# Essays on Two-Sided Markets and Venture Capitalist Compensation

Ph.D. Thesis in Economics


Keke Sun
International Doctorate in Economic Analysis
Departament d'Economia i d'Història Econòmica
Universitat Autònoma de Barcelona


Supervior: Prof. Jacobus Petrus Maria Hurkens
(Sjaak Hurkens)

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**游子吟**

—唐·孟郊

慈母手中线，游子身上衣。
临行密密缝，意恐迟迟归。
谁言寸草心，报得三春晖。

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## Introduction

The first two chapters of this thesis are about two-sided markets. Two-sided markets are economic platforms that bring two interdependent groups of users together and enable certain interactions between them. The main characteristic of two-sided markets is the indirect network externalities where one group's benefits of joining the platform depends on the number of users from the other group on the same platform. These two chapters are inspired by casual observations from the smartphone operating system (OS) industry. Major platform sponsors, i.e. Apple and Google, have brought their previously successful firm recipes to this new industry; and this has a significant bearing on their current platform strategies. Apple bundles its OS platform with the high-end in-house handset while Google adopts the ad-sponsored business model, introducing adverts on consumers side and collect revenues from developers and advertisers. The first two chapters study the duopolistic competition between platforms with different business models that connect consumers and application developers.

In the first chapter, I study the impact of pure bundling and the level of consumer information about developer subscription prices on duopolistic platform competition. I find that, in the presence of asymmetric network externalities, pure bundling emerges as a profit-maximizing strategy when platforms subsidize the low-externality side (consumers) and make profits on the high-externality side (developers). Bundling can be used as a tool to enhance the "divide-and-conquer" nature of platform's pricing strategies, and is more effective in stimulating consumer demand the larger proportion of informed consumers. I also find that consumer information intensifies price competition. Consequently, bundling and more consumer information improve consumer welfare, but bundling is less likely to emerge as the fraction of informed consumers increases.

In the second chapter, I study the platform's choice of business model between the pure subscription-based and the hybrid ad-sponsored business models when facing a subscription-based rival. Under the hybrid ad-sponsored business model, the platform adjusts its consumer subscription price to compensate consumers for the disutility induced by advertising, affecting consumer and developer demands, hence platform profits. I find that, when consumers have increasing marginal disutility towards advertising, the ad-sponsored business model is more profitable when facing a subscription-based rival, no matter whether platforms set the subscription prices simultaneously on both sides or set the subscription prices on developer's side before consumer's side. Consumers are better off when the platform chooses the hybrid ad-sponsored business model, and are worse off when both platforms set the subscription prices on developer's side before consumer's side.

The third chapter of the thesis contributes to the literature on the venture capital limited partnership agreements. The invisible component of the venture capitalist (VC) compensation is the value-of-distribution rules that determine when the VC receives his share of profits and often generate an interest-free loan between the limited partners (LPs) and the VC. This chapter explores how the distribution rules affect the VC's incentives on the timing of starting and exit decision of investments. We provide the first-best outcomes where the roles of capital provider and decision makers coincide as a benchmark, then compare the investment decisions under different distribution rules with the first-best outcomes. The distribution rules we look into are the Escrow contract, the Return First contract and the Payback contract (Litvak, 2009). Under the Escrow contract, the VC's share of profits goes to an escrow account when each investment is realized. The VC only receives payments at the fund liquidation date and the interest of this account goes to the LPs, which generates an interest-free loan from the VC to the LPs. Under the Return First contract, the VC receives no distributions until the invested capital has been fully paid back to the LPs for each investment. After this threshold, the VC can receive his share of the profits at each exit date. There should be no interest-free loan between the two parties once the invested capital is returned. Under the Payback contract, the VC receives his share of the revenues at the investment exit date and pays back the invested capital back to the LPs when the fund liquidates. This type of contract generates an interest-free loan from the LPs to the VC. The results we find are the following. If there is only one project under consideration, both the first-best investment duration (given a certain level of the carried interest) and the starting date can be attained under the Return First contract because there is no interest-free loan between the two parties. If there are two investment projects under consideration, fixing the project that starts first to be normal, only the Escrow contract can restore the first-best investment durations given a certain level of the carried interest for the VC, but not the first-best starting dates. The first-best starting dates are possible to be attained only under the Payback contract. Our results indicate that, regarding the investment durations, using a certain level of the carry can overcome the distortions induced by the interest-free loan from the VC to the LPs, but not the distortion induced by the fact that the VC does not return the invested capital at the exit date.

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# Chapter 1

# Bundling, information and platform competition

## 1.1 Introduction

A smartphone operating system (OS) platform accommodates applications and makes the interactions between consumers and application developers possible. This paper is motivated by the phenomenal growth in the smartphone industry. As in other two-sided industries, an important decision for firms, with consequences for competition and welfare, is whether to bundle the operating system with the hardware. Apple is currently a major competitor both in the smartphone and the OS markets and its success in hardware has a significant bearing on its success in this industry (Kenney and Pon, 2011). Bundling with a best-selling handset certainly adds to the platform's appeal for consumers[1]. Such bundling practice is also common in industries like the video game industry, where major competitors like Sony and Microsoft bundle the OS platforms with their in-house consoles.

In this paper, we emphasize yet another characteristic of these industries: asymmetry of information of the different players. Developers are industry-insiders; they are usually informed about all subscription prices and have good predictions of participation decisions on both sides of the platform. In contrast, not every consumer knows the fixed fees or royalties that the platforms charge to developers. Similarly, newspaper readers may not be aware of how much the newspaper charges advertisers for listing ads.

[1] The iPhone has been the top-selling mobile phone in the U.S. Source: `http://www.bizjournals.com/boston/blog/mass_roundup/2013/02/apple-top-selling-us-mobile-phone.html`, accessed September, 2014.

The goal of this work is to develop a theoretical model to analyze the bundling strategy for platforms when information may be less than perfect for some users on different sides of the platform. We address two main research questions. First, when does a platform practice bundling as a profitable entry accommodation strategy? Second, how does the level of consumer information affect platform competition and the bundling decision?

We consider a two-stage game in which one of the platforms makes a strategic decision, namely, whether to bundle with an in-house handset, in addition to competing through adjustment of tactical variables which are subscription prices. In the first stage, this platform decides whether to bundle with its in-house handset[2], laying the groundwork for the competitive interactions down the line. In the second stage, the platforms decide subscription prices simultaneously, and competition takes place. We do not consider bundling to be an act of predation, but rather a commitment to an aggressive pricing strategy. The platform sells the bundle at a discount, relative to separate selling, to stimulate consumer and developer demand.

Within the framework of the Hotelling model, two platforms compete for single-homing consumers and multi-homing developers. Departing from the standard setting of full information and responsive expectations for all users in the two-sided markets literature, we assume that some consumers are uninformed about developer subscription prices and hold passive expectations about developer participation, whereas the remaining consumers and all developers are informed about all subscription prices and hold responsive expectations.

We find that, in the presence of asymmetric network externalities, price competition can lead to consumer prices being strategic substitutes when platform preferences are small relative to the benefits of attracting an additional consumer. Therefore, bundling, as a commitment to an aggressive pricing strategy, may be in the interest of the firm and detrimental to the rival when platforms subsidize the low-externality side (consumers) and make profits on the high-externality side (developers). Bundling can be used as a tool to enhance the "divide-and-conquer" nature of the pricing strategies. When consumers are heterogeneous with respect to the valuation of the handset, bundling can also emerge when consumer prices are strategic complements, even though there is no subsidy for participation. Bundling expands consumer demand for platform adoption as well as for the handset. Through bundling, the platform coordinates the misaligned consumer valuations of the platform and the handset, attracting consumers with a high valuation of the handset. Our results further show that bundling improves consumer welfare by lowering the prices and offering more application variety for the majority of consumers.

Our second set of findings concerns consumer information. We find that, when the frac-

[2]We consider pure bundling. Under pure bundling, the handset and the access to platform $A$ can only be purchased as a bundle.

tion of informed consumers is larger, price competition is more intense. Informed consumers respond to price changes by adjusting their demand and their expectations of developer demands accordingly. Therefore, bundling, deployed as an implicit discount on the consumer subscription price, is more effective to stimulate consumer demand when there are more "responsive" consumers. Consequently, the developer demand is also stimulated through the demand shifting effect. Consumer surplus increases in the fraction of informed consumers, because a larger fraction of informed consumer leads to lower subscription prices and more application variety. Consumer information also has a negative impact on the emergence of bundling: the region in which bundling emerges shrinks as the fraction of informed consumers increases. This is because bundling only emerges when the competing platform increases its consumer price in response to bundling, but more consumer information intensifies competition and pushes the competing platform to be more aggressive.

The remainder of the paper is organized as follows. Section 1.2 briefly discusses the related literature. In Section 1.3, we set up the duopoly model of platform competition. In Section 1.4, we investigate the bundling strategy of the platform when consumers are homogeneous with respect to the valuation of the handset. We compare two scenarios, depending on whether the platform practices or does not practice bundling. Section 1.5 studies the bundling strategy when consumers are heterogeneous with respect to the valuation of the handset. Section 1.6 concludes.

## 1.2 Relationship to the Literature

This work contributes to the literature on two-sided platforms. A large share of this literature studies pricing in the presence of network effects[3]. The literature shows that the structure of equilibrium prices depends on the relative size of demand elasticities and indirect network externalities on each side, the marginal costs of serving each side, whether the market structure has single-homing users on each side or takes the form of a competitive bottleneck, that is having single-homing users on one side and multi-homing users on the other side. This works studies the impact of bundling in the framework of two-sided platforms. Pure bundling is usually considered as an act of predation. Whinston (1990) shows that pure bundling reduces equilibrium profits of all firms; hence, it is usually adopted to deter entry or drive the rival out of the market. However, in two-sided markets, this may not be the case. Our paper fits this theme by considering bundling as a tool to stimulate consumer demand. The present work is closely related to Farhi and Hagiu (2008) and Amelio and Jullien (2012). The former study shows that a subsidy on one side may lead

[3] See Caillaud and Jullien (2003), Rochet and Tirole (2003, 2006), Armstrong (2006), Armstrong and Wright (2007), Hagiu (2006), Weyl (2010).

to fundamentally new strategic configurations in oligopoly. Farhi and Hagiu (2008) present the conditions upon which a cost-reducing investment by intermediaries may be a successful entry accommodation strategy and may also benefit its rival. A possible interpretation is that this reduction results from a tying strategy. Prices are not necessarily strategic complements in competition, the effects of cost-reducing investments on prices are ambiguous and platforms may earn negative margin on one side. Amelio and Jullien (2012) investigate the effects of tying of independent goods, with single-homing users on both sides of the platform. With a non-negativity constraint, tying works as an implicit subsidy. In the monopoly case, the platform prefers tying as it is a way to subsidize users that have low network externality. Tying leads to higher participation, higher consumer surplus as well as profits. In the duopoly case, tying on one side makes a platform more or less competitive on the other side depending on externalities of the two sides. The impact of tying on platforms' profits also depends on the relative levels of externalities. Total consumer surplus increases in case of high asymmetry in the network externalities between two sides. Tying is used to implement second-degree price discrimination to help a network to coordinate the customers' participation. In Amelio and Jullien (2012), pure bundling arises if the value of the good is below its cost so that selling the good alone is not profitable. In this work, we show that pure bundling works as a commitment device to allow the platform to be more aggressive. The key features differentiating our work from this line of literature are: firstly, we study the effect of information asymmetry, namely, the level of consumer information, on platform's bundling strategy; secondly, we allow consumers to be heterogeneous with respect to the valuation of the handset. We believe that these features bring our analysis closer to reality. Using the one side single-homing and one side multi-homing, we show that bundling always hurt the rival, which differs from Farhi and Hagiu (2008) and Amelio and Jullien (2012). We also show that without the non-negativity constraint, tying makes no difference from untying when consumers have the homogeneous valuation of the handset tied and the consumer market is fixed-sized.

Our work is also about information asymmetry across the two sides of the platform. The main characteristic of two-sided platforms is the bilateral indirect network externalities where one group's benefit from joining the platform depends on the size of the other group that joins the same platform, which gives rise to a "chicken-and-egg" problem (Armstrong, 2006; Hagiu, 2006). The majority of the existing literature on two-sided platform pricing assumes that all users have full information about all prices and preferences, which implies that all users can perfectly predict other's participation decision. In reality, platform users, especially consumers, may not be able to observe all prices or perfectly anticipate the impact of price changes on demands. Hurkens and López (2014) suggest that passive expectation should be a plausible alternative for responsive expectation in a market with (direct or indirect) network externalities. Passive expectations, first introduced by Katz and Shapiro (1985), are fulfilled in equilibrium. Consumers with passive expectations use price information on consumer's side to fixate their expectation of developer demand and do not respond to any price changes on the other side of the platform. In the present work, we allow users on the two sides to have different levels of information. We assume developers are always well-informed; they are informed about all prices and hold responsive expectations about consumer participation. Consumers are not necessarily as well-informed as developers, not every consumer is aware of how much the platforms charge developers for listing their applications. Therefore, we assume there is a fraction of consumers who are informed about developer prices and hold responsive expectations about developer participation while the remaining consumers are uninformed and hold passive expectations. In this spirit, the present paper is very close to Hagiu and Hałaburda (2014). They study the effect of different levels of information on two-sided platform profits, under both monopoly and competition. They assume that developers always hold responsive expectations while all users hold passive or responsive expectations. They show that responsive expectation amplifies the effect of price reductions. A monopoly platform can exploit the demand increases due to user's responsive expectation, so it prefers facing more informed users. While more information intensifies price competition, competing platforms are affected negatively when users are well informed. Our symmetric competition subgame is a replica of Hagiu and Hałaburda (2014)'s hybrid scenario in which some consumers are informed while others are uninformed and hold passive expectations. We reach the same conclusion that more information intensifies price competition regardless of the bundling decision. There is one key difference between Hagiu and Hałaburda (2014) and our analysis. They focus on the impact of different user expectations on equilibrium allocations in the context of monopoly and duopoly markets, whereas we are interested in the impact of the level of consumer information on platform's bundling decision in a duopoly setting, because our work models the competition between smartphone OS platforms where bundling has a significant bearing.

## 1.3 The Model

### 1.3.1 Platforms

Consider two platforms competing for both consumers and developers, indexed by $T$=$A$, $B$. Let $p_T^C$ and $p_T^D$ denote the subscription prices platform $T$ charges to consumers and developers, respectively. We assume that the platforms have zero marginal cost of serving these two groups of users, which is consistent with the literature of information goods and the reality of digital media industry, where large fixed costs and very low marginal costs are observed[4]. We allow for negative prices, as it is possible for platforms to subsidize one side of the market. The number of consumers and developers on platform $T$ are denoted by $n_T^C$ and $n_T^D$, respectively. We allow single-homing on one side and multi-homing on the other side. To be more specific, we assume that each consumer decides in favor of only one platform while developers can design applications for both platforms.

We extend the standard Hotelling model by allowing the duopoly to serve two groups of users on each side of the market. The unit transportation cost for consumers towards each end is $t$, which is the platform differentiation parameter. Platform $A$ and $B$ are exogenously located at $x = 0$ and $x = 1$, respectively. Platform $T$'s profit maximization problem is

$$\max_{p_T^C, p_T^D} \quad \pi_T = p_T^C n_T^C + p_T^D n_T^D,$$

where $T = A, B$.

We assume one of the platforms (without loss of generality, platform $A$) has a in-house killer handset with quality $z$ and marginal cost $C$, for instance, the iPhone by Apple. For calculation simplicity, we normalize this marginal cost $C = 0$. Platform $A$ is a monopolist in the high-end handset market, which vertically differentiated from the other handsets. Platform $A$ can decide whether to bundle with this handset or not.

### 1.3.2 Consumers

There is mass 1 of consumers uniformly distributed along the unit interval, each of whom chooses at most one platform to join. The consumers have identical intrinsic values of two platforms, equivalent to $v$, which is assumed to be large enough so that the whole market is covered. Consumers have a taste for application variety. Every consumer's utility of participating on a platform depends on the total number of developers on the same platform. Consumers have identical utility gain from application variety; parameter $\theta$ is used to capture this direction of network externalities. More specifically, the availability of each additional developer positively generates additional utility $\theta$ for consumers, i.e. $\theta>0$. We ignore the potential positive direct externalities among consumers[5]. The consumer who locates at $x$ decides joining platform $A$ or $B$ by comparing utilities $v+\theta n_A^{D^e} - p_A^C - tx$ from platform $A$ and $v+\theta n_B^{D^e} - p_B^C - t(1-x)$ from platform $B$. Following Hagiu and Hałaburda (2014), we assume there are two types of consumers: a fraction $\lambda$ of consumers is informed about developer

[4]This assumption is made for calculation simplicity. Assuming platforms have the marginal cost of $c^C$ and $c^D$ of serving consumer's side and developer's side complicates the calculations but do not change the qualitative results of the model.

[5]The potential positive externalities indicate that consumers may derive positive utilities from the number of other consumers on the same platform.

subscription prices and holds responsive expectations about developer participation when choosing between two platforms, $0\leq\lambda\leq1$. The expectations of these consumers match the realized developer demand, i.e., $n_T^{D^e} = n_T^D$. The remaining fraction $1-\lambda$ of consumers is uninformed about developer prices and holds passive expectations. They do not adjust their expectation of developer demand in response to price changes on developer's side[6]. Therefore, the realized consumer demand of each platform is

$$n_T^C = \frac{1}{2} + \frac{p_{-T}^C - p_T^C}{2t} + \frac{\lambda\theta n_T^D - \lambda\theta n_{-T}^D}{2t} + \frac{(1-\lambda)\theta n_T^{D^e} - (1-\lambda)\theta n_{-T}^{D^e}}{2t}, \tag{1.1}$$

where $T = A, B$.

For now, we assume consumers are homogeneous with respect to the valuation of the handset; they have identical marginal utility of the quality of platform $A$'s in-house handset $\phi$=1, and each buys at most one copy.

### 1.3.3 Developers

There is mass 1 of potential developers; each developer lists one application on one platform. Assume that developers form responsive expectations of consumer demand. Developers are industry insiders, they are aware of consumer's preference, thus, can perfectly predict consumer participation. Developers differ in the cost of listing applications, denoted by $y$, and are uniformly distributed along the segment $[0, 1]$. Each developer gains additional utility of $\beta$ from each consumer who has access to its application. The revenue for a developer who lists on platform $T$ is given by $\beta n_T^C$ when the number of consumers who participate in platform $T$ is $n_T^C$. We assume all applications are independent of one another, so the potential negative direct network externalities are ignored. The utility of developer $y$ from joining platform $T$ is

$$u_T^D = \beta n_T^C - p_T^D - y,$$

where $T = A, B$. We assume that developers can multi-home[7] and there are no economies of scope in multihoming. Therefore, the decision of joining a platform is independent of the joining decision of the other platform. That is, a $y$-type developer will join platform $T$ if $u_T^D(y) = \beta n_T^C - p_T^D - y\geq0$. So, the developer demand of each platform is

$$n_T^D = \beta n_T^C - p_T^D.$$

[6]Hurkens and López (2014) offers a clear illustration of the difference between passive and responsive expectations.

[7]Some recent survey shows that on average mobile developers use 2.6 mobile platforms (VisionMobile, 2013).

Developers care more about the network benefits of reaching out to the widest population of consumers than they do about the cost of multi-homing since there is no standalone value for developers to join the platforms. We study a case of "competitive bottlenecks" (Armstrong, 2006): there is a high level of competition on consumer's side, and platforms make low profits on this side, but there is no competition for providing applications to consumers.

We assume that the following conditions hold throughout this paper:
**Assumption A1.** $\beta > 2\theta$.

We assume developers care more about consumers than consumers do about developers. This level of asymmetry between the two directions of network effect guarantees the existence of the situation where platforms engage in divide-and-conquer strategies.
**Assumption A2.** $t > \underline{t} = \frac{\beta^2}{6} + \frac{\theta\beta}{6} + \frac{\theta^2\lambda}{6} + \frac{\theta\beta\lambda}{2}$.

With these two assumptions, the conditions for unique and stable equilibrium ($t > \frac{\beta^2}{6} + \frac{\theta^2\lambda^2}{6} + \frac{2\theta\beta\lambda}{3}$) and second order condition ($t > \theta\beta\lambda$) are satisfied, and both platforms make positive profits in equilibrium, so that they remain active in the market[8].

As we are interested in the impact of bundling on platform competition, we assume platform $A$'s bundling decision cannot drive its rival out of the market:
**Assumption A3.** $0 < z < \overline{z} = 3t - 3\underline{t}$.

When $z \geq \overline{z}$, platform $A$ would always bundle the platform with the handset to push the rival out of the market.

The next condition rules out the corner solution that the developer demand for each platform is 1:
**Assumption A4.** $\theta + \beta < 2$.

We propose a two-stage game. The timing of the game is as follows: In Stage 1, platform $A$ makes the strategic decision whether to bundle with its in-house handset or not. The decision is publicly observable. In Stage 2, two platforms simultaneously decide on subscription prices for consumers and developers, and competition takes place.

[8]Notice that $\frac{\beta^2}{6} + \frac{\theta\beta}{6} + \frac{\theta^2\lambda}{6} + \frac{\theta\beta\lambda}{2} - (\frac{\beta^2}{6} + \frac{\theta^2\lambda^2}{6} + \frac{2\theta\beta\lambda}{3}) = (\frac{\theta\beta}{6} + \frac{\theta^2\lambda}{6})(1-\lambda) \geq 0$ and $\frac{\beta^2}{6} + \frac{\theta\beta}{6} + \frac{\theta^2\lambda}{6} + \frac{\theta\beta\lambda}{2} - \theta\beta\lambda = \frac{\beta^2}{6} + \frac{\theta\beta}{6} + \frac{\theta^2\lambda}{6} - \frac{\theta\beta\lambda}{2} \geq \frac{\beta^2\lambda}{6} + \frac{\theta\beta\lambda}{6} + \frac{\theta^2\lambda}{6} + \frac{\theta\beta\lambda}{2} = \frac{\lambda}{6}(\beta - \theta)^2 \geq 0$. Therefore, once Assumption $A2$ is satisfied, both $t > \frac{\beta^2}{6} + \frac{\theta^2\lambda^2}{6} + \frac{2\theta\beta\lambda}{3}$ and $t > \theta\beta\lambda$ hold.

## 1.4 Platform Competition

We analyze the bundling strategy of the platform by comparing the no bundling and bundling scenarios. We also compare bundling with tying. The difference between pure bundling and tying is that, the tied good is still available on a stand-alone basis under tying, which means that, under tying, consumers on platform $B$ can still purchase the handset (see Tirole, 2005).

### 1.4.1 Symmetric Competition

We first derive the competition outcomes when platform $A$ doesn't bundle with its in-house handset as the benchmark case. Platforms engage in symmetric competition as they both make profits through subscription. Platform $T$'s profit function is given by

$$\max_{p_T^C, p_T^D} \quad \pi_T = p_T^C n_T^C + p_T^D n_T^D,$$

where $T = A, B$. Platform $A$ also has revenue $z$ stemming from the in-house handset.

A fraction $\lambda$ of consumers is informed about all subscription prices; they make the participation decision upon subscription prices for both consumers and developers. The remaining consumers are only informed about prices on consumer's side; they make the participation decision upon consumer subscription prices and their expectations about developer participation for each platform. Thus, the realized consumer demand for each platform is

$$n_T^C = \frac{1}{2} + \frac{p_{-T}^C - p_T^C}{2t - 2\theta\beta\lambda} + \frac{\theta(1-\lambda)n_T^{D^e} - \theta(1-\lambda)n_{-T}^{D^e}}{2t - 2\theta\beta\lambda} + \frac{\theta\lambda p_{-T}^D - \theta\lambda p_T^D}{2t - 2\theta\beta\lambda}, \tag{1.2}$$

where $T = A, B$.

**Proposition 1.** *When two platforms engage in symmetric competition, the competition equilibrium outcomes are as follows:*

$$p_T^{C^*} = t - \frac{\beta^2}{4} - \frac{3\theta\beta\lambda}{4}, \qquad n_T^{C^*} = \frac{1}{2},$$

$$p_T^{D^*} = \frac{\beta}{4} - \frac{\theta\lambda}{4}, \qquad n_T^{D^*} = \frac{\beta}{4} + \frac{\theta\lambda}{4},$$

$$\pi_A^* = \frac{t}{2} - \frac{\theta^2\lambda^2}{16} - \frac{3\theta\beta\lambda}{8} - \frac{\beta^2}{16} + z,$$

*and*

$$\pi_B^* = \frac{t}{2} - \frac{\theta^2\lambda^2}{16} - \frac{3\theta\beta\lambda}{8} - \frac{\beta^2}{16},$$

*where $T = A, B$.*

*Proof.* See Appendix. □

The equilibrium consumer subscription price is the standard Hotelling price with zero marginal cost ($t$) adjusted downwards by $\frac{\beta^2}{4} + \frac{3\theta\beta\lambda}{4}$. The adjustment term, which measures the benefits to the platform from attracting an additional consumer, can be decomposed into two factors $\beta(\frac{\beta}{4} + \frac{3\theta\lambda}{4})$. The first factor $\beta$ means the platform attracts $\beta$ additional developers when it has an additional consumer. The second factor $\frac{\beta}{4} + \frac{3\theta\lambda}{4}$ is the profit that the platform can earn from an additional developer. The additional developer pays a subscription price $\frac{\beta}{4} - \frac{\theta\lambda}{4}$ to the platform, also attracts $\theta\lambda$ informed consumers because only informed consumers would adjust their expectations of developer demand according to price changes. The platforms decide their pricing strategies on consumer's side by comparing platform preferences with the benefits of attracting one extra consumer. The larger network externalities ($\beta$ and $\theta$) are, the lower price is charged on the consumer's side.

The equilibrium developer subscription price is the monopoly pricing $\frac{\beta}{4}$[9] adjusted downwards by $\frac{\theta\lambda}{4}$, where $\frac{\theta\lambda}{4}$ is the extra benefit that an extra developer brings to the platform from attracting informed consumers. When $\beta$ is large, developers attach a high value to consumer participation, and platforms have incentives to lower consumer prices or even subsidize consumers for participation. So that the platforms can charge higher prices on developer's side. The equilibrium developer price increases with developer's network externalities. When $\theta$ is large, consumers attach a high value to developer participation, and platforms have incentives to lower developer prices to encourage participation, the equilibrium developer price decreases with consumer's network externalities.

**Corollary 1.** *The subscription prices on both sides of the platform are negatively affected by the fraction of informed consumers while developer participation is positively affected by it. The platform profits decrease in the fraction of informed consumers.*

When informed consumers are offered a lower price, they anticipate that consumer demand would increase and developer demand would increase accordingly. This intensifies price competition (Hagiu and Hałaburda, 2014). Indeed, the intensity of competition increases in the fraction of informed consumers.

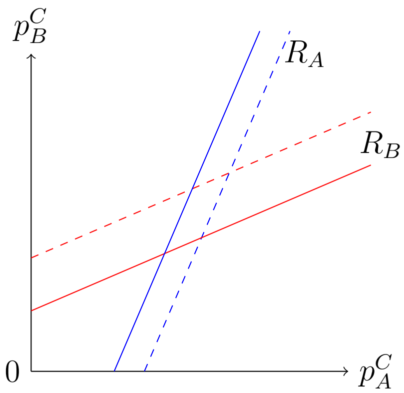
[9] If all consumers are uninformed about developer subscription prices and hold passive expectations about developer demand, platforms exploit monopoly power on developer's side and charge developer subscription price $\frac{\beta}{2}p_T^C$, the equilibrium developer subscription price is $\frac{\beta}{4}$.

The best response function on consumer's side is

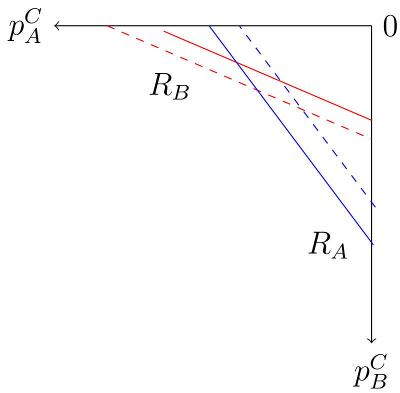
$$
\begin{aligned}
p_T^C(p_{-T}^C) = & \frac{(4t-\beta^2-3\theta\beta\lambda)(4t-\theta^2\lambda^2-3\theta\beta\lambda)}{\gamma} p_{-T}^C \\
& + \frac{(4t-\beta^2-3\theta\beta\lambda)(4t^2+\theta^2\beta^2\lambda^2-5t\theta\beta\lambda)}{\gamma} \\
& + \frac{(4t-\beta^2-3\theta\beta\lambda)\theta(1-\lambda)(4t-3\theta\beta\lambda)(n_T^{D^e}-n_{-T}^{D^e})}{\gamma},
\end{aligned}
\tag{1.3}
$$

where $\gamma = 32t^2 - 4t\theta^2\lambda^2 - 44t\theta\beta\lambda - 4t\beta^2 + 3\theta^3\beta\lambda^3 + 14\theta^2\beta^2\lambda^2 + 3\theta\beta^3\lambda$.

Depending on whether the platforms charge consumers positive prices or subsidize consumers for participation, we have two cases. When $t > \frac{\beta^2}{4} + \frac{3\theta\beta\lambda}{4}$, the platforms charge consumers positive prices for participation, this is the case where platform preferences are larger than the benefits of attracting an extra consumer. The best response curves on consumer's side are upward-sloping (see the dashed lines in Figure 1.1($a$)), the consumer prices of the two platforms are strategic complements ($\frac{\partial p_T^C(p_{-T}^C)}{\partial p_{-T}^C} > 0$). This is the case we often see in one-sided market. When $\underline{t} < t < \frac{\beta^2}{4} + \frac{3\theta\beta\lambda}{4}$,platform preferences are small relative to the benefits of attracting an extra consumer. This is the case where platforms subsidize consumers, which is the low-externality side, and earns a positive margin on developer's side, which is the high-externality side. This is often seen in two-sided markets as the "divide-and-conquer" strategy(Caillaud and Jullien, 2003). The best response curves on consumer's side are downward-sloping (see the dashed lines in Figure 1.1($b$)), and the consumer prices are strategic substitutes ($\frac{\partial p_T^C(p_{-T}^C)}{\partial p_{-T}^C} < 0$) (Besanko et al., 2000).




(a) Strategic Complements




(b) Strategic Substitutes

Figure 1.1: Best response curves on consumer side under bundling

### 1.4.2 Bundling

We assume platform $A$ sets $p_A$ as the price for the bundled products, then $p_A^C = p_A - z$ is the implicit subscription price for consumers. Under pure bundling, neither the in-house handset nor the access to platform A would be available on a standalone basis. Platform $A$ would now charge a lower price for the bundled products, relative to separate selling. Under bundling, platform $A$ has more incentives to lower the consumer price, a fall of $p_A^C$ not only encourages consumer participation, but also stimulates demand of the handset. The marginal consumer locating at $x$ derives utility $v + z - (p_A^C + z) - tx + \theta n_A^{D^e}$ from purchasing the bundle and $v - p_B^C - t(1-x) + \theta n_B^{D^e}$ from joining platform $B$. Again, a fraction $\lambda$ of consumers is informed about developer subscription prices and holds responsive expectations about developer participation, i.e., $n_T^{D^e} = n_T^D$; while the remaining consumers are uninformed and hold passive expectations. Therefore, the consumer demand of each platform is the same as Eq. (1.1). Platform $A$'s profit maximization problem evolves to

$$\max_{p_A^C, p_A^D} \quad \pi_A = p_A n_A^C + p_A^D n_A^D = (p_A^C + z) n_A^C + p_A^D n_A^D.$$

Platform $B$'s profit maximization problem is unchanged.

The following proposition characterizes the equilibrium prices and allocations in the bundling scenario.

**Proposition 2.** *When platform A bundles with its in-house handset and consumers are homogeneous with respect to the valuation of the handset, the equilibrium outcomes are as follows:*

$$p_A^{C^*} = t - \frac{\beta^2}{4} - \frac{3\theta\beta\lambda}{4} - \frac{z(8t - 2\theta\beta - \beta^2 - 2\theta^2\lambda - 3\theta\beta\lambda)}{12(t - \underline{t})},$$

$$p_B^{C^*} = t - \frac{\beta^2}{4} - \frac{3\theta\beta\lambda}{4} - \frac{z(4t - \beta^2 - 3\theta\beta\lambda)}{12(t - \underline{t})},$$

$$n_A^{C^*} = \frac{1}{2} + \frac{z}{6(t - \underline{t})}, \qquad n_B^{C^*} = \frac{1}{2} - \frac{z}{6(t - \underline{t})},$$

$$p_A^{D^*} = (\frac{\beta - \theta\lambda}{2})(\frac{1}{2} + \frac{z}{6(t - \underline{t})}), \qquad n_A^{D^*} = (\frac{\beta + \theta\lambda}{2})(\frac{1}{2} + \frac{z}{6(t - \underline{t})}),$$

$$p_B^{D^*} = (\frac{\beta - \theta\lambda}{2})(\frac{1}{2} - \frac{z}{6(t - \underline{t})}), \qquad n_B^{D^*} = (\frac{\beta + \theta\lambda}{2})(\frac{1}{2} - \frac{z}{6(t - \underline{t})}),$$

$$\pi_A^* = \frac{8t - \beta^2 - \theta^2\lambda^2 - 6\theta\beta\lambda}{16} \frac{(6(t - \underline{t}) + 2z)^2}{36(t - \underline{t})^2}$$

*and*

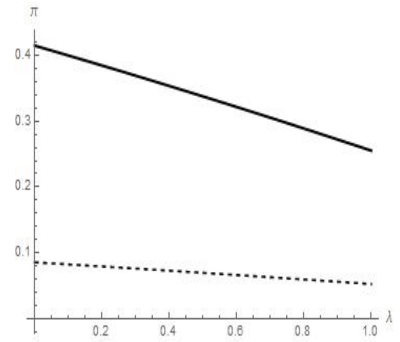
$$\pi_B^* = \frac{8t - \beta^2 - \theta^2\lambda^2 - 6\theta\beta\lambda}{16}\frac{(6(t - \underline{t}) - 2z)^2}{36(t - \underline{t})^2}.$$

*Proof.* See Appendix. □

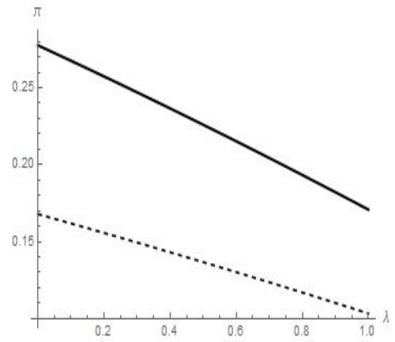
Under bundling, only consumers on platform $A$ purchase the handset. Platform $A$ has to lower its consumer price to stimulate the demand for the bundled products. Thus, it manages to steal some consumers from its rival. The higher value of the bundling handset, the more leverage platform $A$ has on consumers. However, the cut on platform $A$'s consumer price dominates the increment on its consumer demand. Hence, platform $A$ suffers a loss on consumer's side. Yet, the direction of change on platform $B$'s consumer subscription price is ambiguous, depending on the strategic relationship between consumer subscription prices. When consumer prices are strategic complements (resp. substitutes), platform $B$'s consumer price goes down (resp. up). The directions of changes on profits from developer's side for both platforms are clear. As a fall on $p_A^C$ shifts the consumer demand toward platform $A$, platform $A$ (resp. $B$) becomes more (resp. less) attractive on developer's side through network effects. This effect increases with $\beta$, which determines the sensitivity of developer demand to the demand on consumer's side.

