RELIABILITY MODELING AND ANALYSIS OF WIND TURBINE SYSTEMS AND WIND FARMS IN BULK POWER SYSTEMS

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RELIABILITY MODELING AND ANALYSIS OF WIND TURBINE SYSTEMS AND WIND FARMS IN BULK POWER SYSTEMS

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To my beloved parents Dr. Jun Zhao, Dr. Ying Sun, my wife Dr. Bin Liu, and my upcoming son

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SUMMARY

The global trend towards sustainability has called for more integration of renewable energy sources into power systems, among which wind energy takes significant proportion. With the increasing penetration of wind energy, the reliable and economical operation of the bulk power systems with wind farms has become a challenge due to the intermittency of wind energy. This challenge has pushed system planners and operators to seek for methods to analyze the reliability of wind farms specifically from the generation output perspective. This dissertation aims at presenting reliability analysis methods for wind turbine systems and wind farms. Specifically, two main problems are addressed: a) Reliability modeling of wind turbine systems; b) Reliability analysis of wind farms. Reliability analysis of wind farms is based upon the reliability modeling of wind turbine systems. In both of the problems addressed, state-space-based probabilistic models are presented. A specific case study is presented each for the reliability model of wind turbine systems and wind farms.

The reliability model of the wind turbine system and the reliability model of the wind farm presented in this dissertation bring contribution to the planning and operation of bulk power systems with wind farm integration. The developed models can provide the system operator with clear reliability indices in terms of generation states of wind turbine systems and wind farms along with their probability, duration and frequency. These reliability analysis results serve as essential considerations of generation output in bulk power systems with large penetration of fluctuating wind power. The system planners and operators are thereafter able to take the wind farm generation output into account

when performing adequacy assessment for system level reliability analysis, and can compute system reliability metrics when given the load and traditional generation profiles.

Conclusions and the major contributions of this research are presented at the end. In addition future research directions are discussed to address the greater issues associated with wind energy.

CHAPTER 1 Introduction and Objectives of Research

1.1 Introduction

Building smart electric grids has become a global trend. The incentives that serve as the motivation of this trend are as follows: a) the need to mitigate unnecessary production of pollutants; b) the requirement of improving existing power grids towards a more flexible, economical and reliable network; c) the call for more integration of various types of sustainable and renewable energy. The concerns of these incentives have led to the presented research in this dissertation.

Wind power is the most developed type of renewable energy that is being integrated to a large proportion in the power systems. The annual report from World Wind Energy Association [1] has provided that in year 2012 the total worldwide wind energy capacity have increased to 282 GW, which has met more than 3% of the world's electricity demand. Some countries such as Denmark have their energy supply substantially coming from wind. The large integration of wind energy has called for a secure operation of the bulk power system with wind penetration. Reliability problems are the essential concerns in this process. Therefore in this presented research, the reliability modeling and analysis of wind turbine systems and wind farms are presented.

1.2 Problem Statement

To better address the intermittency of wind energy from individual wind turbine systems and wind farms, the reliability analysis in regard of the generation output of wind turbine systems and wind farms are of essential need. The system operators and planners will require the clear picture of what a wind farm or individual wind turbine systems would generate in different scenarios, and how these generation outputs transition to each other in a probabilistic manner.

This research provides the reliability model and analysis of wind turbine systems and wind farms with the result of generation output. The reliability modeling and analysis follow the state-space-based probabilistic manner using the Markov models, and come up with the generation states and transitions with their probability, duration and transitions.

CHAPTER 2 Review of Literature and Previous Work

Integration of wind power into the power grid has been going on for a couple of decades and there is a plethora of literature addressing various aspects of the technology. Here we focus on reliability modeling and probabilistic analysis of wind farms.

There has been some peer research about reliability modeling of wind turbine systems and wind farms. Most of the models proposed in literature present the wind turbine systems as stable-output elements, and the reliability modeling comes primarily in the sense of wind speed modeling and regression such as in [2] – [11]. This has motivated the research of component-based reliability analysis of wind turbine system in this presented research. In addition, the reliability modeling of wind turbine systems is mostly involved in the process of reliability analysis of wind farms in peer research such as in [2] – [14], which has mixed and hidden the uncertainty caused by specific wind turbine system.

Methods proposed to deal with wind farm reliability analysis and wind turbine system reliability analysis include frequency-based approach in [2], probabilistic approach in [3] – [8], time series method in [9] and integrated approach with load information in [10] – [12]. In these existing methods proposed by peer researchers, wind speed is accounted either as following a regressed probabilistic distribution in [2], [4] –[7], or as following a correlated distribution in respect to load information in [9], [10], [17]. Wind angle is considered in [3] as a changing parameter in its Monte Carlo simulation, but the effects of changed angle would not be clearly presented from the simulation result. Some peer research in [19] – [28] considered wind angle in the simulation sections, but the modeling of wind angle is not shown. Other modeling approaches in [4] – [18] have not been able to take wind angle into account in reliability analysis. These facts have motivated the integration of wind angle in this presented research, where wind angle along with wind speed are extracted from historical wind data into discrete states.

In the modeling of wind turbine systems and wind farms, the geographical unsymmetrical locations of wind turbine systems are also taken into account, which is the wake effect. Wake effect is the phenomena of inter-impact between the wind turbine systems on the generation output of the wind turbine systems. More specifically, wake effect presents the influence of upstream wind turbines on the downstream turbines in the direction of wind. Wake effect has been an important problem in the reliability modeling and analysis of wind farms in some peer research in [3], [17] – [29]. The wind turbines located at downstream of wind are generating less than those at upstream, which results in the imbalance and decrease of wind generation. Several models have been proposed to formulate the wake effect for reliability analysis in [17]-[28]. Jensen model and Lissaman model are the ones used most frequently, while some other models also exist in [22], [23], [26] which consider other factors beyond geographical difference such as the dust effect in some countries. Jensen model is more applicable for flat terrain while Lissaman model is suitable for fluctuating terrain. Wake effect can also be caused by different size of wind turbine systems especially different blade diameters in [21], [26], [28]. Since a wind farm in the United States typically uses identical wind turbines systems within the farm, wake effects caused by dust effect or by difference in blade size are often ignored in research. In this presented research, both Jensen model and Lissaman model are considered, and the integrated formulation of the two is also shown based on primarily Lissaman model for fluctuating terrains. But it is noteworthy that the specific selection of model for wake effect does not influence the use of the reliability analysis method presented in this dissertation. The difference of terrain altitude where wind turbine systems stand is given in the geographical information provided for the wind turbine systems in a wind farm, and then the terrain altitudes are accounted in the generation calculation of these wind turbine systems when considering the wake effect model.

Energy storage elements are commonly installed in modern wind turbine systems for smoothing out the generation of wind generators to a certain extent. Energy storage has been considered in some wind farm reliability modeling approaches [8]-[10], [12]. Most of the impacts of energy storage considered in peer research are from the perspective of voltage and frequency stability, and energy storage elements are mostly modeled as constant-output devices. This existing lack of flexibility has motivated the consideration of energy storage elements in this presented research to treat them as components that follow Markov models. In this research, energy storage elements are modeled as part of the wind turbine systems and they are also subject to failure.

The outputs of the reliability analysis of wind turbine systems and wind farms are mostly the reliability indices in [2]-[14], [18]-[21], [29], [30], such as Expected Generated Wind Energy (EGWE). The indices can come from analytical modeling in [4], [6], [29], [31], or Monte Carlo simulation of continuously changing generation-states considering changing of wind speed in [2], [3], [15], [18]. However, from the point of view of the bulk power system, reliability indices are not enough when considering short-term transmission planning and operation. Transitions and duration of individual generation states of wind turbine systems have been considered in [3] and [7] in the simulation process but they cannot be presented in terms of number values. This is due to the integration of regressed wind speed and also the neglecting of the impact of every individual wind turbine systems in a wind farm. This fact has motivated the involvement of presentable output-state-space along with the transitions among the output states in this presented research. In this research, the generation output of each individual wind turbine system is modeled by the state-space in terms of discrete generation ranges. The analysis of the states in the generation state-space includes the calculation of their probability, duration, and transitions among the generation ranges. Similarly for a wind farm, the output is also in terms of generation ranges and the analysis of probability, duration, and transitions.

To sum up, there has been ongoing peer research that deals with the reliability modeling of wind turbine systems and wind farms, but some problems exist. The major existing deficiencies of the peer research are as follows:

- There is lack of individual modeling of every wind turbine system in terms of generation output in the reliability analysis of a wind farm. Generation outputs of wind turbine systems are considered mostly identical in a wind farm.
- There is lack of specific component analysis of a wind turbine system. Wind turbine systems are mostly treated as whole unique systems with no specific component analysis.
- Wind probabilistic modeling comes mostly from regressed models. Wind angle is seldom considered especially when quantifying wake effect.
- There is lack of probabilistic modeling of energy storage elements. They are mostly considered as constant output elements without failure.
- The reliability analysis results of wind farms are mostly presented in terms of reliability indices such as Expected Generated Wind Energy (EGWE) of an entire farm. The distribution of generation states and the transitions among them are not presented.

The research presented in this dissertation contributes accordingly to the above deficiencies as follows:

- Wind turbine systems in a wind farm have their each specific generation model when doing reliability analysis of the wind farm.
- The reliability modeling of wind turbine systems is component-based, which takes into account the Markov models of all their components.
- Wind state-space is generated based upon historical or forecasted wind data, which include both wind speed and wind directions. Wake effect is formulated and quantified for wind turbine systems.
- Energy storage components are included in the modeling of wind turbine systems, and they have their Markov models.
- The reliability analysis of wind farms results in the generation state-space which provides the generation states, their probability, their duration and their transitions.

CHAPTER 3 Reliability Analysis of Wind Turbine Systems

This chapter presents the reliability model for wind turbine systems (WTS) using state-space-based probabilistic approaches (Markov models). Wind data are categorized into discrete states by discretizing speed and angle. Wake effect is accounted when there are neighboring WTSs.

In the reliability model of WTSs, both the states of WTSs and the wind states are accounted, and the generation states are derived by mapping from of the combined states of WTS states and wind states.

The reliability model of a WTS is finally expressed in terms of the generation states. Each generation state represents a generation range of the wind turbine system. Each generation state has a probability of existence, transition rates to any other state and frequency and duration in the state.

An illustrative example is provided in next chapter to present the use of the presented model.

3.1 Introduction

A representative wind turbine system (WTS) is shown in Figure 3.1.

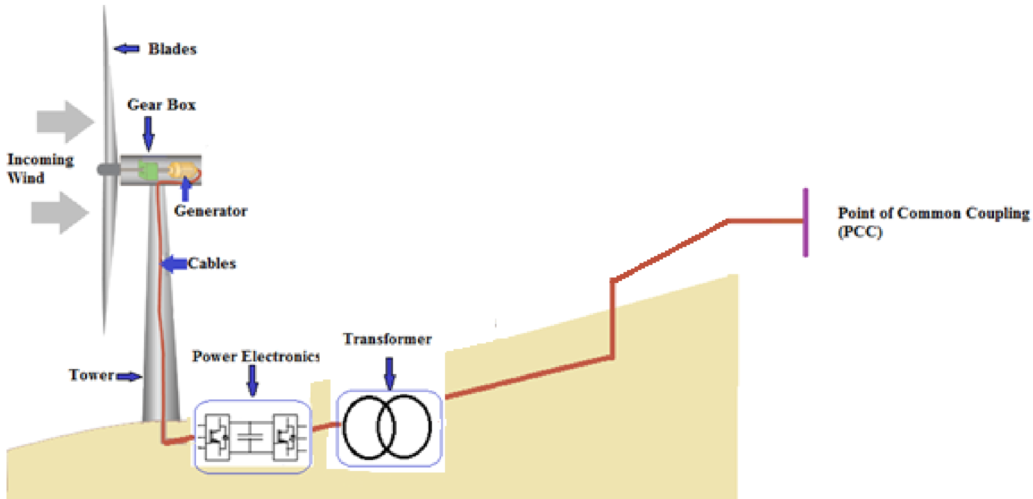


Figure 3.1: Typical Wind Turbine System

A wind turbine system normally consists of the blades, the tower, the gear box, the wind generator, the cables, the power electronics and the transformer. The wind turbine systems in a wind farm are all ultimately connected to the point of common coupling (PCC).

The following assumptions are made for the development of the reliability model of WTS:

1. Each component in a WTS has a Markov model;

2. Each WTS is subject to wake effects from neighboring WTSs;

3. The wind speed and direction data are given;

4. Each component is independent in terms of failure and success states from any other component. This means that the failure of one component in the wind farm is independent of the failure of others;

5. The reliability parameters for each component, namely the failure rate and the repair rate, are available from historical data collection and statistics.

6. The nacelle is adaptive to incoming wind, which means that the blades are always perpendicular to incoming wind.

Based upon the above assumptions, the reliability analysis of a wind turbine system is presented in the following subsections.

There are two factors that impact the generation states of a WTS: a) WTS state space; and b) wind state space:

a) WTS states refer to the states of the WTS determined by the condition of the components in the WTS;

b) The wind states refer to the combination of a wind speed and a wind angle, extracted from given wind data.

The combination of every state in a) and b) results in a state in the combined state space. Effect analysis is performed for every combined state, including the generation output, probability, transition to other combined states, frequency of transitions, and duration. The result of the effect analysis maps the combined state onto a generation range, which belongs to a generation state of the WTS.

In this chapter, section 3.2 provides the model of WTS state space; section 3.3 provides the model of wind state space; section 3.4 presents the combined state space of wind and WTS; section 3.5 presents the generation states derived from the mapping of combined states; and section 3.6 concludes the section.

3.2 WTS state space

The derivation of the states of a WTS is determined by the “up” and “down” conditions of its components. Every combination of the “up” and “down” conditions of the components forms a state of the WTS.

There are four types of generic wind turbine models: (1) Conventional directly connected induction machines, (2) Wound rotor induction generator with variable rotor resistance, (3) Doubly-Fed induction generator, and (4) Full converter interface. Type 3 and type 4 wind turbines are the most commonly used ones, and therefore are the ones to be considered in this report. The following part will introduce the derivation of the state space of Type 3 and Type 4 WTS respectively.

3.2.1 Type 3 WTS State Space

The WTS state space of Type 3 WTS is provided in this subsection.

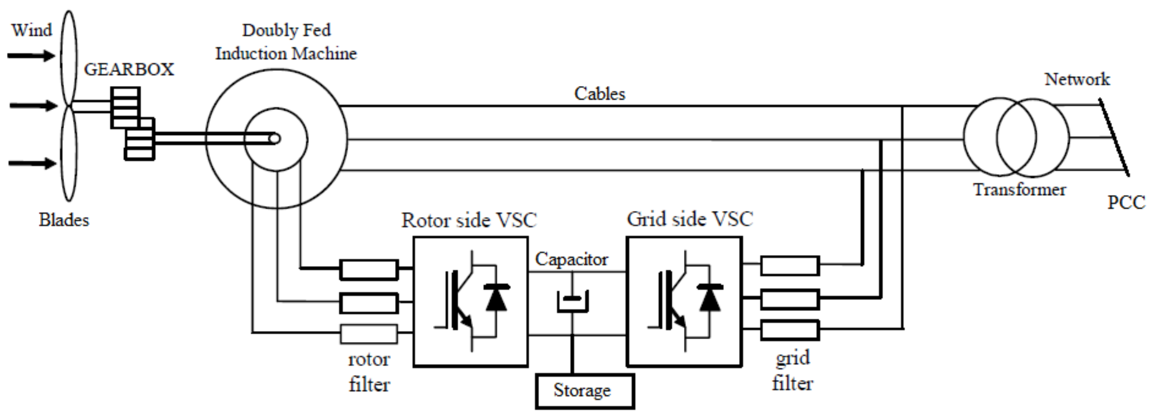


Figure 3.2: Typical Type 3 Wind Turbine System

A typical Type 3 WTS is presented in the figure above. The list of components in this Type 3 WTS is provided in Table 3.1. There are eleven major components in this Type 3 WTS.

Table 3.1: List of Components in Typical Type 3 Wind Turbine System

	Component Name	Abbreviation
1	Blades	B
2	Gear Box	GB
3	Doubly Fed Induction Machine	DFIM
4	Cables	Cab
5	Rotor Side Filter	RSF
6	Rotor Side Voltage Source Converter	RS-VSC
7	Capacitor	Cap
8	Energy Storage(Battery)	Storage
9	Grid Side Voltage Source Converter	GS-VSC
10	Grid Side Filter	GSF
11	Transformer	T

Based on the independent Markov models of every component in Type 3 WTS, each state of Type 3 WTS represents a combination of the conditions of the components. The WTS state space of Type 3 is derived by including all these states.

The Type 3 WTS state space is generated by a computer program developed.

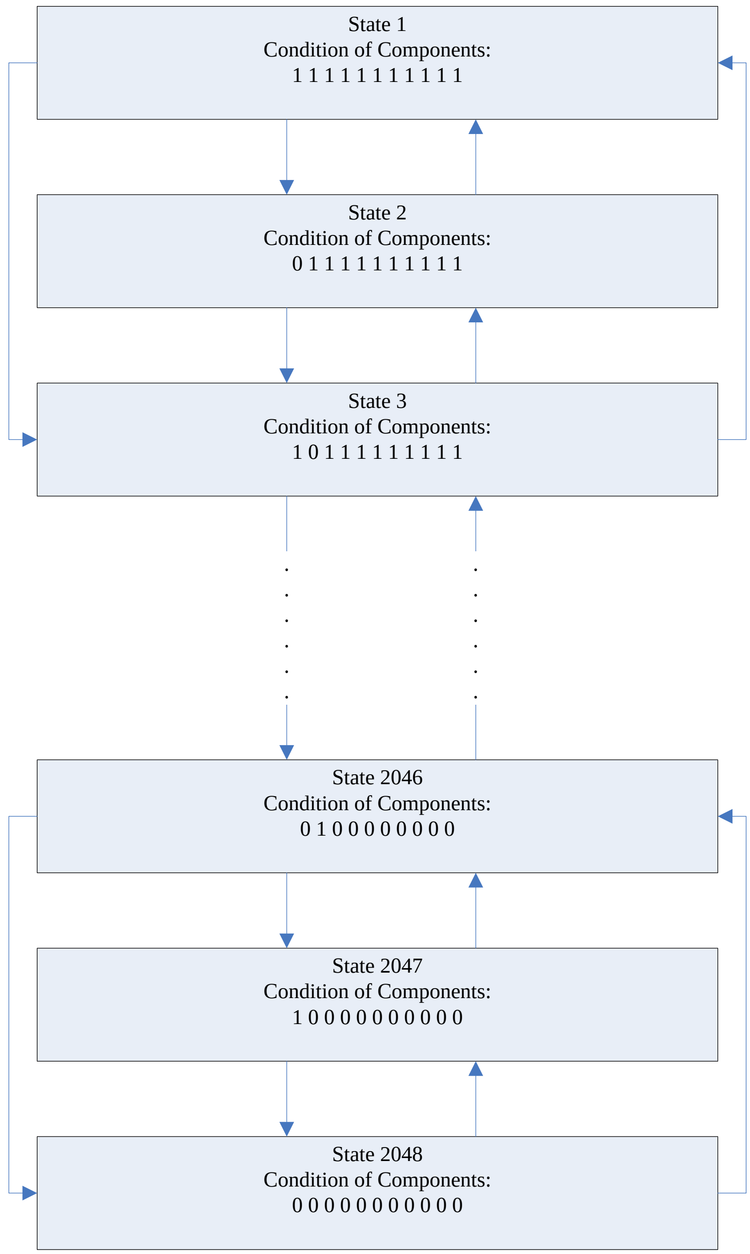


Figure 3.3: Type 3 WTS State Space

The WTS state space will be in the form as shown in Figure 3.3. There are in total $2^{11} = 2048$ states in the state space, and each state represents a combination of the conditions of the components in the Type 3 WTS. In Figure 3.3, the "0"s and "1"s labeled in each state stand for the "down" and "up" conditions of the corresponding components. For example, state 1 represents the all "up" conditions of the eleven components listed in

Table 3.1. The arrows in the figure denote the transitions of the states in the state space. These transitions will be accounted in the effect analysis of the combined state space of WTS states and wind states.

The probability of every state in the Type WTS state space is stored. Using the Markov models of the components, the probability of the conditions of component i are as follows:

For component i, $$P_i(\mathrm{up}) = \frac{\frac{1}{\lambda_i}}{\frac{1}{\lambda_i}+\frac{1}{\mu_i}} = \frac{\mu_i}{\lambda_i+\mu_i} \tag{3.1}$$

$$P_i(\mathrm{down}) = \frac{\frac{1}{\lambda_i}}{\frac{1}{\lambda_i}+\frac{1}{\mu_i}} = \frac{\lambda_i}{\lambda_i+\mu_i} \tag{3.2}$$

where λ_i and μ_i are the failure rate and the repair rate of the component respectively.

The probability of every state in the WTS state space is the multiplication of the probability of the "up" or "down" condition of every component.

Therefore, for Type 3 WTS state space, these values are derived and stored:

1) Probability Vector:

$$P_{\mathrm{Type3}} = [P(\text{WTS state 1});\ P(\text{WTS state 2});\ \cdots; P(\text{WTS state 2048})] \tag{3.3}$$

in which $P(\text{WTS state k}) = \prod_{i=1}^{11} P_i(\text{component up or down in state k})$, where the probability of the components' being up or down are presented in (3.1) and (3.2).

2) Transition Matrix:

$$\lambda_{\mathrm{Type3}} = \begin{bmatrix} \lambda_{1-1} & \lambda_{1-2} & \lambda_{1-3} & \cdots & \lambda_{1-2047} & \lambda_{1-2048} \\ \lambda_{2-1} & \lambda_{2-2} & \lambda_{2-3} & \cdots & \lambda_{2-2047} & \lambda_{2-2048} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \lambda_{2048-1} & \lambda_{2048-2} & \lambda_{2048-3} & \cdots & \lambda_{2048-2047} & \lambda_{2048-2048} \end{bmatrix} \tag{3.4}$$

in which every entry stands for the transition rate between the states numbered with the row index and column index. For example, λ_{2-2047} stands for the transition rate between state 2 and state 2047. There is an assumption made in the derivation of the transition rates, since the simultaneous change of two components' states is high ordered and is very rare. The assumption is only to consider the transitions between the states where only one component changes its state. For example, WTS state 1 represents the "all up" conditions of the 11 components, and it can only transition to the states with "one component down and all others up", which are WTS state 2,3, … ,12. This determined the entry of, λ_{1-3}, …, λ_{1-12} . These entries are filled with the failure rate of the corresponding component. For example, $\lambda_{1-2} = \lambda_{\text{component 1}}$ which is the failure rate of component 1, since the transition from WTS state 1 (Condition of Components: 11111111111) to WTS state 2 (Condition of Components: 011111111111) is caused by the failure of component 1. Similarly, the entry in the matrix between the WTS states whose transition is caused by the change from "0" to "1" of a component is filled with the repair rate of the corresponding component. The diagonal entries in the matrix, λ_{k-k} , has actually no use in the computation and are set to be "1". In this way the transition matrix $\lambda_{\text{Type3}}$is derived and stored.

3.2.2 Type 4 WTS State Space

The WTS state space of Type 4 WTS is provided in this subsection.

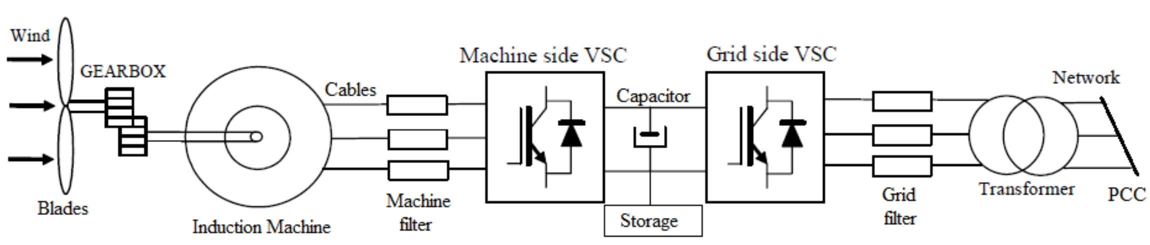


Figure 3.4: Typical Type 4 Wind Turbine System

A typical Type 4 WTS is presented in the figure above. In this Type 4 WTS, the generator is induction and is gear-operated. The list of components in this Type 4 WTS is provided in Table 3.2. There are eleven major components in this Type 4 WTS. Please note that Type 4 WTSs can also have asynchronous or permanent magnet generators, and they can be gear-less.

Table 3.2: List of Components in Typical Type 4 Wind Turbine System

	Component Name	Abbreviation
1	Blades	B
2	Gear Box	GB
3	Induction Machine	IM
4	Cables	Cab
5	Machine Side Filter	MSF
6	Machine Side Voltage Source Converter	MS-VSC
7	Capacitor	Cap
8	Energy Storage(Battery)	Storage
9	Grid Side Voltage Source Converter	GS-VSC
10	Grid Side Filter	GSF
11	Transformer	T

Based on the independent Markov models of every component in Type 4 WTS, each state of Type 4 WTS represents a combination of the “up” or “down” condition of the components. The WTS state space of Type 4 is derived by including all these states. Similarly as in Type 3, the probability of every state in the WTS state space is the multiplication of the probability of the "up" or "down" condition of every component. The Type 4 WTS state space is generated by a computer program developed, and the probability of every state is stored when the state is generated.

The transition rates among the WTS states are also stored. Since the inputs include components reliability parameters in the WTS, which are failure rates and repair rates, the transition rates among WTS states are stored accordingly.

The form of Type 4 WTS state space is presented in Figure 3.5. There are in total $2^{11} = 2048$ states in the state space, and each state represents a combination of the conditions of the components in the Type 4 WTS.

Similarly as presented above for Type 3 WTS state space, the probability vector and the transition matrix are derived and stored for Type 4 WTS state space.

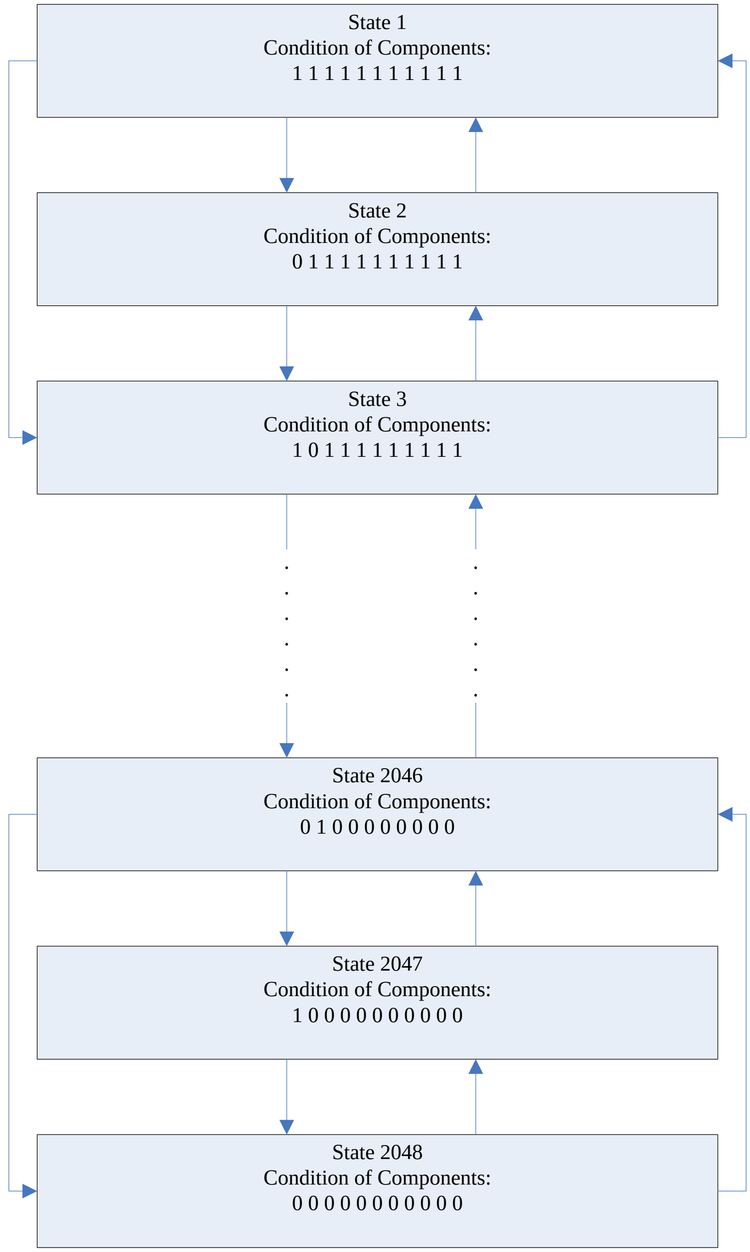


Figure 3.5: Type 4 WTS State Space

3.3 Wind State Space

A wind state space will be in the form as shown in Figure 3.6.

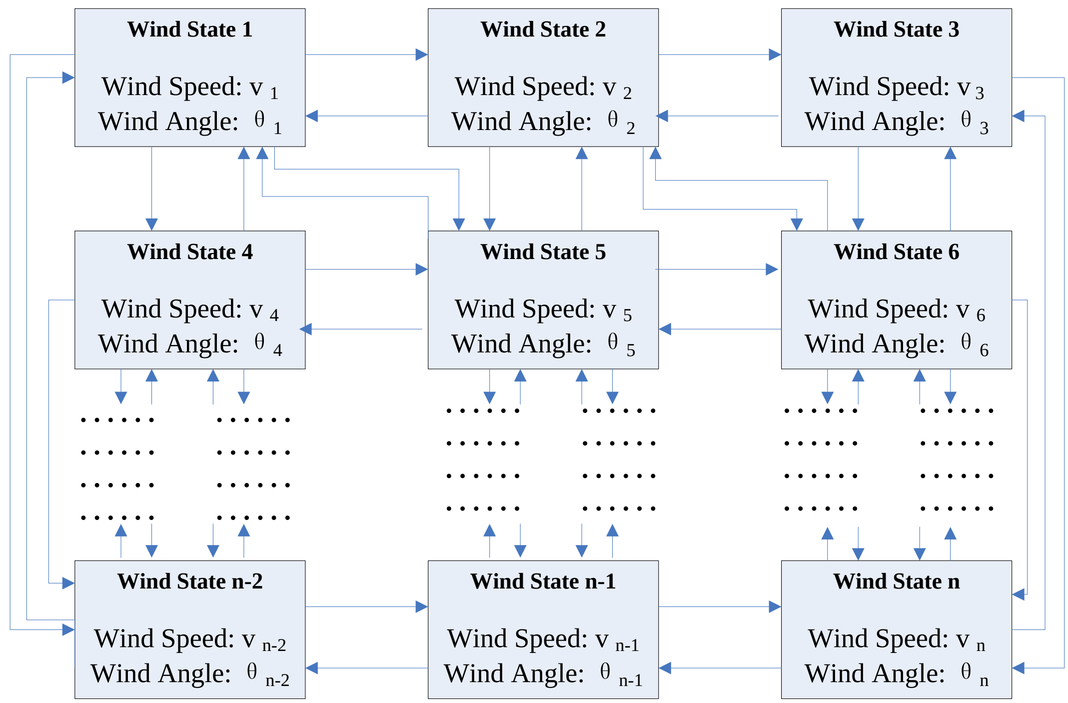


Figure 3.6: Wind State Space

Each state in the state space represents a specific combination of a wind speed and a wind direction. In the figure above, notions with v represent the wind speeds and notions with θ represent the wind direction angles. A wind direction angle is defined to be the angle between the wind flowing direction and the reference direction, in which the reference direction is normally set to be the north direction. There are transitions among the states in the above state space.

Wind data are given in the form as in Table 3.3. These data comes from historical wind record or forecasting, and include both wind speeds and wind direction angles with time information.

Table 3.3: Form of Wind Data Given

Time	Wind Speed (m/s)	Wind Direction (°)
t_1	v_1	θ_1
$t_1 + \Delta t$	v_2	θ_2
$t_1 + 2\Delta t$	v_3	θ_3
$t_1 + 3\Delta t$	v_4	θ_4
$\cdots$	$\cdots$	$\cdots$
$t_1 + (k-1)\Delta t$	v_k	θ_k
$t_1 + k\Delta t$	v_{k+1}	θ_{k+1}

There are $k + 1$ wind data in the table. The time span of the data in the above table is $k\Delta t$. The data are given in chronological order and are the data from consecutive time periods. The wind state space is generated by a computer program from the wind data provided in the form of the above table. The number of wind states depends on the dispersion of the wind data, and the step size of wind speed and wind angle. For example, some coastal cities incur obvious seasonal wind and constant wind speed, which results in very few states extracted from wind data. As an extreme case, Xinglong City in China has had an annual dataset in year 2000 containing wind speed of 18 – 18.5 m/s at most of the time, and 30° – 40° of wind angle during summer seasons while 210 – 220° during winter seasons extracted from its thousands of data. This will result in a wind state space consisting of only $2 \cdot 3 = 6$ states when the step size of wind speed is set as 0.5 m/s and the step size of wind angle is 5°. But most of the time, the dispersion of the wind data is quite wide and therefore there exist numerous wind states.

The derivation steps of a wind state space based upon the given wind data using the computer program are as follows:

Steps:

1) Inputs: Wind data table in the form of Table 3.3; Step size of wind speed; Step size of angle;

   Please note that the step sizes of wind speed and angle are set according to desired accuracy. For example, the step size of wind speed is set in this dissertation as 0.5m/s and the step size of the angle is 5°.

2) Classify every data into a wind state.

   For example, a wind data set with 11.4m/s and 18.9° is classified to be the wind state of 11.5m/s and 20°. This is done by considering the step size 0.5m/s and 5°.

3) The wind state space is determined by including all the wind states derived from step 2.

   For the wind state space, these values are derived and stored:

   a) Probability Vector:

$$P_{\mathrm{Wind}} = [\mathrm{P(Wind\ state\ 1)};\ \mathrm{P(Wind\ state\ 2)};\ \cdots;\mathrm{P(Wind\ state\ n)}] \tag{3.5}$$

The probability of the states follows the frequency principle and is derived from the accumulated counted frequency from the given data.

$$\mathrm{P(Wind\ state\ k)} = \frac{\mathrm{Number\ of\ data\ falling\ into\ state\ k}}{\mathrm{Total\ Number\ of\ Data}}$$

For example, if there are 100 wind data falling into wind state 5 among the total 1000 wind data, then the probability of wind state 5 is 100/1000=0.1.

   b) Transition Matrix:

$$\lambda_{\mathrm{Wind}} = \begin{bmatrix} \lambda_{1-1} & \lambda_{1-2} & \lambda_{1-3} & \cdots & \lambda_{1-(n-1)} & \lambda_{1-n} \\ \lambda_{2-1} & \lambda_{2-2} & \lambda_{2-3} & \cdots & \lambda_{2-(n-1)} & \lambda_{2-n} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \lambda_{n-1} & \lambda_{n-2} & \lambda_{n-3} & \cdots & \lambda_{n-(n-1)} & \lambda_{n-n} \end{bmatrix} \tag{3.6}$$

in which every entry stands for the transition rate between the states numbered with the row index and column index. For example, λ_{2-n} stands for the transition rate between state 2 and state n.

The transition rates are calculated from the frequency of transitions. The transition rates have the relationship with the frequency of transitions as:

$$\lambda_{m-n} = \frac{\mathrm{Frequency}_{m-n}}{\mathrm{P(Wind\ state\ m)}}$$

in which λ_{m-n} is the transition rate from state m to n, and the probability of wind state m is derived in a). Frequency of transitions from state m to state n is defined to be the mean number of transitions from state m to state n per unit time. From the wind data given, the frequency of transitions is extracted by identifying the accumulated number of transitions and then dividing it by the total time span. For example, if the 2nd data is classified as state 10, and if the 3rd data is classified as state 15, then the existing transition from the 2nd data to the 3rd data reflects the transition from state 10 to state 15. This will add one to the number of transitions from state 10 to 15. After accumulating all the transitions, the frequency is derived by dividing the total number of transitions by the total time span: $\frac{n_{mn}}{T}$, in which T is the total time span given by the data and n_{mn} is the total number of transitions from state m to state n.

3.4 Combined State Space of WTS States and Wind States

The combined state space of the above two kinds of states is presented in this subsection.

The combined state space is derived by including all the combined states, and the combined states are obtained by mixing one wind state with one WTS state.

3.4.1 Derivation of Combined State Space

The combined state space is presented in Figure 3.7. Combined state i-j is obtained by taking state i from wind state space, taking state j from WTS state space, and combining them. Please note here that the WTS state space in the Figure can be either Type 3 WTS state space or Type 4 WTS state space. Since Type 3 and Type 4 WTS state space both have 2048 states, the WTS state space in this figure is a general demonstration.

As demonstrated in the figure, the combined state space contains all the combined states of wind states and WTS states. Since there are n states in the wind state space and there are 2048 states in the WTS state space for Type 3 or Type 4 WTS, there are in total $\mathrm{n} \cdot 2048 = 2048\mathrm{n}$ states in the combined state space. These states are labeled from Combined State 1-1 to Combined State n-2048 in the figure.

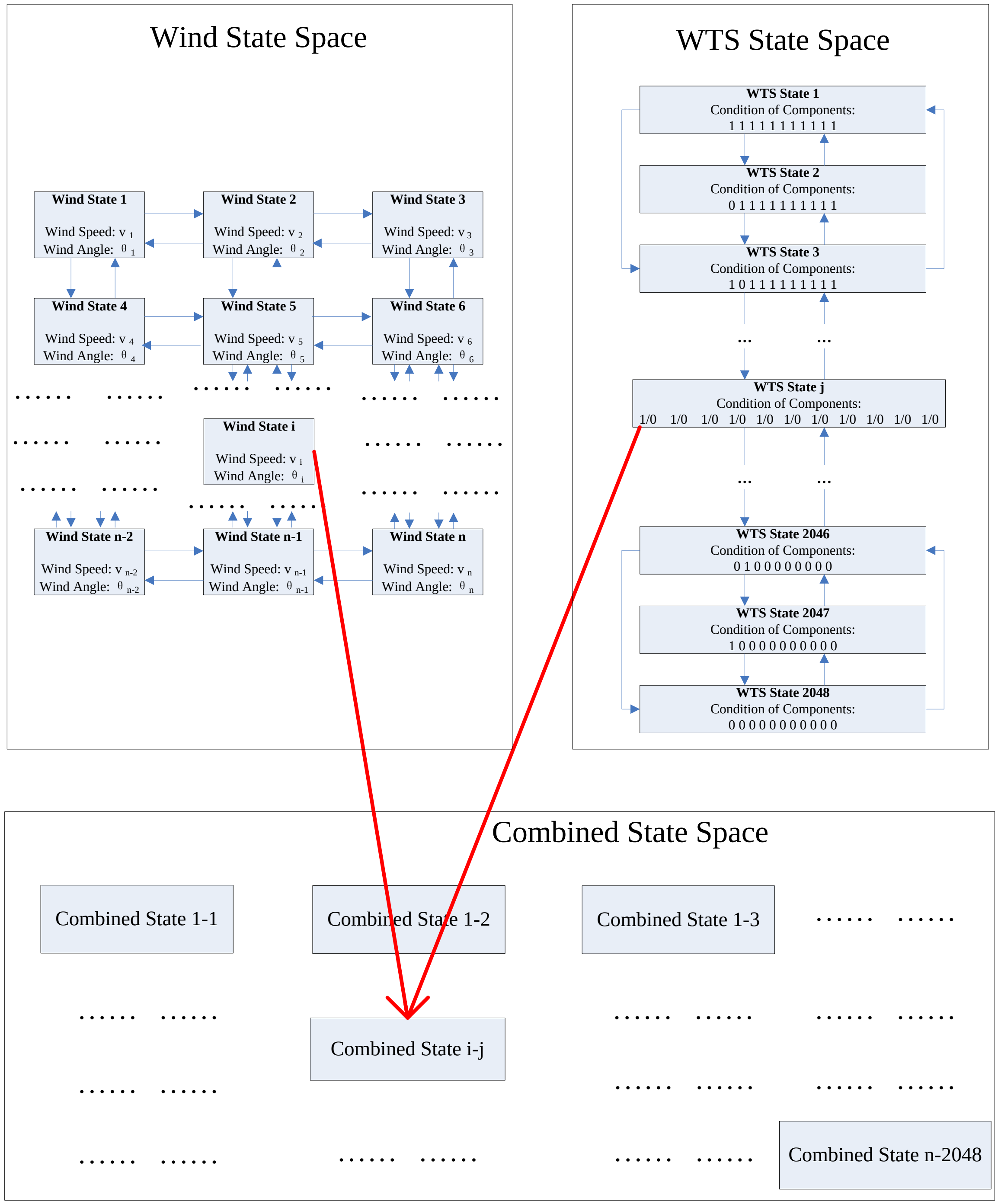


Figure 3.7: Combined State Space of WTS States and Wind States

The effect analysis of every state in the combined state space is then performed, given the WTS state space and the wind state space models.

3.4.2 Effects Analysis of Combined States

With the combination of a state of wind, say i, and a state of WTS, say j, the effects analysis of the combined state i-j is presented in this subsection.

The objective of the effects analysis is to obtain the generation output of the combined states. The inputs needed for the effects analysis of the combined states include:

1) WTS Manufacturer information: Generation curve of the WTS, Blade diameter/radius, tower height, Thrust Coefficient;
2) Geographical parameters of the WTS considered: height/altitude of the location, wake growth rate, wind speed variation factor with height;
3) Geographical parameters of the neighboring WTSs;

The output of the effects analysis is the generation output of the combined state. Wake effect is taken into account when analyzing the generation output.

Wake effect is caused by geographical unsymmetrical distribution of the wind turbines in respect to the incoming wind. When wake effect is taken into account, the equivalent wind speed for each wind turbine is different depending on its geographical location, while the actual wind speed at the farm site is a unique one. This can be understood as an effect that the upstream wind turbines blocked the wind to some extent so that the wind speed at downstream wind turbines is less.

Wake effect only influences the equivalent wind speed at the location of a WTS.

The models that are most commonly used for wake effect are Jensen model and Lissaman model. Jensen model [20] – [23] is to be used primarily in flat terrain, while the Lissaman model [24] – [27] is mostly used in complex terrain. Lissaman model is the one used in this dissertation, since the terrain conditions considered in this dissertation are mostly non-flat. However, please note that the selection of wake effect model does not affect the reliability modeling presented in this dissertation. In other words, the wake effect model can be changed to other ones if needed when applying the reliability analysis presented in this dissertation.

Denoting G as the generation curve in the manufacturer's manual of the WTS according to wind speed, the generation output of the wind turbine is as follows:

$G = G(V_{equivalent})$, in which $V_{equivalent}$ is the equivalent wind speed at the location of the WTS. Given the information of the neighboring WTS which causes the wake effect, the equivalent wind speed of the WTS considered using Lissaman Model is as follows:

$$v_{equivalentx} = v_{wind} \cdot (1 - \sqrt{1 - C_T})\left(\frac{R}{R + KDcos\theta}\right)^2 \cdot (\frac{h + H}{H})^{\alpha} \cdot \left[1 - (\frac{1 - (1 - \sqrt{1 - C_T})\left(\frac{R}{R + KDcos\theta}\right)}{(1 - \sqrt{1 - C_T})\left(\frac{R}{R + KDcos\theta}\right)^2 \cdot \left[(\frac{h + H}{H})^{\alpha}\right]})^2\right]$$

in which: v_{wind} is speed in the wind state, C_T is the thrust coefficient, R is the radius of the blades, α is wind speed variation factor with height, h is the height of WTS, H is the altitude difference of the terrain, D is the distance from the neighboring WTS and K is the wake growth rate depending on the geographical situation. K can be set as 0.075 for onshore and 0.05 for offshore [25]. The angle θ in the formula is calculated as follows:

$$\theta = \theta_{wind} - \theta_{Geographical} = \theta_{wind} - \arctan\frac{Y_{WTS} - Y_{Neighboring\,WTS}}{X_{WTS} - X_{Neighboring\,WTS}}$$

where θ_{wind} is the angle in the wind state, X_{WTS} and Y_{WTS} are the geographical coordinates of the WTS, and $X_{Neighboring\,WTS}$ and $Y_{Neighboring\,WTS}$ are the geographical coordinates of the neighboring WTS which causes the wake effect.

Therefore, the effects analysis results in the generation output of the combined state considering wake effect.

$$\text{Generation (Combined State } i-j) = G[f(v_{\text{wind state i}}, \theta_{\text{wind state i}})] \cdot \varphi(\text{WTS state j}) \quad (3.7)$$

in which G is the generation curve in the manufacturer's manual of the WTS.

