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**Three Essays on Operations Management: Commodity
Market, Sustainability, and Globalization**

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**Three Essays on Operations Management: Commodity
Market, Sustainability, and Globalization**

by

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DISSERTATION

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Dedication

This dissertation is dedicated to my encouraging parents, Hyun Shick Park and Young Hee Jung, who have always been there for me since day one in my life; my lovely wife, Hee Sung Jang, who has always been with me since September 2007; my supportive other parents, Chang Jin Jang and Mi Ae Kim, who have always been there for me since July 2009; and, my eternal mentor, Sridhar Seshadri, who has always been there for me since September 2008.

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Three Essays on Operations Management: Commodity Market, Sustainability, and Globalization

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This dissertation deals with three issues that are important to many firms, namely, volatile commodity prices, environmental regulations, and globalization. In the first essay I study the benefit and the coordination of inventory sharing when there are two existing channels for procurement, i.e., the spot and forward markets. I propose a method for sharing inventory such that the decentralized firms get the same benefit per unit of the sharing transactions regardless of whether the firm is borrowing or lending. The procurement cost gap between the centralized and decentralized cases is dramatically small by using this method. In the second essay, I analyze whether imposing carbon costs to retailers and consumers changes the supply chain design or social welfare. I consider three types of players who want to maximize different objectives and three kinds of competitive settings. Different from previous studies, I show that the supply chain design is changed significantly by imposing carbon costs especially when market competition is medium to high. In the third essay, I consider long-term / short-term strategies of multi-national corporations. For the long-term strategy, I show that the correlation between

the exchange rate and the market demand in a foreign country affects plant location. For the short-term strategy, I show that manufacturers increase the inventory levels as the exchange rate of the country where the plant is located grows weaker. I confirm these results empirically using plant-level data of Korean multi-national corporations provided by the Export-Import Bank of Korea.

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Chapter 1

Introduction

This dissertation deals with three important issues to many firms, i.e., volatile commodity prices, environmental regulations, and globalization. First, commodity prices are notoriously volatile. For instance, the spot price of the crude oil touched \$142.5 in July 2008 and then abruptly dropped to a bottom low at \$37.0 in February 2009; the price gained ground in 2010, exceeding \$100 again in March 2011, and then varied between \$80 and \$110, afterwards. The highly volatile commodity prices pose a substantial challenge to the operations planning of the companies that need direct commodity inputs. For example, Royal Dutch Shell's profit from April to June in 2012 decreased by \$1.2 billion compared to that in 2011 (\$6.6 billion) because of oil price volatility. In addition, due to the price spike of commodities in 2008, Kraft Foods' input materials cost that year increased about \$2 billion from 2007, which represents a 13 percent cost surge. Hence, a good operational strategy that takes commodity prices into account can significantly reduce a firm's cost.

Second, environmental regulations are also becoming important items on companies' daily agenda. Not only are the governments pushing for tighter regulations on energy consumption and pollution control, but the consumers are also giving more preferences to environment-friendly products and socially responsible companies. For instance, the European Union has imposed carbon emission limits while allowing the companies to trade their allowances. British

Columbia, Canada, imposed a carbon tax, at \$20 per metric ton of CO_2 initially and then increased to \$30 per metric ton from July 2012. While it is difficult to determine the exact cost of carbon emission to the society, the Intergovernmental Panel on Climate Change in 2007 suggested that if we impose \$80 per metric ton of CO_2 to large carbon emitters, then we can prevent severe climate change. Several researchers have documented that the estimated carbon cost can range from \$20 to \$ 300 per metric ton of CO_2 . In the United States, while there is no regulation in place yet, the Mandatory Reporting of Greenhouse Gases Rule has been issued by the U.S. Environmental Protection Agency, and large greenhouse gas (GHG) emitters need to report GHG data. Along with the policy change, the public is becoming more receptive to the idea of imposing cost to curb carbon emission, and also more companies have started to report the carbon footprint of their products and services and have been making efforts to reduce their carbon emission. As a result, to stay viable and competitive, companies need to have their operational decisions to the new environment.

Third, globalization has transformed many companies into Multi-national Corporations (MNCs), with either their supply bases and manufacturing functions located in the overseas countries or their markets covering multi-continent. To manage global supply chains, as a long-term strategy, the MNCs invest to other countries and the amount of the investment is enormous. For example, in 2011, the MNCs based in the U.S. invested \$4,156 billion (28% of the U.S. GDP) overseas and the MNCs based in countries outside of the U.S. invested \$2,548 billion (17% of the U.S. GDP) in the U.S. Hence, understanding how firms invest to other countries is important not only to firms who are planning to invest to overseas countries but also to governments who want to attract

Foreign Direct Investment (FDI). In addition, as a short-term strategy, how to efficiently manage the operations is a significantly important question for MNCs due to the increased uncertainties by producing in foreign countries, e.g., volatile exchange rate and economic instability.

In this thesis, we study these three issues from the perspective of Operations Management. In Chapter 2, we study commodity procurement policies in the presence of inventory sharing. In Chapter 3, we analyze whether imposing carbon cost changes the supply chain design and reduces the carbon emissions. Chapter 4 analytically studies the impacts of exchange rates on global supply chain design, and then empirically confirms the analytical findings.

1.1 Overview of Chapter 2

In Chapter 2¹, we study the benefit and the coordination of inventory sharing when there are two existing channels for procurement, i.e., the spot and forward markets. We consider two firms that use a common commodity input to satisfy stochastic demands in a multi-period setting. The firms can procure the commodity as well as sell excess inventory through either the spot or the forward market. The firms can also share the commodity between them when one has leftover inventory while the other has excess demand. We first analyze a benchmark, the centralized case, and show the benefit of inventory sharing. Then, we focus on the more realistic decentralized case. We show that the stochastic prices necessitate more sophisticated and closer coordination for the decentralized case because the benefit per unit of inventory shared changes over time. We propose a method such that the decentralized firms get

¹Joint work with Guoming Lai and Sridhar Seshadri

the same benefit per unit of the sharing transactions regardless of whether the firm is borrowing or lending. The proposed method is fair for the borrower and lender and seems to work very well, i.e., the procurement cost gap between the centralized and decentralized cases is dramatically small in our experiments. We also identify factors that are critical for the benefit of inventory sharing to be substantial which in turn will induce firms to cooperate.

1.2 Overview of Chapter 3

Chapter 3² analyzes whether imposing carbon costs changes the supply chain structure and social welfare. In our model, we consider three players (i.e., a central policymaker, retailers, and consumers) who optimize their own objectives (i.e., social welfare, profits, and net utility, respectively) and three competitive settings (i.e., monopoly, monopolistic competition with symmetric market share, and monopolistic competition with asymmetric market share). While the outcomes of the monopoly case are aligned with the literature where imposing carbon costs neither increases social welfare nor changes the supply chain structure, we find that the outcomes of the monopolistic competition cases can be very different. In particular, for the case of monopolistic competition with symmetric market share, charging carbon cost increases social welfare, reduces carbon emissions, and also significantly changes the number of retailers. These results depend only on the common factors of the industry, e.g., fuel and carbon prices. The outcomes of the case of monopolistic competition with asymmetric market share fall in between the above two cases.

²Joint work with Gerard P. Cachon, Guoming Lai, and Sridhar Seshadri

1.3 Overview of Chapter 4

In Chapter 4³, we consider long-term / short-term strategies of multinational corporations when their home country is an export-oriented country with small domestic markets. For the long-term strategy, when economic growth and strength of the currency are positively (negatively) related, we find that the reason a firm whose home country without a selling market invests more to another country when the sunk cost (labor cost) in the foreign country decreases. On the other hand, a firm whose home country has a selling market invests more in another country when the labor cost (sunk cost) in the foreign country decreases. We verify these results using Foreign Direct Investment Data in Korea from 2002 to 2011. For the short-term strategy, we consider a manufacturer's inventory level decision when the plant and the market are located in two different countries. We show that manufacturers increase the inventory levels as the exchange rate of the country where the plant is located grows weaker. We confirm this result by testing using the plant-level data of Korean multinational corporations provided by the Export-Import Bank of Korea.

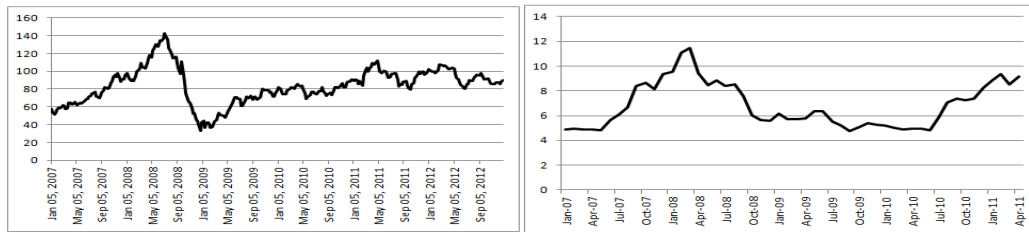
³Joint work with Guoming Lai, Seungrae Lee, and Sridhar Seshadri

Chapter 2

Benefit and Coordination of Sharing Commodity Inventory

2.1 Introduction

Different from other industry supplies, commodities (e.g., crude and wheat) are openly traded and can be procured from both spot markets with immediate delivery and forward markets for future delivery. Commodity prices are however notoriously volatile. For instance, Figure 2.1(a) shows the spot prices of crude oil in Cushing, Oklahoma from January 2007 to December 2012. The spot price of the crude oil touched \$142.5 in July 2008 and then abruptly dropped to a bottom low at \$37.0 in February 2009; the price gained ground in 2010, exceeding \$100 again in March 2011, and then varied between \$80 and \$110, afterwards. Figure 2.1(b) shows the prices of the earliest delivery wheat futures contracts in Kansas City Board of Trade. The futures prices of wheat also show high volatility. The highly volatile commodity prices pose a substantial challenge to the operations planning of the companies that need direct commodity inputs. For instance, Royal Dutch Shell's profit from April to June in 2012 decreased by \$1.2 billion compared to that in 2011 (\$6.6 billion) because of oil price volatility (BBC News 2012). In addition, due to the price spike of commodities in 2008, Kraft Foods' input materials cost that year increased about \$2 billion from 2007, which represents a 13 percent cost surge (Martin 2008). To cope with the risk associated with the commodity



(a) Weekly crude oil spot prices (\$ per barrel) (b) Monthly wheat futures prices (\$ per bushel)

Figure 2.1: Commodity prices

inputs, companies have started making various attempts to mitigate the risk. In one such example, firms share their commodity inventory to reduce the amount of emergent procurement on one hand while better utilizing excess inventory on the other hand (see, e.g., Lindner et al. 2012).

In this chapter, we study what factors prompt firms to engage in such cooperative behavior, whether the coordination is complicated by the stochastic prices in addition to stochastic demand and whether practical solutions can be provided to conduct such transactions in the most beneficial possible way. To answer these questions, we investigate the potential benefit of sharing commodity inventory between two firms. In our study, the firms face stochastic demands that they need to satisfy by using a common commodity input, in a multi-period setting. The firms can procure the input from the commodity markets through forward contracts as well as spot purchases. They can also sell short-term excess inventory to the commodity markets. The commodity prices present a stochastic pattern. With imperfectly correlated demands, scenarios may arise where a firm has excess inventory while the other has inventory shortage. Besides the trades through the commodity markets,

inventory sharing provides an alternative option for the firms to balance their demand and supply. We first study a benchmark, the centralized case, where the inventory sharing decision is made by a central planner. We show the benefit of inventory sharing. We then focus on the decentralized case where the firms make the inventory sharing decision to minimize their own costs.

With stochastic prices, inventory sharing assumes a dimension beyond determining optimal stocking quantities: firms need to decide how to price the inventory sharing transactions. Due to the stochastic nature of spot and forward prices, the benefit per unit of inventory shared changes over time. Thus, the pricing of the sharing transactions becomes more complicated than that with fixed procurement prices. There are many ways to arrange for the payment when firms share inventory. For example, we can assume the borrowing firm pays the spot or forward price of the commodity (which is stochastic) plus the transaction cost. This way the borrower pays all costs. At first this seems an innocuous assumption to make because we expect (at least with identical firms facing independent demand streams) it is equally likely that a firm borrows or lends - thus, the benefit from sharing is symmetric. However, we show that the assumption about the transfer price is quite important because it can affect the stocking decisions as well as the inventory sharing condition. Therefore, attention has to be given to coordination between the firms. In this study, we propose a coordination method under which the inventory sharing condition regarding to the spot and forward prices in the decentralized case becomes the same as that of the centralized case and firms get the same benefits from the sharing transactions. The proposed method is fair for the borrower and lender and seems to work very well, i.e., the gap relative to the centralized solution is negligible in our experiments.

We also identify how the benefits from sharing inventory vary with the parameter values of the demand and the transaction costs. The value of inventory sharing, using our proposed pricing scheme, increases as the mean and variance of demand increase, the correlation between the demand faced by firms decreases or the sharing transaction cost decreases. The value may either increase or decrease as the transaction costs in the spot market increase.

Hence, the contribution of our study is three folds. First, we bridge the gap between the literatures on inventory sharing and commodity procurement with stochastic demands and prices. We derive not only the optimal ordering and selling decisions in the spot and forward markets but also the optimal inventory sharing decisions. Second, we propose a simple method to coordinate inventory sharing, and show that the gap between the centralized and decentralized cases can be dramatically reduced by this method. Third, we identify the critical factors that influence the benefit of inventory sharing.

Besides us, inventory planning of commodities with spot and forward trades has been explored by others in the literature. Seifert et al. (2004) consider a risk-averse newsvendor firm that procures inventory for serving random future demand. They show that besides procurement through forward contracts, utilizing the spot market with instantaneous delivery can lead to a higher payoff for the firm as well as a higher service level. Differently, Secomandi and Kekre (2011) emphasize the benefit of trades in the forward market given its lower transaction cost compared to spot purchases. They analyze a firm's optimal commodity procurement strategy, facing some stochastic future demand, by using both the forward and spot markets with joint evolution of demand forecast and forward price. They characterize the value of the forward procurement option. Based on a newsvendor context with two cor-

related demands, Boyabatli et al. (2011) study the procurement, processing, and production decisions of a meatpacker in the presence of both spot and forward markets for the commodity, fed cattle, input. They solve the meatpacker's optimal contracting portfolio for the commodity. Goel and Gutierrez (2011) study the commodity procurement strategy in a multi-period setting with stochastic demand. They reveal the value of information regarding the marginal convenience yield which can result in a significant procurement cost reduction. Differently, Devalkar et al. (2011) investigate a risk-averse commodity processor's inventory planning problem who purchases commodity from a spot market to produce and sell the output using forward contracts. Besides the above cited studies, other researchers have studied optimal procurement with stochastic input prices. We refer the readers to Haksoz and Seshadri (2007) for an exhaustive survey of such papers. Our work differs from this stream of research studies as we focus on two firms and explore the benefit of inventory sharing between the firms in the presence of both spot and forward markets.

We are however not the first to study inventory sharing. Inventory transshipment, i.e., transshipping one firm's excess inventory to another in need of supply, has been explored from different aspects in the operations management literature. As a seminal work, Krishnan and Rao (1965) study a single-period transshipment problem among multiple retailers in a centralized setting. They explore the inventory decision as well as the value of inventory transshipment. The follow-up studies investigate the benefits of inventory transshipment in a centralized system with multi-period settings, pooling effects and other factors (see, e.g., Tagaras 1989, Robinson 1990). More recent studies extend the prior works to supply chain settings. Dong and Rudi (2004)

consider a supply chain with a single manufacturer and multiple retailers. They analyze the benefits of transshipment for the cases with both exogenous and endogenous wholesale prices. Similarly, Wee and Dada (2005) focus on a case with one common warehouse that serves multiple retailers. They show the optimal transshipment policy of the retailers and analyze when having the common warehouse is beneficial. Çömez et al. (2011) study inventory sharing between two retail stores where the stores replenish a product periodically by considering multiple transshipments per cycle, positive delivery times, and backorder costs. They show partial transshipment by holding back inventory can reduce the retailer's cost.

There are also studies that explore transshipment between independent decision makers. Rudi et al. (2001) consider a setting with two independent retailers that decide whether to agree on transshipment. They show coordinating transshipment prices exist by which both retailers can be better off. Hu et al. (2007) extend Rudi et al. (2001) by considering uncertain capacity. Anupindi et al. (2001) consider the benefit of common warehousing. They demonstrate that coordinating allocation mechanism exists under which the retailers can use the common warehouses not only to reduce the distribution cost for their own retail stores but also to reduce the cost of transshipment. Granot and Sošić (2003) extend Anupindi et al. (2001) by allowing the retailers to decide the amount of sharing. They show retailers might withhold their leftover inventories. In contrast, with a similar framework but allowing the retailers to interact repeatedly, Huang and Sošić (2010) reveal that the retailers may share all the residuals of their inventories if the discount factor is sufficiently large. Differently, Çömez et al. (2012) explore the optimal transshipment strategies and inventory policies for the retailers who compete against each other. This

literature however generally assumes that the selling price or the procurement cost of the underlying product is constant. Differently, in our study, the spot and forward prices of the commodity follow stochastic processes and the firms can use both spot and forward purchases.

The remainder of this chapter is organized as follows. §2.2 describes the problem setting. In §2.3, we analyze the optimal procurement and inventory sharing policies, and propose a coordination method for the decentralized case. We evaluate the value of inventory sharing in §2.4, and discuss several extensions in §2.5. We conclude in §2.6.

2.2 Model Setting

We consider a periodic review inventory planning problem of two firms, denoted by i and j , in a time horizon of T periods. The inventory decisions of the firms are made at the beginning of each period, $t \in \{1, 2, \dots, T\}$, whose goals are to minimize the expected cost-to-go.

2.2.1 Market Demand

The firms face a stochastic demand in each period from their customers. The demand, when it arises, always needs to be satisfied within the same period. That is, we assume the demand cannot be backlogged. Such an assumption is appropriate if the customers are impatient while the premium from the sales is significantly high. We denote the demand for firm i in period t , by D_{it} , which is a non-negative random variable. We use x_{it} to denote the beginning inventory level of firm i in period t .

2.2.2 Spot and Forward Transactions

To satisfy demand, the firms need a common commodity input which they can procure from both the spot and forward markets. The firms can also sell their excess commodity inventory to the spot and forward markets in any period. The prices of the commodity in the spot and forward markets follow some stochastic processes, denoted by S_t and F_t , respectively. We assume that the prices always realize at the beginning of each period and they satisfy the Markov property as well as the property of $E_t[S_{t+1}|S_t] = F_t$, where $E_t[A|B]$ is used to denote the conditional expectation of A given the event B . The firms are price takers. Similar assumptions have been made in the literature (see, e.g., Goel and Gutierrez 2011, Secomandi and Kekre 2011).

We assume the delivery lead time of a purchase or a sale in the spot market is zero. To trade in the spot market, the firms incur a constant transaction cost τ_b per unit of purchase and τ_s per unit of sale. Thus, the unit purchasing cost to buy the commodity from the spot market is $S_t + \tau_b$; whereas, the unit sales revenue from selling in the spot market is $S_t - \tau_s$. The transaction costs in our model are mainly the transportation costs. We assume the firms' locations are close enough so that they incur the same transaction costs when they trade in the spot market.

In contrast to the spot market, we assume the trade in the forward market is always associated with a one-period delay (see, Goel and Gutierrez 2011 for a similar assumption). That is, any purchase or sale in the forward market will be delivered at the beginning of the next period and the payment associated with the contract will also be transferred at that time. We normalize the transaction cost of any forward trades to zero, which reflects the fact that it is relatively inexpensive to trade in the forward market than in the spot

market. This assumption has been made in the literature (see, e.g., Goel and Gutierrez 2011). Hence, if a firm purchases (sells) one unit of commodity through a forward trade in period t , then the firm will receive (deliver) the commodity and pay (obtain) F_t at the beginning of period $t + 1$. We use β to denote the time discount factor per period, which is assumed to be a constant. Therefore, the unit present cost of purchasing (gain of selling) from the forward market is βF_t . We let a_{it} and u_{it} denote firm i 's spot and forward trading actions, respectively, in period t . Here, a positive action corresponds to a purchase, a negative action to a sale, and zero is a do-nothing action. Notice that we allow the firms to sell in the forward market, which eases analysis. This assumption however is not critical for our findings. In fact, in the entire numerical study, we conduct, sales in the forward market do not arise under the optimal policies. Finally, we do not allow short-selling in our model.

2.2.3 Inventory Sharing

Carrying the inventory of the commodity is costly. We assume a holding cost, h , for each unit of commodity held per period. The cost is charged at the beginning of a period. Instead of holding or selling the excess commodity inventory, we assume a firm can lend the inventory to the other party if there is such a need; likewise, instead of buying from the markets, a firm can also borrow inventory from the other party if it is available. We call such a transaction the inventory sharing transaction. Notice that because the demand always needs to be satisfied in each period, without the inventory sharing transaction, if a firm is short of inventory, it would have to buy the excess amount of commodity from the spot market given there is a one-period lead time from the forward market. Compared to the transactions in the forward market, inven-

tory sharing is more flexible. Therefore, we assume that a firm can either buy from the spot market or borrow from the other party to satisfy the current period excess demand, which will always be delivered within the same period. Sharing inventory however is not transaction cost free. We denote a sharing transaction cost by τ_o per unit.

In the decentralized inventory sharing case, we assume that the borrowing firm will return the same amount of commodity to the lending firm at the beginning of next period. Specifically, we assume that the borrowing firm enters into a forward contract for that amount of commodity and has it delivered to the other party in the next period. Given the transaction cost in the forward market is zero, this is equivalent to making a cash payment to the lender. It is also clear that using spot purchases is dominated by using forward purchases to compensate the lender because of the difference of the transaction costs. In addition, we assume the sharing transaction cost τ_o is paid by the borrower and thus the total unit cost of borrowing is $\tau_o + \beta F_t$. In §2.3.3, we will consider a transfer pricing scheme to coordinate the inventory sharing decisions. We let b_{it} denote firm i 's inventory sharing action in period t . As previously, a positive action corresponds to borrowing, a negative action to lending, and zero is a do-nothing action. Inventory sharing will arise only if it is beneficial for both of the firms. Finally, we assume that a firm can borrow at most the amount that is short for satisfying the current period demand. In other words, firms are prevented from borrowing to sell in the spot and forward markets.

2.2.4 Assumptions

To facilitate analysis, we make the following assumptions.

Assumption 2.2.1. *Demand is independent of the spot and forward prices.*

Assumption 2.2.1 greatly simplifies the analysis. Such an assumption is not uncommon in the literature (see, e.g., Nascimento and Powell 2009 and Goel and Gutierrez 2011). Note that this assumption is appropriate for a short horizon problem like ours. According to Krichene (2002), the short run price elasticity of the U.S. crude oil demand is -0.02; that is, if the crude oil price increases by 1%, the crude oil demand decreases only by 0.02% in a short horizon.

In addition, we make the following two assumptions to assure the value function is finite in each period. Similar assumptions are made in the literature (see Secomandi 2010).

Assumption 2.2.2. $0 < \beta < 1$

Assumption 2.2.3. $\mathbb{E}_t[D_{i,t+l}] < \infty$, $\mathbb{E}_t[S_{t+l}D_{i,t+l}] < \infty$, $\forall \mathbb{P}_t \in \mathcal{R}_+^3$, $l = 0, 1, \dots, T - t$, $i = 1, 2$, $\forall t$.

Finally, we define the concept of convenience yield as $C_t \equiv S_t + \tau_b + h - \beta F_t$, which will be useful in our analysis. As Goel and Gutierrez (2011) point out, $S_t + h - \beta F_t$ is always non-negative; otherwise, to buy (and hold) commodity from the spot market and then sell it in the forward market would lead to an arbitrage profit. From the fact that $\tau_b \geq 0$, we make the following assumption which is justified with a similar reason. Assumption 2.2.4 is guaranteed.

Assumption 2.2.4. $C_t \geq 0$, $\forall t$.

Note that our definition of the convenience yield is slightly different from Hull (2005), where the convenience yield is defined as “*ownership of the*

physical asset enables a manufacturer to keep a production process running and perhaps profit from temporary shortages.” That is, there exists a benefit not from entering a forward contract but from owning physical inventory. Differently, in our model, we assume a firm’s demand is realized at the beginning of each period, and thus there is no “*temporary shortages*” during a period. Hence, holding forward contracts is more beneficial than holding physical inventory because firms can reduce the holding cost.

2.2.5 Timeline

Figure 2.2 details the timeline of the problem. At the beginning of period t , the commodity ordered (or lent) in period $t - 1$ arrives; then the demand, the spot price as well as the forward price are realized. Second, the two firms make their inventory decisions. If the demand of a firm is less than its beginning inventory ($D_{it} \leq x_{it}$), then the firm decides whether to sell the excess inventory ($x_{it} - D_{it}$) in the spot or forward market, to lend the inventory to the other party, or to hold the inventory for future use. In contrast, if the demand is more than the beginning inventory ($D_{it} > x_{it}$), then the firm decides whether to borrow inventory from the other party or procure from the spot market to meet the excess demand ($D_{it} - x_{it}$). Finally, the firms decide the ordering quantities in the forward market. In period T (the last period), any excess demand is satisfied by immediate purchase from the spot market, and likewise, any excess inventory is sold to the spot market.

2.3 Model & Analysis

We analyze the centralized solution as a benchmark and the decentralized case in §2.3.1 and §2.3.2, respectively. In §2.3.3, we discuss a stochastic

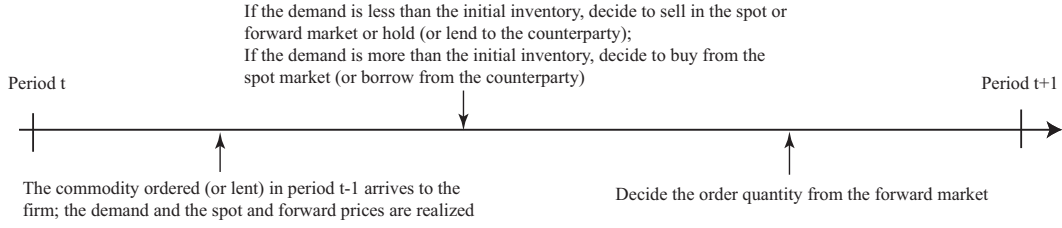


Figure 2.2: The timeline of the model

transfer price to improve the benefit of inventory sharing for the decentralized case relative to the centralized solution.

2.3.1 Benchmark: The Centralized Solution

In this subsection, we analyze the centralized solution as a benchmark. The amount of inventory that can be shared is limited to the minimum of the amount of excess inventory of one firm and the amount of shortage of the other. Therefore, the domain of inventory sharing can be defined as $\mathcal{B}_{ijt} := [-\min\{(x_{it} - D_{it})^+, (D_{jt} - x_{jt})^+\}, \min\{(D_{it} - x_{it})^+, (x_{jt} - D_{jt})^+\}]$. For notational convenience, we define $\mathbf{D}_t := (D_{it}, D_{jt})$ and $\mathbf{x}_t := (x_{it}, x_{jt})$ to represent demands and the beginning inventory levels of two firms in period t , respectively. Let $\hat{\mathbb{P}}_t := (F_t, S_t, \mathbf{D}_t)$ denote the information set in period t . In addition, given that short-selling is not allowed in our model, the total amount of selling in each market shall be limited to the amount of inventory that is available. However, we do not limit the amount of purchases. Therefore, the domain of a firm's trading transactions can be defined as $\mathcal{O}_{it} := [-(x_{it} - D_{it})^+, \infty)$. We define firm i 's immediate cost function as

follows: $\hat{v}_{it}(a_{it}, u_{it}, b_{it}; \mathbf{x}_t, \hat{\mathbb{P}}_t) :=$

$$\begin{cases} (S_t - \tau_s)a_{it} + \beta F_t u_{it} + h((x_{it} - D_{it})^+ + a_{it}) + (\beta F_t + h)b_{it} \\ \quad \text{if } a_{it} \leq 0 \text{ and } b_{it} \leq 0. \\ (S_t + \tau_b)a_{it} + \beta F_t u_{it} + h(a_{it} + x_{it} - D_{it})^+ + (\beta F_t + h)b_{it} \\ \quad \text{if } a_{it} > 0 \text{ and } b_{it} \leq 0, \\ (S_t + \tau_b)a_{it} + \beta F_t u_{it} + h(a_{it} + x_{it} - D_{it})^+ + (\tau_o + \beta F_t)b_{it} \\ \quad \text{if } a_{it} \geq 0 \text{ and } b_{it} > 0, \end{cases}$$

and firm j 's immediate cost function $\hat{v}_{jt}(a_{jt}, u_{jt}, b_{jt}; \mathbf{x}_t, \hat{\mathbb{P}}_t)$ is analogous. In the centralized model, the optimal actions are decided to minimize the total cost of the two firms, and thus the model can be formulated as follows:

$$\begin{aligned} & V_T^C(\mathbf{x}_T, \hat{\mathbb{P}}_T) \\ & \equiv (S_T + \tau_b)\{(D_{iT} - x_{iT})^+ + (D_{jT} - x_{jT})^+\} \\ & \quad - (S_T - \tau_s)\{(x_{iT} - D_{iT})^+ + (x_{jT} - D_{jT})^+\}, \forall (\mathbf{x}_T, \hat{\mathbb{P}}_T) \in \mathcal{R}_+^6 \\ & V_t^C(\mathbf{x}_t, \hat{\mathbb{P}}_t) \\ & \equiv \\ & \min_{a_{it}, u_{it}, a_{it} + u_{it} + b_{it} \in \mathcal{O}_{it}, b_{it} \in \mathcal{B}_{ijt}, a_{jt}, u_{jt}, a_{jt} + u_{jt} + b_{jt} \in \mathcal{O}_{jt}, b_{jt} \in \mathcal{B}_{jit}, a_{it} + b_{it} \geq D_{it} - x_{it}, a_{jt} + b_{jt} \geq D_{jt} - x_{jt}} \\ & \hat{v}_{it}(a_{it}, u_{it}, b_{it}; \mathbf{x}_t, \hat{\mathbb{P}}_t) + \hat{v}_{jt}(a_{jt}, u_{jt}, b_{jt}; \mathbf{x}_t, \hat{\mathbb{P}}_t) + \beta \mathbb{E}_t[V_{t+1}^C(\mathbf{x}_{t+1}, \hat{\mathbb{P}}_{t+1})], \\ & \forall (\mathbf{x}_t, \hat{\mathbb{P}}_t) \in \mathcal{R}_+^6, \forall t \in \{1, 2, \dots, T-1\} \end{aligned}$$

where

$$x_{i,t+1} = x_{it} - D_{it} + a_{it} + u_{it} + b_{it} \quad \text{and} \quad x_{j,t+1} = x_{jt} - D_{jt} + a_{jt} + u_{jt} + b_{jt}.$$

Proposition 2.3.1 shows the optimal actions.

Proposition 2.3.1. *In each period t , a unique pair of the optimal order-up-to levels, $(x_{i,t+1}^C, x_{j,t+1}^C)$ exists that minimizes $F_t(x_{i,t+1}+x_{j,t+1})+\mathbb{E}_t[V_{t+1}^C(\mathbf{x}_{t+1}, \hat{\mathbb{P}}_{t+1})]$, and the optimal trading actions follow: (i) If $C_t \geq \tau_o$, then $b_{it}^C = -b_{jt}^C = \min\{(D_{it} - x_{it})^+, (x_{jt} - D_{jt})^+\} - \min\{(x_{it} - D_{it})^+, (D_{jt} - x_{jt})^+\}$; otherwise, $b_{it}^C = b_{jt}^C = 0$; (ii) If $C_t \geq \tau_b + \tau_s$, then $a_{it}^C = D_{it} - x_{it} - b_{it}^C$; otherwise, $a_{it}^C = (D_{it} - x_{it} - b_{it}^C)^+$; (iii) $u_{it}^C = x_{i,t+1}^C - (x_{it} - D_{it}) - a_{it}^C - b_{it}^C$.*

From Proposition 2.3.1, we observe that the convenience yield plays an important role in the decisions. First, given the optimal inventory sharing actions, the optimal trading actions in the spot and forward markets can be derived as the following. If a firm still has excess inventory after satisfying its own demand and also the other firm's borrowing request, the firm will sell all leftover inventory to the spot market when $C_t \geq \tau_b + \tau_s$ (i.e., $S_t - \tau_s + h - \beta F_t > 0$) and choose not to trade in the spot market otherwise. This is because if $C_t \geq \tau_b + \tau_s$, then it is beneficial for the firm to sell the leftover inventory to the spot market in period t to avoid any holding cost and, at the same time, purchase commodity from the forward market up to the optimal order-up-to level for the next period (the commodity purchased in the forward market will be delivered in the next period and thus the firm will not incur any holding cost). In contrast, if a firm is still short of inventory after using its own inventory and also the inventory borrowed from the other firm, the firm will buy immediately from the spot market for the amount of shortage. Then, the firms will trade in the forward market to increase the beginning inventories of the next period to the equilibrium order-up-to levels, after taking the borrowing and lending actions into account.

The optimal inventory sharing actions can be derived as the following. The cost reduction by borrowing inventory from the other firm is the cost to

buy directly from the spot market, $S_t + \tau_b$, less the total unit cost to borrow and return the inventory, $\tau_o + \beta F_t$. That is, $C_t - h - \tau_o$ is the unit cost reduction for the borrowing firm. At the same time, the cost reduction by lending excess inventory to the other firm is the gain from lending, $h + \beta F_t$, less the gain from selling the inventory to the spot market, $S_t - \tau_s + h$, or selling it to the forward market, βF_t . Notice that the gain from the selling inventory to the spot market is greater than that to the forward market if $C_t \geq \tau_b + \tau_s$ (i.e., $S_t - \tau_s + h - \beta F_t \geq 0$). Hence, when $C_t \geq \tau_b + \tau_s$, the unit cost reduction for the lending firm is $\tau_b + \tau_s + h - C_t$; otherwise, the unit cost reduction is h . Hence, if $C_t \geq \tau_b + \tau_s$, then the total cost reduction by inventory sharing for the centralized case is $\tau_b + \tau_s - \tau_o$ per unit; otherwise, the total cost reduction is $C_t - \tau_o$. Since $\tau_b + \tau_s - \tau_o$ is always positive by assumption, inventory sharing will arise in the centralized case as long as $C_t \geq \tau_o$.

Note that from Proposition 2.3.1, the following two equations should be satisfied simultaneously to find the optimal ordering levels for two firms, $(x_{i,t+1}^C, x_{j,t+1}^C)$:

$$\begin{aligned}
& -\tau_b + G_i(x_{i,t+1}^C) \{(\tau_b + \tau_s) - \mathbb{E}_t[FW_{t+1}]\} \\
& -\mathbb{E}_t[IS_{t+1}] \{P_{iL}(x_{i,t+1}^C, x_{j,t+1}^C) - P_{iB}(x_{i,t+1}^C, x_{j,t+1}^C)\} = 0 \\
& -\tau_b + G_j(x_{j,t+1}^C) \{(\tau_b + \tau_s) - \mathbb{E}_t[FW_{t+1}]\} \\
& -\mathbb{E}_t[IS_{t+1}] \{P_{jL}(x_{i,t+1}^C, x_{j,t+1}^C) - P_{jB}(x_{i,t+1}^C, x_{j,t+1}^C)\} = 0
\end{aligned}$$

where $\mathbb{E}_t[FW_{t+1}] := \mathbb{E}_t[(\tau_b + \tau_s - C_{t+1})\mathbb{I}_{\{C_{t+1} < \tau_b + \tau_s\}} | C_t]$ denotes the expected unit cost reduction by selling to the forward market, $\mathbb{E}_t[IS_{t+1}] := \mathbb{E}_t[(C_{t+1} - \tau_o)\mathbb{I}_{\{\tau_o \leq C_{t+1} < \tau_b + \tau_s\}} | C_t] + \mathbb{E}_t[(\tau_b + \tau_s - \tau_o)\mathbb{I}_{\{C_{t+1} \geq \tau_b + \tau_s\}} | C_t]$ denotes the expected total cost reduction per unit by inventory sharing, G_i denotes the cumulative distribution function of D_{it} , and $P_{iL}(x_{it}, x_{jt}) := Pr(x_{it} > D_i, D_i + D_j > x_{it} +$

x_{jt}), $P_{iB}(x_{it}, x_{jt}) := Pr(x_{it} < D_i, D_i + D_j < x_{it} + x_{jt})$, $P_{jL}(x_{it}, x_{jt}) := Pr(x_{jt} > D_j, D_i + D_j > x_{it} + x_{jt})$, $P_{jB}(x_{it}, x_{jt}) := Pr(x_{jt} < D_j, D_i + D_j < x_{it} + x_{jt})$.

2.3.2 Decentralized Inventory Sharing

In the centralized case, inventory sharing can arise as long as it can reduce the total cost of the two firms. Whereas, if the decisions are decentralized, then inventory sharing will arise only if both firms can benefit from sharing. Specifically, firm i 's problem with inventory sharing can be formulated as follows (which is nested with firm j 's problem due to inventory sharing):

$$\begin{aligned} \hat{V}_{iT}(\mathbf{x}_T, \hat{\mathbb{P}}_t) &\equiv (S_T + \tau_b)(D_{iT} - x_{iT})^+ \\ &\quad - (S_T - \tau_s)(x_{iT} - D_{iT})^+, \forall (\mathbf{x}_T, \hat{\mathbb{P}}_T) \in \mathcal{R}_+^6 \\ \hat{V}_{it}(\mathbf{x}_t, \hat{\mathbb{P}}_t) &\equiv \min_{a_{it}, u_{it}, a_{it} + u_{it} + b_{it} \in \mathcal{O}_{it}, b_{it} \in \mathcal{B}_{ijt}, a_{it} + b_{it} \geq D_{it} - x_{it}} \\ &\quad \hat{v}_{it}(a_{it}, u_{it}, b_{it}; \mathbf{x}_t, \hat{\mathbb{P}}_t) + \beta \mathbb{E}_t[\hat{V}_{it+1}(\mathbf{x}_{t+1}, \hat{\mathbb{P}}_{t+1})], \\ &\quad \forall (\mathbf{x}_t, \hat{\mathbb{P}}_t) \in \mathcal{R}_+^6, \forall t \in \{1, 2, \dots, T-1\} \end{aligned}$$

where

$$x_{i,t+1} = x_{it} - D_{it} + a_{it} + u_{it} + b_{it}.$$

Note that when inventory sharing is executed, the borrowing firm enters two types of forward contracts: one for an amount of u_{it} delivered to the own firm and the other for an amount of b_{it} delivered to the lending firm in the next period.

The presence of the inventory sharing transaction makes the decisions of the two firms interact with each other. We assume firms make these decisions independently of one another. Additionally, the inventory of the other firm is

not observable to the firm making the decision. Thus, beyond optimization, obtaining a Nash equilibrium between the two firms' strategies is necessary.

In the following, we analyze the two firms' inventory policies when they can share inventory with each other. That is, a Nash equilibrium of the two firms' inventory policies can be obtained, which is shown below.

Proposition 2.3.2. *In each period t , there exists a unique pair of order-up-to levels, $(\hat{x}_{i,t+1}^*, \hat{x}_{j,t+1}^*)$, such that given $\hat{x}_{j,t+1}^*, \hat{x}_{i,t+1}^*$ minimizes $F_t x_{i,t+1} + E_t[\hat{V}_{i,t+1}(x_{i,t+1}, \hat{x}_{j,t+1}^*, \hat{\mathbb{P}}_{t+1})]$ and given $\hat{x}_{i,t+1}^*, \hat{x}_{j,t+1}^*$ minimizes $F_t x_{j,t+1} + E_t[\hat{V}_{j,t+1}(x_{j,t+1}, \hat{x}_{i,t+1}^*, \hat{\mathbb{P}}_{t+1})]$. In addition, in each period of t , there exists a unique Nash equilibrium of the two firms' inventory policies: (i) If $h + \tau_o \leq C_t \leq \tau_b + \tau_s + h$, then $\hat{b}_{it}^* = -\hat{b}_{jt}^* = \min\{(D_{it} - x_{it})^+, (x_{jt} - D_{jt})^+\} - \min\{(x_{it} - D_{it})^+, (D_{jt} - x_{jt})^+\}$; otherwise, $\hat{b}_{it}^* = \hat{b}_{jt}^* = 0$; (ii) If $C_t \geq \tau_b + \tau_s$, then $\hat{a}_{it}^* = D_{it} - x_{it} - \hat{b}_{it}^*$; otherwise, $\hat{a}_{it}^* = (D_{it} - x_{it} - \hat{b}_{it}^*)^+$; (iii) $\hat{u}_{it}^* = \hat{x}_{i,t+1}^* - (x_{it} - D_{it}) - \hat{a}_{it}^* - \hat{b}_{it}^*$.*

As we have explained below Proposition 2.3.1, the unit cost reduction for the borrowing firm is $C_t - h - \tau_o$. When selling to the spot market is more beneficial than selling to the forward market, i.e., $C_t \geq \tau_b + \tau_s$, the unit cost reduction for the lending firm is $\tau_b + \tau_s + h - C_t$; otherwise, the unit cost reduction is h . In other words, $h + \tau_o \leq C_t$ acts as a necessary condition in order for a firm to benefit from inventory borrowing and $C_t \leq \tau_b + \tau_s + h$ serves as a necessary condition in order for a firm to be willing to lend inventory to the other firm given $h > 0$. Hence, the necessary condition for inventory sharing is the following: $h + \tau_o \leq C_t \leq \tau_b + \tau_s + h$. Recall that in the centralized case, inventory sharing will arise if $C_t \geq \tau_o$. Hence, the range of the convenience yield under which inventory sharing can arise for the centralized is greater than

that for the decentralized case. Moreover, inventory sharing can take place only if one firm is short of inventory while the other firm has excess inventory. Therefore, the amount of inventory sharing is limited to the minimum of the excess demand and the excess inventory at the two firms. The optimal trading actions in the spot and forward markets can be derived using the same logic that has been applied to Proposition 2.3.1.

We note that from Proposition 2.3.2, the following two equations are satisfied simultaneously and can be used to find the optimal ordering levels for the two firms, $(\hat{x}_{i,t+1}^*, \hat{x}_{j,t+1}^*)$:

$$\begin{aligned}
& -\tau_b + G_i(\hat{x}_{i,t+1}^*)\{(\tau_b + \tau_s) - \mathbb{E}_t[FW_{t+1}]\} - \mathbb{E}_t[L_{t+1}]P_{iL}(\hat{x}_{i,t+1}^*, \hat{x}_{j,t+1}^*) \\
& + \mathbb{E}_t[B_{t+1}]P_{iB}(\hat{x}_{i,t+1}^*, \hat{x}_{j,t+1}^*) = 0 \\
& -\tau_b + G_j(\hat{x}_{j,t+1}^*)\{(\tau_b + \tau_s) - \mathbb{E}_t[FW_{t+1}]\} - \mathbb{E}_t[L_{t+1}]P_{jL}(\hat{x}_{i,t+1}^*, \hat{x}_{j,t+1}^*) \\
& + \mathbb{E}_t[B_{t+1}]P_{jB}(\hat{x}_{i,t+1}^*, \hat{x}_{j,t+1}^*) = 0,
\end{aligned}$$

where $\mathbb{E}_t[B_{t+1}] := \mathbb{E}_t[(C_{t+1} - h - \tau_o)\mathbb{I}_{\{C_{t+1} \in (h+\tau_o, h+\tau_b+\tau_s)\}} | C_t]$ and $\mathbb{E}_t[L_{t+1}] := \mathbb{E}_t[h\mathbb{I}_{\{C_{t+1} \in (h+\tau_o, \tau_b+\tau_s)\}} + (h+\tau_b+\tau_s - C_{t+1})\mathbb{I}_{\{C_{t+1} \in (\max\{h+\tau_o, \tau_b+\tau_s\}, \tau_b+\tau_s+h)\}} | C_t]$ denote the expected unit cost reductions by borrowing and lending transactions, respectively.

Notice that since we normalize the transaction cost in the forward market to zero, we can match the optimal order-up-to level for period $t + 1$ (i.e., $\hat{x}_{i,t+1}^*$) regardless of the initial inventory level and demand in period t (i.e., x_{it} and D_{it}), and $\hat{x}_{i,t+1}^*$ is independent of the later optimal ordering levels, i.e., $\{\hat{x}_{i,t+2}^*, \hat{x}_{i,t+3}^*, \dots, \hat{x}_{iT}^*\}$. The same argument holds in the centralized case.

2.3.3 Coordination: Stochastic Transfer Price

In the above subsections, we have shown the centralized and decentralized actions. Obviously, inventory sharing will be more beneficial if both firms follow the centralized solution. However, with decentralized decision making, the firms each may “myopically” deviate from the centralized solution because one would benefit more by deviating if the other sticks to the centralized solution (we show an example in §2.4.3). As a result, the centralized solution becomes unattainable and some coordination methods might be needed.

In the transshipment literature, a few coordinating mechanisms have been explored to improve the efficiency of the decentralized systems. For instance, one stream of research considers design of transshipment price, by which one firm makes a specific amount of payment to the other for the shared inventory. However, even though *deterministic* transshipment price is easy to implement, as shown by Hu et al. (2007), the coordinating transshipment price may not always exist. Moreover, a deterministic transshipment price will not coordinate in our model due to changing prices. Some other approaches which guarantee decentralized firms choose the centralized decisions have also been proposed, but they are relatively complex. For example, Hanany et al. (2010) propose the transshipment fund mechanism. The mechanism *adjusts* the transshipment price (or the transfer price) in three stages per period to induce decentralized firms to decide the centralized ordering levels with a third party financial entity. This mechanism might be beneficial in our setting, but it is difficult to implement.

In this subsection, we propose a specific coordinating mechanism for our context, a stochastic transfer price. This mechanism is easy to implement, similar to the transshipment price proposed in Hu et al. (2007), but

it allows the transfer price to change corresponding to the commodity prices, which is similar to the transshipment fund mechanism proposed in Hanany et al. (2010). We let $\tau_{k,t}$ be the per unit payment from the lending firm to the borrowing firm in period t . Lemma 2.3.3 characterizes the stochastic transfer pricing schemes which make the convenience yield condition of the decentralized inventory sharing transaction the same as that of the centralized inventory sharing transaction (i.e., $C_t \geq \tau_o$).