**Corollary 2.** *When platform A bundles with its in-house handset, its implicit consumer price decreases with the fraction of informed consumers while its demands on both sides of the platform increase with it. Platform B's developer price, demands on both sides of the platform and total profits are negatively affected by the fraction of informed consumers.*

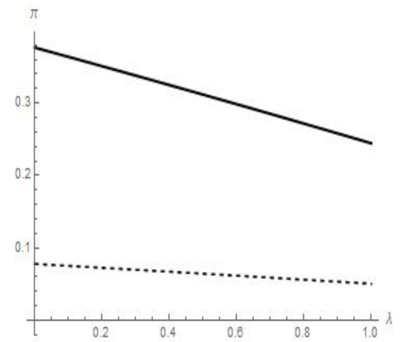
This corollary has significant empirical implications. It indicates that bundling is a more effective tool to stimulate consumer demand when there are more informed consumers. The effect of a discount on platform $A$'s consumer price is amplified. Platform $A$'s demands on both sides of the market reach the highest levels when all consumers are informed. Platform $B$ suffers the largest loss when all consumers are informed. The effect of the fraction of the informed consumers on platform $A$'s profits is ambiguous. Figure 1.2 illustrates how platform profits under bundling change with the fraction of informed consumers $\lambda$, for certain values $\theta$, $\beta$ and $z$. All graphs have parameter $t = 0.5$. The solid line depicts $\pi_A$ and the dotted line depicts $\pi_B$.



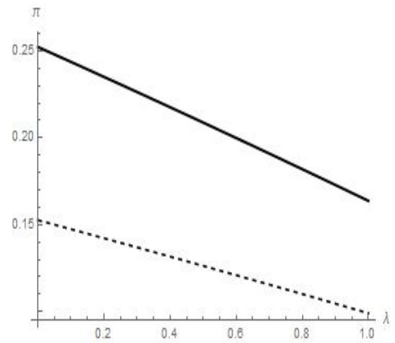

(a) $\theta = 0.3$, $\beta = 0.7$, $z = \frac{3}{4}(3t - 3\underline{t})$




(c) $\theta = 0.3$, $\beta = 0.7$, $z = \frac{1}{4}(3t - 3\underline{t})$




(b) $\theta = 0.2$, $\beta = 0.9$, $z = \frac{3}{4}(3t - 3\underline{t})$




(d) $\theta = 0.2$, $\beta = 0.9$, $z = \frac{1}{4}(3t - 3\underline{t})$

Figure 1.2: Platform profits as functions of the level of consumer information

The system of best response functions on consumer's side is as follows:

$$\begin{aligned} p_A^C(p_B^C) = & \frac{(4t - \beta^2 - 3\theta\beta\lambda)(4t - \theta^2\lambda^2 - 3\theta\beta\lambda)}{\gamma} p_B^C \\ & + \frac{(4t - \beta^2 - 3\theta\beta\lambda)(4t^2 + \theta^2\beta^2\lambda^2 - 5t\theta\beta\lambda)}{\gamma} \\ & + \frac{(4t - \beta^2 - 3\theta\beta\lambda)\theta(1-\lambda)(4t - 3\theta\beta\lambda)(n_A^{D^e} - n_B^{D^e})}{\gamma} \\ & - \frac{16t^2 - 4t\theta^2\lambda^2 - 20t\theta\beta\lambda + 3\theta^3\beta\lambda^3 + 5\theta^2\beta^2\lambda^2}{\gamma} z, \end{aligned} \tag{1.4}$$

and

$$p_B^C(p_A^C) = \frac{(4t-\beta^2-3\theta\beta\lambda)(4t-\theta^2\lambda^2-3\theta\beta\lambda)}{\gamma}p_A^C + \frac{(4t-\beta^2-3\theta\beta\lambda)(4t^2+\theta^2\beta^2\lambda^2-5t\theta\beta\lambda)}{\gamma} + \frac{(4t-\beta^2-3\theta\beta\lambda)\theta(1-\lambda)(4t-3\theta\beta\lambda)(n_B^{D^e}-n_A^{D^e})}{\gamma} - \frac{\theta^2\lambda^2(4t-\beta^2-3\theta\beta\lambda)}{\gamma}z. \tag{1.5}$$

Compared to Eq. (1.3), there are two effects determining the movements of the best response curves. The terms proportional to $z$ represent the impact of bundling on consumer prices. Bundling has a direct impact on consumer prices: all consumers observe the changes on consumer prices. It also has an indirect impact on consumer prices: informed consumers anticipate the impact of bundling on developer's participation decisions. The terms proportional to $n_A^{D^e} - n_B^{D^e}$ represent the impact of bundling on perceived platform quality in terms of application variety for uninformed consumers. Following Amelio and Jullien (2012), we separate the impact of $p_A^C$ on the derivative of platform profits as follows:

$$\frac{\partial}{\partial p_A^C}(\frac{\partial \pi_A}{\partial p_A^D}) = \frac{\partial n_A^C}{\partial p_A^D} + \frac{\partial n_A^D}{\partial p_A^C} = -\frac{\theta\lambda}{2t-2\theta\beta\lambda} - \frac{\beta}{2t-2\theta\beta\lambda}, \tag{1.6}$$

and

$$\frac{\partial}{\partial p_A^C}(\frac{\partial \pi_B}{\partial p_B^D}) = \frac{\partial n_B^D}{\partial p_A^C} = \frac{\beta}{2t-2\theta\beta\lambda}. \tag{1.7}$$

The term $-\frac{\beta}{2t-2\theta\beta\lambda}$ in Eq. (1.6) captures the fact that a fall of $p_A^C$ shifts the consumer demand towards platform $A$. As a result, platform $A$ becomes more attractive for developers. The best response curve of platform $A$ shifts upwards because its perceived quality has improved for consumers. Similarly, the term $\frac{\beta}{2t-2\theta\beta\lambda}$ in Eq. (1.7) indicates that a fall of $p_A^C$ makes platform $B$ less attractive for developers.

The term $-\frac{\theta\lambda}{2t-2\theta\beta\lambda}$ in Eq. (1.6) captures the other direction of the demand shifting effect. A fall of $p_A^D$ increases the developer demand for platform $A$, which improves platform quality in terms of application variety. Therefore, the consumer demand shifts upwards. Note that this direction of effect is discounted because only informed consumers adjust their expectations of developer demand according to the price change. The sensitivity of this direction of demand shifting effect depends on both consumer's network externalities and the fraction of informed consumers. The higher fraction of informed consumers there is, the more sensitive this direction of demand shifting effect is.

In the same fashion as before, we further discuss the impact of bundling decision depending on whether the platforms charge consumers positive prices or subsidize consumers for participation.

#### 1.4.2.1 Case I. Strategic Complements

When $t > \frac{\beta^2}{4} + \frac{3\theta\beta\lambda}{4}$, the best response curves are again upward-sloping, indicating that the consumer prices are strategic complements. The best response curves are shown in Figure 1.1($a$). Compare to the dashed lines in Figure 1.1($a$), we see that, under bundling, the response curve of platform $A$ moves to the left and the curve of platform $B$ shifts downwards. Through bundling, platform $A$ offers a discount on consumer subscription price, so the response curve of platform $A$ moves downwards, but this effect is dampened by consumer's expectations of more application variety on platform $A$. Under bundling, platform $A$ has a higher consumer demand, the demand shifting effect indicates that it also has a higher developer demand, the perceived quality of platform $A$ increases and the perceived quality of platform $B$ decreases. Platform $A$'s best response curve has the tendency to move upwards. Also, when consumer prices are strategic complements, platform $B$ lowers its price in response to bundling.

Pure bundling, works as a commitment device, has both a direct and a strategic effect on the platform's profits (Besanko et al., 2000). The direct effect of the commitment is its impact on the platform profits if the rival's behavior does not change, and the strategic effect takes into account how the commitment changes the tactical decisions of rivals and, ultimately, the market equilibrium (Besanko et al., 2000). We decompose the effect of $z$ on platform $A$'s own profits into a direct effect and strategic effects on both sides of the platform.

$$\frac{d\pi_A}{dz} = \frac{\partial \pi_A}{\partial z} + \frac{\partial \pi_A}{\partial p_B^C}\frac{dp_B^{C^*}}{dz} + \frac{\partial \pi_A}{\partial p_B^D}\frac{dp_B^{D^*}}{dz}$$

Note that the direct effect is $\frac{\partial \pi_A}{\partial z} = n_A^C$. It indicates that platform $A$ suffers a loss on the handset sales under bundling compared to the no bundling case. The term $\frac{\partial \pi_A}{\partial p_B^C}\frac{dp_B^{C^*}}{dz}$ represents the strategic effect of bundling on consumer's side:

$$\frac{\partial \pi_A}{\partial p_B^C}\frac{dp_B^{C^*}}{dz} = (p_A^C + z + \beta p_A^D)\frac{\partial n_A^C}{\partial p_B^C}\left(-\frac{4t - \beta^2 - 3\theta\beta\lambda}{12(t - \underline{t})}\right) < 0.$$

The intuition is that bundling drives the rival to set the consumer price low when prices are strategic complements, it intensifies competition on consumer's side. The last term $\frac{\partial \pi_A}{\partial p_B^D}\frac{dp_B^{D^*}}{dz}$

represents the strategic effect of bundling on developer's side:

$$\frac{\partial \pi_A}{\partial p_B^D}\frac{dp_B^{D^*}}{dz} = (p_A^C + z + \beta p_A^D)\frac{\partial n_A^C}{\partial p_B^D}(-\frac{\beta - \theta\lambda}{12(t - \underline{t})}) < 0.$$

Bundling has a strategic effect on developer's side because informed consumers adjust their expectations about developer participation due to bundling. Bundling makes the competing platform less attractive for developers through the demand shifting effect. Consequently, the competing platform has to lower its developer price. In this case, bundling intensifies competition on both sides of the platform. The speed of platform $A$'s profit increasing in the value of the handset drops from 1 (no bundling) to a speed slower than $n_A^C$ (under bundling) (see Figure 1.3($a$)). Bundling cannot be profitable for platform $A$ in this case.

Bundling has strategic effects on platform $B$'s profits:

$$\frac{d\pi_B}{dz} = \frac{\partial \pi_B}{\partial p_A^C}\frac{dp_A^{C^*}}{dz} + \frac{\partial \pi_B}{\partial p_A^D}\frac{dp_A^{D^*}}{dz}.$$

The first term of strategic effect also concerns the effect of bundling on consumer's side:

$$\frac{\partial \pi_B}{\partial p_A^C}\frac{dp_A^{C^*}}{dz} = (p_B^C\frac{\partial n_B^C}{\partial p_A^C} + p_B^D\beta\frac{\partial n_B^C}{\partial p_A^C})(-\frac{8t - \beta^2 - 2\theta\beta - 2\theta^2\lambda - 3\theta\beta\lambda}{12(t - \underline{t})}) < 0.$$

Under bundling, platform $A$ sets a low subscription price, platform $B$ has to lower its price in response. Platform $A$'s bundling decision leads to a more competitive environment on consumer's side. On developer's side, the strategic effect of bundling is:

$$\frac{\partial \pi_B}{\partial p_A^D}\frac{dp_A^{D^*}}{dz} = (p_B^C\frac{\partial n_B^C}{\partial p_A^D} + p_B^D\beta\frac{\partial n_B^C}{\partial p_A^D})(\frac{\beta - \theta\lambda}{12(t - \underline{t})}) > 0.$$

Bundling makes platform $A$ more attractive to developers, which increase its developer subscription price. Thus, there is room for platform $B$ to increase its developer subscription price as well. Bundling softens competition on this side of the platform. The over all effect of z on platform $B$'s profits are as follows:

$$\frac{d\pi_B}{dz} = n_B^C(-\frac{8t - \beta^2 - 2\theta\beta - 2\theta^2\lambda - 3\theta\beta\lambda}{12(t - \underline{t})}) + \theta\lambda n_B^C(\frac{\beta - \theta\lambda}{12(t - \underline{t})}) < 0.$$

Bundling is detrimental to platform $B$'s profit in this case.

#### 1.4.2.2 Case II. Divide-and-Conquer

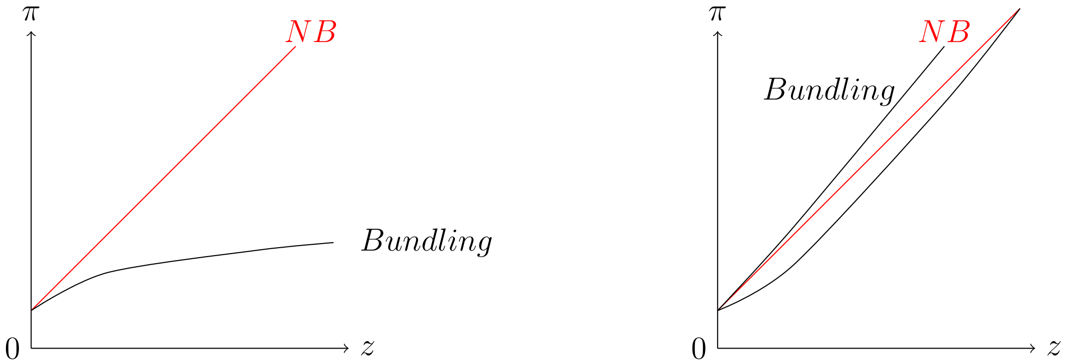
When $\underline{t} < t < \frac{\beta^2}{4} + \frac{3\theta\beta\lambda}{4}$, the best response curves are downward-sloping and consumer subscription prices are again strategic substitutes. Both platforms subsidize consumers for participation. The changes on equilibrium consumer prices are shown in Figure 1.1($b$). Compare to the dashed lines in Figure 1.1($b$), the response curve of platform $A$ shifts downwards under bundling. Through bundling, platform $A$ increases the subsidy for consumer participation, so the response curve of platform $A$ moves downwards. In response, platform $B$ reduces its consumer subsidy because consumer prices are strategic substitutes. Platform $B$'s best response curve moves upwards. The demand shifting effect indicates that platform $A$ has higher developer participation under bundling. The perceived quality of platform $A$ increases, platform $A$ increases subsidy for consumer participation to compete very fiercely for consumer demand because the benefit of attracting one consumer is larger than platform preferences. The best response curve of platform $A$ moves downwards further. Uninformed consumers expect platform $B$ to offer less application variety, the perceived quality of platform $B$ drops. The demand shifting effect indicates that platform $B$ is less attractive to developers. Platform $B$ cuts subsidy for consumer participation further, its best response curve moves upwards further.

We investigate the impact of $z$ on platform $A$'s profits:

$$\frac{d\pi_A}{dz} = n_A^C + n_A^C \frac{-4t + \beta^2 + 2\theta\beta\lambda + \theta^2\lambda^2}{12(t - \underline{t})}.$$

The strategic effect on consumer's side is positive. platform $B$'s response to bundling is to reduce its consumer subsidy, which softens competition on this side of the platform. When platform preferences are small, it is possible that the positive strategic effect on consumer's side dominates the negative strategic effect on developer's side. Although the handset is only sold to consumers on platform $A$ under bundling, platform $A$'s profit increases in the value of the handset faster than $n_A^C$ when consumer prices are strategic substitutes. As depicted in Figure 1.3($b$), the speed of platform $A$'s profits increasing in the value of the handset could be strictly faster than 1, which means that bundling would be profitable for any value of $z$, or bundling could be profitable only when the value of the handset is significant. In all, bundling can be profitable when consumers prices are strategic substitutes.

The overall effect of $z$ on platform $B$'s profits is negative, platform $B$ cannot gain any profits from the rival's bundling practice.




(a) Strategic Complements (b) Strategic Substitutes

Figure 1.3: The impact of bundling on platform profits

Platform $A$ determines its bundling strategy by comparing the profits in two subgames. Let $t_1 = \frac{3\beta^2+4\theta\beta+6\theta\beta\lambda+4\theta^2\lambda-\theta^2\lambda^2}{16}$, $z_1 = \frac{(6t-6\underline{t})(16t+\theta^2\lambda^2-4\theta^2\lambda-6\theta\beta\lambda-4\theta\beta-3\beta^2)}{8t-\theta^2\lambda^2-6\theta\beta\lambda-\beta^2}$ and $t_2 = \frac{5\beta^2+8\theta\beta+6\theta\beta\lambda+8\theta^2\lambda-3\theta^2\lambda^2}{24}$. The following proposition states platform $A$'s bundling strategy.

**Proposition 3.** *When consumers are homogeneous with respect to the valuation of platform A's in-house handset,*
*(i) platform A always chooses to bundle with the handset for all* $z < \overline{z}$ *when* $\underline{t} < t \leq t_1$*;*
*(ii) platform A bundles if the value of the handset is high, i.e.,* $z_1 \leq z < \overline{z}$*, when* $t_1 < t \leq t_2$*;*
*(iii) platform A never practices bundling when* $t > t_2$*,*
*Bundling always hurts the rival.*

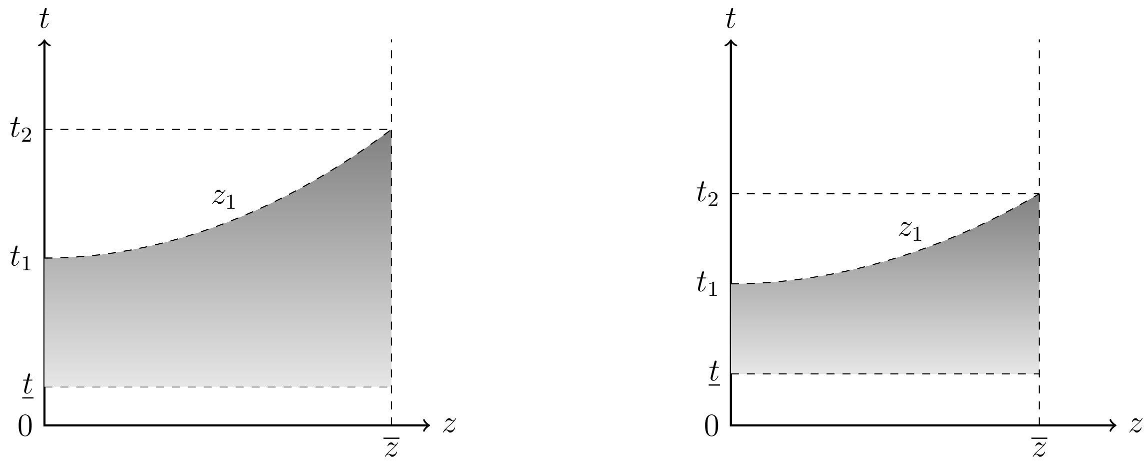
*Proof.* See Appendix. □

It is worth commenting that when platform preferences are small relative to the network externalities, a small extra consumer demand can lead to significant profits on developer's side, so platform $A$ is willing to bundle with the handset even if it can only steal small consumer demand from the rival. When the platform preferences are medium, to recoup the loss on consumer's side due to bundling, platform $A$ needs to have a great consumer demand. Therefore, platform $A$ would practice bundling only when bundling can steal a large consumer demand from the rival, that is to say, the value of bundled handset needs to be significant. When platform preferences are large relative to the network externalities, platform $A$ can never recoup the loss on consumer's side given a fixed-sized consumer market; bundling never occurs. Bundling works as a commitment to an aggressive pricing strategy and it only emergence when platforms subsidize consumers for participation, therefore, bundling can be used as tool to enhance the "divide-and-conquer" nature of the pricing strategies.

The following corollary reveals the impact of the level of consumer information on the bundling strategy.

**Corollary 3.** *The set of parameters upon which bundling emerges shrinks as the fraction of informed consumers increases.*

Bundling emerges only when the platforms engage in divide-and-conquer strategies, where the competing platform reduces its consumer subsidy in response to bundling. However, a larger fraction of informed consumers intensifies competition, pushing the competing platform to increase its consumer subsidy. In Figure 1.4, the grey area represents the region in which bundling would emerge. Bundling is less likely to occur when there is a large fraction of informed consumers.




(a) Bundling strategy: low $\lambda$

(b) Bundling strategy: high $\lambda$

Figure 1.4: The impact of consumer information on bundling strategy

### 1.4.3 Welfare Analysis

Now we address the issue how platform $A$'s bundling practice affects consumer surplus. The equilibrium consumer surplus in two scenarios are as follows:

$$
\begin{aligned}
CS_{symmetric} &= \int_0^{n_A^{C^*}} (v + \theta n_A^{D^*} - tx - p_A^{C^*})dx + \int_{1-n_B^{C^*}}^{1} (v + \theta n_B^{D^*} - t(1-x) - p_B^{C^*})dx \\
&= v - \frac{5t}{4} + \frac{\beta^2}{4} + \frac{\theta\beta}{4} + \frac{\theta^2\lambda}{4} + \frac{3\theta\beta\lambda}{4}.
\end{aligned}
$$

$$
\begin{aligned}
CS_{bundling} &= \int_0^{n_A^{C^*}} (v + \theta n_A^{D^*} - tx + z - z - p_A^{C^*})dx + \int_{1-n_B^{C^*}}^{1} (v + \theta n_B^{D^*} - t(1-x) - p_B^{C^*})dx \\
&= v - \frac{5t}{4} + \frac{\beta^2}{4} + \frac{\theta\beta}{4} + \frac{\theta^2\lambda}{4} + \frac{3\theta\beta\lambda}{4} + \frac{z}{2} + \frac{tz^2}{(6t-6\underline{t})^2}
\end{aligned}
$$

**Corollary 4.** *Consumer surplus is positively affected by the fraction of informed consumers. Under bundling, consumer surplus is positively affected by the value of the handset.*

A higher level of consumer information leads to lower subscription prices and higher developer participation, resulting in greater consumer surplus. Also, both consumer and developer participation on platform $A$ is positively affected by the value of the bundling handset while platform $A$'s consumer subscription price is negatively affected. The surplus of the majority of consumers increases with the value of the handset. Therefore, in general, consumer surplus increases with it.

Bundling has indeed one negative and two positive effects on consumer welfare. On the one hand, the unequal-split of consumer demand between two platforms increases total transportation cost, which reduces consumer welfare. The larger the difference in consumer demand between the two platforms, the larger adverse welfare effect of bundling. On the other hand, there are two positive welfare effects of bundling coming from the fact that the majority of consumers enjoy a lower subscription price and more application variety, which dominates the negative effect on consumer surplus caused by lower subsidy and less application variety for consumers on platform $B$. The change on consumer surplus due to platform $A$'s bundling decision is

$$
\begin{aligned}
\Delta CS &= CS_{bundling} - CS_{symmetric} \\
&= \frac{z}{2} + \frac{tz^2}{(6t-6\underline{t})^2} > 0
\end{aligned}
$$

**Proposition 4.** *When consumers are homogeneous with respect to the valuation of the bundling handset, platform $A$'s bundling decision unambiguously improves consumer welfare.*

### 1.4.4 Tying

If platform $A$ practices tying, it still sells the handset to consumers on platform $B$ and extracts full surplus of the handset from them. Platform A's maximization problem now evolves to

$$
\max_{p_A^C, p_A^D} \quad \pi_A = (p_A^C + z)n_A^C + p_A^D n_A^D + n_B^C z = p_A^C n_A^C + p_A^D n_A^D + z.
$$

**Proposition 5.** *When consumers are homogeneous with respect to the valuation of platform A's in-house handset and platform A extracts full surplus from the fixed-sized handset market, tying makes no difference from untying.*

## 1.5 Extension: Heterogeneous Consumer Valuation of the Handset

Now we modify our setting regarding consumer's valuation of the handset. Let consumer's location on the unit interval be $x$ and the marginal utility of the quality of platform $A$'s in-house handset be $\phi$. The pair $(x, \phi)$ defines a consumer type. Both $x$ and $\phi$ are distributed independently and uniformly on $[0, 1]$. Type-$\phi$ consumer's utility from the handset is

$$U^{hs} = \phi z - p^{hs}.$$

Without bundling, platform $A$ sells the handset at monopoly price $p^{hs} = \frac{z}{2}$, and the demand for this handset is $D(p^{hs}) = \frac{1}{2}$. Consumers with high marginal utility $\phi \geq \overline{\phi} = \frac{1}{2}$ purchase (Figure 1.5($a$)). Platform $A$ earns revenue $\pi^{hs} = \frac{z}{4}$ from the unbundled handset.

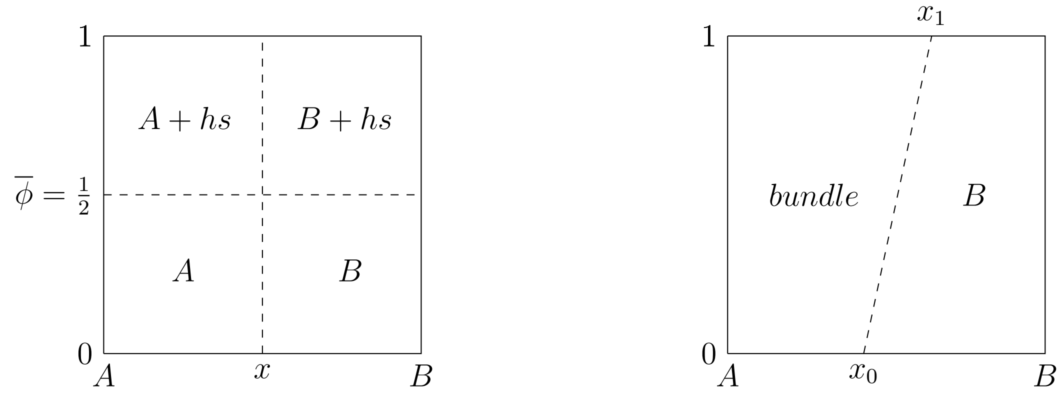
Again, we assume platform $A$ sets $p_A$ as the price for the bundled products, where $p_A = p_A^C + \frac{z}{2}$. Consumers with the heterogeneous marginal utility of the handset quality derive different levels of utility from purchasing the bundled products (Figure 1.5($b$)). For instance, consumer $(x, 0)$ derives utility $v - p_A + \theta n_A^{D^e} - tx$ from purchasing the bundle and $v - p_B^C + \theta n_B^{D^e} - t(1-x)$ from joining platform $B$, the marginal consumer with 0 marginal utility for the handset quality is

$$x_0 = \frac{1}{2} + \frac{p_B^C - p_A^C}{2t} - \frac{z}{4t} + \frac{\lambda\theta n_A^{D^e} - \lambda\theta n_B^{D^e}}{2t} + \frac{(1-\lambda)\theta n_A^{D^e} - (1-\lambda)\theta n_B^{D^e}}{2t}.$$

Similarly, consumer $(x, 1)$ derives utility $v - p_A + z + \theta n_A^{D^e} - tx$ from purchasing the bundle and $v - p_B^C + \theta n_B^{D^e} - t(1-x)$ from joining platform $B$, the marginal consumer with the highest marginal utility of the handset quality is

$$x_1 = \frac{1}{2} + \frac{p_B^C - p_A^C}{2t} + \frac{z}{4t} + \frac{\lambda\theta n_A^{D^e} - \lambda\theta n_B^{D^e}}{2t} + \frac{(1-\lambda)\theta n_A^{D^e} - (1-\lambda)\theta n_B^{D^e}}{2t}.$$

The realized consumer demand of each platform is the same as Eq. (1.1).




(a) No Bundling (b) Bundling

Figure 1.5: Consumers are heterogeneous with repsect to the valuation of the handset

Platform $A$'s profit maximization problem evolves to

$$\max_{p_A^C, p_A^D} \quad \pi_A = p_A n_A^C + p_A^D n_A^D = (p_A^C + \frac{z}{2}) n_A^C + p_A^D n_A^D.$$

**Proposition 6.** *When platform A bundles with its in-house handset and consumer's marginal utility of the handset quality is uniformly distributed over* $[0, 1]$*, the equilibrium outcomes are as follows:*

$$p_A^{C^*} = t - \frac{\beta^2}{4} - \frac{3\theta\beta\lambda}{4} - \frac{z(8t - 2\theta\beta - \beta^2 - 2\theta^2\lambda - 3\theta\beta\lambda)}{24(t - \underline{t})},$$

$$p_B^{C^*} = t - \frac{\beta^2}{4} - \frac{3\theta\beta\lambda}{4} - \frac{z(4t - \beta^2 - 3\theta\beta\lambda)}{24(t - \underline{t})},$$

$$n_A^{C^*} = \frac{1}{2} + \frac{z}{12(t - \underline{t})}, \qquad n_B^{C^*} = \frac{1}{2} - \frac{z}{12(t - \underline{t})},$$

$$p_A^{D^*} = \frac{\beta - \theta\lambda}{2}(\frac{1}{2} + \frac{z}{12(t - \underline{t})}), \qquad n_A^{D^*} = \frac{\beta + \theta\lambda}{2}(\frac{1}{2} + \frac{z}{12(t - \underline{t})}),$$

$$p_B^{D^*} = \frac{\beta - \theta\lambda}{2}(\frac{1}{2} - \frac{z}{12(t - \underline{t})}), \qquad n_B^{D^*} = \frac{\beta + \theta\lambda}{2}(\frac{1}{2} - \frac{z}{12(t - \underline{t})}),$$

$$\pi_A^* = \frac{8t - \beta^2 - \theta^2\lambda^2 - 6\theta\beta\lambda}{16} \frac{(6t - 6\underline{t} + z)^2}{(6t - 6\underline{t})^2},$$

*and*

$$\pi_B^* = \frac{8t - \beta^2 - \theta^2\lambda^2 - 6\theta\beta\lambda}{16} \frac{(6t - 6\underline{t} - z)^2}{(6t - 6\underline{t})^2}.$$

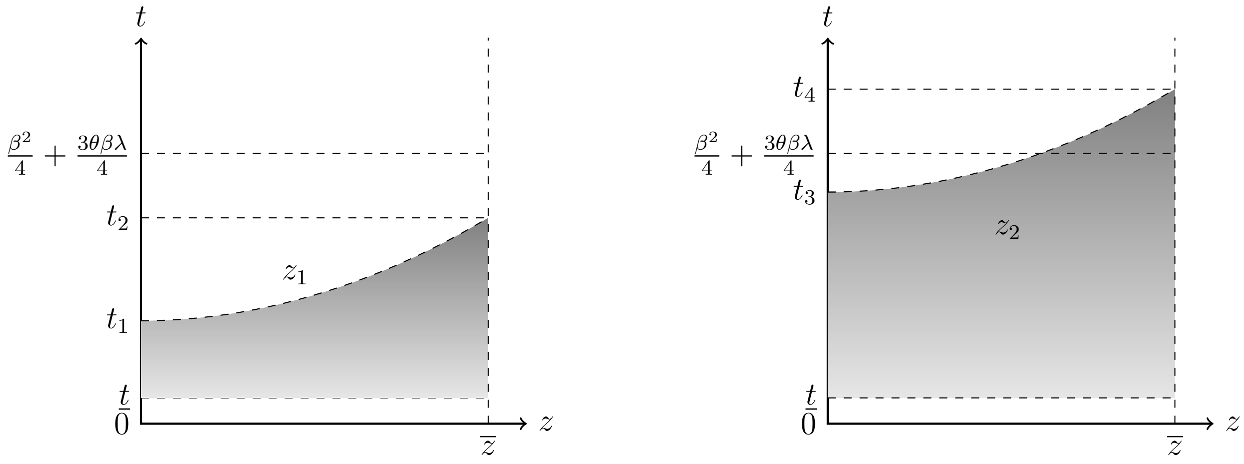
*Proof.* See Appendix. □

Let $t_3 = \frac{2\theta\beta+\beta^2+2\theta^2\lambda-\theta^2\lambda^2}{4}$, $z_2 = \frac{(6t-6\underline{t})(8t-4\theta\beta-2\beta^2-4\theta^2\lambda+2\theta^2\lambda^2)}{8t-\beta^2-\theta^2\lambda^2-6\theta\beta\lambda}$ and $t_4 = \frac{8\theta\beta+3\beta^2+8\theta^2\lambda-5\theta^2\lambda^2-6\theta\beta\lambda}{8}$. We use the following proposition to identify the bundling strategy for platform $A$ when consumer's valuation of the handset is uniformly distributed along $[0, 1]$.

**Proposition 7.** *When consumer's marginal utility of the quality of platform A's in-house handset is uniformly distributed over* $[0, 1]$,
(*i*) *platform A chooses to bundle with the handset for all* $z < \overline{z}$ *when* $\underline{t} < t \leq t_3$;
(*ii*) *platform A practices bundling iff* $z_2 \leq z < \overline{z}$ *when* $t_3 < t \leq t_4$;
(*iii*) *platform A never practices bundling when* $t > t_4$,
*Bundling always hurts the rival.*

*Proof.* See Appendix. □

The set of parameters upon which bundling emerges is strictly larger than the case where consumers are homogeneous with respect to the valuation of the handset. Again, the set of parameters upon which bundling emerges shrinks as the fraction of informed consumers increases. Notice that bundling can be profitable even when consumer subscription prices are strategic complements. When all consumers are uninformed and hold passive expectations, bundling can be profitable even when consumer subscription prices are strategic complements regardless of the value of the handset. We compare the regions in which bundling emerges when consumers are homogeneous or heterogeneous with respect to the valuation of the handset (see Figure 1.6).




(a) homogeneous valuation of the handset (b) heterogeneous valuation of the handset

Figure 1.6: Bundling strategy when consumers are homogeneous and heterogeneous with respect to the valuation handset

Indeed, the overall effect of $z$ on platform $A$'s profit is:

$$\frac{d\pi_A}{dz} = n_A^C + (p_A^C + \frac{z}{2} + \beta p_A^D)\frac{\partial n_A^C}{\partial p_B^C}(-\frac{4t-\beta^2-3\theta\beta\lambda}{12(t-\underline{t})}) + (p_A^C + \frac{z}{2} + \beta p_A^D)\frac{\partial n_A^C}{\partial p_B^D}(-\frac{\beta-\theta\lambda}{24(t-\underline{t})}).$$

When consumer prices are strategic substitutes, the strategic effects are positive, $\frac{d\pi_A}{dz} > n_A^C$, platform $A$'s profit increases in value of the handset at a rate faster than $n_A^C$, also bundling expands consumer demand ($n_A^C > \frac{1}{2}$), bundling is profitable. When consumer prices are strategic complements, although platform $A$'s profit increases in value of the handset at a rate slower than $n_A^C$, given the expanded consumer demand for the bundle, bundling still can be profitable.

From Figure 1.5($b$), we see a difference in consumer demand between consumers with a high valuation of the handset and the ones with a low valuation. In fact, under bundling, more consumers with a high valuation of the handset ($\phi \geq \overline{\phi} = \frac{1}{2}$) join platform $A$ than the ones with a low valuation ($\Delta n_A^C = n_{A(\phi\geq\overline{\phi})}^C - n_{A(\phi<\overline{\phi})}^C = \frac{z}{8t}$), and the differnce in demand increases with the value of the handset. Through bundling, platform $A$ coordinates the misaligned consumer valuations of the platform and the handset, targeting consumers with a high valuation of the handset for participation.

