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Student Rating of the Usefulness of Teacher-Provided Strategies for Simplifying Expressions and Solving Equations: How might student understanding of equals and equivalence be impacted by these strategies?

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by

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Dedication

This dissertation is dedicated to the following people:

- To my parents, Bill and Evajo York, whose living example of lifelong learning has inspired me since Day 1.
- To my husband, Ronald York-Hammons, for being a wonderful surprise in the second half of my life. Your love has brightened every day.
- To my children, Charisa and Damon, for loving me at my most crabby moments.
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Student Rating of the Usefulness of Teacher-Provided Strategies for Simplifying Expressions and Solving Equations: How might student understanding of equals and equivalence be impacted by these strategies?

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Almost twenty years ago Texas implemented a functions-based approach (FBA) to teaching algebra. This approach emphasized the use of nearly all multiple representations, use of a graphing calculator to explore graphs, and modeling of linear and quadratic functions. This interpretation of FBA in conjunction with curriculum placing the teaching of simplifying expressions and solving equations close in sequence may contribute to student confounding of the rules for simplifying and solving.

The purpose of this exploratory qualitative study was to explore student rating of the usefulness of teacher-provided and function-based approach (FBA) strategies for simplifying expressions and solving equations in Algebra. The subjects of this study were two algebra teachers and their respective algebra students. The teachers, who taught both Algebra 1 and Algebra 2, were at a high school campus located in an urban district.

A researcher created survey based on teacher-provided strategies used by participating teachers was administered to 100 students and 22 teachers. The teacher survey results were used as a professional basis for comparing students' results. Descriptive statistics were used to create graphical representations of students by course groups and identify students who confounded rules.

Student FBA preferences and course groups were used to identify 18 student interviewees. Student and teacher interviews were used to corroborate survey results. Participating teachers identified and commented on areas of concern from the survey results. Both teachers approved of the low percentages of students rating FBA strategies as useful but were concerned about higher percentages of students (30% or greater) confounding rules or not realizing the usefulness of relevant sub-strategies. Neither teachers nor students were aware of benefits of graphing calculator use in simplifying.

Students, regardless of course group or FBA preference, justified the use of teacher-provided strategies with symbolic manipulation and changed FBA ratings to less likely. There were few student references to equivalence and equality that were supported by FBA.

These results are important for algebraic instruction in Texas. Texas has mandated use of graphing calculator on 8th grade Mathematics STAAR exam. Recognizing the benefits of a complete FBA along with effective use of graphing technology may prevent this type of confounding.

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Chapter 1 Introduction

BACKGROUND

Although many research studies (Britt & Irwin, 2005; Demby, 2005; Dowker et al., 1996; Fujii, 2003; Kieran, 1992; Linsell, 2009; Steinle et al., 2009) on students' difficulties with simplifying expressions and solving equations have been published, none have explored student rating of the usefulness of teacher provided strategies for simplifying expressions or solving equations. Kieran (1992) reviewed the research literature that addressed Algebra students' belief that "mathematics is rule based, and … learning mathematics is mostly memorizing" (Kieran, p. 390). The research studies included in Kieran's review centered on content, the way mathematics is taught, and student approaches to Algebraic tasks. However, much of this research was completed prior to the use of graphing technology in the mathematics classroom.

Thirty-seven percent of the 334, 234 Algebra 1 students in Texas in the spring of 2006 did not receive credit for the course (Schneider, 2007). That means that 123,666 students in Texas failed Algebra I in that year alone. For the 2011-2012 school year, STAAR Algebra 1 EOC results indicated that 83% of Algebra 1 students passed the state exam. However, this is a "phase in" standard for the first year of implementation for freshmen. Looking at what would normally be the cut off for passing; only 39% of Algebra 1 students would have passed. This is similar to the Texas Assessment of Knowledge and Skills (TAKS) Exit Level cut off in 2004 for juniors. At that time students were only required to correctly answer 24 out of 60 questions on the mathematics TAKS exam. The results of the STAAR Algebra 1 EOC results for 2012 for each reporting category indicate that 60% of the test items were answered correctly. When disaggregating by LEP, the percentage dropped to 49%. The lowest scoring category is *Linear Equations and Inequalities* (TEA, 2012). When one considers that "nearly 70% of a standard introductory Algebra curriculum centers on only three big topics …: [1] equivalence (of functions) [2] equals (as one kind of comparison of functions) and a systemic engagement of linear function" (Stroup, n.d.). Perhaps the students who confound equivalence and equals when solving equations may be the same group of students who are not passing Algebra. A Functions Based Algebra (FBA) would be the best strategy to address this problem.

The Standards (NCTM, 1989) state, "The concept of function is an important unifying idea in mathematics. Functions, which are special correspondences between the elements of two sets, are common throughout the curriculum." (p. 154) Kaput (1998) listed five forms of Algebraic reasoning:

1. Algebra as Generalizing and Formalizing Patterns and Constraints, especially, but not exclusively, Algebra as Generalized Arithmetic Reasoning and Algebra as Generalized Quantitative Reasoning;
2. Algebra as Syntactically-Guided Manipulation of Formalisms;
3. Algebra as the Study of Structures and Systems Abstracted from Computations and Relations;
4. Algebra as the Study of Functions, Relations , and Joint Variation; and
5. Algebra as a Cluster of (a) Modeling and (b) Phenomena-Controlling Languages. (p. 26)

Yerushalmy (2000) provided a description of a FBA to Algebra. This description includes representing functions in various forms. The curriculum that Algebra students were exposed to included, "(1) Emergence of the concept of function throughout modeling, (2) manipulating function expressions and function comparisons (equations and inequalities), and (3) exploring families of functions and specifically linear and quadratic functions" (p. 126).

In Texas, FBA has been fraught with misconceptions. Typically, Algebra teachers' misconception of what constitutes a functions-based approach to Algebra arose from professional development provided to teachers across the state via the Regional Education Service Center. Modeling through real-world situations was intended to hold the interest of weaker students and to be used only as needed. This may explain why as a result was not emphasized in daily lessons (Stroup, 2009). The professional development series TEXTEAMS, more specifically Algebra 1: 2000 and Beyond, was initially proposed for teacher training only (Charles A. Dana Center, 2000). However, the inclusion of a few student activities in the training may have encouraged the misconception that the training was indeed curriculum for Algebra 1.

Although the Texas Education Agency mandated the integral use of graphing technology in Algebra 1 in 1998, some Algebra teachers still prefer that students use paper and pencil before using a graphing calculator. This may be due to a concern that students might have an unfair advantage by using graphing calculators (Brown et al., 2007) or the belief that students must learn by doing math symbolically by hand first (Dewey, Kinzel, & Singletary, 2009).

PURPOSE OF THE STUDY

The purpose of this dissertation study is to explore a possible relationship between students' rating of the usefulness of teacher provided strategies for simplifying expressions and solving equations and students' understanding of equivalence and equals. This study is designed to explore student preference for various mathematical procedures that have been demonstrated to the students in their current Algebra course. These mathematical procedures, or teacher-provided strategies as they are referred to in the student survey, refer to symbolic manipulations used by the Algebra teachers to explain to students how to simplify expressions and solve equations. This study will explore student preferences for using these strategies by allowing students to rate the usefulness of each strategy in correctly simplifying an expression or solving an equation. The math problems include problems students would have encountered early in the year in Algebra 1.

There is a body of literature devoted to student ability to solve and simplify where answers were judged as correct or incorrect. Researchers identified various strategies that students use when asked to simplify or solve and code those strategies (Kieran, 1992). More recent research includes results related to the implementation of Cognitive Tutor Algebra 1. This complete curriculum program combined classroom activities and practice sets in a text to be used in conjunction with a computer software program. The computer software actually tracked student choice for steps taken in simplifying and solving. (Ritter et al, 2007) Although the Cognitive Tutor Algebra 1 software tracks student use of the strategies used for simplifying and solving, these choices are from a drop down list and do not indicate student perception of usefulness of a particular strategy. The software can only determine if students continue to use the strategies to avoid negative feedback and continue to the next set of problems. There is no research related to how students perceive the usefulness of strategies demonstrated in class by teachers to simplify or solve.

This study is intended to fill the gap in the research related to student perception about the usefulness of strategies for simplifying and solving and the possible relationship to students' understanding of equivalence and equals. This dissertation is a descriptive and exploratory study that will answer the following research questions:

1. Does selecting the strategy "examining graphical representations or tables" either with a calculator/computer or pencil/pen and paper as useful relate to students selecting strategies that differentiate between equivalence and equals?
2. How does the student usefulness rating of teacher provided strategies relate to student understanding of equivalence and equals?

A successful descriptive study is one in which the researcher identifies commonalities within groups or courses as well as differences between course members' preferences for strategy use. If students can identify situations in which they perceive function based approaches as useful in Algebra, the researcher will have a better vision of the issues surrounding student understanding of equality and equivalence in Algebra. This understanding might drive future studies.

SIGNIFICANCE OF THE STUDY

Algebra 1 is the "gatekeeper" course to advanced mathematics, and students who fail Algebra 1 are unlikely to complete advanced mathematics courses during their high school education and are less likely to graduate with a college degree (Achieve Inc., 2006; U.S. Department of Education, 2008). Algebra teachers with a limited vision of FBA may be preventing students from understanding a major part of the Algebra curriculum—equivalence and equals. Teacher understanding of students' rating of the usefulness of strategies currently used in teaching Algebra may open avenues for future professional development between the local university and districts. Professional development aimed at providing further instruction in FBA may enable teachers to prevent the confounding of rules for simplifying and solving, which may lead to higher passing rates for Algebra students. If students experience success in Algebra 1 they may be more likely to continue taking more advanced mathematics during their high school career.

Conceptual Framework

Students who apply rules for simplifying and solving without understanding will tend to confound those rules. The FBA promoted in Texas has focused on Multiple representations (verbal, concrete, pictorial, tabular, symbolic, graphical) as the main feature of FBA. Without the emphasis on the other features of FBA, teachers continue to focus on an approach characterized by symbolic manipulation. A complete FBA to teaching Algebra provides students with the necessary tools to differentiate between rules used for simplifying expressions and solving equations. Chapter 2 will outline the components necessary for a complete FBA.

Brief Description of the Methodology

This sequential, qualitative study required the observation of two high school math teachers teaching both Algebra 1 and Algebra 2. The observations provided the researcher with the strategies to include on the student survey as well as the problems presented to the students during instruction on units involving simplifying expressions and solving equations. Algebra students were given a survey to determine their ratings of the usefulness of teacher-provided strategies for simplifying and solving situations. Survey responses are displayed as divergent bar charts and data will be examined to determine the percentages of students choosing appropriate strategies for the problem situation. My study explored whether students who selected a FBA strategy as useful also selected strategies that were appropriate for the problem situation. The final goal of the study was to determine if the strategies students choose are an indication of their understanding of equivalence or equals.

The survey results were used to identify students to interview from each subgroup. Interviews were semi-structured and open ended. Once the results of the surveys were analyzed and the interviews coded and analyzed, the Algebra teachers were interviewed about the results. The teacher interviews were semi-structured and open ended.

Chapter 2 Literature Review

INTRODUCTION

There have been major issues with the implementation of FBA. One of these issues deals with conflicting visions as to what Algebra in school should include. Another issue concerns conflicting accounts of what defines FBA. These issues are compounded by the professional development opportunities available to teachers, specifically in Texas, since the 1990s. In addition, Algebra teachers in Texas have grappled with the mandated integration of handheld technology in the Algebra 1 curriculum brought about by the implementation of the Texas Essential Knowledge and Skills (TEKS) in 1998. Texas' requirement to have *highly qualified mathematics teachers* teaching core subjects may have inadvertently compounded issues as well. High school mathematics teachers and pre-service teachers with higher mathematics education are likely to follow the symbolic-precedence view of Algebra development (Nathan & Koedinger, 2000; Nathan & Petrosino, 2003).

The intent of this literature review is to frame the issues associated with defining FBA and the history of implementation of a version of FBA in Texas. Issues related to defining FBA might provide an account of students' confounding the rules for simplifying expressions and solving equations. Understanding these issues may provide insight into models for local reform that would help Algebra teachers prevent the confounding of rules.

ALGEBRA IN SCHOOLS

Kaput (1995) stated, "Despite the fact that we all use one word "Algebra," there is no one Algebra, no monolith" (p. 5). If according to Kaput, there is "no one Algebra," how do we define Algebra? What should Algebra be all about? There are different stances regarding how Algebra is understood. These include a structural stance as identified by Piaget (1973), the somewhat conflicted stance proposed by The Algebra Working Group in the document "A Framework for Constructing a Vision of Algebra: a Discussion Document" (NCTM, 1997), a symbolic stance one might consider typical of the traditional Algebra curriculum, Algebra as modeling, and functions as rules expressing co-variation between dependent quantities.

Structural Stance.

Piaget (1973) indicated that among the earliest structures was the *mathematical group*. "A mathematical group is a system consisting of a set of elements together with an operation or rule of combination." (p. 18) Students in early elementary mathematics classrooms are provided opportunities to develop mathematical understanding of such topics as identities ($n + 0 = n$) and inverses (subtraction: $n - n = 0$).

NCTM: The Algebra Working Group.

The framework described as "Building a Dynamic View of Algebra" by the Algebra Working Group in a report for NCTM (1997) outlines the varying beliefs of the group (which included university faculty, public and private school teachers, as well as the liaison to the NCTM board) about what is important in understanding Algebra. It would appear that the group was unable to resolve conflicting viewpoints and decided to include all viewpoints. According to NCTM (1997) standards, mathematics should include problem solving, reasoning, communication, and making connections. Opportunities would be provided to students to apply these processes with contextual settings. The themes used to organize the curriculum include functions and relations, modeling, structure, as well as language and representation (NCTM, 1997). The theme of functions and relations would allow students to explore tables, graphs, and equations that we typically associate with Algebra. Modeling would allow students to explore natural phenomena to determine if there is an underlying relationship and if so, represent it symbolically (in upper grades). The theme of structure would include the algorithms that some mathematicians consider the focus of Algebra. The theme of language and representation refer to the idea that "Algebra can be seen as a language—with 'dialects' of literal symbols, graphs, tables, words, diagrams, and other visual displays" (NCTM, 1997). This is where the Algebra Working Group includes the idea of multiple representations (including concrete models as well as pictorial representations) and emphasizes the importance of students being able to explain the concepts illustrated by one representation that might not be seen when exploring a different representation. Consider a linear function in standard form. If students graphed points that are solutions to that linear function on a coordinate grid, the students might be able to tell you more about the rate of change in the graph of the line. That information might not be apparent in the standard symbolic form. Although the members of the group suggested that educators reflect on which themes are most appropriate at a particular grade level, they suggested that students should be able to function within multiple themes (NCTM, 1997). The group referenced few technology sources considering the inclusion of graphing utilities in grades 9-12 Algebra section of the Curriculum and Evaluation Standards (NCTM, 1989).

Algebra as Syntactically-Guided Manipulation of Formalisms.

Currently Algebra as taught to secondary students is characterized by the manipulation of symbols according to "syntactic principles and conventions" in addition to a few short problems (Kaput, 1995). In 2006 several groups of K-16 mathematics educators and mathematicians met to discuss "common principles that can serve as models for improvement" in various levels of Algebra. The working group for Intermediate Algebra "took the position that the primary purpose of an intermediate Algebra course is to prepare college-intending students for further work in mathematics. . .. Traditionally it covers many of the symbolic manipulation skills that students need to do well in calculus and in science and engineering courses" (Katz, 2007). The following ideas make up the central core of Algebra:

> *The idea of an Algebraic expression*: Recognizing that the symbols in an expression stand for numbers and that expressions represent calculations with numbers.
>
> *The idea of an equation*: Understanding that an equation is an assertion of equality between two expressions.
>
> *The relation between form and function of an Algebraic expression*: Recognizing that different forms of Algebraic expressions and equations reveal different properties of the objects they represent (functions, graphs, solutions). Performing Algebraic manipulations as a strategic choice rather than obedience to a command (simplify, expand, factor).
>
> *Solving equations as a process of reasoning*: Understanding that the steps in solving an equation constitute a series of mathematical deductions with logical justifications; understanding the difference between transformations on expressions that preserve value, and operations on equations that preserve equality. (p. 20)

Algebra as Modeling.

Schwartz and Yerushalmy (1995) provided a definition of modeling in Algebra. They stated:

> There are many people, we among them, who will argue that mathematics has the importance it has in the school curriculum because it provides people with a set of analytic tools for dealing with the quantitative aspects of their world. Doing so requires people to mathematize the situations that they wish to analyze. … The first step in this process is to move from the perceptions and measurements of the actual situation to a verbal description of the elements and relationships that hold among them. The task then becomes one of formalizing this verbal representation of the situation. This procedure is called modeling. (p. 29)

Their use of a software environment called "The Algebra Sketchbook" allows students to connect graphs and verbal descriptions of the problem. Kaput (1995) stated, "In modeling, we begin with phenomena and attempt to mathematize them."

Co-variation.

Yerushalmy (2000) described a process in which a student "starts with graphic qualitative descriptions of variations, moves on to numeric descriptions of differences, and uses numeric and graphical images of rate of change as objects upon which understanding of function can be built" (p. 126). Examples of problems explored in her research study were "break-even" problems. A functions based approach to Algebra would highlight how "functions can be used to model co-variation – i.e., how one variable is related to, or co-varies with, another variable" (Stroup, Carmona & Davis, 2011).

Curriculum.

In the past, mathematics educators who have experienced teaching Algebra in the secondary classroom using both traditional approaches and later a reformed approach to teaching Algebra experienced conflict in determining what should be included in their Algebra curriculum. Consider the experience of one such teacher, Daniel Chazan (1999).

In the 1980s, Chazan taught Algebra via a traditional approach using a Dolciani and Wooten text. The instruction for these college bound students was teacher centered. Students unquestionably accepted Chazan's algorithmic methods as the accepted method to solve problems. The topics had little connection other than that to other topics yet to be covered in future mathematics courses. In comparison, during the 1990s, Chazan team-taught Algebra to students not likely to attend college using what he considered a *functions based approach* according to NCTM (p. 125). Using this *functions based approach*, Chazan noticed discrepancies in what was advocated by the NCTM 1989 Curriculum and Evaluation Standards compared to what his students would be expected to learn in their course. One such discrepancy was the recommendation to decrease emphasis on traditional symbolic manipulations, including simplifying radical expressions and factoring, without providing any insight into ways in which to address the work requiring manipulation of symbols still in the Algebra curriculum (Chazan, 1999, p. 130). What is this *functions based approach* that Chazan is referring to?

FUNCTION BASED APPROACH TO ALGEBRA

One view of a FBA to Algebra was promoted, inadvertently, by the Charles A. Dana Center in their 1996 TEXTEAMS Algebra I Institute and TEXTEAMS Algebra 1: 2000 and Beyond Institute (Dana Center, 2000). Features of the institute included multiple representations, integration of manipulative materials, and graphing technology as well as rich connections within and outside mathematics (Dana Center, 2000). The first mentioned and possibly most emphasized is the multiple representation feature. Multiple representations such as verbal, concrete, pictorial, tabular, symbolic and graphical were used in each of the activities in the training. The majority of the activities focused on linear and quadratic functions and equations. Although the math content was covered in great depth, the focus was on exploring each of the representations and their connections to that content. This emphasis by the TEXTEAMS Institute (requiring the use of nearly all representations in the modeling of a real world application) defines FBA in Texas. Unfortunately, this small part of the FBA defines FBA in Texas schools.

There is a "long history of attempts to use the idea of function as an organizing principle for the mathematics curriculum, including and especially Algebra" (Fey, as cited in Kaput, 1995, p. 77). As early as 1908[1], Klein (2007) stated, "We, who are called the reformers, would put the function concept at the very center of instruction because, of all the concepts of the mathematics of the past two centuries, this one plays the leading role" (p. 4). This instruction would be introduced early and make use of the graphical representation (Klein, 2007). Prior to the availability of the graphing calculator, Gel'fand, Glagoleva, and Shnol (1969) provided instruction to ninth grade students on the topic of constructing graphs by considering features of functions. Functions, including the linear fraction function, power functions and rational functions, typically not covered in the current Texas Algebra I curriculum, are presented by exploring patterns from less complex functions and applying transformations to those functions. For areas of graphs of functions that may prove difficult to plot, the authors used small tables of x and y values that show a more accurate graph of the function. One such example is the graph of $y = \frac{1}{1+x^2}$. Gel'fand et al. used values of the argument, x, between 0 and 1 to plot a more accurate graph (pp. 9-10). This FBA, although without the benefit of a graphing utility, focuses on the graphical and symbolic representations along with the relationships or patterns observed on one representation when the other is transformed. Tabular representations are used when they can provide additional insight or a more accurate

[1] In the introduction of the reprint from 2007, the publishers indicate that the wording of the text is for the most part from the original lithograph volume of 1908.

graph. Problems are devoid of context. One might reason that the patterns observed as a result of acting on these objects (functions) is engaging enough that context is not necessary to maintain student interest.

Current research (Chazan, 1993; Schwartz & Yerushalmy, 1991; Stroup, Carmona, & Davis, 2005; Stroup, 2007, 2009, 2010, n.d.; Yerushalmy, 2000) provides additional insight into FBA. FBA is a rigorous alternative approach to the formalisms of Algebra (Stroup, 2009; see Appendix A). Schwartz and Yerushalmy (1991, p.47) stated that "the objects of school mathematics are numbers, shapes and functions." In Algebra, as well as high level mathematics, the function is a central concept (Chazan, 1993; Schwartz & Yerushalmy, 1991; Yerushalmy, 2000). In an introductory Algebra course, students will be introduced to expressions and equations for the majority of the first 2 years of Algebra. Functions are given comparatively little interest (Chazan, 1993).

FBA requires a different understanding of the terms equation and function. Functions are rules expressing co-variation between dependent quantities. Equations are one type of comparison of functions (Chazan, 1993; Schwartz & Yerushalmy, 1991; Stroup et al., 2005; Stroup, 2007, 2009, 2010, n.d.; Yerushalmy, 2000). The graphical and symbolic representations of the functions being compared would be observed to solve equations. Student understanding is supported by graphing technologies such as computer software and handheld graphing calculators. When solving equations, one would simply find the values of the shared domain for which the compared functions intersect. This type of visual has been supported by the use of graphing technologies, either computer software for graphing or handheld graphing calculators. Students observe both the graphic and symbolic form of the functions being compared (Chazan, 1993; Stroup et al, 2005; Stroup, n.d.). "In addition, FBA can help students understand more advanced topics including roots, properties of higher order polynomials, and composite functions by having them think in terms of combining functions using operations like +, - , * and ÷" (Stroup, 2010). Rather than factoring quadratic expressions in isolation, students would explore the patterns resulting from multiplying two linear functions. Schwartz and Yerushalmy (1991) extended this idea of solving equations to a comparison of functions in two variables. For example, solving $x^2 + y^2 = x + 4$ in a traditional Algebra course would require a symbolic manipulation of the equation along with completing the square in order to put the equation in a standard form for an ellipse. However, using the concepts of FBA, one would compare the graphs of two functions, $F(x, y) = x^2 + y^2$ and $G(x, y) = x + 4$ to determine where the intersection of the functions occurred. In this instance the projection of the intersection of these functions results in an ellipse in the *XY* plane (Schwartz & Yerushalmy, 1991, p. 50).

Although modeling is an important concept in FBA, the kind of modeling activity advocated by FBA is relatively uncommon. This type of modeling is the writing of functions that describe the relationship between dependent quantities (Schwartz & Yerushalmy, 1991). Students should be familiar with modeling that might involve quadratic functions in situations such as area and uniformly accelerated motion or absolute value functions such as apply to distance on a number line. Students should be familiar with the typical symbolic and graphical representations associated with these more common models (Schwarz & Yerushalmy, 1991). Modeling real world contexts using technology helps students to distinguish rate concepts from amount concepts. The concepts may be represented in complex graphs. "Interpreting these complex graphs foreshadows the study of calculus ideas later in the curriculum" (Stroup, n.d.).

IMPLEMENTATION IN TEXAS

The 74th Texas Legislature in 1995 established a required curriculum for kindergarten through Grade 12 that included a foundation in mathematics as well as other areas in the curriculum. This was in addition to the phasing out of low-level mathematics courses and increasing graduation requirements. Additionally, the State Board of Education was directed to identify essential knowledge and skills for each subject area. Teachers, parents, business representatives, etc. participated in this process. The Texas Essential Knowledge and Skills (TEKS) were adopted in July 1996. The TEKS for high school (including Algebra I and Geometry) required the use of graphing calculators (TEA, 2002). In 1994, Texas received a 4- year NSF grant to support the Texas Statewide Systemic Initiative (Texas SSI) at the Charles A. Dana Center at The University of Texas at Austin. The Dana Center was "designated as the Center for Educator Development (CED) for both mathematics and science" (Seeley, 1998). One of the early products of the CED was the Mathematics and Science TEKS Framework Toolkits delivered electronically through the World Wide Web (Seeley, 1998). The CED provides the following via the toolkits:

> Access to instructional resources, professional development opportunities, and a variety of TEKS-specific materials, including links to resources listed below:
>
> - Clarifying activities, clarifying lessons, and clarifying discussions that elaborate the mathematics TEKS;
> - Snapshots and Vistas that elaborate the science TEKS;
> - Quality professional development experiences in mathematics available statewide through the Texas Teachers Empowered for Achievement in Mathematics (TEXTEAM) program which, in Spring 1998, with the addition of science professional development experiences, will expand into Texas

Teachers Empowered for Achievement in Mathematics and Science (TEXTEAMS).

- A variety of resources particular to specific needs and initiatives, including various Algebra resources and an Advanced Placement Vertical Teams Toolkit for preparing students for advanced mathematical courses. (Seeley, 1998)

NSF provided $2 million annually for this initiative. TEA provided $1 million annually in matching funds to the Texas SSI (TEA, 1998). The TEXTEAMS professional development implemented a trainer of trainers model using the 20 Regional Education Service Center (ESC) content specialists to disseminate the institutes to districts within each region. One of the early TEXTEAMS trainings for high school teachers was the 1996 TEXTEAMS Algebra I Institute. This institute as well as the follow-up, TEXTEAMS Algebra I: 2000 and Beyond, were used to deepen teacher content knowledge for Algebra I. The introduction to TEXTEAMS Algebra I: 2000 and Beyond stated that both the institutes "assert that 'Algebra for All' is a realistic and attainable goal" (The Charles A. Dana Center, 2000). Who constitutes this "all"? Who wasn't taking Algebra I that TEA and The Dana Center felt should be included? This "all" was understood to include students exempted from the Algebra 1 requirement. Although it might include students with weak mathematics background or students not considered to be college bound. The institutes were designed to assist teachers in experiencing the type of "real world situations" that the institute believes would motivate these students, "especially for weaker students" (Stroup, 2010).

Similarly, "the primary goal of Equity 2000 is to close the gap in the college-going and success rates between minority and nonminority, and advantaged and disadvantaged students" (The College Board, 2000, p. 3). In the Equity 2000 report from the Madison, WI site, there were varying responses by teachers to the idea of teaching "Algebra for all."

- Algebra for all was not a rallying cry for all teachers. I'd say that there has been a substantial number of teachers who've persisted in the sentiment that Algebra is not for everyone. Their numbers vary from building to building from a small group to a rather large and entrenched group.
- It's clear [that] many teachers had their expectations challenged; not all changed their views.
- Even in the well-intended spirit of Equity 2000, people operate in an entrenched mind-set. Teachers are still having some difficulty in shifting their paradigms. . . . The veteran teachers will attend in-service and come back and do the same thing. (Ham & Walker, 1999, pp. 41-42)

If this is the response by teachers in an urban district after 5 years of teacher support (i.e., financial, curricular, professional development, coaching in the classroom) and student support (i.e., tutoring, summer classes, and additional counseling from school counselors), how well could the TEXTEAMS institutes have prepared teachers across Texas who did not have any support other than 5 days of training? Even though the ESC's for each region trained teachers for most districts that were ESC member districts, there were seldom opportunities for the ESC mathematics specialists to drive out to the campus and provide classroom support. Funding of the ESC's would prohibit this type of travel, especially for the more rural areas of the state. I believe that the above attitudes toward "Algebra for all" exist in many of our schools. Although Texas teachers currently have access to graphing technology and curricular materials, they do not all have access to mentors or mathematics specialists to assist them in implementing a FBA on a daily basis. Neither is it typical for Algebra I teachers to have common planning times during the day to collaborate with other Algebra I teachers at their campus, assuming the campus is large enough to have more than one Algebra I teacher.

The TEXTEAMS institutes were not developed to function as curriculum for Algebra. They were developed to provide activities that would strengthen teachers' mathematics content in a way that was consistent with the NCTM Principles and Standards. These activities explored mathematics content deeper than what would be expected for Algebra I in the classroom. Participant activities were not intended to be used with students in the Algebra I classroom.

> "This institute is not meant as a scope and sequence for the Algebra I course, nor is it a set of student activities for use in a classroom without careful thought and modification on the part of a knowledgeable teacher."
>
> (Dana Center, 2001)

The institutes provided several student activities that could be used by teachers in their own classrooms. This combination of participant-only activities and student activities may have led some participants to believe that all of the activities were for classroom use. Teachers may have also assumed that these activities were instances of a FBA. The participant activities in TEXTEAMS Algebra I: 2000 and Beyond provided experience with multiple representations of contextually based problems. The representations included concrete and pictorial representations as well as tables, graphs and equations. Participants were expected to have had experience with Algebra tiles in several applications and graphing calculators (Dana Center, 2001). Participants were encouraged to explore each of these representations for the activities presented. Consider the overview for Activity 2.1 Identifying Patterns, "Overview: Participants represent linear relationships among quantities using concrete models, tables, diagrams, written descriptions, and Algebraic forms (Dana Center, 2001).

Due to the use of the "Trainer of Trainers Model" to provide training to Texas teachers via each of the 20 ESC personnel, one unintended result of using all representations in each activity may have been the misinterpretation by many participants (some of whom would later become trainers in their districts) that all representations should be included when teaching students similar activities in Algebra I classrooms. The TEXTEAMS Algebra I: 2000 and Beyond Institute acknowledged that the training is based in part on the Principles and Standards for School Mathematics (NCTM, 2000). This fact, in connection with the fact that the TEKS for high school courses such as Algebra I require the use of graphing calculators (TEA, 2002), may have caused many educators to see the approach provided in the TEXTEAMS training as defining the FBA to learning and teaching Algebra. In fact, the Texas Education Service Center Curriculum Collaborative (TESCCC), developers of the CSCOPE curriculum documents, specifically referred to the approach of the TEXTEAMS institutes as function based when describing their own Algebra I curriculum. The stated, "This functional approach is not unique to CSCOPE and has been promoted by the National Council of Teachers of Mathematics, the Dana Center in the development of the TEXTEAMS institutes"

(TESCCC, 2009). The TESCCC continued to describe this approach to include the following methods:

> After connecting the representations of one set of data, students begin to look at two sets of data and their commonalities. This gives rise to the idea of equality and leads to solving equations using tables and graphs. Then students use the properties of Algebra to transform and solve linear equations and inequalities, and to determine reasonable solutions to problem situations with a focus on application to real-life situations. (TESCCC, 2009)

Just how many of these representations of that one set of data are students required to connect? The TEXTEAMS Algebra Institute material itself lists the first and possibly most important feature of TEXTEAMS as, "**Multiple representations (verbal, concrete, pictorial, tabular, symbolic, graphical)**. Mathematical ideas will be represented in many different formats. This helps both teachers and students understand mathematical relationships in different ways" (Dana Center, 2001). Participants would be likely to surmise that nearly all representations should be explored and related as they had their own students work through student activities. Teachers may have misconstrued the use of nearly all representation being used in any given problem situation as a necessary feature of FBA. The 5-day training was limited to topics of linear and non-linear (quadratic and exponential) functions although the activities included solving both linear and quadratic equations. One indication that the training was not following a functions-based approach comes from Activity 3.1, "**Overview**: Participants solve linear equations with concrete models and make connections between the concrete model, abstract, and symbolic representations" (Dana Center, 2001).

Nowhere in this activity or the ones used to solve quadratic equations is the idea of comparing two functions on a common domain used to find the solution(s) to the equations as instructed in a FBA. However, this idea is used to instruct participants in the solving linear inequalities in one variable in Activity 3.3. Algebra teachers throughout the state could be confused about what constitutes a FBA to teaching Algebra. Modeling activities are limited to the concept of function as explored in the sections of the institute exploring functions (linear, quadratic, and exponential) but not sections exploring solving equations.

Some student activities were accompanied by an assessment at the end of the activity. These were linear motion, linear parent function, y-intercept, rates of change, and out for a stretch (data collection, Dana Center, 2001). Although sample assessments were provided, they were for the most part open-ended. These types of assessments were not typical in any way to the Algebra I End of Course (EOC) Exam in use in Texas at the time. Although the Dana Center had hopes that these types of assessments would in fact appear on the EOC exams (P. U. Treisman, personal communication, April 15, 2014). The TAKS test had few assessment items related to the approach emphasized in TEXTEAMS when one compares the amount of time and effort needed to explore connections between all forms of representations in these real world situations (Stroup, 2010). As a result, teachers may feel that this type of instruction is less valuable when it is not emphasized on the Texas state standardized tests that are required for graduation.

One might wonder if this disconnect between the material as presented in the TEXTEAMS institutes and what is tested on the TAKS test and/or Algebra I EOC exam had to have happened. TEA contracted with Harcourt Educational Measurement and

NCS Pearson to create test items for TAKS and selected committees to review the test items. According to TEA (2003):

> Committees made up of assessment and curriculum content area specialists from TEA reviewed the items in preparation for external educator reviews. From September to December 2001, external educator review committees were convened in Austin to review TAKS items for all subject areas and grade levels. Overall, 29 TAKS item review meetings were attended by 583 educators from around the state.
>
> TEA seeks recommendations for item-review committee members from superintendents and other district administrators, district curriculum specialists, ESC executive directors and staff members, subject-area specialists in TEA's Curriculum Division, and other agency divisions. Nomination forms are provided to districts and education service centers by TEA's Student Assessment Division and can be found on the TEA website. In partnership with TEA, Pearson builds the educator review committees and selects committee members based on their established expertise in a particular subject area. Committee members represent the 20 ESC regions of Texas and the major ethnic groups in the state as well as the various types of districts (such as urban, suburban, rural, large, and small districts).