- $f(v_{\text{wind state i}}, \theta_{\text{wind state i}})$

  $$= v_{\text{wind state i}} \cdot \left(1-\sqrt{1-C_T}\right)\left(\frac{R}{R+KD\cos\theta}\right)^2 \cdot \left(\frac{h+H}{H}\right)^{\alpha} \cdot \left[1-\left(\frac{1-\left(1-\sqrt{1-C_T}\right)\left(\frac{R}{R+KD\cos\theta}\right)}{\left(1-\sqrt{1-C_T}\right)\left(\frac{R}{R+KD\cos\theta}\right)^2 \cdot \left[\left(\frac{h+H}{H}\right)^{\alpha}\right]}\right)^2\right]$$

  is the determination function of the equivalent wind speed considering wake effect, where $v_{\text{wind state i}}$ is the wind speed in wind state i, and

  $$\theta = \theta_{\text{wind state i}} - \theta_{\text{Geographical}} = \theta_{\text{wind}} - arctan\frac{Y_{WTS} - Y_{Neighboring\ WTS}}{X_{WTS} - X_{Neighboring\ WTS}}$$

  where $\theta_{\text{wind state i}}$ is the wind direction angle in wind state i. X_{WTS} and Y_{WTS} are the geographical coordinates of the WTS, and $X_{Neighboring\ WTS}$ and $Y_{Neighboring\ WTS}$ are the geographical coordinates of the neighboring WTS which causes the wake effect. C_T is the thrust coefficient, R is the radius of the blades, α is wind speed variation factor with height, h is the height of WTS, H is the relative altitude of the terrain, $H = H_{WTS} - H_{Neighboring\ WTS}$, D is the distance from the neighboring WTS and K is the wake growth rate.

- φ(WTS state j) is the impact factor of WTS state on the generation output. This factor is determined by the conditions of the components in the WTS. For example, when all the components in the WTS are "up", the WTS can successfully generate and transmit 100% of wind power to PCC; when a critical component is "down" such as the transformer, the WTS can transmit 0% of wind power to PCC. The determination of this impact factor is as follows:

a) Type 3 WTS

Essential Components: Component 1,2,3,4,9,10,11 in Table 3.1, which are blades, gear box, DFIM, cables, grid side VSC, grid filter and transformer. When any one of these essential components fails, the WTS transmit 0% wind power to PCC and therefore $\varphi(\text{WTS state j}) = 0$ in this case;

When none of the essential components fails, there are two scenarios:

When component 8 fails, which is the energy storage component, only when components 5(rotor side filter), 6(rotor side VSC), 7(capacitor) are all up can energy be transmitted, and the transmitted percentage is 100%, where $\varphi(\text{WTS state j}) = 1$;

When component 8 is up, the failure of any of 5,6,7 or the combination of them results in a reduced transmitted energy. This reduced ratio is calculated and simulated in several peer literature [16][17][18], and the ratio is typically based upon the size and control algorithms of the storage component. In most research, the ratio is within a range between 0.9 and 1.1 in a limited time frame. In this dissertation, the ratio is assumed to be 0.9, where $\varphi(\text{WTS state j}) = 0.9$.

The logic among the 11 components is presented in the figure below as an analogy in circuits. Any series components' failure will result in the failure of the circuit.

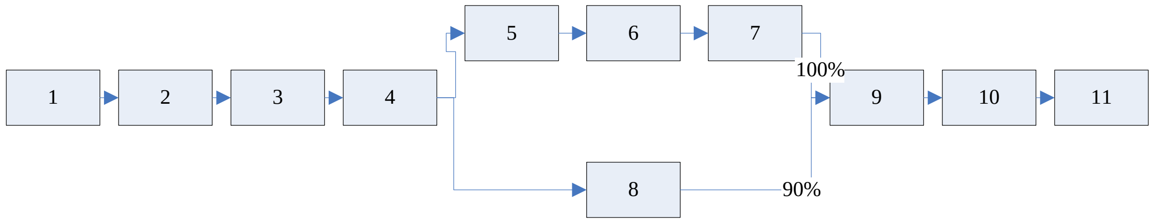


Figure 3.8: Logic of Components in Generation of Type 3 WTS

b) Type 4 WTS

Essential Components: Component 9,10,11 in Table 3.2, which are grid side VSC, grid filter and transformer. When any one of these essential components fails, the WTS transmit 0% wind power to PCC and therefore $\varphi(\text{WTS state j}) = 0$ in this case;

When none of the essential components fails, there are two scenarios:

When component 8 fails, which is the energy storage component, only when components 1(blades), 2(Gear Box), 3(Induction Machine), 4(Cables), 5(machine side filter), 6(machine side VSC), 7(capacitor) are all up can energy be transmitted, and the transmitted percentage is 100%, where $\varphi(\text{WTS state j}) = 1$;

When component 8 is up, the failure of any of 1,2,3,4,5,6,7 or the combination of them results in the energy storage supplying case. The ratio of power delivered by the storage component is calculated and simulated in peer literature [19], and the ratio is typically based upon the size and control algorithms of the storage component. It is assumed in this dissertation that 100% of power can be served and transmitted in energy storage supplying case, where $\varphi(\text{WTS state j}) = 1$.

The logic among the 11 components is presented in the figure below as an analogy in circuits.

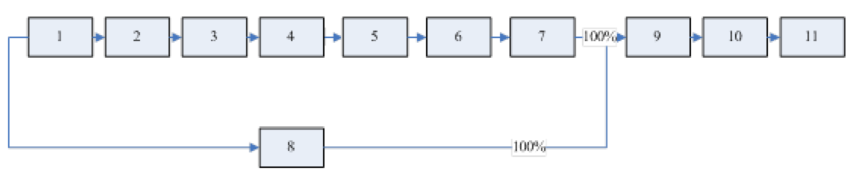


Figure 3.9: Logic of Components in Generation of Type 4 WTS

The impact factor $\varphi(\text{WTS state j})$ is determined using the above logics for Type 3 and Type 4 WTS. This is done in the computer program by judging the conditions of the components in the WTS state with the stored logic in the program. For example, when determining the impact factor of WTS state 2 which represents the conditions of the eleven components as 01111111111, essential components are first judged and the “0” condition of essential component 1 results in the impact factor $\varphi(\text{WTS state j}) = 0$.

Given the function of $f(\mathrm{v}_{\text{wind state i}}, \theta_{\text{wind state i}})$ using wind state i and the impact factor of WTS state j, the generation output is derived using (3.7) above. The generation output serves as the result of the effects analysis of the combined state.

3.4.3 Probability and Transitions of Combined States

For each of the combined states, the probability and transition rates are calculated using the probability vectors and transition matrixes of wind state space and WTS state space.

a) Probability

The probability of the combined state is derived by multiplying the probability of the WTS state and the wind state.

$$\mathrm{P}(\text{combined state}) = \mathrm{P}(\text{wind state i}) \cdot \mathrm{P}(\text{WTS state j}) \tag{3.8}$$

in which the probability of wind state *i* is the *i*th element in the probability vector (3.5) obtained when generating the wind state space; the probability of WTS state *j* is the *j*th element in the probability vector (3.3) when generating the WTS state space for Type 3 WTS, and similar for Type 4 WTS state space.

Therefore for the combined state space, the probability vector is derived as follows:

$$\mathrm{P}_{\text{Combined}} = [\mathrm{P}(\text{combined state 1});\ \mathrm{P}(\text{combined state 2});\ \cdots; \mathrm{P}(\text{combined state } 2048 \cdot \mathrm{n})] \tag{3.9}$$

in which the probability of every combined state is derived using (3.8) above.

b) Transitions

The attributes associated with transitions that are considered in this subsection include the transition rates and frequency of transitions to other generation states. In general, the relationship between the frequency and the transition rate is as follows:

$$\mathrm{Frequency}_{m-n} = \lambda_{mn} \cdot \mathrm{P(state\ m)} \tag{3.10}$$

in which λ_{mn} is the transition rate from state m to n.

In the combined state space of WTS states and wind states, the transitions between states can be caused by: a) transitions of states in WTS state space; or b) transitions of states in wind state space. According to the assumption that high order simultaneous transitions in more than one state spaces is not considered, the transitions in the combined state space is caused by either a) or b) but not both at a time.

For the combined state space, the transition matrix is derived as follows:

$$\lambda_{\mathrm{Combined}} = \begin{bmatrix} \lambda_{1-1} & \lambda_{1-2} & \lambda_{1-3} & \cdots & \lambda_{1-(2048n-1)} & \lambda_{1-2048n} \\ \lambda_{2-1} & \lambda_{2-2} & \lambda_{2-3} & \cdots & \lambda_{2-(2048n-1)} & \lambda_{2-2048n} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \lambda_{2048n-1} & \lambda_{2048n-2} & \lambda_{2048n-3} & \cdots & \lambda_{2048n-(2048n-1)} & \lambda_{2048n-2048n} \end{bmatrix} \tag{3.11}$$

in which every entry stands for the transition rate between the states numbered with the row index and column index. The dimension of the matrix is 2048n*2048n.

This transition matrix is derived given the wind state space transition matrix (3.6) and the WTS state space transition matrix (3.4). The process of obtaining the transition matrix for the combined state space is as follows:

1) For every entry λ_{p-q}, retrieve from the combined state space the two corresponding combined states – Combined state p and Combined state q.
2) Extract the WTS states and wind states that were combined to obtain the Combined state p and q. The result will show that Combined state p is derived by combining

wind state p_w and WTS state p_{WTS}, and Combined state q is derived by combining wind state q_w and WTS state q_{WTS}.

3) Judge if $p_w = q_w$. If yes, then the transition in the combined state space is caused by the transition in WTS state space, and the entry λ_{p-q} is filled by the corresponding entry (p_{WTS}, q_{WTS}) in the WTS state space transition matrix (3.4): $\lambda_{combined\ p-q} = \lambda_{WTS\ p_{WTS}-q_{WTS}}$. If no, go to next step.

4) Judge if $p_{WTS} = q_{WTS}$. If yes, then the transition in the combined state space is caused by the transition in wind state space, and the entry λ_{p-q} is filled by the corresponding entry (p_W, q_W) in the wind state space transition matrix (3.6): $\lambda_{combined\ p-q} = \lambda_{wind\ p_w-q_w}$. If no, $\lambda_{p-q} = 0$, which is due to the assumption that high-order transitions are not considered so that the WTS and wind state cannot transition simultaneously.

For example, the entry $\lambda_{combined\ 1-2}$ is decided by analyzing the combined state 1 and 2. Combined state 1 is derived with Wind state 1 and WTS state 1; Combined state 2 is derived with Wind state 1 and WTS state 2. By judging the two wind states, it is found that the two combined states have the same wind state and therefore the transition is caused by transition in WTS state space. Therefore, $\lambda_{combined\ 1-2} = \lambda_{WTS\ 1-2}$, in which $\lambda_{WTS\ 1-2}$ is the entry (1,2) in the WTS transition matrix in (3.4).

These steps apply to all the combined state space transition matrix entries and the matrix is derived.

3.5 Generation States of WTS

The generation states of a wind turbine system refer to the possible generating ranges, and these states are the desired reliability analysis result in this dissertation. The attributes of the generation states are derived from the mapping of the combined state space in last subsection. The effects analysis and the above presented calculation results of the combined states are used to derive the attributes of the generation states. The generation output of the combined states, which is presented in effects analysis part of combined states above, is used to map the combined states to the generating states. After mapping, the generating states are treated as events which consist of their mapped combined states. The probability, transitions and duration of the generation states are then calculated using the results in the subsection above.

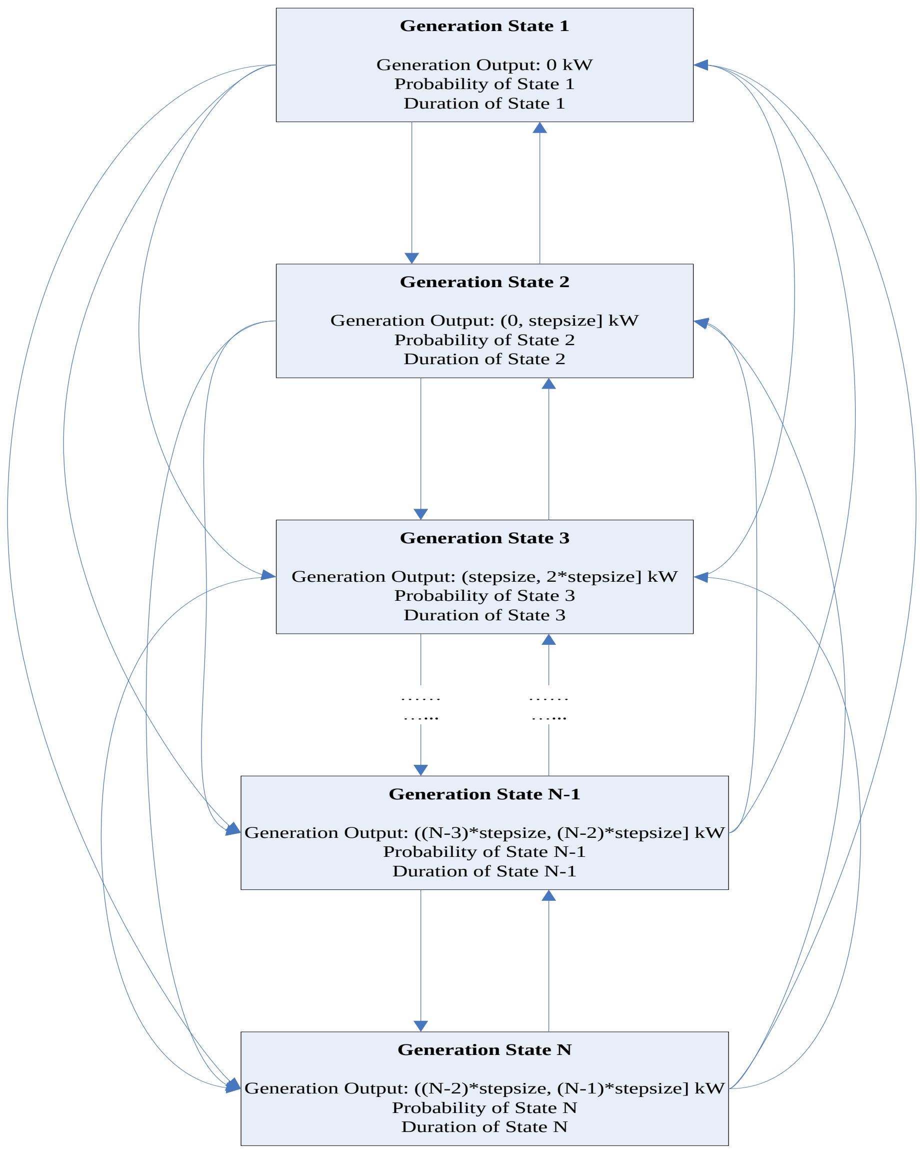


Figure 3.10: Generation States of WTS

By setting a step size for generation output, the generation states are defined by dividing the possible generation capacity of a wind turbine system into ranges. For example, by setting step size to be 5kW, a Type 3 WTS of 2MW can have different generation ranges as:

0, (0,5kW], (5kW, 10kW], …,(1990kW, 1995kW], (1995kW, 2000kW].

In the reliability analysis presented in this dissertation, each of the range forms a generation state. Therefore for the above example, there are $1 + \frac{2000}{5} = 401$ generation states. The reliability analysis result of the wind turbine system is presented in terms of these generation states and the transitions among them, as shown in Figure 3.10. In the figure, there are N generation states. The corresponding generation outputs of the generation states are from 0 to the full capacity of the WTS. Since the first state indicate the zero generation case, the total number of generation states, N, has the relationship with the capacity as of: $\mathrm{N} = \frac{\text{WTS Capacity}}{\text{Step Size}} + 1$.

From last subsection, the combined states of wind and WTS are given, and the effect analysis of the combined states is performed resulting in the generation outputs, probability, transitions and duration of the combined states. The generation states in this subsection are mapped from the effect analysis result of the combined state. The relationship between the combined state and the mapped generation state is presented in Figure 3.11.

The mapping of a combined state to the generation state is done right after the effects analysis of the combined states. As the output of the effects analysis, the generation output of the combined state has been derived. This generation output is then classified to fit into one of the generation ranges. When the classification is done, the combined state has been mapped with this generation range. For example in the figure, the combined state i-j is mapped to the mth generation state. This is done by fitting the generation output of the combined state i-j to a generation range. For instance the generation output of the combined state i-j fits into the generation range ((m-2)*stepsize, (m-1)*stepsize] kW, the combined state i-j is then mapped to this generation range, which is labeled as generation state m.

Since each of the generation state represents a generation range, the mapping of the combined states results in enabling the generating states to be event consisting of the mapped combined states. Given all the mappings of the combined states to the generation states, the values associated with the generation states are derived given the effects analysis result of the combined states. These values are calculated using the theory of events.

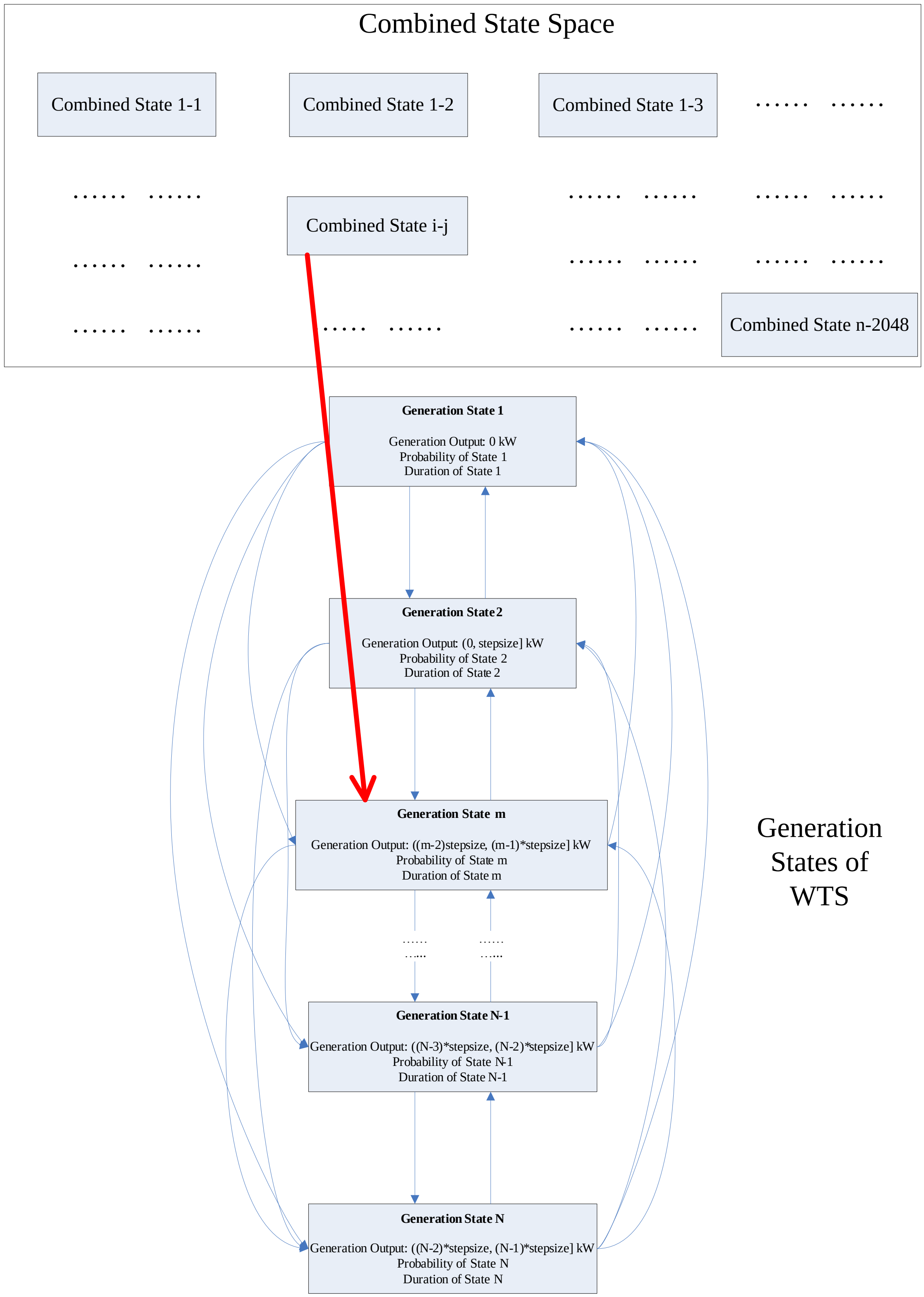


Figure 3.11: Mapping of Combined State to Generation State of WTS

These values include the probability of the generation state, transitions to other generation states, and duration of the generation state.

- Probability of Generation States

The probability of the generation state is derived as follows:

$$\mathrm{P(U)} = \sum_{\mathrm{i \in U}} \mathrm{P(Combined\ State\ i)}$$

in which U stands for the generation range considered, and combined state i represents all the combined state that is mapped to generation range U. Given the results of the combined state space, $\mathrm{P(Combined\ State\ i)}$ is the *i*th element in the probability vector (3.9) of the combined state space.

- Transition Rates of Generation States

The transition rate from generation range U to V is formulated as follows:

$$\mathrm{Transition\ Rate_{U-V}} = \sum_{\mathrm{i \in U}} \sum_{\mathrm{j \in V}} \lambda_{\mathrm{ij}}$$

in which λ_{ij} is the transition rate from state i in range U to state j in range V.

Given the results of the combined state space, λ_{ij} is the (i,j) entry in the combined state space transition matrix (3.11).

- Frequency of Transitions of Generation States

The frequency from generation range U to V is formulated as follows:

$$\mathrm{Frequency_{U-V}} = \sum_{\mathrm{j \in V}} \sum_{\mathrm{i \in U}} \mathrm{Frequency_{i-j}} = \sum_{\mathrm{j \in V}} \sum_{\mathrm{i \in U}} \mathrm{P(Combined\ State\ i)} \cdot \lambda_{\mathrm{ij}} = \sum_{\mathrm{i \in U}} [\mathrm{P(Combined\ State\ i)} \cdot \sum_{\mathrm{j \in V}} \lambda_{\mathrm{ij}}]$$

in which λ_{ij} is the transition rate from state i in range U to state j in range V.

Given the results of the combined state space, $\mathrm{P(Combined\ State\ i)}$ is the *i*th element in the probability vector (3.9) of the combined state space, and λ_{ij} is the (i,j) entry in the combined state space transition matrix (3.11).

- Duration of Generation States

The Duration of generation range U is as follows:

$$\mathrm{Duration\ (U)} = \frac{\mathrm{P(U)}}{\sum_{\mathrm{V}} \mathrm{Frequency}_{\mathrm{U-V}}}$$

in which the probability and the frequency results are the ones demonstrated above.

The mapping and the calculation of the above listed attributes of the generation states are performed by a developed computer program. An illustrative example is shown in next section to demonstrate the use of the presented models.

3.6 Summary and Discussion

This chapter presents the reliability model of the wind turbine systems. Computer programs have been created to obtain the reliability model.

The reliability model of each WTS takes into account the wind states from given wind data and the WTS states. WTS states are derived by taking all the combinations of the states of the individual components in a WTS. Wind states are derived from wind data by extracting combinations of wind speed and wind direction angle.

The combined states are generated by mixing one wind state and one WTS state. Effect analysis is performed for every combined state, resulting in the generation output of the combined state. Wake effect is accounted when there are neighboring WTSs in the effect analysis of generation output. The probability and transition rates are calculated for the combined states. Combined states are then mapped to generation states of the WTS, each of which represents a generation range of the WTS. The reliability analysis result is provided finally with the generation states and the calculation results of their probability, duration, and the transitions among them.

Given the reliability model of the wind turbine systems, the reliability analysis of a wind farm can be performed in the following chapter.

CHAPTER 4 Example WTS Reliability Analysis

This chapter provides an example of the application of the above reliability models.

4.1 WTS System and Wind Data Description

The WTS is Type 3 and the turbine is Vestas 80. The capacity level it is 2.0 MVA. The generation versus wind speed curve is provided in Figure 4.1, which is obtained from the manufacturer's product brochure [28].

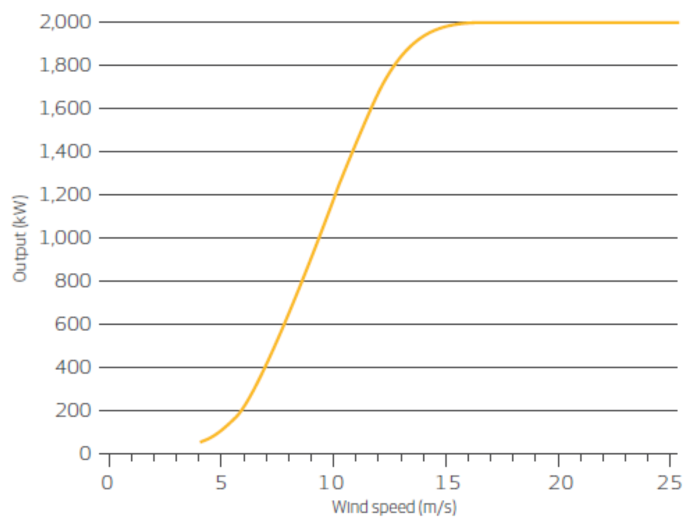


Figure 4.1: Generation Curve of the WTS in the Illustrative Example [28]

The parameters of the Vestas 80 WTS are as shown in Table 4.1.

Table 4.1: Parameters of the WTS in the Illustrative Example

Item	Value
Blade Diameter	78m
Tower Height	67m
Thrust Coefficient C_T	0.9
Wake Growth Rate	K=0.05
Wind Speed Variation Factor	$\alpha = 1$
Geographical Coordinate X	1500m
Geographical Coordinate Y	3000m
Geographical Altitude of Terrain H (base of tower)	5m (Reference level is PCC point)

A nearby WTS that is causing wake effect on the presented WTS has the following parameters:

Table 4.2: Parameters of a Nearby WTS Causing Wake Effect

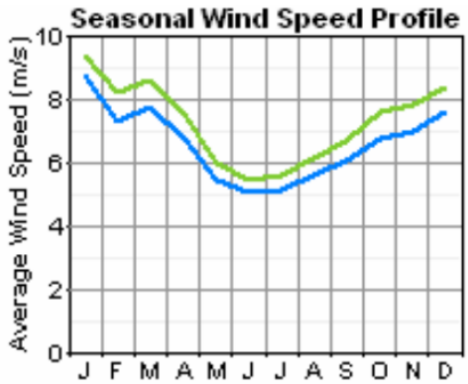
Item	Value
Geographical Coordinate Xw	1000m
Geographical Coordinate Yw	2500m
Geographical Altitude of Terrain Hw (base of tower)	2m (Reference level is PCC point)

The list of components and their reliability parameters of the WTS in this example is provided in Table 4.3. Failure rate values are in term of per year, since they indicates the general occurrence of a failure within a year; repair rate values are in term of per hour, since the values are normally obtained by the repair duration given in term of hours.

Table 4.3: List of Components and Their Reliability Parameters in the WTS

	Component Name	Failure Rate (/year)	Repair Rate (/hour)
1	Blades	$\lambda_1 = 0.1$	$\mu_1 = 0.02$
2	Gear Box	$\lambda_2 = 0.2$	$\mu_2 = 0.01$
3	DFIM	$\lambda_3 = 0.1$	$\mu_3 = 0.01$
4	Cables	$\lambda_4 = 0.2$	$\mu_4 = 0.02$
5	Rotor Side Filter	$\lambda_5 = 0.1$	$\mu_5 = 0.02$
6	Rotor Side VSC	$\lambda_6 = 0.1$	$\mu_6 = 0.03$
7	Capacitor	$\lambda_7 = 0.1$	$\mu_7 = 0.02$
8	Energy Storage/Battery	$\lambda_8 = 0.6$	$\mu_8 = 0.1$
9	Grid Side VSC	$\lambda_9 = 0.2$	$\mu_9 = 0.02$
10	Grid Side Filter	$\lambda_{10} = 0.1$	$\mu_{10} = 0.02$
11	Transformer	$\lambda_{11} = 0.1$	$\mu_{11} = 0.01$

Wind data are from Alaska Energy Authority [33] with the wind speeds at 50 meter height and wind directions. The wind data contains the wind information in year 2004 with 8760 data sets. The time step is one hour. Figure 4.2 provides the statistics of the wind data used. The green line represents the data statistics for wind speed at 50m height which is utilized in this case study, and the blue line shows the data statistics for wind speed at 30m height. The wind direction statistics shown on the right of the figure indicate that the major direction of the wind in this location is northeast, but the directions certainly vary in different seasons to some extent.



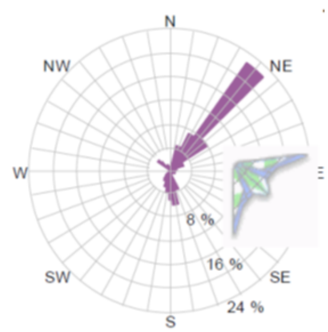


Figure 4.2: Wind Speed and Direction used in Example Reliability Analysis [33]

4.2 Wind State Space

Given the wind data of one year, wind states are extracted from these data by identifying the combinations of wind speed and angles.

The wind speed from the given data ranges from 0.40 m/s to 30.85 m/s, and the wind angle from the given data ranges from 0° to 359°.

The step size of wind speed is set as 5m/s.

The step size of wind direction angle is set as 90°.

There are totally $6 \cdot 4 = 24$ wind states.

Figure 4.3: Wind State Space in the Example

In Figure 4.3 above, each wind state represents a combination of wind speed and direction. The probability of each wind state is calculated and labeled in the wind state box in the figure. The probability of the states follows the frequency principle and is derived from the accumulated counted frequency from the given data.

$$\mathrm{P(Wind\ state\ k)} = \frac{\text{Number of data falling into state k}}{\text{Total Number of Data}} = \frac{\text{Number of data falling into state k}}{8760}$$

The transition rate matrix is also obtained as follows:

$\lambda_{\mathrm{Wind}} =$

1.000	0.148	0.042	0.163	0.040	0.008	0.000	0.008	0.001	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.111	1.000	0.084	0.030	0.001	0.022	0.006	0.004	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.049	0.131	1.000	0.143	0.003	0.007	0.056	0.005	0.000	0.000	0.004	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.091	0.034	0.075	1.000	0.005	0.002	0.005	0.051	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.259	0.041	0.015	0.056	1.000	0.056	0.020	0.096	0.046	0.005	0.000	0.010	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.056	0.306	0.050	0.019	0.069	1.000	0.069	0.006	0.006	0.025	0.006	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.011	0.030	0.181	0.047	0.005	0.022	1.000	0.036	0.000	0.000	0.055	0.003	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.025	0.005	0.015	0.206	0.035	0.003	0.017	1.000	0.003	0.003	0.000	0.035	0.000	0.000	0.000	0.002	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.030	0.000	0.000	0.152	0.000	0.000	0.121	1.000	0.091	0.030	0.121	0.000	0.000	0.000	0.030	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.071	0.000	0.071	0.000	0.357	0.214	0.000	0.000	1.000	0.071	0.000	0.000	0.143	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.060	0.030	0.000	0.015	0.239	0.015	0.015	0.015	1.000	0.030	0.000	0.000	0.015	0.000	0.000	0.000	0.015	0.000	0.000	0.000	0.000	0.000
0.000	0.012	0.000	0.049	0.025	0.000	0.025	0.296	0.037	0.000	0.012	1.000	0.000	0.000	0.000	0.012	0.000	0.000	0.000	0.012	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000	0.000	1.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.333	0.000	0.000	0.000	1.000	0.333	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.333	0.000	0.000	0.000	0.000	0.333	0.333	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.200	0.000	0.200	0.000	0.000	0.000	0.000	0.600	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.500	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.962
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.963	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000

in which the row index and the column index represent the wind state index. For example, the entry (10,20) represents the transition rate from wind state 10 to wind state 20. The dimension of the transition rate matrix is 24*24.

The transition rates are calculated from the frequency of transitions. From the wind data given, the frequency of transitions is extracted by identifying the accumulated number of transitions and then dividing it by the total time span.

$$\lambda_{\mathrm{m-n}} = \frac{\mathrm{Frequency}_{\mathrm{m-n}}}{\mathrm{P(Wind\ state\ m)}} = \frac{\frac{n_{mn}}{T}}{\mathrm{P(Wind\ state\ m)}} = \frac{\frac{n_{mn}}{8760}}{\mathrm{P(Wind\ state\ m)}}$$

in which $\lambda_{\mathrm{m-n}}$ is the transition rate from state m to n, T is the total time span given by the data which is 8760 hours, $n_{mn}$is the total number of transitions from state m to state n, and the probability of wind state m is derived above.

4.3 WTS State Space

Given the component list for Type 3 WTS, the WTS states are generated using a computer program developed in this research. The WTS state space is in the form as shown in Figure 3.3 in above sections. There are in total $2^{11} = 2048$ states in the state space, and each state represents a combination of the conditions of the components in the Type 3 WTS.

The probability vector of the WTS state space is obtained.

$P_{WTS} = [\mathrm{P(WTS\ state\ 1)};\ \mathrm{P(WTS\ state\ 2)};\ \cdots ; \mathrm{P(WTS\ state\ 2048)}]$

For example, the sub-vector of the probability values for the first ten WTS states is as follows:

$P_{WTS}(\mathrm{WTS\ state\ 1}\ , 2, 3, \dots\ 10) = [\mathrm{P(WTS\ state\ 1)};\ \mathrm{P(WTS\ state\ 2)};\ \cdots ; \mathrm{P(WTS\ state\ 10)}] =$

[0.989860055 0.001129977 0.000564989 0.000000645 0.001129977 0.000001290 0.000000645 0.000000001 0.000677986 0.000000774]

The transition matrix is also obtained:

$$\lambda_{WTS} = \begin{bmatrix} \lambda_{1-1} & \lambda_{1-2} & \lambda_{1-3} & \cdots & \lambda_{1-2047} & \lambda_{1-2048} \\ \lambda_{2-1} & \lambda_{2-2} & \lambda_{2-3} & \cdots & \lambda_{2-2047} & \lambda_{2-2048} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \lambda_{2048-1} & \lambda_{2048-2} & \lambda_{2048-3} & \cdots & \lambda_{2048-2047} & \lambda_{2048-2048} \end{bmatrix}$$

in which every entry stands for the transition rate between the states numbered with the row index and column index. The dimension of the transition matrix is 2048*2048.

For example, the sub-matrix of the transition rates between the first ten WTS states is as follows:

$$\lambda_{WTS}(\text{between WTS state } 1-10) = \begin{bmatrix} \lambda_{1-1} & \lambda_{1-2} & \lambda_{1-3} & \cdots & \lambda_{1-9} & \lambda_{1-10} \\ \lambda_{2-1} & \lambda_{2-2} & \lambda_{2-3} & \cdots & \lambda_{2-9} & \lambda_{2-10} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \lambda_{10-1} & \lambda_{10-2} & \lambda_{10-3} & \cdots & \lambda_{10-9} & \lambda_{10-10} \end{bmatrix} =$$

$$\begin{pmatrix} 1.00000 & 0.00001 & 0.00001 & 0.00000 & 0.00002 & 0.00000 & 0.00000 & 0.00000 & 0.00007 & 0.00000 \\ 0.01000 & 1.00000 & 0.00000 & 0.00001 & 0.00000 & 0.00002 & 0.00000 & 0.00000 & 0.00000 & 0.00007 \\ 0.02000 & 0.00000 & 1.00000 & 0.00001 & 0.00000 & 0.00000 & 0.00002 & 0.00000 & 0.00000 & 0.00000 \\ 0.00000 & 0.02000 & 0.01000 & 1.00000 & 0.00000 & 0.00000 & 0.00000 & 0.00002 & 0.00000 & 0.00000 \\ 0.02000 & 0.00000 & 0.00000 & 0.00000 & 1.00000 & 0.00001 & 0.00001 & 0.00000 & 0.00000 & 0.00000 \\ 0.00000 & 0.02000 & 0.00000 & 0.00000 & 0.01000 & 1.00000 & 0.00000 & 0.00001 & 0.00000 & 0.00000 \\ 0.00000 & 0.00000 & 0.02000 & 0.00000 & 0.02000 & 0.00000 & 1.00000 & 0.00001 & 0.00000 & 0.00000 \\ 0.00000 & 0.00000 & 0.00000 & 0.02000 & 0.00000 & 0.02000 & 0.01000 & 1.00000 & 0.00000 & 0.00000 \\ 0.10000 & 0.00000 & 0.00000 & 0.00000 & 0.00000 & 0.00000 & 0.00000 & 0.00000 & 1.00000 & 0.00001 \\ 0.00000 & 0.10000 & 0.00000 & 0.00000 & 0.00000 & 0.00000 & 0.00000 & 0.00000 & 0.01000 & 1.00000 \end{pmatrix}$$

4.4 Combined State Space of WTS/Wind

Given the wind states and the WTS states, the combined states are derived using the developed computer program. There are in total $24 \cdot 2048 = 49152$ states in the combined state space.

The effects analysis for every combined state is performed. The inputs for the effects analysis are listed in the above subsection as the system inputs, the output of the effects analysis is the generation outputs of the combined state.

As described in the modeling sections, wake effect is taken into account when analyzing the generation output. Wake effect only influences the equivalent wind speed at the location of a WTS.

Denoting G as the generation curve in the manufacturer's manual of the WTS in Figure 4.1, an approximation of the generation curve is made as follows:

$$G_{\text{example}}(v) = \begin{cases} 0 & v < 5 \\ 200 \cdot (v-5) & 5 \le v \le 15 \\ 2000 & v > 15 \end{cases}$$

in which v is the equivalent wind speed at the location of the WTS.

Given the information of the neighboring WTS which causes the wake effect, the equivalent wind speed of the WTS considered using Lissaman Model is as follows:

$$\mathrm{v}_{\mathrm{equivalentx}} =$$

$$\mathrm{v}_{\mathrm{wind}} \cdot \left(1-\sqrt{1-0.9}\right)\left(\frac{39}{39+0.05\cdot 3354\cdot\cos\theta}\right)^{2} \cdot \left(\frac{67+3}{3}\right)^{1} \cdot \left[1-\left(\frac{1-\left(1-\sqrt{1-0.9}\right)\left(\frac{39}{39+0.05\cdot 3354\cdot\cos\theta}\right)}{\left(1-\sqrt{1-0.9}\right)\left(\frac{39}{39+0.05\cdot 3354\cdot\cos\theta}\right)^{2}\cdot\left[\left(\frac{67+3}{3}\right)^{1}\right]}\right)^{2}\right]$$

in which: $\mathrm{v}_{\mathrm{wind}}$ is speed in the wind state. The angle θ in the formula is calculated as follows:

$$\theta = \theta_{\mathrm{wind}} - \theta_{\mathrm{Geographical}} = \theta_{\mathrm{wind}} - \arctan\frac{3000-2500}{1500-1000}$$

where θ_{wind} is the angle in the wind state.

The generation output of every one of the 49152 combined states is calculated using the above functions. These generation output results are then used for mapping to generation states. Probability vector of the combined state space is calculated and stored:

$$\mathrm{P}_{\mathrm{Combined}} = [\mathrm{P}(\text{combined state 1});\ \mathrm{P}(\text{combined state 2});\ \cdots;\mathrm{P}(\text{combined state }49152)]$$

For example, the sub-vector of the probability values for the first ten combined states is as follows:

$$\mathrm{P}_{\mathrm{Combined}}(\text{Combined state }1\,,2,3,\ldots\ 10) =$$

$$[\mathrm{P}(\text{Combined state 1});\ \mathrm{P}(\text{Combined state 2});\ \cdots;\mathrm{P}(\text{Combined state 10})] =$$

[0.163128937 0.000186220 0.000093110 0.000000106 0.000186220 0.000000213 0.000000106 0.000000000 0.000111732 0.000000128]

Transition matrix of combined states is also calculated and stored.

$$\lambda_{\mathrm{Combined}}$$

$$= \begin{bmatrix} \lambda_{1-1} & \lambda_{1-2} & \lambda_{1-3} & \cdots & \lambda_{1-(2048\mathrm{n}-1)} & \lambda_{1-2048\mathrm{n}} \\ \lambda_{2-1} & \lambda_{2-2} & \lambda_{2-3} & \cdots & \lambda_{2-(2048\mathrm{n}-1)} & \lambda_{2-2048\mathrm{n}} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \lambda_{2048\mathrm{n}-1} & \lambda_{2048\mathrm{n}-2} & \lambda_{2048\mathrm{n}-3} & \cdots & \lambda_{2048\mathrm{n}-(2048\mathrm{n}-1)} & \lambda_{2048\mathrm{n}-2048\mathrm{n}} \end{bmatrix}$$

in which every entry stands for the transition rate between the states numbered with the row index and column index. The dimension of the transition matrix is $49152 * 49152$.

For example, the sub-matrix of the transition rates between the first ten combined states is as follows:

$$\lambda_{\text{Combined}}(\text{between Combined state } 1-10) =$$

$$\begin{bmatrix} \lambda_{1-1} & \lambda_{1-2} & \lambda_{1-3} & \cdots & \lambda_{1-9} & \lambda_{1-10} \\ \lambda_{2-1} & \lambda_{2-2} & \lambda_{2-3} & \cdots & \lambda_{2-9} & \lambda_{2-10} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \lambda_{10-1} & \lambda_{10-2} & \lambda_{10-3} & \cdots & \lambda_{10-9} & \lambda_{10-10} \end{bmatrix} =$$

1.00000	0.00001	0.00001	0.00000	0.00002	0.00000	0.00000	0.00000	0.00007	0.00000
0.01000	1.00000	0.00000	0.00001	0.00000	0.00002	0.00000	0.00000	0.00000	0.00007
0.02000	0.00000	1.00000	0.00001	0.00000	0.00000	0.00002	0.00000	0.00000	0.00000
0.00000	0.02000	0.01000	1.00000	0.00000	0.00000	0.00000	0.00002	0.00000	0.00000
0.02000	0.00000	0.00000	0.00000	1.00000	0.00001	0.00001	0.00000	0.00000	0.00000
0.00000	0.02000	0.00000	0.00000	0.01000	1.00000	0.00000	0.00001	0.00000	0.00000
0.00000	0.00000	0.02000	0.00000	0.02000	0.00000	1.00000	0.00001	0.00000	0.00000
0.00000	0.00000	0.00000	0.02000	0.00000	0.02000	0.01000	1.00000	0.00000	0.00000
0.10000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	1.00000	0.00001
0.00000	0.10000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.01000	1.00000

With the effects analysis and these calculation results of the combined states, the generation states of WTS are obtained.

4.5 Generation States of WTS

As discussed in the sections above, these combined states are mapped to the generation states which represent generation ranges. In this example, the step-size of the generation ranges is selected to be 500 kW. For the considered 2MW Type 3 WTS, there are totally 5 generation states. Using the above model and the developed computer program, the combined states are mapped to the generation states and the attributes associated with the generation states are calculated. These attributes include the probability of the generation state, transition rates, frequency of transitions, and duration of the generation state. The mapping and the calculations are as follows:

The mapping of a combined state to the generation state is done right after the effects analysis of the combined states. From the output of the effects analysis, the generation output of the combined state has been derived. This generation output is then classified to fit into one of the generation ranges. For example, the combined state 100-150 is mapped to the 2nd generation state, since the effects analysis result of the combined state 100-150 gives out the generation output of that state is 388.6 kW. This generation output fits into the generation range (0, 500] kW, which is represented by the 2nd generation state.

After mapping the combined states, the calculations for the generation states are initiated. The calculations include the probability, transition rates, frequency and duration. Each of the generation states is considered to be an event, which consists the mapped combined states. The number of combined states mapped with each of the generation state is listed in the table below.

Table 4.4: Number of Combined States Mapped with Generation States

Generation State ID	Number of Combined States Mapped
1	28408
2	9516
3	5101
4	4105
5	2022

Given all the mappings of the combined states to the generation states, the values associated with the generation states are calculated, given the effects analysis result of the combined states. These values are calculated using the theory of events.

The probability of the generation state is derived as follows:

$$\mathrm{P(U)} = \sum_{\mathrm{i \in U}} \mathrm{P(Combined\ State\ i)}$$

in which U stands for the generation range considered.

The probability vector of the generation states is as follows:

$$\mathrm{P_{Generation\ States'}} = \begin{pmatrix} \mathrm{P(Generation\ state\ (1))} \\ \mathrm{P(Generation\ state\ (2))} \\ \mathrm{P(Generation\ state\ (3))} \\ \mathrm{P(Generation\ state\ (4))} \\ \mathrm{P(Generation\ state\ (5))} \end{pmatrix} = \begin{pmatrix} 0.66 \\ 0.15 \\ 0.14 \\ 0.04 \\ 0.01 \end{pmatrix}$$

The transition rate from generation range U to V is calculated as follows:

$$\mathrm{Transition\ Rate_{U-V}} = \sum_{\mathrm{i \in U}} \sum_{\mathrm{j \in V}} \lambda_{\mathrm{ij}}$$

in which λ_{ij} is the transition rate from state i in range U to state j in range V.

