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Estimating a Three-Level Latent Variable Regression Model with Cross-Classified Multiple Membership Data

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Estimating a Three-Level Latent Variable Regression Model with Cross-Classified Multiple Membership Data

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The current study proposed a new model, termed the cross-classified multiple membership latent variable regression (CCMM-LVR) model, to be utilized for multiple membership data structures (for example, in the presence of student mobility across schools) that provides an extension to the three-level latent variable regression model (HM3-LVR). The HM3-LVR model is beneficial for testing more flexible, directional hypotheses about growth trajectory parameters and handles pure clustering of participants within higher-level units. However, the HM3-LVR model involves the assumption that students remain in the same cluster (school) throughout the duration of the time period of interest. The CCMM-LVR model, on the other hand, appropriately models the participants' changing clusters over time.

The first purpose of this study was to demonstrate use and interpretation of the CCMM-LVR model and its parameters with a large-scale longitudinal dataset that had a multiple membership data structure (i.e., student mobility). The impact of ignoring mobility in the real data was investigated by comparing parameter estimates, standard error estimates, and model fit indices for the two estimating models (CCMM-LVR and HM3-LVR). The second purpose of the dissertation was to conduct a simulation study to try to understand the source of potential differences between the two estimating models and find out which model's estimates were closer to the truth given the conditions investigated. The manipulated conditions in the simulation study included the mobility rate, number of clustering units, number of individuals (i.e., students) per cluster (here, school), and number of measurement occasions per individual. The outcomes investigated in the simulation study included relative parameter bias, relative standard error bias, root mean square error, and coverage rates of the 95% credible intervals.

Substantial bias was found across conditions for both models, but the CCMM-LVR model resulted in the least amount of relative parameter bias and more efficient estimates of the parameters, especially for larger numbers of clustering units. The results of the real data and simulation studies are discussed, along with the implications for applied researchers for when to consider using the CCMM-LVR model versus the misspecified HM3-LVR model.

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Chapter 1: Introduction

In educational research, evaluating changes in student achievement over time is an essential research endeavor. Many studies examine student growth over time in order to have more than a one-time snapshot of student performance. Examples include school progress monitoring, educational interventions, program evaluations, and school effects that examine the differences in school implementations on student progress. Typical analyses for these types of studies use hierarchical growth curve modeling (GCM), where students' scores at different time-points are nested within each individual student resulting in two-level data that can be analyzed using the conventional hierarchical linear modeling (HLM). Beyond modeling the longitudinal student data, additional complexities inherent in educational research studies can be handled using HLM. Some educational research studies evaluate student growth for clusters of students sampled from schools. For these kinds of data, a three-level GCM would be utilized, where students' scores are modeled as nested within students, who are then nested within schools.

There is another type of growth analysis, called latent variable regression (LVR) modeling, that extends GCM by modeling the prediction of a student's growth rate by the student's initial status (i.e., start of time for a study). The argument is that it is important to study the expected differences in growth rates while taking into account the levels or variation in student achievement at the initial status (Seltzer, Choi, & Thum, 2003). Student-level predictors can also be included in the model to evaluate the interaction between the student characteristics and the effect of initial status on growth rate. As an example, an interaction may indicate that differences in growth rates between treatment and control groups in an educational intervention program may vary as a function of the students' initial status values. Along the same lines in a three-level LVR model, school-level characteristics could be included as predictors of the effect of initial status on growth rate. The coefficient representing the prediction of growth rate by initial status could be modeled as varying across schools (random effects). Therefore, more hypotheses about growth trajectories can be tested by using the more flexible LVR framework.

Choi and Seltzer (2010) presented a fully Bayesian approach to estimate a three-level LVR model with purely clustered data, and conducted a small simulation study to evaluate the differences in the choice of prior distributions for the level-2 random effects' variance components. Using fully Bayesian estimation can provide more precise and robust estimates for parameters in a more complex model, such as with LVR models, as long as prior distributions are appropriately chosen. Choi and Seltzer (2010) found that uniform priors, which are analogous to inverse-Pareto distributions for the level-2 LVR variance components, resulted in less bias than using default inverse-gamma distributions. These results assist future researchers when selecting the best prior distributions to use for a three-level LVR analysis.

Research evaluating student growth over time clustered within some unit (i.e., school, class, district, etc.), such as a three-level LVR analysis, can lead to additional intricacies, including student mobility in educational research. Students can change classrooms, teachers, schools, or districts within a study, especially if the study continues for multiple years. Students switched schools or moved at rates ranging between 12% and 38.5% between 2005 and 2010, with 25% of students relocating within the same county (see, for example, Ihrke & Faber, 2012; U.S. Census Bureau, 2013; U.S. Government Accounting Office, 2010). Also, within a school, students can change classes and teachers on a semester or yearly basis. A report by the U.S. Government Accounting Office (2010) found that certain school characteristics, such as low-income area, proportion of English language proficiency, and percentage of students receiving special education, tend to be associated with the school rates of mobility. These non-trivial student mobility rates produce problems for the LVR structure, because students who move will then be associated with a different cluster.

Grady and Beretvas (2010) introduced a cross-classified multiple membership growth curve model (CCMM-GCM) for modeling, as an example, academic achievement trajectories in the presence of student mobility. Previous research has demonstrated that incorrect model specification in the presence of student mobility can negatively impact parameter estimates. Previous simulation studies have shown that model misspecification can lead to inaccurate estimates of between-schools variance components and standard errors of the fixed effects (Chung & Beretvas, 2012; Grady, 2010; Luo & Kwok, 2009; Luo & Kwok, 2012; Meyers & Beretvas, 2006). The purpose of this research is then to investigate the estimation of a three-level latent variable regression model with cross-classified multiple membership data, resulting in the utilization of a CCMM-LVR model.

Parameter estimates for the CCMM-LVR model were compared to those estimated assuming a three-level LVR model that ignores mobility by only recognizing the first school attended (HM3-LVR). The dissertation is comprised of two studies, a real data analysis and simulation study. The real data analysis compared the two models with a large-scale longitudinal dataset containing mobile students. The second study is a simulation study that examined differences in parameter estimates from the two models to discover the impact of ignoring student mobility. The real data are from the Longitudinal Study of American Youth (LSAY) (Miller, 1987-1994) and contained three yearly measurement occasions from ninth through eleventh grade. Results from that analysis are presented including the parameter and standard error estimates, along with fit indices, from fitting the baseline unconditional and conditional versions of the CCMM-LVR and HM3-LVR models. The parameter estimate values from the real data analysis were used to help inform generating parameter values in the simulation study. In addition, some of the real dataset's characteristics were used in designing the simulation study's conditions.

A simulation study is conducted because true population parameters are known and design factors can be manipulated to assess their impact on the resulting estimates. The conditions that were manipulated for the simulation study include the percentage of mobile students, number of schools, number of students per school, and number of measurement occasions. Relative parameter bias, relative standard error bias, root mean square error, and coverage rates were used to evaluate the estimation of model parameters under the various manipulated conditions for the two models, CCMM-LVR and the HM3-LVR.

This study is important due to the increased educational research with longitudinal data, where subject mobility is likely to arise, and the benefit of knowing which model is best for researchers' circumstances based on the current study's findings. For policymakers and educational interventionists, it is also important for them to be aware that student growth analyses based on a model that does not handle mobility would model the students' growth as only being affected by one school, rather than all schools attended. Evaluating school- or student-level characteristics with a misspecified model impacts the validity of resulting statistical inferences and associated conclusions (Chung & Beretvas, 2012; Luo & Kwok, 2009; Luo & Kwok, 2012; Meyers & Beretvas, 2006). In addition, the results from this study are applicable not only to educational research, but to any applied research entailing a three-level data structure with mobility of individuals across clusters.

Chapter 2: Literature Review

In this chapter, a general discussion of growth curve modeling will first be presented. Following this discussion, subsequent sections will expand on this topic by covering latent variable regression modeling, growth curve modeling with mobile individuals, and lastly latent variable regression modeling with mobile individuals.

GROWTH CURVE MODELING

Individual change is a topic that has been studied for many years within the context of multilevel modeling, particularly in the educational context where, for example, studies have assessed students' rate of growth in reading comprehension (Bryk & Raudenbush, 1987; Seltzer, Frank, & Bryk, 1994), student trajectories in math achievement (Bryk & Raudenbush, 1987), as well as teacher-reported student aggressiveness over time within an intervention program (Muthén & Curran, 1997), among many other examples. With GCMs, various student demographics can be integrated into the models to assess how these student characteristics relate to differences in change over time. Additional hierarchical clustering levels (for example, schools or classrooms) can also easily be incorporated into the GCM, and characteristics describing these higher level clustering units can be investigated as predictors of differences in student achievement trajectories (Bryk & Raudenbush, 1988). These are just examples within the education context, but many other fields of applied social and behavioral science research also employ GCMs (see for example, Francis, Fletcher, Stuebing, Davidson, & Thompson, 1991; Horney, Osgood, & Marshall, 1995; Huttenlocher, Haight, Bryk, Seltzer, & Lyons, 1991; Raudenbush, & Chan, 1993).

For GCMs, researchers need to utilize measures with scales equated using item response theory, which are more appropriately suited for evaluating change across time (Embretson & Reise, 2000; Seltzer et al., 1994). As for the design of the study, more than two time-points of data need to be collected in order to obtain a more adequate description of individual growth (Bryk & Raudenbush, 1987; Bryk & Weisberg, 1977; Rogosa, Brandt, & Zimowski, 1982). The following section will discuss the two-level baseline unconditional GCM.

Two-Level Baseline Unconditional GCM

A model of individual change in some outcome over time can be constructed using a two-level hierarchical linear model. Data for these models are assumed to have repeated observations (level-1) nested within each individual (level-2). Raudenbush and Bryk's (2002) formulation will be used here. At level 1, the baseline unconditional model is

$$Y_{ti} = \pi_{0i} + \pi_{1i} TIME_{ti} + e_{ti}, \qquad (1)$$

where Y_{ti} is the observed score at time t for individual i, $TIME_{ti}$ is the time-point for individual i at time t, π_{0i} is the intercept parameter for individual i when $TIME_{ti}$ equals zero, π_{1i} is the slope parameter for individual i representing the expected change over a specified period of time, and e_{ti} is the error (i.e., random effect). The errors are assumed to be independent and normally distributed with a mean of zero and constant variance σ^2, which is a simpler error structure but the one that is most frequently assumed (Raudenbush & Bryk, 2002). This model can be used even when the spacing between time and the number of scores are different across people.

At level-2 in the baseline unconditional model, the intercept and slope parameters are typically permitted to vary across individuals as follows:

$$\begin{cases} \pi_{0i} = \beta_{00} + r_{0i} \\ \pi_{1i} = \beta_{10} + r_{1i} \end{cases}, \qquad (2)$$

with β_{00} as the average outcome across individuals when $TIME_{ti}$ equals zero, β_{10} is the average growth rate for individuals, r_{0i} is the residual for the intercept for individual i, and r_{1i} is the residual for the growth rate for individual i.

The level-2 residuals (i.e., random effects) are assumed bivariate normally distributed with means of zero and variances of τ_{00} and τ_{11} for r_{0i} and r_{1i}, respectively, and with covariance τ_{01}. τ_{00} is the variance of the intercept residuals and τ_{11} is the variance of growth rate residuals. Larger estimates for τ_{11} suggest that there is a lot of variability across individuals in their growth rates. Likewise for τ_{00}, large estimates reflect that individuals' scores when $TIME_{ti}$ equals zero vary a lot around the fixed effect parameter, β_{00}. Last, larger positive estimates of τ_{01} indicate that as scores when $TIME_{ti}$ equals zero increase, the growth rates increase. Negative large τ_{01} estimates would imply that the growth rates are stronger for individuals with lower scores when $TIME_{ti}$ equals zero.

With GCMs, the location of the time variable, *TIME*, is vital to the interpretation of the intercept, level-2 coefficients of the intercept, level-2 variance of the intercept, and the covariance between the intercept and the slope. Initial status is π_{0i} when $TIME_{ti}$ represents the amount of time that has passed since the starting point of data collection for a study. $TIME_{ti}$ can also denote the period of time from the last point of data collection, in which case π_{0i} would be the final status, or even represent the period of time from the mid-point of data collection and be the mid-point status.

To explain using an example, let's suppose students were assessed in 6th, 7th, and 8th grade. To represent π_{0i} as initial status, the level-1 formula would utilize ($TIME_{ti} - 6$), where $TIME_{ti}$ represents the grade level. For final status, the level-1 equation would use ($TIME_{ti} - 8$), and the equation would utilize ($TIME_{ti} - 7$) to model mid-point status. Using ($TIME_{ti} - 6$) in the level-1 equation in Equation 1, the intercept is interpreted as the initial status, the level-2 β_{00} coefficient is the average initial status across individuals, the level-2 variance τ_{00} is defined as the variance in initial status, and the covariance is between the initial status and growth rate. Similarly, different centering of the time variable [e.g., ($TIME_{ti} - 7$) or ($TIME_{ti} - 8$)] in the level-1 equation changes interpretation of the resulting coefficients by replacing initial status with mid-point status or final status, respectively. Thus, from here on, the time variable, *TIME*, in the level-1 equation (Equation 1) will be assumed centered such that the intercept represents initial status.

With this two-level baseline unconditional growth model, the correlation of change with initial status can also be computed. To compute the correlation of change with initial status, the equation is

$$\rho(\pi_{0i}, \pi_{1i}) = \tau_{01} / \sqrt{\tau_{00} \times \tau_{11}}, \tag{3}$$

which is simply the correlation between individuals' initial status, π_{0i}, and their linear growth in the outcome, π_{1i}. Similar to a Pearson correlation coefficient, the values can range between –1 and 1. The next section describes the extension to this baseline unconditional model that includes the addition of predictors to the model.

Two-Level Conditional GCM

The two-level conditional GCM incorporates individual-level characteristics or treatment indicators as predictors to explain variability found in the baseline unconditional model. These types of models can be used to evaluate questions about the effect of individual characteristics on the initial status and growth rate. Equation 1 would remain the same, but Equation 2 would be modified to become:

$$\begin{cases} \pi_{0i} = \beta_{00} + \beta_{01} X_i + r_{0i} \\ \pi_{1i} = \beta_{10} + \beta_{11} X_i + r_{1i} \end{cases}, \tag{4}$$

where X_i represents an individual-level variable about some characteristic of individual *i*. β_{00} is now the average initial status across individuals when X_i equals zero, and β_{01} is the change in initial status for one unit change in X_i. This definition of β_{01} assumes that X_i is a continuous explanatory variable, but when X_i is an indicator variable, β_{01} is defined as the contrast in outcomes for someone with $X_i = 1$ versus a case with $X_i = 0$. β_{10} is the average growth rate when X_i equals zero and β_{11} is the change in growth rate for a one unit change in X_i. Again, this definition of β_{11} assumes X_i is continuous, but when X_i is an indicator β_{11} represents the contrast in growth rates between someone with $X_i = 1$ versus someone for whom $X_i = 0$. For the level-2 random effects, r_{0i} is now the intercept residual for individual *i* when X_i equals zero, and r_{1i} is the distinct slope residual for individual *i* when X_i equals zero. The same assumptions hold for the level-2 residuals (i.e., random effects) as are made with the baseline unconditional model, however, τ_{00} is defined as the variance in initial status remaining after controlling for level-2 predictor variable X_i, and τ_{11} is the variance in growth rates remaining after controlling for X_i.

More complex growth curve models could contain additional growth trajectory parameters including, for example, a quadratic term. The initial decision about the functional form of the GCM should be based on the inspection of the data. These trajectories could look linear or non-linear, and if non-linear, then the number of “bends” in the trajectories would help identify the appropriate order of the polynomial to be modeled. Other level-1 predictors, besides the time variable, could be included in the model. In addition, more complex level-1 error assumptions can be presumed, such as dependence on individual characteristics or estimation of separate level-1 error variances for each time point. For the current study, only simple level-1 error structures (described previously) and linear growth forms without time-varying covariates will be considered, however, more information can be found in Raudenbush and Bryk (2002). The following section will describe the extension of the two-level GCM to the three-level GCM.

Three-Level Baseline Unconditional GCM

Researchers commonly gather repeated measures (level-1) on individuals (level-2) who are clustered within higher level organizational units (level-3). In education, common organizational units include schools, classrooms, or districts. Bryk and Raudenbush (1988) illustrated this approach with longitudinal data using five measurement occasions between the spring of first grade through the spring of third grade. Other fields of research in the social sciences may have to consider organizational units such as regions, neighborhoods, hospitals, and clinics.

Adding a third level to the GCM, the data structure entails repeated measures nested within individuals who would then be nested within organizational units. Therefore, the individual growth trajectories represent the level-1 model, variation in growth parameters among individuals within an organizational unit is captured in the level-2 model, and variation among the organizational units is modeled in the level-3 model. With these types of models, studies can assess questions about how characteristics of these organizational units (as well as about the individuals themselves) influence individuals' growth trajectory parameters.

There are important reasons for incorporating the third level representing organizational units into a GCM that resolve potential issues with aggregation bias, misestimated standard errors, and heterogeneity of regression (Raudenbush & Bryk, 2002). Aggregation bias can occur when variations in characteristics of an organization affect individual scores in addition to the effect the individual's characteristics have on their scores. In educational research, for example, differences in school socioeconomic status can affect student achievement above and beyond the student's socioeconomic status. Misestimated standard errors can ensue when the dependence within an organizational unit is not taken into account. This dependence is inherent to many situations of individuals nested within organizational units because the individuals have shared experiences within an organization. Heterogeneity of regression happens when the relationship between individual's characteristics and their scores differ across the organizational units. Recognition of a third level of clustering in a dataset through use of a three-level GCM provides one way to resolve aggregation bias, misestimated standard errors, and heterogeneity of regression.

As with the previous section on the two-level GCM, the three-level GCM discussion will begin with the baseline unconditional model. The baseline unconditional level-1 model equation, with the measurement occasions at time *t* for individual *i* within the organization unit *j*, is

$$Y_{tij} = \pi_{0ij} + \pi_{1ij} TIME + e_{tij}, \tag{5}$$

where Y_{tij} is the observed score at time *t* for individual *i* within organizational unit *j*, π_{0ij} is the intercept parameter for individual *i* within organization *j*, π_{1ij} is the slope parameter for individual *i* within organization *j*, $TIME_{tij}$ is the time-point for individual *i* within organization *j* at time *t*, and e_{tij} is the error. As before, the errors are assumed to be independent and normally distributed with a mean of zero and constant variance σ^2. The difference between Equation 1 and Equation 5 is the inclusion of the additional third level subscript *j* to represent the organizational unit. This will also be the same difference between the level-2 model from the two-level and the three-level baseline unconditional GCMs.

The level-2 model of the baseline unconditional three-level GCM is

$$\begin{cases} \pi_{0ij} = \beta_{00j} + r_{0ij} \\ \pi_{1ij} = \beta_{10j} + r_{1ij} \end{cases}, \tag{6}$$

where β_{00j} is the mean initial status (i.e., when $TIME_{tij}$ equals zero) across individuals within organization *j*, β_{10j} is the mean growth rate for individuals within organization *j*, r_{0ij} is individual-level residual for the mean initial status for organization *j*, and r_{1ij} is the individual-level residual for the mean growth rate for organization *j*. The level-2 residuals are again assumed bivariate normally distributed with means of zero, but with variances of

$\tau_{\pi 00}$ and $\tau_{\pi 11}$ for r_{0ij} and r_{1ij}, respectively, and with covariance $\tau_{\pi 01}$. $\tau_{\pi 00}$ is the variance of initial status within the organizational units and $\tau_{\pi 11}$ is the variance of growth rates within the organizations. Another assumption is that the variability among individuals within the *j* organizational units is the same.

The third level of the baseline unconditional three-level GCM is represented as

$$\begin{cases} \beta_{00j} = \gamma_{000} + u_{00j} \\ \beta_{10j} = \gamma_{100} + u_{10j} \end{cases}, \tag{7}$$

with γ_{000} representing the overall mean initial status across individuals and organizations, γ_{100} is the overall mean growth rate across individuals and organizations, u_{00j} is the residual for organization *j* for the overall mean initial status, and u_{10j} is the residual for organization *j* for the overall mean growth rate. The level-3 residuals are assumed bivariate normally distributed with means of zero, but with variances of $\tau_{\beta 00}$ and $\tau_{\beta 11}$ for u_{00j} and u_{10j}, respectively, and with covariance $\tau_{\beta 01}$. $\tau_{\beta 00}$ is the variance of initial status among the organizational units and $\tau_{\beta 11}$ is the variance of growth rates among the organizations. The following discussion will describe the formulation of the conditional three-level GCM, which includes level-2 and level-3 predictors in the model.

Three-Level Conditional GCM

The three-level conditional GCM allows estimation of the distinct effects of individual and organizational characteristics on individual outcomes. The level-1 equation remains the same as Equation 5. The conditional level-2 model is

$$\begin{cases} \pi_{0ij} = \beta_{00j} + \beta_{01j} X_{ij} + r_{0\mathrm{ij}} \\ \pi_{1ij} = \beta_{10j} + \beta_{11j} X_{ij} + r_{1ij} \end{cases}, \tag{8}$$

where β_{00j} is the mean initial status across individuals within organization j when X_{ij} equals zero and β_{01j} is the change in initial status within organization j for one unit change (or contrast) in X_{ij}. β_{10j} is the mean growth rate within organization j when X_{ij} equals zero and β_{11j} is the change in growth rate within organization j for one unit change (or contrast) in X_{ij}.

For the level-2 random effects, r_{0ij} is now the individual-level residual for the mean initial status for organization j when X_{ij} equals zero, and r_{1ij} is the individual-level residual for the mean growth rate for organization j when X_{ij} equals zero. The same assumptions hold for the conditional level-2 random effects as in the baseline unconditional two-level GCM. However, $\tau_{\pi 00}$ is now defined as the variance in initial status remaining within the organizational units after including level-2 predictor variables and $\tau_{\pi 11}$ is the variance in growth rates remaining within the organizations after including the level-2 predictor variables.