Although educators were asked to apply to participate in the review process, when you consider that these 29 review meetings included TAKS committees for all subject areas from grades 3-Exit level, you realize that 583 people is not a great number of representatives from the approximately 1000 school districts in the state. It is possible that the Algebra I representatives on the committees mirrored the traditional approach to teaching Algebra and as such, preferred to keep questions that would be typical of that type of instruction. In addition, the application form for consideration to serve on the review committee requires a series of links to access. Unless this link were sent to the educator by their regional ESC content specialist, it is unlikely the educator would find the application form on their own. It would then be up to TEA representatives to determine who was an acceptable choice. These committee members most likely mirrored those who were invited to write the TEKS.

With as fractured a view of FBA as has been outlined for Texas, there would be little chance that a particular committee would be comprised of mathematics educators who were knowledgeable about the types of questions that would best fit the FBA. As we have progressed to the next level of state standardized testing, State of Texas Assessments of Academic Readiness or STAAR (TEA, 2010), the concern is that the assessments will move further from a FBA. With the requirement of NCLB that states track annual progress of students, concern is that the STAAR EOC tests and the TAKS are based upon a psychometric formula that places students on a curve based on performance of the items. Students would be unlikely to score much higher on these exams even with intervention due to the way that the items are selected (Stroup, 2009).

Given this fractured implementation of FBA in Texas, I suspect that teachers have regressed to teaching *rules for simplifying and solving*. Algebra teachers may have taken modeling real-world situations as a means for motivating students and as a result used modeling only as needed (Stroup, 2009, see Appendix A). The notion that an FBA requirement to use nearly all multiple representations for modeling problems caused teachers to exert more effort and time to these problems is incorrect. Texas state testing did not emphasize this type of modeling and as a result teachers used these techniques infrequently in their day to day instruction. At the heart of this issue may be the fact that many teachers believe that students have more difficulty with verbally presented problems than with symbolic equations (Nathan & Koedinger, 2000).

My preliminary study in 2009 was designed to explore this issue. I wanted to explore the possibility that Algebra 1 students were confounding rules for simplifying and solving although the state standards from for over ten years have promoted a FBA that should help students differentiate these rules. I created a four-question survey after observing the students' Algebra 1 teacher teaching lessons on simplifying expressions and solving equations. The teacher-provided strategies were listed on the survey for each problem. In addition to the teacher-provided strategy, four FBA strategies were listed. The four problems were taken from the text that the district was currently using. Eighty Algebra 1 students were in attendance the day of the survey; 63 surveys were valid. The results of the survey indicated that students confounded traditional strategies that require the manipulation of variables to solve equations with the strategies typically used to simplify expressions (see Appendix C). Students were also unlikely to choose relevant sub-strategies related to solving equations. Few students indicated that they would use strategies that would be identified with a FBA to Algebra. And those students were even less likely to use those same strategies with technology such as a calculator or computer. This preliminary study raised questions that are now the subject of my dissertation. These questions are outlined in Chapter 3.

Chapter 3 Methodology

INTRODUCTION

The purpose of this research study was to explore a possible relationship between students' rating of the usefulness of teacher provided strategies for simplifying expressions and solving equations and their understanding or equivalence (of functions) and equality (as one kind of comparison of functions). In addition, supplementary questions were posed concerning student characteristics by LEP (Limited English Proficiency) status and SpEd (Special Education) status to explore potential differences in understanding or use of strategies.

RESEARCH DESIGN

I implemented a qualitative design in which a sample of students and participating classroom teachers were interviewed to provide explanations of the usefulness ratings students selected for the strategies as well as provide students the option to change the usefulness ratings of those strategies. The data from surveys of district Algebra teachers and campus Algebra students as well as interviews with students and teachers were triangulated (Stake, 1995). The search for additional interpretations of students' and teachers' preferences of strategy and understanding of equals and equivalence helped me address possible researcher bias.

The interview protocol was developed from the results of a pilot study done in 2009 in which Algebra 1 students were surveyed but not interviewed. Figure 3.1 illustrates the research design.

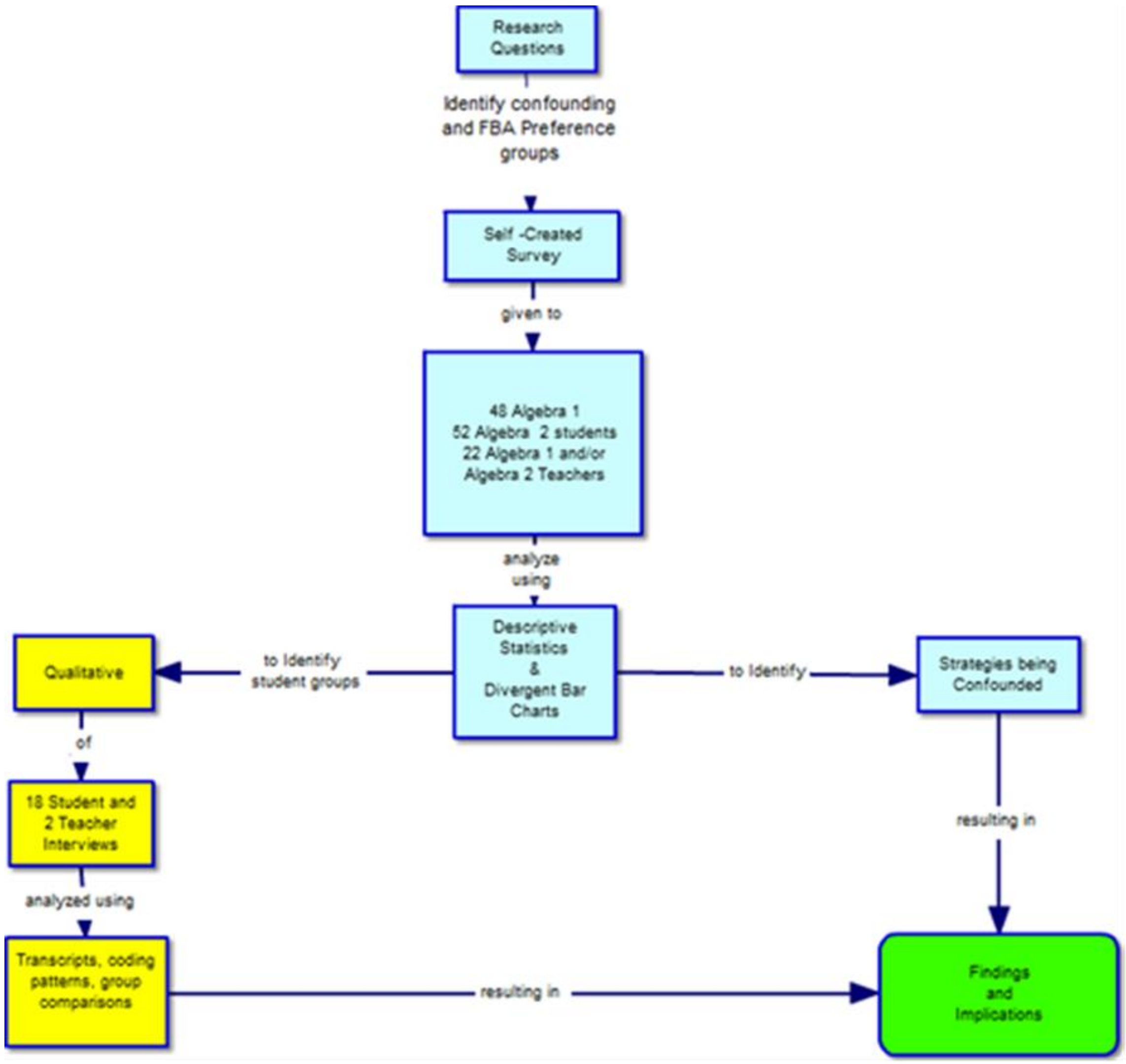


Figure 3.1: Research design of a Qualitative Study of Student Rating of the Usefulness of Teacher-Provided Strategies for Simplifying and Solving.

The results of this pilot study (see Appendix C) showed that students were confounding rules for simplifying and solving in ways that indicated that students might not understand the difference between equivalence and equals. There are four ways in which this confounding will be discussed:

- Confounding I: Students indicate that solving strategies are useful in problems that only involve simplifying expressions.
- Confounding II: Students select strategies that are not relevant to the problem situation as useful in either simplifying or solving.
- Confounding III: Students do not identify potentially relevant sub-strategies as useful when solving an equation.
- Confounding IV: The student indicates a different understanding of the use of a teacher-provided strategy from the ones that the algebra teacher(s) intended.

Confounding I occurred when some students rated the strategy *Do the same thing to both sides* as useful when simplifying the expression 3x+2-2x-1 in Question 2. There is no equal sign in the expression which would indicate that there were a "both sides" to which students might do the same thing.

Some students were selecting irrelevant strategies as useful. An example of this type of confounding occurred when students selected the strategy Apply the Distributive Property as useful in Questions 2 on the survey. The simplification of the expression 3x+2-2x-1 doesn't include the use of parenthesis to indicate the multiplication of a binomial term by either a variable or constant such as one would see in Question 3 from the survey: -7(x+2) +4x. In Question 3 the students would be expected to indicate that Apply the Distributive Property would be a useful strategy because the student would use this strategy to multiply both terms in the parentheses by -7. This type of confounding was coded as Confounding II. The survey indicated that confounding might happening in other ways as well. These other types of confounding include:

Confounding III is indicted when the student does not identify potentially relevant sub-strategies for solving when solving an equation. This is considered a type of confounding because students who don't identify these strategies as being relevant only to solving equations may be likely to use these strategies when simplifying expressions. Although not indicating a strategy for solving might plausibly have other causes, the analysis of data suggests that this absence fits with the overall profile of confounding. If it is present then confounding is likely or, equivalently, if there is not confounding that it is unlikely students will omit the relevant strategy.

Confounding IV is indicated when the student has a different understanding of the use of a teacher-provided strategy from the ones that their algebra teacher intended. This type of confounding may occur in conjunction with Confounding II. This happened with student selection of Apply the Distributive Property in Question 2 from the survey. This will be identified in specific examples in Chapter 4. As with Confounding III, Confounding IV might plausibly have other sources. This possibility notwithstanding, one of the findings of this research is that the particular forms of interpreting the strategies fit within the larger framework of confounding. If these alternative interpretations are present then it is likely they are confounding. If the student is not confounding, then these alternative interpretations are unlikely to be found.

Confounding IV is not coded if a student simply identifies a strategy by the wrong name. It is possible for a student to confuse the names of teacher-provided strategies.

This type of confusion would NOT be coded as any category of confounding. This appeared to occur during the interview with an LEP student. This student had used the phrase “subtract a number from x” to indicate “substitute a number for x.” When asked to clarify with an example the student stated, “I’m sorry, I am still learning English, do you know?” By observing the work of the student it was clear that the student was using a correct strategy but used the wrong name for the strategy they were able to use.

If a student exhibits one or more of the four categories of confounding for any of the teacher-provided strategies which both surveyed and interviewed teachers agreed were useful for a question the student is coded as confounding for that question. This means, equivalently, if a student is not confounding for a question then the student will not exhibit, or is highly unlikely to exhibit any of these specific forms of confounding. It is possible for a student to seem to exhibit Confounding II and Confounding IV but still not be confounding for the question overall. When this occurred, however, the interviews suggested this was not the case. One example that illustrates this difficulty with coding was having students select the strategy *cancel* as useful for simplifying the expression 3x+2-2x-1 in Question 2 from the survey. Students appeared to select a strategy which was irrelevant to the problem. However, when the students were interviewed they indicated a different understanding of the strategy *cancel*. This example is explored further in Chapter 4.

Research Questions

This research study explored the following research questions:

1. Does selecting the strategy "examine graphs or tables" either with a calculator/computer or pencil/pen and paper as "Definitely would use" or "Likely to use" relate to students selecting strategies that differentiate between equivalence and equals?
2. How does the student usefulness rating of teacher-provided strategies relate to student understanding of equivalence and equals?

This descriptive study explored patterns between and among groups as well as Algebra 1 and Algebra 2 course groups. These patterns will inform future research.

Instrumentation

I used two instruments to gather data for this study: a Simplifying Expressions and Solving Equations survey and interviews with student work artifacts. The two instruments were used sequentially. The survey was used to identify groups of students based on confounding of strategies. This survey was adapted from a preliminary study, completed 2009, with 80 Algebra 1 students in an urban school. STEM education faculty, Clinical Faculty, and graduate students validated the survey. The interviews helped me gain a deeper understanding of Algebra 1 and Algebra 2 students' preferences for strategy use in simplifying and solving and allowed me to identify patterns among groups and courses.

Survey

A researcher-created survey containing seven Likert type items was given to Algebra 1 and Algebra 2 students to rate teacher-provided strategies used in their Algebra classes as evidenced by observations of Teachers A and B prior to students taking the survey. Both teachers were observed on five separate occasions to identify the strategies that the teachers demonstrated as useful in the process of simplifying expressions or solving equations. The total observation time was approximately 10 hours. A copy of this survey is included in Appendix B. Questions 2 through 7 used a 4-point Likert-type rating with the following response choices: *Definitely would NOT use*, *Unlikely to use*, *Likely to use*, or *Definitely would use for each strategy listed*.

The first question on the survey required students to type in an identification code to continue the survey. Questions 2 and 3 required students to rate seven teacher-provided strategies and four function-based approach strategies as to the usefulness of that strategy in correctly simplifying an expression. Questions 4 and 5 asked students to rate seven teacher-provided strategies and four function-based strategies as to the usefulness of that strategy in correctly solving an equation. The online survey randomized the order of questions two through five as well as the 11 strategies for each question. This was done to reduce the question-order effect (Schuman & Presser, 1981). The response choices appeared in the same order throughout the survey. The sixth question asked students to rate the usefulness of a table of values in correctly simplifying an expression. The seventh and final question on the survey asked students to rate the usefulness of a graph of intersecting lines in correctly solving an equation. The survey was intended to identify student preferences and the usefulness of teacher-provided strategies as well as function-based strategies.

The teacher survey had the same questions and format of the student survey with the exception of providing open-ended responses for each question because these teachers completed the survey anonymously and were not interviewed about their ratings. The purpose of the teacher survey is to provide a professional opinion against which to compare student survey ratings for confounding of simplifying and solving strategies.

Interview Protocol

After the survey results were analyzed and groups of students identified, 18 student interviews were completed. The respondents were selected "purposively …to obtain instances of all dissimilar forms" (Weiss, 1994, p. 23). According to Merriam (1998), "Purposeful sampling is based on the assumption that the investigator wants to discover, understand, and gain insight and therefore must select a sample from which the most can be learned" (p. 61). Students considered most characteristic of their group were chosen to be interviewed. Students who were identified as possible participants for the interview process were checked against consent/assent forms to verify that they had permission to be audio-taped during the interview process. Of the 18 interviews, nine were conducted with Algebra 2 students and nine were conducted with Algebra 1 and Pre-AP (PAP) Algebra 1 students combined. The interview transcriptions were coded and organized by groups identified from the survey. Each interview was a semi-structured, open-ended interview (Merriam, 1998) lasting between 20 and 45 minutes depending on student availability during the Algebra class period. The interview protocol included the following questions:

1. How did you make decisions about the ratings you chose for each strategy for the survey overall?
2. I would like for you to work through question number ___. As you work the problem, please talk out loud about your work and indicate when you use any of the following strategies (Student is provided with blank copy of survey with the question and strategies listed on the same page).
3. How do you know you are finished with the problem?
4. How do you know your answer is correct?
4. Please explain why you feel that ______ strategy is useful for problem # ____.
5. Please explain why you feel that ____ strategy is not useful for problem # ____.

Student responses were coded for words or phrases that linked to equals and/or equivalence statements. Prior to working on the problems from the survey, students were provided a choice of writing utensil such as regular or mechanical pencil, color pen, or marker, lined notebook paper, grid paper, ruler, and a TI graphing calculator from a classroom set available to students on a daily basis in class. Students were encouraged to use whatever means at their disposal to work through the problems.

Once the survey data were analyzed, the algebra teachers were interviewed. Teacher A was shown the survey results of the Algebra 1 students first followed by the results of the Algebra 2 students. Teacher B was asked about the Algebra 2 student survey results followed by the Algebra 1 student results. This was done to reduce question-order effects (Schuman & Presser, 1981). Each teacher was asked the following questions in the semi-structured, face-to-face interview:

1. Nearly ______% of the Algebra 1 class chose ____ as a useful strategy for simplifying when that strategy is typically useful only for solving. What reasons might you suggest for that ranking?
2. Based on the percent frequency graphs of Algebra 1 results, what do you see that surprises you? Why do you think this might have happened?
3. Based on the percent frequency graphs of Algebra 1 results, what do you see that you expected? Why do you think this might have happened?
4. What do you find surprising about student responses for their rating choices? Why do you think the student might respond this way?
(Questions 1-4 are repeated for the survey results for the Algebra 2 students)
5. Less than ____% of the Algebra 1 class see ____ as useful for any of the problems while ___% of the Algebra 2 classes see the same strategy as useful. What reasons might you suggest for those rankings
6. How do you use the graphing calculator in explanations or demonstrations?

DATA COLLECTION

Over the course of the spring 2013 semester, I conducted ten class observations, five for each teacher, in both teacher's Algebra 1 and Algebra 2 classes. I surveyed 100 Algebra 1 and Algebra 2 students from a sample of 198 students after completing class observations. I also surveyed 22 Algebra 1 and/or Algebra 2 teachers from the same district as the two participating teachers. Two teacher interviews, one for each participating teacher, were completed after the student survey results were analyzed. Data were displayed in diverging stacked bar charts for ease of viewing. The charts shown to Teachers A and B during the interview process did not include the teacher survey results. Teachers A and B were asked to identify the strategies that they thought were useful when identifying student results as either an area of concern or agreement. During the interviews both Teachers A and B were found to agree with the results from the teacher surveys when identifying which teacher-provided strategies were useful for each question.

Data Sources

Sources, selection criteria, and setting for survey participants (qualitative).

After gaining Internal Review Board (IRB) approval (IRB # 2012-09-0065) from The University of Texas at Austin and the local school district (R13.18), I contacted campus principals at a local urban public school district. I chose to contact campuses with a student population that I felt appeared to be undergoing a change in demographics. I was unable to secure a meeting with the principals from the other three campuses I contacted. Two principals agreed to meetings with me to discuss my research study. Only one principal agreed to allow me to conduct the study at that campus. I met with the campus mathematics department chair to explain my research its parameters. The mathematics department chair provided me with email contacts for the two teachers who taught both Algebra 1 and Algebra 2 at this campus. I outlined the requirements of the study with the teachers and they agreed to participate. The campus chosen for this study experienced demographic changes in the recent past due to other campuses not meeting AYP. Students from those campuses have the opportunity to transfer to this or other campuses within the district. The target population in this study was students enrolled in Algebra 1 or Algebra 2 at this campus. I was allowed to address the students and distribute the research parental permission forms and student assent forms on my initial visit to the classrooms. The forms were returned within 2 weeks and I began class observations immediately; I was not allowed to video tape the observations until student forms were returned. The two teachers were selected specifically because of their dual teaching of Algebra 1 and Algebra 2. Although my pilot study only surveyed Algebra 1 students, the FBA used by Algebra 1 teachers in Texas may have focused mainly on multiple representations. In order to determine if this might be happening in Algebra 1, I included Algebra 2 students in this study. The Algebra 2 standards in Texas require the exploration of functions including "$f(x)=\sqrt{x}$, $f(x)=1/x$, $f(x)=x^3$, $f(x)=\sqrt[3]{x}$, $f(x)=b^x$, $f(x)=|x|$, and $f(x)=\log_b(x)$ where b is 2, 10, and e" graphically and symbolically." (TEA, §111.40. Algebra II (c)(2)(A), 2012). Algebra 2 students would be more likely to have been exposed to FBA due to the very nature of the course.

The sources of data consisted of a subset (n = 100) of each teacher's Algebra 1 and Algebra 2 classes (N = 198). Selection criteria for participation in the student survey were the return of both a signed Parental Permission Form and Student Assent Form by the student. Copies of these forms are included in the APPENDIX D. In cooperation with each teacher, a date was set for the survey to take place in the math computer lab at the high school campus during the students' respective Algebra class time. Students were provided with a form which listed the Survey website, the Student Last and First Name, as well as an ID code used to answer Question 1 of the survey. This ID code would later be used to identify students to interview. Students were also provided with a pencil and graphing calculator. The graphing calculator was one that students had access to in algebra class. Students were encouraged to use the blank portion of the form as scratch paper to assist with selecting strategy ratings.

The two instructors had 198 actively enrolled students. See Table 3.1 for a breakdown of students by course and instructor. The table also indicates the number of students who returned all necessary forms and completed the survey. A total of 101

Algebra 1 students and 97 Algebra 2 students were enrolled in the eight classes that were observed for Teacher A and Teacher B. Of the 101 Algebra 1 students, 53 returned the signed parental consent form and student assent form. Of these 53 students 48 attended the day of the online survey and successfully completed the survey. Of the 97 Algebra 2 students, 58 returned the parental consent and student assent form. Of these 58 students, 52 attended the day of the online survey and successfully completed the survey. Student confidentiality for the survey was provided by assigning the participants a unique numeric passcode to access the survey. The same unique numeric code was used in the qualitative analysis.

Table 3.1 Student Breakdown by Course, Teacher, and Completion of Survey

	Algebra 1		PAP Algebra 1		Algebra 2	
	Total Students	Forms Returned	Total students	Forms Returned	Total students	Forms Returned
Teacher A	26	6	54	30	31	14
Teacher B	21	17		*	66	44
Students Completing Survey		22		26		52

Note. * Course not taught by Teacher B.

Demographics

The high school campus was part of an urban public school district. There were approximately 2200 students in grades 9-12. The mathematics department has 20 teachers. Nine teachers are assigned sections of Algebra 1 or PAP Algebra 1 serving 435 students. Five teachers are assigned sections of Algebra 2 serving 364 students.

Teacher A received a B.S. in Pure Mathematics and was ending the third year of teaching mathematics. This teacher taught Algebra 1, PAP Algebra 1 and Algebra 2 for all 3 years. Teacher B received a B.S. in Mathematics and was ending the sixth year of teaching. This teacher taught Algebra 1 and Algebra 2 for 5 years. Both teachers were certified to teach grades 8-12 by a nationally acclaimed, secondary STEM teacher-preparation program. Demographics for the teachers who completed the survey online can be seen in Appendix E.

The demographic characteristic of students taking the survey include course, grade in school (i.e., 9, 10, 11 or 12), numerical grade in course for both the fall 2012 and spring 2013 semester, ELL status, and SpEd status. This information was provided by the district central office after the end of the school year when all surveys and interviews were completed. There were some discrepancies between this chart and the completed surveys because some of the students in Table 3.2 did not take both semesters of Algebra in the 2012-2013 school year. Some students who took the survey were not enrolled at the campus by the end of the year. As a result, I have no demographic information for those six students.

Table 3.2: Student Demographics for participating Algebra 1, PAP Algebra 1, and Algebra 2 Students

Course	Number of students in each Grade	Frequency Course Grade Fall 2012	Percent by Grade Fall 2012	Frequency Course Grade Spring 2013	Percent by Grade Spring 2013	LEP Status	SpEd Students
18 * Algebra 1	18-Grade 9	9-As 4-Bs 3-Cs 0-Fs	50% 22% 17%	7-As 7-Bs 1-Cs 3-Fs	39% 39% 6% 17%	One-1	1
24** PAP Algebra 1	24-Grade 9	11-As 8-Bs 3-Cs 1 -F	46% 33% 13% 4%	9-As 9-Bs 2-Cs 4-Fs	38% 38% 8% 17%		
52 Algebra 2	1- Grade 10 11- Grade 11 40- Grade 12	14-As 16-Bs 20-Cs 2-Fs	27% 31% 38% 4%	20-As 14-Bs 14-Cs 4-Fs	38% 27% 27% 8%	4-S 2-F	3

* Missing data for four Algebra 1 students who took the survey earlier.
**Missing data for two PAP Algebra 1 students who took the survey earlier.
NOTE: 1=Student is currently receiving LEP services; F=Student no longer receiving LEP services but is in first year of monitoring; S=Student no longer receiving LEP services but is in second year of monitoring

Selection Criteria for Interview Participants

Student survey ratings for each strategy by problem were uploaded to MS Excel from the survey monkey website. Results for the Algebra 1 and PAP Algebra 1 students were separated from the Algebra 2 students for comparing data for the two courses. Students who selected either *Likely to use* or *Definitely would use* for any of the four FBA strategies on each simplifying or solving problem were given an FBA point for each of those rating selections up to four points per problem. Students who accumulated any points for FBA ratings were classified as FBA (students who appeared to prefer a functions-based approach to Algebra). Students who accumulated zero FBA points for the simplifying and solving problems were identified as Non-FBA (students who appeared to indicate that a functions-based approach was not useful).

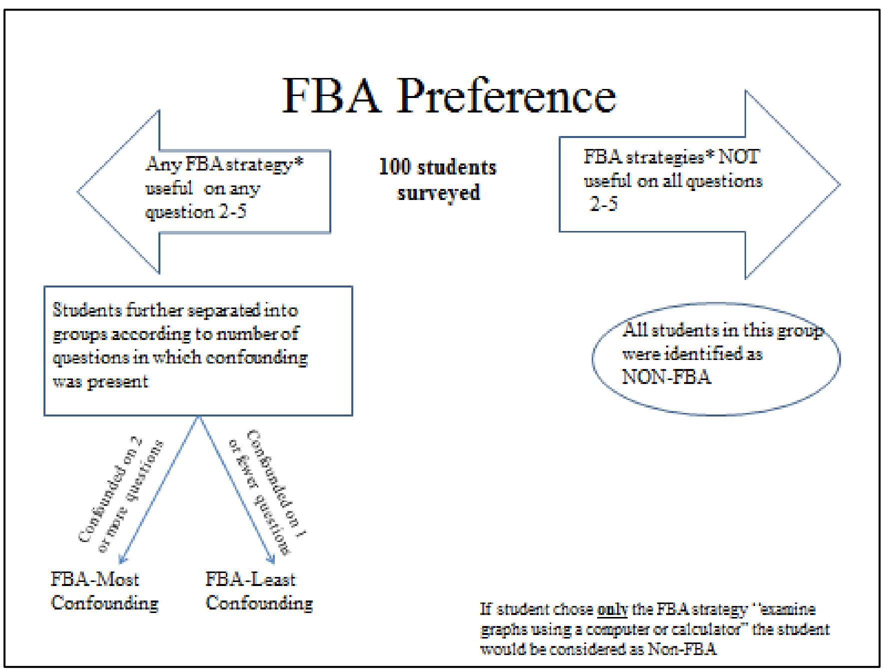


Figure 3.2: Identification of students taking the survey by FBA preference.

The student strategy rating for each teacher-provided strategy was coded in the same fashion. These student ratings were compared to the ratings selected by the Algebra 1 and/or Algebra 2 teachers who completed the survey online.

Students who rated a teacher-provided strategy as *Likely to use* or *Definitely would use* when the teachers rated the strategy *Unlikely to use* or *Definitely would NOT use*; or as *Unlikely to use* or *Definitely would NOT use* when the teachers rated the strategy as *Likely to use* or *Definitely would use*, were classified as confounding that strategy. Only the teacher-provided strategies that Teacher A, Teacher B and the surveyed teachers agreed on were used to identify students as confounding for a particular question from the survey. Agreement between the surveyed teachers and interviewed teachers was reached when the percentage of surveyed teachers indicating that a teacher-provided strategy was useful was greater than or equal to 75 percent and the interviewed teachers also indicated the strategy was useful. If the surveyed teachers were not in agreement on a particular strategy (less than 75 percent indicated the strategy was useful), that strategy was not used in identifying students as confounding. The FBA group was split between students who confounded strategies for three or four of the simplifying and solving problems as opposed to confounding zero, one, or two of the four simplifying and solving problems. The former was labeled as the FBA-Most Confounding group (FBA-MC). The latter was labeled as the FBA-Least Confounding group (FBA-LC). The Non-FBA group included four students who would be considered "most confounding," but the number of these students was too few to separate for interviewing purposes.

Although attempts were made to stratify the sampling, obstacles such as student absences, testing schedules along with an electrical outage made it impossible to ensure each group was represented by percentage in the overall interviews. Table 3.6 shows the frequencies of interviews for each group by course. Demographic data for the interviewees can be seen in Appendix E.

Table 3.3: Frequency of Interviews for Each Group by Course

	FBA-Confounding	FBA-No Confounding	Non-FBA
Algebra 1 and PAP Algebra 1 combined	4	3	2
Algebra 2	3	3	3
TOTAL	7	6	5

METHODS FOR DATA ANALYSES

The following section explains the analysis procedures used for the survey data and qualitative data, including ethical considerations.

Survey Analysis Procedures

For my dissertation study, I was specifically interested in determining if students who indicated a preference for FBA strategies on the survey showed less confounding of strategies for simplifying and solving. I analyzed the raw survey data relating to FBA strategies for each of the simplifying and solving problems to identify students who selected *Likely to use* or *Definitely would use* for those strategies. I also analyzed the raw survey data by question to create diverging stacked bar charts for each course, Algebra 1 and Algebra 2, to explore similarities and differences in the percentage of ratings choices for the two course groups. A separate but similar teacher survey given to teachers in the same district at a later date was used to provide a professional reference for strategies against which to compare the two course groups.

Data organization and management.

The raw survey data results were available for download from a secure and licensed account (via a password protected, personal paid account) held at the Survey Monkey Website. After the student interviews were completed on February 25, 2013, I accessed the data and closed the student survey. I downloaded the student results in an Excel spreadsheet with comma delimited values. Data were organized by course (Algebra 1 or Algebra 2) and then by group (FBA-MC, FBA-LC, Non-FBA) for each course. For each question, tallies of responses were calculated for the four usefulness ratings for individual strategies. Excel software was used to tally the results and create the diverging stacked bar charts of results for both courses as well as the teacher results.

Qualitative Analysis Procedures

Interviews were digitally audio-recorded then transcribed and coded in a systematic method to develop themes.

Audio recordings.

I recorded all 18 student interviews and both teacher interviews on a handheld, Zoom H1 Handheld Handy Stereo Digital Audio Recorder. Each file was saved on the device and then downloaded to my HP ENVY laptop and converted to a MP3 file. I transcribed each file verbatim using Windows Media Player. Each interview was typed into a Microsoft Word document. Students are referred to by their ID code from the survey. I am referred to as P throughout the transcriptions. Before importing documents into the NVivo software, I edited out any speech patterns and pauses or repetitions that detracted from the student explanations in the interviews.

I videotaped 5 of 10 classroom observations. I transcribed field notes from the other five observations. Observations of Teacher A and Teacher B were coded for use of strategies included in the student survey. Tallies were made each time a teacher used the strategy. An example of how each strategy was being applied was copied into the field notes.

Coding.

I downloaded each transcribed interview into NVivo 10 software for coding. NVivo is a qualitative data analysis (QDA) computer software. Each interview was coded by course (Algebra 1 or Algebra 2) and group (FBA-MC, FBA-LC, or Non-FBA). The coding followed the interview protocol. Student interviews were coded into categories related to the purpose of the question and labeled by category describing the type of response to the question posed. I used the software to look for patterns in responses between and among both courses and groups. Tallies were kept for the number of responses for each category as an additional comparison of groups.

After I coded the first three interviews, another STEM graduate student coded the same interviews independently. The coding structure was updated and descriptions for each node and sub-node were used to further explain the conditions for each node to be referenced.

Human Subjects Protection

Site approval was granted by the district IRB and high school principal. The human subjects protection plan was approved by the IRB at The University of Texas at Austin. The procedures for recruitment of subjects included an explanation of the purpose of the study and the use of the data by the researcher to the groups involved. A cover letter with complete details and a consent form to be signed by parents was given to each individual student. It was emphasized that participation was strictly voluntary; the incentive for student return of signed forms and completion of survey was entry in a drawing for one of two IPOD Nanos to be given away by random drawing for each of the participating teachers' students. Students completing a face-to-face interview were given a choice of a $10 gift card. No penalties were imposed for non-participation or withdrawal from the study. Parental consent and student assent for each participant were obtained, and the confidentiality of individual students was fully protected. Interviews were conducted during class time. All information on the audio tape addressed the student by the survey code used earlier. No student names were used on the audio files or transcripts. Only teacher participants had access to the results of the survey; teachers only received the diverging stacked bar charts. Teachers did not see individual student data. Teachers were provided a gift card when all teacher obligations for the study were fulfilled. Risk to participants was minimal. All data and findings are reported using ID code numbers and pseudonyms for the teachers.

Limitations of the Study

The study was limited to two algebra teachers and their students in an urban high school. The results may not be typical of all Texas high schools. The small sample of students may make comparisons between groups difficult. The observations of the teachers may not allow the researcher to uncover all possible strategies that teacher might use for a given problem situation. The original study was to include survey results disaggregated by LEP and SpEd status. However, the numbers of each group were too small to provide any meaningful pattern exploration.

The original research agreement with the participating teachers allowed for the student interviews to take up to 1 hour. After the first interview took 45 minutes, Teacher B indicated dissatisfaction with the time this student missed from class. The two teachers met to discuss an alteration to the time allotment for the interviews. I was subsequently allowed 30 minutes per student after the teacher addressed the class for the day and completed any additional instruction for the class to begin work on their daily assignment. This caused a change in the original protocol for the student interviews. Rather than having students work through all four of the problems from the survey, I limited the work to two problems. For some students, I was unable to ask for an explanation of their ratings for both Questions 6 and 7. This further restricted my access to information that might provide a link to student understanding of equivalence and equals. The campus experienced an electrical blackout the day of the teacher interviews. I was able to interview the teachers in a corner room with a window providing enough light so that we could view the diverging stacked bar charts from the Algebra 1 and

Algebra 2 groups. Both teachers indicated that the interview needed to take place on the designated day due to rapidly approaching state testing dates. Neither teacher was available to continue the interview at a later date.

Chapter 4 Results

Introduction

This chapter organizes and reports the main results of the study including relevant survey and qualitative data. The purpose for collecting this data was to explore student preference for teacher-provided strategies as well as FBA strategies for simplifying expressions and solving equations. This qualitative study explores the strategy preferences of 100 Algebra 1 and Algebra 2 students along with explanations provided by 18 of these students and concerns about strategy ratings expressed by the two participating teachers.

Teacher observations

I observed the two participating teachers five times each prior to the student survey. The observations lasted between 45 minutes and 90 minutes depending on block day (A, B, or C) or type of lesson. All Algebra 1 classes were double blocked. The teacher met with all Algebra 1 classes everyday although not always at the same time of day. For example, one of the Algebra 1 classes met during third period on A-day but sixth period on B-day. A typical A-day is one in which the teacher introduces a new topic. The teacher works through sample problems provided on district worksheets with the students and then allows the students to complete additional problems on the worksheet or similar problems from the textbook. The B-days are separated into two parts: review of homework and additional homework. The beginning of class is used to briefly review the assignment from the day before and answer student questions about specific problems. The remainder of the period is used by students to complete additional assignments with problems similar to those demonstrated on the A-day. Algebra 2 classes were not double blocked. The teacher would see the students in Algebra 2 every other day for 90 minutes. These students would be introduced to new content most class days.