The transition matrix of the generation states of WTS is as follows:

$$\lambda_{\mathrm{Generation\ States}} = \begin{bmatrix} 1 & 0.68 & 0.62 & 0.23 & 0.46 \\ 0.31 & 1 & 0.25 & 0.08 & 0.14 \\ 0.55 & 0.69 & 1 & 0.58 & 0.03 \\ 0.88 & 0.64 & 0.31 & 1 & 0.09 \\ 0.73 & 0.66 & 0.37 & 0.38 & 1 \end{bmatrix}$$

The frequency from generation range U to V is calculated as follows:

$$\mathrm{Frequency_{U-V}} = \sum_{\mathrm{i \in U}} [\mathrm{P(Combined\ State\ i)} \cdot \sum_{\mathrm{j \in V}} \lambda_{\mathrm{ij}}]$$

in which λ_{ij} is the transition rate from state i in range U to state j in range V.

The frequency matrix of the generation states of WTS is as follows:

$$\lambda_{\text{Generation States}} = \begin{bmatrix} 1 & 0.86 & 0.45 & 0.09 & 0.56 \\ 0.95 & 1 & 0.02 & 0.08 & 0.04 \\ 0.54 & 0.31 & 1 & 0.04 & 0.05 \\ 0.08 & 0.64 & 0.06 & 1 & 0.01 \\ 0.38 & 0.01 & 0.01 & 0.01 & 1 \end{bmatrix}$$

The Duration of generation range U is as follows:

$$\text{Duration (U)} = \frac{\text{P(U)}}{\sum_{\text{V}} \text{Frequency}_{\text{U-V}}}$$

in which the probability and the frequency results are the ones demonstrated above.

The duration vector of the generation states is as follows, in the unit of hour:

$$\text{Duration}_{\text{Generation States}} = \begin{pmatrix} \text{Duration(Generation state 1)} \\ \text{Duration(Generation state 2)} \\ \text{Duration(Generation state 3)} \\ \text{Duration(Generation state 4)} \\ \text{Duration(Generation state 5)} \end{pmatrix} = \begin{pmatrix} 1.58 \\ 0.13 \\ 0.08 \\ 0.02 \\ 0.01 \end{pmatrix}$$

The end results of the calculations are provided in Figure 4.4. The numbers labeled on the connection links are the transition rates. The probability and the duration of the generation state are listed in the state boxes. The duration results are in term of hour.

It can be found from the result that the probability of generating more than 500kW is relatively smaller. This is caused by the fact that the wind speed at the location of the wind turbine system is limited – the majority data are less than 10m/s as presented in Figure 4.2 of the wind data.

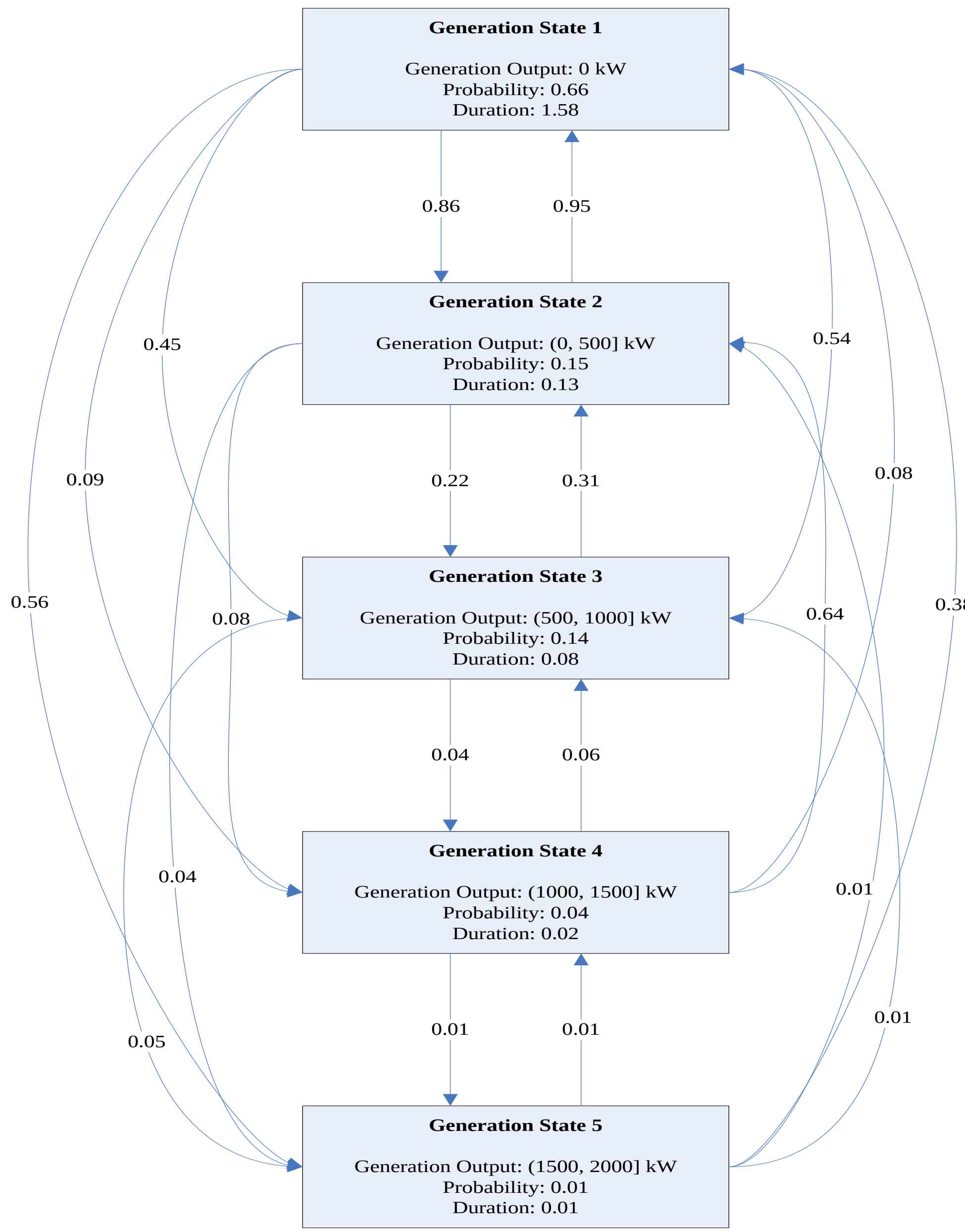


Figure 4.4: Generation States of the Example WTS

CHAPTER 5 Reliability Analysis of Wind Farm

This chapter presents the reliability analysis of the wind farm given the reliability models of individual wind turbine systems in last chapters.

5.1 Introduction

A wind farm is a connection of multiple wind turbine systems through distribution lines. The wind farm is connected to the power grid at the point of common coupling. The point of common coupling may be located in a substation.

The following assumptions are made in the development of the wind farm reliability model:

1) The WTS state model of each individual wind turbine system is given;
2) The wind state model is given;
3) Each distribution line has a Markov model, and the distribution lines are independent of each other in terms of success and failure;
4) The WTSs in a wind farm are identical in terms of manufacture related characteristics, including generation curve, heights and radius of blades, parameters of components, etc.

The wind farm reliability analysis is performed based on the WTS states, wind states, and the distribution line states. The generation states of wind turbines in a wind farm are dependent with each other upon wind states. Therefore, the method used in this chapter follows the procedure as below:

Step 1: Define WTS state space based on the methods in last chapter;

Step 2: Perform the effect analysis over the WTS state space to achieve at the delivery ratio. This was done in the effect analysis part of the combined states in last chapter. In the analysis of wind farms, the key consideration of wind turbine systems is the delivery ratio, which is defined as the ratio between deliverable electricity power with wind power:

$$\varphi(\text{WTS state i}) = \frac{\text{Electric Power}}{\text{Wind Power}}$$

This ratio is the impact factor described in last chapter. For type 3 WTS, this ratio can be 1, 0.9 or 0. For type 4 WTS, this ratio can be 1 or 0.

Step 3: Derive the wind power by wind state and generation curve of the WTSs.

Step 4: For each state of the conditions of distribution lines, identify the existence of paths to PCC for every WTS. If there is a path for a WTS, the generation of it will count in the total generation. If there is no path for a WTS to PCC, the generation of it will not count in the total generation of the wind farm.

Step 5: For each state of the conditions of distribution lines, derive the total generation of the wind farm given the combined state of wind state and all WTS states. This total generation is calculated using the wind power and delivery ratio of each WTS. Each combined state will have a generation output as total generation of the wind farm.

Step 6: Derive the generation output state space for all the states of distribution lines. The result is the expected generation state space of the wind farm.

Since for an entire wind farm, the impact of wake effect is limited and therefore is not considered in this chapter. This results in some simplifications compared to last chapter:

1) Wind states contain only wind speed, since the consideration of wind direction is only for wake effect concerns;

2) The generation of each WTS is the multiplication of wind power and delivery ratio, where the wind power is obtained by wind state and the generation curve, and the delivery ratio is given. This applies to all the WTSs in the wind farm.

3) Since WTSs are identical in parameters and the state space of WTS states, the calculation of total generation output of a wind farm can utilize the combinatorial number in math. For example, a wind farm with 20 Type 3 WTSs can have 19 WTSs in their "all up" WTS states which have the delivery ratio as $\varphi(\text{WTS state i}) = \frac{\text{Electric Power}}{\text{Wind Power}} = 1$. Then the total generation of all these states who have 19 WTSs in "all up" has the same result, and the number of these states are $C_{20}^{19} = 20$. These 20 states fall into the same category since they have the same impact on the total generation of the wind farm. Similar combinatorial rules will apply to other combinations of WTS states.

This chapter describes the model, calculation process and format of results of wind farm reliability analysis. In this chapter, section 5.2 presents the state space of the distribution lines; section 5.3 presents the WTS states and their mapping onto delivery ratio states of all the WTSs in the wind farm; section 5.4 provides the derivation of the combined state space of WTS delivery ratio states, wind states and distribution line states, and the effect analysis of the combined states; section 5.5 provides generation states and the mapping from the combined states to generation states; and section 5.6 concludes the chapter.

5.2 State Space of In-Farm Distribution Lines

This section provides the demonstration of the state space of in-farm distribution lines.

Based on the independent Markov models of every distribution line in the wind farm, each state of distribution lines represents a combination of the conditions of the distribution lines. The distribution line state space is derived by including all these states.

The distribution line state space will be in the form as shown in Figure 5.1. There are in total 2^n states in the state space since each distribution line has a two state Markov model, and each state represents a combination of the conditions of the lines. In Figure 5.1, the “0”s and “1”s labeled in each state stand for the “down” and “up” conditions of the corresponding distribution line. For example, state 1 represents the all “up” conditions of the lines. The arrows in the figure denote the transitions of the states in the state space. These transitions will be accounted in the effect analysis of the combined state space in the next section.

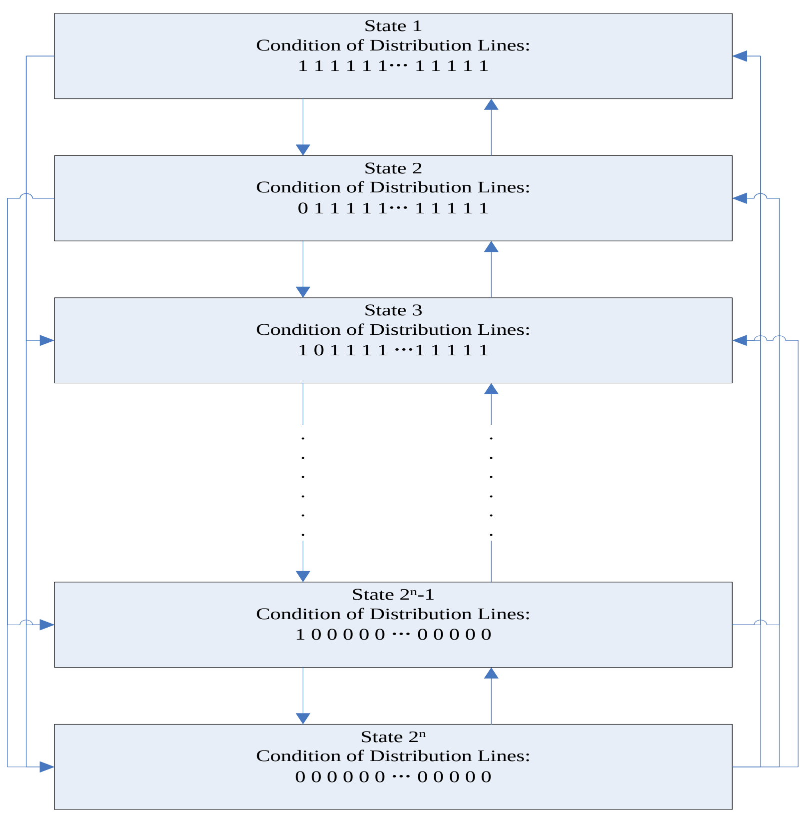


Figure 5.1: State Space Distribution Lines

The probability of every state in the state space of distribution lines is stored. Using the Markov models of the components, the probability of the conditions of component i are as follows:

$$P(\text{Lineup}) = \frac{\mu_{\text{Line}}}{\lambda_{\text{Line}}+\mu_{\text{Line}}} \tag{5.1}$$

$$P(\text{Linedown}) = \frac{\lambda_{\text{Line}}}{\lambda_{\text{Line}}+\mu_{\text{Line}}} \tag{5.2}$$

in which λ_{Line} is the failure rate and μ_{Line} is the repair rate of the corresponding distribution line. The probability of every state in the state space is the multiplication of the probability of the "up" or "down" condition of every distribution line.

5.3 WTS states and delivery ratio states

As described in last chapter especially in Figure 3.3 and Figure 3.5, there are twelve components in each Type 3 or Type 4 WTS. The total number of WTS states for each WTS is $2^{11} = 2048$. For these WTS states, an effect analysis is performed to derive the delivery ratio of the WTS state. The delivery ratio is defined as the ratio between deliverable electricity power with wind power $\varphi(\text{WTS state i}) = \frac{\text{Electric Power}}{\text{Wind Power}}$. This ratio is the impact factor described in last chapter. For type 3 WTS, this ratio can be 1, 0.9 or 0. For type 4 WTS, this ratio can be 1 or 0. This factor is determined by the conditions of the components in the WTS. For example, when all the components in the WTS are "up", the WTS can successfully generate and transmit 100% of wind power to PCC; when a critical component is "down" such as the transformer, the WTS can transmit 0% of wind power to PCC. The determination of this ratio is as follows:

Type 3 WTS:

Essential Components: Component 1,2,3,4,9,10,11 in Table 3.1, which are blades, gear box, DFIM, cables, grid side VSC, grid filter and transformer. When any one of these essential components fails, the WTS transmit 0% wind power to PCC and therefore $\varphi(\text{WTS state i}) = 0$ in this case;

When none of the essential components fails, there are two scenarios:

When component 8 fails, which is the energy storage component, only when components 5(rotor side filter), 6(rotor side VSC), 7(capacitor) are all up can energy be transmitted, and the transmitted percentage is 100%, where $\varphi(\text{WTS state i}) = 1$;

When component 8 is up, the failure of any of 5,6,7 or the combination of them results in a reduced transmitted energy, which is normally 90% of wind power[16], where $\varphi(\text{WTS state i}) = 0.9$.

Type 4 WTS

Essential Components: Component 9,10,11 in Table 3.2, which are grid side VSC, grid filter and transformer. When any one of these essential components fails, the WTS transmit 0% wind power to PCC and therefore $\varphi(\text{WTS state i}) = 0$ in this case;

When none of the essential components fails, there are two scenarios:

When component 8 fails, which is the energy storage component, only when components 1(blades), 2(Gear Box), 3(Induction Machine), 4(Cables), 5(machine side filter), 6(machine side VSC), 7(capacitor) are all up can energy be transmitted, and the transmitted percentage is 100%, where $\varphi(\text{WTS state i}) = 1$;

When component 8 is up, the failure of any of 1,2,3,4,5,6,7 or the combination of them results in the energy storage supplying case. It is assumed that 100% of power can be served and transmitted in energy storage supplying case, where $\varphi(\text{WTS state i}) = 1$.

The delivery ratio $\varphi(\text{WTS state})$ is determined using the above logics for Type 3 and Type 4 WTS. This is done in the computer program by judging the conditions of the components in the WTS state with the stored logic in the program. For example, when determining the impact factor of WTS state 2 which represents the conditions of the eleven components as 01111111111, essential components are first judged and the "0" condition of essential component 1 results in the impact factor $\varphi(\text{WTS state i}) = 0$.

This process serves as the effect analysis of the WTS states for each WTS. In this way, each WTS state will have one of the three options (1, 0.9 or 0) for Type 3 WTS as the delivery ratio, and each WTS state will have one of the two options (1 or 0) for Type 4 WTS as the delivery ratio.

By analyzing all the $2^{11} = 2048$ WTS states in the WTS state space, the delivery ratio states and the attributes of them will be obtained. These attributes include the probability, transitions and duration in each delivery ratio states. Figure 5.2 presents the delivery ratio states for Type 3 WTS. Figure 5.3 presents the delivery ratio states for Type 4 WTS.

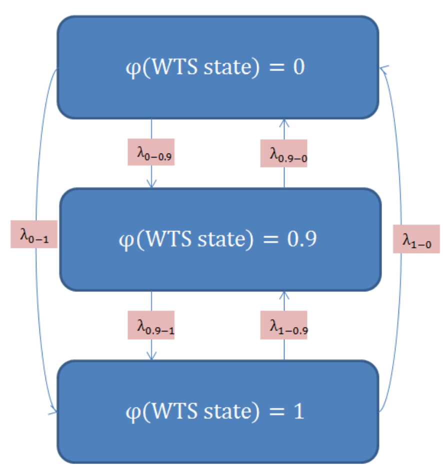


Figure 5.2: Delivery Ratio States of Type 3 WTS

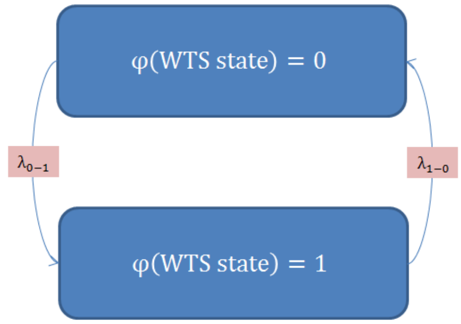


Figure 5.3: Delivery Ratio States of Type 4 WTS

The probability, transitions and duration of each delivery ratio state are calculated using event analysis.

The probability of a delivery ratio state is derived as follows:

$$P(U) = \sum_{i \in U} P(\text{WTS State i})$$

in which U stands for the delivery ratio considered, and WTS state *i* represents the WTS state that has the delivery ratio as U. Given the results of the WTS state space, P(WTS State i) is the *i*th element in the probability vector (3.3) of WTS state space.

The transition rate from delivery ratio state U to V is formulated as follows:

$$\text{Transition Rate}_{U-V} = \sum_{i \in U} \sum_{j \in V} \lambda_{ij}$$

in which λ_{ij} is the transition rate from state i in U to state j in V.

Given the results of the WTS state space, λ_{ij} is the (i,j) entry in the WTS state space transition matrix (3.4).

The frequency from delivery ratio state U to V is formulated as follows:

$$\text{Frequency}_{U-V} = \sum_{j \in V} \sum_{i \in U} \text{Frequency}_{i-j} = \sum_{j \in V} \sum_{i \in U} P(\text{WTS State i}) \cdot \lambda_{ij} = \sum_{i \in U} [P(\text{WTS State i}) \cdot \sum_{j \in V} \lambda_{ij}]$$

in which λ_{ij} is the transition rate from state i in U to state j in V.

Given the results of the WTS state space, $P(\text{WTS State i})$ is the *i*th element in the probability vector (3.3) of the WTS state space, and λ_{ij} is the (i,j) entry in the WTS state space transition matrix (3.4).

The Duration of delivery ratio state U is as follows:

$$\text{Duration (U)} = \frac{\text{P(U)}}{\sum_{\text{V}} \text{Frequency}_{\text{U-V}}}$$

in which the probability and the frequency results are the ones demonstrated above.

With the above analysis of WTS states, each WTS has the delivery ratio states as shown in Figure 5.2 or Figure 5.3.

5.4 Combined State Space of WTS, Wind and Distribution Line States

The combined state space of the WTS states, wind states, and distribution line states, is presented in this subsection.

5.4.1 Derivation of Combined State Space

The combined states are derived by mixing one state from the delivery ratio states of every WTS, one state from wind state space, and one state from distribution line state space. Figure 5.4 presents the composition of a combined state.

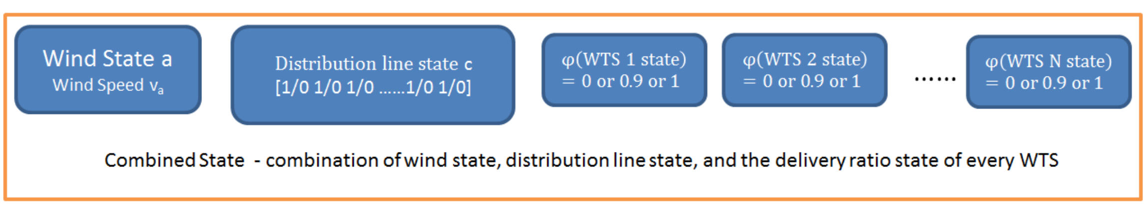


Figure 5.4: Combined States of WTS, Wind and Distribution Line States

Figure 5.5 provides the illustrative combined state space.

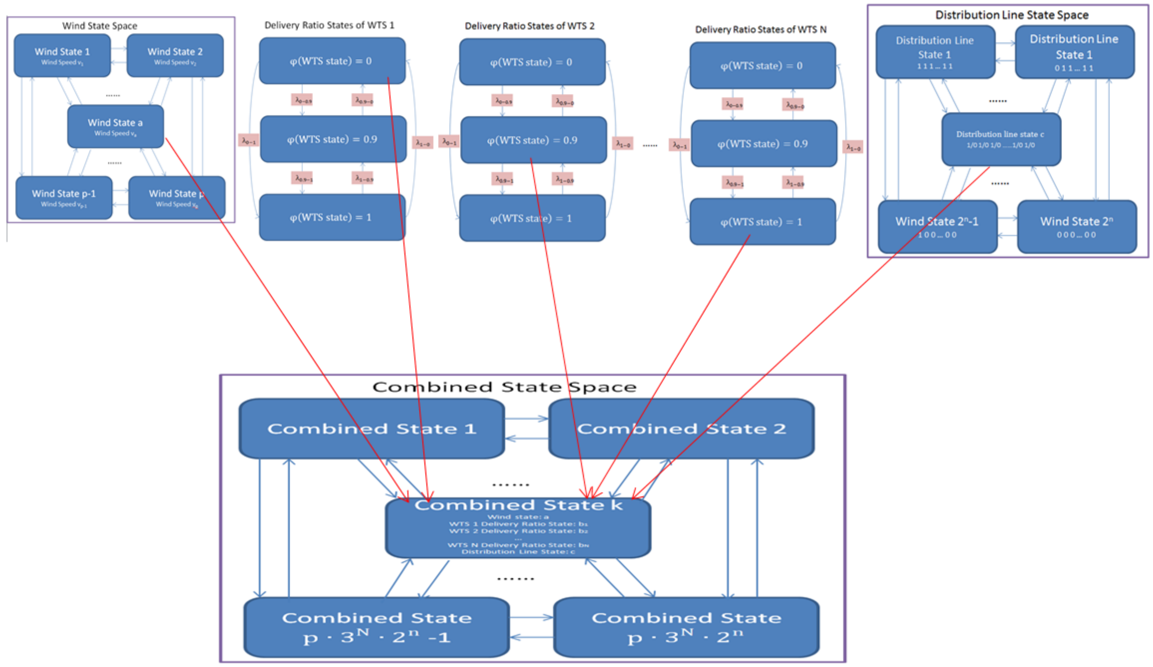


Figure 5.5: Combined State Space of WTS, Wind and Distribution Line States

The WTS delivery ratio model for every WTS is the one presented in last subsection. The wind state space model is the presented wind model in last chapters. However, since wake effect is not considered in this chapter, the wind states contain only wind speeds. Wind directions are not reflected in the wind states since they do not impact the generation of the wind turbine systems when wake effect is not considered.

The combining of states here follows similar process as the combining process in the modeling of WTS in last chapter. The difference is that the combining of states in WTS analysis is for WTS and wind states, while the combining here is for multiple WTS states, wind states, and distribution line states. Furthermore, the WTS states are represented by the delivery ratio states of every WTS in here.

As demonstrated in Figure above, the combined state space contains all the combined states of delivery ratio states of every WTS, wind states, and distribution line states.

The total number of states in the combined state space is derived as follows:

Assume there are *p* states in the wind state space extracted from wind data; there are *N* Type 3 WTSs and *n* distribution lines in the wind farm. Then the number of delivery ratio states for every WTS is three, as presented in Figure 5.2. The number of distribution line states is 2^n. Therefore, the total number of combined states in the combined state space is $p \cdot 3^N \cdot 2^n$.

Similarly, when the WTSs in the wind farm are Type 4, the total number of combined states are derived with the assumption that there are *p* states in the wind state space extracted from wind data, there are *M* Type 4 WTSs and there are *n* distribution lines in the wind farm. The number of delivery ratio states for every WTS is two, as presented in Figure 5.3. The number of distribution line states is 2^n. Therefore, the total number of combined states in the combined state space is $p \cdot 2^M \cdot 2^n$.

The effects analysis of every combined state in the combined state space is then performed, given the WTS delivery ratio states, the wind state space and the distribution line state space models. The effect analysis includes the calculation of generation outputs, probability, transitions and duration of the combined state.

5.4.2 Effects Analysis of Combined States

The effects analysis of the combined states is presented in this subsection.

The objective of the effects analysis is to obtain the attributes that are associated with the combined states. The effects analysis is performed for all the combined states.

The inputs needed for the effects analysis of the combined states include:

The topology and connections of all WTSs and distribution lines in the wind farm;

WTS Manufacturer information: Generation curve of every WTS;

Components reliability parameters in every WTS: failure rates and repair rates of every component in the WTS. These parameters are used in deriving the delivery ratio states of the WTS as shown in Figure 5.2 and 5.3.

Reliability parameters of the distribution lines: failure rates and repair rates of the distribution lines in the wind farm.

The output of the effects analysis is the generation output of the combined state.

The generation output of a combined state in the combined state space of the wind farm is determined by the generation of each WTS and the state of the distribution lines. When the distribution line which connects a WTS to PCC is "up", the transmitted generation output depends only on the state of the WTS. When the distribution line which connects a WTS to PCC is "down", the transmitted generation output of the WTS to PCC is 0, no matter what state the WTS is in. In this way, the generation of the wind farm is as follows:

$$\text{Wind Farm Generation} = \sum_{\text{i}=1}^{\text{\# of WTSs}} \text{Gen(WTS i)} \cdot \text{Link(WTS i)} \tag{5.3}$$

in which Gen(WTS i) is the generation output of the *i*th WTS and link(i) is the connection status determined by the distribution line state:

$$\text{Link(WTS i)} = \begin{cases} 1 & \text{when there is a link from WTS i to PCC} \\ 0 & \text{when there is no link from WTS i to PCC} \end{cases} \tag{5.4}$$

The generation output of the *i*th WTS is determined by the wind state and the delivery ratio state of that WTS.

As shown in Figure 5.4, the combined states contain the wind state, WTS delivery ratio state, and the distribution line state. Assuming there are p wind states, N Type 3 WTSs and n distribution lines in the wind farm, a combined state can be denoted in general term by involving: wind state a, delivery ratio state b_1 for WTS 1, delivery ratio state b_2 for WTS 2, …, delivery ratio state b_N for WTS N, distribution line state c. where a is in the range between 1 and p, b_1, b_2, …, b_N are in the range between 1 and 3 for Type 3 WTS or between 1 and 2 for Type 4 WTS, and c is between 1 and 2^{n}. Table 5.1 provides the composition of a general combined state.

Table 5.1: The composition of combined state

Composition	Wind State	Delivery Ratio State for WTS1	Delivery Ratio State for WTS2	……	Delivery Ratio State for WTSN	Distribution Line State
State	a	b_1	b_2	……	b_N	c
Range	{1,2, …p}	Type 3:{1,2,3} Type 4: {1,2}	Type 3:{1,2,3} Type 4: {1,2}	Type 3:{1,2,3} Type 4: {1,2}	Type 3:{1,2,3} Type 4: {1,2}	{1,2,…, 2^{n}}

The effect analysis is performed following the steps below.

Step 1: Connection analysis of every WTS given distribution line state c. This step targets at getting the connection status of every WTS. The method used in find the connection is the search of path based on the given topology matrix of WTSs and distribution lines in the wind farm. With the program, the connection status Link(WTS i) of the *i*th WTS can be obtained given the distribution line state c. The connection status of every WTS can be 1 or 0, representing if it is connected to PCC or not under this distribution line state.

Step 2: Derive the wind power by the wind state a and the generation curve of the WTSs. By assuming that the WTSs in the wind farm are identical in manufacture, the WTSs have the identical generation curve G. For every WTS, the maximum generation power which is the wind power is as follows:

$$\text{Wind Power/Maximum Generation Power} = \mathrm{G}[\mathrm{v}_{\text{wind state a}}] \,, \tag{5.5}$$

in which G is the generation curve in the manufacturer's manual of the WTS, $v_{\text{wind state a}}$ is the wind speed in wind state a.

Step 3: Derive the generation of every WTS. The generation of each WTS is determined by the wind power and the delivery ratio, as shown in the following formula:

$$\text{Gen(WTS i)} = \text{Wind Power} \cdot \text{delivery ratio i} = \text{G}[v_{\text{wind state a}}] \cdot \varphi(b_i)\ , \tag{5.6}$$

in which $\varphi(b_i)$ is the delivery ratio state of WTS i.

Step 4: Obtain the wind farm generation. The wind farm generation is the sum of the generation of each WTS which is connected to PCC. The formula is as (5.3) and is deducted as follows:

$$\text{Wind Farm Generation} = \sum_{\text{i=1}}^{\text{\# of WTSs}} \text{Gen(WTS i)} \cdot \text{Link(WTS i)}$$

$$= \textstyle\sum_{\text{i=1}}^{\text{\# of WTSs}} \text{Wind Power} \cdot \text{delivery ratio i} \cdot \text{Link(WTS i)}$$

$$= \textstyle\sum_{\text{i=1}}^{\text{\# of WTSs}} \text{G}[v_{\text{wind state a}}] \cdot \varphi(b_i) \ \cdot \text{Link(WTS i)}\ , \tag{5.7}$$

in which G is the generation curve in the manufacturer's manual of the WTS, $v_{\text{wind state a}}$ is the wind speed in wind state a, $\varphi(b_i)$ is the delivery ratio state of WTS i, Link(WTS i) is the connection status from WTS i to PCC as defined in (5.4) under the distribution line state c.

Figure 5.6 provides the overview of the effect analysis of the combined states.

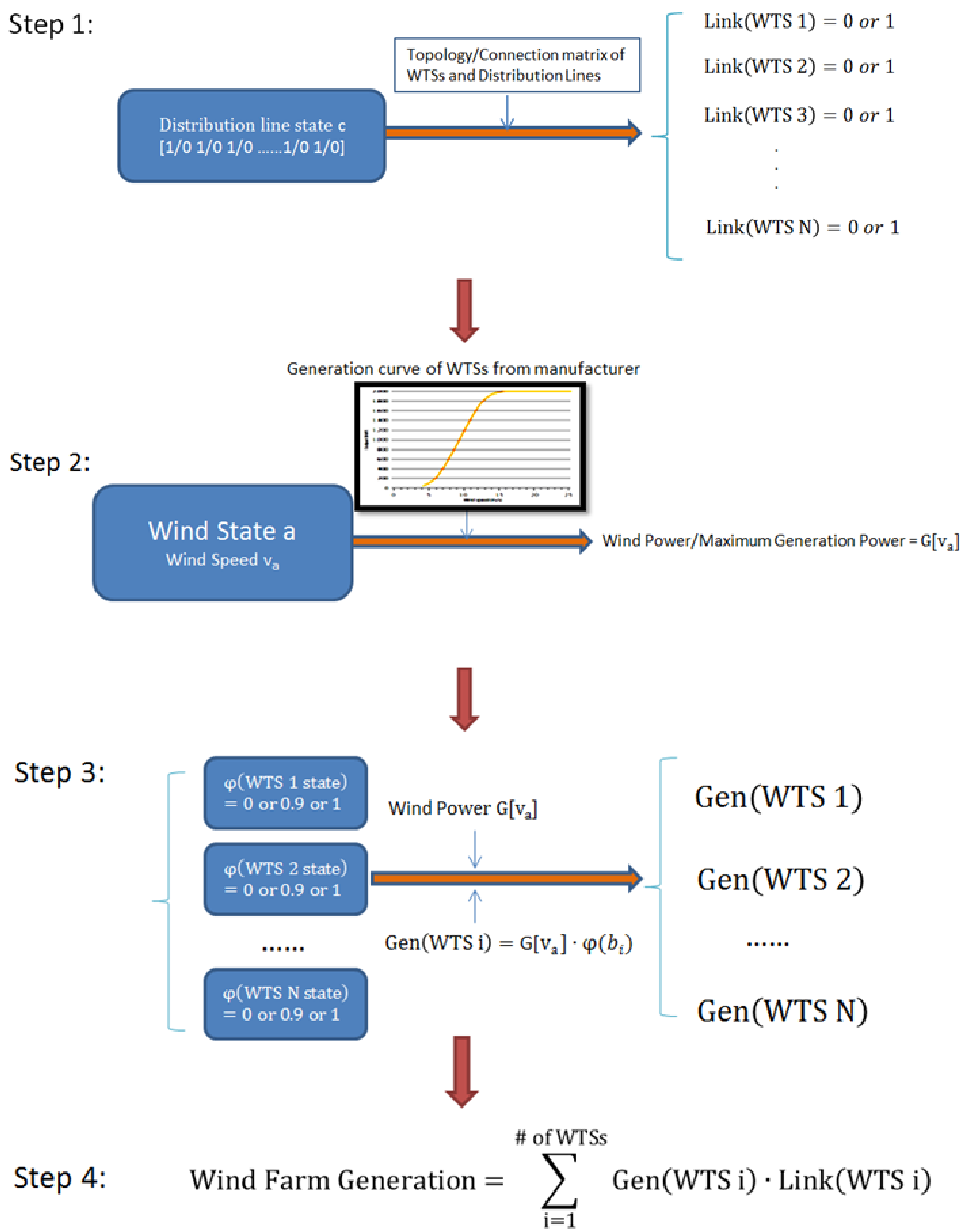


Figure 5.6: Effect Analysis of Combined States

5.4.3 Probability, Transitions and Duration of Combined States

a) Probability

The probability of the combined state is derived by multiplying the probability of all the WTS states with the wind state and the distribution line state.

$$\mathrm{P(combined\ state)} = \left[\prod_{j=1}^{\#\ \mathrm{of\ WTS}} \mathrm{P(WTS\ delivery\ ratio\ state)}\right] \cdot \mathrm{P(distribution\ line\ state)} \cdot \mathrm{P(wind\ state)}$$

b) Transitions

In the combined state space, the transitions between states can be caused by: 1) transitions of WTS delivery ratio states; or 2) transitions of states in wind state space; or 3) transitions of states in distribution line state space. The transition rate in 1) is the result from Figures 5.2 and 5.3 and the entry in the transition rate matrix of the delivery ratio states. The transition rate in 2) is the result from wind transition rate matrix (3.6). The transition rate in 3) is the failure rate or repair rate of the changed distribution line, depending on if the transition is from success to failure or from failure to success.

The transition rate and frequency of transitions of every generation state takes into account all transitions to other generation states.

Given the probability derived in a) and the calculation result of transition rates, the frequency of transitions is derived by (3.4):

$$\mathrm{Frequency}_{m-n} = \frac{n_{mn}}{T} = \frac{n_{mn}}{T_m} \cdot \frac{T_m}{T} = \lambda_{mn} \cdot \frac{T_m}{T} = \lambda_{mn} \cdot \mathrm{P(combined\ state\ m)}$$

in which the probability is the effect analysis result in the last step.

c) Duration

Similarly as in the previous chapter, the duration of state *m* in the combined state space of the wind farm is defined as the ratio of the probability of the state with the sum of frequency values leaving the state.

$$\mathrm{Duration}_{\mathrm{State\ m}} = \frac{\mathrm{P(State\ m)}}{\sum_{\mathrm{n}} \mathrm{Frequency}_{\mathrm{m-j}}}$$

Given the effect analysis result of probability and frequency of transitions above, the duration of the combined state can be derived thereafter.

5.5 Generation States

The generation states of a wind farm refer to the possible generating ranges, and these states are the desired reliability analysis result in this dissertation. The attributes of the generation states are derived from the mapping of the combined state space in last subsection.

By setting a step size for generation output, the generation states are defined by dividing the possible generation capacity of a wind farm into ranges, which is similar approach as in the WTS generation state analysis sections.

In the reliability analysis presented in this dissertation, each of the range forms a generation state. The reliability analysis result of the wind farm is presented in terms of these generation states and the transitions among them.

From last subsection, the combined states are given, and the effect analysis and the calculations of the combined states are performed resulting in the generation outputs, probability, transitions and duration of the combined states. The generation states in this subsection are mapped from the effect analysis result of the combined state, which follows similar procedure as presented in the WTS generation state analysis sections. The determination of the mapped-to generation state is by the judging of the generation output of the combined state.

Given all the mappings of the combined states to the generation states, the values associated with the generation states are derived using the effect analysis result of the combined states. These values include the probability of the generation state, transitions to other generation states, and duration of the generation state.

The probability of the generation state is derived as follows:

$$\mathrm{P(U)} = \sum_{\mathrm{i \in U}} \mathrm{P(Combined\ State\ i)}$$

in which U stands for the generation range considered, and combined state i represents all the combined state that is mapped to generation range U.

The transition rate from generation range U to V is formulated as follows:

$$\mathrm{Transition\ Rate}_{\mathrm{U-V}} = \sum_{\mathrm{i \in U}} \sum_{\mathrm{j \in V}} \lambda_{\mathrm{ij}}$$

in which λ_{ij} is the transition rate from state i in range U to state j in range V.

The frequency from generation range U to V is formulated as follows:

$$\mathrm{Frequency}_{\mathrm{U-V}} = \sum_{\mathrm{j \in V}} \sum_{\mathrm{i \in U}} \mathrm{Frequency}_{\mathrm{i-j}} = \sum_{\mathrm{j \in V}} \sum_{\mathrm{i \in U}} \mathrm{P(Combined\ State\ i)} \cdot \lambda_{\mathrm{ij}}$$

$$= \sum_{\mathrm{i \in U}} \left[\mathrm{P(Combined\ State\ i)} \cdot \sum_{\mathrm{j \in V}} \lambda_{\mathrm{ij}}\right]$$

in which λ_{ij} is the transition rate from state i in range U to state j in range V.

The Duration of generation range U is as follows:

$$\mathrm{Duration\ (U)} = \frac{\mathrm{P(U)}}{\sum_{\mathrm{V}} \mathrm{Frequency}_{\mathrm{U-V}}}$$

in which the probability and the frequency results are the ones demonstrated above.

These results are for the base method. With the result of the base method, dependent-error method can be compared with.

5.6 Chapter Summary

This chapter presents the reliability analysis of wind farms. The presented method for wind farm analysis utilizes the WTS state spaces of every WTS in the wind farm, the wind state space, and the distribution line state space in the reliability analysis. In order to truncate the state space for ease of computation, the WTS delivery ratio states are derived first given the WTS state space. The combined state space is derived by combining the WTS delivery ratio states of each WTS, wind states, and distribution line states. Effect analysis is performed for all combined states to derive the generation output, probability and transitions of the combined states. The combined states are then mapped to the generation states of the wind farm. The results of the reliability model of a wind farm are associated with the generation states of the wind farm, which include the probability, transition rates to other states/ranges, frequency to other states/ranges, and duration.

The reliability analysis results of wind farms serves as critical input for transmission and load feeding planning of bulk power systems.

CHAPTER 6 Example Wind Farm Reliability Analysis

This chapter provides an example of the application of the above reliability models for wind farm.

6.1 Wind Farm System Description

The system configuration is provided in Figure 6.1. There are in total 24 WTSs in this wind farm.

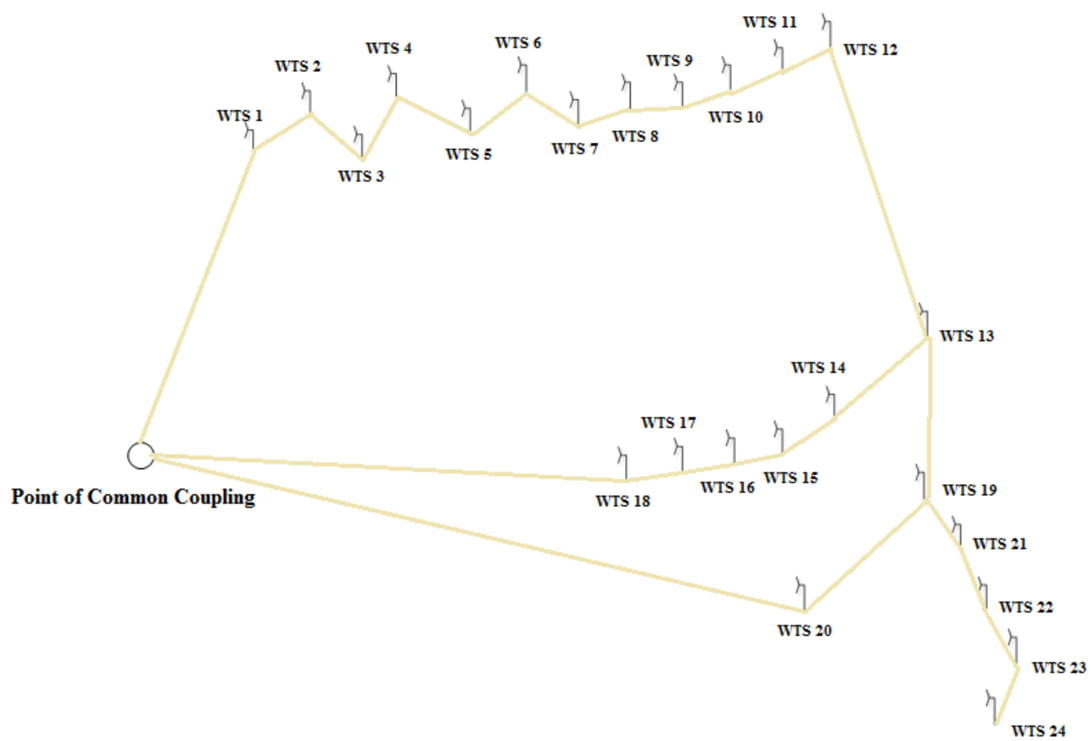


Figure 6.1: System Configuration of the Example Wind Farm

6.1.1 WTS Information

The WTSs in the wind farm are Type 3 and the turbines are Vestas 80. The capacity level it is 2.0 MVA. The generation versus wind speed curve is provided in Figure 6.2, which is obtained from the manufacturer's product brochure [28].

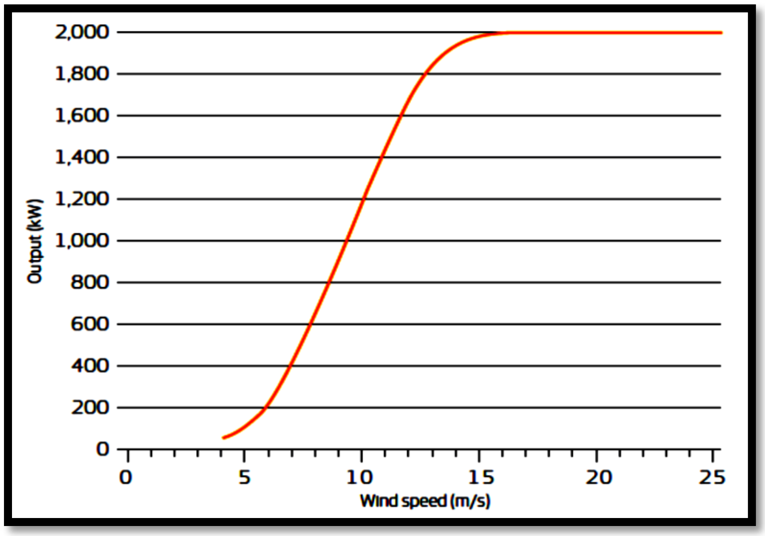


Figure 6.2: Generation Curve of the WTSs in the Example Wind Farm [28]

The list of components and their reliability parameters of the WTS in this example is provided in Table 6.1. Failure rate values are in term of per year, since they indicates the general occurrence of a failure within a year; repair rate values are in term of per hour, since the values are normally obtained by the repair duration given in term of hours.