The formulation for the third level of the three-level conditional GCM with the influence of X_{ij} assumed as fixed is as follows:

$$\begin{cases} \beta_{00j} = \gamma_{000} + \gamma_{001} Z_j + u_{00j} \\ \beta_{01j} = \gamma_{010} \\ \beta_{10j} = \gamma_{100} + \gamma_{101} Z_j + u_{10j} \\ \beta_{11j} = \gamma_{110} \end{cases}, \tag{9}$$

where Z_j represents an organizational level variable about some characteristic of organization *j*, which can be viewed as a predictor of the organization effect. Here, γ_{000} represents the overall mean initial status across individuals and organizations when X_{ij} and Z_j equal zero and γ_{001} represents the change in initial status for one unit change in Z_j (if Z_j is continuous) or the contrast in initial status between Zj = 1 and Z_j = 0 (if Z_j is an indicator) when X_{ij} equals zero. Now, γ_{100} represents the overall mean growth rate across individuals and organizations when X_{ij} and Z_j equal zero and γ_{101} represents the change in growth rates for one unit change or contrast in Z_j when X_{ij} equals zero. For the random effects, u_{00j} is the residual for organization *j* for the overall mean initial status when X_{ij} and Z_j equal zero and u_{10j} is the residual for organization *j* for the overall mean growth rate when X_{ij} and Z_j equal zero. The level-3 residuals follow the same assumptions as the baseline unconditional level-2 GCM, but $\tau_{\beta 00}$ is now the variance of initial status among the organizational units after including individual and organizational variables and $\tau_{\beta 11}$ is the variance of growth rates among the organizations after including individual and organizational variables.

GCM provides a very useful modeling technique for longitudinal data; however, an extension of this modeling framework, called latent variable regression, adds potentially even more valuable information by allowing assessment of directional influences of one growth parameter on another rather than solely modeling the covariance between the parameters. The next section introduces latent variable regression and explains the kinds of research questions that can be answered when combined with a GCM.

LATENT VARIABLE REGRESSION MODELING

Growth curve modeling focuses exclusively on changes in individual outcomes over time by estimating growth rates, whereas the use of latent variable regression modeling in the GCM context extends this notion by allowing modeling of, for example, the prediction of an individual's growth rate (latent growth parameter) by the individual's initial status. The rationale behind this type of growth analysis is to study the expected differences in growth rates holding constant initial status. In particular with educational research using longitudinal data, the argument is that it is frequently important to take into account the levels or variation in student achievement at the initial status (i.e., start of time for the study). Modeling the expected change in growth rates for one unit change in initial status can allow for more questions to be addressed in longitudinal studies.

As a simple illustrative example to help convey the concepts behind latent variable regression modeling, suppose individuals were measured each year on math achievement from 1st grade to 5th grade. If we believe that there is a positive relationship between initial status and growth rate, Figure 1 would demonstrate that scenario. The four expected math achievement trajectories for students show the pattern that as first grade scores increase so do the slopes of the trajectories.

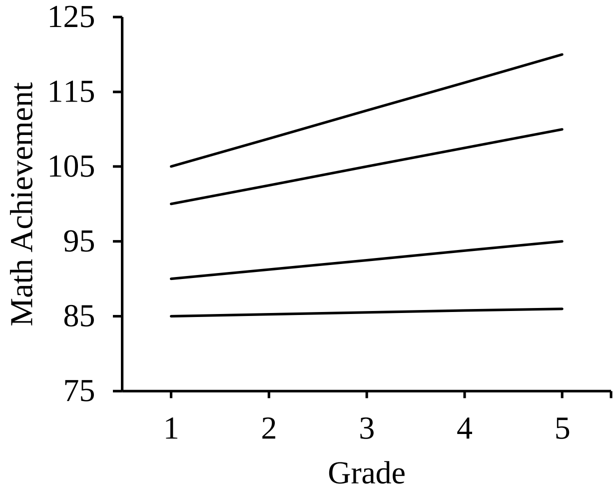

Figure 1. *Positive relationship between initial math achievement score and growth rate.*

Figure 2 would be the situation were no relationship exists between first grade outcomes and the growth rates. Regardless of what an individual scored in first grade, their growth rate is the same across those initial outcomes.

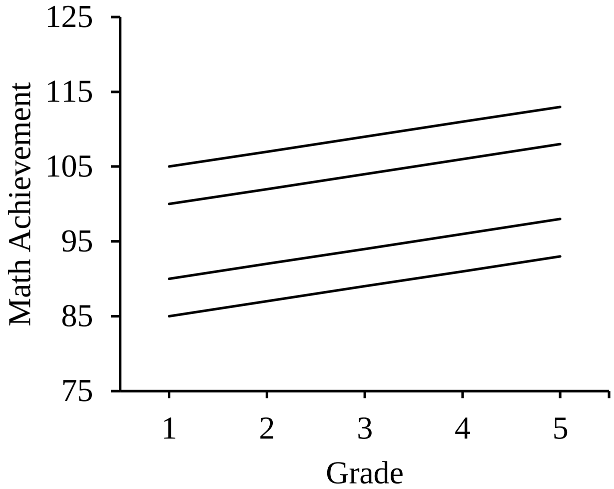

Figure 2. *No relationship between initial math achievement score and growth rate.*

In Figure 3, the four expected trajectories display a negative relationship between initial status and growth rate. This means that as the initial statuses increase, the slopes of the trajectories decrease.

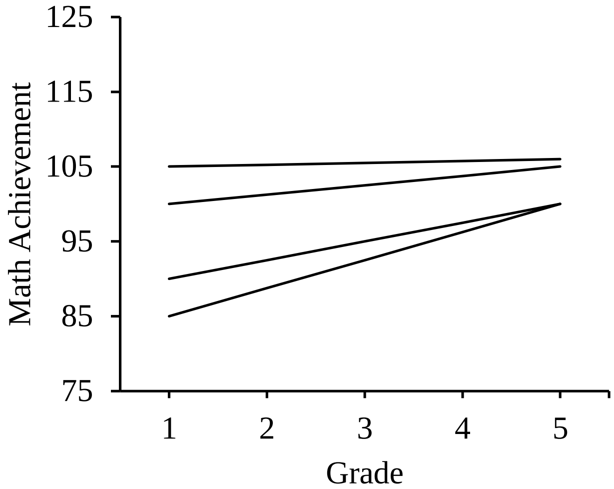

Figure 3*. Negative relationship between initial math achievement score and growth rate.*

All three figures have the same mean growth rate, but they have very different relationships between initial status and growth. This relationship between initial status and growth rate is provided with typical GCM. What is not provided is the answer to “What is the expected change in growth rates for one unit change in initial status?” In Figure 1, assuming the mean initial status is 95, the expected growth rate is 3.0 points per grade for students who scored 10 points above this mean. For students who scored 85 points in first grade (10 points below the mean), their expected growth from first to fifth grade is 0.2 points per grade. In Figure 2, the expected growth rate is 1.6 points per grade level, regardless of what the students scored in first grade. For Figure 3, the results are opposite to those in Figure 1, where students who scored 105 points in first grade have an expected growth rate of 0.2 points per grade and students who scored 10 points below the mean initial status have an expected growth rate of 3 points per grade.

By including individuals' characteristics in the LVR model, the researcher could examine interactions between the characteristics and the effect of initial status on growth rate. For example, it might be the case that there are differences between the genders in the prediction of growth rates by initial status. The prediction of growth by initial status might be stronger for females than for males. This hypothesis cannot be directly tested using a conventional GCM although this interaction effect can be tested using an LVR model that incorporates gender as a predictor of the coefficient representing the influence of initial status on growth rate (Seltzer et al., 2003). Alternatively, there might be an interaction effect in that differences in growth rates between educational intervention and control groups may vary as a function of individuals' initial status values (Choi & Seltzer, 2010; Muthén & Curran, 1997). In addition, the LVR coefficient representing the prediction of growth rate by initial status could be modeled as varying across organizational units, as well as a function of organizational characteristics in the model (Choi & Seltzer, 2010). The three-level LVR model could be used to evaluate these kinds of research questions which cannot be assessed using a typical GCM. Given these reasons, use of the more flexible LVR framework is beneficial for testing more flexible hypotheses about growth trajectory parameters.

Demonstrations of how to estimate the LVR model using multilevel modeling software (such as HLM7 software; Raudenbush, Bryk, Cheong, Congdon, & du Toit, 2011) have allowed the use of this type of growth modeling to be more accessible to applied researchers and practitioners. Muthén and Curran (1997), Raudenbush and Bryk (2002, Chapter 11), Svärdsudd and Blomqvist (1978), and Adler, Adam, and Arenberg (1990) have demonstrated use of the LVR modeling technique in the context of randomized intervention studies, gender differences in high school math achievement, longitudinal male blood pressure related to older age, and cognitive functioning associated with aging, respectively. All of these studies employed the maximum likelihood estimation procedure for obtaining the parameter estimates. Seltzer et al. (2003) showed how to use fully Bayesian estimation with their demonstration of a two-level LVR model in a school performance setting. The reasoning behind the authors' use of Bayesian estimation was that it works better than maximum likelihood for estimating fixed effects and covariance components as long as the choice of prior distributions is carefully considered. Maximum likelihood estimation can be problematic particularly when the number of clustering units is small or when the hierarchical data structure is unbalanced. For a more detailed explanation of Bayesian inference for hierarchical linear models, please refer to Raudenbush and Bryk (2002, Chapter 13).

It is important to note as well that latent variable regression modeling is not just isolated to the hierarchical linear modeling (HLM) context, but originated from the structural equation modeling framework. Raudenbush and Sampson (1999) extended LVRs to the HLM framework by demonstrating a method for integrating LVRs into the HLM context using maximum likelihood estimation for the parameters. In structural equation modeling (SEM), latent relationships are specified among model parameters, which is equivalent to what is accomplished when using latent variable regression modeling. The HLM and SEM frameworks are equivalent in many ways and in particular with the conventional GCM, where the within-person (level 1) model in HLM is the measurement model in SEM and the between-person (level 2) model is the structural model. Figure 4 displays how a two-level baseline unconditional GCM (see Equations 1 and 2) would be represented using an SEM path diagram.

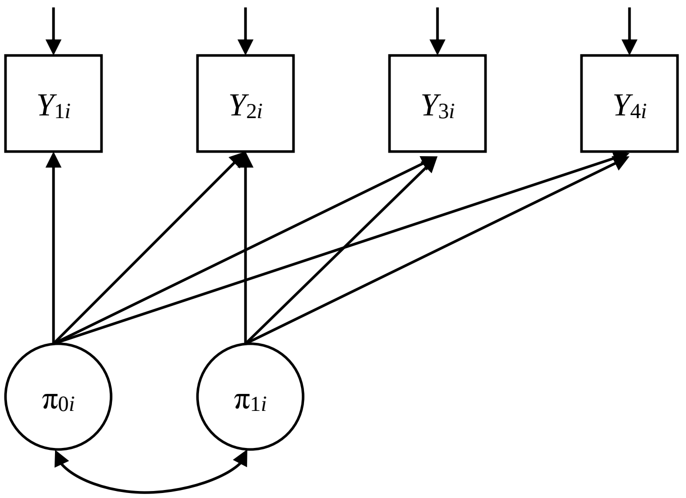

Figure 4. *Linear growth model with four measurement occasions.*

And Figure 5 shows how a two-level baseline unconditional LVR model would be presented.

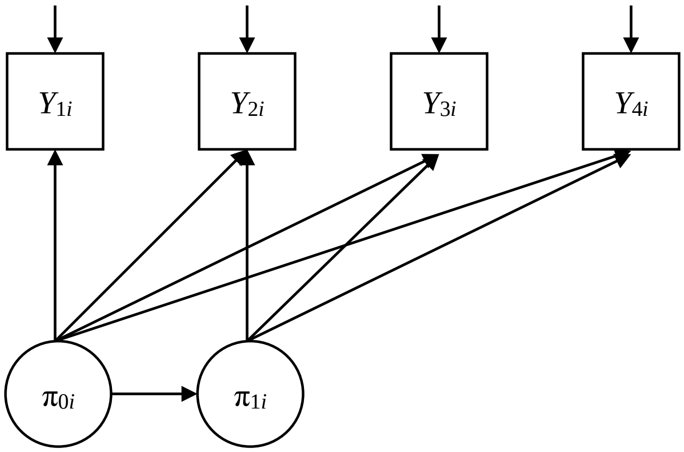

Figure 5. *Linear growth model for four measurement occasions using LVR parameterization.*

Note that in Figure 4, the intercept and slope are modeled as co-varying, while in Figure 5 the intercept is modeled as a predictor of the slope. The SEM framework can easily be used for growth curve analysis, but for the current study the HLM framework will be utilized because it offers more flexibility for handling non-purely nested data structures that will be discussed in a later section. The next section will provide information about the formulation of the two-level baseline unconditional LVR model.

Two-Level Baseline Unconditional LVR Model

In an LVR model, initial status is included as a predictor of growth rate, whereas in a GCM, initial status and growth rate are modeled as co-varying. This means that with LVR, it is possible to model moderation of initial status' prediction of the slope by some individual participant characteristic. For example, a researcher might be interested in testing the effect of an educational treatment intervention on math achievement. A typical

GCM analysis could reveal differences in growth between treatment groups, but the LVR model could explore additional research questions including, for example, that these treatment effects on the slope differ based on where a student scored at the beginning of the study (i.e., pretest). The overall treatment effect may reveal a positive impact on growth, but the LVR model could then show that the treatment effect is more positively substantial for students who score lower at pretest than those who score higher.

As with the formulation of the GCM, explanation of the formulation of the LVR model will begin with the baseline unconditional two-level model (which is termed HM2-LVR). Level 1, the within-individuals model, is the same as in Equation 1. The differences between the GCM and latent variable regression model occur at level 2, where π_{0i} is included as a predictor of π_{1i}. For level 2, the HM2-LVR equation is

$$\begin{cases} \pi_{0i} = \beta_{00} + r_{0i} \\ \pi_{1i} = \beta_{10} + b(\pi_{0i} - \beta_{00}) + r_{1i} \end{cases}, \tag{10}$$

where β_{10} is the mean growth rate for individual *i* at the mean on initial status, *b* is the latent variable regression coefficient that represents the change in the growth rate (π_{1i}) for one unit increase in initial status (π_{0i}), and r_{1i} is the slope residual for individual *i* at the mean on initial status. The random effects are assumed normally distributed with means of zero and variances of τ_{00} and τ_{11} for r_{0i} and r_{1i}, respectively, and covariance τ_{01} equals zero. The same definition holds for τ_{00} as in the two-level baseline unconditional GCM section, but τ_{11} is now the variance in growth rates remaining after taking into account differences in initial status. Notice that π_{0i} is centered around β_{00} in order for β_{10} to not represent the mean growth rate for someone whose initial status value is zero, which provides a more useful interpretation of β_{10}. In addition, the centering of π_{0i} around β_{00} in LVR models decreases the amount of autocorrelations among the samples generated by the Markov Chain Monte Carlo (MCMC) sampling with Bayesian estimation. Expanding on the baseline unconditional two-level LVR model, the conditional LVR model will be described in the next section.

Two-Level Conditional LVR Model

The two-level conditional LVR model has the same level-1 formula as Equation 1. The level-2 conditional formula with individual characteristic variable X_i is then represented as

$$\begin{cases} \pi_{0i} = \beta_{00} + \beta_{01} X_i + r_{0i} \\ \pi_{1i} = \beta_{10} + \beta_{11} X_i + b(\pi_{0i}) + r_{1i} \end{cases}, \tag{11}$$

where β_{10} is the mean growth rate for individual i at the mean on initial status and for whom X_i equals zero, β_{11} is the change in growth rate for one unit change (or the contrast) in X_i *holding constant initial status*, and b is the change in the growth rate for one unit increase in initial status when X_i equals zero. The level-2 random effect r_{1i} is the slope residual for individual i at the mean on initial status and for whom X_i equals zero. The same assumptions hold for the level-2 random effects and τ_{00} is still defined as before with the two-level conditional GCM, however, τ_{11} is now the variance in growth rates remaining after taking into account differences in initial status and including individual variables. In LVR modeling, the covariance of r_{0i} and r_{1i} is assumed to be unrelated (i.e., equal to zero) because that covariance is instead modeled by incorporating π_{0i} as a predictor of π_{1i}. In the next section, a discussion of the baseline unconditional three-level LVR model is provided.

Three-Level Baseline Unconditional LVR Model

Choi and Seltzer (2010) extended the two-level LVR model to a three-level LVR model using a fully Bayesian approach. The three-level LVR model allows handling of the dependence of individuals clustered within organizations (such as schools, classrooms, etc.). The LVR coefficient that designates the effect of initial status on growth within the organizations can be modeled as varying across organizations. Assessment of this variation permits evaluation of organizational differences in the LVR coefficient as well as assessment of factors that might influence the effect of initial status on growth.

Level 1 of the three-level baseline unconditional LVR model (termed HM3-LVR) is the same as Equation 5 in the discussion of the three-level baseline unconditional GCM, and involves the same assumptions. Following the same notation as Choi and Seltzer (2010), the formulation for level 2 is

$$\begin{cases} \pi_{0ij} = \beta_{00j} + r_{0ij} \\ \pi_{1ij} = \beta_{10j} + \mathrm{Bw}_j \left(\pi_{0ij} - \beta_{00j}\right) + r_{1ij} \end{cases}, \tag{12}$$

where β_{10j} is the mean growth rate for organization j for an individual at the mean on initial status, and Bw_j is the LVR coefficient that represents the change in the growth rate for one unit increase in initial status *within* organization j. The latter coefficient is termed the within-organization initial status on growth effect. The random effects are assumed normally distributed with means of zero and variances $\tau_{\pi 0j}$ and $\tau_{\pi 1j}$ for r_{0ij} and r_{1ij}, respectively, and $\mathrm{Cov}(r_{0ij}, r_{1ij}) = 0$. $\tau_{\pi 1j}$ is now the variance in growth rates remaining after taking into account differences in initial status within the organizations. Note that different from the presentation of the three-level GCM, the three-level LVR model presented in Choi and Seltzer (2010) allowed the variability in residuals among individuals within organizational units to differ, but the current study will not model that same variability among individuals within the organizations. Instead, constant variances, $\tau_{\pi 0j}$ and $\tau_{\pi 1j}$, will be assumed across organizations.

The level-3 baseline unconditional LVR model is

$$\begin{cases} \beta_{00j} = \gamma_{000} + u_{00j} \\ \beta_{10j} = \gamma_{100} + \mathrm{Bb}\left(\beta_{00j} - \gamma_{000}\right) + u_{10j} \\ \mathrm{Bw}_j = \mathrm{Bw\_0} + \mathrm{Bw\_1}\left(\beta_{00j} - \gamma_{000}\right) + u_{\mathrm{Bw}j} \end{cases}, \tag{13}$$

where Bb is an LVR coefficient that represents the change in growth rate for one unit increase in mean initial status *across* organizations, and γ_{100} is then the mean growth rate across organizations for organization *j* at the grand mean on initial status. Bw_0 is another LVR coefficient that is the effect of initial status on growth for organization *j* at the grand mean on initial status, and Bw_1 is the change in the effect of initial status on growth for one unit increase in mean initial status for organization *j*. The three random effects are assumed multivariate normally distributed with means of zero and a 3 by 3 covariance matrix $\mathbf{T}_{\mathrm{u}}$, which is

$$\mathbf{T}_{\mathrm{u}} = \begin{bmatrix} \tau_{\beta 00} & 0 & 0 \\ 0 & \tau_{\beta 10} & \tau_{\beta 10,\mathrm{Bw}} \\ 0 & \tau_{\mathrm{Bw},\beta 10} & \tau_{\mathrm{Bw}} \end{bmatrix}, \tag{14}$$

where $\tau_{\beta 00}$ is defined as before, $\tau_{\beta 10}$ is the variance in growth rates remaining between the organizational units after taking into account organization mean initial status, and τ_{Bw} is the variance in within-organization initial status on growth effects remaining between the organizational units after taking into account organization mean initial status. Note also that the Cov(u_{00j}, u_{10j}) = 0 and the Cov(u_{00j}, $u_{\mathrm{Bw}j}$) = 0 because β_{00j} is used as a predictor of β_{10j}. The conditional model extension of the baseline unconditional three-level LVR model will be discussed next.

Three-Level Conditional LVR Model

The baseline unconditional HM3-LVR can easily be extended to provide the conditional model. Equation 5 would still represent the level-1 model, then including predictors X_{ij} and Z_j at levels two and three, respectively, the level-2 model becomes

$$\begin{cases} \pi_{0ij} = \beta_{00j} + \beta_{01j} X_{ij} + r_{0ij} \\ \pi_{1ij} = \beta_{10j} + \mathrm{Bw}_j \left(\pi_{0ij} - \beta_{00j}\right) + \beta_{11j} X_{ij} + r_{1ij} \end{cases}, \tag{15}$$

where β_{10j} is the mean growth rate for organization j for an individual at the mean on initial status and for whom X_{ij} equals zero, Bw_j represents the change in growth rate for one unit increase in initial status for organization j when X_{ij} equals zero, and β_{11j} is the change in growth rate within organization j for one unit change (or contrast) in X_{ij} holding constant initial status for organization j. The same assumptions hold for the random effects as for the baseline unconditional model, but now $\tau_{\pi 1j}$ is the variance in growth rates remaining after taking into account differences in initial status within the organizations and in individual variables.

For simplicity's sake, here, the effect of the individual-level variable will be assumed fixed in the level-3 model which is then expressed as follows:

$$\begin{cases} \beta_{00j} = \gamma_{000} + \gamma_{001} Z_j + u_{00j} \\ \beta_{01j} = \gamma_{010} \\ \beta_{10j} = \gamma_{100} + \text{Bb}\left(\beta_{00j} - \gamma_{000}\right) + \gamma_{101} Z_j + u_{10j} \\ \beta_{11j} = \gamma_{110} \\ \text{Bw}_j = \text{Bw\_0} + \text{Bw\_1}\left(\beta_{00j} - \gamma_{000}\right) + \text{Bw\_2}\left(Z_j\right) + u_{\text{Bw}j} \end{cases} , \quad (16)$$

where γ_{100} represents the overall mean growth rate across individuals and organizations for organizations at the grand mean on initial status when X_{ij} and Z_j equal zero, Bb represents the change in growth rate for one unit increase in mean initial status across organizations when X_{ij} and Z_j equal zero, and γ_{101} represents the change in growth rates for one unit change or contrast in Z_j when X_{ij} equals zero for organizations at the grand mean on initial status. Bw_0 is the effect of initial status on growth for organization *j* at the grand mean on initial status when X_{ij} and Z_j equal zero, Bw_1 is the change in the effect of initial status on growth for one unit increase in mean initial status for organization *j* when X_{ij} and Z_j equal zero, and Bw_2 is the expected change in the effect of initial status on growth for organization *j* for one unit change or contrast in Z_j for an organization at the grand mean on initial status and when X_{ij} equals zero.