### 1.5.1 Tying

If platform $A$ decides to practice tying instead of bundling, the handset is still available to consumers on platform $B$. Among these consumers, only those with high marginal utility of the handset quality would purchase, i.e., $\phi \geq \overline{\phi} = \frac{1}{2}$. So, consumers with high marginal utility of the handset quality make participation decision by comparing the utility of buying the bundle from platform $A$ with buying the access to platform $B$ plus the handset from platform $A$ (see Figure 8). The marginal consumer with high marginal utility of the handset quality locates at

$$x_{\phi \geq \overline{\phi}} = \frac{1}{2} + \frac{p_B^C - p_A^C}{2t} + \frac{\lambda\theta n_A^D - \lambda\theta n_B^D}{2t} + \frac{(1-\lambda)\theta n_A^{D^e} - (1-\lambda)\theta n_B^{D^e}}{2t}.$$

The consumers with low marginal utility of the handset quality have a demand

$$n^C_{A(\phi<\overline{\phi})} = \frac{1}{2} - \frac{z}{4t} + \frac{p_B^C - p_A^C}{2t} + \frac{\lambda\theta n_A^D - \lambda\theta n_B^D}{2t} + \frac{(1-\lambda)\theta n_A^{D^e} - (1-\lambda)\theta n_B^{D^e}}{2t}$$

for the bundled products and

$$n^C_{B(\phi<\overline{\phi})} = \frac{1}{2} + \frac{z}{4t} + \frac{p_B^C - p_A^C}{2t} + \frac{\lambda\theta n_A^D - \lambda\theta n_B^D}{2t} + \frac{(1-\lambda)\theta n_A^{D^e} - (1-\lambda)\theta n_B^{D^e}}{2t}$$

for the access to platform $B$. Therefore, the realized consumer demands are

$$n_A^C = \frac{1}{2} + \frac{p_B^C - p_A^C}{2t} - \frac{z}{16t} + \frac{\lambda\theta n_A^{D^e} - \lambda\theta n_B^{D^e}}{2t} + \frac{(1-\lambda)\theta n_A^{D^e} - (1-\lambda)\theta n_B^{D^e}}{2t}$$

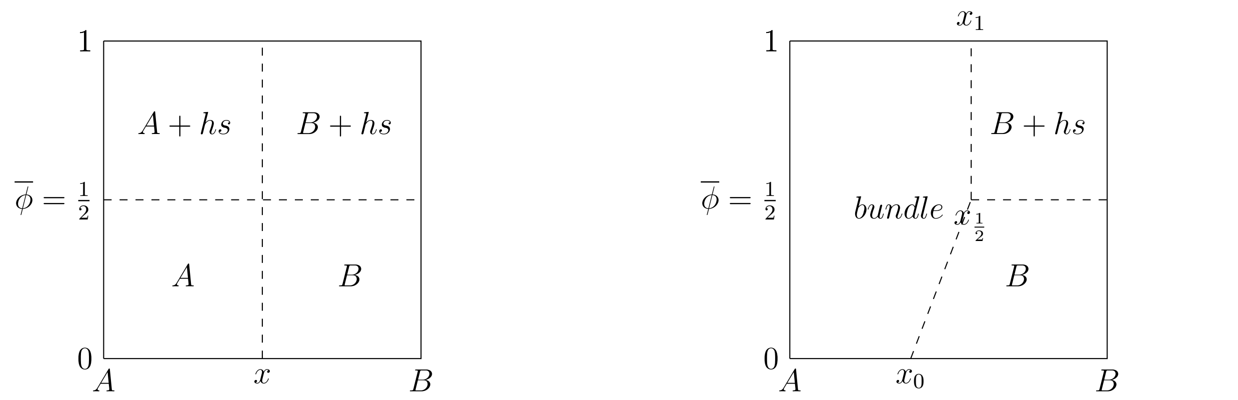
and

$$n_B^C = \frac{1}{2} + \frac{p_A^C - p_B^C}{2t} + \frac{z}{16t} + \frac{\lambda\theta n_B^{D^e} - \lambda\theta n_B^{D^e}}{2t} + \frac{(1-\lambda)\theta n_B^{D^e} - (1-\lambda)\theta n_A^{D^e}}{2t}.$$

Platform $A$'s profit maximization problem is

$$\max_{p_A^C, p_A^D} \quad \pi_A = p_A n_A^C + p_A^D n_A^D + \frac{z}{2} n^{hs} = (p_A^C + \frac{z}{2}) n_A^C + p_A^D n_A^D + \frac{z}{2} n^{hs},$$

where $n^{hs} = \frac{1}{2}(\frac{1}{2} + \frac{p_A^C - p_B^C}{2t} + \frac{\lambda\theta n_B^{D^e} - \lambda\theta n_A^{D^e}}{2t} + \frac{(1-\lambda)\theta n_B^{D^e} - (1-\lambda)\theta n_A^{D^e}}{2t})$; it is the consumer demand of handset from platform $B$.




(a) No Tying (b) Tying

Figure 1.7: Tying when consumers are heterogeneous w.r.t. the valuation of the handset

**Proposition 8.** *When platform A practices tying with its in-house handset and consumer's marginal utility of the handset quality is uniformly distributed over* $[0, 1]$, *the equilibrium outcomes are as follows:*

$$p_A^{C^*} = t - \frac{\beta^2}{4} - \frac{3\theta\beta\lambda}{4} - \frac{z(20t - 4\theta\beta - 3\beta^2 - 4\theta^2\lambda - 9\theta\beta\lambda)}{16(6t - 6\underline{t})},$$

$$p_B^{C^*} = t - \frac{\beta^2}{4} - \frac{3\theta\beta\lambda}{4} - \frac{z(4t - \beta^2 - 3\theta\beta\lambda)}{16(6t - 6\underline{t})},$$

$$n_A^{C^*} = \frac{1}{2} + \frac{z}{8(6t - 6\underline{t})}, \qquad n_B^{C^*} = \frac{1}{2} - \frac{z}{8(6t - 6\underline{t})},$$

$$p_A^{D^*} = \frac{\beta - \theta\lambda}{2}(\frac{1}{2} + \frac{z}{8(6t - 6\underline{t})}), \qquad n_A^{D^*} = \frac{\beta + \theta\lambda}{2}(\frac{1}{2} + \frac{z}{8(6t - 6\underline{t})}),$$

$$p_B^{D^*} = \frac{\beta - \theta\lambda}{2}(\frac{1}{2} - \frac{z}{8(6t - 6\underline{t})}), \qquad n_B^{D^*} = \frac{\beta + \theta\lambda}{2}(\frac{1}{2} - \frac{z}{8(6t - 6\underline{t})}),$$

$$n^{hs^*} = \frac{1}{4} - \frac{z(8t - 6\underline{t})}{32t(6t - 6\underline{t})},$$

$$\pi_A^* = (p_A^{C^*} + \frac{z}{2})n_A^{C^*} + p_A^{D^*} n_A^{D^*} + n^{hs^*}\frac{z}{2}$$

*and*

$$\pi_B^* = \frac{(8t - \beta^2 - \theta^2\lambda^2 - 6\theta\beta\lambda)}{256}\frac{(24t - 24\underline{t} - z)^2}{(6t - 6\underline{t})^2}.$$

*Proof.* See Appendix. □

**Corollary 5.** *Under tying, the implicit consumer subscription price of platform A, hence the price for the bundle, is higher relative to the bundling case, its consumer demand for the bundle and developer subscription price as well as developer participation are lower relative to the bundling case. Platform A makes less profit through subscription under tying than bundling. Tying also hurts the rival, but platform B is better off than under bundling.*

Under tying, platform $A$ has fewer incentives to offer a discount on consumer subscription price relative to the bundling case. This is because the demand from consumers with high marginal utility of the handset acts less sensitively to a fall of $p_A^C$. Under bundling, when consumers with a high valuation of the handset choose between the bundle and platform $B$, a discount on subscription price and the utility from consuming the handset make the bundle more attractive among these consumers. Bundling induces more consumers with a high valuation of the handset to purchase the bundle ($\Delta n^C_{A(\phi \geq \overline{\phi})} = n^C_{A(\phi \geq \overline{\phi})}(bundling) - n^C_{A(\phi \geq \overline{\phi})}(tying) = \frac{z(12t-6\underline{t})}{32t(6t-6\underline{t})} > 0$). This suggests that, relative to tying scheme, bundling is more effective not only to stimulate consumer demand but also to target certain consumers for participation.

## 1.6 Concluding Remarks

This work studies how bundling practice and the level of consumer information about developer subscription prices affect platform competition. In this paper, bundling is a commitment to an aggressive pricing strategy; it is deployed to stimulate consumer demand. We show that bundling can be beneficial to the bundling platform and detrimental to the rival when platforms engage in divide-and-conquer strategies given consumers are homogeneous with respect to the valuation of the bundling handset. Once we assume that consumers are heterogeneous with respect to the valuation of the handset, the set of parameters upon which bundling emerges is strictly larger than the previous case. Bundling is more effective to target consumers with a high valuation of the handset. A larger fraction of informed consumers intensifies price competition. Informed consumers respond to price changes by adjusting their own demand as well as the expectation of developer demand. This amplifies the effect of a discount on consumer subscription prices. Therefore, bundling is more effective to stimulate consumer demand when there are more informed consumers. Bundling is less likely to emerge when there is a larger fraction of informed consumers. We further show that bundling and more information increase consumer welfare by lowering subscription prices and improving platform quality in terms of application variety.

Our results offer clear strategy and policy recommendations. From a strategy perspective, both platforms have incentives to affect consumer's knowledge regarding developer subscription prices. Both platforms have incentives to withhold the information because a high level of consumer information intensifies price competition on both sides. Also, bundling is less likely to occur when there is a higher level of consumer information. However, when bundling does occur, the two platforms may have different attitudes towards consumer information. The bundling platform prefers a high level of consumer information because bundling is more effective to stimulate consumer demand. The competing platform wishes to withhold the information as it gets worse off as the level of consumer information increases. Because bundling works as a commitment to an aggressive pricing strategy and it emerges when the platforms subsidize consumers for participation, this work shows that bundling can be used as a tool to enhance the "divide-and-conquer" nature of pricing strategies.

From a public policy perspective, our results concern bundling and information disclosure. In conventional one-sided markets, bundling is usually considered to be anti-competitive by competition authorities as it's adopted either for price discrimination or foreclosure reasons, but analyzing a two-sided market using one-sided market logic may lead to policy errors (Wright, 2004). Due to the existence of (positive) network externalities, consumer surplus increases with the number of developers on the same platform. Bundling does not affect only the consumer subscription prices but also the perceived quality of platforms as it affects developer participation. We have shown that pure bundling improves consumer welfare mainly because it offers a lower subscription price and more application variety to the majority of consumers. For the same reason, even when bundling implements second-degree price discrimination, bundling still improves consumer welfare. Also, information disclosure unambiguously improves consumer surplus by lowering subscription prices on both sides of the platform and improving developer participation. Thus, information disclosure should be encouraged or mandated for consumer's sake.

# Appendix

**Proof of Proposition 1**

A fraction $\lambda$ of consumers is informed about developer subscription prices and holds responsive expectations, while the remaining fraction $1-\lambda$ of consumers is uninformed about developer prices and holds passive expectations, the consumer demand for platform $T$ is

$$n_T^C = \frac{1}{2} + \frac{p_{-T}^C - p_T^C}{2t} + \lambda\frac{\theta n_T^D - \theta n_{-T}^D}{2t} + (1-\lambda)\frac{\theta n_T^{D^e} - \theta n_{-T}^{D^e}}{2t}, \tag{1.8}$$

and the developer demand of each platform is

$$n_T^D = \beta n_T^C - p_T^D, \tag{1.9}$$

where $T = A, B$. As the fraction $\lambda$ of consumers is informed about the developer prices and the structure of developer demand, we substitute Eq. (1.9) to Eq. (1.8) for $n_T^D$ and $n_{-T}^D$, and solve for consumer demand as a function of subscription prices on both sides of the platform and uninformed consumers' expectations on developer demand of each platform. The realized consumer demand of platform $T$ is

$$n_T^C = \frac{1}{2} + \frac{p_{-T}^C - p_T^C}{2t - 2\theta\beta\lambda} + \frac{\theta(1-\lambda)n_T^{D^e} - \theta(1-\lambda)n_{-T}^{D^e}}{2t - 2\theta\beta\lambda} + \frac{\theta\lambda p_{-T}^D - \theta\lambda p_T^D}{2t - 2\theta\beta\lambda},$$

and the developer demand is

$$n_T^D = \beta(\frac{1}{2} + \frac{p_{-T}^C - p_T^C}{2t - 2\theta\beta\lambda} + \frac{\theta(1-\lambda)n_T^{D^e} - \theta(1-\lambda)n_{-T}^{D^e}}{2t - 2\theta\beta\lambda} + \frac{\theta\lambda p_{-T}^D - \theta\lambda p_T^D}{2t - 2\theta\beta\lambda}) - p_T^D.$$

Platform $T$'s profit maximization problem is

$$\max_{p_T^C, p_T^D} \quad \pi_T = p_T^C n_T^C + p_T^D n_T^D.$$

Taking the first order conditions of the profit function in $p_T^C$ and $p_T^D$ and solving for $p_T^C$ and $p_T^D$ as functions of uninformed consumers' expectations on developer demands, we obtain:

$$p_T^C = \frac{(4t - \beta^2 - 3\theta\beta\lambda)(6t - \beta^2 - \theta^2\lambda^2 + 2\theta(1-\lambda)(n_T^{D^e} - n_{-T}^{D^e}) - 4\theta\beta\lambda)}{24t - 4\beta^2 - 16\theta\beta\lambda - 4\theta^2\lambda^2} \tag{1.10}$$

and

$$p_T^D = \frac{(\beta - \theta\lambda)(6t - \beta^2 - \theta^2\lambda^2 + 2\theta(1-\lambda)(n_T^{D^e} - n_{-T}^{D^e}) - 4\theta\beta\lambda)}{24t - 4\beta^2 - 16\theta\beta\lambda - 4\theta^2\lambda^2}. \tag{1.11}$$

Substituting Eqs. (1.10) and (1.11) to demand functions, we obtain demand functions on both sides of the platform as functions of uninformed consumers' expectations on developer demand:

$$n_T^C = \frac{6t - \beta^2 - \theta^2\lambda^2 + 2\theta(1-\lambda)(n_T^{D^e} - n_{-T}^{D^e}) - 4\theta\beta\lambda}{12t - 2\beta^2 - 8\theta\beta\lambda - 2\theta^2\lambda^2}$$

and

$$n_T^D = \frac{(\beta + \theta\lambda)(6t - \beta^2 - \theta^2\lambda^2 + 2\theta(1-\lambda)(n_T^{D^e} - n_{-T}^{D^e}) - 4\theta\beta\lambda)}{24t - 4\beta^2 - 16\theta\beta\lambda - 4\theta^2\lambda^2}.$$

In equilibrium, uninformed consumers' expectations are fulfilled. Imposing $n_T^D = n_T^{D^e}$, we obtain equilibrium prices and allocations:

$$p_T^{C^*} = t - \frac{\beta^2}{4} - \frac{3\theta\beta\lambda}{4}, \qquad n_T^{C^*} = \frac{1}{2},$$
$$p_T^{D^*} = \frac{\beta}{4} - \frac{\theta\lambda}{4}, \qquad \text{and} \qquad n_T^{D^*} = \frac{\beta}{4} + \frac{\theta\lambda}{4}.$$

**Proof of Proposition 2**

Platform $A$ sets the price for the bundle $p_A = p_A^C + p^{hs}$, where $p^{hs} = z$. The marginal consumer locates at

$$x = \frac{1}{2} + \frac{p_B^C - p_A^C}{2t} + \lambda\frac{\theta n_A^D - \theta n_B^D}{2t} + (1-\lambda)\frac{\theta n_A^{D^e} - \theta n_B^{D^e}}{2t}.$$

Therefore, the consumer demands are

$$n_A^C = \frac{1}{2} + \frac{p_B^C - p_A^C}{2t} + \lambda\frac{\theta n_A^D - \theta n_B^D}{2t} + (1-\lambda)\frac{\theta n_A^{D^e} - \theta n_B^{D^e}}{2t}, \tag{1.12}$$

$$n_B^C = \frac{1}{2} + \frac{p_A^C - p_B^C}{2t} + \lambda\frac{\theta n_B^D - \theta n_A^D}{2t} + (1-\lambda)\frac{\theta n_B^{D^e} - \theta n_A^{D^e}}{2t}. \tag{1.13}$$

The developer demand of each platform is

$$n_T^D = \beta n_T^C - p_T^D. \tag{1.14}$$

As the fraction $\lambda$ of consumers is informed about the developer prices and the structure of developer demand, we substitute Eq. (1.14) to Eqs. (1.12) and (1.13) for $n_A^D$ and $n_B^D$, and solve for consumer demand as a function of subscription prices on both sides of the platform and uninformed consumers' expectations on developer demands:

$$n_A^C = \frac{1}{2} + \frac{p_B^C - p_A^C}{2t - 2\theta\beta\lambda} + \frac{\theta(1-\lambda)n_A^{D^e} - \theta(1-\lambda)n_B^{D^e}}{2t - 2\theta\beta\lambda} + \frac{\theta\lambda p_B^D - \theta\lambda p_A^D}{2t - 2\theta\beta\lambda},$$

and

$$n_B^C = \frac{1}{2} + \frac{p_A^C - p_B^C}{2t - 2\theta\beta\lambda} + \frac{\theta(1-\lambda)n_B^{D^e} - \theta(1-\lambda)n_A^{D^e}}{2t - 2\theta\beta\lambda} + \frac{\theta\lambda p_A^D - \theta\lambda p_B^D}{2t - 2\theta\beta\lambda}.$$

Platform $A$'s profit maximization problem is

$$\max_{p_A^C, p_A^D} \quad \pi_A = (p_A^C + p^{hs})n_A^C + p_A^D n_A^D = (p_A^C + z)n_A^C + p_A^D n_A^D.$$

Platform $B$'s profit maximization problem remains the same.
Taking the first order conditions of the profit function in $p_T^C$ and $p_T^D$ and solving for $p_T^C$ and $p_T^D$ as functions of uninformed consumers' expectations, we obtain:

$$\begin{aligned} p_A^C = &\frac{(4t - \beta^2 - 3\theta\beta\lambda)(6t - \beta^2 - \theta^2\lambda^2 + 2\theta(1-\lambda)(n_A^{D^e} - n_B^{D^e}) - 4\theta\beta\lambda)}{24t - 4\beta^2 - 16\theta\beta\lambda - 4\theta^2\lambda^2} \\ &- \frac{(8t - \beta^2 - 2\theta^2\lambda^2 - 5\theta\beta\lambda)z}{12t - 2\beta^2 - 8\theta\beta\lambda - 2\theta^2\lambda^2}, \end{aligned} \tag{1.15}$$

$$p_B^C = \frac{(4t - \beta^2 - 3\theta\beta\lambda)(6t - \beta^2 - \theta^2\lambda^2 + 2\theta(1-\lambda)(n_B^{D^e} - n_A^{D^e}) - 4\theta\beta\lambda - 2z)}{24t - 4\beta^2 - 16\theta\beta\lambda - 4\theta^2\lambda^2}, \tag{1.16}$$

$$p_A^D = \frac{(\beta - \theta\lambda)(6t - \beta^2 - \theta^2\lambda^2 + 2\theta(1-\lambda)(n_A^{D^e} - n_B^{D^e}) - 4\theta\beta\lambda + 2z)}{24t - 4\beta^2 - 16\theta\beta\lambda - 4\theta^2\lambda^2}, \tag{1.17}$$

and

$$p_T^D = \frac{(\beta - \theta\lambda)(6t - \beta^2 - \theta^2\lambda^2 + 2\theta(1-\lambda)(n_B^{D^e} - n_A^{D^e}) - 4\theta\beta\lambda - 2z)}{24t - 4\beta^2 - 16\theta\beta\lambda - 4\theta^2\lambda^2}. \tag{1.18}$$

Substituting Eqs. (1.15) to (1.18) four functions of subscription prices to demand functions, we obtain demand functions on both sides of the market as functions of uninformed consumers' expectations on developer demand:

$$n_A^C = \frac{6t - \beta^2 - \theta^2\lambda^2 + 2\theta(1-\lambda)(n_A^{D^e} - n_B^{D^e}) - 4\theta\beta\lambda + 2z}{12t - 2\beta^2 - 8\theta\beta\lambda - 2\theta^2\lambda^2},$$

$$n_B^C = \frac{6t - \beta^2 - \theta^2\lambda^2 + 2\theta(1-\lambda)(n_B^{D^e} - n_A^{D^e}) - 4\theta\beta\lambda - 2z}{12t - 2\beta^2 - 8\theta\beta\lambda - 2\theta^2\lambda^2},$$

$$n_A^D = \frac{(\beta + \theta\lambda)(6t - \beta^2 - \theta^2\lambda^2 + 2\theta(1-\lambda)(n_A^{D^e} - n_B^{D^e}) - 4\theta\beta\lambda + 2z)}{24t - 4\beta^2 - 16\theta\beta\lambda - 4\theta^2\lambda^2},$$

and

$$n_B^D = \frac{(\beta+\theta\lambda)(6t-\beta^2-\theta^2\lambda^2+2\theta(1-\lambda)(n_B^{D^e}-n_A^{D^e})-4\theta\beta\lambda-2z)}{24t-4\beta^2-16\theta\beta\lambda-4\theta^2\lambda^2}.$$

In equilibrium, uninformed consumers' expectations are fulfilled. Imposing $n_A^D = n_A^{D^e}$ and $n_B^D = n_B^{D^e}$, we obtain equilibrium prices and allocations:

$$p_A^{C^*} = t - \frac{\beta^2}{4} - \frac{3\theta\beta\lambda}{4} - \frac{z}{2}\frac{(8t-2\theta\beta-\beta^2-2\theta^2\lambda-3\theta\beta\lambda)}{(6t-\theta\beta-\beta^2-\theta^2\lambda-3\theta\beta\lambda)},$$

$$p_B^{C^*} = t - \frac{\beta^2}{4} - \frac{3\theta\beta\lambda}{4} - \frac{z}{2}\frac{(4t-\beta^2-3\theta\beta\lambda)}{(6t-\theta\beta-\beta^2-\theta^2\lambda-3\theta\beta\lambda)},$$

$$n_A^{C^*} = \frac{1}{2} + \frac{z}{6t-\theta\beta-\beta^2-\theta^2\lambda-3\theta\beta\lambda},$$

$$n_B^{C^*} = \frac{1}{2} - \frac{z}{6t-\theta\beta-\beta^2-\theta^2\lambda-3\theta\beta\lambda},$$

$$p_A^{D^*} = (\frac{\beta-\theta\lambda}{2})(\frac{1}{2} + \frac{z}{6t-\theta\beta-\beta^2-\theta^2\lambda-3\theta\beta\lambda}),$$

$$n_A^{D^*} = (\frac{\beta+\theta\lambda}{2})(\frac{1}{2} + \frac{z}{6t-\theta\beta-\beta^2-\theta^2\lambda-3\theta\beta\lambda}),$$

$$p_B^{D^*} = (\frac{\beta-\theta\lambda}{2})(\frac{1}{2} - \frac{z}{6t-\theta\beta-\beta^2-\theta^2\lambda-3\theta\beta\lambda}),$$

$$n_B^{D^*} = (\frac{\beta+\theta\lambda}{2})(\frac{1}{2} - \frac{z}{6t-\theta\beta-\beta^2-\theta^2\lambda-3\theta\beta\lambda}).$$

**Proof of Proposition 3**

Let $z_1 = \frac{(6t-\beta^2-\theta\beta-\theta^2\lambda-3\theta\beta\lambda)(16t+\theta^2\lambda^2-4\theta^2\lambda-6\theta\beta\lambda-4\theta\beta-3\beta^2)}{8t-\theta^2\lambda^2-6\theta\beta\lambda-\beta^2}$, $t_1 = \frac{3\beta^2+4\theta\beta+6\theta\beta\lambda+4\theta^2\lambda-\theta^2\lambda^2}{16}$, and $t_2 = \frac{5\beta^2+8\theta\beta+6\theta\beta\lambda+8\theta^2\lambda-3\theta^2\lambda^2}{24}$.

When consumers are homogeneous with respect to the valuation of the handset, bundling leads to change on platform $A$'s profit

$$\begin{aligned}\Delta\pi_A &= \pi_A^{BUN} - \pi_A^{NB} \\ &= -\frac{z}{4}(\frac{16t+\theta^2\lambda^2-4\theta^2\lambda-6\theta\beta\lambda-4\theta\beta-3\beta^2}{6t-\theta\beta-\beta^2-\theta^2\lambda-3\theta\beta\lambda} - \frac{(8t-\theta^2\lambda^2-6\theta\beta\lambda-\beta^2)z}{(6t-\theta\beta-\beta^2-\theta^2\lambda-3\theta\beta\lambda)^2}).\end{aligned}$$

Platform $A$ would choose bundling iff $\Delta\pi_A \geq 0 \Rightarrow z \geq z_1$.
Either $(i)$ $z_1 < 0$ or
$(ii)$ $0 < z_1 < \overline{z}$ holds.

$(i)$ When $\underline{t} < t \leq t_1$, $z_1 \leq 0$, $z > z_1$ holds as $z > 0$.
Therefore, when $\underline{t} < t \leq t_1$, $\Delta\pi_A > 0$.
$(ii)$ When $t_1 < t \leq t_2$, $0 < z_1 < \overline{z}$ holds. $\Delta\pi_A \geq 0$ when $z_1 \leq z < \overline{z}$.
When $t > t_2$, $\Delta\pi_A \geq 0$ when $z \geq z_1$, which contradicts assumption $A3$.

Bundling leads to change on platform $B$'s profit $\Delta\pi_B = -\frac{z(8t-\theta^2\lambda^2-6\theta\beta\lambda-\beta^2)}{4}\frac{(6t-\beta^2-\theta\beta-\theta^2\lambda-3\theta\beta\lambda-z)}{(6t-\beta^2-\theta\beta-\theta^2\lambda-3\theta\beta\lambda)^2}$.
As $\Delta p_B^D = -\frac{z(\beta-\theta\lambda)}{2(6t-\theta\beta-\beta^2-\theta^2\lambda-3\theta\beta\lambda)} < 0$ and $\Delta n_B^D = -\frac{z(\beta+\theta\lambda)}{2(6t-\theta\beta-\beta^2-\theta^2\lambda-3\theta\beta\lambda)} < 0$.
$\Delta\pi_B^D = -\frac{z(\beta^2-\theta^2\lambda^2)}{4}\frac{(6t-\beta^2-\theta\beta-\theta^2\lambda-3\theta\beta\lambda-z)}{(6t-\beta^2-\theta\beta-\theta^2\lambda-3\theta\beta\lambda)^2} < 0$. Therefore, $(6t-\beta^2-\theta\beta-\theta^2\lambda-3\theta\beta\lambda-z) > 0$.
As $\pi_B^* = \frac{8t-\beta^2-\theta^2\lambda^2-6\theta\beta\lambda}{16}\frac{(6t-\theta\beta-\beta^2-\theta^2\lambda+3\theta\beta\lambda-2z)^2}{(6t-\theta\beta-\beta^2-\theta^2\lambda-3\theta\beta\lambda)^2}$, $8t-\beta^2-\theta^2\lambda^2-6\theta\beta\lambda > 0$ also holds.
Therefore, $\Delta\pi_B < 0$.

**Proof of Proposition 6**

Platform $A$ sets the price for the bundle $p_A = p_A^C + p^{hs}$, where $p^{hs} = \frac{z}{2}$. The marginal consumer of type $(x, 0)$, whose marginal utility of the handset quality $\phi = 0$, locates at

$$x_0 = \frac{1}{2} - \frac{z}{4t} + \frac{p_B^C - p_A^C}{2t} + \lambda\frac{\theta n_A^D - \theta n_B^D}{2t} + (1-\lambda)\frac{\theta n_A^{D^e} - \theta n_B^{D^e}}{2t}.$$

The marginal consumer with the highest marginal utility of the handset quality $\phi = 1$, locates at

$$x_1 = \frac{1}{2} + \frac{z}{4t} + \frac{p_B^C - p_A^C}{2t} + \lambda\frac{\theta n_A^D - \theta n_B^D}{2t} + (1-\lambda)\frac{\theta n_A^{D^e} - \theta n_B^{D^e}}{2t}.$$

The consumer demands for platform $A$ is $n_A^C = \frac{x_0+x_1}{2}$. Therefore, the realized consumer demands are:

$$n_A^C = \frac{1}{2} + \frac{p_B^C - p_A^C}{2t} + \lambda\frac{\theta n_A^D - \theta n_B^D}{2t} + (1-\lambda)\frac{\theta n_A^{D^e} - \theta n_B^{D^e}}{2t}, \tag{1.19}$$

$$n_B^C = \frac{1}{2} + \frac{p_A^C - p_B^C}{2t} + \lambda\frac{\theta n_B^D - \theta n_A^D}{2t} + (1-\lambda)\frac{\theta n_B^{D^e} - \theta n_A^{D^e}}{2t}. \tag{1.20}$$

The developer demand of each platform is

$$n_T^D = \beta n_T^C - p_T^D, \tag{1.21}$$

where $T = A, B$. As the fraction $\lambda$ of consumers is informed about the developer prices and the structure of developer demand, we substitute Eq. (1.21) to Eqs. (1.19) and (1.20) for $n_A^D$ and $n_B^D$, and solve for consumer demand as a function of subscription prices on both sides of the platform and uninformed consumers' expectations:

$$n_A^C = \frac{1}{2} + \frac{p_B^C - p_A^C}{2t - 2\theta\beta\lambda} + \frac{\theta(1-\lambda)n_A^{D^e} - \theta(1-\lambda)n_B^{D^e}}{2t - 2\theta\beta\lambda} + \frac{\theta\lambda p_B^D - \theta\lambda p_A^D}{2t - 2\theta\beta\lambda},$$

and

$$n_B^C = \frac{1}{2} + \frac{p_A^C - p_B^C}{2t - 2\theta\beta\lambda} + \frac{\theta(1-\lambda)n_B^{D^e} - \theta(1-\lambda)n_A^{D^e}}{2t - 2\theta\beta\lambda} + \frac{\theta\lambda p_A^D - \theta\lambda p_B^D}{2t - 2\theta\beta\lambda}.$$

Platform $A$'s profit maximization problem is

$$\max_{p_A^C, p_A^D} \quad \pi_A = (p_A^C + p^{hs})n_A^C + p_A^D n_A^D = (p_A^C + \frac{z}{2})n_A^C + p_A^D n_A^D.$$

Platform $B$'s profit maximization problem remains the same.
Taking the first order conditions of the profit function in $p_T^C$ and $p_T^D$ and solving for $p_T^C$ and $p_T^D$ as functions of uninformed consumers' expectations on developer demands, we obtain:

$$\begin{aligned} p_A^C = & \frac{(4t - \beta^2 - 3\theta\beta\lambda)(6t - \beta^2 - \theta^2\lambda^2 + 2\theta(1-\lambda)(n_A^{D^e} - n_B^{D^e}) - 4\theta\beta\lambda)}{24t - 4\beta^2 - 16\theta\beta\lambda - 4\theta^2\lambda^2} \\ & - \frac{(8t - \beta^2 - 2\theta^2\lambda^2 - 5\theta\beta\lambda)z}{24t - 4\beta^2 - 16\theta\beta\lambda - 4\theta^2\lambda^2}, \end{aligned} \tag{1.22}$$

$$p_B^C = \frac{(4t - \beta^2 - 3\theta\beta\lambda)(6t - \beta^2 - \theta^2\lambda^2 + 2\theta(1-\lambda)(n_B^{D^e} - n_A^{D^e}) - 4\theta\beta\lambda - z)}{24t - 4\beta^2 - 16\theta\beta\lambda - 4\theta^2\lambda^2}, \tag{1.23}$$

$$p_A^D = \frac{(\beta - \theta\lambda)(6t - \beta^2 - \theta^2\lambda^2 + 2\theta(1-\lambda)(n_A^{D^e} - n_B^{D^e}) - 4\theta\beta\lambda + z)}{24t - 4\beta^2 - 16\theta\beta\lambda - 4\theta^2\lambda^2}, \tag{1.24}$$

and

$$p_B^D = \frac{(\beta - \theta\lambda)(6t - \beta^2 - \theta^2\lambda^2 + 2\theta(1-\lambda)(n_B^{D^e} - n_A^{D^e}) - 4\theta\beta\lambda - z)}{24t - 4\beta^2 - 16\theta\beta\lambda - 4\theta^2\lambda^2}. \tag{1.25}$$

Substituting Eqs. (1.22) to (1.25) four price functions to demand functions, we obtain demand functions on both sides of the platform as functions of uninformed consumers' expectations:

$$n_A^C = \frac{6t - \beta^2 - \theta^2\lambda^2 + 2\theta(1-\lambda)(n_A^{D^e} - n_B^{D^e}) - 4\theta\beta\lambda + z}{12t - 2\beta^2 - 8\theta\beta\lambda - 2\theta^2\lambda^2},$$

$$n_B^C = \frac{6t - \beta^2 - \theta^2\lambda^2 + 2\theta(1-\lambda)(n_B^{D^e} - n_A^{D^e}) - 4\theta\beta\lambda - z}{12t - 2\beta^2 - 8\theta\beta\lambda - 2\theta^2\lambda^2},$$

$$n_A^D = \frac{(\beta + \theta\lambda)(6t - \beta^2 - \theta^2\lambda^2 + 2\theta(1-\lambda)(n_A^{D^e} - n_B^{D^e}) - 4\theta\beta\lambda + z)}{24t - 4\beta^2 - 16\theta\beta\lambda - 4\theta^2\lambda^2},$$

and

$$n_B^D = \frac{(\beta + \theta\lambda)(6t - \beta^2 - \theta^2\lambda^2 + 2\theta(1-\lambda)(n_B^{D^e} - n_A^{D^e}) - 4\theta\beta\lambda - z)}{24t - 4\beta^2 - 16\theta\beta\lambda - 4\theta^2\lambda^2}.$$

In equilibrium, uninformed consumers' expectations are fulfilled. Imposing $n_A^D = n_A^{D^e}$ and $n_B^D = n_B^{D^e}$, we obtain equilibrium prices and allocations:

$$p_A^{C^*} = t - \frac{\beta^2}{4} - \frac{3\theta\beta\lambda}{4} - \frac{z(8t - 2\theta\beta - \beta^2 - 2\theta^2\lambda - 3\theta\beta\lambda)}{4(6t - \theta\beta - \beta^2 - \theta^2\lambda - 3\theta\beta\lambda)},$$

$$p_B^{C^*} = t - \frac{\beta^2}{4} - \frac{3\theta\beta\lambda}{4} - \frac{z(4t - \beta^2 - 3\theta\beta\lambda)}{4(6t - \theta\beta - \beta^2 - \theta^2\lambda - 3\theta\beta\lambda)},$$

$$n_A^{C^*} = \frac{1}{2} + \frac{z}{2(6t - \theta\beta - \beta^2 - \theta^2\lambda - 3\theta\beta\lambda)},$$

$$n_B^{C^*} = \frac{1}{2} - \frac{z}{2(6t - \theta\beta - \beta^2 - \theta^2\lambda - 3\theta\beta\lambda)},$$

$$p_A^{D^*} = (\frac{\beta - \theta\lambda}{2})(\frac{1}{2} + \frac{z}{2(6t - \theta\beta - \beta^2 - \theta^2\lambda - 3\theta\beta\lambda)}),$$

$$n_A^{D^*} = (\frac{\beta + \theta\lambda}{2})(\frac{1}{2} + \frac{z}{2(6t - \theta\beta - \beta^2 - \theta^2\lambda - 3\theta\beta\lambda)}),$$

$$p_B^{D^*} = (\frac{\beta - \theta\lambda}{2})(\frac{1}{2} - \frac{z}{2(6t - \theta\beta - \beta^2 - \theta^2\lambda - 3\theta\beta\lambda)}),$$

$$n_B^{D^*} = (\frac{\beta + \theta\lambda}{2})(\frac{1}{2} - \frac{z}{2(6t - \theta\beta - \beta^2 - \theta^2\lambda - 3\theta\beta\lambda)}).$$

**Proof of Proposition 7**

Let $z_2 = \frac{(6t-\beta^2-\theta\beta-\theta^2\lambda-3\theta\beta\lambda)(8t+2\theta^2\lambda^2-4\theta^2\lambda-4\theta\beta-2\beta^2)}{8t-\theta^2\lambda^2-6\theta\beta\lambda-\beta^2}$, $t_3 = \frac{\beta^2+2\theta\beta+2\theta^2\lambda-\theta^2\lambda^2}{4}$, and $t_4 = \frac{3\beta^2+8\theta\beta-6\theta\beta\lambda+8\theta^2\lambda-5\theta^2\lambda^2}{8}$.