Teacher A

Teacher A's room was split in half with short rows facing the center of the room. The doc cam was in the center aisle facing a screen on the back wall. Teacher A worked through the problems using the doc cam. Few students asked any questions about the problems as the teacher worked through the worksheet. The teacher did not ask the students to complete any of the problems or provide answers to any part of the problems being worked regardless of content. Teacher A would move around the room to answer student questions while the students completed the homework assignments. Some students worked together on the assignments others worked on their own. A few students visited rather than work on the assignment.

Teacher B

Teacher B's room was composed of long rows of desks facing the front of the room where there was a screen for the doc cam and the teacher desk. Teacher B worked through the problems with the students. However, Teacher B asked the students to complete parts of the problem as the teacher worked through them. Teacher B commented on how or why each process was used but did not ask the students to come up with the process for a problem on their own until students were given the homework assignment. Most often after completing the demonstration, Teacher B would sit at the teacher desk. Students were comfortable coming up to the teacher desk to ask questions. Some students would call out to the teacher to come to their desk to help them. Most students worked in groups and would check their answers with each other. If the group couldn't come to a consensus, the teacher was asked about the answer. Teacher B also used the blackboard on the side wall when reviewing content. The teacher followed the same pattern of questioning when using the blackboard. The content on the blackboard was left up during the period as students worked on their assignment.

During those observations I was tallied the use of teacher-provided strategies in both Algebra 1 and Algebra 2. The seven teacher-provided strategies were identified by observations of Teacher A and Teacher B as they taught their respective algebra classes. Table 3.1 shows the strategies and number of times the teachers were observed using each strategy over the observations completed prior to giving students the survey.

Table 4.1 Number of times a teacher strategy was used by Teacher A or B

STRATEGY	Times Used (over all observations)
Combine Like Terms	30
Apply the Distributive Property (Distribute)	22
Do the Same thing to Both Sides	19
Cancel	13
Multiply Both Terms by a Number or Variable	7
Do the Opposite	8
Substitute a number for x (plug in)	6

Examples of the use of each teacher-provided strategy as seen during the classroom observations are provided in the following paragraphs.

Combine Like Terms. The teacher was adding polynomials and instructed students to underline like terms and then combine the like terms. One example was $(6x^2 + 3x) + (2x^2 + 6x)$. The teacher had the students underline the $6x^2 and\ 2x^2$ once and then underline the $3x\ and\ 6x$ twice to indicate which terms were like terms. The teacher then indicates that the answer would be $8x^2 + 9x$.

There were examples of using this strategy when solving if the variable occurred in two different terms on the same side of the equal sign. The teacher was solving the equation -12y-12+5y=2. The teacher instructed the students to combine the like terms -12y and 5y to yield -7y-12=5.

Apply the Distributive Property (Distribute). The teacher was solving the equation $x + 6 = 8(x - 1)$. The teacher instructs the students to distribute the 8 across the x and the minus 1 to get $x + 6 = 8x - 8$

The teacher was solving the equation $2^{x-1} = (2^3)^{x-3}$. The teacher instructed the students to distribute the exponent 3 to the x and the -3 in order to have $2^{x-1} = 2^{3x-9}$

During interviews both Teacher A and B indicated that the distributive property would be used only when parentheses were in the problem. This may have caused confounding II and IV as evidenced in the student interviews. Students had rated this strategy as useful. The surveyed teachers rated this strategy as not useful. The interviewed teachers also indicated that this strategy was not useful since there were no parentheses in the problem. However, when the students were interviewed they indicated a different understanding of what Apply the Distributive Property meant to them.

Students indicated that they were using the distributive property when checking their answer by substituting a value of x into the original expression. The student substitutes the value of 1 in for each x in the expression 3x+2-2x-1 by placing parentheses around the 1 each time so that the expression looks like 3(1)+2-2(1)-1. The student indicated that the multiplying of 3(1) was an example of using the distributive property. Teachers A and B had been observed using this technique of using parentheses to emphasize where the numerical value for the constant should be placed.

Do the Same Thing to Both Sides. This strategy was used by teachers when solving equations and inequalities. The teacher was solving the equation 3x-10=2. The teacher instructs the students to first add 10 to both sides and then divide both sides by 3.

The teacher was solving the inequality $\sqrt{2x+7}-6>-1$. The teacher instructed the students to add 6 to both sides then square both sides followed by subtracting 7 from both sides. Finally the students were instructed to divide both sides by 2.

When solving simple equations the surveyed teachers as well as Teachers A and B indicated that the strategies Combine Like Terms and Do the same thing to both sides were used in combination. An example of this was provided during an observation when the teacher was solving the equation 35x=25x+50. The teacher instructed the students to subtract 25x from both sides to cancel the 25x on the right side of the equal sign. When the teacher wrote the -25x under the 35x, the teacher emphasized that the placement was because the 35x and -25x were like terms. This process of 35x-25x resulting in 10x was considered combining like terms.

<u>Cancel;</u> The teacher was simplifying the polynomial expression $(4c^5+8c^2-2c-2)-(c^3-2c+5)$. The teacher instructs the students to distribute the minus to all the terms in the second set of parenthesis resulting in $4c^5+8c^2-2c-2-c^3+2c-5$. The teacher then explains to the students that the -2c and the +2c cancel. The teacher was multiplying binomials such as $(2x-3)(2x+3)$ to show students that the resulting middle terms would cancel. The teacher explained that the -6x and +6x would result in zero so that $4x^2-6x+6x-9$ would be written as $4x^2-9$.

<u>Multiply Both Terms by a Number or Variable:</u> The teacher was factoring the expression $5(a-3)-2a(3-a)$. The teacher instructs the students to factor out a negative one from the $(3-a)$ so that the expression would be $5(a-3)-2a(-1)(a-3)$. The

teacher then instructs the students to multiply both terms (the a and the minus 3) by negative one to verify that factoring the negative one "flipped" the signs.

The teacher was solving a system of equations using elimination. The equations were $3x + 2y = 2 \; and \; x + 4y = -4$ The teacher instructed the students to multiply the terms in the equation $x + 4y = -4$ by negative three so that they could cancel the x's.

<u>Do the Opposite:</u> The teacher solved the equation $x - 3 = x^2 - 10 + 25$ by instructing the students to do the opposite of minus three and positive x in order to have a zero on the left side of the equal sign. The teacher subtracts x and adds 3 to both sides of the equation.

<u>Substitute a number for x (plug in).</u> The teacher had solved the equation $\sqrt{x-3} + 5 = x$ and came up with x={4,7}. The students were instructed to substitute each of these values of x back into the original equation separately to check the solutions. The students are left to verify that the value x=7 is a solution because the answer ends up being 7=7. However, the value x=4 is not a solution because the answer ends up being 6=4.

The usefulness of each of these teacher-provided strategies as rated by the surveyed teachers was consistent with the ratings expressed by Teachers A and B during the teacher interviews. In order for the strategy ratings to be consistent at least 75 percent of the surveyed teachers had to be in agreement with the ratings of the two interviewed teachers.

SURVEY

A self-created survey, updated from the pilot study done in 2009, was used to identify student preferences for strategies for simplifying expressions and solving equations as well as to determine if students were confounding rules for simplifying and solving. A copy of the updated survey is in Appendix B. The survey was also used to group students according to preference for FBA strategies for simplifying and solving

The survey was followed by face-to-face interviews with 18 of the students who had completed the survey. These students were selected based on their preferences and extent to which they exhibited the characteristic preferences of their group. The qualitative design allowed me to explore student reasoning behind the strategy ratings as well as provide the students an opportunity to change ratings for each strategy, if desired, after working through problems from the survey.

SURVEY RESULTS

The focus of this research was to explore the strategies that students found useful when simplifying and solving as well as which strategies students confounded. Students were interviewed to explore patterns of response between and among courses and groups.

Student survey results by course were compared to teacher results to determine what strategies were being confounded for simplifying and solving. The student interviews provided an insight as to whether the students were actually confounding strategies of if their understanding of the use of the strategy might impact the rating selected. Teacher interviews were conducted to allow instructors the opportunity to explain student choices for strategies that were being confounded or used inappropriately.

I analyzed the results of the 100 student surveys and 22 teacher surveys. To determine if students were confounding strategies for simplifying or solving, the overall results of the survey for each question of students segregated by course was compared to the teacher survey results for each question. The results were displayed as diverging stacked bar charts to more easily identify differences in percentages of respondents who indicate the rating is useful or not useful. (Robbins & Heiberger, 2011; Schmid, 1983) This data helped to determine the strategies to select when identifying a student as confounding.

Strategies Confounded by Students by Course

Patterns from the diverging stacked bar charts indicate that both Algebra 1 students and Algebra 2 students are confounding strategies for each question when compared to the teacher ratings.

Survey Question 2

Survey question 2 results, as seen in Figure 4.1, indicate that over 80% of both Algebra 1 and Algebra 2 students are rating combine like terms as useful; a strategy that teachers indicate as appropriate for simplifying the expression 3x+2-2x-1. Students from both courses considered the strategy, Apply the distributive property, useful although it is irrelevant to this problem. This strategy was selected by 31% of the Algebra 1 students compared to only 20 % of the Algebra 2 students. Both Teacher A and B were concerned about the percentage of Algebra 1 students that rated this strategy as useful. Neither teacher could provide an explanation as to why students would do so. Teacher B felt this was disconcerting since “there were no parentheses in the problem.” In response to being asked if the percentage of Algebra 2 students was cause for concern, Teacher B stated, “I feel like the bottom fourth of my class would choose the wrong answer because they are unsure what they need to do.”

Students from both courses appear to confound the following properties: do the same thing to both sides, cancel, multiply both terms by a number or variable, do the opposite or substitute a number for x. Although there is a small percentage of teachers (9%) who indicate that they would be *Likely to use* the strategy substitute a number for x, this percentage is too small to omit this strategy from use in determining confounding of strategies by students.

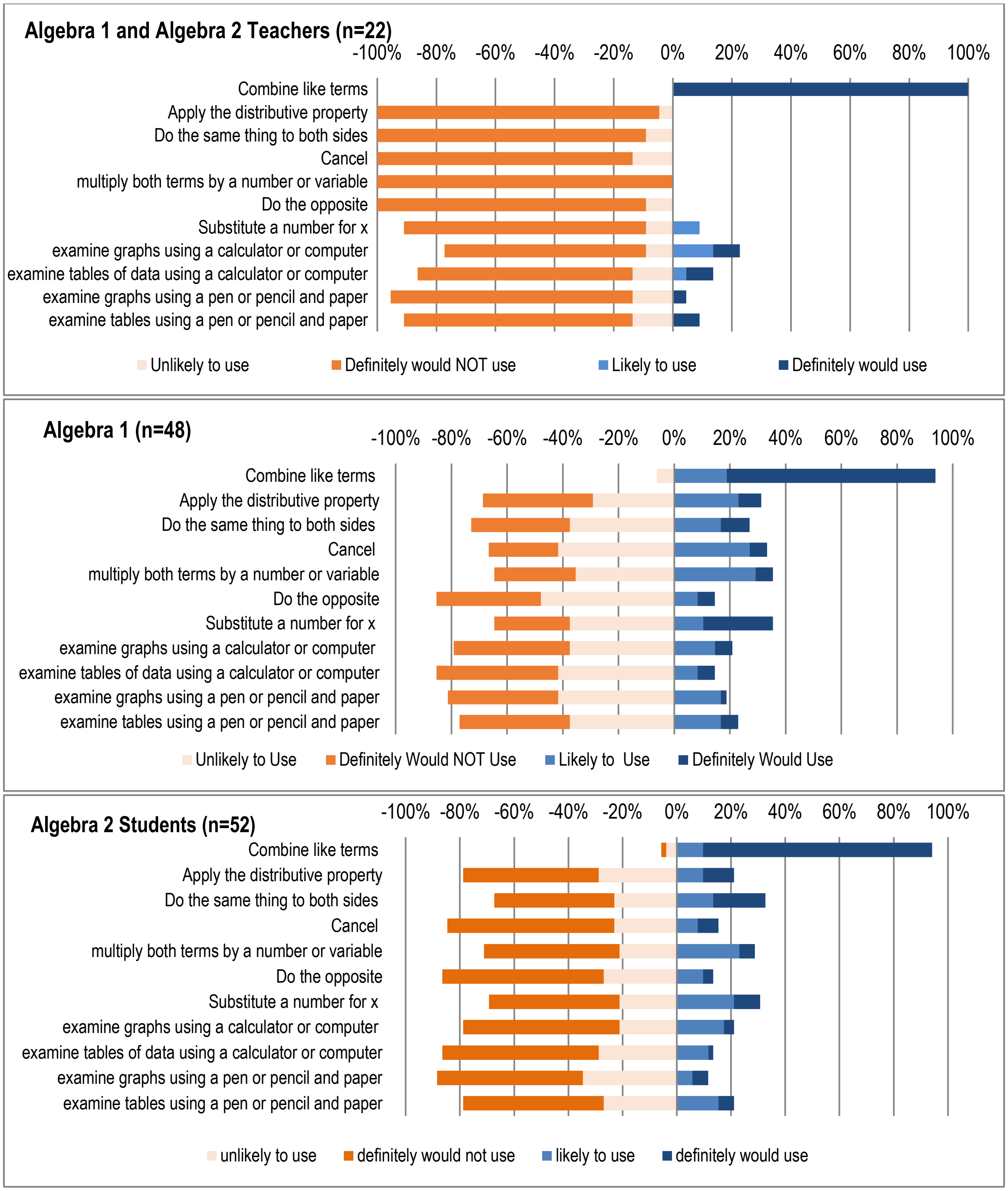


Figure 4.1: Results by group for survey question 2: Rate the following strategies based on how useful the strategy would be in correctly simplify the expression: $3x + 2 – 2x – 1$

Teacher B was concerned about the percentage of Algebra 2 students who selected substitute a number for x as useful saying, “The only time that we talk about substituting a number in for x is if you’ve got an equation and you solve and you are checking your answer. But there’s not even anything to solve here.” Teacher A and Teacher B were concerned about the percentages of students who rated Do the same thing to both sides as well as Multiply both terms by a number or variable as useful. Teacher B provided an example of when students should be using those strategies “When solving systems of equations by elimination. You know, multiplying everything by a number to maybe cancel some things out or if you’ve got, you know, x/2=7 or something like that and multiplying both sides of an equation, but that falls under do the same thing to both sides.”

Between 15% and 20% of the Algebra 1 students rated FBA strategies as useful in simplifying. This is similar to the percentages for the Algebra 2 students. Teachers A and B were not surprised that the students would rate examine graphs using a calculator or computer as useful because both teachers use the graphing calculators to explore functions throughout the year. But the teachers were surprised that any of the students would think that examining tables using a pen or pencil and paper would be useful. The teachers stated, “We do almost all exploration using a graphing calculator.” Both Teacher A and B indicated that the low percentage of students rating the FBA strategies as useful was appropriate. Teacher A stated, “They are all talking about graphs and tables although that doesn't really apply to this at all.” Teacher B agreed saying, “Even

Algebra 2 students might say graphs just because this is a safety net for students when they aren't sure what they are doing."

Survey Question 3

Survey question 3 results, as seen in Figure 4.2, indicate that almost 90% of the Algebra 1 students and 80% of the Algebra 2 students selected combine like terms as a useful strategy for simplifying the expression -7(x+2) + 4x. Almost all of the teachers surveyed (95%) agree. Both Teacher A and B agreed that students should see this strategy as useful. Neither of the teachers was concerned with the small number of students who indicated this strategy was not useful. However the percentages for another useful strategy according to the surveyed teachers (100%), apply the distributive property, were less for Algebra 1 students (77%) but higher for Algebra 2 students (90%). Neither Teacher A nor Teacher B was concerned with the percentage of Algebra 1 students who indicated that this strategy was not useful. Teacher B suggested, "Maybe some kids just forgot the name distributive property."

Approximately 30% of the Algebra 1 students and 40% of the Algebra 2 students confounded the strategy do the same thing to both sides. Both Teacher A and Teacher B were concerned about these percentages. When I asked the two teachers to tell me at what percent they became concerned, Teacher A stated, "Since this is out of 50, I would expect up to maybe 5 or so to just be guessing or not have much of an idea.

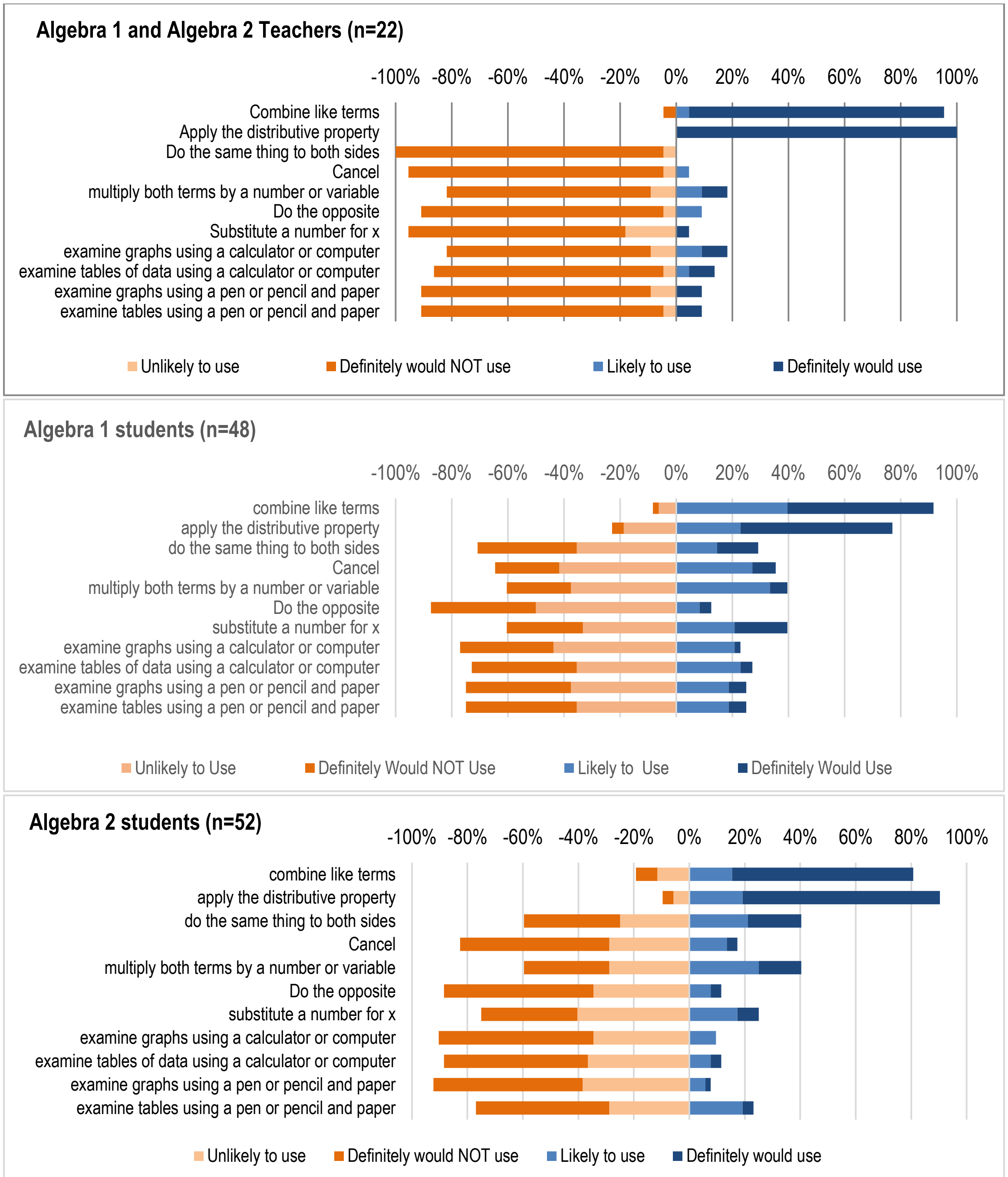


Figure 4.2: Results by group for survey question 3: Rate the following strategies based on how useful the strategy would be in correctly simplifying the expression -7(x+2) + 4x

But once it climbs up here where you're hitting the 20 or 30 % mark, you're talking about well more than that close to 20 students or something that are saying "doing the same things to both sides" when there's not even an equation." Teacher B replied, "It kind of concerns me this right here that over 30% of kids said do the same thing to both sides when there's not a both sides."

There were slight differences among surveyed teachers as to whether the following strategies were useful or not: cancel, do the opposite, and substitute a number for x. Both Teacher A and B were concerned about the percentages of students rating substitute a number for x as useful. Teacher A was concerned more with the Algebra 1 students saying, "Substitute a number for x ... I sort of think this is the thing kids fall back on when they don't know what else to do and they start trying things." Teacher B was more concerned about the Algebra 2 students and commented, "We talk about substituting a number in for x if you've got an equation and you solve and you are checking your answer. There's nothing to solve [in question 3]." The strategy multiply both terms by a number or variable was considered useful by almost 20% of the teachers. When determining if students confounded strategies, this strategy was not used for question 3 due to conflicting teacher ratings. However, this strategy would be important to examine in the student interviews. Teacher A mentions, "Multiply both terms by a number or variable is most likely has to be an equation or doing a fraction or something, but this one could apply more to distributive if you are thinking of it like that. I can see why the Algebra 1 students would select that strategy as useful."

Although the FBA strategy rated as useful by the highest percentage of teachers was examine graphs using a calculator or computer, less than 20% of the teachers selected a rating of *Likely to use* or *Definitely would use*. An overwhelming 65% indicated they *Definitely would NOT use* this strategy in correctly simplifying the expression. The percent of Algebra 1 students who rated the FBA strategies as useful was similar to those for question 2. With the exception of the FBA strategy, examine tables using a pen or pencil and paper, less than 10% of the Algebra 2 students rated the FBA strategies as useful. Teacher A was surprised that any of the students would rate examine tables using a pen or pencil and paper as useful and comments, "I don't know why this last one has a lot more 'examine tables.' No one said definitely graph on a computer. But this definitely with the tables doesn't make sense to me. I don't know what they might be thinking."

Survey Question 4

Question 4 on the survey asks students to rate the usefulness of strategies in correctly solving the equation 2x-5=x+4. The survey results for this question are shown in Figure 4.3. According to the teacher survey results, combine like terms (77%) and do the opposite (73%) were useful. Teacher A thought that even though about 30% of the students indicated that the strategy combine like terms was not useful that it's because "They are doing it but not thinking about it that way. When they do it they are

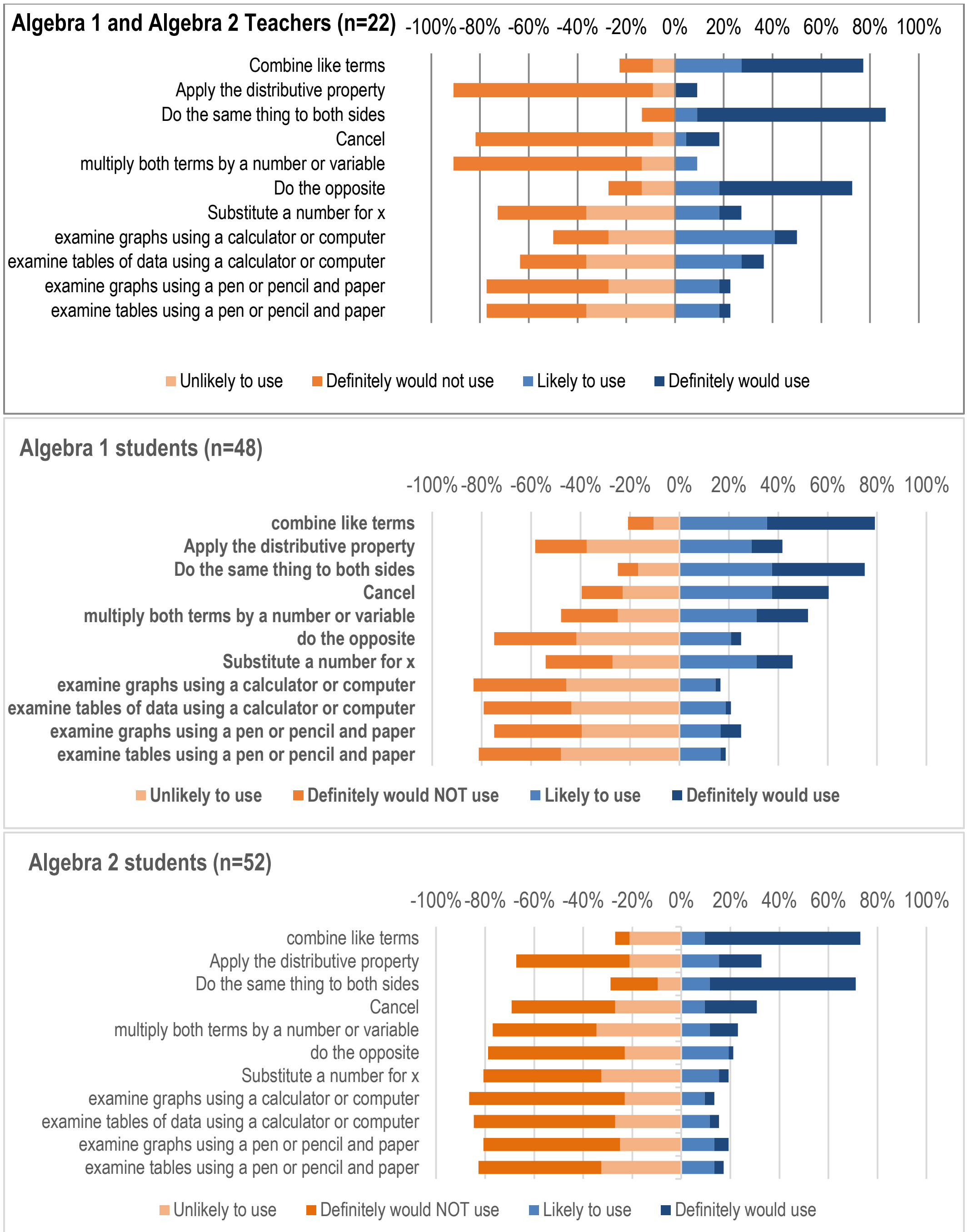


Figure 4.3: Results by group for survey question 4: Rate the following strategies based on how useful the strategy would be in correctly solving the equation 2x - 5 = x + 4

subtracting x on both sides." Teacher A was also surprised that the student percentages were lower for the strategy do the opposite. This teacher had "expected all of the students to use this strategy since this is the undoing of all those operations."

Teachers (86%) indicated that the strategy do the same thing to both sides was useful in solving. Teacher A was concerned that only 70% of the students thought this strategy was useful and commented, "It seems like everyone should have said they would do that."

Although almost 10% percent of teachers rated the strategy apply the distributive property as *Definitely would use*, 80% indicated they *Definitely would NOT use* this strategy. Both Teacher A and B were concerned that over 40% of the Algebra 1 and 30% of the Algebra 2 students thought this strategy was useful. Neither teacher could provide an explanation as to why students would choose that rating. Teacher B repeated the concern, "There aren't even any parentheses in this problem."

Over 25% of teachers indicated that they would be *Likely to use* or *Definitely would use* the strategy substitute a number for x when solving. Due to the conflicting teacher ratings, this strategy was not used for indicating that students were confounding strategies. Teacher A indicated that substitute a number for x "makes more sense for question 4. But I wouldn't have students do it for this equation, only for more complicated ones." This strategy was discussed with the students in the interviews.

As in survey question 2, 40% of Algebra 1 students and over 30% of Algebra 2 students indicated that they were *Likely to use* or *Definitely would use* the irrelevant strategy apply the distributive property. Both Teacher A and B were concerned about the percent of students indicating this strategy was useful. Teacher B stated, “I can’t explain why a third of the kids said they would apply the distributive property to something that obviously doesn’t apply to at all.”

Over 75% of the Algebra 1 students rated the strategies combine like terms and do the same things to both sides as useful. The percentage of Algebra 2 students was very close to those of the Algebra 1 students for these two strategies. However much lower percentages of both groups selected the strategy do the opposite as useful. Teacher B explains, “The Algebra 2 students have done this type of problem so often that they aren’t thinking of it as do the opposite and then combine like terms anymore when they are moving the terms to the other side. They’re just thinking do the same thing to both sides.” Teacher A agrees, “This problem is too simple.”

Other strategies that students appear to be confounding according to the teacher survey results include cancel and multiply both terms by a number or variable. The percentage of Algebra 1 students rating these strategies as useful was almost twice that of the Algebra 2 students.

A greater percentage of teachers (50%) indicated that the FBA strategy examine graphs using a calculator or computer were useful in solving compared to the simplifying questions 2 and 3. The other FBA strategies were rated as useful by between 20% and 40% of the teachers. Between 16% and 25% of Algebra 1 students rated the FBA strategies as useful with the strategy examine graphs using a pen or pencil and paper having the highest percentage of Algebra 1 students rating it as useful. Teacher B stated, “I’m really surprised at this whole graphing using pen and pencil when I feel like all the other Algebra I teachers and I, we show them how to use the calculator a lot. I'm really surprised that this would be the case." Less than 20% of Algebra 2 students indicated any one FBA strategy as useful for solving equations. These percentages are close to those of the Algebra 2 group for survey questions 2 and 3.

Survey Question 5

Survey question 5 asked students to rate the usefulness of strategies in correctly solving the equation x-4-3x=3+2x+1. The results of this survey question are seen in Figure 4.4. The teacher survey results indicate that the following strategies are useful: combine like terms (100%), do the same thing to both sides (77%), do the opposite (77%). There is conflict between teachers as to the usefulness of the following strategies: cancel, multiply both terms by a number or variable, and substitute a number for x. Since 23% of teachers rated each of these strategies as useful in solving these strategies were omitted when determining if students were confounding rules. Students would be asked about these strategies in the interviews.

Almost 90% of both Algebra 1 and Algebra 2 students rated the strategy combine like terms as useful. However smaller percentages of Algebra 1 students (77%) and Algebra 2 students (73%) indicated that the strategy do the same thing to both sides was useful in solving this problem. Students appear to be confounding the strategy do the opposite since less than 20% of the students in both courses rated this strategy as useful.

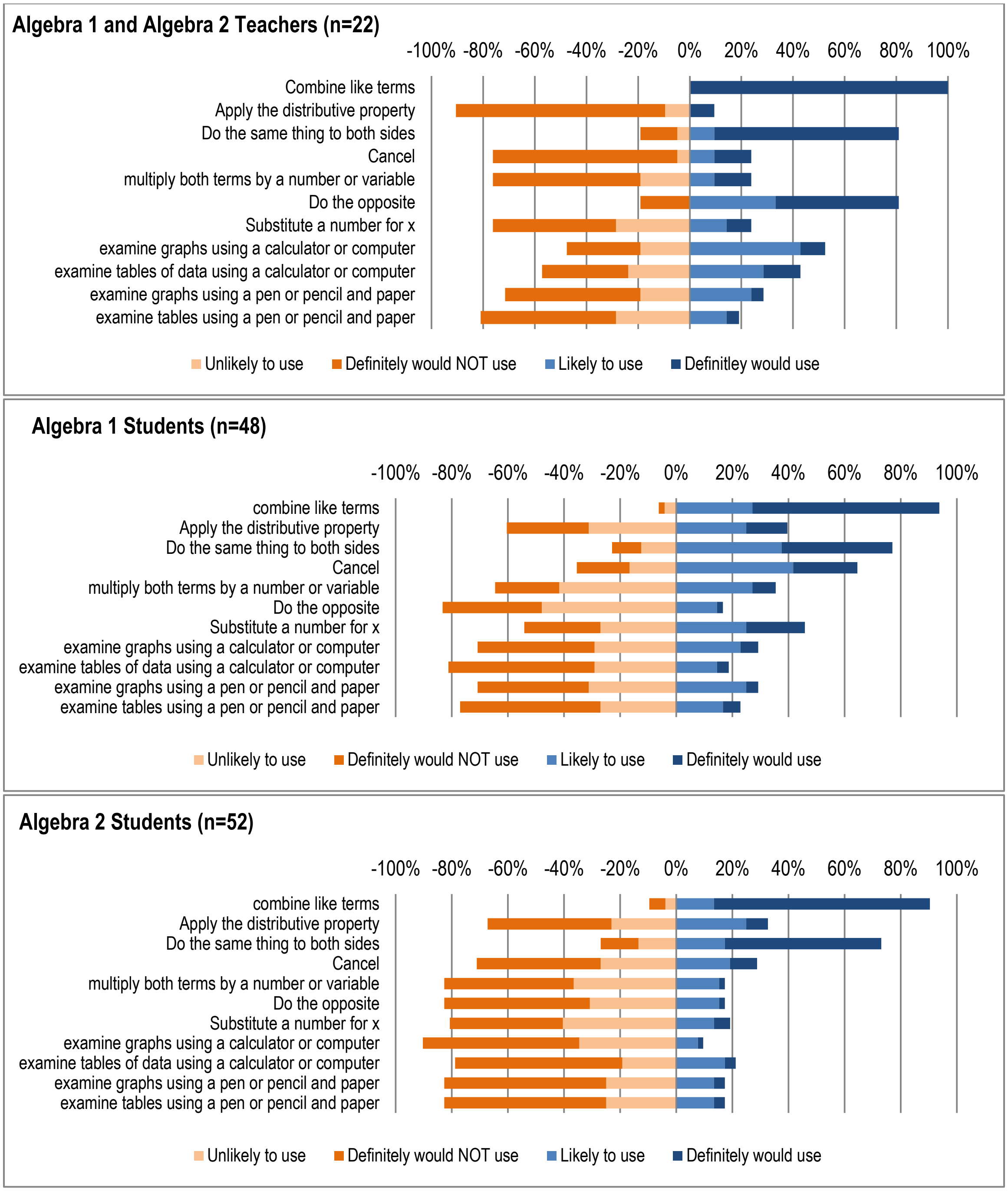


Figure 4.4: Results by group for survey question 5: Rate the following strategies based on how useful the strategy would be in correctly solving the equation $x - 4 - 3x = 3 + 2x + 1$

The irrelevant strategy, apply the distributive property, was indicated as useful by 40% of the Algebra 1 students and 60% of the Algebra 2 students. Teacher A was confounded by these percentages "This distributive property one again, keeps coming up. And a third of them are saying they're definitely or maybe going to use it is weird." Teacher B once again finds this difficult to explain as well "I was surprised by yes, the distributive property because there's got to be some sort of parentheses there."