The same assumptions are made about the distributions of the three random effects as for the baseline unconditional three-level LVR model. Now, $\tau_{\beta 10}$ is the variance in growth rates remaining between the organizational units after taking into account organization mean initial status and including individual and organizational variables, and τ_{Bw} is the variance in within-organization initial status on growth effects remaining between the organizational units after taking into account organization mean initial status and including individual and organizational variables. The next section will describe methodological research focused on the three-level LVR model.

Methodological Research with the Three-Level LVR Model

The only methodological work found that assesses estimation of the HM3-LVR was a small simulation study conducted in Choi and Seltzer (2010). Choi and Seltzer (2010) discussed and demonstrated the use of a fully Bayesian approach to estimating the HM3-LVR model. One of the advantages of using a fully Bayesian approach is that it can provide point and interval estimates for parameters for a variety of more complex models, such as the HM3-LVR family of models. It is also noted in Raudenbush and Bryk (2002, Chapter 13) that fully Bayesian estimates are robust and more precise to scenarios with unbalanced hierarchical data structures or smaller numbers of clustering units as compared to other estimation procedures, such as maximum likelihood estimation, as long as appropriate prior distributions were used.

One condition in the Choi and Seltzer (2010) simulation that was manipulated was the type of prior used for the level-2 variance components in the Bayesian estimation procedure. Previous studies had commonly used either uniform or default inverse gamma (DIG) priors for scalar random effects variance components when the covariances between random effects are set to zero (Gelman, Carlin, Stern, & Rubin, 2004; Seltzer, 1993). However, research has indicated that there are some issues with the use of DIG priors for scalar variances in regard to biased estimates and insufficient coverage of true values (see Browne & Draper, 2006; Spiegelhalter, 2001; Gelman, 2006). Choi and Seltzer (2010) wanted to demonstrate the consequences of the choice of priors for the level-2 variance components. They conducted the simulation with 300 replications using the baseline unconditional three-level LVR model. The authors based their generating parameter values using model estimates based on a corresponding analysis using the Longitudinal Study of America Youth data. Each replicated dataset in the simulation study had 8,585 time-point observations within 2,628 students nested within 45 schools. The uniform priors set for the level-2 scalar variance components resulted in less relative bias as well as the highest coverage rates for the 95% credible interval estimates of the following fixed effects: γ_{000}, Bb, Bw_0, and Bw_0 (see Equation 13). Using the DIG priors for the level-2 variance components led to slightly better results for estimates of γ_{100}, but using the uniform priors led to reasonable results. Based on these results, the authors recommended utilizing uniform priors for the level-2 and level-3 scalar variance components as well as for all of the fixed effects, and inverse-Wishart priors for the level-3 residuals' covariance matrix. The uniform priors for the fixed effects are functionally the same as specifying a normal distribution with a mean of zero and variance of 100,000, and the uniform priors placed on the scalar variance components are analogous to using inverse-Pareto(1, 0.0001) distributions (Choi & Seltzer, 2010).

Thus far, a summarization of growth modeling techniques has been presented that assumes a purely hierarchical data structure, whereas the following section will discuss techniques for growth modeling when data structures are not perfectly hierarchical.

GROWTH CURVE MODELING WITH MOBILE INDIVIDUALS

All of the three-level growth modeling techniques previously discussed involved a purely hierarchical data structure, where measurement occasions were assumed nested within individuals who were themselves nested within a single organization for the entire duration of the study. In reality, this purely clustered data structure may not always hold, especially in educational studies where students can move to different schools or classrooms over time. In the educational context, suppose there exists a pure hierarchical structure, where students attended the same school for the entire three years of the study, which is depicted as a network graph in Figure 7.

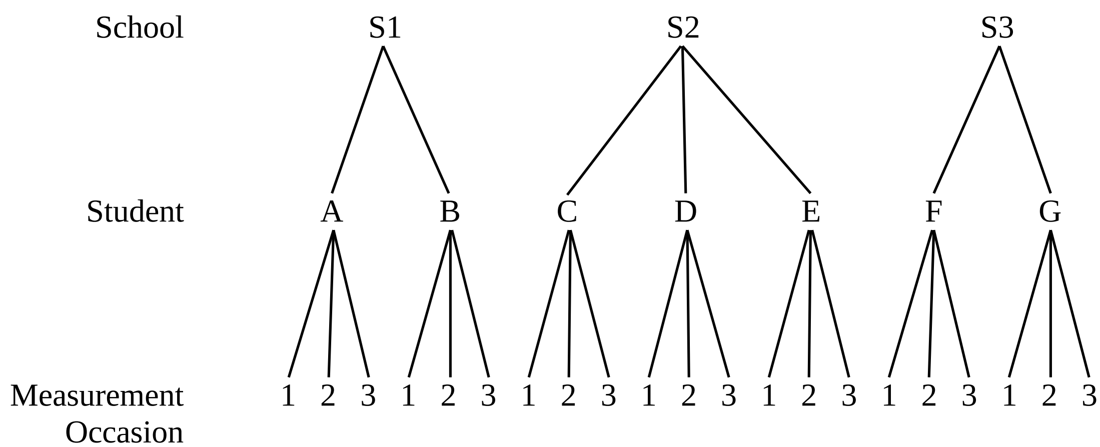

Figure 7. *Network graph of pure three-level clustering of measurement occasions (level 1) within students (level 2) within schools (level 3).*

For the data depicted in Figure 7, each student remained in the same cluster (here, school) for all three years. An alternative view of the pure three-level hierarchical growth curve data structure is presented in Figure 8. This figure will be helpful when explaining cross-classified and multiple-membership data structures in the following sections.

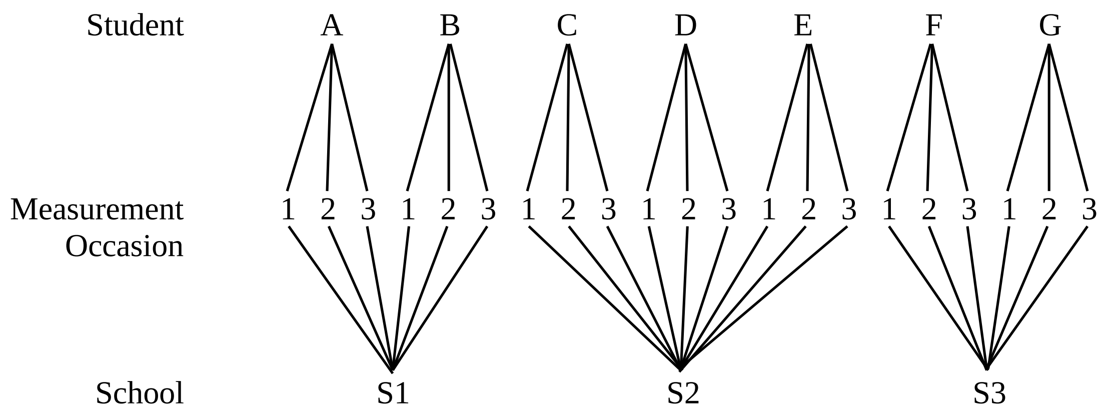

Figure 8. *Alternate depiction of network graph of pure three-level clustering of measurement occasions (level 1) within students (level 2) within schools (level 3).*

The reason why these pure nested structures do not hold in real-world educational datasets is because usually a subset of students tends to move for a variety of reasons. According to the Ihrke and Faber (2012) report titled *Geographical Mobility: 2005 to 2010*, 38.5% of people aged 5 to 17 years moved within those years. More specifically, 25% of people between the ages 5 and 17 relocated within the same county. From 2012 to 2013, 12% of people between the ages 5 and 17 years old moved, with 69% of those moves occurring within the same county (U.S. Census Bureau, 2013). A report by the U.S. Government Accounting Office (2010) found that 13% of students changed schools four or more times between kindergarten and 8th grade, and 11.5% of schools had high rates of mobility. The same report found that schools with higher rates of mobile students tend to be in low-income areas, have a higher percentage of students receiving special education, and have a higher proportion of students with limited English language proficiency. However, not all student relocations are based on a residential move, because students can change classes or teachers on a semester or yearly basis and school closures cause students to switch schools. Irrespective of the reason for mobility, students are changing schools at non-trivial rates, which produces problems for the use of three-level GCMs or LVR models based on an assumption of purely clustered data.

Previous research has suggested two ways to handle student mobility when using the GCM. However, a study has yet to present how to incorporate the modeling of student mobility when interested in using the LVR model. Use of the two GCMs model options for mobile students' data is presented in the next section. The first model is termed the cross-classified growth curve model and has been studied by Raudenbush and Bryk (2002, Chapter 12) and Luo and Kwok (2012). The second model is called a cross-classified multiple membership growth curve model, which was presented in Grady and Beretvas's study (2010).

Cross-Classified Growth Curve Modeling

In the situation where an individual did not remain in the same organization (third-level unit) during the duration of the study, the resulting nested data structure can no longer be assumed to entail a pure clustering. Instead, the data structure can be conceived of as a cross-classified structure. Figure 9 displays a simplified example of such cross-classification where, for instance, Student B who attended School 1 at the first two measurement occasions, moved to School 2 for the third measurement occasion. Student E attended School 2 at the first time-point and was at School 3 for the second and third measurement occasions. Another circumstance depicted is Student C, who was at School 2 for the first time-point then left for School 1 and returned to School 2 for the last measurement occasion. Notice also the occurrence of lines “crossing” that was not present in the purely clustered dataset depicted in Figure 8.

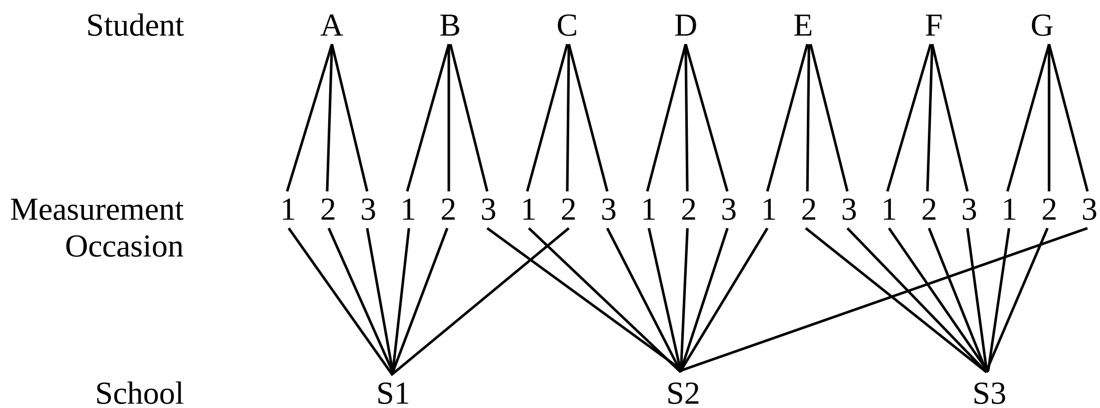

Figure 9. *Network graph of cross-classification of measurement occasions (level 1) within students (level 2) and schools (level 2).*

This type of data structure can easily be handled using a cross-classified GCM, where the random effects of each school (or organization) attended is incorporated into the typical GCM model for the intercept at level 2. Using the baseline unconditional cross-classified GCM presented in Luo and Kwok (2012), level 1 is

$$Y_{t(ij)} = \pi_{0(ij)} + \pi_{1(ij)} TIME_{t(ij)} + e_{t(ij)} \tag{17}$$

and level 2 is

$$\begin{cases} \pi_{0(ij)} = \beta_{00} + r_{0i} + u_{0j} \\ \pi_{1(ij)} = \beta_{10} + r_{1i} \end{cases}, \tag{18}$$

where the parentheses around *i* and *j* signify cross-classification between an individual and an organization. These additional organizational random effects (u_{0j}) are called “deflections” in Raudenbush and Bryk (2002), meaning that they modify the individual’s growth trajectory due to encountering each organization. According to Raudenbush and Bryk (2002), no organization-level random effects are included in the growth rate model at level 2 because the estimation of the effect of an organizational random effect on the growth rate should not be allowed unless the individual has two consecutive measurement occasions in the same organization. It is not included in the model because the model assumes separate organizational random effects at each time-point, even if a student went to the same school for all measurement occasions. Therefore, under this parameterization, the set of schools attended for mobile students are modeled as affecting only the intercept (i.e., the first time-point). Organization-level characteristics can also be incorporated into this model, again for just the intercept model at level 2.

An issue arises with this type of GCM with cross-classified individuals because the unique effect of each organization is assumed to be the same across measurement occasions (Luo & Kwok, 2012; Raudenbush & Bryk, 2002). For example, using Figure 9, the contribution of School 2 and its characteristics for Student E would only remain for the first measurement occasion, while only the effect of School 3 is incorporated into the model for time-points two and three. Raudenbush and Bryk (2002) do present a modification to the cross-classified GCM that would allow for cumulative organizational effects using a dummy-coded variable associated with each organization and time-point for the organizational random effects (and possible organizational characteristics) in the intercept model of level 2. However, their data are yearly assessments nested within students nested within teachers, so every student has a different teacher each year. This again means that different organizational random effects are assumed at each time-point, so no organizational random effects are incorporated into the growth rate model at level 2. The following section discusses the other growth curve model that could be used to handle complications introduced by mobile individuals.

Cross-Classified Multiple Membership Growth Curve Modeling

A GCM was introduced that was designed to handle mobility across clustering units that does not involve the assumption that the organization's effect is the same across time like with the cross-classified GCM. For this reason, the following model is used as the model to handle mobility in the current study. This model is termed the cross-classified multiple membership growth curve model (CCMM-GCM), and was presented by Grady and Beretvas (2010). The model is intended for researchers using the initial status as the interpretation of the intercept (see previous discussion on centering). The CCMM-GCM model is a combination of cross-classified and multiple membership random effects models because individuals are cross-classified by their first organization and the subsequent organization or organizations attended (which results in the possible multiple membership portion). The cross-classified component is required because at the initial status the individual has only been affiliated with the first organization, therefore all organizations attended should not be modeled as contributing to an individual's outcome at the first measurement occasion. The CCMM-GCM will allow an individual's intercept to vary across the first organization attended, and their growth rate can be modeled as varying across the set of organizations attended across the duration of the study.

The scenario for a cross-classified multiple membership data structure is depicted in Figure 10 as a simple example with the two higher levels, where every student attended their first school, and in years two through three their subsequent school could remain the same (as with Student A) or their subsequent school could change (as with Student C). Student C attended School 1 in year one, but then attended School 4 at some point in years two through three. Notice that dashed lines are now used to signify a change in school in subsequent years, because students are modeled as members of all the schools they attended. Therefore, the school effect on the slope in the model considers all of the schools attended by a student. This is accomplished with weighting the effects of organization-level units on the individual growth. Next, a discussion of the baseline unconditional three-level CCMM-GCM model is provided.

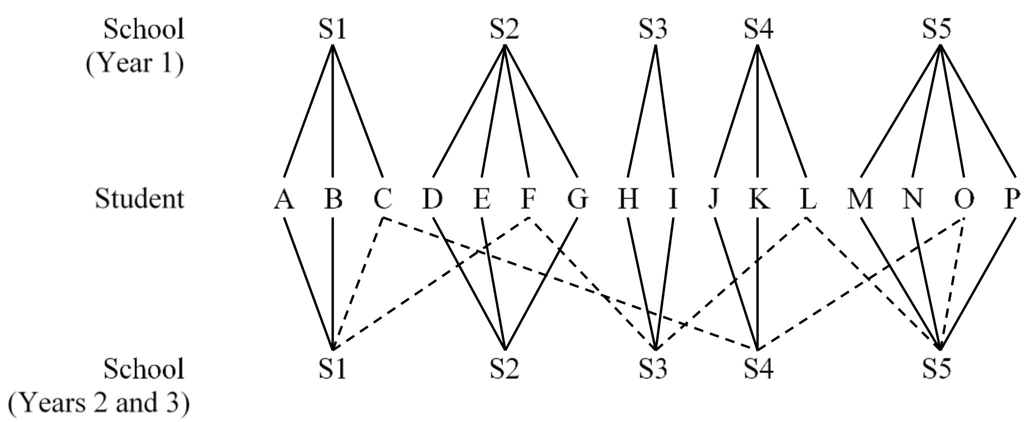

Figure 10. *Network graph of a cross-classified multiple membership structure where students (level 2) are cross-classified by the first and subsequent school attended (level 3) with some students attending multiple schools.*

Baseline Unconditional CCMM-GCM

Using the parentheses as before to signify cross-classification and then the brackets to denote multiple membership, the level-1 equation for the three-level baseline unconditional CCMM-GCM is

$$Y_{ti(j_1,\{j_2\})} = \pi_{0i(j_1,\{j_2\})} + \pi_{1i(j_1,\{j_2\})} TIME_{ti(j_1,\{j_2\})} + e_{ti(j_1,\{j_2\})}, \tag{19}$$

where the level-1 residual follows the same assumptions as before. Note that j_1 represents the first organization attended and $\{j_2\}$ represents the subsequent set of organizations attended. Level 2 is

$$\begin{cases} \pi_{0i(j_1,\{j_2\})} = \beta_{00(j_1,\{j_2\})} + r_{0i(j_1,\{j_2\})} \\ \pi_{1i(j_1,\{j_2\})} = \beta_{10(j_1,\{j_2\})} + r_{1i(j_1,\{j_2\})} \end{cases}, \tag{20}$$

with variances τ_{r00} and τ_{r11} for the residuals $r_{0i(j_1,\{j_2\})}$ and $r_{1i(j_1,\{j_2\})}$, respectively, and covariance τ_{r01} following the same assumptions as previously discussed in the GCM section. The level-3 equation is

$$\begin{cases} \beta_{00(j_1,\{j_2\})} = \gamma_{0000} + u_{00j_10} \\ \beta_{10(j_1,\{j_2\})} = \gamma_{1000} + u_{10j_10} + \sum_{h\in\{j_2\}} w_{tih} u_{100h} \end{cases}, \tag{21}$$

where the residuals u_{00j_10}, u_{10j_10}, and $u_{100\{j_2\}}$ are normally distributed with means of zero and variances $\tau_{u_{j_1}00}$, $\tau_{u_{j_1}11}$, and $\tau_{u_{\{j_2\}}11}$, respectively. The Cov(u_{00j_10}, u_{10j_10}) equals $\tau_{u_{j_1}01}$. $\tau_{u_{j_1}00}$ is the variance of initial status among the first organizational units, $\tau_{u_{j_1}11}$ is the variance of growth rates among the first organizations, and $\tau_{u_{\{j_2\}}11}$ is the variance of growth rates among the subsequent set of organizations attended.

The weight w_{tih} is assigned to each individual i who attended organization h at each time-point t, and the sum of the weights must equal to one. As an example, suppose Student C attended School 1 in the first two years and School 4 in the last two years of a study, then Student C's assigned weights would be one-third and two-thirds for Schools 1 and 4, respectively. Consequently, for student C the residual $u_{100\{j_2\}}$ for the subset of schools attended is weighted at level 3 for the slope equation. If a student was not mobile, such as Student A, or if the subsequent schools attended were the same (but different from the first) then the weight assigned for w_{tih} would be one.

As previously mentioned, the incorporation of u_{00j_10} into the intercept equation at level 3 allows the initial status outcomes to vary only across the first organization attended. The incorporation of the cross-classification of the first organization and subsequent organizations into the slope at level 3 allows the growth rate to vary across all organizations attended, meaning that the estimation of the slope is based on all organizations attended across the entire time of the study. Therefore, the effect of the organizations on the slope does not diminish over time; rather, it is cumulative. For the same mobile Student C, the slope would be a function of Schools 1 and 4 ($u_{1010} + {}^1\!/_3\, u_{1001} + {}^2\!/_3\, u_{1004}$), but then for non-mobile Student A, it would be a function of School 1 only ($u_{1010} + u_{1001}$). It should be noted that the effect of attending the first school and effect of attending the subsequent set of schools (even if they are the same, as with Student A) on the growth are allowed to differ, because the School 1 effect on growth at the first time-point may be different than the School 1 effect on growth for the successive time-points. The next section will present the conditional CCMM-GCM.

Conditional CCMM-GCM

For reasons discussed previously, it is important to include individual and organizational variables into a growth analysis. For the CCMM-GCM, the conditional level-2 model is

$$\begin{cases} \pi_{0i(j_1,\{j_2\})} = \beta_{00(j_1,\{j_2\})} + \beta_{01(j_1,\{j_2\})} X_{i(j_1,\{j_2\})} + r_{0i(j_1,\{j_2\})} \\ \pi_{1i(j_1,\{j_2\})} = \beta_{10(j_1,\{j_2\})} + \beta_{11(j_1,\{j_2\})} X_{i(j_1,\{j_2\})} + r_{1i(j_1,\{j_2\})} \end{cases} \tag{22}$$

and the conditional level-3 model is

$$\begin{cases} \beta_{00(j_1,\{j_2\})} = \gamma_{0000} + \gamma_{0010} Z_{j_1} + u_{00j_1 0} \\ \beta_{01(j_1,\{j_2\})} = \gamma_{0100} \\ \beta_{10(j_1,\{j_2\})} = \gamma_{1000} + \gamma_{1010} Z_{j_1} + u_{10j_1 0} + \sum_{h \in \{j_2\}} \left[w_{tih} \left(\gamma_{1001} Z_h + u_{100h} \right) \right] \\ \beta_{11(j_1,\{j_2\})} = \gamma_{1100} \end{cases}, \tag{23}$$

where $X_{i(j_1,\{j_2\})}$ is an individual variable modeled as fixed across organizations (for the sake of simplicity), Z_{j_1} is an organizational variable for the first organization attended, and $Z_{\{j_2\}}$ is an organization-level variable for the subsequent organizations attended that is weighted according to time spent within each subsequent organization. Here, γ_{0010} is the change in initial status for one unit change or contrast in the first organization's Z_{j_1} value when $X_{i(j_1,\{j_2\})}$ is zero and γ_{1010} is the change in growth rates for one unit change or contrast in the first organization's Z_{j_1} value when $X_{i(j_1,\{j_2\})}$ is zero. γ_{1001} is the change in growth rates for one unit change or contrast in the *weighted average* of the $Z_{\{j_2\}}$ values from all of the subsequent organizations attended when $X_{i(j_1,\{j_2\})}$ is zero. The focus will now turn to methodological work with hierarchical linear model that handles mobility to discuss those research findings.