When consumer's valuation of the handset is uniformly distributed along $[0,1]$, bundling leads to change on platform $A$'s profit

$$\begin{aligned}\Delta\pi_A &= \pi_A^{BUN} - \pi_A^{NB} \\ &= -\frac{z}{16}(\frac{8t + 2\theta^2\lambda^2 - 4\theta^2\lambda - 4\theta\beta - 2\beta^2}{6t - \theta\beta - \beta^2 - \theta^2\lambda - 3\theta\beta\lambda} - \frac{8t - \theta^2\lambda^2 - 6\theta\beta\lambda - \beta^2)z}{(6t - \theta\beta - \beta^2 - \theta^2\lambda - 3\theta\beta\lambda)^2}).\end{aligned}$$

Platform $A$ would choose bundling iff $\Delta\pi_A \geq 0 \Rightarrow z \geq z_2$.
Either $(i)$ $z_2 < 0$ or
$(ii)$ $0 < z_2 < \overline{z}$ holds.
$(i)$ When $\underline{t} < t \leq t_3$, $z_2 < 0$, $z > z_2$ holds as $z > 0$. Therefore, when $\underline{t} < t \leq t_3$, $\Delta\pi_A > 0$.
$(ii)$ When $t_3 < t \leq t_4$, $0 < z_2 < \overline{z}$ holds. $\Delta\pi_A \geq 0$ when $z_2 \leq z < \overline{z}$.

Bundling leads to change on platform $B$'s profit
$\Delta\pi_B = -\frac{z(8t-\theta^2\lambda^2-6\theta\beta\lambda-\beta^2)}{16}\frac{(12t-2\beta^2-2\theta\beta-2\theta^2\lambda-6\theta\beta\lambda-z)}{(6t-\beta^2-\theta\beta-\theta^2\lambda-3\theta\beta\lambda)^2}$.
$(8t-\theta^2\lambda^2-6\theta\beta\lambda-\beta^2) > 0$ and $(12t-2\beta^2-2\theta\beta-2\theta^2\lambda-6\theta\beta\lambda-z) > 0$,therefore, $\Delta\pi_B < 0$.

**Proof of Proposition 8**

Consumers with high marginal utility of the handset quality, i.e., $\phi \geq \overline{\phi} = \frac{1}{2}$, have the demand for the bundle from platform $A$

$$n^C_{A(\phi\geq\frac{1}{2})} = \frac{1}{2}(\frac{1}{2} + \frac{p^C_B - p^C_A}{2t} + \lambda\frac{\theta n^D_A - \theta n^D_B}{2t} + (1-\lambda)\frac{\theta n^{D^e}_A - \theta n^{D^e}_B}{2t}),$$

and the demand for access to platform $B$ plus handset from platform $A$

$$n^C_{B(\phi\geq\frac{1}{2})} = \frac{1}{2}(\frac{1}{2} + \frac{p^C_A - p^C_B}{2t} + \lambda\frac{\theta n^D_B - \theta n^D_A}{2t} + (1-\lambda)\frac{\theta n^{D^e}_B - \theta n^{D^e}_A}{2t}).$$

Consumers with low marginal utility of the handset quality, i.e., $\phi < \overline{\phi} = \frac{1}{2}$, have the demand for the bundle from platform $A$

$$n^C_{A(\phi<\frac{1}{2})} = \frac{1}{2}(\frac{1}{2} + \frac{p^C_B - p^C_A}{2t} - \frac{z}{8t} + \lambda\frac{\theta n^D_A - \theta n^D_B}{2t} + (1-\lambda)\frac{\theta n^{D^e}_A - \theta n^{D^e}_B}{2t}),$$

and the demand for access to platform $B$ plus handset from platform $A$

$$n^C_{B(\phi<\frac{1}{2})} = \frac{1}{2}(\frac{1}{2} + \frac{p^C_A - p^C_B}{2t} + \frac{z}{8t} + \lambda\frac{\theta n^D_B - \theta n^D_A}{2t} + (1-\lambda)\frac{\theta n^{D^e}_B - \theta n^{D^e}_A}{2t}).$$

There, the aggregate consume demand for the bundle is

$$n^C_A = \frac{1}{2} + \frac{p^C_B - p^C_A}{2t} - \frac{z}{16t} + \lambda\frac{\theta n^D_A - \theta n^D_B}{2t} + (1-\lambda)\frac{\theta n^{D^e}_A - \theta n^{D^e}_B}{2t}, \tag{1.26}$$

and for platform $B$ is

$$n^C_B = \frac{1}{2} + \frac{p^C_A - p^C_B}{2t} + \frac{z}{16t} + \lambda\frac{\theta n^D_B - \theta n^D_A}{2t} + (1-\lambda)\frac{\theta n^{D^e}_B - \theta n^{D^e}_A}{2t}. \tag{1.27}$$

The developer demand of each platform is

$$n^D_T = \beta n^C_T - p^D_T, \tag{1.28}$$

where $T = A, B$. As the fraction $\lambda$ of consumers is informed about the developer prices and the structure of developer demand, we substitute Eq. (1.28) to Eqs. (1.26) and (1.27) for $n^D_A$

and $n_B^D$, and solve for consumer demand as a function of subscription prices on both sides of the platform and uninformed consumers' expectations:

$$n_A^C = \frac{1}{2} + \frac{p_B^C - p_A^C}{2t - 2\theta\beta\lambda} - \frac{z}{16t - 16\theta\beta\lambda} + \frac{\theta(1-\lambda)n_A^{D^e} - \theta(1-\lambda)n_B^{D^e}}{2t - 2\theta\beta\lambda} + \frac{\theta\lambda p_B^D - \theta\lambda p_A^D}{2t - 2\theta\beta\lambda},$$

and

$$n_B^C = \frac{1}{2} + \frac{p_A^C - p_B^C}{2t - 2\theta\beta\lambda} + \frac{z}{16t - 16\theta\beta\lambda} + \frac{\theta(1-\lambda)n_B^{D^e} - \theta(1-\lambda)n_A^{D^e}}{2t - 2\theta\beta\lambda} + \frac{\theta\lambda p_A^D - \theta\lambda p_B^D}{2t - 2\theta\beta\lambda}.$$

Also, the demand for the handset from consumers on platform $B$ is

$$n_B^{hs} = \frac{1}{4} + \frac{p_A^C - p_B^C}{4t - 4\theta\beta\lambda} + \frac{z}{32t^2 - 32t\theta\beta\lambda} + \frac{\theta(1-\lambda)n_B^{D^e} - \theta(1-\lambda)n_A^{D^e}}{4t - 4\theta\beta\lambda} + \frac{\theta\lambda p_A^D - \theta\lambda p_B^D}{4t - 4\theta\beta\lambda}.$$

Platform $A$'s profit maximization problem is

$$\max_{p_A^C, p_A^D} \quad \pi_A = (p_A^C + p^{hs})n_A^C + p_A^D n_A^D + p^{hs} n_B^{hs} = (p_A^C + \frac{z}{2})n_A^C + p_A^D n_A^D + \frac{z}{2} n_B^{hs}.$$

Platform $B$'s profit maximization problem remains the same.
Taking the first order conditions of the profit function in $p_T^C$ and $p_T^D$ and solving for $p_T^C$ and $p_T^D$ as functions of uninformed consumers' expectations on developer demands, we obtain:

$$p_A^C = t - \frac{\beta^2}{4} - \frac{3\theta\beta\lambda}{4} + \frac{(4t - \beta^2 - 3\theta\beta\lambda)(8\theta(1-\lambda)(n_A^{D^e} - n_B^{D^e})}{96t - 16\beta^2 - 64\theta\beta\lambda - 16\theta^2\lambda^2} + \frac{(20t - 3\beta^2 - 4\theta^2\lambda^2 - 13\theta\beta\lambda)z}{96t - 16\beta^2 - 64\theta\beta\lambda - 16\theta^2\lambda^2}, \tag{1.29}$$

$$p_B^C = \frac{(4t - \beta^2 - 3\theta\beta\lambda)(24t - 4\beta^2 - 4\theta^2\lambda^2 - 16\theta\beta\lambda + 8\theta(1-\lambda)(n_B^{D^e} - n_A^{D^e}) - z)}{96t - 16\beta^2 - 64\theta\beta\lambda - 16\theta^2\lambda^2}, \tag{1.30}$$

$$p_A^D = \frac{(\beta - \theta\lambda)(24t - 4\beta^2 - 4\theta^2\lambda^2 + 8\theta(1-\lambda)(n_A^{D^e} - n_B^{D^e}) - 16\theta\beta\lambda + z)}{96t - 16\beta^2 - 64\theta\beta\lambda - 16\theta^2\lambda^2}, \tag{1.31}$$

and

$$p_B^D = \frac{(\beta - \theta\lambda)(24t - 4\beta^2 - 4\theta^2\lambda^2 + 8\theta(1-\lambda)(n_B^{D^e} - n_A^{D^e}) - 16\theta\beta\lambda - z)}{96t - 16\beta^2 - 64\theta\beta\lambda - 16\theta^2\lambda^2}. \tag{1.32}$$

Substituting Eqs. (1.29) to (1.32) four price functions to demand functions, we obtain demand functions on both sides of the platform as functions of uninformed consumers' expectations:

$$n_A^C = \frac{24t - 4\beta^2 - 4\theta^2\lambda^2 + 8\theta(1-\lambda)(n_A^{D^e} - n_B^{D^e}) - 16\theta\beta\lambda + z}{48t - 8\beta^2 - 32\theta\beta\lambda - 8\theta^2\lambda^2},$$

$$n_B^C = \frac{24t - 4\beta^2 - 4\theta^2\lambda^2 + 8\theta(1-\lambda)(n_B^{D^e} - n_A^{D^e}) - 16\theta\beta\lambda - z}{48t - 8\beta^2 - 32\theta\beta\lambda - 8\theta^2\lambda^2},$$

$$n_A^D = \frac{(\beta+\theta\lambda)(24t - 4\beta^2 - 4\theta^2\lambda^2 + 8\theta(1-\lambda)(n_A^{D^e} - n_B^{D^e}) - 16\theta\beta\lambda + z)}{96t - 16\beta^2 - 64\theta\beta\lambda - 16\theta^2\lambda^2},$$

and

$$n_B^D = \frac{(\beta+\theta\lambda)(24t - 4\beta^2 - 4\theta^2\lambda^2 + 8\theta(1-\lambda)(n_A^{D^e} - n_B^{D^e}) - 16\theta\beta\lambda - z)}{96t - 16\beta^2 - 64\theta\beta\lambda - 16\theta^2\lambda^2}.$$

In equilibrium, uninformed consumers' expectations are fulfilled. Imposing $n_A^D = n_A^{D^e}$ and $n_B^D = n_B^{D^e}$, we obtain equilibrium prices and allocations:

$$p_A^{C^*} = t - \frac{\beta^2}{4} - \frac{3\theta\beta\lambda}{4} - \frac{z(20t - 4\theta\beta - 3\beta^2 - 4\theta^2\lambda - 9\theta\beta\lambda)}{16(6t - \theta\beta - \beta^2 - \theta^2\lambda - 3\theta\beta\lambda)},$$

$$p_B^{C^*} = t - \frac{\beta^2}{4} - \frac{3\theta\beta\lambda}{4} - \frac{z(4t - \beta^2 - 3\theta\beta\lambda)}{16(6t - \theta\beta - \beta^2 - \theta^2\lambda - 3\theta\beta\lambda)},$$

$$n_A^{C^*} = \frac{1}{2} + \frac{z}{8(6t - \theta\beta - \beta^2 - \theta^2\lambda - 3\theta\beta\lambda)},$$

$$n_B^{C^*} = \frac{1}{2} - \frac{z}{8(6t - \theta\beta - \beta^2 - \theta^2\lambda - 3\theta\beta\lambda)},$$

$$p_A^{D^*} = (\frac{\beta - \theta\lambda}{2})(\frac{1}{2} + \frac{z}{8(6t - \theta\beta - \beta^2 - \theta^2\lambda - 3\theta\beta\lambda)}),$$

$$n_A^{D^*} = (\frac{\beta + \theta\lambda}{2})(\frac{1}{2} + \frac{z}{8(6t - \theta\beta - \beta^2 - \theta^2\lambda - 3\theta\beta\lambda)}),$$

$$p_B^{D^*} = (\frac{\beta - \theta\lambda}{2})(\frac{1}{2} - \frac{z}{8(6t - \theta\beta - \beta^2 - \theta^2\lambda - 3\theta\beta\lambda)}),$$

$$n_B^{D^*} = (\frac{\beta + \theta\lambda}{2})(\frac{1}{2} - \frac{z}{8(6t - \theta\beta - \beta^2 - \theta^2\lambda - 3\theta\beta\lambda)})$$

and

$$n^{hs^*} = \frac{1}{4} - \frac{z(8t - 6\underline{t})}{32t(6t - 6\underline{t})}.$$

# Chapter 2

# Hybrid ad-sponsored platform

## 2.1 Introduction

Ad-sponsored business models have been widely adopted by companies ranging from newspapers to application software. Companies offer free or discounted products or service to consumers in exchange for listing adverts. For instance, some free newspapers have a significant amount of adverts, while some newspapers have fewer adverts and charge positive prices to the readers. In the smartphone operating system (OS) industry, Google, as an advertising company[1], introduces adverts on consumers side and collect revenues from application developers and advertisers[2]. These casual observations indicate that, when competing with rivals, platforms does not only make tactical decisions about subscription prices but also make strategic decisions about their business models.

How does a platform choose between the subscription-based and the ad-sponsored business models? The goal of this work is to develop a theoretical model to analyze the platform's strategic decision between the subscription-based and the ad-sponsored business models when competing with a subscription-based rival. We consider a four-stage game in which one of the platforms (without loss of generality, platform $B$) makes the strategic decision about its business model. In the first stage, platform $B$ chooses its business model, laying the groundwork for the competitive interactions down the line. In the second stage, both platforms decide subscription prices simultaneously on both consumer's and developer's sides. In the third stage, developers make participation decisions, and in the last stage, consumers

[1]The total advertising revenues make up more than 90 percent of Google's total revenues in 2013 and 2014. See `https://investor.google.com/financial/tables.html`, accessed May 2015.

[2]Google shares the advertising revenues with Android application developers if they incorporate banner adverts in their applications.

make participation decisions.

The ad-sponsored business model in this paper is "hybrid" as platform $B$ does not commit to free consumer access when it adopts the ad-sponsored business model. Under this business model, platform $B$ adjusts its consumer subscription price to compensate consumers for the perceived platform quality drop due to advertising, which affects consumer and developer participation decisions, hence platform profits. In the setting of single-homing consumers and multi-homing developers, our analysis shows that when consumers have increasing marginal disutility towards advertising, it is profitable for platform $B$ to adopt the hybrid ad-sponsored business model regardless of the degree of network externalities, and it is detrimental to the rival. Our results may help to explain why ad-sponsored business models are widely used even consumers dislike adverts.

This paper also features another characteristic of this industry: most members of one side (developers) of the market arrive before most members of the other side (consumers). This is probably because it takes time and effort for developers to design compatible applications for certain platforms, so platforms have to get developers on board to ensure the application availability at launch (Hagiu, 2006). We ask the following question: will platforms set the subscription prices on both sides simultaneously or set the subscription prices on developer's side before consumer's side?

We find that, if two sides have symmetric network externalities, setting the subscription prices simultaneously on both sides makes no difference from setting the subscription prices on developer's side before consumer's side, regardless of platform $B$'s choice of business models. This is because the platforms charge the developer subscription prices based on the difference between two directions of network externalities: if the developer's side is the high-externality side, platforms charge positive prices on this side; if the developer's side is the low-externality side, platforms subsidize developers for participation. If two sides have symmetric network externalities, the developer subscription prices are fixated to marginal costs. If two sides have asymmetric network externalities, platforms may have the incentives to set the subscription prices on two sides in a sequential fashion rather than setting them simultaneously. When both platforms compete through the subscription-based business model, platforms exploit more profits at the expense of consumers if the subscription prices on developer's side are set before the prices on consumer's side. When competing through different business models, the ad-sponsored business model is less effective in expanding consumer demand when the developer subscription prices are set before consumer's.

The remainder of the paper is organized as follows: Section 2.2 discusses the related literature. Section 2.3 presents our model setup. Section 2.4 analyzes the duopolistic platform competition. Section 2.5 offers an extension where the developer subscription prices are set before the consumer subscription prices. Section 2.6 concludes.

## 2.2 Relationship to the Literature

The present paper is about ad-sponsored platforms where advertising is usually considered to be a nuisance. The ad-sponsored platform is distinctive because it raises the issue of both positive and negative indirect network effects because Anderson and Coate (2005) study competition between TV stations that attract viewers and advertisers. The market structure takes a form of competitive bottleneck: viewers single-home and advertisers multi-home. Gabszewicz et al. (2005) examines an ad-sponsored monopolist's pricing decisions when consumers can be either ad-avoiders or ad-lovers. When the majority of the consumers are ad-lovers, advertising implies a lower consumer subscription price only if the ad-attraction is weak; when the majority of the consumers are ad-avoiders, the consumer subscription price is always lower relative to the price without advertising. Reisinger (2012) studies the duopolistic competition where platforms offer consumers free access and charge advertisers. Additional to competing for consumers, there is also Bertrand competition for advertisers. He shows that platforms charge the advertising price above marginal cost even there is Bertrand competition. This is because advertisers generate negative effects on consumers, platforms are restricted from attracting all advertisers by lowering prices. Our work is close to Casadesus-Masanell and Zhu (2010). They study the competition between an ad-sponsored entrant and an incumbent that offer vertically differentiated products. The incumbent can change its business models after observing the rival's entry decision. They show that the emergence of an ad-sponsored entrant does not necessarily intensify competition, because the ad-sponsored entrant can target non-adopters of the incumbent's product. The entrant avoids competition by expanding the consumer's market. For the most part, this literature has focused on platforms that connect consumers and advertisers. We contribute by including developers as the third groups of users in a duopoly setting. The consumers are attracted to a platform by the application variety it provides and are discouraged by the presence of advertising. Hence, the presence of advertising affects the platform's pricing strategies on both consumer's and developer's side.

This paper fits into the literature in strategy that studies competitive interactions between organizations with different business models. While the competition between a proprietary firm and an open source firm has featured in a fairly large body of work, most of these papers use the prominent example of Windows VS. Linux. Economides and Katsamakas (2005) study the incentives to invest in OS, and compare proprietary OS with open-source OS, e.g., Windows Vs. Linux. Regarding investment incentives, the difference between proprietary OS and open-source OS is: investments are made by the platform sponsor if the OS is proprietary and investments are are made by the application developers and users if the OS is open-source. They show the level of investment in applications is larger for an open source operating system. Economides and Katsamakas (2006) compare a proprietary platform (Windows) with an open source platform (Linux). They show that, when a system based on an open source platform with an independent proprietary application competes with a proprietary system, the proprietary system is likely to dominate the open source platform industry both in terms of market share and profitability. Their results may explain the dominance of Microsoft in the PC OS market. Most of the literature focuses on firms with exogenously given business models. More recently, Casadesus-Masanell and Llanes (2011) examine a profit-maximizing firm's choice of business models competing with a free open-source rival. In our work, we also allow one of the platforms to choose its business model. The strategic decision of introducing adverts works similarly to a commitment of offering a discount on consumer subscription price: the platform offers a discount on the consumer subscription price which expands consumer and developer demand, while advertising reduces the perceived platform quality which discourages consumer and developer participation. Therefore, introducing ads does not necessarily expand consumer demand or developer demand.

This paper also touches upon the issue of platform pricing strategies where users on one side (developers) arrive before the users on the other side (consumers). Unlike most literature in two-sided platforms, which assumes that two sides of users arrive at the same time, Hagiu (2006) argues that there are some two-sided markets where most members of one side of the market arrive before most members of the other side. Thus, a sequential-move game should be a more realistic model for these categories of two-sided markets. High technology-based OS platforms usually need to have developers on board before the launch on consumer's side because developers might need significant time to make applications specific for the platforms they have decided to support (Hagiu, 2006). Taking the recently introduced Apple Pay for example, Apple had partnered with some major credit card corporations long before the launch of Apple Pay[3]. This paper intends to offer some insights into platforms' incentives regarding the order of setting prices on each side when two sides' users arrive in a sequential fashion.

## 2.3 The Model

### 2.3.1 The timing of the game

In the main model, we use a four-stage game, which is a modification of Casadesus-Masanell and Llanes (2011)'s generic two-stage game as a general framework for the study of competition through business models. To be more precise, the timing of the game is as follows: In

[3]See `http://www.theverge.com/2014/9/10/6132153/apple-pay-was-this-weeks-most-revolutionary-product`, accessed December, 2014.

Stage 1, platform $B$ chooses its advertising intensity. In Stage 2, two platforms simultaneously set the subscription prices for both consumers and developers. In Stage 3, developers make participation decisions. In Stage 4, consumers make participation decisions[4]. All the decisions are publicly observable. The concept of the equilibrium is subgame-perfect Nash equilibrium; we solve this game by backward induction.

### 2.3.2 Platforms

Consider two platforms competing for both consumers and developers, indexed by $T$=$A$, $B$. Let $p_T^C$ and $p_T^D$ denote the subscription prices platform $T$ charges to consumers and developers, respectively. We assume that the platforms have zero marginal cost of serving these two groups of users, which is consistent with the literature of information goods and the reality of digital media industry, where large fixed costs and very low marginal costs are observed. We allow for negative prices, as it is possible for platforms to subsidize one side of the market. The mass of consumers and developers on platform $T$ are denoted by $n_T^C$ and $n_T^D$, respectively. We allow single-homing on one side and multi-homing on the other side: we assume that each consumer decides in favor of only one platform while developers can design applications for both platforms.

We extend the standard Hotelling model by allowing the duopoly to serve two groups of users on each side of the market. The unit transportation cost for consumers towards each end is $t$, which is the platform differentiation parameter. Platform $A$ and $B$ are exogenously located at $x = 0$ and $x = 1$, respectively. Platform $A$ is subscription-based, its profit function is

$$\pi_A = p_A^C n_A^C + p_A^D n_A^D.$$

Platform $B$ can choose between the subscription-based and the ad-sponsored business models by deciding its advertising intensity, which is $d \geq 0$. Its profit function is

$$\pi_B = p_B^C n_B^C + p_B^D n_B^D + r d n_B^C,$$

where $r > 0$ is the exogenous advertising rate per capita. Following Casadesus-Masanell and Zhu (2010), we assume that the advertising fee charged for each advert $r n_B^C$ is a linear function of consumer demand of platform $B$. When platform $B$ collects partial revenue from advertising, the larger the number of consumers, the more valuable the platform is for the advertisers. Platform $B$'s choice of business models is simplified as the choice of its advertising intensity. If platform $B$ sets the advertising intensity $d = 0$, it is subscription-based; and if platform $B$ sets the advertising intensity $d > 0$, it is ad-sponsored.

[4]The equilibrium outcomes of this game are the same as those of a three-stage game where developers and consumers arrive simultaneously in Stage 3 as long as all consumers hold responsive expectations about developer participation.

### 2.3.3 Developers

There is a unit mass of potential developers; each developer lists at most one application on each platform. Developers differ in the cost of listing applications, denoted by $y$, and are uniformly distributed along the segment $[0, 1]$. Each developer gains additional utility of $\beta$ from each consumer who has access to its application. Assume that developers form responsive expectations about consumer participation, their expectations match the realized consumer demands, i.e., $n_T^{C^e} = n_T^C$ for any given subscription price pair $(p_T^C, p_T^D)$ (Hagiu and Hałaburda, 2014). We do not consider the potential negative direct network externalities among developers. The revenue for a developer who lists on platform $T$ is given by $\beta n_T^C$ when the mass of consumers who participate in platform $T$ is $n_T^C$. The utility of developer $y$ from joining platform $T$ is

$$u_T^D = \beta n_T^C - p_T^D - y,$$

where $T = A, B$. We assume that developers can multi-home and there are no economies of scope in multi-homing. Therefore, the decision of joining one platform is independent of the decision to join the other. That is, a $y$-type developer will join platform $T$ if $u_T^D(y) = \beta n_T^C - p_T^D - y \geq 0$. So, the developer demands are

$$n_T^D = \beta n_T^C - p_T^D.$$

Developers care more about the network benefits of reaching out to the widest population of consumers than they do about the cost of multi-homing since there is no standalone value for developers to join the platforms. We study a case of "competitive bottlenecks" (Armstrong, 2006).

### 2.3.4 Consumers

There is a unit mass of consumers uniformly distributed along the unit interval, each of whom chooses at most one platform to join. The consumers have identical intrinsic values for two platforms, equal to $v$, which is assumed to be large enough so that the whole market is covered. Consumers have a taste for application variety. Every consumer's utility of participating on a platform depends on the total number of developers on the same platform. Consumers have identical utility gain from application variety; parameter $\theta$ is used to capture this direction of network externalities. More specifically, the availability of each extra developer positively generates additional utility $\theta > 0$ for consumers. The consumer who locates at $x$ chooses between platform $A$ or $B$ by comparing utilities $v + \theta n_A^D - p_A^C - tx$ from joining platform $A$ and $v + \theta n_B^D - p_B^C - t(1 - x) - \alpha d^2$ from joining platform $B$. Advertising is considered to be a nuisance for consumers. Following Casadesus-Masanell and Zhu (2010), we assume the disutility of the adverts is $\alpha d^2$, it implies that the marginal disutility of adverts increases with the intensity. $\alpha > 0$ is the disutility parameter, identical to all consumers. Therefore, the realized consumer demand of each platform is

$$n_A^C = \frac{1}{2} + \frac{p_B^C + \alpha d^2 - p_A^C}{2t} + \frac{\theta n_A^D - \theta n_B^D}{2t},$$

and

$$n_B^C = \frac{1}{2} + \frac{p_A^C - \alpha d^2 - p_B^C}{2t} + \frac{\theta n_B^D - \theta n_A^D}{2t}.$$

We assume that the following conditions hold throughout this paper:
**Assumption A5.** $t > \underline{t} = \frac{\theta^2}{6} + \frac{2\theta\beta}{3} + \frac{\beta^2}{6}$.

This assumption ensures a unique and stable equilibrium. With this assumption, the second order condition ($t > \theta\beta$) is satisfied, and both platforms make positive profits in equilibrium ($t > \frac{\theta^2}{8} + \frac{3\theta\beta}{4} + \frac{\beta^2}{8}$), so that they remain active in the market[5].

We also want to guarantee that both platforms remain active in the market when platform $B$ has positive advertising intensity by assuming that:
**Assumption A6.** $\frac{r^2}{2\alpha} < 6t - 6\underline{t}$.

The following condition ensures that we rule out the corner solution that the developer demands are 1:
**Assumption A7.** $0 < \beta < 1$ *and* $0 < \theta < 1$.

## 2.4 Platform Competition

We analyze a model where one of the platforms (platform $B$) chooses between the subscription-based and the hybrid ad-sponsored business models when competing with the subscription-based rival (platform $A$).

Platform $B$'s profit maximization problem reads

$$\max_{p_B^C, p_B^D, d} \quad \pi_B = p_B^C n_B^C + p_B^D n_B^D + r d n_B^C,$$

where $d \geq 0$.

Developers have all the price information and form responsive expectations about consumer participation, their expectations perfectly match the realized consumer demand for any given

[5]If $\theta \neq \beta$, $\underline{t} > \frac{\theta^2}{8} + \frac{3\theta\beta}{4} + \frac{\beta^2}{8} > \theta\beta$. If $\theta = \beta$, $\underline{t} = \frac{\theta^2}{8} + \frac{3\theta\beta}{4} + \frac{\beta^2}{8} = \theta\beta$. Therefore, once Assumption $A1$ is satisfied, both $t > \theta\beta$ and $t > \frac{\theta^2}{8} + \frac{3\theta\beta}{4} + \frac{\beta^2}{8}$ are satisfied.

prices, i.e., $n_T^{C^e} = n_T^C$. So, the realized developer demand is

$$n_T^D = \beta n_T^C - p_T^D,$$

where $T = A, B$.

All consumers arrive after the developers, they can observe developers' participation decisions. The consumer demands are:

$$n_A^C = \frac{1}{2} + \frac{p_B^C + \alpha d^2 - p_A^C}{2t} + \frac{\theta n_A^D - \theta n_B^D}{2t},$$

and

$$n_B^C = \frac{1}{2} + \frac{p_A^C - \alpha d^2 - p_B^C}{2t} + \frac{\theta n_B^D - \theta n_A^D}{2t}.$$

Ultimately, consumers make the participation decision upon the subscription prices for both consumers and developers. Thus, the realized consumer demands are

$$n_A^C = \frac{1}{2} + \frac{p_B^C + \alpha d^2 - p_A^C}{2t - 2\theta\beta} + \frac{\theta p_B^D - \theta p_A^D}{2t - 2\theta\beta},$$

and

$$n_B^C = \frac{1}{2} + \frac{p_A^C - \alpha d^2 - p_B^C}{2t - 2\theta\beta} + \frac{\theta p_A^D - \theta p_B^D}{2t - 2\theta\beta}.$$

Platform $B$ makes the strategic decision of its business model by setting the advertising intensity $d$, which lays the groundwork for the competitive interactions down the line. The following proposition characterizes the competition outcomes of Stage 2 as functions of the advertising intensity $d$.

**Proposition 9.** *When two platforms simultaneously set the subscription prices on both sides, given platform $B$'s chooses the advertising intensity $d$ in Stage 1, the competition outcomes of Stage 2 can be written as functions of $d$:*

$$p_A^C(d) = \frac{(4t - 3\theta\beta - \beta^2)(6t - 6\underline{t} - 2rd + 2\alpha d^2)}{4(6t - 6\underline{t})},$$

$$p_B^C(d) = \frac{(4t - 3\theta\beta - \beta^2)(6t - 6\underline{t} + 2rd - 2\alpha d^2)}{4(6t - 6\underline{t})} - rd,$$

$$n_A^C(d) = \frac{1}{2} - \frac{rd - \alpha d^2}{6t - 6\underline{t}}, \qquad n_B^C(d) = \frac{1}{2} + \frac{rd - \alpha d^2}{6t - 6\underline{t}},$$

$$p_A^D(d) = \frac{\beta - \theta}{2}(\frac{1}{2} - \frac{rd - \alpha d^2}{6t - 6\underline{t}}), \qquad p_B^D(d) = \frac{\beta - \theta}{2}(\frac{1}{2} + \frac{rd - \alpha d^2}{6t - 6\underline{t}}),$$

*and* $n_A^D(d) = \frac{\beta+\theta}{2}(\frac{1}{2} - \frac{rd-\alpha d^2}{6t-6\underline{t}})$, $n_B^D(d) = \frac{\beta+\theta}{2}(\frac{1}{2} + \frac{rd-\alpha d^2}{6t-6\underline{t}})$,
*resulting in platform profits*

$$\pi_A(d) = \frac{(8t-\theta^2-6\theta\beta-\beta^2)(6t-6\underline{t}-2rd+2\alpha d^2)^2}{16(6t-6\underline{t})^2}$$

*and*

$$\pi_B(d) = \frac{(8t-\theta^2-6\theta\beta-\beta^2)(6t-6\underline{t}+2rd-2\alpha d^2)^2}{16(6t-6\underline{t})^2}.$$

*Proof.* See Appendix. □

Platforms optimally set the subscription prices on consumer's side according to the following response function given the platform B's advertising intensity $d$:

$$p_A^C(d, p_B^C, p_A^D, p_B^D) = \frac{t-\theta\beta}{2} + \frac{1}{2}p_B^C + \frac{\alpha d^2}{2} + \frac{\theta(p_B^D - p_A^D)}{2} - \frac{\beta}{2}p_A^D$$

and

$$p_B^C(d, p_A^C, p_A^D, p_B^D) = \frac{t-\theta\beta}{2} + \frac{1}{2}p_A^C - \frac{rd+\alpha d^2}{2} + \frac{\theta(p_A^D - p_B^D)}{2} - \frac{\beta}{2}p_B^D.$$

The best response functions show that the platform A adjusts its consumer subscription prices by taking into account the advertising disutility for each consumer on platform $B$, and platform $B$ adjusts its consumer subscription price by comparing the disutility it imposes on each consumer due to advertising and the advertising revenue per capita.

We separate the impact of $d$ on the derivative of platform A's profit as follows:

$$\frac{\partial}{\partial d}(\frac{\partial \pi_A}{\partial p_A^C}) = \frac{\partial n_A^C}{\partial d} = \frac{\alpha d}{t-\theta\beta} \tag{2.1}$$

and

$$\frac{\partial}{\partial d}(\frac{\partial \pi_A}{\partial p_A^D}) = \frac{\partial n_A^D}{\partial d} = \beta\frac{\alpha d}{t-\theta\beta}. \tag{2.2}$$

The term $\frac{\alpha d}{t-\theta\beta}$ in Eq. (2.1) captures the fact that increasing the advertising intensity on platform $B$ increases the consumer demand of platform $A$ because adverts create disutility for consumers, which decreases the consumer demand of platform $B$. This effect of advertising on consumer demand of platform $A$ increases in both the advertising intensity $d$ and $\alpha$, which measure consumer's disutility arising from advertising. The term $\beta\frac{\alpha d}{t-\theta\beta}$ in Eq. (2.2) indicates that the effect of advertising on developer demand of platform $A$ works through the demand shifting effect. Increasing the advertising intensity $d$ on platform $B$ increases the consumer demand of platform $A$, which makes platform $A$ more valuable to developers. Therefore, increasing the advertising intensity increases the developer demand of platform $A$. This effect increases in $\beta$, which measures how much developers value consumer participation, along with the advertising intensity $d$ and consumer's disutility parameter $\alpha$.

Similarly, the impact of $d$ on the derivative of platform B's profit can be separated as follows:

$$\frac{\partial}{\partial d}(\frac{\partial \pi_B}{\partial p_B^C}) = r\frac{\partial n_B^C}{\partial p_B^C} + \frac{\partial n_B^C}{\partial d} = -\frac{r}{2t-2\theta\beta} - \frac{\alpha d}{t-\theta\beta}. \tag{2.3}$$

and

$$\frac{\partial}{\partial d}(\frac{\partial \pi_B}{\partial p_B^D}) = r\frac{\partial n_B^C}{\partial p_B^D} + \frac{\partial n_B^D}{\partial d} = -\frac{r\theta}{2t-2\theta\beta} - \beta\frac{\alpha d}{t-\theta\beta}. \tag{2.4}$$

The term $-\frac{r}{2t-2\theta\beta}$ in Eq. (2.3) shows that increasing the advertising intensity decreases consumer demand of platform $B$, which causes loses on advertising revenue, where $r$ is the advertising rate per capita. The term $-\frac{r\theta}{2t-2\theta\beta}$ in Eq. (2.4) shows that increasing $d$ loses developer demand through the demand shifting effect. A fall of application variety has an impact on platform $B$'s consumer demand, which induces a loss on advertising. The term $-\frac{\alpha d}{t-\theta\beta}$ in Eq. (2.3) captures the fact that increasing the advertising intensity $d$ decreases the consumer demand of platform $B$ because adverts create disutility for consumers. This negative effect grows with both the advertising intensity $d$ and consumer's disutility parameter $\alpha$. The term $-\beta\frac{\alpha d}{t-\theta\beta}$ in Eq. (2.4) captures the fact that increasing the advertising intensity $d$ decreases the consumer demand of platform $B$, which makes platform $B$ less valuable to developers. Therefore, through the demand shifting effect, increasing the advertising intensity decreases the developer demand of platform $B$. This negative effect grows in $\beta$, which measures how much developers value consumer participation, besides the advertising intensity $d$ and consumer's disutility parameter $\alpha$.