Lemma 2.3.3. *The convenience yield condition of the decentralized inventory sharing transaction becomes the same as that of the centralized inventory sharing transaction, i.e., $C_t \geq \tau_o$, if we set the stochastic transfer price in period t , $\tau_{k,t}$, as follows: i) if $C_t \geq \tau_b + \tau_s$, then $\tau_{k,t}$ is in-between $(h + \tau_o - C_t)$ and $(\tau_b + \tau_s + h - C_t)$, i.e., $\tau_{k,t} \in \{h + \tau_o - C_t, \tau_b + \tau_s + h - C_t\}$; ii) if $\tau_o \leq C_t < \tau_b + \tau_s$, then $\tau_{k,t}$ is in-between $(h + \tau_o - C_t)$ and h , i.e., $\tau_{k,t} \in \{h + \tau_o - C_t, h\}$. Specifically, the stochastic transfer price $\tau_{k,t}$ makes inventory sharing benefit to each firm be the same in every period by setting $\tau_{k,t}$ as follows: iii) if $C_t \geq \tau_b + \tau_s$, then $\tau_{k,t} = -C_t + h + \frac{\tau_b + \tau_s + \tau_o}{2}$; iv) if $\tau_o \leq C_t < \tau_b + \tau_s$, then $\tau_{k,t} = -\frac{C_t}{2} + h + \frac{\tau_o}{2}$.*

Lemma 2.3.3 shows that there are various ways to set the stochastic transfer price which make the convenience yield condition of the decentralized inventory sharing transaction the same as that of the centralized inventory sharing transaction. The setting affects the unit inventory sharing benefit for both the lending and the borrowing firms. For example, if $C_t \geq \tau_b + \tau_s$ and $\tau_{k,t} = \tau_b + \tau_s + h - C_t$, then all the benefit (i.e., $\tau_b + \tau_s - \tau_o$) belongs to the borrowing firm. Numerically, there is not much difference by selecting different $\tau_{k,t}$ within the range shown in Lemma 2.3.3. Hence, we select the *fair* division. That is, when $C_t \geq \tau_b + \tau_s$ and $\tau_o \leq C_t < \tau_b + \tau_s$, we set $\tau_{k,t} = -C_t + h + \frac{\tau_b + \tau_s + \tau_o}{2}$ and $\tau_{k,t} = -\frac{C_t}{2} + h + \frac{\tau_o}{2}$, and the two firms' sharing benefits become $\frac{\tau_b + \tau_s - \tau_o}{2}$ and

$\frac{C_t - \tau_o}{2}$, respectively. Therefore, the two firms will be willing to share inventory as long as $C_t \geq \tau_o$ (the condition becomes identical to that of the centralized case) and the benefits the firms gain from inventory sharing become equal in each period.

Notice that Lemma 2.3.3 shows that the stochastic transfer price makes the range of the convenience yield under which inventory sharing can arise in the decentralized case be the same as that in the centralized case. However, we should note that the benefits of inventory sharing in the two cases are different due to the different optimal ordering levels in the two cases. To find the optimal ordering levels of the decentralized case $(\hat{x}_{i,t+1}^*, \hat{x}_{j,t+1}^*)$, we derive the following two equations which should be satisfied simultaneously:

$$\begin{aligned} & -\tau_b + G_i(\hat{x}_{i,t+1}^*)\{(\tau_b + \tau_s) - \mathbb{E}_t[FW_{t+1}]\} \\ & - \frac{\mathbb{E}_t[IS_{t+1}]}{2}\{P_{iL}(\hat{x}_{i,t+1}^*, \hat{x}_{j,t+1}^*) - P_{iB}(\hat{x}_{i,t+1}^*, \hat{x}_{j,t+1}^*)\} = 0 \\ & -\tau_b + G_j(\hat{x}_{j,t+1}^*)\{(\tau_b + \tau_s) - \mathbb{E}_t[FW_{t+1}]\} \\ & - \frac{\mathbb{E}_t[IS_{t+1}]}{2}\{P_{jL}(\hat{x}_{i,t+1}^*, \hat{x}_{j,t+1}^*) - P_{jB}(\hat{x}_{i,t+1}^*, \hat{x}_{j,t+1}^*)\} = 0. \end{aligned}$$

The above conditions are different from the conditions to find the optimal ordering levels in the centralized case (see §2.3.1) because, in any period $t + 1$, the unit cost reduction by inventory sharing is IS_{t+1} in the centralized case but $\frac{IS_{t+1}}{2}$ in the decentralized case.

2.4 Value of Inventory Sharing

In this section, we assess the value of inventory sharing, which is defined as the procurement cost difference without inventory sharing relative to with inventory sharing. Notice that we can easily get the optimal actions in the

model without inventory sharing by letting the amount of sharing quantity, b_{it} (or b_{jt}), be zero for all t .

2.4.1 Experiment Setting

Market Demand. We assume that the demands faced by the two firms in each period follow a bivariate normal distribution and the demands across different time periods are independent. We consider a planning horizon of 20 periods, each of which corresponds to two weeks (i.e., total 40 weeks). The detailed information of the demand distributions is presented in Table 2.1. Note that the coefficient of variation of the demand is chosen in the range from 0.1 to 0.2, which is typical for the large refinery firms in the U.S. (according to the data from the U.S. Energy Information Administration).

Transaction Costs. The spot transaction costs depend on distance. Trench (2001) reports that the transportation cost of gasoline by pipelines (for the forward transactions) from Houston to Chicago was \$0.84 per barrel; whereas, the transportation cost of trucking (for the spot transactions) increases dramatically with distance. Hence, we set the selling and buying transaction costs for spot trades range from \$1.0 to \$1.4 per barrel. We vary the transaction cost of inventory sharing from \$0.2 to \$0.6 per barrel.

Discount Factor and Holding Cost. The 4-week T-bill rate from January 2006 to December 2010 varied from 0.02% to 5.13%. Thus, we set the time discount rate for two weeks to 1.5%; i.e., β equals 0.985. The holding cost in general consists of the opportunity cost and the physical cost. Considering the typical handling and storage cost for crude oil (\$0.3 per barrel per month according to Bohn 1996) and the average spot price (\$80), we set the holding cost h at \$1.35 per barrel for two weeks.

Table 2.1: Demand and operational costs parameters

Parameter	Description (unit)	Estimate
(μ_i, σ_i^2)	mean and variance of demand for firm i (barrels in two weeks)	$(100, 20^2)$
(μ_j, σ_j^2)	mean and variance of demand for firm j (barrels in two weeks)	$(100, 10^2), (100, 20^2),$ $(200, 20^2), (200, 40^2)$
ρ	correlation coefficient between the two demands	0, 0.3, 0.5, 0.7, 0.9
τ_b	buying transaction cost in the spot market (\$ per barrel)	1.0, 1.2, 1.4
τ_s	selling transaction cost in the spot market (\$ per barrel)	1.0, 1.2, 1.4
τ_o	transaction cost in inventory sharing (\$ per barrel)	0.2, 0.4, 0.6
h	holding cost (\$ per barrel per two weeks)	1.35
β	discount factor (per barrel per two weeks)	0.985

Price Model. Our numerical study assumes that the evolution of the crude oil spot price and that of the forward price follow the model given in Schwartz and Smith (2000). In particular, the log spot price, $\ln(S_t)$, decomposes into a short-term deviation of the price, χ_t and an equilibrium price level, ξ_t ; i.e., $\ln(S_t) = \chi_t + \xi_t$. The short-term deviation of the price, χ_t , follows the Ornstein-Uhlenbeck process: $d\chi_t = (-\kappa\chi_t - \lambda_\chi)dt + \sigma_\chi dZ_\chi$, where κ , λ_χ , and σ_χ are the mean-reversion rate, the risk premium, and the volatility in the short-term factor (χ_t), respectively; the long-term deviation of the price, ξ_t , follows $d\xi_t = \mu_\xi^* dt + \sigma_\xi dZ_\xi$, where μ_ξ^* and σ_ξ are the drift rate and the volatility in the long-term factor (ξ_t), respectively. dZ_χ and dZ_ξ are the standard Brownian motion increments that satisfy $dZ_\chi dZ_\xi = \rho_{\chi\xi} dt$. We use the parameters calibrated in Schwartz and Smith (2000) in our experiments (see Table 2.2) and set the initial short-term deviation χ_0 to 0.17 and the initial equilibrium price level ξ_0 to 4.15.

Table 2.2: Price parameters

Parameter	Description	Estimate
κ	Short-term mean-reversion rate	1.49
σ_χ	Short-term volatility	0.286
λ_χ	Short-term risk premium	0.157
μ_ξ^*	Equilibrium risk-neutral drift rate	0.0115
σ_ξ	Equilibrium volatility	0.145
$\rho_{\chi\xi}$	Correlation coefficient between dZ_χ and dZ_ξ	0.3

2.4.2 Value of Inventory Sharing in the Centralized Case

Here, we investigate the value of inventory sharing in the centralized case which will serve as a benchmark for the later analysis. The magnitude of the value of inventory sharing depends on the business environment. We examine the impacts of the parameters including the mean and variance of the counterparty's demand, the correlation of the two firms' demands, and the per-unit transaction costs. Figure 2.3 presents the experiment results (the caption of the figure shows the base values of the parameters and we vary one parameter at a time reflected in each plot). Note that the percentage of procurement cost reduction shown in our numerical study is relatively small due to the large base cost (i.e., the raw material procurement cost). However, given the nature of this industry with relatively thin profit margins, the small amount of procurement cost reduction by inventory sharing as reported in our study can still significantly increase the profitability of the firms. Therefore, to have a better representation, we estimated the annual procurement cost of typical U.S. oil refinery firms with comparable mean demands as in our experiment and converted the percentage cost reduction to absolute cost saving as reported in the figures ¹.

¹Chevron's net refinery input in 2011 was 652,255,500 barrels. We assume that the average oil price is \$80 per barrel. Hence, the approximate annual raw material procurement cost is $652,255,500 * \$80 = \$52,180,400,000$. Since we consider two-size firms, with mean of demands 100 and 200, we assume the firm with mean of demand 100 incurs $\$52,180,400,000/2 = \$26,090,200,000$ per year, and the firm with mean of demand 200 incurs $\$52,180,400,000$ per year. Note that since the estimated procurement cost does not include the spot transaction and holding costs, the actual procurement cost would be higher than our estimation.

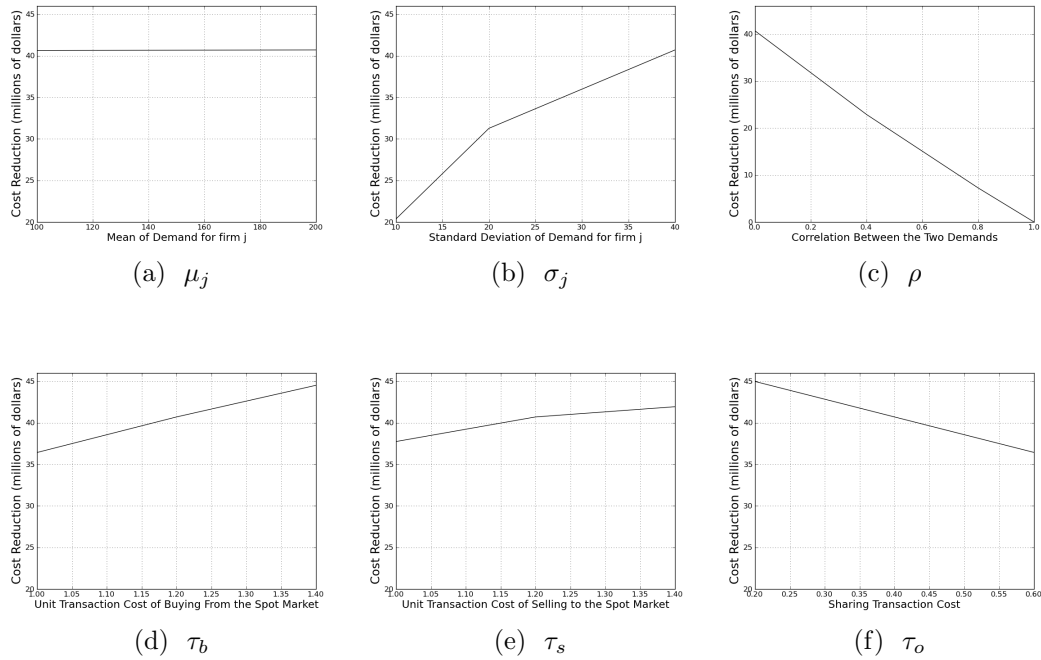


Figure 2.3: Demonstration of the value of inventory sharing in the centralized case depending on the parameter values of the two demands. The base values of the parameters are $\mu_i = 100, \mu_j = 200, \sigma_i = 20, \sigma_j = 40, \rho = 0, \tau_b = \tau_s = 1.2, \tau_o = 0.4, h = 1.35$. We change one parameter at a time while keeping the other parameters fixed at the base values.

2.4.2.1 Effect of Demand.

We observe in our experiments that a change in the mean of the counterparty's demand has little impact on the value of inventory sharing (see Figure 2.3(a)). This is because with a normally distributed demand, an increase of the mean results in a corresponding increase of the order-up-to level. As a result, the probabilities of shortage and overage do not change significantly. In contrast, an increase of the volatility of the counterparty's demand can influence the value of inventory sharing substantially. For instance, Figure 2.3(b) shows that as the standard deviation of the counterparty's demand increases from 10 to 40, the value of inventory sharing increases by \$20.3 million. Intuitively, as the counterparty's demand becomes more volatile, the chance of mismatch between its demand and supply will increase, which will thus increase the probability of inventory sharing. In addition, in order for inventory sharing to occur, one firm needs to have excess inventory while the other firm needs to have excess demand. Therefore, inventory sharing will be more beneficial if the demands of the two firms are less positively correlated (or more negatively correlated). Figure 2.3(c) demonstrates the effect of the correlation. As ρ increases from 0 to 1, the value of inventory sharing reduces by nearly \$40.7 million.

2.4.2.2 Effect of the Transaction Costs.

Changes in transaction costs also affect the value of inventory sharing. As we have explained below Proposition 2.3.1, the range of the convenience yield under which inventory sharing can arise is $C_t \geq \tau_o$. Furthermore, relative to trading in the spot market, the magnitude of saving from sharing inventory depends on the convenience yield; that is, when $\tau_o \leq C_t < \tau_b + \tau_s$, the per unit

saving is $C_t - \tau_o$, and when $C_t \geq \tau_b + \tau_s$, the per unit savings is $\tau_b + \tau_s - \tau_o$. Hence, as τ_b increases or τ_o decreases, the value of inventory sharing increases because an increase of τ_b or a decrease of τ_o increases not only the per unit cost savings but also the range of the convenience yield under which inventory sharing can arise. For instance, Figures 2.3(d) and 2.3(f) demonstrate that as τ_b increases from 1.0 to 1.4 and as τ_o decreases from 0.6 to 0.2, the values of inventory sharing increase about \$8.1 million and \$8.5 million, respectively. The effect of an increase of τ_s on the value of inventory sharing is not analytically clear because as τ_s increases, the condition under which inventory sharing can occur (i.e., $C_t \geq \tau_b + \tau_s$) becomes tighter although the cost saving $\tau_b + \tau_s - \tau_o$ increases. In our numerical study, as τ_s increases, the value of inventory sharing mostly increases. For example, Figure 2.3(e) shows that as τ_s increases from 1.0 to 1.4, the value of inventory sharing increases by \$4.2 million.

2.4.3 Gaps between the values of inventory sharing

In this subsection, we compare the gap between the centralized and decentralized cases. Before examining the gap, we first check whether the centralized solution can be installed when the firms make decentralized decisions. Our numerical experiment shows that when one firm uses the policy under the centralized solution, the other firm will benefit by deviating from the centralized solution. As a consequence, a prisoner's dilemma can arise, as we show in Table 2.3. Even though the firms can obtain a higher profit under the centralized solution, it is not implementable in equilibrium.

Now let us investigate the performances of the stochastic transfer price. In our entire numerical study, we observe that the stochastic transfer price significantly reduces the gap between the values of inventory sharing of the

Table 2.3: Value of inventory sharing (millions of dollars). The borrowing firm pays all the sharing transaction cost τ_o . The values of the parameters are $\mu_i = 100, \sigma_i = 20, \mu_j = 100, \sigma_j = 20, \rho = 0, \tau_b = \tau_s = 1.2, \tau_o = 0.4, h = 1.35$.

		Firm j	
		Centralized	Deviated
Firm i	Centralized	(\$31.26, \$31.26)	(\$30.10, \$31.89)
	Deviated	(\$31.89, \$30.10)	(\$30.72, \$30.72)

centralized and decentralized cases. As an example, Table 2.4 shows the gap is about \$2,900 under the stochastic transfer price; whereas, the gap is about \$670,500 when a borrowing firms pays all the sharing transaction cost. Notice that the assumption of the borrower pays all the sharing transaction cost τ_o seems an innocuous because we expect it is equally likely a firm borrows or lends, and thus the benefit from sharing is symmetric. However, the assumption about the transfer price is quite important because it affects the stocking decisions as well as the range of the convenience yield under which inventory sharing can arise. Therefore, attention has to be given to coordination between the firms. We note that calculating the stochastic transfer price is simple, so it is easy to implement. Also, under the stochastic transfer price, the benefits from inventory sharing are shared fairly between the two firms, while it is generally not true under other mechanisms.

We additionally conduct a sensitivity analysis of the gap with respect to the system parameters. Notice that the range of the convenience yield under which inventory sharing can arise is the same for the centralized case and the decentralized case with stochastic transfer price, i.e., $C_t \geq \tau_o$. In other words, the convenience yield condition to the inventory sharing transactions does not affect the gap. Hence, the gap is caused due to two reasons: First, as the

Table 2.4: Gap of the value of inventory sharing (thousands of dollars). The values of the parameters are $\mu_i = 100, \sigma_i = 20, \mu_j = 200, \sigma_j = 40, \rho = 0, \tau_b = \tau_s = 1.2, \tau_o = 0.4, h = 1.35$.

	Default ($\tau_{k,t} = 0$ for all t)	Stochastic Transfer Price
Gap to the Centralized Case	\$670.5	\$2.9

value of inventory sharing in the centralized case increases, the gap increases. We converted the percentage cost reduction to absolute cost saving. Hence, although the percentage of the gap reduction is the same, the gap reduction in dollars increases as the absolute cost saving in the centralized case increases. Second, the gap is affected by the difference of the ordering levels due to the different first order conditions for the ordering levels. Recall that the first order conditions for firm i in the centralized and decentralized cases, respectively, can be written as:

$$\begin{aligned}
 & \underbrace{-\tau_b + G_i(x_{i,t+1})\{(\tau_b + \tau_s) - \mathbb{E}_t[FW_{t+1}]\}}_{\text{Common Term}} \\
 & - \underbrace{\mathbb{E}_t[IS_{t+1}]\{P_{iL}(x_{i,t+1}, x_{j,t+1}) - P_{iB}(x_{i,t+1}, x_{j,t+1})\}}_{\text{Different Term in the Centralized Case}} = 0, \\
 & \underbrace{-\tau_b + G_i(x_{i,t+1})\{(\tau_b + \tau_s) - \mathbb{E}_t[FW_{t+1}]\}}_{\text{Common Term}} \\
 & - \underbrace{\frac{\mathbb{E}_t[IS_{t+1}]}{2}\{P_{iL}(x_{i,t+1}, x_{j,t+1}) - P_{iB}(x_{i,t+1}, x_{j,t+1})\}}_{\text{Different Term in the Decentralized Sharing Case}} = 0.
 \end{aligned}$$

Hence, the only difference in the first order conditions between the two cases is the term in front of the probability differences, i.e., $\mathbb{E}_t[IS_{t+1}]$ in the centralized case and $\frac{\mathbb{E}_t[IS_{t+1}]}{2}$ in the decentralized case. Hence, the gap increases as the common term in the first order conditions decreases and the different term in

the conditions increases. Notice that the common term is always positive.

2.4.3.1 Effect of Demand.

The changes of the gaps due to demand factors are explained by the first reason, i.e., as the value of inventory sharing in the centralized case increases, the gap increases. From Figures 2.3 and 2.4, we observe that when the parameter value related to demand changes, the gap increases as the value of inventory sharing in the centralized case increases. That is, the gap is almost stable when the mean of demand changes; whereas, the gap increases as the variance of demand increases or as the correlation between the two demands decreases. Notice that the change in demand terms does not affect $\mathbb{E}_t[IS_{t+1}]$ and has a negligible effect on the difference of the lending and borrowing probabilities (i.e., $P_{iL}(x_{i,t+1}, x_{j,t+1}) - P_{iB}(x_{i,t+1}, x_{j,t+1})$). Hence, the difference of the ordering levels changes little as the demand factors change.

2.4.3.2 Effect of Transaction Costs.

The changes in transaction costs also affect the gap between the values of inventory sharing of the centralized and decentralized cases. An increase of τ_b decreases the common term in the first order condition and increases $\mathbb{E}_t[IS_{t+1}]$. In addition, as τ_b increases, the value of inventory sharing in the centralized case increases (see Figure 2.3(d)). Hence, as τ_b increases, the gap increases. For instance, Figure 2.4(d) illustrates that as τ_b increases from 1.0 to 1.4, the gap increases about \$21,715. Differently, as τ_o increases, the common term in the first order condition remains the same while $\mathbb{E}_t[IS_{t+1}]$ decreases. Moreover, as τ_o increases, the value of inventory sharing in the centralized case decreases (see Figure 2.3(f)). Hence, as τ_o increases, the gap decreases.

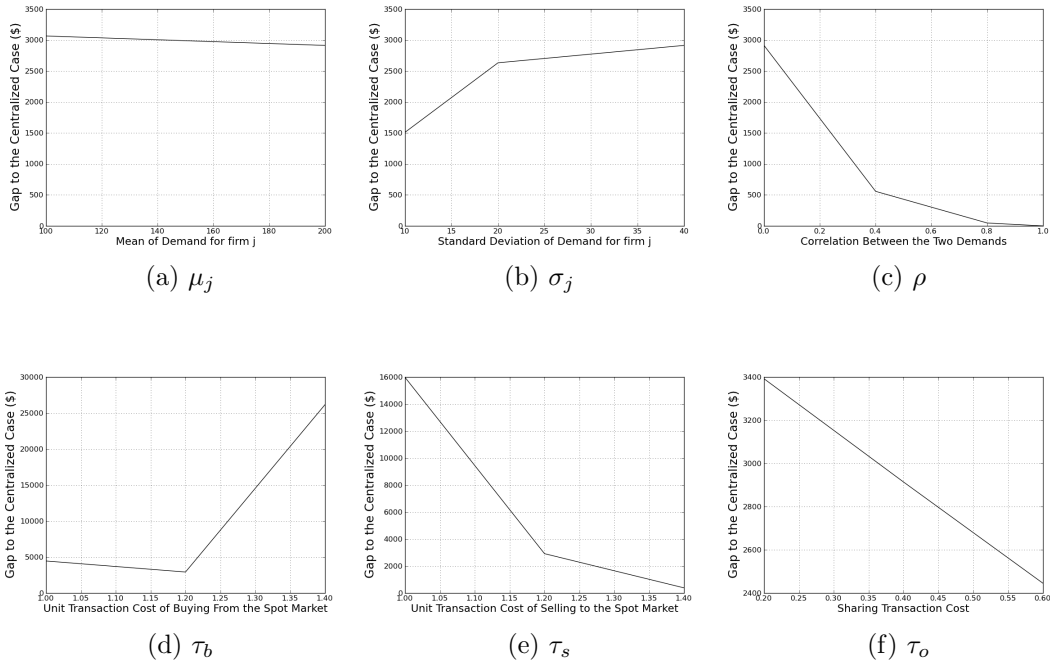


Figure 2.4: Demonstration of the gap of the value of inventory sharing between the decentralized case and the centralized one depending on the parameter values of the transaction costs. The stochastic pricing scheme is used. The base values of the parameters are $\mu_i = 100, \mu_j = 200, \sigma_i = 20, \sigma_j = 40, \rho = 0, \tau_b = \tau_s = 1.2, \tau_o = 0.4, h = 1.35$. We change one parameter at a time while keeping the other parameters fixed at the base values.

For example, Figure 2.4(f) shows that as τ_o increases from 0.2 to 0.6, the gap decreases about \$947. The gap with respect to τ_s is not analytically clear because an increase of τ_s increases both the common term and $\mathbb{E}_t[IS_{t+1}]$. In our numerical study, we observe that as τ_s increases, the gap mostly decreases. For example, in Figure 2.4(e), as τ_s increases from 1.0 to 1.4, the gap decreases about \$15,594. Notice that in Figures 2.3(e) and 2.4(e), as τ_s increases, the value of inventory sharing in the centralized case increases; whereas the gap decreases. This observation implies that the inventory level difference has greater impact on the gap than the magnitude of the value of inventory sharing in the centralized case.

2.4.4 Transaction Volumes

It is also interesting to examine the volumes of transactions that drive the inventory sharing value. We find in our experiments that the firms use the forward market most frequently to procure the commodity. In particular, in the experiments presented in Table 2.5, the two firms buy about 88% and 12% of the commodity from the forward and spot markets, respectively, to satisfy their demands, and they share about 1% of their total purchased commodity with each other. The volume of selling in the spot market is about 2% of the total purchased commodity.

It is interesting to note three things. First, we do not observe the firms sell their inventory in the forward market in our entire numerical study. An important reason for this is that the coefficient of variation of demand is relatively small in our experiments (which matches the data in practice). A firm will sell in the forward market only if the leftover inventory in one period exceeds the optimal order-up-to level of the next period. Such an

Table 2.5: Transaction quantities (%) (The values of the parameters are $\mu_i = 100, \sigma_i = 20, \mu_j = 200, \sigma_j = 40, \rho = 0, \tau_o = 0.4, h = 1.35$)

(τ_b, τ_s)	Buy From Forward	For-	Buy From Spot	Inventory Sharing	Sell to Forward	For-	Sell to Spot
(1.0, 1.4)	88.28%, 85.26%		11.72%, 14.74%	1.39%, 0.68%	0.00%, 0.00%		1.16%, 0.57%
(1.2, 1.2)	89.25%, 87.39%		10.75%, 12.61%	1.26%, 0.61%	0.00%, 0.00%		2.10%, 1.02%
(1.4, 1.0)	89.98%, 89.65%		10.02%, 10.95%	1.07%, 0.52%	0.00%, 0.00%		3.30%, 1.58%

event will rarely occur, with a small coefficient of variation and relatively high order-up-to levels. Second, we observe that the volume of inventory sharing transactions is small relative to the volume of total purchased commodity. Such a result arises because in general the chance for one firm to have a large inventory shortage and, at the same time, for the other firm to have much leftover inventory is small. Therefore, inventory sharing should be considered as a supporting tool, instead of playing a major role, to balance demand with supply. Third, the value of inventory sharing increases as the sharing quantity increases which is proportional to a firm's demand. The relatively small sharing transaction percentage compared to other procurement options might not result in sufficient benefit for small firms to share inventory with other firms. Hence, we expect that large companies (e.g., major petroleum or food processing) share inventory, but small firms to rarely share inventory unless the sharing transaction cost is negligible (example, borrowing salt from a neighbor).

2.5 Extensions & Discussion

In this section, we discuss several extensions of our main analysis. In particular, we investigate the coordination mechanisms with fixed proportional transfer price and sequential decision making in §2.5.1. We consider the case with positive forward transaction costs in §2.5.2. Finally, we discuss the properties of the value of inventory sharing under a different price model in §2.5.3.

2.5.1 Transfer Prices and Sequence of Decisions

In this subsection, we consider the case in which each firm pays a fraction of the sharing transaction cost and the case where the two firms make decisions as in a Stackelberg competition model.

Fixed Share of Transaction Cost. Let us consider the case in which each firm pays a fraction of the sharing transaction cost. By sharing the cost, the benefits may be distributed more evenly, which can perhaps result in a better outcome. Specifically, we assume that the borrowing firm pays $\tau_o - \tau_k$ to the lending firm where τ_o is the inventory sharing transaction cost. That is, the lending firm will take a charge of τ_k for lending the inventory to the borrowing firm. τ_k can be any value from zero to τ_o . Here, we assume τ_k is fixed in the whole time horizon of T periods.

In §2.3.2, we have explained that the necessary condition for inventory sharing to arise is: $h + \tau_o \leq C_t \leq \tau_b + \tau_s + h$, where $h + \tau_o \leq C_t$ represents the condition under which the borrowing firm is willing to borrow and $C_t \leq \tau_b + \tau_s + h$ is the condition under which the lending firm is willing to lend. Applying the same logic, when the lending firm shares a part of the transaction cost, we can derive the new necessary condition for inventory sharing: $h + \tau_o - \tau_k \leq C_t \leq \tau_b + \tau_s + h - \tau_k$. Notice that with an extra parameter τ_k , we

can adjust the sharing condition. In particular, for specific distributions of the convenience yield, one might find appropriate τ_k that makes inventory sharing more likely occur and thus achieves more benefits for the two firms. We summarize the conditions and benefits under the fixed share of transaction cost and the stochastic transfer price in Table 2.6.

Table 2.6: Summary of the Transfer Prices

	Fixed Share of Transaction Cost	
	$C_t < \tau_b + \tau_s$	$C_t \geq \tau_b + \tau_s$
Transfer Price	$\tau_k \in [h + \tau_o - C_t, \min\{\tau_b + \tau_s + h - C_t, h\}]$	
Convenience Yield Condition	$h + \tau_o - \tau_k \leq C_t \leq (\tau_b + \tau_s + h) - \tau_k$	
Saving to the Borrower	$(C_t - h - \tau_o) + \tau_k$	
Saving to the Lender	$h - \tau_k$	$(\tau_b + \tau_s + h - C_t) - \tau_k$
	Stochastic Transfer Price	
	$C_t < \tau_b + \tau_s$	$C_t \geq \tau_b + \tau_s$
Transfer Price	$-\frac{C_t}{2} + h + \frac{\tau_o}{2}$	$-C_t + h + \frac{\tau_b + \tau_s + \tau_o}{2}$
Convenience Yield Condition	$C_t \geq \tau_o$	
Saving to the Borrower	$\frac{C_t - \tau_o}{2}$	$\frac{\tau_b + \tau_s - \tau_o}{2}$
Saving to the Lender	$\frac{C_t - \tau_o}{2}$	$\frac{\tau_b + \tau_s - \tau_o}{2}$

Sequential Decision Making. So far, we assume that firms make decisions simultaneously without observing the other firm's inventory position as in a Cournot competition model. Here, we consider a Stackelberg type of solution, i.e., firms make decisions one after the other and the follower can observe the inventory position of the leader. Even though the assumption that inventory is visible to one but not the other is somewhat unrealistic, it might provide benefits to the leader and improve the inventory decisions.

When the firms make their decisions sequentially, we assume firm i moves first without loss of generality. Given firm i 's ordering level $x_{i,t+1}$, firm

j 's first order condition is

$$\begin{aligned}
& -\tau_b + G_j(x_{j,t+1})\{(\tau_b + \tau_s) - \mathbb{E}_t[FW_{t+1}]\} - \mathbb{E}_t[L_{t+1}]P_{jL}(x_{i,t+1}, x_{j,t+1}) \\
& + \mathbb{E}_t[B_{t+1}]P_{jB}(x_{i,t+1}, x_{j,t+1}) = 0.
\end{aligned}$$

Then, by backward induction, we can derive firm i 's first order condition (note that $x_{j,t+1}$ is the function of $x_{i,t+1}$):

$$\begin{aligned}
& -\tau_b + G_i(x_{i,t+1})[\beta(\tau_b + \tau_s) - \beta\mathbb{E}_t[FW_{t+1}]] \\
& - \mathbb{E}_t[L_{t+1}] \left(P_{iL}(x_{i,t+1}, x_{j,t+1}) - \frac{\partial x_{j,t+1}}{\partial x_{i,t+1}} P_{jB}(x_{i,t+1}, x_{j,t+1}) \right) \\
& + \beta\mathbb{E}_t[B_{t+1}] \left(P_{iB}(x_{i,t+1}, x_{j,t+1}) - \frac{\partial x_{j,t+1}}{\partial x_{i,t+1}} P_{jL}(x_{i,t+1}, x_{j,t+1}) \right) = 0.
\end{aligned}$$

The existence of an equilibrium can be also shown in this case. Notice that $\mathbb{E}_t[B_{t+1}]$ and $\mathbb{E}_t[L_{t+1}]$, the expected unit cost reductions by borrowing and lending, depend on the transfer price which can be either fixed or stochastic.

Comparing the Value of Inventory Sharing. Now let us investigate the performances of four possible combinations (see Table 2.7) with different sequences of decision making and transfer prices. In our entire numerical study, we observe that the case with simultaneous decision making and stochastic transfer price dominates the other cases. As an example, Table 2.7 shows the gap between the centralized solution and the case with simultaneous decision making and stochastic transfer price is about \$2,900 which is the least among all the combinations. We find that when the two firms move sequentially, the leader can perform better than the follower, but the total gain is less than that when they move simultaneously.

Table 2.7: Gap of the value of inventory sharing (thousands of dollars). In the sequential decision, firm i decides the order-up-to level in advance. The values of the parameters are $\mu_i = 100, \sigma_i = 20, \mu_j = 200, \sigma_j = 40, \rho = 0, \tau_b = \tau_s = 1.2, \tau_o = 0.4, h = 1.35$.

	Fixed Share of Transaction Cost ($\tau_k = 0.2$)	Stochastic Transfer Price
Sequential Decision	\$117.5	\$3.9
Simultaneous Decision	\$114.5	\$2.9

2.5.2 Forward Transaction Costs

In this subsection, we consider the case with positive forward transaction costs. We denote the buying and selling transaction costs in the forward market by τ_{FB} and τ_{FS} , respectively. By adding the forward transaction costs to our model, we redefine the convenience yield concept as $C_t^F \equiv S_t + \tau_b + h - \beta(F_t - \tau_{FS})$. The non-negative convenience yield assumption still holds to prohibit an arbitrage profit.

When we analyze the model with the forward transaction costs, we assume that the leftover inventory level in the current period is always lower than the optimal order-up-to level for the next period as we observe in Table 2.5 in §2.4.4. Proposition 2.5.1 shows the optimal actions with forward transaction costs when firms decide ordering levels simultaneously and the borrowing firm pays all the sharing transaction cost.

Proposition 2.5.1. *Suppose $(x_{it} - D_{it})^+$ is less than $x_{i,t+1}^*$, $\hat{x}_{i,t+1}^*$, and $x_{i,t+1}^C \forall t$, and $h \geq \beta(\tau_{FB} + \tau_{FS})$. In each period t , firms are willing to share inventory if $h + \tau_o + \beta(\tau_{FB} + \tau_{FS}) \leq C_t^F \leq \tau_b + \tau_s + h + \beta(\tau_{FB} + \tau_{FS})$ in the case of decentralized inventory sharing, and if $C_t^F \geq \tau_o$ in the case of centralized inventory sharing. Moreover, firms sell the rest of inventory to the spot market*

if $C_t^F \geq \tau_b + \tau_s$; otherwise, they sell it to the forward market.

From Proposition 2.5.1, we observe that with forward transaction costs, firms' optimal actions continue to depend on the convenience yield, and the convenience yield conditions for sharing inventory and selling it to the spot market are similar to those in the model with zero forward transaction costs. Note that although we assume $h \geq \beta(\tau_{FB} + \tau_{FS})$ which guarantees that the benefit from selling leftover inventory to the forward market is greater than that from holding inventory in Proposition 2.5.1, we can similarly obtain the optimal actions without that assumption.

We also examined the gap of the value of inventory sharing between the centralized and decentralized cases under the four pricing schemes. Table 2.8 shows an example. As in the model with zero forward transaction cost, the case with simultaneous decision making and stochastic transfer price dominates the other three cases.

Table 2.8: Gap of the value of inventory sharing (thousands of dollars). In the sequential decision, firm i decides in advance. The values of the parameters are $\mu_i = 100, \sigma_i = 20, \mu_j = 200, \sigma_j = 40, \rho = 0, \tau_b = \tau_s = 1.2, \tau_{FB} = \tau_{FS} = 0.2, \tau_o = 0.4, h = 1.35$.

	Fixed Transfer Price ($\tau_k = 0.2$)	Stochastic Transfer Price
Sequential Decision	\$1,580.3	\$79.5
Simultaneous Decision	\$1,558.1	\$48.3

2.5.3 Price Model of Gibson and Schwartz (1990)

In this subsection, we adopt the commodity price model developed by Gibson and Schwartz (1990) to approximately assess the properties of the

value of inventory sharing. This price model captures directly the spot price (S_t) and the instantaneous net convenience yield (δ_t). Under the assumption that δ_t measures the price difference between the spot and forward prices, the distribution of convenience yield in period $t + 1$ is the following:

$$(C_{t+1}|S_t, F_t) \sim N(\mu_c, \sigma_c^2),$$

where $\mu_c = (\alpha - \frac{\lambda}{\kappa})(1 - e^{-\kappa\Delta t}) + (S_t - \beta F_t)e^{-\kappa\Delta t} + \tau_b + h$ and $\sigma_c^2 = \frac{\sigma_\delta^2}{2\kappa}(1 - e^{-2\kappa\Delta t})$. Notice that κ , λ , $\alpha - \lambda/\kappa$, σ_δ , and Δt denote the mean-reversion rate, the risk premium, the asymptotic mean, the volatility in the instantaneous net convenience yield factor, and the duration of each time period, respectively². With this model, we can analytically derive two factors that directly affect the value of inventory sharing, i.e., the probability that the convenience yield satisfies the inventory sharing condition and the expected unit cost reduction by sharing inventory as follows. We denote the marginal probability density and cumulative distribution functions of the standard normal distribution by $\phi(\cdot)$ and $\Phi(\cdot)$, respectively.

Lemma 2.5.2. *In the centralized case and the decentralized case with stochastic transfer price, the probability that the convenience yield satisfies the inventory sharing condition during period $t + 1$ given C_t is $1 - \Phi\left(\frac{\tau_o - \mu_c}{\sigma_c}\right)$. The probability increases as the risk premium λ decreases, the buying transaction*

²As discussed in Schwartz and Smith (2000), the price models in Gibson and Schwartz (1990) and Schwartz and Smith (2000) can be converted from one to the other but each has its own advantages and disadvantages in this study. Using the model in Schwartz and Smith (2000), by taking exponential of the sample log spot and forward prices, we can simulate the evolution of the convenience yield (as we have explored in §2.4), but the model is too complex to derive analytical results; whereas, under the model in Gibson and Schwartz (1990), the instantaneous net convenience yield δ_t (which is a good proxy for $S_t - \beta F_t$) is concise to derive analytical results (i.e., Lemmas 2.5.2 and 2.5.3), but the evolution of δ_t is not exactly that of $S_t - \beta F_t$.

cost in the spot market τ_b increases, or, the sharing transaction cost τ_o decreases. The probability is independent of the buying transaction cost in the spot market τ_s and may either increase or decrease as the standard deviation of the convenience yield σ_δ increases.

Lemma 2.5.3. *In the centralized case and the decentralized case with stochastic transfer price, the expected unit cost reduction by sharing inventory during period $t + 1$ given C_t is $(\tau_b + \tau_s - \tau_o) - \sigma_c \cdot \left\{ \phi\left(\frac{\tau_b + \tau_s - \mu_c}{\sigma_c}\right) - \phi\left(\frac{\tau_o - \mu_c}{\sigma_c}\right) \right\} - (\tau_b + \tau_s - \mu_c) \cdot \Phi\left(\frac{\tau_b + \tau_s - \mu_c}{\sigma_c}\right) + (\tau_o - \mu_c) \cdot \Phi\left(\frac{\tau_o - \mu_c}{\sigma_c}\right)$. The expected unit cost reduction increases as the risk premium λ decreases, the buying transaction cost in the spot market τ_b or the buying transaction cost in the spot market τ_s increases, or, the sharing transaction cost τ_o decreases. The expected reduction may either increase or decrease as the standard deviation of the convenience yield σ_δ increases.*

From Lemmas 2.5.2 and 2.5.3, we can obtain the following three insights regarding the value of inventory sharing (Recall that the unit cost reductions by sharing inventory are zero, $C_t - \tau_o$, and $\tau_b + \tau_s - \tau_o$ when $C_t < \tau_o$, $\tau_o \leq C_t < \tau_b + \tau_s$, and $C_t \geq \tau_b + \tau_s$, respectively). First, the value of inventory sharing increases as the risk premium λ decreases because a decrease of λ increases the asymptotic mean of the instantaneous net convenience yield (i.e., $\alpha - \lambda/\kappa$), and the increase of the mean increases both the frequency of the inventory sharing transactions and the unit cost reduction by sharing inventory.

Second, an increase of the standard deviation of the instantaneous net convenience yield σ_δ may either increase or decrease the value of inventory sharing, which depends on the magnitudes of the mean of the convenience yield μ_c , the sum of spot transaction costs $\tau_b + \tau_s$, and the sharing transaction

cost τ_o . If the mean of C_t is larger (smaller) than $\tau_b + \tau_s$ (τ_o), then the increased variance of C_t increases the probability that C_t is less (greater) than $\tau_b + \tau_s$ or τ_o , and it will decrease (increase) the frequency of sharing or the unit cost reduction by sharing inventory. In addition, we expect if the mean of C_t is in-between $\tau_b + \tau_s$ and τ_o , the effect of the increased variance of C_t on the value of inventory sharing is negligible.

Third, changes in the transaction cost of procurement from the spot market and the sharing transaction cost affect the value of inventory sharing similarly as we have observed in §2.4. As τ_b increases, τ_s increases, or τ_o decreases, the value of inventory sharing increases because an increase of τ_b , an increase of τ_s , or a decrease of τ_o increases not only the unit cost savings but also the frequency of the inventory sharing transactions.

2.6 Conclusion

In this study, we explore the benefit of inventory sharing between firms that use a common commodity for their production to satisfy their demands, in the presence of both spot and forward markets. We characterize the optimal inventory and transaction decisions for the cases with centralized and decentralized inventory sharing, and reveal the gap of the values of inventory sharing between the two cases. One might think that the setting of the transfer price would not affect the value of inventory sharing since the borrowing and lending transactions would tend to cancel out in the long-run, especially in the case of inventory sharing between symmetric firms. However, the transfer price affects not only the condition for inventory sharing but also the ordering levels. We show that under the proposed stochastic transfer price, the benefit of inventory sharing in the decentralized case is close to that in the central-

ized case, and the gap can be large under other transfer price schemes. The stochastic transfer price scheme might be useful in practice because 1) it is easy to implement; 2) it shares the benefits fairly between the two firms; and 3) the cost reduction is significant.

We also identify how the benefits from sharing inventory vary with the parameters of the demand process and the transaction costs. The value of inventory sharing, using our proposed pricing scheme, increases as the mean and the variance of demand process increase, the correlation between the demand faced by the two firms decreases, or the sharing transaction cost decreases. The value may either increase or decrease as the transaction costs in the spot market increase.

Therefore, our work extends the existing literature to investigate firms' procurement strategies utilizing both the commodity markets and inventory sharing, which bridges the gap between the literatures on inventory sharing and commodity procurement with stochastic demands and prices. While the findings from our work are interesting and useful for practice, we believe more studies can be conducted along this line of research. For instance, it is interesting to consider scenarios where the demand and price processes are correlated, to consider more than two firms, or to consider the issues of transportation and logistics for commodity inventory sharing among firms.

Chapter 3

Supply Chain Design and Carbon Penalty: Monopoly vs. Monopolistic Competition

3.1 Introduction

Carbon emission has received significant attention in the recent years, which is believed to be one of the global warming contributors. Central planners are passing various regulations on carbon emission. For instance, the European Union has imposed carbon emission limits while allowing the companies to trade their allowances (EC 2005). British Columbia, Canada, imposed a carbon tax (BC 2008), at \$20 per metric ton of CO_2 initially and then increased to \$30 per metric ton from July 2012. While it is difficult to determine the exact cost of carbon emission to the society, the Intergovernmental Panel on Climate Change in 2007 (IPCC 2007) suggested that if we impose \$80 per metric ton of CO_2 to large carbon emitters, then we can prevent severe climate change. Several research also documents that the estimated carbon cost can range from \$20 to \$ 300 per metric ton of CO_2 (Tol 2008, Frank 2012, and John and Hope 2012). In the United States, while there is no regulation in place yet, the Mandatory Reporting of Greenhouse Gases Rule has been issued by the U.S. Environmental Protection Agency, and large greenhouse gas (GHG) emitters need to report GHG data (EPA 2008). Along with the policy change, the public is becoming more receptive to the idea of imposing cost to curb carbon emission, and also more companies started to report the carbon

footprint of their products and services and have been making efforts to reduce their carbon emission. As such, a natural question surfaces whether imposing carbon emission cost will influence the supply chain structure and consumers' purchasing behavior, and if so, how to optimally determine the charge of carbon emission cost over the companies and consumers from a central planner's perspective.

To address this question, we focus on the "last mile" supply chain in this study, i.e., ranging from retail stores to consumers. We follow the main model settings as of those in Cachon (2013). However, instead of minimizing the total costs, we consider the problem of maximizing social welfare with three types of players, i.e., a central policymaker, retailers, and consumers, and also allow the selling price to be determined endogenously. In addition, we consider three competitive settings: monopoly, monopolistic competition with symmetric market share, and monopolistic competition with asymmetric market share.

Retailers and consumers maximize their own profits and net utilities, which however incur negative externalities for the society by generating carbon emissions. Increased carbon emissions can increase the average temperature on Earth, influence the patterns and amounts of precipitation, reduce the ice and snow coverage, and raise the sea levels (EPA 2012). The central policymaker in our study is to maximize the social welfare by balancing the retailers' and consumers' self-interest with the negative externalities. More specifically, the central policymaker decides the carbon emission cost recovery rates to be imposed on the retailers and consumers, and we define the social welfare as the sum of the retailers' total profits and the customers' total net utilities less the portion of carbon cost that is not recovered from the consumers

and retailers. Charging the carbon costs on the retailers' and consumers' transportation activities may change their operations and shopping decisions, thereby influencing the total carbon emission and social welfare.

While our results show that charging carbon costs does not affect much the supply chain structure and the social welfare in the monopoly case (which is aligned with the literature, Cachon 2013), it can however change the supply chain structure and carbon emission significantly when there is competition. In particular, we find that in the case of monopolistic competition with symmetric market share, imposing carbon costs on the retailers' and consumers' transportation activities can substantially change the number of retail stores in the market and the total carbon emission. The negative effect on the social welfare would be large if the carbon emission recovery rates are not properly charged. These results depend only on the industry common factors, i.e., the fuel efficiency and the carbon cost. The case of monopolistic competition with asymmetric market share falls in between the monopoly case and the case of monopolistic competition with symmetric market share. We show that the benefit of properly imposing carbon costs becomes more significant as the competition level increases. Furthermore, we find a good fit of the data from the U.S. retail industry to the case of monopolistic competition with asymmetric market share.