Almost 65% of Algebra 1 students indicated that the strategy cancel was useful. These students may be confounding this strategy when compared to teacher preference. Approximately 30% of the Algebra 2 students appear to be confounding this strategy. Teacher B provided an interpretation for cancel that students might be using "When I think of cancel, there's nothing left. The students cancel the -2x on the left after combining. "

A little over 35% of the Algebra 1 students indicated the strategy multiply both terms by a number or variable was useful. Whereas only 17% of the Algebra 2 students indicated this strategy was useful. There appears to be less confounding of this strategy by Algebra 2 students.

The teachers had the highest percentages for indicating that the FBA strategies would be useful in solving (18%-50%) with examine graphs using a calculator or computer receiving the highest percentage of teachers rating the strategy as useful. The percentages of Algebra 1 students indicating the FBA strategies as useful varied from 18% to 29% with examine graphs (using either tool) having the highest percentage of students rating it as useful. This is a little higher than the percentages of Algebra 1 students rating the FBA strategies as useful for question 4 which is also a problem involving solving.

The percentage of Algebra 2 students who rated any of the FBA strategies as useful were comparable to the results for the other survey questions (21% or less). The highest percentage of Algebra 2 students (21%) rated the FBA strategy examine tables of data using a calculator or computer as useful. Teacher B believes that these percentages should be low saying, "There's only one [FBA strategy] that's barely above 20% and I think that's good to see because none of those would be something that I would say to use." Teacher A agreed, "There's not really a graph to look at for this. The pen and pencil one is worrisome. I can't explain that. I think there are a lot of situations where we say 'Let's take a look at the graph.' That's useful and I think it's ingrained in the students so I'm not surprised at this chunk that say *Likely to use* or *Definitely would use*."

Identifying FBA and Non-FBA Students

Students were given a zero though three ranking for each of the four questions involving simplifying or solving in the survey. A zero ranking indicated that the student rated zero of the four FBA strategies as either *Likely to use* or *Definitely would use* with the exception of "Examine graphs using a computer or calculator" because this is a strategy used in Algebra that teachers taking the survey referred to as "tricks used to pass TAKS." If this is the only FBA strategy rated either *Likely to use* or *Definintely would use*, the ranking for that student's question is still a zero. A ranking of three indicated that the student rated three of the other four FBA strategies as either *Likely to use* or *Definitely would use*. Students who received a zero ranking for all four survey questions were designated as Non-FBA. Students who received a ranking of one, two, or three on any question were designated as FBA. There were a total of 52 students designated as FBA and 48 students designated as Non-FBA. These groups were further differentiated by course. Of the Non- FBA group (n=48), 23 were Algebra 1 or PAP Algebra 1 students while 25 were Algebra 2 students. Of the FBA group (n=52), 25 were Algebra 1 or PAP Algebra 1 students while 27 were Algebra 2 students.

Identifying Confounding Students

Teacher-provided strategies for each question were identified as appropriate to use if 18 or more (82%) of the teachers surveyed selected a rating of either Likely to use or definitely would use. The strategy was identified as either irrelevant or inappropriate if four or less (18%) of the teachers surveyed selected a rating of either Likely to use or definitely would use. Student ratings were compared to the teacher ratings to identify ratings that matched for all selected teacher-provided strategies. If a student's rating did not match the teacher rating for two or more strategies, the student was classified as "confounding" for that survey question. If the student's ratings matched the teacher ratings for all or all but one of the selected teacher-provided strategies, the student was classified as "non-confounding" for that survey question. The FBA group was split between students who confounded strategies for three or four of the simplifying and solving problems as opposed to confounding zero, one or two of the four simplifying and solving problems. The former was labeled as the FBA-Most Confounding group (FBA-MC). The latter was labeled as the FBA-Least Confounding group (FBA-LC). The Non-FBA group had four students who were considered "most confounding", but the number of these students was too few to separate out for interviewing purposes. The Algebra 1 and PAP Algebra 1 course group had 13 FBA-MC members, 12 FBA-LC members, and 23 Non-FBA members. The Algebra 2 course group has 14 FBA-MC members, 13 FBA-LC members and 25 Non-FBA members.

INTERVIEWS (QUALITATIVE)

Of the 18 students interviewed seven were from the FBA-MC group. Of those seven, three were in Algebra 1/PAP Algebra 1. The FBA-LC group consisted of two Algebra 1/PAP Algebra 1 students and three Algebra 2 students. The Non-FBA group consisted of three Algebra 1/PAP Algebra 1 students and three Algebra 2 students. Eight of the students were in Teacher A's algebra classes. Nine of the students were in Teacher B's algebra classes.

CODING METHODS

In response to the concern that qualitative research articles "often provide only very brief excerpts of the qualitative data to illustrate the coding scheme" (Hammer and Berland, 2014, p38) To meet Schoenfeld's (1992) standards for novel research I

- Describe the method in sufficient detail that readers who wish to can apply the method.
- Provide a body of data that is large enough to allow readers to (a) analyze it on their own terms, to see if their sense of what happened in it agrees with the author's, and (b) employ the author's method and see if it produces the author's analyses. (p. 181)

Once the description of the codes has been illustrated and made clear from working through the accounts from transcripts, student work artifacts, teacher interviews and student and teacher survey data, I will provide a table of counts of the relative frequency of occurrences of specific forms of strategy use by students and teachers.

To address my research question 2, I structured my analysis of the study data in terms of the three forms of analysis associated with the systematic design of grounded theory: open coding, axial coding, and selective coding (Creswell, 2002, p.441).

Open Coding

Survey data indicated that students were confounding rules for simplifying expressions and solving equations. These results provided open categories for the first round of coding. Student work was coded as symbolic manipulation, confounding rules for simplifying and solving, FBA or errors. To be considered as symbolic manipulation the student "moves letters and numbers" according to teacher-provided strategies but without reasoning. Skemp (2006) refers to this use of "rules without reasoning" as instrumental understanding. Student dialogue used to explain work or justify use of a particular strategy was coded as symbolic manipulation if the phrases used by the student indicated "x" as an unknown (i.e. "letters" as opposed to "just numbers")

As discussed earlier (see pages 28-31) student work was coded as confounding if the student indicated any of the following types of confounding for any of the questions 2 through 5 from the survey:

- Confounding I: Students indicate that solving strategies are useful in problems that only involve simplifying expressions.
- Confounding II: Students select irrelevant strategies as useful in either simplifying or solving.
- Confounding III: Students do not identify potentially relevant sub-strategies as useful when solving an equation.

- Confounding IV: The student indicates a different understanding of the use of a teacher-provided strategy from the ones that the algebra teacher(s) intended.

If a student exhibits one or more of the four categories of confounding for any of the teacher-provided strategies which both surveyed and interviewed teachers agreed were useful for a question the student is coded as confounding for that question. If a student is coded as NOT confounding for a question then the student did not exhibit any of the four categories of confounding for the teacher-provided strategies or the student is at least unlikely to exhibit any of those types of confounding.

Student work and dialogue were coded FBA if the student refers to the specific use of FBA strategies (examine tables or graphs using computers or calculators or examine tables or graphs using pen or pencil and paper) in a way that indicates a function-based understanding. As an example, students who indicated that the graphs of two expressions would overlap (or be on top of the other) would be coded as FBA. This type of dialogue would indicate that the students would perceive tables of values for x, the original expression and the simplified expression as a means for identifying the two expressions as "everywhere the same." Students who indicated that the intersection of the graphs of two functions would be the solution to an equation in one variable were coded as FBA.

Errors in student work included arithmetic errors such as multiplying or adding integers, incorrect use of order of operations, and combining terms that are not like terms.

An additional coding category, knowing the answer is correct, was included in the open coding. Students who are taught by teachers following a standard, non-functions based, algebra curriculum tend to confound the rules for simplifying and solving and as a result do not seem to be aware of ways of checking their results (Stroup, n.d.). Properties of this category included the following: asking the teacher, reviewing my work, I don't know, and substituting a value for x. Although substituting a value for x would be useful for checking an answer found by solving an equation, this strategy would not be as useful when simplifying an expression.

Axial Coding

Each of the categories for the open coding was examined. Specific to my research question 2, I pursued relationships between the categories for symbolic manipulation and FBA strategies.

Selective Coding

After reviewing the teacher transcripts and the first three student transcripts I realized that the students from the FBA group (both FBA-MC and FBA-LC) were changing FBA ratings or providing examples of using graphs or tables that had nothing to do with the questions from the survey. I revised my coding scheme to include student responses that indicated when graphs and tables were useful as well as what it was about the survey questions that caused students to perceive those strategies as less useful. I then referred back to the transcripts from the teacher interviews to code instances that indicated when graphs and tables were useful as well as comments indicating that FBA strategies were not useful. Themes that emerged served as the basis for the grounded theory related to student confounding of rules for simplifying and solving.

Survey Ratings Overall

Students were shown a blank copy of the survey that they had taken one month prior to the interview. Students were first asked how they made decisions about the ratings for strategies on the survey overall. The FBA-MC group responses referred mostly to logical processes whereas the Non-FBA group had more responses that fell within the category "teacher strategies and prior experience." The Non-FBA group is consistent in this category of responses throughout the interview process.

Regardless of group or course, the responses for logical processes were similar as shown in Table 4.2. Table 4.3 provides sample student responses for the group preferring to use teacher strategies and prior experience. There was one student, a FBA-MC student in Algebra 2, who indicated how ratings were done for each question but still had no response after being asked a second time about an explanation for overall survey ratings.

It is important to note that another FBA-MC student indicated that they simply guessed to select ratings. Originally the idea to ask students how they selected ratings overall is because I believed that many students might say they guessed and I would want to have the student work through the problem and review their strategies. The protocol remained the same even though more students came up with "logical processes' and "teacher strategies or experience" as their means for rating strategies.

Table 4. 2 Sample Student Responses for Logical Processes

	Algebra 1	Algebra 2
FBA-MC	113: I guess I just kind of tried to think of which one I would use first or the one that would be easiest for me or that we've gone over more thoroughly.	None
FBA-LC	117: Sometimes using process of elimination from what I thought would work the best. There were a few that I wasn't quite sure about came up with different ways to get a better idea how to particularly use certain equation formula and I narrowed it down to different ones	212: I gave my strategies based on how I knew where I should start first from the easiest equation to the hardest one. And how I go by that is just by calculating… making sure like to see it in my head before I put down any answer.
Non-FBA	310: Usually I work out the problem how I would normally do and then I would go back and say, oh I used this property or this principle and then I'd put like, very likely or definitely would use it. If I didn't use it I would say definitely would not.	213: I think I kind of just imagined if I were to actually sit there and work it out and I went through not all the equations, but imagined some of them that I'm like more familiar with and whether or not it would make sense to use them or not.

Table 4. 3 Sample student response for teacher strategies and prior experience

	Algebra 1	Algebra 2
FBA-MC	101: whenever the teacher's explaining it, he tells us step by step and tells us which one is easier to go with, which makes more sense so that's how I picked.	1080215: Well, I thought back to Algebra I and basic Math and what I've learned from Algebra 1 because this is pretty basic.
FBA-LC	301: I based it off my past experience. Not just this year but since kindergarten, what I've learned. Different strategies work for different people, so I just try to think of which ones I've been using most of my life and rated those as probably the ones I'd use most.	209: I made my ratings by what I usually use best, like what I understand. P: When you say what you understand, where does that understanding come from? 209: The explanations that the teachers give us.
Non-FBA	103: I looked at them a little closer and then the options on there were, most of them were what he taught us and some we didn't learn about so I just picked the ones that were easier to me in class.	214: I just went about choosing it by like the way we were taught.

SIMPLIFYING EXPRESSIONS

Students were asked to "correctly simplify the expression" for question 2 or 3 or both. As they simplified the expression they were asked to "talk out loud" about their thinking and reasoning. Most students chose to use pencil and paper to complete this task although they were given a graphing calculator to use as well as notebook paper and grid paper. The student work artifacts were was coded as either symbolically correct; exhibiting confounding or containing arithmetic or other errors. The FBA groups contained the only students who used rules for solving although both questions 2 and 3 asked students to "correctly simplify the expression."

Students from all three groups regardless of course used symbolic manipulation correctly for question 2. However, student references to the terms variable and constant differed from student to student even within the same group. The only student to use the term variable was an Algebra 2 student who used the term "right variables.' When asked to explain what the student meant by "right variables" the student uses a phrase similar to ones other students had used such as "both have an x", "both have an x on the back", and "like terms." No student regardless of group or course referred to the term constant. The students referred to the constant in the expression as "the ones that don't have an x" "only numbers", "regular numbers", or "normal numbers."

A response from an Algebra 1 student in the FBA -MC group is presented below. This student's response is similar to the others who simplified the expression correctly. The corresponding work is shown in Figure 4.5.

111: So what I said trying to make the problem smaller, so you want to add the x's. That would give you only x because 3x-2x is just x and then you would add the numbers, which is 2-1. That would be just 1, I guess, I don't know.
P: You drew a line from the 3x to the -2x saying that you put those together. So how do you know that the 3x and the -2x can go together?
111: Because there are no parentheses or square root symbol. There are just minus and plus every single place so … it's simple.
P: You knew the +2 and the -1 went together. Is there a way that you know that they don't go with the 3x and the -2x, that they're separate?
111: That's because *they don't have an x* with them. They are *only numbers.*

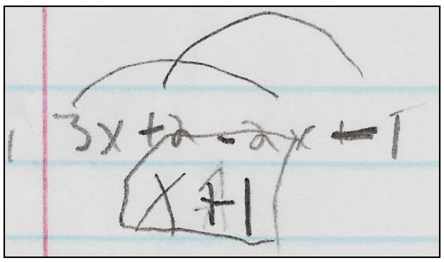


Figure 4.5: Student 111's work for Question 2

Symbolically correct manipulation: The student work and dialogue refer only to moving letters and numbers. I have italicized the words in student 111's dialogue that indicate that this student is thinking of x as an unknown rather than a variable. The entire dialogue and corresponding work were coded as one instance of symbolically correct manipulation.

Two students from the FBA-MC group, both taking Algebra 2, confounded the rules for simplifying and solving when working through question 2. Both students were following a teacher provided strategy of "do the same thing to both sides" although there is no equal sign in the expression. Since both of these students were identified as confounding based on their survey results it isn't surprising that they apply solving rules to a question asking them to simplify. The difference between these two responses is that student 201 applied the rule to both constants and variables (if not consistently). Student

1080215 applied the solving rule only to the constant terms but then combined like terms when encountering variable terms. It isn't apparent if this is because of proximity of variable terms once the constant term +2 has been eliminated. The student responses and corresponding work are shown below.

> 201: Well, first, what do you mean solve for x? You mean like subtract the 2, because this becomes the negative 2 and you subtract it. I think you also subtract it from here. Well, was this one 2 negative [points to the 2 in the 3x+2]?
> P: It is 3x+2
> 201: Okay, so like this x… so 3x-2x and then what do you mean subtract 2x … I feel like I'm doing it wrong. And then I like freak out about it. I'm doing this thing wrong and I think you like subtract and then… but I'm not sure if you… because 1=x and you would just plug it in.

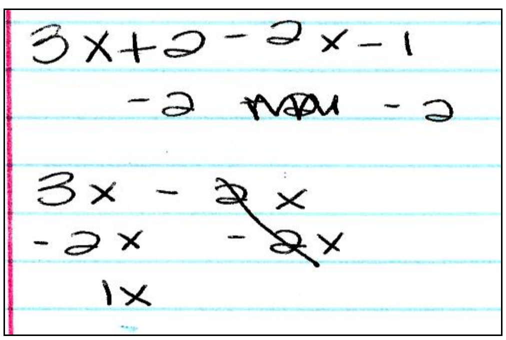


Figure 4.6: Student 201's work for Question 2

Confounding I: The student begins the dialogue by indicating that they were asked to solve even when the instructions said to simplify. The student writes the -2 under both the +2 and the -1. This work was also coded as one instance of Confounding I. I did not code the instance of subtracting 2x even though the student wrote -2x under the 3x and the -2x because the student isn't actually cancelling the -2x term. The student work at this point resembles combining like terms.

1080215: All right, so the first thing that I'm going to do is subtract the 2 from the left side and do the same thing on the other. And the 3x is going to stay there so I'll have3x-2x-3 and then 3x-2x is just x and then you leave the -3.
P: So the first thing you did, you said you're subtracting 2.
1080215: Right. From both sides.
P: From both sides and so why are you subtracting 2?
1080215: So that I can get everything, like I want all my *x's* on one side and my *regular numbers* on the other. … I simplified as much as I can, but I don't know how to explain that. … I can combine like terms: the 3x and -2x. I combined those and then there is the -3 there's nothing else I can combine with since there's nothing else like that from the equation.

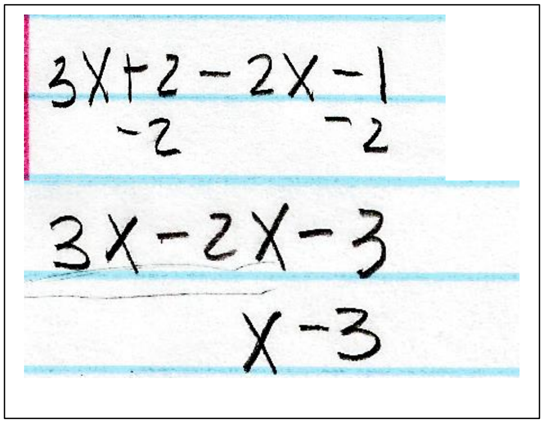


Figure 4.7: Student 1080215's work for Question 2

Confounding I: The student applies a rule for solving when subtracting two from both the +2 and -1. The dialogue was coded as one instance of Confounding I.

Symbolically correct manipulation: The student combines like terms in the last step of simplifying. I did not code the use of the word "equation" in the dialogue because the student leaves the answer as an expression.

An Algebra 2 student from the FBA-LC group also confounded the rules for simplifying and solving. However, this student actually changes the expression to an equation before solving. This student's work is shown in Figure 4.8.

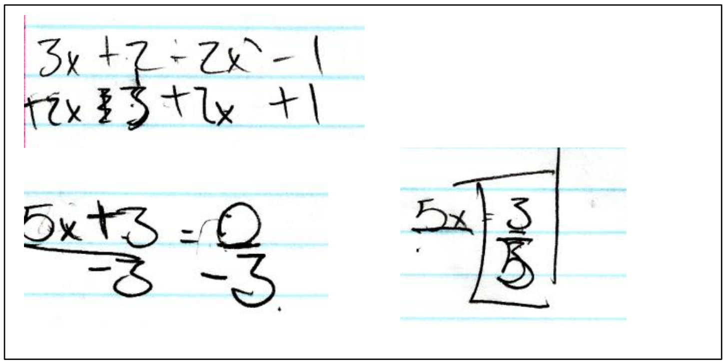


Figure 4.8: Student 212's work for Question 2

> 212: All right, well, the first thing you would do is you'd want to add this one to this side over here and to make this 3. You'd want to add all the common numbers and x's before you make your first choice on what you do after that. Oh, hold on. [mumbling] And I guess that's it because you can't really go past that right there, I don't think, unless I'm wrong. I don't know if I can simplify any further… other than maybe trying to get a fraction…If they had given me a number.
> P: What number would that be?
> 212: Like one. I could bring 3 to this side because we have an x on the opposite side …you can't divide 5 into 3 so it would be three-fifths, but you can't have [a fraction that isn't a number like one].

Confounding I: This student eliminates the -1 and -2x terms as if they are in an equation. The student said, "add this one to this side over here" which indicates that the student interprets the problem as one that uses solving strategies. The student then sets the expression equal to zero to create an equation to solve. This dialogue was counted as two instances of Confounding I because the two manipulations were different in nature. This would not be Confounding II or III because the strategies the student used would be relevant if this were an equation. This is not an example of Confounding IV since the student uses the strategies in a way that would be consistent with teacher understanding if the problem had been an equation such as $3x + 2 = -2x + 1$.

The five students who worked question 3 correctly using symbolic manipulation had work that was similar. The explanation that each student provided as they worked the problem was very different. I have included two of those responses to show the difference in those students' explanations. Both students were in Algebra 1. The first student is from the FBA-MC group. This student's work is shown in Figure 4.9. Although this student ends up with the correct answer, the use of the equal sign is confusing. This student appears to want to "solve" but realizes that this can't be done as the problem stands.

> 113: All right. And then you would combine like terms. You would combine...oh, there is an x there. The -7x and the 4x and you would get -3x-14= whatever goes at the other side of the equal thing. I think I did that right.
> P: What does the equal sign that you put at the end of the original expression mean to you?
> 113: I don't know. It just kind of makes...it makes more sense to me because it's like you're setting it up to be solved, you're simplifying it so that it can be solved more easily, even though it really can't because there's nothing on the other side.

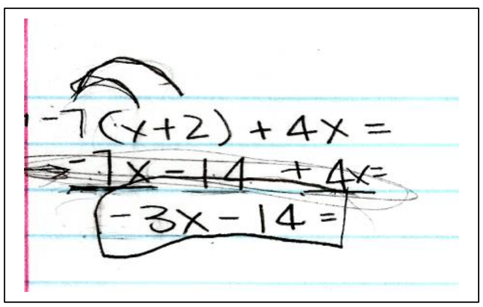


Figure 4.9: Student 113's work for Question 3

Symbolically correct manipulation: This student actually completes all manipulations correctly so the work is coded this way. The dialogue referring to the use of an equal sign is not coded as confounding because the student verbalizes thoughts that lead up to realizing that the problem is simplifying rather than solving.

The second student is in the FBA-LC group. This student's work is shown in Figure 4.10.

> 308: Okay, well, because of PEMDAS, which is the order if which, you know, things you do in an equation, I know P for parentheses comes first. So you're going to multiply, well, it's only in the parentheses to *figure* out so the second thing you're going to do is to multiply what's outside of the parentheses to what's inside of the parentheses. So I multiply the -7 to the x and the 2 that's inside there. It becomes -7x-14 and then you'd add on the +4x at the end. And then since you have to add together the two things with the same variables so the Xs and the things without the x's, you'd add the -7x with the 4x, which would give you -3x. Yes, -3x. And then you'd have the -14 and that's as far as you can simplify it.
> P: And so you mentioned PEMDAS at the very beginning and you said that inside the parentheses there wasn't anything you could do so how do you know that?
> 308: Because the variables were not the same so you cannot take x and add it to 2. So you have to leave them alone. But like say if it was 2x +3x, then you could add them together.
> P: If they were inside the parentheses together?
> 308: If they have the same variables, like you know, if it has no x at the end or if they both have x at the end or they're both x's, then you can add them together, but if they're not, then you can't add the

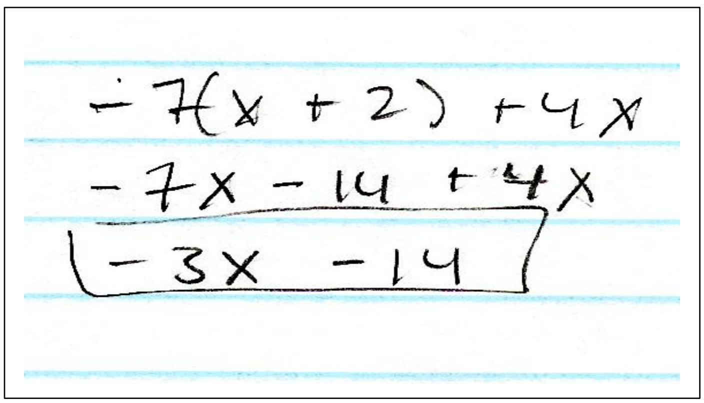


Figure 4.10: Student 308's work for Question 3

Symbolically correct manipulation: The student work was coded to indicate that the student understands the x as an unknown. The student does refer to both the x and the constant as variables. I did not code this as confounding.

The student mentions that PEMDAS refers to the order in which you do things in an equation the student refers to this process while applying it to simplifying an expression.

This was coded as an instance of symbolically correct manipulation. The student recognizes that the terms in the parentheses are not like terms.

The two students from the FBA-MC group who confounded the rules for simplifying and solving did so quite differently. Both students used the distributive property first, but the Algebra 1 student, as shown in Figure 4.11, applied a solving rule. This student indicates that the answer should still be an expression.

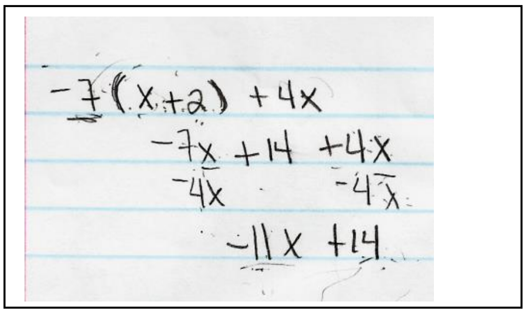


Figure 4.11: Student 101's work for question 3

Symbolically correct manipulation: The student applies the distributive property.

*Error: The student made an arithmetic error multiplying -7*2.*

Confounding I: The student subtracts 4x from both the 4x and the 7x as if the student were solving an equation.

The Algebra 2 student, shown in Figure 4.12, sets the expression equal to zero and actually solves for x. The student comments after completing the problem, "I always set everything equal to zero."

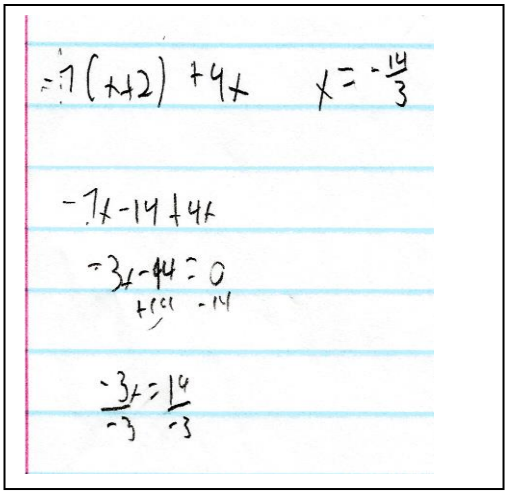


Figure 4.12: Student 216’s work for Question 3

Symbolically correct manipulation: The student applies the distributive property correctly.

Confounding I: The student sets the resulting expression equal to zero. I only coded this as one instance for the dialogue and work combined. Once the student has changed the expression to an equation, I no longer coded for symbolically correct manipulation

Student 117, an Algebra 1 student from the FBA-LC group, appears to have difficulties differentiating between constants and variables as well as appropriate symbolic manipulations for variables and constants. This student’s work is shown in Figure 4.13. The student dialogue mentions PEMDAS as a strategy and proceeds to combine terms in the parentheses. The student maintains this as a viable strategy even after further questioning. This strategy was not demonstrated by either Teacher A or Teacher B during any of the observations. However, one of the teachers who took the survey made the following comment, “Order of Operations... PEMDAS. I would have my students distribute first to simplify the grouping symbols (PEMDAS) followed by combining like terms."

> 117: Well, what I would do first is… since this is dealing with letters or I can't come up with the word right now… or I don't know what to call the x, but my first idea would be PEMDAS, Parenthesis, Exponents, Multiply, Divide, Add, Subtract, but when I see the x+2 and then the +4x after that, I don't feel that is correct. So I would add the x+2 and get 2x and have the -7 but I still have the parentheses. I would later then combine the 2x and the 4x and get 6x and the -7+6x and then… I don't know if I'm doing this totally right. I have a feeling there are later steps, but I just don't know what they are right now. It's not coming to me.
> P: And so if you were to explain to someone how you know that you can combine those, is there a way that you can explain to them that you know that you can do that?
> 117: Maybe. Because they are both x's and they are both positive. There's no exponents after the x or in front of the 2 and 4 then you can combine the first x and the amount of x's, so 4x and x+2 or 2x and you can get 6x because they are all x's.

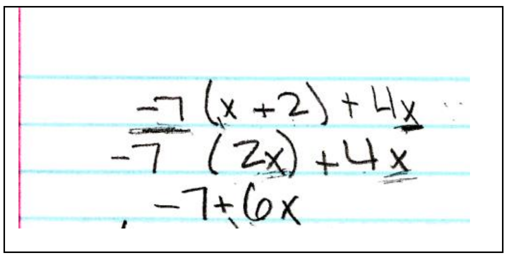


Figure 4.13: Student 117's Work for Question 3

Error: Student combines unlike terms. Student mentions use of PEMDAS but combines like terms prior to multiplication. This work with dialogue was counted as two instances of error.

Only one of the five students in the Non-FBA group worked through question 3. This student incorrectly applies the distributive property. The student had rated this strategy *Definitely would use*, but was unable to apply it correctly to the expression when simplifying as shown in Figure 4.14. The student said, "I would get -7 distributed over to the 2 and get -7."

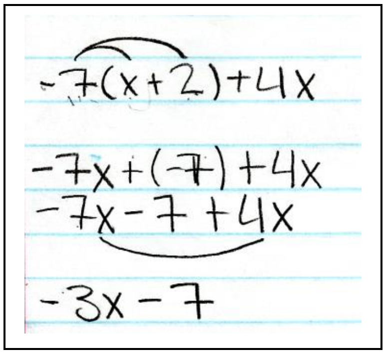


Figure 4.14: Student 103's work for Question 3

Symbolically correct manipulation: This student applies the distributive property and then combines like terms. This was coded as one instance of symbolic manipulation.

Error: The student incorrectly multiplies the -7(2). This was an arithmetic error.

Knowing the Answer is Correct

When students indicated that they were finished simplifying, they were asked, "How do you know your answer is correct?" Student responses were most often categorized as ask the teacher, check by substituting a value for x, or looking back over work.

Ask Teachers or Classmates

The FBA-LC group had no responses that fit this category. The five student responses in this category included statements such as "my teacher puts a checkmark next to it" or "I'll ask my friends what they got." Students from both Algebra 1 and Algebra 2 had similar responses regardless of FBA preference.

Substitute a value for x

When interviewing teachers, initially neither teacher could come up with a reason as to why students would use this strategy for either question 2 or 3. After being confronted by the percentage of students rating this strategy as useful, Teacher A came to the conclusion, "The only way I can see it being useful is if they said what if x is 1 and they plug it in the original and see what they get and then maybe they do some work and they simplify it and they do the same number in their simplified version and see if they get the same thing." This explanation is echoed by the students.

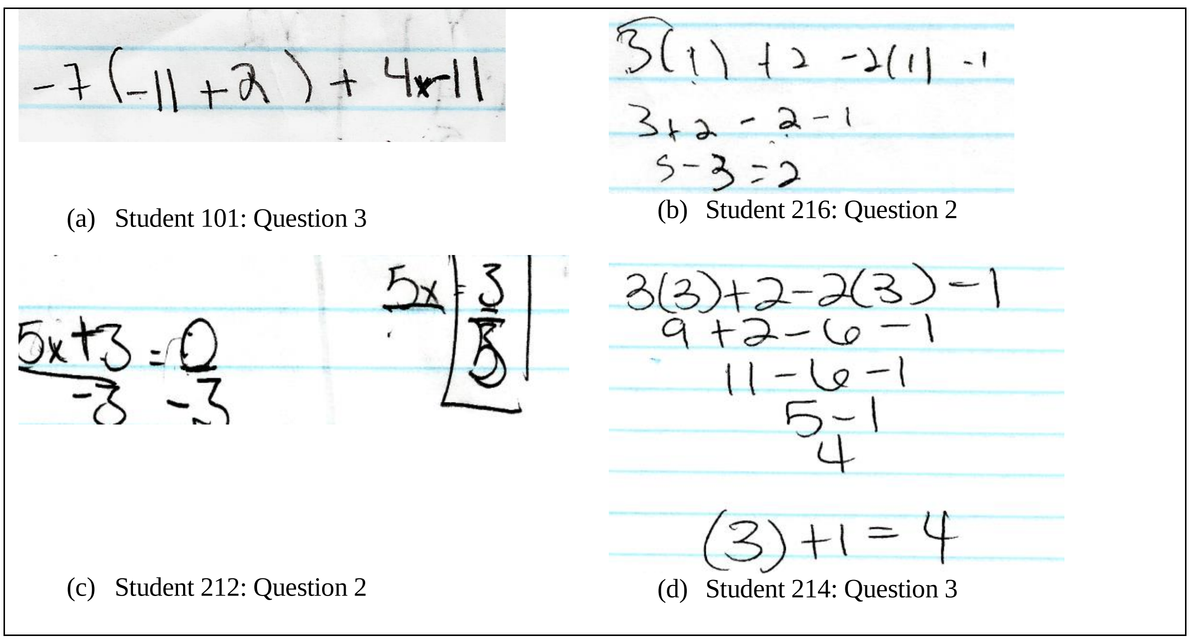
Two students from the FBA-MC group indicated that they could use substitution to determine if their answer was correct. The Algebra 1 student's answer was -11x+14. This student indicated that they should be able to substitute -11 in for x to check their work. This student's response is provided below along with student work in Figure 4.15(a). The student doesn't finish checking this work after substituting in, but is sure that they would end up with -11.

> 101: I don't know if I'm sure, but I'll try because that's what I think. I think that's what it does.
> P: You have replaced both of the X's with…
> 101: -11.
> P: what would you end up with that would tell you if you were right or not?
> 101: You would end up with this, once you distribute these two and then add these and then simplify, then you would get this [points to -11x+14]. You would get the -11

Confounding I: This dialogue was coded as confounding rules. The student indicates that the result of substitution would be equal to the value of x used in the substitution. The student is confounding this with the idea that substituting the solved value of x back in to an equation would result in an equation with equal numerical values on each side of the equal sign.

The Algebra 2 student from this group had confounded simplifying with solving and came up with the answer x=1. This student substitutes the value x=1 into the original equation and comes up with 2=2 as a check that the work is correct. The substitution is shown in Figure 4.15(b).

Confounding I: The student begins by substituting the value x=1 into the original expression and then changes to an equation. The student doesn't refer back to the answer x=1 that they found by simplifying.



(a) Student 101: Question 3

(b) Student 216: Question 2

(c) Student 212: Question 2

(d) Student 214: Question 3

Figure 4.15: Student work for substitution to check answer.

An Algebra 2 student in the FBA-LC group has also confounded simplifying with solving. Based on the student's verbal response, the student believes that whatever number you substitute for x should be the same as the answer. However, when this student checks the value x=1 against the answer found by working the problem the student indicates that you can't have x=3/5 if someone gives you x=1. I was unable to probe further about this student's understanding of checking by substitution. The dialogue with this student, shown below, is somewhat confusing. The work this student refers to is shown in Figure 4.15(c).

