sheet of paper to help them solve problems, if needed.

Strategy Usage Test

To answer research question 2, the participant's strategy usage data was collected three times (i.e., before, in the middle of, and after the intervention phase) across the study and analyzed to determine whether participants used more the doubling strategy to solve the multiplication fact problems across the study. The participants were asked to complete 10 multiplication problems (e.g., 3×4, 8×7, 6×8) randomly selected from the target multiplication number facts for this study. To compare changes in participants' strategy usage across the study, three assessment forms (A, B, C) including the same 10 questions, but assigned differently, were used. Prior to the test, the participants were informed that they would be asked to explain their strategies for solving the problems in the probe. The strategy usage assessment form included enough blank space for the participants to use to solve the problems if needed. The participants were asked to solve each problem in 30 seconds. The investigator observed participants' strategy usage while solving the problem. After the 30 seconds, the participants were immediately asked to describe how they solved the problem (Mabbott & Bisanz, 2008). The participant's answer was audio-recorded. The investigator listened to the student's audio-recorded description before making a decision on a strategy used for solving each problem. The audio-recordings also were used for any further verification or analysis and checking inter-scorer agreement. Based on the observation of the participant's behavior, written solutions on the probe and verbal self-description, the investigator recorded the participant's strategy usage on the researcher-developed observation form (Mabbott & Bisanz, 2008).

The observation form was developed based on previous literatures (Geary, 2004; Mabbott & Bisanz, 2008; Sherin & Fuson, 2005). A taxonomy of strategies for single-digit multiplication (Sherin & Fuson, 2005, p. 355), procedures for solving simple multiplication problems (Mabbott & Bisanz, 2008, p. 18) and children's characteristics in arithmetic (Geary, 2004, p. 7) were combined and modified for the strategy categories on the form. The taxonomy of strategies for single-digit multiplication developed by Sherin and Fuson (2005) was primarily used. The following are the eight strategy categories on the observation form: (a) Guess: participants noted they guessed the answer (Mabbott & Bisanz, 2008), (b) Count-all: participants counted from 1 to the product including paper-based count-all (e.g., 4 × 3: count after drawing four groups of 3 dots), finger-based count-all (count aloud from one to total with finger assistance) and rhythmic counting with fingers (count out loud while highlighting at each multiple of the group size verbally as student's fingers keep track of the number of groups) (Geary, 2004; Sherin & Fuson, 2005), (c) Additive calculation: participants solve multiplication facts using understanding of addition including repeated addition (e.g., 4 × 3 : 4 + 4 = 8 and 8 + 4 = 12) or collapse groups and add (e.g., 4 × 8: drawing 4 groups of 8 tallies, adding two pairs of 4 to get 8 (4 + 4 = 8), and then adding two pairs of 8 (8 + 8 = 16) (Mabbott & Bisanz, 2008; Sherin & Fuson, 2005), (d) Count-by: participants used sequenced counting to solve the multiplication fact (e.g., 2 × 4: counting by 2, 4, 6, 8) using a drawing (e.g., 2 × 3: drawing / / / and counting each / as 2, 4, 6), written numbers (e.g., 5 × 3: writing 5 5 5 and counting each 5 as 5, 10, 15), and fingers (Mabbott & Bisanz, 2008; Sherin & Fuson, 2005), (e) Pattern-based: participants use rule-based strategies (e.g., ×0s: $0 \times N = 0$, ×1s: $1 \times N = N$) (Mabbot & Bisanz, 2008; Sherin & Fuson, 2005), (f) Learned product (also known as automatic retrieval): participants recall the product automatically (Geary, 2004; Mabbott & Bisanz, 2008; Sherin & Fuson, 2005); when participants answered immediately (in 3-4 seconds), they were considered to be using the learned product strategy meaning they knew the answer automatically, (g) Hybrids: participants use mixed strategies such as doubling strategies (e.g., $7 \times 4 = 28$, $7 \times 2 = 14$, $7 \times 2 = 14$, $14 + 14 = 28$: using split factors, retrieval and additive calculation) (Geary, 2004; Mabbott & Bisanz, 2008; Sherin & Fuson, 2005), and (h): Other: an ambiguous answer that does not fit any category (Mabbott & Bisanz, 2008).

Participants' strategy use for each multiplication fact problem was coded according to the eight categories; the investigator placed "×" marks on the corresponding category for each problem. The investigator also wrote specific notes or comments about participants' strategy usage for each problem if needed. The accuracy of solving the problems in the probe (i.e., the number of correct items divided by the number of items attempted) was recorded (Kroesbergen et al., 2004). In addition, the percentage of use of a doubling strategy (i.e., the total number of times the doubling strategy was used divided by the total number of items (10) in the probe and multiplied by 100) was recorded. Specifically, for recording the use of the doubling strategy, even though the investigator marked "X" in the Hybrids category when participants used mixed strategies, it was noted what specific strategies were used. Sherin and Fuson (2005), found that children typically develop and employ more mature strategies for solving multiplication fact problems as they get old; the researchers also found that they normally deploy simpler strategies (e.g., count-all, simple additive calculation) first and then later progress to more advanced strategies (e.g., learned product, hybrids). The strategy usage test is located in Appendix B and the student's strategy usage observation form is located in Appendix C.

Inter-Scorer Agreement

The investigator scored all tests (i.e., the pre-test, daily probes, the maintenance tests, and strategy usage tests) across baseline, intervention and maintenance phases. Then, two other scorers independently scored the tests to determine inter-scorer agreement; all tests were double-scored by the scorers. One of the scorers independently scored the pre-test, daily probes, and the maintenance tests and another scorer independently scored the strategy usage tests. For both scorers, inter-scorer agreement (between the investigator and each scorer) was 99% and a discussion between the scorers and the investigator occurred until they reached 100% agreement. Inter-scorer agreement was calculated by using the formula, number of agreements of participant's responses divided by the number of agreements plus disagreements multiplied by 100 (Cihak & Bowlin, 2009; Haydon et al., 2012). According to Neuendorf (2002), a percentage value of 90 or greater would be considered as a highly acceptable inter-scorer agreement.

MATERIALS

Table 3.5 represents the materials that were used for this study. During the baseline and maintenance phases, participants were asked to work only on 2-minute daily probes, so a stopwatch for timing, test forms (the pre-test and the maintenance tests), blank sheets, pencil and eraser were used. During the intervention phase, across all instructional periods (i.e., TDI, independent practice using an iPad application, daily probes and graphing daily progress), scripted lessons, a stopwatch for timing, pencils and erasers were used. The lesson plans were adapted from the Tier 2 Mathematics Intervention Teacher Lesson Booklet (module: multiplication and division fact strategies) developed by The Meadow Center for Preventing Educational Risk (University of Texas System/Texas Education Agency, 2012).

Teacher Directed Instruction (TDI)

During TDI (i.e., warm-up, modeling, guided practice), student worksheets were used for all intervention sessions and mini wipe boards, makers, board erasers, 3 × 5 inch fact practice cards, and manipulative devices (i.e., connected cubes) were used in corresponding lessons. In addition, addition fact tables (e.g., when participants struggled with adding numbers or correcting errors in adding numbers when using a doubling strategy), multiplication fact tables (e.g., when reviewing easy facts such as ×2s and ×5s), and a Math Ready poster (for reminding participants of the actions) were used if needed.

Independent Practice using an iPad Application

For independent practice using an iPad application, one application, Math Evolve, (Zephyr Games, 2012) was installed and used with a headphone. The headphone was used only when other teachers stayed in the classroom during tutoring time. Another iPad with the application installed was prepared as a back up in case any technical problems (e.g., low battery) occurred.

Math Evolve is an educational game-type drill and practice application designed to build basic fact fluency including addition, subtraction, multiplication and division. The application costs $2.50 and has a customer rating, 5 out of 5. Math Evolve allows users to create a personal ID so that individual performance data can be documented and teachers and parents can get individual progress monitoring data. The application also allows users to customize settings (e.g., speed, sound, the number of problems to be solved) and save their preferences. Students need to solve problems to fight against enemies coming down from the top of the screen to the bottom (see Figure 3.3). There are two modes, a story mode and a practice mode. The story mode does not allow users to select exact multiplication number facts to practice, so for this study, participants only worked in the practice mode. If participants responded incorrectly, error correction opportunities were provided. If participants skipped a problem, they were asked to solve the problem again immediately. If participants responded incorrectly twice consecutively, correct answers were given and they were asked to insert the correct answer.

The Math Evolve application was selected because the features included progress-monitoring data, immediate and corrective feedback, multiple opportunities to practice, and customizable settings; all of which are considered to be effective practices for students with LD (Boone & Higgins, 2007; Matthew, Tsurusaki, & Basham, 2011; Okolo, 1992). Math Evolve is a game-type drill and practice application and according to Okolo (1992), game-type CBI programs are effective for helping students with LD to increase motivation to practice more problems. In addition, similar results were observed in the pilot study; students with LD liked working with Math Evolve. In the pilot study, the students worked with two applications, Math Evolve (game-type drill and practice application) and Math Drills (flash card type drill and practice application). It was often observed that students were more excited and eager to practice when they used Math Evolve than Math Drills.

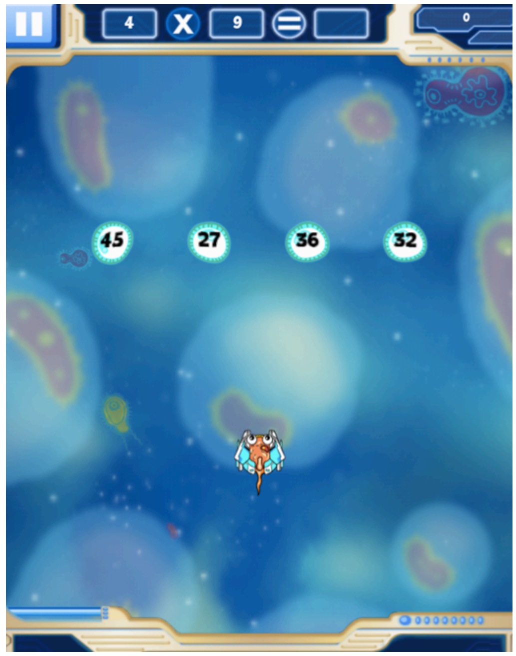


Figure 3. 3: Math Evolve

Daily Probes and Graphing Daily Data

For daily probes and the graphing daily data period, daily probes (Forms A-E) with blank sheets (for solving problems in daily probes if needed), color pens (for scoring and graphing daily progress), My Daily Progress forms, and stickers, as reinforcements were used.

Instruction Period	Materials
All periods	Scripted lessons, Stopwatch, Pencils, Erasers
TDI (i.e., warm-up, modeling, guided practice)	Mini wipe boards, Markers, Board erasers, Student worksheets, Fact practice cards, Connected cubes, Addition facts table, Multiplication fact table, Math Ready poster
Independent practice using an iPad application	iPads with an application (Math Evolve), Headphone
Daily probes and graphing daily data	Daily probes, Blank sheets, Color pens, My Daily Progress form, Stickers

Table 3. 5: Materials

PROCEDURE

Once the participants were selected, the study adhered to the following procedures: (a) pre-training, (b) baseline, (c) intervention and (d) maintenance. The timeline for this study is shown in Table 3.6.

Timeline	
Phase	Things to do
Pre-training (30 min)	✓ Orientation ✓ Training on iPad and Math Evolve app ✓ Behavior management (Math Ready)
Baseline (10-30 min)	✓ 2 minute-daily probe; no instruction provided ✓ Strategy usage test (last baseline session) ✓ 3-8 baseline data points across participants
Intervention (30 min)	✓ 15 intervention sessions (lessons 1-5: ×4s; lessons 6-10: ×8s; lesson 11-15: a mix of ×4s and ×8s) ✓ 2 minute-daily probe (at the end of each session) ✓ Strategy usage tests (8^{th} and after 15^{th} intervention sessions) ✓ Social validity interview (after 15^{th} intervention sessions)
Maintenance (10 min)	✓ Maintenance tests; no instruction provided

Table 3. 6: Timeline for the Study

Pre-Training

Prior to the study, an investigator provided 30 minutes of pre-training to the participants. Participants were given a brief overview of the study (e.g., purpose of the study, what participants would be asked to do during the study). If participants had any questions, the investigator answered them.

iPad and Math Evolve application

Participants were provided training on using an iPad and Math Evolve application. The investigator explicitly showed how to use the iPad and the application, participants tried them out with her guidance, and then participants tried it by themselves.

The participants first were provided with basic training on using the iPad including (a) turning on and turning off the iPad (e.g., click home button and slide to unlock), (b) go back home (i.e., click home button), and (c) wearing a headphone and adjusting the volume (i.e., click volume button). Then, participants were guided to create their own personal ID for Math Evolve application. The application allowed for the set-up of personalized settings for each user, so participants first created their own ID (i.e., save their names and/or their pictures) so that each participant's progress data could be recorded. For training on using the Math Evolve application, specific procedure was developed. For using the application, the participants were trained to do the following tasks: (a) find Math Evolve (e.g., recognize the app), (b) check your name, (c) click "Practice" mode, (d) click "Start Practice" (e) solve the problem (e.g., move the alien around to hit the correct answer), (f) try to enter the correct answer using the keypad if "tutor (error correction opportunity)" comes up and then move to the next question. The investigator monitored while participants tried out the devices to confirm if participants could use the devices independently (e.g., being able to follow the direction to use each application without assistance). If participants needed extra support (e.g., skipped tasks, made three mistakes (e.g., clicked incorrect buttons) consecutively to do a task, took more than 10 seconds to move to the next task, asked help directly), the investigator provided additional training.

Behavior management

It is well known that a teacher's ability to manage students' behavior and organize classrooms is important to yield positive outcomes in academic learning (Oliver &

Reschly, 2007). When teaching students with disabilities, behavior management is critical for minimizing distractions and off-task activities so that students can maximize their engagement in academic learning (Witzel & Mercer, 2003). Bryant and colleagues (2011) used a behavior management system called Math Ready, in their study investigating the effects of a Tier 2 early preventative intervention on mathematics performance of first grade students with mathematical difficulties. Math Ready consists of five behavior management rules: (a) Eyes on activity: you should watch your teacher when she/he is talking and you should pay attention to a handout if you are asked to work on it, (b) Mouth quiet: you should not talk unless a teacher asks you a question, (c) Hands on table: sit up straight and put your hands on the table in front of you, (d) Ears listening: listen carefully to what your teacher says, and (e) Ready to learn: you are only math ready when you are ready to focus and work hard, so be ready to learn. In pre-training, an investigator introduced these rules to participants. A poster (see Appendix D) was given to participants and the direct instruction approach was used to instruct all the five behaviors (i.e., eyes on the teacher, listening, ready to learn, mouth quiet, and hands on table). An investigator first provided modeling on each behavior and next asked participants to follow each behavior guided by the investigator. The investigator asked questions and reminded participants of each behavior (e.g., “What is the first behavior?” “Eyes-on activity” “What do you need to do?” “Put my eyes on the teacher when the teacher is talking.”), participants were asked to answer the questions and showed how to act. Then, whenever the investigator said, “Math Ready” and participants were asked show how to act all five behaviors. If participants did not act “Math Ready” independently (e.g., miss behaviors, ask for help directly), additional training was provided. It was expected that participants demonstrate these appropriate behaviors whenever the investigator said “Math Ready!” during the intervention. At the beginning of or after intervention sessions, the investigator reminded participants of these rules if needed.

Baseline Phase

During the baseline phase, participants only worked on a 2-minute daily probe containing 60 single-digit multiplication number facts (i.e., ×4s and ×8s). Participants were assessed with the alternate forms (A-E). A blank sheet of paper was given to participants so that they could use it to solve the problems, if needed. Participants were prompted to try to do their best to solve as many problems as they could during the 2 minutes, however, no instruction was provided during baseline sessions. The baseline phase occurred until a consistent pattern (“sufficiently consistent level and variability, with little or no trend”, Kratochwill et al., 2010, p. 19) of participant’s responses in the daily probes was observed for at least three sessions (Horner et al., 2005). In addition, in the last session in the baseline phase, after completing the 2-minute daily probe; participants were asked to complete the strategy usage test.

Intervention Phases

Staggered intervention was introduced across the participants (Kratochwill et al., 2010). James entered the first intervention phase after a consistently low and stable pattern was observed in his baseline data (he demonstrated the lowest level of baseline data among participants), while other participants remained in the baseline. Because all other participants also showed a consistently low and stable baseline data pattern, it was decided Kate would enter the intervention phase after James; it was convenient to introduce the intervention in this manner because Kate and James were attending the same school. The following was the participants' order to enter the intervention phase: James, Kate, Amy and Perry.

During the intervention phase, participants were provided a total of fifteen 30-minute intervention sessions. In the middle of the intervention phase (after lesson 8), participants were asked to work on the strategy usage test to investigate if participants had improved the use of mature strategies for solving ×4s and ×8s. To complete the test, it took more than 10 minutes, so in session 9, participants were asked to complete the strategy usage test only; no intervention was provided. Lesson 9 was provided in session 10.

Explicit, strategic TDI with iPad application practice was designed to help 5th grade students with LD who have IEP goals in mathematics to improve fluency on single-digit multiplication number facts (i.e., ×4s and ×8s). To design an effective intervention for teaching target multiplication number facts to students with LD, critical features of effective research-based instruction, including explicit instruction, strategy instruction, the Concrete-Representative-Abstract (CRA) technique, CBI, and distribution of data and feedback to students, were employed to design the intervention (Gerten et al., 2009; Jayanthi et al., 2008; Kroesbergen & Van Luit, 2003; Seo & Bryant, 2009; Steedly et al., 2008). The intervention was designed to teach single-digit ×4s and ×8s, only hard facts (Woodward, 2006), especially 4×3, 4×4, 4×6, 4×7, 4×8, 4×9, 8×3, 8×4, 8×6, 8

× 7, 8 × 8, 8 × 9 and commutative property facts. The target fact skills were broken down into smaller sets (i.e., 2 new facts including commutative property facts for each lesson) as well as sequenced logically (e.g., teaching ×4s before ×8s) to be taught to participants (Archer & Hughes, 2011). Lessons 1 through 5 focused on ×4s, lessons 6 through 10 focused on ×8s and lessons 11 through 15 focused on a mix of ×4s and ×8s. According to Woodward (2006), a systematic review of facts is one of the critical approaches to developing computational fluency, so the target facts were repeatedly reviewed systematically. New facts for each fact strand (i.e., ×4s) were introduced over 3 lessons, and participants reviewed all possible facts in the fact strand for 2 lessons before moving to the next fact strand (i.e., ×8s). All facts learned were reviewed over 5 lessons designed to teach a mix of ×4s and ×8s. In addition, participants had cumulative review opportunities in each lesson (e.g., practice new facts and facts previously taught with iPad applications). Participants were provided a total of 15 lessons (i.e., 5 lessons for ×4s, 5 lessons for ×8s, and 5 lessons for a mix). The lessons were developed to support participants to master the target multiplication number fact skills (Archer & Hughes, 2011). The target aim of this study was that participants reached the mastery level (i.e., score at 40 or greater CD/M) (Deno & Mirkin, 1977) on multiplication number facts (i.e., ×4s and ×8s) through the 15 lessons (Archer & Hughes, 2011). A detailed lesson sequence is presented in Table 3.7 and example lesson plans are provided in Appendix E.

<table>
<tr><th>Lesson</th><th>Facts</th><th>Instructional Content</th><th>Review</th><th>Visual Representations</th></tr>
<tr><td>1</td><td>×4s</td><td>4 × 3, 4 × 4,
commutative property facts</td><td rowspan="12"><u>Prerequisite skills/ knowledge</u>

✓ Counting by 2s
✓ Doubling number
✓ Easy facts (×0s, ×1s, ×2s, ×5s)
✓ Addition (e.g. ×4s: 8+8, 12+12, ×8s: 8+8+8+8, 12+12+12+12)

<u>Skills/knowledge taught in prior lessons</u>

✓ Facts & Vocabularies taught in previous lessons
✓ Doubling strategy</td><td rowspan="2">Concrete-Abstract</td></tr>
<tr><td>2</td><td>×4s</td><td>4 × 6, 4 × 7,
commutative property facts</td></tr>
<tr><td>3</td><td>×4s</td><td>4 × 8, 4 × 9,
commutative property facts</td><td rowspan="2">Representational-Abstract</td></tr>
<tr><td>4</td><td>×4s</td><td>Review</td></tr>
<tr><td>5</td><td>×4s</td><td>Review</td><td>Abstract</td></tr>
<tr><td>6</td><td>×8s</td><td>8 × 3, 8 × 4
commutative property facts</td><td rowspan="2">Concrete-Abstract</td></tr>
<tr><td>7</td><td>×8s</td><td>8 × 6, 8 × 7
commutative property facts</td></tr>
<tr><td>8</td><td>×8s</td><td>8 × 8, 8 × 9,
commutative property facts</td><td rowspan="2">Representational-Abstract</td></tr>
<tr><td>9</td><td>×8s</td><td>Review</td></tr>
<tr><td>10</td><td>×8s</td><td>Review</td><td>Abstract</td></tr>
<tr><td>11-12</td><td>Mix</td><td>×4s & ×8s</td><td>Representational-Abstract</td></tr>
<tr><td>13-15</td><td>Mix</td><td>×4s & ×8s</td><td>Abstract</td></tr>
</table>

Table 3. 7: Lesson Sequence

In addition, lessons were systematically organized (Archer & Hughes, 2011); the instructional routine for each lesson consisted of (a) warm-up (3 minutes): reviewed prerequisite skills and knowledge and/or skills and knowledge taught in previous lessons,

(b) modeling (8 minutes): provided modeling designed to develop both conceptual and procedural understanding of a doubling strategy to solve ×4s and ×8s, (c) guided practice (7 minutes): practiced the target facts under the investigator's guidance, (d) independent practice using an iPad application (5 minutes): practiced the target facts independently using the Math Evolve application, (e) daily probe (2 minutes): completed daily probes, and (f) graphing daily data and providing feedback (3 minutes): scored probe, created a graph daily data, and obtained feedback on the performance. Each intervention session lasted 30 minutes. Throughout all lessons, participants were asked to respond frequently, their performance was monitored carefully, and immediate and corrective feedback was provided (Archer & Hughes, 2011).