Platform $B$ determines its advertising intensity by weighing the aforementioned effects on both consumer's and developer's side against the advertising rate $rn_B^C$. Adopting the hybrid ad-sponsored business model appears to be similar to committing to a discount on consumer's side. Committing to a discount on consumer's side expands the consumer demand; hence, it also expands demand on developer's side. However, when having a positive advertising intensity, there is a trade-off between the consumer subscription price, application variety and advertising rate, because advertising generates negative externalities for consumers, hence developers. Therefore, introducing adverts does not necessarily expand consumer demand or developer demand.

We identify 2 scenarios depending on platform $B$'s choice of the advertising intensity when two platforms simultaneously set the subscription prices on two sides:

*Scenario* 1. Platform $B$ sets the advertising intensity to be 0 in Stage 1. This can be the case where adverts are prohibited or platform $B$ does not want to list adverts for reputational concerns.

*Scenario* 2. Platform $B$ sets the advertising intensity to be $d^*$ in Stage 1, which maximizes its anticipated equilibrium profits.

### 2.4.1 Subscription-Based Business Model

In *Scenario* 1, we derive the equilibrium outcomes where platform $B$ chooses the subscription-based business model by setting the advertising intensity $d = 0$ in Stage 1.

**Proposition 10.** *When two platforms simultaneously set the subscription prices on both sides, the equilibrium outcomes of Scenario* 1 *are as follows:*

$$p_T^{C^*} = t - \frac{\beta^2}{4} - \frac{3\theta\beta}{4}, \qquad n_T^{C^*} = \frac{1}{2},$$

$$p_T^{D^*} = \frac{\beta}{4} - \frac{\theta}{4}, \qquad n_T^{D^*} = \frac{\beta}{4} + \frac{\theta}{4},$$

$$\pi_T^* = \frac{t}{2} - \frac{\theta^2}{16} - \frac{3\theta\beta}{8} - \frac{\beta^2}{16},$$

*where $T = A, B$.*

*Proof.* See Appendix. □

The equilibrium consumer subscription price is the standard Hotelling price with zero marginal cost ($t$) adjusted downwards by $\frac{\beta^2}{4} + \frac{3\theta\beta}{4}$. The adjustment term, which measures the benefits of attracting an extra consumer, can be decomposed into two parts $\beta(\frac{\beta}{4} + \frac{3\theta}{4})$. The factor $\beta$ means the platform attracts $\beta$ extra developers when it has an extra consumer. The term $\frac{\beta}{4} + \frac{3\theta}{4}$ is the profit that the platform can earn from an extra developer. The extra developer pays a subscription price $\frac{\beta}{4} - \frac{\theta}{4}$ to the platform, also attracts $\theta$ consumers because developer demand changes according to price changes. The equilibrium consumer subscription price decreases in both $\beta$ and $\theta$. The larger network externalities are, the lower the price charged on the consumer's side. If the benefits of attracting one extra consumer are large compared to platform preferences, the platforms subsidize consumers for participation.

The equilibrium developer subscription price is the monopoly price $\frac{\beta}{4}$ adjusted downwards by $\frac{\theta}{4}$, where $\frac{\theta}{4}$ is the extra benefit that an extra developer brings to the platform from attracting consumers. The equilibrium developer subscription price increases in $\beta$ and decreases in $\theta$. The platforms set the developer subscription prices based on the difference between two sides' network externalities. If the developer's side is the high-externality side, i.e., $\beta$ is large, developers attach a high value to consumer participation, platforms have incentives to lower the consumer subscription prices or even subsidize consumers for participation, so they can charge higher prices on developer's side. If the consumer's side is the high-externality side, i.e., $\theta$ is large, consumers attach a high value to developer participation, and platforms have incentives to lower the developer subscription prices to encourage participation.

Platforms' pricing strategies exhibit the "divide-and-conquer" nature (Caillaud and Jullien, 2003), subsidizing the low-externality side and making profits on the high-externality side. The following table summarizes platforms' pricing strategies.

Table 2.1: Subscription prices on two sides of *Scenario* 1

| | $\beta > \theta$ | $\beta < \theta$ | $\beta = \theta$ |
|---|---|---|---|
| consumer subscription price $p_T^{C*}$ | $< 0$ if $\underline{t} < t < \frac{\beta^2}{4} + \frac{3\theta\beta}{4}$<br>$> 0$ if $t > \frac{\beta^2}{4} + \frac{3\theta\beta}{4}$ | $> 0$ | $> 0$ |
| developer subscription price $p_T^{D*}$ | $> 0$ | $< 0$ | 0 |

Let us now consider the consumer welfare. The equilibrium consumer surplus of Scenario 1 is

$$\begin{aligned} CS(1) &= \int_0^{n_A^{C*}} (v + \theta n_A^{D*} - tx - p_A^{C*})dx + \int_{1-n_B^{C*}}^{1} (v + \theta n_B^{D*} - t(1-x) - p_B^{C*})dx \\ &= v - \frac{5t}{4} + \frac{\beta^2}{4} + \theta\beta + \frac{\theta^2}{4}. \end{aligned}$$

The consumer surplus increases in both directions of network externalities ($\beta$ and $\theta$). Consumers derive higher utility if they attach a higher value to developer participation, i.e., larger $\theta$. Developer demand is greater in the equilibrium if developers attach a higher value to consumer participation, i.e., larger $\beta$, and more application variety improves consumer welfare.

### 2.4.2 Ad-sponsored Business Model

Now we investigate *Scenario* 2 where platform $B$ chooses the hybrid ad-sponsored business model, setting the advertising intensity $d > 0$. In Stage 1, platform $B$ chooses the optimal positive advertising intensity $d$ to maximize its anticipated profits[6].

**Proposition 11.** *When two platforms simultaneously set the subscription prices on two sides, the equilibrium outcomes of Scenario 2 are as follows:*

$$d^* = \frac{r}{2\alpha},$$

$$p_A^{C^*} = \frac{(4t - \beta^2 - 3\theta\beta)(2\alpha(6t - 6\underline{t}) - r^2)}{8\alpha(6t - 6\underline{t})},$$

$$p_B^{C^*} = \frac{(4t - \beta^2 - 3\theta\beta)(2\alpha(6t - 6\underline{t}) + r^2)}{8\alpha(6t - 6\underline{t})} - \frac{r^2}{2\alpha},$$

$$n_A^{C^*} = \frac{2\alpha(6t - 6\underline{t}) - r^2}{4\alpha(6t - 6\underline{t})}, \qquad n_B^{C^*} = \frac{2\alpha(6t - 6\underline{t}) + r^2}{4\alpha(6t - 6\underline{t})},$$

$$p_A^{D^*} = \frac{\beta - \theta}{2}\left(\frac{2\alpha(6t - 6\underline{t}) - r^2}{4\alpha(6t - 6\underline{t})}\right), \qquad p_B^{D^*} = \frac{\beta - \theta}{2}\left(\frac{2\alpha(6t - 6\underline{t}) + r^2}{4\alpha(6t - 6\underline{t})}\right),$$

$$n_A^{D^*} = \frac{\beta + \theta}{2}\left(\frac{2\alpha(6t - 6\underline{t}) - r^2}{4\alpha(6t - 6\underline{t})}\right), \qquad n_B^{D^*} = \frac{\beta + \theta}{2}\left(\frac{2\alpha(6t - 6\underline{t}) + r^2}{4\alpha(6t - 6\underline{t})}\right),$$

*resulting in platform profits*

$$\pi_A^* = \frac{(8t - \theta^2 - 6\theta\beta - \beta^2)(2\alpha(6t - 6\underline{t}) - r^2)^2}{64\alpha^2(6t - 6\underline{t})^2},$$

$$\pi_B^* = \frac{(8t - \theta^2 - 6\theta\beta - \beta^2)(2\alpha(6t - 6\underline{t}) + r^2)^2}{64\alpha^2(6t - 6\underline{t})^2}.$$

*Proof.* See Appendix. □

When competing through different business models, platforms still can deploy the "divide-and-conquer" strategy. If $\beta > \theta$, platforms subsidize consumer's side when the benefits of bringing an extra consumer are larger than platform preferences, and make profits on developer's side. If $\theta > \beta$, platforms subsidize developers and make profits on consumer's side. Platform $B$ offers a discount to compensate consumers for the perceived platform quality drop induced by advertising, so it steals consumers from the rival, which increases its developer demand. The larger discount platform $B$'s offers on consumer subscription price, the higher profits it makes.

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[6]We rule out the corner solution $d = \frac{r + \sqrt{2\alpha(6t - 6\underline{t}) + r^2}}{2\alpha}$. This is the case where the intensity of advertising is so high that platform $B$ loses all consumer demand and is pushed out of the market.

Table 2.2: Changes in equilibrium outcomes due to advertising (*Scenario* 2−*Scenario* 1)

| | $\beta > \theta$ | $\beta < \theta$ | $\beta = \theta$ |
|---|---|---|---|
| $\Delta p_A^{C*}$ | $>0$ if $\underline{t} < t < \frac{\beta^2}{4} + \frac{3\theta\beta}{4}$<br>$<0$ if $t > \frac{\beta^2}{4} + \frac{3\theta\beta}{4}$ | $<0$ | $<0$ |
| $\Delta p_B^{C*}$ | $<0$ | $>0$ if $\underline{t} < t < \frac{\theta^2}{5} + \frac{13\theta\beta}{20} + \frac{3\beta^2}{20}$<br>$<0$ if $t > \frac{\theta^2}{5} + \frac{13\theta\beta}{20} + \frac{3\beta^2}{20}$ | $<0$ |
| $\Delta n_A^{C*}$ | $<0$ | $<0$ | $<0$ |
| $\Delta n_B^{C*}$ | $>0$ | $>0$ | $>0$ |
| $\Delta p_A^{D*}$ | $<0$ | $>0$ | 0 |
| $\Delta p_B^{D*}$ | $>0$ | $<0$ | 0 |
| $\Delta n_A^{D*}$ | $<0$ | $<0$ | $<0$ |
| $\Delta n_B^{D*}$ | $>0$ | $>0$ | $>0$ |
| $\Delta \pi_A^*$ | $<0$ | $<0$ | $<0$ |
| $\Delta \pi_B^*$ | $>0$ | $>0$ | $>0$ |

If the developer's side is the high-externality side ($\beta > \theta$), platform $B$ lowers its consumer subscription price to compensate consumers for the disutility caused by advertising. Platform $A$'s reaction on consumer's side depends on whether the platforms are subsidizing consumers or not. If both platforms charge positive consumer subscription prices, platform $A$ would follow platform $B$'s price movement, lowering its consumer subscription prices as consumer prices are strategic complements ($t > \frac{3\theta\beta}{4} + \frac{\beta^2}{4}$); if both platforms subsidize consumers, platform $A$ would move against platform $B$'s pricing strategy, cutting its subsidy as consumer prices are strategic substitutes ($\underline{t} < t < \frac{3\theta\beta}{4} + \frac{\beta^2}{4}$) (Besanko et al., 2000). Platform $B$ expands it consumers demand, so it can charge a higher developer subscription price and gains more developer demand; while platform $A$ has to lower its developer subscription price because it is now less valuable to developers, still loses some developer demand.

If the consumer's side is the high-externality side ($\beta < \theta$), consumers attach a high value to developer participation. Platform $B$ lowers its developer subscription price to expand developer demand. Therefore, platform $B$ becomes more valuable to consumers even in the presence of adverts. When the network externalities are strong compared to platform preferences ($\underline{t} < t < \frac{\theta^2}{5} + \frac{13\theta\beta}{20} + \frac{3\beta^2}{20}$), this is the case where a small extra developer demand can lead to a large extra consumer demand, platform $B$ can even increases its consumer subscription prices. When the network externalities are weak compared to platform preferences, platform $B$ lowers its consumer subscription price to compensate for the disutility caused by advertising. Given a smaller consumer demand, platform $A$ is better off to reduce its subsidy on developer's side.

If $\beta = \theta$, platforms set the developer subscription price at the marginal cost, both platforms charge lower the consumer subscription prices to compete for consumers. This is because both platforms' revenue rely on their consumer demands. Platform $A$ only collects revenue from consumers subscription while platform $B$ collects revenue from consumer subscription and its advertising revenue (which also depends on its consumer demand). Platform $B$ can over-compensate consumers for disutility caused by advertising to expand its consumer demand, hence, developer demand.

From a welfare standpoint, consumers are affected by platform $B$'s adverts through different channels. Firstly, consumers on platform $B$ suffer utility loss because they view adverts as a nuisance. Secondly. both platforms adjust the consumer subscription prices, which affects all consumers. Thirdly. consumers are affected by the changes in developer demands.

**Proposition 12.** *When consumers have increasing marginal disutility towards advertising, consumers are better off when platform B adopts the hybrid ad-sponsored business model.*

*Proof.* See Appendix. □

## 2.5 Extension: Sequential Price Setting

In this section, we are interested in a game where both platforms set the developer subscription prices before the consumer subscription prices. Thus, we propose the following four-stage game: In Stage 1, platform $B$ chooses its advertising intensity. In Stage $2a$, two platforms simultaneously set the developer subscription prices. In Stage $2b$, two platforms simultaneously set the consumer subscription prices. In Stage 3, developers make participation decisions. In Stage 4, consumers make participation decisions[7]. All the decisions are publicly observable.

Let $t_1 = \frac{\theta^2}{9} + \frac{7\theta\beta}{9} + \frac{\beta^2}{9}$. The following proposition characterizes the competition outcomes of Stage 2 as functions of advertising intensity $d$.

**Proposition 13.** *When two platforms set the developer subscription prices before the consumer subscription prices, given platform B chooses the advertising intensity d in Stage* 1*, the competition outcomes of Stage* 2 *can be written as functions of d:*

$$p_A^C(d) = \frac{6t - 5\theta\beta + \beta^2}{3}\left(\frac{3t - 3t_1 + \alpha d^2 - rd}{6t - 6t_1}\right),$$
$$p_B^C(d) = \frac{6t - 5\theta\beta + \beta^2}{3}\left(\frac{3t - 3t_1 - \alpha d^2 + rd}{6t - 6t_1}\right) - rd,$$

[7] The equilibrium outcomes of this game are the same as those of a three-stage game where developers and consumers arrive simultaneously in Stage 3 as long as all consumers hold responsive expectations

$$n_A^C(d) = \frac{3t - 3t_1 + \alpha d^2 - rd}{6t - 6t_1},$$
$$n_B^C(d) = \frac{3t - 3t_1 - \alpha d^2 + rd}{6t - 6t_1},$$
$$p_A^D(d) = \frac{\beta - \theta}{3}(\frac{3t - 3t_1 + \alpha d^2 - rd}{6t - 6t_1}),$$
$$p_B^D(d) = \frac{\beta - \theta}{3}(\frac{3t - 3t_1 - \alpha d^2 + rd}{6t - 6t_1}),$$
$$n_A^D(d) = \frac{2\beta + \theta}{3}(\frac{3t - 3t_1 + \alpha d^2 - rd}{6t - 6t_1}),$$
$$n_B^D(d) = \frac{2\beta + \theta}{3}(\frac{3t - 3t_1 - \alpha d^2 + rd}{6t - 6t_1}),$$

*resulting in platform profits*

$$\pi_A(d) = \frac{(18t - \theta^2 - 16\theta\beta - \beta^2)(t - t_1 + \alpha d^2 - rd)^2}{(4(9t - 9t_1))^2},$$

$$\pi_B(d) = \frac{(18t - \theta^2 - 16\theta\beta - \beta^2)(6t - 6t_1 - \alpha d^2 + rd)^2}{(4(9t - 9t_1))^2}.$$

*Proof.* See Appendix. □

We also identify 2 scenarios depending on platform $B$'s choice of advertising intensity when two platforms set the developer subscription prices before the consumer subscription prices:

*Scenario 1'.* Platform $B$ sets the advertising intensity to be 0 in Stage 1.

*Scenario 2'.* Platform $B$ sets the advertising intensity to be $d^*$ in Stage 1, which maximizes its anticipated equilibrium profits.

### 2.5.1 Subscription-based Business Model

We first derive the subgame equilibrium where platform $B$ sets advertising intensity $d = 0$. The following proposition characterizes the equilibrium outcomes when platform $B$ sets $d = 0$ in Stage 1.

**Proposition 14.** *When two platforms set the developer subscription prices before the consumer subscription prices, the equilibrium outcomes of Scenario* $1'$ *are as follows:*

$$p_T^{C^*} = t - \frac{\beta^2}{6} - \frac{5\theta\beta}{6}, \qquad n_T^{C^*} = \frac{1}{2},$$

$$p_T^{D^*} = \frac{\beta}{6} - \frac{\theta}{6}, \qquad n_T^{D^*} = \frac{\beta}{3} + \frac{\theta}{6},$$

$$\pi_T^* = \frac{t}{2} - \frac{\theta^2}{36} - \frac{4\theta\beta}{9} - \frac{\beta^2}{36},$$

*where* $T = A, B$.

*Proof.* See Appendix. □

In this scenario, the equilibrium consumer subscription price is also the standard Hotelling price with zero marginal cost, which is $t$, adjusted downwards by the benefits of attracting an extra consumer. However, the benefits of attracting an extra consumer are different from those in *Scenario* 1. The platform attracts $\beta$ extra developers when it has an extra consumer. Each extra developer pays a subscription price $\frac{\beta}{6} - \frac{\theta}{6}$ to the platform, also attracts $\theta$ extra consumers. Platform subsidize consumers if the benefits of attracting one extra consumer are large compared to platform preferences.

Again, platforms set the developer subscription prices based on the difference between two directions of network externalities. If two sides have symmetric network externalities ($\beta = \theta$), platforms set the developer subscription prices at the marginal cost. If the developer's side is the high-externality side, platforms charge positive prices on this side; if the developer's side is the low-externality side, platforms subsidize developers for participation.

Similarly, platforms deploy the "divide-and-conquer" strategy to overcome the coordination problem. The following table summarizes the platforms' pricing strategies in *Scenario* $1'$.

Table 2.3: Subscription prices on two sides of *Scenario* $1'$

| | $\beta > \theta$ | $\beta < \theta$ | $\beta = \theta$ |
|---|---|---|---|
| consumer subscription price $p_T^{C^*}$ | $< 0$ if $\underline{t} < t < \frac{\beta^2}{6} + \frac{5\theta\beta}{6}$<br>$> 0$ if $t > \frac{\beta^2}{6} + \frac{5\theta\beta}{6}$ | $> 0$ | $> 0$ |
| developer subscription price $p_T^{D^*}$ | $> 0$ | $< 0$ | 0 |

We compare the equilibrium outcomes of *Scenario* $1' withthoseof Scenario$ 1. The changes in equilibrium outcomes are summarized in the following table:

Table 2.4: Changes in equilibrium outcomes due to the order of price setting (*Scenario* $1'$−*Scenario* 1)

| | $\beta > \theta$ | $\beta < \theta$ | $\beta = \theta$ |
|---|---|---|---|
| $\Delta p_T^{C*}$ | $> 0$ | $< 0$ | 0 |
| $\Delta p_T^{D*}$ | $< 0$ | $> 0$ | 0 |
| $\Delta n_T^{D*}$ | $> 0$ | $< 0$ | 0 |
| $\Delta \pi_T^*$ | $> 0$ | $> 0$ | 0 |
| $\Delta CS$ | $< 0$ | $< 0$ | 0 |

If two sides have symmetric network externalities ($\beta = \theta$), the platforms fixate the developer subscription prices at marginal cost, 0. Given that symmetric platforms compete for market share on the fixed-sized consumer's side, setting the developer subscription prices before the consumer subscription prices results in the same equilibrium outcomes as setting the subscription prices on both side simultaneously.

If two sides have asymmetric network externalities ($\beta \neq \theta$) and competing platforms could coordinate the order of price setting, they could set the subscription prices on developer's side before consumer's side to increase platform profits. Price competition is intensified on the high-externality side and is softened on the low-externality side. The intuition behind this is as follows: when setting the prices on two sides simultaneously, the network externalities are fully internalized; but setting the subscription prices on developer's side before consumer's side dampens the two-sidedness, the network externalities are partially internalized, so platforms would compete more aggressively for users on the more profitable side (the high-externality side), and the low-externality side becomes less valuable.

### 2.5.2 Ad-sponsored Business Model

Now we investigate the subgame where platform $B$ chooses the hybrid ad-sponsored business model, i.e., $d > 0$. In Stage 1, platform $B$ chooses the optimal positive advertising intensity $d$ to maximize its anticipated profits.

**Proposition 15.** *When two platforms set the developer subscription prices before the consumer subscription prices, the equilibrium outcomes of Scenario* $2'$ *are as follows:*

$$d^* = \frac{r}{2\alpha},$$

$$p_A^{C*} = \frac{(6t - 5\theta\beta - \beta^2)(4\alpha(3t - 3t_1) - r^2)}{24\alpha(3t - 3t_1)},$$

$$p_B^{C*} = \frac{(6t - 5\theta\beta - \beta^2)(4\alpha(3t - 3t_1) + r^2)}{24\alpha(3t - 3t_1)} - \frac{r^2}{2\alpha},$$

$$n_A^{C*} = \frac{4\alpha(3t - 3t_1) - r^2}{8\alpha(3t - 3t_1)}, \qquad n_B^{C*} = \frac{4\alpha(3t - 3t_1) + r^2}{8\alpha(3t - 3t_1)},$$

$$p_A^{D*} = \frac{\beta - \theta}{3}(\frac{4\alpha(3t - 3t_1) - r^2}{8\alpha(3t - 3t_1)}), \qquad p_B^{D*} = \frac{\beta - \theta}{3}(\frac{4\alpha(3t - 3t_1) + r^2}{8\alpha(3t - 3t_1)}),$$

$$n_A^{D*} = \frac{\theta + 2\beta}{3}(\frac{4\alpha(3t - 3t_1) - r^2}{8\alpha(3t - 3t_1)}), \qquad n_B^{D*} = \frac{\theta + 2\beta}{3}(\frac{4\alpha(3t - 3t_1) + r^2}{8\alpha(3t - 3t_1)}),$$

*resulting in platform profits*

$$\pi_A^* = \frac{(18t - \theta^2 - 16\theta\beta - \beta^2)(4\alpha(3t - 3t_1) - r^2)^2}{576\alpha^2(3t - 3t_1)^2},$$

$$\pi_B^* = \frac{(18t - \theta^2 - 16\theta\beta - \beta^2)(4\alpha(3t - 3t_1) + r^2)^2}{576\alpha^2(3t - 3t_1)^2}.$$

*Proof.* See Appendix. □

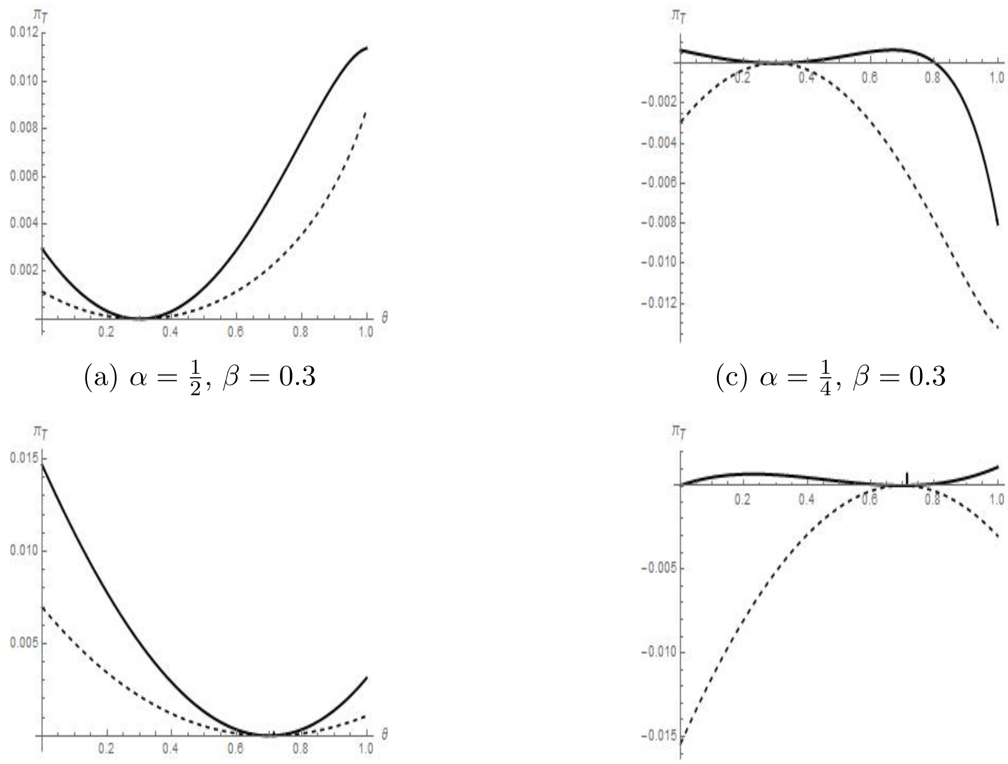
The next table presents the changes in prices and demands between *Scenario* 2′ and *Scenario* 2 due to the change on the order of price setting.

Table 2.5: Changes in equilibrium outcomes due to the order of price setting (*Scenario* 2′−*Scenario* 2)

<table>
<tr><th></th><th>$\beta > \theta$</th><th>$\beta < \theta$</th><th>$\beta = \theta$</th></tr>
<tr><td>$\Delta p_A^{C*}$</td><td></td><td></td><td>0</td></tr>
<tr><td>$\Delta p_B^{C*}$</td><td>$> 0$</td><td>$< 0$</td><td>0</td></tr>
<tr><td>$\Delta n_A^{C*}$</td><td colspan="2">$> 0$</td><td>0</td></tr>
<tr><td>$\Delta n_B^{C*}$</td><td colspan="2">$< 0$</td><td>0</td></tr>
<tr><td>$\Delta p_A^{D*}$</td><td></td><td></td><td>0</td></tr>
<tr><td>$\Delta p_B^{D*}$</td><td>$< 0$</td><td>$> 0$</td><td>0</td></tr>
<tr><td>$\Delta n_A^{D*}$</td><td></td><td></td><td>0</td></tr>
<tr><td>$\Delta n_B^{D*}$</td><td></td><td>$< 0$</td><td>0</td></tr>
</table>

The order of price setting does not change the equilibrium advertising intensity, but it does have an impact on consumer demand. The ad-sponsored business model is less effective in expanding consumer demand when platforms set the subscription prices on developer's side before consumer's side. As explained before, this is because when platforms set price simultaneously on both sides, the externalities are fully internalized, which better coordinates the demands from both sides.

Again, if two sides have symmetric network externalities ($\beta = \theta$), the equilibrium outcomes of Scenario 2 and those of Scenario 2′ coincide. But if two sides have asymmetric network externalities ($\beta \neq \theta$), we could only tell that, for platform $B$, the price competition is intensified on the high-externality side and softened on the low-externality side. For platform $A$, the effect on subscription prices is ambiguous when mixed with the effect of facing a hybrid ad-sponsored rival.




(a) $\alpha = \frac{1}{2}$, $\beta = 0.3$ (c) $\alpha = \frac{1}{4}$, $\beta = 0.3$

(b) $\alpha = \frac{1}{2}$, $\beta = 0.7$ (d) $\alpha = \frac{1}{4}$, $\beta = 0.7$

Figure 2.1: The differences on platform profits due to the order of the price setting as functions of $\theta$ (*Scenario* 2′−*Scenario* 2).

Figure 2.1 illustrates the differences on platform profits as functions of $\theta$, for different values of $\beta$ and $\alpha$ when platform $B$ is ad-sponsored (*Scenario* 2′−*Scenario* 2). The dotted line represents $\Delta\pi_A$ and the solid line represents $\Delta\pi_B$. $\Delta\pi_A$ and $\Delta\pi_B$ both reach zero when $\theta = \beta$. All graphs have parameters $t = \frac{1}{2}$ and $r^2 = \frac{3}{4}(6t - 6\underline{t})$. We reach to the following proposition:

**Proposition 16.** *In the presence of symmetric network externalities ($\beta = \theta$), if the consumer's market size is fixed, platforms are indifferent between setting the developer subscription prices before the consumer subscription prices and setting the subscription prices on both side simultaneously, regardless of platform $B$'s business model.*

*Proof.* See Appendix. □

From the perspective of consumer welfare, under the hybrid ad-sponsored business model, the majority of consumers (consumers on platform $B$) are better off because of a lower subscription price (which over-compensates the disutility caused by advertising) and more application variety, while the minority of consumers (consumers on platform $A$) are worse off, and the unequal split of consumer demand increase the aggregate transportation cost. Similarly, setting the subscription prices on developer's side before consumer's side creates both positive and negative effects on consumer welfare. In general, the intensified price competition on the high-externality side benefits consumers either by offering more application variety (if developer's side is the high-externality side) or offering lower consumer subscription prices (if consumer's side is the high-externality side); while the softened price competition on the low-externality side harms consumers either by reducing application variety (if developer's side is the low-externality side) or charging higher consumer subscription prices (if consumer's side is the low-externality side). The following proposition characterizes how consumer surplus is affected by platform $B$'s choice of business models and the order of price setting by both platforms.

**Proposition 17.** *In a game where both platforms set the developer subscription prices before the consumer subscription prices, consumers are better off when platform $B$ adopts the hybrid ad-sponsored business model. They are worse off when platforms set the developers subscription prices before the consumer subscription prices relative to the case where platforms set the subscription prices on both sides simultaneously, regardless of platform $B$'s business model.*

*Proof.* See Appendix. □

## 2.6 Conclusion

This paper analyzes the platform's strategic choices of business models between the pure subscription-based and the hybrid ad-sponsored business models when competing with a subscription-based rival. The fact that high technology-based platforms usually have one side of users (developers) arriving before the other side users (consumers) is also feature in this paper. We show that, in the setting of single-homing consumers and multi-homing developers, when consumers have increasing marginal disutility towards advertising, it is profitable to adopt the hybrid ad-sponsored business model independent of the degree of network externalities, and it is detrimental to the rival. This result holds no matter whether the platforms set the subscription prices on both sides simultaneously or set the developer subscription prices before the consumer subscription prices.

We shed light on platforms' incentives to set the subscription prices on each side in a sequential fashion rather than simultaneously. We show that, when competing through the subscription-based business model, setting the developer subscription prices before consumer prices intensifies price competition on the high-externality side and softens competition on the low-externality side. Platforms exploit more profits at the expense of consumers. When two platforms compete through different business models, price competition is also intensified on the high-externality side and softened on the low-externality side for platform $B$, but for platform $A$, the effect on subscription prices is ambiguous when mixed with the effect of facing a hybrid ad-sponsored rival.

From a welfare standpoint, it improves consumer welfare when platform $B$ adopts the hybrid ad-sponsored business model no matter whether platforms set the subscription prices on both sides simultaneously or set the developer subscription prices before the consumer subscription prices. We also show that consumers are better of if platforms set the subscription prices on both sides simultaneously.

We notice that a dual business model where the platform offers consumers both the pure subscription-based service and the pure ad-sponsored service is often used in reality. For instance, on most Chinese online video platforms, the viewers can either pay subscription fees to watch the video contents or watch some adverts in exchange for watching the video contents for free. A pressing step for future research is to allow the platform to have the choice of adopting the dual business model. When competing with a pure subscription-based rival, the dual business model seems to have a competitive advantage. However, the analysis of the competition between the hybrid ad-sponsored model and the dual business model could provide new insights into platform's business strategies. We leave this analysis for future research.