> 212: I would say if there was more information than this … maybe go back in and plug them in to see if what the numbers they gave me and the numbers that I got matched. … Let's see, like they put 3x+2
> P: You were saying if you were given a number? Like 1?
> 212: Yes, like 1.
> P: Okay, so if I say I give you a number 1, how would that help?
> 212: We have 0 [points to the right side of the equal sign], so what we would do is we would subtract this [points to the 3 in the 5x+3], which would be 3 to bring to this side because we got to have an x on the opposite side of *a regular number*. So it would be 5x=3. The thing is, is that you can't divide 5 into 3 so it would be 3 over 5 [3/5], which would give you your answer, and you can't [do that].

The one student in the Non-FBA group, an Algebra 2 student, seems surprised that this strategy of substituting a value for x actually does allow the student to verify that the work that was done is correct. The student stated, " I don't know if this will apply, but say x is equal to 3, so 3(3+2)-2(3-1) would be 9+2-6-1, which equals 11-6-5, and 5-1 is 4 and this would be 4 if you plugged 3 into x+1. So that's I guess how you check it. I guess I figured that out just then." The work shown by this student is Figure 4.15(d). This student realized this strategy would work when reviewing strategy ratings later in the interview.

Symbolically Correct Manipulation: The student work done in checking the work is also coded as symbolically correct. I did not code this as use of FBA strategy because the substitution was for only one value of x.

Look back over work

The three students from the FBA-LC group as well as one student from the Non-FBA group had similar responses for looking back over their work regardless of whether they were in Algebra 1 or Algebra 2. These students would "make sure I didn't make any small multiplying mistakes" or "double check … make sure you did the addition right"

Review of Teacher-Provided Strategy Ratings

Strategies from the survey have been separated by classification as teacher-provided strategies or function-based strategies. Students were first asked to indicate which of the teacher-provided strategies that they used as they were working the problem. After that students were asked to "Please review your strategy ratings and indicate which ones you used and which ones you would like to change." When given the opportunity to change their ratings of the strategies selected for questions 2 and 3, five students indicated that they would not change any of their ratings. It is not surprising that four of these students are in the Non-FBA group. These students had commented earlier that they used what they had been taught by their teacher to rate the strategies for the survey from the beginning. Later in the interview I would ask these students to show me where they had used the strategies that they indicated were useful. I also asked them for an example of when they would use the strategies that they had rated as not useful.

The one Algebra 2 student from the FBA-MC group reviewed these strategies for question 2 and stated that all ratings were correct. Although this student confounded strategies when compared to the teacher survey data, the student was able to provide explanations for each of the strategies when asked.

No Change to Teacher-provided Strategies

Students were asked to indicate where they had used a strategy in the process of correctly simplifying the expression if they indicated that they would not change their rating for a particular strategy. If they changed their rating to *Unlikely to use* or *Definitely would NOT use* students were asked if they could give an example of when that strategy would be useful. Not all students were able to come up with examples when asked.

Apply the distributive property

For the FBA-MC group, two students (both in Algebra 1) were able to identify their appropriate use of the distributive property in simplifying the expression -7(x+2) +4x from question 3 when asked.

Symbolically correct manipulation: Each of these responses justifying use of a strategy was coded as symbolically correct.

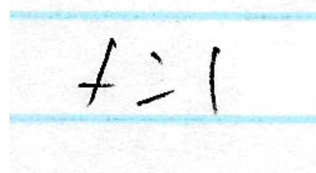
However, for question 2, simplify the expression 3x-2-2x+1, one Algebra 2 student indicated that the distributive property was used. This was done as part of the process used by the student when verifying that the answer was correct. The student refers to "distribute" as both the substitution of the value of one for x in the original expression and the multiplication of the 3 and the 1 as the student is calculating the results of the substitution. This student's response and work are shown in Figure 4.16. Even though the student has found the answer to be x=1, the student relates this situation to one in which they might substitute the solved value for x back into the original equation to determine if the two sides of the equation end up with equal numerical values.

The student changes from an expression to an equation when completing the calculation for 5-3.

<table>
<tr>
<td>P: What about applying the distributive property [for question 2]?

216: If you have to.

P: If you have to. Okay.

216: So if you distribute the 1[student draws curve connecting the 3 and the 1 in parentheses], if you get x=1 [student's answer is shown x=1], you distribute back in… that's… we got x so that would be 3x+2-2x-1 and that's the same answer… same question. You get back to the same question. So I know my answer is right because if I don't have the correct… if I don't get to the same original problem, then I did something wrong, but since I got to the original problem, it worked. Yes.</td>
<td>

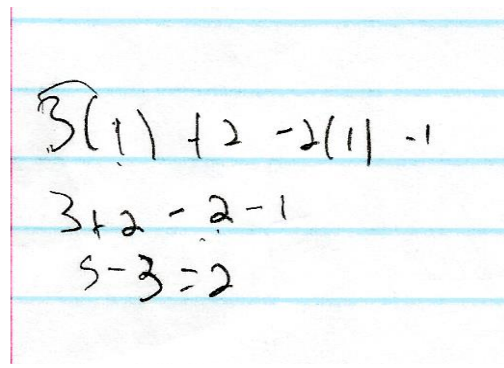

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Figure 4.16: Student 216's explanation and work for use of distributive property

Confounding II & IV: This student uses the irrelevant strategy Apply the Distributive Property. This student's has a different definition of this strategy than the teachers.

Cancel

Two of the three Algebra 1 students in the FBA-MC group who supported the use of cancelling as a strategy indicated that this was done as part of the combining like terms. These students mentioned "when I subtracted the common terms" and "I combined and cancelled the ones with the x's." The third student had a different definition of cancel as illustrated in the dialogue and work shown in Figure 4.17.

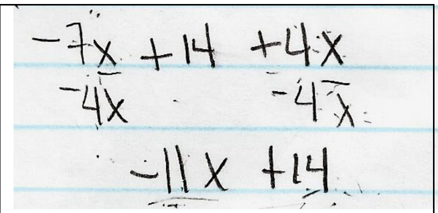
101: Yes, because cancel is kind of like just canceling this 4x and subtracting -4x so they would cancel each other and it would come here and add to this and make this. P: Okay. Where you have the plus 4x and you wrote the -4x underneath it, that was your cancel. How do you know they cancel? 101: Because if you go to your calculator …4+-4 it would be 0.	

Figure 4.17: Student 101’s dialogue and work for justifying the use of cancelling in Question 3.

The two Algebra 1 students from the FBA-LC group who responded to the rating for the strategy cancel for question 3 rated the strategy as *Unlikely to use* and indicated why that strategy would not be used. Student 117 appears to be referring to solving systems of equations.

> 117: Correct. You don’t really have anything to cancel out. You only have x as your variable and there’s no y. If there were a y you would be able to cancel out all the Ys or all the Xs and then solve
>
> .
>
> 308: Do you mean like canceling things out in this case? Okay, I think not in this one specifically, but I think there are problems like this one that I would use canceling, so…
> P: Could you show me an example of one you in which you would use canceling?
> 308: Instead if it was -7x and positive 4x , if it was -7x and positive 7x, then I would cancel out both of them because they cancel each other out, because they both add up to 0.
> P: So if that second line had read -7x-14+7x, you would use canceling on which two terms?
> 308: The -7x and the 7x..

Teachers A and B eventually came up a possible reason that students would rate cancel as useful in questions 2 and 3. Their examples match the explanation by an Algebra 2 student from the Non-FBA group provided for question 2 below:

> P: You said that you would be *Likely to use* the strategy cancel. Can you show me where that might have happened?
> 205: Well, I guess like when you combine them, they cancel out. So like if it was 3x they both cancel out so that there’s one left.
> P: Okay, and so you drew the 1’s, three 1’sand two 1’s over here. These were minuses, is that why you’re marking through them?
> 205: Yes.

This student used the work shown in Figure 4.18 to illustrate their point about cancelling.

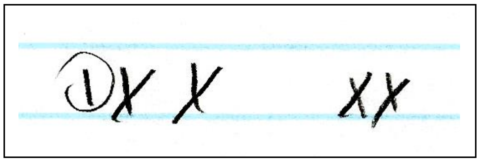


Figure 4.18: Student 205's work to illustrate use of cancelling.

Symbolically correct manipulation: Each instance for canceling above was coded as symbolically correct. Although the students each have a different idea of what cancelling means, the explanations fit teacher descriptions from the interviews that cancelling results in 0 or cancelling can be from making zero pairs.

Combine like terms

All students from all three FBA preference groups were able to explain their reasoning behind rating "combine like terms" as *Likely to use* or *Definitely would use*. All but one of the explanations was similar in referring either to the variable, the constants or both. Students made comments such as the following: "When I did these both have the x's, the -7-4."; "Subtracting the x's and the digits."; " The combining like terms like x and 3x.";and " I had to combine the x's because they were both like terms, the -7x and the 4x."

Symbolically correct manipulation: Each instance above was coded as symbolically correct as a justification for use of the strategy combine like terms.

An Algebra 1student from the FBA-LC group had an explanation different from the others. This student had a different understanding of the how strategy combine like terms is to be applied when simplifying expressions. This student appeared to be having difficulty transitioning the use of order of operations (PEMDAS) from elementary/middle school to Algebra.

P: So for combine like terms you said you were *Likely to use* this strategy.
117: Yes, I combined the 2x or the x+2 to get 2x and the 2x plus the 4x and got 6x.
P: And so if you were to explain to someone how you know that you can combine those, is there a way that you can explain to them that you know that you can do that?
117: Maybe. They are both x's and they are both positive. There's no exponents after the x or in front of the 2 and 4 then you can combine the first x and the amount of' x's, so 4x and x+2 or 2x and you can get 6x because they are all x's

Error: This student's explanation was coded as an error for combining of terms x and 2.

Do the opposite

An Algebra 2 student from the FBA-MC group was the only student to provide an explanation as to what the strategy do the opposite meant. This student confounded solving with simplifying. As a result, the student was able to justify the use of the strategy do the opposite as part of their solving process. The student dialogue and work are shown in Figure 4.19.

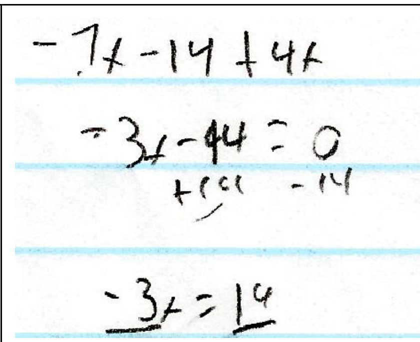
216: Do the opposite. I definitely would use because I did the opposite. I added 14 instead of subtracting [student points to place after he set expression = zero].	

Figure 4. 19: Student use of strategy do the opposite when asked to simplify

Symbolically Correct: Once the student converts the expression to an equation, the justification of the use of the property is coded as a symbolically correct manipulation.

Do the same thing to both sides

Two students from the FBA-MC maintained that they *Definitely would use* this strategy. Both students confounded solving with simplifying as they completed the task to "correctly simplify the expression."

> 101: Yes, because right here it did -4 and also here -4
> P: You wrote -4x under the… -7x and -4x under the +4x that was doing the same thing to both sides.
>
> 216: Do the same thing to both sides, like I did. [Student points to where the +14 is written on both sides of the student's equation for question 3

Confounding I: Each of these student justifications for use of strategy were coded as confounding I. The students maintained the treatment of the expression as an equation.

One student from the FBA-LC group, explained when this strategy might be useful since the student had rated this strategy as *Unlikely to use*. The student refers to the type of symbol necessary to indicate the use of solving strategies rather than simplifying strategies.

> 117: Correct, because there is no equal sign and there is not another equation on the other side per se. To do it on both sides you have to have an equal sign or a greater than or less than, of which none is present here, so I would be *Unlikely to use* this strategy.

Symbolically correct manipulation: The student provides an example of a situation in which this strategy would be used correctly. The student response is coded symbolically correct manipulation because the example fits the description for symbolic rule use.

Multiply both terms by a number or variable

The two students from the FBA-MC group had differing ideas as to what this strategy involved. The Algebra 2 student, refer back to Figure 4.12, considers that this strategy is the same as applying the distributive property.

> 216: And multiply both times a number or variable.
> P: You rated this as *Likely to use* for question 3.
> 216: Yes, I would likely use it because I multiply the 7 to the x+2. Multiply both times a number or variable, which I do with the negative 7.

Symbolically correct manipulation: This student's justification is coded as symbolically correct because teacher interviews indicated that the teachers believed that the students might be thinking of this strategy being the same as the distributive property.

However, the Algebra 1 student appears to link this strategy uniquely to solving equations.

> P: Your rating for "multiply both terms by a number or variable" was definitely would not use.
> 113: Multiply both terms by a number or variable. Probably not because you're not solving it, you're just simplifying it, so unlikely is correct [for question 3].

Symbolically correct manipulation: This student's explanation was also coded as symbolically correct. Both Teacher A and B originally linked this strategy to solving an equation with a fractional coefficient or solving rational equations and inequalities before extending the strategy to include Apply the distributive property.

Substitute a number for x

An Algebra 2 student in the FBA-MC group used the graphing calculator to substitute a value of one for x in the expression for question 2 to justify the use of this strategy in simplifying the expression. The student indicates that because “one equals x” you should “just plug it in” to the original expression even though their answer was shown as 1x. The display on the calculator screen was 3(1+2-2)1-1 which resulted in an answer of two. . When asked what the n “2” means to them, the student states, “That it equals 2. Because I knew we were solving. Yes, because we simplified the expression.”

Confounding I: This student’s justification for use of substitution is coded as confounding rules. The student maintains that x=1 although their answer was 1x. The student also appears to be confused as to whether they have solved or simplified. The dialogue was counted as one instance of confounding I.

Another Algebra 2 student indicated that this strategy would be useful “but I’m not going to do the work just for time.” An Algebra 1 student from the same group who had chosen the rating *Unlikely to use* for question 3 indicated a situation in which this strategy would be useful. The student dialogue and work are shown in Figure 4.20. This Algebra 1 student appears to referring to solving systems of equations; a topic recently covered in class.

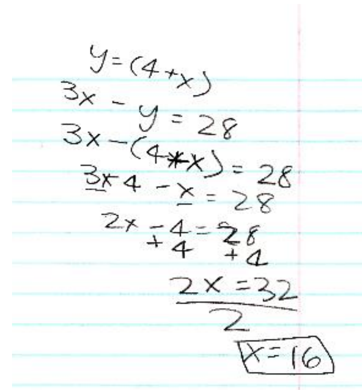
113: Substitute a number for x. Unlikely. I'm going to leave that as unlikely because that would be something you would do if you wanted to solve it, not simplify. …If you used substitution. P: What sort of example are you thinking about when you say use substitution? 113: If it said like y=4+x or something and then the next one is like 3x-y=28 or something, you would substitute the y for 4+x so it would be 3x-4+x … I did not do that right. 4+x=28 and then you would make it 3x-4 … 3x-4-x=28 and then you combine the like terms so it would be 2x-4=28 and then you would add the 4 on both sides and it would be 2x=32and then it would be … divide that whole thing by 2 and then x would equal 16.	

Figure 4. 20: Student dialogue and work for example of use of substitute a number for x

Error: This explanation was coded as an error. Although the student is providing an example of substitution, the example substitutes an expression involving x rather than a value for x.

This dialogue was not coded as confounding. Although the student states that substitution is only for solving, that statement is in agreement with the teacher ratings from the survey.

Two students, one each in Algebra 1 and Algebra 2, from the FBA-LC group justified the use of substitution for question 2. The students indicated that one could choose a value of x and substitute that value into the original equation. Neither of the students referred back to their answer for comparison of the calculated value. Figure 4.21 includes the dialogue and work for the Algebra 1 student. If the student had used this process in the answer the student might have recognized an error had been made.

117: Yes. Okay, $-7(x+2)+4x$ you could change out x to say… let's say x will be 2. Then you'd have $-7*(2+2)+4*2$. You could *solve* now and get an actual answer with no variables. So 2+2 would be 4 parenthesis +4 times 2 is 8 , then 4+8 is 12,[student erases the 12] -7… not 4+8. -7 times 4 would be -28 plus 8 equal -20 and you could get that possible answer.

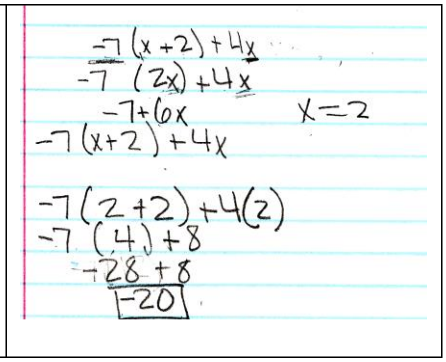


Figure 4. 21: Example of FBA-LC dialogue and work for question 2 use of substitute a number for x

Confounding I: Both student explanations were coded as confounding I. Although the substitution was done correctly for the original expression, neither student uses substitution in the answer to verify that the values are identical. In solving, students would use the answer to substitute back into the original equation.

Changed Ratings for Teacher-provided Strategies

As students were asked to work through the problem many of them realized they used some properties not rated as useful or didn't use properties rated as useful. Students from the FBA-MC group changed more strategy ratings than FBA-LC and Non-FBA combined.

Apply the distributive property

Although only one student mentions changing the strategy apply the distributive property, the reference is important. This Algebra 1 student initially wants to use the distributive property due to the parenthesis in question 3 but then proceeds to combine terms inside the parentheses although they are not like terms, refer to Figure 4.13.

> 117: When I thought of distributive, I thought of distributing the -7. I'm not positive this is the correct way or a valid reason. However, I was thinking at the time I could distribute the -7 to the parentheses and now I'm not so sure about this. I'm thinking I'm going to change that to *Unlikely to use* this strategy.

Error: This student's explanation was coded as an error. The student originally believes that the distributive property should be used, but due to possible issues with transference of PEMDAS strategies from middle school finally decides that this property would not be used.

Cancel

The two students from the FBA-MC group change their ratings for this strategy but for different reasons. The Algebra 1 student indicates that this strategy is one that would more likely be used with an equation.

> 113: Cancel. I definitely would not use this strategy. [Student changes D*efinitely would use* to D*efinitely would **NOT** use*] I don't know why you would need to really, because you're not moving it to the other side.

Symbolically Correct Manipulation: This response is coded as symbolically correct. The student provides a reason that depends on manipulation to justify the use of this property.

Another Algebra 1 student, refer to Figure 4.5, from this group is not sure at first about the meaning of this strategy but ultimately decides that what they did was different from cancelling. This student said, "Well, I kind of canceled a 2x, but that's not here, that's subtracting." The Algebra 2 student from this group had confounded solving and simplifying. This student whose work is shown in Figure 4.6 provides the following explanation:

201: What does it mean by cancel?
P: What comes to mind?
201: Oh, like you cancel like the numbers?
P: And what does that mean if you say you cancel the numbers?
201: Well, it becomes like zero [student changes rating from unlikely to likely].

Symbolically Correct Manipulation: Both students' responses were coded as symbolically correct. The explanations fit the teacher description of this strategy.

Although two students from the Non-FBA group indicated that they would change the rating for this strategy, only the Algebra 1 student gave a reason for this change.

P: So what would you change that to?
103: *Likely to use* this strategy.
P: Can you tell me where you used that in the problem?
103: Right here where you cancel out the 2x and it would get you 1x over here.

Symbolically correct manipulation: This response is coded as symbolically correct. Teachers A and B provided an instance of appropriate use of cancelling as a strategy when simplifying.

Combine like terms

The only students who changed their ratings for this strategy were students in the FBA-MC group. One of the Algebra 1 students indicated that the rating should change from *Likely to use* to *Definitely would use*. The other Algebra 1 student was surprised about their original rating saying, "I'm not sure why I said would not, because I did." This student then showed me where this strategy was used in the problem. The Algebra 2 student, refer back to Figure 4.6 to see student work, also changes the rating to likely and provided the following explanation:

P: Okay. You saw yourself combining like terms and so where did you do that?
201: When like over here when like we subtracted.
P: And what things were you subtracting?
201: The negative 2 because the positive 2 becomes a negative 2
P: And where else did you subtract 2?
201: Over here [student subtracts 2 from both the +2 and the -1 in the expression].
P: So why over there?
201: Because it's by itself. It doesn't have an x next to it. It's [the -1] by itself. It doesn't have an x next to it

Confounding IV: The student was using a strategy which they referred to as combining like terms but which the teachers would refer to as cancel.

Do the opposite

Students from the FBA-MC group changed their rating for this strategy to *Unlikely to use*. One of the Algebra 1 students has a reason for the change in rating although the student is not confident in their understanding of the strategy.

113: Do the opposite is unlikely because there is only one side [student changes likely to unlikely]. I think when I was doing this I thought maybe that I needed to solve it? I don't know.
P: Oh, and so you feel like you chose some strategies that went more with solving?
113: Yes, I feel like I was a little confused, that's why.

An Algebra 2 student from the Non-FBA group changes the rating for this strategy to unlikely saying, "Not in this problem. There aren't two sides."

Symbolically correct manipulation: Both responses were coded as symbolically correct sine the students realize this strategy would require that the problem "have two sides."

Do the same thing to both sides

The two Algebra 1 students from the FBA-MC group both changed the rating to *Unlikely to use* without providing any explanation.

These verbal responses were not coded as symbolically correct manipulation because the students did not provide any explanation for the change

An Algebra 2 student from the Non-FBA group is confident when changing the rating to definitely would NOT use. This student indicates that this strategy is a strategy to be used with solving rather than simplifying.

> 213: These [do the same thing to both sides; cancel] I'd probably change because if it's just a simplifying thing I have to do, there's nothing that I have to do on both sides.

Multiply both by a number or variable

Students from all three FBA preference groups changed ratings for this strategy. The three students in the FBA-MC group stated that they did not use the strategy in survey question 2. One of the Algebra 1 students specifically refers to parenthesis being needed in order to use this strategy.

> P: You chose *Likely to use* for this strategy. Did you see yourself using this strategy for [question 2]?
> 111: No, I don't use any here. But sometimes, if there were parentheses, yes.

Symbolically correct manipulation: This response was coded as symbolically correct. The student provided an explanation as to what is needed in order to use this strategy. The other two responses were not coded because no explanation was given.

An Algebra 2 student from the FBA-LC group is specific in regards to how to interpret this strategy as used in question 3.

> 209: I guess that's just the same thing as distributive property, but it's just worded differently?
> P: Okay, do you want to leave that as unlikely or do you want to change it?
> 209: Well, I mean, it's doing the same thing basically. It's just worded differently. So if I had to pick, I'd think I would still stick with apply the distributive property.

Symbolically correct manipulation: This response was coded as symbolically correct. The student explanation fits Teacher A's description of a possibility of students seeing this strategy as identical to Apply the distributive property.

Although two students from the Non-FBA group changed the rating for this strategy only the Algebra 2 student indicated a reason for this change saying, ", That one is probably a no because there's no multiplying [in question 2]." This student was unable to provide an example of when this strategy would be useful.

Substitute a value for x

The Algebra 1 student from the FBA-LC group finds it difficult to explain the change in rating other than because it wasn't used during the simplifying process. This student replied, "I definitely didn't see any point where I would have put the x in here for anything."

The two Algebra 2 students from the Non-FBA group were explaining when this strategy would be useful. Although both explanations seem similar in how the process would occur, the first student indicates that this process is the opposite of substituting a number for x. Student 213 changes the rating to *Unlikely to use* whereas student 214 changes the rating to *Likely to use.*

> P: You chose D*efinitely would use* for the strategy substitute a number for *x*.
> 213: Substitute. I don't know. That didn't happen in this problem.
> P: And so can you think of when you would substitute a number for *x*? What sort of situation you might have?
> 213: I know like the opposite of doing that, like putting a number in for *x*, but…
> P: Okay, so what would that look like, putting a number in for *x*?
> 213: Like for when I went… you asked how would I know if this original problem or this was correct. You would take this 9 and substitute it in for the *x*.
> P: And so you said putting a number in for *x*. And that's kind of like the opposite of substitute a number for x?
> 213: Yes.

Error: This student dialogue was coded as error. The student describes substituting a value for x but refers to the process as the "opposite of substitute." I didn't code this as confounding rules.

Student 214 discovers that the two expressions should be equivalent but only explores one value of x.

> 214: if you had a number to substitute for x, I guess you could do that too if it was like one of those problems where you just plug in the answer and see if you get what you get. Do you know what I'm talking about?
> P: Can you show me an example or make one up to show substitute a number for x?
> 214: I don't know if this will apply, but say x is equal to 3, so 3(3+2)-2(3-1) would be 9+2-6-1 , which equals 11-6-5 , and 5-1 is 4 and this would be 4 if you plugged 3 into x+1. So that's I guess how you check it. I guess I figured that out just then.

Symbolically correct manipulation: The student substitutes in both the original expression and answer. The student only uses one value for x rather than showing that the two expressions must be equivalent for all values of x.

Review of Function Based Approach (FBA) Strategies for Simplifying

No Change to FBA Ratings

Students were asked about the four FBA strategies separately from the teacher-provided strategies. These strategies were strategies that I did not observe either teacher use during the observations. All students from the Non-FBA group indicated that they would not change any of the FBA ratings. Fewer than half of the students in the FBA groups were willing to stay with original ratings for FBA strategies for questions 2 and 3 which asked students to correctly simplify an expression.

Regardless of course, the two students in the FBA-MC group provided a reason for rating FBA strategies as *Definitely would NOT use* or *Unlikely to use*. Students indicated the need to either be given a graph to begin with or being asked for a graph.

P: Can you elaborate on why those would not be useful?
1080215: Well, you could put it into a graph if you were making it into a graph, but for this purpose *I was just solving the equation* so all I needed to use were like properties such as like the distributive property and just like adding like terms. So using a graph… I mean, it might be helpful to others, but for me that would just confuse me more.

Confounding I: The student uses the phrase "solving the equation." This student had used a solving strategy when they subtracted 2 from both the +2 and -1 in the expression. However, the student did use the combine like terms strategy when combining the 3x-2x to end up with an x. But the dialogue indicated that the student considered this an equation rather than an expression.

P: The next one there says examine graphs using a calculator or computer.
113: Probably not that one either because there's no graph [in question 3], so just an equation.

No graph in problem: I coded this reference under the category Why FBA is not useful. The student indicates that the instructions would have had to have been to make a graph.

An Algebra 1 student from the FBA-LC group states, "There's no y or another variable, that's the word I was looking for, and usually for graphing you have to have two, a y axis and x axis in order to graph ."

The Algebra 2 student from the FBA-MC group explained how a calculator might be useful, "Because [my answer] is a fraction … the calculator will help because it can reduce and find the common denominator." The Algebra 2 student from the FBA-LC group maintained that the strategy examine graphs using a graphing calculator (as well as with pen/pencil and paper) would be useful even though the student explanation refers to a quadratic function rather than the expression from question 2.

212: Maybe if it was giving me an x value or like this would be 0 and maybe it was like it gave me a little bit more information where it was like you know, 5x and then over here it was , like x squared [student writes x squared in front of the -5x+3 where they had been working through question 2] that and then what you would do is you would take x squared and then you would plug this into the calculator to bring up a graph.

P: Okay, oh, you want to show me how that would work?

212: Turning the calculator one, should bring up y and then in y you just put x squared -5x+3 and that's what it brings up.

P: Oh, okay, and so which one would you be looking at?

212: The …

P: I see you're tracing your finger along that [graph that is a curve].

212: The curve, yes.

P: Okay, and so how would that help you?

212: Help me just to know that my equation is good. It's positive because it's going up instead of going down …. It's all positive; there's going to be no negatives in the equation.

P: Okay, and so if it didn't have an x squared then you probably wouldn't graph it?

212: I wouldn't graph it.

P: You rated using a pencil or paper to examine graphs *Definitely would use*." How would you explain your rating for that strategy?

212: Yes. The same kind of thing.

Why FBA is not useful: This response was coded this way for both tables and graphs because the student example did not relate to the expression from question 2. The student indicates that the graphs and tables would be useful only if the question had involved quadratic.

Changed FBA Ratings

Students were given the opportunity to explain how they used a particular FBA strategy or change the strategy. The Non-FBA students maintained the less useful ratings for all FBA strategies. Three times as many students from the FBA-MC group changed their strategies towards less likely than those who changed to more likely.

Changed towards less likely

The six responses from the FBA-MC group are similar to those for the students who did not want to change their ratings in the previous section. Students mention the following: “because to simplify an equation I don’t think graphs are going to help”; “When it’s y = x squared +3 or something like that”; and “There’s no table. It’s just an equation.”

An Algebra 1 student from the FBA-LC group repeats the explanation given before, “There’s no y or another variable, that’s the word I was looking for, and usually for graphing you have to have two, a y axis and x axis in order to graph .” Another Algebra 1 student from this group changes both examine tables and examine graphs to *Unlikely to use* because “To simplify an equation I don’t think graphs are going to help with that. The same with tables.”

Changed towards more likely

Both students from the FBA-MC group were Algebra 1 students. These students had different ideas as to how the graph would be useful. Student 101 uses the graphing calculator to support the change in rating to *Likely to use.* This student made errors in simplifying and the table values from the calculator provided evidence of that error. Refer back to Figures 4.11 and 4.15(b) to see this student’s work simplifying and substituting by hand.

> 101: It would just be y equals -7(x+2) + 4*x* and graph and you could go to the second graph [student goes to table view on calculator screen] and get your points there and kind of see what equals like… no, wait, I don’t think I’m doing this right.
> P: What’s confusing you, or what is it that makes you think it’s not right?
> 101: Because I think that whenever you graph it, you look for the point like the *x*, which is -11 and then you would find it here and then you would get 19, but I got 14.
> 101: 19. So I think I made a mistake. I thought I would get 14.

This student's response was not coded as FBA because the student is only looking at one value of x for the original expression and comparing that to the student answer.

This dialogue was coded as confounding rules. The student interprets their answer, -11x+14, as x=-11 for the purpose of substitution and expects that the answer should be 14 as if the original expression had been an equation.

The other Algebra 1 student maintains the rating *Likely to use* because the student knows the calculator is a useful tool even though the student doesn't see either strategy as useful in the problem situation.

> 111: Okay, this one, examine the graph … using a calculator or paper? Computer? You don't have to do it for this one, but I would be *Likely to use* this strategy because I can do my graph with a calculator, but I put *Definitely would use* because I don't have to do it here so it would still be yes.
> P: Can you tell me the type of problem where you know you would use graph or tables?
> 111: Like there's some problems they give you a graph and you have to find a point. You would use that one.

This student's response was not coded as FBA. The student provided an example of when they think that that graphs are useful. However, this example doesn't have anything to do with simplifying an expression even though the student perceives graphs as useful.

SOLVING EQUATIONS

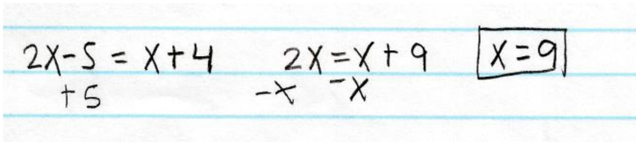
Students were asked to work through at least one problem that tasked the students with correctly solving a given equation in one variable. Students were asked to talk out loud about the process as they worked. Almost all students used pencil and paper although they were provided with a graphing calculator and grid paper.

Question 4

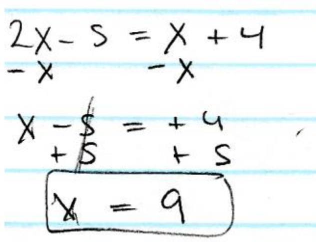
Of the 12 students asked to work question 4, 11 were able to solve the equation correctly. Students regardless of FBA preference group or course used similar symbolically correct methods to solve the equation for question 4: 2x-5=x+4. All but one student either subtracted x from or added 5 to both sides as the first step. All students worked the problem on notebook paper. Most students used phrases such as "move x over to the other side"; "when you move it over a negative becomes a positive"; and "move the variable and the number to the other side." A Non-FBA Algebra 1 student is the only student who uses the term cancel saying, "I would subtract x on this side with the +4, [the subtracted x] would cancel [the x] over here."

Symbolically correct manipulation: Each instance of student work was coded as one instance of symbolically correct manipulation for the question although students may have applied more than one teacher-provided strategy correctly.

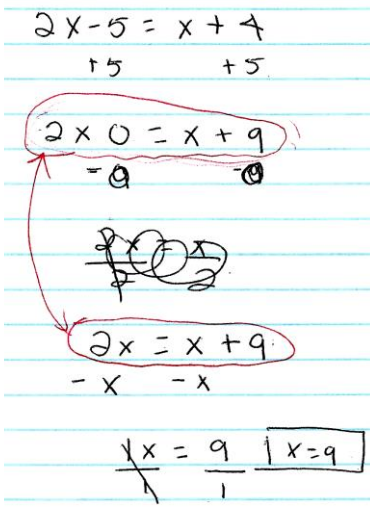
The one Algebra 2 student from the FBA-MC group had difficulty with solving but ended up with a correct answer after being allowed to talk through the process. I have included three examples of work in Figure 4.22. Figures 4.22 (a) and (b) show the most used symbolic manipulation. Figure 4.22(C) shows the work of the Algebra 1 student who followed a more circuitous route to solving.



(a) Student 215 adds five first



(b) Student 308 subtracts x first

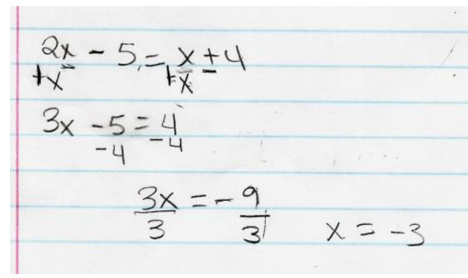


(c) Student 201 is confused by the 0

Figure 4. 22: Student work from each group for survey Question 4.

The only student making errors when solving was an Algebra 1 student in the FBA-MC group. The student eventually comes up with two different answers after working the problem different ways upon being asked to justify work. The dialogue and work for this student is shown in Figure 4.23. The student appeared to be confused about whether to add or subtract x. This student inadvertently places the minus nine on the right side of the equation in both solutions.

101: Okay, so it's $2x - 5 = x + 4$. So here you would just be combining these.
P: Okay and the "these" you are pointing to are the 2*x* and the *x*.
101: You would want to get it on this side so you would subtract the x… no, add the x, sorry. You would add the x so then it would be $3x - 5 = 4$. So then you would subtract the 4 and then you would do the -5-4 and then you would get -9 so then it would be $3x = -9$ and you could divide 3 here. You could divide it so that…
P: What is the "it?"
101: The -9 and then you would get $x = -3$.
P: How do you know it's a plus *x*? You said at first minus and then you changed it to plus.
101: Well, I always get confused on these since this is a plus, you would add it, I don't know, because if you wanted to cancel, you would subtract it then. … So yes, you would subtract the x and then it would just be $x - 5 = 4$ and then since this was a plus and if you were to subtract by 4, it would be a 0, so that's what you want to do, you want to cancel this so you could do the same here. So -4 and then here…
P: And so how did you know to cancel the 4?
101: Because you want to combine it to the -5 so you want to cancel this out here so you would just leave it here and…
P: So you want to cancel out the 4 that's over here on the right and combine that -4 that you're writing down with the -5.
101: So then after you do $-5 - 4$ it would be $x = -9$ and that is my answer.