Table 6.1: List of Components and Their Reliability Parameters in the WTS

	Component Name	Failure Rate (/year)	Repair Rate (/hour)
1	Blades	$\lambda_1 = 0.1$	$\mu_1 = 0.02$
2	Gear Box	$\lambda_2 = 0.2$	$\mu_2 = 0.01$
3	DFIM	$\lambda_3 = 0.1$	$\mu_3 = 0.01$
4	Cables	$\lambda_4 = 0.2$	$\mu_4 = 0.02$
5	Rotor Side Filter	$\lambda_5 = 0.1$	$\mu_5 = 0.02$
6	Rotor Side VSC	$\lambda_6 = 0.1$	$\mu_6 = 0.03$
7	Capacitor	$\lambda_7 = 0.1$	$\mu_7 = 0.02$
8	Energy Storage/Battery	$\lambda_8 = 0.6$	$\mu_8 = 0.1$
9	Grid Side VSC	$\lambda_9 = 0.2$	$\mu_9 = 0.02$
10	Grid Side Filter	$\lambda_{10} = 0.1$	$\mu_{10} = 0.02$
11	Transformer	$\lambda_{11} = 0.1$	$\mu_{11} = 0.01$

6.1.2 Distribution Lines Information

The connection of distribution lines are shown in the wind farm configuration in Figure 6.1. The reliability parameters of the distribution lines are as follows in Table 6.2.

Table 6.2: Reliability Parameters of the Distribution Lines

	Failure Rate (/year)	Repair Rate (/hour)
Parameters of Distribution Lines	$\lambda_{Line} = 0.08$	$\mu_{Line} = 0.02$

6.1.3 Wind Information

Wind data is from Alaska Energy Authority [33] with the wind speeds at 50 meter height and wind directions. The wind data contains the wind information in year 2004 with 8760 data sets. The time step is one hour. Figure 6.3 provides the statistics of the wind data used. The wind speed data used here are the same as used in wind turbine system case study, but the wind direction is not considered in this case. The green line represents the wind speed at 50m height and the blue line shows the wind speed at 30m height. The data at 50m height are the ones used in this case study.

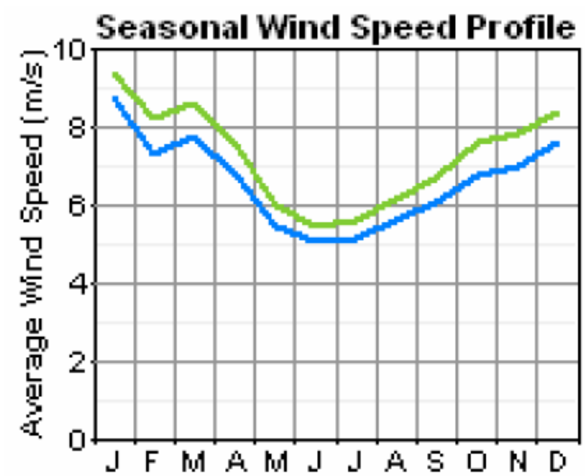


Figure 6.3: Wind Speed Data used in Example Reliability Analysis of Wind Farm [33]

6.2 Distribution Line State Space

Given the parameters of distribution lines, distribution line states are generated using a computer program developed in this research. The state space is in the form as shown in Figure 5.1 in above sections. There are in total $2^{26} = 67108864$ states in the state space, and each state represents a combination of the conditions of the distribution lines.

The probability vector of the distribution line state space is obtained.

$$P_{line} = [P(\text{line state }1);\ P(\text{line state }2);\ \cdots; P(\text{line state }67108864)]$$

For example, the sub-vector of the probability values for the first ten distribution line states is as follows:

$$P_{line}(\text{line state } 1, 2, 3, \dots\ 10) = [P(\text{line state } 1);\ P(\text{line state } 2);\ \cdots; P(\text{line state } 10)] =$$

$$[\,0.98820 \quad 0.00045 \quad 0.00045 \quad 0.00045 \quad 0.00045 \quad 0.00045 \quad 0.00045 \quad 0.00045 \quad 0.00045 \quad 0.00045\,]$$

The transition matrix is in the format as follows:

$$\lambda_{line} =$$

$$\begin{bmatrix} \lambda_{1-1} & \lambda_{1-2} & \lambda_{1-3} & \cdots & \lambda_{1-67108863} & \lambda_{1-67108864} \\ \lambda_{2-1} & \lambda_{2-2} & \lambda_{2-3} & \cdots & \lambda_{2-67108863} & \lambda_{2-67108864} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \lambda_{67108864-1} & \lambda_{67108864-2} & \lambda_{67108864-3} & \cdots & \lambda_{67108864-67108863} & \lambda_{67108864-67108864} \end{bmatrix},$$

in which every entry stands for the transition rate between the states numbered with the row index and column index. The dimension of the transition matrix is 67108864* 67108864. Since this transition matrix is huge, in the computation process the matrix is not necessary to be stored. Based on the assumption that there are no simultaneous failure of two or more distribution lines, the transition in this distribution line state space is simple and intuitive – the transition is caused by the failure or repairing of one distribution line. In this way the use of the transition rates in this matrix follows a judgment of binary state combinations: when λ_{p-q} is requested, the binary number of p and q is compared to see if there is only one digit difference. If yes, which mean there is only one distribution line that had the state change, then λ_{p-q} equals the failure rate of the line when the line changes from 1 to 0, or the repair rate of the line when the line changes from 0 to 1. In this manner, the transition matrix is virtual and the entries in the matrix are either the failure rate of the line or the repair rate of the line, and are sparsely distributed.

6.3 WTS State Space and Delivery Ratio States

Given the component list for Type 3 WTS, the WTS states are generated using a computer program developed in this research. The WTS state space is in the form as shown in Figure 3.3 in above sections. There are in total $2^{11} = 2048$ states in the state space, and each state represents a combination of the conditions of the components in the Type 3 WTS.

Using the method presented in Chapter 5, the delivery ratio states are derived from the WTS state space.

By analyzing all the $2^{11} = 2048$ WTS states in the WTS state space, the delivery ratio states and the attributes of them are obtained. These attributes include the probability, transitions and duration in each delivery ratio states. Figure 6.4 presents the delivery ratio states.

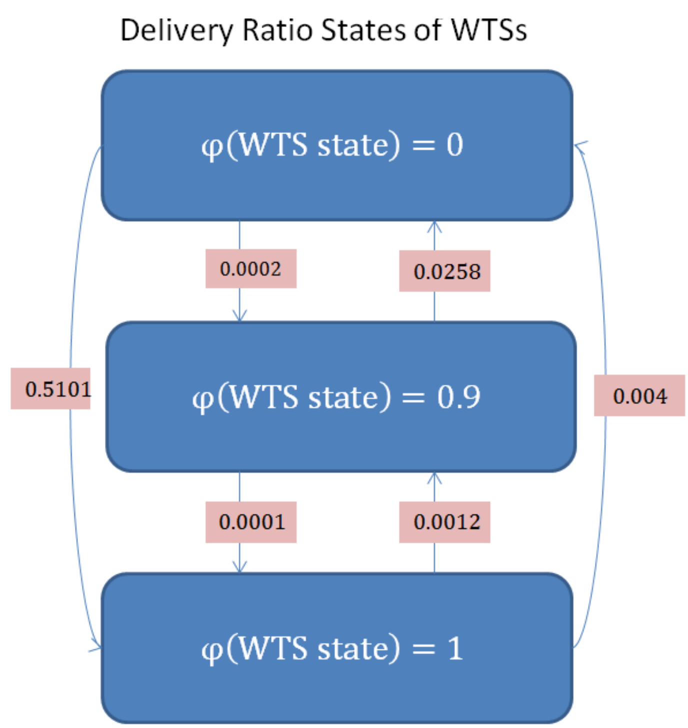


Figure 6.4: Delivery Ratio States of WTSs

The probability vector of the WTS delivery ratio states is obtained.

$$P_{\text{WTS delivery ratio states}} = [P(\text{delivery ratio} = 0);\ P(\text{delivery ratio} = 0.9);\ P(\text{delivery ratio} = 1)]$$

The probability of a delivery ratio state is derived as follows:

$$P(U) = \sum_{i \in U} P(\text{WTS State i})$$

in which U stands for the delivery ratio considered, and WTS state *i* represents the WTS state that has the delivery ratio as U. Given the results of the WTS state space, $P(\text{WTS State i})$ is the *i*th element in the probability vector (3.3) of WTS state space.

The probability vector result is as follows:

$$P_{\text{WTS delivery ratio states}} = [P(\text{delivery ratio} = 0);\ P(\text{delivery ratio} = 0.9);\ P(\text{delivery ratio} = 1)]$$

$$= [0.0089; 0.0012;\ 0.9899]$$

The transition matrix is also obtained.

$$\lambda_{\text{WTS delivery ratio states}} = \begin{bmatrix} \lambda_{0-0} & \lambda_{0-0.9} & \lambda_{0-1} \\ \lambda_{0.9-0} & \lambda_{0.9-0.9} & \lambda_{0.9-1} \\ \lambda_{1-0} & \lambda_{1-0.9} & \lambda_{1-1} \end{bmatrix}$$

in which every entry stands for the transition rate between the states numbered with the row index and column index. The dimension of the transition matrix is 3*3.

The transition rate from delivery ratio state U to V is formulated as follows:

$$\text{Transition Rate}_{U-V} = \sum_{i \in U} \sum_{j \in V} \lambda_{ij}$$

in which λ_{ij} is the transition rate from state i in U to state j in V.

Given the results of the WTS state space, λ_{ij} is the (i,j) entry in the WTS state space transition matrix (3.4).

The transition rate result is as follows:

$$\lambda_{\text{WTS delivery ratio states}} = \begin{bmatrix} \lambda_{0-0} & \lambda_{0-0.9} & \lambda_{0-1} \\ \lambda_{0.9-0} & \lambda_{0.9-0.9} & \lambda_{0.9-1} \\ \lambda_{1-0} & \lambda_{1-0.9} & \lambda_{1-1} \end{bmatrix} = \begin{bmatrix} 1 & 0.0002 & 0.5101 \\ 0.0258 & 1 & 0.0001 \\ 0.0040 & 0.0012 & 1 \end{bmatrix}$$

6.4 Wind State Space

The analysis of the wind data is performed using the developed computer program. Given the wind data of one year, wind states are extracted from these data by identifying the combinations of wind speed and angles.

The wind speed from the given data ranges from 0.40 m/s to 30.85 m/s. The step size of wind speed states is set as 3m/s. There are totally eight wind states as presented in Figure 6.5. The speeds between 21m/s and 30m/s are classified to be the eighth state because it is beyond the cut-off speed of the WTS given the generation curve in Figure 6.2.

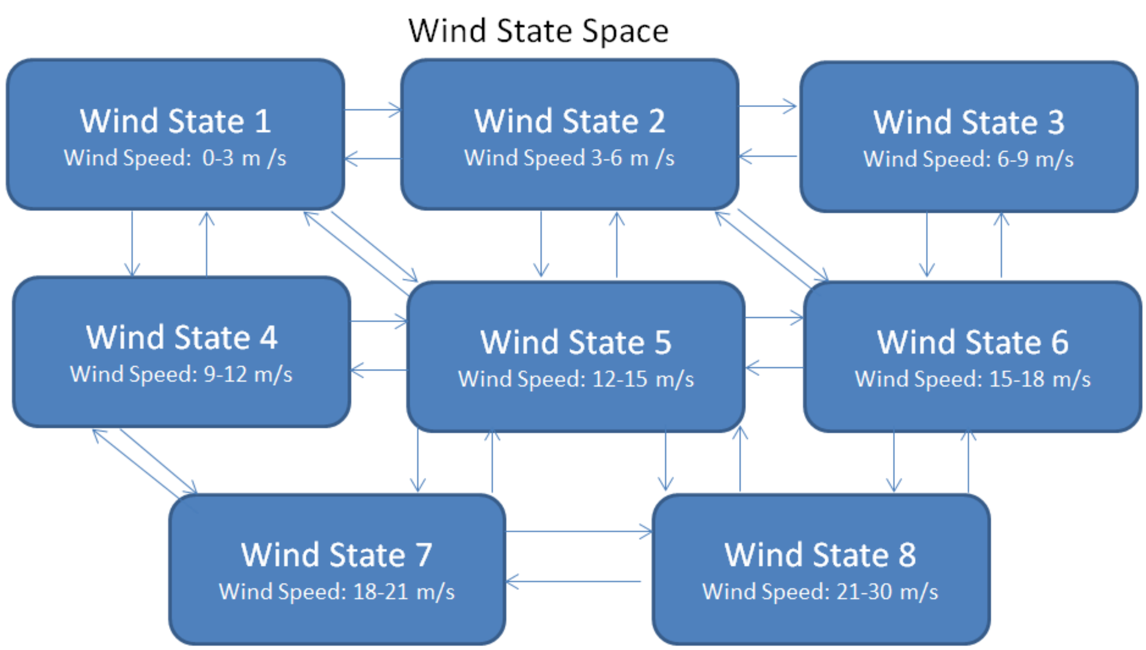


Figure 6.5: Wind State Space in Wind Farm Example

The probability of each wind state is calculated following the frequency principle and is derived from the accumulated counted frequency from the given data.

$$\mathrm{P(Wind\ state\ k)} = \frac{\text{Number of data falling into state k}}{\text{Total Number of Data}}$$

$$= \frac{\text{Number of data falling into state k}}{8760}$$

The probability vector result of the wind states is as follows:

$$P(Wind) = [0.2154; 0.1735; 0.2626; 0.1726; 0.1082; 0.0454; 0.0211; 0.0012]$$

The transition rates are calculated from the frequency of transitions. From the wind data given, the frequency of transitions is extracted by identifying the accumulated number of transitions and then dividing it by the total time span.

$$\lambda_{\mathrm{m-n}} = \frac{\mathrm{Frequency_{m-n}}}{\mathrm{P(Wind\ state\ m)}} = \frac{\frac{\mathrm{n_{mn}}}{\mathrm{T}}}{\mathrm{P(Wind\ state\ m)}} = \frac{\frac{\mathrm{n_{mn}}}{8760}}{\mathrm{P(Wind\ state\ m)}}$$

in which $\lambda_{\mathrm{m-n}}$ is the transition rate from state m to n, T is the total time span given by the data which is 8760 hours, $\mathrm{n_{mn}}$is the total number of transitions from state m to state n, and the probability of wind state m is derived above.

6.5 Combined State Space

Given the wind states, distribution line states and the WTS delivery ratio states, the combined states are derived using the developed computer program. There are in total $8 \cdot 3^{24} \cdot 2^{26}$ states in the combined state space. Figure 6.6 presents the derivation of the combined state space.

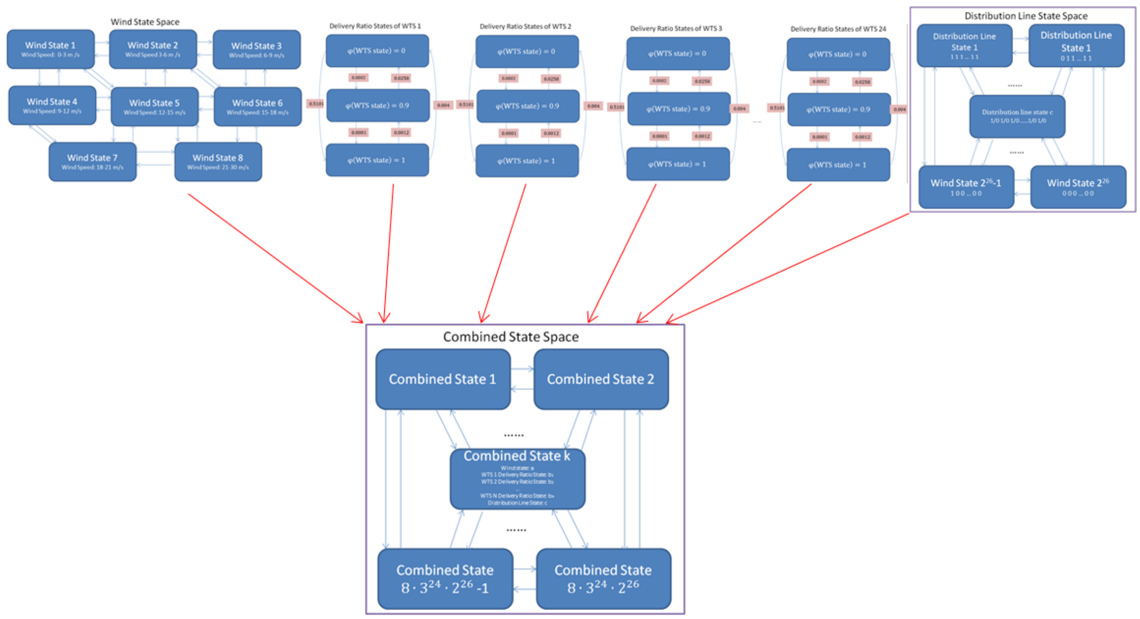


Figure 6.6: Combined State Space in the Example

For each of the combined state, the composition is provided as in Figure 5.4. Each combined state contains a wind state, 24 WTS delivery ratio states, and a distribution line state. The effects analysis for every combined state is performed.

Denoting G as the generation curve in the manufacturer's manual of the WTS in Figure 6.2, an approximation of the generation curve is made as follows:

$$G_{example}(v) = \begin{cases} 0 & v < 5 \\ 200 \cdot (v - 5) & 5 \leq v \leq 15 \\ 2000 & v > 15 \end{cases}$$

in which v is the equivalent wind speed at the location of the WTS.

The process of effect analysis of the combined states is presented in Figure 6.7.

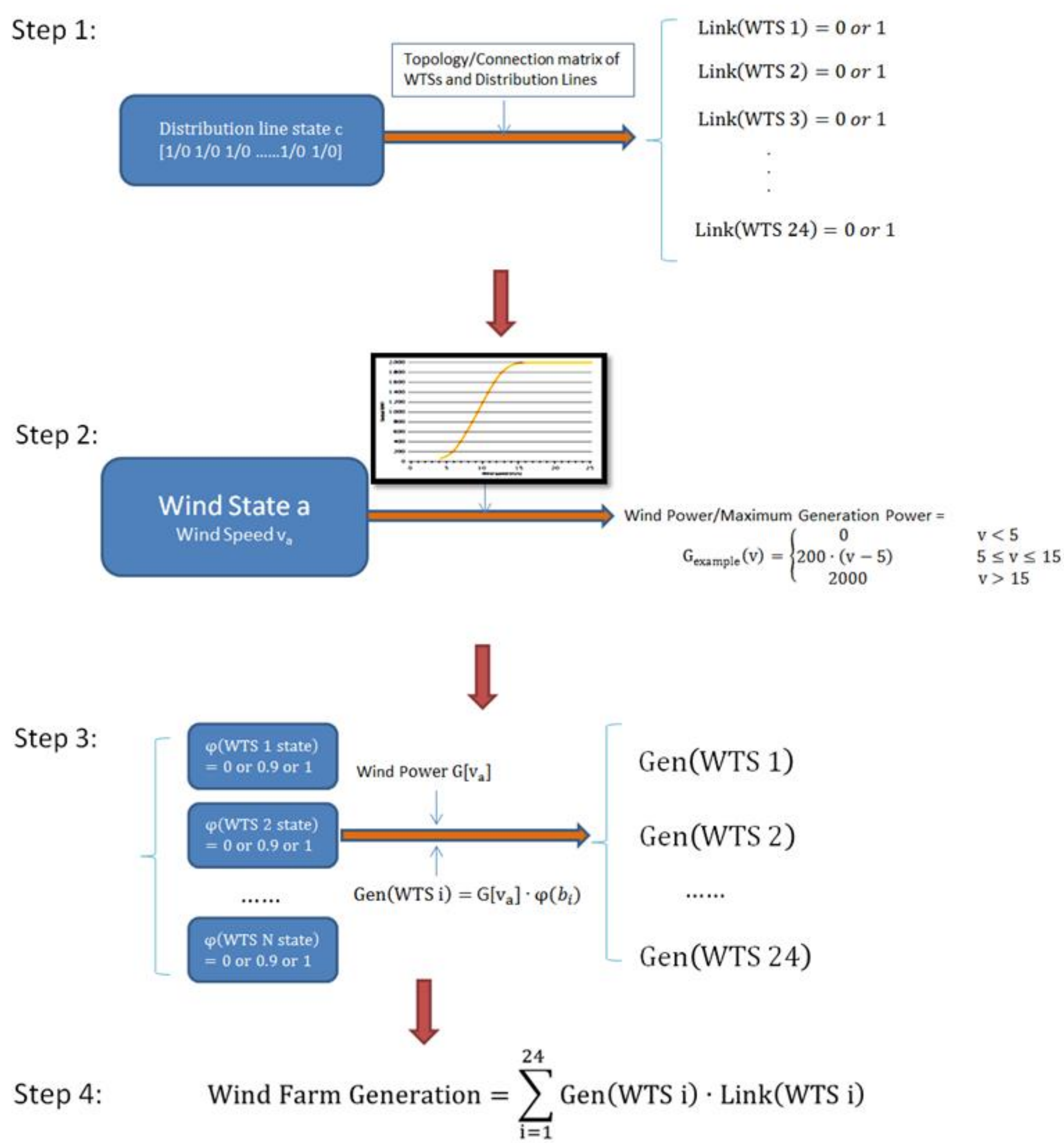


Figure 6.7: Effect Analysis of Combined States in the Example

Combinatory methods are used in the effect analysis process. For the combined states with the same wind state and distribution line state, the combination of delivery ratio states of the 24 WTSs can be categorized into several combinatory scenarios depending on the number of WTSs in the same delivery ratio state. For example, the combined states with 1 WTS in delivery ratio 0, 2 WTS in delivery ratio 0.9, and 21 WTSs in delivery ratio

1 have the same effect when the wind and distribution line state are identical. The 21 WTSs in this case would be one of the C_{24}^{21} combinations. Using combinatory methods, the state space is pruned and the calculation is simplified.

6.6 Generation States of Wind Farm

The derived combined states are mapped to the generation states which represent generation ranges. In this example, the step-size of the generation ranges is selected to be 1000 kW. For the considered wind farm with 24 Type 3 – 2MW WTS, there are totally 49 generation states. Using the above model and the developed computer program, the combined states are mapped to the generation states and the attributes associated with the generation states are calculated. These attributes include the probability of the generation state, transition rates, frequency of transitions, and duration of the generation state. Given all the mappings of the combined states to the generation states, the values associated with the generation states are calculated based on the effects analysis result of the combined states. These values are calculated using the theory of events.

The wind farm reliability analysis result is shown in Figure 6.8. The duration results are in term of hour.

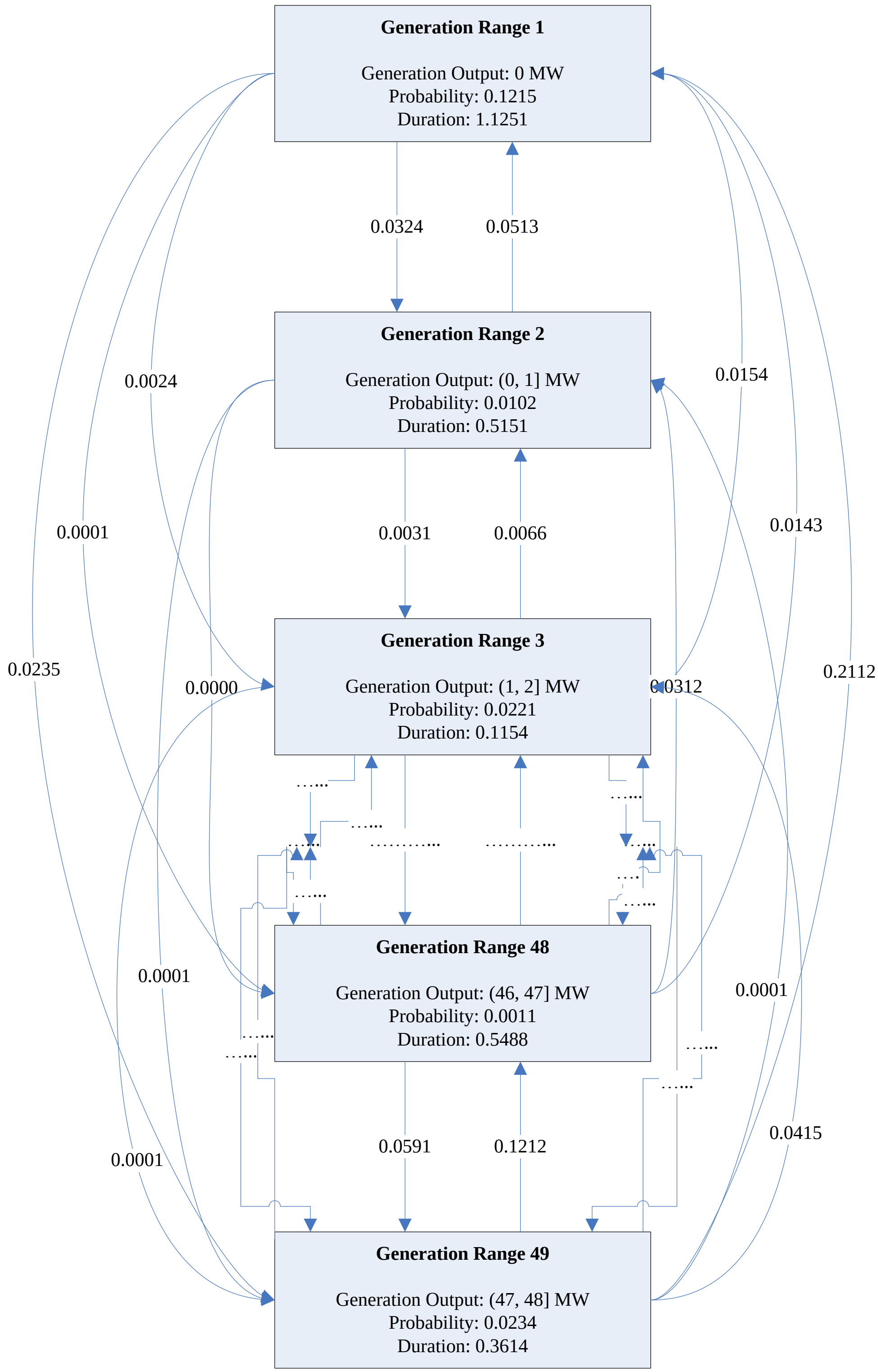


Figure 6.8: Generation States of the Example Wind Farm

The probability values and the duration values of the 49 ranges are presented in Table 6.3. Each range is specified with a probability value and a duration value.

Table 6.3: Probability and duration of the 49 Generation States in the Example

State Index	Generation Range (MW)	Probability	Duration (hour)
1	0	0.1215	1.1251
2	(0,1]	0.0102	0.5151
3	(1,2]	0.0221	0.1154
4	(2,3]	0.0178	0.2952
5	(3,4]	0.0204	0.1799
6	(4,5]	0.0154	0.1743
7	(5,6]	0.0346	0.256
8	(6,7]	0.0355	0.4882
9	(7,8]	0.0363	0.1042
10	(8,9]	0.0174	0.444
11	(9,10]	0.0121	0.155
12	(10,11]	0.0317	0.2766
13	(11,12]	0.0121	0.495
14	(12,13]	0.03	0.0456
15	(13,14]	0.0014	0.4467
16	(14,15]	0.0035	0.4092
17	(15,16]	0.0143	0.3117
18	(16,17]	0.0003	0.0461
19	(17,18]	0.0387	0.4833
20	(18,19]	0.0197	0.338
21	(19,20]	0.0298	0.3904
22	(20,21]	0.0099	0.4421
23	(21,22]	0.0239	0.4929
24	(22,23]	0.001	0.175
25	(23,24]	0.0199	0.3176
26	(24,25]	0.0268	0.364
27	(25,26]	0.0237	0.5994
28	(26,27]	0.0143	0.3378

29	(27,28]	0.0095	0.5502
30	(28,29]	0.0011	0.3462
31	(29,30]	0.0153	0.3973
32	(30,31]	0.0138	0.1115
33	(31,32]	0.0252	0.0697
34	(32,33]	0.0087	0.4652
35	(33,34]	0.0225	0.4261
36	(34,35]	0.0155	0.3697
37	(35,36]	0.0031	0.4701
38	(36,37]	0.0046	0.4373
39	(37,38]	0.0299	0.2466
40	(38,39]	0.0307	0.2159
41	(39,40]	0.0081	0.0349
42	(40,41]	0.0269	0.1218
43	(41,42]	0.0323	0.461
44	(42,43]	0.0007	0.5188
45	(43,44]	0.025	0.1903
46	(44,45]	0.0295	0.0122
47	(45,46]	0.0288	0.3096
48	(46,47]	0.0011	0.5488
49	(47,48]	0.0234	0.3614

The transition rates between the 49 states are derived, and a sub-matrix of the transition rates between the first 10 states is presented in Table 6.4. The entry in element (i,j) stands for the transition rate from state i to state j. Generation state i represents the generation output (i-2, i-1] MW when i is larger than 1. When i is 1, generation state 1 represents the generation output 0MW.

Table 6.4: Transition Rates among the First Ten Generation States in the Example

1	0.0006	5E-05	0.0038	0.0009	0.0057	0.005	0.0059	0.0007	0.0042
0.001	1	6E-05	0.0007	0.0005	0.006	0.0026	0.0074	0.0049	0.0009
0.0003	0.0001	1	0.003	0.0034	0.0057	0	0.0031	0.0063	0.0023
0.0024	0.0002	0.0076	1	0.0014	0.0022	0.0004	0.0076	0.0015	0.0013
0.0031	0.0037	0.0028	0.0042	1	0.0008	0.0018	0.0046	0.006	0.0042
3E-05	0.0078	0.005	0.0046	0.0023	1	0.0016	0.0045	0.0044	2E-05
0.0007	0.0013	0.0065	0.0058	0.0076	0.0079	1	0.0026	0.0057	0.0009
0.0039	0.0005	0.007	0.0042	0.0043	0.002	0.001	1	0.0024	0.0077
0.0036	0.0078	0.0066	0.0045	0.0078	0.0075	0.0027	0.0063	1	0.0053
0.0037	0.0061	0.0022	0.0004	0.0058	0.0077	0.0068	0.0023	0.0025	1

The frequency values of transitions between the 49 generation states are also derived with the transition rate result and the probability vector.

Therefore, the end results of the wind farm reliability analysis are the properties of each state and the properties of each generation state. The properties include the probability, transition rates to other states/ranges, frequency to other states/ranges, and duration.

CHAPTER 7 State Sequence Method in Wind Farm Reliability Analysis Considering Wake Effect

This section provides another method in reliability analysis of wind farms when the turbines are able to be modeled by discrete generation states. The method presented utilizes state sequence of generation output as the tool to perform reliability analysis for wind farms.

The differentiation between this method and the state-space method presented in last chapters is primarily the application scenarios. Both of the methods make use of the generation states of every individual wind turbines, but the method to be presented in this chapter focuses more on the scenario when the number of wind turbines in a wind farm is relatively small, and the reliability analysis targets mostly at obtaining the reliability indices of the wind farm. In addition, wake effect is more convenient to be taken into account in this method. Since the method utilizes the state sequences generated by the WTS analysis in a software environment, it would be easy for the method to be generalized and scaled up using computer platforms. While the state-space probabilistic method proposed in previous chapters serves as the widely applied algorithm for all wind farm analysis.

7.1 Introduction

There have been a few probabilistic methods presented dealing with the reliability analysis of wind farms by peer researchers. [33] has proposed the model of wind farms based on wind turbine state models, and has provided the expressions of basic reliability indices such as Expected Generated Wind Energy (EGWE). The adequacy assessment in

[33] has considered the probabilistic attributes of the turbines in an analytic way. Similarly, a probabilistic aggregation model has been proposed in [34]. The scenarios of having identical wind turbines and different wind turbines, and their respective impacts on the wind farm modeling have been assessed in [35].

Wake effect has been an important problem in the modeling and reliability analysis of wind farms. Wake effect is caused by different geographical distribution of wind turbines. The wind turbines located at downstream of wind are generating less than those at upstream, which results in the imbalance and decrease of wind generation. Several models have been proposed to simplify the wake effect for reliability analysis [23]-[39]. In this dissertation, the specific choice of model of wake effect does not influence the use of the state sequence method proposed.

The state sequence method has been investigated and used in some of the reliability analysis problems for conventional generation sites [40]. The ease of using the state sequence method has enabled faster modeling and computation.

In this dissertation, a state sequence method is presented for wind farms. The method utilizes the generation probability series of wind turbines, in which the uncertainties are involved. When wake effect is considered, the operation of the probability state sequences remains unchanged. The only modification needed in wake case is on the representing states of the total output for the sum sequence. This brings computation efficiency and the convenience to edit the parameters. Reliability analysis is performed using this method, and the required indices are thereafter calculated.

In this dissertation, Section II presents the math definition and operations of state sequences; Section III provides the modeling of wind farm and wake effect; Section IV demonstrates in detail the presented probability state sequence method; Section V presents a case study and Section VI comes with the conclusion.

7.2 State Sequence

State sequence is defined mathematically as a series of values in accordance to the sequence numbers which are non-negative integers 0,1,2,…n. The sequence is named "state sequence" because the values represent the states in their respective sequence location. A state sequence x(i) is as presented in Table 7.1.

Table 7.1: Typical State Sequence

i	0	1	…	n
x	x(0)	x(1)	…	x(n)

For example, a generation site consisting of 3 generators with 5MW power each has a generation state sequence P(i) in Table 7.2.

Table 7.2: Example State Sequence

i	0	1	2	3
P(MW)	0	5	10	15

State sequence can be used to represent a variety of value series associated with the non-negative integers. In this dissertation, the probability values are listed in state sequence, while the states in the sequences can represent the generation capacity at the same time.

7.3 Arithmetic Operations of State Sequences

Let x(i) be a state sequence from state 0 to m, as shown in Table 7.3.

Table 7.3: State Sequence x(i)

i	0	1	…	m
x	x(0)	x(1)	…	x(m)

Let y(j) be another state sequence from state 0 to n, as shown in Table 7.4.

Table 7.4: State Sequence y(i)

j	0	1	…	n
y	y(0)	y(1)	…	y(n)

Define the sum ($\hat{+}$) of two state sequences, $= x \hat{+} y$, as the state sequence with its each state as:

$$z(k) = \sum_{i+j=k} [x(i) \cdot y(j)]$$

where $i=0,1,2,\ldots,m$ and $j=0,1,2,\ldots,n$

The length of the sum state sequence z is: $m+n$, which in other words, $k=0,1,2,\ldots,(m+n)$.

The sum sequence is as provided in Table 7.5.

Table 7.5: Sum Sequence of x and y

k	0	1	…	m+n
z	x(0)y(0)	x(0)y(1)+x(1)y(0)	…	x(m)y(n)

Define the subtraction ($\hat{-}$) of two state sequences, $= x \hat{-} y$, as the state sequence with its each state as:

$$w(k) = \sum_{i-j=k}[x(i) \cdot y(j)] \qquad \text{when k>0}$$

$$\sum_{i \leq j}[x(i) \cdot y(j)] \qquad \text{when k=0}$$

where $i=0,1,2,\ldots,m$ and $j=0,1,2,\ldots,n$

The length of the subtraction state sequence w is m, which in other words, $k=0,1,2,\ldots,m$.

The presented probability state sequence method (PSS) will be introduced and formulated in the following section.

7.4 Probability State Sequence Method for Wind Turbines

For each wind turbine, a probability state sequence is generated based on the generation states. [41] has explained the derivation of the discrete states for a wind turbine. In this dissertation, the focus is the PSS method used for reliability analysis when the wind turbines are modeled with discrete generation states. The methodology of how to model the wind turbines with discrete probabilities is not discussed in detail in this dissertation. The reason that a wind turbine is modeled by finite discrete generation states is the simplicity for reliability analysis. This is similar to the methods applied to a traditional generator for reliability, which normally considers the “up”, “derated”, and “down” states of the generator.

For wind turbines, the probabilities of the discrete generation states represent wind turbine unavailability, which can be caused independently by many factors such as Forced Outage Rate (FOR), mechanical failure rate, and accumulated aging effect. Turbine-to-turbine wind speed differences will likely be small between turbines at the same site;

hence, wind speed is not an independent variable. The PSS in this dissertation assumes a particular wind velocity *v* at a site. In some of the parallel work, the accumulated effect of wind speed variation on wind turbines over a period of time can be modeled as independent factors for different turbines, for which PSS can also be applied when taking wind speed effect into account. For this dissertation, it is assumed that the independency of the state sequences of turbines is resulted from the physical attributes of the turbines such as FORs rather than wind speed effect.

For example, a wind turbine with 2MW capacity can have equivalent output states 0, 0.5, 1, 1.5 and 2MW. The step size is 0.5MW in this case as an example, and the step size along with the number of states is arbitrary which is set according to the required accuracy of the analysis [41].

In this way every wind turbine is modeled with a probability sequence. The probability sequence of a 2MW turbine as an example is provided in Table 7.6.

Table 7.6: Wind Turbine Probability Sequence

i	0	1	2	3	4
x_i	0.1	0.2	0.3	0.3	0.1

where state i represents the output state with 0.5i MW. In the following sections, the represented output states will be provided with the sequence for display, but the represented output states are not into operation.

7.5 Probability State Sequence Method in Reliability Analysis

This section introduces the PSS method in wind farm reliability analysis. The approach of using PSS in reliability analysis does not depend on any specific models, and the impact of considering wake effect will be provided in the next section.

With the operations of state sequences defined in sections above, the output sequence of a wind farm consisting of identical wind turbines is represented as G:

$$G = x \hat{+} y \hat{+} z \hat{+} \dots$$

where x, y, z,…are the state sequences of wind turbines. When wake effect is not considered, these state sequences are identical with same representing output states. For example, if turbines x and y have the following state sequences x(i) and y(i) as shown in Table 7.7.

Table 7.7: Wind Turbine Sequence x and y

i	0	1	2	3	4
Represented P_{out}(MW)	0	0.5	1	1.5	2
x_i	0.1	0.2	0.3	0.3	0.1

i	0	1	2	3	4
Represented P_{out}(MW)	0	0.5	1	1.5	2
y_i	0.1	0.2	0.3	0.3	0.1

The wind farm with only turbines x and y has the output probability sequence $G = x \hat{+} y$ as shown in Table 7.8.

Table 7.8: Sum Sequence of x and y

i	0	1	2	3	4	5	6	7	8
Represented P_{out}(MW)	0	0.5	1	1.5	2	2.5	3	3.5	4
G_i	0.01	0.04	0.1	0.18	0.23	0.22	0.15	0.06	0.01

This sequence G provides the total output distribution of the wind farm.

Reliability indices can thereafter be calculated as follows:

Expected Generated Wind Energy (EGWE) is the expectation of the sequence G in respect to the represented output of the states, which are 0.5*i*.

In this case, EGWE is as follows:

EGWE = $\sum_{i=0}^{8} G_i \cdot 0.5 \cdot i \cdot T$, where T is the time interval.

Another important index that is used in reliability evaluation is the Expected Energy Not Supplied (EENS). This index requires the load profile information. Suppose the load profile is also a probability sequence *L(i)*, and the step size is the same as the wind turbine sequences, *i.e.*, 0.5 in this case. The un-served load state sequence, *U*, is the subtraction of L and G as follows:

$$U = L \mathbin{\hat{-}} G$$

which provides the probability sequence of un-served load states. The index EENS is the expectation of the sequence U in respect to the representing load states, which is:

EENS = $\sum_{i=0}^{N_L} U_i \cdot 0.5 \cdot i \cdot T$, where N_L is the length of sequence *L*, and T is the time interval.

7.6 Probability State Sequence Method Considering Wake Effect

Wake effect is modeled in the sections above as a modification of wind speed in each turbine. By not considering much of the wind speed beyond the cut-off point, the modification of the wind speed does not change the probability sequence of a turbine without wake effect. For example, when not considering the wake effect, if the probability sequence of a turbine is as the sequence *x* above with 0.5MW step size, the consideration of wake effect will bring about the equivalent wind speed v_x as in equation

(1). The capacity is therefore modified to be $F(v_x)$ from the original $F(v)$. Suppose $F(v_x)/F(v) = a$, then the original state sequence with 0.5 MW step size is modified to be the ones shown in Table 7.9.

Table 7.9: State Sequence x when Considering Wake Effect

i	0	1	2	3	4
Represented P_{out}(MW)	0·a	0.5·a	1·a	1.5·a	2·a
x_i	0.1	0.2	0.3	0.3	0.1

in which only the represented outputs are modified. Please note that this discrete state sequence is considered to be representing all the states of a wind turbine, including the ones after cut-off point by equivalent states. If specific problems for wind speeds beyond cut-off point are to be considered, the solution is to be edited by adding the full capacity by the end of the state sequence, which is not included in this dissertation.

An essential problem with summing the modified state sequences is that the step sizes are now changed for each wind turbine. For example, if $G = x \hat{+} y$ where x is the modified sequence above, then for state G(2), the probability state should be equal to G(2) = x(0)y(2)+x(1)y(1)+x(2)y(0). However, the part in the summation expression: x(0)y(2) represents the total output state of 0+1 = 1 MW, x(1)y(1) represents the total output state of 0.5·a+0.5 MW, and x(2)y(0) represents 1·a + 0 =a MW. This has come up with the problem that the state in the sum sequence does not represent a unique output state.

The solution to this problem is to modify the representing states in the sum sequence. Note that the probability of the identical turbine sequences has not been changed when integrating the wake effect, x(0)y(2) = x(2)y(0), and this enables the state G(2) to represent the output state (a+1)/2. Since the sequence is finally used for reliability analysis, the merging of the representing output states does not influence either the total expectation of the sum sequence or the subtraction with load sequence. The general method when considering wake effect and the modification of the representing states is described as below:

If G is the sum sequence of sequence x,y,z…, and if the step sizes of the representing states of x,y,z… are a,b,c…, then the sum operation of the sequences is still performed without consideration of modification, while the final representing states of G have the step size of average(a,b,c…).

Using a 3 sequence (x,y,z) example to explain the above method, denote the representing step size of the 3 sequences as a,b,c, and the sum sequence $G = x \hat{+} y \hat{+} z$ has the state G(3)= x(0)y(0)z(3) + x(0)y(1)z(2) + x(0)y(2)z(1) + x(0)y(3)z(0) + x(1)y(0)z(2) + x(1)y(1)z(1) + x(1)y(2)z(0) + x(2)y(0)z(1) + x(2)y(1)z(0) + x(3)y(0)z(0).

Since x(k)=y(k)=z(k), k=0,1,2,3, the representing states of 3a, 3b and 3c have the same probability; the states of b+2c, 2b+c, a+2c, a+2b, 2a+c and 2a+b have the same probability. Based on the symmetrical attribute of the sequences, it has enabled the merging of the representing states into a+b+c, with the probability G(3). Similarly, the representing state of G(2) is 2(a+b+c)/3, and the representing state of G(1) is (a+b+c)/3. This illustrates that the sum sequence still has the same states as before, while the representing states are modified to have the step size (a+b+c)/3.

This method enables the including of wake effect by only modifying the representing state in the sum sequence, without the need to change the sequence states. This method can increase the computation efficiency, and can bring about ease of coding when doing the reliability analysis or changing parameters during the analysis.

7.7 Case Study

A small wind system with 5 wind turbines is studied as a demonstration of the method presented. The connection of the 5 wind turbines is provided in Figure 7.1. The parallel connection has enabled the independent transmission to PCC from the turbines.

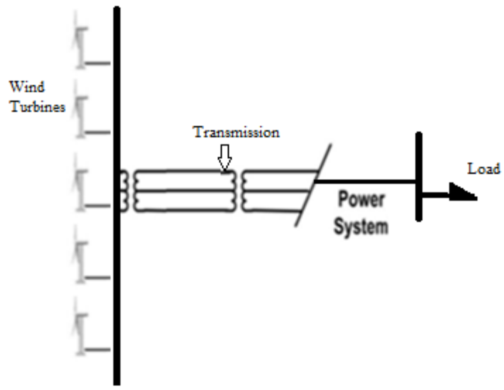


Figure 7.1: Case Study System

The 5 wind turbine probability state sequences, *p, q, w, x, y*, and their corresponding representing states are provided in Table 7.10. Since the turbines are identical, their probability sequences are the same, with different representing states caused by the wake effect.

Table 7.10: The 5 Sequences Used in Case Study

i	0	1	2
Sequence p,q,w,x,y	0.2	0.6	0.2
Representing States of p	0MW	1MW	2MW
Representing States of q	0MW	1MW	2MW
Representing States of w	0MW	0.6MW	1.2MW
Representing States of x	0MW	0.5MW	1MW
Representing States of y	0MW	0.4MW	0.8MW

The sum sequence $G = p \hat{+} q \hat{+} w \hat{+} x \hat{+} y$ is shown in Table 7.11.

Table 7.11: Sum Sequence in the Case Study

i	0	1	…	10
Represented P_{out}(MW)	0·a	1·a	…	10 ·a
G_i	0.00032	0.00096	…	0.00032

where step size of representing states:

a= (1+1+0.6+0.5+0.4)/5 = 0.7.

The reliability index EGWE = $\sum_{i=0}^{10} G_i \cdot 0.7 \cdot i \cdot T$. By letting T=1 hour, the index EGWE = 2.5 MWh.

Suppose the load sequence *L* has the load profile as shown in Table 7.12.