Methodological Research with HLM that Handles Mobility

The sole methodological study conducted with the CCMM-GCM is the Grady (2010) dissertation and the only known simulation study using the cross-classified GCM is in Luo and Kwok (2012). Grady (2010) used the CCMM-GCM proposed in Grady and Beretvas (2010) to evaluate the effect of disregarding a cross-classified multiple membership data structure on the accuracy of parameter estimates. The comparison was made between the two different approaches, the CCMM-GCM and the *first school* GCM, with both the baseline unconditional and conditional versions of each model using Markov Chain Monte Carlo (MCMC) estimation (which is a Bayesian method). The other manipulated conditions were the percentage of mobile students (10%, 20%), the mean number of students per school (20, 40), and the number of measurement occasions (3, 5). The total number of schools was set at 50, and 100 replicated datasets were generated. The maximum number of school changes for the mobile students was set at two.

Results from the Grady (2010) study found that ignoring the multiple-membership data structure led to inaccurate parameter estimates for the between-schools variance in growth rates. The conclusions drawn upon results from the *first school* GCM would mislead researchers because the between-schools variance in growth rates was reallocated to the between-first-schools variance in growth rates. This means that the individual's growth rate would be modeled as only having been affected by the first school attended.

Luo and Kwok (2012) compared the *first school* GCM to the cross-classified GCM to also investigate the effect of ignoring mobile students. Their study manipulated the number of schools (25, 50), the number of students per school (50, 100), the percentage of mobile students (5%, 20%, 35%), the student-level variances and covariance $\left(\begin{bmatrix} .20 & .05 \\ .05 & .10 \end{bmatrix}, \begin{bmatrix} .10 & .025 \\ .025 & .05 \end{bmatrix}\right)$ and the school variance (0.1, 0.2). 200 replications were conducted using four measurement occasions. The authors split their simulation study, where the first study assumed mobile students only switched schools once at the same time, and the second study randomly switched students at each measurement occasion using the specific condition of 35% mobility rate, 50 schools, 100 students per school, school variance 0.2, and student-level covariance matrix $\begin{bmatrix} .10 & .025 \\ .025 & .05 \end{bmatrix}$.

The authors found that the pattern of mobility based on the different studies played a huge role on the impact of the direction and magnitude of the relative biases. For all conditions, the *first school* GCM redistributed to other levels the school variance, which underestimated the school-level variance component. In study 1, the redistributed variance was added to the student level, while in study 2 the variance was added to both the student and repeated measures levels which then led to underestimated student-level variance components for the intercept. In addition, the standard errors of the intercept and the coefficient of the school-level predictor were underestimated for all conditions. Under study 1, no other substantial bias was found in the standard errors of the other fixed effects, but in study 2 positive relative bias was found with the fixed effects using the *first school* GCM. Overall, model misspecification had a higher influence when the school-level variance was larger (0.2) with the smaller student-level variances and covariance $\left(\begin{bmatrix} .10 & .025 \\ .025 & .05 \end{bmatrix}\right)$.

Other methodological research has been conducted with multilevel models, but not necessarily growth curve models, that use the cross-classified or multiple membership random effects modeling approaches for handling mobile individuals. Meyers and Beretvas (2006) and Luo and Kwok (2009) evaluated cross-classified random effects models (CCREMs) against misspecified models that ignored or deleted mobile individuals. Their studies found little differences between approaches in the fixed effects estimates, where no differences were found with the predictors associated with the crossed factor that was not ignored in the misspecified HLMs. Standard errors of the parameter estimates of the predictors associated with the ignored crossed factor were underestimated with the misspecified models, while standard error estimates were overestimated with predictors at the lower level. Bias in the standard errors of the parameter estimates was exacerbated when the correlation between crossed factors was zero and also when the sample size of the ignored crossed-factor was larger (50 vs. 30). Model misspecification also led to overestimation of variance components between levels of the crossed factor that was not ignored in the HLMs as well as the level-1 variance component, because the ignored cross-factor variance was reapportioned to these other levels.

Chung and Beretvas (2012) compared the multiple membership random effects model (MMREM) to the misspecified HLM that ignored mobility. This study demonstrated that bias resulted in the estimate of the higher level's predictor as well as in the variance component estimates of both levels. The conditions manipulated in their study were the number of organizational units (50 vs. 100), number of individuals per organizational unit (30 vs. 60), intra-class correlation (0.05, 0.15, and 0.25), percentage of mobile individuals (10% vs. 20%), and number of moves mobile individuals made (2 vs. 3). The purpose of the current study is next presented, which ties together all of the research and techniques previously discussed in this chapter.

STATEMENT OF PURPOSE

As mentioned earlier, no previous research has suggested a latent variable regression model that handles individual mobility across clustering units. Mobile individuals are encountered frequently in longitudinal studies, especially in educational research. Students switch schools or move at rates ranging between 12% and 38.5% over the span of a few years (see, for example, Ihrke & Faber, 2012; U.S. Census Bureau, 2013; U.S. Government Accounting Office, 2010). A growth curve model has been derived to handle mobility, which is the cross-classified multiple membership GCM. Previous simulation studies have shown that model misspecification can lead to inaccurate estimates of between-schools variance components and standard errors of the fixed effects. The pattern of the mobility, where individuals were allowed to randomly change organizational units at any time-point, made a large impact on the results. Therefore, it is vital to appropriately model a cross-classified multiple membership data structure. By appropriately handling mobility and taking into account the expected differences in growth rates holding constant initial status, this study evaluated the cross-classified multiple membership LVR model using a real data set and a simulation study to test the extremes of certain conditions on the parameter estimates of the correctly and incorrectly specified models. This newly proposed model, termed the cross-classified multiple membership latent variable regression (CCMM-LVR) model, is defined in the following section.

Latent Variable Regression Modeling with Mobile Individuals

The following is the model specification of the newly proposed CCMM-LVR model. The level-2 formulation of the baseline unconditional CCMM-LVR model is

$$\begin{cases} \pi_{0i(j_1,\{j_2\})} = \beta_{00(j_1,\{j_2\})} + r_{0i(j_1,\{j_2\})} \\ \pi_{1i(j_1,\{j_2\})} = \beta_{10(j_1,\{j_2\})} + \mathrm{Bw}_{(j_1,\{j_2\})}\left(\pi_{0i(j_1,\{j_2\})} - \beta_{00(j_1,\{j_2\})}\right) + r_{1i(j_1,\{j_2\})} \end{cases} \tag{24}$$

and at level 3 the model is

$$\begin{cases} \beta_{00(j_1,\{j_2\})} = \gamma_{0000} + u_{00j_10} \\ \beta_{10(j_1,\{j_2\})} = \gamma_{1000} + \mathrm{Bb}\left(\beta_{00(j_1,\{j_2\})} - \gamma_{0000}\right) + u_{10j_10} + \sum_{h\in\{j_2\}} w_{tih}u_{100h}, \\ \mathrm{Bw}_{(j_1,\{j_2\})} = \mathrm{Bw\_0} + \mathrm{Bw\_1}\left(\beta_{00(j_1,\{j_2\})} - \gamma_{0000}\right) + u_{\mathrm{Bw}j_1} \end{cases} \tag{25}$$

where Bb is now the LVR coefficient that captures the change in growth rate for one unit increase in *first organization* j_1 mean initial status across *first organizations*, and γ_{100} is then the mean growth rate across organizations for *first organization* j_1 at the grand mean on initial status. Bw_0 is the effect of initial status on growth for *first organization* j_1 at the grand mean on initial status, and Bw_1 is the change in the effect of initial status on growth for one unit increase in mean initial status for *first organization* j_1. Also, γ_{1000} is now the mean growth rate across first and subsequent organizations for *first organization* j_1 at the grand mean on initial status.

The level-1 errors $e_{ti(j_1,\{j_2\})}$ are assumed to be normally distributed with a mean of zero and variance σ^2. The level-2 random effects are assumed normally distributed with means of zero and variances τ_{r00} and τ_{r11} for $r_{0i(j_1,\{j_2\})}$ and $r_{1i(j_1,\{j_2\})}$, respectively, and $\mathrm{Cov}(r_{0i(j_1,\{j_2\})}, r_{1i(j_1,\{j_2\})}) = 0$. The four level-3 random effects are assumed multivariate normally distributed with means of zero and a 4 by 4 covariance matrix $\mathbf{T}_\mathrm{u}$, which is defined as

$$\mathbf{T}_\mathrm{u} = \begin{bmatrix} \tau_{u_{j_1}00} & 0 & 0 & 0 \\ 0 & \tau_{u_{j_1}11} & 0 & \tau_{u_{j_1}\mathrm{Bw}11} \\ 0 & 0 & \tau_{u_{\{j_2\}}11} & 0 \\ 0 & \tau_{u_{j_1}11\mathrm{Bw}} & 0 & \tau_{u_{j_1}\mathrm{Bw}} \end{bmatrix}, \quad (26)$$

where $\tau_{u_{j_1}00}$ is defined as before, $\tau_{u_{j_1}11}$ is the variance in growth rates remaining among the first organizations after taking into account the first organization mean initial status, and $\tau_{u_{\{j_2\}}11}$ is the variance in growth rates remaining among the set of organizations attended after taking into account the first organization mean initial status, and $\tau_{u_{j_1}\mathrm{Bw}}$ is the variance in within-first-organization initial status on growth effects remaining between the first organizational units after taking into account the first organization mean initial status. Additionally, individual and organizational predictors can be incorporated into the model (as previously demonstrated), which will be presented in the following chapter when discussing the method.

For the current study, parameter estimates for the CCMM-LVR model were compared to those estimated assuming a three-level LVR model that ignores mobility by only recognizing the first school attended (HM3-LVR). The first study was a real data analysis that compared results for the two models that were estimating using a large-scale longitudinal dataset containing mobile students. The second was a simulation study that examined differences in relative parameter bias, relative standard error bias, root mean square error, and coverage rates from the two models to discover the impact of ignoring student mobility. The parameter estimate values from the real data analysis were used to help inform generating parameter values in the simulation study. In addition, some of the real dataset's characteristics were used in designing the simulation study's conditions. A simulation study was conducted because true population parameters are known and design factors can be manipulated to assess their impact on the resulting estimates. The conditions that were manipulated for the simulation study included the percentage of mobile students, number of schools, number of students per school, and number of measurement occasions. The real data analysis method and its results along with the details of the simulation study method will be presented in the next chapter.

Chapter 3: Method

This dissertation was comprised of two studies, one using a large-scale longitudinal real dataset and the other a simulation. The real data study investigated the differences in parameter and standard error estimates as well as model fit using two models, the HM3-LVR and the CCMM-LVR models, on a dataset that includes student mobility. The HM3-LVR is a three-level latent variable regression model that ignores student mobility by only modeling the first school students attended, while the CCMM-LVR model handles the multiple membership data structure. The simulation study examined the differences in results from the two models to discover the impact of ignoring multiple membership operationalized, here, as student mobility across schools.

REAL DATA STUDY

This section describes the real data study that was used to compare the parameter and standard error estimates from fitting the HM3-LVR and CCMM-LVR models to the dataset that included mobile students.

Data

The data used for the analysis of the real data study is from the Longitudinal Study of American Youth (LSAY) conducted from 1987 to 1994 (Miller, 1987-1994), which is a longitudinal study that investigated student achievement in mathematics and science from seventh through twelfth grade in the United States. This data is structured in a hierarchical manner, where measurement occasions are nested within students who are then nested within schools. The data also has a multiple membership structure because students switched schools throughout the duration of data collection. In the dataset for the students who entered the study in the fall of 1989 in ninth grade in the originally sampled high schools (schools 201-309), there is a total of 1,941 students and 45 schools. Three measurement occasions were used from the dataset, where students were tested at the beginning of each grade year: fall of ninth, tenth, and eleventh grades. Therefore, students without school identifiers at each of the three measurement occasions were removed, which then left 1,803 students. In addition, one school did not participate at every time-point in the study and was removed from the analysis leaving a dataset with 1,744 students and 44 schools.

Measures

For the current study, the three measurement occasions were used to examine student achievement in environmental sciences over time. This means that the achievement scores from fall of ninth (1989), tenth (1990), and eleventh (1991) grades were included as dependent measures for the models. The environmental sciences achievement scores are based on items from the National Assessment of Educational Progress (NAEP, 1986), that is scaled using an item response theory (IRT) model. Using IRT scaled scores allows for the scores to be compared across students and across time. Students who had scores for at least one of the measurement occasions were included in the analysis.

Level-2 and Level-3 Predictors

The level-2 (student level) predictor that was included in the models is gender, where $FEMALE_{i(j_1,\{j_2\})}$ = 1 for females and equals 0 for males. The level-3 (school level) predictor that was incorporated is school type, which is a dichotomous variable ($URBAN_{j_1}$

and $URBAN_{\{j_2\}}$) with values 1 for urban schools and 0 for non-urban schools. Students in the sample missing values on level-2 or level-3 predictors were removed from the analysis, which left a final sample total of 1,698 students.

Student Mobility

There were 194 (11.4%) students considered mobile from the sample of 1,698 students. Out of those mobile students, 64 (33.0%) changed schools only between the first and second measurement occasions, 100 (51.5%) changed schools solely between the second and third measurement occasions, and 30 (15.5%) switched schools twice between the first and second as well as the second and third time-points.

Analyses

The two models, HM3-LVR that ignores mobility and CCMM-LVR that handles student mobility, were fit to the sample using both a baseline unconditional model and a conditional model. The purpose was to compare differences in the parameter and standard error estimates as well as the deviance information criterion (DIC; Spiegelhalter, Best, Carlin, & van der Linde, 2002) values obtained from these models. In order to assess that a linear growth model was the functional form most appropriate, an inspection of a large number of individual growth trajectories was conducted.

The baseline unconditional HM3-LVR model fit to the data was exactly the same as in Equations 5, 12, and 13 for levels one, two, and three, respectively. The $TIME_{tij}$ variable was assigned values of 0, 1, and 2 for the ninth, tenth, and eleventh grade measurement occasions, respectively. In addition, this model ignored any school changes made by students, and used their first school attended as the school identifier for all three measurement occasions. The first school was chosen for this study because in research studies, especially those using randomized control trials or cluster randomized trials, school identifier information is more likely known from the initial measurement occasion in the study, and identifiers for schools are typically missing for mobile students whose outcome scores might be missing at later time points are typically missing.

The conditional HM3-LVR model is specified as follows for level one:

$$SCORE_{tij} = \pi_{0ij} + \pi_{1ij}TIME_{tij} + e_{tij}, \tag{27}$$

for level two,

$$\begin{cases} \pi_{0ij} = \beta_{00j} + \beta_{01j}\left(FEMALE_{ij} - \overline{FEMALE}_{..}\right) + r_{0ij} \\ \pi_{1ij} = \beta_{10j} + \mathrm{Bw}_j\left(\pi_{0ij} - \beta_{00j}\right) + \beta_{11j}\left(FEMALE_{ij} - \overline{FEMALE}_{..}\right) + r_{1ij} \end{cases}, \tag{28}$$

and for level three,

$$\begin{cases} \beta_{00j} = \gamma_{000} + \gamma_{001}URBAN_j + u_{00j} \\ \beta_{01j} = \gamma_{010} \\ \beta_{10j} = \gamma_{100} + \mathrm{Bb}\left(\beta_{00j} - \gamma_{000}\right) + \gamma_{101}URBAN_j + u_{10j} \\ \beta_{11j} = \gamma_{110} \\ \mathrm{Bw}_j = \mathrm{Bw\_0} + \mathrm{Bw\_1}\left(\beta_{00j} - \gamma_{000}\right) + \mathrm{Bw\_2}\left(URBAN_j\right) + u_{\mathrm{Bw}j} \end{cases}. \tag{29}$$

Note that for simplicity's sake, the gender effect is modeled as fixed across schools for the intercept and slope. The $TIME_{tij}$ variable took on the same values as the baseline unconditional HM3-LVR model, in order for the intercept to take on the meaning as initial status.

The baseline unconditional CCMM-LVR model that handles mobility was fit to the data using Equations 19, 20, and 21. The conditional CCMM-LVR model that was used to estimate the parameters and standard errors is as follows for level one:

$$SCORE_{ti(j_1,\{j_2\})} = \pi_{0i(j_1,\{j_2\})} + \pi_{1i(j_1,\{j_2\})} TIME_{ti(j_1,\{j_2\})} + e_{ti(j_1,\{j_2\})}, \quad (30)$$

for level two

$$\begin{cases} \pi_{0i(j_1,\{j_2\})} = \beta_{00(j_1,\{j_2\})} + \beta_{01(j_1,\{j_2\})}\left(FEMALE_{i(j_1,\{j_2\})} - \overline{FEMALE}_{..}\right) + r_{0i(j_1,\{j_2\})} \\ \pi_{1i(j_1,\{j_2\})} = \beta_{10(j_1,\{j_2\})} + \mathrm{Bw}_{(j_1,\{j_2\})}\left(\pi_{0i(j_1,\{j_2\})} - \beta_{00(j_1,\{j_2\})}\right) \\ \qquad + \beta_{11(j_1,\{j_2\})}\left(FEMALE_{i(j_1,\{j_2\})} - \overline{FEMALE}_{..}\right) + r_{1i(j_1,\{j_2\})} \end{cases}, \quad (31)$$

and at level three the model is

$$\begin{cases} \beta_{00(j_1,\{j_2\})} = \gamma_{0000} + \gamma_{0010} URBAN_{j_1} + u_{00j_1 0} \\ \beta_{01(j_1,\{j_2\})} = \gamma_{0100} \\ \beta_{10(j_1,\{j_2\})} = \gamma_{1000} + \mathrm{Bb}\left(\beta_{00(j_1,\{j_2\})} - \gamma_{0000}\right) + \gamma_{1010} URBAN_{j_1} \\ \qquad + u_{10j_1 0} + \sum_{h\in\{j_2\}}\left[w_{tih}\left(\gamma_{1001} URBAN_h + u_{100h}\right)\right] \\ \beta_{11(j_1,\{j_2\})} = \gamma_{1100} \\ \mathrm{Bw}_{(j_1,\{j_2\})} = \mathrm{Bw\_0} + \mathrm{Bw\_1}\left(\beta_{00(j_1,\{j_2\})} - \gamma_{0000}\right) + \mathrm{Bw\_2}\left(URBAN_{j_1}\right) + u_{\mathrm{Bw}j_1} \end{cases}. \quad (32)$$

Once again, for the sake of simplicity, the student-level predictor was modeled as fixed. The weights that were used for both the baseline unconditional and conditional CCMM-LVR models were based on how long a student was a member of a school. If a student did not change schools or their subsequent school remained the same from the second through third measurement occasions, then their weight assigned was one. If a student only changed schools between the second and third measurement occasions, the subsequent schools attended had a weight of one-half for the first school and a weight of one-half for the second school. This same weighting scenario applied to a student who changed schools between each time-point, where a weight of one-half was associated with each subsequent school attended.

All four models were fit using R software (version 3.1.0; R Core Team, 2014) with the package R2jags (version 0.04-03; Su & Yajima, 2014), which is the R interface to the Just Another Gibbs Sampler (JAGS) MCMC software (Plummer, 2013). JAGS (version 3.4.0) is open sourced and works by having the user specify any kind of statistical model, then JAGS uses the Gibbs sampler to determine the appropriate Markov Chain Monte Carlo (MCMC) arrangement for analyzing the statistical model. The prior specification set for all of the fixed effects parameters was a normal distribution with a mean of 0 and a very large variance of 100,000 (or 0.00001 in terms of precision for JAGS). The priors were set to the inverse-Pareto(1, 0.0001) distribution for the scalar variance components and the inverse-Wishart distribution for the variance-covariance matrix associated with the β_{10j} and Bw_j level-3 equations, which is recommended based on the simulation from Choi and Seltzer (2010). To determine the burn-in period and number of iterations for convergence, an examination of the trace plots, autocorrelation function plots, and Gelman-Rubin statistics was conducted. The examination indicated one chain with a burn-in period of 10,000 iterations and an additional 50,000 iterations, for a total of 60,000 iterations, was optimal.

In addition to comparing parameter and standard errors estimates across models, comparisons between model fit were made. For MCMC estimation, the DIC was utilized, where smaller DIC values indicate better fit. The DIC fit index is defined as

$$\mathrm{DIC} = \overline{D} + p_D, \tag{33}$$

where $\overline{D}$ is the posterior mean deviance and p_D is the effective number of parameters in the model. The next section describes the results from the real data analysis.

Results

This section summarizes the results from the real data analysis previously described. The descriptive statistics are presented for the sample utilized in the real data analysis, as well as the results for the fixed and random effects estimated assuming the baseline unconditional and conditional models for the two types of models (CCMM-LVR and HM3-LVR).

Descriptive Statistics

Descriptive statistics are provided in Table 1 for the environmental sciences achievement scores at each of the three measurement occasions.

Table 1

Descriptive Statistics for Environmental Sciences Achievement Scores at Each Measurement Occasion

Outcome	Variable Name	*M*	*SD*	*N*
Score at Time 1	Y_{1ij}	59.7	13.01	1,666
Score at Time 2	Y_{2ij}	63.2	13.20	1,668
Score at Time 3	Y_{3ij}	65.8	13.48	1,534

In Table 2, the descriptive statistics are displayed for the level-2 (student-level) and level-3 (school-level) predictors, gender and school type, respectively, from the real data sample.

Table 2

Descriptive Statistics for Gender and School Type in the Real Data Analysis

Predictor	*N*	Percentage
Gender		
Female	848	49.94%
Male	850	50.06%
School type		
Urban	12	27.27%
Non-urban	32	72.73%

The following two sections discuss the parameter and standard error estimates for the baseline unconditional models comparing the CCMM-LVR and the HM3-LVR.

Baseline Unconditional Fixed Effects

In Table 3, the fixed effects parameter estimates for the two models, CCMM-LVR and HM3-LVR, are presented for the baseline unconditional versions.

Table 3

Fixed Effects Parameter and Standard Error Estimates for the Baseline Unconditional CCMM-LVR and HM3-LVR Models

	Estimating Model					
	CCMM-LVR			HM3-LVR		
Parameter	Coeff.	Est.	(*SE*)	Coeff.	Est.	(*SE*)
Model for intercept						
Grand mean	γ_{0000}	59.285	(0.849)	γ_{000}	59.222	(0.820)
Model for slope						
Grand mean	γ_{1000}	2.755	(0.197)	γ_{100}	2.832	(0.194)
School mean initial status	Bb	–0.041	(0.043)	Bb	–0.039	(0.042)
Model for Bw						
Grand mean	Bw_0	0.005	(0.013)	Bw_0	0.007	(0.013)
School mean initial status	Bw_1	0.003	(0.003)	Bw_1	0.003	(0.003)

Note. CCMM-LVR = cross-classified multiple membership latent variable regression; HM3-LVR = three-level latent variable regression; Coeff. = coefficient; Est. = parameter estimate; *SE* = standard error estimate.