Warm-up

First of all, at the beginning of each lesson, the lesson goals were stated to the participants (Archer & Hughes, 2011). The 3-minute warm-up phase was designed to activate participants' prerequisite knowledge and skills (e.g., counting by 2s, doubling numbers, easy facts (i.e., ×0s, ×1s, ×2s, ×5s), addition frequently used for solving target multiplication number facts (e.g., 4×3: $6 + 6$ and 8 x 6: $12 + 12 + 12 + 12$) and review knowledge and skills taught in previous lessons (e.g., facts and vocabularies taught in former lessons) so they could be prepared to gain new mathematical knowledge and skills (Rosenshine & Stevens, 1986). A multiplication table (for reviewing easy facts), 3×5 flashcards (for reviewing facts previously taught), and an addition fact table (for if participants struggle with addition when doubling numbers) were used.

Modeling

In the 8-minute modeling phase, the investigator introduced the doubling strategy and modeled how to use the strategy to solve ×4s and ×8s by using a think-aloud approach. The modeling phase was designed to improve both conceptual and procedural understanding of the doubling strategy (Rittle-Johnson, Siegler, & Alibali, 2001). Explicit step-by-step modeling on the steps of the strategy and how to apply the strategy to solve ×4s and ×8s facts was provided to participants (Archer & Hughes, 2011; Jayanthi et al., 2008). The steps of the doubling strategy are (a) break apart 4 to 2 and 2 (for ×4s) or break apart 8 to 2, 2, 2, 2 (for ×8s), (b) multiply 2 with the other factor, and (c) add the products. For example, the following were steps to solve 4 × 3 with the doubling strategy: (a) break apart 4 to 2 and 2, (b) multiply 2 with 3 (2 x 3 and 2 x 3), and (c) add the product (6 + 6). The doubling strategy poster was given to participants as a reminder and the participants were asked to verbalize each step followed by the investigator (Wood et al., 1998). The investigator used the think-aloud approach when providing modeling how to use the strategy to solve ×4s and ×8s to help participants understand procedural processes. In addition, the investigator had participants verbalize the steps of the doubling strategy during the learning of the steps and when solving the problems. The student think-aloud approach is recommended as an effective strategy to teach mathematics for students with LD (Gersten et al., 2009; Krosesbergen & Van Luitt, 2003; NCTM, 2007; NMAP, 2008). The approach can help students with LD to use the effective strategy step-by-step rather than impulsively attempt to solve the problems in ineffective ways (Jayanthi et al., 2008). The investigator also provided an explanation as to why the doubling strategy could be used for solving ×4s and ×8s and both adequate examples (e.g., 4×3, 6×8) and non-examples (e.g.. 5×9, 6×7) that the doubling strategy could be used to solve were provided (Archer & Hughes, 2011; NMAP, 2008).

Visual representation has been known as an effective strategy to teach basic math skills (Manalo, Bunnell, & Stillman, 2000). Visual representations such as a strategy poster, an addition fact table, and a multiplication fact table were used to support the learning of participants across intervention phases. For the modeling phase, Concrete-Representational-Abstract (CRA) techniques were used to help participants to understand why and how the doubling strategy works to solve ×4s and ×8s. The CRA has been proved an effective approach to teach mathematics to students with LD; it especially helps students develop thorough conceptual understanding of new math concepts and skills (Witzel et al., 2003). The investigator showed modeling with the material (e.g., connected cubes) first, and then participants were asked to do what the investigator modeled (Mancl et al., 2011). On the concrete level, participants represented the fact as an array using connected cubes. For example, to understand breaking apart 4 as 2 and 2, participants broke 4 connected cubes into 2 and 2. To explain solving 4×3 using a doubling strategy, four connected cubes consisting of 3 cubes (4×3) were provided; participants broke them into two groups of 2 connected cubes consisting of 3 cubes (2×3 and 2×3). Participants then counted each group (2×3) and summed up both groups ($6 + 6$). At the representational (semi-concrete) level, participants represented the facts as an array using a picture of dots. For example, to explain how to solve problems 8×3, a picture of eight columns of 3 dots (8×3) was provided; participants made a loop with four groups of 2 columns of 3 dots (2 × 3, 2 × 3, 2 × 3, 2 × 3). Each group was counted, and participants added all four groups (6 + 6 + 6 + 6). In addition, in the abstract level, participants were provided only numbers and symbols. Participants were asked to use a doubling strategy step-by-step to solve the multiplication number fact equation without connected cubes or pictures of dots. For each lesson teaching ×4s (lesson 1-5) and ×8s (lesson 6-10), participants used the concrete materials with abstract materials in the first two lessons, representational materials with abstract materials in the next two lessons and only abstract materials in the last lesson. However, for lessons teaching a mix of ×4s and ×8s (lesson 11-15), the concrete materials were not used; only representational and abstract materials (lesson 11-12) and abstract materials (lesson 13-15) were used.

Guided practice

Following the modeling phase, participants solved multiplication problems on a guided practice activity sheet using the doubling strategy for 7 minutes. The guided practice activity sheet included new facts introduced including commutative property facts as well as facts taught in previous lessons for the same fact strand. For example, for lesson 7, new facts (8 × 6, 8 × 7) and ×8s facts taught in previous lessons (e.g., 8 × 3, 8 × 4) were included, but ×4s was not be included. The guided practice activity was designed to help participants have practice opportunities to develop proficiency in using the doubling strategy to solve ×4s and ×8s under the investigator's close guidance. The investigator monitored the participant's work closely (Archer & Hughes, 2011), so that they could be able to receive timely and corrective feedback, cues, prompts, and guidance while working on the guided practice activity sheet (NMAP, 2008; Rosenshine & Steve, 1986). The addition fact table was used if participants struggled in adding numbers when doubling (e.g., for correcting errors in addition). An extra blank sheet also was provided if participants wanted to use it for solving the problem. The investigator re-taught (e.g., provide modeling for each step), if it was needed (e.g., participants had difficulties using a doubling strategy to solve the target multiplication number facts).

Independent practice with an iPad application

After guided practice, participants practiced single-digit multiplication number facts (i.e., ×4s and ×8s) independently for 5 minutes using the iPad application, Math Evolve. Across interventions, single digit multiplication number facts, ×4s (i.e., $4 \times 1 - 4 \times 9$) and ×8s (i.e., $8 \times 1 - 8 \times 9$) were practiced. According to Wilson et al. (1996), effective CBI related to TDI (Woodward et al., 1986), provides sufficient opportunities to practice (Salisbury, 1990), and provides corrective and immediate feedback (Colins, Carnine, & Gersten, 1987). First of all, participants practiced multiplication number facts strongly related to facts taught in corresponding TDI in each lesson. Second, the application provided sufficient cumulative and distributed practice opportunities to participants (see Table 3.8). Distributed practice refers to multiple opportunities to practice a skill over time and cumulative practice is a method for providing distributed practice by including practice opportunities that address both previously and newly acquired skills (Archer & Charles, 2011). Thus, prior to each intervention, the investigator selected exact multiplication fact sets to be practiced for each lesson in the applications. The multiplication fact sets included new facts taught in each lesson as well as reviewed easy facts (e.g., 4×2, 1×8) and facts taught in previous lessons. However, while learning ×8s, participants did not practice ×4s so that they could focus on practicing more ×8s equations (Sweller, 1988). In addition, two iPads were used to practice commutative property facts. Participants practiced with the first iPad selecting facts starting with a 4 or 8 (e.g., 4 × 3, 8 × 6) during the first two-and-a-half-minutes and then practiced with the second iPad selecting commutative property facts (e.g., 3 × 4, 6 × 8) during the rest of the two-and-a-half-minutes.

Lessons	New Facts	Known Facts (i.e., easy facts & facts taught in previous lessons)
1 (×4s)	4 × 3, 4 × 4, Commutative property facts	4 × 1, 4 × 2, 4 × 5, Commutative property facts
2 (×4s)	4 × 6, 4 × 7, Commutative property facts	4 × 1 – 4 × 5, Commutative property facts
3 (×4s)	4 × 8, 4 × 9, Commutative property facts	4 × 1 – 4 × 7, Commutative property facts
4 (×4s review)	None	4 × 1 – 4 × 9, Commutative property facts
5 (×4s review)	None	4 × 1 – 4 × 9, Commutative property facts
6 (×8s)	8 × 3, 8 × 4, Commutative property facts	8 × 1, 8 × 2, 8 × 5 Commutative property facts
7 (×8s)	8 × 6, 8 × 7, Commutative property facts	8 × 1 – 8 × 5 Commutative property facts
8 (×8s)	8 × 8, 8 × 9, Commutative property facts	8 × 1 – 8 × 7, Commutative property facts
9 (×8s review)	None	8 × 1 – 8 × 9, Commutative property facts
10 (×8s review)	None	8 × 1 – 8 × 9, Commutative property facts
11-15 (×4s and ×8s)	None	4 × 1 – 4 × 9, 8 × 1 – 8 × 9, Commutative property facts

Table 3. 8: Facts Practiced with Math Evolve iPad Application

In addition, the application provided immediate and corrective feedback. If participants responded correctly, immediate feedback was provided (e.g., green check mark and sound, see Figure 3.4). If participants completed all multiplication fact sets assigned, immediate feedback was provided on their performance (e.g., “Training Completed!”). Error detection and correction features were also turned on. Math Evolve provided error correction opportunities to participants and the correct answer was given if participants made a mistake twice in a row (tutor mode, see Figure 3.5). Only after participants inserted the correct answer, they could go back to solve the next problem. Moreover, the investigator set up settings for the application in regard to the number of problems, theme, color, sound, speed, arrangement of problems, and error correction assistance (see Figures 3.6) so that all participants practiced solving multiplication fact problems under the same conditions. Participants were given headphones to wear so that the sound of the application did not distract others and they were able to adjust sound volume while they worked in the application. When there were only the investigator and participants in the classroom, they did not wear headphones. The investigator closely monitored participants’ performance (Archer & Hughes, 2011) and provided guidance, or technical support, if needed.

Figure 3. 4: Feedback for correct answers in Math Evolve

Figure 3. 5: Error correction feature in Math Evolve

Figure 3. 6: Customizable features of Math Evolve

Daily probes and graphing daily data

Following the independent practice with the iPad application, participants worked on a 2-minute daily probe (Lembke & Stecker, 2007). Participants were asked to solve as many multiplication number fact problems (i.e., ×4s and ×8s) as they could for 2 minutes. As soon as the participants completed the daily probe, the probe was scored immediately. The participants corrected their own work with a color pen as the investigator dictated the answer keys (Woodward, 2006); they were asked to circle any incorrect items and wrote correct answers. The total number of correct items per 2 minutes was calculated and written on the top of the probe form. The investigator reviewed incorrect items with students and specified consistent error patterns if observed. Monitoring their own progress can be effective to help students with LD to manage and take responsibility in their learning progress as a learner (Bandura, 1986; Steedly et al., 2008). Thus, participants recorded the total number of correct items per 2 minutes and graphed the number on the daily progress chart, My Progress Graph (i.e., drawing bar graphs with a color pen, see Appendix F). The investigator provided specific and positive feedback on their daily performance (e.g., “You solved 5 more problems than yesterday”) to the participants (Mercer & Miller, 1992). After providing feedback, a small character sticker was given to participants as reinforcement. A total of 5 minutes was spent for a daily probe, review of incorrect items, graphing daily data, and providing feedback and reinforcement.

Maintenance Phase

The maintenance effect of explicit, strategic TDI with iPad application practice on single-digit multiplication number facts (i.e., ×4s and ×8s) was measured at 2 weeks after the intervention was terminated. To determine if explicit, strategic TDI with iPad application practice influenced the participants’ fluency on multiplication number facts (i.e., ×4s and ×8s) over time, participants worked on 2-minute daily probes. The daily probes with alternate forms (A-E) that were used during the baseline and intervention phases also were utilized for the maintenance phase. For example, if the participants used Form B in the last intervention session, Form C and Form D were used for the maintenance tests. The average score between the two tests was recorded. No instruction was provided in the maintenance phase.

Fidelity of Implementation

The fidelity checklist form consisted of two main components including observation information (e.g., date, time, observed lesson) and a fidelity checklist. The checklist contained 20 items that assessed procedural fidelity on materials, instruction (i.e., warm-up, modeling, guided practice, independent practice), feedback/support, daily probes/graphing daily data, and behavior management. Fidelity was measured using 3-level points (0: behaviors absent or not observed, 1: inconsistent level of implementation, 2: high level of implementation). To calculate the fidelity of implementation, the points of behaviors observed by the investigator was divided by the total possible points (total possible points: 40) of all planned behaviors in the checklist and multiplied by 100 (Cihack & Bowlin, 2009). The form also contained space for taking notes and for any other comments. The fidelity checklist form is represented in Appendix G.

To assess the fidelity of the implementation for this study, at least 20% of all interventions were observed or checked with audio files recorded for each lesson by other researchers. Three doctoral students assessed the fidelity of implementation for a total of 12 intervention sessions randomly selected for each participant (3 lessons for James, 3 lessons for Kate, 3 lessons for Amy, and 3 lessons for Perry). Each of them assessed the fidelity for a total of four lessons; the four lessons included one of each participant's lessons (1 lesson for James, 1 lesson for Kate, 1 lesson for Amy and 1 lesson for Perry).

The fidelity of implementation was calculated at 98%, overall. Specifically, the fidelity was 98% for James, 98% for Kate, 100% for Amy, and 98% for Perry. A common minimum acceptable percentage for fidelity of implementation is 80% (James

Bell Association [JBA], 2009). In addition, the fidelity checklist results and comments were discussed with the investigator after each observation for assessing procedural fidelity.

Social Validity

After completing the intervention phase, participants were interviewed to identify their perspectives toward intervention, explicit, strategic TDI with iPad application practice. The investigator read aloud all 20 interview questions and participants also verbally responded to the questions. The questionnaires consisted of 16 5-point Likert-scale (1: strongly disagree, 2: disagree, 3: neither, 4: agree, 5: strongly agree) questions and four open-ended questions. The questions were designed to determine participants' perspective towards (a) explicit, strategic TDI, (b) independent practice using iPads, (c) daily probes and graphing daily data and (d) overall intervention. For example, the 5-point Likert scale questions addressed participants' thoughts about the doubling strategy, graphing daily data, Math Ready and their use of iPads. In addition, participants responded to the following four open-ended questions: (a) What method do you prefer to practice multiplication facts (flashcards/worksheets vs. iPad application) Why?, (b) What did you like best about our tutoring time? Why?, (d) What aspects of tutoring time did you dislike? Why?, and (e) Do you have any other comments or suggestions regarding tutoring time? A detailed description of each questionnaire is given in Appendix H.

DATA ANALYSIS

Research Question 1 and 3

Visual analysis

The traditional approach to analyze data to determine the effect of the independent variable on dependent variables in a single-case design is a systematic visual analysis of participants' response on dependent variables within and across phases (e.g., baseline, intervention, maintenance) of the study (Parsonson & Baer, 1978). The visual analysis is used to determine if there is evidence in the casual relation between the independent variable and outcomes as well as the strength of the relation (Kennedy, 2005). According to Parsonson and Bear (1978), the visual analysis involves four steps to follow and six features to consider. The four steps include (a) assessing if data in the baseline phase determine a predictable pattern, (b) identifying if sufficient data and consistency exist to determine a predictable pattern of data within each phase, (c) comparing data between adjacent or similar phases to document if the change of the independent variable is related to predicted change in the pattern of outcomes, and (d) assessing data across all phases to determine if experimental control (e.g., at least three data points across time) exists. In addition, the visual analysis requires consideration of the following six features (i.e., level, trend, variability, immediacy of effect, overlap, and consistency of data patterns across similar phases) to investigate patterns within and between phases to see if there is a causal relation between independent variables and outcomes and the patterns of data meet the evidence standards of the causal relation (Horner et al., 2005; Kennedy, 2005; Kratochwill et al., 2010; Parsonson & Bear, 1978).

First of all, level means “the mean score for the data within a phase” (Kratochwill et al., 2010, p. 18). Second, trend is “the slope of the best-fitting straight line for the data within a phase” (Kratochwill et al., 2010, p. 18). Third, variability denotes “the range of standard deviation of data about the best-fitting straight line" (Kratochwill et al., 2010, p. 18). Fourth, immediacy of effect means “the change in level between the last three data points in one phase and the first three data points of the next” (Kratochwill et al., 2010, p. 18). Fifth, overlap is “the proportion of data from one phase that overlaps with data from the previous phase” (Kratochwill et al., 2010, p. 18). Lastly, consistency of data patterns across similar phases denotes “looking at data from all phases within the same condition and examining the extent to which there is consistency in the data patterns from phases with the same conditions” (Kratochwill et al., 2010, p. 19). An assessment of level, trends and variability within a phase allows us to identify observed patterns of participants’ performance on dependent variables as well as to predict the pattern of their performance in time (Furlong & Wampold, 1981). In addition, more immediate change in levels indicates a stronger effect of an independent variable on dependent variables (Kratochwill et al., 2010). The smaller proportion of data overlapped indicates a larger effect of independent variables on dependent variables (Kratochwill et al., 2010) and the higher consistency means there exists a stronger causal relation between an independent variable and dependent variables (Horner et al., 2005; Kratochwill et al., 2010).

Therefore, all six features were considered to determine whether a strong causal relation between explicit, strategic TDI with iPad application practice and participants’ fluency on multiplication number facts (i.e., ×4s and ×8s) exists (Kratochwill et al., 2010). Microsoft Excel was used for the visual analysis (e.g., developing graphs with data, computing the level, finding slopes for the best-fitting straight line, computing standard deviation).

Effect sizes

Even though researchers tend to rely heavily on a visual analysis approach to interpret data in a single-case design and no methods and standards for effect size calculation is agreed for single-case design studies, several useful quantitative approaches have been suggested such as percentage of non-overlapping data points (PND) (Kratochwill et al., 2010). Moreover, Parker and colleagues (2011) recently developed and suggested a new approach, Tau-U, to analyze data in a single-case design. To demonstrate the magnitude and strength of the effect of explicit, strategic TDI with iPad application practice on multiplication facts fluency performance, two methods, PND and Tau-U were utilized. It would be beneficial to use multiple approaches to confirm the results of the data analysis.

First of all, PND was utilized to determine the effect size (Scruggs, Mastropieri, & Casto, 1987) of the intervention on multiplication facts fluency. The PND method has been commonly used in computing the effect size of single-case design (Scruggs & Mastropoeri, 2001). The PND was calculated using the number of the intervention data points that exceeds the highest baseline point divided by the total number of the intervention data points (Scruggs et al., 1987). A value of PND of less than 50% is interpreted as a zero effect, 50-69% as a small effect, 70-89% as a medium effect and over 90% as a large effect (Scruggs, Mastropieri, Cook, & Escobar, 1986; Scruggs & Mastropieri, 1998).

In addition, Parker and colleagues (2011) recently suggested a new approach to calculate effect size for a single-case design study. Tau-U is a method combining non-overlap between phases and trends from within the intervention phase while controlling for undesirable trends in the baseline (Parker et al., 2011). Unlike other non-overlap analyses such as PND, Tau-U is sensitive to data trends (Parker et al., 2011). Tau-U provides more statistical power than any other non-overlap methods, decreases the ceiling effect unlikely to other non-overlap methods and Tau-U also provides confidence intervals and *p*-values (Parker et al., 2011). In addition, data can be easily computed by being entered on a free website (http://www.singlecaseresearch.org) developed by Vannest, Parker and Gonen (2011). Tau-U can be useful to answer the following four questions: (a) "What is the improvement trend during Phase B?", (b) "What is the improvement in non-overlapping data between Phase A and B?", (c) "What is the overall client improvement in A versus B contrast plus Phase B trend?", and (d) "What is the overall client improvement, controlling of preexisting improvement trend?" (Parker et al., 2011, p. 292). For this study, Tau-U data was used to answer the second question, "What is the improvement in non-overlapping data between phase A (baseline) and B (intervention)?" The value of Tau is interpreted as "the percent of data that improve over time" or "the percent of nonoverlap between phases" (Parker et al., 2011, p. 289). For example, .70 of Tau (p = .03) for non-overlapping data between phase A and B can be interpreted as there is 70% improvement from phase A to B or 70% nonoveralp between phase A and B, which is statistically significant ($p < .05$) (Parker et al., 2011). The web-based Tau-U calculator (http://www.singlecaseresearch.org) developed by Vannest and colleagues (2011) was used to compute Tau.

In sum, in order to identify the degree of the effect of explicit, strategic TDI with iPad application practice on the multiplication fact performance of 5th grade students with LD who have IEP goals in mathematics, the six features of visual analysis (i.e., level, trend, variability, immediacy of the effect, overlap, and consistency of the data pattern across similar phases) were examined as well as two effect sizes (i.e., PND and Tau-U) were computed.

Research Question 2

First of all, to analyze participants' accuracy data on the strategy usage test, their accuracy was computed using the following formula, total number of correct items divided by number of items attempted in the test, multiplied by 100. Then the accuracy data (%) across three tests (baseline, in the middle of, and after the intervention phase) were compared to determine if participants improved their accuracy on solving the fact problems. In addition, to analyze participants' doubling strategy usage, the percentage of their doubling strategy usage for solving fact problems on the strategy test was computed using the following formula, total number of doubling strategy used divided by total number of items in the test, multiplied by 100. Then the percentage data across three tests (baseline, in the middle of, and after the intervention phase) were compared to determine if participants improved their doubling strategy usage across the study.