Table 7.12: Load Sequence L

j	0	1	2
Represented Load(MW)	0	0.7	1.4
L_i	0.2	0.5	0.3

Then the reliability index EENS is calculated based on the subtraction sequence $U = L \hat{-} G$ and can be expressed as:

EENS = $\sum_{i=0}^{2} U_i \cdot 0.7 \cdot i \cdot T$ = 0.00048 MWh when T is 1hour.

The presented methods are demonstrated in this case study, and it can be found that the computation efficiency is improved. In addition, if parameters and wake effect profiles are to be changed, it can be easily realized by re-modifying the representing states.

7.8 Conclusion

A probability state sequence method used for reliability analysis of wind farms considering wake effect is presented in this chapter. This method utilizes the independent probability distribution of each wind turbine, and sums easily the identical wind turbines. When wake effect is considered, the original identical wind turbines do not have the same outputs, but they can be still modeled with the same probability state sequences with certain weighed corrections. The reliability analysis in wake effect case only calls for a modification of the representing states for the sum sequence. Similar methods can be applied when doing subtraction to get the un-served energy profile. By using the probability state sequence, the reliability analysis can come with higher computation efficiency and can be subject to easy change of parameters.

CHAPTER 8 Reliability Assessment of Alternate Wind Farm Configurations

This chapter presents the analysis results of alternate wind farm configurations. The reliability analysis over these configurations utilizes probabilistic methods, and comes up with the reliability indices for different configurations.

The relationship and differentiation between the previously proposed state-space probabilistic method and the method in this chapter is provided in Chapter 9. Additional case study is made to demonstrate the connection and difference between the two methods. In general, the state-space probabilistic method proposed in previous chapters serves as a widely applied algorithm for wind farm analysis, and can come up with all the information needed for system planners and operators, including probability, transition rate, frequency and duration. The purpose of the state-space probabilistic method is to provide the system planners and operators with as much information as possible for grid level analysis in bulk power systems which has wind penetration. The analysis presented in this chapter, however, focuses on primarily the assessment over alternate configurations from the reliability perspective, resulting in typical generation states and the probability of the states. Reliability indices are thereafter calculated given the derived generation states, which serve as the major metrics for assessment and comparison among the alternate configurations.

8.1 Introduction

Alternative transmission configurations have been formed by the combination of AC transmission at nominal power frequency, HVDC transmission, and low frequency AC transmission. There have been some literatures discussing the transmission means of wind energy. The application and comparison of AC, HVDC and low-frequency transmission in transmitting wind energy is shown in detail in [45] – [58]. The selection of transmission methods are also influenced by other factors regarding wind generation, such as the estimated wind power to be delivered. When considering wind generation, two main issues should be considered: (a) the cost for transmission of wind power from remote sites where large wind farms can be developed is relatively high, and (b) the unpredictability and associated substantial variations of wind energy that results in low capacity credits from the operation of wind farms.

Low frequency transmission is presented for the purpose of decreasing the cost of transmission and making the wind farm a more reliable power source so that the capacity credit can be increased [45]. Reference [45] proposes a total of 8 different alternative topologies for wind farm configurations and associated transmission for interconnection to the power grid.

8.2 Configurations of Alternate Wind Farm and Interconnections

This section describes the 8 alternative configurations for wind farms and interconnections. Generally, the configurations differ in two major aspects: (a) in-farm connection topology and transmission; (b) out-of-farm transmission and connection.

A combination of in-farm and out-of-farm options makes a configuration unique. The 8 configurations listed in [45] have the following combinations:

- Config. 1: AC Wind Farm (WF), AC Transmission
- Config. 2: AC WF, DC Transmission

- Config. 3: DC Series WF, AC Transmission
- Config. 4: DC Parallel WF, AC Transmission
- Config. 5: DC Series WF(Single Branch), LFAC Trans.
- Config. 6: DC Parallel WF, LFAC Transmission
- Config. 7: DC Series WF(Multiple-Branch), LFAC Trans.
- Config. 8: DC Parallel WF, LFAC Network Transmission

The figures of these configurations will be shown in the following sectons with individual reliability analysis.

The general formulation of the reliability approach is provided in section 8.2, followed by the individual analysis of each of the 8 alternate configurations. For each configuration, the reliability analysis model is developed first, considering both the structural reliability model (full capacity model) and wind variability model. In addition, for each alternative configuration, a 30 wind turbine example is also provided. The reliability parameters used in each of the examples were assumed since actual reliability data are not available. The assumed parameters are shown in Appendix.

8.3 Approach Description

The reliability analysis of any wind farm configuration is performed by assuming that each component of the configuration has a two state Markov model. In other words, each component is characterized with two basic parameters: failure rate and repair rate. Assumed values for these parameters are shown in Appendix. Using these models for each component, the overall reliability of the wind farm is computed. In order to separate and quantify the effect of equipment failures from the variability of the wind energy resource, the overall problem of reliability analysis is separated into two sub-problems: (a) structural reliability model (full capacity model), and (b) wind variability model. The definition and relationship between these two models is provided below.

Structural Reliability Model: This model assumes that the wind energy is 100% reliable. This means that if the wind farm equipment/apparatus are available, the output of the wind farm will be equal to the full capacity of the wind farm. For this reason, we shall also refer to this model as full capacity model.

In this case, wind speed variation is not considered. This is achieved by assuming that all wind turbines will generate their full capacity, e.g., a WTS V80-2MW will always generate 2MW, if available. Generally speaking, the generation capacity of the wind turbines is denoted as G in the following analysis, and some assumptions are made:

- All wind turbines are identical, i.e., they have the same generation capacity;
- All components in the configurations are 2-states components, i.e., they can only be either "up" or "down" states.
- Storage components are not considered in the reliability analysis.
- No transmission constraints are considered.

For each configuration, the specific formulation of reliability analysis calculation is shown in the following sections. Based on the assumptions above, the reliability analysis in full capacity case of each configuration will follow the procedure as:

- Component identification and classification of the configuration;
- Reliability modeling of each component
- Reliability modeling of connections, e.g, parallel or series lines

- Generation capacity probabilistic distribution at PCC
- Reliability indices calculation

Formulations of reliability calculation will be provided after the analysis, and a 30 wind turbine case study will be provided in each section as an example.

Wind Variability Model: This model includes the wind variability as well as the structural reliability model of the wind farm.

In this case, wind speed is considered so that the wind turbines are generating at whatever wind power is available. The reliability index will consider the wind speed distribution.

Wind speed is an important factor that influences the reliability indices especially the adequacy assessment result of the wind farm. The output of a wind turbine is essentially influenced by the wind speed. This subsection provides a general description of the analysis procedure for calculating reliability indices when considering wind speed.

Reference [41] has provided a good analysis of the relationship of wind speed and wind turbine output power. A typical probability distribution of wind speeds is shown in Figure 8.1 created from historical hourly wind data, and a typical WTS power output curve as a function of wind speed is shown in Figure 8.2 for a WTS V80-2MW. Since wind speed is continuously changing, a probabilistic way is used to assess it and to calculate the expected value. In this chapter of analysis, wake effect is ignored.

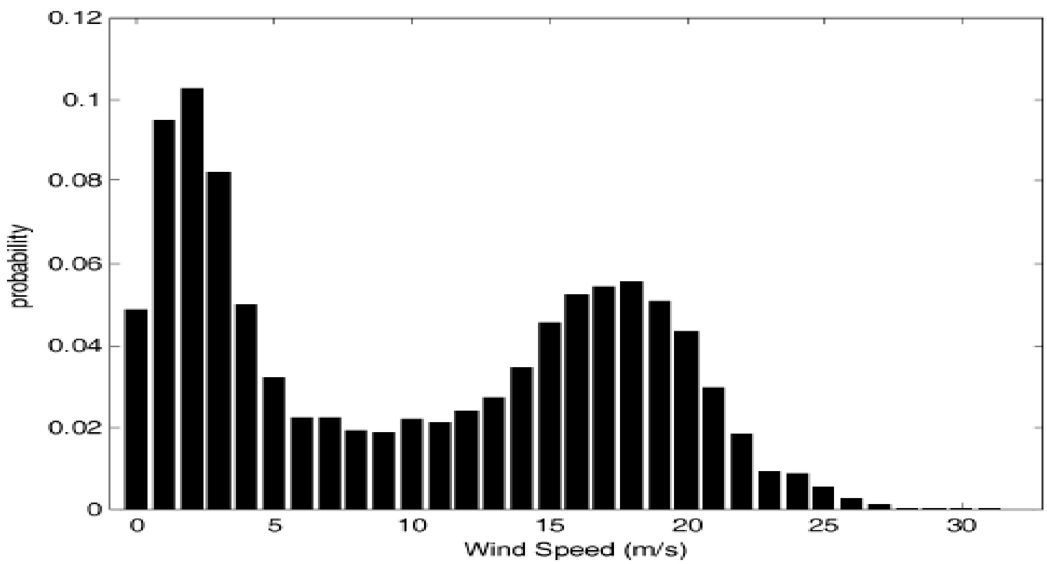


Figure 8.1: Wind Speed Probabilistic Distribution [41]

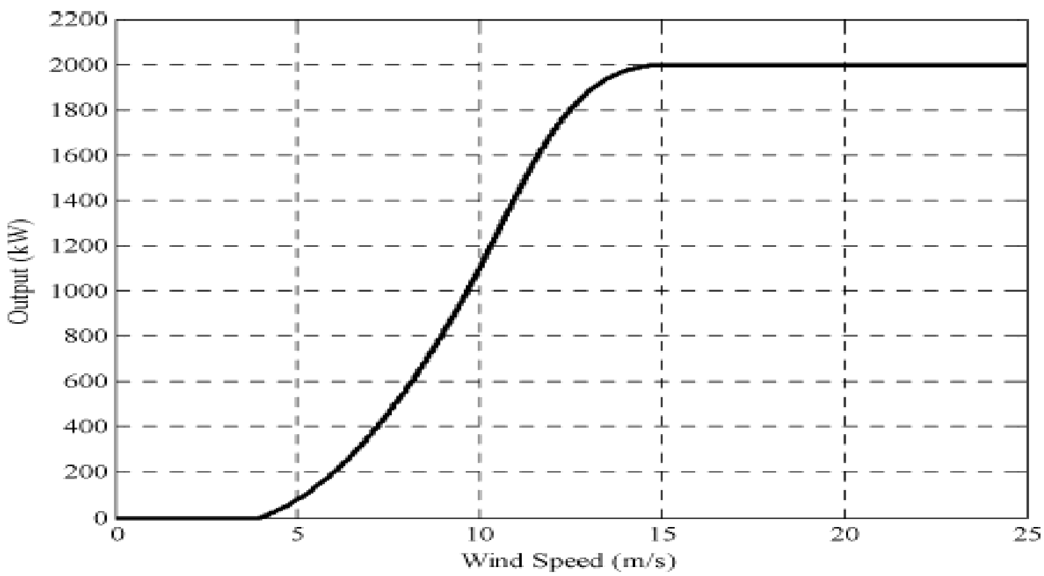


Figure 8.2: A Typical Wind Turbine Output Considering Wind Speed Variation [41]

Figure 8.2 provides the output of wind turbine as a function of the wind speed. When the turbine/generator is in its "up" state, the specific output of the turbine/generator, G(v), is provided from the mapping shown in Figure 8.2, *i.e.* from the wind speed. For each speed v, its probability P(v) is as shown in Figure 8.1.

The expected value of the output of a single turbine is:

$$E(G) = \int_v [G(v) \cdot P(v)] \tag{8.1}$$

In case of discrete wind speed data, the integral becomes the summation:

$$E(G) = \sum_v [G(v) \cdot P(v)] \tag{8.2}$$

The relationship shown in Figure 8.2 can be simplified by approximating the given function with a piecewise linear function, *i.e.*, after cut-in point and before cut-out point of wind speed, the output power of a wind turbine is almost a linear function of the speed, which results in the expression of output function:

$$G(v) = \begin{cases} 0 & v < 5 \\ 200(v-5) & 5 \le v \le 15 \\ 2000 & v > 15 \end{cases} \tag{8.3}$$

which is then used in the evaluation of the integral (8.1) or summation (8.2) above.

Using the single turbine generating system analysis, the effect of wind speed variation is modeled as a modification to the constant generation capacity *G* in the full capacity case, *i.e.*, *G* is substitute by *E(G)* provided by (8.1) or (8.2) in wind variable model.

In the following sections, the wind speed variation case is analyzed for every configuration, provided after the full capacity case.

8.4 Reliability Analysis of Wind Farm Configuration 1

The first configuration is shown in Figure 8.3. In this configuration, the wind turbine systems (WTS) generate AC at variable frequency dictated by the wind speed. This is a simple configuration, without low frequency transmission taken into account.

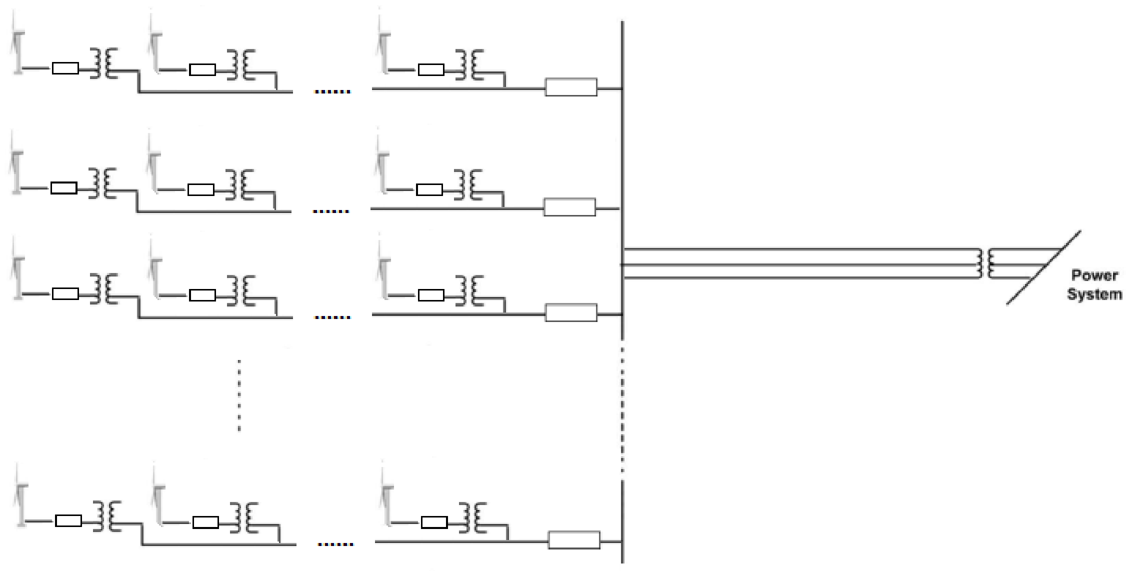


Figure 8.3: Wind Farm Configuration 1

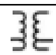
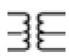
In this configuration, there are m parallel circuits of wind generators, each consisting of $n_1, n_2, \ldots, n_m$ series wind turbines respectively. The wind generator output is AC and all the wind generators in a line are connected in series.

This section describes the reliability analysis of wind farm configuration 1 assuming the wind turbines are working in full capacity condition and in wind speed variation condition. The full capacity case refers to the situation that wind is blowing all the time so that the wind turbines are generating at their full capacity. The wind speed variation case refers to the situation that the wind turbines generate capacity based on the wind speed variation.

The reliability analysis in full capacity case is provided first. The objective of reliability analysis is to provide the probability distribution function of power supply at the point of common coupling (PCC) assuming that the availability of wind energy is 100%. The end result will be in the form of a cumulative probability function of the available power at PCC. This result can be served as the planning and operation index of the power system operator. The wind speed variation case is provided after the full capacity case.

A list of components in Configuration 1 and their quantities is shown in Table 8.1.

Table 8.1: Components and Their Quantities in Configuration 1

	Icon	Component	Total Number of Component
1		Wind Turbine	$N = n_1 + n_2 + \cdots + n_m$
2		Small Switch	$N = n_1 + n_2 + \cdots + n_m$
3		Small Transformer	$N = n_1 + n_2 + \cdots + n_m$
4		In Farm AC Transmission Line	$N = n_1 + n_2 + \cdots + n_m$
5		Large Switch	m
6		AC Bus	1
7		60Hz AC Transmission Line	1
8		Large Substation Transformer	1

In order to simplify the presentation, the following assumptions are made: (a) all the elements of the same component type are identical, *i.e.*, have the same attributes and parameters. For example, there are $N = n_1 + n_2 + \cdots + n_m$ wind turbines in the configuration, and the assumption is that they have identical failure rates and repair rates; (b) every component is assumed to be a two-state element, which means it only has “up” and “down” states. “Up” state represents the successfully working state while the “down” state represents the failure state of the component; and (c) no transmission constraints are taken into account. It should be understood that these assumptions are not necessary in the computer model and analysis.

The reliability parameters of all the components, which include the failure rate and the repair rate, are listed in Appendix. The full capacity of each wind turbine is denoted as *G*.

In each parallel circuit, a number of components are connected in series. The components are “dependent” on each other’s success state from the reliability point of view, *i.e.*, if anyone in the line fails, that line will fail.

Each of the parallel circuits performs “independently” from the others. If one circuit fails, the other circuits will still be able to operate. At the AC collection bus and subsequent transmission part, the components are connected in series. These components are “essential” to the transmission system of the wind farm. If any of the components fails, the wind farm will not be able to transmit any energy to the power grid. The calculation of generation capacity levels and their corresponding probabilities are provided as below.

8.4.1 Reliability Analysis of Configuration 1 in Full Capacity Case

Single Component Probabilistic Model: For component *i*, the duration of success is $\frac{1}{\lambda_i}$ and the duration of failure is $\frac{1}{\mu_i}$. The probabilities of this component’s being success and failure are:

$$P_i(Success) = \frac{\frac{1}{\lambda_i}}{\frac{1}{\lambda_i}+\frac{1}{\mu_i}} = \frac{\mu_i}{\lambda_i+\mu_i} \tag{8.4}$$

$$P_i(Failure) = \frac{\frac{1}{\lambda_i}}{\frac{1}{\lambda_i}+\frac{1}{\mu_i}} = \frac{\lambda_i}{\lambda_i+\mu_i} \tag{8.5}$$

Probabilistic Model of Circuit *i*: The *i*th circuit (of the *m* parallel circuits) is considered. The *i*th circuit will be successful in transmitting energy only when all the components in the line are in their "up" state due to that all generators are serially connected through each parallel line. Therefore, the states of the *i*th circuit and the power transmitted in each state are shown in Table 8.2.

Table 8.2: States of the *i*th Circuit and Power Transmitted in Configuration 1

State	Power Transmitted	Component Status	Probability
Up	$n_i . G$	All components "up"	$P_{line\ i}(Up)$
Down	0	At least one "down"	$P_{line\ i}(Down)$

Line up and line down probabilities can be easily obtained by multiplying the up state probabilities of the components which are connected in series. The success and failure probabilities of each component can be computed from following formula, in the format of (8.4) and (8.5).

Wind Turbine: $(WT\ up) = \frac{\mu_{WT}}{\lambda_{WT}+\mu_{WT}}$; $P(WT\ down) = \frac{\lambda_{WT}}{\lambda_{WT}+\mu_{WT}}$

Small Switch: $(sSW\ up) = \frac{\mu_{sSw}}{\lambda_{sSw}+\mu_{sSw}}$; $P(WT\ down) = \frac{\lambda_{sSw}}{\lambda_{sSw}+\mu_{sSw}}$

Small Transformer: $(sT\ up) = \frac{\mu_{sT}}{\lambda_{sT}+\mu_{sT}}$; $P(T_k\ down) = \frac{\lambda_{sT}}{\lambda_{sT}+\mu_{sT}}$

In Farm AC Line: $P(FAC\ up) = \frac{\mu_{FAC}}{\lambda_{FAC}+\mu_{FAC}}$; $P(WT\ down) = \frac{\lambda_{FAC}}{\lambda_{FAC}+\mu_{FAC}}$

Large Switch: $P(lSW\ up) = \frac{\mu_{lSw}}{\lambda_{lSw}+\mu_{lSw}}$; $P(WT\ down) = \frac{\lambda_{lSw}}{\lambda_{lSw}+\mu_{lSw}}$

Therefore the probability of up state and down state of the *i*th line can be computed from formulas shown below.

$$P_{line\ i}(Up) = [(P(WT\ up) \cdot P(sSw\ up) \cdot P(sT\ up) \cdot P(FAC\ up)]^{n_i} \cdot P(lSW\ up) \qquad (8.6)$$

$$P_{line\ i}(Down) = 1 - P_{line\ i}(Up) \qquad i = 1,2, \ldots, m \qquad (8.7)$$

which can be easily obtained by putting the same factors together assuming that in each category components have the same failure rate of λ and the same repair rate of μ.

$$P_{line\ i}(Up) = [(P(WT\ up) \cdot P(sSw\ up) \cdot P(sT\ up) \cdot P(FAC\ up)]^{n_i} \cdot P(lSW\ up)$$

$$= [\frac{\mu_{WT}}{\lambda_{WT}+\mu_{WT}} \cdot \frac{\mu_{sSw}}{\lambda_{sSw}+\mu_{sSw}} \cdot \frac{\mu_{sT}}{\lambda_{sT}+\mu_{sT}} \cdot \frac{\mu_{FAC}}{\lambda_{FAC}+\mu_{FAC}}]^{n_i} \cdot \frac{\mu_{lSw}}{\lambda_{lSw}+\mu_{lSw}} \tag{8.8}$$

$$P\ (line\ i\ down) = 1 - [\frac{\mu_{WT}}{\lambda_{WT}+\mu_{WT}} \cdot \frac{\mu_{sSw}}{\lambda_{sSw}+\mu_{sSw}} \cdot \frac{\mu_{sT}}{\lambda_{sT}+\mu_{sT}} \cdot \frac{\mu_{FAC}}{\lambda_{FAC}+\mu_{FAC}}]^{n_i} \cdot \frac{\mu_{lSw}}{\lambda_{lSw}+\mu_{lSw}} \tag{8.9}$$

Probabilistic Model of m Parallel Circuits: Since the m circuits are in parallel, the total generating states depend on the success or failure status of each line. For example, if it is the case that $\mathrm{n}_1 = \mathrm{n}_2 = \cdots = \mathrm{n}_\mathrm{m} = \mathrm{n}$, the generation capacity states of the m lines are shown in Table 8.3.

Table 8.3: States of the *m* Parallel Circuits (Lines)

State	Generation Capacity	Probability
1	0	$P_{parallel\_0}$
2	$1 \cdot n \cdot G$	$P_{parallel\_1nG}$
3	$2 \cdot n \cdot G$	$P_{parallel\_2nG}$
…	…	…
m	$(m-1) \cdot n \cdot G$	$P_{parallel\_(m-1)nG}$
m+1	$m \cdot n \cdot G$	$P_{parallel\_mnG}$

The probability of the states is calculated using combinatorial analysis. The *(j+1)* state, which has the generation capacity of $j \cdot n \cdot G$, has the probability:

$$P_{parallel\_jng} = C_m^j \cdot P_{line\ i}^j(up) \cdot P_{line\ i}^{m-j}(down) \tag{8.10}$$

where $C_m^j = \frac{m!}{j!(m-j)!}$ is the combinatorial number.

Probabilistic Model of the Entire Configuration: Given the probabilistic model of the generating section above, i.e., the m parallel circuits (lines), the states of the whole configuration are determined by the generating section in series with the transmission section. As an example, the states and corresponding probability when $\mathrm{n}_1 = \mathrm{n}_2 = \cdots = \mathrm{n}_\mathrm{m} = \mathrm{n}$ are shown in Table 8.4.

Table 8.4: States of Entire Configuration and Probabilities

State	Generation Capacity	Probability
1	0	P_{system_0}
2	$1 \cdot n \cdot G$	P_{system_1nG}
3	$2 \cdot n \cdot G$	P_{system_2nG}
…	…	…
m	$(m\text{-}1) \cdot n \cdot G$	$P_{system\_(m-1)nG}$
$m+1$	$m \cdot n \cdot G$	P_{system_mnG}

The success and failure probabilities of components in transmission section can be computed from the following formula, in the format of (8.4) and (8.5).

AC Bus: $P(ACB\ up) = \frac{\mu_{ACB}}{\lambda_{ACB}+\mu_{ACB}}$; $P(ACB\ down) = \frac{\lambda_{ACB}}{\lambda_{ACB}+\mu_{ACB}}$

60 Hz AC Line: $P(60AC\ up) = \frac{\mu_{60AC}}{\lambda_{60AC}+\mu_{60AC}}$; $P(WT\ down) = \frac{\lambda_{60AC}}{\lambda_{60AC}+\mu_{60AC}}$

Large Substation Transformer: $(lT\ up) = \frac{\mu_{lT}}{\lambda_{lT}+\mu_{lT}}$; $P(lT\ down) = \frac{\lambda_{lT}}{\lambda_{lT}+\mu_{lT}}$

When $j \neq 0$, the probability of the *(j+1)*th state is:

$$P_{system\_jnG} = P_{parallel\_jnG} \cdot P(ACB\ up) \cdot P(60AC\ up) \cdot P(lT\ up)$$

$$= P_{parallel\_jnG} \cdot \frac{\mu_{ACB}}{\lambda_{ACB}+\mu_{ACB}} \cdot \frac{\mu_{60AC}}{\lambda_{60AC}+\mu_{60AC}} \cdot \frac{\mu_{lT}}{\lambda_{lT}+\mu_{lT}}$$

$$= C_m^j \cdot P_{line\ i}^j(up) \cdot P_{line\ i}^{m-j}(down) \cdot \frac{\mu_{ACB}}{\lambda_{ACB}+\mu_{ACB}} \cdot \frac{\mu_{60AC}}{\lambda_{60AC}+\mu_{60AC}} \cdot \frac{\mu_{lT}}{\lambda_{lT}+\mu_{lT}}$$

$$= \frac{m!}{j!(m-j)!} \cdot \left\{\left[\frac{\mu_{WT}}{\lambda_{WT}+\mu_{WT}} \cdot \frac{\mu_{sSw}}{\lambda_{sSw}+\mu_{sSw}} \cdot \frac{\mu_{sT}}{\lambda_{sT}+\mu_{sT}} \cdot \frac{\mu_{FAC}}{\lambda_{FAC}+\mu_{FAC}}\right]^{n_i} \cdot \frac{\mu_{lSw}}{\lambda_{lSw}+\mu_{lSw}}\right\}^j \cdot \{1 - \left[\frac{\mu_{WT}}{\lambda_{WT}+\mu_{WT}} \cdot \frac{\mu_{sSw}}{\lambda_{sSw}+\mu_{sSw}} \cdot \frac{\mu_{sT}}{\lambda_{sT}+\mu_{sT}} \cdot \frac{\mu_{FAC}}{\lambda_{FAC}+\mu_{FAC}}\right]^{n_i} \cdot \frac{\mu_{lSw}}{\lambda_{lSw}+\mu_{lSw}}\}^{m-j} \cdot \frac{\mu_{ACB}}{\lambda_{ACB}+\mu_{ACB}} \cdot \frac{\mu_{60AC}}{\lambda_{60AC}+\mu_{60AC}} \cdot \frac{\mu_{lT}}{\lambda_{lT}+\mu_{lT}} \quad (8.11)$$

This probability is denoted as *P(j+1)*, for all $j \neq 0$.

When $j = 0$, the probability *P(0+1)=P(1)* can be calculated as:

$$P(1) = 1 - \sum_{w=1}^{m} P(w+1) \quad (8.12)$$

where P(w+1) is determined by (8.11)

Therefore in this simple case, the energy levels of the configuration shown in Figure 8.3 at the coupling point with the power grid have a list of states and corresponding probability shown in Table 8.5.

Table 8.5: States and Corresponding Probabilities

State	Generation Capacity	Probability
1	0	$P(1)$
2	$1 \cdot n \cdot G$	$P(2)$
3	$2 \cdot n \cdot G$	$P(3)$
…	…	…
m	$(m-1) \cdot n \cdot G$	$P(\mathrm{m})$
$m+1$	$m \cdot n \cdot G$	$P(\mathrm{m}+1)$

where the probabilities are determined by (8.11) and (8.12).

Expected Generated Wind Energy (EGWE): In this part, the reliability index of EGWE is calculated given the probability analysis as above. EGWE is the essential index to be considered for this configuration. Some of other indices that are important are shown in next part.

Generally, if we know the output states and the probability of each state such as in Table 8.5, EGWE is the expectation of the probabilistic output (in one hour):

$$EGWE = \sum_{i=1}^{number\ of\ states} G_i \cdot P(i) \tag{8.13}$$

where G_i is the generation capacity of the ith state.

Note here that EGWE is in term of energy (MWh), and (8.13) gives the result in term of power. Since operating time is easy to be measured, the required EGWE can be determined by multiplying (8.13) by the operating time (in hours).

For the general case, the assumption of $\mathrm{n}_1 = \mathrm{n}_2 = \cdots = \mathrm{n}_\mathrm{m} = n$ should be removed; each of the m parallel circuits has two states: (a) either transmitting full capacity or (b) 0. For line i, the expected transmitted capacity to the collector AC bus is:

$$EGWE_{line\ i} = n_i \cdot G \cdot P_{line\ i}(up) + 0 \cdot P_{line\ i}(down) = n_i \cdot G \cdot P_{line\ i}(up) \tag{8.14}$$

The total expected available energy at point of common coupling is the index of EGWE:

$$EGWE = \left[\sum_{i=1}^{m} n_i \cdot G \cdot P_{line\ i}(up)\right] \cdot P(ACB\ up) \cdot P(60AC\ up) \cdot P(lT\ up) + 0$$

$$= \left[\sum_{i=1}^{m} n_i \cdot G \cdot P_{line\ i}(up)\right] \cdot P(ACB\ up) \cdot P(60AC\ up) \cdot P(lT\ up)$$

$$= (\sum_{i=1}^{m}\{n_i \cdot G \cdot \left[\frac{\mu_{WT}}{\lambda_{WT}+\mu_{WT}} \cdot \frac{\mu_{sSw}}{\lambda_{sSw}+\mu_{sSw}} \cdot \frac{\mu_{sT}}{\lambda_{sT}+\mu_{sT}} \cdot \frac{\mu_{FAC}}{\lambda_{FAC}+\mu_{FAC}}\right]^{n_i} \cdot \frac{\mu_{lSw}}{\lambda_{lSw}+\mu_{lSw}}\}) \cdot \frac{\mu_{ACB}}{\lambda_{ACB}+\mu_{ACB}} \cdot \frac{\mu_{60AC}}{\lambda_{60AC}+\mu_{60AC}} \cdot \frac{\mu_{lT}}{\lambda_{lT}+\mu_{lT}} \tag{8.15}$$

8.4.2 Reliability Analysis of Configuration 1 Considering Wind Speed

Using the single turbine generating system analysis, the effect of wind speed variation is modeled as a modification to the constant generation capacity *G* in the full capacity case.

The total output expectation at a certain point of time is still in the form of $EGWE = (\sum_{i=1}^{m}\{n_i \cdot G \cdot \left[\frac{\mu_{WT}}{\lambda_{WT}+\mu_{WT}} \cdot \frac{\mu_{sSw}}{\lambda_{sSw}+\mu_{sSw}} \cdot \frac{\mu_{sT}}{\lambda_{sT}+\mu_{sT}} \cdot \frac{\mu_{FAC}}{\lambda_{FAC}+\mu_{FAC}}\right]^{n_i} \cdot \frac{\mu_{lSw}}{\lambda_{lSw}+\mu_{lSw}}\}) \cdot \frac{\mu_{ACB}}{\lambda_{ACB}+\mu_{ACB}} \cdot \frac{\mu_{60AC}}{\lambda_{60AC}+\mu_{60AC}} \cdot \frac{\mu_{lT}}{\lambda_{lT}+\mu_{lT}}$ as in equation (8.15), but *G* is changed into G(v) at that time point as the function of the wind speed. For a given period of time with the wind speed probability distribution as Figure 8.3, the expectation of output energy will be:

$$EGWE = E(G) = \sum_{v}[G(v) \cdot P(v)] \cdot (\sum_{i=1}^{m}\{n_i \cdot \left[\frac{\mu_{WT}}{\lambda_{WT}+\mu_{WT}} \cdot \frac{\mu_{sSw}}{\lambda_{sSw}+\mu_{sSw}} \cdot \frac{\mu_{sT}}{\lambda_{sT}+\mu_{sT}} \cdot \frac{\mu_{FAC}}{\lambda_{FAC}+\mu_{FAC}}\right]^{n_i} \cdot \frac{\mu_{lSw}}{\lambda_{lSw}+\mu_{lSw}}\}) \cdot \frac{\mu_{ACB}}{\lambda_{ACB}+\mu_{ACB}} \cdot \frac{\mu_{60AC}}{\lambda_{60AC}+\mu_{60AC}} \cdot \frac{\mu_{lT}}{\lambda_{lT}+\mu_{lT}} \tag{8.16}$$

It is shown from (8.16) that the influence of wind speed on the calculation of the expectation is simply a modification by substituting the constant turbine output by the expectation of that turbine output.

The expected value of the total output is the index of EGWE in a single second. This index is an averaging assessment of the output power of the wind farm, which can be used as reference for power system planning and operation control. The next section will include the calculation of most of the other indices.

8.4.3 Reliability Indices Calculation

This section describes the calculation method of most reliability indices associated with the wind farm, taking configuration 1 as the analysis objective. The definition and the formula of these indices, including EGWE, are shown in Table 8.6.

Table 8.6: Definition and Expression of Reliability indices

Reliability Index	*Expression*
Installed wind power (IWP)	IWP=∑ nominal power of turbines in wind farm
Installed wind energy (IWE)	IWE = Installed Capacity ·Number of Operating Hours
Expected available wind energy (EAWE)	EAWE=∑ Energy produced by turbines Note: components' possible failure is not included
Expected generated wind energy (EGWE)	EGWE= ∑ Energy effectively available by turbines Note: components's failure is considered
Capacity factor (CF)	CF=EGWE / IWE
Generation Ratio (GR)	GR= power delivered to PCC / IWP wind speed and failure of components are considered.

Expected Generated Wind Energy (EGWE) and Expected Available Wind Energy (EAWE):

In this part, the reliability indices of EGWE and EAWE will be calculated.

EGWE is discussed much in detail in the sections above, including the calculation methods both in full capacity case and wind speed variation case. Formula (8.15) and (8.16) have provided the expression of EGWE.

EAWE stands for *Expected Available Wind Energy,* which is the index to evaluate the available wind power, *i.e.*, the available wind energy in a second. Commonly, wind speed variation is considered.

$$EAWE = \sum Energy\ produced\ by\ turbines = N \cdot E(G) \qquad (8.17)$$

where E(G) is the expected turbine output determined by (8.1) or (8.2). In this case, the wind speed variation is considered, but the transmission failure due to the failure of components is not considered. This is how EAWE is different from EGWE.

Installed Wind Power (IWP), Installed Wind Energy(IWE), Capacity Factor (CF), and Generation Ratio (GR):

In this part, the reliability indices of IWP, IWE, CF and GR are calculated based on the result of EGWE and the probability analysis provided above.

Given the definitions and expressions in Table 13.2.6, the indices are calculated for configuration 1 as:

$$\mathrm{IWP} = \sum nominal\ power\ of\ turbines\ in\ wind\ farm = N \cdot G \qquad (8.18)$$

where $\mathrm{N} = \mathrm{n}_1 + \mathrm{n}_2 + \cdots + \mathrm{n}_\mathrm{m}$ is the total number of wind turbines in the farm, and G is the nominal capacity of a single wind turbine in full capacity case.

$$\mathrm{IWE} = \sum \text{Installed Capacity } \cdot \text{\# of Operating Hours} = IWP \cdot t = N \cdot G \cdot t \quad (8.19)$$

where t is the number of operating time in hours.

$$CF = EGWE\,/\,IWE \qquad (8.20)$$

$$GR = (EGWE/t)/\,IWP \qquad (8.21)$$

where EGWE is the index calculated by (8.11) with both wind speed variation and transmission failure considered.

8.4.4 Case Study: A 30 WT Case of Configuration 1

An example reliability analysis is shown in this section, using the configuration in Figure 8.4. The parameters of the configuration are: (a) Number of parallel lines m=3; (b) Number of wind turbines in each Line: $n_1 = n_2 = n_3 = 10$; (c) Generating Capacity of each Turbine: G = 2MW. Wind speed distribution and the wind turbine output function are shown in Figures 8.1 and 8.2. The configuration in this case study is shown in Figure 8.4.

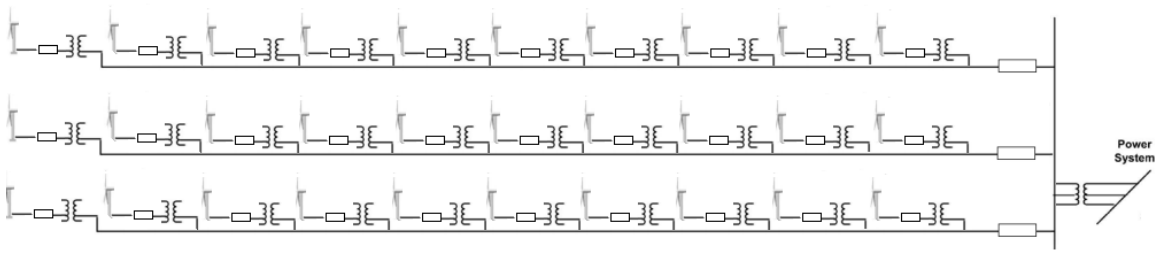


Figure 8.4: A 30 WT Case Study Configuration for Configuration 1

Reliability parameters used for this example calculation is shown in Appendix.

Full Capacity Case: In full capacity case, the calculation result of the output probabilistic distribution is shown in Table 8.7. The essential reliability index EGWE = 49.42 MWh in one hour.

Table 8.7: Case Study Output States and Probabilities

State	Generation Capacity(MW)	Probability
1	0	0.06291222
2	8	0.00861636
3	16	0.08026351
4	24	0.33229952

5	32	0.51590836

The other reliability indices are calculated as follows:

$EAWE = \sum Energy\ produced\ by\ turbines = N \cdot E(G) = 30 \cdot 2 = 60$ MW in one hour.

Note here the indices of EGWE and EAWE is calculated in one hour basis. In order to get the indices within a certain period, the operating time should be multiplied. For example, if the indices for a year is needed, the calculation of EGWE and EAWE should be $25.8374MW \cdot 8760\ hours$ and $60MW \cdot 8760\ hours$ respectively. IWP = $\sum nominal\ power\ of\ turbines\ in\ wind\ farm = N \cdot G = 30 \cdot 2 = 60MW$

IWE $= IWP \cdot t = N \cdot G \cdot t = 60MW \cdot t$ =60 MWh in one hour

where t is the number of operating time in hours.

CF=EGWE / IWE = 0.824

*GR= EGWE / IWP=*49.42/60= 82.4%

Wind Speed Variation Case: When wind speed is considered, the single wind turbine output function is shown in (8.1). From the wind speed probability distribution, the expected output of a single wind turbine becomes EGWE = $\sum_v [G(v) \cdot P(v)] = 0.93$MWh. From (13.2.13), EGWE = $\sum_v [G(v) \cdot P(v)] \cdot \sum_{i=1}^{m} [n_i \cdot \left(\prod_{k=1}^{4} \frac{\mu_k}{\lambda_k + \mu_k}\right)^{n_i} \cdot \frac{\mu_5}{\lambda_5 + \mu_5}] \cdot \prod_{q=6}^{12} \frac{\mu_q}{\lambda_q + \mu_q}$=22.98MWh, which is the reliability index EGWE in one hour.

The result shows the expected generation capacity that can be transmitted to the point of common coupling. This expected value can be used for system planning and operation.

$EAWE = \sum Energy\ produced\ by\ turbines = N \cdot E(G) = 30 \cdot 0.93 = 27.9$ MWh in one hour.

IWP = $\sum nominal\ power\ of\ turbines\ in\ wind\ farm = N \cdot G = 30 \cdot 2 = 60MW$

IWE $= IWP \cdot t = N \cdot G \cdot t = 60MW \cdot t$ = 60MWh in one hour, where t is the number of operating hours.

$CF = EGWE \,/\, IWE =$ 22.98/60 = 0.383

$GR = EGWE \,/\, IWP =$ 22.98/60= 38.3%

The results of the reliability analysis for alternate configuration 1 are summarized in Table 8.8.

Table 8.8: Summary of Reliability Indices for Alternate Configuration 1

Reliability and Cost Index	*Computed Value*
Installed wind power (IWP)	60 MW
Installed wind energy (IWE)	60 MWh in one hour
Expected available wind energy (EAWE)	27.9 MWh in one hour
Expected generated wind energy (EGWE)	22.98MWh in one hour
Capacity factor (CF)	0.383
Generation Ratio (GR)	38.3 %

8.5 Reliability Analysis of Wind Farm Configuration 2

This section provides the reliability analysis and case study result for configuration 2.

Figure 8.5 shows configuration 2. Configuration 2 has AC Wind Farm and DC Transmission.

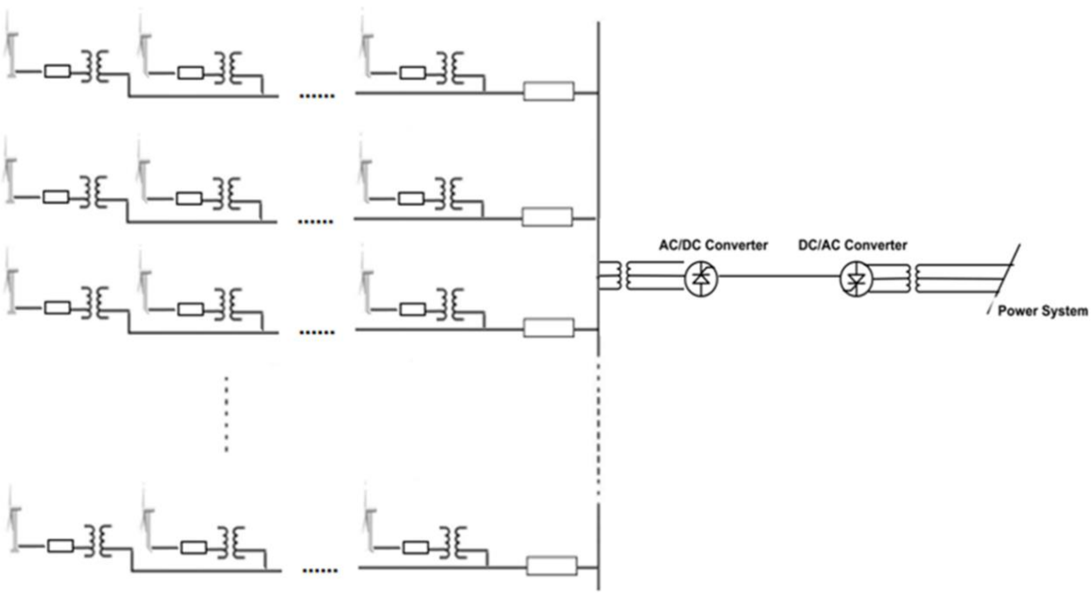


Figure 8.5: Wind Farm Configuration 2

The formulation of the reliability analysis for this configuration is as below. The probabilistic methods used for this formulation are exactly the ones shown in the analysis of configuration 1.

8.5.1 Probabilistic Model of Circuit *i*

This configuration has *m* parallel lines, each line consisting of *n* wind turbines. The probability of the states of each line is provided as below.

$$P_{line\ i}(Up) = \left[P_{WT}^{n_i}(up) \cdot P_{sSW}^{n_i}(up) \cdot P_{sST}^{n_i}(up) \cdot P_{FAC}^{n_i}(up)\right] \cdot P_{lSW}(up)$$

$$= \left[\frac{\mu_{WT}}{\lambda_{WT}+\mu_{WT}} \cdot \frac{\mu_{sSw}}{\lambda_{sSw}+\mu_{sSw}} \cdot \frac{\mu_{sT}}{\lambda_{sT}+\mu_{sT}} \cdot \frac{\mu_{FAC}}{\lambda_{FAC}+\mu_{FAC}}\right]^{n_i} \cdot \frac{\mu_{lSw}}{\lambda_{lSw}+\mu_{lSw}}$$

$P_{line\ i}(Down) = 1 - P_{line\ i}(Up)$

$$= 1 - \left[\frac{\mu_{WT}}{\lambda_{WT}+\mu_{WT}} \cdot \frac{\mu_{sSw}}{\lambda_{sSw}+\mu_{sSw}} \cdot \frac{\mu_{sT}}{\lambda_{sT}+\mu_{sT}} \cdot \frac{\mu_{FAC}}{\lambda_{FAC}+\mu_{FAC}}\right]^{n_i} \cdot \frac{\mu_{lSw}}{\lambda_{lSw}+\mu_{lSw}}$$

8.5.2 Probabilistic Model of m Parallel Circuits

The *m* lines are in parallel, and the probability of having *j* lines up is shown as below.

$P_{parallel\_jng} = C_m^j \cdot P_{line\ i}^j(up) \cdot P_{line\ i}^{m-j}(down)$

where $C_m^j = \frac{m!}{j!(m-j)!}$ is the combinatorial number.