To assist in understanding the coefficients from the CCMM-LVR and HM3-LVR models, the CCMM-LVR results will be fully interpreted along with a presentation of graphical explanations. From Table 3, the grand mean of the initial status (i.e., intercept) is 59.285 for the CCMM-LVR model, and the grand mean of the growth (i.e., slope) is 2.755. The Bb coefficient is negative, which indicates that the growth rate for a school with a higher mean initial status will be lower than the growth rate for a school with a lower mean initial status. To demonstrate visually, consider three schools, where School 1 is two standard deviations (*SD*s, 10.32 points) below the grand mean initial status, School 2 is at the grand mean initial status, and School 3 is two *SD*s above the grand mean initial status. Expected school growth rates are calculated using the grand mean growth rate, γ_{1000} (2.755), and the between-schools effect of initial status on growth, Bb (−0.041). Therefore, the expected growth rate for students in School 1 would be 3.18 points per grade [i.e., 2.755 + (−0.041 × −10.23)], for School 2 it would be 2.76 points per grade, and for School 3 it would be 2.33 points per grade. Figure 11 displays the expected growth rates for the three schools, where the negative relationship between school mean initial status and school mean growth rate can be seen.

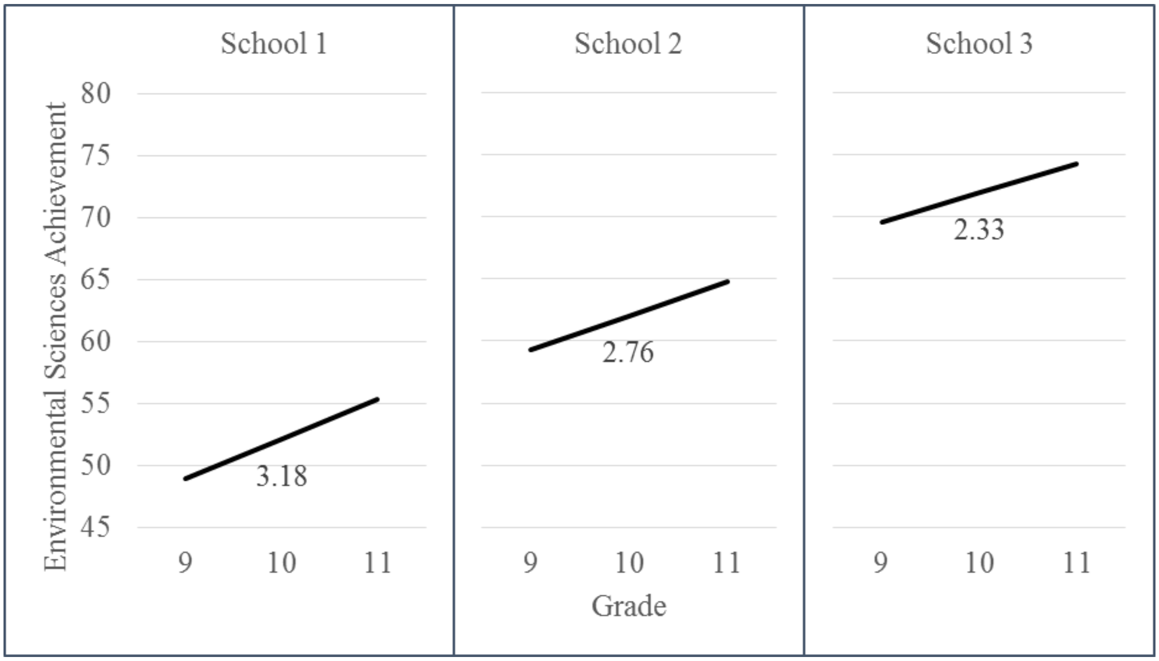

Figure 11. *Expected growth trajectories for Schools 1, 2 and 3 with mean initial status values that are two SDs below the grand mean intercept, at the grand mean intercept, and two SDs above the grand mean intercept, respectively*.

To help visualize the expected growth rates *within schools*, consider three students from each of the previous three schools who are, respectively, two *SD*s (22.23 points) below their school's mean initial status, at their school's mean initial status, and two *SD*s above their school's mean initial status. The expected growth trajectories within a school are based on the growth (2.755), Bb (−0.041), Bw_0 (0.005), and Bb_1 (0.003) parameter estimate values. Figure 12 displays the expected growth trajectories for the three students within each of the three schools. The expected growth rates increase as the students' initial statuses increase within School 2 and School 3. For School 1, the students' expected growth rates decrease as the values for initial status increase. For Student C within School 1, for example, the expected growth rate is calculated by adding the school's expected growth rate (3.18) with the value from the model for Bw [i.e., $(0.005 \times -22.23) + (0.003 \times -10.32 \times -22.23) = 0.57$] to obtain $3.18 + 0.57 = 3.75$.

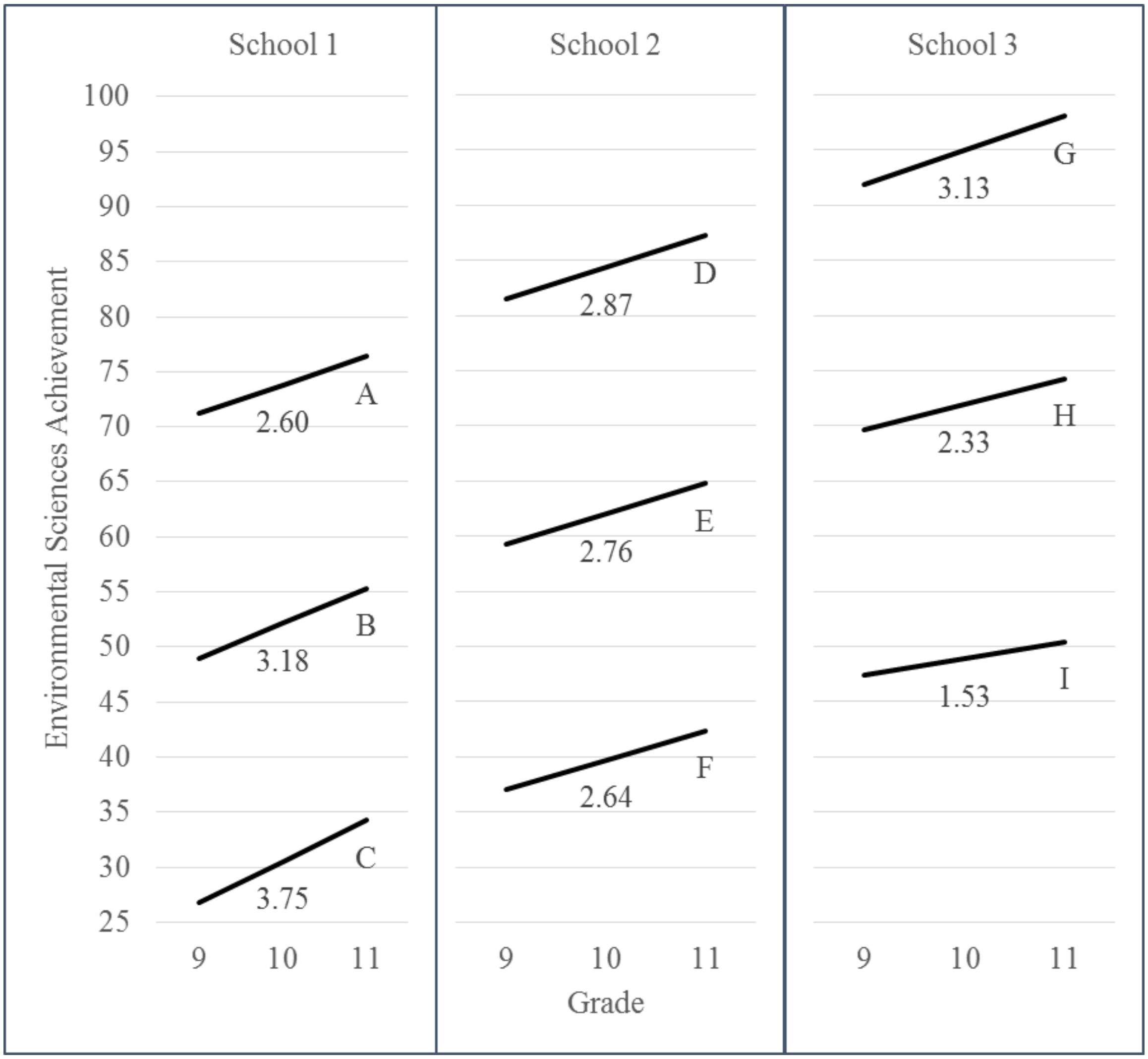

Figure 12. *Expected growth trajectories for three students within Schools 1, 2, and 3 with initial status values that are two SDs below their school's mean initial status, at their school's mean initial status, and two SDs above their school's mean initial status, respectively.*

For the baseline unconditional fixed effects parameters in Table 3, there were minimal differences between the two models' estimates. The fixed effects standard error estimates also revealed very few differences between the two types of baseline unconditional models. The values of the Bw_0 parameter estimates differed (0.005 versus 0.007 for the CCMM-LVR and HM3-LVR models, respectively).

Baseline Unconditional Random Effects

Table 4 provides the results for the variance components estimates and standard error estimates for the baseline unconditional versions of the CCMM-LVR and HM3-LVR models.

Table 4

Random Effects Parameter and Standard Error Estimates for Baseline Unconditional CCMM-LVR and HM3-LVR Models

	Estimating Model					
	CCMM-LVR			HM3-LVR		
Parameter	Coeff.	Est.	(*SE*)	Coeff.	Est.	(*SE*)
Level-1 variance between						
Measures	σ^2	24.185	(0.856)	σ^2	24.189	(0.857)
Intercept variance between						
Students	τ_{r00}	123.528	(5.020)	τ_{r00}	123.279	(4.999)
1st schools	$\tau_{u_{j1}00}$	26.603	(7.828)	τ_{u00}	26.589	(7.247)
Slope variance between						
Students	τ_{r11}	3.437	(0.739)	τ_{r11}	3.596	(0.725)
1st schools	$\tau_{u_{j1}11}$	0.302	(0.197)	τ_{u11}	0.936	(0.300)
Subsequent schools	$\tau_{u_{\{j2\}}11}$	1.009	(0.494)	—	—	—
Bw variance between						
1st schools	$\tau_{u_{j1}\mathrm{Bw}}$	0.002	(0.001)	$\tau_{u\mathrm{Bw}}$	0.001	(0.001)

Note. — = not applicable; CCMM-LVR = cross-classified multiple membership latent variable regression; HM3-LVR = three-level latent variable regression; Coeff. = coefficient; Est. = parameter estimate; *SE* = standard error estimate.

For the baseline unconditional model's level-1 variance, parameter and standard error estimates, σ^2, were very similar for the two models, CCMM-LVR and HM3-LVR. Parameter and standard error estimates of the between-students intercept variance (τ_{r00}), as well as between-first-schools intercept variances ($\tau_{u_{j1}00}$ and τ_{u00}), were similar for the two models as well. The values of the between-students slope variance, τ_{r11}, along with associated standard error estimates, were similar for the CCMM-LVR and HM3-LVR models. The between-first-schools Bw variance, $\tau_{u_{j1}\mathrm{Bw}}$ and $\tau_{u\mathrm{Bw}}$, and associated standard error was slightly larger for the CCMM-LVR model (0.002 and its *SE* = 0.00095) than for the HM3-LVR model (0.001 and its *SE* = 0.00081).

Large differences were found in the estimates of the between-first-schools slope variance, $\tau_{u_{j1}11}$ and τ_{u11}, with values of 0.302 and 0.936 for the CCMM-LVR and HM3-LVR models, respectively. Under the CCMM-LVR model, the between-schools slope variance is partitioned into the between-first-schools slope variance ($\tau_{u_{j1}11}$) and the between-subsequent-schools slope variance ($\tau_{u_{\{j2\}}11}$). The parameter estimate of the between-first-schools slope variance, $\tau_{u_{j1}11}$, for the CCMM-LVR model was less than one-third of the estimate, τ_{u11}, for the HM3-LVR model, while the associated standard error estimates were smaller for the CCMM-LVR model (0.197 versus 0.300). The difference in the parameter estimates, $\tau_{u_{j1}11}$ and τ_{u11}, was captured in the parameter estimate of the between-subsequent-schools slope variance, $\tau_{u_{\{j2\}}11}$ (1.009).

The next two sections discuss the parameter and standard error estimates for the conditional CCMM-LVR and HM3-LVR models.

Conditional Fixed Effects

In Table 5, the estimates of the conditional models' fixed effects for the two models, CCMM-LVR and HM3-LVR, are presented.

Table 5

Fixed Effects Parameter and Standard Error Estimates for Conditional CCMM-LVR and HM3-LVR Models that Include Level-2 and Level-3 Predictors

	Estimating Model					
	CCMM-LVR			HM3-LVR		
Parameter	Coeff.	Est.	(*SE*)	Coeff.	Est.	(*SE*)
Model for intercept						
Grand mean	γ_{0000}	60.098	(0.949)	γ_{000}	60.043	(0.988)
FEMALE	γ_{0100}	–1.241	(0.588)	γ_{010}	–1.274	(0.604)
Sch1_URBAN	γ_{0010}	–3.066	(1.935)	γ_{001}	–2.936	(1.858)
Model for slope						
Grand mean	γ_{1000}	2.921	(0.215)	γ_{100}	3.018	(0.216)
School mean initial status	Bb	–0.058	(0.043)	Bb	–0.053	(0.043)
FEMALE	γ_{1100}	–0.326	(0.209)	γ_{110}	–0.334	(0.219)
Sch1_URBAN	γ_{1010}	–1.220	(0.842)	γ_{101}	–0.877	(0.417)
SubSch_URBAN	γ_{1001}	0.418	(0.824)	—	—	—
Model for Bw						
Grand mean	Bw_0	0.009	(0.014)	Bw_0	0.009	(0.015)
School mean initial status	Bw_1	0.003	(0.003)	Bw_1	0.002	(0.003)
Sch1_URBAN	Bw_2	–0.011	(0.028)	Bw_2	–0.012	(0.026)

Note. — = not applicable; CCMM-LVR = cross-classified multiple membership latent variable regression; HM3-LVR = three-level latent variable regression; Coeff. = coefficient; Est. = parameter estimate; *SE* = standard error estimate; *FEMALE* = whether student was female; *Sch1_URBAN* = whether first school was rural; *SubSch_URBAN* = weighted average of school type being rural.

The parameter and standard error estimates in the model for the intercept were similar across the CCMM-LVR and HM3-LVR models. A substantial difference was found in estimates of the effect of the first school's type (urban or not) on the slope, while all other estimate in the model for the slope were similar. The HM3-LVR model resulted in a much weaker parameter estimate (γ_{101} = –0.877) of the effect of the first school urbanicity on the slope with a smaller standard error estimate (0.417) as compared with the CCMM-LVR model's estimates (γ_{1010} = –1.220 and its *SE* = 0.842). The difference in the parameter estimates, γ_{1010} and γ_{101}, is reflected in the parameter estimate, γ_{1001}, for the effect of the weighted average subsequent schools' type on the slope in the CCMM-LVR model (0.418). The Bw_2 parameter estimates from the model for Bw slightly differed for the CCMM-LVR (–0.011) and HM3-LVR (–0.012) models. The values for the Bw_0 and Bw_1 parameter estimates and all of the standard error estimates in the model for Bw were very similar between the two estimating models.

Conditional Random Effects

The parameter and standard error estimates for the variance components of the random effects for the conditional CCMM-LVR and HM3-LVR models are displayed in Table 6.

Table 6

Random Effects Parameter and Standard Error Estimates for Conditional CCMM-LVR and HM3-LVR Models that Include Level-2 and Level-3 Predictors

	Estimating Model					
	CCMM-LVR			HM3-LVR		
Parameter	Coeff.	Est.	(*SE*)	Coeff.	Est.	(*SE*)
Level-1 variance between						
Measures	σ^2	24.144	(0.849)	σ^2	24.160	(0.830)
Intercept variance between						
Students	τ_{r00}	123.301	(5.184)	τ_{r00}	123.203	(5.256)
1st schools	$\tau_{u_{j1}00}$	26.126	(7.512)	τ_{u00}	26.380	(7.798)
Slope variance between						
Students	τ_{r11}	3.424	(0.739)	τ_{r11}	3.601	(0.727)
1st schools	$\tau_{u_{j1}11}$	0.298	(0.183)	τ_{u11}	0.875	(0.292)
Subsequent schools	$\tau_{u_{\{j2\}}11}$	0.890	(0.424)	—	—	—
Bw variance between						
1st schools	$\tau_{u_{j1}\mathrm{Bw}}$	0.002	(0.001)	$\tau_{u\mathrm{Bw}}$	0.002	(0.001)

Note. — = not applicable; CCMM-LVR = cross-classified multiple membership latent variable regression; HM3-LVR = three-level latent variable regression; Coeff. = coefficient; Est. = parameter estimate; *SE* = standard error estimate.

The pattern of results for the random effects variance component estimates for the conditional models were generally similar to the results from the baseline unconditional models. The parameter and standard error estimates of the level-1 variance, σ^2, for the conditional models were very similar for the CCMM-LVR and HM3-LVR models. Parameter and standard error estimates of the between-students intercept variance (τ_{r00})

and between-first-schools intercept variance ($\tau_{u_{j1}00}$ and τ_{u00}) were also similar for the two models. The between-students slope variance estimates, τ_{r11}, along with their standard error estimates, were similar for the CCMM-LVR and HM3-LVR models as well. The between-first-schools Bw variance estimates, $\tau_{u_{j1}\text{Bw}}$ and $\tau_{u\text{Bw}}$, and associated standard error estimates were identical to the third decimal place.

The parameter estimates of the between-first-schools slope variance, $\tau_{u_{j1}11}$ and τ_{u11}, revealed some large differences, with values of 0.298 and 0.875 for the CCMM-LVR and HM3-LVR models, respectively. Under the CCMM-LVR model, the between-schools slope variance is partitioned into the between-first-schools slope variance ($\tau_{u_{j1}11}$) and the between-subsequent-schools slope variance ($\tau_{u_{\{j2\}}11}$). The parameter estimate of the between-first-schools slope variance, $\tau_{u_{j1}11}$, for the CCMM-LVR model was about a third of the size of the value for the HM3-LVR model estimate, τ_{u11}, while the associated standard error estimate was smaller for the CCMM-LVR model (0.183 versus 0.292). The difference between the parameter estimates, $\tau_{u_{j1}11}$ and τ_{u11}, was reflected in the parameter estimate, $\tau_{u_{\{j2\}}11}$ (0.890).

The next section discusses the fit index results for the baseline unconditional and conditional models comparing the CCMM-LVR and HM3-LVR models.

Fit Index

The DIC values for the baseline unconditional and conditional CCMM-LVR and HM3-LVR models are presented in Table 7.

Table 7

Deviance Information Criterion Values for the Baseline Unconditional and Conditional Models

	Estimating Model	
	CCMM-LVR	HM3-LVR
Model Type	DIC	DIC
Baseline unconditional	39,129.7	39,241.4
Conditional	38,871.9	38,736.4

Note. DIC = deviance information criterion.

The DIC values were lower for the CCMM-LVR model than the HM3-LVR model for the baseline unconditional model that was estimated, while the opposite result was found for the conditional models. The magnitude of the difference was 111.7 for the baseline unconditional models and 135.5 for the conditional models. Also, for the CCMM-LVR and HM3-LVR models, the DIC values for the baseline unconditional models were larger than for the conditional models.

As noted, some differences were found for the CCMM-LVR and HM3-LVR models' estimates in the real data analysis. While it is possible to hypothesize the source of some of these differences, it is not clear with real data which model's estimates are actually closer to the true values. The simulation study helps to figure out which models' estimates are closer to the truth and to understand how and why these differences have occurred. The following section describes the simulation study that used parameter estimate results and other characteristics of the real dataset to generate datasets and establish conditions.

SIMULATION STUDY

While the real data study provided valuable information about demonstrating how to specify the newly formed CCMM-LVR model with real data, it did not provide the answer to which model's estimates are closer to truth. To achieve these goals, a simulation study was accompanied, where true population parameters are known and design factors can be manipulated to assess their impact on resulting estimates.

Conditions

Several conditions were manipulated to evaluate their effect on parameter recovery. A total of four conditions (i.e., factors) were manipulated in this study. The first condition is the percentage of mobile students, which has two values that were manipulated (10%, 20%). The second factor is the number of schools, which is the level-3 unit, and has two values that were investigated (50, 100). The third factor is the number of students per school, i.e., the number of level-2 units per level-3 unit, and has two values (50, 100). The final factor is the number of measurement occasions, which also has two values (3, 4). The overview of the manipulated conditions is presented in Table 8. The next sections provide more information about each of the conditions.

Table 8

Combinations of Design Factors

Manipulated Conditions			
Mobility Rate	Number of Schools	Number of Students Per School	Number of Measurement Occasions
10%	50	40	3
10%	50	40	4
10%	50	80	3
10%	50	80	4
10%	100	40	3
10%	100	40	4
10%	100	80	3
10%	100	80	4
20%	50	40	3
20%	50	40	4
20%	50	80	3
20%	50	80	4
20%	100	40	3
20%	100	40	4
20%	100	80	3
20%	100	80	4

Mobility Rate

The two values chosen for the percentage of mobile students were 10% and 20%. These mobility rates reflect low and medium percentages of mobility in educational longitudinal studies. A student is considered mobile when they change schools between two measurement occasions. Recent geographical mobility trends in the United States discovered that between 12% and 38.5% of students (people aged 5 to 17) relocated in the span of two to five years (Ihrke & Faber, 2012; U.S. Census Bureau, 2013; U.S. Government Accounting Office, 2010). In addition, rates similar to these have been used in other educational mobility simulation studies (Chung & Beretvas, 2012; Grady, 2010; Luo & Kwok, 2012). Last, the low mobility rate value is similar to the mobility rate found in the real data analysis (11.4%).

Number of Schools

The two values chosen for the number of schools, which are the level-3 units, were 50 and 100. Chung and Beretvas (2012) estimated a multiple membership multilevel model and found that utilizing 50 higher-level clustering units led to reasonable parameter recovery, while using 100 clustering units led to better parameter recovery. Grady (2010) set a fixed number of level-3 units to 50 for the CCMM-GCM. Therefore, the two values selected were 50 and 100 level-3 units. The lower value is also close to the number of organizational units (schools) found in the real data analysis, which was 44.