Our study is related to the recent literature that investigate how the new carbon emission regulations influence firms' operational decisions. Benjaafar et al. (2010) study a lot-sizing problem with carbon cap, carbon tax, and carbon cap-and-trade. They show that carbon emission can be reduced not just by investing in energy-efficient technologies but also by adjusting the operational decisions. To support and complement the numerical findings in

Benjaafar et al. (2010), Chen et al. (2011) and Hua et al. (2011) consider economic order quantity (EOQ) models with carbon cap and carbon cap-and-trade constraints, respectively. Chen et al. (2011) show that with the carbon cap regulation, the relative decrease of carbon emission can be much higher than the relative increase of costs, and similarly, Hua et al. (2011) find that the carbon emission decreases as the carbon price increases when the carbon cap-and-trade system is implemented. Differently, Hoen et al. (2010) consider the transportation mode selection decision when carbon emission regulations are imposed. They find that neither carbon tax nor carbon cap-and-trade will change the firms' transportation mode. Similar to us, Cachon (2013) studies the "last mile" supply chain structure with an imposed carbon tax. He shows that the current supply chain structure is robust. We endogenize the retailers' store opening, stocking and pricing decisions and the consumers' shopping decisions with various retail competition. We show that imposing carbon cost can significantly change the supply chain structure and social welfare.

The remainder of this chapter is organized as follows. §3.2 describes the problem and §3.3 analyzes each player's optimal decisions in three competitive settings. The carbon penalty which measures the negative effect by not charging the optimal carbon cost is investigated in §3.4, and the impact of carbon cost on the supply chain structure is presented in §3.5. We compare our results with Cachon (2013) in §3.6 and conclude in §3.7.

3.2 Problem Description

In this section, we explain the problem settings regarding each player, transportation, and competition. Note that, as a default case, we consider carbon costs emitted by retailers' and consumers' transportation activities (we

additionally consider carbon costs emitted by cooling and heating activities in retail stores in §3.6.2).

3.2.1 Players

Central Policymaker. In our problem, retailers distribute products to their retail stores and consumers travel to the retail stores to buy the products. These actions emit carbon which is costly to society. The cost of emission is assumed to be recovered by imposing a fee (i.e., a carbon tax) on the fuel price. The fee is proportional to the amount of carbon emission when retailers and consumers consume fuel. For example, if the carbon cost is \$80 per metric ton of CO_2 and the amount of carbon emission per unit of gasoline is $2.325 \text{ kg}CO_2$ per liter (Cachon 2013), then the gasoline price increase due to the carbon tax is \$0.186 per liter, i.e., \$0.70 per gallon, when the carbon cost is fully recovered. Note that we only consider a carbon tax because the carbon tax is easy to implement (Frank 2012) and it is more effective than any other carbon-related regulations, such as carbon cap and carbon cap-and-trade (Bauman and Hsu 2012).

The role of a central policymaker is to decide the fractions of carbon emission cost recovered from the consumers and retailers. We use α_c and α_r to denote the recovery fractions from the consumers and retailers, respectively. We assume α_c and α_r are nonnegative; that is, no subsidy should be provided. Furthermore, the policymaker can charge at most the full carbon emission cost, i.e., α_c and α_r are bounded by one. The policymaker's problem is to maximize social welfare, $SW(\alpha_c, \alpha_r)$:

$$\max_{\alpha_c, \alpha_r \in [0,1]} SW(\alpha_c, \alpha_r),$$

where $SW(\alpha_c, \alpha_r)$

:= the retailers' total profit + the customers' total utility – carbon cost that is not recovered from the consumers and retailers.

Consumers. The consumers are located evenly in the whole market area. We normalize the consumer density to one. Except for their locations, the consumers are identical. Below, we explain the consumer's problem. Each consumer's demand rate is λ_c . The consumer obtains a utility u_c per unit of consumption. To buy products, consumer i travels in straight lines to the closest store of a retailer j , and chooses the purchasing quantity per trip, $q_c^{j,i}$. Let τ_c be the transportation cost per unit distance, $d_c^{j,i}$ the round-trip distance to the store, and h_c the cost of holding one unit product per unit time. Then, we can formulate consumer i 's problem as one of maximizing her utility, $U_c^{j,i}(q_c^{j,i})$:

$$\max_{q_c^{j,i}} U_c^{j,i}(q_c^{j,i}) \equiv \lambda_c(u_c - p^j) - \frac{\lambda_c \tau_c d_c^{j,i}}{q_c^{j,i}} - \frac{h_c q_c^{j,i}}{2}.$$

Retailers. The entire market area is a which is fixed to a single polygonal region. Let k be the number of retailers. Before establishing a product distribution plan to the retail stores, retailer $j \in \{1, 2, \dots, k\}$ chooses the number of retail stores n^j , serving area $r^j (\leq a)$, and unit selling price p^j given a fraction m^j of the consumers prefer to shop at retailer j 's stores. We assume retailer j 's consumers are uniformly distributed in the entire market area a and retailer j 's retail stores are also uniformly located in retailer j 's serving area r^j .

We follow Cachon (2013)'s store configuration assumption, i.e., the store configuration forms a Voronoi diagram which consists of a single regular polygon. A retail store is located in the center of mass of the regular polygon. We let b be the shortest distance from the the center of mass to the

side of the regular polygon and θ be the smallest angle formed by the shortest line from the center of mass to the side of the regular polygon and the shortest line from the center of mass to the vertex of the regular polygon, i.e., $\theta = \pi/s$, where s is the number of sides in the regular polygon. Then,

$$r^j = \frac{1}{2}(b^j)^2(\tan \theta)(2s)n^j \leq a,$$

where $\frac{1}{2}(b^j)^2(\tan \theta)(2s)$ is the area of the regular polygon. In addition, retailer j 's demand rate, λ_r^j is the following

$$\lambda_r^j = r^j m^j \lambda_c$$

because we normalize the consumer density to one. In other words, the demand rate per unit area is $m^j \lambda_c$ and the total area served is r^j . Retailer j 's selling price p^j is the price that makes the utility of the farthest customer from the nearest store equal to zero, i.e., the utility of the consumer at the vertex of the polygon is zero. That is,

$$p^j = u_c - \frac{\tau_c}{q_c^{j,F}} 2\sqrt{(b^j)^2 + (b^j)^2(\tan \theta)} - \frac{h_c q_c^{j,F}}{2\lambda_c}$$

where $2\sqrt{(b^j)^2 + (b^j)^2(\tan \theta)}$ is the round-trip distance from the center of mass to the to the vertex of the regular polygon and F denotes the farthest consumer. Notice that in §3.3.2.1, we show under reasonable alternative selling price settings, the improved profitability of the retailer is negligible. We assume the procurement cost g_r is the same for all retailers. Therefore, retailer j 's revenue and procurement cost per unit time are $\lambda_r^j p^j$ and $\lambda_r^j g_r$, respectively.

Besides the above decisions, retailer j needs to choose the distribution quantity per trip q_r^j . We follow the same assumptions as those in Cachon

(2013): 1) retailer j has a single warehouse and a single vehicle; 2) the warehouse is co-located with one of the n^j stores and the point where the distribution trip starts and ends; 3) transportation of the products from the supplier to retailer j 's warehouse is not considered; 4) every distribution trip covers all retail stores; 5) the vehicle travels in straight lines and the delivery time is zero; 6) retailer j 's distance of a distribution trip d_r^j is $2b^j n^j$ which is the minimum distance to travel into and out of every regular polygon under the assumption that there are two or more retail stores. Let τ_r be the transportation cost per unit distance and h_r the retailer's cost of holding one unit of product per unit time. Then, we can formulate retailer j 's problem as one of maximizing his profit rate, $Z^j(q_r^j, n^j, r^j, p^j)$:

$$\max_{q_r^j, n^j, r^j} Z^j(q_r^j, n^j, r^j, p^j) \equiv \lambda_r^j(p^j - g_r) - \frac{\lambda_r^j \tau_r d_r^j}{q_r^j} - \frac{h_r q_r^j}{2}$$

subject to

$$\begin{aligned} r^j &= \frac{1}{2}(b^j)^2(\tan \theta)(2s)n^j \leq a \\ \lambda_r^j &= r^j m^j \lambda_c \\ p^j &= u_c - \frac{\tau_c}{q_c^{j,F}} 2\sqrt{(b^j)^2 + (b^j)^2(\tan \theta)} - \frac{h_c q_c^{j,F}}{2\lambda_c} \\ d_r^j &= 2b^j n^j \end{aligned}$$

3.2.2 Transportation Cost

In our study, the transportation cost consists of not only those components considered in Cachon (2013) but also the carbon recovery rate. Let c denote a consumer's vehicle and r denote a retailer's vehicle. The per-unit transportation cost of a vehicle of type $t \in \{c, r\}$ follows:

$$\tau_t(\alpha_t) := v_t + (p_t + \alpha_t c_t e) f_t,$$

where v_t is the non-fuel variable cost per unit of distance (\$ per km); p_t is per unit fuel cost (\$ per $liter(l)$); α_t is the fraction of carbon emission cost recovered; c_t is the amount of carbon emission per unit of fuel ($kgCO_2$ per l); e is the carbon cost per unit of emissions (\$ per $kgCO_2$); and f_t is the amount of fuel necessary to transport the vehicle per unit distance (l per km). For simplicity, we use τ_c and τ_r to denote $\tau_c(\alpha_c)$ and $\tau_r(\alpha_r)$, respectively.

Notice that changing the recovery rates of the carbon emission costs, α_c and α_r , can influence the consumers' and the retailers' decisions. Thus, the central policymaker can alter the recovery rates to maximize social welfare.

3.2.3 Competition

Besides the monopoly case, we consider two kinds of competitive settings: monopolistic competition with symmetric market share and monopolistic competition with asymmetric market share.

3.2.3.1 Monopoly.

In the monopoly setting, there is a single retailer in the market whose stores are uniformly located in his serving area.

3.2.3.2 Monopolistic Competition.

Recall that we assume only a fraction m^j of the consumers prefer to shop at retailer j 's stores and retailer j 's customers are uniformly distributed in the entire market area a . Hence, the retailers serve overlapping regions and might even have their stores co-located with each other. *In the case of monopolistic competition with symmetric market share*, we assume that each retailer has the same market share, and retailers will keep entering the market

until the profit becomes zero. In other words, $m^j = 1/k$ for all j where k is the number of retailers in the market, and the profit to each retailer is zero. For analytical simplicity, we assume that $k \in \mathcal{R}$. *In the case of monopolistic competition with asymmetric market share*, we assume that the number of retailers in the market, k , is a fixed integer and retailers have different market shares. For analytical simplicity, we assume the number of retailers and the market shares are given. We elaborate on the market share model in §3.4 and verify it using the real retail data set, see Table 3.3.

3.2.4 Timeline

Figure 3.1 details the timeline of the model. In the first stage, the central policymaker decides the fractions of carbon emission cost recovered from the consumers and the retailers. In the second stage, the retailers decide their number of stores, selling price, and serving area. In the third stage (the last stage), the consumers decide their purchasing quantity per trip and the retailers decide their delivery quantity per trip. This timeline reflects the sequence of decisions in a greenfield setting. However, given that retailers have already made stage 2 decisions, the timeline makes sense if entry and exit decisions are relatively costless.

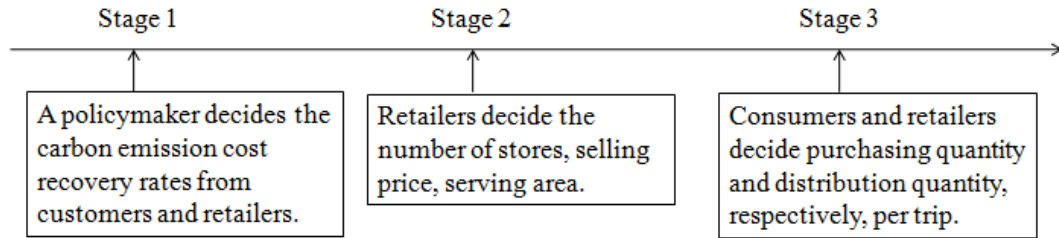


Figure 3.1: The timeline of the model

3.3 Optimal Decisions

We use the superscripts M , S , and A to denote the association with the cases of monopoly, monopolistic competition with symmetric market share, and monopolistic competition with asymmetric market share, respectively. For notational convenience, let $\phi_{c1} := 2^{1/2}s^{-1/4}(\tan \theta)^{-1/4}(1+(\tan \theta)^2)^{1/4}$ and $\phi_r := 2^{1/2}s^{-1/4}(\tan \theta)^{-1/4}$, where s is the number of sides in a regular polygon and θ is π/s . We make the following assumption to ensure participation by a retailer.

Assumption 3.3.1.

$$\tau_c(1)\tau_r(1) \leq \frac{\lambda_c^2(u_c - g_r)^4}{2^6(\phi_{c1}\phi_r)^2(h_ch_r)}.$$

Assumption 3.3.1 will guarantee: (1) the monopolist's profit is always non-negative; (2) the number of retailers is at least one in the model of monopolistic competition with symmetric market share; (3) the number of retailers is at least one, and each retailer's profit is always non-negative in the asymmetric market share case (see the proofs of Propositions 3.3.1, 3.3.2, and 3.3.3).

3.3.1 Third Stage

In this stage, the customers decide their purchasing quantity per shopping trip, $q_c^{j,i}$, to maximize their utility, and the retailers decide their distribution quantity per trip to each store, q_r^j , to maximize their profits. By the EOQ formula (see Cachon and Terwiesch 2009), the optimal purchasing quantity and the associated utility of customer i are:

$$q_c^{j,i,l} = \sqrt{\frac{2\lambda_c\tau_c d_c^{j,i}}{h_c}} \quad \text{and} \quad U_c^{j,i}(q_c^{j,i,l}) = \lambda_c(u_c - p^j) - \sqrt{2\lambda_c\tau_c d_c^{j,i} h_c},$$

where $l \in \{M, S, A\}$. Similarly, retailer j 's optimal delivery quantity and the associated profit are:

$$q_r^{j,l} = \sqrt{\frac{2\lambda_r^j \tau_r d_r^j}{h_r}} \quad \text{and} \quad Z^j(q_r^{j,l}) = \lambda_r^j(p^j - g_r) - \sqrt{2\lambda_r^j \tau_r d_r^j h_r}.$$

3.3.2 Second Stage

Given the fractions of carbon emission cost recovery (α_c, α_r) , retailer j decides the selling price p^j , number of retail stores n^j , and serving market area r^j .

3.3.2.1 Monopoly.

Proposition 3.3.1 gives the monopolist's optimal decisions in the second stage.

Proposition 3.3.1. *In the monopoly case, (i) the whole market area a is served, i.e., $r^M = a$; (ii) the optimal number of retail stores is $n^M = a \left(\frac{\phi_{e1}}{\phi_r}\right)^2 \left(\frac{\tau_c h_c}{\tau_r h_r}\right)$; (iii) the optimal unit selling price is $p^M = u_c - \left(\frac{2\phi_{c1}\phi_r}{\lambda_c}\right)^{1/2} (\tau_c h_c \tau_r h_r)^{1/4}$; (iv) the demand rate is $\lambda_r^M = a\lambda_c$; (v) the distance of a distribution trip is $d_r^M = \phi_r^2 (an^M)^{1/2}$.*

Proposition 3.3.1 shows that the monopolist always serves the whole market area (i.e., $r^M = a$), and the number of retail stores is linearly proportional to the market area. The increased carbon emission cost which is included in τ_c and τ_r decreases the selling price, but may or may not increase the number of retail stores. We remark upon three things on the market coverage result. First, the retailer will cover every consumer or no consumer since both the retailer's revenue and cost are linearly proportional to the number

of retail stores. Second, the whole market area is served. Third, the retailer could alternately decide not to cover every customer *within* a polygon. For example, the coverage area could be a circle within the polygon. In this case, if the area covered is a circle strictly within the polygon, then the polygon can be shrunk until the circle touches the midpoints of each side. Thus, without loss of generality the actual optimal price will lie somewhere between the price to entice the customer at the vertex (our assumption) and the price to entice the customer at the midpoint of a side of the polygon. Extensive numerical investigations show that setting the price in that way (to cover all customers within a circle inscribed in the polygon) does not change the profitability of the retailer by more than 1.3% when the polygon is a hexagon.

3.3.2.2 Monopolistic Competition with Symmetric Market Share.

The competitors enter the market until each retailer's profit becomes zero, and that decides the number of retailers k . We let tk be the total number of retail stores in the market. Proposition 3.3.2 shows the optimal actions and equilibrium in the symmetric case.

Proposition 3.3.2. *In monopolistic competition with symmetric market share, (i) all the market area is served, i.e., $r^{j,S} = a \forall j$; (ii) the optimal number of retailers in equilibrium is $k^S = \frac{\lambda_c^2(u_c - g_r)^4}{2^6(\phi_{c1}\phi_r)^2(\tau_c\tau_r h_c h_r)}$; (iii) the optimal number of retail stores for each retailer is $n^{j,S} = a \frac{2^6 \phi_{c1}^4 (\tau_c h_c)^2}{\lambda_c^2 (u_c - g_r)^4}$; (iv) the total number of retail stores is $tk^S = a \left(\frac{\phi_{c1}}{\phi_r}\right)^2 \left(\frac{\tau_c h_c}{\tau_r h_r}\right)$; (v) the optimal unit selling price is $p^S = \frac{1}{2}(u_c + g_r)$; (vi) the demand rate for each retailer is $\lambda_r^{j,S} = \left(\frac{a}{k}\right) \lambda_c$; (vii) the distance of a distribution trip for each retailer is $d_r^{j,S} = \phi_r^2 (an^{j,S})^{1/2}$.*

As in the monopoly case, the retailers in the symmetric market share case also serve the whole market area (i.e., $r^{j,S} = a \forall j$). As the carbon

emission cost increases, the number of stores per retailer increases; whereas, the number of retailers decreases. Hence, we infer that the increased carbon cost decreases competition. We remark on two things. First, the number of stores per retailer is independent of the retailer's per-unit transportation and holding costs but depends on the per-unit procurement cost. Second, the retail price is independent of the transportation costs, τ_c and τ_r , and thus the price is also independent of the carbon emission cost, e , and the optimal recovery rates, α_c^S and α_r^S . Notice that the number of retailers is greater than or equal to one.

3.3.2.3 Monopolistic Competition with Asymmetric Market Share.

In the asymmetric market share case, the retailers enter the market (sequentially, that is $j = 1, 2, 3, \dots$) until the profit is negative for the retailer that next enters the market. We assume that the number of retailers k^A (which is an integer value) is given and retailer j 's market share m^j is also given for all j . For notational convenience, we assume the market share of retailers k^A , m^{k^A} , is the smallest. Proposition 3.3.3 shows the optimal actions in the asymmetric case.

Proposition 3.3.3. *In monopolistic competition with asymmetric market share, (i) all the market area is served, i.e., $r^{j,A} = a \forall j$; (ii) the optimal number of stores for retailer j is $n^{j,A} = (am^j) \left(\frac{\phi_{c1}}{\phi_r}\right)^2 \left(\frac{\tau_c h_c}{\tau_r h_r}\right)$; (iii) the optimal unit selling price for retailer j is $p^{j,A} = u_c - \left(\frac{2\phi_{c1}\phi_r}{\lambda_c}\right)^{1/2} (\tau_c h_c \tau_r h_r)^{1/4} (m^j)^{-1/4}$; (iv) the demand rate for retailer j is $\lambda_r^{j,A} = (am^j)\lambda_c$; (v) the distance of a distribution trip for retailer j is $d_r^{j,A} = \phi_r^2 (an^{j,A})^{1/2}$.*

From Proposition 3.3.3, if $m^1 = 1$, then the analytic results are the same

as those in the monopoly case; whereas if m^j is the same for all j , then the analytic results are the same as those in the case of monopolistic competition with symmetric market share. Notice that the profit of each retailer is non-negative with the optimal actions if the following inequality holds:

$$\frac{1}{m^{k^A}} \geq \frac{\lambda_c^2(u_c - g_r)^4}{2^6(\phi_{c1}\phi_r)^2(\tau_c\tau_r h_c h_r)},$$

and $\frac{\lambda_c^2(u_c - g_r)^4}{2^6(\phi_{c1}\phi_r)^2(\tau_c\tau_r h_c h_r)}$ equals the number of retailers in equilibrium in the symmetric case, that is, k^S . Recall that k^S is greater than or equal to one. Hence, the above condition implies that the number of retailer is at least one and each retailer's profit is always non-negative because the retailer's profit increases as its market share increases (see the proof of Proposition 3.3.3).

3.3.3 First Stage

The central policymaker decides the fractions of carbon emission cost recovery (α_c, α_r) to maximize social welfare. For notational convenience, let $\phi_{c2} := \left(\frac{4\sqrt{2} \int_0^{\tan\theta} (1+t^2)^{1/4} dt}{\tan\theta} \right) s^{-1/4} (\tan\theta)^{-1/4}$. Proposition 3.3.4 shows the optimal carbon recovery rate to maximize social welfare.

Proposition 3.3.4. *In the monopoly case and the case of monopolistic competition with asymmetric market share, if $(v_r + p_r f_r) > \frac{3\phi_{c1}}{(8\phi_{c2} - 4\phi_{c1})} e f_r c_r$ and $(v_c + p_c f_c) > \frac{3\phi_{c2}}{(8\phi_{c1} - 4\phi_{c2})} e f_c c_c$, then the optimal carbon recovery rate from consumers is less than or equal to one, while that from the retailer(s) is one, i.e., $(\alpha_c^M = \alpha_c^A \leq 1, \alpha_r^M = \alpha_r^A = 1)$. In the case of monopolistic competition with symmetric market share, both the optimal carbon recovery rates from consumers and retailers are one, i.e., $\alpha_c^S = \alpha_r^S = 1$.*

The optimal carbon recovery rate from retailers is one for each case. In the monopoly case and the case of monopolistic competition with asym-

metric market share, by fully charging the carbon emission recovery rates to consumers (i.e., $\alpha_c = 1$), carbon emissions might decrease and thus social welfare increases. However, by charging only a fraction of carbon emission recovery rates to consumers, the consumer's transportation cost and the number of retail stores decrease. This makes social welfare increase because of less frequent travel by customers and less transportation by retailers. Notice that the carbon recovery rates in the monopoly and the asymmetric cases are the same because, in the asymmetric case, retailers act as if they are monopolists since the profits are positive with the fixed fraction of market share. In the numerical experiments replicating realistic parameters, more often than not the central policymaker stops short of the boundary value of one and decides to partially recover the carbon cost for maximizing welfare. Differently, in the case of monopolistic competition with symmetric market share, consumers pay fully when they emit carbon. The market is competitive, so the policymaker does not need to charge only a fraction of carbon emission cost to customers any more. Analytically, the consumers' total utility is independent of the transportation costs, τ_c and τ_r , because the consumers' total holding and transportation costs (i.e., sum of $\sqrt{2\lambda_c\tau_c d_c^{j,i}} h_c$ for all j and i) as well as the optimal selling price p^S are independent of τ_c and τ_r . Hence, with each retailer's zero profit, social welfare can be maximized by reducing the carbon cost as much as possible, and it is achieved by increasing the carbon recovery rate up to the upper bound, i.e., $\alpha_c^S = 1$. Note that the conditions in the Proposition 3.3.4, i.e., $(v_r + p_r f_r) > \frac{3\phi_{c1}}{(8\phi_{c2} - 4\phi_{c1})} e f_r c_r$ and $(v_c + p_c f_c) > \frac{3\phi_{c2}}{(8\phi_{c1} - 4\phi_{c2})} e f_c c_c$, are sufficient conditions that social welfare is jointly concave in α_c and α_r in the monopoly and asymmetric cases. We note that these conditions hold when the carbon cost is between \$0 and \$0.6 per metric ton of $kgCO_2$ (i.e., between \$0 and \$600 per metric ton of CO_2) under our parameter values (see Table 3.1

in §3.4). In our numerical study, even if the carbon cost is greater than \$0.6 per metric ton of $kgCO_2$, the uniqueness of the solution is still obtained.

3.4 Carbon Penalty

In this section, we simulate how social welfare and carbon emissions change by imposing the carbon cost. To examine the changes, we use the carbon penalty concept. The carbon penalty is the decrease in social welfare or increase in carbon emissions that occurs due to not fully charging the true cost of emissions. In other words, the penalty measures the negative effect by not charging the optimal carbon cost. We define the two following carbon penalties:

$$\text{Social Welfare Penalty (SWP)} := \{SW(\alpha_c^l, \alpha_r^l) - SW(0, 0)\} / SW(\alpha_c^l, \alpha_r^l)$$

$$\text{Total Emission Penalty (TEP)} := \{TE(0, 0) - TE(\alpha_c^l, \alpha_r^l)\} / TE(\alpha_c^l, \alpha_r^l)$$

where $l \in \{M, S, A\}$ and $TE(\alpha_c, \alpha_r)$ denotes the amount of total carbon emissions when the carbon recovery rates from consumers and retailers are α_c and α_r , respectively.

For the parameters that are defined in Cachon (2013), we use the same values as those in Cachon (2013). We set the additional parameter values for our numerical study when the values are not set in Cachon (2013), i.e., a , u_c , λ_c , g_r , h_c , and h_r . Table 3.1 shows the parameter values that are used in our numerical study. We remark on four things: First, according to Census Bureau (2008), the average annual food expenditure by a household in 2008 was \$6,443, and adjusting for inflation of 2.5%, the average weekly food expenditure by household, i.e., demand rate per customer in 2012, is estimated as \$136.77 per week, i.e., $\lambda_c = 136.77$. Second, although we let the market

size be 300, the penalties are constant regardless of the market size because social welfare and emissions are linearly proportional to the market size. That is, when we calculate the penalties, the market size is canceled out. Third, it is hard to estimate parameter values for the following four factors: 1) gross utility per unit of consumption; 2) unit procurement cost to retailers; 3) unit holding costs to retailers; 4) unit holding costs to consumers. As a default, we choose those parameter values as shown in Table 3.1. However, even if we use other parameter values for the four factors, the main result remains the same. Fourth, we assume the Voronoi diagram generated by the retail stores consists of hexagons, i.e., $s = 6$. Note that even if we change s to $s = 3$ (triangle) or $s = 4$ (square), the main results remain the same.

Table 3.1: Parameters

Parameter	Description (unit)	Estimate
a	market area	300
u_c	consumer utility per unit of consumption (\$ per unit)	0.3
λ_c	consumer's demand rate (\$ per week)	136.77
g_r	retailer's per unit procurement cost (\$ per unit)	0.25
(h_c, h_r)	unit holding cost (\$ per unit per week)	(0.04, 0.06)
(v_c, v_r)	non-fuel variable cost per unit of distance (\$ per km)	(0.0804, 0.484)
(f_c, f_r)	the amount of fuel consumption per unit of distance (l per km)	(0.111, 0.392)
(p_c, p_r)	per unit of fuel cost (\$ per l)	(0.98, 1.05)
(c_c, c_r)	the amount of carbon emission per unit of fuel ($kgCO_2$ per l)	(2.325, 2.669)

For the numerical study in the case of monopolistic competition with asymmetric market share, we assume that the market share of the j^{th} largest retailer equals

$$m^j := \left(\frac{(1 - \gamma)^{j-1}}{k} \right) (1 - (j - k)\gamma),$$

where $\gamma \in (0, 1)$ denotes the degree of market share asymmetry. This special form allows us to vary the analysis from the monopoly case to the case of mo-

nopolistic competition with symmetric market share by changing γ . When γ goes to zero, the model converges to the symmetric market share case; whereas, when γ goes to one, the model converges to the monopoly case. Table 3.2 shows a few examples of asymmetric market shares with different k and γ . For example, if $k = 2$ and $\gamma = 0.1$, then the larger retailer's market share is 0.55, and the smaller retailer's market share is 0.45.

Table 3.2: Asymmetric Market Share

k	$\gamma = 0.1$			$\gamma = 0.5$			$\gamma = 0.9$		
	2	3	4	2	3	4	2	3	4
m^1	0.55	0.40	0.33	0.75	0.67	0.63	0.95	0.93	0.93
m^2	0.45	0.33	0.27	0.25	0.25	0.25	0.05	0.06	0.07
m^3	N/A	0.27	0.22	N/A	0.08	0.09	N/A	0.00	0.00
m^4	N/A	N/A	0.18	N/A	N/A	0.03	N/A	N/A	0.00

We validate whether the m^j function is appropriate to reflect the observed data. Table 3.3 shows the market shares of the U.S. major supermarket chains and the estimated market shares of those chains using the m^j function in each market area. γ is the value that minimizes sum of squared errors. For example, in the Pacific Coast area, the market share of Walmart is 20%, and the estimated market share of Walmart is 17% with $\gamma = 0.079$. The m^j function works well in most market areas. Notice that we assume Costco, Kroger, Safeway, Target, and Walmart represent the U.S. supermarket chain industry. We calculate the sales of each supermarket chain in each area by multiplying the number of retail stores in each area by the sales per retail store. The number of retail stores in each area is found in each firm's annual report or website. The sales per retail store is estimated by dividing total sales by the total number of retail stores of each retailer. In addition, we divide the

U.S. market into ten areas, and each area has similar geographic features¹ (AL 2013).

Note that, in the monopolistic competition case, consumers' choice among retailers can be explained by the choice model which is widely used to explain consumer choice behavior among products, e.g., the multinomial logit model (Honhon et al. 2010). If the choice probabilities are equal, then we obtain the case of monopolistic competition with symmetric market share. Otherwise, any distribution is possible to explain consumer choice behavior. In our study, we use the m^j function to explain the asymmetric market share driven by the unequal probabilities in consumer choice. By changing γ , we smoothly capture consumer choice behavior from the symmetric case to the monopoly case, and the function fits well with real data as we observe in Table 3.3.

We simulate how social welfare and total carbon emission change by imposing the carbon cost. Figures 3.2(a), 3.2(b), and 3.2(c) show the changes in the social welfare penalty, total emission penalty, and difference of the numbers of retailers (i.e., number of retailers without charging carbon cost minus that with charging optimal carbon cost) as the carbon price increases from zero to \$1,000 per metric ton of CO_2 . "Monopoly", "Symmetric", and "gamma"

¹We divide the U.S. market into ten areas, and each area has similar geographic features (see www.learner.org). In detail, Pacific Coast area includes California, Oregon, Washington, and Alaska; Mountain area includes Nevada, Utah, Colorado, Wyoming, Idaho, and Montana; Southwest area includes Arizona, New Mexico, Texas, and Oklahoma; Heartland area includes Minnesota, North Dakota, South Dakota, Nebraska, Kansas, Iowa, and Missouri; Southeast area includes Arkansas, Louisiana, Alabama, Mississippi, Georgia, South Carolina, and Florida; Midwest area includes Michigan, Wisconsin, Illinois, Indiana, and Ohio; Appalachian area includes Kentucky, Virginia, West Virginia, North Carolina, and Tennessee; Mid-Atlantic area includes New York, New Jersey, Pennsylvania, Maryland, and Delaware; New England area includes Massachusetts, New Hampshire, Maine, Vermont, Rhode Island, and Connecticut.

Table 3.3: Asymmetric Market Share Example in Supermarket Chain Industry

	Costco	Kroger	Safeway	Target	Walmart
Pacific Coast (AK, CA, OR, WA)					
Market Share	26%	20%	21%	12%	20%
$m^j(\gamma = 0.079, k = 5)$	26%	20%	23%	14%	17%
Mountain					
Market Share	16%	25%	11%	9%	38%
$m^j(\gamma = 0.196, k = 5)$	18%	26%	12%	8%	36%
Southwest					
Market Share	8%	16%	8%	11%	58%
$m^j(\gamma = 0.275, k = 5)$	10%	26%	6%	16%	42%
Heartland					
Market Share	6%	9%	1%	18%	66%
$m^j(\gamma = 0.545, k = 5)$	3%	9%	1%	24%	64%
Southeast					
Market Share	7%	12%	N/A	12%	69%
$m^j(\gamma = 0.386, k = 4)$	6%	27%	N/A	13%	54%
Midwest					
Market Share	8%	24%	2%	14%	52%
$m^j(\gamma = 0.390, k = 5)$	6%	26%	3%	13%	51%
Appalachian					
Market Share	7%	22%	2%	11%	59%
$m^j(\gamma = 0.429, k = 5)$	5%	26%	2%	12%	54%
Mid-Atlantic					
Market Share	17%	N/A	7%	19%	57%
$m^j(\gamma = 0.339, k = 4)$	15%	N/A	7%	28%	50%
New England					
Market Share	14%	N/A	N/A	20%	67%
$m^j(\gamma = 0.399, k = 3)$	12%	N/A	N/A	28%	60%
Hawaii					
Market Share	47%	N/A	21%	5%	27%
$m^j(\gamma = 0.359, k = 4)$	52%	N/A	14%	7%	28%

represent the cases of monopoly, monopolistic competition with symmetric market share, and monopolistic competition with asymmetric market share, respectively. Notice that, in the numerical study, we assume that the number of retailer in the symmetric case is an integer as that in the asymmetric case. Table 3.4 shows the detailed numbers in the cases of monopoly, symmetric market share, and asymmetric market share. In the asymmetric case, we report the penalties when γ equals 0.1, 0.3, and 0.5 because the actual γ seems to range from 0.1 to 0.5 as we observe in Table 3.3. Note that, the monopolist's profit is less than zero when the carbon price is over about \$1,160 per metric ton of CO_2 , and, in the case of monopolistic competition with symmetric market share, the number of retailers is one when the carbon price is over \$985 per metric ton of CO_2 .

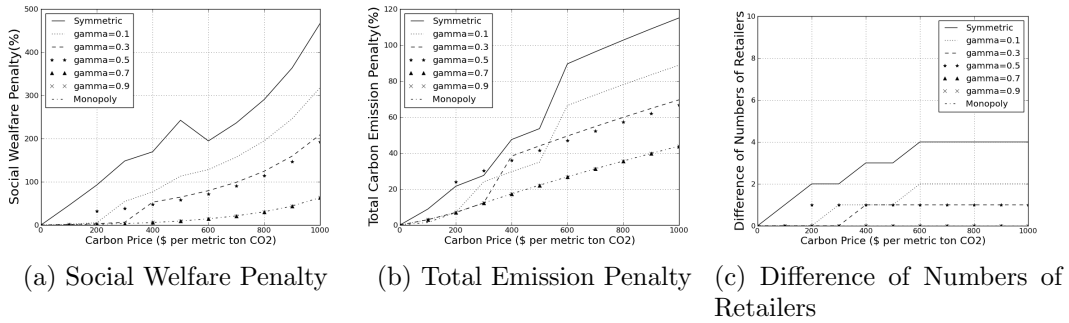


Figure 3.2: Social Welfare Penalty, Total Emission Penalty, and Difference of Numbers of Retailers

Under monopoly, when the carbon price is very high (i.e., over \$600 per metric ton of CO_2), then the social welfare penalty is significant (i.e., over 10%). However, under the reasonable range of the carbon price, i.e., between \$20 and \$300 per metric ton of CO_2 , (see, IPCC 2007, Tol 2008, Frank 2012,

Table 3.4: Social Welfare Penalty (SWP), Total Emission Penalty (TEP), and Difference of Numbers of Retailers (DNR)

Carbon Price (metric ton of CO_2)	Monopoly (SWP, TEP, DNR)	$\gamma = 0.5$ (SWP, TEP, DNR)	$\gamma = 0.3$ (SWP, TEP, DNR)
\$100	(0.7%, 1.4%, 0)	(1.1%, 3.0%, 0)	(1.1%, 3.0%, 0)
\$300	(1.6%, 7.0%, 0)	(32.4%, 24.0%, 1)	(2.9%, 6.96%, 0)
\$500	(3.2%, 12.2%, 0)	(39.1%, 30.1%, 1)	(6.3%, 12.2%, 0)
\$1,000	(9.3%, 22.1%, 0)	(58.4%, 41.6%, 1)	(64.4%, 44.0%, 1)
\$1,000	(63.3%, 43.8%, 0)	(193.1%, 66.7%, 1)	(208.9%, 69.5%, 1)
Carbon Price (metric ton of CO_2)	$\gamma = 0.1$ (SWP, TEP, DNR)	Symmetric (SWP, TEP, DNR)	
\$100	(1.8%, 3.0%, 0)	(45.4%, 8.87%, 1)	\$200
\$200	(5.3%, 7.0%, 0)	(92.6%, 21.5%, 2)	
\$300	(54.6%, 24.0%, 1)	(148.2%, 27.5%, 2)	
\$500	(113.1%, 34.9%, 1)	(242.4%, 53.5%, 3)	
\$1,000	(317.9%, 88.8%, 2)	(466.3%, 115.0%, 4)	

and John and Hope 2012), the social welfare penalty is negligible (i.e., less than 3.2%) although the total emissions penalty is significant when the carbon price is \$300 per metric ton of CO_2 (i.e., 12.2%). Notice that the purpose of charging carbon cost is to increase social welfare. Hence, although charging carbon costs decreases carbon emissions, the central policymaker does not need to impose carbon costs. In contrast, in the case of monopolistic competition with symmetric market share, the two penalties are significant under the reasonable range of the carbon price, e.g., the social welfare and total carbon emission penalties are about 148.2% and 27.5%, respectively when the carbon price is \$300 per metric ton of CO_2 . Notice that the social welfare penalty could be greater than 100% if social welfare is less than zero because of the carbon emission cost which is not recovered. In the case of monopolistic competition with asymmetric market share, the penalties are in-between the cases

of monopoly and monopolistic competition with asymmetric market share. The penalties in the asymmetric market share case are affected by the two factors, i.e., as either the difference of the numbers of retailers increases or γ decreases, the penalties become significant. For example, when the difference of the number of retailers is zero, the social welfare penalties are less than about 5%; whereas when the difference of the number of retailers is greater than or equal to one, the social welfare penalties are more than about 30% when carbon price exceeds \$300. In addition, when the carbon price is \$500 per metric ton of CO_2 , the social welfare penalty increases as γ decreases although the difference of the number of retailers are the same. Notice that the total emission penalty also shows the similar pattern. Overall, by imposing the optimal carbon recover rates, we can not only increase social welfare but also reduce carbon emissions when market competition is high.

One may suspect that the significant carbon penalty in the symmetric case might come from our industry specific parameter values, e.g., demand rate or holding cost. Proposition 3.4.1, however, rules out such possibilities.

Proposition 3.4.1. *In monopolistic competition with symmetric market share, (i) the social welfare penalty is $\frac{e}{2(1-\frac{\phi_{c2}}{\phi_{c1}})} \left(\frac{\phi_{c2}}{\phi_{c1}} f_c c_c \tau_c(0)^{-1} + f_r c_r \tau_r(0)^{-1} \right)$; (ii) the total emission penalty is $\frac{\left(\frac{\phi_{c2}}{\phi_{c1}} f_c c_c \tau_c(0)^{-1} + f_r c_r \tau_r(0)^{-1} \right) - \left(\frac{\phi_{c2}}{\phi_{c1}} f_c c_c \tau_c(1)^{-1} + f_r c_r \tau_r(1)^{-1} \right)}{\frac{\phi_{c2}}{\phi_{c1}} f_c c_c \tau_c(1)^{-1} + f_r c_r \tau_r(1)^{-1}}$. Hence, the social welfare and total emission penalties are independent of industry specific parameter values, i.e., a , u_c , λ_c , g_r , h_c , and h_r .*

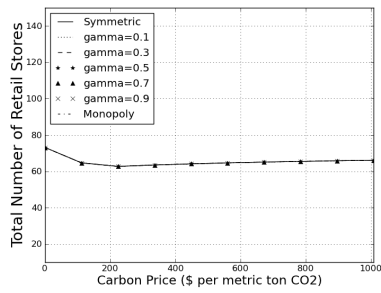
Proposition 3.4.1 shows that the social welfare and total emissions penalties in the symmetric case are independent of industry specific parameter values. In other words, the two penalties only depend on industry common factors, e.g., fuel and carbon prices. Notice that in the monopoly and

asymmetric market share cases, the two penalties are also independent of the market size a because social welfare and emissions are linearly proportional to the market size. That is, when we calculate the penalties, the market size is canceled out. However, the penalties change with respect to other industry specific parameter values, such as, u_c , λ_c , g_r , h_c , and h_r . In our numerical study, the two penalties become more significant as either the retailer's profit or the consumer's utility decreases, i.e., u_c and λ_c decrease, and g_r , h_c and h_r increase. That is, as market competition becomes more intense, the carbon penalties become more significant. Notice that in Proposition 3.4.1, we assume the number of retailers is a real number. The values of the carbon penalties when the number of retailers is a real number are almost the same as those when the number of retailers is an integer.

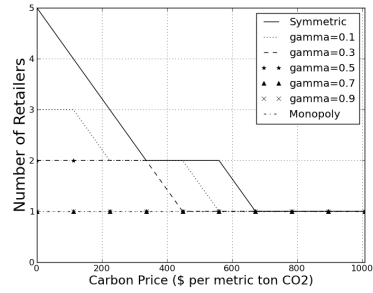
3.5 Supply Chain Design and Carbon Cost

In this section, we discuss the impact of carbon cost on supply chain design, such as total number of retail stores, number of retailers, total carbon emissions, and social welfare. We assume that the optimal carbon recovery rates are always imposed upon consumers and retailers. We vary the carbon emission price from zero to \$1,000 per metric ton of CO_2 . For other parameter values, we use the values that are used in §3.4 (see Table 3.1). We also use the m^j function which is defined in §3.4 to represent the market share of each firm in the case of monopolistic competition with asymmetric market share.

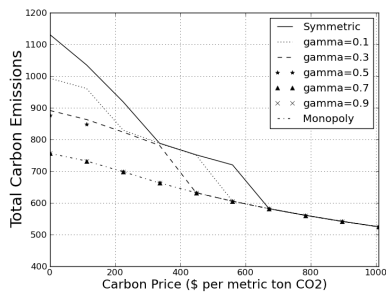
In the monopoly case (see Figure 3.3), the increased carbon cost increases the number of retail stores, but the changes are not significant although carbon emissions and social welfare decrease significantly. Hence, the supply chain design in monopoly is robust even if the cost of carbon emission is high.



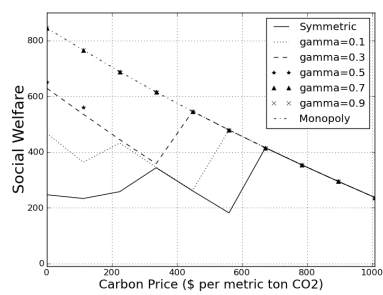
(a) Total Number of Retail Stores



(b) Number of Retailers



(c) Total Carbon Emissions



(d) Social Welfare

Figure 3.3: Supply Chain Design and Carbon Cost

Note that as the carbon price increases, the number of retail stores decreases and then increases. This happens due to the restriction that recovery rates are between zero and one. If there is no constraint on the rates, then the optimal carbon cost recovery rate imposed over customers would become negative when the carbon cost is very low, and the number of retail stores would increase as the carbon cost increases.

In the case of monopolistic competition with symmetric market share, as the carbon price increases, the number of retailers and total carbon emissions decrease significantly. Hence, as the carbon price increases, the supply chain design changes significantly, i.e., it results in decreased number of competitors, and the reduction of carbon emission is also significant by imposing the optimal carbon recovery rates. Note that the total number of retail store in the symmetric case is the same as that in the monopoly case (see Propositions 3.3.1 and 3.3.2).

In the case of monopolistic competition with the asymmetric market share, when γ is high, the changes to the supply chain are similar to those in the monopoly case; whereas, when γ is low, the changes to the supply chain are similar to those in the case of monopolistic competition with symmetric market share.

We remark upon two things. First, social welfare in the monopoly case is the greatest and that in the case of monopolistic competition with symmetric competition is the least. This is because, as the market changes from monopolistic competition with symmetric competition to monopoly, the retailer profit increases more than the decrease in consumer utility. Second, total carbon emission is the least in the monopoly case and the greatest in the symmetric case. This comes from our assumption that the retailer's cus-

tomers are uniformly distributed in the entire market area and the distribution system of each retailer is independent. Hence, the monopolist's distribution system is the most efficient and the emissions in the monopoly case is less than that in other cases, while the symmetric retailers' is the least efficient and the emissions in the symmetric case is more than that in other cases. Notice that the U.S. nationwide retailers, e.g., Kroger and Walmart, have their own distribution systems (Kroger 2010 and Walmart 2011). Although the assumption that each retailer has its own distribution system is reasonable, we explain the effect of relaxing the assumption by using a third-party logistics provider in §3.6.1.2.

3.6 Comparison with Cachon (2013)

We discuss why our results are different from Cachon (2013) in §3.6.1. Then, we consider retail space costs in §3.6.2. The main different result from Cachon (2013) is that imposing carbon cost to consumers and retailers could make significant changes in the supply chain design. In the monopoly case, the changes in the number of retail stores and in social welfare are not significant although total carbon emissions can be reduced. Since the purpose of charging carbon costs is to increase social welfare, the central policymaker does not need to impose carbon costs and thus, charging carbon costs is pointless from a social welfare perspective which is the same result as in Cachon (2013). However, in the monopolistic competition case, as the carbon cost increases, the changes in social welfare, carbon emissions, and the supply chain design are significant. Therefore, we expect that by imposing carbon cost, the supply chain design will change, and the impact of the change becomes more significant when the market is more competitive.

3.6.1 Comparing the Two Penalties: Social Welfare and Total Costs Penalties

In this subsection, we explain why the two problems (i.e., maximizing social welfare and minimizing total costs) yield different results by comparing two penalties (i.e., social welfare penalty for this study and total costs penalty for Cachon 2013). We recall and define the two penalties as follows:

$$\text{Social Welfare Penalty} := \{SW(\alpha_c^l, \alpha_r^l) - SW(0, 0)\} / SW(\alpha_c^l, \alpha_r^l)$$

$$\text{Total Costs Penalty} := \{TC(0, 0) - TC(\alpha_c^l, \alpha_r^l)\} / TC(\alpha_c^l, \alpha_r^l)$$

where $l \in \{M, S, A\}$ and $TC(\alpha_c, \alpha_r)$ denotes the total costs when the carbon recovery rates from consumers and retailers are α_c and α_r , respectively.

3.6.1.1 Different Denominators and Competition.

The two main reasons of the difference are different denominators in the two penalties and considering competition. Recall that social welfare is “retailers’ total profit + customers’ net utility – carbon cost that is not recovered from the consumers and retailers”. In addition, retailers’ total profit is “revenue – procurement cost – distribution cost”, and customers’ net utility is “gross utility – procurement cost – travel cost”. Since retailer’s revenue is the same as customers’ procurement cost and every customer is always covered by retailers, retailers’ procurement cost and consumers’ gross utility are constant. Hence, social welfare can be re-written as “social welfare = constant term – cost term” where “constant term” equals to “customers’ gross utility – retailers’ procurement cost” and “cost term” equals to “retailers’ distribution cost + consumers’ travel cost + carbon cost that is not recovered from the consumers and retailers”. Therefore, the numerator in the social welfare penalty (i.e., social welfare decrease by not charging optimal carbon cost) is

the same as the numerator in the total costs penalty (i.e., total costs increase by not charging optimal carbon cost).

However, the denominators in the penalties (i.e., social welfare and total costs with optimal carbon recovery rates) are different. If a denominator in one penalty is less than that in the other penalty, then the penalty is more significant than the other since the numerators in the two penalties are the same. In the case of monopolistic competition with symmetric market share, we can analytically show that social welfare is always less than the total costs and the difference increases as the number of retailers increases. In the case of symmetric market share with positive profit, we can show that as the number of retailers increases, social welfare decreases and the total costs increases. Hence, we can infer that as competition increases, the total costs will eventually become higher than social welfare, and the difference will also increase. Moreover, as the carbon costs increase, the total costs will go up but social welfare will decrease in any case. Therefore, the social welfare penalty is high under competition.