## Appendix

**Proof of Proposition 9**

In Stage 4, consumers make their participation decisions given $p_T^C$ set in Stage 2 and $n_T^D$ realized in Stage 3, as well as the advertising intensity $d$ chosen by platform $B$ in Stage 1. The consumer demands are

$$n_A^C = \frac{1}{2} + \frac{p_B^C + \alpha d^2 - p_A^C}{2t} + \frac{\theta n_A^D - \theta n_B^D}{2t},$$

and

$$n_B^C = \frac{1}{2} + \frac{p_A^C - \alpha d^2 - p_B^C}{2t} + \frac{\theta n_B^D - \theta n_A^D}{2t}.$$

In Stage 3, The developer demand is

$$n_T^D = \beta n_T^{C^e} - p_T^D,$$

where $n_T^{C^e} = n_T^C$ for any given price pair $(p_T^C, p_T^D)$, $T = A, B$. The implied consumer demands for each platform are:

$$n_A^C = \frac{1}{2} + \frac{p_B^C + \alpha d^2 - p_A^C}{2t - 2\theta\beta} + \frac{\theta p_B^D - \theta p_A^D}{2t - 2\theta\beta}$$

and

$$n_B^C = \frac{1}{2} + \frac{p_A^C - \alpha d^2 - p_B^C}{2t - 2\theta\beta} + \frac{\theta p_A^D - \theta p_B^D}{2t - 2\theta\beta}.$$

In Stage 2, given platform $B$'s advertising intensity $d$, the platforms choose the subscription prices on two sides simultaneously. Platform $A$'s profit maximization problem is

$$\max_{p_A^C, p_A^D} \quad \pi_A = p_A^C n_A^C + p_A^D n_A^D,$$

and platform $B$'s profit maximization problem is

$$\max_{p_B^C, p_B^D} \quad \pi_B = p_B^C n_B^C + p_B^D n_B^D + r d n_B^C.$$

Taking the first order conditions in $p_T^C$ and $p_T^D$, given $d$, we obtain the following first order condition equations:

$$p_A^C(d, p_B^C, p_A^D, p_B^D) = \frac{t - \theta\beta}{2} + \frac{1}{2}p_B^C - \frac{\theta + \beta}{2}p_A^D + \frac{\theta}{2}p_B^D + \frac{\alpha d^2}{2},$$

$$p_A^D(d, p_A^C, p_B^C, p_B^D) = \frac{\beta(t - \theta\beta) - (\theta + \beta)p_A^C + \beta p_B^C + \theta\beta p_B^D}{4t - 2\theta\beta} + \frac{\alpha d^2 \beta}{4t - 2\theta\beta},$$

$$p_B^C(d, p_A^C, p_A^D, p_B^D) = \frac{t-\theta\beta}{2} + \frac{1}{2}p_A^C - \frac{\theta+\beta}{2}p_B^D + \frac{\theta}{2}p_A^D - \frac{rd+\alpha d^2}{2}$$

and

$$p_B^D(d, p_A^C, p_B^C, p_B^D) = \frac{\beta(t-\theta\beta) - (\theta+\beta)p_B^C + \beta p_A^C + \theta\beta p_A^D}{4t-2\theta\beta} - \frac{\alpha d^2\beta + \theta rd}{4t-2\theta\beta}.$$

Solving the system of the first order equations, we obtain prices and demands as functions of $d$:

$$p_A^C(d) = \frac{(4t-3\theta\beta-\beta^2)(6t-6\underline{t}-2rd+2\alpha d^2)}{4(6t-6\underline{t})},$$

$$p_B^C(d) = \frac{(4t-3\theta\beta-\beta^2)(6t-6\underline{t}+2rd-2\alpha d^2)}{4(6t-6\underline{t})} - rd,$$

$$n_A^C(d) = \frac{1}{2} - \frac{rd-\alpha d^2}{6t-6\underline{t}}, \qquad n_B^C(d) = \frac{1}{2} + \frac{rd-\alpha d^2}{6t-6\underline{t}},$$

$$p_A^D(d) = \frac{\beta-\theta}{2}(\frac{1}{2} - \frac{rd-\alpha d^2}{6t-6\underline{t}}), \qquad p_B^D(d) = \frac{\beta-\theta}{2}(\frac{1}{2} + \frac{rd-\alpha d^2}{6t-6\underline{t}}),$$

and

$$n_A^D(d) = \frac{\beta+\theta}{2}(\frac{1}{2} - \frac{rd-\alpha d^2}{6t-6\underline{t}}), \qquad n_B^D(d) = \frac{\beta+\theta}{2}(\frac{1}{2} + \frac{rd-\alpha d^2}{6t-6\underline{t}}),$$

resulting in platform profits

$$\pi_A(d) = \frac{(8t-\theta^2-6\theta\beta-\beta^2)(6t-6\underline{t}-2rd+2\alpha d^2)^2}{16(6t-6\underline{t})^2}$$

and

$$\pi_B(d) = \frac{(8t-\theta^2-6\theta\beta-\beta^2)(6t-6\underline{t}+2rd-2\alpha d^2)^2}{16(6t-6\underline{t})^2}.$$

**Proof of Proposition 11**

In Proposition 1, we have obtain the prices and demands as functions of advertising intensity $d$:

$$p_A^C(d) = \frac{(4t-3\theta\beta-\beta^2)(6t-6\underline{t}-2rd+2\alpha d^2)}{4(6t-6\underline{t})},$$

$$p_B^C(d) = \frac{(4t-3\theta\beta-\beta^2)(6t-6\underline{t}+2rd-2\alpha d^2)}{4(6t-6\underline{t})} - rd,$$

$$n_A^C(d) = \frac{1}{2} - \frac{rd-\alpha d^2}{6t-6\underline{t}}, \qquad n_B^C(d) = \frac{1}{2} + \frac{rd-\alpha d^2}{6t-6\underline{t}},$$

$$p_A^D(d) = \frac{\beta-\theta}{2}(\frac{1}{2} - \frac{rd-\alpha d^2}{6t-6\underline{t}}), \qquad p_B^D(d) = \frac{\beta-\theta}{2}(\frac{1}{2} + \frac{rd-\alpha d^2}{6t-6\underline{t}}),$$

and $n_A^D(d) = \frac{\beta+\theta}{2}(\frac{1}{2} - \frac{rd - \alpha d^2}{6t - 6\underline{t}})$, $n_B^D(d) = \frac{\beta+\theta}{2}(\frac{1}{2} + \frac{rd - \alpha d^2}{6t - 6\underline{t}})$,
resulting in platform profits

$$\pi_A(d) = \frac{(8t - \theta^2 - 6\theta\beta - \beta^2)(6t - 6\underline{t} - 2rd + 2\alpha d^2)^2}{16(6t - 6\underline{t})^2}$$

and

$$\pi_B(d) = \frac{(8t - \theta^2 - 6\theta\beta - \beta^2)(6t - 6\underline{t} + 2rd - 2\alpha d^2)^2}{16(6t - 6\underline{t})^2}.$$

In Satge 1, platform $B$ chooses the advertising intensity $d$ to maximize its anticipated profits, we obtain the market-sharing equilibrium advertising intensity $d^* = \frac{r}{2\alpha}$. Therefore the equilibrium prices and allocations are as follows:

$$p_A^{C^*} = \frac{(4t - \beta^2 - 3\theta\beta)(2\alpha(6t - 6\underline{t}) - r^2)}{8\alpha(6t - 6\underline{t})},$$

$$p_B^{C^*} = \frac{(4t - \beta^2 - 3\theta\beta)(2\alpha(6t - 6\underline{t}) + r^2)}{8\alpha(6t - 6\underline{t})} - \frac{r^2}{2\alpha},$$

$$n_A^{C^*} = \frac{2\alpha(6t - 6\underline{t}) - r^2}{4\alpha(6t - 6\underline{t})}, \qquad n_B^{C^*} = \frac{2\alpha(6t - 6\underline{t}) + r^2}{4\alpha(6t - 6\underline{t})},$$

$$p_A^{D^*} = \frac{\beta - \theta}{2}(\frac{2\alpha(6t - 6\underline{t}) - r^2}{4\alpha(6t - 6\underline{t})}), \qquad p_B^{D^*} = \frac{\beta - \theta}{2}(\frac{2\alpha(6t - 6\underline{t}) + r^2}{4\alpha(6t - 6\underline{t})}),$$

$$n_A^{D^*} = \frac{\beta + \theta}{2}(\frac{2\alpha(6t - 6\underline{t}) - r^2}{4\alpha(6t - 6\underline{t})}), \qquad n_B^{D^*} = \frac{\beta + \theta}{2}(\frac{2\alpha(6t - 6\underline{t}) + r^2}{4\alpha(6t - 6\underline{t})}),$$

resulting in platform profits

$$\pi_A^* = \frac{(8t - \theta^2 - 6\theta\beta - \beta^2)(2\alpha(6t - 6\underline{t}) - r^2)^2}{64\alpha^2(6t - 6\underline{t})^2},$$

$$\pi_B^* = \frac{(8t - \theta^2 - 6\theta\beta - \beta^2)(2\alpha(6t - 6\underline{t}) + r^2)^2}{64\alpha^2(6t - 6\underline{t})^2}.$$

**Proof of Proposition 12**

In Scenario 2, the equilibrium consumer surplus is

$$CS(2) = v - \frac{5}{4}t + \frac{\beta^2}{4} + \theta\beta + \frac{\theta^2}{4} + \frac{r^2(r^2 t + 2\alpha(6t - 6\underline{t})^2)}{16\alpha^2(6t - 6\underline{t})^2},$$

Therefore, the difference in equilibrium consumer surplus due to platform $B$'s choice of business models is

$$\Delta CS = CS(2) - CS(1) = \frac{r^2(r^2 t + 2\alpha(6t - 6\underline{t})^2)}{16\alpha^2(6t - 6\underline{t})^2} > 0.$$

**Proof of Proposition 13**

In Stage 4, given that the advertising intensity and all subscription prices are set, developers have already made their participation decisions, the consumer demands are:

$$n_A^C = \frac{1}{2} + \frac{p_B^C + \alpha d^2 - p_A^C}{2t} + \frac{\theta n_A^D - \theta n_B^D}{2t},$$

$$n_B^C = \frac{1}{2} + \frac{p_A^C - \alpha d^2 - p_B^C}{2t} + \frac{\theta n_B^D - \theta n_A^D}{2t},$$

where the developer demands

$$n_A^D = \beta n_A^{C^e} - p_A^D$$

and

$$n_B^D = \beta n_B^{C^e} - p_B^D$$

are realized in Stage 3. Developers have the information about the advertising intensity $d$ and the subscription prices on both sides, their expectations about consumer demand match the realized consumer demand. Hence, the implied consumer demands are:

$$n_A^C = \frac{1}{2} + \frac{p_B^C + \alpha d^2 - p_A^C}{2t - 2\theta\beta} + \frac{\theta p_B^D - \theta p_A^D}{2t - 2\theta\beta},$$

and

$$n_B^C = \frac{1}{2} + \frac{p_A^C - \alpha d^2 - p_B^C}{2t - 2\theta\beta} + \frac{\theta p_A^D - \theta p_B^D}{2t - 2\theta\beta}.$$

In Stage 2$b$, given the advertising intensity and the developer subscription prices are already set, the platforms choose the consumer subscription prices to maximize profits. Platform $A$'s profit maximization problem is

$$\max_{p_A^C} \quad \pi_A = p_A^C n_A^C + p_A^D(\beta n_A^C - p_A^D),$$

and platform $B$'s profit maximization problem is

$$\max_{p_B^C} \quad \pi_B = p_B^C n_B^C + p_B^D(\beta n_B^C - p_B^D) + rdn_B^C.$$

Taking the first-order condition in the consumer subscription prices, we obtain the following first order condition equations:

$$p_A^C(d, p_B^C, p_A^D, p_B^D) = \frac{t - \theta\beta}{2} + \frac{p_B^C}{2} - \frac{\theta + \beta}{2}p_A^D + \frac{\theta}{2}p_B^D + \frac{\alpha d^2}{2}$$

and

$$p_B^C(d, p_A^C, p_A^D, p_B^D) = \frac{t - \theta\beta}{2} + \frac{p_B^C}{2} - \frac{\theta + \beta}{2}p_A^D + \frac{\theta}{2}p_B^D - \frac{\alpha d^2}{2} - \frac{rd}{2}.$$

Therefore, we obtain the consumer subscription prices and two sided demands as functions of the advertising intensity and the developer subscription prices:

$$p_A^C(d, p_A^D, p_B^D) = t - \theta\beta - \frac{\theta + 2\beta}{3} p_A^D + \frac{\theta - \beta}{3} p_B^D + \frac{\alpha d^2 - rd}{3},$$

$$p_B^C(d, p_A^D, p_B^D) = t - \theta\beta - \frac{\theta + 2\beta}{3} p_B^D + \frac{\theta - \beta}{3} p_A^D - \frac{\alpha d^2 + 2rd}{3},$$

$$n_A^C(d, p_A^D, p_B^D) = \frac{1}{2} - \frac{\theta - \beta}{6t - 6\theta\beta} p_A^D + \frac{\theta - \beta}{6t - 6\theta\beta} p_B^D + \frac{\alpha d^2 - rd}{6t - 6\theta\beta},$$

$$n_B^C(d, p_A^D, p_B^D) = \frac{1}{2} - \frac{\theta - \beta}{6t - 6\theta\beta} p_B^D + \frac{\theta - \beta}{6t - 6\theta\beta} p_A^D - \frac{\alpha d^2 - rd}{6t - 6\theta\beta},$$

$$n_A^D(d, p_A^D, p_B^D) = \beta(\frac{1}{2} - \frac{\theta - \beta}{6t - 6\theta\beta} p_A^D + \frac{\theta - \beta}{6t - 6\theta\beta} p_B^D + \frac{\alpha d^2 - rd}{6t - 6\theta\beta}) - p_A^D,$$

and

$$n_B^D(d, p_A^D, p_B^D) = \beta(\frac{1}{2} - \frac{\theta - \beta}{6t - 6\theta\beta} p_B^D + \frac{\theta - \beta}{6t - 6\theta\beta} p_A^D - \frac{\alpha d^2 - rd}{6t - 6\theta\beta}) - p_B^D.$$

In Stage $2a$, platforms set the developer subscription prices to maximize its profits. Taking the first-order condition in the developer subscription prices, we obtain the system of best response functions:

$$p_A^D(d, p_B^D) = \frac{3(t - \theta\beta)(\beta - \theta)}{18t - \theta^2 - 16\theta\beta - \beta^2} + \frac{d(\alpha d - r)(\beta - \theta)}{18t - \theta^2 - 16\theta\beta - \beta^2} - \frac{(\beta - \theta)^2 p_B^D}{18t - \theta^2 - 16\theta\beta - \beta^2},$$

and

$$p_B^D(d, p_A^D) = \frac{3(t - \theta\beta)(\beta - \theta)}{18t - \theta^2 - 16\theta\beta - \beta^2} - \frac{d(\alpha d - r)(\beta - \theta)}{18t - \theta^2 - 16\theta\beta - \beta^2} - \frac{(\beta - \theta)^2 p_A^D}{18t - \theta^2 - 16\theta\beta - \beta^2}.$$

Let $t_1 = \frac{\theta^2}{9} - \frac{7\theta\beta}{9} - \frac{\beta^2}{9}$. It's straightforward to obtain the prices and demands as functions of advertising intensity $d$:

$$p_A^C(d) = \frac{6t - 5\theta\beta + \beta^2}{3} (\frac{3t - 3t_1 + \alpha d^2 - rd}{6t - 6t_1}),$$

$$p_B^C(d) = \frac{6t - 5\theta\beta + \beta^2}{3} (\frac{3t - 3t_1 - \alpha d^2 + rd}{6t - 6t_1}) - rd,$$

$$n_A^C(d) = \frac{3t - 3t_1 + \alpha d^2 - rd}{6t - 6t_1},$$

$$n_B^C(d) = \frac{3t - 3t_1 - \alpha d^2 + rd}{6t - 6t_1},$$

$$p_A^D(d) = \frac{\beta - \theta}{3}(\frac{3t - 3t_1 + \alpha d^2 - rd}{6t - 6t_1}),$$

$$p_B^D(d) = \frac{\beta - \theta}{3}(\frac{3t - 3t_1 - \alpha d^2 + rd}{6t - 6t_1}),$$

$$n_A^D(d) = \frac{2\beta + \theta}{3}(\frac{3t - 3t_1 + \alpha d^2 - rd}{6t - 6t_1}),$$

$$n_B^D(d) = \frac{2\beta + \theta}{3}(\frac{3t - 3t_1 - \alpha d^2 + rd}{6t - 6t_1}),$$

resulting in platform profits

$$\pi_A(d) = \frac{(18t - \theta^2 - 16\theta\beta - \beta^2)(t - t_1 + \alpha d^2 - rd)^2}{(4(9t - 9t_1))^2},$$

$$\pi_B(d) = \frac{(18t - \theta^2 - 16\theta\beta - \beta^2)(6t - 6t_1 - \alpha d^2 + rd)^2}{(4(9t - 9t_1))^2}.$$

**Proof of Proposition 15**

In Proposition 5, we present the prices and demands as functions of $d$:

$$p_A^C(d) = \frac{6t - 5\theta\beta + \beta^2}{3}(\frac{3t - 3t_1 + \alpha d^2 - rd}{6t - 6t_1}),$$

$$p_B^C(d) = \frac{6t - 5\theta\beta + \beta^2}{3}(\frac{3t - 3t_1 - \alpha d^2 + rd}{6t - 6t_1}) - rd,$$

$$n_A^C(d) = \frac{3t - 3t_1 + \alpha d^2 - rd}{6t - 6t_1},$$

$$n_B^C(d) = \frac{3t - 3t_1 - \alpha d^2 + rd}{6t - 6t_1},$$

$$p_A^D(d) = \frac{\beta - \theta}{3}(\frac{3t - 3t_1 + \alpha d^2 - rd}{6t - 6t_1}),$$

$$p_B^D(d) = \frac{\beta - \theta}{3}(\frac{3t - 3t_1 - \alpha d^2 + rd}{6t - 6t_1}),$$

$$n_A^D(d) = \frac{2\beta + \theta}{3}(\frac{3t - 3t_1 + \alpha d^2 - rd}{6t - 6t_1}),$$

$$n_B^D(d) = \frac{2\beta+\theta}{3}(\frac{3t-3t_1-\alpha d^2+rd}{6t-6t_1}),$$

resulting in platform profits

$$\pi_A(d) = \frac{(18t-\theta^2-16\theta\beta-\beta^2)(t-t_1+\alpha d^2-rd)^2}{(4(9t-9t_1))^2},$$

$$\pi_B(d) = \frac{(18t-\theta^2-16\theta\beta-\beta^2)(6t-6t_1-\alpha d^2+rd)^2}{(4(9t-9t_1))^2}.$$

In Stage 1, platform $B$ chooses the optimal advertising intensity $d$ to maximize its anticipated profits. The market-sharing equilibrium advertising intensity is $d^* = \frac{r}{2\alpha}$, and the equilibrium prices and allocations are as follows:

$$p_A^{C*} = \frac{(6t-5\theta\beta-\beta^2)(4\alpha(3t-3t_1)-r^2)}{24\alpha(3t-3t_1)},$$

$$p_B^{C*} = \frac{(6t-5\theta\beta-\beta^2)(4\alpha(3t-3t_1)+r^2)}{24\alpha(3t-3t_1)} - \frac{r^2}{2\alpha},$$

$$n_A^{C*} = \frac{4\alpha(3t-3t_1)-r^2}{8\alpha(3t-3t_1)}, \qquad n_B^{C*} = \frac{4\alpha(3t-3t_1)+r^2}{8\alpha(3t-3t_1)},$$

$$p_A^{D*} = \frac{\beta-\theta}{3}(\frac{4\alpha(3t-3t_1)-r^2}{8\alpha(3t-3t_1)}), \qquad p_B^{D*} = \frac{\beta-\theta}{3}(\frac{4\alpha(3t-3t_1)+r^2}{8\alpha(3t-3t_1)}),$$

$$n_A^{D*} = \frac{\theta+2\beta}{3}(\frac{4\alpha(3t-3t_1)-r^2}{8\alpha(3t-3t_1)}), \qquad n_B^{D*} = \frac{\theta+2\beta}{3}(\frac{4\alpha(3t-3t_1)+r^2}{8\alpha(3t-3t_1)}),$$

resulting in platform profits

$$\pi_A^* = \frac{(18t-\theta^2-16\theta\beta-\beta^2)(4\alpha(3t-3t_1)-r^2)^2}{576\alpha^2(3t-3t_1)^2},$$

$$\pi_B^* = \frac{(18t-\theta^2-16\theta\beta-\beta^2)(4\alpha(3t-3t_1)+r^2)^2}{576\alpha^2(3t-3t_1)^2}.$$

**Proof of Proposition 16**

When platform $B$ is subscription-based, the differences on equilibrium outcomes due to the change of the order of price setting are as follows (*Scenario* 1′-*Scenario* 1):

$$\Delta p_T^{C*} = \frac{\beta(\beta-\theta)}{12}, \qquad \Delta n_T^{C*} = 0,$$

$$\Delta p_T^{D*} = \frac{(\theta-\beta)}{12}, \qquad \Delta n_T^{D*} = \frac{\beta-\theta}{12},$$

$$\Delta\pi_T^* = \frac{5(\beta-\theta)^2}{144},$$

where $T = A, B$.

In the presence of symmetric network externalities ($\beta = \theta$), the difference in equilibrium outcomes become

$$\Delta p_T^{C*} = 0, \qquad \Delta n_T^{C*} = 0,$$
$$\Delta p_T^{D*} = 0, \qquad \Delta n_T^{D*} = 0,$$
$$\Delta \pi_T^* = 0,$$

where $T = A, B$.

When platform $B$ is ad-sponsored, the differences on equilibrium prices and allocations due to the change of the order of price setting are as follows (*Scenario* 2′-*Scenario* 2):

$$\Delta p_A^{C*} = \frac{(\theta-\beta)(-3(2\theta+\beta)(t-\theta\beta)r^2 + \beta(6t-6\underline{t})(9t-9t_1))}{12(6t-6\underline{t})(9t-9t_1)},$$
$$\Delta p_B^{C*} = \frac{(\beta-\theta)(3(2\theta+\beta)(t-\theta\beta)r^2 + \beta(6t-6\underline{t})(9t-9t_1))}{12(6t-6\underline{t})(9t-9t_1)},$$
$$\Delta n_A^{C*} = \frac{r^2(\beta-\theta)^2}{4(6t-6\underline{t})(9t-9t_1)}, \qquad \Delta n_B^{C*} = -\frac{r^2(\beta-\theta)^2}{4(6t-6\underline{t})(9t-9t_1)},$$
$$\Delta p_A^{D*} = \frac{(\theta-\beta)(-9(t-\theta\beta)r^2 + (6t-6\underline{t})(9t-9t_1))}{12(6t-6\underline{t})(9t-9t_1)},$$
$$\Delta p_B^{D*} = \frac{(\theta-\beta)(9(t-\theta\beta)r^2 + (6t-6\underline{t})(9t-9t_1))}{12(6t-6\underline{t})(9t-9t_1)},$$
$$\Delta n_A^{D*} = \frac{(\beta-\theta)((-9t+6\theta\beta+3\beta^2)r^2 + (6t-6\underline{t})(9t-9t_1))}{12(6t-6\underline{t})(9t-9t_1)},$$
$$\Delta n_B^{D*} = \frac{(\beta-\theta)((9t-6\theta\beta-3\beta^2)r^2 + (6t-6\underline{t})(9t-9t_1))}{12(6t-6\underline{t})(9t-9t_1)}.$$

In the presence of symmetric network externalities ($\beta = \theta$), the difference in equilibrium outcomes become

$$\Delta p_A^{C*} = 0, \qquad \Delta p_B^{C*} = 0,$$
$$\Delta n_A^{C*} = 0, \qquad \Delta n_B^{C*} = 0,$$
$$\Delta p_A^{D*} = 0, \qquad \Delta p_B^{D*} = 0,$$
$$\Delta n_A^{D*} = 0, \qquad \Delta n_B^{D*} = 0,$$

resulting in the differences on platform profits

$$\Delta \pi_A^* = 0, \qquad \Delta \pi_B^* = 0.$$

**Proof of Proposition 17**

In a game where platforms set the developer subscription prices before the consumer subscription prices:
When platform $B$ is subscription-based (Scenario $1'$), the equilibrium consumer surplus is

$$CS(1') = v - \frac{5}{4}t + \frac{\beta^2}{6} + \frac{7}{6}\theta\beta + \frac{\theta^2}{6}.$$

When platform $B$ is ad-sponsored (Scenario $2'$), the equilibrium consumer surplus is

$$CS(2') = v - \frac{5}{4}t + \frac{\beta^2}{6} + \frac{7}{6}\theta\beta + \frac{\theta^2}{6} + \frac{r^2(9r^2t + 8\alpha(9t - 9t_1)^2)}{64\alpha^2(9t - 9t_1)^2}.$$

The change in consumer surplus due to platform $B$'s choice of business model is

$$\Delta CS = CS(2') - CS(1') = \frac{r^2(9r^2t + 8\alpha(9t - 9t_1)^2)}{64\alpha^2(9t - 9t_1)^2} > 0.$$

Therefore, consumers are better off in the presence of adverts when platforms set the developer subscription prices before the consumer subscription prices.

Recall that in the game where platforms set the subscription prices on both sides simultaneously, when platform $B$ is subscription-based, the equilibrium consumer surplus is

$$CS(1) = v - \frac{5}{4}t + \frac{\beta^2}{4} + \theta\beta + \frac{\theta^2}{4}.$$

Therefore, the difference in equilibrium consumer surplus due to the order of price setting is

$$\Delta CS = CS(1') - CS(1) = -\frac{(\theta - \beta)^2}{12} < 0.$$

Therefore, gievn platform $B$ is subscription-based, consumers are worse off when platforms set the developer subscription prices before the consumer subscription prices.

The equilibrium consumer surplus of Scenario $2'$ is

$$CS(2') = v - \frac{5}{4}t + \frac{\beta^2}{6} + \frac{7}{6}\theta\beta + \frac{\theta^2}{6} + \frac{r^2(9r^2t + 8\alpha(9t - 9t_1)^2)}{64\alpha^2(9t - 9t_1)^2},$$

Recall that the equilibrium consumer surplus of Scenario 2 is

$$CS(2) = v - \frac{5}{4}t + \frac{\beta^2}{4} + \theta\beta + \frac{\theta^2}{4} + \frac{r^2(r^2t + 2\alpha(6t - 6\underline{t})^2)}{16\alpha^2(6t - 6\underline{t})^2}.$$

Therefore, the difference in equilibrium consumer surplus is

$$\Delta CS = CS(2') - CS(2) = -\frac{(\theta-\beta)^2}{12} + \frac{r^4 t(\theta-\beta)^2(5\theta^2+26\theta\beta+5\beta^2-36t)}{64\alpha^2(9t-9t_1)^2(6t-6\underline{t})^2}.$$

$5\theta^2+26\theta\beta+5\beta^2-36t<0$ for all $t>\underline{t}$, so $\Delta CS = CS(2') - CS(2) < 0$. So, gievn platform $B$ is ad-sponsored, consumers are worse off when platforms set the developer subscription prices before the consumer subscription prices.

We conclude that consumers are worse off when platforms set the developer subscription prices before the consumer subscription prices regardless of platform $B$'s choice of business model.

# Chapter 3

# How venture capitalist compensation affects investment decisions

*joint with Nuno Alvim*

## 3.1 Introduction

A venture capital fund is typically formed under the limited partnership agreement. The agreement rules the behavior of the investors and the venture capitalist (VC) in the relationship over the entire life of the fund. Individual and institutional investors serve as the limited partners (LPs), and the VC runs the fund as the general partner. The partnership agreement explicitly specifies the terms of VC compensation, which consist of both visible and invisible components. The visible components are management fee and carried interest (Da Rin et al., 2011), which have received full attention from both the industry insiders and academic scholars[1]. The invisible component is the value-of-distribution rules that determine when the VC receives his carry (Litvak, 2009). A venture fund makes several investments at different times throughout its life, and each investment has its exit date. The VC can either receive his carry at the investment exit date or wait until the fund liquidates. A recent study suggests that when the VC receives his carry becomes an important issue: early distribution of the carries reduces the costs of outside borrowing and creates correct incentives for the VC who control investment decisions (Litvak, 2009). Given the roles of capital provider and decision maker are separated, the VC and the LPs potentially have a conflict of interests and distort their valuation of the same projects: the VC only receives a share of profits but incurs costs of monitoring the projects. Identifying and analyzing these conflicts under different types of the value-of-distribution rules is the main goal of this paper.

[1]See Gompers and Lerner (1996), Gompers and Lerner (1999), Metrick and Yasuda (2010).

This paper theoretically explores how the value-of-distribution rules affect the VC's incentives on the timing of investment and exit decisions, hence the duration, of investments. The value-of-distribution rules we look into are "Escrow, all interest to fund" (the Escrow contract), "Return all capital contributions first" (the Return First contract) and "Payback with no interest note" (the Payback contract) (Litvak, 2009). Under the Escrow contract, the VC's share of profits goes to an escrow account when each investment is realized. The VC only receives payments at the fund liquidation date and the interest of this account goes to the LPs, which generates an interest-free loan from the VC to the LPs. Under the Return First contract, the VC receives no distributions until the invested capital has been fully paid back to the LPs for each investment. After this threshold, the VC can receive his carry at each exit date. There should be no interest-free loan between the two parties once the invested capital is returned. Under the Payback contract, the VC receives his share of the revenues at the investment exit date and pays back the invested capital back to the LPs without interest when the fund liquidates. This type of contract generates an interest-free loan from the LPs to the VC.

We provide the first-best outcomes where the roles of capital provider and decision makers coincide as a benchmark, then compare the investment decisions under different value-of-distribution rules with the first-best outcomes. The results we find are the following. If there is only one project under consideration, the first-best exit date arrives when the marginal cost of staying in the investment equals to the marginal benefits of staying, and the investor wants to start the investment as early as possible. Under the Escrow contract, the first-best investment duration can be attained give a certain level of the carried interest for the VC, but the VC would postpone the starting date of the investment because he wants to shorten the period during which he practically lends an interest-free loan to the LPs. Under the Payback contract, there is practically an interest-free loan from the LPs to the VC, the VC would start the project early and exit from the project early to keep the loan period long. So, under the Payback contract, the optimal starting date can be attained but the duration of the project is always shorter than the optimal. Given a certain level of the carried interest, both the first-best investment duration and the starting date can be attained under the Return First contract because there is no interest-free loan between the two parties. If there are two investment projects under consideration, fixing the project that starts first to be normal, the project that ends first incurs higher marginal monitoring cost because the investor has convex monitoring cost when there are two active projects. Therefore, the project that ends first has a shorter first-best duration than that of the one project case, and the project that ends second has the same first-best duration as that of the one project case. The investor wants to start both project as early as possible. Only the Escrow contract can restore the first-best investment durations given a certain level of the carried interest for the VC, but cannot achieve the first-best starting dates. The first-best starting dates are possible to be attained only under the Payback contract. Our results indicate that, regarding the investment durations, using a certain level of carried interest can overcome the distortions induced by the interest-free loan, but not the distortion induced by the fact that the VC does not return the invested capital at the exit date.

To the best of our knowledge, this paper is the first attempt to analyze the impact of the value-of-distribution rules on investment decisions. The main contribution of the paper lies in highlighting the impact of the interest-free loan between the VC and the LPs generated under different types of distribution rules. The fact that the VC can only keep his share of profits instead of revenues makes monitoring costs relatively larger pushes for an early exit from investment projects. The interest-free loan from the LPs to the VC pushes for an early exit while the interest-free loan from the VC to the LPs has a contradicting effect. With these incentives, we study the efficiency of different distribution rules. We show that the Return First contract can provide first-best outcomes when there is only one project. In Litvak (2009)'s sample, the Return First contract is the most popular one, used by almost half of the funds. However, when there are two projects under consideration, the Escrow contract, which is the least-VC friendly contract, can restore the first-best durations given a certain level of the carried interest. In Litvak (2009)'s sample, the Escrow contract is the least used one. We try to provide an answer to the question why Return First contract is popular than the others although it cannot provide first-best investment durations when there are two projects. Gompers and Lerner (1996) find empirical evidence suggest that the price of venture capital services shift if the demand for venture funds changes while the supply of fund managers remains fixed in the short-run.

This paper is related to the literature on the conflict of interests between the VC and the LPs in a venture capital fund. Compared with the extensive literature on the contracts between the VC and their portfolio enterprises, little light has been shed on the contracts between the VC and the LPs of venture funds. To the best of our knowledge, there are only a few research papers studying the elements of venture capital limited partnership agreement. Kandel et al. (2011) explore the cost of the limited life span of venture capital funds, featuring the VC's informational advantage over the LPs and outsiders. They offer the first-best outcome where no limited life span is imposed as the benchmark. They show that the limited life span creates two types of "myopia" on the VC's investment decisions and project choices relative to the first-best case: firstly, the VC may prefer to continue bad projects and sell them as unfinished good ones as long as the outsiders cannot distinguish them, therefore, the first-best outcome is not likely to achieve; secondly, the VC does not monitor good but delay-prone projects if he is not sufficiently compensated. Banal-Estañol and Ippolito (2012) focus on default penalties of the committed capital of private equity funds. They argue that commitment by the LPs can reduce the cost of screening general partners by limiting ex-post renegotiation, and show that the degree of commitment bear implications for the investment size of the fund and on the fee structure of the general partners.

This paper also relates to several studies on VC compensation, mostly empirical. The LPs cannot get involved in the daily management of the funds. Therefore, the VC compensation is the most important contractual mechanism for aligning the incentives of the LPs and the VC (Gompers and Lerner, 1999). Gompers and Lerner (1999) empirically examine VC compensation with a dataset containing 419 partnerships dated from 1978 to 1992. In their sample, They observe that the VC compensation is higher for older and larger funds; the pay of new VCs is less sensitive to performance because reputational concerns induce them to work hard Gompers and Lerner (1999) report that 81 percent of the funds in their sample use the classic 20 percent carry for the VC. Metrick and Yasuda (2010) study both venture capital and leveraged buyout funds. They use a sample of 238 funds raised between 1993 and 2006, 95 percent of which use the classic 20 percent carry. In both papers, only the visible components of VC compensation (management fee and carried interest) are considered, the value-of-distribution rules are ignored.

Our study is inspired by Litvak (2009), which is the first study that pays attention to the value-of-distribution rules. Litvak (2009) suggests that the value-of-distribution rules should be the third element of VC compensation, along with the visible components (management fee and carried interest). Through the interest-free loan generated by the value-of-distribution rules, the VC almost always captures a higher fraction of funds' profits than the nominal carry percentage. Using a hand-collected dataset[2], she shows that the timing of distribution matters because it is highly valuable to the VC: a shift from the most popular distribution rule to the second most popular rule can affect the VC compensation as much or more than common variations in management fee or carried interest. Also, more complex management fee provisions predict lower total compensation while common proxies for VC quality predict higher levels of the visible components of VC compensation (carried interest and management fee) but offers no prediction of the levels of invisible component (the value-of-capital distribution rules).

This paper proceeds as follows. Section 3.2 introduces the model. The first-best outcomes are presented in Section 3.3 as the benchmark cases. The impact of different types distribution rules is analyzed in Section 3.4. The last section concludes.

## 3.2 The Model

There are projects that each needs an investment $F$. The value of the project depends on the duration of the investment $d$ in two ways. Firstly: longer duration increases the value of a normal project $V(d)$ in a decreasing rate, i.e., $V'(d) > 0$ and $V''(d) < 0$. Secondly: the

[2]Litvak (2009)'s sample consists of partnership agreements of 68 venture capital funds, raised by 28 venture capital firms.

project may either remain normal or fail. The event of a failed project follows a Poisson process with intensity parameter $\lambda_F$ . A failed project pays 0. We also consider that if a project has shown to be a failure, the uncertainty is realized[3]. It means that a project can only change state if it is normal. Therefore, the probability that a project fails at any given moment $t$ is $\lambda_F e^{-\lambda_F t}$. The revenue of a normal project can either be sufficient to compensate the invested capital, i.e. $V(d) > F$, or not, i.e., $V(d) < F$. If it cannot compensate the invested capital, i.e. $V(d) < F$, the VC will terminate the project immediately. So, we only consider the case $V(d) > F$.

The VC is risk neutral, cash poor and cares about his monetary returns. Time is continuous and is discounted at rate $r$. The VC has to monitor the project, due to his limited time and energy, we consider that there is a convex cost of doing so: when there are two active investment projects at the same time, the VC will need more personnel to monitor these investments just to keep the monitoring intensity at the same level as that of one project. The monitoring costs incurred to the VC up to time $t$ is

$$c(d_a, d_b, t) = \delta \int_a^t (\Phi_a(d_a, t) + \Phi_b(d_b, t))^2 d\tau,$$

$$0 \leq a \leq b \leq t,$$

where $\Phi_i(d_i, t) = 1$ if project $i$ is active at time $t$ and 0 otherwise. The subscripts $a$ and $b$ indicate the starting date of each project. The VC decides when to invest in the project and when to exit, hence its duration. We present all the expressions in future value evaluated at $T$, which is the fund liquidate date, it allows a nature comparison between different types of the value-of-distribution rules that the VC and the LPs sign.

We assume that the following conditions hold throughout this paper:
**Assumption A8.** $V'(d) - \lambda_F V(d) > rV(d)$.

The marginal benefits of staying in an investment project are greater than the opportunity costs of staying.
**Assumption A9.** $r(\delta + V(d_N)(r + \lambda_F)) > e^{d_N(r+\lambda_F)}(r(F(r + \lambda_F) + \delta) + 2\delta(r + \lambda_F))$.

We assume that the investor's marginal opportunity costs of investing are smaller than his marginal costs of staying in the investment, so that we can ignore the question whether to invest or not, and focus on the starting and exit decisions. This condition ensures that the investor invests[4].

[3]Venture funds invest in portfolio enterprises that have a novel technology or business model in some high-technology industries, such as biotechnology, IT, software, etc; they offer high potential but high risk (Sahlman, 1990). Once the portfolio enterprises fail, venture funds usually just write them down or off. The amazing story of how Steve Jobs brought Apple from near bankruptcy to billions is rather rare.

[4]The VC does not own any capital, he receives a share of profits only if he makes investments. So we focus on the VC's starting and exit decisions of investments

## 3.3 First-best outcomes

We derive the first-best outcomes where the roles of capital provider and investment decision maker coincide as a benchmark. The investment projects have the aforementioned characteristics.

### 3.3.1 One Project

We start with the case where there is only one investment project under consideration. For now, we assume this project starts at date 0, which is also the starting date of the venture fund. The investor must evaluate whether to exit from or remain in the investment at each point in time. If the project fails, the investor will terminate the investment immediately. There will be no revenue, but further monitoring cost can be avoided by exiting. At date 0, the expected profits of having a project with duration $d$, as long as it remains normal, are

$$\begin{aligned} E(\Pi(d_N)) = {} & e^{-\lambda_F d_N}(e^{r(T-d_N)}V(d_N) - \int_0^{d_N} \delta e^{r(T-\tau)} d\tau) \\ & + \int_0^{d_N} \lambda_F e^{-\lambda_F t}(-\int_0^t \delta e^{r(T-\tau)} d\tau) dt - e^{rT} F. \end{aligned} \tag{3.1}$$

The first part of the first two terms concerns the probability of being at each specific state after duration $d$ and the second part (in brackets) concerns the future value of the payoff (net of monitoring costs) of each possible state, minus the invested capital of the project, evaluated at date $T$.