Number of Students per School

The values selected for the number of level-2 units per level-3 unit, which represents, here, the average school size, were 40 and 80. The average number of students (level-2 units) per school (level-3 unit) was 39 in the real dataset. The value of 40 was based on this average, and an upper average school size was chosen for the higher value, which is based on doubling the real-data average. Based on these two condition values along with the number of schools values, the total number of students simulated was 2,000 for the 40 level-2 units condition for each of 50 level-3 units, 4,000 for the 40 level-2 units per 100 level-3 units condition, 4,000 for the 80 level-2 units per 50 level-3 units condition, and 8,000 for the 80 level-2 units per 100 level-3 units condition. These student total values resemble previous student totals found in growth analysis demonstrations and simulation studies, such as Seltzer et al. (2003), Choi and Seltzer (2010), Grady and Beretvas (2010), and Luo and Kwok (2012).

Number of Measurement Occasions

The two values, 3 and 4, were chosen for the number of measurement occasions per simulated student. Longitudinal studies involve a minimum of three measurement occasions (see Bryk & Raudenbush, 1987; Bryk & Weisberg, 1977; Rogosa et al., 1982), but each additional time-point adds cost to a study and measures become less stable over longer periods of time. As an example, the studies by Grady and Beretvas (2010) used three time-points, Luo and Kwok (2012) used four time-points, and Choi and Seltzer (2010) used four measurement occasions. In addition, Grady (2010) utilized three and five measurement occasions for the simulation study, and the real data analysis conducted here consisted of three time-points.

Data Generation

For each combination of the conditions, 1,000 datasets were generated for a total of 16,000 datasets with the 16 conditions estimated by the two models. The data were generated and estimated in R using the R2jags package and JAGS (version 3.4.0) software. Each model was estimated using MCMC estimation with 50,000 iterations and a burn-in period of 10,000 iterations, with the following details provided below.

Generating Models

All of the simulated datasets were designed to have a three-level multiple membership structure, where level-1 represents measurement occasion, level-2 is the student level, and level-3 is the school-level unit. Reflecting the proportions in the real data, 30% of the schools were designated as non-mobile and 70% were designated as mobile. As an example using the 100 level-3 units condition, schools 1-70 contained mobile students and schools 71-100 did not have mobile students. Therefore, if a student was randomly assigned to one of the non-mobile schools, then this student was not mobile. If a student was randomly assigned to one of the mobile schools, then there was a chance, depending on the mobility rate condition (10% or 20%), that this student could have been randomly selected as a mobile student. Therefore, the selection of mobile students was random and based on the mobility rate, but once selected as mobile, the students were associated only with the designated mobile schools. The subsequent school or schools attended also only occurred within this mobile designated group of schools. For example, if a student from the mobile School 4 was selected as mobile, then that student was assigned mobile School 5 (one plus first school identifier number) as the next school attended. If a student was selected as mobile from mobile School 70, then the subsequent school move was to School 1.

The pattern of mobility also varied to reflect what occurred in the real data. Of the mobile students, 33% were randomly selected to change schools once between measurement occasions one and two, 51.5% were randomly selected to change schools once between measurement occasions two and three, and 15.5% were randomly selected to change schools twice between measurement occasions one and two as well as between time-points two and three. This pattern was the same regardless of the measurement occasions condition, meaning that in the four measurement occasions condition no students changed schools between time-points three and four. Note that students who were randomly selected to change schools twice attended mobile schools S1, S1 + 1, and S2 + 2, unless they were randomly assigned to School 69 or School 70 (in the 100 level-3 units condition, as a demonstration). These students attended mobile Schools 69, 70, and 1 and Schools 70, 1, and 2, respectively.

No weights were used for the HM3-LVR model, but weights were used for the measurement occasions after the first time-point in the CCMM-LVR model. A weight of one was assigned to the single school attended by each of the non-mobile students, as well as to mobile students who only changed schools once between measurement occasions one and two. For the three measurement occasions condition, multiple membership weights of one-half were assigned to each of the two schools attended, respectively, for the mobile students who changed schools once between time-points two and three as well as for those who changed schools twice. For the four measurement occasions condition, mobile students who changed schools between time-points two and three as well as those who changed schools twice were assigned weights of one-third and two-thirds associated with the last two schools attended, respectively.

The baseline unconditional data generating model was a three-level cross-classified multiple membership latent variable regression model, where level-1 is

$$Y_{ti(j_1,\{j_2\})} = \pi_{0i(j_1,\{j_2\})} + \pi_{1i(j_1,\{j_2\})} TIME_{ti(j_1,\{j_2\})} + e_{ti(j_1,\{j_2\})}, \tag{34}$$

level two is

$$\begin{cases} \pi_{0i(j_1,\{j_2\})} = \beta_{00(j_1,\{j_2\})} + r_{0i(j_1,\{j_2\})} \\ \pi_{1i(j_1,\{j_2\})} = \beta_{10(j_1,\{j_2\})} + \text{Bw}_{(j_1,\{j_2\})}\left(\pi_{0i(j_1,\{j_2\})} - \beta_{00(j_1,\{j_2\})}\right) + r_{1i(j_1,\{j_2\})} \end{cases}, \tag{35}$$

and level three is

$$\begin{cases} \beta_{00(j_1,\{j_2\})} = 59.285 + u_{00j_10} \\ \beta_{10(j_1,\{j_2\})} = 2.755 - 0.041\left(\beta_{00(j_1,\{j_2\})} - 59.285\right) + u_{10j_10} + \sum_{h\in\{j_2\}} w_{tih}u_{100h} \\ \text{Bw}_{(j_1,\{j_2\})} = 0.005 + 0.003\left(\beta_{00(j_1,\{j_2\})} - 59.285\right) + u_{\text{Bw}j_1} \end{cases}. \tag{36}$$

The time variable $TIME_{ti(j_1,\{j_2\})}$ took on the values 0, 1, and 2 for the three measurement occasions condition, and took on the additional value of 3 for the four measurement occasions condition.

Fixed Effects

The true fixed effect values for the baseline unconditional generating models were set to 59.285 for the intercept (γ_{0000}), 2.755 for the slope (γ_{1000}), –0.041 for the effect of Bb on the slope, 0.005 for the effect of Bw_0 on the Bw model, and 0.003 for the effect of Bw_1 on the Bw model. The true (generating) values for these fixed effects were obtained from the real data analysis.

Random Effects

For the baseline unconditional generating models, the level-1 error $e_{ti(j_1,\{j_2\})}$ was sampled from a normal distribution with a mean of zero and a variance (σ^2) of 24.185. The level-2 residuals $r_{0i(j_1,\{j_2\})}$ and $r_{1i(j_1,\{j_2\})}$ were each generated from normal distributions with means of zero and variances of 123.528 ad 3.437, respectively. The covariance of $r_{0i(j_1,\{j_2\})}$

and $r_{1i(j_1,\{j_2\})}$ was generated to be zero. The level-3 residual u_{00j_10} was generated from a normal distribution with a mean of zero and variance set to 26.603, and it was not generated to co-vary (covariance set to zero) with any of the other level-3 residuals. The level-3 residual $u_{100\{j_2\}}$ was generated from a normal distribution with a mean of zero and variance set to 1.009, and it also did not co-vary with any of the other level-3 residuals. The other level-3 residuals, u_{10j_10} and $u_{\text{Bw}j_1}$, were generated from a multivariate normal distribution with means of zero and variances set to 0.302 and 0.002, respectively. The covariance of u_{10j_10} and $u_{\text{Bw}j_1}$ was set to 0.002. Similar to the fixed effects generating values, the true values set for the variance components of the random effects were based on the real data analysis results.

In addition, the real data has revealed that the mobile schools have lower intercept residuals on average than the non-mobile schools for the baseline unconditional CCMM-LVR model. This difference was about half a standard deviation between the means. Therefore, in order to mimic the real data and have student mobility that was not completely at random, the means of the u_{00j_10} residuals in the data generations were generated to be –0.774 and 1.805 for the mobile and non-mobile schools, respectively. Note that the overall mean of the school-level residuals for the intercept was still zero.

Estimating Models

The baseline unconditional CCMM-LVR model from Equations 19, 24, and 25 was used to estimate the baseline unconditional model that handles student mobility. The HM3-

LVR baseline unconditional model was estimated from the model presented in Equations 5, 12, and 13, but it ignored the subsequent schools attended by the mobile students.

Estimation Procedure

The R2jags R package (version 0.04-03; Su & Yajima, 2014) was used to interface with JAGS software (version 3.4.0; Plummer, 2013) to estimate the two models. JAGS software estimates user-specified statistical models by using the Gibbs sampler to determine the suitable Markov Chain Monte Carlo (MCMC) estimation scheme. The estimation of the models for the 16,000 generated datasets was made possible by the high performance computing resources provided by the Texas Advanced Computing Center (TACC) at The University of Texas at Austin.

Similar to the real data analysis, Normal(0, 100,000) priors were placed on the fixed effects, inverse-Pareto(1, 0.0001) priors for the scalar variance components, and an inverse-Wishart prior for the variance-covariance matrix at level-3 for equations β_{10j} and Bw_j, which were decided based on the simulation results from Choi and Seltzer (2010). An examination of autocorrelation function plots, trace plots, and Gelman-Rubin statistics was done to inform selection of the appropriate burn-in period and number of iterations for convergence. One chain with a burn-in period of 10,000 iterations and an additional 50,000 iterations, for a total of 60,000 iterations, was needed.

Analyses

The analyses for the simulation study compared differences in the parameter estimates of the fixed effects and random effects variance components from the two models, HM3-LVR and CCMM-LVR. The following descriptions are provided about the relative parameter bias, relative standard error bias, root mean square error, and coverage rates that were computed and analyzed.

Relative Parameter Bias

Relative parameter bias was calculated for the fixed effects and the random effects variance components using the following formula,

$$B(\hat{\theta}) = \frac{\hat{\theta} - \theta}{\theta}, \tag{37}$$

where $\hat{\theta}$ is the estimate of the parameter, θ. Positive relative parameter bias would indicate that the parameter was overestimated, while a negative value would reveal underestimation of that parameter. Relative parameter bias values between –0.05 and 0.05 would be considered acceptable based on Hoogland and Boomsma's criteria (1998). These 1,000 bias values were averaged across replications for each condition and then compared across models.

Relative Standard Error Bias

Relative bias of the standard errors was computed for the fixed effects estimates using

$$B(\hat{S}_{\hat{\theta}}) = \frac{\hat{S}_{\hat{\theta}} - \hat{S}_{\hat{\theta}\_\mathrm{EMP}}}{\hat{S}_{\hat{\theta}\_\mathrm{EMP}}}, \tag{38}$$

where $\hat{S}_{\hat{\theta}}$ is the estimated standard error of the fixed effects estimates and $\hat{S}_{\hat{\theta}\_\mathrm{EMP}}$ is the true (empirical) value of the standard error of the fixed effects estimates, which was obtained by calculating the standard deviation of the 1,000 parameter estimates (the $\hat{\theta}$s) for each condition. Likewise for relative parameter bias, relative standard error values are interpreted in the same manner, but a range of values from –0.1 to 0.1 was considered acceptable (Hoogland & Boomsma, 1998). Relative standard error bias for the random effects variance components were not calculated because their distributions can be positively skewed and truncated at zero, especially for smaller values (Fears, Benichou, & Gail, 1996). These 1,000 bias values were averaged across replications for each condition and then compared across models.

Root Mean Square Error

The root mean square error (RMSE) was also calculated using

$$\text{RMSE} = \sqrt{\left(\bar{\bar{\theta}} - \theta\right)^2 + \hat{S}^2{}_{\hat{\theta}\_\text{EMP}}}, \quad (39)$$

where $\bar{\bar{\theta}}$ is the mean of a parameter estimate across the 1,000 replications for each condition. The RMSE values were compared across the two models being estimated with the smaller values signifying less bias and variation in parameter estimates.

Coverage Rates

For the fixed effects parameter estimates, the proportion of the 95% credible intervals that included the true parameter value was tallied for each condition and model. Ideally, the coverage rates of the 95% credible intervals should be around 0.95.

Chapter 4: Simulation Study Results

This chapter presents the results from the simulation study that investigated differences in relative parameter bias, relative standard error bias, root mean square error, and coverage rates for the baseline unconditional model parameters estimated using the CCMM-LVR and HM3-LVR models. The CCMM-LVR model handles student mobility across clustering units, while the HM3-LVR model assumes the students remained in their first school (cluster) for the entire duration of the study. The following sections summarize the results.

RELATIVE PARAMETER BIAS

The relative parameter bias was computed for estimates of the fixed effects, including the following model-specific (CCMM-LVR and HM3-LVR, respectively) parameters: the intercept (γ_{0000} and γ_{000}), slope (γ_{1000} and γ_{100}), Bb coefficient, Bw_0 coefficient, and Bw_1 coefficient. Relative parameter bias was also calculated for the estimates of the random effects variance components including the following parameters: the level-one variance (σ^2), between-students intercept variance (τ_{r00}), between-students slope variance (τ_{r11}), between-first-schools intercept variance ($\tau_{u_{j1}00}$ and τ_{u00}), between-first-schools slope variance ($\tau_{u_{j1}11}$ and τ_{u11}), between-first-schools Bw variance ($\tau_{u_{j1}\text{Bw}}$ and $\tau_{u\text{Bw}}$), and between-subsequent-schools slope variance ($\tau_{u_{\{j2\}}11}$). Substantial relative parameter bias would be indicated by values smaller than –0.05 or larger than 0.05, according to Hoogland and Boomsma's criteria (1998).

Fixed Effects

This section presents the relative parameter bias for the 16 conditions for each of the fixed effects parameters estimated using the two models, CCMM-LVR and HM3-LVR.

Intercept, γ_{0000} and γ_{000}

Table 9 provides the relative parameter bias for the intercept estimates, γ_{0000} and γ_{000}, for the CCMM-LVR and HM3-LVR models, respectively, across conditions. No substantial bias was found for either model's estimates of the intercept under any of the conditions.

Table 9

Relative Parameter Bias of the Intercept and Slope Estimates

Condition				Parameter and Estimating Model			
				γ_{0000}	γ_{000}	γ_{1000}	γ_{100}
Mobility Rate	Schools	Students per School	Time-points	CCMM-LVR	HM3-LVR	CCMM-LVR	HM3-LVR
10%	50	40	3	0.000	0.000	0.002	0.002
			4	0.000	0.000	0.001	0.001
		80	3	0.000	0.000	0.001	0.001
			4	0.000	0.000	−0.002	−0.002
	100	40	3	0.000	0.000	0.001	0.001
			4	0.000	0.000	0.001	0.001
		80	3	0.000	0.000	0.000	0.000
			4	0.000	0.000	−0.001	−0.001
20%	50	40	3	0.000	0.000	−0.002	−0.002
			4	−0.001	−0.001	0.000	0.000
		80	3	0.000	0.000	−0.001	−0.001
			4	0.000	0.000	0.001	0.001
	100	40	3	0.000	0.000	0.003	0.003
			4	0.000	0.000	0.001	0.001
		80	3	0.000	0.000	0.001	0.001
			4	0.000	0.000	0.001	0.001

Note. CCMM-LVR = cross-classified multiple membership latent variable regression; HM3-LVR = three-level latent variable regression.

Slope, γ_{1000} and γ_{100}

Table 9 provides the relative parameter bias for the slope estimates, γ_{1000} and γ_{100}, for the CCMM-LVR and HM3-LVR models, respectively. Similar to the results for the intercept estimates, no substantial bias was found for estimates of the slope under any of the conditions.

Bb *Coefficient*

The relative parameter bias values are provided in Table 10 for the Bb coefficient estimates across conditions for the two estimating models. No substantial bias was found for the estimates of the Bb coefficient under any of the conditions for both estimating models.

Table 10

Relative Parameter Bias of the Bb, Bw_0, and Bw_1 Coefficient Estimates

Condition				Parameter and Estimating Model					
				Bb		Bw_0		Bw_1	
Mobility Rate	Schools	Students per School	Time-points	CCMM-LVR	HM3-LVR	CCMM-LVR	HM3-LVR	CCMM-LVR	HM3-LVR
10%	50	40	3	−0.010	−0.016	***0.145***	***0.142***	0.004	0.000
			4	−0.014	0.010	***0.071***	***0.076***	0.018	0.013
		80	3	0.029	0.033	−0.011	−0.014	0.022	0.017
			4	0.024	0.032	−0.014	−0.008	−0.015	−0.022
	100	40	3	−0.010	−0.014	−0.037	−0.045	0.019	0.015
			4	−0.012	−0.003	−0.046	−0.049	0.021	0.016
		80	3	−0.012	−0.005	0.036	0.032	0.015	0.011
			4	−0.002	−0.003	−0.003	−0.004	0.024	0.017
20%	50	40	3	−0.004	−0.003	***0.114***	***0.109***	−0.001	−0.010
			4	−0.034	−0.041	0.040	0.036	0.015	0.002
		80	3	−0.004	0.028	−0.010	−0.014	0.048	0.040
			4	−0.021	−0.014	0.000	−0.004	0.028	0.015
	100	40	3	−0.034	−0.032	***0.061***	***0.063***	0.006	−0.001
			4	−0.020	−0.011	0.027	0.036	−0.006	−0.017
		80	3	−0.031	−0.044	***0.069***	***0.071***	−0.005	−0.014
			4	−0.019	−0.010	−0.001	0.000	−0.012	−0.023

Note. CCMM-LVR = cross-classified multiple membership latent variable regression; HM3-LVR = three-level latent variable regression; italicized and bolded values indicate substantial bias.

Bw_0 *Coefficient*

The relative parameter bias for the Bw_0 coefficient estimates across conditions for the CCMM-LVR and HM3-LVR models are presented in Table 10. Substantial positive relative parameter bias was found under a few of the conditions (5 out of the 16 conditions) for both estimating models. Slightly more positive bias was generally found for the conditions with three measurement occasions, where the Bw_0 coefficient was overestimated by 6.1% to 14.5% for the CCMM-LVR model and by 6.3% to 14.2% for the HM3-LVR model.

Bw_1 *Coefficient*

Table 10 displays the relative parameter bias for the Bw_1 coefficient estimates for the estimating models across the 16 conditions. No substantial bias was found under any of the conditions for the two models for the Bw_1 coefficient estimates.

Random Effects

This section provides the relative parameter bias results for the random effects variance component estimates.

Level-One Variance, σ^2

The relative parameter bias for estimates of the level-one variance, σ^2, is presented in Table 11. No substantial bias was found for the level-one variance component estimates under any of the 16 conditions for both models.

Table 11

Relative Parameter Bias of the Level-One and Level-Two Variance Component Estimates

Condition				Parameter and Estimating Model					
				σ^2		τ_{r00}		τ_{r11}	
Mobility Rate	Schools	Students per School	Time-points	CCMM-LVR	HM3-LVR	CCMM-LVR	HM3-LVR	CCMM-LVR	HM3-LVR
10%	50	40	3	0.003	0.003	0.001	0.001	0.004	0.037
			4	0.001	0.001	0.002	0.002	0.003	0.039
		80	3	0.000	0.000	0.002	0.002	0.008	0.039
			4	0.000	0.000	0.000	0.000	0.005	0.041
	100	40	3	0.001	0.001	0.001	0.001	0.005	0.038
			4	0.001	0.001	0.001	0.001	0.004	0.039
		80	3	0.001	0.001	0.000	0.000	0.001	0.032
			4	0.001	0.001	0.001	0.001	0.000	0.034
20%	50	40	3	0.003	0.003	0.002	0.002	−0.003	***0.051***
			4	0.001	0.001	0.002	0.002	0.007	***0.067***
		80	3	0.001	0.001	0.000	0.000	0.010	***0.062***
			4	0.001	0.001	0.002	0.002	0.003	***0.062***
	100	40	3	0.000	0.000	0.000	0.000	0.006	***0.060***
			4	0.000	0.000	0.000	0.000	0.004	***0.063***
		80	3	0.000	0.000	0.000	0.000	0.006	***0.056***
			4	0.000	0.000	0.000	0.000	0.002	***0.061***

Note. CCMM-LVR = cross-classified multiple membership latent variable regression; HM3-LVR = three-level latent variable regression; italicized and bolded values indicate substantial bias.

Between-Students Intercept Variance, τ_{r00}

Table 11 provides the relative parameter bias for the estimates of the between-students intercept variance, τ_{r00}. Neither model resulted in substantially biased estimates of the between-students intercept variance under any of the conditions.

Between-Students Slope Variance, τ_{r11}

The relative parameter bias is presented in Table 11 for estimates of the between-students slope variance, τ_{r11}, across the 16 conditions for the CCMM-LVR and HM3-LVR models. The results revealed slightly substantial positive bias across the 20% mobility rate conditions for the HM3-LVR model, where the values ranged from 0.051 to 0.067.

Between-First-Schools Intercept Variance, $\tau_{u_{j1}00}$ and τ_{u00}

Table 12 provides the relative parameter bias for estimates of the between-first-schools intercept variance, $\tau_{u_{j1}00}$ and τ_{u00}. For both the CCMM-LVR and HM3-LVR models, substantial positive bias was discovered in estimates of the between-first-schools intercept variance component estimates across conditions. The bias was the same to the third decimal point for both models, with the between-first-schools intercept variance overestimated by degrees between 9.1% and 16.4%.

Table 12

Relative Parameter Bias of the Level-Three Variance Component Estimates

Condition				Parameter and Estimating Model						
				$\tau_{u_{j1}00}$	τ_{u00}	$\tau_{u_{j1}11}$	τ_{u11}	$\tau_{u_{j1}Bw}$	τ_{uBw}	$\tau_{u\{j2\}11}$
Mobility Rate	Schools	Students per School	Time-points	CCMM-LVR	HM3-LVR	CCMM-LVR	HM3-LVR	CCMM-LVR	HM3-LVR	CCMM-LVR
10%	50	40	3	***0.140***	***0.139***	***0.222***	***3.071***	−0.009	−0.047	***0.108***
			4	***0.147***	***0.147***	***0.108***	***3.010***	−0.032	***−0.052***	***0.115***
		80	3	***0.164***	***0.164***	***0.114***	***3.097***	−0.006	−0.023	0.***131***
			4	***0.148***	***0.148***	***0.059***	***3.048***	−0.006	−0.014	***0.137***
	100	40	3	***0.091***	***0.091***	0.049	***2.949***	***−0.052***	***−0.076***	***0.072***
			4	***0.094***	***0.094***	−0.010	***2.944***	−0.018	−0.026	***0.084***
		80	3	***0.106***	***0.107***	−0.013	***2.970***	−0.011	−0.018	***0.080***
			4	***0.092***	***0.092***	−0.008	***2.939***	0.004	0.002	***0.067***
20%	50	40	3	***0.151***	***0.151***	***0.145***	***2.745***	−0.011	−0.044	***0.133***
			4	***0.158***	***0.158***	***0.077***	***2.659***	−0.032	−0.048	***0.133***
		80	3	***0.152***	***0.152***	***0.084***	***2.788***	0.003	−0.013	***0.145***
			4	***0.143***	***0.143***	0.026	***2.633***	0.019	0.016	***0.122***
	100	40	3	***0.100***	***0.100***	0.020	***2.661***	***−0.053***	***−0.074***	***0.093***
			4	***0.093***	***0.093***	−0.009	***2.554***	−0.028	−0.034	***0.073***
		80	3	***0.092***	***0.092***	−0.023	***2.600***	−0.008	−0.013	***0.069***
			4	***0.092***	***0.092***	0.002	***2.561***	0.007	0.008	***0.066***

Note. CCMM-LVR = cross-classified multiple membership latent variable regression; HM3-LVR = three-level latent variable regression; italicized and bolded values indicate substantial bias.