Notice that when we explain the total costs above, the retailer's procurement cost is included not in the "cost term" but in the "constant term" since Cachon (2013) does not consider the retailer's procurement cost and our goal in this subsection is comparing the two studies. If we add the retailer's procurement cost to the "cost term", then the total costs penalty should be much lower.

3.6.1.2 Independent Distribution Systems.

In §3.6.1.1, we explain that the social welfare penalty becomes significant as the number of retailers increases since the denominator of the penalty

(i.e., social welfare) decreases as the number of retailer increases. Social welfare in the monopolistic competition case is less than that in the monopoly case because we assume that each retailer owns and operates its own distribution system to support retail stores which are located all over the market area. Hence, the retailer's supply chain system in the competition case becomes less efficient than that in the monopoly case. The above assumption reflects the practice in the real world, e.g., major supermarket chains (at least the top five chains) are located all over the U.S. (see Table 3.3) and they have their own distribution systems (from the supermarket chains' annual reports). We checked how the carbon penalty changes if retailers use a third-party logistics (3PL) company to distribute their products. We can show that if the retailers are symmetric and use a common 3PL company to distribute their products, then the total number of retail stores and the selling price become the same as those in the monopoly case. That is, the retailers act as if they are monopolists. Hence, the carbon penalty becomes the same as the monopoly case even if there are multiple retailers. This result implies that using the 3PL company's distribution system could be an alternative option to reduce carbon emissions rather than imposing carbon costs although it might be hard to implement.

In summary, the social welfare penalty is more significant than the total costs penalty due to the different denominators in the two penalties under the two assumptions that competition is high and each retailer has its own distribution system to support retail stores which are located all over the market area. Since the two assumptions fit the current retail industry situations well, if we impose the carbon price, then we can not only increase social welfare but also decrease carbon emissions.

3.6.2 Extension: Retail Space Costs

Now we additionally consider retail space costs. Similar to the transportation costs, the retail space costs consists of not only those considered in Cachon (2013) but also the carbon recovery rate. Notice that, in Cachon (2013), the unit of the space cost is \$ per m^2 per period, but since in our study, we use the concept of the unit space cost per period, the unit is changed to \$ per period.

The per-unit retail space cost follows:

$$v_s + (p_s + \alpha_s c_s e) f_s,$$

where v_s is the variable cost of the per-unit retail space cost per period (\$ per period); p_s is per unit energy cost (\$ per unit energy); α_s is the fraction of carbon emission cost recovered; c_s is the amount of carbon emission per unit energy usage ($kgCO_2$ per unit energy); e is the carbon cost per unit of emissions (\$ per $kgCO_2$); and f_s is the amount of energy necessary to hold a unit of product per period (energy usage per period). Note that both the retail space and holding costs are proportional to the inventory levels. Hence, for notational convenience, we use $\hat{h}_r(\alpha_s)$ to denote the retail space and holding costs, i.e., $\hat{h}_r(\alpha_s) := (h_r + v_s) + (p_s + \alpha_s c_s e) f_s$. Similar to the transportation costs, we use \hat{h}_r to denote $\hat{h}_r(\alpha_s)$ for simplicity.

With the retail space costs, the policymaker's problem is to maximize social welfare, $SW(\alpha_c, \alpha_r, \alpha_s)$:

$$\max_{\alpha_c, \alpha_r, \alpha_s \in [0,1]} SW(\alpha_c, \alpha_r, \alpha_s).$$

Consumer i 's problem is the same as before. That is, consumer i 's problem is as follows:

$$\max_{q_c^{j,i}} U_c^{j,i}(q_c^{j,i}) \equiv \lambda_c(u_c - p^j) - \frac{\lambda_c \tau_c d_c^{j,i}}{q_c^{j,i}} - \frac{h_c q_c^{j,i}}{2}.$$

Since the retail space and holding cost \hat{h}_r is linearly proportionally to inventory level, we formulate retailer j 's problem as one of maximizing his profit rate, $Z^j(q_r^j, n^j, r^j, p^j)$:

$$\max_{q_r^j, n^j, r^j} Z^j(q_r^j, n^j, r^j, p^j) \equiv \lambda_r^j(p^j - g_r) - \frac{\lambda_r^j \tau_r d_r^j}{q_r^j} - \frac{\hat{h}_r q_r^j}{2}$$

subject to

$$\begin{aligned} r^j &= \frac{1}{2}(b^j)^2(\tan \theta)(2s)n^j \leq a \\ \lambda_r^j &= r^j m^j \lambda_c \\ p^j &= u_c - \frac{\tau_c}{q_c^{j,F}} 2\sqrt{(b^j)^2 + (b^j)^2(\tan \theta)} - \frac{h_c q_c^{j,F}}{2\lambda_c} \\ d_r^j &= 2b^j n^j \end{aligned}$$

We examine how social welfare and carbon emission change by considering retail space costs. By substituting \hat{h}_r for h_r in §3.3, the optimal decisions in the second and third stages with retail space costs can be derived. We set that the carbon recovery rates equal to one for all cases. Notice that we find that there exists a unique solution which satisfies the first order conditions without considering the bounds of the carbon recovery rates (between zero and one), and the solutions are $\alpha_c = 1$, $\alpha_r > 1$, and $\alpha_s > 1$ in the cases of monopoly and monopolistic competition with asymmetric market share. We use the same parameter values as those in Table 3.1. For \hat{h}_r , we follow Cachon (2012) by assuming 100 units can be stored per m^2 , the variable cost is \$1.84/100/52 per week per unit; the energy consumption cost is \$0.197/100/52 per week per unit; the amount of carbon emission is 1.087/100/52 $kgCO_2$ per week per unit. For example, if the carbon price is \$200 per metric ton of CO_2 and the carbon recovery rate is one, then, the retail space cost is \$0.0004 per

week per unit, and \hat{h}_r , i.e., the sum of unit holding cost and space cost to retailers, is \$0.0404 per week per unit .

Figures 3.4(a), 3.4(b), and 3.4(c) show the changes in the social welfare penalty, the total carbon emission penalty, and the difference of numbers of retailers, respectively, as the carbon price increases from zero to \$500 per metric ton of CO_2 (see Table 3.5 for the detailed numbers). Note that when the carbon price is over about \$585 per metric ton of CO_2 , the monopolist's profit is less than zero and the number of retailer is one in the case of monopolistic competition with symmetric market share. Similar to Cachon (2013), the penalties tend to become more significant by considering retail space costs especially in the asymmetric case. For example, when γ equals to 0.1 and the carbon price is \$300 per metric ton of CO_2 , by considering retail space costs, the social welfare and total emissions penalties increase by 15.1% and 10.4%, respectively although the difference of numbers of retailers are the same (see Tables 3.4 and 3.5).

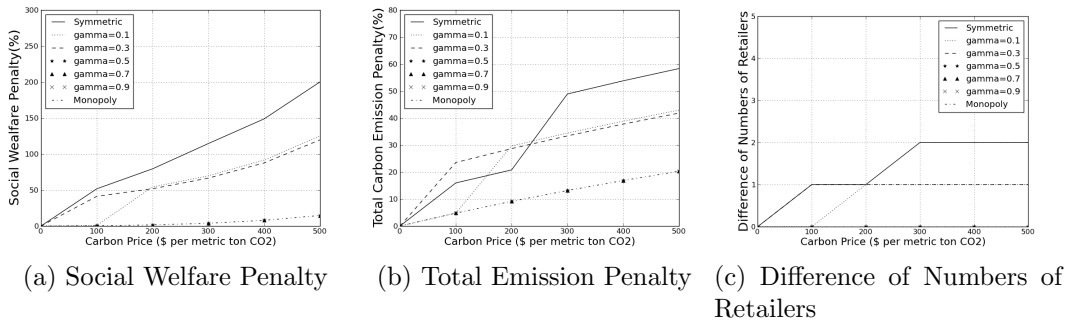


Figure 3.4: Social Welfare, Carbon Emissions Penalties, and Difference of Numbers of Retailers with Retail Space Costs

Table 3.5: Social Welfare Penalty (SWP), Total Emission Penalty (TEP) and Difference of Numbers of Retailers (DNR) with Retail Space Costs

Carbon Price (metric ton of CO_2)	Monopoly (SWP, TEP, DNR)	$\gamma = 0.5$ (SWP, TEP, DNR)	$\gamma = 0.3$ (SWP, TEP, DNR)
\$100	(0.0%, 4.7%, 0)	(0.3%, 4.8%, 0)	(41.2%, 23.5%, 1)
\$200	(1.5%, 9.1%, 0)	(1.5%, 9.1%, 0)	(51.8%, 28.7%, 1)
\$300	(3.8%, 13.1%, 0)	(3.8%, 13.1%, 0)	(66.5%, 33.4%, 1)
\$500	(14.6%, 20.3%, 0)	(14.6%, 20.3%, 0)	(119.6%, 41.8%, 1)
Carbon Price (metric ton of CO_2)	$\gamma = 0.1$ (SWP, TEP, DNR)	Symmetric (SWP, TEP, DNR)	
\$100	(0.7%, 4.8%, 0)	(51.6%, 16.0%, 1)	
\$200	(54.4%, 29.7%, 1)	(79.4%, 20.8%, 1)	
\$300	(69.7%, 34.4%, 1)	(114.5%, 48.9%, 2)	
\$500	(125.0%, 42.9%, 1)	(200.1%, 58.3%, 2)	

3.7 Discussion

In this section, we discuss the implications of our findings for policy-making. We have shown that the nature of competition can significantly change not only the impact of the carbon tax policy but also the optimal value of the carbon tax. However, the majority of retail industries fall in the asymmetric market share case. In the asymmetric case, the optimal carbon tax rate is independent of the degree of competition. Hence, the policy maker can impose the carbon tax easily. The optimal suggested carbon tax rates are one for retailers (i.e., fully recover the negative impact caused by retailers' activities) and less than one for consumers (i.e., partially recover the negative impact caused by retailers' activities). We have also shown that imposing the carbon tax tends to make the industry less competitive. This inefficiency is caused due to individual retail companies conducting their own logistics op-

erations. This study focused on the last mile issue, therefore such a result is to be expected. However, as we discussed in Section 6.1.2, if retailers use a common 3PL company to distribute their products, then their logistics operations might become more efficient and competition will be intensified again. Therefore, provision of suitable incentives to 3PL companies might be a priority policy issue. While we believe our work is useful for practice (especially for policy-making), we hope more studies can enrich this line of research. For instance, the following limitations in our analysis can be relaxed and the resulting configurations studied: the simplified spatial model to represent the location of retail stores, the exogenous retailer choice by consumers, and the limited players in supply chain (i.e., the last-mile supply chain).

Chapter 4

Exchange Rate and Global Supply Chain Design: Long-term and Short-term Strategies

4.1 Introduction

Globalization has transformed many companies into Multi-national Corporations (MNCs), with either their supply bases and manufacturing functions located in the overseas countries or their markets covering multi-continent. To manage global supply chains, as a long-term strategy, the MNCs invest to other countries and the amount of the investment is enormous. For example, according to BEA (2013), in 2011, the MNCs based in the U.S. invested \$4,156 billion overseas countries (28% of the U.S. GDP) and the MNCs based in countries outside of the U.S. invested \$2,548 billion (17% of the U.S. GDP) in the U.S. Hence, understanding how firms invest outside their home countries is important not only to firms who are planning to invest to overseas countries but also to governments who want to attract Foreign Direct Investment (FDI). We specifically focus on the FDI of export-oriented countries with small domestic markets, e.g., Korea¹ and Taiwan. In addition, as a short-term strategy, how to efficiently manage the operations is an important question for MNCs due to the increased uncertainties involved in producing in foreign countries, e.g., volatile exchange rate and economic instability (see, Dornier et al. 1998).

¹According to World Bank Database, the amount of exports of good and services in Korea is 56% of GDP of Korea.

In this study, we investigate both long-term and short-term strategies of MNCs, i.e., location decision and inventory level decision, respectively. For the long-term strategy, we consider two types of firms whose headquarters are located in two different countries. The firms produce an identical product, and compete with each other by selling the product in the same market. The market is located in one of the two countries, and the firms want to maximize their own profits in the currencies of countries where their headquarters are located. We analytically show that the relationship between the economic growth of a country where the market is located and the currency of the other country is a critical factor to decide the firms' locations especially when the uncertainty in the future economic growth is high. That is, we find that when economic growth and strength of the currency are positively (negatively) related, we find that the reason a firm whose home country without a selling market invests more in another country is because the sunk cost (labor cost) in the foreign country decreases. On the other hand, a firm whose home country with a selling market invests more to another country is because the labor cost (sunk cost) in the foreign country decreases. Then, we validate our analytical findings using the data obtained from the Export-Import Bank of Korea and the Ministry of Trade, Industry, and Energy. The dataset includes outward FDI and inward FDI of Korea between 2002 and 2011.

For the short-term strategy, we study a manufacturer's inventory level decision when the plant and the market are located in two different countries. We show that manufacturers increase the inventory levels as the exchange rate of the country where the plant is located grows weaker. We confirm this result by testing using the plant-level data of Korean multinational corporations provided by the Export-Import Bank of Korea. The dataset provides information

on individual plants between 2002 and 2010 established by firms that are listed on the Korean Stock Exchange (KSE).

Hence, the contribution of our study is three fold. First, we analytically discuss the location decision as well as the inventory decision from the perspective of the MNCs whose home country is an export-oriented country, such as Korea and Taiwan. Second, we empirically validate our analytical findings using the unique data set obtained from the Export-Import Bank of Korea and the Ministry of Trade, Industry, and Energy. Third, we provide several interesting managerial insights. That is, the export-oriented country can increase inward FDI by adjusting its labor policies, such as enabling flexible labor force. On the other hand, a country who has big market (e.g., the U.S. and China) can increase inward FDI by reducing costs related to the sunk cost, such as lowering investment barriers. In addition, by adjusting inventory level depending on the exchange rate, firms can reduce costs.

Besides us, the global supply chain design problem has been explored by others in the literature. Kogut and Kulatilaka (1994), analytically, study the option value of switching a production base from one country to the other country. The value depends on the real exchange rate. Notice that Kogut and Kulatilaka (1994)'s study is similar to ours, but they do not consider competition. Kazaz et al. (2005) consider the production and allocation problems when a manufacturer serves two markets in foreign countries. The production decision (i.e., total production quantity for the two markets) is made before exchange rates of the foreign countries are realized, but the allocation decision can be made after the exchange rates are realized. They show that the degree of correlation between the exchange rates affects the production decision. We note that Kazaz et al. (2005) consider nominal exchange rates, but we consider

real exchange rates.

In addition to the analytical studies, there are several papers which empirically analyze the multinational firms' location decision. Campa and Guillen (1999) study the degree of internalization of exports (e.g., an alliance with a foreign partner and an exporter's own distribution) using a survey data set from manufacturing exporting firms in a middle-income country (i.e., Spain). They consider the ownership factor (e.g., the level of firm's intangible asset) and the location factors (e.g., the competitor's home country), and show that the location factors are independent of the degree of internalization. Chung and Alcacer (2002) study whether the R&D intensity attracts the FDI, use the data of the inward FDI transactions by the Organization for Economic Cooperation and Development (OECD) nations. They show that companies in research-intensive industries (e.g., pharmaceutical industry) are more willing to invest to regions with higher R&D intensity. Nachum et al. (2008) define three types of proximity measures, i.e., knowledge, markets, and resources, and test whether the proximity to other countries affects location decision of MNCs. Using a data set of U.S. MLCs' investments to foreign countries, the authors show that the effect of proximity to location decision is positive. Notice that our study is different from others since we consider the relationship between the economic uncertainty and the location decision as well as the relationship between the exchange rate and the inventory level.

The remainder of this chapter is organized as follows. §4.2 and §4.3 analyze the long-term and short-term strategies, respectively. We conclude in §4.4.

4.2 Long-term Strategy

In this section, we investigate a MNC's long-term strategy, i.e., location decision. §4.2.1 describes the model setting and §4.2.2 explains four possible equilibriums between two types of MNCs. We provide the analytical results in §4.2.3 and empirically validate the findings in §4.2.4.

4.2.1 Model Setting

We consider two types of manufacturers who produce an identical product, and call manufacturers U and K . The headquarters of manufacturers U and K are located in countries U and K , respectively. The two manufacturers produce an identical product, and they sell in a market in country U , i.e., there is only one market. Each manufacturer wants to maximize its own profit in the currency of the country where its headquarter is located.

Exchange Rate. We let X_t be the nominal exchange rate between two currencies in period t . X_t denotes the amount of money in the currency of country K per unit of money in the currency of country U . That is, an increase of X_t means the currency of country K becomes weaker than the currency of country U .

Demand and Exchange Rate Processes. The evolutions of demand and exchange rate are related to the economic growth of country U . Given demand in period $t - 1$, D_{t-1} , an increase of D_t compared to D_{t-1} is positively related to G_t , i.e., $D_t = D_{t-1} + \beta G_t$ and $\beta > 0$, where G_t is the economic growth of country U in period t . Similarly, given X_{t-1} , an increase of X_t compared to X_{t-1} is $X_t = X_{t-1} + \alpha G_t$, but α can be either positive or negative. In period $t - 1$, we assume that the economic growth in period t follows the normal distribution, i.e., $G_t \sim N(\mu_t, \sigma_t^2)$.

Selling Price. The selling price of the product in period t , P_t , is represented by the currency of country U . We assume a linear selling price, i.e., $P_t = D_t - q_t^U - q_t^K$, where q_t^U and q_t^K denote the production quantities by manufacturers U and K in period t , respectively. If a manufacturer's plant is located in country K , then the manufacturer has to pay a transportation cost to ship its products from country K to country U . We let τ be the unit transportation cost from country K to country U .

Unit Production Cost. The unit production cost is the sum of two unit costs, i.e., the technology related cost and the labor related cost. We assume the technology related costs depend on the type of the manufacturer; whereas the labor related costs depend on the location of the manufacturer. In other words, we let c^U and c^K be the technology related costs of manufacturers U and K , respectively, and let w^U and w^K be the labor related costs of countries U and K , respectively. The currency units of c^U and w^U are country U 's currency and those of c^K and w^K are country K 's. The relation between c^U and c^K or that between w^U and w^K cannot be expressed only using the nominal exchange rate X_t because the labor cost in one country could be much lower than the other under the same currency unit. Hence, we introduce θ^T and θ^W to adjust the technology difference between the two manufacturers and the labor cost difference between the two countries, respectively. θ^T and θ^W are defined as follows:

$$\theta^T := X_t \cdot \frac{c^U}{c^K} \quad \text{and} \quad \theta^W := X_t \cdot \frac{w^U}{w^K}.$$

For example, $\theta^T < 1$ means the technology related cost of manufacturer U is lower than that of manufacturer K , and $\theta^W > 1$ means the labor related cost of country U is higher than that of country K .

Transportation Cost & Sunk Cost. If a manufacturer's plant is located in

country K , then the manufacturer has to pay the transportation cost to ship its products from country K to country U . We let τ be the unit transportation cost from country K to country U in country U 's currency (which is similar to Kazaz et al. 2005). In addition, each manufacturer can move from one country to the other, but moving its plant is not free. We let S^U and S^K be the sunk costs to move from country K to country U and country U to country K in the currencies of countries U and K , respectively. Notice that the sunk cost is unit independent.

Decisions. Each manufacturer makes two decisions to maximize its own profit in the currency of a country where its headquarter is located. First, each manufacturers decides its plant location. Notice that by moving its plants, the total cost is changed due to the labor and transportation costs. Second, based on the location decision, manufacturers U and K decide the production quantities in period t , q_t^U and q_t^K , respectively.

4.2.2 Four Possible Equilibriums

In this subsection, we first formulate a general case given location decisions, and then show the expected profits depending on the two manufacturers' location decisions.

Let us formulate a generalized case given location decisions. We let A and B be the sum of unit production and transportation costs of manufacturer U and K , respectively. In addition, we let SC^U and SC^K be the sunk costs to move plants to countries U and K , respectively. Then, manufacture U 's problem in period t given the location decision can be formulated as:

$$\Pi^U = \max_{q_t^U, y^U} V_t^U(q_t^U, y^U; q_t^K, y^K) = \mathbb{E}[P_t | P_{t-1}] q_t^U - A q_t^U - SC^U y^U,$$

where $P_t = D_t - q_t^U - q_t^K$. Similarly, manufacture K 's problem given the location decision can be formulated as:

$$\Pi^K = \max_{q_t^K, y^K} V_t^K(q_t^K, y^K; q_t^U, y^U) = \mathbb{E}[X_t P_t | X_{t-1}, P_{t-1}] q_t^K - B q_t^K - SC^K y^K.$$

The first derivatives of V_t^U and V_t^K with respect to q_t^U and q_t^K , respectively, are

$$\begin{aligned} \frac{\partial V_t^U}{\partial q_t^U} &= \mathbb{E}[D_t] - 2q_t^U - q_t^K - A \\ \frac{\partial V_t^K}{\partial q_t^K} &= \mathbb{E}[X_t D_t] - \mathbb{E}[X_t] q_t^U - 2\mathbb{E}[X_t] q_t^K - B. \end{aligned}$$

Hence, the manufacturers' optimal production quantities $(q_t^{U,*}, q_t^{K,*})$ are

$$\begin{aligned} q_t^{U,*} &= \frac{1}{3} \left\{ 2\mathbb{E}[D_t] - \frac{\mathbb{E}[X_t D_t]}{\mathbb{E}[X_t]} - 2A + \frac{B}{\mathbb{E}[X_t]} \right\} \\ q_t^{K,*} &= \frac{1}{3} \left\{ -\mathbb{E}[D_t] + \frac{2\mathbb{E}[X_t D_t]}{\mathbb{E}[X_t]} + A - \frac{2B}{\mathbb{E}[X_t]} \right\}, \end{aligned}$$

and the expected selling price given the optimal production quantities is

$$\mathbb{E}[P_t | P_{t-1}] = \frac{1}{3} \left\{ 2\mathbb{E}[D_t] - \frac{\mathbb{E}[X_t D_t]}{\mathbb{E}[X_t]} + A + \frac{B}{\mathbb{E}[X_t]} \right\}.$$

Therefore, the expected profits for manufacturers U and K , respectively, are

$$\begin{aligned} \Pi_t^U &= (q_t^{U,*})^2 - SCU, \\ \Pi_t^K &= \mathbb{E}[X_t] (q_t^{K,*})^2 - SCK. \end{aligned}$$

Now, let us investigate profits depending on location decisions. As a default, each manufacturer's plant is located in the country where its head-quarter is located, and we call it Case 1. That is, in Case 1, manufacturers U and K are located in countries U and K , respectively. Three more possible

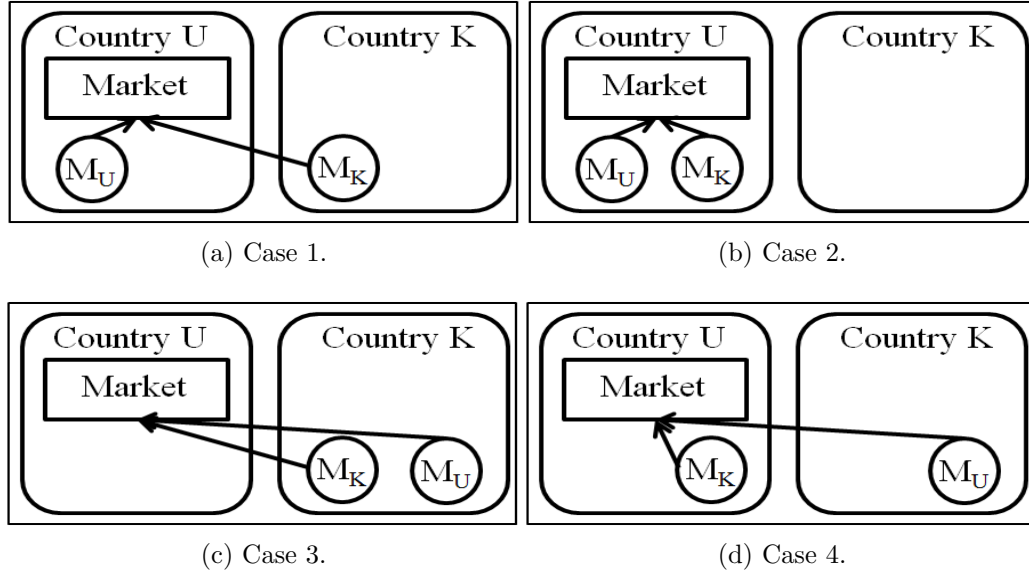


Figure 4.1: Four Possible Cases

cases as well as Case 1 are presented in Figure 4.1, where M_U and M_K stand for manufacturers U and K , respectively. For example, in Case 2, the two manufacturers are located in country U .

The generalized unit costs A and B are affected by the location of plants. For manufacturer U , if its plant is located in country U , then A is replaced by $c^U + w^U$; otherwise, A is replaced by $c^U + w^K/X_t + \tau$. Notice that w^K/X_t is the unit labor related cost by producing in country K in the currency of country U . Similarly, for manufacturer K , if its plant is located in country U , then B is replaced by $c^K + X_t w^U$; otherwise, B is replaced by $c^K + w^K + \tau$. Notice that $X_t w^U$ is the unit labor related cost by producing in country U in the currency of country K .

The generalized sunk costs SCU and SCK are also affected by the

location of plants. If manufacturer U decides to move its plant to country K , then SCU is S^K/X_t ; otherwise, SCU is zero. Similarly, if manufacturer K decides to move its plant to country U , then SCK is $X_t S^U$; otherwise, SCK is zero. Then, we can get each manufacturer's profit in each case and it is summarized in Table 4.1.

Table 4.1: Profits in Each Case

Cases	Profits
Case 1. U	$\frac{1}{9} \left\{ 2\mathbb{E}[D_t] - \frac{\mathbb{E}[X_t D_t]}{\mathbb{E}[X_t]} - 2(c^U + w^U) + \left(\frac{c^U}{\theta^T} + \frac{w^U}{\theta^W} + \tau \right) \right\}^2$
Case 1. K	$\frac{\mathbb{E}[X_t]}{9} \left\{ -\mathbb{E}[D_t] + \frac{2\mathbb{E}[X_t D_t]}{\mathbb{E}[X_t]} + (c^U + w^U) - 2 \left(\frac{c^U}{\theta^T} + \frac{w^U}{\theta^W} + \tau \right) \right\}^2$
Case 2. U	$\frac{1}{9} \left\{ 2\mathbb{E}[D_t] - \frac{\mathbb{E}[X_t D_t]}{\mathbb{E}[X_t]} - 2(c^U + w^U) + \left(\frac{c^U}{\theta^T} + w^U \right) \right\}^2$
Case 2. K	$\frac{\mathbb{E}[X_t]}{9} \left\{ -\mathbb{E}[D_t] + \frac{2\mathbb{E}[X_t D_t]}{\mathbb{E}[X_t]} + (c^U + w^U) - 2 \left(\frac{c^U}{\theta^T} + w^U \right) \right\}^2 - x_0 S^U$
Case 3. U	$\frac{1}{9} \left\{ 2\mathbb{E}[D_t] - \frac{\mathbb{E}[X_t D_t]}{\mathbb{E}[X_t]} - 2 \left(c^U + \frac{w^U}{\theta^W} + \tau \right) + \left(\frac{c^U}{\theta^T} + \frac{w^U}{\theta^W} + \tau \right) \right\}^2 - \frac{S^K}{x_0}$
Case 3. K	$\frac{\mathbb{E}[X_t]}{9} \left\{ -\mathbb{E}[D_t] + \frac{2\mathbb{E}[X_t D_t]}{\mathbb{E}[X_t]} + \left(c^U + \frac{w^U}{\theta^W} + \tau \right) - 2 \left(\frac{c^U}{\theta^T} + \frac{w^U}{\theta^W} + \tau \right) \right\}^2$
Case 4. U	$\frac{1}{9} \left\{ 2\mathbb{E}[D_t] - \frac{\mathbb{E}[X_t D_t]}{\mathbb{E}[X_t]} - 2 \left(c^U + \frac{w^U}{\theta^W} + \tau \right) + \left(\frac{c^U}{\theta^T} + w^U \right) \right\}^2 - \frac{S^K}{x_0}$
Case 4. K	$\frac{\mathbb{E}[X_t]}{9} \left\{ -\mathbb{E}[D_t] + \frac{2\mathbb{E}[X_t D_t]}{\mathbb{E}[X_t]} + \left(c^U + \frac{w^U}{\theta^W} + \tau \right) - 2 \left(\frac{c^U}{\theta^T} + w^U \right) \right\}^2 - x_0 S^U$

4.2.3 Analysis

We investigate an equilibrium of two manufacturers' location decisions. The equilibrium depends on the values of various parameters. We consider four major factors that affect the location decisions. First, the exchange rate X_t and the economic growth G_t can be positive and negative, and it is represented by α . In other words, if α is negative, then the currency of country K becomes stronger as the economic growth of country U increases, and vice versa. Second, the labor- and transportation-cost differences between the two countries is also important. If $w^K/X_t + \tau$ is less than w^U , then production in country K can reduce cost, ceteris paribus. Third, the sunk cost to build a

plant in country K , i.e., S^K , can be (almost) zero or positive. Notice that as X_t increases, then S^K goes to zero. Fourth, the sunk cost to build a plant in country U , i.e., S^U , can be negative or positive. Notice a negative S^U is due to the benefit of producing in the local plants, e.g., no export tax, shorter lead time, etc. Therefore, we need to consider 16 different scenarios.

4.2.3.1 No Uncertainty in Economic Growth.

We first consider an equilibrium when there is no uncertainty in economic growth, and the result is summarized in Lemma 4.2.1.

Lemma 4.2.1. *Suppose that there is no uncertainty in economic growth. (i) If $w^K/X_t + \tau < w^U$, $S^U \geq 0$, and $S^K = 0$, then both manufacturers U and K produce in country K , i.e., Case 3. (ii) If $w^K/X_t + \tau \geq w^U$ and $S^U < 0$, then both manufacturers U and K produce in country U , i.e., Case 2. (iii) In other scenarios, the equilibrium depends on the parameter values.*

Intuitively, if the labor cost in country U is greater than the sum of labor cost in country K and transportation cost from country K to country U as well as the sunk cost to build a plant in country U is positive, then both manufacturers want to produce in country K . Similarly, if the labor cost in country U is less than the sum of labor cost in country K and transportation cost from country K to country U as well as the sunk costs to build a plant in countries U and K are negative and zero, respectively, then both manufacturers want to produce in country U . In other words, manufacturers produce in a country where the labor cost is low enough to compensate the transportation cost and the sunk cost is low.

4.2.3.2 High Uncertainty in Economic Growth.

Now let us consider an equilibrium when the uncertainty in economic growth is high. For notational convenience, let

$$\begin{aligned} \sigma_U^2 &:= -\frac{(X_0 + \alpha\mu_t)}{2\alpha\beta} \\ &\cdot \left[\left(\frac{9S^K}{2x_0} \right) \left\{ \frac{1}{w^U - \left(\frac{w^U}{\theta^W} + \tau \right)} \right\} - 4(D_0 + \beta\mu_t) \right] \\ &- \frac{(X_0 + \alpha\mu_t)}{2\alpha\beta} \\ &\cdot \left[+2 \left(2c^U - \frac{c^U}{\theta^T} + \max \left\{ w^U, \frac{w^U}{\theta^W} + \tau \right\} \right) \right] \\ &- \left\{ \frac{X_0 D_0 + (\alpha D_0 + \beta X_0)\mu_t + \alpha\beta\mu_t^2}{\alpha\beta} \right\}, \end{aligned}$$

and

$$\begin{aligned} \sigma_K^2 &:= -\left(\frac{9x_0 S^U}{8\alpha\beta} \right) \left\{ \frac{1}{w^U - \left(\frac{w^U}{\theta^W} + \tau \right)} \right\} \\ &+ \frac{(X_0 + \alpha\mu_t)}{2\alpha\beta} \left\{ (D_0 + \beta\mu_t) - \left(c^U - \frac{2c^U}{\theta^T} - \max \left\{ w^U, \frac{w^U}{\theta^W} + \tau \right\} \right) \right\} \\ &- \left\{ \frac{X_0 D_0 + (\alpha D_0 + \beta X_0)\mu_t + \alpha\beta\mu_t^2}{\alpha\beta} \right\}. \end{aligned}$$

Propositions 4.2.2 and 4.2.3 show equilibriums when $\alpha < 0$ and $\alpha \geq 0$, respectively.

Proposition 4.2.2. *Suppose that $\sigma_t^2 \geq \max\{\sigma_U^2, \sigma_K^2\}$ and α is negative. (i) If $w^K/X_t + \tau < w^U$ and $S^U \geq 0$, then both manufacturers U and K produce in country K , i.e., Case 3. (ii) If $w^K/X_t + \tau < w^U$ and $S^U < 0$, then manufacturers U and K produce in countries K and U , respectively, i.e., Case 4. (iii) If $w^K/X_t + \tau \geq w^U$ and $S^U \geq 0$, then manufacturers U and K produce*

in countries U and K , respectively, i.e., Case 1. (iv) If $w^K/X_t + \tau \geq w^U$ and $S^U < 0$, then both manufacturers U and K produce in country U , i.e., Case 2.

Proposition 4.2.3. *Suppose that $\sigma_t^2 \geq \max\{\sigma_U^2, \sigma_K^2\}$ and α is positive. (i) If $w^K/X_t + \tau < w^U$ and $S^K > 0$, then manufacturers U and K produce in countries U and K , respectively, i.e., Case 1. (ii) If $w^K/X_t + \tau < w^U$, $S^K = 0$ and $S^U < 0$, then both manufacturers U and K produce in country K , i.e., Case 3. (iii) If $w^K/X_t + \tau \geq w^U$, then both manufacturers U and K produce in country U , i.e., Case 2.*

Propositions 4.2.2 and 4.2.3 are summarized in Table 4.2. Notice that a number in parenthesis in Table 4.2 represents each scenario. First, the equilibriums in scenarios (1), (2), (11), (12), (15), and (16) are explained below Lemma 4.2.1. That is, whether the uncertainty in the economic growth is high or not, the equilibriums remain the same in those scenarios.

Second, let us compare scenarios (5) and (6). In these scenarios, the labor cost in country K is low enough to compensate the transportation cost and the sunk cost in country U is positive. Hence, manufacturer K does not need to move its plant and the equilibrium is either Case 1 or Case 3. When the sign of α is negative, the manufacturer U 's profit increase by moving to country K increases as the uncertainty in the economic growth increases since $\mathbb{E}[X_t D_t]$ decreases as the magnitude of the shock increases (see, Table 4.1). Hence, if $\alpha < 0$, the equilibrium becomes Case 3. Similarly, the equilibrium becomes Case 1 if $\alpha > 0$.

Third, in scenarios (3) and (4), the labor cost in country K is not low enough to compensate the transportation cost. Hence, manufacturer U does not need to move its plant and the equilibrium is either Case 1 or Case 2. When

Table 4.2: Equilibriums With an Economic Shock

		$w^K/X_t + \tau < w^U$		$w^K/X_t + \tau \geq w^U$	
		$\alpha < 0$	$\alpha \geq 0$	$\alpha < 0$	$\alpha \geq 0$
$S^U \geq 0$	$S^K = 0$	(1) Case 3	(2) Case 3	(3) Case 1	(4) Case 2
	$S^K > 0$	(5) Case 3	(6) Case 1	(7) Case 1	(8) Case 2
$S^U < 0$	$S^K = 0$	(9) Case 4	(10) Case 3	(11) Case 2	(12) Case 2
	$S^K > 0$	(13) Case 4	(14) Case 1	(15) Case 2	(16) Case 2

the sign of α is positive, the manufacturer K 's profit increase by moving to country U increases as the uncertainty in the economic growth increases since $\mathbb{E}[X_t D_t]$ increases as the magnitude of the shock increases (see, Table 4.1). Hence, if $\alpha > 0$, the equilibrium becomes Case 2. Similarly, the equilibrium becomes Case 1 if $\alpha < 0$. We can explain scenarios (7) and (8) in a similar argument.

Fourth, the labor cost in country K is low enough to compensate the transportation cost and the sunk cost in country K is negligible in scenarios (9) and (10). Hence, manufacturer U is willing to move its plant to country K and the equilibrium is either Case 4 or Case 3. When the sign of α is positive, the manufacturer K 's profit increase by moving to country U increases since $\mathbb{E}[X_t D_t]$ increases as the uncertainty in the economic growth increases (see, Table 4.1). Hence, if $\alpha > 0$, the equilibrium becomes Case 3. Similarly, the equilibrium becomes Case 4 if $\alpha < 0$.

Fifth, let us compare scenarios (13) and (14). In these scenarios, the labor cost in country K is low enough to compensate the transportation cost. In addition the sunk cost in country K is not negligible. and the sunk cost in country U is negative. Hence, when the sign of α is negative, manufacturer U is willing to move its plant to country K and manufacturer K is willing to

move its plant to country U . That is the equilibrium is Case 4. Similarly, the equilibrium becomes Case 1 if $\alpha > 0$.

4.2.4 Empirical Analysis

In this section, we characterize the data and variables for empirical analysis. With these measures, we build hypotheses based on our theoretical framework and provide an empirical specification to test our analytical findings.

4.2.4.1 Data Description.

To test our analytical findings, we require a data that provide bilateral FDI between countries. However, since obtaining such information is not available, we use data on outward FDI and inward FDI of Korean firms for the analysis. The main data source on outward FDI from Korea is obtained from Overseas Direct Investment Statistics from the Export-Import Bank of Korea. This data includes the full list of Korean worldwide investment from 2002 to 2011 and the status on foreign investment is disaggregated by three-digit Korean Standard Industrial Classification (KSIC) industry sectors and by destination countries in a given year. To make it consistent with our theoretical framework, we aggregate the data within three-digit KSIC manufacturing industry sectors for each year and measure outward FDI from Korea by using data on the amount of investment and number of reported plant establishment in destination countries. Hence, we expect that Korean manufacturers invest more in the foreign country as their investment amount and number of reported establishment increase in that country.

On the other hand, the main data source on inward FDI to Korea is

obtained from the Ministry of Trade, Industry, and Energy of Korea. This data include the full list of worldwide investment into Korea for between 1980 and 2011 and the investment is disaggregated by three-digit KSIC industry sectors and by the source country in a given year. Consistent with outward FDI, we aggregate the data within three-digit KSIC manufacturing industry sectors for the year between 2002 and 2011. We measure inward FDI to Korea by using the data on the amount of investment arrived in Korea and number of reported establishments in Korea. Therefore, an increase in the investment amount or number of new establishment can be interpreted that foreign manufacturers invest more in Korea.

Our model focuses on the likelihood of firms to invest abroad and its link to the relationship between the home currency and the economic growth of a host country (α), sunk costs of FDI – S^j stands for sunk costs associated with Korean outward FDI in country j while S^k represents sunk costs associated with inward FDI to Korea by foreign investors – and the labor costs between home and foreign countries (w^k, w^j). For the empirical specification, we estimate α as a correlation coefficient between the real exchange rate of Korea currency and the GDP growth of a host country. Hence, we expect negative α in countries where Korean currency is stronger than the GDP growth of the country.

To capture the labor costs between home and foreign countries, using the data on earnings per hour across countries from International Labor Organization leads to reduced number of observations. Instead, we follow prior literature that analyze the relationship between country's labor cost and GDP per capita by using real GDP per capita as a proxy for labor cost and measure the labor cost difference between home and foreign countries. In particular,

we construct real GDP per capita ratio between Korea and host countries (w^k/w^j). Thus, we expect an increase in a ratio as higher labor cost incurred from Korea than from host country.

To capture the sunk cost of FDI, we use two measures for the analysis. First, following Carr et al. (2001), we measure sunk cost of FDI as investment barriers or investment costs in host countries and compute a simple average of nine indices of perceived impediments to foreign investment, which is obtained from the World Competitiveness Report of the World Economic Forum. These indices are calculated on a scale from zero to 100 in which a higher number implies higher investment costs.² Second, following the previous results that show investment costs are more likely to be larger in countries with high political risk (see, Campos et al. 1999 and Asiedu 2006), we use data on estimating the perceptions of the quality of public services, the quality of the civil service and the degree of its independence from political pressures, the quality of policy formulation and implementation, and the credibility of the government's commitment to such policies. This estimate of governance ranges from -2.5 to 2.5 in which a higher number indicates that country is politically stable where data is obtained from the World Government Indicators from World Bank. Therefore, we expect a low investment cost or barrier as country is more politically stable. To represent the difference in sunk costs between countries, we construct the ratio between Korea and host country for each measure. Thus, an increase in investment cost ratio or a decrease in political stability ratio

²These indices include restrictions on the ability to acquire control in a domestic company, limitations on the ability to employ foreign skilled labor, restraints on negotiating joint ventures, strict controls on hiring and firing practices, market dominance by a small number of enterprises, an absence of fair administration of justice, difficulties in acquiring local bank credit, restrictions on access to local and foreign capital markets, and inadequate protection of intellectual property.

indicates a lower sunk cost of FDI in foreign countries than Korea.

We also incorporate other control variables that affect firm’s FDI decision. Previous trade literature examines foreign market size and transportation costs as determinants of firms choosing between exports and FDI to serve foreign markets such that firms are more likely to undertake FDI in countries with large market size or high transportation costs from home (see, Markusen 1984 and Brainard 1997). Thus, we add country’s market size measured by real GDP, transportation costs from Korea by computing a ratio of Cost, Insurance, Freight imports into the country to Free on Board imports, and country’s infrastructure level by computing the simple average of three communication indices – telephone users, mobile users, and internet users per 100 habitants – where data are obtained from World Development Indicators 2012 and IMF Direction of Trade Statistics. Summary statistics of variables are provided in Table 4.3.

Table 4.3: Descriptive summary statistics

Variable	Mean	Standard deviation	Minimum	Maximum
Investment cost	38.89	24.26	0.86	94.81
Political stability estimate	0.194	1.011	-2.34	1.99
Real GDP per capita (in thousand)	10.01	13.38	0.83	108.11
Real GDP (in billion)	361.61	1267.25	0.07	11744.22
Infrastructure (per 100 people)	40.65	28.1	0.22	115.24
Trade cost	1.07	0.91	0.004	25.99
Investment amount (Outward, in million)	141.34	500.3	0.56	5945.72
New plants (Outward)	34.22	180.3	0	2298
Investment amount (Inward, million)	106.07	380.7	0.89	4717.78
New plants (Inward)	29.7	84.9	0	672

4.2.4.2 Hypotheses.

The signs of α are negative for all the countries (except for Argentina), i.e., as the GDPs of foreign countries increase Korean currency becomes stronger. The reason is that Korean economy is highly dependent on foreign countries, i.e., the ratio of export to GDP is high. Hence, we only test the analytical results when α is negative.

Lemma 4.2.1 and Proposition 4.2.2 can be summarized as follows. When there is no uncertainty in the economic growth (i.e., no economic shock), the manufacturers invest abroad more as the sunk cost as well as the labor cost becomes lower. When the uncertainty in the economic growth is very high (i.e., an economic shock), regardless of the production cost (including the labor and transportation costs) difference, manufacturer K is more willing to move its plant to country U as the sunk cost in country U becomes lower than that in country K . On the other hand, regardless of the sunk cost difference between two countries, manufacturer U is more willing to move its plant to country K as the production cost in country K becomes lower than that in country U . Hence, we can derive the following two hypotheses.

Hypothesis 4.2.4. *When there is no economic shock, home (foreign) manufacturers invest abroad as foreign (home) countries incur lower sunk costs and lower labor costs than home (foreign) country.*

Hypothesis 4.2.5. *When there is an economic shock, home manufacturers' production location choices are affected by sunk cost differences while foreign manufacturers' location choices depend on the labor cost differences.*

Table 4.4: Firm's Long-term strategy (Outward FDI)

	Outward FDI	
	Pre-crisis (2002-2007)	
	Investment amount	Investment number
Investment cost	0.026*** (0.009)	0.153 (0.104)
Political stability	-0.041 (0.037)	-0.192** (0.075)
Real GDP per capita	-0.002 (0.013)	0.052* (0.028)
Real GDP	0.969*** (0.135)	0.816*** (0.134)
Infrastructure	0.027** (0.011)	0.012 (0.01)
Trade cost	0.967*** (0.257)	0.197*** (0.072)
Investment cost*Real GDP per capita	0.0009*** (0.0003)	0.007* (0.003)
Political stability*Real GDP per capita	-0.001*** (0.0002)	-0.014*** (0.005)
Year Fixed effects	Yes	Yes
Country Fixed effects	Yes	Yes
Observations	1014	1014
R-squared	0.4081	0.6181
	Outward FDI	
	Full sample	
	Investment amount	Investment number
Investment cost	0.241* (0.136)	0.212*** (0.06)
Political stability	-0.197** (0.09)	-0.159 (0.148)
Real GDP per capita	-0.03 (0.056)	-0.024 (0.045)
Real GDP	1.19*** (0.152)	0.431*** (0.134)
Infrastructure	0.018 (0.019)	0.003 (0.006)
Trade cost	0.134* (0.072)	0.492*** (0.183)
Investment cost*2008	0.035** (0.015)	0.044 (0.122)
Investment cost*2009	0.239* (0.133)	0.213** (0.089)
Political stability*2008	-0.467*** (0.111)	-0.094 (0.193)
Political stability*2009	-0.179** (0.085)	-0.458** (0.225)
Real GDP per capita*2008	0.061 (0.125)	-0.019 (0.079)
Real GDP per capita*2009	0.201 (0.134)	0.074 (0.064)
Year Fixed effects	Yes	Yes
Country Fixed effects	Yes	Yes
Observations	1690	1690
R-squared	0.5730	0.7124

4.2.4.3 Results.

We first estimate Korean firm's FDI decision as a balanced panel with country fixed effects for the period 2002 to 2011. All regressions include industry- and year-fixed effects to control for the possible time trends and for any unobserved systematic differences across industry sectors. The standard errors have been corrected for heteroskedasticity at the country level.

The results from estimating Korean outward FDI are presented in Table 4.4. To examine the effects of economic shock and other country characteristics on outward FDI, we first estimate firm's decision using data only up to 2007, which avoids the global financial crisis period between 2008 and 2009, and present the results in first two columns. We then incorporate global crisis period by estimating with the full sample period and construct interaction variables between crisis periods and sunk cost difference variables and report the results in last two columns. For each specification, we provide the results from using data on investment amount and new plant establishments as a dependent variable. Between non-economic shock periods, columns 1 and 2 in Table 4.4 show that firms are likely to invest abroad in countries that incur relatively lower sunk cost than Korea. In particular, we can see that firms' investment amount are positively and significantly affected by country's investment barrier differences (0.026), while the number of establishing plants are negatively and significantly associated with government effectiveness (-0.192). This indicates that firms are likely to invest abroad as foreign country has relatively low investment costs or is more politically stable.