8.5.3 Probabilistic Model of the Entire Configuration

The output states at the point of common coupling (PCC) are similar to the ones in configuration 1. At PCC, there are *m+1* states, the delivered capacity of which is similar to that in the analysis of Configuration 1. The 1st state represents the situation when there is no capacity successfully delivered.

When $j \neq 0$, the probability of the *(j+1)*th state is:

$P_{system\_jnG} = P_{parallel\_jnG} \cdot P(ACB\ up) \cdot P(lT\ up) \cdot P(lAC/DC\ up) \cdot P(DC\ up) \cdot P(DC/AC\ up) \cdot P(lT\ up)$

$= \mathrm{P}_{\mathrm{parallel\_jnG}} \cdot \frac{\mu_{ACB}}{\lambda_{ACB}+\mu_{ACB}} \cdot \frac{\mu_{lT}}{\lambda_{lT}+\mu_{lT}} \cdot \frac{\mu_{lAC/DC}}{\lambda_{lAC/DC}+\mu_{lAC/DC}} \cdot \frac{\mu_{DC}}{\lambda_{DC}+\mu_{DC}} \cdot \frac{\mu_{DC/AC}}{\lambda_{DC/AC}+\mu_{DC/AC}} \cdot \frac{\mu_{lT}}{\lambda_{lT}+\mu_{lT}}$

$= C_m^j \cdot P_{line\ i}^j(up) \cdot P_{line\ i}^{m-j}(down) \cdot \frac{\mu_{ACB}}{\lambda_{ACB}+\mu_{ACB}} \cdot \frac{\mu_{lT}}{\lambda_{lT}+\mu_{lT}} \cdot \frac{\mu_{lAC/DC}}{\lambda_{lAC/DC}+\mu_{lAC/DC}} \cdot \frac{\mu_{DC}}{\lambda_{DC}+\mu_{DC}} \cdot \frac{\mu_{DC/AC}}{\lambda_{DC/AC}+\mu_{DC/AC}} \cdot \frac{\mu_{lT}}{\lambda_{lT}+\mu_{lT}}$

$= \frac{m!}{j!(m-j)!} \cdot \left\{\left[\frac{\mu_{WT}}{\lambda_{WT}+\mu_{WT}} \cdot \frac{\mu_{sSw}}{\lambda_{sSw}+\mu_{sSw}} \cdot \frac{\mu_{sT}}{\lambda_{sT}+\mu_{sT}} \cdot \frac{\mu_{FAC}}{\lambda_{FAC}+\mu_{FAC}}\right]^{n_i} \cdot \frac{\mu_{lSw}}{\lambda_{lSw}+\mu_{lSw}}\right\}^j \cdot \{1 - \left[\frac{\mu_{WT}}{\lambda_{WT}+\mu_{WT}} \cdot \frac{\mu_{sSw}}{\lambda_{sSw}+\mu_{sSw}} \cdot \frac{\mu_{sT}}{\lambda_{sT}+\mu_{sT}} \cdot \frac{\mu_{FAC}}{\lambda_{FAC}+\mu_{FAC}}\right]^{n_i} \cdot \frac{\mu_{lSw}}{\lambda_{lSw}+\mu_{lSw}}\}^{m-j} \cdot \frac{\mu_{ACB}}{\lambda_{ACB}+\mu_{ACB}} \cdot \frac{\mu_{lT}}{\lambda_{lT}+\mu_{lT}} \cdot \frac{\mu_{lAC/DC}}{\lambda_{lAC/DC}+\mu_{lAC/DC}} \cdot \frac{\mu_{DC}}{\lambda_{DC}+\mu_{DC}} \cdot \frac{\mu_{DC/AC}}{\lambda_{DC/AC}+\mu_{DC/AC}} \cdot \frac{\mu_{lT}}{\lambda_{lT}+\mu_{lT}}$

This probability is denoted as *P(j+1)*, for all $j \neq 0$.

When $j = 0$, the probability *P(0+1)=P(1)* can be calculated as:

$$P(1) = 1 - \sum_{w=1}^{m} P(w+1)$$

8.5.4 Expected Generated Wind Energy (EGWE)

The total expected available energy at point of common coupling is the index of EGWE:

$$EGWE = \left[\sum_{i=1}^{m} n_i \cdot G \cdot P_{line\ i}(up)\right] \cdot P(ACB\ up) \cdot P(lT\ up) \cdot P(lAC/DC\ up) \cdot P(DC\ up) \cdot P(DC/AC\ up) \cdot P(lT\ up) + 0$$

$$= \left[\sum_{i=1}^{m} n_i \cdot G \cdot P_{line\ i}(up)\right] \cdot P(ACB\ up) \cdot P(lT\ up) \cdot P(lAC/DC\ up) \cdot P(DC\ up) \cdot P(DC/AC\ up) \cdot P(lT\ up)$$

$$= (\sum_{i=1}^{m}\{n_i \cdot G \cdot \left[\frac{\mu_{WT}}{\lambda_{WT}+\mu_{WT}} \cdot \frac{\mu_{sSw}}{\lambda_{sSw}+\mu_{sSw}} \cdot \frac{\mu_{sT}}{\lambda_{sT}+\mu_{sT}} \cdot \frac{\mu_{FAC}}{\lambda_{FAC}+\mu_{FAC}}\right]^{n_i} \cdot \frac{\mu_{lSw}}{\lambda_{lSw}+\mu_{lSw}}\}) \cdot \frac{\mu_{ACB}}{\lambda_{ACB}+\mu_{ACB}} \cdot \frac{\mu_{lT}}{\lambda_{lT}+\mu_{lT}} \cdot \frac{\mu_{lAC/DC}}{\lambda_{lAC/DC}+\mu_{lAC/DC}} \cdot \frac{\mu_{DC}}{\lambda_{DC}+\mu_{DC}} \cdot \frac{\mu_{DC/AC}}{\lambda_{DC/AC}+\mu_{DC/AC}} \cdot \frac{\mu_{lT}}{\lambda_{lT}+\mu_{lT}}$$

8.5.5 Case Study

A 30 turbine case is also studied for configuration 2. The parameters of the connection in the configuration are:

- m=3, which means there are 3 parallel lines;
- n=10, which means there are 10 turbines in each line.

The formulation of the calculation is as below.

$$EGWE = \left[\sum_{i=1}^{m} n_i \cdot G \cdot P_{line\ i}(up)\right] \cdot P(ACB\ up) \cdot P(lT\ up) \cdot P(lAC/DC\ up) \cdot P(DC\ up) \cdot P(DC/AC\ up) \cdot P(lT\ up) + 0$$

$$= \left[\sum_{i=1}^{m} n_i \cdot G \cdot P_{line\ i}(up)\right] \cdot P(ACB\ up) \cdot P(lT\ up) \cdot P(lAC/DC\ up) \cdot P(DC\ up) \cdot P(DC/AC\ up) \cdot P(lT\ up)$$

$$= (\sum_{i=1}^{m}\{n_i \cdot G \cdot \left[\frac{\mu_{WT}}{\lambda_{WT}+\mu_{WT}} \cdot \frac{\mu_{sSw}}{\lambda_{sSw}+\mu_{sSw}} \cdot \frac{\mu_{sT}}{\lambda_{sT}+\mu_{sT}} \cdot \frac{\mu_{FAC}}{\lambda_{FAC}+\mu_{FAC}}\right]^{n_i} \cdot \frac{\mu_{lSw}}{\lambda_{lSw}+\mu_{lSw}}\}) \cdot \frac{\mu_{ACB}}{\lambda_{ACB}+\mu_{ACB}}$$

$$\cdot \frac{\mu_{lT}}{\lambda_{lT}+\mu_{lT}} \cdot \frac{\mu_{\frac{lAC}{DC}}}{\lambda_{\frac{lAC}{DC}}+\mu_{\frac{lAC}{DC}}} \cdot \frac{\mu_{DC}}{\lambda_{DC}+\mu_{DC}} \cdot \frac{\mu_{\frac{DC}{AC}}}{\lambda_{\frac{DC}{AC}}+\mu_{\frac{DC}{AC}}} \cdot \frac{\mu_{lT}}{\lambda_{lT}+\mu_{lT}}$$

$= 24.1052 \cdot G,$

IWP $= 30 \cdot$ G=60 MW,

IWE $= 30 \cdot \mathrm{G} \cdot \mathrm{t} = 60\mathrm{MW} \cdot \mathrm{t}$ =60 MWh in an hour.

In full capacity case,

EGWE=48.21 MWh in an hour,

EAWE = 60 MWh in an hour,

$\mathrm{CF} = \frac{\mathrm{EGWE}}{\mathrm{IWE}} = \frac{48.21}{60} = 0.8035,$

$\mathrm{GR} = \frac{\frac{\mathrm{EGWE}}{\mathrm{t}}}{IWP} = \frac{48.21}{60} = 80.35\%.$

In wind speed variation case,

EGWE=22.42 MWh in an hour,

EAWE = 27.9 MWh in an hour,

$\mathrm{CF} = \frac{\mathrm{EGWE}}{\mathrm{IWE}} = \quad 0.3736,$

$\mathrm{GR} = \frac{\frac{\mathrm{EGWE}}{\mathrm{t}}}{IWP} = \frac{22.42}{60} = 37.36\%.$

The results of the reliability analysis for alternate configuration 2 are summarized in Table 8.9.

Table 8.9: Summary of Reliability Indices for Alternate Configuration 2

Reliability and Cost Index	*Computed Value*
Installed wind power (IWP)	60 MW
Installed wind energy (IWE)	60 MWh in one hour
Expected available wind energy (EAWE)	27.9 MWh in one hour
Expected generated wind energy (EGWE)	22.42MWh in one hour
Capacity factor (CF)	0.374
Generation Ratio (GR)	37.4%

8.6 Reliability Analysis of Wind Farm Configuration 3

This section describes the reliability analysis and case study result for configuration 3. Figure 8.6 shows configuration 3. Configuration 3 has DC Series Wind Farm and AC Transmission.

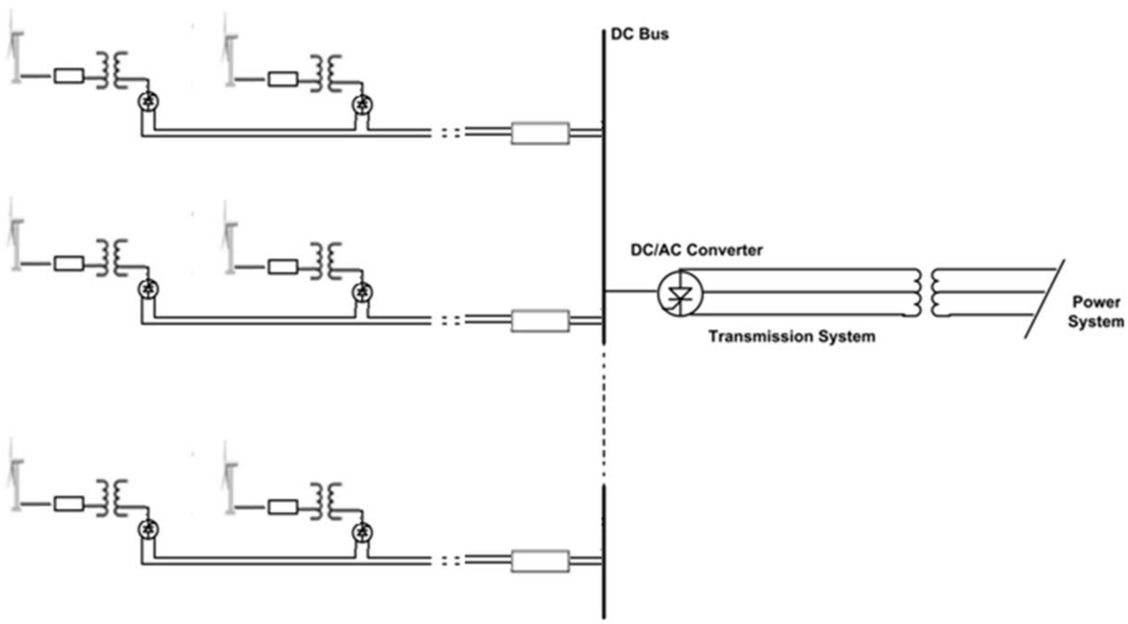


Figure 8.6: Wind Farm Configuration 3

The formulation of the reliability analysis for this configuration is as below. The probabilistic methods used for this formulation is exactly the ones shown in the analysis of configuration 1.

8.6.1 Probabilistic Model of Circuit *i*

This configuration has m parallel lines, each line consisting of n wind turbines. The probability of the states of each line is provided as below.

$$P_{line\ i}(Up) = \left[P_{WT}^{n_i}(up) \cdot P_{sSW}^{n_i}(up) \cdot P_{sST}^{n_i}(up) \cdot P_{sAC/DC}^{n_i}(up) \cdot P_{FDC}^{n_i}(up)\right] \cdot P_{lSW}(up)$$

$$= \left[\frac{\mu_{WT}}{\lambda_{WT}+\mu_{WT}} \cdot \frac{\mu_{sSw}}{\lambda_{sSw}+\mu_{sSw}} \cdot \frac{\mu_{sT}}{\lambda_{sT}+\mu_{sT}} \cdot \frac{\mu_{sAC/DC}}{\lambda_{sAC/DC}+\mu_{sAC/DC}} \cdot \frac{\mu_{FDC}}{\lambda_{FDC}+\mu_{FDC}}\right]^{n_i} \cdot \frac{\mu_{lSw}}{\lambda_{lSw}+\mu_{lSw}}$$

$$P_{line\ i}(Down) = 1 - P_{line\ i}(Up)$$

$$= 1 - \left[\frac{\mu_{WT}}{\lambda_{WT}+\mu_{WT}} \cdot \frac{\mu_{sSw}}{\lambda_{sSw}+\mu_{sSw}} \cdot \frac{\mu_{sT}}{\lambda_{sT}+\mu_{sT}} \cdot \frac{\mu_{AC/DC}}{\lambda_{AC/DC}+\mu_{AC/DC}} \cdot \frac{\mu_{FDC}}{\lambda_{FDC}+\mu_{FDC}}\right]^{n_i} \cdot \frac{\mu_{lSw}}{\lambda_{lSw}+\mu_{lSw}}$$

8.6.2 Probabilistic Model of m Parallel Circuits

The *m* lines are in parallel, and the probability of having *j* lines up is shown as below.

$P_{parallel\_jng} = C_m^j \cdot P_{line\ i}^j(up) \cdot P_{line\ i}^{m-j}(down)$, where $C_m^j = \frac{m!}{j!(m-j)!}$ is the combinatorial number

8.6.3 Probabilistic Model of the Entire Configuration

The output states at the point of common coupling (PCC) are similar to the ones in configuration 1. At PCC, there are *m+1* states, the delivered capacity of which is similar to that in the analysis of Configuration 1. The 1st state represents the situation when there is no capacity successfully delivered.

When $j \neq 0$, the probability of the *(j+1)*th state is:

$$P_{system\_jnG} = P_{parallel\_jnG} \cdot P(DCB\ up) \cdot P(DC/AC\ up) \cdot P(60AC\ up) \cdot P(lT\ up)$$

$$= \mathrm{P}_{\mathrm{parallel\_jnG}} \cdot \frac{\mu_{DCB}}{\lambda_{DCB}+\mu_{DCB}} \cdot \frac{\mu_{DC/AC}}{\lambda_{DC/AC}+\mu_{DC/AC}} \cdot \frac{\mu_{60AC}}{\lambda_{60AC}+\mu_{60AC}} \cdot \frac{\mu_{lT}}{\lambda_{lT}+\mu_{lT}}$$

$$= C_m^j \cdot P_{line\ i}^j(up) \cdot P_{line\ i}^{m-j}(down) \cdot \frac{\mu_{DCB}}{\lambda_{DCB}+\mu_{DCB}} \cdot \frac{\mu_{DC/AC}}{\lambda_{DC/AC}+\mu_{DC/AC}} \cdot \frac{\mu_{60AC}}{\lambda_{60AC}+\mu_{60AC}} \cdot \frac{\mu_{lT}}{\lambda_{lT}+\mu_{lT}}$$

$$= \frac{m!}{j!(m-j)!} \cdot \left\{\left[\frac{\mu_{WT}}{\lambda_{WT}+\mu_{WT}} \cdot \frac{\mu_{sSw}}{\lambda_{sSw}+\mu_{sSw}} \cdot \frac{\mu_{sT}}{\lambda_{sT}+\mu_{sT}} \cdot \frac{\mu_{sAC/DC}}{\lambda_{sAC/DC}+\mu_{sAC/DC}} \cdot \frac{\mu_{FDC}}{\lambda_{FDC}+\mu_{FDC}}\right]^{n_i} \cdot \frac{\mu_{lSw}}{\lambda_{lSw}+\mu_{lSw}}\right\}^j \cdot \{1 - \left[\frac{\mu_{WT}}{\lambda_{WT}+\mu_{WT}} \cdot \frac{\mu_{sSw}}{\lambda_{sSw}+\mu_{sSw}} \cdot \frac{\mu_{sT}}{\lambda_{sT}+\mu_{sT}} \cdot \frac{\mu_{sAC/DC}}{\lambda_{sAC/DC}+\mu_{sAC/DC}} \cdot \frac{\mu_{FDC}}{\lambda_{FDC}+\mu_{FDC}}\right]^{n_i} \cdot \frac{\mu_{lSw}}{\lambda_{lSw}+\mu_{lSw}}\}^{m-j} \cdot \frac{\mu_{DCB}}{\lambda_{DCB}+\mu_{DCB}} \cdot \frac{\mu_{DC/AC}}{\lambda_{DC/AC}+\mu_{DC/AC}} \cdot \frac{\mu_{60AC}}{\lambda_{60AC}+\mu_{60AC}} \cdot \frac{\mu_{lT}}{\lambda_{lT}+\mu_{lT}}$$

This probability is denoted as *P(j+1)*, for all $j \neq 0$.

When $j = 0$, the probability *P(0+1)=P(1)* can be calculated as:

$P(1) = 1 - \sum_{w=1}^{m} P(w+1).$

8.6.4 Expected Generated Wind Energy (EGWE)

The total expected available energy at point of common coupling is the index of EGWE:

$$EGWE = \left[\sum_{i=1}^{m} n_i \cdot G \cdot P_{line\ i}(up)\right] \cdot P(DCB\ up) \cdot P(DC/AC\ up) \cdot P(60AC\ up) \cdot P(lT\ up) + 0$$

$$= \left[\sum_{i=1}^{m} n_i \cdot G \cdot P_{line\ i}(up)\right] \cdot P(DCB\ up) \cdot P(DC/AC\ up) \cdot P(60AC\ up) \cdot P(lT\ up)$$

$$= \left(\sum_{i=1}^{m}\{n_i \cdot G \cdot \left[\frac{\mu_{WT}}{\lambda_{WT}+\mu_{WT}} \cdot \frac{\mu_{sSw}}{\lambda_{sSw}+\mu_{sSw}} \cdot \frac{\mu_{sT}}{\lambda_{sT}+\mu_{sT}} \cdot \frac{\mu_{sAC/DC}}{\lambda_{sAC/DC}+\mu_{sAC/DC}} \cdot \frac{\mu_{FDC}}{\lambda_{FDC}+\mu_{FDC}}\right]^{n_i} \cdot \frac{\mu_{lSw}}{\lambda_{lSw}+\mu_{lSw}}\}\right) \cdot \frac{\mu_{DCB}}{\lambda_{DCB}+\mu_{DCB}} \cdot$$

$$\frac{\mu_{DC/AC}}{\lambda_{DC/AC}+\mu_{DC/AC}} \cdot \frac{\mu_{60AC}}{\lambda_{60AC}+\mu_{60AC}} \cdot \frac{\mu_{lT}}{\lambda_{lT}+\mu_{lT}}$$

8.6.5 Case Study

A 30 turbine case is also studied for configuration 3. The parameters of the connection in the configuration are:

- m=3, which means there are 3 parallel lines;
- n=10, which means there are 10 turbines in each line.

The formulation of the calculation is as below.

$$EGWE = \left[\sum_{i=1}^{m} n_i \cdot G \cdot P_{line\ i}(up)\right] \cdot P(DCB\ up) \cdot P(DC/AC\ up) \cdot P(60AC\ up) \cdot P(lT\ up) + 0$$

$$= \left[\sum_{i=1}^{m} n_i \cdot G \cdot P_{line\ i}(up)\right] \cdot P(DCB\ up) \cdot P(DC/AC\ up) \cdot P(60AC\ up) \cdot P(lT\ up)$$

$$= \left(\sum_{i=1}^{m}\{n_i \cdot G \cdot \left[\frac{\mu_{WT}}{\lambda_{WT}+\mu_{WT}} \cdot \frac{\mu_{sSw}}{\lambda_{sSw}+\mu_{sSw}} \cdot \frac{\mu_{sT}}{\lambda_{sT}+\mu_{sT}} \cdot \frac{\mu_{sAC/DC}}{\lambda_{sAC/DC}+\mu_{sAC/DC}} \cdot \frac{\mu_{FDC}}{\lambda_{FDC}+\mu_{FDC}}\right]^{n_i} \cdot \frac{\mu_{lSw}}{\lambda_{lSw}+\mu_{lSw}}\}\right) \cdot \frac{\mu_{DCB}}{\lambda_{DCB}+\mu_{DCB}} \cdot$$

$$\frac{\mu_{DC/AC}}{\lambda_{DC/AC}+\mu_{DC/AC}} \cdot \frac{\mu_{60AC}}{\lambda_{60AC}+\mu_{60AC}} \cdot \frac{\mu_{lT}}{\lambda_{lT}+\mu_{lT}}$$

$$= 22.3897\text{G},$$

$\text{IWP} = 30 \cdot \text{G}$=60 MW,

$\text{IWE} = 30 \cdot \text{G} \cdot \text{t} = 60\text{MW} \cdot \text{t}$ =60 MWh in an hour.

In full capacity case,

EGWE=44.77 MWh in an hour,

EAWE = 60 MWh in an hour,

$\text{CF} = \frac{\text{EGWE}}{\text{IWE}} = \frac{44.77}{60} = 0.7462,$

$\text{GR} = \frac{\frac{\text{EGWE}}{\text{t}}}{IWP} = \frac{44.77}{60} = 74.62\%.$

In wind speed variation case,

EGWE=20.82 MWh in an hour,

EAWE = 27.9 MWh in an hour,

$\text{CF} = \frac{\text{EGWE}}{\text{IWE}} = \quad 0.347,$

$\text{GR} = \frac{\frac{\text{EGWE}}{\text{t}}}{IWP} = \frac{22.42}{60} = 34.7\%.$

The results of the reliability analysis for alternate configuration 3 are summarized in Table 8.10.

Table 8.10: Summary of Reliability Indices for Alternate Configuration 3

Reliability and Cost Index	*Computed Value*
Installed wind power (IWP)	60 MW
Installed wind energy (IWE)	60 MWh in one hour
Expected available wind energy (EAWE)	27.9 MWh in one hour
Expected generated wind energy (EGWE)	20.82MWh in one hour
Capacity factor (CF)	0.347
Generation Ratio (GR)	34.7%

8.7 Reliability Analysis of Wind Farm Configuration 4

This section provides the reliability analysis and case study result for configuration 4.

Figure 8.7 shows configuration 4. Configuration 4 has DC Parallel Wind Farm and AC Transmission.

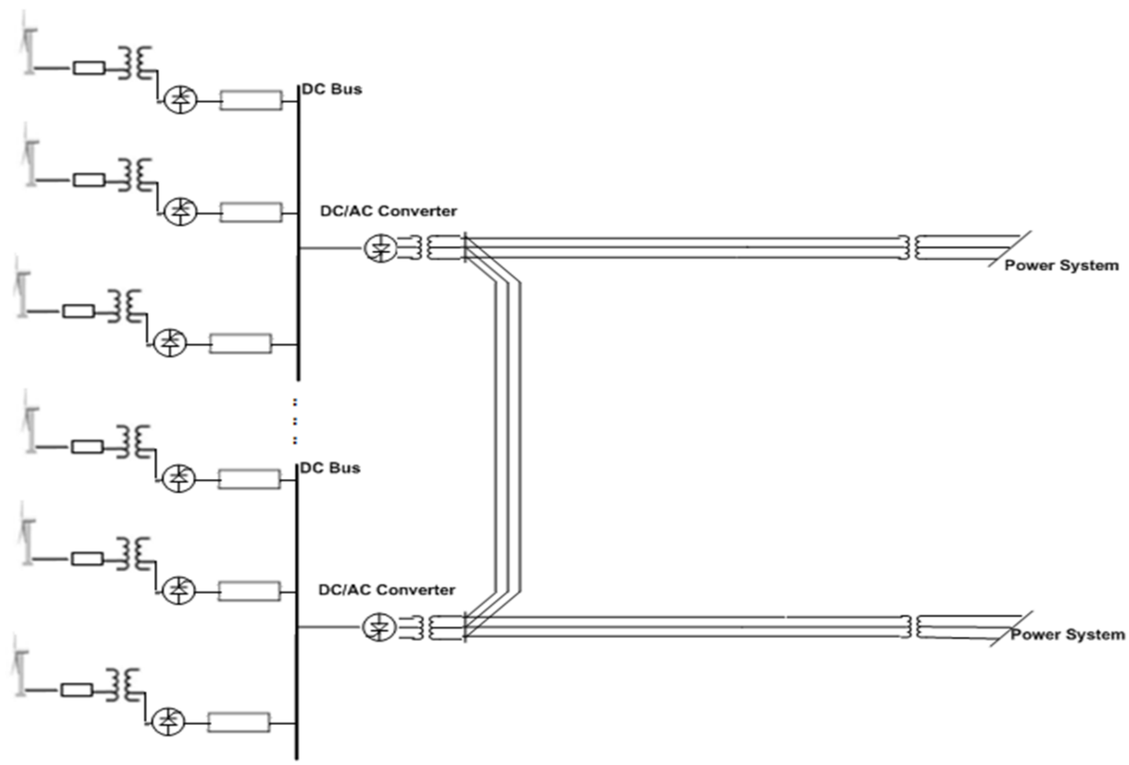


Figure 8.7: Wind Farm Configuration 4

The formulation of the reliability analysis for this configuration is as below. The probabilistic methods used for this formulation is exactly the ones shown in the analysis of configuration 1.

8.7.1 Probabilistic Model of Circuit *i*

In this configuration, there are assumed to be m DC buses, each bus consist of *n* parallel connected wind turbines.

$$P_{line\ i}(Up) = P(WT\ up) \cdot P(sSw\ up) \cdot P(sT\ up) \cdot P(sAC/DC\ up) \cdot P(FDC\ up) \cdot P_{lSW}(up)$$

$$= \frac{\mu_{WT}}{\lambda_{WT}+\mu_{WT}} \cdot \frac{\mu_{sSw}}{\lambda_{sSw}+\mu_{sSw}} \cdot \frac{\mu_{sT}}{\lambda_{sT}+\mu_{sT}} \cdot \frac{\mu_{sAC/DC}}{\lambda_{sAC/DC}+\mu_{sAC/DC}} \cdot \frac{\mu_{FDC}}{\lambda_{FDC}+\mu_{FDC}} \cdot \frac{\mu_{lSw}}{\lambda_{lSw}+\mu_{lSw}}$$

$$P_{line\ i}(Down) = 1 - P_{line\ i}(Up)$$

$$= 1 - \frac{\mu_{WT}}{\lambda_{WT}+\mu_{WT}} \cdot \frac{\mu_{sSw}}{\lambda_{sSw}+\mu_{sSw}} \cdot \frac{\mu_{sT}}{\lambda_{sT}+\mu_{sT}} \cdot \frac{\mu_{sAC/DC}}{\lambda_{sAC/DC}+\mu_{sAC/DC}} \cdot \frac{\mu_{FDC}}{\lambda_{FDC}+\mu_{FDC}} \cdot \frac{\mu_{lSw}}{\lambda_{lSw}+\mu_{lSw}}$$

8.7.2 Probabilistic Model of m Buses

$P_{parallel\_jng} = C_{mn}^{j} \cdot P_{line\ i}^{j}(up) \cdot P_{line\ i}^{mn-j}(down)$, where $C_{mn}^{j} = \frac{mn!}{j!(mn-j)!}$ is the combinatorial number.

8.7.3 Probabilistic Model of the Entire Configuration

The configuration contains *m* buses and each bus contains *n* wind turbines. This case is more complicated than the previous configurations.

Some assumptions need to be made.

- Transmission Constraints are not considered.
- All DC buses are identical.
- All inter-transmission lines, which are the lines connecting the transmission lines, are identical.
- All transmission lines, which are the lines connecting the DC bus with the point of common coupling, are identical.

With these assumptions, the probability of a wind turbine generation's successful transmitting to point of common coupling (PCC), is the multiplication of the probability of its successful transmitting to the DC bus with its successful transmitting from DC bus to PCC. In addition, the probability of the successful transmission for each turbine is identical.

The successful transmission from a DC bus to PCC contains many cases. However, the unsuccessful transmission from a DC bus to PCC, is limited to 2 cases: a) all transmission lines fail, and b) the transmission line connected to this DC bus fails, and at the same time, the inter-transmission lines connected to this DC bus fail.

Therefore, the power reaches the transmission line can be delivered to the power system unless all of the transmission is not successful.

The probability of at least one transmission line works is

$$P_{Tran}(up) = 1 - (1 - P(60AC\ up) \cdot P(lT\ up))^m$$

The probability of the DC bus, converter and 20Hz transformer will work is

$$P_{DCT}(up) = \ P(DCB\ up) \cdot P(Conv\ up) \cdot P(T\ up)$$

$$P_{DCT}(down) = \ 1 - P(DCB\ up) \cdot P(Conv\ up) \cdot P(T\ up)$$

For the convenience of computation, the probability of the internal transmission line that can deliver the power to transmission line is:

$$P_{Intel60}(up) = \frac{\mu_{Inter60}}{\lambda_{Inter60} + \mu_{Inter60}}$$

So the probability of the right part of this configuration will work is

$$P_{right} = P_{Tran}(up) \cdot P_{Intel}(up)$$

For $0 < j \leq n$

$$\mathrm{P}_{\mathrm{system\_inG}} = P_{right} \cdot [C_m^m \cdot P_{DCT}^m(up) \cdot C_{mn}^j \cdot P_{line\ i}^j(up) \cdot P_{line\ i}^{mn-j}(down)$$

$$+ C_m^{m-1} \cdot P_{DCT}^{m-1}(up) \cdot P_{DCT}^1(down) \cdot C_{(m-1)n}^j \cdot P_{line\ i}^j(up) \cdot P_{line\ i}^{mn-j}(down)$$

$$+ \cdots + C_m^1 \cdot P_{DCT}^1(up) \cdot P_{DCT}^{m-1}(down) \cdot C_n^j \cdot P_{line\ i}^j(up) \cdot P_{line\ i}^{mn-j}(down)]$$

For $\mathrm{n} < j \leq n + n$

⋮

⋮

For $(\mathrm{m} - 1)\mathrm{n} < j \leq mn$

$$\mathrm{P}_{\mathrm{system\_inG}} = P_{right} \cdot [C_m^m \cdot P_{DCT}^m(up) \cdot C_{mn}^j \cdot P_{line\ i}^j(up) \cdot P_{line\ i}^{mn-j}(down)$$

8.7.4 Expected Generated Wind Energy (EGWE)

$$EGWE = \sum_{i=1}^{number\ of\ states} G_i \cdot P(i)$$

For the general case, each of the *mn* parallel circuits has two states: (a) either transmitting full capacity or (b) 0. For line *i*, the expected transmitted capacity to the collector AC bus is:

$$EGWE_{line\ i} = G \cdot P_{line\ i}(up) + 0 \cdot P_{line\ i}(down) = G \cdot P_{line\ i}(up)$$

The total expected available energy at point of common coupling is the index of EGWE:

$$EGWE = mn \cdot G \cdot P_{line\ i}(up) \cdot P(DCT\ up) \cdot P(right\ up) + 0$$

$$= mn \cdot G \cdot P_{line\ i}(up) \cdot P(DCB\ up) \cdot P(Conv\ up) \cdot P(T\ up) \cdot P_{Intel60}(up) \cdot [1 - \left(1 - P(60AC\ up) \cdot P(lT\ up)\right)^m]$$

$$= mn \cdot G \cdot \frac{\mu_{WT}}{\lambda_{WT}+\mu_{WT}} \cdot \frac{\mu_{sSw}}{\lambda_{sSw}+\mu_{sSw}} \cdot \frac{\mu_{sT}}{\lambda_{sT}+\mu_{sT}} \cdot \frac{\mu_{sAC/DC}}{\lambda_{sAC/DC}+\mu_{sAC/DC}} \cdot \frac{\mu_{FDC}}{\lambda_{FDC}+\mu_{FDC}} \cdot \frac{\mu_{lSw}}{\lambda_{lSw}+\mu_{lSw}} \cdot \frac{\mu_{DCB}}{\lambda_{DCB}+\mu_{DCB}} \cdot \frac{\mu_{DC/AC}}{\lambda_{DC/AC}+\mu_{DC/AC}} \cdot \frac{\mu_{lT}}{\lambda_{lT}+\mu_{lT}} \cdot \frac{\mu_{Inter60}}{\lambda_{Inter60}+\mu_{Inter60}} \cdot \left[1 - (1 - \frac{\mu_{60AC}}{\lambda_{60AC}+\mu_{60AC}} \cdot \frac{\mu_{lT}}{\lambda_{lT}+\mu_{lT}})^m\right]$$

8.7.5 Case Study

A 30 turbine case is also studied for configuration 4. The parameters of the connection in the configuration are:

1) m=3, which means there are 3 buses;
2) n=10, which means there are 10 turbines on each bus.

The formulation of the calculation is as below.

$$EGWE = mn * G \cdot P_{line\ i}(up) \cdot P(DCT\ up) \cdot P(right\ up) + 0$$

$$= mn * G \cdot P_{line\ i}(up) \cdot P(DCB\ up) \cdot P(Conv\ up) \cdot P(T\ up) \cdot P_{Intel60}(up) \cdot [1 - (1 - P(60AC\ up) \cdot P(lT\ up))^m]$$

$$= mn \cdot G \cdot \frac{\mu_{WT}}{\lambda_{WT}+\mu_{WT}} \cdot \frac{\mu_{sSw}}{\lambda_{sSw}+\mu_{sSw}} \cdot \frac{\mu_{sT}}{\lambda_{sT}+\mu_{sT}} \cdot \frac{\mu_{s\frac{AC}{DC}}}{\lambda_{s\frac{AC}{DC}}+\mu_{s\frac{AC}{DC}}} \cdot \frac{\mu_{FDC}}{\lambda_{FDC}+\mu_{FDC}} \cdot \frac{\mu_{lSw}}{\lambda_{lSw}+\mu_{lSw}} \cdot \frac{\mu_{DCB}}{\lambda_{DCB}+\mu_{DCB}} \cdot \frac{\mu_{\frac{DC}{AC}}}{\lambda_{\frac{DC}{AC}}+\mu_{\frac{DC}{AC}}} \cdot \frac{\mu_{lT}}{\lambda_{lT}+\mu_{lT}} \cdot \frac{\mu_{Inter60}}{\lambda_{Inter60}+\mu_{Inter60}} \cdot \left[1 - \left(1 - \frac{\mu_{60AC}}{\lambda_{60AC}+\mu_{60AC}} \cdot \frac{\mu_{lT}}{\lambda_{lT}+\mu_{lT}}\right)^m\right]$$

$$= 28.1884G,$$

$\mathrm{IWP} = 30 \cdot \mathrm{G}$=60 MW,

$\mathrm{IWE} = 30 \cdot \mathrm{G} \cdot \mathrm{t} = 60\mathrm{MW} \cdot \mathrm{t}$ =60 MWh in an hour.

In full capacity case,

EGWE=56.37 MWh in an hour,

EAWE = 60 MWh in an hour,

$$\mathrm{CF} = \frac{\mathrm{EGWE}}{\mathrm{IWE}} = \frac{56.37}{60} = 0.9396,$$

$$\mathrm{GR} = \frac{\frac{\mathrm{EGWE}}{\mathrm{t}}}{IWP} = \frac{56.37}{60} = 93.96\%.$$

In wind speed variation case,

EGWE=26.22 MWh in an hour,

EAWE = 27.9 MWh in an hour,

$$\mathrm{CF} = \frac{\mathrm{EGWE}}{\mathrm{IWE}} = \quad 0.437,$$

$$\mathrm{GR} = \frac{\frac{\mathrm{EGWE}}{\mathrm{t}}}{IWP} = \frac{26.22}{60} = 43.7\%.$$

The results of the reliability analysis for alternate configuration 4 are summarized in Table 8.11.

Table 8.11: Summary of Reliability Indices for Alternate Configuration 4

Reliability and Cost Index	*Computed Value*
Installed wind power (IWP)	60 MW
Installed wind energy (IWE)	60 MWh in one hour
Expected available wind energy (EAWE)	27.9 MWh in one hour
Expected generated wind energy (EGWE)	26.22MWh in one hour
Capacity factor (CF)	0.437
Generation Ratio (GR)	43.7%

8.8 Reliability Analysis of Wind Farm Configuration 5

This section provides the reliability analysis and case study result for configuration 5.

Figure 8.8 gives out configuration 5. Configuration 5 has DC Series Wind Farm and LFAC Transmission (single branch).

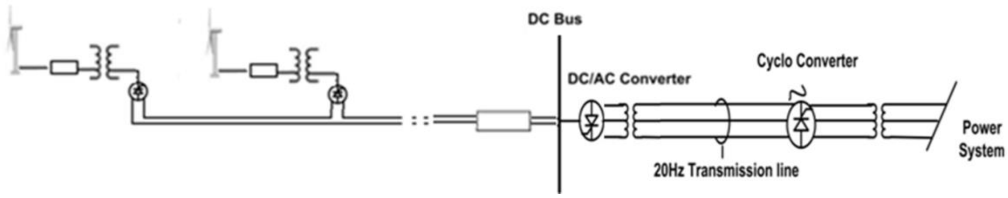


Figure 8.8: Wind Farm Configuration 5

The formulation of the reliability analysis for this configuration is as below. The probabilistic methods used for this formulation is exactly the ones shown in the analysis of configuration 1.

8.8.1 Probabilistic Model of the Entire Configuration

Since the generators are in series, the system won't be up unless all the components are up. Therefore, the number of wind turbine power that is delivered successfully, j ,is either 0 or m.

When $j = m$, the probability of the state is:

$$P_{system\_jnG} = \left[P_{WT}^{mn}(up) \cdot P_{sSW}^{mn}(up) \cdot P_{sST}^{mn}(up) \cdot P_{sAC/DC}^{mn}(up) \cdot P_{FDC}^{mn}(up)\right] \cdot P(lSW\ up) \cdot P(DCB\ up) \cdot P(DC/AC\ up) \cdot P(20T\ up) \cdot P(20AC\ up) \cdot P(Conv\ up) \cdot P(lT\ up)$$

$$= \left[\frac{\mu_{WT}}{\lambda_{WT}+\mu_{WT}} \cdot \frac{\mu_{sSw}}{\lambda_{sSw}+\mu_{sSw}} \cdot \frac{\mu_{sT}}{\lambda_{sT}+\mu_{sT}} \cdot \frac{\mu_{sAC/DC}}{\lambda_{sAC/DC}+\mu_{sAC/DC}} \cdot \frac{\mu_{FDC}}{\lambda_{FDC}+\mu_{FDC}}\right]^{mn} \cdot \frac{\mu_{lSw}}{\lambda_{lSw}+\mu_{lSw}} \cdot \frac{\mu_{DCB}}{\lambda_{DCB}+\mu_{DCB}} \cdot \frac{\mu_{DC/AC}}{\lambda_{DC/AC}+\mu_{DC/AC}} \cdot \frac{\mu_{20T}}{\lambda_{20T}+\mu_{20T}} \cdot \frac{\mu_{20AC}}{\lambda_{20AC}+\mu_{20AC}} \cdot \frac{\mu_{Conv}}{\lambda_{Conv}+\mu_{Conv}} \cdot \frac{\mu_{lT}}{\lambda_{lT}+\mu_{lT}}$$

When $j = 0$, the probability *P(0+1)=P(1)* can be calculated as:

$$P(1) = 1 - \sum_{w=1}^{m} P(w+1)$$

8.8.2 Expected Generated Wind Energy (EGWE)

The total expected available energy at point of common coupling is the index of EGWE:

$$EGWE = mn \cdot G \cdot \left[P_{WT}^{mn}(up) \cdot P_{sSW}^{mn}(up) \cdot P_{sST}^{mn}(up) \cdot P_{\frac{sAC}{DC}}^{mn}(up) \cdot P_{FDC}^{mn}(up)\right] \cdot P(lSW\ up) \cdot P(DCB\ up) \cdot P\left(\frac{DC}{AC}up\right) \cdot P(20T\ up) \cdot P(20AC\ up) \cdot P(Conv\ up) \cdot P(lT\ up)$$

$$= mn \cdot G \cdot \left[\frac{\mu_{WT}}{\lambda_{WT}+\mu_{WT}} \cdot \frac{\mu_{sSw}}{\lambda_{sSw}+\mu_{sSw}} \cdot \frac{\mu_{sT}}{\lambda_{sT}+\mu_{sT}} \cdot \frac{\mu_{sAC/DC}}{\lambda_{sAC/DC}+\mu_{sAC/DC}} \cdot \frac{\mu_{FDC}}{\lambda_{FDC}+\mu_{FDC}}\right]^{mn} \cdot \frac{\mu_{lSw}}{\lambda_{lSw}+\mu_{lSw}} \cdot \frac{\mu_{DCB}}{\lambda_{DCB}+\mu_{DCB}} \cdot \frac{\mu_{DC/AC}}{\lambda_{DC/AC}+\mu_{DC/AC}} \cdot \frac{\mu_{20T}}{\lambda_{20T}+\mu_{20T}} \cdot \frac{\mu_{20AC}}{\lambda_{20AC}+\mu_{20AC}} \cdot \frac{\mu_{Conv}}{\lambda_{Conv}+\mu_{Conv}} \cdot \frac{\mu_{lT}}{\lambda_{lT}+\mu_{lT}}$$

8.8.3 Case Study

A 30 turbine case is also studied for configuration 5. The parameters of the connection in the configuration are:

1) m=1, which means there are 1 parallel lines;
2) n=30, which means there are 30 turbines in each line.

The formulation of the calculation is as below.

$$EGWE = mn \cdot G \cdot \left[P_{WT}^{mn}(up) \cdot P_{sSW}^{mn}(up) \cdot P_{sST}^{mn}(up) \cdot P_{\frac{sAC}{DC}}^{mn}(up) \cdot P_{FDC}^{mn}(up)\right] \cdot P(lSW\ up) \cdot P(DCB\ up) \cdot P\left(\frac{DC}{AC}up\right) \cdot P(20T\ up) \cdot P(20AC\ up) \cdot P(Conv\ up) \cdot P(lT\ up)$$

$$= mn \cdot G \cdot \left[\frac{\mu_{WT}}{\lambda_{WT}+\mu_{WT}} \cdot \frac{\mu_{sSw}}{\lambda_{sSw}+\mu_{sSw}} \cdot \frac{\mu_{sT}}{\lambda_{sT}+\mu_{sT}} \cdot \frac{\mu_{\frac{sAC}{DC}}}{\lambda_{\frac{sAC}{DC}}+\mu_{\frac{sAC}{DC}}} \cdot \frac{\mu_{FDC}}{\lambda_{FDC}+\mu_{FDC}}\right]^{mn} \cdot \frac{\mu_{lSw}}{\lambda_{lSw}+\mu_{lSw}} \cdot \frac{\mu_{DCB}}{\lambda_{DCB}+\mu_{DCB}} \cdot \frac{\mu_{\frac{DC}{AC}}}{\lambda_{\frac{DC}{AC}}+\mu_{\frac{DC}{AC}}} \cdot \frac{\mu_{20T}}{\lambda_{20T}+\mu_{20T}} \cdot \frac{\mu_{20AC}}{\lambda_{20AC}+\mu_{20AC}} \cdot \frac{\mu_{Conv}}{\lambda_{Conv}+\mu_{Conv}} \cdot \frac{\mu_{lT}}{\lambda_{lT}+\mu_{lT}}$$

$= 13.2399\text{G}$,

$\mathrm{IWP} = 30 \cdot \mathrm{G}$=60 MW,

$\mathrm{IWE} = 30 \cdot \mathrm{G} \cdot \mathrm{t} = 60\mathrm{MW} \cdot \mathrm{t}$ =60 MWh in an hour.