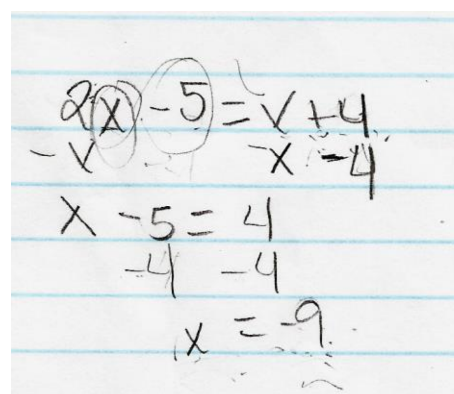


Figure 4. 23: Algebra 1 student dialogue and work for survey Question 4

Error: Student 101's dialogue and work was coded as an error. The student writes the -9 on the right hand side of the equal sign rather than leaving the -9 on the left side of the equal sign in both examples. This work counted as two separate instances of error.

Confounding IV: The student adds x to both sides of the equation rather than the subtracting x. This student incorrectly applies the do the opposite strategy.

The Algebra 2 student in the FBA-MC group ends up with an answer x=0 but realizes that this value is not correct. This student is unable to work back through the problem to determine where the error might have occurred. The dialogue and work for the Algebra 2 student is shown in Figure 4.24.

201: Well, assume like this one we'll solve for *x* too. Do I solve it? Okay.
201: Subtracting the *x*. Wait. So I add 5. Wait, no. I subtract the 4 so [the right side of the equation] can become 0.
P: And so you're using the calculator to…?
201: To subtract the -5-4. And that equals -1. I subtract now *x* because you don't have another number. So you don't want *x* and you divide by 1. But I don't think that's right. Well, it's not right.
P: You're saying that you don't think that's right so what is it that's making you think it's not right?
201: Because it's not 0. It doesn't make sense. When you plug it in, 0 for x, you would get -5, but that [the right hand side] would be -4. That's not right. So that wouldn't be the answer, because it has to equal the same.

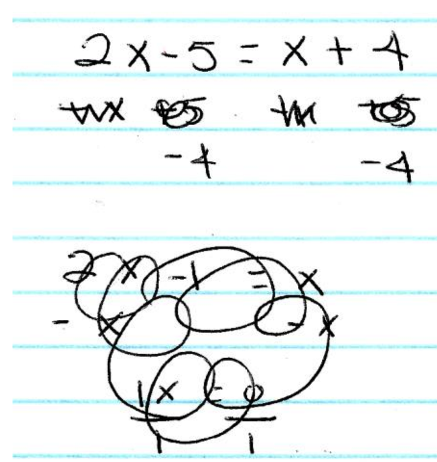


Figure 4. 24: Algebra 2 student dialogue and work for survey question 4

Error: This student's work was coded as two instances of errors. The first error was due to the student's arithmetic error in combining the -5 and the -4 to get -1. The second error occurred after subtracting x from both sides. The student ignores the -1 on the right side of the equation.

Symbolically correct: The student does recognize that an error has been made by substitution of the value of x from the answer. The student is following a teacher-

provided strategy of substituting a value of x in the original equation as a way to check the answer. This dialogue has been italicized in Figure. 4.24.

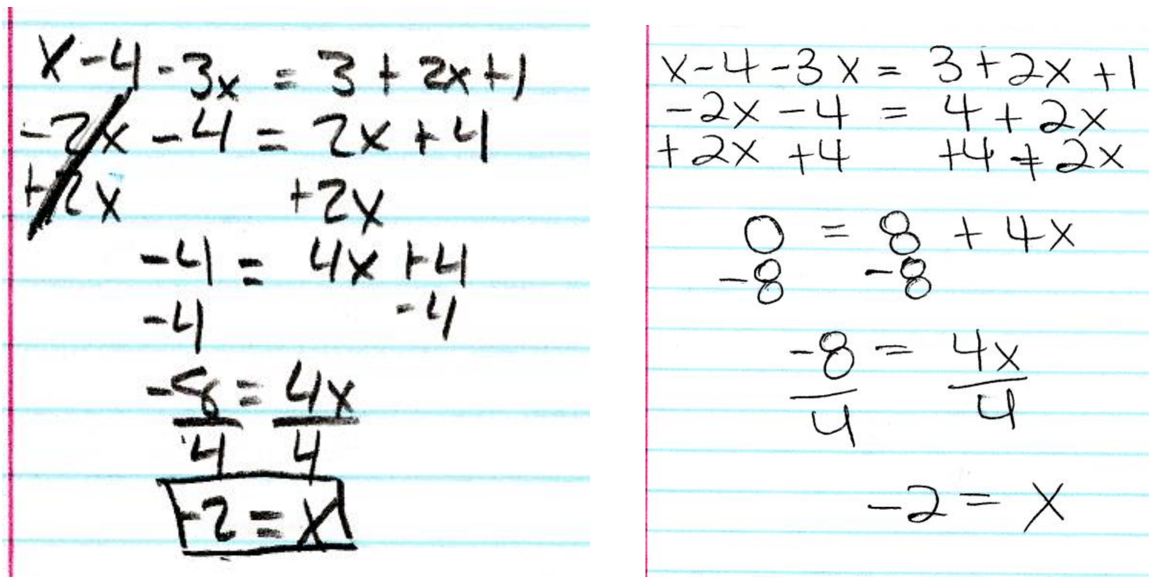
Question 5

Of the nine students asked to work survey question 5 seven students correctly used symbolic manipulation. The work for four of the seven students regardless of FBA preference group or course was done similarly to student 215, an Algebra 2 student, whose work is shown in Figure 4.25 (a). One of the four, an Algebra 1 student, divided all terms by two after combining like terms for both sides "to make it simpler" before continuing to solve. The other three students, all Algebra 2 students, set the equation equal to zero after combining like terms. A sample of this type of work by student 214 is shown in Figure 4.25(b). This student is confused as to how the problem should be worked after combining like terms on both sides. The student states, "I don't know if it wants me to move everything over to this side because the terms all cancel" and "I don't know if I'm supposed to be setting equal to zero, finding points, or graphing it or what." The student eventually decides "it's best if I set this equation to zero and solve." This might be a result of the Algebra 2 classes solving rational inequalities as well as quadratics prior to the interviews. The instructors had both used the strategy of setting the expression equal to zero.

Symbolically correct manipulation: Each instance of a correct solution to question 5 was coded as symbolically correct even though students may have used more than one strategy.

All students preferred to eliminate the negative x term. An Algebra 1 student from the Non-FBA group provides this explanation for that preference:

> "I like working with positive numbers rather than negative numbers. If I had subtracted the 2x from the right side of the equation it would have given me a -4, which would still work, but it's just a lot nicer to work with positive numbers, in my opinion.



(a) Student 215 work solving (b) Student 214 setting equation equal to zero

Figure 4. 25: Student work samples for solving survey Question 5

The FBA-LC and Non-FBA students make the same error when substituting a value for x. This leads both students to conclude the solution -2=x is incorrect. When multiplying the 3(-2), the product is -6, but the student subtracts a 6 rather than a -6. The student from the FBA-LC group states, "There is no solution" since the -12 is not equal to 0." The student from the Non-FBA group states, "That would be -8 … the other would not be -8. I guess I'm wrong."

Error: Both instances were coded separately as arithmetic errors.

Two other students have issues with integer coefficients causing these students to have incorrect answers. When combining like terms, x and -3x, the Algebra 2 student from the FBA-LC group incorrectly writes the result as -4x. The remaining symbolic manipulations are correct. The student isn't confident in the answer because the answer is a fraction. The student indicates the work is complete by saying, "I'm guessing." The Algebra 1 student from the Non-FBA group combines the x, -3x and the 2x that was subtracted from both sides incorrectly to result in a -6x rather than a -4x. This causes the student to end up with -6x=8. At this point the student says, "I don't exactly know how to solve the rest of this. The numbers are just confusing me."

Error: Both instances were coded separately as errors. Both were arithmetic errors using integer coefficients.

Knowing Answer Is Correct

After solving each equation for survey questions 4 and 5, students were asked how they would know if the answer was correct. The majority of the students interviewed (78%) indicated that they would check their answer by substituting the value they found for x back into the original equation regardless of FBA preference group or course. All but one of the students opting to check by substitution did so using the original equation.

Symbolically correct manipulation: Each of these responses was coded as symbolically correct. This type of substitution is used by teachers to check answers for solving equations, although Teacher A indicated, "Not for question 4, but for more complex equations."

An Algebra 1 student from the Non-FBA group substituted the value 9 for x not in the original equation but in the equation x-5=4 that was created by eliminating the x on the right hand side. The student indicated that "it doesn't matter where you substitute the number in; you just need to end up with 9=9."

Error: This response was coded as an error. If the student had made an error in the first symbolic manipulation, the answers might not have been equal after substitution of the value of x.

Students were unable to explain why this might work, only that they knew substitution was the correct process. The only student who was unable to explain how one would know if their answer was correct was still able to correctly solve the equation. This Algebra 1 student said, "I guess you could put it in a calculator if you wanted to, but I'm not sure how to do that with the equals in the middle so just …? Okay, I could not put it in that calculator. I'm sorry, I don't know." Another Algebra 1 student from the FBA-MC group stated, "Well, this problem was kind of simple so I'm kind of sure of myself, but I would still ask my friends and if they don't know either, I would just ask the teacher and he would help me." Two of the students from the FBA-LC group gave the same response as for simplifying expressions. The students indicated they would review their work.

No Change in Strategy Ratings for Solving

Students were asked to review the strategies they used for solving equations for survey questions 4 and 5. Two students from the Non-FBA group, one from each course, indicated that they would not change any of the ratings for the teacher-provided strategies.

These responses would be coded when students were asked to justify ratings for various strategies.

Apply the Distributive Property

An Algebra 2 student, from the FBA-MC group, indicated that the distributive property was used in question 4. It appears the student is using the terms distribute and substitute interchangeably. The student uses the same explanation to justify the use of the strategy "substitute a number for x" for both questions 4 and 5. The student said, "I put a number in for x." The student refers to the multiplication 2(9) as applying the distributive property. This is echoed by another Algebra 2 student in the Non-FBA group who said, "Yes, well, distributing is also multiplying. It's the same thing."

Confounding IV: Students understand the use of this strategy different from the teachers.

The FBA-LC and Non-FBA students indicated that they did not use the distributive property because there were no parentheses in either question 4 or 5.

Cancel

With the exception of one Algebra 2 student from the FBA-LC group, the other seven justifications for cancel were referring to the process in which the student added the opposite of a term to eliminate that term from one side of the equation and "make [the term] go away." The Algebra 2 student who was thinking of this differently said, "If there's something in common on each side you can just cancel them. For example, if you had -2x+x+3 = -2x+3x+5, the -2x's on each side would cancel."

Symbolically correct manipulation: Although this type of cancel was not part of questions 4 or 5, the student provides an explanation that met the initial description of the use of cancel by both Teachers A and B.

Combine Like Terms

The eleven students who justified their use of the strategy combine like terms explained the process in two different ways. The students who worked question 5 indicated that they combined the x and -3x as well as the 3 and 1. Several students mentioned the use of the strategy combine like terms in question 4 when they were eliminating either a variable term or constant term from one side of the equation. One Algebra 2 student from the FBA-MC group responded as follows:

> 216: Combine like terms, no… yes, I did.
> P: Oh, so you had a question about this? Can you see where you did combine like terms?
> 216: Yes, I did. I combined the x and the -x [student points to work for question 4 where the student wrote -x on both sides of the equation].

Symbolically correct manipulation: Each of these instances was coded as an instance pf symbolically correct manipulation. Students indicated characteristics such as same letter as justification.

Do the Opposite

Two of the four FBA-MC group students who referred to using the strategy do the opposite explained that they were doing the opposite when they were eliminating a term “from one side of the equation. The dialogue and work by one of these Algebra 2 students is shown in Figure 4.26.

P: Can you tell me where you did the opposite on question 2?
216: I added 5.
P: That was the opposite of what?
216: Of subtracting 5.

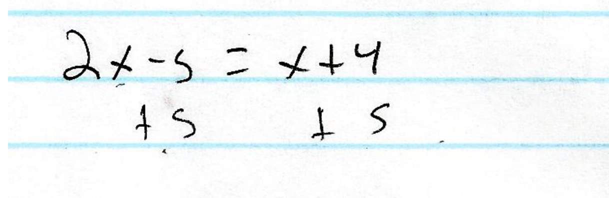


Figure 4.26: Student explanation for use of the strategy do the opposite.

The third student from this group, an Algebra 1 student, stated that the strategy do the opposite was not used in question 4 although the student’s work is very similar to the work of student 216 in Figure 4.25. The student had selected a rating of *Definitely would NOT use* for the do the opposite strategy. The student had mentioned earlier, “I’m still learning English.” It is not apparent from the interview whether the student has a different understanding of how this teacher-provided strategy is used in the course or if the strategy held no meaning for the student. The student was aware that the strategy do the same thing to both sides was useful. The student was not able to give a specific example of when you would use the strategy do the opposite. If the student had been able to correctly associate the name of the strategy (do the opposite) with the work that was done the student work would have fit with the strategy.

Confounding III: The student doesn’t indicate usefulness of potentially relevant sub-strategy even though the student knows to subtract x (the opposite of a +x) from and add 5 (the opposite of -5) to both sides of the equation. I am unable to determine if this is an *example of Confounding IV since the student is unable to give me an example of when the strategy would be used.*

The fourth student in this group, an Algebra 1 student, mentions using this strategy when test-taking.

> 101: Do the opposite, well, yes, here when we were trying to like plug it back in when we got the *x* minus -9 here, but then it led me to another thing, but if like in a test you were to get multiple choice, you could look at the answers and plug those in and see if it would work out.
> P: And so you're relating that to where you plug in answer choices?
> 101: Yes.

Confounding IV: The student explanation does not meet the teacher description of the rule for using this strategy.

Do the Same Thing to Both Sides

With the exception of one Algebra 1 student from the FBA-LC group, students had the same explanation for the strategy do the same thing to both sides. All eight, regardless of group or course, indicated places in the problem where they either added or subtracted a variable or constant from both sides in order to eliminate that particular term. The student who was the exception provided the explanation and work shown in Figure 4.27. This student was not asked to work question 5 and did not use the strategy when solving question 4 even though the student used the strategy and rated it as *Likely to use.* The student said, "So I take the x and on the right side and subtract it from both sides…"

Confounding IV: I coded the dialogue as one instance of confounding rules. The student used the property but then indicates that the strategy was not used.

However the student was able to provide an example of when the strategy should be used.

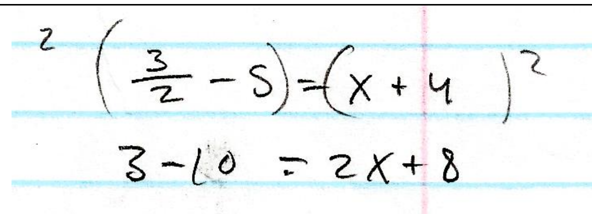
308: And for doing the same things to both sides, in [question 3] I didn't find myself using it, but I could see if I had a fraction on one side… like I would say, this was like, you know… P: You can write me another example down here that's simple that would be… 308: So if it was like $\frac{3}{2} - 5 = x + 4$ and if I wanted to get rid of the 2, then to both sides I would multiply them both by 2, so then this would be $3 - 10 = 2x + 8$ so I guess that's if you do the same thing to both sides. I guess keep it likely.	

Figure 4.27: Student explanation and example of when to use the strategy Do the same thing to both sides.

Symbolically correct manipulation: The work and dialogue in Figure 4.26 is coded as symbolically correct. The student applies the strategy as indicated by participating teachers as a means to "remove the denominator" in problems involving fractions.

Multiply Both Terms by a Number or Variable

An Algebra 1 student from the Non-FBA group indicated that the selected rating of *Unlikely to use* should stay the same for both questions 4 and 5, but was unable to give an example of when that strategy might be useful.

Neither of these responses was coded as symbolically correct or confounding rules because neither an explanation nor example was provided by the student.

Substitute a Number for x

The two responses from the FBA-MC group stated that the substitution was done as part of the process of checking their answer. One of the two responses from the Algebra 1 student in the Non-FBA group matched the explanation from the other two groups. This student was able to solve the equation from question 4 symbolically. The student substitutes this value for x back in to the original equation and indicates that the rating should remain as *Definitely would use*. However, this same student was unable to solve the equation from question 5. Since the student was unable to solve the equation, there was no opportunity to substitute the value of x found by solving the equation. The student then indicates that this strategy was not used in question 5 and leaves the rating as *Unlikely to use*.

Confounding III: This student's response for question 5 was coded as confounding III because the student was unable to identify this strategy as a potentially relevant sub-strategy.

Changed Strategy Ratings for Solving

Students were asked to indicate teacher-provided strategies for which they wanted to change the rating. There are 15 instances of changed ratings for the FBA-MC group compared to five for the FBA-LC group and 10 for the Non-FBA group. This is not surprising since the FBA-MC group did tend to confound the strategies for solving and simplifying. Students were asked to give examples of when a particular strategy might be used if they indicated that the strategy was not useful for a particular problem.

Apply the Distributive Property

Three of the four FBA-MC students changed their rating for apply the distributive property to *Definitely would not use*. One of the Algebra 1 students gave no reason for the change. Two of those students (one Algebra 1 and one Algebra 2) mentioned that there were no parentheses.

Symbolically correct manipulation: Two of the responses were coded as symbolically correct because the students were able to provide an explanation of what was necessary for use of this strategy.

The fourth student in this group changed the rating from *Unlikely to use* to *Likely to use* because the student used the distributive property "whenever I plugged [the value for x] in."

This student's response was not coded as confounding rules because the student is confusing the name of the strategy by referring to substitution as the distributive property.

The responses from the Non-FBA group were very similar. Two students, one in Algebra 1 and one in Algebra 2, changed the rating from *Likely to use* to *Definitely would not use*. The Algebra 2 student explained the reasoning behind the ratings change as follows:

> 213: Distribute. I guess now that I'm thinking about it, when you distribute, it would be like you had something in parentheses and then… you'd have a 2 out here and then an x+3 and then distributing would be like putting the 2 with the x and then multiplying the 2 with the 3, … and I didn't do that on [question 4].

The Algebra 2 student who indicated that the distributive property had been used referred back the work done when substituting the value of x obtained from solving the equation for question 5 back into the original equation. The student specifically mentions, "Distributing is also the same thing as multiplying." The student was consistent in the use of this strategy for both questions 4 and 5.

Confounding IV: This dialogue was coded as two separate instances of confounding IV because the student has a different understanding of the use of this strategy.

Cancel

Five of the six responses for changing the rating for the strategy cancel to *Likely to use* or *Definitely would use* were the same as the ones mentioned by the students who did not change their ratings. These students, regardless of course and group, explained that they were using the strategy cancel when they were eliminating a term in the equation.

An Algebra 1 student from the Non-FBA group never used the word "cancel" to describe any of the symbolic manipulations used in simplifying or solving. When asked about the rating for this strategy the student simply replied, "It's not something I did." The student could not give an example of when they thought the strategy cancel would be useful.

Confounding III: This student did not recognize potentially relevant sub-strategies.

Combine Like Terms

Both Algebra 1 and Algebra 2 students from the FBA-MC group as well as the Algebra 1 student from the FBA-LC group changed their ratings of the strategy combine like terms to *Definitely would use*. The students used the example of elimination of a term when solving to explain the reason for the rating change.

111: Combine like terms? That's when we subtracted x. Like terms are could be like letters.

201: When I put the -x on both sides under the 2x and the x

308: Combining like terms, yes, because it's the same as the other one. You know, anything that is an x you would add it together.

Do the Opposite

Students from both the FBA-MC group and the Non-FBA group changed the rating of this strategy to *Likely to use*. Both students referred to the process in which they eliminated a term when solving.

201: Whenever you're like a positive *x* so it becomes a negative *x*.
P: In order to do what?
201: To subtract it. [Student points at the x]
P: So where you had this $2x = x + 9$ and you wrote a –*x* below the *x* on that right hand side, you said that's doing the opposite.
201: Yes, then adding.

103: Right here when I did the opposite for this side instead of adding 2x, I subtracted it.

Do the Same Thing to Both Sides

The four students responding to this strategy were surprised that they had rated this strategy *Unlikely to use*. Regardless of FBA preference group or course all changed their ratings to either *Likely to use* or *Definitely would use*.

One student simply states, "There aren't two sides on this one." The other student isn't so sure about the reason behind the change in rating.

P: You chose *Likely to use* for do the same thing to both sides. Where did you use that?
319: Would that be when you do that, or no?
P: You're pointing to where you put the –x under the x on the right and under the 2x on the left and you feel like that's doing the same thing to both sides?
319: Oh, never mind. No. I don't think so.

The student applied solving strategies correctly in their work. The student initially indicates that this strategy was used. The student actually pointed to where they used this strategy correctly although they were not sure of their example. I did not code this as confounding even though the student, "Oh, never mind."

Multiply both terms by a Number or Variable

An Algebra 2 student from the FBA-LC group provides an example of when this strategy would be used. The student has extended the use of this strategy to division as shown in Figure 4.28. This student changed their rating from *Unlikely to use* to *Definitely would use*.

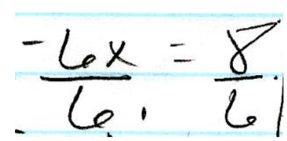
212: The numbers and the variables right here, multiplying them and dividing them. [Student points to the 6 in the denominator on both sides.	

Figure 4.28: Student dialogue and work showing the use of the strategy multiply both terms by a number or variable.

Symbolically correct manipulation: Although the student used division rather than multiplication, the example is similar to one provided by Teacher A. This teacher commented, "To remove a coefficient of 2, they might multiply by one-half."

The other two students changed their ratings from *Likely to use* to *Unlikely to use*. Neither the Algebra 1 student in the FBA-MC nor the Algebra 2 student in the Non-FBA group could provide an example of an equation in which this strategy would be useful.

Substitute a Number for x

Four of the five students who changed the rating for this strategy changed the rating to *Definitely would use*. Each of these students regardless of group or course cited using this strategy to check the answer as the reason for changing the rating.

> 205: Okay, well, substituting a number for x, I would use that if I was checking my answer.
>
> 319: Could that be after solving?
> P: It could be. So you would use substitute a number for x after solving.
> 319: Yes
>
> 209: Well, to check my answer I substituted a number for x.

Symbolically correct manipulation: These responses were coded as symbolically correct manipulation with the exception of student 209. The students perceive substitution as a process for checking correctness of answer rather. There is no indication of equals as one type of comparison of functions.

Error: Student 209 substitutes the value for x obtained from solving the equation but makes an arithmetic error in calculations by hand to end up with -12 = 0. The student then concludes the answer x= -2 is incorrect.

The one Algebra 1 student from the FBA-LC group seemed to be confused about whether the strategy was referring to a number being substituted in for x as opposed to substituting the x for a number. The student had actually substituted the value of 9 for x in the equation 2x-5=x+4 in order to check the correctness of the solution.

> 308: Substituting a number for x… I don't think so. I mean, I guess when I was going back I kind of put it, you know, put the x in there, but I wouldn't substitute the x for any number. So I guess I would be an unlikely.

Confounding IV: The student appeared to think that the strategy indicated that you could substitute x in place of a number.

Unsure of Meaning of Teacher-provided Strategies

Two of these strategies, Do the Opposite and Multiply both terms by a number or variable, were confusing to eight of the students when simplifying. This occurred in all three FBA preference groups.

Do the Opposite

One of the Algebra 1 students from the FBA-MC group rated this strategy *Likely to use*. When asked to show where the strategy was used in question 2 and 3, the student states, "I didn't know what that meant. " The other Algebra 1 student from this group thinks this might be a solving strategy but isn't sure.

> P: You said you definitely would not use this strategy.
> 113: I don't even know what that is. I mean, unless he like calls it something else. I don't know what do the opposite would be.
> P: What does "do the opposite" bring to mind?
> 113: I mean, all I can think of is doing the same to one side of the equation, but this is kind of a one-sided thing, so I don't really know.

The two Algebra 1 students from the FBA-LC group rated this strategy *Unlikely to use*. One of these students indicated that this strategy might mean the same as inverse although the student was unable to provide an example of when to use an inverse. The Algebra 2 student from this group who rated this strategy *definitely would NOT use* simply stated, "It did not make sense to me." Algebra 2 students in the Non-FBA group admitted that they "did not use the strategy in this problem." and "I don't know what it means."

Multiply both terms by a number or variable

The Algebra 1 student from the FBA-MC group had rated this strategy *Likely to use*. When asked to show where this strategy was used in question 3, the student asks for verification of the meaning of this strategy first, but then decides to stick with the rating. Refer back to Figure 4.11 to see the student work for this problem.

> 101: What is this, multiply both terms by a number or variable? Does that mean to distribute?
> P: What do you think? What did it mean to you?
> 101: I thought it meant like multiplying this all together.
> P: Multiplying… so give me an example of what you mean when you say multiply it all together.
> 101: I thought it meant like you would distribute -7 to all of these.
> P: Oh, distribute it to all of the terms so the -7 would get multiplied …
> 101: By x, by 2 and then…
> P: Do you want to change your rating?
> 101: No.

There were students who were unsure of the meaning of these same strategies in addition to DO the same thing to both sides in reference to solving for questions 4 and 5. Table 4.16 shows sample student responses from each group in reference to these strategies. Even students who provided an example of how the strategy might be used (or was used) were not confident in their response.

Unsure of strategy: Each of these responses was coded as an instance of unsure of strategy.

Table 4. 4 Student responses to strategies they were unsure of for use with survey questions 4 and 5

Do The Opposite	**Do the Same Thing to Both Sides**	**Multiply Both Terms by a Number or Variable.**
209: What does it mean to do the opposite? P: What do you think it might mean? 209: Like adding maybe? P: Adding what? 209: Like wherever there's a negative that's where you would add them?	P: You said you were likely to do the same thing to both sides. Where did you use that? 319: Would that be when you do that, or no? P: Oh, you're pointing to where you put the –x under the x on the right and under the 2x on the left and you feel like that's doing the same thing to both sides? 319: Oh, never mind. No. I don't think so.	319: Did I use that? P: Multiply both terms by a number or variable… and you're asking if that's what you were doing when you were plugging it in? Does that make sense that that's what you were doing when you were plugging it in? 319: I think so.

Function Based Approach (FBA) Strategies for Solving

No Change to FBA Ratings

Three FBA-MC group students kept the rating of *Unlikely to use* or *Definitely would NOT use* for the FBA strategies when asked about solving. Two of the Non-FBA students specifically stated they would keep these same ratings. An Algebra 1 student from the FBA-MC group seems to remember a way that graphs might be useful when solving, but when attempting to show how this might work, the student is unable to do so. The student decides to stay with the rating of *Unlikely to use*. The student refers back to this example when indicating that the other FBA ratings would still be unlikely as well.

101: The graph… you could graph it, but I don't know. You could graph it, but I don't know how you would get the answer. If you had a table you could, but I'm not really sure. [student starts to enter data on the graphing calculator]
P: So you think you could put it in the calculator?
101: In the calculator, yes.
P: So if you were going to put it in a calculator, where do you think you would go and what would you do?

101: Well, I would go to *y*= and then plug in the expression, which is $2x - 5$… oh, well, you would first have to move this to the side because on the calculator there no equals. It's supposed to be simplified already when you put it in the calculator.
P: Oh, okay. So since you don't have the equal sign on your calculator, you're saying this needs to be simplified?
101: Simplified where like you could plug it in to the computer.
P: And so you're saying you would need to get this as +4 over to the other side first?
101: I'm pretty sure. Yes. I don't think I would use the calculator because… no.
P: You don't think you would? You've put a check by *Unlikely to use*.

No graph or table in problem: The student dialogue was coded as why FBA strategies are not useful. The student indicates that you would need to have a table. This was coded as one instance. The references to graphing were not coded because the student was unable to explain what it would be about the graph that would be helpful due to not being able to input the equation into the calculator.

The Algebra 1 student from the Non-FBA group supported the rating of *Unlikely to use* for all FBA strategies on question 4 and both "examine graphs using a pencil and paper" and "examine tables of data using a calculator or computer" on question 5.

P: What about your rating for examine graphs?
103: I wouldn't use them because there's no way to graph this.
P: There's no way to graph any of it, or is that only the original equation?
103: Yes, there's no way to graph it because there is no equation.

The student also uses this line of reasoning for maintain the rating of *Unlikely to use* for the other FBA strategies. The student did not differentiate between tables and graphs for the strategies although the student was asked about each strategy separately.

No graph or table in problem: The dialogue for each student was coded as one instance of why FBA strategies are not useful.

Changed FBA Ratings for Solving

Students were also asked about the ratings for the FBA strategies in relationship to solving equations. The eleven responses for changing the rating for an FBA strategy were to change the rating to *Unlikely to use* or *Definitely would NOT use*. There were four possible FBA strategies for both questions 4 and 5. Students who indicated that the strategies were less likely than originally rated often changed the ratings for both tables and graphs regardless of whether using a calculator or pencil and paper. Two Algebra 2 students from the FBA-LC group, student responses referred to the fact that the problem situation didn't include or ask for a graph or table. One of the Algebra 2 students said, "You could use tables or graphs for like when they give you the coordinates."

No graph or table in problem: The student refers to being given coordinates. I coded this as no table in the problem under the category Why FBA strategies are not useful.

Another Algebra 2 student mentions tables being useful when working with quadratics.

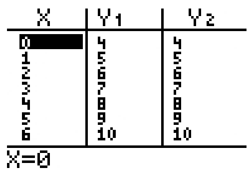
> P: Since you changed "examine tables" to "definitely would not use" when would you use tables?
> 212: If we're using tables and graphs we would be dealing with things like x squared and long equations dealing with x squared like x^2 +3x-6.

Useful only for quadratics: This student response is coded as a single instance of why FBA is not useful.

Student and teacher rating for usefulness of a specific FBA strategy for simplifying

Question 6 provided an example of a FBA strategy using a table of values which would be helpful in allowing a student to determine if they have correctly simplified an expression. The question from the survey is shown in Figure 4.29.

Use the table below to answer Question 6. In the table, Y1= 2x+3-x+1 and Y2= x+4 .



6. Rate the usefulness of the table as a strategy in correctly simplifying the expression 2x + 3 – x + 1?

Figure 4.29: Survey Question 6

The survey results for the Algebra 1 and 2 teachers (n=22), Algebra 1 students (n=48), and Algebra 2 students (n=52) are shown in Figure 4.30. Although the teacher percentage (36%) for ratings of useful is lower than the student percentages, it is the highest teacher percentage given to an FBA strategy for simplifying an expression.

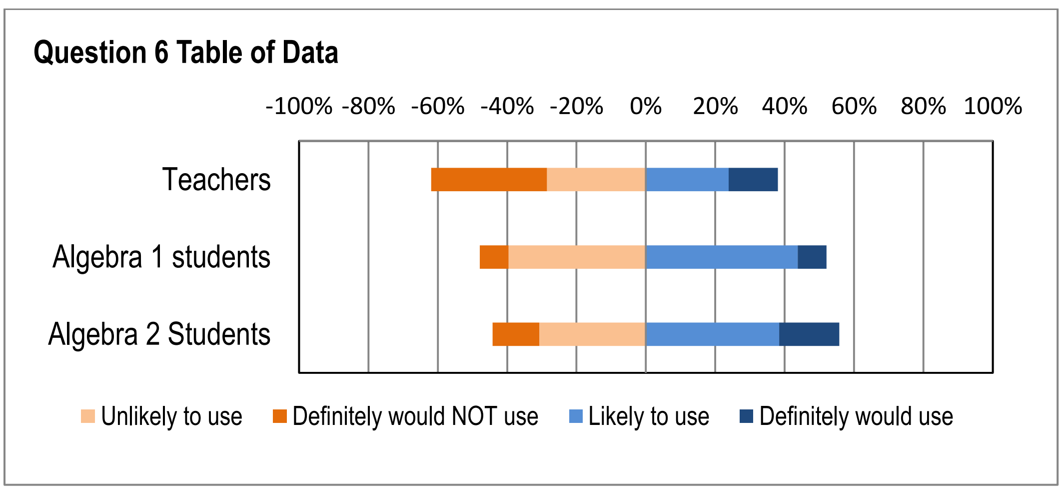


Figure 4.30: Percent response for teachers and student course groups for survey Question 6

The percentages from the student survey (approximately 50%) are very close to the percentages expressed by the 16 students who were asked to explain the rating choice for question 6. However, the reasons several of the students gave for the ratings depended mostly on Y1.

The FBA-MC student rating the usefulness of the table as *definitely would NOT use* stated that tables would require the use of both an x and a y variable. An Algebra 2 student in the FBA-MC group states, "I would use a calculator but you have to have a y. We are solving for x and it has nothing to do with y."

No y: Each student response is coded as one instance in the category Why FBA strategies are not useful.

An Algebra 1 student in the Non-FBA group started to use the graphing calculator to show how the table would be useful. However, the student ended up creating a table and sketching a graph from that table of data. This student's dialogue and work are shown in Figure 4.31.

No table in problem: This student response was coded as why FBA strategies are not useful. The student comes up with an example of when tables and groups could be useful. This fits with Teacher A's and B's description of exploring graphs when they are introduced. The student only references the pattern from the values in the table. There is no reference to how this is related to the graph or the symbolic representation of the function.

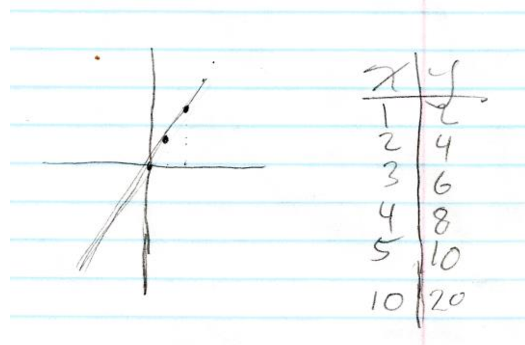
309: If you had a graph it would be very helpful, because just like a calculator you can input those numbers onto a graph or you could use a calculator to do that for you. But with a graph, the numbers are all organized perfectly with Xs and Ys just like what would be needed for a graph. I'm going to use the calculator and I'm going to input this table. I'm not sure how to use calculators very well, but I'll just write these on my paper… if you had a table with x and y and you had x is 1, 2, 3, 4, and 5, those are your Xs and Ys were say… 2, 4, 6, 8, and 10, then you could put those into the graph as coordinates. The x coordinate and y coordinate would correspond with each other, the ones they were next to. The first number on x is 1 and the first number on y is 2. So for x it would be a positive 1, which would be 1 moved to the right and for y, a positive 2, which would be 2 spaces up. Then you continue with this pattern with x as positive 2 and the y as positive 4 to continue the pattern until you find your line, which would be a linear equation.	