Research Question 4

To analyze the social validity interview data, first of all, mean scores (5-point Likert scale) of closed-ended questions for each section (i.e., explicit, strategic TDI, independent practice using an iPad application, daily probe/graphing daily data, overall intervention) in the social validity interview form were computed across all participants. Then the mean score for each section was used to determine participants' perspectives toward each instructional component. For example, a 4.7 mean score for overall intervention was interpreted as participants' perspectives toward the overall intervention was favoring a strongly positive direction (between 4: agree and 5: strongly agree). In addition, participants' responses to open-ended questions were also recorded to understand their thoughts about what method they preferred for practicing facts (flashcards/worksheets vs. iPad application), what they liked and disliked about tutoring time, and any suggestions regarding tutoring time.

Chapter 4: Results

The purpose of this study was to investigate the effects of explicit, strategic TDI with iPad application practice on the multiplication fact performance of 5^{th} grade students with LD who had mathematics IEP goals as well as demonstrated lack of fluency on target multiplication facts. The study investigated the following research questions:

1. What is the effect of explicit, strategic TDI with iPad application practice on the fluency of 5^{th} grade students with LD, who have mathematics IEP goals, on single-digit multiplication number facts, ×4s and ×8s?
2. What is the effect of explicit, strategic TDI with iPad application practice on the use of a doubling strategy for solving single-digit multiplication number facts, ×4s and ×8s, among 5^{th} grade students with LD, who have mathematics IEP goals?
3. How do 5^{th} grade students with LD, who have mathematics IEP goals, maintain their fluency in single-digit multiplication number facts, ×4s and ×8s, 2 weeks following explicit, strategic TDI with iPad application practice?
4. What are the perspectives of 5^{th} grade students with LD, who have mathematics IEP goals, toward explicit, strategic TDI with iPad application practice on learning multiplication facts, ×4s and ×8s?

In this chapter, the results will be organized according to each research question.

RESEARCH QUESTION 1

Research question 1 examined the effect of explicit, strategic TDI with iPad application practice on the multiplication fact fluency of participants. To assess participants' performance on the target multiplication facts (i.e., ×4s and ×8s), DC/M on the daily probes was recorded. To understand the effect of explicit, strategic TDI with iPad application practice on the target multiplication fact fluency of participants, a visual analysis of the data was conducted and effect sizes were computed. Figure 4.1 shows DC/M scores across baseline, intervention, and maintenance phase across all four participants.

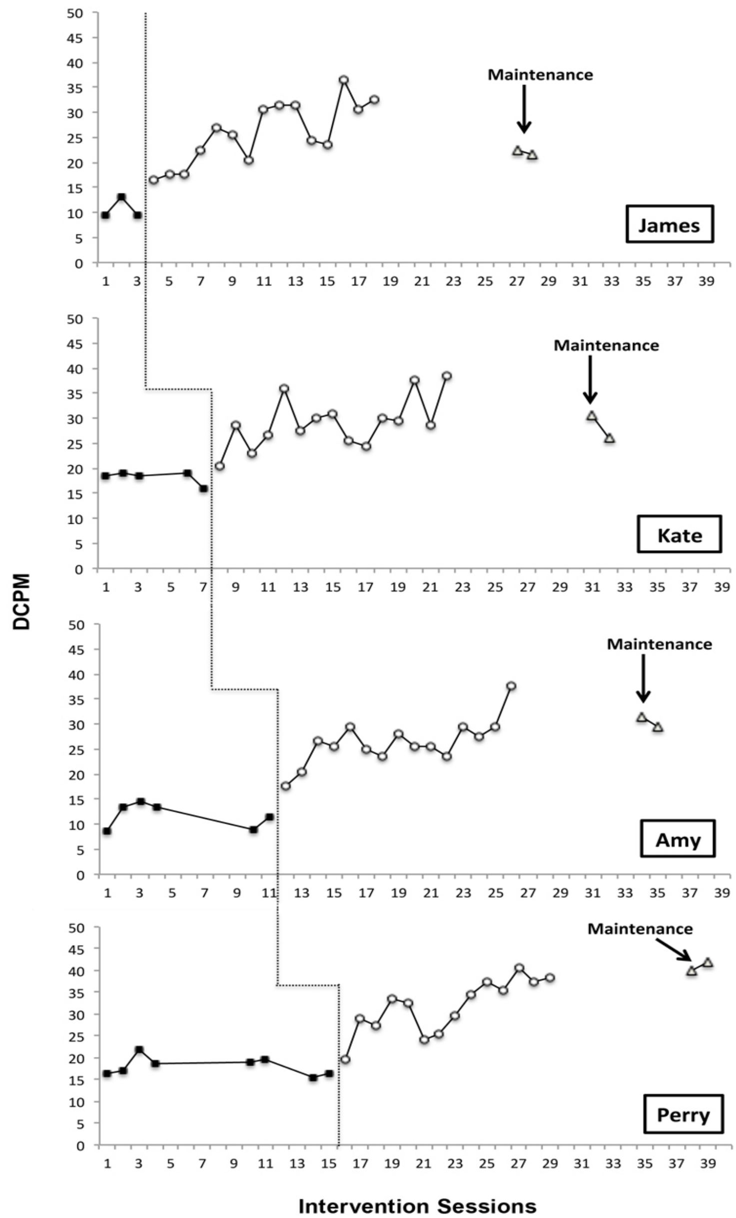


Figure 4. 1: DC/M scores across the baseline, intervention, and maintenance phases for participants

Visual Analysis

For visual inspection of the data, six features (i.e., level, trend, variability, immediacy of effect, overlap, and consistency of data patterns across similar phases) recommended by the What Works Clearinghouse (Kratochwill et al., 2010) were conducted and the results were analyzed to determine whether a causal relation existed between the explicit, strategic TDI with iPad application practice (Independent Variable) and participants' fluency on multiplication number facts (i.e., ×4s and ×8s) (Dependent Variable). Table 4.1 shows level and trend data for the participants and Table 4.2 depicts variability (i.e., the fluctuation of data around the mean score indicated by standard deviation and range of data), immediacy of effect and overlap of data between the baseline and intervention phase for participants.

Participants	Level			Trend	
	Baseline	Intervention	Maintenance	Baseline	Intervention
James	10.67	25.87	22	0	1.09
Kate	18.20	29.13	28.25	-0.29	0.65
Amy	11.75	26.30	30.5	-0.16	0.74
Perry	18.29	32.50	41	-0.21	1.22

Table 4. 1: Level and Trend Data for Participants

Participants	Variability Standard Deviation (Range)		Immediacy of effect (%)	Overlap
	Baseline	Intervention		
James	2.02 (9.5-13)	6.21 (16.5-36.5)	65.00	No
Kate	1.02 (16-19)	5.12 (20.5-38.5)	61.70	No
Amy	2.30 (8.5-14.5)	4.56 (17.5-37.5)	101.70	No
Perry	2.11 (16.5-22)	6.55 (19.5-42.5)	81.70	Yes

Table 4. 2: Variability, Immediacy of effect, Overlap data for participants

James

In terms of level, his mean DC/M on probes were 10.67 and 25.87 during the baseline and intervention phases, respectively (see Figure 4.2). James had the lowest level data for all phases among participants. His scores were constant at low and stable levels during the baseline phase (M = 10.06 DC/M, range 9.5 to 13 DC/M). After the intervention was implemented, his fluency scores immediately increased (M = 25.87 DC/M, range 16.5 to 36.5 DC/M) and stayed high and at increasing levels for the rest of the intervention phase. The change of levels between two phases was 15.20 DC/M and his mean DC/M for maintenance phase was 22 DC/M. The level data shows that James improved his fluency on target multiplication facts across the study and maintained fluency skills 2 weeks after the intervention was completed even though the level of the maintenance phase was lower than the level of the intervention phase (3.87 DC/M lower).

The trend showed that there was no directional pattern as determined by a slope of zero for the baseline data (see Figure 4.3). However, James showed an upward performance trend of 1.09 for the intervention data. Based on the trend data, it was evident that James's performance did not change during the baseline phase, but his DC/M increased by 1.09 across the intervention phase. James's variability data represented a standard deviation of 2.02 DC/M with a range of 9.5 to 13 DC/M (around the mean 10.67 DC/M) for the baseline and a standard deviation of 6.21 DC/M with a range of 16.5 to 36.5 DC/M (around the mean 25.87 DC/M) for intervention (see Figure 4.4). James demonstrated a frustration level (scored lower than 20 DC/M) on fact fluency during baseline, but he demonstrated instructional level (scored at 20 or greater DC/M) on fluency during the intervention and maintenance phases. It was noted that his fact fluency performance closely approached mastery level (score of 40 or greater DC/M); his highest fluency score was 36.5 DC/M. In addition, 65% of the immediacy of effect was observed through the comparison of the level of baseline data points and the level of the first three intervention data points (see Figure 4. 5); there was no overlap between baseline and intervention data (see Figure 4. 6). Lastly, James's data pattern across baseline, intervention, and maintenance phases showed consistency with other participants' data pattern except for the baseline trend (while other participants showed a decreasing pattern, his pattern had no directional trend) (see Figure 4.7).

Kate

Kate's fluency scores were initially low during baseline and were constantly at low and stable levels during the baseline phase (M = 18.20 DC/M, range 16 to 19 DC/M).

Her scores promptly increased after the intervention was implemented and continued at relatively high and increasing levels for the rest of the intervention phase (*M* = 29.13 DC/M, range 20.5 to 38.5 DC/M). Kate's data showed a level of 18.20 DC/M for the baseline phase and 29.13 DC/M for the intervention phase. The change in levels between baseline and intervention data was 10.93 DC/M; the level for maintenance data was 28.25 DC/M (see Figure 4. 2). The level data showed that she improved her fluency on the targeted multiplication facts across the phases. She also maintained her fluency skills during maintenance phase after the intervention was removed even though the level of the maintenance phase was slightly lower (0.88 DC/M lower) than the level of the intervention phase. The trend data was -0.29 and 0.65 during the baseline and intervention phases, respectively; it was evident that Kate's fluency score decreased by 0.29 DC/M during the baseline phase while her performance increased by 0.65 DC/M during the intervention phase (see Figure 4.3). Regarding variability, the standard deviation was 1.02 DC/M (range: 16-19 DC/M) during baseline and 5.12 DC/M (range: 20.5 – 38.5 DC/M) during intervention (see Figure 4.4). The standard deviation for the baseline phase was the lowest among participants. Her fact fluency scores were at the frustration level during baseline phase, but her fluency scores improved across the study. Although her fluency scores did not reach the mastery level, her highest score (38.5 DC/M) was close to mastery. In addition, Kate's data showed a 61.70% immediacy effect (see Figure 4.5); no overlap between baseline and intervention data was observed (see Figure 4.6). Lastly, Kate's data pattern across baseline, intervention, and maintenance phases showed consistency with other participants' data pattern (see Figure 4.7).

Amy

According to Amy's data, the level was 11.75 and 26.30 DC/M for the baseline and intervention phases, respectively. Her scores were initially low and continued at low and relatively stable levels during the baseline phase (*M* = 11.75 DC/M, range 8.5 to 14.5 DC/M). In addition, her scores promptly increased after the intervention was implemented and continued at relatively high and increasing levels for the rest of the intervention phase (*M* = 26.50 DC/M, range 17.5 to 37.5 DC/M) (see Figure 4.2). Amy's data showed that she improved her fact fluency score across the study (the change of levels between baseline and intervention data was 14.55 DC/M). She also maintained her fluency skills two weeks after the intervention was completed; the level of maintenance data was even higher than the level of intervention (4.2 DC/M higher). In addition, Amy's data demonstrated a downward trend (-0.16) during the baseline phase, but an upward trend (0.74) during the intervention phase (see Figure 4.3). It was evident that her fluency score decreased by 0.16 DC/M during baseline and increased by 0.74 DC/M during intervention. Amy's variability data documented a standard deviation of 2.3 DC/M (around the mean level 11.75 DC/M) with a range of 8.5 and 14.5 DC/M during her baseline. For intervention data, the variability demonstrated a standard deviation of 4.56 DC/M with a range of 17.5 and 37.5 DC/M. Her baseline variability data was the highest but her intervention variability data was the lowest among all of the participants (see Figure 4.4). Amy's fact fluency was at the frustration level (11.75 DC/M) during the baseline phase, but her fluency improved across the study; her fact fluency was at the instructional level during both intervention and maintenance phases. It was noted that her fact fluency performance closely reached the mastery level; her highest fluency score was 37.5 DC/M. In addition, 101.70% of the immediacy of effect was observed; she showed the largest immediacy of effect among participants when compared to the last three baseline data points and the first three intervention data points (see Figure 4.5); there was no data point overlap between the baseline and intervention phase (see Figure 4.6). Lastly, her data pattern across baseline, intervention, and maintenance phases showed consistency with other participants' data pattern (see Figure 4.7).

Perry

Perry's data level across all phases (baseline, intervention, maintenance) was the highest among all participants (see Figure 4.2). His data level was 18.29 and 32.50 DC/M during the baseline and intervention phases, respectively (see Figure 4.2); the change of level between baseline and intervention data was 14.21 DC/M. Perry's fluency scores were low initially and were constant at low and stable levels throughout the baseline phase (M = 18.29 DC/M, range 16.5 to 22 DC/M). In addition, after the intervention was introduced, his scores increased and continued at relatively high and increasing levels for the rest of the intervention phase (M = 32.50 DC/M, range 19.5 to 42.5 DC/M). According to his level data, Perry demonstrated a frustration level on fact fluency during baseline, but he demonstrated an instructional level on fluency during intervention. His fluency score even reached the mastery level during the intervention phase twice, session 12 (40.5 DC/M) and session 15 (42.5 DC/M). The level of maintenance data was 41 DC/M; he maintained his fluency skills two weeks after the intervention was completed. The level of maintenance data was higher than the level of intervention (8.5 DC/M higher); he demonstrated mastery level on fluency during the maintenance phase. The level data showed Perry improved his fact fluency across the study and maintained the intervention gains after the removal of the intervention. In session 6, his fluency score dropped significantly from 32.5 to 24 DC/M. He was sick and absent from school for three days in a row between session 5 and 6. Regarding the trend, a downward trend (-0.21) was demonstrated during the baseline phase while an upward trend (1.22) was demonstrated during the intervention (see Figure 4.3). It was evident that his fluency score decreased by 0.21 DC/M during the baseline phase while his score increased by 1.22 DC/M during the intervention phase. His data demonstrated the most pronounced increasing trend pattern among the participants. Regarding variability, the standard deviation was 2.11 DC/M (range: between 16.5 and 22 DC/M) during baseline and 6.55 DC/M (range: 19.5 and 42.5 DC/M) (see Figure 4.4). In addition, 81.70% of the immediacy of effect was observed (see Figure 4.5). One data point overlapped between baseline and intervention data; his highest baseline data point was 19.5 DC/M and his first intervention data score was 19 DC/M (see Figure 4.6). Lastly, Perry's data pattern across baseline, intervention, and maintenance phases showed consistency with other participants' data pattern except his fluency performance for both the intervention and maintenance phases were higher (reached the mastery level) than the other participants (at the instructional level) (see Figure 4.7).

Summary

In sum, all of the participants demonstrated basic multiplication fact fluency at the frustration level during the baseline phase, but they improved their fact fluency ability during the intervention phase. One of the participants' (Perry) fluency scores reached the mastery level during the intervention phase and maintained at the mastery level for fluency in the maintenance phase. In addition, the other three participants demonstrated fluency at the instructional level during both the intervention and maintenance phases; their highest fluency scores were close to the mastery level (James: 36.5 DC/M, Kate: 37.5 DC/M, Amy: 38.5 DC/M).

Regarding data trends, all participants except James (no directional pattern) demonstrated a downward trend pattern during baseline while all of them showed an upward trend pattern during the intervention phase. Based on the trend data, it was evident that their fluency score had no or decreased changes during baseline while the score improved during the intervention phase.

Regarding variability, all participants' data showed a standard deviation between 1.02 and 2.30 DC/M (with a range of 8.5 and 22 DC/M) during the baseline phase and a standard deviation, between 4.56 and 6.55 (with a range of 16.5 and 42.5 DC/M) during the intervention phase. Participants demonstrated an immediacy of effect between 61.70% and 101.70% when comparing the last three baseline data points and the first three intervention data points. In addition, it was shown that there was no overlapped data point between baseline and intervention for all participants except Perry (one data point overlapped). All participants' data patterns across similar phases (each baseline, intervention and maintenance) appear to be consistent overall. Based on a visual analysis of the data, it was shown that there was a causal relation between explicit, strategic TDI with iPad application practice and the target fact performance of all participants.