In full capacity case,

EGWE=26.47 MWh in an hour,

EAWE = 60 MWh in an hour,

$$\mathrm{CF} = \frac{\mathrm{EGWE}}{\mathrm{IWE}} = \frac{26.47}{60} = 0.4413,$$

$$\mathrm{GR} = \frac{\frac{\mathrm{EGWE}}{\mathrm{t}}}{IWP} = \frac{26.47}{60} = 44.13\%.$$

In wind speed variation case,

EGWE=12.31 MWh in an hour,

EAWE = 27.9 MWh in an hour,

$$\mathrm{CF} = \frac{\mathrm{EGWE}}{\mathrm{IWE}} = \quad 0.205,$$

$$\mathrm{GR} = \frac{\frac{\mathrm{EGWE}}{\mathrm{t}}}{IWP} = \frac{22.42}{60} = 20.5\%.$$

The results of the reliability analysis for alternate configuration 5 are summarized in Table 8.12.

Table 8.12: Summary of Reliability Indices for Alternate Configuration 5

Reliability and Cost Index	*Computed Value*
Installed wind power (IWP)	60 MW
Installed wind energy (IWE)	60 MWh in one hour
Expected available wind energy (EAWE)	27.9 MWh in one hour
Expected generated wind energy (EGWE)	12.31MWh in one hour
Capacity factor (CF)	0.205
Generation Ratio (GR)	20.5%

8.9 Reliability Analysis of Wind Farm Configuration 6

This section provides the reliability analysis and case study result for configuration 6.

Figure 8.9 gives out configuration 6. Configuration 6 has DC Parallel Wind Farm, and LFAC Transmission (single branch).

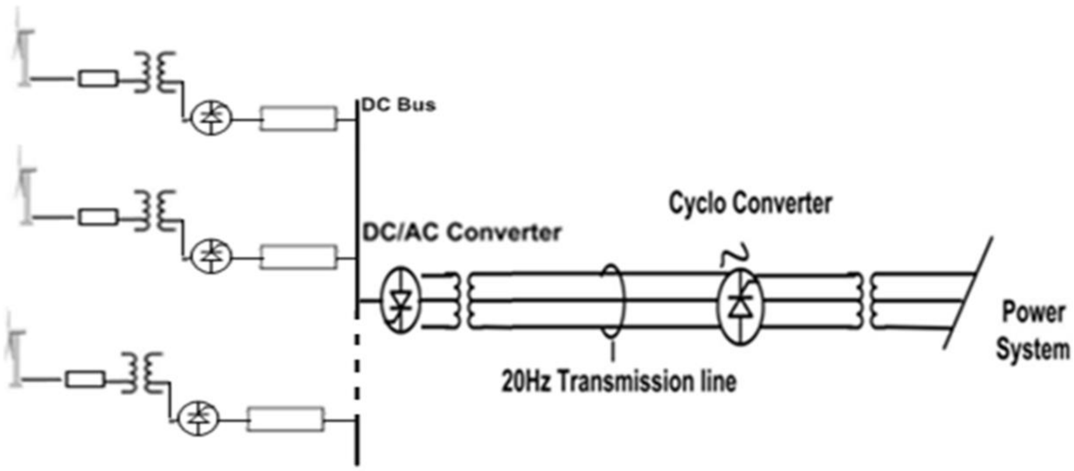


Figure 8.9: Wind Farm Configuration 6

The formulation of the reliability analysis for this configuration is as below. The probabilistic methods used for this formulation is exactly the ones shown in the analysis of configuration 1.

8.9.1 Probabilistic Model of Circuit *i*

The probabilities of the states of the *i*th circuit are developed.

$$P_{line\ i}(Up) = P(WT\ up) \cdot P(sSw\ up) \cdot P(sT\ up) \cdot P(sAC/DC\ up) \cdot P(FDC\ up) \cdot P_{lSW}(up)$$

$$= \frac{\mu_{WT}}{\lambda_{WT}+\mu_{WT}} \cdot \frac{\mu_{sSw}}{\lambda_{sSw}+\mu_{sSw}} \cdot \frac{\mu_{sT}}{\lambda_{sT}+\mu_{sT}} \cdot \frac{\mu_{sAC/DC}}{\lambda_{sAC/DC}+\mu_{sAC/DC}} \cdot \frac{\mu_{FDC}}{\lambda_{FDC}+\mu_{FDC}} \cdot \frac{\mu_{lSw}}{\lambda_{lSw}+\mu_{lSw}}$$

$$P_{line\ i}(Down) = 1 - P_{line\ i}(Up)$$

$$= 1 - \frac{\mu_{WT}}{\lambda_{WT}+\mu_{WT}} \cdot \frac{\mu_{sSw}}{\lambda_{sSw}+\mu_{sSw}} \cdot \frac{\mu_{sT}}{\lambda_{sT}+\mu_{sT}} \cdot \frac{\mu_{sAC/DC}}{\lambda_{sAC/DC}+\mu_{sAC/DC}} \cdot \frac{\mu_{FDC}}{\lambda_{FDC}+\mu_{FDC}} \cdot \frac{\mu_{lSw}}{\lambda_{lSw}+\mu_{lSw}}$$

8.9.2 Probabilistic Model of m Parallel Circuits

$P_{parallel\_jng} = C_m^j \cdot P_{line\ i}^j(up) \cdot P_{line\ i}^{m-j}(down)$, where $C_m^j = \frac{m!}{j!(m-j)!}$ is the combinatorial number.

8.9.3 Probabilistic Model of the Entire Configuration

The output states at the point of common coupling (PCC) are similar to the ones in configuration 1. At PCC, there are *m+1* states, the delivered capacity of which is similar to that in the analysis of Configuration 1. The 1st state represents the situation when there is no capacity successfully delivered.

When $j \neq 0$, the probability of the *(j+1)*th state is:

$$P_{system\_jnG} = P_{parallel\_jnG} \cdot P(DCB\ up) \cdot P(DC/AC\ up) \cdot P(20T\ up) \cdot P(20AC\ up) \cdot P(Conv\ up) \cdot P(lT\ up) = \mathrm{P_{parallel\_jnG}} \cdot \frac{\mu_{DCB}}{\lambda_{DCB}+\mu_{DCB}} \cdot \frac{\mu_{DC/AC}}{\lambda_{DC/AC}+\mu_{DC/AC}} \cdot \frac{\mu_{20T}}{\lambda_{20T}+\mu_{20T}} \cdot \frac{\mu_{20AC}}{\lambda_{20AC}+\mu_{20AC}} \cdot \frac{\mu_{Conv}}{\lambda_{Conv}+\mu_{Conv}} \cdot \frac{\mu_{lT}}{\lambda_{lT}+\mu_{lT}}$$

$$= C_m^j \cdot P_{line\ i}^j(up) \cdot P_{line\ i}^{m-j}(down) \cdot \frac{\mu_{DCB}}{\lambda_{DCB}+\mu_{DCB}} \cdot \frac{\mu_{DC/AC}}{\lambda_{DC/AC}+\mu_{DC/AC}} \cdot \frac{\mu_{20T}}{\lambda_{20T}+\mu_{20T}} \cdot \frac{\mu_{20AC}}{\lambda_{20AC}+\mu_{20AC}} \cdot \frac{\mu_{Conv}}{\lambda_{Conv}+\mu_{Conv}} \cdot \frac{\mu_{lT}}{\lambda_{lT}+\mu_{lT}}$$

$$= \frac{m!}{j!(m-j)!} \cdot \left\{\frac{\mu_{WT}}{\lambda_{WT}+\mu_{WT}} \cdot \frac{\mu_{sSw}}{\lambda_{sSw}+\mu_{sSw}} \cdot \frac{\mu_{sT}}{\lambda_{sT}+\mu_{sT}} \cdot \frac{\mu_{sAC/DC}}{\lambda_{sAC/DC}+\mu_{sAC/DC}} \cdot \frac{\mu_{FDC}}{\lambda_{FDC}+\mu_{FDC}} \cdot \frac{\mu_{lSw}}{\lambda_{lSw}+\mu_{lSw}}\right\}^j \cdot \{1 - \frac{\mu_{WT}}{\lambda_{WT}+\mu_{WT}} \cdot \frac{\mu_{sSw}}{\lambda_{sSw}+\mu_{sSw}} \cdot \frac{\mu_{sT}}{\lambda_{sT}+\mu_{sT}} \cdot \frac{\mu_{sAC/DC}}{\lambda_{sAC/DC}+\mu_{sAC/DC}} \cdot \frac{\mu_{FDC}}{\lambda_{FDC}+\mu_{FDC}} \cdot \frac{\mu_{lSw}}{\lambda_{lSw}+\mu_{lSw}}\}^{m-j} \cdot \frac{\mu_{DCB}}{\lambda_{DCB}+\mu_{DCB}} \cdot \frac{\mu_{DC/AC}}{\lambda_{DC/AC}+\mu_{DC/AC}} \cdot \frac{\mu_{20T}}{\lambda_{20T}+\mu_{20T}} \cdot \frac{\mu_{20AC}}{\lambda_{20AC}+\mu_{20AC}} \cdot \frac{\mu_{Conv}}{\lambda_{Conv}+\mu_{Conv}} \cdot \frac{\mu_{lT}}{\lambda_{lT}+\mu_{lT}}$$

This probability is denoted as *P(j+1)*, for all $j \neq 0$.

When $j = 0$, the probability *P(0+1)=P(1)* can be calculated as:

$$P(1) = 1 - \sum_{w=1}^{m} P(w+1)$$

8.9.4 Expected Generated Wind Energy (EGWE)

The total expected available energy at point of common coupling is the index of EGWE:

$EGWE = \left[\sum_{i=1}^{m} n_i \cdot G \cdot P_{line\ i}(up)\right] \cdot P(DCB\ up) \cdot P(DC/AC\ up) \cdot P(20T\ up) \cdot P(20AC\ up) \cdot$

$P(Conv\ up) \cdot P(lT\ up) + 0$

$= \left[\sum_{i=1}^{m} n_i \cdot G \cdot P_{line\ i}(up)\right] \cdot P(DCB\ up) \cdot P(DC/AC\ up) \cdot P(20T\ up) \cdot P(20AC\ up) \cdot P(Conv\ up) \cdot$

$P(lT\ up)$

$= (\sum_{i=1}^{m}\{n_i \cdot G \cdot \frac{\mu_{WT}}{\lambda_{WT}+\mu_{WT}} \cdot \frac{\mu_{sSw}}{\lambda_{sSw}+\mu_{sSw}} \cdot \frac{\mu_{sT}}{\lambda_{sT}+\mu_{sT}} \cdot \frac{\mu_{\frac{sAC}{DC}}}{\lambda_{\frac{sAC}{DC}}+\mu_{\frac{sAC}{DC}}} \cdot \frac{\mu_{FDC}}{\lambda_{FDC}+\mu_{FDC}} \cdot \frac{\mu_{lSw}}{\lambda_{lSw}+\mu_{lSw}}) \cdot \frac{\mu_{DCB}}{\lambda_{DCB}+\mu_{DCB}} \cdot \frac{\mu_{DC/AC}}{\lambda_{DC/AC}+\mu_{DC/AC}} \cdot \frac{\mu_{20T}}{\lambda_{20T}+\mu_{20T}} \cdot$

$\frac{\mu_{20AC}}{\lambda_{20AC}+\mu_{20AC}} \cdot \frac{\mu_{Conv}}{\lambda_{Conv}+\mu_{Conv}} \cdot \frac{\mu_{lT}}{\lambda_{lT}+\mu_{lT}}$

8.9.5 Case Study

A 30 turbine case is also studied for configuration 6. The parameters of the connection in the configuration are:

1) m=30, which means there are 30 parallel lines;
2) n=1, which means there are 1 turbines in each line.

The formulation of the calculation is as below.

$EGWE = \left[\sum_{i=1}^{m} n_i \cdot G \cdot P_{line\ i}(up)\right] \cdot P(DCB\ up) \cdot P(DC/AC\ up) \cdot P(20T\ up) \cdot P(20AC\ up) \cdot$

$P(Conv\ up) \cdot P(lT\ up) + 0$

$= \left[\sum_{i=1}^{m} n_i \cdot G \cdot P_{line\ i}(up)\right] \cdot P(DCB\ up) \cdot P(DC/AC\ up) \cdot P(20T\ up) \cdot P(20AC\ up) \cdot P(Conv\ up) \cdot$

$P(lT\ up)$

$= (\sum_{i=1}^{m}\{n_i \cdot G \cdot \frac{\mu_{WT}}{\lambda_{WT}+\mu_{WT}} \cdot \frac{\mu_{sSw}}{\lambda_{sSw}+\mu_{sSw}} \cdot \frac{\mu_{sT}}{\lambda_{sT}+\mu_{sT}} \cdot \frac{\mu_{sAC/DC}}{\lambda_{sAC/DC}+\mu_{sAC/DC}} \cdot \frac{\mu_{FDC}}{\lambda_{FDC}+\mu_{FDC}} \cdot \frac{\mu_{lSw}}{\lambda_{lSw}+\mu_{lSw}}) \cdot \frac{\mu_{DCB}}{\lambda_{DCB}+\mu_{DCB}} \cdot$

$\frac{\mu_{DC/AC}}{\lambda_{DC/AC}+\mu_{DC/AC}} \cdot \frac{\mu_{20T}}{\lambda_{20T}+\mu_{20T}} \cdot \frac{\mu_{20AC}}{\lambda_{20AC}+\mu_{20AC}} \cdot \frac{\mu_{Conv}}{\lambda_{Conv}+\mu_{Conv}} \cdot \frac{\mu_{lT}}{\lambda_{lT}+\mu_{lT}}$

=27.8086G,

IWP $= 30 \cdot$ G=60 MW,

IWE $= 30 \cdot$ G $\cdot$ t $=$ 60MW $\cdot$ t =60 MWh in an hour.

In full capacity case,

EGWE=55.61 MWh in an hour,

EAWE = 60 MWh in an hour,

$\mathrm{CF} = \frac{\mathrm{EGWE}}{\mathrm{IWE}} = \frac{55.61}{60} = 0.927,$

$\mathrm{GR} = \frac{\frac{\mathrm{EGWE}}{\mathrm{t}}}{IWP} = \frac{55.61}{60} = 92.7\%.$

In wind speed variation case,

EGWE=25.86 MWh in an hour

EAWE = 27.9 MWh in an hour

$\mathrm{CF} = \frac{\mathrm{EGWE}}{\mathrm{IWE}} = \quad 0.431,$

$\mathrm{GR} = \frac{\frac{\mathrm{EGWE}}{\mathrm{t}}}{IWP} = \frac{22.42}{60} = 43.1\%.$

The results of the reliability analysis for alternate configuration 6 are summarized in Table 8.13.

Table 8.13: Summary of Reliability Indices for Alternate Configuration 6

Reliability and Cost Index	*Computed Value*
Installed wind power (IWP)	60 MW
Installed wind energy (IWE)	60 MWh in one hour
Expected available wind energy (EAWE)	27.9 MWh in one hour
Expected generated wind energy (EGWE)	25.86MWh in one hour
Capacity factor (CF)	0.431
Generation Ratio (GR)	43.1%

8.10 Reliability Analysis of Wind Farm Configuration 7

This section provides the reliability analysis and case study result for configuration 7.

Figure 8.10 gives out configuration 7. Configuration 7 has DC Series Wind Farm, LFAC Transmission (multiple branches).

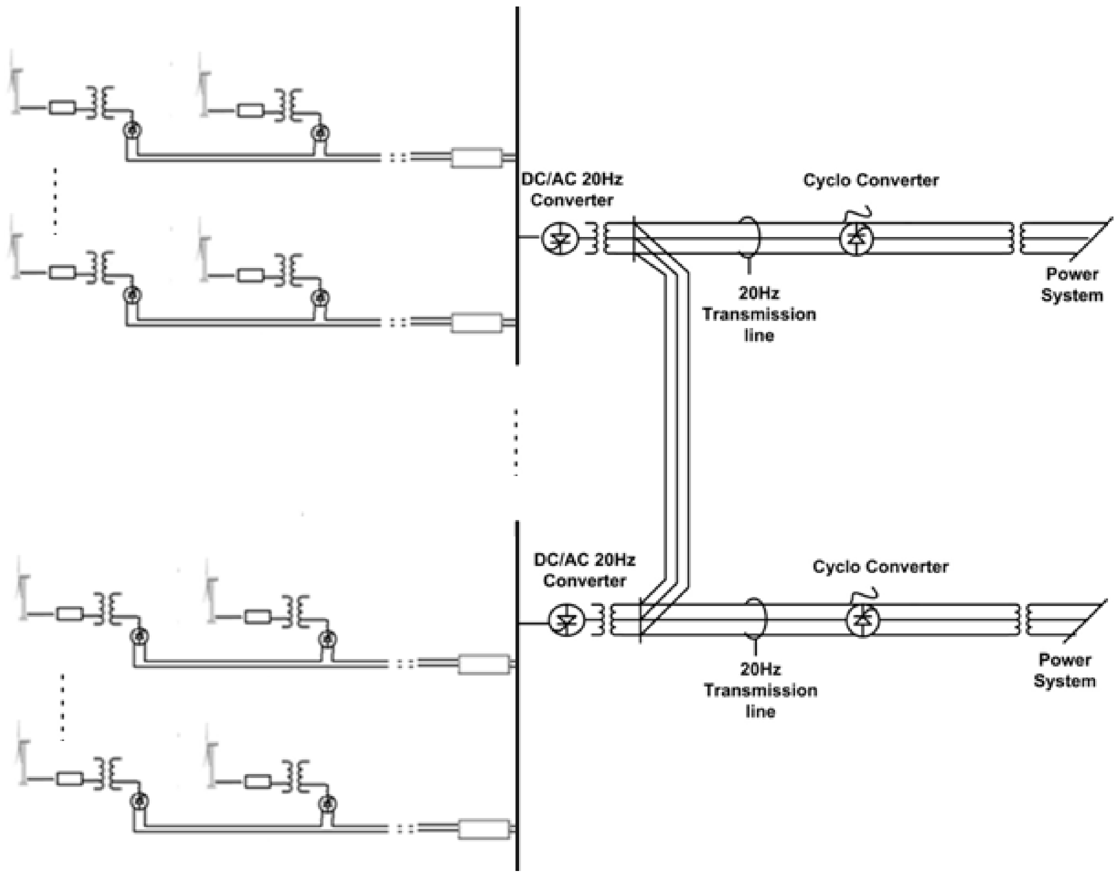


Figure 8.10: Wind Farm Configuration 7

The formulation of the reliability analysis for this configuration is as below. The probabilistic methods used for this formulation is exactly the ones shown in the analysis of configuration 1.

8.10.1 Probabilistic Model of Circuit *i*

$$P_{line\ i}(Up) = \left[P_{WT}^{n_i}(up) \cdot P_{sSW}^{n_i}(up) \cdot P_{sST}^{n_i}(up) \cdot P_{sAC/DC}^{n_i}(up) \cdot P_{FDC}^{n_i}(up)\right] \cdot P_{lSW}(up)$$

$$= \left[\frac{\mu_{WT}}{\lambda_{WT}+\mu_{WT}} \cdot \frac{\mu_{sSw}}{\lambda_{sSw}+\mu_{sSw}} \cdot \frac{\mu_{sT}}{\lambda_{sT}+\mu_{sT}} \cdot \frac{\mu_{sAC/DC}}{\lambda_{sAC/DC}+\mu_{sAC/DC}} \cdot \frac{\mu_{FDC}}{\lambda_{FDC}+\mu_{FDC}}\right]^{n_i} \cdot \frac{\mu_{lSw}}{\lambda_{lSw}+\mu_{lSw}}$$

$$P_{line\ i}(Down) = 1 - P_{line\ i}(Up)$$

$$= 1 - \left[\frac{\mu_{WT}}{\lambda_{WT}+\mu_{WT}} \cdot \frac{\mu_{sSw}}{\lambda_{sSw}+\mu_{sSw}} \cdot \frac{\mu_{sT}}{\lambda_{sT}+\mu_{sT}} \cdot \frac{\mu_{sAC/DC}}{\lambda_{sAC/DC}+\mu_{sAC/DC}} \cdot \frac{\mu_{FDC}}{\lambda_{FDC}+\mu_{FDC}} \cdot \frac{\mu_{lSw}}{\lambda_{lSw}+\mu_{lSw}}\right]^{n_i} \cdot \frac{\mu_{lSw}}{\lambda_{lSw}+\mu_{lSw}}$$

8.10.2 Probabilistic Model of m Parallel Circuits

$P_{parallel\_jng} = C_m^j \cdot P_{line\ i}^j(up) \cdot P_{line\ i}^{m-j}(down)$, where $C_m^j = \frac{m!}{j!(m-j)!}$ is the combinatorial number

8.10.3 Probabilistic Model of the Entire Configuration

This configuration has network transmission system. The probability method used to analyze the network is exactly the one to analyze a single branch. Since in the network transmission case, the variables related to the number of wind turbines are more than that of a single branch, by including the number of buses, the number of parallel circuits, and the number of turbines on a single circuit. This variability makes the formulation of the probability of this entire configuration complicated, by involving multiple combinatorial numbers. In this subsection, the single branch case is analyzed to provide the method that is to be used for more complex cases.

When the transmission system contains only one branch, the output states at the point of common coupling (PCC) are similar to the ones in configuration 1. At PCC, there are *m+1* states, the delivered capacity of which is similar to that in the analysis of

Configuration 1. The 1st state represents the situation when there is no capacity successfully delivered.

When $j \neq 0$, the probability of the *(j+1)*th state is:

$P_{system\_jnG} = P_{parallel\_jnG} \cdot P(DCB\ up) \cdot P(DC/AC\ up) \cdot P(20T\ up) \cdot P(20AC\ up) \cdot P(Conv\ up) \cdot P(lT\ up)$

$= \mathrm{P}_{\mathrm{parallel\_jnG}} \cdot \frac{\mu_{DCB}}{\lambda_{DCB}+\mu_{DCB}} \cdot \frac{\mu_{DC/AC}}{\lambda_{DC/AC}+\mu_{DC/AC}} \cdot \frac{\mu_{20T}}{\lambda_{20T}+\mu_{20T}} \cdot \frac{\mu_{20AC}}{\lambda_{20AC}+\mu_{20AC}} \cdot \frac{\mu_{Conv}}{\lambda_{Conv}+\mu_{Conv}} \cdot \frac{\mu_{lT}}{\lambda_{lT}+\mu_{lT}}$

$= C_m^j \cdot P_{line\ i}^j(up) \cdot P_{line\ i}^{m-j}(down)\ \cdot \frac{\mu_{DCB}}{\lambda_{DCB}+\mu_{DCB}} \cdot \frac{\mu_{DC/AC}}{\lambda_{DC/AC}+\mu_{DC/AC}} \cdot \frac{\mu_{20T}}{\lambda_{20T}+\mu_{20T}} \cdot \frac{\mu_{20AC}}{\lambda_{20AC}+\mu_{20AC}} \cdot \frac{\mu_{Conv}}{\lambda_{Conv}+\mu_{Conv}} \cdot \frac{\mu_{lT}}{\lambda_{lT}+\mu_{lT}}$

$= \frac{m!}{j!(m-j)!} \cdot \left\{\left[\frac{\mu_{WT}}{\lambda_{WT}+\mu_{WT}} \cdot \frac{\mu_{sSw}}{\lambda_{sSw}+\mu_{sSw}} \cdot \frac{\mu_{sT}}{\lambda_{sT}+\mu_{sT}} \cdot \frac{\mu_{sAC/DC}}{\lambda_{sAC/DC}+\mu_{sAC/DC}} \cdot \frac{\mu_{FDC}}{\lambda_{FDC}+\mu_{FDC}} \cdot \frac{\mu_{lSw}}{\lambda_{lSw}+\mu_{lSw}}\right]^{n_i} \cdot \frac{\mu_{lSw}}{\lambda_{lSw}+\mu_{lSw}}\right\}^j \cdot \{1 -$

$\left[\frac{\mu_{WT}}{\lambda_{WT}+\mu_{WT}} \cdot \frac{\mu_{sSw}}{\lambda_{sSw}+\mu_{sSw}} \cdot \frac{\mu_{sT}}{\lambda_{sT}+\mu_{sT}} \cdot \frac{\mu_{sAC/DC}}{\lambda_{sAC/DC}+\mu_{sAC/DC}} \cdot \frac{\mu_{FDC}}{\lambda_{FDC}+\mu_{FDC}} \cdot \frac{\mu_{lSw}}{\lambda_{lSw}+\mu_{lSw}}\right]^{n_i} \cdot \frac{\mu_{lSw}}{\lambda_{lSw}+\mu_{lSw}}\}^{m-j} \cdot \frac{\mu_{DCB}}{\lambda_{DCB}+\mu_{DCB}} \cdot$

$\frac{\mu_{DC/AC}}{\lambda_{DC/AC}+\mu_{DC/AC}} \cdot \frac{\mu_{20T}}{\lambda_{20T}+\mu_{20T}} \cdot \frac{\mu_{20AC}}{\lambda_{20AC}+\mu_{20AC}} \cdot \frac{\mu_{Conv}}{\lambda_{Conv}+\mu_{Conv}} \cdot \frac{\mu_{lT}}{\lambda_{lT}+\mu_{lT}}$

This probability is denoted as *P(j+1)*, for all $j \neq 0$.

When $j = 0$, the probability *P(0+1)=P(1)* can be calculated as:

$$P(1) = 1 - \sum_{w=1}^{m} P(w+1)$$

8.10.4 Expected Generated Wind Energy (EGWE)

The total expected available energy at point of common coupling is the index of EGWE:

$EGWE = \left[\sum_{i=1}^{m} n_i \cdot G \cdot P_{line\ i}(up)\right] \cdot P(DCB\ up) \cdot P(DC/AC\ up) \cdot P(20T\ up) \cdot P(20AC\ up) \cdot$

$P(Conv\ up) \cdot P(lT\ up) + 0$

$= \left[\sum_{i=1}^{m} n_i \cdot G \cdot P_{line\ i}(up)\right] \cdot P(DCB\ up) \cdot P(DC/AC\ up) \cdot P(20T\ up) \cdot P(20AC\ up) \cdot P(Conv\ up) \cdot$

$P(lT\ up)$

$= (\sum_{i=1}^{m}\{n_i \cdot G \cdot \left[\frac{\mu_{WT}}{\lambda_{WT}+\mu_{WT}} \cdot \frac{\mu_{sSw}}{\lambda_{sSw}+\mu_{sSw}} \cdot \frac{\mu_{sT}}{\lambda_{sT}+\mu_{sT}} \cdot \frac{\mu_{sAC/DC}}{\lambda_{sAC/DC}+\mu_{sAC/DC}} \cdot \frac{\mu_{FDC}}{\lambda_{FDC}+\mu_{FDC}}\right]^{n_i} \cdot \frac{\mu_{lSw}}{\lambda_{lSw}+\mu_{lSw}}\}) \cdot \frac{\mu_{DCB}}{\lambda_{DCB}+\mu_{DCB}} \cdot$

$\frac{\mu_{DC/AC}}{\lambda_{DC/AC}+\mu_{DC/AC}} \cdot \frac{\mu_{20T}}{\lambda_{20T}+\mu_{20T}} \cdot \frac{\mu_{20AC}}{\lambda_{20AC}+\mu_{20AC}} \cdot \frac{\mu_{Conv}}{\lambda_{Conv}+\mu_{Conv}} \cdot \frac{\mu_{lT}}{\lambda_{lT}+\mu_{lT}}$

8.10.5 Case Study

A 30 turbine case is also studied for configuration 7. The parameters of the connection in the configuration are:

1) m=3, which means there are 3 parallel lines;
2) n=10, which means there are 10 turbines in each line.

The formulation of the calculation is as below.

$$EGWE = \left[\sum_{i=1}^{m} n_i \cdot G \cdot P_{line\ i}(up)\right] \cdot P(DCB\ up) \cdot P(DC/AC\ up) \cdot P(20T\ up) \cdot P(20AC\ up) \cdot P(Conv\ up) \cdot P(lT\ up) + 0$$

$$= \left[\sum_{i=1}^{m} n_i \cdot G \cdot P_{line\ i}(up)\right] \cdot P(DCB\ up) \cdot P(DC/AC\ up) \cdot P(20T\ up) \cdot P(20AC\ up) \cdot P(Conv\ up) \cdot P(lT\ up)$$

$$= \left(\sum_{i=1}^{m}\left\{n_i \cdot G \cdot \left[\frac{\mu_{WT}}{\lambda_{WT}+\mu_{WT}} \cdot \frac{\mu_{sSw}}{\lambda_{sSw}+\mu_{sSw}} \cdot \frac{\mu_{sT}}{\lambda_{sT}+\mu_{sT}} \cdot \frac{\mu_{\frac{sAC}{DC}}}{\lambda_{\frac{sAC}{DC}}+\mu_{\frac{sAC}{DC}}} \cdot \frac{\mu_{FDC}}{\lambda_{FDC}+\mu_{FDC}}\right]^{n_i} \cdot \frac{\mu_{lSw}}{\lambda_{lSw}+\mu_{lSw}}\right\}\right) \cdot \frac{\mu_{DCB}}{\lambda_{DCB}+\mu_{DCB}} \cdot \frac{\mu_{\frac{DC}{AC}}}{\lambda_{\frac{DC}{AC}}+\mu_{\frac{DC}{AC}}} \cdot \frac{\mu_{20T}}{\lambda_{20T}+\mu_{20T}} \cdot \frac{\mu_{20AC}}{\lambda_{20AC}+\mu_{20AC}} \cdot \frac{\mu_{Conv}}{\lambda_{Conv}+\mu_{Conv}} \cdot \frac{\mu_{lT}}{\lambda_{lT}+\mu_{lT}}$$

$$= 22.088G,$$

IWP $= 30 \cdot \mathrm{G}$=60 MW,

IWE $= 30 \cdot \mathrm{G} \cdot \mathrm{t} = 60\mathrm{MW} \cdot \mathrm{t}$ =60 MWh in an hour.

In full capacity case,

EGWE=44.17 MWh in an hour,

EAWE = 60 MWh in an hour,

$\mathrm{CF} = \frac{\mathrm{EGWE}}{\mathrm{IWE}} = \frac{44.17}{60} = 0.7362,$

$\mathrm{GR} = \frac{\frac{\mathrm{EGWE}}{\mathrm{t}}}{IWP} = \frac{44.17}{60} = 73.62\%.$

In wind speed variation case,

EGWE=20.54 MWh in an hour,

EAWE = 27.9 MWh in an hour,

$\mathrm{CF} = \frac{\mathrm{EGWE}}{\mathrm{IWE}} = \quad 0.342,$

$\mathrm{GR} = \frac{\frac{\mathrm{EGWE}}{\mathrm{t}}}{IWP} = \frac{22.42}{60} = 34.2\%.$

The results of the reliability analysis for alternate configuration 7 are summarized in Table 8.14.

Table 8.14: Summary of Reliability Indices for Alternate Configuration 7

Reliability and Cost Index	*Computed Value*
Installed wind power (IWP)	60 MW
Installed wind energy (IWE)	60 MWh in one hour
Expected available wind energy (EAWE)	27.9 MWh in one hour
Expected generated wind energy (EGWE)	20.54MWh in one hour
Capacity factor (CF)	0.342
Generation Ratio (GR)	34.2%

8.11 Reliability Analysis of Wind Farm Configuration 8

This section provides the reliability analysis and case study result for configuration 8.

Figure 8.11 gives out configuration 8. Configuration 8 has DC Parallel Wind Farm, LFAC Transmission (multiple branches).

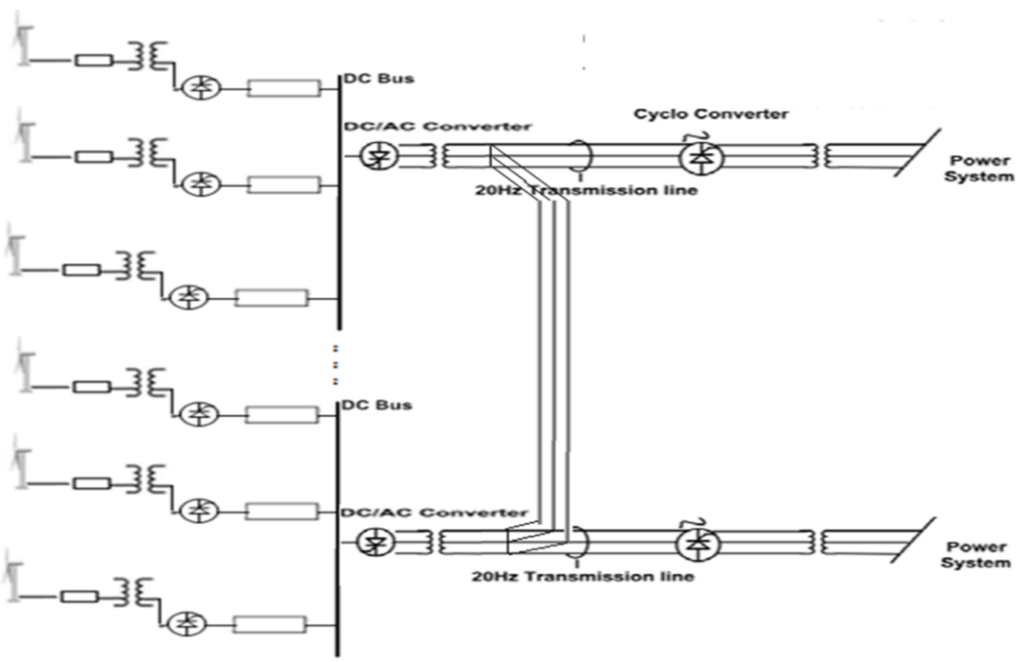


Figure 8.11: Wind Farm Configuration 8

The formulation of the reliability analysis for this configuration is as below. The probabilistic methods used for this formulation is exactly the ones shown in the analysis of configuration 1.

8.11.1 Probabilistic Model of Circuit *i*

The probabilities of the states of the *i*th circuit are provided.

$$P_{line\ i}(Up) = P(WT\ up) \cdot P(sSw\ up) \cdot P(sT\ up) \cdot P(sAC/DC\ up) \cdot P(FDC\ up) \cdot P_{lSW}(up)$$

$$= \frac{\mu_{WT}}{\lambda_{WT}+\mu_{WT}} \cdot \frac{\mu_{sSw}}{\lambda_{sSw}+\mu_{sSw}} \cdot \frac{\mu_{sT}}{\lambda_{sT}+\mu_{sT}} \cdot \frac{\mu_{sAC/DC}}{\lambda_{sAC/DC}+\mu_{sAC/DC}} \cdot \frac{\mu_{FDC}}{\lambda_{FDC}+\mu_{FDC}} \cdot \frac{\mu_{lSw}}{\lambda_{lSw}+\mu_{lSw}}$$

$P_{line\ i}(Down) = 1 - P_{line\ i}(Up)$

$$= 1 - \frac{\mu_{WT}}{\lambda_{WT}+\mu_{WT}} \cdot \frac{\mu_{sSw}}{\lambda_{sSw}+\mu_{sSw}} \cdot \frac{\mu_{sT}}{\lambda_{sT}+\mu_{sT}} \cdot \frac{\mu_{sAC/DC}}{\lambda_{sAC/DC}+\mu_{sAC/DC}} \cdot \frac{\mu_{FDC}}{\lambda_{FDC}+\mu_{FDC}} \cdot \frac{\mu_{lSw}}{\lambda_{lSw}+\mu_{lSw}}$$

8.11.2 Probabilistic Model of m Parallel Circuits

$P_{parallel\_jng} = C_{mn}^{j} \cdot P_{line\ i}^{j}(up) \cdot P_{line\ i}^{mn-j}(down)$, where $C_{mn}^{j} = \frac{mn!}{j!(mn-j)!}$ is the combinatorial number.

8.11.3 Probabilistic Model of the Entire Configuration

The configuration contains *m* buses and each bus contains *n* wind turbines. This case is more complicated so that some assumptions need to be made. Firstly, a single AC transmission line is supposed to be able to transmit the whole capacity. Therefore, the power reaches the transmission line can be derived to the power system unless all of the transmission lines are broken down.

The probability of at least one transmission line works is

$$P_{Tran} = 1 - (1 - P(20AC\ up) \cdot P(Conv\ up) \cdot P(lT\ up))^{m}$$

The probability of the DC bus, converter and 20Hz transformer will work is

$$P_{DC\_Con\_T} = P(DCB\ up) \cdot P(Conv\ up) \cdot P(20T\ up)$$

$$P_{DCT}(down) = 1 - P(DCB\ up) \cdot P(Conv\ up) \cdot P(20T\ up)$$

For the convenience of computation, the probability of the internal transmission line that can deliver the power to transmission line is

$$P_{Intel20}(up) = \frac{\mu_{Inter20}}{\lambda_{Inter20} + \mu_{Inter20}}$$

So the probability of the right part of this configuration will work is

$$P_{right} = P_{Tran}(up) \cdot P_{Intel}(up)$$

For $0 < j \le n$

$$\mathrm{P}_{\text{system\_inG}} = P_{right} \cdot [C_m^m \cdot P_{DCT}^m(up) \cdot C_{mn}^j \cdot P_{line\ i}^j(up) \cdot P_{line\ i}^{mn-j}(down)$$

$$+ C_m^{m-1} \cdot P_{DCT}^{m-1}(up) \cdot P_{DCT}^1(down) \cdot C_{(m-1)n}^j \cdot P_{line\ i}^j(up) \cdot P_{line\ i}^{mn-j}(down)$$

$$+ \cdots + C_m^1 \cdot P_{DCT}^1(up) \cdot P_{DCT}^{m-1}(down) \cdot C_n^j \cdot P_{line\ i}^j(up) \cdot P_{line\ i}^{mn-j}(down)]$$

For $\mathrm{n} < j \le n + n$

⋮

⋮

For $(\mathrm{m}-1)\mathrm{n} < j \le mn$

$$\mathrm{P}_{\mathrm{system}_{\mathrm{inG}}} = P_{right} \cdot [C_m^m \cdot P_{DCT}^m(up) \cdot C_{mn}^j \cdot P_{line\ i}^j(up) \cdot P_{line\ i}^{mn-j}(down)$$

8.11.4 Expected Generated Wind Energy (EGWE)

$$EGWE = \sum_{i=1}^{number\ of\ states} G_i \cdot P(i)$$

For the general case, each of the *mn* parallel circuits has two states: (a) either transmitting full capacity or (b) 0. For line *i*, the expected transmitted capacity to the collector AC bus is:

$$EGWE = mn * G \cdot P_{line\ i}(up) \cdot P(DCT\ up) \cdot P(right\ up) + 0$$

$$= mn * G \cdot P_{line\ i}(up) \cdot P(DCB\ up) \cdot P(Conv\ up) \cdot P(T\ up) \cdot P_{Intel20}(up) \cdot [1 - (1 - P(20AC\ up) \cdot P(Conv\ up) \cdot P(lT\ up))^m]$$

$$= mn \cdot G \cdot \frac{\mu_{WT}}{\lambda_{WT}+\mu_{WT}} \cdot \frac{\mu_{sSw}}{\lambda_{sSw}+\mu_{sSw}} \cdot \frac{\mu_{sT}}{\lambda_{sT}+\mu_{sT}} \cdot \frac{\mu_{sAC/DC}}{\lambda_{sAC/DC}+\mu_{sAC/DC}} \cdot \frac{\mu_{FDC}}{\lambda_{FDC}+\mu_{FDC}} \cdot \frac{\mu_{lSw}}{\lambda_{lSw}+\mu_{lSw}} \cdot \frac{\mu_{DCB}}{\lambda_{DCB}+\mu_{DCB}} \cdot \frac{\mu_{DC/AC}}{\lambda_{DC/AC}+\mu_{DC/AC}} \cdot \frac{\mu_{20T}}{\lambda_{20T}+\mu_{20T}} \cdot \frac{\mu_{Inter20}}{\lambda_{Inter20}+\mu_{Inter20}} \cdot \left[1 - (1 - \frac{\mu_{20AC}}{\lambda_{20AC}+\mu_{20AC}} \cdot \frac{\mu_{Conv}}{\lambda_{Conv}+\mu_{Conv}} \cdot \frac{\mu_{lT}}{\lambda_{lT}+\mu_{lT}})^m\right]$$

8.11.5 Case Study

A 30 turbine case is also studied for configuration 8. The parameters of the connection in the configuration are:

1) m=3, which means there are 3 parallel lines;
2) n=10, which means there are 10 turbines in each line.

The formulation of the calculation is as below.

$$EGWE = mn * G \cdot P_{line\ i}(up) \cdot P(DCT\ up) \cdot P(right\ up) + 0$$

$$= mn * G \cdot P_{line\ i}(up)\ \cdot P(DCB\ up) \cdot P(Conv\ up) \cdot P(T\ up) \cdot P_{Intel20}(up) \cdot [1 - (1 - P(20AC\ up) \cdot P(Conv\ up) \cdot P(lT\ up))^m]$$

$$= mn \cdot G \cdot \frac{\mu_{WT}}{\lambda_{WT}+\mu_{WT}} \cdot \frac{\mu_{sSw}}{\lambda_{sSw}+\mu_{sSw}} \cdot \frac{\mu_{sT}}{\lambda_{sT}+\mu_{sT}} \cdot \frac{\mu_{s\frac{AC}{DC}}}{\lambda_{s\frac{AC}{DC}}+\mu_{s\frac{AC}{DC}}} \cdot \frac{\mu_{FDC}}{\lambda_{FDC}+\mu_{FDC}} \cdot \frac{\mu_{lSw}}{\lambda_{lSw}+\mu_{lSw}} \cdot \frac{\mu_{DCB}}{\lambda_{DCB}+\mu_{DCB}} \cdot \frac{\mu_{\frac{DC}{AC}}}{\lambda_{\frac{DC}{AC}}+\mu_{\frac{DC}{AC}}} \cdot \frac{\mu_{20T}}{\lambda_{20T}+\mu_{20T}} \cdot \frac{\mu_{Inter20}}{\lambda_{Inter20}+\mu_{Inter20}} \cdot \left[1 - \left(1 - \frac{\mu_{20AC}}{\lambda_{20AC}+\mu_{20AC}} \cdot \frac{\mu_{Conv}}{\lambda_{Conv}+\mu_{Conv}} \cdot \frac{\mu_{lT}}{\lambda_{lT}+\mu_{lT}}\right)^m\right]$$

= 28.2625G,

IWP $= 30 \cdot \mathrm{G}$=60 MW,

IWE $= 30 \cdot \mathrm{G} \cdot \mathrm{t} = 60\mathrm{MW} \cdot$ t =60 MWh in an hour.

In full capacity case,

EGWE=56.53 MWh in an hour,

EAWE = 60 MWh in an hour,

$$\mathrm{CF} = \frac{\mathrm{EGWE}}{\mathrm{IWE}} = \frac{56.53}{60} = 0.942,$$

$$\mathrm{GR} = \frac{\frac{\mathrm{EGWE}}{\mathrm{t}}}{IWP} = \frac{56.53}{60} = 94.2\%.$$

In wind speed variation case,

EGWE=26.27 MWh in an hour,

EAWE = 27.9 MWh in an hour,

$\mathrm{CF} = \frac{\mathrm{EGWE}}{\mathrm{IWE}} = \quad 0.438,$

$\mathrm{GR} = \frac{\frac{\mathrm{EGWE}}{\mathrm{t}}}{IWP} = \frac{22.42}{60} = 43.8\%.$

The results of the reliability analysis for alternate configuration 8 are summarized in Table 8.15.

Table 8.15: Summary of Reliability Indices for Alternate Configuration 8

Reliability and Cost Index	*Computed Value*
Installed wind power (IWP)	60 MW
Installed wind energy (IWE)	60 MWh in one hour
Expected available wind energy (EAWE)	27.9 MWh in one hour
Expected generated wind energy (EGWE)	26.27MWh in one hour
Capacity factor (CF)	0.438
Generation Ratio (GR)	43.8%

8.12 Conclusions

Chapter 8 provides the general reliability analysis methods for the alternate configurations of wind farms. For each configuration, a specific modeling and analyzing process has been shown in a subsection. A 30 wind turbine case study is made after the analysis of every configuration. The calculation results have been provided and compared. Table 8.16 provides the summary of the essential reliability indices results from the case studies of the 8 configurations.

Table 8.16: Summary of Reliability Indices for the 8 Alternate Configurations

Configuration	EGWE	IWP	CF
1	22.98	60	0.383
2	22.42	60	0.374
3	20.82	60	0.347
4	26.22	60	0.437
5	12.31	60	0.205
6	25.86	60	0.431
7	20.54	60	0.342
8	26.27	60	0.438

It can be understood that the decision of alternate wind farm or transmission technology selection is not completely based on the reliability analysis, but these results are of significant value as the assessment criteria. For planning engineer of power systems, the EGWE indices are the ones to be used when taking wind farm into account in the generation planning, and the CF indices represents the proportion of expected usage of wind energy comparing to the total installed capacity of wind turbine systems. These proportions reflect the cost concerns in the wind farm. Cost analysis has been presented in separate research results as shown in [45]. The cost analysis of the wind farm configurations includes the operational cost which is proportional to power loss on the lines, and the acquisition cost of all the apparatus in the wind farm. The cost is most of the time a tradeoff with the reliability, which means in general that the desiring of higher reliability indices of generation would require higher cost. The relationship between cost and reliability provides the system planners clear idea of the selection of wind farm configurations, based upon the expected adequacy assessment of the bulk power system being planned and the budget of system construction. The reliability analysis of the alternate configurations in this chapter provides the probabilistic estimation of the input when performing adequacy assessment in the bulk system by incorporating traditional generation, renewable generation especially wind farms, and load profiles in the planning for the bulk power system.