The investor solves the following profit maximization problem:

$$\max_{d_N} \quad E(\Pi(d_N)).$$

The investor's strategy is determined by a first order condition that depends on whether the optimal exit date of a normal project has passed or not.

$$\begin{aligned} \frac{\partial E(\Pi(d_N))}{\partial d_N} = {} & e^{-\lambda_F d_N + r(T-d_N)}(-\lambda_F - r)V(d_N) + e^{-\lambda_F d_N + r(T-d_N)}V'(d_N) \\ & - e^{-\lambda_F d_N}(e^{r(T-d_N)} - (e^{rT} - e^{r(T-d_N)})\lambda_F)\delta \\ & - \frac{\lambda_F \delta}{r}(e^{rT - d_N\lambda_F} - e^{r(T-d_N) - d_N\lambda_F}). \end{aligned}$$

If the project remains normal, then the exit condition is

$$rV(d_N^*) + \delta = V'(d_N^*) - \lambda_F V(d_N^*). \tag{3.2}$$

On the left hand side we have the marginal costs of staying in the investment project, which consist of the interest opportunity costs and the increase in monitoring cost. On the right hand side lies the marginal benefits of staying, which include not only the growth of a regular project, but also the possibility that a project fails and all the revenue is lost.

We then ask when the investor wants to start the project if it does not necessarily starts at date 0. Assume the investor starts the investment at date $a$, which may or may not be 0. The investor's expected profits are

$$\begin{aligned} E(\Pi(d_N, a)) = e^{-\lambda_F d_N}(e^{r(T-d_N-a)}V(d_N) - \int_a^{a+d_N} \delta e^{r(T-\tau)} d\tau) \\ + \int_0^{d_N} \lambda_F e^{-\lambda_F t}(- \int_a^{a+t} \delta e^{r(T-\tau)} d\tau) dt - e^{r(T-a)} F. \end{aligned}$$

The investor makes the starting decision by taking the first derivatives of the expected profits of starting date $a$. The derivative with respect to $a$ is

$$\frac{\partial E(\Pi(d_N, a))}{\partial a} = -re^{r(-a)} E(\Pi(d_N)) < 0.$$

The next proposition characterizes the investor's starting decision if there is only one investment project under consideration.

**Proposition 18.** *If there is only one investment project, the investor who makes the investment decision would like to start the project as early as possible, at date* 0.

*Proof.* See Appendix. □

#### 3.3.1.1 Two Projects

Now we consider the case where the investor needs to make investment decisions for two projects. The investor will have to decide when to start financing and when to exit from each investment. We refer to the project that starts first, at date $a$, as project $A$ and refer to the one that starts later at date $b$ as project $B$. The existence of project $B$, which may eventually overlap with project $A$, changes the investor's monitoring costs. To better demonstrate how the exit and starting decisions are affected by the existence of the other project, we fixate project $A$ to be normal. Project $B$ could either be normal or fail. Let $d_a$

and $d_b$ be the durations of project $A$ and $B$, respectively. The investor's expected profits at date 0 are

$$\begin{aligned} E(\Pi(d_a, d_b, a, b)) = & e^{-\lambda_F d_b}(e^{r(T-d_a-a)}V(d_a) + e^{r(T-d_b-b)}V(d_b) \\ & - \delta(\int_a^b e^{r(T-\tau)}d\tau + \int_b^{d1N} 4e^{r(T-\tau)}d\tau + \int_{d1N}^{d2N} e^{r(T-\tau)}d\tau)) \\ & + \int_0^{d_b} \lambda_F e^{-\lambda_F t}(e^{r(T-d_a-a)}V(d_a) \\ & - \delta(\int_a^b e^{r(T-\tau)}d\tau + \int_b^{d1F} 4e^{r(T-\tau)}d\tau + \int_{d1F}^{d2F} e^{r(T-\tau)}d\tau))dt \\ & - e^{r(T-a)}F - e^{r(T-b)}F, \end{aligned}$$

$$0 \leq a \leq b \leq t,$$

where the number of the state designation indicates the project ends first or second, respectively; the letter of the state designation refers to the state of project $B$. To be more specific, $d1N = \min\{a + d_a, b + d_b\}$ denotes the exit date of the project that ends first in the state that project $B$ stays normal and $d2N = \max\{a + d_a, b + d_b\}$ denotes the exit date of the project that ends second in the state that project $B$ stays normal. In the case that project $B$ fails, the exit date of the project that ends first is $d1F = \min\{a + d_a, b + t\}$ and the exit date of the project that ends second $d2F = \max\{a + d_a, b + t\}$.

In this expression, we include the payoff and the investment of both projects. We also take into account the additional monitoring costs. Here we allow for any ending dates of the projects. It may be the case that project $A$ ends before project $B$, or the other way around. We identify the following 2 scenarios:

*Scenario* 1. Project $A$ ends before project $B$.

*Scenario* 2. Project $B$ ends before project $A$.

#### 3.3.1.2 Exit decisions

If a project fails, the investor will terminate it immediately. There will be no revenue, but further monitoring costs can be avoided. So the investor only intentionally decides the exit timing for a normal project. The presence of the other project increases the monitoring costs and consequently changes the exit decision of the current one, depending on whether the other one is still active by the time the current one exits.

#### 3.3.1.2.1 Scenario 1

We first consider the scenario in which project $A$ ends before project $B$, which means that project $B$ is still active by the exit time of project $A$. The exit condition for project $A$ is

$$rV(d_a^*) + 3\delta = V'(d_a^*). \tag{3.3}$$

On the left hand side, the marginal costs of staying in the investment consist of the opportunity costs of staying in the project and the increase in the monitoring cost. On the right hand side, the marginal benefits of not exiting are the growth of a normal project. If project $A$ were the only investment project under consideration, its exit condition would have been $rV(d_a^*) + \delta = V'(d_a^*)$. We notice that the existence of an active project $B$ significantly increases the marginal monitoring costs, which pushes project $A$ towards an early exit.

Given that project $A$ has already ended by the exit time of project $B$, the exit condition for project $B$ is

$$rV(d_b^*) + \delta = V'(d_b^*) - \lambda_F V(d_b^*), \tag{3.4}$$

which is the same as that of the one project case.

#### 3.3.1.2.2 Scenario 2

In the scenario where project $B$ ends first. Given that there is no other active project by the exit time of project $A$, the exit condition for project $A$ is

$$rV(d_a^*) + \delta = V'(d_a^*), \tag{3.5}$$

which is the exit condition for a normal project.

The exit condition for project $B$ in this scenario is

$$rV(d_b^*) + 3\delta = V'(d_b^*) - \lambda_F V(d_b^*). \tag{3.6}$$

Again, on the left hand side, we have the marginal costs of staying in the investment. The marginal monitoring costs increases given the existence of an active project $A$, which pushes for an early exit. On the right hand side, the marginal benefits of staying in the investment include the growth of a normal project and the possibility that the project fails. Therefore, the duration of project $B$ is shorter than that of the one project case.

The following proposition characterizes the first-best durations of the two projects given that project $A$ remains normal.

**Proposition 19.** *Given that project A remains normal, the project that ends first has a shorter duration than that of the one project case, and the project that ends second has the same duration as that of the one project case.*

The existence of the other active project generates externalities on the one that ends first through the monitoring costs. The investor will exit early from the project that ends first to shorten the overlapping period between two projects.

#### 3.3.1.3 Starting decisions

Then we look into the question when the investor starts the two projects. The investor's profit maximization problem reads

$$\max_{a,b} \quad E(\Pi(d_a, d_b, a, b)).$$

The investor makes the starting decisions by taking the first derivatives of starting date $a$ and $b$. The following proposition characterizes the investor's starting decisions of both projects.

**Proposition 20.** *Given that project A stays normal, when there are two projects under consideration, the investor wants to start both projects as early as possible, i.e., at date* 0.

*Proof.* See Appendix. □

## 3.4 Contracts

In this section, we study the investment and exit decisions under different types of the value-of-distribution rules. We first study the case where there is only one investment project under consideration and then move to the case where there are two investment projects.

### 3.4.1 One Project

In reality, the VC usually is not the owner of capital. Wealthy institutional and individual investors put up a fund and hire the VC to run it. The partnership agreement specifies the committed capital, the fund liquidation date, the management fee, the size of the carried interest for the VC, and the value-of-distribution rules.

The committed capital is the total amount that the LPs make available for the fund. Therefore, the capital is owned and retained by the LPs but is available to the VC. For simplicity, we assume the committed capital is available from date 0, and the amount is large enough. Now there is only one project under consideration, so only the invested capital for each project $F$ is relevant. The fund liquidation date $T$ is the date when the fund ends and all payments are finalized. The fund typically has a contractually limited life of 10 years with a provision for an extension of 1 to 3 years. We assume that all exit dates take place before the fund liquidates, so the limit life span does not affect the VC's exit decisions. The VC earns his payments through two channels: firstly, the management fees, which usually is set as a percentage of the total committed capital; secondly, the carried interest, which is the $\alpha$ percentage of the fund profits on invested capital. The final feature of the contracts is the value-of-distribution rules describing when each party receives the payments they are entitled to.

Our interest is to understand the impact of value-of-distribution rules on the VC's incentives on the timing of the starting and the exit decisions, hence the duration, of investments. In order to do so, we keep the invested capital for one single project $F$, the fund liquidation date $T$ and the carried interest $\alpha$ constant to see how the VC's decisions change with different types of the value-of-distribution rules.

Some typical value-of-distribution rules can be fully described by using the following pair of parameters $(\beta, \gamma)$. $\beta$ indicates whether the VC is entitled to some of the investment profits at the exit date, which occurs before the liquidation date $T$. $\beta$ takes the value of either 0 or 1. Therefore, $\beta = 1$ means that the VC receives some profits before the fund liquidates. $\gamma$ is the share of the invested capital $F$ that the VC has to return to the LPs at the exit date. $\gamma$ may either be a constant or a function of investment revenues.

The fact that VC compensation terms are relatively standardized[5], at least in its basic structure, makes it meaningful to compare different types of the value-of-distribution rules. We consider three types of the value-of-distribution rules. They generate an interest-free loan either from the VC to the LPs (the Escrow contract), or from the LPs to the VC (the Payback Contract), or no interest-free loan between two parties (the Return First contract).

The first one is the Escrow contract. Under this contract, the profits are distributed to the LPs throughout the fund's life. the VC's share goes to an escrow account, and the interest of this account belongs to the LPs. The VC only receives payments when the fund liquidates. In this case, the amount of the invested capital that is paid back to the LPs at the exit date of each investment is irrelevant because all revenues are held by the LPs in any case. In practical terms, the VC makes an interest-free loan of his carry to the LPs during the period of time between the investment exit date and the fund liquidation date. In this model, the

[5]See Axelrad and Wright (1997); Litvak (2009).

Escrow contract implies $\beta = 0$ and $\gamma = 1$.

The second one is the Payback contract. The VC receives $\alpha$ share of the revenues at the exit date of each investment but is only entitled to $\alpha$ share of the realized profits. The VC returns the $\alpha$ share of the invested capital to the LPs without interest, and the repayment usually happens at the fund liquidation date (Litvak, 2009). In practical terms, the LPs make an interest-free loan to the VC between the investment exit date and the fund liquidation date, and the amount is equivalent to the $\alpha$ share of the invested capital. In this model, the Payback contract implies $\beta = 1$ and $\gamma = 0$.

The third one is the Return First contract. For each investment, the VC receives no profits distributions until total invested capital has been paid back to the LPs. After this threshold, the VC can receive his carry. In the model, it implies $\beta = 1$ because the VC can receive his carry as long as the revenue exceeds the invested capital. It also implies $\gamma = \min\{\frac{V(d)}{F}, 1\}$: if the project generates enough revenue to compensate for the invested capital, all invested capital $F$ is returned to the LPs at the exit date, i.e., $\gamma = 1$; if the project does not generate enough revenue to compensate for the invested capital, all revenue is returned to the LPs at the exit date, i.e., $\gamma = \frac{V(d)}{F}$.

Assume the investment project starts at date $a$ with the aforementioned characteristics. We analyze the investment decisions the VC makes under different contracts and compare them with the first-best decisions.

We first define the payoff function of the VC. If the investment is profitable, the VC will get paid according to the contract. If the investment is not profitable, the VC will deliver all project revenues to the LPs. Therefore, the compensation scheme for the VC is

$$\pi = e^{\beta r(T-d)}\alpha(V(d) - \gamma F) - \alpha F(1-\gamma).$$

If the project starts at date $a$, in case the revenues of the project in the normal state can compensate the invested capital, i.e., $V(d_N) > F$, the expected payoff of the VC is

$$\begin{aligned} E(\pi(d_N, a)) = e^{-\lambda_F d_N}(e^{\beta r(T-d_N-a)}\alpha(V(d_N) - \gamma F) - \int_a^{a+d_N} \delta e^{r(T-\tau)} d\tau) \\ + \int_0^{d_N} \lambda_F e^{-\lambda_F t}(-\int_a^{a+t} \delta e^{r(T-\tau)} d\tau) dt - \alpha F(1-\gamma), \end{aligned}$$

where $d_N$ is the duration of a project if it remains normal. The first part of each term concerns the probability that either of the possible states is achieved, and the second part concerns the correspondent payoff.

The VC solves the following profit maximization problem:

$$\max_{d_N, a} \quad E(\pi(d_N, a)).$$

#### 3.4.1.1 Exit Decisions

The VC's exit strategy is defined by a first order condition that depends whether the optimal exit date of a normal project has passed or not.

$$\begin{aligned}\frac{\partial E(\pi(d_N,a))}{\partial d_N} &= e^{-\lambda_F d_N+\beta r(T-d_N-a)}(-\lambda_F-\beta r)\alpha(V(d_N)-\gamma F)+e^{-\lambda_F d_N+\beta r(T-d_N-a)}V'(d_N)\\ &-e^{-\lambda_F d_N-ra}(e^{r(T-d_N)}-(e^{rT}-e^{r(T-d_N)})\lambda_F)\frac{\delta}{r}\\ &-\frac{\lambda_F\delta e^(-ra)}{r}(e^{rT-d_N\lambda_F}-e^{r(T-d_N)-d_N\lambda_F}).\end{aligned}$$

For $V(d_N) > F$, the exit condition for an interior solution of a normal project is

$$e^{\beta r(T-d_N-a)}\beta r\alpha(V(d_N)-\gamma F)+\delta e^{r(T-d_N-a)} = e^{\beta r(T-d_N-a)}\alpha V'(d_N)-e^{\beta r(T-d_N-a)}\lambda_F\alpha(V(d_N)-\gamma F).$$

On the left hand side of the exit condition, again we have the marginal costs of staying in the investment which comprise the interest opportunity cost and the marginal monitoring cost. On the right hand side, the marginal benefits of staying in the investment consist of the VC's share of the growth of a normal project and his loss in the case that the project fails and all revenue is lost. For all contracts, the fact that the VC only keeps $\alpha$ share of the profits makes the monitoring costs relatively larger. It is the effort incentive distortion that pushes for an early exit from the investment.

Under the Escrow contract, the exit condition is

$$\delta e^{r(T-d_N^E-a)} = \alpha V'(d_N^E)-\lambda_F\alpha(V(d_N^E)-F).$$

Under the Escrow contract, there are distortions leading towards the opposite directions. The VC faces no interest opportunity cost because he cannot receive anything until the fund liquidates, which makes the VC stay in the investment longer. Notice that in the case that the project fails, the VC only loses his share of profits, which also makes him stay in the investment longer. However, there are the effort incentive distortion and the timing distortion both pushing for an early exit. The timing distortion arises from the fact that the VC only gets paid at $T$ increases the marginal cost of staying longer ($e^{r(T-a-d_N^E)} > 1$).

Under the Payback contract, the exit condition is

$$r\alpha V(d_N^P)+\delta = \alpha V'(d_N^P)-\lambda_F\alpha V(d_N^P).$$

In this case, the only distortion is the effort incentive distortion that pushes for an early exit. Therefore, the duration under the Payback contract is shorter than the optimal.

Under the Return First contract, the exit condition is

$$r\alpha(V(d_N^R) - F) + \delta = \alpha V'(d_N^R) - \lambda_F \alpha(V(d_N^R) - F).$$

Now there are distortions leading towards the opposite directions. On the one hand we still observe the effort incentive distortion, which leads to an early exit; on the other hand the interest opportunity cost of not exiting is now smaller and the loss in the case that the project fails is also smaller; these effects make him stay in the investment longer.

The following proposition characterizes the project duration under the three types of distribution contracts.

**Proposition 21.** *If there is only one investment project under consideration and the revenue of this project can compensate for the invested capital ($V(d) > F$), there exists an* $\underline{\alpha}^E \equiv \frac{e^{r(T-a-d_N^E)}(V'(d_N^*) - \lambda_F V(d_N^*) - rV(d_N^*))}{V'(d_N^E) - \lambda_F(V(d_N^E) - F)}$ *such that the duration under the Escrow contract coincides with the first-best duration; the duration of the project under the Payback contract is shorter than the first-best duration; there exists an* $\underline{\alpha}^R \equiv \frac{V'(d_N^*) - \lambda_F V(d_N^*) - rV(d_N^*)}{V'(d_N^R) - (r+\lambda_F)(V(d_N^R) - F)}$ *such that the duration under the Return First contract coincides with the first-best duration.*

*Proof.* See Appendix. □

The previous proposition indicates that if the LPs are seeking to achieve the efficient project duration, they cannot rely on the Payback contract. The Escrow contract and the Return First contract can achieve the optimal duration.

**Corollary 6.** *If* $\alpha < \underline{\alpha}^E$*, the duration under the Escrow contract is shorter than the first-best duration.*

This corollary follows directly from the fact that the VC's benefits of staying in the investment project under Escrow contract are smaller if the carried interest $\alpha < \underline{\alpha}^E$, which indicates an early exit.

**Corollary 7.** *If* $\alpha < \underline{\alpha}^R$*, the duration under the Return First contract is shorter than the first-best duration.*

*Proof.* See Appendix. □

**Corollary 8.** *For a same share* $\alpha$*, the duration under the Payback contract is shorter than that of the Return First contract.*

This corollary follows directly from the fact that the interest opportunity costs and the VC's loss in case the project fails under the Return First contract are smaller than those of the Payback contract, which makes the VC stays longer in the investment.

#### 3.4.1.2 Starting decisions

Now we investigate when the VC starts the project under different contracts. The VC makes the starting decision by taking the first derivative of his expected payoff with respect to starting date $a$:

$$\begin{aligned}\frac{\partial E(\pi(d_N,a))}{\partial a} &= e^{-\lambda_F d_N+\beta r(T-d_N-a)}(-\beta r)\alpha(V(d_N)-\gamma F) \\ &+ \delta e^{-\lambda_F d_N+r(T-d_N-a)}(-1+e^{dr}) \\ &+ \delta\frac{e^{r(T-a)}r+e^{r(T-d_N-a)-d\lambda_F}\lambda_F}{r+\lambda_F} - \delta e^{r(T-a)-d_N\lambda_F}.\end{aligned}$$

The next proposition characterizes the VC's starting decisions under different distribution contracts if there is only one investment project under consideration.

**Proposition 22.** *If there is only one investment project, under both the Payback contract and the Return First contract, the VC wants to start the project as early as possible, i.e., at date* 0*, which coincides the optimal starting date. Under the Escrow contract, the VC would start late.*

*Proof.* See Appendix. □

If there is only one project under consideration, we conclude the impact of the distribution contracts, essentially the interest-free loan, on the VC's investment decisions as follows. The Escrow contract can restore the first-best investment duration given a certain level of the carried interest for the VC but cannot restore the first-best starting date. Under this contract, the VC would postpone the starting date to shorten the period between the investment exit date and the fund liquidation date, which is exactly the period during which the VC practically lends an interest-free loan to the LPs. The Payback contract cannot restore the first-best investment duration but can restore the first-best starting date. Under this contract, the VC would start the project as early as possible and the duration of the project is shorter than optimal, because the VC has the incentives to get the interest-free loan from the LPs as early as possible and keep the duration of this loan long. Only the Return First contract can restore both the first-best investment duration, given a certain level of the carried interest for the VC, and the starting date because there is no interest-free loan to distort the VC's decisions. It is worth noting that the Return First contract is the most popular distribution contract for both cash distributions and securities distribution in Litvak (2009)'s sample.

### 3.4.2 Two Projects

Now there are two projects under consideration. We assume that project $A$ starts at date $a$ and stays normal, project $B$ starts later at date $b$ and can either stay normal or fail. Let $d_a$ and $d_b$ be the duration of projects that start at $a$ and $b$, respectively. We need to identify how much of the invested capital is returned to the LPs at each exit date.

The Escrow contract implies $\beta = 0$, $\gamma_a = \gamma_b = 1$: the VC will only receive his carry at the fund liquidation date $T$, and all the revenue goes to the LPs at each exit date. Under this contract, it does not matter how much of the invested capital is paid by each project. There are practically two interest-free loans from the VC to the LPs. The first interest-free loan exists between the first exit date and the fund liquidation date, and the amount of the loan is equal to the VC's carry from the project that ends first. The second interest-free loan exists between the second exit date and the fund liquidation date, and the amount is equal to the VC's carry from the project that ends second.

The Payback contract implies $\beta = 1$, $\gamma_a = \gamma_b = 0$: the VC receives some revenue at each exit date and only returns the invested capital at the fund liquidation date $T$. Under this contract, there are two interest-free loans from the LPs to the VC in practical terms. The VC receives the first interest-free loan at the first exit date, and the amount of the loan equals to his carry of the invested capital from the project that ends first. The VC receives the second interest-free loan at the second exit date, and the amount equals to his carry of the invested capital from the project that ends second. Both interest-free loans end at the fund liquidation date.

Among theses distribution contracts, the Return First contract is the only contract under which the exit decision of the project that ends second depends on the revenue generated by the project that ends first. The Return First contract suggests $\beta = 1$, which means that the VC can receive some profits before $T$ as long as the invested capital is returned to the LPs. Depending on whether the revenue of one normal project can compensate for the invested capital of two projects, we further identify the following two cases.

Firstly, if both projects remain normal until their optimal exit decisions then there will be enough revenue for the fund to have profits, however, one normal project cannot generate enough revenue to compensate for the invested capital of two projects, i.e., $F < V(d_N) < 2F$. In this case, the revenue of the project that ends first is all returned to the LPs, the revenue of the project that ends later is partly returned to the LPs to compensate for the remaining invested capital, and the VC can receive his share of profits from both projects from the rest of the revenue. In our model, it implies that $\gamma_a = \frac{V(d_a)}{F}$, $\gamma_b = \frac{2F-V(d_a)}{F}$ in Scenario 1, and $\gamma_a = \frac{2F-V(d_b)}{F}$, $\gamma_b = \frac{V(d_b)}{F}$ in Scenario 2. In practical terms, the VC makes an interest-free loan to the LPs at the first exit date, and the amount of the loan is equal to the VC's carry from the project that ends first. This interest free loan ends at the second exit date.

Secondly, the revenue of one normal project can compensate for the invested capital of two projects, i.e., $V(d_N) > 2F$. In this case, the LPs' invested capital are fully compensated at the first exit date, and the VC is entitled to his share of the remaining revenue. At the second exit date, the VC does not return any invested capital to the LPs. Therefore, it generates an interest-free loan from the VC to the LPs at the first exit date, and the amount of the loan is $\alpha F$[6]. This interest-free loan ends at the second exit date. In our model, it implies that $\beta = 1$, $\gamma_a = 2$ and $\gamma_b = 0$ in Scenario 1, and $\beta = 1$, $\gamma_a = 0$ and $\gamma_b = 2$ in Scenario 2.

If there are two investment projects under consideration, the VC's payoff function is:

$$\pi = e^{\beta r(T-d_a-a)}\alpha(V(d_a) - \gamma_a F) + e^{\beta r(T-d_b-b)}\alpha(V(d_b) - \gamma_b F) - \alpha F(1-\gamma_a) - \alpha F(1-\gamma_b).$$

If these investments are profitable, then the VC will get paid according to the types of the value-of-distribution rules. If these investments are not profitable, the VC will have to deliver all project revenues to the LPs.

Given that project $A$ remains normal, the VC's expected payoff function is:

$$\begin{aligned}
E(\Pi(d_a, d_b, a, b)) = {} & e^{-\lambda_F d_b}(e^{\beta r(T-d_a-a)}\alpha(V(d_a) - \gamma_a F) + e^{\beta r(T-d_b-b)}\alpha(V(d_b) - \gamma_b F) \\
& - \delta(\int_a^b e^{r(T-\tau)}d\tau + \int_b^{dlN} 4e^{r(T-\tau)}d\tau + \int_{d1N}^{d2N} e^{r(T-\tau)}d\tau)) \\
& + \int_0^{d_b} \lambda_F e^{-\lambda_F t}(e^{\beta r(T-d_a-a)}\alpha(V(d_a) - \gamma_a F) \\
& - \delta(\int_a^b e^{r(T-\tau)}d\tau + \int_b^{d1F} 4e^{r(T-\tau)}d\tau + \int_{d1F}^{d2F} e^{r(T-\tau)}d\tau))dt \\
& - \alpha F(1-\gamma_a) - \alpha F(1-\gamma_b),
\end{aligned}$$

where the number 1 or 2 of the state designation indicates the project ends first or second, respectively; the letter of the state designation refer to the states of project $B$, either to be normal $N$ or a failure $F$. For instance, $d1N = \min\{a + d_a, b + d_b\}$ denotes the first exit date in the case project $B$ stays normal and $d2N = \min\{a + d_a, b + d_b\}$ denotes the second exit date in the case that project $B$ stays normal.

The VC maximizes his expected payoff by deciding over the starting and exit dates of both projects, he solves

$$\max_{d_a, d_b, a, b} \quad E(\Pi(d_a, d_b, a, b))$$

[6] At the first exit date, the VC is entitle to his share of profits $\alpha(V(d_a) - F)$ in Scenario 1 and $\alpha(V(d_b) - F)$ in Scenario 2. However, the VC has to return the invested capital for the two projects at the first exit date, he can only receive $\alpha(V(d_a) - 2F)$ in Scenario 1 and $\alpha(V(d_b) - 2F)$ in Scenario 2.

$$0 \leq a \leq b \leq t.$$

#### 3.4.2.1 Exit decisions

Now we discuss the exit decisions of both projects under different types of contracts. When deciding whether to exit from or to stay in a project, the VC has to consider the possible states of the other project. For the purpose of presentation, the general exit condition for project $A$ and $B$ in two scenarios are presented in the appendix.

The effort incentive distortion exists under all the contracts. It arises from the fact that the VC can only keep $\alpha$ share of profits makes the monitoring costs relatively larger, so it pushes for an early exit. For the project that ends first, the existence of a second active project increases the marginal monitoring costs, which also pushes for an early exit.

##### 3.4.2.1.1 Escrow contract

We first discuss the exit decisions of the two projects under the Escrow contract. In Scenario 1, the exit condition for project $A$ is

$$3\delta e^{r(T-d_a^E-a)} = \alpha V'(d_a^E).$$

Under the Escrow contract, there are no opportunity costs for the VC to stay in project $A$, which pushes the VC to stay longer. By the exit date of project $A$, project $B$ is still active, which increases the marginal monitoring costs of project $A$. Given that project $A$ stays normal, the marginal benefits of staying in this investment are the VC's share of the growth of this project.

The exit condition for project $B$ is

$$\delta e^{r(T-d_b^E-b)} = \alpha V'(d_b^E) - \lambda_F \alpha (V(d_b^E) - F),$$

which is the exit condition for one single project under the same contract. The timing distortion exists for both projects because the VC only gets paid at the fund liquidation date, which makes the monitoring costs more expensive for the VC, pushing for an early exit from project $B$.

In Scenario 2, the exit condition for project $A$ is

$$\delta e^{r(T-d_a^E-a)} = \alpha V'(d_a^E),$$

and the exit condition for project $B$ is

$$3\delta e^{r(T-d_b^E-b)} = \alpha V'(d_b^E) - \lambda_F \alpha(V(d_b^E) - F).$$

In the scenario where project $B$ ends first, again, the VC receives nothing at the exit dates under the Escrow contract, so there are no opportunity costs for him to stay in the projects. This fact pushes the VC to stay longer in the projects. By the exit date of project $B$, the existence of an active project $A$ increases the marginal monitoring costs of project $B$. The exit condition of project $A$ is the same as the exit condition of one normal project under the same contract. The timing distortion exists for both projects because the VC only gets paid at the fund liquidation date, which makes monitoring more expensive and pushes for an early exit.

We can conclude that the exit conditions for the two projects under the Escrow contract are essentially the same as those of the one project case under the same contract. The difference is that due to the existence of the other active project, the marginal monitoring costs for the project that ends first increase. Therefore, for the project that ends first, the effects pushing towards an early exit should be stronger than those of the one project case under the same contract. However, there should be a certain level of the carried interest for the VC so that the first-best durations can be attained.

#### 3.4.2.1.2 Payback contract

In Scenario 1, the exit conditions for project $A$ and $B$ are as follows:

$$r\alpha V(d_a^P) + 3\delta = \alpha V'(d_a^P)$$

and

$$r\alpha V(d_b^P) + \delta = \alpha V'(d_b^P) - \lambda_F \alpha V(d_b^P).$$

In this scenario, the exit condition of project $B$ is the same as that of the one project case under the same contract. Project $A$'s marginal monitoring costs increase because project $B$ is still active at the exit date of project $A$. The effort incentive distortion is the only distortion present under this contract for both projects. Therefore, both projects have shorter durations relative to the first-best durations in the same scenario.

In Scenario 2, the exit conditions for project $A$ and $B$ are as follows:

$$r\alpha V(d_a^P) + \delta = \alpha V'(d_a^P)$$

and

$$r\alpha V(d_b^P) + 3\delta = \alpha V'(d_b^P) - \lambda_F \alpha V(d_b^P).$$

Again, the effort incentive distortion is the only distortion present in this scenario, and it pushes the VC to exit early from the investment projects. So both projects have shorter durations relative to the first-best durations in the same scenario.

We can conclude that under the Payback contract, the exit conditions for both projects are essentially the same as that of the one project case under the same contract. The effort incentive distortion is the only distortion present under the Payback contract, the durations of the projects are shorter than optimal. The only difference is that the existence of the other active project creates externalities on the marginal monitoring costs for the project that ends first. Therefore, the duration of the project that ends first is shorter relative to that of the one project case under the same contract.

#### 3.4.2.1.3 Return First contract

We first consider the case where one normal project cannot generate enough revenue to compensate for the invested capital of two projects, i.e., $F < V(d_N) < 2F$.

In Scenario 1, the exit condition for project $A$ is:

$$3\delta = \alpha V'(d_a^R).$$

In this scenario, at the exit date of project $A$, all revenue is returned to the LPs to compensate for the invested capital and VC receives nothing. In practical terms, the interest-free loan from the VC to the LPs starts from this date, and the amount of this loan is equal to VC's carry from project $A$. Therefore, the exit condition for project $A$ is very similar to the exit condition for project $A$ the Escrow contract in the same scenario. However, the interest-free loan ends at the exit date of project $B$ under the Return First contract. Therefore, the timing distortion does not exist under this contract, the marginal monitoring costs are smaller than those under the Escrow contract.

The exit condition for project $B$ is:

$$r\alpha(V(d_b^R) + V(d_a^R) - 2F) + \delta = \alpha V'(d_b^R) - \lambda_F \alpha(V(d_b^R) + V(d_a^R) - 2F).$$

At the second exit date, a part of the revenue generated by project $B$ is returned to the LPs to compensate for the remaining invested capital, and the VC receives his carry from the remaining revenue. Therefore, the opportunity costs of staying in project $B$ are smaller than those of Eq. (3.4), which pushes the VC to stay longer in this project. The marginal benefits of staying in project $B$ are larger because the loss for the VC is relatively small in case project $B$ fails later, which also pushes the VC to stay longer in the investment. The effort incentive distortion pushes for an early exit.

In Scenario 2, the exit condition for project $B$ under the Return First contract is:

$$3\delta = \alpha V'(d_b^R).$$

All revenue from project $B$ is returned to the LPs to compensate for the invested capital at the first exit date. In practical terms, the interest-free loan from the VC to the LPs starts from the first exit date, and the amount of this loan is equal to VC's carry from project $B$. Therefore, the marginal costs of staying in project $B$ are the marginal monitoring costs. The fact that project $A$ is still active when project $B$ exits contributes to the large marginal monitoring costs project $B$ bears. The VC's marginal benefits of staying in project $B$ are the VC's share of the growth of this project. Because all the revenue from this project goes to the LPs at this exit date, the possibility that the project fails later does not affect the VC at this exit date.

The exit condition for project $A$ in this scenario is:

$$r\alpha(V(d_b^R) + V(d_a^R) - 2F) + \delta = \alpha V'(d_a^R).$$

At the second exit date, a part of the revenue of project $A$ is returned to the LPs to compensate for the remaining invested capital, and the VC receives his carry from the remaining revenue. Therefore, the opportunity costs of staying in project $A$ are smaller than those of Eq. (3.5), which pushes the VC to stay longer in project $A$. The effort incentive distortion has a contradicting effect, pushing for an early exit.

To sum up, in the case where the revenue of one normal project cannot compensate for the invested capital of two projects, i.e., $F < V(d_N) < 2F$, there are distortions pushing towards contradicting directions regarding the exit timing of the projects, there should be a certain level of the carried interest for the VC so that the first-best durations can be attained.

Then we consider the case where the revenue of one normal project can compensate for the invested capital of two projects, i.e., $V(d_N) > 2F$.

In Scenario 1, the exit conditions for project $A$ under the Return First contract is:

$$r\alpha(V(d_a) - 2F) + 3\delta = \alpha V'(d_a).$$

The invested capital of both projects are returned to the LPs and the VC receives his carry of the remaining profits ($\alpha(V(d_a) - 2F)$) from project $A$ at the first exit date. In practical terms, there is an interest-free loan from the VC to the LPs starts from the first exit date, and the amount of this loan is $\alpha F$. Therefore, the opportunity costs of staying in project $A$ are smaller, which pushes the VC to stay longer in the project. However, The effort incentive distortion pushes for an early exit.

The exit condition for project $B$ is:

$$r\alpha V(d_b) + \delta = \alpha V'(d_a) - \alpha\lambda_F V(d_b).$$

Given that all the invested capital has already been returned to the LPs, the VC does not return any invested capital at the second exit date. Therefore, the exit condition for project $B$ is the same as that of the one project case under the Payback contract. The only distortion present is the effort incentive distortion, so the duration of project $B$ is shorter than optimal in this scenario.

In Scenario 2, the exit conditions for project $A$ and $B$ under the Return First contract are as follows:

$$r\alpha V(d_a) + \delta = \alpha V'(d_a),$$

and

$$r\alpha(V(d_b) - 2F) + 3\delta = \alpha V'(d_b) - \alpha\lambda_F(V(d_b) - 2F).$$

In the scenario where project $B$ ends first, the invested capital of both projects are returned to the LPs and the VC receives his carry $\alpha(V(d_b) - 2F)$ at the first exit date. At the second exit date, the VC does not return the invested capital to the LPs. The exit condition for project $A$ is the same as that of the one single project case under the Payback contract. Therefore, the duration of project $A$ is shorter than optimal in this scenario.