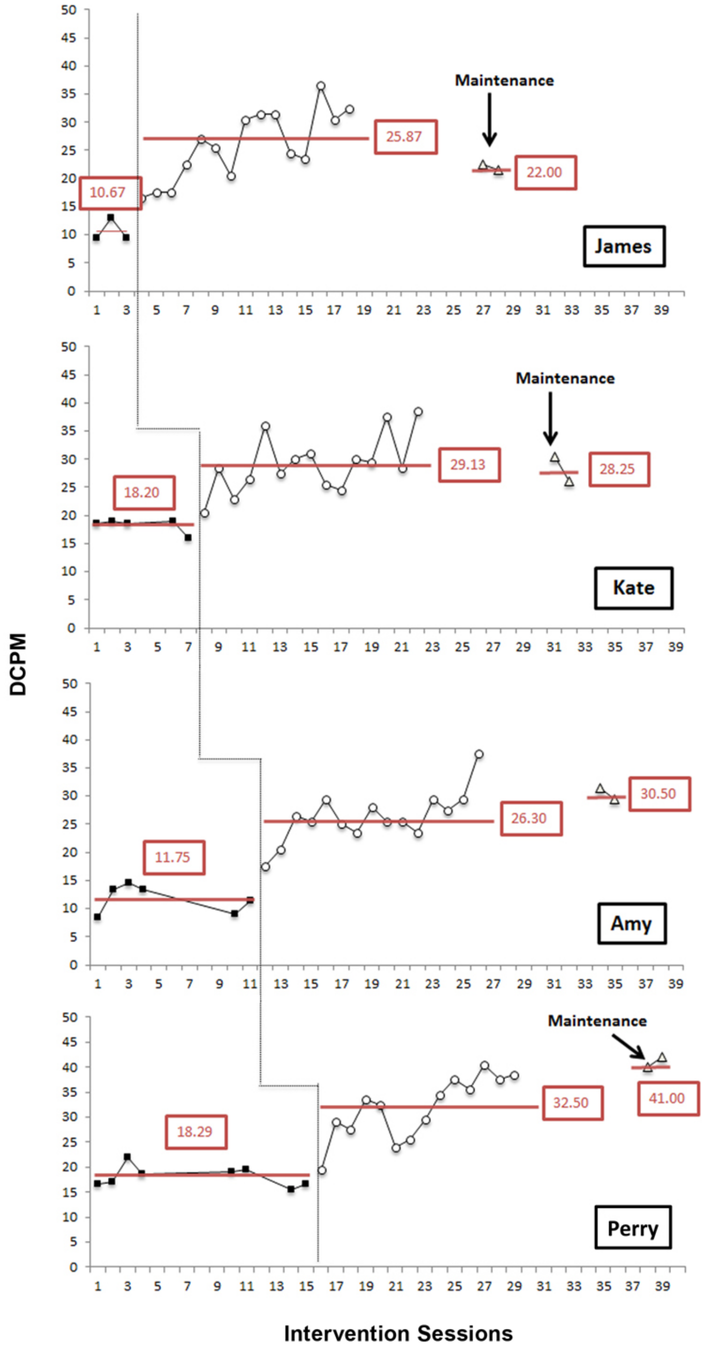


Figure 4. 2: Level data across participants

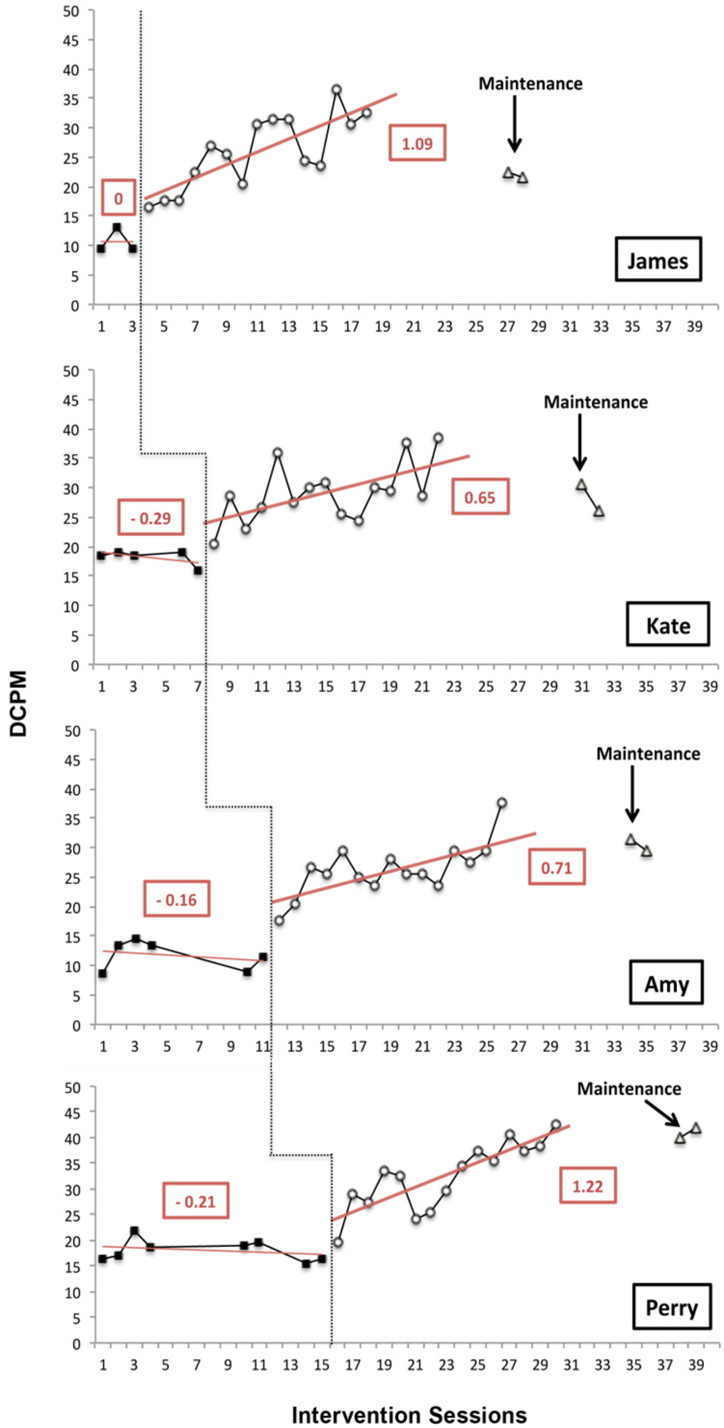


Figure 4. 3: Trend data across participants

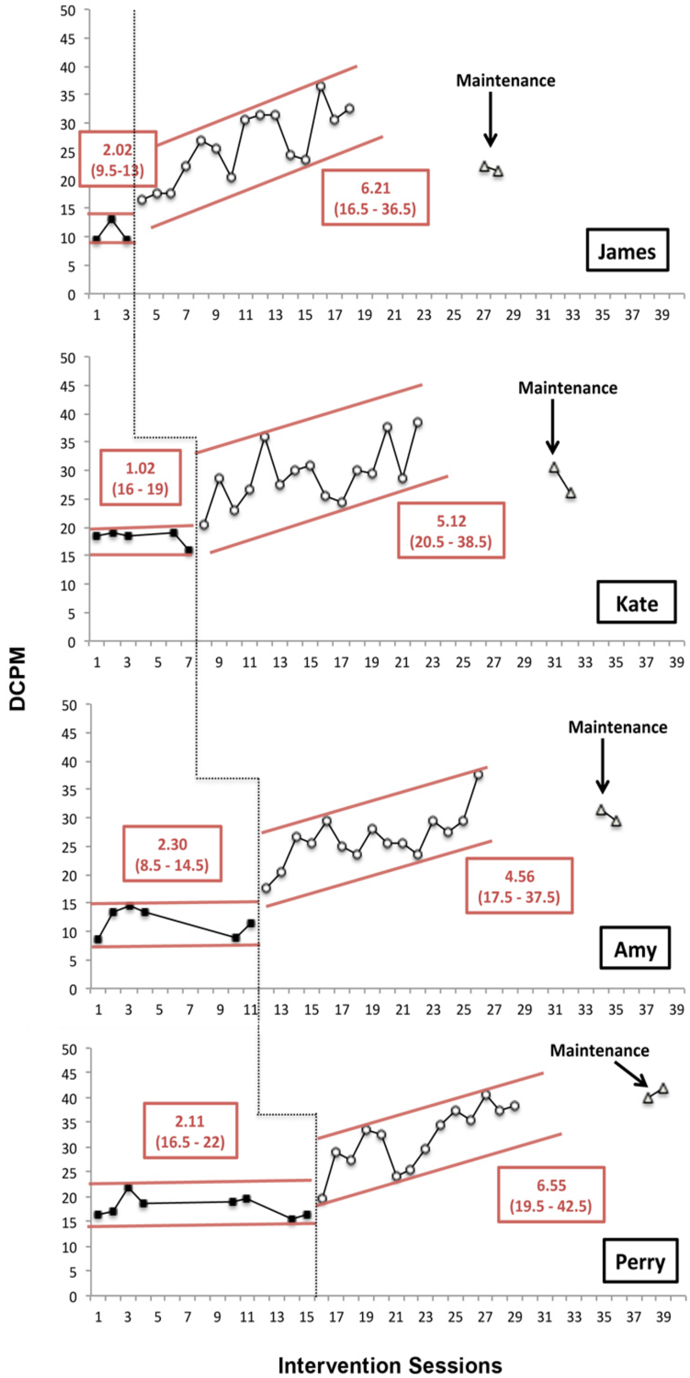


Figure 4. 4: Variability data across participants

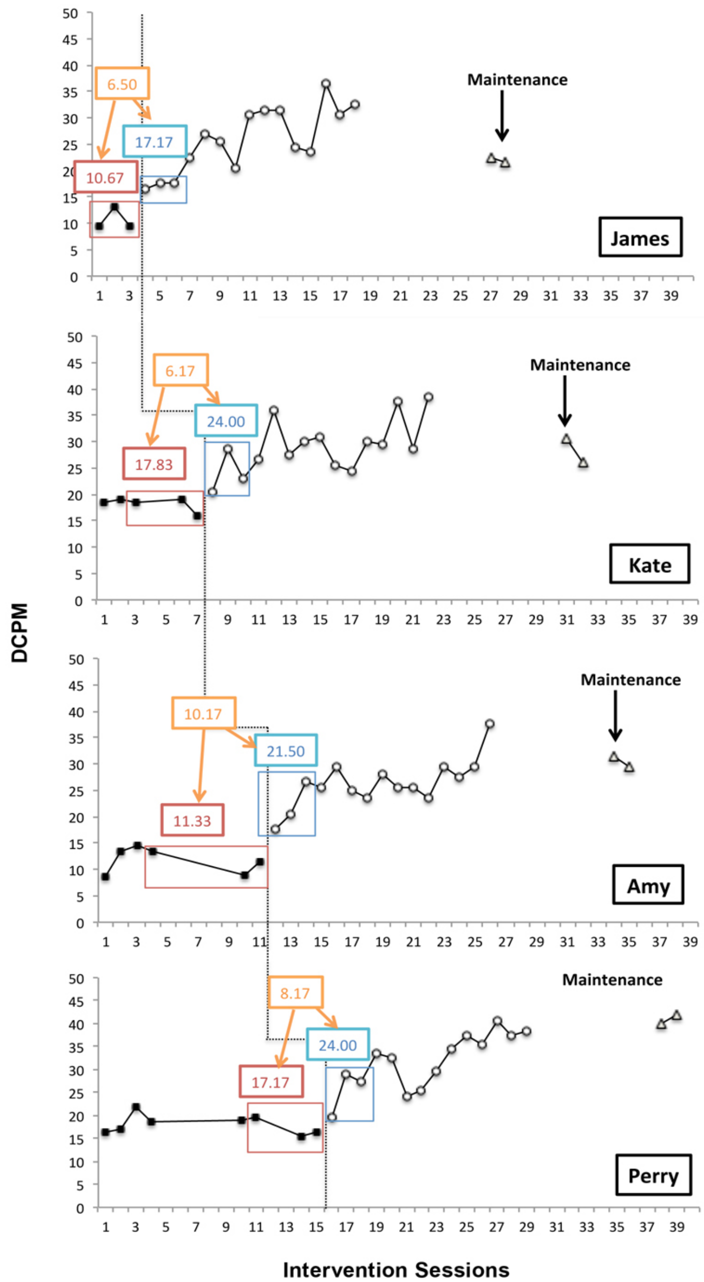


Figure 4. 5: Immediacy of effect data across participants

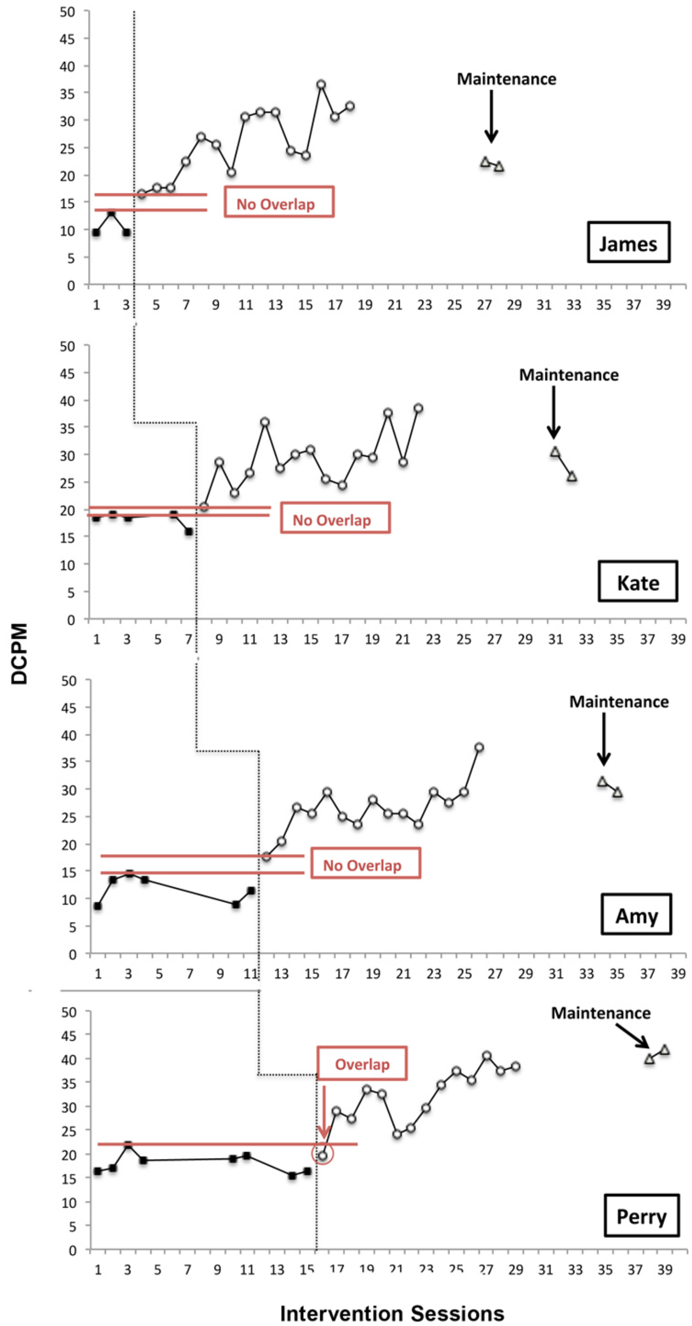


Figure 4. 6: Overlap data between baseline and intervention phase across participants

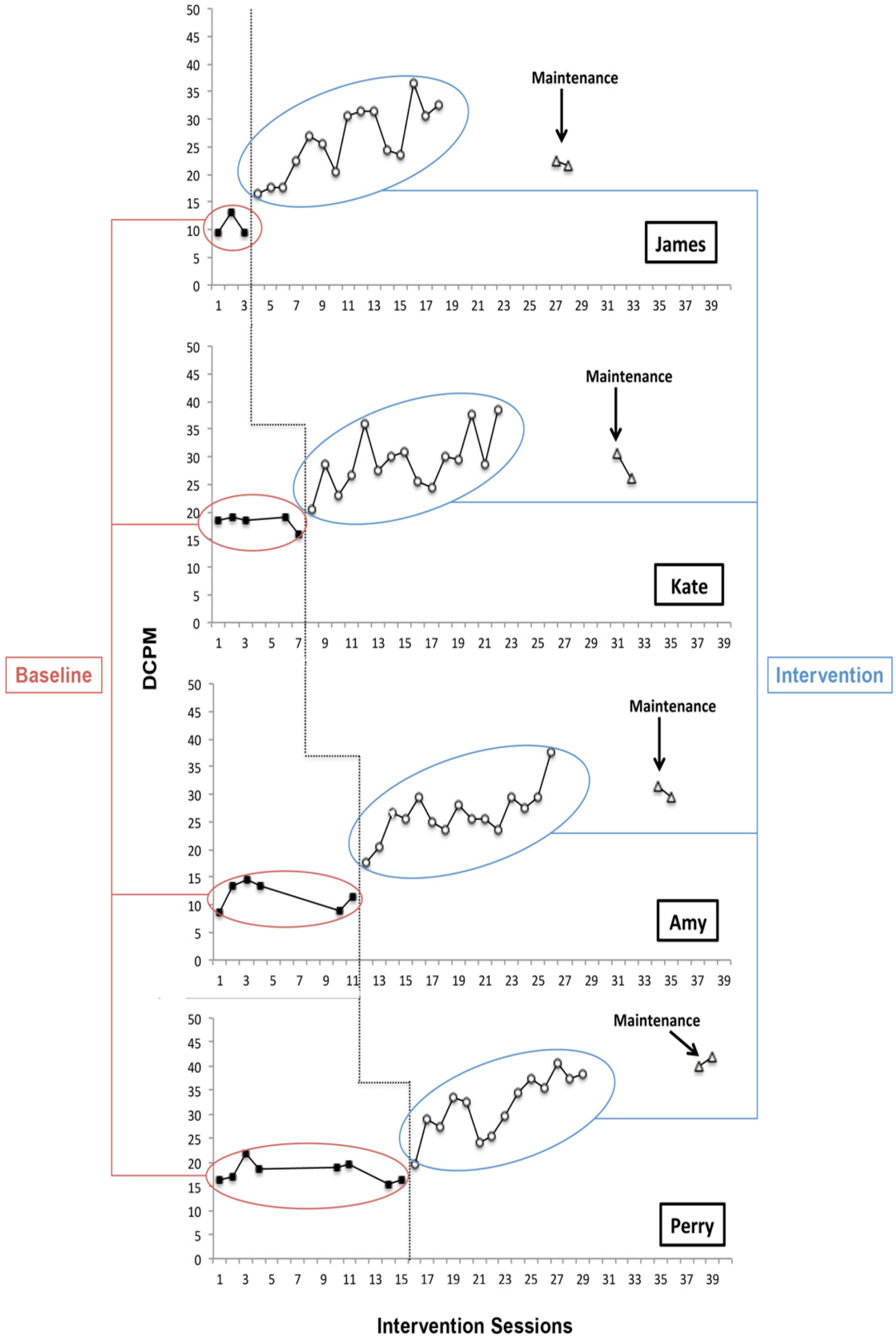


Figure 4. 7: Consistency of data patterns across similar phases across participants

Effect Sizes

In addition to the visual inspection, two effect sizes, PND and Tau-U, were computed to investigate the magnitude and strength of the effect of explicit, strategic TDI with iPad application practice on participants' multiplication facts fluency performance. Table 4.3 shows the results of PND and Tau-U computed for all participants.

Participants	PND (%)	Tau-U		
		Tau	$CI_{90\%}$	*p*-value
James	100	1.00	0.38<>1.62	<.01 (0.0077)
Kate	100	1.00	0.50<>1.50	<.01 (0.0011)
Amy	100	1.00	0.53<>1.47	<.01 (0.0005)
Perry	93.33	0.98	0.55<>1.40	<.01 (0.0002)

Note. PND = percentage of non-overlapping data; CI = confidence interval.

Table 4. 3: PND and Tau-U

James

From baseline to the intervention phase, James' data demonstrated a 100% improvement; there was no overlapped data point between baseline and intervention phases (PND: 100%, Tau: 1.00, $p < .01$). The PND value (100%) can be interpreted as a large effect (Scruggs & Mastropieri, 1998) of explicit, strategic TDI with iPad application practice on his fluency skills. In addition, the Tau value (1.00) was interpreted, as there was 100% improvement from baseline to intervention phase or there was a 100% nonoverlap between baseline and intervention data points. According to confidence interval (CI) data, there was 90% certainty that the true effect size of 1.00 lies between

0.038 and 1.62 (CI_{90}: 0.38<>1.62) (Neyman, 1935). In addition, the results were statistically significant (p = 0.0077).

Kate

According to PND (100%) data, Kate improved her fluency score 100% between baseline and intervention phase; there was no overlap data point between the phases. The PND value demonstrated a large effect of explicit, strategic TDI with iPad application practice on the target fact fluency performance of Kate. In addition, the Tau value (Tau: 1.00, $p < .01$) showed that her fluency score improved 100% from baseline to intervention phase or there was a 100% nonoverlap in data points between the two phases. The CI data showed there was 90% certainty that the true effect size of 1.00 lies between 0.50 and 1.50 (CI_{90}: 0.50<>1.50) (Neyman, 1935). Moreover, the results were statistically significant (p = 0.0011).

Amy

Amy's PND data represents there was a 100% nonoverlap in data points between baseline and intervention and her fluency score improved 100% between the two phases. The PND value demonstrated a large effect of explicit, strategic TDI with iPad application on her target fact fluency performance. In addition, the Tau value (Tau: 1.00, $p < .01$) shows there was 100% improvement from baseline and intervention phase or there was a 100% nonoverlap in data points between the two phases. According to the CI value, there was 90% certainty that the true effect size of 1.00 lies between 0.55 and 1.540 (CI_{90} : 0.55<>1.40) (Neyman, 1935). The p value of Tau demonstrated the results were statistically significant (p = 0.0005).

Perry

According to the PND data, Perry's fluency score improved 93.33% from baseline to intervention phase; 93.33% of data points were not overlapped between the two phases. Approximately, 6.67% of the data overlapped; one of fifteen intervention data points overlapped with the highest baseline data point. The PND value (93.33%) demonstrated a large effect of explicit, strategic TDI with iPad application practice on Perry's fact fluency performance. The Tau value (0.98) can be interpreted, as there was 98% improvement from baseline to intervention phases or 98% nonoverlap between the two phases. Regarding the CI reported, there was 90% certainty that the true effect size of 0.98 lies between 0.55 and 1.40 (CI_{90}: 0.55<>1.40) (Neyman, 1935). The p value (0.0002) also indicated the result was statistically significant.

Summary

To summarize, both effect sizes computed indicated there was a large effect of explicit, strategic TDI with iPad application practice on the targeted multiplication fact fluency performance of all participants. According to their PND and Tau-U data, all students showed no overlapped data points between baseline and intervention phase except Perry (one intervention data point overlapped with the highest baseline data point). The range of the PND values was between 93.33 and 100 (their fluency scores improved between 93.33% and 100% from baseline and intervention phase) and the range of Tau values was between 0.98 and 1.00 (their fluency scores improved between 98% and 100% from baseline and intervention phase). In addition, the results were statistically significant (all $p < .01$). In addition, the web-based Tau-U calculator was able to compute an overall, averaged effect size for all participants. The overall Tau computed across all participants was 0.99 (there was 99% improvement from baseline and intervention phase) with 90% CI ranging between 0.74 and 1.25 (CI_{90}: 0.74<>1.25). Based on both visual analysis and effect sizes computed, it was evident that there was a causal relation between explicit, strategic TDI with iPad application practice and the targeted multiplication fact fluency of all participants. In addition, the effect of the intervention was large and statistically significant.

RESEARCH QUESTION 2

Research question 2 investigated the effect of explicit, strategic TDI with iPad application practice on the use of a doubling strategy for solving target multiplication facts among participants. Participants were asked to work on the strategy usage test three times (before, in the middle of, and after intervention phase) across the study. For research question 2, specifically, the following two facets were examined: (a) accuracy of solving multiplication facts problems on the strategy usage test, and (b) percentage of the use of a doubling strategy when solving ten facts problems on the strategy usage test.

Accuracy on the Strategy Usage Tests

Table 4.4 shows the accuracy on the strategy usage tests for all participants across the study. It was determined that participants improved in accuracy over time; the average accuracy percentage across participants was 65% (before intervention), 82.5% (in the middle of the intervention), and 97.5% (after intervention). James' accuracy percentage was 40% (before intervention), 70% (the middle of intervention), and 90% (after intervention). He improved his accuracy percentage 30% from baseline to the middle of the intervention and 20% from the middle of the intervention to after the intervention. Kate's accuracy percentage was 70% (before intervention), 90% (the middle of intervention), and 100% (after intervention). She improved her accuracy 20% from baseline to the middle of the intervention and 10% from the middle of the intervention to after the intervention. Amy's accuracy percentage was 70% (before intervention), 70% (the middle of intervention), and 100% (after intervention). She did not improve her accuracy between baseline and the middle of intervention, but improved 30% from the middle of the intervention to after intervention. In addition, Perry's accuracy percentage was 80% (before intervention), 100% (the middle of intervention), and 100% (after intervention). He improved 20% from before intervention to the middle of intervention. He solved the fact problems 100% both in the middle of intervention and after intervention, but his speed of solving problems became much faster after the intervention. Except for James (90%), all participants solved all 10 problems correctly (100%) after intervention. In addition, it was observed all participants were able to recall the facts automatically when working on the test after intervention.

Participant	Accuracy (%)		
	Before intervention	Middle of intervention	After intervention
James	40%	70%	90%
Kate	70%	90%	100%
Amy	70%	70%	100%
Perry	80%	100%	100%

Table 4. 4: Accuracy on Strategy Usage Test across Participants

Percentage of Doubling Strategy Usage

It was also examined whether participants improved in their use of the doubling strategy to solve multiplication fact problems across the study. Table 4.5 represents percentage of using the doubling strategy and Table 4.6 shows the percentage of all strategies used for solving problems on the strategy usage test. Overall, it was determined none of the participants used a doubling strategy before intervention; participants used a variety of strategies (e.g., guess, count-by, count-all, additive calculation, learned product). In the middle of the intervention, it was observed that all participants except Perry used only two strategies, the doubling strategy (30-40%) and the learned product strategy (60-70%); Perry used only the learned product strategy (100%); he did not use the doubling strategy across the study. After intervention, it was determined none of the participants used the doubling strategy. All of them only used the learned product strategy (100%). Even though none of them used the doubling strategy, all participants were able to recall the facts automatically.

Participant	Before intervention	Middle of intervention	After intervention
James	0%	40% (60% Learned product)	0% (100% Learned product)
Kate	0%	30% (70% Learned product)	0% (100% Learned product)
Amy	0%	40% (60% Learned product)	0% (100% Learned product)
Perry	0%	0% (100% Learned product)	0% (100% Learned product)

Table 4. 5: Percentage of Using the Doubling Strategy

Based on the strategy usage test data, it was demonstrated that participants improved in their use of the doubling strategy from before intervention to the middle of intervention except Perry; they used a mix of the doubling strategy and the learned product strategy. In addition, none of them used the doubling strategy after intervention; all of them used only the learned product strategy. According to Sherin and Fuson (2005), the learned product and the doubling strategy (hybrids) could be considered as more mature strategies than strategies such as count-all and simple additive calculation because the strategies are more complicated and children develop and employ the strategies more frequently as they get older. It was documented that explicit, strategic TDI with iPad application practice helped the participants improve use of more mature strategies such as the doubling strategy and the learned product strategy.