The interaction variable between real GDP per capita ratio and investment barrier ratio indicates that if Korea has relatively high labor costs, firms are more likely to invest in foreign countries that incur low sunk costs of FDI.

Alternatively, coefficients on interaction term between political stability ratio and real GDP per capita ratio are negative and significant, implying that firms invest more in countries with relatively low labor costs and low sunk costs that result from high political stability. These results support our first hypothesis where home manufacturers prefer producing abroad if home incurs high labor and sunk costs when there is no economic shock. For other covariates, our results are consistent with prior trade literature that firms tend to invest in countries with large market size and low transportation costs.

Examining outward FDI using the full sample of period, columns 3 and 4 in Table 4.4 show that firm's FDI decision is negatively and significantly associated with foreign country's sunk costs (positive and significant coefficients on investment cost ratio, while negative and significant coefficients on political stability ratio for both dependent variables). Analyzing the effects of economic shock and cost differences on firm's FDI, we first construct binary variable to capture economic shock. Using the fact that global financial crisis occurred between 2008 and 2009, binary variables, 2008 and 2009, are equal to 1 if there was outward FDI in year 2008 and 2009. Interacting crisis variables with alternative measures for sunk cost and labor cost differences between Korea and foreign countries, coefficients on interaction terms between crisis variable and real GDP per capita ratio are statistically insignificant implying that firm's FDI during the crisis period is not affected by labor cost differences. On the other hand, interaction terms between investment cost and crisis dummies, and between political stability and crisis dummies are positively and negatively associated with outward FDI, respectively. These coefficients indicate that in crisis periods, firms invest more in foreign countries that incur relatively low sunk costs, which support our second hypothesis that manufacturers tend to

produce abroad as foreign country has low sunk costs when there is an economic shock.³ For other controls, we still find that country's market size and scale of transportation costs are significantly and positively associated with firm's FDI decision.

The results from Table 4.4 support our hypothesis by showing that between the periods without economic shock, the firm's FDI decision is affected by the link between sunk cost and labor cost differences across countries. In particular, we find that in countries with low labor costs, firms are more likely to produce through FDI as countries incur lower sunk costs. Alternatively, between periods with economic shock, results show that labor cost differences are not determinant of firm's FDI. Instead, firms are affected by sunk cost differences such that they prefer to invest in countries with low sunk costs during the economic crisis.

Table 4.5 reports the results from estimating inward FDI to Korea from other foreign countries with country characteristics by dividing into pre-global financial crisis periods and full sample of periods involving financial crisis. All the variables are same as from the previous specification except for their interpretation. For instance, all the ratio between home and foreign countries represent the difference between Korea as a host country and foreign country as an investing (source) country, while other country characteristics such as real GDP, transportation cost, and infrastructure now capture that of Korea over the sample period. Analyzing the effects of independent variables on

³We also included interaction variables between year dummy variables and alternative measures for sunk cost differences. The results shows that except for interaction term between investment cost and year 2003 from the specification with investment amount as a dependent variable, all coefficients are statistically insignificant. we do not report these interaction variables in Table 4 to save space.

Table 4.5: Firm's Long-term strategy (Inward FDI)

	Inward FDI	
	Pre-crisis (2002-2007)	
	Investment amount	Investment number
Investment cost	-0.019*** (0.007)	-0.007*** (0.003)
Political stability	0.462*** (0.137)	0.099* (0.059)
Real GDP per capita	-0.034 (0.022)	0.024 (0.015)
Real GDP	0.779 (0.544)	0.41 (0.394)
Infrastructure	0.032*** (0.009)	0.004 (0.005)
Trade cost	0.001 (0.005)	0.0056 (0.006)
Investment cost*Real GDP per capita	-0.003*** (0.001)	-0.0007** (0.0003)
Political stability*Real GDP per capita	0.019** (0.007)	-0.006 (0.004)
Year Fixed effects	Yes	Yes
Country Fixed effects	Yes	Yes
Observations	744	744
R-squared	0.5255	0.4889
	Inward FDI	
	Full sample	
	Investment amount	Investment number
Investment cost	-0.079 (0.064)	-0.018*** (0.005)
Political stability	-0.138 (0.137)	0.006 (0.018)
Real GDP per capita	-0.181* (0.093)	0.024 (0.033)
Real GDP	0.647 (0.386)	0.389** (0.132)
Infrastructure	0.021 (0.015)	0.02*** (0.005)
Trade cost	-0.015 (0.439)	0.098* (0.055)
Investment cost*2008	0.582 (0.375)	-0.096 (0.172)
Investment cost*2009	-0.241 (0.226)	0.364 (0.53)
Political stability*2008	-0.248 (0.499)	0.102 (0.147)
Political stability*2009	-0.262 (0.215)	0.036 (0.026)
Real GDP per capita*2008	-0.464*** (0.151)	-0.058** (0.026)
Real GDP per capita*2009	-1.896** (0.829)	-0.054* (0.028)
Year Fixed effects	Yes	Yes
Country Fixed effects	Yes	Yes
Observations	1240	1240
R-squared	0.476	0.6354

Korean inward FDI before the crisis, the results in columns 1 and 2 in Table 4.5 show that foreign countries tend to invest more in Korea as Korea has relatively low sunk cost. For example, a fall in investment cost ratio between Korea and source country of 10 percentage points increases source country's foreign investment by 0.2%. Consistent with outward FDI case, labor cost differences do not seem to be a determinant of inward FDI.

Interacting between alternative measures for sunk costs and real GDP per capita, the coefficients imply that if Korea has relatively higher labor cost, foreign firms invest more as Korea has an advantage in sunk cost differences with source country, which supports our first hypothesis on foreign manufacturers' production location choices when there is no economic shock. In contrast to the previous specification, other variables capturing characteristics of Korea do not have significant effects on inward FDI. Examining the full sample of period including global financial crisis, columns 3 and 4 Table 4.5 show that sunk cost differences and labor cost differences by themselves have insignificant effects on foreign manufacturer's FDI decision. However, when differences are interacted with economic shock variables – binary variables that are equal to 1 if year is 2008 and 2009, respectively – the coefficients on interaction terms between real GDP per capita and crisis dummies are negative and significant. This indicates that in crisis period, foreign manufacturers are likely to invest more in Korea as Korea has relatively low labor costs. Alternatively, statistically insignificant coefficients on interaction terms between alternative measures for sunk cost differences and crisis dummies suggest that sunk cost differences do not affect foreign manufacturer's FDI decision in Korea during economic shock.⁴ These results also support our second hypothesis

⁴Consistent with previous results, we add interaction terms between cost difference mea-

that foreign manufacturers choose production locations on the basis of labor cost differences between home and host countries when there is an economic shock.

4.3 Short-term Strategy

In this section, we consider a manufacturer's short-term strategy, i.e., inventory decision with respect to the exchange rate and the transportation cost.

4.3.1 Model Setting

We consider two types of models, i.e., the economic order quantity (EOQ) model and the newsvendor model.

4.3.1.1 EOQ Model.

We let D^E denote the constant demand rate in the EOQ model. The unit holding cost is h^K in the currency of country K . There are two types of transportation costs, i.e., the transportation cost in country K (e.g., shipping cost from a plant to a port in country K) and that in country U (e.g., shipping cost from a port to a customer in country U). We denote the transportation costs per shipment in countries U and K by t^U and t^K in the currency of countries U and K , respectively. Notice that considering per unit transportation cost does not change the decision because of the constant demand rate. We

asures and year dummies. The estimates show that sunk cost differences have significant effects on firm's FDI decision between 2004 and 2006, and after 2010, while labor cost differences are not significant in pre- and post-crisis periods. We do not report coefficients on other interaction terms in Table 5 to save space.

let $q^{E,U}$ and $q^{E,K}$ denote the quantities per shipment in the plants of manufacturers U and K , respectively. Then, manufacture U 's cost is:

$$\left(\frac{h^K/X}{2}\right)q^{E,U} + \frac{D^E(t^U + t^K/X)}{q^{E,U}}.$$

where $\frac{h^K/X}{2}$ is the average holding cost and D^E/q^U is the number of shipments. Similarly, manufacture K 's cost is:

$$\left(\frac{h^K}{2}\right)q^{E,K} + \frac{D^E(t^U X + t^K)}{q^{E,K}}.$$

4.3.1.2 Newsvendor Model.

In the newsvendor model, demand D is random. We let $G(\cdot)$ denote the cumulative distribution of D . We let p and c denote the unit selling price in the currency of country U and the unit production cost in the currency of country K , respectively. Similar to the EOQ model, there are two types of transportation costs, i.e., the transportation cost in country K and that in country U . We denote the per unit transportation costs in countries U and K by τ^U and τ^K in the currency of countries U and K , respectively. Notice that considering the transportation cost per shipment does not change the decision because of the one-time selling opportunity. In addition, for simplicity, we do not consider the salvage value and the shortage cost. We let $q^{N,U}$ and $q^{N,K}$ denote the quantities per shipment in the plants of manufacturers U and K , respectively. Then, manufacture U 's expected cost is:

$$(c/X + \tau^U + \tau^K/X)q^{N,U} - p\mathbb{E}[\min\{D, q^{N,U}\}].$$

Similarly, manufacture K 's expected cost is:

$$(c + X\tau^U + \tau^K)q^{N,K} - Xp\mathbb{E}[\min\{D, q^{N,K}\}].$$

4.3.2 Analysis

Proposition 4.3.1 shows the relationship between the exchange rate and the inventory level.

Proposition 4.3.1. *From the EOQ and the newsvendor model, both the inventory levels of manufacturers U and K increases as the the exchange rate increases. That is, q^U and q^K increase as X increases.*

In the EOQ model, when manufacturer U produces in country K , as the currency of country K becomes weaker, the transportation cost in country U becomes relatively expensive because both the holding cost and the transportation cost in country K decreases and the transportation cost in country K remains the same. Hence, manufacturer U increase the quantity per shipment as the currency of country K becomes weaker, and it increases the inventory level of manufacturer U . Similarly, when manufacturer K produces in country K , as the currency of country K becomes weaker, the transportation cost in country U becomes expensive while the holding cost and the transportation cost in country K remain the same. Hence, manufacturer K also increases the quantity per shipment as the currency of country K becomes weaker, and it increases the inventory level of manufacturer K .

In the newsvendor model, when manufacturer U produces in country K , as the currency of country K becomes weaker, the unit underage cost (i.e., $p - (c/X + \tau^U + \tau^K/X)$) increases and the sum of unit overage and underage costs (i.e., p) remain the same. Hence, manufacturer U produce more as the currency of country K becomes weaker, and it increases the inventory level of manufacturer U . Similarly, when manufacturer K produces in country K , as the currency of country K becomes weaker, the relative unit underage

cost increases and the relative sum of unit overage and underage costs (i.e., p) remain the same. Hence, manufacturer K also produce more as the currency of country K becomes weaker, and it increases the inventory level of manufacturer K . More intuitively, as the currency of country K becomes weaker, the relative unit underage cost increases while the relative unit overage cost decreases. Hence, the inventory level increases as the currency of country K becomes weaker.

Proposition 4.3.2 shows the relationship between the transportation cost and the inventory level.

Proposition 4.3.2. *From the EOQ model, as the transportation cost per shipment increases, the inventory level increases. From the newsvendor model, as the transportation cost per unit decreases, the inventory level increases.*

Intuitively, in the EOQ model, as the transportation cost per shipment increases, manufacturers increase the amount of shipment to reduce the total cost. Hence, the average inventory levels increase as the transportation cost per shipment increases. In the newsvendor model, an increase of the transportation cost per unit increases the unit overage cost and decreases unit underage cost. Hence, the inventory level increases as the transportation cost per unit decreases.

4.3.3 Empirical Analysis

4.3.3.1 Data Description

In order to test our hypothesis for short-term strategy, we use plant-level data of Korean multinational firms obtained from the Export-Import Bank of Korea. This panel dataset provides information on individual plants

between 2002 and 2010 established by firms that are listed on the Korean Stock Exchange (KSE). All plants are in 92 three-digit KSIC equivalent sectors and located in 169 countries. Our plant-level data are very unique in that they provide information on each plant, divided by its industry sectors and host country in a given year. For our interest, in particular, the data not only provides information on plants' balance sheets but also on total sales which is divided into sales made from local market, sales made from exporting back to Korea, and sales made from exporting to third countries. Furthermore, the dataset contains information on total imports which is also divided into imports from host country, from Korea, and from third countries.

To make it consistent with our model, we first use data on Korean foreign plants from three-digit KSIC manufacturing industry sectors for the year between 2002 and 2007.⁵ Using the information on plant's distribution of sales to different markets, we consider plants that are designed to produce in the host country and export back to home (export-platform FDI). In other words, foreign manufacturing plants that make sales only from Korea are considered for the empirical specification. Thus, the final dataset is an unbalanced panel of around 65 plants per year with a total of 621 observations.

Our analytical findings highlight on the effects of exchange rates and transportation costs on plants' inventory changes. First, to test the link between plants' inventory levels and the exchange rates of host countries, we construct a dependent variable, inventory levels by using data on the amount of inventories a plant has for a given year which is measured in tons. For

⁵Our hypothesis is on testing the effects of exchange rates on plant's inventory levels when there is no economic shock. Hence, our sample period is identical to the pre-crisis period from the previous empirical specification on testing firm's long-term strategy.

independent variables, we construct exchange rates in two measures. First, we use data on real effective exchange rates, obtained from The Brussels-based Think Tank (Bruegel) and from the Bank of International Settlements (BIS). Both real effective exchange rate indices are CPI-based where Bruegel indices are based on year 2007, while BIS indices are based on year 2010. From the based year, therefore, we expect national currency is stronger as exchange rate index decreases. Second, we compute the period average national currency per Korean won by using data from IMF.

Examining the effects of transportation costs on inventory levels, we use two alternative measures for transportation costs. We first consider shipment costs of foreign trade (see, Brainard 1997 and Helpman et al. 2004). These costs are due to costs of shipping products across borders, such as transport and insurance. Therefore, we measure transportation cost per shipment as an ad-valorem measure of freight and insurance, which we compute the ratio of CIF imports into the Korea from the host country to FOB imports using data from IMF Direction of Trade and Statistics. On the other hand, we consider unit costs of foreign trade (see, Anderson and Van Wincoop 2003 and Bergstrand 1985). We interpret these costs from arising due to the physical barriers across countries that are not restricted by barriers created by destination-country governments, such as insurance costs or tariffs. Hence, we measure transportation costs per unit by computing the distance between host country and Korea where the data is obtained from Centre d'Etudes Prospectives et d'Informations Internationales (i.e., France Institute for Research on the International Economy).

For other controls, we include variables that represent plant and host country characteristics. To capture plant characteristics, we add plant size

measured as total employment in the plant, share of material imports from its headquarter located in Korea, and its ownership structure measured as equity share of a plant owned by Korean parent firm. For host country characteristics, we include country's real GDP per capita as a proxy for labor costs using data from World Development Indicators 2012.

4.3.3.2 Hypotheses.

Proposition 4.3.1 shows that regardless of the demand types, both the inventory levels of manufacturers U and K increase as the exchange rate of country K becomes weaker. In addition, Proposition 4.3.2 shows that the inventory levels of manufacturers U and K increase as the transportation cost per shipment increases and the transportation cost per unit decreases. Hence, we derive the following hypotheses.

Hypothesis 4.3.3. *The inventory level of the plant in the foreign country increases as the currency of the foreign country becomes weaker.*

Hypothesis 4.3.4. *The inventory level of the manufacturer's plant in the foreign country increases as the transportation cost per shipment increases and the transportation cost per unit decreases.*

4.3.3.3 Results.

Table 4.6 reports the results from estimating the effects of exchange rates and transportation costs on plants' inventory changes. Year dummies and industry sector dummies are included to control for year- and industry-specific fixed effects. Robust standard errors clustering for host countries are presented in the parenthesis to explain the possible correlated shocks that

Table 4.6: Firm's short-term strategy

Variables	Inventory levels		
	(1)	(2)	(3)
REER (Bruegel)	-0.016*** (0.005)		
REER (BIS)		-0.002*** (0.0005)	
National currency per won			0.024** (0.012)
Inventory level _{t-1}	0.812*** (0.056)	0.82*** (0.03)	0.822*** (0.036)
Plant size	0.132** (0.065)	0.13*** (0.043)	0.135*** (0.046)
Share of imports from HQ	-0.159** (0.076)	-0.011 (0.077)	-0.025 (0.069)
Ownership structure	-0.005** (0.002)	-0.004*** (0.0013)	-0.004*** (0.001)
Real GDP per capita	-0.002 (0.031)	-0.065* (0.036)	-0.084* (0.046)
CIF/FOB ratio	0.083*** (0.013)	0.065*** (0.011)	0.067*** (0.019)
Year fixed effects	Yes	Yes	Yes
Industry fixed effects	Yes	Yes	Yes
Observations	308	578	483
R-squared	0.8487	0.8084	0.7999

Variables	Inventory levels		
	(4)	(5)	(6)
REER (Bruegel)	-0.019*** (0.005)		
REER (BIS)		-0.003*** (0.0005)	
National currency per won			0.023** (0.011)
Inventory level _{t-1}	0.795*** (0.065)	0.816*** (0.027)	0.817*** (0.033)
Plant size	0.152* (0.08)	0.133*** (0.041)	0.139*** (0.044)
Share of imports from HQ	-0.391 (0.238)	-0.01 (0.079)	-0.035 (0.063)
Ownership structure	-0.004* (0.002)	-0.005*** (0.0015)	-0.004*** (0.001)
Real GDP per capita	-0.013 (0.034)	-0.098** (0.042)	-0.117*** (0.043)
Distance	-0.151* (0.088)	-0.143* (0.081)	-0.134** (0.068)
Year fixed effects	Yes	Yes	Yes
Industry fixed effects	Yes	Yes	Yes
Observations	307	578	483
R-squared	0.8528	0.8093	0.8007

Note: Heteroskedasticity consistent standard errors are reported in the parenthesis. ***, **, * denote significance at the 1,5,10 percent, respectively. Time, country and industry dummies are suppressed.

REER stands for real effective change rate.

could affect all plants in the same host country. We also assume an AR(1) process in the error-term, which is equivalent to including a lagged dependent variable but allows for the inclusion of plant-level fixed effects.

The first three columns present the results from using three alternative measures for exchange rates and a measure for transportation cost per shipment (CIF/FOB ratio). Columns 1 and 2 in Table 4.6 show that the coefficients on the real effective exchange rates are negatively and significantly associated with plant's inventory levels. This indicates that inventory levels of plants in the host country increase as foreign currency becomes weaker, which supports Hypothesis 4.3.3. For instance, we can see that a fall in real effective exchange rates of 10 percentage points increases the inventory level by 0.2% when using data from Bruegel and by 0.02% when using data from BIS. In the last column, we use foreign currency per Korean Won as a proxy for exchange rates. Its coefficient is positive and significant at the 5% level, implying that plants are likely to increase inventories as foreign currency becomes weaker which is also consistent with third hypothesis. Furthermore, all three columns show positive and significant coefficients on CIF/FOB ratio. These results indicate that the inventory level of foreign plant increases as transportation cost per shipment rises, which is consistent with Hypothesis 4.3.4.

The last three columns in Table 4.6 report the results from using same proxies for exchange rates and a proxy for transportation cost per unit measured by the distance between host country and Korea. As expected, all columns show similar coefficients on explanatory variables as from the first three columns. Exchange rates still have significant effects on plants' inventories, indicating that plants tend to increase their inventories as foreign currency becomes weaker. However, the coefficients on the distance are negative and

significant, indicating that plants decrease their inventories they are more distant from the destination country, which is Korea in our specification. In other words, plants are more likely to ship their products to the destination country as transportation cost per unit measured by physical barriers across countries decreases, which all support our hypotheses.

Turning to other covariates, it can be seen that plants tend to increase inventories as they are large in size and contain a large volume of inventories from the previous period. It is also interesting to find that as Korea parent firms have small equity share, plants are more likely to increase to inventories as they are more free to make decisions by themselves or in other words, as parent firms have less influence over plant manager's decision due to the weak ownership over the plant, such as when plants are established through joint-venture ownership. Furthermore, the coefficients on real GDP per capita are negative and significant, indicating that plants increase inventories in countries that incur low labor cost of production.

4.4 Conclusion

In this study, we explore the FDI decision as well as inventory decision of multi-national corporations when their home country is one of export-oriented countries with small domestic markets. For the long-term strategy, when economic growth and strength of the currency are positively (negatively) related, we find that the reason a firm whose home country without a selling market invests more to another country is because the sunk cost (labor cost) in the foreign country decreases. On the other hand, a firm whose home country with a selling market invests more to another country is because the labor cost (sunk cost) in the foreign country decreases. We verify these results using

Foreign Direct Investment Data in Korea from 2002 to 2011. For the short-term strategy, we consider a manufacturer's inventory level decision when the plant and the market are located in two different countries. We show that manufacturers increase the inventory levels as the exchange rate of the country where the plant is located grows weaker. We confirm this result by testing using the plant-level data of Korean multinational corporations provided by the Export-Import Bank of Korea.

Hence, a country whose economy is heavily dependent on exports (e.g., Korea or Taiwan) can increase its inward FDI by adjusting its labor policies, such as enabling flexible labor force. On the other hand, a country who has big market (e.g., the U.S. and China) can increase its inward FDI by reducing costs regarding to the sunk cost, such as lowering investment barriers or investment costs. In addition, by adjusting inventory level depending on the exchange rate, firms can increase profit.

While the findings from our work are interesting and useful for practice, we believe more studies can be conducted along this line of research. For instance, it is interesting to consider how the free trade agreements among countries can affect firms' long-term and short-term strategies.

Appendices

.1 Proofs of Chapter 2.

We denote the marginal probability density and cumulative distribution functions of D_{it} by $g_i(\cdot)$ and $G_i(\cdot)$, and the joint density and cumulative distribution functions of (D_{it}, D_{jt}) as $\hat{g}(\cdot)$ and $\hat{G}(\cdot)$. We define $BQ_{it}(x_{it}, x_{jt}) := \min\{(D_{it} - x_{it})^+, (x_{jt} - D_{jt})^+\}$ to measure the maximum borrowing quantity for firm i from firm j in period t or, identically, the maximum lending quantity from firm j to firm i . In addition, we define $\mathbb{P}_t := (F_t, S_t, D_{it})$. For notational convenience, let $M_{it}(x_{i,t+1}, \mathbb{P}_t) := \beta F_t x_{i,t+1} + \beta \mathbb{E}_t[V_{i,t+1}(x_{i,t+1}, \mathbb{P}_{t+1})]$ and $M_{it}^U(\mathbf{x}_{t+1}, \hat{\mathbb{P}}_t) := \beta F_t x_{i,t+1} + \beta \mathbb{E}_t[V_{i,t+1}^U(\mathbf{x}_{t+1}, \hat{\mathbb{P}}_{t+1})]$ where $V_{it}(x_{it}, \mathbb{P}_t)$ and $V_{it}^U(\mathbf{x}_t, \hat{\mathbb{P}}_t)$ denote the minimum cost of firm i without and with sharing inventory, respectively. Notice that $V_{it}^U(\mathbf{x}_t, \hat{\mathbb{P}}_t)$ represents the generalized cost function of firm i with sharing inventory (i.e., centralized or decentralized).

We first present Lemmas A1, A2, and A3. Lemma A1 will be used for the proof of Lemma A2. Lemma A2 will be used for the proofs of Propositions 1 and 2 as well as Lemma A3. To prove Propositions 1 and 2, Lemma A3 will be also used.

Lemma A1 $\beta \mathbb{E}_t[V_{it}(x_{it}, \mathbb{P}_t)]$ is finite for finite $(x_{it}, \mathbb{P}_t) \forall t$.

Proof of Lemma A1. Suppose (x_{it}, \mathbb{P}_t) is finite. Let $\bar{V}_{it}(x_{it}, \mathbb{P}_t)$ be the total expected cost when the entire procurement is done through the spot market; i.e., $\bar{V}_{iT}(x_{iT}, \mathbb{P}_T) = V_{iT}(x_{iT}, \mathbb{P}_T)$ and

$$\begin{aligned} \bar{V}_{it}(x_{it}, \mathbb{P}_t) &= (S_t + \tau_b)(D_{it} - x_{it})^+ + h(x_{it} - D_{it})^+ + \beta \mathbb{E}_t[\bar{V}_{t+1}(x_{i,t+1}, \mathbb{P}_{t+1})], \\ &\forall (x_{it}, \mathbb{P}_t) \in \mathcal{R}_+^4 \end{aligned}$$

where $x_{i,t+1} = (x_{it} - D_{it})^+$. Also, $x_{i,l+1} \leq x_{il}$ for $l = t, t+1, \dots, T-1$. Then,

by Assumptions 2 and 3,

$$\begin{aligned}\bar{V}_{it}(x_{it}, \mathbb{P}_t) &= \sum_{l=t}^T \mathbb{E}_t[\beta^{l-t}(S_l + \tau_b)(D_{il} - x_{il})^+] + \sum_{l=t}^T \mathbb{E}_t[\beta^{l-t}h(x_{il} - D_{il})^+] \\ &\leq \sum_{l=t}^T \beta^{l-t} \mathbb{E}_t[S_l D_{il}] + \tau_b \sum_{l=t}^T \beta^{l-t} \mathbb{E}_t[D_{il}] + h \sum_{l=t}^T \beta^{l-t} x_{it} < \infty\end{aligned}$$

Since $\bar{V}_{it}(x_{it}, \mathbb{P}_t) \geq V_{it}(x_{it}, \mathbb{P}_t)$, $V_{it}(x_{it}, \mathbb{P}_t)$ is finite $\forall t$. So is $\beta \mathbb{E}_t[V_t(x_{it}, \mathbb{P}_t)]$. \square

Lemma A2 In each period t , the cost function $V_{it}(x, \mathbb{P}_t)$ is convex in finite $x \in R_+$ for any given finite $\mathbb{P}_t \in R_+^3$. In addition, a unique $x_{i,t+1}^*$ exists that minimizes $F_t x_{i,t+1} + \mathbb{E}_t[V_{i,t+1}(x_{i,t+1}, \mathbb{P}_t)]$, and the optimal trading actions in the spot and forward markets follow: (i) $a_{it}^* = D_{it} - x_{it}$ if $C_t \geq \tau_b + \tau_s$, and $a_{it}^* = (D_{it} - x_{it})^+$ otherwise; (ii) $u_{it}^* = x_{i,t+1}^* - (x_{it} - D_{it}) - a_{it}^*$.

Proof of Lemma A2. Using backward induction, we show the convexity property for the model without sharing inventory.

In the last period T , since $\beta \mathbb{E}_T[V_{i,T+1}(x_{i,T+1}, \mathbb{P}_{T+1})] = 0$ and $V_{iT}(x_{iT}, \mathbb{P}_T) = (S_T + \tau_b)(D_{iT} - x_{iT})^+ - (S_T - \tau_s)(x_{iT} - D_{iT})^+$, $\beta \mathbb{E}_T[V_{i,T+1}(x_{i,T+1}, \mathbb{P}_{T+1})]$ and $V_{iT}(x_{iT}, \mathbb{P}_T)$ are convex in $x_{i,T+1}$ and x_{iT} , respectively.

Suppose the convexity property holds in periods $t+1, \dots, T-1$. Let $x_{i,t+1}^\phi \equiv \phi x_{i,t+1}^1 + (1-\phi)x_{i,t+1}^2$ for arbitrary $\phi \in [0, 1]$ and $x_{i,t+1}^1, x_{i,t+1}^2 \in \mathcal{R}_+$. The convexity of $V_{i,t+1}(x_{i,t+1}, \mathbb{P}_{t+1})$ in $x_{i,t+1}$ implies that

$$V_{i,t+1}(x_{i,t+1}^\phi, \mathbb{P}_{t+1}) \leq \phi V_{i,t+1}(x_{i,t+1}^1, \mathbb{P}_{t+1}) + (1-\phi)V_{i,t+1}(x_{i,t+1}^2, \mathbb{P}_{t+1}).$$

Taking expectations and discounting on both sides of the above inequality,

$$\beta \mathbb{E}_t[V_{i,t+1}(x_{i,t+1}^\phi, \mathbb{P}_{t+1})] \leq \phi \beta \mathbb{E}_t[V_{i,t+1}(x_{i,t+1}^1, \mathbb{P}_{t+1})] + (1-\phi) \beta \mathbb{E}_t[V_{i,t+1}(x_{i,t+1}^2, \mathbb{P}_{t+1})]$$

since $\beta\mathbb{E}_t[V_{i,t+1}(x_{i,t+1}, \mathbb{P}_{t+1})]$ is finite with finite $(x_{i,t+1}, \mathbb{P}_{t+1}) \in \mathcal{R}_+^4$ by Lemma A1. Hence, $\beta F_t x_{i,t+1} + \beta\mathbb{E}_t[V_{i,t+1}(x_{i,t+1}, \mathbb{P}_{t+1})]$ is convex in finite $x_{i,t+1} \in \mathcal{R}_+$ for any given finite $\mathbb{P}_t \in \mathcal{R}_+^3$ and there exists a unique solution, $x_{i,t+1}^*$, that minimizes $\beta F_t x_{i,t+1} + \beta\mathbb{E}_t[V_{i,t+1}(x_{i,t+1}, \mathbb{P}_{t+1})]$.

We show that $V_{it}(x_{it}, \mathbb{P}_t)$ is convex in $x_{it} \in \mathcal{R}_+$ in three steps. First, we show $x_{i,t+1}^*$ is nonnegative; second, we derive the optimal trading actions; finally, we establish the convexity result.

Step 1. $x_{i,t+1}^*$ is nonnegative: Let $x'_{i,t+1} \in \mathcal{R}_-$ and we have

$$\begin{aligned} M_{it}(0) &= \beta\mathbb{E}_t[(S_{t+1} + \tau_b)D_{i,t+1}] \\ &\quad + \min_{u_{i,t+1} \in \mathcal{R}_+} [\beta F_{t+1}u_{i,t+1} + \beta\mathbb{E}_{t+1}[V_{i,t+2}(u_{i,t+1}, \mathbb{P}_{t+2})]] \\ M_{it}(x'_{i,t+1}) &= \beta F_t x'_{i,t+1} + \beta\mathbb{E}_t[(S_{t+1} + \tau_b)(D_{i,t+1} - x'_{i,t+1})] \\ &\quad + \min_{u_{i,t+1} \in \mathcal{R}_+} [\beta F_{t+1}u_{i,t+1} + \beta\mathbb{E}_{t+1}[V_{i,t+2}(u_{i,t+1}, \mathbb{P}_{t+2})]] \end{aligned}$$

Hence, $M_{it}(0) - M_{it}(x'_{i,t+1}) = -\beta F_t x'_{i,t+1} + \beta\mathbb{E}_t[(S_{t+1} + \tau_b)x'_{i,t+1}] = -x'_{i,t+1}(\beta F_t - \beta\mathbb{E}_t[S_{t+1} + \tau_b])$. Given $F_t = \mathbb{E}_t[S_{t+1}]$, $M_{it}(0) - M_{it}(x'_{i,t+1}) \leq 0$, which implies $x_{i,t+1}^* \geq 0$.

Step 2. the optimal trading actions in period t : When $x_{it} < D_{it}$, $v_{it}(a_{it}, u_{it}; x_{it}, \mathbb{P}_t) = (S_t + \tau_b)a_{it} + \beta F_t u_{it} + h(a_{it} + x_{it} - D_{it})^+$. Due to the non-negative convenience yield property ($S_t + \tau_b + h \geq \beta F_t$), firm i buys sufficient in the spot market to meet the excess demand and orders in the forward market to install the optimal order-up-to level for period $t+1$, $x_{i,t+1}^*$. Hence, $a_{it}^* = -x_{it} + D_{it}$ and $u_{it}^* = x_{i,t+1}^*$ if $x_{it} < D_{it}$. When $x_{it} \geq D_{it}$, $v_{it}(a_{it}, u_{it}; x_{it}, \mathbb{P}_t) = (S_t - \tau_s + h)a_{it} + \beta F_t u_{it} + h(x_{it} - D_{it})$. If firm i sells the excess inventory in the spot market, then the per-unit revenue is $S_t - \tau_s + h$ and the per-unit cost is βF_t . Hence, if $C_t \geq \tau_b + \tau_s$ (or identically, $S_t - \tau_s + h \geq \beta F_t$), then $a_{it}^* = -x_{it} + D_{it}$ and $u_{it}^* = x_{i,t+1}^*$; otherwise, $a_{it}^* = 0$ and $u_{it}^* = x_{i,t+1}^* - (x_{it} - D_{it})$.

Step 3. $V_{it}(x_{it}, \mathbb{P}_t)$ is convex in $x_{it} \in \mathcal{R}_+$: If $C_t \geq \tau_b + \tau_s$, then, by Step 2,

$$\begin{aligned} V_{it}(x_{it}, \mathbb{P}_t) &= (S_t + \tau_b)(D_{it} - x_{it})^+ - (S_t - \tau_s)(x_{it} - D_{it})^+ \\ &\quad + \beta F_t x_{i,t+1}^* + \beta \mathbb{E}_t[V_{i,t+1}(x_{i,t+1}^*, \mathbb{P}_{t+1})]. \end{aligned}$$

By $-(S_t + \tau_b) \leq -(S_t - \tau_s)$, $(S_t + \tau_b)(D_{it} - x)^+ - (S_t - \tau_s)(x_{it} - D_{it})^+$ is convex in x_{it} . Hence, $V_{it}(x_{it}, \mathbb{P}_t)$ is convex in x_{it} . If $C_t < \tau_b + \tau_s$, then, by Step 2,

$$\begin{aligned} V_{it}(x_{it}, \mathbb{P}_t) &= (S_t + \tau_b)(D_{it} - x_{it})^+ + (h - \beta F_t)(x_{it} - D_{it})^+ + \beta F_t x_{i,t+1}^* \\ &\quad + \beta \mathbb{E}_t[V_{i,t+1}(x_{i,t+1}^*, \mathbb{P}_{t+1})]. \end{aligned}$$

By the assumption of nonnegative convenience yield, $S_t + \tau_b + h \geq \beta F_t$; i.e., $-(S_t + \tau_b) \leq (h - \beta F_t)$, which asserts that $V_{it}(x_{it}, \mathbb{P}_t)$ is convex in x_{it} . Note that $x_{i,t+1}^*$ is independent of x_{it} .

Hence, by induction, $\beta \mathbb{E}_t[V_{t+1}(x_{it}, \mathbb{P}_{t+1})]$ and $V_{it}(x_{it}, \mathbb{P}_t)$ are convex in finite $x_{it} \in \mathcal{R}_+$ for any given finite $\mathbb{P}_t \in \mathcal{R}_+^3$, $\forall t$. \square

Lemma A3 In each period t , given the optimal inventory sharing decisions and the optimal actions in the spot market, there exists a unique pair of order-up-to levels, $(x_{i,t+1}^U, x_{j,t+1}^U)$, such that given $x_{j,t+1}^U$, $x_{i,t+1}^U$ minimizes $F_t x_{i,t+1} + E_t[V_{i,t+1}^U(x_{i,t+1}, x_{j,t+1}^U, \hat{\mathbb{P}}_{t+1})]$ and given $x_{i,t+1}^U$, $x_{j,t+1}^U$ minimizes $F_t x_{j,t+1} + E_t[V_{j,t+1}^U(x_{j,t+1}, x_{i,t+1}^U, \hat{\mathbb{P}}_{t+1})]$.

Proof of Lemma A3. We prove this lemma by backward induction.

In period $(T - 1)$, firm i derives x_{iT}^U to minimize $M_{i,T-1}^U(\mathbf{x}_T, \hat{\mathbb{P}}_T)$. By Assumption 1,

$$\begin{aligned} M_{i,T-1}^U(\mathbf{x}_T, \hat{\mathbb{P}}_T) &= \beta F_{T-1} x_{iT} + \beta \mathbb{E}_{T-1}[S_T + \tau_b] \mathbb{E}_{T-1}[(D_{iT} - x_{iT})^+] \\ &\quad - \beta \mathbb{E}_{T-1}[S_T - \tau_s] \mathbb{E}_{T-1}[(x_{iT} - D_{iT})^+]. \end{aligned}$$

Taking the first derivative of $M_{i,T-1}^U(\mathbf{x}_T, \hat{\mathbb{P}}_T)$ with respect to x_{iT} yields:

$$\frac{\partial M_{i,T-1}^U(\mathbf{x}_T, \hat{\mathbb{P}}_T)}{\partial x_{iT}} = \beta F_{T-1} - \beta \mathbb{E}_{T-1}[S_T + \tau_b](1 - G_i(x_{iT})) - \beta \mathbb{E}_{T-1}[S_T - \tau_s]G_i(x_{iT}).$$

Hence, given $\mathbb{E}_{T-1}[S_T] = F_{T-1}$, firm i 's optimal order-up-to level for period T satisfies the following equation:

$$G_i(x_{iT}^U) = \frac{\beta \mathbb{E}_{T-1}[S_T + \tau_b] - \beta F_{T-1}}{\beta \mathbb{E}_{T-1}[\tau_b + \tau_s]} = \frac{\tau_b}{\tau_b + \tau_s}.$$

The analysis for firm j is analogous. Therefore, there exists a unique pair of order-up-to levels in period $(T-1)$, $(x_{iT}^U, x_{jT}^U) = (G_i^{-1}(\frac{\tau_b}{\tau_b + \tau_s}), G_j^{-1}(\frac{\tau_b}{\tau_b + \tau_s}))$ where $G_i^{-1}(\cdot)$ and $G_j^{-1}(\cdot)$ are inverse functions of $G_i(\cdot)$ and $G_j(\cdot)$, respectively.

Suppose a unique pair of order-up-to levels exists in periods $t+1, \dots, T-2$; i.e., $(x_{i,t+2}^U, x_{j,t+2}^U), \dots, (x_{i,T-1}^U, x_{j,T-1}^U)$ exist. We show the existence of a unique pair of order-up-to levels in period t , $(x_{i,t+1}^U, x_{j,t+1}^U)$ by showing that the two best-response functions, $x_{i,t+1}^U(x_{j,t+1})$ and $x_{j,t+1}^U(x_{i,t+1})$, are monotone and the absolute value of the slope is less than 1 (see Fudenberg and Tirole 1991).

From the optimal actions in Lemma A2, we can derive the expectation of the immediate cost function for period $t+1$ in the model of without sharing inventory as the following:

$$\mathbb{E}_t[m_{i,t+1}(x_{i,t+1})] + \mathbb{E}_t[\beta F_{t+1} x_{i,t+2}^U]$$

where

$$\begin{aligned} \mathbb{E}_t[m_{i,t+1}(x_{i,t+1})] &:= \mathbb{E}_t[(S_{t+1} + \tau_b)]\mathbb{E}_t[(D_{i,t+1} - x_{i,t+1})^+] \\ &\quad - \mathbb{E}_t[(\beta F_{t+1} - h)\mathbb{I}_{\{C_{t+1} < \tau_b + \tau_s\}}|C_t]\mathbb{E}_t[(x_{i,t+1} - D_{i,t+1})^+] \\ &\quad - \mathbb{E}_t[(S_{t+1} - \tau_s)\mathbb{I}_{\{C_{t+1} \geq \tau_b + \tau_s\}}|C_t]\mathbb{E}_t[(x_{i,t+1} - D_{i,t+1})^+]. \end{aligned}$$

Then, the expectation of the immediate cost function for period $t + 1$ in the model of with sharing inventory follows

$$\begin{aligned} & \mathbb{E}_t[m_{i,t+1}(x_{i,t+1})] + \mathbb{E}_t[\beta F_{t+1} \hat{x}_{i,t+2}^*] - \mathbb{E}_t[GB_{t+1}] \mathbb{E}_t[BQ_{i,t+1}(x_{i,t+1}, x_{j,t+1})] \\ & - \mathbb{E}_t[GL_{t+1}] \mathbb{E}_t[BQ_{j,t+1}(x_{j,t+1}, x_{i,t+1})], \end{aligned}$$

where $\mathbb{E}_t[GB_{t+1}]$ and $\mathbb{E}_t[GL_{t+1}]$ denote the expected unit cost reduction by borrowing and lending, respectively. Hence,

$$\begin{aligned} M_{it}^U(x_{i,t+1}, x_{j,t+1}) &= \beta F_t x_{i,t+1} + \beta \mathbb{E}_t[m_{i,t+1}(x_{i,t+1})] + \beta \mathbb{E}_t[M_{i,t+1}^U(x_{i,t+2}^U, x_{j,t+2}^U)] \\ & - \beta \mathbb{E}_t[GB_{t+1}] \mathbb{E}_t[BQ_{i,t+1}(x_{i,t+1}, x_{j,t+1})] \\ & - \beta \mathbb{E}_t[GL_{t+1}] \mathbb{E}_t[BQ_{j,t+1}(x_{j,t+1}, x_{i,t+1})]. \end{aligned}$$

The first order condition of $M_{it}^U(x_{i,t+1}, x_{j,t+1})$ with respect to $x_{i,t+1}$ satisfies

$$\begin{aligned} \frac{\partial M_{it}^U(x_{i,t+1}, x_{j,t+1})}{\partial x_{i,t+1}} &= \beta F_t + \frac{\partial \beta \mathbb{E}_t[m_{i,t+1}(x_{i,t+1})]}{\partial x_{i,t+1}} \\ & - \beta \mathbb{E}_t[GB_{t+1}] \frac{\partial \mathbb{E}_t[BQ_{i,t+1}(x_{i,t+1}, x_{j,t+1})]}{\partial x_{i,t+1}} \\ & - \beta \mathbb{E}_t[GL_{t+1}] \frac{\partial \mathbb{E}_t[BQ_{j,t+1}(x_{j,t+1}, x_{i,t+1})]}{\partial x_{i,t+1}} = 0. \end{aligned}$$

Using implicit differentiation, the slope of firm i 's best-response function follows

$$\frac{\partial x_{i,t+1}^U(x_{j,t+1})}{\partial x_{j,t+1}} = - \frac{\frac{\partial^2 M_{it}^U(x_{i,t+1}, x_{j,t+1})}{\partial x_{i,t+1} \partial x_{j,t+1}}}{\frac{\partial^2 M_{it}^U(x_{i,t+1}, x_{j,t+1})}{\partial^2 x_{i,t+1}}},$$

where

$$\begin{aligned}
\frac{\partial^2 M_{it}^U(x_{i,t+1}, x_{j,t+1})}{\partial x_{i,t+1} \partial x_{j,t+1}} &= -\beta \mathbb{E}_t[GB_{t+1}] \frac{\partial^2 \mathbb{E}_t[BQ_{i,t+1}(x_{i,t+1}, x_{j,t+1})]}{\partial x_{i,t+1} \partial x_{j,t+1}} \\
&\quad -\beta \mathbb{E}_t[GL_{t+1}] \frac{\partial^2 \mathbb{E}_t[BQ_{j,t+1}(x_{j,t+1}, x_{i,t+1})]}{\partial x_{i,t+1} \partial x_{j,t+1}}, \\
\frac{\partial^2 M_{it}^U(x_{i,t+1}, x_{j,t+1})}{\partial^2 x_{i,t+1}} &= \frac{\partial^2 \beta \mathbb{E}_t[m_{i,t+1}(x_{i,t+1})]}{\partial^2 x_{i,t+1}} \\
&\quad -\beta \mathbb{E}_t[GB_{t+1}] \frac{\partial^2 \mathbb{E}_t[BQ_{i,t+1}(x_{i,t+1}, x_{j,t+1})]}{\partial^2 x_{i,t+1}} \\
&\quad -\beta \mathbb{E}_t[GL_{t+1}] \frac{\partial^2 \mathbb{E}_t[BQ_{j,t+1}(x_{j,t+1}, x_{i,t+1})]}{\partial^2 x_{i,t+1}}.
\end{aligned}$$

We can show that $\frac{\partial^2 M_{it}^U(x_{i,t+1}, x_{j,t+1})}{\partial x_{i,t+1} \partial x_{j,t+1}} \geq 0$ and $\frac{\partial^2 M_{it}^U(x_{i,t+1}, x_{j,t+1})}{\partial^2 x_{i,t+1}} \geq \frac{\partial^2 M_{it}^U(x_{i,t+1}, x_{j,t+1})}{\partial x_{i,t+1} \partial x_{j,t+1}}$. Thus, $\frac{\partial x_{i,t+1}^U(x_{j,t+1})}{\partial x_{j,t+1}}$ is non-positive and the absolute value of $\frac{\partial x_{i,t+1}^U(x_{j,t+1})}{\partial x_{j,t+1}}$ is less than 1. The same properties for $x_{j,t+1}^U(x_{i,t+1})$ can be shown. Hence, by induction, there exists a unique pair of order-up-to levels, $(x_{i,t+1}^U, x_{j,t+1}^U) \forall t$. \square

Proof of Proposition 1. The optimal inventory sharing actions can be derived as the following. The cost reduction by borrowing inventory from the other firm is the cost to buy directly from the spot market, $S_t + \tau_b$, less the total unit cost to borrow and return the inventory, $\tau_o + \beta F_t$. That is, $C_t - h - \tau_o$ is the unit cost reduction for the borrowing firm. At the same time, the cost reduction by lending excess inventory to the other firm is the gain from lending, $h + \beta F_t$, less the gain from selling the inventory to the spot market, $S_t - \tau_s + h$, or selling it to the forward market, βF_t . Notice that the gain from the selling inventory to the spot market is greater than that to the forward market if $C_t \geq \tau_b + \tau_s$ (i.e., $S_t - \tau_s + h - \beta F_t \geq 0$). Hence, when $C_t \geq \tau_b + \tau_s$, the unit cost reduction for the lending firm is $\tau_b + \tau_s + h - C_t$; otherwise, the unit cost reduction is h . Hence, if $C_t \geq \tau_b + \tau_s$, then the total

cost reduction by inventory sharing for the centralized case is $\tau_b + \tau_s - \tau_o$ per unit; otherwise, the total cost reduction is $C_t - \tau_o$. Since $\tau_b + \tau_s - \tau_o$ is always positive by assumption, inventory sharing will arise in the centralized case as long as $C_t \geq \tau_o$.