Between-First-Schools Slope Variance, $\tau_{u_{j1}11}$ and τ_{u11}

The relative parameter bias is presented in Table 12 for the between-first-schools slope variance, $\tau_{u_{j1}11}$ and τ_{u11}, across the 16 conditions for the CCMM-LVR and HM3-LVR models, respectively. Very substantial positive relative parameter bias was found under all of the conditions for the HM3-LVR model, ranging from 255.4% up to 309.7%. The CCMM-LVR model consistently resulted in much less bias than the HM3-LVR model across conditions, and substantial bias was generally found only under the conditions with 50 schools with relative parameter bias values ranging from 5.9% to 22.2%.

Between-First-Schools Bw Variance, $\tau_{u_{j1}\mathrm{Bw}}$ and $\tau_{u\mathrm{Bw}}$

The relative parameter bias of the between-first-schools Bw variance, $\tau_{u_{j1}\mathrm{Bw}}$ and $\tau_{u\mathrm{Bw}}$, is provided in Table 12 for the two estimating models across conditions. Substantially negatively biased estimates of the between-first-schools Bw variance were found in a small number of conditions, including under the conditions with 100 schools, 40 students per school, and 3 time-points for the CCMM-LVR and HM3-LVR models. The between-first-schools Bw variance was underestimated by 5.2% to 5.3% for the CCMM-LVR model and by 7.4% to 7.6% for the HM3-LVR model. For the HM3-LVR model, slightly substantial negative bias was also found under the 10% mobility, 50 schools, 40 students per school, and 4 time-points condition (−0.052).

Between-Subsequent-Schools Slope Variance, $\tau_{u_{\{j2\}}11}$

Table 12 presents the relative parameter bias for estimates of the between-subsequent-schools slope variance, $\tau_{u_{\{j2\}}11}$, across the conditions for the CCMM-LVR model. The between-subsequent-schools slope variance was not estimated under the HM3-LVR model. Substantial positive relative parameter bias was found across all conditions, however, more bias was found for the conditions with 50 schools (0.108 to 0.145) than for the conditions with 100 schools (0.066 to 0.093).

RELATIVE STANDARD ERROR BIAS

The relative standard error bias was computed for estimates of the fixed effects including the following model-specific (CCMM-LVR and HM3-LVR, respectively) parameters: the intercept (γ_{0000} and γ_{000}), slope (γ_{1000} and γ_{100}), Bb coefficient, Bw_0 coefficient, and Bw_1 coefficient. Relative standard error bias values outside the range of –0.1 to 0.1 were considered substantially biased (Hoogland & Boomsma, 1998).

Fixed Effects

This section presents the relative standard error bias for the 16 conditions for each of the fixed effects parameters estimated using each of the two models.

Intercept, γ_{0000} and γ_{000}

Table 13 provides the relative standard error bias for the intercept estimates, γ_{0000} and γ_{000}, for the two estimating models across conditions. For only one condition (the 10% mobility rate, 50 schools, 40 students per school, and 3 time-points condition), the CCMM-LVR and HM3-LVR models resulted in very slightly substantial positive bias for the intercept estimates. The standard error of the intercept was overestimated by 10.8% under the CCMM-LVR model and by 10.5% under the HM3-LVR model.

Table 13

Relative Standard Error Bias of the Intercept and Slope Estimates

Condition				Parameter and Estimating Model			
				γ_{0000}	γ_{000}	γ_{1000}	γ_{100}
Mobility Rate	Schools	Students per School	Time-points	CCMM-LVR	HM3-LVR	CCMM-LVR	HM3-LVR
10%	50	40	3	***0.108***	***0.105***	0.069	0.000
			4	0.056	0.054	0.091	0.017
		80	3	0.076	0.076	−0.003	−0.071
			4	0.056	0.056	0.056	−0.018
	100	40	3	0.015	0.014	0.043	−0.014
			4	0.041	0.039	−0.009	−0.066
		80	3	0.023	0.022	0.070	0.011
			4	0.064	0.063	0.035	−0.026
20%	50	40	3	0.048	0.049	0.048	−0.048
			4	0.061	0.060	0.038	−0.067
		80	3	0.042	0.040	0.079	−0.023
			4	0.035	0.034	0.014	−0.090
	100	40	3	0.040	0.040	0.048	−0.039
			4	0.029	0.028	0.011	−0.083
		80	3	0.061	0.060	0.002	−0.088
			4	0.013	0.013	0.051	−0.052

Note. CCMM-LVR = cross-classified multiple membership latent variable regression; HM3-LVR = three-level latent variable regression; italicized and bolded values indicate substantial bias.

***Slope,* γ_{1000} *and* γ_{100}**

Table 13 provides the relative standard error bias for the slope estimates, γ_{1000} and γ_{100}, for the CCMM-LVR and HM3-LVR models, respectively. Table 13 indicates no substantial bias for the standard error estimates of the slope under any of the conditions.

Bb *Coefficient*

The relative standard error bias in presented in Table 14 for the Bb coefficient estimates across conditions for the CCMM-LVR and HM3-LVR models. Substantial positive relative standard error bias was found for the HM3-LVR model estimates under 12 of the 16 conditions. The bias values ranged from 0.128 to 0.283, meaning that the HM3-LVR model overestimated the standard error of the Bb coefficient by 12.8% to 28.3%.

Table 14

Relative Standard Error Bias of the Bb, Bw_0, and Bw_1 Coefficient Estimates

Condition				Parameter and Estimating Model					
				Bb		Bw_0		Bw_1	
Mobility Rate	Schools	Students per School	Time-points	CCMM-LVR	HM3-LVR	CCMM-LVR	HM3-LVR	CCMM-LVR	HM3-LVR
10%	50	40	3	0.046	0.083	0.049	0.043	−0.001	−0.006
			4	−0.015	0.089	−0.012	−0.015	−0.080	−0.082
		80	3	0.062	***0.174***	0.008	0.003	−0.015	−0.015
			4	0.050	***0.263***	0.036	0.036	0.024	0.023
	100	40	3	0.002	0.050	0.020	0.019	−0.023	−0.024
			4	0.007	***0.128***	−0.003	−0.003	−0.029	−0.029
		80	3	0.020	***0.139***	0.009	0.009	−0.016	−0.016
			4	−0.024	***0.184***	0.045	0.045	0.014	0.015
20%	50	40	3	−0.031	0.039	0.004	0.003	0.019	0.019
			4	0.013	***0.168***	0.021	0.020	−0.038	−0.035
		80	3	0.035	0.***197***	0.067	0.062	−0.011	−0.012
			4	0.021	***0.283***	−0.020	−0.017	−0.058	−0.055
	100	40	3	0.052	***0.139***	−0.011	−0.013	0.021	0.021
			4	0.016	***0.175***	0.016	0.018	−0.056	−0.053
		80	3	0.024	***0.189***	−0.050	−0.049	0.014	0.014
			4	0.002	***0.255***	−0.032	−0.030	−0.006	−0.001

Note. CCMM-LVR = cross-classified multiple membership latent variable regression; HM3-LVR = three-level latent variable regression; italicized and bolded values indicate substantial bias.

Bw_0 ***Coefficient***

The relative standard error bias for the Bw_0 coefficient estimates across conditions for the CCMM-LVR and HM3-LVR models are presented in Table 14. Substantial bias was not found for the models under any of the conditions.

Bw_1 ***Coefficient***

Table 14 displays the relative standard error bias for the Bw_1 coefficient estimates. No substantial bias was found under any of the conditions for the two estimating models for the standard error estimates of the Bw_1 coefficient.

ROOT MEAN SQUARE ERROR

The root mean square error (RMSE) was calculated for estimates of the fixed effects including the following model-specific (CCMM-LVR and HM3-LVR, respectively) parameters: the intercept (γ_{0000} and γ_{000}), slope (γ_{1000} and γ_{100}), Bb coefficient, Bw_0 coefficient, and Bw_1 coefficient. RMSE was also computed for the estimates of the random effects variance components: the level-one variance (σ^2), between-students intercept variance (τ_{r00}), between-students slope variance (τ_{r11}), between-first-schools intercept variance ($\tau_{u_{j1}00}$ and τ_{u00}), between-first-schools slope variance ($\tau_{u_{j1}11}$ and τ_{u11}), between-first-schools Bw variance ($\tau_{u_{j1}\mathrm{Bw}}$ and $\tau_{u\mathrm{Bw}}$), and between-subsequent-schools slope variance ($\tau_{u_{\{j2\}}11}$). Smaller RMSE values between the two estimating models would indicate less bias and variation (i.e., more efficiency) in the parameter estimates. Note that RMSE was not computed for the between-subsequent-schools slope variance, $\tau_{u_{\{j2\}}11}$, because no comparison could be made between models due to the HM3-LVR model not estimating that parameter.

Fixed Effects

This section presents the RMSE values for each of the 16 unique conditions for each of the fixed effects parameters for the two estimating models, CCMM-LVR and HM3-LVR.

Intercept, γ_{0000} and γ_{000}

Table 15 provides the RMSE values for the intercept estimates, γ_{0000} and γ_{000}, for the two estimating models across conditions. The outcomes indicated that there was no difference in RMSE values (at least to the third decimal place) estimating the intercept across conditions.

Table 15

Root Mean Square Error of the Intercept and Slope Estimates

Condition				Parameter and Estimating Model			
				γ_{0000}	γ_{000}	γ_{1000}	γ_{100}
Mobility Rate	Schools	Students per School	Time-points	CCMM-LVR	HM3-LVR	CCMM-LVR	HM3-LVR
10%	50	40	3	0.741	0.741	0.187	0.187
			4	0.778	0.778	0.172	0.172
		80	3	0.749	0.749	0.189	0.189
			4	0.757	0.757	0.172	0.172
	100	40	3	0.561	0.561	0.131	0.131
			4	0.548	0.548	0.131	0.131
		80	3	0.545	0.545	0.120	0.120
			4	0.521	0.521	0.120	0.120
20%	50	40	3	0.784	0.784	0.191	0.191
			4	0.778	0.778	0.181	0.181
		80	3	0.770	0.770	0.174	0.174
			4	0.771	0.771	0.177	0.177
	100	40	3	0.550	0.550	0.131	0.131
			4	0.554	0.554	0.127	0.127
		80	3	0.523	0.523	0.128	0.128
			4	0.546	0.546	0.118	0.118

Note. CCMM-LVR = cross-classified multiple membership latent variable regression; HM3-LVR = three-level latent variable regression.

Slope, γ_{1000} *and* γ_{100}

Table 15 provides the RMSE values for the slope estimates, γ_{1000} and γ_{100}, for the CCMM-LVR and HM3-LVR models, respectively. The models resulted in the same values (at least to the third decimal place) for estimating the slope across conditions.

Bb *Coefficient*

The RMSE values are provided in Table 16 for the Bb coefficient estimates across conditions for the CCMM-LVR and HM3-LVR models. The results indicated that there was no difference in RMSE values for the models (at least to the third decimal place) when estimating the Bb coefficient across conditions.

Table 16

Root Mean Square Error of the Bb, Bw_0, and Bw_1 Coefficient Estimates

Condition				Parameter and Estimating Model					
				Bb		Bw_0		Bw_1	
Mobility Rate	Schools	Students per School	Time-points	CCMM-LVR	HM3-LVR	CCMM-LVR	HM3-LVR	CCMM-LVR	HM3-LVR
10%	50	40	3	0.035	0.035	0.011	0.011	0.002	0.002
			4	0.032	0.032	0.010	0.010	0.002	0.002
		80	3	0.029	0.029	0.010	0.010	0.002	0.002
			4	0.026	0.026	0.008	0.008	0.001	0.001
	100	40	3	0.025	0.025	0.008	0.008	0.002	0.002
			4	0.021	0.021	0.007	0.007	0.001	0.001
		80	3	0.020	0.020	0.007	0.007	0.001	0.001
			4	0.019	0.019	0.006	0.006	0.001	0.001
20%	50	40	3	0.035	0.035	0.012	0.012	0.002	0.002
			4	0.029	0.029	0.009	0.009	0.002	0.002
		80	3	0.027	0.027	0.009	0.009	0.002	0.002
			4	0.024	0.024	0.009	0.009	0.002	0.002
	100	40	3	0.022	0.022	0.008	0.008	0.001	0.001
			4	0.020	0.020	0.007	0.007	0.001	0.001
		80	3	0.019	0.019	0.007	0.007	0.001	0.001
			4	0.017	0.017	0.006	0.006	0.001	0.001

Note. CCMM-LVR = cross-classified multiple membership latent variable regression; HM3-LVR = three-level latent variable regression.

Bw_0 *Coefficient*

The RMSE values for the Bw_0 coefficient estimates across conditions for the CCMM-LVR and HM3-LVR models are presented in Table 16. No differences were found (at least to the third decimal place) between the estimating models in RMSE values when estimating the Bw_0 coefficient across conditions.

Bw_1 *Coefficient*

Table 16 displays the RMSE values for the Bw_1 coefficient estimates for the estimating models, CCMM-LVR and HM3-LVR, for the 16 conditions. The models resulted in the same RMSE values (at least to the third decimal place) when estimating the Bw_1 coefficient across the conditions.

Random Effects

This section provides the root mean square error (RMSE) for the random effects variance component estimates. As noted before, the RMSE is not presented for the HM3-LVR estimates of the between-subsequent-schools slope variance because this parameter is not estimated under that model, and therefore, no model comparisons in RMSE values could be made.

Level-One Variance, σ^2

The RMSE values for the level-one variance component estimate, σ^2, are presented in Table 17 for the 16 conditions and for each of the two estimating models, CCMM-LVR and HM3-LVR. No differences between models were found (at least to the third decimal place) in the RMSE values when estimating the level-one variance across conditions.

Table 17

Root Mean Square Error (RMSE) of the Level-One and Level-Two Variance Component Estimates

Condition				Parameter and Estimating Model					
				σ^2		τ_{r00}		τ_{r11}	
Mobility Rate	Schools	Students per School	Time-points	CCMM-LVR	HM3-LVR	CCMM-LVR	HM3-LVR	CCMM-LVR	HM3-LVR
10%	50	40	3	0.762	0.762	4.732	4.732	0.630	***0.642***
			4	0.542	0.542	4.271	4.271	0.292	***0.321***
		80	3	0.516	0.516	3.292	3.292	0.443	***0.461***
			4	0.394	0.394	3.213	3.213	0.205	***0.248***
	100	40	3	0.541	0.541	3.340	3.340	0.436	***0.455***
			4	0.364	0.364	3.308	3.308	0.209	***0.249***
		80	3	0.391	0.391	2.302	2.302	0.315	***0.333***
			4	0.277	0.277	2.193	2.193	0.144	***0.187***
20%	50	40	3	0.803	0.803	4.711	***4.712***	0.615	***0.639***
			4	0.547	0.547	4.356	4.356	0.299	***0.377***
		80	3	0.535	0.535	3.369	3.369	0.444	***0.490***
			4	0.386	0.386	3.170	3.170	0.209	***0.298***
	100	40	3	0.540	0.540	3.241	3.241	0.468	***0.510***
			4	0.375	0.375	3.209	3.209	0.197	***0.292***
		80	3	0.376	0.376	2.404	2.404	0.321	***0.374***
			4	0.266	0.266	2.175	2.175	0.147	***0.257***

Note. CCMM-LVR = cross-classified multiple membership latent variable regression; HM3-LVR = three-level latent variable regression; italicized and bolded values indicate larger RMSE values in the estimate.

Between-Students Intercept Variance, τ_{r00}

Table 17 provides the RMSE values by condition and for the two estimating models, CCMM-LVR and HM3-LVR, for estimates of the between-students intercept variance, τ_{r00}. The models resulted in the same RMSE values (at least to the third decimal place) when estimating the between-students intercept variance in all but one condition. However, the difference in values was very slight between the CCMM-LVR (4.711) and HM3-LVR (4.712) models.

Between-Students Slope Variance, τ_{r11}

The RMSE values are displayed in Table 17 for estimates of the between-students slope variance, τ_{r11}, across the 16 conditions for the CCMM-LVR and HM3-LVR models. Smaller RMSE values were found for estimates of the between-students slope variance with the CCMM-LVR model than the HM3-LVR model across conditions.

Between-First-Schools Intercept Variance, $\tau_{u_{j1}00}$ and τ_{u00}

Table 18 contains the RMSE values for estimates of the between-first-schools intercept variance, $\tau_{u_{j1}00}$ and τ_{u00}, for the CCMM-LVR and HM3-LVR models, respectively, for each condition. The results indicated differences in RMSE values when estimating the between-first-schools intercept variance, for 12 of the 16 conditions. The differences were very slight (less than 0.01), and the model resulting in the better RMSE values differed across the 12 conditions.

Table 18

Root Mean Square Error (RMSE) of the Level-Three Variance Component Estimates

Condition				Parameter and Estimating Model					
				$\tau_{u_{j1}00}$	τ_{u00}	$\tau_{u_{j1}11}$	τ_{u11}	$\tau_{u_{j1}\mathrm{Bw}}$	$\tau_{u\mathrm{Bw}}$
Mobility Rate	Schools	Students per School	Time-points	CCMM-LVR	HM3-LVR	CCMM-LVR	HM3-LVR	CCMM-LVR	HM3-LVR
10%	50	40	3	***7.774***	7.766	0.140	***0.935***	0.001	0.001
			4	***7.987***	7.985	0.127	***0.917***	0.001	0.001
		80	3	7.783	***7.785***	0.130	***0.944***	0.001	0.001
			4	7.756	7.756	0.115	***0.927***	0.001	0.001
	100	40	3	***5.335***	5.334	0.113	***0.898***	0.001	0.001
			4	5.273	5.273	0.106	***0.895***	0.001	0.001
		80	3	5.230	***5.233***	0.106	***0.903***	0.000	0.000
			4	***4.968***	4.967	0.094	***0.893***	0.000	0.000
20%	50	40	3	8.200	***8.202***	0.134	***0.839***	0.001	0.001
			4	7.851	7.851	0.123	***0.812***	0.001	0.001
		80	3	7.529	***7.531***	0.124	***0.851***	0.001	0.001
			4	7.540	***7.543***	0.108	***0.802***	0.001	0.001
	100	40	3	***5.493***	5.486	0.112	***0.811***	0.001	0.001
			4	***5.367***	5.363	0.097	***0.778***	0.001	0.001
		80	3	4.987	4.987	0.098	***0.791***	0.000	0.000
			4	5.066	***5.073***	0.082	***0.778***	0.000	0.000

Note. CCMM-LVR = cross-classified multiple membership latent variable regression; HM3-LVR = three-level latent variable regression; italicized and bolded values indicate larger RMSE values in the estimate.

Between-First-Schools Slope Variance, $\tau_{u_{j1}11}$ and τ_{u11}

The RMSE values are presented in Table 18 for estimates of the between-first-schools slope variance, $\tau_{u_{j1}11}$ and τ_{u11}, across the 16 conditions for the CCMM-LVR and HM3-LVR models, respectively. Smaller RMSE values were found for estimates of the between-first-schools slope variance for the CCMM-LVR model under all conditions.

Between-First-Schools Bw Variance, $\tau_{u_{j1}\mathrm{Bw}}$ and $\tau_{u\mathrm{Bw}}$

The RMSE values for estimates of the between-first-schools Bw variance, $\tau_{u_{j1}\mathrm{Bw}}$ and $\tau_{u\mathrm{Bw}}$, are provided in Table 18 for the two estimating models across conditions. Table 18 revealed that no differences were found (at least to the third decimal place) between the models in RMSE values for estimates of the between-first-schools Bw variance.

COVERAGE RATES

The coverage rates for the 95% credible intervals were calculated for estimates of the fixed effects including the following model-specific (CCMM-LVR and HM3-LVR, respectively) parameters: the intercept (γ_{0000} and γ_{000}), slope (γ_{1000} and γ_{100}), Bb coefficient, Bw_0 coefficient, and Bw_1 coefficient. The closer the coverage rate to the nominal 95% rate the better the credible intervals are functioning.

Fixed Effects

This section presents the coverage rates across conditions for each of the fixed effects parameters for the two estimating models, CCMM-LVR and HM3-LVR.

Intercept, γ_{0000} and γ_{000}

Table 19 provides the coverage rates for the intercept estimates, γ_{0000} and γ_{000}, for the two estimating models for each condition. All coverage rates for the CCMM-LVR and HM3-LVR models were relatively close to 95% for each condition, and were similar between the two estimating models, ranging from 94.7% to 97.1% and from 94.4% to 97.0% for the CCMM-LVR and HM3-LVR models, respectively.

Table 19

Coverage Rates of the Intercept and Slope Estimates

Condition				Parameter and Estimating Model			
				γ_{0000}	γ_{000}	γ_{1000}	γ_{100}
Mobility Rate	Schools	Students per School	Time-points	CCMM-LVR	HM3-LVR	CCMM-LVR	HM3-LVR
10%	50	40	3	0.971	0.970	0.965	0.946
			4	0.952	0.950	0.967	0.953
		80	3	0.966	0.968	0.955	0.930
			4	0.952	0.949	0.961	0.949
	100	40	3	0.948	0.950	0.960	0.947
			4	0.953	0.951	0.950	0.934
		80	3	0.947	0.950	0.957	0.946
			4	0.958	0.960	0.954	0.943
20%	50	40	3	0.948	0.945	0.958	0.927
			4	0.960	0.961	0.963	0.939
		80	3	0.955	0.955	0.955	0.942
			4	0.950	0.955	0.948	0.919
	100	40	3	0.951	0.947	0.955	0.930
			4	0.950	0.949	0.949	0.928
		80	3	0.959	0.958	0.944	0.923
			4	0.949	0.944	0.961	0.932

Note. CCMM-LVR = cross-classified multiple membership latent variable regression; HM3-LVR = three-level latent variable regression.