CHAPTER 9 Example State-Space Probabilistic Reliability Analysis of Alternate Wind Farms

Chapter 3, 4, 5 and 6 provided the state-space probabilistic reliability analysis of wind turbines systems and wind farms. The proposed state-space probabilistic method serves as a widely applied algorithm for wind farm analysis, and provides all the information needed from a wind farm for system planners and operators, including probability, transition rate, frequency and duration. The purpose of the state-space probabilistic method is to provide the system planners and operators with as much information as possible for grid level analysis in bulk power systems which has wind penetration. The analysis presented in last chapter (Chapter 8), however, focuses on primarily the assessment over alternate configurations from the reliability perspective, resulting in typical generation states and the probability of the states. Reliability indices are thereafter calculated given the derived generation states, which serve as the major metrics for assessment and comparison among the alternate configurations.

The relationship and differentiation between the previously proposed state-space probabilistic method and the method in Chapter 8 is demonstrated in this chapter using an additional case study of one of the alternate wind farm configurations.

9.1 Example Alternate Wind Farm System Description

The system configuration is provided in Figure 9.1. Here the alternate wind farm configuration 5 in last chapter is used as the example wind farm. Configuration 5 has DC Series Wind Farm and LFAC Transmission (single branch). There are in total 30 WTSs in this wind farm. This is consistent with the example calculation shown in last chapter.

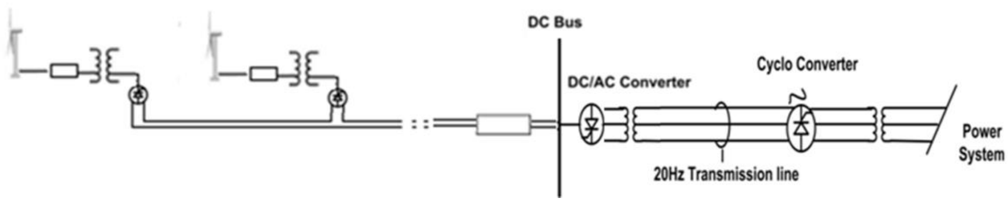


Figure 9.1: Example Wind Farm Configuration with LFAC

9.1.1 WTS Information

The WTSs in the wind farm have the capacity level of 2.0 MVA. The generation versus wind speed curve is provided in Figure 9.2.

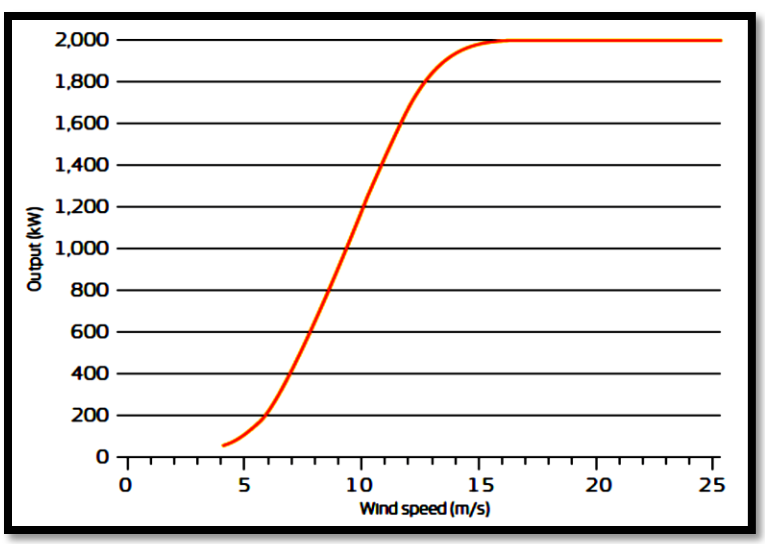


Figure 9.2: Generation Curve of the WTSs in the Example Alternate Wind Farm [28]

The list of components and their reliability parameters in this example is provided in Table 9.1. Failure rate values are in term of per year, since they indicates the general occurrence of a failure within a year; repair rate values are in term of per hour, since the values are normally obtained by the repair duration given in term of hours.

Table 9.1: List of Components and Their Reliability Parameters

Component	Failure Rate	Typical Value of Failure Rate (/year)	Repair Rate	Typical Value of Repair Rate (/hour)
Blade	λ_{WT}	$\lambda_{WT} = 0.402$	μ_{WT}	$\mu_{WT} = 0.0079$
Small Switch	λ_{sSW}	$\lambda_{sSW} = 0.0061$	μ_{sSW}	$\mu_{sSW} = 0.0017$
Small Transformer	λ_{sT}	$\lambda_{sT} = 0.003$	μ_{sT}	$\mu_{sT} = 0.0006$
Small AC/DC Converter	$\lambda_{sAC/DC}$	$\lambda_{sAC/DC} = 0.0298$	$\mu_{sAC/DC}$	$\mu_{sAC/DC} = 0.0003$
In Farm DC Transmission Line	λ_{FDC}	$\lambda_{FDC} = 0.0141$	μ_{FDC}	$\mu_{FDC} = 0.0003$
Large Switch	λ_{lSW}	$\lambda_{lSW} = 0.0096$	μ_{lSW}	$\mu_{lSW} = 0.0010$
DC Bus	λ_{DCB}	$\lambda_{DCB} = 0.000125$	μ_{DCB}	$\mu_{DCB} = 0.0084$
DC/AC Converter	$\lambda_{DC/AC}$	$\lambda_{DC/AC} = 0.0298$	$\mu_{DC/AC}$	$\mu_{DC/AC} = 0.0003$
20 Hz Transformer	λ_{20T}	$\lambda_{20T} = 0.0032$	μ_{20T}	$\mu_{20T} = 0.0004$
20Hz AC Transmission Line	λ_{20AC}	$\lambda_{20AC} = 0.0075$	μ_{20AC}	$\mu_{20AC} = 0.0003$
Cyclo Conveter	λ_{Conv}	$\lambda_{Conv} = 0.0298$	μ_{Conv}	$\mu_{Conv} = 0.0003$
Large/ Substation Transformer	λ_{lT}	$\lambda_{lT} = 0.0032$	μ_{lT}	$\mu_{lT} = 0.0001$

9.1.2 Wind Information

Wind data is from Alaska Energy Authority [33] with the wind speeds at 50 meter height and wind directions. The wind data contains the wind information in year 2004 with 8760 data sets. The time step is one hour. Figure 9.3 provides the statistics of the wind data used. The wind speed data used here are the same as used in wind turbine system case study, but the wind direction is not considered in this case. The green line represents the wind speed at 50m height and the blue line shows the wind speed at 30m height. The data at 50m height are the ones used in this case study.

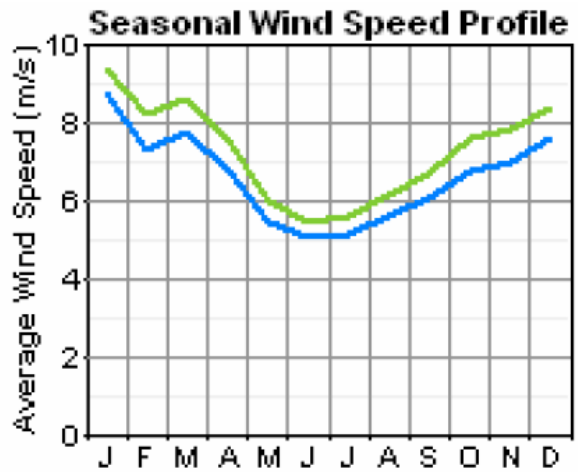


Figure 9.3: Wind Speed Data used in Example Reliability Analysis of Wind Farm [33]

9.2 Distribution Line State Space

Given the parameters of distribution lines, distribution line states are generated using a computer program developed in this research. The state space is in the form as shown in Figure 5.1 in above sections. There are totally 31 distribution lines in the farm. There are in total $2^{31} = 2147483648$ states in the state space, and each state represents a combination of the conditions of the distribution lines.

The probability vector o-f the distribution line state space is obtained.

$P_{\text{line}} = [\text{P(line state 1)};\ \text{P(line state 2)};\ \cdots;\text{P(line state } 2147483648)]$

For example, the sub-vector of the probability values for the first ten distribution line states is as follows:

$P_{\text{line}}(\text{line state 1}\ ,2,3,\ldots\ 10) = [\text{P(line state 1)};\ \text{P(line state 2)};\ \cdots;\text{P(line state 10)}] =$

$$[\,0.976 \quad 0.00021 \quad 0.00021 \quad 0.00021 \quad 0.00021 \quad 0.00021 \quad 0.00021 \quad 0.00021 \quad 0.00021 \quad 0.00021\,]$$

The transition matrix is in the format as follows:

$$\lambda_{line} =$$

$$\begin{bmatrix} \lambda_{1-1} & \lambda_{1-2} & \lambda_{1-3} & \cdots & \lambda_{1-2147483648} & \lambda_{1-2147483648} \\ \lambda_{2-1} & \lambda_{2-2} & \lambda_{2-3} & \cdots & \lambda_{2-2147483648} & \lambda_{2-2147483648} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \lambda_{2147483648-1} & \lambda_{2147483648-2} & \lambda_{2147483648-3} & \cdots & \lambda_{2147483648-2147483647} & \lambda_{2147483648-2147483648} \end{bmatrix}$$

in which every entry stands for the transition rate between the states numbered with the row index and column index. The dimension of the transition matrix is $2147483648*2147483648$. Since this transition matrix is huge, in the computation process the matrix is not necessary to be stored. Based on the assumption that there are no simultaneous failure of two or more distribution lines, the transition in this distribution line state space is simple and intuitive – the transition is caused by the failure or repairing of one distribution line. In this way the use of the transition rates in this matrix follows a judgment of binary state combinations: when λ_{p-q} is requested, the binary number of p and q is compared to see if there is only one digit difference. If yes, which mean there is only one distribution line that had the state change, then λ_{p-q} equals the failure rate of the line when the line changes from 1 to 0, or the repair rate of the line when the line changes from 0 to 1. In this manner, the transition matrix is virtual and the entries in the matrix are either the failure rate of the line or the repair rate of the line, and are sparsely distributed.

9.3 WTS State Space and Delivery Ratio States

Given the component list for the WTSs, the WTS states are generated using a computer program developed in this research. The WTS state space is in the form as shown in Figure 3.3 in above sections, but the components differ. In this example, the wind turbine systems are simplified to contain only the blade, small switch and a transformer. There are in total $2^3 = 8$ states in the state space, and each state represents a combination of the conditions of the components in the WTS.

Using the method presented in Chapter 5, the delivery ratio states are derived from the WTS state space. By analyzing all the $2^3 = 8$ WTS states in the WTS state space, the delivery ratio states and the attributes of them are obtained. These attributes include the probability, transitions and duration in each delivery ratio states. Figure 9.4 presents the delivery ratio states.

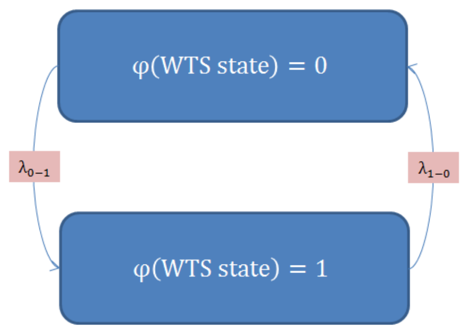


Figure 9.4: Delivery Ratio States of WTSs

The probability vector of the WTS delivery ratio states is obtained.

$$P_{\text{WTS delivery ratio states}} = [P(\text{delivery ratio} = 0);\ P(\text{delivery ratio} = 1)]$$

The probability of a delivery ratio state is derived as follows:

$$P(U) = \sum_{i \in U} P(\text{WTS State i})$$

in which U stands for the delivery ratio considered, and WTS state *i* represents the WTS state that has the delivery ratio as U. Given the results of the WTS state space, P(WTS State i) is the *i*th element in the probability vector (3.3) of WTS state space.

The probability vector result is as follows:

$$\mathrm{P_{WTS\ delivery\ ratio\ states}} = [\mathrm{P(delivery\ ratio} = 0);\ \mathrm{P(delivery\ ratio} = 1)] = [0.0014; 0.9986]$$

The transition matrix is also obtained.

$$\lambda_{\mathrm{WTS\ delivery\ ratio\ states}} = \begin{bmatrix} \lambda_{0-0} & \lambda_{0-1} \\ \lambda_{1-0} & \lambda_{1-1} \end{bmatrix}$$

in which every entry stands for the transition rate between the states numbered with the row index and column index. The dimension of the transition matrix is 2*2.

The transition rate from delivery ratio state U to V is formulated as follows:

$$\mathrm{Transition\ Rate}_{\mathrm{U-V}} = \sum_{\mathrm{i \in U}} \sum_{\mathrm{j \in V}} \lambda_{\mathrm{ij}}$$

in which λ_{ij} is the transition rate from state i in U to state j in V.

Given the results of the WTS state space, λ_{ij} is the (i,j) entry in the WTS state space transition matrix (3.4).

The transition rate result is as follows:

$$\lambda_{\mathrm{WTS\ delivery\ ratio\ states}} = \begin{bmatrix} \lambda_{0-0} & \lambda_{0-1} \\ \lambda_{1-0} & \lambda_{1-1} \end{bmatrix} = \begin{bmatrix} 1 & 0.0001 \\ 0.0001 & 1 \end{bmatrix}$$

9.4 Wind State Space

The analysis of the wind data is performed using the developed computer program. Given the wind data of one year, wind states are extracted from these data by identifying the combinations of wind speed and angles.

The wind speed from the given data ranges from 0.40 m/s to 30.85 m/s. The step size of wind speed states is set as 3m/s. There are totally eight wind states as presented in Figure 9.5. The speeds between 21m/s and 30m/s are classified to be the eighth state because it is beyond the cut-off speed of the WTS given the generation curve in Figure 9.2.

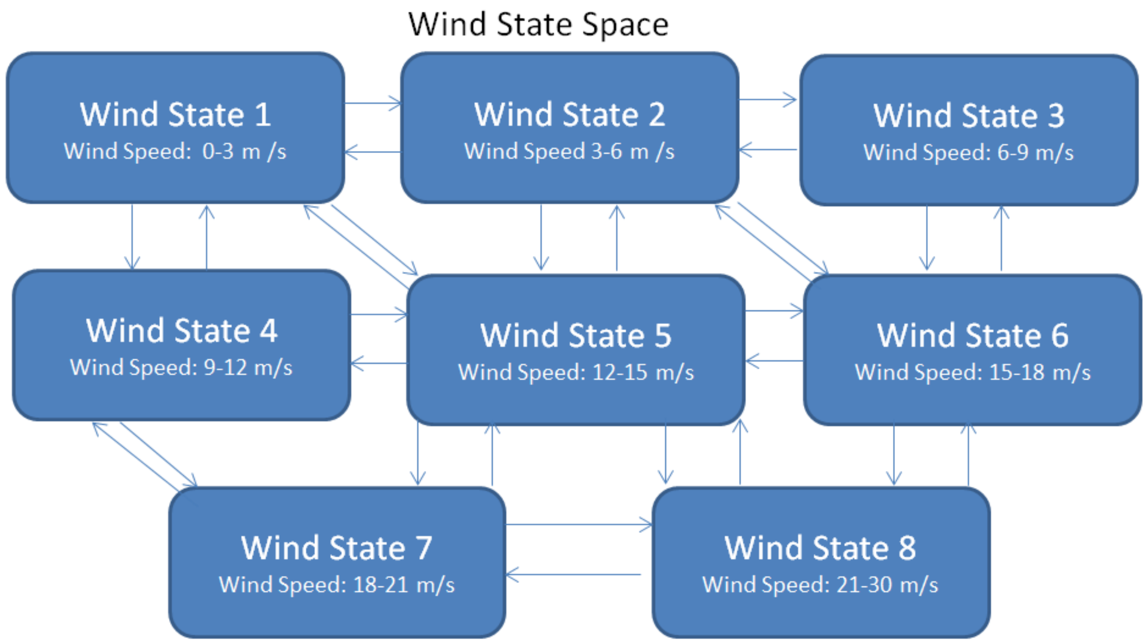


Figure 9.5: Wind State Space in Wind Farm Example

The probability of each wind state is calculated following the frequency principle and is derived from the accumulated counted frequency from the given data.

$$\mathrm{P(Wind\ state\ k)} = \frac{\mathrm{Number\ of\ data\ falling\ into\ state\ k}}{\mathrm{Total\ Number\ of\ Data}}$$

$$= \frac{\mathrm{Number\ of\ data\ falling\ into\ state\ k}}{8760}$$

The probability vector result of the wind states is as follows:

$$P(Wind) = [0.2154; 0.1735; 0.2626; 0.1726; 0.1082; 0.0454; 0.0211; 0.0012]$$

The transition rates are calculated from the frequency of transitions. From the wind data given, the frequency of transitions is extracted by identifying the accumulated number of transitions and then dividing it by the total time span.

$$\lambda_{\mathrm{m-n}} = \frac{\mathrm{Frequency_{m-n}}}{\mathrm{P(Wind\ state\ m)}} = \frac{\frac{\mathrm{n_{mn}}}{\mathrm{T}}}{\mathrm{P(Wind\ state\ m)}} = \frac{\frac{\mathrm{n_{mn}}}{8760}}{\mathrm{P(Wind\ state\ m)}}$$

in which λ_{m-n} is the transition rate from state m to n, T is the total time span given by the data which is 8760 hours, n_{mn} is the total number of transitions from state m to state n, and the probability of wind state m is derived above.

9.5 Combined State Space

The other components in the configuration, the large switch, DC bus, DC/AC converter, 20Hz transformer, 20Hz AC transmission line, cyclo-converter, and the large transformer are all two states components. Given the wind states, distribution line states and the WTS delivery ratio states, the combined states are derived using the developed computer program with all the other components. There are in total $8 \cdot 2^{30} \cdot 2^{31} \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$ states in the combined state space. Figure 9.6 presents the derivation of the combined state space.

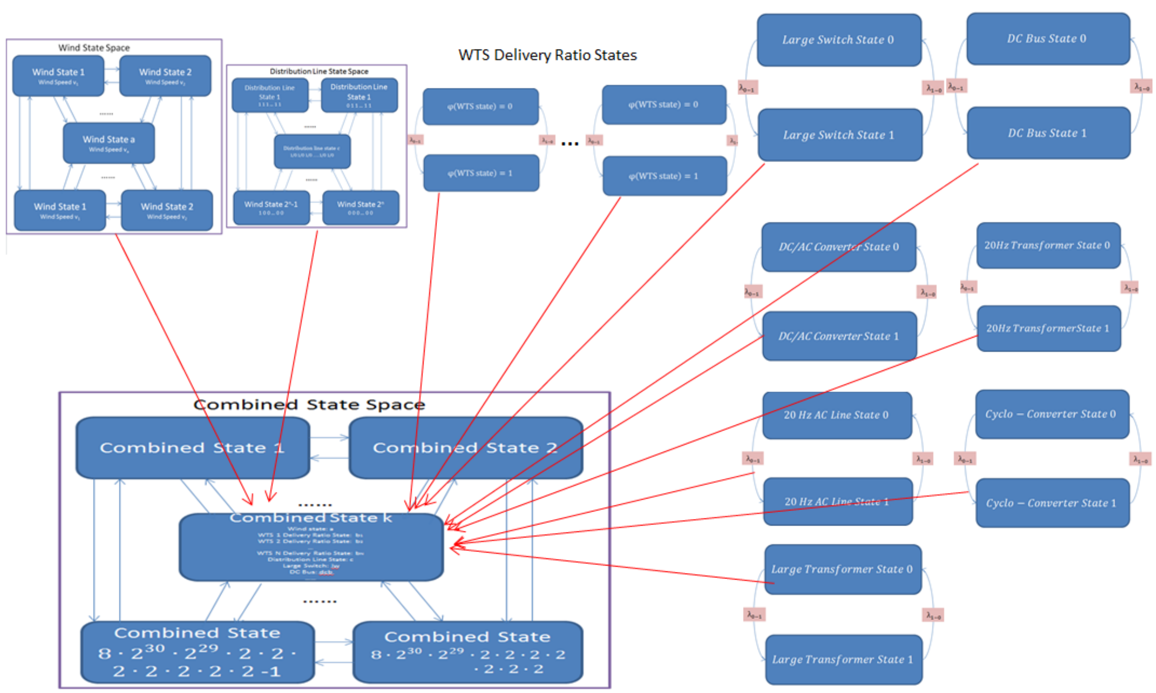


Figure 9.6: Combined State Space in the Example

For each of the combined state, the composition is provided as in Figure 5.4. Each combined state contains a wind state, 30 WTS delivery ratio states, an in-farm distribution line state, a the large switch state, a DC bus state, a DC/AC converter state, a 20Hz transformer state, a 20Hz AC transmission line state, a cyclo-converter state, and a large transformer state . The effects analysis for every combined state is performed.

Denoting G as the generation curve in the manufacturer's manual of the WTS in Figure 9.2, an approximation of the generation curve is made as follows:

$$G_{\mathrm{example}}(v) = \begin{cases} 0 & v < 5 \\ 200 \cdot (v - 5) & 5 \leq v \leq 15 \\ 2000 & v > 15 \end{cases}$$

in which v is the equivalent wind speed at the location of the WTS.

The process of effect analysis of the combined states is presented in Figure 9.7.

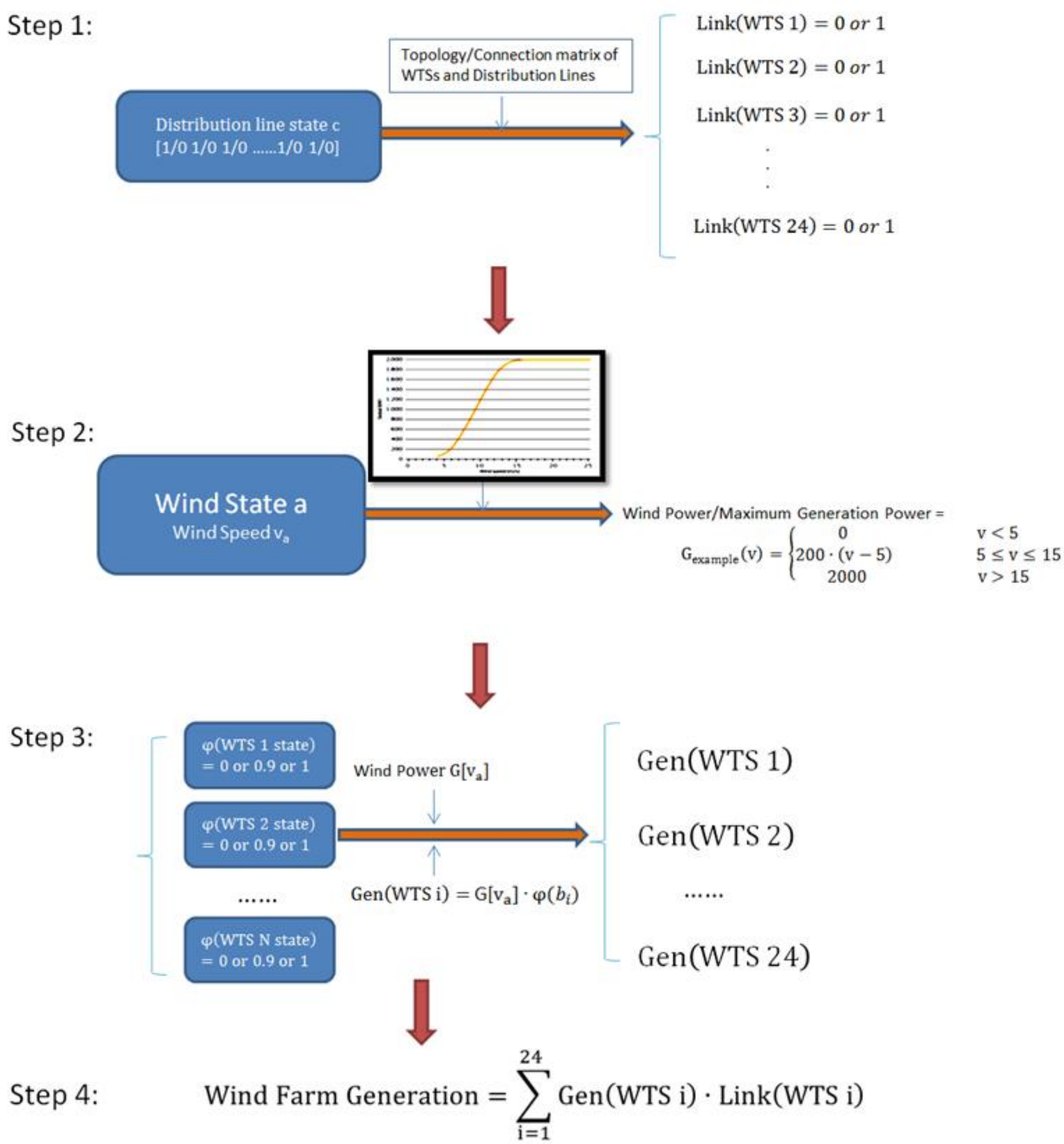


Figure 9.7: Effect Analysis of Combined States in the Example

Combinatory methods are used in the effect analysis process. For the combined states with the same wind state and distribution line state, the combination of delivery ratio states of the 30 WTSs can be categorized into several combinatory scenarios depending on the number of WTSs in the same delivery ratio state.

9.6 Generation States of the Alternate Configuration

The derived combined states are mapped to the generation states which represent generation ranges. In this example, the step-size of the generation ranges is selected to be 2000 kW. For the considered wind farm with 30 2MW WTS, there are totally 31 generation states. Using the above model and the developed computer program, the combined states are mapped to the generation states and the attributes associated with the generation states are calculated. These attributes include the probability of the generation state, transition rates, frequency of transitions, and duration of the generation state. Given all the mappings of the combined states to the generation states, the values associated with the generation states are calculated based on the effects analysis result of the combined states. These values are calculated using the theory of events.

The wind farm reliability analysis result is shown in Figure 9.8.

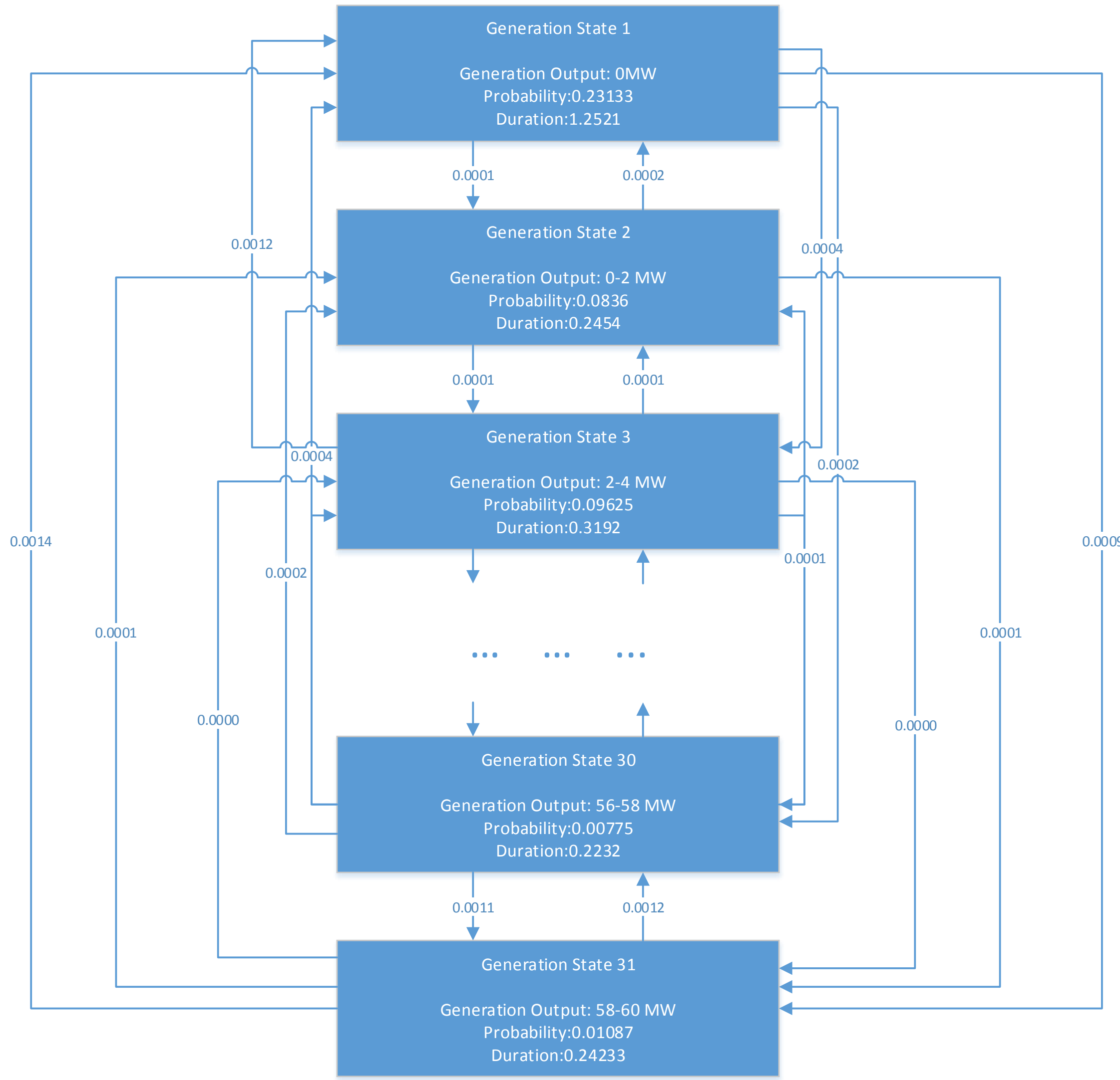


Figure 9.8: Generation States of the Example Alternate Wind Farm

The probability values and the duration values of the 31 ranges are presented in Table 9.3. Each range is specified with a probability value and a duration value.

Table 9.2: Probability and duration of the 31 Generation States in the Example

State Index	Generation Range (MW)	Probability	Duration (h)
1	0	0.23133	1.2521
2	(0,2]	0.0836	0.2454
3	(2,4]	0.09625	0.3192
4	(4,6]	0.0975	0.0081
5	(6,8]	0.0887	0.1288

6	(8,10]	0.00325	0.22667
7	(10,12]	0.067	0.2662
8	(12,14]	0.02145	0.2895
9	(14,16]	0.01115	0.0675
10	(16,18]	0.02825	0.1655
11	(18,20]	0.02435	0.1896
12	(20,22]	0.02925	0.31967
13	(22,24]	0.01235	0.0958
14	(24,26]	0.02225	0.29233
15	(26,28]	0.0283	0.19467
16	(28,30]	0.0114	0.1263
17	(30,32]	0.01665	0.30567
18	(32,34]	0.00125	0.21967
19	(34,36]	0.02725	0.26357
20	(36,38]	0.00775	0.14867
21	(38,40]	0.01405	0.18177
22	(40,42]	0.00875	0.13743
23	(42,44]	0.01095	0.03643
24	(44,46]	0.00925	0.12933
25	(46,48]	0.0075	0.0006
26	(48,50]	0.00775	0.10523
27	(50,52]	0.00275	0.1645
28	(52,54]	0.00905	0.01856
29	(54,56]	0.00205	0.24833
30	(56,58]	0.00775	0.2232
31	(58,60]	0.01087	0.24233

The transition rates between the 31 states are derived, and a sub-matrix of the transition rates between the first 10 states is presented in Table 9.4. The entry in element (i,j) stands for the transition rate from state i to state j. Generation state i represents the generation output $(i\text{-}2,\ i\text{-}1]$ MW when i is larger than 1. When i is 1, generation state 1 represents the generation output 0MW.

Table 9.3: Transition Rates among the First Ten Gen. States in the Example

1	0.0001	0.0004	0.0001	0.0001	0.0001	0	0	0	0
0.0002	1	0.0001	0.0001	0	0	0	0	0	0
0.0012	0.0001	1	0.0002	0.0001	0	0	0	0	0
0.0001	0.0001	0.0001	1	0.0002	0.0001	0.0001	0.0001	0	0
0.0001	0.0001	0	0.0001	1	0	0	0	0	0
0.0001	0.0001	0	0	0.0001	1	0	0	0	0
0.0001	0	0	0	0	0	1	0	0	0
0.0001	0	0	0	0	0.0001	0	1	0.0001	0.0001
0.0001	0	0	0	0	0	0	0	1	0.0001
0	0	0	0	0	0	0	0	0.0001	1

The frequency values of transitions between the 31 generation states are also derived with the transition rate result and the probability vector.

Therefore, the end results of the wind farm reliability analysis are the properties of each state and the properties of each generation state. The properties include the probability, transition rates to other states/ranges, frequency to other states/ranges, and duration.

9.7 Conclusion

This chapter provides an example of the reliability analysis of a low frequency transmission configuration wind farm. The reliability analysis follows the manner of the state space probabilistic analysis Chapter 3, 4, 5 and 6. The state-space probabilistic method serves as a widely applied algorithm for wind farm analysis, and can come up with all the information needed from a wind farm for system planners and operators, including probability, transition rate, frequency and duration. The purpose of the state-space probabilistic method is to provide the system planners and operators with as much information as possible for grid level analysis in bulk power systems which has wind penetration.

The analysis presented in last chapter (Chapter 8), however, focuses on primarily the assessment over alternate configurations from the reliability perspective, resulting in typical generation states and the probability of the states. Reliability indices are thereafter calculated given the derived generation states, which serve as the major metrics for assessment and comparison among the alternate configurations.

Given the analysis from this chapter and Chapter 8, some comparisons and verification can be made towards the results. The EGWE value of this example configuration is 12.31 MWh in an hour as presented in Chapter 8. With the results shown in Table 9.2 in this chapter, a similar estimation can be made using the average value of the generation state range. Table 9.4 presents the EGWE calculation given the probability values of Table 9.2 and the generation state ranges.

Table 9.4: Generation Ranges and Probability to Calculate EGWE

State Index	Generation Range (MW)	Average Generation (MW)	Probability
1	0	0	0.23133
2	(0,2]	1	0.0836
3	(2,4]	3	0.09625
4	(4,6]	5	0.0975
5	(6,8]	7	0.0887
6	(8,10]	9	0.00325
7	(10,12]	11	0.067
8	(12,14]	13	0.02145
9	(14,16]	15	0.01115
10	(16,18]	17	0.02825
11	(18,20]	19	0.02435
12	(20,22]	21	0.02925
13	(22,24]	23	0.01235
14	(24,26]	25	0.02225
15	(26,28]	27	0.0283

16	(28,30]	29	0.0114
17	(30,32]	31	0.01665
18	(32,34]	33	0.00125
19	(34,36]	35	0.02725
20	(36,38]	37	0.00775
21	(38,40]	39	0.01405
22	(40,42]	41	0.00875
23	(42,44]	43	0.01095
24	(44,46]	45	0.00925
25	(46,48]	47	0.0075
26	(48,50]	49	0.00775
27	(50,52]	51	0.00275
28	(52,54]	53	0.00905
29	(54,56]	55	0.00205
30	(56,58]	57	0.00775
31	(58,60]	59	0.01087

The index EGWE is derived by the summation of the product of average generation with its probability. This is the expectation value of the generation state outputs.

$$EGWE = 0 \cdot 0.23133 + 1 \cdot 0.0836 + 3 \cdot 0.09625 + \cdots + 57 \cdot 0.00775 + 59 \cdot 0.01087 = 12.32493$$

This EGWE result verifies the EGWE derived in Chapter 8 as 12.31 MWh in an hour. This again indicates that the state space method is a widely applied method for the wind farms and the results can provide more indices for the system planners and operators. The connection and the relationship between the method proposed in Chapter 8 and the general state space method is that the method in Chapter 8 is a probability-based approach which focuses primarily on analyzing the generation output for alternate wind farms. The planners can utilize the generation output results for determination of the wind farm configuration. Meanwhile, the state space method can still be applied to these alternate wind farms for more detailed analysis.

CHAPTER 10 SUMMARY, CONTRIBUTIONS AND FUTURE DIRECTIONS

10.1 Summary

This dissertation provides the modeling of wind turbine systems (WTS) and wind farms. The WTS reliability model provides the generation state space of a WTS. The generation states are derived from the combinations of the wind states from given wind data and the condition states of each component in a WTS. Wake effect is accounted when there are neighboring WTSs. The results of the reliability model of a WTS are associated with the generation ranges of the WTS, which include the probability, transition rates to other states/ranges, frequency of transitions to other states/ranges, and duration.

The generation model of the wind farm is derived by combining the wind states, WTS states and the distribution line states. The results of the reliability model of a wind farm are associated with the generation ranges of the wind farm, which include the probability, transition rates to other states/ranges, frequency to other states/ranges, and duration.

10.2 Contributions

The presented reliability models are applicable for any kind of wind turbine system and wind farms, and the analysis results can serve as substantial inputs for wind farm planning. Specifically, most contributions are presented in the publications made during the Ph.D study:

- Developed the component state space models of wind turbine systems, as presented in publication 5, and 8 in Chapter 11;

- Developed probability models and performed reliability analysis for alternated wind farm configurations, as presented in publication 2 and 3 in Chapter 11;
- Analyzed the trade-offs between reliability indices and cost, as presented in publication 6 and 7 in Chapter 11;
- Demonstrated the application and extended methods of the reliability models, as presented in publication 1 and 4 in Chapter 11.

The reliability analysis results of wind farms serves as critical input for transmission planning or operation of bulk power systems. The generation states of the wind farm impact the reliability of the transmission system in terms of adequacy of generation. The results of the generation states of wind farms presented in this research provides much more information to system operation and planning in comparison with simply providing some reliability indices. The probability and transitions of the generation states of the wind farm quantifies the fluctuation of the wind energy, and are therefore more effective information given to system planners and operators. The models and the computer programs can become a platform that is suitable for reliability analysis of any wind farms. Wind farm owners will be able to perform analytical and numerical estimation over the generation of wind farms following the methods presented in this dissertation, and will thereafter be able to more precisely place their bids in the energy market or reserve market. Bulk power system operators will be able to obtain the clear idea of the generation probabilistic profiles of the wind farms and the transitions of the generation states so as to perform the operational dispatch with clearer indices.

10.3 Future Directions

Future research in this area can primarily include the load profile probabilistic modeling and the adequacy assessment of the bulk power system when considering traditional generators, probabilistic wind farm generation models as presented in this dissertation, and load probabilistic profiles. This adequacy assessment will come up with the probability and transitional results of the generation-load balance, and the quantification of spinning/non-spinning reserve need and frequency reserve need. In addition, the outage management in regions with high penetration of wind farms can integrate these probabilistic analytics in the Outage Management Systems (OMS) and Energy Management Systems (EMS) to perform more accurate control and dispatch over the resources. Specially, the future directions will include:

- Bulk power system reliability modeling considering wind farm reliability models and load models;
- Cost and operation analysis of bulk power system with wind farms considering energy and reserve markets;
- Regional power system planning and wind farm planning analysis using wind farm reliability models.

CHAPTER 11 Publications

The research presented in this dissertation has been published in several papers as follows:

1. Sakis Meliopoulos, Evangelos Polymeneas, Zhenyu Tan, Renke Huang, Dongbo Zhao, "Advanced Distribution Management System," IEEE Transactions on Smart Grid, Vol. 4, No. 4, pp. 2109 – 2117, December 2013
2. Dongbo Zhao, Sakis Meliopoulos, George Cokkinides, Ramazan Caglar, "Reliability Analysis of Alternate Wind Energy Farms and Interconnections," Proc. 12th International Conf. on Probabilistic Methods Applied to Power Systems (PMAPS), Istanbul, Turkey, 2012.
3. Dongbo Zhao, Sakis Meliopoulos, Rui Fan, Zhenyu Tan, Yongnam Cho, "Reliability Evaluation with Cost Analysis of Alternate Wind Energy Farms and Interconnections," 44th North American Power Symposium (NAPS), Urbana, IL, 2012
4. Dongbo Zhao, Sakis Meliopoulos, Zhenyu Tan, Rui Fan, "Probability State Sequence Method for Reliability Analysis of Wind Farms Considering Wake Effect," 45th North American Power Symposium (NAPS), KS, 2013
5. Zhenyu Tan, Liangyi Sun, Dongbo Zhao, Sakis Meliopoulos, "Dynamic Modeling of Doubly Fed Induction Machine during Unbalanced Voltage Dips with Control Effect Formulation," IEEE PES General Meeting, BC, Canada, 2013
6. Dongbo Zhao, Sakis Meliopoulos, Zhenyu Tan, Aniemi Umana, Rui Fan, "A Market-based Operation Method for Distribution System with Distributed

Generation and Demand Response," 45th North American Power Symposium (NAPS), KS, 2013

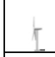
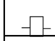
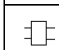
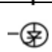
7. Dongbo Zhao, Sakis Meliopoulos, Liangyi Sun, "Cost Analysis and Optimal kV Level Selection of Alternate Wind Farms," 45th North American Power Symposium (NAPS), KS, 2013

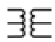
8. Rui Fan, Dongbo Zhao, Zhenyu Tan, Liangyi Sun, Sakis Meliopoulos, “State-space Based Modeling and Sensitivity Analysis of Doubly Fed Induction Machine,” 45th North American Power Symposium (NAPS), KS, 2013

APPENDIX

This appendix provides the symbols and typical values of component reliability parameters. The list shows all the components that appear in the 8 wind farm configurations discussed above. These parameters are used for the example calculation of the 8 configurations provided in the previous subsections.

Table A.1: Reliability Parameters of Components

Icon	Component	Failure Rate	Typical Value of Failure Rate (/year)	Repair Rate	Typical Value of Repair Rate (/hour)
	Wind Turbine	λ_{WT}	$\lambda_{WT} = 0.402$	μ_{WT}	$\mu_{WT} = 0.0079$
	Small Switch	λ_{sSW}	$\lambda_{sSW} = 0.0061$	μ_{sSW}	$\mu_{sSW} = 0.0017$
	Small Transformer	λ_{sT}	$\lambda_{sT} = 0.003$	μ_{sT}	$\mu_{sT} = 0.0006$
	Small AC/DC Converter	$\lambda_{sAC/DC}$	$\lambda_{sAC/DC} = 0.0298$	$\mu_{sAC/DC}$	$\mu_{sAC/DC} = 0.0003$
	Large AC/DC Converter	$\lambda_{lAC/DC}$	$\lambda_{lAC/DC} = 0.0298$	$\mu_{lAC/DC}$	$\mu_{lAC/DC} = 0.0003$
	In Farm AC Transmission Line	λ_{FAC}	$\lambda_{FAC} = 0.0189$	μ_{FAC}	$\mu_{FAC} = 0.0004$
	In Farm DC Transmission Line	λ_{FDC}	$\lambda_{FDC} = 0.0141$	μ_{FDC}	$\mu_{FDC} = 0.0003$
	Large Switch	λ_{lSW}	$\lambda_{lSW} = 0.0096$	μ_{lSW}	$\mu_{lSW} = 0.0010$
	AC Bus	λ_{ACB}	$\lambda_{ACB} = 0.000125$	μ_{ACB}	$\mu_{ACB} = 0.0084$
	DC Bus	λ_{DCB}	$\lambda_{DCB} = 0.000125$	μ_{DCB}	$\mu_{DCB} = 0.0084$
	DC/AC Converter	$\lambda_{DC/AC}$	$\lambda_{DC/AC} = 0.0298$	$\mu_{DC/AC}$	$\mu_{DC/AC} = 0.0003$
	20 Hz Transformer	λ_{20T}	$\lambda_{20T} = 0.0032$	μ_{20T}	$\mu_{20T} = 0.0001$
	DC Transmission Line	λ_{DC}	$\lambda_{DC} = 0.0123$	μ_{DC}	$\mu_{DC} = 0.0003$
	20Hz AC Transmission Line	λ_{20AC}	$\lambda_{20AC} = 0.0075$	μ_{20AC}	$\mu_{20AC} = 0.0004$

	Cyclo Conveter	λ_{Conv}	$\lambda_{\text{Conv}} = 0.0298$	μ_{Conv}	$\mu_{\text{Conv}} = 0.0003$
	60Hz AC Transmission Line	$\lambda_{60\text{AC}}$	$\lambda_{60\text{AC}} = 0.0141$	$\mu_{60\text{AC}}$	$\mu_{60\text{AC}} =0.0003$
	Large/ Substation Transformer	λ_{lT}	$\lambda_{\text{lT}} = 0.0032$	μ_{lT}	$\mu_{\text{lT}} = 0.0001$
	20Hz Inter-Transmission Line	λ_{Inter20}	$\lambda_{\text{Inter20}}=0.0075$	μ_{Inter20}	$\mu_{\text{Inter20}} = 0.0003$
	60Hz Inter-Transmission Line	λ_{Inter60}	$\lambda_{\text{Inter60}}=0.0141$	μ_{Inter60}	$\mu_{\text{Inter60}} = 0.0003$

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