Figure 4.31: Student dialogue and work to explain when a table of values would be useful

All four students regardless of group or course, who rated question 6 as *Likely to use* or *Definitely would use* for use with a calculator referred to only one of the functions. Two of the students in the FBA group and a Non-FBA student focused on the pattern created by table of x and y1 values. Each of these students typed the functions Y1 and Y2 in the graphing calculator and then looked at a table of values for those functions. The students recreated the table as shown in question 6. These students neither indicated that the Y2 function was necessary nor provided a reason as to how the Y2 function might be useful. An Algebra 2 student mentions, "They didn't use the x+4 in the question." An Algebra 1 student gives an example of the type of problem that would need both a Y1 and Y2. However, the student is describing a situation involving solving rather than simplifying.

308: Whenever you have a problem when it says there's a fee, you know. You have to pay so much per hour and then there's also an additional fee of like $10 and if it was different for two different stores or something. Then you know y1 would be one store, y2 would be another store and then you'd plug in, you know, x could be how many hours and you'd plug in how many hours you took. And it shows a pattern going down and like how many, like how the cost is to get, you know, I'd rather go here because it's cheaper than to go there.

Another FBA Algebra 2 student gives an example of how a table would but useful but uses a quadratic function, y equal x squared plus 4, to create a table of values. The student states, "The table lets me see which points would be on the line. You have [the points] (0, 4) and (1, 5). You can also see the negatives." When I asked if there were specific points of interest, the student replied, "I can't think of any specific ones, but you can find any point on the graph by using a table."

Of the six students who used pencil and paper to explain the rating of *Definitely would NOT* use or *Unlikely to use* for question 6, students (one from each FBA-MC, FBA-LC, and Non-FBA group) stated that "I was too tired at the end of the survey"; "I don't know how to table"; or "tables confuse me" as reasons for choosing those ratings. The other two students from FBA-MC group, both in Algebra 1, gave responses similar to those from the students who preferred to use a graphing calculator. These students focused on the pattern they could see for the values listed in Y1. Neither of these students could explain why Y2 was part of the question. The one Non-FBA student, an Algebra 2 student, was confused by the fact that the values for Y1 and Y2 were the same for each value of x. The student explains, "It should be two lines because there is a Y1 and a Y2. There's like a comparison you are supposed to make with them. Maybe they are parallel or perpendicular."

FBA useful for systems of equations: The student is referring to solving systems of equations. This coding was one instance of Why FBA is not useful in survey problems.

The Algebra 1 student in the FBA-MC group recognizes that there is an "add 4" pattern for both Y1 and Y2. However this student sees no connection between Y1 and Y2 other than that both have the same pattern.

FBA useful for linear functions: This student is referring to a characteristic of some linear functions. Early in the year students plot points from tables and create rules from patterns in the values from the table.

Two of the students provided explanations that align with a FBA strategy recognizing the equivalence of the two expressions identified in question 6. An Algebra 2 student from the FBA-LC group indicates that the values in the table have to "come from an equation because they aren't just getting the numbers out of nowhere." The student goes on further to say, "the numbers are the same for both Y1 and Y2" When asked if the student would like to sketch what they are thinking since the numbers are the same for Y1 and Y2, the student replies, "This would be like an overlap of both lines."

FBA Strategy: This student's response is coded as an FBA strategy. The student realizes that the graphs of the two expressions are "everywhere the same."

The student plots a point at (0, 4), another point at (1, 5), and then plots the remaining points from the table. The sketch drawn by this student is shown in Figure 4.32.

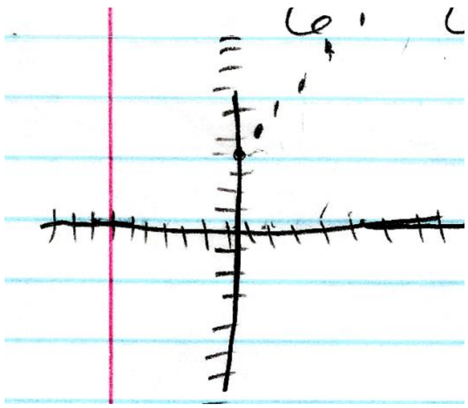


Figure 4.32: Student sketch using the table of values from Question 6.

The Algebra 1 student from the Non-FBA group indicated that Y2 was a simplified form of Y1.

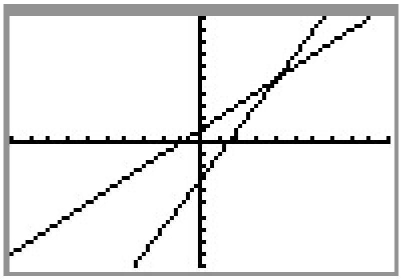
> 310: Well, you can see that like for each x term both y terms, Y1 and Y2 are the same, so you can assume because it's for all of the x's and not just one that since both y terms are the same for all of them that the equations must be equal.
>
> P: So how does that help you in simplifying the expression?
>
> 310: Because Y2 is simpler than Y1. You can assume… like because they are exactly the same you can assume that Y1 is Y2, which is simpler than what Y1 originally was.

FBA Strategy: This student's response is coded as an FBA strategy. The student realizes that Y2 is the simplified form of Y1 because the table of values is exactly the same.

Student and teacher rating for useful ness of a specific FBA strategy for solving

Question 7 provided an example of a FBA strategy using a graph of two functions which is useful in allowing a student to determine if they have correctly solved an equation. The question from the survey is shown in Figure 4.33.

Use the graph below to answer Question 7. In the graph, one line is the graph of
Y1=2x-3 and the other line is the graph of Y2=x+1



7. Rate the usefulness of the graph as a strategy for correctly solving the equation 2x-3=x+1

Figure 4.33: Survey Question 7

The survey results for the Algebra 1 and 2 teachers (n=22), Algebra 1 students (n=48), and Algebra 2 students (n=52) are shown in Figure 4.34. The teacher percentage for rating the strategy as useful (81 %) is higher than both of the student group percentages. However, the student course percentages (65% for Algebra 1 and 55% for Algebra 2) for rating the strategy as useful are higher than any of the course percentages for FBA strategies for questions 2 through 5 on the survey.

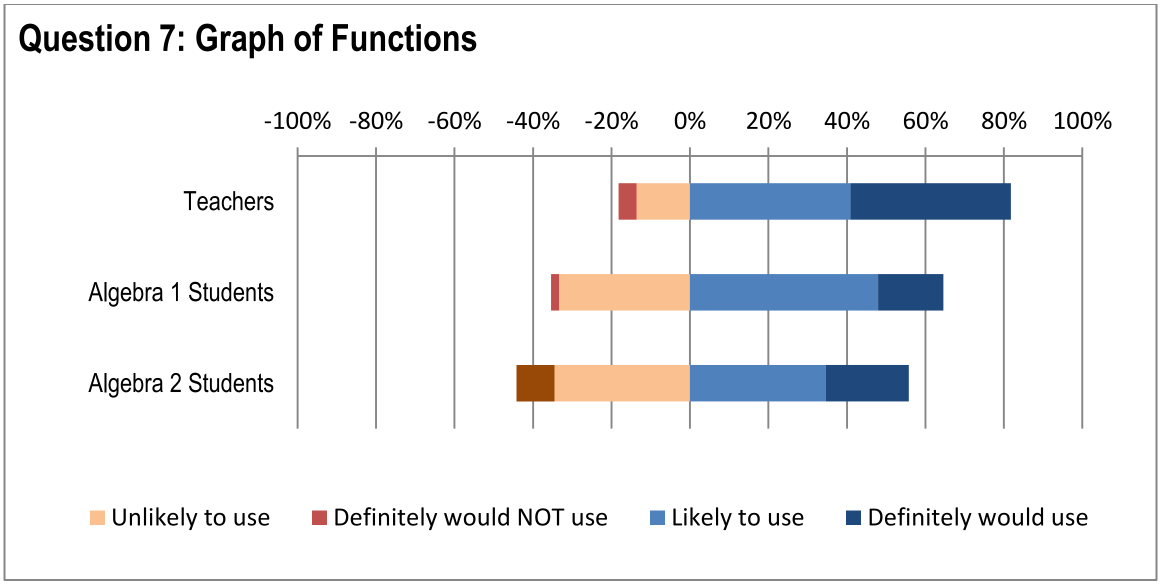


Figure 4.34: Percent response for teachers and student course groups for survey question 7

Although the 12 students who were asked about the rating selection for question are split on usefulness ratings, the survey shows a slightly greater percentage by course groups overall.

The two FBA students, both in Algebra 1, who rated this strategy *Unlikely to use* provided examples of when graphs might be useful in solving equations with a graphing calculator. The FBA-MC student describes using the graphing calculator to solve systems of equations by graphing to see where the two lines intersect. The FBA-LC student mentions parabolas. The student types the equation x^2+3x-5 in Y1 and views the graph of the function. This student points to the y intercept and states, "It crosses at negative 5 just like what was in my equation."

Of the eight students who indicated this strategy would be useful if graphing by hand, five changed their rating to *Unlikely to use*. Four of these students felt it was easier to just solve by hand. The fifth student used the graph paper to graph the two equations to verify that the point of intersection was (4, 5) but then indicated that the strategy wasn't useful for this question. Two other students could not explain their reason for rating the strategy as useful. Only one student, an Algebra 2 student in the FBA-MC group, mentions that the point of intersection of the two lines is the solution to the equation. However, when asked the meaning of the point (4, 5) the student couldn't identify what each value represented or how this point was related to the equation to be solved.

FBA Strategy: This student's response was coded as an FBA strategy because the student realized that the solution to the equation in question 7 was the intersection of the two lines.

The two Algebra 1 students who rated the strategy as *Unlikely to use* when working with pencil or pen and paper were able to provide reasons for the rating. The FBA-LC student indicated that a better example would be one in which you would be solving system of equations such as 2x-4y=12 and x=9. This student solves the system by hand to find the point (9, 3/2) and then pointed to where this point would be on the graph from question 7.

Systems of Equations: This student's response was coded under the category Why FBA is not useful. The student doesn't see the equation in question 7 as a comparison of two functions.

The Non-FBA student indicated that it was too difficult to see the actual point of intersection just by looking at the graph saying, "It's not very precise on where it crosses and it gets pixelated and similar in the middle. You could estimate where it is they cross exactly, but it's a lot more precise to just simplify the equation than looking at the graph."

Equals: The student realizes that the equation is a comparison of two functions but is unable to find a precise value for the intersection. If the graph had been enlarged the student may have been able to distinguish the solution.

Comparative Results from Data Analyses

Based on the coding described in Chapter 3, Table 4.5 shows overall results from my data analyses including the number of student interviews, comments coded, and the occurrences of symbolic manipulation, errors, confounding, equivalence, equals , instances in which students were unsure of the meaning of the strategy, examples of use of FBA strategies, and explanations as to why FBA strategies were not useful. The table serves to illustrate the scope of my efforts to examine student responses for evidence of understanding of equivalence or equals as opposed to symbolic manipulation. The data is from the 18 student interviews segregated by FBA preference. Table 4.5 also compares the incidence of student comments in general to the incidence of the areas of concern that are the focus of this study.

Table 4.5: Instances of student support of survey ratings and work from interviews

			Symbolic Manipulation		Errors in work		Confound rules		Equivalence (everywhere the same)	Equals (a type of comparison of functions)	Unsure of strategy or I Don't Know	example of use of FBA strategy	Explanations as to why FBA strategies are not useful		
	Student Interview (n)	Coded Student comments	Simplify	Solve	Simplify	Solve	Simplify	Solve					No y	No graph or table in problem	useful only for linear and quadratic
FBA-MC	7	231	33	43	2	2	22	4	2	5	4	2	1	6	6
FBA-LC	5	102	17	18	2	2	12	3	2	2	5	0	3	3	3
Non-FBA	6	176	19	27	1	2	8	5	2	4	6	0	1	3	2
Total	18	509	69	88	5	6	42	12	6	11	15	2	5	12	11

Chapter 5 Discussion

SUMMARY

This study was designed to explore student preference for the use of teacher-provided strategies for simplifying expressions and solving equations. Exploration of these preferences was used to identify students who were confounding strategies for simplifying and solving as compared to Algebra 1 and 2 teachers. Student groups were identified according to course (Algebra 1 or Algebra 2) and FBA preference, FBA or Non-FBA. Students identified as FBA were further separated as most confounding (FBA-MC) or least confounding (FBA-LC). Student interviews were used to identify patterns between and among course groups as well as FBA preference groups. Teacher surveys were used to identify possible reasons for student preferences.

QUESTION 1

The first research question guiding this investigation was does selecting the strategy "examine graphs or tables" either with a calculator/computer or pencil/pen and paper as *definitely would use* or *likely to use* relate to students selecting strategies that differentiate between equivalence and equals?

Once students were identified as FBA or Non-FBA, I determined if the FBA group members were selecting strategies that differentiate between "equivalence (everywhere the same) and equals (one type of comparison of functions)" (Stroup, n.d.).

The results of the survey showed that the FBA group contained more students who were confounding the teacher-provided strategies than the Non-FBA group when compared to the results of the teacher surveys. Only four of the 47 Non-FBA students would be considered as 'most confounding' compared to 27 students in the FBA group. There was more confounding for solving equations than simplifying expressions. Students did not recognize potentially helpful strategies such as do the opposite, multiply both terms by a number or variable, and substitute a number for x as useful in solving equations.

Texas' implementation and definition of FBA in public schools provides insight into how these results may have occurred. Due to statewide professional development focusing on the use of nearly all multiple representations for each instance of problem solving (Dana Center, 2001) might have led algebra teachers to see this as an operating definition of FBA. as well as modeling being used only to motivate weaker students (Stroup, n.d.), one could conclude that a complete FBA (as later defined by Stroup) was not taught to the high school students being studied. This would result in students not selecting FBA strategies as useful.

It is also possible that student preference for examining tables and graphs may not be related to FBA ideas at all. Students may rate these strategies as useful for reasons that have nothing to do with understanding equivalence and equality. The survey used forced choice, 4-Point Likert items. This was done to prevent a high percentage of students opting out for a neutral position on usefulness (Schuman & Presser, 1981). However, it is possible that students' responses are skewed towards a "more useful" rating. One of the benefits of interviewing students was to determine if students' ratings would change once they worked through the problems. This complementarity model of triangulation "produces a fuller picture of the empirical domain under study which would not be the case if only one single method were applied." (Erzberger & Kelle, 2003, pp. 469-470). The interviews provided insight into how students' made decisions about ratings on the survey. It also allowed me to compare student and teacher explanations of the same data.

Almost four times as many Algebra 1 students as Algebra 2 students rated the inappropriate strategy of applying the distributive property as useful for Question 2. An overwhelming percentage of Algebra 2 students rated the strategy as *Definitely would NOT use*. This is consistent with intensity results for students who are more familiar with the content. Teacher A mentioned that one expected difference between course groups would be that the Algebra 2 results would be "more crystallized." Teacher A indicated that this meant that the Algebra 2 students would have higher percentages of students selecting the polar positions. Survey data supports this observation. Unfortunately, this was true even when the students confounded strategies such as apply the distributive property and substitute a number for x for Question 2 and do the opposite for Question 4. When grading assignments it's not difficult to determine that a student used an inappropriate strategy or confounded a strategy for simplifying and/or solving. But knowing that a student is thinking about an inappropriate strategy being useful is more difficult to determine. Teachers should listen to what their students are saying. Schwartz (1989) explained:

> As a direct consequence of the realization that mathematics is not a spectator sport, roles in classrooms must and do shift. It is no longer possible for teachers to serve as ex cathedra authorities, nor is the authority of the text beyond question. Teachers and students must and do learn to listen carefully to and assess the quality of one another's arguments. Furthermore, teachers who work with their students and their subject matter in a way that allows students to explore their own understanding often come to think of themselves as people who continue to learn the subject they are teaching. (p. 57)

If Teacher B was aware of the benefits of a complete FBA it is likely that he would have been able to answer this question, "How do you know when students are thinking that an inappropriate strategy is useful? Teacher B commented, "We only talk about a property like distributive property when it is there like for -7(x+2). I don't know when they are thinking distribute when it's not the right method. I don't see a way to find out if they are thinking that way." This teacher might say, "I know when this is happening because students in my class are encouraged to reflect on and communicate mathematical understanding."

Students from both the FBA and Non-FBA groups used symbolic manipulations to simplify and solve. From observations and interviews it is apparent that both Teacher A and B follow a symbolic-precedence view of Algebra development. Nathan and

Koedinger (2000) and Nathan and Petrosino (2003) stated that teachers with a symbol precedence view of Algebra development believe that students are better able to work problems symbolically.

Low percentages of both Algebra 1 and Algebra 2 students rated the FBA strategies as useful. Teachers A and B were pleased with these results. Both teachers mentioned that none of the FBA strategies were useful and students should not rate them as *Likely to use* and especially not *Definitely would use* for problems involving simplifying expressions. Neither teacher could understand why students chose to explore graphs or tables using pen or pencil and paper. Teacher B stated, “We spend a lot of time showing students how to use the graphing calculator. I can’t imagine why anyone would rate those strategies as useful for any of the problems.” When asked about the tables in Question 6, Teacher B further stated, “That would *never* be something that I would try to show them. I feel like combining like terms is such a simple concept that there’s not really a need for the calculator.” Dewey, Kinzel, and Singletary (2009) indicated that teachers function under the belief that students must be able to learn by doing symbolically by hand first. Teacher A and B were observed to teach according to this belief and supported that observation with comments indicating that the FBA strategies from the survey were NOT useful.

Limitations

The researcher created survey was validated by STEM faculty, graduate students, and current Algebra teachers in the field. Due to the need for observation of teachers in the classroom, it is unlikely that preferences for the specific teacher –provided strategies could be generalized to other campuses and/or schools. Teachers who completed the survey commented on different wording for the strategies such as "add a zero pair" rather than do the opposite. However, the examples of FBA strategies could be used with all campuses. Examining graphs and tables to assist with understanding of strategies for simplifying and solving are one feature of FBA (Stroup, n.d.).

I observed the two algebra teachers noting when and how they were using each strategy for each class. I believed that the students would be familiar with all of the strategies based on my observations. After exploring the survey data in this study, I believe that an additional rating option of "I don't understand this strategy" should be included with each strategy on the survey. This would allow students to rate usefulness only for the strategies with which they were familiar. The survey provided a description of students' preferences but these preferences needed verification. It was not feasible to interview all 100 students, so I interviewed a sample, including students from each group, to explore student reasoning used to support ratings.

Additional Research

Extended time observing teachers and students in the algebra classrooms could be used to more accurately represent the strategies that the teachers use with students when simplifying expressions and solving equations. Surveying additional students from other campuses and possibly other districts might provide insight as to whether the confounding of strategies occurs for different populations. Texas mandated the use of graphing calculators for use on the Grade 8 STAAR Mathematics exam (TEA, 2014). Teachers certified to teach Algebra 1 to eighth grade students for high school credit may have the following certifications: Elementary Mathematics (Grades 1-8) and Mathematics: Grades 4-8.(TEA, 2013) It is unlikely that these certifications include requirements related to FBA or the effective use of graphing calculators. Research targeted for grades 6-8 as teachers begin teaching the algebraic reasoning standards new to those grades might allow for the development of professional development to address the potential confounding of simplifying and solving rules apparent in this study.

QUESTION 2

The second question explored in this study was how does the student usefulness rating of teacher-provided strategies relate to student understanding of equivalence and equals? Although the survey indicated two distinct groups of students according to preference for FBA strategies, interviews were necessary to provide further insight into student understanding of what Stroup (n.d.) referred to as equivalence (everywhere the same) and equals (as one kind of comparison of functions) .

Texas required that graphing calculators be an integral part of the Algebra TEKS since 1996 (TEA, 2002). However, many teachers in Texas public schools have not used the graphing calculators other than to show multiple representations of linear and quadratic functions in Algebra 1. Algebra 2 teachers are required to explore and determine characteristics of more complex functions such as higher order polynomials, square root, logarithms and power functions. I expected the Algebra 2 students who used graphing calculators to explore transformations of more complex functions to be more likely to refer to graphical or tabular representations to support strategy ratings and methods for simplifying and solving than Algebra 1 students. The Algebra 2 group provided similar explanations to the Algebra 1 group regardless of FBA group. All but two students worked the problems by hand although students were given a graphing calculator for use during the interview. The two students who tried to use the calculator were unable to figure out how to use it to get a solution. Students showed more confounding of solving strategies than simplifying strategies. This might be due to the fact that more of the teacher-provided strategies were related to solving. Students frequently did not see themselves as using a particular strategy. However, when asked, many students could identify when a particular strategy was used (or how someone might have used that strategy even if that student did not think they used the strategy). Students had more difficulty correctly simplifying expressions than solving equations. Furthermore, Algebra 2 students had more difficulty simplifying than did Algebra 1 students.

With few exceptions, regardless of course or FBA preference, students supported strategy ratings with the following comments:

- Put the ones with x's together and the ones that are just numbers together
- That's what we were taught
- You can't combine variables and numbers because they aren't compatible
- You have parentheses so you have to multiply.
- I combined them so [the expression] is more simpler (or shorter).
- 14 and -7x wouldn't combine, it would just be something weird
- PEMDAS!
- You can't graph this. There is no equation.
- You have to get the x all by itself on one side of the equals sign
- Move them over to the other side and change the sign (it cleans up the equation)
- I'm subtracting x to simplify the equation.
- It's easier to see what to do next with fewer terms
- If you don't do it to both sides it doesn't work I guess.
- I'd put this one in the calculator. I don't know how to do that with the equals in the middle.
- I'm not good at explaining this. I don't know.

Students were unaware of how tables and graphs could be used to support the strategies much less provide a way to know if the simplified expression was the same as the original expression. Stroup (2009) stated that students have difficulties as they move beyond elementary mathematics to Algebra. These include " 'x' moves from unknown to variable" and "Order of operation issues appear e.g.,('PEMDAS' – Do you 'do' the parenthesis first or the multiplication?)." Based on student responses, it is apparent that these students are having issues with these transitions. These issues are compounded by the fact that instructors are following a symbol precedence view as well as focusing on graphs and tables only to show multiple representations. Teacher A and B stated, "We use the graphing calculator to explore characteristics of new functions as we encounter them." Teacher A specifically mentioned, "We do growth and decay and quadratics in Algebra 1. In Algebra 2 we have several more complicated functions. We use pencil and paper on day one and then follow up with the calculator."

Although some students indicated that they could use substitution to check the simplification of the expression for Questions 2 and 3, only one student used both the original expression and the simplified expression to do so. This student was extremely surprised that this strategy would work even when checking with just one value of x. Many of the students had no way to check their work. One Algebra 2 student from the FBA-LC group, noted, "This would be like an overlap of both lines maybe." This student appears to have a developing conception of FBA as described by Stroup (n.d.). Stroup provided an example of how graphs and tables could be used to help students verify that the simplified form of an expression is equivalent to the original expression.

> If the expression x + x + 3 is *equivalent* to the expression 2x + 3, then the function f(x) = x + x + 3 and the (simplified) function g(x) = 2x + 3, when assigned to Y1 and Y2 on the calculator, will have graphs that are everywhere coincident. They will also have paired values in the tables that are, for any values in the domain, the same. Students will say "the graphs" are "on top of each other."

> Even though in symbolic form the functions Y1 = x + x + 3 and Y2 = 2x + 3 may not look the same; the multiple representations available on the graphing calculator allow students to confirm that the graphs and tables are, in fact, everywhere the same.

Most of the students appearing to perceive a FBA as useful changed their ratings to less useful when asked to explain how these strategies were useful in Questions 2, 3, and 6, which involved simplifying an expression.

Student explanations for a table of values being useful referred to looking for patterns in Y1 with little to no regard to Y2. Some students noticed the constant rate of change for question 6 from the survey saying, "It goes up by 4 each time. It's same for Y2." Some were confused that Y2 was a part of the FBA strategy. These students indicated that Y2 wasn't necessary. One of the Algebra 2 students from the FBA-MC commented, "They didn't include the x+4 [Y2] in the question." Other students focused on the values from the table showing that when the values of *x* were substituted in the functions Y1 and Y2 you would end up with the values in the Y1 and Y2 lists.

Students, regardless of course, had much greater percentages for graphs being useful for Question 7 (approximately 55%) as opposed to Questions 4 and 5 (less than 20%). Even the Non-FBA group (54%) perceived the strategy used in Question 7 as useful. Several students mentioned solving systems of equations. This is not surprising because students learned about this topic prior to the interviews. An Algebra 1 student from the FBA-MC group actually solved the equation as a system by graphing and came up with the coordinate for the intersection. However, the student maintained that the graph of intersecting lines was not useful. This student provided examples where this strategy would be useful, “We tried to tell how much money like comparing costs, how much the economy grows or how water goes down or stuff like that.” The student did not see the original equation as a comparison of functions. Other students mentioned that the graphs would be useful to “find the slope of a line”; “find points where it’s crossing [y-axis]”; or “bar graphs relating distance and time.”

An Algebra 2 student from the FBA-MC group mentioned, “My class has been doing ‘wave’ graphs.” These “wave” graphs are what Teacher A used to describe the solution to rational inequalities. The Algebra 2 classes had been studying this topic close to the time the interviews were being completed. The sample problem $\frac{x+3}{x-1} > 0$ was demonstrated by Teacher B. A sketch of the problem solution is shown in Figure 5.1. The teacher required the students to find critical points and mark those on a number line being careful to use an open or closed circle for each critical point. “The critical point for the denominator is 1. Put a star because the denominator is special. No matter what, the denominator critical point will be an open circle. Can the denominator equal zero?” One student replied, “No.” The teacher continued,” So we can’t close the circle.” The teacher then identified the critical point -3 from the numerator.

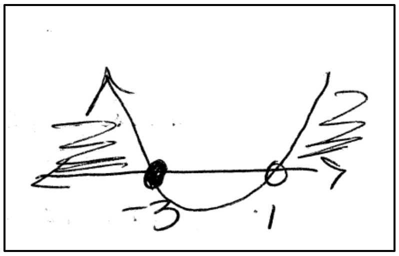


Figure 5.1: Sketch of Teacher B's solution to rational inequality.

Students were told, "In a wave, we always start from the right. You have +, + so we start at the top. So there's no bouncing. You always shade between the curve and the zero line. So it is above or below." A student asked, "Do we shade above?" The teacher replied, "Yes, shade above." There was no y-axis indicated on the graphical representation of the solution to this inequality even though students were to shade in the areas above or below the number line. There was no other form shown for the solution except for the sketch in Figure 5.1. There was no discussion about either the graphical or tabular representations of the function $f(x) = \frac{x+3}{x-1}$ to assist students in understanding why the solution might make sense. A graph of f(x) using a graphing handheld can be seen in Figure 5.2. A corresponding table of values can be seen in Figure 5.3. Although the teacher indicated that the value of x=-3 should be included in the solution set, the table of values shows that f(3)=0 and the inequality asks for values of x for which f(x) > 0. The table does indicate that the value of the function at x=1 is undefined. Students could explore the graph of the function as well as the table of values for the function to determine the solution set for the rational inequality.

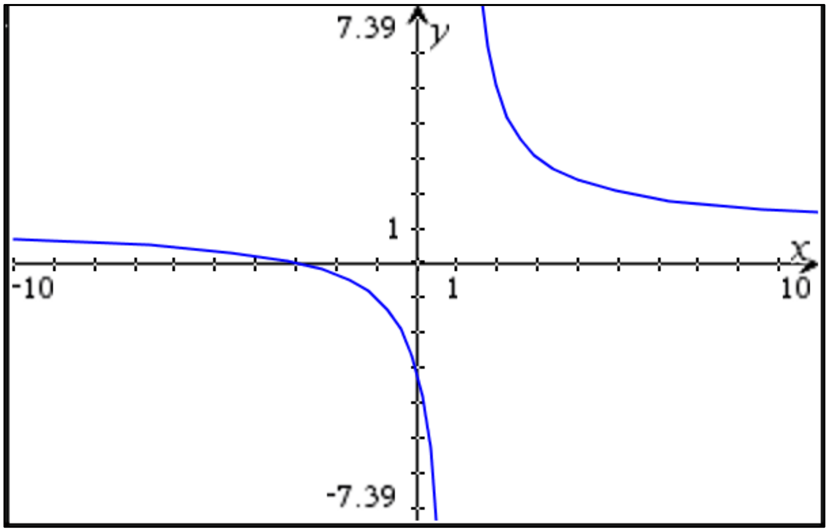


Figure 5.2: Graph of $f(x) = \frac{x+3}{x-1}$

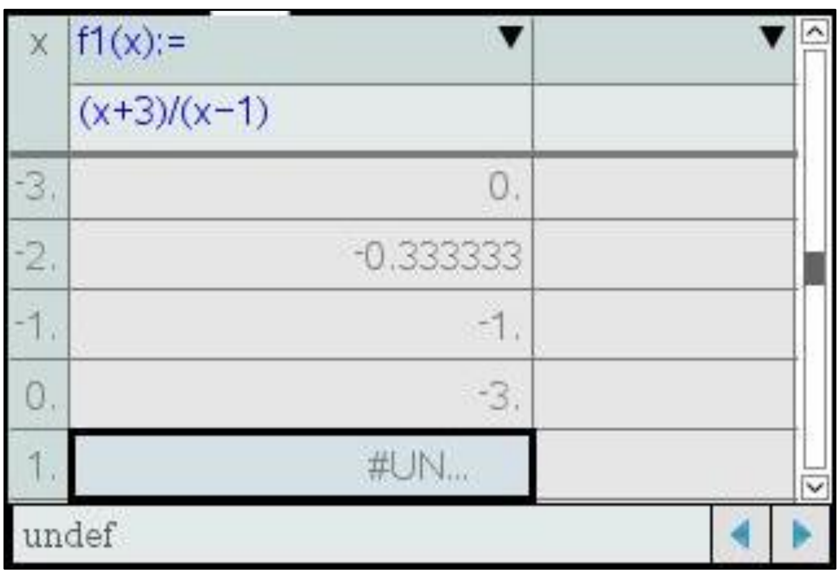


Figure 5.3: Table of values for $f(x) = \frac{x+3}{x-1}$

The students did not use the graphing calculator during the lesson. The graph on the number line was actually represented as a quadratic. This could be extremely confusing if students were trying to make connections to the quadratic functions they had explored earlier in the year. This type of sketch also provides no information as to why one critical point is open and the other is closed. The use of the "wave" strategy also suggests confusion of $f(x) = (x+3) * (x-1)$and $f(x) = \frac{x+3}{x-1}$ which is a type of "confounding" on the part of the teacher (but not one of the forms discussed herein).

This lesson would fit Skemp's (2006) definition of instrumental understanding, "It is what I have in the past described as 'rules without reasons', without realizing that for many *pupils and their teachers* the possession of such a rule, and the ability to use it, was what they meant by 'understanding' " (p. 89).

Although the wave graphs that the Algebra 2 student referred to were not linear, the situation this student described is similar to one described by Schwartz and Yerushalmy (1991) in which:

> Equations, inequalities and identities involving functions in one variable are all comparisons of two functions of the form F(x) compared via < or = or > to G(x). They can all be represented graphically by plotting the left and right sides of the comparison on the same set of axes. The solution set, if any of the comparison is then immediately evident. Moreover, in such an environment, it is possible to explore easily operations that may be carried out on the symbolic representation of these two functions being compared. … some of these operations will preserve the solution set and some will not. (p.14)

This type of exploration of functions is more easily done with the graphing calculator available to all students on a daily basis in Algebra class. Calculators can be an effective learning tool. It all depends on how they are used in the Algebra classroom. The effective use of technology in the mathematics classroom depends on the teacher.

> Technology is not a panacea. As with any teaching tool, it can be used well or poorly. Teachers should use technology to enhance their students' learning opportunities by selecting or creating mathematical tasks that take advantage of what technology can do efficiently and well—graphing, visualizing, and computing.
>
> Technology can help teachers connect the development of skills and procedures to the more general development of mathematical understanding. As some skills that were once considered essential are rendered less necessary by technological tools, students can be asked to work at higher levels of generalization or abstraction. (NCTM, 2000, pp. 25-26)

I did not observe Teacher A and B using the graphing calculator to connect the symbolic manipulation to changes in either graphical or numerical changes. These teachers appeared to subscribe to a philosophy in direct opposition to that of Dugdale (1993):

> The easy manipulation of graphical representations allowed by current function-plotting tools has raised the possibility of visual representations of functions playing a more important role in mathematical reasoning, investigation, and argument. Relationships among functions can be readily observed, conjectures can be made and tested, and reasoning can be refined through graphical investigation. (p 115)

Teacher A and B both referred to learning "tricks" to the graphing calculator. When asked "Where did you learn how to use the graphing calculator?" Teacher B stated:

> Some while I was in college. At UTeach we did a lot. We learned a lot of tricks and tips in classes such as Functions and Modeling. … I can't come up with the names now, but we learned a lot of tricks and whenever I started teaching here, a lot of the other teachers had more tricks to show us. I feel like I have learned most all of the tricks and behind the scenes things to give the kids.

This opinion was shared by one of the surveyed teachers who commented in response to Question 7, "This would not be my first choice because I want my students to be able to solve. Using a graphing calculator is a "short cut" trick and strategy to be used before state testing in my opinion." This teacher had a BS in Engineering.

Compare this to the statement from a LEP and SpEd teacher, "This is a good visual tool for helping students understand the connection between the algebra and geometry components of equations. It is a crucial component of helping students understand that the solution is the where the two lines intersect." This teacher appeared to agree with Skemp's (2006) definition of relational understanding "knowing not only what method worked but why." This type of understanding would possibly enable students to "to adapt the method to a new problem."

Limitations

My student sample was small (*n* = 18) and from a targeted population. I only surveyed and interviewed the students from two teachers teaching at a campus that hired 20 teachers in the math department. The results of this study would not be generalizable to a larger population. I would be unable to indicate what other Algebra 1 or Algebra 2 students might provide as explanations at other campuses within this urban district much less throughout the state of Texas. This study was an exploratory study used to describe patterns among and between courses and FBA groups. I uncovered how various students understand the usefulness of both teacher-provided strategies and FBA strategies.