***Slope,* γ_{1000} *and* γ_{100}**

Table 19 provides the coverage rates for the slope estimates, γ_{1000} and γ_{100}, for the CCMM-LVR and HM3-LVR models, respectively. Table 19 reveals that the coverage rates for the slope estimates were close to 95% across conditions for the CCMM-LVR model (94.4% to 96.7%) and slightly under 95% for the HM3-LVR model (91.9% to 95.3%). The HM3-LVR model consistently resulted in slightly lower coverage rates (M = 93.7%) than the CCMM-LVR model (M = 95.6%) by about 2% on average.

Bb *Coefficient*

The coverage rates are provided in Table 20 for the Bb coefficient estimates across conditions for the CCMM-LVR and HM3-LVR models. No coverage rates were found to be far from 95% for estimates of the Bb coefficient under any of the conditions for both models, ranging from 94.0% to 95.8% and from 93.9% to 96.2% for the CCMM-LVR and HM3-LVR models, respectively.

Table 20

Coverage Rates of the Bb, Bw_0, and Bw_1 Coefficient Estimates

Condition				Parameter and Estimating Model					
				Bb		Bw_0		Bw_1	
Mobility Rate	Schools	Students per School	Time-points	CCMM-LVR	HM3-LVR	CCMM-LVR	HM3-LVR	CCMM-LVR	HM3-LVR
10%	50	40	3	0.951	0.944	0.967	0.964	0.944	0.940
			4	0.950	0.939	0.944	0.942	0.914	0.913
		80	3	0.957	0.962	0.943	0.936	0.941	0.941
			4	0.958	0.954	0.947	0.947	0.941	0.945
	100	40	3	0.952	0.959	0.953	0.957	0.944	0.946
			4	0.949	0.956	0.948	0.947	0.949	0.947
		80	3	0.954	0.953	0.951	0.951	0.950	0.954
			4	0.940	0.942	0.961	0.956	0.951	0.954
20%	50	40	3	0.951	0.942	0.946	0.949	0.947	0.951
			4	0.944	0.954	0.950	0.943	0.938	0.936
		80	3	0.958	0.945	0.965	0.961	0.945	0.942
			4	0.957	0.947	0.941	0.943	0.933	0.936
	100	40	3	0.957	0.960	0.956	0.949	0.955	0.953
			4	0.958	0.956	0.948	0.947	0.929	0.924
		80	3	0.954	0.950	0.937	0.939	0.953	0.957
			4	0.949	0.953	0.938	0.947	0.932	0.934

Note. CCMM-LVR = cross-classified multiple membership latent variable regression; HM3-LVR = three-level latent variable regression.

Bw_0 *Coefficient*

The coverage rates for the Bw_0 coefficient estimates for each condition for the CCMM-LVR and HM3-LVR models are presented in Table 20. Coverage rates of the Bw_0 estimates were very close to 95% for both estimating models across conditions, ranging from 93.7% to 96.7% and from 93.6% to 96.4% for the CCMM-LVR and HM3-LVR models, respectively.

Bw_1 *Coefficient*

Table 20 displays the coverage rates for the Bw_1 coefficient estimates for the models, CCMM-LVR and HM3-LVR, for each of the 16 conditions. Coverage rates were found to be slightly under 95% but similar for the two estimating models, ranging from 93.7% to 96.7% and from 93.6% to 96.4% for the CCMM-LVR and HM3-LVR models, respectively.

Chapter 5: Discussion

This dissertation was comprised of two studies including a real data analysis and a simulation study. Both of these studies compared results when estimating two different models, the CCMM-LVR and HM3-LVR models. The HM3-LVR is a three-level latent variable regression model that ignores mobility by only modeling one of the multiple clusters associated with some participants (the first school attended, in the example used in this dissertation), while the CCMM-LVR model handles the student mobility found in multiple membership data structures. The HM3-LVR model is a useful extension to the typical HM3-GCM because it can model the LVR coefficients as varying across clusters, can examine interactions between the participant and/or cluster characteristics and the initial status on growth effect, and controls for differences in initial status among participants and among clusters. However, the HM3-LVR model cannot handle the participant mobility that is typically encountered in large-scale longitudinal studies, which is why the CCMM-LVR model was proposed in this study. The following section contains a discussion of the real data analysis and simulation results, followed by a discussion of the limitations and directions for future research. Lastly, the conclusion and implications of this study are discussed.

Summary of Real Data Analysis and Simulation Study Results

The real data study demonstrated interpretation of estimates of parameters for the newly proposed CCMM-LVR model when applied to a longitudinal dataset that included student mobility. The results revealed differences in parameter estimate values and model fit between the two estimating models. Some of the values for the parameter estimates and characteristics of the real dataset that was analyzed were used when designing the associated simulation study. The purpose of the simulation study was to determine which baseline unconditional model, CCMM-LVR or HM3-LVR, resulted in parameter estimates that were closer to the truth and to assess how well the true parameter values could be recovered under a variety of design conditions. Several outcomes were evaluated to identify the differences in parameter estimates, such as relative parameter bias, relative standard error bias, root mean square error, and coverage rates. Conditions were manipulated to try to understand why these differences occurred, including the student mobility rate, number of level-3 units (schools), number of level-2 units (students) per level-3 unit, and number of measurement occasions. The following sections describe the results from both the real data analysis and simulation study.

Baseline Unconditional Fixed Effects

In the baseline unconditional models from the real data analysis, fixed effects estimates of the Bw_0 coefficient were smaller for the CCMM-LVR model than the HM3-LVR model (see Table 3). Differences between multiple membership and typical multilevel models that ignore the multiple membership structure for baseline unconditional fixed effects estimates have not frequently been found in previous research (Chung & Beretvas, 2012; Grady, 2010; Luo & Kwok, 2009; Luo & Kwok, 2012; Meyers & Beretvas, 2006). However, in a study by Smith (2012), substantial positive relative parameter bias was found when estimating the level-1 predictor for the MMREM and HLM models, which was attributed to model misspecification. From the simulation results in this dissertation, the difference in the Bw_0 coefficient estimates could be explained by the trouble experienced by both estimating models in terms of parameter recovery for the Bw_0 coefficient in the conditions with three measurement occasions. The substantial positive relative parameter bias found for both estimating models was not exceptionally large (ranging from 6.1% to 14.5%), but does indicate that more work needs to be conducted to further investigate whether fewer time-points for an LVR model causes overestimation of the Bw_0 coefficients. No other fixed effects estimates from the CCMM-LVR and HM3-LVR models were associated with substantial relative parameter bias.

Substantial relative standard error bias was only found for fixed effects estimates of the Bb coefficient for the baseline unconditional HM3-LVR model across most of the conditions (see Table 14). In particular, the conditions with the larger number of students per school (80) resulted in more positive bias (M = 21.1%) than the 40 students per school conditions (M = 10.9%). Interestingly, this indicates that as the number of level-2 units increases, the HM3-LVR model will more likely overestimate the standard error of the Bb coefficient. Meyers and Beretvas (2006) also found in their simulation study that the larger number of units per cluster conditions resulted in misestimated standard errors of the fixed effects with HLM. However, the direction of bias found in Meyers and Beretvas (2006) was negative rather than positive. The explanation given was that HLM may result in more biased estimates of the fixed effects' standard errors as the number of participants per cluster increases, which is termed the design effect in cluster sampling (Kalton, 1983). This finding would potentially impact the use of an HM3-LVR model with multiple membership data and larger numbers of participants per cluster, because inferences about the Bb parameter would be misleading due to inflated Type II error rates (i.e., decreased power).

Differences between the CCMM-LVR and HM3-LVR models in standard error estimates of the Bb coefficient were not found in the real data analysis, but the smaller number of students per school (*M* = 39) may account for this result. Future research should further look into the impact of cluster sample sizes on the standard errors of the fixed effects, which may also help explain why the directional difference in bias was found. No other substantial relative standard error bias was found for the fixed effects, and coverage rates of the 95% credible intervals of the fixed effects were close to 95%.

Conditional Fixed Effects

For the fixed effects parameters estimated using the two conditional models in the real data analysis, the effect of first-school type on the slope (γ_{1010} and γ_{101}) differed between the models (see Table 4). The HM3-LVR model's parameter estimate for the effect of first-school type on the slope, γ_{101}, was weaker but statistically significant from zero, while the parameter estimate, γ_{1010}, was not statistically significant under the CMM-LVR model due to the larger standard error estimate. The real data analysis results from Grady and Beretvas (2010) found that their standard error estimates of the level-3 predictor associated with the first school attended were similarly larger for the CCMM-GCM than for the HM3-GCM that ignored student mobility. . The HM3-LVR model in the current study seems to capture the sum of the effect of first-school type on the slope and the effect of subsequent-school type on the slope, whereas the CCMM-LVR model breaks down the effect of urbanicity on the slope into the subcomponents (i.e., first-school and subsequent-school).

This difference between the CCMM-LVR and HM3-LVR models' estimates was observed when the conditional models' results were compared, because the effect on the slope of the first-school urbanicity (γ_{1010}) as well as the subsequent-school urbanicity (γ_{1001}) were both estimated with the CCMM-LVR model, while only the first-school urbanicity effect (γ_{101}) on the slope was estimated under the HM3-LVR model. Grady (2010) examined the difference in estimates of the effect of a first-school-level predictor on the slope between the CCMM-GCM and HM3-GCM models with a simulation study, and found positively biased parameter estimates for the HM3-GCM model. This was attributed to the HM3-GCM model incorporating the effect of the subsequent-school predictor on the slope into the parameter estimate of the effect of the first-school predictor on the slope, which then resulted in positive bias. The findings of substantial positive bias from Grady (2010) could explain why the estimate of the effect of first-school urbanicity on the slope (γ_{101}) was larger for the HM3-LVR model than the parameter estimate, γ_{1010}, from the CCMM-LVR model in the real data analysis.

Baseline Unconditional and Conditional Random Effects

For the random effects variance component estimates from both the baseline unconditional and conditional models in the real data analysis, estimates of the between-first-schools slope variance, $\tau_{u_{j1}11}$ and τ_{u11}, were substantially smaller for the CCMM-LVR than for the HM3-LVR model (see Tables 5 and 6). This difference demonstrates how the two models parameterized the between-schools slope variance, because the HM3-LVR model only estimated a single level-3 slope variance (the between-first-schools slope variance, τ_{u11}) while the CCMM-LVR model partitioned that variance into the between-first-schools slope variance ($\tau_{u_{j1}11}$) and the between-subsequent-schools slope variance ($\tau_{u_{\{j2\}}11}$). The substantially larger values found for the between-first-schools slope variance from the HM3-LVR models in the real data analysis match what previous research has revealed for the variance component estimates at the cluster level when comparing a multiple membership model to a typical multilevel model that ignores the multiple membership data structure (Chung & Beretvas, 2012; Grady, 2010; Grady & Beretvas, 2010; Luo & Kwok, 2009; Luo & Kwok, 2012; Meyers & Beretvas, 2006). In addition, the pattern of results for the level-3 slope variance from the real data analysis matches those found in the current simulation study, where the between-first-schools slope variance (τ_{u11}) was substantially overestimated for the HM3-model as compared to the CCMM-LVR model's parameter estimate, $\tau_{u_{j1}11}$ (see Table 12). The HM3-LVR model also resulted in much larger RMSE values (see Table 18) when estimating the between-first-schools slope variance.

For the other level-3 slope variance in the CCMM-LVR model, $\tau_{u_{\{j2\}}11}$, the simulation study found substantial positive relative parameter bias across conditions, but positive bias was larger for the conditions with 50 schools than 100 schools. Both estimating models also resulted in substantial positive relative parameter bias for estimates of the between-first-schools intercept variance, $\tau_{u_{j1}00}$ and τ_{u00}, where larger positive bias was found for the conditions with 50 versus 100 schools. In addition, slightly substantial positive bias was found for the conditions with 50 schools for the between-first-schools slope variance ($\tau_{u_{j1}11}$) in the CCMM-LVR model, but no substantial bias was found in this estimate for the conditions with 100 schools. Grady (2010) found that both CCMM-GCM and HM3-GCM models had difficulty recovering the level-3 variance component values, where positive bias was found under several conditions for the between-first-schools intercept and slope variances. This finding is similar to those found in the current simulation for the level-3 slope and intercept variance component estimates. The results from the current simulation study appear to indicate that 100 clustering units may not be sufficient when estimating the between-subsequent-schools variance parameter ($\tau_{u_{\{j2\}}11}$) using the simplest CCMM-LVR model examined here.

From Table 12, there is also indication that larger numbers of clustering units should lead to better parameter recovery of estimates of the between-first-schools intercept variance ($\tau_{u_{j1}00}$ and τ_{u00}), but use of 100 clustering units may not be enough for both estimating models. However, using 100 clustering units appears to be sufficient for the CCMM-LVR in terms of reasonable parameter recovery for estimates of the between-first-schools slope variance, $\tau_{u_{j1}11}$, while 50 clustering units seems to generally not to be sufficient. In previous research (Browne & Draper, 2006; Chung & Beretvas, 2012), the use of MCMC estimation in multilevel modeling was found to lead to overestimated variance component estimates for units at the highest level in the model when the number of clustering units was small. This led to the authors' recommendation to use a minimum of 50 or 100 clusters. However, for LVR modeling of multiple membership data structures, the results from this simulation study seem to indicate that more than 100 clustering units are needed for sufficient parameter recovery of the between-first schools intercept variance and the between-subsequent-schools slope variance.

Another finding from the simulation study was that the between-students slope variance, τ_{r11}, was slightly overestimated by 5.1% to 6.7% for the HM3-LVR model under the 20% mobility rate conditions (see Table 11). Better RMSE values were found when estimating the between-students slope variance, τ_{r11}, for the CCMM-LVR model. It seems that higher rates of mobility negatively impact estimation of the between-students slope variance (τ_{r11}) for the HM3-LVR model. In the real data analysis (see Tables 4 and 6), it also appears that a small amount of the between-subsequent-schools variance ($\tau_{u_{\{j2\}}11}$) not estimated in the HM3-LVR model is reallocated to the between-students slope variances (τ_{r11}), because the between-students slope variance in the CCMM-LVR model is slightly smaller than the corresponding HM3-LVR model estimates. The findings for the between-students slope variance from the real data analysis and simulation study are similar to the pattern of results found in the simulation study by Grady (2010) and Luo and Kwok (2012), where higher mobility rates led to slight increases in positive relative parameter bias of estimates of the between-students slope variance (τ_{r11}).

Model Fit

Model fit between the CCMM-LVR and HM3-LVR models was evaluated using the DIC values (see Table 7). The results from the real data analysis indicated that the model fit was better with the CCMM-LVR model for the baseline unconditional models. For the conditional models estimated using the real dataset, the model fit was better for the HM3-LVR model. The difference in better model fit between CCMM-LVR and HM3-LVR could indicate that overall model fit is not very different between the CCMM-LVR and HM3-LVR models, which may coincide with known inconsistencies and weak theoretical justifications in model fit results when utilizing the DIC (Spiegelhalter, Best, Carlin, & van der Linde, 2014). The DIC values were not examined in the simulation study, but given the discrepancies of fit values for the real data analysis, future simulation studies should investigate the DIC for comparing model fit.

LIMITATIONS AND DIRECTIONS FOR FUTURE RESEARCH

This study proposed a newly parameterized model, the CCMM-LVR, and investigated the impact of ignoring student mobility when fitting a three-level latent variable regression model. Of course, not every possible condition was examined in the simulation study conducted in this dissertation and additional conditions should be investigated. The current simulation study only investigated the baseline conditional CCMM-LVR and HM3-LVR models, however, future simulation research should include predictors at the different levels in the models to compare parameter recovery across the two estimating models.

Several assumptions were made in this dissertation, including use of a linear growth model. Obviously, other possible growth forms should also be evaluated. The time variable was coded such that the intercept in the models represented initial status. Researchers might be interested in other coding possibilities (for example, final instead of initial status) that would change the parameterization of the growth model (see Grady & Beretvas, 2010). In addition, fixed measurement occasions were assumed in this study. Homogeneous variances were also assumed across the measurement occasions, but separate variances at level one could be estimated for each time-point. In addition, homogeneous variances across level-3 units were assumed here, whereas this assumption was not made in the Choi and Seltzer (2010) HM3-LVR real data analysis. This would be another direction for future research with the HM3-LVR model, although even more problems may be encountered with some of the parameter estimates given that the more parsimonious model evaluated in this study led to some issues with parameter recovery.

Another issue inherent with multiple membership data structures is the school (or cluster) identifiers. In many datasets several of these are missing, especially for those students who have changed schools. A study by Smith (2012) evaluated several ways of handling missing identifiers in a two-level multiple membership data structure. The results indicated that using a multiple membership model (MMREM) and either deleting cases with missing identifiers or assigning pseudo identifiers would be preferred to a typical HLM model with regard to relative parameter bias. Hill and Goldstein (1998) introduced a method to be used when the highest-level units were cross-classified. The authors demonstrated their procedure using a real dataset, but did not use a simulation study to empirically investigate parameter recovery. Generally, minimal differences were found between results from using their proposed procedure with a real dataset versus deleting the cases with missing classroom identifiers. In the real data analysis in this dissertation, students with missing school identifiers were removed from the analysis, which led to the removal of about 10% of the sample. If identifiers are not missing at random, then this could bias the results when researchers have simply deleted cases with missing cluster identifiers. However, the intention of the real data analysis was not to make substantive conclusions about the variables that were examined but rather to demonstrate estimation of the CCMM-LVR model and compare its results with that of the HM3-LVR model. Future methodological research should investigate additional ways of handling missing identifiers in multiple membership data structures.

CONCLUSIONS AND IMPLICATIONS

Many longitudinal datasets are evaluated in the field of education, and in social science research more generally, where growth curve modeling is utilized to assess achievement over time. In these applied fields, there are many examples in which participants are clustered within higher-level contexts (such as students within schools, patients within hospitals, residents within neighborhoods, etc.). An extension to the typical growth curve model is the latent variable regression model, which can account for variation in student achievement at the initial status when studying expected differences in growth rates, as an example in the education field. Additionally, higher-level clustering units can be incorporated in an LVR model, where the initial status on growth effect can be modeled as varying across those clusters and the model allows testing of more flexible hypotheses about the influence of initial status on growth and of factors that might influence that relationship.

In education research, the three-level LVR model would be useful to researchers examining school performance because it can investigate whether student achievement at initial status predicts growth. Including student- or school-level predictors in the HM3-LVR model could inform policymakers and educational interventionists whether the program has differential effects on growth based on where the students score at the first measurement occasion. When testing treatment or program outcomes, as well as school performance, the HM3-LVR model adjusts for differences among students and among schools in achievement scores at the start of a study.

When conducting a typical LVR analysis using a higher-level clustering unit, such as schools, the assumption is made that the participant (or student) remains in the same higher-level cluster for the duration of analysis. However, there are many scenarios in which the participants change contexts over time, which results in a multiple membership data structure. This study evaluated the impact of ignoring multiple membership structures when a researcher is interested in estimating a three-level latent variable regression model.

Using the misspecified HM3-LVR model can lead to substantial relative parameter bias, substantial relative standard error bias, and larger root mean square error values. Overestimation of the estimates of the student-level and school-level variance in the growth rates, as well as for the standard error estimates of the Bb coefficient, occurred for the HM3-LVR model. This suggests that ignoring student mobility across schools can lead to inaccurate conclusions about the relevant parameters. When estimating an HM3-LVR model, inferences about the Bb coefficient could be incorrect due to the inflated Type II error rates found across most conditions. The effect of the schools attended on the slope is also misleading when the HM3-LVR model is estimated, because only the first school attended is modeled as influencing student growth, whereas in reality the subsequent schools attended also impact student achievement over time. Given the interest that policymakers and educational researchers have in evaluating students' growth over time, the level-3 residuals are frequently used to determine value added by the level-3 unit, which is typically the school or classroom. By inappropriately modeling the student mobility across, for example, schools or classrooms, incorrect conclusions could be drawn about the "value added" by the clustering unit. The current study did not explore recovery of residuals but it is possible that if the residuals are poorly estimated then this will negatively influence their use as value-added measures. Future research should explore how well the residuals in the CCMM-LVR and HM3-LVR models are recovered.

A researcher solely interested in investigating the fixed effects from an HM3-LVR model may find that the simulation results from this study would indicate that parameter recovery was reasonable for the fixed effects. Slightly substantial positive bas was found for the Bw_0 parameter and the standard error of the Bb parameter, but RMSE values were nearly identical for both estimating models and coverage rates of the fixed effects' credible intervals were all very close to 95%. However, the results from the conditional fixed effects in the real data analysis suggest that including a cluster-level predictor associated with first- and subsequent-clusters could reveal differences in the magnitude of that effect on the slope and in its associated standard error. It has been found in Grady (2010) that the model ignoring participant mobility substantially overestimates the effect of the first clusters' characteristic on the slope, which seems to correspond with what was found in the current study's conditional fixed effects results from the real data analysis. Therefore, practitioners examining the results from an HM3-LVR model that includes school-level characteristics in the model should be careful when interpreting the effects on student growth achievement associated with school-level predictors. In addition, school-specific residuals were not assessed in this study, but the HM3-LVR model results revealed substantially overestimated between-first-clusters slope variance which seems to indicate that the school-specific growth residuals might also be inaccurate using the HM3-LVR model. The between-first-schools slope variance was found to be overestimated across all conditions in this study, which implies that inaccurate decisions might be made about schools with as little as 10% mobility.

Overall, it would be recommended for researchers to utilize the CCMM-LVR model over the HM3-LVR model when analyzing three-level multiple membership data that contains participants moving to different higher-level clustering units over time. In addition, the simulation findings suggest that a minimum of 100 clustering units should be utilized when estimating the CCMM-LVR model in order to avoid overestimation of the level-3 variance component estimates. It also appears that datasets with more than three measurement occasions per individual should be analyzed using the CCMM-LVR model so as to prevent overestimating the Bw_0 parameter.

In summary, future research should continue focusing on finding ways to best handle and assess the impact of ignoring mobility across clusters. This work has provided a first exploration of an extension to the flexible three-level latent variable regression model that could provide the foundation for future research intended to identify optimal solutions for handling multiple membership data structures that are so common in applied educational and social science research.

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