Hence, $\mathbb{E}_t[IS_{t+1}] (= \mathbb{E}_t[(C_{t+1} - \tau_o)\mathbb{I}_{\{\tau_o \leq C_{t+1} < \tau_b + \tau_s\}}|C_t] + \mathbb{E}_t[(\tau_b + \tau_s - \tau_o)\mathbb{I}_{\{C_{t+1} \geq \tau_b + \tau_s\}}|C_t])$ denotes the expected total cost reduction per unit by inventory sharing. We can show the existence of the unique pair of order-up-to levels by substituting $\mathbb{E}_t[IS_t]$ into both $\mathbb{E}_t[GB_t]$ and $\mathbb{E}_t[GL_t]$ as well as $V_{it}^C(x_{it}, x_{jt}^U, \hat{\mathbb{P}}_t)$ into $V_{it}^U(x_{it}, x_{jt}^U, \hat{\mathbb{P}}_t)$ for all t in Lemma A3. In addition, the optimal actions in the spot and forward markets are analogous to Lemma A2. \square

Proof of Proposition 2. As we have explained in Proposition 1, the unit cost reduction by borrowing is $C_t - h - \tau_o$. In addition, if $C_t \geq \tau_b + \tau_s$, the unit cost reduction by lending is $\tau_b + \tau_s + h - C_t$; otherwise, the unit cost reduction is h . Since h is positive, firms share inventory when $h + \tau_o \leq C_t \leq \tau_b + \tau_s + h$. Hence, $\mathbb{E}_t[B_{t+1}] (= \mathbb{E}_t[(C_{t+1} - h - \tau_o)\mathbb{I}_{\{C_{t+1} \in (h + \tau_o, h + \tau_b + \tau_s)\}}|C_t])$ and $\mathbb{E}_t[L_{t+1}] (= \mathbb{E}_t[h\mathbb{I}_{\{C_{t+1} \in (h + \tau_o, \tau_b + \tau_s)\}} + (h + \tau_b + \tau_s - C_{t+1})\mathbb{I}_{\{C_{t+1} \in (\max\{h + \tau_o, \tau_b + \tau_s\}, \tau_b + \tau_s + h)\}}|C_t])$ denote the expected unit cost reductions by borrowing and lending transactions, respectively. We can show the existence of the unique pair of order-up-to levels can be shown by substituting $\mathbb{E}_t[B_{t+1}]$ and $\mathbb{E}_t[L_{t+1}]$ into $\mathbb{E}_t[GB_{t+1}]$ and $\mathbb{E}_t[GL_{t+1}]$, respectively, as well as $\hat{V}_{it}(x_{it}, x_{jt}^U, \hat{\mathbb{P}}_t)$ into $V_{it}^U(x_{it}, x_{jt}^U, \hat{\mathbb{P}}_t)$ for all t in Lemma A3. In addition, the optimal actions in the spot and forward markets are analogous to Lemma A2. \square

Proof of Lemma 1. We show Lemma 1 as follows:

i) $C_t \geq \tau_b + \tau_s$

The lending and borrowing firms' unit cost reductions by sharing inventory are $\tau_b + \tau_s + h - C_t - \tau_{k,t}$ and $C_t - h - \tau_o + \tau_{k,t}$, respectively. Hence, if $h + \tau_o - C_t \leq \tau_{k,t} \leq \tau_b + \tau_s + h - C_t$, then the two firms are willing to share inventory. Notice that by setting $\tau_{k,t} = -C_t + h + \frac{\tau_b + \tau_s + \tau_o}{2}$, each firm's sharing benefit becomes $\frac{\tau_b + \tau_s - \tau_o}{2}$.

ii) $\tau_o \leq C_t < \tau_b + \tau_s$

The lending and borrowing firms' unit cost reductions by sharing inventory are $h - \tau_{k,t}$ and $C_t - h - \tau_o + \tau_{k,t}$, respectively. Hence, if $h + \tau_o - C_t \leq \tau_{k,t} \leq h$, then the two firms are willing to share inventory. Notice that by setting $\tau_{k,t} = -\frac{C_t}{2} + h + \frac{\tau_o}{2}$, each firm's sharing benefit becomes $\frac{C_t - \tau_o}{2}$.

By i) and ii), Lemma 1 is proved. \square

Proof of Proposition 3. Let us first analyze the optimal actions without sharing inventory. Since the leftover inventory is always less than the optimal order-up-to level, the cost if firms hold leftover inventory is $h(x_{it} - D_{it})^+ + \beta(F_t + \tau_{FB})(x_{i,t+1}^* - (x_{it} - D_{it})^+)$; the cost if firms sell leftover inventory to the forward market is $-\beta(F_t - \tau_{FS})(x_{it} - D_{it})^+ + \beta(F_t + \tau_{FB})x_{i,t+1}^*$; the cost if firms sell leftover inventory to the spot market is $-(S_t - \tau_S + h)(x_{it} - D_{it})^+ + \beta(F_t + \tau_{FB})x_{i,t+1}^*$. Hence, the cost of selling leftover inventory to the forward market is always beneficial than holding it because we assume $h > \beta(\tau_{FB} + \tau_{FS})$. Moreover, since the cost difference between selling to the forward and spot markets is $\{(S_t - \tau_S + h) - \beta(F_t - \tau_{FS})\}(x_{it} - D_{it})^+ = (C_t^F - \tau_b - \tau_s)(x_{it} - D_{it})^+$, firms sell leftover inventory to the spot market if $C_t^F \geq \tau_b + \tau_s$; otherwise, firms sell it to the forward market.

With the inventory sharing option, the unit cost reduction by bor-

rowing is $(S_t + \tau_b) - (\tau_o + \beta(F_t + \tau_{FB})) = C_t^F - h - \tau_o - \beta(\tau_{FB} + \tau_{FS})$. The unit cost reduction by lending is $(h + \beta(F_t + \tau_{FB})) - (S_t + h - \tau_s) = (\tau_b + \tau_s + h + \beta(\tau_{FB} + \tau_{FS})) - C_t^F$ if $C_t^F \geq \tau_b + \tau_s$; otherwise, the cost reduction is $(h + \beta(F_t + \tau_{FB})) - \beta(F_t - \tau_{FS}) = h + \beta(\tau_{FB} + \tau_{FS})$. Hence, in the case of inter-firm inventory sharing, firms are willing to share inventory if $h + \tau_o + \beta(\tau_{FB} + \tau_{FS}) \leq C_t^F \leq \tau_b + \tau_s + h + \beta(\tau_{FB} + \tau_{FS})$. In the centralized inventory sharing case, the unit cost reduction is $\tau_b + \tau_s - \tau_o$ if $C_t^F \geq \tau_b + \tau_s$; the unit cost reduction is $C_t^F - \tau_o$ if $\tau_o \leq C_t^F < \tau_b + \tau_s$. Hence, the inventory sharing condition in the centralized case is $C_t^F \geq \tau_o$. \square

Proof of Lemma 2. The probability that the convenience yield satisfies the inventory sharing condition during period $t + 1$ given C_t is

$$P[C_{t+1} > \tau_o | C_t] = 1 - \Phi\left(\frac{\tau_o - \mu_c}{\sigma_c}\right).$$

The first derivative of the above probability with respect to λ , σ_δ , τ_b , τ_s , and

τ_o are the following:

$$\begin{aligned}
\frac{\partial P[C_{t+1} > \tau_o | C_t]}{\partial \lambda} &= \phi\left(\frac{\tau_o - \mu_c}{\sigma_c}\right) \cdot \left(\frac{1}{\sigma_c}\right) \cdot \frac{\partial \mu_c}{\partial \lambda} \\
&= -\phi\left(\frac{\tau_o - \mu_c}{\sigma_c}\right) \cdot \left(\frac{1}{\sigma_c}\right) \cdot \frac{(1 - e^{-\kappa t})}{\kappa} \\
\frac{\partial P[C_{t+1} > \tau_o | C_t]}{\partial \sigma_\delta} &= \phi\left(\frac{\tau_o - \mu_c}{\sigma_c}\right) \cdot \left(\frac{\tau_o - \mu_c}{\sigma_c^2}\right) \cdot \frac{\partial \sigma_c}{\partial \sigma_\delta} \\
&= \phi\left(\frac{\tau_o - \mu_c}{\sigma_c}\right) \cdot \left(\frac{\tau_o - \mu_c}{\sigma_c^2}\right) \cdot \sqrt{\frac{1 - e^{-2\kappa t}}{2\kappa}}, \\
\frac{\partial P[C_{t+1} > \tau_o | C_t]}{\partial \tau_b} &= \phi\left(\frac{\tau_o - \mu_c}{\sigma_c}\right) \cdot \left(\frac{1}{\sigma_c}\right) \cdot \frac{\partial \mu_c}{\partial \tau_b} = \phi\left(\frac{\tau_o - \mu_c}{\sigma_c}\right) \cdot \left(\frac{1}{\sigma_c}\right), \\
\frac{\partial P[C_{t+1} > \tau_o | C_t]}{\partial \tau_s} &= \phi\left(\frac{\tau_o - \mu_c}{\sigma_c}\right) \cdot \left(\frac{1}{\sigma_c}\right) \cdot \frac{\partial \mu_c}{\partial \tau_s} = 0, \\
\frac{\partial P[C_{t+1} > \tau_o | C_t]}{\partial \tau_o} &= -\phi\left(\frac{\tau_o - \mu_c}{\sigma_c}\right) \cdot \left(\frac{1}{\sigma_c}\right) \cdot \frac{\partial(\tau_o - \mu_c)}{\partial \tau_o} \\
&= -\phi\left(\frac{\tau_o - \mu_c}{\sigma_c}\right) \cdot \left(\frac{1}{\sigma_c}\right) \cdot \square
\end{aligned}$$

Proof of Lemma 3. Let us denote $EPVIS_{t+1}$ be the expected per-unit

value of inventory sharing during period $t + 1$ given C_t .

$$\begin{aligned}
EPVIS_{t+1} &= \mathbb{E}_t[(C_{t+1} - \tau_o)\mathbb{I}_{\{\tau_o \leq C_t < \tau_b + \tau_s\}} | C_t] \\
&\quad + \mathbb{E}_t[(\tau_b + \tau_s - \tau_o)\mathbb{I}_{\{C_t \geq \tau_b + \tau_s\}} | C_t] \\
&= \int_{\tau_o}^{\tau_b + \tau_s} (C_{t+1} - \tau_o) dC_{t+1} + \int_{\tau_b + \tau_s}^{\infty} (\tau_b + \tau_s - \tau_o) dC_{t+1} \\
&= \int_{\tau_o}^{\tau_b + \tau_s} C_{t+1} dC_{t+1} - \tau_o \int_{\tau_o}^{\tau_b + \tau_s} dC_{t+1} \\
&\quad + (\tau_b + \tau_s - \tau_o) \int_{\tau_b + \tau_s}^{\infty} dC_{t+1} \\
&= \int_{\tau_o}^{\tau_b + \tau_s} C_{t+1} dC_{t+1} \\
&\quad - \tau_o \left\{ \Phi \left(\frac{\tau_b + \tau_s - \mu_c}{\sigma_c} \right) - \Phi \left(\frac{\tau_o - \mu_c}{\sigma_c} \right) \right\} \\
&\quad + (\tau_b + \tau_s - \tau_o) \left(1 - \Phi \left(\frac{\tau_b + \tau_s - \mu_c}{\sigma_c} \right) \right) \\
&= \int_0^{\tau_b + \tau_s} C_{t+1} dC_{t+1} - \int_0^{\tau_o} C_{t+1} dC_{t+1} \\
&\quad - \tau_o \left\{ \Phi \left(\frac{\tau_b + \tau_s - \mu_c}{\sigma_c} \right) - \Phi \left(\frac{\tau_o - \mu_c}{\sigma_c} \right) \right\} \\
&\quad + (\tau_b + \tau_s - \tau_o) \left(1 - \Phi \left(\frac{\tau_b + \tau_s - \mu_c}{\sigma_c} \right) \right).
\end{aligned}$$

Since C_t is non-negative $\forall t$,

$$\int_0^{\tau_b + \tau_s} C_{t+1} dC_{t+1} = \int_{-\infty}^{\tau_b + \tau_s} C_{t+1} dC_{t+1}.$$

In addition, by letting $u_{t+1} = (C_{t+1} - \mu_c)/\sigma_c$,

$$\begin{aligned}
\int_0^{\tau_b + \tau_s} C_{t+1} dC_{t+1} &= \int_{-\infty}^{\tau_b + \tau_s} C_{t+1} \cdot \frac{1}{\sigma_c \sqrt{2\pi}} e^{-\frac{(C_{t+1} - \mu_c)^2}{2\sigma_c^2}} dC_{t+1} \\
&= \int_{-\infty}^{\frac{\tau_b + \tau_s - \mu_c}{\sigma_c}} (\sigma_c u_{t+1} + \mu_c) \frac{1}{\sqrt{2\pi}} e^{-\frac{u_{t+1}^2}{2}} du_{t+1} \\
&= -\frac{\sigma_c}{\sqrt{2\pi}} e^{-\frac{1}{2} \left(\frac{\tau_b + \tau_s - \mu_c}{\sigma_c} \right)^2} + \mu_c \Phi \left(\frac{\tau_b + \tau_s - \mu_c}{\sigma_c} \right).
\end{aligned}$$

Similarly,

$$\int_0^{\tau_o} C_{t+1} dC_{t+1} = -\frac{\sigma_c}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\tau_o - \mu_c}{\sigma_c}\right)^2} + \mu_c \Phi\left(\frac{\tau_o - \mu_c}{\sigma_c}\right).$$

Hence,

$$\begin{aligned} EPVIS_{t+1} &= -\frac{\sigma_c}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\tau_b + \tau_s - \mu_c}{\sigma_c}\right)^2} + \frac{\sigma_c}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\tau_o - \mu_c}{\sigma_c}\right)^2} \\ &\quad + \mu_c \Phi\left(\frac{\tau_b + \tau_s - \mu_c}{\sigma_c}\right) \\ &\quad - \mu_c \Phi\left(\frac{\tau_o - \mu_c}{\sigma_c}\right) - \tau_o \left\{ \Phi\left(\frac{\tau_b + \tau_s - \mu_c}{\sigma_c}\right) - \Phi\left(\frac{\tau_o - \mu_c}{\sigma_c}\right) \right\} \\ &\quad + (\tau_b + \tau_s - \tau_o) \left(1 - \Phi\left(\frac{\tau_b + \tau_s - \mu_c}{\sigma_c}\right) \right) \\ &= -\sigma_c \left\{ \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\tau_b + \tau_s - \mu_c}{\sigma_c}\right)^2} - \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\tau_o - \mu_c}{\sigma_c}\right)^2} \right\} \\ &\quad - (\tau_b + \tau_s - \mu_c) \cdot \Phi\left(\frac{\tau_b + \tau_s - \mu_c}{\sigma_c}\right) \\ &\quad + (\tau_o - \mu_c) \cdot \Phi\left(\frac{\tau_o - \mu_c}{\sigma_c}\right) \\ &\quad + (\tau_b + \tau_s - \tau_o) \end{aligned}$$

The first derivative of $EPVIS_{t+1}$ with respect to λ , σ_δ , τ_b , τ_s , and τ_o are the

following:

$$\begin{aligned}
& \frac{\partial EPVIS_{t+1}}{\partial \lambda} \\
&= -\frac{\sigma_c}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\tau_b + \tau_s - \mu_c}{\sigma_c}\right)^2} \cdot \left(-\frac{\tau_b + \tau_s - \mu_c}{\sigma_c}\right) \cdot \left(-\frac{1}{\sigma_c}\right) \cdot \frac{\partial \mu_c}{\partial \lambda} \\
&+ \frac{\sigma_c}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\tau_o - \mu_c}{\sigma_c}\right)^2} \cdot \left(-\frac{\tau_o - \mu_c}{\sigma_c}\right) \cdot \left(-\frac{1}{\sigma_c}\right) \cdot \frac{\partial \mu_c}{\partial \lambda} \\
&+ \Phi\left(\frac{\tau_b + \tau_s - \mu_c}{\sigma_c}\right) \cdot \frac{\partial \mu_c}{\partial \lambda} \\
&- (\tau_b + \tau_s - \mu_c) \cdot \phi\left(\frac{\tau_b + \tau_s - \mu_c}{\sigma_c}\right) \cdot \left(-\frac{1}{\sigma_c}\right) \cdot \frac{\partial \mu_c}{\partial \lambda} \\
&- \Phi\left(\frac{\tau_o - \mu_c}{\sigma_c}\right) \cdot \frac{\partial \mu_c}{\partial \lambda} \\
&+ (\tau_o - \mu_c) \cdot \phi\left(\frac{\tau_o - \mu_c}{\sigma_c}\right) \cdot \left(-\frac{1}{\sigma_c}\right) \cdot \frac{\partial \mu_c}{\partial \lambda} \\
&= -\left\{ \Phi\left(\frac{\tau_b + \tau_s - \mu_c}{\sigma_c}\right) - \Phi\left(\frac{\tau_o - \mu_c}{\sigma_c}\right) \right\} \cdot \frac{(1 - e^{-\kappa t})}{\kappa}
\end{aligned}$$

$$\begin{aligned}
& \frac{\partial EPVIS_{t+1}}{\partial \sigma_\delta} \\
&= -\frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{1}{2}\left(\frac{\tau_b + \tau_s - \mu_c}{\sigma_c}\right)^2} \cdot \frac{\partial \sigma_c}{\partial \sigma_\delta} \\
&\quad - \frac{1}{\sqrt{2\pi}} \cdot \sigma_c \cdot e^{-\frac{1}{2}\left(\frac{\tau_b + \tau_s - \mu_c}{\sigma_c}\right)^2} \cdot (\tau_b + \tau_s - \mu_c)^2 \sigma_c^{-3} \cdot \frac{\partial \sigma_c}{\partial \sigma_\delta} \\
&\quad + \frac{1}{\sqrt{2\pi}} \cdot \left(e^{-\frac{1}{2}\left(\frac{\tau_o - \mu_c}{\sigma_c}\right)^2} + \sigma_c \cdot e^{-\frac{1}{2}\left(\frac{\tau_o - \mu_c}{\sigma_c}\right)^2} \cdot (\tau_o - \mu_c)^2 \sigma_c^{-3} \right) \cdot \frac{\partial \sigma_c}{\partial \sigma_\delta} \\
&\quad - (\tau_b + \tau_s - \mu_c) \cdot \phi\left(\frac{\tau_b + \tau_s - \mu_c}{\sigma_c}\right) \cdot (\tau_b + \tau_s - \mu_c)(-\sigma_c^2) \cdot \frac{\partial \sigma_c}{\partial \sigma_\delta} \\
&\quad + (\tau_o - \mu_c) \cdot \phi\left(\frac{\tau_o - \mu_c}{\sigma_c}\right) \cdot (\tau_o - \mu_c)(-\sigma_c^2) \cdot \frac{\partial \sigma_c}{\partial \sigma_\delta} \\
&= -\left\{ \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{1}{2}\left(\frac{\tau_b + \tau_s - \mu_c}{\sigma_c}\right)^2} - \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{1}{2}\left(\frac{\tau_o - \mu_c}{\sigma_c}\right)^2} \right\} \cdot \sqrt{\frac{1 - e^{-2\kappa t}}{2\kappa}} \\
&= -\left\{ \phi\left(\frac{\tau_b + \tau_s - \mu_c}{\sigma_c}\right) - \phi\left(\frac{\tau_o - \mu_c}{\sigma_c}\right) \right\} \cdot \sqrt{\frac{1 - e^{-2\kappa t}}{2\kappa}} \\
& \frac{\partial EPVIS_{t+1}}{\partial \tau_b} \\
&= \frac{\sigma_c}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\tau_o - \mu_c}{\sigma_c}\right)^2} \sigma_c^{-2} (\tau_o - \mu_c) \left(-\frac{\partial \mu_c}{\partial \tau_b} \right) \\
&\quad - \frac{\partial \mu_c}{\partial \tau_b} \cdot \Phi\left(\frac{\tau_o - \mu_c}{\sigma_c}\right) + (\tau_o - \mu_c) \phi\left(\frac{\tau_o - \mu_c}{\sigma_c}\right) \sigma_c^{-1} \left(-\frac{\partial \mu_c}{\partial \tau_b} \right) + 1 \\
&= 1 - \Phi\left(\frac{\tau_o - \mu_c}{\sigma_c}\right) \\
& \frac{\partial EPVIS_{t+1}}{\partial \tau_s} \\
&= -\frac{\sigma_c}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\tau_b + \tau_s - \mu_c}{\sigma_c}\right)^2} \cdot \left(-\frac{\tau_b + \tau_s - \mu_c}{\sigma_c} \right) \cdot \left(\frac{1}{\sigma_c} \right) \\
&\quad - \left\{ \Phi\left(\frac{\tau_b + \tau_s - \mu_c}{\sigma_c}\right) + (\tau_b + \tau_s - \mu_c) \cdot \phi\left(\frac{\tau_b + \tau_s - \mu_c}{\sigma_c}\right) \cdot \frac{1}{\sigma_c} \right\} \\
&\quad + 1 \\
&= 1 - \Phi\left(\frac{\tau_b + \tau_s - \mu_c}{\sigma_c}\right)
\end{aligned}$$

$$\begin{aligned}
& \frac{\partial EPVIS_{t+1}}{\partial \tau_o} \\
&= \frac{\sigma_c}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\tau_o - \mu_c}{\sigma_c}\right)^2} \sigma_c^{-2} (\tau_o - \mu_c) + \Phi\left(\frac{\tau_o - \mu_c}{\sigma_c}\right) \\
&+ (\tau_o - \mu_c) \phi\left(\frac{\tau_o - \mu_c}{\sigma_c}\right) \sigma_c^{-1} - 1 \\
&= -\left(1 - \Phi\left(\frac{\tau_o - \mu_c}{\sigma_c}\right)\right) \cdot \square
\end{aligned}$$

.2 Proofs of Chapter 3.

In the proofs of the monopoly case, we omit the notation j for notational convenience since there is only one store in the market. Similarly, in the proofs of the monopolistic competition with symmetric market share case, we also omit the notation j for notational convenience since the retailers are symmetric.

Proof of Proposition 1. We prove Proposition 1 in two steps. First, we set the selling price; second, we derive the optimal number of stores and serving area. From §3.1, the monopolist's profit is

$$Z = \lambda_r(p - g_r) - \sqrt{2\lambda_r\tau_r d_r h_r}. \quad (1)$$

Step 1. Selling Price: Given b , the farthest customer's round-trip distance is

$$d_c^F = 2\sqrt{b^2 + b^2(\tan \theta)^2} \quad (2)$$

and thus, from Equation (2), the farthest customer's utility is

$$\begin{aligned} U_c^F &= \lambda_c(u_c - p) - \sqrt{2\lambda_c\tau_c d_c^F h_c} \\ &= \lambda_c(u_c - p) - \sqrt{2\lambda_c\tau_c h_c (2b)^{1/2} (1 + (\tan \theta)^2)^{1/4}}. \end{aligned} \quad (3)$$

Since we assume that the selling price is the price that makes the farthest customer's utility be zero, from Equation (3), the selling price is

$$p = u_c - \sqrt{\frac{2\tau_c h_c d_c^F}{\lambda_c}} \quad (4)$$

$$= u_c - \sqrt{\frac{2\tau_c h_c}{\lambda_c}} (2b)^{1/2} (1 + (\tan \theta)^2)^{1/4}. \quad (5)$$

Step 2. Number of Stores and Market Area: The market area covered by the monopolist r is

$$r = (1/2)(b^2 \tan \theta)2sn = (\tan \theta)sb^2n, \quad (6)$$

where $\frac{1}{2}b^2(\tan \theta)(2s)$ is the area of the regular polygon.

Hence, from Equation (6), the monopolist's demand rate λ_r is

$$\lambda_r = r\lambda_c = (\tan \theta)sb^2n\lambda_c \quad (7)$$

because we normalize the consumer density to one.

By following the distribution assumption in Cachon (2013), the monopolist's distribution distance is

$$d_r = 2bn. \quad (8)$$

By substituting Equations (5), (7), and (8) into the monopolist's profit function Z (Equation (1)),

$$\begin{aligned} Z(n, b) &= \lambda_r(p - g_r) - \sqrt{2\lambda_r\tau_r d_r h_r} \\ &= (\tan \theta)sb^2n\lambda_c \left(u_c - \sqrt{\frac{2\tau_c h_c}{\lambda_c}}(2b)^{1/2}(1 + (\tan \theta)^2)^{1/4} - g_r \right) \\ &\quad - \sqrt{2(\tan \theta)sb^2n\lambda_c\tau_r h_r} \sqrt{2bn} \\ &= n \left[(\tan \theta)sb^2 \left\{ \lambda_c(u_c - g_r) - 2\sqrt{\lambda_c\tau_c h_c}(b)^{1/2}(1 + (\tan \theta)^2)^{1/4} \right\} \right] \\ &\quad - n \left[\sqrt{2(\tan \theta)sb^2\lambda_c\tau_r h_r} \sqrt{2b} \right]. \end{aligned}$$

If

$$\begin{aligned} &(\tan \theta)sb^2 \left\{ \lambda_c(u_c - g_r) - 2\sqrt{\lambda_c\tau_c h_c}(b)^{1/2}(1 + (\tan \theta)^2)^{1/4} \right\} \\ &\geq \sqrt{2(\tan \theta)sb^2\lambda_c\tau_r h_r} \sqrt{2b}, \end{aligned}$$

then the monopolist wants to increase the number of retail stores to as many as possible. But, the retailer has the restriction that the maximum market area is a . Thus, $r = a$ and

$$n = \frac{a}{(\tan \theta)sb^2} \quad (9)$$

By substituting Equation (9) into Equation (7), we obtain

$$\lambda_r = (\tan \theta)sb^2n\lambda_c = (\tan \theta)sb^2\frac{a}{(\tan \theta)sb^2}\lambda_c = a\lambda_c \quad (10)$$

From Equation (9), we derive

$$b = \left(\frac{a}{n}\right)^{1/2} (s \tan \theta)^{-1/2} \quad (11)$$

By substituting Equation (11) into Equations (2) and (8), we obtain

$$\begin{aligned} d_c^F &= 2\sqrt{b^2 + b^2(\tan \theta)} = 2\left(\frac{a}{n}\right)^{1/2} (s \tan \theta)^{-1/2}\sqrt{1 + (\tan \theta)} \\ &= \phi_{c1}^2 \left(\frac{a}{n}\right)^{1/2} \end{aligned} \quad (12)$$

$$d_r = 2bn = 2\left(\frac{a}{n}\right)^{1/2} (s \tan \theta)^{-1/2}n = \phi_r^2 (an)^{1/2} \quad (13)$$

where $\phi_{c1} = 2^{1/2}s^{-1/4}(\tan \theta)^{-1/4}(1 + (\tan \theta)^2)^{1/4}$ and $\phi_r = 2^{1/2}s^{-1/4}(\tan \theta)^{-1/4}$.

By substituting Equation (12) into Equation (4), we derive

$$p = u_c - \sqrt{\frac{2\tau_ch_cd_c^F}{\lambda_c}} = u_c - \phi_{c1}\sqrt{\frac{2\tau_ch_c}{\lambda_c}} \left(\frac{a}{n}\right)^{1/4}. \quad (14)$$

Hence, we can re-write the retailer's maximum profit as a function of n using Equations (10), (13) and (14):

$$\begin{aligned} Z(n) &= \lambda_r(p - g_r) - \sqrt{2\lambda_r\tau_r d_r h_r} \\ &= a\lambda_c \left((u_c - g_r) - \phi_{c1}\sqrt{\frac{2\tau_ch_c}{\lambda_c}} \left(\frac{a}{n}\right)^{1/4} \right) \\ &\quad - \sqrt{2a\lambda_c\tau_r h_r}\phi_r(an)^{1/4}. \end{aligned} \quad (15)$$

Deriving the first order condition of $Z(n)$ with respect to n , we obtain:

$$\frac{\partial Z(n)}{\partial n} = \frac{1}{4}a\lambda_c\phi_{c1}\sqrt{\frac{2\tau_ch_c}{\lambda_c}}a^{1/4}n^{-5/4} - \frac{1}{4}\sqrt{2a\lambda_c\tau_rh_r}\phi_r a^{1/4}n^{-3/4} = 0$$

and consequently, the optimal n^M follows:

$$n^M = a \left(\frac{\phi_{c1}}{\phi_r} \right)^2 \left(\frac{\tau_ch_c}{\tau_rh_r} \right). \quad (16)$$

By substituting Equation (16) into Equation (15), the retailer's profit is

$$\begin{aligned} Z^M &= a\lambda_c(u_c - g_r) - a\lambda_c\phi_{c1}\sqrt{\frac{2\tau_ch_c}{\lambda_c}}a^{1/4} * a^{-1/4} \left(\frac{\phi_{c1}}{\phi_r} \right)^{-1/2} \left(\frac{\tau_ch_c}{\tau_rh_r} \right)^{-1/4} \\ &\quad - \sqrt{2a\lambda_c\tau_rh_r}\phi_r a^{1/4} * a^{1/4} \left(\frac{\phi_{c1}}{\phi_r} \right)^{1/2} \left(\frac{\tau_ch_c}{\tau_rh_r} \right)^{1/4} \\ &= a\lambda_c(u_c - g_r) - a\sqrt{2\lambda_c}(\phi_{c1}\phi_r)^{1/2}(h_ch_r)^{1/4}(\tau_c\tau_r)^{1/4} \\ &\quad - a\sqrt{2\lambda_c}(\phi_{c1}\phi_r)^{1/2}(h_ch_r)^{1/4}(\tau_c\tau_r)^{1/4} \\ &= a\lambda_c(u_c - g_r) - 2a\sqrt{2\lambda_c}(\phi_{c1}\phi_r)^{1/2}(h_ch_r)^{1/4}(\tau_c\tau_r)^{1/4}. \end{aligned}$$

By Assumption 1, Z^M is always non-negative.

In addition, by substituting Equation (16) into Equation (14), and by substituting Equation (9) into Equation (6), the monopolist's optimal selling price is

$$\begin{aligned} p^M &= u_c - \phi_{c1}\sqrt{\frac{2\tau_ch_c}{\lambda_c}} \left(\frac{a}{n^M} \right)^{1/4} \\ &= u_c - \phi_{c1}\sqrt{\frac{2\tau_ch_c}{\lambda_c}}a^{1/4} * a^{-1/4} \left(\frac{\phi_{c1}}{\phi_r} \right)^{-1/2} \left(\frac{\tau_ch_c}{\tau_rh_r} \right)^{-1/4} \\ &= u_c - \left(\frac{2\phi_{c1}\phi_r}{\lambda_c} \right)^{1/2} (\tau_ch_c\tau_rh_r)^{1/4} \end{aligned}$$

and the monopolist's optimal serving area is

$$r^M = (\tan \theta)sb^2n = (\tan \theta)sb^2 \left(\frac{a}{(\tan \theta)sb^2} \right) = a$$

□

Proof of Proposition 2. We prove Proposition 2 in three steps. First, we set the selling price; second, we derive the optimal number of stores; finally, the number of retailers are derived.

From §3.1, each retailer's profit in the symmetric market share case is

$$Z = \lambda_r(p - g_r) - \sqrt{2\lambda_r\tau_r d_r h_r}. \quad (17)$$

Step 1. Selling Price: Given b , the farthest customer's round-trip distance is

$$d_c^F = 2\sqrt{b^2 + b^2(\tan \theta)^2} \quad (18)$$

and thus, from Equation (18), the farthest customer's utility is

$$\begin{aligned} U_c^F &= \lambda_c(u_c - p) - \sqrt{2\lambda_c\tau_c d_c^F h_c} \\ &= \lambda_c(u_c - p) - \sqrt{2\lambda_c\tau_c h_c}(2b)^{1/2}(1 + (\tan \theta)^2)^{1/4}. \end{aligned} \quad (19)$$

Since we assume that the selling price is the price that makes the farthest customer's utility be zero, from Equation (19), the selling price is

$$p = u_c - \sqrt{\frac{2\tau_c h_c d_c^F}{\lambda_c}} \quad (20)$$

$$= u_c - \sqrt{\frac{2\tau_c h_c}{\lambda_c}}(2b)^{1/2}(1 + (\tan \theta)^2)^{1/4}. \quad (21)$$

Step 2. Number of Stores and Market Area: The market area covered by each retailer r is

$$r = (1/2)(b^2 \tan \theta)2sn = (\tan \theta)sb^2n, \quad (22)$$

where $\frac{1}{2}b^2(\tan \theta)(2s)$ is the area of the regular polygon.

Hence, from Equation (22), the symmetric retailer's demand rate λ_r is

$$\lambda_r = \left(\frac{r}{k}\right) \lambda_c = \left(\frac{(\tan \theta)sb^2n}{k}\right) \lambda_c \quad (23)$$

where k is the number of symmetric retailers.

By following the distribution assumption in Cachon (2013), the symmetric retailer's distribution distance is

$$d_r = 2bn. \quad (24)$$

By substituting Equations (21), (23), and (24) into the retailer's profit function Z (Equation (17)),

$$\begin{aligned} Z(n, b) &= \lambda_r(p - g_r) - \sqrt{2\lambda_r\tau_r d_r h_r} \\ &= \left(\frac{(\tan \theta)sb^2n}{k}\right) \lambda_c \left(u_c - \sqrt{\frac{2\tau_c h_c}{\lambda_c}}(2b)^{1/2}(1 + (\tan \theta)^2)^{1/4} - g_r\right) \\ &\quad - \sqrt{2\left(\frac{(\tan \theta)sb^2n}{k}\right) \lambda_c \tau_r h_r \sqrt{2bn}} \\ &= n \left[\frac{(\tan \theta)sb^2}{k} \left\{ \lambda_c(u_c - g_r) - 2\sqrt{\lambda_c \tau_c h_c}(b)^{1/2}(1 + (\tan \theta)^2)^{1/4} \right\} \right] \\ &\quad - \left[\sqrt{2\frac{(\tan \theta)sb^2}{k} \lambda_c \tau_r h_r \sqrt{2b}} \right]. \end{aligned}$$

If

$$\begin{aligned} &\left[\frac{(\tan \theta)sb^2}{k} \left\{ \lambda_c(u_c - g_r) - 2\sqrt{\lambda_c \tau_c h_c}(b)^{1/2}(1 + (\tan \theta)^2)^{1/4} \right\} \right] \\ &\geq \left[\sqrt{2\frac{(\tan \theta)sb^2}{k} \lambda_c \tau_r h_r \sqrt{2b}} \right], \end{aligned}$$

then each retailer wants to increase the number of retail stores to as many as possible. But, the retailer has the restriction that the maximum market area is a . Thus $r = a$ and

$$n = \frac{a}{(\tan \theta)sb^2} \quad (25)$$

By substituting Equation (25) into Equation (23), we obtain

$$\lambda_r = \left(\frac{(\tan \theta)sb^2n}{k} \right) \lambda_c = \left(\frac{(\tan \theta)sb^2}{k} \right) \frac{a}{(\tan \theta)sb^2} \lambda_c = \left(\frac{a}{k} \right) \lambda_c \quad (26)$$

From Equation (25), we derive

$$b = \left(\frac{a}{n} \right)^{1/2} (s \tan \theta)^{-1/2} \quad (27)$$

By substituting Equation (27) into Equations (18) and (24), we obtain

$$\begin{aligned} d_c^F &= 2\sqrt{b^2 + b^2(\tan \theta)} = 2 \left(\frac{a}{n} \right)^{1/2} (s \tan \theta)^{-1/2} \sqrt{1 + (\tan \theta)} \\ &= \phi_{c1}^2 \left(\frac{a}{n} \right)^{1/2} \end{aligned} \quad (28)$$

$$d_r = 2bn = 2 \left(\frac{a}{n} \right)^{1/2} (s \tan \theta)^{-1/2} n = \phi_r^2 (an)^{1/2} \quad (29)$$

where $\phi_{c1} = 2^{1/2}s^{-1/4}(\tan \theta)^{-1/4}(1 + (\tan \theta)^2)^{1/4}$ and $\phi_r = 2^{1/2}s^{-1/4}(\tan \theta)^{-1/4}$.

By substituting Equation (28) into Equation (20), we derive

$$p = u_c - \sqrt{\frac{2\tau_c h_c d_c^F}{\lambda_c}} = u_c - \phi_{c1} \sqrt{\frac{2\tau_c h_c}{\lambda_c}} \left(\frac{a}{n} \right)^{1/4}. \quad (30)$$

Hence, we can re-write each retailer's maximum profit as a function of n using Equations (26), (29) and (30):

$$\begin{aligned} Z(n) &= \lambda_r(p - g_r) - \sqrt{2\lambda_r\tau_r d_r h_r} \\ &= \left(\frac{a}{k} \right) \lambda_c \left((u_c - g_r) - \phi_{c1} \sqrt{\frac{2\tau_c h_c}{\lambda_c}} \left(\frac{a}{n} \right)^{1/4} \right) \\ &\quad - \sqrt{2 \left(\frac{a}{k} \right) \lambda_c \tau_r h_r \phi_r (an)^{1/4}}. \end{aligned} \quad (31)$$

Deriving the first order condition of $Z(n)$ with respect to n , we obtain:

$$\frac{\partial Z(n)}{\partial n} = \frac{1}{4}a\lambda_c\phi_{c1}\sqrt{\frac{2\tau_ch_c}{\lambda_c}}a^{1/4}k^{-1}n^{-5/4} - \frac{1}{4}\sqrt{2a\lambda_c\tau_r h_r}\phi_r a^{1/4}k^{-1/2}n^{-3/4} = 0$$

and consequently, the optimal n^S follows:

$$n^S = \left(\frac{a}{k}\right) \left(\frac{\phi_{c1}}{\phi_r}\right)^2 \left(\frac{\tau_ch_c}{\tau_r h_r}\right). \quad (32)$$

Step 3. Number of Retailers: Given α_c and α_r , by substituting Equation (32) into Equation (31), we obtain:

$$\begin{aligned} Z(k) &= \left(\frac{a}{k}\right) \left(\lambda_c(u_c - g_r) - \phi_{c1}\sqrt{2\lambda_c\tau_ch_c} \left(\frac{a}{n^S}\right)^{1/4} \right) \\ &\quad - \phi_r \sqrt{2\left(\frac{a}{k}\right) \lambda_c\tau_r h_r (an^S)^{1/4}} \\ &= \left(\frac{a}{k}\right) \lambda_c(u_c - g_r) \\ &\quad - \phi_{c1}\sqrt{2\lambda_c\tau_ch_c}a^{5/4}k^{-1} * a^{-1/4}k^{1/4} \left(\frac{\phi_{c1}}{\phi_r}\right)^{-1/2} \left(\frac{\tau_ch_c}{\tau_r h_r}\right)^{-1/4} \\ &\quad - \phi_r \sqrt{2\lambda_c\tau_r h_r}a^{3/4}k^{-1/2} * a^{1/4}k^{-1/4} \left(\frac{\phi_{c1}}{\phi_r}\right)^{1/2} \left(\frac{\tau_ch_c}{\tau_r h_r}\right)^{1/4} \\ &= \left(\frac{a}{k}\right) \lambda_c^{1/2} \left\{ \lambda_c^{1/2}(u_c - g_r) - 2^{3/2}(\phi_{c1}\phi_r)^{1/2}(\tau_ch_c\tau_r h_r)^{1/4}k^{1/4} \right\}. \end{aligned}$$

The number of retailers, k , shall satisfy $Z(k) = 0$. Therefore,

$$k^S = \frac{\lambda_c^2(u_c - g_r)^4}{2^6(\phi_{c1}\phi_r)^2(\tau_c\tau_r h_c h_r)} \quad (33)$$

Substituting Equation (33) into Equation (32), the number of retail stores per retailer is

$$\begin{aligned} n^S &= \left(\frac{a}{k}\right) \left(\frac{\phi_{c1}}{\phi_r}\right)^2 \left(\frac{\tau_ch_c}{\tau_r h_r}\right) \\ &= a \left(\frac{\phi_{c1}}{\phi_r}\right)^2 \left(\frac{\tau_ch_c}{\tau_r h_r}\right) \lambda_c^{-2}(u_c - g_r)^{-4} 2^6(\phi_{c1}\phi_r)^2(\tau_c\tau_r h_c h_r) \\ &= a \frac{2^6\phi_{c1}^4(\tau_ch_c)^2}{\lambda_c^2(u_c - g_r)^4} \end{aligned} \quad (34)$$

From Equations (33) and (34), the total number of retail stores is

$$tk^S = n^S k^S = a \frac{2^6 \phi_{c1}^4 (\tau_c h_c)^2}{\lambda_c^2 (u_c - g_r)^4} * \frac{\lambda_c^2 (u_c - g_r)^4}{2^6 (\phi_{c1} \phi_r)^2 (\tau_c \tau_r h_c h_r)} = a \left(\frac{\phi_{c1}}{\phi_r} \right)^2 \left(\frac{\tau_c h_c}{\tau_r h_r} \right).$$

In addition, by substituting Equation (34) into Equation (30), and by substituting Equation (25) into Equation (22), each retailer's optimal selling price is

$$\begin{aligned} p^S &= u_c - \phi_{c1} \sqrt{\frac{2\tau_c h_c}{\lambda_c}} \left(\frac{a}{n^S} \right)^{1/4} \\ &= u_c - \phi_{c1} \sqrt{\frac{2\tau_c h_c}{\lambda_c}} a^{1/4} * a^{-1/4} 2^{-3/2} \phi_{c1}^{-1} (\tau_c h_c)^{-1/2} \lambda_c^{1/2} (u_c - g_r) \\ &= u_c - \frac{1}{2} (u_c - g_r) \\ &= \frac{1}{2} (u_c + g_r) \end{aligned} \tag{35}$$

and each retailer's optimal serving area is

$$r^S = (\tan \theta) s b^2 n = (\tan \theta) s b^2 \left(\frac{a}{(\tan \theta) s b^2} \right) = a$$

□

Proof of Proposition 3. We prove Proposition 3 in three steps. First, we set the selling price; second, we derive the optimal number of stores; finally, the number of retailers are derived.

From §3.1, retailer j 's profit is

$$Z^j = \lambda_r^j (p^j - g_r) - \sqrt{2\lambda_r^j \tau_r d_r^j h_r}. \tag{36}$$

Step 1. Selling Price: Given b^j , the farthest customer's round-trip distance is

$$d_c^{j,F} = 2\sqrt{(b^j)^2 + (b^j)^2 (\tan \theta)^2} \tag{37}$$

and thus, from Equation (37), the farthest customer's utility is

$$\begin{aligned} U_c^{j,F} &= \lambda_c(u_c - p^j) - \sqrt{2\lambda_c\tau_c d_c^{j,F} h_c} \\ &= \lambda_c(u_c - p^j) - \sqrt{2\lambda_c\tau_c h_c} (2b^j)^{1/2} (1 + (\tan \theta)^2)^{1/4}. \end{aligned} \quad (38)$$

Since we assume that the selling price is the price that makes the farthest customer's utility be zero, from Equation (38), the selling price is

$$p^j = u_c - \sqrt{\frac{2\tau_c h_c d_c^{j,F}}{\lambda_c}} \quad (39)$$

$$= u_c - \sqrt{\frac{2\tau_c h_c}{\lambda_c}} (2b^j)^{1/2} (1 + (\tan \theta)^2)^{1/4}. \quad (40)$$

Step 2. Number of Stores and Market Area: The market area covered by retailer j r^j is

$$r^j = (1/2)(b^j)^2(\tan \theta)2sn^j = (\tan \theta)s(b^j)^2n^j, \quad (41)$$

where $\frac{1}{2}(b^j)^2(\tan \theta)(2s)$ is the area of the regular polygon.

Hence, from Equation (41), retailer j 's demand rate λ_r^j is

$$\lambda_r^j = (r^j m^j) \lambda_c = (\tan \theta)s(b^j)^2n^j m^j \lambda_c \quad (42)$$

where m^j is the market share of retailer j .

By following the distribution assumption in Cachon (2013), retailer j 's distribution distance is

$$d_r^j = 2b^j n^j. \quad (43)$$

By substituting Equations (40), (42), and (43) into retailer j 's profit function Z (Equation (36)),

$$\begin{aligned}
Z^j(n^j, b^j) &= \lambda_r^j(p^j - g_r) - \sqrt{2\lambda_r\tau_r d_r^j h_r} \\
&= (\tan \theta) s(b^j)^2 n^j m^j \lambda_c \left(u_c - \sqrt{\frac{2\tau_c h_c}{\lambda_c}} (2b^j)^{1/2} (1 + (\tan \theta)^2)^{1/4} - g_r \right) \\
&\quad - \sqrt{2(\tan \theta) s(b^j)^2 n^j m^j \lambda_c \tau_r h_r} \sqrt{2b^j n^j} \\
&= n^j \cdot (\tan \theta) s(b^j)^2 m^j \cdot \lambda_c (u_c - g_r) \\
&\quad - 2n^j \cdot (\tan \theta) s(b^j)^2 m^j \cdot \sqrt{\lambda_c \tau_c h_c} (b^j)^{1/2} (1 + (\tan \theta)^2)^{1/4} \\
&\quad - \sqrt{2(\tan \theta) s(b^j)^2 m^j \lambda_c \tau_r h_r} \sqrt{2b^j}.
\end{aligned}$$

If

$$\begin{aligned}
&\left[(\tan \theta) s(b^j)^2 m^j \left\{ \lambda_c (u_c - g_r) - 2\sqrt{\lambda_c \tau_c h_c} (b^j)^{1/2} (1 + (\tan \theta)^2)^{1/4} \right\} \right] \\
&\geq \left[\sqrt{2(\tan \theta) s(b^j)^2 m^j \lambda_c \tau_r h_r} \sqrt{2b^j} \right],
\end{aligned}$$

then retailer j wants to increase the number of retail stores to as many as possible. But, the retailer has the restriction that the maximum market area is a . Thus, $r^j = a$ and

$$n^j = \frac{a}{(\tan \theta) s(b^j)^2} \quad (44)$$

By substituting Equation (44) into Equation (42), we obtain

$$\lambda_r^j = (\tan \theta) s(b^j)^2 n^j m^j \lambda_c = (\tan \theta) s(b^j)^2 m^j \lambda_c * \frac{a}{(\tan \theta) s(b^j)^2} = (am^j) \lambda_c \quad (45)$$

From Equation (44), we derive

$$b^j = \left(\frac{a}{n^j} \right)^{1/2} (s \tan \theta)^{-1/2} \quad (46)$$

By substituting Equation (46) into Equations (37) and (43), we obtain

$$\begin{aligned} d_c^{j,F} &= 2\sqrt{(b^j)^2 + (b^j)^2(\tan \theta)} = 2\left(\frac{a}{n^j}\right)^{1/2} (s \tan \theta)^{-1/2} \sqrt{1 + (\tan \theta)} \\ &= \phi_{c1}^2 \left(\frac{a}{n^j}\right)^{1/2} \end{aligned} \quad (47)$$

$$d_r^j = 2b^j n^j = 2\left(\frac{a}{n^j}\right)^{1/2} (s \tan \theta)^{-1/2} n^j = \phi_r^2 (an^j)^{1/2} \quad (48)$$

where $\phi_{c1} = 2^{1/2} s^{-1/4} (\tan \theta)^{-1/4} (1 + (\tan \theta)^2)^{1/4}$ and $\phi_r = 2^{1/2} s^{-1/4} (\tan \theta)^{-1/4}$.