Participant	Percentage of Strategy Usage		
	Before intervention	Middle of intervention	After intervention
James	Learned product: 40% Other (I don't know): 40% Count-by: 20%	Learned product: 60% Hybrids (doubling strategy): 40%	Learned product: 100%
Kate	Additive calculation: 70% Learned product: 30%	Learned product: 70% Hybrids (doubling strategy): 30%	Learned product: 100%
Amy	Count-by: 30% Other (I don't know): 30% Hybrids: 20% (Learned product + count-all: 10% & Count all + Count-by: 10%) Count-all: 10%	Learned product: 60% Hybrids (doubling strategy): 40%	Learned product: 100%
Perry	Learned product: 60% Hybrids (learned product + additive calculation): 30% Guess: 10%	Learned product: 100%	Learned product: 100%

Table 4. 6: Percentage of Strategy Usage for Strategy Usage Tests

James

Before intervention, it was observed James used the learned product strategy (40%), count-by (20%), and others (I do not know; 40%). He did not use the doubling strategy before intervention. He solved four problems (i.e., 4 × 4, 3 × 8, 6 × 4, and 4 × 7) correctly with the learned product strategy. It was reported he used the count-by strategy for solving two problems (3 × 4 and 4 × 9) (e. g., to solve 4 × 3, he counted by 3 (3, 6, 9, 12)), but he could not answer correctly. He also tried to solve the rest of the four problems (8 × 7, 6 × 8, 8 × 8, and 9 × 8), but he could not answer the problems; he said, "I do not know," when he was asked what strategy he tried to use for solving the problems.

In the middle of the intervention, it was observed he used only two strategies, the doubling strategy and the learned product. He did not use the count-by strategy he used before the intervention phase for solving problems at this time point. He improved the use of doubling strategy; he used the strategy for solving four problems (8 × 8, 6 × 8, 4 × 7, and 9 × 8). However, he could only answer two of them correctly (6 × 8, 4 × 7); for the other two problems (8 × 8, 9 × 8), he needed more time to answer using the strategy. He also tried to use the learned product strategy for solving six problems (4 × 9, 6 × 4, 3 × 8, 4 × 4, 8 × 7, 3 × 4). He was able to solve five of them correctly; he could not answer 3 × 8 in 30 seconds.

After intervention, he only used the learned product strategy for solving all ten problems. It was observed he recalled the facts quickly and effortlessly. He solved nine out of ten problems correctly. However, he made a mistake to recall a correct answer for

4 × 9; he thought it was 24. He did not use the doubling strategy at this time point. Table 4. 7. shows examples of James' strategy usage for solving problems on the strategy usage tests across the study.

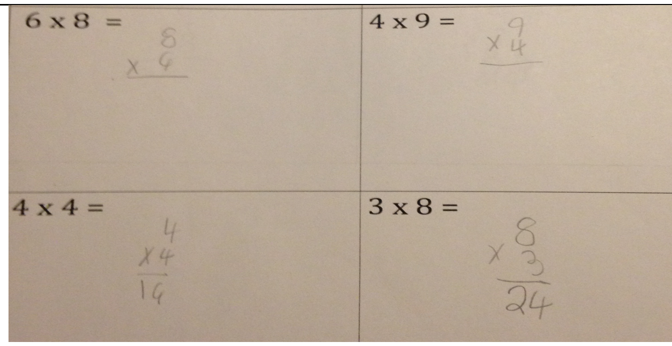
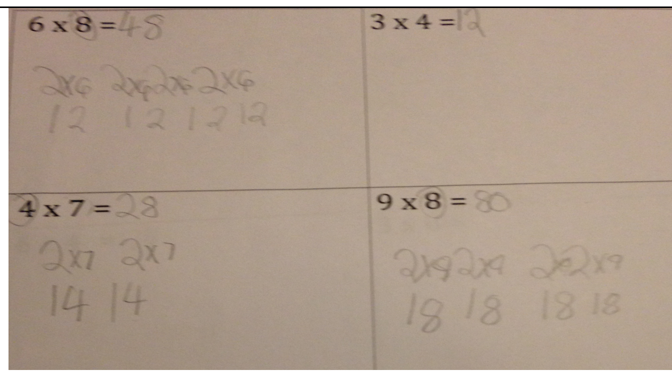
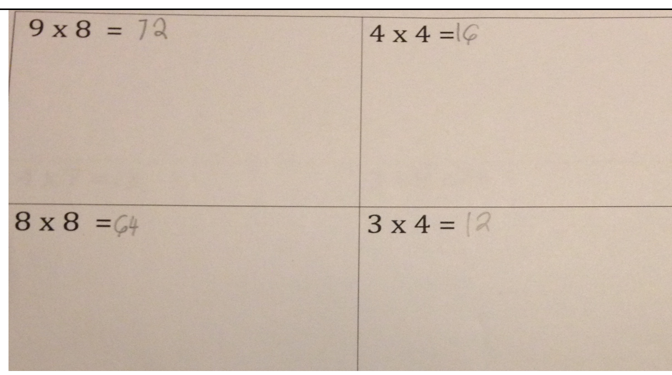
Before intervention	
Middle of intervention	
After intervention	

Table 4. 7: Examples of James' Strategy Usage

Kate

Before intervention, it was observed Kate used two strategies including the additive calculation strategy (70%) and learned product strategy (30%) for solving problems. She did not use the doubling strategy before intervention. She used additive calculation strategy for solving seven out of ten problems (8 × 7, 6 × 8, 4 × 9, 8 × 8, 6 × 4, 9 × 8, 4 × 7). For example, to solve 6 × 8, she wrote "6" six times (6, 6, 6, 6, 6, 6), added two 6s (12, 12, 12), and then summed the three "12" (12 + 12 + 12 = 48, see Table 4. 8). She was able to answer correctly only four (6 × 8, 4 × 9, 6 × 4, 4 × 7) out of the seven problems using the additive calculation strategy; she needed more time to answer using the strategy for the other three problems (8 × 7, 8 × 8, 9 × 8). Kate also used the learned product strategy to solve three problems (3 × 4, 4 × 4, 3 × 8) and she solved all three problems correctly.

In the middle of the intervention, it was determined she used two strategies (learned product (70%) and the doubling strategy (30%)) to solve problems. She did not use the additive calculation strategy she often used before the intervention at this time point. Kate improved her use of the doubling strategy. She used the strategy for solving three problems (8 × 8, 6 × 8, 9 × 8) and solved them all correctly. She also used the product strategy for solving seven problems (4 × 9, 6 × 4, 3 × 8, 4 × 4, 8 × 7, 3 × 4, 4 × 7). She was able to recall the facts correctly except one fact (3 × 8).

After intervention, Kate used only one strategy, the learned product strategy. She was able to recall all facts quickly and effortlessly. She solved all ten problems correctly.

She did not use the doubling strategy at this time. Table 4.8. shows examples of Kate's strategy usage for solving problems on the strategy usage test,

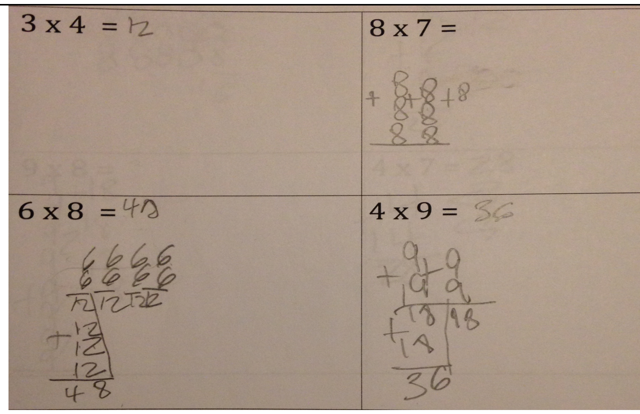
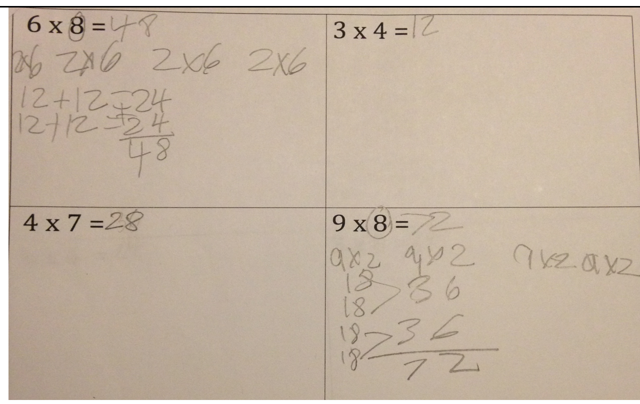
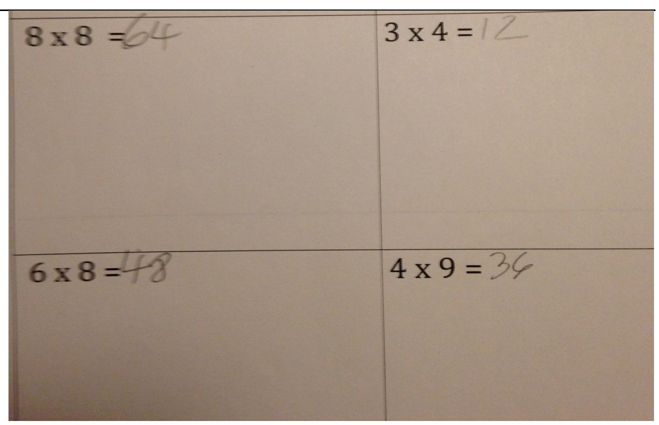
Before intervention	
Middle of intervention	
After intervention	

Table 4. 8: Examples of Kate's Strategy Usage

Amy

Before intervention, it was determined Amy used a variety of strategies including count-by (30%), hybrids (learned product + count-all: 10% and count all + count-by: 10%), count-all (10%) and others (I do not know; 30%). She used the count-by strategy for solving three problems (3 × 4, 4 × 9, 3 × 8). For example, to solve 3 × 4, she said, "I drew a number line and count by 4, 8, and 12 (see Table 4.9)." By using the count-by strategy, she solved all three problems correctly. She used two different hybrid strategies (combined strategies). First of all, for solving 4 × 4, she used a combined strategy with learned product and count-all. She explained she knew 4 × 3 = 12 (learned product) and then added four to the 12 (12 + 1+ 1 + 1+ 1; count-all). Second, for solving 6 × 4, she used a combined strategy with count-all and count-by. She said she drew six dots first and then counted all dots (••••••: 6). Then she counted by 6 (i.e., 6, 12, 18, 24). She solved the two problems correctly with the hybrid strategies. For solving 4 × 7, she used the count-all strategy; she explained, "I drew dots (7 lines of 4 dots) and counted all." She solved the problem with the strategy correctly. She was not able to solve three problems (8 × 8, 8 × 7, 6 × 8) and she said, "I do not know," when she was asked what strategies were used for trying to solve the problems.

In the middle of the intervention, she used only two strategies (i.e., learned product and doubling strategy) to solve the problems. She no longer used the strategies (e.g., count-by or count-all) that she used before the intervention at this point. She improved her use of the doubling strategy; she tried to use the strategy to solve four out of ten problems (3 × 8, 8 × 7, 6 × 8, 8 × 8). However, she solved only one of them (8 × 8) correctly; although she used the strategy correctly, she was not able to come up with the answers in 30 seconds. In addition, she used the learned product strategy for solving six problems (4 × 9, 6 × 4, 4 × 4, 3 × 4, 4 × 7, 9 × 8) correctly.

After the intervention, she used only the learned product strategy for solving all of the problems. She was able to recall the facts quickly without effort. She solved them all correctly. Table 4.9 represents examples of Amy's strategy usage.

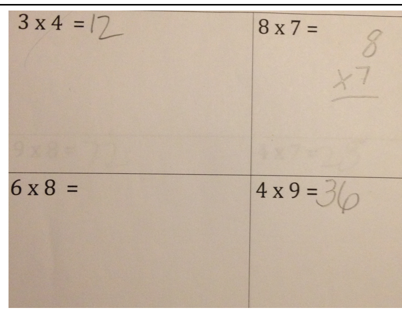
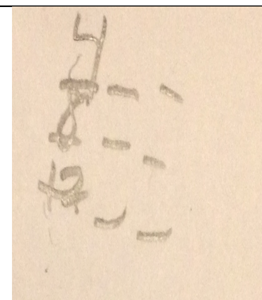
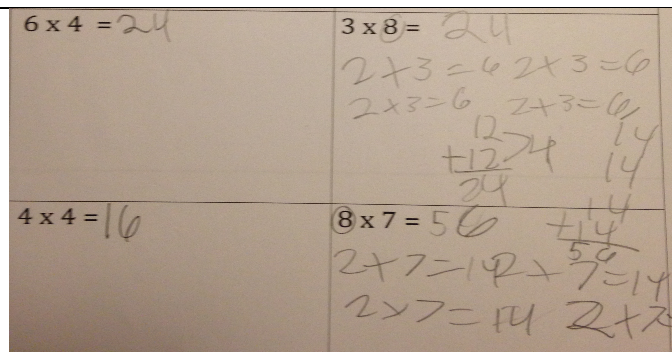
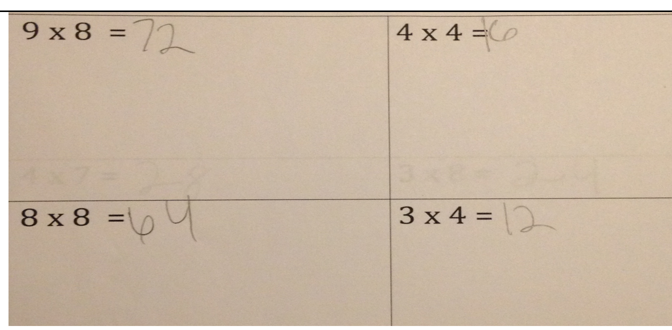
Before intervention	 
Middle of intervention	
After intervention	

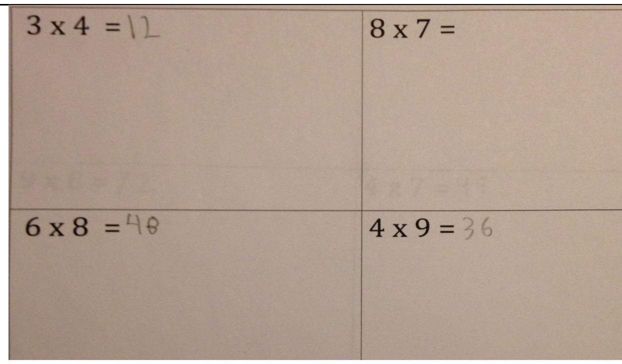
Table 4. 9: Examples of Amy' Strategy Usage

Perry

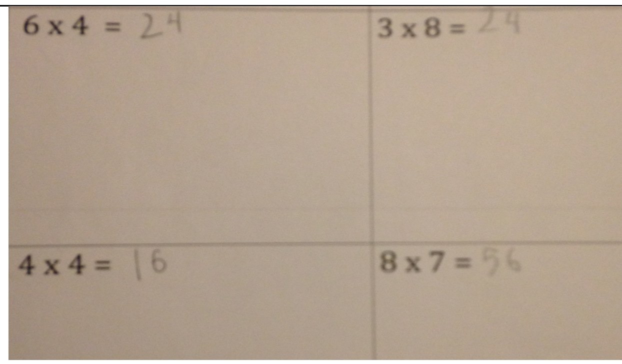
Before the intervention, Perry used three strategies (learned product, hybrids, and guess) to solve problems. First of all, he was able to recall six facts (3×4, 4×9, 4×4, 8×8, 6×4, 9×8) correctly. He tried to use the hybrid strategy (learned product + additive calculation) to solve three problems (6×8, 3×8, 8×7). For example, to solve 6×8, he said, "I knew $8 \times 2 = 16$ (learned product), so I tried to add up $16 + 16 + 16$ (additive calculation)." He solved two problems correctly using the strategy, but was not able to answer to 8×7. In addition, he said he just guessed to solve 4×7 and was not able to come up with an answer in time.

In the middle of the intervention and after the intervention, he only used the learned product strategy. He was able to recall all ten facts correctly in 30 seconds. Unlike the other three participants, it was not observed he improved using the doubling strategy in the middle of the intervention; he did not use the strategy to solve the problems on the strategy usage test across the study. According to his social validity interview, he did not like using the doubling strategy; he thought it took too long to solve the problems. Also, he was able to reach 100% accuracy using the learned product strategy even in the middle of the intervention while none of the other participants reached 100% accuracy. Table 4.10 shows examples of Perry's strategy usage to solve problems on the strategy usage tests.

Before intervention



Middle of intervention



After intervention

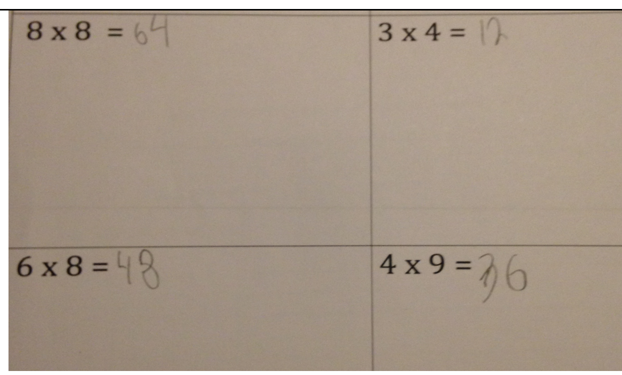


Table 4. 10: Examples of Perry' Strategy Usage

RESEARCH QUESTION 3

Research Question 3 investigated how the participants maintained their fluency in the target multiplication, two weeks following explicit, strategic TDI with iPad application practice. The result (see Table 4.2 and Figure 4.3) showed that three of the participants maintained their fluency scores that gained through the intervention at the instructional level (range of between 21.5 and 31.5 DC/M). In addition, one of the participants (Perry) maintained his fluency scores at the mastery level (range of between 40 and 42 DC/M).

James

According to his level data, it was determined his fact fluency skills were maintained at the instructional level (22 DC/M), 2 weeks following intervention phase. His fluency score in maintenance phase decreased by 3.87 DC/M compared to his level of intervention data (25.87 DC/M). Among participants, his DC/M score dropped the most compared to the level of intervention data. However, his fluency skill was at the frustration level during baseline and he improved his skills to the instructional level during the intervention phase. In addition, his fluency scores remained at the instructional level following the removal of the intervention during maintenance.

Kate

The level of Kate's data in the maintenance phase was 28.25 DC/M. When compared to the level of intervention phase (29.13 DC/M), her fluency score declined by 0.88 DC/M. However, her fluency performance was at the frustration level (18.20 DC/M) during baseline phase, she improved her fluency to the instructional level during the intervention phase and she maintained the intervention gains at the instructional level during the maintenance phase, after the intervention was removed.

Amy

Amy's level of data for the maintenance phase was 30.5 DC/M; she maintained her fluency score at the instructional level following the removal of the intervention during maintenance. Her maintenance performance was 4.2 DC/M higher compared to the level of her intervention data (26.3 DC/M). She demonstrated her fluency performance at the frustration level (11.75 DC/M) during baseline phase, at the instructional level during intervention phase and it was determined her intervention gains were maintained at the instructional level.

Perry

It was demonstrated that Perry's level of maintenance performance was 41 DC/M; his fluency was maintained at the mastery level during the maintenance phase, after the intervention was removed. His level of data in the maintenance phase was 8.5 DC/M higher than the level of intervention data (32.5 DC/M). Even though his level of intervention data was at the instructional level, he reached the mastery level twice during the intervention phase (40.5 and 42.5 DC/M). He demonstrated the fact fluency at the frustration level (18.29 DC/M) during baseline and his fluency reached the mastery level during the intervention phase. Moreover, he maintained the intervention gains at the mastery level.

RESEARCH QUESTION 4

Research question 4 examined the perspectives of participants toward explicit, strategic TDI with iPad application practice on learning target multiplication facts. To understand their perspectives toward intervention, interviews were conducted after the intervention phase was completed. Participants were asked to answer a total of 20 questions (16 five-point Likert scale questions (1: strong disagree, 2: disagree, 3: neither, 4: agree, 5: strongly agree) and four open-ended questions) to express their thoughts toward the intervention in terms of explicit, strategic TDI, independent practice using an iPad application, daily probes and graphing daily data and overall intervention.

Explicit, Strategic Teacher-Directed Instruction

Table 4.4 represents overall participants' perspectives toward explicit, strategic TDI. First of all, the average Likert-scale to the question whether participants liked the cdoubling strategy was 3.5, between "neither" and "agree." James and Kate strongly favored the doubling strategy (1 point), Amy did not like or disliked it (neither, 3 points), and Perry strongly disliked it (5 points). Perry said he did not like it because it took a long time for him to use the doubling strategy to solve fact problems. During intervention sessions, it was often observed he did not like to use the doubling strategy. For the three other questions (question 2-4, see Table 4.11) regarding the doubling strategy, the average point was between 4 and 4.5 (agree and strongly agree) across all four participants. Overall, participants thought the doubling strategy was easy to learn and use in order to solve multiplication facts (average points: 4), the strategy helped them do better on multiplication facts (average points: 4.5) and they wanted to recommend the use of the strategy to their friends (average points: 4); Amy said she already introduced the doubling strategy to her friends and recommended that them used it. It was determined that all students expressed positive perspectives toward using the doubling strategy except Perry. Perry did not think the strategy was easy to learn and use to solve the facts (2 points) and did not want to recommend it to his friends (1 point). However, he thought the strategy helped him to do better on the multiplication facts (4 points). Regarding the use of the behavior management system, Math Ready, participants thought it helped them to focus on their tasks (4.5 point).