We can conclude that, if the revenue of a normal project can compensate for the invested capital of two projects, i.e., $V(d_N) > 2F$, all invested capital is returned to the LPs at the first exit date, which generates an interest-free loan from the VC to the LPs. At the second exit date, the VC does not return any invested capital to the LPs, the exit condition of the project that ends second is the same as the exit condition under the Payback contract. Hence, the duration of the project that ends second is shorter than optimal.

The carried interest is usually measured as a flat percentage of a fund's profits on invested capital[7], so the LPs cannot set the level of the carried interest for the VC to attain the optimal duration under the Return First contract.

The key characteristic of the Return First contract is that there is no interest-free loan between the two parties once the invested capital is returned. The interest-free loan from the VC to the LPs ends no matter whether the revenue of a normal project can compensate for the invested capital of two projects or not. In the case where the revenue of a normal project cannot compensate for the invested capital of two projects ($F < V(d_N) < 2F$), it is possible to attain the first-best duration for the project that ends second given a certain

[7]See Gompers and Lerner (1999), Metrick and Yasuda (2010), Litvak (2009) and Da Rin et al. (2011).

level of carried interest[8]. However, in the case where the revenue of a normal project can compensate for the invested capital of two projects ($V(d_N) > 2F$), the project that ends second has a shorter duration than the optimal. The difference between two cases is that: in the former case, the VC returns some invested capital at each exit date; while in the latter case, the VC does not return any invested capital at the second exit date.

The following proposition characterizes the investment durations under different types of contracts when there are two projects under consideration, given project $A$ stays normal.

**Proposition 23.** *If there are two investment projects under consideration, given that project A stays normal, the first-best investment durations can be attained under the Escrow contract given a certain level of the carried interest, the first-best investment durations cannot be attained under the Payback or the Return First contracts.*

*Proof.* See Appendix. □

The previous proposition has significant implications regarding the distortions in investment durations. At first glance, the Return First contract appears to be fair in the sense that no interest-free loan between the two parties exists once the invested capital is returned to the LPs. We recall that, if there is only one investment project and the revenue of a normal project can compensate for the invested capital ($V(d) > F$), under the Return First contract, the VC returns the invested capital and receives his carry at the exit date. The first-best duration can be attained by setting a certain level of the carried interest. However, this contract fails to attain the first-best durations in the case where the VC does not return the invested capital to the LPs at some exit date. Under the Escrow contract, the VC returns the invested capital to the LPs at each exit date, the optimal duration can be attained given a certain level of the carried interest even in the presence of the interest-free loans from the VC to the LPs. Under the Payback contract, the VC does not return the invested capital to the LPs at any exit date, the optimal duration cannot be attained in the presence of the interest-free loans from the LPs to the VC. These findings indicate that, regarding the investment durations, using a certain level of the carried interest can overcome the distortions induced by the interest-free loan, but not the distortion induced by the fact that the VC does not return the invested capital at exit date.

[8] In Scenario 1, the lowest level of carried interest to attain the first-best duration for the project that ends second (project $B$) is $\underline{\alpha_B^R} = \frac{(V'(d_b^*)-\lambda_F V(d_b^*)-rV(d_b^*))}{V'(d_b^R)-(r+\lambda_F)(V(d_b^R)+V(d_a^R)-2F)}$; in Scenario 2, the lowest level of carried interest to attain the first-best duration for the project that ends second (project $A$) is $\underline{\alpha_A^R} = \frac{V'(d_a^*)-rV(d_a^*)}{V'(d_a^R)-r(V(d_b^R)+V(d_a^R)-2F)}$. Therefore, the LPs can choose the higher one between $\underline{\alpha_A^R}$ and $\underline{\alpha_B^R}$ so that the first-best duration of project that ends second can be attained.

In Litvak (2009)'s sample, the Escrow contract is the least popular contract[9].

#### 3.4.2.2 Starting decisions

##### 3.4.2.2.1 Scenario 1

If project $A$ ends first, the first derivative of the VC's expected payoff with respect to $a$ is:

$$\frac{\partial E(\pi(d_a, d_b, a, b))}{\partial a} = -e^{\beta r(T-a-d_a)}\beta r\alpha(V(d_a) - \gamma_a F) - e^{r(T-a-d_a)}3\delta + e^{r(T-a)}\delta.$$

The VC's starting decisions of project $A$ under all the three contracts, given that project $A$ stays normal and ends first, are summarized in the following table.

Table 3.1: Starting decision of project $A$

| Scenario 1 | Starting as early as possible $\frac{\partial E(\pi(d_a,d_b,a,b))}{\partial a} < 0$ | Starting as late as possible $\frac{\partial E(\pi(d_a,d_b,a,b))}{\partial a} > 0$ |
|---|---|---|
| Escrow | $3\delta > e^{rd_a^E}\delta$ | $3\delta < e^{rd_a^E}\delta$ |
| Payback | $r\alpha V(d_a^P) + 3\delta > e^{rd_a^P}\delta$ | $r\alpha V(d_a^P) + 3\delta < e^{rd_a^P}\delta$ |
| RF($F < V(d_N) < 2F$) | $3\delta > e^{rd_a^R}\delta$ | $3\delta < e^{rd_a^R}\delta$ |
| RF($V(d_N) > 2F$) | $r\alpha(V(d_a^R) - 2F) + 3\delta > e^{rd_a^R}\delta$ | $r\alpha(V(d_a^R) - 2F) + 3\delta < e^{rd_a^R}\delta$ |

Under the Escrow contract and the Return First contract ($F < V(d_N) < 2F$), the VC makes the starting decision of project $A$ by comparing the increase on the marginal monitoring costs with the marginal opportunity costs of monitoring. The increase on the marginal monitoring costs is caused by the overlapping of two active projects. If the increase on the marginal monitoring costs is larger, the VC will start project $A$ as early as possible to shorten the overlapping period. Under the Payback contract and the Return First contract ($V(d_N) > 2F$), the VC compares his marginal costs of staying in the project with the marginal opportunity costs of monitoring. If his marginal costs of staying in project $A$ are larger, the VC starts project $A$ as early as possible, i.e., at date 0.

For the purpose of presentation, we only present the starting decisions of project $B$ under different types of contracts. Under the Escrow contract, the first derivative of the VC's

[9]There are one fund using the Escrow contract for cash distributions and one fund using it for securities distributions out of 68 venture funds (See Litvak (2009)).

expected payoff with respect to $b$ is:

$$\frac{\partial E(\pi(d_a^E, d_b^E, a, b))}{\partial b} = \frac{e^{r(T-b-d_b^E)-d_b^E\lambda_F}}{r+\lambda_F}\delta((-1+3e^{(r+\lambda_F)d_b^E})r + 2e^{(r+\lambda_F)d_b^E}\lambda_F) > 0.$$

So, under the Escrow contract, the VC will start project $B$ late in Scenario 1. Given project $B$ ends second, the VC wants to postpone its starting date to shorten the period of the second interest-free loan to the LPs. Therefore, the optimal starting date of project $B$ cannot be attained under the Escrow contract.

Recall that the Payback contract implies that $\beta = 1$ and $\gamma_b = 0$, the Return First contract in the case $V(d_N) > 2F$ also implies $\beta = 1$ and $\gamma_b = 0$. So under the Payback contract and the Return First contract ($V(d_N) > 2F$), the VC will start project $B$ as early as possible if $r((r+\lambda_F)\alpha V(d_b) + \delta) > e^{(r+\lambda_F)d_b}(r\delta + (r+\lambda_F)2\delta)$, and will delay project $B$ if $r((r+\lambda_F)\alpha V(d_b) + \delta) < e^{(r+\lambda_F)d_b}(r\delta + (r+\lambda_F)2\delta)$.

Under the Return First contract ($F < V(d_N) < 2F$), the VC will start project $B$ as early as possible if $r((r+\lambda_F)\alpha(V(d_b^R) + V(d_a^R) - 2F) + \delta) > e^{(r+\lambda_F)d_b^R}(r\delta + (r+\lambda_F)2\delta)$, and will delay project $B$ if $r((r+\lambda_F)\alpha(V(d_b^R) + V(d_a^R) - 2F) + \delta) < e^{(r+\lambda_F)d_b^R}(r\delta + (r+\lambda_F)2\delta)$.

#### 3.4.2.2.2 Scenario 2

If project $B$ ends first, the first derivative of the VC's expected payoff with respect to $a$ is:

$$\frac{\partial E(\pi(d_a, d_b, a, b))}{\partial a} = -e^{\beta r(T-a-d_a)}\beta r\alpha(V(d_a) - \gamma_a F) - e^{r(T-a-d_a)}\delta + e^{r(T-a)}\delta.$$

The Escrow contract implies that $\beta = 0$ and $\gamma_a = 1$. Under the Escrow contract, the first derivative of the VC's expected payoff with respect to $a$ is:

$$\frac{\partial E(\pi(d_a^E, d_b^E, a, b))}{\partial a} = e^{r(T-a-d_a^E)}(-1 + e^{rd_a^E})\delta > 0.$$

In this scenario, under the Escrow contract, the VC will delay project $A$ to shorten the period of the second interest-free loan.

The VC's starting decisions of project $A$ under the Payback and the Return First contracts, given that project $B$ ends first, are summarized in the following table.

Table 3.2: Starting decision of project $A$

| Scenario 2 | Starting as early as possible $\frac{\partial E(\pi(d_a,d_b,a,b))}{\partial a} < 0$ | Starting as late as possible $\frac{\partial E(\pi(d_a,d_b,a,b))}{\partial a} > 0$ |
|---|---|---|
| Payback | $r\alpha V(d_a^P) + \delta > e^{rd_a^P}\delta$ | $r\alpha V(d_a^P) + \delta < e^{rd_a^P}\delta$ |
| RF($F < V(d_N) < 2F$) | $r\alpha(V(d_a^R) + V(d_b^R) - 2F) + \delta > e^{rd_a^R}\delta$ | $r\alpha(V(d_a^R) + V(d_b^R) - 2F) + \delta < e^{rd_a^R}\delta$ |
| RF($V(d_N) > 2F$) | $r\alpha V(d_a^R) + \delta > e^{rd_a^R}\delta$ | $r\alpha V(d_a^R) + \delta < e^{rd_a^R}\delta$ |

The VC compares his marginal costs of staying in the project with the marginal opportunity costs of monitoring. If his marginal costs of staying in project $A$ are larger, the VC starts project $A$ as early as possible, i.e., at date 0.

Under the Payback contract, the VC will start project $B$ as soon as possible when $\alpha V(d_b^P)(r+\lambda_F)+3\delta > e^{d_b^P(r+\lambda_F)}3\delta$; and will delay project $B$ when $\alpha V(d_b^P)(r+\lambda_F)+3\delta < e^{d_b^P(r+\lambda_F)}3\delta$.

Under the Escrow contract, the first derivative of the VC's expected payoff with respect to $b$ is:

$$\frac{\partial E(\pi(d_a^E, d_b^E, a, b))}{\partial b} = \frac{e^{r(T-b-d_b^E)-d_b^E\lambda_F}(-1 + e^{d_b^E(r+\lambda_F)})r3\delta}{r + \lambda_F} > 0.$$

So the VC will delay project $B$ under the Escrow contract to shorten the period of the first interest-free loan.

In the case where the revenue of a normal project cannot compensate for the invested capital of two project ($F < V(d_N) < 2F$), under the Return First contracts, the first derivative of the VC's expected payoff with respect to $b$ is:

$$\frac{\partial E(\pi(d_a^R, d_b^R, a, b))}{\partial b} = \frac{e^{r(T-b-d_b^R)-d_b^R\lambda_F}(-1 + e^{d_b^R(r+\lambda_F)})r3\delta}{r + \lambda_F} > 0.$$

In this case, at the first exit date, the VC receives nothing under the Return First contract. So he will delay project $B$ to shorten the period of the interest-free loan to the LPs. Therefore, under the Return First contract, the first-best starting date of both projects cannot be achieved.

In the case where the revenue of a normal project cannot compensate for the invested capital of two project ($V(d_N) > 2F$), the VC will start project $B$ as soon as possible when $\alpha(V(d_b^R)-2F)(r+\lambda_F)+3\delta > e^{d_b^R(r+\lambda_F)}3\delta$; and will delay project $B$ when $\alpha(V(d_b^R)-2F)(r+\lambda_F)+3\delta < e^{d_b^R(r+\lambda_F)}3\delta$.

The following proposition summarizes the VC's starting decisions of both projects given project $A$ stays normal.

**Proposition 24.** *If there are two investment projects under consideration and given that project A stays normal, the first-best starting dates of both projects are possible to attain only under the Payback contract. Under the Payback contract, the VC will starts both projects as early as possible if the VC's marginal costs of staying in the projects are larger than the marginal opportunity costs of monitoring.*

## 3.5 Conclusion

In this paper, we study the impact of distribution rules on the VC's investment and divestment decisions, hence, the investment durations. We show that, when there is only one investment project under consideration, under the Return First contract, the LPs can choose a certain level of the carried interest for the VC to attain the first-best investment duration, and the VC wants to start the project as early as possible, which coincide with the first-best starting date because there is no interest-free loan between two parties. However, when there are two projects under consideration and project $A$ stays normal , there is an interest-free loan between two parties under all three types of contracts. The Escrow contract can restore the first-best durations given a certain level of the carry but cannot attain the first-best starting dates. These results imply that, regarding the investment durations, using a certain level of the carry can overcome the distortions induced by the interest-free loan from the VC to the LPs, but not the distortion induced by the fact that the VC does not return the invested capital at the exit date. To the best of our knowledge, this paper is the first attempt to analyze the impact of the distribution rules on the VC's investment decisions.

We intend to provide some theoretical explanation to the question why the Return First contract is more commonly used than others. We show that the first-best duration and starting date can be attained under the Return First contract if there is only one investment project under consideration. However, if there are two investment projects under consideration and given that project $A$ stays normal, neither the first-best durations nor the first-best starting dates can be attained under the Return First contract. The Escrow contract, under which the first-best durations of projects can be attained, is the least used distribution contract in Litvak (2009)'s sample. We believe this is because the price of venture capital services is influenced by the the supply and demand of venture capitalist (Gompers and Lerner, 1996).

The main focus of this paper is the VC's incentives on the timing of investment decisions. For future research, it will be interesting to move our focus from the VC's incentives on timing to the incentives on monitoring projects. The VC monitors the investments and provides managerial consulting to the portfolio enterprises, this feature is highlighted by Brander et al. (2002) and Casamatta (2003). It would be interesting if further studies could capture the effect of monitoring on the failing rate of projects or on the project revenue. This idea is left for being explored soon.

# Appendix

**Proof of Proposition 18**
The expected profits of the investment project is

$$\begin{aligned}
E(\Pi(d_N,a)) &= e^{-\lambda_F d_N}(e^{r(T-d_N-a)}V(d_N) - \int_a^{a+d_N} \delta e^{r(T-\tau)}d\tau) \\
&+ \int_0^{d_N} \lambda_F e^{-\lambda_F t}(-\int_a^{a+t} \delta e^{r(T-\tau)}d\tau)dt - e^{r(T-a)}F \\
&= e^{-\lambda_F d_N + r(T-a-d_N)}V(d_N) - e^{-\lambda_F d_N}(e^{r(T-a)} - e^{r(T-a-d_N)})\frac{\delta}{r} \\
&- \frac{\delta}{r}(\frac{e^{r(T-a-d_N)-d_N\lambda_F}(e^{rd_N(r+\lambda_F)} + \lambda_F - e^{rd_N}(r+\lambda_F))}{r+\lambda_F}) - e^{r(T-a)}F \\
&= e^{-\lambda_F d_N + r(T-a-d_N)}V(d_N) + e^{-\lambda_F d_N + r(T-a-d_N)}\frac{\delta}{r} \\
&- \frac{\delta}{r}(\frac{e^{r(T-a-d_N)-d_N\lambda_F}(e^{rd_N(r+\lambda_F)} + \lambda_F)}{r+\lambda_F}) - e^{r(T-a)}F \\
&= e^{r(-a)}E(\Pi(d_N)).
\end{aligned}$$

The derivative with respect to $a$ is

$$\frac{\partial E(\Pi(d_N,a))}{\partial a} = -re^{r(-a)}E(\Pi(d_N)) < 0.$$

Therefore, the investor would like to start the project as early as possible, at date 0.

**Proof of Proposition 20**
In Scenario 1, the first derivative of the investor's expected profits with respect to starting date $a$ is

$$\frac{\partial E(\Pi(d_a,d_b,a,b))}{\partial a} = e^{r(T-a-d_a)}(-rV(d_a) - 3\delta + e^{rd_a}(rF+\delta)).$$

Notice that $rV(d_a)+3\delta$ is the marginal costs of staying in project $A$. Project $A$ is the one that ends first, it bears larger marginal monitoring costs due to the existence of an active project $B$. $e^{rd_a}(rF+\delta)$ is the opportunity costs of investing in the project. The opportunity costs of investing take into account both the invested capital and the monitoring costs, which are the monitoring costs of one single project. Given that project $A$ stays normal, Assumption A9 suggests $\delta + rV(d_a) > e^{rd_a}(rF+\delta) + 3\delta)$, so $\frac{\partial E(\Pi(d_a,d_b,a,b))}{\partial a} < 0$ in this scenario.

The first derivative with respect to $b$ is

$$\frac{\partial E(\Pi(d_a, d_b, a, b))}{\partial b} = \frac{e^{r(T-b-d_b)-d_b\lambda_F}(-r(\delta + V(d_b)(r+\lambda_F)) + e^{d_b(r+\lambda_F)}(rF(r+\lambda_F) + \delta(3r+2\lambda_F)))}{r+\lambda_F}. \tag{3.7}$$

Similarly, $\delta+V(d_b)(r+\lambda_F)$ is the marginal costs for the investor to stay in project $B$. Notice that the investor takes into account the possibility that project $B$ can fail later. The investor's opportunity costs of investing consist of two parts: the first part $e^{d_b(r+\lambda_F)}(F(r+\lambda_F)+\delta)$ is the opportunity costs of investing in project $B$, the second part $e^{d_b(r+\lambda_F)}2\frac{\delta}{r}(r+\lambda_F)$ is the opportunity costs of the increase on project $A$'s marginal monitoring costs caused by an active project $B$. By Assumption A9, $\frac{\partial E(\Pi(d_a,d_b,a,b))}{\partial b} < 0$ in this scenario.

In Scenario 2, the first derivative for starting date $a$ is

$$\frac{\partial E(\Pi(d_a, d_b, a, b))}{\partial a} = e^{r(T-a-d_a)}(-rV(d_a) - \delta + e^{rd_a}(rF+\delta)),$$

Again, $rV(d_a)+\delta$ is the marginal costs of staying the project. In this scenario, Project $A$ is the one that ends second, the fact that project $B$ has already ended does not affect project $A$'s marginal monitoring costs. $e^{rd_a}(rF+\delta)$ is the opportunity costs of investing in the project. Given that project $A$ stays normal, Assumption A9 suggests $\delta+rV(d_a) > e^{rd_a}(rF+\delta)+3\delta)$, so $\frac{\partial E(\Pi(d_a,d_b,a,b))}{\partial a} < 0$ in this scenario.

The first derivative with respect to starting date $b$ is

$$\frac{\partial E(\Pi(d_a, d_b, a, b))}{\partial b} = \frac{e^{r(T-b-d_b)-d_b\lambda_F}r(-3\delta - V(d_b)(r+\lambda_F) + e^{d_b(r+\lambda_F)}(F(r+\lambda_F)+3\delta))}{r+\lambda_F}. \tag{3.8}$$

For project $B$, $3\delta+V(d_b)(r+\lambda_F)$ is the marginal costs for the investor to stay in the project, and $e^{d_b(r+\lambda_F)}(F(r+\lambda_F)+3\delta)r+\lambda_F$ is the opportunity costs of investing. By Assumption A9, $\frac{\partial E(\Pi(d_a,d_b,a,b))}{\partial b} < 0$ in this scenario.

Therefore, the investor wants to start both projects as early as possible.

**Proof of Proposition 21**

The optimal duration of the project is determined by the exit condition

$$rV(d_N^*) + \delta = V'(d_N^*) - \lambda_F V(d_N^*) \Longleftrightarrow \delta = V'(d_N^*) - \lambda_F V(d_N^*) - rV(d_N^*).$$

Under the Escrow contract, the duration of the project is determined by the following exit condition

$$\delta e^{r(T-d_N^E-a)} = \alpha^E V'(d_N^E) - \lambda_F \alpha^E (V(d_N^E) - F) \Longleftrightarrow \delta = \alpha^E \frac{V'(d_N^E) - \lambda_F(V(d_N^E) - F)}{e^{r(T-d_N^E-a)}}.$$

Therefore,

$$V'(d_N^*) - \lambda_F V(d_N^*) - rV(d_N^*) = \alpha^E \frac{V'(d_N^E) - \lambda_F(V(d_N^E) - F)}{e^{r(T-d_N^E-a)}}.$$

Therefore,

$$\underline{\alpha}^E \equiv \frac{e^{r(T-d_N^E-a)}(V'(d_N^*) - \lambda_F V(d_N^*) - rV(d_N^*))}{V'(d_N^E) - \lambda_F(V(d_N^E) - F)}.$$

Under the Return First contract, the duration of the project is determined by the following exit condition

$$r\alpha^R(V(d_N^R)-F)+\delta = \alpha^R V'(d_N^R)-\lambda_F\alpha^R(V(d_N^R)-F) \Longleftrightarrow \delta = \alpha^R V'(d_N^R)-(r+\lambda_F)\alpha^R(V(d_N^R)-F).$$

Therefore,

$$V'(d_N^*) - \lambda_F V(d_N^*) - rV(d_N^*) = \alpha^R(V'(d_N^R) - (r + \lambda_F)(V(d_N^R) - F)).$$

Therefore,

$$\underline{\alpha}^R \equiv \frac{V'(d_N^*) - \lambda_F V(d_N^*) - rV(d_N^*)}{V'(d_N^R) - (r + \lambda_F)(V(d_N^R) - F)}.$$

**Proof of Corollary 7**
Under the Return First contract, the duration of the project is determined by the following exit condition

$$r\alpha^R(V(d_N^R)-F)+\delta = \alpha^R V'(d_N^R)-\lambda_F\alpha^R(V(d_N^R)-F) \Longleftrightarrow \delta = \alpha^R(V'(d_N^R)-(r+\lambda_F)(V(d_N^R)-F)).$$

When $\alpha = \alpha^R$, the optimal duration is obtained, $\delta = \underline{\alpha}^R(V'(d_N^R) - (r + \lambda_F)(V(d_N^R) - F))$. When $\alpha < \alpha^R$, the marginal monitoring costs becomes relatively larger, so the duration of the project becomes shorter.

**Proof of Proposition 22**
The VC intends maximizes his expected payoff, which reads

$$E(\pi(d_N, a)) = e^{-\lambda_F d_N}(e^{\beta r(T-d_N-a)}\alpha(V(d_N) - \gamma F) - \int_a^{a+d_N} \delta e^{r(T-\tau)} d\tau)$$
$$+ \int_0^{d_N} \lambda_F e^{-\lambda_F t}(- \int_a^{a+t} \delta e^{r(T-\tau)} d\tau) dt - \alpha F(1 - \gamma),$$

where $(\beta, \gamma)$ take different sets of values under different value-of-distribution rules.

$$
\begin{aligned}
E(\Pi(d_N, a)) &= e^{-\lambda_F d_N}(e^{\beta r(T-d_N-a)}\alpha(V(d_N)-\gamma F) - \int_a^{a+d_N}\delta e^{r(T-\tau)}d\tau) \\
&+ \int_0^{d_N}\lambda_F e^{-\lambda_F t}(-\int_a^{a+t}\delta e^{r(T-\tau)}d\tau)dt - \alpha F(1-\gamma) \\
&= e^{-\lambda_F d_N+\beta r(T-a-d_N)}\alpha(V(d_N)-\gamma F) - e^{-\lambda_F d_N}(e^{r(T-a)} - e^{r(T-a-d_N)})\frac{\delta}{r} \\
&- \frac{\delta}{r}(\frac{e^{r(T-a-d_N)-d_N\lambda_F}(e^{d_N(r+\lambda_F)}r + \lambda_F - e^{rd_N}(r+\lambda_F))}{r+\lambda_F}) - \alpha F(1-\gamma).
\end{aligned}
$$

The derivative with respect to $a$ is

$$
\begin{aligned}
\frac{\partial E(\Pi(d_N, a))}{\partial a} &= e^{-\lambda_F d_N+\beta r(T-a-d_N)}(-\beta r)\alpha(V(d_N)-\gamma F) + \delta e^{-\lambda_F d_N+r(T-a-d_N)}(-1+e^{dr}) \\
&+ \delta(\frac{e^{r(T-a-d_N)-d_N\lambda_F}(e^{d_N(r+\lambda_F)}r + \lambda_F - e^{rd_N}(r+\lambda_F))}{r+\lambda_F}) \\
&= e^{-\lambda_F d_N+\beta r(T-a-d_N)}(-\beta r)\alpha(V(d_N)-\gamma F) - \delta e^{-\lambda_F d_N+r(T-a-d_N)} \\
&+ \delta\frac{e^{r(T-a)}r + e^{r(T-d_N-a)-d_N\lambda_F}\lambda_F}{r+\lambda_F}.
\end{aligned}
$$

Under the Escrow contract, $(\beta, \gamma)$ take the set of value $(0, 1)$, so the derivative with respect to $a$ is

$$
\frac{\partial E(\Pi(d_N^E, a))}{\partial a} = -\delta e^{-\lambda_F d_N^E+r(T-d_N^E-a)} + \delta\frac{e^{r(T-a)}r + e^{r(T-d_N^E-a)-d_N^E\lambda_F}\lambda_F}{r+\lambda_F} = \frac{(-e^{-rd_N^E-d_N^E\lambda_F}+1)\delta e^{r(T-a)}}{r+\lambda_F} > 0
$$

Therefore, under the Escrow contract, the VC would like to start the project as late as possible.

Under the Payback contract, $(\beta, \gamma)$ take the set of value $(1, 0)$, so the derivative with respect to $a$ is

$$
\frac{\partial E(\Pi(d_N^P, a))}{\partial a} = e^{-\lambda_F d_N^P+r(T-a-d_N^P)}(-r)\alpha V(d_N^P) - \delta e^{-\lambda_F d_N^P+r(T-a-d_N^P)} + \delta\frac{e^{r(T-a)}r + e^{r(T-d_N^P-a)-d_N^P\lambda_F}\lambda_F}{r+\lambda_F}.
$$

Notice that VC's expected payoff under the Payback contract is

$$
\begin{aligned}
E(\pi(d_N^P, a)) &= e^{-\lambda_F d_N^P}(e^{r(T-d_N^P-a)}\alpha V(d_N^P) - \int_a^{a+d_N^P} \delta e^{r(T-\tau)} d\tau) \\
&+ \int_0^{d_N^P} \lambda_F e^{-\lambda_F t}(-\int_a^{a+t} \delta e^{r(T-\tau)} d\tau) dt - \alpha F \\
&= e^{-\lambda_F d_N^P + r(T-a-d_N^P)}\alpha V(d_N^P) + e^{-\lambda_F d_N^P + r(T-a-d_N^P)}\frac{\delta}{r} \\
&- \frac{\delta}{r}(\frac{e^{r(T-a-d_N^P)-d_N^P\lambda_F}(e^{d_N^P(r+\lambda_F)}r + \lambda_F)}{r+\lambda_F}) - \alpha F.
\end{aligned}
$$

$$
\frac{\partial E(\Pi(d_N^P, a))}{\partial a} = -rE[\pi(d_N^P, a) + \alpha F] < 0.
$$

Therfore, under the Payback contract, the VC wants to start the project as early as possible.

Under the Return First contract, $(\beta, \gamma)$ take the set of value $(1, 1)$, so the derivative with respect to $a$ is

$$
\begin{aligned}
\frac{\partial E(\Pi(d_N^R, a))}{\partial a} &= e^{-\lambda_F d_N^R + r(T-a-d_N^R)}(-r)\alpha(V(d_N^R) - F) - \delta e^{-\lambda_F d_N^R + r(T-a-d_N^R)} \\
&+ \delta \frac{e^{r(T-a)}r + e^{r(T-d_N^R-a)-d_N^R\lambda_F}\lambda_F}{r+\lambda_F}.
\end{aligned}
$$

Recall that VC's expected payoff under the Return First contract is

$$
\begin{aligned}
E(\pi(d_N^R, a)) &= e^{-\lambda_F d_N^R}(e^{r(T-d_N^R-a)}\alpha(V(d_N^R) - F) - \int_a^{a+d_N^R} \delta e^{r(T-\tau)} d\tau) \\
&+ \int_0^{d_N^R} \lambda_F e^{-\lambda_F t}(-\int_a^{a+t} \delta e^{r(T-\tau)} d\tau) dt, \\
&= e^{-\lambda_F d_N^R + r(T-a-d_N^R)}\alpha(V(d_N^R) - F) + e^{-\lambda_F d_N^R + r(T-a-d_N^R)}\frac{\delta}{r} \\
&- \frac{\delta}{r}(\frac{e^{r(T-a-d_N^R)-d_N^R\lambda_F}(e^{d_N^R(r+\lambda_F)}r + \lambda_F)}{r+\lambda_F}).
\end{aligned}
$$

$$
\frac{\partial E(\Pi(d_N^R, a))}{\partial a} = -rE[\pi(d_N^R, a)] < 0.
$$

Therfore, under the Return First contract, the VC wants to start the project as early as possible.

**Exit conditions for two projects under different contracts**
In Scenario 1, given that project $A$ remains normal, the VC's expected payoff function is:

$$\begin{aligned}E(\Pi(d_a,d_b,a,b)) &= e^{-\lambda_F d_b}(e^{\beta r(T-d_a-a)}\alpha(V(d_a)-\gamma_a F)+e^{\beta r(T-d_b-b)}\alpha(V(d_b)-\gamma_b F)\\ &-\delta(\int_a^b e^{r(T-\tau)}d\tau+\int_b^{a+d_a}4e^{r(T-\tau)}d\tau+\int_{a+d_a}^{b+d_b}e^{r(T-\tau)}d\tau))\\ &+\int_0^{d_b}\lambda_F e^{-\lambda_F t}(e^{\beta r(T-d_a-a)}\alpha(V(d_a)-\gamma_a F)\\ &-\delta(\int_a^b e^{r(T-\tau)}d\tau+\int_b^{a+d_a}4e^{r(T-\tau)}d\tau+\int_{a+d_a}^{b+t}e^{r(T-\tau)}d\tau))dt\\ &-\alpha F(1-\gamma_a)-\alpha F(1-\gamma_b).\end{aligned}$$

The VC's exit strategy of project $A$ (respectively, $B$) is determined by the first order condition $\frac{\partial E(\Pi(d_a,d_b,a,b))}{\partial d_a}$ (respectively, $\frac{\partial E(\Pi(d_a,d_b,a,b))}{\partial d_b}$).

In Scenario 1, the following exit condition determines the duration of project $A$, which stays normal and ends first,

$$e^{\beta r(T-d_a-a)}\beta r\alpha(V(d_a)-\gamma_a F)+3\delta e^{r(T-d_a-a)}=e^{\beta r(T-d_a-a)}\alpha V'(d_a),$$

and the following exit condition determines the VC's exit strategy of project $B$:

$$e^{\beta r(T-d_b-b)}\beta r\alpha(V(d_b)-\gamma_b F)+\delta e^{r(T-d_b-b)}=e^{\beta r(T-d_b-b)}\alpha V'(d_b)-e^{\beta r(T-d_b-b)}\lambda_F\alpha(V(d_b)-\gamma_b F).$$

In Scenario 2, given that project $A$ remains normal, the VC's expected payoff function is:

$$\begin{aligned}E(\Pi(d_a,d_b,a,b)) &= e^{-\lambda_F d_b}(e^{\beta r(T-d_a-a)}\alpha(V(d_a)-\gamma_a F)+e^{\beta r(T-d_b-b)}\alpha(V(d_b)-\gamma_b F)\\ &-\delta(\int_a^b e^{r(T-\tau)}d\tau+\int_b^{b+d_b}4e^{r(T-\tau)}d\tau+\int_{b+d_b}^{a+d_a}e^{r(T-\tau)}d\tau))\\ &+\int_0^{d_b}\lambda_F e^{-\lambda_F t}(e^{\beta r(T-d_a-a)}\alpha(V(d_a)-\gamma_a F)\\ &-\delta(\int_a^b e^{r(T-\tau)}d\tau+\int_b^{b+t}4e^{r(T-\tau)}d\tau+\int_{b+t}^{a+d_a}e^{r(T-\tau)}d\tau))dt\\ &-\alpha F(1-\gamma_a)-\alpha F(1-\gamma_b).\end{aligned}$$

So, the following two exit conditions determine the duration of project $A$ and $B$, respectively:

$$e^{(\beta r-r)(T-d_a-a)}\beta r\alpha(V(d_a)-\gamma_a F)+\delta=e^{(\beta r-r)(T-d_a-a)}\alpha V'(d_a),$$

and

$$\beta r\alpha(V(d_b)-\gamma_b F)+3\delta e^{(r-\beta r)(T-d_b-b)}=\alpha V'(d_b)-\lambda_F\alpha(V(d_b)-\gamma_b F).$$

**Proof of Proposition 23**

In Scenario 1, the first-best duration of project $A$ is determined by the following exit condition:

$$rV(d_a^*)+3\delta=V'(d_a^*).$$

So, we obtain $\delta=\frac{V'(d_a^*)-rV(d_a^*)}{3}$.

While the exit condition for project $A$ under the Escrow contract in the same scenario is:

$$3\delta e^{r(T-d_a^E-a)}=\alpha V'(d_a^E).$$

We obtain $\delta=\frac{\alpha V'(d_a^E)}{3e^{r(T-d_a^E-a)}}$. From $\frac{V'(d_a^*)-rV(d_a^*)}{3}=\frac{\alpha V'(d_a^E)}{3e^{r(T-d_a^E-a)}}$, we find the lowest level of carried interest which can attain the first-best duration of project $A$ in Scenario 1:

$$\underline{\alpha_A^E}=\frac{e^{r(T-a-d_a^E)}(V'(d_a^*)-rV(d_a^*))}{V'(d_a^E)}.$$

In Scenario 1, the first-best duration of project $B$ is determined by the following exit condition:

$$rV(d_b^*)+\delta=V'(d_b^*)-\lambda_F V(d_b^*),$$

we obtain $\delta=V'(d_b^*)-(\lambda_F+r)V(d_b^*)$.

While the exit condition for project $B$ under the Escrow contract in the same scenario is:

$$\delta e^{r(T-d_b^E-b)}=\alpha V'(d_b^E)-\lambda_F\alpha(V(d_b^E)-F),$$

we obtain $\delta=\frac{\alpha V'(d_b^E)-\lambda_F\alpha(V(d_b^E)-F)}{e^{r(T-d_b^E-b)}}$. From $V'(d_b^*)-(\lambda_F+r)V(d_b^*)=\frac{\alpha V'(d_b^E)-\lambda_F\alpha(V(d_b^E)-F)}{e^{r(T-d_b^E-b)}}$, we find the lowest level of carried interest which can attain the first-best duration of project $B$ in Scenario 1:

$$\underline{\alpha_B^E}=\frac{e^{r(T-b-d_b^E)}(V'(d_b^*)-\lambda_F V(d_b^*)-rV(d_b^*))}{V'(d_b^E)-\lambda_F(V(d_b^E)-F)}.$$

In Scenario 2, we also find the levels of carried interest that can attain the first-best durations for project $A$ and $B$ are the same as those in Scenario 1. Because the carried interest is a flat percentage of investment profits, the LPs can choose the higher one between $\underline{\alpha_A^E}$ and $\underline{\alpha_B^E}$ so that the first-best durations of two projects can be attained.

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