By substituting Equation (47) into Equation (39), we derive

$$p^j = u_c - \sqrt{\frac{2\tau_c h_c d_c^{j,F}}{\lambda_c}} = u_c - \phi_{c1} \sqrt{\frac{2\tau_c h_c}{\lambda_c}} \left(\frac{a}{n^j}\right)^{1/4}. \quad (49)$$

Hence, we can re-write the retailer's maximum profit as a function of n^j using Equations (45), (48) and (49):

$$\begin{aligned} Z^j(n^j) &= \lambda_r(p^j - g_r) - \sqrt{2\lambda_r^j \tau_r d_r^j h_r} \\ &= (am^j)\lambda_c \left((u_c - g_r) - \phi_{c1} \sqrt{\frac{2\tau_c h_c}{\lambda_c}} \left(\frac{a}{n^j}\right)^{1/4} \right) \\ &\quad - \sqrt{2(am^j)\lambda_c \tau_r h_r} \phi_r (an^j)^{1/4}. \end{aligned} \quad (50)$$

Deriving the first order condition of $Z^j(n^j)$ with respect to n^j , we obtain:

$$\frac{\partial Z(n^j)}{\partial n^j} = \frac{1}{4} a \lambda_c \phi_{c1} \sqrt{\frac{2\tau_c h_c}{\lambda_c}} a^{1/4} m^j n^{-5/4} - \frac{1}{4} \sqrt{2a\lambda_c \tau_r h_r} \phi_r a^{1/4} (m^j)^{1/2} n^{-3/4} = 0$$

and consequently, the optimal $n^{j,A}$ follows:

$$n^{j,A} = (am^j) \left(\frac{\phi_{c1}}{\phi_r}\right)^2 \left(\frac{\tau_c h_c}{\tau_r h_r}\right). \quad (51)$$

By substituting Equation (51) into Equation (49) and by substituting Equation (44) into Equation (41), retailer j 's optimal selling price is

$$\begin{aligned}
p^{j,A} &= u_c - \phi_{c1} \sqrt{\frac{2\tau_c h_c}{\lambda_c}} \left(\frac{a}{n^{j,A}} \right)^{1/4} \\
&= u_c - \phi_{c1} \sqrt{\frac{2\tau_c h_c}{\lambda_c}} a^{1/4} * (am^j)^{-1/4} \left(\frac{\phi_{c1}}{\phi_r} \right)^{-1/2} \left(\frac{\tau_c h_c}{\tau_r h_r} \right)^{-1/4} \\
&= u_c - 2^{1/2} (\phi_{c1} \phi_r)^{1/2} (\tau_c h_c \tau_r h_r)^{1/4} \lambda_c^{-1/2} (m^j)^{-1/4}.
\end{aligned}$$

and retailer j 's optimal serving area is

$$r^{j,A} = (\tan \theta) s(b^j)^2 n^j = (\tan \theta) s(b^j)^2 \left(\frac{a}{(\tan \theta) s(b^j)^2} \right) = a$$

Step 3. Retailer Participation: Given α_c and α_r , by substituting Equation (51) into Equation (50), retailer j 's profit is:

$$\begin{aligned}
Z^j &= (am^j) \lambda_c \left((u_c - g_r) - \phi_{c1} \sqrt{\frac{2\tau_c h_c}{\lambda_c}} \left(\frac{a}{n^j} \right)^{1/4} \right) \\
&\quad - \sqrt{2} (am^j) \lambda_c \tau_r h_r \phi_r (an^j)^{1/4} \\
&= (am^j) \lambda_c (u_c - g_r) \\
&\quad - (am^j) \lambda_c \phi_{c1} \sqrt{\frac{2\tau_c h_c}{\lambda_c}} a^{1/4} * (am^j)^{-1/4} \left(\frac{\phi_{c1}}{\phi_r} \right)^{-1/2} \left(\frac{\tau_c h_c}{\tau_r h_r} \right)^{-1/4} \\
&\quad - \sqrt{2} (am^j) \lambda_c \tau_r h_r \phi_r a^{1/4} * (am^j)^{1/4} \left(\frac{\phi_{c1}}{\phi_r} \right)^{1/2} \left(\frac{\tau_c h_c}{\tau_r h_r} \right)^{1/4} \\
&= (am^j) \lambda_c^{1/2} \\
&\quad * \left\{ \lambda_c^{1/2} (u_c - g_r) - 2^{3/2} (\phi_{c1} \phi_r)^{1/2} (\tau_c h_c \tau_r h_r)^{1/4} (m^j)^{-1/4} \right\}. \quad (52)
\end{aligned}$$

Hence, the retailer's profit increases as the market share increases. Let retailer k^A is the retailer who has the smallest market share. Then, from Equation (52), retailer k^A 's profit is

$$Z^{k^A} = \left(am^{k^A} \right) \lambda_c^{1/2} \left\{ \lambda_c^{1/2} (u_c - g_r) - 2^{3/2} (\phi_{c1} \phi_r)^{1/2} (\tau_c h_c \tau_r h_r)^{1/4} (m^{k^A})^{-1/4} \right\},$$

and retailer k^A 's profit is non-negative if

$$\frac{1}{m^{k^A}} \geq \frac{\lambda_c^2(u_c - g_r)^4}{2^6(\phi_{c1}\phi_r)^2(\tau_c\tau_r h_c h_r)} = k^S.$$

k^S is greater than or equal to one by Assumption 1. Since the $(k^A)^{th}$ entering retailer's market share in the asymmetric case is less than or equal to the market share in the symmetric case, i.e., $m^{k^A} \leq 1/k^S$, the $(k^A)^{th}$ entering retailer's profit is non-negative. \square

Proof of Proposition 4. We show the optimal carbon recovery rates in the cases of monopoly, monopolistic competition with symmetric market share, and monopolistic competition with asymmetric market share in the following.

Case 1. Monopoly

Step 1. Total Consumer Utility. Total consumer utility is

$$\begin{aligned} & 2sn^M \int_0^b \int_0^{x \tan \theta} \left(\lambda_c(u_c - p^M) - \sqrt{2\lambda_c\tau_c d_c^{i,M} h_c} \right) dy dx \\ &= 2sn^M \int_0^b \int_0^{x \tan \theta} \lambda_c(u_c - p^M) dy dx \\ & - 2sn^M \int_0^b \int_0^{x \tan \theta} \sqrt{2\lambda_c\tau_c d_c^{i,M} h_c} dy dx, \end{aligned} \quad (53)$$

where $d_c^{i,M} = 2\sqrt{x^2 + y^2}$ which is consumer i 's round trip distance to the closest retail store.

Since $(\tan \theta)b^2 = \frac{a}{sn^M}$ from Equation (9), we have:

$$\begin{aligned} & 2sn^M \int_0^b \int_0^{x \tan \theta} \lambda_c(u_c - p^M) dy dx \\ &= 2sn^M \int_0^b \lambda_c(u_c - p^M) x(\tan \theta) dx \\ &= 2sn^M \lambda_c(u_c - p^M)(\tan \theta)(1/2)b^2 = sn^M \lambda_c(u_c - p^M) \left(\frac{a}{sn^M} \right) \\ &= a\lambda_c(u_c - p^M). \end{aligned} \quad (54)$$

In addition, since $sn^M = \frac{a}{b^2 \tan \theta}$ (from Equation (9)),

$$\begin{aligned}
& 2sn^M \int_0^b \int_0^{x \tan \theta} \sqrt{d_c^{i,M}} dy dx \\
&= \frac{a}{(1/2)b^2 \tan \theta} \int_0^b \int_0^{x \tan \theta} \sqrt{d_c^{i,M}} dy dx \\
&= \frac{a}{(1/2)b^2 \tan \theta} \int_0^b \int_0^{x \tan \theta} 2^{1/2}(x^2 + y^2)^{1/4} dy dx. \tag{55}
\end{aligned}$$

Using a change of variables (see Cachon 2013) and by Equation (11), Equation (55) becomes

$$\begin{aligned}
& 2sn^M \int_0^b \int_0^{x \tan \theta} \sqrt{d_c^{i,M}} dy dx \\
&= \frac{a}{(1/2)b^2 \tan \theta} \int_0^b \int_0^{x \tan \theta} 2^{1/2}(x^2 + y^2)^{1/4} dy dx \\
&= a \left(\frac{4\sqrt{2} \int_0^{\tan \theta} (1+t^2)^{1/4} dt}{5 \tan \theta} \right) b^{1/2} \\
&= a \left(\frac{4\sqrt{2} \int_0^{\tan \theta} (1+t^2)^{1/4} dt}{5 \tan \theta} \right) s^{-1/4} (\tan \theta)^{-1/4} \left(\frac{a}{n^M} \right)^{1/4} \\
&= \phi_{c2} a^{5/4} (n^M)^{-1/4} \tag{56}
\end{aligned}$$

where $\phi_{c2} = \left(\frac{4\sqrt{2} \int_0^{\tan \theta} (1+t^2)^{1/4} dt}{5 \tan \theta} \right) s^{-1/4} (\tan \theta)^{-1/4}$.

By substituting Equations (54) and (56) into Equation (53),

$$\begin{aligned}
& 2sn^M \int_0^b \int_0^{x \tan \theta} \left(\lambda_c(u_c - p^M) - \sqrt{2\lambda_c \tau_c d_c^{i,M} h_c} \right) \\
&= a\lambda_c(u_c - p^M) - \sqrt{2\lambda_c \tau_c h_c} \phi_{c2} a^{5/4} (n^M)^{-1/4} \tag{57}
\end{aligned}$$

Step 2. Monopolist's Profit. From Equations (1), (10) and (13), the monopolist's profit is

$$Z^M = a\lambda_c(p^M - g_r) - \sqrt{2a\lambda_c \tau_r h_r} \phi_r (an^M)^{1/4}. \tag{58}$$

Step 3. Carbon Emission Costs. Consumer i emits $f_c c_c d_c^{i,M}$ amount of carbon per travel and the frequency of travel is $\frac{\lambda_c}{q_c^{i,M}}$. Similarly, the retailer emits $f_r c_r d_r$ amount of carbon per distribution and the frequency of distribution is $\frac{a\lambda_c}{q_r^M}$. Hence, the sum of the carbon emissions costs from consumers and the monopolist is

$$\begin{aligned} & (1 - \alpha_c)e \left(2sn^M \int_0^b \int_0^{x \tan \theta} f_c c_c d_c^{i,M} \frac{\lambda_c}{q_c^{i,M}} dy dx \right) \\ & + (1 - \alpha_r)e \left(f_r c_r d_r \frac{a\lambda_c}{q_r^M} \right) \end{aligned} \quad (59)$$

where α_c and α_r denote the carbon cost recovery rates from consumers and retailers, respectively.

Since $q_c^{i,M} = \sqrt{\frac{2\lambda_c \tau_c d_c^{i,M}}{h_c}}$ and $q_r^M = \sqrt{\frac{2a\lambda_c \tau_r d_r^M}{h_r}}$ (from §3.1), and $\sqrt{d_r^M} = \phi_r (an^M)^{1/4}$ (from Equation (13)), we have

$$\begin{aligned} f_c c_c d_c^{i,M} \frac{\lambda_c}{q_c^{i,M}} &= f_c c_c d_c^{i,M} \lambda_c \sqrt{\frac{h_c}{2\lambda_c \tau_c d_c^{i,M}}} = f_c c_c \sqrt{\frac{\lambda_c h_c}{2\tau_c}} \sqrt{d_c^{i,M}}, \quad (60) \\ f_r c_r d_r^M \frac{a\lambda_c}{q_r^M} &= f_r c_r d_r^M a\lambda_c \sqrt{\frac{h_r}{2a\lambda_c \tau_r d_r^M}} \\ &= f_r c_r \sqrt{\frac{a\lambda_c h_r}{2\tau_r}} \sqrt{d_r^M} = f_r c_r \sqrt{\frac{a\lambda_c h_r}{2\tau_r}} \phi_r (an^M)^{1/4}. \quad (61) \end{aligned}$$

By substituting Equations (60) and (61) into Equation (59), and by

$2sn^M \int_0^b \int_0^{x \tan \theta} \sqrt{d_c^{i,M}} dy dx = \phi_c a^{5/4} (n^M)^{-1/4}$ (from Step 2), the carbon emis-

sion costs becomes

$$\begin{aligned}
& (1 - \alpha_c)e \left(2sn^M \int_0^b \int_0^{x \tan \theta} f_c c_c d_c^{i,M} \frac{\lambda_c}{q_c^{i,M}} dy dx \right) + (1 - \alpha_r)e \left(f_r c_r d_r \frac{a \lambda_c}{q_r^M} \right) \\
= & (1 - \alpha_c)e f_c c_c \sqrt{\frac{\lambda_c h_c}{2\tau_c}} 2sn^M \int_0^b \int_0^{x \tan \theta} \sqrt{d_c^{i,M}} dy dx \\
& + (1 - \alpha_r)e f_r c_r \sqrt{\frac{a \lambda_c h_r}{2\tau_r}} \phi_r (an^M)^{1/4} \\
= & (1 - \alpha_c)e f_c c_c \sqrt{\frac{\lambda_c h_c}{2\tau_c}} \phi_{c2} a^{5/4} (n^M)^{-1/4} \\
& + (1 - \alpha_r)e f_r c_r \sqrt{\frac{a \lambda_c h_r}{2\tau_r}} \phi_r (an^M)^{1/4}. \tag{62}
\end{aligned}$$

By Steps 1, 2, and 3 (Equations (57), (58), and (62)), social welfare in the monopoly case is

$$\begin{aligned}
SW(\alpha_c, \alpha_r) &= a \lambda_c (u_c - p^M) - \sqrt{2 \lambda_c \tau_c h_c} \phi_{c2} a^{5/4} (n^M)^{-1/4} + a \lambda_c (p^M - g_r) \\
&\quad - \sqrt{2 a \lambda_c \tau_r h_r} \phi_r (an^M)^{1/4} \\
&\quad - (1 - \alpha_c)e f_c c_c \sqrt{\frac{\lambda_c h_c}{2\tau_c}} \phi_{c2} a^{5/4} (n^M)^{-1/4} \\
&\quad - (1 - \alpha_r)e f_r c_r \sqrt{\frac{a \lambda_c h_r}{2\tau_r}} \phi_r (an^M)^{1/4} \\
= & a \lambda_c (u_c - g_r) \\
&\quad - \phi_{c2} \left(\sqrt{2 \lambda_c \tau_c h_c} + (1 - \alpha_c)e f_c c_c \sqrt{\frac{\lambda_c h_c}{2\tau_c}} \right) a^{5/4} (n^M)^{-1/4} \\
&\quad - \phi_r \left(\sqrt{2 a \lambda_c \tau_r h_r} + (1 - \alpha_r)e f_r c_r \sqrt{\frac{a \lambda_c h_r}{2\tau_r}} \right) (an^M)^{1/4}. \tag{63}
\end{aligned}$$

The first derivatives of $SW(\alpha_c, \alpha_r)$ with respect to α_c and α_r are

$$\begin{aligned}
& \frac{\partial SW(\alpha_c, \alpha_r)}{\partial \alpha_c} \\
&= 2^{-5/2} (\lambda_c h_c)^{1/2} \tau_c^{-3/2} e f_c c_c a^{5/4} (n^M)^{-1/4} \\
& * \left\{ 3(1 - \alpha_c) \phi_{c2} e f_c c_c + 2\tau_c (\phi_{c2} - \phi_{c1}) - (1 - \alpha_r) \phi_{c1} e f_r c_r \tau_c \tau_r^{-1} \right\}, \\
& \frac{\partial SW(\alpha_c, \alpha_r)}{\partial \alpha_r} \\
&= 2^{-5/2} \left(\frac{\phi_r}{\phi_{c1}} \right) e f_r c_r (\lambda_c h_r)^{1/2} \tau_r^{-3/2} a^{3/4} (n^M)^{1/4} \\
& * \left\{ -2\phi_{c2} \tau_r - \phi_{c2} (1 - \alpha_c) e f_c c_c \tau_c^{-1} \tau_r + 3\phi_{c1} (1 - \alpha_r) e f_r c_r + 2\phi_{c1} \tau_r \right\}.
\end{aligned}$$

We obtain the unconstrained solution to the first-order conditions:

$$\alpha_c = \frac{-\phi_{c1}(v_c + f_c p_c) + \phi_{c2}(v_c + f_c p_c + 2e c_c f_c)}{e c_c f_c (\phi_{c1} + \phi_{c2})}$$

and

$$\alpha_r = \frac{-\phi_{c2}(v_r + f_r p_r) + \phi_{c1}(v_r + f_r p_r + 2e c_r f_r)}{e c_r f_r (\phi_{c1} + \phi_{c2})}.$$

From the fact that $\phi_{c1} \geq \phi_{c2}$, we can verify that the unconstrained (α_c, α_r) satisfy $\alpha_c \leq 1$ and $\alpha_r \geq 1$. In addition, if $v_r + p_r f_r \geq \frac{3\phi_{c1}}{8\phi_{c2} - 4\phi_{c1}} e f_r c_r$ and $v_c + p_c f_c \geq \frac{3\phi_{c2}}{8\phi_{c1} - 4\phi_{c2}} e f_c c_c$, then $SW(\alpha_c, \alpha_r)$ is jointly concave. Therefore, $\alpha_c^M \leq 1$ and $\alpha_r^M = 1$ if $v_r + p_r f_r \geq \frac{3\phi_{c1}}{8\phi_{c2} - 4\phi_{c1}} e f_r c_r$ and $v_c + p_c f_c \geq \frac{3\phi_{c2}}{8\phi_{c1} - 4\phi_{c2}} e f_c c_c$.

Case 2. Monopolistic Competition with Symmetric Market Share

Step 1. Total Consumer Utility. Total consumer utility is

$$\begin{aligned}
& 2sn^S (1/k^S) \int_0^b \int_0^{x \tan \theta} \left(\lambda_c (u_c - p^S) - \sqrt{2\lambda_c \tau_c d_c^{i,S} h_c} \right) dy dx * k^S \\
&= 2sn^S \int_0^b \int_0^{x \tan \theta} \lambda_c (u_c - p^S) dy dx \\
&- 2sn^S \int_0^b \int_0^{x \tan \theta} \sqrt{2\lambda_c \tau_c d_c^{i,S} h_c} dy dx.
\end{aligned} \tag{64}$$

where $d_c^{i,S} = 2\sqrt{x^2 + y^2}$ which is consumer i 's round trip distance to the closest retail store.

Since $(\tan \theta)b^2 = \frac{a}{sn^S}$ from Equation (25), we have:

$$\begin{aligned}
& 2sn^S \int_0^b \int_0^{x \tan \theta} \lambda_c(u_c - p^S) dy dx \\
&= 2sn^S \int_0^b \lambda_c(u_c - p^S) x (\tan \theta) dx \\
&= 2sn^S \lambda_c(u_c - p^S) (\tan \theta) (1/2)b^2 = sn^S \lambda_c(u_c - p^S) \left(\frac{a}{sn^S} \right) \\
&= a \lambda_c(u_c - p^S).
\end{aligned} \tag{65}$$

In addition, since $sn^S = \frac{a}{b^2 \tan \theta}$ (from Equation (25)),

$$2sn^S \int_0^b \int_0^{x \tan \theta} \sqrt{d_c^{i,S}} dy dx \tag{66}$$

$$\begin{aligned}
&= \frac{a}{(1/2)b^2 \tan \theta} \int_0^b \int_0^{x \tan \theta} \sqrt{d_c^{i,S}} dy dx \\
&= \frac{a}{(1/2)b^2 \tan \theta} \int_0^b \int_0^{x \tan \theta} 2^{1/2} (x^2 + y^2)^{1/4} dy dx.
\end{aligned} \tag{67}$$

Using a change of variables (see Cachon 2013) and by Equation (27), Equation (67) becomes

$$\begin{aligned}
& 2sn^S \int_0^b \int_0^{x \tan \theta} \sqrt{d_c^{i,S}} dy dx \\
&= \frac{a}{(1/2)b^2 \tan \theta} \int_0^b \int_0^{x \tan \theta} 2^{1/2} (x^2 + y^2)^{1/4} dy dx \\
&= a \left(\frac{4\sqrt{2} \int_0^{\tan \theta} (1+t^2)^{1/4} dt}{5 \tan \theta} \right) b^{1/2} \\
&= a \left(\frac{4\sqrt{2} \int_0^{\tan \theta} (1+t^2)^{1/4} dt}{5 \tan \theta} \right) s^{-1/4} (\tan \theta)^{-1/4} \left(\frac{a}{n^S} \right)^{1/4} \\
&= \phi_{c2} a^{5/4} (n^S)^{-1/4}
\end{aligned} \tag{68}$$

where $\phi_{c2} = \left(\frac{4\sqrt{2} \int_0^{\tan \theta} (1+t^2)^{1/4} dt}{\tan \theta} \right) s^{-1/4} (\tan \theta)^{-1/4}$.

By substituting Equations (65) and (68) into Equation (64),

$$\begin{aligned} & 2sn^S \int_0^b \int_0^{x \tan \theta} \left(\lambda_c (u_c - p^S) - \sqrt{2\lambda_c \tau_c d_c^{i,S} h_c} \right) \\ &= a\lambda_c (u_c - p^S) - \sqrt{2\lambda_c \tau_c h_c} \phi_{c2} a^{5/4} (n^S)^{-1/4} \end{aligned} \quad (69)$$

Step 2. Retailers Profit. In the symmetric market share case, each retailer's profit is zero.

Step 3. Carbon Emission Costs. Consumer i emits $f_c c_c d_c^{i,S}$ amount of carbon per travel and the frequency of travel is $\frac{\lambda_c}{q_c^{i,S}}$. Similarly, the retailer emits $f_r c_r d_r$ amount of carbon per distribution and the frequency of distribution is $\frac{(a/k^S)\lambda_c}{q_r^S}$. Hence, the sum of the carbon emissions costs from consumers and the symmetric retailers is

$$\begin{aligned} & (1 - \alpha_c) e \left(2sn^S (1/k^S) \int_0^b \int_0^{x \tan \theta} f_c c_c d_c^{i,S} \frac{\lambda_c}{q_c^{i,S}} dy dx \right) * k^S \\ & + (1 - \alpha_r) e \left(f_r c_r d_r \frac{(a/k^S)\lambda_c}{q_r^S} \right) * k^S \end{aligned} \quad (70)$$

where α_c and α_r denote the carbon cost recovery rates from consumers and retailers, respectively.

Since $q_c^{i,S} = \sqrt{\frac{2\lambda_c \tau_c d_c^{i,S}}{h_c}}$ and $q_r^S = \sqrt{\frac{2(a/k^S)\lambda_c \tau_r d_r^S}{h_r}}$ (from §3.1), and $\sqrt{d_r^S} = \phi_r (an^S)^{1/4}$ (from Equation (29)), we have

$$f_c c_c d_c^{i,S} \frac{\lambda_c}{q_c^{i,S}} = f_c c_c d_c^{i,S} \lambda_c \sqrt{\frac{h_c}{2\lambda_c \tau_c d_c^{i,S}}} = f_c c_c \sqrt{\frac{\lambda_c h_c}{2\tau_c}} \sqrt{d_c^{i,S}}, \quad (71)$$

$$\begin{aligned} f_r c_r d_r^S \frac{a\lambda_c}{q_r^S} &= f_r c_r d_r^S a \lambda_c \sqrt{\frac{h_r}{2(a/k^S)\lambda_c \tau_r d_r^S}} = f_r c_r \sqrt{\frac{ak^S \lambda_c h_r}{2\tau_r}} \sqrt{d_r^S} \\ &= f_r c_r \sqrt{\frac{ak^S \lambda_c h_r}{2\tau_r}} \phi_r (an^S)^{1/4}. \end{aligned} \quad (72)$$

By substituting Equations (71) and (72) into Equation (70), and by $2sn^S \int_0^b \int_0^{x \tan \theta} \sqrt{d_c^{i,S}} dy dx = \phi_{c2} a^{5/4} (n^S)^{-1/4}$ (from Step 2), the carbon emission costs becomes

$$\begin{aligned}
& (1 - \alpha_c) e \left(2sn^S \int_0^b \int_0^{x \tan \theta} f_c c_c d_c^{i,S} \frac{\lambda_c}{q_c^{i,S}} dy dx \right) + (1 - \alpha_r) e \left(f_r c_r d_r \frac{a \lambda_c}{q_r^S} \right) \\
= & (1 - \alpha_c) e f_c c_c \sqrt{\frac{\lambda_c h_c}{2\tau_c}} 2sn^S \int_0^b \int_0^{x \tan \theta} \sqrt{d_c^{i,S}} dy dx \\
& + (1 - \alpha_r) e f_r c_r \sqrt{\frac{ak^S \lambda_c h_r}{2\tau_r}} \phi_r (an^S)^{1/4} \\
= & (1 - \alpha_c) e f_c c_c \sqrt{\frac{\lambda_c h_c}{2\tau_c}} \phi_{c2} a^{5/4} (n^S)^{-1/4} \\
& + (1 - \alpha_r) e f_r c_r \sqrt{\frac{ak^S \lambda_c h_r}{2\tau_r}} \phi_r (an^S)^{1/4}. \tag{73}
\end{aligned}$$

By Steps 1, 2, and 3 (Equations (69) and (73)), and by $p^S = \frac{u_c + g_r}{2}$ (Equation (35)), we obtain

$$\begin{aligned}
SW(\alpha_c, \alpha_r) &= a \lambda_c (u_c - p^S) - \sqrt{2\lambda_c \tau_c h_c} \phi_{c2} a^{5/4} (n^S)^{-1/4} \\
&\quad - (1 - \alpha_c) e f_c c_c \sqrt{\frac{\lambda_c h_c}{2\tau_c}} \phi_{c2} a^{5/4} (n^S)^{-1/4} \\
&\quad - (1 - \alpha_r) e f_r c_r \sqrt{\frac{ak^S \lambda_c h_r}{2\tau_r}} \phi_r (an^S)^{1/4} \\
= & \frac{a \lambda_c}{2} (u_c - g_r) - \sqrt{2\lambda_c \tau_c h_c} \phi_{c2} a^{5/4} (n^S)^{-1/4} \\
&\quad - (1 - \alpha_c) e f_c c_c \sqrt{\frac{\lambda_c h_c}{2\tau_c}} \phi_{c2} a^{5/4} (n^S)^{-1/4} \\
&\quad - (1 - \alpha_r) e f_r c_r \sqrt{\frac{ak^S \lambda_c h_r}{2\tau_r}} \phi_r (an^S)^{1/4} \tag{74}
\end{aligned}$$

By substituting Equations (32) and (33) into Equation (74), we derive

$$\begin{aligned}
& SW(\alpha_c, \alpha_r) \\
&= \frac{a\lambda_c}{2}(u_c - g_r) \\
&\quad - \sqrt{2\lambda_c\tau_ch_c\phi_{c2}}a^{5/4} * a^{-1/4}2^{-3/2}\phi_{c1}^{-1}(\tau_ch_c)^{-1/2}\lambda_c^{1/2}(u_c - g_r) \\
&\quad - (1 - \alpha_c)ef_c c_c \sqrt{\frac{\lambda_ch_c}{2\tau_c}}\phi_{c2}a^{5/4} * a^{-1/4}2^{-3/2}\phi_{c1}^{-1}(\tau_ch_c)^{-1/2}\lambda_c^{1/2}(u_c - g_r) \\
&\quad - (1 - \alpha_r)ef_r c_r \sqrt{\frac{a\lambda_ch_r}{2\tau_r}}\phi_r a^{1/4} * a^{1/4}2^{3/2}\phi_{c1}(\tau_ch_c)^{1/2}\lambda_c^{-1/2}(u_c - g_r)^{-1} \\
&\quad * \frac{\lambda_c(u_c - g_r)^2}{2^3(\phi_{c1}\phi_r)(\tau_c\tau_r h_c h_r)^{1/2}} \\
&= \frac{a\lambda_c}{2}(u_c - g_r) \left(1 - \frac{\phi_{c2}}{\phi_{c1}}\right) \\
&\quad - \frac{a\phi_{c2}}{4\phi_{c1}}ef_c c_c \lambda_c(u_c - g_r)(1 - \alpha_c)\tau_c^{-1} - \frac{a}{4}ef_r c_r \lambda_c(u_c - g)(1 - \alpha_r)\tau_r^{-1} \quad (75)
\end{aligned}$$

From

$$\frac{\partial\tau_c}{\partial\alpha_c} = ec_c f_c \text{ and } \frac{\partial\tau_r}{\partial\alpha_r} = ec_r f_r,$$

$$\begin{aligned}
\frac{\partial SW(\alpha_c, \alpha_r)}{\partial\alpha_c} &= -\frac{a\phi_{c2}}{4\phi_{c1}}ef_c c_c \lambda_c(u_c - g_r)\{-\tau_c^{-1} - (1 - \alpha_c)\tau_c^{-2}ef_c c_c\} \\
&= \frac{a\phi_{c2}}{4\phi_{c1}}ef_c c_c \lambda_c(u_c - g_r)\tau_c^{-2}\{\tau_c + (1 - \alpha_c)ef_c c_c\} \\
&= \frac{a\phi_{c2}}{4\phi_{c1}}ef_c c_c \lambda_c(u_c - g_r)\tau_c^{-2}\{v_c + p_c f_c + \alpha_c ec_c f_c + (1 - \alpha_c)ef_c c_c\} \\
&= \frac{a\phi_{c2}}{4\phi_{c1}}ef_c c_c \lambda_c(u_c - g_r)\tau_c^{-2}(v_c + p_c f_c + ef_c c_c)
\end{aligned}$$

and

$$\begin{aligned}
\frac{\partial SW(\alpha_c, \alpha_r)}{\partial \alpha_r} &= -\frac{a}{4} e f_r c_r \lambda_c (u_c - g) \{-\tau_r^{-1} - (1 - \alpha_r) \tau_r^{-2} e f_r c_r\} \\
&= \frac{a}{4} e f_r c_r \lambda_c (u_c - g_r) \tau_r^{-2} \{\tau_r + (1 - \alpha_r) e f_r c_r\} \\
&= \frac{a}{4} e f_r c_r \lambda_c (u_c - g_r) \tau_r^{-2} \{v_r + p_r f_r + \alpha_r e c_r f_r + (1 - \alpha_r) e f_r c_r\} \\
&= \frac{a}{4} e f_r c_r \lambda_c (u_c - g_r) \tau_r^{-2} (v_r + p_r f_r + e c_r f_r).
\end{aligned}$$

Consequently, we derive

$$\frac{\partial SW(\alpha_c, \alpha_r)}{\partial \alpha_c} > 0 \quad \text{and} \quad \frac{\partial SW(\alpha_c, \alpha_r)}{\partial \alpha_r} > 0$$

which lead to $\alpha_c^S = 1$ and $\alpha_r^S = 1$.

Case 3. Monopolistic Competition with Asymmetric Market Share

Step 1. Total Utility of Retailer j 's Customers. Total utility of retailer j 's customers is

$$\begin{aligned}
& 2sn^{j,A} m^j \int_0^{b^j} \int_0^{x \tan \theta} \left(\lambda_c (u_c - p^{j,A}) - \sqrt{2\lambda_c \tau_c d_c^{j,i,A} h_c} \right) dy dx \\
&= 2sn^{j,A} m^j \int_0^{b^j} \int_0^{x \tan \theta} \lambda_c (u_c - p^{j,A}) dy dx \\
&- 2sn^{j,A} m^j \int_0^{b^j} \int_0^{x \tan \theta} \sqrt{2\lambda_c \tau_c d_c^{j,i,A} h_c} dy dx.
\end{aligned} \tag{76}$$

where $d_c^{j,i,A} = 2\sqrt{x^2 + y^2}$ which is consumer i 's round trip distance to retailer j 's closest retail stores.

Since $(\tan \theta)(b^j)^2 = \frac{a}{sn^{j,A}}$ from Equation (44), we have:

$$2sn^{j,A}m^j \int_0^{b^j} \int_0^{x \tan \theta} \lambda_c(u_c - p^{j,A}) dy dx \quad (77)$$

$$\begin{aligned} &= 2sn^{j,A}m^j \int_0^{b^j} \lambda_c(u_c - p^{j,A}) x (\tan \theta) dx \\ &= 2sn^{j,A}m^j \lambda_c(u_c - p^{j,A}) (\tan \theta) (1/2) (b^j)^2 \\ &= sn^{j,A}m^j \lambda_c(u_c - p^{j,A}) \left(\frac{a}{sn^{j,A}} \right) \\ &= (am^j) \lambda_c(u_c - p^{j,A}). \end{aligned} \quad (78)$$

In addition, since $sn^{j,A} = \frac{a}{(b^j)^2 \tan \theta}$ (from Equation (44)),

$$\begin{aligned} &2sn^{j,A}m^j \int_0^{b^j} \int_0^{x \tan \theta} \sqrt{d_c^{i,j,A}} dy dx \\ &= \frac{am^j}{(1/2)(b^j)^2 \tan \theta} \int_0^{b^j} \int_0^{x \tan \theta} \sqrt{d_c^{i,j,A}} dy dx \\ &= \frac{am^j}{(1/2)(b^j)^2 \tan \theta} \int_0^{b^j} \int_0^{x \tan \theta} 2^{1/2} (x^2 + y^2)^{1/4} dy dx. \end{aligned} \quad (79)$$

Using a change of variables (see Cachon 2013) and by Equation (46), Equation (79) becomes

$$\begin{aligned} &2sn^{j,A}m^j \int_0^{b^j} \int_0^{x \tan \theta} \sqrt{d_c^{i,j,A}} dy dx \\ &= \frac{am^j}{(1/2)(b^j)^2 \tan \theta} \int_0^{b^j} \int_0^{x \tan \theta} 2^{1/2} (x^2 + y^2)^{1/4} dy dx \\ &= am^j \left(\frac{4\sqrt{2} \int_0^{\tan \theta} (1+t^2)^{1/4} dt}{5 \tan \theta} \right) (b^j)^{1/2} \\ &= am^j \left(\frac{4\sqrt{2} \int_0^{\tan \theta} (1+t^2)^{1/4} dt}{5 \tan \theta} \right) s^{-1/4} (\tan \theta)^{-1/4} \left(\frac{a}{n^{j,A}} \right)^{1/4} \\ &= \phi_{c2} a^{5/4} m^j (n^{j,A})^{-1/4} \end{aligned} \quad (80)$$

where $\phi_{c2} = \left(\frac{4\sqrt{2}}{5} \int_0^{\tan \theta} \frac{(1+t^2)^{1/4} dt}{\tan \theta} \right) s^{-1/4} (\tan \theta)^{-1/4}$.

By substituting Equations (78) and (80) into Equation (76),

$$\begin{aligned} & 2sn^{j,A}m^j \int_0^{b^j} \int_0^{x \tan \theta} \left(\lambda_c(u_c - p^{j,A}) - \sqrt{2\lambda_c\tau_c d_c^{j,i,A} h_c} \right) dydx \\ & = (am^j)\lambda_c(u_c - p^{j,A}) - \sqrt{2\lambda_c\tau_c h_c \phi_{c2} a^{5/4} m^j (n^{j,A})^{-1/4}} \end{aligned} \quad (81)$$

Step 2. Monopolist's Profit. From Equations (36), (45) and (48), retailer j 's profit is

$$Z^{j,A} = (am^j)\lambda_c(p^{j,A} - g_r) - \sqrt{2(am^j)\lambda_c\tau_r h_r \phi_r (an^{j,A})^{1/4}}. \quad (82)$$

Step 3. Carbon Emission Costs. Retailer j 's consumer i emits $f_c c_c d_c^{j,i,A}$ amount of carbon per travel and the frequency of travel is $\frac{\lambda_c}{q_c^{j,i,A}}$. Similarly, the retailer emits $f_r c_r d_r$ amount of carbon per distribution and the frequency of distribution is $\frac{am^j \lambda_c}{q_r^{j,A}}$. Hence, the sum of the carbon emissions costs from consumers and the monopolist is

$$\begin{aligned} & (1 - \alpha_c)e \left(2sn^{j,A}m^j \int_0^{b^j} \int_0^{x \tan \theta} f_c c_c d_c^{j,i,A} \frac{\lambda_c}{q_c^{j,i,A}} dydx \right) \\ & + (1 - \alpha_r)e \left(f_r c_r d_r \frac{am^j \lambda_c}{q_r^{j,A}} \right) \end{aligned} \quad (83)$$

where α_c and α_r denote the carbon cost recovery rates from consumers and retailers, respectively.

Since $q_c^{j,i,A} = \sqrt{\frac{2\lambda_c\tau_c d_c^{j,i,A}}{h_c}}$ and $q_r^{j,A} = \sqrt{\frac{2(am^j)\lambda_c\tau_r d_r^{j,A}}{h_r}}$ (from §3.1), and $\sqrt{d_r^{j,A}} =$

$\phi_r(an^{j,A})^{1/4}$ (from Equation (48)), we have

$$f_c c_c d_c^{j,i,A} \frac{\lambda_c}{q_c^{j,i,A}} = f_c c_c d_c^{j,i,A} \lambda_c \sqrt{\frac{h_c}{2\lambda_c \tau_c d_c^{j,i,A}}} = f_c c_c \sqrt{\frac{\lambda_c h_c}{2\tau_c}} \sqrt{d_c^{j,i,A}}, \quad (84)$$

$$\begin{aligned} f_r c_r d_r^j \frac{am^j \lambda_c}{q_r^{j,A}} &= f_r c_r d_r^j am^j \lambda_c \sqrt{\frac{h_r}{2am^j \lambda_c \tau_r d_r^j}} = f_r c_r \sqrt{\frac{am^j \lambda_c h_r}{2\tau_r}} \sqrt{d_r^j} \\ &= f_r c_r \sqrt{\frac{am^j \lambda_c h_r}{2\tau_r}} \phi_r(an^{j,A})^{1/4}. \end{aligned} \quad (85)$$

By substituting Equations (84) and (85) into Equation (83), and by $2sn^{j,A}m^j \int_0^{b^j} \int_0^{x \tan \theta} \sqrt{d_c^{j,i,A}} dy dx = \phi_{c2} a^{5/4} m^j (n^{j,A})^{-1/4}$ (from Step 2), the carbon emission costs becomes

$$\begin{aligned} &(1 - \alpha_c) e \left(2sn^{j,A}m^j \int_0^{b^j} \int_0^{x \tan \theta} f_c c_c d_c^{j,i,A} \frac{\lambda_c}{q_c^{j,i,A}} dy dx \right) \\ &+ (1 - \alpha_r) e \left(f_r c_r d_r^j \frac{am^j \lambda_c}{q_r^{j,A}} \right) \\ &= (1 - \alpha_c) e f_c c_c \sqrt{\frac{\lambda_c h_c}{2\tau_c}} 2sn^{j,A}m^j \int_0^{b^j} \int_0^{x \tan \theta} \sqrt{d_c^{j,i,A}} dy dx \\ &+ (1 - \alpha_r) e f_r c_r \sqrt{\frac{am^j \lambda_c h_r}{2\tau_r}} \phi_r(an^{j,A})^{1/4} \\ &= (1 - \alpha_c) e f_c c_c \sqrt{\frac{\lambda_c h_c}{2\tau_c}} \phi_{c2} a^{5/4} m^j (n^{j,A})^{-1/4} \\ &+ (1 - \alpha_r) e f_r c_r \sqrt{\frac{am^j \lambda_c h_r}{2\tau_r}} \phi_r(an^{j,A})^{1/4} \end{aligned} \quad (86)$$

By Steps 1, 2, and 3 (Equations (81), (82), and (86)), social welfare in the asymmetric case is

$$SW(\alpha_c, \alpha_r) = \sum_{j=1}^{k^A} SW^j(\alpha_c, \alpha_r)$$

where

$$\begin{aligned}
& SW^j(\alpha_c, \alpha_r) \\
&= (am^j)\lambda_c(u_c - p^{j,A}) - \phi_{c2}\sqrt{2\lambda_c\tau_ch_c}a^{5/4}(m^j)(n^{j,A})^{-1/4} \\
&+ (am^j)\lambda_c(p^{j,A} - g_r) - \phi_r\sqrt{2\lambda_c\tau_rh_r}a^{3/4}(m^j)^{1/2}(n^{j,A})^{1/4} \\
&- (1 - \alpha_c)ef_c c_c \sqrt{\frac{\lambda_ch_c}{2\tau_c}}\phi_{c2}a^{5/4}(m^j)(n^{j,A})^{-1/4} \\
&- (1 - \alpha_r)ef_r c_r \sqrt{\frac{\lambda_ch_r}{2\tau_r}}\phi_r a^{3/4}(m^j)^{1/2}(n^{j,A})^{1/4} \\
&= (am^j)\lambda_c(u_c - g_r) \\
&- \phi_{c2}\left(\sqrt{2\lambda_c\tau_ch_c} + (1 - \alpha_c)ef_c c_c \sqrt{\frac{\lambda_ch_c}{2\tau_c}}\right)a^{5/4}(m^j)(n^{j,A})^{-1/4} \\
&- \phi_r\left(\sqrt{2\lambda_c\tau_rh_r} + (1 - \alpha_r)ef_r c_r \sqrt{\frac{\lambda_ch_r}{2\tau_r}}\right)a^{3/4}(m^j)^{1/2}(n^{j,A})^{1/4} \quad (87)
\end{aligned}$$

Therefore, we can show that $\alpha_c^A = \alpha_c^M$ and $\alpha_r^A = \alpha_r^M$.

By Cases 1, 2 and 3, Proposition 4 is proved. \square

Proof of Proposition 5. In the following, the social welfare and total emissions penalties in the case of monopolistic competition with symmetric market share are derived.

Social Welfare Penalty (SWP). From Equation (75),

$$\begin{aligned}
& SW(\alpha_c, \alpha_r) \\
&= \frac{a\lambda_c}{2}(u_c - g_r)\left(1 - \frac{\phi_{c2}}{\phi_{c1}}\right) - \frac{a\phi_{c2}}{4\phi_{c1}}ef_c c_c \lambda_c(u_c - g_r)(1 - \alpha_c)\tau_c^{-1} \\
&- \frac{a}{4}ef_r c_r \lambda_c(u_c - g_r)(1 - \alpha_r)\tau_r^{-1}.
\end{aligned}$$

Hence, the social welfare penalty is

$$\begin{aligned}
SWP(\alpha_c^S, \alpha_r^S) &= \frac{SWP(1, 1) - SWP(0, 0)}{SWP(1, 1)} \\
&= \frac{\frac{a\phi_{c2}}{4\phi_{c1}} e f_c c_c \lambda_c (u_c - g_r) \tau_c(0)^{-1} + \frac{a}{4} e f_r c_r \lambda_c (u_c - g_r) \tau_r(0)^{-1}}{\frac{a\lambda_c}{2} (u_c - g_r) \left(1 - \frac{\phi_{c2}}{\phi_{c1}}\right)} \\
&= \frac{e}{2 \left(1 - \frac{\phi_{c2}}{\phi_{c1}}\right)} \left(\frac{\phi_{c2}}{\phi_{c1}} f_c c_c \tau_c(0)^{-1} + f_r c_r \tau_r(0)^{-1} \right).
\end{aligned}$$

Total Emission Penalty (TEP). From Equation (75),

$$\begin{aligned}
TE(\alpha_c, \alpha_r) &= \frac{a\phi_{c2}}{4\phi_{c1}} f_c c_c \lambda_c (u_c - g_r) \tau_c^{-1} + \frac{a}{4} f_r c_r \lambda_c (u_c - g_r) \tau_r^{-1} \\
&= \left(\frac{a\lambda_c (u_c - g_r)}{4} \right) \left(\frac{\phi_{c2}}{\phi_{c1}} f_c c_c \tau_c^{-1} + f_r c_r \tau_r^{-1} \right).
\end{aligned}$$

Hence, the total emission penalty is

$$\begin{aligned}
TEP(\alpha_c^S, \alpha_r^S) &= \frac{TEP(0, 0) - TEP(1, 1)}{TEP(1, 1)} \\
&= \left(\frac{a\lambda_c (u_c - g_r)}{4} \right) \\
&\quad * \frac{\left\{ \left(\frac{\phi_{c2}}{\phi_{c1}} f_c c_c \tau_c(0)^{-1} + f_r c_r \tau_r(0)^{-1} \right) - \left(\frac{\phi_{c2}}{\phi_{c1}} f_c c_c \tau_c(1)^{-1} + f_r c_r \tau_r(1)^{-1} \right) \right\}}{\left(\frac{a\lambda_c (u_c - g_r)}{4} \right) \left(\frac{\phi_{c2}}{\phi_{c1}} f_c c_c \tau_c(1)^{-1} + f_r c_r \tau_r(1)^{-1} \right)} \\
&= \frac{\left(\frac{\phi_{c2}}{\phi_{c1}} f_c c_c \tau_c(0)^{-1} + f_r c_r \tau_r(0)^{-1} \right) - \left(\frac{\phi_{c2}}{\phi_{c1}} f_c c_c \tau_c(1)^{-1} + f_r c_r \tau_r(1)^{-1} \right)}{\frac{\phi_{c2}}{\phi_{c1}} f_c c_c \tau_c(1)^{-1} + f_r c_r \tau_r(1)^{-1}}.
\end{aligned}$$

□

.3 Proofs of Chapter 4.

We prove Lemma 1, Proposition 1, and Proposition 2 at the same time by considering 16 scenarios.

Proofs of Lemma 1, Proposition 1, and Proposition 2.

(i) Equilibriums in Scenario (1) and Scenario (2)

Given that manufacturer U is located in country U , manufacturer K 's profits by staying in country K and moving to country U , respectively, are

$$\begin{aligned} & \frac{\mathbb{E}[X_t]}{9} \left\{ -\mathbb{E}[D_t] + \frac{2\mathbb{E}[X_t D_t]}{\mathbb{E}[X_t]} + (c^U + w^U) - 2 \left(\frac{c^U}{\theta^T} + \frac{w^U}{\theta^W} + \tau \right) \right\}^2 \text{ and} \\ & \frac{\mathbb{E}[X_t]}{9} \left\{ -\mathbb{E}[D_t] + \frac{2\mathbb{E}[X_t D_t]}{\mathbb{E}[X_t]} + (c^U + w^U) - 2 \left(\frac{c^U}{\theta^T} + w^U \right) \right\}^2 - x_0 S^U. \end{aligned}$$

Since $\frac{w^U}{\theta^W} + \tau < w^U$ and $S^U > 0$, manufacturer K stays in country K . Then, given that manufacturer K is located in country K , since $S^K = 0$, manufacturer U 's profits by staying in country U and moving to country K , respectively, are

$$\begin{aligned} & \frac{1}{9} \left\{ 2\mathbb{E}[D_t] - \frac{\mathbb{E}[X_t D_t]}{\mathbb{E}[X_t]} - 2(c^U + w^U) + \left(\frac{c^U}{\theta^T} + \frac{w^U}{\theta^W} + \tau \right) \right\}^2 \text{ and} \\ & \frac{1}{9} \left\{ 2\mathbb{E}[D_t] - \frac{\mathbb{E}[X_t D_t]}{\mathbb{E}[X_t]} - 2 \left(c^U + \frac{w^U}{\theta^W} + \tau \right) + \left(\frac{c^U}{\theta^T} + \frac{w^U}{\theta^W} + \tau \right) \right\}^2. \end{aligned}$$

Hence, manufacturer U moves to country K because $\frac{w^U}{\theta^W} + \tau < w^U$. Then, given that manufacturer U is located in country K , manufacturer K 's profits by staying in country K and moving to country U , respectively, are

$$\begin{aligned} & \frac{\mathbb{E}[X_t]}{9} \left\{ -\mathbb{E}[D_t] + \frac{2\mathbb{E}[X_t D_t]}{\mathbb{E}[X_t]} + \left(c^U + \frac{w^U}{\theta^W} + \tau \right) - 2 \left(\frac{c^U}{\theta^T} + \frac{w^U}{\theta^W} + \tau \right) \right\}^2 \text{ and} \\ & \frac{\mathbb{E}[X_t]}{9} \left\{ -\mathbb{E}[D_t] + \frac{2\mathbb{E}[X_t D_t]}{\mathbb{E}[X_t]} + \left(c^U + \frac{w^U}{\theta^W} + \tau \right) - 2 \left(\frac{c^U}{\theta^T} + w^U \right) \right\}^2 - x_o S^U. \end{aligned}$$

Since $\frac{w^U}{\theta^W} + \tau < w^U$ and $S^U > 0$, manufacturer K stays in country K . Therefore, both manufacturers U and K produce in country K , i.e., the equilibrium is Case 3.