Additional Research

In this study, I was the only interviewer. To validate coding constructs, additional interviewers would be needed. This would also allow for inter-rater reliability for class room observations. This would also address issues of researcher bias.

IMPLICATIONS AND CONCLUSIONS

Implications

Although this study was narrow in scope, limited to 100 Algebra 1 and Algebra 2 students from only two teachers at the campus over one semester, I believe that the findings from this research have important implications. A FBA is consistent with not only the NCTM standards vision of curriculum and methodology reform but with the more recent Curriculum Focal Points for grades K-8. In response to these standards, Texas revised the TEKS for mathematics in response to curriculum focal points as well as to meet career and college readiness standards. Algebraic reasoning TEKS have been moved to grades 6-8. Additionally the state is requiring the use of graphing calculators on the 8th grade mathematics STAAR. This study adds to the body of knowledge about student preference and use of both teacher-provided strategies and FBA strategies. In addition, the study revealed teacher preference for FBA strategies and calculator usage in general for Algebra 1 and Algebra 2 students. Both course groups initially showed low percentages of usefulness for FBA strategies for simplifying and solving. When interviewed, students in the FBA group changed those ratings to less useful. This observation as well as the teachers' reluctance to use FBA strategies indicates that these issues may also occur in middle school as the content is moved to lower grades and becomes part of state standardized testing.

Future research

This study showed evidence of confounding of rules for simplifying and solving by Algebra 1 and Algebra 2 students. Student comments indicated an instrumental understanding of this content. Teacher interviews and surveys indicated a preference for a symbolic approach and a minimized role for the graphing calculator. Further research is needed determine the issues underlying these preferences in learning and teaching. This research should provide a three prong approach that includes Pre-Service Teachers, Induction, and teacher professional development. This collaboration is outlined in Figure 5.2. The collaboration includes university faculty from the teacher preparation program and the mathematics department, district curriculum personnel, campus administrative personnel, Mathematics Chair of Departments at campuses, pre-service teachers at the university, and students at local district.

Pre-Service Teachers

The local university graduates approximately 80 secondary STEM teachers each year. There have been over 800 graduates of this certification program since its beginning in 1997. According to data from 2003 through 2007, over half of those graduates were certified mathematics teachers. These graduates secured positions in middle schools (grades 6-8) as well as high schools. Courses required by UTeach graduates for certification included Foundations, Functions & Regression Models (formerly Functions and Modeling), and Classroom Interactions.(UTeach, n.d.) Research addressing why graduates such as Teachers A and B chose not to use methods learned from those classes in their Algebra classes might provide insight into how the Pre-Service curriculum components impact teaching decisions.

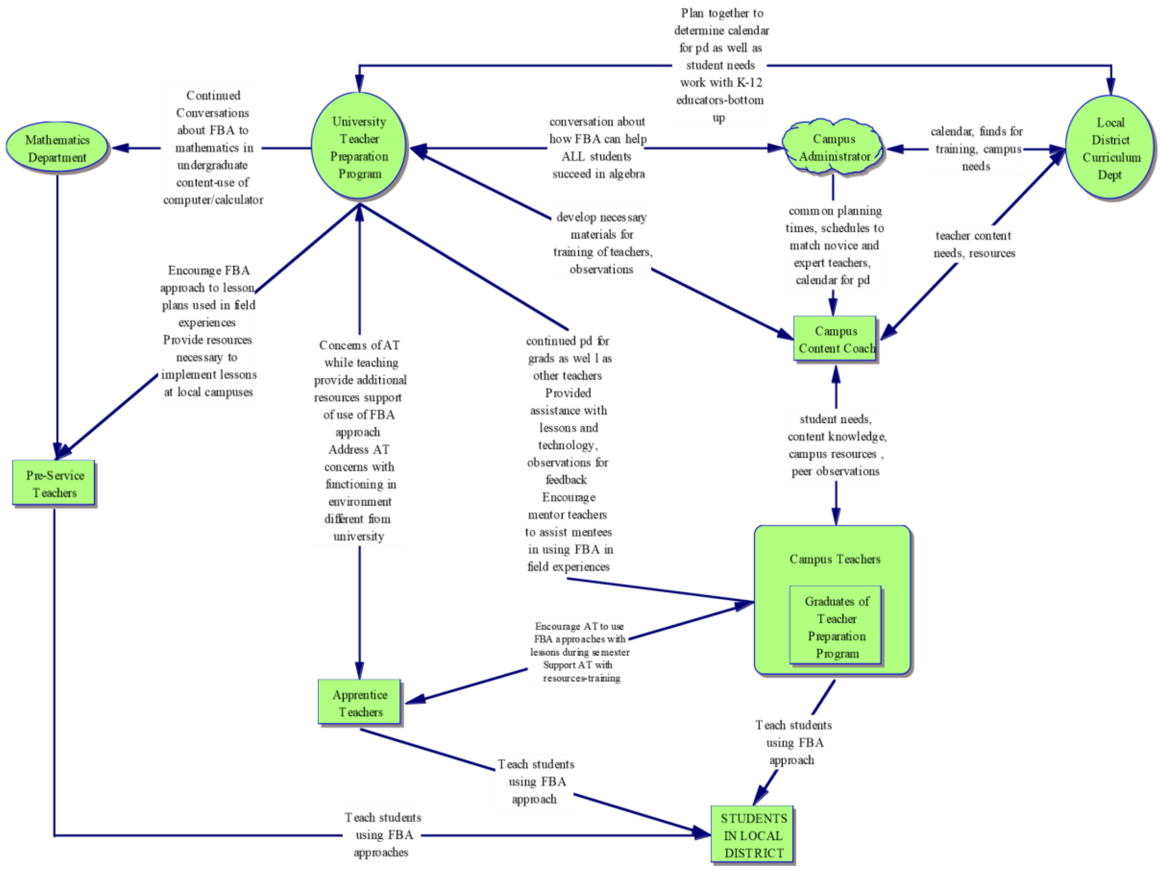


Figure 5.4: Outline of possible collaboration between the university and local school district.

Induction Teachers

Teachers A and B had completed 5 years or less in the classroom. Although these teachers received induction support as novice teachers, not all graduates of certification programs receive such support. Research is needed to determine how best to support novice teachers as they use FBA in their mathematics classrooms. Research could also provide information on why induction teachers such as Teachers A and B focus on a symbolic approach and relegate the use of the graphing calculator to exploring characteristics of new functions as they are introduced.

Professional Development

The district provides curricular materials to all campuses. These materials are used at each campus to teach content. During the classroom observations, both Teacher A and B used these materials. Collaboration between university faculty and the Math department chairs, campus administrative personnel for the various campuses in the district and district curriculum personnel could provide an arena for the development of curricular materials that would support a FBA for all mathematics courses. Professional development could be designed to help teachers implement a complete FBA. This type of collaboration embraces the commitments of TEXTEAMS but uses a different, more explicit model for engaging teachers in FBA in a way informed by the findings of this investigation.

Conclusions

The statement "Algebra for All" has come under greater scrutiny with the most recent changes in degree programs in Texas. Students not pursuing a STEM endorsement or Distinguished Level of Achievement will only be required to complete Algebra 1, Geometry and one other mathematics, not necessarily Algebra 2, to graduate. (TEA, 2014, HB5) The results of this study indicate that even for those students who explore more complicated functions use of strategies related to a FBA is not evident. Students often were unable to explain why a particular strategy was used in terms other than it was what they had been taught.

Many Texas high school students will not be required to take Algebra 2 under the current graduation guidelines. It is imperative that these students be taught using a complete FBA to meet the demands of College and Career Readiness standards as well as TEKS. Students have to be problem solvers and have the ability to work with quantitative information. A FBA has the potential to allow students to develop a deeper relational understanding of Algebra and mathematics in general. There were glimpses of FBA conceptions in a few of the students' responses. Teachers could build on the student discovery of finding a way to determine if two expressions were equivalent as well as student understanding that the graphs of those two expressions "would overlap." The results of this study could help Algebra teachers understand that the current symbolic approach is causing half of their students to confound rules for simplifying and solving, and is posing problems for students' communication of mathematical understanding.

Appendices

APPENDIX A: RECENT HISTORY OF FBA

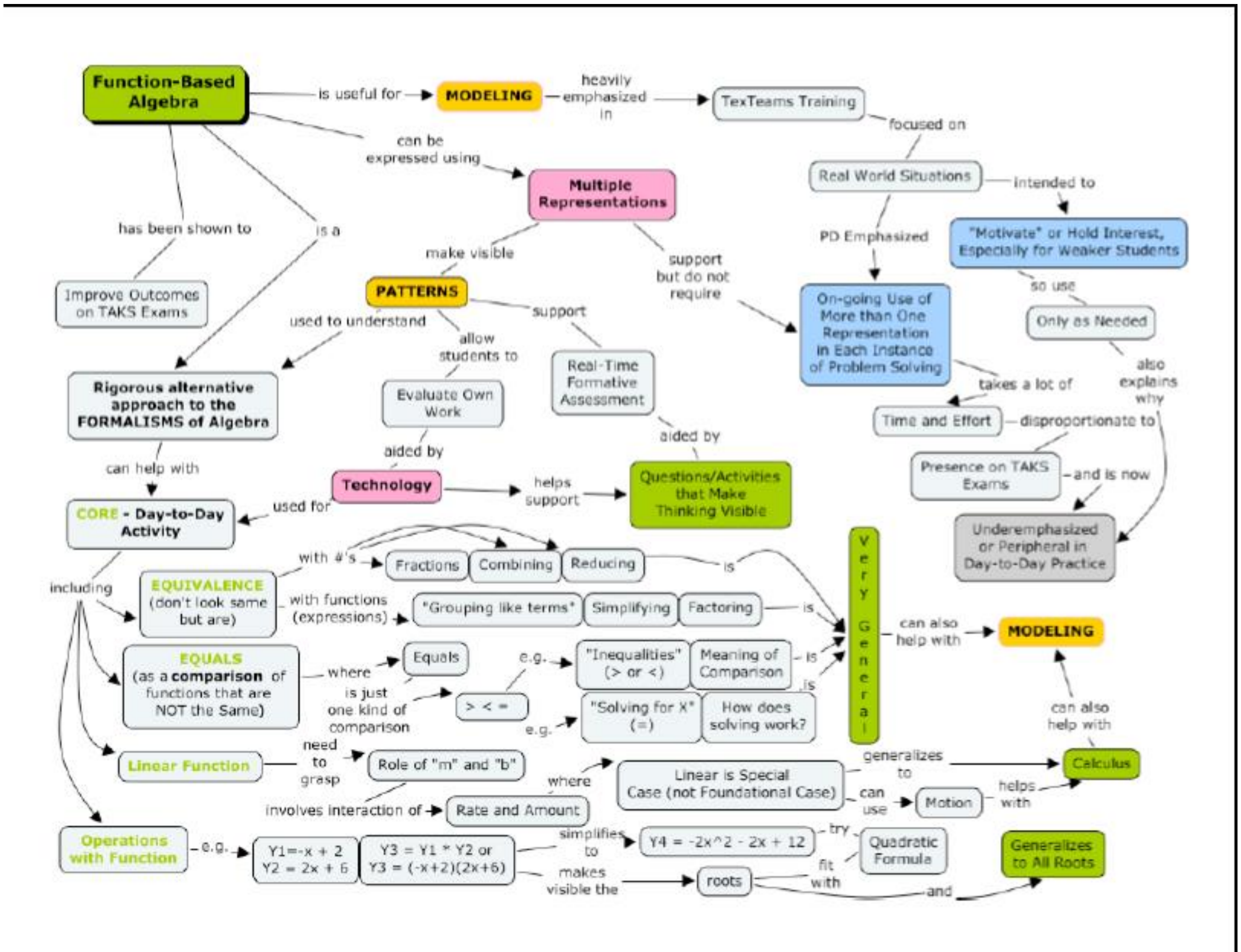


Figure A. 1 Recent History of FBA: Concept Map (Stroup, 2009)

APPENDIX B: 2013 SURVEY FOR DISSERTATION STUDY

Algebra Survey

The purpose of this survey is to have a better understanding of the strategies that are most useful or important to Algebra students when they are simplifying expressions or solving equations. Participants may select more than one strategy as being useful.

Your Algebra teacher has provided a set of problems for your class to complete. Please rate the strategies provided as to how useful you think they would be to a student who correctly simplifies or solves the problem

Algebra Survey

1. Please enter your id number from the scratch sheet you were provided.

2. Rate the following stategies based on how useful the strategy would be in correctly simplifying the expression 3x + 2 - 2x - 1

	Definitely would NOT use the strategy	Unlikely to use this strategy	Likely to use this strategy	Definitely would use this strategy
multiply both terms by a number or variable	○	○	○	○
Apply the distributive property	○	○	○	○
examine tables using a pen or pencil and paper	○	○	○	○
examine graphs using a calculator or computer	○	○	○	○
examine graphs using a pen or pencil and paper	○	○	○	○
Combine like terms	○	○	○	○
Substitute a number for x	○	○	○	○
examine tables of data using a calculator or computer	○	○	○	○
Do the opposite	○	○	○	○
Do the same thing to both sides	○	○	○	○
Cancel	○	○	○	○

3. Rate the following stategies based on how useful the strategy would be in correctly simplifying the expression - 7(x + 2) +4x

	Definitely would NOT use the strategy	Unlikely to use this strategy	Likely to use this strategy	Definitely would use this strategy
multiply both terms by a number or variable	○	○	○	○
Do the same thing to both sides	○	○	○	○
Cancel	○	○	○	○
Do the opposite	○	○	○	○
examine tables using a pen or pencil and paper	○	○	○	○
Apply the distributive property	○	○	○	○
examine tables of data using a calculator or computer	○	○	○	○
Combine like terms	○	○	○	○
examine graphs using a calculator or computer	○	○	○	○
examine graphs using a pen or pencil and paper	○	○	○	○
Substitute a number for x	○	○	○	○

4. Rate the following stategies based on how useful the strategy would be in correctly solving the equation 2x - 5 = x + 4

	Definitely would not use the strategy	Unlikely to use this strategy	Likely to use this strategy	Definitely would use this strategy
Substitute a number for x	○	○	○	○
Do the opposite	○	○	○	○
Cancel	○	○	○	○
Apply the distributive property	○	○	○	○
multiply both terms by a number or variable	○	○	○	○
Do the same thing to both sides	○	○	○	○
examine graphs using a pen or pencil and paper	○	○	○	○
examine tables of data using a calculator or computer	○	○	○	○
examine graphs using a calculator or computer	○	○	○	○
Combine like terms	○	○	○	○
examine tables using a pen or pencil and paper	○	○	○	○

5. Rate the following stategies based on how useful the strategy would be in correctly solving the equation x - 4 - 3x = 3 + 2x + 1

	Definitely would NOT use the strategy	Unlikely to use this strategy	Likely to use this strategy	Definitley would use this strategy
examine tables using a pen or pencil and paper	○	○	○	○
Combine like terms	○	○	○	○
examine graphs using a pen or pencil and paper	○	○	○	○
Do the opposite	○	○	○	○
Substitute a number for x	○	○	○	○
multiply both terms by a number or variable	○	○	○	○
Apply the distributive property	○	○	○	○
examine tables of data using a calculator or computer	○	○	○	○
Do the same thing to both sides	○	○	○	○
examine graphs using a calculator or computer	○	○	○	○
Cancel	○	○	○	○

Use the table below to answer Question 6. In the table, Y1= 2x+3-x+1 and Y2= x+4.

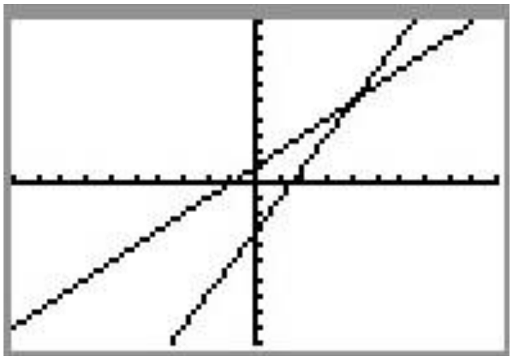
X	Y1	Y2
0	4	4
1	5	5
2	6	6
3	7	7
4	8	8
5	9	9
6	10	10

X=0

6. Rate the usefulness of the table as a strategy in correctly simplifying the expression 2x+3-x+1?

Definitely would NOT use the strategy | Unlikely to use this strategy | Likely to use this strategy | Definitely wou

Use the graph below to answer Question 7. In the graph, one line is the graph of Y1=2x-3 and the other line is the graph of Y2=x+1



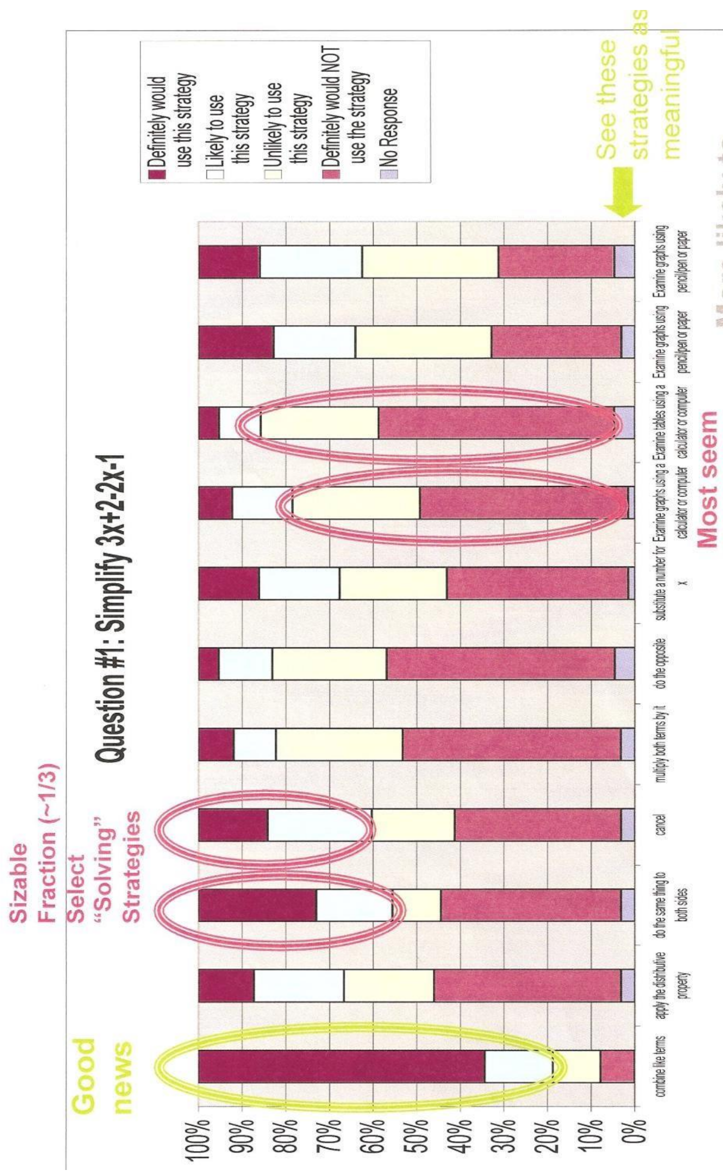
7. Rate the usefulness of the graph as a strategy for correctly solving the equation 2x-3=x+1

Definitely would NOT use the strategy	Unlikely to use this strategy	Likely to use this strategy	Definitely wou

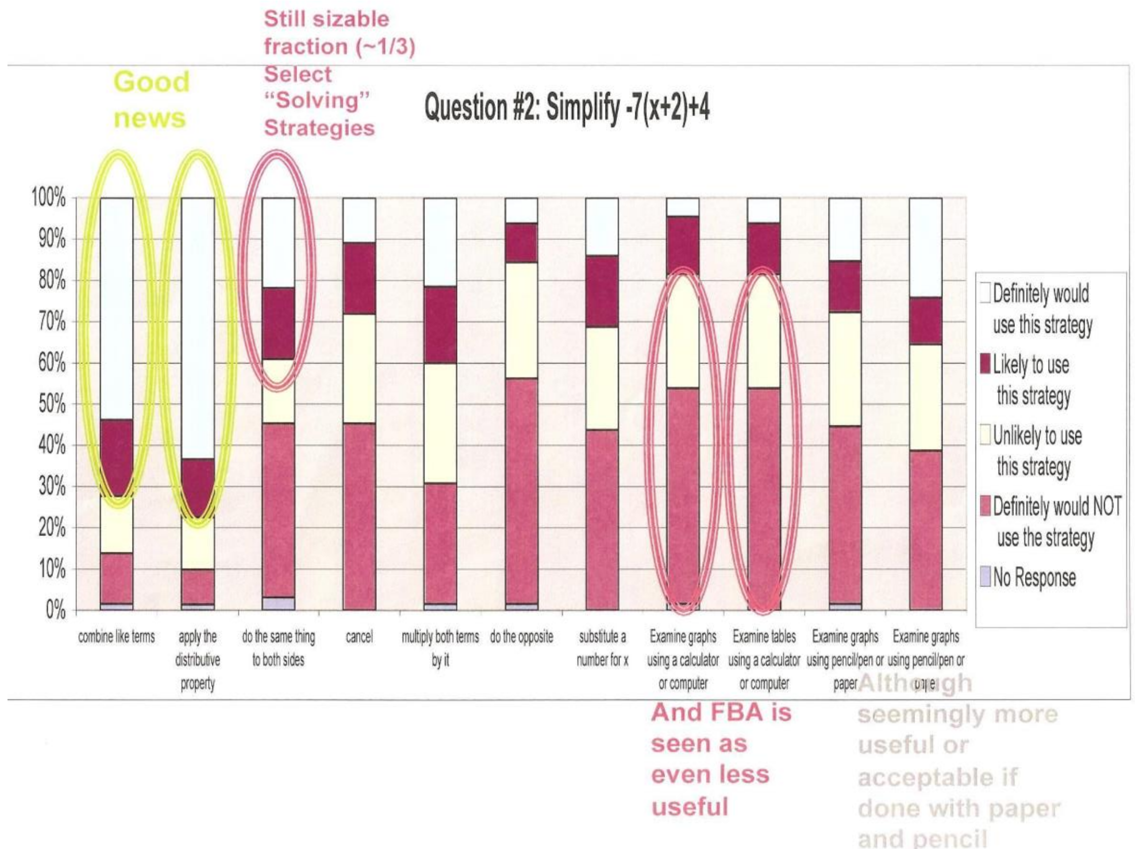
END OF SURVEY

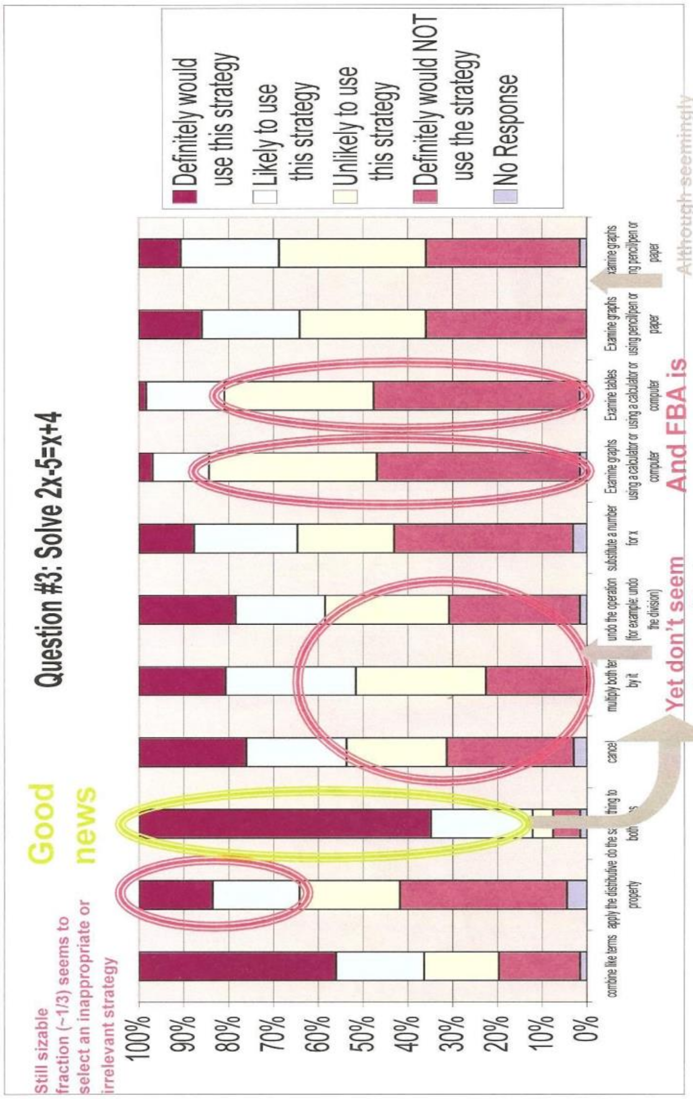
You have completed the survey. Thank you for your participation in this research study.

APPENDIX C: RESULTS OF PILOT STUDY SURVEY



Question #1: Simplify 3x+2-2x-1
Definitely would use this strategy
Likely to use this strategy
Unlikely to use this strategy
Definitely would NOT use the strategy
No Response
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
combine like terms
apply the distributive property
do the same thing to both sides
cancel
multiply both terms by it
do the opposite
substitute a number for x
Examine graphs using a calculator or computer
Examine tables using a calculator or computer
Examine graphs using pencil/pen or paper
Examine graphs using pencil/pen or paper
Good news
Sizable Fraction (~1/3) Select "Solving" Strategies
Most seem unaware of how calculator (FBA) might help
More likely to view paper version as useful
See these strategies as meaningful


Question #2: Simplify -7(x+2)+4
Good news
Still sizable fraction (~1/3) Select "Solving" Strategies
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
combine like terms
apply the distributive property
do the same thing to both sides
cancel
multiply both terms by it
do the opposite
substitute a number for x
Examine graphs using a calculator or computer
Examine tables using a calculator or computer
Examine graphs using pencil/pen or paper
Examine graphs using pencil/pen or
Definitely would use this strategy
Likely to use this strategy
Unlikely to use this strategy
Definitely would NOT use the strategy
No Response
And FBA is seen as even less useful
Although seemingly more useful or acceptable if done with paper and pencil



Question #3: Solve 2x-5=x+4
Definitely would use this strategy
Likely to use this strategy
Unlikely to use this strategy
Definitely would NOT use the strategy
No Response
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
combine like terms
apply the distributive property
cancel
undo the operation (for example: undo the division)
substitute a number for x
Examine graphs using a calculator or computer
Examine tables using a calculator or computer
Examine graphs using pencil/pen or paper
Still sizable fraction (~1/3) seems to select an inappropriate or irrelevant strategy
Good news
Yet don't seem to be as able to connect to potentially relevant sub-strategies
And FBA is EVEN LESS USEFUL for solving than for simplifying
Although seemingly more appropriate if done with paper and pencil


Question #4: Solve x-4-3x=3+2x+1
Definitely would use this strategy
Likely to use this strategy
Unlikely to use this strategy
Definitely would NOT use the strategy
No Response
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
combine like terms
apply the distributive property
cancel
do the opposite
substitute a number for x
Examine graphs using a calculator or computer
Examine tables using a calculator or computer
Examine graphs using pencil/pen or paper
Examine graphs using pencil/pen or paper
Still sizable fraction (~1/3) seems to select an inappropriate or irrelevant strategy
Good news
Again, seemingly not connected to potentially relevant sub-strategies
And FBA not seen as useful
But somehow it is more likely to be useful if you graph or create tables using pencil and paper

APPENDIX D: CONSENT AND ASSENT FORMS FOR STUDENT PARTICIPATION

Study Number: 2012-09-0065
Approval Date: 11/20/12 Expires: 11/19/13

Parent/Guardian Consent Form

The project titled " **Student rating of the usefulness of teacher strategies for simplifying and solving in Algebra: How might this impact student understanding of equals and equivalence?"** is to be conducted at your child's school with the purpose of **determining how students rate the usefulness of strategies provided in class to simplify expressions and solve equations.**

Survey data will be used as a springboard to interviews with students to explore how their understanding of equals (as one kind of comparison of functions) and equivalence might be influenced by these strategies.

If you agree to allow your child to participate in the study, you are agreeing to the conditions listed below with the understanding that you may withdraw your child from the project at any time, and that your child may choose not to answer any questions that he/she does not want to answer.

1. Students will complete an online survey lasting between 60-90 minutes during Algebra class time. Various students will be selected to participate in an audiotaped interview about their survey answers. The interview will last no more than 45 minutes. The interview will take place during Algebra class. The total amount of time in Algebra class devoted to this study will not exceed 2 hours and 15 minutes.

2. The researcher will request individual student data from Austin ISD including student ID number, LEP status, Special Education status and teacher assigned grade for the algebra course. This information will be recorded and analyzed. The research team will use this information **ONLY** to review and analyze survey data separated by these sub-groups; no identifying information will be included with this data.

3. The researcher plans to observe and videotape your child's algebra class approximately ten times for a total of no more than 15 hours. It is possible that the student may be videotaped during the algebra class.

   ☐ YES, the researcher may videotape my child during algebra class.

   ☐ No, the researcher may **NOT** videotape my child during algebra class.

4. The researcher will provide each student participating in the survey a code and password to access the online survey. The record of this information will be kept in a locked filing cabinet in an office protected by an electronic entry system accessible only by high-assurance identification card coded specifically for that office. Only the research team will have access to the survey data and videotapes. Your child's teacher will have access only to whole group survey and interview results. Your child's teacher will not have access to individual student data.

5. The survey data will be presented in percent stacked bar graph form. Student interviews will be coded for phrases related to ideas of equals and equivalence.

No student, teacher, campus or district names will be used. Pseudonyms will be used for students and teachers.

6. There is a minimal risk of loss of confidentiality. The individual student data will be kept in a locked file cabinet in an office protected by an electronic entry system accessible only by high-assurance identity cards coded to that office. Audiotapes will be erased upon completion of the research study.

   ☐ YES, the researcher may audiotape my child during the interview.

   ☐ No, the researcher may **NOT** audiotape my child during the interview.

7. Your consent is optional. Your decision whether or not to allow your child to participate will not prejudice you or your child's present or future relations with The University of Texas at Austin, ☐, or your child's school or teacher. If you decide to let your child participate, your child is free to discontinue participation at any time without prejudice. If your child participates, you can get information about the project and copies of any surveys given to your child by contacting Prudence Cain by phone at 512-232-2697 or by email to pcain@austin.utexas.edu. You may contact The University of Texas at Austin concerning IRB# 2012-09-0065 by calling the IRB Office at (512) 471-8871. As an alternative method of contact, an email may be sent to **orsc@uts.cc.utexas.edu** or a letter sent to IRB Administrator, P.O. Box 7426, Mail Code A3200, Austin, TX 78713.

8. You understand that while this project has been reviewed by ☐ and by the principal at my child's school, ☐ is not conducting project activities.

9. A copy of this signed agreement will remain in your child's permanent school folder.

Your signature below indicates that you have read the information provided and have decided to allow your child to participate in the project titled, **"Student rating of the usefulness of teacher strategies for simplifying and solving in Algebra: How might this impact student understanding of equals and equivalence?"** to be conducted at your child's school.

Name of Student: ______________________________

Signature ____
Parent/Guardian Date

Signature ____
Principal Date

Signature ____
Researcher Date

Study Number: 2012-09-0065
Approval Date: 11/20/12 Expires: 11/19/13

STUDENT ASSENT FORM
How do Algebra students rate the usefulness of the strategies their Algebra teacher uses for simplifying and solving?

I agree to be in a study about simplifying expressions and solving equations. This study was explained to my (mother/father/parents/guardian) and (she/he/they) said that I could be in it. The only people who will know about what I say and do in the study will be the people in charge of the study and my Algebra teacher. My Algebra teacher will not have access to my survey data or interview data. My Algebra teacher will only have access to whole group data.

I understand that the researcher will observe and videotape my Algebra teacher during Algebra class for up to ten days of class. It is possible that I may be videotaped during class as well.

☐ I **will** allow the researcher to videotape me during Algebra class.

☐ I will **NOT** allow the researcher to videotape me during class.

In the study I will be asked to complete a survey that may take up to 90 minutes of my Algebra class. The survey will ask me to tell how useful I think the strategies my teacher uses to simplify expressions and solve equations would be for a student who works problems like ones we see in Algebra class. I may be asked to participate in an interview, which could take up an additional 45 minutes of Algebra class. I will be asked questions about how I decided on the usefulness of the strategies. I may also be asked to work a problem from the survey. I understand that the researcher will audiotape my responses during the interview.

☐ I **will** allow the researcher to audiotape me during the interview.

☐ I will **NOT** allow the researcher to audiotape me during the interview.

It is possible that I could use up to 2 hours and 15 minutes of Algebra class time to complete the survey and participate in the interview for this study.

Writing my name on this page means that the page was read (by me/to me) and that I agree to be in the study. I know what will happen to me. If I decide to quit the study, all I have to do is tell the person in charge. I can say no to being in the study and I can quit at any time. If there are any questions I do not want to answer, I can choose not to answer them. I will not get in trouble with my teacher, school or parents if I do not agree to be in this study.

____________________________ ____________
Child's Signature Date

____________________________ ____________
Signature of Researcher Date

APPENDIX E: ADDITIONAL DEMOGRAPHICS

Teacher Survey Demographics (n=22*)			
Degree	**Years of Experience (Y)**	**Years teaching Algebra 1 or Algebra 2**	**Certification**
9-BA/BS 9-MA/MS 1-ABD 1-PhD	Y < 5 : 3 5≤Y<10 : 3 10≤Y<15 : 8 Y≥ 20 : 6	Y < 5 : 3 5≤Y<10 : 12 10≤Y<15 : 1 Y≥ 20 : 6	Secondary Math : 20 SpEd: 4 ESL : 1

*Two teachers completed the online survey but did not provide all demographic information

Student Interviewee Demographics (n=18)

GROUP	Course	Students by Grade	Course Grade Fall 2012	Course Grade Spring 2013	LEP Status	SpEd
FBA-MC n=7	4- Alg 1 3-Alg 2	4- Grade 9 1- Grade 11 2- Grade 12	2-As 3-Bs 1-C 1-F	1-A 4-Bs 1-C 1-F	One-1	
FBA-LC n=5	2-Alg 1 3-Alg 2	3- Grade 9 2- Grade 12	4-As 1-F	3-A 1-B 1-C		1
Non-FBA n=6	3- Alg 1 3-Alg 2	3- Grade 9 1- Grade 11 2- Grade 12	3-As 1-B 2-Cs	3-As 1-B 2-Cs		

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