Explicit, Strategic Teacher-Directed Instruction	Average scale
1. Do you like the doubling strategy we used?	3.5 *
2. Do you think the doubling strategy was easy to learn and use to solve multiplication number facts?	4*
3. Do you think the doubling strategy helped you to do better in multiplication number facts?	4.5*
4. Do you want to recommend using the doubling strategy to your friends?	4*
5. Do you think "Math Ready" helped you to focus on your tasks?	4.5*

Table 4. 11: Perspectives of Participants toward Explicit, Strategic TDI

**Note*: Rating options were 1 = strongly disagree, 2 = disagree, 3 = neither, 4 = agree, 5 = strongly agree.

Independent Practice using an iPad Application

It was determined that all participants had positive perspectives toward using an iPad application to practice the multiplication facts (average score between 4 and 4.75, see Table 4.12). According to the average scale points, participants liked using the iPad to practice facts (4.75), they thought using the iPad helped them learn the facts (4.5) and motivate them to practice facts (4), and they wanted to recommend using the iPad for practice facts to their friends (4). Even though all participants had positive views on using iPads for practice, James and Kate appeared to like using iPads more than Amy and Perry when comparing the points. In addition, it was interesting that even though all participants liked using the iPad application to practice multiplication facts, they preferred using flashcards or worksheets to the iPad application. For question 10, which asked what they prefer between flashcards or practice worksheets vs. the iPad application to practice facts, all of them except Perry selected flashcards or worksheets. The students were also asked to list the order in which they liked the methods. To assess the overall preferences across all participants, a different score was assigned; 3 points for the best method, 2 points for the second-best method, and 1 point for the least favored method. In addition, the point score was summed up across participants for each practice method (results: flashcard (9 points), worksheet (8 points), iPad application (5 points).

Per the results, it was determined that James and Kate selected flashcards as the best method to practice facts, followed by worksheets, and iPads. Both of them said using flashcards helped them to practice the facts more than the other methods. Amy selected worksheets as the best method to practice followed by the iPad application and flashcards. She did not have a specific reason for the order. In addition, Perry selected the iPad application as the best method, followed by flashcards, and worksheets; he said using the iPad to practice was fun. Overall, it was determined that the participants preferred the practice methods in the following order: (a) flashcards, (b) practice worksheets and (c) the iPad application.

Independent Practice using an iPad Application	Average scale
6. Did you like using an iPad to practice multiplication number facts?	4.75*
7. Do you think using an iPad helped you learn multiplication number facts?	4.5*
8. Do you think using an iPad helped you motivate practice multiplication number facts?	4*
9. Do you want to recommend using an iPad for practicing multiplication number facts for your friends?	4*
10. What do you prefer for practicing multiplication facts: (a) flashcards or practice worksheets or (b) using an iPad app? Why?	1. Flashcard 2. Worksheet 3. iPad application

Table 4. 12: Perspectives of Participants toward Independent Practice Using an iPad App

**Note*: Rating options were 1 = strongly disagree, 2 = disagree, 3 = neither, 4 = agree, 5 = strongly agree.

Daily Probe and Graphing Daily Data

Participants were also asked their thoughts toward the daily probes and graphing their daily data. Figure 4.8 represents participants' My Progress Graph in which they graphed the number of correct items in the daily probes at the end of each session during the intervention phase. Overall, participants expressed positive perspectives toward graphing daily data and teachers' feedback on their performance and effort for the daily probes (between 4.5 and 4.75 points, see Table 4. 13). They thought graphing their daily data and teachers' feedback on their performance motivated them to learn the facts and practice harder (4.75 points).

Daily Probe and Graphing Daily Data	Average scale
11. Did you like graphing your daily data?	4.5*
12. Did you like the teacher's feedback on your performance and effort?	4.75*
13. Do you think graphing your daily data and teacher's feedback motivated you to learn and work harder?	4.75*

Table 4. 13: Perspectives of Participants toward Daily Probes and Graphing Daily Data

**Note*: Rating options were 1 = strongly disagree, 2 = disagree, 3 = neither, 4 = agree, 5 = strongly agree.

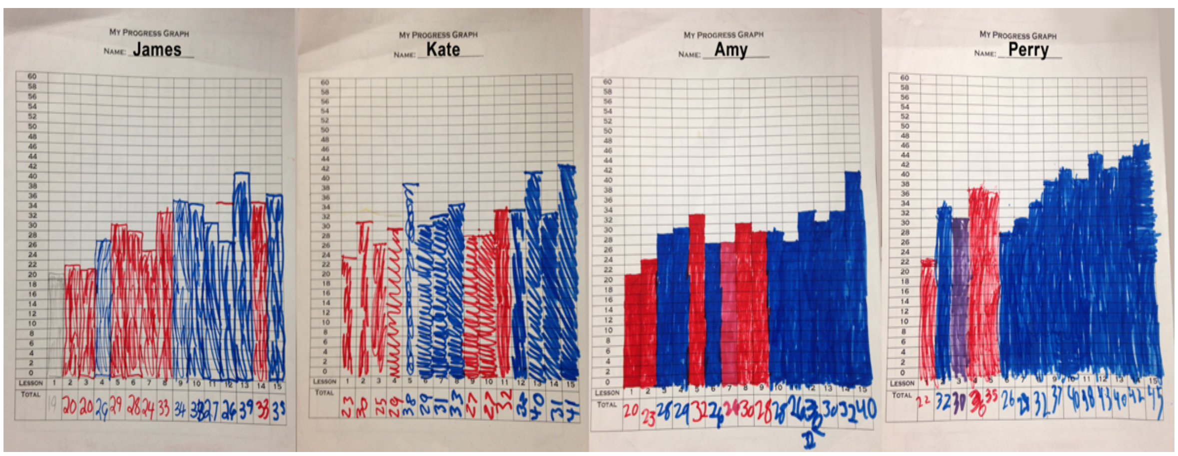


Figure 4. 8: My Progress Graph for participants

Overall Intervention

In addition, some questions were asked to determine participants' perspectives toward the overall intervention. It was demonstrated that participants expressed positive perspectives toward the overall intervention (average points were between 4.5 and 4.75). They enjoyed the intervention time (4.75), thought the intervention helped them to learn multiplication facts (4.75), be motivated/engaged in learning the facts (4.75), and be focused on the tasks (learning facts; 4.75). For the open-ended question #18 (see Table 4.14), James and Kate said the aspect of the intervention session they liked best was learning target facts (×4s and ×8s). According to Kate, she was not good at the facts before intervention, but she was good at the facts after the intervention. Amy liked writing on a mini wipe board when practicing facts using the doubling strategy. In addition, Perry liked using an iPad application to practice facts the best; he said it was fun. For open-ended question 19, to examine what aspects of the intervention session they disliked, James and Amy said there were none. Kate did not like that she had to be removed from the special class; she was provided with tutoring between 3:15-3:45 p.m., so she had to be out of her last class 10 minutes early. In addition, Perry did not like working on practice worksheets.

In sum, based on the results of the social validity interviews, it was determined that participants' perspectives toward the intervention (regarding explicit, strategic TDI, independent practice using an iPad application, providing teachers' feedback on daily probes and graphing daily data, and overall intervention) were positive.

Overall Intervention	Average scale
14. Did you enjoy our tutoring time?	4.75*
15. Do you think tutoring time helped you learn multiplication facts?	4.75*
16. Do you think tutoring time helped you to be motivated and engaged you in learning multiplication facts?	4.75*
17. Do you think tutoring time helped you to be focused on task?	4.5*
18. What did you like best about our tutoring time? Why?	• Learning ×4s and ×8s • iPads: fun to use • Writing on the board
19. What aspects of tutoring time did you dislike? Why?	• Worksheet • Early out of specials • None
20. Do you have any other comments/suggestions regarding tutoring time?	• None

Table 4. 14: Perspectives of Participants toward Overall Intervention

**Note*: Rating options were 1 = strongly disagree, 2 = disagree, 3 = neither, 4 = agree, 5 = strongly agree.

SUMMARY OF THE CHAPTER

To sum up this chapter, for research question 1, based on visual analysis of data and effect sizes computed, it was determined explicit, strategic TDI with iPad application practice was effective on the improvement of multiplication fact performance of 5th grade students with LD, who have mathematics IEP goals; a statistically significant large effect of intervention was detected. All students demonstrated their fact fluency at the frustration level during baseline, but improved the instructional level of fluency during the intervention phase. Perry reached the mastery level of fluency, and the highest fluency scores of the other three participants were close to the mastery level. For research question 2, a data analysis was conducted on the accuracy and percentage of doubling strategy usage of participants on the strategy usage tests across the study. It was determined that all participants except Perry improved using the doubling strategy from baseline to the middle of the intervention, but they did not use the strategy after the intervention. All of them were able to recall the facts quickly and effortlessly after the intervention. For research question 3, it was detected all participants maintained intervention gains 2 weeks following the intervention. Perry maintained his fact performance at the mastery level while the three other participants maintained their performance at the instructional level. At last, for research question 4, all participants expressed positive perspectives toward explicit, strategic TDI with iPad application practice. Regarding the use of the doubling strategy, except Perry, all students favored using the strategy. All participants liked using the iPad application to practice, getting their teacher's feedback on their daily probes and graphing their daily data. Interestingly, even though all participants expressed their preference for using iPads to practice, most of them preferred using flashcards or worksheets to using the iPads.

Chapter 5: Discussion

The purpose of this study was to examine the effects of explicit, strategic TDI with iPad application practice on the multiplication fact performance of 5^{th} grade students with LD, who have mathematics IEP goals. Four students with LD who have mathematics IEP goals and demonstrated a lack of the target multiplication fact skills (×4s and ×8s) participated in this study.

The goal of the intervention was to improve the participants' fluency on multiplication facts, which is an essential foundation skill for developing a higher level of mathematics (e.g., algebra, fractions) (NGA/CCSSO, 2010; NMAP, 2008). In addition, for upper level elementary students, instruction on multiplication fact skills has been emphasized (NGA/CCSSO, 2010; NMAP, 2008). The intervention was designed with effective instructional strategies (e.g., explicit, strategic instruction, CBI, CRA, teaching logically sequenced or ranged mathematics skills, providing data and feedback on student performance) for teaching mathematics for students with LD recommended by previous literature (e.g., Cihak & Bowlin, 2009; Gersten et al., 2009; Kroesbergen & Van Luitt, 2003; NMAP, 2008; Nordness et al., 2011; Swanson, Hoskyn, & Lee, 1999). Participants were taught a doubling strategy explicitly to solve the target multiplication facts and had independent practice opportunities using an iPad application. Each intervention session consisted of the following instructional routine: (a) warm-up, (b) modeling, (c), guided practice, (d) independent practice using an iPad application, (e) 2-minute daily probes, and (f) graphing daily data, and providing feedback on the student performance. Fifteen 30-minute, one-to-one intervention sessions were provided.

In order to investigate the effects of the intervention, a single case design, a multiple probe baseline design across participants, was employed. During the baseline phase, participants worked on 2-minute daily probes, during the intervention phase, 15 intervention sessions were provided, and in the maintenance phase, 2 weeks following the intervention phase, participants were asked to work on two daily probes to check the maintenance effect of the intervention. Participants' DC/M on the daily probes across the study was measured to examine the effect of intervention on multiplication fact fluency. In addition, participants were assessed with the strategy usage test three times across the study (before, in the middle of, and after the intervention) to investigate the effects of the intervention on participants' doubling strategy usage. After the intervention, a social validity interview was conducted to understand the perspectives of participants toward the intervention. In this chapter, results in relation to the four research questions are discussed. In addition, the limitations of this study, suggestions for future research, and implications for practice are presented.

RESEARCH QUESTION 1

Research question 1 investigated the effect of explicit, strategic TDI with iPad application practice on the fact performance of the participants. To examine the effect, both visual analysis and computing effect sizes were conducted.

As a result of the visual analysis, it was determined that explicit, strategic TDI with iPad application practice had a causal relation with participants' multiplication fact fluency performance (Kratochwill et al., 2010). All participants demonstrated fact fluency DC/M on daily probes at the frustration level during the baseline phase but they improved fluency performance significantly from the baseline to the intervention phase (change of level- James: 15.20, Kate: 10.93, Amy: 14.55, Perry: 14.21 DC/M) and demonstrated fluency at the instructional level. It was indicated that the level of participants' fluency performance changed after explicit, strategic TDI with iPad application practice was introduced. Even Perry's two highest intervention fluency scores reached the mastery level and the other three participants' highest scores were close to the mastery level. In addition, all participants maintained intervention gains 2 weeks following the intervention phase. In the maintenance phase, three participants maintained their fluency scores at the instructional level while one of the participants (Perry) demonstrated his fluency at the mastery level. Perry was the only one of the participants who reached the mastery level during both the intervention and the maintenance phase. According to his FIE data, his mathematics scores on the standardized test (Kaufman Test of Educational Achievement II) was in the average range. It might have an impact on the results. However, Amy's mathematics standardized score was in the average range, but she did not reach the mastery level. James and Kate had mathematics-standardized scores (Woodcock-Johnson III Test of Achievement) lower than the average range. In addition, according to Perry's baseline data and strategy usage test before intervention, it seemed like Perry was knowledgeable at facts more than the other participants, although he was very slow to recall the facts; this might have had an impact on the results. It was also found that all participants had no or a decreasing trend during baseline, but only after the intervention was introduced, the trend moved upward. Moreover, except Perry (one intervention data point overlapped with the highest baseline data point), all participants had no overlapped data points between the baseline and intervention phase; the range of immediacy effect of intervention for participants was between 61.70 and 101.70%. Based on the data, it was indicated that the intervention rather than any other external factors strongly influenced participants' improved performance (Kratochwill et al., 2010).

In addition, the effect sizes (PND and Tau-U) computed indicated there was a significant large effect of explicit, strategic TDI with iPad application practice on improving the target fact fluency of all participants (Parket et al., 2011; Scruggs & Mastropieri, 1998). According to the effect size data (PND: James, Kate, Amy: 100%, Perry: 93.33%; Tau: James, Kate, Amy: 1.0, Perry: 0.98), all participants improved their fluency scores (PND: between 93.33 and 100%, Tau: between 98 and 100%) from baseline to intervention phase. The Tau-U data also demonstrated the result was statistically significant (all $p < .01$).

According to both the visual analysis and effect sizes computed, it was indicated that explicit, strategic TDI with iPad application practice was effective for supporting participants with LD who have IEP goals in mathematics to improve their multiplication fact fluency. The participants' data showed that before the intervention, all participants' multiplication fact fluency performance was at the frustration level. It was consistent with findings in previous literature; many students with LD experience difficulties with developing basic fact automaticity and computation fluency in multiplication (e.g., Mabbott & Bisanz, 2008; Mazzocco, Devlin, & McKenny, 2008; Rotem & Henik, 2013). However, after the intervention was introduced, all participants improved their fact fluency performance. The intervention, explicit, strategic TDI with iPad application practice, was designed with various effective instructional variables (e.g., explicit, strategic instruction, CBI, CRA, providing student data and teacher feedback) recommended by previous studies (Gersten et al., 2009; NMAP, 2008; Seo & Bryant, 2009; Swanson, Hoskyn, & Lee, 1999). The instructional variable features embedded in the intervention may be possible factors that account for the results of this study.

Instructional Features of the Intervention

Explicit, strategic TDI

Previous research has consistently recommended the use of explicit instruction and strategy instruction as effective instructional methods for teaching students with LD various mathematics skills (Gersten et al., 2009; Glover et al., 2010; Iseman & Naglieri, 2011; Kroesbergen & Van Luitt, 2003; Montague & Dietz, 2009; Swanson, 1999). In particular, it was indicated that combined instruction between explicit instruction and strategy instruction could be more effective than any other instructional approaches for teaching students with LD; it is also effective for helping them to maintain and generalize the skills obtained (Swanson, 1999; Swanson & Hoskyn, 1998).

One of the important instructional features of the intervention for this study was explicit, strategic TDI. Participants were provided explicit, strategic instruction (e.g., systemically organized instructional routine (modeling, guided practice, independent practice), systematic review of prerequisite and target skills, close monitoring of student performance, providing immediate and corrective feedback, teaching a doubling strategy) (Archer & Hughes, 2010; Swanson, 1999; Montague & Dietz, 20009; Woodward, 2006) during intervention sessions. The use of combined instruction, explicit, strategic instruction, may be possible factors that can account for the results of this study. The findings of this study were consistent with previous studies (e.g., Flores et al., 2006; Kelly et al., 1990; Mancl et al., 2012; McIntrye et al., 1991; Tournaki, 1993; Wood et al., 1993); the studies reported a statistically significant large effect of explicit, strategic instruction for supporting students with LD to improve basic mathematics skills as well as maintaining the skills after the removal of the intervention.

In addition, the use of the CRA technique during modeling to help participants develop a thorough conceptual understanding of why and how the doubling strategy works to solve the target multiplication facts (Witzel et al., 2003) might have an impact on the results of this study. The findings of this study was consistent with previous studies (e.g., Butler et al., 2003; Harris et al., 1995; Mancl et al., 2012; Morin & Miller, 1998; Peterson et al. 1998), which determined the positive effects of using CRA for teaching students with LD various mathematics skills including basic facts. In particular, the results of this study were consistent with a study by Macnl et al. (2012), which examined the effects of the use of explicit, strategic instruction with a CRA technique to teach elementary students with LD addition and subtraction computation and word problem solving skills. The study indicated explicit, strategic instruction using CRA was significantly effective (PND: 95.73%) for helping students with LD to improve target skills; all students even reached mastery criterion as well as maintained the skills over time. Additional use of the CRA technique might boost the effects of explicit, strategic TDI for teaching multiplication facts skills to students with LD.

Independent practice with an iPad application

In addition, independent practice with an iPad application, one of the important instructional variables in the intervention, may be a possible factor attributed to the findings of this study. Previous literature has reported that the use of technology (e.g., computer programs, tablet computers and applications) could be effective for helping students with LD enhance their mathematics skills (e.g., Chiang, 1986; McDermott & Watkins, 1983) as well as enhance their motivation, attention in learning, and time on task (Fisher, 1983; Okolo, 1992; Okolo et al., 1993). The findings of this study were consistent with previous research (e.g., Bryant et al., in press; Cihak & Bowlin, 2009; Irish, 2002; Koscinski & Gast, 1993a; Nordness et al., 2011; Wilson et al., 1996), which determined the positive effects of CBI or using tablet computers on teaching mathematics for students with LD; the studies reported the instructions were effective for helping students with LD to improve their mathematics skills, maintain the skills gained over time, and keep them motivated and engaged in learning.

In particular, the iPad application used for this study, Math Evolve included instructional variables (e.g., providing multiple opportunities to practice, immediate and corrective feedback, and progress monitoring data), which are considered to be effective practices for students with LD (Boone & Higgins, 2007; Matthew, Tsurusaki, & Basham, 2011). As Clark (1983) determined, instructional variables embedded in technology-based programs are critical for effective instruction. The effective instructional design of Math Evolve also might impact on the findings of this study. In addition, as indicated in previous studies (e.g., Okolo, 1992), using a game-type drill and practice application could be effective for helping students with LD to increase motivation to practice more problems. The finding of this study also revealed tablet computers such as iPads could be successfully integrated in TDI and independent practice with iPads could be effective for students with LD to learn mathematics as reported in recent research (e.g., Bryant et al., in press; Cihak & Bowlin, 2009; Nordness et al., 2011).

Moreover, combined instruction between TDI and using an iPad might boost the effects of the intervention for teaching students with LD multiplication fact skills, as indicated in previous studies (e.g., Bryant et al., in press; Howell et al., 1987; Powell et al., 2009). Howell et al. (1987) reported when CBI is combined with TDI, it could be more effective for teaching multiplication facts for students with LD compared to CBI only (Combined: PND of 85% vs. CBI only: PND of 50%). In addition, Powell et al. (2009) also reported on the positive effects of using CBI combined with TDI on enhancing mathematics fact retrieval for elementary students with LD.

Graphing daily data and providing teacher feedback

Moreover, graphing daily data and providing teacher feedback on their performance during the intervention may be one of the possible factors attributed to the findings of this study. Previous literature reported providing assessment data to students (e.g., graph, charts, and scores) and ongoing feedback on their performance could be effective for teaching mathematics skills to students with LD (e.g., Burns, 2005; Flores et al., 2006; Gersten et al., 2009; Krosebergen & Van Luit, 2003). For example, in a study by Burns (2005), students graphed their scores and were provided feedback. A significantly large effect of the intervention was detected; PND reported for all participants was 100%. In addition, according to the social validity interviews for this study, participants strongly liked graphing their daily data as well as getting teacher's feedback on their performance; they also said it motivated them to learn and work harder.

In addition, the instructional components (e.g., explicit strategic TDI, iPad application practice) in the intervention were rooted in operant conditioning theory and information processing theory. According to operant conditioning theory, students can learn best when instruction is directly focused on content to be taught and manipulated (Gurganusm, 2007). Explicit, strategic TDI with iPad application practice was designed to focus on the specific content to be taught and provide stimuli and reinforcements in a systemically structured manner in order to increase participants' fact skills. In addition, according to information processing theory, learning strategies are effective for helping students to learn and store information in their longer-term memory meaningfully (Silva, 2004). Based on the theories, the instructional components of the intervention might have an impact on the positive findings of this study.

RESEARCH QUESTION 2

Research question 2 investigated the effect of explicit, strategic TDI with iPad application practice in participants' doubling strategy usage to solve target multiplication facts. First of all, it was found all participants improved accuracy on the strategy usage tests across the study (James: 40-90%, Kate: 70-100%, Amy: 70-100%, Perry: 80-100%). Regarding the percentage of doubling strategy usage, it was indicated that overall, the participants improved the use of a doubling strategy from before the intervention phase to the middle of the intervention phase. Three of the participants improved in the use of a doubling strategy (James: 0-40%, Kate: 0-30%, Amy: 0-40%), but Perry did not improve the use of a doubling strategy. Participants used a variety of strategies (e.g., count-all, count-by, additive calculation, guess) before the intervention, they used a mix of the doubling strategy and the learned product strategy in the middle of the intervention, and all of them only used the learned product strategy after the intervention. Even though they did not improve the use of the doubling strategy from the middle of the intervention to after the intervention, they were able to recall the facts quickly and effortlessly after the intervention as hoped in the results of this study.