(ii) Equilibriums in Scenario (5) and Scenario (6)

As we have shown in (i), given that manufacturer U is located in country U , manufacturer K stays in country K . Then, given that manufacturer K is located in country K , manufacturer U 's profits by staying in country U and moving to country K , respectively, are

$$\frac{1}{9} \left\{ 2\mathbb{E}[D_t] - \frac{\mathbb{E}[X_t D_t]}{\mathbb{E}[X_t]} - 2(c^U + w^U) + \left(\frac{c^U}{\theta^T} + \frac{w^U}{\theta^W} + \tau \right) \right\}^2 \text{ and}$$

$$\frac{1}{9} \left\{ 2\mathbb{E}[D_t] - \frac{\mathbb{E}[X_t D_t]}{\mathbb{E}[X_t]} - 2 \left(c^U + \frac{w^U}{\theta^W} + \tau \right) + \left(\frac{c^U}{\theta^T} + \frac{w^U}{\theta^W} + \tau \right) \right\}^2 - \frac{S^K}{x_0}.$$

Then, the manufacturer U 's profit increase by moving to county K is

$$\begin{aligned} & \frac{1}{9} \left\{ 2\mathbb{E}[D_t] - \frac{\mathbb{E}[X_t D_t]}{\mathbb{E}[X_t]} - 2 \left(c^U + \frac{w^U}{\theta^W} + \tau \right) + \left(\frac{c^U}{\theta^T} + \frac{w^U}{\theta^W} + \tau \right) \right\}^2 - \frac{S^K}{x_0} \\ & - \frac{1}{9} \left\{ 2\mathbb{E}[D_t] - \frac{\mathbb{E}[X_t D_t]}{\mathbb{E}[X_t]} - 2(c^U + w^U) + \left(\frac{c^U}{\theta^T} + \frac{w^U}{\theta^W} + \tau \right) \right\}^2 \\ & = \frac{2}{9} \left\{ 4\mathbb{E}[D_t] - 2 \frac{\mathbb{E}[X_t D_t]}{\mathbb{E}[X_t]} - 2 \left(2c^U + w^U - \frac{c^U}{\theta^T} \right) \right\} \left\{ w^U - \left(\frac{w^U}{\theta^W} + \tau \right) \right\} \\ & - \frac{S^K}{x_0}. \end{aligned}$$

Since $\mathbb{E}[X_t] = X_0 + \alpha\mu_t$, $\mathbb{E}[D_t] = D_0 + \beta\mu_t$, and $\mathbb{E}[X_t D_t] = X_0 D_0 + (\alpha D_0 + \beta X_0)\mu_t + \alpha\beta(\mu_t^2 + \sigma_t^2)$, the manufacturer U 's profit increase by moving to county K is non-negative if

$$\begin{aligned} \alpha\beta\sigma_t^2 \leq & -\frac{(X_0 + \alpha\mu_t)}{2} \left(\frac{9S^K}{2x_0} \right) \left\{ \frac{1}{w^U - \left(\frac{w^U}{\theta^W} + \tau \right)} \right\} \\ & + \frac{(X_0 + \alpha\mu_t)}{2} \left\{ 4(D_0 + \beta\mu_t) + 2 \left(2c^U + w^U - \frac{c^U}{\theta^T} \right) \right\} \\ & - \{ X_0 D_0 + (\alpha D_0 + \beta X_0)\mu_t + \alpha\beta\mu_t^2 \}. \end{aligned}$$

Hence, when α is negative, manufacturer U moves to country K if

$$\begin{aligned} \sigma_t^2 \geq & -\frac{(X_0 + \alpha\mu_t)}{2\alpha\beta} \left(\frac{9S^K}{2x_0} \right) \left\{ \frac{1}{w^U - \left(\frac{w^U}{\theta^W} + \tau \right)} \right\} \\ & + \frac{(X_0 + \alpha\mu_t)}{2\alpha\beta} \left\{ 4(D_0 + \beta\mu_t) - 2 \left(2c^U + w^U - \frac{c^U}{\theta^T} \right) \right\} \\ & - \left\{ \frac{X_0 D_0 + (\alpha D_0 + \beta X_0)\mu_t + \alpha\beta\mu_t^2}{\alpha\beta} \right\}, \end{aligned}$$

and manufacturer K stays in country K even if manufacturer U moves to country U , i.e., the equilibrium is Case 3. Similarly, when α is positive, manufacturer U stays in country U if

$$\begin{aligned} \sigma_t^2 \geq & -\frac{(X_0 + \alpha\mu_t)}{2\alpha\beta} \left(\frac{9S^K}{2x_0} \right) \left\{ \frac{1}{w^U - \left(\frac{w^U}{\theta^W} + \tau \right)} \right\} \\ & + \frac{(X_0 + \alpha\mu_t)}{2\alpha\beta} \left\{ 4(D_0 + \beta\mu_t) - 2 \left(2c^U + w^U - \frac{c^U}{\theta^T} \right) \right\} \\ & - \left\{ \frac{X_0 D_0 + (\alpha D_0 + \beta X_0)\mu_t + \alpha\beta\mu_t^2}{\alpha\beta} \right\}, \end{aligned}$$

and manufacturer K stays in country K , i.e., the equilibrium is Case 1. Notice that regardless of the sign of α , if σ_t^2 is not high enough, the equilibrium depends on various parameter values.

(iii) Equilibriums in Scenario (9) and Scenario (10)

Given that manufacturer U is located in country U , manufacturer K 's profits by staying in country K and moving to country U , respectively, are

$$\begin{aligned} & \frac{\mathbb{E}[X_t]}{9} \left\{ -\mathbb{E}[D_t] + \frac{2\mathbb{E}[X_t D_t]}{\mathbb{E}[X_t]} + (c^U + w^U) - 2 \left(\frac{c^U}{\theta^T} + \frac{w^U}{\theta^W} + \tau \right) \right\}^2 \text{ and} \\ & \frac{\mathbb{E}[X_t]}{9} \left\{ -\mathbb{E}[D_t] + \frac{2\mathbb{E}[X_t D_t]}{\mathbb{E}[X_t]} + (c^U + w^U) - 2 \left(\frac{c^U}{\theta^T} + w^U \right) \right\}^2 - x_0 S^U. \end{aligned}$$

Then, the manufacturer K 's profit increase by moving to county U is

$$\begin{aligned}
& \frac{\mathbb{E}[X_t]}{9} \left\{ -\mathbb{E}[D_t] + \frac{2\mathbb{E}[X_t D_t]}{\mathbb{E}[X_t]} + (c^U + w^U) - 2 \left(\frac{c^U}{\theta^T} + w^U \right) \right\}^2 - x_0 S^U \\
& - \frac{\mathbb{E}[X_t]}{9} \left\{ -\mathbb{E}[D_t] + \frac{2\mathbb{E}[X_t D_t]}{\mathbb{E}[X_t]} + (c^U + w^U) - 2 \left(\frac{c^U}{\theta^T} + \frac{w^U}{\theta^W} + \tau \right) \right\}^2 \\
& = -\frac{2\mathbb{E}[X_t]}{9} \left\{ -2\mathbb{E}[D_t] + \frac{4\mathbb{E}[X_t D_t]}{\mathbb{E}[X_t]} \right\} \left\{ w^U - \left(\frac{w^U}{\theta^W} + \tau \right) \right\} \\
& - \frac{2\mathbb{E}[X_t]}{9} \left\{ 2 \left(c^U - 2\frac{c^U}{\theta^T} - \frac{w^U}{\theta^W} - \tau \right) \right\} \left\{ w^U - \left(\frac{w^U}{\theta^W} + \tau \right) \right\} - x_0 S^U.
\end{aligned}$$

Hence, the manufacturer K 's profit increase by moving to county U is non-negative if

$$\begin{aligned}
\alpha\beta\sigma_t^2 \leq & - \left(\frac{9x_0 S^U}{8} \right) \left\{ \frac{1}{w^U - \left(\frac{w^U}{\theta^W} + \tau \right)} \right\} \\
& + \frac{(X_0 + \alpha\mu_t)}{2} \left\{ (D_0 + \beta\mu_t) - \left(c^U - \frac{2c^U}{\theta^T} - \frac{w^U}{\theta^W} - \tau \right) \right\} \\
& - \left\{ X_0 D_0 + (\alpha D_0 + \beta X_0)\mu_t + \alpha\beta\mu_t^2 \right\}.
\end{aligned}$$

(iii-1) $\alpha < 0$

Manufacturer K moves to country U if

$$\begin{aligned}
\sigma_t^2 \geq & - \left(\frac{9x_0 S^U}{8\alpha\beta} \right) \left\{ \frac{1}{w^U - \left(\frac{w^U}{\theta^W} + \tau \right)} \right\} \\
& + \frac{(X_0 + \alpha\mu_t)}{2\alpha\beta} \left\{ (D_0 + \beta\mu_t) - \left(c^U - \frac{2c^U}{\theta^T} - \frac{w^U}{\theta^W} - \tau \right) \right\} \\
& - \left\{ \frac{X_0 D_0 + (\alpha D_0 + \beta X_0)\mu_t + \alpha\beta\mu_t^2}{\alpha\beta} \right\}.
\end{aligned}$$

Then, given that manufacturer K moves to country U , manufacturer U 's profit

by staying in country U and moving to country K , respectively, are

$$\frac{1}{9} \left\{ 2\mathbb{E}[D_t] - \frac{\mathbb{E}[X_t D_t]}{\mathbb{E}[X_t]} - 2(c^U + w^U) + \left(\frac{c^U}{\theta^T} + w^U \right) \right\}^2 \text{ and}$$

$$\frac{1}{9} \left\{ 2\mathbb{E}[D_t] - \frac{\mathbb{E}[X_t D_t]}{\mathbb{E}[X_t]} - 2 \left(c^U + \frac{w^U}{\theta^W} + \tau \right) + \left(\frac{c^U}{\theta^T} + w^U \right) \right\}^2.$$

Since $\frac{w^U}{\theta^W} + \tau < w^U$, manufacturer U moves to country K . Then, given that manufacturer U moves to country K , manufacturer K 's profit by staying in country K and moving to country U , respectively, are

$$\frac{\mathbb{E}[X_t]}{9} \left\{ -\mathbb{E}[D_t] + \frac{2\mathbb{E}[X_t D_t]}{\mathbb{E}[X_t]} + \left(c^U + \frac{w^U}{\theta^W} + \tau \right) - 2 \left(\frac{c^U}{\theta^T} + \frac{w^U}{\theta^W} + \tau \right) \right\}^2 \text{ and}$$

$$\frac{\mathbb{E}[X_t]}{9} \left\{ -\mathbb{E}[D_t] + \frac{2\mathbb{E}[X_t D_t]}{\mathbb{E}[X_t]} + \left(c^U + \frac{w^U}{\theta^W} + \tau \right) - 2 \left(\frac{c^U}{\theta^T} + w^U \right) \right\}^2 - x_0 S^U.$$

Then, the manufacturer K 's profit increase by moving to county U is

$$\begin{aligned} & \frac{\mathbb{E}[X_t]}{9} \left\{ -\mathbb{E}[D_t] + \frac{2\mathbb{E}[X_t D_t]}{\mathbb{E}[X_t]} + \left(c^U + \frac{w^U}{\theta^W} + \tau \right) - 2 \left(\frac{c^U}{\theta^T} + w^U \right) \right\}^2 - x_0 S^U \\ & - \frac{\mathbb{E}[X_t]}{9} \left\{ -\mathbb{E}[D_t] + \frac{2\mathbb{E}[X_t D_t]}{\mathbb{E}[X_t]} + \left(c^U + \frac{w^U}{\theta^W} + \tau \right) - 2 \left(\frac{c^U}{\theta^T} + \frac{w^U}{\theta^W} + \tau \right) \right\}^2 \\ & = -\frac{2\mathbb{E}[X_t]}{9} \left\{ -2\mathbb{E}[D_t] + \frac{4\mathbb{E}[X_t D_t]}{\mathbb{E}[X_t]} \right\} \left\{ w^U - \left(\frac{w^U}{\theta^W} + \tau \right) \right\} \\ & - \frac{2\mathbb{E}[X_t]}{9} \left\{ 2 \left(c^U - w^U - \frac{2c^U}{\theta^T} \right) \right\} \left\{ w^U - \left(\frac{w^U}{\theta^W} + \tau \right) \right\} - x_0 S^U. \end{aligned}$$

Hence, the manufacturer K 's profit increase by moving to county U is non-negative if

$$\begin{aligned} \alpha\beta\sigma_t^2 \leq & - \left(\frac{9x_0 S^U}{8} \right) \left\{ \frac{1}{w^U - \left(\frac{w^U}{\theta^W} + \tau \right)} \right\} \\ & + \frac{(X_0 + \alpha\mu_t)}{2} \left\{ (D_0 + \beta\mu_t) - \left(c^U - w^U - \frac{2c^U}{\theta^T} \right) \right\} \\ & - \{ X_0 D_0 + (\alpha D_0 + \beta X_0)\mu_t + \alpha\beta\mu_t^2 \}. \end{aligned}$$

Since $\alpha < 0$, manufacturer K moves to country U if

$$\begin{aligned} \sigma_t^2 \geq & - \left(\frac{9x_0 S^U}{8\alpha\beta} \right) \left\{ \frac{1}{w^U - \left(\frac{w^U}{\theta^w} + \tau \right)} \right\} \\ & + \frac{(X_0 + \alpha\mu_t)}{2\alpha\beta} \left\{ (D_0 + \beta\mu_t) - \left(c^U - w^U - \frac{2c^U}{\theta^T} \right) \right\} \\ & - \left\{ \frac{X_0 D_0 + (\alpha D_0 + \beta X_0)\mu_t + \alpha\beta\mu_t^2}{\alpha\beta} \right\}. \end{aligned}$$

Given that manufacturer K moves to country U , manufacturer U moves to country K . Hence, the equilibrium is Case 4 if

$$\begin{aligned} \sigma_t^2 \geq & - \left(\frac{9x_0 S^U}{8\alpha\beta} \right) \left\{ \frac{1}{w^U - \left(\frac{w^U}{\theta^w} + \tau \right)} \right\} \\ & + \frac{(X_0 + \alpha\mu_t)}{2\alpha\beta} \left\{ (D_0 + \beta\mu_t) - \left(c^U - \frac{2c^U}{\theta^T} - \frac{w^U}{\theta^w} - \tau \right) \right\} \\ & - \left\{ \frac{X_0 D_0 + (\alpha D_0 + \beta X_0)\mu_t + \alpha\beta\mu_t^2}{\alpha\beta} \right\} \end{aligned}$$

and

$$\begin{aligned} \sigma_t^2 \geq & - \left(\frac{9x_0 S^U}{8\alpha\beta} \right) \left\{ \frac{1}{w^U - \left(\frac{w^U}{\theta^w} + \tau \right)} \right\} \\ & + \frac{(X_0 + \alpha\mu_t)}{2\alpha\beta} \left\{ (D_0 + \beta\mu_t) - \left(c^U - w^U - \frac{2c^U}{\theta^T} \right) \right\} \\ & - \left\{ \frac{X_0 D_0 + (\alpha D_0 + \beta X_0)\mu_t + \alpha\beta\mu_t^2}{\alpha\beta} \right\}. \end{aligned}$$

(iii-2) $\alpha > 0$

Manufacturer K stays in country K if

$$\begin{aligned} \sigma_t^2 \geq & - \left(\frac{9x_0 S^U}{8\alpha\beta} \right) \left\{ \frac{1}{w^U - \left(\frac{w^U}{\theta^W} + \tau \right)} \right\} \\ & + \frac{(X_0 + \alpha\mu_t)}{2\alpha\beta} \left\{ (D_0 + \beta\mu_t) - \left(c^U - \frac{2c^U}{\theta^T} - \frac{w^U}{\theta^W} - \tau \right) \right\} \\ & - \left\{ \frac{X_0 D_0 + (\alpha D_0 + \beta X_0)\mu_t + \alpha\beta\mu_t^2}{\alpha\beta} \right\}. \end{aligned}$$

Then, given that manufacturer K stays in country K , manufacturer U 's profit by staying in country U and moving to country K , respectively, are

$$\begin{aligned} & \frac{1}{9} \left\{ 2\mathbb{E}[D_t] - \frac{\mathbb{E}[X_t D_t]}{\mathbb{E}[X_t]} - 2(c^U + w^U) + \left(\frac{c^U}{\theta^T} + \frac{w^U}{\theta^W} + \tau \right) \right\}^2 \text{ and} \\ & \frac{1}{9} \left\{ 2\mathbb{E}[D_t] - \frac{\mathbb{E}[X_t D_t]}{\mathbb{E}[X_t]} - 2 \left(c^U + \frac{w^U}{\theta^W} + \tau \right) + \left(\frac{c^U}{\theta^T} + \frac{w^U}{\theta^W} + \tau \right) \right\}^2. \end{aligned}$$

Since $\frac{w^U}{\theta^W} + \tau < w^U$, manufacturer U moves to country K . Then, given that manufacturer U moves to country K , manufacturer K 's profit by staying in country K and moving to country U , respectively, are

$$\begin{aligned} & \frac{\mathbb{E}[X_t]}{9} \left\{ -\mathbb{E}[D_t] + \frac{2\mathbb{E}[X_t D_t]}{\mathbb{E}[X_t]} + \left(c^U + \frac{w^U}{\theta^W} + \tau \right) - 2 \left(\frac{c^U}{\theta^T} + \frac{w^U}{\theta^W} + \tau \right) \right\}^2 \text{ and} \\ & \frac{\mathbb{E}[X_t]}{9} \left\{ -\mathbb{E}[D_t] + \frac{2\mathbb{E}[X_t D_t]}{\mathbb{E}[X_t]} + \left(c^U + \frac{w^U}{\theta^W} + \tau \right) - 2 \left(\frac{c^U}{\theta^T} + w^U \right) \right\}^2 - x_0 S^U. \end{aligned}$$

Then, similar to (iii-1), since $\alpha > 0$, manufacturer K stays in country K if

$$\begin{aligned} \sigma_t^2 \geq & - \left(\frac{9x_0 S^U}{8\alpha\beta} \right) \left\{ \frac{1}{w^U - \left(\frac{w^U}{\theta^W} + \tau \right)} \right\} \\ & + \frac{(X_0 + \alpha\mu_t)}{2\alpha\beta} \left\{ (D_0 + \beta\mu_t) - \left(c^U - w^U - \frac{2c^U}{\theta^T} \right) \right\} \\ & - \left\{ \frac{X_0 D_0 + (\alpha D_0 + \beta X_0)\mu_t + \alpha\beta\mu_t^2}{\alpha\beta} \right\}. \end{aligned}$$

Given that manufacturer K stays in country K , manufacturer U moves to country K . Hence, the equilibrium is Case 3 if

$$\begin{aligned} \sigma_t^2 \geq & - \left(\frac{9x_0 S^U}{8\alpha\beta} \right) \left\{ \frac{1}{w^U - \left(\frac{w^U}{\theta^W} + \tau \right)} \right\} \\ & + \frac{(X_0 + \alpha\mu_t)}{2\alpha\beta} \left\{ (D_0 + \beta\mu_t) - \left(c^U - \frac{2c^U}{\theta^T} - \frac{w^U}{\theta^w} - \tau \right) \right\} \\ & - \left\{ \frac{X_0 D_0 + (\alpha D_0 + \beta X_0)\mu_t + \alpha\beta\mu_t^2}{\alpha\beta} \right\} \end{aligned}$$

and

$$\begin{aligned} \sigma_t^2 \geq & - \left(\frac{9x_0 S^U}{8\alpha\beta} \right) \left\{ \frac{1}{w^U - \left(\frac{w^U}{\theta^W} + \tau \right)} \right\} \\ & + \frac{(X_0 + \alpha\mu_t)}{2\alpha\beta} \left\{ (D_0 + \beta\mu_t) - \left(c^U - w^U - \frac{2c^U}{\theta^T} \right) \right\} \\ & - \left\{ \frac{X_0 D_0 + (\alpha D_0 + \beta X_0)\mu_t + \alpha\beta\mu_t^2}{\alpha\beta} \right\}. \end{aligned}$$

(iv) Equilibriums in Scenario (13) and Scenario (14)

(iv-1) $\alpha < 0$

Similar to (iii-1), when $\alpha < 0$, given that manufacturer U is located in country U , manufacturer K moves to country U if

$$\begin{aligned} \sigma_t^2 \geq & - \left(\frac{9x_0 S^U}{8\alpha\beta} \right) \left\{ \frac{1}{w^U - \left(\frac{w^U}{\theta^W} + \tau \right)} \right\} \\ & + \frac{(X_0 + \alpha\mu_t)}{2\alpha\beta} \left\{ (D_0 + \beta\mu_t) - \left(c^U - \frac{2c^U}{\theta^T} - \frac{w^U}{\theta^w} - \tau \right) \right\} \\ & - \left\{ \frac{X_0 D_0 + (\alpha D_0 + \beta X_0)\mu_t + \alpha\beta\mu_t^2}{\alpha\beta} \right\}. \end{aligned}$$

Then, given that manufacturer K moves to country U , manufacturer U 's profit

by staying in country U and moving to country K , respectively, are

$$\frac{1}{9} \left\{ 2\mathbb{E}[D_t] - \frac{\mathbb{E}[X_t D_t]}{\mathbb{E}[X_t]} - 2(c^U + w^U) + \left(\frac{c^U}{\theta^T} + w^U \right) \right\}^2 \quad \text{and}$$

$$\frac{1}{9} \left\{ 2\mathbb{E}[D_t] - \frac{\mathbb{E}[X_t D_t]}{\mathbb{E}[X_t]} - 2 \left(c^U + \frac{w^U}{\theta^W} + \tau \right) + \left(\frac{c^U}{\theta^T} + w^U \right) \right\}^2 - \frac{S^K}{x_0}.$$

Then, the manufacturer U 's profit increase by moving to county K is

$$\begin{aligned} & \frac{1}{9} \left\{ 2\mathbb{E}[D_t] - \frac{\mathbb{E}[X_t D_t]}{\mathbb{E}[X_t]} - 2 \left(c^U + \frac{w^U}{\theta^W} + \tau \right) + \left(\frac{c^U}{\theta^T} + w^U \right) \right\}^2 - \frac{S^K}{x_0} \\ & - \frac{1}{9} \left\{ 2\mathbb{E}[D_t] - \frac{\mathbb{E}[X_t D_t]}{\mathbb{E}[X_t]} - 2(c^U + w^U) + \left(\frac{c^U}{\theta^T} + w^U \right) \right\}^2 \\ & = \frac{2}{9} \left\{ 4\mathbb{E}[D_t] - 2 \frac{\mathbb{E}[X_t D_t]}{\mathbb{E}[X_t]} \right\} \left\{ w^U - \left(\frac{w^U}{\theta^W} + \tau \right) \right\} \\ & - \frac{2}{9} \left\{ 2 \left(2c^U + \frac{w^U}{\theta^W} + \tau - \frac{c^U}{\theta^T} \right) \right\} \left\{ w^U - \left(\frac{w^U}{\theta^W} + \tau \right) \right\} - \frac{S^K}{x_0}. \end{aligned}$$

Since $\alpha < 0$, manufacturer U moves to county K if

$$\begin{aligned} \sigma_t^2 \geq & - \frac{(X_0 + \alpha\mu_t)}{2\alpha\beta} \left[\left(\frac{9S^K}{2x_0} \right) \left\{ \frac{1}{w^U - \left(\frac{w^U}{\theta^W} + \tau \right)} \right\} \right] \\ & + \frac{(X_0 + \alpha\mu_t)}{2\alpha\beta} \left[4(D_0 + \beta\mu_t) - 2 \left(2c^U + \frac{w^U}{\theta^W} + \tau - \frac{c^U}{\theta^T} \right) \right] \\ & - \left\{ \frac{X_0 D_0 + (\alpha D_0 + \beta X_0)\mu_t + \alpha\beta\mu_t^2}{\alpha\beta} \right\}. \end{aligned}$$

Then, given that manufacturer U moves to county K , by (iii-1), manufacturer K moves to country U if

$$\begin{aligned} \sigma_t^2 \geq & - \left(\frac{9x_0 S^U}{8\alpha\beta} \right) \left\{ \frac{1}{w^U - \left(\frac{w^U}{\theta^W} + \tau \right)} \right\} \\ & + \frac{(X_0 + \alpha\mu_t)}{2\alpha\beta} \left\{ (D_0 + \beta\mu_t) - \left(c^U - w^U - \frac{2c^U}{\theta^T} \right) \right\} \\ & - \left\{ \frac{X_0 D_0 + (\alpha D_0 + \beta X_0)\mu_t + \alpha\beta\mu_t^2}{\alpha\beta} \right\}. \end{aligned}$$

Therefore, if

$$\begin{aligned}
\sigma_t^2 &\geq -\left(\frac{9x_0S^U}{8\alpha\beta}\right)\left\{\frac{1}{w^U - \left(\frac{w^U}{\theta^W} + \tau\right)}\right\} \\
&+ \frac{(X_0 + \alpha\mu_t)}{2\alpha\beta}\left\{(D_0 + \beta\mu_t) - \left(c^U - \frac{2c^U}{\theta^T} - \frac{w^U}{\theta^W} - \tau\right)\right\} \\
&- \left\{\frac{X_0D_0 + (\alpha D_0 + \beta X_0)\mu_t + \alpha\beta\mu_t^2}{\alpha\beta}\right\}, \\
\sigma_t^2 &\geq -\frac{(X_0 + \alpha\mu_t)}{2\alpha\beta}\left[\left(\frac{9S^K}{2x_0}\right)\left\{\frac{1}{w^U - \left(\frac{w^U}{\theta^W} + \tau\right)}\right\}\right] \\
&+ \frac{(X_0 + \alpha\mu_t)}{2\alpha\beta}\left[4(D_0 + \beta\mu_t) - 2\left(2c^U + \frac{w^U}{\theta^W} + \tau - \frac{c^U}{\theta^T}\right)\right] \\
&- \left\{\frac{X_0D_0 + (\alpha D_0 + \beta X_0)\mu_t + \alpha\beta\mu_t^2}{\alpha\beta}\right\} \text{ and} \\
\sigma_t^2 &\geq -\left(\frac{9x_0S^U}{8\alpha\beta}\right)\left\{\frac{1}{w^U - \left(\frac{w^U}{\theta^W} + \tau\right)}\right\} \\
&+ \frac{(X_0 + \alpha\mu_t)}{2\alpha\beta}\left\{(D_0 + \beta\mu_t) - \left(c^U - w^U - \frac{2c^U}{\theta^T}\right)\right\} \\
&- \left\{\frac{X_0D_0 + (\alpha D_0 + \beta X_0)\mu_t + \alpha\beta\mu_t^2}{\alpha\beta}\right\},
\end{aligned}$$

then the equilibrium is Case 4.

(iv-2) $\alpha > 0$

Similar to (iii-2), when $\alpha > 0$, given that manufacturer U is located in country

U , manufacturer K stays in country K if

$$\begin{aligned} \sigma_t^2 \geq & - \left(\frac{9x_0 S^U}{8\alpha\beta} \right) \left\{ \frac{1}{w^U - \left(\frac{w^U}{\theta^W} + \tau \right)} \right\} \\ & + \frac{(X_0 + \alpha\mu_t)}{2\alpha\beta} \left\{ (D_0 + \beta\mu_t) - \left(c^U - \frac{2c^U}{\theta^T} - \frac{w^U}{\theta^W} - \tau \right) \right\} \\ & - \left\{ \frac{X_0 D_0 + (\alpha D_0 + \beta X_0)\mu_t + \alpha\beta\mu_t^2}{\alpha\beta} \right\}. \end{aligned}$$

Then, given that manufacturer K stays in country K , manufacturer U 's profit by staying in country U and moving to country K , respectively, are

$$\begin{aligned} & \frac{1}{9} \left\{ 2\mathbb{E}[D_t] - \frac{\mathbb{E}[X_t D_t]}{\mathbb{E}[X_t]} - 2(c^U + w^U) + \left(\frac{c^U}{\theta^T} + \frac{w^U}{\theta^W} + \tau \right) \right\}^2 \text{ and} \\ & \frac{1}{9} \left\{ 2\mathbb{E}[D_t] - \frac{\mathbb{E}[X_t D_t]}{\mathbb{E}[X_t]} - 2 \left(c^U + \frac{w^U}{\theta^W} + \tau \right) + \left(\frac{c^U}{\theta^T} + \frac{w^U}{\theta^W} + \tau \right) \right\}^2 - \frac{S^K}{x_0}. \end{aligned}$$

Then, by (ii), manufacturer U stays in country U if

$$\begin{aligned} \sigma_t^2 \geq & - \frac{(X_0 + \alpha\mu_t)}{2\alpha\beta} \left[\left(\frac{9S^K}{2x_0} \right) \left\{ \frac{1}{w^U - \left(\frac{w^U}{\theta^W} + \tau \right)} \right\} \right] \\ & + \frac{(X_0 + \alpha\mu_t)}{2\alpha\beta} \left[4(D_0 + \beta\mu_t) - 2 \left(2c^U + w^U - \frac{c^U}{\theta^T} \right) \right] \\ & - \left\{ \frac{X_0 D_0 + (\alpha D_0 + \beta X_0)\mu_t + \alpha\beta\mu_t^2}{\alpha\beta} \right\}. \end{aligned}$$

Therefore, if

$$\begin{aligned}
\sigma_t^2 &\geq - \left(\frac{9x_0 S^U}{8\alpha\beta} \right) \left\{ \frac{1}{w^U - \left(\frac{w^U}{\theta^w} + \tau \right)} \right\} \\
&\quad + \frac{(X_0 + \alpha\mu_t)}{2\alpha\beta} \left\{ (D_0 + \beta\mu_t) - \left(c^U - \frac{2c^U}{\theta^T} - \frac{w^U}{\theta^w} - \tau \right) \right\} \\
&\quad - \left\{ \frac{X_0 D_0 + (\alpha D_0 + \beta X_0)\mu_t + \alpha\beta\mu_t^2}{\alpha\beta} \right\} \text{ and} \\
\sigma_t^2 &\geq - \frac{(X_0 + \alpha\mu_t)}{2\alpha\beta} \left[\left(\frac{9S^K}{2x_0} \right) \left\{ \frac{1}{w^U - \left(\frac{w^U}{\theta^w} + \tau \right)} \right\} \right] \\
&\quad + \frac{(X_0 + \alpha\mu_t)}{2\alpha\beta} \left[4(D_0 + \beta\mu_t) - 2 \left(2c^U + w^U - \frac{c^U}{\theta^T} \right) \right] \\
&\quad - \left\{ \frac{X_0 D_0 + (\alpha D_0 + \beta X_0)\mu_t + \alpha\beta\mu_t^2}{\alpha\beta} \right\},
\end{aligned}$$

then the equilibrium is Case 1.

(v) Equilibriums in Scenario (3) and Scenario (4)

(v-1) $\alpha < 0$

Similar to (iii-2), given that manufacturer U is located in country U , manufacturer K stays in country K if

$$\begin{aligned}
\sigma_t^2 &\geq - \left(\frac{9x_0 S^U}{8\alpha\beta} \right) \left\{ \frac{1}{w^U - \left(\frac{w^U}{\theta^w} + \tau \right)} \right\} \\
&\quad + \frac{(X_0 + \alpha\mu_t)}{2\alpha\beta} \left\{ (D_0 + \beta\mu_t) - \left(c^U - \frac{2c^U}{\theta^T} - \frac{w^U}{\theta^w} - \tau \right) \right\} \\
&\quad - \left\{ \frac{X_0 D_0 + (\alpha D_0 + \beta X_0)\mu_t + \alpha\beta\mu_t^2}{\alpha\beta} \right\}.
\end{aligned}$$

Then, given that manufacturer K stays in country K , manufacturer U 's profit

by staying in country U and moving to country K , respectively, are

$$\frac{1}{9} \left\{ 2\mathbb{E}[D_t] - \frac{\mathbb{E}[X_t D_t]}{\mathbb{E}[X_t]} - 2(c^U + w^U) + \left(\frac{c^U}{\theta^T} + \frac{w^U}{\theta^W} + \tau \right) \right\}^2 \text{ and}$$

$$\frac{1}{9} \left\{ 2\mathbb{E}[D_t] - \frac{\mathbb{E}[X_t D_t]}{\mathbb{E}[X_t]} - 2 \left(c^U + \frac{w^U}{\theta^W} + \tau \right) + \left(\frac{c^U}{\theta^T} + \frac{w^U}{\theta^W} + \tau \right) \right\}^2 .$$

Since $\frac{w^U}{\theta^W} + \tau > w^U$, manufacturer U stays in country U . Hence, the equilibrium is Case 1 if

$$\sigma_t^2 \geq - \left(\frac{9x_0 S^U}{8\alpha\beta} \right) \left\{ \frac{1}{w^U - \left(\frac{w^U}{\theta^W} + \tau \right)} \right\}$$

$$+ \frac{(X_0 + \alpha\mu_t)}{2\alpha\beta} \left\{ (D_0 + \beta\mu_t) - \left(c^U - \frac{2c^U}{\theta^T} - \frac{w^U}{\theta^W} - \tau \right) \right\}$$

$$- \left\{ \frac{X_0 D_0 + (\alpha D_0 + \beta X_0)\mu_t + \alpha\beta\mu_t^2}{\alpha\beta} \right\} .$$

(v-2) $\alpha > 0$

Similar to (iii-1), manufacturer K moves to country U if

$$\sigma_t^2 \geq - \left(\frac{9x_0 S^U}{8\alpha\beta} \right) \left\{ \frac{1}{w^U - \left(\frac{w^U}{\theta^W} + \tau \right)} \right\}$$

$$+ \frac{(X_0 + \alpha\mu_t)}{2\alpha\beta} \left\{ (D_0 + \beta\mu_t) - \left(c^U - \frac{2c^U}{\theta^T} - \frac{w^U}{\theta^W} - \tau \right) \right\}$$

$$- \left\{ \frac{X_0 D_0 + (\alpha D_0 + \beta X_0)\mu_t + \alpha\beta\mu_t^2}{\alpha\beta} \right\} .$$

Then, given that manufacturer K moves to country U , manufacturer U 's profit by staying in country U and moving to country K , respectively, are

$$\frac{1}{9} \left\{ 2\mathbb{E}[D_t] - \frac{\mathbb{E}[X_t D_t]}{\mathbb{E}[X_t]} - 2(c^U + w^U) + \left(\frac{c^U}{\theta^T} + w^U \right) \right\}^2 \text{ and}$$

$$\frac{1}{9} \left\{ 2\mathbb{E}[D_t] - \frac{\mathbb{E}[X_t D_t]}{\mathbb{E}[X_t]} - 2 \left(c^U + \frac{w^U}{\theta^W} + \tau \right) + \left(\frac{c^U}{\theta^T} + w^U \right) \right\}^2 .$$

Since $\frac{w^U}{\theta^W} + \tau > w^U$, manufacturer U stays in country U . Hence, the equilibrium is Case 2 if

$$\begin{aligned} \sigma_t^2 \geq & - \left(\frac{9x_0 S^U}{8\alpha\beta} \right) \left\{ \frac{1}{w^U - \left(\frac{w^U}{\theta^W} + \tau \right)} \right\} \\ & + \frac{(X_0 + \alpha\mu_t)}{2\alpha\beta} \left\{ (D_0 + \beta\mu_t) - \left(c^U - \frac{2c^U}{\theta^T} - \frac{w^U}{\theta^w} - \tau \right) \right\} \\ & - \left\{ \frac{X_0 D_0 + (\alpha D_0 + \beta X_0)\mu_t + \alpha\beta\mu_t^2}{\alpha\beta} \right\}. \end{aligned}$$

(vi) Equilibriums in Scenario (7) and Scenario (8)

(vi-1) $\alpha < 0$

By (v-1), given that manufacturer U is located in country U , manufacturer K stays in country K if

$$\begin{aligned} \sigma_t^2 \geq & - \left(\frac{9x_0 S^U}{8\alpha\beta} \right) \left\{ \frac{1}{w^U - \left(\frac{w^U}{\theta^W} + \tau \right)} \right\} \\ & + \frac{(X_0 + \alpha\mu_t)}{2\alpha\beta} \left\{ (D_0 + \beta\mu_t) - \left(c^U - \frac{2c^U}{\theta^T} - \frac{w^U}{\theta^w} - \tau \right) \right\} \\ & - \left\{ \frac{X_0 D_0 + (\alpha D_0 + \beta X_0)\mu_t + \alpha\beta\mu_t^2}{\alpha\beta} \right\}. \end{aligned}$$

Then, similar to (ii), manufacturer U stays in country U if

$$\begin{aligned} \sigma_t^2 \geq & - \frac{(X_0 + \alpha\mu_t)}{2\alpha\beta} \left[\left(\frac{9S^K}{2x_0} \right) \left\{ \frac{1}{w^U - \left(\frac{w^U}{\theta^W} + \tau \right)} \right\} \right] \\ & + \frac{(X_0 + \alpha\mu_t)}{2\alpha\beta} \left[4(D_0 + \beta\mu_t) - 2 \left(2c^U + w^U - \frac{c^U}{\theta^T} \right) \right] \\ & - \left\{ \frac{X_0 D_0 + (\alpha D_0 + \beta X_0)\mu_t + \alpha\beta\mu_t^2}{\alpha\beta} \right\}. \end{aligned}$$

Hence, the equilibrium is Case 1 if

$$\begin{aligned} \sigma_t^2 \geq & - \left(\frac{9x_0 S^U}{8\alpha\beta} \right) \left\{ \frac{1}{w^U - \left(\frac{w^U}{\theta^w} + \tau \right)} \right\} \\ & + \frac{(X_0 + \alpha\mu_t)}{2\alpha\beta} \left\{ (D_0 + \beta\mu_t) - \left(c^U - \frac{2c^U}{\theta^T} - \frac{w^U}{\theta^w} - \tau \right) \right\} \\ & - \left\{ \frac{X_0 D_0 + (\alpha D_0 + \beta X_0)\mu_t + \alpha\beta\mu_t^2}{\alpha\beta} \right\} \end{aligned}$$

and

$$\begin{aligned} \sigma_t^2 \geq & - \frac{(X_0 + \alpha\mu_t)}{2\alpha\beta} \left[\left(\frac{9S^K}{2x_0} \right) \left\{ \frac{1}{w^U - \left(\frac{w^U}{\theta^w} + \tau \right)} \right\} \right] \\ & + \frac{(X_0 + \alpha\mu_t)}{2\alpha\beta} \left[4(D_0 + \beta\mu_t) - 2 \left(2c^U + w^U - \frac{c^U}{\theta^T} \right) \right] \\ & - \left\{ \frac{X_0 D_0 + (\alpha D_0 + \beta X_0)\mu_t + \alpha\beta\mu_t^2}{\alpha\beta} \right\}. \end{aligned}$$

(vi-2) $\alpha > 0$

By (v-2), given that manufacturer U is located in country U , manufacturer K moves to country U if

$$\begin{aligned} \sigma_t^2 \geq & - \left(\frac{9x_0 S^U}{8\alpha\beta} \right) \left\{ \frac{1}{w^U - \left(\frac{w^U}{\theta^w} + \tau \right)} \right\} \\ & + \frac{(X_0 + \alpha\mu_t)}{2\alpha\beta} \left\{ (D_0 + \beta\mu_t) - \left(c^U - \frac{2c^U}{\theta^T} - \frac{w^U}{\theta^w} - \tau \right) \right\} \\ & - \left\{ \frac{X_0 D_0 + (\alpha D_0 + \beta X_0)\mu_t + \alpha\beta\mu_t^2}{\alpha\beta} \right\}. \end{aligned}$$

Then, similar to (iv-1), manufacturer U stays in country U if

$$\begin{aligned} \sigma_t^2 \geq & -\frac{(X_0 + \alpha\mu_t)}{2\alpha\beta} \left[\left(\frac{9S^K}{2x_0} \right) \left\{ \frac{1}{w^U - \left(\frac{w^U}{\theta^W} + \tau \right)} \right\} \right] \\ & + \frac{(X_0 + \alpha\mu_t)}{2\alpha\beta} \left[4(D_0 + \beta\mu_t) - 2 \left(2c^U + \frac{w^U}{\theta^W} + \tau - \frac{c^U}{\theta^T} \right) \right] \\ & - \left\{ \frac{X_0 D_0 + (\alpha D_0 + \beta X_0)\mu_t + \alpha\beta\mu_t^2}{\alpha\beta} \right\}. \end{aligned}$$

Hence, the equilibrium is Case 2 if

$$\begin{aligned} \sigma_t^2 \geq & -\left(\frac{9x_0 S^U}{8\alpha\beta} \right) \left\{ \frac{1}{w^U - \left(\frac{w^U}{\theta^W} + \tau \right)} \right\} \\ & + \frac{(X_0 + \alpha\mu_t)}{2\alpha\beta} \left\{ (D_0 + \beta\mu_t) - \left(c^U - \frac{2c^U}{\theta^T} - \frac{w^U}{\theta^W} - \tau \right) \right\} \\ & - \left\{ \frac{X_0 D_0 + (\alpha D_0 + \beta X_0)\mu_t + \alpha\beta\mu_t^2}{\alpha\beta} \right\} \end{aligned}$$

and

$$\begin{aligned} \sigma_t^2 \geq & -\frac{(X_0 + \alpha\mu_t)}{2\alpha\beta} \left[\left(\frac{9S^K}{2x_0} \right) \left\{ \frac{1}{w^U - \left(\frac{w^U}{\theta^W} + \tau \right)} \right\} \right] \\ & + \frac{(X_0 + \alpha\mu_t)}{2\alpha\beta} \left[4(D_0 + \beta\mu_t) - 2 \left(2c^U + \frac{w^U}{\theta^W} + \tau - \frac{c^U}{\theta^T} \right) \right] \\ & - \left\{ \frac{X_0 D_0 + (\alpha D_0 + \beta X_0)\mu_t + \alpha\beta\mu_t^2}{\alpha\beta} \right\}. \end{aligned}$$

(vii) Equilibriums in Scenarios (11), (12), (15) and (16)

Given that manufacturer U is located in country U , manufacturer K 's profits by staying in country K and moving to country U , respectively, are

$$\begin{aligned} & \frac{\mathbb{E}[X_t]}{9} \left\{ -\mathbb{E}[D_t] + \frac{2\mathbb{E}[X_t D_t]}{\mathbb{E}[X_t]} + (c^U + w^U) - 2 \left(\frac{c^U}{\theta^T} + \frac{w^U}{\theta^W} + \tau \right) \right\}^2 \text{ and} \\ & \frac{\mathbb{E}[X_t]}{9} \left\{ -\mathbb{E}[D_t] + \frac{2\mathbb{E}[X_t D_t]}{\mathbb{E}[X_t]} + (c^U + w^U) - 2 \left(\frac{c^U}{\theta^T} + w^U \right) \right\}^2 - x_0 S^U. \end{aligned}$$

Since $\frac{w^U}{\theta^W} + \tau > w^U$ and $S^U < 0$, manufacturer K moves to country U . Given that manufacturer K moves to country U , manufacturer U 's profits by staying in country U and moving to country K , respectively, are

$$\frac{1}{9} \left\{ 2\mathbb{E}[D_t] - \frac{\mathbb{E}[X_t D_t]}{\mathbb{E}[X_t]} - 2(c^U + w^U) + \left(\frac{c^U}{\theta^T} + w^U \right) \right\}^2 \text{ and}$$

$$\frac{1}{9} \left\{ 2\mathbb{E}[D_t] - \frac{\mathbb{E}[X_t D_t]}{\mathbb{E}[X_t]} - 2 \left(c^U + \frac{w^U}{\theta^W} + \tau \right) + \left(\frac{c^U}{\theta^T} + w^U \right) \right\}^2 - \frac{S^K}{x_0}.$$

Since $\frac{w^U}{\theta^W} + \tau > w^U$ and $S^K \geq 0$, manufacturer U stays in country U . Therefore, the equilibrium is Case 2. \square

Proof of Propositions 3 and 4.

(i) EOQ Model

Manufacture U 's and K 's costs, respectively, are

$$\left(\frac{h^K/X}{2} \right) q^{E,U} + \frac{D^E(t^U + t^K/X)}{q^{E,U}} \text{ and } \left(\frac{h^K}{2} \right) q^{E,K} + \frac{D^E(t^U X + t^K)}{q^{E,K}}.$$

From the EOQ formula,

$$q^{E,U,*} = \sqrt{\frac{2D^E(t^U + t^K/X)}{h^K/X}} \text{ and } q^{E,K,*} = \sqrt{\frac{2D^E(t^U X + t^K)}{h^K}}.$$

Hence, both $q^{E,U,*}$ and $q^{E,K,*}$ increase as X increases. In addition, both $q^{E,U,*}$ and $q^{E,K,*}$ increase as t^U or t^K increases.

(ii) Newvendor Model

For manufacturer U , the unit underage and overage costs, respectively, are

$$p - (c/X + \tau^U + \tau^K/X) \text{ and } (c/X + \tau^U + \tau^K/X).$$

Similarly, for manufacturer K , the unit underage and overage costs, respectively, are

$$Xp - (c + X\tau^U + \tau^K) \text{ and } (c + X\tau^U + \tau^K).$$

Then, by the newsvendor formula, both optimal quantities satisfy

$$G(q^{E,U,*}) = G(q^{E,K,*}) = \frac{p - (c/X + \tau^U + \tau^K/X)}{p}.$$

Hence, both $q^{N,U,*}$ and $q^{N,K,*}$ increase as X increases; whereas, both $q^{N,U,*}$ and $q^{N,K,*}$ increase as τ^U or τ^K decreases. \square

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