The findings of the study were consistent with previous research; that is, explicit, strategic instruction could be an effective instructional approach for helping students with LD to not only improve their mathematics skills but also learn and use more mature, efficient strategies for mathematics (e.g., Iseman & Naglieri, 2011; Tournaki, 1993; Van Houten, 1993; Van Luit & Naglieri, 1999; Woodward, 2006). First of all, teaching the doubling strategy explicitly for solving targeted multiplication facts (i.e., ×4s and ×8s) may be possible factors contribute to help participants with LD to improve their accuracy on the strategy usage tests across the study. In particular, it was also consistent with findings of Woodward's study (2006) and Wood et al. (1998), which taught the doubling strategy during intervention. Both studies reported it was effective for students with LD to learn and use the strategy as well as achieve automaticity in multiplication facts. In addition, as found in previous literature (e.g., Geary, 2011; Montague & Applegate, 1993; Rosenzweig et al., 2011), in this study, it was observed that students with LD often use developmentally immature, inefficient strategies (e.g., count-all, simple additive calculation) rather than more mature strategies (e.g., doubling strategy, learned product strategy) for solving multiplication facts before intervention (Geary et al., 2007; Sherin & Fuson, 2005). However, they improved using the more mature, efficient strategies (doubling strategy, learned product strategy) across the study. According to Sherin and Fuson (2005), the learned product and the doubling strategy could be considered as more mature strategies than strategies such as count-all and simple additive calculation because the strategies are more complicated and students usually develop and employ the strategies more frequently as they get older. In addition, according to information processing theory rooted in the intervention, learning strategies are important for effective learning (Silava, 2004) and teaching strategies meet the needs of students with LD who have difficulties with selecting effective strategies or using the strategies effectively and efficiently (Swanson, 1990, 1993).

Thus, it was documented that explicit, strategic TDI with iPad application practice could be an effective instructional method for supporting students with LD who have mathematics IEP goals to improve fact accuracy as well as the use of more mature strategies for solving multiplication facts. However, it was observed that one of the participants (Perry) did not improve in his use of a doubling strategy across the study. Even though his fluency was at the frustration level during the baseline phase, it was documented he already used the learned product strategy for solving 60% of the problems in the strategy usage test before the intervention phase. He was also able to recall all facts correctly in the test (used the learned product strategy for solving all of the problems correctly) in the middle of the intervention phase while none of the other participants were able to recall all the facts correctly. Unlikely working on daily probes (participants were asked to solve problems for 2 minutes as fast/many as they could), participants had 30 seconds for solving one fact problem. It seemed like Perry knew more facts than the other participants, but his speed of recalling the facts was slow. The social validity interview and observation during the intervention sessions revealed he disliked using the doubling strategy; he thought it took a long time to solve the facts. Interestingly, according to the social validity interview, although he strongly disliked it, he thought the strategy helped him to do better with the facts. It seemed he was reluctant to use the doubling strategy because he was able to recall many of the facts taught even though he could not recall quickly. For students like Perry (students who know facts, but need to increase the speed of recall), it might be more effective to use instructional strategies such as practice with flashcards or an iPad application, so they could improve speed of recalling the facts rather than learn the doubling strategy. According to the social validity interview, he strongly liked using the iPad application to practice the facts and he thought it helped him learn the facts.

RESEARCH QUESTION 3

Research question 3 examined the maintenance effect of explicit, strategic TDI with iPad application practice on the fact fluency performance of the participants. The result of the study indicated all participants demonstrated fluency scores at the frustration level before the intervention phase, but they improved their scores through the intervention and maintained intervention gains in 2 weeks after the intervention phase is completed. Thus, it was determined that explicit, strategic TDI with iPad application practice was helpful for students with LD, who have mathematics IEP goals, to maintain the fact skills obtained after the intervention was removed. The finding was consistent with previous research (e.g., Irish, 2002; Iseman & Naglieri, 2011; McIntyre et al., 1991; Nordness et al., 2011; Woodward, 2006); explicit, strategic instruction and using tablet computers could be effective for helping students with LD not only improve mathematics skills but also maintain the skills over time. In addition, Flores et al. (2006) reported strategic instruction with CRA also was effective for supporting students with LD to maintain multiplication fact skills obtained. Thus, the instructional components (e.g., explicit, strategic instruction, using tablet computers, CRA) of the intervention could be a possible factor that contributed to the findings of this study. Moreover, according to the information processing theory, learning strategies can help learners store information in their long-term memory meaningfully (Silva, 2004). Learning the doubling strategy might help participants to maintain the skills over time.

In particular, three of the participants maintained their fluency scores at the instructional level (range of between 21.5 and 31.5 DC/M) and one of the participants (Perry) maintained his fluency at the mastery level (range of between 40 and 42). Interestingly, for James and Kate, the level of maintenance data points were lower than the level of intervention data points, but for Amy and Perry, the level of maintenance data points were higher than the level of intervention data points. The maintenance effect of this study should be interpreted cautiously because it was found that Amy and Perry were provided some instruction on multiplication facts (×0s - ×12s) in their special education mathematics class during the maintenance phase while James and Kate were not provided instruction on target multiplication fact skills. This might have impacted the results.

RESEARCH QUESTION 4

Research question 4 investigated the perspectives of participants toward explicit, strategic TDI with iPad application. The social validity interview results revealed that participants had positive perspectives toward the intervention overall.

Explicit, Strategic TDI

First of all, regarding learning and using the doubling strategy, participants' perspectives varied; James and Kate strongly liked the strategy, Amy neither liked or disliked it, and Perry strongly disliked it. Perry said it took a long time to solve problems using the strategy. As discussed before, it was observed that Perry knew many of the target facts taught, but his speed of recalling the answer was slow. However, trying to recall the facts probably was faster than using the doubling strategy for him. That might be the reason why he strongly disliked using the strategy. In addition, for students like Perry (who knew facts, but whose speed of recalling the answer is slow), instruction that could support them to improve the speed of recall would be better than teaching strategies for solving the facts. However, except Perry, all participants thought the strategy was easy to learn and use to solve multiplication facts, it helped them to do better in the facts, and they wanted to recommended using the strategy to their friends (average score: 4 out of 5). Even though Perry did not like the strategy, he thought it helped him to do better on multiplication facts. In addition, Amy said she already taught the strategy to her friends and Amy's teacher said Amy showed her how to use the strategy to solve the fact; the teacher said Amy really liked the strategy. The interview results were consistent with findings reported in Flores et al. (2006); all participants with LD but one expressed positive perspectives toward the strategy instruction. Participants reported it was easy to learn a cognitive strategy, the strategy helped them to improve their multiplication fact skills, and they were able to use the strategy for solving unknown facts that they could not recall quickly. Interestingly, similar to Perry's case, one of the participants noted he disliked the strategy because it took too long even though he improved his fact skills through the strategic instruction as he reached mastery criterion. Based on the interview data, teaching the doubling strategy could be promising to support students with LD to improve their fact fluency on target multiplication facts; teaching the cognitive strategy might have enhanced their multiplication fact performance as indicated in previous research (e.g., Flores et al., 2006; Van Houten, 1993; Woodward, 2006).

Independent Practice Using an iPad Application

All participants also had positive perspectives toward using an iPad application to practice the multiplication facts. Participants strongly liked using the iPad application to practice facts, they thought the application helped them learn the facts, motivated practice, and wanted to recommend it to their friends (average score: 4.3 out of 5). The finding was consistent with results of previous research (e.g., Bryant et al., in press; Cihak & Bowlin, 2009; Irish, 2002; Nordness et al., 2011); use of technology-based programs (e.g., computer software, tablet computers and applications) could motivate students with LD to practice mathematics skills more frequently and support them to improve their mathematics performance. Based on the interview data, it was determined using iPads could be promising to teach multiplication facts for students with LD. It was also observed that James and Kate liked using iPads more than Amy and Perry. This might be because Amy and Perry were attending a school that all students have their own iPad (1:1 iPad program) and they often used their iPads during their classes. Thus, for James and Kate, using iPads might be more exciting and motivating them to learn than Amy and Perry who used to use iPads to learn in their classes.

Interestingly, although the participants decidedly liked using iPads to practice facts, when they were asked which approach they preferred for practice among flashcards, worksheets, and iPads, flashcards were cited as the best approach, followed by worksheets and then iPads. Only one of the participants (Perry) selected iPads as the best practice method. Participants who selected flashcards as the best method said it helped them to practice the facts more than other approaches. The finding was aligned with the pilot study previously conducted (Bryant et al., in press). Although participants with LD liked iPad application-based instruction (AI), most participants preferred teacher-directed instruction (TDI) or combined instruction (CI) to AI to learn multiplication facts; they also thought CI or TDI helped them to be more engaged in learning, learn the most facts, and be on-task than AI. They seemed to like the interaction they had with the interventionists and the positive reinforcement they received during TDI or CI. Similarly, in this study, using flashcards was the approach they interacted with the investigator the most and gained her feedback than using worksheets or the iPad application. Technology tools such as iPads could provide multiple opportunities to practice mathematics facts as indicated in previous research (e.g., Howell et al., 1987; Irish, 2002; Nordness et al, 2011). However, educators should be cautious when using technology-based lessons. It might be more effective to integrate technology into their TDI rather than using technology alone to teach students with LD as indicated in previous research (Howell et al., 1987). In addition, if teachers would like to provide technology-based lessons, it is required to identify if the technology programs include effective instructional strategies for teaching students with LD (Woodward & Rieth, 1997).

Graphing Daily Data and Providing Teacher Feedback

All participants also strongly liked graphing their daily performance in a My Progress Graph and obtaining the investigator's feedback on their performance and effort (average score: 4.7 out of 5). They thought it motivated them to learn and work harder. As indicated in previous research (Gersten et al., 2009; Hattie & Timperly, 2007), providing performance data and feedback on their performance to students with LD could be effective for teaching mathematics for students with LD.

Overall Intervention

In addition, all participants expressed strongly positive perspectives toward the overall intervention (average score: 4.7 out of 5). Participants answered that they enjoyed the intervention sessions; they thought the intervention helped them learn facts and focus on tasks; and they were engaged/motivated in learning. The findings in regards to the social validity questions indicated that participants with LD seemed to be very positive about explicit, strategic TDI with iPad application practice overall.

LIMITATIONS

There are four particular limitations in this study that need to be considered when interpreting the findings. Frist of all, explicit, strategic TDI with iPad application practice was a multi-component intervention. It was designed with several empirical validated instructional strategies (e.g., explicit, strategic instruction, iPad application practice, CRA) for teaching mathematics for students with LD (Gersten et al., 2009; Kroesbergen & Van Luitt, 2003; Nordness et al., 2011; Swanson & Hoskyn, 1998). Even though it was indicated that the intervention was effective for teaching multiplication facts for students with LD, it is difficult to identify the effects of each instructional component on the participants' fact fluency performance.

Second, only one short-term maintenance effect (2 weeks following the intervention phase) of the intervention was assessed. It was not possible to assess a longer-term interval maintenance effect (e.g., 4 weeks following the intervention phase) because when the intervention phase for the last participant was completed, there were not enough school days left to assess the longer-term maintenance effect. The short period (2 weeks) might have an impact on the positive results on the maintenance effect of the intervention.

Third, to assess participants' fluency performance, only research-developed CBM was used. Even though the daily probes were developed based on recommendation for CBM design (Hosp et al, 2007; Shapiro, 2010), the reliability and validity of the daily probes were not adequately assessed.

Last, it was found that Amy and Perry were provided some instruction on target multiplication fact skills in their special education mathematics class during the maintenance phase while James and Kate were not provided any instruction; this might have had an impact on the maintenance effect data.

FUTURE RESEARCH

The findings of this study suggest several proposals for future research. First, it is necessary to conduct studies to identify the effect of each instructional component of explicit, strategic TDI with iPad application practice. Research on the effects of explicit, strategic instruction for teaching students with LD has been conducted over the past decades. However, more research is required to investigate the effects of using iPads for teaching students with LD. The finding of this study indicated iPads could be promising learning tools for teaching mathematics for students with LD, but there has been little empirical research on this topic. Explicit, strategic TDI with iPad application practice was a multi-component intervention, so future research should examine the effect of using iPads solely for teaching facts; future research should compare the effects of using flashcards versus using an iPad application to practice. For this study, iPads were used to practice multiplication facts. More research is required to examine the use iPads for teaching other mathematics skills (e.g., division, algebra, fraction, and problem solving skills) or for teaching different types of students (e.g., secondary students with LD or students with different type of disabilities). For this study, especially, the Math Evolve application was selected. It is needed to investigate the effects of other mathematics applications available in the market for teaching students with LD. Future research should also include measuring participants' engagement and motivation in learning when using iPads to investigate if there is a relation between students' engagement/motivation and their performance. In addition, Math Evolve is a game-type application. Previous CBI research (Howell et al., 1987; Okolo, 1992) reported that drill and practice type CBI (without distracting features) could be more effective than game-type CBI for teaching mathematics for students with LD. It is needed to compare the effects of game type iPad applications versus drill and practice type applications for teaching students with LD.

Second, for this study, only a one-time short-term interval maintenance effect (2 weeks following intervention phase) was assessed. It is necessary to investigate a longer-term interval (e.g., 4 weeks or 8 weeks) maintenance effect of the intervention.

Third, only researcher-developed daily probes were used to assess participants' fluency performance. The internal consistency reliability of the daily probes should be measured. Moreover, future research should include additional standardized tests to assess participants' performance. It could provide more convincing results and assess how participants could generalize the skills taught through intervention.

Fourth, to measure the maintenance effect of the intervention accurately, future research should verify that no instruction on target skills was provided during the maintenance phase; other external variables that could impact the maintenance effect should be controlled.

Last, when assessing participants' percentage of strategy usage, participants were asked to solve each problem in the strategy usage test in 30 seconds. However, it was not timed exactly to see how long it took to solve the problem using the strategies. If the time was measured, the information would have helped to understand their strategy usages better. For examples, Perry used the learned product strategy for solving all problems in the strategy usage test both in the middle of intervention and after intervention and he solved all problems correctly. If how long it took to solve the problems was measured, we could better understand if his speed of recall of the facts increased from the middle of the intervention to after the intervention. Thus, future research should measure how long participants would spend to solve the problems using the strategies in order to better understand their strategy usages.

IMPLICATIONS FOR PRACTICE

This study has several implications for practice. First, the findings of this study suggested explicit, strategic TDI using an iPad application could be effective instruction for teaching multiplication facts for students with LD, who have mathematics IEP goals. The intervention was designed based on evidence-based instructional strategies (e.g., explicit instruction, strategic instruction, CRA, CBI, graphing student data and providing feedback) (Gersten et al., 2009; Nordness et al., 2011; Swanson & Hoskyn, 1998) and the result of this study indicated these strategies could be effective for teaching mathematics for students with LD. Teachers can use these instructional methods in their classes to support students with LD to learn mathematics; for example, teaching the doubling strategy explicitly and using iPads could help the students to improve multiplication fact skills.

Second, even though the findings of this study indicated effects of teaching the doubling strategy for teaching multiplication facts (×4s and ×8s), teachers should cautiously recognize that the strategy might not be effective for all students with LD. For students who already know facts, but need to improve the speed of recall like Perry, other instruction that is helpful for increasing speed might be more appropriate than teaching a cognitive strategy. Even though it was documented that cognitive strategy instruction is effective for teaching students with LD, teachers should identify students' needs first to see if the strategy would be beneficial for them.

Finally, even though iPad applications like Math Evolve could provide multiple opportunities for practicing fact skills to students with LD and motivate and engage them in learning however, when teachers plan to use technology-based lessons, they should be cautious. Technology is a vehicle to deliver instruction; instructional strategies embedded in the technology programs are important for effective instruction (Clark, 1983; Woodward & Rieth, 1997). The findings of this study also support previous literature (e.g., Bryant et al., in press; Howell et al., 1987); integrating technology into TDI might be more effective than using technology solely to provide instruction. In addition, teachers should be aware of the importance of instructional components embedded in technology programs. When selecting technology programs (e.g., iPad applications), teachers should evaluate if the programs include effective instructional components for teaching students with LD.

SUMMARY

In sum, the purpose of this study was to examine the effect of explicit, strategic TDI with iPad application practice on the multiplication fact fluency performance of 5th grade students with LD who have IEP goals in mathematics. The results of the study indicated that explicit, strategic instruction with iPad application practice could be effective for teaching students with LD the multiplication fact skills (×4s and ×8s). The needs of students with LD to improve multiplication facts and the importance of instruction in the facts (NGA/CCSSO, 2010; NCTM, 2008) warranted this research. A single case design, multiple probe design across participants was applied; participants were provided fifteen 30 minute-individual interventions. Participants' fact fluency performance was measured with 2-minute daily probes and their strategy usage was also assessed. In 2 weeks after the intervention was completed, the maintenance effect of the intervention was also measured. To understand their perspectives toward the intervention, social validity interviews were conducted after the intervention.

In the results, it was found that all participants improved fact fluency scores through intervention and maintained intervention gains 2 weeks following the intervention; even the fact fluency of one of the participants reached the mastery level during the intervention phase and maintained a mastery level of fluency. Both the visual inspection of the data and the effect sizes computed indicated the effect of the intervention on the participants' fact fluency performance was large and statistically significant. It was also observed that most participants (three out of four) also improved the doubling strategy usage from before the intervention to the middle of the intervention. After the intervention, all participants were able to recall the facts automatically. In addition, participants favored the intervention; they liked the doubling strategy, the iPad application practice, graphing daily data, and obtaining their teacher's feedback on their performance. In conclusion, the participants thought the intervention helped them to learn the target multiplication facts.

Appendix A

Pre-test, Daily Probes, Maintenace Test

Name: ______________________________ Date: ____________ Scorer 1: __________ Scorer 2: __________A

4	2	1	4	3	4	7	4	8	4
x 0	x 4	x 4	x 5	x 4	x 6	x 4	x 4	x 4	x 3
6	9	4	7	3	4	4	4	4	7
x 4	x 4	x 4	x 4	x 4	x 6	x 9	x 8	x 4	x 4
0	8	5	1	8	3	7	9	6	8
x 8	x 2	x 8	x 8	x 4	x 8	x 8	x 8	x 8	x 8
8	3	8	4	8	8	8	6	9	7
x 6	x 8	x 9	x 8	x 7	x 8	x 3	x 8	x 8	x 8
4	4	8	8	4	8	6	8	8	4
x 9	x 3	x 4	x 8	x 7	x 6	x 4	x 9	x 7	x 4
3	9	4	3	4	8	8	9	7	8
x 8	x 4	x 8	x 4	x 4	x 7	x 6	x 8	x 4	x 8

Appendix B

Strategy Usage Test: Student Probe

Name: ______________________ Date: __________________________

3 x 4 =	8 x 7 =
6 x 8 =	4 x 9 =
4 x 4 =	3 x 8 =

8 x 8 =	6 x 4 =
9 x 8 =	4 x 7 =

Appendix C

Student's Strategy Usage Observation Form

Name: ________________________ Date: ______________________

Strategy	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Guess										
Count-all										
Additive calculation										
Count-by										
Pattern-based										
Learned Product										
Hybrids										
Other										
Answer: Correct/Incorrect										

- **Accuracy: (# of correct items/# of items attempted) x 100:** ________
- **Percentage of Strategy Usage**
  : (a total number of use of each strategy/10) x 100
  - Guess: _________ Count-all: __________ Additive calculation: _________
  - Count-by: ____________ Pattern-based: ______________
  - Learned product: ____________ Hybrids: ___________ Other: ______

Comments or Notes

1. ____
   ____
   ____

2. ____
   ____
   ____

3. ____
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4. ____
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5. ____
____
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6. ____
____
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7. ____
____
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8. ____
____
____

9. ____
____
____

10. ____
____
____

Appendix D

Math Ready Poster

(Bryant et al., 2011)

Appendix E

Example Lesson Plans

Lesson 1: Teaching the ×4s using the Doubling Strategy

Lesson Objectives	The students will • Use the doubling strategy with facts including 4 as a factor to build fluency with multiplication facts
Instructional Content	4×3, 4×4, and commutative property facts
Total Time	30 minutes
Vocabulary	**Factors**: Numbers multiplied by another number **Product:** The answer to a multiplication problem **Doubling:** A number multiply by 2 or add same number **Commutative Property of Multiplication:** The order of the factors does not change the product
Instructional Materials	Connected cubes, Mini whiteboards with markers, Doubling strategy poster, Addition fact table, Multiplication table, Scripted lesson, Student worksheets, Daily probe, Color pen, My progress chart, Pencil and Eraser

Warm-up: 3 min

State a lesson goal: Today, we will learn a new strategy, the doubling strategy, to use when multiplying numbers by 4.

Vocabulary & Review:
First, let's review multiplication vocabulary.

When you multiply two numbers, each number that you multiply is called a factor. *(Use student worksheet page 1)* And the answer you get when you multiply two numbers is called a product. What are the two numbers called? *(Factors)* what is the answer to the multiplication problem? *(Product)*.

Use the examples on the student worksheet *(page 1)* to practice the vocabularies. For example:

- What is missing: the product or the factor?
- What numbers are missing?

Review easy facts using a multiplication table (×0s, ×1s, ×2s, ×5s).
Review counting by ×2s (2, 4, 6, 8, 10, so forth).

Modeling: 8 min

1. Introduce a doubling strategy

*(Have a student use a wipe board and show the students the doubling strategy poster).*We will use a "doubling strategy" to solve problems including "4" as a factor. You can use this strategy when you cannot recall the answer quickly. *(Provide examples (e.g., 4 × 3) and non-examples (e.g., 5 × 9) for using the doubling strategy).*

Let's practice the doubling strategy. *(Write the fact 4 × 3 on the wipe board)*

What strategy should we use for solving 4 × 3? *(Doubling strategy)*
Why? *(Includes 4 as a factor)*
What is "doubling"? How do you double a number? You can double a number by multiplying 2 or by adding the same number. For example, how can you double the number 3? *(3 times 2 or 3 plus 3)*
(Let students provide more examples of doubling numbers).

These are the steps for using the doubling strategy *(point to the doubling strategy poster).* Let's read the steps out loud *(Read aloud each step and let the students repeat each step).*

2. Show modeling for how to solve problems using the doubling strategy

(Have a student use connected cubes)
The first step is "Break apart 4 to 2 and 2" *(Take 4 connected cubes).*
How many cubes do we have? *(4).*
Let's break 4 in half. How do you break 4 into 2 equal groups? *(2 and 2).*
How many 2s in 4? *(2).*
We can use the doubling strategy for solving multiplication facts including 4 because 4 can be broken apart to equal groups of 2s.

(Provide 4 groups of connected cubes; each consists of 3 cubes).
We have 4 groups of 3 cubes. We can write a multiplication equation as 4 x 3 *(write on the board and let the students write on his/her wipe board).*
Let's break the cubes in half. *(show how to break the cubes into two groups of cubes and let the students break the cubes followed by modeling).* We broke 4 to 2 and 2, into equal groups of 2s.

What is the second step? *(multiply each 2 with the other factor)*
What is the other factor? *(3)*
Look at the cubes we broke. We have two groups; the first group has 2 groups of 3 cubes and the next group also has 2 groups of 3 cubes.
(Write 2 x 3, 2 x 3 on the board and let the students follow to write on his/her board). We multiply each 2 with the other factor.

What is the third step*? (add the products)*
Let's count cubes for each group *(6 and 6).*
To solve 4 × 3, we will sum up both groups. *(6 + 6).*
What is the product? *(12)* What does 4 × 3 equal? *(12).*
What is the equation when you reverse the factors? (3 × 4)
What is the product? *(12).* The order of the factors does not change the product. We call it the commutative property of multiplication.
Use other examples *(e.g., 4 × 2=8, 4 × 1=4).*

Review the doubling strategy: Read aloud the steps and have a student read aloud. Follow the steps to solve 4 x 4 with 16 connected cubes.
If students have difficulties with addition when doubling numbers (e.g., 8 + 8), provide a 100s chart to support addition.

Guided Practice: 7 min

Let students complete the guided practice sheet (student worksheet page 3-5). Provide immediate and corrective feedback/guidance while students work on the problems. Provide each student a 100s chart if they are struggling in adding numbers when doubling. Provide an extra blank paper if students need it to solve the problem.

Independent Practice: 5 min

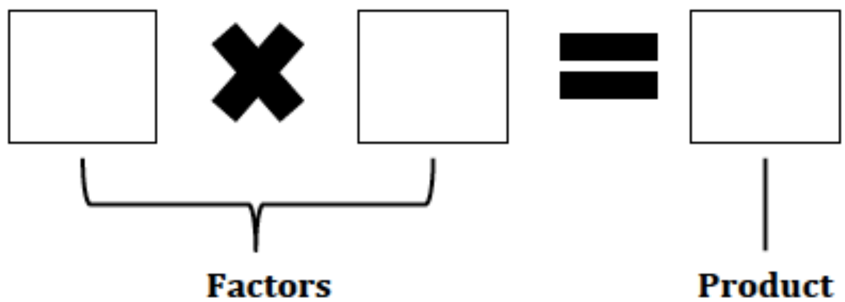
Prior to the lesson, only multiplication facts they learned today including easy facts (4×1, 4×2, 4×3, 4×4, 4×5 and the commutative property facts) set up in the *Math Drills or Math Evolve* app (check schedule for each student). Students will practice facts independently.

Daily Probe & Graphing Daily Data: 5 min

Administer the 2-minute daily probe. Have students write their name and date.
"You will have 2 minutes to solve as many problems as you can. If you do not know an answer, skip it and continue to the next problem. Do your best work. When I say begin, you will start. Are you ready? Let's begin.
Let's stop. Pencils down." *(after 2 minutes)*

Dictate the answer key and let a students scores the probe immediately. Review the incorrect items immediately. Have students write the total number of correct problems and graph the number on their own My Daily Progress Chart. Provide feedback on their performance and provide a sticker as reinforcement.

Lesson 1 Student worksheets



Factor × Factor = Product

1) 4 × 2= 8

2) 4 × 5 = ____

3) ____ × 1 = 4

4) 4 × ____ = 0

Doubling Strategy for x4s

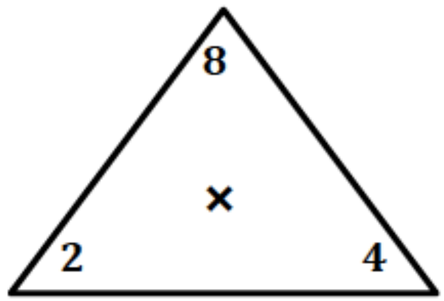
Step 1.) **Break apart the 4 to 2 & 2**

Step 2.) **Multiply each 2 with the other factor**

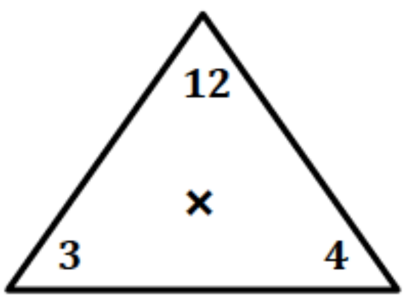
Step 3.) **Add the products**

1. Fill in the blanks.

Example



Factors: 2 , 4
Product: 8
Equations: 2 × 4 = 8 and 4 × 2 = 8

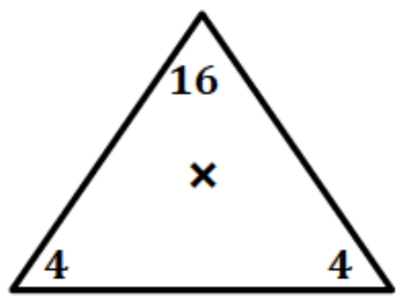


Factors: ________, ______

Product: ________

Equations: ______________________

____



Factors: ________, _______

Product: _______

Equations: ____________________

____

2. Use the doubling strategy to solve.

a. **4 × 3** = ________

= (2 × _____) + (2 × _____)

= __________ + ___________ = _________

b. **4 × 4** = _________

= (2 × ______) + (2 × _______)

= ___________ + _____________ = _________

c. **3 × 4** = _________

= (______× 2) + (_____ × 2)

= _____________ + ______________ = _________

d. **4 × 4** = _________

= (_______× ______) + (______× ______)

= ______________ + _____________ = _________

3. Solve the problems.

$4 \times 3 =$ ______

____

____

____

$4 \times 4 =$ ______

____

____

____

$4 \times 4 =$ ______

____

____

____

$3 \times 4 =$ ______

____

____

____

$$\begin{array}{r} 4 \\ \times\ 3 \\ \hline \end{array} \qquad \begin{array}{r} 4 \\ \times\ 4 \\ \hline \end{array} \qquad \begin{array}{r} 3 \\ \times\ 4 \\ \hline \end{array} \qquad \begin{array}{r} 2 \\ \times\ 4 \\ \hline \end{array} \qquad \begin{array}{r} 4 \\ \times\ 1 \\ \hline \end{array}$$

$$\begin{array}{r} 3 \\ \times\ 4 \\ \hline \end{array} \qquad \begin{array}{r} 5 \\ \times\ 4 \\ \hline \end{array} \qquad \begin{array}{r} 4 \\ \times\ 4 \\ \hline \end{array} \qquad \begin{array}{r} 4 \\ \times\ 3 \\ \hline \end{array} \qquad \begin{array}{r} 4 \\ \times\ 0 \\ \hline \end{array}$$

Lesson 8: Teaching the × 8s using the Doubling Strategy

Lesson Objectives	The students will • Use the doubling strategy with factors including 8 as a factor to build fluency with multiplication facts
Instructional Content	8 × 8, 8 × 9 & Commutative property facts
Total Time	30 minutes
Vocabulary	**Factors:** Numbers multiplied by another number **Product:** The answer to a multiplication problem **Doubling:** A number multiplied by 2 or by adding the same number **Commutative Property of Multiplication:** The order of the factors does not change the product **Column:** a vertical arrangement of items **Array:** a set of numbers or objects that will follow a specific pattern, often in rows, columns or a matrix
Instructional Materials	Whiteboards with markers, Doubling strategy poster, Addition fact table, Student worksheets, My Progress Chart, Color pen, Pencils, Eraser, Timer, Scripted lessons, Daily probe

Warm-up: 3 min

State a lesson goal: Today, we will keep learning new facts including 8 as a factor and using the doubling strategy.

Activate prior knowledge:
Let's review the doubling strategy for ×8s *(Display doubling strategy poster)*.
What is the first step? *(Break apart 8 to 2,2,2,*2)
The second step? *(Multiply each 2 to the other factor)*
The last step? *(Add products)* Pick the facts where you can use the doubling strategy *(provide examples on the board e.g., 8 × 3, 6 × 7, 9 × 5, provide more examples).*

Let's review the new vocabulary we learned in the last class.
What is a column? *(a vertical row of items)*
What is an array? *(a set of numbers or objects that will follow a specific pattern)*

Let's review some addition we need to use for solving ×8s.
(e.g., 6+6+6+6, 8+8+8+8, 12+12+12+12).

Let's review the facts we previously learned in the last class.
Use 3 x 5 flashcards written containing facts taught previously. Have students read the equation and give the answer. If students have difficulties with recalling the number, have them use a doubling strategy on the board to solve the facts.

Modeling: 8 min

1. **Provide modeling with pictorial materials on 8 × 8 using a doubling strategy**

    (Have students turn to page 3 in the student booklet)
    We are going to solve 8 × 8.
    How many columns do we have? *(8)* How many dots for each column? *(8)*
    We will divide 8 groups into equal numbers of 2. How many 2s in 8? *(4)*
    Let's make a loop for each group of 2 columns (*show how to make a loop and let a student follow)*

    What is the first step of the doubling strategy? *(Break apart the 8 as 2, 2, 2, 2)*

We broke these dots into four groups of 2 columns with 8 dots. *(Have a student write down "4 groups of 2 columns of 8" below the picture)*

What is the second step? *(multiply 2 by the other factor).*
Look at the first group. We have 2 columns of 8 dots.
How can we write a multiplication equation for this? (*2 × 8). (Write down 2 × 8 and go through all 4 groups)*
We have four "2 × 8" What is 2 × 8? *(16)*
(If a student has a problem, let him/her use counting by ×2s)

What is the third step? *(add the products).* What is the product? *(16)*
Let's add all of them. *(16+16+16+16).* How many? *(64)*
(If students have difficulties with counting, let them use hundred charts)
So 8 × 8 equals? *(64) (Have a student write down the answer)*

2. Provide modeling with pictorial materials on 8 × 9 using a doubling strategy

(Have a student go to page 4) We are going to solve 8 × 9.
How many columns do we have? *(8)* How many dots for each column? *(9)*
We will divide 8 groups into equal numbers of 2. How many 2s are in 8? *(4)*
Let's make a loop each group of 2 columns (*show how to make a loop and let a student follow)*

What is the first step of the doubling strategy? *(Break apart the 8 as 2, 2, 2, 2)*
We broke these dots into four groups of 2 columns with 9 dots. *(Have a student write down "4 groups of 2 columns of 9" below the picture)*

What is the second step? *(multiply 2 by the other factor).*
Look at the first group. We have 2 columns of 9 dots.
How can we write a multiplication equation for this? (*2 × 9).* (*Write down 2 × 9 and go through all 4 groups)*
We have four "2 × 9" What is 2 × 9? *(18)*
(If a student has a problem, let him/her use counting by ×2s)

What is the third step? *(add the products).* What is the product? *(18)*
Let's add all of them. *(18+18+18+18).* How many? *(72)*
(If students have difficulties with counting, let them use the hundred chart.)
So 8 × 9 equals? *(72) (Have a student write down the answer)*

Look at the bottom and fill out the blank. 8 times what number equals 72? *(9)*
What is 9 × 8? (72) What is this called? *(the commutative property).*

A change in the order of factors does not change the product.

Write problems (e.g., 8 × 9, 8 × 8, 9 × 8) on the board to show how to use the doubling strategy to solve the problem without pictorial materials. Have a student follow the steps on his/her own board.

Guided Practice: 7 min

Let students complete the guided practice sheet *(student worksheet p. 5-6)*. Provide immediate and corrective feedback/guidance while students work on the problems. Provide each student a hundreds chart if they are struggling with adding numbers when doubling. Provide an extra blank paper if students need to use it to solve a problem.

Independent Practice: 5 min

Prior to the lesson, only cover multiplication facts they learned today (8 × 8, 8 × 9 and the commutative property facts, as well as facts taught in previous ×8s lessons) set up in the *Math Drills or Math Evolve application (check student schedule)*. The students practice the facts independently.

Daily Probe & Graphing Daily Data: 5 min

Administer the 2-minute daily probe. Have students write their name and date.
You will have 2 minutes to solve as many problems as you can. If you do not know an answer, skip it and continue to the next problem. Do your best work. When I say begin, you will start. Are you ready? Let's begin.
Let's stop. Pencils down *(after 2 minutes)*.

Dictate answer keys, have students score the probe immediately. Review the incorrect items immediately. Have students write down the total number of corrected problems and graph the number on their own My Daily Progress Chart. Provide feedback on their performance or progress and provide a sticker as reinforcement.

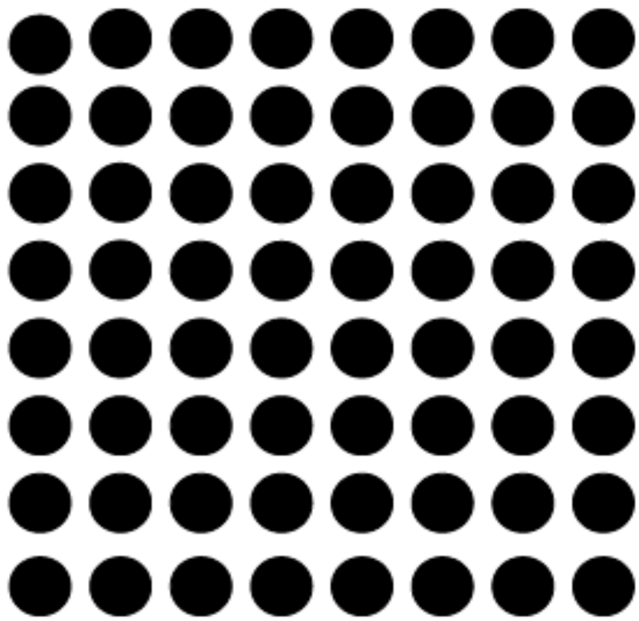
Doubling Strategy for x8s

Step 1.) **Break apart the 8 to 2, 2, 2, 2**

Step 2.) **Multiply each 2 with the other factor**

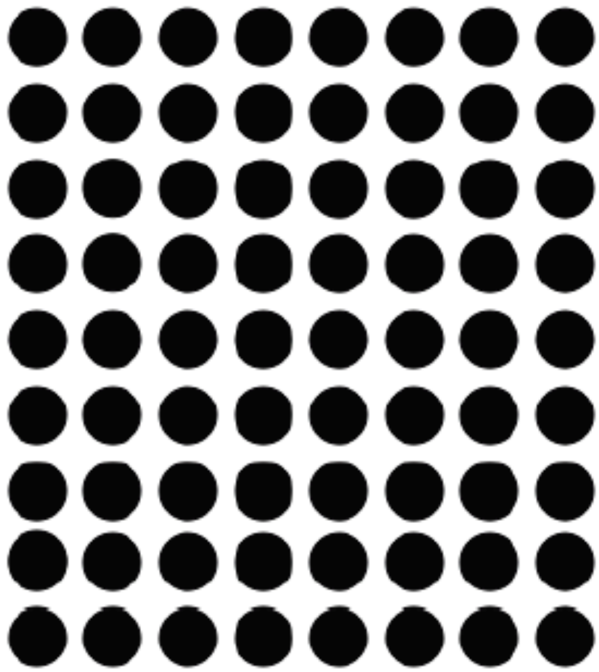
Step 3.) **Add the products**

8×8



$8 \times 8 =$ ______

8×9



____

____

____

$8 \times$ _____ $= 72$

Use the doubling strategy to solve

a. $8 \times 8 =$

(____ × ____) + (____ × ____) + (____ × ____) + (____ × ____)

= ______ + ______ + ______ + ______ =

b. $8 \times 9 =$

(____ × ____) + (____ × ____) + (____ × ____) + (____ × ____)

= ______ + ______ + ______ + ______ =

c. $9 \times 8 =$

(____ × ____) + (____ × ____) + (____ × ____) + (____ × ____)

= ______ + ______ + ______ + ______ =

2. Use the doubling strategy to solve.

8 × 9= ______

8 × 8 = ______

9 × 8= ______

6 × 8 = ______

8 × 3= ______

4 × 8 = ______

Appendix F

My Progress Graph

Name: ________________

60															
58															
56															
54															
52															
50															
48															
46															
44															
42															
40															
38															
36															
34															
32															
30															
28															
26															
24															
22															
20															
18															
16															
14															
12															
10															
8															
6															
4															
2															
0															
Lesson	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Total															

Appendix G

Fidelity of Implementation Checklist

A. Observation Information

Date: ___________________ Time: _________ - __________

Observed lesson: ________________ Name of Observer: ____________________

Name of Student observed: ________________

B. Fidelity Checklist Direction:

Please provide points for each element to identify if a teacher followed instructional procedures of implementation. Please use the following rating scales:

- 0 points: element is absent or not observed
- 1 point: inconsistent level of implementation is observed
- 2 points: high level of implementation observed

	Level of implementation		
Materials			
Teacher and student materials are ready and used appropriately	0	1	2
A timer was used to adhere to time limits in each lesson phase	0	1	2
Instruction: Warm-up			
The objective of the lesson was stated to the students	0	1	2
The teacher completed the review of prior knowledge/skills	0	1	2
3 minutes were spent to adhere to the time limit	0	1	2
Instruction: Modeling			
The teacher followed the proper modeling steps	0	1	2
8 minutes were spent to adhere to the time limit	0	1	2
Instruction: Guided practice			
A student completed the guided practice sheet	0	1	2
The teacher provided an ongoing check and support for the student while he/she worked on the sheet	0	1	2
7 minutes were spent to adhere to the time limit	0	1	2
Instruction: Independent practice			
The student completed independent practice with an iPad application	0	1	2
Teacher provided appropriate support and feedback when the	0	1	2

student had iPad-related technical questions or problems			
5 minutes were spent to adhere to the time limit	0	1	2
Instruction: Feedback/Support			
Teacher provided ongoing support and immediate/corrective feedback across instruction	0	1	2
Daily probes & Graphing daily data			
The student completed the probes	0	1	2
The student completed graphing his/her daily data	0	1	2
Teacher provided feedback to the student on his/her daily data	0	1	2
2 minutes for daily probes was spent to adhere to the time limit	0	1	2
3 minutes for reviewing incorrect items, graphing daily data, and providing feedback was spent to adhere to the time limit	0	1	2
Behavior management			
Teacher used "Math Ready" appropriately across instruction	0	1	2

Fidelity of Implementation:

(Total points obtained __________ ÷ 40) × 100 = ___________%

Other comments:

Appendix H

Social Validity Questionnaire

Name: ________________________ Date: ___________________________

* ***Directions***: *Read aloud the questions to students and mark their responses. Circle one of the five scales to the right. Write down students' response to the open-ended questions.*

1: Strongly disagree, 2: Disagree, 3: Neither, 4. Agree, 5. Strongly Agree

Explicit, Strategic Teacher-Directed Instruction					
1. Do you like the doubling strategy we used?	**1**	**2**	**3**	**4**	**5**
2. Do you think the doubling strategy was easy to learn and use to solve multiplication number facts?	**1**	**2**	**3**	**4**	**5**
3. Do you think the doubling strategy helped you to do better in multiplication number facts?	**1**	**2**	**3**	**4**	**5**
4. Do you want to recommend using the doubling strategy to your friends?	**1**	**2**	**3**	**4**	**5**
5. Do you think "Math Ready" helped you to focus on your tasks?	**1**	**2**	**3**	**4**	**5**
Independent Practice using an iPad Application					
6. Did you like using an iPad to practice multiplication number facts?	**1**	**2**	**3**	**4**	**5**
7. Do you think using an iPads helped you learn multiplication number facts?	**1**	**2**	**3**	**4**	**5**
8. Do you think using an iPad helped you motivate practice multiplication number facts?	**1**	**2**	**3**	**4**	**5**
9. Do you want to recommend using an iPad for practicing multiplication number facts for your friends?	**1**	**2**	**3**	**4**	**5**
10. What do you prefer for practicing multiplication facts: (a) flashcards and practice worksheets or (b) using an iPads app? Why?	Open-ended question				

Daily Probe and Graphing Daily Data					
11. Did you like graphing your daily data?	**1**	**2**	**3**	**4**	**5**
12. Did you like the teacher's feedback on your performance and effort?	**1**	**2**	**3**	**4**	**5**
13. Do you think graphing your daily data and teacher feedback motivated you to learn and work harder?	**1**	**2**	**3**	**4**	**5**
Overall Intervention					
14. Did you enjoy our tutoring time?	**1**	**2**	**3**	**4**	**5**
15. Do you think tutoring time helped you learn multiplication facts?	**1**	**2**	**3**	**4**	**5**
16. Do you think tutoring time helped you to be motivated and engaged you in learning multiplication facts?	**1**	**2**	**3**	**4**	**5**
17. Do you think tutoring time helped you to be focused on task?	**1**	**2**	**3**	**4**	**5**
18. What did you like best about our tutoring time? Why?	Open-ended question				
19. What aspects of tutoring time did you dislike? Why?	Open-ended question				
20. Do you have any other comments/suggestions regarding tutoring time?	Open-ended question				

Open-ended questions

10. What do you prefer for practicing multiplication facts: (a) flashcards and practice worksheets or (b) using an iPads app? And why?

____

____

18. What did you like best about our tutoring time? Why?

____

____

19. What aspects of tutoring time did you dislike? Why?

20. Do you have any other comments/suggestions regarding tutoring time?

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